Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.2-a+b-x-^m-c+d-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [1603]. This is test number [1].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1603)	0.00 (0)
Mathematica	100.00 (1603)	0.00 (0)
Fricas	100.00 (1603)	0.00 (0)
Maple	95.57 (1532)	4.43 (71)
Maxima	82.84 (1328)	17.16 (275)
Giac	79.60 (1276)	20.40 (327)
Mupad	77.42 (1241)	22.58 (362)
Sympy	71.62 (1148)	% 28.38 (455)
IntegrateAlgebraic	63.38 (1016)	36.62 (587)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

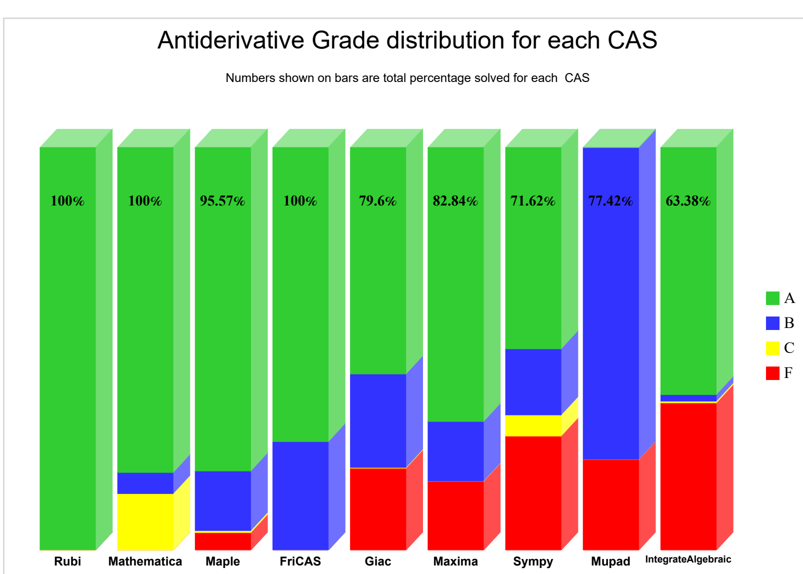
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

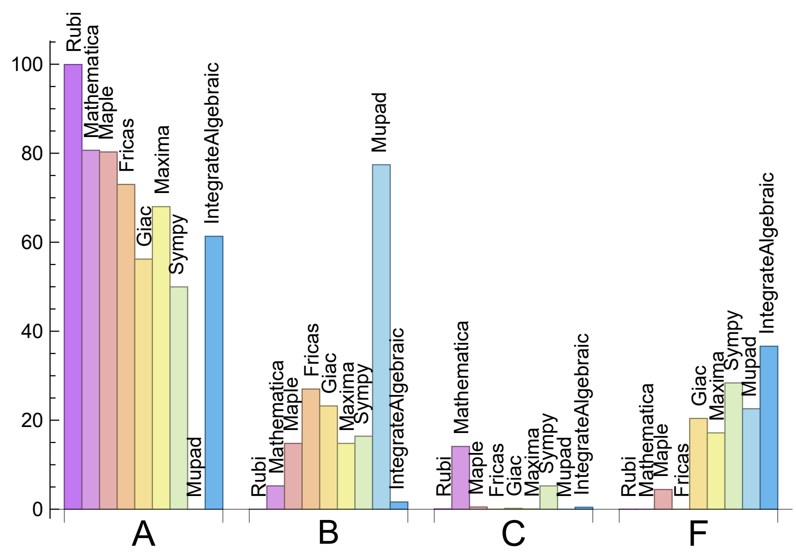
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.94	0.00	0.06	0.00
Mathematica	80.66	5.24	14.10	0.00
Maple	80.29	14.78	0.50	4.43
Fricas	72.99	27.01	0.00	0.00
Maxima	68.00	14.78	0.06	17.16
IntegrateAlgebraic	61.32	1.62	0.44	36.62
Giac	56.21	23.21	0.19	20.40
Sympy	49.97	16.41	5.24	28.38
Mupad	N/A	77.42	0.00	22.58

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	71	100.00 %	0.00 %	0.00 %
Fricas	0	0.00 %	0.00 %	0.00 %
IntegrateAlgebraic	587	100.00 %	0.00 %	0.00 %
Giac	327	63.00 %	15.60 %	21.41 %
Maxima	275	55.64 %	0.00 %	44.36 %
Sympy	455	62.20 %	37.36 %	0.44 %
Mupad	362	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

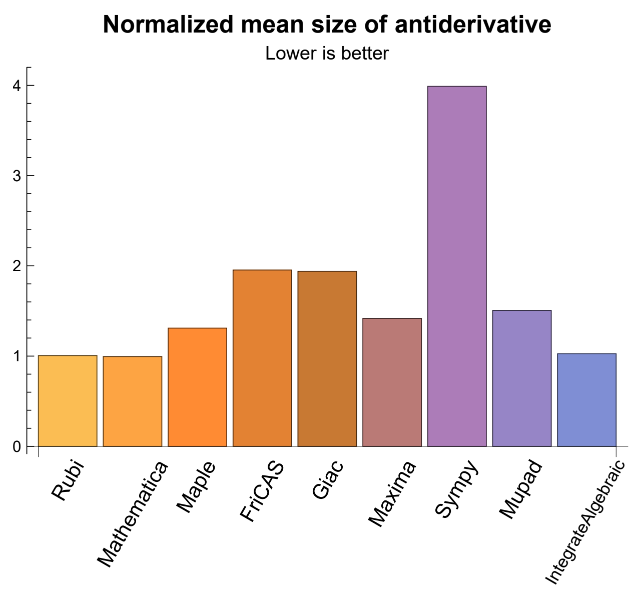
1.3 Performance

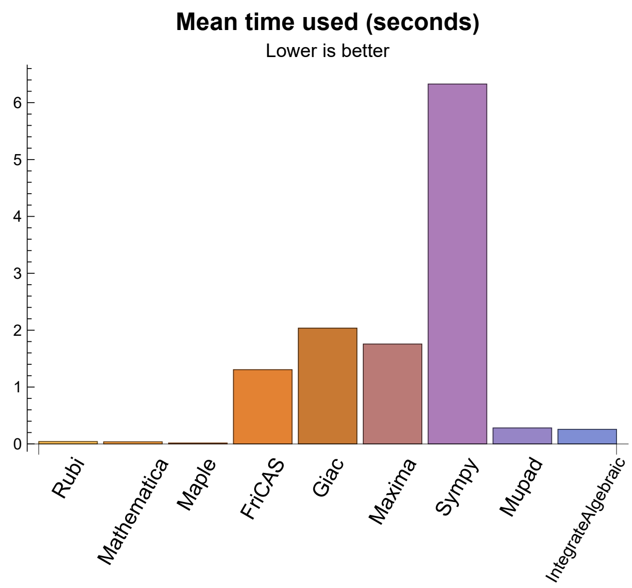
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	75.73	1.00	63.00	1.00
Mathematica	0.04	73.35	0.99	41.00	0.85
Maple	0.02	106.43	1.31	53.00	0.93
Maxima	1.76	107.90	1.42	56.00	0.99
Fricas	1.31	202.55	1.95	77.00	1.35
Sympy	6.33	271.67	3.99	92.00	1.67
Giac	2.04	159.13	1.94	65.00	1.07
Mupad	0.28	123.92	1.51	52.00	0.97
IntegrateAlgebraic	0.26	77.16	1.02	57.00	0.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

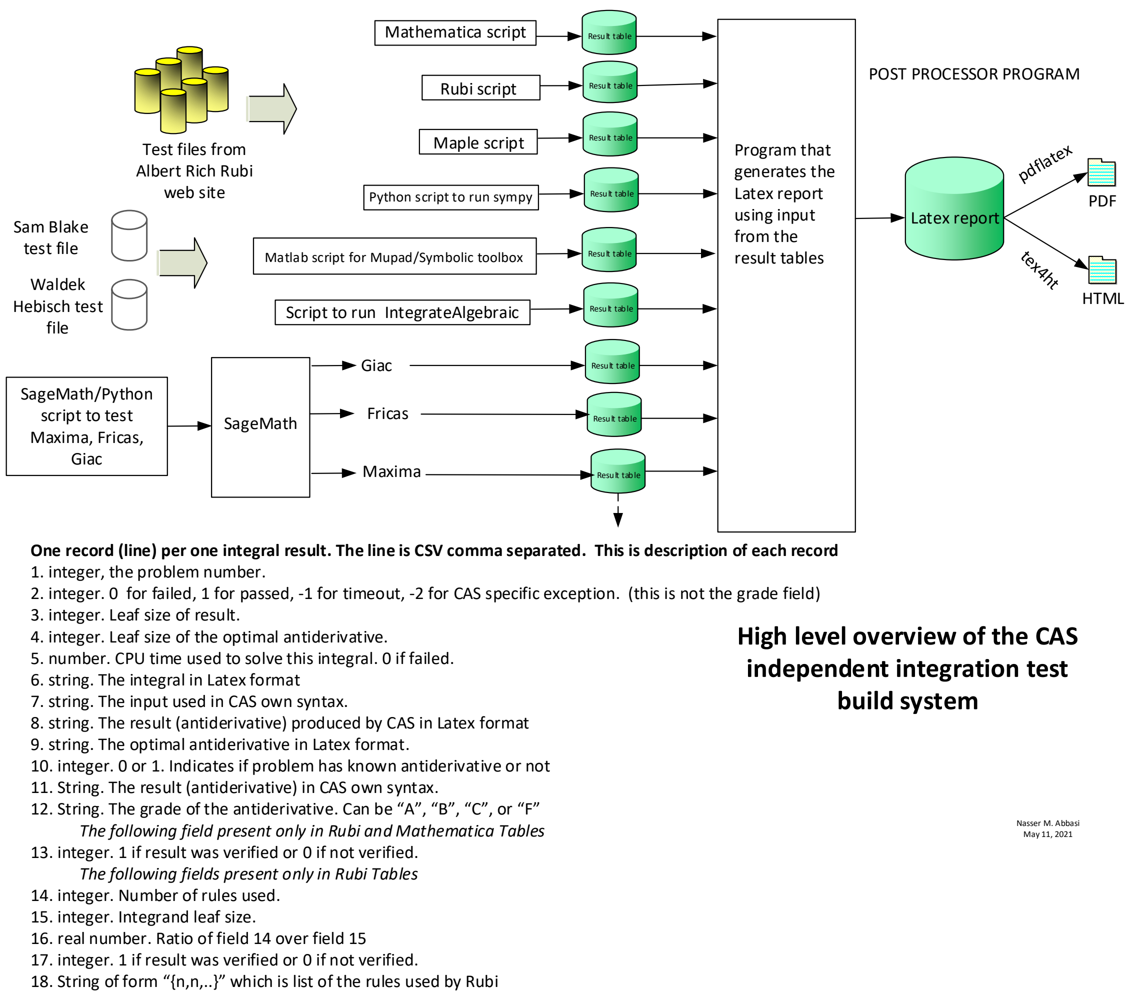
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 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1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { }

C grade: { 369 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 292, 293, 294, 295, 296, 298, 300, 301, 302, 303, 304, 307, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 325, 326, 328, 330, 331, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 351, 352, 353, 354, 355, 359, 360, 368, 369, 370, 371, 372, 373, 374, 375, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 392, 393, 394, 395, 396, 399, 400, 401, 402, 406, 407, 408, 409, 410, 413, 414, 415, 416, 420, 421, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 458, 459, 464, 469, 470, 471, 472, 478, 479, 484, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 527, 528, 529, 530, 533, 534, 535, 536, 539, 540, 541, 542, 545, 546, 547, 548, 551, 552, 553, 554, 557, 558, 559, 560, 563, 564, 565, 566, 569, 570, 571, 572, 573, 574, 575, 576, 579, 580, 581, 582, 583, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 598, 599, 600, 601, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 675, 677, 679, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1104, 1105, 1106, 1107, 1110, 1111, 1112, 1115, 1116, 1117, 1120, 1121, 1122, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1163, 1164, 1165, 1166, 1176, 1177, 1180, 1181, 1184, 1190, 1191, 1192, 1205, 1209, 1210, 1211, 1212, 1213, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1258, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1289, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1302, 1305, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1318, 1319, 1320, 1321, 1322, 1323, 1328, 1329, 1330, 1331, 1332, 1333, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1409, 1410, 1411, 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B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 1130, 1140, 1152, 1153, 1162, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1178, 1179, 1182, 1183, 1185, 1186, 1187, 1188, 1189, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1256, 1257, 1259, 1260, 1417, 1418, 1420, 1421, 1422, 1428, 1433, 1434, 1436, 1442, 1444, 1452 }

C grade: { 290, 291, 297, 299, 305, 306, 308, 318, 319, 320, 321, 323, 324, 327, 329, 332, 333, 341, 342, 348, 349, 350, 356, 357, 358, 361, 362, 363, 364, 365, 366, 367, 376, 377, 383, 384, 390, 391, 397, 398, 403, 404, 405, 411, 412, 417, 418, 419, 422, 423, 453, 454, 455, 456, 457, 460, 461, 462, 463, 465, 466, 467, 468, 473, 474, 475, 476, 477, 480, 481, 482, 483, 485, 486, 487, 488, 525, 526, 531, 532, 537, 538, 543, 544, 549, 550, 555, 556, 561, 562, 567, 568, 577, 578, 584, 596, 597, 602, 616, 617, 623, 635, 636, 641, 674, 676, 678, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 1012, 1013, 1026, 1027, 1028, 1047, 1048, 1049, 1057, 1058, 1059, 1101, 1102, 1103, 1108, 1109, 1113, 1114, 1118, 1119, 1123, 1277, 1278, 1279, 1280, 1288, 1290, 1291, 1292, 1300, 1301, 1303, 1304, 1306, 1315, 1316, 1317, 1324, 1325, 1326, 1327, 1334, 1335, 1336, 1337, 1369, 1370, 1379, 1380, 1381, 1396, 1397, 1398, 1406, 1407, 1408, 1453, 1454, 1455, 1460, 1461, 1462, 1467, 1468, 1469, 1474, 1475, 1476, 1481, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1497, 1498, 1499, 1504, 1505, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1547, 1548, 1549 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 573, 574, 575, 576, 577, 580, 581, 582, 583, 586, 587, 588, 589, 590, 591, 592, 594, 595, 596, 599, 600, 601, 604, 605, 606, 607, 608, 609, 610, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 642, 643, 644, 645, 646, 647, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1011, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1070, 1071, 1072, 1073, 1074, 1075, 1080, 1081, 1082, 1083, 1084, 1088, 1089, 1090, 1091, 1092, 1094, 1095, 1096, 1097, 1098, 1099, 1104, 1105, 1106, 1107, 1110, 1111, 1112, 1115, 1116, 1117, 1120, 1121, 1122, 1125, 1126, 1127, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1157, 1158, 1164, 1165, 1166, 1176, 1205, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1240, 1241, 1242, 1243, 1244, 1245, 1250, 1251, 1252, 1253, 1254, 1255, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1282, 1283, 1284, 1285, 1286, 1289, 1290, 1291, 1292, 1294, 1295, 1296, 1297, 1298, 1302, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1352, 1353, 1358, 1360, 1361, 1362, 1363, 1364, 1371, 1372, 1373, 1374, 1382, 1383, 1384, 1385, 1389, 1391, 1392, 1393, 1394, 1395, 1400, 1401, 1402, 1403, 1404, 1410, 1411, 1412, 1413, 1414, 1424, 1435, 1438, 1456, 1457, 1458, 1459, 1463, 1464, 1465, 1466, 1470, 1471, 1472, 1473, 1477, 1478, 1479, 1480, 1486, 1487, 1488, 1489, 1493, 1494, 1495, 1496, 1500, 1501, 1502, 1503, 1506, 1507, 1508, 1509, 1515, 1516, 1517, 1518, 1522, 1523, 1524, 1525, 1529, 1530, 1531, 1532, 1536, 1537, 1538, 1539, 1543, 1544, 1545, 1546, 1550, 1551, 1552, 1553, 1554, 1557, 1560, 1561, 1563, 1564, 1565, 1566, 1569, 1570, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 199, 212, 213, 226, 227, 228, 243, 244, 442, 516, 517, 543, 544, 572, 578, 584, 593, 597, 602, 611, 617, 623, 632, 635, 636, 641, 648, 649, 699, 700, 701, 947, 948, 949, 965, 984, 997, 998, 999, 1000, 1009, 1010, 1012, 1013, 1022, 1036, 1040, 1041, 1049, 1050, 1060, 1068, 1069, 1076, 1077, 1078, 1079, 1085, 1086, 1087, 1093, 1100, 1124, 1130, 1131, 1140, 1141, 1148, 1152, 1153, 1154, 1155, 1156, 1159, 1160, 1161, 1162, 1163, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1237, 1238, 1239, 1246, 1247, 1248, 1249, 1256, 1257, 1258, 1259, 1260, 1261, 1269, 1281, 1287, 1288, 1293, 1299, 1300, 1301, 1307, 1318, 1327, 1328, 1347, 1354, 1355, 1356, 1357, 1365, 1366, 1367, 1368, 1375, 1376, 1377, 1378, 1386, 1387, 1388, 1390, 1405, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1436, 1437, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1555, 1556, 1558, 1559, 1562, 1567, 1568, 1571, 1572 }

C grade: { 1102, 1108, 1113, 1114, 1118, 1119, 1123, 1481 }

F grade: { 369, 501, 531, 532, 555, 556, 579, 585, 598, 603, 1101, 1103, 1109, 1359, 1369, 1370, 1379, 1380, 1381, 1396, 1397, 1398, 1399, 1406, 1407, 1408, 1409, 1453, 1454, 1455, 1460, 1461, 1462, 1467, 1468, 1469, 1474, 1475, 1476, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1497, 1498, 1499, 1504, 1505, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1547, 1548, 1549 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 217, 218, 219, 220, 221, 222, 223, 224, 225, 230, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 525, 526, 527, 528, 529, 530, 531, 532, 537, 538, 539, 540, 541, 542, 543, 544, 549, 550, 551, 552, 553, 554, 555, 556, 562, 563, 565, 566, 567, 568, 569, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 711, 714, 715, 716, 717, 722, 723, 724, 725, 726, 730, 731, 732, 733, 734, 735, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 770, 771, 772, 773, 774, 778, 779, 780, 781, 786, 787, 789, 790, 791, 792, 793, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 815, 816, 817, 818, 819, 820, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 839, 840, 841, 842, 843, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 924, 931, 932, 933, 934, 939, 940, 941, 942, 952, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 966, 967, 969, 970, 971, 972, 973, 974, 975, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1154, 1158, 1159, 1160, 1161, 1176, 1205, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1238, 1239, 1240, 1241, 1242, 1243, 1248, 1249, 1250, 1251, 1252, 1265, 1269, 1270, 1271, 1272, 1273, 1274, 1281, 1282, 1283, 1284, 1285, 1286, 1293, 1294, 1295, 1296, 1297, 1298, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1318, 1319, 1320, 1321, 1322, 1323, 1328, 1329, 1330, 1331, 1332, 1333, 1338, 1339, 1341, 1342, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1424, 1426, 1427, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1438, 1440, 1441, 1443, 1447, 1450, 1451, 1556, 1557, 1559, 1560, 1561, 1573, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 68, 73, 82, 83, 90, 105, 115, 116, 132, 133, 146, 147, 148, 186, 199, 212, 213, 214, 215, 216, 226, 227, 228, 229, 231, 232, 233, 243, 244, 489, 490, 491, 492, 505, 506, 507, 508, 521, 522, 523, 524, 533, 534, 535, 536, 545, 546, 547, 548, 557, 558, 559, 560, 561, 564, 570, 571, 572, 608, 609, 610, 611, 648, 788, 794, 813, 821, 838, 844, 845, 846, 875, 876, 877, 930, 935, 936, 937, 938, 943, 944, 945, 946, 947, 948, 949, 950, 951, 957, 965, 968, 976, 984, 1001, 1002, 1003, 1004, 1005, 1013, 1014, 1015, 1016, 1017, 1018, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1055, 1056, 1059, 1060, 1061, 1128, 1130, 1131, 1140, 1141, 1148, 1151, 1152, 1153, 1155, 1156, 1157, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1244, 1245, 1246, 1247, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1266, 1267, 1268, 1307, 1340, 1347, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1425, 1428, 1437, 1439, 1442, 1444, 1446, 1449, 1554, 1555, 1558 }

C grade: { 958 }

F grade: { 708, 709, 710, 712, 713, 718, 719, 720, 721, 727, 728, 729, 736, 737, 766, 767, 768, 769, 775, 776, 777, 782, 783, 784, 785, 814, 920, 921, 922, 923, 925, 926, 927, 928, 929, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1275, 1276, 1277, 1278, 1279, 1280, 1287, 1288, 1289, 1290, 1291, 1292, 1299, 1300, 1301, 1302, 1303, 1304, 1313, 1314, 1315, 1316, 1317, 1324, 1325, 1326, 1327, 1334, 1335, 1336, 1337, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1445, 1448, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 204, 205, 206, 207, 208, 209, 210, 211, 222, 223, 230, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 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696, 698, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 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1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1125, 1126, 1127, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1158, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1240, 1241, 1242, 1243, 1250, 1251, 1252, 1271, 1272, 1273, 1274, 1275, 1276, 1287, 1288, 1299, 1300, 1301, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1319, 1320, 1321, 1322, 1323, 1324, 1329, 1330, 1331, 1332, 1343, 1344, 1345, 1346, 1348, 1349, 1350, 1351, 1354, 1355, 1356, 1357, 1358, 1365, 1366, 1367, 1368, 1369, 1376, 1377, 1378, 1379, 1386, 1387, 1388, 1389, 1391, 1397, 1398, 1400, 1417, 1418, 1419, 1420, 1422, 1424, 1426, 1427, 1429, 1435, 1436, 1438, 1440, 1441, 1443, 1446, 1450, 1453, 1463, 1470, 1475, 1477, 1481, 1493, 1500, 1506, 1510, 1536, 1543, 1550, 1554, 1557, 1560, 1561, 1564, 1565, 1569, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 37, 38, 42, 68, 73, 82, 83, 90, 105, 106, 115, 116, 132, 133, 134, 146, 147, 148, 186, 199, 201, 202, 203, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 243, 244, 295, 303, 314, 315, 316, 355, 356, 388, 410, 411, 418, 442, 586, 604, 625, 643, 648, 684, 685, 686, 687, 689, 690, 691, 692, 693, 694, 695, 697, 699, 700, 701, 930, 935, 936, 937, 938, 943, 944, 945, 946, 947, 948, 949, 950, 951, 957, 965, 968, 976, 999, 1000, 1001, 1013, 1014, 1015, 1029, 1030, 1031, 1032, 1040, 1041, 1050, 1060, 1061, 1087, 1093, 1101, 1124, 1130, 1131, 1140, 1141, 1148, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1238, 1239, 1244, 1245, 1246, 1247, 1248, 1249, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1302, 1303, 1304, 1314, 1315, 1316, 1317, 1318, 1325, 1326, 1327, 1328, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1347, 1352, 1353, 1359, 1360, 1361, 1362, 1363, 1364, 1370, 1371, 1372, 1373, 1374, 1375, 1380, 1381, 1382, 1383, 1384, 1385, 1390, 1392, 1393, 1394, 1395, 1396, 1399, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1421, 1423, 1425, 1428, 1430, 1431, 1432, 1433, 1434, 1437, 1439, 1442, 1444, 1445, 1447, 1448, 1449, 1451, 1452, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1464, 1465, 1466, 1467, 1468, 1469, 1471, 1472, 1473, 1474, 1476, 1478, 1479, 1480, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1494, 1495, 1496, 1497, 1498, 1499, 1501, 1502, 1503, 1504, 1505, 1507, 1508, 1509, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1537, 1538, 1539, 1540, 1541, 1542, 1544, 1545, 1546, 1547, 1548, 1549, 1551, 1552, 1553, 1555, 1556, 1558, 1559, 1562, 1563, 1566, 1567, 1568, 1570, 1571, 1572 }

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F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 224, 225, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 289, 290, 291, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 330, 333, 338, 339, 340, 341, 342, 346, 347, 349, 350, 352, 353, 354, 355, 359, 361, 363, 364, 370, 374, 381, 387, 388, 395, 402, 409, 415, 416, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 497, 498, 499, 500, 501, 503, 505, 506, 507, 508, 509, 511, 513, 514, 515, 516, 519, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 569, 570, 571, 572, 573, 574, 577, 578, 579, 580, 581, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 608, 609, 610, 611, 612, 613, 616, 617, 618, 619, 620, 625, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 666, 667, 668, 674, 675, 676, 677, 678, 679, 680, 681, 684, 685, 686, 687, 699, 700, 701, 702, 703, 704, 705, 706, 707, 712, 713, 714, 715, 716, 717, 718, 721, 722, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 743, 744, 745, 746, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 770, 771, 772, 773, 774, 775, 778, 779, 780, 781, 782, 783, 786, 787, 793, 799, 800, 801, 805, 806, 807, 808, 809, 855, 863, 871, 877, 924, 931, 932, 933, 934, 939, 940, 941, 942, 946, 953, 954, 955, 956, 959, 960, 961, 962, 963, 964, 966, 969, 970, 971, 973, 974, 975, 977, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 1000, 1001, 1006, 1007, 1011, 1012, 1020, 1021, 1024, 1025, 1026, 1035, 1036, 1037, 1038, 1039, 1042, 1047, 1048, 1049, 1051, 1052, 1057, 1061, 1062, 1063, 1087, 1091, 1092, 1093, 1094, 1096, 1098, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1142, 1143, 1145, 1146, 1147, 1149, 1158, 1159, 1160, 1161, 1229, 1230, 1231, 1232, 1233, 1237, 1238, 1239, 1240, 1241, 1242, 1248, 1249, 1250, 1251, 1270, 1271, 1272, 1273, 1274, 1275, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1350, 1351, 1419, 1424, 1426, 1438, 1440, 1446, 1447, 1449, 1450, 1451, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

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C grade: { 328, 331, 332, 362, 368, 369, 375, 376, 377, 382, 383, 384, 389, 390, 391, 396, 397, 398, 399, 400, 401, 403, 404, 405, 410, 411, 412, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 517, 532, 544, 556, 568, 605, 644, 661, 662, 663, 664, 665, 669, 670, 671, 672, 673, 698, 958, 1043, 1044, 1045, 1046, 1058, 1059, 1060, 1071, 1072, 1073, 1074, 1079, 1080, 1081, 1082, 1097, 1099, 1100, 1349, 1421, 1428, 1431, 1433, 1435, 1442, 1444, 1481 }

F grade: { 365, 366, 438, 445, 682, 683, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 708, 709, 710, 711, 719, 720, 729, 741, 742, 747, 748, 754, 767, 768, 769, 777, 785, 789, 790, 791, 795, 796, 797, 802, 803, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 869, 870, 872, 873, 874, 875, 876, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 1018, 1019, 1032, 1033, 1034, 1067, 1068, 1069, 1070, 1075, 1076, 1077, 1078, 1083, 1084, 1085, 1089, 1090, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1256, 1257, 1258, 1259, 1277, 1278, 1279, 1280, 1288, 1289, 1290, 1291, 1292, 1300, 1301, 1302, 1303, 1304, 1314, 1315, 1316, 1317, 1325, 1326, 1327, 1335, 1336, 1337, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1420, 1422, 1423, 1425, 1427, 1429, 1430, 1432, 1434, 1436, 1437, 1439, 1441, 1443, 1445, 1448, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 113, 114, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 296, 297, 298, 299, 304, 305, 306, 307, 308, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 374, 375, 376, 377, 381, 382, 383, 384, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 495, 496, 503, 504, 511, 512, 519, 520, 574, 575, 576, 577, 583, 595, 613, 614, 615, 616, 634, 647, 648, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 703, 706, 707, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 746, 747, 748, 754, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 794, 795, 796, 802, 810, 811, 812, 813, 814, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 832, 835, 836, 837, 838, 839, 840, 843, 844, 845, 846, 847, 848, 851, 852, 853, 854, 855, 859, 860, 861, 862, 863, 867, 868, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 884, 888, 889, 931, 932, 933, 934, 935, 936, 937, 939, 940, 941, 942, 943, 944, 945, 951, 952, 953, 955, 956, 957, 959, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 985, 986, 987, 988, 989, 990, 991, 992, 993, 995, 997, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1008, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1054, 1055, 1056, 1057, 1058, 1067, 1071, 1086, 1087, 1091, 1092, 1093, 1094, 1095, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1142, 1143, 1144, 1145, 1146, 1147, 1149, 1150, 1151, 1157, 1158, 1160, 1161, 1164, 1165, 1166, 1176, 1205, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1240, 1241, 1242, 1243, 1244, 1248, 1249, 1250, 1251, 1252, 1262, 1263, 1264, 1265, 1274, 1275, 1276, 1277, 1278, 1287, 1288, 1289, 1290, 1291, 1299, 1300, 1301, 1302, 1303, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1320, 1321, 1322, 1323, 1324, 1325, 1330, 1331, 1332, 1333, 1334, 1345, 1346, 1348, 1349, 1350, 1351, 1352, 1353, 1358, 1386, 1387, 1388, 1389, 1390, 1397, 1398, 1399, 1400, 1410, 1416, 1417, 1418, 1419, 1420, 1422, 1423, 1424, 1425, 1426, 1427, 1429, 1430, 1431, 1432, 1433, 1434, 1437, 1438, 1439, 1440, 1441, 1443, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1561, 1573, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 37, 38, 73, 82, 90, 105, 115, 116, 132, 133, 146, 147, 148, 212, 226, 227, 243, 244, 284, 285, 286, 292, 293, 294, 295, 300, 301, 302, 303, 309, 310, 311, 312, 313, 314, 315, 316, 371, 372, 373, 378, 379, 380, 385, 386, 387, 388, 435, 442, 494, 502, 510, 518, 573, 578, 579, 580, 581, 582, 584, 585, 586, 587, 588, 589, 594, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 612, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 699, 700, 701, 702, 704, 705, 742, 869, 870, 881, 882, 883, 885, 886, 890, 920, 922, 930, 938, 946, 947, 948, 949, 950, 954, 964, 984, 994, 996, 998, 1006, 1007, 1009, 1019, 1020, 1021, 1022, 1023, 1024, 1041, 1049, 1050, 1051, 1052, 1053, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1068, 1069, 1070, 1072, 1073, 1074, 1075, 1080, 1081, 1082, 1083, 1084, 1085, 1088, 1089, 1090, 1096, 1097, 1098, 1099, 1128, 1130, 1131, 1140, 1141, 1148, 1152, 1153, 1154, 1155, 1156, 1159, 1162, 1163, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1236, 1237, 1238, 1239, 1245, 1246, 1247, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1304, 1307, 1317, 1318, 1319, 1326, 1327, 1328, 1329, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1347, 1354, 1355, 1356, 1357, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1391, 1392, 1393, 1394, 1395, 1396, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1411, 1412, 1413, 1414, 1415, 1421, 1428, 1442, 1444, 1554, 1555, 1556, 1557, 1558, 1559, 1560 }

C grade: { 958, 1435, 1436 }

F grade: { 368, 369, 489, 490, 491, 492, 493, 497, 498, 499, 500, 501, 505, 506, 507, 508, 509, 513, 514, 515, 516, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 590, 591, 592, 593, 608, 609, 610, 611, 629, 630, 631, 632, 649, 708, 709, 710, 711, 712, 713, 743, 744, 745, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 791, 792, 793, 797, 798, 799, 800, 801, 803, 804, 805, 806, 807, 808, 809, 815, 816, 817, 823, 824, 825, 826, 833, 834, 841, 842, 849, 850, 856, 857, 858, 864, 865, 866, 887, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 921, 923, 924, 925, 926, 927, 928, 929, 1076, 1077, 1078, 1079, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549, 1550, 1551, 1552, 1553, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1574, 1575, 1576 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 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709, 710, 711, 712, 713, 717, 718, 721, 727, 728, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 747, 748, 749, 750, 751, 752, 753, 755, 756, 757, 758, 759, 760, 761, 792, 793, 798, 799, 800, 801, 804, 805, 806, 807, 808, 809, 855, 863, 871, 877, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 998, 999, 1001, 1002, 1003, 1004, 1005, 1014, 1015, 1016, 1017, 1018, 1029, 1030, 1031, 1032, 1033, 1034, 1036, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1051, 1052, 1053, 1054, 1055, 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1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1307, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1318, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1328, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1357, 1358, 1360, 1361, 1362, 1363, 1364, 1371, 1372, 1373, 1374, 1382, 1383, 1384, 1385, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1400, 1401, 1402, 1403, 1404, 1405, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1421, 1422, 1423, 1424, 1425, 1426, 1427, 1428, 1429, 1430, 1431, 1432, 1433, 1434, 1435, 1436, 1437, 1438, 1439, 1440, 1441, 1442, 1443, 1444, 1445, 1446, 1447, 1448, 1449, 1450, 1451, 1452, 1456, 1457, 1458, 1459, 1470, 1471, 1472, 1473, 1486, 1487, 1488, 1489, 1500, 1501, 1502, 1503, 1515, 1516, 1517, 1518, 1522, 1523, 1524, 1525, 1529, 1530, 1531, 1532, 1536, 1537, 1538, 1539, 1550, 1551, 1552, 1553, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

C grade: { }

F grade: { 369, 489, 490, 493, 497, 498, 501, 505, 506, 509, 513, 514, 517, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 577, 578, 579, 584, 585, 590, 591, 596, 597, 598, 602, 603, 608, 609, 616, 617, 618, 623, 624, 629, 630, 635, 636, 637, 641, 642, 714, 715, 716, 719, 720, 722, 723, 724, 725, 726, 729, 730, 731, 732, 733, 734, 738, 746, 754, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 794, 795, 796, 797, 802, 803, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 856, 857, 858, 859, 860, 861, 862, 864, 865, 866, 867, 868, 869, 870, 872, 873, 874, 875, 876, 878, 879, 880, 994, 995, 996, 997, 1000, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1035, 1037, 1038, 1039, 1047, 1048, 1049, 1050, 1057, 1058, 1059, 1060, 1068, 1069, 1076, 1077, 1084, 1085, 1091, 1101, 1102, 1103, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1123, 1354, 1355, 1356, 1359, 1365, 1366, 1367, 1368, 1369, 1370, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1386, 1387, 1388, 1396, 1397, 1398, 1399, 1406, 1407, 1408, 1409, 1453, 1454, 1455, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1519, 1520, 1521, 1526, 1527, 1528, 1533, 1534, 1535, 1540, 1541, 1542, 1543, 1544, 1545, 1546, 1547, 1548, 1549 }

2.1.9 IntegrateAlgebraic

A grade: { 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 37, 38, 39, 40, 41, 42, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 436, 437, 438, 439, 440, 441, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 994, 995, 996, 997, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1010, 1011, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1023, 1024, 1025, 1026, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1091, 1092, 1094, 1095, 1096, 1097, 1100, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304, 1305, 1306, 1308, 1309, 1310, 1311, 1312, 1313, 1314, 1315, 1316, 1317, 1319, 1320, 1321, 1322, 1323, 1324, 1325, 1326, 1327, 1329, 1330, 1331, 1332, 1333, 1334, 1335, 1336, 1337, 1338, 1339, 1340, 1341, 1342, 1343, 1344, 1345, 1346, 1347, 1348, 1349, 1350, 1351, 1352, 1353, 1354, 1355, 1356, 1357, 1358, 1359, 1360, 1361, 1362, 1363, 1364, 1365, 1366, 1367, 1368, 1369, 1370, 1371, 1372, 1373, 1374, 1375, 1376, 1377, 1378, 1379, 1380, 1381, 1382, 1383, 1384, 1385, 1386, 1387, 1388, 1389, 1390, 1391, 1392, 1393, 1394, 1395, 1396, 1397, 1398, 1399, 1400, 1401, 1402, 1403, 1404, 1405, 1406, 1407, 1408, 1409, 1410, 1411, 1412, 1413, 1414, 1415, 1416, 1417, 1418, 1419, 1420, 1422, 1423, 1425, 1426, 1427, 1429, 1430, 1432, 1436, 1440, 1441, 1443, 1445, 1446, 1447, 1448, 1450, 1451, 1452, 1453, 1454, 1455, 1456, 1457, 1458, 1459, 1460, 1461, 1462, 1463, 1464, 1465, 1466, 1467, 1468, 1469, 1470, 1471, 1472, 1473, 1474, 1475, 1476, 1477, 1478, 1479, 1480, 1481, 1482, 1483, 1484, 1485, 1486, 1487, 1488, 1489, 1490, 1491, 1492, 1493, 1494, 1495, 1496, 1497, 1498, 1499, 1500, 1501, 1502, 1503, 1504, 1505, 1506, 1507, 1508, 1509, 1510, 1511, 1512, 1513, 1514, 1515, 1516, 1517, 1518, 1519, 1520, 1521, 1522, 1523, 1524, 1525, 1526, 1527, 1528, 1529, 1530, 1531, 1532, 1533, 1534, 1535, 1536, 1537, 1538, 1539, 1540, 1542, 1543, 1544, 1545, 1546, 1547, 1549, 1550, 1551, 1552, 1553, 1596, 1597, 1598, 1599, 1600, 1601, 1602, 1603 }

B grade: { 315, 998, 1009, 1022, 1041, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1093, 1269, 1307, 1318, 1328, 1421, 1428, 1431, 1433, 1434, 1437, 1439, 1442, 1444 }

C grade: { 999, 1012, 1027, 1040, 1049, 1059, 1449 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 34, 35, 36, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 368, 369, 428, 435, 442, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 1098, 1099, 1101, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1424, 1435, 1438, 1541, 1548, 1554, 1555, 1556, 1557, 1558, 1559, 1560, 1561, 1562, 1563, 1564, 1565, 1566, 1567, 1568, 1569, 1570, 1571, 1572, 1573, 1574, 1575, 1576, 1577, 1578, 1579, 1580, 1581, 1582, 1583, 1584, 1585, 1586, 1587, 1588, 1589, 1590, 1591, 1592, 1593, 1594, 1595 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	0	1	1	0
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.005	0.420	1.406	0.008	1.087	0.041	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	0	1	1	0
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.002	0.415	1.443	0.023	1.067	0.005	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.413	1.448	0.016	1.037	0.006	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	3	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	1.00	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.419	1.257	0.015	1.114	0.004	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	5	5	5	4	3	3	5	3	3	0
N.S.	1	1.00	1.00	0.80	0.60	0.60	1.00	0.60	0.60	0.00
time (sec)	N/A	0.000	0.000	0.000	0.422	1.295	0.015	1.158	0.008	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.000	0.434	1.458	0.015	0.836	0.002	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	3	3	3	4	3	3	2	3	3	0
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.67	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.436	0.679	0.016	0.747	0.002	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	5	4	4	3	4	4	0
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.426	0.861	0.016	1.248	0.002	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	12	11	18	10	11	11	0
N.S.	1	1.00	1.00	0.86	0.79	1.29	0.71	0.79	0.79	0.00
time (sec)	N/A	0.008	0.000	0.000	0.445	1.203	0.055	1.075	0.003	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	3	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.001	0.421	1.310	0.055	1.182	0.118	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	3	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.002	0.416	1.137	0.054	1.183	0.022	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	3	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.000	0.420	1.130	0.055	1.095	0.010	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	3	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.43	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.001	0.415	1.105	0.017	0.953	0.012	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	0	1	1	0
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	0.00
time (sec)	N/A	0.000	0.000	0.000	0.442	1.287	0.015	1.002	0.002	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	2	2	3	2	0
N.S.	1	1.00	1.00	1.50	1.00	1.00	1.00	1.50	1.00	0.00
time (sec)	N/A	0.000	0.000	0.002	0.435	0.995	0.060	0.803	0.036	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	5	5	5	6	5	5	3	5	5	0
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	1.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.000	0.432	1.793	0.058	0.849	0.035	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.001	0.489	1.336	0.058	0.841	0.014	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.000	0.426	1.263	0.059	1.132	0.012	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.000	0.416	1.311	0.060	1.290	0.070	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.001	0.001	0.017	0.428	0.863	0.068	0.900	0.077	0.013

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.002	0.415	1.547	0.059	0.982	0.031	0.003

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.002	0.425	0.932	0.062	1.042	0.029	0.003

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	5	5	5	7
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71	1.00
time (sec)	N/A	0.000	0.001	0.003	0.419	0.771	0.059	1.127	0.031	0.003

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	7
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	1.00
time (sec)	N/A	0.000	0.001	0.002	0.698	1.427	0.061	1.010	0.031	0.003

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	8	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.001	0.616	1.362	0.061	0.924	0.034	0.003

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.001	0.001	0.003	0.559	1.265	0.060	1.051	0.073	0.003

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.003	0.555	1.470	0.061	1.047	0.066	0.003

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.001	0.505	0.699	0.062	0.913	0.065	0.003

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.002	0.480	1.391	0.061	0.848	0.065	0.003

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	7	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.78	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.001	0.456	1.350	0.061	1.136	0.040	0.003

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	5	5	5	7
N.S.	1	1.00	1.00	0.86	0.71	0.71	0.71	0.71	0.71	1.00
time (sec)	N/A	0.000	0.001	0.002	0.521	0.814	0.060	0.949	0.067	0.003

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	7
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	1.00
time (sec)	N/A	0.000	0.001	0.003	0.513	1.424	0.060	1.119	0.073	0.003

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	6	5	5	8	5	5	9
N.S.	1	1.00	1.00	0.67	0.56	0.56	0.89	0.56	0.56	1.00
time (sec)	N/A	0.000	0.001	0.001	0.491	1.106	0.060	0.923	0.051	0.003

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	10	12	11	20	0
N.S.	1	1.00	1.00	1.09	1.00	0.91	1.09	1.00	1.82	0.00
time (sec)	N/A	0.002	0.001	0.003	0.638	1.686	0.063	0.948	0.345	0.003

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	12	13	16	12	17	16	12	0
N.S.	1	1.00	0.75	0.81	1.00	0.75	1.06	1.00	0.75	0.00
time (sec)	N/A	0.003	0.002	0.003	0.537	1.371	0.064	1.037	0.183	0.011

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	22	21	21	19	22	21	0
N.S.	1	1.00	1.00	0.96	0.91	0.91	0.83	0.96	0.91	0.00
time (sec)	N/A	0.014	0.004	0.002	0.665	1.446	0.087	1.071	0.143	0.001

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	104	270	444	93	23
N.S.	1	1.00	1.00	0.87	0.83	4.52	11.74	19.30	4.04	1.00
time (sec)	N/A	0.012	0.020	0.003	0.558	1.484	71.801	1.635	0.183	0.009

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	59	156	195	45	23
N.S.	1	1.00	1.00	0.87	0.83	2.57	6.78	8.48	1.96	1.00
time (sec)	N/A	0.011	0.014	0.002	0.885	0.869	5.270	1.043	0.170	0.011

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	82	19	19	23
N.S.	1	1.00	1.00	0.87	0.83	0.83	3.57	0.83	0.83	1.00
time (sec)	N/A	0.011	0.010	0.003	0.738	1.403	0.444	1.070	0.075	0.010

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	20	19	19	31	19	19	21
N.S.	1	1.00	1.00	0.95	0.90	0.90	1.48	0.90	0.90	1.00
time (sec)	N/A	0.010	0.008	0.004	0.920	1.073	1.782	1.165	0.106	0.010

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	20	19	34	58	19	19	21
N.S.	1	1.00	1.00	0.95	0.90	1.62	2.76	0.90	0.90	1.00
time (sec)	N/A	0.011	0.008	0.002	1.003	1.572	1.795	1.645	0.135	0.013

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	68	102	19	19	23
N.S.	1	1.00	1.00	0.87	0.83	2.96	4.43	0.83	0.83	1.00
time (sec)	N/A	0.011	0.011	0.003	0.848	1.406	6.786	1.317	0.179	0.011

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.008	0.001	0.000	0.895	1.341	0.063	1.216	0.020	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.007	0.001	0.000	0.861	1.452	0.063	1.738	0.019	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.00
time (sec)	N/A	0.006	0.001	0.001	0.831	1.255	0.064	1.172	0.019	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.002	0.000	0.002	0.887	1.360	0.060	1.248	0.017	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	9	8	8	7	9	8	0
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.12	1.00	0.00
time (sec)	N/A	0.003	0.001	0.008	0.843	1.264	0.090	1.057	0.017	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	13	7	12	11	0
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.64	1.09	1.00	0.00
time (sec)	N/A	0.004	0.002	0.018	0.871	1.408	0.111	1.048	0.033	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	15	14	11	11	12	11	11	0
N.S.	1	1.00	0.88	0.82	0.65	0.65	0.71	0.65	0.65	0.00
time (sec)	N/A	0.002	0.001	0.005	1.125	1.316	0.110	1.223	0.023	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.005	0.002	0.004	1.097	1.376	0.131	1.383	0.026	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.005	0.002	0.006	1.051	1.466	0.163	1.679	0.027	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.012	0.002	0.002	1.195	0.749	0.072	1.611	0.077	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	24	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.00
time (sec)	N/A	0.010	0.002	0.000	1.051	1.130	0.073	1.023	0.031	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.009	0.001	0.002	0.989	1.152	0.067	1.414	0.030	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	20	20	19	12	20	0
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	0.00
time (sec)	N/A	0.001	0.001	0.001	1.078	1.218	0.071	1.163	0.029	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	20	20	21	20	0
N.S.	1	1.00	1.00	0.95	0.91	0.91	0.91	0.95	0.91	0.00
time (sec)	N/A	0.006	0.001	0.001	1.115	0.841	0.108	1.350	0.029	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	21	20	24	17	21	20	0
N.S.	1	1.00	1.00	1.05	1.00	1.20	0.85	1.05	1.00	0.00
time (sec)	N/A	0.008	0.001	0.006	1.103	1.366	0.126	1.092	0.066	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	23	21	26	22	22	23	0
N.S.	1	1.00	1.00	0.96	0.88	1.08	0.92	0.92	0.96	0.00
time (sec)	N/A	0.008	0.003	0.007	1.179	0.909	0.167	1.179	0.044	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	26	25	22	22	24	22	22	0
N.S.	1	1.00	1.53	1.47	1.29	1.29	1.41	1.29	1.29	0.00
time (sec)	N/A	0.002	0.006	0.006	1.354	1.294	0.178	1.479	0.035	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.008	0.003	0.005	1.344	0.885	0.187	1.116	0.035	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.008	0.007	0.005	1.329	1.434	0.190	1.191	0.035	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.008	0.003	0.005	1.347	0.719	0.198	1.069	0.034	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.008	0.006	0.006	1.351	1.363	0.214	1.199	0.036	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.018	0.002	0.002	1.320	1.074	0.077	1.218	0.042	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.016	0.002	0.001	1.354	0.686	0.074	1.272	0.041	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	39	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.81	0.00
time (sec)	N/A	0.015	0.002	0.000	1.337	0.951	0.072	1.221	0.039	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	40	35	34	34	36	34	34	0
N.S.	1	1.00	1.33	1.17	1.13	1.13	1.20	1.13	1.13	0.00
time (sec)	N/A	0.009	0.002	0.001	1.378	0.695	0.074	1.206	0.040	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	31	31	32	12	31	0
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21	0.00
time (sec)	N/A	0.002	0.002	0.001	1.386	1.341	0.074	1.364	0.041	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	32	31	31	34	32	31	0
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.97	0.91	0.89	0.00
time (sec)	N/A	0.011	0.003	0.003	1.343	1.334	0.122	0.949	0.035	0.001

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	34	33	32	36	31	33	32	0
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.91	0.97	0.94	0.00
time (sec)	N/A	0.013	0.004	0.006	1.293	1.542	0.130	1.357	0.035	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	32	30	37	32	31	32	0
N.S.	1	1.00	1.00	0.97	0.91	1.12	0.97	0.94	0.97	0.00
time (sec)	N/A	0.013	0.005	0.004	1.344	1.328	0.187	1.177	0.030	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	34	34	37	36	35	34	0
N.S.	1	1.00	1.00	0.92	0.92	1.00	0.97	0.95	0.92	0.00
time (sec)	N/A	0.012	0.004	0.006	1.305	1.006	0.235	1.141	0.071	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	39	36	33	33	36	33	33	0
N.S.	1	1.00	2.29	2.12	1.94	1.94	2.12	1.94	1.94	0.00
time (sec)	N/A	0.002	0.003	0.006	1.334	1.297	0.256	1.286	0.026	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	41	36	35	35	37	35	34	0
N.S.	1	1.00	1.14	1.00	0.97	0.97	1.03	0.97	0.94	0.00
time (sec)	N/A	0.005	0.006	0.005	1.351	1.478	0.247	1.133	0.027	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.013	0.004	0.006	1.355	0.974	0.336	1.152	0.025	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.013	0.003	0.004	1.355	1.026	0.290	1.123	0.026	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	66	57	56	56	63	56	56	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.85	0.00
time (sec)	N/A	0.032	0.003	0.006	1.383	0.841	0.089	0.940	0.025	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	65	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.83	0.00
time (sec)	N/A	0.028	0.002	0.001	1.362	1.287	0.082	1.205	0.023	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	66	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.83	0.00
time (sec)	N/A	0.025	0.002	0.000	1.469	1.327	0.079	1.095	0.024	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	66	57	56	56	63	56	56	0
N.S.	1	1.00	1.03	0.89	0.88	0.88	0.98	0.88	0.88	0.00
time (sec)	N/A	0.026	0.002	0.001	1.397	1.278	0.100	1.047	0.024	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	67	58	57	57	65	57	57	0
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.38	1.21	1.21	0.00
time (sec)	N/A	0.021	0.002	0.002	1.322	1.406	0.077	1.715	0.024	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	67	58	57	57	65	57	57	0
N.S.	1	1.00	2.23	1.93	1.90	1.90	2.17	1.90	1.90	0.00
time (sec)	N/A	0.008	0.002	0.000	1.311	0.797	0.084	0.930	0.023	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	53	53	60	12	53	0
N.S.	1	1.00	1.00	0.93	3.79	3.79	4.29	0.86	3.79	0.00
time (sec)	N/A	0.002	0.002	0.002	1.367	1.208	0.081	1.202	0.024	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	59	54	53	53	60	54	53	0
N.S.	1	1.00	1.00	0.92	0.90	0.90	1.02	0.92	0.90	0.00
time (sec)	N/A	0.018	0.003	0.003	1.372	1.437	0.155	1.116	0.029	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	55	54	59	56	55	54	0
N.S.	1	1.00	1.00	0.95	0.93	1.02	0.97	0.95	0.93	0.00
time (sec)	N/A	0.021	0.005	0.007	1.396	0.969	0.175	1.275	0.029	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	55	53	59	60	54	55	0
N.S.	1	1.00	1.00	0.92	0.88	0.98	1.00	0.90	0.92	0.00
time (sec)	N/A	0.022	0.005	0.005	1.362	0.747	0.204	1.187	0.029	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	60	55	55	59	60	56	55	0
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.93	0.92	0.00
time (sec)	N/A	0.022	0.004	0.006	1.407	1.732	0.258	0.995	0.039	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	57	54	54	59	58	55	54	0
N.S.	1	1.00	1.00	0.95	0.95	1.04	1.02	0.96	0.95	0.00
time (sec)	N/A	0.020	0.005	0.007	1.365	0.807	0.289	1.380	0.077	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	56	56	59	60	57	56	0
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.98	0.93	0.92	0.00
time (sec)	N/A	0.021	0.004	0.007	1.348	1.934	0.357	1.368	0.041	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	65	58	55	55	60	55	55	0
N.S.	1	1.00	3.82	3.41	3.24	3.24	3.53	3.24	3.24	0.00
time (sec)	N/A	0.002	0.004	0.004	1.299	1.136	0.371	0.941	0.039	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	67	58	57	57	61	57	57	0
N.S.	1	1.00	1.86	1.61	1.58	1.58	1.69	1.58	1.58	0.00
time (sec)	N/A	0.005	0.004	0.006	1.356	1.371	0.414	1.116	0.073	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	67	58	57	57	61	57	57	0
N.S.	1	1.00	1.20	1.04	1.02	1.02	1.09	1.02	1.02	0.00
time (sec)	N/A	0.010	0.004	0.006	1.395	1.767	0.431	1.100	0.039	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	58	57	57	61	57	56	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84	0.00
time (sec)	N/A	0.022	0.006	0.003	1.333	1.101	0.450	1.207	0.084	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	61	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.83	0.83	0.00
time (sec)	N/A	0.021	0.004	0.007	1.374	0.979	0.490	1.003	0.082	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	69	58	57	57	61	57	57	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.88	0.83	0.83	0.00
time (sec)	N/A	0.021	0.004	0.006	1.314	0.835	0.582	1.381	0.042	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	58	57	57	61	57	56	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84	0.00
time (sec)	N/A	0.021	0.004	0.006	1.364	0.730	0.616	1.233	0.040	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	58	57	57	61	57	56	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84	0.00
time (sec)	N/A	0.022	0.004	0.004	1.378	1.185	0.625	1.116	0.038	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	80	79	79	94	79	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.00
time (sec)	N/A	0.048	0.003	0.001	1.400	1.286	0.098	0.952	0.153	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	80	79	79	92	79	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.97	0.83	0.83	0.00
time (sec)	N/A	0.040	0.002	0.001	1.299	1.343	0.093	0.992	0.074	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	80	79	79	94	79	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.83	0.00
time (sec)	N/A	0.039	0.002	0.001	1.292	1.325	0.107	1.009	0.068	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	92	79	78	78	90	78	78	0
N.S.	1	1.00	0.96	0.82	0.81	0.81	0.94	0.81	0.81	0.00
time (sec)	N/A	0.040	0.002	0.000	1.389	1.344	0.106	1.083	0.060	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	93	80	79	79	92	79	79	0
N.S.	1	1.00	1.15	0.99	0.98	0.98	1.14	0.98	0.98	0.00
time (sec)	N/A	0.036	0.002	0.001	1.351	1.257	0.099	1.127	0.062	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	93	80	79	79	92	79	79	0
N.S.	1	1.00	1.45	1.25	1.23	1.23	1.44	1.23	1.23	0.00
time (sec)	N/A	0.030	0.002	0.001	1.341	1.327	0.089	1.221	0.104	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	93	80	79	79	92	79	31	0
N.S.	1	1.00	1.98	1.70	1.68	1.68	1.96	1.68	0.66	0.00
time (sec)	N/A	0.024	0.002	0.001	1.327	1.111	0.093	0.911	0.121	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	91	80	79	79	90	79	25	0
N.S.	1	1.00	3.03	2.67	2.63	2.63	3.00	2.63	0.83	0.00
time (sec)	N/A	0.008	0.002	0.001	1.349	0.996	0.090	0.918	0.115	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	75	83	12	75	0
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36	0.00
time (sec)	N/A	0.002	0.001	0.000	1.389	0.891	0.082	1.062	0.060	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	87	76	75	75	88	76	75	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.87	0.86	0.00
time (sec)	N/A	0.027	0.003	0.003	1.338	1.383	0.187	1.301	0.072	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	77	76	81	85	77	76	0
N.S.	1	1.00	1.00	0.90	0.88	0.94	0.99	0.90	0.88	0.00
time (sec)	N/A	0.032	0.004	0.007	1.357	1.314	0.201	1.123	0.055	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	77	75	81	85	76	77	0
N.S.	1	1.00	1.00	0.92	0.89	0.96	1.01	0.90	0.92	0.00
time (sec)	N/A	0.033	0.004	0.007	1.377	1.618	0.252	1.060	0.051	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	77	77	81	87	78	77	0
N.S.	1	1.00	1.00	0.90	0.90	0.94	1.01	0.91	0.90	0.00
time (sec)	N/A	0.032	0.004	0.007	1.329	1.330	0.308	1.091	0.051	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	86	77	77	81	85	78	77	0
N.S.	1	1.00	1.00	0.90	0.90	0.94	0.99	0.91	0.90	0.00
time (sec)	N/A	0.032	0.004	0.008	1.397	1.518	0.328	1.042	0.090	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	84	77	77	81	83	78	77	0
N.S.	1	1.00	1.00	0.92	0.92	0.96	0.99	0.93	0.92	0.00
time (sec)	N/A	0.032	0.004	0.006	1.382	1.642	0.449	1.014	0.106	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	85	76	76	81	82	77	81	0
N.S.	1	1.00	1.00	0.89	0.89	0.95	0.96	0.91	0.95	0.00
time (sec)	N/A	0.031	0.005	0.008	1.384	1.404	0.595	0.995	0.110	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	78	78	81	83	79	78	0
N.S.	1	1.00	1.00	0.88	0.88	0.91	0.93	0.89	0.88	0.00
time (sec)	N/A	0.033	0.004	0.007	1.363	1.332	0.627	1.274	0.068	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	87	80	77	77	83	77	77	0
N.S.	1	1.00	5.12	4.71	4.53	4.53	4.88	4.53	4.53	0.00
time (sec)	N/A	0.002	0.004	0.005	1.295	1.407	0.604	0.923	0.067	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	91	80	79	79	85	79	23	0
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.19	0.64	0.00
time (sec)	N/A	0.005	0.004	0.006	1.314	1.210	0.729	1.007	0.093	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	93	80	79	79	85	79	79	0
N.S.	1	1.00	1.66	1.43	1.41	1.41	1.52	1.41	1.41	0.00
time (sec)	N/A	0.010	0.004	0.005	1.382	1.323	0.762	1.064	0.109	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	93	80	79	79	85	79	79	0
N.S.	1	1.00	1.22	1.05	1.04	1.04	1.12	1.04	1.04	0.00
time (sec)	N/A	0.016	0.004	0.006	1.364	0.941	0.747	1.149	0.106	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	93	80	79	79	85	79	79	0
N.S.	1	1.00	0.97	0.83	0.82	0.82	0.89	0.82	0.82	0.00
time (sec)	N/A	0.026	0.004	0.006	1.445	1.125	0.795	1.147	0.066	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	93	80	79	79	85	79	78	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.91	0.85	0.84	0.00
time (sec)	N/A	0.033	0.006	0.005	1.368	0.863	0.782	1.039	0.066	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	80	79	79	85	79	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.83	0.83	0.00
time (sec)	N/A	0.030	0.004	0.006	1.343	0.823	0.878	0.890	0.070	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	80	79	79	85	79	79	0
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.89	0.83	0.83	0.00
time (sec)	N/A	0.030	0.004	0.005	1.390	1.428	0.878	1.002	0.110	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	132	113	112	112	133	112	112	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85	0.00
time (sec)	N/A	0.076	0.003	0.001	1.326	1.229	0.108	0.978	0.150	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	132	113	112	112	131	112	112	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.85	0.00
time (sec)	N/A	0.059	0.004	0.001	1.340	1.052	0.111	0.987	0.084	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	132	113	112	112	133	112	112	0
N.S.	1	1.00	1.00	0.86	0.85	0.85	1.01	0.85	0.85	0.00
time (sec)	N/A	0.060	0.003	0.000	1.369	0.740	0.104	1.119	0.125	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	125	112	111	111	126	111	111	0
N.S.	1	1.00	0.85	0.76	0.76	0.76	0.86	0.76	0.76	0.00
time (sec)	N/A	0.064	0.003	0.001	1.365	1.087	0.113	1.013	0.087	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	130	113	112	112	131	112	112	0
N.S.	1	1.00	0.98	0.86	0.85	0.85	0.99	0.85	0.85	0.00
time (sec)	N/A	0.057	0.003	0.001	1.337	1.074	0.106	0.836	0.082	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	126	113	112	112	128	112	112	0
N.S.	1	1.00	1.12	1.01	1.00	1.00	1.14	1.00	1.00	0.00
time (sec)	N/A	0.052	0.003	0.000	1.384	0.973	0.102	1.102	0.121	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	132	113	112	112	133	112	112	0
N.S.	1	1.00	1.35	1.15	1.14	1.14	1.36	1.14	1.14	0.00
time (sec)	N/A	0.045	0.003	0.000	1.365	0.717	0.102	1.143	0.123	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	130	113	112	112	131	112	112	0
N.S.	1	1.00	1.60	1.40	1.38	1.38	1.62	1.38	1.38	0.00
time (sec)	N/A	0.039	0.003	0.001	1.292	1.217	0.109	1.055	0.120	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	128	113	112	112	129	112	112	0
N.S.	1	1.00	2.00	1.77	1.75	1.75	2.02	1.75	1.75	0.00
time (sec)	N/A	0.036	0.003	0.002	1.358	1.135	0.118	1.087	0.118	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	126	113	112	112	128	112	31	0
N.S.	1	1.00	2.68	2.40	2.38	2.38	2.72	2.38	0.66	0.00
time (sec)	N/A	0.030	0.003	0.002	1.374	1.120	0.107	1.178	0.070	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	128	113	112	112	129	112	25	0
N.S.	1	1.00	4.27	3.77	3.73	3.73	4.30	3.73	0.83	0.00
time (sec)	N/A	0.008	0.003	0.001	1.345	1.009	0.110	1.444	0.095	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	108	114	12	108	0
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71	0.00
time (sec)	N/A	0.002	0.001	0.000	1.335	1.258	0.115	1.164	0.112	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	122	109	108	108	126	109	108	0
N.S.	1	1.00	1.00	0.89	0.89	0.89	1.03	0.89	0.89	0.00
time (sec)	N/A	0.043	0.004	0.004	1.388	1.557	0.257	1.076	0.079	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	115	110	109	114	117	110	109	0
N.S.	1	1.00	1.00	0.96	0.95	0.99	1.02	0.96	0.95	0.00
time (sec)	N/A	0.047	0.010	0.007	1.352	0.839	0.266	0.960	0.115	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	110	108	114	122	109	110	0
N.S.	1	1.00	1.00	0.92	0.91	0.96	1.03	0.92	0.92	0.00
time (sec)	N/A	0.049	0.005	0.006	1.438	1.414	0.310	0.928	0.071	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	115	110	108	114	119	109	110	0
N.S.	1	1.00	1.00	0.96	0.94	0.99	1.03	0.95	0.96	0.00
time (sec)	N/A	0.047	0.010	0.007	1.368	1.134	0.338	1.139	0.062	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	110	110	114	121	111	110	0
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.02	0.93	0.92	0.00
time (sec)	N/A	0.049	0.008	0.008	1.269	1.565	0.437	1.118	0.098	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	117	110	110	114	121	111	110	0
N.S.	1	1.00	1.00	0.94	0.94	0.97	1.03	0.95	0.94	0.00
time (sec)	N/A	0.052	0.010	0.006	1.399	1.313	0.573	1.145	0.099	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	110	110	114	122	111	110	0
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.03	0.93	0.92	0.00
time (sec)	N/A	0.049	0.005	0.008	1.367	1.201	0.566	1.240	0.055	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	115	110	110	114	119	111	110	0
N.S.	1	1.00	1.00	0.96	0.96	0.99	1.03	0.97	0.96	0.00
time (sec)	N/A	0.050	0.010	0.009	1.298	1.308	0.695	1.100	0.095	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	110	110	114	119	111	110	0
N.S.	1	1.00	1.00	0.92	0.92	0.96	1.00	0.93	0.92	0.00
time (sec)	N/A	0.049	0.005	0.009	1.422	1.143	0.765	1.156	0.068	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	114	109	109	114	117	110	114	0
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00	0.00
time (sec)	N/A	0.053	0.006	0.008	1.398	1.208	0.825	1.211	0.076	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	124	111	111	114	119	112	111	0
N.S.	1	1.00	1.00	0.90	0.90	0.92	0.96	0.90	0.90	0.00
time (sec)	N/A	0.048	0.005	0.009	1.400	1.425	1.013	1.128	0.074	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	114	113	110	110	119	110	110	0
N.S.	1	1.00	6.71	6.65	6.47	6.47	7.00	6.47	6.47	0.00
time (sec)	N/A	0.002	0.010	0.006	1.335	1.265	1.026	1.064	0.134	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	128	113	112	112	121	112	23	0
N.S.	1	1.00	3.56	3.14	3.11	3.11	3.36	3.11	0.64	0.00
time (sec)	N/A	0.005	0.004	0.004	1.393	1.335	0.992	0.963	0.096	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	126	113	112	112	121	112	112	0
N.S.	1	1.00	2.25	2.02	2.00	2.00	2.16	2.00	2.00	0.00
time (sec)	N/A	0.010	0.009	0.005	1.373	1.392	1.223	1.031	0.132	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	128	113	112	112	121	112	112	0
N.S.	1	1.00	1.68	1.49	1.47	1.47	1.59	1.47	1.47	0.00
time (sec)	N/A	0.017	0.008	0.007	1.285	1.286	1.099	0.891	0.094	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	130	113	112	112	121	112	112	0
N.S.	1	1.00	1.35	1.18	1.17	1.17	1.26	1.17	1.17	0.00
time (sec)	N/A	0.025	0.010	0.006	1.361	1.339	1.253	1.517	0.130	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	132	113	112	112	121	112	112	0
N.S.	1	1.00	1.14	0.97	0.97	0.97	1.04	0.97	0.97	0.00
time (sec)	N/A	0.035	0.005	0.006	1.388	0.769	1.268	1.101	0.135	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	126	113	112	112	121	112	112	0
N.S.	1	1.00	0.93	0.83	0.82	0.82	0.89	0.82	0.82	0.00
time (sec)	N/A	0.049	0.010	0.007	1.402	1.139	1.327	1.153	0.134	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	130	113	112	112	121	112	112	0
N.S.	1	1.00	1.00	0.87	0.86	0.86	0.93	0.86	0.86	0.00
time (sec)	N/A	0.047	0.004	0.007	1.362	0.765	1.350	0.975	0.099	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	126	113	112	112	121	112	111	0
N.S.	1	1.00	1.00	0.90	0.89	0.89	0.96	0.89	0.88	0.00
time (sec)	N/A	0.049	0.007	0.007	1.396	1.342	1.424	1.110	0.137	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	14	13	13	12	12	13	11	0
N.S.	1	1.00	0.93	0.87	0.87	0.80	0.80	0.87	0.73	0.00
time (sec)	N/A	0.002	0.001	0.002	1.360	1.486	0.068	1.092	0.021	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	19	18	18	27	22	17	16	0
N.S.	1	1.00	0.95	0.90	0.90	1.35	1.10	0.85	0.80	0.00
time (sec)	N/A	0.004	0.001	0.000	1.374	2.366	0.077	1.204	0.075	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	70	63	64	63	61	65	62	0
N.S.	1	1.00	1.00	0.90	0.91	0.90	0.87	0.93	0.89	0.00
time (sec)	N/A	0.034	0.004	0.003	1.394	1.219	0.163	1.003	0.080	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	57	52	52	52	49	53	51	0
N.S.	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89	0.00
time (sec)	N/A	0.023	0.004	0.003	1.325	1.211	0.152	1.353	0.100	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	41	42	41	37	43	40	0
N.S.	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91	0.00
time (sec)	N/A	0.020	0.004	0.003	1.237	1.168	0.144	1.050	0.038	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	30	29	29	26	30	29	0
N.S.	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94	0.00
time (sec)	N/A	0.014	0.003	0.003	1.374	1.282	0.130	0.938	0.039	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	19	18	17	14	19	18	0
N.S.	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00	0.00
time (sec)	N/A	0.009	0.003	0.001	1.309	1.143	0.122	1.019	0.076	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	0.00
time (sec)	N/A	0.002	0.001	0.000	1.297	1.414	0.070	0.922	0.021	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	19	18	16	10	20	15	0
N.S.	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83	0.00
time (sec)	N/A	0.004	0.004	0.005	1.395	1.246	0.153	1.104	0.085	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	28	26	19	30	25	0
N.S.	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89	0.00
time (sec)	N/A	0.013	0.005	0.008	1.300	1.397	0.195	0.968	0.051	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	41	40	41	31	45	38	0
N.S.	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90	0.00
time (sec)	N/A	0.018	0.005	0.007	1.337	1.301	0.217	1.021	0.058	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	53	51	54	44	56	48	0
N.S.	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86	0.00
time (sec)	N/A	0.021	0.005	0.007	1.353	1.220	0.242	1.070	0.105	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	68	63	62	65	56	67	60	0
N.S.	1	1.00	1.00	0.93	0.91	0.96	0.82	0.99	0.88	0.00
time (sec)	N/A	0.035	0.005	0.007	1.365	1.221	0.275	1.060	0.065	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	77	78	82	96	78	103	83	0
N.S.	1	1.00	0.95	0.96	1.01	1.19	0.96	1.27	1.02	0.00
time (sec)	N/A	0.059	0.024	0.009	1.332	1.310	0.275	1.140	0.140	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	66	67	70	85	71	90	72	0
N.S.	1	1.00	0.92	0.93	0.97	1.18	0.99	1.25	1.00	0.00
time (sec)	N/A	0.042	0.017	0.007	1.327	1.171	0.252	1.222	0.070	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	54	57	59	73	54	79	62	0
N.S.	1	1.00	0.93	0.98	1.02	1.26	0.93	1.36	1.07	0.00
time (sec)	N/A	0.033	0.019	0.009	1.367	1.205	0.215	1.071	0.067	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	43	45	47	62	44	66	50	0
N.S.	1	1.00	0.93	0.98	1.02	1.35	0.96	1.43	1.09	0.00
time (sec)	N/A	0.027	0.013	0.006	1.357	1.216	0.204	1.139	0.079	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	34	36	47	31	50	36	0
N.S.	1	1.00	0.88	1.03	1.09	1.42	0.94	1.52	1.09	0.00
time (sec)	N/A	0.018	0.013	0.006	1.341	1.277	0.173	1.147	0.081	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	24	26	28	20	42	23	0
N.S.	1	1.00	0.87	1.04	1.13	1.22	0.87	1.83	1.00	0.00
time (sec)	N/A	0.012	0.007	0.007	1.320	1.218	0.169	1.049	0.036	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	13	10	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	0.00
time (sec)	N/A	0.002	0.002	0.000	1.376	1.177	0.149	1.175	0.029	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	24	30	28	39	22	38	26	0
N.S.	1	1.00	0.83	1.03	0.97	1.34	0.76	1.31	0.90	0.00
time (sec)	N/A	0.014	0.012	0.008	1.302	1.285	0.222	1.000	0.122	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	35	43	45	63	37	52	45	0
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.24	1.07	0.00
time (sec)	N/A	0.021	0.039	0.008	1.393	1.108	0.303	1.217	0.120	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	53	57	64	86	54	74	57	0
N.S.	1	1.00	0.91	0.98	1.10	1.48	0.93	1.28	0.98	0.00
time (sec)	N/A	0.028	0.050	0.010	1.405	0.554	0.312	1.130	0.110	0.001

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	66	68	73	95	66	90	69	0
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.30	1.00	0.00
time (sec)	N/A	0.035	0.056	0.009	1.410	0.933	0.336	1.015	0.080	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	79	79	86	108	80	104	79	0
N.S.	1	1.00	0.94	0.94	1.02	1.29	0.95	1.24	0.94	0.00
time (sec)	N/A	0.043	0.043	0.012	1.340	1.179	0.394	0.988	0.116	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	89	94	103	129	109	95	91	0
N.S.	1	1.00	0.90	0.95	1.04	1.30	1.10	0.96	0.92	0.00
time (sec)	N/A	0.070	0.027	0.008	1.386	1.038	0.532	0.891	0.234	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	77	83	91	117	92	83	78	0
N.S.	1	1.00	0.90	0.97	1.06	1.36	1.07	0.97	0.91	0.00
time (sec)	N/A	0.053	0.025	0.007	1.395	1.112	0.405	1.153	0.158	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	67	72	81	107	85	73	67	0
N.S.	1	1.00	0.87	0.94	1.05	1.39	1.10	0.95	0.87	0.00
time (sec)	N/A	0.046	0.023	0.007	1.357	1.117	0.359	1.120	0.124	0.001

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	55	61	69	95	70	61	54	0
N.S.	1	1.00	0.86	0.95	1.08	1.48	1.09	0.95	0.84	0.00
time (sec)	N/A	0.036	0.018	0.007	1.332	0.884	0.339	0.947	0.078	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	40	49	57	83	58	44	43	0
N.S.	1	1.00	0.80	0.98	1.14	1.66	1.16	0.88	0.86	0.00
time (sec)	N/A	0.026	0.040	0.007	1.315	0.838	0.310	0.964	0.146	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	33	40	48	61	46	37	46	0
N.S.	1	1.00	0.80	0.98	1.17	1.49	1.12	0.90	1.12	0.00
time (sec)	N/A	0.020	0.013	0.006	1.341	0.655	0.250	1.080	0.093	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	20	27	32	32	32	18	32	0
N.S.	1	1.00	1.18	1.59	1.88	1.88	1.88	1.06	1.88	0.00
time (sec)	N/A	0.002	0.005	0.003	1.392	1.208	0.200	1.142	0.072	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	24	26	12	26	0
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86	0.00
time (sec)	N/A	0.001	0.002	0.000	1.287	1.013	0.209	0.948	0.068	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	37	42	51	80	46	43	43	0
N.S.	1	1.00	0.86	0.98	1.19	1.86	1.07	1.00	1.00	0.00
time (sec)	N/A	0.019	0.029	0.008	1.349	0.868	0.349	1.031	0.100	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	53	56	69	109	66	60	63	0
N.S.	1	1.00	0.93	0.98	1.21	1.91	1.16	1.05	1.11	0.00
time (sec)	N/A	0.029	0.049	0.009	1.318	1.146	0.404	1.074	0.114	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	68	73	86	130	78	73	79	0
N.S.	1	1.00	0.89	0.96	1.13	1.71	1.03	0.96	1.04	0.00
time (sec)	N/A	0.036	0.052	0.010	1.371	1.182	0.406	1.386	0.119	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	79	84	97	141	92	86	91	0
N.S.	1	1.00	0.89	0.94	1.09	1.58	1.03	0.97	1.02	0.00
time (sec)	N/A	0.048	0.068	0.012	1.456	1.096	0.480	0.938	0.128	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	90	94	108	152	102	97	101	0
N.S.	1	1.00	0.93	0.97	1.11	1.57	1.05	1.00	1.04	0.00
time (sec)	N/A	0.052	0.057	0.012	1.348	0.946	0.476	1.008	0.092	0.001

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	101	109	125	162	131	106	103	0
N.S.	1	1.00	0.89	0.96	1.10	1.42	1.15	0.93	0.90	0.00
time (sec)	N/A	0.085	0.036	0.008	1.435	0.863	0.524	1.110	0.370	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	90	98	114	151	119	95	90	0
N.S.	1	1.00	0.86	0.93	1.09	1.44	1.13	0.90	0.86	0.00
time (sec)	N/A	0.070	0.028	0.008	1.395	1.040	0.483	1.013	0.221	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	90	87	102	139	107	83	79	0
N.S.	1	1.00	1.00	0.97	1.13	1.54	1.19	0.92	0.88	0.00
time (sec)	N/A	0.057	0.022	0.009	1.422	0.973	0.484	1.110	0.153	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	68	76	91	129	94	72	66	0
N.S.	1	1.00	0.84	0.94	1.12	1.59	1.16	0.89	0.81	0.00
time (sec)	N/A	0.048	0.024	0.008	1.458	0.792	0.461	0.873	0.125	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	51	64	79	116	82	55	55	0
N.S.	1	1.00	0.78	0.98	1.22	1.78	1.26	0.85	0.85	0.00
time (sec)	N/A	0.036	0.045	0.008	1.405	0.880	0.398	0.864	0.173	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	44	55	70	94	70	46	45	0
N.S.	1	1.00	0.76	0.95	1.21	1.62	1.21	0.79	0.78	0.00
time (sec)	N/A	0.030	0.016	0.006	1.391	1.104	0.306	1.032	0.070	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	31	41	54	54	56	29	56	0
N.S.	1	1.00	1.82	2.41	3.18	3.18	3.29	1.71	3.29	0.00
time (sec)	N/A	0.002	0.011	0.005	1.405	1.042	0.297	1.269	0.085	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	20	27	43	43	44	18	44	0
N.S.	1	1.00	0.67	0.90	1.43	1.43	1.47	0.60	1.47	0.00
time (sec)	N/A	0.013	0.005	0.005	1.374	0.772	0.317	1.008	0.072	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	35	37	12	37	0
N.S.	1	1.00	1.00	0.93	0.86	2.50	2.64	0.86	2.64	0.00
time (sec)	N/A	0.002	0.003	0.002	1.379	1.139	0.265	1.021	0.077	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	48	54	73	124	70	54	60	0
N.S.	1	1.00	0.84	0.95	1.28	2.18	1.23	0.95	1.05	0.00
time (sec)	N/A	0.028	0.034	0.007	1.466	1.071	0.444	1.024	0.128	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	64	69	91	153	90	71	85	0
N.S.	1	1.00	0.91	0.99	1.30	2.19	1.29	1.01	1.21	0.00
time (sec)	N/A	0.039	0.059	0.009	1.401	1.099	0.460	1.160	0.084	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	79	88	108	174	104	86	101	0
N.S.	1	1.00	0.85	0.95	1.16	1.87	1.12	0.92	1.09	0.00
time (sec)	N/A	0.048	0.057	0.012	1.413	0.847	0.557	0.953	0.136	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	88	99	117	183	114	93	113	0
N.S.	1	1.00	0.86	0.97	1.15	1.79	1.12	0.91	1.11	0.00
time (sec)	N/A	0.053	0.055	0.011	1.405	0.743	0.529	0.987	0.104	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	101	110	130	196	128	108	123	0
N.S.	1	1.00	0.86	0.94	1.11	1.68	1.09	0.92	1.05	0.00
time (sec)	N/A	0.070	0.068	0.011	1.390	1.132	0.582	1.000	0.173	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	139	143	180	250	190	128	126	0
N.S.	1	1.00	0.93	0.95	1.20	1.67	1.27	0.85	0.84	0.00
time (sec)	N/A	0.135	0.027	0.011	1.472	1.033	0.931	1.207	1.086	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	128	132	169	239	180	117	115	0
N.S.	1	1.00	0.92	0.95	1.22	1.72	1.29	0.84	0.83	0.00
time (sec)	N/A	0.114	0.032	0.010	1.587	1.117	0.914	1.475	0.553	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	104	121	157	228	165	105	102	0
N.S.	1	1.00	0.81	0.95	1.23	1.78	1.29	0.82	0.80	0.00
time (sec)	N/A	0.089	0.048	0.009	1.564	0.928	0.843	1.069	0.179	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	104	109	145	215	153	88	91	0
N.S.	1	1.00	0.88	0.92	1.23	1.82	1.30	0.75	0.77	0.00
time (sec)	N/A	0.071	0.029	0.009	1.490	1.033	0.820	0.916	0.336	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	77	100	136	193	141	79	81	0
N.S.	1	1.00	0.71	0.92	1.25	1.77	1.29	0.72	0.74	0.00
time (sec)	N/A	0.064	0.024	0.007	1.448	1.078	0.642	1.164	0.106	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	64	87	120	120	128	62	72	0
N.S.	1	1.00	3.76	5.12	7.06	7.06	7.53	3.65	4.24	0.00
time (sec)	N/A	0.002	0.011	0.005	1.434	0.949	0.583	1.083	0.122	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	53	72	109	109	116	51	22	0
N.S.	1	1.00	1.51	2.06	3.11	3.11	3.31	1.46	0.63	0.00
time (sec)	N/A	0.005	0.010	0.006	1.452	0.862	0.573	1.143	0.072	0.001

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	64	42	57	98	98	104	40	48	0
N.S.	1	1.23	0.81	1.10	1.88	1.88	2.00	0.77	0.92	0.00
time (sec)	N/A	0.029	0.010	0.005	1.401	1.120	0.559	1.001	0.068	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	31	42	87	87	92	29	31	0
N.S.	1	1.00	0.66	0.89	1.85	1.85	1.96	0.62	0.66	0.00
time (sec)	N/A	0.021	0.008	0.004	1.451	0.885	0.547	0.997	0.080	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	20	27	76	76	80	18	18	0
N.S.	1	1.00	0.67	0.90	2.53	2.53	2.67	0.60	0.60	0.00
time (sec)	N/A	0.013	0.006	0.007	1.373	0.882	0.503	1.392	0.100	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	68	73	12	70	0
N.S.	1	1.00	1.00	0.93	0.86	4.86	5.21	0.86	5.00	0.00
time (sec)	N/A	0.002	0.003	0.001	1.411	1.198	0.464	1.091	0.064	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	81	90	139	256	141	87	102	0
N.S.	1	1.00	0.82	0.91	1.40	2.59	1.42	0.88	1.03	0.00
time (sec)	N/A	0.049	0.063	0.010	1.512	0.655	0.681	1.014	0.455	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	97	108	157	285	162	104	151	0
N.S.	1	1.00	0.83	0.92	1.34	2.44	1.38	0.89	1.29	0.00
time (sec)	N/A	0.073	0.094	0.010	1.590	1.322	0.797	1.048	0.186	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	112	133	174	306	175	119	167	0
N.S.	1	1.00	0.78	0.92	1.21	2.12	1.22	0.83	1.16	0.00
time (sec)	N/A	0.089	0.073	0.012	1.548	1.326	0.843	1.273	0.209	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	123	144	185	317	187	130	179	0
N.S.	1	1.00	0.78	0.92	1.18	2.02	1.19	0.83	1.14	0.00
time (sec)	N/A	0.103	0.092	0.013	1.592	1.289	0.992	1.254	0.311	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	161	177	234	338	250	149	151	0
N.S.	1	1.00	0.87	0.95	1.26	1.82	1.34	0.80	0.81	0.00
time (sec)	N/A	0.177	0.047	0.012	1.730	1.199	1.545	1.069	0.979	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	177	150	166	223	327	236	138	138	0
N.S.	1	1.00	0.85	0.94	1.26	1.85	1.33	0.78	0.78	0.00
time (sec)	N/A	0.142	0.029	0.010	1.651	0.681	1.477	1.696	0.227	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	159	159	137	154	211	314	224	121	127	0
N.S.	1	1.00	0.86	0.97	1.33	1.97	1.41	0.76	0.80	0.00
time (sec)	N/A	0.121	0.033	0.011	1.660	1.417	1.326	1.287	0.943	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	111	145	202	292	212	112	117	0
N.S.	1	1.00	0.72	0.94	1.31	1.90	1.38	0.73	0.76	0.00
time (sec)	N/A	0.108	0.034	0.007	1.601	1.211	1.105	1.216	0.187	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	97	131	186	186	199	95	107	0
N.S.	1	1.00	5.71	7.71	10.94	10.94	11.71	5.59	6.29	0.00
time (sec)	N/A	0.002	0.018	0.006	1.509	1.078	0.980	1.231	0.141	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	86	117	175	175	187	84	22	0
N.S.	1	1.00	2.46	3.34	5.00	5.00	5.34	2.40	0.63	0.00
time (sec)	N/A	0.005	0.014	0.006	1.592	0.880	0.999	0.962	0.126	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	75	102	164	164	175	73	85	0
N.S.	1	1.00	1.44	1.96	3.15	3.15	3.37	1.40	1.63	0.00
time (sec)	N/A	0.010	0.016	0.007	1.493	0.867	0.913	1.013	0.140	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	64	86	153	153	163	62	71	0
N.S.	1	1.00	0.93	1.25	2.22	2.22	2.36	0.90	1.03	0.00
time (sec)	N/A	0.016	0.015	0.004	1.538	0.976	0.840	0.899	0.078	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	53	72	142	142	151	51	61	0
N.S.	1	1.00	0.65	0.89	1.75	1.75	1.86	0.63	0.75	0.00
time (sec)	N/A	0.040	0.015	0.006	1.455	1.134	0.794	1.219	0.077	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	42	57	131	131	139	40	48	0
N.S.	1	1.00	0.66	0.89	2.05	2.05	2.17	0.62	0.75	0.00
time (sec)	N/A	0.030	0.010	0.005	1.475	0.806	0.698	1.125	0.127	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	31	42	120	120	128	29	31	0
N.S.	1	1.00	0.66	0.89	2.55	2.55	2.72	0.62	0.66	0.00
time (sec)	N/A	0.021	0.011	0.005	1.416	1.174	0.679	1.043	0.150	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	20	27	109	109	116	18	18	0
N.S.	1	1.00	0.67	0.90	3.63	3.63	3.87	0.60	0.60	0.00
time (sec)	N/A	0.014	0.006	0.004	1.421	1.043	0.723	1.045	0.067	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	101	109	12	103	0
N.S.	1	1.00	1.00	0.93	0.86	7.21	7.79	0.86	7.36	0.00
time (sec)	N/A	0.002	0.003	0.001	1.336	1.102	0.671	1.034	0.141	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	127	126	205	388	212	120	145	0
N.S.	1	1.00	0.90	0.89	1.45	2.75	1.50	0.85	1.03	0.00
time (sec)	N/A	0.075	0.102	0.012	1.755	1.076	1.023	1.048	0.762	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	130	147	223	417	233	137	217	0
N.S.	1	1.00	0.82	0.93	1.41	2.64	1.47	0.87	1.37	0.00
time (sec)	N/A	0.125	0.127	0.014	1.705	1.173	1.148	1.014	0.395	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	191	191	145	178	240	438	246	152	233	0
N.S.	1	1.00	0.76	0.93	1.26	2.29	1.29	0.80	1.22	0.00
time (sec)	N/A	0.141	0.111	0.015	1.712	0.610	1.176	1.102	0.438	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	156	189	251	449	258	163	245	0
N.S.	1	1.00	0.79	0.95	1.27	2.27	1.30	0.82	1.24	0.00
time (sec)	N/A	0.165	0.124	0.013	1.720	1.257	1.241	1.349	0.586	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	141	132	132	136	143	133	132	0
N.S.	1	1.00	1.00	0.94	0.94	0.96	1.01	0.94	0.94	0.00
time (sec)	N/A	0.079	0.011	0.010	1.372	1.036	0.914	1.139	0.081	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	132	132	132	121	121	125	131	122	121	0
N.S.	1	1.00	1.00	0.92	0.92	0.95	0.99	0.92	0.92	0.00
time (sec)	N/A	0.070	0.005	0.010	1.373	1.076	0.848	1.277	0.092	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	114	109	109	114	117	110	114	0
N.S.	1	1.00	1.00	0.96	0.96	1.00	1.03	0.96	1.00	0.00
time (sec)	N/A	0.058	0.006	0.000	1.432	1.046	0.865	1.056	0.002	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	109	100	100	103	107	101	100	0
N.S.	1	1.00	1.00	0.92	0.92	0.94	0.98	0.93	0.92	0.00
time (sec)	N/A	0.052	0.005	0.008	1.335	0.948	0.786	1.040	0.082	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	96	91	88	88	95	88	88	0
N.S.	1	1.00	5.65	5.35	5.18	5.18	5.59	5.18	5.18	0.00
time (sec)	N/A	0.002	0.009	0.008	1.337	0.777	0.727	1.168	0.090	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	91	80	79	79	85	79	23	0
N.S.	1	1.00	2.53	2.22	2.19	2.19	2.36	2.19	0.64	0.00
time (sec)	N/A	0.006	0.004	0.000	1.357	0.864	0.705	1.037	0.002	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	80	69	68	68	73	68	68	0
N.S.	1	1.00	1.43	1.23	1.21	1.21	1.30	1.21	1.21	0.00
time (sec)	N/A	0.010	0.009	0.007	1.368	1.132	0.563	1.018	0.098	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	67	58	57	57	61	57	56	0
N.S.	1	1.00	1.00	0.87	0.85	0.85	0.91	0.85	0.84	0.00
time (sec)	N/A	0.025	0.007	0.000	1.362	1.112	0.493	1.011	0.002	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	47	46	46	49	46	46	0
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.88	0.82	0.82	0.00
time (sec)	N/A	0.019	0.008	0.006	1.335	0.808	0.497	1.246	0.035	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	36	35	35	37	35	35	0
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.00
time (sec)	N/A	0.014	0.004	0.004	1.346	0.992	0.379	1.086	0.032	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	24	24	26	24	24	0
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80	0.00
time (sec)	N/A	0.009	0.007	0.005	1.341	1.086	0.270	0.873	0.036	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	14	13	13	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.76	0.00
time (sec)	N/A	0.005	0.002	0.005	1.303	0.834	0.210	0.995	0.029	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	7	7	7	6	5	5	7	5	5	0
N.S.	1	1.00	1.00	0.86	0.71	0.71	1.00	0.71	0.71	0.00
time (sec)	N/A	0.000	0.000	0.002	1.318	0.942	0.074	1.044	0.020	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	134	119	117	120	116	122	114	0
N.S.	1	1.00	1.00	0.89	0.87	0.90	0.87	0.91	0.85	0.00
time (sec)	N/A	0.061	0.006	0.010	1.349	0.865	0.410	1.161	0.127	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	134	135	141	163	139	180	135	0
N.S.	1	1.00	0.92	0.92	0.97	1.12	0.95	1.23	0.92	0.00
time (sec)	N/A	0.090	0.094	0.011	1.398	0.741	0.613	1.284	0.083	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	145	150	163	207	163	152	157	0
N.S.	1	1.00	0.89	0.92	1.00	1.27	1.00	0.93	0.96	0.00
time (sec)	N/A	0.108	0.108	0.014	1.432	1.100	0.678	1.128	0.228	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	15	10	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.88	0.59	0.00
time (sec)	N/A	0.002	0.003	0.006	1.342	0.801	0.117	1.147	0.170	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	12	15	10	0
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.88	0.59	0.00
time (sec)	N/A	0.002	0.003	0.001	1.295	1.159	0.120	1.006	0.143	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	19	18	21	20	20	18	0
N.S.	1	1.00	1.00	0.79	0.75	0.88	0.83	0.83	0.75	0.00
time (sec)	N/A	0.008	0.003	0.007	1.381	0.591	0.139	1.013	0.054	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	24	23	28	26	25	18	0
N.S.	1	1.00	1.00	0.77	0.74	0.90	0.84	0.81	0.58	0.00
time (sec)	N/A	0.010	0.003	0.008	1.294	0.953	0.147	1.072	0.037	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	29	28	33	31	30	24	0
N.S.	1	1.00	1.00	0.76	0.74	0.87	0.82	0.79	0.63	0.00
time (sec)	N/A	0.012	0.003	0.006	1.353	0.570	0.162	1.028	0.088	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	34	33	38	36	35	28	0
N.S.	1	1.00	1.00	0.76	0.73	0.84	0.80	0.78	0.62	0.00
time (sec)	N/A	0.012	0.003	0.006	1.325	0.743	0.166	1.140	0.040	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	23	22	32	19	25	20	0
N.S.	1	1.00	0.93	0.82	0.79	1.14	0.68	0.89	0.71	0.00
time (sec)	N/A	0.009	0.013	0.007	1.367	0.854	0.136	1.114	0.055	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	31	28	31	48	31	40	34	0
N.S.	1	1.00	0.89	0.80	0.89	1.37	0.89	1.14	0.97	0.00
time (sec)	N/A	0.010	0.013	0.008	1.387	1.191	0.148	0.951	0.089	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	36	33	38	59	36	51	31	0
N.S.	1	1.00	0.86	0.79	0.90	1.40	0.86	1.21	0.74	0.00
time (sec)	N/A	0.014	0.013	0.009	1.315	1.071	0.157	0.937	0.042	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	44	38	43	64	41	60	37	0
N.S.	1	1.00	0.90	0.78	0.88	1.31	0.84	1.22	0.76	0.00
time (sec)	N/A	0.016	0.030	0.006	1.308	1.191	0.169	1.232	0.093	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	56	43	48	69	46	69	41	0
N.S.	1	1.00	1.00	0.77	0.86	1.23	0.82	1.23	0.73	0.00
time (sec)	N/A	0.020	0.010	0.009	1.357	0.941	0.175	1.154	0.095	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	29	32	30	50	27	27	29	0
N.S.	1	1.00	0.74	0.82	0.77	1.28	0.69	0.69	0.74	0.00
time (sec)	N/A	0.012	0.020	0.007	1.389	1.081	0.170	1.034	0.125	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	39	37	41	68	41	37	35	0
N.S.	1	1.00	0.85	0.80	0.89	1.48	0.89	0.80	0.76	0.00
time (sec)	N/A	0.015	0.021	0.009	1.391	0.923	0.180	0.894	0.092	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	44	42	48	79	46	43	41	0
N.S.	1	1.00	0.83	0.79	0.91	1.49	0.87	0.81	0.77	0.00
time (sec)	N/A	0.016	0.025	0.009	1.384	0.948	0.181	1.068	0.094	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	49	47	53	84	51	47	47	0
N.S.	1	1.00	0.82	0.78	0.88	1.40	0.85	0.78	0.78	0.00
time (sec)	N/A	0.021	0.019	0.008	1.380	0.753	0.207	1.291	0.046	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	54	52	58	89	56	52	51	0
N.S.	1	1.00	0.81	0.78	0.87	1.33	0.84	0.78	0.76	0.00
time (sec)	N/A	0.022	0.022	0.010	1.385	0.986	0.216	1.133	0.047	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	10	9	6	6	7	7	6	0
N.S.	1	1.00	1.25	1.12	0.75	0.75	0.88	0.88	0.75	0.00
time (sec)	N/A	0.001	0.001	0.001	1.337	1.123	0.074	0.902	0.151	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	9	8	8	8	9	6	0
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.90	0.60	0.00
time (sec)	N/A	0.001	0.001	0.002	1.339	0.982	0.071	1.146	0.077	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	16	13	12	10	14	13	10	0
N.S.	1	1.00	1.14	0.93	0.86	0.71	1.00	0.93	0.71	0.00
time (sec)	N/A	0.002	0.004	0.002	1.251	0.785	0.079	1.285	0.105	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	20	17	17	16	0
N.S.	1	1.00	1.00	0.85	0.80	1.00	0.85	0.85	0.80	0.00
time (sec)	N/A	0.007	0.005	0.000	1.372	1.156	0.081	1.102	0.108	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	21	19	19	14	0
N.S.	1	1.00	1.00	0.86	0.82	0.95	0.86	0.86	0.64	0.00
time (sec)	N/A	0.002	0.004	0.001	1.322	0.889	0.079	0.931	0.052	0.001

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	23	19	19	16	0
N.S.	1	1.00	1.00	0.86	0.82	1.05	0.86	0.86	0.73	0.00
time (sec)	N/A	0.002	0.004	0.001	1.308	0.837	0.080	1.091	0.057	0.001

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	19	18	24	19	19	16	0
N.S.	1	1.00	1.00	0.90	0.86	1.14	0.90	0.90	0.76	0.00
time (sec)	N/A	0.004	0.007	0.000	1.327	0.871	0.092	1.040	0.152	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	22	19	18	24	20	19	14	0
N.S.	1	1.00	1.10	0.95	0.90	1.20	1.00	0.95	0.70	0.00
time (sec)	N/A	0.004	0.009	0.001	1.310	1.013	0.088	1.079	0.175	0.001

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	11	11	8	13	9	0
N.S.	1	1.00	1.00	1.09	1.00	1.00	0.73	1.18	0.82	0.00
time (sec)	N/A	0.002	0.003	0.004	1.305	0.767	0.135	0.992	0.097	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	12	11	11	8	13	9	0
N.S.	1	1.00	1.00	1.00	0.92	0.92	0.67	1.08	0.75	0.00
time (sec)	N/A	0.002	0.003	0.006	1.361	0.619	0.125	1.021	0.036	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	20	19	21	14	21	16	0
N.S.	1	1.00	1.00	1.05	1.00	1.11	0.74	1.11	0.84	0.00
time (sec)	N/A	0.008	0.004	0.007	1.318	0.932	0.177	0.916	0.043	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	18	17	21	14	19	14	0
N.S.	1	1.00	1.00	1.00	0.94	1.17	0.78	1.06	0.78	0.00
time (sec)	N/A	0.010	0.003	0.006	1.333	0.732	0.191	1.144	0.029	0.001

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	15	14	15	10	15	20	0
N.S.	1	1.00	1.00	1.07	1.00	1.07	0.71	1.07	1.43	0.00
time (sec)	N/A	0.009	0.005	0.001	1.325	0.880	0.148	1.251	0.036	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	53	1742	116	56	59
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.61	0.78	0.82
time (sec)	N/A	0.018	0.023	0.015	1.250	1.358	2.910	0.948	0.050	0.062

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	42	666	93	37	45
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.75	0.70	0.85
time (sec)	N/A	0.013	0.016	0.004	1.407	1.047	2.039	1.032	0.046	0.019

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	30	202	66	25	35
N.S.	1	1.00	0.71	0.62	0.76	0.88	5.94	1.94	0.74	1.03
time (sec)	N/A	0.008	0.011	0.003	1.315	1.140	1.393	1.284	0.028	0.012

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	12	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	1.00
time (sec)	N/A	0.001	0.004	0.003	1.336	1.114	0.074	1.058	0.021	0.006

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	35	28	42	73	68	32	27	35
N.S.	1	1.00	1.00	0.80	1.20	2.09	1.94	0.91	0.77	1.00
time (sec)	N/A	0.010	0.009	0.013	2.919	1.114	1.602	1.131	0.094	0.032

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	47	37	47	93	44	41	31	39
N.S.	1	1.00	1.21	0.95	1.21	2.38	1.13	1.05	0.79	1.00
time (sec)	N/A	0.011	0.027	0.012	2.869	1.227	2.221	0.964	0.053	0.052

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	35	53	88	119	97	66	48	55
N.S.	1	1.00	0.54	0.82	1.35	1.83	1.49	1.02	0.74	0.85
time (sec)	N/A	0.018	0.007	0.012	3.027	0.980	4.015	1.136	0.068	0.085

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	35	65	121	145	122	84	66	71
N.S.	1	1.00	0.40	0.75	1.39	1.67	1.40	0.97	0.76	0.82
time (sec)	N/A	0.027	0.007	0.013	3.020	0.982	6.678	1.229	0.107	0.113

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	64	1742	193	56	59
N.S.	1	1.00	0.64	0.60	0.78	0.89	24.19	2.68	0.78	0.82
time (sec)	N/A	0.019	0.024	0.005	1.348	1.234	3.199	1.108	0.048	0.022

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	53	733	156	37	45
N.S.	1	1.00	0.66	0.60	0.77	1.00	13.83	2.94	0.70	0.85
time (sec)	N/A	0.013	0.017	0.006	1.347	0.751	2.175	0.936	0.041	0.020

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	41	80	119	25	46
N.S.	1	1.00	0.71	0.62	0.76	1.21	2.35	3.50	0.74	1.35
time (sec)	N/A	0.008	0.013	0.002	1.347	0.809	0.740	1.240	0.026	0.013

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	28	12	58	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75	1.00
time (sec)	N/A	0.001	0.005	0.001	1.287	0.614	0.072	1.126	0.016	0.006

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	44	38	52	88	71	44	37	50
N.S.	1	1.00	0.90	0.78	1.06	1.80	1.45	0.90	0.76	1.02
time (sec)	N/A	0.014	0.021	0.007	2.937	1.137	2.295	1.077	0.042	0.026

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	33	47	58	102	92	56	42	49
N.S.	1	1.00	0.65	0.92	1.14	2.00	1.80	1.10	0.82	0.96
time (sec)	N/A	0.015	0.007	0.011	2.875	1.042	2.664	1.043	0.100	0.061

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	68	51	86	124	76	64	46	56
N.S.	1	1.00	1.10	0.82	1.39	2.00	1.23	1.03	0.74	0.90
time (sec)	N/A	0.016	0.034	0.012	2.984	1.118	3.283	1.287	0.057	0.102

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	35	63	119	145	124	84	64	71
N.S.	1	1.00	0.42	0.75	1.42	1.73	1.48	1.00	0.76	0.85
time (sec)	N/A	0.022	0.009	0.012	3.047	1.006	5.837	1.161	0.102	0.116

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	75	146	281	56	51
N.S.	1	1.00	0.64	0.60	0.78	1.04	2.03	3.90	0.78	0.71
time (sec)	N/A	0.017	0.027	0.006	1.347	1.003	4.469	1.158	0.050	0.024

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	64	124	233	37	39
N.S.	1	1.00	0.66	0.60	0.77	1.21	2.34	4.40	0.70	0.74
time (sec)	N/A	0.012	0.020	0.005	1.366	1.167	3.770	1.090	0.044	0.023

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	52	102	182	25	57
N.S.	1	1.00	0.71	0.62	0.76	1.53	3.00	5.35	0.74	1.68
time (sec)	N/A	0.009	0.016	0.003	1.371	1.103	2.599	1.152	0.028	0.015

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	39	12	95	12	16
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	5.94	0.75	1.00
time (sec)	N/A	0.001	0.006	0.002	1.341	0.980	0.077	0.983	0.018	0.007

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	50	64	114	97	56	52	66
N.S.	1	1.00	0.89	0.77	0.98	1.75	1.49	0.86	0.80	1.02
time (sec)	N/A	0.021	0.048	0.007	3.017	1.155	4.117	1.027	0.053	0.029

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	33	61	71	126	99	74	58	64
N.S.	1	1.00	0.50	0.92	1.08	1.91	1.50	1.12	0.88	0.97
time (sec)	N/A	0.020	0.009	0.010	2.945	0.803	3.747	1.088	0.115	0.074

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	35	61	101	133	126	80	64	68
N.S.	1	1.00	0.45	0.78	1.29	1.71	1.62	1.03	0.82	0.87
time (sec)	N/A	0.021	0.009	0.010	2.940	1.037	4.300	1.120	0.053	0.111

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	79	63	115	146	104	79	64	68
N.S.	1	1.00	0.98	0.78	1.42	1.80	1.28	0.98	0.79	0.84
time (sec)	N/A	0.023	0.038	0.010	2.985	0.995	5.157	1.130	0.046	0.140

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	35	75	144	167	155	99	79	83
N.S.	1	1.00	0.34	0.73	1.40	1.62	1.50	0.96	0.77	0.81
time (sec)	N/A	0.031	0.009	0.012	2.934	0.958	8.359	1.094	0.111	0.154

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	90	87	116	141	279	781	116	115
N.S.	1	1.00	0.62	0.60	0.79	0.97	1.91	5.35	0.79	0.79
time (sec)	N/A	0.042	0.055	0.006	1.334	1.020	40.296	1.140	0.036	0.038

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	79	76	101	130	257	709	101	101
N.S.	1	1.00	0.62	0.60	0.80	1.02	2.02	5.58	0.80	0.80
time (sec)	N/A	0.035	0.042	0.007	1.343	0.727	36.609	1.324	0.031	0.034

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	68	65	86	119	235	637	86	75
N.S.	1	1.00	0.62	0.59	0.78	1.08	2.14	5.79	0.78	0.68
time (sec)	N/A	0.032	0.038	0.006	1.347	0.911	28.760	1.054	0.027	0.031

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	57	54	71	108	212	565	71	63
N.S.	1	1.00	0.63	0.59	0.78	1.19	2.33	6.21	0.78	0.69
time (sec)	N/A	0.023	0.032	0.006	1.359	1.074	25.700	1.122	0.025	0.028

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	97	190	493	56	51
N.S.	1	1.00	0.64	0.60	0.78	1.35	2.64	6.85	0.78	0.71
time (sec)	N/A	0.018	0.030	0.006	1.293	0.955	20.346	1.102	0.045	0.024

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	86	168	421	36	39
N.S.	1	1.00	0.66	0.60	0.77	1.62	3.17	7.94	0.68	0.74
time (sec)	N/A	0.013	0.022	0.004	1.341	0.956	16.856	1.058	0.040	0.023

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	74	146	347	25	79
N.S.	1	1.00	0.71	0.62	0.76	2.18	4.29	10.21	0.74	2.32
time (sec)	N/A	0.008	0.017	0.003	1.356	0.718	14.851	1.269	0.030	0.015

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	61	12	229	12	16
N.S.	1	1.00	1.00	0.81	0.75	3.81	0.75	14.31	0.75	1.00
time (sec)	N/A	0.002	0.007	0.003	1.336	1.046	0.083	0.991	0.018	0.006

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	78	74	88	158	148	80	76	94
N.S.	1	1.00	0.80	0.76	0.91	1.63	1.53	0.82	0.78	0.97
time (sec)	N/A	0.034	0.085	0.007	2.942	1.180	11.102	1.227	0.037	0.031

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	33	84	97	172	150	104	84	88
N.S.	1	1.00	0.34	0.86	0.99	1.76	1.53	1.06	0.86	0.90
time (sec)	N/A	0.034	0.011	0.010	2.968	0.857	9.922	1.427	0.043	0.070

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	35	86	131	180	184	112	117	92
N.S.	1	1.00	0.31	0.75	1.15	1.58	1.61	0.98	1.03	0.81
time (sec)	N/A	0.035	0.011	0.011	2.952	1.165	8.989	1.104	0.047	0.111

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	35	87	145	178	184	112	131	92
N.S.	1	1.00	0.31	0.76	1.27	1.56	1.61	0.98	1.15	0.81
time (sec)	N/A	0.036	0.011	0.013	2.939	0.927	7.912	1.101	0.121	0.138

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	35	85	155	177	182	110	94	92
N.S.	1	1.00	0.30	0.73	1.34	1.53	1.57	0.95	0.81	0.79
time (sec)	N/A	0.037	0.011	0.013	2.965	0.975	8.547	1.215	0.062	0.168

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	101	87	169	190	158	109	94	92
N.S.	1	1.00	0.85	0.73	1.42	1.60	1.33	0.92	0.79	0.77
time (sec)	N/A	0.039	0.044	0.013	2.965	0.901	10.250	1.190	0.119	0.203

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	35	99	198	211	209	129	109	107
N.S.	1	1.00	0.25	0.70	1.40	1.50	1.48	0.91	0.77	0.76
time (sec)	N/A	0.051	0.011	0.014	3.034	1.269	15.689	1.042	0.133	0.234

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	35	111	229	233	236	144	124	119
N.S.	1	1.00	0.21	0.68	1.40	1.43	1.45	0.88	0.76	0.73
time (sec)	N/A	0.068	0.011	0.014	3.083	1.415	22.200	0.951	0.126	0.256

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	32	31	78	148	31	31	39
N.S.	1	1.00	1.00	0.82	0.79	2.00	3.79	0.79	0.79	1.00
time (sec)	N/A	0.011	0.011	0.007	2.939	1.210	1.737	1.136	0.094	0.021

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	52	35	34	98	121	41	34	42
N.S.	1	1.00	1.24	0.83	0.81	2.33	2.88	0.98	0.81	1.00
time (sec)	N/A	0.010	0.024	0.012	2.942	0.999	2.140	1.081	0.097	0.043

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	38	55	83	124	207	66	54	60
N.S.	1	1.00	0.54	0.77	1.17	1.75	2.92	0.93	0.76	0.85
time (sec)	N/A	0.016	0.009	0.010	2.964	1.091	4.161	1.081	0.103	0.077

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	48	44	43	93	187	43	43	58
N.S.	1	1.00	0.87	0.80	0.78	1.69	3.40	0.78	0.78	1.05
time (sec)	N/A	0.015	0.024	0.007	3.018	0.914	2.463	1.058	0.041	0.028

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	36	48	47	105	197	58	47	55
N.S.	1	1.00	0.63	0.84	0.82	1.84	3.46	1.02	0.82	0.96
time (sec)	N/A	0.014	0.009	0.011	3.001	1.043	2.837	0.959	0.045	0.054

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	72	53	80	129	190	66	52	62
N.S.	1	1.00	1.06	0.78	1.18	1.90	2.79	0.97	0.76	0.91
time (sec)	N/A	0.015	0.035	0.012	3.021	1.039	3.302	0.952	0.097	0.086

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	60	58	57	119	240	57	57	74
N.S.	1	1.00	0.82	0.79	0.78	1.63	3.29	0.78	0.78	1.01
time (sec)	N/A	0.019	0.033	0.006	2.918	1.162	4.241	1.218	0.043	0.029

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	36	64	63	131	245	75	63	72
N.S.	1	1.00	0.49	0.86	0.85	1.77	3.31	1.01	0.85	0.97
time (sec)	N/A	0.020	0.011	0.012	3.049	0.978	3.686	1.020	0.103	0.055

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	38	70	97	139	267	83	69	76
N.S.	1	1.00	0.44	0.81	1.13	1.62	3.10	0.97	0.80	0.88
time (sec)	N/A	0.022	0.010	0.013	2.924	0.898	3.927	1.037	0.095	0.094

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	57	54	71	53	3755	61	71	73
N.S.	1	1.00	0.64	0.61	0.80	0.60	42.19	0.69	0.80	0.82
time (sec)	N/A	0.022	0.028	0.006	1.345	0.719	4.839	1.040	0.024	0.025

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	46	43	56	42	1640	49	56	59
N.S.	1	1.00	0.68	0.63	0.82	0.62	24.12	0.72	0.82	0.87
time (sec)	N/A	0.018	0.026	0.003	1.254	1.040	2.703	1.207	0.046	0.022

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	35	32	41	31	600	37	37	45
N.S.	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.73	0.88
time (sec)	N/A	0.012	0.016	0.005	1.323	0.707	1.767	1.342	0.038	0.020

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	23	21	26	19	162	23	25	24
N.S.	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.78	0.75
time (sec)	N/A	0.008	0.011	0.003	1.320	0.949	1.160	1.006	0.027	0.013

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86	1.00
time (sec)	N/A	0.001	0.003	0.002	1.298	0.938	0.066	0.985	0.018	0.006

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	32	56	24	21	17	23
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74	1.00
time (sec)	N/A	0.007	0.004	0.006	2.936	1.117	1.108	1.244	0.055	0.018

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	47	40	60	93	44	47	33	41
N.S.	1	1.00	1.15	0.98	1.46	2.27	1.07	1.15	0.80	1.00
time (sec)	N/A	0.011	0.065	0.006	2.999	0.860	2.302	1.069	0.111	0.053

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	33	66	92	123	102	69	51	63
N.S.	1	1.00	0.49	0.97	1.35	1.81	1.50	1.01	0.75	0.93
time (sec)	N/A	0.017	0.006	0.006	2.917	1.124	4.366	1.195	0.061	0.078

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	33	90	121	145	129	84	69	71
N.S.	1	1.00	0.37	1.00	1.34	1.61	1.43	0.93	0.77	0.79
time (sec)	N/A	0.025	0.006	0.008	2.916	0.907	7.022	0.908	0.052	0.074

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	57	54	71	63	3606	77	71	63
N.S.	1	1.00	0.67	0.64	0.84	0.74	42.42	0.91	0.84	0.74
time (sec)	N/A	0.024	0.028	0.006	1.329	0.949	4.806	1.252	0.028	0.025

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	45	42	56	51	1538	61	56	49
N.S.	1	1.00	0.68	0.64	0.85	0.77	23.30	0.92	0.85	0.74
time (sec)	N/A	0.017	0.020	0.006	1.375	0.953	2.935	1.102	0.048	0.024

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	34	32	41	40	534	46	35	37
N.S.	1	1.00	0.69	0.65	0.84	0.82	10.90	0.94	0.71	0.76
time (sec)	N/A	0.013	0.016	0.006	1.290	0.843	1.827	1.145	0.040	0.023

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	21	20	26	29	37	29	19	21
N.S.	1	1.00	0.70	0.67	0.87	0.97	1.23	0.97	0.63	0.70
time (sec)	N/A	0.008	0.010	0.003	1.324	0.911	0.669	0.995	0.087	0.011

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	20	12	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86	1.00
time (sec)	N/A	0.001	0.004	0.002	1.341	0.742	0.070	1.238	0.019	0.008

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	30	31	45	110	146	37	30	38
N.S.	1	1.00	0.79	0.82	1.18	2.89	3.84	0.97	0.79	1.00
time (sec)	N/A	0.011	0.005	0.009	2.926	1.028	1.866	1.070	0.043	0.028

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	59	31	55	76	151	73	64	60	52
N.S.	1	1.04	0.54	0.96	1.33	2.65	1.28	1.12	1.05	0.91
time (sec)	N/A	0.017	0.007	0.011	3.012	1.170	3.411	0.905	0.122	0.066

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	85	33	67	108	189	107	80	90	71
N.S.	1	0.98	0.38	0.77	1.24	2.17	1.23	0.92	1.03	0.82
time (sec)	N/A	0.023	0.006	0.013	3.063	1.074	5.983	1.047	0.059	0.099

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	57	54	71	74	3456	75	68	63
N.S.	1	1.00	0.66	0.62	0.82	0.85	39.72	0.86	0.78	0.72
time (sec)	N/A	0.022	0.028	0.005	1.349	0.995	4.575	1.081	0.049	0.029

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	45	43	56	62	163	59	47	47
N.S.	1	1.00	0.66	0.63	0.82	0.91	2.40	0.87	0.69	0.69
time (sec)	N/A	0.017	0.024	0.006	1.253	0.964	1.200	1.032	0.043	0.024

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	35	32	41	52	121	39	35	39
N.S.	1	1.00	0.71	0.65	0.84	1.06	2.47	0.80	0.71	0.80
time (sec)	N/A	0.013	0.017	0.004	1.328	1.269	1.275	0.940	0.084	0.024

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	24	21	26	41	80	20	20	24
N.S.	1	1.00	0.75	0.66	0.81	1.28	2.50	0.62	0.62	0.75
time (sec)	N/A	0.008	0.012	0.003	1.338	0.988	1.128	1.055	0.033	0.013

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	31	14	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	0.75	0.75	1.00
time (sec)	N/A	0.001	0.005	0.002	1.307	0.912	0.075	1.038	0.018	0.007

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	32	43	53	177	697	45	42	49
N.S.	1	1.00	0.59	0.80	0.98	3.28	12.91	0.83	0.78	0.91
time (sec)	N/A	0.015	0.006	0.010	2.912	1.016	2.993	1.003	0.050	0.041

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	80	33	67	89	221	818	65	73	67
N.S.	1	1.08	0.45	0.91	1.20	2.99	11.05	0.88	0.99	0.91
time (sec)	N/A	0.024	0.006	0.012	3.032	1.133	5.596	1.027	0.109	0.071

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	35	80	123	255	464	93	105	83
N.S.	1	1.00	0.33	0.75	1.16	2.41	4.38	0.88	0.99	0.78
time (sec)	N/A	0.034	0.007	0.015	3.001	1.152	8.571	0.994	0.120	0.112

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	58	54	19	19	25
N.S.	1	1.00	1.00	0.80	0.76	2.32	2.16	0.76	0.76	1.00
time (sec)	N/A	0.006	0.005	0.004	2.991	1.400	1.226	0.986	0.048	0.019

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	53	37	46	97	121	43	36	44
N.S.	1	1.00	1.20	0.84	1.05	2.20	2.75	0.98	0.82	1.00
time (sec)	N/A	0.011	0.065	0.007	2.976	0.908	2.458	0.886	0.041	0.043

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	36	59	86	128	216	68	57	69
N.S.	1	1.00	0.49	0.80	1.16	1.73	2.92	0.92	0.77	0.93
time (sec)	N/A	0.017	0.006	0.007	2.957	1.364	4.199	0.965	0.045	0.066

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	33	35	34	124	478	34	34	42
N.S.	1	1.00	0.79	0.83	0.81	2.95	11.38	0.81	0.81	1.00
time (sec)	N/A	0.010	0.007	0.008	2.972	1.265	2.199	1.016	0.095	0.028

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	65	34	54	67	164	156	64	52	59
N.S.	1	1.05	0.55	0.87	1.08	2.65	2.52	1.03	0.84	0.95
time (sec)	N/A	0.016	0.007	0.014	3.022	0.870	3.499	1.013	0.063	0.059

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	93	36	75	104	198	226	81	101	79
N.S.	1	0.98	0.38	0.79	1.09	2.08	2.38	0.85	1.06	0.83
time (sec)	N/A	0.023	0.007	0.014	2.926	0.894	5.567	1.006	0.132	0.100

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	35	49	42	182	0	42	48	55
N.S.	1	1.00	0.58	0.82	0.70	3.03	0.00	0.70	0.80	0.92
time (sec)	N/A	0.015	0.008	0.010	2.938	0.847	0.000	1.057	0.092	0.041

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	88	36	68	82	226	0	66	70	75
N.S.	1	1.09	0.44	0.84	1.01	2.79	0.00	0.81	0.86	0.93
time (sec)	N/A	0.023	0.009	0.013	2.928	0.827	0.000	1.004	0.118	0.067

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	38	92	121	260	1108	97	117	93
N.S.	1	1.00	0.33	0.79	1.04	2.24	9.55	0.84	1.01	0.80
time (sec)	N/A	0.033	0.007	0.015	2.875	0.963	11.219	0.902	0.071	0.119

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	14	78	0	11	0
N.S.	1	1.00	1.00	0.92	0.85	1.08	6.00	0.00	0.85	0.00
time (sec)	N/A	0.006	0.041	0.006	1.875	0.907	84.074	0.000	0.413	0.254

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	F	A	A	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	92	13	0	11	11	73	0	-1	0
N.S.	1	7.08	1.00	0.00	0.85	0.85	5.62	0.00	-0.08	0.00
time (sec)	N/A	0.045	0.015	0.085	1.862	0.943	5.344	0.000	0.000	0.593

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	32	56	24	21	17	23
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74	1.00
time (sec)	N/A	0.008	0.004	0.000	2.991	1.114	1.091	0.847	0.002	0.023

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	53	1742	117	56	51
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.62	0.78	0.71
time (sec)	N/A	0.019	0.024	0.005	1.385	0.835	2.833	0.867	0.052	0.023

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	42	666	92	37	39
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.74	0.70	0.74
time (sec)	N/A	0.012	0.017	0.006	1.304	0.861	1.856	0.834	0.038	0.022

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	30	202	67	25	35
N.S.	1	1.00	0.71	0.62	0.76	0.88	5.94	1.97	0.74	1.03
time (sec)	N/A	0.008	0.012	0.003	1.293	0.911	1.201	1.038	0.028	0.013

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	12	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	1.00
time (sec)	N/A	0.001	0.004	0.003	1.257	0.959	0.065	0.892	0.017	0.006

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	113	85	86	91	180	87	107	116
N.S.	1	1.00	1.24	0.93	0.95	1.00	1.98	0.96	1.18	1.27
time (sec)	N/A	0.047	0.047	0.007	3.078	1.081	2.018	2.374	0.121	0.067

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	33	92	93	139	643	105	117	125
N.S.	1	1.00	0.34	0.95	0.96	1.43	6.63	1.08	1.21	1.29
time (sec)	N/A	0.033	0.006	0.008	2.959	0.756	2.185	2.555	0.067	0.157

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	35	113	139	187	2266	128	196	145
N.S.	1	1.00	0.28	0.89	1.09	1.47	17.84	1.01	1.54	1.14
time (sec)	N/A	0.049	0.007	0.011	2.986	0.979	2.577	2.426	0.230	0.215

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	53	1742	117	56	51
N.S.	1	1.00	0.64	0.60	0.78	0.74	24.19	1.62	0.78	0.71
time (sec)	N/A	0.018	0.025	0.004	1.364	0.821	3.073	1.125	0.045	0.022

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	42	666	92	37	39
N.S.	1	1.00	0.66	0.60	0.77	0.79	12.57	1.74	0.70	0.74
time (sec)	N/A	0.013	0.017	0.006	1.349	0.654	1.938	0.825	0.041	0.022

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	31	202	68	25	35
N.S.	1	1.00	0.71	0.62	0.76	0.91	5.94	2.00	0.74	1.03
time (sec)	N/A	0.008	0.012	0.003	1.350	0.819	1.276	0.892	0.028	0.014

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	12	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	1.00
time (sec)	N/A	0.002	0.004	0.003	1.286	0.841	0.065	1.144	0.018	0.006

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	86	84	85	110	182	86	117	117
N.S.	1	1.00	0.93	0.91	0.92	1.20	1.98	0.93	1.27	1.27
time (sec)	N/A	0.032	0.039	0.003	2.971	0.922	2.056	2.200	0.113	0.063

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	33	92	93	252	643	106	127	125
N.S.	1	1.00	0.35	0.98	0.99	2.68	6.84	1.13	1.35	1.33
time (sec)	N/A	0.033	0.007	0.011	2.982	0.995	2.238	2.292	0.114	0.156

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	35	113	139	350	2266	129	194	147
N.S.	1	1.00	0.28	0.89	1.09	2.76	17.84	1.02	1.53	1.16
time (sec)	N/A	0.047	0.008	0.011	2.983	0.938	2.655	2.470	0.329	0.248

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	64	1844	193	56	51
N.S.	1	1.00	0.64	0.60	0.78	0.89	25.61	2.68	0.78	0.71
time (sec)	N/A	0.018	0.025	0.006	1.357	0.917	3.178	1.223	0.047	0.024

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	53	733	157	37	39
N.S.	1	1.00	0.66	0.60	0.77	1.00	13.83	2.96	0.70	0.74
time (sec)	N/A	0.013	0.018	0.004	1.303	0.771	2.160	0.984	0.042	0.023

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	41	80	118	25	24
N.S.	1	1.00	0.71	0.62	0.76	1.21	2.35	3.47	0.74	0.71
time (sec)	N/A	0.009	0.013	0.003	1.286	1.175	1.493	1.018	0.027	0.014

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	28	12	58	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75	1.00
time (sec)	N/A	0.002	0.005	0.002	1.330	0.864	0.066	1.249	0.018	0.007

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	130	95	96	98	209	97	123	131
N.S.	1	1.00	1.24	0.90	0.91	0.93	1.99	0.92	1.17	1.25
time (sec)	N/A	0.041	0.032	0.006	3.026	0.660	2.388	2.025	0.060	0.064

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	33	103	104	111	719	119	131	135
N.S.	1	1.00	0.31	0.96	0.97	1.04	6.72	1.11	1.22	1.26
time (sec)	N/A	0.042	0.007	0.011	3.028	0.715	2.580	2.348	0.073	0.185

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	35	111	136	162	2266	127	174	146
N.S.	1	1.00	0.28	0.90	1.10	1.31	18.27	1.02	1.40	1.18
time (sec)	N/A	0.044	0.008	0.012	3.055	0.985	2.738	1.940	0.122	0.227

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	46	43	56	42	1640	49	56	51
N.S.	1	1.00	0.64	0.60	0.78	0.58	22.78	0.68	0.78	0.71
time (sec)	N/A	0.018	0.028	0.005	1.315	0.869	2.777	0.903	0.043	0.025

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	35	32	41	31	600	37	37	45
N.S.	1	1.00	0.66	0.60	0.77	0.58	11.32	0.70	0.70	0.85
time (sec)	N/A	0.012	0.020	0.005	1.376	0.757	1.770	0.908	0.038	0.022

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	24	21	26	20	162	25	25	24
N.S.	1	1.00	0.71	0.62	0.76	0.59	4.76	0.74	0.74	0.71
time (sec)	N/A	0.008	0.012	0.003	1.297	0.846	1.157	0.900	0.029	0.014

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	12	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	1.00
time (sec)	N/A	0.002	0.003	0.002	1.317	0.989	0.064	0.973	0.016	0.007

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	66	75	76	213	155	77	99	104
N.S.	1	1.00	0.84	0.95	0.96	2.70	1.96	0.97	1.25	1.32
time (sec)	N/A	0.025	0.013	0.004	2.932	0.870	1.881	2.371	0.086	0.055

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	33	95	106	306	831	109	130	128
N.S.	1	1.00	0.33	0.95	1.06	3.06	8.31	1.09	1.30	1.28
time (sec)	N/A	0.033	0.006	0.006	2.995	0.894	2.204	2.445	0.137	0.160

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	35	117	142	296	2730	130	182	149
N.S.	1	1.00	0.27	0.90	1.09	2.28	21.00	1.00	1.40	1.15
time (sec)	N/A	0.048	0.007	0.006	3.028	0.955	2.586	2.236	0.226	0.153

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	48	45	64	44	4974	57	64	59
N.S.	1	1.00	0.60	0.56	0.80	0.55	62.18	0.71	0.80	0.74
time (sec)	N/A	0.019	0.027	0.004	1.339	0.901	2.977	1.069	0.048	0.024

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	37	34	47	33	1326	43	43	51
N.S.	1	1.00	0.63	0.58	0.80	0.56	22.47	0.73	0.73	0.86
time (sec)	N/A	0.013	0.021	0.005	1.330	0.939	1.897	1.071	0.040	0.022

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	26	23	30	22	486	29	29	26
N.S.	1	1.00	0.68	0.61	0.79	0.58	12.79	0.76	0.76	0.68
time (sec)	N/A	0.009	0.012	0.003	1.340	0.906	1.261	0.979	0.029	0.014

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	12	14	14	18
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.67	0.78	0.78	1.00
time (sec)	N/A	0.001	0.004	0.002	1.317	0.710	0.066	1.017	0.020	0.007

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	35	83	86	285	160	112	117	113
N.S.	1	1.00	0.43	1.01	1.05	3.48	1.95	1.37	1.43	1.38
time (sec)	N/A	0.033	0.011	0.007	3.007	0.942	1.878	2.509	0.095	0.055

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	36	103	116	328	838	144	133	136
N.S.	1	1.00	0.35	1.00	1.13	3.18	8.14	1.40	1.29	1.32
time (sec)	N/A	0.034	0.006	0.007	2.989	1.061	2.236	2.429	0.175	0.139

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	38	128	159	374	2744	167	216	160
N.S.	1	1.00	0.28	0.94	1.17	2.75	20.18	1.23	1.59	1.18
time (sec)	N/A	0.044	0.006	0.008	3.093	0.656	2.615	2.242	0.220	0.128

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	46	43	56	42	1640	49	56	51
N.S.	1	1.00	0.66	0.61	0.80	0.60	23.43	0.70	0.80	0.73
time (sec)	N/A	0.018	0.024	0.006	1.345	0.528	2.787	1.026	0.045	0.026

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	35	32	41	31	600	37	37	45
N.S.	1	1.00	0.69	0.63	0.80	0.61	11.76	0.73	0.73	0.88
time (sec)	N/A	0.012	0.017	0.004	1.314	0.829	1.817	0.897	0.041	0.023

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	23	21	26	19	162	23	25	24
N.S.	1	1.00	0.72	0.66	0.81	0.59	5.06	0.72	0.78	0.75
time (sec)	N/A	0.007	0.011	0.003	1.335	0.904	1.190	0.891	0.029	0.011

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86	1.00
time (sec)	N/A	0.001	0.003	0.003	1.295	0.886	0.063	0.976	0.017	0.007

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	93	76	77	115	150	78	95	105
N.S.	1	1.00	1.16	0.95	0.96	1.44	1.88	0.98	1.19	1.31
time (sec)	N/A	0.024	0.026	0.005	2.874	0.998	1.930	2.317	0.166	0.061

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	31	95	106	166	830	108	122	128
N.S.	1	1.00	0.32	0.97	1.08	1.69	8.47	1.10	1.24	1.31
time (sec)	N/A	0.033	0.005	0.007	2.951	0.692	2.266	2.373	0.130	0.155

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	33	117	142	162	2728	130	175	149
N.S.	1	1.00	0.25	0.90	1.09	1.25	20.98	1.00	1.35	1.15
time (sec)	N/A	0.048	0.006	0.009	3.043	0.816	2.727	2.049	0.131	0.124

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	46	43	56	52	1538	62	56	51
N.S.	1	1.00	0.66	0.61	0.80	0.74	21.97	0.89	0.80	0.73
time (sec)	N/A	0.018	0.022	0.004	1.358	0.859	2.891	1.103	0.053	0.026

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	34	32	41	40	534	46	35	37
N.S.	1	1.00	0.69	0.65	0.84	0.82	10.90	0.94	0.71	0.76
time (sec)	N/A	0.013	0.016	0.006	1.327	0.830	1.882	0.996	0.042	0.026

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	23	20	26	29	41	30	20	23
N.S.	1	1.00	0.72	0.62	0.81	0.91	1.28	0.94	0.62	0.72
time (sec)	N/A	0.009	0.011	0.003	1.303	0.830	0.724	0.934	0.030	0.012

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	20	12	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86	1.00
time (sec)	N/A	0.001	0.004	0.002	1.353	1.055	0.067	0.764	0.021	0.009

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	30	87	88	285	184	89	114	118
N.S.	1	1.00	0.32	0.94	0.95	3.06	1.98	0.96	1.23	1.27
time (sec)	N/A	0.033	0.005	0.009	3.032	0.955	2.215	2.377	0.056	0.075

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	115	31	108	122	407	857	120	173	138
N.S.	1	1.02	0.27	0.96	1.08	3.60	7.58	1.06	1.53	1.22
time (sec)	N/A	0.044	0.006	0.013	2.993	0.829	2.525	2.399	0.068	0.168

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	147	33	131	158	407	2793	140	221	161
N.S.	1	0.99	0.22	0.88	1.06	2.73	18.74	0.94	1.48	1.08
time (sec)	N/A	0.058	0.006	0.014	2.960	1.047	3.185	2.590	0.131	0.264

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	66	87	86	88	138	87	105	102
N.S.	1	1.00	0.93	1.23	1.21	1.24	1.94	1.23	1.48	1.44
time (sec)	N/A	0.031	0.021	0.011	3.089	0.971	2.134	1.032	0.101	0.048

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	68	91	90	92	136	91	108	106
N.S.	1	1.00	0.93	1.25	1.23	1.26	1.86	1.25	1.48	1.45
time (sec)	N/A	0.031	0.022	0.004	2.995	0.865	1.889	0.767	0.128	0.050

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	41	97	94	93	134	95	112	111
N.S.	1	1.00	0.55	1.31	1.27	1.26	1.81	1.28	1.51	1.50
time (sec)	N/A	0.032	0.013	0.009	2.903	0.828	1.824	1.068	0.108	0.050

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	41	101	98	97	139	99	115	115
N.S.	1	1.00	0.54	1.33	1.29	1.28	1.83	1.30	1.51	1.51
time (sec)	N/A	0.028	0.012	0.004	2.922	0.865	1.827	1.003	0.072	0.075

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	95	88	87	86	134	88	101	103
N.S.	1	1.00	1.32	1.22	1.21	1.19	1.86	1.22	1.40	1.43
time (sec)	N/A	0.024	0.025	0.009	2.877	0.756	1.857	1.003	0.141	0.052

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	99	92	91	90	136	92	104	107
N.S.	1	1.00	1.34	1.24	1.23	1.22	1.84	1.24	1.41	1.45
time (sec)	N/A	0.025	0.025	0.003	2.978	0.961	1.921	1.086	0.107	0.048

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	108	96	93	95	134	94	107	110
N.S.	1	1.00	1.46	1.30	1.26	1.28	1.81	1.27	1.45	1.49
time (sec)	N/A	0.023	0.032	0.008	2.951	0.890	1.983	0.988	0.156	0.052

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	112	100	97	99	133	98	110	114
N.S.	1	1.00	1.47	1.32	1.28	1.30	1.75	1.29	1.45	1.50
time (sec)	N/A	0.023	0.031	0.003	2.889	1.153	1.895	1.074	0.160	0.055

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	22	31	25	33	87	43	30	0
N.S.	1	1.00	0.88	1.24	1.00	1.32	3.48	1.72	1.20	0.00
time (sec)	N/A	0.008	0.015	0.003	1.339	0.959	0.299	1.006	0.313	0.014

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	18	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.334	0.846	1.589	0.828	0.091	0.011

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	18	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.293	1.006	0.547	0.880	0.027	0.009

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	16	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.338	0.992	1.621	0.859	0.025	0.008

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	16	13	13	12	17	13	12	20
N.S.	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.63	1.05
time (sec)	N/A	0.004	0.005	0.004	1.316	0.921	0.156	1.081	0.025	0.008

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	14	12	13	12	15	13	11	14
N.S.	1	1.00	0.82	0.71	0.76	0.71	0.88	0.76	0.65	0.82
time (sec)	N/A	0.004	0.005	0.003	1.328	0.548	0.350	0.920	0.028	0.010

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	15	12	11	11	19	11	13	15
N.S.	1	1.00	0.79	0.63	0.58	0.58	1.00	0.58	0.68	0.79
time (sec)	N/A	0.004	0.005	0.002	1.362	0.828	0.562	0.966	0.027	0.012

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	38	87	43	85	299	117	93	0
N.S.	1	1.00	0.88	2.02	1.00	1.98	6.95	2.72	2.16	0.00
time (sec)	N/A	0.014	0.033	0.005	1.357	0.776	0.535	1.102	0.416	0.022

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	29	34	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.94
time (sec)	N/A	0.007	0.008	0.003	1.272	0.708	2.597	1.156	0.103	0.014

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	29	34	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.94
time (sec)	N/A	0.007	0.007	0.004	1.318	0.882	1.031	1.118	0.035	0.013

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	27	0	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.75	0.00	0.67	0.67	0.94
time (sec)	N/A	0.007	0.007	0.004	1.294	0.810	0.000	0.899	0.042	0.011

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	28	25	24	24	32	24	24	34
N.S.	1	1.00	0.82	0.74	0.71	0.71	0.94	0.71	0.71	1.00
time (sec)	N/A	0.007	0.007	0.004	1.381	0.839	0.262	0.917	0.037	0.011

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	27	25	24	23	31	24	24	27
N.S.	1	1.00	0.84	0.78	0.75	0.72	0.97	0.75	0.75	0.84
time (sec)	N/A	0.007	0.008	0.004	1.318	0.891	0.430	1.016	0.035	0.014

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	26	23	23	24	31	23	24	28
N.S.	1	1.00	0.81	0.72	0.72	0.75	0.97	0.72	0.75	0.88
time (sec)	N/A	0.007	0.009	0.004	1.289	0.805	0.593	1.002	0.030	0.017

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	54	170	61	157	663	224	167	0
N.S.	1	1.00	0.89	2.79	1.00	2.57	10.87	3.67	2.74	0.00
time (sec)	N/A	0.020	0.032	0.004	1.358	0.892	0.884	1.027	0.389	0.023

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	40	49	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.011	0.010	0.004	1.343	0.810	3.881	0.990	0.045	0.015

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	40	49	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.012	0.010	0.004	1.339	0.884	1.703	0.991	0.046	0.015

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	38	0	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.00	0.69	0.69	0.92
time (sec)	N/A	0.011	0.010	0.005	1.256	0.822	0.000	1.082	0.042	0.013

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	39	36	35	35	46	35	35	47
N.S.	1	1.00	0.83	0.77	0.74	0.74	0.98	0.74	0.74	1.00
time (sec)	N/A	0.011	0.010	0.004	1.352	0.768	0.445	0.939	0.043	0.013

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	38	36	35	34	44	35	35	38
N.S.	1	1.00	0.84	0.80	0.78	0.76	0.98	0.78	0.78	0.84
time (sec)	N/A	0.011	0.011	0.005	1.356	0.801	0.639	1.032	0.047	0.016

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	38	34	34	34	46	34	35	38
N.S.	1	1.00	0.81	0.72	0.72	0.72	0.98	0.72	0.74	0.81
time (sec)	N/A	0.011	0.012	0.005	1.368	0.904	0.777	1.065	0.039	0.019

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	61	54	54	132	121	59	48	67
N.S.	1	1.00	0.90	0.79	0.79	1.94	1.78	0.87	0.71	0.99
time (sec)	N/A	0.032	0.026	0.009	3.010	0.966	7.295	0.875	0.057	0.047

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	49	43	42	103	105	45	37	53
N.S.	1	1.00	0.92	0.81	0.79	1.94	1.98	0.85	0.70	1.00
time (sec)	N/A	0.017	0.019	0.007	2.941	0.703	1.926	1.238	0.052	0.037

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	32	31	85	92	31	28	40
N.S.	1	1.00	1.00	0.80	0.78	2.12	2.30	0.78	0.70	1.00
time (sec)	N/A	0.012	0.010	0.006	2.971	0.982	0.724	0.989	0.041	0.021

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	19	18	68	94	18	19	29
N.S.	1	1.00	1.00	0.66	0.62	2.34	3.24	0.62	0.66	1.00
time (sec)	N/A	0.008	0.005	0.006	2.926	0.907	1.291	0.947	0.044	0.016

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	25	32	31	93	102	31	28	40
N.S.	1	1.00	0.62	0.80	0.78	2.32	2.55	0.78	0.70	1.00
time (sec)	N/A	0.013	0.005	0.008	2.883	0.954	2.773	0.997	0.044	0.027

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	27	43	41	118	121	41	38	48
N.S.	1	1.00	0.51	0.81	0.77	2.23	2.28	0.77	0.72	0.91
time (sec)	N/A	0.018	0.005	0.011	3.017	0.886	7.834	1.157	0.102	0.039

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	27	54	52	144	139	52	49	61
N.S.	1	1.00	0.40	0.79	0.76	2.12	2.04	0.76	0.72	0.90
time (sec)	N/A	0.023	0.006	0.010	2.915	1.040	24.823	0.994	0.110	0.048

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	27	61	63	161	479	65	58	74
N.S.	1	1.00	0.39	0.87	0.90	2.30	6.84	0.93	0.83	1.06
time (sec)	N/A	0.022	0.005	0.011	2.964	0.925	24.566	0.928	0.114	0.078

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	27	47	49	134	411	46	46	58
N.S.	1	1.00	0.47	0.82	0.86	2.35	7.21	0.81	0.81	1.02
time (sec)	N/A	0.017	0.004	0.011	2.907	0.826	9.176	0.956	0.124	0.072

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	37	37	115	337	36	34	46
N.S.	1	1.00	1.00	0.80	0.80	2.50	7.33	0.78	0.74	1.00
time (sec)	N/A	0.013	0.020	0.010	2.936	0.893	4.445	0.901	0.040	0.059

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	36	35	116	328	35	33	45
N.S.	1	1.00	1.00	0.80	0.78	2.58	7.29	0.78	0.73	1.00
time (sec)	N/A	0.013	0.017	0.009	2.925	0.873	7.527	0.894	0.095	0.054

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	25	48	51	147	434	49	48	54
N.S.	1	1.00	0.45	0.86	0.91	2.62	7.75	0.88	0.86	0.96
time (sec)	N/A	0.017	0.005	0.013	2.943	0.957	17.734	1.006	0.124	0.073

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	27	60	64	184	507	58	58	68
N.S.	1	1.00	0.39	0.87	0.93	2.67	7.35	0.84	0.84	0.99
time (sec)	N/A	0.022	0.005	0.015	2.882	1.005	50.524	0.957	0.150	0.076

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	27	79	86	227	906	77	81	89
N.S.	1	1.00	0.28	0.83	0.91	2.39	9.54	0.81	0.85	0.94
time (sec)	N/A	0.033	0.005	0.015	3.067	0.909	135.242	1.027	0.122	0.132

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	27	66	73	200	816	59	69	76
N.S.	1	1.00	0.33	0.80	0.89	2.44	9.95	0.72	0.84	0.93
time (sec)	N/A	0.023	0.005	0.015	2.941	0.593	53.287	0.953	0.143	0.119

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	59	50	61	185	726	47	58	63
N.S.	1	1.00	0.84	0.71	0.87	2.64	10.37	0.67	0.83	0.90
time (sec)	N/A	0.019	0.035	0.013	2.960	1.009	29.374	0.908	0.131	0.117

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	27	52	64	186	721	52	56	60
N.S.	1	1.00	0.37	0.71	0.88	2.55	9.88	0.71	0.77	0.82
time (sec)	N/A	0.019	0.005	0.009	3.012	0.919	15.275	1.066	0.125	0.107

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	25	53	60	186	712	47	57	63
N.S.	1	1.00	0.36	0.76	0.86	2.66	10.17	0.67	0.81	0.90
time (sec)	N/A	0.019	0.005	0.009	2.956	0.984	25.687	0.862	0.126	0.077

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	25	66	73	214	865	59	70	70
N.S.	1	1.00	0.30	0.80	0.89	2.61	10.55	0.72	0.85	0.85
time (sec)	N/A	0.025	0.005	0.015	2.992	0.719	54.354	1.142	0.149	0.118

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	27	79	86	250	962	71	80	81
N.S.	1	1.00	0.28	0.83	0.91	2.63	10.13	0.75	0.84	0.85
time (sec)	N/A	0.029	0.005	0.016	2.981	0.843	138.083	1.101	0.155	0.120

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	61	54	70	131	116	61	51	67
N.S.	1	1.00	0.90	0.79	1.03	1.93	1.71	0.90	0.75	0.99
time (sec)	N/A	0.024	0.026	0.006	2.942	1.084	7.100	1.034	0.147	0.048

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	49	43	58	103	100	47	37	53
N.S.	1	1.00	0.92	0.81	1.09	1.94	1.89	0.89	0.70	1.00
time (sec)	N/A	0.018	0.019	0.008	2.920	0.669	1.870	0.940	0.114	0.037

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	32	47	83	87	33	28	40
N.S.	1	1.00	1.00	0.80	1.18	2.08	2.18	0.82	0.70	1.00
time (sec)	N/A	0.014	0.010	0.004	3.075	0.758	0.708	1.045	0.112	0.025

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	19	34	67	88	20	19	29
N.S.	1	1.00	1.00	0.66	1.17	2.31	3.03	0.69	0.66	1.00
time (sec)	N/A	0.011	0.006	0.005	3.026	0.980	1.253	1.005	0.125	0.018

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	24	32	47	91	94	33	28	40
N.S.	1	1.00	0.60	0.80	1.18	2.28	2.35	0.82	0.70	1.00
time (sec)	N/A	0.014	0.004	0.007	2.867	0.698	2.757	1.019	0.056	0.027

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	26	43	55	113	112	41	37	48
N.S.	1	1.00	0.49	0.81	1.04	2.13	2.11	0.77	0.70	0.91
time (sec)	N/A	0.018	0.005	0.010	2.960	1.061	7.665	0.981	0.122	0.044

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	26	54	68	143	131	54	48	61
N.S.	1	1.00	0.38	0.79	1.00	2.10	1.93	0.79	0.71	0.90
time (sec)	N/A	0.022	0.005	0.010	2.975	0.898	24.442	0.968	0.129	0.047

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	26	61	81	167	444	69	61	76
N.S.	1	1.00	0.37	0.87	1.16	2.39	6.34	0.99	0.87	1.09
time (sec)	N/A	0.025	0.005	0.012	3.057	0.699	24.751	0.984	0.071	0.075

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	26	49	68	138	381	51	47	56
N.S.	1	1.00	0.46	0.86	1.19	2.42	6.68	0.89	0.82	0.98
time (sec)	N/A	0.018	0.005	0.011	2.919	0.576	9.127	0.978	0.115	0.065

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	61	40	56	123	311	40	35	49
N.S.	1	1.00	1.30	0.85	1.19	2.62	6.62	0.85	0.74	1.04
time (sec)	N/A	0.015	0.016	0.009	2.988	0.839	4.428	1.064	0.112	0.057

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	39	56	122	303	41	34	46
N.S.	1	1.00	1.00	0.85	1.22	2.65	6.59	0.89	0.74	1.00
time (sec)	N/A	0.014	0.020	0.008	2.970	0.577	7.413	0.883	0.055	0.052

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	24	49	69	151	403	52	49	55
N.S.	1	1.00	0.42	0.86	1.21	2.65	7.07	0.91	0.86	0.96
time (sec)	N/A	0.018	0.006	0.013	3.005	0.848	17.544	1.006	0.073	0.064

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	26	60	82	187	471	61	60	69
N.S.	1	1.00	0.37	0.86	1.17	2.67	6.73	0.87	0.86	0.99
time (sec)	N/A	0.022	0.006	0.016	2.998	0.998	50.249	0.930	0.138	0.080

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	26	70	103	227	840	81	83	91
N.S.	1	1.00	0.27	0.72	1.06	2.34	8.66	0.84	0.86	0.94
time (sec)	N/A	0.030	0.006	0.015	2.992	0.786	136.150	1.033	0.141	0.124

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	26	58	90	199	756	63	69	78
N.S.	1	1.00	0.31	0.69	1.07	2.37	9.00	0.75	0.82	0.93
time (sec)	N/A	0.026	0.005	0.012	2.996	0.651	53.453	0.951	0.065	0.119

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	60	52	78	186	673	51	58	65
N.S.	1	1.00	0.83	0.72	1.08	2.58	9.35	0.71	0.81	0.90
time (sec)	N/A	0.021	0.036	0.013	2.989	0.708	29.248	0.916	0.143	0.117

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	26	54	80	183	668	55	57	60
N.S.	1	1.00	0.35	0.72	1.07	2.44	8.91	0.73	0.76	0.80
time (sec)	N/A	0.020	0.005	0.010	2.848	1.046	15.189	0.933	0.137	0.099

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	24	63	77	185	660	51	58	64
N.S.	1	1.00	0.33	0.88	1.07	2.57	9.17	0.71	0.81	0.89
time (sec)	N/A	0.020	0.005	0.009	2.884	1.003	25.674	1.155	0.135	0.086

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	24	58	90	213	802	63	69	71
N.S.	1	1.00	0.29	0.69	1.07	2.54	9.55	0.75	0.82	0.85
time (sec)	N/A	0.024	0.005	0.015	2.930	0.971	54.560	1.046	0.155	0.110

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	26	69	103	249	892	73	80	82
N.S.	1	1.00	0.27	0.71	1.06	2.57	9.20	0.75	0.82	0.85
time (sec)	N/A	0.030	0.004	0.017	2.950	1.014	138.884	1.042	0.165	0.122

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	96	120	178	162	153	0	-1	95
N.S.	1	1.00	0.79	0.98	1.46	1.33	1.25	0.00	-0.01	0.78
time (sec)	N/A	0.042	0.185	0.009	2.950	0.948	11.697	0.000	0.000	0.145

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	85	102	146	141	122	0	-1	82
N.S.	1	1.00	0.87	1.04	1.49	1.44	1.24	0.00	-0.01	0.84
time (sec)	N/A	0.031	0.107	0.005	3.044	0.537	6.383	0.000	0.000	0.095

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	72	81	108	114	97	0	52	68
N.S.	1	1.00	0.97	1.09	1.46	1.54	1.31	0.00	0.70	0.92
time (sec)	N/A	0.023	0.110	0.006	2.962	0.906	3.570	0.000	0.150	0.065

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	62	62	70	93	42	0	41	47
N.S.	1	1.00	1.41	1.41	1.59	2.11	0.95	0.00	0.93	1.07
time (sec)	N/A	0.017	0.086	0.004	2.992	0.947	1.916	0.000	0.683	0.058

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	64	61	54	89	68	0	-1	47
N.S.	1	1.00	1.42	1.36	1.20	1.98	1.51	0.00	-0.02	1.04
time (sec)	N/A	0.017	0.095	0.037	3.004	0.929	1.561	0.000	0.000	0.071

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	15	15	41	33	21	21
N.S.	1	1.00	1.00	0.76	0.71	0.71	1.95	1.57	1.00	1.00
time (sec)	N/A	0.002	0.007	0.004	1.357	1.006	1.461	1.327	0.237	0.020

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	29	24	31	34	65	50	32	40
N.S.	1	1.00	0.66	0.55	0.70	0.77	1.48	1.14	0.73	0.91
time (sec)	N/A	0.005	0.009	0.004	1.287	0.694	4.875	1.102	0.255	0.084

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	40	35	46	45	347	66	43	51
N.S.	1	1.00	0.59	0.51	0.68	0.66	5.10	0.97	0.63	0.75
time (sec)	N/A	0.010	0.011	0.005	1.354	0.760	13.772	0.974	0.264	0.092

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	98	127	170	164	323	0	-1	104
N.S.	1	1.00	0.77	1.00	1.34	1.29	2.54	0.00	-0.01	0.82
time (sec)	N/A	0.042	0.141	0.010	3.010	0.950	11.648	0.000	0.000	0.135

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	87	108	135	142	260	0	-1	91
N.S.	1	1.00	0.85	1.06	1.32	1.39	2.55	0.00	-0.01	0.89
time (sec)	N/A	0.030	0.117	0.006	2.990	1.022	6.325	0.000	0.000	0.116

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	75	86	95	118	207	0	58	78
N.S.	1	1.00	0.97	1.12	1.23	1.53	2.69	0.00	0.75	1.01
time (sec)	N/A	0.024	0.097	0.006	2.974	0.904	3.595	0.000	0.083	0.088

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	65	66	52	94	119	0	43	55
N.S.	1	1.00	1.41	1.43	1.13	2.04	2.59	0.00	0.93	1.20
time (sec)	N/A	0.017	0.090	0.004	2.899	0.982	1.959	0.000	0.593	0.070

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	69	0	35	91	148	0	-1	53
N.S.	1	1.00	1.47	0.00	0.74	1.94	3.15	0.00	-0.02	1.13
time (sec)	N/A	0.016	0.060	0.032	2.928	0.747	1.701	0.000	0.000	0.087

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	23	88	42	21	22
N.S.	1	1.00	1.00	0.77	0.73	1.05	4.00	1.91	0.95	1.00
time (sec)	N/A	0.002	0.007	0.004	1.315	1.023	1.549	1.396	0.243	0.023

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	30	25	33	34	241	61	32	40
N.S.	1	1.00	0.65	0.54	0.72	0.74	5.24	1.33	0.70	0.87
time (sec)	N/A	0.005	0.009	0.005	1.336	0.865	5.006	1.384	0.253	0.104

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	41	36	49	46	707	79	43	52
N.S.	1	1.00	0.58	0.51	0.69	0.65	9.96	1.11	0.61	0.73
time (sec)	N/A	0.010	0.011	0.005	1.351	0.957	26.813	1.336	0.269	0.114

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	70	108	163	140	117	0	-1	85
N.S.	1	1.00	0.65	1.00	1.51	1.30	1.08	0.00	-0.01	0.79
time (sec)	N/A	0.032	0.047	0.008	3.030	0.917	10.124	0.000	0.000	0.096

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	58	93	134	121	90	0	-1	72
N.S.	1	1.00	0.69	1.11	1.60	1.44	1.07	0.00	-0.01	0.86
time (sec)	N/A	0.020	0.034	0.005	3.016	0.918	5.223	0.000	0.000	0.088

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	51	75	98	101	71	0	46	59
N.S.	1	1.00	0.80	1.17	1.53	1.58	1.11	0.00	0.72	0.92
time (sec)	N/A	0.015	0.027	0.004	2.906	1.136	2.912	0.000	0.098	0.055

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	40	58	68	86	37	0	40	46
N.S.	1	1.00	1.00	1.45	1.70	2.15	0.92	0.00	1.00	1.15
time (sec)	N/A	0.008	0.013	0.004	2.959	1.198	1.648	0.000	0.617	0.054

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	59	54	87	48	0	-1	47
N.S.	1	1.00	1.00	1.44	1.32	2.12	1.17	0.00	-0.02	1.15
time (sec)	N/A	0.009	0.012	0.024	2.951	1.025	1.429	0.000	0.000	0.066

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	13	12	12	37	29	18	18
N.S.	1	1.00	1.00	0.72	0.67	0.67	2.06	1.61	1.00	1.00
time (sec)	N/A	0.001	0.005	0.003	1.314	0.781	1.451	1.152	0.208	0.023

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	23	18	26	25	56	42	26	31
N.S.	1	1.00	0.61	0.47	0.68	0.66	1.47	1.11	0.68	0.82
time (sec)	N/A	0.004	0.007	0.004	1.317	1.038	4.717	1.092	0.215	0.071

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	32	27	41	34	270	55	34	40
N.S.	1	1.00	0.54	0.46	0.69	0.58	4.58	0.93	0.58	0.68
time (sec)	N/A	0.008	0.010	0.003	1.274	1.253	13.796	1.111	0.224	0.076

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	71	116	147	141	252	0	-1	94
N.S.	1	1.00	0.63	1.04	1.31	1.26	2.25	0.00	-0.01	0.84
time (sec)	N/A	0.029	0.044	0.009	2.932	1.043	9.916	0.000	0.000	0.129

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	60	100	117	125	196	0	-1	81
N.S.	1	1.00	0.69	1.15	1.34	1.44	2.25	0.00	-0.01	0.93
time (sec)	N/A	0.023	0.035	0.005	2.976	1.030	5.294	0.000	0.000	0.110

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	51	81	81	107	156	0	53	70
N.S.	1	1.00	0.78	1.25	1.25	1.65	2.40	0.00	0.82	1.08
time (sec)	N/A	0.015	0.029	0.005	2.953	1.198	2.942	0.000	0.104	0.076

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	63	49	89	121	0	42	55
N.S.	1	1.00	1.00	1.54	1.20	2.17	2.95	0.00	1.02	1.34
time (sec)	N/A	0.008	0.014	0.003	2.912	1.218	1.711	0.000	0.562	0.064

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	90	35	90	136	0	-1	53
N.S.	1	1.00	1.00	2.14	0.83	2.14	3.24	0.00	-0.02	1.26
time (sec)	N/A	0.009	0.014	0.036	3.013	1.235	1.580	0.000	0.000	0.077

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	18	82	35	18	24
N.S.	1	1.00	1.00	0.74	0.68	0.95	4.32	1.84	0.95	1.26
time (sec)	N/A	0.001	0.005	0.003	1.285	1.292	1.506	1.116	0.224	0.072

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	24	19	28	25	194	48	26	31
N.S.	1	1.00	0.60	0.48	0.70	0.62	4.85	1.20	0.65	0.78
time (sec)	N/A	0.004	0.008	0.004	1.334	0.667	4.909	0.853	0.219	0.084

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	33	28	44	35	554	61	34	41
N.S.	1	1.00	0.53	0.45	0.71	0.56	8.94	0.98	0.55	0.66
time (sec)	N/A	0.008	0.010	0.003	1.312	0.904	24.598	0.975	0.224	0.091

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	107	138	212	184	178	0	-1	108
N.S.	1	1.00	0.75	0.97	1.48	1.29	1.24	0.00	-0.01	0.76
time (sec)	N/A	0.051	0.208	0.007	2.964	1.096	17.714	0.000	0.000	0.144

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	96	120	178	163	153	0	-1	95
N.S.	1	1.00	0.81	1.01	1.50	1.37	1.29	0.00	-0.01	0.80
time (sec)	N/A	0.038	0.125	0.006	2.961	0.698	9.279	0.000	0.000	0.101

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	85	96	144	140	124	0	-1	82
N.S.	1	1.00	0.89	1.01	1.52	1.47	1.31	0.00	-0.01	0.86
time (sec)	N/A	0.029	0.115	0.006	3.022	0.718	5.590	0.000	0.000	0.106

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	69	78	107	119	75	0	-1	66
N.S.	1	1.00	0.97	1.10	1.51	1.68	1.06	0.00	-0.01	0.93
time (sec)	N/A	0.022	0.098	0.006	2.985	1.104	3.173	0.000	0.000	0.093

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	46	71	84	109	92	0	-1	54
N.S.	1	1.00	0.73	1.13	1.33	1.73	1.46	0.00	-0.02	0.86
time (sec)	N/A	0.021	0.011	0.018	2.987	1.331	2.722	0.000	0.000	0.134

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	48	67	67	109	71	0	-1	55
N.S.	1	1.00	0.75	1.05	1.05	1.70	1.11	0.00	-0.02	0.86
time (sec)	N/A	0.021	0.010	0.019	2.926	0.905	3.036	0.000	0.000	0.130

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	110	146	207	185	376	0	-1	117
N.S.	1	1.00	0.74	0.98	1.39	1.24	2.52	0.00	-0.01	0.79
time (sec)	N/A	0.053	0.171	0.007	3.115	1.090	17.693	0.000	0.000	0.177

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	99	127	170	163	323	0	-1	104
N.S.	1	1.00	0.80	1.02	1.37	1.31	2.60	0.00	-0.01	0.84
time (sec)	N/A	0.041	0.145	0.006	2.968	1.116	9.060	0.000	0.000	0.134

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	87	102	133	141	264	0	-1	91
N.S.	1	1.00	0.88	1.03	1.34	1.42	2.67	0.00	-0.01	0.92
time (sec)	N/A	0.031	0.121	0.007	2.990	1.165	5.537	0.000	0.000	0.124

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	71	83	93	119	190	0	-1	75
N.S.	1	1.00	0.96	1.12	1.26	1.61	2.57	0.00	-0.01	1.01
time (sec)	N/A	0.021	0.116	0.005	2.896	1.425	3.212	0.000	0.000	0.099

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	47	0	68	109	197	0	-1	61
N.S.	1	1.00	0.71	0.00	1.03	1.65	2.98	0.00	-0.02	0.92
time (sec)	N/A	0.020	0.011	0.025	2.846	1.150	2.881	0.000	0.000	0.129

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	49	0	49	115	187	0	-1	62
N.S.	1	1.00	0.73	0.00	0.73	1.72	2.79	0.00	-0.01	0.93
time (sec)	N/A	0.022	0.011	0.029	2.859	1.620	3.241	0.000	0.000	0.139

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	78	123	194	156	136	0	-1	95
N.S.	1	1.00	0.62	0.98	1.54	1.24	1.08	0.00	-0.01	0.75
time (sec)	N/A	0.034	0.050	0.006	2.986	1.182	15.671	0.000	0.000	0.125

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	70	108	163	137	117	0	-1	84
N.S.	1	1.00	0.67	1.03	1.55	1.30	1.11	0.00	-0.01	0.80
time (sec)	N/A	0.026	0.032	0.004	3.043	1.308	7.858	0.000	0.000	0.086

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	60	87	132	124	92	0	-1	72
N.S.	1	1.00	0.73	1.06	1.61	1.51	1.12	0.00	-0.01	0.88
time (sec)	N/A	0.016	0.033	0.004	2.993	1.409	4.809	0.000	0.000	0.088

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	48	72	98	105	76	0	-1	59
N.S.	1	1.00	0.79	1.18	1.61	1.72	1.25	0.00	-0.02	0.97
time (sec)	N/A	0.011	0.026	0.005	2.895	1.513	2.816	0.000	0.000	0.077

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	28	72	81	99	73	0	-1	51
N.S.	1	1.00	0.48	1.24	1.40	1.71	1.26	0.00	-0.02	0.88
time (sec)	N/A	0.012	0.005	0.018	2.914	1.228	2.442	0.000	0.000	0.100

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	30	73	67	108	70	0	-1	55
N.S.	1	1.00	0.50	1.22	1.12	1.80	1.17	0.00	-0.02	0.92
time (sec)	N/A	0.014	0.006	0.017	2.972	1.307	2.812	0.000	0.000	0.107

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	79	132	179	157	291	0	-1	104
N.S.	1	1.00	0.60	1.01	1.37	1.20	2.22	0.00	-0.01	0.79
time (sec)	N/A	0.040	0.052	0.006	2.943	0.906	15.279	0.000	0.000	0.161

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	70	116	147	139	252	0	-1	94
N.S.	1	1.00	0.64	1.06	1.35	1.28	2.31	0.00	-0.01	0.86
time (sec)	N/A	0.028	0.045	0.006	2.925	1.120	7.726	0.000	0.000	0.116

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	60	94	115	125	199	0	-1	81
N.S.	1	1.00	0.71	1.12	1.37	1.49	2.37	0.00	-0.01	0.96
time (sec)	N/A	0.016	0.042	0.004	3.042	1.308	4.780	0.000	0.000	0.117

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	49	78	79	107	167	0	-1	69
N.S.	1	1.00	0.78	1.24	1.25	1.70	2.65	0.00	-0.02	1.10
time (sec)	N/A	0.012	0.029	0.004	2.984	1.471	2.863	0.000	0.000	0.102

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	28	97	63	101	160	0	-1	58
N.S.	1	1.00	0.47	1.62	1.05	1.68	2.67	0.00	-0.02	0.97
time (sec)	N/A	0.012	0.005	0.020	2.989	0.745	2.493	0.000	0.000	0.121

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	30	98	49	111	182	0	-1	62
N.S.	1	1.00	0.48	1.58	0.79	1.79	2.94	0.00	-0.02	1.00
time (sec)	N/A	0.013	0.006	0.022	3.004	1.153	2.923	0.000	0.000	0.133

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	118	156	244	206	207	0	-1	121
N.S.	1	1.00	0.72	0.95	1.49	1.26	1.26	0.00	-0.01	0.74
time (sec)	N/A	0.063	0.244	0.005	2.989	0.782	25.936	0.000	0.000	0.123

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	107	138	212	185	180	0	-1	108
N.S.	1	1.00	0.76	0.99	1.51	1.32	1.29	0.00	-0.01	0.77
time (sec)	N/A	0.048	0.138	0.006	2.979	1.346	16.411	0.000	0.000	0.149

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	96	111	176	162	155	0	-1	95
N.S.	1	1.00	0.83	0.96	1.52	1.40	1.34	0.00	-0.01	0.82
time (sec)	N/A	0.038	0.199	0.006	2.981	1.454	9.860	0.000	0.000	0.127

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	80	93	141	141	102	0	-1	79
N.S.	1	1.00	0.87	1.01	1.53	1.53	1.11	0.00	-0.01	0.86
time (sec)	N/A	0.028	0.111	0.006	2.991	1.509	6.229	0.000	0.000	0.111

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	48	84	125	137	126	0	-1	73
N.S.	1	1.00	0.54	0.94	1.40	1.54	1.42	0.00	-0.01	0.82
time (sec)	N/A	0.028	0.012	0.018	2.977	1.387	6.148	0.000	0.000	0.135

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	50	82	100	138	99	0	-1	69
N.S.	1	1.00	0.58	0.95	1.16	1.60	1.15	0.00	-0.01	0.80
time (sec)	N/A	0.026	0.011	0.020	2.949	1.530	5.622	0.000	0.000	0.155

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	120	165	242	208	435	0	-1	130
N.S.	1	1.00	0.70	0.96	1.42	1.22	2.54	0.00	-0.01	0.76
time (sec)	N/A	0.061	0.191	0.007	2.864	1.399	25.960	0.000	0.000	0.305

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	109	146	207	186	379	0	-1	117
N.S.	1	1.00	0.75	1.00	1.42	1.27	2.60	0.00	-0.01	0.80
time (sec)	N/A	0.051	0.149	0.006	3.008	1.135	16.398	0.000	0.000	0.180

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	98	118	168	164	326	0	-1	104
N.S.	1	1.00	0.81	0.98	1.39	1.36	2.69	0.00	-0.01	0.86
time (sec)	N/A	0.038	0.132	0.007	3.046	1.245	9.807	0.000	0.000	0.158

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	82	99	130	142	246	0	-1	88
N.S.	1	1.00	0.85	1.03	1.35	1.48	2.56	0.00	-0.01	0.92
time (sec)	N/A	0.030	0.123	0.005	2.930	0.691	6.228	0.000	0.000	0.133

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	49	0	112	137	267	0	-1	79
N.S.	1	1.00	0.53	0.00	1.20	1.47	2.87	0.00	-0.01	0.85
time (sec)	N/A	0.028	0.012	0.030	2.992	1.302	6.222	0.000	0.000	0.161

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	51	0	84	139	245	0	-1	76
N.S.	1	1.00	0.57	0.00	0.93	1.54	2.72	0.00	-0.01	0.84
time (sec)	N/A	0.030	0.012	0.028	3.000	1.327	5.845	0.000	0.000	0.187

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	86	138	223	172	158	0	-1	105
N.S.	1	1.00	0.60	0.96	1.55	1.19	1.10	0.00	-0.01	0.73
time (sec)	N/A	0.045	0.059	0.005	2.977	1.291	22.960	0.000	0.000	0.106

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	78	123	194	155	138	0	-1	95
N.S.	1	1.00	0.63	1.00	1.58	1.26	1.12	0.00	-0.01	0.77
time (sec)	N/A	0.028	0.048	0.004	2.943	1.316	14.423	0.000	0.000	0.129

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	70	99	161	140	119	0	-1	85
N.S.	1	1.00	0.69	0.97	1.58	1.37	1.17	0.00	-0.01	0.83
time (sec)	N/A	0.022	0.041	0.006	2.986	1.428	8.611	0.000	0.000	0.105

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	57	84	129	123	97	0	-1	70
N.S.	1	1.00	0.72	1.06	1.63	1.56	1.23	0.00	-0.01	0.89
time (sec)	N/A	0.016	0.032	0.004	3.028	1.242	5.458	0.000	0.000	0.101

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	28	81	113	116	94	0	-1	62
N.S.	1	1.00	0.35	1.03	1.43	1.47	1.19	0.00	-0.01	0.78
time (sec)	N/A	0.016	0.007	0.019	2.943	1.236	5.602	0.000	0.000	0.117

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	30	82	96	123	88	0	-1	63
N.S.	1	1.00	0.37	1.01	1.19	1.52	1.09	0.00	-0.01	0.78
time (sec)	N/A	0.017	0.006	0.019	2.945	1.445	5.152	0.000	0.000	0.129

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	150	150	87	148	209	173	337	0	-1	114
N.S.	1	1.00	0.58	0.99	1.39	1.15	2.25	0.00	-0.01	0.76
time (sec)	N/A	0.045	0.059	0.004	2.960	1.280	22.838	0.000	0.000	0.150

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	79	132	179	157	294	0	-1	104
N.S.	1	1.00	0.62	1.03	1.40	1.23	2.30	0.00	-0.01	0.81
time (sec)	N/A	0.029	0.046	0.006	3.013	1.317	14.298	0.000	0.000	0.185

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	71	107	145	141	255	0	-1	94
N.S.	1	1.00	0.67	1.01	1.37	1.33	2.41	0.00	-0.01	0.89
time (sec)	N/A	0.023	0.041	0.005	2.887	1.438	8.592	0.000	0.000	0.151

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	58	91	112	125	209	0	-1	79
N.S.	1	1.00	0.71	1.11	1.37	1.52	2.55	0.00	-0.01	0.96
time (sec)	N/A	0.017	0.034	0.005	2.976	1.261	5.522	0.000	0.000	0.137

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	28	106	96	117	202	0	-1	68
N.S.	1	1.00	0.34	1.29	1.17	1.43	2.46	0.00	-0.01	0.83
time (sec)	N/A	0.017	0.006	0.019	2.923	1.321	5.619	0.000	0.000	0.152

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	30	107	79	126	221	0	-1	70
N.S.	1	1.00	0.36	1.27	0.94	1.50	2.63	0.00	-0.01	0.83
time (sec)	N/A	0.017	0.007	0.020	2.888	0.860	5.346	0.000	0.000	0.168

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	85	102	146	140	128	0	-1	82
N.S.	1	1.00	0.84	1.01	1.45	1.39	1.27	0.00	-0.01	0.81
time (sec)	N/A	0.030	0.161	0.006	3.014	1.332	8.521	0.000	0.000	0.098

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	85	84	112	119	100	0	-1	69
N.S.	1	1.00	1.10	1.09	1.45	1.55	1.30	0.00	-0.01	0.90
time (sec)	N/A	0.022	0.050	0.006	2.867	1.151	4.301	0.000	0.000	0.083

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	68	65	73	91	44	0	44	49
N.S.	1	1.00	1.42	1.35	1.52	1.90	0.92	0.00	0.92	1.02
time (sec)	N/A	0.016	0.039	0.004	2.913	1.471	2.187	0.000	0.548	0.065

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	50	48	41	57	22	0	30	30
N.S.	1	1.00	1.79	1.71	1.46	2.04	0.79	0.00	1.07	1.07
time (sec)	N/A	0.013	0.013	0.004	2.953	1.037	1.097	0.000	0.030	0.040

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	19	33	15	19
N.S.	1	1.00	1.00	0.84	0.79	0.79	1.00	1.74	0.79	1.00
time (sec)	N/A	0.002	0.004	0.004	1.342	0.946	0.913	2.050	0.349	0.020

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	27	22	31	23	42	50	25	29
N.S.	1	1.00	0.61	0.50	0.70	0.52	0.95	1.14	0.57	0.66
time (sec)	N/A	0.005	0.007	0.004	1.299	1.150	1.920	1.902	0.342	0.075

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	40	35	46	34	287	66	36	40
N.S.	1	1.00	0.59	0.51	0.68	0.50	4.22	0.97	0.53	0.59
time (sec)	N/A	0.010	0.009	0.005	1.337	1.210	6.249	1.669	0.353	0.089

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	51	46	61	45	488	82	47	51
N.S.	1	1.00	0.55	0.50	0.66	0.49	5.30	0.89	0.51	0.55
time (sec)	N/A	0.016	0.012	0.004	1.304	1.211	16.137	1.412	0.382	0.096

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	50	119	131	175	105	131	-1	82
N.S.	1	1.00	0.52	1.24	1.36	1.82	1.09	1.36	-0.01	0.85
time (sec)	N/A	0.029	0.010	0.035	2.924	1.413	8.139	92.143	0.000	0.136

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	50	106	92	145	71	115	-1	61
N.S.	1	1.00	0.74	1.56	1.35	2.13	1.04	1.69	-0.01	0.90
time (sec)	N/A	0.022	0.010	0.030	3.019	1.392	3.678	93.123	0.000	0.126

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	64	0	57	119	46	85	-1	50
N.S.	1	1.00	1.33	0.00	1.19	2.48	0.96	1.77	-0.02	1.04
time (sec)	N/A	0.016	0.069	0.035	2.981	1.326	1.780	94.858	0.000	0.084

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	22	17	45	22	19
N.S.	1	1.00	1.00	0.84	0.79	1.16	0.89	2.37	1.16	1.00
time (sec)	N/A	0.002	0.005	0.005	1.321	1.006	0.879	1.109	0.326	0.021

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	25	22	32	34	41	82	39	25
N.S.	1	1.00	0.64	0.56	0.82	0.87	1.05	2.10	1.00	0.64
time (sec)	N/A	0.005	0.008	0.006	1.320	0.889	1.598	1.052	0.386	0.076

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	38	33	50	49	219	98	46	40
N.S.	1	1.00	0.60	0.52	0.79	0.78	3.48	1.56	0.73	0.63
time (sec)	N/A	0.010	0.009	0.005	1.310	1.351	3.983	1.201	0.413	0.108

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	49	44	64	58	348	121	58	49
N.S.	1	1.00	0.56	0.51	0.74	0.67	4.00	1.39	0.67	0.56
time (sec)	N/A	0.017	0.010	0.005	1.309	1.184	11.154	1.213	0.426	0.121

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	50	147	109	214	396	197	-1	78
N.S.	1	1.00	0.55	1.62	1.20	2.35	4.35	2.16	-0.01	0.86
time (sec)	N/A	0.028	0.010	0.049	2.939	1.420	7.610	92.464	0.000	0.153

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	80	0	69	186	328	165	-1	64
N.S.	1	1.00	1.16	0.00	1.00	2.70	4.75	2.39	-0.01	0.93
time (sec)	N/A	0.022	0.121	0.032	2.826	0.724	4.027	105.595	0.000	0.138

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	15	33	42	86	36	21
N.S.	1	1.00	1.00	0.76	0.71	1.57	2.00	4.10	1.71	1.00
time (sec)	N/A	0.002	0.005	0.003	1.328	1.243	1.427	1.631	0.242	0.024

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	29	24	27	43	92	81	54	29
N.S.	1	1.00	0.67	0.56	0.63	1.00	2.14	1.88	1.26	0.67
time (sec)	N/A	0.005	0.009	0.005	1.344	1.494	1.896	1.500	0.400	0.082

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	40	35	46	58	153	159	71	40
N.S.	1	1.00	0.62	0.55	0.72	0.91	2.39	2.48	1.11	0.62
time (sec)	N/A	0.009	0.011	0.007	1.340	1.370	3.972	1.632	0.420	0.104

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	49	44	64	71	337	175	88	51
N.S.	1	1.00	0.58	0.52	0.76	0.85	4.01	2.08	1.05	0.61
time (sec)	N/A	0.015	0.014	0.005	1.268	1.040	7.061	2.238	0.471	0.114

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	88	108	135	141	270	0	-1	91
N.S.	1	1.00	0.84	1.03	1.29	1.34	2.57	0.00	-0.01	0.87
time (sec)	N/A	0.030	0.146	0.007	2.964	1.346	8.432	0.000	0.000	0.118

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	86	89	98	119	214	0	-1	78
N.S.	1	1.00	1.08	1.11	1.22	1.49	2.68	0.00	-0.01	0.98
time (sec)	N/A	0.023	0.048	0.006	2.960	0.772	4.296	0.000	0.000	0.100

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	71	70	56	93	121	0	47	59
N.S.	1	1.00	1.42	1.40	1.12	1.86	2.42	0.00	0.94	1.18
time (sec)	N/A	0.017	0.042	0.006	3.004	1.213	2.278	0.000	0.517	0.075

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	52	51	21	57	54	0	27	38
N.S.	1	1.00	1.79	1.76	0.72	1.97	1.86	0.00	0.93	1.31
time (sec)	N/A	0.013	0.014	0.006	2.883	0.919	1.153	0.000	0.031	0.059

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	46	35	16	20
N.S.	1	1.00	1.00	0.85	0.80	0.80	2.30	1.75	0.80	1.00
time (sec)	N/A	0.002	0.004	0.005	1.341	0.690	0.969	1.277	0.401	0.021

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	28	23	32	22	177	54	26	28
N.S.	1	1.00	0.61	0.50	0.70	0.48	3.85	1.17	0.57	0.61
time (sec)	N/A	0.005	0.007	0.004	1.302	1.771	2.055	1.453	0.350	0.107

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	51	127	118	181	224	154	-1	100
N.S.	1	1.00	0.51	1.27	1.18	1.81	2.24	1.54	-0.01	1.00
time (sec)	N/A	0.030	0.010	0.040	2.922	1.372	8.030	98.066	0.000	0.168

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	51	114	75	152	155	130	-1	81
N.S.	1	1.00	0.72	1.61	1.06	2.14	2.18	1.83	-0.01	1.14
time (sec)	N/A	0.022	0.010	0.030	2.952	1.304	3.704	111.127	0.000	0.142

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	66	0	38	128	102	98	-1	68
N.S.	1	1.00	1.32	0.00	0.76	2.56	2.04	1.96	-0.02	1.36
time (sec)	N/A	0.016	0.075	0.030	3.012	1.207	1.891	113.038	0.000	0.109

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	25	44	53	24	20
N.S.	1	1.00	1.00	0.85	0.80	1.25	2.20	2.65	1.20	1.00
time (sec)	N/A	0.002	0.005	0.003	1.297	1.031	0.943	1.374	0.339	0.024

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	26	23	34	38	112	94	42	37
N.S.	1	1.00	0.63	0.56	0.83	0.93	2.73	2.29	1.02	0.90
time (sec)	N/A	0.005	0.008	0.003	1.305	0.963	1.678	1.441	0.400	0.103

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	39	34	52	51	452	112	48	50
N.S.	1	1.00	0.59	0.52	0.79	0.77	6.85	1.70	0.73	0.76
time (sec)	N/A	0.010	0.009	0.005	1.347	1.201	4.640	1.478	0.432	0.147

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	51	160	94	215	971	221	-1	96
N.S.	1	1.00	0.54	1.68	0.99	2.26	10.22	2.33	-0.01	1.01
time (sec)	N/A	0.029	0.012	0.043	3.018	1.353	8.483	112.518	0.000	0.195

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	82	0	52	188	833	194	-1	82
N.S.	1	1.00	1.14	0.00	0.72	2.61	11.57	2.69	-0.01	1.14
time (sec)	N/A	0.022	0.171	0.031	2.940	1.562	4.496	110.078	0.000	0.173

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	34	95	102	37	22
N.S.	1	1.00	1.00	0.77	0.73	1.55	4.32	4.64	1.68	1.00
time (sec)	N/A	0.002	0.005	0.005	1.364	1.106	1.507	1.633	0.252	0.028

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	30	25	30	44	211	96	56	30
N.S.	1	1.00	0.67	0.56	0.67	0.98	4.69	2.13	1.24	0.67
time (sec)	N/A	0.005	0.009	0.003	1.344	1.251	1.996	1.426	0.406	0.117

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	41	36	50	59	314	189	73	41
N.S.	1	1.00	0.61	0.54	0.75	0.88	4.69	2.82	1.09	0.61
time (sec)	N/A	0.010	0.012	0.005	1.290	1.194	4.265	1.613	0.441	0.139

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	50	45	68	70	688	207	92	50
N.S.	1	1.00	0.57	0.51	0.77	0.80	7.82	2.35	1.05	0.57
time (sec)	N/A	0.016	0.014	0.005	1.329	1.234	13.450	1.697	0.475	0.147

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	60	93	134	124	95	0	-1	73
N.S.	1	1.00	0.68	1.06	1.52	1.41	1.08	0.00	-0.01	0.83
time (sec)	N/A	0.022	0.040	0.005	2.975	1.149	7.397	0.000	0.000	0.090

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	51	78	102	105	75	0	-1	62
N.S.	1	1.00	0.76	1.16	1.52	1.57	1.12	0.00	-0.01	0.93
time (sec)	N/A	0.014	0.028	0.005	2.858	0.996	3.671	0.000	0.000	0.076

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	43	62	70	87	54	0	43	49
N.S.	1	1.00	1.00	1.44	1.63	2.02	1.26	0.00	1.00	1.14
time (sec)	N/A	0.009	0.016	0.004	2.919	1.022	1.934	0.000	0.585	0.060

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	46	41	55	24	0	30	30
N.S.	1	1.00	1.00	1.92	1.71	2.29	1.00	0.00	1.25	1.25
time (sec)	N/A	0.006	0.004	0.004	2.959	1.278	1.022	0.000	0.035	0.040

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	15	29	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.94	1.81	0.75	1.00
time (sec)	N/A	0.001	0.004	0.003	1.355	1.282	0.879	1.108	0.330	0.019

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	23	18	26	17	34	42	17	23
N.S.	1	1.00	0.61	0.47	0.68	0.45	0.89	1.11	0.45	0.61
time (sec)	N/A	0.004	0.006	0.005	1.315	1.108	1.873	1.087	0.316	0.068

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	32	27	41	26	224	55	26	32
N.S.	1	1.00	0.54	0.46	0.69	0.44	3.80	0.93	0.44	0.54
time (sec)	N/A	0.007	0.008	0.006	1.311	0.870	6.104	1.065	0.323	0.073

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	40	35	56	34	374	68	33	40
N.S.	1	1.00	0.50	0.44	0.70	0.42	4.68	0.85	0.41	0.50
time (sec)	N/A	0.012	0.009	0.006	1.295	0.809	15.993	1.025	0.333	0.079

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	30	106	119	152	80	119	-1	72
N.S.	1	1.00	0.35	1.23	1.38	1.77	0.93	1.38	-0.01	0.84
time (sec)	N/A	0.020	0.006	0.028	3.055	1.254	7.099	11.125	0.000	0.118

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	30	100	90	134	58	106	-1	59
N.S.	1	1.00	0.48	1.59	1.43	2.13	0.92	1.68	-0.02	0.94
time (sec)	N/A	0.014	0.006	0.024	2.988	1.333	3.065	10.155	0.000	0.104

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	48	57	117	41	82	-1	50
N.S.	1	1.00	1.00	1.09	1.30	2.66	0.93	1.86	-0.02	1.14
time (sec)	N/A	0.009	0.029	0.112	2.896	1.294	1.565	10.610	0.000	0.075

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	15	44	11	15
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.00	2.93	0.73	1.00
time (sec)	N/A	0.001	0.003	0.004	1.340	1.319	0.862	1.216	0.307	0.021

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	21	18	26	28	34	74	17	21
N.S.	1	1.00	0.66	0.56	0.81	0.88	1.06	2.31	0.53	0.66
time (sec)	N/A	0.003	0.007	0.005	1.335	1.003	1.543	1.123	0.348	0.064

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	32	27	41	39	170	86	37	32
N.S.	1	1.00	0.60	0.51	0.77	0.74	3.21	1.62	0.70	0.60
time (sec)	N/A	0.007	0.008	0.004	1.372	0.644	3.857	1.232	0.380	0.094

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	39	35	56	47	269	107	46	39
N.S.	1	1.00	0.53	0.47	0.76	0.64	3.64	1.45	0.62	0.53
time (sec)	N/A	0.014	0.008	0.004	1.303	1.221	10.971	1.110	0.426	0.100

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	30	136	105	186	308	182	-1	73
N.S.	1	1.00	0.35	1.58	1.22	2.16	3.58	2.12	-0.01	0.85
time (sec)	N/A	0.020	0.007	0.036	3.003	1.350	6.617	10.823	0.000	0.132

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	52	55	69	171	257	154	-1	63
N.S.	1	1.00	0.80	0.85	1.06	2.63	3.95	2.37	-0.02	0.97
time (sec)	N/A	0.014	0.069	0.040	2.971	1.288	3.578	10.772	0.000	0.120

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	13	12	27	27	82	12	18
N.S.	1	1.00	1.00	0.72	0.67	1.50	1.50	4.56	0.67	1.00
time (sec)	N/A	0.001	0.004	0.005	1.328	1.185	1.404	1.224	0.252	0.021

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	23	18	24	32	75	79	42	23
N.S.	1	1.00	0.62	0.49	0.65	0.86	2.03	2.14	1.14	0.62
time (sec)	N/A	0.003	0.006	0.005	1.318	0.814	1.842	1.215	0.357	0.068

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	32	27	40	45	117	145	57	32
N.S.	1	1.00	0.58	0.49	0.73	0.82	2.13	2.64	1.04	0.58
time (sec)	N/A	0.006	0.008	0.004	1.401	1.474	3.889	1.362	0.378	0.095

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	40	35	55	55	257	158	71	40
N.S.	1	1.00	0.56	0.49	0.77	0.77	3.62	2.23	1.00	0.56
time (sec)	N/A	0.009	0.010	0.005	1.276	1.049	6.853	1.272	0.418	0.097

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	61	100	117	125	206	0	-1	82
N.S.	1	1.00	0.67	1.10	1.29	1.37	2.26	0.00	-0.01	0.90
time (sec)	N/A	0.021	0.041	0.005	3.033	1.440	7.511	0.000	0.000	0.105

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	52	84	85	107	163	0	-1	72
N.S.	1	1.00	0.75	1.22	1.23	1.55	2.36	0.00	-0.01	1.04
time (sec)	N/A	0.015	0.029	0.004	2.927	1.344	3.591	0.000	0.000	0.091

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	67	52	90	121	0	46	59
N.S.	1	1.00	1.00	1.49	1.16	2.00	2.69	0.00	1.02	1.31
time (sec)	N/A	0.010	0.016	0.006	2.928	1.280	1.966	0.000	0.520	0.069

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	50	21	56	58	0	27	38
N.S.	1	1.00	1.00	2.08	0.88	2.33	2.42	0.00	1.12	1.58
time (sec)	N/A	0.007	0.004	0.004	2.962	1.294	1.081	0.000	0.032	0.056

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	13	13	39	30	13	17
N.S.	1	1.00	1.00	0.82	0.76	0.76	2.29	1.76	0.76	1.00
time (sec)	N/A	0.001	0.004	0.004	1.351	1.130	0.934	1.283	0.310	0.020

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	24	19	28	18	139	43	19	25
N.S.	1	1.00	0.60	0.48	0.70	0.45	3.48	1.08	0.48	0.62
time (sec)	N/A	0.004	0.006	0.003	1.346	0.874	1.957	1.117	0.292	0.092

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	30	138	101	155	173	136	-1	88
N.S.	1	1.00	0.34	1.55	1.13	1.74	1.94	1.53	-0.01	0.99
time (sec)	N/A	0.021	0.006	0.028	2.991	1.226	7.026	10.853	0.000	0.170

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	30	133	71	138	128	119	-1	75
N.S.	1	1.00	0.46	2.05	1.09	2.12	1.97	1.83	-0.02	1.15
time (sec)	N/A	0.014	0.006	0.028	2.995	1.303	3.199	10.407	0.000	0.152

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	45	67	38	122	92	92	-1	66
N.S.	1	1.00	1.00	1.49	0.84	2.71	2.04	2.04	-0.02	1.47
time (sec)	N/A	0.009	0.038	0.046	2.982	1.042	1.693	10.063	0.000	0.118

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	20	39	50	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.25	2.44	3.12	0.75	1.00
time (sec)	N/A	0.001	0.004	0.005	1.273	1.158	0.931	1.026	0.299	0.025

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	21	18	28	29	90	83	27	29
N.S.	1	1.00	0.62	0.53	0.82	0.85	2.65	2.44	0.79	0.85
time (sec)	N/A	0.003	0.007	0.004	1.259	1.132	1.606	1.086	0.321	0.088

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	33	28	44	40	354	96	38	40
N.S.	1	1.00	0.59	0.50	0.79	0.71	6.32	1.71	0.68	0.71
time (sec)	N/A	0.008	0.008	0.006	1.281	1.265	4.293	1.168	0.362	0.125

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	30	168	86	187	753	200	-1	89
N.S.	1	1.00	0.34	1.89	0.97	2.10	8.46	2.25	-0.01	1.00
time (sec)	N/A	0.022	0.007	0.038	2.990	1.137	6.803	10.724	0.000	0.186

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	53	73	50	173	649	178	-1	79
N.S.	1	1.00	0.79	1.09	0.75	2.58	9.69	2.66	-0.01	1.18
time (sec)	N/A	0.015	0.056	0.041	2.919	1.391	3.700	10.628	0.000	0.168

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	28	65	95	13	26
N.S.	1	1.00	1.00	0.74	0.68	1.47	3.42	5.00	0.68	1.37
time (sec)	N/A	0.001	0.005	0.003	1.382	1.034	1.458	1.252	0.232	0.106

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	24	19	25	33	177	90	45	31
N.S.	1	1.00	0.62	0.49	0.64	0.85	4.54	2.31	1.15	0.79
time (sec)	N/A	0.004	0.008	0.005	1.302	1.310	1.908	1.100	0.357	0.100

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	33	28	42	46	243	170	59	40
N.S.	1	1.00	0.57	0.48	0.72	0.79	4.19	2.93	1.02	0.69
time (sec)	N/A	0.006	0.010	0.004	1.352	1.184	3.995	1.122	0.367	0.120

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	41	36	58	56	529	183	73	48
N.S.	1	1.00	0.55	0.48	0.77	0.75	7.05	2.44	0.97	0.64
time (sec)	N/A	0.010	0.013	0.004	1.351	1.099	12.398	1.250	0.438	0.122

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	25	41	37	27	54	17	31	39
N.S.	1	1.00	0.93	1.52	1.37	1.00	2.00	0.63	1.15	1.44
time (sec)	N/A	0.005	0.009	0.006	3.004	1.516	1.646	1.174	0.570	0.076

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	12	27	14	14	20	6	16	8
N.S.	1	1.00	1.50	3.38	1.75	1.75	2.50	0.75	2.00	1.00
time (sec)	N/A	0.003	0.008	0.004	2.951	1.255	0.971	1.095	0.051	0.037

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	48	21	57	42	0	23	38
N.S.	1	1.00	1.00	2.53	1.11	3.00	2.21	0.00	1.21	2.00
time (sec)	N/A	0.005	0.005	0.007	2.889	1.162	1.060	0.000	0.125	0.054

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	18	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.001	1.296	1.253	2.009	0.977	0.028	0.010

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	18	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.86	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.002	1.351	1.236	1.304	1.045	0.025	0.010

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	16	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.348	1.238	0.446	0.988	0.024	0.009

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	16	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.76	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.368	1.166	1.520	1.027	0.025	0.009

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	17	14	13	13	19	13	13	21
N.S.	1	1.00	0.81	0.67	0.62	0.62	0.90	0.62	0.62	1.00
time (sec)	N/A	0.004	0.005	0.003	1.307	1.303	1.659	1.121	0.024	0.009

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	16	13	13	12	17	13	12	20
N.S.	1	1.00	0.84	0.68	0.68	0.63	0.89	0.68	0.63	1.05
time (sec)	N/A	0.003	0.005	0.002	1.283	1.134	1.494	1.116	0.023	0.009

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	16	14	13	12	17	13	13	16
N.S.	1	1.00	0.84	0.74	0.68	0.63	0.89	0.68	0.68	0.84
time (sec)	N/A	0.004	0.005	0.001	1.336	0.996	0.387	1.126	0.027	0.011

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	12	13	13	17	13	13	17
N.S.	1	1.00	1.00	0.63	0.68	0.68	0.89	0.68	0.68	0.89
time (sec)	N/A	0.004	0.005	0.003	1.299	1.167	0.451	1.101	0.026	0.012

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	29	34	24	24	28
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.78
time (sec)	N/A	0.007	0.008	0.004	1.312	1.302	3.748	1.044	0.045	0.013

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	29	34	24	24	28
N.S.	1	1.00	0.78	0.69	0.67	0.81	0.94	0.67	0.67	0.78
time (sec)	N/A	0.007	0.008	0.005	1.324	1.018	2.646	1.059	0.038	0.013

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	27	34	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.75	0.94	0.67	0.67	0.94
time (sec)	N/A	0.007	0.007	0.003	1.320	1.101	1.062	1.050	0.038	0.013

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	27	2633	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.75	73.14	0.67	0.67	0.94
time (sec)	N/A	0.007	0.007	0.006	1.277	1.147	2.218	1.036	0.035	0.012

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	28	25	24	24	1765	24	24	34
N.S.	1	1.00	0.78	0.69	0.67	0.67	49.03	0.67	0.67	0.94
time (sec)	N/A	0.008	0.008	0.005	1.318	1.104	2.045	1.106	0.038	0.012

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	28	25	24	24	1741	24	24	34
N.S.	1	1.00	0.82	0.74	0.71	0.71	51.21	0.71	0.71	1.00
time (sec)	N/A	0.007	0.008	0.006	1.380	1.331	2.075	0.921	0.034	0.013

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	27	25	24	23	1826	24	24	27
N.S.	1	1.00	0.84	0.78	0.75	0.72	57.06	0.75	0.75	0.84
time (sec)	N/A	0.007	0.008	0.003	1.355	1.287	2.090	1.175	0.037	0.015

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	27	25	24	23	1957	24	24	27
N.S.	1	1.00	0.79	0.74	0.71	0.68	57.56	0.71	0.71	0.79
time (sec)	N/A	0.007	0.008	0.004	1.351	1.207	2.063	1.221	0.036	0.015

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	40	49	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.010	0.011	0.005	1.346	1.254	6.393	1.060	0.043	0.015

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	40	49	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.78	0.96	0.69	0.69	0.92
time (sec)	N/A	0.011	0.010	0.005	1.289	1.213	4.551	1.061	0.044	0.014

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	38	49	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.75	0.96	0.69	0.69	0.92
time (sec)	N/A	0.011	0.010	0.006	1.337	1.396	2.187	1.066	0.044	0.014

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	38	5012	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.75	98.27	0.69	0.69	0.92
time (sec)	N/A	0.011	0.010	0.005	1.349	1.178	3.223	1.161	0.048	0.014

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	36	35	35	6246	35	35	47
N.S.	1	1.00	0.76	0.71	0.69	0.69	122.47	0.69	0.69	0.92
time (sec)	N/A	0.010	0.010	0.005	1.327	0.776	3.191	0.859	0.044	0.014

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	39	36	35	35	6667	35	35	47
N.S.	1	1.00	0.80	0.73	0.71	0.71	136.06	0.71	0.71	0.96
time (sec)	N/A	0.011	0.010	0.006	1.293	1.300	3.171	0.941	0.044	0.015

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	39	36	35	35	4004	35	35	39
N.S.	1	1.00	0.80	0.73	0.71	0.71	81.71	0.71	0.71	0.80
time (sec)	N/A	0.011	0.011	0.004	1.323	0.976	3.261	1.072	0.044	0.018

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	39	36	35	35	3964	35	35	39
N.S.	1	1.00	0.80	0.73	0.71	0.71	80.90	0.71	0.71	0.80
time (sec)	N/A	0.011	0.011	0.005	1.361	1.313	3.238	1.167	0.043	0.018

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	38	122	130	147	241	138	151	150
N.S.	1	1.00	0.30	0.98	1.04	1.18	1.93	1.10	1.21	1.20
time (sec)	N/A	0.070	0.010	0.008	3.003	0.851	47.121	1.037	0.243	0.095

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	140	121	128	116	240	136	126	144
N.S.	1	1.00	1.14	0.98	1.04	0.94	1.95	1.11	1.02	1.17
time (sec)	N/A	0.056	0.058	0.006	3.051	1.419	25.855	1.208	0.068	0.086

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	29	107	114	128	228	118	130	136
N.S.	1	1.00	0.26	0.96	1.03	1.15	2.05	1.06	1.17	1.23
time (sec)	N/A	0.040	0.009	0.007	2.905	1.453	9.084	1.043	0.150	0.079

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	126	108	115	114	219	119	126	135
N.S.	1	1.00	1.16	0.99	1.06	1.05	2.01	1.09	1.16	1.24
time (sec)	N/A	0.039	0.026	0.007	2.926	1.276	6.098	1.147	0.074	0.077

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	27	96	103	313	212	118	120	126
N.S.	1	1.00	0.27	0.96	1.03	3.13	2.12	1.18	1.20	1.26
time (sec)	N/A	0.028	0.007	0.005	2.833	1.298	7.412	1.171	0.113	0.062

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	103	95	102	307	212	117	110	125
N.S.	1	1.00	1.03	0.95	1.02	3.07	2.12	1.17	1.10	1.25
time (sec)	N/A	0.028	0.024	0.006	2.963	1.487	11.348	1.175	0.206	0.071

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	25	104	111	113	218	125	124	134
N.S.	1	1.00	0.23	0.95	1.02	1.04	2.00	1.15	1.14	1.23
time (sec)	N/A	0.038	0.005	0.009	2.963	1.239	25.292	1.211	0.147	0.091

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	27	105	112	147	221	120	138	137
N.S.	1	1.00	0.24	0.95	1.01	1.32	1.99	1.08	1.24	1.23
time (sec)	N/A	0.039	0.005	0.008	2.994	1.425	34.964	1.139	0.072	0.089

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	27	123	133	162	0	135	150	159
N.S.	1	1.00	0.21	0.95	1.03	1.26	0.00	1.05	1.16	1.23
time (sec)	N/A	0.047	0.005	0.011	2.959	1.349	0.000	1.052	0.265	0.183

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	27	123	133	147	0	135	142	156
N.S.	1	1.00	0.22	0.98	1.06	1.18	0.00	1.08	1.14	1.25
time (sec)	N/A	0.047	0.005	0.013	2.932	1.396	0.000	1.069	0.152	0.184

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	27	112	120	394	787	136	142	145
N.S.	1	1.00	0.23	0.97	1.04	3.43	6.84	1.18	1.23	1.26
time (sec)	N/A	0.038	0.004	0.010	2.994	1.316	106.983	1.199	0.236	0.163

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	27	112	120	389	607	136	120	145
N.S.	1	1.00	0.23	0.96	1.03	3.32	5.19	1.16	1.03	1.24
time (sec)	N/A	0.038	0.005	0.011	3.026	1.253	71.720	1.129	0.063	0.168

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	27	120	127	396	774	132	144	144
N.S.	1	1.00	0.23	1.03	1.09	3.41	6.67	1.14	1.24	1.24
time (sec)	N/A	0.041	0.005	0.009	2.964	1.438	79.660	1.111	0.358	0.157

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	25	120	127	387	590	132	134	144
N.S.	1	1.00	0.22	1.06	1.12	3.42	5.22	1.17	1.19	1.27
time (sec)	N/A	0.040	0.005	0.008	3.009	1.193	115.985	1.035	0.223	0.156

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	25	121	132	156	0	145	151	152
N.S.	1	1.00	0.20	0.98	1.06	1.26	0.00	1.17	1.22	1.23
time (sec)	N/A	0.049	0.005	0.012	3.036	1.009	0.000	0.975	0.152	0.174

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	27	121	132	189	0	137	166	155
N.S.	1	1.00	0.21	0.95	1.03	1.48	0.00	1.07	1.30	1.21
time (sec)	N/A	0.050	0.005	0.014	2.943	0.650	0.000	1.006	0.165	0.181

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	27	124	143	506	0	146	165	161
N.S.	1	1.00	0.19	0.89	1.02	3.61	0.00	1.04	1.18	1.15
time (sec)	N/A	0.049	0.005	0.013	2.952	1.028	0.000	1.058	0.172	0.261

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	27	124	143	503	0	146	139	161
N.S.	1	1.00	0.19	0.89	1.02	3.59	0.00	1.04	0.99	1.15
time (sec)	N/A	0.054	0.004	0.012	2.879	1.115	0.000	1.125	0.066	0.252

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	27	132	153	508	0	149	172	164
N.S.	1	1.00	0.19	0.92	1.07	3.55	0.00	1.04	1.20	1.15
time (sec)	N/A	0.050	0.005	0.013	2.922	0.989	0.000	1.064	0.265	0.239

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	27	132	152	501	0	148	146	163
N.S.	1	1.00	0.19	0.92	1.06	3.50	0.00	1.03	1.02	1.14
time (sec)	N/A	0.052	0.005	0.013	2.959	1.207	0.000	1.110	0.235	0.236

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	27	136	151	510	0	143	167	157
N.S.	1	1.00	0.19	0.97	1.08	3.64	0.00	1.02	1.19	1.12
time (sec)	N/A	0.053	0.005	0.007	2.963	0.886	0.000	1.064	0.192	0.158

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	25	136	151	499	0	143	157	157
N.S.	1	1.00	0.18	0.97	1.08	3.56	0.00	1.02	1.12	1.12
time (sec)	N/A	0.050	0.004	0.007	2.997	1.365	0.000	0.999	0.241	0.152

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	25	139	154	211	0	155	174	168
N.S.	1	1.00	0.16	0.91	1.01	1.39	0.00	1.02	1.14	1.11
time (sec)	N/A	0.060	0.005	0.017	2.989	1.286	0.000	1.163	0.087	0.272

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	27	139	154	244	0	150	182	168
N.S.	1	1.00	0.18	0.91	1.01	1.61	0.00	0.99	1.20	1.11
time (sec)	N/A	0.061	0.005	0.017	2.991	0.668	0.000	1.080	0.173	0.270

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	58	62	61	82	243	64	46	58
N.S.	1	1.00	1.00	1.07	1.05	1.41	4.19	1.10	0.79	1.00
time (sec)	N/A	0.021	0.017	0.010	2.909	0.961	2.352	1.169	0.068	0.085

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	166	1535	187	1277	9996	1925	1274	0
N.S.	1	1.00	0.89	8.21	1.00	6.83	53.45	10.29	6.81	0.00
time (sec)	N/A	0.085	0.111	0.007	1.387	1.168	6.929	1.204	1.370	0.180

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	118	782	133	665	4257	992	683	0
N.S.	1	1.00	0.89	5.88	1.00	5.00	32.01	7.46	5.14	0.00
time (sec)	N/A	0.050	0.068	0.006	1.366	1.347	3.337	1.396	0.777	0.051

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	54	170	61	157	663	224	167	0
N.S.	1	1.00	0.89	2.79	1.00	2.57	10.87	3.67	2.74	0.00
time (sec)	N/A	0.018	0.031	0.000	1.299	1.035	0.917	1.067	0.443	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	38	87	43	85	299	117	93	0
N.S.	1	1.00	0.88	2.02	1.00	1.98	6.95	2.72	2.16	0.00
time (sec)	N/A	0.013	0.031	0.000	1.323	1.167	0.546	1.044	0.370	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	22	31	25	33	87	43	30	0
N.S.	1	1.00	0.88	1.24	1.00	1.32	3.48	1.72	1.20	0.00
time (sec)	N/A	0.006	0.014	0.000	1.354	0.792	0.308	1.021	0.304	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	67	126	101	143	1318	226	176	0
N.S.	1	1.00	0.81	1.52	1.22	1.72	15.88	2.72	2.12	0.00
time (sec)	N/A	0.031	0.054	0.007	1.364	1.286	2.326	1.212	0.534	0.026

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	57	73	68	96	597	140	192	0
N.S.	1	1.00	0.95	1.22	1.13	1.60	9.95	2.33	3.20	0.00
time (sec)	N/A	0.019	0.028	0.006	1.374	1.077	1.315	1.074	0.557	0.024

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	36	42	53	201	76	94	0
N.S.	1	1.00	0.85	0.92	1.08	1.36	5.15	1.95	2.41	0.00
time (sec)	N/A	0.012	0.017	0.002	1.311	1.144	0.702	1.051	0.378	0.020

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	17	19	18	20	20	18	18	0
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00	0.00
time (sec)	N/A	0.003	0.009	0.003	1.296	1.231	0.066	1.137	0.198	0.013

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	64	77	0	104	0	0	136	0
N.S.	1	1.00	0.58	0.70	0.00	0.95	0.00	0.00	1.24	0.00
time (sec)	N/A	0.036	0.026	0.006	0.000	0.993	0.000	0.000	0.523	0.026

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	39	44	0	64	0	0	80	0
N.S.	1	1.00	0.61	0.69	0.00	1.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.009	0.016	0.005	0.000	1.226	0.000	0.000	0.447	0.026

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	25	29	0	33	0	0	29	0
N.S.	1	1.00	0.89	1.04	0.00	1.18	0.00	0.00	1.04	0.00
time (sec)	N/A	0.003	0.006	0.003	0.000	0.973	0.000	0.000	0.349	0.026

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	20	22	32	0	0	19	0
N.S.	1	1.00	1.00	1.05	1.16	1.68	0.00	0.00	1.00	0.00
time (sec)	N/A	0.003	0.005	0.003	1.302	0.952	0.000	0.000	0.504	0.035

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	41	0	64	323	0	86	0
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.57	0.00	1.48	0.00
time (sec)	N/A	0.013	0.014	0.005	0.000	1.147	94.815	0.000	0.498	0.025

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	41	0	64	323	0	86	0
N.S.	1	1.00	0.69	0.71	0.00	1.10	5.57	0.00	1.48	0.00
time (sec)	N/A	0.011	0.002	0.000	0.000	1.021	94.488	0.000	0.002	0.041

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	33	22	36	22	-1	24
N.S.	1	1.00	0.69	0.60	0.94	0.63	1.03	0.63	-0.03	0.69
time (sec)	N/A	0.011	0.005	0.006	1.334	0.718	0.435	0.886	0.000	0.020

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	31	22	36	22	-1	24
N.S.	1	1.00	0.69	0.60	0.89	0.63	1.03	0.63	-0.03	0.69
time (sec)	N/A	0.010	0.004	0.002	1.370	0.970	0.339	1.031	0.000	0.020

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	28	22	36	22	-1	24
N.S.	1	1.00	0.69	0.60	0.80	0.63	1.03	0.63	-0.03	0.69
time (sec)	N/A	0.009	0.004	0.004	1.348	0.972	0.268	0.854	0.000	0.018

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	22	19	25	20	34	22	20	22
N.S.	1	1.00	0.67	0.58	0.76	0.61	1.03	0.67	0.61	0.67
time (sec)	N/A	0.008	0.003	0.010	1.316	0.947	0.224	1.047	0.542	0.017

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	24	17	0	16	29	17	14	20
N.S.	1	1.00	0.89	0.63	0.00	0.59	1.07	0.63	0.52	0.74
time (sec)	N/A	0.004	0.005	0.003	0.000	0.920	0.228	1.090	0.192	0.016

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	20	20	0	19	0	17	-1	19
N.S.	1	1.00	0.71	0.71	0.00	0.68	0.00	0.61	-0.04	0.68
time (sec)	N/A	0.005	0.005	0.022	0.000	1.272	0.000	0.925	0.000	0.019

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	20	21	0	20	0	20	-1	24
N.S.	1	1.00	0.62	0.66	0.00	0.62	0.00	0.62	-0.03	0.75
time (sec)	N/A	0.007	0.006	0.008	0.000	1.227	0.000	1.099	0.000	0.022

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	22	19	0	18	36	19	28	24
N.S.	1	1.00	0.85	0.73	0.00	0.69	1.38	0.73	1.08	0.92
time (sec)	N/A	0.004	0.004	0.005	0.000	1.063	0.507	0.971	0.144	0.021

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	24	21	33	24	36	22	-1	24
N.S.	1	1.00	0.65	0.57	0.89	0.65	0.97	0.59	-0.03	0.65
time (sec)	N/A	0.013	0.006	0.003	1.335	0.908	1.161	1.173	0.000	0.021

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	24	21	31	24	36	22	-1	24
N.S.	1	1.00	0.65	0.57	0.84	0.65	0.97	0.59	-0.03	0.65
time (sec)	N/A	0.012	0.006	0.005	1.339	1.085	0.909	0.929	0.000	0.020

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	24	21	28	24	36	22	-1	24
N.S.	1	1.00	0.65	0.57	0.76	0.65	0.97	0.59	-0.03	0.65
time (sec)	N/A	0.011	0.005	0.003	1.353	0.856	0.728	0.993	0.000	0.019

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	22	19	25	24	34	22	-1	22
N.S.	1	1.00	0.59	0.51	0.68	0.65	0.92	0.59	-0.03	0.59
time (sec)	N/A	0.010	0.005	0.003	1.297	0.826	0.558	0.878	0.000	0.019

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	25	18	22	24	31	22	-1	21
N.S.	1	1.00	0.68	0.49	0.59	0.65	0.84	0.59	-0.03	0.57
time (sec)	N/A	0.009	0.002	0.005	1.336	1.087	0.577	1.090	0.000	0.019

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	23	21	0	22	31	22	20	24
N.S.	1	1.00	0.66	0.60	0.00	0.63	0.89	0.63	0.57	0.69
time (sec)	N/A	0.008	0.002	0.003	0.000	1.010	0.572	1.082	0.267	0.020

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	21	20	0	18	32	17	14	23
N.S.	1	1.00	0.72	0.69	0.00	0.62	1.10	0.59	0.48	0.79
time (sec)	N/A	0.004	0.003	0.003	0.000	1.042	0.741	0.989	0.222	0.021

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	21	20	0	21	0	17	-1	23
N.S.	1	1.00	0.70	0.67	0.00	0.70	0.00	0.57	-0.03	0.77
time (sec)	N/A	0.005	0.005	0.003	0.000	0.879	0.000	0.958	0.000	0.022

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	33	28	36	28	-1	24
N.S.	1	1.00	0.59	0.51	0.80	0.68	0.88	0.68	-0.02	0.59
time (sec)	N/A	0.016	0.007	0.003	1.244	1.218	2.543	0.966	0.000	0.025

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	31	28	36	28	-1	24
N.S.	1	1.00	0.59	0.51	0.76	0.68	0.88	0.68	-0.02	0.59
time (sec)	N/A	0.015	0.007	0.003	1.305	0.594	2.097	0.801	0.000	0.022

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	28	28	36	28	-1	24
N.S.	1	1.00	0.59	0.51	0.68	0.68	0.88	0.68	-0.02	0.59
time (sec)	N/A	0.013	0.007	0.003	1.351	0.981	1.742	1.027	0.000	0.022

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	22	19	25	28	34	28	-1	22
N.S.	1	1.00	0.54	0.46	0.61	0.68	0.83	0.68	-0.02	0.54
time (sec)	N/A	0.013	0.006	0.005	1.341	0.987	1.410	0.948	0.000	0.020

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	25	18	22	28	31	28	-1	21
N.S.	1	1.00	0.61	0.44	0.54	0.68	0.76	0.68	-0.02	0.51
time (sec)	N/A	0.012	0.003	0.002	1.402	1.212	1.429	1.074	0.000	0.023

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	21	24	28	31	28	25	24
N.S.	1	1.00	0.56	0.51	0.59	0.68	0.76	0.68	0.61	0.59
time (sec)	N/A	0.011	0.003	0.003	1.283	0.791	1.529	1.103	0.284	0.023

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	27	21	0	28	34	28	25	24
N.S.	1	1.00	0.66	0.51	0.00	0.68	0.83	0.68	0.61	0.59
time (sec)	N/A	0.010	0.003	0.003	0.000	1.076	1.578	0.994	0.265	0.022

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	25	21	0	26	36	28	20	24
N.S.	1	1.00	0.64	0.54	0.00	0.67	0.92	0.72	0.51	0.62
time (sec)	N/A	0.008	0.002	0.004	0.000	0.774	1.623	1.139	0.260	0.023

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	33	25	36	26	-1	27
N.S.	1	1.00	0.69	0.60	0.94	0.71	1.03	0.74	-0.03	0.77
time (sec)	N/A	0.009	0.005	0.003	1.403	1.263	0.606	1.305	0.000	0.021

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	26	23	36	24	23	25
N.S.	1	1.00	0.69	0.60	0.74	0.66	1.03	0.69	0.66	0.71
time (sec)	N/A	0.008	0.004	0.003	1.301	1.017	0.522	1.158	0.251	0.020

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	23	20	22	19	34	22	19	23
N.S.	1	1.00	0.72	0.62	0.69	0.59	1.06	0.69	0.59	0.72
time (sec)	N/A	0.004	0.002	0.003	1.301	1.180	0.462	1.105	0.221	0.020

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	19	18	20	22	0	35	17	26
N.S.	1	1.00	0.66	0.62	0.69	0.76	0.00	1.21	0.59	0.90
time (sec)	N/A	0.005	0.002	0.004	1.325	1.017	0.000	1.321	0.513	0.021

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	23	18	17	23	0	47	22	30
N.S.	1	1.00	0.85	0.67	0.63	0.85	0.00	1.74	0.81	1.11
time (sec)	N/A	0.006	0.006	0.004	1.326	1.006	0.000	0.984	1.222	0.022

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	23	19	19	21	31	0	25	27
N.S.	1	1.00	0.88	0.73	0.73	0.81	1.19	0.00	0.96	1.04
time (sec)	N/A	0.004	0.006	0.003	1.251	1.028	0.544	0.000	0.157	0.022

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	22	21	19	23	36	0	26	27
N.S.	1	1.00	0.63	0.60	0.54	0.66	1.03	0.00	0.74	0.77
time (sec)	N/A	0.007	0.006	0.005	1.279	0.899	0.636	0.000	0.149	0.023

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	24	21	19	23	37	0	26	27
N.S.	1	1.00	0.69	0.60	0.54	0.66	1.06	0.00	0.74	0.77
time (sec)	N/A	0.007	0.005	0.003	1.296	1.099	0.810	0.000	0.149	0.022

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	23	20	32	19	34	25	-1	23
N.S.	1	1.00	0.61	0.53	0.84	0.50	0.89	0.66	-0.03	0.61
time (sec)	N/A	0.006	0.004	0.003	1.307	1.045	0.635	1.014	0.000	0.024

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	21	20	23	22	0	40	30	23
N.S.	1	1.00	0.60	0.57	0.66	0.63	0.00	1.14	0.86	0.66
time (sec)	N/A	0.005	0.004	0.004	1.346	1.112	0.000	0.986	0.320	0.024

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	22	21	21	23	0	47	28	24
N.S.	1	1.00	0.67	0.64	0.64	0.70	0.00	1.42	0.85	0.73
time (sec)	N/A	0.007	0.003	0.003	1.312	1.070	0.000	1.224	0.250	0.025

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	22	17	23	21	34	0	25	20
N.S.	1	1.00	0.76	0.59	0.79	0.72	1.17	0.00	0.86	0.69
time (sec)	N/A	0.004	0.003	0.003	1.346	0.705	0.538	0.000	0.149	0.022

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	25	18	19	23	32	0	26	21
N.S.	1	1.00	0.61	0.44	0.46	0.56	0.78	0.00	0.63	0.51
time (sec)	N/A	0.008	0.008	0.003	1.318	1.761	0.632	0.000	0.158	0.022

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	27	21	19	23	32	0	26	24
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.78	0.00	0.63	0.59
time (sec)	N/A	0.007	0.008	0.004	1.338	1.071	0.769	0.000	0.151	0.023

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	22	21	19	23	36	0	26	24
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.88	0.00	0.63	0.59
time (sec)	N/A	0.008	0.007	0.004	1.278	0.958	0.934	0.000	0.149	0.024

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	19	23	37	0	26	24
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.90	0.00	0.63	0.59
time (sec)	N/A	0.007	0.006	0.004	1.348	1.276	1.164	0.000	0.151	0.024

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	22	21	24	23	0	47	-1	24
N.S.	1	1.00	0.67	0.64	0.73	0.70	0.00	1.42	-0.03	0.73
time (sec)	N/A	0.008	0.005	0.004	1.429	0.789	0.000	1.015	0.000	0.029

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	24	19	26	21	36	0	25	22
N.S.	1	1.00	0.83	0.66	0.90	0.72	1.24	0.00	0.86	0.76
time (sec)	N/A	0.005	0.006	0.003	1.342	1.050	0.931	0.000	0.153	0.025

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	23	23	37	0	26	24
N.S.	1	1.00	0.59	0.51	0.56	0.56	0.90	0.00	0.63	0.59
time (sec)	N/A	0.008	0.004	0.003	1.387	1.093	0.918	0.000	0.151	0.024

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	27	19	23	23	36	0	26	22
N.S.	1	1.00	0.66	0.46	0.56	0.56	0.88	0.00	0.63	0.54
time (sec)	N/A	0.007	0.003	0.003	1.374	0.999	0.916	0.000	0.160	0.022

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	27	18	19	23	32	0	26	21
N.S.	1	1.00	0.66	0.44	0.46	0.56	0.78	0.00	0.63	0.51
time (sec)	N/A	0.008	0.008	0.003	1.334	0.743	1.121	0.000	0.158	0.024

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	27	21	19	23	32	0	26	24
N.S.	1	1.00	0.66	0.51	0.46	0.56	0.78	0.00	0.63	0.59
time (sec)	N/A	0.008	0.007	0.004	1.316	1.030	1.341	0.000	0.162	0.026

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	22	21	19	23	36	0	26	24
N.S.	1	1.00	0.54	0.51	0.46	0.56	0.88	0.00	0.63	0.59
time (sec)	N/A	0.009	0.008	0.005	1.370	1.098	1.644	0.000	0.159	0.025

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	24	21	19	23	37	0	26	24
N.S.	1	1.00	0.59	0.51	0.46	0.56	0.90	0.00	0.63	0.59
time (sec)	N/A	0.008	0.007	0.002	1.342	0.827	1.968	0.000	0.162	0.027

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	54	33	60	35	-1	35
N.S.	1	1.00	0.61	0.56	0.95	0.58	1.05	0.61	-0.02	0.61
time (sec)	N/A	0.015	0.006	0.004	1.395	0.860	0.587	1.078	0.000	0.027

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	52	33	61	35	-1	35
N.S.	1	1.00	0.61	0.56	0.91	0.58	1.07	0.61	-0.02	0.61
time (sec)	N/A	0.015	0.006	0.005	1.348	0.752	0.463	1.042	0.000	0.024

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	49	33	60	35	-1	35
N.S.	1	1.00	0.61	0.56	0.86	0.58	1.05	0.61	-0.02	0.61
time (sec)	N/A	0.013	0.006	0.004	1.311	1.105	0.368	1.110	0.000	0.024

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	55	55	33	30	44	31	60	35	-1	33
N.S.	1	1.00	0.60	0.55	0.80	0.56	1.09	0.64	-0.02	0.60
time (sec)	N/A	0.014	0.007	0.004	1.352	1.117	0.297	0.948	0.000	0.025

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	28	0	27	51	29	-1	31
N.S.	1	1.00	0.96	1.08	0.00	1.04	1.96	1.12	-0.04	1.19
time (sec)	N/A	0.004	0.005	0.003	0.000	1.078	0.297	0.972	0.000	0.025

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	33	33	0	32	0	32	-1	34
N.S.	1	1.00	0.67	0.67	0.00	0.65	0.00	0.65	-0.02	0.69
time (sec)	N/A	0.009	0.009	0.007	0.000	0.762	0.000	1.081	0.000	0.031

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	31	32	0	31	0	31	-1	37
N.S.	1	1.00	0.63	0.65	0.00	0.63	0.00	0.63	-0.02	0.76
time (sec)	N/A	0.011	0.011	0.010	0.000	1.212	0.000	1.015	0.000	0.034

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	36	34	0	33	0	35	-1	38
N.S.	1	1.00	0.67	0.63	0.00	0.61	0.00	0.65	-0.02	0.70
time (sec)	N/A	0.012	0.010	0.010	0.000	1.356	0.000	0.987	0.000	0.041

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	35	32	54	36	60	35	-1	35
N.S.	1	1.00	0.58	0.53	0.90	0.60	1.00	0.58	-0.02	0.58
time (sec)	N/A	0.019	0.008	0.006	1.345	0.843	1.498	1.086	0.000	0.028

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	35	32	52	36	61	35	-1	35
N.S.	1	1.00	0.58	0.53	0.87	0.60	1.02	0.58	-0.02	0.58
time (sec)	N/A	0.017	0.008	0.005	1.329	1.140	1.225	1.085	0.000	0.029

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	35	32	49	36	60	35	-1	35
N.S.	1	1.00	0.58	0.53	0.82	0.60	1.00	0.58	-0.02	0.58
time (sec)	N/A	0.016	0.007	0.005	1.312	1.208	0.971	1.185	0.000	0.025

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	33	30	44	36	60	35	-1	33
N.S.	1	1.00	0.55	0.50	0.73	0.60	1.00	0.58	-0.02	0.55
time (sec)	N/A	0.015	0.007	0.006	1.359	0.934	0.770	1.129	0.000	0.026

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	36	29	40	36	54	35	-1	32
N.S.	1	1.00	0.60	0.48	0.67	0.60	0.90	0.58	-0.02	0.53
time (sec)	N/A	0.015	0.004	0.003	1.296	1.069	0.793	0.951	0.000	0.028

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	34	32	0	34	54	35	-1	35
N.S.	1	1.00	0.59	0.55	0.00	0.59	0.93	0.60	-0.02	0.60
time (sec)	N/A	0.013	0.004	0.004	0.000	1.064	0.805	1.026	0.000	0.029

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	31	0	30	51	29	-1	34
N.S.	1	1.00	0.96	1.15	0.00	1.11	1.89	1.07	-0.04	1.26
time (sec)	N/A	0.004	0.005	0.003	0.000	1.061	0.939	1.224	0.000	0.027

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	34	33	0	35	0	32	-1	37
N.S.	1	1.00	0.65	0.63	0.00	0.67	0.00	0.62	-0.02	0.71
time (sec)	N/A	0.010	0.009	0.005	0.000	1.133	0.000	1.198	0.000	0.033

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	35	32	49	42	60	44	-1	35
N.S.	1	1.00	0.53	0.48	0.74	0.64	0.91	0.67	-0.02	0.53
time (sec)	N/A	0.019	0.008	0.004	1.344	0.943	2.186	1.221	0.000	0.027

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	33	30	44	42	60	44	-1	33
N.S.	1	1.00	0.50	0.45	0.67	0.64	0.91	0.67	-0.02	0.50
time (sec)	N/A	0.017	0.008	0.005	1.317	1.016	1.802	1.048	0.000	0.026

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	36	29	40	42	54	44	-1	32
N.S.	1	1.00	0.55	0.44	0.61	0.64	0.82	0.67	-0.02	0.48
time (sec)	N/A	0.016	0.004	0.004	1.281	0.820	1.823	0.956	0.000	0.027

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	34	32	40	42	54	44	-1	35
N.S.	1	1.00	0.52	0.48	0.61	0.64	0.82	0.67	-0.02	0.53
time (sec)	N/A	0.015	0.005	0.005	1.375	1.110	1.841	0.991	0.000	0.028

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	38	32	0	42	54	44	-1	35
N.S.	1	1.00	0.58	0.48	0.00	0.64	0.82	0.67	-0.02	0.53
time (sec)	N/A	0.014	0.004	0.004	0.000	1.191	1.948	0.942	0.000	0.029

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	36	32	0	40	60	44	-1	35
N.S.	1	1.00	0.56	0.50	0.00	0.62	0.94	0.69	-0.02	0.55
time (sec)	N/A	0.015	0.003	0.005	0.000	1.183	2.007	1.053	0.000	0.031

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	26	31	0	36	56	41	-1	34
N.S.	1	1.00	0.90	1.07	0.00	1.24	1.93	1.41	-0.03	1.17
time (sec)	N/A	0.004	0.006	0.002	0.000	0.974	2.034	1.116	0.000	0.030

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	35	33	0	41	0	41	-1	37
N.S.	1	1.00	0.60	0.57	0.00	0.71	0.00	0.71	-0.02	0.64
time (sec)	N/A	0.011	0.010	0.006	0.000	1.088	0.000	1.082	0.000	0.034

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	54	36	60	41	-1	38
N.S.	1	1.00	0.61	0.56	0.95	0.63	1.05	0.72	-0.02	0.67
time (sec)	N/A	0.013	0.006	0.004	1.345	1.240	0.788	0.943	0.000	0.028

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	47	34	61	38	-1	36
N.S.	1	1.00	0.61	0.56	0.82	0.60	1.07	0.67	-0.02	0.63
time (sec)	N/A	0.012	0.005	0.002	1.322	1.068	0.647	1.252	0.000	0.028

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	31	42	30	56	36	-1	34
N.S.	1	1.00	1.00	1.29	1.75	1.25	2.33	1.50	-0.04	1.42
time (sec)	N/A	0.003	0.002	0.003	1.370	1.157	0.540	1.147	0.000	0.025

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	32	31	35	35	0	50	-1	40
N.S.	1	1.00	0.62	0.60	0.67	0.67	0.00	0.96	-0.02	0.77
time (sec)	N/A	0.010	0.003	0.005	1.348	0.991	0.000	1.118	0.000	0.028

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	34	29	35	34	0	65	-1	43
N.S.	1	1.00	0.72	0.62	0.74	0.72	0.00	1.38	-0.02	0.91
time (sec)	N/A	0.011	0.009	0.003	1.357	1.032	0.000	0.943	0.000	0.031

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	35	34	31	36	0	0	-1	44
N.S.	1	1.00	0.71	0.69	0.63	0.73	0.00	0.00	-0.02	0.90
time (sec)	N/A	0.011	0.008	0.007	1.289	1.091	0.000	0.000	0.000	0.035

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	33	30	33	32	53	0	33	38
N.S.	1	1.00	1.27	1.15	1.27	1.23	2.04	0.00	1.27	1.46
time (sec)	N/A	0.004	0.010	0.005	1.330	1.067	0.662	0.000	0.183	0.030

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	35	32	33	34	61	0	42	38
N.S.	1	1.00	0.61	0.56	0.58	0.60	1.07	0.00	0.74	0.67
time (sec)	N/A	0.013	0.007	0.006	1.326	1.255	0.819	0.000	0.187	0.031

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	26	31	52	30	56	39	-1	34
N.S.	1	1.00	0.96	1.15	1.93	1.11	2.07	1.44	-0.04	1.26
time (sec)	N/A	0.004	0.004	0.003	1.356	1.064	0.804	1.054	0.000	0.028

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	34	33	45	35	0	55	-1	39
N.S.	1	1.00	0.56	0.54	0.74	0.57	0.00	0.90	-0.02	0.64
time (sec)	N/A	0.011	0.007	0.005	1.331	1.178	0.000	1.096	0.000	0.032

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	33	32	42	34	0	69	-1	35
N.S.	1	1.00	0.59	0.57	0.75	0.61	0.00	1.23	-0.02	0.62
time (sec)	N/A	0.012	0.005	0.004	1.379	1.198	0.000	1.083	0.000	0.034

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	34	32	35	36	0	0	-1	38
N.S.	1	1.00	0.59	0.55	0.60	0.62	0.00	0.00	-0.02	0.66
time (sec)	N/A	0.012	0.004	0.004	1.327	1.040	0.000	0.000	0.000	0.036

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	36	27	37	32	53	0	33	32
N.S.	1	1.00	1.24	0.93	1.28	1.10	1.83	0.00	1.14	1.10
time (sec)	N/A	0.004	0.012	0.005	1.361	0.804	0.667	0.000	0.191	0.028

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	38	32	33	34	56	0	42	35
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.85	0.00	0.64	0.53
time (sec)	N/A	0.013	0.009	0.006	1.310	0.989	0.812	0.000	0.193	0.030

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	33	32	33	34	56	0	42	35
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.85	0.00	0.64	0.53
time (sec)	N/A	0.013	0.012	0.005	1.358	1.284	0.982	0.000	0.197	0.032

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	35	32	33	34	61	0	42	35
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.92	0.00	0.64	0.53
time (sec)	N/A	0.014	0.009	0.005	1.293	1.146	1.183	0.000	0.175	0.032

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	33	32	45	34	0	65	-1	35
N.S.	1	1.00	0.59	0.57	0.80	0.61	0.00	1.16	-0.02	0.62
time (sec)	N/A	0.013	0.008	0.005	1.451	0.968	0.000	1.066	0.000	0.037

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	36	34	38	36	0	0	-1	40
N.S.	1	1.00	0.62	0.59	0.66	0.62	0.00	0.00	-0.02	0.69
time (sec)	N/A	0.012	0.009	0.006	1.398	0.842	0.000	0.000	0.000	0.037

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	35	30	44	32	58	0	33	33
N.S.	1	1.00	1.21	1.03	1.52	1.10	2.00	0.00	1.14	1.14
time (sec)	N/A	0.004	0.007	0.004	1.377	0.859	0.957	0.000	0.180	0.030

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	38	30	37	34	61	0	42	33
N.S.	1	1.00	0.58	0.45	0.56	0.52	0.92	0.00	0.64	0.50
time (sec)	N/A	0.013	0.004	0.005	1.357	0.830	0.963	0.000	0.175	0.031

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	38	29	37	34	56	0	42	32
N.S.	1	1.00	0.58	0.44	0.56	0.52	0.85	0.00	0.64	0.48
time (sec)	N/A	0.012	0.012	0.006	1.309	1.489	1.154	0.000	0.176	0.032

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	38	32	33	34	56	0	42	35
N.S.	1	1.00	0.58	0.48	0.50	0.52	0.85	0.00	0.64	0.53
time (sec)	N/A	0.013	0.009	0.004	1.345	0.773	1.399	0.000	0.180	0.032

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	33	32	33	34	56	0	42	35
N.S.	1	1.00	0.50	0.48	0.50	0.52	0.85	0.00	0.64	0.53
time (sec)	N/A	0.013	0.014	0.005	1.369	0.739	1.704	0.000	0.182	0.034

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	35	32	33	34	61	0	42	35
N.S.	1	1.00	0.53	0.48	0.50	0.52	0.92	0.00	0.64	0.53
time (sec)	N/A	0.013	0.009	0.006	1.352	1.201	2.029	0.000	0.180	0.034

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	63	63	128	62	0	81	-1	64
N.S.	1	1.00	0.62	0.62	1.25	0.61	0.00	0.79	-0.01	0.63
time (sec)	N/A	0.036	0.018	0.006	1.570	0.968	0.000	1.106	0.000	0.055

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	52	52	110	51	0	69	-1	54
N.S.	1	1.00	0.65	0.65	1.38	0.64	0.00	0.86	-0.01	0.68
time (sec)	N/A	0.025	0.015	0.008	1.561	1.073	0.000	0.944	0.000	0.048

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	40	40	91	39	0	54	-1	41
N.S.	1	1.00	0.69	0.69	1.57	0.67	0.00	0.93	-0.02	0.71
time (sec)	N/A	0.018	0.011	0.007	1.494	1.189	0.000	1.015	0.000	0.036

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	28	29	74	27	0	37	-1	29
N.S.	1	1.00	0.74	0.76	1.95	0.71	0.00	0.97	-0.03	0.76
time (sec)	N/A	0.012	0.007	0.005	1.451	1.041	0.000	0.962	0.000	0.026

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	21	0	20	0	28	-1	22
N.S.	1	1.00	0.95	0.95	0.00	0.91	0.00	1.27	-0.05	1.00
time (sec)	N/A	0.003	0.004	0.004	0.000	1.171	0.000	0.949	0.000	0.019

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	26	26	24	64	0	0	-1	37
N.S.	1	1.00	0.62	0.62	0.57	1.52	0.00	0.00	-0.02	0.88
time (sec)	N/A	0.007	0.007	0.008	1.376	1.420	0.000	0.000	0.000	0.037

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	32	33	37	31	0	0	-1	44
N.S.	1	1.00	0.52	0.54	0.61	0.51	0.00	0.00	-0.02	0.72
time (sec)	N/A	0.017	0.011	0.011	1.384	0.768	0.000	0.000	0.000	0.039

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	53	51	52	44	0	0	-1	58
N.S.	1	1.00	0.63	0.61	0.62	0.52	0.00	0.00	-0.01	0.69
time (sec)	N/A	0.023	0.015	0.010	1.361	1.154	0.000	0.000	0.000	0.050

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	64	63	124	67	0	81	-1	67
N.S.	1	1.00	0.60	0.59	1.16	0.63	0.00	0.76	-0.01	0.63
time (sec)	N/A	0.032	0.013	0.006	1.619	1.037	0.000	0.998	0.000	0.046

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	53	52	109	55	0	69	-1	57
N.S.	1	1.00	0.63	0.62	1.30	0.65	0.00	0.82	-0.01	0.68
time (sec)	N/A	0.025	0.011	0.005	1.541	1.305	0.000	1.155	0.000	0.044

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	42	40	93	42	0	54	-1	44
N.S.	1	1.00	0.69	0.66	1.52	0.69	0.00	0.89	-0.02	0.72
time (sec)	N/A	0.019	0.005	0.005	1.482	1.421	0.000	1.119	0.000	0.039

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	30	29	75	29	0	37	-1	33
N.S.	1	1.00	0.75	0.72	1.88	0.72	0.00	0.92	-0.02	0.82
time (sec)	N/A	0.012	0.004	0.003	1.479	0.835	0.000	0.998	0.000	0.029

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	21	13	21	0	28	-1	22
N.S.	1	1.00	0.96	0.91	0.57	0.91	0.00	1.22	-0.04	0.96
time (sec)	N/A	0.004	0.003	0.004	1.357	1.045	0.000	1.137	0.000	0.023

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	27	26	24	66	0	0	-1	37
N.S.	1	1.00	0.61	0.59	0.55	1.50	0.00	0.00	-0.02	0.84
time (sec)	N/A	0.007	0.008	0.005	1.380	0.793	0.000	0.000	0.000	0.035

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	34	33	37	33	0	0	-1	44
N.S.	1	1.00	0.53	0.52	0.58	0.52	0.00	0.00	-0.02	0.69
time (sec)	N/A	0.016	0.011	0.006	1.356	1.105	0.000	0.000	0.000	0.041

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	53	51	52	47	0	0	-1	58
N.S.	1	1.00	0.60	0.58	0.59	0.53	0.00	0.00	-0.01	0.66
time (sec)	N/A	0.024	0.015	0.005	1.446	0.913	0.000	0.000	0.000	0.050

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	65	62	66	59	0	0	-1	69
N.S.	1	1.00	0.58	0.55	0.59	0.53	0.00	0.00	-0.01	0.62
time (sec)	N/A	0.031	0.026	0.012	1.449	1.009	0.000	0.000	0.000	0.069

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	76	74	146	91	0	116	-1	79
N.S.	1	1.00	0.54	0.52	1.03	0.64	0.00	0.82	-0.01	0.56
time (sec)	N/A	0.045	0.022	0.006	1.584	1.343	0.000	1.184	0.000	0.062

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	65	63	130	77	0	99	-1	67
N.S.	1	1.00	0.56	0.54	1.11	0.66	0.00	0.85	-0.01	0.57
time (sec)	N/A	0.051	0.006	0.006	1.602	1.190	0.000	1.011	0.000	0.052

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	54	52	114	63	0	84	-1	57
N.S.	1	1.00	0.59	0.57	1.24	0.68	0.00	0.91	-0.01	0.62
time (sec)	N/A	0.030	0.005	0.004	1.562	0.868	0.000	0.960	0.000	0.048

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	42	40	97	48	0	66	-1	44
N.S.	1	1.00	0.63	0.60	1.45	0.72	0.00	0.99	-0.01	0.66
time (sec)	N/A	0.020	0.005	0.006	1.496	1.227	0.000	1.134	0.000	0.039

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	30	29	77	33	0	46	-1	33
N.S.	1	1.00	0.68	0.66	1.75	0.75	0.00	1.05	-0.02	0.75
time (sec)	N/A	0.012	0.004	0.004	1.512	0.676	0.000	1.043	0.000	0.032

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	22	21	13	23	0	34	-1	22
N.S.	1	1.00	0.88	0.84	0.52	0.92	0.00	1.36	-0.04	0.88
time (sec)	N/A	0.004	0.004	0.003	1.359	1.360	0.000	1.102	0.000	0.024

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	28	27	24	70	0	0	-1	37
N.S.	1	1.00	0.58	0.56	0.50	1.46	0.00	0.00	-0.02	0.77
time (sec)	N/A	0.008	0.009	0.006	1.352	1.536	0.000	0.000	0.000	0.037

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	34	33	37	37	0	0	-1	44
N.S.	1	1.00	0.49	0.47	0.53	0.53	0.00	0.00	-0.01	0.63
time (sec)	N/A	0.017	0.012	0.006	1.394	0.936	0.000	0.000	0.000	0.042

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	51	50	142	54	0	81	-1	60
N.S.	1	1.00	0.61	0.60	1.71	0.65	0.00	0.98	-0.01	0.72
time (sec)	N/A	0.024	0.011	0.006	1.519	1.395	0.000	1.153	0.000	0.046

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	39	38	100	42	0	67	-1	47
N.S.	1	1.00	0.64	0.62	1.64	0.69	0.00	1.10	-0.02	0.77
time (sec)	N/A	0.017	0.012	0.004	1.494	1.351	0.000	1.087	0.000	0.040

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	27	27	64	30	0	51	-1	36
N.S.	1	1.00	0.69	0.69	1.64	0.77	0.00	1.31	-0.03	0.92
time (sec)	N/A	0.012	0.007	0.003	1.472	0.947	0.000	1.187	0.000	0.030

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	19	46	23	0	36	-1	25
N.S.	1	1.00	1.00	0.95	2.30	1.15	0.00	1.80	-0.05	1.25
time (sec)	N/A	0.004	0.002	0.005	1.459	0.630	0.000	0.901	0.000	0.019

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	25	24	35	70	0	59	-1	43
N.S.	1	1.00	0.66	0.63	0.92	1.84	0.00	1.55	-0.03	1.13
time (sec)	N/A	0.007	0.004	0.005	1.461	0.945	0.000	0.966	0.000	0.033

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	36	30	37	34	0	91	-1	53
N.S.	1	1.00	0.67	0.56	0.69	0.63	0.00	1.69	-0.02	0.98
time (sec)	N/A	0.016	0.011	0.006	1.370	0.772	0.000	1.194	0.000	0.039

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	52	51	55	47	0	0	-1	67
N.S.	1	1.00	0.68	0.66	0.71	0.61	0.00	0.00	-0.01	0.87
time (sec)	N/A	0.029	0.012	0.005	1.375	0.949	0.000	0.000	0.000	0.052

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	63	62	69	58	0	0	-1	78
N.S.	1	1.00	0.63	0.62	0.69	0.58	0.00	0.00	-0.01	0.78
time (sec)	N/A	0.026	0.012	0.005	1.396	0.840	0.000	0.000	0.000	0.058

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	53	52	162	54	0	86	-1	59
N.S.	1	1.00	0.56	0.55	1.71	0.57	0.00	0.91	-0.01	0.62
time (sec)	N/A	0.027	0.011	0.006	1.790	1.146	0.000	1.181	0.000	0.048

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	41	40	140	42	0	71	-1	46
N.S.	1	1.00	0.59	0.57	2.00	0.60	0.00	1.01	-0.01	0.66
time (sec)	N/A	0.020	0.008	0.005	1.660	0.830	0.000	0.958	0.000	0.042

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	29	29	116	30	0	55	-1	33
N.S.	1	1.00	0.64	0.64	2.58	0.67	0.00	1.22	-0.02	0.73
time (sec)	N/A	0.013	0.007	0.004	1.602	1.111	0.000	1.094	0.000	0.035

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	21	74	23	0	36	-1	22
N.S.	1	1.00	0.96	0.91	3.22	1.00	0.00	1.57	-0.04	0.96
time (sec)	N/A	0.004	0.004	0.002	1.604	1.088	0.000	0.970	0.000	0.025

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	27	26	35	70	0	63	-1	37
N.S.	1	1.00	0.61	0.59	0.80	1.59	0.00	1.43	-0.02	0.84
time (sec)	N/A	0.008	0.007	0.005	1.430	1.291	0.000	1.047	0.000	0.036

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	35	33	51	34	0	91	-1	44
N.S.	1	1.00	0.56	0.52	0.81	0.54	0.00	1.44	-0.02	0.70
time (sec)	N/A	0.017	0.006	0.006	1.481	0.946	0.000	1.080	0.000	0.039

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	51	49	65	47	0	0	-1	58
N.S.	1	1.00	0.57	0.55	0.73	0.53	0.00	0.00	-0.01	0.65
time (sec)	N/A	0.022	0.007	0.006	1.538	1.160	0.000	0.000	0.000	0.053

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	66	59	69	58	0	0	-1	66
N.S.	1	1.00	0.57	0.51	0.60	0.50	0.00	0.00	-0.01	0.57
time (sec)	N/A	0.028	0.015	0.006	1.359	1.048	0.000	0.000	0.000	0.055

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	81	88	135	83	0	96	-1	85
N.S.	1	1.00	0.76	0.83	1.27	0.78	0.00	0.91	-0.01	0.80
time (sec)	N/A	0.039	0.028	0.012	1.516	1.144	0.000	0.997	0.000	0.080

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	70	76	118	72	0	80	-1	74
N.S.	1	1.00	0.82	0.89	1.39	0.85	0.00	0.94	-0.01	0.87
time (sec)	N/A	0.032	0.021	0.012	1.545	0.846	0.000	1.057	0.000	0.064

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	53	62	96	57	0	58	-1	57
N.S.	1	1.00	0.82	0.95	1.48	0.88	0.00	0.89	-0.02	0.88
time (sec)	N/A	0.022	0.020	0.010	1.515	1.016	0.000	0.920	0.000	0.056

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	36	41	79	38	0	46	-1	39
N.S.	1	1.00	0.77	0.87	1.68	0.81	0.00	0.98	-0.02	0.83
time (sec)	N/A	0.016	0.012	0.008	1.467	0.669	0.000	1.061	0.000	0.038

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	23	23	16	23	39	29	22	24
N.S.	1	1.00	0.96	0.96	0.67	0.96	1.62	1.21	0.92	1.00
time (sec)	N/A	0.004	0.007	0.004	1.361	1.252	0.816	1.016	0.165	0.022

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	45	52	38	42	0	0	-1	48
N.S.	1	1.00	0.69	0.80	0.58	0.65	0.00	0.00	-0.02	0.74
time (sec)	N/A	0.018	0.017	0.011	1.318	1.054	0.000	0.000	0.000	0.048

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	57	74	58	60	0	0	-1	59
N.S.	1	1.00	0.66	0.85	0.67	0.69	0.00	0.00	-0.01	0.68
time (sec)	N/A	0.027	0.030	0.013	1.397	1.044	0.000	0.000	0.000	0.068

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	82	95	79	77	0	0	-1	77
N.S.	1	1.00	0.73	0.85	0.71	0.69	0.00	0.00	-0.01	0.69
time (sec)	N/A	0.036	0.024	0.016	1.420	1.162	0.000	0.000	0.000	0.087

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	82	88	132	91	0	96	-1	85
N.S.	1	1.00	0.74	0.79	1.19	0.82	0.00	0.86	-0.01	0.77
time (sec)	N/A	0.037	0.020	0.007	1.625	1.095	0.000	1.174	0.000	0.070

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	71	76	115	79	0	80	-1	74
N.S.	1	1.00	0.80	0.85	1.29	0.89	0.00	0.90	-0.01	0.83
time (sec)	N/A	0.029	0.017	0.005	1.553	1.100	0.000	0.947	0.000	0.060

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	55	62	98	63	0	58	-1	57
N.S.	1	1.00	0.81	0.91	1.44	0.93	0.00	0.85	-0.01	0.84
time (sec)	N/A	0.020	0.007	0.006	1.424	1.263	0.000	0.956	0.000	0.053

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	38	41	80	43	0	46	-1	39
N.S.	1	1.00	0.78	0.84	1.63	0.88	0.00	0.94	-0.02	0.80
time (sec)	N/A	0.015	0.009	0.004	1.480	1.413	0.000	0.961	0.000	0.041

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	23	16	24	44	29	24	24
N.S.	1	1.00	0.96	0.92	0.64	0.96	1.76	1.16	0.96	0.96
time (sec)	N/A	0.004	0.006	0.003	1.335	1.094	2.226	1.143	0.148	0.027

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	46	52	38	47	0	0	-1	48
N.S.	1	1.00	0.68	0.76	0.56	0.69	0.00	0.00	-0.01	0.71
time (sec)	N/A	0.018	0.017	0.006	1.397	1.119	0.000	0.000	0.000	0.047

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	59	74	58	65	0	0	-1	59
N.S.	1	1.00	0.65	0.81	0.64	0.71	0.00	0.00	-0.01	0.65
time (sec)	N/A	0.024	0.022	0.007	1.435	1.162	0.000	0.000	0.000	0.060

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	82	95	79	82	0	0	-1	77
N.S.	1	1.00	0.70	0.81	0.68	0.70	0.00	0.00	-0.01	0.66
time (sec)	N/A	0.032	0.022	0.006	1.332	0.928	0.000	0.000	0.000	0.072

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	80	86	168	85	0	155	-1	91
N.S.	1	1.00	0.75	0.80	1.57	0.79	0.00	1.45	-0.01	0.85
time (sec)	N/A	0.033	0.015	0.006	1.558	0.969	0.000	1.131	0.000	0.069

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	69	74	129	74	0	143	-1	80
N.S.	1	1.00	0.80	0.86	1.50	0.86	0.00	1.66	-0.01	0.93
time (sec)	N/A	0.026	0.012	0.005	1.555	0.986	0.000	1.041	0.000	0.064

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	52	60	88	59	0	127	-1	63
N.S.	1	1.00	0.81	0.94	1.38	0.92	0.00	1.98	-0.02	0.98
time (sec)	N/A	0.020	0.012	0.006	1.450	1.051	0.000	1.133	0.000	0.057

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	35	39	68	40	0	89	-1	45
N.S.	1	1.00	0.81	0.91	1.58	0.93	0.00	2.07	-0.02	1.05
time (sec)	N/A	0.013	0.009	0.004	1.411	1.200	0.000	1.149	0.000	0.041

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	21	25	85	38	25	27
N.S.	1	1.00	1.00	0.95	0.95	1.14	3.86	1.73	1.14	1.23
time (sec)	N/A	0.003	0.003	0.003	1.453	1.078	1.169	1.159	0.156	0.027

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	44	50	61	44	0	86	-1	57
N.S.	1	1.00	0.75	0.85	1.03	0.75	0.00	1.46	-0.02	0.97
time (sec)	N/A	0.016	0.005	0.006	1.460	0.994	0.000	1.084	0.000	0.050

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	60	71	57	62	0	126	-1	68
N.S.	1	1.00	0.77	0.91	0.73	0.79	0.00	1.62	-0.01	0.87
time (sec)	N/A	0.022	0.026	0.006	1.432	0.952	0.000	1.142	0.000	0.067

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	81	95	76	79	0	152	-1	86
N.S.	1	1.00	0.79	0.92	0.74	0.77	0.00	1.48	-0.01	0.83
time (sec)	N/A	0.032	0.018	0.006	1.407	1.284	0.000	1.222	0.000	0.073

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	54	62	149	63	0	127	-1	59
N.S.	1	1.00	0.74	0.85	2.04	0.86	0.00	1.74	-0.01	0.81
time (sec)	N/A	0.021	0.015	0.005	1.611	0.998	0.000	1.264	0.000	0.058

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	37	41	108	44	0	89	-1	39
N.S.	1	1.00	0.76	0.84	2.20	0.90	0.00	1.82	-0.02	0.80
time (sec)	N/A	0.014	0.011	0.003	1.557	1.116	0.000	1.025	0.000	0.046

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	24	23	47	29	90	38	25	24
N.S.	1	1.00	0.96	0.92	1.88	1.16	3.60	1.52	1.00	0.96
time (sec)	N/A	0.004	0.006	0.003	1.483	0.958	1.940	1.164	0.167	0.029

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	46	52	82	48	0	83	-1	48
N.S.	1	1.00	0.68	0.76	1.21	0.71	0.00	1.22	-0.01	0.71
time (sec)	N/A	0.017	0.014	0.006	1.520	1.019	0.000	1.057	0.000	0.052

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	59	74	79	66	0	137	-1	61
N.S.	1	1.00	0.66	0.82	0.88	0.73	0.00	1.52	-0.01	0.68
time (sec)	N/A	0.024	0.011	0.005	1.412	1.209	0.000	1.109	0.000	0.059

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	80	93	98	83	0	152	-1	77
N.S.	1	1.00	0.68	0.79	0.83	0.70	0.00	1.29	-0.01	0.65
time (sec)	N/A	0.032	0.009	0.007	1.463	0.978	0.000	1.027	0.000	0.067

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	97	136	116	153	0	300	214	0
N.S.	1	1.00	0.74	1.04	0.89	1.17	0.00	2.29	1.63	0.00
time (sec)	N/A	0.037	0.068	0.007	1.464	1.243	0.000	1.111	0.348	0.215

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	68	83	80	106	0	200	142	0
N.S.	1	1.00	0.71	0.86	0.83	1.10	0.00	2.08	1.48	0.00
time (sec)	N/A	0.029	0.047	0.006	1.413	1.564	0.000	0.961	0.252	0.199

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	44	46	51	63	0	119	85	0
N.S.	1	1.00	0.70	0.73	0.81	1.00	0.00	1.89	1.35	0.00
time (sec)	N/A	0.017	0.030	0.003	1.450	0.786	0.000	1.144	0.222	0.189

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	29	28	30	0	42	31	0
N.S.	1	1.00	0.97	0.97	0.93	1.00	0.00	1.40	1.03	0.00
time (sec)	N/A	0.006	0.014	0.002	1.395	0.986	0.000	1.011	0.228	0.190

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	132	199	157	233	0	426	307	0
N.S.	1	1.00	0.78	1.18	0.93	1.38	0.00	2.52	1.82	0.00
time (sec)	N/A	0.055	0.077	0.009	1.437	1.340	0.000	1.196	0.414	0.223

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	98	136	116	164	0	300	219	0
N.S.	1	1.00	0.73	1.01	0.86	1.21	0.00	2.22	1.62	0.00
time (sec)	N/A	0.039	0.055	0.007	1.441	1.055	0.000	1.141	0.320	0.214

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	70	83	80	113	0	0	146	0
N.S.	1	1.00	0.71	0.84	0.81	1.14	0.00	0.00	1.47	0.00
time (sec)	N/A	0.028	0.031	0.006	1.426	1.015	0.000	0.000	0.261	0.223

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	46	46	51	68	0	119	88	0
N.S.	1	1.00	0.71	0.71	0.78	1.05	0.00	1.83	1.35	0.00
time (sec)	N/A	0.018	0.008	0.004	1.426	0.835	0.000	0.910	0.231	0.218

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	30	29	28	33	0	42	45	0
N.S.	1	1.00	0.97	0.94	0.90	1.06	0.00	1.35	1.45	0.00
time (sec)	N/A	0.006	0.015	0.001	1.423	1.013	0.000	1.087	0.229	0.210

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	217	217	172	280	203	352	0	640	424	0
N.S.	1	1.00	0.79	1.29	0.94	1.62	0.00	2.95	1.95	0.00
time (sec)	N/A	0.074	0.108	0.009	1.529	0.885	0.000	1.063	0.496	0.232

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	179	179	133	199	157	265	0	0	319	0
N.S.	1	1.00	0.74	1.11	0.88	1.48	0.00	0.00	1.78	0.00
time (sec)	N/A	0.051	0.022	0.007	1.452	1.152	0.000	0.000	0.379	0.245

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	99	136	116	186	0	0	229	0
N.S.	1	1.00	0.69	0.95	0.81	1.30	0.00	0.00	1.60	0.00
time (sec)	N/A	0.042	0.018	0.007	1.454	1.146	0.000	0.000	0.317	0.236

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	70	83	80	127	0	0	154	0
N.S.	1	1.00	0.67	0.79	0.76	1.21	0.00	0.00	1.47	0.00
time (sec)	N/A	0.029	0.045	0.006	1.361	1.045	0.000	0.000	0.265	0.245

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	46	46	51	76	0	0	94	0
N.S.	1	1.00	0.67	0.67	0.74	1.10	0.00	0.00	1.36	0.00
time (sec)	N/A	0.019	0.009	0.002	1.453	1.052	0.000	0.000	0.239	0.234

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	31	29	28	37	0	0	49	0
N.S.	1	1.00	0.94	0.88	0.85	1.12	0.00	0.00	1.48	0.00
time (sec)	N/A	0.006	0.017	0.002	1.388	0.838	0.000	0.000	0.231	0.226

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	96	134	104	158	0	0	186	0
N.S.	1	1.00	0.78	1.09	0.85	1.28	0.00	0.00	1.51	0.00
time (sec)	N/A	0.037	0.040	0.006	1.449	1.056	0.000	0.000	0.369	0.224

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	67	81	83	110	0	0	121	0
N.S.	1	1.00	0.74	0.90	0.92	1.22	0.00	0.00	1.34	0.00
time (sec)	N/A	0.026	0.029	0.004	1.464	1.328	0.000	0.000	0.295	0.219

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	43	44	45	66	0	0	71	0
N.S.	1	1.00	0.73	0.75	0.76	1.12	0.00	0.00	1.20	0.00
time (sec)	N/A	0.016	0.021	0.003	1.487	0.957	0.000	0.000	0.277	0.203

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	27	31	33	0	0	36	0
N.S.	1	1.00	1.00	0.96	1.11	1.18	0.00	0.00	1.29	0.00
time (sec)	N/A	0.005	0.011	0.002	1.437	1.690	0.000	0.000	0.220	0.191

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	98	136	104	168	0	0	201	0
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49	0.00
time (sec)	N/A	0.044	0.041	0.007	1.464	0.989	0.000	0.000	0.404	0.254

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	69	83	83	118	0	0	133	0
N.S.	1	1.00	0.70	0.84	0.84	1.19	0.00	0.00	1.34	0.00
time (sec)	N/A	0.031	0.034	0.005	1.455	1.165	0.000	0.000	0.313	0.248

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	45	46	45	72	0	0	80	0
N.S.	1	1.00	0.69	0.71	0.69	1.11	0.00	0.00	1.23	0.00
time (sec)	N/A	0.019	0.023	0.003	1.467	1.586	0.000	0.000	0.289	0.242

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	30	29	31	37	0	0	42	0
N.S.	1	1.00	0.97	0.94	1.00	1.19	0.00	0.00	1.35	0.00
time (sec)	N/A	0.007	0.014	0.002	1.441	0.912	0.000	0.000	0.229	0.226

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	99	136	104	168	0	0	201	0
N.S.	1	1.00	0.73	1.01	0.77	1.24	0.00	0.00	1.49	0.00
time (sec)	N/A	0.046	0.037	0.006	1.472	0.951	0.000	0.000	0.410	0.271

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	70	83	83	118	0	0	133	0
N.S.	1	1.00	0.71	0.84	0.84	1.19	0.00	0.00	1.34	0.00
time (sec)	N/A	0.031	0.030	0.006	1.451	1.874	0.000	0.000	0.355	0.263

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	46	46	45	72	0	0	80	0
N.S.	1	1.00	0.71	0.71	0.69	1.11	0.00	0.00	1.23	0.00
time (sec)	N/A	0.020	0.020	0.003	1.407	0.919	0.000	0.000	0.285	0.248

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	29	31	37	0	0	42	0
N.S.	1	1.00	1.00	0.94	1.00	1.19	0.00	0.00	1.35	0.00
time (sec)	N/A	0.007	0.012	0.003	1.453	1.488	0.000	0.000	0.230	0.243

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	38	40	39	58	0	0	44	0
N.S.	1	1.00	0.58	0.62	0.60	0.89	0.00	0.00	0.68	0.00
time (sec)	N/A	0.030	0.027	0.003	1.518	1.412	0.000	0.000	0.266	0.462

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	38	40	39	50	0	0	42	0
N.S.	1	1.00	0.62	0.66	0.64	0.82	0.00	0.00	0.69	0.00
time (sec)	N/A	0.029	0.024	0.002	1.554	1.494	0.000	0.000	0.237	0.392

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	38	40	39	44	0	0	39	0
N.S.	1	1.00	0.64	0.68	0.66	0.75	0.00	0.00	0.66	0.00
time (sec)	N/A	0.027	0.020	0.003	1.520	1.520	0.000	0.000	0.212	0.350

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	33	32	32	36	0	0	30	0
N.S.	1	1.00	0.69	0.67	0.67	0.75	0.00	0.00	0.62	0.00
time (sec)	N/A	0.019	0.015	0.001	1.482	0.751	0.000	0.000	0.211	0.396

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	38	40	39	53	0	0	48	0
N.S.	1	1.00	0.58	0.62	0.60	0.82	0.00	0.00	0.74	0.00
time (sec)	N/A	0.036	0.025	0.003	1.506	1.060	0.000	0.000	0.259	0.385

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	38	40	39	53	0	0	47	0
N.S.	1	1.00	0.57	0.60	0.58	0.79	0.00	0.00	0.70	0.00
time (sec)	N/A	0.037	0.027	0.002	1.517	1.209	0.000	0.000	0.275	0.601

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	48	95	64	123	0	0	127	0
N.S.	1	1.00	0.47	0.92	0.62	1.19	0.00	0.00	1.23	0.00
time (sec)	N/A	0.049	0.051	0.004	1.600	1.276	0.000	0.000	0.312	0.854

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	48	95	64	105	0	0	121	0
N.S.	1	1.00	0.49	0.98	0.66	1.08	0.00	0.00	1.25	0.00
time (sec)	N/A	0.045	0.045	0.005	1.551	1.420	0.000	0.000	0.279	0.621

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	72	95	64	94	0	0	116	0
N.S.	1	1.00	0.77	1.01	0.68	1.00	0.00	0.00	1.23	0.00
time (sec)	N/A	0.041	0.043	0.004	1.525	1.546	0.000	0.000	0.264	0.549

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	62	79	57	85	0	0	62	0
N.S.	1	1.00	0.77	0.98	0.70	1.05	0.00	0.00	0.77	0.00
time (sec)	N/A	0.036	0.034	0.005	1.635	1.185	0.000	0.000	0.258	0.622

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	62	83	59	92	0	0	66	0
N.S.	1	1.00	0.67	0.89	0.63	0.99	0.00	0.00	0.71	0.00
time (sec)	N/A	0.045	0.039	0.006	1.583	1.308	0.000	0.000	0.319	0.688

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	105	105	72	95	64	106	0	0	82	0
N.S.	1	1.00	0.69	0.90	0.61	1.01	0.00	0.00	0.78	0.00
time (sec)	N/A	0.054	0.046	0.006	1.569	1.415	0.000	0.000	0.339	0.868

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	32	0	40	0	74	33	0
N.S.	1	1.00	0.97	0.97	0.00	1.21	0.00	2.24	1.00	0.00
time (sec)	N/A	0.010	0.017	0.004	0.000	1.223	0.000	1.261	0.266	0.088

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	34	33	0	40	0	0	34	0
N.S.	1	1.00	1.06	1.03	0.00	1.25	0.00	0.00	1.06	0.00
time (sec)	N/A	0.009	0.019	0.003	0.000	1.227	0.000	0.000	0.235	0.083

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	32	0	38	0	72	33	0
N.S.	1	1.00	0.97	0.97	0.00	1.15	0.00	2.18	1.00	0.00
time (sec)	N/A	0.010	0.015	0.004	0.000	1.508	0.000	1.070	0.217	0.078

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	28	31	0	36	0	0	32	0
N.S.	1	1.00	0.93	1.03	0.00	1.20	0.00	0.00	1.07	0.00
time (sec)	N/A	0.008	0.015	0.004	0.000	1.366	0.000	0.000	0.200	0.072

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	25	27	31	264	0	26	0
N.S.	1	1.00	1.00	0.96	1.04	1.19	10.15	0.00	1.00	0.00
time (sec)	N/A	0.006	0.007	0.003	1.446	1.181	59.694	0.000	0.262	0.080

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	38	0	37	0	0	32	0
N.S.	1	1.00	0.97	1.15	0.00	1.12	0.00	0.00	0.97	0.00
time (sec)	N/A	0.011	0.010	0.003	0.000	1.170	0.000	0.000	0.240	0.064

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	32	32	0	37	0	0	50	0
N.S.	1	1.00	0.91	0.91	0.00	1.06	0.00	0.00	1.43	0.00
time (sec)	N/A	0.011	0.010	0.003	0.000	0.881	0.000	0.000	0.248	0.073

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	33	0	37	0	0	51	0
N.S.	1	1.00	0.97	1.00	0.00	1.12	0.00	0.00	1.55	0.00
time (sec)	N/A	0.010	0.010	0.002	0.000	1.320	0.000	0.000	0.251	0.081

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	39	0	49	0	0	50	0
N.S.	1	1.00	1.00	1.03	0.00	1.29	0.00	0.00	1.32	0.00
time (sec)	N/A	0.010	0.029	0.003	0.000	1.422	0.000	0.000	0.338	0.111

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	40	0	57	0	0	39	0
N.S.	1	1.00	1.00	1.03	0.00	1.46	0.00	0.00	1.00	0.00
time (sec)	N/A	0.010	0.015	0.003	0.000	1.357	0.000	0.000	0.265	0.118

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	31	31	34	31	27	0
N.S.	1	1.00	1.00	0.94	1.82	1.82	2.00	1.82	1.59	0.00
time (sec)	N/A	0.004	0.002	0.002	1.367	1.189	0.121	1.086	0.050	0.000

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	19	18	20	20	20	20	16	0
N.S.	1	1.00	0.83	0.78	0.87	0.87	0.87	0.87	0.70	0.00
time (sec)	N/A	0.006	0.001	0.001	1.239	1.255	0.119	1.194	0.034	0.000

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	9	8	8	7	8	8	0
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00	0.00
time (sec)	N/A	0.002	0.000	0.001	1.270	1.353	0.107	1.004	0.010	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	19	14	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.46	1.08	1.00	0.00
time (sec)	N/A	0.003	0.002	0.001	1.310	1.218	0.112	1.073	0.048	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	19	19	19	15	15	0
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.27	1.00	1.00	0.00
time (sec)	N/A	0.003	0.004	0.000	1.338	1.344	0.177	0.988	0.037	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	47	47	53	15	49	0
N.S.	1	1.00	1.00	0.94	2.76	2.76	3.12	0.88	2.88	0.00
time (sec)	N/A	0.003	0.005	0.001	1.307	1.329	0.288	1.187	0.153	0.001

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	61	61	68	15	63	0
N.S.	1	1.00	1.00	0.94	3.59	3.59	4.00	0.88	3.71	0.00
time (sec)	N/A	0.003	0.006	0.002	1.354	1.358	0.363	0.905	0.064	0.001

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	75	75	83	15	77	0
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	0.88	4.53	0.00
time (sec)	N/A	0.003	0.006	0.002	1.332	0.879	0.422	0.962	0.054	0.000

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	35	35	34	35	27	0
N.S.	1	1.00	1.00	0.94	2.06	2.06	2.00	2.06	1.59	0.00
time (sec)	N/A	0.004	0.003	0.002	1.372	1.251	0.130	1.007	0.156	0.000

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	19	18	21	21	20	21	16	0
N.S.	1	1.00	0.83	0.78	0.91	0.91	0.87	0.91	0.70	0.00
time (sec)	N/A	0.005	0.001	0.001	1.315	1.256	0.118	1.037	0.030	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	9	8	8	7	8	8	0
N.S.	1	1.00	1.00	1.12	1.00	1.00	0.88	1.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.000	1.327	1.188	0.104	1.050	0.009	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	17	14	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.08	1.00	0.00
time (sec)	N/A	0.003	0.002	0.001	1.367	1.274	0.104	0.890	0.142	0.001

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	16	16	12	13	13	0
N.S.	1	1.00	1.00	1.08	1.23	1.23	0.92	1.00	1.00	0.00
time (sec)	N/A	0.003	0.004	0.002	1.381	1.199	0.154	1.001	0.040	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	36	36	44	12	38	0
N.S.	1	1.00	1.00	0.93	2.57	2.57	3.14	0.86	2.71	0.00
time (sec)	N/A	0.003	0.005	0.001	1.344	1.286	0.280	1.136	0.047	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	59	59	68	20	61	0
N.S.	1	1.00	1.00	0.93	3.93	3.93	4.53	1.33	4.07	0.00
time (sec)	N/A	0.003	0.006	0.001	1.346	1.308	0.362	0.978	0.050	0.000

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	75	75	83	15	77	0
N.S.	1	1.00	1.00	0.94	4.41	4.41	4.88	0.88	4.53	0.00
time (sec)	N/A	0.003	0.006	0.000	1.360	1.364	0.419	0.922	0.173	0.000

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	25	27	649	80	212	141	107	0
N.S.	1	1.00	1.04	1.12	27.04	3.33	8.83	5.88	4.46	0.00
time (sec)	N/A	0.009	0.020	0.002	1.809	0.999	2.293	1.132	0.326	0.069

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	114	113	113	124	113	113	0
N.S.	1	1.00	1.00	6.71	6.65	6.65	7.29	6.65	6.65	0.00
time (sec)	N/A	0.004	0.002	0.001	1.313	1.247	0.099	0.957	0.051	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	100	99	99	110	99	99	0
N.S.	1	1.00	1.00	5.88	5.82	5.82	6.47	5.82	5.82	0.00
time (sec)	N/A	0.004	0.002	0.003	1.359	0.982	0.095	0.986	0.041	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	72	71	71	78	71	71	0
N.S.	1	1.00	1.00	4.80	4.73	4.73	5.20	4.73	4.73	0.00
time (sec)	N/A	0.003	0.002	0.002	1.353	1.275	0.084	1.093	0.030	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	48	48	51	48	57	0
N.S.	1	1.00	1.00	0.94	2.82	2.82	3.00	2.82	3.35	0.00
time (sec)	N/A	0.004	0.002	0.000	1.351	1.103	0.102	0.893	0.025	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	37	37	46	18	43	0
N.S.	1	1.00	1.00	0.94	2.18	2.18	2.71	1.06	2.53	0.00
time (sec)	N/A	0.004	0.001	0.002	1.309	1.183	0.107	0.978	0.048	0.000

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	26	26	29	26	24	0
N.S.	1	1.00	1.00	0.94	1.53	1.53	1.71	1.53	1.41	0.00
time (sec)	N/A	0.004	0.001	0.000	1.305	0.709	0.109	1.002	0.036	0.000

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	16	15	15	15	15	15	13	0
N.S.	1	1.00	0.89	0.83	0.83	0.83	0.83	0.83	0.72	0.00
time (sec)	N/A	0.005	0.001	0.001	1.380	1.358	0.109	1.211	0.024	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	5	5	5	6	5	5	3	15	5	0
N.S.	1	1.00	1.00	1.20	1.00	1.00	0.60	3.00	1.00	0.00
time (sec)	N/A	0.001	0.000	0.002	1.399	1.219	0.107	1.111	0.008	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	17	14	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	1.31	1.08	1.00	0.00
time (sec)	N/A	0.004	0.002	0.002	1.311	1.329	0.123	0.955	0.040	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	16	19	19	17	15	19	0
N.S.	1	1.00	1.00	1.07	1.27	1.27	1.13	1.00	1.27	0.00
time (sec)	N/A	0.004	0.002	0.000	1.328	1.216	0.200	1.090	0.046	0.001

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	16	33	33	36	15	35	0
N.S.	1	1.00	1.00	0.94	1.94	1.94	2.12	0.88	2.06	0.00
time (sec)	N/A	0.004	0.004	0.001	1.332	1.362	0.260	1.047	0.146	0.001

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	C	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	6	1	53	11	35	0
N.S.	1	1.00	1.00	0.82	0.21	0.04	1.89	0.39	1.25	0.00
time (sec)	N/A	0.003	0.006	0.003	2.992	1.267	1.461	1.008	0.222	1.781

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	40	45	44	44	44	44	44	0
N.S.	1	1.00	1.05	1.18	1.16	1.16	1.16	1.16	1.16	0.00
time (sec)	N/A	0.012	0.003	0.002	1.228	1.290	0.080	0.998	0.160	0.000

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	42	45	44	44	46	44	44	0
N.S.	1	1.00	1.11	1.18	1.16	1.16	1.21	1.16	1.16	0.00
time (sec)	N/A	0.017	0.002	0.002	1.313	0.729	0.074	1.037	0.048	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	17	16	16	15	16	18	0
N.S.	1	1.00	1.00	0.94	0.89	0.89	0.83	0.89	1.00	0.00
time (sec)	N/A	0.004	0.001	0.001	1.351	1.034	0.063	1.010	0.023	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.002	0.000	0.000	1.336	1.068	0.058	0.859	0.017	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	25	24	23	17	25	23	0
N.S.	1	1.00	1.00	1.09	1.04	1.00	0.74	1.09	1.00	0.00
time (sec)	N/A	0.012	0.005	0.003	1.308	1.305	0.140	0.924	0.046	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	28	35	37	39	29	81	37	0
N.S.	1	1.00	0.88	1.09	1.16	1.22	0.91	2.53	1.16	0.00
time (sec)	N/A	0.017	0.017	0.006	1.308	1.247	0.187	1.114	0.052	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	33	30	30	27	14	13	0
N.S.	1	1.00	1.00	2.54	2.31	2.31	2.08	1.08	1.00	0.00
time (sec)	N/A	0.002	0.008	0.005	1.310	0.679	0.241	0.892	0.148	0.000

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	25	35	54	54	56	23	54	0
N.S.	1	1.00	0.66	0.92	1.42	1.42	1.47	0.61	1.42	0.00
time (sec)	N/A	0.019	0.011	0.007	1.337	1.242	0.307	0.979	0.049	0.000

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	24	35	67	67	73	40	67	0
N.S.	1	1.00	0.63	0.92	1.76	1.76	1.92	1.05	1.76	0.00
time (sec)	N/A	0.019	0.010	0.004	1.349	1.408	0.392	0.953	0.167	0.001

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	35	84	84	88	25	82	0
N.S.	1	1.00	0.71	0.92	2.21	2.21	2.32	0.66	2.16	0.00
time (sec)	N/A	0.019	0.013	0.004	1.312	1.215	0.463	1.035	0.075	0.001

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	68	73	72	72	78	72	72	0
N.S.	1	1.00	1.19	1.28	1.26	1.26	1.37	1.26	1.26	0.00
time (sec)	N/A	0.030	0.003	0.001	1.293	1.140	0.086	1.041	0.032	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	35	34	34	36	34	31	0
N.S.	1	1.00	1.00	0.92	0.89	0.89	0.95	0.89	0.82	0.00
time (sec)	N/A	0.017	0.002	0.001	1.338	0.486	0.078	1.105	0.040	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	40	37	36	36	39	36	36	0
N.S.	1	1.00	1.25	1.16	1.12	1.12	1.22	1.12	1.12	0.00
time (sec)	N/A	0.015	0.002	0.002	1.318	1.129	0.074	1.020	0.049	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	20	20	19	12	20	0
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	0.00
time (sec)	N/A	0.001	0.001	0.002	1.270	1.134	0.064	0.967	0.027	0.000

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	37	37	35	34	31	46	34	0
N.S.	1	1.00	0.86	0.86	0.81	0.79	0.72	1.07	0.79	0.00
time (sec)	N/A	0.014	0.007	0.003	1.289	1.226	0.165	0.875	0.047	0.000

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	35	44	46	57	39	79	46	0
N.S.	1	1.00	0.85	1.07	1.12	1.39	0.95	1.93	1.12	0.00
time (sec)	N/A	0.022	0.029	0.007	1.378	0.602	0.199	1.083	0.149	0.001

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	33	56	61	69	54	46	59	0
N.S.	1	1.00	0.63	1.08	1.17	1.33	1.04	0.88	1.13	0.00
time (sec)	N/A	0.028	0.024	0.007	1.333	1.028	0.306	1.072	0.172	0.001

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	31	52	60	60	61	29	58	0
N.S.	1	1.00	1.11	1.86	2.14	2.14	2.18	1.04	2.07	0.00
time (sec)	N/A	0.005	0.018	0.004	1.333	1.295	0.351	0.984	0.046	0.001

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	35	51	78	78	85	64	76	0
N.S.	1	1.00	0.62	0.91	1.39	1.39	1.52	1.14	1.36	0.00
time (sec)	N/A	0.024	0.012	0.005	1.380	1.156	0.439	1.070	0.052	0.001

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	38	52	95	95	100	36	91	0
N.S.	1	1.00	0.67	0.91	1.67	1.67	1.75	0.63	1.60	0.00
time (sec)	N/A	0.025	0.018	0.004	1.386	1.191	0.505	0.846	0.187	0.001

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	37	52	108	108	117	36	104	0
N.S.	1	1.00	0.63	0.88	1.83	1.83	1.98	0.61	1.76	0.00
time (sec)	N/A	0.029	0.015	0.006	1.420	0.990	0.600	0.934	0.108	0.001

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	42	49	48	52	49	59	48	0
N.S.	1	1.00	0.69	0.80	0.79	0.85	0.80	0.97	0.79	0.00
time (sec)	N/A	0.021	0.006	0.003	1.342	1.372	0.181	1.148	0.049	0.000

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	31	35	34	38	34	45	32	0
N.S.	1	1.00	0.72	0.81	0.79	0.88	0.79	1.05	0.74	0.00
time (sec)	N/A	0.014	0.005	0.003	1.342	1.228	0.153	0.871	0.148	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	19	18	20	15	19	18	0
N.S.	1	1.00	1.00	1.06	1.00	1.11	0.83	1.06	1.00	0.00
time (sec)	N/A	0.010	0.003	0.003	1.339	1.348	0.122	1.013	0.039	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	0.00
time (sec)	N/A	0.002	0.001	0.000	1.326	1.157	0.066	1.055	0.019	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	38	37	28	22	39	17	0
N.S.	1	1.00	1.00	2.24	2.18	1.65	1.29	2.29	1.00	0.00
time (sec)	N/A	0.009	0.006	0.006	1.401	1.228	0.171	1.195	0.170	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	53	58	60	60	48	53	42	0
N.S.	1	1.00	1.26	1.38	1.43	1.43	1.14	1.26	1.00	0.00
time (sec)	N/A	0.029	0.015	0.007	1.256	1.410	0.283	1.030	0.071	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	65	78	82	98	71	69	64	0
N.S.	1	1.00	1.03	1.24	1.30	1.56	1.13	1.10	1.02	0.00
time (sec)	N/A	0.039	0.022	0.007	1.348	1.489	0.381	0.866	0.078	0.000

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	46	53	53	79	51	80	52	0
N.S.	1	1.00	0.85	0.98	0.98	1.46	0.94	1.48	0.96	0.00
time (sec)	N/A	0.031	0.020	0.007	1.294	0.767	0.246	1.119	0.055	0.000

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	33	40	40	61	36	59	39	0
N.S.	1	1.00	0.85	1.03	1.03	1.56	0.92	1.51	1.00	0.00
time (sec)	N/A	0.021	0.017	0.007	1.362	1.254	0.194	1.183	0.168	0.000

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	23	28	28	33	24	54	27	0
N.S.	1	1.00	0.85	1.04	1.04	1.22	0.89	2.00	1.00	0.00
time (sec)	N/A	0.013	0.009	0.005	1.338	1.188	0.172	1.022	0.043	0.000

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	13	10	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	0.00
time (sec)	N/A	0.001	0.002	0.000	1.372	1.278	0.137	1.106	0.024	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	50	56	55	51	44	44	37	0
N.S.	1	1.00	1.22	1.37	1.34	1.24	1.07	1.07	0.90	0.00
time (sec)	N/A	0.028	0.016	0.007	1.376	1.011	0.284	0.953	0.177	0.001

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	74	76	64	76	49	83	46	0
N.S.	1	1.00	1.61	1.65	1.39	1.65	1.07	1.80	1.00	0.00
time (sec)	N/A	0.017	0.024	0.010	1.315	1.154	0.273	1.066	0.181	0.001

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	68	96	108	146	104	81	86	0
N.S.	1	1.00	0.82	1.16	1.30	1.76	1.25	0.98	1.04	0.00
time (sec)	N/A	0.050	0.043	0.011	1.355	1.172	0.512	0.961	0.098	0.001

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	60	113	68	62	289	185	-1	128
N.S.	1	1.00	0.56	1.05	0.63	0.57	2.68	1.71	-0.01	1.19
time (sec)	N/A	0.021	0.052	0.008	3.005	0.761	48.589	1.273	0.000	0.091

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	56	99	54	57	253	115	-1	114
N.S.	1	1.00	0.64	1.12	0.61	0.65	2.88	1.31	-0.01	1.30
time (sec)	N/A	0.015	0.051	0.005	2.965	1.318	21.067	1.061	0.000	0.083

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	50	85	40	52	218	101	-1	100
N.S.	1	1.00	0.74	1.25	0.59	0.76	3.21	1.49	-0.01	1.47
time (sec)	N/A	0.011	0.041	0.006	2.953	1.358	9.031	1.293	0.000	0.078

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	44	71	28	47	168	50	-1	82
N.S.	1	1.00	0.92	1.48	0.58	0.98	3.50	1.04	-0.02	1.71
time (sec)	N/A	0.006	0.038	0.005	2.902	1.356	4.494	1.023	0.000	0.073

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	20	57	17	38	133	42	37	73
N.S.	1	1.00	0.71	2.04	0.61	1.36	4.75	1.50	1.32	2.61
time (sec)	N/A	0.004	0.006	0.005	2.978	0.881	2.728	1.036	0.205	0.059

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	32	42	14	37	100	28	14	45
N.S.	1	1.00	1.52	2.00	0.67	1.76	4.76	1.33	0.67	2.14
time (sec)	N/A	0.004	0.007	0.006	2.901	1.114	1.843	1.014	0.145	0.087

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	36	64	21	48	71	33	-1	39
N.S.	1	1.00	1.57	2.78	0.91	2.09	3.09	1.43	-0.04	1.70
time (sec)	N/A	0.004	0.013	0.030	2.954	1.275	1.619	1.064	0.000	0.045

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	38	33	61	19	34	20
N.S.	1	1.00	1.00	0.75	1.90	1.65	3.05	0.95	1.70	1.00
time (sec)	N/A	0.002	0.005	0.003	1.261	1.254	1.674	0.937	0.270	0.067

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	18	64	53	173	22	50	34
N.S.	1	1.00	0.56	0.44	1.56	1.29	4.22	0.54	1.22	0.83
time (sec)	N/A	0.004	0.009	0.003	1.397	0.920	6.547	1.046	0.244	0.056

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	30	25	95	70	568	29	64	48
N.S.	1	1.00	0.49	0.41	1.56	1.15	9.31	0.48	1.05	0.79
time (sec)	N/A	0.008	0.012	0.003	1.272	1.172	19.923	1.224	0.269	0.066

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	35	30	131	85	1562	35	80	77
N.S.	1	1.00	0.43	0.37	1.62	1.05	19.28	0.43	0.99	0.95
time (sec)	N/A	0.013	0.014	0.003	1.348	1.308	53.780	1.118	0.282	0.069

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	40	35	172	100	3650	42	94	95
N.S.	1	1.00	0.40	0.35	1.70	0.99	36.14	0.42	0.93	0.94
time (sec)	N/A	0.019	0.017	0.004	1.322	0.988	135.085	1.359	0.292	0.074

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	66	127	66	67	325	237	-1	169
N.S.	1	1.00	0.61	1.17	0.61	0.61	2.98	2.17	-0.01	1.55
time (sec)	N/A	0.020	0.058	0.005	3.027	1.159	75.198	1.256	0.000	0.155

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	61	113	52	62	289	185	-1	151
N.S.	1	1.00	0.69	1.27	0.58	0.70	3.25	2.08	-0.01	1.70
time (sec)	N/A	0.013	0.055	0.005	2.904	1.154	32.987	1.330	0.000	0.132

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	55	99	40	57	250	91	-1	133
N.S.	1	1.00	0.80	1.43	0.58	0.83	3.62	1.32	-0.01	1.93
time (sec)	N/A	0.008	0.043	0.005	2.977	1.224	15.248	1.164	0.000	0.112

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	29	85	29	46	214	101	-1	115
N.S.	1	1.00	0.59	1.73	0.59	0.94	4.37	2.06	-0.02	2.35
time (sec)	N/A	0.007	0.011	0.005	2.911	1.189	7.461	1.116	0.000	0.096

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	44	71	28	47	165	66	-1	95
N.S.	1	1.00	0.92	1.48	0.58	0.98	3.44	1.38	-0.02	1.98
time (sec)	N/A	0.006	0.033	0.006	2.967	1.280	4.820	0.920	0.000	0.073

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	37	57	28	40	136	31	-1	68
N.S.	1	1.00	0.79	1.21	0.60	0.85	2.89	0.66	-0.02	1.45
time (sec)	N/A	0.007	0.012	0.004	3.088	1.239	3.275	1.172	0.000	0.060

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	35	72	42	52	100	35	-1	59
N.S.	1	1.00	0.85	1.76	1.02	1.27	2.44	0.85	-0.02	1.44
time (sec)	N/A	0.007	0.006	0.016	2.993	0.725	2.920	1.190	0.000	0.145

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	B	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	37	76	66	71	500	38	-1	55
N.S.	1	1.00	0.90	1.85	1.61	1.73	12.20	0.93	-0.02	1.34
time (sec)	N/A	0.005	0.006	0.020	2.973	1.311	3.698	1.019	0.000	0.062

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	94	52	88	19	50	20
N.S.	1	1.00	1.00	0.75	4.70	2.60	4.40	0.95	2.50	1.00
time (sec)	N/A	0.002	0.006	0.003	1.284	0.996	6.256	1.057	0.252	0.065

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	18	131	69	228	22	64	34
N.S.	1	1.00	0.56	0.44	3.20	1.68	5.56	0.54	1.56	0.83
time (sec)	N/A	0.004	0.010	0.003	1.384	1.148	19.081	1.127	0.268	0.065

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	30	25	172	86	677	29	80	48
N.S.	1	1.00	0.49	0.41	2.82	1.41	11.10	0.48	1.31	0.79
time (sec)	N/A	0.008	0.012	0.003	1.365	0.666	51.652	1.106	0.319	0.070

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	35	30	218	101	1753	35	94	62
N.S.	1	1.00	0.43	0.37	2.69	1.25	21.64	0.43	1.16	0.77
time (sec)	N/A	0.013	0.015	0.002	1.392	1.532	132.965	1.167	0.310	0.076

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	40	35	269	116	0	42	110	76
N.S.	1	1.00	0.40	0.35	2.66	1.15	0.00	0.42	1.09	0.75
time (sec)	N/A	0.019	0.017	0.003	1.380	1.225	0.000	1.231	0.328	0.086

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	75	155	78	77	0	323	-1	205
N.S.	1	1.00	0.58	1.19	0.60	0.59	0.00	2.48	-0.01	1.58
time (sec)	N/A	0.025	0.069	0.006	3.012	0.765	0.000	1.400	0.000	0.193

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	70	141	64	72	360	296	-1	187
N.S.	1	1.00	0.64	1.28	0.58	0.65	3.27	2.69	-0.01	1.70
time (sec)	N/A	0.019	0.061	0.005	2.970	1.155	117.569	1.493	0.000	0.163

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	66	127	52	67	321	143	-1	169
N.S.	1	1.00	0.73	1.41	0.58	0.74	3.57	1.59	-0.01	1.88
time (sec)	N/A	0.012	0.063	0.005	3.006	1.319	53.580	1.143	0.000	0.138

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	34	113	41	51	286	185	-1	151
N.S.	1	1.00	0.49	1.61	0.59	0.73	4.09	2.64	-0.01	2.16
time (sec)	N/A	0.011	0.014	0.004	3.103	1.292	25.761	1.308	0.000	0.120

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	55	99	40	57	246	114	-1	133
N.S.	1	1.00	0.80	1.43	0.58	0.83	3.57	1.65	-0.01	1.93
time (sec)	N/A	0.008	0.045	0.005	3.049	1.064	16.531	1.181	0.000	0.104

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	50	85	40	52	214	101	-1	100
N.S.	1	1.00	0.74	1.25	0.59	0.76	3.15	1.49	-0.01	1.47
time (sec)	N/A	0.009	0.037	0.005	3.060	1.129	9.886	1.045	0.000	0.077

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	44	71	42	47	172	39	-1	84
N.S.	1	1.00	0.66	1.06	0.63	0.70	2.57	0.58	-0.01	1.25
time (sec)	N/A	0.010	0.023	0.005	2.934	1.502	7.503	1.053	0.000	0.067

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	35	77	56	58	139	42	-1	81
N.S.	1	1.00	0.54	1.18	0.86	0.89	2.14	0.65	-0.02	1.25
time (sec)	N/A	0.010	0.007	0.018	2.971	1.113	7.762	1.007	0.000	0.084

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	37	84	99	75	576	44	-1	73
N.S.	1	1.00	0.59	1.33	1.57	1.19	9.14	0.70	-0.02	1.16
time (sec)	N/A	0.010	0.008	0.020	2.971	0.705	7.473	0.977	0.000	0.183

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	37	84	160	91	1608	44	-1	69
N.S.	1	1.00	0.59	1.33	2.54	1.44	25.52	0.70	-0.02	1.10
time (sec)	N/A	0.007	0.009	0.022	3.075	1.181	11.213	0.982	0.000	0.069

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	171	66	116	19	64	20
N.S.	1	1.00	1.00	0.75	8.55	3.30	5.80	0.95	3.20	1.00
time (sec)	N/A	0.002	0.008	0.003	1.355	1.300	19.494	1.108	0.279	0.060

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	18	218	83	282	22	80	34
N.S.	1	1.00	0.56	0.44	5.32	2.02	6.88	0.54	1.95	0.83
time (sec)	N/A	0.004	0.011	0.005	1.403	1.061	53.145	1.227	0.303	0.070

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	30	25	269	100	785	29	94	48
N.S.	1	1.00	0.49	0.41	4.41	1.64	12.87	0.48	1.54	0.79
time (sec)	N/A	0.008	0.016	0.003	1.415	0.973	133.938	1.268	0.310	0.078

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	35	30	325	115	0	35	110	62
N.S.	1	1.00	0.43	0.37	4.01	1.42	0.00	0.43	1.36	0.77
time (sec)	N/A	0.013	0.017	0.002	1.456	1.309	0.000	0.990	0.315	0.081

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	40	35	386	130	0	42	124	76
N.S.	1	1.00	0.40	0.35	3.82	1.29	0.00	0.42	1.23	0.75
time (sec)	N/A	0.019	0.021	0.003	1.393	1.361	0.000	0.877	0.352	0.085

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	45	40	452	145	0	48	140	90
N.S.	1	1.00	0.37	0.33	3.74	1.20	0.00	0.40	1.16	0.74
time (sec)	N/A	0.025	0.022	0.003	1.384	1.162	0.000	0.858	0.370	0.093

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	47	98	42	55	75	42	-1	86
N.S.	1	1.00	0.73	1.53	0.66	0.86	1.17	0.66	-0.02	1.34
time (sec)	N/A	0.012	0.039	0.013	2.989	1.272	33.751	0.699	0.000	0.081

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	91	118	42	48	76	34	55	72
N.S.	1	1.00	1.47	1.90	0.68	0.77	1.23	0.55	0.89	1.16
time (sec)	N/A	0.028	0.102	0.013	3.030	1.323	7.083	0.702	0.151	0.292

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	61	85	56	52	201	101	-1	100
N.S.	1	1.00	0.70	0.98	0.64	0.60	2.31	1.16	-0.01	1.15
time (sec)	N/A	0.018	0.025	0.006	2.993	1.286	14.676	0.758	0.000	0.070

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	54	71	42	47	175	69	-1	84
N.S.	1	1.00	0.81	1.06	0.63	0.70	2.61	1.03	-0.01	1.25
time (sec)	N/A	0.011	0.026	0.005	2.901	1.264	5.643	0.703	0.000	0.067

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	57	28	40	139	44	-1	67
N.S.	1	1.00	1.00	1.21	0.60	0.85	2.96	0.94	-0.02	1.43
time (sec)	N/A	0.007	0.020	0.003	3.023	1.401	2.586	0.697	0.000	0.071

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	30	41	12	36	100	27	12	44
N.S.	1	1.00	1.50	2.05	0.60	1.80	5.00	1.35	0.60	2.20
time (sec)	N/A	0.003	0.013	0.004	3.101	1.281	1.547	0.650	0.119	0.088

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	27	2	22	41	13	22	20
N.S.	1	1.00	1.00	13.50	1.00	11.00	20.50	6.50	11.00	10.00
time (sec)	N/A	0.001	0.005	0.004	2.946	0.831	1.035	0.654	0.078	0.038

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	14	16	23	29	19	13	17
N.S.	1	1.00	1.00	0.82	0.94	1.35	1.71	1.12	0.76	1.00
time (sec)	N/A	0.003	0.003	0.002	2.983	1.148	0.937	0.677	0.283	0.018

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	18	38	39	139	22	43	33
N.S.	1	1.00	0.56	0.44	0.93	0.95	3.39	0.54	1.05	0.80
time (sec)	N/A	0.004	0.007	0.003	3.120	1.198	2.246	0.643	0.309	0.056

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	30	25	64	56	332	29	55	48
N.S.	1	1.00	0.49	0.41	1.05	0.92	5.44	0.48	0.90	0.79
time (sec)	N/A	0.008	0.009	0.004	3.026	1.180	7.892	0.690	0.324	0.059

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	35	30	95	71	595	35	67	62
N.S.	1	1.00	0.43	0.37	1.17	0.88	7.35	0.43	0.83	0.77
time (sec)	N/A	0.014	0.011	0.004	3.029	1.283	22.133	0.664	0.345	0.067

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	40	35	131	86	933	42	80	76
N.S.	1	1.00	0.40	0.35	1.30	0.85	9.24	0.42	0.79	0.75
time (sec)	N/A	0.018	0.012	0.004	3.085	1.224	58.395	0.672	0.363	0.070

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	37	84	70	65	207	81	-1	98
N.S.	1	1.00	0.44	0.99	0.82	0.76	2.44	0.95	-0.01	1.15
time (sec)	N/A	0.016	0.013	0.017	2.839	1.038	17.475	0.748	0.000	0.101

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	37	77	56	58	168	73	-1	82
N.S.	1	1.00	0.57	1.18	0.86	0.89	2.58	1.12	-0.02	1.26
time (sec)	N/A	0.011	0.011	0.017	2.987	1.324	6.987	0.795	0.000	0.087

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	37	71	41	53	133	70	-1	49
N.S.	1	1.00	0.90	1.73	1.00	1.29	3.24	1.71	-0.02	1.20
time (sec)	N/A	0.007	0.010	0.016	2.862	1.390	2.485	0.726	0.000	0.132

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	34	67	21	50	104	55	-1	39
N.S.	1	1.00	1.48	2.91	0.91	2.17	4.52	2.39	-0.04	1.70
time (sec)	N/A	0.003	0.036	0.015	2.863	0.769	1.538	0.691	0.000	0.044

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	16	23	29	43	14	18
N.S.	1	1.00	1.00	0.83	0.89	1.28	1.61	2.39	0.78	1.00
time (sec)	N/A	0.002	0.004	0.004	2.940	1.022	1.199	0.651	0.360	0.018

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	13	15	11	22	65	62	14	34
N.S.	1	1.00	0.72	0.83	0.61	1.22	3.61	3.44	0.78	1.89
time (sec)	N/A	0.002	0.003	0.002	1.338	1.243	1.857	0.719	0.305	0.059

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	30	25	40	54	158	67	42	48
N.S.	1	1.00	0.71	0.60	0.95	1.29	3.76	1.60	1.00	1.14
time (sec)	N/A	0.005	0.007	0.003	1.416	1.225	5.279	0.701	0.322	0.066

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	33	28	79	59	282	73	55	62
N.S.	1	1.00	0.53	0.45	1.27	0.95	4.55	1.18	0.89	1.00
time (sec)	N/A	0.008	0.009	0.003	1.346	1.037	16.833	0.664	0.337	0.077

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	40	35	134	86	423	79	68	76
N.S.	1	1.00	0.49	0.43	1.63	1.05	5.16	0.96	0.83	0.93
time (sec)	N/A	0.013	0.010	0.004	1.375	1.268	44.937	0.695	0.355	0.076

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	45	40	201	91	592	85	80	90
N.S.	1	1.00	0.44	0.39	1.97	0.89	5.80	0.83	0.78	0.88
time (sec)	N/A	0.020	0.012	0.005	1.367	1.248	113.605	0.676	0.363	0.082

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	37	89	125	85	250	127	-1	131
N.S.	1	1.00	0.36	0.86	1.21	0.83	2.43	1.23	-0.01	1.27
time (sec)	N/A	0.021	0.015	0.022	3.009	1.499	45.202	0.889	0.000	0.104

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	C	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	37	84	111	81	207	119	-1	113
N.S.	1	1.00	0.43	0.97	1.28	0.93	2.38	1.37	-0.01	1.30
time (sec)	N/A	0.015	0.012	0.020	3.000	1.270	17.501	0.800	0.000	0.090

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	C	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	37	79	98	75	160	115	-1	61
N.S.	1	1.00	0.59	1.25	1.56	1.19	2.54	1.83	-0.02	0.97
time (sec)	N/A	0.011	0.011	0.021	2.965	1.259	6.461	0.752	0.000	0.171

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	C	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	49	73	66	71	126	102	-1	54
N.S.	1	1.00	1.20	1.78	1.61	1.73	3.07	2.49	-0.02	1.32
time (sec)	N/A	0.006	0.057	0.018	3.011	1.191	3.284	0.703	0.000	0.056

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	15	38	37	65	89	32	20
N.S.	1	1.00	1.00	0.75	1.90	1.85	3.25	4.45	1.60	1.00
time (sec)	N/A	0.002	0.004	0.002	1.324	1.305	1.694	0.708	0.263	0.068

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	23	18	38	38	65	89	33	33
N.S.	1	1.00	0.56	0.44	0.93	0.93	1.59	2.17	0.80	0.80
time (sec)	N/A	0.004	0.005	0.003	2.934	1.346	2.346	0.672	0.310	0.052

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	30	25	38	49	165	108	48	48
N.S.	1	1.00	0.52	0.43	0.66	0.84	2.84	1.86	0.83	0.83
time (sec)	N/A	0.008	0.006	0.003	1.351	1.202	5.397	0.678	0.338	0.066

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	23	23	25	35	279	113	41	62
N.S.	1	1.00	0.53	0.53	0.58	0.81	6.49	2.63	0.95	1.44
time (sec)	N/A	0.005	0.007	0.002	1.391	1.525	9.607	0.683	0.365	0.073

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	40	35	52	84	423	119	75	76
N.S.	1	1.00	0.63	0.56	0.83	1.33	6.71	1.89	1.19	1.21
time (sec)	N/A	0.008	0.011	0.001	1.394	1.269	27.605	0.710	0.377	0.076

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	45	40	91	101	592	125	86	90
N.S.	1	1.00	0.54	0.48	1.10	1.22	7.13	1.51	1.04	1.08
time (sec)	N/A	0.014	0.013	0.002	1.344	1.334	71.007	0.693	0.413	0.081

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	50	45	146	114	0	131	99	104
N.S.	1	1.00	0.49	0.44	1.42	1.11	0.00	1.27	0.96	1.01
time (sec)	N/A	0.019	0.014	0.003	1.421	0.875	0.000	0.737	0.423	0.087

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	114	193	72	201	0	679	-1	206
N.S.	1	1.00	0.90	1.53	0.57	1.60	0.00	5.39	-0.01	1.63
time (sec)	N/A	0.055	0.100	0.011	3.025	1.606	0.000	1.574	0.000	0.380

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	104	143	50	155	0	403	-1	160
N.S.	1	1.00	1.08	1.49	0.52	1.61	0.00	4.20	-0.01	1.67
time (sec)	N/A	0.037	0.082	0.005	3.092	1.400	0.000	1.253	0.000	0.277

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	69	98	28	127	0	173	59	105
N.S.	1	1.00	1.03	1.46	0.42	1.90	0.00	2.58	0.88	1.57
time (sec)	N/A	0.029	0.061	0.006	3.083	1.602	0.000	0.902	0.300	0.187

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	47	57	8	101	85	49	44	43
N.S.	1	1.00	1.09	1.33	0.19	2.35	1.98	1.14	1.02	1.00
time (sec)	N/A	0.024	0.017	0.005	2.991	1.644	3.953	0.764	0.175	0.082

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	25	21	39	82	116	23	47
N.S.	1	1.00	1.00	0.93	0.78	1.44	3.04	4.30	0.85	1.74
time (sec)	N/A	0.003	0.017	0.003	1.315	1.484	4.444	0.701	0.391	0.103

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	42	32	45	57	82	237	62	93
N.S.	1	1.00	0.69	0.52	0.74	0.93	1.34	3.89	1.02	1.52
time (sec)	N/A	0.010	0.028	0.002	1.347	1.084	13.690	0.807	0.415	0.118

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	49	37	67	74	85	333	50	141
N.S.	1	1.00	0.54	0.41	0.74	0.81	0.93	3.66	0.55	1.55
time (sec)	N/A	0.018	0.036	0.003	1.358	1.123	55.151	1.065	0.442	0.125

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	54	42	89	89	0	437	66	187
N.S.	1	1.00	0.45	0.35	0.74	0.74	0.00	3.61	0.55	1.55
time (sec)	N/A	0.028	0.040	0.003	1.401	1.261	0.000	1.563	0.478	0.135

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	120	243	89	232	0	0	-1	215
N.S.	1	1.00	0.89	1.80	0.66	1.72	0.00	0.00	-0.01	1.59
time (sec)	N/A	0.052	0.150	0.012	3.121	1.165	0.000	0.000	0.000	0.328

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	102	109	185	63	193	0	0	-1	169
N.S.	1	1.00	1.07	1.81	0.62	1.89	0.00	0.00	-0.01	1.66
time (sec)	N/A	0.036	0.127	0.006	3.024	1.426	0.000	0.000	0.000	0.240

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	95	127	39	159	0	0	72	114
N.S.	1	1.00	1.40	1.87	0.57	2.34	0.00	0.00	1.06	1.68
time (sec)	N/A	0.027	0.123	0.006	3.086	1.401	0.000	0.000	0.203	0.161

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	48	71	14	108	90	0	53	39
N.S.	1	1.00	1.26	1.87	0.37	2.84	2.37	0.00	1.39	1.03
time (sec)	N/A	0.020	0.014	0.005	2.933	0.828	4.688	0.000	0.177	0.082

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	29	30	25	45	94	115	26	55
N.S.	1	1.00	0.97	1.00	0.83	1.50	3.13	3.83	0.87	1.83
time (sec)	N/A	0.004	0.014	0.003	1.400	0.957	5.179	1.861	0.497	0.118

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	46	45	53	72	94	251	80	101
N.S.	1	1.00	0.69	0.67	0.79	1.07	1.40	3.75	1.19	1.51
time (sec)	N/A	0.011	0.025	0.003	1.426	1.230	15.849	2.380	0.584	0.129

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	57	56	79	98	97	366	111	149
N.S.	1	1.00	0.57	0.56	0.79	0.98	0.97	3.66	1.11	1.49
time (sec)	N/A	0.020	0.031	0.004	1.322	1.403	59.496	2.573	0.650	0.141

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	76	67	105	122	0	487	170	195
N.S.	1	1.00	0.57	0.50	0.79	0.92	0.00	3.66	1.28	1.47
time (sec)	N/A	0.034	0.041	0.003	1.293	1.602	0.000	3.296	0.714	0.148

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	44	134	46	65	0	227	-1	229
N.S.	1	1.00	0.44	1.34	0.46	0.65	0.00	2.27	-0.01	2.29
time (sec)	N/A	0.017	0.034	0.009	2.864	1.210	0.000	1.243	0.000	0.967

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	39	102	34	60	0	125	-1	179
N.S.	1	1.00	0.53	1.38	0.46	0.81	0.00	1.69	-0.01	2.42
time (sec)	N/A	0.010	0.035	0.005	2.859	0.868	0.000	0.984	0.000	0.841

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	30	70	22	52	187	55	44	122
N.S.	1	1.00	0.70	1.63	0.51	1.21	4.35	1.28	1.02	2.84
time (sec)	N/A	0.006	0.010	0.004	3.068	1.135	4.742	1.069	0.256	0.677

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	37	9	28	41	15	40	36
N.S.	1	1.00	1.00	2.85	0.69	2.15	3.15	1.15	3.08	2.77
time (sec)	N/A	0.003	0.016	0.003	2.943	1.093	3.347	0.885	0.051	0.588

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	16	28	12	26	156	71	24	80
N.S.	1	1.00	0.57	1.00	0.43	0.93	5.57	2.54	0.86	2.86
time (sec)	N/A	0.002	0.018	0.004	1.358	1.392	85.283	1.062	0.461	0.732

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	37	35	25	39	0	128	49	334
N.S.	1	1.00	0.65	0.61	0.44	0.68	0.00	2.25	0.86	5.86
time (sec)	N/A	0.006	0.025	0.003	1.277	1.314	0.000	1.017	0.311	0.848

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	42	40	37	49	0	181	66	616
N.S.	1	1.00	0.49	0.47	0.44	0.58	0.00	2.13	0.78	7.25
time (sec)	N/A	0.011	0.032	0.003	1.315	1.257	0.000	1.021	0.452	1.250

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	91	91	80	89	67	62	199	101	-1	115
N.S.	1	1.00	0.88	0.98	0.74	0.68	2.19	1.11	-0.01	1.26
time (sec)	N/A	0.022	0.047	0.009	2.968	1.095	7.482	0.879	0.000	0.093

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	69	61	38	52	124	42	41	78
N.S.	1	1.00	1.35	1.20	0.75	1.02	2.43	0.82	0.80	1.53
time (sec)	N/A	0.010	0.023	0.006	2.964	1.289	3.010	1.019	0.207	0.058

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	12	31	6	32	26	8	31	20
N.S.	1	1.00	1.50	3.88	0.75	4.00	3.25	1.00	3.88	2.50
time (sec)	N/A	0.004	0.010	0.004	3.001	1.354	1.607	1.019	0.178	0.035

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	21	20	30	29	100	53	32	31
N.S.	1	1.00	0.57	0.54	0.81	0.78	2.70	1.43	0.86	0.84
time (sec)	N/A	0.004	0.006	0.004	1.318	1.267	2.302	0.848	0.255	0.047

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	33	30	59	49	282	97	69	61
N.S.	1	1.00	0.42	0.38	0.75	0.62	3.57	1.23	0.87	0.77
time (sec)	N/A	0.013	0.013	0.004	1.337	1.222	9.849	1.096	0.370	0.058

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	16	16	12	22	73	62	22	34
N.S.	1	1.00	0.76	0.76	0.57	1.05	3.48	2.95	1.05	1.62
time (sec)	N/A	0.002	0.005	0.003	1.367	1.555	1.800	0.896	0.364	0.059

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	19	19	15	29	73	82	26	43
N.S.	1	1.00	0.79	0.79	0.62	1.21	3.04	3.42	1.08	1.79
time (sec)	N/A	0.003	0.007	0.003	1.371	1.530	5.162	1.098	0.463	0.084

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	21	19	12	22	90	71	22	0
N.S.	1	1.00	0.81	0.73	0.46	0.85	3.46	2.73	0.85	0.00
time (sec)	N/A	0.002	0.013	0.002	1.332	1.289	20.446	1.044	0.374	0.132

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	19	24	15	29	83	91	26	0
N.S.	1	1.00	0.66	0.83	0.52	1.00	2.86	3.14	0.90	0.00
time (sec)	N/A	0.003	0.018	0.003	1.254	1.211	31.496	1.144	0.321	0.245

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	C	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	39	76	39	108	88	0	56	39
N.S.	1	1.00	1.00	1.95	1.00	2.77	2.26	0.00	1.44	1.00
time (sec)	N/A	0.024	0.020	0.010	1.429	1.272	4.766	0.000	0.218	0.081

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	42	0	0	505	0	0	-1	0
N.S.	1	1.00	0.17	0.00	0.00	2.10	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.254	0.021	0.072	0.000	1.407	0.000	0.000	0.000	0.132

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	70	477	0	194	0	0	-1	126
N.S.	1	1.00	0.27	1.86	0.00	0.76	0.00	0.00	-0.00	0.49
time (sec)	N/A	0.173	0.021	2.284	0.000	1.486	0.000	0.000	0.000	0.470

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	68	0	0	227	0	0	-1	83
N.S.	1	1.00	0.29	0.00	0.00	0.97	0.00	0.00	-0.00	0.36
time (sec)	N/A	0.131	0.023	0.076	0.000	1.499	0.000	0.000	0.000	0.197

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	31	0	32	0	0	38	33
N.S.	1	1.00	1.00	0.94	0.00	0.97	0.00	0.00	1.15	1.00
time (sec)	N/A	0.003	0.014	0.046	0.000	1.713	0.000	0.000	0.549	0.112

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	45	44	0	44	0	0	46	55
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.69	0.82
time (sec)	N/A	0.009	0.021	0.048	0.000	1.570	0.000	0.000	0.666	0.125

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	52	50	0	58	0	0	51	77
N.S.	1	1.00	0.52	0.50	0.00	0.58	0.00	0.00	0.51	0.77
time (sec)	N/A	0.018	0.025	0.054	0.000	1.417	0.000	0.000	0.752	0.129

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	57	55	0	70	0	0	57	99
N.S.	1	1.00	0.43	0.41	0.00	0.53	0.00	0.00	0.43	0.74
time (sec)	N/A	0.028	0.029	0.062	0.000	1.472	0.000	0.000	0.794	0.124

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	70	464	0	204	0	0	-1	128
N.S.	1	1.00	0.27	1.81	0.00	0.80	0.00	0.00	-0.00	0.50
time (sec)	N/A	0.158	0.025	2.142	0.000	1.475	0.000	0.000	0.000	0.601

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	70	0	0	227	0	0	-1	83
N.S.	1	1.00	0.30	0.00	0.00	0.97	0.00	0.00	-0.00	0.36
time (sec)	N/A	0.135	0.023	0.060	0.000	1.411	0.000	0.000	0.000	0.153

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	31	0	31	0	0	-1	31
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.014	0.042	0.000	1.416	0.000	0.000	0.000	0.062

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	45	44	0	44	0	0	-1	54
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	-0.01	0.81
time (sec)	N/A	0.010	0.020	0.053	0.000	1.161	0.000	0.000	0.000	0.114

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	52	50	0	58	0	0	-1	77
N.S.	1	1.00	0.52	0.50	0.00	0.58	0.00	0.00	-0.01	0.77
time (sec)	N/A	0.018	0.024	0.056	0.000	1.400	0.000	0.000	0.000	0.117

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	291	291	70	469	0	244	0	0	-1	150
N.S.	1	1.00	0.24	1.61	0.00	0.84	0.00	0.00	-0.00	0.52
time (sec)	N/A	0.181	0.028	2.013	0.000	1.425	0.000	0.000	0.000	0.901

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	70	459	0	307	0	0	-1	116
N.S.	1	1.00	0.26	1.73	0.00	1.15	0.00	0.00	-0.00	0.44
time (sec)	N/A	0.140	0.022	1.882	0.000	0.844	0.000	0.000	0.000	0.192

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	31	0	32	0	0	-1	33
N.S.	1	1.00	1.00	0.94	0.00	0.97	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.012	0.044	0.000	1.509	0.000	0.000	0.000	0.085

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	38	33	0	36	0	0	-1	55
N.S.	1	1.00	0.58	0.51	0.00	0.55	0.00	0.00	-0.02	0.85
time (sec)	N/A	0.009	0.017	0.049	0.000	1.099	0.000	0.000	0.000	0.105

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	50	44	0	58	0	0	-1	77
N.S.	1	1.00	0.50	0.44	0.00	0.58	0.00	0.00	-0.01	0.77
time (sec)	N/A	0.017	0.023	0.058	0.000	1.514	0.000	0.000	0.000	0.109

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	287	287	70	481	0	240	0	0	-1	146
N.S.	1	1.00	0.24	1.68	0.00	0.84	0.00	0.00	-0.00	0.51
time (sec)	N/A	0.178	0.029	2.037	0.000	1.282	0.000	0.000	0.000	0.709

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	264	264	70	476	0	303	0	0	-1	114
N.S.	1	1.00	0.27	1.80	0.00	1.15	0.00	0.00	-0.00	0.43
time (sec)	N/A	0.139	0.024	2.008	0.000	0.935	0.000	0.000	0.000	0.193

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	31	0	31	0	0	27	31
N.S.	1	1.00	1.00	1.00	0.00	1.00	0.00	0.00	0.87	1.00
time (sec)	N/A	0.003	0.012	0.041	0.000	0.970	0.000	0.000	1.161	0.066

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	38	33	0	36	0	0	40	55
N.S.	1	1.00	0.57	0.49	0.00	0.54	0.00	0.00	0.60	0.82
time (sec)	N/A	0.009	0.018	0.049	0.000	1.496	0.000	0.000	0.597	0.129

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	50	44	0	58	0	0	46	77
N.S.	1	1.00	0.50	0.44	0.00	0.58	0.00	0.00	0.46	0.77
time (sec)	N/A	0.018	0.022	0.057	0.000	0.880	0.000	0.000	0.764	0.135

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	70	490	0	351	0	0	-1	137
N.S.	1	1.00	0.24	1.65	0.00	1.18	0.00	0.00	-0.00	0.46
time (sec)	N/A	0.140	0.025	0.059	0.000	1.486	0.000	0.000	0.000	0.219

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	50	0	45	0	0	38	33
N.S.	1	1.00	1.00	1.52	0.00	1.36	0.00	0.00	1.15	1.00
time (sec)	N/A	0.003	0.012	0.044	0.000	1.429	0.000	0.000	0.546	0.109

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	45	44	0	44	0	0	38	54
N.S.	1	1.00	0.67	0.66	0.00	0.66	0.00	0.00	0.57	0.81
time (sec)	N/A	0.010	0.020	0.045	0.000	1.361	0.000	0.000	0.633	0.141

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	50	44	0	58	0	0	45	77
N.S.	1	1.00	0.50	0.44	0.00	0.58	0.00	0.00	0.45	0.77
time (sec)	N/A	0.018	0.023	0.055	0.000	1.266	0.000	0.000	0.535	0.139

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	57	56	0	54	0	0	56	99
N.S.	1	1.00	0.43	0.42	0.00	0.41	0.00	0.00	0.42	0.74
time (sec)	N/A	0.029	0.030	0.065	0.000	1.490	0.000	0.000	0.687	0.140

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	77	103	167	128	819	256	133	0
N.S.	1	1.00	0.93	1.24	2.01	1.54	9.87	3.08	1.60	0.00
time (sec)	N/A	0.029	0.043	0.007	1.530	1.245	1.295	1.145	0.493	0.100

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	43	47	81	58	245	103	66	0
N.S.	1	1.00	0.81	0.89	1.53	1.09	4.62	1.94	1.25	0.00
time (sec)	N/A	0.017	0.021	0.003	1.396	1.344	0.699	1.050	0.319	0.081

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	84	97	96	97	100	97	88	0
N.S.	1	1.00	2.21	2.55	2.53	2.55	2.63	2.55	2.32	0.00
time (sec)	N/A	0.015	0.019	0.002	1.383	1.203	0.083	1.032	0.190	0.000

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	67	73	69	72	73	72	65	0
N.S.	1	1.00	1.76	1.92	1.82	1.89	1.92	1.89	1.71	0.00
time (sec)	N/A	0.012	0.011	0.000	1.406	1.411	0.078	1.139	0.159	0.000

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	46	49	48	49	49	49	47	0
N.S.	1	1.00	1.21	1.29	1.26	1.29	1.29	1.29	1.24	0.00
time (sec)	N/A	0.027	0.008	0.000	1.300	1.728	0.071	0.993	0.047	0.000

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	25	24	26	26	26	25	0
N.S.	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89	0.00
time (sec)	N/A	0.015	0.005	0.001	1.350	1.558	0.060	0.994	0.035	0.000

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	10	10	8	10	10	0
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83	0.00
time (sec)	N/A	0.002	0.000	0.000	1.350	1.457	0.055	0.903	0.019	0.000

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	32	25	24	20	26	26	0
N.S.	1	1.00	1.00	1.28	1.00	0.96	0.80	1.04	1.04	0.00
time (sec)	N/A	0.017	0.008	0.004	1.251	1.698	0.150	0.955	0.049	0.000

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	31	39	35	39	27	57	31	0
N.S.	1	1.00	0.97	1.22	1.09	1.22	0.84	1.78	0.97	0.00
time (sec)	N/A	0.019	0.012	0.005	1.339	1.682	0.187	1.020	0.171	0.000

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	35	38	38	39	24	39	0
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	0.86	1.39	0.00
time (sec)	N/A	0.004	0.009	0.006	1.362	1.748	0.260	1.062	0.159	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	35	50	50	53	25	52	0
N.S.	1	1.00	0.71	0.92	1.32	1.32	1.39	0.66	1.37	0.00
time (sec)	N/A	0.020	0.009	0.006	1.315	1.754	0.338	0.749	0.165	0.000

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	35	61	61	65	41	63	0
N.S.	1	1.00	0.71	0.92	1.61	1.61	1.71	1.08	1.66	0.00
time (sec)	N/A	0.020	0.010	0.004	1.385	1.202	0.428	0.924	0.041	0.000

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	148	163	156	170	168	170	144	0
N.S.	1	1.00	2.28	2.51	2.40	2.62	2.58	2.62	2.22	0.00
time (sec)	N/A	0.088	0.027	0.002	1.362	1.105	0.097	1.132	0.066	0.000

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	122	125	124	130	133	130	115	0
N.S.	1	1.00	1.88	1.92	1.91	2.00	2.05	2.00	1.77	0.00
time (sec)	N/A	0.064	0.015	0.001	1.344	1.565	0.091	1.132	0.050	0.000

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	79	87	81	89	87	89	74	0
N.S.	1	1.00	1.22	1.34	1.25	1.37	1.34	1.37	1.14	0.00
time (sec)	N/A	0.045	0.011	0.001	1.339	1.333	0.080	0.795	0.166	0.000

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	47	49	48	49	49	49	47	0
N.S.	1	1.00	1.24	1.29	1.26	1.29	1.29	1.29	1.24	0.00
time (sec)	N/A	0.027	0.010	0.001	1.306	1.183	0.071	1.027	0.044	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	20	20	19	12	20	0
N.S.	1	1.00	1.00	0.93	1.43	1.43	1.36	0.86	1.43	0.00
time (sec)	N/A	0.002	0.002	0.000	1.314	1.249	0.061	0.963	0.028	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	43	74	61	63	44	60	62	0
N.S.	1	1.00	0.88	1.51	1.24	1.29	0.90	1.22	1.27	0.00
time (sec)	N/A	0.019	0.018	0.003	1.356	0.839	0.224	1.196	0.190	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	47	86	67	92	60	98	71	0
N.S.	1	1.00	0.92	1.69	1.31	1.80	1.18	1.92	1.39	0.00
time (sec)	N/A	0.035	0.037	0.009	1.372	1.420	0.337	0.937	0.200	0.000

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	49	92	79	99	80	68	77	0
N.S.	1	1.00	0.83	1.56	1.34	1.68	1.36	1.15	1.31	0.00
time (sec)	N/A	0.035	0.025	0.006	1.300	1.144	0.454	1.031	0.197	0.000

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	53	70	84	84	88	59	80	0
N.S.	1	1.00	1.89	2.50	3.00	3.00	3.14	2.11	2.86	0.00
time (sec)	N/A	0.004	0.022	0.006	1.374	1.391	0.597	0.960	0.037	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	56	71	98	98	104	96	39	0
N.S.	1	1.00	0.86	1.09	1.51	1.51	1.60	1.48	0.60	0.00
time (sec)	N/A	0.033	0.019	0.006	1.329	1.490	0.764	1.126	0.193	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	57	71	109	109	116	61	107	0
N.S.	1	1.00	0.88	1.09	1.68	1.68	1.78	0.94	1.65	0.00
time (sec)	N/A	0.033	0.024	0.007	1.350	1.337	0.956	0.870	0.202	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	58	71	120	120	128	61	118	0
N.S.	1	1.00	0.89	1.09	1.85	1.85	1.97	0.94	1.82	0.00
time (sec)	N/A	0.033	0.020	0.005	1.391	1.334	1.159	1.036	0.089	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	235	281	277	303	308	303	261	0
N.S.	1	1.00	2.55	3.05	3.01	3.29	3.35	3.29	2.84	0.00
time (sec)	N/A	0.157	0.075	0.000	1.380	1.137	0.116	1.011	0.240	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	217	229	225	245	243	245	208	0
N.S.	1	1.00	2.36	2.49	2.45	2.66	2.64	2.66	2.26	0.00
time (sec)	N/A	0.114	0.029	0.001	1.318	1.198	0.106	1.116	0.214	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	161	177	167	188	190	188	152	0
N.S.	1	1.00	1.75	1.92	1.82	2.04	2.07	2.04	1.65	0.00
time (sec)	N/A	0.083	0.019	0.002	1.340	1.091	0.097	0.973	0.056	0.000

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	122	125	124	130	133	130	115	0
N.S.	1	1.00	1.88	1.92	1.91	2.00	2.05	2.00	1.77	0.00
time (sec)	N/A	0.064	0.014	0.001	1.340	1.269	0.087	1.039	0.046	0.000

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	67	73	69	72	73	72	65	0
N.S.	1	1.00	1.76	1.92	1.82	1.89	1.92	1.89	1.71	0.00
time (sec)	N/A	0.015	0.008	0.000	1.390	1.087	0.077	0.877	0.032	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	31	31	32	12	31	0
N.S.	1	1.00	1.00	0.93	2.21	2.21	2.29	0.86	2.21	0.00
time (sec)	N/A	0.002	0.001	0.000	1.346	1.182	0.065	0.997	0.037	0.000

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	74	133	114	116	83	115	118	0
N.S.	1	1.00	1.01	1.82	1.56	1.59	1.14	1.58	1.62	0.00
time (sec)	N/A	0.028	0.031	0.004	1.327	1.787	0.303	1.031	0.201	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	72	149	118	173	102	167	123	0
N.S.	1	1.00	0.96	1.99	1.57	2.31	1.36	2.23	1.64	0.00
time (sec)	N/A	0.055	0.051	0.008	1.317	1.416	0.503	0.991	0.215	0.000

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	114	160	125	188	128	112	130	0
N.S.	1	1.00	1.46	2.05	1.60	2.41	1.64	1.44	1.67	0.00
time (sec)	N/A	0.052	0.042	0.009	1.373	1.343	0.821	0.952	0.821	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	80	166	142	176	148	118	138	0
N.S.	1	1.00	0.93	1.93	1.65	2.05	1.72	1.37	1.60	0.00
time (sec)	N/A	0.050	0.041	0.006	1.353	1.510	1.126	0.944	0.254	0.001

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	91	122	143	143	155	159	135	0
N.S.	1	1.00	3.25	4.36	5.11	5.11	5.54	5.68	4.82	0.00
time (sec)	N/A	0.003	0.031	0.007	1.392	1.542	1.497	0.965	0.072	0.000

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	97	121	160	160	172	114	39	0
N.S.	1	1.00	1.67	2.09	2.76	2.76	2.97	1.97	0.67	0.00
time (sec)	N/A	0.010	0.035	0.006	1.467	1.647	1.961	1.001	0.082	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	97	122	171	171	184	114	165	0
N.S.	1	1.00	1.05	1.33	1.86	1.86	2.00	1.24	1.79	0.00
time (sec)	N/A	0.050	0.031	0.006	1.439	1.487	2.536	0.985	0.222	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	97	122	182	182	196	114	176	0
N.S.	1	1.00	1.05	1.33	1.98	1.98	2.13	1.24	1.91	0.00
time (sec)	N/A	0.049	0.032	0.006	1.445	1.457	3.123	0.947	0.114	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	97	122	193	193	207	114	187	0
N.S.	1	1.00	1.05	1.33	2.10	2.10	2.25	1.24	2.03	0.00
time (sec)	N/A	0.046	0.036	0.006	1.522	1.437	3.954	0.864	0.232	0.000

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	993	1033	1023	1175	1163	1175	997	0
N.S.	1	1.00	4.96	5.16	5.12	5.88	5.82	5.88	4.98	0.00
time (sec)	N/A	0.676	0.147	0.002	1.431	0.865	0.232	1.046	0.554	0.000

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	897	925	921	1050	1046	1050	892	0
N.S.	1	1.00	4.48	4.62	4.60	5.25	5.23	5.25	4.46	0.00
time (sec)	N/A	0.574	0.111	0.002	1.396	1.282	0.214	1.006	0.357	0.000

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	785	817	807	924	935	924	781	0
N.S.	1	1.00	3.92	4.08	4.04	4.62	4.68	4.62	3.90	0.00
time (sec)	N/A	0.454	0.086	0.003	1.335	1.250	0.194	1.009	0.402	0.000

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	684	709	706	798	796	798	683	0
N.S.	1	1.00	3.95	4.10	4.08	4.61	4.60	4.61	3.95	0.00
time (sec)	N/A	0.435	0.081	0.001	1.317	1.201	0.178	0.953	0.256	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	144	144	574	601	594	670	673	670	570	0
N.S.	1	1.00	3.99	4.17	4.12	4.65	4.67	4.65	3.96	0.00
time (sec)	N/A	0.360	0.076	0.002	1.384	1.092	0.164	1.279	0.214	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	473	493	489	546	549	546	470	0
N.S.	1	1.00	3.97	4.14	4.11	4.59	4.61	4.59	3.95	0.00
time (sec)	N/A	0.279	0.054	0.001	1.479	1.307	0.146	1.215	0.313	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	360	385	376	420	427	420	356	0
N.S.	1	1.00	3.91	4.18	4.09	4.57	4.64	4.57	3.87	0.00
time (sec)	N/A	0.218	0.042	0.002	1.373	1.253	0.131	1.259	0.273	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	261	277	273	294	303	294	249	0
N.S.	1	1.00	4.02	4.26	4.20	4.52	4.66	4.52	3.83	0.00
time (sec)	N/A	0.159	0.030	0.001	1.363	1.206	0.116	1.240	0.107	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	151	169	163	169	178	169	143	0
N.S.	1	1.00	3.97	4.45	4.29	4.45	4.68	4.45	3.76	0.00
time (sec)	N/A	0.016	0.015	0.000	1.376	1.566	0.100	1.297	0.079	0.000

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	75	83	12	75	0
N.S.	1	1.00	1.00	0.93	0.86	5.36	5.93	0.86	5.36	0.00
time (sec)	N/A	0.002	0.001	0.000	1.315	1.285	0.076	1.298	0.057	0.000

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	304	539	460	462	408	497	509	0
N.S.	1	1.00	1.80	3.19	2.72	2.73	2.41	2.94	3.01	0.00
time (sec)	N/A	0.073	0.148	0.007	1.395	1.171	0.802	1.298	0.217	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	388	571	467	632	428	567	841	0
N.S.	1	1.00	2.07	3.05	2.50	3.38	2.29	3.03	4.50	0.00
time (sec)	N/A	0.235	0.123	0.011	1.403	1.580	1.445	1.281	0.241	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	389	599	473	703	447	477	690	0
N.S.	1	1.00	2.10	3.24	2.56	3.80	2.42	2.58	3.73	0.00
time (sec)	N/A	0.217	0.132	0.014	1.538	1.202	2.949	1.265	0.266	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	199	622	484	739	474	470	559	0
N.S.	1	1.00	1.06	3.33	2.59	3.95	2.53	2.51	2.99	0.00
time (sec)	N/A	0.213	0.109	0.014	1.628	1.509	6.124	1.318	0.289	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	173	641	494	754	500	660	512	0
N.S.	1	1.00	0.93	3.43	2.64	4.03	2.67	3.53	2.74	0.00
time (sec)	N/A	0.198	0.112	0.015	1.734	1.575	22.438	1.287	0.772	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	389	656	504	732	524	463	508	0
N.S.	1	1.00	2.15	3.62	2.78	4.04	2.90	2.56	2.81	0.00
time (sec)	N/A	0.191	0.153	0.014	1.817	1.399	97.193	1.381	0.339	0.000

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	390	666	516	692	0	459	517	0
N.S.	1	1.00	2.10	3.58	2.77	3.72	0.00	2.47	2.78	0.00
time (sec)	N/A	0.172	0.204	0.013	1.799	1.513	0.000	1.298	0.369	0.000

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	308	672	534	624	0	466	461	0
N.S.	1	1.00	1.59	3.46	2.75	3.22	0.00	2.40	2.38	0.00
time (sec)	N/A	0.156	0.163	0.010	1.648	1.416	0.000	1.281	0.353	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	353	464	509	509	0	489	571	0
N.S.	1	1.00	12.61	16.57	18.18	18.18	0.00	17.46	20.39	0.00
time (sec)	N/A	0.003	0.125	0.008	1.650	1.395	0.000	1.290	0.169	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	367	464	548	548	0	496	39	0
N.S.	1	1.00	6.33	8.00	9.45	9.45	0.00	8.55	0.67	0.00
time (sec)	N/A	0.009	0.126	0.009	1.709	1.410	0.000	1.272	0.146	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	371	464	559	559	0	496	600	0
N.S.	1	1.00	4.17	5.21	6.28	6.28	0.00	5.57	6.74	0.00
time (sec)	N/A	0.022	0.123	0.008	1.731	1.496	0.000	1.311	0.446	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	369	464	570	570	0	496	548	0
N.S.	1	1.00	3.08	3.87	4.75	4.75	0.00	4.13	4.57	0.00
time (sec)	N/A	0.033	0.122	0.005	1.812	0.711	0.000	1.303	0.518	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	371	464	581	581	0	496	559	0
N.S.	1	1.00	2.46	3.07	3.85	3.85	0.00	3.28	3.70	0.00
time (sec)	N/A	0.047	0.129	0.008	1.757	1.417	0.000	1.264	0.229	0.000

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	369	463	592	592	0	496	570	0
N.S.	1	1.00	1.86	2.34	2.99	2.99	0.00	2.51	2.88	0.00
time (sec)	N/A	0.153	0.127	0.009	1.745	1.229	0.000	1.237	0.399	0.000

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	371	464	603	603	0	496	581	0
N.S.	1	1.00	1.86	2.32	3.02	3.02	0.00	2.48	2.90	0.00
time (sec)	N/A	0.143	0.127	0.008	1.835	1.353	0.000	1.285	1.238	0.000

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	371	464	614	614	0	496	592	0
N.S.	1	1.00	1.86	2.32	3.07	3.07	0.00	2.48	2.96	0.00
time (sec)	N/A	0.140	0.128	0.006	1.911	1.099	0.000	1.288	2.196	0.001

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	275	275	1817	1891	1877	2186	2088	2186	1847	0
N.S.	1	1.00	6.61	6.88	6.83	7.95	7.59	7.95	6.72	0.00
time (sec)	N/A	1.465	0.290	0.003	1.546	1.332	0.372	1.314	0.984	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	1702	1741	1740	2010	1965	2010	1702	0
N.S.	1	1.00	6.10	6.24	6.24	7.20	7.04	7.20	6.10	0.00
time (sec)	N/A	1.275	0.226	0.004	1.575	1.112	0.344	1.352	1.026	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	1539	1591	1581	1833	1775	1833	1549	0
N.S.	1	1.00	5.52	5.70	5.67	6.57	6.36	6.57	5.55	0.00
time (sec)	N/A	1.112	0.173	0.002	1.555	0.946	0.313	1.343	0.689	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	250	250	1397	1441	1437	1656	1598	1656	1404	0
N.S.	1	1.00	5.59	5.76	5.75	6.62	6.39	6.62	5.62	0.00
time (sec)	N/A	1.042	0.185	0.003	1.508	1.166	0.296	1.298	0.788	0.000

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	1241	1291	1283	1478	1428	1478	1253	0
N.S.	1	1.00	5.52	5.74	5.70	6.57	6.35	6.57	5.57	0.00
time (sec)	N/A	0.899	0.161	0.002	1.512	1.171	0.273	1.290	0.707	0.000

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	1105	1141	1135	1302	1280	1302	1106	0
N.S.	1	1.00	5.52	5.70	5.68	6.51	6.40	6.51	5.53	0.00
time (sec)	N/A	0.766	0.136	0.001	1.520	1.082	0.246	1.260	0.613	0.000

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	939	991	977	1124	1088	1124	953	0
N.S.	1	1.00	5.52	5.83	5.75	6.61	6.40	6.61	5.61	0.00
time (sec)	N/A	0.673	0.124	0.001	1.439	1.065	0.227	1.296	0.532	0.000

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	811	841	835	948	940	948	806	0
N.S.	1	1.00	5.55	5.76	5.72	6.49	6.44	6.49	5.52	0.00
time (sec)	N/A	0.529	0.088	0.003	1.465	0.776	0.209	1.316	0.341	0.000

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	660	691	686	771	748	771	664	0
N.S.	1	1.00	5.55	5.81	5.76	6.48	6.29	6.48	5.58	0.00
time (sec)	N/A	0.437	0.079	0.002	1.423	1.100	0.183	1.270	0.426	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	511	541	535	594	586	594	495	0
N.S.	1	1.00	5.55	5.88	5.82	6.46	6.37	6.46	5.38	0.00
time (sec)	N/A	0.349	0.065	0.002	1.324	1.054	0.162	1.275	0.231	0.000

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	358	391	384	417	415	417	348	0
N.S.	1	1.00	5.51	6.02	5.91	6.42	6.38	6.42	5.35	0.00
time (sec)	N/A	0.251	0.046	0.001	1.332	0.944	0.145	1.263	0.320	0.000

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	220	241	240	241	248	241	208	0
N.S.	1	1.00	5.79	6.34	6.32	6.34	6.53	6.34	5.47	0.00
time (sec)	N/A	0.016	0.029	0.001	1.434	1.158	0.119	1.265	0.129	0.000

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	108	114	12	108	0
N.S.	1	1.00	1.00	0.93	0.86	7.71	8.14	0.86	7.71	0.00
time (sec)	N/A	0.002	0.001	0.001	1.371	0.985	0.088	1.278	0.080	0.000

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	591	1022	866	868	799	961	979	0
N.S.	1	1.00	2.45	4.24	3.59	3.60	3.32	3.99	4.06	0.00
time (sec)	N/A	0.098	0.339	0.009	1.523	1.282	1.415	1.345	0.130	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	708	1066	874	1124	816	1012	3475	0
N.S.	1	1.00	2.74	4.13	3.39	4.36	3.16	3.92	13.47	0.00
time (sec)	N/A	0.473	0.240	0.016	1.395	1.307	2.653	1.265	0.352	0.000

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	708	1105	881	1233	843	924	3299	0
N.S.	1	1.00	2.70	4.22	3.36	4.71	3.22	3.53	12.59	0.00
time (sec)	N/A	0.444	0.241	0.018	1.621	1.262	5.671	1.252	0.378	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	427	1141	891	1316	867	907	2219	0
N.S.	1	1.00	1.66	4.42	3.45	5.10	3.36	3.52	8.60	0.00
time (sec)	N/A	0.441	0.179	0.022	1.730	1.247	32.528	1.261	0.385	0.000

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	359	1172	903	1365	0	1168	1494	0
N.S.	1	1.00	1.37	4.47	3.45	5.21	0.00	4.46	5.70	0.00
time (sec)	N/A	0.422	0.196	0.020	1.932	1.255	0.000	1.379	0.381	0.000

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	260	260	305	1199	912	1395	0	883	1141	0
N.S.	1	1.00	1.17	4.61	3.51	5.37	0.00	3.40	4.39	0.00
time (sec)	N/A	0.421	0.214	0.023	2.253	1.270	0.000	1.339	0.396	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	265	1222	925	1386	0	878	997	0
N.S.	1	1.00	1.01	4.66	3.53	5.29	0.00	3.35	3.81	0.00
time (sec)	N/A	0.387	0.219	0.021	2.451	1.228	0.000	1.290	0.422	0.000

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	239	1241	934	1362	0	872	950	0
N.S.	1	1.00	0.93	4.81	3.62	5.28	0.00	3.38	3.68	0.00
time (sec)	N/A	0.365	0.249	0.022	2.444	1.398	0.000	1.258	0.428	0.000

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	258	258	712	1256	945	1296	0	871	946	0
N.S.	1	1.00	2.76	4.87	3.66	5.02	0.00	3.38	3.67	0.00
time (sec)	N/A	0.341	0.317	0.019	2.584	1.387	0.000	1.292	0.263	0.000

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	257	257	708	1266	957	1216	0	867	955	0
N.S.	1	1.00	2.75	4.93	3.72	4.73	0.00	3.37	3.72	0.00
time (sec)	N/A	0.311	0.424	0.018	2.362	1.228	0.000	1.248	0.502	0.000

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	271	271	591	1271	975	1107	0	874	866	0
N.S.	1	1.00	2.18	4.69	3.60	4.08	0.00	3.23	3.20	0.00
time (sec)	N/A	0.287	0.361	0.013	2.014	1.220	0.000	1.357	0.555	0.000

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	665	866	920	920	0	951	1066	0
N.S.	1	1.00	23.75	30.93	32.86	32.86	0.00	33.96	38.07	0.00
time (sec)	N/A	0.003	0.285	0.007	2.128	1.349	0.000	1.355	0.458	0.000

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	684	867	986	986	0	961	39	0
N.S.	1	1.00	11.79	14.95	17.00	17.00	0.00	16.57	0.67	0.00
time (sec)	N/A	0.010	0.281	0.009	2.167	1.291	0.000	1.322	0.394	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	690	867	997	997	0	961	1098	0
N.S.	1	1.00	7.75	9.74	11.20	11.20	0.00	10.80	12.34	0.00
time (sec)	N/A	0.020	0.294	0.009	2.208	1.258	0.000	1.283	0.475	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	692	867	1008	1008	0	961	1109	0
N.S.	1	1.00	5.77	7.22	8.40	8.40	0.00	8.01	9.24	0.00
time (sec)	N/A	0.030	0.287	0.009	2.160	1.316	0.000	1.393	1.295	0.000

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	690	867	1019	1019	0	961	1120	0
N.S.	1	1.00	4.57	5.74	6.75	6.75	0.00	6.36	7.42	0.00
time (sec)	N/A	0.044	0.288	0.009	2.257	1.123	0.000	1.319	2.279	0.001

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	694	867	1030	1030	0	961	1131	0
N.S.	1	1.00	3.81	4.76	5.66	5.66	0.00	5.28	6.21	0.00
time (sec)	N/A	0.063	0.280	0.009	2.266	1.261	0.000	1.311	0.584	0.000

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	213	213	690	867	1041	1041	0	961	1142	0
N.S.	1	1.00	3.24	4.07	4.89	4.89	0.00	4.51	5.36	0.00
time (sec)	N/A	0.079	0.323	0.010	2.293	1.314	0.000	1.322	0.658	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	694	867	1052	1052	0	961	1153	0
N.S.	1	1.00	2.84	3.55	4.31	4.31	0.00	3.94	4.73	0.00
time (sec)	N/A	0.105	0.279	0.009	2.545	1.191	0.000	1.403	12.020	0.001

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	273	273	692	866	1063	1063	0	961	1164	0
N.S.	1	1.00	2.53	3.17	3.89	3.89	0.00	3.52	4.26	0.00
time (sec)	N/A	0.284	0.282	0.009	2.454	1.303	0.000	1.310	25.721	0.001

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	692	867	1074	1074	0	961	1175	0
N.S.	1	1.00	2.48	3.11	3.85	3.85	0.00	3.44	4.21	0.00
time (sec)	N/A	0.272	0.288	0.009	2.509	1.233	0.000	1.308	0.799	0.000

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	692	867	1085	1085	0	961	1186	0
N.S.	1	1.00	2.48	3.11	3.89	3.89	0.00	3.44	4.25	0.00
time (sec)	N/A	0.271	0.301	0.008	2.457	1.286	0.000	1.301	1.036	0.001

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	122	122	167	302	258	259	209	273	280	0
N.S.	1	1.00	1.37	2.48	2.11	2.12	1.71	2.24	2.30	0.00
time (sec)	N/A	0.053	0.072	0.005	1.350	1.090	0.500	1.260	0.074	0.000

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	115	209	177	179	136	184	189	0
N.S.	1	1.00	1.17	2.13	1.81	1.83	1.39	1.88	1.93	0.00
time (sec)	N/A	0.038	0.044	0.005	1.405	1.193	0.392	1.228	0.218	0.000

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	133	114	115	83	116	118	0
N.S.	1	1.00	1.00	1.80	1.54	1.55	1.12	1.57	1.59	0.00
time (sec)	N/A	0.030	0.028	0.004	1.301	0.667	0.299	1.200	0.065	0.000

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	43	74	60	62	44	60	62	0
N.S.	1	1.00	0.86	1.48	1.20	1.24	0.88	1.20	1.24	0.00
time (sec)	N/A	0.021	0.017	0.003	1.354	1.483	0.219	1.210	0.225	0.000

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	25	32	26	25	20	27	25	0
N.S.	1	1.00	0.96	1.23	1.00	0.96	0.77	1.04	0.96	0.00
time (sec)	N/A	0.018	0.008	0.003	1.319	0.929	0.148	1.200	0.201	0.000

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	0.00
time (sec)	N/A	0.001	0.001	0.001	1.304	0.901	0.062	1.261	0.022	0.000

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	26	37	36	26	128	46	25	0
N.S.	1	1.00	0.72	1.03	1.00	0.72	3.56	1.28	0.69	0.00
time (sec)	N/A	0.008	0.013	0.006	1.360	1.331	0.328	1.243	0.258	0.000

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	53	57	92	93	233	78	46	0
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.37	0.81	0.00
time (sec)	N/A	0.032	0.025	0.008	1.363	1.265	0.682	1.316	0.142	0.001

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	67	81	202	242	381	165	182	0
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.01	2.22	0.00
time (sec)	N/A	0.045	0.066	0.009	1.411	1.364	1.064	1.294	0.161	0.000

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	228	326	264	373	231	339	327	0
N.S.	1	1.00	1.75	2.51	2.03	2.87	1.78	2.61	2.52	0.00
time (sec)	N/A	0.139	0.075	0.010	1.398	1.211	0.886	1.267	0.245	0.000

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	104	165	230	183	267	155	245	203	0
N.S.	1	1.00	1.59	2.21	1.76	2.57	1.49	2.36	1.95	0.00
time (sec)	N/A	0.100	0.057	0.009	1.357	0.903	0.680	1.265	0.073	0.000

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	114	149	117	172	102	166	123	0
N.S.	1	1.00	1.52	1.99	1.56	2.29	1.36	2.21	1.64	0.00
time (sec)	N/A	0.062	0.036	0.009	1.357	1.816	0.506	1.264	0.077	0.000

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	47	86	67	92	60	98	71	0
N.S.	1	1.00	0.92	1.69	1.31	1.80	1.18	1.92	1.39	0.00
time (sec)	N/A	0.039	0.037	0.007	1.351	1.158	0.338	1.246	0.236	0.000

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	39	34	37	27	57	32	0
N.S.	1	1.00	1.00	1.26	1.10	1.19	0.87	1.84	1.03	0.00
time (sec)	N/A	0.021	0.011	0.006	1.337	1.310	0.185	1.262	0.041	0.000

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	12	13	10	12	12	0
N.S.	1	1.00	1.00	1.08	1.00	1.08	0.83	1.00	1.00	0.00
time (sec)	N/A	0.002	0.003	0.001	1.285	1.150	0.128	1.229	0.187	0.000

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	53	58	90	92	233	77	47	0
N.S.	1	1.00	0.95	1.04	1.61	1.64	4.16	1.38	0.84	0.00
time (sec)	N/A	0.032	0.029	0.008	1.321	1.207	0.684	1.342	0.292	0.000

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	66	82	208	241	406	153	74	0
N.S.	1	1.00	0.81	1.01	2.57	2.98	5.01	1.89	0.91	0.00
time (sec)	N/A	0.050	0.071	0.008	1.429	1.238	1.113	1.207	0.334	0.000

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	98	109	386	494	634	216	330	0
N.S.	1	1.00	0.90	1.00	3.54	4.53	5.82	1.98	3.03	0.00
time (sec)	N/A	0.076	0.074	0.013	1.555	1.234	1.717	1.347	0.396	0.000

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	303	464	364	548	340	362	441	0
N.S.	1	1.00	1.92	2.94	2.30	3.47	2.15	2.29	2.79	0.00
time (sec)	N/A	0.202	0.113	0.011	1.474	1.259	2.154	1.276	0.274	0.000

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	230	346	271	416	258	264	291	0
N.S.	1	1.00	1.73	2.60	2.04	3.13	1.94	1.98	2.19	0.00
time (sec)	N/A	0.121	0.073	0.009	1.476	1.205	1.650	1.288	0.099	0.000

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	167	245	191	291	185	183	196	0
N.S.	1	1.00	1.62	2.38	1.85	2.83	1.80	1.78	1.90	0.00
time (sec)	N/A	0.087	0.056	0.008	1.386	0.771	1.246	1.354	0.099	0.000

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	114	160	125	188	128	112	130	0
N.S.	1	1.00	1.46	2.05	1.60	2.41	1.64	1.44	1.67	0.00
time (sec)	N/A	0.055	0.039	0.009	1.340	1.403	0.827	1.276	0.109	0.000

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	48	92	80	100	80	69	77	0
N.S.	1	1.00	0.81	1.56	1.36	1.69	1.36	1.17	1.31	0.00
time (sec)	N/A	0.038	0.025	0.006	1.327	1.437	0.450	1.299	0.228	0.000

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	26	35	38	38	39	24	39	0
N.S.	1	1.00	0.93	1.25	1.36	1.36	1.39	0.86	1.39	0.00
time (sec)	N/A	0.003	0.009	0.004	1.359	0.987	0.263	1.251	0.029	0.000

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	24	26	12	26	0
N.S.	1	1.00	1.00	0.93	0.86	1.71	1.86	0.86	1.86	0.00
time (sec)	N/A	0.002	0.003	0.000	1.335	1.238	0.182	1.203	0.024	0.000

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	67	81	202	242	381	165	183	0
N.S.	1	1.00	0.82	0.99	2.46	2.95	4.65	2.01	2.23	0.00
time (sec)	N/A	0.046	0.053	0.010	1.452	1.197	1.074	1.360	0.299	0.000

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	97	108	386	495	632	217	329	0
N.S.	1	1.00	0.88	0.98	3.51	4.50	5.75	1.97	2.99	0.00
time (sec)	N/A	0.074	0.105	0.013	1.594	1.611	1.720	1.265	0.395	0.000

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	128	140	594	760	881	345	542	0
N.S.	1	1.00	0.90	0.98	4.15	5.31	6.16	2.41	3.79	0.00
time (sec)	N/A	0.102	0.116	0.012	1.551	1.182	2.419	1.277	0.526	0.001

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	232	232	584	1035	786	1093	0	723	784	0
N.S.	1	1.00	2.52	4.46	3.39	4.71	0.00	3.12	3.38	0.00
time (sec)	N/A	0.357	0.271	0.017	2.198	0.956	0.000	1.321	0.257	0.000

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	209	209	474	845	649	852	0	581	649	0
N.S.	1	1.00	2.27	4.04	3.11	4.08	0.00	2.78	3.11	0.00
time (sec)	N/A	0.278	0.204	0.013	1.959	0.972	0.000	1.268	0.430	0.000

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	194	194	308	672	535	625	0	467	460	0
N.S.	1	1.00	1.59	3.46	2.76	3.22	0.00	2.41	2.37	0.00
time (sec)	N/A	0.209	0.162	0.009	1.662	1.120	0.000	1.298	0.383	0.000

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	271	357	398	398	0	369	378	0
N.S.	1	1.00	9.68	12.75	14.21	14.21	0.00	13.18	13.50	0.00
time (sec)	N/A	0.003	0.090	0.007	1.606	1.380	0.000	1.282	0.146	0.000

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	205	265	326	326	354	271	39	0
N.S.	1	1.00	3.53	4.57	5.62	5.62	6.10	4.67	0.67	0.00
time (sec)	N/A	0.011	0.060	0.008	1.579	1.312	54.908	1.317	0.277	0.000

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	144	186	247	247	267	184	237	0
N.S.	1	1.00	1.62	2.09	2.78	2.78	3.00	2.07	2.66	0.00
time (sec)	N/A	0.019	0.048	0.006	1.517	1.064	9.645	1.294	0.110	0.000

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	94	122	182	182	196	114	176	0
N.S.	1	1.00	1.02	1.33	1.98	1.98	2.13	1.24	1.91	0.00
time (sec)	N/A	0.057	0.029	0.007	1.469	0.957	3.101	1.240	0.099	0.000

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	55	71	131	131	139	61	129	0
N.S.	1	1.00	0.85	1.09	2.02	2.02	2.14	0.94	1.98	0.00
time (sec)	N/A	0.040	0.024	0.005	1.456	1.256	1.391	1.222	0.086	0.000

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	35	94	94	100	25	96	0
N.S.	1	1.00	0.71	0.92	2.47	2.47	2.63	0.66	2.53	0.00
time (sec)	N/A	0.022	0.010	0.006	1.380	1.203	0.731	1.309	0.226	0.000

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	79	85	12	81	0
N.S.	1	1.00	1.00	0.93	0.86	5.64	6.07	0.86	5.79	0.00
time (sec)	N/A	0.002	0.003	0.000	1.365	1.172	0.456	1.257	0.223	0.000

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	196	192	1418	1589	1776	703	1299	0
N.S.	1	1.00	0.97	0.95	7.02	7.87	8.79	3.48	6.43	0.00
time (sec)	N/A	0.169	0.097	0.016	2.975	1.545	4.488	1.333	0.873	0.000

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	231	231	213	223	1881	2264	2336	714	1738	0
N.S.	1	1.00	0.92	0.97	8.14	9.80	10.11	3.09	7.52	0.00
time (sec)	N/A	0.270	0.243	0.020	3.883	1.461	7.745	1.370	1.386	0.000

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	276	276	254	265	2399	3016	2917	1029	2224	0
N.S.	1	1.00	0.92	0.96	8.69	10.93	10.57	3.73	8.06	0.00
time (sec)	N/A	0.356	0.205	0.020	5.223	1.588	20.658	1.493	1.911	0.000

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	156	156	123	273	259	338	314	641	137	315
N.S.	1	1.00	0.79	1.75	1.66	2.17	2.01	4.11	0.88	2.02
time (sec)	N/A	0.063	0.145	0.008	1.425	1.215	5.117	1.378	0.081	0.096

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	101	186	181	245	223	470	112	213
N.S.	1	1.00	0.78	1.44	1.40	1.90	1.73	3.64	0.87	1.65
time (sec)	N/A	0.052	0.095	0.007	1.365	1.692	4.195	1.249	0.225	0.069

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	79	116	118	164	146	322	87	132
N.S.	1	1.00	0.79	1.16	1.18	1.64	1.46	3.22	0.87	1.32
time (sec)	N/A	0.036	0.063	0.005	1.374	1.164	3.336	1.275	0.065	0.052

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	61	63	68	99	85	200	68	72
N.S.	1	1.00	0.86	0.89	0.96	1.39	1.20	2.82	0.96	1.01
time (sec)	N/A	0.025	0.038	0.007	1.350	1.394	2.695	1.291	0.240	0.038

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	30	27	33	46	36	100	29	33
N.S.	1	1.00	0.71	0.64	0.79	1.10	0.86	2.38	0.69	0.79
time (sec)	N/A	0.014	0.019	0.004	1.294	1.432	2.123	1.372	0.043	0.021

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	12	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.75	0.75	0.75	1.00
time (sec)	N/A	0.002	0.004	0.002	1.350	1.278	0.060	1.342	0.021	0.006

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	62	92	0	143	61	62	50	72
N.S.	1	1.00	1.00	1.48	0.00	2.31	0.98	1.00	0.81	1.16
time (sec)	N/A	0.053	0.038	0.011	0.000	1.362	4.389	1.265	0.066	0.069

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	69	64	0	232	573	72	61	91
N.S.	1	1.00	0.99	0.91	0.00	3.31	8.19	1.03	0.87	1.30
time (sec)	N/A	0.030	0.081	0.013	0.000	1.435	58.580	1.380	0.242	0.215

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	52	111	0	456	0	126	135	125
N.S.	1	1.00	0.47	1.01	0.00	4.15	0.00	1.15	1.23	1.14
time (sec)	N/A	0.078	0.016	0.013	0.000	1.059	0.000	1.331	0.302	0.423

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	52	170	0	785	0	207	207	176
N.S.	1	1.00	0.36	1.16	0.00	5.38	0.00	1.42	1.42	1.21
time (sec)	N/A	0.098	0.014	0.015	0.000	1.092	0.000	1.350	0.374	0.753

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	182	182	52	248	0	1176	0	311	297	226
N.S.	1	1.00	0.29	1.36	0.00	6.46	0.00	1.71	1.63	1.24
time (sec)	N/A	0.123	0.015	0.016	0.000	0.886	0.000	1.386	0.217	1.067

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	52	337	0	1673	0	432	401	317
N.S.	1	1.00	0.24	1.55	0.00	7.67	0.00	1.98	1.84	1.45
time (sec)	N/A	0.151	0.016	0.018	0.000	1.345	0.000	1.473	0.490	1.515

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	123	273	259	418	763	1084	137	315
N.S.	1	1.00	0.78	1.73	1.64	2.65	4.83	6.86	0.87	1.99
time (sec)	N/A	0.053	0.146	0.007	1.346	1.366	26.419	1.532	0.244	0.099

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	101	186	181	311	559	807	112	213
N.S.	1	1.00	0.78	1.44	1.40	2.41	4.33	6.26	0.87	1.65
time (sec)	N/A	0.042	0.097	0.007	1.355	1.071	20.089	1.403	0.242	0.075

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	79	116	118	216	386	566	87	132
N.S.	1	1.00	0.79	1.16	1.18	2.16	3.86	5.66	0.87	1.32
time (sec)	N/A	0.034	0.069	0.006	1.359	0.777	14.380	1.311	0.249	0.058

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	61	63	68	137	240	360	68	72
N.S.	1	1.00	0.86	0.89	0.96	1.93	3.38	5.07	0.96	1.01
time (sec)	N/A	0.024	0.043	0.006	1.378	1.239	9.606	1.413	0.059	0.039

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	30	27	33	69	146	192	29	33
N.S.	1	1.00	0.71	0.64	0.79	1.64	3.48	4.57	0.69	0.79
time (sec)	N/A	0.014	0.021	0.003	1.374	1.270	0.666	1.227	0.215	0.024

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	28	12	58	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.75	0.75	3.62	0.75	1.00
time (sec)	N/A	0.001	0.005	0.002	1.359	1.196	0.061	1.244	0.017	0.007

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	77	167	0	188	82	105	93	90
N.S.	1	1.00	0.90	1.94	0.00	2.19	0.95	1.22	1.08	1.05
time (sec)	N/A	0.046	0.073	0.009	0.000	1.144	14.700	1.287	0.075	0.124

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	50	148	0	210	0	113	109	107
N.S.	1	1.00	0.59	1.74	0.00	2.47	0.00	1.33	1.28	1.26
time (sec)	N/A	0.038	0.014	0.013	0.000	1.736	0.000	1.299	0.105	0.257

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	90	121	0	383	0	108	135	116
N.S.	1	1.00	0.90	1.21	0.00	3.83	0.00	1.08	1.35	1.16
time (sec)	N/A	0.048	0.100	0.013	0.000	1.005	0.000	1.357	0.284	0.379

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	52	163	0	666	0	185	209	166
N.S.	1	1.00	0.38	1.20	0.00	4.90	0.00	1.36	1.54	1.22
time (sec)	N/A	0.057	0.017	0.015	0.000	1.375	0.000	1.400	0.338	0.620

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	52	222	0	1043	0	285	296	226
N.S.	1	1.00	0.30	1.29	0.00	6.06	0.00	1.66	1.72	1.31
time (sec)	N/A	0.074	0.016	0.017	0.000	1.545	0.000	1.467	0.371	1.008

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	52	300	0	1492	0	410	398	317
N.S.	1	1.00	0.25	1.44	0.00	7.17	0.00	1.97	1.91	1.52
time (sec)	N/A	0.093	0.017	0.017	0.000	1.308	0.000	1.392	0.473	1.984

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	123	273	259	497	1292	1599	137	315
N.S.	1	1.00	0.78	1.73	1.64	3.15	8.18	10.12	0.87	1.99
time (sec)	N/A	0.051	0.114	0.005	1.362	1.287	43.079	1.522	0.267	0.107

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	101	186	181	377	960	1204	112	213
N.S.	1	1.00	0.78	1.44	1.40	2.92	7.44	9.33	0.87	1.65
time (sec)	N/A	0.043	0.110	0.007	1.332	1.379	33.636	1.447	0.234	0.078

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	100	79	116	118	268	549	857	87	132
N.S.	1	1.00	0.79	1.16	1.18	2.68	5.49	8.57	0.87	1.32
time (sec)	N/A	0.031	0.072	0.007	1.402	1.541	4.609	1.579	0.076	0.060

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	61	63	68	174	355	558	68	72
N.S.	1	1.00	0.86	0.89	0.96	2.45	5.00	7.86	0.96	1.01
time (sec)	N/A	0.023	0.046	0.007	1.381	1.231	3.579	1.764	0.067	0.044

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	30	27	33	93	194	306	29	33
N.S.	1	1.00	0.71	0.64	0.79	2.21	4.62	7.29	0.69	0.79
time (sec)	N/A	0.015	0.024	0.003	1.395	1.215	2.363	1.615	0.045	0.027

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	39	12	95	12	16
N.S.	1	1.00	1.00	0.81	0.75	2.44	0.75	5.94	0.75	1.00
time (sec)	N/A	0.001	0.006	0.002	1.296	1.215	0.065	1.781	0.021	0.007

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	105	263	0	290	121	171	130	130
N.S.	1	1.00	0.94	2.35	0.00	2.59	1.08	1.53	1.16	1.16
time (sec)	N/A	0.058	0.155	0.008	0.000	1.247	27.012	1.598	0.083	0.118

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	50	258	0	330	0	181	161	187
N.S.	1	1.00	0.45	2.35	0.00	3.00	0.00	1.65	1.46	1.70
time (sec)	N/A	0.055	0.015	0.014	0.000	1.252	0.000	1.277	0.124	0.367

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	52	238	0	344	0	171	199	155
N.S.	1	1.00	0.44	2.00	0.00	2.89	0.00	1.44	1.67	1.30
time (sec)	N/A	0.049	0.018	0.016	0.000	1.565	0.000	1.242	0.160	0.479

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	119	204	0	563	0	161	222	155
N.S.	1	1.00	0.94	1.62	0.00	4.47	0.00	1.28	1.76	1.23
time (sec)	N/A	0.050	0.152	0.015	0.000	1.197	0.000	0.985	0.365	0.619

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	162	52	246	0	894	0	259	309	226
N.S.	1	1.00	0.32	1.52	0.00	5.52	0.00	1.60	1.91	1.40
time (sec)	N/A	0.070	0.018	0.015	0.000	1.419	0.000	1.091	0.412	1.076

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	198	198	52	305	0	1337	0	380	411	307
N.S.	1	1.00	0.26	1.54	0.00	6.75	0.00	1.92	2.08	1.55
time (sec)	N/A	0.092	0.018	0.018	0.000	1.491	0.000	1.251	0.503	1.429

Problem 1305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	51	30	29	33	104	29	29	35
N.S.	1	1.00	1.46	0.86	0.83	0.94	2.97	0.83	0.83	1.00
time (sec)	N/A	0.008	0.030	0.010	2.970	1.372	1.498	0.962	0.062	0.046

Problem 1306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	28	40	43	46	167	37	45	43
N.S.	1	1.00	0.50	0.71	0.77	0.82	2.98	0.66	0.80	0.77
time (sec)	N/A	0.013	0.005	0.010	3.033	1.122	2.609	1.036	0.041	0.061

Problem 1307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	123	273	283	261	728	283	137	315
N.S.	1	1.00	0.80	1.77	1.84	1.69	4.73	1.84	0.89	2.05
time (sec)	N/A	0.051	0.088	0.007	1.383	1.263	79.908	1.066	0.069	0.099

Problem 1308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	101	186	204	182	532	204	112	213
N.S.	1	1.00	0.80	1.46	1.61	1.43	4.19	1.61	0.88	1.68
time (sec)	N/A	0.041	0.088	0.006	1.381	1.180	56.898	0.959	0.241	0.068

Problem 1309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	79	116	137	115	366	137	87	132
N.S.	1	1.00	0.82	1.21	1.43	1.20	3.81	1.43	0.91	1.38
time (sec)	N/A	0.031	0.056	0.006	1.378	0.936	37.064	1.010	0.264	0.052

Problem 1310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	60	63	82	64	231	82	68	72
N.S.	1	1.00	0.87	0.91	1.19	0.93	3.35	1.19	0.99	1.04
time (sec)	N/A	0.021	0.035	0.005	1.369	1.461	20.940	1.099	0.067	0.038

Problem 1311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	29	26	39	25	121	39	28	32
N.S.	1	1.00	0.72	0.65	0.98	0.62	3.02	0.98	0.70	0.80
time (sec)	N/A	0.013	0.017	0.003	1.345	1.060	4.778	0.884	0.048	0.025

Problem 1312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86	1.00
time (sec)	N/A	0.001	0.003	0.001	1.298	1.224	0.063	0.909	0.021	0.006

Problem 1313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	37	0	119	44	38	38	57
N.S.	1	1.00	1.00	0.79	0.00	2.53	0.94	0.81	0.81	1.21
time (sec)	N/A	0.020	0.015	0.006	0.000	1.204	5.409	0.879	0.273	0.047

Problem 1314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	76	77	0	280	0	87	74	98
N.S.	1	1.00	1.00	1.01	0.00	3.68	0.00	1.14	0.97	1.29
time (sec)	N/A	0.027	0.071	0.010	0.000	1.294	0.000	1.030	0.094	0.204

Problem 1315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	50	115	0	549	0	148	142	124
N.S.	1	1.00	0.44	1.01	0.00	4.82	0.00	1.30	1.25	1.09
time (sec)	N/A	0.037	0.011	0.009	0.000	1.574	0.000	0.933	0.332	0.226

Problem 1316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	50	147	0	884	0	231	218	173
N.S.	1	1.00	0.34	1.00	0.00	6.01	0.00	1.57	1.48	1.18
time (sec)	N/A	0.050	0.011	0.008	0.000	1.383	0.000	1.020	0.395	0.273

Problem 1317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	50	179	0	1325	0	331	307	223
N.S.	1	1.00	0.28	0.99	0.00	7.36	0.00	1.84	1.71	1.24
time (sec)	N/A	0.065	0.012	0.010	0.000	1.171	0.000	1.134	0.455	0.440

Problem 1318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	123	273	267	271	243	350	192	315
N.S.	1	1.00	0.81	1.80	1.76	1.78	1.60	2.30	1.26	2.07
time (sec)	N/A	0.049	0.120	0.006	1.556	1.092	47.937	1.008	0.077	0.071

Problem 1319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	101	186	189	192	168	240	153	213
N.S.	1	1.00	0.82	1.51	1.54	1.56	1.37	1.95	1.24	1.73
time (sec)	N/A	0.037	0.080	0.007	1.353	1.544	32.865	1.082	0.056	0.069

Problem 1320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	78	116	125	124	109	152	114	131
N.S.	1	1.00	0.83	1.23	1.33	1.32	1.16	1.62	1.21	1.39
time (sec)	N/A	0.030	0.056	0.006	1.381	1.261	21.510	1.049	0.082	0.054

Problem 1321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	59	63	75	73	65	84	67	71
N.S.	1	1.00	0.88	0.94	1.12	1.09	0.97	1.25	1.00	1.06
time (sec)	N/A	0.021	0.031	0.005	1.300	1.134	13.292	1.041	0.264	0.041

Problem 1322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	27	26	37	35	60	34	25	29
N.S.	1	1.00	0.71	0.68	0.97	0.92	1.58	0.89	0.66	0.76
time (sec)	N/A	0.014	0.019	0.003	1.332	1.129	0.614	1.027	0.054	0.026

Problem 1323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	20	12	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	1.43	0.86	0.86	0.86	1.00
time (sec)	N/A	0.002	0.004	0.003	1.331	1.205	0.064	1.020	0.024	0.009

Problem 1324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	46	68	0	214	60	69	57	79
N.S.	1	1.00	0.67	0.99	0.00	3.10	0.87	1.00	0.83	1.14
time (sec)	N/A	0.027	0.010	0.010	0.000	1.428	11.487	0.935	0.270	0.087

Problem 1325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	99	99	48	101	0	423	0	143	123	115
N.S.	1	1.00	0.48	1.02	0.00	4.27	0.00	1.44	1.24	1.16
time (sec)	N/A	0.038	0.013	0.014	0.000	1.318	0.000	1.025	0.186	0.288

Problem 1326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	50	179	0	782	0	234	205	163
N.S.	1	1.00	0.36	1.28	0.00	5.59	0.00	1.67	1.46	1.16
time (sec)	N/A	0.052	0.013	0.018	0.000	1.278	0.000	1.030	0.444	0.495

Problem 1327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	173	173	50	292	0	1204	0	326	294	223
N.S.	1	1.00	0.29	1.69	0.00	6.96	0.00	1.88	1.70	1.29
time (sec)	N/A	0.066	0.015	0.022	0.000	1.531	0.000	1.312	0.541	0.677

Problem 1328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	123	273	265	283	196	335	229	315
N.S.	1	1.00	0.81	1.80	1.74	1.86	1.29	2.20	1.51	2.07
time (sec)	N/A	0.048	0.119	0.007	1.380	1.357	59.926	0.913	0.083	0.074

Problem 1329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	125	125	101	186	187	203	136	229	175	213
N.S.	1	1.00	0.81	1.49	1.50	1.62	1.09	1.83	1.40	1.70
time (sec)	N/A	0.039	0.084	0.007	1.462	1.130	43.586	1.016	0.302	0.076

Problem 1330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	96	76	115	122	136	461	141	128	130
N.S.	1	1.00	0.79	1.20	1.27	1.42	4.80	1.47	1.33	1.35
time (sec)	N/A	0.031	0.058	0.007	1.374	1.183	1.436	0.995	0.088	0.067

Problem 1331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	62	62	72	85	265	72	68	72
N.S.	1	1.00	0.93	0.93	1.07	1.27	3.96	1.07	1.01	1.07
time (sec)	N/A	0.021	0.035	0.007	1.394	1.242	1.265	1.105	0.072	0.051

Problem 1332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	29	26	28	46	124	28	29	32
N.S.	1	1.00	0.72	0.65	0.70	1.15	3.10	0.70	0.72	0.80
time (sec)	N/A	0.014	0.020	0.003	1.279	1.113	1.122	1.021	0.246	0.030

Problem 1333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	31	14	12	12	16
N.S.	1	1.00	1.00	0.81	0.75	1.94	0.88	0.75	0.75	1.00
time (sec)	N/A	0.001	0.004	0.004	1.363	1.319	0.066	0.958	0.026	0.009

Problem 1334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	48	90	0	398	83	113	100	97
N.S.	1	1.00	0.52	0.97	0.00	4.28	0.89	1.22	1.08	1.04
time (sec)	N/A	0.039	0.010	0.013	0.000	1.440	13.575	1.156	0.333	0.133

Problem 1335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	124	124	50	125	0	782	0	216	161	157
N.S.	1	1.00	0.40	1.01	0.00	6.31	0.00	1.74	1.30	1.27
time (sec)	N/A	0.050	0.015	0.017	0.000	1.424	0.000	1.111	0.377	0.356

Problem 1336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	52	206	0	1226	0	298	243	223
N.S.	1	1.00	0.31	1.23	0.00	7.34	0.00	1.78	1.46	1.34
time (sec)	N/A	0.062	0.016	0.019	0.000	1.361	0.000	1.198	0.285	0.600

Problem 1337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	52	319	0	1840	0	432	334	304
N.S.	1	1.00	0.26	1.60	0.00	9.20	0.00	2.16	1.67	1.52
time (sec)	N/A	0.134	0.019	0.023	0.000	1.434	0.000	1.199	0.641	0.866

Problem 1338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	18	95	66	637	17	22
N.S.	1	1.00	1.14	1.05	0.82	4.32	3.00	28.95	0.77	1.00
time (sec)	N/A	0.005	0.020	0.004	1.373	1.136	1.215	1.262	0.050	0.052

Problem 1339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	18	75	66	495	17	22
N.S.	1	1.00	1.14	1.05	0.82	3.41	3.00	22.50	0.77	1.00
time (sec)	N/A	0.005	0.012	0.003	1.364	1.159	1.063	1.274	0.034	0.043

Problem 1340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	374	67	73	374	17	22
N.S.	1	1.00	1.14	1.05	17.00	3.05	3.32	17.00	0.77	1.00
time (sec)	N/A	0.004	0.014	0.003	1.460	0.827	1.542	0.973	0.028	0.048

Problem 1341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	18	56	73	266	17	22
N.S.	1	1.00	1.14	1.05	0.82	2.55	3.32	12.09	0.77	1.00
time (sec)	N/A	0.004	0.015	0.003	1.431	1.401	1.646	1.176	0.028	0.056

Problem 1342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	18	45	73	178	17	22
N.S.	1	1.00	1.14	1.05	0.82	2.05	3.32	8.09	0.77	1.00
time (sec)	N/A	0.004	0.014	0.002	1.381	1.435	1.615	0.956	0.028	0.057

Problem 1343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	25	23	18	34	80	106	17	22
N.S.	1	1.00	1.14	1.05	0.82	1.55	3.64	4.82	0.77	1.00
time (sec)	N/A	0.004	0.014	0.004	1.411	1.258	4.108	0.987	0.028	0.056

Problem 1344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	26	23	18	23	53	54	17	22
N.S.	1	1.00	1.18	1.05	0.82	1.05	2.41	2.45	0.77	1.00
time (sec)	N/A	0.004	0.014	0.003	1.393	1.311	8.308	1.070	0.029	0.056

Problem 1345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	24	23	18	18	29	18	17	20
N.S.	1	1.00	1.20	1.15	0.90	0.90	1.45	0.90	0.85	1.00
time (sec)	N/A	0.004	0.008	0.003	1.383	1.189	15.476	1.028	0.030	0.058

Problem 1346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	24	23	18	29	48	18	17	27
N.S.	1	1.00	1.20	1.15	0.90	1.45	2.40	0.90	0.85	1.35
time (sec)	N/A	0.005	0.010	0.002	1.340	1.099	39.429	0.811	0.030	0.058

Problem 1347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	22	21	21	27	22	10	14
N.S.	1	1.00	1.00	1.57	1.50	1.50	1.93	1.57	0.71	1.00
time (sec)	N/A	0.004	0.003	0.008	1.350	1.215	0.659	0.870	0.054	0.016

Problem 1348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	19	18	18	61	18	15	25
N.S.	1	1.00	1.00	0.76	0.72	0.72	2.44	0.72	0.60	1.00
time (sec)	N/A	0.009	0.009	0.006	3.003	1.047	1.124	0.980	0.060	0.030

Problem 1349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	104	84	86	86	170	87	104	115
N.S.	1	1.00	1.24	1.00	1.02	1.02	2.02	1.04	1.24	1.37
time (sec)	N/A	0.038	0.048	0.006	3.003	1.333	2.257	1.045	0.068	0.121

Problem 1350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	18	15	19	19	114	26	14	22
N.S.	1	1.00	0.67	0.56	0.70	0.70	4.22	0.96	0.52	0.81
time (sec)	N/A	0.005	0.009	0.003	1.349	1.240	1.093	0.945	0.255	0.014

Problem 1351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	23	20	28	24	146	38	21	31
N.S.	1	1.00	0.61	0.53	0.74	0.63	3.84	1.00	0.55	0.82
time (sec)	N/A	0.007	0.010	0.003	1.360	1.322	1.518	0.860	0.046	0.016

Problem 1352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	139	106	161	0	570	0	196	204	191
N.S.	1	1.00	0.76	1.16	0.00	4.10	0.00	1.41	1.47	1.37
time (sec)	N/A	0.108	0.076	0.008	0.000	1.265	0.000	1.100	0.211	0.231

Problem 1353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	154	160	0	900	0	207	206	190
N.S.	1	1.00	1.10	1.14	0.00	6.43	0.00	1.48	1.47	1.36
time (sec)	N/A	0.074	0.075	0.007	0.000	1.583	0.000	0.959	0.369	0.213

Problem 1354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	194	858	0	702	0	1107	-1	198
N.S.	1	1.00	0.84	3.73	0.00	3.05	0.00	4.81	-0.00	0.86
time (sec)	N/A	0.160	1.465	0.012	0.000	1.736	0.000	1.838	0.000	0.311

Problem 1355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	192	190	645	0	540	0	726	-1	176
N.S.	1	1.00	0.99	3.36	0.00	2.81	0.00	3.78	-0.01	0.92
time (sec)	N/A	0.095	0.598	0.008	0.000	1.421	0.000	1.639	0.000	0.294

Problem 1356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	151	460	0	410	0	438	-1	154
N.S.	1	1.00	0.98	2.99	0.00	2.66	0.00	2.84	-0.01	1.00
time (sec)	N/A	0.072	0.435	0.008	0.000	1.369	0.000	1.377	0.000	0.261

Problem 1357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	118	305	0	300	0	232	88	129
N.S.	1	1.00	1.02	2.63	0.00	2.59	0.00	2.00	0.76	1.11
time (sec)	N/A	0.055	0.277	0.007	0.000	1.503	0.000	1.254	0.136	0.207

Problem 1358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	117	107	0	236	0	93	260	104
N.S.	1	1.00	1.62	1.49	0.00	3.28	0.00	1.29	3.61	1.44
time (sec)	N/A	0.038	0.104	0.006	0.000	1.322	0.000	1.095	4.005	0.306

Problem 1359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	99	0	0	241	0	131	-1	66
N.S.	1	1.00	1.50	0.00	0.00	3.65	0.00	1.98	-0.02	1.00
time (sec)	N/A	0.033	0.227	0.082	0.000	1.477	0.000	1.192	0.000	0.100

Problem 1360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	65	0	152	27	32
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	4.75	0.84	1.00
time (sec)	N/A	0.003	0.012	0.005	0.000	1.530	0.000	1.433	0.717	0.038

Problem 1361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	175	0	447	127	57
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	6.77	1.92	0.86
time (sec)	N/A	0.009	0.020	0.006	0.000	2.318	0.000	1.444	0.822	0.101

Problem 1362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	337	0	689	203	83
N.S.	1	1.00	0.76	1.04	0.00	3.34	0.00	6.82	2.01	0.82
time (sec)	N/A	0.016	0.039	0.009	0.000	3.931	0.000	1.582	0.966	0.104

Problem 1363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	532	0	989	292	109
N.S.	1	1.00	0.87	1.26	0.00	3.91	0.00	7.27	2.15	0.80
time (sec)	N/A	0.029	0.054	0.011	0.000	13.035	0.000	2.041	1.184	0.111

Problem 1364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	170	256	0	781	0	1345	397	117
N.S.	1	1.00	0.99	1.50	0.00	4.57	0.00	7.87	2.32	0.68
time (sec)	N/A	0.041	0.077	0.013	0.000	27.264	0.000	2.383	1.432	0.141

Problem 1365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	187	853	0	702	0	1740	-1	197
N.S.	1	1.00	0.82	3.76	0.00	3.09	0.00	7.67	-0.00	0.87
time (sec)	N/A	0.130	1.780	0.006	0.000	1.522	0.000	2.297	0.000	0.413

Problem 1366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	189	189	193	640	0	534	0	1071	-1	176
N.S.	1	1.00	1.02	3.39	0.00	2.83	0.00	5.67	-0.01	0.93
time (sec)	N/A	0.090	0.567	0.007	0.000	1.352	0.000	1.892	0.000	0.343

Problem 1367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	152	459	0	410	0	576	-1	153
N.S.	1	1.00	1.01	3.04	0.00	2.72	0.00	3.81	-0.01	1.01
time (sec)	N/A	0.068	0.445	0.006	0.000	1.479	0.000	1.606	0.000	0.239

Problem 1368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	109	308	0	306	0	233	-1	134
N.S.	1	1.00	0.96	2.73	0.00	2.71	0.00	2.06	-0.01	1.19
time (sec)	N/A	0.051	0.291	0.007	0.000	1.024	0.000	1.246	0.000	0.176

Problem 1369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	71	0	0	311	0	204	-1	159
N.S.	1	1.00	0.72	0.00	0.00	3.17	0.00	2.08	-0.01	1.62
time (sec)	N/A	0.047	0.053	0.079	0.000	1.392	0.000	1.499	0.000	0.557

Problem 1370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	73	0	0	325	0	455	-1	85
N.S.	1	1.00	0.79	0.00	0.00	3.53	0.00	4.95	-0.01	0.92
time (sec)	N/A	0.040	0.050	0.079	0.000	2.061	0.000	1.744	0.000	0.140

Problem 1371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	104	0	374	27	32
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	11.69	0.84	1.00
time (sec)	N/A	0.003	0.015	0.005	0.000	2.261	0.000	1.695	0.796	0.055

Problem 1372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	235	0	1024	178	51
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	15.52	2.70	0.77
time (sec)	N/A	0.008	0.025	0.005	0.000	3.925	0.000	2.122	0.926	0.122

Problem 1373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	426	0	1394	268	73
N.S.	1	1.00	0.76	1.04	0.00	4.22	0.00	13.80	2.65	0.72
time (sec)	N/A	0.016	0.045	0.007	0.000	13.562	0.000	2.937	1.113	0.136

Problem 1374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	649	0	1823	376	95
N.S.	1	1.00	0.87	1.26	0.00	4.77	0.00	13.40	2.76	0.70
time (sec)	N/A	0.028	0.064	0.011	0.000	28.364	0.000	3.282	1.333	0.150

Problem 1375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	262	262	209	1089	0	882	0	3120	-1	220
N.S.	1	1.00	0.80	4.16	0.00	3.37	0.00	11.91	-0.00	0.84
time (sec)	N/A	0.149	2.529	0.008	0.000	1.425	0.000	3.533	0.000	0.504

Problem 1376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	187	848	0	702	0	1962	-1	198
N.S.	1	1.00	0.83	3.79	0.00	3.13	0.00	8.76	-0.00	0.88
time (sec)	N/A	0.119	1.468	0.008	0.000	0.894	0.000	2.326	0.000	0.429

Problem 1377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	191	641	0	540	0	1083	-1	175
N.S.	1	1.00	1.03	3.45	0.00	2.90	0.00	5.82	-0.01	0.94
time (sec)	N/A	0.086	0.580	0.008	0.000	1.301	0.000	1.926	0.000	0.285

Problem 1378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	139	465	0	412	0	446	-1	160
N.S.	1	1.00	0.94	3.14	0.00	2.78	0.00	3.01	-0.01	1.08
time (sec)	N/A	0.066	0.407	0.008	0.000	1.250	0.000	1.431	0.000	0.198

Problem 1379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	71	0	0	439	0	287	-1	151
N.S.	1	1.00	0.51	0.00	0.00	3.18	0.00	2.08	-0.01	1.09
time (sec)	N/A	0.063	0.068	0.083	0.000	1.423	0.000	2.018	0.000	0.281

Problem 1380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	73	0	0	475	0	650	-1	227
N.S.	1	1.00	0.57	0.00	0.00	3.71	0.00	5.08	-0.01	1.77
time (sec)	N/A	0.062	0.062	0.080	0.000	1.933	0.000	2.186	0.000	0.992

Problem 1381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	120	120	73	0	0	463	0	1025	-1	119
N.S.	1	1.00	0.61	0.00	0.00	3.86	0.00	8.54	-0.01	0.99
time (sec)	N/A	0.052	0.070	0.082	0.000	2.685	0.000	2.477	0.000	0.141

Problem 1382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	138	0	706	27	32
N.S.	1	1.00	1.00	0.84	0.00	4.31	0.00	22.06	0.84	1.00
time (sec)	N/A	0.003	0.018	0.003	0.000	3.847	0.000	2.541	0.971	0.057

Problem 1383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	295	0	1826	229	57
N.S.	1	1.00	0.70	0.82	0.00	4.47	0.00	27.67	3.47	0.86
time (sec)	N/A	0.009	0.031	0.005	0.000	13.617	0.000	3.803	1.140	0.124

Problem 1384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	513	0	2316	333	73
N.S.	1	1.00	0.76	1.04	0.00	5.08	0.00	22.93	3.30	0.72
time (sec)	N/A	0.016	0.052	0.007	0.000	31.126	0.000	4.592	1.355	0.165

Problem 1385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	765	0	2868	459	95
N.S.	1	1.00	0.87	1.26	0.00	5.62	0.00	21.09	3.38	0.70
time (sec)	N/A	0.028	0.074	0.010	0.000	55.759	0.000	6.088	1.615	0.148

Problem 1386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	189	650	0	542	0	268	-1	172
N.S.	1	1.00	1.03	3.55	0.00	2.96	0.00	1.46	-0.01	0.94
time (sec)	N/A	0.097	0.656	0.007	0.000	1.099	0.000	1.229	0.000	0.198

Problem 1387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	148	150	465	0	412	0	198	-1	160
N.S.	1	1.00	1.01	3.14	0.00	2.78	0.00	1.34	-0.01	1.08
time (sec)	N/A	0.072	0.524	0.009	0.000	1.017	0.000	1.270	0.000	0.212

Problem 1388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	119	308	0	306	0	139	-1	134
N.S.	1	1.00	1.05	2.73	0.00	2.71	0.00	1.23	-0.01	1.19
time (sec)	N/A	0.050	0.365	0.007	0.000	1.362	0.000	0.945	0.000	0.191

Problem 1389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	103	107	0	235	0	97	261	106
N.S.	1	1.00	1.41	1.47	0.00	3.22	0.00	1.33	3.58	1.45
time (sec)	N/A	0.035	0.268	0.006	0.000	0.768	0.000	1.115	3.803	0.287

Problem 1390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	77	76	0	178	0	50	45	42
N.S.	1	1.00	1.83	1.81	0.00	4.24	0.00	1.19	1.07	1.00
time (sec)	N/A	0.026	0.065	0.007	0.000	1.110	0.000	1.097	0.288	0.088

Problem 1391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	66	26	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	2.20	0.87	1.00
time (sec)	N/A	0.003	0.009	0.006	0.000	0.916	0.000	1.043	0.732	0.033

Problem 1392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	121	71	56
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	1.83	1.08	0.85
time (sec)	N/A	0.008	0.017	0.004	0.000	0.916	0.000	1.080	0.894	0.100

Problem 1393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	75	105	0	251	0	227	133	83
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	2.25	1.32	0.82
time (sec)	N/A	0.016	0.031	0.007	0.000	1.315	0.000	1.270	1.005	0.104

Problem 1394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	116	171	0	419	0	386	209	109
N.S.	1	1.00	0.85	1.26	0.00	3.08	0.00	2.84	1.54	0.80
time (sec)	N/A	0.028	0.047	0.008	0.000	2.891	0.000	1.468	1.192	0.111

Problem 1395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	168	256	0	638	0	596	303	135
N.S.	1	1.00	0.98	1.50	0.00	3.73	0.00	3.49	1.77	0.79
time (sec)	N/A	0.041	0.066	0.012	0.000	11.400	0.000	1.568	1.370	0.114

Problem 1396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	73	0	0	603	0	279	-1	173
N.S.	1	1.00	0.42	0.00	0.00	3.47	0.00	1.60	-0.01	0.99
time (sec)	N/A	0.089	0.067	0.079	0.000	1.732	0.000	1.385	0.000	0.337

Problem 1397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	73	0	0	441	0	201	-1	151
N.S.	1	1.00	0.53	0.00	0.00	3.20	0.00	1.46	-0.01	1.09
time (sec)	N/A	0.067	0.055	0.082	0.000	1.505	0.000	1.260	0.000	0.243

Problem 1398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	73	0	0	311	0	137	-1	120
N.S.	1	1.00	0.74	0.00	0.00	3.17	0.00	1.40	-0.01	1.22
time (sec)	N/A	0.047	0.048	0.087	0.000	1.048	0.000	1.273	0.000	0.563

Problem 1399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	95	0	0	241	0	96	-1	66
N.S.	1	1.00	1.44	0.00	0.00	3.65	0.00	1.45	-0.02	1.00
time (sec)	N/A	0.032	0.349	0.088	0.000	1.731	0.000	1.223	0.000	0.102

Problem 1400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	47	26	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	1.57	0.87	1.00
time (sec)	N/A	0.003	0.008	0.005	0.000	1.291	0.000	1.038	0.741	0.033

Problem 1401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	42	52	0	125	0	142	71	46
N.S.	1	1.00	0.68	0.84	0.00	2.02	0.00	2.29	1.15	0.74
time (sec)	N/A	0.009	0.017	0.004	0.000	1.554	0.000	1.222	0.858	0.112

Problem 1402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	75	105	0	273	0	368	141	73
N.S.	1	1.00	0.74	1.04	0.00	2.70	0.00	3.64	1.40	0.72
time (sec)	N/A	0.018	0.030	0.009	0.000	1.919	0.000	1.544	1.064	0.125

Problem 1403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	114	170	0	455	0	830	227	93
N.S.	1	1.00	0.84	1.25	0.00	3.35	0.00	6.10	1.67	0.68
time (sec)	N/A	0.027	0.042	0.010	0.000	2.452	0.000	2.459	1.312	0.134

Problem 1404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	166	256	0	689	0	1518	337	117
N.S.	1	1.00	0.97	1.50	0.00	4.03	0.00	8.88	1.97	0.68
time (sec)	N/A	0.043	0.061	0.012	0.000	8.186	0.000	4.771	1.500	0.141

Problem 1405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	206	206	226	356	0	955	0	2438	454	139
N.S.	1	1.00	1.10	1.73	0.00	4.64	0.00	11.83	2.20	0.67
time (sec)	N/A	0.057	0.080	0.013	0.000	15.843	0.000	8.708	1.959	0.154

Problem 1406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	73	0	0	879	0	500	-1	194
N.S.	1	1.00	0.36	0.00	0.00	4.31	0.00	2.45	-0.00	0.95
time (sec)	N/A	0.109	0.097	0.083	0.000	2.899	0.000	2.418	0.000	0.338

Problem 1407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	73	0	0	657	0	380	-1	172
N.S.	1	1.00	0.43	0.00	0.00	3.86	0.00	2.24	-0.01	1.01
time (sec)	N/A	0.079	0.082	0.088	0.000	2.244	0.000	2.141	0.000	0.275

Problem 1408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F(-2)	B	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	73	0	0	475	0	276	-1	166
N.S.	1	1.00	0.57	0.00	0.00	3.71	0.00	2.16	-0.01	1.30
time (sec)	N/A	0.058	0.078	0.082	0.000	1.624	0.000	1.972	0.000	0.978

Problem 1409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	92	92	111	0	0	325	0	181	-1	85
N.S.	1	1.00	1.21	0.00	0.00	3.53	0.00	1.97	-0.01	0.92
time (sec)	N/A	0.040	0.550	0.086	0.000	1.487	0.000	1.434	0.000	0.136

Problem 1410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	65	0	51	130	32
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	1.59	4.06	1.00
time (sec)	N/A	0.003	0.010	0.005	0.000	1.146	0.000	1.147	0.561	0.037

Problem 1411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	53	0	118	0	126	127	57
N.S.	1	1.00	0.70	0.80	0.00	1.79	0.00	1.91	1.92	0.86
time (sec)	N/A	0.008	0.015	0.004	0.000	1.317	0.000	1.019	0.897	0.094

Problem 1412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	78	104	0	273	0	373	132	73
N.S.	1	1.00	0.80	1.06	0.00	2.79	0.00	3.81	1.35	0.74
time (sec)	N/A	0.017	0.031	0.007	0.000	2.079	0.000	1.480	1.030	0.123

Problem 1413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	118	169	0	447	0	670	224	92
N.S.	1	1.00	0.87	1.25	0.00	3.31	0.00	4.96	1.66	0.68
time (sec)	N/A	0.028	0.052	0.010	0.000	2.418	0.000	2.063	1.289	0.138

Problem 1414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	170	256	0	715	0	1203	346	117
N.S.	1	1.00	0.99	1.49	0.00	4.16	0.00	6.99	2.01	0.68
time (sec)	N/A	0.044	0.067	0.010	0.000	8.263	0.000	3.918	1.531	0.150

Problem 1415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	207	233	356	0	999	0	1964	478	139
N.S.	1	1.00	1.13	1.72	0.00	4.83	0.00	9.49	2.31	0.67
time (sec)	N/A	0.061	0.082	0.015	0.000	19.331	0.000	7.760	1.908	0.165

Problem 1416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	86	48	31	0	24	50	28
N.S.	1	1.00	1.00	4.53	2.53	1.63	0.00	1.26	2.63	1.47
time (sec)	N/A	0.006	0.010	0.009	1.361	1.049	0.000	1.018	0.308	0.052

Problem 1417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	39	66	33	27	0	23	47	25
N.S.	1	1.00	2.05	3.47	1.74	1.42	0.00	1.21	2.47	1.32
time (sec)	N/A	0.005	0.012	0.009	1.366	0.811	0.000	1.055	0.339	0.051

Problem 1418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	39	66	33	27	0	23	43	25
N.S.	1	1.00	2.05	3.47	1.74	1.42	0.00	1.21	2.26	1.32
time (sec)	N/A	0.005	0.011	0.008	1.389	0.876	0.000	0.965	0.326	0.053

Problem 1419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	34	60	32	25	15	21	33	25
N.S.	1	1.00	2.00	3.53	1.88	1.47	0.88	1.24	1.94	1.47
time (sec)	N/A	0.004	0.010	0.007	1.330	0.952	1.265	0.936	0.309	0.041

Problem 1420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	39	64	33	27	0	23	44	25
N.S.	1	1.00	2.05	3.37	1.74	1.42	0.00	1.21	2.32	1.32
time (sec)	N/A	0.005	0.011	0.008	1.316	0.966	0.000	1.032	0.318	0.051

Problem 1421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	25	57	26	26	75	23	50	25
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	4.55	2.27
time (sec)	N/A	0.003	0.005	0.007	1.329	1.122	4.199	1.075	0.304	0.051

Problem 1422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	39	66	33	27	0	23	46	25
N.S.	1	1.00	2.05	3.47	1.74	1.42	0.00	1.21	2.42	1.32
time (sec)	N/A	0.005	0.011	0.009	1.375	0.729	0.000	0.995	0.292	0.050

Problem 1423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	66	33	28	0	23	47	25
N.S.	1	1.00	1.00	4.40	2.20	1.87	0.00	1.53	3.13	1.67
time (sec)	N/A	0.004	0.004	0.007	1.380	1.131	0.000	1.012	0.293	0.053

Problem 1424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	10	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00	0.00
time (sec)	N/A	0.001	0.001	0.001	1.319	0.996	0.062	0.952	0.257	0.000

Problem 1425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	66	33	28	0	23	43	25
N.S.	1	1.00	1.00	4.40	2.20	1.87	0.00	1.53	2.87	1.67
time (sec)	N/A	0.004	0.004	0.006	1.386	0.952	0.000	0.925	0.286	0.053

Problem 1426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	36	58	32	25	20	21	37	25
N.S.	1	1.00	1.89	3.05	1.68	1.32	1.05	1.11	1.95	1.32
time (sec)	N/A	0.005	0.009	0.006	1.382	0.984	1.349	1.067	0.280	0.042

Problem 1427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	41	65	30	28	0	23	44	25
N.S.	1	1.00	1.95	3.10	1.43	1.33	0.00	1.10	2.10	1.19
time (sec)	N/A	0.006	0.009	0.007	1.682	1.077	0.000	1.023	0.286	0.054

Problem 1428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	25	57	26	26	75	23	50	25
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	4.55	2.27
time (sec)	N/A	0.002	0.003	0.000	1.361	0.866	4.282	0.959	0.002	0.001

Problem 1429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	41	66	33	28	0	23	50	25
N.S.	1	1.00	1.95	3.14	1.57	1.33	0.00	1.10	2.38	1.19
time (sec)	N/A	0.006	0.011	0.007	1.242	1.047	0.000	0.958	0.283	0.052

Problem 1430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	22	65	21	44	0	18	44	26
N.S.	1	1.00	1.38	4.06	1.31	2.75	0.00	1.12	2.75	1.62
time (sec)	N/A	0.014	0.013	0.012	2.990	0.827	0.000	1.049	0.084	0.053

Problem 1431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	56	9	31	76	15	44	26
N.S.	1	1.00	1.00	5.09	0.82	2.82	6.91	1.36	4.00	2.36
time (sec)	N/A	0.003	0.009	0.007	3.033	0.751	4.440	0.907	0.078	0.051

Problem 1432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	22	66	19	43	0	18	40	26
N.S.	1	1.00	1.38	4.12	1.19	2.69	0.00	1.12	2.50	1.62
time (sec)	N/A	0.013	0.013	0.010	3.008	1.067	0.000	1.050	0.320	0.052

Problem 1433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	51	58	18	26	24	18	34	24
N.S.	1	1.00	5.10	5.80	1.80	2.60	2.40	1.80	3.40	2.40
time (sec)	N/A	0.010	0.014	0.004	3.140	1.122	1.277	0.934	0.293	0.041

Problem 1434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	49	66	21	44	0	13	41	26
N.S.	1	1.00	4.45	6.00	1.91	4.00	0.00	1.18	3.73	2.36
time (sec)	N/A	0.010	0.015	0.009	3.014	1.088	0.000	1.172	0.303	0.048

Problem 1435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	28	26	16	1	53	12	47	0
N.S.	1	1.00	0.97	0.90	0.55	0.03	1.83	0.41	1.62	0.00
time (sec)	N/A	0.004	0.009	0.004	1.355	0.990	1.980	0.968	0.069	0.079

Problem 1436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	53	66	21	44	0	23	47	26
N.S.	1	1.00	2.04	2.54	0.81	1.69	0.00	0.88	1.81	1.00
time (sec)	N/A	0.014	0.016	0.010	3.012	0.815	0.000	1.071	0.297	0.050

Problem 1437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	70	33	30	0	25	49	59
N.S.	1	1.00	1.00	4.38	2.06	1.88	0.00	1.56	3.06	3.69
time (sec)	N/A	0.005	0.007	0.009	1.357	1.069	0.000	1.037	0.312	0.060

Problem 1438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	13	11	11	8	12	11	0
N.S.	1	1.00	1.00	1.08	0.92	0.92	0.67	1.00	0.92	0.00
time (sec)	N/A	0.001	0.001	0.001	1.423	0.673	0.066	1.102	0.034	0.000

Problem 1439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	70	33	30	0	25	45	59
N.S.	1	1.00	1.00	4.38	2.06	1.88	0.00	1.56	2.81	3.69
time (sec)	N/A	0.005	0.007	0.008	1.327	0.912	0.000	0.853	0.311	0.055

Problem 1440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	37	64	32	27	53	23	39	27
N.S.	1	1.00	1.85	3.20	1.60	1.35	2.65	1.15	1.95	1.35
time (sec)	N/A	0.005	0.008	0.007	1.356	0.703	1.337	1.025	0.282	0.041

Problem 1441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	40	70	33	30	0	25	46	27
N.S.	1	1.00	1.82	3.18	1.50	1.36	0.00	1.14	2.09	1.23
time (sec)	N/A	0.007	0.011	0.007	1.383	0.766	0.000	1.095	0.285	0.056

Problem 1442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	27	61	26	28	78	25	52	27
N.S.	1	1.00	2.25	5.08	2.17	2.33	6.50	2.08	4.33	2.25
time (sec)	N/A	0.003	0.005	0.006	1.274	1.018	4.625	1.104	0.294	0.052

Problem 1443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	40	69	30	30	0	25	52	27
N.S.	1	1.00	1.82	3.14	1.36	1.36	0.00	1.14	2.36	1.23
time (sec)	N/A	0.007	0.011	0.007	1.369	1.118	0.000	1.277	0.292	0.054

Problem 1444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	C	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	25	57	26	26	75	23	40	25
N.S.	1	1.00	2.27	5.18	2.36	2.36	6.82	2.09	3.64	2.27
time (sec)	N/A	0.002	0.005	0.006	1.377	0.839	4.209	1.040	0.324	0.053

Problem 1445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	41	100	0	175	0	57	66	57
N.S.	1	1.00	0.95	2.33	0.00	4.07	0.00	1.33	1.53	1.33
time (sec)	N/A	0.015	0.025	0.018	0.000	0.899	0.000	0.972	0.495	0.115

Problem 1446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	31	48	41	26	44	23	30	30
N.S.	1	1.00	1.41	2.18	1.86	1.18	2.00	1.05	1.36	1.36
time (sec)	N/A	0.004	0.009	0.007	2.869	0.976	1.029	0.923	0.439	0.042

Problem 1447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	57	28	46	58	30	43	35
N.S.	1	1.00	1.00	2.19	1.08	1.77	2.23	1.15	1.65	1.35
time (sec)	N/A	0.008	0.009	0.007	3.041	1.074	1.092	1.260	0.122	0.073

Problem 1448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	67	118	0	176	0	58	63	61
N.S.	1	1.00	1.60	2.81	0.00	4.19	0.00	1.38	1.50	1.45
time (sec)	N/A	0.016	0.050	0.043	0.000	0.847	0.000	1.079	0.515	0.120

Problem 1449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	14	27	14	14	26	8	16	24
N.S.	1	1.00	1.40	2.70	1.40	1.40	2.60	0.80	1.60	2.40
time (sec)	N/A	0.007	0.010	0.005	2.993	0.982	0.992	1.099	0.290	0.041

Problem 1450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	31	21	21	44	13	27	38
N.S.	1	1.00	1.00	1.55	1.05	1.05	2.20	0.65	1.35	1.90
time (sec)	N/A	0.007	0.004	0.006	3.119	1.072	1.003	1.085	0.295	0.070

Problem 1451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	45	39	11	44	58	21	40	36
N.S.	1	1.00	1.73	1.50	0.42	1.69	2.23	0.81	1.54	1.38
time (sec)	N/A	0.009	0.012	0.006	3.000	0.772	1.069	0.934	0.080	0.055

Problem 1452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F(-2)	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	103	84	0	185	0	54	44	43
N.S.	1	1.00	2.40	1.95	0.00	4.30	0.00	1.26	1.02	1.00
time (sec)	N/A	0.028	0.081	0.009	0.000	0.728	0.000	1.162	0.343	0.093

Problem 1453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	219	219	73	0	0	717	0	0	-1	294
N.S.	1	1.00	0.33	0.00	0.00	3.27	0.00	0.00	-0.00	1.34
time (sec)	N/A	0.088	0.034	0.060	0.000	0.885	0.000	0.000	0.000	0.495

Problem 1454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	172	73	0	0	596	0	0	-1	300
N.S.	1	1.00	0.42	0.00	0.00	3.47	0.00	0.00	-0.01	1.74
time (sec)	N/A	0.046	0.025	0.033	0.000	0.978	0.000	0.000	0.000	7.149

Problem 1455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	71	0	0	233	0	0	-1	200
N.S.	1	1.00	0.48	0.00	0.00	1.56	0.00	0.00	-0.01	1.34
time (sec)	N/A	0.033	0.028	0.084	0.000	1.065	0.000	0.000	0.000	0.161

Problem 1456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	65	0	0	92	32
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	2.88	1.00
time (sec)	N/A	0.003	0.013	0.005	0.000	0.992	0.000	0.000	0.713	0.040

Problem 1457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	175	0	0	127	57
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	1.92	0.86
time (sec)	N/A	0.009	0.023	0.006	0.000	1.078	0.000	0.000	1.032	0.106

Problem 1458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	337	0	0	203	73
N.S.	1	1.00	0.76	1.04	0.00	3.34	0.00	0.00	2.01	0.72
time (sec)	N/A	0.017	0.040	0.008	0.000	0.868	0.000	0.000	1.017	0.121

Problem 1459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	533	0	0	293	95
N.S.	1	1.00	0.87	1.26	0.00	3.92	0.00	0.00	2.15	0.70
time (sec)	N/A	0.028	0.058	0.009	0.000	1.110	0.000	0.000	1.152	0.131

Problem 1460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	73	0	0	740	0	0	-1	292
N.S.	1	1.00	0.34	0.00	0.00	3.43	0.00	0.00	-0.00	1.35
time (sec)	N/A	0.090	0.030	0.063	0.000	1.072	0.000	0.000	0.000	0.497

Problem 1461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	73	0	0	618	0	0	-1	297
N.S.	1	1.00	0.43	0.00	0.00	3.61	0.00	0.00	-0.01	1.74
time (sec)	N/A	0.041	0.029	0.035	0.000	1.129	0.000	0.000	0.000	7.305

Problem 1462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	71	0	0	519	0	0	-1	177
N.S.	1	1.00	0.56	0.00	0.00	4.12	0.00	0.00	-0.01	1.40
time (sec)	N/A	0.014	0.029	0.086	0.000	0.715	0.000	0.000	0.000	0.140

Problem 1463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	42	0	0	-1	32
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.012	0.006	0.000	1.320	0.000	0.000	0.000	0.040

Problem 1464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	0	-1	51
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02	0.77
time (sec)	N/A	0.009	0.017	0.004	0.000	1.397	0.000	0.000	0.000	0.159

Problem 1465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	251	0	0	-1	73
N.S.	1	1.00	0.76	1.04	0.00	2.49	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.018	0.034	0.007	0.000	1.588	0.000	0.000	0.000	0.173

Problem 1466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	420	0	0	-1	95
N.S.	1	1.00	0.87	1.26	0.00	3.09	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.028	0.048	0.009	0.000	1.407	0.000	0.000	0.000	0.177

Problem 1467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	216	73	0	0	741	0	0	-1	285
N.S.	1	1.00	0.34	0.00	0.00	3.43	0.00	0.00	-0.00	1.32
time (sec)	N/A	0.080	0.035	0.038	0.000	1.578	0.000	0.000	0.000	0.297

Problem 1468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	73	0	0	619	0	0	-1	298
N.S.	1	1.00	0.43	0.00	0.00	3.66	0.00	0.00	-0.01	1.76
time (sec)	N/A	0.041	0.027	0.033	0.000	1.529	0.000	0.000	0.000	7.896

Problem 1469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	73	0	0	521	0	0	-1	177
N.S.	1	1.00	0.58	0.00	0.00	4.13	0.00	0.00	-0.01	1.40
time (sec)	N/A	0.015	0.030	0.089	0.000	1.236	0.000	0.000	0.000	0.168

Problem 1470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	26	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	1.00
time (sec)	N/A	0.003	0.009	0.005	0.000	0.738	0.000	0.000	0.828	0.050

Problem 1471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	0	71	56
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08	0.85
time (sec)	N/A	0.010	0.017	0.004	0.000	2.109	0.000	0.000	0.977	0.116

Problem 1472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	75	105	0	251	0	0	133	83
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32	0.82
time (sec)	N/A	0.017	0.033	0.007	0.000	1.108	0.000	0.000	1.508	0.115

Problem 1473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	116	171	0	419	0	0	209	95
N.S.	1	1.00	0.85	1.26	0.00	3.08	0.00	0.00	1.54	0.70
time (sec)	N/A	0.030	0.049	0.010	0.000	0.832	0.000	0.000	1.273	0.139

Problem 1474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	241	241	73	0	0	423	0	0	-1	318
N.S.	1	1.00	0.30	0.00	0.00	1.76	0.00	0.00	-0.00	1.32
time (sec)	N/A	0.107	0.062	0.098	0.000	1.042	0.000	0.000	0.000	0.500

Problem 1475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	73	0	0	306	0	0	-1	326
N.S.	1	1.00	0.37	0.00	0.00	1.57	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.071	0.048	0.099	0.000	1.222	0.000	0.000	0.000	11.017

Problem 1476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	73	0	0	233	0	0	-1	200
N.S.	1	1.00	0.49	0.00	0.00	1.56	0.00	0.00	-0.01	1.34
time (sec)	N/A	0.031	0.044	0.082	0.000	1.458	0.000	0.000	0.000	0.160

Problem 1477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	-1	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.007	0.004	0.000	0.760	0.000	0.000	0.000	0.050

Problem 1478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	45	53	0	126	0	0	-1	49
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02	0.74
time (sec)	N/A	0.009	0.020	0.005	0.000	1.212	0.000	0.000	0.000	0.112

Problem 1479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	75	105	0	273	0	0	-1	73
N.S.	1	1.00	0.74	1.04	0.00	2.70	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.018	0.031	0.007	0.000	1.106	0.000	0.000	0.000	0.120

Problem 1480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	116	171	0	456	0	0	-1	95
N.S.	1	1.00	0.85	1.26	0.00	3.35	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.030	0.047	0.009	0.000	1.521	0.000	0.000	0.000	0.134

Problem 1481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	C	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	48	573	0	107	39	0	-1	113
N.S.	1	1.00	0.62	7.44	0.00	1.39	0.51	0.00	-0.01	1.47
time (sec)	N/A	0.014	0.019	0.377	0.000	1.352	2.548	0.000	0.000	0.218

Problem 1482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	73	0	0	2151	0	0	-1	218
N.S.	1	1.00	0.36	0.00	0.00	10.49	0.00	0.00	-0.00	1.06
time (sec)	N/A	0.139	0.059	0.080	0.000	1.557	0.000	0.000	0.000	0.532

Problem 1483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	73	0	0	1468	0	0	-1	189
N.S.	1	1.00	0.44	0.00	0.00	8.79	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.102	0.043	0.045	0.000	1.342	0.000	0.000	0.000	0.382

Problem 1484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	71	0	0	857	0	0	-1	244
N.S.	1	1.00	0.47	0.00	0.00	5.64	0.00	0.00	-0.01	1.61
time (sec)	N/A	0.098	0.047	0.105	0.000	1.040	0.000	0.000	0.000	13.905

Problem 1485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	73	0	0	368	0	0	-1	134
N.S.	1	1.00	0.54	0.00	0.00	2.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.088	0.053	0.092	0.000	1.170	0.000	0.000	0.000	0.209

Problem 1486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	104	0	0	99	32
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	3.09	1.00
time (sec)	N/A	0.003	0.015	0.004	0.000	1.034	0.000	0.000	0.810	0.055

Problem 1487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	235	0	0	178	51
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.70	0.77
time (sec)	N/A	0.009	0.027	0.006	0.000	1.236	0.000	0.000	0.955	0.175

Problem 1488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	426	0	0	268	73
N.S.	1	1.00	0.76	1.04	0.00	4.22	0.00	0.00	2.65	0.72
time (sec)	N/A	0.017	0.049	0.006	0.000	1.523	0.000	0.000	1.126	0.182

Problem 1489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	649	0	0	376	95
N.S.	1	1.00	0.87	1.26	0.00	4.77	0.00	0.00	2.76	0.70
time (sec)	N/A	0.029	0.067	0.013	0.000	1.656	0.000	0.000	1.360	0.194

Problem 1490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	73	0	0	1468	0	0	-1	189
N.S.	1	1.00	0.44	0.00	0.00	8.79	0.00	0.00	-0.01	1.13
time (sec)	N/A	0.102	0.031	0.077	0.000	1.148	0.000	0.000	0.000	0.401

Problem 1491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	73	0	0	814	0	0	-1	176
N.S.	1	1.00	0.57	0.00	0.00	6.41	0.00	0.00	-0.01	1.39
time (sec)	N/A	0.074	0.026	0.042	0.000	1.133	0.000	0.000	0.000	7.844

Problem 1492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	71	0	0	234	0	0	-1	85
N.S.	1	1.00	0.84	0.00	0.00	2.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.058	0.029	0.092	0.000	0.747	0.000	0.000	0.000	0.122

Problem 1493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	42	0	0	-1	32
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.012	0.005	0.000	1.096	0.000	0.000	0.000	0.040

Problem 1494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	0	-1	51
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	-0.02	0.77
time (sec)	N/A	0.009	0.017	0.005	0.000	1.300	0.000	0.000	0.000	0.159

Problem 1495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	252	0	0	-1	73
N.S.	1	1.00	0.76	1.04	0.00	2.50	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.017	0.033	0.009	0.000	2.549	0.000	0.000	0.000	0.165

Problem 1496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	419	0	0	-1	95
N.S.	1	1.00	0.87	1.26	0.00	3.08	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.031	0.049	0.010	0.000	5.098	0.000	0.000	0.000	0.178

Problem 1497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	73	0	0	1457	0	0	-1	182
N.S.	1	1.00	0.44	0.00	0.00	8.72	0.00	0.00	-0.01	1.09
time (sec)	N/A	0.105	0.035	0.046	0.000	1.026	0.000	0.000	0.000	0.273

Problem 1498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	73	0	0	808	0	0	-1	176
N.S.	1	1.00	0.57	0.00	0.00	6.36	0.00	0.00	-0.01	1.39
time (sec)	N/A	0.081	0.028	0.042	0.000	1.416	0.000	0.000	0.000	8.418

Problem 1499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	85	85	73	0	0	234	0	0	-1	85
N.S.	1	1.00	0.86	0.00	0.00	2.75	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.067	0.031	0.092	0.000	1.238	0.000	0.000	0.000	0.116

Problem 1500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	26	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	1.00
time (sec)	N/A	0.003	0.007	0.005	0.000	1.171	0.000	0.000	0.706	0.051

Problem 1501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	0	71	56
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.08	0.85
time (sec)	N/A	0.009	0.015	0.005	0.000	1.214	0.000	0.000	0.868	0.115

Problem 1502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	75	105	0	251	0	0	133	83
N.S.	1	1.00	0.74	1.04	0.00	2.49	0.00	0.00	1.32	0.82
time (sec)	N/A	0.017	0.033	0.007	0.000	0.877	0.000	0.000	1.020	0.118

Problem 1503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	116	171	0	419	0	0	209	109
N.S.	1	1.00	0.85	1.26	0.00	3.08	0.00	0.00	1.54	0.80
time (sec)	N/A	0.028	0.048	0.010	0.000	1.317	0.000	0.000	1.260	0.134

Problem 1504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	73	0	0	857	0	0	-1	200
N.S.	1	1.00	0.48	0.00	0.00	5.64	0.00	0.00	-0.01	1.32
time (sec)	N/A	0.087	0.050	0.106	0.000	1.217	0.000	0.000	0.000	14.156

Problem 1505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	73	0	0	273	0	0	-1	108
N.S.	1	1.00	0.68	0.00	0.00	2.53	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.065	0.044	0.089	0.000	1.716	0.000	0.000	0.000	0.125

Problem 1506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	-1	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.008	0.005	0.000	0.772	0.000	0.000	0.000	0.050

Problem 1507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	45	53	0	126	0	0	-1	49
N.S.	1	1.00	0.68	0.80	0.00	1.91	0.00	0.00	-0.02	0.74
time (sec)	N/A	0.008	0.017	0.006	0.000	1.167	0.000	0.000	0.000	0.112

Problem 1508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	76	105	0	273	0	0	-1	73
N.S.	1	1.00	0.75	1.04	0.00	2.70	0.00	0.00	-0.01	0.72
time (sec)	N/A	0.017	0.033	0.007	0.000	1.386	0.000	0.000	0.000	0.128

Problem 1509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	116	171	0	457	0	0	-1	95
N.S.	1	1.00	0.85	1.26	0.00	3.36	0.00	0.00	-0.01	0.70
time (sec)	N/A	0.027	0.043	0.010	0.000	2.577	0.000	0.000	0.000	0.132

Problem 1510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	279	279	65	0	0	247	0	0	-1	176
N.S.	1	1.00	0.23	0.00	0.00	0.89	0.00	0.00	-0.00	0.63
time (sec)	N/A	0.304	0.037	0.088	0.000	1.107	0.000	0.000	0.000	0.220

Problem 1511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	42	0	0	448	0	0	-1	123
N.S.	1	1.00	0.22	0.00	0.00	2.32	0.00	0.00	-0.01	0.64
time (sec)	N/A	0.138	0.010	0.082	0.000	0.920	0.000	0.000	0.000	0.121

Problem 1512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	427	427	73	0	0	5633	0	0	-1	364
N.S.	1	1.00	0.17	0.00	0.00	13.19	0.00	0.00	-0.00	0.85
time (sec)	N/A	0.643	0.053	0.105	0.000	1.537	0.000	0.000	0.000	0.770

Problem 1513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	378	378	73	0	0	3025	0	0	-1	385
N.S.	1	1.00	0.19	0.00	0.00	8.00	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.497	0.041	0.052	0.000	1.395	0.000	0.000	0.000	12.199

Problem 1514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	73	0	0	663	0	0	-1	256
N.S.	1	1.00	0.22	0.00	0.00	2.00	0.00	0.00	-0.00	0.77
time (sec)	N/A	0.487	0.056	0.097	0.000	1.002	0.000	0.000	0.000	0.238

Problem 1515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	65	0	0	130	32
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06	1.00
time (sec)	N/A	0.003	0.015	0.006	0.000	1.169	0.000	0.000	0.565	0.042

Problem 1516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	175	0	0	137	51
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08	0.77
time (sec)	N/A	0.009	0.034	0.006	0.000	0.930	0.000	0.000	0.750	0.155

Problem 1517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	338	0	0	213	73
N.S.	1	1.00	0.76	1.04	0.00	3.35	0.00	0.00	2.11	0.72
time (sec)	N/A	0.021	0.057	0.009	0.000	0.753	0.000	0.000	0.949	0.165

Problem 1518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	533	0	0	302	95
N.S.	1	1.00	0.87	1.26	0.00	3.92	0.00	0.00	2.22	0.70
time (sec)	N/A	0.034	0.082	0.011	0.000	1.105	0.000	0.000	1.147	0.173

Problem 1519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	427	427	73	0	0	5633	0	0	-1	365
N.S.	1	1.00	0.17	0.00	0.00	13.19	0.00	0.00	-0.00	0.85
time (sec)	N/A	0.642	0.047	0.098	0.000	1.471	0.000	0.000	0.000	0.746

Problem 1520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	378	378	73	0	0	2997	0	0	-1	385
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00	1.02
time (sec)	N/A	0.557	0.037	0.050	0.000	1.254	0.000	0.000	0.000	17.905

Problem 1521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	334	334	73	0	0	755	0	0	-1	258
N.S.	1	1.00	0.22	0.00	0.00	2.26	0.00	0.00	-0.00	0.77
time (sec)	N/A	0.561	0.072	0.092	0.000	1.175	0.000	0.000	0.000	0.330

Problem 1522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	65	0	0	130	32
N.S.	1	1.00	1.00	0.84	0.00	2.03	0.00	0.00	4.06	1.00
time (sec)	N/A	0.003	0.011	0.004	0.000	0.771	0.000	0.000	0.587	0.054

Problem 1523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	175	0	0	137	51
N.S.	1	1.00	0.70	0.82	0.00	2.65	0.00	0.00	2.08	0.77
time (sec)	N/A	0.009	0.032	0.005	0.000	1.139	0.000	0.000	0.744	0.171

Problem 1524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	338	0	0	214	73
N.S.	1	1.00	0.76	1.04	0.00	3.35	0.00	0.00	2.12	0.72
time (sec)	N/A	0.020	0.055	0.008	0.000	1.156	0.000	0.000	0.944	0.183

Problem 1525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	533	0	0	303	95
N.S.	1	1.00	0.87	1.26	0.00	3.92	0.00	0.00	2.23	0.70
time (sec)	N/A	0.033	0.089	0.010	0.000	1.179	0.000	0.000	1.158	0.196

Problem 1526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	424	424	73	0	0	5633	0	0	-1	363
N.S.	1	1.00	0.17	0.00	0.00	13.29	0.00	0.00	-0.00	0.86
time (sec)	N/A	0.552	0.039	0.058	0.000	1.557	0.000	0.000	0.000	0.779

Problem 1527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	403	403	73	0	0	3084	0	0	-1	417
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.526	0.066	0.136	0.000	1.431	0.000	0.000	0.000	23.487

Problem 1528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	358	358	73	0	0	855	0	0	-1	282
N.S.	1	1.00	0.20	0.00	0.00	2.39	0.00	0.00	-0.00	0.79
time (sec)	N/A	0.501	0.085	0.096	0.000	1.463	0.000	0.000	0.000	0.334

Problem 1529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	104	0	0	199	32
N.S.	1	1.00	1.00	0.84	0.00	3.25	0.00	0.00	6.22	1.00
time (sec)	N/A	0.003	0.023	0.006	0.000	1.335	0.000	0.000	0.756	0.055

Problem 1530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	235	0	0	189	51
N.S.	1	1.00	0.70	0.82	0.00	3.56	0.00	0.00	2.86	0.77
time (sec)	N/A	0.009	0.036	0.004	0.000	1.343	0.000	0.000	0.907	0.178

Problem 1531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	427	0	0	278	73
N.S.	1	1.00	0.76	1.04	0.00	4.23	0.00	0.00	2.75	0.72
time (sec)	N/A	0.018	0.060	0.008	0.000	1.370	0.000	0.000	1.144	0.202

Problem 1532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	649	0	0	385	95
N.S.	1	1.00	0.87	1.26	0.00	4.77	0.00	0.00	2.83	0.70
time (sec)	N/A	0.030	0.094	0.010	0.000	1.394	0.000	0.000	1.429	0.230

Problem 1533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	424	424	73	0	0	5633	0	0	-1	363
N.S.	1	1.00	0.17	0.00	0.00	13.29	0.00	0.00	-0.00	0.86
time (sec)	N/A	0.610	0.054	0.056	0.000	2.014	0.000	0.000	0.000	0.791

Problem 1534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	378	378	73	0	0	3025	0	0	-1	388
N.S.	1	1.00	0.19	0.00	0.00	8.00	0.00	0.00	-0.00	1.03
time (sec)	N/A	0.560	0.036	0.051	0.000	1.850	0.000	0.000	0.000	16.568

Problem 1535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	309	309	73	0	0	620	0	0	-1	233
N.S.	1	1.00	0.24	0.00	0.00	2.01	0.00	0.00	-0.00	0.75
time (sec)	N/A	0.509	0.047	0.098	0.000	0.932	0.000	0.000	0.000	0.217

Problem 1536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	42	0	0	27	32
N.S.	1	1.00	1.00	0.84	0.00	1.31	0.00	0.00	0.84	1.00
time (sec)	N/A	0.003	0.013	0.004	0.000	0.700	0.000	0.000	0.763	0.041

Problem 1537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	54	0	118	0	0	127	51
N.S.	1	1.00	0.70	0.82	0.00	1.79	0.00	0.00	1.92	0.77
time (sec)	N/A	0.010	0.027	0.005	0.000	1.381	0.000	0.000	0.860	0.106

Problem 1538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	252	0	0	203	73
N.S.	1	1.00	0.76	1.04	0.00	2.50	0.00	0.00	2.01	0.72
time (sec)	N/A	0.020	0.045	0.008	0.000	1.372	0.000	0.000	1.033	0.117

Problem 1539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	420	0	0	292	95
N.S.	1	1.00	0.87	1.26	0.00	3.09	0.00	0.00	2.15	0.70
time (sec)	N/A	0.030	0.068	0.010	0.000	1.437	0.000	0.000	1.200	0.123

Problem 1540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	424	424	71	0	0	5591	0	0	-1	356
N.S.	1	1.00	0.17	0.00	0.00	13.19	0.00	0.00	-0.00	0.84
time (sec)	N/A	0.549	0.056	0.059	0.000	2.183	0.000	0.000	0.000	0.428

Problem 1541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	378	378	71	0	0	2997	0	0	-1	0
N.S.	1	1.00	0.19	0.00	0.00	7.93	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.479	0.034	0.055	0.000	1.834	0.000	0.000	0.000	106.342

Problem 1542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	309	309	71	0	0	620	0	0	-1	233
N.S.	1	1.00	0.23	0.00	0.00	2.01	0.00	0.00	-0.00	0.75
time (sec)	N/A	0.444	0.042	0.099	0.000	1.678	0.000	0.000	0.000	0.224

Problem 1543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	-1	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.003	0.010	0.004	0.000	1.350	0.000	0.000	0.000	0.054

Problem 1544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	46	53	0	118	0	0	-1	57
N.S.	1	1.00	0.70	0.80	0.00	1.79	0.00	0.00	-0.02	0.86
time (sec)	N/A	0.009	0.016	0.006	0.000	1.441	0.000	0.000	0.000	0.118

Problem 1545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	77	105	0	252	0	0	-1	83
N.S.	1	1.00	0.76	1.04	0.00	2.50	0.00	0.00	-0.01	0.82
time (sec)	N/A	0.018	0.036	0.008	0.000	1.238	0.000	0.000	0.000	0.125

Problem 1546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	118	171	0	420	0	0	-1	109
N.S.	1	1.00	0.87	1.26	0.00	3.09	0.00	0.00	-0.01	0.80
time (sec)	N/A	0.031	0.060	0.010	0.000	1.114	0.000	0.000	0.000	0.132

Problem 1547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	449	449	71	0	0	5690	0	0	-1	389
N.S.	1	1.00	0.16	0.00	0.00	12.67	0.00	0.00	-0.00	0.87
time (sec)	N/A	0.660	0.079	0.124	0.000	2.225	0.000	0.000	0.000	0.811

Problem 1548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	403	403	71	0	0	3084	0	0	-1	0
N.S.	1	1.00	0.18	0.00	0.00	7.65	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.596	0.055	0.126	0.000	2.008	0.000	0.000	0.000	156.135

Problem 1549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	332	332	71	0	0	663	0	0	-1	256
N.S.	1	1.00	0.21	0.00	0.00	2.00	0.00	0.00	-0.00	0.77
time (sec)	N/A	0.541	0.031	0.095	0.000	1.582	0.000	0.000	0.000	0.259

Problem 1550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	27	0	42	0	0	26	30
N.S.	1	1.00	1.00	0.90	0.00	1.40	0.00	0.00	0.87	1.00
time (sec)	N/A	0.003	0.010	0.006	0.000	1.434	0.000	0.000	0.680	0.051

Problem 1551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	45	53	0	126	0	0	72	49
N.S.	1	1.00	0.70	0.83	0.00	1.97	0.00	0.00	1.12	0.77
time (sec)	N/A	0.011	0.027	0.007	0.000	1.330	0.000	0.000	0.834	0.123

Problem 1552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	98	98	77	105	0	273	0	0	132	73
N.S.	1	1.00	0.79	1.07	0.00	2.79	0.00	0.00	1.35	0.74
time (sec)	N/A	0.020	0.040	0.007	0.000	1.569	0.000	0.000	0.960	0.134

Problem 1553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	118	171	0	457	0	0	209	95
N.S.	1	1.00	0.88	1.28	0.00	3.41	0.00	0.00	1.56	0.71
time (sec)	N/A	0.033	0.063	0.012	0.000	1.489	0.000	0.000	1.146	0.141

Problem 1554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	B	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	12	106	17	20	23	11	0
N.S.	1	1.00	1.00	1.09	9.64	1.55	1.82	2.09	1.00	0.00
time (sec)	N/A	0.002	0.013	0.003	1.123	1.189	0.269	0.996	0.462	0.050

Problem 1555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	110	110	94	389	246	497	4058	833	478	0
N.S.	1	1.00	0.85	3.54	2.24	4.52	36.89	7.57	4.35	0.00
time (sec)	N/A	0.055	0.107	0.009	1.166	1.257	4.666	1.036	0.942	0.049

Problem 1556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	67	159	138	235	1506	385	226	0
N.S.	1	1.00	0.86	2.04	1.77	3.01	19.31	4.94	2.90	0.00
time (sec)	N/A	0.032	0.096	0.008	1.147	1.181	2.137	0.970	0.655	0.044

Problem 1557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	41	49	63	83	377	132	88	0
N.S.	1	1.00	0.89	1.07	1.37	1.80	8.20	2.87	1.91	0.00
time (sec)	N/A	0.018	0.033	0.003	1.063	0.726	0.856	0.859	0.484	0.034

Problem 1558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	95	386	246	496	4058	833	478	0
N.S.	1	1.00	0.86	3.48	2.22	4.47	36.56	7.50	4.31	0.00
time (sec)	N/A	0.057	0.110	0.009	1.295	1.035	4.439	0.979	0.913	0.047

Problem 1559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	78	67	159	138	237	1506	385	226	0
N.S.	1	1.00	0.86	2.04	1.77	3.04	19.31	4.94	2.90	0.00
time (sec)	N/A	0.032	0.096	0.008	1.168	1.352	2.089	0.925	0.618	0.045

Problem 1560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	41	46	63	83	377	132	88	0
N.S.	1	1.00	0.87	0.98	1.34	1.77	8.02	2.81	1.87	0.00
time (sec)	N/A	0.018	0.037	0.003	1.169	1.281	0.822	0.999	0.494	0.034

Problem 1561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	17	19	18	20	20	18	18	0
N.S.	1	1.00	0.94	1.06	1.00	1.11	1.11	1.00	1.00	0.00
time (sec)	N/A	0.003	0.011	0.003	1.116	1.254	0.063	1.018	0.379	0.013

Problem 1562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	143	112	322	0	512	0	0	528	0
N.S.	1	1.00	0.78	2.25	0.00	3.58	0.00	0.00	3.69	0.00
time (sec)	N/A	0.063	0.097	0.007	0.000	1.434	0.000	0.000	1.096	0.048

Problem 1563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	86	86	59	127	0	206	0	0	220	0
N.S.	1	1.00	0.69	1.48	0.00	2.40	0.00	0.00	2.56	0.00
time (sec)	N/A	0.012	0.039	0.006	0.000	1.384	0.000	0.000	0.767	0.050

Problem 1564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	36	45	0	60	0	0	102	0
N.S.	1	1.00	0.92	1.15	0.00	1.54	0.00	0.00	2.62	0.00
time (sec)	N/A	0.004	0.017	0.003	0.000	1.323	0.000	0.000	0.559	0.047

Problem 1565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	38	41	0	59	0	0	97	0
N.S.	1	1.00	1.03	1.11	0.00	1.59	0.00	0.00	2.62	0.00
time (sec)	N/A	0.006	0.021	0.004	0.000	1.331	0.000	0.000	0.532	0.047

Problem 1566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	60	123	0	207	0	0	214	0
N.S.	1	1.00	0.75	1.54	0.00	2.59	0.00	0.00	2.68	0.00
time (sec)	N/A	0.020	0.033	0.004	0.000	1.277	0.000	0.000	0.736	0.046

Problem 1567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	113	318	0	509	0	0	525	0
N.S.	1	1.00	0.86	2.43	0.00	3.89	0.00	0.00	4.01	0.00
time (sec)	N/A	0.046	0.084	0.008	0.000	1.015	0.000	0.000	0.992	0.048

Problem 1568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	186	186	195	661	0	959	0	0	944	0
N.S.	1	1.00	1.05	3.55	0.00	5.16	0.00	0.00	5.08	0.00
time (sec)	N/A	0.086	0.124	0.010	0.000	1.405	0.000	0.000	1.642	0.046

Problem 1569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	42	0	58	0	0	98	0
N.S.	1	1.00	1.00	1.17	0.00	1.61	0.00	0.00	2.72	0.00
time (sec)	N/A	0.005	0.013	0.005	0.000	1.323	0.000	0.000	0.556	0.047

Problem 1570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	59	124	0	205	0	0	214	0
N.S.	1	1.00	0.75	1.57	0.00	2.59	0.00	0.00	2.71	0.00
time (sec)	N/A	0.017	0.037	0.005	0.000	1.292	0.000	0.000	0.745	0.047

Problem 1571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	112	319	0	507	0	0	528	0
N.S.	1	1.00	0.86	2.45	0.00	3.90	0.00	0.00	4.06	0.00
time (sec)	N/A	0.037	0.092	0.007	0.000	1.298	0.000	0.000	1.023	0.046

Problem 1572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	195	662	0	954	0	0	945	0
N.S.	1	1.00	1.05	3.58	0.00	5.16	0.00	0.00	5.11	0.00
time (sec)	N/A	0.062	0.129	0.009	0.000	0.925	0.000	0.000	1.609	0.046

Problem 1573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	53	57	92	93	233	78	46	0
N.S.	1	1.00	0.93	1.00	1.61	1.63	4.09	1.37	0.81	0.00
time (sec)	N/A	0.030	0.030	0.008	1.162	1.494	0.694	0.923	0.439	0.001

Problem 1574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	54	57	0	85	0	0	81	0
N.S.	1	1.00	0.57	0.60	0.00	0.89	0.00	0.00	0.85	0.00
time (sec)	N/A	0.036	0.074	0.006	0.000	1.317	0.000	0.000	1.037	0.221

Problem 1575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	46	66	0	84	0	0	119	0
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.23	0.00
time (sec)	N/A	0.022	0.040	0.004	0.000	1.392	0.000	0.000	2.138	0.365

Problem 1576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	46	66	0	84	0	0	142	0
N.S.	1	1.00	0.47	0.68	0.00	0.87	0.00	0.00	1.46	0.00
time (sec)	N/A	0.018	0.066	0.005	0.000	1.397	0.000	0.000	0.850	0.139

Problem 1577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	23	22	22	22	22	22	0
N.S.	1	1.00	1.00	0.82	0.79	0.79	0.79	0.79	0.79	0.00
time (sec)	N/A	0.005	0.000	0.000	1.073	1.167	0.060	1.009	0.037	0.000

Problem 1578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	8	11	10	0
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.73	0.67	0.00
time (sec)	N/A	0.002	0.000	0.000	1.082	1.036	0.054	1.084	0.021	0.000

Problem 1579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	5	9	8	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.45	0.82	0.73	0.00
time (sec)	N/A	0.002	0.000	0.000	0.962	0.965	0.054	0.941	0.017	0.000

Problem 1580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	10	9	9	7	9	7	0
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	0.78	0.00
time (sec)	N/A	0.001	0.000	0.000	1.015	0.967	0.053	0.961	0.029	0.000

Problem 1581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	10	12	12	0
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.71	0.86	0.86	0.00
time (sec)	N/A	0.002	0.000	0.000	1.000	0.713	0.055	1.059	0.020	0.000

Problem 1582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	9	10	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.91	0.00
time (sec)	N/A	0.001	0.000	0.001	1.097	0.718	0.054	1.104	0.019	0.000

Problem 1583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	11	8	11	8	0
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.53	0.73	0.53	0.00
time (sec)	N/A	0.002	0.000	0.001	1.054	1.016	0.055	0.886	0.021	0.000

Problem 1584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	15	14	14	15	14	13	0
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.78	0.72	0.00
time (sec)	N/A	0.003	0.000	0.000	0.971	0.493	0.057	0.848	0.024	0.000

Problem 1585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	17	16	16	12	16	15	0
N.S.	1	1.00	1.00	0.85	0.80	0.80	0.60	0.80	0.75	0.00
time (sec)	N/A	0.003	0.000	0.000	0.973	1.094	0.054	1.098	0.026	0.000

Problem 1586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	13	12	12	10	12	12	0
N.S.	1	1.00	1.00	0.81	0.75	0.75	0.62	0.75	0.75	0.00
time (sec)	N/A	0.002	0.000	0.000	0.984	0.993	0.056	0.959	0.023	0.000

Problem 1587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	14	13	13	12	13	13	0
N.S.	1	1.00	1.00	1.08	1.00	1.00	0.92	1.00	1.00	0.00
time (sec)	N/A	0.002	0.000	0.000	1.145	0.530	0.057	1.005	0.028	0.000

Problem 1588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	21	20	27	20	21	20	0
N.S.	1	1.00	1.00	0.95	0.91	1.23	0.91	0.95	0.91	0.00
time (sec)	N/A	0.004	0.007	0.002	1.034	1.200	0.160	0.848	0.037	0.001

Problem 1589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	17	16	17	15	16	17	0
N.S.	1	1.00	1.00	0.77	0.73	0.77	0.68	0.73	0.77	0.00
time (sec)	N/A	0.003	0.001	0.001	1.084	1.277	0.081	0.809	0.029	0.000

Problem 1590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	17	14	14	11	0
N.S.	1	1.00	1.00	0.93	0.87	1.13	0.93	0.93	0.73	0.00
time (sec)	N/A	0.002	0.002	0.001	1.041	1.228	0.085	1.082	0.029	0.000

Problem 1591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	11	10	11	7	11	10	0
N.S.	1	1.00	1.00	1.10	1.00	1.10	0.70	1.10	1.00	0.00
time (sec)	N/A	0.002	0.002	0.002	1.026	1.018	0.078	1.119	0.033	0.000

Problem 1592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	12	11	12	10	11	12	0
N.S.	1	1.00	1.00	0.80	0.73	0.80	0.67	0.73	0.80	0.00
time (sec)	N/A	0.002	0.001	0.000	0.965	1.306	0.083	1.075	0.025	0.000

Problem 1593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	10	9	0
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.91	0.82	0.00
time (sec)	N/A	0.001	0.001	0.001	0.919	1.193	0.076	0.888	0.027	0.000

Problem 1594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	10	8	11	10	0
N.S.	1	1.00	1.00	0.92	0.85	0.77	0.62	0.85	0.77	0.00
time (sec)	N/A	0.002	0.002	0.001	0.966	1.231	0.076	0.902	0.031	0.000

Problem 1595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	12	11	11	10	12	11	0
N.S.	1	1.00	1.00	0.92	0.85	0.85	0.77	0.92	0.85	0.00
time (sec)	N/A	0.002	0.001	0.000	1.077	1.286	0.076	1.014	0.026	0.000

Problem 1596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	11	11	12	11	11	17
N.S.	1	1.00	1.00	0.71	0.65	0.65	0.71	0.65	0.65	1.00
time (sec)	N/A	0.002	0.002	0.001	1.066	1.276	0.059	1.063	0.030	0.023

Problem 1597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	9	10	9	8	13
N.S.	1	1.00	1.00	0.77	0.69	0.69	0.77	0.69	0.62	1.00
time (sec)	N/A	0.002	0.002	0.000	1.001	1.198	0.057	0.855	0.024	0.012

Problem 1598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	14	11	9	10	12	9	10	19
N.S.	1	1.00	0.93	0.73	0.60	0.67	0.80	0.60	0.67	1.27
time (sec)	N/A	0.001	0.007	0.003	1.113	0.966	0.061	1.077	0.021	0.008

Problem 1599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	14	13	18	14	14	15	22
N.S.	1	1.00	1.00	0.93	0.87	1.20	0.93	0.93	1.00	1.47
time (sec)	N/A	0.002	0.014	0.002	1.015	1.251	0.061	0.955	0.291	0.016

Problem 1600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	14	11	11	10	14	11	12	14
N.S.	1	1.00	0.82	0.65	0.65	0.59	0.82	0.65	0.71	0.82
time (sec)	N/A	0.002	0.005	0.003	1.003	1.200	0.059	1.138	0.027	0.011

Problem 1601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	10	9	11	14	12	11	8	10
N.S.	1	1.00	0.67	0.60	0.73	0.93	0.80	0.73	0.53	0.67
time (sec)	N/A	0.002	0.004	0.003	1.007	1.225	0.061	0.892	0.027	0.010

Problem 1602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	17	16	14	19	16	15	24
N.S.	1	1.00	1.00	0.71	0.67	0.58	0.79	0.67	0.62	1.00
time (sec)	N/A	0.002	0.005	0.001	1.046	1.192	0.062	1.089	0.029	0.009

Problem 1603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	16	15	19	20	16	17	26
N.S.	1	1.00	1.00	0.70	0.65	0.83	0.87	0.70	0.74	1.13
time (sec)	N/A	0.003	0.015	0.001	1.034	1.272	0.062	0.824	0.281	0.014

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [1] had the largest ratio of [1.000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	1	1.000
2	A	1	1	1.00	1	1.000
3	A	1	1	1.00	1	1.000
4	A	1	1	1.00	1	1.000
5	A	1	1	1.00	3	0.333
6	A	1	1	1.00	1	1.000
7	A	1	1	1.00	1	1.000
8	A	1	1	1.00	3	0.333
9	A	1	1	1.00	13	0.077
10	A	1	1	1.00	3	0.333
11	A	1	1	1.00	3	0.333
12	A	1	1	1.00	3	0.333
13	A	1	1	1.00	1	1.000
14	A	1	1	1.00	1	1.000
15	A	1	1	1.00	3	0.333
16	A	1	1	1.00	3	0.333
17	A	1	1	1.00	3	0.333
18	A	1	1	1.00	3	0.333
19	A	1	1	1.00	3	0.333
20	A	1	1	1.00	5	0.200
21	A	1	1	1.00	5	0.200
22	A	1	1	1.00	5	0.200
23	A	1	1	1.00	5	0.200
24	A	1	1	1.00	5	0.200
25	A	1	1	1.00	5	0.200
26	A	1	1	1.00	5	0.200
27	A	1	1	1.00	5	0.200
28	A	1	1	1.00	5	0.200
29	A	1	1	1.00	5	0.200
30	A	1	1	1.00	5	0.200
31	A	1	1	1.00	5	0.200
32	A	1	1	1.00	5	0.200
33	A	1	1	1.00	5	0.200
34	A	1	1	1.00	3	0.333
35	A	1	1	1.00	5	0.200
36	A	2	2	1.00	17	0.118
37	A	2	2	1.00	13	0.154
38	A	2	2	1.00	13	0.154
39	A	2	2	1.00	13	0.154
40	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
41	A	2	2	1.00	13	0.154
42	A	2	2	1.00	13	0.154
43	A	2	1	1.00	9	0.111
44	A	2	1	1.00	9	0.111
45	A	2	1	1.00	7	0.143
46	A	1	0	1.00	5	0.000
47	A	2	1	1.00	9	0.111
48	A	2	1	1.00	9	0.111
49	A	1	1	1.00	9	0.111
50	A	2	1	1.00	9	0.111
51	A	2	1	1.00	9	0.111
52	A	2	1	1.00	11	0.091
53	A	2	1	1.00	11	0.091
54	A	2	1	1.00	9	0.111
55	A	1	1	1.00	7	0.143
56	A	2	1	1.00	11	0.091
57	A	2	1	1.00	11	0.091
58	A	2	1	1.00	11	0.091
59	A	1	1	1.00	11	0.091
60	A	2	1	1.00	11	0.091
61	A	2	1	1.00	11	0.091
62	A	2	1	1.00	11	0.091
63	A	2	1	1.00	11	0.091
64	A	2	1	1.00	11	0.091
65	A	2	1	1.00	11	0.091
66	A	2	1	1.00	11	0.091
67	A	2	1	1.00	9	0.111
68	A	1	1	1.00	7	0.143
69	A	2	1	1.00	11	0.091
70	A	2	1	1.00	11	0.091
71	A	2	1	1.00	11	0.091
72	A	2	1	1.00	11	0.091
73	A	1	1	1.00	11	0.091
74	A	2	2	1.00	11	0.182
75	A	2	1	1.00	11	0.091
76	A	2	1	1.00	11	0.091
77	A	2	1	1.00	11	0.091
78	A	2	1	1.00	11	0.091
79	A	2	1	1.00	11	0.091
80	A	2	1	1.00	11	0.091
81	A	2	1	1.00	11	0.091
82	A	2	1	1.00	9	0.111
83	A	1	1	1.00	7	0.143
84	A	2	1	1.00	11	0.091
85	A	2	1	1.00	11	0.091
86	A	2	1	1.00	11	0.091
87	A	2	1	1.00	11	0.091
88	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
89	A	2	1	1.00	11	0.091
90	A	1	1	1.00	11	0.091
91	A	2	2	1.00	11	0.182
92	A	3	2	1.00	11	0.182
93	A	2	1	1.00	11	0.091
94	A	2	1	1.00	11	0.091
95	A	2	1	1.00	11	0.091
96	A	2	1	1.00	11	0.091
97	A	2	1	1.00	11	0.091
98	A	2	1	1.00	11	0.091
99	A	2	1	1.00	11	0.091
100	A	2	1	1.00	11	0.091
101	A	2	1	1.00	11	0.091
102	A	2	1	1.00	11	0.091
103	A	2	1	1.00	11	0.091
104	A	2	1	1.00	11	0.091
105	A	2	1	1.00	9	0.111
106	A	1	1	1.00	7	0.143
107	A	2	1	1.00	11	0.091
108	A	2	1	1.00	11	0.091
109	A	2	1	1.00	11	0.091
110	A	2	1	1.00	11	0.091
111	A	2	1	1.00	11	0.091
112	A	2	1	1.00	11	0.091
113	A	2	1	1.00	11	0.091
114	A	2	1	1.00	11	0.091
115	A	1	1	1.00	11	0.091
116	A	2	2	1.00	11	0.182
117	A	3	2	1.00	11	0.182
118	A	4	2	1.00	11	0.182
119	A	5	2	1.00	11	0.182
120	A	2	1	1.00	11	0.091
121	A	2	1	1.00	11	0.091
122	A	2	1	1.00	11	0.091
123	A	2	1	1.00	11	0.091
124	A	2	1	1.00	11	0.091
125	A	2	1	1.00	11	0.091
126	A	2	1	1.00	11	0.091
127	A	2	1	1.00	11	0.091
128	A	2	1	1.00	11	0.091
129	A	2	1	1.00	11	0.091
130	A	2	1	1.00	11	0.091
131	A	2	1	1.00	11	0.091
132	A	2	1	1.00	11	0.091
133	A	2	1	1.00	9	0.111
134	A	1	1	1.00	7	0.143
135	A	2	1	1.00	11	0.091
136	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
137	A	2	1	1.00	11	0.091
138	A	2	1	1.00	11	0.091
139	A	2	1	1.00	11	0.091
140	A	2	1	1.00	11	0.091
141	A	2	1	1.00	11	0.091
142	A	2	1	1.00	11	0.091
143	A	2	1	1.00	11	0.091
144	A	2	1	1.00	11	0.091
145	A	2	1	1.00	11	0.091
146	A	1	1	1.00	11	0.091
147	A	2	2	1.00	11	0.182
148	A	3	2	1.00	11	0.182
149	A	4	2	1.00	11	0.182
150	A	5	2	1.00	11	0.182
151	A	6	2	1.00	11	0.182
152	A	7	2	1.00	11	0.182
153	A	2	1	1.00	11	0.091
154	A	2	1	1.00	11	0.091
155	A	1	1	1.00	7	0.143
156	A	1	1	1.00	12	0.083
157	A	2	1	1.00	11	0.091
158	A	2	1	1.00	11	0.091
159	A	2	1	1.00	11	0.091
160	A	2	1	1.00	11	0.091
161	A	2	1	1.00	9	0.111
162	A	1	1	1.00	7	0.143
163	A	3	3	1.00	11	0.273
164	A	2	1	1.00	11	0.091
165	A	2	1	1.00	11	0.091
166	A	2	1	1.00	11	0.091
167	A	2	1	1.00	11	0.091
168	A	2	1	1.00	11	0.091
169	A	2	1	1.00	11	0.091
170	A	2	1	1.00	11	0.091
171	A	2	1	1.00	11	0.091
172	A	2	1	1.00	11	0.091
173	A	2	1	1.00	9	0.111
174	A	1	1	1.00	7	0.143
175	A	2	1	1.00	11	0.091
176	A	2	1	1.00	11	0.091
177	A	2	1	1.00	11	0.091
178	A	2	1	1.00	11	0.091
179	A	2	1	1.00	11	0.091
180	A	2	1	1.00	11	0.091
181	A	2	1	1.00	11	0.091
182	A	2	1	1.00	11	0.091
183	A	2	1	1.00	11	0.091
184	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
185	A	2	1	1.00	11	0.091
186	A	1	1	1.00	9	0.111
187	A	1	1	1.00	7	0.143
188	A	2	1	1.00	11	0.091
189	A	2	1	1.00	11	0.091
190	A	2	1	1.00	11	0.091
191	A	2	1	1.00	11	0.091
192	A	2	1	1.00	11	0.091
193	A	2	1	1.00	11	0.091
194	A	2	1	1.00	11	0.091
195	A	2	1	1.00	11	0.091
196	A	2	1	1.00	11	0.091
197	A	2	1	1.00	11	0.091
198	A	2	1	1.00	11	0.091
199	A	1	1	1.00	11	0.091
200	A	2	1	1.00	9	0.111
201	A	1	1	1.00	7	0.143
202	A	2	1	1.00	11	0.091
203	A	2	1	1.00	11	0.091
204	A	2	1	1.00	11	0.091
205	A	2	1	1.00	11	0.091
206	A	2	1	1.00	11	0.091
207	A	2	1	1.00	11	0.091
208	A	2	1	1.00	11	0.091
209	A	2	1	1.00	11	0.091
210	A	2	1	1.00	11	0.091
211	A	2	1	1.00	11	0.091
212	A	1	1	1.00	11	0.091
213	A	2	2	1.00	11	0.182
214	A	2	1	1.23	11	0.091
215	A	2	1	1.00	11	0.091
216	A	2	1	1.00	9	0.111
217	A	1	1	1.00	7	0.143
218	A	2	1	1.00	11	0.091
219	A	2	1	1.00	11	0.091
220	A	2	1	1.00	11	0.091
221	A	2	1	1.00	11	0.091
222	A	2	1	1.00	11	0.091
223	A	2	1	1.00	11	0.091
224	A	2	1	1.00	11	0.091
225	A	2	1	1.00	11	0.091
226	A	1	1	1.00	11	0.091
227	A	2	2	1.00	11	0.182
228	A	3	2	1.00	11	0.182
229	A	4	2	1.00	11	0.182
230	A	2	1	1.00	11	0.091
231	A	2	1	1.00	11	0.091
232	A	2	1	1.00	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
233	A	2	1	1.00	9	0.111
234	A	1	1	1.00	7	0.143
235	A	2	1	1.00	11	0.091
236	A	2	1	1.00	11	0.091
237	A	2	1	1.00	11	0.091
238	A	2	1	1.00	11	0.091
239	A	2	1	1.00	11	0.091
240	A	2	1	1.00	11	0.091
241	A	2	1	1.00	11	0.091
242	A	2	1	1.00	11	0.091
243	A	1	1	1.00	11	0.091
244	A	2	2	1.00	11	0.182
245	A	3	2	1.00	11	0.182
246	A	2	1	1.00	11	0.091
247	A	2	1	1.00	11	0.091
248	A	2	1	1.00	11	0.091
249	A	2	1	1.00	11	0.091
250	A	2	1	1.00	9	0.111
251	A	1	1	1.00	3	0.333
252	A	2	1	1.00	11	0.091
253	A	2	1	1.00	11	0.091
254	A	2	1	1.00	11	0.091
255	A	3	3	1.00	11	0.273
256	A	3	3	1.00	11	0.273
257	A	2	1	1.00	11	0.091
258	A	2	1	1.00	11	0.091
259	A	2	1	1.00	11	0.091
260	A	2	1	1.00	11	0.091
261	A	2	1	1.00	11	0.091
262	A	2	1	1.00	11	0.091
263	A	2	1	1.00	11	0.091
264	A	2	1	1.00	11	0.091
265	A	2	1	1.00	11	0.091
266	A	2	1	1.00	11	0.091
267	A	2	1	1.00	11	0.091
268	A	2	1	1.00	11	0.091
269	A	2	1	1.00	11	0.091
270	A	2	1	1.00	11	0.091
271	A	1	1	1.00	7	0.143
272	A	1	1	1.00	7	0.143
273	A	1	1	1.00	11	0.091
274	A	1	1	1.00	13	0.077
275	A	1	1	1.00	15	0.067
276	A	1	1	1.00	15	0.067
277	A	1	1	1.00	15	0.067
278	A	1	1	1.00	15	0.067
279	A	3	3	1.00	11	0.273
280	A	3	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
281	A	2	1	1.00	11	0.091
282	A	2	1	1.00	11	0.091
283	A	3	1	1.00	17	0.059
284	A	2	1	1.00	13	0.077
285	A	2	1	1.00	13	0.077
286	A	2	1	1.00	11	0.091
287	A	1	1	1.00	9	0.111
288	A	3	3	1.00	13	0.231
289	A	3	3	1.00	13	0.231
290	A	4	4	1.00	13	0.308
291	A	5	4	1.00	13	0.308
292	A	2	1	1.00	13	0.077
293	A	2	1	1.00	13	0.077
294	A	2	1	1.00	11	0.091
295	A	1	1	1.00	9	0.111
296	A	4	3	1.00	13	0.231
297	A	4	4	1.00	13	0.308
298	A	4	3	1.00	13	0.231
299	A	5	4	1.00	13	0.308
300	A	2	1	1.00	13	0.077
301	A	2	1	1.00	13	0.077
302	A	2	1	1.00	11	0.091
303	A	1	1	1.00	9	0.111
304	A	5	3	1.00	13	0.231
305	A	5	4	1.00	13	0.308
306	A	5	4	1.00	13	0.308
307	A	5	3	1.00	13	0.231
308	A	6	4	1.00	13	0.308
309	A	2	1	1.00	13	0.077
310	A	2	1	1.00	13	0.077
311	A	2	1	1.00	13	0.077
312	A	2	1	1.00	13	0.077
313	A	2	1	1.00	13	0.077
314	A	2	1	1.00	13	0.077
315	A	2	1	1.00	11	0.091
316	A	1	1	1.00	9	0.111
317	A	7	3	1.00	13	0.231
318	A	7	4	1.00	13	0.308
319	A	7	4	1.00	13	0.308
320	A	7	4	1.00	13	0.308
321	A	7	4	1.00	13	0.308
322	A	7	3	1.00	13	0.231
323	A	8	4	1.00	13	0.308
324	A	9	4	1.00	13	0.308
325	A	3	3	1.00	15	0.200
326	A	3	3	1.00	15	0.200
327	A	4	4	1.00	15	0.267
328	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
329	A	4	4	1.00	15	0.267
330	A	4	3	1.00	15	0.200
331	A	5	3	1.00	15	0.200
332	A	5	4	1.00	15	0.267
333	A	5	4	1.00	15	0.267
334	A	2	1	1.00	13	0.077
335	A	2	1	1.00	13	0.077
336	A	2	1	1.00	13	0.077
337	A	2	1	1.00	11	0.091
338	A	1	1	1.00	9	0.111
339	A	2	2	1.00	13	0.154
340	A	3	3	1.00	13	0.231
341	A	4	3	1.00	13	0.231
342	A	5	3	1.00	13	0.231
343	A	2	1	1.00	13	0.077
344	A	2	1	1.00	13	0.077
345	A	2	1	1.00	13	0.077
346	A	2	1	1.00	11	0.091
347	A	1	1	1.00	9	0.111
348	A	3	3	1.00	13	0.231
349	A	4	3	1.04	13	0.231
350	A	5	3	0.98	13	0.231
351	A	2	1	1.00	13	0.077
352	A	2	1	1.00	13	0.077
353	A	2	1	1.00	13	0.077
354	A	2	1	1.00	11	0.091
355	A	1	1	1.00	9	0.111
356	A	4	3	1.00	13	0.231
357	A	5	3	1.08	13	0.231
358	A	6	3	1.00	13	0.231
359	A	2	2	1.00	15	0.133
360	A	3	3	1.00	15	0.200
361	A	4	3	1.00	15	0.200
362	A	3	3	1.00	15	0.200
363	A	4	3	1.05	15	0.200
364	A	5	3	0.98	15	0.200
365	A	4	3	1.00	15	0.200
366	A	5	3	1.09	15	0.200
367	A	6	3	1.00	15	0.200
368	A	2	2	1.00	31	0.065
369	C	5	2	7.08	34	0.059
370	A	3	3	1.00	29	0.103
371	A	2	1	1.00	13	0.077
372	A	2	1	1.00	13	0.077
373	A	2	1	1.00	11	0.091
374	A	1	1	1.00	9	0.111
375	A	5	5	1.00	13	0.385
376	A	5	5	1.00	13	0.385

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
377	A	6	6	1.00	13	0.462
378	A	2	1	1.00	13	0.077
379	A	2	1	1.00	13	0.077
380	A	2	1	1.00	11	0.091
381	A	1	1	1.00	9	0.111
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	13	0.385
384	A	6	6	1.00	13	0.462
385	A	2	1	1.00	13	0.077
386	A	2	1	1.00	13	0.077
387	A	2	1	1.00	11	0.091
388	A	1	1	1.00	9	0.111
389	A	6	5	1.00	13	0.385
390	A	6	6	1.00	13	0.462
391	A	6	5	1.00	13	0.385
392	A	2	1	1.00	13	0.077
393	A	2	1	1.00	13	0.077
394	A	2	1	1.00	11	0.091
395	A	1	1	1.00	9	0.111
396	A	4	4	1.00	13	0.308
397	A	5	5	1.00	13	0.385
398	A	6	5	1.00	13	0.385
399	A	2	1	1.00	15	0.067
400	A	2	1	1.00	15	0.067
401	A	2	1	1.00	13	0.077
402	A	1	1	1.00	11	0.091
403	A	4	4	1.00	15	0.267
404	A	5	5	1.00	15	0.333
405	A	6	5	1.00	15	0.333
406	A	2	1	1.00	13	0.077
407	A	2	1	1.00	13	0.077
408	A	2	1	1.00	11	0.091
409	A	1	1	1.00	9	0.111
410	A	4	4	1.00	13	0.308
411	A	5	5	1.00	13	0.385
412	A	6	5	1.00	13	0.385
413	A	2	1	1.00	13	0.077
414	A	2	1	1.00	13	0.077
415	A	2	1	1.00	11	0.091
416	A	1	1	1.00	9	0.111
417	A	5	5	1.00	13	0.385
418	A	6	5	1.02	13	0.385
419	A	7	5	0.99	13	0.385
420	A	4	4	1.00	17	0.235
421	A	4	4	1.00	18	0.222
422	A	4	4	1.00	19	0.210
423	A	4	4	1.00	20	0.200
424	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
425	A	4	4	1.00	18	0.222
426	A	4	4	1.00	19	0.210
427	A	4	4	1.00	20	0.200
428	A	2	1	1.00	9	0.111
429	A	2	1	1.00	11	0.091
430	A	2	1	1.00	11	0.091
431	A	2	1	1.00	11	0.091
432	A	2	1	1.00	11	0.091
433	A	2	1	1.00	11	0.091
434	A	2	1	1.00	11	0.091
435	A	2	1	1.00	11	0.091
436	A	2	1	1.00	13	0.077
437	A	2	1	1.00	13	0.077
438	A	2	1	1.00	13	0.077
439	A	2	1	1.00	13	0.077
440	A	2	1	1.00	13	0.077
441	A	2	1	1.00	13	0.077
442	A	2	1	1.00	11	0.091
443	A	2	1	1.00	13	0.077
444	A	2	1	1.00	13	0.077
445	A	2	1	1.00	13	0.077
446	A	2	1	1.00	13	0.077
447	A	2	1	1.00	13	0.077
448	A	2	1	1.00	13	0.077
449	A	5	3	1.00	13	0.231
450	A	4	3	1.00	13	0.231
451	A	3	3	1.00	13	0.231
452	A	2	2	1.00	13	0.154
453	A	3	3	1.00	13	0.231
454	A	4	3	1.00	13	0.231
455	A	5	3	1.00	13	0.231
456	A	5	4	1.00	13	0.308
457	A	4	4	1.00	13	0.308
458	A	3	3	1.00	13	0.231
459	A	3	3	1.00	13	0.231
460	A	4	3	1.00	13	0.231
461	A	5	3	1.00	13	0.231
462	A	6	4	1.00	13	0.308
463	A	5	4	1.00	13	0.308
464	A	4	3	1.00	13	0.231
465	A	4	4	1.00	13	0.308
466	A	4	3	1.00	13	0.231
467	A	5	3	1.00	13	0.231
468	A	6	3	1.00	13	0.231
469	A	5	3	1.00	15	0.200
470	A	4	3	1.00	15	0.200
471	A	3	3	1.00	15	0.200
472	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
473	A	3	3	1.00	15	0.200
474	A	4	3	1.00	15	0.200
475	A	5	3	1.00	15	0.200
476	A	5	4	1.00	15	0.267
477	A	4	4	1.00	15	0.267
478	A	3	3	1.00	15	0.200
479	A	3	3	1.00	15	0.200
480	A	4	3	1.00	15	0.200
481	A	5	3	1.00	15	0.200
482	A	6	4	1.00	15	0.267
483	A	5	4	1.00	15	0.267
484	A	4	3	1.00	15	0.200
485	A	4	4	1.00	15	0.267
486	A	4	3	1.00	15	0.200
487	A	5	3	1.00	15	0.200
488	A	6	3	1.00	15	0.200
489	A	7	4	1.00	15	0.267
490	A	6	4	1.00	15	0.267
491	A	5	4	1.00	15	0.267
492	A	4	4	1.00	15	0.267
493	A	4	4	1.00	15	0.267
494	A	1	1	1.00	15	0.067
495	A	2	2	1.00	15	0.133
496	A	3	2	1.00	15	0.133
497	A	7	4	1.00	16	0.250
498	A	6	4	1.00	16	0.250
499	A	5	4	1.00	16	0.250
500	A	4	4	1.00	16	0.250
501	A	4	4	1.00	16	0.250
502	A	1	1	1.00	16	0.062
503	A	2	2	1.00	16	0.125
504	A	3	2	1.00	16	0.125
505	A	6	3	1.00	15	0.200
506	A	5	3	1.00	15	0.200
507	A	4	3	1.00	15	0.200
508	A	3	3	1.00	15	0.200
509	A	3	3	1.00	15	0.200
510	A	1	1	1.00	15	0.067
511	A	2	2	1.00	15	0.133
512	A	3	2	1.00	15	0.133
513	A	6	3	1.00	16	0.188
514	A	5	3	1.00	16	0.188
515	A	4	3	1.00	16	0.188
516	A	3	3	1.00	16	0.188
517	A	3	3	1.00	16	0.188
518	A	1	1	1.00	16	0.062
519	A	2	2	1.00	16	0.125
520	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
521	A	8	4	1.00	15	0.267
522	A	7	4	1.00	15	0.267
523	A	6	4	1.00	15	0.267
524	A	5	4	1.00	15	0.267
525	A	5	5	1.00	15	0.333
526	A	5	4	1.00	15	0.267
527	A	8	4	1.00	16	0.250
528	A	7	4	1.00	16	0.250
529	A	6	4	1.00	16	0.250
530	A	5	4	1.00	16	0.250
531	A	5	5	1.00	16	0.312
532	A	5	4	1.00	16	0.250
533	A	7	3	1.00	15	0.200
534	A	6	3	1.00	15	0.200
535	A	5	3	1.00	15	0.200
536	A	4	3	1.00	15	0.200
537	A	4	4	1.00	15	0.267
538	A	4	3	1.00	15	0.200
539	A	7	3	1.00	16	0.188
540	A	6	3	1.00	16	0.188
541	A	5	3	1.00	16	0.188
542	A	4	3	1.00	16	0.188
543	A	4	4	1.00	16	0.250
544	A	4	3	1.00	16	0.188
545	A	9	4	1.00	15	0.267
546	A	8	4	1.00	15	0.267
547	A	7	4	1.00	15	0.267
548	A	6	4	1.00	15	0.267
549	A	6	5	1.00	15	0.333
550	A	6	5	1.00	15	0.333
551	A	9	4	1.00	16	0.250
552	A	8	4	1.00	16	0.250
553	A	7	4	1.00	16	0.250
554	A	6	4	1.00	16	0.250
555	A	6	5	1.00	16	0.312
556	A	6	5	1.00	16	0.312
557	A	8	3	1.00	15	0.200
558	A	7	3	1.00	15	0.200
559	A	6	3	1.00	15	0.200
560	A	5	3	1.00	15	0.200
561	A	5	4	1.00	15	0.267
562	A	5	4	1.00	15	0.267
563	A	8	3	1.00	16	0.188
564	A	7	3	1.00	16	0.188
565	A	6	3	1.00	16	0.188
566	A	5	3	1.00	16	0.188
567	A	5	4	1.00	16	0.250
568	A	5	4	1.00	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
569	A	6	4	1.00	15	0.267
570	A	5	4	1.00	15	0.267
571	A	4	4	1.00	15	0.267
572	A	3	3	1.00	15	0.200
573	A	1	1	1.00	15	0.067
574	A	2	2	1.00	15	0.133
575	A	3	2	1.00	15	0.133
576	A	4	2	1.00	15	0.133
577	A	6	5	1.00	15	0.333
578	A	5	5	1.00	15	0.333
579	A	4	4	1.00	15	0.267
580	A	1	1	1.00	15	0.067
581	A	2	2	1.00	15	0.133
582	A	3	2	1.00	15	0.133
583	A	4	2	1.00	15	0.133
584	A	6	5	1.00	15	0.333
585	A	5	4	1.00	15	0.267
586	A	1	1	1.00	15	0.067
587	A	2	2	1.00	15	0.133
588	A	3	2	1.00	15	0.133
589	A	4	2	1.00	15	0.133
590	A	6	4	1.00	16	0.250
591	A	5	4	1.00	16	0.250
592	A	4	4	1.00	16	0.250
593	A	3	3	1.00	16	0.188
594	A	1	1	1.00	16	0.062
595	A	2	2	1.00	16	0.125
596	A	6	5	1.00	16	0.312
597	A	5	5	1.00	16	0.312
598	A	4	4	1.00	16	0.250
599	A	1	1	1.00	16	0.062
600	A	2	2	1.00	16	0.125
601	A	3	2	1.00	16	0.125
602	A	6	5	1.00	16	0.312
603	A	5	4	1.00	16	0.250
604	A	1	1	1.00	16	0.062
605	A	2	2	1.00	16	0.125
606	A	3	2	1.00	16	0.125
607	A	4	2	1.00	16	0.125
608	A	5	3	1.00	15	0.200
609	A	4	3	1.00	15	0.200
610	A	3	3	1.00	15	0.200
611	A	2	2	1.00	15	0.133
612	A	1	1	1.00	15	0.067
613	A	2	2	1.00	15	0.133
614	A	3	2	1.00	15	0.133
615	A	4	2	1.00	15	0.133
616	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
617	A	4	4	1.00	15	0.267
618	A	3	3	1.00	15	0.200
619	A	1	1	1.00	15	0.067
620	A	2	2	1.00	15	0.133
621	A	3	2	1.00	15	0.133
622	A	4	2	1.00	15	0.133
623	A	5	4	1.00	15	0.267
624	A	4	3	1.00	15	0.200
625	A	1	1	1.00	15	0.067
626	A	2	2	1.00	15	0.133
627	A	3	2	1.00	15	0.133
628	A	4	2	1.00	15	0.133
629	A	5	3	1.00	16	0.188
630	A	4	3	1.00	16	0.188
631	A	3	3	1.00	16	0.188
632	A	2	2	1.00	16	0.125
633	A	1	1	1.00	16	0.062
634	A	2	2	1.00	16	0.125
635	A	5	4	1.00	16	0.250
636	A	4	4	1.00	16	0.250
637	A	3	3	1.00	16	0.188
638	A	1	1	1.00	16	0.062
639	A	2	2	1.00	16	0.125
640	A	3	2	1.00	16	0.125
641	A	5	4	1.00	16	0.250
642	A	4	3	1.00	16	0.188
643	A	1	1	1.00	16	0.062
644	A	2	2	1.00	16	0.125
645	A	3	2	1.00	16	0.125
646	A	4	2	1.00	16	0.125
647	A	4	4	1.00	15	0.267
648	A	3	3	1.00	15	0.200
649	A	2	2	1.00	16	0.125
650	A	2	1	1.00	11	0.091
651	A	2	1	1.00	11	0.091
652	A	2	1	1.00	11	0.091
653	A	2	1	1.00	11	0.091
654	A	2	1	1.00	11	0.091
655	A	2	1	1.00	11	0.091
656	A	2	1	1.00	11	0.091
657	A	2	1	1.00	11	0.091
658	A	2	1	1.00	13	0.077
659	A	2	1	1.00	13	0.077
660	A	2	1	1.00	13	0.077
661	A	2	1	1.00	13	0.077
662	A	2	1	1.00	13	0.077
663	A	2	1	1.00	13	0.077
664	A	2	1	1.00	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
665	A	2	1	1.00	13	0.077
666	A	2	1	1.00	13	0.077
667	A	2	1	1.00	13	0.077
668	A	2	1	1.00	13	0.077
669	A	2	1	1.00	13	0.077
670	A	2	1	1.00	13	0.077
671	A	2	1	1.00	13	0.077
672	A	2	1	1.00	13	0.077
673	A	2	1	1.00	13	0.077
674	A	6	5	1.00	13	0.385
675	A	6	5	1.00	13	0.385
676	A	5	5	1.00	13	0.385
677	A	5	5	1.00	13	0.385
678	A	4	4	1.00	13	0.308
679	A	4	4	1.00	13	0.308
680	A	5	5	1.00	13	0.385
681	A	5	5	1.00	13	0.385
682	A	6	6	1.00	13	0.462
683	A	6	6	1.00	13	0.462
684	A	5	5	1.00	13	0.385
685	A	5	5	1.00	13	0.385
686	A	5	5	1.00	13	0.385
687	A	5	5	1.00	13	0.385
688	A	6	5	1.00	13	0.385
689	A	6	5	1.00	13	0.385
690	A	6	5	1.00	13	0.385
691	A	6	5	1.00	13	0.385
692	A	6	6	1.00	13	0.462
693	A	6	6	1.00	13	0.462
694	A	6	5	1.00	13	0.385
695	A	6	5	1.00	13	0.385
696	A	7	5	1.00	13	0.385
697	A	7	5	1.00	13	0.385
698	A	5	5	1.00	15	0.333
699	A	2	1	1.00	11	0.091
700	A	2	1	1.00	11	0.091
701	A	2	1	1.00	11	0.091
702	A	2	1	1.00	11	0.091
703	A	2	1	1.00	9	0.111
704	A	2	1	1.00	11	0.091
705	A	2	1	1.00	11	0.091
706	A	2	1	1.00	9	0.111
707	A	1	1	1.00	7	0.143
708	A	3	2	1.00	15	0.133
709	A	2	2	1.00	15	0.133
710	A	1	1	1.00	15	0.067
711	A	1	1	1.00	17	0.059
712	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
713	A	2	2	1.00	19	0.105
714	A	3	2	1.00	18	0.111
715	A	3	2	1.00	18	0.111
716	A	3	2	1.00	16	0.125
717	A	3	2	1.00	15	0.133
718	A	2	1	1.00	18	0.056
719	A	3	2	1.00	18	0.111
720	A	3	2	1.00	18	0.111
721	A	2	2	1.00	18	0.111
722	A	3	2	1.00	18	0.111
723	A	3	2	1.00	18	0.111
724	A	3	2	1.00	16	0.125
725	A	3	2	1.00	15	0.133
726	A	3	2	1.00	18	0.111
727	A	3	2	1.00	18	0.111
728	A	2	1	1.00	18	0.056
729	A	3	2	1.00	18	0.111
730	A	3	2	1.00	18	0.111
731	A	3	2	1.00	18	0.111
732	A	3	2	1.00	16	0.125
733	A	3	2	1.00	15	0.133
734	A	3	2	1.00	18	0.111
735	A	3	2	1.00	18	0.111
736	A	3	2	1.00	18	0.111
737	A	3	2	1.00	18	0.111
738	A	3	2	1.00	18	0.111
739	A	3	2	1.00	18	0.111
740	A	2	1	1.00	16	0.062
741	A	3	2	1.00	15	0.133
742	A	3	2	1.00	18	0.111
743	A	2	2	1.00	18	0.111
744	A	3	2	1.00	18	0.111
745	A	3	2	1.00	18	0.111
746	A	2	1	1.00	18	0.056
747	A	3	2	1.00	18	0.111
748	A	3	2	1.00	16	0.125
749	A	2	2	1.00	15	0.133
750	A	3	2	1.00	18	0.111
751	A	3	2	1.00	18	0.111
752	A	3	2	1.00	18	0.111
753	A	3	2	1.00	18	0.111
754	A	3	2	1.00	18	0.111
755	A	2	2	1.00	18	0.111
756	A	3	2	1.00	16	0.125
757	A	3	2	1.00	15	0.133
758	A	3	2	1.00	18	0.111
759	A	3	2	1.00	18	0.111
760	A	3	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
761	A	3	2	1.00	18	0.111
762	A	3	2	1.00	20	0.100
763	A	3	2	1.00	20	0.100
764	A	3	2	1.00	18	0.111
765	A	3	2	1.00	17	0.118
766	A	2	2	1.00	20	0.100
767	A	3	2	1.00	20	0.100
768	A	3	2	1.00	20	0.100
769	A	3	2	1.00	20	0.100
770	A	3	2	1.00	20	0.100
771	A	3	2	1.00	20	0.100
772	A	3	2	1.00	18	0.111
773	A	3	2	1.00	17	0.118
774	A	3	2	1.00	20	0.100
775	A	3	2	1.00	20	0.100
776	A	2	2	1.00	20	0.100
777	A	3	2	1.00	20	0.100
778	A	3	2	1.00	18	0.111
779	A	3	2	1.00	17	0.118
780	A	3	2	1.00	20	0.100
781	A	3	2	1.00	20	0.100
782	A	3	2	1.00	20	0.100
783	A	3	2	1.00	20	0.100
784	A	2	2	1.00	20	0.100
785	A	3	2	1.00	20	0.100
786	A	3	2	1.00	20	0.100
787	A	3	2	1.00	20	0.100
788	A	2	2	1.00	18	0.111
789	A	3	2	1.00	17	0.118
790	A	3	2	1.00	20	0.100
791	A	3	2	1.00	20	0.100
792	A	2	2	1.00	20	0.100
793	A	3	2	1.00	20	0.100
794	A	2	2	1.00	20	0.100
795	A	3	2	1.00	20	0.100
796	A	3	2	1.00	18	0.111
797	A	3	2	1.00	17	0.118
798	A	2	2	1.00	20	0.100
799	A	3	2	1.00	20	0.100
800	A	3	2	1.00	20	0.100
801	A	3	2	1.00	20	0.100
802	A	3	2	1.00	20	0.100
803	A	3	2	1.00	20	0.100
804	A	2	2	1.00	18	0.111
805	A	3	2	1.00	17	0.118
806	A	3	2	1.00	20	0.100
807	A	3	2	1.00	20	0.100
808	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
809	A	3	2	1.00	20	0.100
810	A	3	2	1.00	20	0.100
811	A	3	2	1.00	20	0.100
812	A	3	2	1.00	18	0.111
813	A	3	2	1.00	17	0.118
814	A	2	2	1.00	20	0.100
815	A	4	4	1.00	20	0.200
816	A	3	2	1.00	20	0.100
817	A	3	2	1.00	20	0.100
818	A	3	2	1.00	18	0.111
819	A	3	2	1.00	17	0.118
820	A	3	2	1.00	20	0.100
821	A	3	2	1.00	20	0.100
822	A	2	2	1.00	20	0.100
823	A	4	4	1.00	20	0.200
824	A	3	2	1.00	20	0.100
825	A	3	2	1.00	20	0.100
826	A	3	2	1.00	20	0.100
827	A	3	2	1.00	17	0.118
828	A	3	2	1.00	20	0.100
829	A	3	2	1.00	20	0.100
830	A	3	2	1.00	20	0.100
831	A	3	2	1.00	20	0.100
832	A	2	2	1.00	20	0.100
833	A	4	4	1.00	20	0.200
834	A	3	2	1.00	20	0.100
835	A	3	2	1.00	20	0.100
836	A	3	2	1.00	20	0.100
837	A	3	2	1.00	20	0.100
838	A	2	2	1.00	18	0.111
839	A	4	4	1.00	17	0.235
840	A	3	2	1.00	20	0.100
841	A	3	2	1.00	20	0.100
842	A	3	2	1.00	20	0.100
843	A	3	2	1.00	20	0.100
844	A	3	2	1.00	20	0.100
845	A	3	2	1.00	20	0.100
846	A	2	2	1.00	20	0.100
847	A	4	4	1.00	20	0.200
848	A	3	2	1.00	18	0.111
849	A	3	2	1.00	17	0.118
850	A	3	2	1.00	20	0.100
851	A	3	2	1.00	20	0.100
852	A	3	2	1.00	20	0.100
853	A	3	2	1.00	18	0.111
854	A	3	2	1.00	17	0.118
855	A	2	2	1.00	20	0.100
856	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
857	A	3	2	1.00	20	0.100
858	A	3	2	1.00	20	0.100
859	A	3	2	1.00	18	0.111
860	A	3	2	1.00	17	0.118
861	A	3	2	1.00	20	0.100
862	A	3	2	1.00	20	0.100
863	A	2	2	1.00	20	0.100
864	A	3	2	1.00	20	0.100
865	A	3	2	1.00	20	0.100
866	A	3	2	1.00	20	0.100
867	A	3	2	1.00	20	0.100
868	A	3	2	1.00	20	0.100
869	A	3	2	1.00	20	0.100
870	A	3	2	1.00	20	0.100
871	A	2	2	1.00	18	0.111
872	A	3	2	1.00	17	0.118
873	A	3	2	1.00	20	0.100
874	A	3	2	1.00	20	0.100
875	A	3	2	1.00	20	0.100
876	A	3	2	1.00	20	0.100
877	A	2	2	1.00	20	0.100
878	A	3	2	1.00	20	0.100
879	A	3	2	1.00	18	0.111
880	A	3	2	1.00	17	0.118
881	A	3	2	1.00	20	0.100
882	A	3	2	1.00	18	0.111
883	A	3	2	1.00	17	0.118
884	A	2	2	1.00	20	0.100
885	A	3	2	1.00	18	0.111
886	A	3	2	1.00	17	0.118
887	A	3	2	1.00	20	0.100
888	A	3	2	1.00	20	0.100
889	A	2	2	1.00	20	0.100
890	A	3	2	1.00	17	0.118
891	A	3	2	1.00	20	0.100
892	A	3	2	1.00	20	0.100
893	A	3	2	1.00	20	0.100
894	A	3	2	1.00	20	0.100
895	A	2	2	1.00	20	0.100
896	A	3	2	1.00	20	0.100
897	A	3	2	1.00	20	0.100
898	A	3	2	1.00	20	0.100
899	A	2	2	1.00	18	0.111
900	A	3	2	1.00	20	0.100
901	A	3	2	1.00	20	0.100
902	A	3	2	1.00	20	0.100
903	A	2	2	1.00	20	0.100
904	A	3	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
905	A	3	2	1.00	20	0.100
906	A	3	2	1.00	20	0.100
907	A	2	2	1.00	20	0.100
908	A	4	3	1.00	20	0.150
909	A	4	3	1.00	20	0.150
910	A	4	3	1.00	20	0.150
911	A	4	3	1.00	20	0.150
912	A	4	3	1.00	20	0.150
913	A	4	3	1.00	20	0.150
914	A	4	3	1.00	22	0.136
915	A	4	3	1.00	22	0.136
916	A	4	3	1.00	22	0.136
917	A	4	3	1.00	22	0.136
918	A	4	3	1.00	22	0.136
919	A	4	3	1.00	22	0.136
920	A	2	2	1.00	22	0.091
921	A	2	2	1.00	22	0.091
922	A	2	2	1.00	20	0.100
923	A	2	2	1.00	19	0.105
924	A	2	2	1.00	22	0.091
925	A	2	2	1.00	20	0.100
926	A	2	2	1.00	22	0.091
927	A	2	2	1.00	22	0.091
928	A	2	2	1.00	25	0.080
929	A	3	3	1.00	27	0.111
930	A	2	2	1.00	20	0.100
931	A	2	1	1.00	20	0.050
932	A	2	2	1.00	20	0.100
933	A	2	2	1.00	20	0.100
934	A	2	2	1.00	18	0.111
935	A	2	2	1.00	20	0.100
936	A	2	2	1.00	20	0.100
937	A	2	2	1.00	20	0.100
938	A	2	2	1.00	20	0.100
939	A	2	1	1.00	20	0.050
940	A	2	2	1.00	20	0.100
941	A	2	2	1.00	20	0.100
942	A	2	2	1.00	18	0.111
943	A	2	2	1.00	20	0.100
944	A	2	2	1.00	20	0.100
945	A	2	2	1.00	20	0.100
946	A	2	2	1.00	18	0.111
947	A	2	2	1.00	18	0.111
948	A	2	2	1.00	18	0.111
949	A	2	2	1.00	16	0.125
950	A	2	2	1.00	18	0.111
951	A	2	2	1.00	18	0.111
952	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
953	A	2	1	1.00	18	0.056
954	A	2	2	1.00	18	0.111
955	A	2	2	1.00	18	0.111
956	A	2	2	1.00	18	0.111
957	A	2	2	1.00	18	0.111
958	A	2	2	1.00	19	0.105
959	A	2	1	1.00	17	0.059
960	A	2	1	1.00	17	0.059
961	A	2	1	1.00	15	0.067
962	A	1	0	1.00	5	0.000
963	A	2	1	1.00	17	0.059
964	A	2	1	1.00	17	0.059
965	A	1	1	1.00	17	0.059
966	A	2	1	1.00	17	0.059
967	A	2	1	1.00	17	0.059
968	A	2	1	1.00	17	0.059
969	A	2	1	1.00	19	0.053
970	A	3	2	1.00	19	0.105
971	A	2	1	1.00	17	0.059
972	A	1	1	1.00	7	0.143
973	A	2	1	1.00	19	0.053
974	A	2	1	1.00	19	0.053
975	A	2	1	1.00	19	0.053
976	A	1	1	1.00	19	0.053
977	A	2	1	1.00	19	0.053
978	A	2	1	1.00	19	0.053
979	A	2	1	1.00	19	0.053
980	A	2	1	1.00	19	0.053
981	A	2	1	1.00	19	0.053
982	A	2	1	1.00	17	0.059
983	A	1	1	1.00	7	0.143
984	A	2	2	1.00	19	0.105
985	A	3	2	1.00	19	0.105
986	A	3	2	1.00	19	0.105
987	A	2	1	1.00	19	0.053
988	A	2	1	1.00	19	0.053
989	A	2	1	1.00	17	0.059
990	A	1	1	1.00	7	0.143
991	A	3	2	1.00	19	0.105
992	A	3	3	1.00	19	0.158
993	A	3	2	1.00	19	0.105
994	A	7	4	1.00	17	0.235
995	A	6	4	1.00	17	0.235
996	A	5	4	1.00	17	0.235
997	A	4	4	1.00	17	0.235
998	A	3	3	1.00	17	0.176
999	A	3	3	1.00	17	0.176
1000	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1001	A	1	1	1.00	17	0.059
1002	A	2	2	1.00	17	0.118
1003	A	3	2	1.00	17	0.118
1004	A	4	2	1.00	17	0.118
1005	A	5	2	1.00	17	0.118
1006	A	7	4	1.00	17	0.235
1007	A	6	4	1.00	17	0.235
1008	A	5	4	1.00	17	0.235
1009	A	4	3	1.00	17	0.176
1010	A	4	4	1.00	17	0.235
1011	A	4	3	1.00	17	0.176
1012	A	4	4	1.00	17	0.235
1013	A	4	3	1.00	17	0.176
1014	A	1	1	1.00	17	0.059
1015	A	2	2	1.00	17	0.118
1016	A	3	2	1.00	17	0.118
1017	A	4	2	1.00	17	0.118
1018	A	5	2	1.00	17	0.118
1019	A	8	4	1.00	17	0.235
1020	A	7	4	1.00	17	0.235
1021	A	6	4	1.00	17	0.235
1022	A	5	3	1.00	17	0.176
1023	A	5	4	1.00	17	0.235
1024	A	5	4	1.00	17	0.235
1025	A	5	3	1.00	17	0.176
1026	A	5	4	1.00	17	0.235
1027	A	5	4	1.00	17	0.235
1028	A	5	3	1.00	17	0.176
1029	A	1	1	1.00	17	0.059
1030	A	2	2	1.00	17	0.118
1031	A	3	2	1.00	17	0.118
1032	A	4	2	1.00	17	0.118
1033	A	5	2	1.00	17	0.118
1034	A	6	2	1.00	17	0.118
1035	A	4	3	1.00	20	0.150
1036	A	3	3	1.00	28	0.107
1037	A	6	3	1.00	17	0.176
1038	A	5	3	1.00	17	0.176
1039	A	4	3	1.00	17	0.176
1040	A	3	3	1.00	17	0.176
1041	A	2	2	1.00	17	0.118
1042	A	1	1	1.00	17	0.059
1043	A	2	2	1.00	17	0.118
1044	A	3	2	1.00	17	0.118
1045	A	4	2	1.00	17	0.118
1046	A	5	2	1.00	17	0.118
1047	A	6	4	1.00	17	0.235
1048	A	5	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1049	A	4	4	1.00	17	0.235
1050	A	3	3	1.00	17	0.176
1051	A	1	1	1.00	17	0.059
1052	A	1	1	1.00	17	0.059
1053	A	2	2	1.00	17	0.118
1054	A	3	2	1.00	17	0.118
1055	A	4	2	1.00	17	0.118
1056	A	5	2	1.00	17	0.118
1057	A	7	4	1.00	17	0.235
1058	A	6	4	1.00	17	0.235
1059	A	5	4	1.00	17	0.235
1060	A	4	3	1.00	17	0.176
1061	A	1	1	1.00	17	0.059
1062	A	2	2	1.00	17	0.118
1063	A	3	2	1.00	17	0.118
1064	A	2	2	1.00	17	0.118
1065	A	3	3	1.00	17	0.176
1066	A	4	3	1.00	17	0.176
1067	A	5	3	1.00	17	0.176
1068	A	6	4	1.00	20	0.200
1069	A	5	4	1.00	20	0.200
1070	A	4	4	1.00	20	0.200
1071	A	3	3	1.00	20	0.150
1072	A	1	1	1.00	20	0.050
1073	A	2	2	1.00	20	0.100
1074	A	3	2	1.00	20	0.100
1075	A	4	2	1.00	20	0.100
1076	A	6	4	1.00	23	0.174
1077	A	5	4	1.00	23	0.174
1078	A	4	4	1.00	23	0.174
1079	A	3	3	1.00	23	0.130
1080	A	1	1	1.00	23	0.043
1081	A	2	2	1.00	23	0.087
1082	A	3	2	1.00	23	0.087
1083	A	4	2	1.00	23	0.087
1084	A	5	3	1.00	19	0.158
1085	A	4	3	1.00	19	0.158
1086	A	3	3	1.00	19	0.158
1087	A	2	2	1.00	19	0.105
1088	A	1	1	1.00	19	0.053
1089	A	2	2	1.00	19	0.105
1090	A	3	2	1.00	19	0.105
1091	A	7	4	1.00	17	0.235
1092	A	5	4	1.00	17	0.235
1093	A	3	3	1.00	17	0.176
1094	A	2	2	1.00	17	0.118
1095	A	4	2	1.00	17	0.118
1096	A	1	1	1.00	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1097	A	1	1	1.00	20	0.050
1098	A	1	1	1.00	17	0.059
1099	A	1	1	1.00	20	0.050
1100	A	3	3	1.00	23	0.130
1101	A	11	8	1.00	20	0.400
1102	A	12	9	1.00	25	0.360
1103	A	11	8	1.00	25	0.320
1104	A	1	1	1.00	25	0.040
1105	A	2	2	1.00	25	0.080
1106	A	3	2	1.00	25	0.080
1107	A	4	2	1.00	25	0.080
1108	A	12	9	1.00	25	0.360
1109	A	11	8	1.00	25	0.320
1110	A	1	1	1.00	25	0.040
1111	A	2	2	1.00	25	0.080
1112	A	3	2	1.00	25	0.080
1113	A	13	10	1.00	25	0.400
1114	A	12	9	1.00	25	0.360
1115	A	1	1	1.00	25	0.040
1116	A	2	2	1.00	25	0.080
1117	A	3	2	1.00	25	0.080
1118	A	13	10	1.00	25	0.400
1119	A	12	9	1.00	25	0.360
1120	A	1	1	1.00	25	0.040
1121	A	2	2	1.00	25	0.080
1122	A	3	2	1.00	25	0.080
1123	A	13	9	1.00	25	0.360
1124	A	1	1	1.00	25	0.040
1125	A	2	2	1.00	25	0.080
1126	A	3	2	1.00	25	0.080
1127	A	4	2	1.00	25	0.080
1128	A	2	1	1.00	19	0.053
1129	A	2	1	1.00	17	0.059
1130	A	2	1	1.00	13	0.077
1131	A	2	1	1.00	13	0.077
1132	A	2	1	1.00	13	0.077
1133	A	2	1	1.00	11	0.091
1134	A	1	0	1.00	5	0.000
1135	A	2	1	1.00	13	0.077
1136	A	2	1	1.00	13	0.077
1137	A	1	1	1.00	13	0.077
1138	A	2	1	1.00	13	0.077
1139	A	2	1	1.00	13	0.077
1140	A	2	1	1.00	15	0.067
1141	A	2	1	1.00	15	0.067
1142	A	2	1	1.00	15	0.067
1143	A	2	1	1.00	13	0.077
1144	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1145	A	2	1	1.00	15	0.067
1146	A	2	1	1.00	15	0.067
1147	A	2	1	1.00	15	0.067
1148	A	1	1	1.00	15	0.067
1149	A	2	1	1.00	15	0.067
1150	A	2	1	1.00	15	0.067
1151	A	2	1	1.00	15	0.067
1152	A	2	1	1.00	15	0.067
1153	A	2	1	1.00	15	0.067
1154	A	2	1	1.00	15	0.067
1155	A	2	1	1.00	15	0.067
1156	A	2	1	1.00	13	0.077
1157	A	1	1	1.00	7	0.143
1158	A	2	1	1.00	15	0.067
1159	A	2	1	1.00	15	0.067
1160	A	2	1	1.00	15	0.067
1161	A	2	1	1.00	15	0.067
1162	A	1	1	1.00	15	0.067
1163	A	2	2	1.00	15	0.133
1164	A	2	1	1.00	15	0.067
1165	A	2	1	1.00	15	0.067
1166	A	2	1	1.00	15	0.067
1167	A	2	1	1.00	15	0.067
1168	A	2	1	1.00	15	0.067
1169	A	2	1	1.00	15	0.067
1170	A	2	1	1.00	15	0.067
1171	A	2	1	1.00	15	0.067
1172	A	2	1	1.00	15	0.067
1173	A	2	1	1.00	15	0.067
1174	A	2	1	1.00	15	0.067
1175	A	2	1	1.00	13	0.077
1176	A	1	1	1.00	7	0.143
1177	A	2	1	1.00	15	0.067
1178	A	2	1	1.00	15	0.067
1179	A	2	1	1.00	15	0.067
1180	A	2	1	1.00	15	0.067
1181	A	2	1	1.00	15	0.067
1182	A	2	1	1.00	15	0.067
1183	A	2	1	1.00	15	0.067
1184	A	2	1	1.00	15	0.067
1185	A	1	1	1.00	15	0.067
1186	A	2	2	1.00	15	0.133
1187	A	3	2	1.00	15	0.133
1188	A	4	2	1.00	15	0.133
1189	A	5	2	1.00	15	0.133
1190	A	2	1	1.00	15	0.067
1191	A	2	1	1.00	15	0.067
1192	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1193	A	2	1	1.00	15	0.067
1194	A	2	1	1.00	15	0.067
1195	A	2	1	1.00	15	0.067
1196	A	2	1	1.00	15	0.067
1197	A	2	1	1.00	15	0.067
1198	A	2	1	1.00	15	0.067
1199	A	2	1	1.00	15	0.067
1200	A	2	1	1.00	15	0.067
1201	A	2	1	1.00	15	0.067
1202	A	2	1	1.00	15	0.067
1203	A	2	1	1.00	15	0.067
1204	A	2	1	1.00	13	0.077
1205	A	1	1	1.00	7	0.143
1206	A	2	1	1.00	15	0.067
1207	A	2	1	1.00	15	0.067
1208	A	2	1	1.00	15	0.067
1209	A	2	1	1.00	15	0.067
1210	A	2	1	1.00	15	0.067
1211	A	2	1	1.00	15	0.067
1212	A	2	1	1.00	15	0.067
1213	A	2	1	1.00	15	0.067
1214	A	2	1	1.00	15	0.067
1215	A	2	1	1.00	15	0.067
1216	A	2	1	1.00	15	0.067
1217	A	1	1	1.00	15	0.067
1218	A	2	2	1.00	15	0.133
1219	A	3	2	1.00	15	0.133
1220	A	4	2	1.00	15	0.133
1221	A	5	2	1.00	15	0.133
1222	A	6	2	1.00	15	0.133
1223	A	7	2	1.00	15	0.133
1224	A	8	2	1.00	15	0.133
1225	A	2	1	1.00	15	0.067
1226	A	2	1	1.00	15	0.067
1227	A	2	1	1.00	15	0.067
1228	A	2	1	1.00	15	0.067
1229	A	2	1	1.00	15	0.067
1230	A	2	1	1.00	15	0.067
1231	A	2	1	1.00	15	0.067
1232	A	2	1	1.00	13	0.077
1233	A	1	1	1.00	7	0.143
1234	A	3	2	1.00	15	0.133
1235	A	2	1	1.00	15	0.067
1236	A	2	1	1.00	15	0.067
1237	A	2	1	1.00	15	0.067
1238	A	2	1	1.00	15	0.067
1239	A	2	1	1.00	15	0.067
1240	A	2	1	1.00	15	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1241	A	2	1	1.00	13	0.077
1242	A	1	1	1.00	7	0.143
1243	A	2	1	1.00	15	0.067
1244	A	2	1	1.00	15	0.067
1245	A	2	1	1.00	15	0.067
1246	A	2	1	1.00	15	0.067
1247	A	2	1	1.00	15	0.067
1248	A	2	1	1.00	15	0.067
1249	A	2	1	1.00	15	0.067
1250	A	2	1	1.00	15	0.067
1251	A	1	1	1.00	13	0.077
1252	A	1	1	1.00	7	0.143
1253	A	2	1	1.00	15	0.067
1254	A	2	1	1.00	15	0.067
1255	A	2	1	1.00	15	0.067
1256	A	2	1	1.00	15	0.067
1257	A	2	1	1.00	15	0.067
1258	A	2	1	1.00	15	0.067
1259	A	1	1	1.00	15	0.067
1260	A	2	2	1.00	15	0.133
1261	A	3	2	1.00	15	0.133
1262	A	2	1	1.00	15	0.067
1263	A	2	1	1.00	15	0.067
1264	A	2	1	1.00	13	0.077
1265	A	1	1	1.00	7	0.143
1266	A	2	1	1.00	15	0.067
1267	A	2	1	1.00	15	0.067
1268	A	2	1	1.00	15	0.067
1269	A	2	1	1.00	17	0.059
1270	A	2	1	1.00	17	0.059
1271	A	2	1	1.00	17	0.059
1272	A	2	1	1.00	17	0.059
1273	A	2	1	1.00	15	0.067
1274	A	1	1	1.00	9	0.111
1275	A	3	3	1.00	17	0.176
1276	A	3	3	1.00	17	0.176
1277	A	4	4	1.00	17	0.235
1278	A	5	4	1.00	17	0.235
1279	A	6	4	1.00	17	0.235
1280	A	7	4	1.00	17	0.235
1281	A	2	1	1.00	17	0.059
1282	A	2	1	1.00	17	0.059
1283	A	2	1	1.00	17	0.059
1284	A	2	1	1.00	17	0.059
1285	A	2	1	1.00	15	0.067
1286	A	1	1	1.00	9	0.111
1287	A	4	3	1.00	17	0.176
1288	A	4	4	1.00	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1289	A	4	3	1.00	17	0.176
1290	A	5	4	1.00	17	0.235
1291	A	6	4	1.00	17	0.235
1292	A	7	4	1.00	17	0.235
1293	A	2	1	1.00	17	0.059
1294	A	2	1	1.00	17	0.059
1295	A	2	1	1.00	17	0.059
1296	A	2	1	1.00	17	0.059
1297	A	2	1	1.00	15	0.067
1298	A	1	1	1.00	9	0.111
1299	A	5	3	1.00	17	0.176
1300	A	5	4	1.00	17	0.235
1301	A	5	4	1.00	17	0.235
1302	A	5	3	1.00	17	0.176
1303	A	6	4	1.00	17	0.235
1304	A	7	4	1.00	17	0.235
1305	A	3	3	1.00	13	0.231
1306	A	4	4	1.00	13	0.308
1307	A	2	1	1.00	17	0.059
1308	A	2	1	1.00	17	0.059
1309	A	2	1	1.00	17	0.059
1310	A	2	1	1.00	17	0.059
1311	A	2	1	1.00	15	0.067
1312	A	1	1	1.00	9	0.111
1313	A	2	2	1.00	17	0.118
1314	A	3	3	1.00	17	0.176
1315	A	4	3	1.00	17	0.176
1316	A	5	3	1.00	17	0.176
1317	A	6	3	1.00	17	0.176
1318	A	2	1	1.00	17	0.059
1319	A	2	1	1.00	17	0.059
1320	A	2	1	1.00	17	0.059
1321	A	2	1	1.00	17	0.059
1322	A	2	1	1.00	15	0.067
1323	A	1	1	1.00	9	0.111
1324	A	3	3	1.00	17	0.176
1325	A	4	3	1.00	17	0.176
1326	A	5	3	1.00	17	0.176
1327	A	6	3	1.00	17	0.176
1328	A	2	1	1.00	17	0.059
1329	A	2	1	1.00	17	0.059
1330	A	2	1	1.00	17	0.059
1331	A	2	1	1.00	17	0.059
1332	A	2	1	1.00	15	0.067
1333	A	1	1	1.00	9	0.111
1334	A	4	3	1.00	17	0.176
1335	A	5	3	1.00	17	0.176
1336	A	6	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1337	A	7	3	1.00	17	0.176
1338	A	2	2	1.00	20	0.100
1339	A	2	2	1.00	20	0.100
1340	A	2	2	1.00	20	0.100
1341	A	2	2	1.00	20	0.100
1342	A	2	2	1.00	20	0.100
1343	A	2	2	1.00	20	0.100
1344	A	2	2	1.00	20	0.100
1345	A	2	2	1.00	20	0.100
1346	A	2	2	1.00	20	0.100
1347	A	2	2	1.00	13	0.154
1348	A	2	2	1.00	17	0.118
1349	A	5	5	1.00	15	0.333
1350	A	2	1	1.00	13	0.077
1351	A	2	1	1.00	15	0.067
1352	A	4	4	1.00	17	0.235
1353	A	4	4	1.00	17	0.235
1354	A	8	4	1.00	19	0.210
1355	A	7	4	1.00	19	0.210
1356	A	6	4	1.00	19	0.210
1357	A	5	4	1.00	19	0.210
1358	A	4	4	1.00	19	0.210
1359	A	4	4	1.00	19	0.210
1360	A	1	1	1.00	19	0.053
1361	A	2	2	1.00	19	0.105
1362	A	3	2	1.00	19	0.105
1363	A	4	2	1.00	19	0.105
1364	A	5	2	1.00	19	0.105
1365	A	8	4	1.00	19	0.210
1366	A	7	4	1.00	19	0.210
1367	A	6	4	1.00	19	0.210
1368	A	5	4	1.00	19	0.210
1369	A	5	5	1.00	19	0.263
1370	A	5	4	1.00	19	0.210
1371	A	1	1	1.00	19	0.053
1372	A	2	2	1.00	19	0.105
1373	A	3	2	1.00	19	0.105
1374	A	4	2	1.00	19	0.105
1375	A	9	4	1.00	19	0.210
1376	A	8	4	1.00	19	0.210
1377	A	7	4	1.00	19	0.210
1378	A	6	4	1.00	19	0.210
1379	A	6	5	1.00	19	0.263
1380	A	6	5	1.00	19	0.263
1381	A	6	4	1.00	19	0.210
1382	A	1	1	1.00	19	0.053
1383	A	2	2	1.00	19	0.105
1384	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1385	A	4	2	1.00	19	0.105
1386	A	7	4	1.00	19	0.210
1387	A	6	4	1.00	19	0.210
1388	A	5	4	1.00	19	0.210
1389	A	4	4	1.00	19	0.210
1390	A	3	3	1.00	19	0.158
1391	A	1	1	1.00	19	0.053
1392	A	2	2	1.00	19	0.105
1393	A	3	2	1.00	19	0.105
1394	A	4	2	1.00	19	0.105
1395	A	5	2	1.00	19	0.105
1396	A	7	5	1.00	19	0.263
1397	A	6	5	1.00	19	0.263
1398	A	5	5	1.00	19	0.263
1399	A	4	4	1.00	19	0.210
1400	A	1	1	1.00	19	0.053
1401	A	2	2	1.00	19	0.105
1402	A	3	2	1.00	19	0.105
1403	A	4	2	1.00	19	0.105
1404	A	5	2	1.00	19	0.105
1405	A	6	2	1.00	19	0.105
1406	A	8	5	1.00	19	0.263
1407	A	7	5	1.00	19	0.263
1408	A	6	5	1.00	19	0.263
1409	A	5	4	1.00	19	0.210
1410	A	1	1	1.00	19	0.053
1411	A	2	2	1.00	19	0.105
1412	A	3	2	1.00	19	0.105
1413	A	4	2	1.00	19	0.105
1414	A	5	2	1.00	19	0.105
1415	A	6	2	1.00	19	0.105
1416	A	2	2	1.00	20	0.100
1417	A	2	2	1.00	19	0.105
1418	A	2	2	1.00	19	0.105
1419	A	2	2	1.00	17	0.118
1420	A	2	2	1.00	19	0.105
1421	A	1	1	1.00	19	0.053
1422	A	2	2	1.00	19	0.105
1423	A	2	2	1.00	19	0.105
1424	A	1	1	1.00	7	0.143
1425	A	2	2	1.00	19	0.105
1426	A	2	2	1.00	17	0.118
1427	A	2	2	1.00	19	0.105
1428	A	1	1	1.00	19	0.053
1429	A	2	2	1.00	19	0.105
1430	A	3	3	1.00	20	0.150
1431	A	2	2	1.00	20	0.100
1432	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1433	A	3	3	1.00	18	0.167
1434	A	3	3	1.00	20	0.150
1435	A	2	2	1.00	20	0.100
1436	A	3	3	1.00	20	0.150
1437	A	2	2	1.00	21	0.095
1438	A	1	1	1.00	8	0.125
1439	A	2	2	1.00	21	0.095
1440	A	2	2	1.00	19	0.105
1441	A	2	2	1.00	21	0.095
1442	A	1	1	1.00	21	0.048
1443	A	2	2	1.00	21	0.095
1444	A	1	1	1.00	19	0.053
1445	A	2	2	1.00	29	0.069
1446	A	2	2	1.00	15	0.133
1447	A	2	2	1.00	19	0.105
1448	A	2	2	1.00	29	0.069
1449	A	3	3	1.00	15	0.200
1450	A	2	2	1.00	15	0.133
1451	A	2	2	1.00	19	0.105
1452	A	3	3	1.00	20	0.150
1453	A	3	2	1.00	19	0.105
1454	A	2	2	1.00	19	0.105
1455	A	2	2	1.00	19	0.105
1456	A	1	1	1.00	19	0.053
1457	A	2	2	1.00	19	0.105
1458	A	3	2	1.00	19	0.105
1459	A	4	2	1.00	19	0.105
1460	A	3	2	1.00	19	0.105
1461	A	2	2	1.00	19	0.105
1462	A	1	1	1.00	19	0.053
1463	A	1	1	1.00	19	0.053
1464	A	2	2	1.00	19	0.105
1465	A	3	2	1.00	19	0.105
1466	A	4	2	1.00	19	0.105
1467	A	3	2	1.00	19	0.105
1468	A	2	2	1.00	19	0.105
1469	A	1	1	1.00	19	0.053
1470	A	1	1	1.00	19	0.053
1471	A	2	2	1.00	19	0.105
1472	A	3	2	1.00	19	0.105
1473	A	4	2	1.00	19	0.105
1474	A	4	3	1.00	19	0.158
1475	A	3	3	1.00	19	0.158
1476	A	2	2	1.00	19	0.105
1477	A	1	1	1.00	19	0.053
1478	A	2	2	1.00	19	0.105
1479	A	3	2	1.00	19	0.105
1480	A	4	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1481	A	2	2	1.00	15	0.133
1482	A	8	6	1.00	19	0.316
1483	A	7	6	1.00	19	0.316
1484	A	7	7	1.00	19	0.368
1485	A	7	6	1.00	19	0.316
1486	A	1	1	1.00	19	0.053
1487	A	2	2	1.00	19	0.105
1488	A	3	2	1.00	19	0.105
1489	A	4	2	1.00	19	0.105
1490	A	7	6	1.00	19	0.316
1491	A	6	6	1.00	19	0.316
1492	A	5	5	1.00	19	0.263
1493	A	1	1	1.00	19	0.053
1494	A	2	2	1.00	19	0.105
1495	A	3	2	1.00	19	0.105
1496	A	4	2	1.00	19	0.105
1497	A	7	6	1.00	19	0.316
1498	A	6	6	1.00	19	0.316
1499	A	5	5	1.00	19	0.263
1500	A	1	1	1.00	19	0.053
1501	A	2	2	1.00	19	0.105
1502	A	3	2	1.00	19	0.105
1503	A	4	2	1.00	19	0.105
1504	A	7	7	1.00	19	0.368
1505	A	6	6	1.00	19	0.316
1506	A	1	1	1.00	19	0.053
1507	A	2	2	1.00	19	0.105
1508	A	3	2	1.00	19	0.105
1509	A	4	2	1.00	19	0.105
1510	A	11	8	1.00	20	0.400
1511	A	11	8	1.00	20	0.400
1512	A	14	9	1.00	19	0.474
1513	A	13	9	1.00	19	0.474
1514	A	13	9	1.00	19	0.474
1515	A	1	1	1.00	19	0.053
1516	A	2	2	1.00	19	0.105
1517	A	3	2	1.00	19	0.105
1518	A	4	2	1.00	19	0.105
1519	A	14	9	1.00	19	0.474
1520	A	13	9	1.00	19	0.474
1521	A	13	9	1.00	19	0.474
1522	A	1	1	1.00	19	0.053
1523	A	2	2	1.00	19	0.105
1524	A	3	2	1.00	19	0.105
1525	A	4	2	1.00	19	0.105
1526	A	14	9	1.00	19	0.474
1527	A	14	10	1.00	19	0.526
1528	A	14	9	1.00	19	0.474

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1529	A	1	1	1.00	19	0.053
1530	A	2	2	1.00	19	0.105
1531	A	3	2	1.00	19	0.105
1532	A	4	2	1.00	19	0.105
1533	A	14	9	1.00	19	0.474
1534	A	13	9	1.00	19	0.474
1535	A	12	8	1.00	19	0.421
1536	A	1	1	1.00	19	0.053
1537	A	2	2	1.00	19	0.105
1538	A	3	2	1.00	19	0.105
1539	A	4	2	1.00	19	0.105
1540	A	14	9	1.00	19	0.474
1541	A	13	9	1.00	19	0.474
1542	A	12	8	1.00	19	0.421
1543	A	1	1	1.00	19	0.053
1544	A	2	2	1.00	19	0.105
1545	A	3	2	1.00	19	0.105
1546	A	4	2	1.00	19	0.105
1547	A	15	10	1.00	19	0.526
1548	A	14	10	1.00	19	0.526
1549	A	13	9	1.00	19	0.474
1550	A	1	1	1.00	19	0.053
1551	A	2	2	1.00	19	0.105
1552	A	3	2	1.00	19	0.105
1553	A	4	2	1.00	19	0.105
1554	A	1	1	1.00	16	0.062
1555	A	2	1	1.00	15	0.067
1556	A	2	1	1.00	15	0.067
1557	A	2	1	1.00	13	0.077
1558	A	2	1	1.00	15	0.067
1559	A	2	1	1.00	15	0.067
1560	A	2	1	1.00	13	0.077
1561	A	1	1	1.00	7	0.143
1562	A	3	2	1.00	19	0.105
1563	A	2	2	1.00	19	0.105
1564	A	1	1	1.00	19	0.053
1565	A	1	1	1.00	19	0.053
1566	A	2	2	1.00	19	0.105
1567	A	3	2	1.00	19	0.105
1568	A	4	2	1.00	19	0.105
1569	A	1	1	1.00	19	0.053
1570	A	2	2	1.00	19	0.105
1571	A	3	2	1.00	19	0.105
1572	A	4	2	1.00	19	0.105
1573	A	3	2	1.00	24	0.083
1574	A	2	2	1.00	28	0.071
1575	A	2	2	1.00	44	0.045
1576	A	2	2	1.00	51	0.039

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1577	A	1	0	1.00	15	0.000
1578	A	1	0	1.00	9	0.000
1579	A	1	0	1.00	5	0.000
1580	A	1	0	1.00	5	0.000
1581	A	1	0	1.00	9	0.000
1582	A	1	0	1.00	9	0.000
1583	A	1	0	1.00	15	0.000
1584	A	1	0	1.00	10	0.000
1585	A	1	0	1.00	10	0.000
1586	A	1	0	1.00	12	0.000
1587	A	1	0	1.00	15	0.000
1588	A	1	0	1.00	17	0.000
1589	A	1	0	1.00	8	0.000
1590	A	1	0	1.00	10	0.000
1591	A	1	0	1.00	11	0.000
1592	A	1	0	1.00	11	0.000
1593	A	1	0	1.00	6	0.000
1594	A	1	0	1.00	11	0.000
1595	A	1	0	1.00	10	0.000
1596	A	1	0	1.00	11	0.000
1597	A	1	0	1.00	7	0.000
1598	A	1	0	1.00	17	0.000
1599	A	1	0	1.00	18	0.000
1600	A	1	0	1.00	11	0.000
1601	A	1	0	1.00	15	0.000
1602	A	1	0	1.00	18	0.000
1603	A	1	0	1.00	20	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int 0\,dx$

Optimal. Leaf size=1

$$0$$

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$0$$

Antiderivative was successfully verified.

```
[In] Int[0,x]
[Out] 0
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 0\,dx = 0$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$0$$

Antiderivative was successfully verified.

```
[In] Integrate[0,x]
[Out] 0
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 0\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[0,x]
[Out] IntegrateAlgebraic[0, x]
```

fricas [A] time = 1.41, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="fricas")
[Out] 0
```

giac [A] time = 1.09, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="giac")
```

```
[Out] 0
```

maple [A] time = 0.00, size = 2, normalized size = 2.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

maxima [A] time = 0.42, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x, algorithm="maxima")
```

```
[Out] 0
```

mupad [B] time = 0.04, size = 1, normalized size = 1.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(0,x)
```

```
[Out] 0
```

sympy [A] time = 0.01, size = 0, normalized size = 0.00

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(0,x)
```

```
[Out] 0
```

3.2 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[1,x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[1,x]
```

```
[Out] x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 1\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1,x]
```

```
[Out] IntegrateAlgebraic[1, x]
```

fricas [A] time = 1.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 1.07, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

maxima [A] time = 0.41, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 0.01, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

sympy [A] time = 0.02, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

3.3 $\int 5\,dx$

Optimal. Leaf size=3

$$5x$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$5x$$

Antiderivative was successfully verified.

```
[In] Int[5,x]
```

```
[Out] 5*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 5\,dx = 5x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5,x]
```

```
[Out] 5*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[5,x]
```

```
[Out] IntegrateAlgebraic[5, x]
```

fricas [A] time = 1.45, size = 3, normalized size = 1.00

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="fricas")
```

```
[Out] 5*x
```

giac [A] time = 1.04, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="giac")
```

```
[Out] 5*x
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x)
```

```
[Out] 5*x
```

maxima [A] time = 0.41, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x, algorithm="maxima")
```

```
[Out] 5*x
```

mupad [B] time = 0.01, size = 3, normalized size = 1.00

$$5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5,x)
```

```
[Out] 5*x
```

sympy [A] time = 0.02, size = 2, normalized size = 0.67

$$5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5,x)
```

```
[Out] 5*x
```

3.4 $\int -2\,dx$

Optimal. Leaf size=3

$$-2x$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$-2x$$

Antiderivative was successfully verified.

```
[In] Int[-2,x]
[Out] -2*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -2\,dx = -2x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[-2,x]
[Out] -2*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2,x]
[Out] IntegrateAlgebraic[-2, x]
```

fricas [A] time = 1.26, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="fricas")
[Out] -2*x
```

giac [A] time = 1.11, size = 3, normalized size = 1.00

$$-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="giac")
```

```
[Out] -2*x
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

maxima [A] time = 0.42, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x, algorithm="maxima")
```

```
[Out] -2*x
```

mupad [B] time = 0.00, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2,x)
```

```
[Out] -2*x
```

sympy [A] time = 0.02, size = 3, normalized size = 1.00

$$-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2,x)
```

```
[Out] -2*x
```

3.5 $\int -\frac{3}{2}\,dx$

Optimal. Leaf size=5

$$-\frac{3x}{2}$$

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[-3/2,x]
[Out] (-3*x)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int -\frac{3}{2}\,dx = -\frac{3x}{2}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/2,x]
[Out] (-3*x)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{3}{2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-3/2,x]
[Out] IntegrateAlgebraic[-3/2, x]
```

fricas [A] time = 1.29, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="fricas")
[Out] -3/2*x
```

giac [**A**] time = 1.16, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="giac")
```

```
[Out] -3/2*x
```

maple [**A**] time = 0.00, size = 4, normalized size = 0.80

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -3/2*x
```

maxima [**A**] time = 0.42, size = 3, normalized size = 0.60

$$-\frac{3}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x, algorithm="maxima")
```

```
[Out] -3/2*x
```

mupad [**B**] time = 0.01, size = 3, normalized size = 0.60

$$-\frac{3\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/2,x)
```

```
[Out] -(3*x)/2
```

sympy [**A**] time = 0.01, size = 5, normalized size = 1.00

$$-\frac{3x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/2,x)
```

```
[Out] -3*x/2
```

3.6 $\int \pi \, dx$

Optimal. Leaf size=3

$$\pi x$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Int[Pi,x]
```

```
[Out] Pi*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \pi \, dx = \pi x$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$\pi x$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi,x]
```

```
[Out] Pi*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \pi \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Pi,x]
```

```
[Out] IntegrateAlgebraic[Pi, x]
```

fricas [A] time = 1.46, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="fricas")
```

```
[Out] pi*x
```

giac [A] time = 0.84, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="giac")
[Out] pi*x
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x)
[Out] Pi*x
```

maxima [A] time = 0.43, size = 3, normalized size = 1.00

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x, algorithm="maxima")
[Out] pi*x
```

mupad [B] time = 0.00, size = 3, normalized size = 1.00

$$\Pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi,x)
[Out] Pi*x
```

sympy [A] time = 0.01, size = 2, normalized size = 0.67

$$\pi x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi,x)
[Out] pi*x
```

3.7 $\int a\,dx$

Optimal. Leaf size=3

$$ax$$

Rubi [A] time = 0.00, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$ax$$

Antiderivative was successfully verified.

[In] Int[a,x]

[Out] a*x

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int a\,dx = ax$$

Mathematica [A] time = 0.00, size = 3, normalized size = 1.00

$$ax$$

Antiderivative was successfully verified.

[In] Integrate[a,x]

[Out] a*x

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int a\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[a,x]

[Out] IntegrateAlgebraic[a, x]

fricas [A] time = 0.68, size = 3, normalized size = 1.00

$$xa$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a,x, algorithm="fricas")

[Out] x*a

giac [A] time = 0.75, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="giac")
```

```
[Out] a*x
```

maple [A] time = 0.00, size = 4, normalized size = 1.33

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x)
```

```
[Out] a*x
```

maxima [A] time = 0.44, size = 3, normalized size = 1.00

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x, algorithm="maxima")
```

```
[Out] a*x
```

mupad [B] time = 0.00, size = 3, normalized size = 1.00

$$a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a,x)
```

```
[Out] a*x
```

sympy [A] time = 0.02, size = 2, normalized size = 0.67

$$ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a,x)
```

```
[Out] a*x
```

3.8 $\int 3a\,dx$

Optimal. Leaf size=4

$$3ax$$

Rubi [A] time = 0.00, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {8}

$$3ax$$

Antiderivative was successfully verified.

```
[In] Int[3*a,x]
[Out] 3*a*x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 3a\,dx = 3ax$$

Mathematica [A] time = 0.00, size = 4, normalized size = 1.00

$$3ax$$

Antiderivative was successfully verified.

```
[In] Integrate[3*a,x]
[Out] 3*a*x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 3a\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3*a,x]
[Out] IntegrateAlgebraic[3*a, x]
```

fricas [A] time = 0.86, size = 4, normalized size = 1.00

$$3xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="fricas")
[Out] 3*x*a
```

giac [A] time = 1.25, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="giac")
```

```
[Out] 3*a*x
```

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x)
```

```
[Out] 3*a*x
```

maxima [A] time = 0.43, size = 4, normalized size = 1.00

$$3\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x, algorithm="maxima")
```

```
[Out] 3*a*x
```

mupad [B] time = 0.00, size = 4, normalized size = 1.00

$$3\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3*a,x)
```

```
[Out] 3*a*x
```

sympy [A] time = 0.02, size = 3, normalized size = 0.75

$$3ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3*a,x)
```

```
[Out] 3*a*x
```

3.9 $\int \frac{\pi}{\sqrt{16-e^2}}\,dx$

Optimal. Leaf size=14

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {8}

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\pi}{\sqrt{16-e^2}}\,dx = \frac{\pi x}{\sqrt{16-e^2}}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x}{\sqrt{16-e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Pi/Sqrt[16 - E^2],x]
[Out] (Pi*x)/Sqrt[16 - E^2]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\pi}{\sqrt{16-e^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Pi/Sqrt[16 - E^2],x]
[Out] IntegrateAlgebraic[Pi/Sqrt[16 - E^2], x]
```

fricas [A] time = 1.20, size = 18, normalized size = 1.29

$$-\frac{\pi x\sqrt{-e^2+16}}{e^2-16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="fricas")
[Out] -pi*x*sqrt(-e^2 + 16)/(e^2 - 16)
```

giac [A] time = 1.08, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^2 + 16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="giac")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

maple [A] time = 0.00, size = 12, normalized size = 0.86

$$\frac{\pi x}{\sqrt{16 - \mathrm{e}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16-exp(2))^(1/2),x)
```

```
[Out] Pi*x/(16-exp(2))^(1/2)
```

maxima [A] time = 0.45, size = 11, normalized size = 0.79

$$\frac{\pi x}{\sqrt{-e^2 + 16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))^(1/2),x, algorithm="maxima")
```

```
[Out] pi*x/sqrt(-e^2 + 16)
```

mupad [B] time = 0.00, size = 11, normalized size = 0.79

$$\frac{\Pi x}{\sqrt{16 - \mathrm{e}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi/(16 - exp(2))^(1/2),x)
```

```
[Out] (Pi*x)/(16 - exp(2))^(1/2)
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{\pi x}{\sqrt{16 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi/(16-exp(2))**(1/2),x)
```

```
[Out] pi*x/sqrt(16 - exp(2))
```

3.10 $\int x^{100}\,dx$

Optimal. Leaf size=7

$$\frac{x^{101}}{101}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Int[x^100,x]

[Out] x^101/101

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{100}\,dx = \frac{x^{101}}{101}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^{101}}{101}$$

Antiderivative was successfully verified.

[In] Integrate[x^100,x]

[Out] x^101/101

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{100}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^100,x]

[Out] IntegrateAlgebraic[x^100, x]

fricas [A] time = 1.31, size = 5, normalized size = 0.71

$$\frac{1}{101}x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^100,x, algorithm="fricas")

[Out] 1/101*x^101

giac [A] time = 1.18, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="giac")
```

```
[Out] 1/101*x^101
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] 1/101*x^101
```

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{101}\,x^{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^100,x, algorithm="maxima")
```

```
[Out] 1/101*x^101
```

mupad [B] time = 0.12, size = 5, normalized size = 0.71

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^100,x)
```

```
[Out] x^101/101
```

sympy [A] time = 0.06, size = 3, normalized size = 0.43

$$\frac{x^{101}}{101}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**100,x)
```

```
[Out] x**101/101
```

3.11 $\int x^3 \, dx$

Optimal. Leaf size=7

$$\frac{x^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3,x]
```

```
[Out] x^4/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^3 \, dx = \frac{x^4}{4}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3,x]
```

```
[Out] x^4/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3,x]
```

```
[Out] IntegrateAlgebraic[x^3, x]
```

fricas [A] time = 1.14, size = 5, normalized size = 0.71

$$\frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4
```

giac [A] time = 1.18, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] 1/4*x^4
```

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4
```

mupad [B] time = 0.02, size = 5, normalized size = 0.71

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3,x)
```

```
[Out] x^4/4
```

sympy [A] time = 0.05, size = 3, normalized size = 0.43

$$\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3,x)
```

```
[Out] x**4/4
```

3.12 $\int x^2\,dx$

Optimal. Leaf size=7

$$\frac{x^3}{3}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2,x]
[Out] x^3/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^2\,dx = \frac{x^3}{3}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2,x]
[Out] x^3/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2,x]
[Out] IntegrateAlgebraic[x^2, x]
```

fricas [A] time = 1.13, size = 5, normalized size = 0.71

$$\frac{1}{3}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="fricas")
[Out] 1/3*x^3
```

giac [A]   time = 1.10, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3
```

maple [A]   time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] 1/3*x^3
```

maxima [A]   time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3
```

mupad [B]   time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2,x)
```

```
[Out] x^3/3
```

sympy [A]   time = 0.05, size = 3, normalized size = 0.43

$$\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2,x)
```

```
[Out] x**3/3
```

3.13 $\int x\,dx$

Optimal. Leaf size=7

$$\frac{x^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {30}

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x,x]
```

```
[Out] x^2/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x\,dx = \frac{x^2}{2}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x,x]
```

```
[Out] x^2/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x,x]
```

```
[Out] IntegrateAlgebraic[x, x]
```

fricas [A] time = 1.10, size = 5, normalized size = 0.71

$$\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2
```

giac [A] time = 0.95, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="giac")
[Out] 1/2*x^2
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
[Out] 1/2*x^2
```

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$\frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x, algorithm="maxima")
[Out] 1/2*x^2
```

mupad [B] time = 0.01, size = 5, normalized size = 0.71

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x,x)
[Out] x^2/2
```

sympy [A] time = 0.02, size = 3, normalized size = 0.43

$$\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x,x)
[Out] x**2/2
```

3.14 $\int 1\,dx$

Optimal. Leaf size=1

$$x$$

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {8}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[1,x]
```

```
[Out] x
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int 1\,dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[1,x]
```

```
[Out] x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int 1\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1,x]
```

```
[Out] IntegrateAlgebraic[1, x]
```

fricas [A] time = 1.29, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="fricas")
```

```
[Out] x
```

giac [A] time = 1.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="giac")
```

```
[Out] x
```

maple [A] time = 0.00, size = 2, normalized size = 2.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

maxima [A] time = 0.44, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x, algorithm="maxima")
```

```
[Out] x
```

mupad [B] time = 0.00, size = 1, normalized size = 1.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1,x)
```

```
[Out] x
```

sympy [A] time = 0.01, size = 0, normalized size = 0.00

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1,x)
```

```
[Out] x
```

3.15 $\int \frac{1}{x}\,dx$

Optimal. Leaf size=2

$$\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x}\,dx = \log(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
[Out] Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1),x]
[Out] IntegrateAlgebraic[x^(-1), x]
```

fricas [A] time = 1.00, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
[Out] log(x)
```

giac [A] time = 0.80, size = 3, normalized size = 1.50

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

maxima [A] time = 0.44, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

mupad [B] time = 0.04, size = 2, normalized size = 1.00

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] log(x)
```

sympy [A] time = 0.06, size = 2, normalized size = 1.00

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

3.16 $\int \frac{1}{x^2} dx$

Optimal. Leaf size=5

$$-\frac{1}{x}$$

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2),x]
[Out] -x^(-1)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2} dx = -\frac{1}{x}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2),x]
[Out] -x^(-1)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-2),x]
[Out] IntegrateAlgebraic[x^(-2), x]
```

fricas [A] time = 1.79, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="fricas")
[Out] -1/x
```

giac [A] time = 0.85, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="giac")
```

```
[Out] -1/x
```

maple [A] time = 0.00, size = 6, normalized size = 1.20

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
```

```
[Out] -1/x
```

maxima [A] time = 0.43, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2,x, algorithm="maxima")
```

```
[Out] -1/x
```

mupad [B] time = 0.03, size = 5, normalized size = 1.00

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2,x)
```

```
[Out] -1/x
```

sympy [A] time = 0.06, size = 3, normalized size = 0.60

$$-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2,x)
```

```
[Out] -1/x
```

3.17 $\int \frac{1}{x^3} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3),x]
[Out] -1/(2*x^2)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^3} \, dx = -\frac{1}{2x^2}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3),x]
[Out] -1/2*1/x^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3),x]
[Out] IntegrateAlgebraic[x^(-3), x]
```

fricas [A] time = 1.34, size = 5, normalized size = 0.71

$$-\frac{1}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="fricas")
[Out] -1/2/x^2
```

giac [A] time = 0.84, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="giac")
[Out] -1/2/x^2
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
[Out] -1/2/x^2
```

maxima [A] time = 0.49, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3,x, algorithm="maxima")
[Out] -1/2/x^2
```

mupad [B] time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3,x)
[Out] -1/(2*x^2)
```

sympy [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3,x)
[Out] -1/(2*x**2)
```

3.18 $\int \frac{1}{x^4} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[x^(-4),x]

[Out] -1/(3*x^3)

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^4} \, dx = -\frac{1}{3x^3}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-4),x]

[Out] -1/3*1/x^3

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4} \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^(-4),x]

[Out] IntegrateAlgebraic[x^(-4), x]

fricas [A] time = 1.26, size = 5, normalized size = 0.71

$$-\frac{1}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4,x, algorithm="fricas")

[Out] -1/3/x^3

giac [A] time = 1.13, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="giac")
```

```
[Out] -1/3/x^3
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/3/x^3
```

maxima [A] time = 0.43, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4,x, algorithm="maxima")
```

```
[Out] -1/3/x^3
```

mupad [B] time = 0.01, size = 5, normalized size = 0.71

$$-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4,x)
```

```
[Out] -1/(3*x^3)
```

sympy [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4,x)
```

```
[Out] -1/(3*x**3)
```

3.19 $\int \frac{1}{x^{100}} \, dx$

Optimal. Leaf size=7

$$-\frac{1}{99x^{99}}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-100),x]
[Out] -1/(99*x^99)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{100}} \, dx = -\frac{1}{99x^{99}}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-100),x]
[Out] -1/99*1/x^99
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{100}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-100),x]
[Out] IntegrateAlgebraic[x^(-100), x]
```

fricas [A] time = 1.31, size = 5, normalized size = 0.71

$$-\frac{1}{99 \, x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="fricas")
[Out] -1/99/x^99
```

giac [**A**] time = 1.29, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="giac")
```

```
[Out] -1/99/x^99
```

maple [**A**] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/99/x^99
```

maxima [**A**] time = 0.42, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^100,x, algorithm="maxima")
```

```
[Out] -1/99/x^99
```

mupad [**B**] time = 0.07, size = 5, normalized size = 0.71

$$-\frac{1}{99\,x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^100,x)
```

```
[Out] -1/(99*x^99)
```

sympy [**A**] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{1}{99x^{99}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**100,x)
```

```
[Out] -1/(99*x**99)
```

3.20 $\int x^{5/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{7/2}}{7}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2),x]
[Out] (2*x^(7/2))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/2}\,dx = \frac{2x^{7/2}}{7}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2),x]
[Out] (2*x^(7/2))/7
```

IntegrateAlgebraic [A] time = 0.01, size = 9, normalized size = 1.00

$$\frac{2x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2),x]
[Out] (2*x^(7/2))/7
```

fricas [A] time = 0.86, size = 5, normalized size = 0.56

$$\frac{2}{7}\,x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="fricas")
[Out] 2/7*x^(7/2)
```

giac [A] time = 0.90, size = 5, normalized size = 0.56

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="giac")
[Out] 2/7*x^(7/2)
```

maple [A] time = 0.02, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
[Out] 2/7*x^(7/2)
```

maxima [A] time = 0.43, size = 5, normalized size = 0.56

$$\frac{2}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2),x, algorithm="maxima")
[Out] 2/7*x^(7/2)
```

mupad [B] time = 0.08, size = 5, normalized size = 0.56

$$\frac{2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2),x)
[Out] (2*x^(7/2))/7
```

sympy [A] time = 0.07, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2),x)
[Out] 2*x**(7/2)/7
```

3.21 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2),x]
[Out] (2*x^(5/2))/5
```

fricas [A] time = 1.55, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

giac [A] time = 0.98, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
[Out] 2/5*x^(5/2)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
[Out] 2/5*x^(5/2)
```

maxima [A] time = 0.42, size = 5, normalized size = 0.56

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
[Out] (2*x^(5/2))/5
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
[Out] 2*x**(5/2)/5
```

3.22 $\int \sqrt{x}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{3/2}}{3}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x],x]
[Out] (2*x^(3/2))/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\,dx = \frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x],x]
[Out] (2*x^(3/2))/3
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x],x]
[Out] (2*x^(3/2))/3
```

fricas [A] time = 0.93, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="fricas")
```

[Out] 2/3*x^(3/2)

giac [A] time = 1.04, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="giac")
[Out] 2/3*x^(3/2)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
[Out] 2/3*x^(3/2)
```

maxima [A] time = 0.42, size = 5, normalized size = 0.56

$$\frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2),x, algorithm="maxima")
[Out] 2/3*x^(3/2)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.56

$$\frac{2\,x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2),x)
[Out] (2*x^(3/2))/3
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2),x)
[Out] 2*x**(3/2)/3
```

3.23 $\int \frac{1}{\sqrt{x}}\,dx$

Optimal. Leaf size=7

$$2\sqrt{x}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.00, size = 7, normalized size = 1.00

$$2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]
```

fricas [A] time = 0.77, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)
```

giac [A] time = 1.13, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

maxima [A] time = 0.42, size = 5, normalized size = 0.71

$$2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.71

$$2\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2),x)
```

```
[Out] 2*x^(1/2)
```

sympy [A] time = 0.06, size = 5, normalized size = 0.71

$$2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2),x)
```

```
[Out] 2*sqrt(x)
```

3.24 $\int \frac{1}{x^{3/2}}\,dx$

Optimal. Leaf size=7

$$-\frac{2}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}}\,dx = -\frac{2}{\sqrt{x}}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-3/2),x]
```

```
[Out] -2/Sqrt[x]
```

fricas [A] time = 1.43, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="fricas")
```

[Out] -2/sqrt(x)

giac [A] time = 1.01, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="giac")
[Out] -2/sqrt(x)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
[Out] -2/x^(1/2)
```

maxima [A] time = 0.70, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2),x, algorithm="maxima")
[Out] -2/sqrt(x)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.71

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2),x)
[Out] -2/x^(1/2)
```

sympy [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2),x)
[Out] -2/sqrt(x)
```

3.25 $\int \frac{1}{x^{5/2}} \, dx$

Optimal. Leaf size=9

$$-\frac{2}{3x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/2),x]
[Out] -2/(3*x^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/2}} \, dx = -\frac{2}{3x^{3/2}}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/2),x]
[Out] -2/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{2}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-5/2),x]
[Out] -2/(3*x^(3/2))
```

fricas [A] time = 1.36, size = 5, normalized size = 0.56

$$-\frac{2}{3 \, x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3/x^(3/2)
```

giac [A] time = 0.92, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="giac")
[Out] -2/3/x^(3/2)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
[Out] -2/3/x^(3/2)
```

maxima [A] time = 0.62, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2),x, algorithm="maxima")
[Out] -2/3/x^(3/2)
```

mupad [B] time = 0.03, size = 5, normalized size = 0.56

$$-\frac{2}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2),x)
[Out] -2/(3*x^(3/2))
```

sympy [A] time = 0.06, size = 8, normalized size = 0.89

$$-\frac{2}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2),x)
[Out] -2/(3*x**(3/2))
```

3.26 $\int x^{5/3} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{8/3}}{8}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{5/3} \, dx = \frac{3x^{8/3}}{8}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{8/3}}{8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3),x]
```

```
[Out] (3*x^(8/3))/8
```

fricas [A] time = 1.26, size = 5, normalized size = 0.56

$$\frac{3}{8} x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/8*x^(8/3)
```

giac [A] time = 1.05, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="giac")
```

```
[Out] 3/8*x^(8/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] 3/8*x^(8/3)
```

maxima [A] time = 0.56, size = 5, normalized size = 0.56

$$\frac{3}{8}\,x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/8*x^(8/3)
```

mupad [B] time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3),x)
```

```
[Out] (3*x^(8/3))/8
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3),x)
```

```
[Out] 3*x**(8/3)/8
```

3.27 $\int x^{4/3}\,dx$

Optimal. Leaf size=9

$$\frac{3x^{7/3}}{7}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3),x]
[Out] (3*x^(7/3))/7
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{4/3}\,dx = \frac{3x^{7/3}}{7}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3),x]
[Out] (3*x^(7/3))/7
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{7/3}}{7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3),x]
[Out] (3*x^(7/3))/7
```

fricas [A] time = 1.47, size = 5, normalized size = 0.56

$$\frac{3}{7}\,x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="fricas")
[Out] 3/7*x^(7/3)
```

giac [A] time = 1.05, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="giac")
```

```
[Out] 3/7*x^(7/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
```

```
[Out] 3/7*x^(7/3)
```

maxima [A] time = 0.56, size = 5, normalized size = 0.56

$$\frac{3}{7}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*x^(7/3)
```

mupad [B] time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3),x)
```

```
[Out] (3*x^(7/3))/7
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3),x)
```

```
[Out] 3*x**(7/3)/7
```

3.28 $\int x^{2/3} \, dx$

Optimal. Leaf size=9

$$\frac{3x^{5/3}}{5}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3),x]
[Out] (3*x^(5/3))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{2/3} \, dx = \frac{3x^{5/3}}{5}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3),x]
[Out] (3*x^(5/3))/5
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{5/3}}{5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3),x]
[Out] (3*x^(5/3))/5
```

fricas [A] time = 0.70, size = 5, normalized size = 0.56

$$\frac{3}{5} \, x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*x^(5/3)
```

giac [A] time = 0.91, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="giac")
[Out] 3/5*x^(5/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
[Out] 3/5*x^(5/3)
```

maxima [A] time = 0.51, size = 5, normalized size = 0.56

$$\frac{3}{5}\,x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3),x, algorithm="maxima")
[Out] 3/5*x^(5/3)
```

mupad [B] time = 0.07, size = 5, normalized size = 0.56

$$\frac{3\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3),x)
[Out] (3*x^(5/3))/5
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3),x)
[Out] 3*x**(5/3)/5
```

3.29 $\int \sqrt[3]{x}\, dx$

Optimal. Leaf size=9

$$\frac{3x^{4/3}}{4}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{x}\, dx = \frac{3x^{4/3}}{4}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{4/3}}{4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3),x]
[Out] (3*x^(4/3))/4
```

fricas [A] time = 1.39, size = 5, normalized size = 0.56

$$\frac{3}{4}\, x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*x^(4/3)
```

giac [A] time = 0.85, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="giac")
```

```
[Out] 3/4*x^(4/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] 3/4*x^(4/3)
```

maxima [A] time = 0.48, size = 5, normalized size = 0.56

$$\frac{3}{4}\,x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/4*x^(4/3)
```

mupad [B] time = 0.06, size = 5, normalized size = 0.56

$$\frac{3\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3),x)
```

```
[Out] (3*x^(4/3))/4
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3),x)
```

```
[Out] 3*x**(4/3)/4
```

3.30 $\int \frac{1}{\sqrt[3]{x}}\, dx$

Optimal. Leaf size=9

$$\frac{3x^{2/3}}{2}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.200, Rules used = {30}

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}}\, dx = \frac{3x^{2/3}}{2}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$\frac{3x^{2/3}}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-1/3),x]
[Out] (3*x^(2/3))/2
```

fricas [A] time = 1.35, size = 5, normalized size = 0.56

$$\frac{3}{2}\, x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*x^(2/3)
```

giac [A] time = 1.14, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="giac")
[Out] 3/2*x^(2/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
[Out] 3/2*x^(2/3)
```

maxima [A] time = 0.46, size = 5, normalized size = 0.56

$$\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3),x, algorithm="maxima")
[Out] 3/2*x^(2/3)
```

mupad [B] time = 0.04, size = 5, normalized size = 0.56

$$\frac{3\,x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3),x)
[Out] (3*x^(2/3))/2
```

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3),x)
[Out] 3*x**(2/3)/2
```

3.31 $\int \frac{1}{x^{2/3}}\,dx$

Optimal. Leaf size=7

$$3\sqrt[3]{x}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{2/3}}\,dx = 3\sqrt[3]{x}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

IntegrateAlgebraic [A] time = 0.00, size = 7, normalized size = 1.00

$$3\sqrt[3]{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-2/3),x]
```

```
[Out] 3*x^(1/3)
```

fricas [A] time = 0.81, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3*x^(1/3)
```

giac [A] time = 0.95, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="giac")
```

```
[Out] 3*x^(1/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

maxima [A] time = 0.52, size = 5, normalized size = 0.71

$$3\,x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3*x^(1/3)
```

mupad [B] time = 0.07, size = 5, normalized size = 0.71

$$3\,x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3),x)
```

```
[Out] 3*x^(1/3)
```

sympy [A] time = 0.06, size = 5, normalized size = 0.71

$$3\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3),x)
```

```
[Out] 3*x**(1/3)
```

3.32 $\int \frac{1}{x^{4/3}}\,dx$

Optimal. Leaf size=7

$$-\frac{3}{\sqrt[3]{x}}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4/3),x]
[Out] -3/x^(1/3)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{4/3}}\,dx = -\frac{3}{\sqrt[3]{x}}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4/3),x]
[Out] -3/x^(1/3)
```

IntegrateAlgebraic [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-4/3),x]
[Out] -3/x^(1/3)
```

fricas [A] time = 1.42, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="fricas")
```

```
[Out] -3/x^(1/3)
```

giac [A] time = 1.12, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="giac")
```

```
[Out] -3/x^(1/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
```

```
[Out] -3/x^(1/3)
```

maxima [A] time = 0.51, size = 5, normalized size = 0.71

$$-\frac{3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3),x, algorithm="maxima")
```

```
[Out] -3/x^(1/3)
```

mupad [B] time = 0.07, size = 5, normalized size = 0.71

$$-\frac{3}{x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3),x)
```

```
[Out] -3/x^(1/3)
```

sympy [A] time = 0.06, size = 7, normalized size = 1.00

$$-\frac{3}{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3),x)
```

```
[Out] -3/x**(1/3)
```

3.33 $\int \frac{1}{x^{5/3}}\,dx$

Optimal. Leaf size=9

$$-\frac{3}{2x^{2/3}}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {30}

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{5/3}}\,dx = -\frac{3}{2x^{2/3}}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.00, size = 9, normalized size = 1.00

$$-\frac{3}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-5/3),x]
```

```
[Out] -3/(2*x^(2/3))
```

fricas [A] time = 1.11, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="fricas")
```

```
[Out] -3/2/x^(2/3)
```

giac [A] time = 0.92, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="giac")
[Out] -3/2/x^(2/3)
```

maple [A] time = 0.00, size = 6, normalized size = 0.67

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
[Out] -3/2/x^(2/3)
```

maxima [A] time = 0.49, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3),x, algorithm="maxima")
[Out] -3/2/x^(2/3)
```

mupad [B] time = 0.05, size = 5, normalized size = 0.56

$$-\frac{3}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3),x)
[Out] -3/(2*x^(2/3))
```

sympy [A] time = 0.06, size = 8, normalized size = 0.89

$$-\frac{3}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3),x)
[Out] -3/(2*x**(2/3))
```

3.34 $\int x^n\,dx$

Optimal. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n\,dx = \frac{x^{1+n}}{1+n}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]
[Out] x^(1 + n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^n,x]
[Out] Defer[IntegrateAlgebraic][x^n, x]
```

fricas [A] time = 1.69, size = 10, normalized size = 0.91

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
[Out] x*x^n/(n + 1)
```

giac [A] time = 0.95, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
[Out] x^(n + 1)/(n + 1)
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
[Out] x^(1+n)/(1+n)
```

maxima [A] time = 0.64, size = 11, normalized size = 1.00

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")
[Out] x^(n + 1)/(n + 1)
```

mupad [B] time = 0.35, size = 20, normalized size = 1.82

$$\begin{cases} \ln(x) & \text{if } n = -1 \\ \frac{x^{n+1}}{n+1} & \text{if } n \neq -1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)
[Out] piecewise(n == -1, log(x), n ~= -1, x^(n + 1)/(n + 1))
```

sympy [A] time = 0.06, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

3.35 $\int (bx)^n \, dx$

Optimal. Leaf size=16

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {32}

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x)^n,x]
```

```
[Out] (b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (bx)^n \, dx = \frac{(bx)^{1+n}}{b(1+n)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 0.75

$$\frac{x(bx)^n}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x)^n,x]
```

```
[Out] (x*(b*x)^n)/(1 + n)
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(b*x)^n, x]
```

fricas [A] time = 1.37, size = 12, normalized size = 0.75

$$\frac{(bx)^n x}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x)^n,x, algorithm="fricas")
```

[Out] (b*x)^n*x/(n + 1)

giac [A] time = 1.04, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^n,x, algorithm="giac")

[Out] (b*x)^(n + 1)/(b*(n + 1))

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{x\,(bx)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^n,x)

[Out] x/(n+1)*(b*x)^n

maxima [A] time = 0.54, size = 16, normalized size = 1.00

$$\frac{(bx)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)^n,x, algorithm="maxima")

[Out] (b*x)^(n + 1)/(b*(n + 1))

mupad [B] time = 0.18, size = 12, normalized size = 0.75

$$\frac{x\,(b\,x)^{n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x)^n,x)

[Out] (x*(b*x)^n)/(n + 1)

sympy [A] time = 0.06, size = 17, normalized size = 1.06

$$\frac{\begin{cases} \frac{(bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x)**n,x)

[Out] Piecewise(((b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(b*x), True))/b

3.36 $\int \frac{1}{\sqrt{-a}+e(c+dx)} dx$

Optimal. Leaf size=23

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {33, 31}

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-a}+ex}\,dx, x, c+dx\right)}{d}\\ &= \frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}+ce+dex\right)}{de}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

```
[Out] Log[Sqrt[-a] + c*e + d*e*x]/(d*e)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a}+e(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[-a] + e*(c + d*x))^(-1),x]
```

[Out] IntegrateAlgebraic[(Sqrt[-a] + e*(c + d*x))^(-1), x]

fricas [A] time = 1.45, size = 21, normalized size = 0.91

$$\frac{\log\left(dex + ce + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="fricas")

[Out] log(d*e*x + c*e + sqrt(-a))/(d*e)

giac [A] time = 1.07, size = 22, normalized size = 0.96

$$\frac{e^{(-1)}\log\left(\left|(dx + c)e + \sqrt{-a}\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="giac")

[Out] e^(-1)*log(abs((d*x + c)*e + sqrt(-a)))/d

maple [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{\ln\left(dex + ce + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*(d*x+c)+(-a)^(1/2)),x)

[Out] ln(c*e+d*e*x+(-a)^(1/2))/d/e

maxima [A] time = 0.67, size = 21, normalized size = 0.91

$$\frac{\log\left((dx + c)e + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*(d*x+c)+(-a)^(1/2)),x, algorithm="maxima")

[Out] log((d*x + c)*e + sqrt(-a))/(d*e)

mupad [B] time = 0.14, size = 21, normalized size = 0.91

$$\frac{\ln\left(\sqrt{-a} + c\,e + d\,e\,x\right)}{d\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-a)^(1/2) + e*(c + d*x)),x)

[Out] log((-a)^(1/2) + c*e + d*e*x)/(d*e)

sympy [A] time = 0.09, size = 19, normalized size = 0.83

$$\frac{\log\left(ce + dex + \sqrt{-a}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*(d*x+c)+(-a)**(1/2)),x)

[Out] log(c*e + d*e*x + sqrt(-a))/(d*e)

3.37 $\int (c+d(a+bx))^{5/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{5/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{5/2}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{7/2}}{7bd}\end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(7/2))/(7*b*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{7/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(5/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(7/2))/(7*b*d)
```

fricas [B] time = 1.48, size = 104, normalized size = 4.52

$$\frac{2\left(b^{3}d^{3}x^{3}+a^{3}d^{3}+3\,a^{2}cd^{2}+3\,ac^{2}d+c^{3}+3\left(ab^{2}d^{3}+b^{2}cd^{2}\right)x^{2}+3\left(a^{2}bd^{3}+2\,abcd^{2}+bc^{2}d\right)x\right)\sqrt{bdx+ad+c}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*d^3*x^3 + a^3*d^3 + 3*a^2*c*d^2 + 3*a*c^2*d + c^3 + 3*(a*b^2*d^3 +
 b^2*c*d^2)*x^2 + 3*(a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d)*x)*sqrt(b*d*x + a*d
 + c)/(b*d)
```

giac [B] time = 1.64, size = 444, normalized size = 19.30

$$\frac{2\left(35\,(bdx+ad+c)^{\frac{3}{2}}a^{2}d^{2}-35\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)a^{2}d^{2}-21\,(bdx+ad+c)^{\frac{5}{2}}ad+70\,(bdx+ad+c)^{\frac{3}{2}}acd-70\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)acd+5\,(bdx+ad+c)^{\frac{7}{2}}-21\,(bdx+ad+c)^{\frac{5}{2}}c+35\,(bdx+ad+c)^{\frac{3}{2}}c^{2}-35\left(3\,\sqrt{bdx+ad+c}\,ad-(bdx+ad+c)^{\frac{3}{2}}+3\,\sqrt{bdx+ad+c}\,c\right)c^{2}+7\left(15\,\sqrt{bdx+ad+c}\,a^{2}d^{2}-10\,(bdx+ad+c)^{\frac{3}{2}}ad+30\,\sqrt{bdx+ad+c}\,acd+3\,(bdx+ad+c)^{\frac{5}{2}}-10\,(bdx+ad+c)^{\frac{3}{2}}c+15\,\sqrt{bdx+ad+c}\,c^{2}\right)ad+7\left(15\,\sqrt{bdx+ad+c}\,a^{2}d^{2}-10\,(bdx+ad+c)^{\frac{3}{2}}ad+30\,\sqrt{bdx+ad+c}\,acd+3\,(bdx+ad+c)^{\frac{5}{2}}-10\,(bdx+ad+c)^{\frac{3}{2}}c+15\,\sqrt{bdx+ad+c}\,c^{2}\right)c\right)}{35\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*(b*d*x + a*d + c)^(3/2)*a^2*d^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d
- (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a^2*d^2 - 21*(b*d*x
+ a*d + c)^(5/2)*a*d + 70*(b*d*x + a*d + c)^(3/2)*a*c*d - 70*(3*sqrt(b*d*x
+ a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*c*d
 + 5*(b*d*x + a*d + c)^(7/2) - 21*(b*d*x + a*d + c)^(5/2)*c + 35*(b*d*x + a
*d + c)^(3/2)*c^2 - 35*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/
2) + 3*sqrt(b*d*x + a*d + c)*c)*c^2 + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 -
 10*(b*d*x + a*d + c)^(3/2)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x
 + a*d + c)^(5/2) - 10*(b*d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)
*c^2)*a*d + 7*(15*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2
)*a*d + 30*sqrt(b*d*x + a*d + c)*a*c*d + 3*(b*d*x + a*d + c)^(5/2) - 10*(b*
d*x + a*d + c)^(3/2)*c + 15*sqrt(b*d*x + a*d + c)*c^2)*c)/(b*d)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2\,(bdx+ad+c)^{\frac{7}{2}}}{7bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(5/2),x)
```

```
[Out] 2/7*(b*d*x+a*d+c)^(7/2)/d/b
```

maxima [A] time = 0.56, size = 19, normalized size = 0.83

$$\frac{2\,((bx+a)d+c)^{\frac{7}{2}}}{7\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*((b*x + a)*d + c)^(7/2)/(b*d)
```

mupad [B] time = 0.18, size = 93, normalized size = 4.04

$$\frac{6\,x\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^2}{7}+\frac{2\,\sqrt{c+d\,\left(a+b\,x\right)}\,{\left(c+a\,d\right)}^3}{7\,b\,d}+\frac{2\,b^2\,d^2\,x^3\,\sqrt{c+d\,\left(a+b\,x\right)}}{7}+\frac{6\,b\,d\,x^2\,\sqrt{c+d\,\left(a+b\,x\right)}\,\left(c+a\,d\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(5/2),x)
```

```
[Out] (6*x*(c + d*(a + b*x))^(1/2)*(c + a*d)^2)/7 + (2*(c + d*(a + b*x))^(1/2)*(c
 + a*d)^3)/(7*b*d) + (2*b^2*d^2*x^3*(c + d*(a + b*x))^(1/2))/7 + (6*b*d*x^2
*(c + d*(a + b*x))^(1/2)*(c + a*d))/7
```

sympy [A] time = 71.80, size = 270, normalized size = 11.74

$$\begin{cases} c^{\frac{5}{2}} x & \text{for}\: b = 0 \wedge d = 0 \\ x \left(a d + c\right)^{\frac{5}{2}} & \text{for}\: b = 0 \\ c^{\frac{5}{2}} x & \text{for}\: d = 0 \\ \frac{2 a^{3} d^{2} \sqrt{a d + b d x + c}}{7 b} + \frac{6 a^{2} d^{2} x \sqrt{a d + b d x + c}}{7} + \frac{6 a^{2} c d \sqrt{a d + b d x + c}}{7 b} + \frac{6 a b d^{2} x^{2} \sqrt{a d + b d x + c}}{7} + \frac{12 a c d x \sqrt{a d + b d x + c}}{7} + \frac{6 a c^{2} \sqrt{a d + b d x + c}}{7 b} + \frac{2 b^{2} d^{2} x^{3} \sqrt{a d + b d x + c}}{7} + \frac{6 b c d x^{2} \sqrt{a d + b d x + c}}{7} + \frac{6 c^{2} x \sqrt{a d + b d x + c}}{7} + \frac{2 c^{3} \sqrt{a d + b d x + c}}{7 b d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((c**(5/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(5/2), Eq(b, 0))
, (c**(5/2)*x, Eq(d, 0)), (2*a**3*d**2*sqrt(a*d + b*d*x + c)/(7*b) + 6*a**2
*d**2*x*sqrt(a*d + b*d*x + c)/7 + 6*a**2*c*d*sqrt(a*d + b*d*x + c)/(7*b) +
6*a*b*d**2*x**2*sqrt(a*d + b*d*x + c)/7 + 12*a*c*d*x*sqrt(a*d + b*d*x + c)/
7 + 6*a*c**2*sqrt(a*d + b*d*x + c)/(7*b) + 2*b**2*d**2*x**3*sqrt(a*d + b*d*
x + c)/7 + 6*b*c*d*x**2*sqrt(a*d + b*d*x + c)/7 + 6*c**2*x*sqrt(a*d + b*d*x
 + c)/7 + 2*c**3*sqrt(a*d + b*d*x + c)/(7*b*d), True))
```

3.38 $\int (c+d(a+bx))^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int (c+d(a+bx))^{3/2}\,dx &= \frac{\text{Subst}\left(\int (c+dx)^{3/2}\,dx,x,a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{5/2}}{5bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + d*(a + b*x))^(5/2))/(5*b*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{5/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(3/2),x]
```

```
[Out] (2*(c + a*d + b*d*x)^(5/2))/(5*b*d)
```

fricas [**B**] time = 0.87, size = 59, normalized size = 2.57

$$\frac{2\left(b^2d^2x^2 + a^2d^2 + 2\,acd + c^2 + 2\left(abd^2 + bcd\right)x\right)\sqrt{bdx + ad + c}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*d^2*x^2 + a^2*d^2 + 2*a*c*d + c^2 + 2*(a*b*d^2 + b*c*d)*x)*sqrt(b*
d*x + a*d + c)/(b*d)
```

giac [**B**] time = 1.04, size = 195, normalized size = 8.48

$$\frac{2\left(30\,\sqrt{bdx + ad + c}\,a^2d^2 - 10\,(bdx + ad + c)^{\frac{3}{2}}ad + 60\,\sqrt{bdx + ad + c}\,acd - 10\left(3\,\sqrt{bdx + ad + c}\,ad - (bdx + ad + c)^{\frac{3}{2}} + 3\,\sqrt{bdx + ad + c}\,c\right)ad + 3\,(bdx + ad + c)^{\frac{5}{2}} - 10\,(bdx + ad + c)^{\frac{3}{2}}c + 30\,\sqrt{bdx + ad + c}\,c^2 - 10\left(3\,\sqrt{bdx + ad + c}\,ad - (bdx + ad + c)^{\frac{3}{2}} + 3\,\sqrt{bdx + ad + c}\,c\right)c\right)}{15\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(30*sqrt(b*d*x + a*d + c)*a^2*d^2 - 10*(b*d*x + a*d + c)^(3/2)*a*d + 6
0*sqrt(b*d*x + a*d + c)*a*c*d - 10*(3*sqrt(b*d*x + a*d + c)*a*d - (b*d*x +
a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c)*a*d + 3*(b*d*x + a*d + c)^(5/2)
 - 10*(b*d*x + a*d + c)^(3/2)*c + 30*sqrt(b*d*x + a*d + c)*c^2 - 10*(3*sqrt
(b*d*x + a*d + c)*a*d - (b*d*x + a*d + c)^(3/2) + 3*sqrt(b*d*x + a*d + c)*c
)*c)/(b*d)
```

maple [**A**] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2\,(bdx + ad + c)^{\frac{5}{2}}}{5bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(3/2),x)
```

```
[Out] 2/5*(b*d*x+a*d+c)^(5/2)/d/b
```

maxima [**A**] time = 0.88, size = 19, normalized size = 0.83

$$\frac{2\,((bx + a)d + c)^{\frac{5}{2}}}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((b*x + a)*d + c)^(5/2)/(b*d)
```

mupad [**B**] time = 0.17, size = 45, normalized size = 1.96

$$\sqrt{c + d\,\left(a + b\,x\right)}\,\left(x\,\left(\frac{4\,c}{5} + \frac{4\,a\,d}{5}\right) + \frac{2\,{\left(c + a\,d\right)}^2}{5\,b\,d} + \frac{2\,b\,d\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(3/2),x)
```

```
[Out] (c + d*(a + b*x))^(1/2)*(x*((4*c)/5 + (4*a*d)/5) + (2*(c + a*d)^2)/(5*b*d)
+ (2*b*d*x^2)/5)
```

sympy [A] time = 5.27, size = 156, normalized size = 6.78

$$\begin{cases} c^{\frac{3}{2}}x & \text{for } b = 0 \wedge d = 0 \\ x(ad+c)^{\frac{3}{2}} & \text{for } b = 0 \\ c^{\frac{3}{2}}x & \text{for } d = 0 \\ \frac{2a^2d\sqrt{ad+bdx+c}}{5b} + \frac{4adx\sqrt{ad+bdx+c}}{5} + \frac{4ac\sqrt{ad+bdx+c}}{5b} + \frac{2bdx^2\sqrt{ad+bdx+c}}{5} + \frac{4cx\sqrt{ad+bdx+c}}{5} + \frac{2c^2\sqrt{ad+bdx+c}}{5bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((c**(3/2)*x, Eq(b, 0) & Eq(d, 0)), (x*(a*d + c)**(3/2), Eq(b, 0))
, (c**(3/2)*x, Eq(d, 0)), (2*a**2*d*sqrt(a*d + b*d*x + c)/(5*b) + 4*a*d*x*s
qrt(a*d + b*d*x + c)/5 + 4*a*c*sqrt(a*d + b*d*x + c)/(5*b) + 2*b*d*x**2*sqr
t(a*d + b*d*x + c)/5 + 4*c*x*sqrt(a*d + b*d*x + c)/5 + 2*c**2*sqrt(a*d + b*
d*x + c)/(5*b*d), True))
```

3.39 $\int \sqrt{c+d(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+d(a+bx)}\,dx &= \frac{\text{Subst}\left(\int \sqrt{c+dx}\,dx, x, a+bx\right)}{b}\\ &= \frac{2(c+d(a+bx))^{3/2}}{3bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(d(a+bx)+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2(ad+bdx+c)^{3/2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*(c + a*d + b*d*x)^(3/2))/(3*b*d)
```

fricas [A] time = 1.40, size = 19, normalized size = 0.83

$$\frac{2\,(bdx + ad + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

giac [A] time = 1.07, size = 19, normalized size = 0.83

$$\frac{2\,(bdx + ad + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2/3*(b*d*x + a*d + c)^(3/2)/(b*d)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$\frac{2\,(bdx + ad + c)^{\frac{3}{2}}}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*(b*x+a))^(1/2),x)
[Out] 2/3*(b*d*x+a*d+c)^(3/2)/d/b
```

maxima [A] time = 0.74, size = 19, normalized size = 0.83

$$\frac{2\,((bx + a)d + c)^{\frac{3}{2}}}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2/3*((b*x + a)*d + c)^(3/2)/(b*d)
```

mupad [B] time = 0.08, size = 19, normalized size = 0.83

$$\frac{2\,\left(c+d\,\left(a+b\,x\right)\right)^{3/2}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*(a + b*x))^(1/2),x)
[Out] (2*(c + d*(a + b*x))^(3/2))/(3*b*d)
```

sympy [A] time = 0.44, size = 82, normalized size = 3.57

$$\begin{cases} \sqrt{c}\,x & \text{for}\: b = 0 \wedge d = 0 \\ x\sqrt{ad + c} & \text{for}\: b = 0 \\ \sqrt{c}\,x & \text{for}\: d = 0 \\ \frac{2a\sqrt{ad+bdx+c}}{3b} + \frac{2x\sqrt{ad+bdx+c}}{3} + \frac{2c\sqrt{ad+bdx+c}}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((sqrt(c)*x, Eq(b, 0) & Eq(d, 0)), (x*sqrt(a*d + c), Eq(b, 0)), (s
qrt(c)*x, Eq(d, 0)), (2*a*sqrt(a*d + b*d*x + c)/(3*b) + 2*x*sqrt(a*d + b*d*
x + c)/3 + 2*c*sqrt(a*d + b*d*x + c)/(3*b*d), True))
```

3.40 $\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*(a + b*x)],x]
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c+d(a+bx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c+dx}}\,dx, x, a+bx\right)}{b}\\ &= \frac{2\sqrt{c+d(a+bx)}}{bd}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{d(a+bx)+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*(a + b*x)],x]
[Out] (2*Sqrt[c + d*(a + b*x)])/(b*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2\sqrt{ad+bdx+c}}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c + d*(a + b*x)],x]
```

```
[Out] (2*Sqrt[c + a*d + b*d*x])/(b*d)
```

fricas [A] time = 1.07, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx + ad + c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

giac [A] time = 1.17, size = 19, normalized size = 0.90

$$\frac{2\sqrt{bdx + ad + c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="giac")
[Out] 2*sqrt(b*d*x + a*d + c)/(b*d)
```

maple [A] time = 0.00, size = 20, normalized size = 0.95

$$\frac{2\sqrt{bdx + ad + c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(1/2),x)
[Out] 2*(b*d*x+a*d+c)^(1/2)/d/b
```

maxima [A] time = 0.92, size = 19, normalized size = 0.90

$$\frac{2\sqrt{(bx + a)d + c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(1/2),x, algorithm="maxima")
[Out] 2*sqrt((b*x + a)*d + c)/(b*d)
```

mupad [B] time = 0.11, size = 19, normalized size = 0.90

$$\frac{2\sqrt{c + d\,(a + b\,x)}}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(1/2),x)
[Out] (2*(c + d*(a + b*x))^(1/2))/(b*d)
```

sympy [A] time = 1.78, size = 31, normalized size = 1.48

$$\begin{cases} \frac{x}{\sqrt{ad+c}} & \text{for}\: b = 0 \\ \frac{x}{\sqrt{c}} & \text{for}\: d = 0 \\ \frac{2\sqrt{c+d(a+bx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(1/2),x)
```

```
[Out] Piecewise((x/sqrt(a*d + c), Eq(b, 0)), (x/sqrt(c), Eq(d, 0)), (2*sqrt(c + d
*(a + b*x))/(b*d), True))
```

3.41 $\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {33, 32}

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{3/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{bd\sqrt{c+d(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{d(a+bx)+c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + d*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2}{bd\sqrt{ad+bdx+c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(-3/2),x]
```

```
[Out] -2/(b*d*Sqrt[c + a*d + b*d*x])
```

fricas [A] time = 1.57, size = 34, normalized size = 1.62

$$-\frac{2\sqrt{bdx+ad+c}}{b^2d^2x+abd^2+bcd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*d*x + a*d + c)/(b^2*d^2*x + a*b*d^2 + b*c*d)
```

giac [A] time = 1.65, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{bdx+ad+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*d*x + a*d + c)*b*d)
```

maple [A] time = 0.00, size = 20, normalized size = 0.95

$$-\frac{2}{\sqrt{bdx+ad+c}\;bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(3/2),x)
[Out] -2/(b*d*x+a*d+c)^(1/2)/d/b
```

maxima [A] time = 1.00, size = 19, normalized size = 0.90

$$-\frac{2}{\sqrt{(bx+a)d+c}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt((b*x + a)*d + c)*b*d)
```

mupad [B] time = 0.13, size = 19, normalized size = 0.90

$$-\frac{2}{b\,d\,\sqrt{c+d\,\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(3/2),x)
[Out] -2/(b*d*(c + d*(a + b*x))^(1/2))
```

sympy [A] time = 1.79, size = 58, normalized size = 2.76

$$\begin{cases} \frac{x}{c^{\frac{3}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{\left(ad+c\right)^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{3}{2}}} & \text{for}\: d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{abd^2+b^2d^2x+bcd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(3/2),x)
```

```
[Out] Piecewise((x/c**(3/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(3/2), Eq(b, 0))
, (x/c**(3/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(a*b*d**2 + b**2*d**2*x
 + b*c*d), True))
```

3.42 $\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=23

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {33, 32}

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 33

```
Int[((a_.) + (b_.)*(u_))^(m_), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Su
bst[Int[(a + b*x)^m, x], x, u], x] /; FreeQ[{a, b, m}, x] && LinearQ[u, x]
&& NeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c+d(a+bx))^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(c+dx)^{5/2}}\,dx, x, a+bx\right)}{b}\\ &= -\frac{2}{3bd(c+d(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(d(a+bx)+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{2}{3bd(ad+bdx+c)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*(a + b*x))^(-5/2),x]
```

```
[Out] -2/(3*b*d*(c + a*d + b*d*x)^(3/2))
```

fricas [B] time = 1.41, size = 68, normalized size = 2.96

$$-\frac{2\sqrt{bdx+ad+c}}{3\left(b^3d^3x^2+a^2bd^3+2\,abcd^2+bc^2d+2\left(ab^2d^3+b^2cd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*d*x + a*d + c)/(b^3*d^3*x^2 + a^2*b*d^3 + 2*a*b*c*d^2 + b*c^2*d
 + 2*(a*b^2*d^3 + b^2*c*d^2)*x)
```

giac [A] time = 1.32, size = 19, normalized size = 0.83

$$-\frac{2}{3\,(bdx+ad+c)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] -2/3/((b*d*x + a*d + c)^(3/2)*b*d)
```

maple [A] time = 0.00, size = 20, normalized size = 0.87

$$-\frac{2}{3\,(bdx+ad+c)^{\frac{3}{2}}\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d*(b*x+a))^(5/2),x)
```

```
[Out] -2/3/(b*d*x+a*d+c)^(3/2)/d/b
```

maxima [A] time = 0.85, size = 19, normalized size = 0.83

$$-\frac{2}{3\,((bx+a)d+c)^{\frac{3}{2}}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3/(((b*x + a)*d + c)^(3/2)*b*d)
```

mupad [B] time = 0.18, size = 19, normalized size = 0.83

$$-\frac{2}{3\,b\,d\,(c+d\,\left(a+b\,x\right))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*(a + b*x))^(5/2),x)
```

```
[Out] -2/(3*b*d*(c + d*(a + b*x))^(3/2))
```

sympy [A] time = 6.79, size = 102, normalized size = 4.43

$$\begin{cases} \frac{x}{c^{\frac{5}{2}}} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{x}{(ad+c)^{\frac{5}{2}}} & \text{for}\: b = 0 \\ \frac{x}{c^{\frac{5}{2}}} & \text{for}\: d = 0 \\ -\frac{2\sqrt{ad+bdx+c}}{3a^2bd^3+6ab^2d^3x+6abcd^2+3b^3d^3x^2+6b^2cd^2x+3bc^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d*(b*x+a))**(5/2),x)
```

```
[Out] Piecewise((x/c**(5/2), Eq(b, 0) & Eq(d, 0)), (x/(a*d + c)**(5/2), Eq(b, 0))
, (x/c**(5/2), Eq(d, 0)), (-2*sqrt(a*d + b*d*x + c)/(3*a**2*b*d**3 + 6*a*b*
*2*d**3*x + 6*a*b*c*d**2 + 3*b**3*d**3*x**2 + 6*b**2*c*d**2*x + 3*b*c**2*d)
, True))
```

3.43 $\int x^3(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x),x]
[Out] (a*x^4)/4 + (b*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)\,dx &= \int \left(ax^3+bx^4\right)\,dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x),x]
[Out] (a*x^4)/4 + (b*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x),x]
[Out] IntegrateAlgebraic[x^3*(a + b*x), x]
```

fricas [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/4*x^4*a
```

giac [A] time = 1.22, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^5+\frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

maxima [A] time = 0.89, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^4\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

3.44 $\int x^2(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)\,dx&=\int\left(ax^2+bx^3\right)\,dx\\&=\frac{ax^3}{3}+\frac{bx^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x),x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x), x]
```

fricas [A] time = 1.45, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="fricas")
[Out] 1/4*x^4*b + 1/3*x^3*a
```

giac [A] time = 1.74, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^4 + \frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a),x)
[Out] 1/3*a*x^3+1/4*b*x^4
```

maxima [A] time = 0.86, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a),x, algorithm="maxima")
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x),x)
[Out] (x^3*(4*a + 3*b*x))/12
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a),x)
[Out] a*x**3/3 + b*x**4/4
```

3.45 $\int x(a+bx)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)\,dx&=\int\left(ax+bx^2\right)dx\\&=\frac{ax^2}{2}+\frac{bx^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x),x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x), x]
```

fricas [A] time = 1.26, size = 13, normalized size = 0.76

$$\frac{1}{3}x^3b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*b + 1/2*x^2*a
```

giac [A] time = 1.17, size = 13, normalized size = 0.76

$$\frac{1}{3}\, bx^3 + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{1}{3} b\, x^3 + \frac{1}{2} a\, x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a),x)
```

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

maxima [A] time = 0.83, size = 13, normalized size = 0.76

$$\frac{1}{3}\, bx^3 + \frac{1}{2}\, ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\, \left(3\, a + 2\, b\, x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x),x)
```

```
[Out] (x^2*(3*a + 2*b*x))/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a),x)
```

```
[Out] a*x**2/2 + b*x**3/3
```

3.46 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx = ax+\frac{bx^2}{2}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
```

```
[Out] a*x + (b*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x,x]
```

```
[Out] IntegrateAlgebraic[a + b*x, x]
```

fricas [A] time = 1.36, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*b + x*a
```

giac [A] time = 1.25, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}b\,x^{2}+a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)
```

```
[Out] a*x+1/2*b*x^2
```

maxima [A] time = 0.89, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

3.47 $\int \frac{a+bx}{x}\,dx$

Optimal. Leaf size=8

$$a\log(x) + bx$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x}\,dx &= \int \left(b+\frac{a}{x}\right)dx\\ &= bx + a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$a\log(x) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x,x]
```

```
[Out] b*x + a*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x, x]
```

fricas [A] time = 1.26, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="fricas")
```

```
[Out] b*x + a*log(x)
```

giac [A] time = 1.06, size = 9, normalized size = 1.12

$$bx + a\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="giac")
[Out] b*x + a*log(abs(x))
```

maple [A] time = 0.01, size = 9, normalized size = 1.12

$$a\ln(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x,x)
[Out] b*x+a*ln(x)
```

maxima [A] time = 0.84, size = 8, normalized size = 1.00

$$bx + a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x, algorithm="maxima")
[Out] b*x + a*log(x)
```

mupad [B] time = 0.02, size = 8, normalized size = 1.00

$$b\,x + a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x,x)
[Out] b*x + a*log(x)
```

sympy [A] time = 0.09, size = 7, normalized size = 0.88

$$a\log(x) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x,x)
[Out] a*log(x) + b*x
```

3.48 $\int \frac{a+bx}{x^2}\,dx$

Optimal. Leaf size=11

$$b\log(x) - \frac{a}{x}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2}\,dx &= \int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx\\ &= -\frac{a}{x} + b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$b\log(x) - \frac{a}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^2,x]
```

```
[Out] -(a/x) + b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x^2, x]
```

fricas [A] time = 1.41, size = 13, normalized size = 1.18

$$\frac{bx\log(x) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="fricas")
[Out] (b*x*log(x) - a)/x
```

giac [A] time = 1.05, size = 12, normalized size = 1.09

$$b\log\left(|x|\right) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="giac")
[Out] b*log(abs(x)) - a/x
```

maple [A] time = 0.02, size = 12, normalized size = 1.09

$$b\ln(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2,x)
[Out] -a/x+b*ln(x)
```

maxima [A] time = 0.87, size = 11, normalized size = 1.00

$$b\log(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2,x, algorithm="maxima")
[Out] b*log(x) - a/x
```

mupad [B] time = 0.03, size = 11, normalized size = 1.00

$$b\,\ln(x) - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^2,x)
[Out] b*log(x) - a/x
```

sympy [A] time = 0.11, size = 7, normalized size = 0.64

$$-\frac{a}{x} + b\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2,x)
[Out] -a/x + b*log(x)
```

3.49 $\int \frac{a+bx}{x^3} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^2}{2ax^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$-\frac{(a+bx)^2}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^3,x]
```

```
[Out] -(a + b*x)^2/(2*a*x^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{x^3} dx = -\frac{(a+bx)^2}{2ax^2}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 0.88

$$-\frac{a}{2x^2} - \frac{b}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^3,x]
```

```
[Out] -1/2*a/x^2 - b/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x^3, x]
```

fricas [A] time = 1.32, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="fricas")
[Out] -1/2*(2*b*x + a)/x^2
```

giac [A] time = 1.22, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="giac")
[Out] -1/2*(2*b*x + a)/x^2
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{x}-\frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3,x)
[Out] -b/x-1/2*a/x^2
```

maxima [A] time = 1.12, size = 11, normalized size = 0.65

$$-\frac{2\,bx+a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3,x, algorithm="maxima")
[Out] -1/2*(2*b*x + a)/x^2
```

mupad [B] time = 0.02, size = 11, normalized size = 0.65

$$-\frac{a+2\,b\,x}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^3,x)
[Out] -(a + 2*b*x)/(2*x^2)
```

sympy [A] time = 0.11, size = 12, normalized size = 0.71

$$\frac{-a-2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3,x)
[Out] (-a - 2*b*x)/(2*x**2)
```

3.50 $\int \frac{a+bx}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^4,x]
```

```
[Out] -a/(3*x^3) - b/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4} dx &= \int \left(\frac{a}{x^4} + \frac{b}{x^3}\right) dx\\ &= -\frac{a}{3x^3} - \frac{b}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{3x^3} - \frac{b}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^4,x]
```

```
[Out] -1/3*a/x^3 - b/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/x^4, x]
```

fricas [A] time = 1.38, size = 13, normalized size = 0.76

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

giac [A] time = 1.38, size = 13, normalized size = 0.76

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{2x^2} - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4,x)
[Out] -1/3*a/x^3-1/2*b/x^2
```

maxima [A] time = 1.10, size = 13, normalized size = 0.76

$$-\frac{3\,bx + 2\,a}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4,x, algorithm="maxima")
[Out] -1/6*(3*b*x + 2*a)/x^3
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{2\,a + 3\,b\,x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^4,x)
[Out] -(2*a + 3*b*x)/(6*x^3)
```

sympy [A] time = 0.13, size = 14, normalized size = 0.82

$$\frac{-2a - 3bx}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4,x)
[Out] (-2*a - 3*b*x)/(6*x**3)
```

3.51 $\int \frac{a+bx}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^5,x]
[Out] -a/(4*x^4) - b/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^5} \, dx &= \int \left(\frac{a}{x^5}+\frac{b}{x^4}\right) \, dx\\ &= -\frac{a}{4x^4}-\frac{b}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{4x^4}-\frac{b}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^5,x]
[Out] -1/4*a/x^4 - b/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^5,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^5, x]
```

fricas [A] time = 1.47, size = 13, normalized size = 0.76

$$-\frac{4\,bx+3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="fricas")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

giac [A] time = 1.68, size = 13, normalized size = 0.76

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="giac")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$-\frac{b}{3x^3} - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^5,x)
[Out] -1/4*a/x^4-1/3*b/x^3
```

maxima [A] time = 1.05, size = 13, normalized size = 0.76

$$-\frac{4\,bx + 3\,a}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^5,x, algorithm="maxima")
[Out] -1/12*(4*b*x + 3*a)/x^4
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{3\,a + 4\,b\,x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^5,x)
[Out] -(3*a + 4*b*x)/(12*x^4)
```

sympy [A] time = 0.16, size = 14, normalized size = 0.82

$$\frac{-3a - 4bx}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**5,x)
[Out] (-3*a - 4*b*x)/(12*x**4)
```

3.52 $\int x^3(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^2\,dx &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^2, x]
```

fricas [A] time = 0.75, size = 24, normalized size = 0.80

$$\frac{1}{6}x^6b^2+\frac{2}{5}x^5ba+\frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^2 + 2/5*x^5*b*a + 1/4*x^4*a^2
```

giac [A] time = 1.61, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6 + \frac{2}{5}ab\,x^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2,x)
```

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

maxima [A] time = 1.20, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

mupad [B] time = 0.08, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^2,x)
```

```
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2,x)
```

```
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

3.53 $\int x^2(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^2\,dx&=\int\left(a^2x^2+2abx^3+b^2x^4\right)dx\\&=\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^2, x]
```

fricas [A] time = 1.13, size = 24, normalized size = 0.80

$$\frac{1}{5}x^5b^2+\frac{1}{2}x^4ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^2 + 1/2*x^4*b*a + 1/3*x^3*a^2
```

giac [A] time = 1.02, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{5}b^2x^5 + \frac{1}{2}ab\,x^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2,x)
```

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

maxima [A] time = 1.05, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2,x)
```

```
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

3.54 $\int x(a+bx)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^2\,dx &= \int \left(a^2x+2abx^2+b^2x^3\right)\,dx\\ &= \frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^2}{2}+\frac{2}{3}abx^3+\frac{b^2x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2)/2 + (2*a*b*x^3)/3 + (b^2*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^2, x]
```

fricas [A] time = 1.15, size = 24, normalized size = 0.80

$$\frac{1}{4}x^4b^2+\frac{2}{3}x^3ba+\frac{1}{2}x^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b^2 + 2/3*x^3*b*a + 1/2*x^2*a^2
```

giac [A] time = 1.41, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$\frac{1}{4}b^2x^4 + \frac{2}{3}ab\,x^3 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2,x)
```

```
[Out] 1/2*a^2*x^2+2/3*a*b*x^3+1/4*b^2*x^4
```

maxima [A] time = 0.99, size = 24, normalized size = 0.80

$$\frac{1}{4}\,b^2x^4 + \frac{2}{3}\,abx^3 + \frac{1}{2}\,a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*x^4 + 2/3*a*b*x^3 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^2}{2} + \frac{2\,a\,b\,x^3}{3} + \frac{b^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^2,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^4)/4 + (2*a*b*x^3)/3
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^2}{2} + \frac{2abx^3}{3} + \frac{b^2x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2,x)
```

```
[Out] a**2*x**2/2 + 2*a*b*x**3/3 + b**2*x**4/4
```

3.55 $\int (a+bx)^2\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2\,dx=\frac{(a+bx)^3}{3b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^2, x]
```

fricas [A] time = 1.22, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3b^2+x^2ba+xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

giac [A] time = 1.16, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(b*x + a)^3/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x)
[Out] 1/3*(b*x+a)^3/b
```

maxima [A] time = 1.08, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

mupad [B] time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

sympy [B] time = 0.07, size = 19, normalized size = 1.36

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

3.56 $\int \frac{(a+bx)^2}{x} dx$

Optimal. Leaf size=22

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x} dx &= \int \left(2ab + \frac{a^2}{x} + b^2x\right) dx \\ &= 2abx + \frac{b^2x^2}{2} + a^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$a^2\log(x) + 2abx + \frac{b^2x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x,x]
```

```
[Out] 2*a*b*x + (b^2*x^2)/2 + a^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x, x]
```

fricas [A] time = 0.84, size = 20, normalized size = 0.91

$$\frac{1}{2}b^2x^2 + 2abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

giac [A] time = 1.35, size = 21, normalized size = 0.95

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="giac")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(abs(x))
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{b^2x^2}{2} + a^2\ln(x) + 2abx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x,x)
[Out] 2*a*b*x+1/2*b^2*x^2+a^2*ln(x)
```

maxima [A] time = 1.11, size = 20, normalized size = 0.91

$$\frac{1}{2}\,b^2x^2 + 2\,abx + a^2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*b^2*x^2 + 2*a*b*x + a^2*log(x)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.91

$$a^2\,\ln(x) + \frac{b^2\,x^2}{2} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x,x)
[Out] a^2*log(x) + (b^2*x^2)/2 + 2*a*b*x
```

sympy [A] time = 0.11, size = 20, normalized size = 0.91

$$a^2\log\left(x\right) + 2abx + \frac{b^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x,x)
[Out] a**2*log(x) + 2*a*b*x + b**2*x**2/2
```

3.57 $\int \frac{(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=20

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2}\,dx &= \int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx\\ &= -\frac{a^2}{x}+b^2x+2ab\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{a^2}{x}+2ab\log(x)+b^2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^2,x]
```

```
[Out] -(a^2/x) + b^2*x + 2*a*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^2, x]
```

fricas [A] time = 1.37, size = 24, normalized size = 1.20

$$\frac{b^2x^2+2\,abx\log(x)-a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)/x
```

giac [A] time = 1.09, size = 21, normalized size = 1.05

$$b^2x + 2\,ab\log\left(|x|\right) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="giac")
[Out] b^2*x + 2*a*b*log(abs(x)) - a^2/x
```

maple [A] time = 0.01, size = 21, normalized size = 1.05

$$2ab\ln(x) + b^2x - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2,x)
[Out] -a^2/x+b^2*x+2*a*b*ln(x)
```

maxima [A] time = 1.10, size = 20, normalized size = 1.00

$$b^2x + 2\,ab\log(x) - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2,x, algorithm="maxima")
[Out] b^2*x + 2*a*b*log(x) - a^2/x
```

mupad [B] time = 0.07, size = 20, normalized size = 1.00

$$b^2\,x - \frac{a^2}{x} + 2\,a\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^2,x)
[Out] b^2*x - a^2/x + 2*a*b*log(x)
```

sympy [A] time = 0.13, size = 17, normalized size = 0.85

$$-\frac{a^2}{x} + 2ab\log\left(x\right) + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2,x)
[Out] -a**2/x + 2*a*b*log(x) + b**2*x
```

3.58 $\int \frac{(a+bx)^2}{x^3} dx$

Optimal. Leaf size=24

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^3,x]
```

```
[Out] -a^2/(2*x^2) - (2*a*b)/x + b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3} dx &= \int \left(\frac{a^2}{x^3} + \frac{2ab}{x^2} + \frac{b^2}{x}\right) dx \\ &= -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^3,x]
```

```
[Out] -1/2*a^2/x^2 - (2*a*b)/x + b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^3, x]
```

fricas [A] time = 0.91, size = 26, normalized size = 1.08

$$\frac{2\,b^2x^2\log(x) - 4\,abx - a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)/x^2
```

giac [A] time = 1.18, size = 22, normalized size = 0.92

$$b^2 \log(|x|) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="giac")
[Out] b^2*log(abs(x)) - 1/2*(4*a*b*x + a^2)/x^2
```

maple [A] time = 0.01, size = 23, normalized size = 0.96

$$b^2 \ln(x) - \frac{2ab}{x} - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3,x)
[Out] -1/2*a^2/x^2-2*a*b/x+b^2*ln(x)
```

maxima [A] time = 1.18, size = 21, normalized size = 0.88

$$b^2 \log(x) - \frac{4\,abx + a^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3,x, algorithm="maxima")
[Out] b^2*log(x) - 1/2*(4*a*b*x + a^2)/x^2
```

mupad [B] time = 0.04, size = 23, normalized size = 0.96

$$b^2\,\ln(x) - \frac{\frac{a^2}{2} + 2\,b\,x\,a}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^3,x)
[Out] b^2*log(x) - (a^2/2 + 2*a*b*x)/x^2
```

sympy [A] time = 0.17, size = 22, normalized size = 0.92

$$b^2 \log(x) + \frac{-a^2 - 4abx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3,x)
[Out] b**2*log(x) + (-a**2 - 4*a*b*x)/(2*x**2)
```

3.59 $\int \frac{(a+bx)^2}{x^4} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^3}{3ax^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^3}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^4,x]
```

```
[Out] -(a + b*x)^3/(3*a*x^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{x^4} dx = -\frac{(a+bx)^3}{3ax^3}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.53

$$-\frac{a^2}{3x^3} - \frac{ab}{x^2} - \frac{b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^4,x]
```

```
[Out] -1/3*a^2/x^3 - (a*b)/x^2 - b^2/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^4, x]
```

fricas [A] time = 1.29, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2 + 3\,abx + a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

giac [A] time = 1.48, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="giac")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

maple [A] time = 0.01, size = 25, normalized size = 1.47

$$-\frac{b^2}{x}-\frac{ab}{x^2}-\frac{a^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4,x)
[Out] -a*b/x^2-b^2/x-1/3*a^2/x^3
```

maxima [A] time = 1.35, size = 22, normalized size = 1.29

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4,x, algorithm="maxima")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/x^3
```

mupad [B] time = 0.04, size = 22, normalized size = 1.29

$$-\frac{\frac{a^2}{3}+a\,b\,x+b^2\,x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^4,x)
[Out] -(a^2/3 + b^2*x^2 + a*b*x)/x^3
```

sympy [A] time = 0.18, size = 24, normalized size = 1.41

$$\frac{-a^2-3abx-3b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4,x)
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*x**3)
```

3.60 $\int \frac{(a+bx)^2}{x^5} dx$

Optimal. Leaf size=30

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^5,x]
```

```
[Out] -a^2/(4*x^4) - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^5} dx &= \int \left(\frac{a^2}{x^5} + \frac{2ab}{x^4} + \frac{b^2}{x^3}\right) dx \\ &= -\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{4x^4} - \frac{2ab}{3x^3} - \frac{b^2}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^5,x]
```

```
[Out] -1/4*a^2/x^4 - (2*a*b)/(3*x^3) - b^2/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^5, x]
```

fricas [A] time = 0.88, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

giac [A] time = 1.12, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="giac")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{2x^2} - \frac{2ab}{3x^3} - \frac{a^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^5,x)
[Out] -1/4*a^2/x^4-2/3*a*b/x^3-1/2*b^2/x^2
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$-\frac{6\,b^2x^2 + 8\,abx + 3\,a^2}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^5,x, algorithm="maxima")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)/x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{4} + \frac{2\,a\,b\,x}{3} + \frac{b^2\,x^2}{2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^5,x)
[Out] -(a^2/4 + (b^2*x^2)/2 + (2*a*b*x)/3)/x^4
```

sympy [A] time = 0.19, size = 26, normalized size = 0.87

$$\frac{-3a^2 - 8abx - 6b^2x^2}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**5,x)
[Out] (-3*a**2 - 8*a*b*x - 6*b**2*x**2)/(12*x**4)
```

3.61 $\int \frac{(a+bx)^2}{x^6}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^6,x]
```

```
[Out] -a^2/(5*x^5) - (a*b)/(2*x^4) - b^2/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^6}\,dx &= \int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx\\ &= -\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{5x^5}-\frac{ab}{2x^4}-\frac{b^2}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^6,x]
```

```
[Out] -1/5*a^2/x^5 - (a*b)/(2*x^4) - b^2/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^6, x]
```

fricas [A] time = 1.43, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2+15\,abx+6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

giac [A] time = 1.19, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="giac")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{3x^3} - \frac{ab}{2x^4} - \frac{a^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^6,x)
[Out] -1/5*a^2/x^5-1/2*a*b/x^4-1/3*b^2/x^3
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$-\frac{10\,b^2x^2 + 15\,abx + 6\,a^2}{30\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^6,x, algorithm="maxima")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)/x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{5} + \frac{a\,b\,x}{2} + \frac{b^2\,x^2}{3}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^6,x)
[Out] -(a^2/5 + (b^2*x^2)/3 + (a*b*x)/2)/x^5
```

sympy [A] time = 0.19, size = 26, normalized size = 0.87

$$\frac{-6a^2 - 15abx - 10b^2x^2}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**6,x)
[Out] (-6*a**2 - 15*a*b*x - 10*b**2*x**2)/(30*x**5)
```

3.62 $\int \frac{(a+bx)^2}{x^7}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^7,x]
[Out] -a^2/(6*x^6) - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^7}\,dx &= \int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx\\ &= -\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$-\frac{a^2}{6x^6}-\frac{2ab}{5x^5}-\frac{b^2}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^7,x]
[Out] -1/6*a^2/x^6 - (2*a*b)/(5*x^5) - b^2/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^7,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^7, x]
```

fricas [A] time = 0.72, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2+24\,abx+10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

giac [A] time = 1.07, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="giac")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{4x^4} - \frac{2ab}{5x^5} - \frac{a^2}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^7,x)
[Out] -1/6*a^2/x^6-2/5*a*b/x^5-1/4*b^2/x^4
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$-\frac{15\,b^2x^2 + 24\,abx + 10\,a^2}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^7,x, algorithm="maxima")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)/x^6
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{6} + \frac{2\,a\,b\,x}{5} + \frac{b^2\,x^2}{4}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^7,x)
[Out] -(a^2/6 + (b^2*x^2)/4 + (2*a*b*x)/5)/x^6
```

sympy [A] time = 0.20, size = 26, normalized size = 0.87

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**7,x)
[Out] (-10*a**2 - 24*a*b*x - 15*b**2*x**2)/(60*x**6)
```

3.63 $\int \frac{(a+bx)^2}{x^8} \, dx$

Optimal. Leaf size=30

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^8,x]
```

```
[Out] -a^2/(7*x^7) - (a*b)/(3*x^6) - b^2/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^8} \, dx &= \int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right) dx\\ &= -\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{7x^7}-\frac{ab}{3x^6}-\frac{b^2}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^8,x]
```

```
[Out] -1/7*a^2/x^7 - (a*b)/(3*x^6) - b^2/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/x^8, x]
```

fricas [A] time = 1.36, size = 24, normalized size = 0.80

$$-\frac{21\,b^2x^2+35\,abx+15\,a^2}{105\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="fricas")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

giac [A] time = 1.20, size = 24, normalized size = 0.80

$$-\frac{21\, b^2 x^2 + 35\, abx + 15\, a^2}{105\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="giac")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{b^2}{5x^5} - \frac{ab}{3x^6} - \frac{a^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^8,x)
[Out] -1/7*a^2/x^7-1/3*a*b/x^6-1/5*b^2/x^5
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$-\frac{21\, b^2 x^2 + 35\, abx + 15\, a^2}{105\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^8,x, algorithm="maxima")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)/x^7
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{7} + \frac{a\, b\, x}{3} + \frac{b^2\, x^2}{5}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^8,x)
[Out] -(a^2/7 + (b^2*x^2)/5 + (a*b*x)/3)/x^7
```

sympy [A] time = 0.21, size = 26, normalized size = 0.87

$$\frac{-15a^2 - 35abx - 21b^2x^2}{105x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**8,x)
[Out] (-15*a**2 - 35*a*b*x - 21*b**2*x**2)/(105*x**7)
```

3.64 $\int x^4(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}a^2bx^6+\frac{a^3x^5}{5}+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^3\,dx&=\int\left(a^3x^4+3a^2bx^5+3ab^2x^6+b^3x^7\right)dx\\&=\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^5}{5}+\frac{1}{2}a^2bx^6+\frac{3}{7}ab^2x^7+\frac{b^3x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^3,x]
```

```
[Out] (a^3*x^5)/5 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7 + (b^3*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^3, x]
```

fricas [A] time = 1.07, size = 35, normalized size = 0.81

$$\frac{1}{8}x^8b^3+\frac{3}{7}x^7b^2a+\frac{1}{2}x^6ba^2+\frac{1}{5}x^5a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^3 + 3/7*x^7*b^2*a + 1/2*x^6*b*a^2 + 1/5*x^5*a^3
```

giac [A] time = 1.22, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{8}b^3x^8 + \frac{3}{7}a\,b^2x^7 + \frac{1}{2}a^2b\,x^6 + \frac{1}{5}a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^3,x)
```

```
[Out] 1/5*a^3*x^5+1/2*a^2*b*x^6+3/7*a*b^2*x^7+1/8*b^3*x^8
```

maxima [A] time = 1.32, size = 35, normalized size = 0.81

$$\frac{1}{8}\,b^3x^8 + \frac{3}{7}\,ab^2x^7 + \frac{1}{2}\,a^2bx^6 + \frac{1}{5}\,a^3x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^3*x^8 + 3/7*a*b^2*x^7 + 1/2*a^2*b*x^6 + 1/5*a^3*x^5
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^5}{5} + \frac{a^2\,b\,x^6}{2} + \frac{3\,a\,b^2\,x^7}{7} + \frac{b^3\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^3,x)
```

```
[Out] (a^3*x^5)/5 + (b^3*x^8)/8 + (a^2*b*x^6)/2 + (3*a*b^2*x^7)/7
```

sympy [A] time = 0.08, size = 37, normalized size = 0.86

$$\frac{a^3x^5}{5} + \frac{a^2bx^6}{2} + \frac{3ab^2x^7}{7} + \frac{b^3x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**3,x)
```

```
[Out] a**3*x**5/5 + a**2*b*x**6/2 + 3*a*b**2*x**7/7 + b**3*x**8/8
```

3.65 $\int x^3(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}a^2bx^5+\frac{a^3x^4}{4}+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^3,x]
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^3\,dx &= \int \left(a^3x^3+3a^2bx^4+3ab^2x^5+b^3x^6\right)dx\\ &= \frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^4}{4}+\frac{3}{5}a^2bx^5+\frac{1}{2}ab^2x^6+\frac{b^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^3,x]
[Out] (a^3*x^4)/4 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2 + (b^3*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^3,x]
[Out] IntegrateAlgebraic[x^3*(a + b*x)^3, x]
```

fricas [A] time = 0.69, size = 35, normalized size = 0.81

$$\frac{1}{7}x^7b^3+\frac{1}{2}x^6b^2a+\frac{3}{5}x^5ba^2+\frac{1}{4}x^4a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^3 + 1/2*x^6*b^2*a + 3/5*x^5*b*a^2 + 1/4*x^4*a^3
```

giac [A] time = 1.27, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{7}b^3x^7 + \frac{1}{2}a\,b^2x^6 + \frac{3}{5}a^2b\,x^5 + \frac{1}{4}a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^3,x)
```

```
[Out] 1/4*a^3*x^4+3/5*a^2*b*x^5+1/2*a*b^2*x^6+1/7*b^3*x^7
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$\frac{1}{7}\,b^3x^7 + \frac{1}{2}\,ab^2x^6 + \frac{3}{5}\,a^2bx^5 + \frac{1}{4}\,a^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*x^7 + 1/2*a*b^2*x^6 + 3/5*a^2*b*x^5 + 1/4*a^3*x^4
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^4}{4} + \frac{3\,a^2\,b\,x^5}{5} + \frac{a\,b^2\,x^6}{2} + \frac{b^3\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^3,x)
```

```
[Out] (a^3*x^4)/4 + (b^3*x^7)/7 + (3*a^2*b*x^5)/5 + (a*b^2*x^6)/2
```

sympy [A] time = 0.07, size = 37, normalized size = 0.86

$$\frac{a^3x^4}{4} + \frac{3a^2bx^5}{5} + \frac{ab^2x^6}{2} + \frac{b^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**3,x)
```

```
[Out] a**3*x**4/4 + 3*a**2*b*x**5/5 + a*b**2*x**6/2 + b**3*x**7/7
```

3.66 $\int x^2(a+bx)^3\,dx$

Optimal. Leaf size=43

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}a^2bx^4+\frac{a^3x^3}{3}+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^3\,dx &= \int\left(a^3x^2+3a^2bx^3+3ab^2x^4+b^3x^5\right)dx\\ &= \frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$\frac{a^3x^3}{3}+\frac{3}{4}a^2bx^4+\frac{3}{5}ab^2x^5+\frac{b^3x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^3,x]
```

```
[Out] (a^3*x^3)/3 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5 + (b^3*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^3, x]
```

fricas [A] time = 0.95, size = 35, normalized size = 0.81

$$\frac{1}{6}x^6b^3+\frac{3}{5}x^5b^2a+\frac{3}{4}x^4ba^2+\frac{1}{3}x^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b^3 + 3/5*x^5*b^2*a + 3/4*x^4*b*a^2 + 1/3*x^3*a^3
```

giac [A] time = 1.22, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$\frac{1}{6}b^3x^6 + \frac{3}{5}a\,b^2x^5 + \frac{3}{4}a^2b\,x^4 + \frac{1}{3}a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^3,x)
```

```
[Out] 1/3*a^3*x^3+3/4*a^2*b*x^4+3/5*a*b^2*x^5+1/6*b^3*x^6
```

maxima [A] time = 1.34, size = 35, normalized size = 0.81

$$\frac{1}{6}\,b^3x^6 + \frac{3}{5}\,ab^2x^5 + \frac{3}{4}\,a^2bx^4 + \frac{1}{3}\,a^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*x^6 + 3/5*a*b^2*x^5 + 3/4*a^2*b*x^4 + 1/3*a^3*x^3
```

mupad [B] time = 0.04, size = 35, normalized size = 0.81

$$\frac{a^3\,x^3}{3} + \frac{3\,a^2\,b\,x^4}{4} + \frac{3\,a\,b^2\,x^5}{5} + \frac{b^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^3,x)
```

```
[Out] (a^3*x^3)/3 + (b^3*x^6)/6 + (3*a^2*b*x^4)/4 + (3*a*b^2*x^5)/5
```

sympy [A] time = 0.07, size = 39, normalized size = 0.91

$$\frac{a^3x^3}{3} + \frac{3a^2bx^4}{4} + \frac{3ab^2x^5}{5} + \frac{b^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**3,x)
```

```
[Out] a**3*x**3/3 + 3*a**2*b*x**4/4 + 3*a*b**2*x**5/5 + b**3*x**6/6
```

3.67 $\int x(a+bx)^3\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^5}{5b^2}-\frac{a(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^3,x]
```

```
[Out] -(a*(a + b*x)^4)/(4*b^2) + (a + b*x)^5/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^3\,dx&=\int\left(-\frac{a(a+bx)^3}{b}+\frac{(a+bx)^4}{b}\right)dx\\&=-\frac{a(a+bx)^4}{4b^2}+\frac{(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.33

$$\frac{a^3x^2}{2}+a^2bx^3+\frac{3}{4}ab^2x^4+\frac{b^3x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^3,x]
```

```
[Out] (a^3*x^2)/2 + a^2*b*x^3 + (3*a*b^2*x^4)/4 + (b^3*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^3, x]
```

fricas [A] time = 0.70, size = 34, normalized size = 1.13

$$\frac{1}{5}x^5b^3+\frac{3}{4}x^4b^2a+x^3ba^2+\frac{1}{2}x^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b^3 + 3/4*x^4*b^2*a + x^3*b*a^2 + 1/2*x^2*a^3
```

giac [A] time = 1.21, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

maple [A] time = 0.00, size = 35, normalized size = 1.17

$$\frac{1}{5}b^3x^5 + \frac{3}{4}a\,b^2x^4 + a^2b\,x^3 + \frac{1}{2}a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^3,x)
```

```
[Out] 1/5*b^3*x^5+3/4*a*b^2*x^4+a^2*b*x^3+1/2*a^3*x^2
```

maxima [A] time = 1.38, size = 34, normalized size = 1.13

$$\frac{1}{5}\,b^3x^5 + \frac{3}{4}\,ab^2x^4 + a^2bx^3 + \frac{1}{2}\,a^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^3*x^5 + 3/4*a*b^2*x^4 + a^2*b*x^3 + 1/2*a^3*x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 1.13

$$\frac{a^3\,x^2}{2} + a^2\,b\,x^3 + \frac{3\,a\,b^2\,x^4}{4} + \frac{b^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^3,x)
```

```
[Out] (a^3*x^2)/2 + (b^3*x^5)/5 + a^2*b*x^3 + (3*a*b^2*x^4)/4
```

sympy [A] time = 0.07, size = 36, normalized size = 1.20

$$\frac{a^3x^2}{2} + a^2bx^3 + \frac{3ab^2x^4}{4} + \frac{b^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**3,x)
```

```
[Out] a**3*x**2/2 + a**2*b*x**3 + 3*a*b**2*x**4/4 + b**3*x**5/5
```

3.68 $\int (a+bx)^3\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^4}{4b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3,x]
[Out] (a + b*x)^4/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^3\,dx=\frac{(a+bx)^4}{4b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3,x]
[Out] (a + b*x)^4/(4*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^3, x]
```

fricas [B] time = 1.34, size = 31, normalized size = 2.21

$$\frac{1}{4}x^4b^3+x^3b^2a+\frac{3}{2}x^2ba^2+xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="fricas")
[Out] 1/4*x^4*b^3 + x^3*b^2*a + 3/2*x^2*b*a^2 + x*a^3
```

giac [A] time = 1.36, size = 12, normalized size = 0.86

$$\frac{(bx+a)^4}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="giac")
[Out] 1/4*(b*x + a)^4/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3,x)
[Out] 1/4*(b*x+a)^4/b
```

maxima [B] time = 1.39, size = 31, normalized size = 2.21

$$\frac{1}{4}\,b^3x^4+ab^2x^3+\frac{3}{2}\,a^2bx^2+a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + a*b^2*x^3 + 3/2*a^2*b*x^2 + a^3*x
```

mupad [B] time = 0.04, size = 31, normalized size = 2.21

$$a^3\,x+\frac{3\,a^2\,b\,x^2}{2}+a\,b^2\,x^3+\frac{b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3,x)
[Out] a^3*x + (b^3*x^4)/4 + (3*a^2*b*x^2)/2 + a*b^2*x^3
```

sympy [B] time = 0.07, size = 32, normalized size = 2.29

$$a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3,x)
[Out] a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4
```

3.69 $\int \frac{(a+bx)^3}{x} dx$

Optimal. Leaf size=35

$$a^3 \log(x) + 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2bx + a^3 \log(x) + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x} dx &= \int \left(3a^2b + \frac{a^3}{x} + 3ab^2x + b^3x^2\right) dx \\ &= 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3} + a^3 \log(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 1.00

$$a^3 \log(x) + 3a^2bx + \frac{3}{2}ab^2x^2 + \frac{b^3x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x,x]
```

```
[Out] 3*a^2*b*x + (3*a*b^2*x^2)/2 + (b^3*x^3)/3 + a^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x, x]
```

fricas [A] time = 1.33, size = 31, normalized size = 0.89

$$\frac{1}{3} b^3x^3 + \frac{3}{2} ab^2x^2 + 3 a^2bx + a^3 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="fricas")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

giac [A] time = 0.95, size = 32, normalized size = 0.91

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 + 3a^2bx + a^3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="giac")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(abs(x))
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{b^3x^3}{3} + \frac{3ab^2x^2}{2} + a^3\ln(x) + 3a^2bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x,x)
[Out] 3*a^2*b*x+3/2*a*b^2*x^2+1/3*b^3*x^3+a^3*ln(x)
```

maxima [A] time = 1.34, size = 31, normalized size = 0.89

$$\frac{1}{3}b^3x^3 + \frac{3}{2}ab^2x^2 + 3a^2bx + a^3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x,x, algorithm="maxima")
[Out] 1/3*b^3*x^3 + 3/2*a*b^2*x^2 + 3*a^2*b*x + a^3*log(x)
```

mupad [B] time = 0.03, size = 31, normalized size = 0.89

$$a^3\ln(x) + \frac{b^3x^3}{3} + \frac{3ab^2x^2}{2} + 3a^2bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x,x)
[Out] a^3*log(x) + (b^3*x^3)/3 + (3*a*b^2*x^2)/2 + 3*a^2*b*x
```

sympy [A] time = 0.12, size = 34, normalized size = 0.97

$$a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x,x)
[Out] a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3
```

3.70 $\int \frac{(a+bx)^3}{x^2}\,dx$

Optimal. Leaf size=34

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a^2b\log(x)-\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^2}\,dx &= \int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)dx\\ &= -\frac{a^3}{x}+3ab^2x+\frac{b^3x^2}{2}+3a^2b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 1.00

$$-\frac{a^3}{x}+3a^2b\log(x)+3ab^2x+\frac{b^3x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^2,x]
```

```
[Out] -(a^3/x) + 3*a*b^2*x + (b^3*x^2)/2 + 3*a^2*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^2, x]
```

fricas [A] time = 1.54, size = 36, normalized size = 1.06

$$\frac{b^3x^3+6\,ab^2x^2+6\,a^2bx\log(x)-2\,a^3}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="fricas")
[Out] 1/2*(b^3*x^3 + 6*a*b^2*x^2 + 6*a^2*b*x*log(x) - 2*a^3)/x
```

giac [A] time = 1.36, size = 33, normalized size = 0.97

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log\left(|x|\right) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="giac")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(abs(x)) - a^3/x
```

maple [A] time = 0.01, size = 33, normalized size = 0.97

$$\frac{b^3x^2}{2} + 3a^2b\ln(x) + 3a\,b^2x - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^2,x)
[Out] -a^3/x+3*a*b^2*x+1/2*b^3*x^2+3*a^2*b*ln(x)
```

maxima [A] time = 1.29, size = 32, normalized size = 0.94

$$\frac{1}{2}\,b^3x^2 + 3\,ab^2x + 3\,a^2b\log(x) - \frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^2,x, algorithm="maxima")
[Out] 1/2*b^3*x^2 + 3*a*b^2*x + 3*a^2*b*log(x) - a^3/x
```

mupad [B] time = 0.03, size = 32, normalized size = 0.94

$$\frac{b^3\,x^2}{2} - \frac{a^3}{x} + 3\,a^2\,b\,\ln(x) + 3\,a\,b^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^2,x)
[Out] (b^3*x^2)/2 - a^3/x + 3*a^2*b*log(x) + 3*a*b^2*x
```

sympy [A] time = 0.13, size = 31, normalized size = 0.91

$$-\frac{a^3}{x} + 3a^2b\log\left(x\right) + 3ab^2x + \frac{b^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**2,x)
[Out] -a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2
```

3.71 $\int \frac{(a+bx)^3}{x^3}\,dx$

Optimal. Leaf size=33

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{x}-\frac{a^3}{2x^2}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^3,x]
```

```
[Out] -a^3/(2*x^2) - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^3}\,dx &= \int \left(b^3+\frac{a^3}{x^3}+\frac{3a^2b}{x^2}+\frac{3ab^2}{x}\right)dx\\ &= -\frac{a^3}{2x^2}-\frac{3a^2b}{x}+b^3x+3ab^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{a^3}{2x^2}-\frac{3a^2b}{x}+3ab^2\log(x)+b^3x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^3,x]
```

```
[Out] -1/2*a^3/x^2 - (3*a^2*b)/x + b^3*x + 3*a*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^3, x]
```

fricas [A] time = 1.33, size = 37, normalized size = 1.12

$$\frac{2\,b^3x^3+6\,ab^2x^2\log(x)-6\,a^2bx-a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="fricas")
[Out] 1/2*(2*b^3*x^3 + 6*a*b^2*x^2*log(x) - 6*a^2*b*x - a^3)/x^2
```

giac [A] time = 1.18, size = 31, normalized size = 0.94

$$b^3x + 3\,ab^2\log\left(|x|\right) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="giac")
[Out] b^3*x + 3*a*b^2*log(abs(x)) - 1/2*(6*a^2*b*x + a^3)/x^2
```

maple [A] time = 0.00, size = 32, normalized size = 0.97

$$3a\,b^2\ln(x) + b^3x - \frac{3a^2b}{x} - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^3,x)
[Out] -1/2*a^3/x^2-3*a^2*b/x+b^3*x+3*a*b^2*ln(x)
```

maxima [A] time = 1.34, size = 30, normalized size = 0.91

$$b^3x + 3\,ab^2\log(x) - \frac{6\,a^2bx + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^3,x, algorithm="maxima")
[Out] b^3*x + 3*a*b^2*log(x) - 1/2*(6*a^2*b*x + a^3)/x^2
```

mupad [B] time = 0.03, size = 32, normalized size = 0.97

$$b^3\,x - \frac{\frac{a^3}{2} + 3\,b\,x\,a^2}{x^2} + 3\,a\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^3,x)
[Out] b^3*x - (a^3/2 + 3*a^2*b*x)/x^2 + 3*a*b^2*log(x)
```

sympy [A] time = 0.19, size = 32, normalized size = 0.97

$$3ab^2\log\left(x\right) + b^3x + \frac{-a^3 - 6a^2bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**3,x)
[Out] 3*a*b**2*log(x) + b**3*x + (-a**3 - 6*a**2*b*x)/(2*x**2)
```

3.72 $\int \frac{(a+bx)^3}{x^4}\,dx$

Optimal. Leaf size=37

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{2x^2}-\frac{a^3}{3x^3}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^4,x]
```

```
[Out] -a^3/(3*x^3) - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^4}\,dx &= \int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^3}+\frac{3ab^2}{x^2}+\frac{b^3}{x}\right)dx\\ &= -\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 37, normalized size = 1.00

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{2x^2}-\frac{3ab^2}{x}+b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^4,x]
```

```
[Out] -1/3*a^3/x^3 - (3*a^2*b)/(2*x^2) - (3*a*b^2)/x + b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^4, x]
```

fricas [A] time = 1.01, size = 37, normalized size = 1.00

$$\frac{6\,b^3x^3\log(x)-18\,ab^2x^2-9\,a^2bx-2\,a^3}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(x) - 18*a*b^2*x^2 - 9*a^2*b*x - 2*a^3)/x^3
```

giac [A] time = 1.14, size = 35, normalized size = 0.95

$$b^3 \log\left(|x|\right) - \frac{18\, ab^2x^2 + 9\, a^2bx + 2\, a^3}{6\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="giac")
```

```
[Out] b^3*log(abs(x)) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

maple [A] time = 0.01, size = 34, normalized size = 0.92

$$b^3 \ln(x) - \frac{3a\, b^2}{x} - \frac{3a^2b}{2x^2} - \frac{a^3}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^4,x)
```

```
[Out] -1/3*a^3/x^3-3/2*a^2*b/x^2-3*a*b^2/x+b^3*ln(x)
```

maxima [A] time = 1.30, size = 34, normalized size = 0.92

$$b^3 \log(x) - \frac{18\, ab^2x^2 + 9\, a^2bx + 2\, a^3}{6\, x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^4,x, algorithm="maxima")
```

```
[Out] b^3*log(x) - 1/6*(18*a*b^2*x^2 + 9*a^2*b*x + 2*a^3)/x^3
```

mupad [B] time = 0.07, size = 34, normalized size = 0.92

$$b^3\, \ln(x) - \frac{\frac{a^3}{3} + \frac{3\, a^2\, b\, x}{2} + 3\, a\, b^2\, x^2}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^4,x)
```

```
[Out] b^3*log(x) - (a^3/3 + 3*a*b^2*x^2 + (3*a^2*b*x)/2)/x^3
```

sympy [A] time = 0.24, size = 36, normalized size = 0.97

$$b^3 \log\left(x\right) + \frac{-2a^3 - 9a^2bx - 18ab^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**4,x)
```

```
[Out] b**3*log(x) + (-2*a**3 - 9*a**2*b*x - 18*a*b**2*x**2)/(6*x**3)
```

3.73 $\int \frac{(a+bx)^3}{x^5}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^4}{4ax^4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^4}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^5,x]
```

```
[Out] -(a + b*x)^4/(4*a*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^3}{x^5}\,dx = -\frac{(a+bx)^4}{4ax^4}$$

Mathematica [B] time = 0.00, size = 39, normalized size = 2.29

$$-\frac{a^3}{4x^4}-\frac{a^2b}{x^3}-\frac{3ab^2}{2x^2}-\frac{b^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^5,x]
```

```
[Out] -1/4*a^3/x^4 - (a^2*b)/x^3 - (3*a*b^2)/(2*x^2) - b^3/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^5, x]
```

fricas [B] time = 1.30, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3+6\,ab^2x^2+4\,a^2bx+a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

giac [B] time = 1.29, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="giac")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

maple [B] time = 0.01, size = 36, normalized size = 2.12

$$-\frac{b^3}{x} - \frac{3a\,b^2}{2x^2} - \frac{a^2b}{x^3} - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^5,x)
```

```
[Out] -a^2*b/x^3-1/4*a^3/x^4-b^3/x-3/2*a*b^2/x^2
```

maxima [B] time = 1.33, size = 33, normalized size = 1.94

$$-\frac{4\,b^3x^3 + 6\,ab^2x^2 + 4\,a^2bx + a^3}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)/x^4
```

mupad [B] time = 0.03, size = 33, normalized size = 1.94

$$-\frac{\frac{a^3}{4} + a^2\,b\,x + \frac{3\,a\,b^2\,x^2}{2} + b^3\,x^3}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^5,x)
```

```
[Out] -(a^3/4 + b^3*x^3 + (3*a*b^2*x^2)/2 + a^2*b*x)/x^4
```

sympy [B] time = 0.26, size = 36, normalized size = 2.12

$$\frac{-a^3 - 4a^2bx - 6ab^2x^2 - 4b^3x^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**5,x)
```

```
[Out] (-a**3 - 4*a**2*b*x - 6*a*b**2*x**2 - 4*b**3*x**3)/(4*x**4)
```

3.74 $\int \frac{(a+bx)^3}{x^6}\,dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^4}{20a^2x^4}-\frac{(a+bx)^4}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^6,x]
```

```
[Out] -(a + b*x)^4/(5*a*x^5) + (b*(a + b*x)^4)/(20*a^2*x^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^6}\,dx &= -\frac{(a+bx)^4}{5ax^5}-\frac{b\int \frac{(a+bx)^3}{x^5}\,dx}{5a}\\ &= -\frac{(a+bx)^4}{5ax^5}+\frac{b(a+bx)^4}{20a^2x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.14

$$-\frac{a^3}{5x^5}-\frac{3a^2b}{4x^4}-\frac{ab^2}{x^3}-\frac{b^3}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^6,x]
```

```
[Out] -1/5*a^3/x^5 - (3*a^2*b)/(4*x^4) - (a*b^2)/x^3 - b^3/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^6,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^6, x]
```

fricas [A] time = 1.48, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="fricas")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

giac [A] time = 1.13, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="giac")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

maple [A] time = 0.00, size = 36, normalized size = 1.00

$$-\frac{b^3}{2x^2} - \frac{a\,b^2}{x^3} - \frac{3a^2b}{4x^4} - \frac{a^3}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^6,x)
[Out] -1/5*a^3/x^5-a*b^2/x^3-3/4*a^2*b/x^4-1/2*b^3/x^2
```

maxima [A] time = 1.35, size = 35, normalized size = 0.97

$$-\frac{10\,b^3x^3 + 20\,ab^2x^2 + 15\,a^2bx + 4\,a^3}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^6,x, algorithm="maxima")
[Out] -1/20*(10*b^3*x^3 + 20*a*b^2*x^2 + 15*a^2*b*x + 4*a^3)/x^5
```

mupad [B] time = 0.03, size = 34, normalized size = 0.94

$$-\frac{\frac{a^3}{5} + \frac{3\,a^2\,b\,x}{4} + a\,b^2\,x^2 + \frac{b^3\,x^3}{2}}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^6,x)
[Out] -(a^3/5 + (b^3*x^3)/2 + a*b^2*x^2 + (3*a^2*b*x)/4)/x^5
```

sympy [A] time = 0.25, size = 37, normalized size = 1.03

$$\frac{-4a^3 - 15a^2bx - 20ab^2x^2 - 10b^3x^3}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**6,x)
```

```
[Out] (-4*a**3 - 15*a**2*b*x - 20*a*b**2*x**2 - 10*b**3*x**3)/(20*x**5)
```

3.75 $\int \frac{(a+bx)^3}{x^7}\,dx$

Optimal. Leaf size=43

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^7}\,dx &= \int \left(\frac{a^3}{x^7}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^5}+\frac{b^3}{x^4}\right)dx\\ &= -\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{6x^6}-\frac{3a^2b}{5x^5}-\frac{3ab^2}{4x^4}-\frac{b^3}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^7,x]
```

```
[Out] -1/6*a^3/x^6 - (3*a^2*b)/(5*x^5) - (3*a*b^2)/(4*x^4) - b^3/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^7, x]
```

fricas [A] time = 0.97, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="fricas")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

giac [A] time = 1.15, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="giac")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

maple [A] time = 0.01, size = 36, normalized size = 0.84

$$-\frac{b^3}{3x^3}-\frac{3a\,b^2}{4x^4}-\frac{3a^2b}{5x^5}-\frac{a^3}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^7,x)
[Out] -1/6*a^3/x^6-3/5*a^2*b/x^5-3/4*a*b^2/x^4-1/3*b^3/x^3
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$-\frac{20\,b^3x^3+45\,ab^2x^2+36\,a^2bx+10\,a^3}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^7,x, algorithm="maxima")
[Out] -1/60*(20*b^3*x^3 + 45*a*b^2*x^2 + 36*a^2*b*x + 10*a^3)/x^6
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{6}+\frac{3\,a^2\,b\,x}{5}+\frac{3\,a\,b^2\,x^2}{4}+\frac{b^3\,x^3}{3}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^7,x)
[Out] -(a^3/6 + (b^3*x^3)/3 + (3*a*b^2*x^2)/4 + (3*a^2*b*x)/5)/x^6
```

sympy [A] time = 0.34, size = 37, normalized size = 0.86

$$\frac{-10a^3-36a^2bx-45ab^2x^2-20b^3x^3}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**7,x)
[Out] (-10*a**3 - 36*a**2*b*x - 45*a*b**2*x**2 - 20*b**3*x**3)/(60*x**6)
```

3.76 $\int \frac{(a+bx)^3}{x^8} dx$

Optimal. Leaf size=43

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2b}{2x^6}-\frac{a^3}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^8,x]
```

```
[Out] -a^3/(7*x^7) - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^3}{x^8}\,dx &= \int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^7}+\frac{3ab^2}{x^6}+\frac{b^3}{x^5}\right) dx \\ &= -\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{7x^7}-\frac{a^2b}{2x^6}-\frac{3ab^2}{5x^5}-\frac{b^3}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^8,x]
```

```
[Out] -1/7*a^3/x^7 - (a^2*b)/(2*x^6) - (3*a*b^2)/(5*x^5) - b^3/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/x^8, x]
```

fricas [A] time = 1.03, size = 35, normalized size = 0.81

$$-\frac{35\,b^3x^3+84\,ab^2x^2+70\,a^2bx+20\,a^3}{140\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

giac [A] time = 1.12, size = 35, normalized size = 0.81

$$-\frac{35\,b^{3}x^{3}+84\,ab^{2}x^{2}+70\,a^{2}bx+20\,a^{3}}{140\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^{3}}{4x^{4}}-\frac{3a\,b^{2}}{5x^{5}}-\frac{a^{2}b}{2x^{6}}-\frac{a^{3}}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^8,x)
```

```
[Out] -1/7*a^3/x^7-1/2*a^2*b/x^6-3/5*a*b^2/x^5-1/4*b^3/x^4
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$-\frac{35\,b^{3}x^{3}+84\,ab^{2}x^{2}+70\,a^{2}bx+20\,a^{3}}{140\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*x^3 + 84*a*b^2*x^2 + 70*a^2*b*x + 20*a^3)/x^7
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^{3}}{7}+\frac{a^{2}\,b\,x}{2}+\frac{3\,a\,b^{2}\,x^{2}}{5}+\frac{b^{3}\,x^{3}}{4}}{x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^8,x)
```

```
[Out] -(a^3/7 + (b^3*x^3)/4 + (3*a*b^2*x^2)/5 + (a^2*b*x)/2)/x^7
```

sympy [A] time = 0.29, size = 37, normalized size = 0.86

$$\frac{-20a^{3}-70a^{2}bx-84ab^{2}x^{2}-35b^{3}x^{3}}{140x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**8,x)
```

```
[Out] (-20*a**3 - 70*a**2*b*x - 84*a*b**2*x**2 - 35*b**3*x**3)/(140*x**7)
```

3.77 $\int x^6(a+bx)^5\,dx$

Optimal. Leaf size=66

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4bx^8+\frac{a^5x^7}{7}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^5\,dx &= \int \left(a^5x^6+5a^4bx^7+10a^3b^2x^8+10a^2b^3x^9+5ab^4x^{10}+b^5x^{11}\right)dx\\ &= \frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.00

$$\frac{a^5x^7}{7}+\frac{5}{8}a^4bx^8+\frac{10}{9}a^3b^2x^9+a^2b^3x^{10}+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^5,x]
```

```
[Out] (a^5*x^7)/7 + (5*a^4*b*x^8)/8 + (10*a^3*b^2*x^9)/9 + a^2*b^3*x^10 + (5*a*b^
4*x^11)/11 + (b^5*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^5, x]
```

fricas [A] time = 0.84, size = 56, normalized size = 0.85

$$\frac{1}{12}x^{12}b^5+\frac{5}{11}x^{11}b^4a+x^{10}b^3a^2+\frac{10}{9}x^9b^2a^3+\frac{5}{8}x^8ba^4+\frac{1}{7}x^7a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^5 + 5/11*x^11*b^4*a + x^10*b^3*a^2 + 10/9*x^9*b^2*a^3 + 5/8*x^8
*b*a^4 + 1/7*x^7*a^5
```

giac [A] time = 0.94, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

maple [A] time = 0.01, size = 57, normalized size = 0.86

$$\frac{1}{12}b^5x^{12}+\frac{5}{11}a\,b^4x^{11}+a^2b^3x^{10}+\frac{10}{9}a^3b^2x^9+\frac{5}{8}a^4b\,x^8+\frac{1}{7}a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^5,x)
```

```
[Out] 1/7*a^5*x^7+5/8*a^4*b*x^8+10/9*a^3*b^2*x^9+a^2*b^3*x^10+5/11*a*b^4*x^11+1/1
2*b^5*x^12
```

maxima [A] time = 1.38, size = 56, normalized size = 0.85

$$\frac{1}{12}\,b^5x^{12}+\frac{5}{11}\,ab^4x^{11}+a^2b^3x^{10}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{8}\,a^4bx^8+\frac{1}{7}\,a^5x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/12*b^5*x^12 + 5/11*a*b^4*x^11 + a^2*b^3*x^10 + 10/9*a^3*b^2*x^9 + 5/8*a^4
*b*x^8 + 1/7*a^5*x^7
```

mupad [B] time = 0.02, size = 56, normalized size = 0.85

$$\frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^8}{8}+\frac{10\,a^3\,b^2\,x^9}{9}+a^2\,b^3\,x^{10}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^5,x)
```

```
[Out] (a^5*x^7)/7 + (b^5*x^12)/12 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^11)/11 + (10*a^3
*b^2*x^9)/9 + a^2*b^3*x^10
```

sympy [A] time = 0.09, size = 63, normalized size = 0.95

$$\frac{a^5x^7}{7}+\frac{5a^4bx^8}{8}+\frac{10a^3b^2x^9}{9}+a^2b^3x^{10}+\frac{5ab^4x^{11}}{11}+\frac{b^5x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**5,x)
```

```
[Out] a**5*x**7/7 + 5*a**4*b*x**8/8 + 10*a**3*b**2*x**9/9 + a**2*b**3*x**10 + 5*a
*b**4*x**11/11 + b**5*x**12/12
```

3.78 $\int x^5(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10}{9}a^2b^3x^9+\frac{5}{4}a^3b^2x^8+\frac{5}{7}a^4bx^7+\frac{a^5x^6}{6}+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^5\,dx&=\int\left(a^5x^5+5a^4bx^6+10a^3b^2x^7+10a^2b^3x^8+5ab^4x^9+b^5x^{10}\right)dx\\&=\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^6}{6}+\frac{5}{7}a^4bx^7+\frac{5}{4}a^3b^2x^8+\frac{10}{9}a^2b^3x^9+\frac{1}{2}ab^4x^{10}+\frac{b^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^5,x]
```

```
[Out] (a^5*x^6)/6 + (5*a^4*b*x^7)/7 + (5*a^3*b^2*x^8)/4 + (10*a^2*b^3*x^9)/9 + (a
*b^4*x^10)/2 + (b^5*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^5, x]
```

fricas [A] time = 1.29, size = 57, normalized size = 0.83

$$\frac{1}{11}x^{11}b^5+\frac{1}{2}x^{10}b^4a+\frac{10}{9}x^9b^3a^2+\frac{5}{4}x^8b^2a^3+\frac{5}{7}x^7ba^4+\frac{1}{6}x^6a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^5 + 1/2*x^10*b^4*a + 10/9*x^9*b^3*a^2 + 5/4*x^8*b^2*a^3 + 5/7*x
^7*b*a^4 + 1/6*x^6*a^5
```

giac [A] time = 1.20, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{7}\,a^4bx^7 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{11}b^5x^{11} + \frac{1}{2}a\,b^4x^{10} + \frac{10}{9}a^2b^3x^9 + \frac{5}{4}a^3b^2x^8 + \frac{5}{7}a^4b\,x^7 + \frac{1}{6}a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^5,x)
```

```
[Out] 1/6*a^5*x^6+5/7*a^4*b*x^7+5/4*a^3*b^2*x^8+10/9*a^2*b^3*x^9+1/2*a*b^4*x^10+1
/11*b^5*x^11
```

maxima [A] time = 1.36, size = 57, normalized size = 0.83

$$\frac{1}{11}\,b^5x^{11} + \frac{1}{2}\,ab^4x^{10} + \frac{10}{9}\,a^2b^3x^9 + \frac{5}{4}\,a^3b^2x^8 + \frac{5}{7}\,a^4bx^7 + \frac{1}{6}\,a^5x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/11*b^5*x^11 + 1/2*a*b^4*x^10 + 10/9*a^2*b^3*x^9 + 5/4*a^3*b^2*x^8 + 5/7*a
^4*b*x^7 + 1/6*a^5*x^6
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^6}{6}+\frac{5\,a^4\,b\,x^7}{7}+\frac{5\,a^3\,b^2\,x^8}{4}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{a\,b^4\,x^{10}}{2}+\frac{b^5\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^5,x)
```

```
[Out] (a^5*x^6)/6 + (b^5*x^11)/11 + (5*a^4*b*x^7)/7 + (a*b^4*x^10)/2 + (5*a^3*b^2
*x^8)/4 + (10*a^2*b^3*x^9)/9
```

sympy [A] time = 0.08, size = 65, normalized size = 0.94

$$\frac{a^5x^6}{6} + \frac{5a^4bx^7}{7} + \frac{5a^3b^2x^8}{4} + \frac{10a^2b^3x^9}{9} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**5,x)
```

```
[Out] a**5*x**6/6 + 5*a**4*b*x**7/7 + 5*a**3*b**2*x**8/4 + 10*a**2*b**3*x**9/9 +
a*b**4*x**10/2 + b**5*x**11/11
```

3.79 $\int x^4(a+bx)^5\,dx$

Optimal. Leaf size=69

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4bx^6+\frac{a^5x^5}{5}+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^5\,dx&=\int\left(a^5x^4+5a^4bx^5+10a^3b^2x^6+10a^2b^3x^7+5ab^4x^8+b^5x^9\right)dx\\&=\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$\frac{a^5x^5}{5}+\frac{5}{6}a^4bx^6+\frac{10}{7}a^3b^2x^7+\frac{5}{4}a^2b^3x^8+\frac{5}{9}ab^4x^9+\frac{b^5x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^5,x]
```

```
[Out] (a^5*x^5)/5 + (5*a^4*b*x^6)/6 + (10*a^3*b^2*x^7)/7 + (5*a^2*b^3*x^8)/4 + (5
*a*b^4*x^9)/9 + (b^5*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^5, x]
```

fricas [A] time = 1.33, size = 57, normalized size = 0.83

$$\frac{1}{10}x^{10}b^5+\frac{5}{9}x^9b^4a+\frac{5}{4}x^8b^3a^2+\frac{10}{7}x^7b^2a^3+\frac{5}{6}x^6ba^4+\frac{1}{5}x^5a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^5 + 5/9*x^9*b^4*a + 5/4*x^8*b^3*a^2 + 10/7*x^7*b^2*a^3 + 5/6*x^
6*b*a^4 + 1/5*x^5*a^5
```

giac [A] time = 1.10, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{1}{10}b^5x^{10}+\frac{5}{9}a\,b^4x^9+\frac{5}{4}a^2b^3x^8+\frac{10}{7}a^3b^2x^7+\frac{5}{6}a^4b\,x^6+\frac{1}{5}a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^5,x)
```

```
[Out] 1/5*a^5*x^5+5/6*a^4*b*x^6+10/7*a^3*b^2*x^7+5/4*a^2*b^3*x^8+5/9*a*b^4*x^9+1/
10*b^5*x^10
```

maxima [A] time = 1.47, size = 57, normalized size = 0.83

$$\frac{1}{10}\,b^5x^{10}+\frac{5}{9}\,ab^4x^9+\frac{5}{4}\,a^2b^3x^8+\frac{10}{7}\,a^3b^2x^7+\frac{5}{6}\,a^4bx^6+\frac{1}{5}\,a^5x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/10*b^5*x^10 + 5/9*a*b^4*x^9 + 5/4*a^2*b^3*x^8 + 10/7*a^3*b^2*x^7 + 5/6*a^
4*b*x^6 + 1/5*a^5*x^5
```

mupad [B] time = 0.02, size = 57, normalized size = 0.83

$$\frac{a^5\,x^5}{5}+\frac{5\,a^4\,b\,x^6}{6}+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{5\,a^2\,b^3\,x^8}{4}+\frac{5\,a\,b^4\,x^9}{9}+\frac{b^5\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^5,x)
```

```
[Out] (a^5*x^5)/5 + (b^5*x^10)/10 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^9)/9 + (10*a^3*b
^2*x^7)/7 + (5*a^2*b^3*x^8)/4
```

sympy [A] time = 0.08, size = 66, normalized size = 0.96

$$\frac{a^5x^5}{5}+\frac{5a^4bx^6}{6}+\frac{10a^3b^2x^7}{7}+\frac{5a^2b^3x^8}{4}+\frac{5ab^4x^9}{9}+\frac{b^5x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**5,x)
```

```
[Out] a**5*x**5/5 + 5*a**4*b*x**6/6 + 10*a**3*b**2*x**7/7 + 5*a**2*b**3*x**8/4 +
5*a*b**4*x**9/9 + b**5*x**10/10
```

3.80 $\int x^3(a+bx)^5\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^7}{7b^4}-\frac{a^3(a+bx)^6}{6b^4}+\frac{(a+bx)^9}{9b^4}-\frac{3a(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^5,x]
```

```
[Out] -(a^3*(a + b*x)^6)/(6*b^4) + (3*a^2*(a + b*x)^7)/(7*b^4) - (3*a*(a + b*x)^8
)/(8*b^4) + (a + b*x)^9/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^5\,dx &= \int\left(-\frac{a^3(a+bx)^5}{b^3}+\frac{3a^2(a+bx)^6}{b^3}-\frac{3a(a+bx)^7}{b^3}+\frac{(a+bx)^8}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^6}{6b^4}+\frac{3a^2(a+bx)^7}{7b^4}-\frac{3a(a+bx)^8}{8b^4}+\frac{(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 66, normalized size = 1.03

$$\frac{a^5x^4}{4}+a^4bx^5+\frac{5}{3}a^3b^2x^6+\frac{10}{7}a^2b^3x^7+\frac{5}{8}ab^4x^8+\frac{b^5x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^5,x]
```

```
[Out] (a^5*x^4)/4 + a^4*b*x^5 + (5*a^3*b^2*x^6)/3 + (10*a^2*b^3*x^7)/7 + (5*a*b^4
*x^8)/8 + (b^5*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^5, x]
```

fricas [A] time = 1.28, size = 56, normalized size = 0.88

$$\frac{1}{9}x^9b^5 + \frac{5}{8}x^8b^4a + \frac{10}{7}x^7b^3a^2 + \frac{5}{3}x^6b^2a^3 + x^5ba^4 + \frac{1}{4}x^4a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^5 + 5/8*x^8*b^4*a + 10/7*x^7*b^3*a^2 + 5/3*x^6*b^2*a^3 + x^5*b*a^
4 + 1/4*x^4*a^5
```

giac [A] time = 1.05, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9 + \frac{5}{8}\,ab^4x^8 + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{3}\,a^3b^2x^6 + a^4bx^5 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{1}{9}b^5x^9 + \frac{5}{8}a\,b^4x^8 + \frac{10}{7}a^2b^3x^7 + \frac{5}{3}a^3b^2x^6 + a^4b\,x^5 + \frac{1}{4}a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^5,x)
```

```
[Out] 1/9*b^5*x^9+5/8*a*b^4*x^8+10/7*a^2*b^3*x^7+5/3*a^3*b^2*x^6+a^4*b*x^5+1/4*a^
5*x^4
```

maxima [A] time = 1.40, size = 56, normalized size = 0.88

$$\frac{1}{9}\,b^5x^9 + \frac{5}{8}\,ab^4x^8 + \frac{10}{7}\,a^2b^3x^7 + \frac{5}{3}\,a^3b^2x^6 + a^4bx^5 + \frac{1}{4}\,a^5x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*x^9 + 5/8*a*b^4*x^8 + 10/7*a^2*b^3*x^7 + 5/3*a^3*b^2*x^6 + a^4*b*x^
5 + 1/4*a^5*x^4
```

mupad [B] time = 0.02, size = 56, normalized size = 0.88

$$\frac{a^5\,x^4}{4} + a^4\,b\,x^5 + \frac{5\,a^3\,b^2\,x^6}{3} + \frac{10\,a^2\,b^3\,x^7}{7} + \frac{5\,a\,b^4\,x^8}{8} + \frac{b^5\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^5,x)
```

```
[Out] (a^5*x^4)/4 + (b^5*x^9)/9 + a^4*b*x^5 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*x^6)/3
 + (10*a^2*b^3*x^7)/7
```

sympy [A] time = 0.10, size = 63, normalized size = 0.98

$$\frac{a^5x^4}{4} + a^4bx^5 + \frac{5a^3b^2x^6}{3} + \frac{10a^2b^3x^7}{7} + \frac{5ab^4x^8}{8} + \frac{b^5x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**5,x)
```

```
[Out] a**5*x**4/4 + a**4*b*x**5 + 5*a**3*b**2*x**6/3 + 10*a**2*b**3*x**7/7 + 5*a*
b**4*x**8/8 + b**5*x**9/9
```

3.81 $\int x^2(a+bx)^5\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^6}{6b^3}+\frac{(a+bx)^8}{8b^3}-\frac{2a(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^5,x]
```

```
[Out] (a^2*(a + b*x)^6)/(6*b^3) - (2*a*(a + b*x)^7)/(7*b^3) + (a + b*x)^8/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^5\,dx&=\int\left(\frac{a^2(a+bx)^5}{b^2}-\frac{2a(a+bx)^6}{b^2}+\frac{(a+bx)^7}{b^2}\right)dx\\&=\frac{a^2(a+bx)^6}{6b^3}-\frac{2a(a+bx)^7}{7b^3}+\frac{(a+bx)^8}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.43

$$\frac{a^5x^3}{3}+\frac{5}{4}a^4bx^4+2a^3b^2x^5+\frac{5}{3}a^2b^3x^6+\frac{5}{7}ab^4x^7+\frac{b^5x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^5,x]
```

```
[Out] (a^5*x^3)/3 + (5*a^4*b*x^4)/4 + 2*a^3*b^2*x^5 + (5*a^2*b^3*x^6)/3 + (5*a*b^
4*x^7)/7 + (b^5*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^5, x]
```

fricas [A] time = 1.41, size = 57, normalized size = 1.21

$$\frac{1}{8}x^8b^5+\frac{5}{7}x^7b^4a+\frac{5}{3}x^6b^3a^2+2x^5b^2a^3+\frac{5}{4}x^4ba^4+\frac{1}{3}x^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^5 + 5/7*x^7*b^4*a + 5/3*x^6*b^3*a^2 + 2*x^5*b^2*a^3 + 5/4*x^4*b*a
^4 + 1/3*x^3*a^5
```

giac [A] time = 1.71, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

maple [A] time = 0.00, size = 58, normalized size = 1.23

$$\frac{1}{8}b^5x^8+\frac{5}{7}a\,b^4x^7+\frac{5}{3}a^2b^3x^6+2a^3b^2x^5+\frac{5}{4}a^4b\,x^4+\frac{1}{3}a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^5,x)
```

```
[Out] 1/8*b^5*x^8+5/7*a*b^4*x^7+5/3*a^2*b^3*x^6+2*a^3*b^2*x^5+5/4*a^4*b*x^4+1/3*a
^5*x^3
```

maxima [A] time = 1.32, size = 57, normalized size = 1.21

$$\frac{1}{8}\,b^5x^8+\frac{5}{7}\,ab^4x^7+\frac{5}{3}\,a^2b^3x^6+2\,a^3b^2x^5+\frac{5}{4}\,a^4bx^4+\frac{1}{3}\,a^5x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/8*b^5*x^8 + 5/7*a*b^4*x^7 + 5/3*a^2*b^3*x^6 + 2*a^3*b^2*x^5 + 5/4*a^4*b*x
^4 + 1/3*a^5*x^3
```

mupad [B] time = 0.02, size = 57, normalized size = 1.21

$$\frac{a^5\,x^3}{3}+\frac{5\,a^4\,b\,x^4}{4}+2\,a^3\,b^2\,x^5+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^7}{7}+\frac{b^5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^5,x)
```

```
[Out] (a^5*x^3)/3 + (b^5*x^8)/8 + (5*a^4*b*x^4)/4 + (5*a*b^4*x^7)/7 + 2*a^3*b^2*x
^5 + (5*a^2*b^3*x^6)/3
```

sympy [A] time = 0.08, size = 65, normalized size = 1.38

$$\frac{a^5x^3}{3}+\frac{5a^4bx^4}{4}+2a^3b^2x^5+\frac{5a^2b^3x^6}{3}+\frac{5ab^4x^7}{7}+\frac{b^5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**5,x)
```

```
[Out] a**5*x**3/3 + 5*a**4*b*x**4/4 + 2*a**3*b**2*x**5 + 5*a**2*b**3*x**6/3 + 5*a
*b**4*x**7/7 + b**5*x**8/8
```

3.82 $\int x(a+bx)^5\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^7}{7b^2}-\frac{a(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^5,x]
```

```
[Out] -(a*(a + b*x)^6)/(6*b^2) + (a + b*x)^7/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^5\,dx &= \int \left(-\frac{a(a+bx)^5}{b}+\frac{(a+bx)^6}{b}\right)dx\\ &= -\frac{a(a+bx)^6}{6b^2}+\frac{(a+bx)^7}{7b^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 67, normalized size = 2.23

$$\frac{a^5x^2}{2}+\frac{5}{3}a^4bx^3+\frac{5}{2}a^3b^2x^4+2a^2b^3x^5+\frac{5}{6}ab^4x^6+\frac{b^5x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^5,x]
```

```
[Out] (a^5*x^2)/2 + (5*a^4*b*x^3)/3 + (5*a^3*b^2*x^4)/2 + 2*a^2*b^3*x^5 + (5*a*b^
4*x^6)/6 + (b^5*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^5, x]
```

fricas [B] time = 0.80, size = 57, normalized size = 1.90

$$\frac{1}{7}x^7b^5+\frac{5}{6}x^6b^4a+2x^5b^3a^2+\frac{5}{2}x^4b^2a^3+\frac{5}{3}x^3ba^4+\frac{1}{2}x^2a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^5 + 5/6*x^6*b^4*a + 2*x^5*b^3*a^2 + 5/2*x^4*b^2*a^3 + 5/3*x^3*b*a
^4 + 1/2*x^2*a^5
```

giac [B] time = 0.93, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^{5}x^{7} + \frac{5}{6}\,ab^{4}x^{6} + 2\,a^{2}b^{3}x^{5} + \frac{5}{2}\,a^{3}b^{2}x^{4} + \frac{5}{3}\,a^{4}bx^{3} + \frac{1}{2}\,a^{5}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

maple [B] time = 0.00, size = 58, normalized size = 1.93

$$\frac{1}{7}b^{5}x^{7} + \frac{5}{6}a\,b^{4}x^{6} + 2a^{2}b^{3}x^{5} + \frac{5}{2}a^{3}b^{2}x^{4} + \frac{5}{3}a^{4}b\,x^{3} + \frac{1}{2}a^{5}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^5,x)
```

```
[Out] 1/7*b^5*x^7+5/6*a*b^4*x^6+2*a^2*b^3*x^5+5/2*a^3*b^2*x^4+5/3*a^4*b*x^3+1/2*a
^5*x^2
```

maxima [B] time = 1.31, size = 57, normalized size = 1.90

$$\frac{1}{7}\,b^{5}x^{7} + \frac{5}{6}\,ab^{4}x^{6} + 2\,a^{2}b^{3}x^{5} + \frac{5}{2}\,a^{3}b^{2}x^{4} + \frac{5}{3}\,a^{4}bx^{3} + \frac{1}{2}\,a^{5}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/7*b^5*x^7 + 5/6*a*b^4*x^6 + 2*a^2*b^3*x^5 + 5/2*a^3*b^2*x^4 + 5/3*a^4*b*x
^3 + 1/2*a^5*x^2
```

mupad [B] time = 0.02, size = 57, normalized size = 1.90

$$\frac{a^5\,x^2}{2}+\frac{5\,a^4\,b\,x^3}{3}+\frac{5\,a^3\,b^2\,x^4}{2}+2\,a^2\,b^3\,x^5+\frac{5\,a\,b^4\,x^6}{6}+\frac{b^5\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^5,x)
```

```
[Out] (a^5*x^2)/2 + (b^5*x^7)/7 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^6)/6 + (5*a^3*b^2*
x^4)/2 + 2*a^2*b^3*x^5
```

sympy [B] time = 0.08, size = 65, normalized size = 2.17

$$\frac{a^{5}x^{2}}{2} + \frac{5a^{4}bx^{3}}{3} + \frac{5a^{3}b^{2}x^{4}}{2} + 2a^{2}b^{3}x^{5} + \frac{5ab^{4}x^{6}}{6} + \frac{b^{5}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**5,x)
```

```
[Out] a**5*x**2/2 + 5*a**4*b*x**3/3 + 5*a**3*b**2*x**4/2 + 2*a**2*b**3*x**5 + 5*a
*b**4*x**6/6 + b**5*x**7/7
```

3.83 $\int (a+bx)^5\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^6}{6b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^5\,dx = \frac{(a+bx)^6}{6b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5,x]
```

```
[Out] (a + b*x)^6/(6*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5, x]
```

fricas [B] time = 1.21, size = 53, normalized size = 3.79

$$\frac{1}{6}x^6b^5 + x^5b^4a + \frac{5}{2}x^4b^3a^2 + \frac{10}{3}x^3b^2a^3 + \frac{5}{2}x^2ba^4 + xa^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="fricas")
```

[Out] 1/6*x^6*b^5 + x^5*b^4*a + 5/2*x^4*b^3*a^2 + 10/3*x^3*b^2*a^3 + 5/2*x^2*b*a^4 + x*a^5

giac [A] time = 1.20, size = 12, normalized size = 0.86

$$\frac{(bx+a)^6}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(b*x + a)^6/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^6}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5,x)
```

```
[Out] 1/6*(b*x+a)^6/b
```

maxima [B] time = 1.37, size = 53, normalized size = 3.79

$$\frac{1}{6}\,b^5x^6 + ab^4x^5 + \frac{5}{2}\,a^2b^3x^4 + \frac{10}{3}\,a^3b^2x^3 + \frac{5}{2}\,a^4bx^2 + a^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^6 + a*b^4*x^5 + 5/2*a^2*b^3*x^4 + 10/3*a^3*b^2*x^3 + 5/2*a^4*b*x^2 + a^5*x
```

mupad [B] time = 0.02, size = 53, normalized size = 3.79

$$a^5\,x + \frac{5\,a^4\,b\,x^2}{2} + \frac{10\,a^3\,b^2\,x^3}{3} + \frac{5\,a^2\,b^3\,x^4}{2} + a\,b^4\,x^5 + \frac{b^5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5,x)
```

```
[Out] a^5*x + (b^5*x^6)/6 + (5*a^4*b*x^2)/2 + a*b^4*x^5 + (10*a^3*b^2*x^3)/3 + (5*a^2*b^3*x^4)/2
```

sympy [B] time = 0.08, size = 60, normalized size = 4.29

$$a^5x + \frac{5a^4bx^2}{2} + \frac{10a^3b^2x^3}{3} + \frac{5a^2b^3x^4}{2} + ab^4x^5 + \frac{b^5x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5,x)
```

```
[Out] a**5*x + 5*a**4*b*x**2/2 + 10*a**3*b**2*x**3/3 + 5*a**2*b**3*x**4/2 + a*b**4*x**5 + b**5*x**6/6
```

3.84 $\int \frac{(a+bx)^5}{x}\,dx$

Optimal. Leaf size=59

$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+5a^4bx+a^5\log(x)+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x,x]

[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x}\,dx &= \int \left(5a^4b+\frac{a^5}{x}+10a^3b^2x+10a^2b^3x^2+5ab^4x^3+b^5x^4\right)dx\\ &= 5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}+a^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 59, normalized size = 1.00

$$a^5\log(x)+5a^4bx+5a^3b^2x^2+\frac{10}{3}a^2b^3x^3+\frac{5}{4}ab^4x^4+\frac{b^5x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x,x]

[Out] 5*a^4*b*x + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^3)/3 + (5*a*b^4*x^4)/4 + (b^5*x^5
)/5 + a^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x,x]

[Out] IntegrateAlgebraic[(a + b*x)^5/x, x]
```

fricas [A] time = 1.44, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

giac [A] time = 1.12, size = 54, normalized size = 0.92

$$\frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(abs(x))
```

maple [A] time = 0.00, size = 54, normalized size = 0.92

$$\frac{b^5x^5}{5} + \frac{5a\,b^4x^4}{4} + \frac{10a^2b^3x^3}{3} + 5a^3b^2x^2 + a^5\ln(x) + 5a^4bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x,x)
```

```
[Out] 5*a^4*b*x+5*a^3*b^2*x^2+10/3*a^2*b^3*x^3+5/4*a*b^4*x^4+1/5*b^5*x^5+a^5*ln(x
)
```

maxima [A] time = 1.37, size = 53, normalized size = 0.90

$$\frac{1}{5}\,b^5x^5 + \frac{5}{4}\,ab^4x^4 + \frac{10}{3}\,a^2b^3x^3 + 5\,a^3b^2x^2 + 5\,a^4bx + a^5\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/5*b^5*x^5 + 5/4*a*b^4*x^4 + 10/3*a^2*b^3*x^3 + 5*a^3*b^2*x^2 + 5*a^4*b*x
+ a^5*log(x)
```

mupad [B] time = 0.03, size = 53, normalized size = 0.90

$$a^5\,\ln(x) + \frac{b^5\,x^5}{5} + \frac{5\,a\,b^4\,x^4}{4} + 5\,a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{3} + 5\,a^4\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x,x)
```

```
[Out] a^5*log(x) + (b^5*x^5)/5 + (5*a*b^4*x^4)/4 + 5*a^3*b^2*x^2 + (10*a^2*b^3*x^
3)/3 + 5*a^4*b*x
```

sympy [A] time = 0.15, size = 60, normalized size = 1.02

$$a^5\log\left(x\right) + 5a^4bx + 5a^3b^2x^2 + \frac{10a^2b^3x^3}{3} + \frac{5ab^4x^4}{4} + \frac{b^5x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x,x)
```

```
[Out] a**5*log(x) + 5*a**4*b*x + 5*a**3*b**2*x**2 + 10*a**2*b**3*x**3/3 + 5*a*b**
4*x**4/4 + b**5*x**5/5
```

3.85 $\int \frac{(a+bx)^5}{x^2} \, dx$

Optimal. Leaf size=58

$$-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$5a^2b^3x^2+10a^3b^2x+5a^4b\log(x)-\frac{a^5}{x}+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^2}\,dx &= \int \left(10a^3b^2+\frac{a^5}{x^2}+\frac{5a^4b}{x}+10a^2b^3x+5ab^4x^2+b^5x^3\right)dx\\ &= -\frac{a^5}{x}+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}+5a^4b\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.00

$$-\frac{a^5}{x}+5a^4b\log(x)+10a^3b^2x+5a^2b^3x^2+\frac{5}{3}ab^4x^3+\frac{b^5x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^2,x]
```

```
[Out] -(a^5/x) + 10*a^3*b^2*x + 5*a^2*b^3*x^2 + (5*a*b^4*x^3)/3 + (b^5*x^4)/4 + 5
*a^4*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^2, x]
```

fricas [A] time = 0.97, size = 59, normalized size = 1.02

$$\frac{3\,b^{5}x^{5} + 20\,ab^{4}x^{4} + 60\,a^{2}b^{3}x^{3} + 120\,a^{3}b^{2}x^{2} + 60\,a^{4}bx\log(x) - 12\,a^{5}}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 + 20*a*b^4*x^4 + 60*a^2*b^3*x^3 + 120*a^3*b^2*x^2 + 60*a^4*
b*x*log(x) - 12*a^5)/x
```

giac [A] time = 1.28, size = 55, normalized size = 0.95

$$\frac{1}{4}\,b^{5}x^{4} + \frac{5}{3}\,ab^{4}x^{3} + 5\,a^{2}b^{3}x^{2} + 10\,a^{3}b^{2}x + 5\,a^{4}b\log\left(|x|\right) - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(ab
s(x)) - a^5/x
```

maple [A] time = 0.01, size = 55, normalized size = 0.95

$$\frac{b^{5}x^{4}}{4} + \frac{5a\,b^{4}x^{3}}{3} + 5a^{2}b^{3}x^{2} + 5a^{4}b\ln(x) + 10a^{3}b^{2}x - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^2,x)
```

```
[Out] -a^5/x+10*a^3*b^2*x+5*a^2*b^3*x^2+5/3*a*b^4*x^3+1/4*b^5*x^4+5*a^4*b*ln(x)
```

maxima [A] time = 1.40, size = 54, normalized size = 0.93

$$\frac{1}{4}\,b^{5}x^{4} + \frac{5}{3}\,ab^{4}x^{3} + 5\,a^{2}b^{3}x^{2} + 10\,a^{3}b^{2}x + 5\,a^{4}b\log(x) - \frac{a^{5}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^5*x^4 + 5/3*a*b^4*x^3 + 5*a^2*b^3*x^2 + 10*a^3*b^2*x + 5*a^4*b*log(x)
 - a^5/x
```

mupad [B] time = 0.03, size = 54, normalized size = 0.93

$$\frac{b^5\,x^4}{4} - \frac{a^5}{x} + 10\,a^3\,b^2\,x + \frac{5\,a\,b^4\,x^3}{3} + 5\,a^4\,b\,\ln(x) + 5\,a^2\,b^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^2,x)
```

```
[Out] (b^5*x^4)/4 - a^5/x + 10*a^3*b^2*x + (5*a*b^4*x^3)/3 + 5*a^4*b*log(x) + 5*a
^2*b^3*x^2
```

sympy [A] time = 0.18, size = 56, normalized size = 0.97

$$-\frac{a^{5}}{x} + 5a^{4}b\log\left(x\right) + 10a^{3}b^{2}x + 5a^{2}b^{3}x^{2} + \frac{5ab^{4}x^{3}}{3} + \frac{b^{5}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**2,x)
```

```
[Out] -a**5/x + 5*a**4*b*log(x) + 10*a**3*b**2*x + 5*a**2*b**3*x**2 + 5*a*b**4*x*
*3/3 + b**5*x**4/4
```

3.86 $\int \frac{(a+bx)^5}{x^3}\,dx$

Optimal. Leaf size=60

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$10a^2b^3x+10a^3b^2\log(x)-\frac{5a^4b}{x}-\frac{a^5}{2x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^3,x]
```

```
[Out] -a^5/(2*x^2) - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^3}\,dx &= \int \left(10a^2b^3+\frac{a^5}{x^3}+\frac{5a^4b}{x^2}+\frac{10a^3b^2}{x}+5ab^4x+b^5x^2\right)dx\\ &= -\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}+10a^3b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{2x^2}-\frac{5a^4b}{x}+10a^3b^2\log(x)+10a^2b^3x+\frac{5}{2}ab^4x^2+\frac{b^5x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^3,x]
```

```
[Out] -1/2*a^5/x^2 - (5*a^4*b)/x + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + (b^5*x^3)/3 +
 10*a^3*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^3, x]
```

fricas [A] time = 0.75, size = 59, normalized size = 0.98

$$\frac{2\,b^5x^5+15\,ab^4x^4+60\,a^2b^3x^3+60\,a^3b^2x^2\log(x)-30\,a^4bx-3\,a^5}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 + 15*a*b^4*x^4 + 60*a^2*b^3*x^3 + 60*a^3*b^2*x^2*log(x) - 30
*a^4*b*x - 3*a^5)/x^2
```

giac [A] time = 1.19, size = 54, normalized size = 0.90

$$\frac{1}{3}\,b^5x^3+\frac{5}{2}\,ab^4x^2+10\,a^2b^3x+10\,a^3b^2\log\left(|x|\right)-\frac{10\,a^4bx+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(abs(x)) - 1/2*(
10*a^4*b*x + a^5)/x^2
```

maple [A] time = 0.00, size = 55, normalized size = 0.92

$$\frac{b^5x^3}{3}+\frac{5a\,b^4x^2}{2}+10a^3b^2\ln(x)+10a^2b^3x-\frac{5a^4b}{x}-\frac{a^5}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^3,x)
```

```
[Out] -1/2*a^5/x^2-5*a^4*b/x+10*a^2*b^3*x+5/2*a*b^4*x^2+1/3*b^5*x^3+10*a^3*b^2*ln
(x)
```

maxima [A] time = 1.36, size = 53, normalized size = 0.88

$$\frac{1}{3}\,b^5x^3+\frac{5}{2}\,ab^4x^2+10\,a^2b^3x+10\,a^3b^2\log(x)-\frac{10\,a^4bx+a^5}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^5*x^3 + 5/2*a*b^4*x^2 + 10*a^2*b^3*x + 10*a^3*b^2*log(x) - 1/2*(10*a^
4*b*x + a^5)/x^2
```

mupad [B] time = 0.03, size = 55, normalized size = 0.92

$$\frac{b^5\,x^3}{3}-\frac{\frac{a^5}{2}+5\,b\,x\,a^4}{x^2}+10\,a^2\,b^3\,x+\frac{5\,a\,b^4\,x^2}{2}+10\,a^3\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^3,x)
```

```
[Out] (b^5*x^3)/3 - (a^5/2 + 5*a^4*b*x)/x^2 + 10*a^2*b^3*x + (5*a*b^4*x^2)/2 + 10
*a^3*b^2*log(x)
```

sympy [A] time = 0.20, size = 60, normalized size = 1.00

$$10a^3b^2\log\left(x\right)+10a^2b^3x+\frac{5ab^4x^2}{2}+\frac{b^5x^3}{3}+\frac{-a^5-10a^4bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**3,x)
```

```
[Out] 10*a**3*b**2*log(x) + 10*a**2*b**3*x + 5*a*b**4*x**2/2 + b**5*x**3/3 + (-a*
*5 - 10*a**4*b*x)/(2*x**2)
```

3.87 $\int \frac{(a+bx)^5}{x^4}\,dx$

Optimal. Leaf size=60

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{x}+10a^2b^3\log(x)-\frac{5a^4b}{2x^2}-\frac{a^5}{3x^3}+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^4,x]
```

```
[Out] -a^5/(3*x^3) - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^4}\,dx &= \int \left(5ab^4+\frac{a^5}{x^4}+\frac{5a^4b}{x^3}+\frac{10a^3b^2}{x^2}+\frac{10a^2b^3}{x}+b^5x\right)dx\\ &= -\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+5ab^4x+\frac{b^5x^2}{2}+10a^2b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 60, normalized size = 1.00

$$-\frac{a^5}{3x^3}-\frac{5a^4b}{2x^2}-\frac{10a^3b^2}{x}+10a^2b^3\log(x)+5ab^4x+\frac{b^5x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^4,x]
```

```
[Out] -1/3*a^5/x^3 - (5*a^4*b)/(2*x^2) - (10*a^3*b^2)/x + 5*a*b^4*x + (b^5*x^2)/2
 + 10*a^2*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^4, x]
```

fricas [A] time = 1.73, size = 59, normalized size = 0.98

$$\frac{3\,b^5x^5 + 30\,ab^4x^4 + 60\,a^2b^3x^3\log(x) - 60\,a^3b^2x^2 - 15\,a^4bx - 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 + 30*a*b^4*x^4 + 60*a^2*b^3*x^3*log(x) - 60*a^3*b^2*x^2 - 15
*a^4*b*x - 2*a^5)/x^3
```

giac [A] time = 1.00, size = 56, normalized size = 0.93

$$\frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log\left(|x|\right) - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="giac")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(abs(x)) - 1/6*(60*a^3*b^2*x^2 + 15
*a^4*b*x + 2*a^5)/x^3
```

maple [A] time = 0.01, size = 55, normalized size = 0.92

$$\frac{b^5x^2}{2} + 10a^2b^3\ln(x) + 5a\,b^4x - \frac{10a^3b^2}{x} - \frac{5a^4b}{2x^2} - \frac{a^5}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^4,x)
```

```
[Out] -1/3*a^5/x^3-5/2*a^4*b/x^2-10*a^3*b^2/x+5*a*b^4*x+1/2*b^5*x^2+10*a^2*b^3*ln
(x)
```

maxima [A] time = 1.41, size = 55, normalized size = 0.92

$$\frac{1}{2}\,b^5x^2 + 5\,ab^4x + 10\,a^2b^3\log(x) - \frac{60\,a^3b^2x^2 + 15\,a^4bx + 2\,a^5}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*b^5*x^2 + 5*a*b^4*x + 10*a^2*b^3*log(x) - 1/6*(60*a^3*b^2*x^2 + 15*a^4*
b*x + 2*a^5)/x^3
```

mupad [B] time = 0.04, size = 55, normalized size = 0.92

$$\frac{b^5\,x^2}{2} - \frac{\frac{a^5}{3} + \frac{5\,a^4\,b\,x}{2} + 10\,a^3\,b^2\,x^2}{x^3} + 10\,a^2\,b^3\,\ln(x) + 5\,a\,b^4\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^4,x)
```

```
[Out] (b^5*x^2)/2 - (a^5/3 + 10*a^3*b^2*x^2 + (5*a^4*b*x)/2)/x^3 + 10*a^2*b^3*log
(x) + 5*a*b^4*x
```

sympy [A] time = 0.26, size = 60, normalized size = 1.00

$$10a^2b^3\log\left(x\right) + 5ab^4x + \frac{b^5x^2}{2} + \frac{-2a^5 - 15a^4bx - 60a^3b^2x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**4,x)
```

```
[Out] 10*a**2*b**3*log(x) + 5*a*b**4*x + b**5*x**2/2 + (-2*a**5 - 15*a**4*b*x - 6
0*a**3*b**2*x**2)/(6*x**3)
```

3.88 $\int \frac{(a+bx)^5}{x^5} dx$

Optimal. Leaf size=57

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}-\frac{5a^4b}{3x^3}-\frac{a^5}{4x^4}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^5,x]
```

```
[Out] -a^5/(4*x^4) - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^5}\,dx &= \int \left(b^5+\frac{a^5}{x^5}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^3}+\frac{10a^2b^3}{x^2}+\frac{5ab^4}{x}\right)dx\\ &= -\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+b^5x+5ab^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 57, normalized size = 1.00

$$-\frac{a^5}{4x^4}-\frac{5a^4b}{3x^3}-\frac{5a^3b^2}{x^2}-\frac{10a^2b^3}{x}+5ab^4\log(x)+b^5x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^5,x]
```

```
[Out] -1/4*a^5/x^4 - (5*a^4*b)/(3*x^3) - (5*a^3*b^2)/x^2 - (10*a^2*b^3)/x + b^5*x
 + 5*a*b^4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^5, x]
```

fricas [A] time = 0.81, size = 59, normalized size = 1.04

$$\frac{12\,b^5x^5 + 60\,ab^4x^4\log(x) - 120\,a^2b^3x^3 - 60\,a^3b^2x^2 - 20\,a^4bx - 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*b^5*x^5 + 60*a*b^4*x^4*log(x) - 120*a^2*b^3*x^3 - 60*a^3*b^2*x^2 -
 20*a^4*b*x - 3*a^5)/x^4
```

giac [A] time = 1.38, size = 55, normalized size = 0.96

$$b^5x + 5\,ab^4\log(|x|) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="giac")
```

```
[Out] b^5*x + 5*a*b^4*log(abs(x)) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a
^4*b*x + 3*a^5)/x^4
```

maple [A] time = 0.01, size = 54, normalized size = 0.95

$$5a\,b^4\ln(x) + b^5x - \frac{10a^2b^3}{x} - \frac{5a^3b^2}{x^2} - \frac{5a^4b}{3x^3} - \frac{a^5}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^5,x)
```

```
[Out] -1/4*a^5/x^4-5/3*a^4*b/x^3-5*a^3*b^2/x^2-10*a^2*b^3/x+b^5*x+5*a*b^4*ln(x)
```

maxima [A] time = 1.37, size = 54, normalized size = 0.95

$$b^5x + 5\,ab^4\log(x) - \frac{120\,a^2b^3x^3 + 60\,a^3b^2x^2 + 20\,a^4bx + 3\,a^5}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^5,x, algorithm="maxima")
```

```
[Out] b^5*x + 5*a*b^4*log(x) - 1/12*(120*a^2*b^3*x^3 + 60*a^3*b^2*x^2 + 20*a^4*b*
x + 3*a^5)/x^4
```

mupad [B] time = 0.08, size = 54, normalized size = 0.95

$$b^5\,x - \frac{\frac{a^5}{4} + \frac{5\,a^4\,b\,x}{3} + 5\,a^3\,b^2\,x^2 + 10\,a^2\,b^3\,x^3}{x^4} + 5\,a\,b^4\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^5,x)
```

```
[Out] b^5*x - (a^5/4 + 5*a^3*b^2*x^2 + 10*a^2*b^3*x^3 + (5*a^4*b*x)/3)/x^4 + 5*a*
b^4*log(x)
```

sympy [A] time = 0.29, size = 58, normalized size = 1.02

$$5ab^4\log(x) + b^5x + \frac{-3a^5 - 20a^4bx - 60a^3b^2x^2 - 120a^2b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**5,x)
```

```
[Out] 5*a*b**4*log(x) + b**5*x + (-3*a**5 - 20*a**4*b*x - 60*a**3*b**2*x**2 - 120
*a**2*b**3*x**3)/(12*x**4)
```

3.89 $\int \frac{(a+bx)^5}{x^6}\,dx$

Optimal. Leaf size=61

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5a^4b}{4x^4}-\frac{a^5}{5x^5}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^6,x]
```

```
[Out] -a^5/(5*x^5) - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^6}\,dx &= \int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^5}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^3}+\frac{5ab^4}{x^2}+\frac{b^5}{x}\right)dx\\ &= -\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 61, normalized size = 1.00

$$-\frac{a^5}{5x^5}-\frac{5a^4b}{4x^4}-\frac{10a^3b^2}{3x^3}-\frac{5a^2b^3}{x^2}-\frac{5ab^4}{x}+b^5\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^6,x]
```

```
[Out] -1/5*a^5/x^5 - (5*a^4*b)/(4*x^4) - (10*a^3*b^2)/(3*x^3) - (5*a^2*b^3)/x^2 -
 (5*a*b^4)/x + b^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^6, x]
```

fricas [A] time = 1.93, size = 59, normalized size = 0.97

$$\frac{60\,b^5x^5\log(x)-300\,ab^4x^4-300\,a^2b^3x^3-200\,a^3b^2x^2-75\,a^4bx-12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^5*x^5*log(x) - 300*a*b^4*x^4 - 300*a^2*b^3*x^3 - 200*a^3*b^2*x^2
 - 75*a^4*b*x - 12*a^5)/x^5
```

giac [A] time = 1.37, size = 57, normalized size = 0.93

$$b^5\log(|x|)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="giac")
```

```
[Out] b^5*log(abs(x)) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 +
 75*a^4*b*x + 12*a^5)/x^5
```

maple [A] time = 0.01, size = 56, normalized size = 0.92

$$b^5\ln(x)-\frac{5a\,b^4}{x}-\frac{5a^2b^3}{x^2}-\frac{10a^3b^2}{3x^3}-\frac{5a^4b}{4x^4}-\frac{a^5}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^6,x)
```

```
[Out] -1/5*a^5/x^5-5/4*a^4*b/x^4-10/3*a^3*b^2/x^3-5*a^2*b^3/x^2-5*a*b^4/x+b^5*ln(
x)
```

maxima [A] time = 1.35, size = 56, normalized size = 0.92

$$b^5\log(x)-\frac{300\,ab^4x^4+300\,a^2b^3x^3+200\,a^3b^2x^2+75\,a^4bx+12\,a^5}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^6,x, algorithm="maxima")
```

```
[Out] b^5*log(x) - 1/60*(300*a*b^4*x^4 + 300*a^2*b^3*x^3 + 200*a^3*b^2*x^2 + 75*a
^4*b*x + 12*a^5)/x^5
```

mupad [B] time = 0.04, size = 56, normalized size = 0.92

$$b^5\,\ln(x)-\frac{\frac{a^5}{5}+\frac{5\,a^4\,b\,x}{4}+\frac{10\,a^3\,b^2\,x^2}{3}+5\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4}{x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^6,x)
```

```
[Out] b^5*log(x) - (a^5/5 + 5*a*b^4*x^4 + (10*a^3*b^2*x^2)/3 + 5*a^2*b^3*x^3 + (5
*a^4*b*x)/4)/x^5
```

sympy [A] time = 0.36, size = 60, normalized size = 0.98

$$b^5\log(x)+\frac{-12a^5-75a^4bx-200a^3b^2x^2-300a^2b^3x^3-300ab^4x^4}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**6,x)
```

```
[Out] b**5*log(x) + (-12*a**5 - 75*a**4*b*x - 200*a**3*b**2*x**2 - 300*a**2*b**3*
x**3 - 300*a*b**4*x**4)/(60*x**5)
```

3.90 $\int \frac{(a+bx)^5}{x^7}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^6}{6ax^6}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^6}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^7,x]
```

```
[Out] -(a + b*x)^6/(6*a*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^5}{x^7}\,dx = -\frac{(a+bx)^6}{6ax^6}$$

Mathematica [B] time = 0.00, size = 65, normalized size = 3.82

$$-\frac{a^5}{6x^6}-\frac{a^4b}{x^5}-\frac{5a^3b^2}{2x^4}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{2x^2}-\frac{b^5}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^7,x]
```

```
[Out] -1/6*a^5/x^6 - (a^4*b)/x^5 - (5*a^3*b^2)/(2*x^4) - (10*a^2*b^3)/(3*x^3) - (
5*a*b^4)/(2*x^2) - b^5/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^7, x]
```

fricas [B] time = 1.14, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5+15\,ab^4x^4+20\,a^2b^3x^3+15\,a^3b^2x^2+6\,a^4bx+a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

giac [**B**] time = 0.94, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

maple [**B**] time = 0.00, size = 58, normalized size = 3.41

$$-\frac{b^5}{x} - \frac{5a\,b^4}{2x^2} - \frac{10a^2b^3}{3x^3} - \frac{5a^3b^2}{2x^4} - \frac{a^4b}{x^5} - \frac{a^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^7,x)
```

```
[Out] -1/6*a^5/x^6-a^4*b/x^5-5/2*a^3*b^2/x^4-b^5/x-10/3*a^2*b^3/x^3-5/2*a*b^4/x^2
```

maxima [**B**] time = 1.30, size = 55, normalized size = 3.24

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/x^6
```

mupad [**B**] time = 0.04, size = 55, normalized size = 3.24

$$-\frac{\frac{a^5}{6} + a^4\,b\,x + \frac{5\,a^3\,b^2\,x^2}{2} + \frac{10\,a^2\,b^3\,x^3}{3} + \frac{5\,a\,b^4\,x^4}{2} + b^5\,x^5}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^7,x)
```

```
[Out] -(a^5/6 + b^5*x^5 + (5*a*b^4*x^4)/2 + (5*a^3*b^2*x^2)/2 + (10*a^2*b^3*x^3)/
3 + a^4*b*x)/x^6
```

sympy [**B**] time = 0.37, size = 60, normalized size = 3.53

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*x**6)
```

3.91 $\int \frac{(a+bx)^5}{x^8} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42a^2x^6} - \frac{(a+bx)^6}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^8,x]
```

```
[Out] -(a + b*x)^6/(7*a*x^7) + (b*(a + b*x)^6)/(42*a^2*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{x^8} dx &= -\frac{(a+bx)^6}{7ax^7} - \frac{b \int \frac{(a+bx)^5}{x^7} dx}{7a} \\ &= -\frac{(a+bx)^6}{7ax^7} + \frac{b(a+bx)^6}{42a^2x^6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.86

$$-\frac{a^5}{7x^7} - \frac{5a^4b}{6x^6} - \frac{2a^3b^2}{x^5} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{3x^3} - \frac{b^5}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^8,x]
```

```
[Out] -1/7*a^5/x^7 - (5*a^4*b)/(6*x^6) - (2*a^3*b^2)/x^5 - (5*a^2*b^3)/(2*x^4) -
(5*a*b^4)/(3*x^3) - b^5/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^8, x]
```

fricas [A] time = 1.37, size = 57, normalized size = 1.58

$$-\frac{21\,b^{5}x^{5}+70\,ab^{4}x^{4}+105\,a^{2}b^{3}x^{3}+84\,a^{3}b^{2}x^{2}+35\,a^{4}bx+6\,a^{5}}{42\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

giac [A] time = 1.12, size = 57, normalized size = 1.58

$$-\frac{21\,b^{5}x^{5}+70\,ab^{4}x^{4}+105\,a^{2}b^{3}x^{3}+84\,a^{3}b^{2}x^{2}+35\,a^{4}bx+6\,a^{5}}{42\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

maple [A] time = 0.01, size = 58, normalized size = 1.61

$$-\frac{b^{5}}{2x^{2}}-\frac{5a\,b^{4}}{3x^{3}}-\frac{5a^{2}b^{3}}{2x^{4}}-\frac{2a^{3}b^{2}}{x^{5}}-\frac{5a^{4}b}{6x^{6}}-\frac{a^{5}}{7x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^8,x)
```

```
[Out] -2*a^3*b^2/x^5-1/7*a^5/x^7-5/2*a^2*b^3/x^4-5/6*a^4*b/x^6-5/3*a*b^4/x^3-1/2*
b^5/x^2
```

maxima [A] time = 1.36, size = 57, normalized size = 1.58

$$-\frac{21\,b^{5}x^{5}+70\,ab^{4}x^{4}+105\,a^{2}b^{3}x^{3}+84\,a^{3}b^{2}x^{2}+35\,a^{4}bx+6\,a^{5}}{42\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*x^5 + 70*a*b^4*x^4 + 105*a^2*b^3*x^3 + 84*a^3*b^2*x^2 + 35*a^
4*b*x + 6*a^5)/x^7
```

mupad [B] time = 0.07, size = 57, normalized size = 1.58

$$-\frac{\frac{a^5}{7}+\frac{5\,a^4\,b\,x}{6}+2\,a^3\,b^2\,x^2+\frac{5\,a^2\,b^3\,x^3}{2}+\frac{5\,a\,b^4\,x^4}{3}+\frac{b^5\,x^5}{2}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^8,x)
```

```
[Out] -(a^5/7 + (b^5*x^5)/2 + (5*a*b^4*x^4)/3 + 2*a^3*b^2*x^2 + (5*a^2*b^3*x^3)/2
 + (5*a^4*b*x)/6)/x^7
```

sympy [B] time = 0.41, size = 61, normalized size = 1.69

$$\frac{-6a^5 - 35a^4bx - 84a^3b^2x^2 - 105a^2b^3x^3 - 70ab^4x^4 - 21b^5x^5}{42x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**8,x)
```

```
[Out] (-6*a**5 - 35*a**4*b*x - 84*a**3*b**2*x**2 - 105*a**2*b**3*x**3 - 70*a*b**4
*x**4 - 21*b**5*x**5)/(42*x**7)
```

3.92 $\int \frac{(a+bx)^5}{x^9} dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^6}{168a^3x^6}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{(a+bx)^6}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^9,x]
```

```
[Out] -(a + b*x)^6/(8*a*x^8) + (b*(a + b*x)^6)/(28*a^2*x^7) - (b^2*(a + b*x)^6)/(
168*a^3*x^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^9} dx &= -\frac{(a+bx)^6}{8ax^8}-\frac{b\int \frac{(a+bx)^5}{x^8} dx}{4a}\\ &= -\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}+\frac{b^2\int \frac{(a+bx)^5}{x^7} dx}{28a^2}\\ &= -\frac{(a+bx)^6}{8ax^8}+\frac{b(a+bx)^6}{28a^2x^7}-\frac{b^2(a+bx)^6}{168a^3x^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.20

$$-\frac{a^5}{8x^8}-\frac{5a^4b}{7x^7}-\frac{5a^3b^2}{3x^6}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{4x^4}-\frac{b^5}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^9,x]
```

```
[Out] -1/8*a^5/x^8 - (5*a^4*b)/(7*x^7) - (5*a^3*b^2)/(3*x^6) - (2*a^2*b^3)/x^5 -
(5*a*b^4)/(4*x^4) - b^5/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^9, x]
```

fricas [A] time = 1.77, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="fricas")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

giac [A] time = 1.10, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="giac")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

maple [A] time = 0.01, size = 58, normalized size = 1.04

$$-\frac{b^5}{3x^3}-\frac{5a\,b^4}{4x^4}-\frac{2a^2b^3}{x^5}-\frac{5a^3b^2}{3x^6}-\frac{5a^4b}{7x^7}-\frac{a^5}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^9,x)
```

```
[Out] -5/7*a^4*b/x^7-1/8*a^5/x^8-5/4*a*b^4/x^4-5/3*a^3*b^2/x^6-1/3*b^5/x^3-2*a^2*
b^3/x^5
```

maxima [A] time = 1.40, size = 57, normalized size = 1.02

$$-\frac{56\,b^5x^5+210\,ab^4x^4+336\,a^2b^3x^3+280\,a^3b^2x^2+120\,a^4bx+21\,a^5}{168\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^9,x, algorithm="maxima")
```

```
[Out] -1/168*(56*b^5*x^5 + 210*a*b^4*x^4 + 336*a^2*b^3*x^3 + 280*a^3*b^2*x^2 + 12
0*a^4*b*x + 21*a^5)/x^8
```

mupad [B] time = 0.04, size = 57, normalized size = 1.02

$$-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x}{7}+\frac{5\,a^3\,b^2\,x^2}{3}+2\,a^2\,b^3\,x^3+\frac{5\,a\,b^4\,x^4}{4}+\frac{b^5\,x^5}{3}}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^9,x)
```

```
[Out] -(a^5/8 + (b^5*x^5)/3 + (5*a*b^4*x^4)/4 + (5*a^3*b^2*x^2)/3 + 2*a^2*b^3*x^3
 + (5*a^4*b*x)/7)/x^8
```

sympy [A] time = 0.43, size = 61, normalized size = 1.09

$$\frac{-21a^5 - 120a^4bx - 280a^3b^2x^2 - 336a^2b^3x^3 - 210ab^4x^4 - 56b^5x^5}{168x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**9,x)
```

```
[Out] (-21*a**5 - 120*a**4*b*x - 280*a**3*b**2*x**2 - 336*a**2*b**3*x**3 - 210*a*
b**4*x**4 - 56*b**5*x**5)/(168*x**8)
```

3.93 $\int \frac{(a+bx)^5}{x^{10}} dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx&=\int\left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\&=-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^10, x]
```

fricas [A] time = 1.10, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

giac [A] time = 1.21, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

maple [A] time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{4x^4}-\frac{a\,b^4}{x^5}-\frac{5a^2b^3}{3x^6}-\frac{10a^3b^2}{7x^7}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/9*a^5/x^9-5/8*a^4*b/x^8-10/7*a^3*b^2/x^7-5/3*a^2*b^3/x^6-a*b^4/x^5-1/4*b
^5/x^4
```

maxima [A] time = 1.33, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

mupad [B] time = 0.08, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x}{8}+\frac{10\,a^3\,b^2\,x^2}{7}+\frac{5\,a^2\,b^3\,x^3}{3}+a\,b^4\,x^4+\frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

sympy [A] time = 0.45, size = 61, normalized size = 0.91

$$\frac{-56a^5-315a^4bx-720a^3b^2x^2-840a^2b^3x^3-504ab^4x^4-126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.94 $\int \frac{(a+bx)^5}{x^{11}} \, dx$

Optimal. Leaf size=69

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5a^4b}{9x^9}-\frac{a^5}{10x^{10}}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^11,x]
```

```
[Out] -a^5/(10*x^10) - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{11}} \, dx &= \int \left(\frac{a^5}{x^{11}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^9}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^7}+\frac{b^5}{x^6}\right) dx\\ &= -\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{10x^{10}}-\frac{5a^4b}{9x^9}-\frac{5a^3b^2}{4x^8}-\frac{10a^2b^3}{7x^7}-\frac{5ab^4}{6x^6}-\frac{b^5}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^11,x]
```

```
[Out] -1/10*a^5/x^10 - (5*a^4*b)/(9*x^9) - (5*a^3*b^2)/(4*x^8) - (10*a^2*b^3)/(7*
x^7) - (5*a*b^4)/(6*x^6) - b^5/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{11}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^11, x]
```

fricas [A] time = 0.98, size = 57, normalized size = 0.83

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="fricas")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

giac [A] time = 1.00, size = 57, normalized size = 0.83

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="giac")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^{5}}{5x^{5}}-\frac{5a\,b^{4}}{6x^{6}}-\frac{10a^{2}b^{3}}{7x^{7}}-\frac{5a^{3}b^{2}}{4x^{8}}-\frac{5a^{4}b}{9x^{9}}-\frac{a^{5}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^11,x)
```

```
[Out] -1/10*a^5/x^10-5/9*a^4*b/x^9-5/4*a^3*b^2/x^8-10/7*a^2*b^3/x^7-5/6*a*b^4/x^6
-1/5*b^5/x^5
```

maple [A] time = 1.37, size = 57, normalized size = 0.83

$$-\frac{252\,b^{5}x^{5}+1050\,ab^{4}x^{4}+1800\,a^{2}b^{3}x^{3}+1575\,a^{3}b^{2}x^{2}+700\,a^{4}bx+126\,a^{5}}{1260\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^11,x, algorithm="maxima")
```

```
[Out] -1/1260*(252*b^5*x^5 + 1050*a*b^4*x^4 + 1800*a^2*b^3*x^3 + 1575*a^3*b^2*x^2
 + 700*a^4*b*x + 126*a^5)/x^10
```

mupad [B] time = 0.08, size = 57, normalized size = 0.83

$$-\frac{\frac{a^{5}}{10}+\frac{5\,a^{4}\,b\,x}{9}+\frac{5\,a^{3}\,b^{2}\,x^{2}}{4}+\frac{10\,a^{2}\,b^{3}\,x^{3}}{7}+\frac{5\,a\,b^{4}\,x^{4}}{6}+\frac{b^{5}\,x^{5}}{5}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^11,x)
```

```
[Out] -(a^5/10 + (b^5*x^5)/5 + (5*a*b^4*x^4)/6 + (5*a^3*b^2*x^2)/4 + (10*a^2*b^3*
x^3)/7 + (5*a^4*b*x)/9)/x^10
```

sympy [A] time = 0.49, size = 61, normalized size = 0.88

$$\frac{-126a^{5}-700a^{4}bx-1575a^{3}b^{2}x^{2}-1800a^{2}b^{3}x^{3}-1050ab^{4}x^{4}-252b^{5}x^{5}}{1260x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**11,x)
```

```
[Out] (-126*a**5 - 700*a**4*b*x - 1575*a**3*b**2*x**2 - 1800*a**2*b**3*x**3 - 105
0*a*b**4*x**4 - 252*b**5*x**5)/(1260*x**10)
```

3.95 $\int \frac{(a+bx)^5}{x^{12}}\,dx$

Optimal. Leaf size=69

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{a^5}{11x^{11}}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^12,x]
```

```
[Out] -a^5/(11*x^11) - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{12}}\,dx &= \int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{11}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^9}+\frac{5ab^4}{x^8}+\frac{b^5}{x^7}\right)dx\\ &= -\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 69, normalized size = 1.00

$$-\frac{a^5}{11x^{11}}-\frac{a^4b}{2x^{10}}-\frac{10a^3b^2}{9x^9}-\frac{5a^2b^3}{4x^8}-\frac{5ab^4}{7x^7}-\frac{b^5}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^12,x]
```

```
[Out] -1/11*a^5/x^11 - (a^4*b)/(2*x^10) - (10*a^3*b^2)/(9*x^9) - (5*a^2*b^3)/(4*x
^8) - (5*a*b^4)/(7*x^7) - b^5/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^12, x]
```

fricas [A] time = 0.83, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="fricas")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

giac [A] time = 1.38, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="giac")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

maple [A] time = 0.01, size = 58, normalized size = 0.84

$$-\frac{b^5}{6x^6} - \frac{5a\,b^4}{7x^7} - \frac{5a^2b^3}{4x^8} - \frac{10a^3b^2}{9x^9} - \frac{a^4b}{2x^{10}} - \frac{a^5}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^12,x)
```

```
[Out] -1/11*a^5/x^11-1/2*a^4*b/x^10-10/9*a^3*b^2/x^9-5/4*a^2*b^3/x^8-5/7*a*b^4/x^
7-1/6*b^5/x^6
```

maxima [A] time = 1.31, size = 57, normalized size = 0.83

$$-\frac{462\,b^5x^5 + 1980\,ab^4x^4 + 3465\,a^2b^3x^3 + 3080\,a^3b^2x^2 + 1386\,a^4bx + 252\,a^5}{2772\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^12,x, algorithm="maxima")
```

```
[Out] -1/2772*(462*b^5*x^5 + 1980*a*b^4*x^4 + 3465*a^2*b^3*x^3 + 3080*a^3*b^2*x^2
 + 1386*a^4*b*x + 252*a^5)/x^11
```

mupad [B] time = 0.04, size = 57, normalized size = 0.83

$$-\frac{\frac{a^5}{11} + \frac{a^4\,b\,x}{2} + \frac{10\,a^3\,b^2\,x^2}{9} + \frac{5\,a^2\,b^3\,x^3}{4} + \frac{5\,a\,b^4\,x^4}{7} + \frac{b^5\,x^5}{6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^12,x)
```

```
[Out] -(a^5/11 + (b^5*x^5)/6 + (5*a*b^4*x^4)/7 + (10*a^3*b^2*x^2)/9 + (5*a^2*b^3*
x^3)/4 + (a^4*b*x)/2)/x^11
```

sympy [A] time = 0.58, size = 61, normalized size = 0.88

$$\frac{-252a^5 - 1386a^4bx - 3080a^3b^2x^2 - 3465a^2b^3x^3 - 1980ab^4x^4 - 462b^5x^5}{2772x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**12,x)
```

```
[Out] (-252*a**5 - 1386*a**4*b*x - 3080*a**3*b**2*x**2 - 3465*a**2*b**3*x**3 - 19
80*a*b**4*x**4 - 462*b**5*x**5)/(2772*x**11)
```

3.96 $\int \frac{(a+bx)^5}{x^{13}} \, dx$

Optimal. Leaf size=67

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5a^4b}{11x^{11}}-\frac{a^5}{12x^{12}}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^13,x]
```

```
[Out] -a^5/(12*x^12) - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{13}}\,dx &= \int \left(\frac{a^5}{x^{13}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{11}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^9}+\frac{b^5}{x^8}\right)dx\\ &= -\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{12x^{12}}-\frac{5a^4b}{11x^{11}}-\frac{a^3b^2}{x^{10}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{8x^8}-\frac{b^5}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^13,x]
```

```
[Out] -1/12*a^5/x^12 - (5*a^4*b)/(11*x^11) - (a^3*b^2)/x^10 - (10*a^2*b^3)/(9*x^9
) - (5*a*b^4)/(8*x^8) - b^5/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^13, x]
```

fricas [A] time = 0.73, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="fricas")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

giac [A] time = 1.23, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="giac")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

maple [A] time = 0.01, size = 58, normalized size = 0.87

$$-\frac{b^5}{7x^7} - \frac{5a\,b^4}{8x^8} - \frac{10a^2b^3}{9x^9} - \frac{a^3b^2}{x^{10}} - \frac{5a^4b}{11x^{11}} - \frac{a^5}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^13,x)
```

```
[Out] -1/12*a^5/x^12-5/11*a^4*b/x^11-a^3*b^2/x^10-10/9*a^2*b^3/x^9-5/8*a*b^4/x^8-
1/7*b^5/x^7
```

maxima [A] time = 1.36, size = 57, normalized size = 0.85

$$-\frac{792\,b^5x^5 + 3465\,ab^4x^4 + 6160\,a^2b^3x^3 + 5544\,a^3b^2x^2 + 2520\,a^4bx + 462\,a^5}{5544\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^13,x, algorithm="maxima")
```

```
[Out] -1/5544*(792*b^5*x^5 + 3465*a*b^4*x^4 + 6160*a^2*b^3*x^3 + 5544*a^3*b^2*x^2
 + 2520*a^4*b*x + 462*a^5)/x^12
```

mupad [B] time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{12} + \frac{5\,a^4\,b\,x}{11} + a^3\,b^2\,x^2 + \frac{10\,a^2\,b^3\,x^3}{9} + \frac{5\,a\,b^4\,x^4}{8} + \frac{b^5\,x^5}{7}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^13,x)
```

```
[Out] -(a^5/12 + (b^5*x^5)/7 + (5*a*b^4*x^4)/8 + a^3*b^2*x^2 + (10*a^2*b^3*x^3)/9
 + (5*a^4*b*x)/11)/x^12
```

sympy [A] time = 0.62, size = 61, normalized size = 0.91

$$\frac{-462a^5 - 2520a^4bx - 5544a^3b^2x^2 - 6160a^2b^3x^3 - 3465ab^4x^4 - 792b^5x^5}{5544x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**13,x)
```

```
[Out] (-462*a**5 - 2520*a**4*b*x - 5544*a**3*b**2*x**2 - 6160*a**2*b**3*x**3 - 34
65*a*b**4*x**4 - 792*b**5*x**5)/(5544*x**12)
```

3.97 $\int \frac{(a+bx)^5}{x^{14}} dx$

Optimal. Leaf size=67

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5a^4b}{12x^{12}}-\frac{a^5}{13x^{13}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^14,x]
```

```
[Out] -a^5/(13*x^13) - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{14}}\,dx &= \int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{13}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^9}\right)dx\\ &= -\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 67, normalized size = 1.00

$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{12x^{12}}-\frac{10a^3b^2}{11x^{11}}-\frac{a^2b^3}{x^{10}}-\frac{5ab^4}{9x^9}-\frac{b^5}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^14,x]
```

```
[Out] -1/13*a^5/x^13 - (5*a^4*b)/(12*x^12) - (10*a^3*b^2)/(11*x^11) - (a^2*b^3)/x
^10 - (5*a*b^4)/(9*x^9) - b^5/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^14, x]
```

fricas [A] time = 1.19, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

giac [A] time = 1.12, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

maple [A] time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{8x^8} - \frac{5a\,b^4}{9x^9} - \frac{a^2b^3}{x^{10}} - \frac{10a^3b^2}{11x^{11}} - \frac{5a^4b}{12x^{12}} - \frac{a^5}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^14,x)
```

```
[Out] -1/13*a^5/x^13-5/12*a^4*b/x^12-10/11*a^3*b^2/x^11-a^2*b^3/x^10-5/9*a*b^4/x^
9-1/8*b^5/x^8
```

maxima [A] time = 1.38, size = 57, normalized size = 0.85

$$-\frac{1287\,b^5x^5 + 5720\,ab^4x^4 + 10296\,a^2b^3x^3 + 9360\,a^3b^2x^2 + 4290\,a^4bx + 792\,a^5}{10296\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1287*b^5*x^5 + 5720*a*b^4*x^4 + 10296*a^2*b^3*x^3 + 9360*a^3*b^2*
x^2 + 4290*a^4*b*x + 792*a^5)/x^13
```

mupad [B] time = 0.04, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{13} + \frac{5\,a^4\,b\,x}{12} + \frac{10\,a^3\,b^2\,x^2}{11} + a^2\,b^3\,x^3 + \frac{5\,a\,b^4\,x^4}{9} + \frac{b^5\,x^5}{8}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^14,x)
```

```
[Out] -(a^5/13 + (b^5*x^5)/8 + (5*a*b^4*x^4)/9 + (10*a^3*b^2*x^2)/11 + a^2*b^3*x^
3 + (5*a^4*b*x)/12)/x^13
```

sympy [A] time = 0.62, size = 61, normalized size = 0.91

$$\frac{-792a^5 - 4290a^4bx - 9360a^3b^2x^2 - 10296a^2b^3x^3 - 5720ab^4x^4 - 1287b^5x^5}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**14,x)
```

```
[Out] (-792*a**5 - 4290*a**4*b*x - 9360*a**3*b**2*x**2 - 10296*a**2*b**3*x**3 - 5
720*a*b**4*x**4 - 1287*b**5*x**5)/(10296*x**13)
```

3.98 $\int x^8(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}a^2b^5x^{14}+\frac{35}{13}a^3b^4x^{13}+\frac{35}{12}a^4b^3x^{12}+\frac{21}{11}a^5b^2x^{11}+\frac{7}{10}a^6bx^{10}+\frac{a^7x^9}{9}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^8(a+bx)^7\,dx&=\int\left(a^7x^8+7a^6bx^9+21a^5b^2x^{10}+35a^4b^3x^{11}+35a^3b^4x^{12}+21a^2b^5x^{13}+7ab^6x^{14}+b^7x^{15}\right)\\&=\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^9}{9}+\frac{7}{10}a^6bx^{10}+\frac{21}{11}a^5b^2x^{11}+\frac{35}{12}a^4b^3x^{12}+\frac{35}{13}a^3b^4x^{13}+\frac{3}{2}a^2b^5x^{14}+\frac{7}{15}ab^6x^{15}+\frac{b^7x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^7,x]
```

```
[Out] (a^7*x^9)/9 + (7*a^6*b*x^10)/10 + (21*a^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/
12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*x^14)/2 + (7*a*b^6*x^15)/15 + (b^7*x
^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x)^7, x]
```

fricas [A] time = 1.29, size = 79, normalized size = 0.83

$$\frac{1}{16}x^{16}b^7 + \frac{7}{15}x^{15}b^6a + \frac{3}{2}x^{14}b^5a^2 + \frac{35}{13}x^{13}b^4a^3 + \frac{35}{12}x^{12}b^3a^4 + \frac{21}{11}x^{11}b^2a^5 + \frac{7}{10}x^{10}ba^6 + \frac{1}{9}x^9a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^7 + 7/15*x^15*b^6*a + 3/2*x^14*b^5*a^2 + 35/13*x^13*b^4*a^3 + 3
5/12*x^12*b^3*a^4 + 21/11*x^11*b^2*a^5 + 7/10*x^10*b*a^6 + 1/9*x^9*a^7
```

giac [A] time = 0.95, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16} + \frac{7}{15}\,ab^6x^{15} + \frac{3}{2}\,a^2b^5x^{14} + \frac{35}{13}\,a^3b^4x^{13} + \frac{35}{12}\,a^4b^3x^{12} + \frac{21}{11}\,a^5b^2x^{11} + \frac{7}{10}\,a^6bx^{10} + \frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

maple [A] time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{16}b^7x^{16} + \frac{7}{15}a\,b^6x^{15} + \frac{3}{2}a^2b^5x^{14} + \frac{35}{13}a^3b^4x^{13} + \frac{35}{12}a^4b^3x^{12} + \frac{21}{11}a^5b^2x^{11} + \frac{7}{10}a^6b\,x^{10} + \frac{1}{9}a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^7,x)
```

```
[Out] 1/9*a^7*x^9+7/10*a^6*b*x^10+21/11*a^5*b^2*x^11+35/12*a^4*b^3*x^12+35/13*a^3
*b^4*x^13+3/2*a^2*b^5*x^14+7/15*a*b^6*x^15+1/16*b^7*x^16
```

maxima [A] time = 1.40, size = 79, normalized size = 0.83

$$\frac{1}{16}\,b^7x^{16} + \frac{7}{15}\,ab^6x^{15} + \frac{3}{2}\,a^2b^5x^{14} + \frac{35}{13}\,a^3b^4x^{13} + \frac{35}{12}\,a^4b^3x^{12} + \frac{21}{11}\,a^5b^2x^{11} + \frac{7}{10}\,a^6bx^{10} + \frac{1}{9}\,a^7x^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^7*x^16 + 7/15*a*b^6*x^15 + 3/2*a^2*b^5*x^14 + 35/13*a^3*b^4*x^13 + 3
5/12*a^4*b^3*x^12 + 21/11*a^5*b^2*x^11 + 7/10*a^6*b*x^10 + 1/9*a^7*x^9
```

mupad [B] time = 0.15, size = 79, normalized size = 0.83

$$\frac{a^7\,x^9}{9} + \frac{7\,a^6\,b\,x^{10}}{10} + \frac{21\,a^5\,b^2\,x^{11}}{11} + \frac{35\,a^4\,b^3\,x^{12}}{12} + \frac{35\,a^3\,b^4\,x^{13}}{13} + \frac{3\,a^2\,b^5\,x^{14}}{2} + \frac{7\,a\,b^6\,x^{15}}{15} + \frac{b^7\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^7,x)
```

```
[Out] (a^7*x^9)/9 + (b^7*x^16)/16 + (7*a^6*b*x^10)/10 + (7*a*b^6*x^15)/15 + (21*a
^5*b^2*x^11)/11 + (35*a^4*b^3*x^12)/12 + (35*a^3*b^4*x^13)/13 + (3*a^2*b^5*
x^14)/2
```

sympy [A] time = 0.10, size = 94, normalized size = 0.99

$$\frac{a^7x^9}{9} + \frac{7a^6bx^{10}}{10} + \frac{21a^5b^2x^{11}}{11} + \frac{35a^4b^3x^{12}}{12} + \frac{35a^3b^4x^{13}}{13} + \frac{3a^2b^5x^{14}}{2} + \frac{7ab^6x^{15}}{15} + \frac{b^7x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**7,x)
```

```
[Out] a**7*x**9/9 + 7*a**6*b*x**10/10 + 21*a**5*b**2*x**11/11 + 35*a**4*b**3*x**1
2/12 + 35*a**3*b**4*x**13/13 + 3*a**2*b**5*x**14/2 + 7*a*b**6*x**15/15 + b*
*7*x**16/16
```

3.99 $\int x^7(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6bx^9+\frac{a^7x^8}{8}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^7(a+bx)^7\,dx&=\int\left(a^7x^7+7a^6bx^8+21a^5b^2x^9+35a^4b^3x^{10}+35a^3b^4x^{11}+21a^2b^5x^{12}+7ab^6x^{13}+b^7x^{14}\right)dx\\&=\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^8}{8}+\frac{7}{9}a^6bx^9+\frac{21}{10}a^5b^2x^{10}+\frac{35}{11}a^4b^3x^{11}+\frac{35}{12}a^3b^4x^{12}+\frac{21}{13}a^2b^5x^{13}+\frac{1}{2}ab^6x^{14}+\frac{b^7x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^7,x]
```

```
[Out] (a^7*x^8)/8 + (7*a^6*b*x^9)/9 + (21*a^5*b^2*x^10)/10 + (35*a^4*b^3*x^11)/11
 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13)/13 + (a*b^6*x^14)/2 + (b^7*x^15
)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x)^7, x]
```

fricas [A] time = 1.34, size = 79, normalized size = 0.83

$$\frac{1}{15}x^{15}b^7+\frac{1}{2}x^{14}b^6a+\frac{21}{13}x^{13}b^5a^2+\frac{35}{12}x^{12}b^4a^3+\frac{35}{11}x^{11}b^3a^4+\frac{21}{10}x^{10}b^2a^5+\frac{7}{9}x^9ba^6+\frac{1}{8}x^8a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^7 + 1/2*x^14*b^6*a + 21/13*x^13*b^5*a^2 + 35/12*x^12*b^4*a^3 +
35/11*x^11*b^3*a^4 + 21/10*x^10*b^2*a^5 + 7/9*x^9*b*a^6 + 1/8*x^8*a^7
```

giac [A] time = 0.99, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

maple [A] time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{15}b^7x^{15}+\frac{1}{2}a\,b^6x^{14}+\frac{21}{13}a^2b^5x^{13}+\frac{35}{12}a^3b^4x^{12}+\frac{35}{11}a^4b^3x^{11}+\frac{21}{10}a^5b^2x^{10}+\frac{7}{9}a^6b\,x^9+\frac{1}{8}a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^7,x)
```

```
[Out] 1/8*a^7*x^8+7/9*a^6*b*x^9+21/10*a^5*b^2*x^10+35/11*a^4*b^3*x^11+35/12*a^3*b
^4*x^12+21/13*a^2*b^5*x^13+1/2*a*b^6*x^14+1/15*b^7*x^15
```

maxima [A] time = 1.30, size = 79, normalized size = 0.83

$$\frac{1}{15}\,b^7x^{15}+\frac{1}{2}\,ab^6x^{14}+\frac{21}{13}\,a^2b^5x^{13}+\frac{35}{12}\,a^3b^4x^{12}+\frac{35}{11}\,a^4b^3x^{11}+\frac{21}{10}\,a^5b^2x^{10}+\frac{7}{9}\,a^6bx^9+\frac{1}{8}\,a^7x^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*x^15 + 1/2*a*b^6*x^14 + 21/13*a^2*b^5*x^13 + 35/12*a^3*b^4*x^12 +
35/11*a^4*b^3*x^11 + 21/10*a^5*b^2*x^10 + 7/9*a^6*b*x^9 + 1/8*a^7*x^8
```

mupad [B] time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^8}{8}+\frac{7\,a^6\,b\,x^9}{9}+\frac{21\,a^5\,b^2\,x^{10}}{10}+\frac{35\,a^4\,b^3\,x^{11}}{11}+\frac{35\,a^3\,b^4\,x^{12}}{12}+\frac{21\,a^2\,b^5\,x^{13}}{13}+\frac{a\,b^6\,x^{14}}{2}+\frac{b^7\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^7,x)
```

```
[Out] (a^7*x^8)/8 + (b^7*x^15)/15 + (7*a^6*b*x^9)/9 + (a*b^6*x^14)/2 + (21*a^5*b^
2*x^10)/10 + (35*a^4*b^3*x^11)/11 + (35*a^3*b^4*x^12)/12 + (21*a^2*b^5*x^13
)/13
```

sympy [A] time = 0.09, size = 92, normalized size = 0.97

$$\frac{a^7x^8}{8}+\frac{7a^6bx^9}{9}+\frac{21a^5b^2x^{10}}{10}+\frac{35a^4b^3x^{11}}{11}+\frac{35a^3b^4x^{12}}{12}+\frac{21a^2b^5x^{13}}{13}+\frac{ab^6x^{14}}{2}+\frac{b^7x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**7,x)
```

```
[Out] a**7*x**8/8 + 7*a**6*b*x**9/9 + 21*a**5*b**2*x**10/10 + 35*a**4*b**3*x**11/
11 + 35*a**3*b**4*x**12/12 + 21*a**2*b**5*x**13/13 + a*b**6*x**14/2 + b**7*
x**15/15
```

3.100 $\int x^6(a+bx)^7\,dx$

Optimal. Leaf size=95

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{7}{4}a^2b^5x^{12}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{2}a^4b^3x^{10}+\frac{7}{3}a^5b^2x^9+\frac{7}{8}a^6bx^8+\frac{a^7x^7}{7}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^6(a+bx)^7\,dx&=\int\left(a^7x^6+7a^6bx^7+21a^5b^2x^8+35a^4b^3x^9+35a^3b^4x^{10}+21a^2b^5x^{11}+7ab^6x^{12}+b^7x^{13}\right)dx\\&=\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$\frac{a^7x^7}{7}+\frac{7}{8}a^6bx^8+\frac{7}{3}a^5b^2x^9+\frac{7}{2}a^4b^3x^{10}+\frac{35}{11}a^3b^4x^{11}+\frac{7}{4}a^2b^5x^{12}+\frac{7}{13}ab^6x^{13}+\frac{b^7x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^7,x]
```

```
[Out] (a^7*x^7)/7 + (7*a^6*b*x^8)/8 + (7*a^5*b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (3
5*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4 + (7*a*b^6*x^13)/13 + (b^7*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^7, x]
```

fricas [A] time = 1.32, size = 79, normalized size = 0.83

$$\frac{1}{14}x^{14}b^7+\frac{7}{13}x^{13}b^6a+\frac{7}{4}x^{12}b^5a^2+\frac{35}{11}x^{11}b^4a^3+\frac{7}{2}x^{10}b^3a^4+\frac{7}{3}x^9b^2a^5+\frac{7}{8}x^8ba^6+\frac{1}{7}x^7a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^7 + 7/13*x^13*b^6*a + 7/4*x^12*b^5*a^2 + 35/11*x^11*b^4*a^3 + 7
/2*x^10*b^3*a^4 + 7/3*x^9*b^2*a^5 + 7/8*x^8*b*a^6 + 1/7*x^7*a^7
```

giac [A] time = 1.01, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^{7}x^{14} + \frac{7}{13}\,ab^{6}x^{13} + \frac{7}{4}\,a^{2}b^{5}x^{12} + \frac{35}{11}\,a^{3}b^{4}x^{11} + \frac{7}{2}\,a^{4}b^{3}x^{10} + \frac{7}{3}\,a^{5}b^{2}x^{9} + \frac{7}{8}\,a^{6}bx^{8} + \frac{1}{7}\,a^{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

maple [A] time = 0.00, size = 80, normalized size = 0.84

$$\frac{1}{14}b^{7}x^{14} + \frac{7}{13}a\,b^{6}x^{13} + \frac{7}{4}a^{2}b^{5}x^{12} + \frac{35}{11}a^{3}b^{4}x^{11} + \frac{7}{2}a^{4}b^{3}x^{10} + \frac{7}{3}a^{5}b^{2}x^{9} + \frac{7}{8}a^{6}b\,x^{8} + \frac{1}{7}a^{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^7,x)
```

```
[Out] 1/7*a^7*x^7+7/8*a^6*b*x^8+7/3*a^5*b^2*x^9+7/2*a^4*b^3*x^10+35/11*a^3*b^4*x^
11+7/4*a^2*b^5*x^12+7/13*a*b^6*x^13+1/14*b^7*x^14
```

maxima [A] time = 1.29, size = 79, normalized size = 0.83

$$\frac{1}{14}\,b^{7}x^{14} + \frac{7}{13}\,ab^{6}x^{13} + \frac{7}{4}\,a^{2}b^{5}x^{12} + \frac{35}{11}\,a^{3}b^{4}x^{11} + \frac{7}{2}\,a^{4}b^{3}x^{10} + \frac{7}{3}\,a^{5}b^{2}x^{9} + \frac{7}{8}\,a^{6}bx^{8} + \frac{1}{7}\,a^{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^7*x^14 + 7/13*a*b^6*x^13 + 7/4*a^2*b^5*x^12 + 35/11*a^3*b^4*x^11 + 7
/2*a^4*b^3*x^10 + 7/3*a^5*b^2*x^9 + 7/8*a^6*b*x^8 + 1/7*a^7*x^7
```

mupad [B] time = 0.07, size = 79, normalized size = 0.83

$$\frac{a^7\,x^7}{7}+\frac{7\,a^6\,b\,x^8}{8}+\frac{7\,a^5\,b^2\,x^9}{3}+\frac{7\,a^4\,b^3\,x^{10}}{2}+\frac{35\,a^3\,b^4\,x^{11}}{11}+\frac{7\,a^2\,b^5\,x^{12}}{4}+\frac{7\,a\,b^6\,x^{13}}{13}+\frac{b^7\,x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^7,x)
```

```
[Out] (a^7*x^7)/7 + (b^7*x^14)/14 + (7*a^6*b*x^8)/8 + (7*a*b^6*x^13)/13 + (7*a^5*
b^2*x^9)/3 + (7*a^4*b^3*x^10)/2 + (35*a^3*b^4*x^11)/11 + (7*a^2*b^5*x^12)/4
```

sympy [A] time = 0.11, size = 94, normalized size = 0.99

$$\frac{a^{7}x^{7}}{7} + \frac{7a^{6}bx^{8}}{8} + \frac{7a^{5}b^{2}x^{9}}{3} + \frac{7a^{4}b^{3}x^{10}}{2} + \frac{35a^{3}b^{4}x^{11}}{11} + \frac{7a^{2}b^{5}x^{12}}{4} + \frac{7ab^{6}x^{13}}{13} + \frac{b^{7}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**7,x)
```

```
[Out] a**7*x**7/7 + 7*a**6*b*x**8/8 + 7*a**5*b**2*x**9/3 + 7*a**4*b**3*x**10/2 +
35*a**3*b**4*x**11/11 + 7*a**2*b**5*x**12/4 + 7*a*b**6*x**13/13 + b**7*x**1
4/14
```

3.101 $\int x^5(a+bx)^7\,dx$

Optimal. Leaf size=96

$$-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^5(a+bx)^8}{8b^6}+\frac{(a+bx)^{13}}{13b^6}-\frac{5a(a+bx)^{12}}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^7,x]
```

```
[Out] -(a^5*(a + b*x)^8)/(8*b^6) + (5*a^4*(a + b*x)^9)/(9*b^6) - (a^3*(a + b*x)^1
0)/b^6 + (10*a^2*(a + b*x)^11)/(11*b^6) - (5*a*(a + b*x)^12)/(12*b^6) + (a
+ b*x)^13/(13*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^7\,dx&=\int\left(-\frac{a^5(a+bx)^7}{b^5}+\frac{5a^4(a+bx)^8}{b^5}-\frac{10a^3(a+bx)^9}{b^5}+\frac{10a^2(a+bx)^{10}}{b^5}-\frac{5a(a+bx)^{11}}{b^5}+\frac{(a+\right.\\&=-\frac{a^5(a+bx)^8}{8b^6}+\frac{5a^4(a+bx)^9}{9b^6}-\frac{a^3(a+bx)^{10}}{b^6}+\frac{10a^2(a+bx)^{11}}{11b^6}-\frac{5a(a+bx)^{12}}{12b^6}+\frac{(a+bx)^1}{13b^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 92, normalized size = 0.96

$$\frac{a^7x^6}{6}+a^6bx^7+\frac{21}{8}a^5b^2x^8+\frac{35}{9}a^4b^3x^9+\frac{7}{2}a^3b^4x^{10}+\frac{21}{11}a^2b^5x^{11}+\frac{7}{12}ab^6x^{12}+\frac{b^7x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^7,x]
```

```
[Out] (a^7*x^6)/6 + a^6*b*x^7 + (21*a^5*b^2*x^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*
b^4*x^10)/2 + (21*a^2*b^5*x^11)/11 + (7*a*b^6*x^12)/12 + (b^7*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^7, x]
```

fricas [A] time = 1.34, size = 78, normalized size = 0.81

$$\frac{1}{13}x^{13}b^7 + \frac{7}{12}x^{12}b^6a + \frac{21}{11}x^{11}b^5a^2 + \frac{7}{2}x^{10}b^4a^3 + \frac{35}{9}x^9b^3a^4 + \frac{21}{8}x^8b^2a^5 + x^7ba^6 + \frac{1}{6}x^6a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^7 + 7/12*x^12*b^6*a + 21/11*x^11*b^5*a^2 + 7/2*x^10*b^4*a^3 + 3
5/9*x^9*b^3*a^4 + 21/8*x^8*b^2*a^5 + x^7*b*a^6 + 1/6*x^6*a^7
```

giac [A] time = 1.08, size = 78, normalized size = 0.81

$$\frac{1}{13}b^7x^{13} + \frac{7}{12}ab^6x^{12} + \frac{21}{11}a^2b^5x^{11} + \frac{7}{2}a^3b^4x^{10} + \frac{35}{9}a^4b^3x^9 + \frac{21}{8}a^5b^2x^8 + a^6bx^7 + \frac{1}{6}a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

maple [A] time = 0.00, size = 79, normalized size = 0.82

$$\frac{1}{13}b^7x^{13} + \frac{7}{12}a\,b^6x^{12} + \frac{21}{11}a^2b^5x^{11} + \frac{7}{2}a^3b^4x^{10} + \frac{35}{9}a^4b^3x^9 + \frac{21}{8}a^5b^2x^8 + a^6b\,x^7 + \frac{1}{6}a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^7,x)
```

```
[Out] 1/13*b^7*x^13+7/12*a*b^6*x^12+21/11*a^2*b^5*x^11+7/2*a^3*b^4*x^10+35/9*a^4*
b^3*x^9+21/8*a^5*b^2*x^8+a^6*b*x^7+1/6*a^7*x^6
```

maxima [A] time = 1.39, size = 78, normalized size = 0.81

$$\frac{1}{13}b^7x^{13} + \frac{7}{12}ab^6x^{12} + \frac{21}{11}a^2b^5x^{11} + \frac{7}{2}a^3b^4x^{10} + \frac{35}{9}a^4b^3x^9 + \frac{21}{8}a^5b^2x^8 + a^6bx^7 + \frac{1}{6}a^7x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^7*x^13 + 7/12*a*b^6*x^12 + 21/11*a^2*b^5*x^11 + 7/2*a^3*b^4*x^10 + 3
5/9*a^4*b^3*x^9 + 21/8*a^5*b^2*x^8 + a^6*b*x^7 + 1/6*a^7*x^6
```

mupad [B] time = 0.06, size = 78, normalized size = 0.81

$$\frac{a^7\,x^6}{6} + a^6\,b\,x^7 + \frac{21\,a^5\,b^2\,x^8}{8} + \frac{35\,a^4\,b^3\,x^9}{9} + \frac{7\,a^3\,b^4\,x^{10}}{2} + \frac{21\,a^2\,b^5\,x^{11}}{11} + \frac{7\,a\,b^6\,x^{12}}{12} + \frac{b^7\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^7,x)
```

```
[Out] (a^7*x^6)/6 + (b^7*x^13)/13 + a^6*b*x^7 + (7*a*b^6*x^12)/12 + (21*a^5*b^2*x
^8)/8 + (35*a^4*b^3*x^9)/9 + (7*a^3*b^4*x^10)/2 + (21*a^2*b^5*x^11)/11
```

sympy [A] time = 0.11, size = 90, normalized size = 0.94

$$\frac{a^7x^6}{6} + a^6bx^7 + \frac{21a^5b^2x^8}{8} + \frac{35a^4b^3x^9}{9} + \frac{7a^3b^4x^{10}}{2} + \frac{21a^2b^5x^{11}}{11} + \frac{7ab^6x^{12}}{12} + \frac{b^7x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**7,x)
```

```
[Out] a**7*x**6/6 + a**6*b*x**7 + 21*a**5*b**2*x**8/8 + 35*a**4*b**3*x**9/9 + 7*a
**3*b**4*x**10/2 + 21*a**2*b**5*x**11/11 + 7*a*b**6*x**12/12 + b**7*x**13/1
3
```

3.102 $\int x^4(a+bx)^7\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{a^4(a+bx)^8}{8b^5}+\frac{(a+bx)^{12}}{12b^5}-\frac{4a(a+bx)^{11}}{11b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^7,x]
```

```
[Out] (a^4*(a + b*x)^8)/(8*b^5) - (4*a^3*(a + b*x)^9)/(9*b^5) + (3*a^2*(a + b*x)^
10)/(5*b^5) - (4*a*(a + b*x)^11)/(11*b^5) + (a + b*x)^12/(12*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^7\,dx&=\int\left(\frac{a^4(a+bx)^7}{b^4}-\frac{4a^3(a+bx)^8}{b^4}+\frac{6a^2(a+bx)^9}{b^4}-\frac{4a(a+bx)^{10}}{b^4}+\frac{(a+bx)^{11}}{b^4}\right)dx\\&=\frac{a^4(a+bx)^8}{8b^5}-\frac{4a^3(a+bx)^9}{9b^5}+\frac{3a^2(a+bx)^{10}}{5b^5}-\frac{4a(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{12}}{12b^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 1.15

$$\frac{a^7x^5}{5}+\frac{7}{6}a^6bx^6+3a^5b^2x^7+\frac{35}{8}a^4b^3x^8+\frac{35}{9}a^3b^4x^9+\frac{21}{10}a^2b^5x^{10}+\frac{7}{11}ab^6x^{11}+\frac{b^7x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^7,x]
```

```
[Out] (a^7*x^5)/5 + (7*a^6*b*x^6)/6 + 3*a^5*b^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^
3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10 + (7*a*b^6*x^11)/11 + (b^7*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^7, x]
```

fricas [A] time = 1.26, size = 79, normalized size = 0.98

$$\frac{1}{12}x^{12}b^7 + \frac{7}{11}x^{11}b^6a + \frac{21}{10}x^{10}b^5a^2 + \frac{35}{9}x^9b^4a^3 + \frac{35}{8}x^8b^3a^4 + 3x^7b^2a^5 + \frac{7}{6}x^6ba^6 + \frac{1}{5}x^5a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^7 + 7/11*x^11*b^6*a + 21/10*x^10*b^5*a^2 + 35/9*x^9*b^4*a^3 + 3
5/8*x^8*b^3*a^4 + 3*x^7*b^2*a^5 + 7/6*x^6*b*a^6 + 1/5*x^5*a^7
```

giac [A] time = 1.13, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12} + \frac{7}{11}\,ab^6x^{11} + \frac{21}{10}\,a^2b^5x^{10} + \frac{35}{9}\,a^3b^4x^9 + \frac{35}{8}\,a^4b^3x^8 + 3\,a^5b^2x^7 + \frac{7}{6}\,a^6bx^6 + \frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

maple [A] time = 0.00, size = 80, normalized size = 0.99

$$\frac{1}{12}b^7x^{12} + \frac{7}{11}a\,b^6x^{11} + \frac{21}{10}a^2b^5x^{10} + \frac{35}{9}a^3b^4x^9 + \frac{35}{8}a^4b^3x^8 + 3a^5b^2x^7 + \frac{7}{6}a^6b\,x^6 + \frac{1}{5}a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^7,x)
```

```
[Out] 1/12*b^7*x^12+7/11*a*b^6*x^11+21/10*a^2*b^5*x^10+35/9*a^3*b^4*x^9+35/8*a^4*
b^3*x^8+3*a^5*b^2*x^7+7/6*a^6*b*x^6+1/5*a^7*x^5
```

maxima [A] time = 1.35, size = 79, normalized size = 0.98

$$\frac{1}{12}\,b^7x^{12} + \frac{7}{11}\,ab^6x^{11} + \frac{21}{10}\,a^2b^5x^{10} + \frac{35}{9}\,a^3b^4x^9 + \frac{35}{8}\,a^4b^3x^8 + 3\,a^5b^2x^7 + \frac{7}{6}\,a^6bx^6 + \frac{1}{5}\,a^7x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^7*x^12 + 7/11*a*b^6*x^11 + 21/10*a^2*b^5*x^10 + 35/9*a^3*b^4*x^9 + 3
5/8*a^4*b^3*x^8 + 3*a^5*b^2*x^7 + 7/6*a^6*b*x^6 + 1/5*a^7*x^5
```

mupad [B] time = 0.06, size = 79, normalized size = 0.98

$$\frac{a^7\,x^5}{5} + \frac{7\,a^6\,b\,x^6}{6} + 3\,a^5\,b^2\,x^7 + \frac{35\,a^4\,b^3\,x^8}{8} + \frac{35\,a^3\,b^4\,x^9}{9} + \frac{21\,a^2\,b^5\,x^{10}}{10} + \frac{7\,a\,b^6\,x^{11}}{11} + \frac{b^7\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^7,x)
```

```
[Out] (a^7*x^5)/5 + (b^7*x^12)/12 + (7*a^6*b*x^6)/6 + (7*a*b^6*x^11)/11 + 3*a^5*b
^2*x^7 + (35*a^4*b^3*x^8)/8 + (35*a^3*b^4*x^9)/9 + (21*a^2*b^5*x^10)/10
```

sympy [A] time = 0.10, size = 92, normalized size = 1.14

$$\frac{a^7x^5}{5} + \frac{7a^6bx^6}{6} + 3a^5b^2x^7 + \frac{35a^4b^3x^8}{8} + \frac{35a^3b^4x^9}{9} + \frac{21a^2b^5x^{10}}{10} + \frac{7ab^6x^{11}}{11} + \frac{b^7x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**7,x)
```

```
[Out] a**7*x**5/5 + 7*a**6*b*x**6/6 + 3*a**5*b**2*x**7 + 35*a**4*b**3*x**8/8 + 35
*a**3*b**4*x**9/9 + 21*a**2*b**5*x**10/10 + 7*a*b**6*x**11/11 + b**7*x**12/
12
```

3.103 $\int x^3(a+bx)^7\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^9}{3b^4}-\frac{a^3(a+bx)^8}{8b^4}+\frac{(a+bx)^{11}}{11b^4}-\frac{3a(a+bx)^{10}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^7,x]
```

```
[Out] -(a^3*(a + b*x)^8)/(8*b^4) + (a^2*(a + b*x)^9)/(3*b^4) - (3*a*(a + b*x)^10)
/(10*b^4) + (a + b*x)^11/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^7\,dx &= \int\left(-\frac{a^3(a+bx)^7}{b^3}+\frac{3a^2(a+bx)^8}{b^3}-\frac{3a(a+bx)^9}{b^3}+\frac{(a+bx)^{10}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^8}{8b^4}+\frac{a^2(a+bx)^9}{3b^4}-\frac{3a(a+bx)^{10}}{10b^4}+\frac{(a+bx)^{11}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 1.45

$$\frac{a^7x^4}{4}+\frac{7}{5}a^6bx^5+\frac{7}{2}a^5b^2x^6+5a^4b^3x^7+\frac{35}{8}a^3b^4x^8+\frac{7}{3}a^2b^5x^9+\frac{7}{10}ab^6x^{10}+\frac{b^7x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^7,x]
```

```
[Out] (a^7*x^4)/4 + (7*a^6*b*x^5)/5 + (7*a^5*b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3
*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3 + (7*a*b^6*x^10)/10 + (b^7*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^7, x]
```

fricas [A] time = 1.33, size = 79, normalized size = 1.23

$$\frac{1}{11}x^{11}b^7 + \frac{7}{10}x^{10}b^6a + \frac{7}{3}x^9b^5a^2 + \frac{35}{8}x^8b^4a^3 + 5x^7b^3a^4 + \frac{7}{2}x^6b^2a^5 + \frac{7}{5}x^5ba^6 + \frac{1}{4}x^4a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*b^7 + 7/10*x^10*b^6*a + 7/3*x^9*b^5*a^2 + 35/8*x^8*b^4*a^3 + 5*x^
7*b^3*a^4 + 7/2*x^6*b^2*a^5 + 7/5*x^5*b*a^6 + 1/4*x^4*a^7
```

giac [A] time = 1.22, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11} + \frac{7}{10}\,ab^6x^{10} + \frac{7}{3}\,a^2b^5x^9 + \frac{35}{8}\,a^3b^4x^8 + 5\,a^4b^3x^7 + \frac{7}{2}\,a^5b^2x^6 + \frac{7}{5}\,a^6bx^5 + \frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

maple [A] time = 0.00, size = 80, normalized size = 1.25

$$\frac{1}{11}b^7x^{11} + \frac{7}{10}a\,b^6x^{10} + \frac{7}{3}a^2b^5x^9 + \frac{35}{8}a^3b^4x^8 + 5a^4b^3x^7 + \frac{7}{2}a^5b^2x^6 + \frac{7}{5}a^6b\,x^5 + \frac{1}{4}a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^7,x)
```

```
[Out] 1/11*b^7*x^11+7/10*a*b^6*x^10+7/3*a^2*b^5*x^9+35/8*a^3*b^4*x^8+5*a^4*b^3*x^
7+7/2*a^5*b^2*x^6+7/5*a^6*b*x^5+1/4*a^7*x^4
```

maxima [A] time = 1.34, size = 79, normalized size = 1.23

$$\frac{1}{11}\,b^7x^{11} + \frac{7}{10}\,ab^6x^{10} + \frac{7}{3}\,a^2b^5x^9 + \frac{35}{8}\,a^3b^4x^8 + 5\,a^4b^3x^7 + \frac{7}{2}\,a^5b^2x^6 + \frac{7}{5}\,a^6bx^5 + \frac{1}{4}\,a^7x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^7*x^11 + 7/10*a*b^6*x^10 + 7/3*a^2*b^5*x^9 + 35/8*a^3*b^4*x^8 + 5*a^
4*b^3*x^7 + 7/2*a^5*b^2*x^6 + 7/5*a^6*b*x^5 + 1/4*a^7*x^4
```

mupad [B] time = 0.10, size = 79, normalized size = 1.23

$$\frac{a^7\,x^4}{4} + \frac{7\,a^6\,b\,x^5}{5} + \frac{7\,a^5\,b^2\,x^6}{2} + 5\,a^4\,b^3\,x^7 + \frac{35\,a^3\,b^4\,x^8}{8} + \frac{7\,a^2\,b^5\,x^9}{3} + \frac{7\,a\,b^6\,x^{10}}{10} + \frac{b^7\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^7,x)
```

```
[Out] (a^7*x^4)/4 + (b^7*x^11)/11 + (7*a^6*b*x^5)/5 + (7*a*b^6*x^10)/10 + (7*a^5*
b^2*x^6)/2 + 5*a^4*b^3*x^7 + (35*a^3*b^4*x^8)/8 + (7*a^2*b^5*x^9)/3
```

sympy [A] time = 0.09, size = 92, normalized size = 1.44

$$\frac{a^7x^4}{4} + \frac{7a^6bx^5}{5} + \frac{7a^5b^2x^6}{2} + 5a^4b^3x^7 + \frac{35a^3b^4x^8}{8} + \frac{7a^2b^5x^9}{3} + \frac{7ab^6x^{10}}{10} + \frac{b^7x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**7,x)
```

```
[Out] a**7*x**4/4 + 7*a**6*b*x**5/5 + 7*a**5*b**2*x**6/2 + 5*a**4*b**3*x**7 + 35*
a**3*b**4*x**8/8 + 7*a**2*b**5*x**9/3 + 7*a*b**6*x**10/10 + b**7*x**11/11
```

3.104 $\int x^2(a+bx)^7\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^8}{8b^3}+\frac{(a+bx)^{10}}{10b^3}-\frac{2a(a+bx)^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^7,x]
```

```
[Out] (a^2*(a + b*x)^8)/(8*b^3) - (2*a*(a + b*x)^9)/(9*b^3) + (a + b*x)^10/(10*b^
3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^7\,dx&=\int\left(\frac{a^2(a+bx)^7}{b^2}-\frac{2a(a+bx)^8}{b^2}+\frac{(a+bx)^9}{b^2}\right)dx\\&=\frac{a^2(a+bx)^8}{8b^3}-\frac{2a(a+bx)^9}{9b^3}+\frac{(a+bx)^{10}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 1.98

$$\frac{a^7x^3}{3}+\frac{7}{4}a^6bx^4+\frac{21}{5}a^5b^2x^5+\frac{35}{6}a^4b^3x^6+5a^3b^4x^7+\frac{21}{8}a^2b^5x^8+\frac{7}{9}ab^6x^9+\frac{b^7x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^7,x]
```

```
[Out] (a^7*x^3)/3 + (7*a^6*b*x^4)/4 + (21*a^5*b^2*x^5)/5 + (35*a^4*b^3*x^6)/6 + 5
*a^3*b^4*x^7 + (21*a^2*b^5*x^8)/8 + (7*a*b^6*x^9)/9 + (b^7*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^7, x]
```

fricas [A] time = 1.11, size = 79, normalized size = 1.68

$$\frac{1}{10}x^{10}b^7+\frac{7}{9}x^9b^6a+\frac{21}{8}x^8b^5a^2+5x^7b^4a^3+\frac{35}{6}x^6b^3a^4+\frac{21}{5}x^5b^2a^5+\frac{7}{4}x^4ba^6+\frac{1}{3}x^3a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*b^7 + 7/9*x^9*b^6*a + 21/8*x^8*b^5*a^2 + 5*x^7*b^4*a^3 + 35/6*x^6
*b^3*a^4 + 21/5*x^5*b^2*a^5 + 7/4*x^4*b*a^6 + 1/3*x^3*a^7
```

giac [A] time = 0.91, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

maple [A] time = 0.00, size = 80, normalized size = 1.70

$$\frac{1}{10}b^7x^{10}+\frac{7}{9}a\,b^6x^9+\frac{21}{8}a^2b^5x^8+5a^3b^4x^7+\frac{35}{6}a^4b^3x^6+\frac{21}{5}a^5b^2x^5+\frac{7}{4}a^6b\,x^4+\frac{1}{3}a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^7,x)
```

```
[Out] 1/10*b^7*x^10+7/9*a*b^6*x^9+21/8*a^2*b^5*x^8+5*a^3*b^4*x^7+35/6*a^4*b^3*x^6
+21/5*a^5*b^2*x^5+7/4*a^6*b*x^4+1/3*a^7*x^3
```

maxima [A] time = 1.33, size = 79, normalized size = 1.68

$$\frac{1}{10}\,b^7x^{10}+\frac{7}{9}\,ab^6x^9+\frac{21}{8}\,a^2b^5x^8+5\,a^3b^4x^7+\frac{35}{6}\,a^4b^3x^6+\frac{21}{5}\,a^5b^2x^5+\frac{7}{4}\,a^6bx^4+\frac{1}{3}\,a^7x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^7*x^10 + 7/9*a*b^6*x^9 + 21/8*a^2*b^5*x^8 + 5*a^3*b^4*x^7 + 35/6*a^4
*b^3*x^6 + 21/5*a^5*b^2*x^5 + 7/4*a^6*b*x^4 + 1/3*a^7*x^3
```

mupad [B] time = 0.12, size = 31, normalized size = 0.66

$$\frac{{\left(a+b\,x\right)}^8\,\left(8\,a^2-64\,a\,b\,x+288\,b^2\,x^2\right)}{2880\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^8*(8*a^2 + 288*b^2*x^2 - 64*a*b*x))/(2880*b^3)
```

sympy [B] time = 0.09, size = 92, normalized size = 1.96

$$\frac{a^7x^3}{3}+\frac{7a^6bx^4}{4}+\frac{21a^5b^2x^5}{5}+\frac{35a^4b^3x^6}{6}+5a^3b^4x^7+\frac{21a^2b^5x^8}{8}+\frac{7ab^6x^9}{9}+\frac{b^7x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**7,x)
```

```
[Out] a**7*x**3/3 + 7*a**6*b*x**4/4 + 21*a**5*b**2*x**5/5 + 35*a**4*b**3*x**6/6 +
 5*a**3*b**4*x**7 + 21*a**2*b**5*x**8/8 + 7*a*b**6*x**9/9 + b**7*x**10/10
```

3.105 $\int x(a+bx)^7\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^9}{9b^2}-\frac{a(a+bx)^8}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^7,x]
```

```
[Out] -(a*(a + b*x)^8)/(8*b^2) + (a + b*x)^9/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^7\,dx&=\int\left(-\frac{a(a+bx)^7}{b}+\frac{(a+bx)^8}{b}\right)dx\\&=-\frac{a(a+bx)^8}{8b^2}+\frac{(a+bx)^9}{9b^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 91, normalized size = 3.03

$$\frac{a^7x^2}{2}+\frac{7}{3}a^6bx^3+\frac{21}{4}a^5b^2x^4+7a^4b^3x^5+\frac{35}{6}a^3b^4x^6+3a^2b^5x^7+\frac{7}{8}ab^6x^8+\frac{b^7x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^7,x]
```

```
[Out] (a^7*x^2)/2 + (7*a^6*b*x^3)/3 + (21*a^5*b^2*x^4)/4 + 7*a^4*b^3*x^5 + (35*a^
3*b^4*x^6)/6 + 3*a^2*b^5*x^7 + (7*a*b^6*x^8)/8 + (b^7*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x*(a + b*x)^7, x]
```

fricas [B] time = 1.00, size = 79, normalized size = 2.63

$$\frac{1}{9}x^9b^7+\frac{7}{8}x^8b^6a+3x^7b^5a^2+\frac{35}{6}x^6b^4a^3+7x^5b^3a^4+\frac{21}{4}x^4b^2a^5+\frac{7}{3}x^3ba^6+\frac{1}{2}x^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^7 + 7/8*x^8*b^6*a + 3*x^7*b^5*a^2 + 35/6*x^6*b^4*a^3 + 7*x^5*b^3*
a^4 + 21/4*x^4*b^2*a^5 + 7/3*x^3*b*a^6 + 1/2*x^2*a^7
```

giac [B] time = 0.92, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6+7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

maple [B] time = 0.00, size = 80, normalized size = 2.67

$$\frac{1}{9}b^7x^9+\frac{7}{8}a\,b^6x^8+3a^2b^5x^7+\frac{35}{6}a^3b^4x^6+7a^4b^3x^5+\frac{21}{4}a^5b^2x^4+\frac{7}{3}a^6b\,x^3+\frac{1}{2}a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^7,x)
```

```
[Out] 1/9*b^7*x^9+7/8*a*b^6*x^8+3*a^2*b^5*x^7+35/6*a^3*b^4*x^6+7*a^4*b^3*x^5+21/4
*a^5*b^2*x^4+7/3*a^6*b*x^3+1/2*a^7*x^2
```

maxima [B] time = 1.35, size = 79, normalized size = 2.63

$$\frac{1}{9}\,b^7x^9+\frac{7}{8}\,ab^6x^8+3\,a^2b^5x^7+\frac{35}{6}\,a^3b^4x^6+7\,a^4b^3x^5+\frac{21}{4}\,a^5b^2x^4+\frac{7}{3}\,a^6bx^3+\frac{1}{2}\,a^7x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b^7*x^9 + 7/8*a*b^6*x^8 + 3*a^2*b^5*x^7 + 35/6*a^3*b^4*x^6 + 7*a^4*b^3*
x^5 + 21/4*a^5*b^2*x^4 + 7/3*a^6*b*x^3 + 1/2*a^7*x^2
```

mupad [B] time = 0.12, size = 25, normalized size = 0.83

$$-\frac{2\left(\frac{a\,{\left(a+b\,x\right)}^8}{16}-\frac{{\left(a+b\,x\right)}^9}{18}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^7,x)
```

```
[Out] -(2*((a*(a + b*x)^8)/16 - (a + b*x)^9/18))/b^2
```

sympy [B] time = 0.09, size = 90, normalized size = 3.00

$$\frac{a^7x^2}{2}+\frac{7a^6bx^3}{3}+\frac{21a^5b^2x^4}{4}+7a^4b^3x^5+\frac{35a^3b^4x^6}{6}+3a^2b^5x^7+\frac{7ab^6x^8}{8}+\frac{b^7x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**7,x)
```

```
[Out] a**7*x**2/2 + 7*a**6*b*x**3/3 + 21*a**5*b**2*x**4/4 + 7*a**4*b**3*x**5 + 35
*a**3*b**4*x**6/6 + 3*a**2*b**5*x**7 + 7*a*b**6*x**8/8 + b**7*x**9/9
```

3.106 $\int (a + bx)^7 \, dx$

Optimal. Leaf size=14

$$\frac{(a + bx)^8}{8b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a + bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a + bx)^7 \, dx = \frac{(a + bx)^8}{8b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a + bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7,x]
```

```
[Out] (a + b*x)^8/(8*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^7 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7, x]
```

fricas [B] time = 0.89, size = 75, normalized size = 5.36

$$\frac{1}{8}x^8b^7 + x^7b^6a + \frac{7}{2}x^6b^5a^2 + 7x^5b^4a^3 + \frac{35}{4}x^4b^3a^4 + 7x^3b^2a^5 + \frac{7}{2}x^2ba^6 + xa^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*b^7 + x^7*b^6*a + 7/2*x^6*b^5*a^2 + 7*x^5*b^4*a^3 + 35/4*x^4*b^3*a^
4 + 7*x^3*b^2*a^5 + 7/2*x^2*b*a^6 + x*a^7
```

giac [A] time = 1.06, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="giac")
```

```
[Out] 1/8*(b*x + a)^8/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^8}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7,x)
```

```
[Out] 1/8*(b*x+a)^8/b
```

maxima [A] time = 1.39, size = 12, normalized size = 0.86

$$\frac{(bx+a)^8}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(b*x + a)^8/b
```

mupad [B] time = 0.06, size = 75, normalized size = 5.36

$$a^7\,x+\frac{7\,a^6\,b\,x^2}{2}+7\,a^5\,b^2\,x^3+\frac{35\,a^4\,b^3\,x^4}{4}+7\,a^3\,b^4\,x^5+\frac{7\,a^2\,b^5\,x^6}{2}+a\,b^6\,x^7+\frac{b^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7,x)
```

```
[Out] a^7*x + (b^7*x^8)/8 + (7*a^6*b*x^2)/2 + a*b^6*x^7 + 7*a^5*b^2*x^3 + (35*a^4
*b^3*x^4)/4 + 7*a^3*b^4*x^5 + (7*a^2*b^5*x^6)/2
```

sympy [B] time = 0.08, size = 83, normalized size = 5.93

$$a^7x+\frac{7a^6bx^2}{2}+7a^5b^2x^3+\frac{35a^4b^3x^4}{4}+7a^3b^4x^5+\frac{7a^2b^5x^6}{2}+ab^6x^7+\frac{b^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7,x)
```

```
[Out] a**7*x + 7*a**6*b*x**2/2 + 7*a**5*b**2*x**3 + 35*a**4*b**3*x**4/4 + 7*a**3*
b**4*x**5 + 7*a**2*b**5*x**6/2 + a*b**6*x**7 + b**7*x**8/8
```

3.107 $\int \frac{(a+bx)^7}{x} dx$

Optimal. Leaf size=87

$$a^7 \log(x) + 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + 7a^6bx + a^7 \log(x) + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x} dx &= \int \left(7a^6b + \frac{a^7}{x} + 21a^5b^2x + 35a^4b^3x^2 + 35a^3b^4x^3 + 21a^2b^5x^4 + 7ab^6x^5 + b^7x^6\right) dx \\
&= 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7} + a^7 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 87, normalized size = 1.00

$$a^7 \log(x) + 7a^6bx + \frac{21}{2}a^5b^2x^2 + \frac{35}{3}a^4b^3x^3 + \frac{35}{4}a^3b^4x^4 + \frac{21}{5}a^2b^5x^5 + \frac{7}{6}ab^6x^6 + \frac{b^7x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x,x]
```

```
[Out] 7*a^6*b*x + (21*a^5*b^2*x^2)/2 + (35*a^4*b^3*x^3)/3 + (35*a^3*b^4*x^4)/4 +
(21*a^2*b^5*x^5)/5 + (7*a*b^6*x^6)/6 + (b^7*x^7)/7 + a^7*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x, x]
```

fricas [A] time = 1.38, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="fricas")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

giac [A] time = 1.30, size = 76, normalized size = 0.87

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="giac")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(abs(x))
```

maple [A] time = 0.00, size = 76, normalized size = 0.87

$$\frac{b^{7}x^{7}}{7}+\frac{7a\,b^{6}x^{6}}{6}+\frac{21a^{2}b^{5}x^{5}}{5}+\frac{35a^{3}b^{4}x^{4}}{4}+\frac{35a^{4}b^{3}x^{3}}{3}+\frac{21a^{5}b^{2}x^{2}}{2}+a^{7}\ln(x)+7a^{6}bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x,x)
```

```
[Out] 7*a^6*b*x+21/2*a^5*b^2*x^2+35/3*a^4*b^3*x^3+35/4*a^3*b^4*x^4+21/5*a^2*b^5*x
^5+7/6*a*b^6*x^6+1/7*b^7*x^7+a^7*ln(x)
```

maxima [A] time = 1.34, size = 75, normalized size = 0.86

$$\frac{1}{7}\,b^{7}x^{7}+\frac{7}{6}\,ab^{6}x^{6}+\frac{21}{5}\,a^{2}b^{5}x^{5}+\frac{35}{4}\,a^{3}b^{4}x^{4}+\frac{35}{3}\,a^{4}b^{3}x^{3}+\frac{21}{2}\,a^{5}b^{2}x^{2}+7\,a^{6}bx+a^{7}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x,x, algorithm="maxima")
```

```
[Out] 1/7*b^7*x^7 + 7/6*a*b^6*x^6 + 21/5*a^2*b^5*x^5 + 35/4*a^3*b^4*x^4 + 35/3*a^
4*b^3*x^3 + 21/2*a^5*b^2*x^2 + 7*a^6*b*x + a^7*log(x)
```

mupad [B] time = 0.07, size = 75, normalized size = 0.86

$$a^7\,\ln(x)+\frac{b^7\,x^7}{7}+\frac{7\,a\,b^6\,x^6}{6}+\frac{21\,a^5\,b^2\,x^2}{2}+\frac{35\,a^4\,b^3\,x^3}{3}+\frac{35\,a^3\,b^4\,x^4}{4}+\frac{21\,a^2\,b^5\,x^5}{5}+7\,a^6\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x,x)
```

```
[Out] a^7*log(x) + (b^7*x^7)/7 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/2 + (35*a^4*b
^3*x^3)/3 + (35*a^3*b^4*x^4)/4 + (21*a^2*b^5*x^5)/5 + 7*a^6*b*x
```

sympy [A] time = 0.19, size = 88, normalized size = 1.01

$$a^{7}\log{\left(x \right)}+7a^{6}bx+\frac{21a^{5}b^{2}x^{2}}{2}+\frac{35a^{4}b^{3}x^{3}}{3}+\frac{35a^{3}b^{4}x^{4}}{4}+\frac{21a^{2}b^{5}x^{5}}{5}+\frac{7ab^{6}x^{6}}{6}+\frac{b^{7}x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x,x)
```

```
[Out] a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**3/3 + 35*a
**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x**7/7
```

3.108 $\int \frac{(a+bx)^7}{x^2} dx$

Optimal. Leaf size=86

$$-\frac{a^7}{x} + 7a^6b\log(x) + 21a^5b^2x + \frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + 21a^5b^2x + 7a^6b\log(x) - \frac{a^7}{x} + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x^2}\,dx &= \int \left(21a^5b^2 + \frac{a^7}{x^2} + \frac{7a^6b}{x} + 35a^4b^3x + 35a^3b^4x^2 + 21a^2b^5x^3 + 7ab^6x^4 + b^7x^5\right) dx \\
&= -\frac{a^7}{x} + 21a^5b^2x + \frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6} + 7a^6b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{x} + 7a^6b\log(x) + 21a^5b^2x + \frac{35}{2}a^4b^3x^2 + \frac{35}{3}a^3b^4x^3 + \frac{21}{4}a^2b^5x^4 + \frac{7}{5}ab^6x^5 + \frac{b^7x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^2,x]
```

```
[Out] -(a^7/x) + 21*a^5*b^2*x + (35*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2
*b^5*x^4)/4 + (7*a*b^6*x^5)/5 + (b^7*x^6)/6 + 7*a^6*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^2, x]
```

fricas [A] time = 1.31, size = 81, normalized size = 0.94

$$\frac{10\,b^7x^7 + 84\,ab^6x^6 + 315\,a^2b^5x^5 + 700\,a^3b^4x^4 + 1050\,a^4b^3x^3 + 1260\,a^5b^2x^2 + 420\,a^6bx\log(x) - 60\,a^7}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*x^7 + 84*a*b^6*x^6 + 315*a^2*b^5*x^5 + 700*a^3*b^4*x^4 + 1050*
a^4*b^3*x^3 + 1260*a^5*b^2*x^2 + 420*a^6*b*x*log(x) - 60*a^7)/x
```

giac [A] time = 1.12, size = 77, normalized size = 0.90

$$\frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log(|x|) - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="giac")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(abs(x)) - a^7/x
```

maple [A] time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7x^6}{6} + \frac{7a\,b^6x^5}{5} + \frac{21a^2b^5x^4}{4} + \frac{35a^3b^4x^3}{3} + \frac{35a^4b^3x^2}{2} + 7a^6b\ln(x) + 21a^5b^2x - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^2,x)
```

```
[Out] -a^7/x+21*a^5*b^2*x+35/2*a^4*b^3*x^2+35/3*a^3*b^4*x^3+21/4*a^2*b^5*x^4+7/5*
a*b^6*x^5+1/6*b^7*x^6+7*a^6*b*ln(x)
```

maxima [A] time = 1.36, size = 76, normalized size = 0.88

$$\frac{1}{6}\,b^7x^6 + \frac{7}{5}\,ab^6x^5 + \frac{21}{4}\,a^2b^5x^4 + \frac{35}{3}\,a^3b^4x^3 + \frac{35}{2}\,a^4b^3x^2 + 21\,a^5b^2x + 7\,a^6b\log(x) - \frac{a^7}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^7*x^6 + 7/5*a*b^6*x^5 + 21/4*a^2*b^5*x^4 + 35/3*a^3*b^4*x^3 + 35/2*a^
4*b^3*x^2 + 21*a^5*b^2*x + 7*a^6*b*log(x) - a^7/x
```

mupad [B] time = 0.05, size = 76, normalized size = 0.88

$$\frac{b^7\,x^6}{6} - \frac{a^7}{x} + 21\,a^5\,b^2\,x + \frac{7\,a\,b^6\,x^5}{5} + 7\,a^6\,b\,\ln(x) + \frac{35\,a^4\,b^3\,x^2}{2} + \frac{35\,a^3\,b^4\,x^3}{3} + \frac{21\,a^2\,b^5\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^2,x)
```

```
[Out] (b^7*x^6)/6 - a^7/x + 21*a^5*b^2*x + (7*a*b^6*x^5)/5 + 7*a^6*b*log(x) + (35
*a^4*b^3*x^2)/2 + (35*a^3*b^4*x^3)/3 + (21*a^2*b^5*x^4)/4
```

sympy [A] time = 0.20, size = 85, normalized size = 0.99

$$-\frac{a^7}{x} + 7a^6b\log(x) + 21a^5b^2x + \frac{35a^4b^3x^2}{2} + \frac{35a^3b^4x^3}{3} + \frac{21a^2b^5x^4}{4} + \frac{7ab^6x^5}{5} + \frac{b^7x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**2,x)
```

```
[Out] -a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*a**3*
b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6
```

3.109 $\int \frac{(a+bx)^7}{x^3} dx$

Optimal. Leaf size=84

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+35a^4b^3x+21a^5b^2\log(x)-\frac{7a^6b}{x}-\frac{a^7}{2x^2}+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^3,x]
```

```
[Out] -a^7/(2*x^2) - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^3} dx &= \int \left(35a^4b^3+\frac{a^7}{x^3}+\frac{7a^6b}{x^2}+\frac{21a^5b^2}{x}+35a^3b^4x+21a^2b^5x^2+7ab^6x^3+b^7x^4\right) dx\\ &= -\frac{a^7}{2x^2}-\frac{7a^6b}{x}+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}+21a^5b^2\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{2x^2}-\frac{7a^6b}{x}+21a^5b^2\log(x)+35a^4b^3x+\frac{35}{2}a^3b^4x^2+7a^2b^5x^3+\frac{7}{4}ab^6x^4+\frac{b^7x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^3,x]
```

```
[Out] -1/2*a^7/x^2 - (7*a^6*b)/x + 35*a^4*b^3*x + (35*a^3*b^4*x^2)/2 + 7*a^2*b^5*
x^3 + (7*a*b^6*x^4)/4 + (b^7*x^5)/5 + 21*a^5*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^3, x]
```

fricas [A] time = 1.62, size = 81, normalized size = 0.96

$$\frac{4\,b^{7}x^{7}+35\,ab^{6}x^{6}+140\,a^{2}b^{5}x^{5}+350\,a^{3}b^{4}x^{4}+700\,a^{4}b^{3}x^{3}+420\,a^{5}b^{2}x^{2}\log(x)-140\,a^{6}bx-10\,a^{7}}{20\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 + 35*a*b^6*x^6 + 140*a^2*b^5*x^5 + 350*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 420*a^5*b^2*x^2*log(x) - 140*a^6*b*x - 10*a^7)/x^2
```

giac [A] time = 1.06, size = 76, normalized size = 0.90

$$\frac{1}{5}\,b^{7}x^{5}+\frac{7}{4}\,ab^{6}x^{4}+7\,a^{2}b^{5}x^{3}+\frac{35}{2}\,a^{3}b^{4}x^{2}+35\,a^{4}b^{3}x+21\,a^{5}b^{2}\log\left(|x|\right)-\frac{14\,a^{6}bx+a^{7}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="giac")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(abs(x)) - 1/2*(14*a^6*b*x + a^7)/x^2
```

maple [A] time = 0.01, size = 77, normalized size = 0.92

$$\frac{b^{7}x^{5}}{5}+\frac{7a\,b^{6}x^{4}}{4}+7a^{2}b^{5}x^{3}+\frac{35a^{3}b^{4}x^{2}}{2}+21a^{5}b^{2}\ln(x)+35a^{4}b^{3}x-\frac{7a^{6}b}{x}-\frac{a^{7}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^3,x)
```

```
[Out] -1/2*a^7/x^2-7*a^6*b/x+35*a^4*b^3*x+35/2*a^3*b^4*x^2+7*a^2*b^5*x^3+7/4*a*b^
6*x^4+1/5*b^7*x^5+21*a^5*b^2*ln(x)
```

maxima [A] time = 1.38, size = 75, normalized size = 0.89

$$\frac{1}{5}\,b^{7}x^{5}+\frac{7}{4}\,ab^{6}x^{4}+7\,a^{2}b^{5}x^{3}+\frac{35}{2}\,a^{3}b^{4}x^{2}+35\,a^{4}b^{3}x+21\,a^{5}b^{2}\log(x)-\frac{14\,a^{6}bx+a^{7}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*b^7*x^5 + 7/4*a*b^6*x^4 + 7*a^2*b^5*x^3 + 35/2*a^3*b^4*x^2 + 35*a^4*b^3
*x + 21*a^5*b^2*log(x) - 1/2*(14*a^6*b*x + a^7)/x^2
```

mupad [B] time = 0.05, size = 77, normalized size = 0.92

$$\frac{b^{7}\,x^{5}}{5}-\frac{\frac{a^{7}}{2}+7\,b\,x\,a^{6}}{x^{2}}+35\,a^{4}\,b^{3}\,x+\frac{7\,a\,b^{6}\,x^{4}}{4}+\frac{35\,a^{3}\,b^{4}\,x^{2}}{2}+7\,a^{2}\,b^{5}\,x^{3}+21\,a^{5}\,b^{2}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^3,x)
```

```
[Out] (b^7*x^5)/5 - (a^7/2 + 7*a^6*b*x)/x^2 + 35*a^4*b^3*x + (7*a*b^6*x^4)/4 + (3
5*a^3*b^4*x^2)/2 + 7*a^2*b^5*x^3 + 21*a^5*b^2*log(x)
```

sympy [A] time = 0.25, size = 85, normalized size = 1.01

$$21a^{5}b^{2}\log\left(x\right)+35a^{4}b^{3}x+\frac{35a^{3}b^{4}x^{2}}{2}+7a^{2}b^{5}x^{3}+\frac{7ab^{6}x^{4}}{4}+\frac{b^{7}x^{5}}{5}+\frac{-a^{7}-14a^{6}bx}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**3,x)
```

```
[Out] 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a**3*b**4*x**2/2 + 7*a**2*b**5*x*
*3 + 7*a*b**6*x**4/4 + b**7*x**5/5 + (-a**7 - 14*a**6*b*x)/(2*x**2)
```

3.110 $\int \frac{(a+bx)^7}{x^4}\,dx$

Optimal. Leaf size=86

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21}{2}a^2b^5x^2-\frac{21a^5b^2}{x}+35a^3b^4x+35a^4b^3\log(x)-\frac{7a^6b}{2x^2}-\frac{a^7}{3x^3}+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^4,x]
```

```
[Out] -a^7/(3*x^3) - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^4}\,dx &= \int \left(35a^3b^4+\frac{a^7}{x^4}+\frac{7a^6b}{x^3}+\frac{21a^5b^2}{x^2}+\frac{35a^4b^3}{x}+21a^2b^5x+7ab^6x^2+b^7x^3\right)dx\\ &= -\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}+35a^4b^3\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{3x^3}-\frac{7a^6b}{2x^2}-\frac{21a^5b^2}{x}+35a^4b^3\log(x)+35a^3b^4x+\frac{21}{2}a^2b^5x^2+\frac{7}{3}ab^6x^3+\frac{b^7x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^4,x]
```

```
[Out] -1/3*a^7/x^3 - (7*a^6*b)/(2*x^2) - (21*a^5*b^2)/x + 35*a^3*b^4*x + (21*a^2*
b^5*x^2)/2 + (7*a*b^6*x^3)/3 + (b^7*x^4)/4 + 35*a^4*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^4, x]
```

fricas [A] time = 1.33, size = 81, normalized size = 0.94

$$\frac{3 b^7 x^7 + 28 a b^6 x^6 + 126 a^2 b^5 x^5 + 420 a^3 b^4 x^4 + 420 a^4 b^3 x^3 \log(x) - 252 a^5 b^2 x^2 - 42 a^6 b x - 4 a^7}{12 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 + 28*a*b^6*x^6 + 126*a^2*b^5*x^5 + 420*a^3*b^4*x^4 + 420*a^
4*b^3*x^3*log(x) - 252*a^5*b^2*x^2 - 42*a^6*b*x - 4*a^7)/x^3
```

giac [A] time = 1.09, size = 78, normalized size = 0.91

$$\frac{1}{4} b^7 x^4 + \frac{7}{3} a b^6 x^3 + \frac{21}{2} a^2 b^5 x^2 + 35 a^3 b^4 x + 35 a^4 b^3 \log(|x|) - \frac{126 a^5 b^2 x^2 + 21 a^6 b x + 2 a^7}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="giac")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(abs(x)) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

maple [A] time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7 x^4}{4} + \frac{7 a b^6 x^3}{3} + \frac{21 a^2 b^5 x^2}{2} + 35 a^4 b^3 \ln(x) + 35 a^3 b^4 x - \frac{21 a^5 b^2}{x} - \frac{7 a^6 b}{2 x^2} - \frac{a^7}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^4,x)
```

```
[Out] -1/3*a^7/x^3-7/2*a^6*b/x^2-21*a^5*b^2/x+35*a^3*b^4*x+21/2*a^2*b^5*x^2+7/3*a
*b^6*x^3+1/4*b^7*x^4+35*a^4*b^3*ln(x)
```

maxima [A] time = 1.33, size = 77, normalized size = 0.90

$$\frac{1}{4} b^7 x^4 + \frac{7}{3} a b^6 x^3 + \frac{21}{2} a^2 b^5 x^2 + 35 a^3 b^4 x + 35 a^4 b^3 \log(x) - \frac{126 a^5 b^2 x^2 + 21 a^6 b x + 2 a^7}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*b^7*x^4 + 7/3*a*b^6*x^3 + 21/2*a^2*b^5*x^2 + 35*a^3*b^4*x + 35*a^4*b^3*
log(x) - 1/6*(126*a^5*b^2*x^2 + 21*a^6*b*x + 2*a^7)/x^3
```

mupad [B] time = 0.05, size = 77, normalized size = 0.90

$$\frac{b^7 x^4}{4} - \frac{\frac{a^7}{3} + \frac{7 a^6 b x}{2} + 21 a^5 b^2 x^2}{x^3} + 35 a^3 b^4 x + \frac{7 a b^6 x^3}{3} + \frac{21 a^2 b^5 x^2}{2} + 35 a^4 b^3 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^4,x)
```

```
[Out] (b^7*x^4)/4 - (a^7/3 + 21*a^5*b^2*x^2 + (7*a^6*b*x)/2)/x^3 + 35*a^3*b^4*x +
 (7*a*b^6*x^3)/3 + (21*a^2*b^5*x^2)/2 + 35*a^4*b^3*log(x)
```

sympy [A] time = 0.31, size = 87, normalized size = 1.01

$$35 a^4 b^3 \log(x) + 35 a^3 b^4 x + \frac{21 a^2 b^5 x^2}{2} + \frac{7 a b^6 x^3}{3} + \frac{b^7 x^4}{4} + \frac{-2 a^7 - 21 a^6 b x - 126 a^5 b^2 x^2}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**4,x)
```

```
[Out] 35*a**4*b**3*log(x) + 35*a**3*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/
3 + b**7*x**4/4 + (-2*a**7 - 21*a**6*b*x - 126*a**5*b**2*x**2)/(6*x**3)
```

3.111 $\int \frac{(a+bx)^7}{x^5} \, dx$

Optimal. Leaf size=86

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+35a^3b^4\log(x)-\frac{7a^6b}{3x^3}-\frac{a^7}{4x^4}+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^5,x]
```

```
[Out] -a^7/(4*x^4) - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^5}\,dx &= \int \left(21a^2b^5+\frac{a^7}{x^5}+\frac{7a^6b}{x^4}+\frac{21a^5b^2}{x^3}+\frac{35a^4b^3}{x^2}+\frac{35a^3b^4}{x}+7ab^6x+b^7x^2\right)dx\\ &= -\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}+35a^3b^4\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 86, normalized size = 1.00

$$-\frac{a^7}{4x^4}-\frac{7a^6b}{3x^3}-\frac{21a^5b^2}{2x^2}-\frac{35a^4b^3}{x}+35a^3b^4\log(x)+21a^2b^5x+\frac{7}{2}ab^6x^2+\frac{b^7x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^5,x]
```

```
[Out] -1/4*a^7/x^4 - (7*a^6*b)/(3*x^3) - (21*a^5*b^2)/(2*x^2) - (35*a^4*b^3)/x +
21*a^2*b^5*x + (7*a*b^6*x^2)/2 + (b^7*x^3)/3 + 35*a^3*b^4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^5, x]
```

fricas [A] time = 1.52, size = 81, normalized size = 0.94

$$\frac{4 b^7 x^7 + 42 a b^6 x^6 + 252 a^2 b^5 x^5 + 420 a^3 b^4 x^4 \log(x) - 420 a^4 b^3 x^3 - 126 a^5 b^2 x^2 - 28 a^6 b x - 3 a^7}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*x^7 + 42*a*b^6*x^6 + 252*a^2*b^5*x^5 + 420*a^3*b^4*x^4*log(x) -
 420*a^4*b^3*x^3 - 126*a^5*b^2*x^2 - 28*a^6*b*x - 3*a^7)/x^4
```

giac [A] time = 1.04, size = 78, normalized size = 0.91

$$\frac{1}{3} b^7 x^3 + \frac{7}{2} a b^6 x^2 + 21 a^2 b^5 x + 35 a^3 b^4 \log(|x|) - \frac{420 a^4 b^3 x^3 + 126 a^5 b^2 x^2 + 28 a^6 b x + 3 a^7}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="giac")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(abs(x)) - 1/12*
(420*a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

maple [A] time = 0.01, size = 77, normalized size = 0.90

$$\frac{b^7 x^3}{3} + \frac{7 a b^6 x^2}{2} + 35 a^3 b^4 \ln(x) + 21 a^2 b^5 x - \frac{35 a^4 b^3}{x} - \frac{21 a^5 b^2}{2 x^2} - \frac{7 a^6 b}{3 x^3} - \frac{a^7}{4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^5,x)
```

```
[Out] -1/4*a^7/x^4-7/3*a^6*b/x^3-21/2*a^5*b^2/x^2-35*a^4*b^3/x+21*a^2*b^5*x+7/2*a
*b^6*x^2+1/3*b^7*x^3+35*a^3*b^4*ln(x)
```

maxima [A] time = 1.40, size = 77, normalized size = 0.90

$$\frac{1}{3} b^7 x^3 + \frac{7}{2} a b^6 x^2 + 21 a^2 b^5 x + 35 a^3 b^4 \log(x) - \frac{420 a^4 b^3 x^3 + 126 a^5 b^2 x^2 + 28 a^6 b x + 3 a^7}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^5,x, algorithm="maxima")
```

```
[Out] 1/3*b^7*x^3 + 7/2*a*b^6*x^2 + 21*a^2*b^5*x + 35*a^3*b^4*log(x) - 1/12*(420*
a^4*b^3*x^3 + 126*a^5*b^2*x^2 + 28*a^6*b*x + 3*a^7)/x^4
```

mupad [B] time = 0.09, size = 77, normalized size = 0.90

$$\frac{b^7 x^3}{3} - \frac{\frac{a^7}{4} + \frac{7 a^6 b x}{3} + \frac{21 a^5 b^2 x^2}{2} + 35 a^4 b^3 x^3}{x^4} + 21 a^2 b^5 x + \frac{7 a b^6 x^2}{2} + 35 a^3 b^4 \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^5,x)
```

```
[Out] (b^7*x^3)/3 - (a^7/4 + (21*a^5*b^2*x^2)/2 + 35*a^4*b^3*x^3 + (7*a^6*b*x)/3)
/x^4 + 21*a^2*b^5*x + (7*a*b^6*x^2)/2 + 35*a^3*b^4*log(x)
```

sympy [A] time = 0.33, size = 85, normalized size = 0.99

$$35 a^3 b^4 \log(x) + 21 a^2 b^5 x + \frac{7 a b^6 x^2}{2} + \frac{b^7 x^3}{3} + \frac{-3 a^7 - 28 a^6 b x - 126 a^5 b^2 x^2 - 420 a^4 b^3 x^3}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**5,x)
```

```
[Out] 35*a**3*b**4*log(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3 + (-3*
a**7 - 28*a**6*b*x - 126*a**5*b**2*x**2 - 420*a**4*b**3*x**3)/(12*x**4)
```

3.112 $\int \frac{(a+bx)^7}{x^6} \, dx$

Optimal. Leaf size=84

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)-\frac{7a^6b}{4x^4}-\frac{a^7}{5x^5}+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^6,x]
```

```
[Out] -a^7/(5*x^5) - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^6}\,dx &= \int \left(7ab^6+\frac{a^7}{x^6}+\frac{7a^6b}{x^5}+\frac{21a^5b^2}{x^4}+\frac{35a^4b^3}{x^3}+\frac{35a^3b^4}{x^2}+\frac{21a^2b^5}{x}+b^7x\right)dx\\ &= -\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+7ab^6x+\frac{b^7x^2}{2}+21a^2b^5\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 84, normalized size = 1.00

$$-\frac{a^7}{5x^5}-\frac{7a^6b}{4x^4}-\frac{7a^5b^2}{x^3}-\frac{35a^4b^3}{2x^2}-\frac{35a^3b^4}{x}+21a^2b^5\log(x)+7ab^6x+\frac{b^7x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^6,x]
```

```
[Out] -1/5*a^7/x^5 - (7*a^6*b)/(4*x^4) - (7*a^5*b^2)/x^3 - (35*a^4*b^3)/(2*x^2) -
 (35*a^3*b^4)/x + 7*a*b^6*x + (b^7*x^2)/2 + 21*a^2*b^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^6, x]
```

fricas [A] time = 1.64, size = 81, normalized size = 0.96

$$\frac{10\,b^7x^7 + 140\,ab^6x^6 + 420\,a^2b^5x^5\log(x) - 700\,a^3b^4x^4 - 350\,a^4b^3x^3 - 140\,a^5b^2x^2 - 35\,a^6bx - 4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*x^7 + 140*a*b^6*x^6 + 420*a^2*b^5*x^5*log(x) - 700*a^3*b^4*x^4
 - 350*a^4*b^3*x^3 - 140*a^5*b^2*x^2 - 35*a^6*b*x - 4*a^7)/x^5
```

giac [A] time = 1.01, size = 78, normalized size = 0.93

$$\frac{1}{2}\,b^7x^2 + 7\,ab^6x + 21\,a^2b^5\log\left(|x|\right) - \frac{700\,a^3b^4x^4 + 350\,a^4b^3x^3 + 140\,a^5b^2x^2 + 35\,a^6bx + 4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="giac")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(abs(x)) - 1/20*(700*a^3*b^4*x^4 +
350*a^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

maple [A] time = 0.01, size = 77, normalized size = 0.92

$$\frac{b^7x^2}{2} + 21a^2b^5\ln(x) + 7a\,b^6x - \frac{35a^3b^4}{x} - \frac{35a^4b^3}{2x^2} - \frac{7a^5b^2}{x^3} - \frac{7a^6b}{4x^4} - \frac{a^7}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^6,x)
```

```
[Out] -1/5*a^7/x^5-7/4*a^6*b/x^4-7*a^5*b^2/x^3-35/2*a^4*b^3/x^2-35*a^3*b^4/x+7*a*
b^6*x+1/2*b^7*x^2+21*a^2*b^5*ln(x)
```

maxima [A] time = 1.38, size = 77, normalized size = 0.92

$$\frac{1}{2}\,b^7x^2 + 7\,ab^6x + 21\,a^2b^5\log(x) - \frac{700\,a^3b^4x^4 + 350\,a^4b^3x^3 + 140\,a^5b^2x^2 + 35\,a^6bx + 4\,a^7}{20\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^6,x, algorithm="maxima")
```

```
[Out] 1/2*b^7*x^2 + 7*a*b^6*x + 21*a^2*b^5*log(x) - 1/20*(700*a^3*b^4*x^4 + 350*a
^4*b^3*x^3 + 140*a^5*b^2*x^2 + 35*a^6*b*x + 4*a^7)/x^5
```

mupad [B] time = 0.11, size = 77, normalized size = 0.92

$$\frac{b^7\,x^2}{2} - \frac{\frac{a^7}{5} + \frac{7\,a^6\,b\,x}{4} + 7\,a^5\,b^2\,x^2 + \frac{35\,a^4\,b^3\,x^3}{2} + 35\,a^3\,b^4\,x^4}{x^5} + 21\,a^2\,b^5\,\ln(x) + 7\,a\,b^6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^6,x)
```

```
[Out] (b^7*x^2)/2 - (a^7/5 + 7*a^5*b^2*x^2 + (35*a^4*b^3*x^3)/2 + 35*a^3*b^4*x^4
+ (7*a^6*b*x)/4)/x^5 + 21*a^2*b^5*log(x) + 7*a*b^6*x
```

sympy [A] time = 0.45, size = 83, normalized size = 0.99

$$21a^2b^5\log(x) + 7ab^6x + \frac{b^7x^2}{2} + \frac{-4a^7 - 35a^6bx - 140a^5b^2x^2 - 350a^4b^3x^3 - 700a^3b^4x^4}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**6,x)
```

```
[Out] 21*a**2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2 + (-4*a**7 - 35*a**6*b*x - 1
40*a**5*b**2*x**2 - 350*a**4*b**3*x**3 - 700*a**3*b**4*x**4)/(20*x**5)
```

3.113 $\int \frac{(a+bx)^7}{x^7} dx$

Optimal. Leaf size=85

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}-\frac{7a^6b}{5x^5}-\frac{a^7}{6x^6}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^7,x]
```

```
[Out] -a^7/(6*x^6) - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^7}\,dx &= \int \left(b^7+\frac{a^7}{x^7}+\frac{7a^6b}{x^6}+\frac{21a^5b^2}{x^5}+\frac{35a^4b^3}{x^4}+\frac{35a^3b^4}{x^3}+\frac{21a^2b^5}{x^2}+\frac{7ab^6}{x}\right)dx\\ &= -\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+b^7x+7ab^6\log(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 85, normalized size = 1.00

$$-\frac{a^7}{6x^6}-\frac{7a^6b}{5x^5}-\frac{21a^5b^2}{4x^4}-\frac{35a^4b^3}{3x^3}-\frac{35a^3b^4}{2x^2}-\frac{21a^2b^5}{x}+7ab^6\log(x)+b^7x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^7,x]
```

```
[Out] -1/6*a^7/x^6 - (7*a^6*b)/(5*x^5) - (21*a^5*b^2)/(4*x^4) - (35*a^4*b^3)/(3*x
^3) - (35*a^3*b^4)/(2*x^2) - (21*a^2*b^5)/x + b^7*x + 7*a*b^6*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^7, x]
```

fricas [A] time = 1.40, size = 81, normalized size = 0.95

$$\frac{60\,b^7x^7 + 420\,ab^6x^6\log(x) - 1260\,a^2b^5x^5 - 1050\,a^3b^4x^4 - 700\,a^4b^3x^3 - 315\,a^5b^2x^2 - 84\,a^6bx - 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 420*a*b^6*x^6*log(x) - 1260*a^2*b^5*x^5 - 1050*a^3*b^4*x
^4 - 700*a^4*b^3*x^3 - 315*a^5*b^2*x^2 - 84*a^6*b*x - 10*a^7)/x^6
```

giac [A] time = 0.99, size = 77, normalized size = 0.91

$$b^7x + 7\,ab^6\log\left(|x|\right) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="giac")
```

```
[Out] b^7*x + 7*a*b^6*log(abs(x)) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 7
00*a^4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

maple [A] time = 0.01, size = 76, normalized size = 0.89

$$7a\,b^6\ln(x) + b^7x - \frac{21a^2b^5}{x} - \frac{35a^3b^4}{2x^2} - \frac{35a^4b^3}{3x^3} - \frac{21a^5b^2}{4x^4} - \frac{7a^6b}{5x^5} - \frac{a^7}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^7,x)
```

```
[Out] -1/6*a^7/x^6-7/5*a^6*b/x^5-21/4*a^5*b^2/x^4-35/3*a^4*b^3/x^3-35/2*a^3*b^4/x
^2-21*a^2*b^5/x+b^7*x+7*a*b^6*ln(x)
```

maxima [A] time = 1.38, size = 76, normalized size = 0.89

$$b^7x + 7\,ab^6\log(x) - \frac{1260\,a^2b^5x^5 + 1050\,a^3b^4x^4 + 700\,a^4b^3x^3 + 315\,a^5b^2x^2 + 84\,a^6bx + 10\,a^7}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^7,x, algorithm="maxima")
```

```
[Out] b^7*x + 7*a*b^6*log(x) - 1/60*(1260*a^2*b^5*x^5 + 1050*a^3*b^4*x^4 + 700*a^
4*b^3*x^3 + 315*a^5*b^2*x^2 + 84*a^6*b*x + 10*a^7)/x^6
```

mupad [B] time = 0.11, size = 81, normalized size = 0.95

$$-\frac{10\,a^7 - 60\,b^7\,x^7 + 315\,a^5\,b^2\,x^2 + 700\,a^4\,b^3\,x^3 + 1050\,a^3\,b^4\,x^4 + 1260\,a^2\,b^5\,x^5 + 84\,a^6\,b\,x - 420\,a\,b^6\,x^6\,\ln(x)}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^7,x)
```

```
[Out] -(10*a^7 - 60*b^7*x^7 + 315*a^5*b^2*x^2 + 700*a^4*b^3*x^3 + 1050*a^3*b^4*x^
4 + 1260*a^2*b^5*x^5 + 84*a^6*b*x - 420*a*b^6*x^6*log(x))/(60*x^6)
```

sympy [A] time = 0.60, size = 82, normalized size = 0.96

$$7ab^6\log\left(x\right) + b^7x + \frac{-10a^7 - 84a^6bx - 315a^5b^2x^2 - 700a^4b^3x^3 - 1050a^3b^4x^4 - 1260a^2b^5x^5}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**7,x)
```

```
[Out] 7*a*b**6*log(x) + b**7*x + (-10*a**7 - 84*a**6*b*x - 315*a**5*b**2*x**2 - 7
00*a**4*b**3*x**3 - 1050*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*x**6)
```

3.114 $\int \frac{(a+bx)^7}{x^8} dx$

Optimal. Leaf size=89

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7a^6b}{6x^6}-\frac{a^7}{7x^7}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^8,x]
```

```
[Out] -a^7/(7*x^7) - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^8}\,dx&=\int \left(\frac{a^7}{x^8}+\frac{7a^6b}{x^7}+\frac{21a^5b^2}{x^6}+\frac{35a^4b^3}{x^5}+\frac{35a^3b^4}{x^4}+\frac{21a^2b^5}{x^3}+\frac{7ab^6}{x^2}+\frac{b^7}{x}\right)dx\\&=-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 89, normalized size = 1.00

$$-\frac{a^7}{7x^7}-\frac{7a^6b}{6x^6}-\frac{21a^5b^2}{5x^5}-\frac{35a^4b^3}{4x^4}-\frac{35a^3b^4}{3x^3}-\frac{21a^2b^5}{2x^2}-\frac{7ab^6}{x}+b^7\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^8,x]
```

```
[Out] -1/7*a^7/x^7 - (7*a^6*b)/(6*x^6) - (21*a^5*b^2)/(5*x^5) - (35*a^4*b^3)/(4*x
^4) - (35*a^3*b^4)/(3*x^3) - (21*a^2*b^5)/(2*x^2) - (7*a*b^6)/x + b^7*Log[x
]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^8, x]
```

fricas [A] time = 1.33, size = 81, normalized size = 0.91

$$\frac{420\,b^7x^7\log(x) - 2940\,ab^6x^6 - 4410\,a^2b^5x^5 - 4900\,a^3b^4x^4 - 3675\,a^4b^3x^3 - 1764\,a^5b^2x^2 - 490\,a^6bx - 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="fricas")
```

```
[Out] 1/420*(420*b^7*x^7*log(x) - 2940*a*b^6*x^6 - 4410*a^2*b^5*x^5 - 4900*a^3*b^
4*x^4 - 3675*a^4*b^3*x^3 - 1764*a^5*b^2*x^2 - 490*a^6*b*x - 60*a^7)/x^7
```

giac [A] time = 1.27, size = 79, normalized size = 0.89

$$b^7\log(|x|) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(x)) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x
^4 + 3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

maple [A] time = 0.01, size = 78, normalized size = 0.88

$$b^7\ln(x) - \frac{7a\,b^6}{x} - \frac{21a^2b^5}{2x^2} - \frac{35a^3b^4}{3x^3} - \frac{35a^4b^3}{4x^4} - \frac{21a^5b^2}{5x^5} - \frac{7a^6b}{6x^6} - \frac{a^7}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^8,x)
```

```
[Out] -1/7*a^7/x^7-7/6*a^6*b/x^6-21/5*a^5*b^2/x^5-35/4*a^4*b^3/x^4-35/3*a^3*b^4/x
^3-21/2*a^2*b^5/x^2-7*a*b^6/x+b^7*ln(x)
```

maxima [A] time = 1.36, size = 78, normalized size = 0.88

$$b^7\log(x) - \frac{2940\,ab^6x^6 + 4410\,a^2b^5x^5 + 4900\,a^3b^4x^4 + 3675\,a^4b^3x^3 + 1764\,a^5b^2x^2 + 490\,a^6bx + 60\,a^7}{420\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^8,x, algorithm="maxima")
```

```
[Out] b^7*log(x) - 1/420*(2940*a*b^6*x^6 + 4410*a^2*b^5*x^5 + 4900*a^3*b^4*x^4 +
3675*a^4*b^3*x^3 + 1764*a^5*b^2*x^2 + 490*a^6*b*x + 60*a^7)/x^7
```

mupad [B] time = 0.07, size = 78, normalized size = 0.88

$$b^7\,\ln(x) - \frac{\frac{a^7}{7} + \frac{7\,a^6\,b\,x}{6} + \frac{21\,a^5\,b^2\,x^2}{5} + \frac{35\,a^4\,b^3\,x^3}{4} + \frac{35\,a^3\,b^4\,x^4}{3} + \frac{21\,a^2\,b^5\,x^5}{2} + 7\,a\,b^6\,x^6}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^8,x)
```

```
[Out] b^7*log(x) - (a^7/7 + 7*a*b^6*x^6 + (21*a^5*b^2*x^2)/5 + (35*a^4*b^3*x^3)/4
 + (35*a^3*b^4*x^4)/3 + (21*a^2*b^5*x^5)/2 + (7*a^6*b*x)/6)/x^7
```

sympy [A] time = 0.63, size = 83, normalized size = 0.93

$$b^7\log(x) + \frac{-60a^7 - 490a^6bx - 1764a^5b^2x^2 - 3675a^4b^3x^3 - 4900a^3b^4x^4 - 4410a^2b^5x^5 - 2940ab^6x^6}{420x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**8,x)
```

```
[Out] b**7*log(x) + (-60*a**7 - 490*a**6*b*x - 1764*a**5*b**2*x**2 - 3675*a**4*b*
*3*x**3 - 4900*a**3*b**4*x**4 - 4410*a**2*b**5*x**5 - 2940*a*b**6*x**6)/(42
0*x**7)
```

3.115 $\int \frac{(a+bx)^7}{x^9}\,dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^8}{8ax^8}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^8}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^9,x]
```

```
[Out] -(a + b*x)^8/(8*a*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^7}{x^9}\,dx = -\frac{(a+bx)^8}{8ax^8}$$

Mathematica [B] time = 0.00, size = 87, normalized size = 5.12

$$-\frac{a^7}{8x^8}-\frac{a^6b}{x^7}-\frac{7a^5b^2}{2x^6}-\frac{7a^4b^3}{x^5}-\frac{35a^3b^4}{4x^4}-\frac{7a^2b^5}{x^3}-\frac{7ab^6}{2x^2}-\frac{b^7}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^9,x]
```

```
[Out] -1/8*a^7/x^8 - (a^6*b)/x^7 - (7*a^5*b^2)/(2*x^6) - (7*a^4*b^3)/x^5 - (35*a^
3*b^4)/(4*x^4) - (7*a^2*b^5)/x^3 - (7*a*b^6)/(2*x^2) - b^7/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^9, x]
```

fricas [B] time = 1.41, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7+28\,ab^6x^6+56\,a^2b^5x^5+70\,a^3b^4x^4+56\,a^4b^3x^3+28\,a^5b^2x^2+8\,a^6bx+a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

giac [B] time = 0.92, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

maple [B] time = 0.00, size = 80, normalized size = 4.71

$$-\frac{b^7}{x} - \frac{7a\,b^6}{2x^2} - \frac{7a^2b^5}{x^3} - \frac{35a^3b^4}{4x^4} - \frac{7a^4b^3}{x^5} - \frac{7a^5b^2}{2x^6} - \frac{a^6b}{x^7} - \frac{a^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^9,x)
```

```
[Out] -7*a^4*b^3/x^5-1/8*a^7/x^8-7/2*a^5*b^2/x^6-35/4*a^3*b^4/x^4-7*a^2*b^5/x^3-b
^7/x-a^6*b/x^7-7/2*a*b^6/x^2
```

maxima [B] time = 1.29, size = 77, normalized size = 4.53

$$-\frac{8\,b^7x^7 + 28\,ab^6x^6 + 56\,a^2b^5x^5 + 70\,a^3b^4x^4 + 56\,a^4b^3x^3 + 28\,a^5b^2x^2 + 8\,a^6bx + a^7}{8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*x^7 + 28*a*b^6*x^6 + 56*a^2*b^5*x^5 + 70*a^3*b^4*x^4 + 56*a^4*b
^3*x^3 + 28*a^5*b^2*x^2 + 8*a^6*b*x + a^7)/x^8
```

mupad [B] time = 0.07, size = 77, normalized size = 4.53

$$-\frac{\frac{a^7}{8} + a^6\,b\,x + \frac{7\,a^5\,b^2\,x^2}{2} + 7\,a^4\,b^3\,x^3 + \frac{35\,a^3\,b^4\,x^4}{4} + 7\,a^2\,b^5\,x^5 + \frac{7\,a\,b^6\,x^6}{2} + b^7\,x^7}{x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^9,x)
```

```
[Out] -(a^7/8 + b^7*x^7 + (7*a*b^6*x^6)/2 + (7*a^5*b^2*x^2)/2 + 7*a^4*b^3*x^3 + (
35*a^3*b^4*x^4)/4 + 7*a^2*b^5*x^5 + a^6*b*x)/x^8
```

sympy [B] time = 0.60, size = 83, normalized size = 4.88

$$\frac{-a^7 - 8a^6bx - 28a^5b^2x^2 - 56a^4b^3x^3 - 70a^3b^4x^4 - 56a^2b^5x^5 - 28ab^6x^6 - 8b^7x^7}{8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**9,x)
```

```
[Out] (-a**7 - 8*a**6*b*x - 28*a**5*b**2*x**2 - 56*a**4*b**3*x**3 - 70*a**3*b**4*
x**4 - 56*a**2*b**5*x**5 - 28*a*b**6*x**6 - 8*b**7*x**7)/(8*x**8)
```

3.116 $\int \frac{(a+bx)^7}{x^{10}} dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8} - \frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}} dx &= -\frac{(a+bx)^8}{9ax^9} - \frac{b\int \frac{(a+bx)^7}{x^9} dx}{9a} \\ &= -\frac{(a+bx)^8}{9ax^9} + \frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9} - \frac{7a^6b}{8x^8} - \frac{3a^5b^2}{x^7} - \frac{35a^4b^3}{6x^6} - \frac{7a^3b^4}{x^5} - \frac{21a^2b^5}{4x^4} - \frac{7ab^6}{3x^3} - \frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{10}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^10, x]
```

fricas [B] time = 1.21, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

giac [B] time = 1.01, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

maple [B] time = 0.01, size = 80, normalized size = 2.22

$$-\frac{b^{7}}{2x^{2}}-\frac{7a\,b^{6}}{3x^{3}}-\frac{21a^{2}b^{5}}{4x^{4}}-\frac{7a^{3}b^{4}}{x^{5}}-\frac{35a^{4}b^{3}}{6x^{6}}-\frac{3a^{5}b^{2}}{x^{7}}-\frac{7a^{6}b}{8x^{8}}-\frac{a^{7}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -7*a^3*b^4/x^5-35/6*a^4*b^3/x^6-1/2*b^7/x^2-7/8*a^6*b/x^8-3*a^5*b^2/x^7-21/
4*a^2*b^5/x^4-7/3*a*b^6/x^3-1/9*a^7/x^9
```

maxima [B] time = 1.31, size = 79, normalized size = 2.19

$$-\frac{36\,b^{7}x^{7}+168\,ab^{6}x^{6}+378\,a^{2}b^{5}x^{5}+504\,a^{3}b^{4}x^{4}+420\,a^{4}b^{3}x^{3}+216\,a^{5}b^{2}x^{2}+63\,a^{6}bx+8\,a^{7}}{72\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

mupad [B] time = 0.09, size = 23, normalized size = 0.64

$$-\frac{(8\,a-b\,x)\,\left(a+b\,x\right)^{8}}{72\,a^{2}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

<u>**sympy**</u> [**B**] time = 0.73, size = 85, normalized size = 2.36

$$\frac{-8a^7 - 63a^6bx - 216a^5b^2x^2 - 420a^4b^3x^3 - 504a^3b^4x^4 - 378a^2b^5x^5 - 168ab^6x^6 - 36b^7x^7}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

3.117 $\int \frac{(a+bx)^7}{x^{11}} \, dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^8}{360a^3x^8}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{(a+bx)^8}{10ax^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^11,x]
```

```
[Out] -(a + b*x)^8/(10*a*x^10) + (b*(a + b*x)^8)/(45*a^2*x^9) - (b^2*(a + b*x)^8)
/(360*a^3*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{11}} \, dx &= -\frac{(a+bx)^8}{10ax^{10}}-\frac{b\int \frac{(a+bx)^7}{x^{10}} \, dx}{5a}\\ &= -\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}+\frac{b^2\int \frac{(a+bx)^7}{x^9} \, dx}{45a^2}\\ &= -\frac{(a+bx)^8}{10ax^{10}}+\frac{b(a+bx)^8}{45a^2x^9}-\frac{b^2(a+bx)^8}{360a^3x^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 1.66

$$-\frac{a^7}{10x^{10}}-\frac{7a^6b}{9x^9}-\frac{21a^5b^2}{8x^8}-\frac{5a^4b^3}{x^7}-\frac{35a^3b^4}{6x^6}-\frac{21a^2b^5}{5x^5}-\frac{7ab^6}{4x^4}-\frac{b^7}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^11,x]
```

```
[Out] -1/10*a^7/x^10 - (7*a^6*b)/(9*x^9) - (21*a^5*b^2)/(8*x^8) - (5*a^4*b^3)/x^7
 - (35*a^3*b^4)/(6*x^6) - (21*a^2*b^5)/(5*x^5) - (7*a*b^6)/(4*x^4) - b^7/(3
*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^11, x]
```

fricas [A] time = 1.32, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

giac [A] time = 1.06, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

maple [A] time = 0.00, size = 80, normalized size = 1.43

$$-\frac{b^7}{3x^3} - \frac{7a\,b^6}{4x^4} - \frac{21a^2b^5}{5x^5} - \frac{35a^3b^4}{6x^6} - \frac{5a^4b^3}{x^7} - \frac{21a^5b^2}{8x^8} - \frac{7a^6b}{9x^9} - \frac{a^7}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^11,x)
```

```
[Out] -21/5*a^2*b^5/x^5-21/8*a^5*b^2/x^8-35/6*a^3*b^4/x^6-1/10*a^7/x^10-5*a^4*b^3
/x^7-7/4*a*b^6/x^4-1/3*b^7/x^3-7/9*a^6*b/x^9
```

maxima [A] time = 1.38, size = 79, normalized size = 1.41

$$-\frac{120\,b^7x^7 + 630\,ab^6x^6 + 1512\,a^2b^5x^5 + 2100\,a^3b^4x^4 + 1800\,a^4b^3x^3 + 945\,a^5b^2x^2 + 280\,a^6bx + 36\,a^7}{360\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*x^7 + 630*a*b^6*x^6 + 1512*a^2*b^5*x^5 + 2100*a^3*b^4*x^4 +
 1800*a^4*b^3*x^3 + 945*a^5*b^2*x^2 + 280*a^6*b*x + 36*a^7)/x^10
```

mupad [B] time = 0.11, size = 79, normalized size = 1.41

$$-\frac{\frac{a^7}{10}+\frac{7\,a^6\,b\,x}{9}+\frac{21\,a^5\,b^2\,x^2}{8}+5\,a^4\,b^3\,x^3+\frac{35\,a^3\,b^4\,x^4}{6}+\frac{21\,a^2\,b^5\,x^5}{5}+\frac{7\,a\,b^6\,x^6}{4}+\frac{b^7\,x^7}{3}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^11,x)
```

```
[Out] -(a^7/10 + (b^7*x^7)/3 + (7*a*b^6*x^6)/4 + (21*a^5*b^2*x^2)/8 + 5*a^4*b^3*x
^3 + (35*a^3*b^4*x^4)/6 + (21*a^2*b^5*x^5)/5 + (7*a^6*b*x)/9)/x^10
```

sympy [A] time = 0.76, size = 85, normalized size = 1.52

$$\frac{-36a^7-280a^6bx-945a^5b^2x^2-1800a^4b^3x^3-2100a^3b^4x^4-1512a^2b^5x^5-630ab^6x^6-120b^7x^7}{360x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**11,x)
```

```
[Out] (-36*a**7 - 280*a**6*b*x - 945*a**5*b**2*x**2 - 1800*a**4*b**3*x**3 - 2100*
a**3*b**4*x**4 - 1512*a**2*b**5*x**5 - 630*a*b**6*x**6 - 120*b**7*x**7)/(36
0*x**10)
```

3.118 $\int \frac{(a+bx)^7}{x^{12}}\,dx$

Optimal. Leaf size=76

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^8}{1320a^4x^8}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{(a+bx)^8}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^12,x]
```

```
[Out] -(a + b*x)^8/(11*a*x^11) + (3*b*(a + b*x)^8)/(110*a^2*x^10) - (b^2*(a + b*x
)^8)/(165*a^3*x^9) + (b^3*(a + b*x)^8)/(1320*a^4*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x^{12}}\,dx &= -\frac{(a+bx)^8}{11ax^{11}}-\frac{(3b)\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}+\frac{\left(3b^2\right)\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^2}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}-\frac{b^3\int \frac{(a+bx)^7}{x^9}\,dx}{165a^3}\\
&= -\frac{(a+bx)^8}{11ax^{11}}+\frac{3b(a+bx)^8}{110a^2x^{10}}-\frac{b^2(a+bx)^8}{165a^3x^9}+\frac{b^3(a+bx)^8}{1320a^4x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 1.22

$$-\frac{a^7}{11x^{11}}-\frac{7a^6b}{10x^{10}}-\frac{7a^5b^2}{3x^9}-\frac{35a^4b^3}{8x^8}-\frac{5a^3b^4}{x^7}-\frac{7a^2b^5}{2x^6}-\frac{7ab^6}{5x^5}-\frac{b^7}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^12,x]
```

```
[Out] -1/11*a^7/x^11 - (7*a^6*b)/(10*x^10) - (7*a^5*b^2)/(3*x^9) - (35*a^4*b^3)/(
8*x^8) - (5*a^3*b^4)/x^7 - (7*a^2*b^5)/(2*x^6) - (7*a*b^6)/(5*x^5) - b^7/(4
*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^12, x]
```

fricas [A] time = 0.94, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

giac [A] time = 1.15, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

maple [A] time = 0.01, size = 80, normalized size = 1.05

$$-\frac{b^7}{4x^4}-\frac{7a\,b^6}{5x^5}-\frac{7a^2b^5}{2x^6}-\frac{5a^3b^4}{x^7}-\frac{35a^4b^3}{8x^8}-\frac{7a^5b^2}{3x^9}-\frac{7a^6b}{10x^{10}}-\frac{a^7}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^12,x)
```

```
[Out] -7/5*a*b^6/x^5-7/3*a^5*b^2/x^9-5*a^3*b^4/x^7-7/2*a^2*b^5/x^6-1/4*b^7/x^4-7/
10*a^6*b/x^10-1/11*a^7/x^11-35/8*a^4*b^3/x^8
```

maxima [A] time = 1.36, size = 79, normalized size = 1.04

$$-\frac{330\,b^7x^7+1848\,ab^6x^6+4620\,a^2b^5x^5+6600\,a^3b^4x^4+5775\,a^4b^3x^3+3080\,a^5b^2x^2+924\,a^6bx+120\,a^7}{1320\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*x^7 + 1848*a*b^6*x^6 + 4620*a^2*b^5*x^5 + 6600*a^3*b^4*x^4
 + 5775*a^4*b^3*x^3 + 3080*a^5*b^2*x^2 + 924*a^6*b*x + 120*a^7)/x^11
```

mupad [**B**] time = 0.11, size = 79, normalized size = 1.04

$$-\frac{\frac{a^7}{11}+\frac{7\,a^6\,b\,x}{10}+\frac{7\,a^5\,b^2\,x^2}{3}+\frac{35\,a^4\,b^3\,x^3}{8}+5\,a^3\,b^4\,x^4+\frac{7\,a^2\,b^5\,x^5}{2}+\frac{7\,a\,b^6\,x^6}{5}+\frac{b^7\,x^7}{4}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^12,x)
```

```
[Out] -(a^7/11 + (b^7*x^7)/4 + (7*a*b^6*x^6)/5 + (7*a^5*b^2*x^2)/3 + (35*a^4*b^3*
x^3)/8 + 5*a^3*b^4*x^4 + (7*a^2*b^5*x^5)/2 + (7*a^6*b*x)/10)/x^11
```

sympy [**A**] time = 0.75, size = 85, normalized size = 1.12

$$\frac{-120a^7-924a^6bx-3080a^5b^2x^2-5775a^4b^3x^3-6600a^3b^4x^4-4620a^2b^5x^5-1848ab^6x^6-330b^7x^7}{1320x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**12,x)
```

```
[Out] (-120*a**7 - 924*a**6*b*x - 3080*a**5*b**2*x**2 - 5775*a**4*b**3*x**3 - 660
0*a**3*b**4*x**4 - 4620*a**2*b**5*x**5 - 1848*a*b**6*x**6 - 330*b**7*x**7)/
(1320*x**11)
```

3.119 $\int \frac{(a+bx)^7}{x^{13}}\,dx$

Optimal. Leaf size=96

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^8}{3960a^5x^8}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{(a+bx)^8}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^13,x]
```

```
[Out] -(a + b*x)^8/(12*a*x^12) + (b*(a + b*x)^8)/(33*a^2*x^11) - (b^2*(a + b*x)^8
)/(110*a^3*x^10) + (b^3*(a + b*x)^8)/(495*a^4*x^9) - (b^4*(a + b*x)^8)/(396
0*a^5*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^7}{x^{13}}\,dx &= -\frac{(a+bx)^8}{12ax^{12}}-\frac{b\int \frac{(a+bx)^7}{x^{12}}\,dx}{3a}\\
&= -\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}+\frac{b^2\int \frac{(a+bx)^7}{x^{11}}\,dx}{11a^2}\\
&= -\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}-\frac{b^3\int \frac{(a+bx)^7}{x^{10}}\,dx}{55a^3}\\
&= -\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b^3(a+bx)^8}{495a^4x^9}+\frac{b^4\int \frac{(a+bx)^7}{x^9}\,dx}{495a^4}\\
&= -\frac{(a+bx)^8}{12ax^{12}}+\frac{b(a+bx)^8}{33a^2x^{11}}-\frac{b^2(a+bx)^8}{110a^3x^{10}}+\frac{b^3(a+bx)^8}{495a^4x^9}-\frac{b^4(a+bx)^8}{3960a^5x^8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 93, normalized size = 0.97

$$-\frac{a^7}{12x^{12}}-\frac{7a^6b}{11x^{11}}-\frac{21a^5b^2}{10x^{10}}-\frac{35a^4b^3}{9x^9}-\frac{35a^3b^4}{8x^8}-\frac{3a^2b^5}{x^7}-\frac{7ab^6}{6x^6}-\frac{b^7}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^13,x]
```

```
[Out] -1/12*a^7/x^12 - (7*a^6*b)/(11*x^11) - (21*a^5*b^2)/(10*x^10) - (35*a^4*b^3
)/(9*x^9) - (35*a^3*b^4)/(8*x^8) - (3*a^2*b^5)/x^7 - (7*a*b^6)/(6*x^6) - b^
7/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^13, x]
```

fricas [A] time = 1.13, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

giac [A] time = 1.15, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

maple [A] time = 0.01, size = 80, normalized size = 0.83

$$-\frac{b^7}{5x^5}-\frac{7a\,b^6}{6x^6}-\frac{3a^2b^5}{x^7}-\frac{35a^3b^4}{8x^8}-\frac{35a^4b^3}{9x^9}-\frac{21a^5b^2}{10x^{10}}-\frac{7a^6b}{11x^{11}}-\frac{a^7}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^13,x)
```

```
[Out] -1/5*b^7/x^5-7/6*a*b^6/x^6-35/8*a^3*b^4/x^8-7/11*a^6*b/x^11-3*a^2*b^5/x^7-3
5/9*a^4*b^3/x^9-21/10*a^5*b^2/x^10-1/12*a^7/x^12
```

maxima [A] time = 1.45, size = 79, normalized size = 0.82

$$-\frac{792\,b^7x^7+4620\,ab^6x^6+11880\,a^2b^5x^5+17325\,a^3b^4x^4+15400\,a^4b^3x^3+8316\,a^5b^2x^2+2520\,a^6bx+330\,a^7}{3960\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*x^7 + 4620*a*b^6*x^6 + 11880*a^2*b^5*x^5 + 17325*a^3*b^4*x
^4 + 15400*a^4*b^3*x^3 + 8316*a^5*b^2*x^2 + 2520*a^6*b*x + 330*a^7)/x^12
```

mupad [B] time = 0.07, size = 79, normalized size = 0.82

$$-\frac{\frac{a^7}{12}+\frac{7\,a^6\,b\,x}{11}+\frac{21\,a^5\,b^2\,x^2}{10}+\frac{35\,a^4\,b^3\,x^3}{9}+\frac{35\,a^3\,b^4\,x^4}{8}+3\,a^2\,b^5\,x^5+\frac{7\,a\,b^6\,x^6}{6}+\frac{b^7\,x^7}{5}}{x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^13,x)
```

```
[Out] -(a^7/12 + (b^7*x^7)/5 + (7*a*b^6*x^6)/6 + (21*a^5*b^2*x^2)/10 + (35*a^4*b^
3*x^3)/9 + (35*a^3*b^4*x^4)/8 + 3*a^2*b^5*x^5 + (7*a^6*b*x)/11)/x^12
```

sympy [A] time = 0.80, size = 85, normalized size = 0.89

$$\frac{-330a^7-2520a^6bx-8316a^5b^2x^2-15400a^4b^3x^3-17325a^3b^4x^4-11880a^2b^5x^5-4620ab^6x^6-792b^7x^7}{3960x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**13,x)
```

```
[Out] (-330*a**7 - 2520*a**6*b*x - 8316*a**5*b**2*x**2 - 15400*a**4*b**3*x**3 - 1
7325*a**3*b**4*x**4 - 11880*a**2*b**5*x**5 - 4620*a*b**6*x**6 - 792*b**7*x*
*7)/(3960*x**12)
```

3.120 $\int \frac{(a+bx)^7}{x^{14}}\,dx$

Optimal. Leaf size=93

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{7a^6b}{12x^{12}}-\frac{a^7}{13x^{13}}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^14,x]
```

```
[Out] -a^7/(13*x^13) - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{14}}\,dx &= \int \left(\frac{a^7}{x^{14}}+\frac{7a^6b}{x^{13}}+\frac{21a^5b^2}{x^{12}}+\frac{35a^4b^3}{x^{11}}+\frac{35a^3b^4}{x^{10}}+\frac{21a^2b^5}{x^9}+\frac{7ab^6}{x^8}+\frac{b^7}{x^7}\right)dx\\ &= -\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 93, normalized size = 1.00

$$-\frac{a^7}{13x^{13}}-\frac{7a^6b}{12x^{12}}-\frac{21a^5b^2}{11x^{11}}-\frac{7a^4b^3}{2x^{10}}-\frac{35a^3b^4}{9x^9}-\frac{21a^2b^5}{8x^8}-\frac{ab^6}{x^7}-\frac{b^7}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^14,x]
```

```
[Out] -1/13*a^7/x^13 - (7*a^6*b)/(12*x^12) - (21*a^5*b^2)/(11*x^11) - (7*a^4*b^3)
/(2*x^10) - (35*a^3*b^4)/(9*x^9) - (21*a^2*b^5)/(8*x^8) - (a*b^6)/x^7 - b^7
/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^14, x]
```

fricas [A] time = 0.86, size = 79, normalized size = 0.85

$$-\frac{1716\, b^7 x^7 + 10296\, a b^6 x^6 + 27027\, a^2 b^5 x^5 + 40040\, a^3 b^4 x^4 + 36036\, a^4 b^3 x^3 + 19656\, a^5 b^2 x^2 + 6006\, a^6 b x + 792\, a^7}{10296\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

giac [A] time = 1.04, size = 79, normalized size = 0.85

$$-\frac{1716\, b^7 x^7 + 10296\, a b^6 x^6 + 27027\, a^2 b^5 x^5 + 40040\, a^3 b^4 x^4 + 36036\, a^4 b^3 x^3 + 19656\, a^5 b^2 x^2 + 6006\, a^6 b x + 792\, a^7}{10296\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

maple [A] time = 0.00, size = 80, normalized size = 0.86

$$-\frac{b^7}{6x^6} - \frac{a\, b^6}{x^7} - \frac{21a^2b^5}{8x^8} - \frac{35a^3b^4}{9x^9} - \frac{7a^4b^3}{2x^{10}} - \frac{21a^5b^2}{11x^{11}} - \frac{7a^6b}{12x^{12}} - \frac{a^7}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^14,x)
```

```
[Out] -1/13*a^7/x^13-7/12*a^6*b/x^12-21/11*a^5*b^2/x^11-7/2*a^4*b^3/x^10-35/9*a^3
*b^4/x^9-21/8*a^2*b^5/x^8-a*b^6/x^7-1/6*b^7/x^6
```

maxima [A] time = 1.37, size = 79, normalized size = 0.85

$$-\frac{1716\, b^7 x^7 + 10296\, a b^6 x^6 + 27027\, a^2 b^5 x^5 + 40040\, a^3 b^4 x^4 + 36036\, a^4 b^3 x^3 + 19656\, a^5 b^2 x^2 + 6006\, a^6 b x + 792\, a^7}{10296\, x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*x^7 + 10296*a*b^6*x^6 + 27027*a^2*b^5*x^5 + 40040*a^3*b^
4*x^4 + 36036*a^4*b^3*x^3 + 19656*a^5*b^2*x^2 + 6006*a^6*b*x + 792*a^7)/x^1
3
```

mupad [B] time = 0.07, size = 78, normalized size = 0.84

$$-\frac{\frac{a^7}{13} + \frac{7\, a^6\, b\, x}{12} + \frac{21\, a^5\, b^2\, x^2}{11} + \frac{7\, a^4\, b^3\, x^3}{2} + \frac{35\, a^3\, b^4\, x^4}{9} + \frac{21\, a^2\, b^5\, x^5}{8} + a\, b^6\, x^6 + \frac{b^7\, x^7}{6}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^14,x)
```

```
[Out] -(a^7/13 + (b^7*x^7)/6 + a*b^6*x^6 + (21*a^5*b^2*x^2)/11 + (7*a^4*b^3*x^3)/
2 + (35*a^3*b^4*x^4)/9 + (21*a^2*b^5*x^5)/8 + (7*a^6*b*x)/12)/x^13
```

sympy [A] time = 0.78, size = 85, normalized size = 0.91

$$\frac{-792a^7 - 6006a^6bx - 19656a^5b^2x^2 - 36036a^4b^3x^3 - 40040a^3b^4x^4 - 27027a^2b^5x^5 - 10296ab^6x^6 - 1716b^7x^7}{10296x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**14,x)
```

```
[Out] (-792*a**7 - 6006*a**6*b*x - 19656*a**5*b**2*x**2 - 36036*a**4*b**3*x**3 -
40040*a**3*b**4*x**4 - 27027*a**2*b**5*x**5 - 10296*a*b**6*x**6 - 1716*b**7
*x**7)/(10296*x**13)
```

3.121 $\int \frac{(a+bx)^7}{x^{15}} dx$

Optimal. Leaf size=95

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7a^6b}{13x^{13}}-\frac{a^7}{14x^{14}}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^15,x]
```

```
[Out] -a^7/(14*x^14) - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{15}}\,dx &= \int \left(\frac{a^7}{x^{15}}+\frac{7a^6b}{x^{14}}+\frac{21a^5b^2}{x^{13}}+\frac{35a^4b^3}{x^{12}}+\frac{35a^3b^4}{x^{11}}+\frac{21a^2b^5}{x^{10}}+\frac{7ab^6}{x^9}+\frac{b^7}{x^8}\right)dx\\ &= -\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{14x^{14}}-\frac{7a^6b}{13x^{13}}-\frac{7a^5b^2}{4x^{12}}-\frac{35a^4b^3}{11x^{11}}-\frac{7a^3b^4}{2x^{10}}-\frac{7a^2b^5}{3x^9}-\frac{7ab^6}{8x^8}-\frac{b^7}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^15,x]
```

```
[Out] -1/14*a^7/x^14 - (7*a^6*b)/(13*x^13) - (7*a^5*b^2)/(4*x^12) - (35*a^4*b^3)/
(11*x^11) - (7*a^3*b^4)/(2*x^10) - (7*a^2*b^5)/(3*x^9) - (7*a*b^6)/(8*x^8)
- b^7/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^15, x]
```

fricas [A] time = 0.82, size = 79, normalized size = 0.83

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

giac [A] time = 0.89, size = 79, normalized size = 0.83

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

maple [A] time = 0.01, size = 80, normalized size = 0.84

$$-\frac{b^{7}}{7x^{7}}-\frac{7a\,b^{6}}{8x^{8}}-\frac{7a^{2}b^{5}}{3x^{9}}-\frac{7a^{3}b^{4}}{2x^{10}}-\frac{35a^{4}b^{3}}{11x^{11}}-\frac{7a^{5}b^{2}}{4x^{12}}-\frac{7a^{6}b}{13x^{13}}-\frac{a^{7}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^15,x)
```

```
[Out] -1/14*a^7/x^14-7/13*a^6*b/x^13-7/4*a^5*b^2/x^12-35/11*a^4*b^3/x^11-7/2*a^3*
b^4/x^10-7/3*a^2*b^5/x^9-7/8*a*b^6/x^8-1/7*b^7/x^7
```

maxima [A] time = 1.34, size = 79, normalized size = 0.83

$$-\frac{3432\,b^{7}x^{7}+21021\,ab^{6}x^{6}+56056\,a^{2}b^{5}x^{5}+84084\,a^{3}b^{4}x^{4}+76440\,a^{4}b^{3}x^{3}+42042\,a^{5}b^{2}x^{2}+12936\,a^{6}bx+1716\,a^{7}}{24024\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*x^7 + 21021*a*b^6*x^6 + 56056*a^2*b^5*x^5 + 84084*a^3*b^
4*x^4 + 76440*a^4*b^3*x^3 + 42042*a^5*b^2*x^2 + 12936*a^6*b*x + 1716*a^7)/x
^14
```

mupad [B] time = 0.07, size = 79, normalized size = 0.83

$$-\frac{\frac{a^{7}}{14}+\frac{7\,a^{6}\,b\,x}{13}+\frac{7\,a^{5}\,b^{2}\,x^{2}}{4}+\frac{35\,a^{4}\,b^{3}\,x^{3}}{11}+\frac{7\,a^{3}\,b^{4}\,x^{4}}{2}+\frac{7\,a^{2}\,b^{5}\,x^{5}}{3}+\frac{7\,a\,b^{6}\,x^{6}}{8}+\frac{b^{7}\,x^{7}}{7}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^15,x)
```

```
[Out] -(a^7/14 + (b^7*x^7)/7 + (7*a*b^6*x^6)/8 + (7*a^5*b^2*x^2)/4 + (35*a^4*b^3*
x^3)/11 + (7*a^3*b^4*x^4)/2 + (7*a^2*b^5*x^5)/3 + (7*a^6*b*x)/13)/x^14
```

sympy [A] time = 0.88, size = 85, normalized size = 0.89

$$\frac{-1716a^7 - 12936a^6bx - 42042a^5b^2x^2 - 76440a^4b^3x^3 - 84084a^3b^4x^4 - 56056a^2b^5x^5 - 21021ab^6x^6 - 3432b^7x^7}{24024x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**15,x)
```

```
[Out] (-1716*a**7 - 12936*a**6*b*x - 42042*a**5*b**2*x**2 - 76440*a**4*b**3*x**3
- 84084*a**3*b**4*x**4 - 56056*a**2*b**5*x**5 - 21021*a*b**6*x**6 - 3432*b*
*7*x**7)/(24024*x**14)
```

3.122 $\int \frac{(a+bx)^7}{x^{16}} \, dx$

Optimal. Leaf size=95

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^16,x]
```

```
[Out] -a^7/(15*x^15) - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{16}} \, dx &= \int \left(\frac{a^7}{x^{16}}+\frac{7a^6b}{x^{15}}+\frac{21a^5b^2}{x^{14}}+\frac{35a^4b^3}{x^{13}}+\frac{35a^3b^4}{x^{12}}+\frac{21a^2b^5}{x^{11}}+\frac{7ab^6}{x^{10}}+\frac{b^7}{x^9}\right) dx\\ &= -\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 95, normalized size = 1.00

$$-\frac{a^7}{15x^{15}}-\frac{a^6b}{2x^{14}}-\frac{21a^5b^2}{13x^{13}}-\frac{35a^4b^3}{12x^{12}}-\frac{35a^3b^4}{11x^{11}}-\frac{21a^2b^5}{10x^{10}}-\frac{7ab^6}{9x^9}-\frac{b^7}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^16,x]
```

```
[Out] -1/15*a^7/x^15 - (a^6*b)/(2*x^14) - (21*a^5*b^2)/(13*x^13) - (35*a^4*b^3)/(
12*x^12) - (35*a^3*b^4)/(11*x^11) - (21*a^2*b^5)/(10*x^10) - (7*a*b^6)/(9*x
^9) - b^7/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{16}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^16, x]
```

fricas [A] time = 1.43, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

giac [A] time = 1.00, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

maple [A] time = 0.00, size = 80, normalized size = 0.84

$$-\frac{b^7}{8x^8}-\frac{7a\,b^6}{9x^9}-\frac{21a^2b^5}{10x^{10}}-\frac{35a^3b^4}{11x^{11}}-\frac{35a^4b^3}{12x^{12}}-\frac{21a^5b^2}{13x^{13}}-\frac{a^6b}{2x^{14}}-\frac{a^7}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^16,x)
```

```
[Out] -1/15*a^7/x^15-1/2*a^6*b/x^14-21/13*a^5*b^2/x^13-35/12*a^4*b^3/x^12-35/11*a
^3*b^4/x^11-21/10*a^2*b^5/x^10-7/9*a*b^6/x^9-1/8*b^7/x^8
```

maxima [A] time = 1.39, size = 79, normalized size = 0.83

$$-\frac{6435\,b^7x^7+40040\,ab^6x^6+108108\,a^2b^5x^5+163800\,a^3b^4x^4+150150\,a^4b^3x^3+83160\,a^5b^2x^2+25740\,a^6bx+3432\,a^7}{51480\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*x^7 + 40040*a*b^6*x^6 + 108108*a^2*b^5*x^5 + 163800*a^3*
b^4*x^4 + 150150*a^4*b^3*x^3 + 83160*a^5*b^2*x^2 + 25740*a^6*b*x + 3432*a^7
)/x^15
```

mupad [B] time = 0.11, size = 79, normalized size = 0.83

$$-\frac{\frac{a^7}{15}+\frac{a^6\,b\,x}{2}+\frac{21\,a^5\,b^2\,x^2}{13}+\frac{35\,a^4\,b^3\,x^3}{12}+\frac{35\,a^3\,b^4\,x^4}{11}+\frac{21\,a^2\,b^5\,x^5}{10}+\frac{7\,a\,b^6\,x^6}{9}+\frac{b^7\,x^7}{8}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^16,x)
```

```
[Out] -(a^7/15 + (b^7*x^7)/8 + (7*a*b^6*x^6)/9 + (21*a^5*b^2*x^2)/13 + (35*a^4*b^
3*x^3)/12 + (35*a^3*b^4*x^4)/11 + (21*a^2*b^5*x^5)/10 + (a^6*b*x)/2)/x^15
```

sympy [A] time = 0.88, size = 85, normalized size = 0.89

$$\frac{-3432a^7 - 25740a^6bx - 83160a^5b^2x^2 - 150150a^4b^3x^3 - 163800a^3b^4x^4 - 108108a^2b^5x^5 - 40040ab^6x^6 - 6435b^7x^7}{51480x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**16,x)
```

```
[Out] (-3432*a**7 - 25740*a**6*b*x - 83160*a**5*b**2*x**2 - 150150*a**4*b**3*x**3
 - 163800*a**3*b**4*x**4 - 108108*a**2*b**5*x**5 - 40040*a*b**6*x**6 - 6435
*b**7*x**7)/(51480*x**15)
```

3.123 $\int x^{11}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab$$

Rubi [A] time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{9}{4}a^2b^8x^{20}+\frac{120}{19}a^3b^7x^{19}+\frac{35}{3}a^4b^6x^{18}+\frac{252}{17}a^5b^5x^{17}+\frac{105}{8}a^6b^4x^{16}+8a^7b^3x^{15}+\frac{45}{14}a^8b^2x^{14}+\frac{10}{13}a^9bx^{13}+\frac{a^{10}x^{12}}{12}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Int[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{11}(a+bx)^{10}\,dx=\int\left(a^{10}x^{11}+10a^9bx^{12}+45a^8b^2x^{13}+120a^7b^3x^{14}+210a^6b^4x^{15}+252a^5b^5x^{16}+210a^4b^6x\right.$$
$$=\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+$$

Mathematica [A] time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{12}}{12}+\frac{10}{13}a^9bx^{13}+\frac{45}{14}a^8b^2x^{14}+8a^7b^3x^{15}+\frac{105}{8}a^6b^4x^{16}+\frac{252}{17}a^5b^5x^{17}+\frac{35}{3}a^4b^6x^{18}+\frac{120}{19}a^3b^7x^{19}+\frac{9}{4}a^2b^8x^{20}+\frac{10}{21}ab^9x^{21}+\frac{b^{10}x^{22}}{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11*(a + b*x)^10,x]
```

```
[Out] (a^10*x^12)/12 + (10*a^9*b*x^13)/13 + (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15
 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (12
0*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)/4 + (10*a*b^9*x^21)/21 + (b^10*x^22)/
22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{11}(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^11*(a + b*x)^10, x]
```

fricas [A] time = 1.23, size = 112, normalized size = 0.85

$$\frac{1}{22}x^{22}b^{10} + \frac{10}{21}x^{21}b^9a + \frac{9}{4}x^{20}b^8a^2 + \frac{120}{19}x^{19}b^7a^3 + \frac{35}{3}x^{18}b^6a^4 + \frac{252}{17}x^{17}b^5a^5 + \frac{105}{8}x^{16}b^4a^6 + 8x^{15}b^3a^7 + \frac{45}{14}x^{14}b^2a^8 + \frac{10}{13}x^{13}ba^9 + \frac{1}{12}x^{12}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*b^10 + 10/21*x^21*b^9*a + 9/4*x^20*b^8*a^2 + 120/19*x^19*b^7*a^3
+ 35/3*x^18*b^6*a^4 + 252/17*x^17*b^5*a^5 + 105/8*x^16*b^4*a^6 + 8*x^15*b^3
*a^7 + 45/14*x^14*b^2*a^8 + 10/13*x^13*b*a^9 + 1/12*x^12*a^10
```

giac [A] time = 0.98, size = 112, normalized size = 0.85

$$\frac{1}{22}b^{10}x^{22} + \frac{10}{21}ab^9x^{21} + \frac{9}{4}a^2b^8x^{20} + \frac{120}{19}a^3b^7x^{19} + \frac{35}{3}a^4b^6x^{18} + \frac{252}{17}a^5b^5x^{17} + \frac{105}{8}a^6b^4x^{16} + 8a^7b^3x^{15} + \frac{45}{14}a^8b^2x^{14} + \frac{10}{13}a^9bx^{13} + \frac{1}{12}a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

maple [A] time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{22}b^{10}x^{22} + \frac{10}{21}a\,b^9x^{21} + \frac{9}{4}a^2b^8x^{20} + \frac{120}{19}a^3b^7x^{19} + \frac{35}{3}a^4b^6x^{18} + \frac{252}{17}a^5b^5x^{17} + \frac{105}{8}a^6b^4x^{16} + 8a^7b^3x^{15} + \frac{45}{14}a^8b^2x^{14} + \frac{10}{13}a^9b\,x^{13} + \frac{1}{12}a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(b*x+a)^10,x)
```

```
[Out] 1/12*a^10*x^12+10/13*a^9*b*x^13+45/14*a^8*b^2*x^14+8*a^7*b^3*x^15+105/8*a^6
*b^4*x^16+252/17*a^5*b^5*x^17+35/3*a^4*b^6*x^18+120/19*a^3*b^7*x^19+9/4*a^2
*b^8*x^20+10/21*a*b^9*x^21+1/22*b^10*x^22
```

maxima [A] time = 1.33, size = 112, normalized size = 0.85

$$\frac{1}{22}b^{10}x^{22} + \frac{10}{21}ab^9x^{21} + \frac{9}{4}a^2b^8x^{20} + \frac{120}{19}a^3b^7x^{19} + \frac{35}{3}a^4b^6x^{18} + \frac{252}{17}a^5b^5x^{17} + \frac{105}{8}a^6b^4x^{16} + 8a^7b^3x^{15} + \frac{45}{14}a^8b^2x^{14} + \frac{10}{13}a^9bx^{13} + \frac{1}{12}a^{10}x^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^10*x^22 + 10/21*a*b^9*x^21 + 9/4*a^2*b^8*x^20 + 120/19*a^3*b^7*x^19
+ 35/3*a^4*b^6*x^18 + 252/17*a^5*b^5*x^17 + 105/8*a^6*b^4*x^16 + 8*a^7*b^3*
x^15 + 45/14*a^8*b^2*x^14 + 10/13*a^9*b*x^13 + 1/12*a^10*x^12
```

mupad [B] time = 0.15, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{12}}{12} + \frac{10\,a^9\,b\,x^{13}}{13} + \frac{45\,a^8\,b^2\,x^{14}}{14} + 8\,a^7\,b^3\,x^{15} + \frac{105\,a^6\,b^4\,x^{16}}{8} + \frac{252\,a^5\,b^5\,x^{17}}{17} + \frac{35\,a^4\,b^6\,x^{18}}{3} + \frac{120\,a^3\,b^7\,x^{19}}{19} + \frac{9\,a^2\,b^8\,x^{20}}{4} + \frac{10\,a\,b^9\,x^{21}}{21} + \frac{b^{10}\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a + b*x)^10,x)
```

```
[Out] (a^10*x^12)/12 + (b^10*x^22)/22 + (10*a^9*b*x^13)/13 + (10*a*b^9*x^21)/21 +
 (45*a^8*b^2*x^14)/14 + 8*a^7*b^3*x^15 + (105*a^6*b^4*x^16)/8 + (252*a^5*b^
5*x^17)/17 + (35*a^4*b^6*x^18)/3 + (120*a^3*b^7*x^19)/19 + (9*a^2*b^8*x^20)
/4
```

sympy [A] time = 0.11, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{12}}{12} + \frac{10a^9bx^{13}}{13} + \frac{45a^8b^2x^{14}}{14} + 8a^7b^3x^{15} + \frac{105a^6b^4x^{16}}{8} + \frac{252a^5b^5x^{17}}{17} + \frac{35a^4b^6x^{18}}{3} + \frac{120a^3b^7x^{19}}{19} + \frac{9a^2b^8x^{20}}{4} + \frac{10ab^9x^{21}}{21} + \frac{b^{10}x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(b*x+a)**10,x)
```

```
[Out] a**10*x**12/12 + 10*a**9*b*x**13/13 + 45*a**8*b**2*x**14/14 + 8*a**7*b**3*x
**15 + 105*a**6*b**4*x**16/8 + 252*a**5*b**5*x**17/17 + 35*a**4*b**6*x**18/
3 + 120*a**3*b**7*x**19/19 + 9*a**2*b**8*x**20/4 + 10*a*b**9*x**21/21 + b**
10*x**22/22
```

3.124 $\int x^{10}(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}$$

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{19}a^2b^8x^{19}+\frac{20}{3}a^3b^7x^{18}+\frac{210}{17}a^4b^6x^{17}+\frac{63}{4}a^5b^5x^{16}+14a^6b^4x^{15}+\frac{60}{7}a^7b^3x^{14}+\frac{45}{13}a^8b^2x^{13}+\frac{5}{6}a^9bx^{12}+\frac{a^{10}x^{11}}{11}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^{10}(a+bx)^{10}\,dx = \int \left(a^{10}x^{10}+10a^9bx^{11}+45a^8b^2x^{12}+120a^7b^3x^{13}+210a^6b^4x^{14}+252a^5b^5x^{15}+210a^4b^6x^{16}\right.$$
$$=\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}$$

Mathematica [A] time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{11}}{11}+\frac{5}{6}a^9bx^{12}+\frac{45}{13}a^8b^2x^{13}+\frac{60}{7}a^7b^3x^{14}+14a^6b^4x^{15}+\frac{63}{4}a^5b^5x^{16}+\frac{210}{17}a^4b^6x^{17}+\frac{20}{3}a^3b^7x^{18}+\frac{45}{19}a^2b^8x^{19}+\frac{1}{2}ab^9x^{20}+\frac{b^{10}x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10*(a + b*x)^10,x]
```

```
[Out] (a^10*x^11)/11 + (5*a^9*b*x^12)/6 + (45*a^8*b^2*x^13)/13 + (60*a^7*b^3*x^14
)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)/4 + (210*a^4*b^6*x^17)/17 + (20*a
^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19 + (a*b^9*x^20)/2 + (b^10*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{10}(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^10*(a + b*x)^10, x]
```

fricas [A] time = 1.05, size = 112, normalized size = 0.85

$$\frac{1}{21}x^{21}b^{10} + \frac{1}{2}x^{20}b^{9}a + \frac{45}{19}x^{19}b^{8}a^{2} + \frac{20}{3}x^{18}b^{7}a^{3} + \frac{210}{17}x^{17}b^{6}a^{4} + \frac{63}{4}x^{16}b^{5}a^{5} + 14x^{15}b^{4}a^{6} + \frac{60}{7}x^{14}b^{3}a^{7} + \frac{45}{13}x^{13}b^{2}a^{8} + \frac{5}{6}x^{12}ba^{9} + \frac{1}{11}x^{11}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*b^10 + 1/2*x^20*b^9*a + 45/19*x^19*b^8*a^2 + 20/3*x^18*b^7*a^3 +
210/17*x^17*b^6*a^4 + 63/4*x^16*b^5*a^5 + 14*x^15*b^4*a^6 + 60/7*x^14*b^3*a
^7 + 45/13*x^13*b^2*a^8 + 5/6*x^12*b*a^9 + 1/11*x^11*a^10
```

giac [A] time = 0.99, size = 112, normalized size = 0.85

$$\frac{1}{21}b^{10}x^{21} + \frac{1}{2}ab^{9}x^{20} + \frac{45}{19}a^{2}b^{8}x^{19} + \frac{20}{3}a^{3}b^{7}x^{18} + \frac{210}{17}a^{4}b^{6}x^{17} + \frac{63}{4}a^{5}b^{5}x^{16} + 14a^{6}b^{4}x^{15} + \frac{60}{7}a^{7}b^{3}x^{14} + \frac{45}{13}a^{8}b^{2}x^{13} + \frac{5}{6}a^{9}bx^{12} + \frac{1}{11}a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

maple [A] time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{21}b^{10}x^{21} + \frac{1}{2}a\,b^{9}x^{20} + \frac{45}{19}a^{2}b^{8}x^{19} + \frac{20}{3}a^{3}b^{7}x^{18} + \frac{210}{17}a^{4}b^{6}x^{17} + \frac{63}{4}a^{5}b^{5}x^{16} + 14a^{6}b^{4}x^{15} + \frac{60}{7}a^{7}b^{3}x^{14} + \frac{45}{13}a^{8}b^{2}x^{13} + \frac{5}{6}a^{9}b\,x^{12} + \frac{1}{11}a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(b*x+a)^10,x)
```

```
[Out] 1/11*a^10*x^11+5/6*a^9*b*x^12+45/13*a^8*b^2*x^13+60/7*a^7*b^3*x^14+14*a^6*b
^4*x^15+63/4*a^5*b^5*x^16+210/17*a^4*b^6*x^17+20/3*a^3*b^7*x^18+45/19*a^2*b
^8*x^19+1/2*a*b^9*x^20+1/21*b^10*x^21
```

maxima [A] time = 1.34, size = 112, normalized size = 0.85

$$\frac{1}{21}b^{10}x^{21} + \frac{1}{2}ab^{9}x^{20} + \frac{45}{19}a^{2}b^{8}x^{19} + \frac{20}{3}a^{3}b^{7}x^{18} + \frac{210}{17}a^{4}b^{6}x^{17} + \frac{63}{4}a^{5}b^{5}x^{16} + 14a^{6}b^{4}x^{15} + \frac{60}{7}a^{7}b^{3}x^{14} + \frac{45}{13}a^{8}b^{2}x^{13} + \frac{5}{6}a^{9}bx^{12} + \frac{1}{11}a^{10}x^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*x^21 + 1/2*a*b^9*x^20 + 45/19*a^2*b^8*x^19 + 20/3*a^3*b^7*x^18 +
210/17*a^4*b^6*x^17 + 63/4*a^5*b^5*x^16 + 14*a^6*b^4*x^15 + 60/7*a^7*b^3*x^
14 + 45/13*a^8*b^2*x^13 + 5/6*a^9*b*x^12 + 1/11*a^10*x^11
```

mupad [B] time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{11}}{11} + \frac{5\,a^{9}\,b\,x^{12}}{6} + \frac{45\,a^{8}\,b^{2}\,x^{13}}{13} + \frac{60\,a^{7}\,b^{3}\,x^{14}}{7} + 14\,a^{6}\,b^{4}\,x^{15} + \frac{63\,a^{5}\,b^{5}\,x^{16}}{4} + \frac{210\,a^{4}\,b^{6}\,x^{17}}{17} + \frac{20\,a^{3}\,b^{7}\,x^{18}}{3} + \frac{45\,a^{2}\,b^{8}\,x^{19}}{19} + \frac{a\,b^{9}\,x^{20}}{2} + \frac{b^{10}\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(a + b*x)^10,x)
```

```
[Out] (a^10*x^11)/11 + (b^10*x^21)/21 + (5*a^9*b*x^12)/6 + (a*b^9*x^20)/2 + (45*a
^8*b^2*x^13)/13 + (60*a^7*b^3*x^14)/7 + 14*a^6*b^4*x^15 + (63*a^5*b^5*x^16)
/4 + (210*a^4*b^6*x^17)/17 + (20*a^3*b^7*x^18)/3 + (45*a^2*b^8*x^19)/19
```

sympy [A] time = 0.11, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{11}}{11} + \frac{5a^{9}bx^{12}}{6} + \frac{45a^{8}b^{2}x^{13}}{13} + \frac{60a^{7}b^{3}x^{14}}{7} + 14a^{6}b^{4}x^{15} + \frac{63a^{5}b^{5}x^{16}}{4} + \frac{210a^{4}b^{6}x^{17}}{17} + \frac{20a^{3}b^{7}x^{18}}{3} + \frac{45a^{2}b^{8}x^{19}}{19} + \frac{ab^{9}x^{20}}{2} + \frac{b^{10}x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(b*x+a)**10,x)
```

```
[Out] a**10*x**11/11 + 5*a**9*b*x**12/6 + 45*a**8*b**2*x**13/13 + 60*a**7*b**3*x*
*14/7 + 14*a**6*b**4*x**15 + 63*a**5*b**5*x**16/4 + 210*a**4*b**6*x**17/17
+ 20*a**3*b**7*x**18/3 + 45*a**2*b**8*x**19/19 + a*b**9*x**20/2 + b**10*x**
21/21
```

3.125 $\int x^9(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}a$$

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5}{2}a^2b^8x^{18}+\frac{120}{17}a^3b^7x^{17}+\frac{105}{8}a^4b^6x^{16}+\frac{84}{5}a^5b^5x^{15}+15a^6b^4x^{14}+\frac{120}{13}a^7b^3x^{13}+\frac{15}{4}a^8b^2x^{12}+\frac{10}{11}a^9bx^{11}+\frac{a^{10}x^{10}}{10}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^9(a+bx)^{10}\,dx=\int\left(a^{10}x^9+10a^9bx^{10}+45a^8b^2x^{11}+120a^7b^3x^{12}+210a^6b^4x^{13}+252a^5b^5x^{14}+210a^4b^6x^{1}\right.$$
$$=\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+$$

Mathematica [A] time = 0.00, size = 132, normalized size = 1.00

$$\frac{a^{10}x^{10}}{10}+\frac{10}{11}a^9bx^{11}+\frac{15}{4}a^8b^2x^{12}+\frac{120}{13}a^7b^3x^{13}+15a^6b^4x^{14}+\frac{84}{5}a^5b^5x^{15}+\frac{105}{8}a^4b^6x^{16}+\frac{120}{17}a^3b^7x^{17}+\frac{5}{2}a^2b^8x^{18}+\frac{10}{19}ab^9x^{19}+\frac{b^{10}x^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*(a + b*x)^10,x]
```

```
[Out] (a^10*x^10)/10 + (10*a^9*b*x^11)/11 + (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^
13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (12
0*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)/2 + (10*a*b^9*x^19)/19 + (b^10*x^20)/
20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^9(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^9*(a + b*x)^10, x]
```

fricas [A] time = 0.74, size = 112, normalized size = 0.85

$$\frac{1}{20}x^{20}b^{10} + \frac{10}{19}x^{19}b^9a + \frac{5}{2}x^{18}b^8a^2 + \frac{120}{17}x^{17}b^7a^3 + \frac{105}{8}x^{16}b^6a^4 + \frac{84}{5}x^{15}b^5a^5 + 15x^{14}b^4a^6 + \frac{120}{13}x^{13}b^3a^7 + \frac{15}{4}x^{12}b^2a^8 + \frac{10}{11}x^{11}ba^9 + \frac{1}{10}x^{10}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*b^10 + 10/19*x^19*b^9*a + 5/2*x^18*b^8*a^2 + 120/17*x^17*b^7*a^3
+ 105/8*x^16*b^6*a^4 + 84/5*x^15*b^5*a^5 + 15*x^14*b^4*a^6 + 120/13*x^13*b^
3*a^7 + 15/4*x^12*b^2*a^8 + 10/11*x^11*b*a^9 + 1/10*x^10*a^10
```

giac [A] time = 1.12, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20} + \frac{10}{19}\,ab^9x^{19} + \frac{5}{2}\,a^2b^8x^{18} + \frac{120}{17}\,a^3b^7x^{17} + \frac{105}{8}\,a^4b^6x^{16} + \frac{84}{5}\,a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}\,a^7b^3x^{13} + \frac{15}{4}\,a^8b^2x^{12} + \frac{10}{11}\,a^9bx^{11} + \frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

maple [A] time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{20}b^{10}x^{20} + \frac{10}{19}a\,b^9x^{19} + \frac{5}{2}a^2b^8x^{18} + \frac{120}{17}a^3b^7x^{17} + \frac{105}{8}a^4b^6x^{16} + \frac{84}{5}a^5b^5x^{15} + 15a^6b^4x^{14} + \frac{120}{13}a^7b^3x^{13} + \frac{15}{4}a^8b^2x^{12} + \frac{10}{11}a^9b\,x^{11} + \frac{1}{10}a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(b*x+a)^10,x)
```

```
[Out] 1/10*a^10*x^10+10/11*a^9*b*x^11+15/4*a^8*b^2*x^12+120/13*a^7*b^3*x^13+15*a^
6*b^4*x^14+84/5*a^5*b^5*x^15+105/8*a^4*b^6*x^16+120/17*a^3*b^7*x^17+5/2*a^2
*b^8*x^18+10/19*a*b^9*x^19+1/20*b^10*x^20
```

maxima [A] time = 1.37, size = 112, normalized size = 0.85

$$\frac{1}{20}\,b^{10}x^{20} + \frac{10}{19}\,ab^9x^{19} + \frac{5}{2}\,a^2b^8x^{18} + \frac{120}{17}\,a^3b^7x^{17} + \frac{105}{8}\,a^4b^6x^{16} + \frac{84}{5}\,a^5b^5x^{15} + 15\,a^6b^4x^{14} + \frac{120}{13}\,a^7b^3x^{13} + \frac{15}{4}\,a^8b^2x^{12} + \frac{10}{11}\,a^9bx^{11} + \frac{1}{10}\,a^{10}x^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^10*x^20 + 10/19*a*b^9*x^19 + 5/2*a^2*b^8*x^18 + 120/17*a^3*b^7*x^17
+ 105/8*a^4*b^6*x^16 + 84/5*a^5*b^5*x^15 + 15*a^6*b^4*x^14 + 120/13*a^7*b^3
*x^13 + 15/4*a^8*b^2*x^12 + 10/11*a^9*b*x^11 + 1/10*a^10*x^10
```

mupad [B] time = 0.12, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^{10}}{10} + \frac{10\,a^9\,b\,x^{11}}{11} + \frac{15\,a^8\,b^2\,x^{12}}{4} + \frac{120\,a^7\,b^3\,x^{13}}{13} + 15\,a^6\,b^4\,x^{14} + \frac{84\,a^5\,b^5\,x^{15}}{5} + \frac{105\,a^4\,b^6\,x^{16}}{8} + \frac{120\,a^3\,b^7\,x^{17}}{17} + \frac{5\,a^2\,b^8\,x^{18}}{2} + \frac{10\,a\,b^9\,x^{19}}{19} + \frac{b^{10}\,x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(a + b*x)^10,x)
```

```
[Out] (a^10*x^10)/10 + (b^10*x^20)/20 + (10*a^9*b*x^11)/11 + (10*a*b^9*x^19)/19 +
 (15*a^8*b^2*x^12)/4 + (120*a^7*b^3*x^13)/13 + 15*a^6*b^4*x^14 + (84*a^5*b^
5*x^15)/5 + (105*a^4*b^6*x^16)/8 + (120*a^3*b^7*x^17)/17 + (5*a^2*b^8*x^18)
/2
```

sympy [A] time = 0.10, size = 133, normalized size = 1.01

$$\frac{a^{10}x^{10}}{10} + \frac{10a^9bx^{11}}{11} + \frac{15a^8b^2x^{12}}{4} + \frac{120a^7b^3x^{13}}{13} + 15a^6b^4x^{14} + \frac{84a^5b^5x^{15}}{5} + \frac{105a^4b^6x^{16}}{8} + \frac{120a^3b^7x^{17}}{17} + \frac{5a^2b^8x^{18}}{2} + \frac{10ab^9x^{19}}{19} + \frac{b^{10}x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(b*x+a)**10,x)
```

```
[Out] a**10*x**10/10 + 10*a**9*b*x**11/11 + 15*a**8*b**2*x**12/4 + 120*a**7*b**3*
x**13/13 + 15*a**6*b**4*x**14 + 84*a**5*b**5*x**15/5 + 105*a**4*b**6*x**16/
8 + 120*a**3*b**7*x**17/17 + 5*a**2*b**8*x**18/2 + 10*a*b**9*x**19/19 + b**
10*x**20/20
```

3.126 $\int x^8(a+bx)^{10}\,dx$

Optimal. Leaf size=147

$$\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{28a^2(a+bx)^{17}}{17b^9}+\frac{(a$$

Rubi [A] time = 0.06, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{28a^2(a+bx)^{17}}{17b^9}-\frac{7a^3(a+bx)^{16}}{2b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{a^8(a+bx)^{11}}{11b^9}+\frac{(a+bx)^{19}}{19b^9}-\frac{4a(a+bx)^{18}}{9b^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8*(a + b*x)^10,x]
```

```
[Out] (a^8*(a + b*x)^11)/(11*b^9) - (2*a^7*(a + b*x)^12)/(3*b^9) + (28*a^6*(a + b
*x)^13)/(13*b^9) - (4*a^5*(a + b*x)^14)/b^9 + (14*a^4*(a + b*x)^15)/(3*b^9)
 - (7*a^3*(a + b*x)^16)/(2*b^9) + (28*a^2*(a + b*x)^17)/(17*b^9) - (4*a*(a
+ b*x)^18)/(9*b^9) + (a + b*x)^19/(19*b^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^8(a+bx)^{10}\,dx=\int\left(\frac{a^8(a+bx)^{10}}{b^8}-\frac{8a^7(a+bx)^{11}}{b^8}+\frac{28a^6(a+bx)^{12}}{b^8}-\frac{56a^5(a+bx)^{13}}{b^8}+\frac{70a^4(a+bx)^{14}}{b^8}-56\right.$$
$$=\frac{a^8(a+bx)^{11}}{11b^9}-\frac{2a^7(a+bx)^{12}}{3b^9}+\frac{28a^6(a+bx)^{13}}{13b^9}-\frac{4a^5(a+bx)^{14}}{b^9}+\frac{14a^4(a+bx)^{15}}{3b^9}-\frac{7a^3(a+}{2b}$$

Mathematica [A] time = 0.00, size = 125, normalized size = 0.85

$$\frac{a^{10}x^9}{9}+a^9bx^{10}+\frac{45}{11}a^8b^2x^{11}+10a^7b^3x^{12}+\frac{210}{13}a^6b^4x^{13}+18a^5b^5x^{14}+14a^4b^6x^{15}+\frac{15}{2}a^3b^7x^{16}+\frac{45}{17}a^2b^8x^{17}+\frac{5}{9}ab^9x^{18}+\frac{b^{10}x^{19}}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*(a + b*x)^10,x]
```

```
[Out] (a^10*x^9)/9 + a^9*b*x^10 + (45*a^8*b^2*x^11)/11 + 10*a^7*b^3*x^12 + (210*a
^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 +
 (45*a^2*b^8*x^17)/17 + (5*a*b^9*x^18)/9 + (b^10*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^8*(a + b*x)^10, x]
```

fricas [A] time = 1.09, size = 111, normalized size = 0.76

$$\frac{1}{19}x^{19}b^{10}+\frac{5}{9}x^{18}b^{9}a+\frac{45}{17}x^{17}b^{8}a^{2}+\frac{15}{2}x^{16}b^{7}a^{3}+14x^{15}b^{6}a^{4}+18x^{14}b^{5}a^{5}+\frac{210}{13}x^{13}b^{4}a^{6}+10x^{12}b^{3}a^{7}+\frac{45}{11}x^{11}b^{2}a^{8}+x^{10}ba^{9}+\frac{1}{9}x^{9}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*b^10 + 5/9*x^18*b^9*a + 45/17*x^17*b^8*a^2 + 15/2*x^16*b^7*a^3 +
14*x^15*b^6*a^4 + 18*x^14*b^5*a^5 + 210/13*x^13*b^4*a^6 + 10*x^12*b^3*a^7 +
 45/11*x^11*b^2*a^8 + x^10*b*a^9 + 1/9*x^9*a^10
```

giac [A] time = 1.01, size = 111, normalized size = 0.76

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}ab^{9}x^{18}+\frac{45}{17}a^{2}b^{8}x^{17}+\frac{15}{2}a^{3}b^{7}x^{16}+14a^{4}b^{6}x^{15}+18a^{5}b^{5}x^{14}+\frac{210}{13}a^{6}b^{4}x^{13}+10a^{7}b^{3}x^{12}+\frac{45}{11}a^{8}b^{2}x^{11}+a^{9}bx^{10}+\frac{1}{9}a^{10}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

maple [A] time = 0.00, size = 112, normalized size = 0.76

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}a\,b^{9}x^{18}+\frac{45}{17}a^{2}b^{8}x^{17}+\frac{15}{2}a^{3}b^{7}x^{16}+14a^{4}b^{6}x^{15}+18a^{5}b^{5}x^{14}+\frac{210}{13}a^{6}b^{4}x^{13}+10a^{7}b^{3}x^{12}+\frac{45}{11}a^{8}b^{2}x^{11}+a^{9}b\,x^{10}+\frac{1}{9}a^{10}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(b*x+a)^10,x)
```

```
[Out] 1/19*b^10*x^19+5/9*a*b^9*x^18+45/17*a^2*b^8*x^17+15/2*a^3*b^7*x^16+14*a^4*b
^6*x^15+18*a^5*b^5*x^14+210/13*a^6*b^4*x^13+10*a^7*b^3*x^12+45/11*a^8*b^2*x
^11+a^9*b*x^10+1/9*a^10*x^9
```

maxima [A] time = 1.36, size = 111, normalized size = 0.76

$$\frac{1}{19}b^{10}x^{19}+\frac{5}{9}ab^{9}x^{18}+\frac{45}{17}a^{2}b^{8}x^{17}+\frac{15}{2}a^{3}b^{7}x^{16}+14a^{4}b^{6}x^{15}+18a^{5}b^{5}x^{14}+\frac{210}{13}a^{6}b^{4}x^{13}+10a^{7}b^{3}x^{12}+\frac{45}{11}a^{8}b^{2}x^{11}+a^{9}bx^{10}+\frac{1}{9}a^{10}x^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^10*x^19 + 5/9*a*b^9*x^18 + 45/17*a^2*b^8*x^17 + 15/2*a^3*b^7*x^16 +
14*a^4*b^6*x^15 + 18*a^5*b^5*x^14 + 210/13*a^6*b^4*x^13 + 10*a^7*b^3*x^12 +
 45/11*a^8*b^2*x^11 + a^9*b*x^10 + 1/9*a^10*x^9
```

mupad [B] time = 0.09, size = 111, normalized size = 0.76

$$\frac{a^{10}\,x^{9}}{9}+a^{9}\,b\,x^{10}+\frac{45\,a^{8}\,b^{2}\,x^{11}}{11}+10\,a^{7}\,b^{3}\,x^{12}+\frac{210\,a^{6}\,b^{4}\,x^{13}}{13}+18\,a^{5}\,b^{5}\,x^{14}+14\,a^{4}\,b^{6}\,x^{15}+\frac{15\,a^{3}\,b^{7}\,x^{16}}{2}+\frac{45\,a^{2}\,b^{8}\,x^{17}}{17}+\frac{5\,a\,b^{9}\,x^{18}}{9}+\frac{b^{10}\,x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(a + b*x)^10,x)
```

```
[Out] (a^10*x^9)/9 + (b^10*x^19)/19 + a^9*b*x^10 + (5*a*b^9*x^18)/9 + (45*a^8*b^2
*x^11)/11 + 10*a^7*b^3*x^12 + (210*a^6*b^4*x^13)/13 + 18*a^5*b^5*x^14 + 14*
a^4*b^6*x^15 + (15*a^3*b^7*x^16)/2 + (45*a^2*b^8*x^17)/17
```

sympy [A] time = 0.11, size = 126, normalized size = 0.86

$$\frac{a^{10}x^{9}}{9}+a^{9}bx^{10}+\frac{45a^{8}b^{2}x^{11}}{11}+10a^{7}b^{3}x^{12}+\frac{210a^{6}b^{4}x^{13}}{13}+18a^{5}b^{5}x^{14}+14a^{4}b^{6}x^{15}+\frac{15a^{3}b^{7}x^{16}}{2}+\frac{45a^{2}b^{8}x^{17}}{17}+\frac{5ab^{9}x^{18}}{9}+\frac{b^{10}x^{19}}{19}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**10,x)
```

```
[Out] a**10*x**9/9 + a**9*b*x**10 + 45*a**8*b**2*x**11/11 + 10*a**7*b**3*x**12 +
210*a**6*b**4*x**13/13 + 18*a**5*b**5*x**14 + 14*a**4*b**6*x**15 + 15*a**3*
b**7*x**16/2 + 45*a**2*b**8*x**17/17 + 5*a*b**9*x**18/9 + b**10*x**19/19
```

3.127 $\int x^7(a+bx)^{10}\,dx$

Optimal. Leaf size=132

$$-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2(a+bx)^{16}}{16b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a}{}$$

Rubi [A] time = 0.06, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21a^2(a+bx)^{16}}{16b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{(a+bx)^{18}}{18b^8}-\frac{7a(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^10,x]
```

```
[Out] -(a^7*(a + b*x)^11)/(11*b^8) + (7*a^6*(a + b*x)^12)/(12*b^8) - (21*a^5*(a +
 b*x)^13)/(13*b^8) + (5*a^4*(a + b*x)^14)/(2*b^8) - (7*a^3*(a + b*x)^15)/(3
*b^8) + (21*a^2*(a + b*x)^16)/(16*b^8) - (7*a*(a + b*x)^17)/(17*b^8) + (a +
 b*x)^18/(18*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7(a+bx)^{10}\,dx=\int\left(-\frac{a^7(a+bx)^{10}}{b^7}+\frac{7a^6(a+bx)^{11}}{b^7}-\frac{21a^5(a+bx)^{12}}{b^7}+\frac{35a^4(a+bx)^{13}}{b^7}-\frac{35a^3(a+bx)^{14}}{b^7}\right.$$
$$=-\frac{a^7(a+bx)^{11}}{11b^8}+\frac{7a^6(a+bx)^{12}}{12b^8}-\frac{21a^5(a+bx)^{13}}{13b^8}+\frac{5a^4(a+bx)^{14}}{2b^8}-\frac{7a^3(a+bx)^{15}}{3b^8}+\frac{21a^2}{}$$

Mathematica [A] time = 0.00, size = 130, normalized size = 0.98

$$\frac{a^{10}x^8}{8}+\frac{10}{9}a^9bx^9+\frac{9}{2}a^8b^2x^{10}+\frac{120}{11}a^7b^3x^{11}+\frac{35}{2}a^6b^4x^{12}+\frac{252}{13}a^5b^5x^{13}+15a^4b^6x^{14}+8a^3b^7x^{15}+\frac{45}{16}a^2b^8x^{16}+\frac{10}{17}ab^9x^{17}+\frac{b^{10}x^{18}}{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^10,x]
```

```
[Out] (a^10*x^8)/8 + (10*a^9*b*x^9)/9 + (9*a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/1
1 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b
^7*x^15 + (45*a^2*b^8*x^16)/16 + (10*a*b^9*x^17)/17 + (b^10*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^7(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^7*(a + b*x)^10, x]
```

fricas [A] time = 1.07, size = 112, normalized size = 0.85

$$\frac{1}{18}x^{18}b^{10}+\frac{10}{17}x^{17}b^{9}a+\frac{45}{16}x^{16}b^{8}a^{2}+8x^{15}b^{7}a^{3}+15x^{14}b^{6}a^{4}+\frac{252}{13}x^{13}b^{5}a^{5}+\frac{35}{2}x^{12}b^{4}a^{6}+\frac{120}{11}x^{11}b^{3}a^{7}+\frac{9}{2}x^{10}b^{2}a^{8}+\frac{10}{9}x^{9}ba^{9}+\frac{1}{8}x^{8}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*b^10 + 10/17*x^17*b^9*a + 45/16*x^16*b^8*a^2 + 8*x^15*b^7*a^3 + 1
5*x^14*b^6*a^4 + 252/13*x^13*b^5*a^5 + 35/2*x^12*b^4*a^6 + 120/11*x^11*b^3*
a^7 + 9/2*x^10*b^2*a^8 + 10/9*x^9*b*a^9 + 1/8*x^8*a^10
```

giac [A] time = 0.84, size = 112, normalized size = 0.85

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}ab^{9}x^{17}+\frac{45}{16}a^{2}b^{8}x^{16}+8a^{3}b^{7}x^{15}+15a^{4}b^{6}x^{14}+\frac{252}{13}a^{5}b^{5}x^{13}+\frac{35}{2}a^{6}b^{4}x^{12}+\frac{120}{11}a^{7}b^{3}x^{11}+\frac{9}{2}a^{8}b^{2}x^{10}+\frac{10}{9}a^{9}bx^{9}+\frac{1}{8}a^{10}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

maple [A] time = 0.00, size = 113, normalized size = 0.86

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}a\,b^{9}x^{17}+\frac{45}{16}a^{2}b^{8}x^{16}+8a^{3}b^{7}x^{15}+15a^{4}b^{6}x^{14}+\frac{252}{13}a^{5}b^{5}x^{13}+\frac{35}{2}a^{6}b^{4}x^{12}+\frac{120}{11}a^{7}b^{3}x^{11}+\frac{9}{2}a^{8}b^{2}x^{10}+\frac{10}{9}a^{9}b\,x^{9}+\frac{1}{8}a^{10}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^10,x)
```

```
[Out] 1/18*b^10*x^18+10/17*a*b^9*x^17+45/16*a^2*b^8*x^16+8*a^3*b^7*x^15+15*a^4*b^
6*x^14+252/13*a^5*b^5*x^13+35/2*a^6*b^4*x^12+120/11*a^7*b^3*x^11+9/2*a^8*b^
2*x^10+10/9*a^9*b*x^9+1/8*a^10*x^8
```

maxima [A] time = 1.34, size = 112, normalized size = 0.85

$$\frac{1}{18}b^{10}x^{18}+\frac{10}{17}ab^{9}x^{17}+\frac{45}{16}a^{2}b^{8}x^{16}+8a^{3}b^{7}x^{15}+15a^{4}b^{6}x^{14}+\frac{252}{13}a^{5}b^{5}x^{13}+\frac{35}{2}a^{6}b^{4}x^{12}+\frac{120}{11}a^{7}b^{3}x^{11}+\frac{9}{2}a^{8}b^{2}x^{10}+\frac{10}{9}a^{9}bx^{9}+\frac{1}{8}a^{10}x^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^10*x^18 + 10/17*a*b^9*x^17 + 45/16*a^2*b^8*x^16 + 8*a^3*b^7*x^15 + 1
5*a^4*b^6*x^14 + 252/13*a^5*b^5*x^13 + 35/2*a^6*b^4*x^12 + 120/11*a^7*b^3*x
^11 + 9/2*a^8*b^2*x^10 + 10/9*a^9*b*x^9 + 1/8*a^10*x^8
```

mupad [B] time = 0.08, size = 112, normalized size = 0.85

$$\frac{a^{10}\,x^8}{8}+\frac{10\,a^9\,b\,x^9}{9}+\frac{9\,a^8\,b^2\,x^{10}}{2}+\frac{120\,a^7\,b^3\,x^{11}}{11}+\frac{35\,a^6\,b^4\,x^{12}}{2}+\frac{252\,a^5\,b^5\,x^{13}}{13}+15\,a^4\,b^6\,x^{14}+8\,a^3\,b^7\,x^{15}+\frac{45\,a^2\,b^8\,x^{16}}{16}+\frac{10\,a\,b^9\,x^{17}}{17}+\frac{b^{10}\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^10,x)
```

```
[Out] (a^10*x^8)/8 + (b^10*x^18)/18 + (10*a^9*b*x^9)/9 + (10*a*b^9*x^17)/17 + (9*
a^8*b^2*x^10)/2 + (120*a^7*b^3*x^11)/11 + (35*a^6*b^4*x^12)/2 + (252*a^5*b^
5*x^13)/13 + 15*a^4*b^6*x^14 + 8*a^3*b^7*x^15 + (45*a^2*b^8*x^16)/16
```

sympy [A] time = 0.11, size = 131, normalized size = 0.99

$$\frac{a^{10}x^{8}}{8}+\frac{10a^{9}bx^{9}}{9}+\frac{9a^{8}b^{2}x^{10}}{2}+\frac{120a^{7}b^{3}x^{11}}{11}+\frac{35a^{6}b^{4}x^{12}}{2}+\frac{252a^{5}b^{5}x^{13}}{13}+15a^{4}b^{6}x^{14}+8a^{3}b^{7}x^{15}+\frac{45a^{2}b^{8}x^{16}}{16}+\frac{10ab^{9}x^{17}}{17}+\frac{b^{10}x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**10,x)
```

```
[Out] a**10*x**8/8 + 10*a**9*b*x**9/9 + 9*a**8*b**2*x**10/2 + 120*a**7*b**3*x**11
/11 + 35*a**6*b**4*x**12/2 + 252*a**5*b**5*x**13/13 + 15*a**4*b**6*x**14 +
8*a**3*b**7*x**15 + 45*a**2*b**8*x**16/16 + 10*a*b**9*x**17/17 + b**10*x**1
8/18
```

3.128 $\int x^6(a+bx)^{10}\,dx$

Optimal. Leaf size=112

$$\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Rubi [A] time = 0.05, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{15}}{b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{a^6(a+bx)^{11}}{11b^7}+\frac{(a+bx)^{17}}{17b^7}-\frac{3a(a+bx)^{16}}{8b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^10,x]
```

```
[Out] (a^6*(a + b*x)^11)/(11*b^7) - (a^5*(a + b*x)^12)/(2*b^7) + (15*a^4*(a + b*x
)^13)/(13*b^7) - (10*a^3*(a + b*x)^14)/(7*b^7) + (a^2*(a + b*x)^15)/b^7 - (
3*a*(a + b*x)^16)/(8*b^7) + (a + b*x)^17/(17*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^6(a+bx)^{10}\,dx=\int\left(\frac{a^6(a+bx)^{10}}{b^6}-\frac{6a^5(a+bx)^{11}}{b^6}+\frac{15a^4(a+bx)^{12}}{b^6}-\frac{20a^3(a+bx)^{13}}{b^6}+\frac{15a^2(a+bx)^{14}}{b^6}-\frac{6a\right.$$
$$=\frac{a^6(a+bx)^{11}}{11b^7}-\frac{a^5(a+bx)^{12}}{2b^7}+\frac{15a^4(a+bx)^{13}}{13b^7}-\frac{10a^3(a+bx)^{14}}{7b^7}+\frac{a^2(a+bx)^{15}}{b^7}-\frac{3a(a+bx}{8b^7}$$

Mathematica [A] time = 0.00, size = 126, normalized size = 1.12

$$\frac{a^{10}x^7}{7}+\frac{5}{4}a^9bx^8+5a^8b^2x^9+12a^7b^3x^{10}+\frac{210}{11}a^6b^4x^{11}+21a^5b^5x^{12}+\frac{210}{13}a^4b^6x^{13}+\frac{60}{7}a^3b^7x^{14}+3a^2b^8x^{15}+\frac{5}{8}ab^9x^{16}+\frac{b^{10}x^{17}}{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^10,x]
```

```
[Out] (a^10*x^7)/7 + (5*a^9*b*x^8)/4 + 5*a^8*b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6
*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)
/7 + 3*a^2*b^8*x^15 + (5*a*b^9*x^16)/8 + (b^10*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^6*(a + b*x)^10, x]
```

fricas [A] time = 0.97, size = 112, normalized size = 1.00

$$\frac{1}{17}x^{17}b^{10} + \frac{5}{8}x^{16}b^9a + 3x^{15}b^8a^2 + \frac{60}{7}x^{14}b^7a^3 + \frac{210}{13}x^{13}b^6a^4 + 21x^{12}b^5a^5 + \frac{210}{11}x^{11}b^4a^6 + 12x^{10}b^3a^7 + 5x^9b^2a^8 + \frac{5}{4}x^8ba^9 + \frac{1}{7}x^7a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*b^10 + 5/8*x^16*b^9*a + 3*x^15*b^8*a^2 + 60/7*x^14*b^7*a^3 + 210/
13*x^13*b^6*a^4 + 21*x^12*b^5*a^5 + 210/11*x^11*b^4*a^6 + 12*x^10*b^3*a^7 +
 5*x^9*b^2*a^8 + 5/4*x^8*b*a^9 + 1/7*x^7*a^10
```

giac [A] time = 1.10, size = 112, normalized size = 1.00

$$\frac{1}{17}b^{10}x^{17} + \frac{5}{8}ab^9x^{16} + 3a^2b^8x^{15} + \frac{60}{7}a^3b^7x^{14} + \frac{210}{13}a^4b^6x^{13} + 21a^5b^5x^{12} + \frac{210}{11}a^6b^4x^{11} + 12a^7b^3x^{10} + 5a^8b^2x^9 + \frac{5}{4}a^9bx^8 + \frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

maple [A] time = 0.00, size = 113, normalized size = 1.01

$$\frac{1}{17}b^{10}x^{17} + \frac{5}{8}a\,b^9x^{16} + 3a^2b^8x^{15} + \frac{60}{7}a^3b^7x^{14} + \frac{210}{13}a^4b^6x^{13} + 21a^5b^5x^{12} + \frac{210}{11}a^6b^4x^{11} + 12a^7b^3x^{10} + 5a^8b^2x^9 + \frac{5}{4}a^9b\,x^8 + \frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^10,x)
```

```
[Out] 1/17*b^10*x^17+5/8*a*b^9*x^16+3*a^2*b^8*x^15+60/7*a^3*b^7*x^14+210/13*a^4*b
^6*x^13+21*a^5*b^5*x^12+210/11*a^6*b^4*x^11+12*a^7*b^3*x^10+5*a^8*b^2*x^9+5
/4*a^9*b*x^8+1/7*a^10*x^7
```

maxima [A] time = 1.38, size = 112, normalized size = 1.00

$$\frac{1}{17}b^{10}x^{17} + \frac{5}{8}ab^9x^{16} + 3a^2b^8x^{15} + \frac{60}{7}a^3b^7x^{14} + \frac{210}{13}a^4b^6x^{13} + 21a^5b^5x^{12} + \frac{210}{11}a^6b^4x^{11} + 12a^7b^3x^{10} + 5a^8b^2x^9 + \frac{5}{4}a^9bx^8 + \frac{1}{7}a^{10}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^10*x^17 + 5/8*a*b^9*x^16 + 3*a^2*b^8*x^15 + 60/7*a^3*b^7*x^14 + 210/
13*a^4*b^6*x^13 + 21*a^5*b^5*x^12 + 210/11*a^6*b^4*x^11 + 12*a^7*b^3*x^10 +
 5*a^8*b^2*x^9 + 5/4*a^9*b*x^8 + 1/7*a^10*x^7
```

mupad [B] time = 0.12, size = 112, normalized size = 1.00

$$\frac{a^{10}\,x^7}{7} + \frac{5\,a^9\,b\,x^8}{4} + 5\,a^8\,b^2\,x^9 + 12\,a^7\,b^3\,x^{10} + \frac{210\,a^6\,b^4\,x^{11}}{11} + 21\,a^5\,b^5\,x^{12} + \frac{210\,a^4\,b^6\,x^{13}}{13} + \frac{60\,a^3\,b^7\,x^{14}}{7} + 3\,a^2\,b^8\,x^{15} + \frac{5\,a\,b^9\,x^{16}}{8} + \frac{b^{10}\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^10,x)
```

```
[Out] (a^10*x^7)/7 + (b^10*x^17)/17 + (5*a^9*b*x^8)/4 + (5*a*b^9*x^16)/8 + 5*a^8*
b^2*x^9 + 12*a^7*b^3*x^10 + (210*a^6*b^4*x^11)/11 + 21*a^5*b^5*x^12 + (210*
a^4*b^6*x^13)/13 + (60*a^3*b^7*x^14)/7 + 3*a^2*b^8*x^15
```

sympy [A] time = 0.10, size = 128, normalized size = 1.14

$$\frac{a^{10}x^7}{7} + \frac{5a^9bx^8}{4} + 5a^8b^2x^9 + 12a^7b^3x^{10} + \frac{210a^6b^4x^{11}}{11} + 21a^5b^5x^{12} + \frac{210a^4b^6x^{13}}{13} + \frac{60a^3b^7x^{14}}{7} + 3a^2b^8x^{15} + \frac{5ab^9x^{16}}{8} + \frac{b^{10}x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**10,x)
```

```
[Out] a**10*x**7/7 + 5*a**9*b*x**8/4 + 5*a**8*b**2*x**9 + 12*a**7*b**3*x**10 + 21
0*a**6*b**4*x**11/11 + 21*a**5*b**5*x**12 + 210*a**4*b**6*x**13/13 + 60*a**
3*b**7*x**14/7 + 3*a**2*b**8*x**15 + 5*a*b**9*x**16/8 + b**10*x**17/17
```

3.129 $\int x^5(a+bx)^{10}\,dx$

Optimal. Leaf size=98

$$-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{a^5(a+bx)^{11}}{11b^6}+\frac{(a+bx)^{16}}{16b^6}-\frac{a(a+bx)^{15}}{3b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^10,x]
```

```
[Out] -(a^5*(a + b*x)^11)/(11*b^6) + (5*a^4*(a + b*x)^12)/(12*b^6) - (10*a^3*(a +
 b*x)^13)/(13*b^6) + (5*a^2*(a + b*x)^14)/(7*b^6) - (a*(a + b*x)^15)/(3*b^6
) + (a + b*x)^16/(16*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5(a+bx)^{10}\,dx &= \int\left(-\frac{a^5(a+bx)^{10}}{b^5}+\frac{5a^4(a+bx)^{11}}{b^5}-\frac{10a^3(a+bx)^{12}}{b^5}+\frac{10a^2(a+bx)^{13}}{b^5}-\frac{5a(a+bx)^{14}}{b^5}+\right.\\ &= -\frac{a^5(a+bx)^{11}}{11b^6}+\frac{5a^4(a+bx)^{12}}{12b^6}-\frac{10a^3(a+bx)^{13}}{13b^6}+\frac{5a^2(a+bx)^{14}}{7b^6}-\frac{a(a+bx)^{15}}{3b^6}+\frac{(a+b}{16b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 132, normalized size = 1.35

$$\frac{a^{10}x^6}{6}+\frac{10}{7}a^9bx^7+\frac{45}{8}a^8b^2x^8+\frac{40}{3}a^7b^3x^9+21a^6b^4x^{10}+\frac{252}{11}a^5b^5x^{11}+\frac{35}{2}a^4b^6x^{12}+\frac{120}{13}a^3b^7x^{13}+\frac{45}{14}a^2b^8x^{14}+\frac{2}{3}ab^9x^{15}+\frac{b^{10}x^{16}}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^10,x]
```

```
[Out] (a^10*x^6)/6 + (10*a^9*b*x^7)/7 + (45*a^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 +
 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/11 + (35*a^4*b^6*x^12)/2 + (120*a^3*b
^7*x^13)/13 + (45*a^2*b^8*x^14)/14 + (2*a*b^9*x^15)/3 + (b^10*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^5*(a + b*x)^10, x]
```

fricas [A] time = 0.72, size = 112, normalized size = 1.14

$$\frac{1}{16}x^{16}b^{10} + \frac{2}{3}x^{15}b^{9}a + \frac{45}{14}x^{14}b^{8}a^{2} + \frac{120}{13}x^{13}b^{7}a^{3} + \frac{35}{2}x^{12}b^{6}a^{4} + \frac{252}{11}x^{11}b^{5}a^{5} + 21x^{10}b^{4}a^{6} + \frac{40}{3}x^{9}b^{3}a^{7} + \frac{45}{8}x^{8}b^{2}a^{8} + \frac{10}{7}x^{7}ba^{9} + \frac{1}{6}x^{6}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*b^10 + 2/3*x^15*b^9*a + 45/14*x^14*b^8*a^2 + 120/13*x^13*b^7*a^3
+ 35/2*x^12*b^6*a^4 + 252/11*x^11*b^5*a^5 + 21*x^10*b^4*a^6 + 40/3*x^9*b^3*
a^7 + 45/8*x^8*b^2*a^8 + 10/7*x^7*b*a^9 + 1/6*x^6*a^10
```

giac [A] time = 1.14, size = 112, normalized size = 1.14

$$\frac{1}{16}b^{10}x^{16} + \frac{2}{3}ab^{9}x^{15} + \frac{45}{14}a^{2}b^{8}x^{14} + \frac{120}{13}a^{3}b^{7}x^{13} + \frac{35}{2}a^{4}b^{6}x^{12} + \frac{252}{11}a^{5}b^{5}x^{11} + 21\,a^{6}b^{4}x^{10} + \frac{40}{3}a^{7}b^{3}x^{9} + \frac{45}{8}a^{8}b^{2}x^{8} + \frac{10}{7}a^{9}bx^{7} + \frac{1}{6}a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

maple [A] time = 0.00, size = 113, normalized size = 1.15

$$\frac{1}{16}b^{10}x^{16} + \frac{2}{3}a\,b^{9}x^{15} + \frac{45}{14}a^{2}b^{8}x^{14} + \frac{120}{13}a^{3}b^{7}x^{13} + \frac{35}{2}a^{4}b^{6}x^{12} + \frac{252}{11}a^{5}b^{5}x^{11} + 21a^{6}b^{4}x^{10} + \frac{40}{3}a^{7}b^{3}x^{9} + \frac{45}{8}a^{8}b^{2}x^{8} + \frac{10}{7}a^{9}b\,x^{7} + \frac{1}{6}a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^10,x)
```

```
[Out] 1/16*b^10*x^16+2/3*a*b^9*x^15+45/14*a^2*b^8*x^14+120/13*a^3*b^7*x^13+35/2*a
^4*b^6*x^12+252/11*a^5*b^5*x^11+21*a^6*b^4*x^10+40/3*a^7*b^3*x^9+45/8*a^8*b
^2*x^8+10/7*a^9*b*x^7+1/6*a^10*x^6
```

maxima [A] time = 1.36, size = 112, normalized size = 1.14

$$\frac{1}{16}b^{10}x^{16} + \frac{2}{3}ab^{9}x^{15} + \frac{45}{14}a^{2}b^{8}x^{14} + \frac{120}{13}a^{3}b^{7}x^{13} + \frac{35}{2}a^{4}b^{6}x^{12} + \frac{252}{11}a^{5}b^{5}x^{11} + 21\,a^{6}b^{4}x^{10} + \frac{40}{3}a^{7}b^{3}x^{9} + \frac{45}{8}a^{8}b^{2}x^{8} + \frac{10}{7}a^{9}bx^{7} + \frac{1}{6}a^{10}x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^10*x^16 + 2/3*a*b^9*x^15 + 45/14*a^2*b^8*x^14 + 120/13*a^3*b^7*x^13
+ 35/2*a^4*b^6*x^12 + 252/11*a^5*b^5*x^11 + 21*a^6*b^4*x^10 + 40/3*a^7*b^3*
x^9 + 45/8*a^8*b^2*x^8 + 10/7*a^9*b*x^7 + 1/6*a^10*x^6
```

mupad [B] time = 0.12, size = 112, normalized size = 1.14

$$\frac{a^{10}\,x^{6}}{6} + \frac{10\,a^{9}\,b\,x^{7}}{7} + \frac{45\,a^{8}\,b^{2}\,x^{8}}{8} + \frac{40\,a^{7}\,b^{3}\,x^{9}}{3} + 21\,a^{6}\,b^{4}\,x^{10} + \frac{252\,a^{5}\,b^{5}\,x^{11}}{11} + \frac{35\,a^{4}\,b^{6}\,x^{12}}{2} + \frac{120\,a^{3}\,b^{7}\,x^{13}}{13} + \frac{45\,a^{2}\,b^{8}\,x^{14}}{14} + \frac{2\,a\,b^{9}\,x^{15}}{3} + \frac{b^{10}\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^10,x)
```

```
[Out] (a^10*x^6)/6 + (b^10*x^16)/16 + (10*a^9*b*x^7)/7 + (2*a*b^9*x^15)/3 + (45*a
^8*b^2*x^8)/8 + (40*a^7*b^3*x^9)/3 + 21*a^6*b^4*x^10 + (252*a^5*b^5*x^11)/1
1 + (35*a^4*b^6*x^12)/2 + (120*a^3*b^7*x^13)/13 + (45*a^2*b^8*x^14)/14
```

sympy [A] time = 0.10, size = 133, normalized size = 1.36

$$\frac{a^{10}x^{6}}{6} + \frac{10a^{9}bx^{7}}{7} + \frac{45a^{8}b^{2}x^{8}}{8} + \frac{40a^{7}b^{3}x^{9}}{3} + 21a^{6}b^{4}x^{10} + \frac{252a^{5}b^{5}x^{11}}{11} + \frac{35a^{4}b^{6}x^{12}}{2} + \frac{120a^{3}b^{7}x^{13}}{13} + \frac{45a^{2}b^{8}x^{14}}{14} + \frac{2ab^{9}x^{15}}{3} + \frac{b^{10}x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**10,x)
```

```
[Out] a**10*x**6/6 + 10*a**9*b*x**7/7 + 45*a**8*b**2*x**8/8 + 40*a**7*b**3*x**9/3
 + 21*a**6*b**4*x**10 + 252*a**5*b**5*x**11/11 + 35*a**4*b**6*x**12/2 + 120
*a**3*b**7*x**13/13 + 45*a**2*b**8*x**14/14 + 2*a*b**9*x**15/3 + b**10*x**1
6/16
```

3.130 $\int x^4(a+bx)^{10}\,dx$

Optimal. Leaf size=81

$$\frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{a^4(a+bx)^{11}}{11b^5}+\frac{(a+bx)^{15}}{15b^5}-\frac{2a(a+bx)^{14}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^10,x]
```

```
[Out] (a^4*(a + b*x)^11)/(11*b^5) - (a^3*(a + b*x)^12)/(3*b^5) + (6*a^2*(a + b*x)
^13)/(13*b^5) - (2*a*(a + b*x)^14)/(7*b^5) + (a + b*x)^15/(15*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{10}\,dx &= \int \left(\frac{a^4(a+bx)^{10}}{b^4}-\frac{4a^3(a+bx)^{11}}{b^4}+\frac{6a^2(a+bx)^{12}}{b^4}-\frac{4a(a+bx)^{13}}{b^4}+\frac{(a+bx)^{14}}{b^4}\right)dx\\ &= \frac{a^4(a+bx)^{11}}{11b^5}-\frac{a^3(a+bx)^{12}}{3b^5}+\frac{6a^2(a+bx)^{13}}{13b^5}-\frac{2a(a+bx)^{14}}{7b^5}+\frac{(a+bx)^{15}}{15b^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 130, normalized size = 1.60

$$\frac{a^{10}x^5}{5}+\frac{5}{3}a^9bx^6+\frac{45}{7}a^8b^2x^7+15a^7b^3x^8+\frac{70}{3}a^6b^4x^9+\frac{126}{5}a^5b^5x^{10}+\frac{210}{11}a^4b^6x^{11}+10a^3b^7x^{12}+\frac{45}{13}a^2b^8x^{13}+\frac{5}{7}ab^9x^{14}+\frac{b^{10}x^{15}}{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^10,x]
```

```
[Out] (a^10*x^5)/5 + (5*a^9*b*x^6)/3 + (45*a^8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*
a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 + (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*
x^12 + (45*a^2*b^8*x^13)/13 + (5*a*b^9*x^14)/7 + (b^10*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^4*(a + b*x)^10, x]
```

fricas [A] time = 1.22, size = 112, normalized size = 1.38

$$\frac{1}{15}x^{15}b^{10} + \frac{5}{7}x^{14}b^{9}a + \frac{45}{13}x^{13}b^{8}a^{2} + 10x^{12}b^{7}a^{3} + \frac{210}{11}x^{11}b^{6}a^{4} + \frac{126}{5}x^{10}b^{5}a^{5} + \frac{70}{3}x^{9}b^{4}a^{6} + 15x^{8}b^{3}a^{7} + \frac{45}{7}x^{7}b^{2}a^{8} + \frac{5}{3}x^{6}ba^{9} + \frac{1}{5}x^{5}a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*b^10 + 5/7*x^14*b^9*a + 45/13*x^13*b^8*a^2 + 10*x^12*b^7*a^3 + 21
0/11*x^11*b^6*a^4 + 126/5*x^10*b^5*a^5 + 70/3*x^9*b^4*a^6 + 15*x^8*b^3*a^7
+ 45/7*x^7*b^2*a^8 + 5/3*x^6*b*a^9 + 1/5*x^5*a^10
```

giac [A] time = 1.06, size = 112, normalized size = 1.38

$$\frac{1}{15}b^{10}x^{15} + \frac{5}{7}ab^{9}x^{14} + \frac{45}{13}a^{2}b^{8}x^{13} + 10a^{3}b^{7}x^{12} + \frac{210}{11}a^{4}b^{6}x^{11} + \frac{126}{5}a^{5}b^{5}x^{10} + \frac{70}{3}a^{6}b^{4}x^{9} + 15a^{7}b^{3}x^{8} + \frac{45}{7}a^{8}b^{2}x^{7} + \frac{5}{3}a^{9}bx^{6} + \frac{1}{5}a^{10}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

maple [A] time = 0.00, size = 113, normalized size = 1.40

$$\frac{1}{15}b^{10}x^{15} + \frac{5}{7}a\,b^{9}x^{14} + \frac{45}{13}a^{2}b^{8}x^{13} + 10a^{3}b^{7}x^{12} + \frac{210}{11}a^{4}b^{6}x^{11} + \frac{126}{5}a^{5}b^{5}x^{10} + \frac{70}{3}a^{6}b^{4}x^{9} + 15a^{7}b^{3}x^{8} + \frac{45}{7}a^{8}b^{2}x^{7} + \frac{5}{3}a^{9}b\,x^{6} + \frac{1}{5}a^{10}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^10,x)
```

```
[Out] 1/15*b^10*x^15+5/7*a*b^9*x^14+45/13*a^2*b^8*x^13+10*a^3*b^7*x^12+210/11*a^4
*b^6*x^11+126/5*a^5*b^5*x^10+70/3*a^6*b^4*x^9+15*a^7*b^3*x^8+45/7*a^8*b^2*x
^7+5/3*a^9*b*x^6+1/5*a^10*x^5
```

maxima [A] time = 1.29, size = 112, normalized size = 1.38

$$\frac{1}{15}b^{10}x^{15} + \frac{5}{7}ab^{9}x^{14} + \frac{45}{13}a^{2}b^{8}x^{13} + 10a^{3}b^{7}x^{12} + \frac{210}{11}a^{4}b^{6}x^{11} + \frac{126}{5}a^{5}b^{5}x^{10} + \frac{70}{3}a^{6}b^{4}x^{9} + 15a^{7}b^{3}x^{8} + \frac{45}{7}a^{8}b^{2}x^{7} + \frac{5}{3}a^{9}bx^{6} + \frac{1}{5}a^{10}x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^10*x^15 + 5/7*a*b^9*x^14 + 45/13*a^2*b^8*x^13 + 10*a^3*b^7*x^12 + 21
0/11*a^4*b^6*x^11 + 126/5*a^5*b^5*x^10 + 70/3*a^6*b^4*x^9 + 15*a^7*b^3*x^8
+ 45/7*a^8*b^2*x^7 + 5/3*a^9*b*x^6 + 1/5*a^10*x^5
```

mupad [B] time = 0.12, size = 112, normalized size = 1.38

$$\frac{a^{10}\,x^5}{5} + \frac{5\,a^9\,b\,x^6}{3} + \frac{45\,a^8\,b^2\,x^7}{7} + 15\,a^7\,b^3\,x^8 + \frac{70\,a^6\,b^4\,x^9}{3} + \frac{126\,a^5\,b^5\,x^{10}}{5} + \frac{210\,a^4\,b^6\,x^{11}}{11} + 10\,a^3\,b^7\,x^{12} + \frac{45\,a^2\,b^8\,x^{13}}{13} + \frac{5\,a\,b^9\,x^{14}}{7} + \frac{b^{10}\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^10,x)
```

```
[Out] (a^10*x^5)/5 + (b^10*x^15)/15 + (5*a^9*b*x^6)/3 + (5*a*b^9*x^14)/7 + (45*a^
8*b^2*x^7)/7 + 15*a^7*b^3*x^8 + (70*a^6*b^4*x^9)/3 + (126*a^5*b^5*x^10)/5 +
 (210*a^4*b^6*x^11)/11 + 10*a^3*b^7*x^12 + (45*a^2*b^8*x^13)/13
```

sympy [A] time = 0.11, size = 131, normalized size = 1.62

$$\frac{a^{10}x^{5}}{5} + \frac{5a^{9}bx^{6}}{3} + \frac{45a^{8}b^{2}x^{7}}{7} + 15a^{7}b^{3}x^{8} + \frac{70a^{6}b^{4}x^{9}}{3} + \frac{126a^{5}b^{5}x^{10}}{5} + \frac{210a^{4}b^{6}x^{11}}{11} + 10a^{3}b^{7}x^{12} + \frac{45a^{2}b^{8}x^{13}}{13} + \frac{5ab^{9}x^{14}}{7} + \frac{b^{10}x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**10,x)
```

```
[Out] a**10*x**5/5 + 5*a**9*b*x**6/3 + 45*a**8*b**2*x**7/7 + 15*a**7*b**3*x**8 +
70*a**6*b**4*x**9/3 + 126*a**5*b**5*x**10/5 + 210*a**4*b**6*x**11/11 + 10*a
**3*b**7*x**12 + 45*a**2*b**8*x**13/13 + 5*a*b**9*x**14/7 + b**10*x**15/15
```

3.131 $\int x^3(a+bx)^{10}\,dx$

Optimal. Leaf size=64

$$-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{12}}{4b^4}-\frac{a^3(a+bx)^{11}}{11b^4}+\frac{(a+bx)^{14}}{14b^4}-\frac{3a(a+bx)^{13}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^10,x]
```

```
[Out] -(a^3*(a + b*x)^11)/(11*b^4) + (a^2*(a + b*x)^12)/(4*b^4) - (3*a*(a + b*x)^
13)/(13*b^4) + (a + b*x)^14/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{10}\,dx &= \int\left(-\frac{a^3(a+bx)^{10}}{b^3}+\frac{3a^2(a+bx)^{11}}{b^3}-\frac{3a(a+bx)^{12}}{b^3}+\frac{(a+bx)^{13}}{b^3}\right)dx\\ &= -\frac{a^3(a+bx)^{11}}{11b^4}+\frac{a^2(a+bx)^{12}}{4b^4}-\frac{3a(a+bx)^{13}}{13b^4}+\frac{(a+bx)^{14}}{14b^4}\end{aligned}$$

Mathematica [A] time = 0.00, size = 128, normalized size = 2.00

$$\frac{a^{10}x^4}{4}+2a^9bx^5+\frac{15}{2}a^8b^2x^6+\frac{120}{7}a^7b^3x^7+\frac{105}{4}a^6b^4x^8+28a^5b^5x^9+21a^4b^6x^{10}+\frac{120}{11}a^3b^7x^{11}+\frac{15}{4}a^2b^8x^{12}+\frac{10}{13}ab^9x^{13}+\frac{b^{10}x^{14}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^10,x]
```

```
[Out] (a^10*x^4)/4 + 2*a^9*b*x^5 + (15*a^8*b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (10
5*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 21*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11
 + (15*a^2*b^8*x^12)/4 + (10*a*b^9*x^13)/13 + (b^10*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^3*(a + b*x)^10, x]
```

fricas [A] time = 1.14, size = 112, normalized size = 1.75

$$\frac{1}{14}x^{14}b^{10} + \frac{10}{13}x^{13}b^9a + \frac{15}{4}x^{12}b^8a^2 + \frac{120}{11}x^{11}b^7a^3 + 21x^{10}b^6a^4 + 28x^9b^5a^5 + \frac{105}{4}x^8b^4a^6 + \frac{120}{7}x^7b^3a^7 + \frac{15}{2}x^6b^2a^8 + 2x^5ba^9 + \frac{1}{4}x^4a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*b^10 + 10/13*x^13*b^9*a + 15/4*x^12*b^8*a^2 + 120/11*x^11*b^7*a^3
 + 21*x^10*b^6*a^4 + 28*x^9*b^5*a^5 + 105/4*x^8*b^4*a^6 + 120/7*x^7*b^3*a^7
 + 15/2*x^6*b^2*a^8 + 2*x^5*b*a^9 + 1/4*x^4*a^10
```

giac [A] time = 1.09, size = 112, normalized size = 1.75

$$\frac{1}{14}b^{10}x^{14} + \frac{10}{13}ab^9x^{13} + \frac{15}{4}a^2b^8x^{12} + \frac{120}{11}a^3b^7x^{11} + 21a^4b^6x^{10} + 28a^5b^5x^9 + \frac{105}{4}a^6b^4x^8 + \frac{120}{7}a^7b^3x^7 + \frac{15}{2}a^8b^2x^6 + 2a^9bx^5 + \frac{1}{4}a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

maple [A] time = 0.00, size = 113, normalized size = 1.77

$$\frac{1}{14}b^{10}x^{14} + \frac{10}{13}a\,b^9x^{13} + \frac{15}{4}a^2b^8x^{12} + \frac{120}{11}a^3b^7x^{11} + 21a^4b^6x^{10} + 28a^5b^5x^9 + \frac{105}{4}a^6b^4x^8 + \frac{120}{7}a^7b^3x^7 + \frac{15}{2}a^8b^2x^6 + 2a^9b\,x^5 + \frac{1}{4}a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^10,x)
```

```
[Out] 1/14*b^10*x^14+10/13*a*b^9*x^13+15/4*a^2*b^8*x^12+120/11*a^3*b^7*x^11+21*a^
4*b^6*x^10+28*a^5*b^5*x^9+105/4*a^6*b^4*x^8+120/7*a^7*b^3*x^7+15/2*a^8*b^2*
x^6+2*a^9*b*x^5+1/4*a^10*x^4
```

maxima [A] time = 1.36, size = 112, normalized size = 1.75

$$\frac{1}{14}b^{10}x^{14} + \frac{10}{13}ab^9x^{13} + \frac{15}{4}a^2b^8x^{12} + \frac{120}{11}a^3b^7x^{11} + 21a^4b^6x^{10} + 28a^5b^5x^9 + \frac{105}{4}a^6b^4x^8 + \frac{120}{7}a^7b^3x^7 + \frac{15}{2}a^8b^2x^6 + 2a^9bx^5 + \frac{1}{4}a^{10}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^10*x^14 + 10/13*a*b^9*x^13 + 15/4*a^2*b^8*x^12 + 120/11*a^3*b^7*x^11
 + 21*a^4*b^6*x^10 + 28*a^5*b^5*x^9 + 105/4*a^6*b^4*x^8 + 120/7*a^7*b^3*x^7
 + 15/2*a^8*b^2*x^6 + 2*a^9*b*x^5 + 1/4*a^10*x^4
```

mupad [B] time = 0.12, size = 112, normalized size = 1.75

$$\frac{a^{10}x^4}{4} + 2a^9bx^5 + \frac{15a^8b^2x^6}{2} + \frac{120a^7b^3x^7}{7} + \frac{105a^6b^4x^8}{4} + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120a^3b^7x^{11}}{11} + \frac{15a^2b^8x^{12}}{4} + \frac{10ab^9x^{13}}{13} + \frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^10,x)
```

```
[Out] (a^10*x^4)/4 + (b^10*x^14)/14 + 2*a^9*b*x^5 + (10*a*b^9*x^13)/13 + (15*a^8*
b^2*x^6)/2 + (120*a^7*b^3*x^7)/7 + (105*a^6*b^4*x^8)/4 + 28*a^5*b^5*x^9 + 2
1*a^4*b^6*x^10 + (120*a^3*b^7*x^11)/11 + (15*a^2*b^8*x^12)/4
```

sympy [B] time = 0.12, size = 129, normalized size = 2.02

$$\frac{a^{10}x^4}{4} + 2a^9bx^5 + \frac{15a^8b^2x^6}{2} + \frac{120a^7b^3x^7}{7} + \frac{105a^6b^4x^8}{4} + 28a^5b^5x^9 + 21a^4b^6x^{10} + \frac{120a^3b^7x^{11}}{11} + \frac{15a^2b^8x^{12}}{4} + \frac{10ab^9x^{13}}{13} + \frac{b^{10}x^{14}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**10,x)
```

```
[Out] a**10*x**4/4 + 2*a**9*b*x**5 + 15*a**8*b**2*x**6/2 + 120*a**7*b**3*x**7/7 +
 105*a**6*b**4*x**8/4 + 28*a**5*b**5*x**9 + 21*a**4*b**6*x**10 + 120*a**3*b
**7*x**11/11 + 15*a**2*b**8*x**12/4 + 10*a*b**9*x**13/13 + b**10*x**14/14
```

3.132 $\int x^2(a+bx)^{10}\,dx$

Optimal. Leaf size=47

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{11}}{11b^3}+\frac{(a+bx)^{13}}{13b^3}-\frac{a(a+bx)^{12}}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^10,x]
```

```
[Out] (a^2*(a + b*x)^11)/(11*b^3) - (a*(a + b*x)^12)/(6*b^3) + (a + b*x)^13/(13*b
^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{10}\,dx&=\int\left(\frac{a^2(a+bx)^{10}}{b^2}-\frac{2a(a+bx)^{11}}{b^2}+\frac{(a+bx)^{12}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{11}}{11b^3}-\frac{a(a+bx)^{12}}{6b^3}+\frac{(a+bx)^{13}}{13b^3}\end{aligned}$$

Mathematica [B] time = 0.00, size = 126, normalized size = 2.68

$$\frac{a^{10}x^3}{3}+\frac{5}{2}a^9bx^4+9a^8b^2x^5+20a^7b^3x^6+30a^6b^4x^7+\frac{63}{2}a^5b^5x^8+\frac{70}{3}a^4b^6x^9+12a^3b^7x^{10}+\frac{45}{11}a^2b^8x^{11}+\frac{5}{6}ab^9x^{12}+\frac{b^{10}x^{13}}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^10,x]
```

```
[Out] (a^10*x^3)/3 + (5*a^9*b*x^4)/2 + 9*a^8*b^2*x^5 + 20*a^7*b^3*x^6 + 30*a^6*b^
4*x^7 + (63*a^5*b^5*x^8)/2 + (70*a^4*b^6*x^9)/3 + 12*a^3*b^7*x^10 + (45*a^2
*b^8*x^11)/11 + (5*a*b^9*x^12)/6 + (b^10*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^2*(a + b*x)^10, x]
```

fricas [B] time = 1.12, size = 112, normalized size = 2.38

$$\frac{1}{13}x^{13}b^{10} + \frac{5}{6}x^{12}b^9a + \frac{45}{11}x^{11}b^8a^2 + 12x^{10}b^7a^3 + \frac{70}{3}x^9b^6a^4 + \frac{63}{2}x^8b^5a^5 + 30x^7b^4a^6 + 20x^6b^3a^7 + 9x^5b^2a^8 + \frac{5}{2}x^4ba^9 + \frac{1}{3}x^3a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*b^10 + 5/6*x^12*b^9*a + 45/11*x^11*b^8*a^2 + 12*x^10*b^7*a^3 + 70
/3*x^9*b^6*a^4 + 63/2*x^8*b^5*a^5 + 30*x^7*b^4*a^6 + 20*x^6*b^3*a^7 + 9*x^5
*b^2*a^8 + 5/2*x^4*b*a^9 + 1/3*x^3*a^10
```

giac [B] time = 1.18, size = 112, normalized size = 2.38

$$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}ab^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8b^2x^5 + \frac{5}{2}a^9bx^4 + \frac{1}{3}a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

maple [B] time = 0.00, size = 113, normalized size = 2.40

$$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}a\,b^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8b^2x^5 + \frac{5}{2}a^9b\,x^4 + \frac{1}{3}a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^10,x)
```

```
[Out] 1/13*b^10*x^13+5/6*a*b^9*x^12+45/11*a^2*b^8*x^11+12*a^3*b^7*x^10+70/3*a^4*b
^6*x^9+63/2*a^5*b^5*x^8+30*a^6*b^4*x^7+20*a^7*b^3*x^6+9*a^8*b^2*x^5+5/2*a^9
*b*x^4+1/3*a^10*x^3
```

maxima [B] time = 1.37, size = 112, normalized size = 2.38

$$\frac{1}{13}b^{10}x^{13} + \frac{5}{6}ab^9x^{12} + \frac{45}{11}a^2b^8x^{11} + 12a^3b^7x^{10} + \frac{70}{3}a^4b^6x^9 + \frac{63}{2}a^5b^5x^8 + 30a^6b^4x^7 + 20a^7b^3x^6 + 9a^8b^2x^5 + \frac{5}{2}a^9bx^4 + \frac{1}{3}a^{10}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^10*x^13 + 5/6*a*b^9*x^12 + 45/11*a^2*b^8*x^11 + 12*a^3*b^7*x^10 + 70
/3*a^4*b^6*x^9 + 63/2*a^5*b^5*x^8 + 30*a^6*b^4*x^7 + 20*a^7*b^3*x^6 + 9*a^8
*b^2*x^5 + 5/2*a^9*b*x^4 + 1/3*a^10*x^3
```

mupad [B] time = 0.07, size = 31, normalized size = 0.66

$$\frac{(a+b\,x)^{11}\,\left(8\,a^2-88\,a\,b\,x+528\,b^2\,x^2\right)}{6864\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^11*(8*a^2 + 528*b^2*x^2 - 88*a*b*x))/(6864*b^3)
```

sympy [B] time = 0.11, size = 128, normalized size = 2.72

$$\frac{a^{10}x^3}{3} + \frac{5a^9bx^4}{2} + 9a^8b^2x^5 + 20a^7b^3x^6 + 30a^6b^4x^7 + \frac{63a^5b^5x^8}{2} + \frac{70a^4b^6x^9}{3} + 12a^3b^7x^{10} + \frac{45a^2b^8x^{11}}{11} + \frac{5ab^9x^{12}}{6} + \frac{b^{10}x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**10,x)
```

```
[Out] a**10*x**3/3 + 5*a**9*b*x**4/2 + 9*a**8*b**2*x**5 + 20*a**7*b**3*x**6 + 30*
a**6*b**4*x**7 + 63*a**5*b**5*x**8/2 + 70*a**4*b**6*x**9/3 + 12*a**3*b**7*x
**10 + 45*a**2*b**8*x**11/11 + 5*a*b**9*x**12/6 + b**10*x**13/13
```

3.133 $\int x(a+bx)^{10}\,dx$

Optimal. Leaf size=30

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{12}}{12b^2}-\frac{a(a+bx)^{11}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^10,x]
[Out] -(a*(a + b*x)^11)/(11*b^2) + (a + b*x)^12/(12*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{10}\,dx&=\int\left(-\frac{a(a+bx)^{10}}{b}+\frac{(a+bx)^{11}}{b}\right)dx\\&=-\frac{a(a+bx)^{11}}{11b^2}+\frac{(a+bx)^{12}}{12b^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 128, normalized size = 4.27

$$\frac{a^{10}x^2}{2}+\frac{10}{3}a^9bx^3+\frac{45}{4}a^8b^2x^4+24a^7b^3x^5+35a^6b^4x^6+36a^5b^5x^7+\frac{105}{4}a^4b^6x^8+\frac{40}{3}a^3b^7x^9+\frac{9}{2}a^2b^8x^{10}+\frac{10}{11}ab^9x^{11}+\frac{b^{10}x^{12}}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^10,x]
[Out] (a^10*x^2)/2 + (10*a^9*b*x^3)/3 + (45*a^8*b^2*x^4)/4 + 24*a^7*b^3*x^5 + 35*
a^6*b^4*x^6 + 36*a^5*b^5*x^7 + (105*a^4*b^6*x^8)/4 + (40*a^3*b^7*x^9)/3 + (
9*a^2*b^8*x^10)/2 + (10*a*b^9*x^11)/11 + (b^10*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^10,x]
[Out] IntegrateAlgebraic[x*(a + b*x)^10, x]
```

fricas [B] time = 1.01, size = 112, normalized size = 3.73

$$\frac{1}{12}x^{12}b^{10}+\frac{10}{11}x^{11}b^9a+\frac{9}{2}x^{10}b^8a^2+\frac{40}{3}x^9b^7a^3+\frac{105}{4}x^8b^6a^4+36x^7b^5a^5+35x^6b^4a^6+24x^5b^3a^7+\frac{45}{4}x^4b^2a^8+\frac{10}{3}x^3ba^9+\frac{1}{2}x^2a^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*b^10 + 10/11*x^11*b^9*a + 9/2*x^10*b^8*a^2 + 40/3*x^9*b^7*a^3 + 1
05/4*x^8*b^6*a^4 + 36*x^7*b^5*a^5 + 35*x^6*b^4*a^6 + 24*x^5*b^3*a^7 + 45/4*
x^4*b^2*a^8 + 10/3*x^3*b*a^9 + 1/2*x^2*a^10
```

giac [B] time = 1.44, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^{9}x^{11}+\frac{9}{2}\,a^{2}b^{8}x^{10}+\frac{40}{3}\,a^{3}b^{7}x^{9}+\frac{105}{4}\,a^{4}b^{6}x^{8}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45}{4}\,a^{8}b^{2}x^{4}+\frac{10}{3}\,a^{9}bx^{3}+\frac{1}{2}\,a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

maple [B] time = 0.00, size = 113, normalized size = 3.77

$$\frac{1}{12}b^{10}x^{12}+\frac{10}{11}a\,b^{9}x^{11}+\frac{9}{2}a^{2}b^{8}x^{10}+\frac{40}{3}a^{3}b^{7}x^{9}+\frac{105}{4}a^{4}b^{6}x^{8}+36a^{5}b^{5}x^{7}+35a^{6}b^{4}x^{6}+24a^{7}b^{3}x^{5}+\frac{45}{4}a^{8}b^{2}x^{4}+\frac{10}{3}a^{9}b\,x^{3}+\frac{1}{2}a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^10,x)
```

```
[Out] 1/12*b^10*x^12+10/11*a*b^9*x^11+9/2*a^2*b^8*x^10+40/3*a^3*b^7*x^9+105/4*a^4
*b^6*x^8+36*a^5*b^5*x^7+35*a^6*b^4*x^6+24*a^7*b^3*x^5+45/4*a^8*b^2*x^4+10/3
*a^9*b*x^3+1/2*a^10*x^2
```

maxima [B] time = 1.35, size = 112, normalized size = 3.73

$$\frac{1}{12}\,b^{10}x^{12}+\frac{10}{11}\,ab^{9}x^{11}+\frac{9}{2}\,a^{2}b^{8}x^{10}+\frac{40}{3}\,a^{3}b^{7}x^{9}+\frac{105}{4}\,a^{4}b^{6}x^{8}+36\,a^{5}b^{5}x^{7}+35\,a^{6}b^{4}x^{6}+24\,a^{7}b^{3}x^{5}+\frac{45}{4}\,a^{8}b^{2}x^{4}+\frac{10}{3}\,a^{9}bx^{3}+\frac{1}{2}\,a^{10}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b^10*x^12 + 10/11*a*b^9*x^11 + 9/2*a^2*b^8*x^10 + 40/3*a^3*b^7*x^9 + 1
05/4*a^4*b^6*x^8 + 36*a^5*b^5*x^7 + 35*a^6*b^4*x^6 + 24*a^7*b^3*x^5 + 45/4*
a^8*b^2*x^4 + 10/3*a^9*b*x^3 + 1/2*a^10*x^2
```

mupad [B] time = 0.09, size = 25, normalized size = 0.83

$$-\frac{2\,\left(\frac{a\,{\left(a+b\,x\right)}^{11}}{22}-\frac{{\left(a+b\,x\right)}^{12}}{24}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^10,x)
```

```
[Out] -(2*((a*(a + b*x)^11)/22 - (a + b*x)^12/24))/b^2
```

sympy [B] time = 0.11, size = 129, normalized size = 4.30

$$\frac{a^{10}x^{2}}{2}+\frac{10a^{9}bx^{3}}{3}+\frac{45a^{8}b^{2}x^{4}}{4}+24a^{7}b^{3}x^{5}+35a^{6}b^{4}x^{6}+36a^{5}b^{5}x^{7}+\frac{105a^{4}b^{6}x^{8}}{4}+\frac{40a^{3}b^{7}x^{9}}{3}+\frac{9a^{2}b^{8}x^{10}}{2}+\frac{10ab^{9}x^{11}}{11}+\frac{b^{10}x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**10,x)
```

```
[Out] a**10*x**2/2 + 10*a**9*b*x**3/3 + 45*a**8*b**2*x**4/4 + 24*a**7*b**3*x**5 +
 35*a**6*b**4*x**6 + 36*a**5*b**5*x**7 + 105*a**4*b**6*x**8/4 + 40*a**3*b**
7*x**9/3 + 9*a**2*b**8*x**10/2 + 10*a*b**9*x**11/11 + b**10*x**12/12
```

3.134 $\int (a+bx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^{11}}{11b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{10}\,dx = \frac{(a+bx)^{11}}{11b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10,x]
```

```
[Out] (a + b*x)^11/(11*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10, x]
```

fricas [B] time = 1.26, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}b^{10} + x^{10}b^{9}a + 5x^{9}b^{8}a^{2} + 15x^{8}b^{7}a^{3} + 30x^{7}b^{6}a^{4} + 42x^{6}b^{5}a^{5} + 42x^{5}b^{4}a^{6} + 30x^{4}b^{3}a^{7} + 15x^{3}b^{2}a^{8} + 5x^{2}ba^{9} + xa^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10,x, algorithm="fricas")
```

[Out] 1/11*x^11*b^10 + x^10*b^9*a + 5*x^9*b^8*a^2 + 15*x^8*b^7*a^3 + 30*x^7*b^6*a^4 + 42*x^6*b^5*a^5 + 42*x^5*b^4*a^6 + 30*x^4*b^3*a^7 + 15*x^3*b^2*a^8 + 5*x^2*b*a^9 + x*a^10

giac [A] time = 1.16, size = 12, normalized size = 0.86

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10,x, algorithm="giac")

[Out] 1/11*(b*x + a)^11/b

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^10,x)

[Out] 1/11*(b*x+a)^11/b

maxima [A] time = 1.33, size = 12, normalized size = 0.86

$$\frac{(bx+a)^{11}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^10,x, algorithm="maxima")

[Out] 1/11*(b*x + a)^11/b

mupad [B] time = 0.11, size = 108, normalized size = 7.71

$$a^{10}\,x+5\,a^9\,b\,x^2+15\,a^8\,b^2\,x^3+30\,a^7\,b^3\,x^4+42\,a^6\,b^4\,x^5+42\,a^5\,b^5\,x^6+30\,a^4\,b^6\,x^7+15\,a^3\,b^7\,x^8+5\,a^2\,b^8\,x^9+a\,b^9\,x^{10}+\frac{b^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^10,x)

[Out] a^10*x + (b^10*x^11)/11 + 5*a^9*b*x^2 + a*b^9*x^10 + 15*a^8*b^2*x^3 + 30*a^7*b^3*x^4 + 42*a^6*b^4*x^5 + 42*a^5*b^5*x^6 + 30*a^4*b^6*x^7 + 15*a^3*b^7*x^8 + 5*a^2*b^8*x^9

sympy [B] time = 0.11, size = 114, normalized size = 8.14

$$a^{10}x+5a^9bx^2+15a^8b^2x^3+30a^7b^3x^4+42a^6b^4x^5+42a^5b^5x^6+30a^4b^6x^7+15a^3b^7x^8+5a^2b^8x^9+ab^9x^{10}+\frac{b^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**10,x)

[Out] a**10*x + 5*a**9*b*x**2 + 15*a**8*b**2*x**3 + 30*a**7*b**3*x**4 + 42*a**6*b**4*x**5 + 42*a**5*b**5*x**6 + 30*a**4*b**6*x**7 + 15*a**3*b**7*x**8 + 5*a**2*b**8*x**9 + a*b**9*x**10 + b**10*x**11/11

3.135 $\int \frac{(a+bx)^{10}}{x} \, dx$

Optimal. Leaf size=122

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9$$

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+10a^9bx+a^{10}\log(x)+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x} \, dx = \int \left(10a^9b+\frac{a^{10}}{x}+45a^8b^2x+120a^7b^3x^2+210a^6b^4x^3+252a^5b^5x^4+210a^4b^6x^5+120a^3b^7\right.$$

$$= 10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a$$

Mathematica [A] time = 0.00, size = 122, normalized size = 1.00

$$a^{10}\log(x)+10a^9bx+\frac{45}{2}a^8b^2x^2+40a^7b^3x^3+\frac{105}{2}a^6b^4x^4+\frac{252}{5}a^5b^5x^5+35a^4b^6x^6+\frac{120}{7}a^3b^7x^7+\frac{45}{8}a^2b^8x^8+\frac{10}{9}ab^9x^9+\frac{b^{10}x^{10}}{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x,x]
```

```
[Out] 10*a^9*b*x + (45*a^8*b^2*x^2)/2 + 40*a^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (2
52*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)
/8 + (10*a*b^9*x^9)/9 + (b^10*x^10)/10 + a^10*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x, x]
```

fricas [A] time = 1.56, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^{9}x^{9} + \frac{45}{8}\,a^{2}b^{8}x^{8} + \frac{120}{7}\,a^{3}b^{7}x^{7} + 35\,a^{4}b^{6}x^{6} + \frac{252}{5}\,a^{5}b^{5}x^{5} + \frac{105}{2}\,a^{6}b^{4}x^{4} + 40\,a^{7}b^{3}x^{3} + \frac{45}{2}\,a^{8}b^{2}x^{2} + 10\,a^{9}bx + a^{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="fricas")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

giac [A] time = 1.08, size = 109, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^{9}x^{9} + \frac{45}{8}\,a^{2}b^{8}x^{8} + \frac{120}{7}\,a^{3}b^{7}x^{7} + 35\,a^{4}b^{6}x^{6} + \frac{252}{5}\,a^{5}b^{5}x^{5} + \frac{105}{2}\,a^{6}b^{4}x^{4} + 40\,a^{7}b^{3}x^{3} + \frac{45}{2}\,a^{8}b^{2}x^{2} + 10\,a^{9}bx + a^{10}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="giac")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(abs(x))
```

maple [A] time = 0.00, size = 109, normalized size = 0.89

$$\frac{b^{10}x^{10}}{10} + \frac{10a\,b^{9}x^{9}}{9} + \frac{45a^{2}b^{8}x^{8}}{8} + \frac{120a^{3}b^{7}x^{7}}{7} + 35a^{4}b^{6}x^{6} + \frac{252a^{5}b^{5}x^{5}}{5} + \frac{105a^{6}b^{4}x^{4}}{2} + 40a^{7}b^{3}x^{3} + \frac{45a^{8}b^{2}x^{2}}{2} + a^{10}\ln(x) + 10a^{9}bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x,x)
```

```
[Out] 10*a^9*b*x+45/2*a^8*b^2*x^2+40*a^7*b^3*x^3+105/2*a^6*b^4*x^4+252/5*a^5*b^5*
x^5+35*a^4*b^6*x^6+120/7*a^3*b^7*x^7+45/8*a^2*b^8*x^8+10/9*a*b^9*x^9+1/10*b
^10*x^10+a^10*ln(x)
```

maxima [A] time = 1.39, size = 108, normalized size = 0.89

$$\frac{1}{10}\,b^{10}x^{10} + \frac{10}{9}\,ab^{9}x^{9} + \frac{45}{8}\,a^{2}b^{8}x^{8} + \frac{120}{7}\,a^{3}b^{7}x^{7} + 35\,a^{4}b^{6}x^{6} + \frac{252}{5}\,a^{5}b^{5}x^{5} + \frac{105}{2}\,a^{6}b^{4}x^{4} + 40\,a^{7}b^{3}x^{3} + \frac{45}{2}\,a^{8}b^{2}x^{2} + 10\,a^{9}bx + a^{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x,x, algorithm="maxima")
```

```
[Out] 1/10*b^10*x^10 + 10/9*a*b^9*x^9 + 45/8*a^2*b^8*x^8 + 120/7*a^3*b^7*x^7 + 35
*a^4*b^6*x^6 + 252/5*a^5*b^5*x^5 + 105/2*a^6*b^4*x^4 + 40*a^7*b^3*x^3 + 45/
2*a^8*b^2*x^2 + 10*a^9*b*x + a^10*log(x)
```

mupad [B] time = 0.08, size = 108, normalized size = 0.89

$$a^{10}\,\ln(x) + \frac{b^{10}\,x^{10}}{10} + \frac{10\,a\,b^{9}\,x^{9}}{9} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{2} + 40\,a^{7}\,b^{3}\,x^{3} + \frac{105\,a^{6}\,b^{4}\,x^{4}}{2} + \frac{252\,a^{5}\,b^{5}\,x^{5}}{5} + 35\,a^{4}\,b^{6}\,x^{6} + \frac{120\,a^{3}\,b^{7}\,x^{7}}{7} + \frac{45\,a^{2}\,b^{8}\,x^{8}}{8} + 10\,a^{9}\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x,x)
```

```
[Out] a^10*log(x) + (b^10*x^10)/10 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/2 + 40*a
^7*b^3*x^3 + (105*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/5 + 35*a^4*b^6*x^6 + (
120*a^3*b^7*x^7)/7 + (45*a^2*b^8*x^8)/8 + 10*a^9*b*x
```

sympy [A] time = 0.26, size = 126, normalized size = 1.03

$$a^{10}\log{(x)} + 10a^{9}bx + \frac{45a^{8}b^{2}x^{2}}{2} + 40a^{7}b^{3}x^{3} + \frac{105a^{6}b^{4}x^{4}}{2} + \frac{252a^{5}b^{5}x^{5}}{5} + 35a^{4}b^{6}x^{6} + \frac{120a^{3}b^{7}x^{7}}{7} + \frac{45a^{2}b^{8}x^{8}}{8} + \frac{10ab^{9}x^{9}}{9} + \frac{b^{10}x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x,x)
```

```
[Out] a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**3 + 105*
a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a**3*b**7
*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10
```

3.136 $\int \frac{(a+bx)^{10}}{x^2}\,dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+45a^8b^2x+10a^9b\log(x)-\frac{a^{10}}{x}+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^2}\,dx &= \int \left(45a^8b^2+\frac{a^{10}}{x^2}+\frac{10a^9b}{x}+120a^7b^3x+210a^6b^4x^2+252a^5b^5x^3+210a^4b^6x^4+120a^3b^7x^5+\right.\\ &= -\frac{a^{10}}{x}+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}a\end{aligned}$$

Mathematica [A] time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{x}+10a^9b\log(x)+45a^8b^2x+60a^7b^3x^2+70a^6b^4x^3+63a^5b^5x^4+42a^4b^6x^5+20a^3b^7x^6+\frac{45}{7}a^2b^8x^7+\frac{5}{4}ab^9x^8+\frac{b^{10}x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^2,x]
```

```
[Out] -(a^10/x) + 45*a^8*b^2*x + 60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4
 + 42*a^4*b^6*x^5 + 20*a^3*b^7*x^6 + (45*a^2*b^8*x^7)/7 + (5*a*b^9*x^8)/4 +
 (b^10*x^9)/9 + 10*a^9*b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^2, x]
```

fricas [A] time = 0.84, size = 114, normalized size = 0.99

$$\frac{28\,b^{10}x^{10} + 315\,ab^{9}x^{9} + 1620\,a^{2}b^{8}x^{8} + 5040\,a^{3}b^{7}x^{7} + 10584\,a^{4}b^{6}x^{6} + 15876\,a^{5}b^{5}x^{5} + 17640\,a^{6}b^{4}x^{4} + 15120\,a^{7}b^{3}x^{3} + 11340\,a^{8}b^{2}x^{2} + 2520\,a^{9}bx\log(x) - 252\,a^{10}}{252\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*x^10 + 315*a*b^9*x^9 + 1620*a^2*b^8*x^8 + 5040*a^3*b^7*x^7 +
 10584*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 17640*a^6*b^4*x^4 + 15120*a^7*b^3*
x^3 + 11340*a^8*b^2*x^2 + 2520*a^9*b*x*log(x) - 252*a^10)/x
```

giac [A] time = 0.96, size = 110, normalized size = 0.96

$$\frac{1}{9}\,b^{10}x^{9} + \frac{5}{4}\,ab^{9}x^{8} + \frac{45}{7}\,a^{2}b^{8}x^{7} + 20\,a^{3}b^{7}x^{6} + 42\,a^{4}b^{6}x^{5} + 63\,a^{5}b^{5}x^{4} + 70\,a^{6}b^{4}x^{3} + 60\,a^{7}b^{3}x^{2} + 45\,a^{8}b^{2}x + 10\,a^{9}b\log\left(|x|\right) - \frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="giac")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(abs(x)) - a^10/x
```

maple [A] time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^{9}}{9} + \frac{5a\,b^{9}x^{8}}{4} + \frac{45a^{2}b^{8}x^{7}}{7} + 20a^{3}b^{7}x^{6} + 42a^{4}b^{6}x^{5} + 63a^{5}b^{5}x^{4} + 70a^{6}b^{4}x^{3} + 60a^{7}b^{3}x^{2} + 10a^{9}b\ln(x) + 45a^{8}b^{2}x - \frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^2,x)
```

```
[Out] -a^10/x+45*a^8*b^2*x+60*a^7*b^3*x^2+70*a^6*b^4*x^3+63*a^5*b^5*x^4+42*a^4*b^
6*x^5+20*a^3*b^7*x^6+45/7*a^2*b^8*x^7+5/4*a*b^9*x^8+1/9*b^10*x^9+10*a^9*b*l
n(x)
```

maxima [A] time = 1.35, size = 109, normalized size = 0.95

$$\frac{1}{9}\,b^{10}x^{9} + \frac{5}{4}\,ab^{9}x^{8} + \frac{45}{7}\,a^{2}b^{8}x^{7} + 20\,a^{3}b^{7}x^{6} + 42\,a^{4}b^{6}x^{5} + 63\,a^{5}b^{5}x^{4} + 70\,a^{6}b^{4}x^{3} + 60\,a^{7}b^{3}x^{2} + 45\,a^{8}b^{2}x + 10\,a^{9}b\log(x) - \frac{a^{10}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^10*x^9 + 5/4*a*b^9*x^8 + 45/7*a^2*b^8*x^7 + 20*a^3*b^7*x^6 + 42*a^4*b
^6*x^5 + 63*a^5*b^5*x^4 + 70*a^6*b^4*x^3 + 60*a^7*b^3*x^2 + 45*a^8*b^2*x +
10*a^9*b*log(x) - a^10/x
```

mupad [B] time = 0.12, size = 109, normalized size = 0.95

$$\frac{b^{10}\,x^{9}}{9} - \frac{a^{10}}{x} + 45\,a^{8}\,b^{2}\,x + \frac{5\,a\,b^{9}\,x^{8}}{4} + 10\,a^{9}\,b\,\ln(x) + 60\,a^{7}\,b^{3}\,x^{2} + 70\,a^{6}\,b^{4}\,x^{3} + 63\,a^{5}\,b^{5}\,x^{4} + 42\,a^{4}\,b^{6}\,x^{5} + 20\,a^{3}\,b^{7}\,x^{6} + \frac{45\,a^{2}\,b^{8}\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^2,x)
```

```
[Out] (b^10*x^9)/9 - a^10/x + 45*a^8*b^2*x + (5*a*b^9*x^8)/4 + 10*a^9*b*log(x) +
60*a^7*b^3*x^2 + 70*a^6*b^4*x^3 + 63*a^5*b^5*x^4 + 42*a^4*b^6*x^5 + 20*a^3*
b^7*x^6 + (45*a^2*b^8*x^7)/7
```

sympy [A] time = 0.27, size = 117, normalized size = 1.02

$$-\frac{a^{10}}{x} + 10a^{9}b\log\left(x\right) + 45a^{8}b^{2}x + 60a^{7}b^{3}x^{2} + 70a^{6}b^{4}x^{3} + 63a^{5}b^{5}x^{4} + 42a^{4}b^{6}x^{5} + 20a^{3}b^{7}x^{6} + \frac{45a^{2}b^{8}x^{7}}{7} + \frac{5ab^{9}x^{8}}{4} + \frac{b^{10}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**2,x)
```

```
[Out] -a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b**3*x**2 + 70*a**6*
b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a**3*b**7*x**6 + 45*
a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9
```

3.137 $\int \frac{(a+bx)^{10}}{x^3}\, dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b}{}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+120a^7b^3x+45a^8b^2\log(x)-\frac{10a^9b}{x}-\frac{a^{10}}{2x^2}+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^3,x]
```

```
[Out] -a^10/(2*x^2) - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^3}\, dx = \int \left(120a^7b^3+\frac{a^{10}}{x^3}+\frac{10a^9b}{x^2}+\frac{45a^8b^2}{x}+210a^6b^4x+252a^5b^5x^2+210a^4b^6x^3+120a^3b^7x^4+\right.$$

$$= -\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+$$

Mathematica [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{2x^2}-\frac{10a^9b}{x}+45a^8b^2\log(x)+120a^7b^3x+105a^6b^4x^2+84a^5b^5x^3+\frac{105}{2}a^4b^6x^4+24a^3b^7x^5+\frac{15}{2}a^2b^8x^6+\frac{10}{7}ab^9x^7+\frac{b^{10}x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^3,x]
```

```
[Out] -1/2*a^10/x^2 - (10*a^9*b)/x + 120*a^7*b^3*x + 105*a^6*b^4*x^2 + 84*a^5*b^5
*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5 + (15*a^2*b^8*x^6)/2 + (10*a*b^
9*x^7)/7 + (b^10*x^8)/8 + 45*a^8*b^2*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^3, x]
```

fricas [A] time = 1.41, size = 114, normalized size = 0.96

$$\frac{7 b^{10}x^{10} + 80 ab^9x^9 + 420 a^2b^8x^8 + 1344 a^3b^7x^7 + 2940 a^4b^6x^6 + 4704 a^5b^5x^5 + 5880 a^6b^4x^4 + 6720 a^7b^3x^3 + 2520 a^8b^2x^2 \log(x) - 560 a^9bx - 28 a^{10}}{56 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*x^10 + 80*a*b^9*x^9 + 420*a^2*b^8*x^8 + 1344*a^3*b^7*x^7 + 294
0*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 5880*a^6*b^4*x^4 + 6720*a^7*b^3*x^3 + 25
20*a^8*b^2*x^2*log(x) - 560*a^9*b*x - 28*a^10)/x^2
```

giac [A] time = 0.93, size = 109, normalized size = 0.92

$$\frac{1}{8} b^{10}x^8 + \frac{10}{7} ab^9x^7 + \frac{15}{2} a^2b^8x^6 + 24 a^3b^7x^5 + \frac{105}{2} a^4b^6x^4 + 84 a^5b^5x^3 + 105 a^6b^4x^2 + 120 a^7b^3x + 45 a^8b^2 \log(|x|) - \frac{20 a^9bx + a^{10}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="giac")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(abs(x)) - 1/2*(20*a^9*b*x + a^10)/x^2
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^8}{8} + \frac{10a\,b^9x^7}{7} + \frac{15a^2b^8x^6}{2} + 24a^3b^7x^5 + \frac{105a^4b^6x^4}{2} + 84a^5b^5x^3 + 105a^6b^4x^2 + 45a^8b^2\ln(x) + 120a^7b^3x - \frac{10a^9b}{x} - \frac{a^{10}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^3,x)
```

```
[Out] -1/2*a^10/x^2-10*a^9*b/x+120*a^7*b^3*x+105*a^6*b^4*x^2+84*a^5*b^5*x^3+105/2
*a^4*b^6*x^4+24*a^3*b^7*x^5+15/2*a^2*b^8*x^6+10/7*a*b^9*x^7+1/8*b^10*x^8+45
*a^8*b^2*ln(x)
```

maxima [A] time = 1.44, size = 108, normalized size = 0.91

$$\frac{1}{8} b^{10}x^8 + \frac{10}{7} ab^9x^7 + \frac{15}{2} a^2b^8x^6 + 24 a^3b^7x^5 + \frac{105}{2} a^4b^6x^4 + 84 a^5b^5x^3 + 105 a^6b^4x^2 + 120 a^7b^3x + 45 a^8b^2 \log(x) - \frac{20 a^9bx + a^{10}}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^10*x^8 + 10/7*a*b^9*x^7 + 15/2*a^2*b^8*x^6 + 24*a^3*b^7*x^5 + 105/2*a
^4*b^6*x^4 + 84*a^5*b^5*x^3 + 105*a^6*b^4*x^2 + 120*a^7*b^3*x + 45*a^8*b^2*
log(x) - 1/2*(20*a^9*b*x + a^10)/x^2
```

mupad [B] time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^8}{8} - \frac{\frac{a^{10}}{2} + 10\,b\,x\,a^9}{x^2} + 120\,a^7\,b^3\,x + \frac{10\,a\,b^9\,x^7}{7} + 105\,a^6\,b^4\,x^2 + 84\,a^5\,b^5\,x^3 + \frac{105\,a^4\,b^6\,x^4}{2} + 24\,a^3\,b^7\,x^5 + \frac{15\,a^2\,b^8\,x^6}{2} + 45\,a^8\,b^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^3,x)
```

```
[Out] (b^10*x^8)/8 - (a^10/2 + 10*a^9*b*x)/x^2 + 120*a^7*b^3*x + (10*a*b^9*x^7)/7
 + 105*a^6*b^4*x^2 + 84*a^5*b^5*x^3 + (105*a^4*b^6*x^4)/2 + 24*a^3*b^7*x^5
+ (15*a^2*b^8*x^6)/2 + 45*a^8*b^2*log(x)
```

sympy [A] time = 0.31, size = 122, normalized size = 1.03

$$45a^8b^2\log(x) + 120a^7b^3x + 105a^6b^4x^2 + 84a^5b^5x^3 + \frac{105a^4b^6x^4}{2} + 24a^3b^7x^5 + \frac{15a^2b^8x^6}{2} + \frac{10ab^9x^7}{7} + \frac{b^{10}x^8}{8} + \frac{-a^{10} - 20a^9bx}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**3,x)
```

```
[Out] 45*a**8*b**2*log(x) + 120*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x
**3 + 105*a**4*b**6*x**4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a
*b**9*x**7/7 + b**10*x**8/8 + (-a**10 - 20*a**9*b*x)/(2*x**2)
```

3.138 $\int \frac{(a+bx)^{10}}{x^4} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5-\frac{45a^8b^2}{x}+210a^6b^4x+120a^7b^3\log(x)-\frac{5a^9b}{x^2}-\frac{a^{10}}{3x^3}+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^4,x]
```

```
[Out] -a^10/(3*x^3) - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^4} dx &= \int \left(210a^6b^4+\frac{a^{10}}{x^4}+\frac{10a^9b}{x^3}+\frac{45a^8b^2}{x^2}+\frac{120a^7b^3}{x}+252a^5b^5x+210a^4b^6x^2+120a^3b^7x^3+45a\right. \\ &= -\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6\end{aligned}$$

Mathematica [A] time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{3x^3}-\frac{5a^9b}{x^2}-\frac{45a^8b^2}{x}+120a^7b^3\log(x)+210a^6b^4x+126a^5b^5x^2+70a^4b^6x^3+30a^3b^7x^4+9a^2b^8x^5+\frac{5}{3}ab^9x^6+\frac{b^{10}x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^4,x]
```

```
[Out] -1/3*a^10/x^3 - (5*a^9*b)/x^2 - (45*a^8*b^2)/x + 210*a^6*b^4*x + 126*a^5*b^
5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2*b^8*x^5 + (5*a*b^9*x^6)/3 +
 (b^10*x^7)/7 + 120*a^7*b^3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^4, x]
```

fricas [A] time = 1.13, size = 114, normalized size = 0.99

$$\frac{3\,b^{10}x^{10} + 35\,ab^{9}x^{9} + 189\,a^{2}b^{8}x^{8} + 630\,a^{3}b^{7}x^{7} + 1470\,a^{4}b^{6}x^{6} + 2646\,a^{5}b^{5}x^{5} + 4410\,a^{6}b^{4}x^{4} + 2520\,a^{7}b^{3}x^{3}\log(x) - 945\,a^{8}b^{2}x^{2} - 105\,a^{9}bx - 7\,a^{10}}{21\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*x^10 + 35*a*b^9*x^9 + 189*a^2*b^8*x^8 + 630*a^3*b^7*x^7 + 1470
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 4410*a^6*b^4*x^4 + 2520*a^7*b^3*x^3*log(x
) - 945*a^8*b^2*x^2 - 105*a^9*b*x - 7*a^10)/x^3
```

giac [A] time = 1.14, size = 109, normalized size = 0.95

$$\frac{1}{7}\,b^{10}x^{7} + \frac{5}{3}\,ab^{9}x^{6} + 9\,a^{2}b^{8}x^{5} + 30\,a^{3}b^{7}x^{4} + 70\,a^{4}b^{6}x^{3} + 126\,a^{5}b^{5}x^{2} + 210\,a^{6}b^{4}x + 120\,a^{7}b^{3}\log\left(|x|\right) - \frac{135\,a^{8}b^{2}x^{2} + 15\,a^{9}bx + a^{10}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="giac")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(abs(x)) - 1/3*(135*
a^8*b^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

maple [A] time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^{7}}{7} + \frac{5a\,b^{9}x^{6}}{3} + 9a^{2}b^{8}x^{5} + 30a^{3}b^{7}x^{4} + 70a^{4}b^{6}x^{3} + 126a^{5}b^{5}x^{2} + 120a^{7}b^{3}\ln(x) + 210a^{6}b^{4}x - \frac{45a^{8}b^{2}}{x} - \frac{5a^{9}b}{x^{2}} - \frac{a^{10}}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^4,x)
```

```
[Out] -1/3*a^10/x^3-5*a^9*b/x^2-45*a^8*b^2/x+210*a^6*b^4*x+126*a^5*b^5*x^2+70*a^4
*b^6*x^3+30*a^3*b^7*x^4+9*a^2*b^8*x^5+5/3*a*b^9*x^6+1/7*b^10*x^7+120*a^7*b^
3*ln(x)
```

maxima [A] time = 1.37, size = 108, normalized size = 0.94

$$\frac{1}{7}\,b^{10}x^{7} + \frac{5}{3}\,ab^{9}x^{6} + 9\,a^{2}b^{8}x^{5} + 30\,a^{3}b^{7}x^{4} + 70\,a^{4}b^{6}x^{3} + 126\,a^{5}b^{5}x^{2} + 210\,a^{6}b^{4}x + 120\,a^{7}b^{3}\log(x) - \frac{135\,a^{8}b^{2}x^{2} + 15\,a^{9}bx + a^{10}}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*b^10*x^7 + 5/3*a*b^9*x^6 + 9*a^2*b^8*x^5 + 30*a^3*b^7*x^4 + 70*a^4*b^6*
x^3 + 126*a^5*b^5*x^2 + 210*a^6*b^4*x + 120*a^7*b^3*log(x) - 1/3*(135*a^8*b
^2*x^2 + 15*a^9*b*x + a^10)/x^3
```

mupad [B] time = 0.06, size = 110, normalized size = 0.96

$$\frac{b^{10}\,x^{7}}{7} - \frac{\frac{a^{10}}{3} + 5\,a^{9}\,b\,x + 45\,a^{8}\,b^{2}\,x^{2}}{x^{3}} + 210\,a^{6}\,b^{4}\,x + \frac{5\,a\,b^{9}\,x^{6}}{3} + 126\,a^{5}\,b^{5}\,x^{2} + 70\,a^{4}\,b^{6}\,x^{3} + 30\,a^{3}\,b^{7}\,x^{4} + 9\,a^{2}\,b^{8}\,x^{5} + 120\,a^{7}\,b^{3}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^4,x)
```

```
[Out] (b^10*x^7)/7 - (a^10/3 + 45*a^8*b^2*x^2 + 5*a^9*b*x)/x^3 + 210*a^6*b^4*x +
(5*a*b^9*x^6)/3 + 126*a^5*b^5*x^2 + 70*a^4*b^6*x^3 + 30*a^3*b^7*x^4 + 9*a^2
*b^8*x^5 + 120*a^7*b^3*log(x)
```

<u>**sympy**</u> **[A]** time = 0.34, size = 119, normalized size = 1.03

$$120a^7b^3\log(x) + 210a^6b^4x + 126a^5b^5x^2 + 70a^4b^6x^3 + 30a^3b^7x^4 + 9a^2b^8x^5 + \frac{5ab^9x^6}{3} + \frac{b^{10}x^7}{7} + \frac{-a^{10} - 15a^9bx - 135a^8b^2x^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**4,x)
```

```
[Out] 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b**6*
x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x**7/
7 + (-a**10 - 15*a**9*b*x - 135*a**8*b**2*x**2)/(3*x**3)
```

3.139 $\int \frac{(a+bx)^{10}}{x^5} dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{2x^2}+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4-\frac{120a^7b^3}{x}+252a^5b^5x+210a^6b^4\log(x)-\frac{10a^9b}{3x^3}-\frac{a^{10}}{4x^4}+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^5,x]
```

```
[Out] -a^10/(4*x^4) - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^5} dx = \int \left(252a^5b^5+\frac{a^{10}}{x^5}+\frac{10a^9b}{x^4}+\frac{45a^8b^2}{x^3}+\frac{120a^7b^3}{x^2}+\frac{210a^6b^4}{x}+210a^4b^6x+120a^3b^7x^2+45\right.$$

$$=-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2a$$

Mathematica [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{4x^4}-\frac{10a^9b}{3x^3}-\frac{45a^8b^2}{2x^2}-\frac{120a^7b^3}{x}+210a^6b^4\log(x)+252a^5b^5x+105a^4b^6x^2+40a^3b^7x^3+\frac{45}{4}a^2b^8x^4+2ab^9x^5+\frac{b^{10}x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^5,x]
```

```
[Out] -1/4*a^10/x^4 - (10*a^9*b)/(3*x^3) - (45*a^8*b^2)/(2*x^2) - (120*a^7*b^3)/x
 + 252*a^5*b^5*x + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 + (45*a^2*b^8*x^4)/4 +
2*a*b^9*x^5 + (b^10*x^6)/6 + 210*a^6*b^4*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^5, x]
```

fricas [A] time = 1.56, size = 114, normalized size = 0.96

$$\frac{2\,b^{10}x^{10}+24\,ab^{9}x^{9}+135\,a^{2}b^{8}x^{8}+480\,a^{3}b^{7}x^{7}+1260\,a^{4}b^{6}x^{6}+3024\,a^{5}b^{5}x^{5}+2520\,a^{6}b^{4}x^{4}\log(x)-1440\,a^{7}b^{3}x^{3}-270\,a^{8}b^{2}x^{2}-40\,a^{9}bx-3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*x^10 + 24*a*b^9*x^9 + 135*a^2*b^8*x^8 + 480*a^3*b^7*x^7 + 1260
*a^4*b^6*x^6 + 3024*a^5*b^5*x^5 + 2520*a^6*b^4*x^4*log(x) - 1440*a^7*b^3*x^
3 - 270*a^8*b^2*x^2 - 40*a^9*b*x - 3*a^10)/x^4
```

giac [A] time = 1.12, size = 111, normalized size = 0.93

$$\frac{1}{6}\,b^{10}x^{6}+2\,ab^{9}x^{5}+\frac{45}{4}\,a^{2}b^{8}x^{4}+40\,a^{3}b^{7}x^{3}+105\,a^{4}b^{6}x^{2}+252\,a^{5}b^{5}x+210\,a^{6}b^{4}\log\left(|x|\right)-\frac{1440\,a^{7}b^{3}x^{3}+270\,a^{8}b^{2}x^{2}+40\,a^{9}bx+3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="giac")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(abs(x)) - 1/12*(1440*a^7*b^3*x^3 +
270*a^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^{6}}{6}+2a\,b^{9}x^{5}+\frac{45a^{2}b^{8}x^{4}}{4}+40a^{3}b^{7}x^{3}+105a^{4}b^{6}x^{2}+210a^{6}b^{4}\ln(x)+252a^{5}b^{5}x-\frac{120a^{7}b^{3}}{x}-\frac{45a^{8}b^{2}}{2x^{2}}-\frac{10a^{9}b}{3x^{3}}-\frac{a^{10}}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^5,x)
```

```
[Out] -1/4*a^10/x^4-10/3*a^9*b/x^3-45/2*a^8*b^2/x^2-120*a^7*b^3/x+252*a^5*b^5*x+1
05*a^4*b^6*x^2+40*a^3*b^7*x^3+45/4*a^2*b^8*x^4+2*a*b^9*x^5+1/6*b^10*x^6+210
*a^6*b^4*ln(x)
```

maxima [A] time = 1.27, size = 110, normalized size = 0.92

$$\frac{1}{6}\,b^{10}x^{6}+2\,ab^{9}x^{5}+\frac{45}{4}\,a^{2}b^{8}x^{4}+40\,a^{3}b^{7}x^{3}+105\,a^{4}b^{6}x^{2}+252\,a^{5}b^{5}x+210\,a^{6}b^{4}\log(x)-\frac{1440\,a^{7}b^{3}x^{3}+270\,a^{8}b^{2}x^{2}+40\,a^{9}bx+3\,a^{10}}{12\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^10*x^6 + 2*a*b^9*x^5 + 45/4*a^2*b^8*x^4 + 40*a^3*b^7*x^3 + 105*a^4*b^
6*x^2 + 252*a^5*b^5*x + 210*a^6*b^4*log(x) - 1/12*(1440*a^7*b^3*x^3 + 270*a
^8*b^2*x^2 + 40*a^9*b*x + 3*a^10)/x^4
```

mupad [B] time = 0.10, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^{6}}{6}-\frac{\frac{a^{10}}{4}+\frac{10\,a^{9}\,b\,x}{3}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{2}+120\,a^{7}\,b^{3}\,x^{3}}{x^{4}}+252\,a^{5}\,b^{5}\,x+2\,a\,b^{9}\,x^{5}+105\,a^{4}\,b^{6}\,x^{2}+40\,a^{3}\,b^{7}\,x^{3}+\frac{45\,a^{2}\,b^{8}\,x^{4}}{4}+210\,a^{6}\,b^{4}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^5,x)
```

```
[Out] (b^10*x^6)/6 - (a^10/4 + (45*a^8*b^2*x^2)/2 + 120*a^7*b^3*x^3 + (10*a^9*b*x
)/3)/x^4 + 252*a^5*b^5*x + 2*a*b^9*x^5 + 105*a^4*b^6*x^2 + 40*a^3*b^7*x^3 +
 (45*a^2*b^8*x^4)/4 + 210*a^6*b^4*log(x)
```

sympy [**A**] time = 0.44, size = 121, normalized size = 1.02

$$210a^6b^4\log(x) + 252a^5b^5x + 105a^4b^6x^2 + 40a^3b^7x^3 + \frac{45a^2b^8x^4}{4} + 2ab^9x^5 + \frac{b^{10}x^6}{6} + \frac{-3a^{10} - 40a^9bx - 270a^8b^2x^2 - 1440a^7b^3x^3}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**5,x)
```

```
[Out] 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105*a**4*b**6*x**2 + 40*a**3*b**7*
x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 + b**10*x**6/6 + (-3*a**10 - 40*
a**9*b*x - 270*a**8*b**2*x**2 - 1440*a**7*b**3*x**3)/(12*x**4)
```

3.140 $\int \frac{(a+bx)^{10}}{x^6}\,dx$

Optimal. Leaf size=117

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Rubi [A] time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}+60a^3b^7x^2+15a^2b^8x^3-\frac{210a^6b^4}{x}+210a^4b^6x+252a^5b^5\log(x)-\frac{5a^9b}{2x^4}-\frac{a^{10}}{5x^5}+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^6,x]
```

```
[Out] -a^10/(5*x^5) - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^6}\,dx &= \int \left(210a^4b^6+\frac{a^{10}}{x^6}+\frac{10a^9b}{x^5}+\frac{45a^8b^2}{x^4}+\frac{120a^7b^3}{x^3}+\frac{210a^6b^4}{x^2}+\frac{252a^5b^5}{x}+120a^3b^7x+45a^2b^8\right.\\ &= -\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\end{aligned}$$

Mathematica [A] time = 0.01, size = 117, normalized size = 1.00

$$-\frac{a^{10}}{5x^5}-\frac{5a^9b}{2x^4}-\frac{15a^8b^2}{x^3}-\frac{60a^7b^3}{x^2}-\frac{210a^6b^4}{x}+252a^5b^5\log(x)+210a^4b^6x+60a^3b^7x^2+15a^2b^8x^3+\frac{5}{2}ab^9x^4+\frac{b^{10}x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^6,x]
```

```
[Out] -1/5*a^10/x^5 - (5*a^9*b)/(2*x^4) - (15*a^8*b^2)/x^3 - (60*a^7*b^3)/x^2 - (
210*a^6*b^4)/x + 210*a^4*b^6*x + 60*a^3*b^7*x^2 + 15*a^2*b^8*x^3 + (5*a*b^9
*x^4)/2 + (b^10*x^5)/5 + 252*a^5*b^5*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^6, x]
```

fricas [A] time = 1.31, size = 114, normalized size = 0.97

$$\frac{2\,b^{10}x^{10} + 25\,ab^{9}x^{9} + 150\,a^{2}b^{8}x^{8} + 600\,a^{3}b^{7}x^{7} + 2100\,a^{4}b^{6}x^{6} + 2520\,a^{5}b^{5}x^{5}\log(x) - 2100\,a^{6}b^{4}x^{4} - 600\,a^{7}b^{3}x^{3} - 150\,a^{8}b^{2}x^{2} - 25\,a^{9}bx - 2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*x^10 + 25*a*b^9*x^9 + 150*a^2*b^8*x^8 + 600*a^3*b^7*x^7 + 2100
*a^4*b^6*x^6 + 2520*a^5*b^5*x^5*log(x) - 2100*a^6*b^4*x^4 - 600*a^7*b^3*x^3
 - 150*a^8*b^2*x^2 - 25*a^9*b*x - 2*a^10)/x^5
```

giac [A] time = 1.14, size = 111, normalized size = 0.95

$$\frac{1}{5}\,b^{10}x^{5} + \frac{5}{2}\,ab^{9}x^{4} + 15\,a^{2}b^{8}x^{3} + 60\,a^{3}b^{7}x^{2} + 210\,a^{4}b^{6}x + 252\,a^{5}b^{5}\log\left(|x|\right) - \frac{2100\,a^{6}b^{4}x^{4} + 600\,a^{7}b^{3}x^{3} + 150\,a^{8}b^{2}x^{2} + 25\,a^{9}bx + 2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="giac")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(abs(x)) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 +
150*a^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

maple [A] time = 0.01, size = 110, normalized size = 0.94

$$\frac{b^{10}x^{5}}{5} + \frac{5a\,b^{9}x^{4}}{2} + 15a^{2}b^{8}x^{3} + 60a^{3}b^{7}x^{2} + 252a^{5}b^{5}\ln(x) + 210a^{4}b^{6}x - \frac{210a^{6}b^{4}}{x} - \frac{60a^{7}b^{3}}{x^{2}} - \frac{15a^{8}b^{2}}{x^{3}} - \frac{5a^{9}b}{2x^{4}} - \frac{a^{10}}{5x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^6,x)
```

```
[Out] -1/5*a^10/x^5-5/2*a^9*b/x^4-15*a^8*b^2/x^3-60*a^7*b^3/x^2-210*a^6*b^4/x+210
*a^4*b^6*x+60*a^3*b^7*x^2+15*a^2*b^8*x^3+5/2*a*b^9*x^4+1/5*b^10*x^5+252*a^5
*b^5*ln(x)
```

maxima [A] time = 1.40, size = 110, normalized size = 0.94

$$\frac{1}{5}\,b^{10}x^{5} + \frac{5}{2}\,ab^{9}x^{4} + 15\,a^{2}b^{8}x^{3} + 60\,a^{3}b^{7}x^{2} + 210\,a^{4}b^{6}x + 252\,a^{5}b^{5}\log(x) - \frac{2100\,a^{6}b^{4}x^{4} + 600\,a^{7}b^{3}x^{3} + 150\,a^{8}b^{2}x^{2} + 25\,a^{9}bx + 2\,a^{10}}{10\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*b^10*x^5 + 5/2*a*b^9*x^4 + 15*a^2*b^8*x^3 + 60*a^3*b^7*x^2 + 210*a^4*b^
6*x + 252*a^5*b^5*log(x) - 1/10*(2100*a^6*b^4*x^4 + 600*a^7*b^3*x^3 + 150*a
^8*b^2*x^2 + 25*a^9*b*x + 2*a^10)/x^5
```

mupad [B] time = 0.10, size = 110, normalized size = 0.94

$$\frac{b^{10}\,x^{5}}{5} - \frac{\frac{a^{10}}{5} + \frac{5\,a^{9}\,b\,x}{2} + 15\,a^{8}\,b^{2}\,x^{2} + 60\,a^{7}\,b^{3}\,x^{3} + 210\,a^{6}\,b^{4}\,x^{4}}{x^{5}} + 210\,a^{4}\,b^{6}\,x + \frac{5\,a\,b^{9}\,x^{4}}{2} + 60\,a^{3}\,b^{7}\,x^{2} + 15\,a^{2}\,b^{8}\,x^{3} + 252\,a^{5}\,b^{5}\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^6,x)
```

```
[Out] (b^10*x^5)/5 - (a^10/5 + 15*a^8*b^2*x^2 + 60*a^7*b^3*x^3 + 210*a^6*b^4*x^4
+ (5*a^9*b*x)/2)/x^5 + 210*a^4*b^6*x + (5*a*b^9*x^4)/2 + 60*a^3*b^7*x^2 + 1
5*a^2*b^8*x^3 + 252*a^5*b^5*log(x)
```

sympy [A] time = 0.57, size = 121, normalized size = 1.03

$$252a^5b^5\log(x) + 210a^4b^6x + 60a^3b^7x^2 + 15a^2b^8x^3 + \frac{5ab^9x^4}{2} + \frac{b^{10}x^5}{5} + \frac{-2a^{10} - 25a^9bx - 150a^8b^2x^2 - 600a^7b^3x^3 - 2100a^6b^4x^4}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**6,x)
```

```
[Out] 252*a**5*b**5*log(x) + 210*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x
**3 + 5*a*b**9*x**4/2 + b**10*x**5/5 + (-2*a**10 - 25*a**9*b*x - 150*a**8*b
**2*x**2 - 600*a**7*b**3*x**3 - 2100*a**6*b**4*x**4)/(10*x**5)
```

3.141 $\int \frac{(a+bx)^{10}}{x^7} dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}+\frac{45}{2}a^2b^8x^2-\frac{252a^5b^5}{x}+120a^3b^7x+210a^4b^6\log(x)-\frac{2a^9b}{x^5}-\frac{a^{10}}{6x^6}+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^7,x]
```

```
[Out] -a^10/(6*x^6) - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^7}\,dx &= \int \left(120a^3b^7+\frac{a^{10}}{x^7}+\frac{10a^9b}{x^6}+\frac{45a^8b^2}{x^5}+\frac{120a^7b^3}{x^4}+\frac{210a^6b^4}{x^3}+\frac{252a^5b^5}{x^2}+\frac{210a^4b^6}{x}+45a^2
\\ &= -\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3\end{aligned}$$

Mathematica [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{6x^6}-\frac{2a^9b}{x^5}-\frac{45a^8b^2}{4x^4}-\frac{40a^7b^3}{x^3}-\frac{105a^6b^4}{x^2}-\frac{252a^5b^5}{x}+210a^4b^6\log(x)+120a^3b^7x+\frac{45}{2}a^2b^8x^2+\frac{10}{3}ab^9x^3+\frac{b^{10}x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^7,x]
```

```
[Out] -1/6*a^10/x^6 - (2*a^9*b)/x^5 - (45*a^8*b^2)/(4*x^4) - (40*a^7*b^3)/x^3 - (
105*a^6*b^4)/x^2 - (252*a^5*b^5)/x + 120*a^3*b^7*x + (45*a^2*b^8*x^2)/2 + (
10*a*b^9*x^3)/3 + (b^10*x^4)/4 + 210*a^4*b^6*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^7, x]
```

fricas [A] time = 1.20, size = 114, normalized size = 0.96

$$\frac{3 b^{10}x^{10} + 40 ab^9x^9 + 270 a^2b^8x^8 + 1440 a^3b^7x^7 + 2520 a^4b^6x^6 \log(x) - 3024 a^5b^5x^5 - 1260 a^6b^4x^4 - 480 a^7b^3x^3 - 135 a^8b^2x^2 - 24 a^9bx - 2 a^{10}}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 + 40*a*b^9*x^9 + 270*a^2*b^8*x^8 + 1440*a^3*b^7*x^7 + 252
0*a^4*b^6*x^6*log(x) - 3024*a^5*b^5*x^5 - 1260*a^6*b^4*x^4 - 480*a^7*b^3*x^
3 - 135*a^8*b^2*x^2 - 24*a^9*b*x - 2*a^10)/x^6
```

giac [A] time = 1.24, size = 111, normalized size = 0.93

$$\frac{1}{4} b^{10}x^4 + \frac{10}{3} ab^9x^3 + \frac{45}{2} a^2b^8x^2 + 120 a^3b^7x + 210 a^4b^6 \log(|x|) - \frac{3024 a^5b^5x^5 + 1260 a^6b^4x^4 + 480 a^7b^3x^3 + 135 a^8b^2x^2 + 24 a^9bx + 2 a^{10}}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="giac")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(abs(x)) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x
^3 + 135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^4}{4} + \frac{10a\,b^9x^3}{3} + \frac{45a^2b^8x^2}{2} + 210a^4b^6\ln(x) + 120a^3b^7x - \frac{252a^5b^5}{x} - \frac{105a^6b^4}{x^2} - \frac{40a^7b^3}{x^3} - \frac{45a^8b^2}{4x^4} - \frac{2a^9b}{x^5} - \frac{a^{10}}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^7,x)
```

```
[Out] -1/6*a^10/x^6-2*a^9*b/x^5-45/4*a^8*b^2/x^4-40*a^7*b^3/x^3-105*a^6*b^4/x^2-2
52*a^5*b^5/x+120*a^3*b^7*x+45/2*a^2*b^8*x^2+10/3*a*b^9*x^3+1/4*b^10*x^4+210
*a^4*b^6*ln(x)
```

maxima [A] time = 1.37, size = 110, normalized size = 0.92

$$\frac{1}{4} b^{10}x^4 + \frac{10}{3} ab^9x^3 + \frac{45}{2} a^2b^8x^2 + 120 a^3b^7x + 210 a^4b^6 \log(x) - \frac{3024 a^5b^5x^5 + 1260 a^6b^4x^4 + 480 a^7b^3x^3 + 135 a^8b^2x^2 + 24 a^9bx + 2 a^{10}}{12 x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*b^10*x^4 + 10/3*a*b^9*x^3 + 45/2*a^2*b^8*x^2 + 120*a^3*b^7*x + 210*a^4*
b^6*log(x) - 1/12*(3024*a^5*b^5*x^5 + 1260*a^6*b^4*x^4 + 480*a^7*b^3*x^3 +
135*a^8*b^2*x^2 + 24*a^9*b*x + 2*a^10)/x^6
```

mupad [B] time = 0.05, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^4}{4} - \frac{\frac{a^{10}}{6} + 2\,a^9\,b\,x + \frac{45\,a^8\,b^2\,x^2}{4} + 40\,a^7\,b^3\,x^3 + 105\,a^6\,b^4\,x^4 + 252\,a^5\,b^5\,x^5}{x^6} + 120\,a^3\,b^7\,x + \frac{10\,a\,b^9\,x^3}{3} + \frac{45\,a^2\,b^8\,x^2}{2} + 210\,a^4\,b^6\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^7,x)
```

```
[Out] (b^10*x^4)/4 - (a^10/6 + (45*a^8*b^2*x^2)/4 + 40*a^7*b^3*x^3 + 105*a^6*b^4*
x^4 + 252*a^5*b^5*x^5 + 2*a^9*b*x)/x^6 + 120*a^3*b^7*x + (10*a*b^9*x^3)/3 +
 (45*a^2*b^8*x^2)/2 + 210*a^4*b^6*log(x)
```

sympy [A] time = 0.57, size = 122, normalized size = 1.03

$$210a^4b^6\log(x) + 120a^3b^7x + \frac{45a^2b^8x^2}{2} + \frac{10ab^9x^3}{3} + \frac{b^{10}x^4}{4} + \frac{-2a^{10} - 24a^9bx - 135a^8b^2x^2 - 480a^7b^3x^3 - 1260a^6b^4x^4 - 3024a^5b^5x^5}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**7,x)
```

```
[Out] 210*a**4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x*
*3/3 + b**10*x**4/4 + (-2*a**10 - 24*a**9*b*x - 135*a**8*b**2*x**2 - 480*a*
*7*b**3*x**3 - 1260*a**6*b**4*x**4 - 3024*a**5*b**5*x**5)/(12*x**6)
```

3.142 $\int \frac{(a+bx)^{10}}{x^8} dx$

Optimal. Leaf size=115

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Rubi [A] time = 0.05, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+120a^3b^7\log(x)-\frac{5a^9b}{3x^6}-\frac{a^{10}}{7x^7}+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^8,x]
```

```
[Out] -a^10/(7*x^7) - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^8}dx &= \int \left(45a^2b^8+\frac{a^{10}}{x^8}+\frac{10a^9b}{x^7}+\frac{45a^8b^2}{x^6}+\frac{120a^7b^3}{x^5}+\frac{210a^6b^4}{x^4}+\frac{252a^5b^5}{x^3}+\frac{210a^4b^6}{x^2}+\frac{120a^3b^7}{x}\right.\\ &= -\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 115, normalized size = 1.00

$$-\frac{a^{10}}{7x^7}-\frac{5a^9b}{3x^6}-\frac{9a^8b^2}{x^5}-\frac{30a^7b^3}{x^4}-\frac{70a^6b^4}{x^3}-\frac{126a^5b^5}{x^2}-\frac{210a^4b^6}{x}+120a^3b^7\log(x)+45a^2b^8x+5ab^9x^2+\frac{b^{10}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^8,x]
```

```
[Out] -1/7*a^10/x^7 - (5*a^9*b)/(3*x^6) - (9*a^8*b^2)/x^5 - (30*a^7*b^3)/x^4 - (7
0*a^6*b^4)/x^3 - (126*a^5*b^5)/x^2 - (210*a^4*b^6)/x + 45*a^2*b^8*x + 5*a*b
^9*x^2 + (b^10*x^3)/3 + 120*a^3*b^7*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^8}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^8, x]
```

fricas [A] time = 1.31, size = 114, normalized size = 0.99

$$\frac{7\,b^{10}x^{10} + 105\,ab^9x^9 + 945\,a^2b^8x^8 + 2520\,a^3b^7x^7\log(x) - 4410\,a^4b^6x^6 - 2646\,a^5b^5x^5 - 1470\,a^6b^4x^4 - 630\,a^7b^3x^3 - 189\,a^8b^2x^2 - 35\,a^9bx - 3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*x^10 + 105*a*b^9*x^9 + 945*a^2*b^8*x^8 + 2520*a^3*b^7*x^7*log(
x) - 4410*a^4*b^6*x^6 - 2646*a^5*b^5*x^5 - 1470*a^6*b^4*x^4 - 630*a^7*b^3*x
^3 - 189*a^8*b^2*x^2 - 35*a^9*b*x - 3*a^10)/x^7
```

giac [A] time = 1.10, size = 111, normalized size = 0.97

$$\frac{1}{3}\,b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log(|x|) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 + 35\,a^9bx + 3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="giac")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(abs(x)) - 1/21*
(4410*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 +
 189*a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

maple [A] time = 0.01, size = 110, normalized size = 0.96

$$\frac{b^{10}x^3}{3} + 5a\,b^9x^2 + 120a^3b^7\ln(x) + 45a^2b^8x - \frac{210a^4b^6}{x} - \frac{126a^5b^5}{x^2} - \frac{70a^6b^4}{x^3} - \frac{30a^7b^3}{x^4} - \frac{9a^8b^2}{x^5} - \frac{5a^9b}{3x^6} - \frac{a^{10}}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^8,x)
```

```
[Out] -1/7*a^10/x^7-5/3*a^9*b/x^6-9*a^8*b^2/x^5-30*a^7*b^3/x^4-70*a^6*b^4/x^3-126
*a^5*b^5/x^2-210*a^4*b^6/x+45*a^2*b^8*x+5*a*b^9*x^2+1/3*b^10*x^3+120*a^3*b^
7*ln(x)
```

maxima [A] time = 1.30, size = 110, normalized size = 0.96

$$\frac{1}{3}\,b^{10}x^3 + 5\,ab^9x^2 + 45\,a^2b^8x + 120\,a^3b^7\log(x) - \frac{4410\,a^4b^6x^6 + 2646\,a^5b^5x^5 + 1470\,a^6b^4x^4 + 630\,a^7b^3x^3 + 189\,a^8b^2x^2 + 35\,a^9bx + 3\,a^{10}}{21\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^8,x, algorithm="maxima")
```

```
[Out] 1/3*b^10*x^3 + 5*a*b^9*x^2 + 45*a^2*b^8*x + 120*a^3*b^7*log(x) - 1/21*(4410
*a^4*b^6*x^6 + 2646*a^5*b^5*x^5 + 1470*a^6*b^4*x^4 + 630*a^7*b^3*x^3 + 189*
a^8*b^2*x^2 + 35*a^9*b*x + 3*a^10)/x^7
```

mupad [B] time = 0.10, size = 110, normalized size = 0.96

$$\frac{b^{10}\,x^3}{3} - \frac{\frac{a^{10}}{7} + \frac{5\,a^9\,b\,x}{3} + 9\,a^8\,b^2\,x^2 + 30\,a^7\,b^3\,x^3 + 70\,a^6\,b^4\,x^4 + 126\,a^5\,b^5\,x^5 + 210\,a^4\,b^6\,x^6}{x^7} + 45\,a^2\,b^8\,x + 5\,a\,b^9\,x^2 + 120\,a^3\,b^7\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^8,x)
```

```
[Out] (b^10*x^3)/3 - (a^10/7 + 9*a^8*b^2*x^2 + 30*a^7*b^3*x^3 + 70*a^6*b^4*x^4 +
126*a^5*b^5*x^5 + 210*a^4*b^6*x^6 + (5*a^9*b*x)/3)/x^7 + 45*a^2*b^8*x + 5*a
*b^9*x^2 + 120*a^3*b^7*log(x)
```

sympy [A] time = 0.70, size = 119, normalized size = 1.03

$$120a^3b^7\log(x) + 45a^2b^8x + 5ab^9x^2 + \frac{b^{10}x^3}{3} + \frac{-3a^{10} - 35a^9bx - 189a^8b^2x^2 - 630a^7b^3x^3 - 1470a^6b^4x^4 - 2646a^5b^5x^5 - 4410a^4b^6x^6}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**8,x)
```

```
[Out] 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 + b**10*x**3/3 + (-3*
a**10 - 35*a**9*b*x - 189*a**8*b**2*x**2 - 630*a**7*b**3*x**3 - 1470*a**6*b
**4*x**4 - 2646*a**5*b**5*x**5 - 4410*a**4*b**6*x**6)/(21*x**7)
```

3.143 $\int \frac{(a+bx)^{10}}{x^9} \, dx$

Optimal. Leaf size=119

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)-\frac{10a^9b}{7x^7}-\frac{a^{10}}{8x^8}+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^9,x]
```

```
[Out] -a^10/(8*x^8) - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^9}\,dx &= \int \left(10ab^9+\frac{a^{10}}{x^9}+\frac{10a^9b}{x^8}+\frac{45a^8b^2}{x^7}+\frac{120a^7b^3}{x^6}+\frac{210a^6b^4}{x^5}+\frac{252a^5b^5}{x^4}+\frac{210a^4b^6}{x^3}+\frac{120a^3b^7}{x^2}+\cdots\right.\\ &= -\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+10ab^9x+\cdots\end{aligned}$$

Mathematica [A] time = 0.01, size = 119, normalized size = 1.00

$$-\frac{a^{10}}{8x^8}-\frac{10a^9b}{7x^7}-\frac{15a^8b^2}{2x^6}-\frac{24a^7b^3}{x^5}-\frac{105a^6b^4}{2x^4}-\frac{84a^5b^5}{x^3}-\frac{105a^4b^6}{x^2}-\frac{120a^3b^7}{x}+45a^2b^8\log(x)+10ab^9x+\frac{b^{10}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^9,x]
```

```
[Out] -1/8*a^10/x^8 - (10*a^9*b)/(7*x^7) - (15*a^8*b^2)/(2*x^6) - (24*a^7*b^3)/x^
5 - (105*a^6*b^4)/(2*x^4) - (84*a^5*b^5)/x^3 - (105*a^4*b^6)/x^2 - (120*a^3
*b^7)/x + 10*a*b^9*x + (b^10*x^2)/2 + 45*a^2*b^8*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^9,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^9, x]
```

fricas [A] time = 1.14, size = 114, normalized size = 0.96

$$\frac{28\,b^{10}x^{10} + 560\,ab^{9}x^{9} + 2520\,a^{2}b^{8}x^{8}\log(x) - 6720\,a^{3}b^{7}x^{7} - 5880\,a^{4}b^{6}x^{6} - 4704\,a^{5}b^{5}x^{5} - 2940\,a^{6}b^{4}x^{4} - 1344\,a^{7}b^{3}x^{3} - 420\,a^{8}b^{2}x^{2} - 80\,a^{9}bx - 7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*x^10 + 560*a*b^9*x^9 + 2520*a^2*b^8*x^8*log(x) - 6720*a^3*b^7
*x^7 - 5880*a^4*b^6*x^6 - 4704*a^5*b^5*x^5 - 2940*a^6*b^4*x^4 - 1344*a^7*b^
3*x^3 - 420*a^8*b^2*x^2 - 80*a^9*b*x - 7*a^10)/x^8
```

giac [A] time = 1.16, size = 111, normalized size = 0.93

$$\frac{1}{2}\,b^{10}x^{2} + 10\,ab^{9}x + 45\,a^{2}b^{8}\log\left(|x|\right) - \frac{6720\,a^{3}b^{7}x^{7} + 5880\,a^{4}b^{6}x^{6} + 4704\,a^{5}b^{5}x^{5} + 2940\,a^{6}b^{4}x^{4} + 1344\,a^{7}b^{3}x^{3} + 420\,a^{8}b^{2}x^{2} + 80\,a^{9}bx + 7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="giac")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(abs(x)) - 1/56*(6720*a^3*b^7*x^7
 + 5880*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^
3 + 420*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

maple [A] time = 0.01, size = 110, normalized size = 0.92

$$\frac{b^{10}x^{2}}{2} + 45a^{2}b^{8}\ln(x) + 10a\,b^{9}x - \frac{120a^{3}b^{7}}{x} - \frac{105a^{4}b^{6}}{x^{2}} - \frac{84a^{5}b^{5}}{x^{3}} - \frac{105a^{6}b^{4}}{2x^{4}} - \frac{24a^{7}b^{3}}{x^{5}} - \frac{15a^{8}b^{2}}{2x^{6}} - \frac{10a^{9}b}{7x^{7}} - \frac{a^{10}}{8x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^9,x)
```

```
[Out] -1/8*a^10/x^8-10/7*a^9*b/x^7-15/2*a^8*b^2/x^6-24*a^7*b^3/x^5-105/2*a^6*b^4/
x^4-84*a^5*b^5/x^3-105*a^4*b^6/x^2-120*a^3*b^7/x+10*a*b^9*x+1/2*b^10*x^2+45
*a^2*b^8*ln(x)
```

maxima [A] time = 1.42, size = 110, normalized size = 0.92

$$\frac{1}{2}\,b^{10}x^{2} + 10\,ab^{9}x + 45\,a^{2}b^{8}\log(x) - \frac{6720\,a^{3}b^{7}x^{7} + 5880\,a^{4}b^{6}x^{6} + 4704\,a^{5}b^{5}x^{5} + 2940\,a^{6}b^{4}x^{4} + 1344\,a^{7}b^{3}x^{3} + 420\,a^{8}b^{2}x^{2} + 80\,a^{9}bx + 7\,a^{10}}{56\,x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^9,x, algorithm="maxima")
```

```
[Out] 1/2*b^10*x^2 + 10*a*b^9*x + 45*a^2*b^8*log(x) - 1/56*(6720*a^3*b^7*x^7 + 58
80*a^4*b^6*x^6 + 4704*a^5*b^5*x^5 + 2940*a^6*b^4*x^4 + 1344*a^7*b^3*x^3 + 4
20*a^8*b^2*x^2 + 80*a^9*b*x + 7*a^10)/x^8
```

mupad [B] time = 0.07, size = 110, normalized size = 0.92

$$\frac{b^{10}\,x^{2}}{2} - \frac{\frac{a^{10}}{8} + \frac{10\,a^{9}\,b\,x}{7} + \frac{15\,a^{8}\,b^{2}\,x^{2}}{2} + 24\,a^{7}\,b^{3}\,x^{3} + \frac{105\,a^{6}\,b^{4}\,x^{4}}{2} + 84\,a^{5}\,b^{5}\,x^{5} + 105\,a^{4}\,b^{6}\,x^{6} + 120\,a^{3}\,b^{7}\,x^{7}}{x^{8}} + 45\,a^{2}\,b^{8}\,\ln(x) + 10\,a\,b^{9}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^9,x)
```

```
[Out] (b^10*x^2)/2 - (a^10/8 + (15*a^8*b^2*x^2)/2 + 24*a^7*b^3*x^3 + (105*a^6*b^4
*x^4)/2 + 84*a^5*b^5*x^5 + 105*a^4*b^6*x^6 + 120*a^3*b^7*x^7 + (10*a^9*b*x)
/7)/x^8 + 45*a^2*b^8*log(x) + 10*a*b^9*x
```

sympy [A] time = 0.76, size = 119, normalized size = 1.00

$$45a^2b^8\log(x) + 10ab^9x + \frac{b^{10}x^2}{2} + \frac{-7a^{10} - 80a^9bx - 420a^8b^2x^2 - 1344a^7b^3x^3 - 2940a^6b^4x^4 - 4704a^5b^5x^5 - 5880a^4b^6x^6 - 6720a^3b^7x^7}{56x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**9,x)
```

```
[Out] 45*a**2*b**8*log(x) + 10*a*b**9*x + b**10*x**2/2 + (-7*a**10 - 80*a**9*b*x
- 420*a**8*b**2*x**2 - 1344*a**7*b**3*x**3 - 2940*a**6*b**4*x**4 - 4704*a**
5*b**5*x**5 - 5880*a**4*b**6*x**6 - 6720*a**3*b**7*x**7)/(56*x**8)
```

3.144 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Rubi [A] time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx = \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{4\ldots}{\ldots}\right.$$

$$= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+1\ldots$$

Mathematica [A] time = 0.01, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^10, x]
```

fricas [A] time = 1.21, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10}+2520\,ab^{9}x^{9}\log(x)-11340\,a^{2}b^{8}x^{8}-15120\,a^{3}b^{7}x^{7}-17640\,a^{4}b^{6}x^{6}-15876\,a^{5}b^{5}x^{5}-10584\,a^{6}b^{4}x^{4}-5040\,a^{7}b^{3}x^{3}-1620\,a^{8}b^{2}x^{2}-315\,a^{9}bx-28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

giac [A] time = 1.21, size = 110, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log\left(|x|\right)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

maple [A] time = 0.01, size = 109, normalized size = 0.96

$$10a\,b^{9}\ln(x)+b^{10}x-\frac{45a^{2}b^{8}}{x}-\frac{60a^{3}b^{7}}{x^{2}}-\frac{70a^{4}b^{6}}{x^{3}}-\frac{63a^{5}b^{5}}{x^{4}}-\frac{42a^{6}b^{4}}{x^{5}}-\frac{20a^{7}b^{3}}{x^{6}}-\frac{45a^{8}b^{2}}{7x^{7}}-\frac{5a^{9}b}{4x^{8}}-\frac{a^{10}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] -1/9*a^10/x^9-5/4*a^9*b/x^8-45/7*a^8*b^2/x^7-20*a^7*b^3/x^6-42*a^6*b^4/x^5-
63*a^5*b^5/x^4-70*a^4*b^6/x^3-60*a^3*b^7/x^2-45*a^2*b^8/x+b^10*x+10*a*b^9*l
n(x)
```

maxima [A] time = 1.40, size = 109, normalized size = 0.96

$$b^{10}x+10\,ab^{9}\log(x)-\frac{11340\,a^{2}b^{8}x^{8}+15120\,a^{3}b^{7}x^{7}+17640\,a^{4}b^{6}x^{6}+15876\,a^{5}b^{5}x^{5}+10584\,a^{6}b^{4}x^{4}+5040\,a^{7}b^{3}x^{3}+1620\,a^{8}b^{2}x^{2}+315\,a^{9}bx+28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

mupad [B] time = 0.08, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9}-b^{10}\,x^{10}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{7}+20\,a^{7}\,b^{3}\,x^{3}+42\,a^{6}\,b^{4}\,x^{4}+63\,a^{5}\,b^{5}\,x^{5}+70\,a^{4}\,b^{6}\,x^{6}+60\,a^{3}\,b^{7}\,x^{7}+45\,a^{2}\,b^{8}\,x^{8}+\frac{5\,a^{9}\,b\,x}{4}-10\,a\,b^{9}\,x^{9}\,\ln(x)}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

sympy [A] time = 0.82, size = 117, normalized size = 1.03

$$10ab^{9}\log(x)+b^{10}x+\frac{-28a^{10}-315a^{9}bx-1620a^{8}b^{2}x^{2}-5040a^{7}b^{3}x^{3}-10584a^{6}b^{4}x^{4}-15876a^{5}b^{5}x^{5}-17640a^{4}b^{6}x^{6}-15120a^{3}b^{7}x^{7}-11340a^{2}b^{8}x^{8}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

3.145 $\int \frac{(a+bx)^{10}}{x^{11}} dx$

Optimal. Leaf size=124

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10a^9b}{9x^9}-\frac{a^{10}}{10x^{10}}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^11,x]
```

```
[Out] -a^10/(10*x^10) - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{11}}dx &= \int \left(\frac{a^{10}}{x^{11}}+\frac{10a^9b}{x^{10}}+\frac{45a^8b^2}{x^9}+\frac{120a^7b^3}{x^8}+\frac{210a^6b^4}{x^7}+\frac{252a^5b^5}{x^6}+\frac{210a^4b^6}{x^5}+\frac{120a^3b^7}{x^4}+\frac{45a^2}{x^3}\right.\\ &= -\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 124, normalized size = 1.00

$$-\frac{a^{10}}{10x^{10}}-\frac{10a^9b}{9x^9}-\frac{45a^8b^2}{8x^8}-\frac{120a^7b^3}{7x^7}-\frac{35a^6b^4}{x^6}-\frac{252a^5b^5}{5x^5}-\frac{105a^4b^6}{2x^4}-\frac{40a^3b^7}{x^3}-\frac{45a^2b^8}{2x^2}-\frac{10ab^9}{x}+b^{10}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^11,x]
```

```
[Out] -1/10*a^10/x^10 - (10*a^9*b)/(9*x^9) - (45*a^8*b^2)/(8*x^8) - (120*a^7*b^3)
/(7*x^7) - (35*a^6*b^4)/x^6 - (252*a^5*b^5)/(5*x^5) - (105*a^4*b^6)/(2*x^4)
 - (40*a^3*b^7)/x^3 - (45*a^2*b^8)/(2*x^2) - (10*a*b^9)/x + b^10*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{11}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^11,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^11, x]
```

fricas [A] time = 1.43, size = 114, normalized size = 0.92

$$\frac{2520\,b^{10}x^{10}\log(x) - 25200\,ab^{9}x^{9} - 56700\,a^{2}b^{8}x^{8} - 100800\,a^{3}b^{7}x^{7} - 132300\,a^{4}b^{6}x^{6} - 127008\,a^{5}b^{5}x^{5} - 88200\,a^{6}b^{4}x^{4} - 43200\,a^{7}b^{3}x^{3} - 14175\,a^{8}b^{2}x^{2} - 2800\,a^{9}bx - 252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^10*x^10*log(x) - 25200*a*b^9*x^9 - 56700*a^2*b^8*x^8 - 10080
0*a^3*b^7*x^7 - 132300*a^4*b^6*x^6 - 127008*a^5*b^5*x^5 - 88200*a^6*b^4*x^4
 - 43200*a^7*b^3*x^3 - 14175*a^8*b^2*x^2 - 2800*a^9*b*x - 252*a^10)/x^10
```

giac [A] time = 1.13, size = 112, normalized size = 0.90

$$b^{10}\log(|x|) - \frac{25200\,ab^{9}x^{9} + 56700\,a^{2}b^{8}x^{8} + 100800\,a^{3}b^{7}x^{7} + 132300\,a^{4}b^{6}x^{6} + 127008\,a^{5}b^{5}x^{5} + 88200\,a^{6}b^{4}x^{4} + 43200\,a^{7}b^{3}x^{3} + 14175\,a^{8}b^{2}x^{2} + 2800\,a^{9}bx + 252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="giac")
```

```
[Out] b^10*log(abs(x)) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3
*b^7*x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43
200*a^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

maple [A] time = 0.01, size = 111, normalized size = 0.90

$$b^{10}\ln(x) - \frac{10a\,b^{9}}{x} - \frac{45a^{2}b^{8}}{2x^{2}} - \frac{40a^{3}b^{7}}{x^{3}} - \frac{105a^{4}b^{6}}{2x^{4}} - \frac{252a^{5}b^{5}}{5x^{5}} - \frac{35a^{6}b^{4}}{x^{6}} - \frac{120a^{7}b^{3}}{7x^{7}} - \frac{45a^{8}b^{2}}{8x^{8}} - \frac{10a^{9}b}{9x^{9}} - \frac{a^{10}}{10x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^11,x)
```

```
[Out] -1/10*a^10/x^10-10/9*a^9*b/x^9-45/8*a^8*b^2/x^8-120/7*a^7*b^3/x^7-35*a^6*b^
4/x^6-252/5*a^5*b^5/x^5-105/2*a^4*b^6/x^4-40*a^3*b^7/x^3-45/2*a^2*b^8/x^2-1
0*a*b^9/x+b^10*ln(x)
```

maxima [A] time = 1.40, size = 111, normalized size = 0.90

$$b^{10}\log(x) - \frac{25200\,ab^{9}x^{9} + 56700\,a^{2}b^{8}x^{8} + 100800\,a^{3}b^{7}x^{7} + 132300\,a^{4}b^{6}x^{6} + 127008\,a^{5}b^{5}x^{5} + 88200\,a^{6}b^{4}x^{4} + 43200\,a^{7}b^{3}x^{3} + 14175\,a^{8}b^{2}x^{2} + 2800\,a^{9}bx + 252\,a^{10}}{2520\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^11,x, algorithm="maxima")
```

```
[Out] b^10*log(x) - 1/2520*(25200*a*b^9*x^9 + 56700*a^2*b^8*x^8 + 100800*a^3*b^7*
x^7 + 132300*a^4*b^6*x^6 + 127008*a^5*b^5*x^5 + 88200*a^6*b^4*x^4 + 43200*a
^7*b^3*x^3 + 14175*a^8*b^2*x^2 + 2800*a^9*b*x + 252*a^10)/x^10
```

mupad [B] time = 0.07, size = 111, normalized size = 0.90

$$b^{10}\ln(x) - \frac{\frac{a^{10}}{10} + \frac{10\,a^{9}\,b\,x}{9} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{8} + \frac{120\,a^{7}\,b^{3}\,x^{3}}{7} + 35\,a^{6}\,b^{4}\,x^{4} + \frac{252\,a^{5}\,b^{5}\,x^{5}}{5} + \frac{105\,a^{4}\,b^{6}\,x^{6}}{2} + 40\,a^{3}\,b^{7}\,x^{7} + \frac{45\,a^{2}\,b^{8}\,x^{8}}{2} + 10\,a\,b^{9}\,x^{9}}{x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^11,x)
```

```
[Out] b^10*log(x) - (a^10/10 + 10*a*b^9*x^9 + (45*a^8*b^2*x^2)/8 + (120*a^7*b^3*x
^3)/7 + 35*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/5 + (105*a^4*b^6*x^6)/2 + 40*a^3
*b^7*x^7 + (45*a^2*b^8*x^8)/2 + (10*a^9*b*x)/9)/x^10
```

sympy [A] time = 1.01, size = 119, normalized size = 0.96

$$b^{10}\log(x) + \frac{-252a^{10} - 2800a^{9}bx - 14175a^{8}b^{2}x^{2} - 43200a^{7}b^{3}x^{3} - 88200a^{6}b^{4}x^{4} - 127008a^{5}b^{5}x^{5} - 132300a^{4}b^{6}x^{6} - 100800a^{3}b^{7}x^{7} - 56700a^{2}b^{8}x^{8} - 25200ab^{9}x^{9}}{2520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**11,x)
```

```
[Out] b**10*log(x) + (-252*a**10 - 2800*a**9*b*x - 14175*a**8*b**2*x**2 - 43200*a
**7*b**3*x**3 - 88200*a**6*b**4*x**4 - 127008*a**5*b**5*x**5 - 132300*a**4*
b**6*x**6 - 100800*a**3*b**7*x**7 - 56700*a**2*b**8*x**8 - 25200*a*b**9*x**
9)/(2520*x**10)
```

3.146 $\int \frac{(a+bx)^{10}}{x^{12}} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^{11}}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^12,x]
```

```
[Out] -(a + b*x)^11/(11*a*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{12}} dx = -\frac{(a+bx)^{11}}{11ax^{11}}$$

Mathematica [B] time = 0.01, size = 114, normalized size = 6.71

$$-\frac{a^{10}}{11x^{11}}-\frac{a^9b}{x^{10}}-\frac{5a^8b^2}{x^9}-\frac{15a^7b^3}{x^8}-\frac{30a^6b^4}{x^7}-\frac{42a^5b^5}{x^6}-\frac{42a^4b^6}{x^5}-\frac{30a^3b^7}{x^4}-\frac{15a^2b^8}{x^3}-\frac{5ab^9}{x^2}-\frac{b^{10}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^12,x]
```

```
[Out] -1/11*a^10/x^11 - (a^9*b)/x^10 - (5*a^8*b^2)/x^9 - (15*a^7*b^3)/x^8 - (30*a
^6*b^4)/x^7 - (42*a^5*b^5)/x^6 - (42*a^4*b^6)/x^5 - (30*a^3*b^7)/x^4 - (15*
a^2*b^8)/x^3 - (5*a*b^9)/x^2 - b^10/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{12}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^12,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^12, x]
```

fricas [B] time = 1.26, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^9x^9+165\,a^2b^8x^8+330\,a^3b^7x^7+462\,a^4b^6x^6+462\,a^5b^5x^5+330\,a^6b^4x^4+165\,a^7b^3x^3+55\,a^8b^2x^2+11\,a^9bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

giac [B] time = 1.06, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

maple [B] time = 0.01, size = 113, normalized size = 6.65

$$-\frac{b^{10}}{x}-\frac{5a\,b^{9}}{x^{2}}-\frac{15a^{2}b^{8}}{x^{3}}-\frac{30a^{3}b^{7}}{x^{4}}-\frac{42a^{4}b^{6}}{x^{5}}-\frac{42a^{5}b^{5}}{x^{6}}-\frac{30a^{6}b^{4}}{x^{7}}-\frac{15a^{7}b^{3}}{x^{8}}-\frac{5a^{8}b^{2}}{x^{9}}-\frac{a^{9}b}{x^{10}}-\frac{a^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^12,x)
```

```
[Out] -1/11*a^10/x^11-42*a^5*b^5/x^6-5*a*b^9/x^2-5*a^8*b^2/x^9-15*a^2*b^8/x^3-30*
a^3*b^7/x^4-42*a^4*b^6/x^5-b^10/x-a^9*b/x^10-30*a^6*b^4/x^7-15*a^7*b^3/x^8
```

maxima [B] time = 1.33, size = 110, normalized size = 6.47

$$-\frac{11\,b^{10}x^{10}+55\,ab^{9}x^{9}+165\,a^{2}b^{8}x^{8}+330\,a^{3}b^{7}x^{7}+462\,a^{4}b^{6}x^{6}+462\,a^{5}b^{5}x^{5}+330\,a^{6}b^{4}x^{4}+165\,a^{7}b^{3}x^{3}+55\,a^{8}b^{2}x^{2}+11\,a^{9}bx+a^{10}}{11\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*x^10 + 55*a*b^9*x^9 + 165*a^2*b^8*x^8 + 330*a^3*b^7*x^7 + 46
2*a^4*b^6*x^6 + 462*a^5*b^5*x^5 + 330*a^6*b^4*x^4 + 165*a^7*b^3*x^3 + 55*a^
8*b^2*x^2 + 11*a^9*b*x + a^10)/x^11
```

mupad [B] time = 0.13, size = 110, normalized size = 6.47

$$-\frac{\frac{a^{10}}{11}+a^{9}\,b\,x+5\,a^{8}\,b^{2}\,x^{2}+15\,a^{7}\,b^{3}\,x^{3}+30\,a^{6}\,b^{4}\,x^{4}+42\,a^{5}\,b^{5}\,x^{5}+42\,a^{4}\,b^{6}\,x^{6}+30\,a^{3}\,b^{7}\,x^{7}+15\,a^{2}\,b^{8}\,x^{8}+5\,a\,b^{9}\,x^{9}+b^{10}\,x^{10}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^12,x)
```

```
[Out] -(a^10/11 + b^10*x^10 + 5*a*b^9*x^9 + 5*a^8*b^2*x^2 + 15*a^7*b^3*x^3 + 30*a
^6*b^4*x^4 + 42*a^5*b^5*x^5 + 42*a^4*b^6*x^6 + 30*a^3*b^7*x^7 + 15*a^2*b^8*
x^8 + a^9*b*x)/x^11
```

sympy [B] time = 1.03, size = 119, normalized size = 7.00

$$\frac{-a^{10}-11a^{9}bx-55a^{8}b^{2}x^{2}-165a^{7}b^{3}x^{3}-330a^{6}b^{4}x^{4}-462a^{5}b^{5}x^{5}-462a^{4}b^{6}x^{6}-330a^{3}b^{7}x^{7}-165a^{2}b^{8}x^{8}-55ab^{9}x^{9}-11b^{10}x^{10}}{11x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**12,x)
```

```
[Out] (-a**10 - 11*a**9*b*x - 55*a**8*b**2*x**2 - 165*a**7*b**3*x**3 - 330*a**6*b
**4*x**4 - 462*a**5*b**5*x**5 - 462*a**4*b**6*x**6 - 330*a**3*b**7*x**7 - 1
65*a**2*b**8*x**8 - 55*a*b**9*x**9 - 11*b**10*x**10)/(11*x**11)
```

3.147 $\int \frac{(a+bx)^{10}}{x^{13}}\,dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^{11}}{132a^2x^{11}}-\frac{(a+bx)^{11}}{12ax^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^13,x]
```

```
[Out] -(a + b*x)^11/(12*a*x^12) + (b*(a + b*x)^11)/(132*a^2*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{13}}\,dx &= -\frac{(a+bx)^{11}}{12ax^{12}}-\frac{b\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{12a}\\ &= -\frac{(a+bx)^{11}}{12ax^{12}}+\frac{b(a+bx)^{11}}{132a^2x^{11}}\end{aligned}$$

Mathematica [B] time = 0.00, size = 128, normalized size = 3.56

$$-\frac{a^{10}}{12x^{12}}-\frac{10a^9b}{11x^{11}}-\frac{9a^8b^2}{2x^{10}}-\frac{40a^7b^3}{3x^9}-\frac{105a^6b^4}{4x^8}-\frac{36a^5b^5}{x^7}-\frac{35a^4b^6}{x^6}-\frac{24a^3b^7}{x^5}-\frac{45a^2b^8}{4x^4}-\frac{10ab^9}{3x^3}-\frac{b^{10}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^13,x]
```

```
[Out] -1/12*a^10/x^12 - (10*a^9*b)/(11*x^11) - (9*a^8*b^2)/(2*x^10) - (40*a^7*b^3
)/(3*x^9) - (105*a^6*b^4)/(4*x^8) - (36*a^5*b^5)/x^7 - (35*a^4*b^6)/x^6 - (
24*a^3*b^7)/x^5 - (45*a^2*b^8)/(4*x^4) - (10*a*b^9)/(3*x^3) - b^10/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^13,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^13, x]
```

fricas [B] time = 1.34, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594\,a^{8}b^{2}x^{2}+120\,a^{9}bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

giac [B] time = 0.96, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594\,a^{8}b^{2}x^{2}+120\,a^{9}bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

maple [B] time = 0.00, size = 113, normalized size = 3.14

$$-\frac{b^{10}}{2x^2}-\frac{10a\,b^9}{3x^3}-\frac{45a^2b^8}{4x^4}-\frac{24a^3b^7}{x^5}-\frac{35a^4b^6}{x^6}-\frac{36a^5b^5}{x^7}-\frac{105a^6b^4}{4x^8}-\frac{40a^7b^3}{3x^9}-\frac{9a^8b^2}{2x^{10}}-\frac{10a^9b}{11x^{11}}-\frac{a^{10}}{12x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^13,x)
```

```
[Out] -35*a^4*b^6/x^6-10/11*a^9*b/x^11-1/12*a^10/x^12-40/3*a^7*b^3/x^9-10/3*a*b^9
/x^3-45/4*a^2*b^8/x^4-1/2*b^10/x^2-24*a^3*b^7/x^5-9/2*a^8*b^2/x^10-36*a^5*b
^5/x^7-105/4*a^6*b^4/x^8
```

maxima [B] time = 1.39, size = 112, normalized size = 3.11

$$-\frac{66\,b^{10}x^{10}+440\,ab^{9}x^{9}+1485\,a^{2}b^{8}x^{8}+3168\,a^{3}b^{7}x^{7}+4620\,a^{4}b^{6}x^{6}+4752\,a^{5}b^{5}x^{5}+3465\,a^{6}b^{4}x^{4}+1760\,a^{7}b^{3}x^{3}+594\,a^{8}b^{2}x^{2}+120\,a^{9}bx+11\,a^{10}}{132\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*x^10 + 440*a*b^9*x^9 + 1485*a^2*b^8*x^8 + 3168*a^3*b^7*x^7
+ 4620*a^4*b^6*x^6 + 4752*a^5*b^5*x^5 + 3465*a^6*b^4*x^4 + 1760*a^7*b^3*x^3
 + 594*a^8*b^2*x^2 + 120*a^9*b*x + 11*a^10)/x^12
```

mupad [B] time = 0.10, size = 23, normalized size = 0.64

$$-\frac{(11\,a-b\,x)\;(a+b\,x)^{11}}{132\,a^2\,x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^13,x)
```

```
[Out] -((11*a - b*x)*(a + b*x)^11)/(132*a^2*x^12)
```

sympy [B] time = 0.99, size = 121, normalized size = 3.36

$$\frac{-11a^{10} - 120a^9bx - 594a^8b^2x^2 - 1760a^7b^3x^3 - 3465a^6b^4x^4 - 4752a^5b^5x^5 - 4620a^4b^6x^6 - 3168a^3b^7x^7 - 1485a^2b^8x^8 - 440ab^9x^9 - 66b^{10}x^{10}}{132x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**13,x)
```

```
[Out] (-11*a**10 - 120*a**9*b*x - 594*a**8*b**2*x**2 - 1760*a**7*b**3*x**3 - 3465
*a**6*b**4*x**4 - 4752*a**5*b**5*x**5 - 4620*a**4*b**6*x**6 - 3168*a**3*b**
7*x**7 - 1485*a**2*b**8*x**8 - 440*a*b**9*x**9 - 66*b**10*x**10)/(132*x**12
)
```

3.148 $\int \frac{(a+bx)^{10}}{x^{14}}\,dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{(a+bx)^{11}}{13ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^14,x]
```

```
[Out] -(a + b*x)^11/(13*a*x^13) + (b*(a + b*x)^11)/(78*a^2*x^12) - (b^2*(a + b*x)
^11)/(858*a^3*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{14}}\,dx &= -\frac{(a+bx)^{11}}{13ax^{13}}-\frac{(2b)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{13a}\\ &= -\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}+\frac{b^2\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{78a^2}\\ &= -\frac{(a+bx)^{11}}{13ax^{13}}+\frac{b(a+bx)^{11}}{78a^2x^{12}}-\frac{b^2(a+bx)^{11}}{858a^3x^{11}}\end{aligned}$$

Mathematica [B] time = 0.01, size = 126, normalized size = 2.25

$$-\frac{a^{10}}{13x^{13}}-\frac{5a^9b}{6x^{12}}-\frac{45a^8b^2}{11x^{11}}-\frac{12a^7b^3}{x^{10}}-\frac{70a^6b^4}{3x^9}-\frac{63a^5b^5}{2x^8}-\frac{30a^4b^6}{x^7}-\frac{20a^3b^7}{x^6}-\frac{9a^2b^8}{x^5}-\frac{5ab^9}{2x^4}-\frac{b^{10}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^14,x]
```

```
[Out] -1/13*a^10/x^13 - (5*a^9*b)/(6*x^12) - (45*a^8*b^2)/(11*x^11) - (12*a^7*b^3
)/x^10 - (70*a^6*b^4)/(3*x^9) - (63*a^5*b^5)/(2*x^8) - (30*a^4*b^6)/x^7 - (
20*a^3*b^7)/x^6 - (9*a^2*b^8)/x^5 - (5*a*b^9)/(2*x^4) - b^10/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^14,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^14, x]
```

fricas [B] time = 1.39, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="fricas")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

giac [B] time = 1.03, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

maple [B] time = 0.00, size = 113, normalized size = 2.02

$$-\frac{b^{10}}{3x^{3}}-\frac{5a\,b^{9}}{2x^{4}}-\frac{9a^{2}b^{8}}{x^{5}}-\frac{20a^{3}b^{7}}{x^{6}}-\frac{30a^{4}b^{6}}{x^{7}}-\frac{63a^{5}b^{5}}{2x^{8}}-\frac{70a^{6}b^{4}}{3x^{9}}-\frac{12a^{7}b^{3}}{x^{10}}-\frac{45a^{8}b^{2}}{11x^{11}}-\frac{5a^{9}b}{6x^{12}}-\frac{a^{10}}{13x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^14,x)
```

```
[Out] -20*a^3*b^7/x^6-1/13*a^10/x^13-70/3*a^6*b^4/x^9-9*a^2*b^8/x^5-1/3*b^10/x^3-
45/11*a^8*b^2/x^11-5/2*a*b^9/x^4-12*a^7*b^3/x^10-5/6*a^9*b/x^12-30*a^4*b^6/
x^7-63/2*a^5*b^5/x^8
```

maxima [B] time = 1.37, size = 112, normalized size = 2.00

$$-\frac{286\,b^{10}x^{10}+2145\,ab^{9}x^{9}+7722\,a^{2}b^{8}x^{8}+17160\,a^{3}b^{7}x^{7}+25740\,a^{4}b^{6}x^{6}+27027\,a^{5}b^{5}x^{5}+20020\,a^{6}b^{4}x^{4}+10296\,a^{7}b^{3}x^{3}+3510\,a^{8}b^{2}x^{2}+715\,a^{9}bx+66\,a^{10}}{858\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*x^10 + 2145*a*b^9*x^9 + 7722*a^2*b^8*x^8 + 17160*a^3*b^7*x
^7 + 25740*a^4*b^6*x^6 + 27027*a^5*b^5*x^5 + 20020*a^6*b^4*x^4 + 10296*a^7*
b^3*x^3 + 3510*a^8*b^2*x^2 + 715*a^9*b*x + 66*a^10)/x^13
```

mupad [B] time = 0.13, size = 112, normalized size = 2.00

$$-\frac{\frac{a^{10}}{13}+\frac{5\,a^9\,b\,x}{6}+\frac{45\,a^8\,b^2\,x^2}{11}+12\,a^7\,b^3\,x^3+\frac{70\,a^6\,b^4\,x^4}{3}+\frac{63\,a^5\,b^5\,x^5}{2}+30\,a^4\,b^6\,x^6+20\,a^3\,b^7\,x^7+9\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{2}+\frac{b^{10}\,x^{10}}{3}}{x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^14,x)
```

```
[Out] -(a^10/13 + (b^10*x^10)/3 + (5*a*b^9*x^9)/2 + (45*a^8*b^2*x^2)/11 + 12*a^7*
b^3*x^3 + (70*a^6*b^4*x^4)/3 + (63*a^5*b^5*x^5)/2 + 30*a^4*b^6*x^6 + 20*a^3
*b^7*x^7 + 9*a^2*b^8*x^8 + (5*a^9*b*x)/6)/x^13
```

sympy [B] time = 1.22, size = 121, normalized size = 2.16

$$\frac{-66a^{10}-715a^9bx-3510a^8b^2x^2-10296a^7b^3x^3-20020a^6b^4x^4-27027a^5b^5x^5-25740a^4b^6x^6-17160a^3b^7x^7-7722a^2b^8x^8-2145ab^9x^9-286b^{10}x^{10}}{858x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**14,x)
```

```
[Out] (-66*a**10 - 715*a**9*b*x - 3510*a**8*b**2*x**2 - 10296*a**7*b**3*x**3 - 20
020*a**6*b**4*x**4 - 27027*a**5*b**5*x**5 - 25740*a**4*b**6*x**6 - 17160*a*
*3*b**7*x**7 - 7722*a**2*b**8*x**8 - 2145*a*b**9*x**9 - 286*b**10*x**10)/(8
58*x**13)
```

3.149 $\int \frac{(a+bx)^{10}}{x^{15}}\,dx$

Optimal. Leaf size=76

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{(a+bx)^{11}}{14ax^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^15,x]
```

```
[Out] -(a + b*x)^11/(14*a*x^14) + (3*b*(a + b*x)^11)/(182*a^2*x^13) - (b^2*(a + b
*x)^11)/(364*a^3*x^12) + (b^3*(a + b*x)^11)/(4004*a^4*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{15}}\,dx &= -\frac{(a+bx)^{11}}{14ax^{14}}-\frac{(3b)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{14a}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}+\frac{\left(3b^2\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{91a^2}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}-\frac{b^3\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{364a^3}\\
&= -\frac{(a+bx)^{11}}{14ax^{14}}+\frac{3b(a+bx)^{11}}{182a^2x^{13}}-\frac{b^2(a+bx)^{11}}{364a^3x^{12}}+\frac{b^3(a+bx)^{11}}{4004a^4x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 128, normalized size = 1.68

$$-\frac{a^{10}}{14x^{14}}-\frac{10a^9b}{13x^{13}}-\frac{15a^8b^2}{4x^{12}}-\frac{120a^7b^3}{11x^{11}}-\frac{21a^6b^4}{x^{10}}-\frac{28a^5b^5}{x^9}-\frac{105a^4b^6}{4x^8}-\frac{120a^3b^7}{7x^7}-\frac{15a^2b^8}{2x^6}-\frac{2ab^9}{x^5}-\frac{b^{10}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^15,x]
```

```
[Out] -1/14*a^10/x^14 - (10*a^9*b)/(13*x^13) - (15*a^8*b^2)/(4*x^12) - (120*a^7*b
^3)/(11*x^11) - (21*a^6*b^4)/x^10 - (28*a^5*b^5)/x^9 - (105*a^4*b^6)/(4*x^8
) - (120*a^3*b^7)/(7*x^7) - (15*a^2*b^8)/(2*x^6) - (2*a*b^9)/x^5 - b^10/(4*
x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^15,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^15, x]
```

fricas [A] time = 1.29, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

giac [A] time = 0.89, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

maple [A] time = 0.01, size = 113, normalized size = 1.49

$$-\frac{b^{10}}{4x^{4}}-\frac{2a\,b^{9}}{x^{5}}-\frac{15a^{2}b^{8}}{2x^{6}}-\frac{120a^{3}b^{7}}{7x^{7}}-\frac{105a^{4}b^{6}}{4x^{8}}-\frac{28a^{5}b^{5}}{x^{9}}-\frac{21a^{6}b^{4}}{x^{10}}-\frac{120a^{7}b^{3}}{11x^{11}}-\frac{15a^{8}b^{2}}{4x^{12}}-\frac{10a^{9}b}{13x^{13}}-\frac{a^{10}}{14x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^15,x)
```

```
[Out] -15/4*a^8*b^2/x^12-15/2*a^2*b^8/x^6-10/13*a^9*b/x^13-28*a^5*b^5/x^9-1/4*b^1
0/x^4-120/11*a^7*b^3/x^11-2*a*b^9/x^5-1/14*a^10/x^14-21*a^6*b^4/x^10-120/7*
a^3*b^7/x^7-105/4*a^4*b^6/x^8
```

maxima [A] time = 1.28, size = 112, normalized size = 1.47

$$-\frac{1001\,b^{10}x^{10}+8008\,ab^{9}x^{9}+30030\,a^{2}b^{8}x^{8}+68640\,a^{3}b^{7}x^{7}+105105\,a^{4}b^{6}x^{6}+112112\,a^{5}b^{5}x^{5}+84084\,a^{6}b^{4}x^{4}+43680\,a^{7}b^{3}x^{3}+15015\,a^{8}b^{2}x^{2}+3080\,a^{9}bx+286\,a^{10}}{4004\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*x^10 + 8008*a*b^9*x^9 + 30030*a^2*b^8*x^8 + 68640*a^3*b^
7*x^7 + 105105*a^4*b^6*x^6 + 112112*a^5*b^5*x^5 + 84084*a^6*b^4*x^4 + 43680
*a^7*b^3*x^3 + 15015*a^8*b^2*x^2 + 3080*a^9*b*x + 286*a^10)/x^14
```

mupad [B] time = 0.09, size = 112, normalized size = 1.47

$$-\frac{\frac{a^{10}}{14}+\frac{10\,a^9\,b\,x}{13}+\frac{15\,a^8\,b^2\,x^2}{4}+\frac{120\,a^7\,b^3\,x^3}{11}+21\,a^6\,b^4\,x^4+28\,a^5\,b^5\,x^5+\frac{105\,a^4\,b^6\,x^6}{4}+\frac{120\,a^3\,b^7\,x^7}{7}+\frac{15\,a^2\,b^8\,x^8}{2}+2\,a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{4}}{x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^15,x)
```

```
[Out] -(a^10/14 + (b^10*x^10)/4 + 2*a*b^9*x^9 + (15*a^8*b^2*x^2)/4 + (120*a^7*b^3
*x^3)/11 + 21*a^6*b^4*x^4 + 28*a^5*b^5*x^5 + (105*a^4*b^6*x^6)/4 + (120*a^3
*b^7*x^7)/7 + (15*a^2*b^8*x^8)/2 + (10*a^9*b*x)/13)/x^14
```

sympy [A] time = 1.10, size = 121, normalized size = 1.59

$$\frac{-286a^{10}-3080a^9bx-15015a^8b^2x^2-43680a^7b^3x^3-84084a^6b^4x^4-112112a^5b^5x^5-105105a^4b^6x^6-68640a^3b^7x^7-30030a^2b^8x^8-8008ab^9x^9-1001b^{10}x^{10}}{4004x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**15,x)
```

```
[Out] (-286*a**10 - 3080*a**9*b*x - 15015*a**8*b**2*x**2 - 43680*a**7*b**3*x**3 -
 84084*a**6*b**4*x**4 - 112112*a**5*b**5*x**5 - 105105*a**4*b**6*x**6 - 686
40*a**3*b**7*x**7 - 30030*a**2*b**8*x**8 - 8008*a*b**9*x**9 - 1001*b**10*x*
*10)/(4004*x**14)
```

3.150 $\int \frac{(a+bx)^{10}}{x^{16}}\,dx$

Optimal. Leaf size=96

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{(a+bx)^{11}}{15ax^{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^16,x]
```

```
[Out] -(a + b*x)^11/(15*a*x^15) + (2*b*(a + b*x)^11)/(105*a^2*x^14) - (2*b^2*(a +
 b*x)^11)/(455*a^3*x^13) + (b^3*(a + b*x)^11)/(1365*a^4*x^12) - (b^4*(a + b
*x)^11)/(15015*a^5*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{16}}\,dx &= -\frac{(a+bx)^{11}}{15ax^{15}}-\frac{(4b)\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{15a}\\
&= -\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}+\frac{\left(2b^2\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{35a^2}\\
&= -\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}-\frac{\left(4b^3\right)\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{455a^3}\\
&= -\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}+\frac{b^4\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{1365a^4}\\
&= -\frac{(a+bx)^{11}}{15ax^{15}}+\frac{2b(a+bx)^{11}}{105a^2x^{14}}-\frac{2b^2(a+bx)^{11}}{455a^3x^{13}}+\frac{b^3(a+bx)^{11}}{1365a^4x^{12}}-\frac{b^4(a+bx)^{11}}{15015a^5x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 130, normalized size = 1.35

$$-\frac{a^{10}}{15x^{15}}-\frac{5a^9b}{7x^{14}}-\frac{45a^8b^2}{13x^{13}}-\frac{10a^7b^3}{x^{12}}-\frac{210a^6b^4}{11x^{11}}-\frac{126a^5b^5}{5x^{10}}-\frac{70a^4b^6}{3x^9}-\frac{15a^3b^7}{x^8}-\frac{45a^2b^8}{7x^7}-\frac{5ab^9}{3x^6}-\frac{b^{10}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^16,x]
```

```
[Out] -1/15*a^10/x^15 - (5*a^9*b)/(7*x^14) - (45*a^8*b^2)/(13*x^13) - (10*a^7*b^3
)/x^12 - (210*a^6*b^4)/(11*x^11) - (126*a^5*b^5)/(5*x^10) - (70*a^4*b^6)/(3
*x^9) - (15*a^3*b^7)/x^8 - (45*a^2*b^8)/(7*x^7) - (5*a*b^9)/(3*x^6) - b^10/
(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^16,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^16, x]
```

fricas [A] time = 1.34, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

giac [A] time = 1.52, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

maple [A] time = 0.01, size = 113, normalized size = 1.18

$$-\frac{b^{10}}{5x^{5}}-\frac{5a\,b^{9}}{3x^{6}}-\frac{45a^{2}b^{8}}{7x^{7}}-\frac{15a^{3}b^{7}}{x^{8}}-\frac{70a^{4}b^{6}}{3x^{9}}-\frac{126a^{5}b^{5}}{5x^{10}}-\frac{210a^{6}b^{4}}{11x^{11}}-\frac{10a^{7}b^{3}}{x^{12}}-\frac{45a^{8}b^{2}}{13x^{13}}-\frac{5a^{9}b}{7x^{14}}-\frac{a^{10}}{15x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^16,x)
```

```
[Out] -5/3*a*b^9/x^6-210/11*a^6*b^4/x^11-70/3*a^4*b^6/x^9-1/5*b^10/x^5-45/13*a^8*
b^2/x^13-5/7*a^9*b/x^14-10*a^7*b^3/x^12-126/5*a^5*b^5/x^10-1/15*a^10/x^15-4
5/7*a^2*b^8/x^7-15*a^3*b^7/x^8
```

maxima [A] time = 1.36, size = 112, normalized size = 1.17

$$-\frac{3003\,b^{10}x^{10}+25025\,ab^{9}x^{9}+96525\,a^{2}b^{8}x^{8}+225225\,a^{3}b^{7}x^{7}+350350\,a^{4}b^{6}x^{6}+378378\,a^{5}b^{5}x^{5}+286650\,a^{6}b^{4}x^{4}+150150\,a^{7}b^{3}x^{3}+51975\,a^{8}b^{2}x^{2}+10725\,a^{9}bx+1001\,a^{10}}{15015\,x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*x^10 + 25025*a*b^9*x^9 + 96525*a^2*b^8*x^8 + 225225*a^3
*b^7*x^7 + 350350*a^4*b^6*x^6 + 378378*a^5*b^5*x^5 + 286650*a^6*b^4*x^4 + 1
50150*a^7*b^3*x^3 + 51975*a^8*b^2*x^2 + 10725*a^9*b*x + 1001*a^10)/x^15
```

mupad [B] time = 0.13, size = 112, normalized size = 1.17

$$-\frac{\frac{a^{10}}{15}+\frac{5\,a^9\,b\,x}{7}+\frac{45\,a^8\,b^2\,x^2}{13}+10\,a^7\,b^3\,x^3+\frac{210\,a^6\,b^4\,x^4}{11}+\frac{126\,a^5\,b^5\,x^5}{5}+\frac{70\,a^4\,b^6\,x^6}{3}+15\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{7}+\frac{5\,a\,b^9\,x^9}{3}+\frac{b^{10}\,x^{10}}{5}}{x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^16,x)
```

```
[Out] -(a^10/15 + (b^10*x^10)/5 + (5*a*b^9*x^9)/3 + (45*a^8*b^2*x^2)/13 + 10*a^7*
b^3*x^3 + (210*a^6*b^4*x^4)/11 + (126*a^5*b^5*x^5)/5 + (70*a^4*b^6*x^6)/3 +
 15*a^3*b^7*x^7 + (45*a^2*b^8*x^8)/7 + (5*a^9*b*x)/7)/x^15
```

sympy [A] time = 1.25, size = 121, normalized size = 1.26

$$\frac{-1001a^{10}-10725a^9bx-51975a^8b^2x^2-150150a^7b^3x^3-286650a^6b^4x^4-378378a^5b^5x^5-350350a^4b^6x^6-225225a^3b^7x^7-96525a^2b^8x^8-25025ab^9x^9-3003b^{10}x^{10}}{15015x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**16,x)
```

```
[Out] (-1001*a**10 - 10725*a**9*b*x - 51975*a**8*b**2*x**2 - 150150*a**7*b**3*x**
3 - 286650*a**6*b**4*x**4 - 378378*a**5*b**5*x**5 - 350350*a**4*b**6*x**6 -
 225225*a**3*b**7*x**7 - 96525*a**2*b**8*x**8 - 25025*a*b**9*x**9 - 3003*b*
*10*x**10)/(15015*x**15)
```

3.151 $\int \frac{(a+bx)^{10}}{x^{17}}\,dx$

Optimal. Leaf size=116

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{(a+bx)^{11}}{16ax^{16}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^17,x]
```

```
[Out] -(a + b*x)^11/(16*a*x^16) + (b*(a + b*x)^11)/(48*a^2*x^15) - (b^2*(a + b*x)
^11)/(168*a^3*x^14) + (b^3*(a + b*x)^11)/(728*a^4*x^13) - (b^4*(a + b*x)^11
)/(4368*a^5*x^12) + (b^5*(a + b*x)^11)/(48048*a^6*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{10}}{x^{17}}\,dx &= -\frac{(a+bx)^{11}}{16ax^{16}}-\frac{(5b)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{16a}\\
&= -\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}+\frac{b^2\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{12a^2}\\
&= -\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}-\frac{b^3\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{56a^3}\\
&= -\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}+\frac{b^4\int \frac{(a+bx)^{10}}{x^{13}}\,dx}{364a^4}\\
&= -\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}-\frac{b^5\int \frac{(a+bx)^{10}}{x^{12}}\,dx}{4368a^5}\\
&= -\frac{(a+bx)^{11}}{16ax^{16}}+\frac{b(a+bx)^{11}}{48a^2x^{15}}-\frac{b^2(a+bx)^{11}}{168a^3x^{14}}+\frac{b^3(a+bx)^{11}}{728a^4x^{13}}-\frac{b^4(a+bx)^{11}}{4368a^5x^{12}}+\frac{b^5(a+bx)^{11}}{48048a^6x^{11}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 132, normalized size = 1.14

$$-\frac{a^{10}}{16x^{16}}-\frac{2a^9b}{3x^{15}}-\frac{45a^8b^2}{14x^{14}}-\frac{120a^7b^3}{13x^{13}}-\frac{35a^6b^4}{2x^{12}}-\frac{252a^5b^5}{11x^{11}}-\frac{21a^4b^6}{x^{10}}-\frac{40a^3b^7}{3x^9}-\frac{45a^2b^8}{8x^8}-\frac{10ab^9}{7x^7}-\frac{b^{10}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^17,x]
```

```
[Out] -1/16*a^10/x^16 - (2*a^9*b)/(3*x^15) - (45*a^8*b^2)/(14*x^14) - (120*a^7*b^
3)/(13*x^13) - (35*a^6*b^4)/(2*x^12) - (252*a^5*b^5)/(11*x^11) - (21*a^4*b^
6)/x^10 - (40*a^3*b^7)/(3*x^9) - (45*a^2*b^8)/(8*x^8) - (10*a*b^9)/(7*x^7)
- b^10/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^17,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^17, x]
```

fricas [A] time = 0.77, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^9x^9+270270\,a^2b^8x^8+640640\,a^3b^7x^7+1009008\,a^4b^6x^6+1100736\,a^5b^5x^5+840840\,a^6b^4x^4+443520\,a^7b^3x^3+154440\,a^8b^2x^2+32032\,a^9bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

giac [A] time = 1.10, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^9x^9+270270\,a^2b^8x^8+640640\,a^3b^7x^7+1009008\,a^4b^6x^6+1100736\,a^5b^5x^5+840840\,a^6b^4x^4+443520\,a^7b^3x^3+154440\,a^8b^2x^2+32032\,a^9bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

maple [A] time = 0.01, size = 113, normalized size = 0.97

$$-\frac{b^{10}}{6x^6}-\frac{10a\,b^9}{7x^7}-\frac{45a^2b^8}{8x^8}-\frac{40a^3b^7}{3x^9}-\frac{21a^4b^6}{x^{10}}-\frac{252a^5b^5}{11x^{11}}-\frac{35a^6b^4}{2x^{12}}-\frac{120a^7b^3}{13x^{13}}-\frac{45a^8b^2}{14x^{14}}-\frac{2a^9b}{3x^{15}}-\frac{a^{10}}{16x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^17,x)
```

```
[Out] -1/6*b^10/x^6-120/13*a^7*b^3/x^13-40/3*a^3*b^7/x^9-252/11*a^5*b^5/x^11-45/1
4*a^8*b^2/x^14-1/16*a^10/x^16-21*a^4*b^6/x^10-2/3*a^9*b/x^15-10/7*a*b^9/x^7
-45/8*a^2*b^8/x^8-35/2*a^6*b^4/x^12
```

maxima [A] time = 1.39, size = 112, normalized size = 0.97

$$-\frac{8008\,b^{10}x^{10}+68640\,ab^9x^9+270270\,a^2b^8x^8+640640\,a^3b^7x^7+1009008\,a^4b^6x^6+1100736\,a^5b^5x^5+840840\,a^6b^4x^4+443520\,a^7b^3x^3+154440\,a^8b^2x^2+32032\,a^9bx+3003\,a^{10}}{48048\,x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*x^10 + 68640*a*b^9*x^9 + 270270*a^2*b^8*x^8 + 640640*a^
3*b^7*x^7 + 1009008*a^4*b^6*x^6 + 1100736*a^5*b^5*x^5 + 840840*a^6*b^4*x^4
+ 443520*a^7*b^3*x^3 + 154440*a^8*b^2*x^2 + 32032*a^9*b*x + 3003*a^10)/x^16
```

mupad [B] time = 0.13, size = 112, normalized size = 0.97

$$-\frac{\frac{a^{10}}{16}+\frac{2\,a^9\,b\,x}{3}+\frac{45\,a^8\,b^2\,x^2}{14}+\frac{120\,a^7\,b^3\,x^3}{13}+\frac{35\,a^6\,b^4\,x^4}{2}+\frac{252\,a^5\,b^5\,x^5}{11}+21\,a^4\,b^6\,x^6+\frac{40\,a^3\,b^7\,x^7}{3}+\frac{45\,a^2\,b^8\,x^8}{8}+\frac{10\,a\,b^9\,x^9}{7}+\frac{b^{10}\,x^{10}}{6}}{x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^17,x)
```

```
[Out] -(a^10/16 + (b^10*x^10)/6 + (10*a*b^9*x^9)/7 + (45*a^8*b^2*x^2)/14 + (120*a
^7*b^3*x^3)/13 + (35*a^6*b^4*x^4)/2 + (252*a^5*b^5*x^5)/11 + 21*a^4*b^6*x^6
 + (40*a^3*b^7*x^7)/3 + (45*a^2*b^8*x^8)/8 + (2*a^9*b*x)/3)/x^16
```

sympy [A] time = 1.27, size = 121, normalized size = 1.04

$$\frac{-3003a^{10}-32032a^9bx-154440a^8b^2x^2-443520a^7b^3x^3-840840a^6b^4x^4-1100736a^5b^5x^5-1009008a^4b^6x^6-640640a^3b^7x^7-270270a^2b^8x^8-68640ab^9x^9-8008b^{10}x^{10}}{48048x^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**17,x)
```

```
[Out] (-3003*a**10 - 32032*a**9*b*x - 154440*a**8*b**2*x**2 - 443520*a**7*b**3*x*
*3 - 840840*a**6*b**4*x**4 - 1100736*a**5*b**5*x**5 - 1009008*a**4*b**6*x**
6 - 640640*a**3*b**7*x**7 - 270270*a**2*b**8*x**8 - 68640*a*b**9*x**9 - 800
8*b**10*x**10)/(48048*x**16)
```

3.152 $\int \frac{(a+bx)^{10}}{x^{18}}\,dx$

Optimal. Leaf size=136

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Rubi [A] time = 0.05, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^6(a+bx)^{11}}{136136a^7x^{11}}+\frac{b^5(a+bx)^{11}}{12376a^6x^{12}}-\frac{3b^4(a+bx)^{11}}{6188a^5x^{13}}+\frac{b^3(a+bx)^{11}}{476a^4x^{14}}-\frac{b^2(a+bx)^{11}}{136a^3x^{15}}+\frac{3b(a+bx)^{11}}{136a^2x^{16}}-\frac{(a+bx)^{11}}{17ax^{17}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^18,x]
```

```
[Out] -(a + b*x)^11/(17*a*x^17) + (3*b*(a + b*x)^11)/(136*a^2*x^16) - (b^2*(a + b
*x)^11)/(136*a^3*x^15) + (b^3*(a + b*x)^11)/(476*a^4*x^14) - (3*b^4*(a + b*
x)^11)/(6188*a^5*x^13) + (b^5*(a + b*x)^11)/(12376*a^6*x^12) - (b^6*(a + b*
x)^11)/(136136*a^7*x^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx = -\frac{(a+bx)^{11}}{17ax^{17}} - \frac{(6b)\int \frac{(a+bx)^{10}}{x^{17}}\,dx}{17a}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} + \frac{\left(15b^2\right)\int \frac{(a+bx)^{10}}{x^{16}}\,dx}{136a^2}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} - \frac{b^3\int \frac{(a+bx)^{10}}{x^{15}}\,dx}{34a^3}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} + \frac{\left(3b^4\right)\int \frac{(a+bx)^{10}}{x^{14}}\,dx}{476a^4}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} - \frac{\left(3b^5\right)\int \frac{(a+bx}{x^{13}}}{3094a^5}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}}$$
$$= -\frac{(a+bx)^{11}}{17ax^{17}} + \frac{3b(a+bx)^{11}}{136a^2x^{16}} - \frac{b^2(a+bx)^{11}}{136a^3x^{15}} + \frac{b^3(a+bx)^{11}}{476a^4x^{14}} - \frac{3b^4(a+bx)^{11}}{6188a^5x^{13}} + \frac{b^5(a+bx)^{11}}{12376a^6x^{12}}$$

Mathematica [A] time = 0.01, size = 126, normalized size = 0.93

$$-\frac{a^{10}}{17x^{17}} - \frac{5a^9b}{8x^{16}} - \frac{3a^8b^2}{x^{15}} - \frac{60a^7b^3}{7x^{14}} - \frac{210a^6b^4}{13x^{13}} - \frac{21a^5b^5}{x^{12}} - \frac{210a^4b^6}{11x^{11}} - \frac{12a^3b^7}{x^{10}} - \frac{5a^2b^8}{x^9} - \frac{5ab^9}{4x^8} - \frac{b^{10}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^18,x]
```

```
[Out] -1/17*a^10/x^17 - (5*a^9*b)/(8*x^16) - (3*a^8*b^2)/x^15 - (60*a^7*b^3)/(7*x
^14) - (210*a^6*b^4)/(13*x^13) - (21*a^5*b^5)/x^12 - (210*a^4*b^6)/(11*x^11
) - (12*a^3*b^7)/x^10 - (5*a^2*b^8)/x^9 - (5*a*b^9)/(4*x^8) - b^10/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^18,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^18, x]
```

fricas [A] time = 1.14, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 + 2199120\,a^6b^4x^4 + 1166880\,a^7b^3x^3 + 408408\,a^8b^2x^2 + 85085\,a^9bx + 8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

giac [A] time = 1.15, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10} + 170170\,ab^9x^9 + 680680\,a^2b^8x^8 + 1633632\,a^3b^7x^7 + 2598960\,a^4b^6x^6 + 2858856\,a^5b^5x^5 + 2199120\,a^6b^4x^4 + 1166880\,a^7b^3x^3 + 408408\,a^8b^2x^2 + 85085\,a^9bx + 8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

maple [A] time = 0.01, size = 113, normalized size = 0.83

$$-\frac{b^{10}}{7x^{7}}-\frac{5a\,b^{9}}{4x^{8}}-\frac{5a^{2}b^{8}}{x^{9}}-\frac{12a^{3}b^{7}}{x^{10}}-\frac{210a^{4}b^{6}}{11x^{11}}-\frac{21a^{5}b^{5}}{x^{12}}-\frac{210a^{6}b^{4}}{13x^{13}}-\frac{60a^{7}b^{3}}{7x^{14}}-\frac{3a^{8}b^{2}}{x^{15}}-\frac{5a^{9}b}{8x^{16}}-\frac{a^{10}}{17x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^18,x)
```

```
[Out] -3*a^8*b^2/x^15-12*a^3*b^7/x^10-5*a^2*b^8/x^9-21*a^5*b^5/x^12-1/17*a^10/x^1
7-5/8*a^9*b/x^16-60/7*a^7*b^3/x^14-210/13*a^6*b^4/x^13-1/7*b^10/x^7-5/4*a*b
^9/x^8-210/11*a^4*b^6/x^11
```

maxima [A] time = 1.40, size = 112, normalized size = 0.82

$$-\frac{19448\,b^{10}x^{10}+170170\,ab^{9}x^{9}+680680\,a^{2}b^{8}x^{8}+1633632\,a^{3}b^{7}x^{7}+2598960\,a^{4}b^{6}x^{6}+2858856\,a^{5}b^{5}x^{5}+2199120\,a^{6}b^{4}x^{4}+1166880\,a^{7}b^{3}x^{3}+408408\,a^{8}b^{2}x^{2}+85085\,a^{9}bx+8008\,a^{10}}{136136\,x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*x^10 + 170170*a*b^9*x^9 + 680680*a^2*b^8*x^8 + 163363
2*a^3*b^7*x^7 + 2598960*a^4*b^6*x^6 + 2858856*a^5*b^5*x^5 + 2199120*a^6*b^4
*x^4 + 1166880*a^7*b^3*x^3 + 408408*a^8*b^2*x^2 + 85085*a^9*b*x + 8008*a^10
)/x^17
```

mupad [B] time = 0.13, size = 112, normalized size = 0.82

$$-\frac{\frac{a^{10}}{17}+\frac{5\,a^9\,b\,x}{8}+3\,a^8\,b^2\,x^2+\frac{60\,a^7\,b^3\,x^3}{7}+\frac{210\,a^6\,b^4\,x^4}{13}+21\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{11}+12\,a^3\,b^7\,x^7+5\,a^2\,b^8\,x^8+\frac{5\,a\,b^9\,x^9}{4}+\frac{b^{10}\,x^{10}}{7}}{x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^18,x)
```

```
[Out] -(a^10/17 + (b^10*x^10)/7 + (5*a*b^9*x^9)/4 + 3*a^8*b^2*x^2 + (60*a^7*b^3*x
^3)/7 + (210*a^6*b^4*x^4)/13 + 21*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/11 + 12*a
^3*b^7*x^7 + 5*a^2*b^8*x^8 + (5*a^9*b*x)/8)/x^17
```

sympy [A] time = 1.33, size = 121, normalized size = 0.89

$$\frac{-8008a^{10}-85085a^{9}bx-408408a^{8}b^{2}x^{2}-1166880a^{7}b^{3}x^{3}-2199120a^{6}b^{4}x^{4}-2858856a^{5}b^{5}x^{5}-2598960a^{4}b^{6}x^{6}-1633632a^{3}b^{7}x^{7}-680680a^{2}b^{8}x^{8}-170170ab^{9}x^{9}-19448b^{10}x^{10}}{136136x^{17}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**18,x)
```

```
[Out] (-8008*a**10 - 85085*a**9*b*x - 408408*a**8*b**2*x**2 - 1166880*a**7*b**3*x
**3 - 2199120*a**6*b**4*x**4 - 2858856*a**5*b**5*x**5 - 2598960*a**4*b**6*x
**6 - 1633632*a**3*b**7*x**7 - 680680*a**2*b**8*x**8 - 170170*a*b**9*x**9 -
 19448*b**10*x**10)/(136136*x**17)
```

3.153 $\int \frac{(a+bx)^{10}}{x^{19}} \, dx$

Optimal. Leaf size=130

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10a^9b}{17x^{17}}-\frac{a^{10}}{18x^{18}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^19,x]
```

```
[Out] -a^10/(18*x^18) - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{19}} \, dx &= \int \left(\frac{a^{10}}{x^{19}}+\frac{10a^9b}{x^{18}}+\frac{45a^8b^2}{x^{17}}+\frac{120a^7b^3}{x^{16}}+\frac{210a^6b^4}{x^{15}}+\frac{252a^5b^5}{x^{14}}+\frac{210a^4b^6}{x^{13}}+\frac{120a^3b^7}{x^{12}}+\frac{45a^2\cdots}{x^{1\cdots}}\right.\\ &= -\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{1\cdots}{9\cdots}\end{aligned}$$

Mathematica [A] time = 0.00, size = 130, normalized size = 1.00

$$-\frac{a^{10}}{18x^{18}}-\frac{10a^9b}{17x^{17}}-\frac{45a^8b^2}{16x^{16}}-\frac{8a^7b^3}{x^{15}}-\frac{15a^6b^4}{x^{14}}-\frac{252a^5b^5}{13x^{13}}-\frac{35a^4b^6}{2x^{12}}-\frac{120a^3b^7}{11x^{11}}-\frac{9a^2b^8}{2x^{10}}-\frac{10ab^9}{9x^9}-\frac{b^{10}}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^19,x]
```

```
[Out] -1/18*a^10/x^18 - (10*a^9*b)/(17*x^17) - (45*a^8*b^2)/(16*x^16) - (8*a^7*b^
3)/x^15 - (15*a^6*b^4)/x^14 - (252*a^5*b^5)/(13*x^13) - (35*a^4*b^6)/(2*x^1
2) - (120*a^3*b^7)/(11*x^11) - (9*a^2*b^8)/(2*x^10) - (10*a*b^9)/(9*x^9) -
b^10/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{19}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^19,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^19, x]
```

fricas [A] time = 0.76, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

giac [A] time = 0.98, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

maple [A] time = 0.01, size = 113, normalized size = 0.87

$$-\frac{b^{10}}{8x^{8}}-\frac{10a\,b^{9}}{9x^{9}}-\frac{9a^{2}b^{8}}{2x^{10}}-\frac{120a^{3}b^{7}}{11x^{11}}-\frac{35a^{4}b^{6}}{2x^{12}}-\frac{252a^{5}b^{5}}{13x^{13}}-\frac{15a^{6}b^{4}}{x^{14}}-\frac{8a^{7}b^{3}}{x^{15}}-\frac{45a^{8}b^{2}}{16x^{16}}-\frac{10a^{9}b}{17x^{17}}-\frac{a^{10}}{18x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^19,x)
```

```
[Out] -1/18*a^10/x^18-10/17*a^9*b/x^17-45/16*a^8*b^2/x^16-8*a^7*b^3/x^15-15*a^6*b
^4/x^14-252/13*a^5*b^5/x^13-35/2*a^4*b^6/x^12-120/11*a^3*b^7/x^11-9/2*a^2*b
^8/x^10-10/9*a*b^9/x^9-1/8*b^10/x^8
```

maxima [A] time = 1.36, size = 112, normalized size = 0.86

$$-\frac{43758\,b^{10}x^{10}+388960\,ab^{9}x^{9}+1575288\,a^{2}b^{8}x^{8}+3818880\,a^{3}b^{7}x^{7}+6126120\,a^{4}b^{6}x^{6}+6785856\,a^{5}b^{5}x^{5}+5250960\,a^{6}b^{4}x^{4}+2800512\,a^{7}b^{3}x^{3}+984555\,a^{8}b^{2}x^{2}+205920\,a^{9}bx+19448\,a^{10}}{350064\,x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*x^10 + 388960*a*b^9*x^9 + 1575288*a^2*b^8*x^8 + 38188
80*a^3*b^7*x^7 + 6126120*a^4*b^6*x^6 + 6785856*a^5*b^5*x^5 + 5250960*a^6*b^
4*x^4 + 2800512*a^7*b^3*x^3 + 984555*a^8*b^2*x^2 + 205920*a^9*b*x + 19448*a
^10)/x^18
```

mupad [B] time = 0.10, size = 112, normalized size = 0.86

$$-\frac{\frac{a^{10}}{18}+\frac{10\,a^{9}\,b\,x}{17}+\frac{45\,a^{8}\,b^{2}\,x^{2}}{16}+8\,a^{7}\,b^{3}\,x^{3}+15\,a^{6}\,b^{4}\,x^{4}+\frac{252\,a^{5}\,b^{5}\,x^{5}}{13}+\frac{35\,a^{4}\,b^{6}\,x^{6}}{2}+\frac{120\,a^{3}\,b^{7}\,x^{7}}{11}+\frac{9\,a^{2}\,b^{8}\,x^{8}}{2}+\frac{10\,a\,b^{9}\,x^{9}}{9}+\frac{b^{10}\,x^{10}}{8}}{x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^19,x)
```

```
[Out] -(a^10/18 + (b^10*x^10)/8 + (10*a*b^9*x^9)/9 + (45*a^8*b^2*x^2)/16 + 8*a^7*
b^3*x^3 + 15*a^6*b^4*x^4 + (252*a^5*b^5*x^5)/13 + (35*a^4*b^6*x^6)/2 + (120
*a^3*b^7*x^7)/11 + (9*a^2*b^8*x^8)/2 + (10*a^9*b*x)/17)/x^18
```

sympy [A] time = 1.35, size = 121, normalized size = 0.93

$$\frac{-19448a^{10}-205920a^{9}bx-984555a^{8}b^{2}x^{2}-2800512a^{7}b^{3}x^{3}-5250960a^{6}b^{4}x^{4}-6785856a^{5}b^{5}x^{5}-6126120a^{4}b^{6}x^{6}-3818880a^{3}b^{7}x^{7}-1575288a^{2}b^{8}x^{8}-388960ab^{9}x^{9}-43758b^{10}x^{10}}{350064x^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**19,x)
```

```
[Out] (-19448*a**10 - 205920*a**9*b*x - 984555*a**8*b**2*x**2 - 2800512*a**7*b**3
*x**3 - 5250960*a**6*b**4*x**4 - 6785856*a**5*b**5*x**5 - 6126120*a**4*b**6
*x**6 - 3818880*a**3*b**7*x**7 - 1575288*a**2*b**8*x**8 - 388960*a*b**9*x**
9 - 43758*b**10*x**10)/(350064*x**18)
```

3.154 $\int \frac{(a+bx)^{10}}{x^{20}} \, dx$

Optimal. Leaf size=126

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{5a^9b}{9x^{18}}-\frac{a^{10}}{19x^{19}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^20,x]
```

```
[Out] -a^10/(19*x^19) - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{10}}{x^{20}} \, dx = \int \left(\frac{a^{10}}{x^{20}}+\frac{10a^9b}{x^{19}}+\frac{45a^8b^2}{x^{18}}+\frac{120a^7b^3}{x^{17}}+\frac{210a^6b^4}{x^{16}}+\frac{252a^5b^5}{x^{15}}+\frac{210a^4b^6}{x^{14}}+\frac{120a^3b^7}{x^{13}}+\frac{45a^2b^8}{x^{12}}\right.$$

$$=-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-$$

Mathematica [A] time = 0.01, size = 126, normalized size = 1.00

$$-\frac{a^{10}}{19x^{19}}-\frac{5a^9b}{9x^{18}}-\frac{45a^8b^2}{17x^{17}}-\frac{15a^7b^3}{2x^{16}}-\frac{14a^6b^4}{x^{15}}-\frac{18a^5b^5}{x^{14}}-\frac{210a^4b^6}{13x^{13}}-\frac{10a^3b^7}{x^{12}}-\frac{45a^2b^8}{11x^{11}}-\frac{ab^9}{x^{10}}-\frac{b^{10}}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^20,x]
```

```
[Out] -1/19*a^10/x^19 - (5*a^9*b)/(9*x^18) - (45*a^8*b^2)/(17*x^17) - (15*a^7*b^3
)/(2*x^16) - (14*a^6*b^4)/x^15 - (18*a^5*b^5)/x^14 - (210*a^4*b^6)/(13*x^13
) - (10*a^3*b^7)/x^12 - (45*a^2*b^8)/(11*x^11) - (a*b^9)/x^10 - b^10/(9*x^9
)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{20}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^20,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^20, x]
```

fricas [A] time = 1.34, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^{9}x^{9} + 3401190\,a^{2}b^{8}x^{8} + 8314020\,a^{3}b^{7}x^{7} + 13430340\,a^{4}b^{6}x^{6} + 14965236\,a^{5}b^{5}x^{5} + 11639628\,a^{6}b^{4}x^{4} + 6235515\,a^{7}b^{3}x^{3} + 2200770\,a^{8}b^{2}x^{2} + 461890\,a^{9}bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

giac [A] time = 1.11, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^{9}x^{9} + 3401190\,a^{2}b^{8}x^{8} + 8314020\,a^{3}b^{7}x^{7} + 13430340\,a^{4}b^{6}x^{6} + 14965236\,a^{5}b^{5}x^{5} + 11639628\,a^{6}b^{4}x^{4} + 6235515\,a^{7}b^{3}x^{3} + 2200770\,a^{8}b^{2}x^{2} + 461890\,a^{9}bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

maple [A] time = 0.01, size = 113, normalized size = 0.90

$$-\frac{b^{10}}{9x^{9}}-\frac{a\,b^{9}}{x^{10}}-\frac{45a^{2}b^{8}}{11x^{11}}-\frac{10a^{3}b^{7}}{x^{12}}-\frac{210a^{4}b^{6}}{13x^{13}}-\frac{18a^{5}b^{5}}{x^{14}}-\frac{14a^{6}b^{4}}{x^{15}}-\frac{15a^{7}b^{3}}{2x^{16}}-\frac{45a^{8}b^{2}}{17x^{17}}-\frac{5a^{9}b}{9x^{18}}-\frac{a^{10}}{19x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^20,x)
```

```
[Out] -1/19*a^10/x^19-5/9*a^9*b/x^18-45/17*a^8*b^2/x^17-15/2*a^7*b^3/x^16-14*a^6*
b^4/x^15-18*a^5*b^5/x^14-210/13*a^4*b^6/x^13-10*a^3*b^7/x^12-45/11*a^2*b^8/
x^11-a*b^9/x^10-1/9*b^10/x^9
```

maxima [A] time = 1.40, size = 112, normalized size = 0.89

$$-\frac{92378\,b^{10}x^{10} + 831402\,ab^{9}x^{9} + 3401190\,a^{2}b^{8}x^{8} + 8314020\,a^{3}b^{7}x^{7} + 13430340\,a^{4}b^{6}x^{6} + 14965236\,a^{5}b^{5}x^{5} + 11639628\,a^{6}b^{4}x^{4} + 6235515\,a^{7}b^{3}x^{3} + 2200770\,a^{8}b^{2}x^{2} + 461890\,a^{9}bx + 43758\,a^{10}}{831402\,x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*x^10 + 831402*a*b^9*x^9 + 3401190*a^2*b^8*x^8 + 83140
20*a^3*b^7*x^7 + 13430340*a^4*b^6*x^6 + 14965236*a^5*b^5*x^5 + 11639628*a^6
*b^4*x^4 + 6235515*a^7*b^3*x^3 + 2200770*a^8*b^2*x^2 + 461890*a^9*b*x + 437
58*a^10)/x^19
```

mupad [B] time = 0.14, size = 111, normalized size = 0.88

$$-\frac{\frac{a^{10}}{19}+\frac{5\,a^9\,b\,x}{9}+\frac{45\,a^8\,b^2\,x^2}{17}+\frac{15\,a^7\,b^3\,x^3}{2}+14\,a^6\,b^4\,x^4+18\,a^5\,b^5\,x^5+\frac{210\,a^4\,b^6\,x^6}{13}+10\,a^3\,b^7\,x^7+\frac{45\,a^2\,b^8\,x^8}{11}+a\,b^9\,x^9+\frac{b^{10}\,x^{10}}{9}}{x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^20,x)
```

```
[Out] -(a^10/19 + (b^10*x^10)/9 + a*b^9*x^9 + (45*a^8*b^2*x^2)/17 + (15*a^7*b^3*x
^3)/2 + 14*a^6*b^4*x^4 + 18*a^5*b^5*x^5 + (210*a^4*b^6*x^6)/13 + 10*a^3*b^7
*x^7 + (45*a^2*b^8*x^8)/11 + (5*a^9*b*x)/9)/x^19
```

sympy [A] time = 1.42, size = 121, normalized size = 0.96

$$\frac{-43758a^{10} - 461890a^9bx - 2200770a^8b^2x^2 - 6235515a^7b^3x^3 - 11639628a^6b^4x^4 - 14965236a^5b^5x^5 - 13430340a^4b^6x^6 - 8314020a^3b^7x^7 - 3401190a^2b^8x^8 - 831402ab^9x^9 - 92378b^{10}x^{10}}{831402x^{19}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**20,x)
```

```
[Out] (-43758*a**10 - 461890*a**9*b*x - 2200770*a**8*b**2*x**2 - 6235515*a**7*b**
3*x**3 - 11639628*a**6*b**4*x**4 - 14965236*a**5*b**5*x**5 - 13430340*a**4*
b**6*x**6 - 8314020*a**3*b**7*x**7 - 3401190*a**2*b**8*x**8 - 831402*a*b**9
*x**9 - 92378*b**10*x**10)/(831402*x**19)
```

3.155 $\int c(a+bx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^2}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {9}

$$\frac{c(a+bx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[c*(a + b*x),x]
[Out] (c*(a + b*x)^2)/(2*b)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int c(a+bx)\,dx=\frac{c(a+bx)^2}{2b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 0.93

$$c\left(ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[c*(a + b*x),x]
[Out] c*(a*x + (b*x^2)/2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int c(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c*(a + b*x),x]
[Out] IntegrateAlgebraic[c*(a + b*x), x]
```

fricas [A] time = 1.49, size = 12, normalized size = 0.80

$$\frac{1}{2}x^2cb+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="fricas")
[Out] 1/2*x^2*c*b + x*c*a
```

giac [A] time = 1.09, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

maple [A] time = 0.00, size = 13, normalized size = 0.87

$$\left(\frac{1}{2}b\,x^2 + ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(b*x+a),x)
```

```
[Out] c*(1/2*b*x^2+a*x)
```

maxima [A] time = 1.36, size = 13, normalized size = 0.87

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c
```

mupad [B] time = 0.02, size = 11, normalized size = 0.73

$$\frac{c\,x\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c*(a + b*x),x)
```

```
[Out] (c*x*(2*a + b*x))/2
```

sympy [A] time = 0.07, size = 12, normalized size = 0.80

$$acx + \frac{bcx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c*(b*x+a),x)
```

```
[Out] a*c*x + b*c*x**2/2
```

3.156 $\int \frac{(c+d)(a+bx)}{e}\,dx$

Optimal. Leaf size=20

$$\frac{(c+d)(a+bx)^2}{2be}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {9}

$$\frac{(c+d)(a+bx)^2}{2be}$$

Antiderivative was successfully verified.

```
[In] Int[((c + d)*(a + b*x))/e,x]
[Out] ((c + d)*(a + b*x)^2)/(2*b*e)
```

Rule 9

```
Int[(a_)*((b_) + (c_.)*(x_)), x_Symbol] :> Simp[(a*(b + c*x)^2)/(2*c), x] /
; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{(c+d)(a+bx)}{e}\,dx = \frac{(c+d)(a+bx)^2}{2be}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{(c+d)\left(ax+\frac{bx^2}{2}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c + d)*(a + b*x))/e,x]
[Out] ((c + d)*(a*x + (b*x^2)/2))/e
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+d)(a+bx)}{e}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c + d)*(a + b*x))/e,x]
[Out] IntegrateAlgebraic[((c + d)*(a + b*x))/e, x]
```

fricas [A] time = 2.37, size = 27, normalized size = 1.35

$$\frac{(bc+bd)x^2+2\,(ac+ad)x}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="fricas")
```

```
[Out] 1/2*((b*c + b*d)*x^2 + 2*(a*c + a*d)*x)/e
```

giac [A] time = 1.20, size = 17, normalized size = 0.85

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)(c + d)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="giac")
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)*e^(-1)
```

maple [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{(c+d)\left(\frac{1}{2}b\,x^2 + ax\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d)*(b*x+a)/e,x)
[Out] (c+d)/e*(1/2*b*x^2+a*x)
```

maxima [A] time = 1.37, size = 18, normalized size = 0.90

$$\frac{\left(bx^2 + 2\,ax\right)(c + d)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x, algorithm="maxima")
[Out] 1/2*(b*x^2 + 2*a*x)*(c + d)/e
```

mupad [B] time = 0.07, size = 16, normalized size = 0.80

$$\frac{x\,(c+d)\,(2\,a+b\,x)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d)*(a + b*x))/e,x)
[Out] (x*(c + d)*(2*a + b*x))/(2*e)
```

sympy [A] time = 0.08, size = 22, normalized size = 1.10

$$\frac{x^2\,(bc + bd)}{2e} + \frac{x\,(ac + ad)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d)*(b*x+a)/e,x)
[Out] x**2*(b*c + b*d)/(2*e) + x*(a*c + a*d)/e
```

3.157 $\int \frac{x^5}{a+bx}\,dx$

Optimal. Leaf size=70

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}+\frac{a^4x}{b^5}-\frac{a^5\log(a+bx)}{b^6}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{a+bx}\,dx &= \int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx\\ &= \frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}-\frac{a^5\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 70, normalized size = 1.00

$$-\frac{a^5\log(a+bx)}{b^6}+\frac{a^4x}{b^5}-\frac{a^3x^2}{2b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^4}{4b^2}+\frac{x^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x),x]
```

```
[Out] (a^4*x)/b^5 - (a^3*x^2)/(2*b^4) + (a^2*x^3)/(3*b^3) - (a*x^4)/(4*b^2) + x^5
/(5*b) - (a^5*Log[a + b*x])/b^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x), x]
```

fricas [A] time = 1.22, size = 63, normalized size = 0.90

$$\frac{12\,b^{5}x^{5}-15\,ab^{4}x^{4}+20\,a^{2}b^{3}x^{3}-30\,a^{3}b^{2}x^{2}+60\,a^{4}bx-60\,a^{5}\log\left(bx+a\right)}{60\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*x^5 - 15*a*b^4*x^4 + 20*a^2*b^3*x^3 - 30*a^3*b^2*x^2 + 60*a^4*
b*x - 60*a^5*log(b*x + a))/b^6
```

giac [A] time = 1.00, size = 65, normalized size = 0.93

$$-\frac{a^{5}\log\left(|bx+a|\right)}{b^{6}}+\frac{12\,b^{4}x^{5}-15\,ab^{3}x^{4}+20\,a^{2}b^{2}x^{3}-30\,a^{3}bx^{2}+60\,a^{4}x}{60\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -a^5*log(abs(b*x + a))/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x
^3 - 30*a^3*b*x^2 + 60*a^4*x)/b^5
```

maple [A] time = 0.00, size = 63, normalized size = 0.90

$$\frac{x^{5}}{5b}-\frac{a\,x^{4}}{4b^{2}}+\frac{a^{2}x^{3}}{3b^{3}}-\frac{a^{3}x^{2}}{2b^{4}}-\frac{a^{5}\ln\left(bx+a\right)}{b^{6}}+\frac{a^{4}x}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a),x)
```

```
[Out] a^4*x/b^5-1/2*a^3*x^2/b^4+1/3*a^2*x^3/b^3-1/4*a*x^4/b^2+1/5*x^5/b-a^5*ln(b*
x+a)/b^6
```

maxima [A] time = 1.39, size = 64, normalized size = 0.91

$$-\frac{a^{5}\log\left(bx+a\right)}{b^{6}}+\frac{12\,b^{4}x^{5}-15\,ab^{3}x^{4}+20\,a^{2}b^{2}x^{3}-30\,a^{3}bx^{2}+60\,a^{4}x}{60\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -a^5*log(b*x + a)/b^6 + 1/60*(12*b^4*x^5 - 15*a*b^3*x^4 + 20*a^2*b^2*x^3 -
30*a^3*b*x^2 + 60*a^4*x)/b^5
```

mupad [B] time = 0.08, size = 62, normalized size = 0.89

$$\frac{x^{5}}{5\,b}-\frac{a^{5}\,\ln\left(a+b\,x\right)}{b^{6}}-\frac{a\,x^{4}}{4\,b^{2}}+\frac{a^{4}\,x}{b^{5}}+\frac{a^{2}\,x^{3}}{3\,b^{3}}-\frac{a^{3}\,x^{2}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x),x)
```

```
[Out] x^5/(5*b) - (a^5*log(a + b*x))/b^6 - (a*x^4)/(4*b^2) + (a^4*x)/b^5 + (a^2*x
^3)/(3*b^3) - (a^3*x^2)/(2*b^4)
```

sympy [A] time = 0.16, size = 61, normalized size = 0.87

$$-\frac{a^{5}\log\left(a+bx\right)}{b^{6}}+\frac{a^{4}x}{b^{5}}-\frac{a^{3}x^{2}}{2b^{4}}+\frac{a^{2}x^{3}}{3b^{3}}-\frac{ax^{4}}{4b^{2}}+\frac{x^{5}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a),x)
```

```
[Out] -a**5*log(a + b*x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4) + a**2*x**3/(3*b
**3) - a*x**4/(4*b**2) + x**5/(5*b)
```

3.158 $\int \frac{x^4}{a+bx} \, dx$

Optimal. Leaf size=57

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{43\}$

$$\frac{a^2 x^2}{2b^3} - \frac{a^3 x}{b^4} + \frac{a^4 \log(a+bx)}{b^5} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{a+bx} \, dx &= \int \left(-\frac{a^3}{b^4} + \frac{a^2 x}{b^3} - \frac{a x^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a+bx)} \right) dx \\ &= -\frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4 \log(a+bx)}{b^5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 57, normalized size = 1.00

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{a+bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x), x]
```

fricas [A] time = 1.21, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4 - 4\,ab^3x^3 + 6\,a^2b^2x^2 - 12\,a^3bx + 12\,a^4\log\left(bx + a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

giac [A] time = 1.35, size = 53, normalized size = 0.93

$$\frac{a^4\log\left(\left|bx + a\right|\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

maple [A] time = 0.00, size = 52, normalized size = 0.91

$$\frac{x^4}{4b} - \frac{a\,x^3}{3b^2} + \frac{a^2x^2}{2b^3} + \frac{a^4\ln\left(bx + a\right)}{b^5} - \frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a),x)
```

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

maxima [A] time = 1.33, size = 52, normalized size = 0.91

$$\frac{a^4\log\left(bx + a\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

mupad [B] time = 0.10, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b} + \frac{a^4\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

sympy [A] time = 0.15, size = 49, normalized size = 0.86

$$\frac{a^4\log\left(a + bx\right)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

3.159 $\int \frac{x^3}{a+bx} dx$

Optimal. Leaf size=44

$$-\frac{a^3 \log(a+bx)}{b^4}+\frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2 x}{b^3}-\frac{a^3 \log(a+bx)}{b^4}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{a+bx} dx &= \int \left(\frac{a^2}{b^3}-\frac{a x}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right) dx \\ &= \frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}-\frac{a^3 \log(a+bx)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{a^3 \log(a+bx)}{b^4}+\frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a+bx} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x), x]
```

fricas [A] time = 1.17, size = 41, normalized size = 0.93

$$\frac{2 b^3 x^3-3 a b^2 x^2+6 a^2 b x-6 a^3 \log(bx+a)}{6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="fricas")
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

giac [A] time = 1.05, size = 43, normalized size = 0.98

$$-\frac{a^3 \log\left(|bx+a|\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="giac")
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

maple [A] time = 0.00, size = 41, normalized size = 0.93

$$\frac{x^3}{3b}-\frac{a\,x^2}{2b^2}-\frac{a^3\ln\left(bx+a\right)}{b^4}+\frac{a^2x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a),x)
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

maxima [A] time = 1.24, size = 42, normalized size = 0.95

$$-\frac{a^3 \log\left(bx+a\right)}{b^4}+\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a),x, algorithm="maxima")
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x),x)
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

sympy [A] time = 0.14, size = 37, normalized size = 0.84

$$-\frac{a^3 \log\left(a+bx\right)}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{2b^2}+\frac{x^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a),x)
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

3.160 $\int \frac{x^2}{a+bx} \, dx$

Optimal. Leaf size=31

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+bx} \, dx &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right) dx \\
&= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a+bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x), x]
```

fricas [A] time = 1.28, size = 29, normalized size = 0.94

$$\frac{b^2x^2 - 2\,abx + 2\,a^2 \log\left(bx + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

giac [A] time = 0.94, size = 30, normalized size = 0.97

$$\frac{a^2\log\left(|bx+a|\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

maple [A] time = 0.00, size = 30, normalized size = 0.97

$$\frac{x^2}{2b}+\frac{a^2\ln\left(bx+a\right)}{b^3}-\frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x)
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

maxima [A] time = 1.37, size = 29, normalized size = 0.94

$$\frac{a^2\log\left(bx+a\right)}{b^3}+\frac{bx^2-2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x),x)
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

sympy [A] time = 0.13, size = 26, normalized size = 0.84

$$\frac{a^2\log\left(a+bx\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.161 $\int \frac{x}{a+bx} \, dx$

Optimal. Leaf size=18

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a+bx} \, dx &= \int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right) dx \\ &= \frac{x}{b}-\frac{a\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b}-\frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a+bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x), x]
```

fricas [A] time = 1.14, size = 17, normalized size = 0.94

$$\frac{bx-a\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

giac [A] time = 1.02, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a \log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$-\frac{a \ln\left(bx + a\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x)
```

```
[Out] x/b-a*ln(b*x+a)/b^2
```

maxima [A] time = 1.31, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

mupad [B] time = 0.08, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a + b\,x\right) - b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

sympy [A] time = 0.12, size = 14, normalized size = 0.78

$$-\frac{a \log\left(a + bx\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

3.162 $\int \frac{1}{a+bx} dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx} dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-1), x]
```

fricas [A] time = 1.41, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

giac [**A**] time = 0.92, size = 11, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

maple [**A**] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

maxima [**A**] time = 1.30, size = 10, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

mupad [**B**] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
```

```
[Out] log(a + b*x)/b
```

sympy [**A**] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

3.163 $\int \frac{1}{x(a+bx)} dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)} dx &= \frac{\int \frac{1}{x} dx}{a} - \frac{b \int \frac{1}{a+bx} dx}{a}\\ &= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)),x]
[Out] IntegrateAlgebraic[1/(x*(a + b*x)), x]
```

fricas [A] time = 1.25, size = 16, normalized size = 0.89

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

giac [A] time = 1.10, size = 20, normalized size = 1.11

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{\ln(x)}{a}-\frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
[Out] ln(x)/a-ln(b*x+a)/a
```

maxima [A] time = 1.39, size = 18, normalized size = 1.00

$$-\frac{\log(bx+a)}{a}+\frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

mupad [B] time = 0.09, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)),x)
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

sympy [A] time = 0.15, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
[Out] (log(x) - log(a/b + x))/a
```

3.164 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)), x]
```

fricas [A] time = 1.40, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.97, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(bx+a\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

maxima [A] time = 1.30, size = 28, normalized size = 1.00

$$\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)),x)
```

```
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.20, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.165 $\int \frac{1}{x^3(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)}\,dx &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)), x]
```

fricas [A] time = 1.30, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 1.02, size = 45, normalized size = 1.07

$$-\frac{b^{2}\log\left(|bx+a|\right)}{a^{3}}+\frac{b^{2}\log\left(|x|\right)}{a^{3}}+\frac{2\,abx-a^{2}}{2\,a^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.01, size = 41, normalized size = 0.98

$$\frac{b^{2}\ln(x)}{a^{3}}-\frac{b^{2}\ln\left(bx+a\right)}{a^{3}}+\frac{b}{a^{2}x}-\frac{1}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a),x)
```

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

maxima [A] time = 1.34, size = 40, normalized size = 0.95

$$-\frac{b^{2}\log\left(bx+a\right)}{a^{3}}+\frac{b^{2}\log(x)}{a^{3}}+\frac{2\,bx-a}{2\,a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^{2}}{2}-a\,b\,x}{a^{3}\,x^{2}}-\frac{2\,b^{2}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

sympy [A] time = 0.22, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^{2}x^{2}}+\frac{b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.166 $\int \frac{1}{x^4(a+bx)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)}\,dx &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)), x]
```

fricas [A] time = 1.22, size = 54, normalized size = 0.96

$$\frac{6\,b^{3}x^{3}\log\left(bx+a\right)-6\,b^{3}x^{3}\log(x)-6\,ab^{2}x^{2}+3\,a^{2}bx-2\,a^{3}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

giac [A] time = 1.07, size = 56, normalized size = 1.00

$$\frac{b^{3}\log\left(|bx+a|\right)}{a^{4}}-\frac{b^{3}\log\left(|x|\right)}{a^{4}}-\frac{6\,ab^{2}x^{2}-3\,a^{2}bx+2\,a^{3}}{6\,a^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

maple [A] time = 0.01, size = 53, normalized size = 0.95

$$-\frac{b^{3}\ln(x)}{a^{4}}+\frac{b^{3}\ln\left(bx+a\right)}{a^{4}}-\frac{b^{2}}{a^{3}x}+\frac{b}{2a^{2}x^{2}}-\frac{1}{3a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a),x)
```

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

maxima [A] time = 1.35, size = 51, normalized size = 0.91

$$\frac{b^{3}\log\left(bx+a\right)}{a^{4}}-\frac{b^{3}\log(x)}{a^{4}}-\frac{6\,b^{2}x^{2}-3\,abx+2\,a^{2}}{6\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

mupad [B] time = 0.10, size = 48, normalized size = 0.86

$$\frac{2\,b^{3}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{4}}-\frac{\frac{a^{3}}{3}-\frac{a^{2}\,b\,x}{2}+a\,b^{2}\,x^{2}}{a^{4}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

sympy [A] time = 0.24, size = 44, normalized size = 0.79

$$\frac{-2a^{2}+3abx-6b^{2}x^{2}}{6a^{3}x^{3}}+\frac{b^{3}\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

3.167 $\int \frac{1}{x^5(a+bx)} dx$

Optimal. Leaf size=68

$$\frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx)}{a^5} + \frac{b^3}{a^4x} - \frac{b^2}{2a^3x^2} + \frac{b}{3a^2x^3} - \frac{1}{4ax^4}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$-\frac{b^2}{2a^3x^2} + \frac{b^3}{a^4x} + \frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx)}{a^5} + \frac{b}{3a^2x^3} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/(4*a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x])
/a^5 - (b^4*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)} dx &= \int \left(\frac{1}{ax^5} - \frac{b}{a^2x^4} + \frac{b^2}{a^3x^3} - \frac{b^3}{a^4x^2} + \frac{b^4}{a^5x} - \frac{b^5}{a^5(a+bx)}\right) dx\\ &= -\frac{1}{4ax^4} + \frac{b}{3a^2x^3} - \frac{b^2}{2a^3x^2} + \frac{b^3}{a^4x} + \frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.00

$$\frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx)}{a^5} + \frac{b^3}{a^4x} - \frac{b^2}{2a^3x^2} + \frac{b}{3a^2x^3} - \frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(3*a^2*x^3) - b^2/(2*a^3*x^2) + b^3/(a^4*x) + (b^4*Log[x
])/a^5 - (b^4*Log[a + b*x])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)), x]
```

fricas [A] time = 1.22, size = 65, normalized size = 0.96

$$-\frac{12\, b^{4} x^{4} \log\left(b x + a\right) - 12\, b^{4} x^{4} \log\left(x\right) - 12\, a b^{3} x^{3} + 6\, a^{2} b^{2} x^{2} - 4\, a^{3} b x + 3\, a^{4}}{12\, a^{5} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^4*x^4*log(b*x + a) - 12*b^4*x^4*log(x) - 12*a*b^3*x^3 + 6*a^2*b
^2*x^2 - 4*a^3*b*x + 3*a^4)/(a^5*x^4)
```

giac [A] time = 1.06, size = 67, normalized size = 0.99

$$-\frac{b^{4} \log\left({\left| b x + a \right|}\right)}{a^{5}} + \frac{b^{4} \log\left({\left| x \right|}\right)}{a^{5}} + \frac{12\, a b^{3} x^{3} - 6\, a^{2} b^{2} x^{2} + 4\, a^{3} b x - 3\, a^{4}}{12\, a^{5} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(b*x + a))/a^5 + b^4*log(abs(x))/a^5 + 1/12*(12*a*b^3*x^3 - 6*a
^2*b^2*x^2 + 4*a^3*b*x - 3*a^4)/(a^5*x^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.93

$$\frac{b^{4} \ln \left(x \right)}{a^{5}}-\frac{b^{4} \ln \left(b x +a \right)}{a^{5}}+\frac{b^{3}}{a^{4} x}-\frac{b^{2}}{2 a^{3} x^{2}}+\frac{b}{3 a^{2} x^{3}}-\frac{1}{4 a \,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a),x)
```

```
[Out] -1/4/a/x^4+1/3*b/a^2/x^3-1/2*b^2/a^3/x^2+b^3/a^4/x+b^4*ln(x)/a^5-b^4*ln(b*x
+a)/a^5
```

maxima [A] time = 1.36, size = 62, normalized size = 0.91

$$-\frac{b^{4} \log\left(b x + a\right)}{a^{5}} + \frac{b^{4} \log\left(x\right)}{a^{5}} + \frac{12\, b^{3} x^{3} - 6\, a b^{2} x^{2} + 4\, a^{2} b x - 3\, a^{3}}{12\, a^{4} x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^4*log(b*x + a)/a^5 + b^4*log(x)/a^5 + 1/12*(12*b^3*x^3 - 6*a*b^2*x^2 + 4
*a^2*b*x - 3*a^3)/(a^4*x^4)
```

mupad [B] time = 0.07, size = 60, normalized size = 0.88

$$-\frac{\frac{a^4}{4}-\frac{a^3\,b\,x}{3}+\frac{a^2\,b^2\,x^2}{2}-a\,b^3\,x^3}{a^5\,x^4}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)),x)
```

```
[Out] - (a^4/4 - a*b^3*x^3 + (a^2*b^2*x^2)/2 - (a^3*b*x)/3)/(a^5*x^4) - (2*b^4*at
anh((2*b*x)/a + 1))/a^5
```

sympy [A] time = 0.28, size = 56, normalized size = 0.82

$$\frac{- 3 a^{3} + 4 a^{2} b x - 6 a b^{2} x^{2} + 12 b^{3} x^{3}}{12 a^{4} x^{4}} + \frac{b^{4} \left(\log{\left(x \right)} - \log{\left(\frac{a}{b} + x \right)}\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a),x)
```

```
[Out] (-3*a**3 + 4*a**2*b*x - 6*a*b**2*x**2 + 12*b**3*x**3)/(12*a**4*x**4) + b**4
*(log(x) - log(a/b + x))/a**5
```

3.168 $\int \frac{x^6}{(a+bx)^2} dx$

Optimal. Leaf size=81

$$-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{a^6}{b^7(a+bx)}+\frac{5a^4x}{b^6}-\frac{6a^5\log(a+bx)}{b^7}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^2,x]
```

```
[Out] (5*a^4*x)/b^6 - (2*a^3*x^2)/b^5 + (a^2*x^3)/b^4 - (a*x^4)/(2*b^3) + x^5/(5*
b^2) - a^6/(b^7*(a + b*x)) - (6*a^5*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^2}\,dx &= \int \left(\frac{5a^4}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{b^4}-\frac{2ax^3}{b^3}+\frac{x^4}{b^2}+\frac{a^6}{b^6(a+bx)^2}-\frac{6a^5}{b^6(a+bx)}\right)dx\\ &= \frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}-\frac{a^6}{b^7(a+bx)}-\frac{6a^5\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.95

$$\frac{-\frac{10a^6}{a+bx}-60a^5\log(a+bx)+50a^4bx-20a^3b^2x^2+10a^2b^3x^3-5ab^4x^4+2b^5x^5}{10b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^2,x]
```

```
[Out] (50*a^4*b*x - 20*a^3*b^2*x^2 + 10*a^2*b^3*x^3 - 5*a*b^4*x^4 + 2*b^5*x^5 - (
10*a^6)/(a + b*x) - 60*a^5*Log[a + b*x])/(10*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^2, x]
```

fricas [A] time = 1.31, size = 96, normalized size = 1.19

$$\frac{2\,b^6x^6-3\,ab^5x^5+5\,a^2b^4x^4-10\,a^3b^3x^3+30\,a^4b^2x^2+50\,a^5bx-10\,a^6-60\,\left(a^5bx+a^6\right)\log\left(bx+a\right)}{10\,\left(b^8x+ab^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^6*x^6 - 3*a*b^5*x^5 + 5*a^2*b^4*x^4 - 10*a^3*b^3*x^3 + 30*a^4*b^2
*x^2 + 50*a^5*b*x - 10*a^6 - 60*(a^5*b*x + a^6)*log(b*x + a))/(b^8*x + a*b^
7)
```

giac [A] time = 1.14, size = 103, normalized size = 1.27

$$-\frac{(bx+a)^5\left(\frac{15\,a}{bx+a}-\frac{50\,a^2}{(bx+a)^2}+\frac{100\,a^3}{(bx+a)^3}-\frac{150\,a^4}{(bx+a)^4}-2\right)}{10\,b^7}+\frac{6\,a^5\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^7}-\frac{a^6}{(bx+a)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/10*(b*x + a)^5*(15*a/(b*x + a) - 50*a^2/(b*x + a)^2 + 100*a^3/(b*x + a)^
3 - 150*a^4/(b*x + a)^4 - 2)/b^7 + 6*a^5*log(abs(b*x + a)/((b*x + a)^2*abs(
b)))/b^7 - a^6/((b*x + a)*b^7)
```

maple [A] time = 0.01, size = 78, normalized size = 0.96

$$\frac{x^5}{5b^2}-\frac{a\,x^4}{2b^3}+\frac{a^2x^3}{b^4}-\frac{2a^3x^2}{b^5}-\frac{a^6}{(bx+a)\,b^7}-\frac{6a^5\ln(bx+a)}{b^7}+\frac{5a^4x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^2,x)
```

```
[Out] 5*a^4*x/b^6-2*a^3*x^2/b^5+a^2*x^3/b^4-1/2*a*x^4/b^3+1/5*x^5/b^2-a^6/b^7/(b*
x+a)-6*a^5*ln(b*x+a)/b^7
```

maxima [A] time = 1.33, size = 82, normalized size = 1.01

$$-\frac{a^6}{b^8x+ab^7}-\frac{6\,a^5\log(bx+a)}{b^7}+\frac{2\,b^4x^5-5\,ab^3x^4+10\,a^2b^2x^3-20\,a^3bx^2+50\,a^4x}{10\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^6/(b^8*x + a*b^7) - 6*a^5*log(b*x + a)/b^7 + 1/10*(2*b^4*x^5 - 5*a*b^3*x
^4 + 10*a^2*b^2*x^3 - 20*a^3*b*x^2 + 50*a^4*x)/b^6
```

mupad [B] time = 0.14, size = 83, normalized size = 1.02

$$\frac{x^5}{5\,b^2}-\frac{6\,a^5\,\ln(a+b\,x)}{b^7}-\frac{a\,x^4}{2\,b^3}+\frac{5\,a^4\,x}{b^6}+\frac{a^2\,x^3}{b^4}-\frac{2\,a^3\,x^2}{b^5}-\frac{a^6}{b\,\left(x\,b^7+a\,b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^2,x)
```

```
[Out] x^5/(5*b^2) - (6*a^5*log(a + b*x))/b^7 - (a*x^4)/(2*b^3) + (5*a^4*x)/b^6 +
(a^2*x^3)/b^4 - (2*a^3*x^2)/b^5 - a^6/(b*(a*b^6 + b^7*x))
```

sympy [A] time = 0.28, size = 78, normalized size = 0.96

$$-\frac{a^6}{ab^7+b^8x}-\frac{6a^5\log(a+bx)}{b^7}+\frac{5a^4x}{b^6}-\frac{2a^3x^2}{b^5}+\frac{a^2x^3}{b^4}-\frac{ax^4}{2b^3}+\frac{x^5}{5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**2,x)
```

```
[Out] -a**6/(a*b**7 + b**8*x) - 6*a**5*log(a + b*x)/b**7 + 5*a**4*x/b**6 - 2*a**3
*x**2/b**5 + a**2*x**3/b**4 - a*x**4/(2*b**3) + x**5/(5*b**2)
```

3.169 $\int \frac{x^5}{(a+bx)^2}\,dx$

Optimal. Leaf size=72

$$\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}-\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{2b^4}+\frac{a^5}{b^6(a+bx)}-\frac{4a^3x}{b^5}+\frac{5a^4\log(a+bx)}{b^6}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^2,x]
```

```
[Out] (-4*a^3*x)/b^5 + (3*a^2*x^2)/(2*b^4) - (2*a*x^3)/(3*b^3) + x^4/(4*b^2) + a^
5/(b^6*(a + b*x)) + (5*a^4*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^2}\,dx &= \int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a^5}{b^5(a+bx)^2}+\frac{5a^4}{b^5(a+bx)}\right)dx\\ &= -\frac{4a^3x}{b^5}+\frac{3a^2x^2}{2b^4}-\frac{2ax^3}{3b^3}+\frac{x^4}{4b^2}+\frac{a^5}{b^6(a+bx)}+\frac{5a^4\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.92

$$\frac{\frac{12a^5}{a+bx}+60a^4\log(a+bx)-48a^3bx+18a^2b^2x^2-8ab^3x^3+3b^4x^4}{12b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^2,x]
```

```
[Out] (-48*a^3*b*x + 18*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4 + (12*a^5)/(a + b*x
) + 60*a^4*Log[a + b*x])/(12*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^2, x]
```

fricas [A] time = 1.17, size = 85, normalized size = 1.18

$$\frac{3\,b^{5}x^{5}-5\,ab^{4}x^{4}+10\,a^{2}b^{3}x^{3}-30\,a^{3}b^{2}x^{2}-48\,a^{4}bx+12\,a^{5}+60\,\left(a^{4}bx+a^{5}\right)\log\left(bx+a\right)}{12\,\left(b^{7}x+ab^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*x^5 - 5*a*b^4*x^4 + 10*a^2*b^3*x^3 - 30*a^3*b^2*x^2 - 48*a^4*b*
x + 12*a^5 + 60*(a^4*b*x + a^5)*log(b*x + a))/(b^7*x + a*b^6)
```

giac [A] time = 1.22, size = 90, normalized size = 1.25

$$-\frac{(bx+a)^{4}\left(\frac{20\,a}{bx+a}-\frac{60\,a^{2}}{(bx+a)^{2}}+\frac{120\,a^{3}}{(bx+a)^{3}}-3\right)}{12\,b^{6}}-\frac{5\,a^{4}\log\left(\frac{|bx+a|}{(bx+a)^{2}|b|}\right)}{b^{6}}+\frac{a^{5}}{(bx+a)b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/12*(b*x + a)^4*(20*a/(b*x + a) - 60*a^2/(b*x + a)^2 + 120*a^3/(b*x + a)^
3 - 3)/b^6 - 5*a^4*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^6 + a^5/((b*x +
 a)*b^6)
```

maple [A] time = 0.01, size = 67, normalized size = 0.93

$$\frac{x^{4}}{4b^{2}}-\frac{2a\,x^{3}}{3b^{3}}+\frac{3a^{2}x^{2}}{2b^{4}}+\frac{a^{5}}{\left(bx+a\right)b^{6}}+\frac{5a^{4}\ln\left(bx+a\right)}{b^{6}}-\frac{4a^{3}x}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2,x)
```

```
[Out] -4*a^3*x/b^5+3/2*a^2*x^2/b^4-2/3*a*x^3/b^3+1/4*x^4/b^2+a^5/b^6/(b*x+a)+5*a^
4*ln(b*x+a)/b^6
```

maxima [A] time = 1.33, size = 70, normalized size = 0.97

$$\frac{a^{5}}{b^{7}x+ab^{6}}+\frac{5\,a^{4}\log\left(bx+a\right)}{b^{6}}+\frac{3\,b^{3}x^{4}-8\,ab^{2}x^{3}+18\,a^{2}bx^{2}-48\,a^{3}x}{12\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^5/(b^7*x + a*b^6) + 5*a^4*log(b*x + a)/b^6 + 1/12*(3*b^3*x^4 - 8*a*b^2*x^
3 + 18*a^2*b*x^2 - 48*a^3*x)/b^5
```

mupad [B] time = 0.07, size = 72, normalized size = 1.00

$$\frac{x^{4}}{4\,b^{2}}+\frac{5\,a^{4}\,\ln\left(a+b\,x\right)}{b^{6}}-\frac{2\,a\,x^{3}}{3\,b^{3}}-\frac{4\,a^{3}\,x}{b^{5}}+\frac{3\,a^{2}\,x^{2}}{2\,b^{4}}+\frac{a^{5}}{b\,\left(x\,b^{6}+a\,b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^2,x)
```

```
[Out] x^4/(4*b^2) + (5*a^4*log(a + b*x))/b^6 - (2*a*x^3)/(3*b^3) - (4*a^3*x)/b^5
+ (3*a^2*x^2)/(2*b^4) + a^5/(b*(a*b^5 + b^6*x))
```

<u>**sympy**</u> **[A]** time = 0.25, size = 71, normalized size = 0.99

$$\frac{a^5}{ab^6 + b^7 x} + \frac{5a^4 \log{(a + bx)}}{b^6} - \frac{4a^3 x}{b^5} + \frac{3a^2 x^2}{2b^4} - \frac{2ax^3}{3b^3} + \frac{x^4}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2,x)
```

```
[Out] a**5/(a*b**6 + b**7*x) + 5*a**4*log(a + b*x)/b**6 - 4*a**3*x/b**5 + 3*a**2*
x**2/(2*b**4) - 2*a*x**3/(3*b**3) + x**4/(4*b**2)
```

3.170 $\int \frac{x^4}{(a+bx)^2} dx$

Optimal. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^2}\,dx &= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\ &= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}-12a^3\log(a+bx)+9a^2bx-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^2, x]
```

fricas [A] time = 1.20, size = 73, normalized size = 1.26

$$\frac{b^{4}x^{4}-2\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}+9\,a^{3}bx-3\,a^{4}-12\,\left(a^{3}bx+a^{4}\right)\log\left(bx+a\right)}{3\,\left(b^{6}x+ab^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

giac [A] time = 1.07, size = 79, normalized size = 1.36

$$-\frac{(bx+a)^{3}\left(\frac{6\,a}{bx+a}-\frac{18\,a^{2}}{(bx+a)^{2}}-1\right)}{3\,b^{5}}+\frac{4\,a^{3}\log\left(\frac{|bx+a|}{(bx+a)^{2}|b|}\right)}{b^{5}}-\frac{a^{4}}{(bx+a)b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/b^5 + 4*a^3*log(a
bs(b*x + a)/((b*x + a)^2*abs(b)))/b^5 - a^4/((b*x + a)*b^5)
```

maple [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{x^{3}}{3b^{2}}-\frac{a\,x^{2}}{b^{3}}-\frac{a^{4}}{\left(bx+a\right)b^{5}}-\frac{4a^{3}\ln\left(bx+a\right)}{b^{5}}+\frac{3a^{2}x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2,x)
```

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

maxima [A] time = 1.37, size = 59, normalized size = 1.02

$$-\frac{a^{4}}{b^{6}x+ab^{5}}-\frac{4\,a^{3}\log\left(bx+a\right)}{b^{5}}+\frac{b^{2}x^{3}-3\,abx^{2}+9\,a^{2}x}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

mupad [B] time = 0.07, size = 62, normalized size = 1.07

$$\frac{x^{3}}{3\,b^{2}}-\frac{4\,a^{3}\,\ln\left(a+b\,x\right)}{b^{5}}-\frac{a\,x^{2}}{b^{3}}+\frac{3\,a^{2}\,x}{b^{4}}-\frac{a^{4}}{b\,\left(x\,b^{5}+a\,b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

sympy [A] time = 0.22, size = 54, normalized size = 0.93

$$-\frac{a^{4}}{ab^{5}+b^{6}x}-\frac{4a^{3}\log\left(a+bx\right)}{b^{5}}+\frac{3a^{2}x}{b^{4}}-\frac{ax^{2}}{b^{3}}+\frac{x^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

3.171 $\int \frac{x^3}{(a+bx)^2} dx$

Optimal. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^2,x]
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^2} dx &= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx\\ &= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^2,x]
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[x^3/(a + b*x)^2, x]
```

fricas [A]   time = 1.22, size = 62, normalized size = 1.35

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log\left(bx + a\right)}{2\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

giac [A]   time = 1.14, size = 66, normalized size = 1.43

$$-\frac{(bx+a)^2\left(\frac{6\,a}{bx+a} - 1\right)}{2\,b^4} - \frac{3\,a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4} + \frac{a^3}{(bx+a)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b*x + a)^2*(6*a/(b*x + a) - 1)/b^4 - 3*a^2*log(abs(b*x + a)/((b*x + a
)^2*abs(b)))/b^4 + a^3/((b*x + a)*b^4)
```

maple [A]   time = 0.01, size = 45, normalized size = 0.98

$$\frac{x^2}{2b^2} + \frac{a^3}{(bx+a)\,b^4} + \frac{3a^2\ln\left(bx+a\right)}{b^4} - \frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2,x)
```

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

maxima [A]   time = 1.36, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x + ab^4} + \frac{3\,a^2\log\left(bx+a\right)}{b^4} + \frac{bx^2 - 4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

mupad [B]   time = 0.08, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2} + \frac{3\,a^2\,\ln\left(a + b\,x\right)}{b^4} + \frac{a^3}{b\,\left(x\,b^4 + a\,b^3\right)} - \frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

sympy [A]   time = 0.20, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2\log\left(a + bx\right)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

3.172 $\int \frac{x^2}{(a+bx)^2} dx$

Optimal. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^2,x]
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^2}\,dx &= \int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx\\ &= \frac{x}{b^2}-\frac{a^2}{b^3(a+bx)}-\frac{2a\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx}-2a\log(a+bx)+bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^2,x]
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[x^2/(a + b*x)^2, x]
```

fricas [A] time = 1.28, size = 47, normalized size = 1.42

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

giac [A] time = 1.15, size = 50, normalized size = 1.52

$$\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} + \frac{bx + a}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^3 + (b*x + a)/b^3 - a^2/((b*x
+ a)*b^3)
```

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$-\frac{a^2}{(bx + a)\,b^3} - \frac{2a\ln\left(bx + a\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2,x)
```

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

maxima [A] time = 1.34, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

mupad [B] time = 0.08, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a + b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^2,x)
```

```
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

sympy [A] time = 0.17, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log\left(a + bx\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2,x)
```

```
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

3.173 $\int \frac{x}{(a+bx)^2} \, dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^2} \, dx &= \int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx \\ &= \frac{a}{b^2(a+bx)} + \frac{\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx} + \log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^2, x]
```

fricas [A] time = 1.22, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a) + a}{b^3x + ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

giac [A] time = 1.05, size = 42, normalized size = 1.83

$$-\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="giac")
[Out] -(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b
```

maple [A] time = 0.01, size = 24, normalized size = 1.04

$$\frac{a}{(bx+a)\,b^2}+\frac{\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2,x)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

maxima [A] time = 1.32, size = 26, normalized size = 1.13

$$\frac{a}{b^3x+ab^2}+\frac{\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

mupad [B] time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln(a+b\,x)}{b^2}+\frac{a}{b^2\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^2,x)
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

sympy [A] time = 0.17, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

3.174 $\int \frac{1}{(a+bx)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-2), x]
```

fricas [A] time = 1.18, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 1.18, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
[Out] -1/b/(b*x+a)
```

maxima [A] time = 1.38, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
[Out] -1/((b*x + a)*b)
```

mupad [B] time = 0.03, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

sympy [A] time = 0.15, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
[Out] -1/(a*b + b**2*x)
```

3.175 $\int \frac{1}{x(a+bx)^2}\,dx$

Optimal. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^2),x]
```

```
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^2),x]
```

```
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^2), x]
```

fricas [A] time = 1.28, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

giac [A] time = 1.00, size = 38, normalized size = 1.31

$$b\left(\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2b}+\frac{1}{(bx+a)ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] b*(log(abs(-a/(b*x + a) + 1))/(a^2*b) + 1/((b*x + a)*a*b))
```

maple [A] time = 0.01, size = 30, normalized size = 1.03

$$\frac{1}{(bx+a)\,a}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2,x)
```

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

maxima [A] time = 1.30, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.12, size = 26, normalized size = 0.90

$$\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^2),x)
```

```
[Out] 1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2
```

sympy [A] time = 0.22, size = 22, normalized size = 0.76

$$\frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2,x)
```

```
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

3.176 $\int \frac{1}{x^2(a+bx)^2} dx$

Optimal. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2} dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^2), x]
```

fricas [A] time = 1.11, size = 63, normalized size = 1.50

$$-\frac{2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(bx + a\right) + 2\left(b^2x^2 + abx\right)\log(x)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

giac [A] time = 1.22, size = 52, normalized size = 1.24

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a} + 1\right|\right)}{a^3} - \frac{b}{(bx + a)a^2} + \frac{b}{a^3\left(\frac{a}{bx+a} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

maple [A] time = 0.01, size = 43, normalized size = 1.02

$$-\frac{b}{(bx + a)\,a^2} - \frac{2b\ln(x)}{a^3} + \frac{2b\ln\left(bx + a\right)}{a^3} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

maxima [A] time = 1.39, size = 45, normalized size = 1.07

$$-\frac{2\,bx + a}{a^2bx^2 + a^3x} + \frac{2\,b\log\left(bx + a\right)}{a^3} - \frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

mupad [B] time = 0.12, size = 45, normalized size = 1.07

$$\frac{2\,b\,\ln\left(\frac{a+b\,x}{x}\right)}{a^3} - \frac{1}{a\,x\,\left(a + b\,x\right)} - \frac{2\,b}{a^2\,\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^2),x)
```

```
[Out] (2*b*log((a + b*x)/x))/a^3 - 1/(a*x*(a + b*x)) - (2*b)/(a^2*(a + b*x))
```

sympy [A] time = 0.30, size = 37, normalized size = 0.88

$$\frac{-a - 2bx}{a^3x + a^2bx^2} + \frac{2b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.177 $\int \frac{1}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{b^2}{a^3(a+bx)}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}+\frac{2b}{a^3x}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^2),x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 -
 (3*b^2*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{2a^2x^2}+\frac{2b}{a^3x}+\frac{b^2}{a^3(a+bx)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx}-\frac{a}{x^2}+\frac{4b}{x}\right)-6b^2\log(a+bx)+6b^2\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^2),x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a +
b*x])/(2*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^2), x]
```

fricas [A] time = 0.55, size = 86, normalized size = 1.48

$$\frac{6\,ab^2x^2 + 3\,a^2bx - a^3 - 6\left(b^3x^3 + ab^2x^2\right)\log\left(bx + a\right) + 6\left(b^3x^3 + ab^2x^2\right)\log(x)}{2\left(a^4bx^3 + a^5x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

giac [A] time = 1.13, size = 74, normalized size = 1.28

$$\frac{3\,b^2\log\left(\left|-\frac{a}{bx+a} + 1\right|\right)}{a^4} + \frac{b^2}{(bx + a)a^3} - \frac{\frac{6\,ab^2}{bx+a} - 5\,b^2}{2\,a^4\left(\frac{a}{bx+a} - 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3*b^2*log(abs(-a/(b*x + a) + 1))/a^4 + b^2/((b*x + a)*a^3) - 1/2*(6*a*b^2/(
b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2)
```

maple [A] time = 0.01, size = 57, normalized size = 0.98

$$\frac{b^2}{(bx + a)\,a^3} + \frac{3b^2\ln(x)}{a^4} - \frac{3b^2\ln\left(bx + a\right)}{a^4} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^2,x)
```

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

maxima [A] time = 1.40, size = 64, normalized size = 1.10

$$\frac{6\,b^2x^2 + 3\,abx - a^2}{2\left(a^3bx^3 + a^4x^2\right)} - \frac{3\,b^2\log\left(bx + a\right)}{a^4} + \frac{3\,b^2\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

mupad [B] time = 0.11, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^2\,x^2}{a^3} - \frac{1}{2\,a} + \frac{3\,b\,x}{2\,a^2}}{b\,x^3 + a\,x^2} - \frac{6\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^2),x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

<u>**sympy**</u> **[A]** time = 0.31, size = 54, normalized size = 0.93

$$\frac{-a^2 + 3abx + 6b^2x^2}{2a^4x^2 + 2a^3bx^3} + \frac{3b^2\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

3.178 $\int \frac{1}{x^4(a+bx)^2} \, dx$

Optimal. Leaf size=69

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(a^3-2a^2bx+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^2), x]
```

fricas [A] time = 0.93, size = 95, normalized size = 1.38

$$-\frac{12\,ab^{3}x^{3}+6\,a^{2}b^{2}x^{2}-2\,a^{3}bx+a^{4}-12\,\left(b^{4}x^{4}+ab^{3}x^{3}\right)\log\left(bx+a\right)+12\,\left(b^{4}x^{4}+ab^{3}x^{3}\right)\log(x)}{3\,\left(a^{5}bx^{4}+a^{6}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

giac [A] time = 1.02, size = 90, normalized size = 1.30

$$-\frac{4\,b^{3}\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{5}}-\frac{b^{3}}{(bx+a)a^{4}}-\frac{\frac{30\,ab^{3}}{bx+a}-\frac{18\,a^{2}b^{3}}{(bx+a)^{2}}-13\,b^{3}}{3\,a^{5}\left(\frac{a}{bx+a}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -4*b^3*log(abs(-a/(b*x + a) + 1))/a^5 - b^3/((b*x + a)*a^4) - 1/3*(30*a*b^3
/(b*x + a) - 18*a^2*b^3/(b*x + a)^2 - 13*b^3)/(a^5*(a/(b*x + a) - 1)^3)
```

maple [A] time = 0.01, size = 68, normalized size = 0.99

$$-\frac{b^{3}}{(bx+a)\,a^{4}}-\frac{4b^{3}\ln(x)}{a^{5}}+\frac{4b^{3}\ln\left(bx+a\right)}{a^{5}}-\frac{3b^{2}}{a^{4}x}+\frac{b}{a^{3}x^{2}}-\frac{1}{3a^{2}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^2,x)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

maxima [A] time = 1.41, size = 73, normalized size = 1.06

$$-\frac{12\,b^{3}x^{3}+6\,ab^{2}x^{2}-2\,a^{2}bx+a^{3}}{3\,\left(a^{4}bx^{4}+a^{5}x^{3}\right)}+\frac{4\,b^{3}\log\left(bx+a\right)}{a^{5}}-\frac{4\,b^{3}\log(x)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

mupad [B] time = 0.08, size = 69, normalized size = 1.00

$$\frac{8\,b^{3}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{5}}-\frac{\frac{1}{3\,a}+\frac{2\,b^{2}\,x^{2}}{a^{3}}+\frac{4\,b^{3}\,x^{3}}{a^{4}}-\frac{2\,b\,x}{3\,a^{2}}}{b\,x^{4}+a\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^2),x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

sympy [**A**] time = 0.34, size = 66, normalized size = 0.96

$$\frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

3.179 $\int \frac{1}{x^5(a+bx)^2}\,dx$

Optimal. Leaf size=84

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^2), x]
```

fricas [A] time = 1.18, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4 + 30\,a^2b^3x^3 - 10\,a^3b^2x^2 + 5\,a^4bx - 3\,a^5 - 60\left(b^5x^5 + ab^4x^4\right)\log\left(bx + a\right) + 60\left(b^5x^5 + ab^4x^4\right)\log(x)}{12\left(a^6bx^5 + a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

giac [A] time = 0.99, size = 104, normalized size = 1.24

$$\frac{5\,b^4\log\left(\left|-\frac{a}{bx+a} + 1\right|\right)}{a^6} + \frac{b^4}{(bx + a)a^5} - \frac{\frac{260\,ab^4}{bx+a} - \frac{300\,a^2b^4}{(bx+a)^2} + \frac{120\,a^3b^4}{(bx+a)^3} - 77\,b^4}{12\,a^6\left(\frac{a}{bx+a} - 1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^4*log(abs(-a/(b*x + a) + 1))/a^6 + b^4/((b*x + a)*a^5) - 1/12*(260*a*b^
4/(b*x + a) - 300*a^2*b^4/(b*x + a)^2 + 120*a^3*b^4/(b*x + a)^3 - 77*b^4)/(
a^6*(a/(b*x + a) - 1)^4)
```

maple [A] time = 0.01, size = 79, normalized size = 0.94

$$\frac{b^4}{(bx + a)\,a^5} + \frac{5b^4\ln(x)}{a^6} - \frac{5b^4\ln\left(bx + a\right)}{a^6} + \frac{4b^3}{a^5x} - \frac{3b^2}{2a^4x^2} + \frac{2b}{3a^3x^3} - \frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

maxima [A] time = 1.34, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4 + 30\,ab^3x^3 - 10\,a^2b^2x^2 + 5\,a^3bx - 3\,a^4}{12\left(a^5bx^5 + a^6x^4\right)} - \frac{5\,b^4\log\left(bx + a\right)}{a^6} + \frac{5\,b^4\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

mupad [B] time = 0.12, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4} - \frac{5\,b^2\,x^2}{6\,a^3} - \frac{1}{4\,a} + \frac{5\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{12\,a^2}}{b\,x^5 + a\,x^4} - \frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

sympy [**A**] time = 0.39, size = 80, normalized size = 0.95

$$\frac{-3a^4 + 5a^3bx - 10a^2b^2x^2 + 30ab^3x^3 + 60b^4x^4}{12a^6x^4 + 12a^5bx^5} + \frac{5b^4\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

3.180 $\int \frac{x^7}{(a+bx)^3} \, dx$

Optimal. Leaf size=99

$$\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

Rubi [A] time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}+\frac{15a^4x}{b^7}-\frac{21a^5\log(a+bx)}{b^8}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^3,x]
```

```
[Out] (15*a^4*x)/b^7 - (5*a^3*x^2)/b^6 + (2*a^2*x^3)/b^5 - (3*a*x^4)/(4*b^4) + x^
5/(5*b^3) + a^7/(2*b^8*(a + b*x)^2) - (7*a^6)/(b^8*(a + b*x)) - (21*a^5*Log
[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(a+bx)^3}\,dx &= \int \left(\frac{15a^4}{b^7}-\frac{10a^3x}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^3}{b^4}+\frac{x^4}{b^3}-\frac{a^7}{b^7(a+bx)^3}+\frac{7a^6}{b^7(a+bx)^2}-\frac{21a^5}{b^7(a+bx)}\right)dx\\
&= \frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{x^5}{5b^3}+\frac{a^7}{2b^8(a+bx)^2}-\frac{7a^6}{b^8(a+bx)}-\frac{21a^5\log(a+bx)}{b^8}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 89, normalized size = 0.90

$$\frac{\frac{10a^7}{(a+bx)^2}-\frac{140a^6}{a+bx}-420a^5\log(a+bx)+300a^4bx-100a^3b^2x^2+40a^2b^3x^3-15ab^4x^4+4b^5x^5}{20b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^3,x]
```

```
[Out] (300*a^4*b*x - 100*a^3*b^2*x^2 + 40*a^2*b^3*x^3 - 15*a*b^4*x^4 + 4*b^5*x^5
+ (10*a^7)/(a + b*x)^2 - (140*a^6)/(a + b*x) - 420*a^5*Log[a + b*x])/(20*b^
8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^3, x]
```

fricas [A] time = 1.04, size = 129, normalized size = 1.30

$$\frac{4\,b^{7}x^{7}-7\,ab^{6}x^{6}+14\,a^{2}b^{5}x^{5}-35\,a^{3}b^{4}x^{4}+140\,a^{4}b^{3}x^{3}+500\,a^{5}b^{2}x^{2}+160\,a^{6}bx-130\,a^{7}-420\left(a^{5}b^{2}x^{2}+2\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{20\left(b^{10}x^{2}+2\,ab^{9}x+a^{2}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*x^7 - 7*a*b^6*x^6 + 14*a^2*b^5*x^5 - 35*a^3*b^4*x^4 + 140*a^4*b
^3*x^3 + 500*a^5*b^2*x^2 + 160*a^6*b*x - 130*a^7 - 420*(a^5*b^2*x^2 + 2*a^6
*b*x + a^7)*log(b*x + a))/(b^10*x^2 + 2*a*b^9*x + a^2*b^8)
```

giac [A] time = 0.89, size = 95, normalized size = 0.96

$$-\frac{21\,a^{5}\log\left(|bx+a|\right)}{b^{8}}-\frac{14\,a^{6}bx+13\,a^{7}}{2\,(bx+a)^{2}b^{8}}+\frac{4\,b^{12}x^{5}-15\,ab^{11}x^{4}+40\,a^{2}b^{10}x^{3}-100\,a^{3}b^{9}x^{2}+300\,a^{4}b^{8}x}{20\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -21*a^5*log(abs(b*x + a))/b^8 - 1/2*(14*a^6*b*x + 13*a^7)/((b*x + a)^2*b^8)
 + 1/20*(4*b^12*x^5 - 15*a*b^11*x^4 + 40*a^2*b^10*x^3 - 100*a^3*b^9*x^2 + 3
00*a^4*b^8*x)/b^15
```

maple [A] time = 0.01, size = 94, normalized size = 0.95

$$\frac{x^{5}}{5b^{3}}-\frac{3a\,x^{4}}{4b^{4}}+\frac{2a^{2}x^{3}}{b^{5}}+\frac{a^{7}}{2\left(bx+a\right)^{2}b^{8}}-\frac{5a^{3}x^{2}}{b^{6}}-\frac{7a^{6}}{\left(bx+a\right)b^{8}}-\frac{21a^{5}\ln\left(bx+a\right)}{b^{8}}+\frac{15a^{4}x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^3,x)
```

```
[Out] 15*a^4*x/b^7-5*a^3*x^2/b^6+2*a^2*x^3/b^5-3/4*a*x^4/b^4+1/5*x^5/b^3+1/2*a^7/
b^8/(b*x+a)^2-7*a^6/b^8/(b*x+a)-21*a^5*ln(b*x+a)/b^8
```

maxima [A] time = 1.39, size = 103, normalized size = 1.04

$$-\frac{14\,a^{6}bx+13\,a^{7}}{2\left(b^{10}x^{2}+2\,ab^{9}x+a^{2}b^{8}\right)}-\frac{21\,a^{5}\log\left(bx+a\right)}{b^{8}}+\frac{4\,b^{4}x^{5}-15\,ab^{3}x^{4}+40\,a^{2}b^{2}x^{3}-100\,a^{3}bx^{2}+300\,a^{4}x}{20\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(14*a^6*b*x + 13*a^7)/(b^10*x^2 + 2*a*b^9*x + a^2*b^8) - 21*a^5*log(b*
x + a)/b^8 + 1/20*(4*b^4*x^5 - 15*a*b^3*x^4 + 40*a^2*b^2*x^3 - 100*a^3*b*x^
2 + 300*a^4*x)/b^7
```

mupad [B] time = 0.23, size = 91, normalized size = 0.92

$$-\frac{\frac{7\,a\,{\left(a+b\,x\right)}^4}{4}-\frac{{\left(a+b\,x\right)}^5}{5}-7\,a^2\,{\left(a+b\,x\right)}^3+\frac{35\,a^3\,{\left(a+b\,x\right)}^2}{2}+\frac{7\,a^6}{a+b\,x}-\frac{a^7}{2\,{\left(a+b\,x\right)}^2}+21\,a^5\,\ln\left(a+b\,x\right)-35\,a^4\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^3,x)
```

```
[Out] -((7*a*(a + b*x)^4)/4 - (a + b*x)^5/5 - 7*a^2*(a + b*x)^3 + (35*a^3*(a + b*
x)^2)/2 + (7*a^6)/(a + b*x) - a^7/(2*(a + b*x)^2) + 21*a^5*log(a + b*x) - 3
5*a^4*b*x)/b^8
```

sympy [A] time = 0.53, size = 109, normalized size = 1.10

$$-\frac{21a^5\log(a+bx)}{b^8}+\frac{15a^4x}{b^7}-\frac{5a^3x^2}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^4}{4b^4}+\frac{-13a^7-14a^6bx}{2a^2b^8+4ab^9x+2b^{10}x^2}+\frac{x^5}{5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**3,x)
```

```
[Out] -21*a**5*log(a + b*x)/b**8 + 15*a**4*x/b**7 - 5*a**3*x**2/b**6 + 2*a**2*x**
3/b**5 - 3*a*x**4/(4*b**4) + (-13*a**7 - 14*a**6*b*x)/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + x**5/(5*b**3)
```

3.181 $\int \frac{x^6}{(a+bx)^3} dx$

Optimal. Leaf size=86

$$-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2x^2}{b^5}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}-\frac{10a^3x}{b^6}+\frac{15a^4\log(a+bx)}{b^7}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^3,x]
```

```
[Out] (-10*a^3*x)/b^6 + (3*a^2*x^2)/b^5 - (a*x^3)/b^4 + x^4/(4*b^3) - a^6/(2*b^7*
(a + b*x)^2) + (6*a^5)/(b^7*(a + b*x)) + (15*a^4*Log[a + b*x])/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^3}\,dx &= \int \left(-\frac{10a^3}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{b^4}+\frac{x^3}{b^3}+\frac{a^6}{b^6(a+bx)^3}-\frac{6a^5}{b^6(a+bx)^2}+\frac{15a^4}{b^6(a+bx)}\right)dx\\ &= -\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{x^4}{4b^3}-\frac{a^6}{2b^7(a+bx)^2}+\frac{6a^5}{b^7(a+bx)}+\frac{15a^4\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.90

$$\frac{-\frac{2a^6}{(a+bx)^2}+\frac{24a^5}{a+bx}+60a^4\log(a+bx)-40a^3bx+12a^2b^2x^2-4ab^3x^3+b^4x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^3,x]
```

```
[Out] (-40*a^3*b*x + 12*a^2*b^2*x^2 - 4*a*b^3*x^3 + b^4*x^4 - (2*a^6)/(a + b*x)^2
 + (24*a^5)/(a + b*x) + 60*a^4*Log[a + b*x])/(4*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^3, x]
```

fricas [A] time = 1.11, size = 117, normalized size = 1.36

$$\frac{b^6x^6 - 2\,ab^5x^5 + 5\,a^2b^4x^4 - 20\,a^3b^3x^3 - 68\,a^4b^2x^2 - 16\,a^5bx + 22\,a^6 + 60\left(a^4b^2x^2 + 2\,a^5bx + a^6\right)\log\left(bx + a\right)}{4\left(b^9x^2 + 2\,ab^8x + a^2b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*x^6 - 2*a*b^5*x^5 + 5*a^2*b^4*x^4 - 20*a^3*b^3*x^3 - 68*a^4*b^2*x^
2 - 16*a^5*b*x + 22*a^6 + 60*(a^4*b^2*x^2 + 2*a^5*b*x + a^6)*log(b*x + a))/
(b^9*x^2 + 2*a*b^8*x + a^2*b^7)
```

giac [A] time = 1.15, size = 83, normalized size = 0.97

$$\frac{15\,a^4\log\left(|bx + a|\right)}{b^7} + \frac{12\,a^5bx + 11\,a^6}{2\,(bx + a)^2b^7} + \frac{b^9x^4 - 4\,ab^8x^3 + 12\,a^2b^7x^2 - 40\,a^3b^6x}{4\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 15*a^4*log(abs(b*x + a))/b^7 + 1/2*(12*a^5*b*x + 11*a^6)/((b*x + a)^2*b^7)
+ 1/4*(b^9*x^4 - 4*a*b^8*x^3 + 12*a^2*b^7*x^2 - 40*a^3*b^6*x)/b^12
```

maple [A] time = 0.01, size = 83, normalized size = 0.97

$$\frac{x^4}{4b^3} - \frac{a\,x^3}{b^4} - \frac{a^6}{2\,(bx + a)^2\,b^7} + \frac{3a^2x^2}{b^5} + \frac{6a^5}{(bx + a)\,b^7} + \frac{15a^4\ln\left(bx + a\right)}{b^7} - \frac{10a^3x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^3,x)
```

```
[Out] -10*a^3*x/b^6+3*a^2*x^2/b^5-a*x^3/b^4+1/4*x^4/b^3-1/2*a^6/b^7/(b*x+a)^2+6*a
^5/b^7/(b*x+a)+15*a^4*ln(b*x+a)/b^7
```

maxima [A] time = 1.40, size = 91, normalized size = 1.06

$$\frac{12\,a^5bx + 11\,a^6}{2\left(b^9x^2 + 2\,ab^8x + a^2b^7\right)} + \frac{15\,a^4\log\left(bx + a\right)}{b^7} + \frac{b^3x^4 - 4\,ab^2x^3 + 12\,a^2bx^2 - 40\,a^3x}{4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^5*b*x + 11*a^6)/(b^9*x^2 + 2*a*b^8*x + a^2*b^7) + 15*a^4*log(b*x
+ a)/b^7 + 1/4*(b^3*x^4 - 4*a*b^2*x^3 + 12*a^2*b*x^2 - 40*a^3*x)/b^6
```

mupad [B] time = 0.16, size = 78, normalized size = 0.91

$$\frac{\frac{(a+b\,x)^4}{4} - 2\,a\,(a + b\,x)^3 + \frac{15\,a^2\,(a+b\,x)^2}{2} + \frac{6\,a^5}{a+b\,x} - \frac{a^6}{2\,(a+b\,x)^2} + 15\,a^4\,\ln\left(a + b\,x\right) - 20\,a^3\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^4/4 - 2*a*(a + b*x)^3 + (15*a^2*(a + b*x)^2)/2 + (6*a^5)/(a + b*
x) - a^6/(2*(a + b*x)^2) + 15*a^4*log(a + b*x) - 20*a^3*b*x)/b^7
```

sympy [A] time = 0.40, size = 92, normalized size = 1.07

$$\frac{15a^4\log\left(a+bx\right)}{b^7}-\frac{10a^3x}{b^6}+\frac{3a^2x^2}{b^5}-\frac{ax^3}{b^4}+\frac{11a^6+12a^5bx}{2a^2b^7+4ab^8x+2b^9x^2}+\frac{x^4}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**3,x)
```

```
[Out] 15*a**4*log(a + b*x)/b**7 - 10*a**3*x/b**6 + 3*a**2*x**2/b**5 - a*x**3/b**4
 + (11*a**6 + 12*a**5*b*x)/(2*a**2*b**7 + 4*a*b**8*x + 2*b**9*x**2) + x**4/
(4*b**3)
```

3.182 $\int \frac{x^5}{(a+bx)^3} \, dx$

Optimal. Leaf size=77

$$\frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6} + \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3}$$

Rubi [A] time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$\frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} + \frac{6a^2x}{b^5} - \frac{10a^3\log(a+bx)}{b^6} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^3,x]
```

```
[Out] (6*a^2*x)/b^5 - (3*a*x^2)/(2*b^4) + x^3/(3*b^3) + a^5/(2*b^6*(a + b*x)^2) - (5*a^4)/(b^6*(a + b*x)) - (10*a^3*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^3} \, dx &= \int \left(\frac{6a^2}{b^5} - \frac{3ax}{b^4} + \frac{x^2}{b^3} - \frac{a^5}{b^5(a+bx)^3} + \frac{5a^4}{b^5(a+bx)^2} - \frac{10a^3}{b^5(a+bx)}\right) dx \\ &= \frac{6a^2x}{b^5} - \frac{3ax^2}{2b^4} + \frac{x^3}{3b^3} + \frac{a^5}{2b^6(a+bx)^2} - \frac{5a^4}{b^6(a+bx)} - \frac{10a^3\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 0.87

$$\frac{\frac{3a^5}{(a+bx)^2} - \frac{30a^4}{a+bx} - 60a^3\log(a+bx) + 36a^2bx - 9ab^2x^2 + 2b^3x^3}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^3,x]
```

```
[Out] (36*a^2*b*x - 9*a*b^2*x^2 + 2*b^3*x^3 + (3*a^5)/(a + b*x)^2 - (30*a^4)/(a + b*x) - 60*a^3*Log[a + b*x])/(6*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^3, x]
```

fricas [A] time = 1.12, size = 107, normalized size = 1.39

$$\frac{2\,b^5x^5 - 5\,ab^4x^4 + 20\,a^2b^3x^3 + 63\,a^3b^2x^2 + 6\,a^4bx - 27\,a^5 - 60\left(a^3b^2x^2 + 2\,a^4bx + a^5\right)\log\left(bx + a\right)}{6\left(b^8x^2 + 2\,ab^7x + a^2b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*x^5 - 5*a*b^4*x^4 + 20*a^2*b^3*x^3 + 63*a^3*b^2*x^2 + 6*a^4*b*x
- 27*a^5 - 60*(a^3*b^2*x^2 + 2*a^4*b*x + a^5)*log(b*x + a))/(b^8*x^2 + 2*a*
b^7*x + a^2*b^6)
```

giac [A] time = 1.12, size = 73, normalized size = 0.95

$$-\frac{10\,a^3\log\left(|bx + a|\right)}{b^6} - \frac{10\,a^4bx + 9\,a^5}{2\,(bx + a)^2b^6} + \frac{2\,b^6x^3 - 9\,ab^5x^2 + 36\,a^2b^4x}{6\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -10*a^3*log(abs(b*x + a))/b^6 - 1/2*(10*a^4*b*x + 9*a^5)/((b*x + a)^2*b^6)
+ 1/6*(2*b^6*x^3 - 9*a*b^5*x^2 + 36*a^2*b^4*x)/b^9
```

maple [A] time = 0.01, size = 72, normalized size = 0.94

$$\frac{x^3}{3b^3} + \frac{a^5}{2\,(bx + a)^2\,b^6} - \frac{3a\,x^2}{2b^4} - \frac{5a^4}{(bx + a)\,b^6} - \frac{10a^3\ln\left(bx + a\right)}{b^6} + \frac{6a^2x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^3,x)
```

```
[Out] 6*a^2*x/b^5-3/2*a*x^2/b^4+1/3*x^3/b^3+1/2*a^5/b^6/(b*x+a)^2-5*a^4/b^6/(b*x+
a)-10*a^3*ln(b*x+a)/b^6
```

maxima [A] time = 1.36, size = 81, normalized size = 1.05

$$-\frac{10\,a^4bx + 9\,a^5}{2\left(b^8x^2 + 2\,ab^7x + a^2b^6\right)} - \frac{10\,a^3\log\left(bx + a\right)}{b^6} + \frac{2\,b^2x^3 - 9\,abx^2 + 36\,a^2x}{6\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(10*a^4*b*x + 9*a^5)/(b^8*x^2 + 2*a*b^7*x + a^2*b^6) - 10*a^3*log(b*x
+ a)/b^6 + 1/6*(2*b^2*x^3 - 9*a*b*x^2 + 36*a^2*x)/b^5
```

mupad [B] time = 0.12, size = 67, normalized size = 0.87

$$-\frac{\frac{5\,a\,(a+b\,x)^2}{2} - \frac{(a+b\,x)^3}{3} + \frac{5\,a^4}{a+b\,x} - \frac{a^5}{2\,(a+b\,x)^2} + 10\,a^3\,\ln\left(a + b\,x\right) - 10\,a^2\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^3,x)
```

```
[Out] -((5*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (5*a^4)/(a + b*x) - a^5/(2*(a + b*x
)^2) + 10*a^3*log(a + b*x) - 10*a^2*b*x)/b^6
```

sympy [**A**] time = 0.36, size = 85, normalized size = 1.10

$$-\frac{10a^3\log(a+bx)}{b^6}+\frac{6a^2x}{b^5}-\frac{3ax^2}{2b^4}+\frac{-9a^5-10a^4bx}{2a^2b^6+4ab^7x+2b^8x^2}+\frac{x^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**3,x)
```

```
[Out] -10*a**3*log(a + b*x)/b**6 + 6*a**2*x/b**5 - 3*a*x**2/(2*b**4) + (-9*a**5 -
 10*a**4*b*x)/(2*a**2*b**6 + 4*a*b**7*x + 2*b**8*x**2) + x**3/(3*b**3)
```

3.183 $\int \frac{x^4}{(a+bx)^3} \, dx$

Optimal. Leaf size=64

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Rubi [A] time = 0.04, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}-\frac{3ax}{b^4}+\frac{x^2}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^3,x]
```

```
[Out] (-3*a*x)/b^4 + x^2/(2*b^3) - a^4/(2*b^5*(a + b*x)^2) + (4*a^3)/(b^5*(a + b*
x)) + (6*a^2*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^3}\,dx &= \int \left(-\frac{3a}{b^4}+\frac{x}{b^3}+\frac{a^4}{b^4(a+bx)^3}-\frac{4a^3}{b^4(a+bx)^2}+\frac{6a^2}{b^4(a+bx)}\right)dx\\ &= -\frac{3ax}{b^4}+\frac{x^2}{2b^3}-\frac{a^4}{2b^5(a+bx)^2}+\frac{4a^3}{b^5(a+bx)}+\frac{6a^2\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.86

$$\frac{-\frac{a^4}{(a+bx)^2}+\frac{8a^3}{a+bx}+12a^2\log(a+bx)-6abx+b^2x^2}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^3,x]
```

```
[Out] (-6*a*b*x + b^2*x^2 - a^4/(a + b*x)^2 + (8*a^3)/(a + b*x) + 12*a^2*Log[a +
b*x])/(2*b^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^3, x]
```

fricas [A] time = 0.88, size = 95, normalized size = 1.48

$$\frac{b^4x^4 - 4\,ab^3x^3 - 11\,a^2b^2x^2 + 2\,a^3bx + 7\,a^4 + 12\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)\log\left(bx + a\right)}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*x^4 - 4*a*b^3*x^3 - 11*a^2*b^2*x^2 + 2*a^3*b*x + 7*a^4 + 12*(a^2*b
^2*x^2 + 2*a^3*b*x + a^4)*log(b*x + a))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

giac [A] time = 0.95, size = 61, normalized size = 0.95

$$\frac{6\,a^2\log\left(|bx + a|\right)}{b^5} + \frac{b^3x^2 - 6\,ab^2x}{2\,b^6} + \frac{8\,a^3bx + 7\,a^4}{2\,(bx + a)^2b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 6*a^2*log(abs(b*x + a))/b^5 + 1/2*(b^3*x^2 - 6*a*b^2*x)/b^6 + 1/2*(8*a^3*b*
x + 7*a^4)/((b*x + a)^2*b^5)
```

maple [A] time = 0.01, size = 61, normalized size = 0.95

$$-\frac{a^4}{2\,(bx + a)^2\,b^5} + \frac{x^2}{2b^3} + \frac{4a^3}{(bx + a)\,b^5} + \frac{6a^2\ln\left(bx + a\right)}{b^5} - \frac{3ax}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^3,x)
```

```
[Out] -3*a*x/b^4+1/2*x^2/b^3-1/2*a^4/b^5/(b*x+a)^2+4*a^3/b^5/(b*x+a)+6*a^2*ln(b*x
+a)/b^5
```

maxima [A] time = 1.33, size = 69, normalized size = 1.08

$$\frac{8\,a^3bx + 7\,a^4}{2\left(b^7x^2 + 2\,ab^6x + a^2b^5\right)} + \frac{6\,a^2\log\left(bx + a\right)}{b^5} + \frac{bx^2 - 6\,ax}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*b*x + 7*a^4)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5) + 6*a^2*log(b*x + a
)/b^5 + 1/2*(b*x^2 - 6*a*x)/b^4
```

mupad [B] time = 0.08, size = 54, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^2}{2} + \frac{4\,a^3}{a+b\,x} - \frac{a^4}{2\,(a+b\,x)^2} + 6\,a^2\,\ln\left(a + b\,x\right) - 4\,a\,b\,x}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^3,x)
```

```
[Out] ((a + b*x)^2/2 + (4*a^3)/(a + b*x) - a^4/(2*(a + b*x)^2) + 6*a^2*log(a + b*
x) - 4*a*b*x)/b^5
```

sympy [A] time = 0.34, size = 70, normalized size = 1.09

$$\frac{6a^2\log{(a+bx)}}{b^5}-\frac{3ax}{b^4}+\frac{7a^4+8a^3bx}{2a^2b^5+4ab^6x+2b^7x^2}+\frac{x^2}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**3,x)
```

```
[Out] 6*a**2*log(a + b*x)/b**5 - 3*a*x/b**4 + (7*a**4 + 8*a**3*b*x)/(2*a**2*b**5
+ 4*a*b**6*x + 2*b**7*x**2) + x**2/(2*b**3)
```

3.184 $\int \frac{x^3}{(a+bx)^3} \, dx$

Optimal. Leaf size=50

$$\frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$\frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4} + \frac{x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^3,x]
```

```
[Out] x/b^3 + a^3/(2*b^4*(a + b*x)^2) - (3*a^2)/(b^4*(a + b*x)) - (3*a*Log[a + b*
x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a+bx)^3} \, dx &= \int \left(\frac{1}{b^3} - \frac{a^3}{b^3(a+bx)^3} + \frac{3a^2}{b^3(a+bx)^2} - \frac{3a}{b^3(a+bx)}\right) dx \\
&= \frac{x}{b^3} + \frac{a^3}{2b^4(a+bx)^2} - \frac{3a^2}{b^4(a+bx)} - \frac{3a\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 40, normalized size = 0.80

$$-\frac{\frac{a^2(5a+6bx)}{(a+bx)^2} + 6a\log(a+bx) - 2bx}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^3,x]
```

```
[Out] -1/2*(-2*b*x + (a^2*(5*a + 6*b*x))/(a + b*x)^2 + 6*a*Log[a + b*x])/b^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^3, x]
```

fricas [A] time = 0.84, size = 83, normalized size = 1.66

$$\frac{2\,b^3x^3+4\,ab^2x^2-4\,a^2bx-5\,a^3-6\left(ab^2x^2+2\,a^2bx+a^3\right)\log\left(bx+a\right)}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*x^3 + 4*a*b^2*x^2 - 4*a^2*b*x - 5*a^3 - 6*(a*b^2*x^2 + 2*a^2*b*x
 + a^3)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

giac [A] time = 0.96, size = 44, normalized size = 0.88

$$\frac{x}{b^3}-\frac{3\,a\log\left(|bx+a|\right)}{b^4}-\frac{6\,a^2bx+5\,a^3}{2\,(bx+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] x/b^3 - 3*a*log(abs(b*x + a))/b^4 - 1/2*(6*a^2*b*x + 5*a^3)/((b*x + a)^2*b^
4)
```

maple [A] time = 0.01, size = 49, normalized size = 0.98

$$\frac{a^3}{2\,(bx+a)^2\,b^4}-\frac{3a^2}{(bx+a)\,b^4}-\frac{3a\ln\left(bx+a\right)}{b^4}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^3,x)
```

```
[Out] x/b^3+1/2*a^3/b^4/(b*x+a)^2-3*a^2/b^4/(b*x+a)-3*a*ln(b*x+a)/b^4
```

maxima [A] time = 1.31, size = 57, normalized size = 1.14

$$-\frac{6\,a^2bx+5\,a^3}{2\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{x}{b^3}-\frac{3\,a\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*a^2*b*x + 5*a^3)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) + x/b^3 - 3*a*log(
b*x + a)/b^4
```

mupad [B] time = 0.15, size = 43, normalized size = 0.86

$$-\frac{3\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{3\,a^2}{a+b\,x}-\frac{a^3}{2\,{\left(a+b\,x\right)}^2}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^3,x)
```

```
[Out] -(3*a*log(a + b*x) - b*x + (3*a^2)/(a + b*x) - a^3/(2*(a + b*x)^2))/b^4
```

sympy [A] time = 0.31, size = 58, normalized size = 1.16

$$-\frac{3a\log\left(a+bx\right)}{b^4}+\frac{-5a^3-6a^2bx}{2a^2b^4+4ab^5x+2b^6x^2}+\frac{x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**3,x)
```

```
[Out] -3*a*log(a + b*x)/b**4 + (-5*a**3 - 6*a**2*b*x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + x/b**3
```

3.185 $\int \frac{x^2}{(a+bx)^3} dx$

Optimal. Leaf size=41

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^3,x]
```

```
[Out] -a^2/(2*b^3*(a + b*x)^2) + (2*a)/(b^3*(a + b*x)) + Log[a + b*x]/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^3}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^3}-\frac{2a}{b^2(a+bx)^2}+\frac{1}{b^2(a+bx)}\right)dx\\ &= -\frac{a^2}{2b^3(a+bx)^2}+\frac{2a}{b^3(a+bx)}+\frac{\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.80

$$\frac{\frac{a(3a+4bx)}{(a+bx)^2}+2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^3,x]
```

```
[Out] ((a*(3*a + 4*b*x))/(a + b*x)^2 + 2*Log[a + b*x])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^3, x]
```

fricas [A] time = 0.65, size = 61, normalized size = 1.49

$$\frac{4\,abx + 3\,a^2 + 2\left(b^2x^2 + 2\,abx + a^2\right)\log\left(bx + a\right)}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a))/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3)
```

giac [A] time = 1.08, size = 37, normalized size = 0.90

$$\frac{\log\left(|bx + a|\right)}{b^3} + \frac{4\,ax + \frac{3\,a^2}{b}}{2\,(bx + a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^3 + 1/2*(4*a*x + 3*a^2/b)/((b*x + a)^2*b^2)
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$-\frac{a^2}{2\,(bx + a)^2\,b^3} + \frac{2a}{(bx + a)\,b^3} + \frac{\ln\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^3,x)
```

```
[Out] -1/2*a^2/b^3/(b*x+a)^2+2*a/b^3/(b*x+a)+ln(b*x+a)/b^3
```

maxima [A] time = 1.34, size = 48, normalized size = 1.17

$$\frac{4\,abx + 3\,a^2}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} + \frac{\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*x + 3*a^2)/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) + log(b*x + a)/b^3
```

mupad [B] time = 0.09, size = 46, normalized size = 1.12

$$\frac{\ln\left(a + b\,x\right)}{b^3} + \frac{\frac{3\,a^2}{2\,b^3} + \frac{2\,a\,x}{b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^3,x)
```

```
[Out] log(a + b*x)/b^3 + ((3*a^2)/(2*b^3) + (2*a*x)/b^2)/(a^2 + b^2*x^2 + 2*a*b*x
)
```

sympy [A] time = 0.25, size = 46, normalized size = 1.12

$$\frac{3a^2 + 4abx}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2 + 4*a*b*x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + log(a + b*x)/
b**3
```

3.186 $\int \frac{x}{(a+bx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2a(a+bx)^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {37}

$$\frac{x^2}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x)^3,x]

[Out] x^2/(2*a*(a + b*x)^2)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x}{(a+bx)^3}\,dx = \frac{x^2}{2a(a+bx)^2}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.18

$$-\frac{a+2bx}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x)^3,x]

[Out] -1/2*(a + 2*b*x)/(b^2*(a + b*x)^2)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(a + b*x)^3,x]

[Out] IntegrateAlgebraic[x/(a + b*x)^3, x]

fricas [B] time = 1.21, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

giac [A] time = 1.14, size = 18, normalized size = 1.06

$$-\frac{2\,bx+a}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x + a)/((b*x + a)^2*b^2)
```

maple [A] time = 0.00, size = 27, normalized size = 1.59

$$\frac{a}{2\,(bx+a)^2\,b^2}-\frac{1}{(bx+a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^3,x)
```

```
[Out] 1/2*a/b^2/(b*x+a)^2-1/b^2/(b*x+a)
```

maxima [B] time = 1.39, size = 32, normalized size = 1.88

$$-\frac{2\,bx+a}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

mupad [B] time = 0.07, size = 32, normalized size = 1.88

$$-\frac{\frac{a}{2\,b^2}+\frac{x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^3,x)
```

```
[Out] -(a/(2*b^2) + x/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

sympy [B] time = 0.20, size = 32, normalized size = 1.88

$$\frac{-a-2bx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**3,x)
```

```
[Out] (-a - 2*b*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

3.187 $\int \frac{1}{(a+bx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2b(a+bx)^2}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3),x]
```

```
[Out] -1/(2*b*(a + b*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3}\,dx = -\frac{1}{2b(a+bx)^2}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3),x]
```

```
[Out] -1/2*1/(b*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-3), x]
```

fricas [A] time = 1.01, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(b^3x^2 + 2\,ab^2x + a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [A] time = 0.95, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(bx+a)^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3,x)
[Out] -1/2/b/(b*x+a)^2
```

maxima [A] time = 1.29, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(bx+a)^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3,x, algorithm="maxima")
[Out] -1/2/((b*x + a)^2*b)
```

mupad [B] time = 0.07, size = 26, normalized size = 1.86

$$-\frac{1}{2\,a^2\,b+4\,a\,b^2\,x+2\,b^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^3,x)
[Out] -1/(2*a^2*b + 2*b^3*x^2 + 4*a*b^2*x)
```

sympy [B] time = 0.21, size = 26, normalized size = 1.86

$$-\frac{1}{2a^2b+4ab^2x+2b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3,x)
[Out] -1/(2*a**2*b + 4*a*b**2*x + 2*b**3*x**2)
```

3.188 $\int \frac{1}{x(a+bx)^3}\,dx$

Optimal. Leaf size=43

$$-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{a^2(a+bx)}+\frac{1}{2a(a+bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^2(a+bx)}-\frac{\log(a+bx)}{a^3}+\frac{\log(x)}{a^3}+\frac{1}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^3),x]
```

```
[Out] 1/(2*a*(a + b*x)^2) + 1/(a^2*(a + b*x)) + Log[x]/a^3 - Log[a + b*x]/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x}-\frac{b}{a(a+bx)^3}-\frac{b}{a^2(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx\\ &= \frac{1}{2a(a+bx)^2}+\frac{1}{a^2(a+bx)}+\frac{\log(x)}{a^3}-\frac{\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.86

$$\frac{\frac{a(3a+2bx)}{(a+bx)^2}-2\log(a+bx)+2\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^3),x]
```

```
[Out] ((a*(3*a + 2*b*x))/(a + b*x)^2 + 2*Log[x] - 2*Log[a + b*x])/(2*a^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^3), x]
```

fricas [A] time = 0.87, size = 80, normalized size = 1.86

$$\frac{2\,abx + 3\,a^2 - 2\left(b^2x^2 + 2\,abx + a^2\right)\log\left(bx + a\right) + 2\left(b^2x^2 + 2\,abx + a^2\right)\log(x)}{2\left(a^3b^2x^2 + 2\,a^4bx + a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*b*x + 3*a^2 - 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a) + 2*(b^2*x^
2 + 2*a*b*x + a^2)*log(x))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)
```

giac [A] time = 1.03, size = 43, normalized size = 1.00

$$-\frac{\log\left(|bx + a|\right)}{a^3} + \frac{\log\left(|x|\right)}{a^3} + \frac{2\,abx + 3\,a^2}{2\left(bx + a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^3 + log(abs(x))/a^3 + 1/2*(2*a*b*x + 3*a^2)/((b*x + a)
^2*a^3)
```

maple [A] time = 0.01, size = 42, normalized size = 0.98

$$\frac{1}{2\left(bx + a\right)^2a} + \frac{1}{\left(bx + a\right)a^2} + \frac{\ln(x)}{a^3} - \frac{\ln\left(bx + a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^3,x)
```

```
[Out] 1/2/a/(b*x+a)^2+1/a^2/(b*x+a)+ln(x)/a^3-ln(b*x+a)/a^3
```

maxima [A] time = 1.35, size = 51, normalized size = 1.19

$$\frac{2\,bx + 3\,a}{2\left(a^2b^2x^2 + 2\,a^3bx + a^4\right)} - \frac{\log\left(bx + a\right)}{a^3} + \frac{\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*x + 3*a)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) - log(b*x + a)/a^3 + log(
x)/a^3
```

mupad [B] time = 0.10, size = 43, normalized size = 1.00

$$\frac{\frac{1}{a^2+b\,x\,a} - \frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a} + \frac{1}{2\,a\,\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^3),x)
```

```
[Out] (1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2)
```

sympy [A] time = 0.35, size = 46, normalized size = 1.07

$$\frac{3a + 2bx}{2a^4 + 4a^3bx + 2a^2b^2x^2} + \frac{\log\left(x\right) - \log\left(\frac{a}{b} + x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**3,x)
```

```
[Out] (3*a + 2*b*x)/(2*a**4 + 4*a**3*b*x + 2*a**2*b**2*x**2) + (log(x) - log(a/b
+ x))/a**3
```

3.189 $\int \frac{1}{x^2(a+bx)^3}\,dx$

Optimal. Leaf size=57

$$-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{2b}{a^3(a+bx)}-\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{2b}{a^3(a+bx)}-\frac{b}{2a^2(a+bx)^2}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}-\frac{1}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^3),x]

[Out] -(1/(a^3*x)) - b/(2*a^2*(a + b*x)^2) - (2*b)/(a^3*(a + b*x)) - (3*b*Log[x])
/a^4 + (3*b*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^2}-\frac{3b}{a^4x}+\frac{b^2}{a^2(a+bx)^3}+\frac{2b^2}{a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{a^3x}-\frac{b}{2a^2(a+bx)^2}-\frac{2b}{a^3(a+bx)}-\frac{3b\log(x)}{a^4}+\frac{3b\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.93

$$-\frac{\frac{a(2a^2+9abx+6b^2x^2)}{x(a+bx)^2}-6b\log(a+bx)+6b\log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^3),x]

[Out] -1/2*((a*(2*a^2 + 9*a*b*x + 6*b^2*x^2))/(x*(a + b*x)^2) + 6*b*Log[x] - 6*b*
Log[a + b*x])/a^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^3),x]

[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^3), x]
```

fricas [A] time = 1.15, size = 109, normalized size = 1.91

$$-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3-6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log\left(bx+a\right)+6\left(b^3x^3+2\,ab^2x^2+a^2bx\right)\log(x)}{2\left(a^4b^2x^3+2\,a^5bx^2+a^6x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(6*a*b^2*x^2 + 9*a^2*b*x + 2*a^3 - 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)
*log(b*x + a) + 6*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*log(x))/(a^4*b^2*x^3 +
2*a^5*b*x^2 + a^6*x)
```

giac [A] time = 1.07, size = 60, normalized size = 1.05

$$\frac{3\,b\log\left(|bx+a|\right)}{a^4}-\frac{3\,b\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2+9\,a^2bx+2\,a^3}{2\left(bx+a\right)^2a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3*b*log(abs(b*x + a))/a^4 - 3*b*log(abs(x))/a^4 - 1/2*(6*a*b^2*x^2 + 9*a^2*
b*x + 2*a^3)/((b*x + a)^2*a^4*x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.98

$$-\frac{b}{2\left(bx+a\right)^2a^2}-\frac{2b}{\left(bx+a\right)a^3}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln\left(bx+a\right)}{a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^3,x)
```

```
[Out] -1/a^3/x-1/2*b/a^2/(b*x+a)^2-2*b/a^3/(b*x+a)-3*b*ln(x)/a^4+3*b*ln(b*x+a)/a^
4
```

maxima [A] time = 1.32, size = 69, normalized size = 1.21

$$-\frac{6\,b^2x^2+9\,abx+2\,a^2}{2\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}+\frac{3\,b\log\left(bx+a\right)}{a^4}-\frac{3\,b\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(6*b^2*x^2 + 9*a*b*x + 2*a^2)/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x) + 3*
b*log(b*x + a)/a^4 - 3*b*log(x)/a^4
```

mupad [B] time = 0.11, size = 63, normalized size = 1.11

$$\frac{6\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{1}{a}+\frac{3\,b^2\,x^2}{a^3}+\frac{9\,b\,x}{2\,a^2}}{a^2\,x+2\,a\,b\,x^2+b^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^3),x)
```

```
[Out] (6*b*atanh((2*b*x)/a + 1))/a^4 - (1/a + (3*b^2*x^2)/a^3 + (9*b*x)/(2*a^2))/
(a^2*x + b^2*x^3 + 2*a*b*x^2)
```

sympy [A] time = 0.40, size = 66, normalized size = 1.16

$$\frac{-2a^2 - 9abx - 6b^2x^2}{2a^5x + 4a^4bx^2 + 2a^3b^2x^3} + \frac{3b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**3,x)
```

```
[Out] (-2*a**2 - 9*a*b*x - 6*b**2*x**2)/(2*a**5*x + 4*a**4*b*x**2 + 2*a**3*b**2*x
**3) + 3*b*(-log(x) + log(a/b + x))/a**4
```

3.190 $\int \frac{1}{x^3(a+bx)^3} \, dx$

Optimal. Leaf size=76

$$\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b^2}{a^4(a+bx)}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}-\frac{1}{2a^3x^2}$$

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3b^2}{a^4(a+bx)}+\frac{b^2}{2a^3(a+bx)^2}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}+\frac{3b}{a^4x}-\frac{1}{2a^3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^3),x]
```

```
[Out] -1/(2*a^3*x^2) + (3*b)/(a^4*x) + b^2/(2*a^3*(a + b*x)^2) + (3*b^2)/(a^4*(a
+ b*x)) + (6*b^2*Log[x])/a^5 - (6*b^2*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^3} \, dx &= \int \left(\frac{1}{a^3x^3}-\frac{3b}{a^4x^2}+\frac{6b^2}{a^5x}-\frac{b^3}{a^3(a+bx)^3}-\frac{3b^3}{a^4(a+bx)^2}-\frac{6b^3}{a^5(a+bx)}\right) dx\\ &= -\frac{1}{2a^3x^2}+\frac{3b}{a^4x}+\frac{b^2}{2a^3(a+bx)^2}+\frac{3b^2}{a^4(a+bx)}+\frac{6b^2\log(x)}{a^5}-\frac{6b^2\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.89

$$\frac{\frac{a\left(-a^3+4a^2bx+18ab^2x^2+12b^3x^3\right)}{x^2(a+bx)^2}-12b^2\log(a+bx)+12b^2\log(x)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^3 + 4*a^2*b*x + 18*a*b^2*x^2 + 12*b^3*x^3))/(x^2*(a + b*x)^2) + 12*
b^2*Log[x] - 12*b^2*Log[a + b*x])/(2*a^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^3), x]
```

fricas [A] time = 1.18, size = 130, normalized size = 1.71

$$\frac{12\,ab^3x^3+18\,a^2b^2x^2+4\,a^3bx-a^4-12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log\left(bx+a\right)+12\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\log(x)}{2\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4 - 12*(b^4*x^4 + 2*a*b^
3*x^3 + a^2*b^2*x^2)*log(b*x + a) + 12*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2
)*log(x))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

giac [A] time = 1.39, size = 73, normalized size = 0.96

$$-\frac{6\,b^2\log\left(|bx+a|\right)}{a^5}+\frac{6\,b^2\log\left(|x|\right)}{a^5}+\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(bx^2+ax\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -6*b^2*log(abs(b*x + a))/a^5 + 6*b^2*log(abs(x))/a^5 + 1/2*(12*b^3*x^3 + 18
*a*b^2*x^2 + 4*a^2*b*x - a^3)/((b*x^2 + a*x)^2*a^4)
```

maple [A] time = 0.01, size = 73, normalized size = 0.96

$$\frac{b^2}{2\left(bx+a\right)^2a^3}+\frac{3b^2}{\left(bx+a\right)a^4}+\frac{6b^2\ln(x)}{a^5}-\frac{6b^2\ln\left(bx+a\right)}{a^5}+\frac{3b}{a^4x}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^3,x)
```

```
[Out] -1/2/a^3/x^2+3*b/a^4/x+1/2*b^2/a^3/(b*x+a)^2+3*b^2/a^4/(b*x+a)+6*b^2*ln(x)/
a^5-6*b^2*ln(b*x+a)/a^5
```

maxima [A] time = 1.37, size = 86, normalized size = 1.13

$$\frac{12\,b^3x^3+18\,ab^2x^2+4\,a^2bx-a^3}{2\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}-\frac{6\,b^2\log\left(bx+a\right)}{a^5}+\frac{6\,b^2\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*b^3*x^3 + 18*a*b^2*x^2 + 4*a^2*b*x - a^3)/(a^4*b^2*x^4 + 2*a^5*b*x^
3 + a^6*x^2) - 6*b^2*log(b*x + a)/a^5 + 6*b^2*log(x)/a^5
```

mupad [B] time = 0.12, size = 79, normalized size = 1.04

$$\frac{\frac{9\,b^2\,x^2}{a^3}-\frac{1}{2\,a}+\frac{6\,b^3\,x^3}{a^4}+\frac{2\,b\,x}{a^2}}{a^2\,x^2+2\,a\,b\,x^3+b^2\,x^4}-\frac{12\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^3),x)
```

```
[Out] ((9*b^2*x^2)/a^3 - 1/(2*a) + (6*b^3*x^3)/a^4 + (2*b*x)/a^2)/(a^2*x^2 + b^2*
x^4 + 2*a*b*x^3) - (12*b^2*atanh((2*b*x)/a + 1))/a^5
```

sympy [A] time = 0.41, size = 78, normalized size = 1.03

$$\frac{-a^3+4a^2bx+18ab^2x^2+12b^3x^3}{2a^6x^2+4a^5bx^3+2a^4b^2x^4}+\frac{6b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**3,x)
```

```
[Out] (-a**3 + 4*a**2*b*x + 18*a*b**2*x**2 + 12*b**3*x**3)/(2*a**6*x**2 + 4*a**5*
b*x**3 + 2*a**4*b**2*x**4) + 6*b**2*(log(x) - log(a/b + x))/a**5
```

3.191 $\int \frac{1}{x^4(a+bx)^3}\,dx$

Optimal. Leaf size=89

$$-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}-\frac{4b^3}{a^5(a+bx)}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{4b^3}{a^5(a+bx)}-\frac{b^3}{2a^4(a+bx)^2}-\frac{6b^2}{a^5x}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/(3*a^3*x^3) + (3*b)/(2*a^4*x^2) - (6*b^2)/(a^5*x) - b^3/(2*a^4*(a + b*x)
^2) - (4*b^3)/(a^5*(a + b*x)) - (10*b^3*Log[x])/a^6 + (10*b^3*Log[a + b*x])
/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^4}-\frac{3b}{a^4x^3}+\frac{6b^2}{a^5x^2}-\frac{10b^3}{a^6x}+\frac{b^4}{a^4(a+bx)^3}+\frac{4b^4}{a^5(a+bx)^2}+\frac{10b^4}{a^6(a+bx)}\right)dx\\ &= -\frac{1}{3a^3x^3}+\frac{3b}{2a^4x^2}-\frac{6b^2}{a^5x}-\frac{b^3}{2a^4(a+bx)^2}-\frac{4b^3}{a^5(a+bx)}-\frac{10b^3\log(x)}{a^6}+\frac{10b^3\log(a+bx)}{a^6}\end{aligned}$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.89

$$-\frac{\frac{a\left(2a^4-5a^3bx+20a^2b^2x^2+90ab^3x^3+60b^4x^4\right)}{x^3(a+bx)^2}-60b^3\log(a+bx)+60b^3\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^3),x]
```

```
[Out] -1/6*((a*(2*a^4 - 5*a^3*b*x + 20*a^2*b^2*x^2 + 90*a*b^3*x^3 + 60*b^4*x^4))/
(x^3*(a + b*x)^2) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a^6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^3), x]
```

fricas [A] time = 1.10, size = 141, normalized size = 1.58

$$-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5-60\,\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\log\left(bx+a\right)+60\,\left(b^5x^5+2\,ab^4x^4+a^2b^3x^3\right)\log(x)}{6\,\left(a^6b^2x^5+2\,a^7bx^4+a^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(60*a*b^4*x^4 + 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5 -
60*(b^5*x^5 + 2*a*b^4*x^4 + a^2*b^3*x^3)*log(b*x + a) + 60*(b^5*x^5 + 2*a*b
^4*x^4 + a^2*b^3*x^3)*log(x))/(a^6*b^2*x^5 + 2*a^7*b*x^4 + a^8*x^3)
```

giac [A] time = 0.94, size = 86, normalized size = 0.97

$$\frac{10\,b^3\log\left(|bx+a|\right)}{a^6}-\frac{10\,b^3\log\left(|x|\right)}{a^6}-\frac{60\,ab^4x^4+90\,a^2b^3x^3+20\,a^3b^2x^2-5\,a^4bx+2\,a^5}{6\,(bx+a)^2a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 10*b^3*log(abs(b*x + a))/a^6 - 10*b^3*log(abs(x))/a^6 - 1/6*(60*a*b^4*x^4 +
 90*a^2*b^3*x^3 + 20*a^3*b^2*x^2 - 5*a^4*b*x + 2*a^5)/((b*x + a)^2*a^6*x^3)
```

maple [A] time = 0.01, size = 84, normalized size = 0.94

$$-\frac{b^3}{2\,(bx+a)^2\,a^4}-\frac{4b^3}{(bx+a)\,a^5}-\frac{10b^3\ln(x)}{a^6}+\frac{10b^3\ln\left(bx+a\right)}{a^6}-\frac{6b^2}{a^5x}+\frac{3b}{2a^4x^2}-\frac{1}{3a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^3,x)
```

```
[Out] -1/3/a^3/x^3+3/2*b/a^4/x^2-6*b^2/a^5/x-1/2*b^3/a^4/(b*x+a)^2-4*b^3/a^5/(b*x
+a)-10*b^3*ln(x)/a^6+10*b^3*ln(b*x+a)/a^6
```

maxima [A] time = 1.46, size = 97, normalized size = 1.09

$$-\frac{60\,b^4x^4+90\,ab^3x^3+20\,a^2b^2x^2-5\,a^3bx+2\,a^4}{6\,\left(a^5b^2x^5+2\,a^6bx^4+a^7x^3\right)}+\frac{10\,b^3\log\left(bx+a\right)}{a^6}-\frac{10\,b^3\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(60*b^4*x^4 + 90*a*b^3*x^3 + 20*a^2*b^2*x^2 - 5*a^3*b*x + 2*a^4)/(a^5*
b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3) + 10*b^3*log(b*x + a)/a^6 - 10*b^3*log(x)/
a^6
```

mupad [B] time = 0.13, size = 91, normalized size = 1.02

$$\frac{20\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^6}-\frac{\frac{1}{3\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{15\,b^3\,x^3}{a^4}+\frac{10\,b^4\,x^4}{a^5}-\frac{5\,b\,x}{6\,a^2}}{a^2\,x^3+2\,a\,b\,x^4+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^3),x)
```

```
[Out] (20*b^3*atanh((2*b*x)/a + 1))/a^6 - (1/(3*a) + (10*b^2*x^2)/(3*a^3) + (15*b
^3*x^3)/a^4 + (10*b^4*x^4)/a^5 - (5*b*x)/(6*a^2))/(a^2*x^3 + b^2*x^5 + 2*a*
b*x^4)
```

sympy [A] time = 0.48, size = 92, normalized size = 1.03

$$\frac{-2a^4 + 5a^3bx - 20a^2b^2x^2 - 90ab^3x^3 - 60b^4x^4}{6a^7x^3 + 12a^6bx^4 + 6a^5b^2x^5} + \frac{10b^3\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**3,x)
```

```
[Out] (-2*a**4 + 5*a**3*b*x - 20*a**2*b**2*x**2 - 90*a*b**3*x**3 - 60*b**4*x**4)/
(6*a**7*x**3 + 12*a**6*b*x**4 + 6*a**5*b**2*x**5) + 10*b**3*(-log(x) + log(
a/b + x))/a**6
```

3.192 $\int \frac{1}{x^5(a+bx)^3}\,dx$

Optimal. Leaf size=97

$$\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{5b^4}{a^6(a+bx)}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}-\frac{3b^2}{a^5x^2}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Rubi [A] time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b^2}{a^5x^2}+\frac{5b^4}{a^6(a+bx)}+\frac{b^4}{2a^5(a+bx)^2}+\frac{10b^3}{a^6x}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx)}{a^7}+\frac{b}{a^4x^3}-\frac{1}{4a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^3),x]
```

```
[Out] -1/(4*a^3*x^4) + b/(a^4*x^3) - (3*b^2)/(a^5*x^2) + (10*b^3)/(a^6*x) + b^4/(
2*a^5*(a + b*x)^2) + (5*b^4)/(a^6*(a + b*x)) + (15*b^4*Log[x])/a^7 - (15*b^
4*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^5}-\frac{3b}{a^4x^4}+\frac{6b^2}{a^5x^3}-\frac{10b^3}{a^6x^2}+\frac{15b^4}{a^7x}-\frac{b^5}{a^5(a+bx)^3}-\frac{5b^5}{a^6(a+bx)^2}-\frac{15b^5}{a^7(a+bx)}\right)dx\\ &= -\frac{1}{4a^3x^4}+\frac{b}{a^4x^3}-\frac{3b^2}{a^5x^2}+\frac{10b^3}{a^6x}+\frac{b^4}{2a^5(a+bx)^2}+\frac{5b^4}{a^6(a+bx)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+b}{a^7}\end{aligned}$$

Mathematica [A] time = 0.06, size = 90, normalized size = 0.93

$$\frac{\frac{a\left(-a^5+2a^4bx-5a^3b^2x^2+20a^2b^3x^3+90ab^4x^4+60b^5x^5\right)}{x^4(a+bx)^2}-60b^4\log(a+bx)+60b^4\log(x)}{4a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^3),x]
```

```
[Out] ((a*(-a^5 + 2*a^4*b*x - 5*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 90*a*b^4*x^4 + 60*
b^5*x^5))/(x^4*(a + b*x)^2) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(4*a^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^3), x]
```

fricas [A] time = 0.95, size = 152, normalized size = 1.57

$$\frac{60\,ab^5x^5 + 90\,a^2b^4x^4 + 20\,a^3b^3x^3 - 5\,a^4b^2x^2 + 2\,a^5bx - a^6 - 60\left(b^6x^6 + 2\,ab^5x^5 + a^2b^4x^4\right)\log\left(bx + a\right) + 60\left(b^6x^6 + 2\,ab^5x^5 + a^2b^4x^4\right)\log(x)}{4\left(a^7b^2x^6 + 2\,a^8bx^5 + a^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(60*a*b^5*x^5 + 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5
*b*x - a^6 - 60*(b^6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(b*x + a) + 60*(b^
6*x^6 + 2*a*b^5*x^5 + a^2*b^4*x^4)*log(x))/(a^7*b^2*x^6 + 2*a^8*b*x^5 + a^9
*x^4)
```

giac [A] time = 1.01, size = 97, normalized size = 1.00

$$-\frac{15\,b^4\log\left(|bx + a|\right)}{a^7} + \frac{15\,b^4\log\left(|x|\right)}{a^7} + \frac{60\,ab^5x^5 + 90\,a^2b^4x^4 + 20\,a^3b^3x^3 - 5\,a^4b^2x^2 + 2\,a^5bx - a^6}{4\left(bx + a\right)^2a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15*b^4*log(abs(b*x + a))/a^7 + 15*b^4*log(abs(x))/a^7 + 1/4*(60*a*b^5*x^5
+ 90*a^2*b^4*x^4 + 20*a^3*b^3*x^3 - 5*a^4*b^2*x^2 + 2*a^5*b*x - a^6)/((b*x
+ a)^2*a^7*x^4)
```

maple [A] time = 0.01, size = 94, normalized size = 0.97

$$\frac{b^4}{2\left(bx + a\right)^2a^5} + \frac{5b^4}{\left(bx + a\right)a^6} + \frac{15b^4\ln(x)}{a^7} - \frac{15b^4\ln\left(bx + a\right)}{a^7} + \frac{10b^3}{a^6x} - \frac{3b^2}{a^5x^2} + \frac{b}{a^4x^3} - \frac{1}{4a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^3,x)
```

```
[Out] -1/4/a^3/x^4+b/a^4/x^3-3*b^2/a^5/x^2+10*b^3/a^6/x+1/2*b^4/a^5/(b*x+a)^2+5*b
^4/a^6/(b*x+a)+15*b^4*ln(x)/a^7-15*b^4*ln(b*x+a)/a^7
```

maxima [A] time = 1.35, size = 108, normalized size = 1.11

$$\frac{60\,b^5x^5 + 90\,ab^4x^4 + 20\,a^2b^3x^3 - 5\,a^3b^2x^2 + 2\,a^4bx - a^5}{4\left(a^6b^2x^6 + 2\,a^7bx^5 + a^8x^4\right)} - \frac{15\,b^4\log\left(bx + a\right)}{a^7} + \frac{15\,b^4\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(60*b^5*x^5 + 90*a*b^4*x^4 + 20*a^2*b^3*x^3 - 5*a^3*b^2*x^2 + 2*a^4*b*x
 - a^5)/(a^6*b^2*x^6 + 2*a^7*b*x^5 + a^8*x^4) - 15*b^4*log(b*x + a)/a^7 + 1
5*b^4*log(x)/a^7
```

mupad [B] time = 0.09, size = 101, normalized size = 1.04

$$\frac{\frac{5\,b^3\,x^3}{a^4} - \frac{5\,b^2\,x^2}{4\,a^3} - \frac{1}{4\,a} + \frac{45\,b^4\,x^4}{2\,a^5} + \frac{15\,b^5\,x^5}{a^6} + \frac{b\,x}{2\,a^2}}{a^2\,x^4 + 2\,a\,b\,x^5 + b^2\,x^6} - \frac{30\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^3),x)
```

```
[Out] ((5*b^3*x^3)/a^4 - (5*b^2*x^2)/(4*a^3) - 1/(4*a) + (45*b^4*x^4)/(2*a^5) + (
15*b^5*x^5)/a^6 + (b*x)/(2*a^2))/(a^2*x^4 + b^2*x^6 + 2*a*b*x^5) - (30*b^4*
atanh((2*b*x)/a + 1))/a^7
```

sympy [**A**] time = 0.48, size = 102, normalized size = 1.05

$$\frac{-a^5 + 2a^4bx - 5a^3b^2x^2 + 20a^2b^3x^3 + 90ab^4x^4 + 60b^5x^5}{4a^8x^4 + 8a^7bx^5 + 4a^6b^2x^6} + \frac{15b^4\left(\log\left(x\right) - \log\left(\frac{a}{b} + x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x+a)**3,x)
```

```
[Out] (-a**5 + 2*a**4*b*x - 5*a**3*b**2*x**2 + 20*a**2*b**3*x**3 + 90*a*b**4*x**4
 + 60*b**5*x**5)/(4*a**8*x**4 + 8*a**7*b*x**5 + 4*a**6*b**2*x**6) + 15*b**4
*(log(x) - log(a/b + x))/a**7
```

3.193 $\int \frac{x^8}{(a+bx)^4} dx$

Optimal. Leaf size=114

$$-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Rubi [A] time = 0.09, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}+\frac{35a^4x}{b^8}-\frac{56a^5\log(a+bx)}{b^9}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^4,x]
```

```
[Out] (35*a^4*x)/b^8 - (10*a^3*x^2)/b^7 + (10*a^2*x^3)/(3*b^6) - (a*x^4)/b^5 + x^
5/(5*b^4) - a^8/(3*b^9*(a + b*x)^3) + (4*a^7)/(b^9*(a + b*x)^2) - (28*a^6)/
(b^9*(a + b*x)) - (56*a^5*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx)^4}\,dx &= \int \left(\frac{35a^4}{b^8}-\frac{20a^3x}{b^7}+\frac{10a^2x^2}{b^6}-\frac{4ax^3}{b^5}+\frac{x^4}{b^4}+\frac{a^8}{b^8(a+bx)^4}-\frac{8a^7}{b^8(a+bx)^3}+\frac{28a^6}{b^8(a+bx)^2}-\ldots\right.\\ &= \frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{x^5}{5b^4}-\frac{a^8}{3b^9(a+bx)^3}+\frac{4a^7}{b^9(a+bx)^2}-\frac{28a^6}{b^9(a+bx)}-\ldots\end{aligned}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.89

$$\frac{-\frac{5a^8}{(a+bx)^3}+\frac{60a^7}{(a+bx)^2}-\frac{420a^6}{a+bx}-840a^5\log(a+bx)+525a^4bx-150a^3b^2x^2+50a^2b^3x^3-15ab^4x^4+3b^5x^5}{15b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^4,x]
```

```
[Out] (525*a^4*b*x - 150*a^3*b^2*x^2 + 50*a^2*b^3*x^3 - 15*a*b^4*x^4 + 3*b^5*x^5
- (5*a^8)/(a + b*x)^3 + (60*a^7)/(a + b*x)^2 - (420*a^6)/(a + b*x) - 840*a^
5*Log[a + b*x])/(15*b^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^4,x]
```

[Out] IntegrateAlgebraic[x^8/(a + b*x)^4, x]

fricas [A] time = 0.86, size = 162, normalized size = 1.42

$$\frac{3 b^8x^8 - 6 ab^7x^7 + 14 a^2b^6x^6 - 42 a^3b^5x^5 + 210 a^4b^4x^4 + 1175 a^5b^3x^3 + 1005 a^6b^2x^2 - 255 a^7bx - 365 a^8 - 840 \left(a^5b^3x^3 + 3 a^6b^2x^2 + 3 a^7bx + a^8\right) \log(bx + a)}{15 \left(b^{12}x^3 + 3 ab^{11}x^2 + 3 a^2b^{10}x + a^3b^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^8*x^8 - 6*a*b^7*x^7 + 14*a^2*b^6*x^6 - 42*a^3*b^5*x^5 + 210*a^4*b
^4*x^4 + 1175*a^5*b^3*x^3 + 1005*a^6*b^2*x^2 - 255*a^7*b*x - 365*a^8 - 840*
(a^5*b^3*x^3 + 3*a^6*b^2*x^2 + 3*a^7*b*x + a^8)*log(b*x + a))/(b^12*x^3 + 3
*a*b^11*x^2 + 3*a^2*b^10*x + a^3*b^9)
```

giac [A] time = 1.11, size = 106, normalized size = 0.93

$$-\frac{56 a^5 \log(|bx + a|)}{b^9} - \frac{84 a^6b^2x^2 + 156 a^7bx + 73 a^8}{3 (bx + a)^3b^9} + \frac{3 b^{16}x^5 - 15 ab^{15}x^4 + 50 a^2b^{14}x^3 - 150 a^3b^{13}x^2 + 525 a^4b^{12}x}{15 b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -56*a^5*log(abs(b*x + a))/b^9 - 1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)
/((b*x + a)^3*b^9) + 1/15*(3*b^16*x^5 - 15*a*b^15*x^4 + 50*a^2*b^14*x^3 - 1
50*a^3*b^13*x^2 + 525*a^4*b^12*x)/b^20
```

maple [A] time = 0.01, size = 109, normalized size = 0.96

$$\frac{x^5}{5b^4} - \frac{a x^4}{b^5} - \frac{a^8}{3 (bx + a)^3 b^9} + \frac{10a^2x^3}{3b^6} + \frac{4a^7}{(bx + a)^2 b^9} - \frac{10a^3x^2}{b^7} - \frac{28a^6}{(bx + a) b^9} - \frac{56a^5 \ln(bx + a)}{b^9} + \frac{35a^4x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^4,x)
```

```
[Out] 35*a^4*x/b^8-10*a^3*x^2/b^7+10/3*a^2*x^3/b^6-a*x^4/b^5+1/5*x^5/b^4-1/3*a^8/
b^9/(b*x+a)^3+4*a^7/b^9/(b*x+a)^2-28*a^6/b^9/(b*x+a)-56*a^5*ln(b*x+a)/b^9
```

maxima [A] time = 1.44, size = 125, normalized size = 1.10

$$-\frac{84 a^6b^2x^2 + 156 a^7bx + 73 a^8}{3 \left(b^{12}x^3 + 3 ab^{11}x^2 + 3 a^2b^{10}x + a^3b^9\right)} - \frac{56 a^5 \log(bx + a)}{b^9} + \frac{3 b^4x^5 - 15 ab^3x^4 + 50 a^2b^2x^3 - 150 a^3bx^2 + 525 a^4x}{15 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(84*a^6*b^2*x^2 + 156*a^7*b*x + 73*a^8)/(b^12*x^3 + 3*a*b^11*x^2 + 3*a
^2*b^10*x + a^3*b^9) - 56*a^5*log(b*x + a)/b^9 + 1/15*(3*b^4*x^5 - 15*a*b^3
*x^4 + 50*a^2*b^2*x^3 - 150*a^3*b*x^2 + 525*a^4*x)/b^8
```

mupad [B] time = 0.37, size = 103, normalized size = 0.90

$$-\frac{2 a (a + b x)^4 - \frac{(a+b x)^5}{5} - \frac{28 a^2 (a+b x)^3}{3} + 28 a^3 (a + b x)^2 + \frac{28 a^6}{a+b x} - \frac{4 a^7}{(a+b x)^2} + \frac{a^8}{3 (a+b x)^3} + 56 a^5 \ln(a + b x) - 70 a^4 b x}{b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^4,x)
```

```
[Out] -(2*a*(a + b*x)^4 - (a + b*x)^5/5 - (28*a^2*(a + b*x)^3)/3 + 28*a^3*(a + b*
x)^2 + (28*a^6)/(a + b*x) - (4*a^7)/(a + b*x)^2 + a^8/(3*(a + b*x)^3) + 56*
a^5*log(a + b*x) - 70*a^4*b*x)/b^9
```

sympy [A] time = 0.52, size = 131, normalized size = 1.15

$$-\frac{56a^5\log(a+bx)}{b^9}+\frac{35a^4x}{b^8}-\frac{10a^3x^2}{b^7}+\frac{10a^2x^3}{3b^6}-\frac{ax^4}{b^5}+\frac{-73a^8-156a^7bx-84a^6b^2x^2}{3a^3b^9+9a^2b^{10}x+9ab^{11}x^2+3b^{12}x^3}+\frac{x^5}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**4,x)
```

```
[Out] -56*a**5*log(a + b*x)/b**9 + 35*a**4*x/b**8 - 10*a**3*x**2/b**7 + 10*a**2*x
**3/(3*b**6) - a*x**4/b**5 + (-73*a**8 - 156*a**7*b*x - 84*a**6*b**2*x**2)/
(3*a**3*b**9 + 9*a**2*b**10*x + 9*a*b**11*x**2 + 3*b**12*x**3) + x**5/(5*b*
*4)
```

3.194 $\int \frac{x^7}{(a+bx)^4} dx$

Optimal. Leaf size=105

$$\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}-\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Rubi [A] time = 0.07, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{5a^2x^2}{b^6}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}-\frac{20a^3x}{b^7}+\frac{35a^4\log(a+bx)}{b^8}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^4,x]
```

```
[Out] (-20*a^3*x)/b^7 + (5*a^2*x^2)/b^6 - (4*a*x^3)/(3*b^5) + x^4/(4*b^4) + a^7/(
3*b^8*(a + b*x)^3) - (7*a^6)/(2*b^8*(a + b*x)^2) + (21*a^5)/(b^8*(a + b*x))
 + (35*a^4*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^4}dx &= \int \left(-\frac{20a^3}{b^7}+\frac{10a^2x}{b^6}-\frac{4ax^2}{b^5}+\frac{x^3}{b^4}-\frac{a^7}{b^7(a+bx)^4}+\frac{7a^6}{b^7(a+bx)^3}-\frac{21a^5}{b^7(a+bx)^2}+\frac{35a^4}{b^7(a+bx)}\right)dx\\ &= -\frac{20a^3x}{b^7}+\frac{5a^2x^2}{b^6}-\frac{4ax^3}{3b^5}+\frac{x^4}{4b^4}+\frac{a^7}{3b^8(a+bx)^3}-\frac{7a^6}{2b^8(a+bx)^2}+\frac{21a^5}{b^8(a+bx)}+\frac{35a^4\log(a+bx)}{b^8}\end{aligned}$$

Mathematica [A] time = 0.03, size = 90, normalized size = 0.86

$$\frac{\frac{4a^7}{(a+bx)^3}-\frac{42a^6}{(a+bx)^2}+\frac{252a^5}{a+bx}+420a^4\log(a+bx)-240a^3bx+60a^2b^2x^2-16ab^3x^3+3b^4x^4}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^4,x]
```

```
[Out] (-240*a^3*b*x + 60*a^2*b^2*x^2 - 16*a*b^3*x^3 + 3*b^4*x^4 + (4*a^7)/(a + b*
x)^3 - (42*a^6)/(a + b*x)^2 + (252*a^5)/(a + b*x) + 420*a^4*Log[a + b*x])/(
12*b^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^4}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^4, x]
```

fricas [A] time = 1.04, size = 151, normalized size = 1.44

$$\frac{3\,b^{7}x^{7}-7\,ab^{6}x^{6}+21\,a^{2}b^{5}x^{5}-105\,a^{3}b^{4}x^{4}-556\,a^{4}b^{3}x^{3}-408\,a^{5}b^{2}x^{2}+222\,a^{6}bx+214\,a^{7}+420\left(a^{4}b^{3}x^{3}+3\,a^{5}b^{2}x^{2}+3\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{12\left(b^{11}x^{3}+3\,ab^{10}x^{2}+3\,a^{2}b^{9}x+a^{3}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^7*x^7 - 7*a*b^6*x^6 + 21*a^2*b^5*x^5 - 105*a^3*b^4*x^4 - 556*a^4*
b^3*x^3 - 408*a^5*b^2*x^2 + 222*a^6*b*x + 214*a^7 + 420*(a^4*b^3*x^3 + 3*a^
5*b^2*x^2 + 3*a^6*b*x + a^7)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2
*b^9*x + a^3*b^8)
```

giac [A] time = 1.01, size = 95, normalized size = 0.90

$$\frac{35\,a^{4}\log\left(|bx+a|\right)}{b^{8}}+\frac{126\,a^{5}b^{2}x^{2}+231\,a^{6}bx+107\,a^{7}}{6\left(bx+a\right)^{3}b^{8}}+\frac{3\,b^{12}x^{4}-16\,ab^{11}x^{3}+60\,a^{2}b^{10}x^{2}-240\,a^{3}b^{9}x}{12\,b^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 35*a^4*log(abs(b*x + a))/b^8 + 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7
)/((b*x + a)^3*b^8) + 1/12*(3*b^12*x^4 - 16*a*b^11*x^3 + 60*a^2*b^10*x^2 -
240*a^3*b^9*x)/b^16
```

maple [A] time = 0.01, size = 98, normalized size = 0.93

$$\frac{x^{4}}{4b^{4}}+\frac{a^{7}}{3\left(bx+a\right)^{3}b^{8}}-\frac{4a\,x^{3}}{3b^{5}}-\frac{7a^{6}}{2\left(bx+a\right)^{2}b^{8}}+\frac{5a^{2}x^{2}}{b^{6}}+\frac{21a^{5}}{\left(bx+a\right)b^{8}}+\frac{35a^{4}\ln\left(bx+a\right)}{b^{8}}-\frac{20a^{3}x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^4,x)
```

```
[Out] -20*a^3*x/b^7+5*a^2*x^2/b^6-4/3*a*x^3/b^5+1/4*x^4/b^4+1/3*a^7/b^8/(b*x+a)^3
-7/2*a^6/b^8/(b*x+a)^2+21*a^5/b^8/(b*x+a)+35*a^4*ln(b*x+a)/b^8
```

maxima [A] time = 1.39, size = 114, normalized size = 1.09

$$\frac{126\,a^{5}b^{2}x^{2}+231\,a^{6}bx+107\,a^{7}}{6\left(b^{11}x^{3}+3\,ab^{10}x^{2}+3\,a^{2}b^{9}x+a^{3}b^{8}\right)}+\frac{35\,a^{4}\log\left(bx+a\right)}{b^{8}}+\frac{3\,b^{3}x^{4}-16\,ab^{2}x^{3}+60\,a^{2}bx^{2}-240\,a^{3}x}{12\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(126*a^5*b^2*x^2 + 231*a^6*b*x + 107*a^7)/(b^11*x^3 + 3*a*b^10*x^2 + 3*
a^2*b^9*x + a^3*b^8) + 35*a^4*log(b*x + a)/b^8 + 1/12*(3*b^3*x^4 - 16*a*b^2
*x^3 + 60*a^2*b*x^2 - 240*a^3*x)/b^7
```

mupad [B] time = 0.22, size = 90, normalized size = 0.86

$$\frac{\frac{{\left(a+b\,x\right)}^4}{4}-\frac{7\,a\,{\left(a+b\,x\right)}^3}{3}+\frac{21\,a^2\,{\left(a+b\,x\right)}^2}{2}+\frac{21\,a^5}{a+b\,x}-\frac{7\,a^6}{2\,{\left(a+b\,x\right)}^2}+\frac{a^7}{3\,{\left(a+b\,x\right)}^3}+35\,a^4\,\ln\left(a+b\,x\right)-35\,a^3\,b\,x}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^4/4 - (7*a*(a + b*x)^3)/3 + (21*a^2*(a + b*x)^2)/2 + (21*a^5)/(a
 + b*x) - (7*a^6)/(2*(a + b*x)^2) + a^7/(3*(a + b*x)^3) + 35*a^4*log(a + b*
x) - 35*a^3*b*x)/b^8
```

sympy [A] time = 0.48, size = 119, normalized size = 1.13

$$\frac{35a^4\log(a+bx)}{b^8} - \frac{20a^3x}{b^7} + \frac{5a^2x^2}{b^6} - \frac{4ax^3}{3b^5} + \frac{107a^7 + 231a^6bx + 126a^5b^2x^2}{6a^3b^8 + 18a^2b^9x + 18ab^{10}x^2 + 6b^{11}x^3} + \frac{x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**4,x)
```

```
[Out] 35*a**4*log(a + b*x)/b**8 - 20*a**3*x/b**7 + 5*a**2*x**2/b**6 - 4*a*x**3/(3
*b**5) + (107*a**7 + 231*a**6*b*x + 126*a**5*b**2*x**2)/(6*a**3*b**8 + 18*a
**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*x**3) + x**4/(4*b**4)
```

3.195 $\int \frac{x^6}{(a+bx)^4} dx$

Optimal. Leaf size=90

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}+\frac{10a^2x}{b^6}-\frac{20a^3\log(a+bx)}{b^7}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^4}\,dx &= \int \left(\frac{10a^2}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{b^4}+\frac{a^6}{b^6(a+bx)^4}-\frac{6a^5}{b^6(a+bx)^3}+\frac{15a^4}{b^6(a+bx)^2}-\frac{20a^3}{b^6(a+bx)}\right)dx\\ &= \frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 90, normalized size = 1.00

$$-\frac{a^6}{3b^7(a+bx)^3}+\frac{3a^5}{b^7(a+bx)^2}-\frac{15a^4}{b^7(a+bx)}-\frac{20a^3\log(a+bx)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{x^3}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^4,x]
```

```
[Out] (10*a^2*x)/b^6 - (2*a*x^2)/b^5 + x^3/(3*b^4) - a^6/(3*b^7*(a + b*x)^3) + (3
*a^5)/(b^7*(a + b*x)^2) - (15*a^4)/(b^7*(a + b*x)) - (20*a^3*Log[a + b*x])/
b^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^4, x]
```

fricas [A] time = 0.97, size = 139, normalized size = 1.54

$$\frac{b^6x^6 - 3\,ab^5x^5 + 15\,a^2b^4x^4 + 73\,a^3b^3x^3 + 39\,a^4b^2x^2 - 51\,a^5bx - 37\,a^6 - 60\left(a^3b^3x^3 + 3\,a^4b^2x^2 + 3\,a^5bx + a^6\right)\log\left(bx + a\right)}{3\left(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(b^6*x^6 - 3*a*b^5*x^5 + 15*a^2*b^4*x^4 + 73*a^3*b^3*x^3 + 39*a^4*b^2*x
^2 - 51*a^5*b*x - 37*a^6 - 60*(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*x + a^
6)*log(b*x + a))/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2*b^8*x + a^3*b^7)
```

giac [A] time = 1.11, size = 83, normalized size = 0.92

$$-\frac{20\,a^3\log\left(|bx + a|\right)}{b^7} - \frac{45\,a^4b^2x^2 + 81\,a^5bx + 37\,a^6}{3\,(bx + a)^3b^7} + \frac{b^8x^3 - 6\,ab^7x^2 + 30\,a^2b^6x}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -20*a^3*log(abs(b*x + a))/b^7 - 1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/
((b*x + a)^3*b^7) + 1/3*(b^8*x^3 - 6*a*b^7*x^2 + 30*a^2*b^6*x)/b^12
```

maple [A] time = 0.01, size = 87, normalized size = 0.97

$$-\frac{a^6}{3\,(bx + a)^3\,b^7} + \frac{x^3}{3b^4} + \frac{3a^5}{(bx + a)^2\,b^7} - \frac{2a\,x^2}{b^5} - \frac{15a^4}{(bx + a)\,b^7} - \frac{20a^3\ln\left(bx + a\right)}{b^7} + \frac{10a^2x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^4,x)
```

```
[Out] 10*a^2*x/b^6-2*a*x^2/b^5+1/3*x^3/b^4-1/3*a^6/b^7/(b*x+a)^3+3*a^5/b^7/(b*x+a
)^2-15*a^4/b^7/(b*x+a)-20*a^3*ln(b*x+a)/b^7
```

maxima [A] time = 1.42, size = 102, normalized size = 1.13

$$-\frac{45\,a^4b^2x^2 + 81\,a^5bx + 37\,a^6}{3\left(b^{10}x^3 + 3\,ab^9x^2 + 3\,a^2b^8x + a^3b^7\right)} - \frac{20\,a^3\log\left(bx + a\right)}{b^7} + \frac{b^2x^3 - 6\,abx^2 + 30\,a^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(45*a^4*b^2*x^2 + 81*a^5*b*x + 37*a^6)/(b^10*x^3 + 3*a*b^9*x^2 + 3*a^2
*b^8*x + a^3*b^7) - 20*a^3*log(b*x + a)/b^7 + 1/3*(b^2*x^3 - 6*a*b*x^2 + 30
*a^2*x)/b^6
```

mupad [B] time = 0.15, size = 79, normalized size = 0.88

$$-\frac{3\,a\,(a + b\,x)^2 - \frac{(a+b\,x)^3}{3} + \frac{15\,a^4}{a+b\,x} - \frac{3\,a^5}{(a+b\,x)^2} + \frac{a^6}{3\,(a+b\,x)^3} + 20\,a^3\,\ln\left(a + b\,x\right) - 15\,a^2\,b\,x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^4,x)
```

```
[Out] -(3*a*(a + b*x)^2 - (a + b*x)^3/3 + (15*a^4)/(a + b*x) - (3*a^5)/(a + b*x)^
2 + a^6/(3*(a + b*x)^3) + 20*a^3*log(a + b*x) - 15*a^2*b*x)/b^7
```

sympy [A] time = 0.48, size = 107, normalized size = 1.19

$$-\frac{20a^3\log\left(a+bx\right)}{b^7}+\frac{10a^2x}{b^6}-\frac{2ax^2}{b^5}+\frac{-37a^6-81a^5bx-45a^4b^2x^2}{3a^3b^7+9a^2b^8x+9ab^9x^2+3b^{10}x^3}+\frac{x^3}{3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**4,x)
```

```
[Out] -20*a**3*log(a + b*x)/b**7 + 10*a**2*x/b**6 - 2*a*x**2/b**5 + (-37*a**6 - 8
1*a**5*b*x - 45*a**4*b**2*x**2)/(3*a**3*b**7 + 9*a**2*b**8*x + 9*a*b**9*x**
2 + 3*b**10*x**3) + x**3/(3*b**4)
```

3.196 $\int \frac{x^5}{(a+bx)^4}\,dx$

Optimal. Leaf size=81

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}-\frac{4ax}{b^5}+\frac{x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^4,x]
```

```
[Out] (-4*a*x)/b^5 + x^2/(2*b^4) + a^5/(3*b^6*(a + b*x)^3) - (5*a^4)/(2*b^6*(a +
b*x)^2) + (10*a^3)/(b^6*(a + b*x)) + (10*a^2*Log[a + b*x])/b^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^4}\,dx &= \int \left(-\frac{4a}{b^5}+\frac{x}{b^4}-\frac{a^5}{b^5(a+bx)^4}+\frac{5a^4}{b^5(a+bx)^3}-\frac{10a^3}{b^5(a+bx)^2}+\frac{10a^2}{b^5(a+bx)}\right)dx\\ &= -\frac{4ax}{b^5}+\frac{x^2}{2b^4}+\frac{a^5}{3b^6(a+bx)^3}-\frac{5a^4}{2b^6(a+bx)^2}+\frac{10a^3}{b^6(a+bx)}+\frac{10a^2\log(a+bx)}{b^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.84

$$\frac{\frac{2a^5}{(a+bx)^3}-\frac{15a^4}{(a+bx)^2}+\frac{60a^3}{a+bx}+60a^2\log(a+bx)-24abx+3b^2x^2}{6b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^4,x]
```

```
[Out] (-24*a*b*x + 3*b^2*x^2 + (2*a^5)/(a + b*x)^3 - (15*a^4)/(a + b*x)^2 + (60*a
^3)/(a + b*x) + 60*a^2*Log[a + b*x])/(6*b^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^4, x]
```

fricas [A] time = 0.79, size = 129, normalized size = 1.59

$$\frac{3\,b^5x^5-15\,ab^4x^4-63\,a^2b^3x^3-9\,a^3b^2x^2+81\,a^4bx+47\,a^5+60\left(a^2b^3x^3+3\,a^3b^2x^2+3\,a^4bx+a^5\right)\log\left(bx+a\right)}{6\left(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^5*x^5 - 15*a*b^4*x^4 - 63*a^2*b^3*x^3 - 9*a^3*b^2*x^2 + 81*a^4*b*x
 + 47*a^5 + 60*(a^2*b^3*x^3 + 3*a^3*b^2*x^2 + 3*a^4*b*x + a^5)*log(b*x + a)
)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*b^7*x + a^3*b^6)
```

giac [A] time = 0.87, size = 72, normalized size = 0.89

$$\frac{10\,a^2\log\left(|bx+a|\right)}{b^6}+\frac{b^4x^2-8\,ab^3x}{2\,b^8}+\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\,(bx+a)^3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 10*a^2*log(abs(b*x + a))/b^6 + 1/2*(b^4*x^2 - 8*a*b^3*x)/b^8 + 1/6*(60*a^3*
b^2*x^2 + 105*a^4*b*x + 47*a^5)/((b*x + a)^3*b^6)
```

maple [A] time = 0.01, size = 76, normalized size = 0.94

$$\frac{a^5}{3\,(bx+a)^3\,b^6}-\frac{5a^4}{2\,(bx+a)^2\,b^6}+\frac{x^2}{2b^4}+\frac{10a^3}{(bx+a)\,b^6}+\frac{10a^2\ln\left(bx+a\right)}{b^6}-\frac{4ax}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^4,x)
```

```
[Out] -4*a*x/b^5+1/2*x^2/b^4+1/3*a^5/b^6/(b*x+a)^3-5/2*a^4/b^6/(b*x+a)^2+10*a^3/b
^6/(b*x+a)+10*a^2*ln(b*x+a)/b^6
```

maxima [A] time = 1.46, size = 91, normalized size = 1.12

$$\frac{60\,a^3b^2x^2+105\,a^4bx+47\,a^5}{6\left(b^9x^3+3\,ab^8x^2+3\,a^2b^7x+a^3b^6\right)}+\frac{10\,a^2\log\left(bx+a\right)}{b^6}+\frac{bx^2-8\,ax}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*a^3*b^2*x^2 + 105*a^4*b*x + 47*a^5)/(b^9*x^3 + 3*a*b^8*x^2 + 3*a^2*
b^7*x + a^3*b^6) + 10*a^2*log(b*x + a)/b^6 + 1/2*(b*x^2 - 8*a*x)/b^5
```

mupad [B] time = 0.12, size = 66, normalized size = 0.81

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{10\,a^3}{a+b\,x}-\frac{5\,a^4}{2\,(a+b\,x)^2}+\frac{a^5}{3\,(a+b\,x)^3}+10\,a^2\,\ln\left(a+b\,x\right)-5\,a\,b\,x}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^4,x)
```

```
[Out] ((a + b*x)^2/2 + (10*a^3)/(a + b*x) - (5*a^4)/(2*(a + b*x)^2) + a^5/(3*(a +
 b*x)^3) + 10*a^2*log(a + b*x) - 5*a*b*x)/b^6
```

<u>**sympy**</u> **[A]** time = 0.46, size = 94, normalized size = 1.16

$$\frac{10a^2\log{(a+bx)}}{b^6}-\frac{4ax}{b^5}+\frac{47a^5+105a^4bx+60a^3b^2x^2}{6a^3b^6+18a^2b^7x+18ab^8x^2+6b^9x^3}+\frac{x^2}{2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**4,x)
```

```
[Out] 10*a**2*log(a + b*x)/b**6 - 4*a*x/b**5 + (47*a**5 + 105*a**4*b*x + 60*a**3*
b**2*x**2)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) +
x**2/(2*b**4)
```

3.197 $\int \frac{x^4}{(a+bx)^4} dx$

Optimal. Leaf size=65

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}+\frac{x}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^4,x]
```

```
[Out] x/b^4 - a^4/(3*b^5*(a + b*x)^3) + (2*a^3)/(b^5*(a + b*x)^2) - (6*a^2)/(b^5*
(a + b*x)) - (4*a*Log[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^4} dx &= \int \left(\frac{1}{b^4}+\frac{a^4}{b^4(a+bx)^4}-\frac{4a^3}{b^4(a+bx)^3}+\frac{6a^2}{b^4(a+bx)^2}-\frac{4a}{b^4(a+bx)}\right) dx\\ &= \frac{x}{b^4}-\frac{a^4}{3b^5(a+bx)^3}+\frac{2a^3}{b^5(a+bx)^2}-\frac{6a^2}{b^5(a+bx)}-\frac{4a\log(a+bx)}{b^5}\end{aligned}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.78

$$-\frac{\frac{a^2(13a^2+30abx+18b^2x^2)}{(a+bx)^3}+12a\log(a+bx)-3bx}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^4,x]
```

```
[Out] -1/3*(-3*b*x + (a^2*(13*a^2 + 30*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 12*a*Lo
g[a + b*x])/b^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^4, x]
```

fricas [A] time = 0.88, size = 116, normalized size = 1.78

$$\frac{3\,b^{4}x^{4}+9\,ab^{3}x^{3}-9\,a^{2}b^{2}x^{2}-27\,a^{3}bx-13\,a^{4}-12\,\left(ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}+3\,a^{3}bx+a^{4}\right)\log\left(bx+a\right)}{3\,\left(b^{8}x^{3}+3\,ab^{7}x^{2}+3\,a^{2}b^{6}x+a^{3}b^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^4*x^4 + 9*a*b^3*x^3 - 9*a^2*b^2*x^2 - 27*a^3*b*x - 13*a^4 - 12*(a*
b^3*x^3 + 3*a^2*b^2*x^2 + 3*a^3*b*x + a^4)*log(b*x + a))/(b^8*x^3 + 3*a*b^7
*x^2 + 3*a^2*b^6*x + a^3*b^5)
```

giac [A] time = 0.86, size = 55, normalized size = 0.85

$$\frac{x}{b^{4}}-\frac{4\,a\log\left(|bx+a|\right)}{b^{5}}-\frac{18\,a^{2}b^{2}x^{2}+30\,a^{3}bx+13\,a^{4}}{3\,\left(bx+a\right)^{3}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] x/b^4 - 4*a*log(abs(b*x + a))/b^5 - 1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a
^4)/((b*x + a)^3*b^5)
```

maple [A] time = 0.01, size = 64, normalized size = 0.98

$$-\frac{a^{4}}{3\left(bx+a\right)^{3}b^{5}}+\frac{2a^{3}}{\left(bx+a\right)^{2}b^{5}}-\frac{6a^{2}}{\left(bx+a\right)b^{5}}-\frac{4a\ln\left(bx+a\right)}{b^{5}}+\frac{x}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^4,x)
```

```
[Out] x/b^4-1/3*a^4/b^5/(b*x+a)^3+2*a^3/b^5/(b*x+a)^2-6*a^2/b^5/(b*x+a)-4*a*ln(b*
x+a)/b^5
```

maxima [A] time = 1.41, size = 79, normalized size = 1.22

$$-\frac{18\,a^{2}b^{2}x^{2}+30\,a^{3}bx+13\,a^{4}}{3\,\left(b^{8}x^{3}+3\,ab^{7}x^{2}+3\,a^{2}b^{6}x+a^{3}b^{5}\right)}+\frac{x}{b^{4}}-\frac{4\,a\log\left(bx+a\right)}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(18*a^2*b^2*x^2 + 30*a^3*b*x + 13*a^4)/(b^8*x^3 + 3*a*b^7*x^2 + 3*a^2*
b^6*x + a^3*b^5) + x/b^4 - 4*a*log(b*x + a)/b^5
```

mupad [B] time = 0.17, size = 55, normalized size = 0.85

$$-\frac{4\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{6\,a^{2}}{a+b\,x}-\frac{2\,a^{3}}{\left(a+b\,x\right)^{2}}+\frac{a^{4}}{3\,\left(a+b\,x\right)^{3}}}{b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^4,x)
```

```
[Out] -(4*a*log(a + b*x) - b*x + (6*a^2)/(a + b*x) - (2*a^3)/(a + b*x)^2 + a^4/(3
*(a + b*x)^3))/b^5
```

sympy [A] time = 0.40, size = 82, normalized size = 1.26

$$-\frac{4a\log{(a+bx)}}{b^5}+\frac{-13a^4-30a^3bx-18a^2b^2x^2}{3a^3b^5+9a^2b^6x+9ab^7x^2+3b^8x^3}+\frac{x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**4,x)
```

```
[Out] -4*a*log(a + b*x)/b**5 + (-13*a**4 - 30*a**3*b*x - 18*a**2*b**2*x**2)/(3*a*
*3*b**5 + 9*a**2*b**6*x + 9*a*b**7*x**2 + 3*b**8*x**3) + x/b**4
```

3.198 $\int \frac{x^3}{(a+bx)^4} \, dx$

Optimal. Leaf size=58

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^4,x]
```

```
[Out] a^3/(3*b^4*(a + b*x)^3) - (3*a^2)/(2*b^4*(a + b*x)^2) + (3*a)/(b^4*(a + b*x
)) + Log[a + b*x]/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^4} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^4}+\frac{3a^2}{b^3(a+bx)^3}-\frac{3a}{b^3(a+bx)^2}+\frac{1}{b^3(a+bx)}\right) dx\\ &= \frac{a^3}{3b^4(a+bx)^3}-\frac{3a^2}{2b^4(a+bx)^2}+\frac{3a}{b^4(a+bx)}+\frac{\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.76

$$\frac{\frac{a\left(11a^2+27abx+18b^2x^2\right)}{(a+bx)^3}+6\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^4,x]
```

```
[Out] ((a*(11*a^2 + 27*a*b*x + 18*b^2*x^2))/(a + b*x)^3 + 6*Log[a + b*x])/(6*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^4, x]
```

fricas [A] time = 1.10, size = 94, normalized size = 1.62

$$\frac{18\,ab^{2}x^{2}+27\,a^{2}bx+11\,a^{3}+6\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\log\left(bx+a\right)}{6\left(b^{7}x^{3}+3\,ab^{6}x^{2}+3\,a^{2}b^{5}x+a^{3}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*
b*x + a^3)*log(b*x + a))/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

giac [A] time = 1.03, size = 46, normalized size = 0.79

$$\frac{\log\left(|bx+a|\right)}{b^{4}}+\frac{18\,abx^{2}+27\,a^{2}x+\frac{11\,a^{3}}{b}}{6\left(bx+a\right)^{3}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^4 + 1/6*(18*a*b*x^2 + 27*a^2*x + 11*a^3/b)/((b*x + a)^3
*b^3)
```

maple [A] time = 0.01, size = 55, normalized size = 0.95

$$\frac{a^{3}}{3\left(bx+a\right)^{3}b^{4}}-\frac{3a^{2}}{2\left(bx+a\right)^{2}b^{4}}+\frac{3a}{\left(bx+a\right)b^{4}}+\frac{\ln\left(bx+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^4,x)
```

```
[Out] 1/3*a^3/b^4/(b*x+a)^3-3/2*a^2/b^4/(b*x+a)^2+3*a/b^4/(b*x+a)+ln(b*x+a)/b^4
```

maxima [A] time = 1.39, size = 70, normalized size = 1.21

$$\frac{18\,ab^{2}x^{2}+27\,a^{2}bx+11\,a^{3}}{6\left(b^{7}x^{3}+3\,ab^{6}x^{2}+3\,a^{2}b^{5}x+a^{3}b^{4}\right)}+\frac{\log\left(bx+a\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3)/(b^7*x^3 + 3*a*b^6*x^2 + 3*a^2*b^5
*x + a^3*b^4) + log(b*x + a)/b^4
```

mupad [B] time = 0.07, size = 45, normalized size = 0.78

$$\frac{\ln\left(a+b\,x\right)+\frac{3\,a}{a+b\,x}-\frac{3\,a^{2}}{2\,\left(a+b\,x\right)^{2}}+\frac{a^{3}}{3\,\left(a+b\,x\right)^{3}}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^4,x)
```

```
[Out] (log(a + b*x) + (3*a)/(a + b*x) - (3*a^2)/(2*(a + b*x)^2) + a^3/(3*(a + b*x
)^3))/b^4
```

sympy [A] time = 0.31, size = 70, normalized size = 1.21

$$\frac{11a^{3}+27a^{2}bx+18ab^{2}x^{2}}{6a^{3}b^{4}+18a^{2}b^{5}x+18ab^{6}x^{2}+6b^{7}x^{3}}+\frac{\log\left(a+bx\right)}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3 + 27*a**2*b*x + 18*a*b**2*x**2)/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + log(a + b*x)/b**4
```

3.199 $\int \frac{x^2}{(a+bx)^4} \, dx$

Optimal. Leaf size=17

$$\frac{x^3}{3a(a+bx)^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^3}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^4,x]
```

```
[Out] x^3/(3*a*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^2}{(a+bx)^4} \, dx = \frac{x^3}{3a(a+bx)^3}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.82

$$-\frac{a^2+3abx+3b^2x^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*a*b*x + 3*b^2*x^2)/(b^3*(a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^4, x]
```

fricas [B] time = 1.04, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

giac [A] time = 1.27, size = 29, normalized size = 1.71

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/((b*x + a)^3*b^3)
```

maple [B] time = 0.00, size = 41, normalized size = 2.41

$$-\frac{a^2}{3\,(bx+a)^3\,b^3}+\frac{a}{(bx+a)^2\,b^3}-\frac{1}{(bx+a)\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^4,x)
```

```
[Out] a/b^3/(b*x+a)^2-1/3*a^2/b^3/(b*x+a)^3-1/b^3/(b*x+a)
```

maxima [B] time = 1.40, size = 54, normalized size = 3.18

$$-\frac{3\,b^2x^2+3\,abx+a^2}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3
*b^3)
```

mupad [B] time = 0.09, size = 56, normalized size = 3.29

$$-\frac{a^2+3\,a\,b\,x+3\,b^2\,x^2}{3\,a^3\,b^3+9\,a^2\,b^4\,x+9\,a\,b^5\,x^2+3\,b^6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^4,x)
```

```
[Out] -(a^2 + 3*b^2*x^2 + 3*a*b*x)/(3*a^3*b^3 + 3*b^6*x^3 + 9*a^2*b^4*x + 9*a*b^5
*x^2)
```

sympy [B] time = 0.30, size = 56, normalized size = 3.29

$$\frac{-a^2-3abx-3b^2x^2}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**4,x)
```

```
[Out] (-a**2 - 3*a*b*x - 3*b**2*x**2)/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**
2 + 3*b**6*x**3)
```

3.200 $\int \frac{x}{(a+bx)^4} dx$

Optimal. Leaf size=30

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^4,x]
```

```
[Out] a/(3*b^2*(a + b*x)^3) - 1/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^4}\,dx &= \int \left(-\frac{a}{b(a+bx)^4}+\frac{1}{b(a+bx)^3}\right) dx\\ &= \frac{a}{3b^2(a+bx)^3}-\frac{1}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+3bx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b^2*(a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^4, x]
```

fricas [A] time = 0.77, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="fricas")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

giac [A] time = 1.01, size = 18, normalized size = 0.60

$$-\frac{3\,bx+a}{6\,(bx+a)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="giac")
[Out] -1/6*(3*b*x + a)/((b*x + a)^3*b^2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$\frac{a}{3\,(bx+a)^3\,b^2}-\frac{1}{2\,(bx+a)^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^4,x)
[Out] 1/3*a/b^2/(b*x+a)^3-1/2/b^2/(b*x+a)^2
```

maxima [A] time = 1.37, size = 43, normalized size = 1.43

$$-\frac{3\,bx+a}{6\,\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/6*(3*b*x + a)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2)
```

mupad [B] time = 0.07, size = 44, normalized size = 1.47

$$-\frac{\frac{a}{6\,b^2}+\frac{x}{2\,b}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^4,x)
[Out] -(a/(6*b^2) + x/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

sympy [A] time = 0.32, size = 44, normalized size = 1.47

$$\frac{-a-3bx}{6a^3b^2+18a^2b^3x+18ab^4x^2+6b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**4,x)
[Out] (-a - 3*b*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6*b**5*x**3)
```

3.201 $\int \frac{1}{(a+bx)^4} dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(a+bx)^3}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4),x]
[Out] -1/(3*b*(a + b*x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4} dx = -\frac{1}{3b(a+bx)^3}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4),x]
[Out] -1/3*1/(b*(a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4),x]
[Out] IntegrateAlgebraic[(a + b*x)^(-4), x]
```

fricas [B] time = 1.14, size = 35, normalized size = 2.50

$$-\frac{1}{3\left(b^4x^3 + 3\,ab^3x^2 + 3\,a^2b^2x + a^3b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
```

giac [A] time = 1.02, size = 12, normalized size = 0.86

$$-\frac{1}{3(bx+a)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="giac")
[Out] -1/3/((b*x + a)^3*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{3(bx+a)^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4,x)
[Out] -1/3/b/(b*x+a)^3
```

maxima [A] time = 1.38, size = 12, normalized size = 0.86

$$-\frac{1}{3(bx+a)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4,x, algorithm="maxima")
[Out] -1/3/((b*x + a)^3*b)
```

mupad [B] time = 0.08, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b+9a^2b^2x+9ab^3x^2+3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^4,x)
[Out] -1/(3*a^3*b + 3*b^4*x^3 + 9*a^2*b^2*x + 9*a*b^3*x^2)
```

sympy [B] time = 0.27, size = 37, normalized size = 2.64

$$-\frac{1}{3a^3b+9a^2b^2x+9ab^3x^2+3b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4,x)
[Out] -1/(3*a**3*b + 9*a**2*b**2*x + 9*a*b**3*x**2 + 3*b**4*x**3)
```

3.202 $\int \frac{1}{x(a+bx)^4} \, dx$

Optimal. Leaf size=57

$$-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{3a(a+bx)^3}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^3(a+bx)}+\frac{1}{2a^2(a+bx)^2}-\frac{\log(a+bx)}{a^4}+\frac{\log(x)}{a^4}+\frac{1}{3a(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^4),x]
```

```
[Out] 1/(3*a*(a + b*x)^3) + 1/(2*a^2*(a + b*x)^2) + 1/(a^3*(a + b*x)) + Log[x]/a^
4 - Log[a + b*x]/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x}-\frac{b}{a(a+bx)^4}-\frac{b}{a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{b}{a^4(a+bx)}\right)dx\\ &= \frac{1}{3a(a+bx)^3}+\frac{1}{2a^2(a+bx)^2}+\frac{1}{a^3(a+bx)}+\frac{\log(x)}{a^4}-\frac{\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.84

$$\frac{\frac{a\left(11a^2+15abx+6b^2x^2\right)}{(a+bx)^3}-6\log(a+bx)+6\log(x)}{6a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^4),x]
```

```
[Out] ((a*(11*a^2 + 15*a*b*x + 6*b^2*x^2))/(a + b*x)^3 + 6*Log[x] - 6*Log[a + b*x
])/(6*a^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^4), x]
```

fricas [B] time = 1.07, size = 124, normalized size = 2.18

$$\frac{6\,ab^2x^2 + 15\,a^2bx + 11\,a^3 - 6\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log\left(bx + a\right) + 6\left(b^3x^3 + 3\,ab^2x^2 + 3\,a^2bx + a^3\right)\log(x)}{6\left(a^4b^3x^3 + 3\,a^5b^2x^2 + 3\,a^6bx + a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b^2*x^2 + 15*a^2*b*x + 11*a^3 - 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b
*x + a^3)*log(b*x + a) + 6*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*log(x)
)/(a^4*b^3*x^3 + 3*a^5*b^2*x^2 + 3*a^6*b*x + a^7)
```

giac [A] time = 1.02, size = 54, normalized size = 0.95

$$-\frac{\log\left(|bx + a|\right)}{a^4} + \frac{\log\left(|x|\right)}{a^4} + \frac{6\,ab^2x^2 + 15\,a^2bx + 11\,a^3}{6\left(bx + a\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^4 + log(abs(x))/a^4 + 1/6*(6*a*b^2*x^2 + 15*a^2*b*x +
11*a^3)/((b*x + a)^3*a^4)
```

maple [A] time = 0.01, size = 54, normalized size = 0.95

$$\frac{1}{3\left(bx + a\right)^3a} + \frac{1}{2\left(bx + a\right)^2a^2} + \frac{1}{\left(bx + a\right)a^3} + \frac{\ln(x)}{a^4} - \frac{\ln\left(bx + a\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^4,x)
```

```
[Out] 1/3/a/(b*x+a)^3+1/2/a^2/(b*x+a)^2+1/a^3/(b*x+a)+ln(x)/a^4-ln(b*x+a)/a^4
```

maxima [A] time = 1.47, size = 73, normalized size = 1.28

$$\frac{6\,b^2x^2 + 15\,abx + 11\,a^2}{6\left(a^3b^3x^3 + 3\,a^4b^2x^2 + 3\,a^5bx + a^6\right)} - \frac{\log\left(bx + a\right)}{a^4} + \frac{\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*b^2*x^2 + 15*a*b*x + 11*a^2)/(a^3*b^3*x^3 + 3*a^4*b^2*x^2 + 3*a^5*b*
x + a^6) - log(b*x + a)/a^4 + log(x)/a^4
```

mupad [B] time = 0.13, size = 60, normalized size = 1.05

$$\frac{\frac{\frac{1}{a^2+b\,x\,a}-\frac{\ln\left(\frac{a+b\,x}{x}\right)}{a^2}}{a}+\frac{1}{2\,a\,{\left(a+b\,x\right)}^2}}{a}+\frac{1}{3\,a\,{\left(a+b\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^4),x)
```

```
[Out] ((1/(a^2 + a*b*x) - log((a + b*x)/x)/a^2)/a + 1/(2*a*(a + b*x)^2))/a + 1/(3
*a*(a + b*x)^3)
```

sympy [A] time = 0.44, size = 70, normalized size = 1.23

$$\frac{11a^2 + 15abx + 6b^2x^2}{6a^6 + 18a^5bx + 18a^4b^2x^2 + 6a^3b^3x^3} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**4,x)
```

```
[Out] (11*a**2 + 15*a*b*x + 6*b**2*x**2)/(6*a**6 + 18*a**5*b*x + 18*a**4*b**2*x**
2 + 6*a**3*b**3*x**3) + (log(x) - log(a/b + x))/a**4
```

3.203 $\int \frac{1}{x^2(a+bx)^4}\,dx$

Optimal. Leaf size=70

$$-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{3b}{a^4(a+bx)}-\frac{1}{a^4x}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}$$

Rubi [A] time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{3b}{a^4(a+bx)}-\frac{b}{a^3(a+bx)^2}-\frac{b}{3a^2(a+bx)^3}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}-\frac{1}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -(1/(a^4*x)) - b/(3*a^2*(a + b*x)^3) - b/(a^3*(a + b*x)^2) - (3*b)/(a^4*(a
+ b*x)) - (4*b*Log[x])/a^5 + (4*b*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^4}\,dx &= \int \left(\frac{1}{a^4x^2}-\frac{4b}{a^5x}+\frac{b^2}{a^2(a+bx)^4}+\frac{2b^2}{a^3(a+bx)^3}+\frac{3b^2}{a^4(a+bx)^2}+\frac{4b^2}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{a^4x}-\frac{b}{3a^2(a+bx)^3}-\frac{b}{a^3(a+bx)^2}-\frac{3b}{a^4(a+bx)}-\frac{4b\log(x)}{a^5}+\frac{4b\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 64, normalized size = 0.91

$$-\frac{\frac{a\left(3a^3+22a^2bx+30ab^2x^2+12b^3x^3\right)}{x(a+bx)^3}-12b\log(a+bx)+12b\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(3*a^3 + 22*a^2*b*x + 30*a*b^2*x^2 + 12*b^3*x^3))/(x*(a + b*x)^3)
+ 12*b*Log[x] - 12*b*Log[a + b*x])/a^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^4), x]
```

fricas [B] time = 1.10, size = 153, normalized size = 2.19

$$-\frac{12 \, a b^{3} x^{3} + 30 \, a^{2} b^{2} x^{2} + 22 \, a^{3} b x + 3 \, a^{4} - 12 \, {\left(b^{4} x^{4} + 3 \, a b^{3} x^{3} + 3 \, a^{2} b^{2} x^{2} + a^{3} b x\right)} \log\left(b x + a\right) + 12 \, {\left(b^{4} x^{4} + 3 \, a b^{3} x^{3} + 3 \, a^{2} b^{2} x^{2} + a^{3} b x\right)} \log\left(x\right)}{3 \, {\left(a^{5} b^{3} x^{4} + 3 \, a^{6} b^{2} x^{3} + 3 \, a^{7} b x^{2} + a^{8} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 30*a^2*b^2*x^2 + 22*a^3*b*x + 3*a^4 - 12*(b^4*x^4 + 3*
a*b^3*x^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(b*x + a) + 12*(b^4*x^4 + 3*a*b^3*x
^3 + 3*a^2*b^2*x^2 + a^3*b*x)*log(x))/(a^5*b^3*x^4 + 3*a^6*b^2*x^3 + 3*a^7*
b*x^2 + a^8*x)
```

giac [A] time = 1.16, size = 71, normalized size = 1.01

$$\frac{4 \, b \log\left({\left| b x + a \right|}\right)}{a^{5}} - \frac{4 \, b \log\left({\left| x \right|}\right)}{a^{5}} - \frac{12 \, a b^{3} x^{3} + 30 \, a^{2} b^{2} x^{2} + 22 \, a^{3} b x + 3 \, a^{4}}{3 \, {\left(b x + a\right)}^{3} a^{5} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 4*b*log(abs(b*x + a))/a^5 - 4*b*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 30*a^
2*b^2*x^2 + 22*a^3*b*x + 3*a^4)/((b*x + a)^3*a^5*x)
```

maple [A] time = 0.01, size = 69, normalized size = 0.99

$$-\frac{b}{3 \left(b x +a \right)^{3} a^{2}}-\frac{b}{\left(b x +a \right)^{2} a^{3}}-\frac{3 b}{\left(b x +a \right) a^{4}}-\frac{4 b \ln \left(x \right)}{a^{5}}+\frac{4 b \ln \left(b x +a \right)}{a^{5}}-\frac{1}{a^{4} x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^4,x)
```

```
[Out] -1/a^4/x-1/3*b/a^2/(b*x+a)^3-b/a^3/(b*x+a)^2-3*b/a^4/(b*x+a)-4*b*ln(x)/a^5+
4*b*ln(b*x+a)/a^5
```

maxima [A] time = 1.40, size = 91, normalized size = 1.30

$$-\frac{12 \, b^{3} x^{3} + 30 \, a b^{2} x^{2} + 22 \, a^{2} b x + 3 \, a^{3}}{3 \, {\left(a^{4} b^{3} x^{4} + 3 \, a^{5} b^{2} x^{3} + 3 \, a^{6} b x^{2} + a^{7} x\right)}} + \frac{4 \, b \log\left(b x + a\right)}{a^{5}} - \frac{4 \, b \log\left(x\right)}{a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 30*a*b^2*x^2 + 22*a^2*b*x + 3*a^3)/(a^4*b^3*x^4 + 3*a^5*
b^2*x^3 + 3*a^6*b*x^2 + a^7*x) + 4*b*log(b*x + a)/a^5 - 4*b*log(x)/a^5
```

mupad [B] time = 0.08, size = 85, normalized size = 1.21

$$\frac{8\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{a}+\frac{10\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}+\frac{22\,b\,x}{3\,a^2}}{a^3\,x+3\,a^2\,b\,x^2+3\,a\,b^2\,x^3+b^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^4),x)
```

```
[Out] (8*b*atanh((2*b*x)/a + 1))/a^5 - (1/a + (10*b^2*x^2)/a^3 + (4*b^3*x^3)/a^4
+ (22*b*x)/(3*a^2))/(a^3*x + b^3*x^4 + 3*a^2*b*x^2 + 3*a*b^2*x^3)
```

sympy [A] time = 0.46, size = 90, normalized size = 1.29

$$\frac{-3a^3-22a^2bx-30ab^2x^2-12b^3x^3}{3a^7x+9a^6bx^2+9a^5b^2x^3+3a^4b^3x^4}+\frac{4b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**4,x)
```

```
[Out] (-3*a**3 - 22*a**2*b*x - 30*a*b**2*x**2 - 12*b**3*x**3)/(3*a**7*x + 9*a**6*
b*x**2 + 9*a**5*b**2*x**3 + 3*a**4*b**3*x**4) + 4*b*(-log(x) + log(a/b + x)
)/a**5
```

3.204 $\int \frac{1}{x^3(a+bx)^4} dx$

Optimal. Leaf size=93

$$\frac{10b^2\log(x)}{a^6} - \frac{10b^2\log(a+bx)}{a^6} + \frac{6b^2}{a^5(a+bx)} + \frac{4b}{a^5x} + \frac{3b^2}{2a^4(a+bx)^2} - \frac{1}{2a^4x^2} + \frac{b^2}{3a^3(a+bx)^3}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{6b^2}{a^5(a+bx)} + \frac{3b^2}{2a^4(a+bx)^2} + \frac{b^2}{3a^3(a+bx)^3} + \frac{10b^2\log(x)}{a^6} - \frac{10b^2\log(a+bx)}{a^6} + \frac{4b}{a^5x} - \frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^4),x]
```

```
[Out] -1/(2*a^4*x^2) + (4*b)/(a^5*x) + b^2/(3*a^3*(a + b*x)^3) + (3*b^2)/(2*a^4*(
a + b*x)^2) + (6*b^2)/(a^5*(a + b*x)) + (10*b^2*Log[x])/a^6 - (10*b^2*Log[a
 + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^4} dx &= \int \left(\frac{1}{a^4x^3} - \frac{4b}{a^5x^2} + \frac{10b^2}{a^6x} - \frac{b^3}{a^3(a+bx)^4} - \frac{3b^3}{a^4(a+bx)^3} - \frac{6b^3}{a^5(a+bx)^2} - \frac{10b^3}{a^6(a+bx)}\right) dx \\ &= -\frac{1}{2a^4x^2} + \frac{4b}{a^5x} + \frac{b^2}{3a^3(a+bx)^3} + \frac{3b^2}{2a^4(a+bx)^2} + \frac{6b^2}{a^5(a+bx)} + \frac{10b^2\log(x)}{a^6} - \frac{10b^2\log(a+\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.85

$$\frac{\frac{a\left(-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4\right)}{x^2(a+bx)^3} - 60b^2\log(a+bx) + 60b^2\log(x)}{6a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^4 + 15*a^3*b*x + 110*a^2*b^2*x^2 + 150*a*b^3*x^3 + 60*b^4*x^4))/(
x^2*(a + b*x)^3) + 60*b^2*Log[x] - 60*b^2*Log[a + b*x])/(6*a^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^4),x]
```

[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^4), x]

fricas [A] time = 0.85, size = 174, normalized size = 1.87

$$\frac{60\,ab^{4}x^{4} + 150\,a^{2}b^{3}x^{3} + 110\,a^{3}b^{2}x^{2} + 15\,a^{4}bx - 3\,a^{5} - 60\,\left(b^{5}x^{5} + 3\,ab^{4}x^{4} + 3\,a^{2}b^{3}x^{3} + a^{3}b^{2}x^{2}\right)\log\left(bx + a\right) + 60\,\left(b^{5}x^{5} + 3\,ab^{4}x^{4} + 3\,a^{2}b^{3}x^{3} + a^{3}b^{2}x^{2}\right)\log\left(x\right)}{6\,\left(a^{6}b^{3}x^{5} + 3\,a^{7}b^{2}x^{4} + 3\,a^{8}bx^{3} + a^{9}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*a*b^4*x^4 + 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5
- 60*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(b*x + a) + 6
0*(b^5*x^5 + 3*a*b^4*x^4 + 3*a^2*b^3*x^3 + a^3*b^2*x^2)*log(x))/(a^6*b^3*x^
5 + 3*a^7*b^2*x^4 + 3*a^8*b*x^3 + a^9*x^2)
```

giac [A] time = 0.95, size = 86, normalized size = 0.92

$$-\frac{10\,b^{2}\log\left(|bx + a|\right)}{a^{6}} + \frac{10\,b^{2}\log\left(|x|\right)}{a^{6}} + \frac{60\,ab^{4}x^{4} + 150\,a^{2}b^{3}x^{3} + 110\,a^{3}b^{2}x^{2} + 15\,a^{4}bx - 3\,a^{5}}{6\,\left(bx + a\right)^{3}a^{6}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -10*b^2*log(abs(b*x + a))/a^6 + 10*b^2*log(abs(x))/a^6 + 1/6*(60*a*b^4*x^4
+ 150*a^2*b^3*x^3 + 110*a^3*b^2*x^2 + 15*a^4*b*x - 3*a^5)/((b*x + a)^3*a^6*
x^2)
```

maple [A] time = 0.01, size = 88, normalized size = 0.95

$$\frac{b^{2}}{3\left(bx + a\right)^{3}a^{3}} + \frac{3b^{2}}{2\left(bx + a\right)^{2}a^{4}} + \frac{6b^{2}}{\left(bx + a\right)a^{5}} + \frac{10b^{2}\ln(x)}{a^{6}} - \frac{10b^{2}\ln\left(bx + a\right)}{a^{6}} + \frac{4b}{a^{5}x} - \frac{1}{2a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^4,x)
```

```
[Out] -1/2/a^4/x^2+4*b/a^5/x+1/3*b^2/a^3/(b*x+a)^3+3/2*b^2/a^4/(b*x+a)^2+6*b^2/a^
5/(b*x+a)+10*b^2*ln(x)/a^6-10*b^2*ln(b*x+a)/a^6
```

maxima [A] time = 1.41, size = 108, normalized size = 1.16

$$\frac{60\,b^{4}x^{4} + 150\,ab^{3}x^{3} + 110\,a^{2}b^{2}x^{2} + 15\,a^{3}bx - 3\,a^{4}}{6\,\left(a^{5}b^{3}x^{5} + 3\,a^{6}b^{2}x^{4} + 3\,a^{7}bx^{3} + a^{8}x^{2}\right)} - \frac{10\,b^{2}\log\left(bx + a\right)}{a^{6}} + \frac{10\,b^{2}\log(x)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*b^4*x^4 + 150*a*b^3*x^3 + 110*a^2*b^2*x^2 + 15*a^3*b*x - 3*a^4)/(a^
5*b^3*x^5 + 3*a^6*b^2*x^4 + 3*a^7*b*x^3 + a^8*x^2) - 10*b^2*log(b*x + a)/a^
6 + 10*b^2*log(x)/a^6
```

mupad [B] time = 0.14, size = 101, normalized size = 1.09

$$\frac{\frac{55\,b^{2}\,x^{2}}{3\,a^{3}} - \frac{1}{2\,a} + \frac{25\,b^{3}\,x^{3}}{a^{4}} + \frac{10\,b^{4}\,x^{4}}{a^{5}} + \frac{5\,b\,x}{2\,a^{2}}}{a^{3}\,x^{2} + 3\,a^{2}\,b\,x^{3} + 3\,a\,b^{2}\,x^{4} + b^{3}\,x^{5}} - \frac{20\,b^{2}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^4),x)
```

```
[Out] ((55*b^2*x^2)/(3*a^3) - 1/(2*a) + (25*b^3*x^3)/a^4 + (10*b^4*x^4)/a^5 + (5*
b*x)/(2*a^2))/(a^3*x^2 + b^3*x^5 + 3*a^2*b*x^3 + 3*a*b^2*x^4) - (20*b^2*ata
nh((2*b*x)/a + 1))/a^6
```

sympy [A] time = 0.56, size = 104, normalized size = 1.12

$$\frac{-3a^4+15a^3bx+110a^2b^2x^2+150ab^3x^3+60b^4x^4}{6a^8x^2+18a^7bx^3+18a^6b^2x^4+6a^5b^3x^5}+\frac{10b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**4,x)
```

```
[Out] (-3*a**4 + 15*a**3*b*x + 110*a**2*b**2*x**2 + 150*a*b**3*x**3 + 60*b**4*x**
4)/(6*a**8*x**2 + 18*a**7*b*x**3 + 18*a**6*b**2*x**4 + 6*a**5*b**3*x**5) +
10*b**2*(log(x) - log(a/b + x))/a**6
```

3.205 $\int \frac{1}{x^4(a+bx)^4} dx$

Optimal. Leaf size=102

$$-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}-\frac{10b^3}{a^6(a+bx)}-\frac{10b^2}{a^6x}-\frac{2b^3}{a^5(a+bx)^2}+\frac{2b}{a^5x^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{1}{3a^4x^3}$$

Rubi [A] time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{10b^3}{a^6(a+bx)}-\frac{2b^3}{a^5(a+bx)^2}-\frac{b^3}{3a^4(a+bx)^3}-\frac{10b^2}{a^6x}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log(a+bx)}{a^7}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/(3*a^4*x^3) + (2*b)/(a^5*x^2) - (10*b^2)/(a^6*x) - b^3/(3*a^4*(a + b*x)^
3) - (2*b^3)/(a^5*(a + b*x)^2) - (10*b^3)/(a^6*(a + b*x)) - (20*b^3*Log[x])
/a^7 + (20*b^3*Log[a + b*x])/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^4} dx &= \int \left(\frac{1}{a^4x^4}-\frac{4b}{a^5x^3}+\frac{10b^2}{a^6x^2}-\frac{20b^3}{a^7x}+\frac{b^4}{a^4(a+bx)^4}+\frac{4b^4}{a^5(a+bx)^3}+\frac{10b^4}{a^6(a+bx)^2}+\frac{20b^4}{a^7(a+bx)}\right) d\\ &= -\frac{1}{3a^4x^3}+\frac{2b}{a^5x^2}-\frac{10b^2}{a^6x}-\frac{b^3}{3a^4(a+bx)^3}-\frac{2b^3}{a^5(a+bx)^2}-\frac{10b^3}{a^6(a+bx)}-\frac{20b^3\log(x)}{a^7}+\frac{20b^3\log}{a}\end{aligned}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.86

$$-\frac{\frac{a\left(a^5-3a^4bx+15a^3b^2x^2+110a^2b^3x^3+150ab^4x^4+60b^5x^5\right)}{x^3(a+bx)^3}-60b^3\log(a+bx)+60b^3\log(x)}{3a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^4),x]
```

```
[Out] -1/3*((a*(a^5 - 3*a^4*b*x + 15*a^3*b^2*x^2 + 110*a^2*b^3*x^3 + 150*a*b^4*x^
4 + 60*b^5*x^5))/(x^3*(a + b*x)^3) + 60*b^3*Log[x] - 60*b^3*Log[a + b*x])/a
^7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^4),x]
```

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^4), x]

fricas [A] time = 0.74, size = 183, normalized size = 1.79

$$-\frac{60\,ab^5x^5+150\,a^2b^4x^4+110\,a^3b^3x^3+15\,a^4b^2x^2-3\,a^5bx+a^6-60\,\left(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3\right)\log\left(bx+a\right)+60\,\left(b^6x^6+3\,ab^5x^5+3\,a^2b^4x^4+a^3b^3x^3\right)\log\left(x\right)}{3\,\left(a^7b^3x^6+3\,a^8b^2x^5+3\,a^9bx^4+a^{10}x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(60*a*b^5*x^5 + 150*a^2*b^4*x^4 + 110*a^3*b^3*x^3 + 15*a^4*b^2*x^2 - 3
*a^5*b*x + a^6 - 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*l
og(b*x + a) + 60*(b^6*x^6 + 3*a*b^5*x^5 + 3*a^2*b^4*x^4 + a^3*b^3*x^3)*log(
x))/(a^7*b^3*x^6 + 3*a^8*b^2*x^5 + 3*a^9*b*x^4 + a^10*x^3)
```

giac [A] time = 0.99, size = 93, normalized size = 0.91

$$\frac{20\,b^3\log\left(|bx+a|\right)}{a^7}-\frac{20\,b^3\log\left(|x|\right)}{a^7}-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,\left(bx^2+ax\right)^3a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 20*b^3*log(abs(b*x + a))/a^7 - 20*b^3*log(abs(x))/a^7 - 1/3*(60*b^5*x^5 + 1
50*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4*b*x + a^5)/((b*x^2
+ a*x)^3*a^6)
```

maple [A] time = 0.01, size = 99, normalized size = 0.97

$$-\frac{b^3}{3\left(bx+a\right)^3a^4}-\frac{2b^3}{\left(bx+a\right)^2a^5}-\frac{10b^3}{\left(bx+a\right)a^6}-\frac{20b^3\ln(x)}{a^7}+\frac{20b^3\ln\left(bx+a\right)}{a^7}-\frac{10b^2}{a^6x}+\frac{2b}{a^5x^2}-\frac{1}{3a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^4,x)
```

```
[Out] -1/3/a^4/x^3+2*b/a^5/x^2-10*b^2/a^6/x-1/3*b^3/a^4/(b*x+a)^3-2*b^3/a^5/(b*x+
a)^2-10*b^3/a^6/(b*x+a)-20*b^3*ln(x)/a^7+20*b^3*ln(b*x+a)/a^7
```

maxima [A] time = 1.40, size = 117, normalized size = 1.15

$$-\frac{60\,b^5x^5+150\,ab^4x^4+110\,a^2b^3x^3+15\,a^3b^2x^2-3\,a^4bx+a^5}{3\,\left(a^6b^3x^6+3\,a^7b^2x^5+3\,a^8bx^4+a^9x^3\right)}+\frac{20\,b^3\log\left(bx+a\right)}{a^7}-\frac{20\,b^3\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(60*b^5*x^5 + 150*a*b^4*x^4 + 110*a^2*b^3*x^3 + 15*a^3*b^2*x^2 - 3*a^4
*b*x + a^5)/(a^6*b^3*x^6 + 3*a^7*b^2*x^5 + 3*a^8*b*x^4 + a^9*x^3) + 20*b^3*
log(b*x + a)/a^7 - 20*b^3*log(x)/a^7
```

mupad [B] time = 0.10, size = 113, normalized size = 1.11

$$\frac{40\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^7}-\frac{\frac{1}{3\,a}+\frac{5\,b^2\,x^2}{a^3}+\frac{110\,b^3\,x^3}{3\,a^4}+\frac{50\,b^4\,x^4}{a^5}+\frac{20\,b^5\,x^5}{a^6}-\frac{b\,x}{a^2}}{a^3\,x^3+3\,a^2\,b\,x^4+3\,a\,b^2\,x^5+b^3\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^4),x)
```

```
[Out] (40*b^3*atanh((2*b*x)/a + 1))/a^7 - (1/(3*a) + (5*b^2*x^2)/a^3 + (110*b^3*x
^3)/(3*a^4) + (50*b^4*x^4)/a^5 + (20*b^5*x^5)/a^6 - (b*x)/a^2)/(a^3*x^3 + b
^3*x^6 + 3*a^2*b*x^4 + 3*a*b^2*x^5)
```

sympy [A] time = 0.53, size = 114, normalized size = 1.12

$$\frac{-a^5+3a^4bx-15a^3b^2x^2-110a^2b^3x^3-150ab^4x^4-60b^5x^5}{3a^9x^3+9a^8bx^4+9a^7b^2x^5+3a^6b^3x^6}+\frac{20b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**4,x)
```

```
[Out] (-a**5 + 3*a**4*b*x - 15*a**3*b**2*x**2 - 110*a**2*b**3*x**3 - 150*a*b**4*x
**4 - 60*b**5*x**5)/(3*a**9*x**3 + 9*a**8*b*x**4 + 9*a**7*b**2*x**5 + 3*a**
6*b**3*x**6) + 20*b**3*(-log(x) + log(a/b + x))/a**7
```

3.206 $\int \frac{1}{x^5(a+bx)^4}\,dx$

Optimal. Leaf size=117

$$\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{15b^4}{a^7(a+bx)}+\frac{20b^3}{a^7x}+\frac{5b^4}{2a^6(a+bx)^2}-\frac{5b^2}{a^6x^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{5b^2}{a^6x^2}+\frac{15b^4}{a^7(a+bx)}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{b^4}{3a^5(a+bx)^3}+\frac{20b^3}{a^7x}+\frac{35b^4\log(x)}{a^8}-\frac{35b^4\log(a+bx)}{a^8}+\frac{4b}{3a^5x^3}-\frac{1}{4a^4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a + b*x)^4),x]
```

```
[Out] -1/(4*a^4*x^4) + (4*b)/(3*a^5*x^3) - (5*b^2)/(a^6*x^2) + (20*b^3)/(a^7*x) +
 b^4/(3*a^5*(a + b*x)^3) + (5*b^4)/(2*a^6*(a + b*x)^2) + (15*b^4)/(a^7*(a +
 b*x)) + (35*b^4*Log[x])/a^8 - (35*b^4*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5(a+bx)^4}\,dx = \int \left(\frac{1}{a^4x^5}-\frac{4b}{a^5x^4}+\frac{10b^2}{a^6x^3}-\frac{20b^3}{a^7x^2}+\frac{35b^4}{a^8x}-\frac{b^5}{a^5(a+bx)^4}-\frac{5b^5}{a^6(a+bx)^3}-\frac{15b^5}{a^7(a+bx)^2}-\cdots\right)$$
$$= -\frac{1}{4a^4x^4}+\frac{4b}{3a^5x^3}-\frac{5b^2}{a^6x^2}+\frac{20b^3}{a^7x}+\frac{b^4}{3a^5(a+bx)^3}+\frac{5b^4}{2a^6(a+bx)^2}+\frac{15b^4}{a^7(a+bx)}+\frac{35b^4\log\cdots}{a^8}$$

Mathematica [A] time = 0.07, size = 101, normalized size = 0.86

$$\frac{\frac{a\left(-3a^6+7a^5bx-21a^4b^2x^2+105a^3b^3x^3+770a^2b^4x^4+1050ab^5x^5+420b^6x^6\right)}{x^4(a+bx)^3}-420b^4\log(a+bx)+420b^4\log(x)}{12a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a + b*x)^4),x]
```

```
[Out] ((a*(-3*a^6 + 7*a^5*b*x - 21*a^4*b^2*x^2 + 105*a^3*b^3*x^3 + 770*a^2*b^4*x^
4 + 1050*a*b^5*x^5 + 420*b^6*x^6))/(x^4*(a + b*x)^3) + 420*b^4*Log[x] - 420
*b^4*Log[a + b*x])/(12*a^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(a + b*x)^4),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(a + b*x)^4), x]
```

fricas [A] time = 1.13, size = 196, normalized size = 1.68

$$\frac{420\,ab^{6}x^{6}+1050\,a^{2}b^{5}x^{5}+770\,a^{3}b^{4}x^{4}+105\,a^{4}b^{3}x^{3}-21\,a^{5}b^{2}x^{2}+7\,a^{6}bx-3\,a^{7}-420\left(b^{7}x^{7}+3\,ab^{6}x^{6}+3\,a^{2}b^{5}x^{5}+a^{3}b^{4}x^{4}\right)\log\left(bx+a\right)+420\left(b^{7}x^{7}+3\,ab^{6}x^{6}+3\,a^{2}b^{5}x^{5}+a^{3}b^{4}x^{4}\right)\log\left(x\right)}{12\left(a^{8}b^{3}x^{7}+3\,a^{9}b^{2}x^{6}+3\,a^{10}bx^{5}+a^{11}x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(420*a*b^6*x^6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3
- 21*a^5*b^2*x^2 + 7*a^6*b*x - 3*a^7 - 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b
^5*x^5 + a^3*b^4*x^4)*log(b*x + a) + 420*(b^7*x^7 + 3*a*b^6*x^6 + 3*a^2*b^5
*x^5 + a^3*b^4*x^4)*log(x))/(a^8*b^3*x^7 + 3*a^9*b^2*x^6 + 3*a^10*b*x^5 + a
^11*x^4)
```

giac [A] time = 1.00, size = 108, normalized size = 0.92

$$-\frac{35\,b^{4}\log\left(|bx+a|\right)}{a^{8}}+\frac{35\,b^{4}\log\left(|x|\right)}{a^{8}}+\frac{420\,ab^{6}x^{6}+1050\,a^{2}b^{5}x^{5}+770\,a^{3}b^{4}x^{4}+105\,a^{4}b^{3}x^{3}-21\,a^{5}b^{2}x^{2}+7\,a^{6}bx-3\,a^{7}}{12\,(bx+a)^{3}a^{8}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -35*b^4*log(abs(b*x + a))/a^8 + 35*b^4*log(abs(x))/a^8 + 1/12*(420*a*b^6*x^
6 + 1050*a^2*b^5*x^5 + 770*a^3*b^4*x^4 + 105*a^4*b^3*x^3 - 21*a^5*b^2*x^2 +
 7*a^6*b*x - 3*a^7)/((b*x + a)^3*a^8*x^4)
```

maple [A] time = 0.01, size = 110, normalized size = 0.94

$$\frac{b^{4}}{3\,(bx+a)^{3}\,a^{5}}+\frac{5b^{4}}{2\,(bx+a)^{2}\,a^{6}}+\frac{15b^{4}}{(bx+a)\,a^{7}}+\frac{35b^{4}\ln(x)}{a^{8}}-\frac{35b^{4}\ln\left(bx+a\right)}{a^{8}}+\frac{20b^{3}}{a^{7}x}-\frac{5b^{2}}{a^{6}x^{2}}+\frac{4b}{3a^{5}x^{3}}-\frac{1}{4a^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x+a)^4,x)
```

```
[Out] -1/4/a^4/x^4+4/3*b/a^5/x^3-5*b^2/a^6/x^2+20*b^3/a^7/x+1/3*b^4/a^5/(b*x+a)^3
+5/2*b^4/a^6/(b*x+a)^2+15*b^4/a^7/(b*x+a)+35*b^4*ln(x)/a^8-35*b^4*ln(b*x+a)
/a^8
```

maxima [A] time = 1.39, size = 130, normalized size = 1.11

$$\frac{420\,b^{6}x^{6}+1050\,ab^{5}x^{5}+770\,a^{2}b^{4}x^{4}+105\,a^{3}b^{3}x^{3}-21\,a^{4}b^{2}x^{2}+7\,a^{5}bx-3\,a^{6}}{12\left(a^{7}b^{3}x^{7}+3\,a^{8}b^{2}x^{6}+3\,a^{9}bx^{5}+a^{10}x^{4}\right)}-\frac{35\,b^{4}\log\left(bx+a\right)}{a^{8}}+\frac{35\,b^{4}\log(x)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(420*b^6*x^6 + 1050*a*b^5*x^5 + 770*a^2*b^4*x^4 + 105*a^3*b^3*x^3 - 21
*a^4*b^2*x^2 + 7*a^5*b*x - 3*a^6)/(a^7*b^3*x^7 + 3*a^8*b^2*x^6 + 3*a^9*b*x^
5 + a^10*x^4) - 35*b^4*log(b*x + a)/a^8 + 35*b^4*log(x)/a^8
```

mupad [B] time = 0.17, size = 123, normalized size = 1.05

$$\frac{\frac{35\,b^{3}\,x^{3}}{4\,a^{4}}-\frac{7\,b^{2}\,x^{2}}{4\,a^{3}}-\frac{1}{4\,a}+\frac{385\,b^{4}\,x^{4}}{6\,a^{5}}+\frac{175\,b^{5}\,x^{5}}{2\,a^{6}}+\frac{35\,b^{6}\,x^{6}}{a^{7}}+\frac{7\,b\,x}{12\,a^{2}}}{a^{3}\,x^{4}+3\,a^{2}\,b\,x^{5}+3\,a\,b^{2}\,x^{6}+b^{3}\,x^{7}}-\frac{70\,b^{4}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + b*x)^4),x)
```

[Out] ((35*b^3*x^3)/(4*a^4) - (7*b^2*x^2)/(4*a^3) - 1/(4*a) + (385*b^4*x^4)/(6*a^5) + (175*b^5*x^5)/(2*a^6) + (35*b^6*x^6)/a^7 + (7*b*x)/(12*a^2))/(a^3*x^4 + b^3*x^7 + 3*a^2*b*x^5 + 3*a*b^2*x^6) - (70*b^4*atanh((2*b*x)/a + 1))/a^8

sympy [A] time = 0.58, size = 128, normalized size = 1.09

$$\frac{-3a^6 + 7a^5bx - 21a^4b^2x^2 + 105a^3b^3x^3 + 770a^2b^4x^4 + 1050ab^5x^5 + 420b^6x^6}{12a^{10}x^4 + 36a^9bx^5 + 36a^8b^2x^6 + 12a^7b^3x^7} + \frac{35b^4\left(\log(x) - \log\left(\frac{a}{b} + x\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(b*x+a)**4,x)

[Out] (-3*a**6 + 7*a**5*b*x - 21*a**4*b**2*x**2 + 105*a**3*b**3*x**3 + 770*a**2*b**4*x**4 + 1050*a*b**5*x**5 + 420*b**6*x**6)/(12*a**10*x**4 + 36*a**9*b*x**5 + 36*a**8*b**2*x**6 + 12*a**7*b**3*x**7) + 35*b**4*(log(x) - log(a/b + x))/a**8

3.207 $\int \frac{x^{10}}{(a+bx)^7} dx$

Optimal. Leaf size=150

$$-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{84a^3}{b^{10}}$$

Rubi [A] time = 0.14, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{14a^2x^2}{b^9}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx)^3}-\frac{105a^6}{b^{11}(a+bx)^2}+\frac{252a^5}{b^{11}(a+bx)}-\frac{84a^3x}{b^{10}}+\frac{210a^4\log(a+bx)}{b^{11}}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^7,x]
```

```
[Out] (-84*a^3*x)/b^10 + (14*a^2*x^2)/b^9 - (7*a*x^3)/(3*b^8) + x^4/(4*b^7) - a^1
0/(6*b^11*(a + b*x)^6) + (2*a^9)/(b^11*(a + b*x)^5) - (45*a^8)/(4*b^11*(a +
 b*x)^4) + (40*a^7)/(b^11*(a + b*x)^3) - (105*a^6)/(b^11*(a + b*x)^2) + (25
2*a^5)/(b^11*(a + b*x)) + (210*a^4*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx)^7} dx = \int \left(-\frac{84a^3}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{b^8}+\frac{x^3}{b^7}+\frac{a^{10}}{b^{10}(a+bx)^7}-\frac{10a^9}{b^{10}(a+bx)^6}+\frac{45a^8}{b^{10}(a+bx)^5}-\frac{120a^7}{b^{10}(a+bx)^4}\right.$$
$$=-\frac{84a^3x}{b^{10}}+\frac{14a^2x^2}{b^9}-\frac{7ax^3}{3b^8}+\frac{x^4}{4b^7}-\frac{a^{10}}{6b^{11}(a+bx)^6}+\frac{2a^9}{b^{11}(a+bx)^5}-\frac{45a^8}{4b^{11}(a+bx)^4}+\frac{40a^7}{b^{11}(a+bx}$$

Mathematica [A] time = 0.03, size = 139, normalized size = 0.93

$$\frac{2131a^{10}+10266a^9bx+18105a^8b^2x^2+11540a^7b^3x^3-3945a^6b^4x^4-9138a^5b^5x^5-4043a^4b^6x^6+2520a^4(a+bx)^6\log(a+bx)-360a^3b^7x^7+45a^2b^8x^8-10ab^9x^9+3b^{10}x^{10}}{12b^{11}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^7,x]
```

```
[Out] (2131*a^10 + 10266*a^9*b*x + 18105*a^8*b^2*x^2 + 11540*a^7*b^3*x^3 - 3945*a
^6*b^4*x^4 - 9138*a^5*b^5*x^5 - 4043*a^4*b^6*x^6 - 360*a^3*b^7*x^7 + 45*a^2
*b^8*x^8 - 10*a*b^9*x^9 + 3*b^10*x^10 + 2520*a^4*(a + b*x)^6*Log[a + b*x])/
(12*b^11*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x)^7,x]
```

[Out] IntegrateAlgebraic[x^10/(a + b*x)^7, x]

fricas [A] time = 1.03, size = 250, normalized size = 1.67

$$\frac{3\,b^{10}x^{10} - 10\,ab^{9}x^{9} + 45\,a^{2}b^{8}x^{8} - 360\,a^{3}b^{7}x^{7} - 4043\,a^{4}b^{6}x^{6} - 9138\,a^{5}b^{5}x^{5} - 3945\,a^{6}b^{4}x^{4} + 11540\,a^{7}b^{3}x^{3} + 18105\,a^{8}b^{2}x^{2} + 10266\,a^{9}bx + 2131\,a^{10} + 2520\left(a^{4}b^{6}x^{6} + 6\,a^{5}b^{5}x^{5} + 15\,a^{6}b^{4}x^{4} + 20\,a^{7}b^{3}x^{3} + 15\,a^{8}b^{2}x^{2} + 6\,a^{9}bx + a^{10}\right)\log\left(bx + a\right)}{12\left(b^{17}x^{6} + 6\,ab^{16}x^{5} + 15\,a^{2}b^{15}x^{4} + 20\,a^{3}b^{14}x^{3} + 15\,a^{4}b^{13}x^{2} + 6\,a^{5}b^{12}x + a^{6}b^{11}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*x^10 - 10*a*b^9*x^9 + 45*a^2*b^8*x^8 - 360*a^3*b^7*x^7 - 4043*
a^4*b^6*x^6 - 9138*a^5*b^5*x^5 - 3945*a^6*b^4*x^4 + 11540*a^7*b^3*x^3 + 181
05*a^8*b^2*x^2 + 10266*a^9*b*x + 2131*a^10 + 2520*(a^4*b^6*x^6 + 6*a^5*b^5*
x^5 + 15*a^6*b^4*x^4 + 20*a^7*b^3*x^3 + 15*a^8*b^2*x^2 + 6*a^9*b*x + a^10)*
log(b*x + a))/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3
+ 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11)
```

giac [A] time = 1.21, size = 128, normalized size = 0.85

$$\frac{210\,a^{4}\log\left(\left|bx + a\right|\right)}{b^{11}} + \frac{3024\,a^{5}b^{5}x^{5} + 13860\,a^{6}b^{4}x^{4} + 25680\,a^{7}b^{3}x^{3} + 23985\,a^{8}b^{2}x^{2} + 11274\,a^{9}bx + 2131\,a^{10}}{12\,(bx + a)^{6}b^{11}} + \frac{3\,b^{21}x^{4} - 28\,ab^{20}x^{3} + 168\,a^{2}b^{19}x^{2} - 1008\,a^{3}b^{18}x}{12\,b^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*a^4*log(abs(b*x + a))/b^11 + 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4
 + 25680*a^7*b^3*x^3 + 23985*a^8*b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/((b*x
 + a)^6*b^11) + 1/12*(3*b^21*x^4 - 28*a*b^20*x^3 + 168*a^2*b^19*x^2 - 1008*
a^3*b^18*x)/b^28
```

maple [A] time = 0.01, size = 143, normalized size = 0.95

$$-\frac{a^{10}}{6\,(bx + a)^{6}\,b^{11}} + \frac{2a^{9}}{(bx + a)^{5}\,b^{11}} - \frac{45a^{8}}{4\,(bx + a)^{4}\,b^{11}} + \frac{x^{4}}{4b^{7}} + \frac{40a^{7}}{(bx + a)^{3}\,b^{11}} - \frac{7a\,x^{3}}{3b^{8}} - \frac{105a^{6}}{(bx + a)^{2}\,b^{11}} + \frac{14a^{2}x^{2}}{b^{9}} + \frac{252a^{5}}{(bx + a)\,b^{11}} + \frac{210a^{4}\ln\left(bx + a\right)}{b^{11}} - \frac{84a^{3}x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^7,x)
```

```
[Out] -84*a^3*x/b^10+14*a^2*x^2/b^9-7/3*a*x^3/b^8+1/4*x^4/b^7-1/6*a^10/b^11/(b*x+
a)^6+2*a^9/b^11/(b*x+a)^5-45/4*a^8/b^11/(b*x+a)^4+40*a^7/b^11/(b*x+a)^3-105
*a^6/b^11/(b*x+a)^2+252*a^5/b^11/(b*x+a)+210*a^4*ln(b*x+a)/b^11
```

maxima [A] time = 1.47, size = 180, normalized size = 1.20

$$\frac{3024\,a^{5}b^{5}x^{5} + 13860\,a^{6}b^{4}x^{4} + 25680\,a^{7}b^{3}x^{3} + 23985\,a^{8}b^{2}x^{2} + 11274\,a^{9}bx + 2131\,a^{10}}{12\left(b^{17}x^{6} + 6\,ab^{16}x^{5} + 15\,a^{2}b^{15}x^{4} + 20\,a^{3}b^{14}x^{3} + 15\,a^{4}b^{13}x^{2} + 6\,a^{5}b^{12}x + a^{6}b^{11}\right)} + \frac{210\,a^{4}\log\left(bx + a\right)}{b^{11}} + \frac{3\,b^{3}x^{4} - 28\,ab^{2}x^{3} + 168\,a^{2}bx^{2} - 1008\,a^{3}x}{12\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3024*a^5*b^5*x^5 + 13860*a^6*b^4*x^4 + 25680*a^7*b^3*x^3 + 23985*a^8*
b^2*x^2 + 11274*a^9*b*x + 2131*a^10)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15
*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 210*a
^4*log(b*x + a)/b^11 + 1/12*(3*b^3*x^4 - 28*a*b^2*x^3 + 168*a^2*b*x^2 - 100
8*a^3*x)/b^10
```

mupad [B] time = 1.09, size = 126, normalized size = 0.84

$$\frac{\frac{(a+b\,x)^4}{4} - \frac{10\,a\,(a+b\,x)^3}{3} + \frac{45\,a^2\,(a+b\,x)^2}{2} + \frac{252\,a^5}{a+b\,x} - \frac{105\,a^6}{(a+b\,x)^2} + \frac{40\,a^7}{(a+b\,x)^3} - \frac{45\,a^8}{4\,(a+b\,x)^4} + \frac{2\,a^9}{(a+b\,x)^5} - \frac{a^{10}}{6\,(a+b\,x)^6} + 210\,a^4\,\ln\left(a+b\,x\right) - 120\,a^3\,b\,x}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^4/4 - (10*a*(a + b*x)^3)/3 + (45*a^2*(a + b*x)^2)/2 + (252*a^5)/
(a + b*x) - (105*a^6)/(a + b*x)^2 + (40*a^7)/(a + b*x)^3 - (45*a^8)/(4*(a +
 b*x)^4) + (2*a^9)/(a + b*x)^5 - a^10/(6*(a + b*x)^6) + 210*a^4*log(a + b*x
) - 120*a^3*b*x)/b^11
```

sympy [A] time = 0.93, size = 190, normalized size = 1.27

$$\frac{210a^4\log(a+bx)}{b^{11}} - \frac{84a^3x}{b^{10}} + \frac{14a^2x^2}{b^9} - \frac{7ax^3}{3b^8} + \frac{2131a^{10} + 11274a^9bx + 23985a^8b^2x^2 + 25680a^7b^3x^3 + 13860a^6b^4x^4 + 3024a^5b^5x^5}{12a^6b^{11} + 72a^5b^{12}x + 180a^4b^{13}x^2 + 240a^3b^{14}x^3 + 180a^2b^{15}x^4 + 72ab^{16}x^5 + 12b^{17}x^6} + \frac{x^4}{4b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**7,x)
```

```
[Out] 210*a**4*log(a + b*x)/b**11 - 84*a**3*x/b**10 + 14*a**2*x**2/b**9 - 7*a*x**
3/(3*b**8) + (2131*a**10 + 11274*a**9*b*x + 23985*a**8*b**2*x**2 + 25680*a*
*7*b**3*x**3 + 13860*a**6*b**4*x**4 + 3024*a**5*b**5*x**5)/(12*a**6*b**11 +
 72*a**5*b**12*x + 180*a**4*b**13*x**2 + 240*a**3*b**14*x**3 + 180*a**2*b**
15*x**4 + 72*a*b**16*x**5 + 12*b**17*x**6) + x**4/(4*b**7)
```

3.208 $\int \frac{x^9}{(a+bx)^7} dx$

Optimal. Leaf size=139

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2}{b^9}$$

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{63a^5}{b^{10}(a+bx)^2}-\frac{126a^4}{b^{10}(a+bx)}+\frac{28a^2x}{b^9}-\frac{84a^3\log(a+bx)}{b^{10}}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^7,x]
```

```
[Out] (28*a^2*x)/b^9 - (7*a*x^2)/(2*b^8) + x^3/(3*b^7) + a^9/(6*b^10*(a + b*x)^6)
 - (9*a^8)/(5*b^10*(a + b*x)^5) + (9*a^7)/(b^10*(a + b*x)^4) - (28*a^6)/(b^
10*(a + b*x)^3) + (63*a^5)/(b^10*(a + b*x)^2) - (126*a^4)/(b^10*(a + b*x))
- (84*a^3*Log[a + b*x])/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^7} dx &= \int \left(\frac{28a^2}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{b^7}-\frac{a^9}{b^9(a+bx)^7}+\frac{9a^8}{b^9(a+bx)^6}-\frac{36a^7}{b^9(a+bx)^5}+\frac{84a^6}{b^9(a+bx)^4}-\frac{126a^5}{b^9(a+bx\right.\\ &= \frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{x^3}{3b^7}+\frac{a^9}{6b^{10}(a+bx)^6}-\frac{9a^8}{5b^{10}(a+bx)^5}+\frac{9a^7}{b^{10}(a+bx)^4}-\frac{28a^6}{b^{10}(a+bx)^3}+\frac{}{b^{10}(}\end{aligned}$$

Mathematica [A] time = 0.03, size = 128, normalized size = 0.92

$$-\frac{2509a^9+12534a^8bx+23775a^7b^2x^2+19100a^6b^3x^3+1725a^5b^4x^4-6870a^4b^5x^5-3665a^3b^6x^6+2520a^3(a+bx)^6\log(a+bx)-360a^2b^7x^7+45ab^8x^8-10b^9x^9}{30b^{10}(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^7,x]
```

```
[Out] -1/30*(2509*a^9 + 12534*a^8*b*x + 23775*a^7*b^2*x^2 + 19100*a^6*b^3*x^3 + 1
725*a^5*b^4*x^4 - 6870*a^4*b^5*x^5 - 3665*a^3*b^6*x^6 - 360*a^2*b^7*x^7 + 4
5*a*b^8*x^8 - 10*b^9*x^9 + 2520*a^3*(a + b*x)^6*Log[a + b*x])/(b^10*(a + b*
x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x)^7, x]
```

fricas [A] time = 1.12, size = 239, normalized size = 1.72

$$\frac{10\,b^{9}x^{9}-45\,ab^{8}x^{8}+360\,a^{2}b^{7}x^{7}+3665\,a^{3}b^{6}x^{6}+6870\,a^{4}b^{5}x^{5}-1725\,a^{5}b^{4}x^{4}-19100\,a^{6}b^{3}x^{3}-23775\,a^{7}b^{2}x^{2}-12534\,a^{8}bx-2509\,a^{9}-2520\left(a^{3}b^{6}x^{6}+6\,a^{4}b^{5}x^{5}+15\,a^{5}b^{4}x^{4}+20\,a^{6}b^{3}x^{3}+15\,a^{7}b^{2}x^{2}+6\,a^{8}bx+a^{9}\right)\log\left(bx+a\right)}{30\left(b^{16}x^{6}+6\,ab^{15}x^{5}+15\,a^{2}b^{14}x^{4}+20\,a^{3}b^{13}x^{3}+15\,a^{4}b^{12}x^{2}+6\,a^{5}b^{11}x+a^{6}b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(10*b^9*x^9 - 45*a*b^8*x^8 + 360*a^2*b^7*x^7 + 3665*a^3*b^6*x^6 + 6870
*a^4*b^5*x^5 - 1725*a^5*b^4*x^4 - 19100*a^6*b^3*x^3 - 23775*a^7*b^2*x^2 - 1
2534*a^8*b*x - 2509*a^9 - 2520*(a^3*b^6*x^6 + 6*a^4*b^5*x^5 + 15*a^5*b^4*x^
4 + 20*a^6*b^3*x^3 + 15*a^7*b^2*x^2 + 6*a^8*b*x + a^9)*log(b*x + a))/(b^16*
x^6 + 6*a*b^15*x^5 + 15*a^2*b^14*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 +
6*a^5*b^11*x + a^6*b^10)
```

giac [A] time = 1.48, size = 117, normalized size = 0.84

$$-\frac{84\,a^{3}\log\left(|bx+a|\right)}{b^{10}}-\frac{3780\,a^{4}b^{5}x^{5}+17010\,a^{5}b^{4}x^{4}+31080\,a^{6}b^{3}x^{3}+28710\,a^{7}b^{2}x^{2}+13374\,a^{8}bx+2509\,a^{9}}{30\,(bx+a)^{6}b^{10}}+\frac{2\,b^{14}x^{3}-21\,ab^{13}x^{2}+168\,a^{2}b^{12}x}{6\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -84*a^3*log(abs(b*x + a))/b^10 - 1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4
 + 31080*a^6*b^3*x^3 + 28710*a^7*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/((b*x
+ a)^6*b^10) + 1/6*(2*b^14*x^3 - 21*a*b^13*x^2 + 168*a^2*b^12*x)/b^21
```

maple [A] time = 0.01, size = 132, normalized size = 0.95

$$\frac{a^{9}}{6\,(bx+a)^{6}\,b^{10}}-\frac{9a^{8}}{5\,(bx+a)^{5}\,b^{10}}+\frac{9a^{7}}{(bx+a)^{4}\,b^{10}}-\frac{28a^{6}}{(bx+a)^{3}\,b^{10}}+\frac{x^{3}}{3b^{7}}+\frac{63a^{5}}{(bx+a)^{2}\,b^{10}}-\frac{7a\,x^{2}}{2b^{8}}-\frac{126a^{4}}{(bx+a)\,b^{10}}-\frac{84a^{3}\ln\left(bx+a\right)}{b^{10}}+\frac{28a^{2}x}{b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^7,x)
```

```
[Out] 28*a^2*x/b^9-7/2*a*x^2/b^8+1/3*x^3/b^7+1/6*a^9/b^10/(b*x+a)^6-9/5*a^8/b^10/
(b*x+a)^5+9*a^7/b^10/(b*x+a)^4-28*a^6/b^10/(b*x+a)^3+63*a^5/b^10/(b*x+a)^2-
126*a^4/b^10/(b*x+a)-84*a^3*ln(b*x+a)/b^10
```

maxima [A] time = 1.59, size = 169, normalized size = 1.22

$$-\frac{3780\,a^{4}b^{5}x^{5}+17010\,a^{5}b^{4}x^{4}+31080\,a^{6}b^{3}x^{3}+28710\,a^{7}b^{2}x^{2}+13374\,a^{8}bx+2509\,a^{9}}{30\left(b^{16}x^{6}+6\,ab^{15}x^{5}+15\,a^{2}b^{14}x^{4}+20\,a^{3}b^{13}x^{3}+15\,a^{4}b^{12}x^{2}+6\,a^{5}b^{11}x+a^{6}b^{10}\right)}-\frac{84\,a^{3}\log\left(bx+a\right)}{b^{10}}+\frac{2\,b^{2}x^{3}-21\,abx^{2}+168\,a^{2}x}{6\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(3780*a^4*b^5*x^5 + 17010*a^5*b^4*x^4 + 31080*a^6*b^3*x^3 + 28710*a^7
*b^2*x^2 + 13374*a^8*b*x + 2509*a^9)/(b^16*x^6 + 6*a*b^15*x^5 + 15*a^2*b^14
*x^4 + 20*a^3*b^13*x^3 + 15*a^4*b^12*x^2 + 6*a^5*b^11*x + a^6*b^10) - 84*a^
3*log(b*x + a)/b^10 + 1/6*(2*b^2*x^3 - 21*a*b*x^2 + 168*a^2*x)/b^9
```

mupad [B] time = 0.55, size = 115, normalized size = 0.83

$$-\frac{\frac{9\,a\,(a+b\,x)^{2}}{2}-\frac{(a+b\,x)^{3}}{3}+\frac{126\,a^{4}}{a+b\,x}-\frac{63\,a^{5}}{(a+b\,x)^{2}}+\frac{28\,a^{6}}{(a+b\,x)^{3}}-\frac{9\,a^{7}}{(a+b\,x)^{4}}+\frac{9\,a^{8}}{5\,(a+b\,x)^{5}}-\frac{a^{9}}{6\,(a+b\,x)^{6}}+84\,a^{3}\,\ln\left(a+b\,x\right)-36\,a^{2}\,b\,x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^7,x)
```

```
[Out] -((9*a*(a + b*x)^2)/2 - (a + b*x)^3/3 + (126*a^4)/(a + b*x) - (63*a^5)/(a +
 b*x)^2 + (28*a^6)/(a + b*x)^3 - (9*a^7)/(a + b*x)^4 + (9*a^8)/(5*(a + b*x)
^5) - a^9/(6*(a + b*x)^6) + 84*a^3*log(a + b*x) - 36*a^2*b*x)/b^10
```

sympy [A] time = 0.91, size = 180, normalized size = 1.29

$$-\frac{84a^3\log(a+bx)}{b^{10}}+\frac{28a^2x}{b^9}-\frac{7ax^2}{2b^8}+\frac{-2509a^9-13374a^8bx-28710a^7b^2x^2-31080a^6b^3x^3-17010a^5b^4x^4-3780a^4b^5x^5}{30a^6b^{10}+180a^5b^{11}x+450a^4b^{12}x^2+600a^3b^{13}x^3+450a^2b^{14}x^4+180ab^{15}x^5+30b^{16}x^6}+\frac{x^3}{3b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**7,x)
```

```
[Out] -84*a**3*log(a + b*x)/b**10 + 28*a**2*x/b**9 - 7*a*x**2/(2*b**8) + (-2509*a
**9 - 13374*a**8*b*x - 28710*a**7*b**2*x**2 - 31080*a**6*b**3*x**3 - 17010*
a**5*b**4*x**4 - 3780*a**4*b**5*x**5)/(30*a**6*b**10 + 180*a**5*b**11*x + 4
50*a**4*b**12*x**2 + 600*a**3*b**13*x**3 + 450*a**2*b**14*x**4 + 180*a*b**1
5*x**5 + 30*b**16*x**6) + x**3/(3*b**7)
```

3.209 $\int \frac{x^8}{(a+bx)^7}\,dx$

Optimal. Leaf size=128

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Rubi [A] time = 0.09, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{56a^3}{b^9(a+bx)}+\frac{28a^2\log(a+bx)}{b^9}-\frac{7ax}{b^8}+\frac{x^2}{2b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^7,x]
```

```
[Out] (-7*a*x)/b^8 + x^2/(2*b^7) - a^8/(6*b^9*(a + b*x)^6) + (8*a^7)/(5*b^9*(a +
b*x)^5) - (7*a^6)/(b^9*(a + b*x)^4) + (56*a^5)/(3*b^9*(a + b*x)^3) - (35*a^
4)/(b^9*(a + b*x)^2) + (56*a^3)/(b^9*(a + b*x)) + (28*a^2*Log[a + b*x])/b^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{(a+bx)^7}\,dx &= \int \left(-\frac{7a}{b^8}+\frac{x}{b^7}+\frac{a^8}{b^8(a+bx)^7}-\frac{8a^7}{b^8(a+bx)^6}+\frac{28a^6}{b^8(a+bx)^5}-\frac{56a^5}{b^8(a+bx)^4}+\frac{70a^4}{b^8(a+bx)^3}-\frac{56}{b^8(a\right.\\ &= -\frac{7ax}{b^8}+\frac{x^2}{2b^7}-\frac{a^8}{6b^9(a+bx)^6}+\frac{8a^7}{5b^9(a+bx)^5}-\frac{7a^6}{b^9(a+bx)^4}+\frac{56a^5}{3b^9(a+bx)^3}-\frac{35a^4}{b^9(a+bx)^2}+\frac{5}{b^9(}\end{aligned}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 0.81

$$\frac{-\frac{5a^8}{(a+bx)^6}+\frac{48a^7}{(a+bx)^5}-\frac{210a^6}{(a+bx)^4}+\frac{560a^5}{(a+bx)^3}-\frac{1050a^4}{(a+bx)^2}+\frac{1680a^3}{a+bx}+840a^2\log(a+bx)-210abx+15b^2x^2}{30b^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^7,x]
```

```
[Out] (-210*a*b*x + 15*b^2*x^2 - (5*a^8)/(a + b*x)^6 + (48*a^7)/(a + b*x)^5 - (21
0*a^6)/(a + b*x)^4 + (560*a^5)/(a + b*x)^3 - (1050*a^4)/(a + b*x)^2 + (1680
*a^3)/(a + b*x) + 840*a^2*Log[a + b*x])/(30*b^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x)^7, x]
```

fricas [A] time = 0.93, size = 228, normalized size = 1.78

$$\frac{15\,b^{8}x^{8} - 120\,ab^{7}x^{7} - 1035\,a^{2}b^{6}x^{6} - 1170\,a^{3}b^{5}x^{5} + 3375\,a^{4}b^{4}x^{4} + 10100\,a^{5}b^{3}x^{3} + 10725\,a^{6}b^{2}x^{2} + 5298\,a^{7}bx + 1023\,a^{8} + 840\left(a^{2}b^{6}x^{6} + 6\,a^{3}b^{5}x^{5} + 15\,a^{4}b^{4}x^{4} + 20\,a^{5}b^{3}x^{3} + 15\,a^{6}b^{2}x^{2} + 6\,a^{7}bx + a^{8}\right)\log\left(bx + a\right)}{30\left(b^{15}x^{6} + 6\,ab^{14}x^{5} + 15\,a^{2}b^{13}x^{4} + 20\,a^{3}b^{12}x^{3} + 15\,a^{4}b^{11}x^{2} + 6\,a^{5}b^{10}x + a^{6}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(15*b^8*x^8 - 120*a*b^7*x^7 - 1035*a^2*b^6*x^6 - 1170*a^3*b^5*x^5 + 33
75*a^4*b^4*x^4 + 10100*a^5*b^3*x^3 + 10725*a^6*b^2*x^2 + 5298*a^7*b*x + 102
3*a^8 + 840*(a^2*b^6*x^6 + 6*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 20*a^5*b^3*x^3
+ 15*a^6*b^2*x^2 + 6*a^7*b*x + a^8)*log(b*x + a))/(b^15*x^6 + 6*a*b^14*x^5
+ 15*a^2*b^13*x^4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*
b^9)
```

giac [A] time = 1.07, size = 105, normalized size = 0.82

$$\frac{28\,a^{2}\log\left(|bx + a|\right)}{b^{9}} + \frac{b^{7}x^{2} - 14\,ab^{6}x}{2\,b^{14}} + \frac{1680\,a^{3}b^{5}x^{5} + 7350\,a^{4}b^{4}x^{4} + 13160\,a^{5}b^{3}x^{3} + 11970\,a^{6}b^{2}x^{2} + 5508\,a^{7}bx + 1023\,a^{8}}{30\,(bx + a)^{6}b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 28*a^2*log(abs(b*x + a))/b^9 + 1/2*(b^7*x^2 - 14*a*b^6*x)/b^14 + 1/30*(1680
*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b^2*x^2 + 5
508*a^7*b*x + 1023*a^8)/((b*x + a)^6*b^9)
```

maple [A] time = 0.01, size = 121, normalized size = 0.95

$$-\frac{a^{8}}{6\,(bx + a)^{6}\,b^{9}} + \frac{8a^{7}}{5\,(bx + a)^{5}\,b^{9}} - \frac{7a^{6}}{(bx + a)^{4}\,b^{9}} + \frac{56a^{5}}{3\,(bx + a)^{3}\,b^{9}} - \frac{35a^{4}}{(bx + a)^{2}\,b^{9}} + \frac{x^{2}}{2b^{7}} + \frac{56a^{3}}{(bx + a)\,b^{9}} + \frac{28a^{2}\ln\left(bx + a\right)}{b^{9}} - \frac{7ax}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^7,x)
```

```
[Out] -7*a*x/b^8+1/2*x^2/b^7-1/6*a^8/b^9/(b*x+a)^6+8/5*a^7/b^9/(b*x+a)^5-7*a^6/b^
9/(b*x+a)^4+56/3*a^5/b^9/(b*x+a)^3-35*a^4/b^9/(b*x+a)^2+56*a^3/b^9/(b*x+a)+
28*a^2*ln(b*x+a)/b^9
```

maxima [A] time = 1.56, size = 157, normalized size = 1.23

$$\frac{1680\,a^{3}b^{5}x^{5} + 7350\,a^{4}b^{4}x^{4} + 13160\,a^{5}b^{3}x^{3} + 11970\,a^{6}b^{2}x^{2} + 5508\,a^{7}bx + 1023\,a^{8}}{30\left(b^{15}x^{6} + 6\,ab^{14}x^{5} + 15\,a^{2}b^{13}x^{4} + 20\,a^{3}b^{12}x^{3} + 15\,a^{4}b^{11}x^{2} + 6\,a^{5}b^{10}x + a^{6}b^{9}\right)} + \frac{28\,a^{2}\log\left(bx + a\right)}{b^{9}} + \frac{bx^{2} - 14\,ax}{2\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(1680*a^3*b^5*x^5 + 7350*a^4*b^4*x^4 + 13160*a^5*b^3*x^3 + 11970*a^6*b
^2*x^2 + 5508*a^7*b*x + 1023*a^8)/(b^15*x^6 + 6*a*b^14*x^5 + 15*a^2*b^13*x^
4 + 20*a^3*b^12*x^3 + 15*a^4*b^11*x^2 + 6*a^5*b^10*x + a^6*b^9) + 28*a^2*lo
g(b*x + a)/b^9 + 1/2*(b*x^2 - 14*a*x)/b^8
```

mupad [B] time = 0.18, size = 102, normalized size = 0.80

$$\frac{\frac{(a+b\,x)^{2}}{2} + \frac{56\,a^{3}}{a+b\,x} - \frac{35\,a^{4}}{(a+b\,x)^{2}} + \frac{56\,a^{5}}{3\,(a+b\,x)^{3}} - \frac{7\,a^{6}}{(a+b\,x)^{4}} + \frac{8\,a^{7}}{5\,(a+b\,x)^{5}} - \frac{a^{8}}{6\,(a+b\,x)^{6}} + 28\,a^{2}\,\ln\left(a + b\,x\right) - 8\,a\,b\,x}{b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^7,x)
```

```
[Out] ((a + b*x)^2/2 + (56*a^3)/(a + b*x) - (35*a^4)/(a + b*x)^2 + (56*a^5)/(3*(a
 + b*x)^3) - (7*a^6)/(a + b*x)^4 + (8*a^7)/(5*(a + b*x)^5) - a^8/(6*(a + b*
x)^6) + 28*a^2*log(a + b*x) - 8*a*b*x)/b^9
```

sympy [A] time = 0.84, size = 165, normalized size = 1.29

$$\frac{28a^2 \log(a+bx)}{b^9} - \frac{7ax}{b^8} + \frac{1023a^8 + 5508a^7bx + 11970a^6b^2x^2 + 13160a^5b^3x^3 + 7350a^4b^4x^4 + 1680a^3b^5x^5}{30a^6b^9 + 180a^5b^{10}x + 450a^4b^{11}x^2 + 600a^3b^{12}x^3 + 450a^2b^{13}x^4 + 180ab^{14}x^5 + 30b^{15}x^6} + \frac{x^2}{2b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**7,x)
```

```
[Out] 28*a**2*log(a + b*x)/b**9 - 7*a*x/b**8 + (1023*a**8 + 5508*a**7*b*x + 11970
*a**6*b**2*x**2 + 13160*a**5*b**3*x**3 + 7350*a**4*b**4*x**4 + 1680*a**3*b*
*5*x**5)/(30*a**6*b**9 + 180*a**5*b**10*x + 450*a**4*b**11*x**2 + 600*a**3*
b**12*x**3 + 450*a**2*b**13*x**4 + 180*a*b**14*x**5 + 30*b**15*x**6) + x**2
/(2*b**7)
```

3.210 $\int \frac{x^7}{(a+bx)^7} dx$

Optimal. Leaf size=118

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Rubi [A] time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+bx)}-\frac{7a\log(a+bx)}{b^8}+\frac{x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^7,x]
```

```
[Out] x/b^7 + a^7/(6*b^8*(a + b*x)^6) - (7*a^6)/(5*b^8*(a + b*x)^5) + (21*a^5)/(4
*b^8*(a + b*x)^4) - (35*a^4)/(3*b^8*(a + b*x)^3) + (35*a^3)/(2*b^8*(a + b*x
)^2) - (21*a^2)/(b^8*(a + b*x)) - (7*a*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^7}{(a+bx)^7} dx = \int \left(\frac{1}{b^7}-\frac{a^7}{b^7(a+bx)^7}+\frac{7a^6}{b^7(a+bx)^6}-\frac{21a^5}{b^7(a+bx)^5}+\frac{35a^4}{b^7(a+bx)^4}-\frac{35a^3}{b^7(a+bx)^3}+\frac{21a^2}{b^7(a+bx)}\right.$$
$$=\frac{x}{b^7}+\frac{a^7}{6b^8(a+bx)^6}-\frac{7a^6}{5b^8(a+bx)^5}+\frac{21a^5}{4b^8(a+bx)^4}-\frac{35a^4}{3b^8(a+bx)^3}+\frac{35a^3}{2b^8(a+bx)^2}-\frac{21a^2}{b^8(a+b}$$

Mathematica [A] time = 0.03, size = 104, normalized size = 0.88

$$-\frac{669a^7+3594a^6bx+7725a^5b^2x^2+8200a^4b^3x^3+4050a^3b^4x^4+360a^2b^5x^5-360ab^6x^6+420a(a+bx)^6\log(a+bx)-60b^7x^7}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7 + 3594*a^6*b*x + 7725*a^5*b^2*x^2 + 8200*a^4*b^3*x^3 + 4050*
a^3*b^4*x^4 + 360*a^2*b^5*x^5 - 360*a*b^6*x^6 - 60*b^7*x^7 + 420*a*(a + b*x
)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^7, x]
```

fricas [A] time = 1.03, size = 215, normalized size = 1.82

$$\frac{60\,b^{7}x^{7}+360\,ab^{6}x^{6}-360\,a^{2}b^{5}x^{5}-4050\,a^{3}b^{4}x^{4}-8200\,a^{4}b^{3}x^{3}-7725\,a^{5}b^{2}x^{2}-3594\,a^{6}bx-669\,a^{7}-420\left(ab^{6}x^{6}+6\,a^{2}b^{5}x^{5}+15\,a^{3}b^{4}x^{4}+20\,a^{4}b^{3}x^{3}+15\,a^{5}b^{2}x^{2}+6\,a^{6}bx+a^{7}\right)\log\left(bx+a\right)}{60\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*x^7 + 360*a*b^6*x^6 - 360*a^2*b^5*x^5 - 4050*a^3*b^4*x^4 - 820
0*a^4*b^3*x^3 - 7725*a^5*b^2*x^2 - 3594*a^6*b*x - 669*a^7 - 420*(a*b^6*x^6
+ 6*a^2*b^5*x^5 + 15*a^3*b^4*x^4 + 20*a^4*b^3*x^3 + 15*a^5*b^2*x^2 + 6*a^6*
b*x + a^7)*log(b*x + a))/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^
3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8)
```

giac [A] time = 0.92, size = 88, normalized size = 0.75

$$\frac{x}{b^{7}}-\frac{7\,a\log\left(|bx+a|\right)}{b^{8}}-\frac{1260\,a^{2}b^{5}x^{5}+5250\,a^{3}b^{4}x^{4}+9100\,a^{4}b^{3}x^{3}+8085\,a^{5}b^{2}x^{2}+3654\,a^{6}bx+669\,a^{7}}{60\left(bx+a\right)^{6}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] x/b^7 - 7*a*log(abs(b*x + a))/b^8 - 1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x
^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^2*x^2 + 3654*a^6*b*x + 669*a^7)/((b*x +
a)^6*b^8)
```

maple [A] time = 0.01, size = 109, normalized size = 0.92

$$\frac{a^{7}}{6\left(bx+a\right)^{6}b^{8}}-\frac{7a^{6}}{5\left(bx+a\right)^{5}b^{8}}+\frac{21a^{5}}{4\left(bx+a\right)^{4}b^{8}}-\frac{35a^{4}}{3\left(bx+a\right)^{3}b^{8}}+\frac{35a^{3}}{2\left(bx+a\right)^{2}b^{8}}-\frac{21a^{2}}{\left(bx+a\right)b^{8}}-\frac{7a\ln\left(bx+a\right)}{b^{8}}+\frac{x}{b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^7,x)
```

```
[Out] x/b^7+1/6*a^7/b^8/(b*x+a)^6-7/5*a^6/b^8/(b*x+a)^5+21/4*a^5/b^8/(b*x+a)^4-35
/3*a^4/b^8/(b*x+a)^3+35/2*a^3/b^8/(b*x+a)^2-21*a^2/b^8/(b*x+a)-7*a*ln(b*x+a
)/b^8
```

maxima [A] time = 1.49, size = 145, normalized size = 1.23

$$-\frac{1260\,a^{2}b^{5}x^{5}+5250\,a^{3}b^{4}x^{4}+9100\,a^{4}b^{3}x^{3}+8085\,a^{5}b^{2}x^{2}+3654\,a^{6}bx+669\,a^{7}}{60\left(b^{14}x^{6}+6\,ab^{13}x^{5}+15\,a^{2}b^{12}x^{4}+20\,a^{3}b^{11}x^{3}+15\,a^{4}b^{10}x^{2}+6\,a^{5}b^{9}x+a^{6}b^{8}\right)}+\frac{x}{b^{7}}-\frac{7\,a\log\left(bx+a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(1260*a^2*b^5*x^5 + 5250*a^3*b^4*x^4 + 9100*a^4*b^3*x^3 + 8085*a^5*b^
2*x^2 + 3654*a^6*b*x + 669*a^7)/(b^14*x^6 + 6*a*b^13*x^5 + 15*a^2*b^12*x^4
+ 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a^6*b^8) + x/b^7 - 7*a*
log(b*x + a)/b^8
```

mupad [B] time = 0.34, size = 91, normalized size = 0.77

$$-\frac{7\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{21\,a^{2}}{a+b\,x}-\frac{35\,a^{3}}{2\left(a+b\,x\right)^{2}}+\frac{35\,a^{4}}{3\left(a+b\,x\right)^{3}}-\frac{21\,a^{5}}{4\left(a+b\,x\right)^{4}}+\frac{7\,a^{6}}{5\left(a+b\,x\right)^{5}}-\frac{a^{7}}{6\left(a+b\,x\right)^{6}}}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^7,x)
```

```
[Out] -(7*a*log(a + b*x) - b*x + (21*a^2)/(a + b*x) - (35*a^3)/(2*(a + b*x)^2) +
(35*a^4)/(3*(a + b*x)^3) - (21*a^5)/(4*(a + b*x)^4) + (7*a^6)/(5*(a + b*x)^
5) - a^7/(6*(a + b*x)^6))/b^8
```

sympy [A] time = 0.82, size = 153, normalized size = 1.30

$$-\frac{7a\log{(a+bx)}}{b^8}+\frac{-669a^7-3654a^6bx-8085a^5b^2x^2-9100a^4b^3x^3-5250a^3b^4x^4-1260a^2b^5x^5}{60a^6b^8+360a^5b^9x+900a^4b^{10}x^2+1200a^3b^{11}x^3+900a^2b^{12}x^4+360ab^{13}x^5+60b^{14}x^6}+\frac{x}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**7,x)
```

```
[Out] -7*a*log(a + b*x)/b**8 + (-669*a**7 - 3654*a**6*b*x - 8085*a**5*b**2*x**2 -
 9100*a**4*b**3*x**3 - 5250*a**3*b**4*x**4 - 1260*a**2*b**5*x**5)/(60*a**6*
b**8 + 360*a**5*b**9*x + 900*a**4*b**10*x**2 + 1200*a**3*b**11*x**3 + 900*a
**2*b**12*x**4 + 360*a*b**13*x**5 + 60*b**14*x**6) + x/b**7
```

3.211 $\int \frac{x^6}{(a+bx)^7}\,dx$

Optimal. Leaf size=109

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{\log(a+bx)}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^7,x]
```

```
[Out] -a^6/(6*b^7*(a + b*x)^6) + (6*a^5)/(5*b^7*(a + b*x)^5) - (15*a^4)/(4*b^7*(a
 + b*x)^4) + (20*a^3)/(3*b^7*(a + b*x)^3) - (15*a^2)/(2*b^7*(a + b*x)^2) +
(6*a)/(b^7*(a + b*x)) + Log[a + b*x]/b^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^6}{(a+bx)^7}\,dx = \int \left(\frac{a^6}{b^6(a+bx)^7}-\frac{6a^5}{b^6(a+bx)^6}+\frac{15a^4}{b^6(a+bx)^5}-\frac{20a^3}{b^6(a+bx)^4}+\frac{15a^2}{b^6(a+bx)^3}-\frac{6a}{b^6(a+bx)^2}+\frac{}{b^6(a}\right.$$
$$= -\frac{a^6}{6b^7(a+bx)^6}+\frac{6a^5}{5b^7(a+bx)^5}-\frac{15a^4}{4b^7(a+bx)^4}+\frac{20a^3}{3b^7(a+bx)^3}-\frac{15a^2}{2b^7(a+bx)^2}+\frac{6a}{b^7(a+bx)}+\frac{1}{}$$

Mathematica [A] time = 0.02, size = 77, normalized size = 0.71

$$\frac{\frac{a\left(147a^5+822a^4bx+1875a^3b^2x^2+2200a^2b^3x^3+1350ab^4x^4+360b^5x^5\right)}{(a+bx)^6}+60\log(a+bx)}{60b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^7,x]
```

```
[Out] ((a*(147*a^5 + 822*a^4*b*x + 1875*a^3*b^2*x^2 + 2200*a^2*b^3*x^3 + 1350*a*b
^4*x^4 + 360*b^5*x^5))/(a + b*x)^6 + 60*Log[a + b*x])/(60*b^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^7, x]
```

fricas [A] time = 1.08, size = 193, normalized size = 1.77

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6+60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(bx+a\right)}{60\left(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6 + 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20
*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a))/(b^13*x^6 +
6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b
^8*x + a^6*b^7)
```

giac [A] time = 1.16, size = 79, normalized size = 0.72

$$\frac{\log\left(|bx+a|\right)}{b^7}+\frac{360\,ab^4x^5+1350\,a^2b^3x^4+2200\,a^3b^2x^3+1875\,a^4bx^2+822\,a^5x+\frac{147\,a^6}{b}}{60\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^7 + 1/60*(360*a*b^4*x^5 + 1350*a^2*b^3*x^4 + 2200*a^3*b
^2*x^3 + 1875*a^4*b*x^2 + 822*a^5*x + 147*a^6/b)/((b*x + a)^6*b^6)
```

maple [A] time = 0.01, size = 100, normalized size = 0.92

$$-\frac{a^6}{6\,(bx+a)^6\,b^7}+\frac{6a^5}{5\,(bx+a)^5\,b^7}-\frac{15a^4}{4\,(bx+a)^4\,b^7}+\frac{20a^3}{3\,(bx+a)^3\,b^7}-\frac{15a^2}{2\,(bx+a)^2\,b^7}+\frac{6a}{(bx+a)\,b^7}+\frac{\ln\left(bx+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^7,x)
```

```
[Out] -1/6*a^6/b^7/(b*x+a)^6+6/5*a^5/b^7/(b*x+a)^5-15/4*a^4/b^7/(b*x+a)^4+20/3*a^
3/b^7/(b*x+a)^3-15/2*a^2/b^7/(b*x+a)^2+6*a/b^7/(b*x+a)+ln(b*x+a)/b^7
```

maxima [A] time = 1.45, size = 136, normalized size = 1.25

$$\frac{360\,ab^5x^5+1350\,a^2b^4x^4+2200\,a^3b^3x^3+1875\,a^4b^2x^2+822\,a^5bx+147\,a^6}{60\left(b^{13}x^6+6\,ab^{12}x^5+15\,a^2b^{11}x^4+20\,a^3b^{10}x^3+15\,a^4b^9x^2+6\,a^5b^8x+a^6b^7\right)}+\frac{\log\left(bx+a\right)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(360*a*b^5*x^5 + 1350*a^2*b^4*x^4 + 2200*a^3*b^3*x^3 + 1875*a^4*b^2*x^
2 + 822*a^5*b*x + 147*a^6)/(b^13*x^6 + 6*a*b^12*x^5 + 15*a^2*b^11*x^4 + 20*
a^3*b^10*x^3 + 15*a^4*b^9*x^2 + 6*a^5*b^8*x + a^6*b^7) + log(b*x + a)/b^7
```

mupad [B] time = 0.11, size = 81, normalized size = 0.74

$$\frac{\ln\left(a+b\,x\right)+\frac{6\,a}{a+b\,x}-\frac{15\,a^2}{2\,{\left(a+b\,x\right)}^2}+\frac{20\,a^3}{3\,{\left(a+b\,x\right)}^3}-\frac{15\,a^4}{4\,{\left(a+b\,x\right)}^4}+\frac{6\,a^5}{5\,{\left(a+b\,x\right)}^5}-\frac{a^6}{6\,{\left(a+b\,x\right)}^6}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^7,x)
```

```
[Out] (log(a + b*x) + (6*a)/(a + b*x) - (15*a^2)/(2*(a + b*x)^2) + (20*a^3)/(3*(a
 + b*x)^3) - (15*a^4)/(4*(a + b*x)^4) + (6*a^5)/(5*(a + b*x)^5) - a^6/(6*(a
 + b*x)^6))/b^7
```

sympy [A] time = 0.64, size = 141, normalized size = 1.29

$$\frac{147a^6 + 822a^5bx + 1875a^4b^2x^2 + 2200a^3b^3x^3 + 1350a^2b^4x^4 + 360ab^5x^5}{60a^6b^7 + 360a^5b^8x + 900a^4b^9x^2 + 1200a^3b^{10}x^3 + 900a^2b^{11}x^4 + 360ab^{12}x^5 + 60b^{13}x^6} + \frac{\log(a + bx)}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**7,x)
```

```
[Out] (147*a**6 + 822*a**5*b*x + 1875*a**4*b**2*x**2 + 2200*a**3*b**3*x**3 + 1350
*a**2*b**4*x**4 + 360*a*b**5*x**5)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a*
*4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**
5 + 60*b**13*x**6) + log(a + b*x)/b**7
```

3.212 $\int \frac{x^5}{(a+bx)^7} dx$

Optimal. Leaf size=17

$$\frac{x^6}{6a(a+bx)^6}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^6}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^7,x]
```

```
[Out] x^6/(6*a*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^5}{(a+bx)^7} dx = \frac{x^6}{6a(a+bx)^6}$$

Mathematica [B] time = 0.01, size = 64, normalized size = 3.76

$$-\frac{a^5 + 6a^4bx + 15a^3b^2x^2 + 20a^2b^3x^3 + 15ab^4x^4 + 6b^5x^5}{6b^6(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^7,x]
```

```
[Out] -1/6*(a^5 + 6*a^4*b*x + 15*a^3*b^2*x^2 + 20*a^2*b^3*x^3 + 15*a*b^4*x^4 + 6*
b^5*x^5)/(b^6*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^7, x]
```

fricas [B] time = 0.95, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\left(b^{12}x^6 + 6\,ab^{11}x^5 + 15\,a^2b^{10}x^4 + 20\,a^3b^9x^3 + 15\,a^4b^8x^2 + 6\,a^5b^7x + a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

giac [B] time = 1.08, size = 62, normalized size = 3.65

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\,(bx+a)^6b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/((b*x + a)^6*b^6)
```

maple [B] time = 0.00, size = 87, normalized size = 5.12

$$\frac{a^5}{6\,(bx+a)^6\,b^6} - \frac{a^4}{(bx+a)^5\,b^6} + \frac{5a^3}{2\,(bx+a)^4\,b^6} - \frac{10a^2}{3\,(bx+a)^3\,b^6} + \frac{5a}{2\,(bx+a)^2\,b^6} - \frac{1}{(bx+a)\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^7,x)
```

```
[Out] 1/6*a^5/b^6/(b*x+a)^6+5/2*a/b^6/(b*x+a)^2-a^4/b^6/(b*x+a)^5+5/2*a^3/b^6/(b*
x+a)^4-10/3*a^2/b^6/(b*x+a)^3-1/b^6/(b*x+a)
```

maxima [B] time = 1.43, size = 120, normalized size = 7.06

$$-\frac{6\,b^5x^5 + 15\,ab^4x^4 + 20\,a^2b^3x^3 + 15\,a^3b^2x^2 + 6\,a^4bx + a^5}{6\left(b^{12}x^6 + 6\,ab^{11}x^5 + 15\,a^2b^{10}x^4 + 20\,a^3b^9x^3 + 15\,a^4b^8x^2 + 6\,a^5b^7x + a^6b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6*(6*b^5*x^5 + 15*a*b^4*x^4 + 20*a^2*b^3*x^3 + 15*a^3*b^2*x^2 + 6*a^4*b*
x + a^5)/(b^12*x^6 + 6*a*b^11*x^5 + 15*a^2*b^10*x^4 + 20*a^3*b^9*x^3 + 15*a
^4*b^8*x^2 + 6*a^5*b^7*x + a^6*b^6)
```

mupad [B] time = 0.12, size = 72, normalized size = 4.24

$$\frac{\frac{5\,a}{2\,{\left(a+b\,x\right)}^2}-\frac{1}{a+b\,x}-\frac{10\,a^2}{3\,{\left(a+b\,x\right)}^3}+\frac{5\,a^3}{2\,{\left(a+b\,x\right)}^4}-\frac{a^4}{{\left(a+b\,x\right)}^5}+\frac{a^5}{6\,{\left(a+b\,x\right)}^6}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^7,x)
```

```
[Out] ((5*a)/(2*(a + b*x)^2) - 1/(a + b*x) - (10*a^2)/(3*(a + b*x)^3) + (5*a^3)/(
2*(a + b*x)^4) - a^4/(a + b*x)^5 + a^5/(6*(a + b*x)^6))/b^6
```

sympy [B] time = 0.58, size = 128, normalized size = 7.53

$$\frac{-a^5 - 6a^4bx - 15a^3b^2x^2 - 20a^2b^3x^3 - 15ab^4x^4 - 6b^5x^5}{6a^6b^6 + 36a^5b^7x + 90a^4b^8x^2 + 120a^3b^9x^3 + 90a^2b^{10}x^4 + 36ab^{11}x^5 + 6b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**7,x)
```

```
[Out] (-a**5 - 6*a**4*b*x - 15*a**3*b**2*x**2 - 20*a**2*b**3*x**3 - 15*a*b**4*x**
4 - 6*b**5*x**5)/(6*a**6*b**6 + 36*a**5*b**7*x + 90*a**4*b**8*x**2 + 120*a*
*3*b**9*x**3 + 90*a**2*b**10*x**4 + 36*a*b**11*x**5 + 6*b**12*x**6)
```

3.213 $\int \frac{x^4}{(a+bx)^7} \, dx$

Optimal. Leaf size=35

$$\frac{x^5}{30a^2(a+bx)^5} + \frac{x^5}{6a(a+bx)^6}$$

Rubi [A] time = 0.00, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {45, 37}

$$\frac{x^5}{30a^2(a+bx)^5} + \frac{x^5}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^7,x]
```

```
[Out] x^5/(6*a*(a + b*x)^6) + x^5/(30*a^2*(a + b*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(a+bx)^7} \, dx &= \frac{x^5}{6a(a+bx)^6} + \frac{\int \frac{x^4}{(a+bx)^6} \, dx}{6a} \\ &= \frac{x^5}{6a(a+bx)^6} + \frac{x^5}{30a^2(a+bx)^5} \end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 1.51

$$-\frac{a^4 + 6a^3bx + 15a^2b^2x^2 + 20ab^3x^3 + 15b^4x^4}{30b^5(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^7,x]
```

```
[Out] -1/30*(a^4 + 6*a^3*b*x + 15*a^2*b^2*x^2 + 20*a*b^3*x^3 + 15*b^4*x^4)/(b^5*(
a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^7, x]
```

fricas [B] time = 0.86, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\left(b^{11}x^6+6\,ab^{10}x^5+15\,a^2b^9x^4+20\,a^3b^8x^3+15\,a^4b^7x^2+6\,a^5b^6x+a^6b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

giac [A] time = 1.14, size = 51, normalized size = 1.46

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\,(bx+a)^6b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/((b*x
+ a)^6*b^5)
```

maple [B] time = 0.01, size = 72, normalized size = 2.06

$$-\frac{a^4}{6\,(bx+a)^6\,b^5}+\frac{4a^3}{5\,(bx+a)^5\,b^5}-\frac{3a^2}{2\,(bx+a)^4\,b^5}+\frac{4a}{3\,(bx+a)^3\,b^5}-\frac{1}{2\,(bx+a)^2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^7,x)
```

```
[Out] -1/6*a^4/b^5/(b*x+a)^6-1/2/b^5/(b*x+a)^2-3/2*a^2/b^5/(b*x+a)^4+4/5*a^3/b^5/
(b*x+a)^5+4/3*a/b^5/(b*x+a)^3
```

maxima [B] time = 1.45, size = 109, normalized size = 3.11

$$-\frac{15\,b^4x^4+20\,ab^3x^3+15\,a^2b^2x^2+6\,a^3bx+a^4}{30\left(b^{11}x^6+6\,ab^{10}x^5+15\,a^2b^9x^4+20\,a^3b^8x^3+15\,a^4b^7x^2+6\,a^5b^6x+a^6b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(15*b^4*x^4 + 20*a*b^3*x^3 + 15*a^2*b^2*x^2 + 6*a^3*b*x + a^4)/(b^11*
x^6 + 6*a*b^10*x^5 + 15*a^2*b^9*x^4 + 20*a^3*b^8*x^3 + 15*a^4*b^7*x^2 + 6*a
^5*b^6*x + a^6*b^5)
```

mupad [B] time = 0.07, size = 22, normalized size = 0.63

$$\frac{x^5\,(6\,a+b\,x)}{30\,a^2\,(a+b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^7,x)
```

```
[Out] (x^5*(6*a + b*x))/(30*a^2*(a + b*x)^6)
```

sympy [B] time = 0.57, size = 116, normalized size = 3.31

$$\frac{-a^4-6a^3bx-15a^2b^2x^2-20ab^3x^3-15b^4x^4}{30a^6b^5+180a^5b^6x+450a^4b^7x^2+600a^3b^8x^3+450a^2b^9x^4+180ab^{10}x^5+30b^{11}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**7,x)
```

```
[Out] (-a**4 - 6*a**3*b*x - 15*a**2*b**2*x**2 - 20*a*b**3*x**3 - 15*b**4*x**4)/(3
0*a**6*b**5 + 180*a**5*b**6*x + 450*a**4*b**7*x**2 + 600*a**3*b**8*x**3 + 4
50*a**2*b**9*x**4 + 180*a*b**10*x**5 + 30*b**11*x**6)
```

3.214 $\int \frac{x^3}{(a+bx)^7} \, dx$

Optimal. Leaf size=52

$$\frac{x^4}{60a^3(a+bx)^4}+\frac{x^4}{15a^2(a+bx)^5}+\frac{x^4}{6a(a+bx)^6}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^7,x]
```

```
[Out] a^3/(6*b^4*(a + b*x)^6) - (3*a^2)/(5*b^4*(a + b*x)^5) + (3*a)/(4*b^4*(a + b
*x)^4) - 1/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^7} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^7}+\frac{3a^2}{b^3(a+bx)^6}-\frac{3a}{b^3(a+bx)^5}+\frac{1}{b^3(a+bx)^4}\right) dx\\ &= \frac{a^3}{6b^4(a+bx)^6}-\frac{3a^2}{5b^4(a+bx)^5}+\frac{3a}{4b^4(a+bx)^4}-\frac{1}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.81

$$-\frac{a^3+6a^2bx+15ab^2x^2+20b^3x^3}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3 + 6*a^2*b*x + 15*a*b^2*x^2 + 20*b^3*x^3)/(b^4*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^7, x]
```

fricas [B] time = 1.12, size = 98, normalized size = 1.88

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

giac [A] time = 1.00, size = 40, normalized size = 0.77

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,(bx+a)^6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/((b*x + a)^6*b^4)
```

maple [A] time = 0.00, size = 57, normalized size = 1.10

$$\frac{a^3}{6\,(bx+a)^6\,b^4}-\frac{3a^2}{5\,(bx+a)^5\,b^4}+\frac{3a}{4\,(bx+a)^4\,b^4}-\frac{1}{3\,(bx+a)^3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^7,x)
```

```
[Out] 1/6*a^3/b^4/(b*x+a)^6-3/5*a^2/b^4/(b*x+a)^5+3/4*a/b^4/(b*x+a)^4-1/3/b^4/(b*
x+a)^3
```

maxima [B] time = 1.40, size = 98, normalized size = 1.88

$$-\frac{20\,b^3x^3+15\,ab^2x^2+6\,a^2bx+a^3}{60\,\left(b^{10}x^6+6\,ab^9x^5+15\,a^2b^8x^4+20\,a^3b^7x^3+15\,a^4b^6x^2+6\,a^5b^5x+a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)/(b^10*x^6 + 6*a*b^9*x^5
 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4
)
```

mupad [B] time = 0.07, size = 48, normalized size = 0.92

$$\frac{\frac{3\,a}{4\,(a+b\,x)^4}-\frac{1}{3\,(a+b\,x)^3}-\frac{3\,a^2}{5\,(a+b\,x)^5}+\frac{a^3}{6\,(a+b\,x)^6}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^7,x)
```

```
[Out] ((3*a)/(4*(a + b*x)^4) - 1/(3*(a + b*x)^3) - (3*a^2)/(5*(a + b*x)^5) + a^3/
(6*(a + b*x)^6))/b^4
```

sympy [B] time = 0.56, size = 104, normalized size = 2.00

$$\frac{-a^3 - 6a^2bx - 15ab^2x^2 - 20b^3x^3}{60a^6b^4 + 360a^5b^5x + 900a^4b^6x^2 + 1200a^3b^7x^3 + 900a^2b^8x^4 + 360ab^9x^5 + 60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**7,x)
```

```
[Out] (-a**3 - 6*a**2*b*x - 15*a*b**2*x**2 - 20*b**3*x**3)/(60*a**6*b**4 + 360*a*
*5*b**5*x + 900*a**4*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 +
 360*a*b**9*x**5 + 60*b**10*x**6)
```

3.215 $\int \frac{x^2}{(a+bx)^7} dx$

Optimal. Leaf size=47

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^7,x]
```

```
[Out] -a^2/(6*b^3*(a + b*x)^6) + (2*a)/(5*b^3*(a + b*x)^5) - 1/(4*b^3*(a + b*x)^4
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^7} dx &= \int \left(\frac{a^2}{b^2(a+bx)^7}-\frac{2a}{b^2(a+bx)^6}+\frac{1}{b^2(a+bx)^5}\right) dx\\ &= -\frac{a^2}{6b^3(a+bx)^6}+\frac{2a}{5b^3(a+bx)^5}-\frac{1}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+6abx+15b^2x^2}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2 + 6*a*b*x + 15*b^2*x^2)/(b^3*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^7, x]
```

fricas [B]   time = 0.88, size = 87, normalized size = 1.85

$$-\frac{15\,b^{2}x^{2}+6\,abx+a^{2}}{60\,\left(b^{9}x^{6}+6\,ab^{8}x^{5}+15\,a^{2}b^{7}x^{4}+20\,a^{3}b^{6}x^{3}+15\,a^{4}b^{5}x^{2}+6\,a^{5}b^{4}x+a^{6}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

giac [A]   time = 1.00, size = 29, normalized size = 0.62

$$-\frac{15\,b^{2}x^{2}+6\,abx+a^{2}}{60\,(bx+a)^{6}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/((b*x + a)^6*b^3)
```

maple [A]   time = 0.00, size = 42, normalized size = 0.89

$$-\frac{a^{2}}{6\,(bx+a)^{6}\,b^{3}}+\frac{2a}{5\,(bx+a)^{5}\,b^{3}}-\frac{1}{4\,(bx+a)^{4}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^7,x)
```

```
[Out] -1/6*a^2/b^3/(b*x+a)^6+2/5*a/b^3/(b*x+a)^5-1/4/b^3/(b*x+a)^4
```

maxima [B]   time = 1.45, size = 87, normalized size = 1.85

$$-\frac{15\,b^{2}x^{2}+6\,abx+a^{2}}{60\,\left(b^{9}x^{6}+6\,ab^{8}x^{5}+15\,a^{2}b^{7}x^{4}+20\,a^{3}b^{6}x^{3}+15\,a^{4}b^{5}x^{2}+6\,a^{5}b^{4}x+a^{6}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*x^2 + 6*a*b*x + a^2)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4
+ 20*a^3*b^6*x^3 + 15*a^4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

mupad [B]   time = 0.08, size = 31, normalized size = 0.66

$$-\frac{8\,a^{2}+48\,a\,b\,x+120\,b^{2}\,x^{2}}{480\,b^{3}\,(a+b\,x)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^7,x)
```

```
[Out] -(8*a^2 + 120*b^2*x^2 + 48*a*b*x)/(480*b^3*(a + b*x)^6)
```

sympy [B]   time = 0.55, size = 92, normalized size = 1.96

$$\frac{-a^{2}-6abx-15b^{2}x^{2}}{60a^{6}b^{3}+360a^{5}b^{4}x+900a^{4}b^{5}x^{2}+1200a^{3}b^{6}x^{3}+900a^{2}b^{7}x^{4}+360ab^{8}x^{5}+60b^{9}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**7,x)
```

```
[Out] (-a**2 - 6*a*b*x - 15*b**2*x**2)/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4
*b**5*x**2 + 1200*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 6
0*b**9*x**6)
```

3.216 $\int \frac{x}{(a+bx)^7} dx$

Optimal. Leaf size=30

$$\frac{a}{6b^2(a+bx)^6}-\frac{1}{5b^2(a+bx)^5}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{6b^2(a+bx)^6}-\frac{1}{5b^2(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^7,x]
```

```
[Out] a/(6*b^2*(a + b*x)^6) - 1/(5*b^2*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^7} dx &= \int \left(-\frac{a}{b(a+bx)^7}+\frac{1}{b(a+bx)^6}\right) dx\\ &= \frac{a}{6b^2(a+bx)^6}-\frac{1}{5b^2(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+6bx}{30b^2(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^7,x]
```

```
[Out] -1/30*(a + 6*b*x)/(b^2*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^7, x]
```

fricas [B] time = 0.88, size = 76, normalized size = 2.53

$$-\frac{6\,bx+a}{30\,\left(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

giac [A] time = 1.39, size = 18, normalized size = 0.60

$$-\frac{6\,bx+a}{30\,(bx+a)^6b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/30*(6*b*x + a)/((b*x + a)^6*b^2)
```

maple [A] time = 0.01, size = 27, normalized size = 0.90

$$\frac{a}{6\,(bx+a)^6\,b^2}-\frac{1}{5\,(bx+a)^5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^7,x)
```

```
[Out] 1/6*a/b^2/(b*x+a)^6-1/5/b^2/(b*x+a)^5
```

maxima [B] time = 1.37, size = 76, normalized size = 2.53

$$-\frac{6\,bx+a}{30\,\left(b^8x^6+6\,ab^7x^5+15\,a^2b^6x^4+20\,a^3b^5x^3+15\,a^4b^4x^2+6\,a^5b^3x+a^6b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/30*(6*b*x + a)/(b^8*x^6 + 6*a*b^7*x^5 + 15*a^2*b^6*x^4 + 20*a^3*b^5*x^3
+ 15*a^4*b^4*x^2 + 6*a^5*b^3*x + a^6*b^2)
```

mupad [B] time = 0.10, size = 18, normalized size = 0.60

$$-\frac{a+6\,b\,x}{30\,b^2\,(a+b\,x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^7,x)
```

```
[Out] -(a + 6*b*x)/(30*b^2*(a + b*x)^6)
```

sympy [B] time = 0.50, size = 80, normalized size = 2.67

$$\frac{-a-6bx}{30a^6b^2+180a^5b^3x+450a^4b^4x^2+600a^3b^5x^3+450a^2b^6x^4+180ab^7x^5+30b^8x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**7,x)
```

```
[Out] (-a - 6*b*x)/(30*a**6*b**2 + 180*a**5*b**3*x + 450*a**4*b**4*x**2 + 600*a**
3*b**5*x**3 + 450*a**2*b**6*x**4 + 180*a*b**7*x**5 + 30*b**8*x**6)
```

3.217 $\int \frac{1}{(a+bx)^7} dx$

Optimal. Leaf size=14

$$-\frac{1}{6b(a+bx)^6}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-7),x]
```

```
[Out] -1/(6*b*(a + b*x)^6)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^7} dx = -\frac{1}{6b(a+bx)^6}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{6b(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-7),x]
```

```
[Out] -1/6*1/(b*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-7),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-7), x]
```

fricas [B] time = 1.20, size = 68, normalized size = 4.86

$$-\frac{1}{6\left(b^7x^6 + 6\,ab^6x^5 + 15\,a^2b^5x^4 + 20\,a^3b^4x^3 + 15\,a^4b^3x^2 + 6\,a^5b^2x + a^6b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/6/(b^7*x^6 + 6*a*b^6*x^5 + 15*a^2*b^5*x^4 + 20*a^3*b^4*x^3 + 15*a^4*b^3*
x^2 + 6*a^5*b^2*x + a^6*b)
```

giac [A] time = 1.09, size = 12, normalized size = 0.86

$$-\frac{1}{6(bx+a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{6(bx+a)^6 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^7,x)
```

```
[Out] -1/6/b/(b*x+a)^6
```

maxima [A] time = 1.41, size = 12, normalized size = 0.86

$$-\frac{1}{6(bx+a)^6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/6/((b*x + a)^6*b)
```

mupad [B] time = 0.06, size = 70, normalized size = 5.00

$$-\frac{1}{6\,a^6\,b+36\,a^5\,b^2\,x+90\,a^4\,b^3\,x^2+120\,a^3\,b^4\,x^3+90\,a^2\,b^5\,x^4+36\,a\,b^6\,x^5+6\,b^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^7,x)
```

```
[Out] -1/(6*a^6*b + 6*b^7*x^6 + 36*a^5*b^2*x + 36*a*b^6*x^5 + 90*a^4*b^3*x^2 + 12
0*a^3*b^4*x^3 + 90*a^2*b^5*x^4)
```

sympy [B] time = 0.46, size = 73, normalized size = 5.21

$$-\frac{1}{6a^6b+36a^5b^2x+90a^4b^3x^2+120a^3b^4x^3+90a^2b^5x^4+36ab^6x^5+6b^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**7,x)
```

```
[Out] -1/(6*a**6*b + 36*a**5*b**2*x + 90*a**4*b**3*x**2 + 120*a**3*b**4*x**3 + 90
*a**2*b**5*x**4 + 36*a*b**6*x**5 + 6*b**7*x**6)
```

3.218 $\int \frac{1}{x(a+bx)^7} dx$

Optimal. Leaf size=99

$$-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{6a(a+bx)^6}$$

Rubi [A] time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^6(a+bx)}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{5a^2(a+bx)^5}-\frac{\log(a+bx)}{a^7}+\frac{\log(x)}{a^7}+\frac{1}{6a(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^7),x]
```

```
[Out] 1/(6*a*(a + b*x)^6) + 1/(5*a^2*(a + b*x)^5) + 1/(4*a^3*(a + b*x)^4) + 1/(3*
a^4*(a + b*x)^3) + 1/(2*a^5*(a + b*x)^2) + 1/(a^6*(a + b*x)) + Log[x]/a^7 -
 Log[a + b*x]/a^7
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x(a+bx)^7} dx = \int \left(\frac{1}{a^7x}-\frac{b}{a(a+bx)^7}-\frac{b}{a^2(a+bx)^6}-\frac{b}{a^3(a+bx)^5}-\frac{b}{a^4(a+bx)^4}-\frac{b}{a^5(a+bx)^3}-\frac{b}{a^6(a+b$$
$$=\frac{1}{6a(a+bx)^6}+\frac{1}{5a^2(a+bx)^5}+\frac{1}{4a^3(a+bx)^4}+\frac{1}{3a^4(a+bx)^3}+\frac{1}{2a^5(a+bx)^2}+\frac{1}{a^6(a+bx)}+$$

Mathematica [A] time = 0.06, size = 81, normalized size = 0.82

$$\frac{\frac{a\left(147a^5+522a^4bx+855a^3b^2x^2+740a^2b^3x^3+330ab^4x^4+60b^5x^5\right)}{(a+bx)^6}-60\log(a+bx)+60\log(x)}{60a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^7),x]
```

```
[Out] ((a*(147*a^5 + 522*a^4*b*x + 855*a^3*b^2*x^2 + 740*a^2*b^3*x^3 + 330*a*b^4*
x^4 + 60*b^5*x^5))/(a + b*x)^6 + 60*Log[x] - 60*Log[a + b*x])/(60*a^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^7), x]
```

fricas [B] time = 0.66, size = 256, normalized size = 2.59

$$\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6-60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(bx+a\right)+60\left(b^6x^6+6\,ab^5x^5+15\,a^2b^4x^4+20\,a^3b^3x^3+15\,a^4b^2x^2+6\,a^5bx+a^6\right)\log\left(x\right)}{60\left(a^7b^6x^6+6\,a^8b^5x^5+15\,a^9b^4x^4+20\,a^{10}b^3x^3+15\,a^{11}b^2x^2+6\,a^{12}bx+a^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a*b^5*x^5 + 330*a^2*b^4*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 +
522*a^5*b*x + 147*a^6 - 60*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3
*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x + a^6)*log(b*x + a) + 60*(b^6*x^6 + 6
*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 + 6*a^5*b*x +
 a^6)*log(x))/(a^7*b^6*x^6 + 6*a^8*b^5*x^5 + 15*a^9*b^4*x^4 + 20*a^10*b^3*x
^3 + 15*a^11*b^2*x^2 + 6*a^12*b*x + a^13)
```

giac [A] time = 1.01, size = 87, normalized size = 0.88

$$-\frac{\log\left(|bx+a|\right)}{a^7}+\frac{\log\left(|x|\right)}{a^7}+\frac{60\,ab^5x^5+330\,a^2b^4x^4+740\,a^3b^3x^3+855\,a^4b^2x^2+522\,a^5bx+147\,a^6}{60\left(bx+a\right)^6a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^7 + log(abs(x))/a^7 + 1/60*(60*a*b^5*x^5 + 330*a^2*b^4
*x^4 + 740*a^3*b^3*x^3 + 855*a^4*b^2*x^2 + 522*a^5*b*x + 147*a^6)/((b*x + a
)^6*a^7)
```

maple [A] time = 0.01, size = 90, normalized size = 0.91

$$\frac{1}{6\left(bx+a\right)^6a}+\frac{1}{5\left(bx+a\right)^5a^2}+\frac{1}{4\left(bx+a\right)^4a^3}+\frac{1}{3\left(bx+a\right)^3a^4}+\frac{1}{2\left(bx+a\right)^2a^5}+\frac{1}{\left(bx+a\right)a^6}+\frac{\ln(x)}{a^7}-\frac{\ln\left(bx+a\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^7,x)
```

```
[Out] 1/6/a/(b*x+a)^6+1/5/a^2/(b*x+a)^5+1/4/a^3/(b*x+a)^4+1/3/a^4/(b*x+a)^3+1/2/a
^5/(b*x+a)^2+1/a^6/(b*x+a)+ln(x)/a^7-ln(b*x+a)/a^7
```

maxima [A] time = 1.51, size = 139, normalized size = 1.40

$$\frac{60\,b^5x^5+330\,ab^4x^4+740\,a^2b^3x^3+855\,a^3b^2x^2+522\,a^4bx+147\,a^5}{60\left(a^6b^6x^6+6\,a^7b^5x^5+15\,a^8b^4x^4+20\,a^9b^3x^3+15\,a^{10}b^2x^2+6\,a^{11}bx+a^{12}\right)}-\frac{\log\left(bx+a\right)}{a^7}+\frac{\log(x)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(60*b^5*x^5 + 330*a*b^4*x^4 + 740*a^2*b^3*x^3 + 855*a^3*b^2*x^2 + 522*
a^4*b*x + 147*a^5)/(a^6*b^6*x^6 + 6*a^7*b^5*x^5 + 15*a^8*b^4*x^4 + 20*a^9*b
^3*x^3 + 15*a^10*b^2*x^2 + 6*a^11*b*x + a^12) - log(b*x + a)/a^7 + log(x)/a
^7
```

mupad [B] time = 0.45, size = 102, normalized size = 1.03

$$-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{15\,b^2\,x^2}{2\,{\left(a+b\,x\right)}^2}+\frac{20\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{15\,b^4\,x^4}{4\,{\left(a+b\,x\right)}^4}+\frac{6\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{b^6\,x^6}{6\,{\left(a+b\,x\right)}^6}+\frac{6\,b\,x}{a+b\,x}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^7),x)
```

```
[Out] -(log((a + b*x)/x) - (15*b^2*x^2)/(2*(a + b*x)^2) + (20*b^3*x^3)/(3*(a + b*
x)^3) - (15*b^4*x^4)/(4*(a + b*x)^4) + (6*b^5*x^5)/(5*(a + b*x)^5) - (b^6*x
^6)/(6*(a + b*x)^6) + (6*b*x)/(a + b*x))/a^7
```

sympy [A] time = 0.68, size = 141, normalized size = 1.42

$$\frac{147a^5 + 522a^4bx + 855a^3b^2x^2 + 740a^2b^3x^3 + 330ab^4x^4 + 60b^5x^5}{60a^{12} + 360a^{11}bx + 900a^{10}b^2x^2 + 1200a^9b^3x^3 + 900a^8b^4x^4 + 360a^7b^5x^5 + 60a^6b^6x^6} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**7,x)
```

```
[Out] (147*a**5 + 522*a**4*b*x + 855*a**3*b**2*x**2 + 740*a**2*b**3*x**3 + 330*a*
b**4*x**4 + 60*b**5*x**5)/(60*a**12 + 360*a**11*b*x + 900*a**10*b**2*x**2 +
 1200*a**9*b**3*x**3 + 900*a**8*b**4*x**4 + 360*a**7*b**5*x**5 + 60*a**6*b*
*6*x**6) + (log(x) - log(a/b + x))/a**7
```

3.219 $\int \frac{1}{x^2(a+bx)^7} dx$

Optimal. Leaf size=117

$$-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{6b}{a^7(a+bx)}-\frac{1}{a^7x}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+$$

Rubi [A] time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{6b}{a^7(a+bx)}-\frac{5b}{2a^6(a+bx)^2}-\frac{4b}{3a^5(a+bx)^3}-\frac{3b}{4a^4(a+bx)^4}-\frac{2b}{5a^3(a+bx)^5}-\frac{b}{6a^2(a+bx)^6}-\frac{7b\log(x)}{a^8}+\frac{7b\log(a+bx)}{a^8}-\frac{1}{a^7x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -(1/(a^7*x)) - b/(6*a^2*(a + b*x)^6) - (2*b)/(5*a^3*(a + b*x)^5) - (3*b)/(4
*a^4*(a + b*x)^4) - (4*b)/(3*a^5*(a + b*x)^3) - (5*b)/(2*a^6*(a + b*x)^2) -
 (6*b)/(a^7*(a + b*x)) - (7*b*Log[x])/a^8 + (7*b*Log[a + b*x])/a^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^7} dx &= \int \left(\frac{1}{a^7x^2}-\frac{7b}{a^8x}+\frac{b^2}{a^2(a+bx)^7}+\frac{2b^2}{a^3(a+bx)^6}+\frac{3b^2}{a^4(a+bx)^5}+\frac{4b^2}{a^5(a+bx)^4}+\frac{5b^2}{a^6(a+bx)^3}+\frac{}{a}\right.\\ &= -\frac{1}{a^7x}-\frac{b}{6a^2(a+bx)^6}-\frac{2b}{5a^3(a+bx)^5}-\frac{3b}{4a^4(a+bx)^4}-\frac{4b}{3a^5(a+bx)^3}-\frac{5b}{2a^6(a+bx)^2}-\frac{6b}{a^7(a+}\end{aligned}$$

Mathematica [A] time = 0.09, size = 97, normalized size = 0.83

$$-\frac{\frac{a(60a^6+1029a^5bx+3654a^4b^2x^2+5985a^3b^3x^3+5180a^2b^4x^4+2310ab^5x^5+420b^6x^6)}{x(a+bx)^6}-420b\log(a+bx)+420b\log(x)}{60a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^7),x]
```

```
[Out] -1/60*((a*(60*a^6 + 1029*a^5*b*x + 3654*a^4*b^2*x^2 + 5985*a^3*b^3*x^3 + 51
80*a^2*b^4*x^4 + 2310*a*b^5*x^5 + 420*b^6*x^6))/(x*(a + b*x)^6) + 420*b*Log
[x] - 420*b*Log[a + b*x])/a^8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^7), x]
```

fricas [B] time = 1.32, size = 285, normalized size = 2.44

$$-\frac{420\,ab^{6}x^{6}+2310\,a^{2}b^{5}x^{5}+5180\,a^{3}b^{4}x^{4}+5985\,a^{4}b^{3}x^{3}+3654\,a^{5}b^{2}x^{2}+1029\,a^{6}bx+60\,a^{7}-420\left(b^{7}x^{7}+6\,ab^{6}x^{6}+15\,a^{2}b^{5}x^{5}+20\,a^{3}b^{4}x^{4}+15\,a^{4}b^{3}x^{3}+6\,a^{5}b^{2}x^{2}+a^{6}bx\right)\log\left(bx+a\right)+420\left(b^{7}x^{7}+6\,ab^{6}x^{6}+15\,a^{2}b^{5}x^{5}+20\,a^{3}b^{4}x^{4}+15\,a^{4}b^{3}x^{3}+6\,a^{5}b^{2}x^{2}+a^{6}bx\right)\log\left(x\right)}{60\left(a^{8}b^{6}x^{7}+6\,a^{9}b^{5}x^{6}+15\,a^{10}b^{4}x^{5}+20\,a^{11}b^{3}x^{4}+15\,a^{12}b^{2}x^{3}+6\,a^{13}bx^{2}+a^{14}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(420*a*b^6*x^6 + 2310*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x
^3 + 3654*a^5*b^2*x^2 + 1029*a^6*b*x + 60*a^7 - 420*(b^7*x^7 + 6*a*b^6*x^6
+ 15*a^2*b^5*x^5 + 20*a^3*b^4*x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*
x)*log(b*x + a) + 420*(b^7*x^7 + 6*a*b^6*x^6 + 15*a^2*b^5*x^5 + 20*a^3*b^4*
x^4 + 15*a^4*b^3*x^3 + 6*a^5*b^2*x^2 + a^6*b*x)*log(x))/(a^8*b^6*x^7 + 6*a^
9*b^5*x^6 + 15*a^10*b^4*x^5 + 20*a^11*b^3*x^4 + 15*a^12*b^2*x^3 + 6*a^13*b*
x^2 + a^14*x)
```

giac [A] time = 1.05, size = 104, normalized size = 0.89

$$\frac{7\,b\log\left(|bx+a|\right)}{a^{8}}-\frac{7\,b\log\left(|x|\right)}{a^{8}}-\frac{420\,ab^{6}x^{6}+2310\,a^{2}b^{5}x^{5}+5180\,a^{3}b^{4}x^{4}+5985\,a^{4}b^{3}x^{3}+3654\,a^{5}b^{2}x^{2}+1029\,a^{6}bx+60\,a^{7}}{60\,(bx+a)^{6}a^{8}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 7*b*log(abs(b*x + a))/a^8 - 7*b*log(abs(x))/a^8 - 1/60*(420*a*b^6*x^6 + 231
0*a^2*b^5*x^5 + 5180*a^3*b^4*x^4 + 5985*a^4*b^3*x^3 + 3654*a^5*b^2*x^2 + 10
29*a^6*b*x + 60*a^7)/((b*x + a)^6*a^8*x)
```

maple [A] time = 0.01, size = 108, normalized size = 0.92

$$-\frac{b}{6\,(bx+a)^{6}\,a^{2}}-\frac{2b}{5\,(bx+a)^{5}\,a^{3}}-\frac{3b}{4\,(bx+a)^{4}\,a^{4}}-\frac{4b}{3\,(bx+a)^{3}\,a^{5}}-\frac{5b}{2\,(bx+a)^{2}\,a^{6}}-\frac{6b}{(bx+a)\,a^{7}}-\frac{7b\ln(x)}{a^{8}}+\frac{7b\ln\left(bx+a\right)}{a^{8}}-\frac{1}{a^{7}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^7,x)
```

```
[Out] -1/a^7/x-1/6*b/a^2/(b*x+a)^6-2/5*b/a^3/(b*x+a)^5-3/4*b/a^4/(b*x+a)^4-4/3*b/
a^5/(b*x+a)^3-5/2*b/a^6/(b*x+a)^2-6*b/a^7/(b*x+a)-7*b*ln(x)/a^8+7*b*ln(b*x+
a)/a^8
```

maxima [A] time = 1.59, size = 157, normalized size = 1.34

$$-\frac{420\,b^{6}x^{6}+2310\,ab^{5}x^{5}+5180\,a^{2}b^{4}x^{4}+5985\,a^{3}b^{3}x^{3}+3654\,a^{4}b^{2}x^{2}+1029\,a^{5}bx+60\,a^{6}}{60\left(a^{7}b^{6}x^{7}+6\,a^{8}b^{5}x^{6}+15\,a^{9}b^{4}x^{5}+20\,a^{10}b^{3}x^{4}+15\,a^{11}b^{2}x^{3}+6\,a^{12}bx^{2}+a^{13}x\right)}+\frac{7\,b\log\left(bx+a\right)}{a^{8}}-\frac{7\,b\log(x)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(420*b^6*x^6 + 2310*a*b^5*x^5 + 5180*a^2*b^4*x^4 + 5985*a^3*b^3*x^3 +
 3654*a^4*b^2*x^2 + 1029*a^5*b*x + 60*a^6)/(a^7*b^6*x^7 + 6*a^8*b^5*x^6 + 1
5*a^9*b^4*x^5 + 20*a^10*b^3*x^4 + 15*a^11*b^2*x^3 + 6*a^12*b*x^2 + a^13*x)
+ 7*b*log(b*x + a)/a^8 - 7*b*log(x)/a^8
```

mupad [B] time = 0.19, size = 151, normalized size = 1.29

$$\frac{14\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{8}}-\frac{\frac{1}{a}+\frac{609\,b^{2}\,x^{2}}{10\,a^{3}}+\frac{399\,b^{3}\,x^{3}}{4\,a^{4}}+\frac{259\,b^{4}\,x^{4}}{3\,a^{5}}+\frac{77\,b^{5}\,x^{5}}{2\,a^{6}}+\frac{7\,b^{6}\,x^{6}}{a^{7}}+\frac{343\,b\,x}{20\,a^{2}}}{a^{6}\,x+6\,a^{5}\,b\,x^{2}+15\,a^{4}\,b^{2}\,x^{3}+20\,a^{3}\,b^{3}\,x^{4}+15\,a^{2}\,b^{4}\,x^{5}+6\,a\,b^{5}\,x^{6}+b^{6}\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^7),x)
```

```
[Out] (14*b*atanh((2*b*x)/a + 1))/a^8 - (1/a + (609*b^2*x^2)/(10*a^3) + (399*b^3*
x^3)/(4*a^4) + (259*b^4*x^4)/(3*a^5) + (77*b^5*x^5)/(2*a^6) + (7*b^6*x^6)/a
^7 + (343*b*x)/(20*a^2))/(a^6*x + b^6*x^7 + 6*a^5*b*x^2 + 6*a*b^5*x^6 + 15*
a^4*b^2*x^3 + 20*a^3*b^3*x^4 + 15*a^2*b^4*x^5)
```

sympy [A] time = 0.80, size = 162, normalized size = 1.38

$$\frac{-60a^{6}-1029a^{5}bx-3654a^{4}b^{2}x^{2}-5985a^{3}b^{3}x^{3}-5180a^{2}b^{4}x^{4}-2310ab^{5}x^{5}-420b^{6}x^{6}}{60a^{13}x+360a^{12}bx^{2}+900a^{11}b^{2}x^{3}+1200a^{10}b^{3}x^{4}+900a^{9}b^{4}x^{5}+360a^{8}b^{5}x^{6}+60a^{7}b^{6}x^{7}}+\frac{7b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**7,x)
```

```
[Out] (-60*a**6 - 1029*a**5*b*x - 3654*a**4*b**2*x**2 - 5985*a**3*b**3*x**3 - 518
0*a**2*b**4*x**4 - 2310*a*b**5*x**5 - 420*b**6*x**6)/(60*a**13*x + 360*a**1
2*b*x**2 + 900*a**11*b**2*x**3 + 1200*a**10*b**3*x**4 + 900*a**9*b**4*x**5
+ 360*a**8*b**5*x**6 + 60*a**7*b**6*x**7) + 7*b*(-log(x) + log(a/b + x))/a*
*8
```

3.220 $\int \frac{1}{x^3(a+bx)^7}\,dx$

Optimal. Leaf size=144

$$\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{21b^2}{a^8(a+bx)}+\frac{7b}{a^8x}+\frac{15b^2}{2a^7(a+bx)^2}-\frac{1}{2a^7x^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+$$

Rubi [A] time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$\frac{21b^2}{a^8(a+bx)}+\frac{15b^2}{2a^7(a+bx)^2}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{b^2}{6a^3(a+bx)^6}+\frac{28b^2\log(x)}{a^9}-\frac{28b^2\log(a+bx)}{a^9}+\frac{7b}{a^8x}-\frac{1}{2a^7x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^7),x]
```

```
[Out] -1/(2*a^7*x^2) + (7*b)/(a^8*x) + b^2/(6*a^3*(a + b*x)^6) + (3*b^2)/(5*a^4*(
a + b*x)^5) + (3*b^2)/(2*a^5*(a + b*x)^4) + (10*b^2)/(3*a^6*(a + b*x)^3) +
(15*b^2)/(2*a^7*(a + b*x)^2) + (21*b^2)/(a^8*(a + b*x)) + (28*b^2*Log[x])/a
^9 - (28*b^2*Log[a + b*x])/a^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^7}\,dx = \int \left(\frac{1}{a^7x^3}-\frac{7b}{a^8x^2}+\frac{28b^2}{a^9x}-\frac{b^3}{a^3(a+bx)^7}-\frac{3b^3}{a^4(a+bx)^6}-\frac{6b^3}{a^5(a+bx)^5}-\frac{10b^3}{a^6(a+bx)^4}-\frac{1}{a^7(a$$

$$= -\frac{1}{2a^7x^2}+\frac{7b}{a^8x}+\frac{b^2}{6a^3(a+bx)^6}+\frac{3b^2}{5a^4(a+bx)^5}+\frac{3b^2}{2a^5(a+bx)^4}+\frac{10b^2}{3a^6(a+bx)^3}+\frac{15b^2}{2a^7(a+$$

Mathematica [A] time = 0.07, size = 112, normalized size = 0.78

$$\frac{\frac{a\left(-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7\right)}{x^2(a+bx)^6}-840b^2\log(a+bx)+840b^2\log(x)}{30a^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^7),x]
```

```
[Out] ((a*(-15*a^7 + 120*a^6*b*x + 2058*a^5*b^2*x^2 + 7308*a^4*b^3*x^3 + 11970*a^
3*b^4*x^4 + 10360*a^2*b^5*x^5 + 4620*a*b^6*x^6 + 840*b^7*x^7))/(x^2*(a + b*
x)^6) + 840*b^2*Log[x] - 840*b^2*Log[a + b*x])/(30*a^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^7),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^7), x]
```

fricas [B] time = 1.33, size = 306, normalized size = 2.12

$$\frac{840\,ab^{7}x^{7} + 4620\,a^{2}b^{6}x^{6} + 10360\,a^{3}b^{5}x^{5} + 11970\,a^{4}b^{4}x^{4} + 7308\,a^{5}b^{3}x^{3} + 2058\,a^{6}b^{2}x^{2} + 120\,a^{7}bx - 15\,a^{8} - 840\,\left(b^{8}x^{8} + 6\,ab^{7}x^{7} + 15\,a^{2}b^{6}x^{6} + 20\,a^{3}b^{5}x^{5} + 15\,a^{4}b^{4}x^{4} + 6\,a^{5}b^{3}x^{3} + a^{6}b^{2}x^{2}\right)\log\left(bx + a\right) + 840\,\left(b^{8}x^{8} + 6\,ab^{7}x^{7} + 15\,a^{2}b^{6}x^{6} + 20\,a^{3}b^{5}x^{5} + 15\,a^{4}b^{4}x^{4} + 6\,a^{5}b^{3}x^{3} + a^{6}b^{2}x^{2}\right)\log\left(x\right)}{30\,\left(a^{9}b^{6}x^{8} + 6\,a^{10}b^{5}x^{7} + 15\,a^{11}b^{4}x^{6} + 20\,a^{12}b^{3}x^{5} + 15\,a^{13}b^{2}x^{4} + 6\,a^{14}bx^{3} + a^{15}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/30*(840*a*b^7*x^7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*
x^4 + 7308*a^5*b^3*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8 - 840*(b^8
*x^8 + 6*a*b^7*x^7 + 15*a^2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a
^5*b^3*x^3 + a^6*b^2*x^2)*log(b*x + a) + 840*(b^8*x^8 + 6*a*b^7*x^7 + 15*a^
2*b^6*x^6 + 20*a^3*b^5*x^5 + 15*a^4*b^4*x^4 + 6*a^5*b^3*x^3 + a^6*b^2*x^2)*
log(x))/(a^9*b^6*x^8 + 6*a^10*b^5*x^7 + 15*a^11*b^4*x^6 + 20*a^12*b^3*x^5 +
 15*a^13*b^2*x^4 + 6*a^14*b*x^3 + a^15*x^2)
```

giac [A] time = 1.27, size = 119, normalized size = 0.83

$$-\frac{28\,b^{2}\log\left(\left|bx + a\right|\right)}{a^{9}} + \frac{28\,b^{2}\log\left(\left|x\right|\right)}{a^{9}} + \frac{840\,ab^{7}x^{7} + 4620\,a^{2}b^{6}x^{6} + 10360\,a^{3}b^{5}x^{5} + 11970\,a^{4}b^{4}x^{4} + 7308\,a^{5}b^{3}x^{3} + 2058\,a^{6}b^{2}x^{2} + 120\,a^{7}bx - 15\,a^{8}}{30\,\left(bx + a\right)^{6}a^{9}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -28*b^2*log(abs(b*x + a))/a^9 + 28*b^2*log(abs(x))/a^9 + 1/30*(840*a*b^7*x^
7 + 4620*a^2*b^6*x^6 + 10360*a^3*b^5*x^5 + 11970*a^4*b^4*x^4 + 7308*a^5*b^3
*x^3 + 2058*a^6*b^2*x^2 + 120*a^7*b*x - 15*a^8)/((b*x + a)^6*a^9*x^2)
```

maple [A] time = 0.01, size = 133, normalized size = 0.92

$$\frac{b^{2}}{6\left(bx + a\right)^{6}a^{3}} + \frac{3b^{2}}{5\left(bx + a\right)^{5}a^{4}} + \frac{3b^{2}}{2\left(bx + a\right)^{4}a^{5}} + \frac{10b^{2}}{3\left(bx + a\right)^{3}a^{6}} + \frac{15b^{2}}{2\left(bx + a\right)^{2}a^{7}} + \frac{21b^{2}}{\left(bx + a\right)a^{8}} + \frac{28b^{2}\ln\left(x\right)}{a^{9}} - \frac{28b^{2}\ln\left(bx + a\right)}{a^{9}} + \frac{7b}{a^{8}x} - \frac{1}{2a^{7}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^7,x)
```

```
[Out] -1/2/a^7/x^2+7*b/a^8/x+1/6*b^2/a^3/(b*x+a)^6+3/5*b^2/a^4/(b*x+a)^5+3/2*b^2/
a^5/(b*x+a)^4+10/3*b^2/a^6/(b*x+a)^3+15/2*b^2/a^7/(b*x+a)^2+21*b^2/a^8/(b*x
+a)+28*b^2*ln(x)/a^9-28*b^2*ln(b*x+a)/a^9
```

maxima [A] time = 1.55, size = 174, normalized size = 1.21

$$\frac{840\,b^{7}x^{7} + 4620\,ab^{6}x^{6} + 10360\,a^{2}b^{5}x^{5} + 11970\,a^{3}b^{4}x^{4} + 7308\,a^{4}b^{3}x^{3} + 2058\,a^{5}b^{2}x^{2} + 120\,a^{6}bx - 15\,a^{7}}{30\,\left(a^{8}b^{6}x^{8} + 6\,a^{9}b^{5}x^{7} + 15\,a^{10}b^{4}x^{6} + 20\,a^{11}b^{3}x^{5} + 15\,a^{12}b^{2}x^{4} + 6\,a^{13}bx^{3} + a^{14}x^{2}\right)} - \frac{28\,b^{2}\log\left(bx + a\right)}{a^{9}} + \frac{28\,b^{2}\log\left(x\right)}{a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] 1/30*(840*b^7*x^7 + 4620*a*b^6*x^6 + 10360*a^2*b^5*x^5 + 11970*a^3*b^4*x^4
+ 7308*a^4*b^3*x^3 + 2058*a^5*b^2*x^2 + 120*a^6*b*x - 15*a^7)/(a^8*b^6*x^8
+ 6*a^9*b^5*x^7 + 15*a^10*b^4*x^6 + 20*a^11*b^3*x^5 + 15*a^12*b^2*x^4 + 6*a
^13*b*x^3 + a^14*x^2) - 28*b^2*log(b*x + a)/a^9 + 28*b^2*log(x)/a^9
```

mupad [B] time = 0.21, size = 167, normalized size = 1.16

$$\frac{\frac{343\,b^{2}\,x^{2}}{5\,a^{3}} - \frac{1}{2\,a} + \frac{1218\,b^{3}\,x^{3}}{5\,a^{4}} + \frac{399\,b^{4}\,x^{4}}{a^{5}} + \frac{1036\,b^{5}\,x^{5}}{3\,a^{6}} + \frac{154\,b^{6}\,x^{6}}{a^{7}} + \frac{28\,b^{7}\,x^{7}}{a^{8}} + \frac{4\,b\,x}{a^{2}}}{a^{6}\,x^{2} + 6\,a^{5}\,b\,x^{3} + 15\,a^{4}\,b^{2}\,x^{4} + 20\,a^{3}\,b^{3}\,x^{5} + 15\,a^{2}\,b^{4}\,x^{6} + 6\,a\,b^{5}\,x^{7} + b^{6}\,x^{8}} - \frac{56\,b^{2}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^7),x)
```

```
[Out] ((343*b^2*x^2)/(5*a^3) - 1/(2*a) + (1218*b^3*x^3)/(5*a^4) + (399*b^4*x^4)/a
^5 + (1036*b^5*x^5)/(3*a^6) + (154*b^6*x^6)/a^7 + (28*b^7*x^7)/a^8 + (4*b*x
)/a^2)/(a^6*x^2 + b^6*x^8 + 6*a^5*b*x^3 + 6*a*b^5*x^7 + 15*a^4*b^2*x^4 + 20
*a^3*b^3*x^5 + 15*a^2*b^4*x^6) - (56*b^2*atanh((2*b*x)/a + 1))/a^9
```

sympy [A] time = 0.84, size = 175, normalized size = 1.22

$$\frac{-15a^7+120a^6bx+2058a^5b^2x^2+7308a^4b^3x^3+11970a^3b^4x^4+10360a^2b^5x^5+4620ab^6x^6+840b^7x^7}{30a^{14}x^2+180a^{13}bx^3+450a^{12}b^2x^4+600a^{11}b^3x^5+450a^{10}b^4x^6+180a^9b^5x^7+30a^8b^6x^8}+\frac{28b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**7,x)
```

```
[Out] (-15*a**7 + 120*a**6*b*x + 2058*a**5*b**2*x**2 + 7308*a**4*b**3*x**3 + 1197
0*a**3*b**4*x**4 + 10360*a**2*b**5*x**5 + 4620*a*b**6*x**6 + 840*b**7*x**7)
/(30*a**14*x**2 + 180*a**13*b*x**3 + 450*a**12*b**2*x**4 + 600*a**11*b**3*x
**5 + 450*a**10*b**4*x**6 + 180*a**9*b**5*x**7 + 30*a**8*b**6*x**8) + 28*b*
*2*(log(x) - log(a/b + x))/a**9
```

3.221 $\int \frac{1}{x^4(a+bx)^7}\,dx$

Optimal. Leaf size=157

$$-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}-\frac{56b^3}{a^9(a+bx)}-\frac{28b^2}{a^9x}-\frac{35b^3}{2a^8(a+bx)^2}+\frac{7b}{2a^8x^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{1}{3a^7x^3}-\frac{5b^3}{2a^6(a+bx)^4}-\ldots$$

Rubi [A] time = 0.10, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{56b^3}{a^9(a+bx)}-\frac{35b^3}{2a^8(a+bx)^2}-\frac{20b^3}{3a^7(a+bx)^3}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{b^3}{6a^4(a+bx)^6}-\frac{28b^2}{a^9x}-\frac{84b^3\log(x)}{a^{10}}+\frac{84b^3\log(a+bx)}{a^{10}}+\frac{7b}{2a^8x^2}-\frac{1}{3a^7x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/(3*a^7*x^3) + (7*b)/(2*a^8*x^2) - (28*b^2)/(a^9*x) - b^3/(6*a^4*(a + b*x
)^6) - (4*b^3)/(5*a^5*(a + b*x)^5) - (5*b^3)/(2*a^6*(a + b*x)^4) - (20*b^3)
/(3*a^7*(a + b*x)^3) - (35*b^3)/(2*a^8*(a + b*x)^2) - (56*b^3)/(a^9*(a + b*
x)) - (84*b^3*Log[x])/a^10 + (84*b^3*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^7}\,dx &= \int \left(\frac{1}{a^7x^4}-\frac{7b}{a^8x^3}+\frac{28b^2}{a^9x^2}-\frac{84b^3}{a^{10}x}+\frac{b^4}{a^4(a+bx)^7}+\frac{4b^4}{a^5(a+bx)^6}+\frac{10b^4}{a^6(a+bx)^5}+\frac{20b^4}{a^7(a+bx)^4}+\ldots\right)dx\\ &= -\frac{1}{3a^7x^3}+\frac{7b}{2a^8x^2}-\frac{28b^2}{a^9x}-\frac{b^3}{6a^4(a+bx)^6}-\frac{4b^3}{5a^5(a+bx)^5}-\frac{5b^3}{2a^6(a+bx)^4}-\frac{20b^3}{3a^7(a+bx)^3}-\ldots\end{aligned}$$

Mathematica [A] time = 0.09, size = 123, normalized size = 0.78

$$-\frac{\frac{a\left(10a^8-45a^7bx+360a^6b^2x^2+6174a^5b^3x^3+21924a^4b^4x^4+35910a^3b^5x^5+31080a^2b^6x^6+13860ab^7x^7+2520b^8x^8\right)}{x^3(a+bx)^6}-2520b^3\log(a+bx)+2520b^3\log(x)}{30a^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^7),x]
```

```
[Out] -1/30*((a*(10*a^8 - 45*a^7*b*x + 360*a^6*b^2*x^2 + 6174*a^5*b^3*x^3 + 21924
*a^4*b^4*x^4 + 35910*a^3*b^5*x^5 + 31080*a^2*b^6*x^6 + 13860*a*b^7*x^7 + 25
20*b^8*x^8))/(x^3*(a + b*x)^6) + 2520*b^3*Log[x] - 2520*b^3*Log[a + b*x])/a
^10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^7}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^7),x]

[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^7), x]

fricas [B] time = 1.29, size = 317, normalized size = 2.02

$$-\frac{2520\,ab^{8}x^{8}+13860\,a^{2}b^{7}x^{7}+31080\,a^{3}b^{6}x^{6}+35910\,a^{4}b^{5}x^{5}+21924\,a^{5}b^{4}x^{4}+6174\,a^{6}b^{3}x^{3}+360\,a^{7}b^{2}x^{2}-45\,a^{8}bx+10\,a^{9}-2520\,\left(b^{9}x^{9}+6\,ab^{8}x^{8}+15\,a^{2}b^{7}x^{7}+20\,a^{3}b^{6}x^{6}+15\,a^{4}b^{5}x^{5}+6\,a^{5}b^{4}x^{4}+a^{6}b^{3}x^{3}\right)\log\left(bx+a\right)+2520\,\left(b^{9}x^{9}+6\,ab^{8}x^{8}+15\,a^{2}b^{7}x^{7}+20\,a^{3}b^{6}x^{6}+15\,a^{4}b^{5}x^{5}+6\,a^{5}b^{4}x^{4}+a^{6}b^{3}x^{3}\right)\log\left(x\right)}{30\,\left(a^{10}b^{6}x^{9}+6\,a^{11}b^{5}x^{8}+15\,a^{12}b^{4}x^{7}+20\,a^{13}b^{3}x^{6}+15\,a^{14}b^{2}x^{5}+6\,a^{15}bx^{4}+a^{16}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="fricas")

[Out] -1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9 - 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(b*x + a) + 2520*(b^9*x^9 + 6*a*b^8*x^8 + 15*a^2*b^7*x^7 + 20*a^3*b^6*x^6 + 15*a^4*b^5*x^5 + 6*a^5*b^4*x^4 + a^6*b^3*x^3)*log(x))/(a^10*b^6*x^9 + 6*a^11*b^5*x^8 + 15*a^12*b^4*x^7 + 20*a^13*b^3*x^6 + 15*a^14*b^2*x^5 + 6*a^15*b*x^4 + a^16*x^3)

giac [A] time = 1.25, size = 130, normalized size = 0.83

$$\frac{84\,b^{3}\log\left(|bx+a|\right)}{a^{10}}-\frac{84\,b^{3}\log\left(|x|\right)}{a^{10}}-\frac{2520\,ab^{8}x^{8}+13860\,a^{2}b^{7}x^{7}+31080\,a^{3}b^{6}x^{6}+35910\,a^{4}b^{5}x^{5}+21924\,a^{5}b^{4}x^{4}+6174\,a^{6}b^{3}x^{3}+360\,a^{7}b^{2}x^{2}-45\,a^{8}bx+10\,a^{9}}{30\,(bx+a)^{6}a^{10}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="giac")

[Out] 84*b^3*log(abs(b*x + a))/a^10 - 84*b^3*log(abs(x))/a^10 - 1/30*(2520*a*b^8*x^8 + 13860*a^2*b^7*x^7 + 31080*a^3*b^6*x^6 + 35910*a^4*b^5*x^5 + 21924*a^5*b^4*x^4 + 6174*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 45*a^8*b*x + 10*a^9)/((b*x + a)^6*a^10*x^3)

maple [A] time = 0.01, size = 144, normalized size = 0.92

$$-\frac{b^{3}}{6\,(bx+a)^{6}\,a^{4}}-\frac{4b^{3}}{5\,(bx+a)^{5}\,a^{5}}-\frac{5b^{3}}{2\,(bx+a)^{4}\,a^{6}}-\frac{20b^{3}}{3\,(bx+a)^{3}\,a^{7}}-\frac{35b^{3}}{2\,(bx+a)^{2}\,a^{8}}-\frac{56b^{3}}{(bx+a)\,a^{9}}-\frac{84b^{3}\ln(x)}{a^{10}}+\frac{84b^{3}\ln(bx+a)}{a^{10}}-\frac{28b^{2}}{a^{9}x}+\frac{7b}{2a^{8}x^{2}}-\frac{1}{3a^{7}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(b*x+a)^7,x)

[Out] -1/3/a^7/x^3+7/2*b/a^8/x^2-28*b^2/a^9/x-1/6*b^3/a^4/(b*x+a)^6-4/5*b^3/a^5/(b*x+a)^5-5/2*b^3/a^6/(b*x+a)^4-20/3*b^3/a^7/(b*x+a)^3-35/2*b^3/a^8/(b*x+a)^2-56*b^3/a^9/(b*x+a)-84*b^3*ln(x)/a^10+84*b^3*ln(b*x+a)/a^10

maxima [A] time = 1.59, size = 185, normalized size = 1.18

$$-\frac{2520\,b^{8}x^{8}+13860\,ab^{7}x^{7}+31080\,a^{2}b^{6}x^{6}+35910\,a^{3}b^{5}x^{5}+21924\,a^{4}b^{4}x^{4}+6174\,a^{5}b^{3}x^{3}+360\,a^{6}b^{2}x^{2}-45\,a^{7}bx+10\,a^{8}}{30\,\left(a^{9}b^{6}x^{9}+6\,a^{10}b^{5}x^{8}+15\,a^{11}b^{4}x^{7}+20\,a^{12}b^{3}x^{6}+15\,a^{13}b^{2}x^{5}+6\,a^{14}bx^{4}+a^{15}x^{3}\right)}+\frac{84\,b^{3}\log\left(bx+a\right)}{a^{10}}-\frac{84\,b^{3}\log\left(x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x+a)^7,x, algorithm="maxima")

[Out] -1/30*(2520*b^8*x^8 + 13860*a*b^7*x^7 + 31080*a^2*b^6*x^6 + 35910*a^3*b^5*x^5 + 21924*a^4*b^4*x^4 + 6174*a^5*b^3*x^3 + 360*a^6*b^2*x^2 - 45*a^7*b*x + 10*a^8)/(a^9*b^6*x^9 + 6*a^10*b^5*x^8 + 15*a^11*b^4*x^7 + 20*a^12*b^3*x^6 + 15*a^13*b^2*x^5 + 6*a^14*b*x^4 + a^15*x^3) + 84*b^3*log(b*x + a)/a^10 - 84*b^3*log(x)/a^10

mupad [B] time = 0.31, size = 179, normalized size = 1.14

$$\frac{168\,b^{3}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{10}}-\frac{\frac{1}{3\,a}+\frac{12\,b^{2}\,x^{2}}{a^{3}}+\frac{1029\,b^{3}\,x^{3}}{5\,a^{4}}+\frac{3654\,b^{4}\,x^{4}}{5\,a^{5}}+\frac{1197\,b^{5}\,x^{5}}{a^{6}}+\frac{1036\,b^{6}\,x^{6}}{a^{7}}+\frac{462\,b^{7}\,x^{7}}{a^{8}}+\frac{84\,b^{8}\,x^{8}}{a^{9}}-\frac{3\,b\,x}{2\,a^{2}}}{a^{6}\,x^{3}+6\,a^{5}\,b\,x^{4}+15\,a^{4}\,b^{2}\,x^{5}+20\,a^{3}\,b^{3}\,x^{6}+15\,a^{2}\,b^{4}\,x^{7}+6\,a\,b^{5}\,x^{8}+b^{6}\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^7),x)
```

```
[Out] (168*b^3*atanh((2*b*x)/a + 1))/a^10 - (1/(3*a) + (12*b^2*x^2)/a^3 + (1029*b
^3*x^3)/(5*a^4) + (3654*b^4*x^4)/(5*a^5) + (1197*b^5*x^5)/a^6 + (1036*b^6*x
^6)/a^7 + (462*b^7*x^7)/a^8 + (84*b^8*x^8)/a^9 - (3*b*x)/(2*a^2))/(a^6*x^3
+ b^6*x^9 + 6*a^5*b*x^4 + 6*a*b^5*x^8 + 15*a^4*b^2*x^5 + 20*a^3*b^3*x^6 + 1
5*a^2*b^4*x^7)
```

sympy [A] time = 0.99, size = 187, normalized size = 1.19

$$\frac{-10a^8+45a^7bx-360a^6b^2x^2-6174a^5b^3x^3-21924a^4b^4x^4-35910a^3b^5x^5-31080a^2b^6x^6-13860ab^7x^7-2520b^8x^8}{30a^{15}x^3+180a^{14}bx^4+450a^{13}b^2x^5+600a^{12}b^3x^6+450a^{11}b^4x^7+180a^{10}b^5x^8+30a^9b^6x^9}+\frac{84b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**7,x)
```

```
[Out] (-10*a**8 + 45*a**7*b*x - 360*a**6*b**2*x**2 - 6174*a**5*b**3*x**3 - 21924*
a**4*b**4*x**4 - 35910*a**3*b**5*x**5 - 31080*a**2*b**6*x**6 - 13860*a*b**7
*x**7 - 2520*b**8*x**8)/(30*a**15*x**3 + 180*a**14*b*x**4 + 450*a**13*b**2*
x**5 + 600*a**12*b**3*x**6 + 450*a**11*b**4*x**7 + 180*a**10*b**5*x**8 + 30
*a**9*b**6*x**9) + 84*b**3*(-log(x) + log(a/b + x))/a**10
```

3.222 $\int \frac{x^{12}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=186

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\cdots$$

Rubi [A] time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\frac{99a^8}{b^{13}(a+bx)^5}+\frac{198a^7}{b^{13}(a+bx)^4}-\frac{308a^6}{b^{13}(a+bx)^3}+\frac{396a^5}{b^{13}(a+bx)^2}-\frac{495a^4}{b^{13}(a+bx)}+\frac{55a^2x}{b^{12}}-\frac{220a^3\log(a+bx)}{b^{13}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^12/(a + b*x)^10,x]
```

```
[Out] (55*a^2*x)/b^12 - (5*a*x^2)/b^11 + x^3/(3*b^10) - a^12/(9*b^13*(a + b*x)^9)
 + (3*a^11)/(2*b^13*(a + b*x)^8) - (66*a^10)/(7*b^13*(a + b*x)^7) + (110*a^
9)/(3*b^13*(a + b*x)^6) - (99*a^8)/(b^13*(a + b*x)^5) + (198*a^7)/(b^13*(a
+ b*x)^4) - (308*a^6)/(b^13*(a + b*x)^3) + (396*a^5)/(b^13*(a + b*x)^2) - (
495*a^4)/(b^13*(a + b*x)) - (220*a^3*Log[a + b*x])/b^13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{12}}{(a+bx)^{10}}\,dx &= \int \left(\frac{55a^2}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{b^{10}}+\frac{a^{12}}{b^{12}(a+bx)^{10}}-\frac{12a^{11}}{b^{12}(a+bx)^9}+\frac{66a^{10}}{b^{12}(a+bx)^8}-\frac{220a^9}{b^{12}(a+bx)^7}+\cdots\right.\\ &= \frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{12}}{9b^{13}(a+bx)^9}+\frac{3a^{11}}{2b^{13}(a+bx)^8}-\frac{66a^{10}}{7b^{13}(a+bx)^7}+\frac{110a^9}{3b^{13}(a+bx)^6}-\cdots\end{aligned}$$

Mathematica [A] time = 0.05, size = 161, normalized size = 0.87

$$-\frac{35201a^{12}+289089a^{11}bx+1031616a^{10}b^2x^2+2074464a^9b^3x^3+2529576a^8b^4x^4+1831032a^7b^5x^5+638568a^6b^6x^6-58968a^5b^7x^7-139482a^4b^8x^8-43218a^3b^9x^9+27720a^3(a+bx)^9\log(a+bx)-2772a^2b^{10}x^{10}+252ab^{11}x^{11}-42b^{12}x^{12}}{126b^{13}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12/(a + b*x)^10,x]
```

```
[Out] -1/126*(35201*a^12 + 289089*a^11*b*x + 1031616*a^10*b^2*x^2 + 2074464*a^9*b
^3*x^3 + 2529576*a^8*b^4*x^4 + 1831032*a^7*b^5*x^5 + 638568*a^6*b^6*x^6 - 5
8968*a^5*b^7*x^7 - 139482*a^4*b^8*x^8 - 43218*a^3*b^9*x^9 - 2772*a^2*b^10*x
^10 + 252*a*b^11*x^11 - 42*b^12*x^12 + 27720*a^3*(a + b*x)^9*Log[a + b*x])/
(b^13*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{12}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^12/(a + b*x)^10,x]

[Out] IntegrateAlgebraic[x^12/(a + b*x)^10, x]

fricas [A] time = 1.20, size = 338, normalized size = 1.82

$$\frac{42 b^{12}x^{12} - 252 ab^{11}x^{11} + 2772 a^2b^{10}x^{10} + 43218 a^3b^9x^9 + 139482 a^4b^8x^8 + 58968 a^5b^7x^7 - 638568 a^6b^6x^6 - 1831032 a^7b^5x^5 - 2529576 a^8b^4x^4 - 2074464 a^9b^3x^3 - 1031616 a^{10}b^2x^2 - 289089 a^{11}bx - 35201 a^{12} - 27720 \left(a^3b^9x^9 + 9 a^4b^8x^8 + 36 a^5b^7x^7 + 84 a^6b^6x^6 + 126 a^7b^5x^5 + 126 a^8b^4x^4 + 84 a^9b^3x^3 + 36 a^{10}b^2x^2 + 9 a^{11}bx + a^{12}\right)\log(bx+a)}{126 \left(b^{22}x^9 + 9 ab^{21}x^8 + 36 a^2b^{20}x^7 + 84 a^3b^{19}x^6 + 126 a^4b^{18}x^5 + 126 a^5b^{17}x^4 + 84 a^6b^{16}x^3 + 36 a^7b^{15}x^2 + 9 a^8b^{14}x + a^9b^{13}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b*x+a)^10,x, algorithm="fricas")

[Out] 1/126*(42*b^12*x^12 - 252*a*b^11*x^11 + 2772*a^2*b^10*x^10 + 43218*a^3*b^9*x^9 + 139482*a^4*b^8*x^8 + 58968*a^5*b^7*x^7 - 638568*a^6*b^6*x^6 - 1831032*a^7*b^5*x^5 - 2529576*a^8*b^4*x^4 - 2074464*a^9*b^3*x^3 - 1031616*a^10*b^2*x^2 - 289089*a^11*b*x - 35201*a^12 - 27720*(a^3*b^9*x^9 + 9*a^4*b^8*x^8 + 36*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 126*a^7*b^5*x^5 + 126*a^8*b^4*x^4 + 84*a^9*b^3*x^3 + 36*a^10*b^2*x^2 + 9*a^11*b*x + a^12)*log(b*x + a))/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13)

giac [A] time = 1.07, size = 149, normalized size = 0.80

$$-\frac{220 a^3 \log(|bx+a|)}{b^{13}} - \frac{62370 a^4b^8x^8 + 449064 a^5b^7x^7 + 1435896 a^6b^6x^6 + 2652804 a^7b^5x^5 + 3089394 a^8b^4x^4 + 2318316 a^9b^3x^3 + 1093356 a^{10}b^2x^2 + 296019 a^{11}bx + 35201 a^{12}}{126 (bx+a)^9 b^{13}} + \frac{b^{20}x^3 - 15 ab^{19}x^2 + 165 a^2b^{18}x}{3 b^{30}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b*x+a)^10,x, algorithm="giac")

[Out] -220*a^3*log(abs(b*x + a))/b^13 - 1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/((b*x + a)^9*b^13) + 1/3*(b^20*x^3 - 15*a*b^19*x^2 + 165*a^2*b^18*x)/b^30

maple [A] time = 0.01, size = 177, normalized size = 0.95

$$-\frac{a^{12}}{9 (bx+a)^9 b^{13}} + \frac{3a^{11}}{2 (bx+a)^8 b^{13}} - \frac{66a^{10}}{7 (bx+a)^7 b^{13}} + \frac{110a^9}{3 (bx+a)^6 b^{13}} - \frac{99a^8}{(bx+a)^5 b^{13}} + \frac{198a^7}{(bx+a)^4 b^{13}} - \frac{308a^6}{(bx+a)^3 b^{13}} + \frac{x^3}{3b^{10}} + \frac{396a^5}{(bx+a)^2 b^{13}} - \frac{5a x^2}{b^{11}} - \frac{495a^4}{(bx+a) b^{13}} - \frac{220a^3 \ln(bx+a)}{b^{13}} + \frac{55a^2x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^12/(b*x+a)^10,x)

[Out] 55*a^2*x/b^12-5*a*x^2/b^11+1/3*x^3/b^10-1/9*a^12/b^13/(b*x+a)^9+3/2*a^11/b^13/(b*x+a)^8-66/7*a^10/b^13/(b*x+a)^7+110/3*a^9/b^13/(b*x+a)^6-99*a^8/b^13/(b*x+a)^5+198*a^7/b^13/(b*x+a)^4-308*a^6/b^13/(b*x+a)^3+396*a^5/b^13/(b*x+a)^2-495*a^4/b^13/(b*x+a)-220*a^3*ln(b*x+a)/b^13

maxima [A] time = 1.73, size = 234, normalized size = 1.26

$$-\frac{62370 a^4b^8x^8 + 449064 a^5b^7x^7 + 1435896 a^6b^6x^6 + 2652804 a^7b^5x^5 + 3089394 a^8b^4x^4 + 2318316 a^9b^3x^3 + 1093356 a^{10}b^2x^2 + 296019 a^{11}bx + 35201 a^{12}}{126 \left(b^{22}x^9 + 9 ab^{21}x^8 + 36 a^2b^{20}x^7 + 84 a^3b^{19}x^6 + 126 a^4b^{18}x^5 + 126 a^5b^{17}x^4 + 84 a^6b^{16}x^3 + 36 a^7b^{15}x^2 + 9 a^8b^{14}x + a^9b^{13}\right)} - \frac{220 a^3 \log(bx+a)}{b^{13}} + \frac{b^2x^3 - 15 abx^2 + 165 a^2x}{3 b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^12/(b*x+a)^10,x, algorithm="maxima")

[Out] -1/126*(62370*a^4*b^8*x^8 + 449064*a^5*b^7*x^7 + 1435896*a^6*b^6*x^6 + 2652804*a^7*b^5*x^5 + 3089394*a^8*b^4*x^4 + 2318316*a^9*b^3*x^3 + 1093356*a^10*b^2*x^2 + 296019*a^11*b*x + 35201*a^12)/(b^22*x^9 + 9*a*b^21*x^8 + 36*a^2*b^20*x^7 + 84*a^3*b^19*x^6 + 126*a^4*b^18*x^5 + 126*a^5*b^17*x^4 + 84*a^6*b^16*x^3 + 36*a^7*b^15*x^2 + 9*a^8*b^14*x + a^9*b^13) - 220*a^3*log(b*x + a)/b^13 + 1/3*(b^2*x^3 - 15*a*b*x^2 + 165*a^2*x)/b^12

mupad [B] time = 0.98, size = 151, normalized size = 0.81

$$-\frac{6\,a\,{\left(a+b\,x\right)}^2-\frac{{\left(a+b\,x\right)}^3}{3}+\frac{495\,a^4}{a+b\,x}-\frac{396\,a^5}{{\left(a+b\,x\right)}^2}+\frac{308\,a^6}{{\left(a+b\,x\right)}^3}-\frac{198\,a^7}{{\left(a+b\,x\right)}^4}+\frac{99\,a^8}{{\left(a+b\,x\right)}^5}-\frac{110\,a^9}{3\,{\left(a+b\,x\right)}^6}+\frac{66\,a^{10}}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^{11}}{2\,{\left(a+b\,x\right)}^8}+\frac{a^{12}}{9\,{\left(a+b\,x\right)}^9}+220\,a^3\,\ln\left(a+b\,x\right)-66\,a^2\,b\,x}{b^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12/(a + b*x)^10,x)
```

```
[Out] -(6*a*(a + b*x)^2 - (a + b*x)^3/3 + (495*a^4)/(a + b*x) - (396*a^5)/(a + b*
x)^2 + (308*a^6)/(a + b*x)^3 - (198*a^7)/(a + b*x)^4 + (99*a^8)/(a + b*x)^5
 - (110*a^9)/(3*(a + b*x)^6) + (66*a^10)/(7*(a + b*x)^7) - (3*a^11)/(2*(a +
 b*x)^8) + a^12/(9*(a + b*x)^9) + 220*a^3*log(a + b*x) - 66*a^2*b*x)/b^13
```

sympy [A] time = 1.54, size = 250, normalized size = 1.34

$$-\frac{220a^3\log\left(a+bx\right)}{b^{13}}+\frac{55a^2x}{b^{12}}-\frac{5ax^2}{b^{11}}+\frac{-35201a^{12}-296019a^{11}bx-1093356a^{10}b^2x^2-2318316a^9b^3x^3-3089394a^8b^4x^4-2652804a^7b^5x^5-1435896a^6b^6x^6-449064a^5b^7x^7-62370a^4b^8x^8}{126a^9b^{13}+1134a^8b^{14}x+4536a^7b^{15}x^2+10584a^6b^{16}x^3+15876a^5b^{17}x^4+15876a^4b^{18}x^5+10584a^3b^{19}x^6+4536a^2b^{20}x^7+1134ab^{21}x^8+126b^{22}x^9}+\frac{x^3}{3b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12/(b*x+a)**10,x)
```

```
[Out] -220*a**3*log(a + b*x)/b**13 + 55*a**2*x/b**12 - 5*a*x**2/b**11 + (-35201*a
**12 - 296019*a**11*b*x - 1093356*a**10*b**2*x**2 - 2318316*a**9*b**3*x**3
- 3089394*a**8*b**4*x**4 - 2652804*a**7*b**5*x**5 - 1435896*a**6*b**6*x**6
- 449064*a**5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*a**9*b**13 + 1134*a**8
*b**14*x + 4536*a**7*b**15*x**2 + 10584*a**6*b**16*x**3 + 15876*a**5*b**17*
x**4 + 15876*a**4*b**18*x**5 + 10584*a**3*b**19*x**6 + 4536*a**2*b**20*x**7
 + 1134*a*b**21*x**8 + 126*b**22*x**9) + x**3/(3*b**10)
```

3.223 $\int \frac{x^{11}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=177

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165 ...}{b^{12}(a+ ...}$$

Rubi [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+bx)^5}-\frac{231a^6}{2b^{12}(a+bx)^4}+\frac{154a^5}{b^{12}(a+bx)^3}-\frac{165a^4}{b^{12}(a+bx)^2}+\frac{165a^3}{b^{12}(a+bx)}+\frac{55a^2\log(a+bx)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a + b*x)^10,x]
```

```
[Out] (-10*a*x)/b^11 + x^2/(2*b^10) + a^11/(9*b^12*(a + b*x)^9) - (11*a^10)/(8*b^
12*(a + b*x)^8) + (55*a^9)/(7*b^12*(a + b*x)^7) - (55*a^8)/(2*b^12*(a + b*x
)^6) + (66*a^7)/(b^12*(a + b*x)^5) - (231*a^6)/(2*b^12*(a + b*x)^4) + (154*
a^5)/(b^12*(a + b*x)^3) - (165*a^4)/(b^12*(a + b*x)^2) + (165*a^3)/(b^12*(a
 + b*x)) + (55*a^2*Log[a + b*x])/b^12
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{11}}{(a+bx)^{10}}\,dx &= \int \left(-\frac{10a}{b^{11}}+\frac{x}{b^{10}}-\frac{a^{11}}{b^{11}(a+bx)^{10}}+\frac{11a^{10}}{b^{11}(a+bx)^9}-\frac{55a^9}{b^{11}(a+bx)^8}+\frac{165a^8}{b^{11}(a+bx)^7}-\frac{330a^7}{b^{11}(a+bx)^6} ...\right.\\ &= -\frac{10ax}{b^{11}}+\frac{x^2}{2b^{10}}+\frac{a^{11}}{9b^{12}(a+bx)^9}-\frac{11a^{10}}{8b^{12}(a+bx)^8}+\frac{55a^9}{7b^{12}(a+bx)^7}-\frac{55a^8}{2b^{12}(a+bx)^6}+\frac{66a^7}{b^{12}(a+b ...}\end{aligned}$$

Mathematica [A] time = 0.03, size = 150, normalized size = 0.85

$$\frac{42131a^{11}+351459a^{10}bx+1281096a^9b^2x^2+2656584a^8b^3x^3+3402756a^7b^4x^4+2704212a^6b^5x^5+1220688a^5b^6x^6+190512a^4b^7x^7-77112a^3b^8x^8-36288a^2b^9x^9+27720a^2(a+bx)^9\log(a+bx)-2772ab^{10}x^{10}+252b^{11}x^{11}}{504b^{12}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a + b*x)^10,x]
```

```
[Out] (42131*a^11 + 351459*a^10*b*x + 1281096*a^9*b^2*x^2 + 2656584*a^8*b^3*x^3 +
 3402756*a^7*b^4*x^4 + 2704212*a^6*b^5*x^5 + 1220688*a^5*b^6*x^6 + 190512*a
^4*b^7*x^7 - 77112*a^3*b^8*x^8 - 36288*a^2*b^9*x^9 - 2772*a*b^10*x^10 + 252
*b^11*x^11 + 27720*a^2*(a + b*x)^9*Log[a + b*x])/(504*b^12*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{11}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^11/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^11/(a + b*x)^10, x]
```

fricas [A] time = 0.68, size = 327, normalized size = 1.85

$$\frac{252\,b^{11}x^{11} - 2772\,ab^{10}x^{10} - 36288\,a^2b^9x^9 - 77112\,a^3b^8x^8 + 190512\,a^4b^7x^7 + 1220688\,a^5b^6x^6 + 2704212\,a^6b^5x^5 + 3402756\,a^7b^4x^4 + 2656584\,a^8b^3x^3 + 1281096\,a^9b^2x^2 + 351459\,a^{10}bx + 42131\,a^{11} + 27720\left(a^2b^9x^9 + 9\,a^3b^8x^8 + 36\,a^4b^7x^7 + 84\,a^5b^6x^6 + 126\,a^6b^5x^5 + 126\,a^7b^4x^4 + 84\,a^8b^3x^3 + 36\,a^9b^2x^2 + 9\,a^{10}bx + a^{11}\right)\log\left(bx + a\right)}{504\left(b^{21}x^9 + 9\,ab^{20}x^8 + 36\,a^2b^{19}x^7 + 84\,a^3b^{18}x^6 + 126\,a^4b^{17}x^5 + 126\,a^5b^{16}x^4 + 84\,a^6b^{15}x^3 + 36\,a^7b^{14}x^2 + 9\,a^8b^{13}x + a^9b^{12}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 - 2772*a*b^10*x^10 - 36288*a^2*b^9*x^9 - 77112*a^3*b^8
*x^8 + 190512*a^4*b^7*x^7 + 1220688*a^5*b^6*x^6 + 2704212*a^6*b^5*x^5 + 340
2756*a^7*b^4*x^4 + 2656584*a^8*b^3*x^3 + 1281096*a^9*b^2*x^2 + 351459*a^10*
b*x + 42131*a^11 + 27720*(a^2*b^9*x^9 + 9*a^3*b^8*x^8 + 36*a^4*b^7*x^7 + 84
*a^5*b^6*x^6 + 126*a^6*b^5*x^5 + 126*a^7*b^4*x^4 + 84*a^8*b^3*x^3 + 36*a^9*
b^2*x^2 + 9*a^10*b*x + a^11)*log(b*x + a))/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^
2*b^19*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6
*b^15*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12)
```

giac [A] time = 1.70, size = 138, normalized size = 0.78

$$\frac{55\,a^2\log\left(|bx + a|\right)}{b^{12}} + \frac{b^{10}x^2 - 20\,ab^9x}{2\,b^{20}} + \frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a^7b^4x^4 + 2835756\,a^8b^3x^3 + 1326204\,a^9b^2x^2 + 356499\,a^{10}bx + 42131\,a^{11}}{504\left(bx + a\right)^9b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 55*a^2*log(abs(b*x + a))/b^12 + 1/2*(b^10*x^2 - 20*a*b^9*x)/b^20 + 1/504*(8
3160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 3318084*a^6*b
^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^2*x^2 +
356499*a^10*b*x + 42131*a^11)/((b*x + a)^9*b^12)
```

maple [A] time = 0.01, size = 166, normalized size = 0.94

$$\frac{a^{11}}{9\left(bx + a\right)^9b^{12}} - \frac{11a^{10}}{8\left(bx + a\right)^8b^{12}} + \frac{55a^9}{7\left(bx + a\right)^7b^{12}} - \frac{55a^8}{2\left(bx + a\right)^6b^{12}} + \frac{66a^7}{\left(bx + a\right)^5b^{12}} - \frac{231a^6}{2\left(bx + a\right)^4b^{12}} + \frac{154a^5}{\left(bx + a\right)^3b^{12}} - \frac{165a^4}{\left(bx + a\right)^2b^{12}} + \frac{x^2}{2b^{10}} + \frac{165a^3}{\left(bx + a\right)b^{12}} + \frac{55a^2\ln\left(bx + a\right)}{b^{12}} - \frac{10ax}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x+a)^10,x)
```

```
[Out] -10*a*x/b^11+1/2*x^2/b^10+1/9*a^11/b^12/(b*x+a)^9-11/8*a^10/b^12/(b*x+a)^8+
55/7*a^9/b^12/(b*x+a)^7-55/2*a^8/b^12/(b*x+a)^6+66*a^7/b^12/(b*x+a)^5-231/2
*a^6/b^12/(b*x+a)^4+154*a^5/b^12/(b*x+a)^3-165*a^4/b^12/(b*x+a)^2+165*a^3/b
^12/(b*x+a)+55*a^2*ln(b*x+a)/b^12
```

maxima [A] time = 1.65, size = 223, normalized size = 1.26

$$\frac{83160\,a^3b^8x^8 + 582120\,a^4b^7x^7 + 1823976\,a^5b^6x^6 + 3318084\,a^6b^5x^5 + 3817044\,a^7b^4x^4 + 2835756\,a^8b^3x^3 + 1326204\,a^9b^2x^2 + 356499\,a^{10}bx + 42131\,a^{11}}{504\left(b^{21}x^9 + 9\,ab^{20}x^8 + 36\,a^2b^{19}x^7 + 84\,a^3b^{18}x^6 + 126\,a^4b^{17}x^5 + 126\,a^5b^{16}x^4 + 84\,a^6b^{15}x^3 + 36\,a^7b^{14}x^2 + 9\,a^8b^{13}x + a^9b^{12}\right)} + \frac{55\,a^2\log\left(bx + a\right)}{b^{12}} + \frac{bx^2 - 20\,ax}{2\,b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(83160*a^3*b^8*x^8 + 582120*a^4*b^7*x^7 + 1823976*a^5*b^6*x^6 + 33180
84*a^6*b^5*x^5 + 3817044*a^7*b^4*x^4 + 2835756*a^8*b^3*x^3 + 1326204*a^9*b^
2*x^2 + 356499*a^10*b*x + 42131*a^11)/(b^21*x^9 + 9*a*b^20*x^8 + 36*a^2*b^1
9*x^7 + 84*a^3*b^18*x^6 + 126*a^4*b^17*x^5 + 126*a^5*b^16*x^4 + 84*a^6*b^15
*x^3 + 36*a^7*b^14*x^2 + 9*a^8*b^13*x + a^9*b^12) + 55*a^2*log(b*x + a)/b^1
2 + 1/2*(b*x^2 - 20*a*x)/b^11
```

mupad [B] time = 0.23, size = 138, normalized size = 0.78

$$\frac{\frac{(a+b\,x)^2}{2}+\frac{165\,a^3}{a+b\,x}-\frac{165\,a^4}{(a+b\,x)^2}+\frac{154\,a^5}{(a+b\,x)^3}-\frac{231\,a^6}{2\,(a+b\,x)^4}+\frac{66\,a^7}{(a+b\,x)^5}-\frac{55\,a^8}{2\,(a+b\,x)^6}+\frac{55\,a^9}{7\,(a+b\,x)^7}-\frac{11\,a^{10}}{8\,(a+b\,x)^8}+\frac{a^{11}}{9\,(a+b\,x)^9}+55\,a^2\,\ln(a+b\,x)-11\,a\,b\,x}{b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a + b*x)^10,x)
```

```
[Out] ((a + b*x)^2/2 + (165*a^3)/(a + b*x) - (165*a^4)/(a + b*x)^2 + (154*a^5)/(a
 + b*x)^3 - (231*a^6)/(2*(a + b*x)^4) + (66*a^7)/(a + b*x)^5 - (55*a^8)/(2*
(a + b*x)^6) + (55*a^9)/(7*(a + b*x)^7) - (11*a^10)/(8*(a + b*x)^8) + a^11/
(9*(a + b*x)^9) + 55*a^2*log(a + b*x) - 11*a*b*x)/b^12
```

sympy [A] time = 1.48, size = 236, normalized size = 1.33

$$\frac{55a^2\log(a+bx)}{b^{12}}-\frac{10ax}{b^{11}}+\frac{42131a^{11}+356499a^{10}bx+1326204a^9b^2x^2+2835756a^8b^3x^3+3817044a^7b^4x^4+3318084a^6b^5x^5+1823976a^5b^6x^6+582120a^4b^7x^7+83160a^3b^8x^8}{504a^9b^{12}+4536a^8b^{13}x+18144a^7b^{14}x^2+42336a^6b^{15}x^3+63504a^5b^{16}x^4+63504a^4b^{17}x^5+42336a^3b^{18}x^6+18144a^2b^{19}x^7+4536ab^{20}x^8+504b^{21}x^9}+\frac{x^2}{2b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x+a)**10,x)
```

```
[Out] 55*a**2*log(a + b*x)/b**12 - 10*a*x/b**11 + (42131*a**11 + 356499*a**10*b*x
 + 1326204*a**9*b**2*x**2 + 2835756*a**8*b**3*x**3 + 3817044*a**7*b**4*x**4
 + 3318084*a**6*b**5*x**5 + 1823976*a**5*b**6*x**6 + 582120*a**4*b**7*x**7
+ 83160*a**3*b**8*x**8)/(504*a**9*b**12 + 4536*a**8*b**13*x + 18144*a**7*b*
*14*x**2 + 42336*a**6*b**15*x**3 + 63504*a**5*b**16*x**4 + 63504*a**4*b**17
*x**5 + 42336*a**3*b**18*x**6 + 18144*a**2*b**19*x**7 + 4536*a*b**20*x**8 +
 504*b**21*x**9) + x**2/(2*b**10)
```

3.224 $\int \frac{x^{10}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=159

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{6}{b^{11}(a}$$

Rubi [A] time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{63a^5}{b^{11}(a+bx)^4}-\frac{70a^4}{b^{11}(a+bx)^3}+\frac{60a^3}{b^{11}(a+bx)^2}-\frac{45a^2}{b^{11}(a+bx)}-\frac{10a\log(a+bx)}{b^{11}}+\frac{x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^10/(a + b*x)^10,x]
```

```
[Out] x/b^10 - a^10/(9*b^11*(a + b*x)^9) + (5*a^9)/(4*b^11*(a + b*x)^8) - (45*a^8
)/(7*b^11*(a + b*x)^7) + (20*a^7)/(b^11*(a + b*x)^6) - (42*a^6)/(b^11*(a +
b*x)^5) + (63*a^5)/(b^11*(a + b*x)^4) - (70*a^4)/(b^11*(a + b*x)^3) + (60*a
^3)/(b^11*(a + b*x)^2) - (45*a^2)/(b^11*(a + b*x)) - (10*a*Log[a + b*x])/b^
11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx=\int \left(\frac{1}{b^{10}}+\frac{a^{10}}{b^{10}(a+bx)^{10}}-\frac{10a^9}{b^{10}(a+bx)^9}+\frac{45a^8}{b^{10}(a+bx)^8}-\frac{120a^7}{b^{10}(a+bx)^7}+\frac{210a^6}{b^{10}(a+bx)^6}-\frac{}{b^{10}}\right.$$

$$=\frac{x}{b^{10}}-\frac{a^{10}}{9b^{11}(a+bx)^9}+\frac{5a^9}{4b^{11}(a+bx)^8}-\frac{45a^8}{7b^{11}(a+bx)^7}+\frac{20a^7}{b^{11}(a+bx)^6}-\frac{42a^6}{b^{11}(a+bx)^5}+\frac{}{b^{11}(}$$

Mathematica [A] time = 0.03, size = 137, normalized size = 0.86

$$-\frac{4861a^{10}+41229a^9bx+153576a^8b^2x^2+328104a^7b^3x^3+439236a^6b^4x^4+375732a^5b^5x^5+197568a^4b^6x^6+54432a^3b^7x^7+2268a^2b^8x^8-2268ab^9x^9+2520a(a+bx)^9\log(a+bx)-252b^{10}x^{10}}{252b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10 + 41229*a^9*b*x + 153576*a^8*b^2*x^2 + 328104*a^7*b^3*x^3
 + 439236*a^6*b^4*x^4 + 375732*a^5*b^5*x^5 + 197568*a^4*b^6*x^6 + 54432*a^3
*b^7*x^7 + 2268*a^2*b^8*x^8 - 2268*a*b^9*x^9 - 252*b^10*x^10 + 2520*a*(a +
b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{10}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^10/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^10/(a + b*x)^10, x]
```

fricas [B] time = 1.42, size = 314, normalized size = 1.97

$$\frac{252 \, b^{10} x^{10} + 2268 \, a b^{9} x^{9} - 2268 \, a^{2} b^{8} x^{8} - 54432 \, a^{3} b^{7} x^{7} - 197568 \, a^{4} b^{6} x^{6} - 375732 \, a^{5} b^{5} x^{5} - 439236 \, a^{6} b^{4} x^{4} - 328104 \, a^{7} b^{3} x^{3} - 153576 \, a^{8} b^{2} x^{2} - 41229 \, a^{9} b x - 4861 \, a^{10} - 2520 \, {\left(a b^{9} x^{9} + 9 \, a^{2} b^{8} x^{8} + 36 \, a^{3} b^{7} x^{7} + 84 \, a^{4} b^{6} x^{6} + 126 \, a^{5} b^{5} x^{5} + 126 \, a^{6} b^{4} x^{4} + 84 \, a^{7} b^{3} x^{3} + 36 \, a^{8} b^{2} x^{2} + 9 \, a^{9} b x + a^{10}\right)} \log\left(b x + a\right)}{252 \, {\left(b^{20} x^{9} + 9 \, a b^{19} x^{8} + 36 \, a^{2} b^{18} x^{7} + 84 \, a^{3} b^{17} x^{6} + 126 \, a^{4} b^{16} x^{5} + 126 \, a^{5} b^{15} x^{4} + 84 \, a^{6} b^{14} x^{3} + 36 \, a^{7} b^{13} x^{2} + 9 \, a^{8} b^{12} x + a^{9} b^{11}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2268*a*b^9*x^9 - 2268*a^2*b^8*x^8 - 54432*a^3*b^7*x^
7 - 197568*a^4*b^6*x^6 - 375732*a^5*b^5*x^5 - 439236*a^6*b^4*x^4 - 328104*a
^7*b^3*x^3 - 153576*a^8*b^2*x^2 - 41229*a^9*b*x - 4861*a^10 - 2520*(a*b^9*x
^9 + 9*a^2*b^8*x^8 + 36*a^3*b^7*x^7 + 84*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 12
6*a^6*b^4*x^4 + 84*a^7*b^3*x^3 + 36*a^8*b^2*x^2 + 9*a^9*b*x + a^10)*log(b*x
 + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a
^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*
b^12*x + a^9*b^11)
```

giac [A] time = 1.29, size = 121, normalized size = 0.76

$$\frac{x}{b^{10}} - \frac{10 \, a \log\left({\left| b x + a \right|}\right)}{b^{11}} - \frac{11340 \, a^{2} b^{8} x^{8} + 75600 \, a^{3} b^{7} x^{7} + 229320 \, a^{4} b^{6} x^{6} + 407484 \, a^{5} b^{5} x^{5} + 460404 \, a^{6} b^{4} x^{4} + 337176 \, a^{7} b^{3} x^{3} + 155844 \, a^{8} b^{2} x^{2} + 41481 \, a^{9} b x + 4861 \, a^{10}}{252 \, {\left(b x + a\right)}^{9} b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] x/b^10 - 10*a*log(abs(b*x + a))/b^11 - 1/252*(11340*a^2*b^8*x^8 + 75600*a^3
*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 3
37176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2 + 41481*a^9*b*x + 4861*a^10)/((b*x +
 a)^9*b^11)
```

maple [A] time = 0.01, size = 154, normalized size = 0.97

$$-\frac{a^{10}}{9 \left(b x +a \right)^{9} b^{11}}+\frac{5 a^{9}}{4 \left(b x +a \right)^{8} b^{11}}-\frac{45 a^{8}}{7 \left(b x +a \right)^{7} b^{11}}+\frac{20 a^{7}}{\left(b x +a \right)^{6} b^{11}}-\frac{42 a^{6}}{\left(b x +a \right)^{5} b^{11}}+\frac{63 a^{5}}{\left(b x +a \right)^{4} b^{11}}-\frac{70 a^{4}}{\left(b x +a \right)^{3} b^{11}}+\frac{60 a^{3}}{\left(b x +a \right)^{2} b^{11}}-\frac{45 a^{2}}{\left(b x +a \right) b^{11}}-\frac{10 a \ln \left(b x +a \right)}{b^{11}}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(b*x+a)^10,x)
```

```
[Out] x/b^10-1/9*a^10/b^11/(b*x+a)^9+5/4*a^9/b^11/(b*x+a)^8-45/7*a^8/b^11/(b*x+a)
^7+20*a^7/b^11/(b*x+a)^6-42*a^6/b^11/(b*x+a)^5+63*a^5/b^11/(b*x+a)^4-70*a^4
/b^11/(b*x+a)^3+60*a^3/b^11/(b*x+a)^2-45*a^2/b^11/(b*x+a)-10*a*ln(b*x+a)/b^
11
```

maxima [A] time = 1.66, size = 211, normalized size = 1.33

$$-\frac{11340 \, a^{2} b^{8} x^{8} + 75600 \, a^{3} b^{7} x^{7} + 229320 \, a^{4} b^{6} x^{6} + 407484 \, a^{5} b^{5} x^{5} + 460404 \, a^{6} b^{4} x^{4} + 337176 \, a^{7} b^{3} x^{3} + 155844 \, a^{8} b^{2} x^{2} + 41481 \, a^{9} b x + 4861 \, a^{10}}{252 \, {\left(b^{20} x^{9} + 9 \, a b^{19} x^{8} + 36 \, a^{2} b^{18} x^{7} + 84 \, a^{3} b^{17} x^{6} + 126 \, a^{4} b^{16} x^{5} + 126 \, a^{5} b^{15} x^{4} + 84 \, a^{6} b^{14} x^{3} + 36 \, a^{7} b^{13} x^{2} + 9 \, a^{8} b^{12} x + a^{9} b^{11}\right)}} + \frac{x}{b^{10}} - \frac{10 \, a \log\left(b x + a\right)}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(11340*a^2*b^8*x^8 + 75600*a^3*b^7*x^7 + 229320*a^4*b^6*x^6 + 407484
*a^5*b^5*x^5 + 460404*a^6*b^4*x^4 + 337176*a^7*b^3*x^3 + 155844*a^8*b^2*x^2
 + 41481*a^9*b*x + 4861*a^10)/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*x^7 +
84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x^3 + 3
6*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + x/b^10 - 10*a*log(b*x + a)/b^11
```

mupad [B] time = 0.94, size = 127, normalized size = 0.80

$$-\frac{10\,a\,\ln\left(a+b\,x\right)-b\,x+\frac{45\,a^2}{a+b\,x}-\frac{60\,a^3}{{\left(a+b\,x\right)}^2}+\frac{70\,a^4}{{\left(a+b\,x\right)}^3}-\frac{63\,a^5}{{\left(a+b\,x\right)}^4}+\frac{42\,a^6}{{\left(a+b\,x\right)}^5}-\frac{20\,a^7}{{\left(a+b\,x\right)}^6}+\frac{45\,a^8}{7\,{\left(a+b\,x\right)}^7}-\frac{5\,a^9}{4\,{\left(a+b\,x\right)}^8}+\frac{a^{10}}{9\,{\left(a+b\,x\right)}^9}}{b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10/(a + b*x)^10,x)
```

```
[Out] -(10*a*log(a + b*x) - b*x + (45*a^2)/(a + b*x) - (60*a^3)/(a + b*x)^2 + (70
*a^4)/(a + b*x)^3 - (63*a^5)/(a + b*x)^4 + (42*a^6)/(a + b*x)^5 - (20*a^7)/
(a + b*x)^6 + (45*a^8)/(7*(a + b*x)^7) - (5*a^9)/(4*(a + b*x)^8) + a^10/(9*
(a + b*x)^9))/b^11
```

sympy [A] time = 1.33, size = 224, normalized size = 1.41

$$-\frac{10a\log(a+bx)}{b^{11}}+\frac{-4861a^{10}-41481a^9bx-155844a^8b^2x^2-337176a^7b^3x^3-460404a^6b^4x^4-407484a^5b^5x^5-229320a^4b^6x^6-75600a^3b^7x^7-11340a^2b^8x^8}{252a^9b^{11}+2268a^8b^{12}x+9072a^7b^{13}x^2+21168a^6b^{14}x^3+31752a^5b^{15}x^4+31752a^4b^{16}x^5+21168a^3b^{17}x^6+9072a^2b^{18}x^7+2268ab^{19}x^8+252b^{20}x^9}+\frac{x}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10/(b*x+a)**10,x)
```

```
[Out] -10*a*log(a + b*x)/b**11 + (-4861*a**10 - 41481*a**9*b*x - 155844*a**8*b**2
*x**2 - 337176*a**7*b**3*x**3 - 460404*a**6*b**4*x**4 - 407484*a**5*b**5*x*
*5 - 229320*a**4*b**6*x**6 - 75600*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(
252*a**9*b**11 + 2268*a**8*b**12*x + 9072*a**7*b**13*x**2 + 21168*a**6*b**1
4*x**3 + 31752*a**5*b**15*x**4 + 31752*a**4*b**16*x**5 + 21168*a**3*b**17*x
**6 + 9072*a**2*b**18*x**7 + 2268*a*b**19*x**8 + 252*b**20*x**9) + x/b**10
```

3.225 $\int \frac{x^9}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=154

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a}{b^{10}(a+}$$

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$\frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx)^4}+\frac{28a^3}{b^{10}(a+bx)^3}-\frac{18a^2}{b^{10}(a+bx)^2}+\frac{9a}{b^{10}(a+bx)}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(a + b*x)^10,x]
```

```
[Out] a^9/(9*b^10*(a + b*x)^9) - (9*a^8)/(8*b^10*(a + b*x)^8) + (36*a^7)/(7*b^10*
(a + b*x)^7) - (14*a^6)/(b^10*(a + b*x)^6) + (126*a^5)/(5*b^10*(a + b*x)^5)
 - (63*a^4)/(2*b^10*(a + b*x)^4) + (28*a^3)/(b^10*(a + b*x)^3) - (18*a^2)/(
b^10*(a + b*x)^2) + (9*a)/(b^10*(a + b*x)) + Log[a + b*x]/b^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a^9}{b^9(a+bx)^{10}}+\frac{9a^8}{b^9(a+bx)^9}-\frac{36a^7}{b^9(a+bx)^8}+\frac{84a^6}{b^9(a+bx)^7}-\frac{126a^5}{b^9(a+bx)^6}+\frac{126a^4}{b^9(a+bx)^5}-\right.\\ &= \frac{a^9}{9b^{10}(a+bx)^9}-\frac{9a^8}{8b^{10}(a+bx)^8}+\frac{36a^7}{7b^{10}(a+bx)^7}-\frac{14a^6}{b^{10}(a+bx)^6}+\frac{126a^5}{5b^{10}(a+bx)^5}-\frac{63a^4}{2b^{10}(a+bx}\end{aligned}$$

Mathematica [A] time = 0.03, size = 111, normalized size = 0.72

$$\frac{a\left(7129a^8+61641a^7bx+235224a^6b^2x^2+518616a^5b^3x^3+725004a^4b^4x^4+661500a^3b^5x^5+388080a^2b^6x^6+136080ab^7x^7+22680b^8x^8\right)}{2520b^{10}(a+bx)^9}+\frac{\log(a+bx)}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(a + b*x)^10,x]
```

```
[Out] (a*(7129*a^8 + 61641*a^7*b*x + 235224*a^6*b^2*x^2 + 518616*a^5*b^3*x^3 + 72
5004*a^4*b^4*x^4 + 661500*a^3*b^5*x^5 + 388080*a^2*b^6*x^6 + 136080*a*b^7*x
^7 + 22680*b^8*x^8))/(2520*b^10*(a + b*x)^9) + Log[a + b*x]/b^10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^9/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^9/(a + b*x)^10, x]
```

fricas [B] time = 1.21, size = 292, normalized size = 1.90

$$\frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+235224\,a^7b^2x^2+61641\,a^8bx+7129\,a^9+2520\left(b^9x^9+9\,ab^8x^8+36\,a^2b^7x^7+84\,a^3b^6x^6+126\,a^4b^5x^5+126\,a^5b^4x^4+84\,a^6b^3x^3+36\,a^7b^2x^2+9\,a^8bx+a^9\right)\log\left(bx+a\right)}{2520\left(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x^7+84\,a^3b^{16}x^6+126\,a^4b^{15}x^5+126\,a^5b^{14}x^4+84\,a^6b^{13}x^3+36\,a^7b^{12}x^2+9\,a^8b^{11}x+a^9b^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9 + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 +
 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a
^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a))/(b^19*x^9 + 9*a*b^18*x^8 + 36*a
^2*b^17*x^7 + 84*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^
6*b^13*x^3 + 36*a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10)
```

giac [A] time = 1.22, size = 112, normalized size = 0.73

$$\frac{\log\left(|bx+a|\right)}{b^{10}}+\frac{22680\,ab^7x^8+136080\,a^2b^6x^7+388080\,a^3b^5x^6+661500\,a^4b^4x^5+725004\,a^5b^3x^4+518616\,a^6b^2x^3+235224\,a^7bx^2+61641\,a^8x+\frac{7129\,a^9}{b}}{2520\left(bx+a\right)^9b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b^10 + 1/2520*(22680*a*b^7*x^8 + 136080*a^2*b^6*x^7 + 388
080*a^3*b^5*x^6 + 661500*a^4*b^4*x^5 + 725004*a^5*b^3*x^4 + 518616*a^6*b^2*
x^3 + 235224*a^7*b*x^2 + 61641*a^8*x + 7129*a^9/b)/((b*x + a)^9*b^9)
```

maple [A] time = 0.01, size = 145, normalized size = 0.94

$$\frac{a^9}{9\left(bx+a\right)^9b^{10}}-\frac{9a^8}{8\left(bx+a\right)^8b^{10}}+\frac{36a^7}{7\left(bx+a\right)^7b^{10}}-\frac{14a^6}{\left(bx+a\right)^6b^{10}}+\frac{126a^5}{5\left(bx+a\right)^5b^{10}}-\frac{63a^4}{2\left(bx+a\right)^4b^{10}}+\frac{28a^3}{\left(bx+a\right)^3b^{10}}-\frac{18a^2}{\left(bx+a\right)^2b^{10}}+\frac{9a}{\left(bx+a\right)b^{10}}+\frac{\ln\left(bx+a\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x+a)^10,x)
```

```
[Out] 1/9*a^9/b^10/(b*x+a)^9-9/8*a^8/b^10/(b*x+a)^8+36/7*a^7/b^10/(b*x+a)^7-14*a^
6/b^10/(b*x+a)^6+126/5*a^5/b^10/(b*x+a)^5-63/2*a^4/b^10/(b*x+a)^4+28*a^3/b^
10/(b*x+a)^3-18*a^2/b^10/(b*x+a)^2+9*a/b^10/(b*x+a)+ln(b*x+a)/b^10
```

maxima [A] time = 1.60, size = 202, normalized size = 1.31

$$\frac{22680\,ab^8x^8+136080\,a^2b^7x^7+388080\,a^3b^6x^6+661500\,a^4b^5x^5+725004\,a^5b^4x^4+518616\,a^6b^3x^3+235224\,a^7b^2x^2+61641\,a^8bx+7129\,a^9}{2520\left(b^{19}x^9+9\,ab^{18}x^8+36\,a^2b^{17}x^7+84\,a^3b^{16}x^6+126\,a^4b^{15}x^5+126\,a^5b^{14}x^4+84\,a^6b^{13}x^3+36\,a^7b^{12}x^2+9\,a^8b^{11}x+a^9b^{10}\right)}+\frac{\log\left(bx+a\right)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(22680*a*b^8*x^8 + 136080*a^2*b^7*x^7 + 388080*a^3*b^6*x^6 + 661500*
a^4*b^5*x^5 + 725004*a^5*b^4*x^4 + 518616*a^6*b^3*x^3 + 235224*a^7*b^2*x^2
+ 61641*a^8*b*x + 7129*a^9)/(b^19*x^9 + 9*a*b^18*x^8 + 36*a^2*b^17*x^7 + 84
*a^3*b^16*x^6 + 126*a^4*b^15*x^5 + 126*a^5*b^14*x^4 + 84*a^6*b^13*x^3 + 36*
a^7*b^12*x^2 + 9*a^8*b^11*x + a^9*b^10) + log(b*x + a)/b^10
```

mupad [B] time = 0.19, size = 117, normalized size = 0.76

$$\frac{\ln\left(a+b\,x\right)+\frac{9\,a}{a+b\,x}-\frac{18\,a^2}{{\left(a+b\,x\right)}^2}+\frac{28\,a^3}{{\left(a+b\,x\right)}^3}-\frac{63\,a^4}{2\,{\left(a+b\,x\right)}^4}+\frac{126\,a^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,a^6}{{\left(a+b\,x\right)}^6}+\frac{36\,a^7}{7\,{\left(a+b\,x\right)}^7}-\frac{9\,a^8}{8\,{\left(a+b\,x\right)}^8}+\frac{a^9}{9\,{\left(a+b\,x\right)}^9}}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a + b*x)^10,x)
```

```
[Out] (log(a + b*x) + (9*a)/(a + b*x) - (18*a^2)/(a + b*x)^2 + (28*a^3)/(a + b*x)
^3 - (63*a^4)/(2*(a + b*x)^4) + (126*a^5)/(5*(a + b*x)^5) - (14*a^6)/(a + b
*x)^6 + (36*a^7)/(7*(a + b*x)^7) - (9*a^8)/(8*(a + b*x)^8) + a^9/(9*(a + b*
x)^9))/b^10
```

sympy [A] time = 1.11, size = 212, normalized size = 1.38

$$\frac{7129a^9 + 61641a^8bx + 235224a^7b^2x^2 + 518616a^6b^3x^3 + 725004a^5b^4x^4 + 661500a^4b^5x^5 + 388080a^3b^6x^6 + 136080a^2b^7x^7 + 22680ab^8x^8}{2520a^9b^{10} + 22680a^8b^{11}x + 90720a^7b^{12}x^2 + 211680a^6b^{13}x^3 + 317520a^5b^{14}x^4 + 317520a^4b^{15}x^5 + 211680a^3b^{16}x^6 + 90720a^2b^{17}x^7 + 22680ab^{18}x^8 + 2520b^{19}x^9} + \frac{\log(a + bx)}{b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x+a)**10,x)
```

```
[Out] (7129*a**9 + 61641*a**8*b*x + 235224*a**7*b**2*x**2 + 518616*a**6*b**3*x**3
 + 725004*a**5*b**4*x**4 + 661500*a**4*b**5*x**5 + 388080*a**3*b**6*x**6 +
136080*a**2*b**7*x**7 + 22680*a*b**8*x**8)/(2520*a**9*b**10 + 22680*a**8*b*
*11*x + 90720*a**7*b**12*x**2 + 211680*a**6*b**13*x**3 + 317520*a**5*b**14*
x**4 + 317520*a**4*b**15*x**5 + 211680*a**3*b**16*x**6 + 90720*a**2*b**17*x
**7 + 22680*a*b**18*x**8 + 2520*b**19*x**9) + log(a + b*x)/b**10
```

3.226 $\int \frac{x^8}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=17

$$\frac{x^9}{9a(a+bx)^9}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$\frac{x^9}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a + b*x)^10,x]
```

```
[Out] x^9/(9*a*(a + b*x)^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{x^8}{(a+bx)^{10}}\,dx = \frac{x^9}{9a(a+bx)^9}$$

Mathematica [B] time = 0.02, size = 97, normalized size = 5.71

$$-\frac{a^8+9a^7bx+36a^6b^2x^2+84a^5b^3x^3+126a^4b^4x^4+126a^3b^5x^5+84a^2b^6x^6+36ab^7x^7+9b^8x^8}{9b^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a + b*x)^10,x]
```

```
[Out] -1/9*(a^8 + 9*a^7*b*x + 36*a^6*b^2*x^2 + 84*a^5*b^3*x^3 + 126*a^4*b^4*x^4 +
 126*a^3*b^5*x^5 + 84*a^2*b^6*x^6 + 36*a*b^7*x^7 + 9*b^8*x^8)/(b^9*(a + b*x
)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^8/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^8/(a + b*x)^10, x]
```

fricas [B] time = 1.08, size = 186, normalized size = 10.94

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\left(b^{18}x^{9}+9\,ab^{17}x^{8}+36\,a^{2}b^{16}x^{7}+84\,a^{3}b^{15}x^{6}+126\,a^{4}b^{14}x^{5}+126\,a^{5}b^{13}x^{4}+84\,a^{6}b^{12}x^{3}+36\,a^{7}b^{11}x^{2}+9\,a^{8}b^{10}x+a^{9}b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

giac [B] time = 1.23, size = 95, normalized size = 5.59

$$-\frac{9 b^{8} x^{8} + 36 a b^{7} x^{7} + 84 a^{2} b^{6} x^{6} + 126 a^{3} b^{5} x^{5} + 126 a^{4} b^{4} x^{4} + 84 a^{5} b^{3} x^{3} + 36 a^{6} b^{2} x^{2} + 9 a^{7} b x + a^{8}}{9 (b x + a)^{9} b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/((b*x + a)^9*
b^9)
```

maple [B] time = 0.01, size = 131, normalized size = 7.71

$$-\frac{a^{8}}{9 (b x + a)^{9} b^{9}} + \frac{a^{7}}{(b x + a)^{8} b^{9}} - \frac{4 a^{6}}{(b x + a)^{7} b^{9}} + \frac{28 a^{5}}{3 (b x + a)^{6} b^{9}} - \frac{14 a^{4}}{(b x + a)^{5} b^{9}} + \frac{14 a^{3}}{(b x + a)^{4} b^{9}} - \frac{28 a^{2}}{3 (b x + a)^{3} b^{9}} + \frac{4 a}{(b x + a)^{2} b^{9}} - \frac{1}{(b x + a) b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x+a)^10,x)
```

```
[Out] 28/3*a^5/b^9/(b*x+a)^6+4*a/b^9/(b*x+a)^2-14*a^4/b^9/(b*x+a)^5+a^7/b^9/(b*x+
a)^8-28/3*a^2/b^9/(b*x+a)^3+14*a^3/b^9/(b*x+a)^4-1/9*a^8/b^9/(b*x+a)^9-4*a^
6/b^9/(b*x+a)^7-1/b^9/(b*x+a)
```

maxima [B] time = 1.51, size = 186, normalized size = 10.94

$$-\frac{9 b^{8} x^{8} + 36 a b^{7} x^{7} + 84 a^{2} b^{6} x^{6} + 126 a^{3} b^{5} x^{5} + 126 a^{4} b^{4} x^{4} + 84 a^{5} b^{3} x^{3} + 36 a^{6} b^{2} x^{2} + 9 a^{7} b x + a^{8}}{9 \left(b^{18} x^{9} + 9 a b^{17} x^{8} + 36 a^{2} b^{16} x^{7} + 84 a^{3} b^{15} x^{6} + 126 a^{4} b^{14} x^{5} + 126 a^{5} b^{13} x^{4} + 84 a^{6} b^{12} x^{3} + 36 a^{7} b^{11} x^{2} + 9 a^{8} b^{10} x + a^{9} b^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/(b^18*x^9 + 9
*a*b^17*x^8 + 36*a^2*b^16*x^7 + 84*a^3*b^15*x^6 + 126*a^4*b^14*x^5 + 126*a^
5*b^13*x^4 + 84*a^6*b^12*x^3 + 36*a^7*b^11*x^2 + 9*a^8*b^10*x + a^9*b^9)
```

mupad [B] time = 0.14, size = 107, normalized size = 6.29

$$-\frac{\frac{1}{a+b x} - \frac{4 a}{(a+b x)^{2}} + \frac{28 a^{2}}{3 (a+b x)^{3}} - \frac{14 a^{3}}{(a+b x)^{4}} + \frac{14 a^{4}}{(a+b x)^{5}} - \frac{28 a^{5}}{3 (a+b x)^{6}} + \frac{4 a^{6}}{(a+b x)^{7}} - \frac{a^{7}}{(a+b x)^{8}} + \frac{a^{8}}{9 (a+b x)^{9}}}{b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a + b*x)^10,x)
```

```
[Out] -(1/(a + b*x) - (4*a)/(a + b*x)^2 + (28*a^2)/(3*(a + b*x)^3) - (14*a^3)/(a
+ b*x)^4 + (14*a^4)/(a + b*x)^5 - (28*a^5)/(3*(a + b*x)^6) + (4*a^6)/(a + b
*x)^7 - a^7/(a + b*x)^8 + a^8/(9*(a + b*x)^9))/b^9
```

sympy [B] time = 0.98, size = 199, normalized size = 11.71

$$\frac{-a^8-9a^7bx-36a^6b^2x^2-84a^5b^3x^3-126a^4b^4x^4-126a^3b^5x^5-84a^2b^6x^6-36ab^7x^7-9b^8x^8}{9a^9b^9+81a^8b^{10}x+324a^7b^{11}x^2+756a^6b^{12}x^3+1134a^5b^{13}x^4+1134a^4b^{14}x^5+756a^3b^{15}x^6+324a^2b^{16}x^7+81ab^{17}x^8+9b^{18}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x+a)**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*a**9*b**9 + 81*a**8*b**10*x + 324*a**7*b**11*x**2 + 756*a**6*b**12*x
**3 + 1134*a**5*b**13*x**4 + 1134*a**4*b**14*x**5 + 756*a**3*b**15*x**6 + 3
24*a**2*b**16*x**7 + 81*a*b**17*x**8 + 9*b**18*x**9)
```

3.227 $\int \frac{x^7}{(a+bx)^{10}} dx$

Optimal. Leaf size=35

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^8}{72a^2(a+bx)^8}+\frac{x^8}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a + b*x)^10,x]
```

```
[Out] x^8/(9*a*(a + b*x)^9) + x^8/(72*a^2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(a+bx)^{10}}\,dx &= \frac{x^8}{9a(a+bx)^9}+\frac{\int \frac{x^7}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^8}{9a(a+bx)^9}+\frac{x^8}{72a^2(a+bx)^8}\end{aligned}$$

Mathematica [B] time = 0.01, size = 86, normalized size = 2.46

$$-\frac{a^7+9a^6bx+36a^5b^2x^2+84a^4b^3x^3+126a^3b^4x^4+126a^2b^5x^5+84ab^6x^6+36b^7x^7}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7 + 9*a^6*b*x + 36*a^5*b^2*x^2 + 84*a^4*b^3*x^3 + 126*a^3*b^4*x^4
+ 126*a^2*b^5*x^5 + 84*a*b^6*x^6 + 36*b^7*x^7)/(b^8*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^7/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^7/(a + b*x)^10, x]
```

fricas [B] time = 0.88, size = 175, normalized size = 5.00

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\left(b^{17}x^9+9\,ab^{16}x^8+36\,a^2b^{15}x^7+84\,a^3b^{14}x^6+126\,a^4b^{13}x^5+126\,a^5b^{12}x^4+84\,a^6b^{11}x^3+36\,a^7b^{10}x^2+9\,a^8b^9x+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

giac [B] time = 0.96, size = 84, normalized size = 2.40

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\,(bx+a)^9b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/((b*x + a)^9*b^8)
```

maple [B] time = 0.01, size = 117, normalized size = 3.34

$$\frac{a^7}{9\,(bx+a)^9\,b^8}-\frac{7a^6}{8\,(bx+a)^8\,b^8}+\frac{3a^5}{(bx+a)^7\,b^8}-\frac{35a^4}{6\,(bx+a)^6\,b^8}+\frac{7a^3}{(bx+a)^5\,b^8}-\frac{21a^2}{4\,(bx+a)^4\,b^8}+\frac{7a}{3\,(bx+a)^3\,b^8}-\frac{1}{2\,(bx+a)^2\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x+a)^10,x)
```

```
[Out] -35/6*a^4/b^8/(b*x+a)^6-1/2/b^8/(b*x+a)^2+3*a^5/b^8/(b*x+a)^7+7*a^3/b^8/(b*
x+a)^5-21/4*a^2/b^8/(b*x+a)^4-7/8*a^6/b^8/(b*x+a)^8+7/3*a/b^8/(b*x+a)^3+1/9
*a^7/b^8/(b*x+a)^9
```

maxima [B] time = 1.59, size = 175, normalized size = 5.00

$$-\frac{36\,b^7x^7+84\,ab^6x^6+126\,a^2b^5x^5+126\,a^3b^4x^4+84\,a^4b^3x^3+36\,a^5b^2x^2+9\,a^6bx+a^7}{72\left(b^{17}x^9+9\,ab^{16}x^8+36\,a^2b^{15}x^7+84\,a^3b^{14}x^6+126\,a^4b^{13}x^5+126\,a^5b^{12}x^4+84\,a^6b^{11}x^3+36\,a^7b^{10}x^2+9\,a^8b^9x+a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 84*a*b^6*x^6 + 126*a^2*b^5*x^5 + 126*a^3*b^4*x^4 + 84*a
^4*b^3*x^3 + 36*a^5*b^2*x^2 + 9*a^6*b*x + a^7)/(b^17*x^9 + 9*a*b^16*x^8 + 3
6*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^5*b^12*x^4 + 84
*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

mupad [B] time = 0.13, size = 22, normalized size = 0.63

$$\frac{x^8\,(9\,a+b\,x)}{72\,a^2\,(a+b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a + b*x)^10,x)
```

```
[Out] (x^8*(9*a + b*x))/(72*a^2*(a + b*x)^9)
```

sympy [B] time = 1.00, size = 187, normalized size = 5.34

$$\frac{-a^7-9a^6bx-36a^5b^2x^2-84a^4b^3x^3-126a^3b^4x^4-126a^2b^5x^5-84ab^6x^6-36b^7x^7}{72a^9b^8+648a^8b^9x+2592a^7b^{10}x^2+6048a^6b^{11}x^3+9072a^5b^{12}x^4+9072a^4b^{13}x^5+6048a^3b^{14}x^6+2592a^2b^{15}x^7+648ab^{16}x^8+72b^{17}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x+a)**10,x)
```

```
[Out] (-a**7 - 9*a**6*b*x - 36*a**5*b**2*x**2 - 84*a**4*b**3*x**3 - 126*a**3*b**4
*x**4 - 126*a**2*b**5*x**5 - 84*a*b**6*x**6 - 36*b**7*x**7)/(72*a**9*b**8 +
 648*a**8*b**9*x + 2592*a**7*b**10*x**2 + 6048*a**6*b**11*x**3 + 9072*a**5*
b**12*x**4 + 9072*a**4*b**13*x**5 + 6048*a**3*b**14*x**6 + 2592*a**2*b**15*
x**7 + 648*a*b**16*x**8 + 72*b**17*x**9)
```

3.228 $\int \frac{x^6}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=52

$$\frac{x^7}{252a^3(a+bx)^7}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{9a(a+bx)^9}$$

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^7}{252a^3(a+bx)^7}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a + b*x)^10,x]
```

```
[Out] x^7/(9*a*(a + b*x)^9) + x^7/(36*a^2*(a + b*x)^8) + x^7/(252*a^3*(a + b*x)^7
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(a+bx)^{10}}\,dx &= \frac{x^7}{9a(a+bx)^9}+\frac{2\int \frac{x^6}{(a+bx)^9}\,dx}{9a}\\ &= \frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{\int \frac{x^6}{(a+bx)^8}\,dx}{36a^2}\\ &= \frac{x^7}{9a(a+bx)^9}+\frac{x^7}{36a^2(a+bx)^8}+\frac{x^7}{252a^3(a+bx)^7}\end{aligned}$$

Mathematica [A] time = 0.02, size = 75, normalized size = 1.44

$$-\frac{a^6+9a^5bx+36a^4b^2x^2+84a^3b^3x^3+126a^2b^4x^4+126ab^5x^5+84b^6x^6}{252b^7(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^6 + 9*a^5*b*x + 36*a^4*b^2*x^2 + 84*a^3*b^3*x^3 + 126*a^2*b^4*x^4
 + 126*a*b^5*x^5 + 84*b^6*x^6)/(b^7*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^6/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^6/(a + b*x)^10, x]
```

fricas [B] time = 0.87, size = 164, normalized size = 3.15

$$-\frac{84\,b^{6}x^{6}+126\,ab^{5}x^{5}+126\,a^{2}b^{4}x^{4}+84\,a^{3}b^{3}x^{3}+36\,a^{4}b^{2}x^{2}+9\,a^{5}bx+a^{6}}{252\left(b^{16}x^{9}+9\,ab^{15}x^{8}+36\,a^{2}b^{14}x^{7}+84\,a^{3}b^{13}x^{6}+126\,a^{4}b^{12}x^{5}+126\,a^{5}b^{11}x^{4}+84\,a^{6}b^{10}x^{3}+36\,a^{7}b^{9}x^{2}+9\,a^{8}b^{8}x+a^{9}b^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

giac [A] time = 1.01, size = 73, normalized size = 1.40

$$-\frac{84\,b^{6}x^{6}+126\,ab^{5}x^{5}+126\,a^{2}b^{4}x^{4}+84\,a^{3}b^{3}x^{3}+36\,a^{4}b^{2}x^{2}+9\,a^{5}bx+a^{6}}{252\,(bx+a)^{9}b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/((b*x + a)^9*b^7)
```

maple [B] time = 0.01, size = 102, normalized size = 1.96

$$-\frac{a^{6}}{9\,(bx+a)^{9}\,b^{7}}+\frac{3a^{5}}{4\,(bx+a)^{8}\,b^{7}}-\frac{15a^{4}}{7\,(bx+a)^{7}\,b^{7}}+\frac{10a^{3}}{3\,(bx+a)^{6}\,b^{7}}-\frac{3a^{2}}{(bx+a)^{5}\,b^{7}}+\frac{3a}{2\,(bx+a)^{4}\,b^{7}}-\frac{1}{3\,(bx+a)^{3}\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x+a)^10,x)
```

```
[Out] 10/3*a^3/b^7/(b*x+a)^6-3*a^2/b^7/(b*x+a)^5+3/4*a^5/b^7/(b*x+a)^8+3/2*a/b^7/
(b*x+a)^4-1/3/b^7/(b*x+a)^3-1/9*a^6/b^7/(b*x+a)^9-15/7*a^4/b^7/(b*x+a)^7
```

maxima [B] time = 1.49, size = 164, normalized size = 3.15

$$-\frac{84\,b^{6}x^{6}+126\,ab^{5}x^{5}+126\,a^{2}b^{4}x^{4}+84\,a^{3}b^{3}x^{3}+36\,a^{4}b^{2}x^{2}+9\,a^{5}bx+a^{6}}{252\left(b^{16}x^{9}+9\,ab^{15}x^{8}+36\,a^{2}b^{14}x^{7}+84\,a^{3}b^{13}x^{6}+126\,a^{4}b^{12}x^{5}+126\,a^{5}b^{11}x^{4}+84\,a^{6}b^{10}x^{3}+36\,a^{7}b^{9}x^{2}+9\,a^{8}b^{8}x+a^{9}b^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 126*a*b^5*x^5 + 126*a^2*b^4*x^4 + 84*a^3*b^3*x^3 + 36*
a^4*b^2*x^2 + 9*a^5*b*x + a^6)/(b^16*x^9 + 9*a*b^15*x^8 + 36*a^2*b^14*x^7 +
```

```
 84*a^3*b^13*x^6 + 126*a^4*b^12*x^5 + 126*a^5*b^11*x^4 + 84*a^6*b^10*x^3 +
36*a^7*b^9*x^2 + 9*a^8*b^8*x + a^9*b^7)
```

mupad [B] time = 0.14, size = 85, normalized size = 1.63

$$-\frac{\frac{1}{3\,{\left(a+b\,x\right)}^3}-\frac{3\,a}{2\,{\left(a+b\,x\right)}^4}+\frac{3\,a^2}{{\left(a+b\,x\right)}^5}-\frac{10\,a^3}{3\,{\left(a+b\,x\right)}^6}+\frac{15\,a^4}{7\,{\left(a+b\,x\right)}^7}-\frac{3\,a^5}{4\,{\left(a+b\,x\right)}^8}+\frac{a^6}{9\,{\left(a+b\,x\right)}^9}}{b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a + b*x)^10,x)
```

```
[Out] -(1/(3*(a + b*x)^3) - (3*a)/(2*(a + b*x)^4) + (3*a^2)/(a + b*x)^5 - (10*a^3
)/(3*(a + b*x)^6) + (15*a^4)/(7*(a + b*x)^7) - (3*a^5)/(4*(a + b*x)^8) + a^
6/(9*(a + b*x)^9))/b^7
```

sympy [B] time = 0.91, size = 175, normalized size = 3.37

$$\frac{-a^6-9a^5bx-36a^4b^2x^2-84a^3b^3x^3-126a^2b^4x^4-126ab^5x^5-84b^6x^6}{252a^9b^7+2268a^8b^8x+9072a^7b^9x^2+21168a^6b^{10}x^3+31752a^5b^{11}x^4+31752a^4b^{12}x^5+21168a^3b^{13}x^6+9072a^2b^{14}x^7+2268ab^{15}x^8+252b^{16}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x+a)**10,x)
```

```
[Out] (-a**6 - 9*a**5*b*x - 36*a**4*b**2*x**2 - 84*a**3*b**3*x**3 - 126*a**2*b**4
*x**4 - 126*a*b**5*x**5 - 84*b**6*x**6)/(252*a**9*b**7 + 2268*a**8*b**8*x +
 9072*a**7*b**9*x**2 + 21168*a**6*b**10*x**3 + 31752*a**5*b**11*x**4 + 3175
2*a**4*b**12*x**5 + 21168*a**3*b**13*x**6 + 9072*a**2*b**14*x**7 + 2268*a*b
**15*x**8 + 252*b**16*x**9)
```

3.229 $\int \frac{x^5}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=69

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{x^6}{504a^4(a+bx)^6}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a + b*x)^10,x]
```

```
[Out] x^6/(9*a*(a + b*x)^9) + x^6/(24*a^2*(a + b*x)^8) + x^6/(84*a^3*(a + b*x)^7)
 + x^6/(504*a^4*(a + b*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(a+bx)^{10}}\,dx &= \frac{x^6}{9a(a+bx)^9}+\frac{\int \frac{x^5}{(a+bx)^9}\,dx}{3a}\\ &= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{\int \frac{x^5}{(a+bx)^8}\,dx}{12a^2}\\ &= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{\int \frac{x^5}{(a+bx)^7}\,dx}{84a^3}\\ &= \frac{x^6}{9a(a+bx)^9}+\frac{x^6}{24a^2(a+bx)^8}+\frac{x^6}{84a^3(a+bx)^7}+\frac{x^6}{504a^4(a+bx)^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.93

$$-\frac{a^5+9a^4bx+36a^3b^2x^2+84a^2b^3x^3+126ab^4x^4+126b^5x^5}{504b^6(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^5 + 9*a^4*b*x + 36*a^3*b^2*x^2 + 84*a^2*b^3*x^3 + 126*a*b^4*x^4 +
 126*b^5*x^5)/(b^6*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^5/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^5/(a + b*x)^10, x]
```

fricas [B] time = 0.98, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,\left(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

giac [A] time = 0.90, size = 62, normalized size = 0.90

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,(bx+a)^9b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/((b*x + a)^9*b^6)
```

maple [A] time = 0.00, size = 86, normalized size = 1.25

$$\frac{a^5}{9\,(bx+a)^9\,b^6}-\frac{5a^4}{8\,(bx+a)^8\,b^6}+\frac{10a^3}{7\,(bx+a)^7\,b^6}-\frac{5a^2}{3\,(bx+a)^6\,b^6}+\frac{a}{(bx+a)^5\,b^6}-\frac{1}{4\,(bx+a)^4\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^10,x)
```

```
[Out] -5/3*a^2/b^6/(b*x+a)^6-1/4/b^6/(b*x+a)^4+a/b^6/(b*x+a)^5-5/8*a^4/b^6/(b*x+a
)^8+10/7*a^3/b^6/(b*x+a)^7+1/9*a^5/b^6/(b*x+a)^9
```

maxima [B] time = 1.54, size = 153, normalized size = 2.22

$$-\frac{126\,b^5x^5+126\,ab^4x^4+84\,a^2b^3x^3+36\,a^3b^2x^2+9\,a^4bx+a^5}{504\,\left(b^{15}x^9+9\,ab^{14}x^8+36\,a^2b^{13}x^7+84\,a^3b^{12}x^6+126\,a^4b^{11}x^5+126\,a^5b^{10}x^4+84\,a^6b^9x^3+36\,a^7b^8x^2+9\,a^8b^7x+a^9b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 126*a*b^4*x^4 + 84*a^2*b^3*x^3 + 36*a^3*b^2*x^2 + 9*a
^4*b*x + a^5)/(b^15*x^9 + 9*a*b^14*x^8 + 36*a^2*b^13*x^7 + 84*a^3*b^12*x^6
+ 126*a^4*b^11*x^5 + 126*a^5*b^10*x^4 + 84*a^6*b^9*x^3 + 36*a^7*b^8*x^2 + 9
*a^8*b^7*x + a^9*b^6)
```

mupad [B] time = 0.08, size = 71, normalized size = 1.03

$$\frac{\frac{a}{(a+b\,x)^5}-\frac{1}{4\,(a+b\,x)^4}-\frac{5\,a^2}{3\,(a+b\,x)^6}+\frac{10\,a^3}{7\,(a+b\,x)^7}-\frac{5\,a^4}{8\,(a+b\,x)^8}+\frac{a^5}{9\,(a+b\,x)^9}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a + b*x)^10,x)
```

```
[Out] (a/(a + b*x)^5 - 1/(4*(a + b*x)^4) - (5*a^2)/(3*(a + b*x)^6) + (10*a^3)/(7*
(a + b*x)^7) - (5*a^4)/(8*(a + b*x)^8) + a^5/(9*(a + b*x)^9))/b^6
```

sympy [B] time = 0.84, size = 163, normalized size = 2.36

$$\frac{-a^5-9a^4bx-36a^3b^2x^2-84a^2b^3x^3-126ab^4x^4-126b^5x^5}{504a^9b^6+4536a^8b^7x+18144a^7b^8x^2+42336a^6b^9x^3+63504a^5b^{10}x^4+63504a^4b^{11}x^5+42336a^3b^{12}x^6+18144a^2b^{13}x^7+4536ab^{14}x^8+504b^{15}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**10,x)
```

```
[Out] (-a**5 - 9*a**4*b*x - 36*a**3*b**2*x**2 - 84*a**2*b**3*x**3 - 126*a*b**4*x*
*4 - 126*b**5*x**5)/(504*a**9*b**6 + 4536*a**8*b**7*x + 18144*a**7*b**8*x**
2 + 42336*a**6*b**9*x**3 + 63504*a**5*b**10*x**4 + 63504*a**4*b**11*x**5 +
42336*a**3*b**12*x**6 + 18144*a**2*b**13*x**7 + 4536*a*b**14*x**8 + 504*b**
15*x**9)
```

3.230 $\int \frac{x^4}{(a+bx)^{10}} dx$

Optimal. Leaf size=81

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^10,x]
```

```
[Out] -a^4/(9*b^5*(a + b*x)^9) + a^3/(2*b^5*(a + b*x)^8) - (6*a^2)/(7*b^5*(a + b*
x)^7) + (2*a)/(3*b^5*(a + b*x)^6) - 1/(5*b^5*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{10}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{10}}-\frac{4a^3}{b^4(a+bx)^9}+\frac{6a^2}{b^4(a+bx)^8}-\frac{4a}{b^4(a+bx)^7}+\frac{1}{b^4(a+bx)^6}\right)dx\\ &= -\frac{a^4}{9b^5(a+bx)^9}+\frac{a^3}{2b^5(a+bx)^8}-\frac{6a^2}{7b^5(a+bx)^7}+\frac{2a}{3b^5(a+bx)^6}-\frac{1}{5b^5(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.65

$$-\frac{a^4+9a^3bx+36a^2b^2x^2+84ab^3x^3+126b^4x^4}{630b^5(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^10,x]
```

```
[Out] -1/630*(a^4 + 9*a^3*b*x + 36*a^2*b^2*x^2 + 84*a*b^3*x^3 + 126*b^4*x^4)/(b^5
*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^4/(a + b*x)^10, x]
```

fricas [A] time = 1.13, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,\left(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

giac [A] time = 1.22, size = 51, normalized size = 0.63

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,(bx+a)^9b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/((b*
x + a)^9*b^5)
```

maple [A] time = 0.01, size = 72, normalized size = 0.89

$$-\frac{a^4}{9\,(bx+a)^9\,b^5}+\frac{a^3}{2\,(bx+a)^8\,b^5}-\frac{6a^2}{7\,(bx+a)^7\,b^5}+\frac{2a}{3\,(bx+a)^6\,b^5}-\frac{1}{5\,(bx+a)^5\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^10,x)
```

```
[Out] -1/9*a^4/b^5/(b*x+a)^9+1/2*a^3/b^5/(b*x+a)^8-6/7*a^2/b^5/(b*x+a)^7+2/3*a/b^
5/(b*x+a)^6-1/5/b^5/(b*x+a)^5
```

maxima [A] time = 1.46, size = 142, normalized size = 1.75

$$-\frac{126\,b^4x^4+84\,ab^3x^3+36\,a^2b^2x^2+9\,a^3bx+a^4}{630\,\left(b^{14}x^9+9\,ab^{13}x^8+36\,a^2b^{12}x^7+84\,a^3b^{11}x^6+126\,a^4b^{10}x^5+126\,a^5b^9x^4+84\,a^6b^8x^3+36\,a^7b^7x^2+9\,a^8b^6x+a^9b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 84*a*b^3*x^3 + 36*a^2*b^2*x^2 + 9*a^3*b*x + a^4)/(b^1
4*x^9 + 9*a*b^13*x^8 + 36*a^2*b^12*x^7 + 84*a^3*b^11*x^6 + 126*a^4*b^10*x^5
 + 126*a^5*b^9*x^4 + 84*a^6*b^8*x^3 + 36*a^7*b^7*x^2 + 9*a^8*b^6*x + a^9*b^
5)
```

mupad [B] time = 0.08, size = 61, normalized size = 0.75

$$-\frac{\frac{1}{5\,{\left(a+b\,x\right)}^5}-\frac{2\,a}{3\,{\left(a+b\,x\right)}^6}+\frac{6\,a^2}{7\,{\left(a+b\,x\right)}^7}-\frac{a^3}{2\,{\left(a+b\,x\right)}^8}+\frac{a^4}{9\,{\left(a+b\,x\right)}^9}}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^10,x)
```

```
[Out] -(1/(5*(a + b*x)^5) - (2*a)/(3*(a + b*x)^6) + (6*a^2)/(7*(a + b*x)^7) - a^3
/(2*(a + b*x)^8) + a^4/(9*(a + b*x)^9))/b^5
```

sympy [B] time = 0.79, size = 151, normalized size = 1.86

$$\frac{-a^4 - 9a^3bx - 36a^2b^2x^2 - 84ab^3x^3 - 126b^4x^4}{630a^9b^5 + 5670a^8b^6x + 22680a^7b^7x^2 + 52920a^6b^8x^3 + 79380a^5b^9x^4 + 79380a^4b^{10}x^5 + 52920a^3b^{11}x^6 + 22680a^2b^{12}x^7 + 5670ab^{13}x^8 + 630b^{14}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**10,x)
```

```
[Out] (-a**4 - 9*a**3*b*x - 36*a**2*b**2*x**2 - 84*a*b**3*x**3 - 126*b**4*x**4)/(
630*a**9*b**5 + 5670*a**8*b**6*x + 22680*a**7*b**7*x**2 + 52920*a**6*b**8*x
**3 + 79380*a**5*b**9*x**4 + 79380*a**4*b**10*x**5 + 52920*a**3*b**11*x**6
+ 22680*a**2*b**12*x**7 + 5670*a*b**13*x**8 + 630*b**14*x**9)
```

3.231 $\int \frac{x^3}{(a+bx)^{10}} \, dx$

Optimal. Leaf size=64

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

Rubi [A] time = 0.03, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^10,x]
```

```
[Out] a^3/(9*b^4*(a + b*x)^9) - (3*a^2)/(8*b^4*(a + b*x)^8) + (3*a)/(7*b^4*(a + b
*x)^7) - 1/(6*b^4*(a + b*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{10}} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{10}}+\frac{3a^2}{b^3(a+bx)^9}-\frac{3a}{b^3(a+bx)^8}+\frac{1}{b^3(a+bx)^7}\right) dx\\ &= \frac{a^3}{9b^4(a+bx)^9}-\frac{3a^2}{8b^4(a+bx)^8}+\frac{3a}{7b^4(a+bx)^7}-\frac{1}{6b^4(a+bx)^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.66

$$-\frac{a^3+9a^2bx+36ab^2x^2+84b^3x^3}{504b^4(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^10,x]
```

```
[Out] -1/504*(a^3 + 9*a^2*b*x + 36*a*b^2*x^2 + 84*b^3*x^3)/(b^4*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a+bx)^{10}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^3/(a + b*x)^10, x]
```

fricas [B] time = 0.81, size = 131, normalized size = 2.05

$$-\frac{84\, b^{3} x^{3} + 36\, a b^{2} x^{2} + 9\, a^{2} b x + a^{3}}{504 \left(b^{13} x^{9} + 9\, a b^{12} x^{8} + 36\, a^{2} b^{11} x^{7} + 84\, a^{3} b^{10} x^{6} + 126\, a^{4} b^{9} x^{5} + 126\, a^{5} b^{8} x^{4} + 84\, a^{6} b^{7} x^{3} + 36\, a^{7} b^{6} x^{2} + 9\, a^{8} b^{5} x + a^{9} b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

giac [A] time = 1.12, size = 40, normalized size = 0.62

$$-\frac{84\, b^{3} x^{3} + 36\, a b^{2} x^{2} + 9\, a^{2} b x + a^{3}}{504\, {\left(b x + a\right)}^{9} b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/((b*x + a)^9*b^4)
```

maple [A] time = 0.00, size = 57, normalized size = 0.89

$$\frac{a^{3}}{9 \left(b x +a \right)^{9} b^{4}}-\frac{3 a^{2}}{8 \left(b x +a \right)^{8} b^{4}}+\frac{3 a}{7 \left(b x +a \right)^{7} b^{4}}-\frac{1}{6 \left(b x +a \right)^{6} b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^10,x)
```

```
[Out] 1/9*a^3/b^4/(b*x+a)^9-3/8*a^2/b^4/(b*x+a)^8+3/7*a/b^4/(b*x+a)^7-1/6/b^4/(b*
x+a)^6
```

maxima [B] time = 1.48, size = 131, normalized size = 2.05

$$-\frac{84\, b^{3} x^{3} + 36\, a b^{2} x^{2} + 9\, a^{2} b x + a^{3}}{504 \left(b^{13} x^{9} + 9\, a b^{12} x^{8} + 36\, a^{2} b^{11} x^{7} + 84\, a^{3} b^{10} x^{6} + 126\, a^{4} b^{9} x^{5} + 126\, a^{5} b^{8} x^{4} + 84\, a^{6} b^{7} x^{3} + 36\, a^{7} b^{6} x^{2} + 9\, a^{8} b^{5} x + a^{9} b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*b^3*x^3 + 36*a*b^2*x^2 + 9*a^2*b*x + a^3)/(b^13*x^9 + 9*a*b^12*x
^8 + 36*a^2*b^11*x^7 + 84*a^3*b^10*x^6 + 126*a^4*b^9*x^5 + 126*a^5*b^8*x^4
+ 84*a^6*b^7*x^3 + 36*a^7*b^6*x^2 + 9*a^8*b^5*x + a^9*b^4)
```

mupad [B] time = 0.13, size = 48, normalized size = 0.75

$$\frac{\frac{3\,a}{7\,{\left(a+b\,x\right)}^7}-\frac{1}{6\,{\left(a+b\,x\right)}^6}-\frac{3\,a^2}{8\,{\left(a+b\,x\right)}^8}+\frac{a^3}{9\,{\left(a+b\,x\right)}^9}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^10,x)
```

```
[Out] ((3*a)/(7*(a + b*x)^7) - 1/(6*(a + b*x)^6) - (3*a^2)/(8*(a + b*x)^8) + a^3/
(9*(a + b*x)^9))/b^4
```

sympy [B] time = 0.70, size = 139, normalized size = 2.17

$$\frac{- a^{3} - 9 a^{2} b x - 36 a b^{2} x^{2} - 84 b^{3} x^{3}}{504 a^{9} b^{4} + 4536 a^{8} b^{5} x + 18144 a^{7} b^{6} x^{2} + 42336 a^{6} b^{7} x^{3} + 63504 a^{5} b^{8} x^{4} + 63504 a^{4} b^{9} x^{5} + 42336 a^{3} b^{10} x^{6} + 18144 a^{2} b^{11} x^{7} + 4536 a b^{12} x^{8} + 504 b^{13} x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**10,x)
```

```
[Out] (-a**3 - 9*a**2*b*x - 36*a*b**2*x**2 - 84*b**3*x**3)/(504*a**9*b**4 + 4536*
a**8*b**5*x + 18144*a**7*b**6*x**2 + 42336*a**6*b**7*x**3 + 63504*a**5*b**8
*x**4 + 63504*a**4*b**9*x**5 + 42336*a**3*b**10*x**6 + 18144*a**2*b**11*x**
7 + 4536*a*b**12*x**8 + 504*b**13*x**9)
```

3.232 $\int \frac{x^2}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^10,x]
```

```
[Out] -a^2/(9*b^3*(a + b*x)^9) + a/(4*b^3*(a + b*x)^8) - 1/(7*b^3*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{10}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{10}}-\frac{2a}{b^2(a+bx)^9}+\frac{1}{b^2(a+bx)^8}\right)dx\\ &= -\frac{a^2}{9b^3(a+bx)^9}+\frac{a}{4b^3(a+bx)^8}-\frac{1}{7b^3(a+bx)^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.66

$$-\frac{a^2+9abx+36b^2x^2}{252b^3(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^10,x]
```

```
[Out] -1/252*(a^2 + 9*a*b*x + 36*b^2*x^2)/(b^3*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x^2/(a + b*x)^10, x]
```

fricas [B] time = 1.17, size = 120, normalized size = 2.55

$$-\frac{36\,b^{2}x^{2}+9\,abx+a^{2}}{252\,\left(b^{12}x^{9}+9\,ab^{11}x^{8}+36\,a^{2}b^{10}x^{7}+84\,a^{3}b^{9}x^{6}+126\,a^{4}b^{8}x^{5}+126\,a^{5}b^{7}x^{4}+84\,a^{6}b^{6}x^{3}+36\,a^{7}b^{5}x^{2}+9\,a^{8}b^{4}x+a^{9}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

giac [A] time = 1.04, size = 29, normalized size = 0.62

$$-\frac{36\,b^{2}x^{2}+9\,abx+a^{2}}{252\,(bx+a)^{9}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/((b*x + a)^9*b^3)
```

maple [A] time = 0.00, size = 42, normalized size = 0.89

$$-\frac{a^{2}}{9\,(bx+a)^{9}\,b^{3}}+\frac{a}{4\,(bx+a)^{8}\,b^{3}}-\frac{1}{7\,(bx+a)^{7}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^10,x)
```

```
[Out] -1/9*a^2/b^3/(b*x+a)^9+1/4*a/b^3/(b*x+a)^8-1/7/b^3/(b*x+a)^7
```

maxima [B] time = 1.42, size = 120, normalized size = 2.55

$$-\frac{36\,b^{2}x^{2}+9\,abx+a^{2}}{252\,\left(b^{12}x^{9}+9\,ab^{11}x^{8}+36\,a^{2}b^{10}x^{7}+84\,a^{3}b^{9}x^{6}+126\,a^{4}b^{8}x^{5}+126\,a^{5}b^{7}x^{4}+84\,a^{6}b^{6}x^{3}+36\,a^{7}b^{5}x^{2}+9\,a^{8}b^{4}x+a^{9}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(36*b^2*x^2 + 9*a*b*x + a^2)/(b^12*x^9 + 9*a*b^11*x^8 + 36*a^2*b^10*
x^7 + 84*a^3*b^9*x^6 + 126*a^4*b^8*x^5 + 126*a^5*b^7*x^4 + 84*a^6*b^6*x^3 +
 36*a^7*b^5*x^2 + 9*a^8*b^4*x + a^9*b^3)
```

mupad [B] time = 0.15, size = 31, normalized size = 0.66

$$-\frac{8\,a^{2}+72\,a\,b\,x+288\,b^{2}\,x^{2}}{2016\,b^{3}\,(a+b\,x)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^10,x)
```

```
[Out] -(8*a^2 + 288*b^2*x^2 + 72*a*b*x)/(2016*b^3*(a + b*x)^9)
```

sympy [B] time = 0.68, size = 128, normalized size = 2.72

$$\frac{-a^{2}-9abx-36b^{2}x^{2}}{252a^{9}b^{3}+2268a^{8}b^{4}x+9072a^{7}b^{5}x^{2}+21168a^{6}b^{6}x^{3}+31752a^{5}b^{7}x^{4}+31752a^{4}b^{8}x^{5}+21168a^{3}b^{9}x^{6}+9072a^{2}b^{10}x^{7}+2268ab^{11}x^{8}+252b^{12}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**10,x)
```

```
[Out] (-a**2 - 9*a*b*x - 36*b**2*x**2)/(252*a**9*b**3 + 2268*a**8*b**4*x + 9072*a
**7*b**5*x**2 + 21168*a**6*b**6*x**3 + 31752*a**5*b**7*x**4 + 31752*a**4*b*
*8*x**5 + 21168*a**3*b**9*x**6 + 9072*a**2*b**10*x**7 + 2268*a*b**11*x**8 +
 252*b**12*x**9)
```

3.233 $\int \frac{x}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=30

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^10,x]
```

```
[Out] a/(9*b^2*(a + b*x)^9) - 1/(8*b^2*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{10}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{10}}+\frac{1}{b(a+bx)^9}\right)dx\\ &= \frac{a}{9b^2(a+bx)^9}-\frac{1}{8b^2(a+bx)^8}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.67

$$-\frac{a+9bx}{72b^2(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^10,x]
```

```
[Out] -1/72*(a + 9*b*x)/(b^2*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[x/(a + b*x)^10, x]
```

fricas [B] time = 1.04, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\left(b^{11}x^9+9\,ab^{10}x^8+36\,a^2b^9x^7+84\,a^3b^8x^6+126\,a^4b^7x^5+126\,a^5b^6x^4+84\,a^6b^5x^3+36\,a^7b^4x^2+9\,a^8b^3x+a^9b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

giac [A] time = 1.04, size = 18, normalized size = 0.60

$$-\frac{9\,bx+a}{72\,(bx+a)^9b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(9*b*x + a)/((b*x + a)^9*b^2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$\frac{a}{9\,(bx+a)^9\,b^2}-\frac{1}{8\,(bx+a)^8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^10,x)
```

```
[Out] 1/9*a/b^2/(b*x+a)^9-1/8/b^2/(b*x+a)^8
```

maxima [B] time = 1.42, size = 109, normalized size = 3.63

$$-\frac{9\,bx+a}{72\,\left(b^{11}x^9+9\,ab^{10}x^8+36\,a^2b^9x^7+84\,a^3b^8x^6+126\,a^4b^7x^5+126\,a^5b^6x^4+84\,a^6b^5x^3+36\,a^7b^4x^2+9\,a^8b^3x+a^9b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(9*b*x + a)/(b^11*x^9 + 9*a*b^10*x^8 + 36*a^2*b^9*x^7 + 84*a^3*b^8*x^
6 + 126*a^4*b^7*x^5 + 126*a^5*b^6*x^4 + 84*a^6*b^5*x^3 + 36*a^7*b^4*x^2 + 9
*a^8*b^3*x + a^9*b^2)
```

mupad [B] time = 0.07, size = 18, normalized size = 0.60

$$-\frac{a+9\,b\,x}{72\,b^2\,(a+b\,x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^10,x)
```

```
[Out] -(a + 9*b*x)/(72*b^2*(a + b*x)^9)
```

sympy [B] time = 0.72, size = 116, normalized size = 3.87

$$\frac{-a-9bx}{72a^9b^2+648a^8b^3x+2592a^7b^4x^2+6048a^6b^5x^3+9072a^5b^6x^4+9072a^4b^7x^5+6048a^3b^8x^6+2592a^2b^9x^7+648ab^{10}x^8+72b^{11}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**10,x)
```

```
[Out] (-a - 9*b*x)/(72*a**9*b**2 + 648*a**8*b**3*x + 2592*a**7*b**4*x**2 + 6048*a
**6*b**5*x**3 + 9072*a**5*b**6*x**4 + 9072*a**4*b**7*x**5 + 6048*a**3*b**8*
x**6 + 2592*a**2*b**9*x**7 + 648*a*b**10*x**8 + 72*b**11*x**9)
```

3.234 $\int \frac{1}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{9b(a+bx)^9}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-10),x]
```

```
[Out] -1/(9*b*(a + b*x)^9)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{10}}\,dx = -\frac{1}{9b(a+bx)^9}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{9b(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-10),x]
```

```
[Out] -1/9*1/(b*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-10),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-10), x]
```

fricas [B] time = 1.10, size = 101, normalized size = 7.21

$$-\frac{1}{9\left(b^{10}x^9 + 9\,ab^9x^8 + 36\,a^2b^8x^7 + 84\,a^3b^7x^6 + 126\,a^4b^6x^5 + 126\,a^5b^5x^4 + 84\,a^6b^4x^3 + 36\,a^7b^3x^2 + 9\,a^8b^2x + a^9b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/9/(b^10*x^9 + 9*a*b^9*x^8 + 36*a^2*b^8*x^7 + 84*a^3*b^7*x^6 + 126*a^4*b^
6*x^5 + 126*a^5*b^5*x^4 + 84*a^6*b^4*x^3 + 36*a^7*b^3*x^2 + 9*a^8*b^2*x + a
^9*b)
```

giac [A] time = 1.03, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{9\,(bx+a)^9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^10,x)
```

```
[Out] -1/9/b/(b*x+a)^9
```

maxima [A] time = 1.34, size = 12, normalized size = 0.86

$$-\frac{1}{9\,(bx+a)^9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/9/((b*x + a)^9*b)
```

mupad [B] time = 0.14, size = 103, normalized size = 7.36

$$-\frac{1}{9\,a^9\,b+81\,a^8\,b^2\,x+324\,a^7\,b^3\,x^2+756\,a^6\,b^4\,x^3+1134\,a^5\,b^5\,x^4+1134\,a^4\,b^6\,x^5+756\,a^3\,b^7\,x^6+324\,a^2\,b^8\,x^7+81\,a\,b^9\,x^8+9\,b^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^10,x)
```

```
[Out] -1/(9*a^9*b + 9*b^10*x^9 + 81*a^8*b^2*x + 81*a*b^9*x^8 + 324*a^7*b^3*x^2 +
756*a^6*b^4*x^3 + 1134*a^5*b^5*x^4 + 1134*a^4*b^6*x^5 + 756*a^3*b^7*x^6 + 3
24*a^2*b^8*x^7)
```

sympy [B] time = 0.67, size = 109, normalized size = 7.79

$$-\frac{1}{9a^9b+81a^8b^2x+324a^7b^3x^2+756a^6b^4x^3+1134a^5b^5x^4+1134a^4b^6x^5+756a^3b^7x^6+324a^2b^8x^7+81ab^9x^8+9b^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**10,x)
```

```
[Out] -1/(9*a**9*b + 81*a**8*b**2*x + 324*a**7*b**3*x**2 + 756*a**6*b**4*x**3 + 1
134*a**5*b**5*x**4 + 1134*a**4*b**6*x**5 + 756*a**3*b**7*x**6 + 324*a**2*b*
*8*x**7 + 81*a*b**9*x**8 + 9*b**10*x**9)
```

3.235 $\int \frac{1}{x(a+bx)^{10}}\,dx$

Optimal. Leaf size=141

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3}$$

Rubi [A] time = 0.07, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{a^9(a+bx)}+\frac{1}{2a^8(a+bx)^2}+\frac{1}{3a^7(a+bx)^3}+\frac{1}{4a^6(a+bx)^4}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{8a^2(a+bx)^8}-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{1}{9a(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^10),x]
```

```
[Out] 1/(9*a*(a + b*x)^9) + 1/(8*a^2*(a + b*x)^8) + 1/(7*a^3*(a + b*x)^7) + 1/(6*
a^4*(a + b*x)^6) + 1/(5*a^5*(a + b*x)^5) + 1/(4*a^6*(a + b*x)^4) + 1/(3*a^7
*(a + b*x)^3) + 1/(2*a^8*(a + b*x)^2) + 1/(a^9*(a + b*x)) + Log[x]/a^10 - L
og[a + b*x]/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{10}}\,dx=\int \left(\frac{1}{a^{10}x}-\frac{b}{a(a+bx)^{10}}-\frac{b}{a^2(a+bx)^9}-\frac{b}{a^3(a+bx)^8}-\frac{b}{a^4(a+bx)^7}-\frac{b}{a^5(a+bx)^6}-\frac{b}{a^6(a+bx\right.$$
$$=\frac{1}{9a(a+bx)^9}+\frac{1}{8a^2(a+bx)^8}+\frac{1}{7a^3(a+bx)^7}+\frac{1}{6a^4(a+bx)^6}+\frac{1}{5a^5(a+bx)^5}+\frac{1}{4a^6(a+bx)^4}+$$

Mathematica [A] time = 0.10, size = 127, normalized size = 0.90

$$-\frac{\log(a+bx)}{a^{10}}+\frac{\log(x)}{a^{10}}+\frac{280a^8+315a^7(a+bx)+360a^6(a+bx)^2+420a^5(a+bx)^3+504a^4(a+bx)^4+630a^3(a+bx)^5+840a^2(a+bx)^6+1260a(a+bx)^7+2520(a+bx)^8}{2520a^9(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^10),x]
```

```
[Out] (280*a^8 + 315*a^7*(a + b*x) + 360*a^6*(a + b*x)^2 + 420*a^5*(a + b*x)^3 +
504*a^4*(a + b*x)^4 + 630*a^3*(a + b*x)^5 + 840*a^2*(a + b*x)^6 + 1260*a*(a
 + b*x)^7 + 2520*(a + b*x)^8)/(2520*a^9*(a + b*x)^9) + Log[x]/a^10 - Log[a
+ b*x]/a^10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(a + b*x)^10), x]
```

fricas [B] time = 1.08, size = 388, normalized size = 2.75

$$\frac{2520\,ab^{8}x^{8}+21420\,a^{2}b^{7}x^{7}+80220\,a^{3}b^{6}x^{6}+173250\,a^{4}b^{5}x^{5}+236754\,a^{5}b^{4}x^{4}+210756\,a^{6}b^{3}x^{3}+120564\,a^{7}b^{2}x^{2}+41481\,a^{8}bx+7129\,a^{9}-2520\left(b^{9}x^{9}+9\,ab^{8}x^{8}+36\,a^{2}b^{7}x^{7}+84\,a^{3}b^{6}x^{6}+126\,a^{4}b^{5}x^{5}+126\,a^{5}b^{4}x^{4}+84\,a^{6}b^{3}x^{3}+36\,a^{7}b^{2}x^{2}+9\,a^{8}bx+a^{9}\right)\log\left(bx+a\right)+2520\left(b^{9}x^{9}+9\,ab^{8}x^{8}+36\,a^{2}b^{7}x^{7}+84\,a^{3}b^{6}x^{6}+126\,a^{4}b^{5}x^{5}+126\,a^{5}b^{4}x^{4}+84\,a^{6}b^{3}x^{3}+36\,a^{7}b^{2}x^{2}+9\,a^{8}bx+a^{9}\right)\log\left(x\right)}{2520\left(a^{10}b^{9}x^{9}+9\,a^{11}b^{8}x^{8}+36\,a^{12}b^{7}x^{7}+84\,a^{13}b^{6}x^{6}+126\,a^{14}b^{5}x^{5}+126\,a^{15}b^{4}x^{4}+84\,a^{16}b^{3}x^{3}+36\,a^{17}b^{2}x^{2}+9\,a^{18}bx+a^{19}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*a*b^8*x^8 + 21420*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4
*b^5*x^5 + 236754*a^5*b^4*x^4 + 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 4
1481*a^8*b*x + 7129*a^9 - 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*a^2*b^7*x^7 + 84
*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b^3*x^3 + 36*a^7*
b^2*x^2 + 9*a^8*b*x + a^9)*log(b*x + a) + 2520*(b^9*x^9 + 9*a*b^8*x^8 + 36*
a^2*b^7*x^7 + 84*a^3*b^6*x^6 + 126*a^4*b^5*x^5 + 126*a^5*b^4*x^4 + 84*a^6*b
^3*x^3 + 36*a^7*b^2*x^2 + 9*a^8*b*x + a^9)*log(x))/(a^10*b^9*x^9 + 9*a^11*b
^8*x^8 + 36*a^12*b^7*x^7 + 84*a^13*b^6*x^6 + 126*a^14*b^5*x^5 + 126*a^15*b^
4*x^4 + 84*a^16*b^3*x^3 + 36*a^17*b^2*x^2 + 9*a^18*b*x + a^19)
```

giac [A] time = 1.05, size = 120, normalized size = 0.85

$$-\frac{\log\left(|bx+a|\right)}{a^{10}}+\frac{\log\left(|x|\right)}{a^{10}}+\frac{2520\,ab^{8}x^{8}+21420\,a^{2}b^{7}x^{7}+80220\,a^{3}b^{6}x^{6}+173250\,a^{4}b^{5}x^{5}+236754\,a^{5}b^{4}x^{4}+210756\,a^{6}b^{3}x^{3}+120564\,a^{7}b^{2}x^{2}+41481\,a^{8}bx+7129\,a^{9}}{2520\left(bx+a\right)^{9}a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a^10 + log(abs(x))/a^10 + 1/2520*(2520*a*b^8*x^8 + 21420
*a^2*b^7*x^7 + 80220*a^3*b^6*x^6 + 173250*a^4*b^5*x^5 + 236754*a^5*b^4*x^4
+ 210756*a^6*b^3*x^3 + 120564*a^7*b^2*x^2 + 41481*a^8*b*x + 7129*a^9)/((b*x
 + a)^9*a^10)
```

maple [A] time = 0.01, size = 126, normalized size = 0.89

$$\frac{1}{9\left(bx+a\right)^{9}a}+\frac{1}{8\left(bx+a\right)^{8}a^{2}}+\frac{1}{7\left(bx+a\right)^{7}a^{3}}+\frac{1}{6\left(bx+a\right)^{6}a^{4}}+\frac{1}{5\left(bx+a\right)^{5}a^{5}}+\frac{1}{4\left(bx+a\right)^{4}a^{6}}+\frac{1}{3\left(bx+a\right)^{3}a^{7}}+\frac{1}{2\left(bx+a\right)^{2}a^{8}}+\frac{1}{\left(bx+a\right)a^{9}}+\frac{\ln\left(x\right)}{a^{10}}-\frac{\ln\left(bx+a\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^10,x)
```

```
[Out] 1/9/a/(b*x+a)^9+1/8/a^2/(b*x+a)^8+1/7/a^3/(b*x+a)^7+1/6/a^4/(b*x+a)^6+1/5/a
^5/(b*x+a)^5+1/4/a^6/(b*x+a)^4+1/3/a^7/(b*x+a)^3+1/2/a^8/(b*x+a)^2+1/a^9/(b
*x+a)+ln(x)/a^10-ln(b*x+a)/a^10
```

maxima [A] time = 1.75, size = 205, normalized size = 1.45

$$\frac{2520\,b^{8}x^{8}+21420\,ab^{7}x^{7}+80220\,a^{2}b^{6}x^{6}+173250\,a^{3}b^{5}x^{5}+236754\,a^{4}b^{4}x^{4}+210756\,a^{5}b^{3}x^{3}+120564\,a^{6}b^{2}x^{2}+41481\,a^{7}bx+7129\,a^{8}}{2520\left(a^{9}b^{9}x^{9}+9\,a^{10}b^{8}x^{8}+36\,a^{11}b^{7}x^{7}+84\,a^{12}b^{6}x^{6}+126\,a^{13}b^{5}x^{5}+126\,a^{14}b^{4}x^{4}+84\,a^{15}b^{3}x^{3}+36\,a^{16}b^{2}x^{2}+9\,a^{17}bx+a^{18}\right)}-\frac{\log\left(bx+a\right)}{a^{10}}+\frac{\log\left(x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/2520*(2520*b^8*x^8 + 21420*a*b^7*x^7 + 80220*a^2*b^6*x^6 + 173250*a^3*b^5
*x^5 + 236754*a^4*b^4*x^4 + 210756*a^5*b^3*x^3 + 120564*a^6*b^2*x^2 + 41481
*a^7*b*x + 7129*a^8)/(a^9*b^9*x^9 + 9*a^10*b^8*x^8 + 36*a^11*b^7*x^7 + 84*a
^12*b^6*x^6 + 126*a^13*b^5*x^5 + 126*a^14*b^4*x^4 + 84*a^15*b^3*x^3 + 36*a^
16*b^2*x^2 + 9*a^17*b*x + a^18) - log(b*x + a)/a^10 + log(x)/a^10
```

mupad [B] time = 0.76, size = 145, normalized size = 1.03

$$\frac{1}{9\,a\,{\left(a+b\,x\right)}^9}-\frac{\ln\left(\frac{a+b\,x}{x}\right)-\frac{14\,b^2\,x^2}{{\left(a+b\,x\right)}^2}+\frac{56\,b^3\,x^3}{3\,{\left(a+b\,x\right)}^3}-\frac{35\,b^4\,x^4}{2\,{\left(a+b\,x\right)}^4}+\frac{56\,b^5\,x^5}{5\,{\left(a+b\,x\right)}^5}-\frac{14\,b^6\,x^6}{3\,{\left(a+b\,x\right)}^6}+\frac{8\,b^7\,x^7}{7\,{\left(a+b\,x\right)}^7}-\frac{b^8\,x^8}{8\,{\left(a+b\,x\right)}^8}+\frac{8\,b\,x}{a+b\,x}}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^10),x)
```

```
[Out] 1/(9*a*(a + b*x)^9) - (log((a + b*x)/x) - (14*b^2*x^2)/(a + b*x)^2 + (56*b^
3*x^3)/(3*(a + b*x)^3) - (35*b^4*x^4)/(2*(a + b*x)^4) + (56*b^5*x^5)/(5*(a
+ b*x)^5) - (14*b^6*x^6)/(3*(a + b*x)^6) + (8*b^7*x^7)/(7*(a + b*x)^7) - (b
^8*x^8)/(8*(a + b*x)^8) + (8*b*x)/(a + b*x))/a^10
```

sympy [A] time = 1.02, size = 212, normalized size = 1.50

$$\frac{7129a^8 + 41481a^7bx + 120564a^6b^2x^2 + 210756a^5b^3x^3 + 236754a^4b^4x^4 + 173250a^3b^5x^5 + 80220a^2b^6x^6 + 21420ab^7x^7 + 2520b^8x^8}{2520a^{18} + 22680a^{17}bx + 90720a^{16}b^2x^2 + 211680a^{15}b^3x^3 + 317520a^{14}b^4x^4 + 317520a^{13}b^5x^5 + 211680a^{12}b^6x^6 + 90720a^{11}b^7x^7 + 22680a^{10}b^8x^8 + 2520a^9b^9x^9} + \frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**10,x)
```

```
[Out] (7129*a**8 + 41481*a**7*b*x + 120564*a**6*b**2*x**2 + 210756*a**5*b**3*x**3
 + 236754*a**4*b**4*x**4 + 173250*a**3*b**5*x**5 + 80220*a**2*b**6*x**6 + 2
1420*a*b**7*x**7 + 2520*b**8*x**8)/(2520*a**18 + 22680*a**17*b*x + 90720*a*
*16*b**2*x**2 + 211680*a**15*b**3*x**3 + 317520*a**14*b**4*x**4 + 317520*a*
*13*b**5*x**5 + 211680*a**12*b**6*x**6 + 90720*a**11*b**7*x**7 + 22680*a**1
0*b**8*x**8 + 2520*a**9*b**9*x**9) + (log(x) - log(a/b + x))/a**10
```

3.236 $\int \frac{1}{x^2(a+bx)^{10}}\,dx$

Optimal. Leaf size=158

$$-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{9b}{a^{10}(a+bx)}-\frac{1}{a^{10}x}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{}{3a^5}$$

Rubi [A] time = 0.12, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9b}{a^{10}(a+bx)}-\frac{4b}{a^9(a+bx)^2}-\frac{7b}{3a^8(a+bx)^3}-\frac{3b}{2a^7(a+bx)^4}-\frac{b}{a^6(a+bx)^5}-\frac{2b}{3a^5(a+bx)^6}-\frac{3b}{7a^4(a+bx)^7}-\frac{b}{4a^3(a+bx)^8}-\frac{b}{9a^2(a+bx)^9}-\frac{10b\log(x)}{a^{11}}+\frac{10b\log(a+bx)}{a^{11}}-\frac{1}{a^{10}x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -(1/(a^10*x)) - b/(9*a^2*(a + b*x)^9) - b/(4*a^3*(a + b*x)^8) - (3*b)/(7*a^
4*(a + b*x)^7) - (2*b)/(3*a^5*(a + b*x)^6) - b/(a^6*(a + b*x)^5) - (3*b)/(2
*a^7*(a + b*x)^4) - (7*b)/(3*a^8*(a + b*x)^3) - (4*b)/(a^9*(a + b*x)^2) - (
9*b)/(a^10*(a + b*x)) - (10*b*Log[x])/a^11 + (10*b*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^2}-\frac{10b}{a^{11}x}+\frac{b^2}{a^2(a+bx)^{10}}+\frac{2b^2}{a^3(a+bx)^9}+\frac{3b^2}{a^4(a+bx)^8}+\frac{4b^2}{a^5(a+bx)^7}+\frac{5b^2}{a^6(a+bx\right.\\ &= -\frac{1}{a^{10}x}-\frac{b}{9a^2(a+bx)^9}-\frac{b}{4a^3(a+bx)^8}-\frac{3b}{7a^4(a+bx)^7}-\frac{2b}{3a^5(a+bx)^6}-\frac{b}{a^6(a+bx)^5}-\frac{}{2a}\end{aligned}$$

Mathematica [A] time = 0.13, size = 130, normalized size = 0.82

$$-\frac{\frac{a\left(252a^9+7129a^8bx+41481a^7b^2x^2+120564a^6b^3x^3+210756a^5b^4x^4+236754a^4b^5x^5+173250a^3b^6x^6+80220a^2b^7x^7+21420ab^8x^8+2520b^9x^9\right)}{x(a+bx)^9}-2520b\log(a+bx)+2520b\log(x)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^10),x]
```

```
[Out] -1/252*((a*(252*a^9 + 7129*a^8*b*x + 41481*a^7*b^2*x^2 + 120564*a^6*b^3*x^3
 + 210756*a^5*b^4*x^4 + 236754*a^4*b^5*x^5 + 173250*a^3*b^6*x^6 + 80220*a^2
*b^7*x^7 + 21420*a*b^8*x^8 + 2520*b^9*x^9))/(x*(a + b*x)^9) + 2520*b*Log[x]
 - 2520*b*Log[a + b*x])/a^11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(a + b*x)^10), x]
```

fricas [B] time = 1.17, size = 417, normalized size = 2.64

$$-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+41481\,a^{8}b^{2}x^{2}+7129\,a^{9}bx+252\,a^{10}-2520\left(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx\right)\log\left(bx+a\right)+2520\left(b^{10}x^{10}+9\,ab^{9}x^{9}+36\,a^{2}b^{8}x^{8}+84\,a^{3}b^{7}x^{7}+126\,a^{4}b^{6}x^{6}+126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}+36\,a^{7}b^{3}x^{3}+9\,a^{8}b^{2}x^{2}+a^{9}bx\right)\log\left(x\right)}{252\left(a^{11}b^{9}x^{10}+9\,a^{12}b^{8}x^{9}+36\,a^{13}b^{7}x^{8}+84\,a^{14}b^{6}x^{7}+126\,a^{15}b^{5}x^{6}+126\,a^{16}b^{4}x^{5}+84\,a^{17}b^{3}x^{4}+36\,a^{18}b^{2}x^{3}+9\,a^{19}bx^{2}+a^{20}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/252*(2520*a*b^9*x^9 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4
*b^6*x^6 + 236754*a^5*b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 4
1481*a^8*b^2*x^2 + 7129*a^9*b*x + 252*a^10 - 2520*(b^10*x^10 + 9*a*b^9*x^9
+ 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x^6 + 126*a^5*b^5*x^5 + 84*
a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^9*b*x)*log(b*x + a) + 2520
*(b^10*x^10 + 9*a*b^9*x^9 + 36*a^2*b^8*x^8 + 84*a^3*b^7*x^7 + 126*a^4*b^6*x
^6 + 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 + 36*a^7*b^3*x^3 + 9*a^8*b^2*x^2 + a^
9*b*x)*log(x))/(a^11*b^9*x^10 + 9*a^12*b^8*x^9 + 36*a^13*b^7*x^8 + 84*a^14*
b^6*x^7 + 126*a^15*b^5*x^6 + 126*a^16*b^4*x^5 + 84*a^17*b^3*x^4 + 36*a^18*b
^2*x^3 + 9*a^19*b*x^2 + a^20*x)
```

giac [A] time = 1.01, size = 137, normalized size = 0.87

$$\frac{10\,b\log\left(|bx+a|\right)}{a^{11}}-\frac{10\,b\log\left(|x|\right)}{a^{11}}-\frac{2520\,ab^{9}x^{9}+21420\,a^{2}b^{8}x^{8}+80220\,a^{3}b^{7}x^{7}+173250\,a^{4}b^{6}x^{6}+236754\,a^{5}b^{5}x^{5}+210756\,a^{6}b^{4}x^{4}+120564\,a^{7}b^{3}x^{3}+41481\,a^{8}b^{2}x^{2}+7129\,a^{9}bx+252\,a^{10}}{252\,(bx+a)^{9}a^{11}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 10*b*log(abs(b*x + a))/a^11 - 10*b*log(abs(x))/a^11 - 1/252*(2520*a*b^9*x^9
 + 21420*a^2*b^8*x^8 + 80220*a^3*b^7*x^7 + 173250*a^4*b^6*x^6 + 236754*a^5*
b^5*x^5 + 210756*a^6*b^4*x^4 + 120564*a^7*b^3*x^3 + 41481*a^8*b^2*x^2 + 712
9*a^9*b*x + 252*a^10)/((b*x + a)^9*a^11*x)
```

maple [A] time = 0.01, size = 147, normalized size = 0.93

$$-\frac{b}{9\,(bx+a)^{9}a^{2}}-\frac{b}{4\,(bx+a)^{8}a^{3}}-\frac{3b}{7\,(bx+a)^{7}a^{4}}-\frac{2b}{3\,(bx+a)^{6}a^{5}}-\frac{b}{(bx+a)^{5}a^{6}}-\frac{3b}{2\,(bx+a)^{4}a^{7}}-\frac{7b}{3\,(bx+a)^{3}a^{8}}-\frac{4b}{(bx+a)^{2}a^{9}}-\frac{9b}{(bx+a)\,a^{10}}-\frac{10b\ln(x)}{a^{11}}+\frac{10b\ln(bx+a)}{a^{11}}-\frac{1}{a^{10}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^10,x)
```

```
[Out] -1/a^10/x-1/9*b/a^2/(b*x+a)^9-1/4*b/a^3/(b*x+a)^8-3/7*b/a^4/(b*x+a)^7-2/3*b
/a^5/(b*x+a)^6-b/a^6/(b*x+a)^5-3/2*b/a^7/(b*x+a)^4-7/3*b/a^8/(b*x+a)^3-4*b/
a^9/(b*x+a)^2-9*b/a^10/(b*x+a)-10*b*ln(x)/a^11+10*b*ln(b*x+a)/a^11
```

maxima [A] time = 1.71, size = 223, normalized size = 1.41

$$-\frac{2520\,b^{9}x^{9}+21420\,ab^{8}x^{8}+80220\,a^{2}b^{7}x^{7}+173250\,a^{3}b^{6}x^{6}+236754\,a^{4}b^{5}x^{5}+210756\,a^{5}b^{4}x^{4}+120564\,a^{6}b^{3}x^{3}+41481\,a^{7}b^{2}x^{2}+7129\,a^{8}bx+252\,a^{9}}{252\left(a^{10}b^{9}x^{10}+9\,a^{11}b^{8}x^{9}+36\,a^{12}b^{7}x^{8}+84\,a^{13}b^{6}x^{7}+126\,a^{14}b^{5}x^{6}+126\,a^{15}b^{4}x^{5}+84\,a^{16}b^{3}x^{4}+36\,a^{17}b^{2}x^{3}+9\,a^{18}bx^{2}+a^{19}x\right)}+\frac{10\,b\log\left(bx+a\right)}{a^{11}}-\frac{10\,b\log\left(x\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/252*(2520*b^9*x^9 + 21420*a*b^8*x^8 + 80220*a^2*b^7*x^7 + 173250*a^3*b^6
*x^6 + 236754*a^4*b^5*x^5 + 210756*a^5*b^4*x^4 + 120564*a^6*b^3*x^3 + 41481
*a^7*b^2*x^2 + 7129*a^8*b*x + 252*a^9)/(a^10*b^9*x^10 + 9*a^11*b^8*x^9 + 36
*a^12*b^7*x^8 + 84*a^13*b^6*x^7 + 126*a^14*b^5*x^6 + 126*a^15*b^4*x^5 + 84*
a^16*b^3*x^4 + 36*a^17*b^2*x^3 + 9*a^18*b*x^2 + a^19*x) + 10*b*log(b*x + a)
/a^11 - 10*b*log(x)/a^11
```

mupad [B] time = 0.39, size = 217, normalized size = 1.37

$$\frac{20\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{a}+\frac{4609\,b^2\,x^2}{28\,a^3}+\frac{3349\,b^3\,x^3}{7\,a^4}+\frac{2509\,b^4\,x^4}{3\,a^5}+\frac{1879\,b^5\,x^5}{2\,a^6}+\frac{1375\,b^6\,x^6}{2\,a^7}+\frac{955\,b^7\,x^7}{3\,a^8}+\frac{85\,b^8\,x^8}{a^9}+\frac{10\,b^9\,x^9}{a^{10}}+\frac{7129\,b\,x}{252\,a^2}}{a^9\,x+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^3+84\,a^6\,b^3\,x^4+126\,a^5\,b^4\,x^5+126\,a^4\,b^5\,x^6+84\,a^3\,b^6\,x^7+36\,a^2\,b^7\,x^8+9\,a\,b^8\,x^9+b^9\,x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^10),x)
```

```
[Out] (20*b*atanh((2*b*x)/a + 1))/a^11 - (1/a + (4609*b^2*x^2)/(28*a^3) + (3349*b
^3*x^3)/(7*a^4) + (2509*b^4*x^4)/(3*a^5) + (1879*b^5*x^5)/(2*a^6) + (1375*b
^6*x^6)/(2*a^7) + (955*b^7*x^7)/(3*a^8) + (85*b^8*x^8)/a^9 + (10*b^9*x^9)/a
^10 + (7129*b*x)/(252*a^2))/(a^9*x + b^9*x^10 + 9*a^8*b*x^2 + 9*a*b^8*x^9 +
 36*a^7*b^2*x^3 + 84*a^6*b^3*x^4 + 126*a^5*b^4*x^5 + 126*a^4*b^5*x^6 + 84*a
^3*b^6*x^7 + 36*a^2*b^7*x^8)
```

sympy [A] time = 1.15, size = 233, normalized size = 1.47

$$\frac{-252a^9-7129a^8bx-41481a^7b^2x^2-120564a^6b^3x^3-210756a^5b^4x^4-236754a^4b^5x^5-173250a^3b^6x^6-80220a^2b^7x^7-21420ab^8x^8-2520b^9x^9}{252a^{19}x+2268a^{18}bx^2+9072a^{17}b^2x^3+21168a^{16}b^3x^4+31752a^{15}b^4x^5+31752a^{14}b^5x^6+21168a^{13}b^6x^7+9072a^{12}b^7x^8+2268a^{11}b^8x^9+252a^{10}b^9x^{10}}+\frac{10b\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**10,x)
```

```
[Out] (-252*a**9 - 7129*a**8*b*x - 41481*a**7*b**2*x**2 - 120564*a**6*b**3*x**3 -
 210756*a**5*b**4*x**4 - 236754*a**4*b**5*x**5 - 173250*a**3*b**6*x**6 - 80
220*a**2*b**7*x**7 - 21420*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**19*x + 226
8*a**18*b*x**2 + 9072*a**17*b**2*x**3 + 21168*a**16*b**3*x**4 + 31752*a**15
*b**4*x**5 + 31752*a**14*b**5*x**6 + 21168*a**13*b**6*x**7 + 9072*a**12*b**
7*x**8 + 2268*a**11*b**8*x**9 + 252*a**10*b**9*x**10) + 10*b*(-log(x) + log
(a/b + x))/a**11
```

3.237 $\int \frac{1}{x^3(a+bx)^{10}} \, dx$

Optimal. Leaf size=191

$$\frac{55b^2\log(x)}{a^{12}} - \frac{55b^2\log(a+bx)}{a^{12}} + \frac{45b^2}{a^{11}(a+bx)} + \frac{10b}{a^{11}x} + \frac{18b^2}{a^{10}(a+bx)^2} - \frac{1}{2a^{10}x^2} + \frac{28b^2}{3a^9(a+bx)^3} + \frac{21b^2}{4a^8(a+bx)^4} + \frac{3b^2}{a^7(a+b}$$

Rubi [A] time = 0.14, antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$\frac{45b^2}{a^{11}(a+bx)} + \frac{18b^2}{a^{10}(a+bx)^2} + \frac{28b^2}{3a^9(a+bx)^3} + \frac{21b^2}{4a^8(a+bx)^4} + \frac{3b^2}{a^7(a+bx)^5} + \frac{5b^2}{3a^6(a+bx)^6} + \frac{6b^2}{7a^5(a+bx)^7} + \frac{3b^2}{8a^4(a+bx)^8} + \frac{b^2}{9a^3(a+bx)^9} + \frac{55b^2\log(x)}{a^{12}} - \frac{55b^2\log(a+bx)}{a^{12}} + \frac{10b}{a^{11}x} - \frac{1}{2a^{10}x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^10),x]
```

```
[Out] -1/(2*a^10*x^2) + (10*b)/(a^11*x) + b^2/(9*a^3*(a + b*x)^9) + (3*b^2)/(8*a^
4*(a + b*x)^8) + (6*b^2)/(7*a^5*(a + b*x)^7) + (5*b^2)/(3*a^6*(a + b*x)^6)
+ (3*b^2)/(a^7*(a + b*x)^5) + (21*b^2)/(4*a^8*(a + b*x)^4) + (28*b^2)/(3*a^
9*(a + b*x)^3) + (18*b^2)/(a^10*(a + b*x)^2) + (45*b^2)/(a^11*(a + b*x)) +
(55*b^2*Log[x])/a^12 - (55*b^2*Log[a + b*x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(a+bx)^{10}} \, dx &= \int \left(\frac{1}{a^{10}x^3} - \frac{10b}{a^{11}x^2} + \frac{55b^2}{a^{12}x} - \frac{b^3}{a^3(a+bx)^{10}} - \frac{3b^3}{a^4(a+bx)^9} - \frac{6b^3}{a^5(a+bx)^8} - \frac{10b^3}{a^6(a+bx)^7} - \frac{1}{a^7(}\right.\\ &= -\frac{1}{2a^{10}x^2} + \frac{10b}{a^{11}x} + \frac{b^2}{9a^3(a+bx)^9} + \frac{3b^2}{8a^4(a+bx)^8} + \frac{6b^2}{7a^5(a+bx)^7} + \frac{5b^2}{3a^6(a+bx)^6} + \frac{3b^2}{a^7(a+bx}\end{aligned}$$

Mathematica [A] time = 0.11, size = 145, normalized size = 0.76

$$\frac{\frac{a\left(-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10}\right)}{x^2(a+bx)^9} - 27720b^2\log(a+bx) + 27720b^2\log(x)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^10),x]
```

```
[Out] ((a*(-252*a^10 + 2772*a^9*b*x + 78419*a^8*b^2*x^2 + 456291*a^7*b^3*x^3 + 13
26204*a^6*b^4*x^4 + 2318316*a^5*b^5*x^5 + 2604294*a^4*b^6*x^6 + 1905750*a^3
*b^7*x^7 + 882420*a^2*b^8*x^8 + 235620*a*b^9*x^9 + 27720*b^10*x^10))/(x^2*(
a + b*x)^9) + 27720*b^2*Log[x] - 27720*b^2*Log[a + b*x])/(504*a^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(a+bx)^{10}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(a + b*x)^10), x]
```

fricas [B] time = 0.61, size = 438, normalized size = 2.29

$$\frac{27720\,ab^{10}x^{10} + 235620\,a^{2}b^{9}x^{9} + 882420\,a^{3}b^{8}x^{8} + 1905750\,a^{4}b^{7}x^{7} + 2604294\,a^{5}b^{6}x^{6} + 2318316\,a^{6}b^{5}x^{5} + 1326204\,a^{7}b^{4}x^{4} + 456291\,a^{8}b^{3}x^{3} + 78419\,a^{9}b^{2}x^{2} + 2772\,a^{10}bx - 252\,a^{11} - 27720\,\left(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^{2}b^{9}x^{9} + 84\,a^{3}b^{8}x^{8} + 126\,a^{4}b^{7}x^{7} + 126\,a^{5}b^{6}x^{6} + 84\,a^{6}b^{5}x^{5} + 36\,a^{7}b^{4}x^{4} + 9\,a^{8}b^{3}x^{3} + a^{9}b^{2}x^{2}\right)\log\left(bx + a\right) + 27720\,\left(b^{11}x^{11} + 9\,ab^{10}x^{10} + 36\,a^{2}b^{9}x^{9} + 84\,a^{3}b^{8}x^{8} + 126\,a^{4}b^{7}x^{7} + 126\,a^{5}b^{6}x^{6} + 84\,a^{6}b^{5}x^{5} + 36\,a^{7}b^{4}x^{4} + 9\,a^{8}b^{3}x^{3} + a^{9}b^{2}x^{2}\right)\log\left(x\right)}{504\,\left(a^{12}b^{9}x^{11} + 9\,a^{13}b^{8}x^{10} + 36\,a^{14}b^{7}x^{9} + 84\,a^{15}b^{6}x^{8} + 126\,a^{16}b^{5}x^{7} + 126\,a^{17}b^{4}x^{6} + 84\,a^{18}b^{3}x^{5} + 36\,a^{19}b^{2}x^{4} + 9\,a^{20}bx^{3} + a^{21}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/504*(27720*a*b^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 190575
0*a^4*b^7*x^7 + 2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4
*x^4 + 456291*a^8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11 -
27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^
4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x
^3 + a^9*b^2*x^2)*log(b*x + a) + 27720*(b^11*x^11 + 9*a*b^10*x^10 + 36*a^2*
b^9*x^9 + 84*a^3*b^8*x^8 + 126*a^4*b^7*x^7 + 126*a^5*b^6*x^6 + 84*a^6*b^5*x
^5 + 36*a^7*b^4*x^4 + 9*a^8*b^3*x^3 + a^9*b^2*x^2)*log(x))/(a^12*b^9*x^11 +
 9*a^13*b^8*x^10 + 36*a^14*b^7*x^9 + 84*a^15*b^6*x^8 + 126*a^16*b^5*x^7 + 1
26*a^17*b^4*x^6 + 84*a^18*b^3*x^5 + 36*a^19*b^2*x^4 + 9*a^20*b*x^3 + a^21*x
^2)
```

giac [A] time = 1.10, size = 152, normalized size = 0.80

$$-\frac{55\,b^{2}\log\left(|bx + a|\right)}{a^{12}} + \frac{55\,b^{2}\log\left(|x|\right)}{a^{12}} + \frac{27720\,ab^{10}x^{10} + 235620\,a^{2}b^{9}x^{9} + 882420\,a^{3}b^{8}x^{8} + 1905750\,a^{4}b^{7}x^{7} + 2604294\,a^{5}b^{6}x^{6} + 2318316\,a^{6}b^{5}x^{5} + 1326204\,a^{7}b^{4}x^{4} + 456291\,a^{8}b^{3}x^{3} + 78419\,a^{9}b^{2}x^{2} + 2772\,a^{10}bx - 252\,a^{11}}{504\,{\left(bx + a\right)}^{9}a^{12}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -55*b^2*log(abs(b*x + a))/a^12 + 55*b^2*log(abs(x))/a^12 + 1/504*(27720*a*b
^10*x^10 + 235620*a^2*b^9*x^9 + 882420*a^3*b^8*x^8 + 1905750*a^4*b^7*x^7 +
2604294*a^5*b^6*x^6 + 2318316*a^6*b^5*x^5 + 1326204*a^7*b^4*x^4 + 456291*a^
8*b^3*x^3 + 78419*a^9*b^2*x^2 + 2772*a^10*b*x - 252*a^11)/((b*x + a)^9*a^12
*x^2)
```

maple [A] time = 0.02, size = 178, normalized size = 0.93

$$\frac{b^{2}}{9\left(bx+a\right)^{9}a^{3}} + \frac{3b^{2}}{8\left(bx+a\right)^{8}a^{4}} + \frac{6b^{2}}{7\left(bx+a\right)^{7}a^{5}} + \frac{5b^{2}}{3\left(bx+a\right)^{6}a^{6}} + \frac{3b^{2}}{\left(bx+a\right)^{5}a^{7}} + \frac{21b^{2}}{4\left(bx+a\right)^{4}a^{8}} + \frac{28b^{2}}{3\left(bx+a\right)^{3}a^{9}} + \frac{18b^{2}}{\left(bx+a\right)^{2}a^{10}} + \frac{45b^{2}}{\left(bx+a\right)a^{11}} + \frac{55b^{2}\ln\left(x\right)}{a^{12}} - \frac{55b^{2}\ln\left(bx+a\right)}{a^{12}} + \frac{10b}{a^{11}x} - \frac{1}{2a^{10}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^10,x)
```

```
[Out] -1/2/a^10/x^2+10*b/a^11/x+1/9*b^2/a^3/(b*x+a)^9+3/8*b^2/a^4/(b*x+a)^8+6/7*b
^2/a^5/(b*x+a)^7+5/3*b^2/a^6/(b*x+a)^6+3*b^2/a^7/(b*x+a)^5+21/4*b^2/a^8/(b*
x+a)^4+28/3*b^2/a^9/(b*x+a)^3+18*b^2/a^10/(b*x+a)^2+45*b^2/a^11/(b*x+a)+55*
b^2*ln(x)/a^12-55*b^2*ln(b*x+a)/a^12
```

maxima [A] time = 1.71, size = 240, normalized size = 1.26

$$\frac{27720\,b^{10}x^{10} + 235620\,ab^{9}x^{9} + 882420\,a^{2}b^{8}x^{8} + 1905750\,a^{3}b^{7}x^{7} + 2604294\,a^{4}b^{6}x^{6} + 2318316\,a^{5}b^{5}x^{5} + 1326204\,a^{6}b^{4}x^{4} + 456291\,a^{7}b^{3}x^{3} + 78419\,a^{8}b^{2}x^{2} + 2772\,a^{9}bx - 252\,a^{10}}{504\,\left(a^{11}b^{9}x^{11} + 9\,a^{12}b^{8}x^{10} + 36\,a^{13}b^{7}x^{9} + 84\,a^{14}b^{6}x^{8} + 126\,a^{15}b^{5}x^{7} + 126\,a^{16}b^{4}x^{6} + 84\,a^{17}b^{3}x^{5} + 36\,a^{18}b^{2}x^{4} + 9\,a^{19}bx^{3} + a^{20}x^{2}\right)} - \frac{55\,b^{2}\log\left(bx + a\right)}{a^{12}} + \frac{55\,b^{2}\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] 1/504*(27720*b^10*x^10 + 235620*a*b^9*x^9 + 882420*a^2*b^8*x^8 + 1905750*a^
3*b^7*x^7 + 2604294*a^4*b^6*x^6 + 2318316*a^5*b^5*x^5 + 1326204*a^6*b^4*x^4
 + 456291*a^7*b^3*x^3 + 78419*a^8*b^2*x^2 + 2772*a^9*b*x - 252*a^10)/(a^11*
b^9*x^11 + 9*a^12*b^8*x^10 + 36*a^13*b^7*x^9 + 84*a^14*b^6*x^8 + 126*a^15*b
```

```
^5*x^7 + 126*a^16*b^4*x^6 + 84*a^17*b^3*x^5 + 36*a^18*b^2*x^4 + 9*a^19*b*x^
3 + a^20*x^2) - 55*b^2*log(b*x + a)/a^12 + 55*b^2*log(x)/a^12
```

mupad [B] time = 0.44, size = 233, normalized size = 1.22

$$\frac{\frac{78419\,b^2\,x^2}{504\,a^3}-\frac{1}{2\,a}+\frac{50699\,b^3\,x^3}{56\,a^4}+\frac{36839\,b^4\,x^4}{14\,a^5}+\frac{27599\,b^5\,x^5}{6\,a^6}+\frac{20669\,b^6\,x^6}{4\,a^7}+\frac{15125\,b^7\,x^7}{4\,a^8}+\frac{10505\,b^8\,x^8}{6\,a^9}+\frac{935\,b^9\,x^9}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}+\frac{11\,b\,x}{2\,a^2}}{a^9\,x^2+9\,a^8\,b\,x^3+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^5+126\,a^5\,b^4\,x^6+126\,a^4\,b^5\,x^7+84\,a^3\,b^6\,x^8+36\,a^2\,b^7\,x^9+9\,a\,b^8\,x^{10}+b^9\,x^{11}}-\frac{110\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^10),x)
```

```
[Out] ((78419*b^2*x^2)/(504*a^3) - 1/(2*a) + (50699*b^3*x^3)/(56*a^4) + (36839*b^
4*x^4)/(14*a^5) + (27599*b^5*x^5)/(6*a^6) + (20669*b^6*x^6)/(4*a^7) + (1512
5*b^7*x^7)/(4*a^8) + (10505*b^8*x^8)/(6*a^9) + (935*b^9*x^9)/(2*a^10) + (55
*b^10*x^10)/a^11 + (11*b*x)/(2*a^2))/(a^9*x^2 + b^9*x^11 + 9*a^8*b*x^3 + 9*
a*b^8*x^10 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^5 + 126*a^5*b^4*x^6 + 126*a^4*b^
5*x^7 + 84*a^3*b^6*x^8 + 36*a^2*b^7*x^9) - (110*b^2*atanh((2*b*x)/a + 1))/a
^12
```

sympy [A] time = 1.18, size = 246, normalized size = 1.29

$$\frac{-252a^{10}+2772a^9bx+78419a^8b^2x^2+456291a^7b^3x^3+1326204a^6b^4x^4+2318316a^5b^5x^5+2604294a^4b^6x^6+1905750a^3b^7x^7+882420a^2b^8x^8+235620ab^9x^9+27720b^{10}x^{10}}{504a^{20}x^2+4536a^{19}bx^3+18144a^{18}b^2x^4+42336a^{17}b^3x^5+63504a^{16}b^4x^6+63504a^{15}b^5x^7+42336a^{14}b^6x^8+18144a^{13}b^7x^9+4536a^{12}b^8x^{10}+504a^{11}b^9x^{11}}+\frac{55b^2\left(\log(x)-\log\left(\frac{a}{b}+x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**10,x)
```

```
[Out] (-252*a**10 + 2772*a**9*b*x + 78419*a**8*b**2*x**2 + 456291*a**7*b**3*x**3
+ 1326204*a**6*b**4*x**4 + 2318316*a**5*b**5*x**5 + 2604294*a**4*b**6*x**6
+ 1905750*a**3*b**7*x**7 + 882420*a**2*b**8*x**8 + 235620*a*b**9*x**9 + 277
20*b**10*x**10)/(504*a**20*x**2 + 4536*a**19*b*x**3 + 18144*a**18*b**2*x**4
 + 42336*a**17*b**3*x**5 + 63504*a**16*b**4*x**6 + 63504*a**15*b**5*x**7 +
42336*a**14*b**6*x**8 + 18144*a**13*b**7*x**9 + 4536*a**12*b**8*x**10 + 504
*a**11*b**9*x**11) + 55*b**2*(log(x) - log(a/b + x))/a**12
```

3.238 $\int \frac{1}{x^4(a+bx)^{10}}\,dx$

Optimal. Leaf size=198

$$-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}-\frac{165b^3}{a^{12}(a+bx)}-\frac{55b^2}{a^{12}x}-\frac{60b^3}{a^{11}(a+bx)^2}+\frac{5b}{a^{11}x^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{1}{3a^{10}x^3}-\frac{14b^3}{a^9(a+bx}$$

Rubi [A] time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$-\frac{165b^3}{a^{12}(a+bx)}-\frac{60b^3}{a^{11}(a+bx)^2}-\frac{28b^3}{a^{10}(a+bx)^3}-\frac{14b^3}{a^9(a+bx)^4}-\frac{7b^3}{a^8(a+bx)^5}-\frac{10b^3}{3a^7(a+bx)^6}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{b^3}{2a^5(a+bx)^8}-\frac{b^3}{9a^4(a+bx)^9}-\frac{55b^2}{a^{12}x}-\frac{220b^3\log(x)}{a^{13}}+\frac{220b^3\log(a+bx)}{a^{13}}+\frac{5b}{a^{11}x^2}-\frac{1}{3a^{10}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/(3*a^10*x^3) + (5*b)/(a^11*x^2) - (55*b^2)/(a^12*x) - b^3/(9*a^4*(a + b*
x)^9) - b^3/(2*a^5*(a + b*x)^8) - (10*b^3)/(7*a^6*(a + b*x)^7) - (10*b^3)/(
3*a^7*(a + b*x)^6) - (7*b^3)/(a^8*(a + b*x)^5) - (14*b^3)/(a^9*(a + b*x)^4)
 - (28*b^3)/(a^10*(a + b*x)^3) - (60*b^3)/(a^11*(a + b*x)^2) - (165*b^3)/(a
^12*(a + b*x)) - (220*b^3*Log[x])/a^13 + (220*b^3*Log[a + b*x])/a^13
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(a+bx)^{10}}\,dx &= \int \left(\frac{1}{a^{10}x^4}-\frac{10b}{a^{11}x^3}+\frac{55b^2}{a^{12}x^2}-\frac{220b^3}{a^{13}x}+\frac{b^4}{a^4(a+bx)^{10}}+\frac{4b^4}{a^5(a+bx)^9}+\frac{10b^4}{a^6(a+bx)^8}+\frac{2}{a^7(a}\right.\\ &= -\frac{1}{3a^{10}x^3}+\frac{5b}{a^{11}x^2}-\frac{55b^2}{a^{12}x}-\frac{b^3}{9a^4(a+bx)^9}-\frac{b^3}{2a^5(a+bx)^8}-\frac{10b^3}{7a^6(a+bx)^7}-\frac{10b^3}{3a^7(a+bx)^6}\end{aligned}$$

Mathematica [A] time = 0.12, size = 156, normalized size = 0.79

$$-\frac{-27720b^3\log(a+bx)+\frac{a\left(42a^{11}-252a^{10}bx+2772a^9b^2x^2+78419a^8b^3x^3+456291a^7b^4x^4+1326204a^6b^5x^5+2318316a^5b^6x^6+2604294a^4b^7x^7+1905750a^3b^8x^8+882420a^2b^9x^9+235620ab^{10}x^{10}+27720b^{11}x^{11}\right)}{x^3(a+bx)^9}+27720b^3\log(x)}{126a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*x)^10),x]
```

```
[Out] -1/126*((a*(42*a^11 - 252*a^10*b*x + 2772*a^9*b^2*x^2 + 78419*a^8*b^3*x^3 +
 456291*a^7*b^4*x^4 + 1326204*a^6*b^5*x^5 + 2318316*a^5*b^6*x^6 + 2604294*a
^4*b^7*x^7 + 1905750*a^3*b^8*x^8 + 882420*a^2*b^9*x^9 + 235620*a*b^10*x^10
+ 27720*b^11*x^11))/(x^3*(a + b*x)^9) + 27720*b^3*Log[x] - 27720*b^3*Log[a
+ b*x])/a^13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(a + b*x)^10),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(a + b*x)^10), x]
```

fricas [B] time = 1.26, size = 449, normalized size = 2.27

$$-\frac{27720\,ab^{11}x^{11}+235620\,a^{2}b^{10}x^{10}+882420\,a^{3}b^{9}x^{9}+1905750\,a^{4}b^{8}x^{8}+2604294\,a^{5}b^{7}x^{7}+2318316\,a^{6}b^{6}x^{6}+1326204\,a^{7}b^{5}x^{5}+456291\,a^{8}b^{4}x^{4}+78419\,a^{9}b^{3}x^{3}+2772\,a^{10}b^{2}x^{2}-252\,a^{11}bx+42\,a^{12}-27720\left(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^{2}b^{10}x^{10}+84\,a^{3}b^{9}x^{9}+126\,a^{4}b^{8}x^{8}+126\,a^{5}b^{7}x^{7}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{5}+9\,a^{8}b^{4}x^{4}+a^{9}b^{3}x^{3}\right)\log\left(bx+a\right)+27720\left(b^{12}x^{12}+9\,ab^{11}x^{11}+36\,a^{2}b^{10}x^{10}+84\,a^{3}b^{9}x^{9}+126\,a^{4}b^{8}x^{8}+126\,a^{5}b^{7}x^{7}+84\,a^{6}b^{6}x^{6}+36\,a^{7}b^{5}x^{5}+9\,a^{8}b^{4}x^{4}+a^{9}b^{3}x^{3}\right)\log(x)}{126\left(a^{13}b^{9}x^{12}+9\,a^{14}b^{8}x^{11}+36\,a^{15}b^{7}x^{10}+84\,a^{16}b^{6}x^{9}+126\,a^{17}b^{5}x^{8}+126\,a^{18}b^{4}x^{7}+84\,a^{19}b^{3}x^{6}+36\,a^{20}b^{2}x^{5}+9\,a^{21}bx^{4}+a^{22}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/126*(27720*a*b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 190
5750*a^4*b^8*x^8 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*
b^5*x^5 + 456291*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*
a^11*b*x + 42*a^12 - 27720*(b^12*x^12 + 9*a*b^11*x^11 + 36*a^2*b^10*x^10 +
84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^
7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)*log(b*x + a) + 27720*(b^12*x^12 +
9*a*b^11*x^11 + 36*a^2*b^10*x^10 + 84*a^3*b^9*x^9 + 126*a^4*b^8*x^8 + 126*a
^5*b^7*x^7 + 84*a^6*b^6*x^6 + 36*a^7*b^5*x^5 + 9*a^8*b^4*x^4 + a^9*b^3*x^3)
*log(x))/(a^13*b^9*x^12 + 9*a^14*b^8*x^11 + 36*a^15*b^7*x^10 + 84*a^16*b^6*
x^9 + 126*a^17*b^5*x^8 + 126*a^18*b^4*x^7 + 84*a^19*b^3*x^6 + 36*a^20*b^2*x
^5 + 9*a^21*b*x^4 + a^22*x^3)
```

giac [A] time = 1.35, size = 163, normalized size = 0.82

$$\frac{220\,b^{3}\log\left(|bx+a|\right)}{a^{13}}-\frac{220\,b^{3}\log\left(|x|\right)}{a^{13}}-\frac{27720\,ab^{11}x^{11}+235620\,a^{2}b^{10}x^{10}+882420\,a^{3}b^{9}x^{9}+1905750\,a^{4}b^{8}x^{8}+2604294\,a^{5}b^{7}x^{7}+2318316\,a^{6}b^{6}x^{6}+1326204\,a^{7}b^{5}x^{5}+456291\,a^{8}b^{4}x^{4}+78419\,a^{9}b^{3}x^{3}+2772\,a^{10}b^{2}x^{2}-252\,a^{11}bx+42\,a^{12}}{126\,(bx+a)^{9}a^{13}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] 220*b^3*log(abs(b*x + a))/a^13 - 220*b^3*log(abs(x))/a^13 - 1/126*(27720*a*
b^11*x^11 + 235620*a^2*b^10*x^10 + 882420*a^3*b^9*x^9 + 1905750*a^4*b^8*x^8
 + 2604294*a^5*b^7*x^7 + 2318316*a^6*b^6*x^6 + 1326204*a^7*b^5*x^5 + 456291
*a^8*b^4*x^4 + 78419*a^9*b^3*x^3 + 2772*a^10*b^2*x^2 - 252*a^11*b*x + 42*a^
12)/((b*x + a)^9*a^13*x^3)
```

maple [A] time = 0.01, size = 189, normalized size = 0.95

$$-\frac{b^{3}}{9\,(bx+a)^{9}a^{4}}-\frac{b^{3}}{2\,(bx+a)^{8}a^{5}}-\frac{10b^{3}}{7\,(bx+a)^{7}a^{6}}-\frac{10b^{3}}{3\,(bx+a)^{6}a^{7}}-\frac{7b^{3}}{(bx+a)^{5}a^{8}}-\frac{14b^{3}}{(bx+a)^{4}a^{9}}-\frac{28b^{3}}{(bx+a)^{3}a^{10}}-\frac{60b^{3}}{(bx+a)^{2}a^{11}}-\frac{165b^{3}}{(bx+a)\,a^{12}}-\frac{220b^{3}\ln(x)}{a^{13}}+\frac{220b^{3}\ln\left(bx+a\right)}{a^{13}}-\frac{55b^{2}}{a^{12}x}+\frac{5b}{a^{11}x^{2}}-\frac{1}{3a^{10}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^10,x)
```

```
[Out] -1/3/a^10/x^3+5*b/a^11/x^2-55*b^2/a^12/x-1/9*b^3/a^4/(b*x+a)^9-1/2*b^3/a^5/
(b*x+a)^8-10/7*b^3/a^6/(b*x+a)^7-10/3*b^3/a^7/(b*x+a)^6-7*b^3/a^8/(b*x+a)^5
-14*b^3/a^9/(b*x+a)^4-28*b^3/a^10/(b*x+a)^3-60*b^3/a^11/(b*x+a)^2-165*b^3/a
^12/(b*x+a)-220*b^3*ln(x)/a^13+220*b^3*ln(b*x+a)/a^13
```

maxima [A] time = 1.72, size = 251, normalized size = 1.27

$$-\frac{27720\,b^{11}x^{11}+235620\,ab^{10}x^{10}+882420\,a^{2}b^{9}x^{9}+1905750\,a^{3}b^{8}x^{8}+2604294\,a^{4}b^{7}x^{7}+2318316\,a^{5}b^{6}x^{6}+1326204\,a^{6}b^{5}x^{5}+456291\,a^{7}b^{4}x^{4}+78419\,a^{8}b^{3}x^{3}+2772\,a^{9}b^{2}x^{2}-252\,a^{10}bx+42\,a^{11}}{126\left(a^{12}b^{9}x^{12}+9\,a^{13}b^{8}x^{11}+36\,a^{14}b^{7}x^{10}+84\,a^{15}b^{6}x^{9}+126\,a^{16}b^{5}x^{8}+126\,a^{17}b^{4}x^{7}+84\,a^{18}b^{3}x^{6}+36\,a^{19}b^{2}x^{5}+9\,a^{20}bx^{4}+a^{21}x^{3}\right)}+\frac{220\,b^{3}\log\left(bx+a\right)}{a^{13}}-\frac{220\,b^{3}\log(x)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/126*(27720*b^11*x^11 + 235620*a*b^10*x^10 + 882420*a^2*b^9*x^9 + 1905750
*a^3*b^8*x^8 + 2604294*a^4*b^7*x^7 + 2318316*a^5*b^6*x^6 + 1326204*a^6*b^5*
x^5 + 456291*a^7*b^4*x^4 + 78419*a^8*b^3*x^3 + 2772*a^9*b^2*x^2 - 252*a^10*
b*x + 42*a^11)/(a^12*b^9*x^12 + 9*a^13*b^8*x^11 + 36*a^14*b^7*x^10 + 84*a^1
```

```
5*b^6*x^9 + 126*a^16*b^5*x^8 + 126*a^17*b^4*x^7 + 84*a^18*b^3*x^6 + 36*a^19
*b^2*x^5 + 9*a^20*b*x^4 + a^21*x^3) + 220*b^3*log(b*x + a)/a^13 - 220*b^3*l
og(x)/a^13
```

mupad [B] time = 0.59, size = 245, normalized size = 1.24

$$\frac{440\, b^3 \operatorname{atanh}\left(\frac{2\, b\, x}{a}+1\right)}{a^{13}}-\frac{\frac{1}{3\, a}+\frac{22\, b^2\, x^2}{a^3}+\frac{78419\, b^3\, x^3}{126\, a^4}+\frac{50699\, b^4\, x^4}{14\, a^5}+\frac{73678\, b^5\, x^5}{7\, a^6}+\frac{55198\, b^6\, x^6}{3\, a^7}+\frac{20669\, b^7\, x^7}{a^8}+\frac{15125\, b^8\, x^8}{a^9}+\frac{21010\, b^9\, x^9}{3\, a^{10}}+\frac{1870\, b^{10}\, x^{10}}{a^{11}}+\frac{220\, b^{11}\, x^{11}}{a^{12}}-\frac{2\, b\, x}{a^2}}{a^9\, x^3+9\, a^8\, b\, x^4+36\, a^7\, b^2\, x^5+84\, a^6\, b^3\, x^6+126\, a^5\, b^4\, x^7+126\, a^4\, b^5\, x^8+84\, a^3\, b^6\, x^9+36\, a^2\, b^7\, x^{10}+9\, a\, b^8\, x^{11}+b^9\, x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^10),x)
```

```
[Out] (440*b^3*atanh((2*b*x)/a + 1))/a^13 - (1/(3*a) + (22*b^2*x^2)/a^3 + (78419*
b^3*x^3)/(126*a^4) + (50699*b^4*x^4)/(14*a^5) + (73678*b^5*x^5)/(7*a^6) + (
55198*b^6*x^6)/(3*a^7) + (20669*b^7*x^7)/a^8 + (15125*b^8*x^8)/a^9 + (21010
*b^9*x^9)/(3*a^10) + (1870*b^10*x^10)/a^11 + (220*b^11*x^11)/a^12 - (2*b*x)
/a^2)/(a^9*x^3 + b^9*x^12 + 9*a^8*b*x^4 + 9*a*b^8*x^11 + 36*a^7*b^2*x^5 + 8
4*a^6*b^3*x^6 + 126*a^5*b^4*x^7 + 126*a^4*b^5*x^8 + 84*a^3*b^6*x^9 + 36*a^2
*b^7*x^10)
```

sympy [A] time = 1.24, size = 258, normalized size = 1.30

$$\frac{-42a^{11}+252a^{10}bx-2772a^9b^2x^2-78419a^8b^3x^3-456291a^7b^4x^4-1326204a^6b^5x^5-2318316a^5b^6x^6-2604294a^4b^7x^7-1905750a^3b^8x^8-882420a^2b^9x^9-235620ab^{10}x^{10}-27720b^{11}x^{11}}{126a^{21}x^3+1134a^{20}bx^4+4536a^{19}b^2x^5+10584a^{18}b^3x^6+15876a^{17}b^4x^7+15876a^{16}b^5x^8+10584a^{15}b^6x^9+4536a^{14}b^7x^{10}+1134a^{13}b^8x^{11}+126a^{12}b^9x^{12}}+\frac{220b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**10,x)
```

```
[Out] (-42*a**11 + 252*a**10*b*x - 2772*a**9*b**2*x**2 - 78419*a**8*b**3*x**3 - 4
56291*a**7*b**4*x**4 - 1326204*a**6*b**5*x**5 - 2318316*a**5*b**6*x**6 - 26
04294*a**4*b**7*x**7 - 1905750*a**3*b**8*x**8 - 882420*a**2*b**9*x**9 - 235
620*a*b**10*x**10 - 27720*b**11*x**11)/(126*a**21*x**3 + 1134*a**20*b*x**4
+ 4536*a**19*b**2*x**5 + 10584*a**18*b**3*x**6 + 15876*a**17*b**4*x**7 + 15
876*a**16*b**5*x**8 + 10584*a**15*b**6*x**9 + 4536*a**14*b**7*x**10 + 1134*
a**13*b**8*x**11 + 126*a**12*b**9*x**12) + 220*b**3*(-log(x) + log(a/b + x)
)/a**13
```

3.239 $\int \frac{(a+bx)^{12}}{x^{10}}\,dx$

Optimal. Leaf size=141

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6a$$

Rubi [A] time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{43\}$

$$-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+66a^2b^{10}x+220a^3b^9\log(x)-\frac{3a^{11}b}{2x^8}-\frac{a^{12}}{9x^9}+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12/x^10,x]
```

```
[Out] -a^12/(9*x^9) - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{12}}{x^{10}}\,dx &= \int \left(66a^2b^{10}+\frac{a^{12}}{x^{10}}+\frac{12a^{11}b}{x^9}+\frac{66a^{10}b^2}{x^8}+\frac{220a^9b^3}{x^7}+\frac{495a^8b^4}{x^6}+\frac{792a^7b^5}{x^5}+\frac{924a^6b^6}{x^4}+\frac{792a^5}{x^3}\right.\\ &= -\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+\end{aligned}$$

Mathematica [A] time = 0.01, size = 141, normalized size = 1.00

$$-\frac{a^{12}}{9x^9}-\frac{3a^{11}b}{2x^8}-\frac{66a^{10}b^2}{7x^7}-\frac{110a^9b^3}{3x^6}-\frac{99a^8b^4}{x^5}-\frac{198a^7b^5}{x^4}-\frac{308a^6b^6}{x^3}-\frac{396a^5b^7}{x^2}-\frac{495a^4b^8}{x}+220a^3b^9\log(x)+66a^2b^{10}x+6ab^{11}x^2+\frac{b^{12}x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12/x^10,x]
```

```
[Out] -1/9*a^12/x^9 - (3*a^11*b)/(2*x^8) - (66*a^10*b^2)/(7*x^7) - (110*a^9*b^3)/
(3*x^6) - (99*a^8*b^4)/x^5 - (198*a^7*b^5)/x^4 - (308*a^6*b^6)/x^3 - (396*a
^5*b^7)/x^2 - (495*a^4*b^8)/x + 66*a^2*b^10*x + 6*a*b^11*x^2 + (b^12*x^3)/3
 + 220*a^3*b^9*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{12}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^12/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^12/x^10, x]
```

fricas [A] time = 1.04, size = 136, normalized size = 0.96

$$\frac{42\,b^{12}x^{12} + 756\,ab^{11}x^{11} + 8316\,a^{2}b^{10}x^{10} + 27720\,a^{3}b^{9}x^{9}\log\left(x\right) - 62370\,a^{4}b^{8}x^{8} - 49896\,a^{5}b^{7}x^{7} - 38808\,a^{6}b^{6}x^{6} - 24948\,a^{7}b^{5}x^{5} - 12474\,a^{8}b^{4}x^{4} - 4620\,a^{9}b^{3}x^{3} - 1188\,a^{10}b^{2}x^{2} - 189\,a^{11}bx - 14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="fricas")
```

```
[Out] 1/126*(42*b^12*x^12 + 756*a*b^11*x^11 + 8316*a^2*b^10*x^10 + 27720*a^3*b^9*
x^9*log(x) - 62370*a^4*b^8*x^8 - 49896*a^5*b^7*x^7 - 38808*a^6*b^6*x^6 - 24
948*a^7*b^5*x^5 - 12474*a^8*b^4*x^4 - 4620*a^9*b^3*x^3 - 1188*a^10*b^2*x^2
- 189*a^11*b*x - 14*a^12)/x^9
```

giac [A] time = 1.14, size = 133, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^{3} + 6\,ab^{11}x^{2} + 66\,a^{2}b^{10}x + 220\,a^{3}b^{9}\log\left(|x|\right) - \frac{62370\,a^{4}b^{8}x^{8} + 49896\,a^{5}b^{7}x^{7} + 38808\,a^{6}b^{6}x^{6} + 24948\,a^{7}b^{5}x^{5} + 12474\,a^{8}b^{4}x^{4} + 4620\,a^{9}b^{3}x^{3} + 1188\,a^{10}b^{2}x^{2} + 189\,a^{11}bx + 14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="giac")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(abs(x)) - 1/1
26*(62370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b
^5*x^5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^1
1*b*x + 14*a^12)/x^9
```

maple [A] time = 0.01, size = 132, normalized size = 0.94

$$\frac{b^{12}x^{3}}{3} + 6a\,b^{11}x^{2} + 220a^{3}b^{9}\ln\left(x\right) + 66a^{2}b^{10}x - \frac{495a^{4}b^{8}}{x} - \frac{396a^{5}b^{7}}{x^{2}} - \frac{308a^{6}b^{6}}{x^{3}} - \frac{198a^{7}b^{5}}{x^{4}} - \frac{99a^{8}b^{4}}{x^{5}} - \frac{110a^{9}b^{3}}{3x^{6}} - \frac{66a^{10}b^{2}}{7x^{7}} - \frac{3a^{11}b}{2x^{8}} - \frac{a^{12}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12/x^10,x)
```

```
[Out] -1/9*a^12/x^9-3/2*a^11*b/x^8-66/7*a^10*b^2/x^7-110/3*a^9*b^3/x^6-99*a^8*b^4
/x^5-198*a^7*b^5/x^4-308*a^6*b^6/x^3-396*a^5*b^7/x^2-495*a^4*b^8/x+66*a^2*b
^10*x+6*a*b^11*x^2+1/3*b^12*x^3+220*a^3*b^9*ln(x)
```

maxima [A] time = 1.37, size = 132, normalized size = 0.94

$$\frac{1}{3}\,b^{12}x^{3} + 6\,ab^{11}x^{2} + 66\,a^{2}b^{10}x + 220\,a^{3}b^{9}\log\left(x\right) - \frac{62370\,a^{4}b^{8}x^{8} + 49896\,a^{5}b^{7}x^{7} + 38808\,a^{6}b^{6}x^{6} + 24948\,a^{7}b^{5}x^{5} + 12474\,a^{8}b^{4}x^{4} + 4620\,a^{9}b^{3}x^{3} + 1188\,a^{10}b^{2}x^{2} + 189\,a^{11}bx + 14\,a^{12}}{126\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12/x^10,x, algorithm="maxima")
```

```
[Out] 1/3*b^12*x^3 + 6*a*b^11*x^2 + 66*a^2*b^10*x + 220*a^3*b^9*log(x) - 1/126*(6
2370*a^4*b^8*x^8 + 49896*a^5*b^7*x^7 + 38808*a^6*b^6*x^6 + 24948*a^7*b^5*x^
5 + 12474*a^8*b^4*x^4 + 4620*a^9*b^3*x^3 + 1188*a^10*b^2*x^2 + 189*a^11*b*x
 + 14*a^12)/x^9
```

mupad [B] time = 0.08, size = 132, normalized size = 0.94

$$\frac{b^{12}\,x^3}{3}-\frac{\frac{a^{12}}{9}+\frac{3\,a^{11}\,b\,x}{2}+\frac{66\,a^{10}\,b^2\,x^2}{7}+\frac{110\,a^9\,b^3\,x^3}{3}+99\,a^8\,b^4\,x^4+198\,a^7\,b^5\,x^5+308\,a^6\,b^6\,x^6+396\,a^5\,b^7\,x^7+495\,a^4\,b^8\,x^8}{x^9}+66\,a^2\,b^{10}\,x+6\,a\,b^{11}\,x^2+220\,a^3\,b^9\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12/x^10,x)
```

```
[Out] (b^12*x^3)/3 - (a^12/9 + (66*a^10*b^2*x^2)/7 + (110*a^9*b^3*x^3)/3 + 99*a^8
*b^4*x^4 + 198*a^7*b^5*x^5 + 308*a^6*b^6*x^6 + 396*a^5*b^7*x^7 + 495*a^4*b^
```

```
8*x^8 + (3*a^11*b*x)/2)/x^9 + 66*a^2*b^10*x + 6*a*b^11*x^2 + 220*a^3*b^9*lo
g(x)
```

sympy [A] time = 0.91, size = 143, normalized size = 1.01

$$220a^3b^9\log(x) + 66a^2b^{10}x + 6ab^{11}x^2 + \frac{b^{12}x^3}{3} + \frac{-14a^{12} - 189a^{11}bx - 1188a^{10}b^2x^2 - 4620a^9b^3x^3 - 12474a^8b^4x^4 - 24948a^7b^5x^5 - 38808a^6b^6x^6 - 49896a^5b^7x^7 - 62370a^4b^8x^8}{126x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12/x**10,x)
```

```
[Out] 220*a**3*b**9*log(x) + 66*a**2*b**10*x + 6*a*b**11*x**2 + b**12*x**3/3 + (-
14*a**12 - 189*a**11*b*x - 1188*a**10*b**2*x**2 - 4620*a**9*b**3*x**3 - 124
74*a**8*b**4*x**4 - 24948*a**7*b**5*x**5 - 38808*a**6*b**6*x**6 - 49896*a**
5*b**7*x**7 - 62370*a**4*b**8*x**8)/(126*x**9)
```

3.240 $\int \frac{(a+bx)^{11}}{x^{10}} dx$

Optimal. Leaf size=132

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Rubi [A] time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)-\frac{11a^{10}b}{8x^8}-\frac{a^{11}}{9x^9}+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11/x^10,x]
```

```
[Out] -a^11/(9*x^9) - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^{11}}{x^{10}} dx = \int \left(11ab^{10}+\frac{a^{11}}{x^{10}}+\frac{11a^{10}b}{x^9}+\frac{55a^9b^2}{x^8}+\frac{165a^8b^3}{x^7}+\frac{330a^7b^4}{x^6}+\frac{462a^6b^5}{x^5}+\frac{462a^5b^6}{x^4}+\frac{330a^4b^7}{x^3}\right.$$
$$= -\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}$$

Mathematica [A] time = 0.01, size = 132, normalized size = 1.00

$$-\frac{a^{11}}{9x^9}-\frac{11a^{10}b}{8x^8}-\frac{55a^9b^2}{7x^7}-\frac{55a^8b^3}{2x^6}-\frac{66a^7b^4}{x^5}-\frac{231a^6b^5}{2x^4}-\frac{154a^5b^6}{x^3}-\frac{165a^4b^7}{x^2}-\frac{165a^3b^8}{x}+55a^2b^9\log(x)+11ab^{10}x+\frac{b^{11}x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11/x^10,x]
```

```
[Out] -1/9*a^11/x^9 - (11*a^10*b)/(8*x^8) - (55*a^9*b^2)/(7*x^7) - (55*a^8*b^3)/(
2*x^6) - (66*a^7*b^4)/x^5 - (231*a^6*b^5)/(2*x^4) - (154*a^5*b^6)/x^3 - (16
5*a^4*b^7)/x^2 - (165*a^3*b^8)/x + 11*a*b^10*x + (b^11*x^2)/2 + 55*a^2*b^9*
Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{11}}{x^{10}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^11/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^11/x^10, x]
```

fricas [A] time = 1.08, size = 125, normalized size = 0.95

$$\frac{252\,b^{11}x^{11}+5544\,ab^{10}x^{10}+27720\,a^{2}b^{9}x^{9}\log(x)-83160\,a^{3}b^{8}x^{8}-83160\,a^{4}b^{7}x^{7}-77616\,a^{5}b^{6}x^{6}-58212\,a^{6}b^{5}x^{5}-33264\,a^{7}b^{4}x^{4}-13860\,a^{8}b^{3}x^{3}-3960\,a^{9}b^{2}x^{2}-693\,a^{10}bx-56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="fricas")
```

```
[Out] 1/504*(252*b^11*x^11 + 5544*a*b^10*x^10 + 27720*a^2*b^9*x^9*log(x) - 83160*
a^3*b^8*x^8 - 83160*a^4*b^7*x^7 - 77616*a^5*b^6*x^6 - 58212*a^6*b^5*x^5 - 3
3264*a^7*b^4*x^4 - 13860*a^8*b^3*x^3 - 3960*a^9*b^2*x^2 - 693*a^10*b*x - 56
*a^11)/x^9
```

giac [A] time = 1.28, size = 122, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^{2}+11\,ab^{10}x+55\,a^{2}b^{9}\log\left(|x|\right)-\frac{83160\,a^{3}b^{8}x^{8}+83160\,a^{4}b^{7}x^{7}+77616\,a^{5}b^{6}x^{6}+58212\,a^{6}b^{5}x^{5}+33264\,a^{7}b^{4}x^{4}+13860\,a^{8}b^{3}x^{3}+3960\,a^{9}b^{2}x^{2}+693\,a^{10}bx+56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="giac")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(abs(x)) - 1/504*(83160*a^3*b^8*
x^8 + 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7
*b^4*x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x
^9
```

maple [A] time = 0.01, size = 121, normalized size = 0.92

$$\frac{b^{11}x^{2}}{2}+55a^{2}b^{9}\ln(x)+11a\,b^{10}x-\frac{165a^{3}b^{8}}{x}-\frac{165a^{4}b^{7}}{x^{2}}-\frac{154a^{5}b^{6}}{x^{3}}-\frac{231a^{6}b^{5}}{2x^{4}}-\frac{66a^{7}b^{4}}{x^{5}}-\frac{55a^{8}b^{3}}{2x^{6}}-\frac{55a^{9}b^{2}}{7x^{7}}-\frac{11a^{10}b}{8x^{8}}-\frac{a^{11}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11/x^10,x)
```

```
[Out] -1/9*a^11/x^9-11/8*a^10*b/x^8-55/7*a^9*b^2/x^7-55/2*a^8*b^3/x^6-66*a^7*b^4/
x^5-231/2*a^6*b^5/x^4-154*a^5*b^6/x^3-165*a^4*b^7/x^2-165*a^3*b^8/x+11*a*b^
10*x+1/2*b^11*x^2+55*a^2*b^9*ln(x)
```

maxima [A] time = 1.37, size = 121, normalized size = 0.92

$$\frac{1}{2}\,b^{11}x^{2}+11\,ab^{10}x+55\,a^{2}b^{9}\log(x)-\frac{83160\,a^{3}b^{8}x^{8}+83160\,a^{4}b^{7}x^{7}+77616\,a^{5}b^{6}x^{6}+58212\,a^{6}b^{5}x^{5}+33264\,a^{7}b^{4}x^{4}+13860\,a^{8}b^{3}x^{3}+3960\,a^{9}b^{2}x^{2}+693\,a^{10}bx+56\,a^{11}}{504\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11/x^10,x, algorithm="maxima")
```

```
[Out] 1/2*b^11*x^2 + 11*a*b^10*x + 55*a^2*b^9*log(x) - 1/504*(83160*a^3*b^8*x^8 +
 83160*a^4*b^7*x^7 + 77616*a^5*b^6*x^6 + 58212*a^6*b^5*x^5 + 33264*a^7*b^4*
x^4 + 13860*a^8*b^3*x^3 + 3960*a^9*b^2*x^2 + 693*a^10*b*x + 56*a^11)/x^9
```

mupad [B] time = 0.09, size = 121, normalized size = 0.92

$$\frac{b^{11}\,x^{2}}{2}-\frac{\frac{a^{11}}{9}+\frac{11\,a^{10}\,b\,x}{8}+\frac{55\,a^{9}\,b^{2}\,x^{2}}{7}+\frac{55\,a^{8}\,b^{3}\,x^{3}}{2}+66\,a^{7}\,b^{4}\,x^{4}+\frac{231\,a^{6}\,b^{5}\,x^{5}}{2}+154\,a^{5}\,b^{6}\,x^{6}+165\,a^{4}\,b^{7}\,x^{7}+165\,a^{3}\,b^{8}\,x^{8}}{x^{9}}+55\,a^{2}\,b^{9}\,\ln(x)+11\,a\,b^{10}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11/x^10,x)
```

```
[Out] (b^11*x^2)/2 - (a^11/9 + (55*a^9*b^2*x^2)/7 + (55*a^8*b^3*x^3)/2 + 66*a^7*b
^4*x^4 + (231*a^6*b^5*x^5)/2 + 154*a^5*b^6*x^6 + 165*a^4*b^7*x^7 + 165*a^3*
b^8*x^8 + (11*a^10*b*x)/8)/x^9 + 55*a^2*b^9*log(x) + 11*a*b^10*x
```

sympy [A] time = 0.85, size = 131, normalized size = 0.99

$$55a^2b^9\log(x) + 11ab^{10}x + \frac{b^{11}x^2}{2} + \frac{-56a^{11} - 693a^{10}bx - 3960a^9b^2x^2 - 13860a^8b^3x^3 - 33264a^7b^4x^4 - 58212a^6b^5x^5 - 77616a^5b^6x^6 - 83160a^4b^7x^7 - 83160a^3b^8x^8}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11/x**10,x)
```

```
[Out] 55*a**2*b**9*log(x) + 11*a*b**10*x + b**11*x**2/2 + (-56*a**11 - 693*a**10*
b*x - 3960*a**9*b**2*x**2 - 13860*a**8*b**3*x**3 - 33264*a**7*b**4*x**4 - 5
8212*a**6*b**5*x**5 - 77616*a**5*b**6*x**6 - 83160*a**4*b**7*x**7 - 83160*a
**3*b**8*x**8)/(504*x**9)
```

3.241 $\int \frac{(a+bx)^{10}}{x^{10}}\,dx$

Optimal. Leaf size=114

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Rubi [A] time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}-\frac{5a^9b}{4x^8}-\frac{a^{10}}{9x^9}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10/x^10,x]
```

```
[Out] -a^10/(9*x^9) - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{10}}{x^{10}}\,dx &= \int \left(b^{10}+\frac{a^{10}}{x^{10}}+\frac{10a^9b}{x^9}+\frac{45a^8b^2}{x^8}+\frac{120a^7b^3}{x^7}+\frac{210a^6b^4}{x^6}+\frac{252a^5b^5}{x^5}+\frac{210a^4b^6}{x^4}+\frac{120a^3b^7}{x^3}+\frac{4\text{...}}{\text{...}}\right.\\ &= -\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+b^{10}x+1\text{...}\end{aligned}$$

Mathematica [A] time = 0.01, size = 114, normalized size = 1.00

$$-\frac{a^{10}}{9x^9}-\frac{5a^9b}{4x^8}-\frac{45a^8b^2}{7x^7}-\frac{20a^7b^3}{x^6}-\frac{42a^6b^4}{x^5}-\frac{63a^5b^5}{x^4}-\frac{70a^4b^6}{x^3}-\frac{60a^3b^7}{x^2}-\frac{45a^2b^8}{x}+10ab^9\log(x)+b^{10}x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10/x^10,x]
```

```
[Out] -1/9*a^10/x^9 - (5*a^9*b)/(4*x^8) - (45*a^8*b^2)/(7*x^7) - (20*a^7*b^3)/x^6
 - (42*a^6*b^4)/x^5 - (63*a^5*b^5)/x^4 - (70*a^4*b^6)/x^3 - (60*a^3*b^7)/x^
2 - (45*a^2*b^8)/x + b^10*x + 10*a*b^9*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{10}}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10/x^10, x]
```

fricas [A]   time = 1.05, size = 114, normalized size = 1.00

$$\frac{252\,b^{10}x^{10} + 2520\,ab^{9}x^{9}\log(x) - 11340\,a^{2}b^{8}x^{8} - 15120\,a^{3}b^{7}x^{7} - 17640\,a^{4}b^{6}x^{6} - 15876\,a^{5}b^{5}x^{5} - 10584\,a^{6}b^{4}x^{4} - 5040\,a^{7}b^{3}x^{3} - 1620\,a^{8}b^{2}x^{2} - 315\,a^{9}bx - 28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*x^10 + 2520*a*b^9*x^9*log(x) - 11340*a^2*b^8*x^8 - 15120*a^
3*b^7*x^7 - 17640*a^4*b^6*x^6 - 15876*a^5*b^5*x^5 - 10584*a^6*b^4*x^4 - 504
0*a^7*b^3*x^3 - 1620*a^8*b^2*x^2 - 315*a^9*b*x - 28*a^10)/x^9
```

giac [A]   time = 1.06, size = 110, normalized size = 0.96

$$b^{10}x + 10\,ab^{9}\log(|x|) - \frac{11340\,a^{2}b^{8}x^{8} + 15120\,a^{3}b^{7}x^{7} + 17640\,a^{4}b^{6}x^{6} + 15876\,a^{5}b^{5}x^{5} + 10584\,a^{6}b^{4}x^{4} + 5040\,a^{7}b^{3}x^{3} + 1620\,a^{8}b^{2}x^{2} + 315\,a^{9}bx + 28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="giac")
```

```
[Out] b^10*x + 10*a*b^9*log(abs(x)) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^
7 + 17640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^
3*x^3 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

maple [A]   time = 0.00, size = 109, normalized size = 0.96

$$10a\,b^{9}\ln(x) + b^{10}x - \frac{45a^{2}b^{8}}{x} - \frac{60a^{3}b^{7}}{x^{2}} - \frac{70a^{4}b^{6}}{x^{3}} - \frac{63a^{5}b^{5}}{x^{4}} - \frac{42a^{6}b^{4}}{x^{5}} - \frac{20a^{7}b^{3}}{x^{6}} - \frac{45a^{8}b^{2}}{7x^{7}} - \frac{5a^{9}b}{4x^{8}} - \frac{a^{10}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10/x^10,x)
```

```
[Out] 10*a*b^9*ln(x)+b^10*x-45*a^2*b^8/x-60*a^3*b^7/x^2-70*a^4*b^6/x^3-63*a^5*b^5
/x^4-42*a^6*b^4/x^5-20*a^7*b^3/x^6-45/7*a^8*b^2/x^7-5/4*a^9*b/x^8-1/9*a^10/
x^9
```

maxima [A]   time = 1.43, size = 109, normalized size = 0.96

$$b^{10}x + 10\,ab^{9}\log(x) - \frac{11340\,a^{2}b^{8}x^{8} + 15120\,a^{3}b^{7}x^{7} + 17640\,a^{4}b^{6}x^{6} + 15876\,a^{5}b^{5}x^{5} + 10584\,a^{6}b^{4}x^{4} + 5040\,a^{7}b^{3}x^{3} + 1620\,a^{8}b^{2}x^{2} + 315\,a^{9}bx + 28\,a^{10}}{252\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10/x^10,x, algorithm="maxima")
```

```
[Out] b^10*x + 10*a*b^9*log(x) - 1/252*(11340*a^2*b^8*x^8 + 15120*a^3*b^7*x^7 + 1
7640*a^4*b^6*x^6 + 15876*a^5*b^5*x^5 + 10584*a^6*b^4*x^4 + 5040*a^7*b^3*x^3
 + 1620*a^8*b^2*x^2 + 315*a^9*b*x + 28*a^10)/x^9
```

mupad [B]   time = 0.00, size = 114, normalized size = 1.00

$$-\frac{\frac{a^{10}}{9} - b^{10}\,x^{10} + \frac{45\,a^{8}\,b^{2}\,x^{2}}{7} + 20\,a^{7}\,b^{3}\,x^{3} + 42\,a^{6}\,b^{4}\,x^{4} + 63\,a^{5}\,b^{5}\,x^{5} + 70\,a^{4}\,b^{6}\,x^{6} + 60\,a^{3}\,b^{7}\,x^{7} + 45\,a^{2}\,b^{8}\,x^{8} + \frac{5\,a^{9}\,b\,x}{4} - 10\,a\,b^{9}\,x^{9}\,\ln(x)}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10/x^10,x)
```

```
[Out] -(a^10/9 - b^10*x^10 + (45*a^8*b^2*x^2)/7 + 20*a^7*b^3*x^3 + 42*a^6*b^4*x^4
 + 63*a^5*b^5*x^5 + 70*a^4*b^6*x^6 + 60*a^3*b^7*x^7 + 45*a^2*b^8*x^8 + (5*a
^9*b*x)/4 - 10*a*b^9*x^9*log(x))/x^9
```

sympy [A]   time = 0.86, size = 117, normalized size = 1.03

$$10ab^{9}\log(x) + b^{10}x + \frac{-28a^{10} - 315a^{9}bx - 1620a^{8}b^{2}x^{2} - 5040a^{7}b^{3}x^{3} - 10584a^{6}b^{4}x^{4} - 15876a^{5}b^{5}x^{5} - 17640a^{4}b^{6}x^{6} - 15120a^{3}b^{7}x^{7} - 11340a^{2}b^{8}x^{8}}{252x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10/x**10,x)
```

```
[Out] 10*a*b**9*log(x) + b**10*x + (-28*a**10 - 315*a**9*b*x - 1620*a**8*b**2*x**
2 - 5040*a**7*b**3*x**3 - 10584*a**6*b**4*x**4 - 15876*a**5*b**5*x**5 - 176
40*a**4*b**6*x**6 - 15120*a**3*b**7*x**7 - 11340*a**2*b**8*x**8)/(252*x**9)
```

3.242 $\int \frac{(a+bx)^9}{x^{10}}\,dx$

Optimal. Leaf size=109

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9a^8b}{8x^8}-\frac{a^9}{9x^9}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/x^10,x]
```

```
[Out] -a^9/(9*x^9) - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^9}{x^{10}}\,dx=\int \left(\frac{a^9}{x^{10}}+\frac{9a^8b}{x^9}+\frac{36a^7b^2}{x^8}+\frac{84a^6b^3}{x^7}+\frac{126a^5b^4}{x^6}+\frac{126a^4b^5}{x^5}+\frac{84a^3b^6}{x^4}+\frac{36a^2b^7}{x^3}+\frac{9ab^8}{x^2}+\frac{b^9}{x}$$

$$=-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log($$

Mathematica [A] time = 0.00, size = 109, normalized size = 1.00

$$-\frac{a^9}{9x^9}-\frac{9a^8b}{8x^8}-\frac{36a^7b^2}{7x^7}-\frac{14a^6b^3}{x^6}-\frac{126a^5b^4}{5x^5}-\frac{63a^4b^5}{2x^4}-\frac{28a^3b^6}{x^3}-\frac{18a^2b^7}{x^2}-\frac{9ab^8}{x}+b^9\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/x^10,x]
```

```
[Out] -1/9*a^9/x^9 - (9*a^8*b)/(8*x^8) - (36*a^7*b^2)/(7*x^7) - (14*a^6*b^3)/x^6
- (126*a^5*b^4)/(5*x^5) - (63*a^4*b^5)/(2*x^4) - (28*a^3*b^6)/x^3 - (18*a^2
*b^7)/x^2 - (9*a*b^8)/x + b^9*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^9}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9/x^10, x]
```

fricas [A] time = 0.95, size = 103, normalized size = 0.94

$$\frac{2520\,b^{9}x^{9}\log\left(x\right) - 22680\,ab^{8}x^{8} - 45360\,a^{2}b^{7}x^{7} - 70560\,a^{3}b^{6}x^{6} - 79380\,a^{4}b^{5}x^{5} - 63504\,a^{5}b^{4}x^{4} - 35280\,a^{6}b^{3}x^{3} - 12960\,a^{7}b^{2}x^{2} - 2835\,a^{8}bx - 280\,a^{9}}{2520\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(x) - 22680*a*b^8*x^8 - 45360*a^2*b^7*x^7 - 70560*a
^3*b^6*x^6 - 79380*a^4*b^5*x^5 - 63504*a^5*b^4*x^4 - 35280*a^6*b^3*x^3 - 12
960*a^7*b^2*x^2 - 2835*a^8*b*x - 280*a^9)/x^9
```

giac [A] time = 1.04, size = 101, normalized size = 0.93

$$b^{9}\log\left(|x|\right) - \frac{22680\,ab^{8}x^{8} + 45360\,a^{2}b^{7}x^{7} + 70560\,a^{3}b^{6}x^{6} + 79380\,a^{4}b^{5}x^{5} + 63504\,a^{5}b^{4}x^{4} + 35280\,a^{6}b^{3}x^{3} + 12960\,a^{7}b^{2}x^{2} + 2835\,a^{8}bx + 280\,a^{9}}{2520\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="giac")
```

```
[Out] b^9*log(abs(x)) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b
^6*x^6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*
a^7*b^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

maple [A] time = 0.01, size = 100, normalized size = 0.92

$$b^{9}\ln\left(x\right)-\frac{9a\,b^{8}}{x}-\frac{18a^{2}b^{7}}{x^{2}}-\frac{28a^{3}b^{6}}{x^{3}}-\frac{63a^{4}b^{5}}{2x^{4}}-\frac{126a^{5}b^{4}}{5x^{5}}-\frac{14a^{6}b^{3}}{x^{6}}-\frac{36a^{7}b^{2}}{7x^{7}}-\frac{9a^{8}b}{8x^{8}}-\frac{a^{9}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/x^10,x)
```

```
[Out] -1/9*a^9/x^9-9/8*a^8*b/x^8-36/7*a^7*b^2/x^7-14*a^6*b^3/x^6-126/5*a^5*b^4/x^
5-63/2*a^4*b^5/x^4-28*a^3*b^6/x^3-18*a^2*b^7/x^2-9*a*b^8/x+b^9*ln(x)
```

maxima [A] time = 1.34, size = 100, normalized size = 0.92

$$b^{9}\log\left(x\right) - \frac{22680\,ab^{8}x^{8} + 45360\,a^{2}b^{7}x^{7} + 70560\,a^{3}b^{6}x^{6} + 79380\,a^{4}b^{5}x^{5} + 63504\,a^{5}b^{4}x^{4} + 35280\,a^{6}b^{3}x^{3} + 12960\,a^{7}b^{2}x^{2} + 2835\,a^{8}bx + 280\,a^{9}}{2520\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/x^10,x, algorithm="maxima")
```

```
[Out] b^9*log(x) - 1/2520*(22680*a*b^8*x^8 + 45360*a^2*b^7*x^7 + 70560*a^3*b^6*x^
6 + 79380*a^4*b^5*x^5 + 63504*a^5*b^4*x^4 + 35280*a^6*b^3*x^3 + 12960*a^7*b
^2*x^2 + 2835*a^8*b*x + 280*a^9)/x^9
```

mupad [B] time = 0.08, size = 100, normalized size = 0.92

$$b^{9}\,\ln\left(x\right)-\frac{\frac{a^{9}}{9}+\frac{9\,a^{8}\,b\,x}{8}+\frac{36\,a^{7}\,b^{2}\,x^{2}}{7}+14\,a^{6}\,b^{3}\,x^{3}+\frac{126\,a^{5}\,b^{4}\,x^{4}}{5}+\frac{63\,a^{4}\,b^{5}\,x^{5}}{2}+28\,a^{3}\,b^{6}\,x^{6}+18\,a^{2}\,b^{7}\,x^{7}+9\,a\,b^{8}\,x^{8}}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/x^10,x)
```

```
[Out] b^9*log(x) - (a^9/9 + 9*a*b^8*x^8 + (36*a^7*b^2*x^2)/7 + 14*a^6*b^3*x^3 + (
126*a^5*b^4*x^4)/5 + (63*a^4*b^5*x^5)/2 + 28*a^3*b^6*x^6 + 18*a^2*b^7*x^7 +
 (9*a^8*b*x)/8)/x^9
```

sympy [A] time = 0.79, size = 107, normalized size = 0.98

$$b^9\log(x)+\frac{-280a^9-2835a^8bx-12960a^7b^2x^2-35280a^6b^3x^3-63504a^5b^4x^4-79380a^4b^5x^5-70560a^3b^6x^6-45360a^2b^7x^7-22680ab^8x^8}{2520x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/x**10,x)
```

```
[Out] b**9*log(x) + (-280*a**9 - 2835*a**8*b*x - 12960*a**7*b**2*x**2 - 35280*a**
6*b**3*x**3 - 63504*a**5*b**4*x**4 - 79380*a**4*b**5*x**5 - 70560*a**3*b**6
*x**6 - 45360*a**2*b**7*x**7 - 22680*a*b**8*x**8)/(2520*x**9)
```

3.243 $\int \frac{(a+bx)^8}{x^{10}} dx$

Optimal. Leaf size=17

$$-\frac{(a+bx)^9}{9ax^9}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {37}

$$-\frac{(a+bx)^9}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/x^10,x]
```

```
[Out] -(a + b*x)^9/(9*a*x^9)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^8}{x^{10}} dx = -\frac{(a+bx)^9}{9ax^9}$$

Mathematica [B] time = 0.01, size = 96, normalized size = 5.65

$$-\frac{a^8}{9x^9}-\frac{a^7b}{x^8}-\frac{4a^6b^2}{x^7}-\frac{28a^5b^3}{3x^6}-\frac{14a^4b^4}{x^5}-\frac{14a^3b^5}{x^4}-\frac{28a^2b^6}{3x^3}-\frac{4ab^7}{x^2}-\frac{b^8}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/x^10,x]
```

```
[Out] -1/9*a^8/x^9 - (a^7*b)/x^8 - (4*a^6*b^2)/x^7 - (28*a^5*b^3)/(3*x^6) - (14*a
^4*b^4)/x^5 - (14*a^3*b^5)/x^4 - (28*a^2*b^6)/(3*x^3) - (4*a*b^7)/x^2 - b^8
/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^8}{x^{10}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8/x^10, x]
```

fricas [B] time = 0.78, size = 88, normalized size = 5.18

$$-\frac{9\,b^8x^8+36\,ab^7x^7+84\,a^2b^6x^6+126\,a^3b^5x^5+126\,a^4b^4x^4+84\,a^5b^3x^3+36\,a^6b^2x^2+9\,a^7bx+a^8}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="fricas")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

giac [B] time = 1.17, size = 88, normalized size = 5.18

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="giac")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

maple [B] time = 0.01, size = 91, normalized size = 5.35

$$-\frac{b^{8}}{x}-\frac{4a\,b^{7}}{x^{2}}-\frac{28a^{2}b^{6}}{3x^{3}}-\frac{14a^{3}b^{5}}{x^{4}}-\frac{14a^{4}b^{4}}{x^{5}}-\frac{28a^{5}b^{3}}{3x^{6}}-\frac{4a^{6}b^{2}}{x^{7}}-\frac{a^{7}b}{x^{8}}-\frac{a^{8}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/x^10,x)
```

```
[Out] -28/3*a^2*b^6/x^3-b^8/x-4*a*b^7/x^2-14*a^4*b^4/x^5-1/9*a^8/x^9-a^7*b/x^8-28
/3*a^5*b^3/x^6-14*a^3*b^5/x^4-4*a^6*b^2/x^7
```

maxima [B] time = 1.34, size = 88, normalized size = 5.18

$$-\frac{9\,b^{8}x^{8}+36\,ab^{7}x^{7}+84\,a^{2}b^{6}x^{6}+126\,a^{3}b^{5}x^{5}+126\,a^{4}b^{4}x^{4}+84\,a^{5}b^{3}x^{3}+36\,a^{6}b^{2}x^{2}+9\,a^{7}bx+a^{8}}{9\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/x^10,x, algorithm="maxima")
```

```
[Out] -1/9*(9*b^8*x^8 + 36*a*b^7*x^7 + 84*a^2*b^6*x^6 + 126*a^3*b^5*x^5 + 126*a^4
*b^4*x^4 + 84*a^5*b^3*x^3 + 36*a^6*b^2*x^2 + 9*a^7*b*x + a^8)/x^9
```

mupad [B] time = 0.09, size = 88, normalized size = 5.18

$$-\frac{\frac{a^{8}}{9}+a^{7}\,b\,x+4\,a^{6}\,b^{2}\,x^{2}+\frac{28\,a^{5}\,b^{3}\,x^{3}}{3}+14\,a^{4}\,b^{4}\,x^{4}+14\,a^{3}\,b^{5}\,x^{5}+\frac{28\,a^{2}\,b^{6}\,x^{6}}{3}+4\,a\,b^{7}\,x^{7}+b^{8}\,x^{8}}{x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/x^10,x)
```

```
[Out] -(a^8/9 + b^8*x^8 + 4*a*b^7*x^7 + 4*a^6*b^2*x^2 + (28*a^5*b^3*x^3)/3 + 14*a
^4*b^4*x^4 + 14*a^3*b^5*x^5 + (28*a^2*b^6*x^6)/3 + a^7*b*x)/x^9
```

sympy [B] time = 0.73, size = 95, normalized size = 5.59

$$\frac{-a^{8}-9a^{7}bx-36a^{6}b^{2}x^{2}-84a^{5}b^{3}x^{3}-126a^{4}b^{4}x^{4}-126a^{3}b^{5}x^{5}-84a^{2}b^{6}x^{6}-36ab^{7}x^{7}-9b^{8}x^{8}}{9x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/x**10,x)
```

```
[Out] (-a**8 - 9*a**7*b*x - 36*a**6*b**2*x**2 - 84*a**5*b**3*x**3 - 126*a**4*b**4
*x**4 - 126*a**3*b**5*x**5 - 84*a**2*b**6*x**6 - 36*a*b**7*x**7 - 9*b**8*x*
*8)/(9*x**9)
```

3.244 $\int \frac{(a+bx)^7}{x^{10}}\,dx$

Optimal. Leaf size=36

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$\frac{b(a+bx)^8}{72a^2x^8}-\frac{(a+bx)^8}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/x^10,x]
```

```
[Out] -(a + b*x)^8/(9*a*x^9) + (b*(a + b*x)^8)/(72*a^2*x^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^7}{x^{10}}\,dx &= -\frac{(a+bx)^8}{9ax^9}-\frac{b\int \frac{(a+bx)^7}{x^9}\,dx}{9a}\\ &= -\frac{(a+bx)^8}{9ax^9}+\frac{b(a+bx)^8}{72a^2x^8}\end{aligned}$$

Mathematica [B] time = 0.00, size = 91, normalized size = 2.53

$$-\frac{a^7}{9x^9}-\frac{7a^6b}{8x^8}-\frac{3a^5b^2}{x^7}-\frac{35a^4b^3}{6x^6}-\frac{7a^3b^4}{x^5}-\frac{21a^2b^5}{4x^4}-\frac{7ab^6}{3x^3}-\frac{b^7}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/x^10,x]
```

```
[Out] -1/9*a^7/x^9 - (7*a^6*b)/(8*x^8) - (3*a^5*b^2)/x^7 - (35*a^4*b^3)/(6*x^6) -
 (7*a^3*b^4)/x^5 - (21*a^2*b^5)/(4*x^4) - (7*a*b^6)/(3*x^3) - b^7/(2*x^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/x^10, x]
```

fricas [B] time = 0.86, size = 79, normalized size = 2.19

$$-\frac{36 b^7 x^7 + 168 ab^6 x^6 + 378 a^2 b^5 x^5 + 504 a^3 b^4 x^4 + 420 a^4 b^3 x^3 + 216 a^5 b^2 x^2 + 63 a^6 bx + 8 a^7}{72 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

giac [B] time = 1.04, size = 79, normalized size = 2.19

$$-\frac{36 b^7 x^7 + 168 ab^6 x^6 + 378 a^2 b^5 x^5 + 504 a^3 b^4 x^4 + 420 a^4 b^3 x^3 + 216 a^5 b^2 x^2 + 63 a^6 bx + 8 a^7}{72 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

maple [B] time = 0.00, size = 80, normalized size = 2.22

$$-\frac{b^7}{2x^2} - \frac{7a\,b^6}{3x^3} - \frac{21a^2b^5}{4x^4} - \frac{7a^3b^4}{x^5} - \frac{35a^4b^3}{6x^6} - \frac{3a^5b^2}{x^7} - \frac{7a^6b}{8x^8} - \frac{a^7}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/x^10,x)
```

```
[Out] -1/2*b^7/x^2-7/3*a*b^6/x^3-21/4*a^2*b^5/x^4-7*a^3*b^4/x^5-35/6*a^4*b^3/x^6-
3*a^5*b^2/x^7-7/8*a^6*b/x^8-1/9*a^7/x^9
```

maxima [B] time = 1.36, size = 79, normalized size = 2.19

$$-\frac{36 b^7 x^7 + 168 ab^6 x^6 + 378 a^2 b^5 x^5 + 504 a^3 b^4 x^4 + 420 a^4 b^3 x^3 + 216 a^5 b^2 x^2 + 63 a^6 bx + 8 a^7}{72 x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/x^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*x^7 + 168*a*b^6*x^6 + 378*a^2*b^5*x^5 + 504*a^3*b^4*x^4 + 420
*a^4*b^3*x^3 + 216*a^5*b^2*x^2 + 63*a^6*b*x + 8*a^7)/x^9
```

mupad [B] time = 0.00, size = 23, normalized size = 0.64

$$-\frac{(8\,a - b\,x)\,\left(a + b\,x\right)^8}{72\,a^2\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/x^10,x)
```

```
[Out] -((8*a - b*x)*(a + b*x)^8)/(72*a^2*x^9)
```

sympy [B] time = 0.70, size = 85, normalized size = 2.36

$$\frac{-8a^7-63a^6bx-216a^5b^2x^2-420a^4b^3x^3-504a^3b^4x^4-378a^2b^5x^5-168ab^6x^6-36b^7x^7}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/x**10,x)
```

```
[Out] (-8*a**7 - 63*a**6*b*x - 216*a**5*b**2*x**2 - 420*a**4*b**3*x**3 - 504*a**3
*b**4*x**4 - 378*a**2*b**5*x**5 - 168*a*b**6*x**6 - 36*b**7*x**7)/(72*x**9)
```

3.245 $\int \frac{(a+bx)^6}{x^{10}} \, dx$

Optimal. Leaf size=56

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {45, 37}

$$-\frac{b^2(a+bx)^7}{252a^3x^7}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{(a+bx)^7}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/x^10,x]
```

```
[Out] -(a + b*x)^7/(9*a*x^9) + (b*(a + b*x)^7)/(36*a^2*x^8) - (b^2*(a + b*x)^7)/(
252*a^3*x^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^6}{x^{10}} \, dx &= -\frac{(a+bx)^7}{9ax^9}-\frac{(2b) \int \frac{(a+bx)^6}{x^9} \, dx}{9a}\\
&= -\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}+\frac{b^2 \int \frac{(a+bx)^6}{x^8} \, dx}{36a^2}\\
&= -\frac{(a+bx)^7}{9ax^9}+\frac{b(a+bx)^7}{36a^2x^8}-\frac{b^2(a+bx)^7}{252a^3x^7}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 80, normalized size = 1.43

$$-\frac{a^6}{9x^9}-\frac{3a^5b}{4x^8}-\frac{15a^4b^2}{7x^7}-\frac{10a^3b^3}{3x^6}-\frac{3a^2b^4}{x^5}-\frac{3ab^5}{2x^4}-\frac{b^6}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/x^10,x]
```

```
[Out] -1/9*a^6/x^9 - (3*a^5*b)/(4*x^8) - (15*a^4*b^2)/(7*x^7) - (10*a^3*b^3)/(3*x
^6) - (3*a^2*b^4)/x^5 - (3*a*b^5)/(2*x^4) - b^6/(3*x^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/x^10, x]
```

fricas [A] time = 1.13, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="fricas")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

giac [A] time = 1.02, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="giac")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

maple [A] time = 0.01, size = 69, normalized size = 1.23

$$-\frac{b^6}{3x^3}-\frac{3a\,b^5}{2x^4}-\frac{3a^2b^4}{x^5}-\frac{10a^3b^3}{3x^6}-\frac{15a^4b^2}{7x^7}-\frac{3a^5b}{4x^8}-\frac{a^6}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/x^10,x)
```

```
[Out] -1/3*b^6/x^3-3*a^2*b^4/x^5-15/7*a^4*b^2/x^7-1/9*a^6/x^9-3/4*a^5*b/x^8-10/3*
a^3*b^3/x^6-3/2*a*b^5/x^4
```

maxima [A] time = 1.37, size = 68, normalized size = 1.21

$$-\frac{84\,b^6x^6+378\,ab^5x^5+756\,a^2b^4x^4+840\,a^3b^3x^3+540\,a^4b^2x^2+189\,a^5bx+28\,a^6}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/x^10,x, algorithm="maxima")
```

```
[Out] -1/252*(84*b^6*x^6 + 378*a*b^5*x^5 + 756*a^2*b^4*x^4 + 840*a^3*b^3*x^3 + 54
0*a^4*b^2*x^2 + 189*a^5*b*x + 28*a^6)/x^9
```

mupad [B] time = 0.10, size = 68, normalized size = 1.21

$$-\frac{\frac{a^6}{9}+\frac{3\,a^5\,b\,x}{4}+\frac{15\,a^4\,b^2\,x^2}{7}+\frac{10\,a^3\,b^3\,x^3}{3}+3\,a^2\,b^4\,x^4+\frac{3\,a\,b^5\,x^5}{2}+\frac{b^6\,x^6}{3}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/x^10,x)
```

```
[Out] -(a^6/9 + (b^6*x^6)/3 + (3*a*b^5*x^5)/2 + (15*a^4*b^2*x^2)/7 + (10*a^3*b^3*
x^3)/3 + 3*a^2*b^4*x^4 + (3*a^5*b*x)/4)/x^9
```

sympy [A] time = 0.56, size = 73, normalized size = 1.30

$$\frac{-28a^6 - 189a^5bx - 540a^4b^2x^2 - 840a^3b^3x^3 - 756a^2b^4x^4 - 378ab^5x^5 - 84b^6x^6}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/x**10,x)
```

```
[Out] (-28*a**6 - 189*a**5*b*x - 540*a**4*b**2*x**2 - 840*a**3*b**3*x**3 - 756*a*
*2*b**4*x**4 - 378*a*b**5*x**5 - 84*b**6*x**6)/(252*x**9)
```

3.246 $\int \frac{(a+bx)^5}{x^{10}}\,dx$

Optimal. Leaf size=67

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/x^10,x]
```

```
[Out] -a^5/(9*x^9) - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{x^{10}}\,dx &= \int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^9}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^7}+\frac{5ab^4}{x^6}+\frac{b^5}{x^5}\right)dx\\ &= -\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.00

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{8x^8}-\frac{10a^3b^2}{7x^7}-\frac{5a^2b^3}{3x^6}-\frac{ab^4}{x^5}-\frac{b^5}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/x^10,x]
```

```
[Out] -1/9*a^5/x^9 - (5*a^4*b)/(8*x^8) - (10*a^3*b^2)/(7*x^7) - (5*a^2*b^3)/(3*x^
6) - (a*b^4)/x^5 - b^5/(4*x^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/x^10, x]
```

fricas [A] time = 1.11, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="fricas")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

giac [A] time = 1.01, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="giac")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

maple [A] time = 0.00, size = 58, normalized size = 0.87

$$-\frac{b^5}{4x^4}-\frac{a\,b^4}{x^5}-\frac{5a^2b^3}{3x^6}-\frac{10a^3b^2}{7x^7}-\frac{5a^4b}{8x^8}-\frac{a^5}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/x^10,x)
```

```
[Out] -1/4*b^5/x^4-a*b^4/x^5-5/3*a^2*b^3/x^6-10/7*a^3*b^2/x^7-5/8*a^4*b/x^8-1/9*a
^5/x^9
```

maxima [A] time = 1.36, size = 57, normalized size = 0.85

$$-\frac{126\,b^5x^5+504\,ab^4x^4+840\,a^2b^3x^3+720\,a^3b^2x^2+315\,a^4bx+56\,a^5}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/x^10,x, algorithm="maxima")
```

```
[Out] -1/504*(126*b^5*x^5 + 504*a*b^4*x^4 + 840*a^2*b^3*x^3 + 720*a^3*b^2*x^2 + 3
15*a^4*b*x + 56*a^5)/x^9
```

mupad [B] time = 0.00, size = 56, normalized size = 0.84

$$-\frac{\frac{a^5}{9}+\frac{5\,a^4\,b\,x}{8}+\frac{10\,a^3\,b^2\,x^2}{7}+\frac{5\,a^2\,b^3\,x^3}{3}+a\,b^4\,x^4+\frac{b^5\,x^5}{4}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/x^10,x)
```

```
[Out] -(a^5/9 + (b^5*x^5)/4 + a*b^4*x^4 + (10*a^3*b^2*x^2)/7 + (5*a^2*b^3*x^3)/3
+ (5*a^4*b*x)/8)/x^9
```

sympy [A] time = 0.49, size = 61, normalized size = 0.91

$$\frac{-56a^5-315a^4bx-720a^3b^2x^2-840a^2b^3x^3-504ab^4x^4-126b^5x^5}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/x**10,x)
```

```
[Out] (-56*a**5 - 315*a**4*b*x - 720*a**3*b**2*x**2 - 840*a**2*b**3*x**3 - 504*a*
b**4*x**4 - 126*b**5*x**5)/(504*x**9)
```

3.247 $\int \frac{(a+bx)^4}{x^{10}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/x^10,x]
```

```
[Out] -a^4/(9*x^9) - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{x^{10}}\,dx &= \int \left(\frac{a^4}{x^{10}}+\frac{4a^3b}{x^9}+\frac{6a^2b^2}{x^8}+\frac{4ab^3}{x^7}+\frac{b^4}{x^6}\right)dx\\ &= -\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{a^4}{9x^9}-\frac{a^3b}{2x^8}-\frac{6a^2b^2}{7x^7}-\frac{2ab^3}{3x^6}-\frac{b^4}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/x^10,x]
```

```
[Out] -1/9*a^4/x^9 - (a^3*b)/(2*x^8) - (6*a^2*b^2)/(7*x^7) - (2*a*b^3)/(3*x^6) -
b^4/(5*x^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/x^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/x^10, x]
```

fricas [A] time = 0.81, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="fricas")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

giac [A] time = 1.25, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="giac")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

maple [A] time = 0.01, size = 47, normalized size = 0.84

$$-\frac{b^4}{5x^5}-\frac{2a\,b^3}{3x^6}-\frac{6a^2b^2}{7x^7}-\frac{a^3b}{2x^8}-\frac{a^4}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/x^10,x)
```

```
[Out] -1/9*a^4/x^9-1/2*a^3*b/x^8-6/7*a^2*b^2/x^7-2/3*a*b^3/x^6-1/5*b^4/x^5
```

maxima [A] time = 1.34, size = 46, normalized size = 0.82

$$-\frac{126\,b^4x^4+420\,ab^3x^3+540\,a^2b^2x^2+315\,a^3bx+70\,a^4}{630\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/x^10,x, algorithm="maxima")
```

```
[Out] -1/630*(126*b^4*x^4 + 420*a*b^3*x^3 + 540*a^2*b^2*x^2 + 315*a^3*b*x + 70*a^
4)/x^9
```

mupad [B] time = 0.03, size = 46, normalized size = 0.82

$$-\frac{\frac{a^4}{9}+\frac{a^3\,b\,x}{2}+\frac{6\,a^2\,b^2\,x^2}{7}+\frac{2\,a\,b^3\,x^3}{3}+\frac{b^4\,x^4}{5}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/x^10,x)
```

```
[Out] -(a^4/9 + (b^4*x^4)/5 + (2*a*b^3*x^3)/3 + (6*a^2*b^2*x^2)/7 + (a^3*b*x)/2)/
x^9
```

sympy [A] time = 0.50, size = 49, normalized size = 0.88

$$\frac{-70a^4-315a^3bx-540a^2b^2x^2-420ab^3x^3-126b^4x^4}{630x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/x**10,x)
```

```
[Out] (-70*a**4 - 315*a**3*b*x - 540*a**2*b**2*x**2 - 420*a*b**3*x**3 - 126*b**4*
x**4)/(630*x**9)
```

3.248 $\int \frac{(a+bx)^3}{x^{10}} \, dx$

Optimal. Leaf size=43

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^10,x]
[Out] -a^3/(9*x^9) - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{10}} \, dx &= \int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^9}+\frac{3ab^2}{x^8}+\frac{b^3}{x^7}\right) dx \\ &= -\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 1.00

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{8x^8}-\frac{3ab^2}{7x^7}-\frac{b^3}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^10,x]
[Out] -1/9*a^3/x^9 - (3*a^2*b)/(8*x^8) - (3*a*b^2)/(7*x^7) - b^3/(6*x^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{x^{10}} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/x^10, x]
```

fricas [A] time = 0.99, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="fricas")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

giac [A] time = 1.09, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="giac")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

maple [A] time = 0.00, size = 36, normalized size = 0.84

$$-\frac{b^3}{6x^6}-\frac{3a\,b^2}{7x^7}-\frac{3a^2b}{8x^8}-\frac{a^3}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^10,x)
[Out] -1/9*a^3/x^9-3/8*a^2*b/x^8-3/7*a*b^2/x^7-1/6*b^3/x^6
```

maxima [A] time = 1.35, size = 35, normalized size = 0.81

$$-\frac{84\,b^3x^3+216\,ab^2x^2+189\,a^2bx+56\,a^3}{504\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^10,x, algorithm="maxima")
[Out] -1/504*(84*b^3*x^3 + 216*a*b^2*x^2 + 189*a^2*b*x + 56*a^3)/x^9
```

mupad [B] time = 0.03, size = 35, normalized size = 0.81

$$-\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x}{8}+\frac{3\,a\,b^2\,x^2}{7}+\frac{b^3\,x^3}{6}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^10,x)
[Out] -(a^3/9 + (b^3*x^3)/6 + (3*a*b^2*x^2)/7 + (3*a^2*b*x)/8)/x^9
```

sympy [A] time = 0.38, size = 37, normalized size = 0.86

$$\frac{-56a^3-189a^2bx-216ab^2x^2-84b^3x^3}{504x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**10,x)
[Out] (-56*a**3 - 189*a**2*b*x - 216*a*b**2*x**2 - 84*b**3*x**3)/(504*x**9)
```

3.249 $\int \frac{(a+bx)^2}{x^{10}}\,dx$

Optimal. Leaf size=30

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^10,x]
[Out] -a^2/(9*x^9) - (a*b)/(4*x^8) - b^2/(7*x^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{10}}\,dx &= \int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^9}+\frac{b^2}{x^8}\right)dx\\ &= -\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$-\frac{a^2}{9x^9}-\frac{ab}{4x^8}-\frac{b^2}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^10,x]
[Out] -1/9*a^2/x^9 - (a*b)/(4*x^8) - b^2/(7*x^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x)^2/x^10, x]
```

fricas [A] time = 1.09, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="fricas")
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

giac [A] time = 0.87, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="giac")
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

maple [A] time = 0.00, size = 25, normalized size = 0.83

$$-\frac{b^2}{7x^7}-\frac{ab}{4x^8}-\frac{a^2}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^10,x)
[Out] -1/9*a^2/x^9-1/4*a*b/x^8-1/7*b^2/x^7
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$-\frac{36\,b^2x^2+63\,abx+28\,a^2}{252\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^10,x, algorithm="maxima")
[Out] -1/252*(36*b^2*x^2 + 63*a*b*x + 28*a^2)/x^9
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$-\frac{\frac{a^2}{9}+\frac{a\,b\,x}{4}+\frac{b^2\,x^2}{7}}{x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^10,x)
[Out] -(a^2/9 + (b^2*x^2)/7 + (a*b*x)/4)/x^9
```

sympy [A] time = 0.27, size = 26, normalized size = 0.87

$$\frac{-28a^2-63abx-36b^2x^2}{252x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**10,x)
[Out] (-28*a**2 - 63*a*b*x - 36*b**2*x**2)/(252*x**9)
```

3.250 $\int \frac{a+bx}{x^{10}}\,dx$

Optimal. Leaf size=17

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^10,x]
[Out] -a/(9*x^9) - b/(8*x^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{10}}\,dx &= \int \left(\frac{a}{x^{10}}+\frac{b}{x^9}\right)dx\\ &= -\frac{a}{9x^9}-\frac{b}{8x^8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{a}{9x^9}-\frac{b}{8x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^10,x]
[Out] -1/9*a/x^9 - b/(8*x^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/x^10,x]
[Out] IntegrateAlgebraic[(a + b*x)/x^10, x]
```

fricas [A] time = 0.83, size = 13, normalized size = 0.76

$$-\frac{9\,bx+8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="fricas")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

giac [A] time = 1.00, size = 13, normalized size = 0.76

$$-\frac{9\,bx + 8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="giac")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{b}{8x^8} - \frac{a}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^10,x)
[Out] -1/9*a/x^9-1/8*b/x^8
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$-\frac{9\,bx + 8\,a}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^10,x, algorithm="maxima")
[Out] -1/72*(9*b*x + 8*a)/x^9
```

mupad [B] time = 0.03, size = 13, normalized size = 0.76

$$-\frac{8\,a + 9\,b\,x}{72\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^10,x)
[Out] -(8*a + 9*b*x)/(72*x^9)
```

sympy [A] time = 0.21, size = 14, normalized size = 0.82

$$\frac{-8a - 9bx}{72x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**10,x)
[Out] (-8*a - 9*b*x)/(72*x**9)
```

3.251 $\int \frac{1}{x^{10}}\,dx$

Optimal. Leaf size=7

$$-\frac{1}{9x^9}$$

Rubi [A] time = 0.00, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.333, Rules used = {30}

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-10),x]
```

```
[Out] -1/(9*x^9)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^{10}}\,dx = -\frac{1}{9x^9}$$

Mathematica [A] time = 0.00, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-10),x]
```

```
[Out] -1/9*1/x^9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-10),x]
```

```
[Out] IntegrateAlgebraic[x^(-10), x]
```

fricas [A] time = 0.94, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="fricas")
```

```
[Out] -1/9/x^9
```

giac [A] time = 1.04, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="giac")
```

```
[Out] -1/9/x^9
```

maple [A] time = 0.00, size = 6, normalized size = 0.86

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/9/x^9
```

maxima [A] time = 1.32, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10,x, algorithm="maxima")
```

```
[Out] -1/9/x^9
```

mupad [B] time = 0.02, size = 5, normalized size = 0.71

$$-\frac{1}{9\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10,x)
```

```
[Out] -1/(9*x^9)
```

sympy [A] time = 0.07, size = 7, normalized size = 1.00

$$-\frac{1}{9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10,x)
```

```
[Out] -1/(9*x**9)
```

3.252 $\int \frac{1}{x^{10}(a+bx)}\, dx$

Optimal. Leaf size=134

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Rubi [A] time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/(9*a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5*a
^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*x)
 - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^{10}(a+bx)}\,dx = \int \left(\frac{1}{ax^{10}}-\frac{b}{a^2x^9}+\frac{b^2}{a^3x^8}-\frac{b^3}{a^4x^7}+\frac{b^4}{a^5x^6}-\frac{b^5}{a^6x^5}+\frac{b^6}{a^7x^4}-\frac{b^7}{a^8x^3}+\frac{b^8}{a^9x^2}-\frac{b^9}{a^{10}x}+\frac{b^{10}}{a^{10}(a+bx)}\right.$$
$$= -\frac{1}{9ax^9}+\frac{b}{8a^2x^8}-\frac{b^2}{7a^3x^7}+\frac{b^3}{6a^4x^6}-\frac{b^4}{5a^5x^5}+\frac{b^5}{4a^6x^4}-\frac{b^6}{3a^7x^3}+\frac{b^7}{2a^8x^2}-\frac{b^8}{a^9x}-\frac{b^9\log(x)}{a^{10}}+$$

Mathematica [A] time = 0.01, size = 134, normalized size = 1.00

$$-\frac{b^9\log(x)}{a^{10}}+\frac{b^9\log(a+bx)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)),x]
```

```
[Out] -1/9*1/(a*x^9) + b/(8*a^2*x^8) - b^2/(7*a^3*x^7) + b^3/(6*a^4*x^6) - b^4/(5
*a^5*x^5) + b^5/(4*a^6*x^4) - b^6/(3*a^7*x^3) + b^7/(2*a^8*x^2) - b^8/(a^9*
x) - (b^9*Log[x])/a^10 + (b^9*Log[a + b*x])/a^10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)), x]
```

fricas [A] time = 0.86, size = 120, normalized size = 0.90

$$\frac{2520\,b^9x^9\log(bx+a)-2520\,b^9x^9\log(x)-2520\,ab^8x^8+1260\,a^2b^7x^7-840\,a^3b^6x^6+630\,a^4b^5x^5-504\,a^5b^4x^4+420\,a^6b^3x^3-360\,a^7b^2x^2+315\,a^8bx-280\,a^9}{2520\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(2520*b^9*x^9*log(b*x + a) - 2520*b^9*x^9*log(x) - 2520*a*b^8*x^8 +
1260*a^2*b^7*x^7 - 840*a^3*b^6*x^6 + 630*a^4*b^5*x^5 - 504*a^5*b^4*x^4 + 42
0*a^6*b^3*x^3 - 360*a^7*b^2*x^2 + 315*a^8*b*x - 280*a^9)/(a^10*x^9)
```

giac [A] time = 1.16, size = 122, normalized size = 0.91

$$\frac{b^9\log(|bx+a|)}{a^{10}}-\frac{b^9\log(|x|)}{a^{10}}-\frac{2520\,ab^8x^8-1260\,a^2b^7x^7+840\,a^3b^6x^6-630\,a^4b^5x^5+504\,a^5b^4x^4-420\,a^6b^3x^3+360\,a^7b^2x^2-315\,a^8bx+280\,a^9}{2520\,a^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="giac")
```

```
[Out] b^9*log(abs(b*x + a))/a^10 - b^9*log(abs(x))/a^10 - 1/2520*(2520*a*b^8*x^8
- 1260*a^2*b^7*x^7 + 840*a^3*b^6*x^6 - 630*a^4*b^5*x^5 + 504*a^5*b^4*x^4 -
420*a^6*b^3*x^3 + 360*a^7*b^2*x^2 - 315*a^8*b*x + 280*a^9)/(a^10*x^9)
```

maple [A] time = 0.01, size = 119, normalized size = 0.89

$$-\frac{b^9\ln(x)}{a^{10}}+\frac{b^9\ln(bx+a)}{a^{10}}-\frac{b^8}{a^9x}+\frac{b^7}{2a^8x^2}-\frac{b^6}{3a^7x^3}+\frac{b^5}{4a^6x^4}-\frac{b^4}{5a^5x^5}+\frac{b^3}{6a^4x^6}-\frac{b^2}{7a^3x^7}+\frac{b}{8a^2x^8}-\frac{1}{9a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a),x)
```

```
[Out] -1/9/a/x^9+1/8*b/a^2/x^8-1/7*b^2/a^3/x^7+1/6*b^3/a^4/x^6-1/5*b^4/a^5/x^5+1/
4*b^5/a^6/x^4-1/3*b^6/a^7/x^3+1/2*b^7/a^8/x^2-b^8/a^9/x-b^9*ln(x)/a^10+b^9*
ln(b*x+a)/a^10
```

maxima [A] time = 1.35, size = 117, normalized size = 0.87

$$\frac{b^9\log(bx+a)}{a^{10}}-\frac{b^9\log(x)}{a^{10}}-\frac{2520\,b^8x^8-1260\,ab^7x^7+840\,a^2b^6x^6-630\,a^3b^5x^5+504\,a^4b^4x^4-420\,a^5b^3x^3+360\,a^6b^2x^2-315\,a^7bx+280\,a^8}{2520\,a^9x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^9*log(b*x + a)/a^10 - b^9*log(x)/a^10 - 1/2520*(2520*b^8*x^8 - 1260*a*b^7
*x^7 + 840*a^2*b^6*x^6 - 630*a^3*b^5*x^5 + 504*a^4*b^4*x^4 - 420*a^5*b^3*x^
3 + 360*a^6*b^2*x^2 - 315*a^7*b*x + 280*a^8)/(a^9*x^9)
```

mupad [B] time = 0.13, size = 114, normalized size = 0.85

$$-\frac{280\,a^9+2520\,a\,b^8\,x^8-5040\,b^9\,x^9\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)+360\,a^7\,b^2\,x^2-420\,a^6\,b^3\,x^3+504\,a^5\,b^4\,x^4-630\,a^4\,b^5\,x^5+840\,a^3\,b^6\,x^6-1260\,a^2\,b^7\,x^7-315\,a^8\,b\,x}{2520\,a^{10}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)),x)
```

```
[Out] -(280*a^9 + 2520*a*b^8*x^8 - 5040*b^9*x^9*atanh((2*b*x)/a + 1) + 360*a^7*b^
2*x^2 - 420*a^6*b^3*x^3 + 504*a^5*b^4*x^4 - 630*a^4*b^5*x^5 + 840*a^3*b^6*x
^6 - 1260*a^2*b^7*x^7 - 315*a^8*b*x)/(2520*a^10*x^9)
```

sympy [A] time = 0.41, size = 116, normalized size = 0.87

$$\frac{-280a^8+315a^7bx-360a^6b^2x^2+420a^5b^3x^3-504a^4b^4x^4+630a^3b^5x^5-840a^2b^6x^6+1260ab^7x^7-2520b^8x^8}{2520a^9x^9}+\frac{b^9\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a),x)
```

```
[Out] (-280*a**8 + 315*a**7*b*x - 360*a**6*b**2*x**2 + 420*a**5*b**3*x**3 - 504*a
**4*b**4*x**4 + 630*a**3*b**5*x**5 - 840*a**2*b**6*x**6 + 1260*a*b**7*x**7
- 2520*b**8*x**8)/(2520*a**9*x**9) + b**9*(-log(x) + log(a/b + x))/a**10
```

3.253 $\int \frac{1}{x^{10}(a+bx)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}+\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x}$$

Rubi [A] time = 0.09, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4b^7}{a^9x^2}-\frac{7b^6}{3a^8x^3}+\frac{3b^5}{2a^7x^4}-\frac{b^4}{a^6x^5}+\frac{2b^3}{3a^5x^6}-\frac{3b^2}{7a^4x^7}-\frac{b^9}{a^{10}(a+bx)}-\frac{9b^8}{a^{10}x}-\frac{10b^9\log(x)}{a^{11}}+\frac{10b^9\log(a+bx)}{a^{11}}+\frac{b}{4a^3x^8}-\frac{1}{9a^2x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/(9*a^2*x^9) + b/(4*a^3*x^8) - (3*b^2)/(7*a^4*x^7) + (2*b^3)/(3*a^5*x^6)
- b^4/(a^6*x^5) + (3*b^5)/(2*a^7*x^4) - (7*b^6)/(3*a^8*x^3) + (4*b^7)/(a^9*
x^2) - (9*b^8)/(a^10*x) - b^9/(a^10*(a + b*x)) - (10*b^9*Log[x])/a^11 + (10
*b^9*Log[a + b*x])/a^11
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^2}\,dx &= \int \left(\frac{1}{a^2x^{10}}-\frac{2b}{a^3x^9}+\frac{3b^2}{a^4x^8}-\frac{4b^3}{a^5x^7}+\frac{5b^4}{a^6x^6}-\frac{6b^5}{a^7x^5}+\frac{7b^6}{a^8x^4}-\frac{8b^7}{a^9x^3}+\frac{9b^8}{a^{10}x^2}-\frac{10b^9}{a^{11}x}+\frac{b}{a^{10}(a}\right.\\ &= -\frac{1}{9a^2x^9}+\frac{b}{4a^3x^8}-\frac{3b^2}{7a^4x^7}+\frac{2b^3}{3a^5x^6}-\frac{b^4}{a^6x^5}+\frac{3b^5}{2a^7x^4}-\frac{7b^6}{3a^8x^3}+\frac{4b^7}{a^9x^2}-\frac{9b^8}{a^{10}x}-\frac{b^9}{a^{10}(a+bx}\end{aligned}$$

Mathematica [A] time = 0.09, size = 134, normalized size = 0.92

$$-\frac{\frac{a\left(28a^9-35a^8bx+45a^7b^2x^2-60a^6b^3x^3+84a^5b^4x^4-126a^4b^5x^5+210a^3b^6x^6-420a^2b^7x^7+1260ab^8x^8+2520b^9x^9\right)}{x^9(a+bx)}-2520b^9\log(a+bx)+2520b^9\log(x)}{252a^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^2),x]
```

```
[Out] -1/252*((a*(28*a^9 - 35*a^8*b*x + 45*a^7*b^2*x^2 - 60*a^6*b^3*x^3 + 84*a^5*
b^4*x^4 - 126*a^4*b^5*x^5 + 210*a^3*b^6*x^6 - 420*a^2*b^7*x^7 + 1260*a*b^8*
x^8 + 2520*b^9*x^9))/(x^9*(a + b*x)) + 2520*b^9*Log[x] - 2520*b^9*Log[a + b
*x])/a^11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)^2}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^10*(a + b*x)^2),x]

[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)^2), x]

fricas [A] time = 0.74, size = 163, normalized size = 1.12

$$-\frac{2520\,ab^{9}x^{9}+1260\,a^{2}b^{8}x^{8}-420\,a^{3}b^{7}x^{7}+210\,a^{4}b^{6}x^{6}-126\,a^{5}b^{5}x^{5}+84\,a^{6}b^{4}x^{4}-60\,a^{7}b^{3}x^{3}+45\,a^{8}b^{2}x^{2}-35\,a^{9}bx+28\,a^{10}-2520\left(b^{10}x^{10}+ab^{9}x^{9}\right)\log\left(bx+a\right)+2520\left(b^{10}x^{10}+ab^{9}x^{9}\right)\log\left(x\right)}{252\left(a^{11}bx^{10}+a^{12}x^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="fricas")

[Out] -1/252*(2520*a*b^9*x^9 + 1260*a^2*b^8*x^8 - 420*a^3*b^7*x^7 + 210*a^4*b^6*x^6 - 126*a^5*b^5*x^5 + 84*a^6*b^4*x^4 - 60*a^7*b^3*x^3 + 45*a^8*b^2*x^2 - 35*a^9*b*x + 28*a^10 - 2520*(b^10*x^10 + a*b^9*x^9)*log(b*x + a) + 2520*(b^10*x^10 + a*b^9*x^9)*log(x))/(a^11*b*x^10 + a^12*x^9)

giac [A] time = 1.28, size = 180, normalized size = 1.23

$$-\frac{10\,b^{9}\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^{11}}-\frac{b^{9}}{(bx+a)a^{10}}-\frac{\frac{41481\,ab^{9}}{bx+a}-\frac{155844\,a^{2}b^{9}}{(bx+a)^{2}}+\frac{337176\,a^{3}b^{9}}{(bx+a)^{3}}-\frac{460404\,a^{4}b^{9}}{(bx+a)^{4}}+\frac{407484\,a^{5}b^{9}}{(bx+a)^{5}}-\frac{229320\,a^{6}b^{9}}{(bx+a)^{6}}+\frac{75600\,a^{7}b^{9}}{(bx+a)^{7}}-\frac{11340\,a^{8}b^{9}}{(bx+a)^{8}}-4861\,b^{9}}{252\,a^{11}\left(\frac{a}{bx+a}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="giac")

[Out] -10*b^9*log(abs(-a/(b*x + a) + 1))/a^11 - b^9/((b*x + a)*a^10) - 1/252*(41481*a*b^9/(b*x + a) - 155844*a^2*b^9/(b*x + a)^2 + 337176*a^3*b^9/(b*x + a)^3 - 460404*a^4*b^9/(b*x + a)^4 + 407484*a^5*b^9/(b*x + a)^5 - 229320*a^6*b^9/(b*x + a)^6 + 75600*a^7*b^9/(b*x + a)^7 - 11340*a^8*b^9/(b*x + a)^8 - 4861*b^9)/(a^11*(a/(b*x + a) - 1)^9)

maple [A] time = 0.01, size = 135, normalized size = 0.92

$$-\frac{b^{9}}{(bx+a)\,a^{10}}-\frac{10b^{9}\ln(x)}{a^{11}}+\frac{10b^{9}\ln(bx+a)}{a^{11}}-\frac{9b^{8}}{a^{10}x}+\frac{4b^{7}}{a^{9}x^{2}}-\frac{7b^{6}}{3a^{8}x^{3}}+\frac{3b^{5}}{2a^{7}x^{4}}-\frac{b^{4}}{a^{6}x^{5}}+\frac{2b^{3}}{3a^{5}x^{6}}-\frac{3b^{2}}{7a^{4}x^{7}}+\frac{b}{4a^{3}x^{8}}-\frac{1}{9a^{2}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^10/(b*x+a)^2,x)

[Out] -1/9/a^2/x^9+1/4*b/a^3/x^8-3/7*b^2/a^4/x^7+2/3*b^3/a^5/x^6-b^4/a^6/x^5+3/2*b^5/a^7/x^4-7/3*b^6/a^8/x^3+4*b^7/a^9/x^2-9*b^8/a^10/x-b^9/a^10/(b*x+a)-10*b^9*ln(x)/a^11+10*b^9*ln(b*x+a)/a^11

maxima [A] time = 1.40, size = 141, normalized size = 0.97

$$-\frac{2520\,b^{9}x^{9}+1260\,ab^{8}x^{8}-420\,a^{2}b^{7}x^{7}+210\,a^{3}b^{6}x^{6}-126\,a^{4}b^{5}x^{5}+84\,a^{5}b^{4}x^{4}-60\,a^{6}b^{3}x^{3}+45\,a^{7}b^{2}x^{2}-35\,a^{8}bx+28\,a^{9}}{252\left(a^{10}bx^{10}+a^{11}x^{9}\right)}+\frac{10\,b^{9}\log\left(bx+a\right)}{a^{11}}-\frac{10\,b^{9}\log\left(x\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^10/(b*x+a)^2,x, algorithm="maxima")

[Out] -1/252*(2520*b^9*x^9 + 1260*a*b^8*x^8 - 420*a^2*b^7*x^7 + 210*a^3*b^6*x^6 - 126*a^4*b^5*x^5 + 84*a^5*b^4*x^4 - 60*a^6*b^3*x^3 + 45*a^7*b^2*x^2 - 35*a^8*b*x + 28*a^9)/(a^10*b*x^10 + a^11*x^9) + 10*b^9*log(b*x + a)/a^11 - 10*b^9*log(x)/a^11

mupad [B] time = 0.08, size = 135, normalized size = 0.92

$$\frac{20\,b^{9}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}}-\frac{\frac{1}{9\,a}+\frac{5\,b^{2}\,x^{2}}{28\,a^{3}}-\frac{5\,b^{3}\,x^{3}}{21\,a^{4}}+\frac{b^{4}\,x^{4}}{3\,a^{5}}-\frac{b^{5}\,x^{5}}{2\,a^{6}}+\frac{5\,b^{6}\,x^{6}}{6\,a^{7}}-\frac{5\,b^{7}\,x^{7}}{3\,a^{8}}+\frac{5\,b^{8}\,x^{8}}{a^{9}}+\frac{10\,b^{9}\,x^{9}}{a^{10}}-\frac{5\,b\,x}{36\,a^{2}}}{b\,x^{10}+a\,x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^2),x)
```

```
[Out] (20*b^9*atanh((2*b*x)/a + 1))/a^11 - (1/(9*a) + (5*b^2*x^2)/(28*a^3) - (5*b
^3*x^3)/(21*a^4) + (b^4*x^4)/(3*a^5) - (b^5*x^5)/(2*a^6) + (5*b^6*x^6)/(6*a
^7) - (5*b^7*x^7)/(3*a^8) + (5*b^8*x^8)/a^9 + (10*b^9*x^9)/a^10 - (5*b*x)/(
36*a^2))/(a*x^9 + b*x^10)
```

sympy [A] time = 0.61, size = 139, normalized size = 0.95

$$\frac{-28a^9 + 35a^8bx - 45a^7b^2x^2 + 60a^6b^3x^3 - 84a^5b^4x^4 + 126a^4b^5x^5 - 210a^3b^6x^6 + 420a^2b^7x^7 - 1260ab^8x^8 - 2520b^9x^9}{252a^{11}x^9 + 252a^{10}bx^{10}} + \frac{10b^9\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**2,x)
```

```
[Out] (-28*a**9 + 35*a**8*b*x - 45*a**7*b**2*x**2 + 60*a**6*b**3*x**3 - 84*a**5*b
**4*x**4 + 126*a**4*b**5*x**5 - 210*a**3*b**6*x**6 + 420*a**2*b**7*x**7 - 1
260*a*b**8*x**8 - 2520*b**9*x**9)/(252*a**11*x**9 + 252*a**10*b*x**10) + 10
*b**9*(-log(x) + log(a/b + x))/a**11
```

3.254 $\int \frac{1}{x^{10}(a+bx)^3}\,dx$

Optimal. Leaf size=163

$$-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}-\frac{10b^9}{a^{11}(a+bx)}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)^2}+\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}$$

Rubi [A] time = 0.11, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$\frac{18b^7}{a^{10}x^2}-\frac{28b^6}{3a^9x^3}+\frac{21b^5}{4a^8x^4}-\frac{3b^4}{a^7x^5}+\frac{5b^3}{3a^6x^6}-\frac{6b^2}{7a^5x^7}-\frac{10b^9}{a^{11}(a+bx)}-\frac{b^9}{2a^{10}(a+bx)^2}-\frac{45b^8}{a^{11}x}-\frac{55b^9\log(x)}{a^{12}}+\frac{55b^9\log(a+bx)}{a^{12}}+\frac{3b}{8a^4x^8}-\frac{1}{9a^3x^9}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/(9*a^3*x^9) + (3*b)/(8*a^4*x^8) - (6*b^2)/(7*a^5*x^7) + (5*b^3)/(3*a^6*x
^6) - (3*b^4)/(a^7*x^5) + (21*b^5)/(4*a^8*x^4) - (28*b^6)/(3*a^9*x^3) + (18
*b^7)/(a^10*x^2) - (45*b^8)/(a^11*x) - b^9/(2*a^10*(a + b*x)^2) - (10*b^9)/
(a^11*(a + b*x)) - (55*b^9*Log[x])/a^12 + (55*b^9*Log[a + b*x])/a^12
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{10}(a+bx)^3}\,dx &= \int \left(\frac{1}{a^3x^{10}}-\frac{3b}{a^4x^9}+\frac{6b^2}{a^5x^8}-\frac{10b^3}{a^6x^7}+\frac{15b^4}{a^7x^6}-\frac{21b^5}{a^8x^5}+\frac{28b^6}{a^9x^4}-\frac{36b^7}{a^{10}x^3}+\frac{45b^8}{a^{11}x^2}-\frac{55b^9}{a^{12}x}+\frac{b^{10}}{a^{10}(a+\right.\\ &= -\frac{1}{9a^3x^9}+\frac{3b}{8a^4x^8}-\frac{6b^2}{7a^5x^7}+\frac{5b^3}{3a^6x^6}-\frac{3b^4}{a^7x^5}+\frac{21b^5}{4a^8x^4}-\frac{28b^6}{3a^9x^3}+\frac{18b^7}{a^{10}x^2}-\frac{45b^8}{a^{11}x}-\frac{b^9}{2a^{10}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.11, size = 145, normalized size = 0.89

$$-\frac{\frac{a\left(56a^{10}-77a^9bx+110a^8b^2x^2-165a^7b^3x^3+264a^6b^4x^4-462a^5b^5x^5+924a^4b^6x^6-2310a^3b^7x^7+9240a^2b^8x^8+41580ab^9x^9+27720b^{10}x^{10}\right)}{x^9(a+bx)^2}-27720b^9\log(a+bx)+27720b^9\log(x)}{504a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^10*(a + b*x)^3),x]
```

```
[Out] -1/504*((a*(56*a^10 - 77*a^9*b*x + 110*a^8*b^2*x^2 - 165*a^7*b^3*x^3 + 264*
a^6*b^4*x^4 - 462*a^5*b^5*x^5 + 924*a^4*b^6*x^6 - 2310*a^3*b^7*x^7 + 9240*a
^2*b^8*x^8 + 41580*a*b^9*x^9 + 27720*b^10*x^10))/(x^9*(a + b*x)^2) + 27720*
b^9*Log[x] - 27720*b^9*Log[a + b*x])/a^12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{10}(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^10*(a + b*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^10*(a + b*x)^3), x]
```

fricas [A] time = 1.10, size = 207, normalized size = 1.27

$$-\frac{27720\,ab^{10}x^{10}+41580\,a^{2}b^{9}x^{9}+9240\,a^{3}b^{8}x^{8}-2310\,a^{4}b^{7}x^{7}+924\,a^{5}b^{6}x^{6}-462\,a^{6}b^{5}x^{5}+264\,a^{7}b^{4}x^{4}-165\,a^{8}b^{3}x^{3}+110\,a^{9}b^{2}x^{2}-77\,a^{10}bx+56\,a^{11}-27720\,\left(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9}\right)\log\left(bx+a\right)+27720\,\left(b^{11}x^{11}+2\,ab^{10}x^{10}+a^{2}b^{9}x^{9}\right)\log\left(x\right)}{504\,\left(a^{12}b^{2}x^{11}+2\,a^{13}bx^{10}+a^{14}x^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/504*(27720*a*b^10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4
*b^7*x^7 + 924*a^5*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^
3*x^3 + 110*a^9*b^2*x^2 - 77*a^10*b*x + 56*a^11 - 27720*(b^11*x^11 + 2*a*b^
10*x^10 + a^2*b^9*x^9)*log(b*x + a) + 27720*(b^11*x^11 + 2*a*b^10*x^10 + a^
2*b^9*x^9)*log(x))/(a^12*b^2*x^11 + 2*a^13*b*x^10 + a^14*x^9)
```

giac [A] time = 1.13, size = 152, normalized size = 0.93

$$\frac{55\,b^{9}\log\left(|bx+a|\right)}{a^{12}}-\frac{55\,b^{9}\log\left(|x|\right)}{a^{12}}-\frac{27720\,ab^{10}x^{10}+41580\,a^{2}b^{9}x^{9}+9240\,a^{3}b^{8}x^{8}-2310\,a^{4}b^{7}x^{7}+924\,a^{5}b^{6}x^{6}-462\,a^{6}b^{5}x^{5}+264\,a^{7}b^{4}x^{4}-165\,a^{8}b^{3}x^{3}+110\,a^{9}b^{2}x^{2}-77\,a^{10}bx+56\,a^{11}}{504\,\left(bx+a\right)^{2}a^{12}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 55*b^9*log(abs(b*x + a))/a^12 - 55*b^9*log(abs(x))/a^12 - 1/504*(27720*a*b^
10*x^10 + 41580*a^2*b^9*x^9 + 9240*a^3*b^8*x^8 - 2310*a^4*b^7*x^7 + 924*a^5
*b^6*x^6 - 462*a^6*b^5*x^5 + 264*a^7*b^4*x^4 - 165*a^8*b^3*x^3 + 110*a^9*b^
2*x^2 - 77*a^10*b*x + 56*a^11)/((b*x + a)^2*a^12*x^9)
```

maple [A] time = 0.01, size = 150, normalized size = 0.92

$$-\frac{b^{9}}{2\left(bx+a\right)^{2}a^{10}}-\frac{10b^{9}}{\left(bx+a\right)a^{11}}-\frac{55b^{9}\ln\left(x\right)}{a^{12}}+\frac{55b^{9}\ln\left(bx+a\right)}{a^{12}}-\frac{45b^{8}}{a^{11}x}+\frac{18b^{7}}{a^{10}x^{2}}-\frac{28b^{6}}{3a^{9}x^{3}}+\frac{21b^{5}}{4a^{8}x^{4}}-\frac{3b^{4}}{a^{7}x^{5}}+\frac{5b^{3}}{3a^{6}x^{6}}-\frac{6b^{2}}{7a^{5}x^{7}}+\frac{3b}{8a^{4}x^{8}}-\frac{1}{9a^{3}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^10/(b*x+a)^3,x)
```

```
[Out] -1/9/a^3/x^9+3/8*b/a^4/x^8-6/7*b^2/a^5/x^7+5/3*b^3/a^6/x^6-3*b^4/a^7/x^5+21
/4*b^5/a^8/x^4-28/3*b^6/a^9/x^3+18*b^7/a^10/x^2-45*b^8/a^11/x-1/2*b^9/a^10/
(b*x+a)^2-10*b^9/a^11/(b*x+a)-55*b^9*ln(x)/a^12+55*b^9*ln(b*x+a)/a^12
```

maxima [A] time = 1.43, size = 163, normalized size = 1.00

$$-\frac{27720\,b^{10}x^{10}+41580\,ab^{9}x^{9}+9240\,a^{2}b^{8}x^{8}-2310\,a^{3}b^{7}x^{7}+924\,a^{4}b^{6}x^{6}-462\,a^{5}b^{5}x^{5}+264\,a^{6}b^{4}x^{4}-165\,a^{7}b^{3}x^{3}+110\,a^{8}b^{2}x^{2}-77\,a^{9}bx+56\,a^{10}}{504\,\left(a^{11}b^{2}x^{11}+2\,a^{12}bx^{10}+a^{13}x^{9}\right)}+\frac{55\,b^{9}\log\left(bx+a\right)}{a^{12}}-\frac{55\,b^{9}\log\left(x\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/504*(27720*b^10*x^10 + 41580*a*b^9*x^9 + 9240*a^2*b^8*x^8 - 2310*a^3*b^7
*x^7 + 924*a^4*b^6*x^6 - 462*a^5*b^5*x^5 + 264*a^6*b^4*x^4 - 165*a^7*b^3*x^
3 + 110*a^8*b^2*x^2 - 77*a^9*b*x + 56*a^10)/(a^11*b^2*x^11 + 2*a^12*b*x^10
+ a^13*x^9) + 55*b^9*log(b*x + a)/a^12 - 55*b^9*log(x)/a^12
```

mupad [B] time = 0.23, size = 157, normalized size = 0.96

$$\frac{110\,b^{9}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{12}}-\frac{\frac{1}{9\,a}+\frac{55\,b^{2}\,x^{2}}{252\,a^{3}}-\frac{55\,b^{3}\,x^{3}}{168\,a^{4}}+\frac{11\,b^{4}\,x^{4}}{21\,a^{5}}-\frac{11\,b^{5}\,x^{5}}{12\,a^{6}}+\frac{11\,b^{6}\,x^{6}}{6\,a^{7}}-\frac{55\,b^{7}\,x^{7}}{12\,a^{8}}+\frac{55\,b^{8}\,x^{8}}{3\,a^{9}}+\frac{165\,b^{9}\,x^{9}}{2\,a^{10}}+\frac{55\,b^{10}\,x^{10}}{a^{11}}-\frac{11\,b\,x}{72\,a^{2}}}{a^{2}\,x^{9}+2\,a\,b\,x^{10}+b^{2}\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^10*(a + b*x)^3),x)
```

```
[Out] (110*b^9*atanh((2*b*x)/a + 1))/a^12 - (1/(9*a) + (55*b^2*x^2)/(252*a^3) - (
55*b^3*x^3)/(168*a^4) + (11*b^4*x^4)/(21*a^5) - (11*b^5*x^5)/(12*a^6) + (11
*b^6*x^6)/(6*a^7) - (55*b^7*x^7)/(12*a^8) + (55*b^8*x^8)/(3*a^9) + (165*b^9
*x^9)/(2*a^10) + (55*b^10*x^10)/a^11 - (11*b*x)/(72*a^2))/(a^2*x^9 + b^2*x^
11 + 2*a*b*x^10)
```

sympy [A] time = 0.68, size = 163, normalized size = 1.00

$$\frac{-56a^{10} + 77a^9bx - 110a^8b^2x^2 + 165a^7b^3x^3 - 264a^6b^4x^4 + 462a^5b^5x^5 - 924a^4b^6x^6 + 2310a^3b^7x^7 - 9240a^2b^8x^8 - 41580ab^9x^9 - 27720b^{10}x^{10}}{504a^{13}x^9 + 1008a^{12}bx^{10} + 504a^{11}b^2x^{11}} + \frac{55b^9\left(-\log(x) + \log\left(\frac{a}{b} + x\right)\right)}{a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**10/(b*x+a)**3,x)
```

```
[Out] (-56*a**10 + 77*a**9*b*x - 110*a**8*b**2*x**2 + 165*a**7*b**3*x**3 - 264*a*
*6*b**4*x**4 + 462*a**5*b**5*x**5 - 924*a**4*b**6*x**6 + 2310*a**3*b**7*x**
7 - 9240*a**2*b**8*x**8 - 41580*a*b**9*x**9 - 27720*b**10*x**10)/(504*a**13
*x**9 + 1008*a**12*b*x**10 + 504*a**11*b**2*x**11) + 55*b**9*(-log(x) + log
(a/b + x))/a**12
```

3.255 $\int \frac{1}{x(2+3x)} dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(2+3x)} dx &= \frac{1}{2}\int \frac{1}{x} dx - \frac{3}{2}\int \frac{1}{2+3x} dx \\ &= \frac{\log(x)}{2} - \frac{1}{2}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{2} - \frac{1}{2}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(2 + 3*x)),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(2+3x)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(2 + 3*x)),x]
[Out] IntegrateAlgebraic[1/(x*(2 + 3*x)), x]
```

fricas [A] time = 0.80, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="fricas")
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

giac [A] time = 1.15, size = 15, normalized size = 0.88

$$-\frac{1}{2}\,\log\left(|3\,x+2|\right)+\frac{1}{2}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="giac")
[Out] -1/2*log(abs(3*x + 2)) + 1/2*log(abs(x))
```

maple [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{\ln\left(x\right)}{2}-\frac{\ln\left(3x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2+3*x),x)
[Out] 1/2*ln(x)-1/2*ln(2+3*x)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$-\frac{1}{2}\,\log\left(3\,x+2\right)+\frac{1}{2}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x, algorithm="maxima")
[Out] -1/2*log(3*x + 2) + 1/2*log(x)
```

mupad [B] time = 0.17, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{2}{x}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(3*x + 2)),x)
[Out] -log(2/x + 3)/2
```

sympy [A] time = 0.12, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{2}-\frac{\log\left(x+\frac{2}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2+3*x),x)
[Out] log(x)/2 - log(x + 2/3)/2
```

3.256 $\int \frac{1}{x(4+6x)} dx$

Optimal. Leaf size=17

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)} dx &= \frac{1}{4}\int \frac{1}{x} dx - \frac{3}{2}\int \frac{1}{4+6x} dx \\ &= \frac{\log(x)}{4} - \frac{1}{4}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{\log(x)}{4} - \frac{1}{4}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)),x]
```

```
[Out] Log[x]/4 - Log[2 + 3*x]/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)),x]
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)), x]
```

fricas [A] time = 1.16, size = 13, normalized size = 0.76

$$-\frac{1}{4}\log\left(3x+2\right)+\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="fricas")
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

giac [A] time = 1.01, size = 15, normalized size = 0.88

$$-\frac{1}{4}\log\left(|3x+2|\right)+\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="giac")
[Out] -1/4*log(abs(3*x + 2)) + 1/4*log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\ln(x)}{4}-\frac{\ln\left(3x+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x),x)
[Out] 1/4*ln(x)-1/4*ln(3*x+2)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.76

$$-\frac{1}{4}\log\left(3x+2\right)+\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x, algorithm="maxima")
[Out] -1/4*log(3*x + 2) + 1/4*log(x)
```

mupad [B] time = 0.14, size = 10, normalized size = 0.59

$$-\frac{\ln\left(\frac{4}{x}+6\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)),x)
[Out] -log(4/x + 6)/4
```

sympy [A] time = 0.12, size = 12, normalized size = 0.71

$$\frac{\log\left(x\right)}{4}-\frac{\log\left(x+\frac{2}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x),x)
[Out] log(x)/4 - log(x + 2/3)/4
```

3.257 $\int \frac{1}{x^2(4+6x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/(4*x) - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)}\,dx &= \int \left(\frac{1}{4x^2}-\frac{3}{8x}+\frac{9}{8(2+3x)}\right)dx\\ &= -\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{1}{4x}-\frac{3\log(x)}{8}+\frac{3}{8}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)),x]
```

```
[Out] -1/4*1/x - (3*Log[x])/8 + (3*Log[2 + 3*x])/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)), x]
```

fricas [A] time = 0.59, size = 21, normalized size = 0.88

$$\frac{3\,x\log\left(3\,x+2\right)-3\,x\log\left(x\right)-2}{8\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*x*log(3*x + 2) - 3*x*log(x) - 2)/x
```

giac [A] time = 1.01, size = 20, normalized size = 0.83

$$-\frac{1}{4x}+\frac{3}{8}\log\left(|3x+2|\right)-\frac{3}{8}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/4/x + 3/8*log(abs(3*x + 2)) - 3/8*log(abs(x))
```

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$-\frac{3\ln(x)}{8}+\frac{3\ln\left(3x+2\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x),x)
```

```
[Out] -1/4/x-3/8*ln(x)+3/8*ln(3*x+2)
```

maxima [A] time = 1.38, size = 18, normalized size = 0.75

$$-\frac{1}{4x}+\frac{3}{8}\log\left(3x+2\right)-\frac{3}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/4/x + 3/8*log(3*x + 2) - 3/8*log(x)
```

mupad [B] time = 0.05, size = 18, normalized size = 0.75

$$-\frac{3\ln\left(\frac{x}{6x+4}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)),x)
```

```
[Out] - (3*log(x/(6*x + 4)))/8 - 1/(4*x)
```

sympy [A] time = 0.14, size = 20, normalized size = 0.83

$$-\frac{3\log\left(x\right)}{8}+\frac{3\log\left(x+\frac{2}{3}\right)}{8}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x),x)
```

```
[Out] -3*log(x)/8 + 3*log(x + 2/3)/8 - 1/(4*x)
```

3.258 $\int \frac{1}{x^3(4+6x)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)),x]
[Out] -1/(8*x^2) + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)}\,dx &= \int \left(\frac{1}{4x^3}-\frac{3}{8x^2}+\frac{9}{16x}-\frac{27}{16(2+3x)}\right)dx\\ &= -\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{1}{8x^2}+\frac{3}{8x}+\frac{9\log(x)}{16}-\frac{9}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)),x]
[Out] -1/8*1/x^2 + 3/(8*x) + (9*Log[x])/16 - (9*Log[2 + 3*x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)),x]
[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)), x]
```

fricas [A] time = 0.95, size = 28, normalized size = 0.90

$$-\frac{9\,x^2\log(3\,x+2)-9\,x^2\log(x)-6\,x+2}{16\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/16*(9*x^2*log(3*x + 2) - 9*x^2*log(x) - 6*x + 2)/x^2
```

giac [A] time = 1.07, size = 25, normalized size = 0.81

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log\left(|3\,x+2|\right)+\frac{9}{16}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(abs(3*x + 2)) + 9/16*log(abs(x))
```

maple [A] time = 0.01, size = 24, normalized size = 0.77

$$\frac{9\ln(x)}{16}-\frac{9\ln\left(3x+2\right)}{16}+\frac{3}{8x}-\frac{1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x),x)
```

```
[Out] -1/8/x^2+3/8/x+9/16*ln(x)-9/16*ln(3*x+2)
```

maxima [A] time = 1.29, size = 23, normalized size = 0.74

$$\frac{3\,x-1}{8\,x^2}-\frac{9}{16}\,\log\left(3\,x+2\right)+\frac{9}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/8*(3*x - 1)/x^2 - 9/16*log(3*x + 2) + 9/16*log(x)
```

mupad [B] time = 0.04, size = 18, normalized size = 0.58

$$\frac{\frac{3\,x}{8}-\frac{1}{8}}{x^2}-\frac{9\,\mathrm{atanh}\left(3\,x+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)),x)
```

```
[Out] ((3*x)/8 - 1/8)/x^2 - (9*atanh(3*x + 1))/8
```

sympy [A] time = 0.15, size = 26, normalized size = 0.84

$$\frac{9\log\left(x\right)}{16}-\frac{9\log\left(x+\frac{2}{3}\right)}{16}+\frac{3x-1}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x),x)
```

```
[Out] 9*log(x)/16 - 9*log(x + 2/3)/16 + (3*x - 1)/(8*x**2)
```

3.259 $\int \frac{1}{x^4(4+6x)} \, dx$

Optimal. Leaf size=38

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {44}

$$\frac{3}{16x^2}-\frac{1}{12x^3}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/(12*x^3) + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)} \, dx &= \int \left(\frac{1}{4x^4}-\frac{3}{8x^3}+\frac{9}{16x^2}-\frac{27}{32x}+\frac{81}{32(2+3x)}\right) dx\\ &= -\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$-\frac{1}{12x^3}+\frac{3}{16x^2}-\frac{9}{16x}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)),x]
```

```
[Out] -1/12*1/x^3 + 3/(16*x^2) - 9/(16*x) - (27*Log[x])/32 + (27*Log[2 + 3*x])/32
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)), x]
```

fricas [A] time = 0.57, size = 33, normalized size = 0.87

$$\frac{81\,x^3\log\left(3\,x+2\right)-81\,x^3\log(x)-54\,x^2+18\,x-8}{96\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="fricas")
```

```
[Out] 1/96*(81*x^3*log(3*x + 2) - 81*x^3*log(x) - 54*x^2 + 18*x - 8)/x^3
```

giac [A] time = 1.03, size = 30, normalized size = 0.79

$$-\frac{27\,x^2 - 9\,x + 4}{48\,x^3} + \frac{27}{32}\,\log\left(|3\,x + 2|\right) - \frac{27}{32}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="giac")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(abs(3*x + 2)) - 27/32*log(abs(x))
```

maple [A] time = 0.01, size = 29, normalized size = 0.76

$$-\frac{27\ln(x)}{32} + \frac{27\ln\left(3x + 2\right)}{32} - \frac{9}{16x} + \frac{3}{16x^2} - \frac{1}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x),x)
```

```
[Out] -1/12/x^3+3/16/x^2-9/16/x-27/32*ln(x)+27/32*ln(3*x+2)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.74

$$-\frac{27\,x^2 - 9\,x + 4}{48\,x^3} + \frac{27}{32}\,\log\left(3\,x + 2\right) - \frac{27}{32}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x),x, algorithm="maxima")
```

```
[Out] -1/48*(27*x^2 - 9*x + 4)/x^3 + 27/32*log(3*x + 2) - 27/32*log(x)
```

mupad [B] time = 0.09, size = 24, normalized size = 0.63

$$\frac{27\,\mathrm{atanh}\left(3\,x + 1\right)}{16} - \frac{\frac{9\,x^2}{16} - \frac{3\,x}{16} + \frac{1}{12}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((9*x^2)/16 - (3*x)/16 + 1/12)/x^3
```

sympy [A] time = 0.16, size = 31, normalized size = 0.82

$$-\frac{27\log\left(x\right)}{32} + \frac{27\log\left(x + \frac{2}{3}\right)}{32} + \frac{-27x^2 + 9x - 4}{48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x),x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-27*x**2 + 9*x - 4)/(48*x**3)
```

3.260 $\int \frac{1}{x^5(4+6x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{9}{32x^2}+\frac{1}{8x^3}-\frac{1}{16x^4}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/(16*x^4) + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)}\,dx &= \int \left(\frac{1}{4x^5}-\frac{3}{8x^4}+\frac{9}{16x^3}-\frac{27}{32x^2}+\frac{81}{64x}-\frac{243}{64(2+3x)}\right)dx\\ &= -\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 1.00

$$-\frac{1}{16x^4}+\frac{1}{8x^3}-\frac{9}{32x^2}+\frac{27}{32x}+\frac{81\log(x)}{64}-\frac{81}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)),x]
```

```
[Out] -1/16*1/x^4 + 1/(8*x^3) - 9/(32*x^2) + 27/(32*x) + (81*Log[x])/64 - (81*Log
[2 + 3*x])/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)), x]
```

fricas [A] time = 0.74, size = 38, normalized size = 0.84

$$-\frac{81\,x^{4}\log\left(3\,x+2\right)-81\,x^{4}\log\left(x\right)-54\,x^{3}+18\,x^{2}-8\,x+4}{64\,x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="fricas")
```

```
[Out] -1/64*(81*x^4*log(3*x + 2) - 81*x^4*log(x) - 54*x^3 + 18*x^2 - 8*x + 4)/x^4
```

giac [A] time = 1.14, size = 35, normalized size = 0.78

$$\frac{27\,x^{3}-9\,x^{2}+4\,x-2}{32\,x^{4}}-\frac{81}{64}\,\log\left(\left|3\,x+2\right|\right)+\frac{81}{64}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="giac")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(abs(3*x + 2)) + 81/64*log(a
bs(x))
```

maple [A] time = 0.01, size = 34, normalized size = 0.76

$$\frac{81\ln\left(x\right)}{64}-\frac{81\ln\left(3x+2\right)}{64}+\frac{27}{32x}-\frac{9}{32x^{2}}+\frac{1}{8x^{3}}-\frac{1}{16x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x),x)
```

```
[Out] -1/16/x^4+1/8/x^3-9/32/x^2+27/32/x+81/64*ln(x)-81/64*ln(3*x+2)
```

maxima [A] time = 1.32, size = 33, normalized size = 0.73

$$\frac{27\,x^{3}-9\,x^{2}+4\,x-2}{32\,x^{4}}-\frac{81}{64}\,\log\left(3\,x+2\right)+\frac{81}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x),x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^3 - 9*x^2 + 4*x - 2)/x^4 - 81/64*log(3*x + 2) + 81/64*log(x)
```

mupad [B] time = 0.04, size = 28, normalized size = 0.62

$$\frac{\frac{27\,x^{3}}{32}-\frac{9\,x^{2}}{32}+\frac{x}{8}-\frac{1}{16}}{x^{4}}-\frac{81\,\mathrm{atanh}\left(3\,x+1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)),x)
```

```
[Out] (x/8 - (9*x^2)/32 + (27*x^3)/32 - 1/16)/x^4 - (81*atanh(3*x + 1))/32
```

sympy [A] time = 0.17, size = 36, normalized size = 0.80

$$\frac{81\log\left(x\right)}{64}-\frac{81\log\left(x+\frac{2}{3}\right)}{64}+\frac{27x^{3}-9x^{2}+4x-2}{32x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x),x)
```

```
[Out] 81*log(x)/64 - 81*log(x + 2/3)/64 + (27*x**3 - 9*x**2 + 4*x - 2)/(32*x**4)
```

3.261 $\int \frac{1}{x(4+6x)^2} \, dx$

Optimal. Leaf size=28

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{8(3x+2)}+\frac{\log(x)}{16}-\frac{1}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^2),x]
[Out] 1/(8*(2 + 3*x)) + Log[x]/16 - Log[2 + 3*x]/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^2} \, dx &= \int \left(\frac{1}{16x}-\frac{3}{8(2+3x)^2}-\frac{3}{16(2+3x)}\right) dx\\ &= \frac{1}{8(2+3x)}+\frac{\log(x)}{16}-\frac{1}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$\frac{1}{16}\left(\frac{2}{3x+2}+\log(-6x)-\log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^2),x]
[Out] (2/(2 + 3*x) + Log[-6*x] - Log[4 + 6*x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)^2),x]
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)^2), x]
```

fricas [A] time = 0.85, size = 32, normalized size = 1.14

$$-\frac{(3x+2)\log(3x+2)-(3x+2)\log(x)-2}{16(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="fricas")
[Out] -1/16*((3*x + 2)*log(3*x + 2) - (3*x + 2)*log(x) - 2)/(3*x + 2)
```

giac [A] time = 1.11, size = 25, normalized size = 0.89

$$\frac{1}{8\,(3\,x+2)}+\frac{1}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="giac")
[Out] 1/8/(3*x + 2) + 1/16*log(abs(-2/(3*x + 2) + 1))
```

maple [A] time = 0.01, size = 23, normalized size = 0.82

$$\frac{\ln(x)}{16}-\frac{\ln(3x+2)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^2,x)
[Out] 1/8/(3*x+2)+1/16*ln(x)-1/16*ln(3*x+2)
```

maxima [A] time = 1.37, size = 22, normalized size = 0.79

$$\frac{1}{8\,(3\,x+2)}-\frac{1}{16}\,\log\,(3\,x+2)+\frac{1}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^2,x, algorithm="maxima")
[Out] 1/8/(3*x + 2) - 1/16*log(3*x + 2) + 1/16*log(x)
```

mupad [B] time = 0.06, size = 20, normalized size = 0.71

$$\frac{1}{8\,(3\,x+2)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^2),x)
[Out] 1/(8*(3*x + 2)) - log((6*x + 4)/x)/16
```

sympy [A] time = 0.14, size = 19, normalized size = 0.68

$$\frac{\log\,(x)}{16}-\frac{\log\left(x+\frac{2}{3}\right)}{16}+\frac{1}{24x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**2,x)
[Out] log(x)/16 - log(x + 2/3)/16 + 1/(24*x + 16)
```

3.262 $\int \frac{1}{x^2(4+6x)^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{16x}-\frac{3}{16(3x+2)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] -1/(16*x) - 3/(16*(2 + 3*x)) - (3*Log[x])/16 + (3*Log[2 + 3*x])/16
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^2}-\frac{3}{16x}+\frac{9}{16(2+3x)^2}+\frac{9}{16(2+3x)}\right)dx\\ &= -\frac{1}{16x}-\frac{3}{16(2+3x)}-\frac{3\log(x)}{16}+\frac{3}{16}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.89

$$\frac{1}{16}\left(-\frac{1}{x}-\frac{3}{3x+2}-3\log(x)+3\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] (-x^(-1) - 3/(2 + 3*x) - 3*Log[x] + 3*Log[2 + 3*x])/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^2), x]
```

fricas [A] time = 1.19, size = 48, normalized size = 1.37

$$\frac{3\left(3x^2+2x\right)\log\left(3x+2\right)-3\left(3x^2+2x\right)\log(x)-6x-2}{16\left(3x^2+2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*(3*x^2 + 2*x)*log(3*x + 2) - 3*(3*x^2 + 2*x)*log(x) - 6*x - 2)/(3*x
^2 + 2*x)
```

giac [A] time = 0.95, size = 40, normalized size = 1.14

$$-\frac{3}{16\,(3\,x+2)}+\frac{3}{32\left(\frac{2}{3\,x+2}-1\right)}-\frac{3}{16}\,\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] -3/16/(3*x + 2) + 3/32/(2/(3*x + 2) - 1) - 3/16*log(abs(-2/(3*x + 2) + 1))
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$-\frac{3\ln(x)}{16}+\frac{3\ln(3x+2)}{16}-\frac{1}{16x}-\frac{3}{16\,(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^2,x)
```

```
[Out] -1/16/x-3/16/(3*x+2)-3/16*ln(x)+3/16*ln(3*x+2)
```

maxima [A] time = 1.39, size = 31, normalized size = 0.89

$$-\frac{3\,x+1}{8\left(3\,x^2+2\,x\right)}+\frac{3}{16}\,\log\left(3\,x+2\right)-\frac{3}{16}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x + 1)/(3*x^2 + 2*x) + 3/16*log(3*x + 2) - 3/16*log(x)
```

mupad [B] time = 0.09, size = 34, normalized size = 0.97

$$\frac{3\,\ln\left(\frac{6\,x+4}{x}\right)}{16}-\frac{3}{4\,\left(6\,x+4\right)}-\frac{1}{4\,x\,\left(6\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^2),x)
```

```
[Out] (3*log((6*x + 4)/x))/16 - 3/(4*(6*x + 4)) - 1/(4*x*(6*x + 4))
```

sympy [A] time = 0.15, size = 31, normalized size = 0.89

$$\frac{-3x-1}{24x^2+16x}-\frac{3\log\left(x\right)}{16}+\frac{3\log\left(x+\frac{2}{3}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**2,x)
```

```
[Out] (-3*x - 1)/(24*x**2 + 16*x) - 3*log(x)/16 + 3*log(x + 2/3)/16
```

3.263 $\int \frac{1}{x^3(4+6x)^2} dx$

Optimal. Leaf size=42

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(3x+2)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^2),x]

[Out] -1/(32*x^2) + 3/(16*x) + 9/(32*(2 + 3*x)) + (27*Log[x])/64 - (27*Log[2 + 3*
x])/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^3}-\frac{3}{16x^2}+\frac{27}{64x}-\frac{27}{32(2+3x)^2}-\frac{81}{64(2+3x)}\right)dx\\ &= -\frac{1}{32x^2}+\frac{3}{16x}+\frac{9}{32(2+3x)}+\frac{27\log(x)}{64}-\frac{27}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.86

$$\frac{1}{64}\left(-\frac{2}{x^2}+\frac{12}{x}+\frac{18}{3x+2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^2),x]

[Out] (-2/x^2 + 12/x + 18/(2 + 3*x) + 27*Log[x] - 27*Log[2 + 3*x])/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^2),x]

[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^2), x]
```

fricas [A] time = 1.07, size = 59, normalized size = 1.40

$$\frac{54\,x^2 - 27\,\left(3\,x^3 + 2\,x^2\right)\log\left(3\,x + 2\right) + 27\,\left(3\,x^3 + 2\,x^2\right)\log\left(x\right) + 18\,x - 4}{64\,\left(3\,x^3 + 2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(54*x^2 - 27*(3*x^3 + 2*x^2)*log(3*x + 2) + 27*(3*x^3 + 2*x^2)*log(x)
+ 18*x - 4)/(3*x^3 + 2*x^2)
```

giac [A] time = 0.94, size = 51, normalized size = 1.21

$$\frac{9}{32\,(3\,x + 2)} - \frac{9\left(\frac{12}{3\,x+2} - 5\right)}{128\left(\frac{2}{3\,x+2} - 1\right)^2} + \frac{27}{64}\,\log\left(\left|-\frac{2}{3\,x + 2} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 9/32/(3*x + 2) - 9/128*(12/(3*x + 2) - 5)/(2/(3*x + 2) - 1)^2 + 27/64*log(a
bs(-2/(3*x + 2) + 1))
```

maple [A] time = 0.01, size = 33, normalized size = 0.79

$$\frac{27\ln(x)}{64} - \frac{27\ln(3x + 2)}{64} + \frac{3}{16x} - \frac{1}{32x^2} + \frac{9}{32\,(3x + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^2,x)
```

```
[Out] -1/32/x^2+3/16/x+9/32/(3*x+2)+27/64*ln(x)-27/64*ln(3*x+2)
```

maxima [A] time = 1.31, size = 38, normalized size = 0.90

$$\frac{27\,x^2 + 9\,x - 2}{32\,\left(3\,x^3 + 2\,x^2\right)} - \frac{27}{64}\,\log\left(3\,x + 2\right) + \frac{27}{64}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(27*x^2 + 9*x - 2)/(3*x^3 + 2*x^2) - 27/64*log(3*x + 2) + 27/64*log(x)
```

mupad [B] time = 0.04, size = 31, normalized size = 0.74

$$\frac{\frac{9\,x^2}{32} + \frac{3\,x}{32} - \frac{1}{48}}{x^3 + \frac{2\,x^2}{3}} - \frac{27\,\mathrm{atanh}\left(3\,x + 1\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^2),x)
```

```
[Out] ((3*x)/32 + (9*x^2)/32 - 1/48)/((2*x^2)/3 + x^3) - (27*atanh(3*x + 1))/32
```

sympy [A] time = 0.16, size = 36, normalized size = 0.86

$$\frac{27\log\left(x\right)}{64} - \frac{27\log\left(x + \frac{2}{3}\right)}{64} + \frac{27x^2 + 9x - 2}{96x^3 + 64x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**2,x)
```

```
[Out] 27*log(x)/64 - 27*log(x + 2/3)/64 + (27*x**2 + 9*x - 2)/(96*x**3 + 64*x**2)
```

3.264 $\int \frac{1}{x^4(4+6x)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{3}{32x^2}-\frac{1}{48x^3}-\frac{27}{64x}-\frac{27}{64(3x+2)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] -1/(48*x^3) + 3/(32*x^2) - 27/(64*x) - 27/(64*(2 + 3*x)) - (27*Log[x])/32 +
 (27*Log[2 + 3*x])/32
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^4}-\frac{3}{16x^3}+\frac{27}{64x^2}-\frac{27}{32x}+\frac{81}{64(2+3x)^2}+\frac{81}{32(2+3x)}\right)dx\\ &= -\frac{1}{48x^3}+\frac{3}{32x^2}-\frac{27}{64x}-\frac{27}{64(2+3x)}-\frac{27\log(x)}{32}+\frac{27}{32}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.90

$$\frac{1}{192}\left(-\frac{4\left(81x^3+27x^2-6x+2\right)}{x^3(3x+2)}-162\log(x)+162\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] ((-4*(2 - 6*x + 27*x^2 + 81*x^3))/(x^3*(2 + 3*x)) - 162*Log[x] + 162*Log[2
+ 3*x])/192
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^2), x]
```

fricas [A] time = 1.19, size = 64, normalized size = 1.31

$$-\frac{162\,x^{3}+54\,x^{2}-81\,\left(3\,x^{4}+2\,x^{3}\right)\log\left(3\,x+2\right)+81\,\left(3\,x^{4}+2\,x^{3}\right)\log\left(x\right)-12\,x+4}{96\,\left(3\,x^{4}+2\,x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(162*x^3 + 54*x^2 - 81*(3*x^4 + 2*x^3)*log(3*x + 2) + 81*(3*x^4 + 2*x
^3)*log(x) - 12*x + 4)/(3*x^4 + 2*x^3)
```

giac [A] time = 1.23, size = 60, normalized size = 1.22

$$-\frac{27}{64\,(3\,x+2)}-\frac{9\left(\frac{60}{3\,x+2}-\frac{72}{(3\,x+2)^{2}}-13\right)}{128\left(\frac{2}{3\,x+2}-1\right)^{3}}-\frac{27}{32}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] -27/64/(3*x + 2) - 9/128*(60/(3*x + 2) - 72/(3*x + 2)^2 - 13)/(2/(3*x + 2)
- 1)^3 - 27/32*log(abs(-2/(3*x + 2) + 1))
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-\frac{27\ln(x)}{32}+\frac{27\ln(3x+2)}{32}-\frac{27}{64x}+\frac{3}{32x^{2}}-\frac{1}{48x^{3}}-\frac{27}{64\,(3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^2,x)
```

```
[Out] -1/48/x^3+3/32/x^2-27/64/x-27/64/(3*x+2)-27/32*ln(x)+27/32*ln(3*x+2)
```

maxima [A] time = 1.31, size = 43, normalized size = 0.88

$$-\frac{81\,x^{3}+27\,x^{2}-6\,x+2}{48\left(3\,x^{4}+2\,x^{3}\right)}+\frac{27}{32}\log\left(3\,x+2\right)-\frac{27}{32}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] -1/48*(81*x^3 + 27*x^2 - 6*x + 2)/(3*x^4 + 2*x^3) + 27/32*log(3*x + 2) - 27
/32*log(x)
```

mupad [B] time = 0.09, size = 37, normalized size = 0.76

$$\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{16}-\frac{\frac{9\,x^{3}}{16}+\frac{3\,x^{2}}{16}-\frac{x}{24}+\frac{1}{72}}{x^{4}+\frac{2\,x^{3}}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^2),x)
```

```
[Out] (27*atanh(3*x + 1))/16 - ((3*x^2)/16 - x/24 + (9*x^3)/16 + 1/72)/((2*x^3)/3
 + x^4)
```

sympy [A] time = 0.17, size = 41, normalized size = 0.84

$$-\frac{27\log(x)}{32}+\frac{27\log\left(x+\frac{2}{3}\right)}{32}+\frac{-81x^3-27x^2+6x-2}{144x^4+96x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**2,x)
```

```
[Out] -27*log(x)/32 + 27*log(x + 2/3)/32 + (-81*x**3 - 27*x**2 + 6*x - 2)/(144*x*
*4 + 96*x**3)
```

3.265 $\int \frac{1}{x^5(4+6x)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/(64*x^4) + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5(4+6x)^2}\,dx &= \int \left(\frac{1}{16x^5}-\frac{3}{16x^4}+\frac{27}{64x^3}-\frac{27}{32x^2}+\frac{405}{256x}-\frac{243}{128(2+3x)^2}-\frac{1215}{256(2+3x)}\right)dx\\ &= -\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(2+3x)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{1}{64x^4}+\frac{1}{16x^3}-\frac{27}{128x^2}+\frac{27}{32x}+\frac{81}{128(3x+2)}+\frac{405\log(x)}{256}-\frac{405}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] -1/64*1/x^4 + 1/(16*x^3) - 27/(128*x^2) + 27/(32*x) + 81/(128*(2 + 3*x)) +
(405*Log[x])/256 - (405*Log[2 + 3*x])/256
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^2), x]
```

fricas [A] time = 0.94, size = 69, normalized size = 1.23

$$\frac{810\,x^4+270\,x^3-60\,x^2-405\left(3\,x^5+2\,x^4\right)\log\left(3\,x+2\right)+405\left(3\,x^5+2\,x^4\right)\log(x)+20\,x-8}{256\left(3\,x^5+2\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="fricas")
```

```
[Out] 1/256*(810*x^4 + 270*x^3 - 60*x^2 - 405*(3*x^5 + 2*x^4)*log(3*x + 2) + 405*
(3*x^5 + 2*x^4)*log(x) + 20*x - 8)/(3*x^5 + 2*x^4)
```

giac [A] time = 1.15, size = 69, normalized size = 1.23

$$\frac{81}{128\left(3\,x+2\right)}-\frac{27\left(\frac{520}{3\,x+2}-\frac{1200}{(3\,x+2)^2}+\frac{960}{(3\,x+2)^3}-77\right)}{1024\left(\frac{2}{3\,x+2}-1\right)^4}+\frac{405}{256}\log\left(\left|-\frac{2}{3\,x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="giac")
```

```
[Out] 81/128/(3*x + 2) - 27/1024*(520/(3*x + 2) - 1200/(3*x + 2)^2 + 960/(3*x + 2
)^3 - 77)/(2/(3*x + 2) - 1)^4 + 405/256*log(abs(-2/(3*x + 2) + 1))
```

maple [A] time = 0.01, size = 43, normalized size = 0.77

$$\frac{405\ln(x)}{256}-\frac{405\ln\left(3x+2\right)}{256}+\frac{27}{32x}-\frac{27}{128x^2}+\frac{1}{16x^3}-\frac{1}{64x^4}+\frac{81}{128\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^2,x)
```

```
[Out] -1/64/x^4+1/16/x^3-27/128/x^2+27/32/x+81/128/(3*x+2)+405/256*ln(x)-405/256*
ln(3*x+2)
```

maxima [A] time = 1.36, size = 48, normalized size = 0.86

$$\frac{405\,x^4+135\,x^3-30\,x^2+10\,x-4}{128\left(3\,x^5+2\,x^4\right)}-\frac{405}{256}\log\left(3\,x+2\right)+\frac{405}{256}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(405*x^4 + 135*x^3 - 30*x^2 + 10*x - 4)/(3*x^5 + 2*x^4) - 405/256*log
(3*x + 2) + 405/256*log(x)
```

mupad [B] time = 0.09, size = 41, normalized size = 0.73

$$\frac{\frac{135\,x^4}{128}+\frac{45\,x^3}{128}-\frac{5\,x^2}{64}+\frac{5\,x}{192}-\frac{1}{96}}{x^5+\frac{2\,x^4}{3}}-\frac{405\,\mathrm{atanh}\left(3\,x+1\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^2),x)
```

```
[Out] ((5*x)/192 - (5*x^2)/64 + (45*x^3)/128 + (135*x^4)/128 - 1/96)/((2*x^4)/3 +
 x^5) - (405*atanh(3*x + 1))/128
```

sympy [**A**] time = 0.18, size = 46, normalized size = 0.82

$$\frac{405\log(x)}{256}-\frac{405\log\left(x+\frac{2}{3}\right)}{256}+\frac{405x^4+135x^3-30x^2+10x-4}{384x^5+256x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**2,x)
```

```
[Out] 405*log(x)/256 - 405*log(x + 2/3)/256 + (405*x**4 + 135*x**3 - 30*x**2 + 10
*x - 4)/(384*x**5 + 256*x**4)
```

3.266 $\int \frac{1}{x(4+6x)^3} dx$

Optimal. Leaf size=39

$$\frac{1}{32(3x+2)} + \frac{1}{32(3x+2)^2} + \frac{\log(x)}{64} - \frac{1}{64}\log(3x+2)$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{32(3x+2)} + \frac{1}{32(3x+2)^2} + \frac{\log(x)}{64} - \frac{1}{64}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(4 + 6*x)^3),x]
[Out] 1/(32*(2 + 3*x)^2) + 1/(32*(2 + 3*x)) + Log[x]/64 - Log[2 + 3*x]/64
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(4+6x)^3} dx &= \int \left(\frac{1}{64x} - \frac{3}{16(2+3x)^3} - \frac{3}{32(2+3x)^2} - \frac{3}{64(2+3x)}\right) dx\\ &= \frac{1}{32(2+3x)^2} + \frac{1}{32(2+3x)} + \frac{\log(x)}{64} - \frac{1}{64}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.74

$$\frac{1}{64}\left(\frac{6(x+1)}{(3x+2)^2} + \log(-6x) - \log(6x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(4 + 6*x)^3),x]
[Out] ((6*(1 + x))/(2 + 3*x)^2 + Log[-6*x] - Log[4 + 6*x])/64
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(4+6x)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(4 + 6*x)^3),x]
[Out] IntegrateAlgebraic[1/(x*(4 + 6*x)^3), x]
```

fricas [A] time = 1.08, size = 50, normalized size = 1.28

$$-\frac{(9x^2+12x+4)\log(3x+2) - (9x^2+12x+4)\log(x) - 6x - 6}{64(9x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/64*((9*x^2 + 12*x + 4)*log(3*x + 2) - (9*x^2 + 12*x + 4)*log(x) - 6*x -
6)/(9*x^2 + 12*x + 4)
```

giac [A] time = 1.03, size = 27, normalized size = 0.69

$$\frac{3\,(x+1)}{32\,(3\,x+2)^2}-\frac{1}{64}\,\log\left(|3\,x+2|\right)+\frac{1}{64}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 3/32*(x + 1)/(3*x + 2)^2 - 1/64*log(abs(3*x + 2)) + 1/64*log(abs(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.82

$$\frac{\ln(x)}{64}-\frac{\ln\left(3x+2\right)}{64}+\frac{1}{32\,(3x+2)^2}+\frac{1}{96x+64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(4+6*x)^3,x)
```

```
[Out] 1/32/(3*x+2)^2+1/32/(3*x+2)+1/64*ln(x)-1/64*ln(3*x+2)
```

maxima [A] time = 1.39, size = 30, normalized size = 0.77

$$\frac{3\,(x+1)}{32\left(9\,x^2+12\,x+4\right)}-\frac{1}{64}\,\log\left(3\,x+2\right)+\frac{1}{64}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 3/32*(x + 1)/(9*x^2 + 12*x + 4) - 1/64*log(3*x + 2) + 1/64*log(x)
```

mupad [B] time = 0.13, size = 29, normalized size = 0.74

$$\frac{1}{32\,\left(3\,x+2\right)}-\frac{\ln\left(\frac{6\,x+4}{x}\right)}{64}+\frac{1}{8\,(6\,x+4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(6*x + 4)^3),x)
```

```
[Out] 1/(32*(3*x + 2)) - log((6*x + 4)/x)/64 + 1/(8*(6*x + 4)^2)
```

sympy [A] time = 0.17, size = 27, normalized size = 0.69

$$\frac{3x+3}{288x^2+384x+128}+\frac{\log\left(x\right)}{64}-\frac{\log\left(x+\frac{2}{3}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(4+6*x)**3,x)
```

```
[Out] (3*x + 3)/(288*x**2 + 384*x + 128) + log(x)/64 - log(x + 2/3)/64
```

3.267 $\int \frac{1}{x^2(4+6x)^3} dx$

Optimal. Leaf size=46

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{64x}-\frac{3}{32(3x+2)}-\frac{3}{64(3x+2)^2}-\frac{9\log(x)}{128}+\frac{9}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] -1/(64*x) - 3/(64*(2 + 3*x)^2) - 3/(32*(2 + 3*x)) - (9*Log[x])/128 + (9*Log
[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(4+6x)^3}\,dx &= \int \left(\frac{1}{64x^2}-\frac{9}{128x}+\frac{9}{32(2+3x)^3}+\frac{9}{32(2+3x)^2}+\frac{27}{128(2+3x)}\right)dx\\ &= -\frac{1}{64x}-\frac{3}{64(2+3x)^2}-\frac{3}{32(2+3x)}-\frac{9\log(x)}{128}+\frac{9}{128}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.85

$$\frac{1}{128}\left(-\frac{2\left(27x^2+27x+4\right)}{x(3x+2)^2}-9\log(x)+9\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(4 + 27*x + 27*x^2))/(x*(2 + 3*x)^2) - 9*Log[x] + 9*Log[2 + 3*x])/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^2*(4 + 6*x)^3), x]
```

fricas [A] time = 0.92, size = 68, normalized size = 1.48

$$-\frac{54 x^{2}-9 \left(9 x^{3}+12 x^{2}+4 x\right) \log \left(3 x+2\right)+9 \left(9 x^{3}+12 x^{2}+4 x\right) \log(x)+54 x+8}{128 \left(9 x^{3}+12 x^{2}+4 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(54*x^2 - 9*(9*x^3 + 12*x^2 + 4*x)*log(3*x + 2) + 9*(9*x^3 + 12*x^2
+ 4*x)*log(x) + 54*x + 8)/(9*x^3 + 12*x^2 + 4*x)
```

giac [A] time = 0.89, size = 37, normalized size = 0.80

$$-\frac{27 x^{2}+27 x+4}{64 \left(3 x+2\right)^{2} x}+\frac{9}{128} \log \left(|3 x+2|\right)-\frac{9}{128} \log \left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/((3*x + 2)^2*x) + 9/128*log(abs(3*x + 2)) - 9/128
*log(abs(x))
```

maple [A] time = 0.01, size = 37, normalized size = 0.80

$$-\frac{9 \ln(x)}{128}+\frac{9 \ln \left(3x+2\right)}{128}-\frac{1}{64x}-\frac{3}{64 \left(3x+2\right)^{2}}-\frac{3}{32 \left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(4+6*x)^3,x)
```

```
[Out] -1/64/x-3/64/(3*x+2)^2-3/32/(3*x+2)-9/128*ln(x)+9/128*ln(3*x+2)
```

maxima [A] time = 1.39, size = 41, normalized size = 0.89

$$-\frac{27 x^{2}+27 x+4}{64 \left(9 x^{3}+12 x^{2}+4 x\right)}+\frac{9}{128} \log \left(3 x+2\right)-\frac{9}{128} \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(27*x^2 + 27*x + 4)/(9*x^3 + 12*x^2 + 4*x) + 9/128*log(3*x + 2) - 9/1
28*log(x)
```

mupad [B] time = 0.09, size = 35, normalized size = 0.76

$$\frac{9 \operatorname{atanh}\left(3 x+1\right)}{64}-\frac{\frac{3 x^{2}}{64}+\frac{3 x}{64}+\frac{1}{144}}{x^{3}+\frac{4 x^{2}}{3}+\frac{4 x}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(6*x + 4)^3),x)
```

```
[Out] (9*atanh(3*x + 1))/64 - ((3*x)/64 + (3*x^2)/64 + 1/144)/((4*x)/9 + (4*x^2)/
3 + x^3)
```

<u>**sympy**</u> **[A]** time = 0.18, size = 41, normalized size = 0.89

$$\frac{-27x^2-27x-4}{576x^3+768x^2+256x}-\frac{9\log(x)}{128}+\frac{9\log\left(x+\frac{2}{3}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(4+6*x)**3,x)
```

```
[Out] (-27*x**2 - 27*x - 4)/(576*x**3 + 768*x**2 + 256*x) - 9*log(x)/128 + 9*log(
x + 2/3)/128
```

3.268 $\int \frac{1}{x^3(4+6x)^3} dx$

Optimal. Leaf size=53

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{1}{128x^2}+\frac{9}{128x}+\frac{27}{128(3x+2)}+\frac{9}{128(3x+2)^2}+\frac{27\log(x)}{128}-\frac{27}{128}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] -1/(128*x^2) + 9/(128*x) + 9/(128*(2 + 3*x)^2) + 27/(128*(2 + 3*x)) + (27*L
og[x])/128 - (27*Log[2 + 3*x])/128
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(4+6x)^3} dx &= \int \left(\frac{1}{64x^3}-\frac{9}{128x^2}+\frac{27}{128x}-\frac{27}{64(2+3x)^3}-\frac{81}{128(2+3x)^2}-\frac{81}{128(2+3x)}\right) dx\\ &= -\frac{1}{128x^2}+\frac{9}{128x}+\frac{9}{128(2+3x)^2}+\frac{27}{128(2+3x)}+\frac{27\log(x)}{128}-\frac{27}{128}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.83

$$\frac{1}{128}\left(\frac{2\left(81x^3+81x^2+12x-2\right)}{x^2(3x+2)^2}+27\log(x)-27\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-2 + 12*x + 81*x^2 + 81*x^3))/(x^2*(2 + 3*x)^2) + 27*Log[x] - 27*Log[2
 + 3*x])/128
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3(4+6x)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^3*(4 + 6*x)^3), x]
```

fricas [A] time = 0.95, size = 79, normalized size = 1.49

$$\frac{162\,x^3+162\,x^2-27\left(9\,x^4+12\,x^3+4\,x^2\right)\log\left(3\,x+2\right)+27\left(9\,x^4+12\,x^3+4\,x^2\right)\log(x)+24\,x-4}{128\left(9\,x^4+12\,x^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(162*x^3 + 162*x^2 - 27*(9*x^4 + 12*x^3 + 4*x^2)*log(3*x + 2) + 27*(9
*x^4 + 12*x^3 + 4*x^2)*log(x) + 24*x - 4)/(9*x^4 + 12*x^3 + 4*x^2)
```

giac [A] time = 1.07, size = 43, normalized size = 0.81

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\left(3\,x^2+2\,x\right)^2}-\frac{27}{128}\,\log\left(|3\,x+2|\right)+\frac{27}{128}\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(3*x^2 + 2*x)^2 - 27/128*log(abs(3*x + 2)
) + 27/128*log(abs(x))
```

maple [A] time = 0.01, size = 42, normalized size = 0.79

$$\frac{27\ln(x)}{128}-\frac{27\ln\left(3x+2\right)}{128}+\frac{9}{128x}-\frac{1}{128x^2}+\frac{9}{128\left(3x+2\right)^2}+\frac{27}{128\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(4+6*x)^3,x)
```

```
[Out] -1/128/x^2+9/128/x+9/128/(3*x+2)^2+27/128/(3*x+2)+27/128*ln(x)-27/128*ln(3*
x+2)
```

maxima [A] time = 1.38, size = 48, normalized size = 0.91

$$\frac{81\,x^3+81\,x^2+12\,x-2}{64\left(9\,x^4+12\,x^3+4\,x^2\right)}-\frac{27}{128}\,\log\left(3\,x+2\right)+\frac{27}{128}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(81*x^3 + 81*x^2 + 12*x - 2)/(9*x^4 + 12*x^3 + 4*x^2) - 27/128*log(3*x
 + 2) + 27/128*log(x)
```

mupad [B] time = 0.09, size = 41, normalized size = 0.77

$$\frac{\frac{9\,x^3}{64}+\frac{9\,x^2}{64}+\frac{x}{48}-\frac{1}{288}}{x^4+\frac{4\,x^3}{3}+\frac{4\,x^2}{9}}-\frac{27\,\mathrm{atanh}\left(3\,x+1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(6*x + 4)^3),x)
```

```
[Out] (x/48 + (9*x^2)/64 + (9*x^3)/64 - 1/288)/((4*x^2)/9 + (4*x^3)/3 + x^4) - (2
7*atanh(3*x + 1))/64
```

sympy [A] time = 0.18, size = 46, normalized size = 0.87

$$\frac{27\log(x)}{128}-\frac{27\log\left(x+\frac{2}{3}\right)}{128}+\frac{81x^3+81x^2+12x-2}{576x^4+768x^3+256x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(4+6*x)**3,x)
```

```
[Out] 27*log(x)/128 - 27*log(x + 2/3)/128 + (81*x**3 + 81*x**2 + 12*x - 2)/(576*x
**4 + 768*x**3 + 256*x**2)
```

3.269 $\int \frac{1}{x^4(4+6x)^3} dx$

Optimal. Leaf size=60

$$-\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{9}{256x^2}-\frac{1}{192x^3}-\frac{27}{128x}-\frac{27}{64(3x+2)}-\frac{27}{256(3x+2)^2}-\frac{135\log(x)}{256}+\frac{135}{256}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] -1/(192*x^3) + 9/(256*x^2) - 27/(128*x) - 27/(256*(2 + 3*x)^2) - 27/(64*(2
+ 3*x)) - (135*Log[x])/256 + (135*Log[2 + 3*x])/256
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4(4+6x)^3} dx &= \int \left(\frac{1}{64x^4}-\frac{9}{128x^3}+\frac{27}{128x^2}-\frac{135}{256x}+\frac{81}{128(2+3x)^3}+\frac{81}{64(2+3x)^2}+\frac{405}{256(2+3x)}\right) dx\\ &= -\frac{1}{192x^3}+\frac{9}{256x^2}-\frac{27}{128x}-\frac{27}{256(2+3x)^2}-\frac{27}{64(2+3x)}-\frac{135\log(x)}{256}+\frac{135}{256}\log(2+3x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.82

$$\frac{1}{768}\left(-\frac{2\left(1215x^4+1215x^3+180x^2-30x+8\right)}{x^3(3x+2)^2}-405\log(x)+405\log(3x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] ((-2*(8 - 30*x + 180*x^2 + 1215*x^3 + 1215*x^4))/(x^3*(2 + 3*x)^2) - 405*Lo
g[x] + 405*Log[2 + 3*x])/768
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4(4+6x)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^4*(4 + 6*x)^3), x]
```

fricas [A] time = 0.75, size = 84, normalized size = 1.40

$$-\frac{2430\,x^4+2430\,x^3+360\,x^2-405\left(9\,x^5+12\,x^4+4\,x^3\right)\log\left(3\,x+2\right)+405\left(9\,x^5+12\,x^4+4\,x^3\right)\log(x)-60\,x+16}{768\left(9\,x^5+12\,x^4+4\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/768*(2430*x^4 + 2430*x^3 + 360*x^2 - 405*(9*x^5 + 12*x^4 + 4*x^3)*log(3*
x + 2) + 405*(9*x^5 + 12*x^4 + 4*x^3)*log(x) - 60*x + 16)/(9*x^5 + 12*x^4 +
 4*x^3)
```

giac [A] time = 1.29, size = 47, normalized size = 0.78

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\,(3\,x+2)^2x^3}+\frac{135}{256}\log\left(|3\,x+2|\right)-\frac{135}{256}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/((3*x + 2)^2*x^3) + 135/2
56*log(abs(3*x + 2)) - 135/256*log(abs(x))
```

maple [A] time = 0.01, size = 47, normalized size = 0.78

$$-\frac{135\ln(x)}{256}+\frac{135\ln\left(3x+2\right)}{256}-\frac{27}{128x}+\frac{9}{256x^2}-\frac{1}{192x^3}-\frac{27}{256\left(3x+2\right)^2}-\frac{27}{64\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(4+6*x)^3,x)
```

```
[Out] -1/192/x^3+9/256/x^2-27/128/x-27/256/(3*x+2)^2-27/64/(3*x+2)-135/256*ln(x)+
135/256*ln(3*x+2)
```

maxima [A] time = 1.38, size = 53, normalized size = 0.88

$$-\frac{1215\,x^4+1215\,x^3+180\,x^2-30\,x+8}{384\left(9\,x^5+12\,x^4+4\,x^3\right)}+\frac{135}{256}\log\left(3\,x+2\right)-\frac{135}{256}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/384*(1215*x^4 + 1215*x^3 + 180*x^2 - 30*x + 8)/(9*x^5 + 12*x^4 + 4*x^3)
+ 135/256*log(3*x + 2) - 135/256*log(x)
```

mupad [B] time = 0.05, size = 47, normalized size = 0.78

$$\frac{135\,\mathrm{atanh}\left(3\,x+1\right)}{128}-\frac{\frac{45\,x^4}{128}+\frac{45\,x^3}{128}+\frac{5\,x^2}{96}-\frac{5\,x}{576}+\frac{1}{432}}{x^5+\frac{4\,x^4}{3}+\frac{4\,x^3}{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(6*x + 4)^3),x)
```

```
[Out] (135*atanh(3*x + 1))/128 - ((5*x^2)/96 - (5*x)/576 + (45*x^3)/128 + (45*x^4
)/128 + 1/432)/((4*x^3)/9 + (4*x^4)/3 + x^5)
```

sympy [**A**] time = 0.21, size = 51, normalized size = 0.85

$$-\frac{135\log(x)}{256}+\frac{135\log\left(x+\frac{2}{3}\right)}{256}+\frac{-1215x^4-1215x^3-180x^2+30x-8}{3456x^5+4608x^4+1536x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(4+6*x)**3,x)
```

```
[Out] -135*log(x)/256 + 135*log(x + 2/3)/256 + (-1215*x**4 - 1215*x**3 - 180*x**2
 + 30*x - 8)/(3456*x**5 + 4608*x**4 + 1536*x**3)
```

3.270 $\int \frac{1}{x^5(4+6x)^3}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{27}{256x^2}+\frac{3}{128x^3}-\frac{1}{256x^4}+\frac{135}{256x}+\frac{405}{512(3x+2)}+\frac{81}{512(3x+2)^2}+\frac{1215\log(x)}{1024}-\frac{1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] -1/(256*x^4) + 3/(128*x^3) - 27/(256*x^2) + 135/(256*x) + 81/(512*(2 + 3*x)
^2) + 405/(512*(2 + 3*x)) + (1215*Log[x])/1024 - (1215*Log[2 + 3*x])/1024
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{x^5(4+6x)^3}\,dx = \int \left(\frac{1}{64x^5}-\frac{9}{128x^4}+\frac{27}{128x^3}-\frac{135}{256x^2}+\frac{1215}{1024x}-\frac{243}{256(2+3x)^3}-\frac{1215}{512(2+3x)^2}-\frac{3645}{1024(2+$$
$$= -\frac{1}{256x^4}+\frac{3}{128x^3}-\frac{27}{256x^2}+\frac{135}{256x}+\frac{81}{512(2+3x)^2}+\frac{405}{512(2+3x)}+\frac{1215\log(x)}{1024}-\frac{1215\,\text{l}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.81

$$\frac{\frac{2(3645x^5+3645x^4+540x^3-90x^2+24x-8)}{x^4(3x+2)^2}+1215\log(x)-1215\log(3x+2)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] ((2*(-8 + 24*x - 90*x^2 + 540*x^3 + 3645*x^4 + 3645*x^5))/(x^4*(2 + 3*x)^2)
 + 1215*Log[x] - 1215*Log[2 + 3*x])/1024
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5(4+6x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(x^5*(4 + 6*x)^3), x]
```

fricas [A] time = 0.99, size = 89, normalized size = 1.33

$$\frac{7290\,x^{5}+7290\,x^{4}+1080\,x^{3}-180\,x^{2}-1215\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)\log\left(3\,x+2\right)+1215\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)\log\left(x\right)+48\,x-16}{1024\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(7290*x^5 + 7290*x^4 + 1080*x^3 - 180*x^2 - 1215*(9*x^6 + 12*x^5 + 4
*x^4)*log(3*x + 2) + 1215*(9*x^6 + 12*x^5 + 4*x^4)*log(x) + 48*x - 16)/(9*x
^6 + 12*x^5 + 4*x^4)
```

giac [A] time = 1.13, size = 52, normalized size = 0.78

$$\frac{3645\,x^{5}+3645\,x^{4}+540\,x^{3}-90\,x^{2}+24\,x-8}{512\left(3\,x+2\right)^{2}x^{4}}-\frac{1215}{1024}\,\log\left(\left|3\,x+2\right|\right)+\frac{1215}{1024}\,\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="giac")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/((3*x + 2)^2*x^4)
 - 1215/1024*log(abs(3*x + 2)) + 1215/1024*log(abs(x))
```

maple [A] time = 0.01, size = 52, normalized size = 0.78

$$\frac{1215\ln\left(x\right)}{1024}-\frac{1215\ln\left(3x+2\right)}{1024}+\frac{135}{256x}-\frac{27}{256x^{2}}+\frac{3}{128x^{3}}-\frac{1}{256x^{4}}+\frac{81}{512\left(3x+2\right)^{2}}+\frac{405}{512\left(3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(4+6*x)^3,x)
```

```
[Out] -1/256/x^4+3/128/x^3-27/256/x^2+135/256/x+81/512/(3*x+2)^2+405/512/(3*x+2)+
1215/1024*ln(x)-1215/1024*ln(3*x+2)
```

maxima [A] time = 1.39, size = 58, normalized size = 0.87

$$\frac{3645\,x^{5}+3645\,x^{4}+540\,x^{3}-90\,x^{2}+24\,x-8}{512\left(9\,x^{6}+12\,x^{5}+4\,x^{4}\right)}-\frac{1215}{1024}\,\log\left(3\,x+2\right)+\frac{1215}{1024}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(4+6*x)^3,x, algorithm="maxima")
```

```
[Out] 1/512*(3645*x^5 + 3645*x^4 + 540*x^3 - 90*x^2 + 24*x - 8)/(9*x^6 + 12*x^5 +
 4*x^4) - 1215/1024*log(3*x + 2) + 1215/1024*log(x)
```

mupad [B] time = 0.05, size = 51, normalized size = 0.76

$$\frac{\frac{405\,x^{5}}{512}+\frac{405\,x^{4}}{512}+\frac{15\,x^{3}}{128}-\frac{5\,x^{2}}{256}+\frac{x}{192}-\frac{1}{576}}{x^{6}+\frac{4\,x^{5}}{3}+\frac{4\,x^{4}}{9}}-\frac{1215\,\mathrm{atanh}\left(3\,x+1\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(6*x + 4)^3),x)
```

```
[Out] (x/192 - (5*x^2)/256 + (15*x^3)/128 + (405*x^4)/512 + (405*x^5)/512 - 1/576
)/((4*x^4)/9 + (4*x^5)/3 + x^6) - (1215*atanh(3*x + 1))/512
```

sympy [**A**] time = 0.22, size = 56, normalized size = 0.84

$$\frac{1215\log\left(x\right)}{1024}-\frac{1215\log\left(x+\frac{2}{3}\right)}{1024}+\frac{3645x^5+3645x^4+540x^3-90x^2+24x-8}{4608x^6+6144x^5+2048x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(4+6*x)**3,x)
```

```
[Out] 1215*log(x)/1024 - 1215*log(x + 2/3)/1024 + (3645*x**5 + 3645*x**4 + 540*x*
*3 - 90*x**2 + 24*x - 8)/(4608*x**6 + 6144*x**5 + 2048*x**4)
```

3.271 $\int \frac{1}{2+2x}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2}\log(x+1)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x)^(-1),x]
```

```
[Out] Log[1 + x]/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+2x}\,dx = \frac{1}{2}\log(1+x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.25

$$\frac{1}{2}\log(2x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x)^(-1),x]
```

```
[Out] Log[2 + 2*x]/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+2x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + 2*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(2 + 2*x)^(-1), x]
```

fricas [A] time = 1.12, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1)
```

giac [**A**] time = 0.90, size = 7, normalized size = 0.88

$$\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1))
```

maple [**A**] time = 0.00, size = 9, normalized size = 1.12

$$\frac{\ln\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+2*x),x)
[Out] 1/2*ln(2+2*x)
```

maxima [**A**] time = 1.34, size = 6, normalized size = 0.75

$$\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x, algorithm="maxima")
[Out] 1/2*log(x + 1)
```

mupad [**B**] time = 0.15, size = 6, normalized size = 0.75

$$\frac{\ln\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 2),x)
[Out] log(x + 1)/2
```

sympy [**A**] time = 0.07, size = 7, normalized size = 0.88

$$\frac{\log\left(2x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+2),x)
[Out] log(2*x + 2)/2
```

3.272 $\int \frac{1}{4-6x}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{6}\log(2-3x)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$-\frac{1}{6}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 6*x)^(-1),x]
```

```
[Out] -Log[2 - 3*x]/6
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{4-6x}\,dx = -\frac{1}{6}\log(2-3x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{6}\log(4-6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 6*x)^(-1),x]
```

```
[Out] -1/6*Log[4 - 6*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{4-6x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(4 - 6*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(4 - 6*x)^(-1), x]
```

fricas [A] time = 0.98, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(3*x - 2)
```

giac [A] time = 1.15, size = 9, normalized size = 0.90

$$-\frac{1}{6}\log\left(|3x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(3*x - 2))
```

maple [A] time = 0.00, size = 9, normalized size = 0.90

$$-\frac{\ln\left(-6x+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-6*x),x)
```

```
[Out] -1/6*ln(4-6*x)
```

maxima [A] time = 1.34, size = 8, normalized size = 0.80

$$-\frac{1}{6}\log\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x, algorithm="maxima")
```

```
[Out] -1/6*log(3*x - 2)
```

mupad [B] time = 0.08, size = 6, normalized size = 0.60

$$-\frac{\ln\left(x-\frac{2}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(6*x - 4),x)
```

```
[Out] -log(x - 2/3)/6
```

sympy [A] time = 0.07, size = 8, normalized size = 0.80

$$-\frac{\log\left(6x-4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-6*x),x)
```

```
[Out] -log(6*x - 4)/6
```

3.273 $\int \frac{1}{a+\sqrt{a}x}\,dx$

Optimal. Leaf size=14

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {31}

$$\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[Sqrt[a] + x]/Sqrt[a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{a}x}\,dx=\frac{\log\left(\sqrt{a}+x\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.14

$$\frac{\log\left(\sqrt{a}x+a\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[a]*x]/Sqrt[a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+\sqrt{a}x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + Sqrt[a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + Sqrt[a]*x)^(-1), x]
```

fricas [A] time = 0.79, size = 10, normalized size = 0.71

$$\frac{\log\left(x+\sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="fricas")
[Out] log(x + sqrt(a))/sqrt(a)
```

giac [A] time = 1.28, size = 13, normalized size = 0.93

$$\frac{\log\left(\left|\sqrt{a}x + a\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="giac")
[Out] log(abs(sqrt(a)*x + a))/sqrt(a)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{\ln\left(\sqrt{a}\,x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*a^(1/2)),x)
[Out] ln(a+x*a^(1/2))/a^(1/2)
```

maxima [A] time = 1.25, size = 12, normalized size = 0.86

$$\frac{\log\left(\sqrt{a}x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a^(1/2)),x, algorithm="maxima")
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

mupad [B] time = 0.11, size = 10, normalized size = 0.71

$$\frac{\ln\left(x + \sqrt{a}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a^(1/2)*x),x)
[Out] log(x + a^(1/2))/a^(1/2)
```

sympy [A] time = 0.08, size = 14, normalized size = 1.00

$$\frac{\log\left(\sqrt{a}x + a\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*a**(1/2)),x)
[Out] log(sqrt(a)*x + a)/sqrt(a)
```

3.274 $\int \frac{1}{a+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {31}

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx=\frac{\log\left(a+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}\,x+a\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a + Sqrt[-a]*x]/Sqrt[-a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + Sqrt[-a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a + Sqrt[-a]*x)^(-1), x]
```

fricas [A] time = 1.16, size = 20, normalized size = 1.00

$$-\frac{\sqrt{-a}\log\left(x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(x - sqrt(-a))/a
```

giac [A] time = 1.10, size = 17, normalized size = 0.85

$$\frac{\log\left(\left|\sqrt{-a}\,x + a\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + a))/sqrt(-a)
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{\ln\left(a + \sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x*(-a)^(1/2)),x)
```

```
[Out] ln(a+x*(-a)^(1/2))/(-a)^(1/2)
```

maxima [A] time = 1.37, size = 16, normalized size = 0.80

$$\frac{\log\left(\sqrt{-a}\,x + a\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + a)/sqrt(-a)
```

mupad [B] time = 0.11, size = 16, normalized size = 0.80

$$\frac{\ln\left(x - \sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - (-a)^(1/2))/(-a)^(1/2)
```

sympy [A] time = 0.08, size = 17, normalized size = 0.85

$$\frac{\log\left(a + x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x*(-a)**(1/2)),x)
```

```
[Out] log(a + x*sqrt(-a))/sqrt(-a)
```

3.275 $\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=22

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx=\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[a^2 + Sqrt[-a]*x]/Sqrt[-a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^2+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^2 + Sqrt[-a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^2 + Sqrt[-a]*x)^(-1), x]
```

fricas [A] time = 0.89, size = 21, normalized size = 0.95

$$-\frac{\sqrt{-a}\,\log\left(-\sqrt{-a}\,a+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(-sqrt(-a)*a + x)/a
```

giac [A] time = 0.93, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^2+\sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a^2 + sqrt(-a)*x))/sqrt(-a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+(-a)^(1/2)*x),x)
```

```
[Out] ln(a^2+(-a)^(1/2)*x)/(-a)^(1/2)
```

maxima [A] time = 1.32, size = 18, normalized size = 0.82

$$\frac{\log\left(a^2+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a^2 + sqrt(-a)*x)/sqrt(-a)
```

mupad [B] time = 0.05, size = 14, normalized size = 0.64

$$\frac{\ln\left(x+(-a)^{3/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + (-a)^(3/2))/(-a)^(1/2)
```

sympy [A] time = 0.08, size = 19, normalized size = 0.86

$$\frac{\log\left(a^2+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2 + x*sqrt(-a))/sqrt(-a)
```

3.276 $\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=22

$$\frac{\log \left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log \left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx=\frac{\log \left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log \left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^3 + Sqrt[-a]*x]/Sqrt[-a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a^3+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^3 + Sqrt[-a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a^3 + Sqrt[-a]*x)^(-1), x]
```

fricas [A] time = 0.84, size = 23, normalized size = 1.05

$$-\frac{\sqrt{-a}\,\log \left(-\sqrt{-a}\,a^2+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="fricas")
[Out] -sqrt(-a)*log(-sqrt(-a)*a^2 + x)/a
```

giac [A] time = 1.09, size = 19, normalized size = 0.86

$$\frac{\log\left(\left|a^3+\sqrt{-a}\,x\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="giac")
[Out] log(abs(a^3 + sqrt(-a)*x))/sqrt(-a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\ln\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+(-a)^(1/2)*x),x)
[Out] ln(a^3+(-a)^(1/2)*x)/(-a)^(1/2)
```

maxima [A] time = 1.31, size = 18, normalized size = 0.82

$$\frac{\log\left(a^3+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x*(-a)^(1/2)),x, algorithm="maxima")
[Out] log(a^3 + sqrt(-a)*x)/sqrt(-a)
```

mupad [B] time = 0.06, size = 16, normalized size = 0.73

$$\frac{\ln\left(x-(-a)^{5/2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3 + (-a)^(1/2)*x),x)
[Out] log(x - (-a)^(5/2))/(-a)^(1/2)
```

sympy [A] time = 0.08, size = 19, normalized size = 0.86

$$\frac{\log\left(a^3+x\sqrt{-a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x*(-a)**(1/2)),x)
[Out] log(a**3 + x*sqrt(-a))/sqrt(-a)
```

3.277 $\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 - (-a)^(3/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx=\frac{\log\left(1-(-a)^{3/2}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{\log\left(\sqrt{-a}\,ax+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] Log[1 + Sqrt[-a]*a*x]/Sqrt[-a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{1}{a}+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(-1) + Sqrt[-a]*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(a^(-1) + Sqrt[-a]*x)^(-1), x]
```

fricas [A] time = 0.87, size = 24, normalized size = 1.14

$$-\frac{\sqrt{-a}\,\log\left(a^2x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^2*x - sqrt(-a))/a
```

giac [A] time = 1.04, size = 19, normalized size = 0.90

$$\frac{\log\left(\left|\sqrt{-a}\,x + \frac{1}{a}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a))/sqrt(-a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.90

$$\frac{\ln\left(\sqrt{-a}\,x + \frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a+(-a)^(1/2)*x),x)
```

```
[Out] ln(1/a+(-a)^(1/2)*x)/(-a)^(1/2)
```

maxima [A] time = 1.33, size = 18, normalized size = 0.86

$$\frac{\log\left(\sqrt{-a}\,x + \frac{1}{a}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a)/sqrt(-a)
```

mupad [B] time = 0.15, size = 16, normalized size = 0.76

$$\frac{\ln\left(x - \frac{1}{(-a)^{3/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a + (-a)^(1/2)*x),x)
```

```
[Out] log(x - 1/(-a)^(3/2))/(-a)^(1/2)
```

sympy [A] time = 0.09, size = 19, normalized size = 0.90

$$\frac{\log\left(ax\sqrt{-a} + 1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a+x*(-a)**(1/2)),x)
```

```
[Out] log(a*x*sqrt(-a) + 1)/sqrt(-a)
```

3.278 $\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx$

Optimal. Leaf size=20

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {31}

$$\frac{\log\left((-a)^{5/2}x+1\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] Log[1 + (-a)^(5/2)*x]/Sqrt[-a]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx = \frac{\log\left(1+(-a)^{5/2}x\right)}{\sqrt{-a}}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.10

$$\frac{\log\left(\frac{1}{a^2}+\sqrt{-a}\,x\right)}{\sqrt{-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] Log[a^(-2) + Sqrt[-a]*x]/Sqrt[-a]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{1}{a^2}+\sqrt{-a}\,x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a^(-2) + Sqrt[-a]*x)^(-1),x]
[Out] IntegrateAlgebraic[(a^(-2) + Sqrt[-a]*x)^(-1), x]
```

fricas [A] time = 1.01, size = 24, normalized size = 1.20

$$-\frac{\sqrt{-a}\log\left(a^3x-\sqrt{-a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(-a)*log(a^3*x - sqrt(-a))/a
```

giac [A] time = 1.08, size = 19, normalized size = 0.95

$$\frac{\log\left(\left|\sqrt{-a}\,x + \frac{1}{a^2}\right|\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(sqrt(-a)*x + 1/a^2))/sqrt(-a)
```

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{\ln\left(\sqrt{-a}\;x + \frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2+(-a)^(1/2)*x),x)
```

```
[Out] ln(1/a^2+(-a)^(1/2)*x)/(-a)^(1/2)
```

maxima [A] time = 1.31, size = 18, normalized size = 0.90

$$\frac{\log\left(\sqrt{-a}\,x + \frac{1}{a^2}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a^2+x*(-a)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(sqrt(-a)*x + 1/a^2)/sqrt(-a)
```

mupad [B] time = 0.18, size = 14, normalized size = 0.70

$$\frac{\ln\left(x + \frac{1}{(-a)^{5/2}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/a^2 + (-a)^(1/2)*x),x)
```

```
[Out] log(x + 1/(-a)^(5/2))/(-a)^(1/2)
```

sympy [A] time = 0.09, size = 20, normalized size = 1.00

$$\frac{\log\left(a^2 x\sqrt{-a} + 1\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/a**2+x*(-a)**(1/2)),x)
```

```
[Out] log(a**2*x*sqrt(-a) + 1)/sqrt(-a)
```

3.279 $\int \frac{1}{x(1+bx)}\,dx$

Optimal. Leaf size=11

$$\log(x) - \log(bx + 1)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {36, 29, 31}

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + b*x)),x]
[Out] Log[x] - Log[1 + b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+bx)}\,dx &= -\left(b\int \frac{1}{1+bx}\,dx\right) + \int \frac{1}{x}\,dx\\ &= \log(x) - \log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\log(x) - \log(bx + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + b*x)),x]
[Out] Log[x] - Log[1 + b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(1+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(1 + b*x)),x]
[Out] IntegrateAlgebraic[1/(x*(1 + b*x)), x]
```

fricas [A] time = 0.77, size = 11, normalized size = 1.00

$$-\log(bx+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="fricas")
[Out] -log(b*x + 1) + log(x)
```

giac [A] time = 0.99, size = 13, normalized size = 1.18

$$-\log(|bx+1|)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="giac")
[Out] -log(abs(b*x + 1)) + log(abs(x))
```

maple [A] time = 0.00, size = 12, normalized size = 1.09

$$\ln(x)-\ln(bx+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+1),x)
[Out] ln(x)-ln(b*x+1)
```

maxima [A] time = 1.31, size = 11, normalized size = 1.00

$$-\log(bx+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x, algorithm="maxima")
[Out] -log(b*x + 1) + log(x)
```

mupad [B] time = 0.10, size = 9, normalized size = 0.82

$$-2\,\mathrm{atanh}(2\,b\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + 1)),x)
[Out] -2*atanh(2*b*x + 1)
```

sympy [A] time = 0.14, size = 8, normalized size = 0.73

$$\log(x)-\log\left(x+\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+1),x)
[Out] log(x) - log(x + 1/b)
```

3.280 $\int \frac{1}{x(-1+bx)}\,dx$

Optimal. Leaf size=12

$$\log(1-bx)-\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = $\{36, 29, 31\}$

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(-1+bx)}\,dx &= b\int \frac{1}{-1+bx}\,dx-\int \frac{1}{x}\,dx\\ &= -\log(x)+\log(1-bx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\log(1-bx)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-1 + b*x)),x]
```

```
[Out] -Log[x] + Log[1 - b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(-1+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x*(-1 + b*x)),x]
```

```
[Out] IntegrateAlgebraic[1/(x*(-1 + b*x)), x]
```

fricas [A] time = 0.62, size = 11, normalized size = 0.92

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="fricas")
[Out] log(b*x - 1) - log(x)
```

giac [A] time = 1.02, size = 13, normalized size = 1.08

$$\log(|bx-1|)-\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="giac")
[Out] log(abs(b*x - 1)) - log(abs(x))
```

maple [A] time = 0.01, size = 12, normalized size = 1.00

$$-\ln(x)+\ln(bx-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-1),x)
[Out] ln(b*x-1)-ln(x)
```

maxima [A] time = 1.36, size = 11, normalized size = 0.92

$$\log(bx-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x, algorithm="maxima")
[Out] log(b*x - 1) - log(x)
```

mupad [B] time = 0.04, size = 9, normalized size = 0.75

$$-2\,\mathrm{atanh}(2\,b\,x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - 1)),x)
[Out] -2*atanh(2*b*x - 1)
```

sympy [A] time = 0.13, size = 8, normalized size = 0.67

$$-\log(x)+\log\left(x-\frac{1}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-1),x)
[Out] -log(x) + log(x - 1/b)
```

3.281 $\int \frac{1}{x^2(1+bx)} dx$

Optimal. Leaf size=19

$$-b\log(x) + b\log(bx+1) - \frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x) + b\log(bx+1) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + b*x)),x]
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(1+bx)} dx &= \int \left(\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{1+bx}\right) dx \\ &= -\frac{1}{x} - b\log(x) + b\log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-b\log(x) + b\log(bx+1) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + b*x)),x]
[Out] -x^(-1) - b*Log[x] + b*Log[1 + b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(1+bx)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(1 + b*x)),x]
[Out] IntegrateAlgebraic[1/(x^2*(1 + b*x)), x]
```

fricas [A] time = 0.93, size = 21, normalized size = 1.11

$$\frac{bx\log(bx+1) - bx\log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="fricas")
[Out] (b*x*log(b*x + 1) - b*x*log(x) - 1)/x
```

giac [A] time = 0.92, size = 21, normalized size = 1.11

$$b\log(|bx+1|) - b\log(|x|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="giac")
[Out] b*log(abs(b*x + 1)) - b*log(abs(x)) - 1/x
```

maple [A] time = 0.01, size = 20, normalized size = 1.05

$$-b\ln(x) + b\ln(bx+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+1),x)
[Out] -1/x-b*ln(x)+b*ln(b*x+1)
```

maxima [A] time = 1.32, size = 19, normalized size = 1.00

$$b\log(bx+1) - b\log(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+1),x, algorithm="maxima")
[Out] b*log(b*x + 1) - b*log(x) - 1/x
```

mupad [B] time = 0.04, size = 16, normalized size = 0.84

$$2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)),x)
[Out] 2*b*atanh(2*b*x + 1) - 1/x
```

sympy [A] time = 0.18, size = 14, normalized size = 0.74

$$b\left(-\log(x) + \log\left(x + \frac{1}{b}\right)\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+1),x)
[Out] b*(-log(x) + log(x + 1/b)) - 1/x
```

3.282 $\int \frac{1}{x^2(-1+bx)}\,dx$

Optimal. Leaf size=18

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-1 + b*x)),x]
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(-1+bx)}\,dx &= \int \left(-\frac{1}{x^2} - \frac{b}{x} + \frac{b^2}{-1+bx}\right) dx\\ &= \frac{1}{x} - b\log(x) + b\log(1-bx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-b\log(x) + b\log(1 - bx) + \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-1 + b*x)),x]
[Out] x^(-1) - b*Log[x] + b*Log[1 - b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(-1+bx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(x^2*(-1 + b*x)),x]
[Out] IntegrateAlgebraic[1/(x^2*(-1 + b*x)), x]
```

fricas [A] time = 0.73, size = 21, normalized size = 1.17

$$\frac{bx\log(bx-1) - bx\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="fricas")
[Out] (b*x*log(b*x - 1) - b*x*log(x) + 1)/x
```

giac [A] time = 1.14, size = 19, normalized size = 1.06

$$b\log\left(|bx-1|\right)-b\log\left(|x|\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="giac")
[Out] b*log(abs(b*x - 1)) - b*log(abs(x)) + 1/x
```

maple [A] time = 0.01, size = 18, normalized size = 1.00

$$-b\ln(x)+b\ln\left(bx-1\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-1),x)
[Out] b*ln(b*x-1)+1/x-b*ln(x)
```

maxima [A] time = 1.33, size = 17, normalized size = 0.94

$$b\log\left(bx-1\right)-b\log(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-1),x, algorithm="maxima")
[Out] b*log(b*x - 1) - b*log(x) + 1/x
```

mupad [B] time = 0.03, size = 14, normalized size = 0.78

$$\frac{1}{x}-2\,b\,\mathrm{atanh}\left(2\,b\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - 1)),x)
[Out] 1/x - 2*b*atanh(2*b*x - 1)
```

sympy [A] time = 0.19, size = 14, normalized size = 0.78

$$b\left(-\log\left(x\right)+\log\left(x-\frac{1}{b}\right)\right)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-1),x)
[Out] b*(-log(x) + log(x - 1/b)) + 1/x
```

3.283 $\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx$

Optimal. Leaf size=14

$$b\log(bx+1)-\frac{1}{x}$$

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {44}

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx &= b\log(x)+\int \frac{1}{x^2(1+bx)}\,dx\\ &= b\log(x)+\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right) dx\\ &= -\frac{1}{x}+b\log(1+bx)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$b\log(bx+1)-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] -x^(-1) + b*Log[1 + b*x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{b}{x}+\frac{1}{x^2(1+bx)}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[b/x + 1/(x^2*(1 + b*x)),x]
```

```
[Out] IntegrateAlgebraic[b/x + 1/(x^2*(1 + b*x)), x]
```

fricas [**A**] time = 0.88, size = 15, normalized size = 1.07

$$\frac{bx \log(bx+1) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="fricas")
[Out] (b*x*log(b*x + 1) - 1)/x
```

giac [**A**] time = 1.25, size = 15, normalized size = 1.07

$$b \log(|bx+1|) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="giac")
[Out] b*log(abs(b*x + 1)) - 1/x
```

maple [**A**] time = 0.00, size = 15, normalized size = 1.07

$$b \ln(bx+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b/x+1/x^2/(b*x+1),x)
[Out] -1/x+b*ln(b*x+1)
```

maxima [**A**] time = 1.32, size = 14, normalized size = 1.00

$$b \log(bx+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x^2/(b*x+1),x, algorithm="maxima")
[Out] b*log(b*x + 1) - 1/x
```

mupad [**B**] time = 0.04, size = 20, normalized size = 1.43

$$b\,\ln(x) + 2\,b\,\mathrm{atanh}\left(2\,b\,x+1\right) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x + 1)) + b/x,x)
[Out] b*log(x) + 2*b*atanh(2*b*x + 1) - 1/x
```

sympy [**A**] time = 0.15, size = 10, normalized size = 0.71

$$b \log(bx+1) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b/x+1/x**2/(b*x+1),x)
[Out] b*log(b*x + 1) - 1/x
```

3.284 $\int x^3\sqrt{a+bx}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{2(a+bx)^{9/2}}{9b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a + b*x)^(5/2))/(5*b^4) - (6*a*(
a + b*x)^(7/2))/(7*b^4) + (2*(a + b*x)^(9/2))/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{a+bx}\,dx&=\int\left(-\frac{a^3\sqrt{a+bx}}{b^3}+\frac{3a^2(a+bx)^{3/2}}{b^3}-\frac{3a(a+bx)^{5/2}}{b^3}+\frac{(a+bx)^{7/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{3/2}}{3b^4}+\frac{6a^2(a+bx)^{5/2}}{5b^4}-\frac{6a(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{9/2}}{9b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3))/(315
*b^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.82

$$-\frac{2\left(105a^3(a+bx)^{3/2}-189a^2(a+bx)^{5/2}-35(a+bx)^{9/2}+135a(a+bx)^{7/2}\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*x],x]
```

```
[Out] (-2*(105*a^3*(a + b*x)^(3/2) - 189*a^2*(a + b*x)^(5/2) + 135*a*(a + b*x)^(7
/2) - 35*(a + b*x)^(9/2)))/(315*b^4)
```

fricas [A] time = 1.36, size = 53, normalized size = 0.74

$$\frac{2\left(35\,b^{4}x^{4}+5\,ab^{3}x^{3}-6\,a^{2}b^{2}x^{2}+8\,a^{3}bx-16\,a^{4}\right)\sqrt{bx+a}}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

giac [B] time = 0.95, size = 116, normalized size = 1.61

$$\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a}{b^{3}}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}}{b^{3}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2
 - 35*sqrt(b*x + a)*a^3)*a/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*
a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a
^4)/b^3)/b
```

maple [A] time = 0.02, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\left(-35b^{3}x^{3}+30a\,b^{2}x^{2}-24a^{2}bx+16a^{3}\right)}{315b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)^(3/2)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.25, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^{4}}-\frac{6\,(bx+a)^{\frac{7}{2}}a}{7\,b^{4}}+\frac{6\,(bx+a)^{\frac{5}{2}}a^{2}}{5\,b^{4}}-\frac{2\,(bx+a)^{\frac{3}{2}}a^{3}}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^4 - 6/7*(b*x + a)^(7/2)*a/b^4 + 6/5*(b*x + a)^(5/2)*a
^2/b^4 - 2/3*(b*x + a)^(3/2)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{(a+b\,x)}^{9/2}}{9\,b^{4}}-\frac{2\,a^{3}\,{(a+b\,x)}^{3/2}}{3\,b^{4}}+\frac{6\,a^{2}\,{(a+b\,x)}^{5/2}}{5\,b^{4}}-\frac{6\,a\,{(a+b\,x)}^{7/2}}{7\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^4) - (2*a^3*(a + b*x)^(3/2))/(3*b^4) + (6*a^2*(a +
 b*x)^(5/2))/(5*b^4) - (6*a*(a + b*x)^(7/2))/(7*b^4)
```

sympy [B] time = 2.91, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(49/2)*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a*
*18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b*
*9*x**5 + 315*a**14*b**10*x**6) + 32*a**(49/2)/(315*a**20*b**4 + 1890*a**19
*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**
4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) - 176*a**(47/2)*b*x*sqrt(1
 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300
*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*
b**10*x**6) + 192*a**(47/2)*b*x/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) - 396*a**(45/2)*b**2*x**2*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 480*a**(45/2)*b**2*x**2/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**
18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**
9*x**5 + 315*a**14*b**10*x**6) - 462*a**(43/2)*b**3*x**3*sqrt(1 + b*x/a)/(3
15*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*
x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
+ 640*a**(43/2)*b**3*x**3/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*
b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x
**5 + 315*a**14*b**10*x**6) - 210*a**(41/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*
a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**
3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 4
80*a**(41/2)*b**4*x**4/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**
6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5
 + 315*a**14*b**10*x**6) + 378*a**(39/2)*b**5*x**5*sqrt(1 + b*x/a)/(315*a**
20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 +
 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 192*
a**(39/2)*b**5*x**5/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x
**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 +
315*a**14*b**10*x**6) + 1134*a**(37/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**20
*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4
725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 32*a**
(37/2)*b**6*x**6/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2
 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315
*a**14*b**10*x**6) + 1494*a**(35/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**20*b*
*4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725
*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6) + 1098*a**(
33/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*
a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*
b**9*x**5 + 315*a**14*b**10*x**6) + 430*a**(31/2)*b**9*x**9*sqrt(1 + b*x/a)
/(315*a**20*b**4 + 1890*a**19*b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b*
*7*x**3 + 4725*a**16*b**8*x**4 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**
6) + 70*a**(29/2)*b**10*x**10*sqrt(1 + b*x/a)/(315*a**20*b**4 + 1890*a**19*
b**5*x + 4725*a**18*b**6*x**2 + 6300*a**17*b**7*x**3 + 4725*a**16*b**8*x**4
 + 1890*a**15*b**9*x**5 + 315*a**14*b**10*x**6)
```

3.285 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx&=\int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 45, normalized size = 0.85

$$\frac{2\left(35a^2(a+bx)^{3/2}+15(a+bx)^{7/2}-42a(a+bx)^{5/2}\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(35*a^2*(a + b*x)^(3/2) - 42*a*(a + b*x)^(5/2) + 15*(a + b*x)^(7/2)))/(1
05*b^3)
```

fricas [**A**] time = 1.05, size = 42, normalized size = 0.79

$$\frac{2\left(15\,b^{3}x^{3}+3\,ab^{2}x^{2}-4\,a^{2}bx+8\,a^{3}\right)\sqrt{bx+a}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

giac [**B**] time = 1.03, size = 93, normalized size = 1.75

$$\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a}{b^{2}}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)}{b^{2}}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*
a/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)/b^2)/b
```

maple [**A**] time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{3}{2}}\left(15b^{2}x^{2}-12abx+8a^{2}\right)}{105b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

maxima [**A**] time = 1.41, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{3}}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^{3}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

mupad [**B**] time = 0.05, size = 37, normalized size = 0.70

$$\frac{30\,(a+b\,x)^{7/2}-84\,a\,(a+b\,x)^{5/2}+70\,a^{2}\,(a+b\,x)^{3/2}}{105\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/2),x)
```

```
[Out] (30*(a + b*x)^(7/2) - 84*a*(a + b*x)^(5/2) + 70*a^2*(a + b*x)^(3/2))/(105*b
^3)
```

sympy [**B**]   time = 2.04, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}bx\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{21}{2}}bx}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{19}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{48a^{\frac{19}{2}}b^2x^2}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{17}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{17}{2}}b^3x^3}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{100a^{\frac{15}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{96a^{\frac{13}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{30a^{\frac{11}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

3.286 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx &= \int\left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{3/2}(3bx-2a)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-2*a + 3*b*x))/(15*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 35, normalized size = 1.03

$$-\frac{2\sqrt{a+bx}\left(2a^2-abx-3b^2x^2\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^2 - a*b*x - 3*b^2*x^2))/(15*b^2)
```

fricas [A] time = 1.14, size = 30, normalized size = 0.88

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

giac [B] time = 1.28, size = 66, normalized size = 1.94

$$\frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a}{b}+\frac{3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a/b + (3*(b*x + a)^(5/2) - 10
*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\,(-3bx+2a)}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

maxima [A] time = 1.31, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{10\,a\,{\left(a+b\,x\right)}^{3/2}-6\,{\left(a+b\,x\right)}^{5/2}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/2),x)
```

```
[Out] -(10*a*(a + b*x)^(3/2) - 6*(a + b*x)^(5/2))/(15*b^2)
```

sympy [B] time = 1.39, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

3.287 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

fricas [A] time = 1.11, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

giac [A] time = 1.06, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(b*x + a)^(3/2)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x)
[Out] 2/3*(b*x+a)^(3/2)/b
```

maxima [A] time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,(a+b\,x)^{3/2}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2),x)
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

sympy [A] time = 0.07, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

3.288 $\int \frac{\sqrt{a+bx}}{x}\,dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x}\,dx &= 2\sqrt{a+bx}+a\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 2\sqrt{a+bx}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\
&= 2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 1.00

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 1.11, size = 73, normalized size = 2.09

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}, 2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

giac [A] time = 1.13, size = 32, normalized size = 0.91

$$\frac{2a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

maple [A] time = 0.01, size = 28, normalized size = 0.80

$$-2\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

maxima [A] time = 2.92, size = 42, normalized size = 1.20

$$\sqrt{a}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*sqrt(b
*x + a)
```

mupad [B] time = 0.09, size = 27, normalized size = 0.77

$$2\sqrt{a+bx}-2\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x,x)
```

```
[Out] 2*(a + b*x)^(1/2) - 2*a^(1/2)*atanh((a + b*x)^(1/2)/a^(1/2))
```

sympy [B] time = 1.60, size = 68, normalized size = 1.94

$$-2\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

3.289 $\int \frac{\sqrt{a+bx}}{x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^2}\,dx &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= -\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.21

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x*Sqrt[a +
b*x]))
```

IntegrateAlgebraic [A] time = 0.05, size = 39, normalized size = 1.00

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.23, size = 93, normalized size = 2.38

$$\left[\frac{\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}\,a}{2ax},\frac{\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

giac [A] time = 0.96, size = 41, normalized size = 1.05

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

maple [A] time = 0.01, size = 37, normalized size = 0.95

$$2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/x/b-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 2.87, size = 47, normalized size = 1.21

$$\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) - sq
rt(b*x + a)/x
```

mupad [B] time = 0.05, size = 31, normalized size = 0.79

$$-\frac{\sqrt{a+b\,x}}{x}-\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^2,x)
```

```
[Out] - (a + b*x)^(1/2)/x - (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 2.22, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

3.290 $\int \frac{\sqrt{a+bx}}{x^3} \, dx$

Optimal. Leaf size=65

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{\sqrt{a+bx}}{2x^2} - \frac{b\sqrt{a+bx}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^3,x]
```

```
[Out] -Sqrt[a + b*x]/(2*x^2) - (b*Sqrt[a + b*x])/(4*a*x) + (b^2*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^3}\,dx = -\frac{\sqrt{a+bx}}{2x^2}+\frac{1}{4}b\int \frac{1}{x^2\sqrt{a+bx}}\,dx$$
$$= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b^2\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a}$$
$$= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}-\frac{b\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a}$$
$$= -\frac{\sqrt{a+bx}}{2x^2}-\frac{b\sqrt{a+bx}}{4ax}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.54

$$-\frac{2b^2(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^3,x]
[Out] (-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/(3*a^3
)
```

IntegrateAlgebraic [A] time = 0.08, size = 55, normalized size = 0.85

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{a+bx}\,(2a+bx)}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^3,x]
[Out] -1/4*(Sqrt[a + b*x]*(2*a + b*x))/(a*x^2) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[
a]])/(4*a^(3/2))
```

fricas [A] time = 0.98, size = 119, normalized size = 1.83

$$\left[\frac{\sqrt{a}\,b^2x^2\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(abx+2a^2\right)\sqrt{bx+a}}{8a^2x^2}, -\frac{\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(abx+2a^2\right)\sqrt{bx+a}}{4a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="fricas")
[Out] [1/8*(sqrt(a)*b^2*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(a*b
*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*
x + a)*sqrt(-a)/a) + (a*b*x + 2*a^2)*sqrt(b*x + a))/(a^2*x^2)]
```

giac [A] time = 1.14, size = 66, normalized size = 1.02

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{(bx+a)^{\frac{3}{2}}b^3+\sqrt{bx+a}\,ab^3}{ab^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x + a)^(3/2)*b^
3 + sqrt(b*x + a)*a*b^3)/(a*b^2*x^2))/b
```

maple [A] time = 0.01, size = 53, normalized size = 0.82

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{8a}-\frac{\sqrt{bx+a}}{8}}{b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^3,x)
```

```
[Out] 2*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/x^2/b^2+1/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(3/2))
```

maxima [A] time = 3.03, size = 88, normalized size = 1.35

$$-\frac{b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{3}{2}}b^2+\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) -
 1/4*((b*x + a)^(3/2)*b^2 + sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a - 2*(b*x +
a)*a^2 + a^3)
```

mupad [B] time = 0.07, size = 48, normalized size = 0.74

$$\frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{3/2}}-\frac{{\left(a+b\,x\right)}^{3/2}}{4\,a\,x^2}-\frac{\sqrt{a+b\,x}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^3,x)
```

```
[Out] (b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (a + b*x)^(3/2)/(4*a*x^2
) - (a + b*x)^(1/2)/(4*x^2)
```

sympy [A] time = 4.02, size = 97, normalized size = 1.49

$$-\frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{3}{2}}}{4a\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**3,x)
```

```
[Out] -a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(a/(b
*x) + 1)) - b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**2*asinh(sqrt(a)/(
sqrt(b)*sqrt(x)))/(4*a**(3/2))
```

3.291 $\int \frac{\sqrt{a+bx}}{x^4} dx$

Optimal. Leaf size=87

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{\sqrt{a+bx}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -Sqrt[a + b*x]/(3*x^3) - (b*Sqrt[a + b*x])/(12*a*x^2) + (b^2*Sqrt[a + b*x])
/(8*a^2*x) - (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^4}\,dx &= -\frac{\sqrt{a+bx}}{3x^3}+\frac{1}{6}b\int \frac{1}{x^3\sqrt{a+bx}}\,dx \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}-\frac{b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{8a^2} \\
&= -\frac{\sqrt{a+bx}}{3x^3}-\frac{b\sqrt{a+bx}}{12ax^2}+\frac{b^2\sqrt{a+bx}}{8a^2x}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.40

$$\frac{2b^3(a+bx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(3*a^4)
```

IntegrateAlgebraic [A] time = 0.11, size = 71, normalized size = 0.82

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{5/2}}-\frac{\sqrt{a+bx}\left(3a^2+8a(a+bx)-3(a+bx)^2\right)}{24a^2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(3*a^2 + 8*a*(a + b*x) - 3*(a + b*x)^2))/(a^2*x^3) - (
b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(5/2))
```

fricas [A] time = 0.98, size = 145, normalized size = 1.67

$$\left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx+a}}{48a^3x^3},\ \frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3ab^2x^2-2a^2bx-8a^3\right)\sqrt{bx+a}}{24a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x + a))/(a^3*x^3), 1/24*(3*sqrt(-a)
*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 - 2*a^2*b*x - 8*a^
3)*sqrt(b*x + a))/(a^3*x^3)]
```

giac [A] time = 1.23, size = 84, normalized size = 0.97

$$\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3(bx+a)^{\frac{5}{2}}b^4-8(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}a^2b^4}{a^2b^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(5
/2)*b^4 - 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a^2*b^3*x^3))
/b
```

maple [A] time = 0.01, size = 65, normalized size = 0.75

$$2\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{5}{2}}}+\frac{-\frac{(bx+a)^{\frac{3}{2}}}{6a}+\frac{(bx+a)^{\frac{5}{2}}}{16a^{2}}-\frac{\sqrt{bx+a}}{16}}{b^{3}x^{3}}\right)b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^4,x)
```

```
[Out] 2*b^3*((1/16/a^2*(b*x+a)^(5/2)-1/6*(b*x+a)^(3/2)/a-1/16*(b*x+a)^(1/2))/x^3/
b^3-1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2))
```

maxima [A] time = 3.02, size = 121, normalized size = 1.39

$$\frac{b^{3}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}b^{3}-8\,(bx+a)^{\frac{3}{2}}ab^{3}-3\,\sqrt{bx+a}\,a^{2}b^{3}}{24\left((bx+a)^{3}a^{2}-3\,(bx+a)^{2}a^{3}+3\,(bx+a)a^{4}-a^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
 1/24*(3*(b*x + a)^(5/2)*b^3 - 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a^
2*b^3)/((b*x + a)^3*a^2 - 3*(b*x + a)^2*a^3 + 3*(b*x + a)*a^4 - a^5)
```

mupad [B] time = 0.11, size = 66, normalized size = 0.76

$$\frac{(a+b\,x)^{5/2}}{8\,a^{2}\,x^{3}}-\frac{(a+b\,x)^{3/2}}{3\,a\,x^{3}}-\frac{\sqrt{a+b\,x}}{8\,x^{3}}+\frac{b^{3}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^4,x)
```

```
[Out] (a + b*x)^(5/2)/(8*a^2*x^3) - (a + b*x)^(3/2)/(3*a*x^3) - (a + b*x)^(1/2)/(
8*x^3) + (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(5/2))
```

sympy [A] time = 6.68, size = 122, normalized size = 1.40

$$-\frac{a}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{3}{2}}}{24ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{b^{\frac{5}{2}}}{8a^{2}\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**4,x)
```

```
[Out] -a/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 5*sqrt(b)/(12*x**(5/2)*sqrt(a/(
b*x) + 1)) + b**(3/2)/(24*a*x**(3/2)*sqrt(a/(b*x) + 1)) + b**(5/2)/(8*a**2*
sqrt(x)*sqrt(a/(b*x) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a**(5
/2))
```

3.292 $\int x^3(a+bx)^{3/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{11/2}}{11b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(7/2))/(7*b^4) - (2*a*(
a + b*x)^(9/2))/(3*b^4) + (2*(a + b*x)^(11/2))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{3/2}\,dx &= \int\left(-\frac{a^3(a+bx)^{3/2}}{b^3}+\frac{3a^2(a+bx)^{5/2}}{b^3}-\frac{3a(a+bx)^{7/2}}{b^3}+\frac{(a+bx)^{9/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{5/2}}{5b^4}+\frac{6a^2(a+bx)^{7/2}}{7b^4}-\frac{2a(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{11/2}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{5/2}\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(11
55*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 59, normalized size = 0.82

$$-\frac{2\left(231a^3(a+bx)^{5/2}-495a^2(a+bx)^{7/2}-105(a+bx)^{11/2}+385a(a+bx)^{9/2}\right)}{1155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(3/2),x]
```

```
[Out] (-2*(231*a^3*(a + b*x)^(5/2) - 495*a^2*(a + b*x)^(7/2) + 385*a*(a + b*x)^(9
/2) - 105*(a + b*x)^(11/2)))/(1155*b^4)
```

fricas [A] time = 1.23, size = 64, normalized size = 0.89

$$\frac{2\left(105\,b^{5}x^{5}+140\,ab^{4}x^{4}+5\,a^{2}b^{3}x^{3}-6\,a^{3}b^{2}x^{2}+8\,a^{4}bx-16\,a^{5}\right)\sqrt{bx+a}}{1155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

giac [B] time = 1.11, size = 193, normalized size = 2.68

$$\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{2}}{b^{3}}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)a}{b^{3}}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^{2}-1386\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}-693\,\sqrt{bx+a}\,a^{5}\right)}{b^{3}}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a
^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^3 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)
^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x
 + a)*a^4)*a/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*
x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 69
3*sqrt(b*x + a)*a^5)/b^3)/b
```

maple [A] time = 0.00, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{5}{2}}\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)}{1155b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)^(5/2)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.35, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^{4}}-\frac{2\,(bx+a)^{\frac{9}{2}}a}{3\,b^{4}}+\frac{6\,(bx+a)^{\frac{7}{2}}a^{2}}{7\,b^{4}}-\frac{2\,(bx+a)^{\frac{5}{2}}a^{3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^4 - 2/3*(b*x + a)^(9/2)*a/b^4 + 6/7*(b*x + a)^(7/2)
*a^2/b^4 - 2/5*(b*x + a)^(5/2)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{11/2}}{11\,b^{4}}-\frac{2\,a^{3}\,(a+b\,x)^{5/2}}{5\,b^{4}}+\frac{6\,a^{2}\,(a+b\,x)^{7/2}}{7\,b^{4}}-\frac{2\,a\,(a+b\,x)^{9/2}}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b^4) - (2*a^3*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a
 + b*x)^(7/2))/(7*b^4) - (2*a*(a + b*x)^(9/2))/(3*b^4)
```

sympy [B] time = 3.20, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(3/2),x)
```

```
[Out] -32*a**(51/2)*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*
a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**1
5*b**9*x**5 + 1155*a**14*b**10*x**6) + 32*a**(51/2)/(1155*a**20*b**4 + 6930
*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16
*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 176*a**(49/2)*
b*x*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6
*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**
5 + 1155*a**14*b**10*x**6) + 192*a**(49/2)*b*x/(1155*a**20*b**4 + 6930*a**1
9*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8
*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 396*a**(47/2)*b**2*
x**2*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**
6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x*
*5 + 1155*a**14*b**10*x**6) + 480*a**(47/2)*b**2*x**2/(1155*a**20*b**4 + 69
30*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**
16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) - 462*a**(45/2
)*b**3*x**3*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a*
*18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*
b**9*x**5 + 1155*a**14*b**10*x**6) + 640*a**(45/2)*b**3*x**3/(1155*a**20*b*
*4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17
325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 480*a
**(43/2)*b**4*x**4/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*
x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5
 + 1155*a**14*b**10*x**6) + 1848*a**(41/2)*b**5*x**5*sqrt(1 + b*x/a)/(1155*
a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x
**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
 + 192*a**(41/2)*b**5*x**5/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**
18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b
**9*x**5 + 1155*a**14*b**10*x**6) + 5544*a**(39/2)*b**6*x**6*sqrt(1 + b*x/a
)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**1
7*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**
10*x**6) + 32*a**(39/2)*b**6*x**6/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 8844*a**(37/2)*b**7*x**7*sqrt(1
+ b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 231
00*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a*
*14*b**10*x**6) + 8448*a**(35/2)*b**8*x**8*sqrt(1 + b*x/a)/(1155*a**20*b**4
 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1732
5*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 4840*a*
*(33/2)*b**9*x**9*sqrt(1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17
325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*
a**15*b**9*x**5 + 1155*a**14*b**10*x**6) + 1540*a**(31/2)*b**10*x**10*sqrt(
1 + b*x/a)/(1155*a**20*b**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 2
3100*a**17*b**7*x**3 + 17325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*
a**14*b**10*x**6) + 210*a**(29/2)*b**11*x**11*sqrt(1 + b*x/a)/(1155*a**20*b
**4 + 6930*a**19*b**5*x + 17325*a**18*b**6*x**2 + 23100*a**17*b**7*x**3 + 1
7325*a**16*b**8*x**4 + 6930*a**15*b**9*x**5 + 1155*a**14*b**10*x**6)
```

3.293 $\int x^2(a+bx)^{3/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{9/2}}{9b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(5/2))/(5*b^3) - (4*a*(a + b*x)^(7/2))/(7*b^3) + (2*(a + b
*x)^(9/2))/(9*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{3/2}\,dx &= \int \left(\frac{a^2(a+bx)^{3/2}}{b^2}-\frac{2a(a+bx)^{5/2}}{b^2}+\frac{(a+bx)^{7/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{9/2}}{9b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{5/2}\left(8a^2-20abx+35b^2x^2\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 45, normalized size = 0.85

$$\frac{2\left(63a^2(a+bx)^{5/2}+35(a+bx)^{9/2}-90a(a+bx)^{7/2}\right)}{315b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(3/2),x]
```

```
[Out] (2*(63*a^2*(a + b*x)^(5/2) - 90*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/(3
15*b^3)
```

fricas [A] time = 0.75, size = 53, normalized size = 1.00

$$\frac{2\left(35\,b^{4}x^{4}+50\,ab^{3}x^{3}+3\,a^{2}b^{2}x^{2}-4\,a^{3}bx+8\,a^{4}\right)\sqrt{bx+a}}{315\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

giac [B] time = 0.94, size = 156, normalized size = 2.94

$$\frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a^{2}}{b^{2}}+\frac{18\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a}{b^{2}}+\frac{35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}}{b^{2}}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*a^2/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2
)*a^2 - 35*sqrt(b*x + a)*a^3)*a/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(
7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x +
 a)*a^4)/b^2)/b
```

maple [A] time = 0.01, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{5}{2}}\left(35b^{2}x^{2}-20abx+8a^{2}\right)}{315b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(3/2),x)
```

```
[Out] 2/315*(b*x+a)^(5/2)*(35*b^2*x^2-20*a*b*x+8*a^2)/b^3
```

maxima [A] time = 1.35, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^{3}}-\frac{4\,(bx+a)^{\frac{7}{2}}a}{7\,b^{3}}+\frac{2\,(bx+a)^{\frac{5}{2}}a^{2}}{5\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^3 - 4/7*(b*x + a)^(7/2)*a/b^3 + 2/5*(b*x + a)^(5/2)*a
^2/b^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{70\,(a+b\,x)^{9/2}-180\,a\,(a+b\,x)^{7/2}+126\,a^{2}\,(a+b\,x)^{5/2}}{315\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(3/2),x)
```

```
[Out] (70*(a + b*x)^(9/2) - 180*a*(a + b*x)^(7/2) + 126*a^2*(a + b*x)^(5/2))/(315
*b^3)
```

sympy [B] time = 2.17, size = 733, normalized size = 13.83

$$\frac{16a^{\frac{25}{2}}\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{25}{2}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{40a^{\frac{23}{2}}bx\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{23}{2}}bx}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{30a^{\frac{21}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{48a^{\frac{21}{2}}b^2x^2}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{110a^{\frac{19}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}-\frac{16a^{\frac{19}{2}}b^3x^3}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{380a^{\frac{17}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{516a^{\frac{15}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{310a^{\frac{13}{2}}b^6x^6\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}+\frac{70a^{\frac{11}{2}}b^7x^7\sqrt{1+\frac{bx}{a}}}{315a^8b^3+945a^7b^4x+945a^6b^5x^2+315a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(3/2),x)
```

```
[Out] 16*a**(25/2)*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b*
*5*x**2 + 315*a**5*b**6*x**3) - 16*a**(25/2)/(315*a**8*b**3 + 945*a**7*b**4
*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 40*a**(23/2)*b*x*sqrt(1 + b
*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6
*x**3) - 48*a**(23/2)*b*x/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*
x**2 + 315*a**5*b**6*x**3) + 30*a**(21/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a*
*8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 48*a
**(21/2)*b**2*x**2/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 +
315*a**5*b**6*x**3) + 110*a**(19/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**8*b**
3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) - 16*a**(19/
2)*b**3*x**3/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a*
*5*b**6*x**3) + 380*a**(17/2)*b**4*x**4*sqrt(1 + b*x/a)/(315*a**8*b**3 + 94
5*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 516*a**(15/2)*b*
*5*x**5*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x*
*2 + 315*a**5*b**6*x**3) + 310*a**(13/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*a**
8*b**3 + 945*a**7*b**4*x + 945*a**6*b**5*x**2 + 315*a**5*b**6*x**3) + 70*a*
*(11/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**8*b**3 + 945*a**7*b**4*x + 945*a*
*6*b**5*x**2 + 315*a**5*b**6*x**3)
```

3.294 $\int x(a+bx)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{7/2}}{7b^2}-\frac{2a(a+bx)^{5/2}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(3/2),x]
[Out] (-2*a*(a + b*x)^(5/2))/(5*b^2) + (2*(a + b*x)^(7/2))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{3/2}\,dx &= \int\left(-\frac{a(a+bx)^{3/2}}{b}+\frac{(a+bx)^{5/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{5/2}}{5b^2}+\frac{2(a+bx)^{7/2}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{5/2}(5bx-2a)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(3/2),x]
[Out] (2*(a + b*x)^(5/2)*(-2*a + 5*b*x))/(35*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 46, normalized size = 1.35

$$-\frac{2\sqrt{a+bx}\left(2a^3-a^2bx-8ab^2x^2-5b^3x^3\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[a + b*x]*(2*a^3 - a^2*b*x - 8*a*b^2*x^2 - 5*b^3*x^3))/(35*b^2)
```

fricas [A] time = 0.81, size = 41, normalized size = 1.21

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

giac [B] time = 1.24, size = 119, normalized size = 3.50

$$\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^{2}}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)}{b}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2/b + 14*(3*(b*x + a)^(5/
2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a/b + 3*(5*(b*x + a)^(7/2
) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)/b
)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{5}{2}}\,(-5bx+2a)}{35b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(3/2),x)
```

```
[Out] -2/35*(b*x+a)^(5/2)*(-5*b*x+2*a)/b^2
```

maxima [A] time = 1.35, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{2}}-\frac{2\,(bx+a)^{\frac{5}{2}}a}{5\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^2 - 2/5*(b*x + a)^(5/2)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{14\,a\,{\left(a+b\,x\right)}^{5/2}-10\,{\left(a+b\,x\right)}^{7/2}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(3/2),x)
```

```
[Out] -(14*a*(a + b*x)^(5/2) - 10*(a + b*x)^(7/2))/(35*b^2)
```

sympy [A] time = 0.74, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{4a^{3}\sqrt{a+bx}}{35b^{2}}+\frac{2a^{2}x\sqrt{a+bx}}{35b}+\frac{16ax^{2}\sqrt{a+bx}}{35}+\frac{2bx^{3}\sqrt{a+bx}}{7} & \text{for } b\neq 0 \\ \frac{a^{\frac{3}{2}}x^{2}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-4*a**3*sqrt(a + b*x)/(35*b**2) + 2*a**2*x*sqrt(a + b*x)/(35*b)
+ 16*a*x**2*sqrt(a + b*x)/35 + 2*b*x**3*sqrt(a + b*x)/7, Ne(b, 0)), (a**(3/
2)*x**2/2, True))
```

3.295 $\int (a+bx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{5/2}}{5b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2),x]
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{3/2}\,dx=\frac{2(a+bx)^{5/2}}{5b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2),x]
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2),x]
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

fricas [B] time = 0.61, size = 28, normalized size = 1.75

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

giac [B] time = 1.13, size = 58, normalized size = 3.62

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+30\,\sqrt{bx+a}\,a^2+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 30*sqrt(b*x + a)*a^2 + 10*
((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)^(5/2)/b
```

maxima [A] time = 1.29, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b)
```

sympy [A] time = 0.07, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2),x)
```

```
[Out] 2*(a + b*x)**(5/2)/(5*b)
```

3.296 $\int \frac{(a+bx)^{3/2}}{x}\,dx$

Optimal. Leaf size=49

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x,x]
```

```
[Out] 2*a*Sqrt[a + b*x] + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]
/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{3/2}}{x}\,dx &= \frac{2}{3}(a+bx)^{3/2}+a\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+a^2\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2a\sqrt{a+bx}+\frac{2}{3}(a+bx)^{3/2}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.90

$$\frac{2}{3}\sqrt{a+bx}\,(4a+bx)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.03, size = 50, normalized size = 1.02

$$\frac{2}{3}\left((a+bx)^{3/2}+3a\sqrt{a+bx}\right)-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x,x]
```

```
[Out] (2*(3*a*Sqrt[a + b*x] + (a + b*x)^(3/2)))/3 - 2*a^(3/2)*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]]
```

fricas [A] time = 1.14, size = 88, normalized size = 1.80

$$\left[a^{\frac{3}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+a},\,2\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{3}(bx+4a)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(3/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/3*(b*x + 4*a)*sqr
t(b*x + a), 2*sqrt(-a)*a*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/3*(b*x + 4*a)
*sqrt(b*x + a)]
```

giac [A] time = 1.08, size = 44, normalized size = 0.90

$$\frac{2\,a^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{3}(bx+a)^{\frac{3}{2}}+2\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/3*(b*x + a)^(3/2) + 2*sqr
t(b*x + a)*a
```

maple [A] time = 0.01, size = 38, normalized size = 0.78

$$-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\,a+\frac{2(bx+a)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x+a)^(3/2)-2*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a*(b*x+a)^(1/2
)
```

maxima [A] time = 2.94, size = 52, normalized size = 1.06

$$a^{\frac{3}{2}} \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}+2\,\sqrt{bx+a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] a^(3/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(b*x
 + a)^(3/2) + 2*sqrt(b*x + a)*a
```

mupad [B] time = 0.04, size = 37, normalized size = 0.76

$$2\,a\,\sqrt{a+b\,x}+\frac{2\,(a+b\,x)^{3/2}}{3}-2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x,x)
```

```
[Out] 2*a*(a + b*x)^(1/2) + (2*(a + b*x)^(3/2))/3 - 2*a^(3/2)*atanh((a + b*x)^(1/
2)/a^(1/2))
```

sympy [A] time = 2.29, size = 71, normalized size = 1.45

$$\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{3}+a^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,bx\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x,x)
```

```
[Out] 8*a**(3/2)*sqrt(1 + b*x/a)/3 + a**(3/2)*log(b*x/a) - 2*a**(3/2)*log(sqrt(1
+ b*x/a) + 1) + 2*sqrt(a)*b*x*sqrt(1 + b*x/a)/3
```

3.297 $\int \frac{(a+bx)^{3/2}}{x^2} dx$

Optimal. Leaf size=51

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{(a+bx)^{3/2}}{x}+3b\sqrt{a+bx}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[a + b*x] - (a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/S
qrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^2}\,dx &= -\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3b)\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+\frac{1}{2}(3ab)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}+(3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= 3b\sqrt{a+bx}-\frac{(a+bx)^{3/2}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.65

$$
\frac{2b(a+bx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 0.96

$$
\frac{\sqrt{a+bx}\,(2(a+bx)-3a)}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a + 2*(a + b*x)))/x - 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]]
```

fricas [A] time = 1.04, size = 102, normalized size = 2.00

$$
\left[\frac{3\sqrt{a}\,bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\,(2\,bx-a)\sqrt{bx+a}}{2\,x},\ \frac{3\sqrt{-a}\,bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+(2\,bx-a)\sqrt{bx+a}}{x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b*x
 - a)*sqrt(b*x + a))/x, (3*sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) +
(2*b*x - a)*sqrt(b*x + a))/x]
```

giac [A] time = 1.04, size = 56, normalized size = 1.10

$$
\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\sqrt{bx+a}\,b^2-\frac{\sqrt{bx+a}\,ab}{x}}{b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)*b^2 - sq
rt(b*x + a)*a*b/x)/b
```

maple [A] time = 0.01, size = 47, normalized size = 0.92

$$2\left(\left(-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a+\sqrt{bx+a}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^2,x)
```

```
[Out] 2*b*((b*x+a)^(1/2)+a*(-1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^(1/2)/a^(1
/2))/a^(1/2)))
```

maxima [A] time = 2.87, size = 58, normalized size = 1.14

$$\frac{3}{2}\sqrt{a}\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b-\frac{\sqrt{bx+a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(a)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2*
sqrt(b*x + a)*b - sqrt(b*x + a)*a/x
```

mupad [B] time = 0.10, size = 42, normalized size = 0.82

$$2\,b\,\sqrt{a+b\,x}-3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)-\frac{a\,\sqrt{a+b\,x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^2,x)
```

```
[Out] 2*b*(a + b*x)^(1/2) - 3*a^(1/2)*b*atanh((a + b*x)^(1/2)/a^(1/2)) - (a*(a +
b*x)^(1/2))/x
```

sympy [B] time = 2.66, size = 92, normalized size = 1.80

$$-3\sqrt{a}\,b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{a^{2}}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{a\sqrt{b}}{\sqrt{x}\,\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**2,x)
```

```
[Out] -3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt
(a/(b*x) + 1)) + a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)
/sqrt(a/(b*x) + 1)
```

3.298 $\int \frac{(a+bx)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=62

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

Rubi [A] time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b\sqrt{a+bx}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[a + b*x])/(4*x) - (a + b*x)^(3/2)/(2*x^2) - (3*b^2*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{3/2}}{x^3}\,dx &= -\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)\\
&= -\frac{3b\sqrt{a+bx}}{4x}-\frac{(a+bx)^{3/2}}{2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.10

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqr
t[1 + (b*x)/a]])/(x^2*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 56, normalized size = 0.90

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{\sqrt{a+bx}\,(5(a+bx)-3a)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[a + b*x]*(-3*a + 5*(a + b*x)))/x^2 - (3*b^2*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/(4*Sqrt[a])
```

fricas [A] time = 1.12, size = 124, normalized size = 2.00

$$\left[\frac{3\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(5\,abx+2\,a^2\right)\sqrt{bx+a}}{8\,ax^2},\ \frac{3\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\left(5\,abx+2\,a^2\right)\sqrt{bx+a}}{4\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(5
*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sqrt
(b*x + a)*sqrt(-a)/a) - (5*a*b*x + 2*a^2)*sqrt(b*x + a))/(a*x^2)]
```

giac [A] time = 1.29, size = 64, normalized size = 1.03

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3-3\,\sqrt{bx+a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3
 - 3*sqrt(b*x + a)*a*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 51, normalized size = 0.82

$$2\left(-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{\frac{3\sqrt{bx+a}\,a}{8}-\frac{5(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^3,x)
```

```
[Out] 2*b^2*((-5/8*(b*x+a)^(3/2)+3/8*(b*x+a)^(1/2)*a)/x^2/b^2-3/8*arctanh((b*x+a)
^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 2.98, size = 86, normalized size = 1.39

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,\sqrt{a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^2-3\,\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
1/4*(5*(b*x + a)^(3/2)*b^2 - 3*sqrt(b*x + a)*a*b^2)/((b*x + a)^2 - 2*(b*x +
 a)*a + a^2)
```

mupad [B] time = 0.06, size = 46, normalized size = 0.74

$$\frac{3\,a\,\sqrt{a+b\,x}}{4\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^3,x)
```

```
[Out] (3*a*(a + b*x)^(1/2))/(4*x^2) - (3*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a
^(1/2)) - (5*(a + b*x)^(3/2))/(4*x^2)
```

sympy [A] time = 3.28, size = 76, normalized size = 1.23

$$-\frac{a\sqrt{b}\,\sqrt{\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**3,x)
```

```
[Out] -a*sqrt(b)*sqrt(a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(a/(b*x) + 1)/(4
*sqrt(x)) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a))
```

3.299 $\int \frac{(a+bx)^{3/2}}{x^4} dx$

Optimal. Leaf size=84

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{(a+bx)^{3/2}}{3x^3} - \frac{b\sqrt{a+bx}}{4x^2}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{b\sqrt{a+bx}}{4x^2} - \frac{(a+bx)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] -(b*Sqrt[a + b*x])/(4*x^2) - (b^2*Sqrt[a + b*x])/(8*a*x) - (a + b*x)^(3/2)/
(3*x^3) + (b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{x^4}\,dx = -\frac{(a+bx)^{3/2}}{3x^3} + \frac{1}{2}b\int \frac{\sqrt{a+bx}}{x^3}\,dx$$
$$= -\frac{b\sqrt{a+bx}}{4x^2} - \frac{(a+bx)^{3/2}}{3x^3} + \frac{1}{8}b^2\int \frac{1}{x^2\sqrt{a+bx}}\,dx$$
$$= -\frac{b\sqrt{a+bx}}{4x^2} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{(a+bx)^{3/2}}{3x^3} - \frac{b^3\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a}$$
$$= -\frac{b\sqrt{a+bx}}{4x^2} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{(a+bx)^{3/2}}{3x^3} - \frac{b^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{8a}$$
$$= -\frac{b\sqrt{a+bx}}{4x^2} - \frac{b^2\sqrt{a+bx}}{8ax} - \frac{(a+bx)^{3/2}}{3x^3} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.42

$$\frac{2b^3(a+bx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(5*a^4)
```

IntegrateAlgebraic [A] time = 0.12, size = 71, normalized size = 0.85

$$\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{3/2}} + \frac{\sqrt{a+bx}\left(3a^2-8a(a+bx)-3(a+bx)^2\right)}{24ax^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x]*(3*a^2 - 8*a*(a + b*x) - 3*(a + b*x)^2))/(24*a*x^3) + (b^3*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(3/2))
```

fricas [A] time = 1.01, size = 145, normalized size = 1.73

$$\left[\frac{3\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3ab^2x^2+14a^2bx+8a^3\right)\sqrt{bx+a}}{48a^2x^3}, -\frac{3\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3ab^2x^2+14a^2bx+8a^3\right)\sqrt{bx+a}}{24a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^2*x^3), -1/24*(3*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b^2*x^2 + 14*a^2*b*x + 8
*a^3)*sqrt(b*x + a))/(a^2*x^3)]
```

giac [A] time = 1.16, size = 84, normalized size = 1.00

$$-\frac{\frac{3b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3(bx+a)^{\frac{5}{2}}b^4+8(bx+a)^{\frac{3}{2}}ab^4-3\sqrt{bx+a}\,a^2b^4}{ab^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x + a)^(5/
2)*b^4 + 8*(b*x + a)^(3/2)*a*b^4 - 3*sqrt(b*x + a)*a^2*b^4)/(a*b^3*x^3))/b
```

maple [A] time = 0.01, size = 63, normalized size = 0.75

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16a^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a}{16}-\frac{(bx+a)^{\frac{5}{2}}}{16a}-\frac{(bx+a)^{\frac{3}{2}}}{6}}{b^3x^3}\right)b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^4,x)
```

```
[Out] 2*b^3*((-1/16/a*(b*x+a)^(5/2)-1/6*(b*x+a)^(3/2)+1/16*(b*x+a)^(1/2)*a)/x^3/b
^3+1/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

maxima [A] time = 3.05, size = 119, normalized size = 1.42

$$-\frac{b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}b^3+8\,(bx+a)^{\frac{3}{2}}ab^3-3\,\sqrt{bx+a}\,a^2b^3}{24\left((bx+a)^3a-3\,(bx+a)^2a^2+3\,(bx+a)a^3-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
- 1/24*(3*(b*x + a)^(5/2)*b^3 + 8*(b*x + a)^(3/2)*a*b^3 - 3*sqrt(b*x + a)*a
^2*b^3)/((b*x + a)^3*a - 3*(b*x + a)^2*a^2 + 3*(b*x + a)*a^3 - a^4)
```

mupad [B] time = 0.10, size = 64, normalized size = 0.76

$$\frac{a\sqrt{a+b\,x}}{8\,x^3}-\frac{(a+b\,x)^{5/2}}{8\,a\,x^3}-\frac{(a+b\,x)^{3/2}}{3\,x^3}-\frac{b^3\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)1\mathrm{i}}{8\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^4,x)
```

```
[Out] (a*(a + b*x)^(1/2))/(8*x^3) - (a + b*x)^(5/2)/(8*a*x^3) - (b^3*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*1i)/(8*a^(3/2)) - (a + b*x)^(3/2)/(3*x^3)
```

sympy [A] time = 5.84, size = 124, normalized size = 1.48

$$-\frac{a^2}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{17b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{b^{\frac{5}{2}}}{8a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**4,x)
```

```
[Out] -a**2/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) - 11*a*sqrt(b)/(12*x**(5/2)*sq
rt(a/(b*x) + 1)) - 17*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) - b**(5/2)/(
8*a*sqrt(x)*sqrt(a/(b*x) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*a
**(3/2))
```

3.300 $\int x^3(a+bx)^{5/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2(a+bx)^{13/2}}{13b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a + b*x)^(9/2))/(3*b^4) - (6*a*(
a + b*x)^(11/2))/(11*b^4) + (2*(a + b*x)^(13/2))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{5/2}\,dx&=\int\left(-\frac{a^3(a+bx)^{5/2}}{b^3}+\frac{3a^2(a+bx)^{7/2}}{b^3}-\frac{3a(a+bx)^{9/2}}{b^3}+\frac{(a+bx)^{11/2}}{b^3}\right)dx\\&=-\frac{2a^3(a+bx)^{7/2}}{7b^4}+\frac{2a^2(a+bx)^{9/2}}{3b^4}-\frac{6a(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{13/2}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{7/2}\left(-16a^3+56a^2bx-126ab^2x^2+231b^3x^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-16*a^3 + 56*a^2*b*x - 126*a*b^2*x^2 + 231*b^3*x^3))/(3
003*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}\left(-429a^3+1001a^2(a+bx)-819a(a+bx)^2+231(a+bx)^3\right)}{3003b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-429*a^3 + 1001*a^2*(a + b*x) - 819*a*(a + b*x)^2 + 231
*(a + b*x)^3))/(3003*b^4)
```

fricas [A] time = 1.00, size = 75, normalized size = 1.04

$$\frac{2\left(231\,b^{6}x^{6}+567\,ab^{5}x^{5}+371\,a^{2}b^{4}x^{4}+5\,a^{3}b^{3}x^{3}-6\,a^{4}b^{2}x^{2}+8\,a^{5}bx-16\,a^{6}\right)\sqrt{bx+a}}{3003\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a
^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)/b^4
```

giac [B] time = 1.16, size = 281, normalized size = 3.90

$$\frac{2\left(\frac{429\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{3}}{b^{3}}+\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)a^{2}}{b^{3}}+\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^{2}-1386\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}-693\,\sqrt{bx+a}\,a^{5}\right)a}{b^{3}}+\frac{5\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^{2}-8580\,(bx+a)^{\frac{7}{2}}a^{3}+9009\,(bx+a)^{\frac{5}{2}}a^{4}-6006\,(bx+a)^{\frac{3}{2}}a^{5}+3003\,\sqrt{bx+a}\,a^{6}\right)}{b^{3}}\right)}{15015\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(429*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)
*a^2 - 35*sqrt(b*x + a)*a^3)*a^3/b^3 + 143*(35*(b*x + a)^(9/2) - 180*(b*x +
 a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(
b*x + a)*a^4)*a^2/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)*a/b^3 + 5*(231*(b*x + a)^(13/2) - 1638*(b*x + a)
^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x
 + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b^3)/b
```

maple [A] time = 0.01, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{7}{2}}\left(-231b^{3}x^{3}+126a\,b^{2}x^{2}-56a^{2}bx+16a^{3}\right)}{3003b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(5/2),x)
```

```
[Out] -2/3003*(b*x+a)^(7/2)*(-231*b^3*x^3+126*a*b^2*x^2-56*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.35, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^{4}}-\frac{6\,(bx+a)^{\frac{11}{2}}\,a}{11\,b^{4}}+\frac{2\,(bx+a)^{\frac{9}{2}}a^{2}}{3\,b^{4}}-\frac{2\,(bx+a)^{\frac{7}{2}}a^{3}}{7\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^4 - 6/11*(b*x + a)^(11/2)*a/b^4 + 2/3*(b*x + a)^(9/
2)*a^2/b^4 - 2/7*(b*x + a)^(7/2)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{13/2}}{13\,b^4}-\frac{2\,a^3\,{\left(a+b\,x\right)}^{7/2}}{7\,b^4}+\frac{2\,a^2\,{\left(a+b\,x\right)}^{9/2}}{3\,b^4}-\frac{6\,a\,{\left(a+b\,x\right)}^{11/2}}{11\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(13/2))/(13*b^4) - (2*a^3*(a + b*x)^(7/2))/(7*b^4) + (2*a^2*(a
 + b*x)^(9/2))/(3*b^4) - (6*a*(a + b*x)^(11/2))/(11*b^4)
```

sympy [A] time = 4.47, size = 146, normalized size = 2.03

$$\begin{cases} -\frac{32a^6\sqrt{a+bx}}{3003b^4} + \frac{16a^5x\sqrt{a+bx}}{3003b^3} - \frac{4a^4x^2\sqrt{a+bx}}{1001b^2} + \frac{10a^3x^3\sqrt{a+bx}}{3003b} + \frac{106a^2x^4\sqrt{a+bx}}{429} + \frac{54abx^5\sqrt{a+bx}}{143} + \frac{2b^2x^6\sqrt{a+bx}}{13} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**6*sqrt(a + b*x)/(3003*b**4) + 16*a**5*x*sqrt(a + b*x)/(30
03*b**3) - 4*a**4*x**2*sqrt(a + b*x)/(1001*b**2) + 10*a**3*x**3*sqrt(a + b*
x)/(3003*b) + 106*a**2*x**4*sqrt(a + b*x)/429 + 54*a*b*x**5*sqrt(a + b*x)/1
43 + 2*b**2*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

3.301 $\int x^2(a+bx)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{7/2}}{7b^3}+\frac{2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(7/2))/(7*b^3) - (4*a*(a + b*x)^(9/2))/(9*b^3) + (2*(a + b
*x)^(11/2))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{5/2}\,dx &= \int\left(\frac{a^2(a+bx)^{5/2}}{b^2}-\frac{2a(a+bx)^{7/2}}{b^2}+\frac{(a+bx)^{9/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{9/2}}{9b^3}+\frac{2(a+bx)^{11/2}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{7/2}\left(8a^2-28abx+63b^2x^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(8*a^2 - 28*a*b*x + 63*b^2*x^2))/(693*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.74

$$\frac{2(a+bx)^{7/2}\left(99a^2-154a(a+bx)+63(a+bx)^2\right)}{693b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(99*a^2 - 154*a*(a + b*x) + 63*(a + b*x)^2))/(693*b^3)
```

fricas [**A**] time = 1.17, size = 64, normalized size = 1.21

$$\frac{2\left(63\,b^{5}x^{5}+161\,ab^{4}x^{4}+113\,a^{2}b^{3}x^{3}+3\,a^{3}b^{2}x^{2}-4\,a^{4}bx+8\,a^{5}\right)\sqrt{bx+a}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^4
*b*x + 8*a^5)*sqrt(b*x + a)/b^3
```

giac [**B**] time = 1.09, size = 233, normalized size = 4.40

$$\frac{2\left(\frac{231\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a^{3}}{b^{2}}+\frac{297\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{2}}{b^{2}}+\frac{33\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)a}{b^{2}}+\frac{5\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^{2}-1386\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}-693\,\sqrt{bx+a}\,a^{5}\right)}{b^{2}}\right)}{3465\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*a^3/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(
3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2/b^2 + 33*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a/b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^
4 - 693*sqrt(b*x + a)*a^5)/b^2)/b
```

maple [**A**] time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{7}{2}}\left(63b^{2}x^{2}-28abx+8a^{2}\right)}{693b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(5/2),x)
```

```
[Out] 2/693*(b*x+a)^(7/2)*(63*b^2*x^2-28*a*b*x+8*a^2)/b^3
```

maxima [**A**] time = 1.37, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b^{3}}-\frac{4\,(bx+a)^{\frac{9}{2}}a}{9\,b^{3}}+\frac{2\,(bx+a)^{\frac{7}{2}}a^{2}}{7\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b^3 - 4/9*(b*x + a)^(9/2)*a/b^3 + 2/7*(b*x + a)^(7/2)
*a^2/b^3
```

mupad [**B**] time = 0.04, size = 37, normalized size = 0.70

$$\frac{126\,(a+b\,x)^{11/2}-308\,a\,(a+b\,x)^{9/2}+198\,a^{2}\,(a+b\,x)^{7/2}}{693\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(5/2),x)
```

```
[Out] (126*(a + b*x)^(11/2) - 308*a*(a + b*x)^(9/2) + 198*a^2*(a + b*x)^(7/2))/(6
93*b^3)
```

sympy [A] time = 3.77, size = 124, normalized size = 2.34

$$\begin{cases} \frac{16a^5\sqrt{a+bx}}{693b^3} - \frac{8a^4x\sqrt{a+bx}}{693b^2} + \frac{2a^3x^2\sqrt{a+bx}}{231b} + \frac{226a^2x^3\sqrt{a+bx}}{693} + \frac{46abx^4\sqrt{a+bx}}{99} + \frac{2b^2x^5\sqrt{a+bx}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**5*sqrt(a + b*x)/(693*b**3) - 8*a**4*x*sqrt(a + b*x)/(693*b
**2) + 2*a**3*x**2*sqrt(a + b*x)/(231*b) + 226*a**2*x**3*sqrt(a + b*x)/693
+ 46*a*b*x**4*sqrt(a + b*x)/99 + 2*b**2*x**5*sqrt(a + b*x)/11, Ne(b, 0)), (
a**(5/2)*x**3/3, True))
```

3.302 $\int x(a+bx)^{5/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{9/2}}{9b^2}-\frac{2a(a+bx)^{7/2}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*a*(a + b*x)^(7/2))/(7*b^2) + (2*(a + b*x)^(9/2))/(9*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{5/2}\,dx &= \int \left(-\frac{a(a+bx)^{5/2}}{b}+\frac{(a+bx)^{7/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{7/2}}{7b^2}+\frac{2(a+bx)^{9/2}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{7/2}(7bx-2a)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*(-2*a + 7*b*x))/(63*b^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 57, normalized size = 1.68

$$-\frac{2\sqrt{a+bx}\left(2a^4-a^3bx-15a^2b^2x^2-19ab^3x^3-7b^4x^4\right)}{63b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^4 - a^3*b*x - 15*a^2*b^2*x^2 - 19*a*b^3*x^3 - 7*b^4*
x^4))/(63*b^2)
```

fricas [A] time = 1.10, size = 52, normalized size = 1.53

$$\frac{2\left(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4\right)\sqrt{bx+a}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x
 + a)/b^2
```

giac [B] time = 1.15, size = 182, normalized size = 5.35

$$\frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^3}{b}+\frac{63\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^2}{b}+\frac{27\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a}{b}+\frac{35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4}{b}\right)}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3/b + 63*(3*(b*x + a)^(5
/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2/b + 27*(5*(b*x + a)^
(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^
3)*a/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*
a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{7}{2}}\,(-7bx+2a)}{63b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(5/2),x)
```

```
[Out] -2/63*(b*x+a)^(7/2)*(-7*b*x+2*a)/b^2
```

maxima [A] time = 1.37, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{9}{2}}}{9\,b^2}-\frac{2\,(bx+a)^{\frac{7}{2}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^2 - 2/7*(b*x + a)^(7/2)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{18\,a\,(a+b\,x)^{7/2}-14\,(a+b\,x)^{9/2}}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(5/2),x)
```

```
[Out] -(18*a*(a + b*x)^(7/2) - 14*(a + b*x)^(9/2))/(63*b^2)
```

sympy [A] time = 2.60, size = 102, normalized size = 3.00

$$\begin{cases} -\frac{4a^4\sqrt{a+bx}}{63b^2}+\frac{2a^3x\sqrt{a+bx}}{63b}+\frac{10a^2x^2\sqrt{a+bx}}{21}+\frac{38abx^3\sqrt{a+bx}}{63}+\frac{2b^2x^4\sqrt{a+bx}}{9} & \text{for } b\neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a**4*sqrt(a + b*x)/(63*b**2) + 2*a**3*x*sqrt(a + b*x)/(63*b)
+ 10*a**2*x**2*sqrt(a + b*x)/21 + 38*a*b*x**3*sqrt(a + b*x)/63 + 2*b**2*x**
4*sqrt(a + b*x)/9, Ne(b, 0)), (a**(5/2)*x**2/2, True))
```

3.303 $\int (a+bx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{7/2}}{7b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{5/2}\,dx=\frac{2(a+bx)^{7/2}}{7b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

fricas [B] time = 0.98, size = 39, normalized size = 2.44

$$\frac{2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\sqrt{bx+a}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)/b
```

giac [B] time = 0.98, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2+35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2+7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 3
5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2 + 7*(3*(b*x + a)^(5/2) - 10*(b*
x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^(7/2)/b
```

maxima [A] time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{7/2}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b)
```

sympy [A] time = 0.08, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{7}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2),x)
```

```
[Out] 2*(a + b*x)**(7/2)/(7*b)
```

3.304 $\int \frac{(a+bx)^{5/2}}{x}\,dx$

Optimal. Leaf size=65

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^2\sqrt{a+bx}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[a + b*x] + (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 - 2*a
^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(a+bx)^{5/2}+a\int \frac{(a+bx)^{3/2}}{x}\,dx\\
&= \frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^2\int \frac{\sqrt{a+bx}}{x}\,dx\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+a^3\int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}+\frac{\left(2a^3\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\
&= 2a^2\sqrt{a+bx}+\frac{2}{3}a(a+bx)^{3/2}+\frac{2}{5}(a+bx)^{5/2}-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 0.89

$$-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{3}a\sqrt{a+bx}\,(4a+bx)+\frac{2}{5}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(a + b*x)^(5/2))/5 + (2*a*Sqrt[a + b*x]*(4*a + b*x))/3 - 2*a^(5/2)*ArcTa
nh[Sqrt[a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.03, size = 66, normalized size = 1.02

$$\frac{2}{15}\left(15a^2\sqrt{a+bx}+3(a+bx)^{5/2}+5a(a+bx)^{3/2}\right)-2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[a + b*x] + 5*a*(a + b*x)^(3/2) + 3*(a + b*x)^(5/2)))/15 - 2
*a^(5/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 1.16, size = 114, normalized size = 1.75

$$\left[a^{\frac{5}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a},2\sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{15}\left(3b^2x^2+11abx+23a^2\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(5/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/15*(3*b^2*x^2 + 1
1*a*b*x + 23*a^2)*sqrt(b*x + a), 2*sqrt(-a)*a^2*arctan(sqrt(b*x + a)*sqrt(-
a)/a) + 2/15*(3*b^2*x^2 + 11*a*b*x + 23*a^2)*sqrt(b*x + a)]
```

giac [A] time = 1.03, size = 56, normalized size = 0.86

$$\frac{2a^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{5}(bx+a)^{\frac{5}{2}}+\frac{2}{3}(bx+a)^{\frac{3}{2}}a+2\sqrt{bx+a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/5*(b*x + a)^(5/2) + 2/3*(
b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

maple [A] time = 0.01, size = 50, normalized size = 0.77

$$-2a^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\,a^2+\frac{2(bx+a)^{\frac{3}{2}}a}{3}+\frac{2(bx+a)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x,x)
```

```
[Out] 2/3*a*(b*x+a)^(3/2)+2/5*(b*x+a)^(5/2)-2*a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/
2))+2*a^2*(b*x+a)^(1/2)
```

maxima [A] time = 3.02, size = 64, normalized size = 0.98

$$a^{\frac{5}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{5}(bx+a)^{\frac{5}{2}}+\frac{2}{3}(bx+a)^{\frac{3}{2}}a+2\sqrt{bx+a}\,a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] a^(5/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/5*(b*x
 + a)^(5/2) + 2/3*(b*x + a)^(3/2)*a + 2*sqrt(b*x + a)*a^2
```

mupad [B] time = 0.05, size = 52, normalized size = 0.80

$$\frac{2\,a\,(a+b\,x)^{3/2}}{3}+\frac{2\,(a+b\,x)^{5/2}}{5}+2\,a^2\,\sqrt{a+b\,x}+a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(3/2))/3 + (2*(a + b*x)^(5/2))/5 + 2*a^2*(a + b*x)^(1/2) + a
^(5/2)*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

sympy [A] time = 4.12, size = 97, normalized size = 1.49

$$\frac{46a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{15}+a^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{22a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}\,b^2x^2\sqrt{1+\frac{bx}{a}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x,x)
```

```
[Out] 46*a**(5/2)*sqrt(1 + b*x/a)/15 + a**(5/2)*log(b*x/a) - 2*a**(5/2)*log(sqrt(
1 + b*x/a) + 1) + 22*a**(3/2)*b*x*sqrt(1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*
sqrt(1 + b*x/a)/5
```

3.305 $\int \frac{(a+bx)^{5/2}}{x^2} dx$

Optimal. Leaf size=66

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{5/2}}{x}+\frac{5}{3}b(a+bx)^{3/2}+5ab\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] 5*a*b*Sqrt[a + b*x] + (5*b*(a + b*x)^(3/2))/3 - (a + b*x)^(5/2)/x - 5*a^(3/
2)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^2}\,dx &= -\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5b)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= \frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}(5ab)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}+\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right) \\
&= 5ab\sqrt{a+bx}+\frac{5}{3}b(a+bx)^{3/2}-\frac{(a+bx)^{5/2}}{x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.50

$$
\frac{2b(a+bx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + (b*x)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 64, normalized size = 0.97

$$
\frac{\sqrt{a+bx}\left(-15a^2+10a(a+bx)+2(a+bx)^2\right)}{3x}-5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-15*a^2 + 10*a*(a + b*x) + 2*(a + b*x)^2))/(3*x) - 5*a^(3/2
)*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 0.80, size = 126, normalized size = 1.91

$$
\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{6\,x},\ \frac{15\,\sqrt{-a}\,abx\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(2\,b^2x^2+14\,abx-3\,a^2\right)\sqrt{bx+a}}{3\,x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(2*b^
2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x, 1/3*(15*sqrt(-a)*a*b*x*arctan(s
qrt(b*x + a)*sqrt(-a)/a) + (2*b^2*x^2 + 14*a*b*x - 3*a^2)*sqrt(b*x + a))/x]
```

giac [A] time = 1.09, size = 74, normalized size = 1.12

$$
\frac{\frac{15\,a^2b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2\,(bx+a)^{\frac{3}{2}}b^2+12\,\sqrt{bx+a}\,ab^2-\frac{3\,\sqrt{bx+a}\,a^2b}{x}}{3\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^2*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*(b*x + a)^(3/2)
*b^2 + 12*sqrt(b*x + a)*a*b^2 - 3*sqrt(b*x + a)*a^2*b/x)/b
```

maple [A] time = 0.01, size = 61, normalized size = 0.92

$$2\left(\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a^2+2\sqrt{bx+a}\;a+\frac{(bx+a)^{\frac{3}{2}}}{3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^2,x)
```

```
[Out] 2*b*(1/3*(b*x+a)^(3/2)+2*(b*x+a)^(1/2)*a+a^2*(-1/2*(b*x+a)^(1/2)/b/x-5/2*ar
ctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 2.94, size = 71, normalized size = 1.08

$$\frac{5}{2}\,a^{\frac{3}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b+4\,\sqrt{bx+a}\,ab-\frac{\sqrt{bx+a}\,a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/2*a^(3/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
3*(b*x + a)^(3/2)*b + 4*sqrt(b*x + a)*a*b - sqrt(b*x + a)*a^2/x
```

mupad [B] time = 0.11, size = 58, normalized size = 0.88

$$\frac{2\,b\,(a+b\,x)^{3/2}}{3}-\frac{a^2\,\sqrt{a+b\,x}}{x}+4\,a\,b\,\sqrt{a+b\,x}+a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(3/2))/3 - (a^2*(a + b*x)^(1/2))/x + a^(3/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*5i + 4*a*b*(a + b*x)^(1/2)
```

sympy [A] time = 3.75, size = 99, normalized size = 1.50

$$-\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}-5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{2\sqrt{a}\,b^2x\sqrt{1+\frac{bx}{a}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**2,x)
```

```
[Out] -a**(5/2)*sqrt(1 + b*x/a)/x + 14*a**(3/2)*b*sqrt(1 + b*x/a)/3 + 5*a**(3/2)*
b*log(b*x/a)/2 - 5*a**(3/2)*b*log(sqrt(1 + b*x/a) + 1) + 2*sqrt(a)*b**2*x*s
qrt(1 + b*x/a)/3
```

3.306 $\int \frac{(a+bx)^{5/2}}{x^3} dx$

Optimal. Leaf size=78

$$\frac{15}{4}b^2\sqrt{a+bx} - \frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{(a+bx)^{5/2}}{2x^2} - \frac{5b(a+bx)^{3/2}}{4x}$$

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{15}{4}b^2\sqrt{a+bx} - \frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{(a+bx)^{5/2}}{2x^2} - \frac{5b(a+bx)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[a + b*x])/4 - (5*b*(a + b*x)^(3/2))/(4*x) - (a + b*x)^(5/2)/(2
*x^2) - (15*Sqrt[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^3}\,dx &= -\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(5b)\int \frac{(a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}+\frac{1}{4}(15ab)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right) \\
&= \frac{15}{4}b^2\sqrt{a+bx}-\frac{5b(a+bx)^{3/2}}{4x}-\frac{(a+bx)^{5/2}}{2x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.45

$$
-\frac{2b^2(a+bx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}+1\right)}{7a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 + (b*x)/a])/(7*a^3
)
```

IntegrateAlgebraic [A] time = 0.11, size = 68, normalized size = 0.87

$$
\frac{\sqrt{a+bx}\left(15a^2-25a(a+bx)+8(a+bx)^2\right)}{4x^2}-\frac{15}{4}\sqrt{a}\,b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^2 - 25*a*(a + b*x) + 8*(a + b*x)^2))/(4*x^2) - (15*Sqr
t[a]*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

fricas [A] time = 1.04, size = 133, normalized size = 1.71

$$
\left[\frac{15\sqrt{a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{8x^2},\ \frac{15\sqrt{-a}\,b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(8b^2x^2-9abx-2a^2\right)\sqrt{bx+a}}{4x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(
8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x + a))/x^2, 1/4*(15*sqrt(-a)*b^2*x^2*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (8*b^2*x^2 - 9*a*b*x - 2*a^2)*sqrt(b*x +
a))/x^2]
```

giac [A] time = 1.12, size = 80, normalized size = 1.03

$$
\frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\sqrt{bx+a}\,b^3-\frac{9\,(bx+a)^{\frac{3}{2}}ab^3-7\sqrt{bx+a}\,a^2b^3}{b^2x^2}}{4\,b}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(15*a*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x + a)*b^3
 - (9*(b*x + a)^(3/2)*a*b^3 - 7*sqrt(b*x + a)*a^2*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 61, normalized size = 0.78

$$2\left(\left(-\frac{15\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{\frac{7\sqrt{bx+a}\,a}{8}-\frac{9(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)a+\sqrt{bx+a}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*((b*x+a)^(1/2)+a*((-9/8*(b*x+a)^(3/2)+7/8*(b*x+a)^(1/2)*a)/x^2/b^2-15
/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 2.94, size = 101, normalized size = 1.29

$$\frac{15}{8}\sqrt{a}\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^2-\frac{9\,(bx+a)^{\frac{3}{2}}ab^2-7\sqrt{bx+a}\,a^2b^2}{4\left((bx+a)^2-2\,(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2*sqrt(b*x + a)*b^2 - 1/4*(9*(b*x + a)^(3/2)*a*b^2 - 7*sqrt(b*x + a)*a^2*b
^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

mupad [B] time = 0.05, size = 64, normalized size = 0.82

$$2\,b^2\,\sqrt{a+b\,x}+\frac{7\,a^2\,\sqrt{a+b\,x}}{4\,x^2}-\frac{9\,a\,{\left(a+b\,x\right)}^{3/2}}{4\,x^2}+\frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(a + b*x)^(1/2) + (7*a^2*(a + b*x)^(1/2))/(4*x^2) + (a^(1/2)*b^2*atan
(((a + b*x)^(1/2)*1i)/a^(1/2))*15i)/4 - (9*a*(a + b*x)^(3/2))/(4*x^2)
```

sympy [A] time = 4.30, size = 126, normalized size = 1.62

$$-\frac{15\sqrt{a}\,b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{a^3}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{11a^2\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**3,x)
```

```
[Out] -15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**3/(2*sqrt(b)*x**(5
/2)*sqrt(a/(b*x) + 1)) - 11*a**2*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) - a
*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(a/(b*x) +
 1)
```

3.307 $\int \frac{(a+bx)^{5/2}}{x^4} dx$

Optimal. Leaf size=81

$$-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{(a+bx)^{5/2}}{3x^3}-\frac{5b(a+bx)^{3/2}}{12x^2}$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8x}-\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}-\frac{5b(a+bx)^{3/2}}{12x^2}-\frac{(a+bx)^{5/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(8*x) - (5*b*(a + b*x)^(3/2))/(12*x^2) - (a + b*x)^(
5/2)/(3*x^3) - (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{x^4}\,dx = -\frac{(a+bx)^{5/2}}{3x^3} + \frac{1}{6}(5b)\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$
$$= -\frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3} + \frac{1}{8}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^2}\,dx$$
$$= -\frac{5b^2\sqrt{a+bx}}{8x} - \frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3} + \frac{1}{16}\left(5b^3\right)\int \frac{1}{x\sqrt{a+bx}}\,dx$$
$$= -\frac{5b^2\sqrt{a+bx}}{8x} - \frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3} + \frac{1}{8}\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)$$
$$= -\frac{5b^2\sqrt{a+bx}}{8x} - \frac{5b(a+bx)^{3/2}}{12x^2} - \frac{(a+bx)^{5/2}}{3x^3} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.98

$$-\frac{8a^3+34a^2bx+15b^3x^3\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+59ab^2x^2+33b^3x^3}{24x^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(8*a^3 + 34*a^2*b*x + 59*a*b^2*x^2 + 33*b^3*x^3 + 15*b^3*x^3*Sqrt[1 +
 (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x^3*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 68, normalized size = 0.84

$$-\frac{\sqrt{a+bx}\left(15a^2-40a(a+bx)+33(a+bx)^2\right)}{24x^3} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^4,x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(15*a^2 - 40*a*(a + b*x) + 33*(a + b*x)^2))/x^3 - (5*b
^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*Sqrt[a])
```

fricas [A] time = 0.99, size = 146, normalized size = 1.80

$$\left[\frac{15\,\sqrt{a}\,b^3x^3\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)-2\left(33\,ab^2x^2+26\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{48\,ax^3}, \frac{15\,\sqrt{-a}\,b^3x^3\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)-\left(33\,ab^2x^2+26\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{24\,ax^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(33*a*b^2*x^2 + 26*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a*x^3), 1/24*(15*sqrt(-
a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (33*a*b^2*x^2 + 26*a^2*b*x +
8*a^3)*sqrt(b*x + a))/(a*x^3)]
```

giac [A] time = 1.13, size = 79, normalized size = 0.98

$$\frac{\frac{15\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{33\,(bx+a)^{\frac{5}{2}}b^4-40\,(bx+a)^{\frac{3}{2}}ab^4+15\,\sqrt{bx+a}\,a^2b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (33*(b*x + a)^(5/2)*
b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 15*sqrt(b*x + a)*a^2*b^4)/(b^3*x^3))/b
```

maple [A] time = 0.01, size = 63, normalized size = 0.78

$$2\left(-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{-\frac{5\sqrt{bx+a}\,a^{2}}{16}+\frac{5(bx+a)^{\frac{3}{2}}a}{6}-\frac{11(bx+a)^{\frac{5}{2}}}{16}}{b^{3}x^{3}}\right)b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^4,x)
```

```
[Out] 2*b^3*((-11/16*(b*x+a)^(5/2)+5/6*(b*x+a)^(3/2)*a-5/16*(b*x+a)^(1/2)*a^2)/x^
3/b^3-5/16*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 2.99, size = 115, normalized size = 1.42

$$\frac{5\,b^{3}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,\sqrt{a}}-\frac{33\,(bx+a)^{\frac{5}{2}}b^{3}-40\,(bx+a)^{\frac{3}{2}}ab^{3}+15\,\sqrt{bx+a}\,a^{2}b^{3}}{24\left((bx+a)^{3}-3\,(bx+a)^{2}a+3\,(bx+a)a^{2}-a^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^4,x, algorithm="maxima")
```

```
[Out] 5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a) -
 1/24*(33*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 15*sqrt(b*x + a)
*a^2*b^3)/((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

mupad [B] time = 0.05, size = 64, normalized size = 0.79

$$\frac{5\,a\,(a+b\,x)^{3/2}}{3\,x^{3}}-\frac{5\,a^{2}\,\sqrt{a+b\,x}}{8\,x^{3}}-\frac{11\,(a+b\,x)^{5/2}}{8\,x^{3}}+\frac{b^{3}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{8\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^4,x)
```

```
[Out] (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^(1/2)) - (5*a^2*(a + b*x)^
(1/2))/(8*x^3) - (11*(a + b*x)^(5/2))/(8*x^3) + (5*a*(a + b*x)^(3/2))/(3*x^
3)
```

sympy [A] time = 5.16, size = 104, normalized size = 1.28

$$-\frac{a^{2}\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x^{\frac{5}{2}}}-\frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{12x^{\frac{3}{2}}}-\frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{8\sqrt{x}}-\frac{5b^{3}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**4,x)
```

```
[Out] -a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x**(5/2)) - 13*a*b**(3/2)*sqrt(a/(b*x) +
 1)/(12*x**(3/2)) - 11*b**(5/2)*sqrt(a/(b*x) + 1)/(8*sqrt(x)) - 5*b**3*asin
h(sqrt(a)/(sqrt(b)*sqrt(x)))/(8*sqrt(a))
```

3.308 $\int \frac{(a+bx)^{5/2}}{x^5} \, dx$

Optimal. Leaf size=103

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}} - \frac{5b^3\sqrt{a+bx}}{64ax} - \frac{5b^2\sqrt{a+bx}}{32x^2} - \frac{(a+bx)^{5/2}}{4x^4} - \frac{5b(a+bx)^{3/2}}{24x^3}$$

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{5b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}} - \frac{5b^2\sqrt{a+bx}}{32x^2} - \frac{5b^3\sqrt{a+bx}}{64ax} - \frac{5b(a+bx)^{3/2}}{24x^3} - \frac{(a+bx)^{5/2}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-5*b^2*Sqrt[a + b*x])/(32*x^2) - (5*b^3*Sqrt[a + b*x])/(64*a*x) - (5*b*(a
+ b*x)^(3/2))/(24*x^3) - (a + b*x)^(5/2)/(4*x^4) + (5*b^4*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(64*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^5}\,dx &= -\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{8}(5b)\int \frac{(a+bx)^{3/2}}{x^4}\,dx \\
&= -\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{16}\left(5b^2\right)\int \frac{\sqrt{a+bx}}{x^3}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{1}{64}\left(5b^3\right)\int \frac{1}{x^2\sqrt{a+bx}}\,dx \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^4\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{128a} \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}-\frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\right.}{64a} \\
&= -\frac{5b^2\sqrt{a+bx}}{32x^2}-\frac{5b^3\sqrt{a+bx}}{64ax}-\frac{5b(a+bx)^{3/2}}{24x^3}-\frac{(a+bx)^{5/2}}{4x^4}+\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.34

$$
-\frac{2b^4(a+bx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};\frac{bx}{a}+1\right)}{7a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, 1 + (b*x)/a])/(7*a^5
)
```

IntegrateAlgebraic [A] time = 0.15, size = 83, normalized size = 0.81

$$
\frac{5b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{64a^{3/2}}-\frac{\sqrt{a+bx}\left(15a^3-55a^2(a+bx)+73a(a+bx)^2+15(a+bx)^3\right)}{192ax^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^5,x]
```

```
[Out] -1/192*(Sqrt[a + b*x]*(15*a^3 - 55*a^2*(a + b*x) + 73*a*(a + b*x)^2 + 15*(a
 + b*x)^3))/(a*x^4) + (5*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(64*a^(3/2))
```

fricas [A] time = 0.96, size = 167, normalized size = 1.62

$$
\left[\frac{15\sqrt{a}\,b^4x^4\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^3x^3+118a^2b^2x^2+136a^3bx+48a^4\right)\sqrt{bx+a}}{384a^2x^4},-\frac{15\sqrt{-a}\,b^4x^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15ab^3x^3+118a^2b^2x^2+136a^3bx+48a^4\right)\sqrt{bx+a}}{192a^2x^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/384*(15*sqrt(a)*b^4*x^4*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2
*(15*a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^
2*x^4), -1/192*(15*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*
a*b^3*x^3 + 118*a^2*b^2*x^2 + 136*a^3*b*x + 48*a^4)*sqrt(b*x + a))/(a^2*x^4
)]
```

giac [A] time = 1.09, size = 99, normalized size = 0.96

$$-\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5+73\,(bx+a)^{\frac{5}{2}}ab^5-55\,(bx+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx+a}\,a^3b^5}{ab^4x^4}}{192\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/192*(15*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x + a)^
(7/2)*b^5 + 73*(b*x + a)^(5/2)*a*b^5 - 55*(b*x + a)^(3/2)*a^2*b^5 + 15*sqrt
(b*x + a)*a^3*b^5)/(a*b^4*x^4))/b
```

maple [A] time = 0.01, size = 75, normalized size = 0.73

$$2\left(\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128a^{\frac{3}{2}}}+\frac{-\frac{5\sqrt{bx+a}\,a^2}{128}+\frac{55(bx+a)^{\frac{3}{2}}a}{384}-\frac{5(bx+a)^{\frac{7}{2}}}{128a}-\frac{73(bx+a)^{\frac{5}{2}}}{384}}{b^4x^4}\right)b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^5,x)
```

```
[Out] 2*b^4*((-5/128/a*(b*x+a)^(7/2)-73/384*(b*x+a)^(5/2)+55/384*(b*x+a)^(3/2)*a-
5/128*(b*x+a)^(1/2)*a^2)/x^4/b^4+5/128*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/
2))
```

maxima [A] time = 2.93, size = 144, normalized size = 1.40

$$-\frac{5\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{128\,a^{\frac{3}{2}}}-\frac{15\,(bx+a)^{\frac{7}{2}}b^4+73\,(bx+a)^{\frac{5}{2}}ab^4-55\,(bx+a)^{\frac{3}{2}}a^2b^4+15\,\sqrt{bx+a}\,a^3b^4}{192\left((bx+a)^4a-4\,(bx+a)^3a^2+6\,(bx+a)^2a^3-4\,(bx+a)a^4+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^5,x, algorithm="maxima")
```

```
[Out] -5/128*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
 - 1/192*(15*(b*x + a)^(7/2)*b^4 + 73*(b*x + a)^(5/2)*a*b^4 - 55*(b*x + a)^
(3/2)*a^2*b^4 + 15*sqrt(b*x + a)*a^3*b^4)/((b*x + a)^4*a - 4*(b*x + a)^3*a^
2 + 6*(b*x + a)^2*a^3 - 4*(b*x + a)*a^4 + a^5)
```

mupad [B] time = 0.11, size = 79, normalized size = 0.77

$$\frac{55\,a\,(a+b\,x)^{3/2}}{192\,x^4}-\frac{5\,a^2\,\sqrt{a+b\,x}}{64\,x^4}-\frac{5\,(a+b\,x)^{7/2}}{64\,a\,x^4}-\frac{73\,(a+b\,x)^{5/2}}{192\,x^4}-\frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{64\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^5,x)
```

```
[Out] (55*a*(a + b*x)^(3/2))/(192*x^4) - (5*a^2*(a + b*x)^(1/2))/(64*x^4) - (5*(a
 + b*x)^(7/2))/(64*a*x^4) - (b^4*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(64
*a^(3/2)) - (73*(a + b*x)^(5/2))/(192*x^4)
```

sympy [A] time = 8.36, size = 155, normalized size = 1.50

$$-\frac{a^3}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^2\sqrt{b}}{24x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{127ab^{\frac{3}{2}}}{96x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{133b^{\frac{5}{2}}}{192x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{7}{2}}}{64a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**5,x)
```

```
[Out] -a**3/(4*sqrt(b)*x**(9/2)*sqrt(a/(b*x) + 1)) - 23*a**2*sqrt(b)/(24*x**(7/2)
*sqrt(a/(b*x) + 1)) - 127*a*b**(3/2)/(96*x**(5/2)*sqrt(a/(b*x) + 1)) - 133*
b**(5/2)/(192*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(7/2)/(64*a*sqrt(x)*sqrt(a
/(b*x) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(64*a**(3/2))
```

3.309 $\int x^7(a+bx)^{9/2}\,dx$

Optimal. Leaf size=146

$$-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{2a^2(a+bx)^{21/2}}{b^8}+\frac{2(a+\cdots}{2\cdots}$$

Rubi [A] time = 0.04, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{21/2}}{b^8}-\frac{70a^3(a+bx)^{19/2}}{19b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{2(a+bx)^{25/2}}{25b^8}-\frac{14a(a+bx)^{23/2}}{23b^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6*(a + b*x)^(13/2))/(13*b^8) - (
14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(a + b*x)^(17/2))/(17*b^8) - (70
*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a + b*x)^(21/2))/b^8 - (14*a*(a +
 b*x)^(23/2))/(23*b^8) + (2*(a + b*x)^(25/2))/(25*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^7(a+bx)^{9/2}\,dx=\int\left(-\frac{a^7(a+bx)^{9/2}}{b^7}+\frac{7a^6(a+bx)^{11/2}}{b^7}-\frac{21a^5(a+bx)^{13/2}}{b^7}+\frac{35a^4(a+bx)^{15/2}}{b^7}-\frac{35a^3(a+bx)^{\cdots}}{b^7}\cdots\right.$$

$$=-\frac{2a^7(a+bx)^{11/2}}{11b^8}+\frac{14a^6(a+bx)^{13/2}}{13b^8}-\frac{14a^5(a+bx)^{15/2}}{5b^8}+\frac{70a^4(a+bx)^{17/2}}{17b^8}-\frac{70a^3(a+bx)^{1\cdots}}{19b^8}\cdots$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-2048a^7+11264a^6bx-36608a^5b^2x^2+91520a^4b^3x^3-194480a^3b^4x^4+369512a^2b^5x^5-646646ab^6x^6+1062347b^7x^7\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2048*a^7 + 11264*a^6*b*x - 36608*a^5*b^2*x^2 + 91520*
a^4*b^3*x^3 - 194480*a^3*b^4*x^4 + 369512*a^2*b^5*x^5 - 646646*a*b^6*x^6 +
1062347*b^7*x^7))/(26558675*b^8)
```

IntegrateAlgebraic [A] time = 0.04, size = 115, normalized size = 0.79

$$-\frac{2\left(2414425a^7(a+bx)^{11/2}-14300825a^6(a+bx)^{13/2}+37182145a^5(a+bx)^{15/2}-54679625a^4(a+bx)^{17/2}+48923875a^3(a+bx)^{19/2}-26558675a^2(a+bx)^{21/2}-1062347(a+bx)^{25/2}+8083075a(a+bx)^{23/2}\right)}{26558675b^8}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^7*(a + b*x)^(9/2),x]
```

```
[Out] (-2*(2414425*a^7*(a + b*x)^(11/2) - 14300825*a^6*(a + b*x)^(13/2) + 3718214
5*a^5*(a + b*x)^(15/2) - 54679625*a^4*(a + b*x)^(17/2) + 48923875*a^3*(a +
```

```
b*x)^(19/2) - 26558675*a^2*(a + b*x)^(21/2) + 8083075*a*(a + b*x)^(23/2) -
1062347*(a + b*x)^(25/2)))/(26558675*b^8)
```

fricas [A] time = 1.02, size = 141, normalized size = 0.97

$$\frac{2\left(1062347\,b^{12}x^{12}+4665089\,ab^{11}x^{11}+7759752\,a^{2}b^{10}x^{10}+5810090\,a^{3}b^{9}x^{9}+1659515\,a^{4}b^{8}x^{8}+429\,a^{5}b^{7}x^{7}-462\,a^{6}b^{6}x^{6}+504\,a^{7}b^{5}x^{5}-560\,a^{8}b^{4}x^{4}+640\,a^{9}b^{3}x^{3}-768\,a^{10}b^{2}x^{2}+1024\,a^{11}bx-2048\,a^{12}\right)\sqrt{bx+a}}{26558675\,b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/26558675*(1062347*b^12*x^12 + 4665089*a*b^11*x^11 + 7759752*a^2*b^10*x^10
 + 5810090*a^3*b^9*x^9 + 1659515*a^4*b^8*x^8 + 429*a^5*b^7*x^7 - 462*a^6*b^
6*x^6 + 504*a^7*b^5*x^5 - 560*a^8*b^4*x^4 + 640*a^9*b^3*x^3 - 768*a^10*b^2*
x^2 + 1024*a^11*b*x - 2048*a^12)*sqrt(b*x + a)/b^8
```

giac [B] time = 1.14, size = 781, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/1673196525*(260015*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a^5/b^7 + 76475*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*
a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x
 + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 -
 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^4/b^7 + 72450*(12
155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*
a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230
*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2
)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^3/b^7 + 17
250*(46189*(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)
^(17/2)*a^2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4
- 22221108*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(
b*x + a)^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*
a^9 + 969969*sqrt(b*x + a)*a^10)*a^2/b^7 + 4125*(88179*(b*x + a)^(23/2) - 1
062347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x +
a)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*(b*x + a)^(13/2)*a
^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^(9/2)*a^7 + 4780561
5*(b*x + a)^(7/2)*a^8 - 22309287*(b*x + a)^(5/2)*a^9 + 7436429*(b*x + a)^(3
/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)*a/b^7 + 99*(676039*(b*x + a)^(25/2)
- 8817900*(b*x + a)^(23/2)*a + 53117350*(b*x + a)^(21/2)*a^2 - 195695500*(b
*x + a)^(19/2)*a^3 + 492116625*(b*x + a)^(17/2)*a^4 - 892371480*(b*x + a)^(
15/2)*a^5 + 1201269300*(b*x + a)^(13/2)*a^6 - 1216870200*(b*x + a)^(11/2)*a
^7 + 929553625*(b*x + a)^(9/2)*a^8 - 531173500*(b*x + a)^(7/2)*a^9 + 223092
870*(b*x + a)^(5/2)*a^10 - 67603900*(b*x + a)^(3/2)*a^11 + 16900975*sqrt(b*
x + a)*a^12)/b^7)/b
```

maple [A] time = 0.01, size = 87, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{11}{2}}\left(-1062347b^{7}x^{7}+646646a\,b^{6}x^{6}-369512a^{2}b^{5}x^{5}+194480a^{3}b^{4}x^{4}-91520a^{4}b^{3}x^{3}+36608a^{5}b^{2}x^{2}-11264a^{6}bx+2048a^{7}\right)}{26558675b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(b*x+a)^(9/2),x)
```

```
[Out] -2/26558675*(b*x+a)^(11/2)*(-1062347*b^7*x^7+646646*a*b^6*x^6-369512*a^2*b^
5*x^5+194480*a^3*b^4*x^4-91520*a^4*b^3*x^3+36608*a^5*b^2*x^2-11264*a^6*b*x+
2048*a^7)/b^8
```

maxima [A] time = 1.33, size = 116, normalized size = 0.79

$$\frac{2\,(bx+a)^{\frac{25}{2}}}{25\,b^8}-\frac{14\,(bx+a)^{\frac{23}{2}}a}{23\,b^8}+\frac{2\,(bx+a)^{\frac{21}{2}}a^2}{b^8}-\frac{70\,(bx+a)^{\frac{19}{2}}a^3}{19\,b^8}+\frac{70\,(bx+a)^{\frac{17}{2}}a^4}{17\,b^8}-\frac{14\,(bx+a)^{\frac{15}{2}}a^5}{5\,b^8}+\frac{14\,(bx+a)^{\frac{13}{2}}a^6}{13\,b^8}-\frac{2\,(bx+a)^{\frac{11}{2}}a^7}{11\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/25*(b*x + a)^(25/2)/b^8 - 14/23*(b*x + a)^(23/2)*a/b^8 + 2*(b*x + a)^(21/
2)*a^2/b^8 - 70/19*(b*x + a)^(19/2)*a^3/b^8 + 70/17*(b*x + a)^(17/2)*a^4/b^
8 - 14/5*(b*x + a)^(15/2)*a^5/b^8 + 14/13*(b*x + a)^(13/2)*a^6/b^8 - 2/11*(
b*x + a)^(11/2)*a^7/b^8
```

mupad [B] time = 0.04, size = 116, normalized size = 0.79

$$\frac{2\,(a+b\,x)^{25/2}}{25\,b^8}-\frac{2\,a^7\,(a+b\,x)^{11/2}}{11\,b^8}+\frac{14\,a^6\,(a+b\,x)^{13/2}}{13\,b^8}-\frac{14\,a^5\,(a+b\,x)^{15/2}}{5\,b^8}+\frac{70\,a^4\,(a+b\,x)^{17/2}}{17\,b^8}-\frac{70\,a^3\,(a+b\,x)^{19/2}}{19\,b^8}+\frac{2\,a^2\,(a+b\,x)^{21/2}}{b^8}-\frac{14\,a\,(a+b\,x)^{23/2}}{23\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(25/2))/(25*b^8) - (2*a^7*(a + b*x)^(11/2))/(11*b^8) + (14*a^6
*(a + b*x)^(13/2))/(13*b^8) - (14*a^5*(a + b*x)^(15/2))/(5*b^8) + (70*a^4*(
a + b*x)^(17/2))/(17*b^8) - (70*a^3*(a + b*x)^(19/2))/(19*b^8) + (2*a^2*(a
+ b*x)^(21/2))/b^8 - (14*a*(a + b*x)^(23/2))/(23*b^8)
```

sympy [A] time = 40.30, size = 279, normalized size = 1.91

$$\begin{cases} -\frac{4096a^{12}\sqrt{a+bx}}{26558675b^8}+\frac{2048a^{11}x\sqrt{a+bx}}{26558675b^7}-\frac{1536a^{10}x^2\sqrt{a+bx}}{26558675b^6}+\frac{256a^9x^3\sqrt{a+bx}}{5311735b^5}-\frac{224a^8x^4\sqrt{a+bx}}{5311735b^4}+\frac{1008a^7x^5\sqrt{a+bx}}{26558675b^3}-\frac{84a^6x^6\sqrt{a+bx}}{2414425b^2}+\frac{6a^5x^7\sqrt{a+bx}}{185725b}+\frac{4642a^4x^8\sqrt{a+bx}}{37145}+\frac{956a^3bx^9\sqrt{a+bx}}{2185}+\frac{336a^2b^2x^{10}\sqrt{a+bx}}{575}+\frac{202ab^3x^{11}\sqrt{a+bx}}{575}+\frac{2b^4x^{12}\sqrt{a+bx}}{25} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4096*a**12*sqrt(a + b*x)/(26558675*b**8) + 2048*a**11*x*sqrt(a
+ b*x)/(26558675*b**7) - 1536*a**10*x**2*sqrt(a + b*x)/(26558675*b**6) + 25
6*a**9*x**3*sqrt(a + b*x)/(5311735*b**5) - 224*a**8*x**4*sqrt(a + b*x)/(531
1735*b**4) + 1008*a**7*x**5*sqrt(a + b*x)/(26558675*b**3) - 84*a**6*x**6*sq
rt(a + b*x)/(2414425*b**2) + 6*a**5*x**7*sqrt(a + b*x)/(185725*b) + 4642*a*
*4*x**8*sqrt(a + b*x)/37145 + 956*a**3*b*x**9*sqrt(a + b*x)/2185 + 336*a**2
*b**2*x**10*sqrt(a + b*x)/575 + 202*a*b**3*x**11*sqrt(a + b*x)/575 + 2*b**4
*x**12*sqrt(a + b*x)/25, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

3.310 $\int x^6(a+bx)^{9/2}\,dx$

Optimal. Leaf size=127

$$\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{19/2}}{19b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+}{7}$$

Rubi [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{30a^2(a+bx)^{19/2}}{19b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^6(a+bx)^{11/2}}{11b^7}+\frac{2(a+bx)^{23/2}}{23b^7}-\frac{4a(a+bx)^{21/2}}{7b^7}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5*(a + b*x)^(13/2))/(13*b^7) + (2
*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b*x)^(17/2))/(17*b^7) + (30*a^2*(
a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^(21/2))/(7*b^7) + (2*(a + b*x)^(
23/2))/(23*b^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^6(a+bx)^{9/2}\,dx=\int\left(\frac{a^6(a+bx)^{9/2}}{b^6}-\frac{6a^5(a+bx)^{11/2}}{b^6}+\frac{15a^4(a+bx)^{13/2}}{b^6}-\frac{20a^3(a+bx)^{15/2}}{b^6}+\frac{15a^2(a+bx}{b^6}\right.$$
$$=\frac{2a^6(a+bx)^{11/2}}{11b^7}-\frac{12a^5(a+bx)^{13/2}}{13b^7}+\frac{2a^4(a+bx)^{15/2}}{b^7}-\frac{40a^3(a+bx)^{17/2}}{17b^7}+\frac{30a^2(a+bx)^{1}}{19b^7}$$

Mathematica [A] time = 0.04, size = 79, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(1024a^6-5632a^5bx+18304a^4b^2x^2-45760a^3b^3x^3+97240a^2b^4x^4-184756ab^5x^5+323323b^6x^6\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(1024*a^6 - 5632*a^5*b*x + 18304*a^4*b^2*x^2 - 45760*a^
3*b^3*x^3 + 97240*a^2*b^4*x^4 - 184756*a*b^5*x^5 + 323323*b^6*x^6))/(743642
9*b^7)
```

IntegrateAlgebraic [A] time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\left(676039a^6(a+bx)^{11/2}-3432198a^5(a+bx)^{13/2}+7436429a^4(a+bx)^{15/2}-8748740a^3(a+bx)^{17/2}+5870865a^2(a+bx)^{19/2}+323323(a+bx)^{23/2}-2124694a(a+bx)^{21/2}\right)}{7436429b^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6*(a + b*x)^(9/2),x]
```

```
[Out] (2*(676039*a^6*(a + b*x)^(11/2) - 3432198*a^5*(a + b*x)^(13/2) + 7436429*a^
4*(a + b*x)^(15/2) - 8748740*a^3*(a + b*x)^(17/2) + 5870865*a^2*(a + b*x)^(
19/2) - 2124694*a*(a + b*x)^(21/2) + 323323*(a + b*x)^(23/2)))/(7436429*b^7
)
```

fricas [A] time = 0.73, size = 130, normalized size = 1.02

$$\frac{2\left(323323\,b^{11}x^{11}+1431859\,ab^{10}x^{10}+2406690\,a^{2}b^{9}x^{9}+1826110\,a^{3}b^{8}x^{8}+530959\,a^{4}b^{7}x^{7}+231\,a^{5}b^{6}x^{6}-252\,a^{6}b^{5}x^{5}+280\,a^{7}b^{4}x^{4}-320\,a^{8}b^{3}x^{3}+384\,a^{9}b^{2}x^{2}-512\,a^{10}bx+1024\,a^{11}\right)\sqrt{bx+a}}{7436429\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/7436429*(323323*b^11*x^11 + 1431859*a*b^10*x^10 + 2406690*a^2*b^9*x^9 + 1
826110*a^3*b^8*x^8 + 530959*a^4*b^7*x^7 + 231*a^5*b^6*x^6 - 252*a^6*b^5*x^5
 + 280*a^7*b^4*x^4 - 320*a^8*b^3*x^3 + 384*a^9*b^2*x^2 - 512*a^10*b*x + 102
4*a^11)*sqrt(b*x + a)/b^7
```

giac [B] time = 1.32, size = 709, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/66927861*(22287*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b
*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6
006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^5/b^6 + 52003*(429*(b*x
 + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025
*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^
5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4/b^6 + 6118*(643
5*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2
 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x +
 a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 1
09395*sqrt(b*x + a)*a^8)*a^3/b^6 + 2898*(12155*(b*x + a)^(19/2) - 122265*(b
*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a
^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(
b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a
^8 - 230945*sqrt(b*x + a)*a^9)*a^2/b^6 + 345*(46189*(b*x + a)^(21/2) - 5105
10*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x + a)^(1
5/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a^5 +
22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*(b*x
+ a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^10)*a
/b^6 + 33*(88179*(b*x + a)^(23/2) - 1062347*(b*x + a)^(21/2)*a + 5870865*(b
*x + a)^(19/2)*a^2 - 19684665*(b*x + a)^(17/2)*a^3 + 44618574*(b*x + a)^(15
/2)*a^4 - 72076158*(b*x + a)^(13/2)*a^5 + 85180914*(b*x + a)^(11/2)*a^6 - 7
4364290*(b*x + a)^(9/2)*a^7 + 47805615*(b*x + a)^(7/2)*a^8 - 22309287*(b*x
+ a)^(5/2)*a^9 + 7436429*(b*x + a)^(3/2)*a^10 - 2028117*sqrt(b*x + a)*a^11)
/b^6)/b
```

maple [A] time = 0.01, size = 76, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{11}{2}}\left(323323x^{6}b^{6}-184756a\,x^{5}b^{5}+97240a^{2}x^{4}b^{4}-45760a^{3}x^{3}b^{3}+18304a^{4}x^{2}b^{2}-5632a^{5}xb+1024a^{6}\right)}{7436429b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^(9/2),x)
```

```
[Out] 2/7436429*(b*x+a)^(11/2)*(323323*b^6*x^6-184756*a*b^5*x^5+97240*a^2*b^4*x^4
-45760*a^3*b^3*x^3+18304*a^4*b^2*x^2-5632*a^5*b*x+1024*a^6)/b^7
```

maxima [A] time = 1.34, size = 101, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{23}{2}}}{23\,b^7}-\frac{4\,(bx+a)^{\frac{21}{2}}\,a}{7\,b^7}+\frac{30\,(bx+a)^{\frac{19}{2}}\,a^2}{19\,b^7}-\frac{40\,(bx+a)^{\frac{17}{2}}\,a^3}{17\,b^7}+\frac{2\,(bx+a)^{\frac{15}{2}}\,a^4}{b^7}-\frac{12\,(bx+a)^{\frac{13}{2}}\,a^5}{13\,b^7}+\frac{2\,(bx+a)^{\frac{11}{2}}\,a^6}{11\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/23*(b*x + a)^(23/2)/b^7 - 4/7*(b*x + a)^(21/2)*a/b^7 + 30/19*(b*x + a)^(1
9/2)*a^2/b^7 - 40/17*(b*x + a)^(17/2)*a^3/b^7 + 2*(b*x + a)^(15/2)*a^4/b^7
- 12/13*(b*x + a)^(13/2)*a^5/b^7 + 2/11*(b*x + a)^(11/2)*a^6/b^7
```

mupad [B] time = 0.03, size = 101, normalized size = 0.80

$$\frac{2\,{\left(a+b\,x\right)}^{23/2}}{23\,b^7}+\frac{2\,a^6\,{\left(a+b\,x\right)}^{11/2}}{11\,b^7}-\frac{12\,a^5\,{\left(a+b\,x\right)}^{13/2}}{13\,b^7}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{15/2}}{b^7}-\frac{40\,a^3\,{\left(a+b\,x\right)}^{17/2}}{17\,b^7}+\frac{30\,a^2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^7}-\frac{4\,a\,{\left(a+b\,x\right)}^{21/2}}{7\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(23/2))/(23*b^7) + (2*a^6*(a + b*x)^(11/2))/(11*b^7) - (12*a^5
*(a + b*x)^(13/2))/(13*b^7) + (2*a^4*(a + b*x)^(15/2))/b^7 - (40*a^3*(a + b
*x)^(17/2))/(17*b^7) + (30*a^2*(a + b*x)^(19/2))/(19*b^7) - (4*a*(a + b*x)^
(21/2))/(7*b^7)
```

sympy [A] time = 36.61, size = 257, normalized size = 2.02

$$\begin{cases} \frac{2048a^{11}\sqrt{a+bx}}{7436429b^7}-\frac{1024a^{10}x\sqrt{a+bx}}{7436429b^6}+\frac{768a^9x^2\sqrt{a+bx}}{7436429b^5}-\frac{640a^8x^3\sqrt{a+bx}}{7436429b^4}+\frac{80a^7x^4\sqrt{a+bx}}{1062347b^3}-\frac{72a^6x^5\sqrt{a+bx}}{1062347b^2}+\frac{6a^5x^6\sqrt{a+bx}}{96577b}+\frac{7426a^4x^7\sqrt{a+bx}}{52003}+\frac{25540a^3bx^8\sqrt{a+bx}}{52003}+\frac{1980a^2b^2x^9\sqrt{a+bx}}{3059}+\frac{62ab^3x^{10}\sqrt{a+bx}}{161}+\frac{2b^4x^{11}\sqrt{a+bx}}{23} & \text{for}\ b\neq 0\\ \frac{a^{\frac{9}{2}}x^7}{7} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((2048*a**11*sqrt(a + b*x)/(7436429*b**7) - 1024*a**10*x*sqrt(a +
b*x)/(7436429*b**6) + 768*a**9*x**2*sqrt(a + b*x)/(7436429*b**5) - 640*a**8
*x**3*sqrt(a + b*x)/(7436429*b**4) + 80*a**7*x**4*sqrt(a + b*x)/(1062347*b*
*3) - 72*a**6*x**5*sqrt(a + b*x)/(1062347*b**2) + 6*a**5*x**6*sqrt(a + b*x)
/(96577*b) + 7426*a**4*x**7*sqrt(a + b*x)/52003 + 25540*a**3*b*x**8*sqrt(a
+ b*x)/52003 + 1980*a**2*b**2*x**9*sqrt(a + b*x)/3059 + 62*a*b**3*x**10*sqr
t(a + b*x)/161 + 2*b**4*x**11*sqrt(a + b*x)/23, Ne(b, 0)), (a**(9/2)*x**7/7
, True))
```

3.311 $\int x^5(a+bx)^{9/2}\,dx$

Optimal. Leaf size=110

$$-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Rubi [A] time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{2(a+bx)^{21/2}}{21b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4*(a + b*x)^(13/2))/(13*b^6) - (
4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a + b*x)^(17/2))/(17*b^6) - (10*
a*(a + b*x)^(19/2))/(19*b^6) + (2*(a + b*x)^(21/2))/(21*b^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^5(a+bx)^{9/2}\,dx=\int\left(-\frac{a^5(a+bx)^{9/2}}{b^5}+\frac{5a^4(a+bx)^{11/2}}{b^5}-\frac{10a^3(a+bx)^{13/2}}{b^5}+\frac{10a^2(a+bx)^{15/2}}{b^5}-\frac{5a(a+bx)^{17/2}}{b^5}\right.$$
$$=-\frac{2a^5(a+bx)^{11/2}}{11b^6}+\frac{10a^4(a+bx)^{13/2}}{13b^6}-\frac{4a^3(a+bx)^{15/2}}{3b^6}+\frac{20a^2(a+bx)^{17/2}}{17b^6}-\frac{10a(a+bx)^{19/2}}{19b^6}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.62

$$\frac{2(a+bx)^{11/2}\left(-256a^5+1408a^4bx-4576a^3b^2x^2+11440a^2b^3x^3-24310ab^4x^4+46189b^5x^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-256*a^5 + 1408*a^4*b*x - 4576*a^3*b^2*x^2 + 11440*a^2
*b^3*x^3 - 24310*a*b^4*x^4 + 46189*b^5*x^5))/(969969*b^6)
```

IntegrateAlgebraic [A] time = 0.03, size = 75, normalized size = 0.68

$$\frac{2(a+bx)^{11/2}\left(-88179a^5+373065a^4(a+bx)-646646a^3(a+bx)^2+570570a^2(a+bx)^3-255255a(a+bx)^4+46189(a+bx)^5\right)}{969969b^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-88179*a^5 + 373065*a^4*(a + b*x) - 646646*a^3*(a + b*
x)^2 + 570570*a^2*(a + b*x)^3 - 255255*a*(a + b*x)^4 + 46189*(a + b*x)^5))/
(969969*b^6)
```

fricas [A] time = 0.91, size = 119, normalized size = 1.08

$$\frac{2\left(46189\,b^{10}x^{10}+206635\,ab^{9}x^{9}+351780\,a^{2}b^{8}x^{8}+271414\,a^{3}b^{7}x^{7}+80773\,a^{4}b^{6}x^{6}+63\,a^{5}b^{5}x^{5}-70\,a^{6}b^{4}x^{4}+80\,a^{7}b^{3}x^{3}-96\,a^{8}b^{2}x^{2}+128\,a^{9}bx-256\,a^{10}\right)\sqrt{bx+a}}{969969\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/969969*(46189*b^10*x^10 + 206635*a*b^9*x^9 + 351780*a^2*b^8*x^8 + 271414*
a^3*b^7*x^7 + 80773*a^4*b^6*x^6 + 63*a^5*b^5*x^5 - 70*a^6*b^4*x^4 + 80*a^7*
b^3*x^3 - 96*a^8*b^2*x^2 + 128*a^9*b*x - 256*a^10)*sqrt(b*x + a)/b^6
```

giac [B] time = 1.05, size = 637, normalized size = 5.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/2909907*(4199*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqr
t(b*x + a)*a^5)*a^5/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2
)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^
(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^4/b^5 + 45
22*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)
*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^3/b^5
 + 266*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)
^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 87
5160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3
/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2/b^5 + 63*(12155*(b*x + a)^(19/2) -
122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)
^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 +
2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a
)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a/b^5 + 3*(46189*(b*x + a)^(21/2) -
 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^2 - 7759752*(b*x +
a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 22221108*(b*x + a)^(11/2)*a
^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)^(7/2)*a^7 + 8729721*
(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 969969*sqrt(b*x + a)*a^
10)/b^5)/b
```

maple [A] time = 0.01, size = 65, normalized size = 0.59

$$-\frac{2\left(bx+a\right)^{\frac{11}{2}}\left(-46189b^{5}x^{5}+24310a\,b^{4}x^{4}-11440a^{2}b^{3}x^{3}+4576a^{3}b^{2}x^{2}-1408a^{4}bx+256a^{5}\right)}{969969b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^(9/2),x)
```

```
[Out] -2/969969*(b*x+a)^(11/2)*(-46189*b^5*x^5+24310*a*b^4*x^4-11440*a^2*b^3*x^3+
4576*a^3*b^2*x^2-1408*a^4*b*x+256*a^5)/b^6
```

maxima [A] time = 1.35, size = 86, normalized size = 0.78

$$\frac{2\left(bx+a\right)^{\frac{21}{2}}}{21\,b^{6}}-\frac{10\left(bx+a\right)^{\frac{19}{2}}a}{19\,b^{6}}+\frac{20\left(bx+a\right)^{\frac{17}{2}}a^{2}}{17\,b^{6}}-\frac{4\left(bx+a\right)^{\frac{15}{2}}a^{3}}{3\,b^{6}}+\frac{10\left(bx+a\right)^{\frac{13}{2}}a^{4}}{13\,b^{6}}-\frac{2\left(bx+a\right)^{\frac{11}{2}}a^{5}}{11\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/21*(b*x + a)^(21/2)/b^6 - 10/19*(b*x + a)^(19/2)*a/b^6 + 20/17*(b*x + a)^
(17/2)*a^2/b^6 - 4/3*(b*x + a)^(15/2)*a^3/b^6 + 10/13*(b*x + a)^(13/2)*a^4/
b^6 - 2/11*(b*x + a)^(11/2)*a^5/b^6
```

mupad [B] time = 0.03, size = 86, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{21/2}}{21\,b^6}-\frac{2\,a^5\,(a+b\,x)^{11/2}}{11\,b^6}+\frac{10\,a^4\,(a+b\,x)^{13/2}}{13\,b^6}-\frac{4\,a^3\,(a+b\,x)^{15/2}}{3\,b^6}+\frac{20\,a^2\,(a+b\,x)^{17/2}}{17\,b^6}-\frac{10\,a\,(a+b\,x)^{19/2}}{19\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(21/2))/(21*b^6) - (2*a^5*(a + b*x)^(11/2))/(11*b^6) + (10*a^4
*(a + b*x)^(13/2))/(13*b^6) - (4*a^3*(a + b*x)^(15/2))/(3*b^6) + (20*a^2*(a
 + b*x)^(17/2))/(17*b^6) - (10*a*(a + b*x)^(19/2))/(19*b^6)
```

sympy [A] time = 28.76, size = 235, normalized size = 2.14

$$\begin{cases} -\frac{512a^{10}\sqrt{a+bx}}{969969b^6}+\frac{256a^9x\sqrt{a+bx}}{969969b^5}-\frac{64a^8x^2\sqrt{a+bx}}{323323b^4}+\frac{160a^7x^3\sqrt{a+bx}}{969969b^3}-\frac{20a^6x^4\sqrt{a+bx}}{138567b^2}+\frac{6a^5x^5\sqrt{a+bx}}{46189b}+\frac{2098a^4x^6\sqrt{a+bx}}{12597}+\frac{3796a^3bx^7\sqrt{a+bx}}{6783}+\frac{1640a^2b^2x^8\sqrt{a+bx}}{2261}+\frac{170ab^3x^9\sqrt{a+bx}}{399}+\frac{2b^4x^{10}\sqrt{a+bx}}{21} & \text{for}\ b\neq 0 \\ \frac{a^{\frac{9}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-512*a**10*sqrt(a + b*x)/(969969*b**6) + 256*a**9*x*sqrt(a + b*x
)/(969969*b**5) - 64*a**8*x**2*sqrt(a + b*x)/(323323*b**4) + 160*a**7*x**3*
sqrt(a + b*x)/(969969*b**3) - 20*a**6*x**4*sqrt(a + b*x)/(138567*b**2) + 6*
a**5*x**5*sqrt(a + b*x)/(46189*b) + 2098*a**4*x**6*sqrt(a + b*x)/12597 + 37
96*a**3*b*x**7*sqrt(a + b*x)/6783 + 1640*a**2*b**2*x**8*sqrt(a + b*x)/2261
+ 170*a*b**3*x**9*sqrt(a + b*x)/399 + 2*b**4*x**10*sqrt(a + b*x)/21, Ne(b,
0)), (a**(9/2)*x**6/6, True))
```

3.312 $\int x^4(a+bx)^{9/2}\,dx$

Optimal. Leaf size=91

$$\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{2a^4(a+bx)^{11/2}}{11b^5}+\frac{2(a+bx)^{19/2}}{19b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*(a + b*x)^(13/2))/(13*b^5) + (4*
a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b*x)^(17/2))/(17*b^5) + (2*(a + b
*x)^(19/2))/(19*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^4(a+bx)^{9/2}\,dx&=\int\left(\frac{a^4(a+bx)^{9/2}}{b^4}-\frac{4a^3(a+bx)^{11/2}}{b^4}+\frac{6a^2(a+bx)^{13/2}}{b^4}-\frac{4a(a+bx)^{15/2}}{b^4}+\frac{(a+bx)^{17/2}}{b^4}\right)d\\&=\frac{2a^4(a+bx)^{11/2}}{11b^5}-\frac{8a^3(a+bx)^{13/2}}{13b^5}+\frac{4a^2(a+bx)^{15/2}}{5b^5}-\frac{8a(a+bx)^{17/2}}{17b^5}+\frac{2(a+bx)^{19/2}}{19b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.63

$$\frac{2(a+bx)^{11/2}\left(128a^4-704a^3bx+2288a^2b^2x^2-5720ab^3x^3+12155b^4x^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(128*a^4 - 704*a^3*b*x + 2288*a^2*b^2*x^2 - 5720*a*b^3*
x^3 + 12155*b^4*x^4))/(230945*b^5)
```

IntegrateAlgebraic [A] time = 0.03, size = 63, normalized size = 0.69

$$\frac{2(a+bx)^{11/2}\left(20995a^4-71060a^3(a+bx)+92378a^2(a+bx)^2-54340a(a+bx)^3+12155(a+bx)^4\right)}{230945b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(20995*a^4 - 71060*a^3*(a + b*x) + 92378*a^2*(a + b*x)^
2 - 54340*a*(a + b*x)^3 + 12155*(a + b*x)^4))/(230945*b^5)
```

fricas [A] time = 1.07, size = 108, normalized size = 1.19

$$\frac{2\left(12155\,b^{9}x^{9}+55055\,ab^{8}x^{8}+95238\,a^{2}b^{7}x^{7}+75086\,a^{3}b^{6}x^{6}+23063\,a^{4}b^{5}x^{5}+35\,a^{5}b^{4}x^{4}-40\,a^{6}b^{3}x^{3}+48\,a^{7}b^{2}x^{2}-64\,a^{8}bx+128\,a^{9}\right)\sqrt{bx+a}}{230945\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/230945*(12155*b^9*x^9 + 55055*a*b^8*x^8 + 95238*a^2*b^7*x^7 + 75086*a^3*b
^6*x^6 + 23063*a^4*b^5*x^5 + 35*a^5*b^4*x^4 - 40*a^6*b^3*x^3 + 48*a^7*b^2*x
^2 - 64*a^8*b*x + 128*a^9)*sqrt(b*x + a)/b^5
```

giac [B] time = 1.12, size = 565, normalized size = 6.21

$$\frac{2\left(\frac{46189\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^{2}-420(bx+a)^{\frac{3}{2}}a^{3}+315\sqrt{bx+a}\,a^{4}\right)a^{5}}{b^{4}}+\frac{104975\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^{2}-1386(bx+a)^{\frac{5}{2}}a^{3}+1155(bx+a)^{\frac{3}{2}}a^{4}-693\sqrt{bx+a}\,a^{5}\right)a^{4}}{b^{4}}+\frac{48450\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^{2}-8580(bx+a)^{\frac{7}{2}}a^{3}+9009(bx+a)^{\frac{5}{2}}a^{4}-6006(bx+a)^{\frac{3}{2}}a^{5}+3003\sqrt{bx+a}\,a^{6}\right)a^{3}}{b^{4}}+\frac{22610\left(429(bx+a)^{\frac{15}{2}}-3465(bx+a)^{\frac{13}{2}}a+12285(bx+a)^{\frac{11}{2}}a^{2}-25025(bx+a)^{\frac{9}{2}}a^{3}+32175(bx+a)^{\frac{7}{2}}a^{4}-27027(bx+a)^{\frac{5}{2}}a^{5}+15015(bx+a)^{\frac{3}{2}}a^{6}-6435\sqrt{bx+a}\,a^{7}\right)a^{2}}{b^{4}}+\frac{665\left(6435(bx+a)^{\frac{17}{2}}-58344(bx+a)^{\frac{15}{2}}a+235620(bx+a)^{\frac{13}{2}}a^{2}-556920(bx+a)^{\frac{11}{2}}a^{3}+850850(bx+a)^{\frac{9}{2}}a^{4}-875160(bx+a)^{\frac{7}{2}}a^{5}+612612(bx+a)^{\frac{5}{2}}a^{6}-291720(bx+a)^{\frac{3}{2}}a^{7}+109395\sqrt{bx+a}\,a^{8}\right)a}{b^{4}}+\frac{63\left(12155(bx+a)^{\frac{19}{2}}-122265(bx+a)^{\frac{17}{2}}a+554268(bx+a)^{\frac{15}{2}}a^{2}-1492260(bx+a)^{\frac{13}{2}}a^{3}+2645370(bx+a)^{\frac{11}{2}}a^{4}-3233230(bx+a)^{\frac{9}{2}}a^{5}+2771340(bx+a)^{\frac{7}{2}}a^{6}-1662804(bx+a)^{\frac{5}{2}}a^{7}+692835(bx+a)^{\frac{3}{2}}a^{8}-230945\sqrt{bx+a}\,a^{9}\right)}{b^{4}}\right)}{14549535\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/14549535*(46189*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x +
a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^5/b^4 + 1
04975*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)
*a^5)*a^4/b^4 + 48450*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3/b^4 + 22610*(429*
(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 2
5025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2
)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2/b^4 + 665*(
6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*
a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*
x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7
+ 109395*sqrt(b*x + a)*a^8)*a/b^4 + 63*(12155*(b*x + a)^(19/2) - 122265*(b*
x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^
3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b
*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^
8 - 230945*sqrt(b*x + a)*a^9)/b^4)/b
```

maple [A] time = 0.01, size = 54, normalized size = 0.59

$$\frac{2\,(bx+a)^{\frac{11}{2}}\left(12155x^{4}b^{4}-5720a\,x^{3}b^{3}+2288a^{2}x^{2}b^{2}-704a^{3}xb+128a^{4}\right)}{230945b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^(9/2),x)
```

```
[Out] 2/230945*(b*x+a)^(11/2)*(12155*b^4*x^4-5720*a*b^3*x^3+2288*a^2*b^2*x^2-704*
a^3*b*x+128*a^4)/b^5
```

maxima [A] time = 1.36, size = 71, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{19}{2}}}{19\,b^{5}}-\frac{8\,(bx+a)^{\frac{17}{2}}a}{17\,b^{5}}+\frac{4\,(bx+a)^{\frac{15}{2}}a^{2}}{5\,b^{5}}-\frac{8\,(bx+a)^{\frac{13}{2}}a^{3}}{13\,b^{5}}+\frac{2\,(bx+a)^{\frac{11}{2}}a^{4}}{11\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/19*(b*x + a)^(19/2)/b^5 - 8/17*(b*x + a)^(17/2)*a/b^5 + 4/5*(b*x + a)^(15
/2)*a^2/b^5 - 8/13*(b*x + a)^(13/2)*a^3/b^5 + 2/11*(b*x + a)^(11/2)*a^4/b^5
```

mupad [B] time = 0.02, size = 71, normalized size = 0.78

$$\frac{2\,{\left(a+b\,x\right)}^{19/2}}{19\,b^5}+\frac{2\,a^4\,{\left(a+b\,x\right)}^{11/2}}{11\,b^5}-\frac{8\,a^3\,{\left(a+b\,x\right)}^{13/2}}{13\,b^5}+\frac{4\,a^2\,{\left(a+b\,x\right)}^{15/2}}{5\,b^5}-\frac{8\,a\,{\left(a+b\,x\right)}^{17/2}}{17\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(19/2))/(19*b^5) + (2*a^4*(a + b*x)^(11/2))/(11*b^5) - (8*a^3*
(a + b*x)^(13/2))/(13*b^5) + (4*a^2*(a + b*x)^(15/2))/(5*b^5) - (8*a*(a + b
*x)^(17/2))/(17*b^5)
```

sympy [A] time = 25.70, size = 212, normalized size = 2.33

$$\begin{cases} \frac{256a^9\sqrt{a+bx}}{230945b^5} - \frac{128a^8x\sqrt{a+bx}}{230945b^4} + \frac{96a^7x^2\sqrt{a+bx}}{230945b^3} - \frac{16a^6x^3\sqrt{a+bx}}{46189b^2} + \frac{14a^5x^4\sqrt{a+bx}}{46189b} + \frac{46126a^4x^5\sqrt{a+bx}}{230945} + \frac{13652a^3bx^6\sqrt{a+bx}}{20995} + \frac{1332a^2b^2x^7\sqrt{a+bx}}{1615} + \frac{154ab^3x^8\sqrt{a+bx}}{323} + \frac{2b^4x^9\sqrt{a+bx}}{19} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^5}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((256*a**9*sqrt(a + b*x)/(230945*b**5) - 128*a**8*x*sqrt(a + b*x)/
(230945*b**4) + 96*a**7*x**2*sqrt(a + b*x)/(230945*b**3) - 16*a**6*x**3*sqr
t(a + b*x)/(46189*b**2) + 14*a**5*x**4*sqrt(a + b*x)/(46189*b) + 46126*a**4
*x**5*sqrt(a + b*x)/230945 + 13652*a**3*b*x**6*sqrt(a + b*x)/20995 + 1332*a
**2*b**2*x**7*sqrt(a + b*x)/1615 + 154*a*b**3*x**8*sqrt(a + b*x)/323 + 2*b*
*4*x**9*sqrt(a + b*x)/19, Ne(b, 0)), (a**(9/2)*x**5/5, True))
```

3.313 $\int x^3(a+bx)^{9/2}\,dx$

Optimal. Leaf size=72

$$-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{2(a+bx)^{17/2}}{17b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*(a + b*x)^(13/2))/(13*b^4) - (2
*a*(a + b*x)^(15/2))/(5*b^4) + (2*(a + b*x)^(17/2))/(17*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{9/2}\,dx &= \int\left(-\frac{a^3(a+bx)^{9/2}}{b^3}+\frac{3a^2(a+bx)^{11/2}}{b^3}-\frac{3a(a+bx)^{13/2}}{b^3}+\frac{(a+bx)^{15/2}}{b^3}\right)dx\\ &= -\frac{2a^3(a+bx)^{11/2}}{11b^4}+\frac{6a^2(a+bx)^{13/2}}{13b^4}-\frac{2a(a+bx)^{15/2}}{5b^4}+\frac{2(a+bx)^{17/2}}{17b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.64

$$\frac{2(a+bx)^{11/2}\left(-16a^3+88a^2bx-286ab^2x^2+715b^3x^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-16*a^3 + 88*a^2*b*x - 286*a*b^2*x^2 + 715*b^3*x^3))/(
12155*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}\left(-1105a^3+2805a^2(a+bx)-2431a(a+bx)^2+715(a+bx)^3\right)}{12155b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-1105*a^3 + 2805*a^2*(a + b*x) - 2431*a*(a + b*x)^2 +
715*(a + b*x)^3))/(12155*b^4)
```

fricas [A] time = 0.96, size = 97, normalized size = 1.35

$$\frac{2\left(715\,b^{8}x^{8}+3289\,ab^{7}x^{7}+5808\,a^{2}b^{6}x^{6}+4714\,a^{3}b^{5}x^{5}+1515\,a^{4}b^{4}x^{4}+5\,a^{5}b^{3}x^{3}-6\,a^{6}b^{2}x^{2}+8\,a^{7}bx-16\,a^{8}\right)\sqrt{bx+a}}{12155\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/12155*(715*b^8*x^8 + 3289*a*b^7*x^7 + 5808*a^2*b^6*x^6 + 4714*a^3*b^5*x^5
 + 1515*a^4*b^4*x^4 + 5*a^5*b^3*x^3 - 6*a^6*b^2*x^2 + 8*a^7*b*x - 16*a^8)*s
qrt(b*x + a)/b^4
```

giac [B] time = 1.10, size = 493, normalized size = 6.85

$$\frac{2\left(\frac{21879\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{5}}{b^{3}}+\frac{12155\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)a^{4}}{b^{3}}+\frac{11050\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^{2}-1386\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}-693\,\sqrt{bx+a}\,a^{5}\right)a^{3}}{b^{3}}+\frac{2550\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^{2}-8580\,(bx+a)^{\frac{7}{2}}a^{3}+9009\,(bx+a)^{\frac{5}{2}}a^{4}-6006\,(bx+a)^{\frac{3}{2}}a^{5}+3003\,\sqrt{bx+a}\,a^{6}\right)a^{2}}{b^{3}}+\frac{595\left(429\,(bx+a)^{\frac{15}{2}}-3465\,(bx+a)^{\frac{13}{2}}a+12285\,(bx+a)^{\frac{11}{2}}a^{2}-25025\,(bx+a)^{\frac{9}{2}}a^{3}+32175\,(bx+a)^{\frac{7}{2}}a^{4}-27027\,(bx+a)^{\frac{5}{2}}a^{5}+15015\,(bx+a)^{\frac{3}{2}}a^{6}-6435\,\sqrt{bx+a}\,a^{7}\right)a}{b^{3}}+\frac{7\left(6435\,(bx+a)^{\frac{17}{2}}-58344\,(bx+a)^{\frac{15}{2}}a+235620\,(bx+a)^{\frac{13}{2}}a^{2}-556920\,(bx+a)^{\frac{11}{2}}a^{3}+850850\,(bx+a)^{\frac{9}{2}}a^{4}-875160\,(bx+a)^{\frac{7}{2}}a^{5}+612612\,(bx+a)^{\frac{5}{2}}a^{6}-291720\,(bx+a)^{\frac{3}{2}}a^{7}+109395\,\sqrt{bx+a}\,a^{8}\right)}{b^{3}}\right)}{765765\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/765765*(21879*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3
/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^5/b^3 + 12155*(35*(b*x + a)^(9/2) - 180*(
b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*
sqrt(b*x + a)*a^4)*a^4/b^3 + 11050*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/
2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^
(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3/b^3 + 2550*(231*(b*x + a)^(13/2) - 1
638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^
3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a
)*a^6)*a^2/b^3 + 595*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 1228
5*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a/b^3 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 23
5620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^
(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 29172
0*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)/b^3)/b
```

maple [A] time = 0.01, size = 43, normalized size = 0.60

$$-\frac{2\,(bx+a)^{\frac{11}{2}}\left(-715b^{3}x^{3}+286a\,b^{2}x^{2}-88a^{2}bx+16a^{3}\right)}{12155b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(9/2),x)
```

```
[Out] -2/12155*(b*x+a)^(11/2)*(-715*b^3*x^3+286*a*b^2*x^2-88*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.29, size = 56, normalized size = 0.78

$$\frac{2\,(bx+a)^{\frac{17}{2}}}{17\,b^{4}}-\frac{2\,(bx+a)^{\frac{15}{2}}a}{5\,b^{4}}+\frac{6\,(bx+a)^{\frac{13}{2}}a^{2}}{13\,b^{4}}-\frac{2\,(bx+a)^{\frac{11}{2}}a^{3}}{11\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/17*(b*x + a)^(17/2)/b^4 - 2/5*(b*x + a)^(15/2)*a/b^4 + 6/13*(b*x + a)^(13
/2)*a^2/b^4 - 2/11*(b*x + a)^(11/2)*a^3/b^4
```

mupad [B] time = 0.04, size = 56, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{17/2}}{17\,b^{4}}-\frac{2\,a^{3}\,(a+b\,x)^{11/2}}{11\,b^{4}}+\frac{6\,a^{2}\,(a+b\,x)^{13/2}}{13\,b^{4}}-\frac{2\,a\,(a+b\,x)^{15/2}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(17/2))/(17*b^4) - (2*a^3*(a + b*x)^(11/2))/(11*b^4) + (6*a^2*
(a + b*x)^(13/2))/(13*b^4) - (2*a*(a + b*x)^(15/2))/(5*b^4)
```

sympy [A] time = 20.35, size = 190, normalized size = 2.64

$$\begin{cases} -\frac{32a^8\sqrt{a+bx}}{12155b^4} + \frac{16a^7x\sqrt{a+bx}}{12155b^3} - \frac{12a^6x^2\sqrt{a+bx}}{12155b^2} + \frac{2a^5x^3\sqrt{a+bx}}{2431b} + \frac{606a^4x^4\sqrt{a+bx}}{2431} + \frac{9428a^3bx^5\sqrt{a+bx}}{12155} + \frac{1056a^2b^2x^6\sqrt{a+bx}}{1105} + \frac{46ab^3x^7\sqrt{a+bx}}{85} + \frac{2b^4x^8\sqrt{a+bx}}{17} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-32*a**8*sqrt(a + b*x)/(12155*b**4) + 16*a**7*x*sqrt(a + b*x)/(1
2155*b**3) - 12*a**6*x**2*sqrt(a + b*x)/(12155*b**2) + 2*a**5*x**3*sqrt(a +
 b*x)/(2431*b) + 606*a**4*x**4*sqrt(a + b*x)/2431 + 9428*a**3*b*x**5*sqrt(a
 + b*x)/12155 + 1056*a**2*b**2*x**6*sqrt(a + b*x)/1105 + 46*a*b**3*x**7*sqr
t(a + b*x)/85 + 2*b**4*x**8*sqrt(a + b*x)/17, Ne(b, 0)), (a**(9/2)*x**4/4,
True))
```

3.314 $\int x^2(a+bx)^{9/2}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{11/2}}{11b^3}+\frac{2(a+bx)^{15/2}}{15b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*a^2*(a + b*x)^(11/2))/(11*b^3) - (4*a*(a + b*x)^(13/2))/(13*b^3) + (2*(a
 + b*x)^(15/2))/(15*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{9/2}\,dx&=\int\left(\frac{a^2(a+bx)^{9/2}}{b^2}-\frac{2a(a+bx)^{11/2}}{b^2}+\frac{(a+bx)^{13/2}}{b^2}\right)dx\\&=\frac{2a^2(a+bx)^{11/2}}{11b^3}-\frac{4a(a+bx)^{13/2}}{13b^3}+\frac{2(a+bx)^{15/2}}{15b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{11/2}\left(8a^2-44abx+143b^2x^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(8*a^2 - 44*a*b*x + 143*b^2*x^2))/(2145*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.74

$$\frac{2(a+bx)^{11/2}\left(195a^2-330a(a+bx)+143(a+bx)^2\right)}{2145b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(195*a^2 - 330*a*(a + b*x) + 143*(a + b*x)^2))/(2145*b^
3)
```

fricas [B] time = 0.96, size = 86, normalized size = 1.62

$$\frac{2\left(143\,b^{7}x^{7}+671\,ab^{6}x^{6}+1218\,a^{2}b^{5}x^{5}+1030\,a^{3}b^{4}x^{4}+355\,a^{4}b^{3}x^{3}+3\,a^{5}b^{2}x^{2}-4\,a^{6}bx+8\,a^{7}\right)\sqrt{bx+a}}{2145\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/2145*(143*b^7*x^7 + 671*a*b^6*x^6 + 1218*a^2*b^5*x^5 + 1030*a^3*b^4*x^4 +
 355*a^4*b^3*x^3 + 3*a^5*b^2*x^2 - 4*a^6*b*x + 8*a^7)*sqrt(b*x + a)/b^3
```

giac [B] time = 1.06, size = 421, normalized size = 7.94

$$\frac{2\left(\frac{3003\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)a^{5}}{b^{2}}+\frac{6435\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)a^{4}}{b^{2}}+\frac{1430\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^{2}-420\,(bx+a)^{\frac{3}{2}}a^{3}+315\,\sqrt{bx+a}\,a^{4}\right)a^{3}}{b^{2}}+\frac{650\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^{2}-1386\,(bx+a)^{\frac{5}{2}}a^{3}+1155\,(bx+a)^{\frac{3}{2}}a^{4}-693\,\sqrt{bx+a}\,a^{5}\right)a^{2}}{b^{2}}+\frac{75\left(231\,(bx+a)^{\frac{13}{2}}-1638\,(bx+a)^{\frac{11}{2}}a+5005\,(bx+a)^{\frac{9}{2}}a^{2}-8580\,(bx+a)^{\frac{7}{2}}a^{3}+9009\,(bx+a)^{\frac{5}{2}}a^{4}-6006\,(bx+a)^{\frac{3}{2}}a^{5}+3003\,\sqrt{bx+a}\,a^{6}\right)a}{b^{2}}+\frac{7\left(429\,(bx+a)^{\frac{15}{2}}-3465\,(bx+a)^{\frac{13}{2}}a+12285\,(bx+a)^{\frac{11}{2}}a^{2}-25025\,(bx+a)^{\frac{9}{2}}a^{3}+32175\,(bx+a)^{\frac{7}{2}}a^{4}-27027\,(bx+a)^{\frac{5}{2}}a^{5}+15015\,(bx+a)^{\frac{3}{2}}a^{6}-6435\,\sqrt{bx+a}\,a^{7}\right)}{b^{2}}\right)}{45045\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*
a^2)*a^5/b^2 + 6435*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a
)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^4/b^2 + 1430*(35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 3
15*sqrt(b*x + a)*a^4)*a^3/b^2 + 650*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9
/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)
^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2/b^2 + 75*(231*(b*x + a)^(13/2) - 16
38*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3
 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)
*a^6)*a/b^2 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*
x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 -
 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)
*a^7)/b^2)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.60

$$\frac{2\,(bx+a)^{\frac{11}{2}}\left(143b^{2}x^{2}-44abx+8a^{2}\right)}{2145b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(9/2),x)
```

```
[Out] 2/2145*(b*x+a)^(11/2)*(143*b^2*x^2-44*a*b*x+8*a^2)/b^3
```

maxima [A] time = 1.34, size = 41, normalized size = 0.77

$$\frac{2\,(bx+a)^{\frac{15}{2}}}{15\,b^{3}}-\frac{4\,(bx+a)^{\frac{13}{2}}a}{13\,b^{3}}+\frac{2\,(bx+a)^{\frac{11}{2}}a^{2}}{11\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*x + a)^(15/2)/b^3 - 4/13*(b*x + a)^(13/2)*a/b^3 + 2/11*(b*x + a)^(1
1/2)*a^2/b^3
```

mupad [B] time = 0.04, size = 36, normalized size = 0.68

$$\frac{\frac{2\,{(a+b\,x)}^{15/2}}{15}-\frac{4\,a\,{(a+b\,x)}^{13/2}}{13}+\frac{2\,a^2\,{(a+b\,x)}^{11/2}}{11}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(9/2),x)
```

```
[Out] ((2*(a + b*x)^(15/2))/15 - (4*a*(a + b*x)^(13/2))/13 + (2*a^2*(a + b*x)^(11
/2))/11)/b^3
```

sympy [A] time = 16.86, size = 168, normalized size = 3.17

$$\begin{cases} \frac{16a^7\sqrt{a+bx}}{2145b^3} - \frac{8a^6x\sqrt{a+bx}}{2145b^2} + \frac{2a^5x^2\sqrt{a+bx}}{715b} + \frac{142a^4x^3\sqrt{a+bx}}{429} + \frac{412a^3bx^4\sqrt{a+bx}}{429} + \frac{812a^2b^2x^5\sqrt{a+bx}}{715} + \frac{122ab^3x^6\sqrt{a+bx}}{195} + \frac{2b^4x^7\sqrt{a+bx}}{15} & \text{for } b \neq 0 \\ \frac{a^{\frac{9}{2}}x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((16*a**7*sqrt(a + b*x)/(2145*b**3) - 8*a**6*x*sqrt(a + b*x)/(2145
*b**2) + 2*a**5*x**2*sqrt(a + b*x)/(715*b) + 142*a**4*x**3*sqrt(a + b*x)/42
9 + 412*a**3*b*x**4*sqrt(a + b*x)/429 + 812*a**2*b**2*x**5*sqrt(a + b*x)/71
5 + 122*a*b**3*x**6*sqrt(a + b*x)/195 + 2*b**4*x**7*sqrt(a + b*x)/15, Ne(b,
 0)), (a**(9/2)*x**3/3, True))
```

3.315 $\int x(a+bx)^{9/2}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{13/2}}{13b^2}-\frac{2a(a+bx)^{11/2}}{11b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*a*(a + b*x)^(11/2))/(11*b^2) + (2*(a + b*x)^(13/2))/(13*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{9/2}\,dx&=\int\left(-\frac{a(a+bx)^{9/2}}{b}+\frac{(a+bx)^{11/2}}{b}\right)dx\\&=-\frac{2a(a+bx)^{11/2}}{11b^2}+\frac{2(a+bx)^{13/2}}{13b^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{11/2}(11bx-2a)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*(-2*a + 11*b*x))/(143*b^2)
```

IntegrateAlgebraic [B] time = 0.01, size = 79, normalized size = 2.32

$$-\frac{2\sqrt{a+bx}\left(2a^6-a^5bx-35a^4b^2x^2-90a^3b^3x^3-100a^2b^4x^4-53ab^5x^5-11b^6x^6\right)}{143b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(9/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(2*a^6 - a^5*b*x - 35*a^4*b^2*x^2 - 90*a^3*b^3*x^3 - 100*
a^2*b^4*x^4 - 53*a*b^5*x^5 - 11*b^6*x^6))/(143*b^2)
```

fricas [B] time = 0.72, size = 74, normalized size = 2.18

$$\frac{2\left(11\,b^6x^6+53\,ab^5x^5+100\,a^2b^4x^4+90\,a^3b^3x^3+35\,a^4b^2x^2+a^5bx-2\,a^6\right)\sqrt{bx+a}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/143*(11*b^6*x^6 + 53*a*b^5*x^5 + 100*a^2*b^4*x^4 + 90*a^3*b^3*x^3 + 35*a^
4*b^2*x^2 + a^5*b*x - 2*a^6)*sqrt(b*x + a)/b^2
```

giac [B] time = 1.27, size = 347, normalized size = 10.21

$$\frac{2\left(\frac{3003\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)a^5}{b}+\frac{3003\left(3(bx+a)^{\frac{5}{2}}-10(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}\,a^2\right)a^4}{b}+\frac{2574\left(5(bx+a)^{\frac{7}{2}}-21(bx+a)^{\frac{5}{2}}a+35(bx+a)^{\frac{3}{2}}a^2-35\sqrt{bx+a}\,a^3\right)a^3}{b}+\frac{286\left(35(bx+a)^{\frac{9}{2}}-180(bx+a)^{\frac{7}{2}}a+378(bx+a)^{\frac{5}{2}}a^2-420(bx+a)^{\frac{3}{2}}a^3+315\sqrt{bx+a}\,a^4\right)a^2}{b}+\frac{65\left(63(bx+a)^{\frac{11}{2}}-385(bx+a)^{\frac{9}{2}}a+990(bx+a)^{\frac{7}{2}}a^2-1386(bx+a)^{\frac{5}{2}}a^3+1155(bx+a)^{\frac{3}{2}}a^4-693\sqrt{bx+a}\,a^5\right)a}{b}+\frac{3\left(231(bx+a)^{\frac{13}{2}}-1638(bx+a)^{\frac{11}{2}}a+5005(bx+a)^{\frac{9}{2}}a^2-8580(bx+a)^{\frac{7}{2}}a^3+9009(bx+a)^{\frac{5}{2}}a^4-6006(bx+a)^{\frac{3}{2}}a^5+3003\sqrt{bx+a}\,a^6\right)}{b}\right)}{9009\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/9009*(3003*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^5/b + 3003*(3*(b*x + a
)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^4/b + 2574*(5*(b*x
 + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x +
 a)*a^3)*a^3/b + 286*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
 + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2/b +
65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 -
 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^
5)*a/b + 3*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)
^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*
x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{2\,(bx+a)^{\frac{11}{2}}\,(-11bx+2a)}{143b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(9/2),x)
```

```
[Out] -2/143*(b*x+a)^(11/2)*(-11*b*x+2*a)/b^2
```

maxima [A] time = 1.36, size = 26, normalized size = 0.76

$$\frac{2\,(bx+a)^{\frac{13}{2}}}{13\,b^2}-\frac{2\,(bx+a)^{\frac{11}{2}}\,a}{11\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*x + a)^(13/2)/b^2 - 2/11*(b*x + a)^(11/2)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{26\,a\,(a+b\,x)^{11/2}-22\,(a+b\,x)^{13/2}}{143\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(9/2),x)
```

```
[Out] -(26*a*(a + b*x)^(11/2) - 22*(a + b*x)^(13/2))/(143*b^2)
```

sympy [A] time = 14.85, size = 146, normalized size = 4.29

$$\begin{cases} -\frac{4a^6\sqrt{a+bx}}{143b^2}+\frac{2a^5x\sqrt{a+bx}}{143b}+\frac{70a^4x^2\sqrt{a+bx}}{143}+\frac{180a^3bx^3\sqrt{a+bx}}{143}+\frac{200a^2b^2x^4\sqrt{a+bx}}{143}+\frac{106ab^3x^5\sqrt{a+bx}}{143}+\frac{2b^4x^6\sqrt{a+bx}}{13} & \text{for } b\neq 0 \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(9/2),x)
```

```
[Out] Piecewise((-4*a**6*sqrt(a + b*x)/(143*b**2) + 2*a**5*x*sqrt(a + b*x)/(143*b
) + 70*a**4*x**2*sqrt(a + b*x)/143 + 180*a**3*b*x**3*sqrt(a + b*x)/143 + 20
0*a**2*b**2*x**4*sqrt(a + b*x)/143 + 106*a*b**3*x**5*sqrt(a + b*x)/143 + 2*
b**4*x**6*sqrt(a + b*x)/13, Ne(b, 0)), (a**(9/2)*x**2/2, True))
```

3.316 $\int (a+bx)^{9/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{11/2}}{11b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2),x]
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{9/2}\,dx = \frac{2(a+bx)^{11/2}}{11b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2),x]
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(a+bx)^{11/2}}{11b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2),x]
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

fricas [B] time = 1.05, size = 61, normalized size = 3.81

$$\frac{2\left(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5\right)\sqrt{bx+a}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x +
 a^5)*sqrt(b*x + a)/b
```

giac [B] time = 0.99, size = 229, normalized size = 14.31

$$\frac{2\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4+1155\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^4+462\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a^3+198\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2+11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a\right)}{693\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 2/693*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^
2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 + 1155*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a^4 + 462*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)
*a + 15*sqrt(b*x + a)*a^2)*a^3 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2 + 11*(35*(b*x + a)
^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3
/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2),x)
```

```
[Out] 2/11*(b*x+a)^(11/2)/b
```

maxima [A] time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(bx+a)^{\frac{11}{2}}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/11*(b*x + a)^(11/2)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(a+b\,x\right)}^{11/2}}{11\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2),x)
```

```
[Out] (2*(a + b*x)^(11/2))/(11*b)
```

sympy [A] time = 0.08, size = 12, normalized size = 0.75

$$\frac{2\,(a+bx)^{\frac{11}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2),x)
```

```
[Out] 2*(a + b*x)**(11/2)/(11*b)
```

3.317 $\int \frac{(a+bx)^{9/2}}{x} \, dx$

Optimal. Leaf size=97

$$-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2a^4\sqrt{a+bx}+\frac{2}{3}a^3(a+bx)^{3/2}+\frac{2}{5}a^2(a+bx)^{5/2}-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2}{7}a(a+bx)^{7/2}+\frac{2}{9}(a+bx)^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x,x]
```

```
[Out] 2*a^4*Sqrt[a + b*x] + (2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5
 + (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 - 2*a^(9/2)*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x}\,dx = \frac{2}{9}(a+bx)^{9/2} + a\int \frac{(a+bx)^{7/2}}{x}\,dx$$
$$= \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^2\int \frac{(a+bx)^{5/2}}{x}\,dx$$
$$= \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^3\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^4\int \frac{\sqrt{a+bx}}{x}\,dx$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + a^5\int \frac{1}{x\sqrt{a+}}$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} + \frac{\left(2a^5\right)\text{Subst}}{}$$
$$= 2a^4\sqrt{a+bx} + \frac{2}{3}a^3(a+bx)^{3/2} + \frac{2}{5}a^2(a+bx)^{5/2} + \frac{2}{7}a(a+bx)^{7/2} + \frac{2}{9}(a+bx)^{9/2} - 2a^{9/2}\tanh^{-1}$$

Mathematica [**A**] time = 0.09, size = 78, normalized size = 0.80

$$\frac{2}{315}\sqrt{a+bx}\left(563a^4+506a^3bx+408a^2b^2x^2+185ab^3x^3+35b^4x^4\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(563*a^4 + 506*a^3*b*x + 408*a^2*b^2*x^2 + 185*a*b^3*x^3 +
 35*b^4*x^4))/315 - 2*a^(9/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [**A**] time = 0.03, size = 94, normalized size = 0.97

$$\frac{2}{315}\left(315a^4\sqrt{a+bx}+105a^3(a+bx)^{3/2}+63a^2(a+bx)^{5/2}+35(a+bx)^{9/2}+45a(a+bx)^{7/2}\right)-2a^{9/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x,x]
```

```
[Out] (2*(315*a^4*Sqrt[a + b*x] + 105*a^3*(a + b*x)^(3/2) + 63*a^2*(a + b*x)^(5/2
) + 45*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/315 - 2*a^(9/2)*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]]
```

fricas [**A**] time = 1.18, size = 158, normalized size = 1.63

$$\left[a^{\frac{9}{2}}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a},\ 2\sqrt{-a}a^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\frac{2}{315}\left(35b^4x^4+185ab^3x^3+408a^2b^2x^2+506a^3bx+563a^4\right)\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="fricas")
```

```
[Out] [a^(9/2)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2/315*(35*b^4*x^4 +
 185*a*b^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a), 2*
sqrt(-a)*a^4*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2/315*(35*b^4*x^4 + 185*a*b
^3*x^3 + 408*a^2*b^2*x^2 + 506*a^3*b*x + 563*a^4)*sqrt(b*x + a)]
```

giac [A] time = 1.23, size = 80, normalized size = 0.82

$$\frac{2\,a^{5}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2}{9}\,(bx+a)^{\frac{9}{2}}+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}a+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}a^{2}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a^{3}+2\,\sqrt{bx+a}\,a^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2/9*(b*x + a)^(9/2) + 2/7*(
b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x + a)^(3/2)*a^3 + 2*sq
rt(b*x + a)*a^4
```

maple [A] time = 0.01, size = 74, normalized size = 0.76

$$-2a^{\frac{9}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;a^{4}+\frac{2\,(bx+a)^{\frac{3}{2}}\,a^{3}}{3}+\frac{2\,(bx+a)^{\frac{5}{2}}\,a^{2}}{5}+\frac{2\,(bx+a)^{\frac{7}{2}}\,a}{7}+\frac{2\,(bx+a)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x,x)
```

```
[Out] 2/3*a^3*(b*x+a)^(3/2)+2/5*a^2*(b*x+a)^(5/2)+2/7*a*(b*x+a)^(7/2)+2/9*(b*x+a)
^(9/2)-2*a^(9/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+2*a^4*(b*x+a)^(1/2)
```

maxima [A] time = 2.94, size = 88, normalized size = 0.91

$$a^{\frac{9}{2}}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{9}\,(bx+a)^{\frac{9}{2}}+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}a+\frac{2}{5}\,(bx+a)^{\frac{5}{2}}a^{2}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}a^{3}+2\,\sqrt{bx+a}\,a^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x,x, algorithm="maxima")
```

```
[Out] a^(9/2)*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/9*(b*x
 + a)^(9/2) + 2/7*(b*x + a)^(7/2)*a + 2/5*(b*x + a)^(5/2)*a^2 + 2/3*(b*x +
a)^(3/2)*a^3 + 2*sqrt(b*x + a)*a^4
```

mupad [B] time = 0.04, size = 76, normalized size = 0.78

$$\frac{2\,a\,(a+b\,x)^{7/2}}{7}+\frac{2\,(a+b\,x)^{9/2}}{9}+2\,a^{4}\,\sqrt{a+b\,x}+\frac{2\,a^{3}\,(a+b\,x)^{3/2}}{3}+\frac{2\,a^{2}\,(a+b\,x)^{5/2}}{5}+a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\;1\mathrm{i}}{\sqrt{a}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x,x)
```

```
[Out] (2*a*(a + b*x)^(7/2))/7 + (2*(a + b*x)^(9/2))/9 + 2*a^4*(a + b*x)^(1/2) + (
2*a^3*(a + b*x)^(3/2))/3 + (2*a^2*(a + b*x)^(5/2))/5 + a^(9/2)*atan(((a + b
*x)^(1/2)*1i)/a^(1/2))*2i
```

sympy [A] time = 11.10, size = 148, normalized size = 1.53

$$\frac{1126a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{315}+a^{\frac{9}{2}}\log\left(\frac{bx}{a}\right)-2a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{1012a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{315}+\frac{272a^{\frac{5}{2}}b^{2}x^{2}\sqrt{1+\frac{bx}{a}}}{105}+\frac{74a^{\frac{3}{2}}b^{3}x^{3}\sqrt{1+\frac{bx}{a}}}{63}+\frac{2\sqrt{a}\,b^{4}x^{4}\sqrt{1+\frac{bx}{a}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x,x)
```

```
[Out] 1126*a**(9/2)*sqrt(1 + b*x/a)/315 + a**(9/2)*log(b*x/a) - 2*a**(9/2)*log(sq
rt(1 + b*x/a) + 1) + 1012*a**(7/2)*b*x*sqrt(1 + b*x/a)/315 + 272*a**(5/2)*b
**2*x**2*sqrt(1 + b*x/a)/105 + 74*a**(3/2)*b**3*x**3*sqrt(1 + b*x/a)/63 + 2
*sqrt(a)*b**4*x**4*sqrt(1 + b*x/a)/9
```

3.318 $\int \frac{(a+bx)^{9/2}}{x^2} dx$

Optimal. Leaf size=98

$$-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+9a^3b\sqrt{a+bx}+3a^2b(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$3a^2b(a+bx)^{3/2}+9a^3b\sqrt{a+bx}-9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{(a+bx)^{9/2}}{x}+\frac{9}{7}b(a+bx)^{7/2}+\frac{9}{5}ab(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] 9*a^3*b*Sqrt[a + b*x] + 3*a^2*b*(a + b*x)^(3/2) + (9*a*b*(a + b*x)^(5/2))/5
 + (9*b*(a + b*x)^(7/2))/7 - (a + b*x)^(9/2)/x - 9*a^(7/2)*b*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^2}\,dx &= -\frac{(a+bx)^{9/2}}{x} + \frac{1}{2}(9b)\int \frac{(a+bx)^{7/2}}{x}\,dx \\
&= \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}(9ab)\int \frac{(a+bx)^{5/2}}{x}\,dx \\
&= \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}\left(9a^2b\right)\int \frac{(a+bx)^{3/2}}{x}\,dx \\
&= 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}\left(9a^3b\right)\int \frac{\sqrt{a+bx}}{x}\,dx \\
&= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \frac{1}{2}\left(9a^4b\right) \\
&= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} + \left(9a^4\right)\text{Sul} \\
&= 9a^3b\sqrt{a+bx} + 3a^2b(a+bx)^{3/2} + \frac{9}{5}ab(a+bx)^{5/2} + \frac{9}{7}b(a+bx)^{7/2} - \frac{(a+bx)^{9/2}}{x} - 9a^{7/2}b\,\text{tar}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.34

$$\frac{2b(a+bx)^{11/2}\,{}_2F_1\left(2,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (2*b*(a + b*x)^(11/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x)/a])/(11*a^
2)
```

IntegrateAlgebraic [A] time = 0.07, size = 88, normalized size = 0.90

$$\frac{\sqrt{a+bx}\left(-315a^4+210a^3(a+bx)+42a^2(a+bx)^2+18a(a+bx)^3+10(a+bx)^4\right)}{35x} - 9a^{7/2}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^2,x]
```

```
[Out] (Sqrt[a + b*x]*(-315*a^4 + 210*a^3*(a + b*x) + 42*a^2*(a + b*x)^2 + 18*a*(a
 + b*x)^3 + 10*(a + b*x)^4))/(35*x) - 9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]
```

fricas [A] time = 0.86, size = 172, normalized size = 1.76

$$\left[\frac{315\,a^{\frac{7}{2}}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4\right)\sqrt{bx+a}}{70\,x}, \frac{315\sqrt{-a}\,a^3bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(10\,b^4x^4+58\,ab^3x^3+156\,a^2b^2x^2+388\,a^3bx-35\,a^4\right)\sqrt{bx+a}}{35\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/70*(315*a^(7/2)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(10
*b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x
+ a))/x, 1/35*(315*sqrt(-a)*a^3*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (10*
b^4*x^4 + 58*a*b^3*x^3 + 156*a^2*b^2*x^2 + 388*a^3*b*x - 35*a^4)*sqrt(b*x +
 a))/x]
```

giac [A] time = 1.43, size = 104, normalized size = 1.06

$$\frac{\frac{315\,a^{4}b^{2}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\,(bx+a)^{\frac{7}{2}}b^{2}+28\,(bx+a)^{\frac{5}{2}}ab^{2}+70\,(bx+a)^{\frac{3}{2}}a^{2}b^{2}+280\,\sqrt{bx+a}\,a^{3}b^{2}-\frac{35\,\sqrt{bx+a}\,a^{4}b}{x}}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/35*(315*a^4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 10*(b*x + a)^(7
/2)*b^2 + 28*(b*x + a)^(5/2)*a*b^2 + 70*(b*x + a)^(3/2)*a^2*b^2 + 280*sqrt(
b*x + a)*a^3*b^2 - 35*sqrt(b*x + a)*a^4*b/x)/b
```

maple [A] time = 0.01, size = 84, normalized size = 0.86

$$2\left(\left(-\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{\sqrt{bx+a}}{2bx}\right)a^{4}+4\sqrt{bx+a}\;a^{3}+(bx+a)^{\frac{3}{2}}\,a^{2}+\frac{2\,(bx+a)^{\frac{5}{2}}\,a}{5}+\frac{(bx+a)^{\frac{7}{2}}}{7}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^2,x)
```

```
[Out] 2*b*(1/7*(b*x+a)^(7/2)+2/5*a*(b*x+a)^(5/2)+a^2*(b*x+a)^(3/2)+4*(b*x+a)^(1/2
)*a^3+a^4*(-1/2*(b*x+a)^(1/2)/b/x-9/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2
)))
```

maxima [A] time = 2.97, size = 97, normalized size = 0.99

$$\frac{9}{2}\,a^{\frac{7}{2}}b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{7}\,(bx+a)^{\frac{7}{2}}b+\frac{4}{5}\,(bx+a)^{\frac{5}{2}}ab+2\,(bx+a)^{\frac{3}{2}}a^{2}b+8\,\sqrt{bx+a}\,a^{3}b-\frac{\sqrt{bx+a}\,a^{4}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^2,x, algorithm="maxima")
```

```
[Out] 9/2*a^(7/2)*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
7*(b*x + a)^(7/2)*b + 4/5*(b*x + a)^(5/2)*a*b + 2*(b*x + a)^(3/2)*a^2*b + 8
*sqrt(b*x + a)*a^3*b - sqrt(b*x + a)*a^4/x
```

mupad [B] time = 0.04, size = 84, normalized size = 0.86

$$\frac{2\,b\,(a+b\,x)^{7/2}}{7}-\frac{a^{4}\,\sqrt{a+b\,x}}{x}+\frac{4\,a\,b\,(a+b\,x)^{5/2}}{5}+8\,a^{3}\,b\,\sqrt{a+b\,x}+2\,a^{2}\,b\,(a+b\,x)^{3/2}+a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^2,x)
```

```
[Out] (2*b*(a + b*x)^(7/2))/7 - (a^4*(a + b*x)^(1/2))/x + a^(7/2)*b*atan(((a + b*
x)^(1/2)*1i)/a^(1/2))*9i + (4*a*b*(a + b*x)^(5/2))/5 + 8*a^3*b*(a + b*x)^(1
/2) + 2*a^2*b*(a + b*x)^(3/2)
```

sympy [A] time = 9.92, size = 150, normalized size = 1.53

$$-\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{x}+\frac{388a^{\frac{7}{2}}b\sqrt{1+\frac{bx}{a}}}{35}+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx}{a}\right)}{2}-9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx}{a}}+1\right)+\frac{156a^{\frac{5}{2}}b^{2}x\sqrt{1+\frac{bx}{a}}}{35}+\frac{58a^{\frac{3}{2}}b^{3}x^{2}\sqrt{1+\frac{bx}{a}}}{35}+\frac{2\sqrt{a}\,b^{4}x^{3}\sqrt{1+\frac{bx}{a}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**2,x)
```

```
[Out] -a**(9/2)*sqrt(1 + b*x/a)/x + 388*a**(7/2)*b*sqrt(1 + b*x/a)/35 + 9*a**(7/2
)*b*log(b*x/a)/2 - 9*a**(7/2)*b*log(sqrt(1 + b*x/a) + 1) + 156*a**(5/2)*b**
2*x*sqrt(1 + b*x/a)/35 + 58*a**(3/2)*b**3*x**2*sqrt(1 + b*x/a)/35 + 2*sqrt(
a)*b**4*x**3*sqrt(1 + b*x/a)/7
```

3.319 $\int \frac{(a+bx)^{9/2}}{x^3}\,dx$

Optimal. Leaf size=114

$$-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{4}a^2b^2\sqrt{a+bx}+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$\frac{63}{4}a^2b^2\sqrt{a+bx}-\frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{63}{20}b^2(a+bx)^{5/2}+\frac{21}{4}ab^2(a+bx)^{3/2}-\frac{(a+bx)^{9/2}}{2x^2}-\frac{9b(a+bx)^{7/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (63*a^2*b^2*Sqrt[a + b*x])/4 + (21*a*b^2*(a + b*x)^(3/2))/4 + (63*b^2*(a +
b*x)^(5/2))/20 - (9*b*(a + b*x)^(7/2))/(4*x) - (a + b*x)^(9/2)/(2*x^2) - (6
3*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^3}\,dx = -\frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{4}(9b)\int \frac{(a+bx)^{7/2}}{x^2}\,dx$$
$$= -\frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{8}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x}\,dx$$
$$= \frac{63}{20}b^2(a+bx)^{5/2} - \frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{8}\left(63ab^2\right)\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= \frac{21}{4}ab^2(a+bx)^{3/2} + \frac{63}{20}b^2(a+bx)^{5/2} - \frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{8}\left(63a^2b^2\right)\int \frac{\sqrt{a+bx}}{x}$$
$$= \frac{63}{4}a^2b^2\sqrt{a+bx} + \frac{21}{4}ab^2(a+bx)^{3/2} + \frac{63}{20}b^2(a+bx)^{5/2} - \frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{8}($$
$$= \frac{63}{4}a^2b^2\sqrt{a+bx} + \frac{21}{4}ab^2(a+bx)^{3/2} + \frac{63}{20}b^2(a+bx)^{5/2} - \frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} + \frac{1}{4}($$
$$= \frac{63}{4}a^2b^2\sqrt{a+bx} + \frac{21}{4}ab^2(a+bx)^{3/2} + \frac{63}{20}b^2(a+bx)^{5/2} - \frac{9b(a+bx)^{7/2}}{4x} - \frac{(a+bx)^{9/2}}{2x^2} - \frac{63}{4}a$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.31

$$-\frac{2b^2(a+bx)^{11/2}\,{}_2F_1\left(3,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (-2*b^2*(a + b*x)^(11/2)*Hypergeometric2F1[3, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^3)
```

IntegrateAlgebraic [A] time = 0.11, size = 92, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(315a^4-525a^3(a+bx)+168a^2(a+bx)^2+24a(a+bx)^3+8(a+bx)^4\right)}{20x^2} - \frac{63}{4}a^{5/2}b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^3,x]
```

```
[Out] (Sqrt[a + b*x]*(315*a^4 - 525*a^3*(a + b*x) + 168*a^2*(a + b*x)^2 + 24*a*(a
 + b*x)^3 + 8*(a + b*x)^4))/(20*x^2) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/4
```

fricas [A] time = 1.16, size = 180, normalized size = 1.58

$$\left[\frac{315\,a^{\frac{5}{2}}b^2x^2\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4\right)\sqrt{bx+a}}{40\,x^2},\ \frac{315\,\sqrt{-a}\,a^2b^2x^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(8\,b^4x^4+56\,ab^3x^3+288\,a^2b^2x^2-85\,a^3bx-10\,a^4\right)\sqrt{bx+a}}{20\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/40*(315*a^(5/2)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt(b*
x + a))/x^2, 1/20*(315*sqrt(-a)*a^2*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (8*b^4*x^4 + 56*a*b^3*x^3 + 288*a^2*b^2*x^2 - 85*a^3*b*x - 10*a^4)*sqrt
(b*x + a))/x^2]
```

giac [A] time = 1.10, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\,(bx+a)^{\frac{5}{2}}b^3 + 40\,(bx+a)^{\frac{3}{2}}ab^3 + 240\,\sqrt{bx+a}\,a^2b^3 - \frac{5\left(17\,(bx+a)^{\frac{3}{2}}a^3b^3 - 15\,\sqrt{bx+a}\,a^4b^3\right)}{b^2x^2}}{20\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/20*(315*a^3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 8*(b*x + a)^(5/
2)*b^3 + 40*(b*x + a)^(3/2)*a*b^3 + 240*sqrt(b*x + a)*a^2*b^3 - 5*(17*(b*x
+ a)^(3/2)*a^3*b^3 - 15*sqrt(b*x + a)*a^4*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 86, normalized size = 0.75

$$2\left(\left(-\frac{63\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}} + \frac{\frac{15\sqrt{bx+a}\,a}{8} - \frac{17(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}\right)a^3 + 6\sqrt{bx+a}\,a^2 + (bx+a)^{\frac{3}{2}}\,a + \frac{(bx+a)^{\frac{5}{2}}}{5}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^3,x)
```

```
[Out] 2*b^2*(1/5*(b*x+a)^(5/2)+(b*x+a)^(3/2)*a+6*(b*x+a)^(1/2)*a^2+a^3*((-17/8*(b
*x+a)^(3/2)+15/8*(b*x+a)^(1/2)*a)/x^2/b^2-63/8*arctanh((b*x+a)^(1/2)/a^(1/2
))/a^(1/2)))
```

maxima [A] time = 2.95, size = 131, normalized size = 1.15

$$\frac{63}{8}\,a^{\frac{5}{2}}b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right) + \frac{2}{5}\,(bx+a)^{\frac{5}{2}}b^2 + 2\,(bx+a)^{\frac{3}{2}}ab^2 + 12\,\sqrt{bx+a}\,a^2b^2 - \frac{17\,(bx+a)^{\frac{3}{2}}a^3b^2 - 15\,\sqrt{bx+a}\,a^4b^2}{4\left((bx+a)^2 - 2\,(bx+a)a + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^3,x, algorithm="maxima")
```

```
[Out] 63/8*a^(5/2)*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) +
 2/5*(b*x + a)^(5/2)*b^2 + 2*(b*x + a)^(3/2)*a*b^2 + 12*sqrt(b*x + a)*a^2*b
^2 - 1/4*(17*(b*x + a)^(3/2)*a^3*b^2 - 15*sqrt(b*x + a)*a^4*b^2)/((b*x + a)
^2 - 2*(b*x + a)*a + a^2)
```

mupad [B] time = 0.05, size = 117, normalized size = 1.03

$$\frac{2\,b^2\,(a+b\,x)^{5/2}}{5} + \frac{\frac{15\,a^4\,b^2\,\sqrt{a+b\,x}}{4} - \frac{17\,a^3\,b^2\,(a+b\,x)^{3/2}}{4}}{(a+b\,x)^2 - 2\,a\,(a+b\,x) + a^2} + 12\,a^2\,b^2\,\sqrt{a+b\,x} + 2\,a\,b^2\,(a+b\,x)^{3/2} + \frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^3,x)
```

```
[Out] (2*b^2*(a + b*x)^(5/2))/5 + ((15*a^4*b^2*(a + b*x)^(1/2))/4 - (17*a^3*b^2*(
a + b*x)^(3/2))/4)/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + 12*a^2*b^2*(a + b*
x)^(1/2) + (a^(5/2)*b^2*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/4 + 2*a*b^2
*(a + b*x)^(3/2)
```

sympy [A] time = 8.99, size = 184, normalized size = 1.61

$$-\frac{63a^{\frac{5}{2}}b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{4} - \frac{a^5}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}} - \frac{19a^4\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}} + \frac{203a^3b^{\frac{3}{2}}}{20\sqrt{x}\sqrt{\frac{a}{bx}+1}} + \frac{86a^2b^{\frac{5}{2}}\sqrt{x}}{5\sqrt{\frac{a}{bx}+1}} + \frac{16ab^{\frac{7}{2}}x^{\frac{3}{2}}}{5\sqrt{\frac{a}{bx}+1}} + \frac{2b^{\frac{9}{2}}x^{\frac{5}{2}}}{5\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**3,x)
```

```
[Out] -63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - a**5/(2*sqrt(b)*x**(
5/2)*sqrt(a/(b*x) + 1)) - 19*a**4*sqrt(b)/(4*x**(3/2)*sqrt(a/(b*x) + 1)) +
203*a**3*b**(3/2)/(20*sqrt(x)*sqrt(a/(b*x) + 1)) + 86*a**2*b**(5/2)*sqrt(x)
/(5*sqrt(a/(b*x) + 1)) + 16*a*b**(7/2)*x**(3/2)/(5*sqrt(a/(b*x) + 1)) + 2*b
**(9/2)*x**(5/2)/(5*sqrt(a/(b*x) + 1))
```

3.320 $\int \frac{(a+bx)^{9/2}}{x^4}\,dx$

Optimal. Leaf size=114

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{21b^2(a+bx)^{5/2}}{8x}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 208}

$$-\frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\frac{21b^2(a+bx)^{5/2}}{8x}+\frac{35}{8}b^3(a+bx)^{3/2}+\frac{105}{8}ab^3\sqrt{a+bx}-\frac{(a+bx)^{9/2}}{3x^3}-\frac{3b(a+bx)^{7/2}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (105*a*b^3*Sqrt[a + b*x])/8 + (35*b^3*(a + b*x)^(3/2))/8 - (21*b^2*(a + b*x
)^(5/2))/(8*x) - (3*b*(a + b*x)^(7/2))/(4*x^2) - (a + b*x)^(9/2)/(3*x^3) -
(105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/8
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^4}\,dx = -\frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{2}(3b)\int \frac{(a+bx)^{7/2}}{x^3}\,dx$$
$$= -\frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{8}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^2}\,dx$$
$$= -\frac{21b^2(a+bx)^{5/2}}{8x} - \frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{16}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x}\,dx$$
$$= \frac{35}{8}b^3(a+bx)^{3/2} - \frac{21b^2(a+bx)^{5/2}}{8x} - \frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{16}\left(105ab^3\right)\int \frac{\sqrt{a+bx}}{x}$$
$$= \frac{105}{8}ab^3\sqrt{a+bx} + \frac{35}{8}b^3(a+bx)^{3/2} - \frac{21b^2(a+bx)^{5/2}}{8x} - \frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{16}(1$$
$$= \frac{105}{8}ab^3\sqrt{a+bx} + \frac{35}{8}b^3(a+bx)^{3/2} - \frac{21b^2(a+bx)^{5/2}}{8x} - \frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} + \frac{1}{8}(10$$
$$= \frac{105}{8}ab^3\sqrt{a+bx} + \frac{35}{8}b^3(a+bx)^{3/2} - \frac{21b^2(a+bx)^{5/2}}{8x} - \frac{3b(a+bx)^{7/2}}{4x^2} - \frac{(a+bx)^{9/2}}{3x^3} - \frac{105}{8}a$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.31

$$\frac{2b^3(a+bx)^{11/2}\,{}_2F_1\left(4,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (2*b^3*(a + b*x)^(11/2)*Hypergeometric2F1[4, 11/2, 13/2, 1 + (b*x)/a])/(11*
a^4)
```

IntegrateAlgebraic [A] time = 0.14, size = 92, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(-315a^4+840a^3(a+bx)-693a^2(a+bx)^2+144a(a+bx)^3+16(a+bx)^4\right)}{24x^3} - \frac{105}{8}a^{3/2}b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^4,x]
```

```
[Out] (Sqrt[a + b*x]*(-315*a^4 + 840*a^3*(a + b*x) - 693*a^2*(a + b*x)^2 + 144*a*
(a + b*x)^3 + 16*(a + b*x)^4))/(24*x^3) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/8
```

fricas [A] time = 0.93, size = 178, normalized size = 1.56

$$\left[\frac{315\,a^{\frac{3}{2}}b^3x^3\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4\right)\sqrt{bx+a}}{48\,x^3},\ \frac{315\sqrt{-a}\,ab^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(16\,b^4x^4+208\,ab^3x^3-165\,a^2b^2x^2-50\,a^3bx-8\,a^4\right)\sqrt{bx+a}}{24\,x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/48*(315*a^(3/2)*b^3*x^3*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2
*(16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt(b
*x + a))/x^3, 1/24*(315*sqrt(-a)*a*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a)
 + (16*b^4*x^4 + 208*a*b^3*x^3 - 165*a^2*b^2*x^2 - 50*a^3*b*x - 8*a^4)*sqrt
(b*x + a))/x^3]
```

giac [A] time = 1.10, size = 112, normalized size = 0.98

$$\frac{\frac{315\,a^2b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+16\,(bx+a)^{\frac{3}{2}}b^4+192\,\sqrt{bx+a}\,ab^4-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^4-280\,(bx+a)^{\frac{3}{2}}a^3b^4+123\,\sqrt{bx+a}\,a^4b^4}{b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/24*(315*a^2*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 16*(b*x + a)^(3
/2)*b^4 + 192*sqrt(b*x + a)*a*b^4 - (165*(b*x + a)^(5/2)*a^2*b^4 - 280*(b*x
 + a)^(3/2)*a^3*b^4 + 123*sqrt(b*x + a)*a^4*b^4)/(b^3*x^3))/b
```

maple [A] time = 0.01, size = 87, normalized size = 0.76

$$2\left(\left(-\frac{105\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{16\sqrt{a}}+\frac{-\frac{41\sqrt{bx+a}\,a^2}{16}+\frac{35(bx+a)^{\frac{3}{2}}a}{6}-\frac{55(bx+a)^{\frac{5}{2}}}{16}}{b^3x^3}\right)a^2+4\sqrt{bx+a}\;a+\frac{(bx+a)^{\frac{3}{2}}}{3}\right)b^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^4,x)
```

```
[Out] 2*b^3*(1/3*(b*x+a)^(3/2)+4*(b*x+a)^(1/2)*a+a^2*((-55/16*(b*x+a)^(5/2)+35/6*
(b*x+a)^(3/2)*a-41/16*(b*x+a)^(1/2)*a^2)/x^3/b^3-105/16*arctanh((b*x+a)^(1/
2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 2.94, size = 145, normalized size = 1.27

$$\frac{105}{16}\,a^{\frac{3}{2}}b^3\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}b^3+8\,\sqrt{bx+a}\,ab^3-\frac{165\,(bx+a)^{\frac{5}{2}}a^2b^3-280\,(bx+a)^{\frac{3}{2}}a^3b^3+123\,\sqrt{bx+a}\,a^4b^3}{24\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^4,x, algorithm="maxima")
```

```
[Out] 105/16*a^(3/2)*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))
 + 2/3*(b*x + a)^(3/2)*b^3 + 8*sqrt(b*x + a)*a*b^3 - 1/24*(165*(b*x + a)^(5
/2)*a^2*b^3 - 280*(b*x + a)^(3/2)*a^3*b^3 + 123*sqrt(b*x + a)*a^4*b^3)/((b*
x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2 - a^3)
```

mupad [B] time = 0.12, size = 131, normalized size = 1.15

$$\frac{2\,b^3\,(a+b\,x)^{3/2}}{3}+\frac{\frac{41\,a^4\,b^3\,\sqrt{a+b\,x}}{8}-\frac{35\,a^3\,b^3\,(a+b\,x)^{3/2}}{3}+\frac{55\,a^2\,b^3\,(a+b\,x)^{5/2}}{8}}{3\,a\,(a+b\,x)^2-3\,a^2\,(a+b\,x)-(a+b\,x)^3+a^3}+8\,a\,b^3\,\sqrt{a+b\,x}+\frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,105\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^4,x)
```

```
[Out] (2*b^3*(a + b*x)^(3/2))/3 + ((41*a^4*b^3*(a + b*x)^(1/2))/8 - (35*a^3*b^3*(
a + b*x)^(3/2))/3 + (55*a^2*b^3*(a + b*x)^(5/2))/8)/(3*a*(a + b*x)^2 - 3*a^
2*(a + b*x) - (a + b*x)^3 + a^3) + (a^(3/2)*b^3*atan(((a + b*x)^(1/2)*1i)/a
^(1/2))*105i)/8 + 8*a*b^3*(a + b*x)^(1/2)
```

sympy [A] time = 7.91, size = 184, normalized size = 1.61

$$-\frac{105a^{\frac{3}{2}}b^3\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{8}-\frac{a^5}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{29a^4\sqrt{b}}{12x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{215a^3b^{\frac{3}{2}}}{24x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{43a^2b^{\frac{5}{2}}}{24\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{28ab^{\frac{7}{2}}\sqrt{x}}{3\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}x^{\frac{3}{2}}}{3\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**4,x)
```

```
[Out] -105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/8 - a**5/(3*sqrt(b)*x**
(7/2)*sqrt(a/(b*x) + 1)) - 29*a**4*sqrt(b)/(12*x**(5/2)*sqrt(a/(b*x) + 1))
- 215*a**3*b**(3/2)/(24*x**(3/2)*sqrt(a/(b*x) + 1)) + 43*a**2*b**(5/2)/(24*
sqrt(x)*sqrt(a/(b*x) + 1)) + 28*a*b**(7/2)*sqrt(x)/(3*sqrt(a/(b*x) + 1)) +
2*b**(9/2)*x**(3/2)/(3*sqrt(a/(b*x) + 1))
```

3.321 $\int \frac{(a+bx)^{9/2}}{x^5} dx$

Optimal. Leaf size=116

$$\frac{315}{64} b^4 \sqrt{a+bx} - \frac{315}{64} \sqrt{a}\, b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{105 b^3 (a+bx)^{3/2}}{64x} - \frac{21 b^2 (a+bx)^{5/2}}{32x^2} - \frac{(a+bx)^{9/2}}{4x^4} - \frac{3b(a+bx)^{7/2}}{8x^3}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {47, 50, 63, 208}

$$-\frac{21 b^2 (a+bx)^{5/2}}{32x^2} - \frac{105 b^3 (a+bx)^{3/2}}{64x} + \frac{315}{64} b^4 \sqrt{a+bx} - \frac{315}{64} \sqrt{a}\, b^4 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{(a+bx)^{9/2}}{4x^4} - \frac{3b(a+bx)^{7/2}}{8x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (315*b^4*Sqrt[a + b*x])/64 - (105*b^3*(a + b*x)^(3/2))/(64*x) - (21*b^2*(a
+ b*x)^(5/2))/(32*x^2) - (3*b*(a + b*x)^(7/2))/(8*x^3) - (a + b*x)^(9/2)/(4
*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/64
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^5}\,dx &= -\frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{8}(9b)\int \frac{(a+bx)^{7/2}}{x^4}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{16}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^3}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{64}\left(105b^3\right)\int \frac{(a+bx)^{3/2}}{x^2}\,dx \\
&= -\frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{128}\left(315b^4\right)\int \frac{\sqrt{a+}}{x} \\
&= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{128}(\\
&= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} + \frac{1}{64}(\\
&= \frac{315}{64}b^4\sqrt{a+bx} - \frac{105b^3(a+bx)^{3/2}}{64x} - \frac{21b^2(a+bx)^{5/2}}{32x^2} - \frac{3b(a+bx)^{7/2}}{8x^3} - \frac{(a+bx)^{9/2}}{4x^4} - \frac{315}{64}\sqrt{}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.30

$$
-\frac{2b^4(a+bx)^{11/2}\,{}_2F_1\left(5,\frac{11}{2};\frac{13}{2};\frac{bx}{a}+1\right)}{11a^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (-2*b^4*(a + b*x)^(11/2)*Hypergeometric2F1[5, 11/2, 13/2, 1 + (b*x)/a])/(11
*a^5)
```

IntegrateAlgebraic [A] time = 0.17, size = 92, normalized size = 0.79

$$
\frac{\sqrt{a+bx}\left(315a^4-1155a^3(a+bx)+1533a^2(a+bx)^2-837a(a+bx)^3+128(a+bx)^4\right)}{64x^4} - \frac{315}{64}\sqrt{a}\,b^4\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^5,x]
```

```
[Out] (Sqrt[a + b*x]*(315*a^4 - 1155*a^3*(a + b*x) + 1533*a^2*(a + b*x)^2 - 837*a
*(a + b*x)^3 + 128*(a + b*x)^4))/(64*x^4) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/64
```

fricas [A] time = 0.98, size = 177, normalized size = 1.53

$$
\left[\frac{315\sqrt{a}b^4x^4\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(128b^4x^4-325ab^3x^3-210a^2b^2x^2-88a^3bx-16a^4\right)\sqrt{bx+a}}{128x^4},\ \frac{315\sqrt{-a}b^4x^4\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(128b^4x^4-325ab^3x^3-210a^2b^2x^2-88a^3bx-16a^4\right)\sqrt{bx+a}}{64x^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/128*(315*sqrt(a)*b^4*x^4*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) +
2*(128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*sqr
t(b*x + a))/x^4, 1/64*(315*sqrt(-a)*b^4*x^4*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (128*b^4*x^4 - 325*a*b^3*x^3 - 210*a^2*b^2*x^2 - 88*a^3*b*x - 16*a^4)*s
qrt(b*x + a))/x^4]
```

giac [A] time = 1.22, size = 110, normalized size = 0.95

$$\frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+128\,\sqrt{bx+a}\,b^5-\frac{325\,(bx+a)^{\frac{7}{2}}ab^5-765\,(bx+a)^{\frac{5}{2}}a^2b^5+643\,(bx+a)^{\frac{3}{2}}a^3b^5-187\,\sqrt{bx+a}\,a^4b^5}{b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/64*(315*a*b^5*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x + a)
*b^5 - (325*(b*x + a)^(7/2)*a*b^5 - 765*(b*x + a)^(5/2)*a^2*b^5 + 643*(b*x
+ a)^(3/2)*a^3*b^5 - 187*sqrt(b*x + a)*a^4*b^5)/(b^4*x^4))/b
```

maple [A] time = 0.01, size = 85, normalized size = 0.73

$$2\left(\left(-\frac{315\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128\sqrt{a}}+\frac{\frac{187\sqrt{bx+a}\,a^3}{128}-\frac{643(bx+a)^{\frac{3}{2}}a^2}{128}+\frac{765(bx+a)^{\frac{5}{2}}a}{128}-\frac{325(bx+a)^{\frac{7}{2}}}{128}}{b^4x^4}\right)a+\sqrt{bx+a}\right)b^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*((b*x+a)^(1/2)+a*((-325/128*(b*x+a)^(7/2)+765/128*(b*x+a)^(5/2)*a-643
/128*(b*x+a)^(3/2)*a^2+187/128*(b*x+a)^(1/2)*a^3)/x^4/b^4-315/128*arctanh((
b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 2.96, size = 155, normalized size = 1.34

$$\frac{315}{128}\sqrt{a}\,b^4\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+2\sqrt{bx+a}\,b^4-\frac{325\,(bx+a)^{\frac{7}{2}}ab^4-765\,(bx+a)^{\frac{5}{2}}a^2b^4+643\,(bx+a)^{\frac{3}{2}}a^3b^4-187\sqrt{bx+a}\,a^4b^4}{64\left((bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2-4\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^5,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*b^4*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))
) + 2*sqrt(b*x + a)*b^4 - 1/64*(325*(b*x + a)^(7/2)*a*b^4 - 765*(b*x + a)^(
5/2)*a^2*b^4 + 643*(b*x + a)^(3/2)*a^3*b^4 - 187*sqrt(b*x + a)*a^4*b^4)/((b
*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2 - 4*(b*x + a)*a^3 + a^4)
```

mupad [B] time = 0.06, size = 94, normalized size = 0.81

$$2\,b^4\sqrt{a+b\,x}+\frac{187\,a^4\sqrt{a+b\,x}}{64\,x^4}-\frac{643\,a^3\,(a+b\,x)^{3/2}}{64\,x^4}+\frac{765\,a^2\,(a+b\,x)^{5/2}}{64\,x^4}-\frac{325\,a\,(a+b\,x)^{7/2}}{64\,x^4}+\frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)315\mathrm{i}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^5,x)
```

```
[Out] 2*b^4*(a + b*x)^(1/2) + (187*a^4*(a + b*x)^(1/2))/(64*x^4) - (643*a^3*(a +
b*x)^(3/2))/(64*x^4) + (765*a^2*(a + b*x)^(5/2))/(64*x^4) + (a^(1/2)*b^4*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*315i)/64 - (325*a*(a + b*x)^(7/2))/(64*x^4
)
```

sympy [A] time = 8.55, size = 182, normalized size = 1.57

$$-\frac{315\sqrt{a}\,b^4\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{64}-\frac{a^5}{4\sqrt{b}\,x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{13a^4\sqrt{b}}{8x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{149a^3b^{\frac{3}{2}}}{32x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{535a^2b^{\frac{5}{2}}}{64x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{197ab^{\frac{7}{2}}}{64\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2b^{\frac{9}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**5,x)
```

```
[Out] -315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/64 - a**5/(4*sqrt(b)*x**
(9/2)*sqrt(a/(b*x) + 1)) - 13*a**4*sqrt(b)/(8*x**(7/2)*sqrt(a/(b*x) + 1)) -
 149*a**3*b**(3/2)/(32*x**(5/2)*sqrt(a/(b*x) + 1)) - 535*a**2*b**(5/2)/(64*
x**(3/2)*sqrt(a/(b*x) + 1)) - 197*a*b**(7/2)/(64*sqrt(x)*sqrt(a/(b*x) + 1))
 + 2*b**(9/2)*sqrt(x)/sqrt(a/(b*x) + 1)
```

3.322 $\int \frac{(a+bx)^{9/2}}{x^6} dx$

Optimal. Leaf size=119

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{(a+bx)^{9/2}}{5x^5}-\frac{9b(a+bx)^{7/2}}{40x^4}$$

Rubi [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{21b^3(a+bx)^{3/2}}{64x^2}-\frac{21b^2(a+bx)^{5/2}}{80x^3}-\frac{63b^4\sqrt{a+bx}}{128x}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}-\frac{9b(a+bx)^{7/2}}{40x^4}-\frac{(a+bx)^{9/2}}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] (-63*b^4*Sqrt[a + b*x])/(128*x) - (21*b^3*(a + b*x)^(3/2))/(64*x^2) - (21*b
^2*(a + b*x)^(5/2))/(80*x^3) - (9*b*(a + b*x)^(7/2))/(40*x^4) - (a + b*x)^(
9/2)/(5*x^5) - (63*b^5*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(128*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^6}\,dx = -\frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{10}(9b)\int \frac{(a+bx)^{7/2}}{x^5}\,dx$$
$$= -\frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{80}\left(63b^2\right)\int \frac{(a+bx)^{5/2}}{x^4}\,dx$$
$$= -\frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{32}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^3}\,dx$$
$$= -\frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}\left(63b^4\right)\int \frac{\sqrt{a+bx}}{x^2}$$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{256}($$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} + \frac{1}{128}($$
$$= -\frac{63b^4\sqrt{a+bx}}{128x} - \frac{21b^3(a+bx)^{3/2}}{64x^2} - \frac{21b^2(a+bx)^{5/2}}{80x^3} - \frac{9b(a+bx)^{7/2}}{40x^4} - \frac{(a+bx)^{9/2}}{5x^5} - \frac{63b^5\,\mathrm{t}}{}$$

Mathematica [A] time = 0.04, size = 101, normalized size = 0.85

$$-\frac{128a^5 + 784a^4bx + 2024a^3b^2x^2 + 2858a^2b^3x^3 + 315b^5x^5\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right) + 2455ab^4x^4 + 965b^5x^5}{640x^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] -1/640*(128*a^5 + 784*a^4*b*x + 2024*a^3*b^2*x^2 + 2858*a^2*b^3*x^3 + 2455*
a*b^4*x^4 + 965*b^5*x^5 + 315*b^5*x^5*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b
*x)/a]])/(x^5*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.20, size = 92, normalized size = 0.77

$$-\frac{\sqrt{a+bx}\left(315a^4 - 1470a^3(a+bx) + 2688a^2(a+bx)^2 - 2370a(a+bx)^3 + 965(a+bx)^4\right)}{640x^5} - \frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{128\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^6,x]
```

```
[Out] -1/640*(Sqrt[a + b*x]*(315*a^4 - 1470*a^3*(a + b*x) + 2688*a^2*(a + b*x)^2
- 2370*a*(a + b*x)^3 + 965*(a + b*x)^4))/x^5 - (63*b^5*ArcTanh[Sqrt[a + b*x
]/Sqrt[a]])/(128*Sqrt[a])
```

fricas [A] time = 0.90, size = 190, normalized size = 1.60

$$\left[\frac{315\,\sqrt{a}b^5x^5\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right) - 2\left(965\,ab^4x^4 + 1490\,a^2b^3x^3 + 1368\,a^3b^2x^2 + 656\,a^4bx + 128\,a^5\right)\sqrt{bx+a}}{1280\,ax^5},\ \frac{315\,\sqrt{-a}b^5x^5\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right) - \left(965\,ab^4x^4 + 1490\,a^2b^3x^3 + 1368\,a^3b^2x^2 + 656\,a^4bx + 128\,a^5\right)\sqrt{bx+a}}{640\,ax^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(315*sqrt(a)*b^5*x^5*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) -
 2*(965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 656*a^4*b*x + 128
*a^5)*sqrt(b*x + a))/(a*x^5), 1/640*(315*sqrt(-a)*b^5*x^5*arctan(sqrt(b*x +
 a)*sqrt(-a)/a) - (965*a*b^4*x^4 + 1490*a^2*b^3*x^3 + 1368*a^3*b^2*x^2 + 65
6*a^4*b*x + 128*a^5)*sqrt(b*x + a))/(a*x^5)]
```

giac [A] time = 1.19, size = 109, normalized size = 0.92

$$\frac{\frac{315\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^6-2370\,(bx+a)^{\frac{7}{2}}ab^6+2688\,(bx+a)^{\frac{5}{2}}a^2b^6-1470\,(bx+a)^{\frac{3}{2}}a^3b^6+315\,\sqrt{bx+a}\,a^4b^6}{b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/640*(315*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (965*(b*x + a)^(9/
2)*b^6 - 2370*(b*x + a)^(7/2)*a*b^6 + 2688*(b*x + a)^(5/2)*a^2*b^6 - 1470*(
b*x + a)^(3/2)*a^3*b^6 + 315*sqrt(b*x + a)*a^4*b^6)/(b^5*x^5))/b
```

maple [A] time = 0.01, size = 87, normalized size = 0.73

$$2\left(-\frac{63\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{256\sqrt{a}}+\frac{-\frac{63\sqrt{bx+a}\,a^4}{256}+\frac{147(bx+a)^{\frac{3}{2}}a^3}{128}-\frac{21(bx+a)^{\frac{5}{2}}a^2}{10}+\frac{237(bx+a)^{\frac{7}{2}}a}{128}-\frac{193(bx+a)^{\frac{9}{2}}}{256}}{b^5x^5}\right)b^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^6,x)
```

```
[Out] 2*b^5*((-193/256*(b*x+a)^(9/2)+237/128*(b*x+a)^(7/2)*a-21/10*(b*x+a)^(5/2)*
a^2+147/128*(b*x+a)^(3/2)*a^3-63/256*(b*x+a)^(1/2)*a^4)/x^5/b^5-63/256*arct
anh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

maxima [A] time = 2.97, size = 169, normalized size = 1.42

$$\frac{63\,b^5\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{256\,\sqrt{a}}-\frac{965\,(bx+a)^{\frac{9}{2}}b^5-2370\,(bx+a)^{\frac{7}{2}}ab^5+2688\,(bx+a)^{\frac{5}{2}}a^2b^5-1470\,(bx+a)^{\frac{3}{2}}a^3b^5+315\,\sqrt{bx+a}\,a^4b^5}{640\left((bx+a)^5-5\,(bx+a)^4a+10\,(bx+a)^3a^2-10\,(bx+a)^2a^3+5\,(bx+a)a^4-a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^6,x, algorithm="maxima")
```

```
[Out] 63/256*b^5*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
 - 1/640*(965*(b*x + a)^(9/2)*b^5 - 2370*(b*x + a)^(7/2)*a*b^5 + 2688*(b*x
+ a)^(5/2)*a^2*b^5 - 1470*(b*x + a)^(3/2)*a^3*b^5 + 315*sqrt(b*x + a)*a^4*b
^5)/((b*x + a)^5 - 5*(b*x + a)^4*a + 10*(b*x + a)^3*a^2 - 10*(b*x + a)^2*a^
3 + 5*(b*x + a)*a^4 - a^5)
```

mupad [B] time = 0.12, size = 94, normalized size = 0.79

$$\frac{147\,a^3\,(a+b\,x)^{3/2}}{64\,x^5}-\frac{63\,a^4\,\sqrt{a+b\,x}}{128\,x^5}-\frac{193\,(a+b\,x)^{9/2}}{128\,x^5}-\frac{21\,a^2\,(a+b\,x)^{5/2}}{5\,x^5}+\frac{237\,a\,(a+b\,x)^{7/2}}{64\,x^5}+\frac{b^5\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)63\mathrm{i}}{128\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^6,x)
```

```
[Out] (147*a^3*(a + b*x)^(3/2))/(64*x^5) - (63*a^4*(a + b*x)^(1/2))/(128*x^5) - (
193*(a + b*x)^(9/2))/(128*x^5) - (21*a^2*(a + b*x)^(5/2))/(5*x^5) + (b^5*at
an(((a + b*x)^(1/2)*1i)/a^(1/2))*63i)/(128*a^(1/2)) + (237*a*(a + b*x)^(7/2
))/(64*x^5)
```

sympy [A] time = 10.25, size = 158, normalized size = 1.33

$$-\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^{\frac{9}{2}}}-\frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{40x^{\frac{7}{2}}}-\frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{80x^{\frac{5}{2}}}-\frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}{64x^{\frac{3}{2}}}-\frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{128\sqrt{x}}-\frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{128\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**6,x)
```

```
[Out] -a**4*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**(9/2)) - 41*a**3*b**(3/2)*sqrt(a/(b*x
) + 1)/(40*x**(7/2)) - 171*a**2*b**(5/2)*sqrt(a/(b*x) + 1)/(80*x**(5/2)) -
149*a*b**(7/2)*sqrt(a/(b*x) + 1)/(64*x**(3/2)) - 193*b**(9/2)*sqrt(a/(b*x)
+ 1)/(128*sqrt(x)) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(128*sqrt(a))
```

3.323 $\int \frac{(a+bx)^{9/2}}{x^7}\,dx$

Optimal. Leaf size=141

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{(a+bx)^{9/2}}{6x^6}-\frac{3b(a+bx)^{7/2}}{20x^5}$$

Rubi [A] time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 208}

$$\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}}-\frac{21b^4\sqrt{a+bx}}{256x^2}-\frac{7b^3(a+bx)^{3/2}}{64x^3}-\frac{21b^2(a+bx)^{5/2}}{160x^4}-\frac{21b^5\sqrt{a+bx}}{512ax}-\frac{3b(a+bx)^{7/2}}{20x^5}-\frac{(a+bx)^{9/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-21*b^4*Sqrt[a + b*x])/(256*x^2) - (21*b^5*Sqrt[a + b*x])/(512*a*x) - (7*b
^3*(a + b*x)^(3/2))/(64*x^3) - (21*b^2*(a + b*x)^(5/2))/(160*x^4) - (3*b*(a
 + b*x)^(7/2))/(20*x^5) - (a + b*x)^(9/2)/(6*x^6) + (21*b^6*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(512*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{9/2}}{x^7}\,dx &= -\frac{(a+bx)^{9/2}}{6x^6} + \frac{1}{4}(3b)\int \frac{(a+bx)^{7/2}}{x^6}\,dx \\
&= -\frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+bx)^{9/2}}{6x^6} + \frac{1}{40}\left(21b^2\right)\int \frac{(a+bx)^{5/2}}{x^5}\,dx \\
&= -\frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+bx)^{9/2}}{6x^6} + \frac{1}{64}\left(21b^3\right)\int \frac{(a+bx)^{3/2}}{x^4}\,dx \\
&= -\frac{7b^3(a+bx)^{3/2}}{64x^3} - \frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+bx)^{9/2}}{6x^6} + \frac{1}{128}\left(21b^4\right)\int \frac{\sqrt{a+bx}}{x^3} \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2} - \frac{7b^3(a+bx)^{3/2}}{64x^3} - \frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+bx)^{9/2}}{6x^6} + \frac{1}{512}(21 \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2} - \frac{21b^5\sqrt{a+bx}}{512ax} - \frac{7b^3(a+bx)^{3/2}}{64x^3} - \frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+}{6} \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2} - \frac{21b^5\sqrt{a+bx}}{512ax} - \frac{7b^3(a+bx)^{3/2}}{64x^3} - \frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+}{6} \\
&= -\frac{21b^4\sqrt{a+bx}}{256x^2} - \frac{21b^5\sqrt{a+bx}}{512ax} - \frac{7b^3(a+bx)^{3/2}}{64x^3} - \frac{21b^2(a+bx)^{5/2}}{160x^4} - \frac{3b(a+bx)^{7/2}}{20x^5} - \frac{(a+}{6}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.25

$$
-\frac{2b^6(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},7;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] (-2*b^6*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 7, 13/2, 1 + (b*x)/a])/(11
*a^7)
```

IntegrateAlgebraic [A] time = 0.23, size = 107, normalized size = 0.76

$$
\frac{21b^6\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{512a^{3/2}} - \frac{\sqrt{a+bx}\left(315a^5 - 1785a^4(a+bx) + 4158a^3(a+bx)^2 - 5058a^2(a+bx)^3 + 3335a(a+bx)^4 + 315(a+bx)^5\right)}{7680ax^6}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^7,x]
```

```
[Out] -1/7680*(Sqrt[a + b*x]*(315*a^5 - 1785*a^4*(a + b*x) + 4158*a^3*(a + b*x)^2
 - 5058*a^2*(a + b*x)^3 + 3335*a*(a + b*x)^4 + 315*(a + b*x)^5))/(a*x^6) +
(21*b^6*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(512*a^(3/2))
```

fricas [A] time = 1.27, size = 211, normalized size = 1.50

$$
\left[\frac{315\sqrt{a}b^6x^6\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) - 2\left(315ab^5x^5 + 4910a^2b^4x^4 + 11432a^3b^3x^3 + 12144a^4b^2x^2 + 6272a^5bx + 1280a^6\right)\sqrt{bx+a}}{15360a^2x^6}, -\frac{315\sqrt{-a}b^6x^6\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \left(315ab^5x^5 + 4910a^2b^4x^4 + 11432a^3b^3x^3 + 12144a^4b^2x^2 + 6272a^5bx + 1280a^6\right)\sqrt{bx+a}}{7680a^2x^6}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/15360*(315*sqrt(a)*b^6*x^6*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
- 2*(315*a*b^5*x^5 + 4910*a^2*b^4*x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x
^2 + 6272*a^5*b*x + 1280*a^6)*sqrt(b*x + a))/(a^2*x^6), -1/7680*(315*sqrt(-
```

a)*b^6*x^6*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^5*x^5 + 4910*a^2*b^4 *x^4 + 11432*a^3*b^3*x^3 + 12144*a^4*b^2*x^2 + 6272*a^5*b*x + 1280*a^6)*sqr t(b*x + a))/(a^2*x^6)]

giac [A] time = 1.04, size = 129, normalized size = 0.91

$$-\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{315\,(bx+a)^{\frac{11}{2}}b^7+3335\,(bx+a)^{\frac{9}{2}}ab^7-5058\,(bx+a)^{\frac{7}{2}}a^2b^7+4158\,(bx+a)^{\frac{5}{2}}a^3b^7-1785\,(bx+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{bx+a}\,a^5b^7}{ab^6x^6}}{7680\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/7680*(315*b^7*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*x +
a)^(11/2)*b^7 + 3335*(b*x + a)^(9/2)*a*b^7 - 5058*(b*x + a)^(7/2)*a^2*b^7 +
 4158*(b*x + a)^(5/2)*a^3*b^7 - 1785*(b*x + a)^(3/2)*a^4*b^7 + 315*sqrt(b*x
 + a)*a^5*b^7)/(a*b^6*x^6))/b
```

maple [A] time = 0.01, size = 99, normalized size = 0.70

$$2\left(\frac{21\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024a^{\frac{3}{2}}}+\frac{-\frac{21\sqrt{bx+a}\,a^4}{1024}+\frac{119(bx+a)^{\frac{3}{2}}a^3}{1024}-\frac{693(bx+a)^{\frac{5}{2}}a^2}{2560}+\frac{843(bx+a)^{\frac{7}{2}}a}{2560}-\frac{21(bx+a)^{\frac{11}{2}}}{1024a}-\frac{667(bx+a)^{\frac{9}{2}}}{3072}}{b^6x^6}\right)b^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^7,x)
```

```
[Out] 2*b^6*((-21/1024/a*(b*x+a)^(11/2)-667/3072*(b*x+a)^(9/2)+843/2560*(b*x+a)^(
7/2)*a-693/2560*(b*x+a)^(5/2)*a^2+119/1024*(b*x+a)^(3/2)*a^3-21/1024*(b*x+a
)^(1/2)*a^4)/x^6/b^6+21/1024*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

maxima [A] time = 3.03, size = 198, normalized size = 1.40

$$-\frac{21\,b^6\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{1024\,a^{\frac{3}{2}}}-\frac{315\,(bx+a)^{\frac{11}{2}}b^6+3335\,(bx+a)^{\frac{9}{2}}ab^6-5058\,(bx+a)^{\frac{7}{2}}a^2b^6+4158\,(bx+a)^{\frac{5}{2}}a^3b^6-1785\,(bx+a)^{\frac{3}{2}}a^4b^6+315\,\sqrt{bx+a}\,a^5b^6}{7680\left((bx+a)^6a-6\,(bx+a)^5a^2+15\,(bx+a)^4a^3-20\,(bx+a)^3a^4+15\,(bx+a)^2a^5-6\,(bx+a)a^6+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^7,x, algorithm="maxima")
```

```
[Out] -21/1024*b^6*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/
2) - 1/7680*(315*(b*x + a)^(11/2)*b^6 + 3335*(b*x + a)^(9/2)*a*b^6 - 5058*(
b*x + a)^(7/2)*a^2*b^6 + 4158*(b*x + a)^(5/2)*a^3*b^6 - 1785*(b*x + a)^(3/2
)*a^4*b^6 + 315*sqrt(b*x + a)*a^5*b^6)/((b*x + a)^6*a - 6*(b*x + a)^5*a^2 +
 15*(b*x + a)^4*a^3 - 20*(b*x + a)^3*a^4 + 15*(b*x + a)^2*a^5 - 6*(b*x + a)
*a^6 + a^7)
```

mupad [B] time = 0.13, size = 109, normalized size = 0.77

$$\frac{119\,a^3\,(a+b\,x)^{3/2}}{512\,x^6}-\frac{21\,a^4\,\sqrt{a+b\,x}}{512\,x^6}-\frac{667\,(a+b\,x)^{9/2}}{1536\,x^6}-\frac{693\,a^2\,(a+b\,x)^{5/2}}{1280\,x^6}-\frac{21\,(a+b\,x)^{11/2}}{512\,a\,x^6}+\frac{843\,a\,(a+b\,x)^{7/2}}{1280\,x^6}-\frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)21\mathrm{i}}{512\,a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^7,x)
```

```
[Out] (119*a^3*(a + b*x)^(3/2))/(512*x^6) - (21*a^4*(a + b*x)^(1/2))/(512*x^6) -
(667*(a + b*x)^(9/2))/(1536*x^6) - (693*a^2*(a + b*x)^(5/2))/(1280*x^6) - (
21*(a + b*x)^(11/2))/(512*a*x^6) - (b^6*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*
21i)/(512*a^(3/2)) + (843*a*(a + b*x)^(7/2))/(1280*x^6)
```

sympy [A] time = 15.69, size = 209, normalized size = 1.48

$$-\frac{a^5}{6\sqrt{b}\,x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{59a^4\sqrt{b}}{60x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1151a^3b^{\frac{3}{2}}}{480x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{2947a^2b^{\frac{5}{2}}}{960x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{8171ab^{\frac{7}{2}}}{3840x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1045b^{\frac{9}{2}}}{1536x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{21b^{\frac{11}{2}}}{512a\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{512a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**7,x)
```

```
[Out] -a**5/(6*sqrt(b)*x**(13/2)*sqrt(a/(b*x) + 1)) - 59*a**4*sqrt(b)/(60*x**(11/
2)*sqrt(a/(b*x) + 1)) - 1151*a**3*b**(3/2)/(480*x**(9/2)*sqrt(a/(b*x) + 1))
 - 2947*a**2*b**(5/2)/(960*x**(7/2)*sqrt(a/(b*x) + 1)) - 8171*a*b**(7/2)/(3
840*x**(5/2)*sqrt(a/(b*x) + 1)) - 1045*b**(9/2)/(1536*x**(3/2)*sqrt(a/(b*x)
 + 1)) - 21*b**(11/2)/(512*a*sqrt(x)*sqrt(a/(b*x) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*sqrt(x)))/(512*a**(3/2))
```

3.324 $\int \frac{(a+bx)^{9/2}}{x^8} \, dx$

Optimal. Leaf size=163

$$-\frac{9b^7 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}+\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{(a+bx)^{9/2}}{7x^7}-\frac{3b\ldots}{\ldots}$$

Rubi [A] time = 0.07, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {47, 51, 63, 208}

$$\frac{9b^6\sqrt{a+bx}}{1024a^2x}-\frac{9b^7 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}}-\frac{3b^5\sqrt{a+bx}}{512ax^2}-\frac{3b^4\sqrt{a+bx}}{128x^3}-\frac{3b^3(a+bx)^{3/2}}{64x^4}-\frac{3b^2(a+bx)^{5/2}}{40x^5}-\frac{3b(a+bx)^{7/2}}{28x^6}-\frac{(a+bx)^{9/2}}{7x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (-3*b^4*Sqrt[a + b*x])/(128*x^3) - (3*b^5*Sqrt[a + b*x])/(512*a*x^2) + (9*b
^6*Sqrt[a + b*x])/(1024*a^2*x) - (3*b^3*(a + b*x)^(3/2))/(64*x^4) - (3*b^2*
(a + b*x)^(5/2))/(40*x^5) - (3*b*(a + b*x)^(7/2))/(28*x^6) - (a + b*x)^(9/2
)/(7*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{x^8}\,dx = -\frac{(a+bx)^{9/2}}{7x^7} + \frac{1}{14}(9b)\int \frac{(a+bx)^{7/2}}{x^7}\,dx$$
$$= -\frac{3b(a+bx)^{7/2}}{28x^6} - \frac{(a+bx)^{9/2}}{7x^7} + \frac{1}{8}\left(3b^2\right)\int \frac{(a+bx)^{5/2}}{x^6}\,dx$$
$$= -\frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - \frac{(a+bx)^{9/2}}{7x^7} + \frac{1}{16}\left(3b^3\right)\int \frac{(a+bx)^{3/2}}{x^5}\,dx$$
$$= -\frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - \frac{(a+bx)^{9/2}}{7x^7} + \frac{1}{128}\left(9b^4\right)\int \frac{\sqrt{a+bx}}{x^4}\,dx$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - \frac{(a+bx)^{9/2}}{7x^7} + \frac{1}{256}\left(3b^5\right.$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^5\sqrt{a+bx}}{512ax^2} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+bx)^{7/2}}{28x^6} - \frac{(a+bx)}{7x^7}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^5\sqrt{a+bx}}{512ax^2} + \frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+b}{28x^6}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^5\sqrt{a+bx}}{512ax^2} + \frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+b}{28x^6}$$
$$= -\frac{3b^4\sqrt{a+bx}}{128x^3} - \frac{3b^5\sqrt{a+bx}}{512ax^2} + \frac{9b^6\sqrt{a+bx}}{1024a^2x} - \frac{3b^3(a+bx)^{3/2}}{64x^4} - \frac{3b^2(a+bx)^{5/2}}{40x^5} - \frac{3b(a+b}{28x^6}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.21

$$\frac{2b^7(a+bx)^{11/2}\,{}_2F_1\left(\frac{11}{2},8;\frac{13}{2};\frac{bx}{a}+1\right)}{11a^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] (2*b^7*(a + b*x)^(11/2)*Hypergeometric2F1[11/2, 8, 13/2, 1 + (b*x)/a])/(11*
a^8)
```

IntegrateAlgebraic [A] time = 0.26, size = 119, normalized size = 0.73

$$-\frac{9b^7\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{1024a^{5/2}} - \frac{\sqrt{a+bx}\left(315a^6 - 2100a^5(a+bx) + 5943a^4(a+bx)^2 - 9216a^3(a+bx)^3 + 8393a^2(a+bx)^4 + 2100a(a+bx)^5 - 315(a+bx)^6\right)}{35840a^2x^7}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/x^8,x]
```

```
[Out] -1/35840*(Sqrt[a + b*x]*(315*a^6 - 2100*a^5*(a + b*x) + 5943*a^4*(a + b*x)^
2 - 9216*a^3*(a + b*x)^3 + 8393*a^2*(a + b*x)^4 + 2100*a*(a + b*x)^5 - 315*
(a + b*x)^6))/(a^2*x^7) - (9*b^7*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(1024*a^(5
/2))
```

fricas [A] time = 1.42, size = 233, normalized size = 1.43

$$\left[\frac{315\sqrt{a}b^7x^7\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right) + 2\left(315ab^6x^6 - 210a^2b^5x^5 - 14168a^3b^4x^4 - 39056a^4b^3x^3 - 44928a^5b^2x^2 - 24320a^6bx - 5120a^7\right)\sqrt{bx+a}}{71680a^3x^7}, \frac{315\sqrt{-a}b^7x^7\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) + \left(315ab^6x^6 - 210a^2b^5x^5 - 14168a^3b^4x^4 - 39056a^4b^3x^3 - 44928a^5b^2x^2 - 24320a^6bx - 5120a^7\right)\sqrt{bx+a}}{35840a^3x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/71680*(315*sqrt(a)*b^7*x^7*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(315*a*b^6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^
3 - 44928*a^5*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7),
 1/35840*(315*sqrt(-a)*b^7*x^7*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (315*a*b^
6*x^6 - 210*a^2*b^5*x^5 - 14168*a^3*b^4*x^4 - 39056*a^4*b^3*x^3 - 44928*a^5
*b^2*x^2 - 24320*a^6*b*x - 5120*a^7)*sqrt(b*x + a))/(a^3*x^7)]
```

giac [A] time = 0.95, size = 144, normalized size = 0.88

$$\frac{\frac{315\,b^8\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{315\,(bx+a)^{\frac{13}{2}}b^8-2100\,(bx+a)^{\frac{11}{2}}ab^8-8393\,(bx+a)^{\frac{9}{2}}a^2b^8+9216\,(bx+a)^{\frac{7}{2}}a^3b^8-5943\,(bx+a)^{\frac{5}{2}}a^4b^8+2100\,(bx+a)^{\frac{3}{2}}a^5b^8-315\,\sqrt{bx+a}\,a^6b^8}{a^2b^7x^7}}{35840\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/35840*(315*b^8*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b*x
+ a)^(13/2)*b^8 - 2100*(b*x + a)^(11/2)*a*b^8 - 8393*(b*x + a)^(9/2)*a^2*b^
8 + 9216*(b*x + a)^(7/2)*a^3*b^8 - 5943*(b*x + a)^(5/2)*a^4*b^8 + 2100*(b*x
 + a)^(3/2)*a^5*b^8 - 315*sqrt(b*x + a)*a^6*b^8)/(a^2*b^7*x^7))/b
```

maple [A] time = 0.01, size = 111, normalized size = 0.68

$$2\left(-\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2048a^{\frac{5}{2}}}+\frac{-\frac{9\sqrt{bx+a}\,a^4}{2048}+\frac{15(bx+a)^{\frac{3}{2}}a^3}{512}-\frac{849(bx+a)^{\frac{5}{2}}a^2}{10240}+\frac{9(bx+a)^{\frac{7}{2}}a}{70}-\frac{15(bx+a)^{\frac{11}{2}}}{512a}+\frac{9(bx+a)^{\frac{13}{2}}}{2048a^2}-\frac{1199(bx+a)^{\frac{9}{2}}}{10240}}{b^7x^7}\right)b^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/x^8,x)
```

```
[Out] 2*b^7*((9/2048/a^2*(b*x+a)^(13/2)-15/512*(b*x+a)^(11/2)/a-1199/10240*(b*x+a
)^(9/2)+9/70*(b*x+a)^(7/2)*a-849/10240*(b*x+a)^(5/2)*a^2+15/512*(b*x+a)^(3/
2)*a^3-9/2048*(b*x+a)^(1/2)*a^4)/x^7/b^7-9/2048*arctanh((b*x+a)^(1/2)/a^(1/
2))/a^(5/2))
```

maxima [A] time = 3.08, size = 229, normalized size = 1.40

$$\frac{9\,b^7\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2048\,a^{\frac{5}{2}}}+\frac{315\,(bx+a)^{\frac{13}{2}}b^7-2100\,(bx+a)^{\frac{11}{2}}ab^7-8393\,(bx+a)^{\frac{9}{2}}a^2b^7+9216\,(bx+a)^{\frac{7}{2}}a^3b^7-5943\,(bx+a)^{\frac{5}{2}}a^4b^7+2100\,(bx+a)^{\frac{3}{2}}a^5b^7-315\,\sqrt{bx+a}\,a^6b^7}{35840\left((bx+a)^7a^2-7\,(bx+a)^6a^3+21\,(bx+a)^5a^4-35\,(bx+a)^4a^5+35\,(bx+a)^3a^6-21\,(bx+a)^2a^7+7\,(bx+a)a^8-a^9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/x^8,x, algorithm="maxima")
```

```
[Out] 9/2048*b^7*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
 + 1/35840*(315*(b*x + a)^(13/2)*b^7 - 2100*(b*x + a)^(11/2)*a*b^7 - 8393*(
b*x + a)^(9/2)*a^2*b^7 + 9216*(b*x + a)^(7/2)*a^3*b^7 - 5943*(b*x + a)^(5/2
)*a^4*b^7 + 2100*(b*x + a)^(3/2)*a^5*b^7 - 315*sqrt(b*x + a)*a^6*b^7)/((b*x
 + a)^7*a^2 - 7*(b*x + a)^6*a^3 + 21*(b*x + a)^5*a^4 - 35*(b*x + a)^4*a^5 +
 35*(b*x + a)^3*a^6 - 21*(b*x + a)^2*a^7 + 7*(b*x + a)*a^8 - a^9)
```

mupad [B] time = 0.13, size = 124, normalized size = 0.76

$$\frac{15\,a^3\,(a+b\,x)^{3/2}}{256\,x^7}-\frac{9\,a^4\,\sqrt{a+b\,x}}{1024\,x^7}-\frac{1199\,(a+b\,x)^{9/2}}{5120\,x^7}-\frac{849\,a^2\,(a+b\,x)^{5/2}}{5120\,x^7}-\frac{15\,(a+b\,x)^{11/2}}{256\,a\,x^7}+\frac{9\,(a+b\,x)^{13/2}}{1024\,a^2\,x^7}+\frac{9\,a\,(a+b\,x)^{7/2}}{35\,x^7}+\frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{1024\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/x^8,x)
```

```
[Out] (15*a^3*(a + b*x)^(3/2))/(256*x^7) - (9*a^4*(a + b*x)^(1/2))/(1024*x^7) - (
1199*(a + b*x)^(9/2))/(5120*x^7) - (849*a^2*(a + b*x)^(5/2))/(5120*x^7) - (
```

```
15*(a + b*x)^(11/2))/(256*a*x^7) + (9*(a + b*x)^(13/2))/(1024*a^2*x^7) + (b
^7*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*9i)/(1024*a^(5/2)) + (9*a*(a + b*x)^(
7/2))/(35*x^7)
```

sympy [A] time = 22.20, size = 236, normalized size = 1.45

$$-\frac{a^5}{7\sqrt{b}\,x^{\frac{15}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{23a^4\sqrt{b}}{28x^{\frac{13}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{541a^3b^{\frac{3}{2}}}{280x^{\frac{11}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5249a^2b^{\frac{5}{2}}}{2240x^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{6653ab^{\frac{7}{2}}}{4480x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{1027b^{\frac{9}{2}}}{2560x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{11}{2}}}{1024ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{9b^{\frac{13}{2}}}{1024a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{9b^7\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{1024a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/x**8,x)
```

```
[Out] -a**5/(7*sqrt(b)*x**(15/2)*sqrt(a/(b*x) + 1)) - 23*a**4*sqrt(b)/(28*x**(13/
2)*sqrt(a/(b*x) + 1)) - 541*a**3*b**(3/2)/(280*x**(11/2)*sqrt(a/(b*x) + 1))
 - 5249*a**2*b**(5/2)/(2240*x**(9/2)*sqrt(a/(b*x) + 1)) - 6653*a*b**(7/2)/(
4480*x**(7/2)*sqrt(a/(b*x) + 1)) - 1027*b**(9/2)/(2560*x**(5/2)*sqrt(a/(b*x
) + 1)) + 3*b**(11/2)/(1024*a*x**(3/2)*sqrt(a/(b*x) + 1)) + 9*b**(13/2)/(10
24*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 9*b**7*asinh(sqrt(a)/(sqrt(b)*sqrt(x))
)/(1024*a**(5/2))
```

3.325 $\int \frac{\sqrt{-a+bx}}{x} \, dx$

Optimal. Leaf size=39

$$2\sqrt{bx-a} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2\sqrt{bx-a} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-a+bx}}{x} \, dx &= 2\sqrt{-a+bx} - a\int \frac{1}{x\sqrt{-a+bx}} \, dx \\ &= 2\sqrt{-a+bx} - \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}} \, dx, x, \sqrt{-a+bx}\right)}{b} \\ &= 2\sqrt{-a+bx} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 1.00

$$2\sqrt{bx-a} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 1.00

$$2\sqrt{bx-a}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x,x]
```

```
[Out] 2*Sqrt[-a + b*x] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

fricas [A] time = 1.21, size = 78, normalized size = 2.00

$$\left[\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a},-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(b*x - a),
-2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)]
```

giac [A] time = 1.14, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

maple [A] time = 0.01, size = 32, normalized size = 0.82

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x,x)
```

```
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x-a)^(1/2)
```

maxima [A] time = 2.94, size = 31, normalized size = 0.79

$$-2\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)
```

mupad [B] time = 0.09, size = 31, normalized size = 0.79

$$2\sqrt{b\,x-a}-2\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x,x)
```

```
[Out] 2*(b*x - a)^(1/2) - 2*a^(1/2)*atan((b*x - a)^(1/2)/a^(1/2))
```

sympy [B] time = 1.74, size = 148, normalized size = 3.79

$$\begin{cases} -2i\sqrt{a}\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2ia}{\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{bx}-1}}-\frac{2i\sqrt{b}\,\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 2\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2a}{\sqrt{b}\,\sqrt{x}\,\sqrt{-\frac{a}{bx}+1}}+\frac{2\sqrt{b}\,\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x,x)
```

```
[Out] Piecewise((-2*I*sqrt(a)*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*I*a/(sqrt(b)*s
qrt(x)*sqrt(a/(b*x) - 1)) - 2*I*sqrt(b)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b
*x)) > 1), (2*sqrt(a)*asin(sqrt(a)/(sqrt(b)*sqrt(x))) - 2*a/(sqrt(b)*sqrt(x
)*sqrt(-a/(b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

3.326 $\int \frac{\sqrt{-a+bx}}{x^2}\,dx$

Optimal. Leaf size=42

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-a+bx}}{x^2}\,dx &= -\frac{\sqrt{-a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -\frac{\sqrt{-a+bx}}{x}+\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -\frac{\sqrt{-a+bx}}{x}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.24

$$\frac{-bx\sqrt{1-\frac{bx}{a}}\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)+a-bx}{x\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] (a - b*x - b*x*Sqrt[1 - (b*x)/a]*ArcTanh[Sqrt[1 - (b*x)/a]])/(x*Sqrt[-a + b
*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[-a + b*x]/x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.00, size = 98, normalized size = 2.33

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{2ax},\ \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\sqrt{bx-a}\,a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqrt(
b*x - a)*a)/(a*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - sqrt(b*x -
a)*a)/(a*x)]
```

giac [A] time = 1.08, size = 41, normalized size = 0.98

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}\,b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)*b/x)/b
```

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^2,x)
```

```
[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)-(b*x-a)^(1/2)/x
```

maxima [A] time = 2.94, size = 34, normalized size = 0.81

$$\frac{b \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{bx-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] b*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - sqrt(b*x - a)/x
```

mupad [B] time = 0.10, size = 34, normalized size = 0.81

$$\frac{b \operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{b\,x-a}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^2,x)
```

```
[Out] (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2) - (b*x - a)^(1/2)/x
```

sympy [A] time = 2.14, size = 121, normalized size = 2.88

$$\begin{cases} -\frac{ia}{\sqrt{b}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{i\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{ib \operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{\sqrt{x}} - \frac{b \operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**2,x)
```

```
[Out] Piecewise((-I*a/(sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(sqrt(x)*s
qrt(a/(b*x) - 1)) + I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*
x)) > 1), (-sqrt(b)*sqrt(-a/(b*x) + 1)/sqrt(x) - b*asin(sqrt(a)/(sqrt(b)*sq
rt(x)))/sqrt(a), True))
```

3.327 $\int \frac{\sqrt{-a+bx}}{x^3}\,dx$

Optimal. Leaf size=71

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}}-\frac{\sqrt{bx-a}}{2x^2}+\frac{b\sqrt{bx-a}}{4ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -Sqrt[-a + b*x]/(2*x^2) + (b*Sqrt[-a + b*x])/(4*a*x) + (b^2*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/(4*a^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-a+bx}}{x^3}\,dx = -\frac{\sqrt{-a+bx}}{2x^2} + \frac{1}{4}b\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$$
$$= -\frac{\sqrt{-a+bx}}{2x^2} + \frac{b\sqrt{-a+bx}}{4ax} + \frac{b^2\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a}$$
$$= -\frac{\sqrt{-a+bx}}{2x^2} + \frac{b\sqrt{-a+bx}}{4ax} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{4a}$$
$$= -\frac{\sqrt{-a+bx}}{2x^2} + \frac{b\sqrt{-a+bx}}{4ax} + \frac{b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{3/2}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.54

$$\frac{2b^2(bx-a)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};1-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] (2*b^2*(-a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 - (b*x)/a])/(3*a^3
)
```

IntegrateAlgebraic [A] time = 0.08, size = 60, normalized size = 0.85

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{3/2}} - \frac{(2a-bx)\sqrt{bx-a}}{4ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-a + b*x]/x^3,x]
```

```
[Out] -1/4*((2*a - b*x)*Sqrt[-a + b*x])/(a*x^2) + (b^2*ArcTan[Sqrt[-a + b*x]/Sqrt
[a]])/(4*a^(3/2))
```

fricas [A] time = 1.09, size = 124, normalized size = 1.75

$$\left[-\frac{\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(abx-2a^2\right)\sqrt{bx-a}}{8a^2x^2}, \frac{\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(abx-2a^2\right)\sqrt{bx-a}}{4a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(
a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2), 1/4*(sqrt(a)*b^2*x^2*arctan(sqrt(b
*x - a)/sqrt(a)) + (a*b*x - 2*a^2)*sqrt(b*x - a))/(a^2*x^2)]
```

giac [A] time = 1.08, size = 66, normalized size = 0.93

$$\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{(bx-a)^{\frac{3}{2}}b^3-\sqrt{bx-a}\,ab^3}{ab^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + ((b*x - a)^(3/2)*b^3 - sqr
t(b*x - a)*a*b^3)/(a*b^2*x^2))/b
```

maple [A] time = 0.01, size = 55, normalized size = 0.77

$$\frac{b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{3}{2}}} - \frac{\sqrt{bx-a}}{4x^2} + \frac{(bx-a)^{\frac{3}{2}}}{4a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(1/2)/x^3,x)
```

```
[Out] 1/4/x^2/a*(b*x-a)^(3/2)-1/4*(b*x-a)^(1/2)/x^2+1/4*b^2*arctan((b*x-a)^(1/2)/
a^(1/2))/a^(3/2)
```

maxima [A] time = 2.96, size = 83, normalized size = 1.17

$$\frac{b^2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{3}{2}}} + \frac{(bx-a)^{\frac{3}{2}} b^2 - \sqrt{bx-a}\,ab^2}{4\left((bx-a)^2 a + 2\,(bx-a)a^2 + a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + 1/4*((b*x - a)^(3/2)*b^2 -
sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a + 2*(b*x - a)*a^2 + a^3)
```

mupad [B] time = 0.10, size = 54, normalized size = 0.76

$$\frac{b^2 \operatorname{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{3/2}} - \frac{\sqrt{b\,x-a}}{4\,x^2} + \frac{(b\,x-a)^{3/2}}{4\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(1/2)/x^3,x)
```

```
[Out] (b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(3/2)) - (b*x - a)^(1/2)/(4*x^2) +
 (b*x - a)^(3/2)/(4*a*x^2)
```

sympy [A] time = 4.16, size = 207, normalized size = 2.92

$$\begin{cases} -\frac{ia}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}} + \frac{3i\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}} - \frac{ib^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{\frac{a}{bx}-1}} + \frac{ib^2 \operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{a}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}} - \frac{3\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}} + \frac{b^{\frac{3}{2}}}{4a\sqrt{x}\sqrt{-\frac{a}{bx}+1}} - \frac{b^2 \operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(1/2)/x**3,x)
```

```
[Out] Piecewise((-I*a/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(4*x**
(3/2)*sqrt(a/(b*x) - 1)) - I*b**(3/2)/(4*a*sqrt(x)*sqrt(a/(b*x) - 1)) + I*b
**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(3/2)), Abs(a/(b*x)) > 1), (a/(2
*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x)
 + 1)) + b**(3/2)/(4*a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b**2*asin(sqrt(a)/(sqr
t(b)*sqrt(x)))/(4*a**(3/2)), True))
```

3.328 $\int \frac{(-a+bx)^{3/2}}{x} \, dx$

Optimal. Leaf size=55

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2a\sqrt{bx-a}+\frac{2}{3}(bx-a)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x,x]
```

```
[Out] -2*a*Sqrt[-a + b*x] + (2*(-a + b*x)^(3/2))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b
*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(-a+bx)^{3/2}}{x} \, dx &= \frac{2}{3}(-a+bx)^{3/2}-a\int \frac{\sqrt{-a+bx}}{x} \, dx\\ &= -2a\sqrt{-a+bx}+\frac{2}{3}(-a+bx)^{3/2}+a^2\int \frac{1}{x\sqrt{-a+bx}} \, dx\\ &= -2a\sqrt{-a+bx}+\frac{2}{3}(-a+bx)^{3/2}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}} \, dx,x,\sqrt{-a+bx}\right)}{b}\\ &= -2a\sqrt{-a+bx}+\frac{2}{3}(-a+bx)^{3/2}+2a^{3/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.87

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}(bx-4a)\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x,x]
```

```
[Out] (2*(-4*a + b*x)*Sqrt[-a + b*x])/3 + 2*a^(3/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]
]
```

IntegrateAlgebraic [A] time = 0.03, size = 58, normalized size = 1.05

$$2a^{3/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{2}{3}\left(3a\sqrt{bx-a}-(bx-a)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x,x]
```

```
[Out] (-2*(3*a*Sqrt[-a + b*x] - (-a + b*x)^(3/2)))/3 + 2*a^(3/2)*ArcTan[Sqrt[-a +
 b*x]/Sqrt[a]]
```

fricas [A] time = 0.91, size = 93, normalized size = 1.69

$$\left[\sqrt{-a}\,a\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+\frac{2}{3}\sqrt{bx-a}\,(bx-4a),2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{bx-a}\,(bx-4a)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/3*sqrt(b*x -
a)*(b*x - 4*a), 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*sqrt(b*x - a)
*(b*x - 4*a)]
```

giac [A] time = 1.06, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

maple [A] time = 0.01, size = 44, normalized size = 0.80

$$2a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}\,a+\frac{2\,(bx-a)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x,x)
```

```
[Out] 2/3*(b*x-a)^(3/2)+2*a^(3/2)*arctan((b*x-a)^(1/2)/a^(1/2))-2*a*(b*x-a)^(1/2)
```

maxima [A] time = 3.02, size = 43, normalized size = 0.78

$$2\,a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}-2\,\sqrt{bx-a}\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2*a^(3/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2) - 2*sqrt(b*x
- a)*a
```

mupad [B] time = 0.04, size = 43, normalized size = 0.78

$$2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-2\,a\,\sqrt{b\,x-a}\,+\frac{2\,(b\,x-a)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x,x)
```

```
[Out] 2*a^(3/2)*atan((b*x - a)^(1/2)/a^(1/2)) - 2*a*(b*x - a)^(1/2) + (2*(b*x - a
)^(3/2))/3
```

sympy [C] time = 2.46, size = 187, normalized size = 3.40

$$\begin{cases} -\frac{8a^{\frac{3}{2}}\sqrt{-1+\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-2a^{\frac{3}{2}}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{a}\,bx\sqrt{-1+\frac{bx}{a}}}{3} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{8ia^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{3}-ia^{\frac{3}{2}}\log\left(\frac{bx}{a}\right)+2ia^{\frac{3}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,bx\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x,x)
```

```
[Out] Piecewise((-8*a**(3/2)*sqrt(-1 + b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**
(3/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) - 2*a**(3/2)*asin(sqrt(a)/(sqrt(b)*sqrt(
x))) + 2*sqrt(a)*b*x*sqrt(-1 + b*x/a)/3, Abs(b*x/a) > 1), (-8*I*a**(3/2)*sq
rt(1 - b*x/a)/3 - I*a**(3/2)*log(b*x/a) + 2*I*a**(3/2)*log(sqrt(1 - b*x/a)
+ 1) + 2*I*sqrt(a)*b*x*sqrt(1 - b*x/a)/3, True))
```

3.329 $\int \frac{(-a+bx)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=57

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$-\frac{(bx-a)^{3/2}}{x}+3b\sqrt{bx-a}-3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] 3*b*Sqrt[-a + b*x] - (-a + b*x)^(3/2)/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(-a+bx)^{3/2}}{x^2}\,dx = -\frac{(-a+bx)^{3/2}}{x} + \frac{1}{2}(3b)\int \frac{\sqrt{-a+bx}}{x}\,dx$$
$$= 3b\sqrt{-a+bx} - \frac{(-a+bx)^{3/2}}{x} - \frac{1}{2}(3ab)\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= 3b\sqrt{-a+bx} - \frac{(-a+bx)^{3/2}}{x} - (3a)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= 3b\sqrt{-a+bx} - \frac{(-a+bx)^{3/2}}{x} - 3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.63

$$\frac{2b(bx-a)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};1-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] (2*b*(-a + b*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 - (b*x)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.05, size = 55, normalized size = 0.96

$$\frac{\sqrt{bx-a}\,(2(bx-a)+3a)}{x} - 3\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x^2,x]
```

```
[Out] (Sqrt[-a + b*x]*(3*a + 2*(-a + b*x)))/x - 3*Sqrt[a]*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]]
```

fricas [A] time = 1.04, size = 105, normalized size = 1.84

$$\left[\frac{3\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\,(2\,bx+a)\sqrt{bx-a}}{2\,x}, -\frac{3\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-(2\,bx+a)\sqrt{bx-a}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(2*b
*x + a)*sqrt(b*x - a))/x, -(3*sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) - (
2*b*x + a)*sqrt(b*x - a))/x]
```

giac [A] time = 0.96, size = 58, normalized size = 1.02

$$-\frac{3\sqrt{a}\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-2\sqrt{bx-a}\,b^2-\frac{\sqrt{bx-a}\,ab}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] -(3*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) - 2*sqrt(b*x - a)*b^2 - sqrt(
b*x - a)*a*b/x)/b
```

maple [A] time = 0.01, size = 48, normalized size = 0.84

$$-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b+\frac{\sqrt{bx-a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x-a)^(1/2)+a*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/
2)
```

maxima [A] time = 3.00, size = 47, normalized size = 0.82

$$-3\sqrt{a}\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\,b+\frac{\sqrt{bx-a}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] -3*sqrt(a)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b + sqrt(b*x -
 a)*a/x
```

mupad [B] time = 0.04, size = 47, normalized size = 0.82

$$2\,b\,\sqrt{b\,x-a}+\frac{a\,\sqrt{b\,x-a}}{x}-3\,\sqrt{a}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^2,x)
```

```
[Out] 2*b*(b*x - a)^(1/2) + (a*(b*x - a)^(1/2))/x - 3*a^(1/2)*b*atan((b*x - a)^(1
/2)/a^(1/2))
```

sympy [B] time = 2.84, size = 197, normalized size = 3.46

$$\begin{cases} -3i\sqrt{a}\,b\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{ia^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{ia\sqrt{b}}{\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{3}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ 3\sqrt{a}\,b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{a^2}{\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{a\sqrt{b}}{\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{3}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**2,x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x))) + I*a**2/(sqrt(b
)*x**(3/2)*sqrt(a/(b*x) - 1)) + I*a*sqrt(b)/(sqrt(x)*sqrt(a/(b*x) - 1)) - 2
*I*b**(3/2)*sqrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (3*sqrt(a)*b*asin
(sqrt(a)/(sqrt(b)*sqrt(x))) - a**2/(sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) -
a*sqrt(b)/(sqrt(x)*sqrt(-a/(b*x) + 1)) + 2*b**(3/2)*sqrt(x)/sqrt(-a/(b*x) +
 1), True))
```

3.330 $\int \frac{(-a+bx)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=68

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 205}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{(bx-a)^{3/2}}{2x^2}-\frac{3b\sqrt{bx-a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] (-3*b*Sqrt[-a + b*x])/(4*x) - (-a + b*x)^(3/2)/(2*x^2) + (3*b^2*ArcTan[Sqrt
[-a + b*x]/Sqrt[a]])/(4*Sqrt[a])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{3/2}}{x^3}\,dx &= -\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\int \frac{\sqrt{-a+bx}}{x^2}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{8}\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{1}{4}(3b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)\\
&= -\frac{3b\sqrt{-a+bx}}{4x}-\frac{(-a+bx)^{3/2}}{2x^2}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 1.06

$$-\frac{2a^2+3b^2x^2\sqrt{1-\frac{bx}{a}}\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)-7abx+5b^2x^2}{4x^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(2*a^2 - 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 - (b*x)/a]*ArcTanh[Sqr
t[1 - (b*x)/a]])/(x^2*Sqrt[-a + b*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 62, normalized size = 0.91

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{\sqrt{bx-a}\,(5(bx-a)+3a)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(3/2)/x^3,x]
```

```
[Out] -1/4*(Sqrt[-a + b*x]*(3*a + 5*(-a + b*x)))/x^2 + (3*b^2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/(4*Sqrt[a])
```

fricas [A] time = 1.04, size = 129, normalized size = 1.90

$$\left[-\frac{3\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(5abx-2a^2\right)\sqrt{bx-a}}{8ax^2},\ \frac{3\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(5abx-2a^2\right)\sqrt{bx-a}}{4ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(sq
rt(b*x - a)/sqrt(a)) - (5*a*b*x - 2*a^2)*sqrt(b*x - a))/(a*x^2)]
```

giac [A] time = 0.95, size = 66, normalized size = 0.97

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^3+3\,\sqrt{bx-a}\,ab^3}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - (5*(b*x - a)^(3/2)*b^3 +
 3*sqrt(b*x - a)*a*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 53, normalized size = 0.78

$$\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\sqrt{a}}-\frac{3\sqrt{bx-a}\,a}{4x^2}-\frac{5\,(bx-a)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(3/2)/x^3,x)
```

```
[Out] -5/4*(b*x-a)^(3/2)/x^2-3/4/x^2*(b*x-a)^(1/2)*a+3/4*b^2*arctan((b*x-a)^(1/2)
/a^(1/2))/a^(1/2)
```

maxima [A] time = 3.02, size = 80, normalized size = 1.18

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,(bx-a)^{\frac{3}{2}}b^2+3\,\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2+2\,(bx-a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a) - 1/4*(5*(b*x - a)^(3/2)*b^2
+ 3*sqrt(b*x - a)*a*b^2)/((b*x - a)^2 + 2*(b*x - a)*a + a^2)
```

mupad [B] time = 0.10, size = 52, normalized size = 0.76

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,\sqrt{a}}-\frac{5\,(b\,x-a)^{3/2}}{4\,x^2}-\frac{3\,a\,\sqrt{b\,x-a}}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(3/2)/x^3,x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(1/2)) - (5*(b*x - a)^(3/2))/(4*
x^2) - (3*a*(b*x - a)^(1/2))/(4*x^2)
```

sympy [A] time = 3.30, size = 190, normalized size = 2.79

$$\begin{cases} \frac{ia^2}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{7ia\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{5ib^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{2x^{\frac{3}{2}}}-\frac{5b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{4\sqrt{x}}-\frac{3b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(3/2)/x**3,x)
```

```
[Out] Piecewise((I*a**2/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 7*I*a*sqrt(b)/(4
*x**(3/2)*sqrt(a/(b*x) - 1)) + 5*I*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) +
 3*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), Abs(a/(b*x)) > 1),
(a*sqrt(b)*sqrt(-a/(b*x) + 1)/(2*x**(3/2)) - 5*b**(3/2)*sqrt(-a/(b*x) + 1)/
(4*sqrt(x)) - 3*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*sqrt(a)), True))
```

3.331 $\int \frac{(-a+bx)^{5/2}}{x} dx$

Optimal. Leaf size=73

$$-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2a^2\sqrt{bx-a}-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 205}

$$2a^2\sqrt{bx-a}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{2}{3}a(bx-a)^{3/2}+\frac{2}{5}(bx-a)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x,x]
```

```
[Out] 2*a^2*Sqrt[-a + b*x] - (2*a*(-a + b*x)^(3/2))/3 + (2*(-a + b*x)^(5/2))/5 -
2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(-a+bx)^{5/2}}{x}\,dx &= \frac{2}{5}(-a+bx)^{5/2}-a\int \frac{(-a+bx)^{3/2}}{x}\,dx\\
&= -\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}+a^2\int \frac{\sqrt{-a+bx}}{x}\,dx\\
&= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-a^3\int \frac{1}{x\sqrt{-a+bx}}\,dx\\
&= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-\frac{\left(2a^3\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{b}\\
&= 2a^2\sqrt{-a+bx}-\frac{2}{3}a(-a+bx)^{3/2}+\frac{2}{5}(-a+bx)^{5/2}-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.82

$$\frac{2}{15}\sqrt{bx-a}\left(23a^2-11abx+3b^2x^2\right)-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*Sqrt[-a + b*x]*(23*a^2 - 11*a*b*x + 3*b^2*x^2))/15 - 2*a^(5/2)*ArcTan[Sq
rt[-a + b*x]/Sqrt[a]]
```

IntegrateAlgebraic [A] time = 0.03, size = 74, normalized size = 1.01

$$\frac{2}{15}\left(15a^2\sqrt{bx-a}+3(bx-a)^{5/2}-5a(bx-a)^{3/2}\right)-2a^{5/2}\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x,x]
```

```
[Out] (2*(15*a^2*Sqrt[-a + b*x] - 5*a*(-a + b*x)^(3/2) + 3*(-a + b*x)^(5/2)))/15
- 2*a^(5/2)*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

fricas [A] time = 1.16, size = 119, normalized size = 1.63

$$\left[\sqrt{-a}\,a^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+\frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a},-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{15}\left(3b^2x^2-11abx+23a^2\right)\sqrt{bx-a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(-a)*a^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2/15*(3*b^2*x
^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a), -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt
(a)) + 2/15*(3*b^2*x^2 - 11*a*b*x + 23*a^2)*sqrt(b*x - a)]
```

giac [A] time = 1.22, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

maple [A] time = 0.01, size = 58, normalized size = 0.79

$$-2a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\;a^2-\frac{2\,(bx-a)^{\frac{3}{2}}\,a}{3}+\frac{2\,(bx-a)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x,x)
```

```
[Out] -2/3*a*(b*x-a)^(3/2)+2/5*(b*x-a)^(5/2)-2*a^(5/2)*arctan((b*x-a)^(1/2)/a^(1/
2))+2*a^2*(b*x-a)^(1/2)
```

maxima [A] time = 2.92, size = 57, normalized size = 0.78

$$-2\,a^{\frac{5}{2}}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{5}\,(bx-a)^{\frac{5}{2}}-\frac{2}{3}\,(bx-a)^{\frac{3}{2}}a+2\,\sqrt{bx-a}\,a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] -2*a^(5/2)*arctan(sqrt(b*x - a)/sqrt(a)) + 2/5*(b*x - a)^(5/2) - 2/3*(b*x -
 a)^(3/2)*a + 2*sqrt(b*x - a)*a^2
```

mupad [B] time = 0.04, size = 57, normalized size = 0.78

$$\frac{2\,(b\,x-a)^{5/2}}{5}-\frac{2\,a\,(b\,x-a)^{3/2}}{3}-2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)+2\,a^2\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x,x)
```

```
[Out] (2*(b*x - a)^(5/2))/5 - (2*a*(b*x - a)^(3/2))/3 - 2*a^(5/2)*atan((b*x - a)^
(1/2)/a^(1/2)) + 2*a^2*(b*x - a)^(1/2)
```

sympy [C] time = 4.24, size = 240, normalized size = 3.29

$$\begin{cases} \frac{46a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+2a^{\frac{5}{2}}\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)-\frac{22a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}}{15}+\frac{2\sqrt{a}b^2x^2\sqrt{-1+\frac{bx}{a}}}{5} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{46ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{15}+ia^{\frac{5}{2}}\log\left(\frac{bx}{a}\right)-2ia^{\frac{5}{2}}\log\left(\sqrt{1-\frac{bx}{a}}+1\right)-\frac{22ia^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}}{15}+\frac{2i\sqrt{a}b^2x^2\sqrt{1-\frac{bx}{a}}}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x,x)
```

```
[Out] Piecewise((46*a**(5/2)*sqrt(-1 + b*x/a)/15 + I*a**(5/2)*log(b*x/a) - 2*I*a*
*(5/2)*log(sqrt(b)*sqrt(x)/sqrt(a)) + 2*a**(5/2)*asin(sqrt(a)/(sqrt(b)*sqrt
(x))) - 22*a**(3/2)*b*x*sqrt(-1 + b*x/a)/15 + 2*sqrt(a)*b**2*x**2*sqrt(-1 +
 b*x/a)/5, Abs(b*x/a) > 1), (46*I*a**(5/2)*sqrt(1 - b*x/a)/15 + I*a**(5/2)*
log(b*x/a) - 2*I*a**(5/2)*log(sqrt(1 - b*x/a) + 1) - 22*I*a**(3/2)*b*x*sqrt
(1 - b*x/a)/15 + 2*I*sqrt(a)*b**2*x**2*sqrt(1 - b*x/a)/5, True))
```

3.332 $\int \frac{(-a+bx)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=74

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{x}+\frac{5}{3}b(bx-a)^{3/2}-5ab\sqrt{bx-a}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] -5*a*b*Sqrt[-a + b*x] + (5*b*(-a + b*x)^(3/2))/3 - (-a + b*x)^(5/2)/x + 5*a
^(3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(-a+bx)^{5/2}}{x^2}\,dx = -\frac{(-a+bx)^{5/2}}{x} + \frac{1}{2}(5b)\int \frac{(-a+bx)^{3/2}}{x}\,dx$$
$$= \frac{5}{3}b(-a+bx)^{3/2} - \frac{(-a+bx)^{5/2}}{x} - \frac{1}{2}(5ab)\int \frac{\sqrt{-a+bx}}{x}\,dx$$
$$= -5ab\sqrt{-a+bx} + \frac{5}{3}b(-a+bx)^{3/2} - \frac{(-a+bx)^{5/2}}{x} + \frac{1}{2}\left(5a^2b\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= -5ab\sqrt{-a+bx} + \frac{5}{3}b(-a+bx)^{3/2} - \frac{(-a+bx)^{5/2}}{x} + \left(5a^2\right)\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)$$
$$= -5ab\sqrt{-a+bx} + \frac{5}{3}b(-a+bx)^{3/2} - \frac{(-a+bx)^{5/2}}{x} + 5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.49

$$\frac{2b(bx-a)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] (2*b*(-a + b*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 - (b*x)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.06, size = 72, normalized size = 0.97

$$5a^{3/2}b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right) + \frac{\sqrt{bx-a}\left(-15a^2-10a(bx-a)+2(bx-a)^2\right)}{3x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x^2,x]
```

```
[Out] (Sqrt[-a + b*x]*(-15*a^2 - 10*a*(-a + b*x) + 2*(-a + b*x)^2))/(3*x) + 5*a^(
3/2)*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]]
```

fricas [A] time = 0.98, size = 131, normalized size = 1.77

$$\left[\frac{15\sqrt{-a}\,abx\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{6x}, \frac{15a^{\frac{3}{2}}bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(2b^2x^2-14abx-3a^2\right)\sqrt{bx-a}}{3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*sqrt(-a)*a*b*x*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*(
2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x, 1/3*(15*a^(3/2)*b*x*arctan(
sqrt(b*x - a)/sqrt(a)) + (2*b^2*x^2 - 14*a*b*x - 3*a^2)*sqrt(b*x - a))/x]
```

giac [A] time = 1.02, size = 75, normalized size = 1.01

$$\frac{15a^{\frac{3}{2}}b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2(bx-a)^{\frac{3}{2}}b^2-12\sqrt{bx-a}\,ab^2-\frac{3\sqrt{bx-a}\,a^2b}{x}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(15*a^(3/2)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*(b*x - a)^(3/2)*b^2 -
 12*sqrt(b*x - a)*a*b^2 - 3*sqrt(b*x - a)*a^2*b/x)/b
```

maple [A] time = 0.01, size = 64, normalized size = 0.86

$$5a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-4\sqrt{bx-a}\,ab-\frac{\sqrt{bx-a}\,a^{2}}{x}+\frac{2\,(bx-a)^{\frac{3}{2}}\,b}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^2,x)
```

```
[Out] 2/3*b*(b*x-a)^(3/2)-4*a*b*(b*x-a)^(1/2)-a^2*(b*x-a)^(1/2)/x+5*a^(3/2)*b*arc
tan((b*x-a)^(1/2)/a^(1/2))
```

maxima [A] time = 3.05, size = 63, normalized size = 0.85

$$5\,a^{\frac{3}{2}}b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\frac{2}{3}\,(bx-a)^{\frac{3}{2}}b-4\,\sqrt{bx-a}\,ab-\frac{\sqrt{bx-a}\,a^{2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5*a^(3/2)*b*arctan(sqrt(b*x - a)/sqrt(a)) + 2/3*(b*x - a)^(3/2)*b - 4*sqrt(
b*x - a)*a*b - sqrt(b*x - a)*a^2/x
```

mupad [B] time = 0.10, size = 63, normalized size = 0.85

$$\frac{2\,b\,{\left(b\,x-a\right)}^{3/2}}{3}-\frac{a^2\,\sqrt{b\,x-a}}{x}+5\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)-4\,a\,b\,\sqrt{b\,x-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^2,x)
```

```
[Out] (2*b*(b*x - a)^(3/2))/3 - (a^2*(b*x - a)^(1/2))/x + 5*a^(3/2)*b*atan((b*x -
 a)^(1/2)/a^(1/2)) - 4*a*b*(b*x - a)^(1/2)
```

sympy [C] time = 3.69, size = 245, normalized size = 3.31

$$\begin{cases} -\frac{a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}}{x}-\frac{14a^{\frac{3}{2}}b\sqrt{-1+\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-5a^{\frac{3}{2}}b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2\sqrt{a}\,b^{2}x\sqrt{-1+\frac{bx}{a}}}{3} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{ia^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{x}-\frac{14ia^{\frac{3}{2}}b\sqrt{1-\frac{bx}{a}}}{3}-\frac{5ia^{\frac{3}{2}}b\log\left(\frac{bx}{a}\right)}{2}+5ia^{\frac{3}{2}}b\log\left(\sqrt{1-\frac{bx}{a}}+1\right)+\frac{2i\sqrt{a}\,b^{2}x\sqrt{1-\frac{bx}{a}}}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**2,x)
```

```
[Out] Piecewise((-a**(5/2)*sqrt(-1 + b*x/a)/x - 14*a**(3/2)*b*sqrt(-1 + b*x/a)/3
- 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqrt(b)*sqrt(x)/sqrt(a))
 - 5*a**(3/2)*b*asin(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*sqrt(a)*b**2*x*sqrt(-1
+ b*x/a)/3, Abs(b*x/a) > 1), (-I*a**(5/2)*sqrt(1 - b*x/a)/x - 14*I*a**(3/2)
*b*sqrt(1 - b*x/a)/3 - 5*I*a**(3/2)*b*log(b*x/a)/2 + 5*I*a**(3/2)*b*log(sqr
t(1 - b*x/a) + 1) + 2*I*sqrt(a)*b**2*x*sqrt(1 - b*x/a)/3, True))
```

3.333 $\int \frac{(-a+bx)^{5/2}}{x^3} \, dx$

Optimal. Leaf size=86

$$\frac{15}{4}b^2\sqrt{bx-a}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{2x^2}-\frac{5b(bx-a)^{3/2}}{4x}$$

Rubi [A] time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 205}

$$\frac{15}{4}b^2\sqrt{bx-a}-\frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\frac{(bx-a)^{5/2}}{2x^2}-\frac{5b(bx-a)^{3/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (15*b^2*Sqrt[-a + b*x])/4 - (5*b*(-a + b*x)^(3/2))/(4*x) - (-a + b*x)^(5/2)
/(2*x^2) - (15*Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(-a+bx)^{5/2}}{x^3}\,dx = -\frac{(-a+bx)^{5/2}}{2x^2} + \frac{1}{4}(5b)\int \frac{(-a+bx)^{3/2}}{x^2}\,dx$$
$$= -\frac{5b(-a+bx)^{3/2}}{4x} - \frac{(-a+bx)^{5/2}}{2x^2} + \frac{1}{8}\left(15b^2\right)\int \frac{\sqrt{-a+bx}}{x}\,dx$$
$$= \frac{15}{4}b^2\sqrt{-a+bx} - \frac{5b(-a+bx)^{3/2}}{4x} - \frac{(-a+bx)^{5/2}}{2x^2} - \frac{1}{8}\left(15ab^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx$$
$$= \frac{15}{4}b^2\sqrt{-a+bx} - \frac{5b(-a+bx)^{3/2}}{4x} - \frac{(-a+bx)^{5/2}}{2x^2} - \frac{1}{4}(15ab)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+\right.$$
$$= \frac{15}{4}b^2\sqrt{-a+bx} - \frac{5b(-a+bx)^{3/2}}{4x} - \frac{(-a+bx)^{5/2}}{2x^2} - \frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.44

$$\frac{2b^2(bx-a)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};1-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (2*b^2*(-a + b*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, 1 - (b*x)/a])/(7*a^3
)
```

IntegrateAlgebraic [A] time = 0.09, size = 76, normalized size = 0.88

$$\frac{\sqrt{bx-a}\left(15a^2+25a(bx-a)+8(bx-a)^2\right)}{4x^2} - \frac{15}{4}\sqrt{a}\,b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(5/2)/x^3,x]
```

```
[Out] (Sqrt[-a + b*x]*(15*a^2 + 25*a*(-a + b*x) + 8*(-a + b*x)^2))/(4*x^2) - (15*
Sqrt[a]*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/4
```

fricas [A] time = 0.90, size = 139, normalized size = 1.62

$$\left[\frac{15\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{8x^2}, -\frac{15\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-\left(8b^2x^2+9abx-2a^2\right)\sqrt{bx-a}}{4x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2
*(8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a))/x^2, -1/4*(15*sqrt(a)*b^2*x^2
*arctan(sqrt(b*x - a)/sqrt(a)) - (8*b^2*x^2 + 9*a*b*x - 2*a^2)*sqrt(b*x - a
))/x^2]
```

giac [A] time = 1.04, size = 83, normalized size = 0.97

$$-\frac{15\sqrt{a}\,b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)-8\sqrt{bx-a}\,b^3-\frac{9(bx-a)^{\frac{3}{2}}ab^3+7\sqrt{bx-a}\,a^2b^3}{b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^3*arctan(sqrt(b*x - a)/sqrt(a)) - 8*sqrt(b*x - a)*b^3 -
(9*(b*x - a)^(3/2)*a*b^3 + 7*sqrt(b*x - a)*a^2*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 70, normalized size = 0.81

$$-\frac{15\sqrt{a}\, b^{2} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4}+2\sqrt{bx-a}\, b^{2}+\frac{7\sqrt{bx-a}\, a^{2}}{4x^{2}}+\frac{9\left(bx-a\right)^{\frac{3}{2}} a}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x-a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x-a)^(1/2)+9/4*a/x^2*(b*x-a)^(3/2)+7/4/x^2*(b*x-a)^(1/2)*a^2-15/4*
b^2*arctan((b*x-a)^(1/2)/a^(1/2))*a^(1/2)
```

maxima [A] time = 2.92, size = 97, normalized size = 1.13

$$-\frac{15}{4}\sqrt{a}\, b^{2} \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+2\sqrt{bx-a}\, b^{2}+\frac{9\left(bx-a\right)^{\frac{3}{2}} ab^{2}+7\sqrt{bx-a}\, a^{2}b^{2}}{4\left(\left(bx-a\right)^{2}+2\left(bx-a\right)a+a^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] -15/4*sqrt(a)*b^2*arctan(sqrt(b*x - a)/sqrt(a)) + 2*sqrt(b*x - a)*b^2 + 1/4
*(9*(b*x - a)^(3/2)*a*b^2 + 7*sqrt(b*x - a)*a^2*b^2)/((b*x - a)^2 + 2*(b*x
- a)*a + a^2)
```

mupad [B] time = 0.09, size = 69, normalized size = 0.80

$$2\,b^{2}\,\sqrt{b\,x-a}-\frac{15\,\sqrt{a}\,b^{2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4}+\frac{9\,a\,{\left(b\,x-a\right)}^{3/2}}{4\,x^{2}}+\frac{7\,a^{2}\,\sqrt{b\,x-a}}{4\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x - a)^(5/2)/x^3,x)
```

```
[Out] 2*b^2*(b*x - a)^(1/2) - (15*a^(1/2)*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/4 +
(9*a*(b*x - a)^(3/2))/(4*x^2) + (7*a^2*(b*x - a)^(1/2))/(4*x^2)
```

sympy [A] time = 3.93, size = 267, normalized size = 3.10

$$\begin{cases} -\frac{15i\sqrt{a}\, b^{2} \operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}-\frac{ia^{3}}{2\sqrt{b}\, x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{11ia^{2}\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{iab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{2ib^{\frac{5}{2}}\sqrt{x}}{\sqrt{\frac{a}{bx}-1}} & \text{for}\: \left|\frac{a}{bx}\right|>1 \\ \frac{15\sqrt{a}\, b^{2} \operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4}+\frac{a^{3}}{2\sqrt{b}\, x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{11a^{2}\sqrt{b}}{4x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{ab^{\frac{3}{2}}}{4\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{2b^{\frac{5}{2}}\sqrt{x}}{\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x-a)**(5/2)/x**3,x)
```

```
[Out] Piecewise((-15*I*sqrt(a)*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/4 - I*a**3/(
2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + 11*I*a**2*sqrt(b)/(4*x**(3/2)*sqrt(
a/(b*x) - 1)) - I*a*b**(3/2)/(4*sqrt(x)*sqrt(a/(b*x) - 1)) - 2*I*b**(5/2)*s
```

```
qrt(x)/sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (15*sqrt(a)*b**2*asin(sqrt(a)/
(sqrt(b)*sqrt(x)))/4 + a**3/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - 11*a*
*2*sqrt(b)/(4*x**(3/2)*sqrt(-a/(b*x) + 1)) + a*b**(3/2)/(4*sqrt(x)*sqrt(-a/
(b*x) + 1)) + 2*b**(5/2)*sqrt(x)/sqrt(-a/(b*x) + 1), True))
```

3.334 $\int \frac{x^4}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=89

$$\frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{2a^4\sqrt{a+bx}}{b^5}+\frac{2(a+bx)^{9/2}}{9b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*a^4*Sqrt[a + b*x])/b^5 - (8*a^3*(a + b*x)^(3/2))/(3*b^5) + (12*a^2*(a +
b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))/(7*b^5) + (2*(a + b*x)^(9/2))/(
9*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^4}{b^4\sqrt{a+bx}}-\frac{4a^3\sqrt{a+bx}}{b^4}+\frac{6a^2(a+bx)^{3/2}}{b^4}-\frac{4a(a+bx)^{5/2}}{b^4}+\frac{(a+bx)^{7/2}}{b^4}\right)dx\\ &= \frac{2a^4\sqrt{a+bx}}{b^5}-\frac{8a^3(a+bx)^{3/2}}{3b^5}+\frac{12a^2(a+bx)^{5/2}}{5b^5}-\frac{8a(a+bx)^{7/2}}{7b^5}+\frac{2(a+bx)^{9/2}}{9b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.64

$$\frac{2\sqrt{a+bx}\left(128a^4-64a^3bx+48a^2b^2x^2-40ab^3x^3+35b^4x^4\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(128*a^4 - 64*a^3*b*x + 48*a^2*b^2*x^2 - 40*a*b^3*x^3 + 35
*b^4*x^4))/(315*b^5)
```

IntegrateAlgebraic [A] time = 0.02, size = 73, normalized size = 0.82

$$\frac{2\left(315a^4\sqrt{a+bx}-420a^3(a+bx)^{3/2}+378a^2(a+bx)^{5/2}+35(a+bx)^{9/2}-180a(a+bx)^{7/2}\right)}{315b^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/Sqrt[a + b*x],x]
```

```
[Out] (2*(315*a^4*Sqrt[a + b*x] - 420*a^3*(a + b*x)^(3/2) + 378*a^2*(a + b*x)^(5/
2) - 180*a*(a + b*x)^(7/2) + 35*(a + b*x)^(9/2)))/(315*b^5)
```

fricas [A] time = 0.72, size = 53, normalized size = 0.60

$$\frac{2\left(35\,b^4x^4 - 40\,ab^3x^3 + 48\,a^2b^2x^2 - 64\,a^3bx + 128\,a^4\right)\sqrt{bx + a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 - 40*a*b^3*x^3 + 48*a^2*b^2*x^2 - 64*a^3*b*x + 128*a^4)*s
qrt(b*x + a)/b^5
```

giac [A] time = 1.04, size = 61, normalized size = 0.69

$$\frac{2\left(35\,(bx + a)^{\frac{9}{2}} - 180\,(bx + a)^{\frac{7}{2}}a + 378\,(bx + a)^{\frac{5}{2}}a^2 - 420\,(bx + a)^{\frac{3}{2}}a^3 + 315\,\sqrt{bx + a}\,a^4\right)}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)/b^5
```

maple [A] time = 0.01, size = 54, normalized size = 0.61

$$\frac{2\sqrt{bx + a}\,\left(35x^4b^4 - 40a\,x^3b^3 + 48a^2x^2b^2 - 64a^3xb + 128a^4\right)}{315b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2/315*(b*x+a)^(1/2)*(35*b^4*x^4-40*a*b^3*x^3+48*a^2*b^2*x^2-64*a^3*b*x+128*
a^4)/b^5
```

maxima [A] time = 1.34, size = 71, normalized size = 0.80

$$\frac{2\,(bx + a)^{\frac{9}{2}}}{9\,b^5} - \frac{8\,(bx + a)^{\frac{7}{2}}a}{7\,b^5} + \frac{12\,(bx + a)^{\frac{5}{2}}a^2}{5\,b^5} - \frac{8\,(bx + a)^{\frac{3}{2}}a^3}{3\,b^5} + \frac{2\,\sqrt{bx + a}\,a^4}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x + a)^(9/2)/b^5 - 8/7*(b*x + a)^(7/2)*a/b^5 + 12/5*(b*x + a)^(5/2)*
a^2/b^5 - 8/3*(b*x + a)^(3/2)*a^3/b^5 + 2*sqrt(b*x + a)*a^4/b^5
```

mupad [B] time = 0.02, size = 71, normalized size = 0.80

$$\frac{2\,(a + b\,x)^{9/2}}{9\,b^5} + \frac{2\,a^4\,\sqrt{a + b\,x}}{b^5} - \frac{8\,a^3\,(a + b\,x)^{3/2}}{3\,b^5} + \frac{12\,a^2\,(a + b\,x)^{5/2}}{5\,b^5} - \frac{8\,a\,(a + b\,x)^{7/2}}{7\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(9/2))/(9*b^5) + (2*a^4*(a + b*x)^(1/2))/b^5 - (8*a^3*(a + b*x
)^(3/2))/(3*b^5) + (12*a^2*(a + b*x)^(5/2))/(5*b^5) - (8*a*(a + b*x)^(7/2))
/(7*b^5)
```

sympy [B] time = 4.84, size = 3755, normalized size = 42.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(1/2),x)
```

```
[Out] 256*a**(89/2)*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a
**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**3
5*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**3
2*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(89/
2)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 2432*a**(87/2)*b*x*sqrt(1 + b*x/a)/(31
5*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8
*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*
x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x
**9 + 315*a**30*b**15*x**10) - 2560*a**(87/2)*b*x/(315*a**40*b**5 + 3150*a*
*39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b*
*9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**
12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*
x**10) + 10336*a**(85/2)*b**2*x**2*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a
**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b
**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b*
*12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15
*x**10) - 11520*a**(85/2)*b**2*x**2/(315*a**40*b**5 + 3150*a**39*b**6*x + 1
4175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 7938
0*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 1417
5*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 25840
*a**(83/2)*b**3*x**3*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x +
14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 793
80*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 141
75*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 3072
0*a**(83/2)*b**3*x**3/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**
7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*
x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*
x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 41990*a**(81/2)*b**
4*x**4*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b*
*7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10
*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13
*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 53760*a**(81/2)*b*
*4*x**4/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800
*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a
**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a*
*31*b**14*x**9 + 315*a**30*b**15*x**10) + 46252*a**(79/2)*b**5*x**5*sqrt(1
+ b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 3780
0*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*
a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a
**31*b**14*x**9 + 315*a**30*b**15*x**10) - 64512*a**(79/2)*b**5*x**5/(315*a
**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x*
*3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**
6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9
 + 315*a**30*b**15*x**10) + 35214*a**(77/2)*b**6*x**6*sqrt(1 + b*x/a)/(315*
a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x
**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x*
*6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**
9 + 315*a**30*b**15*x**10) - 53760*a**(77/2)*b**6*x**6/(315*a**40*b**5 + 31
50*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**
36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**3
```

```
3*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b
**15*x**10) + 19632*a**(75/2)*b**7*x**7*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3
150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a*
*36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**
33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*
b**15*x**10) - 30720*a**(75/2)*b**7*x**7/(315*a**40*b**5 + 3150*a**39*b**6*
x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 +
 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 +
 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) +
10860*a**(73/2)*b**8*x**8*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6
*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4
+ 79380*a**35*b**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7
+ 14175*a**32*b**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) -
 11520*a**(73/2)*b**8*x**8/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 9160*a**(71/2)
*b**9*x**9*sqrt(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**3
8*b**7*x**2 + 37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b
**10*x**5 + 66150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b
**13*x**8 + 3150*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 2560*a**(71/2)
*b**9*x**9/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37
800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 6615
0*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150
*a**31*b**14*x**9 + 315*a**30*b**15*x**10) + 8396*a**(69/2)*b**10*x**10*sqr
t(1 + b*x/a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 +
37800*a**37*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66
150*a**34*b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 31
50*a**31*b**14*x**9 + 315*a**30*b**15*x**10) - 256*a**(69/2)*b**10*x**10/(3
15*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b**
8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**11
*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**14*
x**9 + 315*a**30*b**15*x**10) + 5632*a**(67/2)*b**11*x**11*sqrt(1 + b*x/a)/
(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37*b
**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b**
11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b**1
4*x**9 + 315*a**30*b**15*x**10) + 2446*a**(65/2)*b**12*x**12*sqrt(1 + b*x/a
)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**37
*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*b
**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b*
*14*x**9 + 315*a**30*b**15*x**10) + 620*a**(63/2)*b**13*x**13*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10) + 70*a**(61/2)*b**14*x**14*sqrt(1 + b*x/
a)/(315*a**40*b**5 + 3150*a**39*b**6*x + 14175*a**38*b**7*x**2 + 37800*a**3
7*b**8*x**3 + 66150*a**36*b**9*x**4 + 79380*a**35*b**10*x**5 + 66150*a**34*
b**11*x**6 + 37800*a**33*b**12*x**7 + 14175*a**32*b**13*x**8 + 3150*a**31*b
**14*x**9 + 315*a**30*b**15*x**10)
```

3.335 $\int \frac{x^3}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=68

$$-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2(a+bx)^{7/2}}{7b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*a^3*Sqrt[a + b*x])/b^4 + (2*a^2*(a + b*x)^(3/2))/b^4 - (6*a*(a + b*x)^(
5/2))/(5*b^4) + (2*(a + b*x)^(7/2))/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx}} \, dx &= \int \left(-\frac{a^3}{b^3\sqrt{a+bx}}+\frac{3a^2\sqrt{a+bx}}{b^3}-\frac{3a(a+bx)^{3/2}}{b^3}+\frac{(a+bx)^{5/2}}{b^3}\right) dx\\ &= -\frac{2a^3\sqrt{a+bx}}{b^4}+\frac{2a^2(a+bx)^{3/2}}{b^4}-\frac{6a(a+bx)^{5/2}}{5b^4}+\frac{2(a+bx)^{7/2}}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.68

$$\frac{2\sqrt{a+bx}\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 59, normalized size = 0.87

$$-\frac{2\left(35a^3\sqrt{a+bx}-35a^2(a+bx)^{3/2}-5(a+bx)^{7/2}+21a(a+bx)^{5/2}\right)}{35b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*x],x]
```

```
[Out] (-2*(35*a^3*Sqrt[a + b*x] - 35*a^2*(a + b*x)^(3/2) + 21*a*(a + b*x)^(5/2) -
 5*(a + b*x)^(7/2)))/(35*b^4)
```

fricas [A] time = 1.04, size = 42, normalized size = 0.62

$$\frac{2\left(5\,b^{3}x^{3}-6\,ab^{2}x^{2}+8\,a^{2}bx-16\,a^{3}\right)\sqrt{bx+a}}{35\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x + a)/b^4
```

giac [A] time = 1.21, size = 49, normalized size = 0.72

$$\frac{2\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^{2}-35\,\sqrt{bx+a}\,a^{3}\right)}{35\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 3
5*sqrt(b*x + a)*a^3)/b^4
```

maple [A] time = 0.00, size = 43, normalized size = 0.63

$$-\frac{2\sqrt{bx+a}\,\left(-5b^{3}x^{3}+6a\,b^{2}x^{2}-8a^{2}bx+16a^{3}\right)}{35b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.25, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{4}}-\frac{6\,(bx+a)^{\frac{5}{2}}a}{5\,b^{4}}+\frac{2\,(bx+a)^{\frac{3}{2}}a^{2}}{b^{4}}-\frac{2\,\sqrt{bx+a}\,a^{3}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^4 - 6/5*(b*x + a)^(5/2)*a/b^4 + 2*(b*x + a)^(3/2)*a^2
/b^4 - 2*sqrt(b*x + a)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.82

$$\frac{2\,(a+b\,x)^{7/2}}{7\,b^{4}}-\frac{2\,a^{3}\,\sqrt{a+b\,x}}{b^{4}}+\frac{2\,a^{2}\,(a+b\,x)^{3/2}}{b^{4}}-\frac{6\,a\,(a+b\,x)^{5/2}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^4) - (2*a^3*(a + b*x)^(1/2))/b^4 + (2*a^2*(a + b*x
)^(3/2))/b^4 - (6*a*(a + b*x)^(5/2))/(5*b^4)
```

sympy [B] time = 2.70, size = 1640, normalized size = 24.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/2),x)
```

```
[Out] -32*a**(47/2)*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18
*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**
5 + 35*a**14*b**10*x**6) + 32*a**(47/2)/(35*a**20*b**4 + 210*a**19*b**5*x +
 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**1
5*b**9*x**5 + 35*a**14*b**10*x**6) - 176*a**(45/2)*b*x*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(45/2)*b*x/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a*
*17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*
x**6) - 396*a**(43/2)*b**2*x**2*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*
b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 +
210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 480*a**(43/2)*b**2*x**2/(35*a*
*20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 5
25*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 462*a**(4
1/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6) + 640*a**(41/2)*b**3*x**3/(35*a**20*b**4 + 210*a*
*19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**
4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) - 280*a**(39/2)*b**4*x**4*sq
rt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700
*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**
10*x**6) + 480*a**(39/2)*b**4*x**4/(35*a**20*b**4 + 210*a**19*b**5*x + 525*
a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**
9*x**5 + 35*a**14*b**10*x**6) - 42*a**(37/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*
a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 192*a**
(37/2)*b**5*x**5/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 +
700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*
b**10*x**6) + 84*a**(35/2)*b**6*x**6*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a
**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x*
*4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 32*a**(35/2)*b**6*x**6/(3
5*a**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3
 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 94*a*
*(33/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a
**18*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9
*x**5 + 35*a**14*b**10*x**6) + 48*a**(31/2)*b**8*x**8*sqrt(1 + b*x/a)/(35*a
**20*b**4 + 210*a**19*b**5*x + 525*a**18*b**6*x**2 + 700*a**17*b**7*x**3 +
525*a**16*b**8*x**4 + 210*a**15*b**9*x**5 + 35*a**14*b**10*x**6) + 10*a**(2
9/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**20*b**4 + 210*a**19*b**5*x + 525*a**1
8*b**6*x**2 + 700*a**17*b**7*x**3 + 525*a**16*b**8*x**4 + 210*a**15*b**9*x*
*5 + 35*a**14*b**10*x**6)
```

3.336 $\int \frac{x^2}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 45, normalized size = 0.88

$$\frac{2\left(15a^2\sqrt{a+bx}+3(a+bx)^{5/2}-10a(a+bx)^{3/2}\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*(15*a^2*Sqrt[a + b*x] - 10*a*(a + b*x)^(3/2) + 3*(a + b*x)^(5/2)))/(15*b
^3)
```

fricas [A] time = 0.71, size = 31, normalized size = 0.61

$$\frac{2\left(3\,b^{2}x^{2}-4\,abx+8\,a^{2}\right)\sqrt{bx+a}}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

giac [A] time = 1.34, size = 37, normalized size = 0.73

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^{2}\right)}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.63

$$\frac{2\sqrt{bx+a}\,\left(3b^{2}x^{2}-4abx+8a^{2}\right)}{15b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

maxima [A] time = 1.32, size = 41, normalized size = 0.80

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^{3}}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^{3}}+\frac{2\,\sqrt{bx+a}\,a^{2}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,{\left(a+b\,x\right)}^{5/2}-20\,a\,{\left(a+b\,x\right)}^{3/2}+30\,a^2\,\sqrt{a+b\,x}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/2),x)
```

```
[Out] (6*(a + b*x)^(5/2) - 20*a*(a + b*x)^(3/2) + 30*a^2*(a + b*x)^(1/2))/(15*b^3
)
```

sympy [B] time = 1.77, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{19}{2}}bx}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{30a^{\frac{17}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{48a^{\frac{17}{2}}b^2x^2}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{15}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}-\frac{16a^{\frac{15}{2}}b^3x^3}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{10a^{\frac{13}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}+\frac{6a^{\frac{11}{2}}b^5x^5\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x+a)**(1/2),x)

[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)

3.337 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx&=\int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\&=-\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a-bx)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*(2*a - b*x)*Sqrt[a + b*x])/(3*b^2)
```

fricas [A] time = 0.95, size = 19, normalized size = 0.59

$$\frac{2\sqrt{bx+a}\,(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

giac [A] time = 1.01, size = 23, normalized size = 0.72

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.66

$$-\frac{2\sqrt{bx+a}\,\left(-bx+2a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

maxima [A] time = 1.32, size = 26, normalized size = 0.81

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2}-\frac{2\,\sqrt{bx+a}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.78

$$-\frac{6\,a\,\sqrt{a+b\,x}-2\,{(a+b\,x)}^{3/2}}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/2),x)
```

```
[Out] -(6*a*(a + b*x)^(1/2) - 2*(a + b*x)^(3/2))/(3*b^2)
```

sympy [B] time = 1.16, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

3.338 $\int \frac{1}{\sqrt{a+bx}} dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

fricas [A] time = 0.94, size = 12, normalized size = 0.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

giac [A] time = 0.99, size = 12, normalized size = 0.86

$$\frac{2 \sqrt{bx + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{bx + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

maxima [A] time = 1.30, size = 12, normalized size = 0.86

$$\frac{2 \sqrt{bx + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{2 \sqrt{a + b\, x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/2),x)
```

```
[Out] (2*(a + b*x)^(1/2))/b
```

sympy [A] time = 0.07, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a + bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

3.339 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{63, 208\}$

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.12, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

giac [A] time = 1.24, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.01, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 2.94, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 0.06, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.11, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.340 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{ax}-\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}\\
&= -\frac{\sqrt{a+bx}}{ax}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a}\\
&= -\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.15

$$\frac{\sqrt{a+bx}\left(\frac{b\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{\frac{bx}{a}+1}}-\frac{a}{x}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(-(a/x) + (b*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[1 + (b*x)/a]))
/a^2
```

IntegrateAlgebraic [A] time = 0.05, size = 41, normalized size = 1.00

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.86, size = 93, normalized size = 2.27

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x},-\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

giac [A] time = 1.07, size = 47, normalized size = 1.15

$$-\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

maple [A] time = 0.01, size = 40, normalized size = 0.98

$$2\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/x/b+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))
```

maxima [A] time = 3.00, size = 60, normalized size = 1.46

$$-\frac{\sqrt{bx+a}\,b}{(bx+a)a-a^2}-\frac{b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + a)*b/((b*x + a)*a - a^2) - 1/2*b*log((sqrt(b*x + a) - sqrt(a))/
(sqrt(b*x + a) + sqrt(a)))/a^(3/2)
```

mupad [B] time = 0.11, size = 33, normalized size = 0.80

$$\frac{b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+b\,x}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/2)),x)
```

```
[Out] (b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2) - (a + b*x)^(1/2)/(a*x)
```

sympy [A] time = 2.30, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

3.341 $\int \frac{1}{x^3\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{\sqrt{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a + b*x]),x]
```

```
[Out] -Sqrt[a + b*x]/(2*a*x^2) + (3*b*Sqrt[a + b*x])/(4*a^2*x) - (3*b^2*ArcTanh[S
qrt[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{2ax^2}-\frac{(3b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a}\\ &= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{\left(3b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^2}\\ &= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{4a^2}\\ &= -\frac{\sqrt{a+bx}}{2ax^2}+\frac{3b\sqrt{a+bx}}{4a^2x}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.49

$$-\frac{2b^2\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b*x]),x]
[Out] (-2*b^2*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/a^3
```

IntegrateAlgebraic [A] time = 0.08, size = 63, normalized size = 0.93

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}-\frac{5a\sqrt{a+bx}-3(a+bx)^{3/2}}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*x]),x]
[Out] -1/4*(5*a*Sqrt[a + b*x] - 3*(a + b*x)^(3/2))/(a^2*x^2) - (3*b^2*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

fricas [A] time = 1.12, size = 123, normalized size = 1.81

$$\left[\frac{3\sqrt{a}b^2x^2\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3abx-2a^2\right)\sqrt{bx+a}}{8a^3x^2},\ \frac{3\sqrt{-a}b^2x^2\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx-2a^2\right)\sqrt{bx+a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [1/8*(3*sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(3
*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2), 1/4*(3*sqrt(-a)*b^2*x^2*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + (3*a*b*x - 2*a^2)*sqrt(b*x + a))/(a^3*x^2)]
```

giac [A] time = 1.19, size = 69, normalized size = 1.01

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3(bx+a)^{\frac{3}{2}}b^3-5\sqrt{bx+a}ab^3}{a^2b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x + a)^(3/
2)*b^3 - 5*sqrt(b*x + a)*a*b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.01, size = 66, normalized size = 0.97

$$2\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)}{4a}-\frac{\sqrt{bx+a}}{4a\,b^2x^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^2*(-1/4*(b*x+a)^(1/2)/a/x^2/b^2-3/4/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arc
tanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)))
```

maxima [A] time = 2.92, size = 92, normalized size = 1.35

$$\frac{3\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{3}{2}}b^2-5\,\sqrt{bx+a}\,ab^2}{4\left((bx+a)^2a^2-2\,(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) +
1/4*(3*(b*x + a)^(3/2)*b^2 - 5*sqrt(b*x + a)*a*b^2)/((b*x + a)^2*a^2 - 2*(b
*x + a)*a^3 + a^4)
```

mupad [B] time = 0.06, size = 51, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{3/2}}{4\,a^2\,x^2}-\frac{5\,\sqrt{a+b\,x}}{4\,a\,x^2}-\frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/2)),x)
```

```
[Out] (3*(a + b*x)^(3/2))/(4*a^2*x^2) - (5*(a + b*x)^(1/2))/(4*a*x^2) - (3*b^2*at
anh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(5/2))
```

sympy [A] time = 4.37, size = 102, normalized size = 1.50

$$-\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{3b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/2),x)
```

```
[Out] -1/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(4*a*x**(3/2)*sqrt(a/(b
*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) + 1)) - 3*b**2*asinh(sq
rt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2))
```

3.342 $\int \frac{1}{x^4 \sqrt{a+bx}} \, dx$

Optimal. Leaf size=90

$$\frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{\sqrt{a+bx}}{3ax^3}$$

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b^3 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{\sqrt{a+bx}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -Sqrt[a + b*x]/(3*a*x^3) + (5*b*Sqrt[a + b*x])/(12*a^2*x^2) - (5*b^2*Sqrt[a
 + b*x])/(8*a^3*x) + (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{a+bx}}\,dx = -\frac{\sqrt{a+bx}}{3ax^3} - \frac{(5b)\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{6a}$$
$$= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} + \frac{(5b^2)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{8a^2}$$
$$= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^3)\int \frac{1}{x\sqrt{a+bx}}\,dx}{16a^3}$$
$$= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} - \frac{(5b^2)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{8a^3}$$
$$= -\frac{\sqrt{a+bx}}{3ax^3} + \frac{5b\sqrt{a+bx}}{12a^2x^2} - \frac{5b^2\sqrt{a+bx}}{8a^3x} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.37

$$\frac{2b^3\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] (2*b^3*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/a^4
```

IntegrateAlgebraic [A] time = 0.07, size = 71, normalized size = 0.79

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{8a^{7/2}} - \frac{\sqrt{a+bx}\left(33a^2-40a(a+bx)+15(a+bx)^2\right)}{24a^3x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^4*Sqrt[a + b*x]),x]
```

```
[Out] -1/24*(Sqrt[a + b*x]*(33*a^2 - 40*a*(a + b*x) + 15*(a + b*x)^2))/(a^3*x^3)
+ (5*b^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(8*a^(7/2))
```

fricas [A] time = 0.91, size = 145, normalized size = 1.61

$$\left[\frac{15\sqrt{a}b^3x^3\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^2x^2-10a^2bx+8a^3\right)\sqrt{bx+a}}{48a^4x^3}, -\frac{15\sqrt{-a}b^3x^3\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15ab^2x^2-10a^2bx+8a^3\right)\sqrt{bx+a}}{24a^4x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^3*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*
(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x + a))/(a^4*x^3), -1/24*(15*sqr
t(-a)*b^3*x^3*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 - 10*a^2*b*x
 + 8*a^3)*sqrt(b*x + a))/(a^4*x^3)]
```

giac [A] time = 0.91, size = 84, normalized size = 0.93

$$-\frac{\frac{15b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15(bx+a)^{\frac{5}{2}}b^4-40(bx+a)^{\frac{3}{2}}ab^4+33\sqrt{bx+a}\,a^2b^4}{a^3b^3x^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(15*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x + a)
^(5/2)*b^4 - 40*(b*x + a)^(3/2)*a*b^4 + 33*sqrt(b*x + a)*a^2*b^4)/(a^3*b^3*
x^3))/b
```

maple [A] time = 0.01, size = 90, normalized size = 1.00

$$2\left(-\frac{5\left(-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}}-\frac{\sqrt{bx+a}}{2abx}\right)}{4a}-\frac{\sqrt{bx+a}}{4a\,b^{2}x^{2}}\right)}{6a}-\frac{\sqrt{bx+a}}{6a\,b^{3}x^{3}}\right)b^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x+a)^(1/2),x)
```

```
[Out] 2*b^3*(-1/6*(b*x+a)^(1/2)/a/x^3/b^3-5/6/a*(-1/4*(b*x+a)^(1/2)/a/b^2/x^2-3/4
/a*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2))))
```

maxima [A] time = 2.92, size = 121, normalized size = 1.34

$$-\frac{5\,b^{3}\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{16\,a^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{5}{2}}b^{3}-40\,(bx+a)^{\frac{3}{2}}ab^{3}+33\,\sqrt{bx+a}\,a^{2}b^{3}}{24\left((bx+a)^{3}a^{3}-3\,(bx+a)^{2}a^{4}+3\,(bx+a)a^{5}-a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*b^3*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
- 1/24*(15*(b*x + a)^(5/2)*b^3 - 40*(b*x + a)^(3/2)*a*b^3 + 33*sqrt(b*x + a
)*a^2*b^3)/((b*x + a)^3*a^3 - 3*(b*x + a)^2*a^4 + 3*(b*x + a)*a^5 - a^6)
```

mupad [B] time = 0.05, size = 69, normalized size = 0.77

$$\frac{5\,(a+b\,x)^{3/2}}{3\,a^{2}\,x^{3}}-\frac{11\,\sqrt{a+b\,x}}{8\,a\,x^{3}}-\frac{5\,(a+b\,x)^{5/2}}{8\,a^{3}\,x^{3}}-\frac{b^{3}\operatorname{atan}\left(\frac{\sqrt{a+b\,x}\,1\mathrm{i}}{\sqrt{a}}\right)5\mathrm{i}}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*x)^(1/2)),x)
```

```
[Out] (5*(a + b*x)^(3/2))/(3*a^2*x^3) - (11*(a + b*x)^(1/2))/(8*a*x^3) - (5*(a +
b*x)^(5/2))/(8*a^3*x^3) - (b^3*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*5i)/(8*a^
(7/2))
```

sympy [A] time = 7.02, size = 129, normalized size = 1.43

$$-\frac{1}{3\sqrt{b}\,x^{\frac{7}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{\sqrt{b}}{12ax^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{3}{2}}}{24a^{2}x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{5b^{\frac{5}{2}}}{8a^{3}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{5b^{3}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x+a)**(1/2),x)
```

```
[Out] -1/(3*sqrt(b)*x**(7/2)*sqrt(a/(b*x) + 1)) + sqrt(b)/(12*a*x**(5/2)*sqrt(a/(
b*x) + 1)) - 5*b**(3/2)/(24*a**2*x**(3/2)*sqrt(a/(b*x) + 1)) - 5*b**(5/2)/(
8*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) + 5*b**3*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/(8*a**(7/2))
```

3.343 $\int \frac{x^4}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^4)/(b^5*Sqrt[a + b*x]) - (8*a^3*Sqrt[a + b*x])/b^5 + (4*a^2*(a + b*x)
^(3/2))/b^5 - (8*a*(a + b*x)^(5/2))/(5*b^5) + (2*(a + b*x)^(7/2))/(7*b^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{3/2}}\,dx &= \int \left(\frac{a^4}{b^4(a+bx)^{3/2}}-\frac{4a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{4a(a+bx)^{3/2}}{b^4}+\frac{(a+bx)^{5/2}}{b^4}\right)dx\\ &= -\frac{2a^4}{b^5\sqrt{a+bx}}-\frac{8a^3\sqrt{a+bx}}{b^5}+\frac{4a^2(a+bx)^{3/2}}{b^5}-\frac{8a(a+bx)^{5/2}}{5b^5}+\frac{2(a+bx)^{7/2}}{7b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.67

$$\frac{2\left(-128a^4-64a^3bx+16a^2b^2x^2-8ab^3x^3+5b^4x^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-128*a^4 - 64*a^3*b*x + 16*a^2*b^2*x^2 - 8*a*b^3*x^3 + 5*b^4*x^4))/(35*
b^5*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 63, normalized size = 0.74

$$\frac{2\left(-35a^4-140a^3(a+bx)+70a^2(a+bx)^2-28a(a+bx)^3+5(a+bx)^4\right)}{35b^5\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-35*a^4 - 140*a^3*(a + b*x) + 70*a^2*(a + b*x)^2 - 28*a*(a + b*x)^3 + 5
*(a + b*x)^4))/(35*b^5*Sqrt[a + b*x])
```

fricas [A] time = 0.95, size = 63, normalized size = 0.74

$$\frac{2\left(5\,b^{4}x^{4}-8\,ab^{3}x^{3}+16\,a^{2}b^{2}x^{2}-64\,a^{3}bx-128\,a^{4}\right)\sqrt{bx+a}}{35\left(b^{6}x+ab^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*sqrt
(b*x + a)/(b^6*x + a*b^5)
```

giac [A] time = 1.25, size = 77, normalized size = 0.91

$$-\frac{2\,a^{4}}{\sqrt{bx+a}\,b^{5}}+\frac{2\left(5\,(bx+a)^{\frac{7}{2}}b^{30}-28\,(bx+a)^{\frac{5}{2}}ab^{30}+70\,(bx+a)^{\frac{3}{2}}a^{2}b^{30}-140\,\sqrt{bx+a}\,a^{3}b^{30}\right)}{35\,b^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*a^4/(sqrt(b*x + a)*b^5) + 2/35*(5*(b*x + a)^(7/2)*b^30 - 28*(b*x + a)^(5
/2)*a*b^30 + 70*(b*x + a)^(3/2)*a^2*b^30 - 140*sqrt(b*x + a)*a^3*b^30)/b^35
```

maple [A] time = 0.01, size = 54, normalized size = 0.64

$$-\frac{2\left(-5x^{4}b^{4}+8a\,x^{3}b^{3}-16a^{2}x^{2}b^{2}+64a^{3}xb+128a^{4}\right)}{35\sqrt{bx+a}\;b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(3/2),x)
```

```
[Out] -2/35/(b*x+a)^(1/2)*(-5*b^4*x^4+8*a*b^3*x^3-16*a^2*b^2*x^2+64*a^3*b*x+128*a
^4)/b^5
```

maxima [A] time = 1.33, size = 71, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^{5}}-\frac{8\,(bx+a)^{\frac{5}{2}}a}{5\,b^{5}}+\frac{4\,(bx+a)^{\frac{3}{2}}a^{2}}{b^{5}}-\frac{8\,\sqrt{bx+a}\,a^{3}}{b^{5}}-\frac{2\,a^{4}}{\sqrt{bx+a}\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^5 - 8/5*(b*x + a)^(5/2)*a/b^5 + 4*(b*x + a)^(3/2)*a^2
/b^5 - 8*sqrt(b*x + a)*a^3/b^5 - 2*a^4/(sqrt(b*x + a)*b^5)
```

mupad [B] time = 0.03, size = 71, normalized size = 0.84

$$\frac{2\,(a+b\,x)^{7/2}}{7\,b^{5}}-\frac{8\,a^{3}\,\sqrt{a+b\,x}}{b^{5}}+\frac{4\,a^{2}\,(a+b\,x)^{3/2}}{b^{5}}-\frac{2\,a^{4}}{b^{5}\,\sqrt{a+b\,x}}-\frac{8\,a\,(a+b\,x)^{5/2}}{5\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(7/2))/(7*b^5) - (8*a^3*(a + b*x)^(1/2))/b^5 + (4*a^2*(a + b*x
)^(3/2))/b^5 - (2*a^4)/(b^5*(a + b*x)^(1/2)) - (8*a*(a + b*x)^(5/2))/(5*b^5
)
```

sympy [B] time = 4.81, size = 3606, normalized size = 42.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(3/2),x)
```

```
[Out] -256*a**(87/2)*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**
38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**
10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*
x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(87/2)/(35*a**
40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 +
7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200
*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30
*b**15*x**10) - 2432*a**(85/2)*b*x*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**
39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x
**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 2
560*a**(85/2)*b*x/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2
+ 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 735
0*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a*
*31*b**14*x**9 + 35*a**30*b**15*x**10) - 10336*a**(83/2)*b**2*x**2*sqrt(1 +
 b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**
37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b*
*11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*
x**9 + 35*a**30*b**15*x**10) + 11520*a**(83/2)*b**2*x**2/(35*a**40*b**5 + 3
50*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*
b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**1
2*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**1
0) - 25840*a**(81/2)*b**3*x**3*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b
**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4
+ 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1
575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 30720
*a**(81/2)*b**3*x**3/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x*
*2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 +
7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350
*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 41990*a**(79/2)*b**4*x**4*sqrt(
1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*
a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34
*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**
14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(79/2)*b**4*x**4/(35*a**40*b**5
+ 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**
36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b
**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x
**10) - 46182*a**(77/2)*b**5*x**5*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**3
9*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x*
*4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7
+ 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 64
512*a**(77/2)*b**5*x**5/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7
*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5
 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 +
350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 34584*a**(75/2)*b**6*x**6*sq
rt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 42
00*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a*
*34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*
b**14*x**9 + 35*a**30*b**15*x**10) + 53760*a**(75/2)*b**6*x**6/(35*a**40*b*
```

```
*5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*
a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**3
3*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**1
5*x**10) - 17112*a**(73/2)*b**7*x**7*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 30720*a**(73/2)*b**7*x**7/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b
**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x
**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) - 4980*a**(71/2)*b**8*x**8*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 11520*a**(71/2)*b**8*x**8/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) - 340*a**(69/2)*b**9*x**9*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a
**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9
*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x*
*7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) +
 2560*a**(69/2)*b**9*x**9/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b*
*7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x*
*5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8
+ 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 424*a**(67/2)*b**10*x**10*
sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 +
4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*
a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**3
1*b**14*x**9 + 35*a**30*b**15*x**10) + 256*a**(67/2)*b**10*x**10/(35*a**40*
b**5 + 350*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 735
0*a**36*b**9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a*
*33*b**12*x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b*
*15*x**10) + 248*a**(65/2)*b**11*x**11*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350
*a**39*b**6*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b*
*9*x**4 + 8820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*
x**7 + 1575*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
 + 74*a**(63/2)*b**12*x**12*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6
*x + 1575*a**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8
820*a**35*b**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575
*a**32*b**13*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10) + 10*a**(6
1/2)*b**13*x**13*sqrt(1 + b*x/a)/(35*a**40*b**5 + 350*a**39*b**6*x + 1575*a
**38*b**7*x**2 + 4200*a**37*b**8*x**3 + 7350*a**36*b**9*x**4 + 8820*a**35*b
**10*x**5 + 7350*a**34*b**11*x**6 + 4200*a**33*b**12*x**7 + 1575*a**32*b**1
3*x**8 + 350*a**31*b**14*x**9 + 35*a**30*b**15*x**10)
```

3.344 $\int \frac{x^3}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*a^3)/(b^4*Sqrt[a + b*x]) + (6*a^2*Sqrt[a + b*x])/b^4 - (2*a*(a + b*x)^(3
/2))/b^4 + (2*(a + b*x)^(5/2))/(5*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{3/2}}+\frac{3a^2}{b^3\sqrt{a+bx}}-\frac{3a\sqrt{a+bx}}{b^3}+\frac{(a+bx)^{3/2}}{b^3}\right)dx\\ &= \frac{2a^3}{b^4\sqrt{a+bx}}+\frac{6a^2\sqrt{a+bx}}{b^4}-\frac{2a(a+bx)^{3/2}}{b^4}+\frac{2(a+bx)^{5/2}}{5b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.68

$$\frac{2\left(16a^3+8a^2bx-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.02, size = 49, normalized size = 0.74

$$\frac{2\left(5a^3+15a^2(a+bx)-5a(a+bx)^2+(a+bx)^3\right)}{5b^4\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(3/2),x]
```

```
[Out] (2*(5*a^3 + 15*a^2*(a + b*x) - 5*a*(a + b*x)^2 + (a + b*x)^3))/(5*b^4*Sqrt[
a + b*x])
```

fricas [A] time = 0.95, size = 51, normalized size = 0.77

$$\frac{2\left(b^{3}x^{3} - 2\,ab^{2}x^{2} + 8\,a^{2}bx + 16\,a^{3}\right)\sqrt{bx + a}}{5\left(b^{5}x + ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x + a)/(b^5*x + a*b
^4)
```

giac [A] time = 1.10, size = 61, normalized size = 0.92

$$\frac{2\,a^{3}}{\sqrt{bx + a}\,b^{4}} + \frac{2\left((bx + a)^{\frac{5}{2}}b^{16} - 5\,(bx + a)^{\frac{3}{2}}ab^{16} + 15\,\sqrt{bx + a}\,a^{2}b^{16}\right)}{5\,b^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a^3/(sqrt(b*x + a)*b^4) + 2/5*((b*x + a)^(5/2)*b^16 - 5*(b*x + a)^(3/2)*a
*b^16 + 15*sqrt(b*x + a)*a^2*b^16)/b^20
```

maple [A] time = 0.01, size = 42, normalized size = 0.64

$$\frac{\frac{2}{5}b^{3}x^{3} - \frac{4}{5}a\,b^{2}x^{2} + \frac{16}{5}a^{2}bx + \frac{32}{5}a^{3}}{b^{4}\sqrt{bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2/5/(b*x+a)^(1/2)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.38, size = 56, normalized size = 0.85

$$\frac{2\,(bx + a)^{\frac{5}{2}}}{5\,b^{4}} - \frac{2\,(bx + a)^{\frac{3}{2}}a}{b^{4}} + \frac{6\,\sqrt{bx + a}\,a^{2}}{b^{4}} + \frac{2\,a^{3}}{\sqrt{bx + a}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^4 - 2*(b*x + a)^(3/2)*a/b^4 + 6*sqrt(b*x + a)*a^2/b^4
 + 2*a^3/(sqrt(b*x + a)*b^4)
```

mupad [B] time = 0.05, size = 56, normalized size = 0.85

$$\frac{2\,(a + b\,x)^{5/2}}{5\,b^{4}} + \frac{6\,a^{2}\,\sqrt{a + b\,x}}{b^{4}} + \frac{2\,a^{3}}{b^{4}\,\sqrt{a + b\,x}} - \frac{2\,a\,(a + b\,x)^{3/2}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(3/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^4) + (6*a^2*(a + b*x)^(1/2))/b^4 + (2*a^3)/(b^4*(a
 + b*x)^(1/2)) - (2*a*(a + b*x)^(3/2))/b^4
```

sympy [B] time = 2.94, size = 1538, normalized size = 23.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(3/2),x)
```

```
[Out] 32*a**(45/2)*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**
6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*
a**14*b**10*x**6) - 32*a**(45/2)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18
*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5
+ 5*a**14*b**10*x**6) + 176*a**(43/2)*b*x*sqrt(1 + b*x/a)/(5*a**20*b**4 + 3
0*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x
**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 192*a**(43/2)*b*x/(5*a**20
*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**
16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 396*a**(41/2)*b**
2*x**2*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) - 480*a**(41/2)*b**2*x**2/(5*a**20*b**4 + 30*a**19*b**5*x + 75*
a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*
x**5 + 5*a**14*b**10*x**6) + 462*a**(39/2)*b**3*x**3*sqrt(1 + b*x/a)/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 640*a**(39/2)*b
**3*x**3/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b
**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) +
290*a**(37/2)*b**4*x**4*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 7
5*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**
9*x**5 + 5*a**14*b**10*x**6) - 480*a**(37/2)*b**4*x**4/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 92*a**(35/2)*b**5*x**5*sqrt(1
 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*
b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) -
 192*a**(35/2)*b**5*x**5/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x*
*2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**1
4*b**10*x**6) + 16*a**(33/2)*b**6*x**6*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a
**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4
 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) - 32*a**(33/2)*b**6*x**6/(5*a**
20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a
**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*b**10*x**6) + 6*a**(31/2)*b**
7*x**7*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**19*b**5*x + 75*a**18*b**6*x**2
 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 + 30*a**15*b**9*x**5 + 5*a**14*
b**10*x**6) + 2*a**(29/2)*b**8*x**8*sqrt(1 + b*x/a)/(5*a**20*b**4 + 30*a**1
9*b**5*x + 75*a**18*b**6*x**2 + 100*a**17*b**7*x**3 + 75*a**16*b**8*x**4 +
30*a**15*b**9*x**5 + 5*a**14*b**10*x**6)
```

3.345 $\int \frac{x^2}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (-2*a^2)/(b^3*Sqrt[a + b*x]) - (4*a*Sqrt[a + b*x])/b^3 + (2*(a + b*x)^(3/2)
)/(3*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{3/2}}\,dx&=\int \left(\frac{a^2}{b^2(a+bx)^{3/2}}-\frac{2a}{b^2\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b^2}\right)dx\\&=-\frac{2a^2}{b^3\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{3/2}}{3b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.69

$$\frac{2\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.02, size = 37, normalized size = 0.76

$$\frac{2\left(-3a^2-6a(a+bx)+(a+bx)^2\right)}{3b^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2 - 6*a*(a + b*x) + (a + b*x)^2))/(3*b^3*Sqrt[a + b*x])
```

fricas [A] time = 0.84, size = 40, normalized size = 0.82

$$\frac{2\left(b^2x^2 - 4\,abx - 8\,a^2\right)\sqrt{bx+a}}{3\left(b^4x + ab^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x + a)/(b^4*x + a*b^3)
```

giac [A] time = 1.14, size = 46, normalized size = 0.94

$$-\frac{2\,a^2}{\sqrt{bx+a}\,b^3} + \frac{2\left((bx+a)^{\frac{3}{2}}b^6 - 6\,\sqrt{bx+a}\,ab^6\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="giac")
[Out] -2*a^2/(sqrt(b*x + a)*b^3) + 2/3*((b*x + a)^(3/2)*b^6 - 6*sqrt(b*x + a)*a*b
^6)/b^9
```

maple [A] time = 0.01, size = 32, normalized size = 0.65

$$-\frac{2\left(-b^2x^2 + 4abx + 8a^2\right)}{3\sqrt{bx+a}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(3/2),x)
[Out] -2/3/(b*x+a)^(1/2)*(-b^2*x^2+4*a*b*x+8*a^2)/b^3
```

maxima [A] time = 1.29, size = 41, normalized size = 0.84

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^3} - \frac{4\,\sqrt{bx+a}\,a}{b^3} - \frac{2\,a^2}{\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 2/3*(b*x + a)^(3/2)/b^3 - 4*sqrt(b*x + a)*a/b^3 - 2*a^2/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$-\frac{12\,a\,\left(a+b\,x\right) - 2\,{\left(a+b\,x\right)}^2 + 6\,a^2}{3\,b^3\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(3/2),x)
[Out] -(12*a*(a + b*x) - 2*(a + b*x)^2 + 6*a^2)/(3*b^3*(a + b*x)^(1/2))
```

sympy [B] time = 1.83, size = 534, normalized size = 10.90

$$-\frac{16a^{\frac{19}{2}}\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{16a^{\frac{19}{2}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{40a^{\frac{17}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{48a^{\frac{17}{2}}bx}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{30a^{\frac{15}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{48a^{\frac{15}{2}}b^2x^2}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} - \frac{4a^{\frac{13}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{16a^{\frac{13}{2}}b^3x^3}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3} + \frac{2a^{\frac{11}{2}}b^4x^4\sqrt{1+\frac{bx}{a}}}{3a^8b^3+9a^7b^4x+9a^6b^5x^2+3a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(3/2),x)
```

```
[Out] -16*a**(19/2)*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x*
*2 + 3*a**5*b**6*x**3) + 16*a**(19/2)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6
*b**5*x**2 + 3*a**5*b**6*x**3) - 40*a**(17/2)*b*x*sqrt(1 + b*x/a)/(3*a**8*b
**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(17/2)*b
*x/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 30
*a**(15/2)*b**2*x**2*sqrt(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*
b**5*x**2 + 3*a**5*b**6*x**3) + 48*a**(15/2)*b**2*x**2/(3*a**8*b**3 + 9*a**
7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3) - 4*a**(13/2)*b**3*x**3*sqr
t(1 + b*x/a)/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*
x**3) + 16*a**(13/2)*b**3*x**3/(3*a**8*b**3 + 9*a**7*b**4*x + 9*a**6*b**5*x
**2 + 3*a**5*b**6*x**3) + 2*a**(11/2)*b**4*x**4*sqrt(1 + b*x/a)/(3*a**8*b**
3 + 9*a**7*b**4*x + 9*a**6*b**5*x**2 + 3*a**5*b**6*x**3)
```

3.346 $\int \frac{x}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*a)/(b^2*Sqrt[a + b*x]) + (2*Sqrt[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{3/2}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{3/2}}+\frac{1}{b\sqrt{a+bx}}\right)dx\\ &= \frac{2a}{b^2\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 0.70

$$\frac{2(2a+bx)}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(3/2),x]
```

```
[Out] (2*(2*a + b*x))/(b^2*Sqrt[a + b*x])
```

fricas [A] time = 0.91, size = 29, normalized size = 0.97

$$\frac{2\,(bx+2\,a)\sqrt{bx+a}}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x + 2*a)*sqrt(b*x + a)/(b^3*x + a*b^2)
```

giac [A] time = 1.00, size = 29, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{bx+a}}{b}+\frac{a}{\sqrt{bx+a}\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(sqrt(b*x + a)/b + a/(sqrt(b*x + a)*b))/b
```

maple [A] time = 0.00, size = 20, normalized size = 0.67

$$\frac{2bx+4a}{b^2\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(3/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)*(b*x+2*a)/b^2
```

maxima [A] time = 1.32, size = 26, normalized size = 0.87

$$\frac{2\sqrt{bx+a}}{b^2}+\frac{2\,a}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^2 + 2*a/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 0.09, size = 19, normalized size = 0.63

$$\frac{4\,a+2\,b\,x}{b^2\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(3/2),x)
```

```
[Out] (4*a + 2*b*x)/(b^2*(a + b*x)^(1/2))
```

sympy [A] time = 0.67, size = 37, normalized size = 1.23

$$\begin{cases} \frac{4a}{b^2\sqrt{a+bx}}+\frac{2x}{b\sqrt{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(3/2),x)
```

```
[Out] Piecewise((4*a/(b**2*sqrt(a + b*x)) + 2*x/(b*sqrt(a + b*x)), Ne(b, 0)), (x*
*2/(2*a**(3/2)), True))
```

3.347 $\int \frac{1}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=14

$$-\frac{2}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3/2),x]
[Out] -2/(b*Sqrt[a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}} dx = -\frac{2}{b\sqrt{a+bx}}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3/2),x]
[Out] -2/(b*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3/2),x]
[Out] -2/(b*Sqrt[a + b*x])
```

fricas [A] time = 0.74, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{bx+a}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(b^2*x + a*b)
```

giac [A] time = 1.24, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(b*x + a)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{2}{\sqrt{bx+a}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2),x)
[Out] -2/b/(b*x+a)^(1/2)
```

maxima [A] time = 1.34, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{b\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(3/2),x)
[Out] -2/(b*(a + b*x)^(1/2))
```

sympy [A] time = 0.07, size = 12, normalized size = 0.86

$$-\frac{2}{b\sqrt{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2),x)
[Out] -2/(b*sqrt(a + b*x))
```

3.348 $\int \frac{1}{x(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{a+bx}}+\frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{a+bx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{ab}\\ &= \frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.79

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(3/2)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{2}{a\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(3/2)),x]
[Out] 2/(a*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 1.03, size = 110, normalized size = 2.89

$$\left[\frac{(bx+a)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,a}{a^2bx+a^3},\ \frac{2\left((bx+a)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,a\right)}{a^2bx+a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] [((b*x + a)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b
*x + a)*a)/(a^2*b*x + a^3), 2*((b*x + a)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + sqrt(b*x + a)*a)/(a^2*b*x + a^3)]
```

giac [A] time = 1.07, size = 37, normalized size = 0.97

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + 2/(sqrt(b*x + a)*a)
```

maple [A] time = 0.01, size = 31, normalized size = 0.82

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(3/2),x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(3/2)+2/a/(b*x+a)^(1/2)
```

maxima [A] time = 2.93, size = 45, normalized size = 1.18

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{2}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(3/2) + 2/(sqrt(
b*x + a)*a)
```

mupad [B] time = 0.04, size = 30, normalized size = 0.79

$$\frac{2}{a\sqrt{a+b\,x}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(3/2)),x)
```

```
[Out] 2/(a*(a + b*x)^(1/2)) - (2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 1.87, size = 146, normalized size = 3.84

$$\frac{2a^3\sqrt{1+\frac{bx}{a}}}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{a^{\frac{9}{2}}+a^{\frac{7}{2}}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(3/2),x)
```

```
[Out] 2*a**3*sqrt(1 + b*x/a)/(a**(9/2) + a**(7/2)*b*x) + a**3*log(b*x/a)/(a**(9/2
) + a**(7/2)*b*x) - 2*a**3*log(sqrt(1 + b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*
x) + a**2*b*x*log(b*x/a)/(a**(9/2) + a**(7/2)*b*x) - 2*a**2*b*x*log(sqrt(1
+ b*x/a) + 1)/(a**(9/2) + a**(7/2)*b*x)
```

3.349 $\int \frac{1}{x^2(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{a+bx}}-\frac{1}{ax\sqrt{a+bx}}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{a+bx}}{a^2x}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{ax\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x*Sqrt[a + b*x]) - (3*Sqrt[a + b*x])/(a^2*x) + (3*b*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{3/2}}\,dx = \frac{2}{ax\sqrt{a+bx}} + \frac{3\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} - \frac{(3b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^2}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} - \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a^2}$$
$$= \frac{2}{ax\sqrt{a+bx}} - \frac{3\sqrt{a+bx}}{a^2x} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.54

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.07, size = 52, normalized size = 0.91

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a-3(a+bx)}{a^2x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(3/2)),x]
```

```
[Out] (2*a - 3*(a + b*x))/(a^2*x*Sqrt[a + b*x]) + (3*b*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]])/a^(5/2)
```

fricas [A] time = 1.17, size = 151, normalized size = 2.65

$$\left[\frac{3\left(b^2x^2+abx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(3abx+a^2\right)\sqrt{bx+a}}{2\left(a^3bx^2+a^4x\right)}, -\frac{3\left(b^2x^2+abx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3abx+a^2\right)\sqrt{bx+a}}{a^3bx^2+a^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^2 + a*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)
/x) - 2*(3*a*b*x + a^2)*sqrt(b*x + a))/(a^3*b*x^2 + a^4*x), -(3*(b^2*x^2 +
a*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*a*b*x + a^2)*sqrt(b*x
 + a))/(a^3*b*x^2 + a^4*x)]
```

giac [A] time = 0.90, size = 64, normalized size = 1.12

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} - \frac{3\,(bx+a)b-2\,ab}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) - (3*(b*x + a)*b - 2*a*b
)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)*a^2)
```

maple [A] time = 0.01, size = 55, normalized size = 0.96

$$2\left(-\frac{1}{\sqrt{bx+a}\,a^2}-\frac{-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\sqrt{bx+a}}{2bx}}{a^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(3/2),x)
```

```
[Out] 2*b*(-1/a^2/(b*x+a)^(1/2)-1/a^2*(1/2*(b*x+a)^(1/2)/b/x-3/2*arctanh((b*x+a)^
(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 3.01, size = 76, normalized size = 1.33

$$-\frac{3\,(bx+a)b-2\,ab}{(bx+a)^{\frac{3}{2}}a^2-\sqrt{bx+a}\,a^3}-\frac{3\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x + a)*b - 2*a*b)/((b*x + a)^(3/2)*a^2 - sqrt(b*x + a)*a^3) - 3/2*b*
log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2)
```

mupad [B] time = 0.12, size = 60, normalized size = 1.05

$$\frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,b}{a}-\frac{3\,b\,(a+b\,x)}{a^2}}{a\,\sqrt{a+b\,x}-(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(3/2)),x)
```

```
[Out] (3*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(5/2) - ((2*b)/a - (3*b*(a + b*x))/a
^2)/(a*(a + b*x)^(1/2) - (a + b*x)^(3/2))
```

sympy [A] time = 3.41, size = 73, normalized size = 1.28

$$-\frac{1}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^2\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{3b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(3/2),x)
```

```
[Out] -1/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) + 1)) - 3*sqrt(b)/(a**2*sqrt(x)*sqrt(a/
(b*x) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2)
```

3.350 $\int \frac{1}{x^3(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15b^2}{4a^3\sqrt{a+bx}}+\frac{5b}{4a^2x\sqrt{a+bx}}-\frac{1}{2ax^2\sqrt{a+bx}}$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5\sqrt{a+bx}}{2a^2x^2}+\frac{15b\sqrt{a+bx}}{4a^3x}+\frac{2}{ax^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/(2*a^2*x^2) + (15*b*Sqrt[a + b*
x])/(4*a^3*x) - (15*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^3(a+bx)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{a+bx}} + \frac{5\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{a}$$
$$= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} - \frac{(15b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^2}$$
$$= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} + \frac{(15b^2)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^3}$$
$$= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} + \frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{4a^3}$$
$$= \frac{2}{ax^2\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{2a^2x^2} + \frac{15b\sqrt{a+bx}}{4a^3x} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.38

$$\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 71, normalized size = 0.82

$$\frac{8a^2-25a(a+bx)+15(a+bx)^2}{4a^3x^2\sqrt{a+bx}} - \frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(3/2)),x]
```

```
[Out] (8*a^2 - 25*a*(a + b*x) + 15*(a + b*x)^2)/(4*a^3*x^2*Sqrt[a + b*x]) - (15*b
^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

fricas [A] time = 1.07, size = 189, normalized size = 2.17

$$\left[\frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15ab^2x^2+5a^2bx-2a^3\right)\sqrt{bx+a}}{8\left(a^4bx^3+a^5x^2\right)}, \frac{15\left(b^3x^3+ab^2x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15ab^2x^2+5a^2bx-2a^3\right)\sqrt{bx+a}}{4\left(a^4bx^3+a^5x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) +
 2*a)/x) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 +
 a^5*x^2), 1/4*(15*(b^3*x^3 + a*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt
(-a)/a) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x + a))/(a^4*b*x^3 + a^
5*x^2)]
```

giac [A] time = 1.05, size = 80, normalized size = 0.92

$$\frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^3}+\frac{2\,b^2}{\sqrt{bx+a}\,a^3}+\frac{7\,(bx+a)^{\frac{3}{2}}b^2-9\,\sqrt{bx+a}\,ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) + 2*b^2/(sqrt(b*x +
a)*a^3) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^2)/(a^3*b^2*x^2)
```

maple [A] time = 0.01, size = 67, normalized size = 0.77

$$2\left(\frac{1}{\sqrt{bx+a}\;a^3}+\frac{-\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{-\frac{9\sqrt{bx+a}\;a}{8}+\frac{7(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}}{a^3}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(3/2),x)
```

```
[Out] 2*b^2*(1/a^3/(b*x+a)^(1/2)+1/a^3*((7/8*(b*x+a)^(3/2)-9/8*(b*x+a)^(1/2)*a)/x
^2/b^2-15/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 3.06, size = 108, normalized size = 1.24

$$\frac{15\,(bx+a)^2b^2-25\,(bx+a)ab^2+8\,a^2b^2}{4\left((bx+a)^{\frac{5}{2}}a^3-2\,(bx+a)^{\frac{3}{2}}a^4+\sqrt{bx+a}\,a^5\right)}+\frac{15\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(b*x + a)^2*b^2 - 25*(b*x + a)*a*b^2 + 8*a^2*b^2)/((b*x + a)^(5/2)*
a^3 - 2*(b*x + a)^(3/2)*a^4 + sqrt(b*x + a)*a^5) + 15/8*b^2*log((sqrt(b*x +
 a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(7/2)
```

mupad [B] time = 0.06, size = 90, normalized size = 1.03

$$\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,(a+b\,x)^2}{4\,a^3}-\frac{25\,b^2\,(a+b\,x)}{4\,a^2}}{(a+b\,x)^{5/2}-2\,a\,(a+b\,x)^{3/2}+a^2\,\sqrt{a+b\,x}}-\frac{15\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(3/2)),x)
```

```
[Out] ((2*b^2)/a + (15*b^2*(a + b*x)^2)/(4*a^3) - (25*b^2*(a + b*x))/(4*a^2))/((a
 + b*x)^(5/2) - 2*a*(a + b*x)^(3/2) + a^2*(a + b*x)^(1/2)) - (15*b^2*atanh(
(a + b*x)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

sympy [A] time = 5.98, size = 107, normalized size = 1.23

$$-\frac{1}{2a\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}+1}}-\frac{15b^2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(3/2),x)
```

```
[Out] -1/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x**(3/2)*sq
rt(a/(b*x) + 1)) + 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x) + 1)) - 15*b**2
*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2))
```

3.351 $\int \frac{x^4}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^4)/(3*b^5*(a + b*x)^(3/2)) + (8*a^3)/(b^5*Sqrt[a + b*x]) + (12*a^2*Sq
rt[a + b*x])/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5) + (2*(a + b*x)^(5/2))/(5*b
^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{(a+bx)^{5/2}}\,dx&=\int \left(\frac{a^4}{b^4(a+bx)^{5/2}}-\frac{4a^3}{b^4(a+bx)^{3/2}}+\frac{6a^2}{b^4\sqrt{a+bx}}-\frac{4a\sqrt{a+bx}}{b^4}+\frac{(a+bx)^{3/2}}{b^4}\right)dx\\&=-\frac{2a^4}{3b^5(a+bx)^{3/2}}+\frac{8a^3}{b^5\sqrt{a+bx}}+\frac{12a^2\sqrt{a+bx}}{b^5}-\frac{8a(a+bx)^{3/2}}{3b^5}+\frac{2(a+bx)^{5/2}}{5b^5}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.66

$$\frac{2\left(128a^4+192a^3bx+48a^2b^2x^2-8ab^3x^3+3b^4x^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(128*a^4 + 192*a^3*b*x + 48*a^2*b^2*x^2 - 8*a*b^3*x^3 + 3*b^4*x^4))/(15*
b^5*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 63, normalized size = 0.72

$$\frac{2\left(-5a^4+60a^3(a+bx)+90a^2(a+bx)^2-20a(a+bx)^3+3(a+bx)^4\right)}{15b^5(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-5*a^4 + 60*a^3*(a + b*x) + 90*a^2*(a + b*x)^2 - 20*a*(a + b*x)^3 + 3*(
a + b*x)^4))/(15*b^5*(a + b*x)^(3/2))
```

fricas [A] time = 1.00, size = 74, normalized size = 0.85

$$\frac{2\left(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4\right)\sqrt{bx+a}}{15\left(b^7x^2+2\,ab^6x+a^2b^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x + 128*a^4)*sqr
t(b*x + a)/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

giac [A] time = 1.08, size = 75, normalized size = 0.86

$$\frac{2\left(12\,(bx+a)a^3-a^4\right)}{3\,(bx+a)^{\frac{3}{2}}b^5}+\frac{2\left(3\,(bx+a)^{\frac{5}{2}}b^{20}-20\,(bx+a)^{\frac{3}{2}}ab^{20}+90\,\sqrt{bx+a}\,a^2b^{20}\right)}{15\,b^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(b*x + a)*a^3 - a^4)/((b*x + a)^(3/2)*b^5) + 2/15*(3*(b*x + a)^(5/2
)*b^20 - 20*(b*x + a)^(3/2)*a*b^20 + 90*sqrt(b*x + a)*a^2*b^20)/b^25
```

maple [A] time = 0.00, size = 54, normalized size = 0.62

$$\frac{\frac{2}{5}x^4b^4-\frac{16}{15}a\,x^3b^3+\frac{32}{5}a^2x^2b^2+\frac{128}{5}a^3xb+\frac{256}{15}a^4}{(bx+a)^{\frac{3}{2}}\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^(5/2),x)
```

```
[Out] 2/15/(b*x+a)^(3/2)*(3*b^4*x^4-8*a*b^3*x^3+48*a^2*b^2*x^2+192*a^3*b*x+128*a^
4)/b^5
```

maxima [A] time = 1.35, size = 71, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^5}-\frac{8\,(bx+a)^{\frac{3}{2}}a}{3\,b^5}+\frac{12\,\sqrt{bx+a}\,a^2}{b^5}+\frac{8\,a^3}{\sqrt{bx+a}\,b^5}-\frac{2\,a^4}{3\,(bx+a)^{\frac{3}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^5 - 8/3*(b*x + a)^(3/2)*a/b^5 + 12*sqrt(b*x + a)*a^2/
b^5 + 8*a^3/(sqrt(b*x + a)*b^5) - 2/3*a^4/((b*x + a)^(3/2)*b^5)
```

mupad [B] time = 0.05, size = 68, normalized size = 0.78

$$\frac{2\,(a+b\,x)^{5/2}}{5\,b^5}+\frac{8\,a^3\,(a+b\,x)-\frac{2\,a^4}{3}}{b^5\,(a+b\,x)^{3/2}}+\frac{12\,a^2\,\sqrt{a+b\,x}}{b^5}-\frac{8\,a\,(a+b\,x)^{3/2}}{3\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a + b*x)^(5/2),x)
```

```
[Out] (2*(a + b*x)^(5/2))/(5*b^5) + (8*a^3*(a + b*x) - (2*a^4)/3)/(b^5*(a + b*x)^
(3/2)) + (12*a^2*(a + b*x)^(1/2))/b^5 - (8*a*(a + b*x)^(3/2))/(3*b^5)
```

sympy [B] time = 4.57, size = 3456, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**(5/2),x)
```

```
[Out] 256*a**(85/2)*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256*a**(85/2)/(15*a**40*
b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150
*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**
33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**1
5*x**10) + 2432*a**(83/2)*b*x*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**
(83/2)*b*x/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a
**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*
b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14
*x**9 + 15*a**30*b**15*x**10) + 10336*a**(81/2)*b**2*x**2*sqrt(1 + b*x/a)/(
15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x*
*3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 +
 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a
**30*b**15*x**10) - 11520*a**(81/2)*b**2*x**2/(15*a**40*b**5 + 150*a**39*b*
*6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 +
3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675
*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 25840*a*
*(79/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a
**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b
**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13
*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(79/2)*b**3
*x**3/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*
b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11
*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9
 + 15*a**30*b**15*x**10) + 41990*a**(77/2)*b**4*x**4*sqrt(1 + b*x/a)/(15*a*
*40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 +
3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800
*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*
b**15*x**10) - 53760*a**(77/2)*b**4*x**4/(15*a**40*b**5 + 150*a**39*b**6*x
+ 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*
a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**3
2*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 46192*a**(75/
2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*
b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*
x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8
 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 64512*a**(75/2)*b**5*x**5
/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*
x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6
 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15
*a**30*b**15*x**10) + 34664*a**(73/2)*b**6*x**6*sqrt(1 + b*x/a)/(15*a**40*b
**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*
a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**3
3*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15
*x**10) - 53760*a**(73/2)*b**6*x**6/(15*a**40*b**5 + 150*a**39*b**6*x + 675
```

```
*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35
*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**
13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 17392*a**(71/2)*b*
*7*x**7*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*
x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5
+ 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 15
0*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 30720*a**(71/2)*b**7*x**7/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10) + 5540*a**(69/2)*b**8*x**8*sqrt(1 + b*x/a)/(15*a**40*b**5 +
150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*
b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**1
2*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10
) - 11520*a**(69/2)*b**8*x**8/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38
*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10
*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**
8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 1040*a**(67/2)*b**9*x**9
*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 +
1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*
a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31
*b**14*x**9 + 15*a**30*b**15*x**10) - 2560*a**(67/2)*b**9*x**9/(15*a**40*b*
*5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a
**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33
*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*
x**10) + 136*a**(65/2)*b**10*x**10*sqrt(1 + b*x/a)/(15*a**40*b**5 + 150*a**
39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x*
*4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7
+ 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**30*b**15*x**10) - 256
*a**(65/2)*b**10*x**10/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x
**2 + 1800*a**37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 +
 3150*a**34*b**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150
*a**31*b**14*x**9 + 15*a**30*b**15*x**10) + 32*a**(63/2)*b**11*x**11*sqrt(1
 + b*x/a)/(15*a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a*
*37*b**8*x**3 + 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b
**11*x**6 + 1800*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*
x**9 + 15*a**30*b**15*x**10) + 6*a**(61/2)*b**12*x**12*sqrt(1 + b*x/a)/(15*
a**40*b**5 + 150*a**39*b**6*x + 675*a**38*b**7*x**2 + 1800*a**37*b**8*x**3
+ 3150*a**36*b**9*x**4 + 3780*a**35*b**10*x**5 + 3150*a**34*b**11*x**6 + 18
00*a**33*b**12*x**7 + 675*a**32*b**13*x**8 + 150*a**31*b**14*x**9 + 15*a**3
0*b**15*x**10)
```

3.352 $\int \frac{x^3}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=68

$$\frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*a^3)/(3*b^4*(a + b*x)^(3/2)) - (6*a^2)/(b^4*Sqrt[a + b*x]) - (6*a*Sqrt[a
 + b*x])/b^4 + (2*(a + b*x)^(3/2))/(3*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{5/2}} dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{5/2}} + \frac{3a^2}{b^3(a+bx)^{3/2}} - \frac{3a}{b^3\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b^3}\right) dx \\ &= \frac{2a^3}{3b^4(a+bx)^{3/2}} - \frac{6a^2}{b^4\sqrt{a+bx}} - \frac{6a\sqrt{a+bx}}{b^4} + \frac{2(a+bx)^{3/2}}{3b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.66

$$\frac{2\left(-16a^3 - 24a^2bx - 6ab^2x^2 + b^3x^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-16*a^3 - 24*a^2*b*x - 6*a*b^2*x^2 + b^3*x^3))/(3*b^4*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.69

$$\frac{2\left(a^3 - 9a^2(a+bx) - 9a(a+bx)^2 + (a+bx)^3\right)}{3b^4(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(5/2),x]
```

```
[Out] (2*(a^3 - 9*a^2*(a + b*x) - 9*a*(a + b*x)^2 + (a + b*x)^3))/(3*b^4*(a + b*x
)^(3/2))
```

fricas [A] time = 0.96, size = 62, normalized size = 0.91

$$\frac{2\left(b^3x^3 - 6\,ab^2x^2 - 24\,a^2bx - 16\,a^3\right)\sqrt{bx+a}}{3\left(b^6x^2 + 2\,ab^5x + a^2b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*sqrt(b*x + a)/(b^6*x^2 +
2*a*b^5*x + a^2*b^4)
```

giac [A] time = 1.03, size = 59, normalized size = 0.87

$$-\frac{2\left(9\,(bx+a)a^2 - a^3\right)}{3\,(bx+a)^{\frac{3}{2}}b^4} + \frac{2\left((bx+a)^{\frac{3}{2}}b^8 - 9\,\sqrt{bx+a}\,ab^8\right)}{3\,b^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(b*x + a)*a^2 - a^3)/((b*x + a)^(3/2)*b^4) + 2/3*((b*x + a)^(3/2)*b
^8 - 9*sqrt(b*x + a)*a*b^8)/b^12
```

maple [A] time = 0.01, size = 43, normalized size = 0.63

$$-\frac{2\left(-b^3x^3 + 6a\,b^2x^2 + 24a^2bx + 16a^3\right)}{3\,(bx+a)^{\frac{3}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(-b^3*x^3+6*a*b^2*x^2+24*a^2*b*x+16*a^3)/b^4
```

maxima [A] time = 1.25, size = 56, normalized size = 0.82

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^4} - \frac{6\,\sqrt{bx+a}\,a}{b^4} - \frac{6\,a^2}{\sqrt{bx+a}\,b^4} + \frac{2\,a^3}{3\,(bx+a)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^4 - 6*sqrt(b*x + a)*a/b^4 - 6*a^2/(sqrt(b*x + a)*b^4)
 + 2/3*a^3/((b*x + a)^(3/2)*b^4)
```

mupad [B] time = 0.04, size = 47, normalized size = 0.69

$$-\frac{18\,a\,(a+b\,x)^2 + 18\,a^2\,(a+b\,x) - 2\,(a+b\,x)^3 - 2\,a^3}{3\,b^4\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(5/2),x)
```

[Out] -(18*a*(a + b*x)^2 + 18*a^2*(a + b*x) - 2*(a + b*x)^3 - 2*a^3)/(3*b^4*(a + b*x)^(3/2))

sympy [A] time = 1.20, size = 163, normalized size = 2.40

$$\begin{cases} -\frac{32a^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{48a^2bx}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} - \frac{12ab^2x^2}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} + \frac{2b^3x^3}{3ab^4\sqrt{a+bx}+3b^5x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-32*a**3/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 48*
a**2*b*x/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) - 12*a*b**2*x**2
/(3*a*b**4*sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)) + 2*b**3*x**3/(3*a*b**4*
sqrt(a + b*x) + 3*b**5*x*sqrt(a + b*x)), Ne(b, 0)), (x**4/(4*a**(5/2)), Tru
e))
```

3.353 $\int \frac{x^2}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=49

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (-2*a^2)/(3*b^3*(a + b*x)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x]) + (2*Sqrt[a +
b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{5/2}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{5/2}}-\frac{2a}{b^2(a+bx)^{3/2}}+\frac{1}{b^2\sqrt{a+bx}}\right)dx\\ &= -\frac{2a^2}{3b^3(a+bx)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx}}+\frac{2\sqrt{a+bx}}{b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.71

$$\frac{2\left(8a^2+12abx+3b^2x^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 12*a*b*x + 3*b^2*x^2))/(3*b^3*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{2\left(-a^2+6a(a+bx)+3(a+bx)^2\right)}{3b^3(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(5/2),x]
```

```
[Out] (2*(-a^2 + 6*a*(a + b*x) + 3*(a + b*x)^2))/(3*b^3*(a + b*x)^(3/2))
```

fricas [A] time = 1.27, size = 52, normalized size = 1.06

$$\frac{2\left(3\,b^{2}x^{2}+12\,abx+8\,a^{2}\right)\sqrt{bx+a}}{3\left(b^{5}x^{2}+2\,ab^{4}x+a^{2}b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*sqrt(b*x + a)/(b^5*x^2 + 2*a*b^4*x + a^2
*b^3)
```

giac [A] time = 0.94, size = 39, normalized size = 0.80

$$\frac{2\sqrt{bx+a}}{b^{3}}+\frac{2\left(6\,(bx+a)a-a^{2}\right)}{3\,(bx+a)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 2/3*(6*(b*x + a)*a - a^2)/((b*x + a)^(3/2)*b^3)
```

maple [A] time = 0.00, size = 32, normalized size = 0.65

$$\frac{2b^{2}x^{2}+8abx+\frac{16}{3}a^{2}}{b^{3}\,(bx+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(3*b^2*x^2+12*a*b*x+8*a^2)/b^3
```

maxima [A] time = 1.33, size = 41, normalized size = 0.84

$$\frac{2\sqrt{bx+a}}{b^{3}}+\frac{4\,a}{\sqrt{bx+a}\,b^{3}}-\frac{2\,a^{2}}{3\,(bx+a)^{\frac{3}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b^3 + 4*a/(sqrt(b*x + a)*b^3) - 2/3*a^2/((b*x + a)^(3/2)*b^
3)
```

mupad [B] time = 0.08, size = 35, normalized size = 0.71

$$\frac{6\,{(a+b\,x)}^2+12\,a\,\left(a+b\,x\right)-2\,a^2}{3\,b^3\,{(a+b\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(5/2),x)
```

```
[Out] (6*(a + b*x)^2 + 12*a*(a + b*x) - 2*a^2)/(3*b^3*(a + b*x)^(3/2))
```

sympy [A] time = 1.28, size = 121, normalized size = 2.47

$$\begin{cases} \frac{16a^{2}}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}}+\frac{24abx}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}}+\frac{6b^{2}x^{2}}{3ab^{3}\sqrt{a+bx}+3b^{4}x\sqrt{a+bx}} & \text{for}\ b\neq 0\\ \frac{x^{3}}{3a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((16*a**2/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 24*a
*b*x/(3*a*b**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)) + 6*b**2*x**2/(3*a*b
**3*sqrt(a + b*x) + 3*b**4*x*sqrt(a + b*x)), Ne(b, 0)), (x**3/(3*a**(5/2)),
 True))
```

3.354 $\int \frac{x}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(5/2),x]
[Out] (2*a)/(3*b^2*(a + b*x)^(3/2)) - 2/(b^2*Sqrt[a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{5/2}}\,dx&=\int \left(-\frac{a}{b(a+bx)^{5/2}}+\frac{1}{b(a+bx)^{3/2}}\right)dx\\&=\frac{2a}{3b^2(a+bx)^{3/2}}-\frac{2}{b^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(5/2),x]
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{2(2a+3bx)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(5/2),x]
[Out] (-2*(2*a + 3*b*x))/(3*b^2*(a + b*x)^(3/2))
```

fricas [A] time = 0.99, size = 41, normalized size = 1.28

$$-\frac{2\,(3\,bx+2\,a)\sqrt{bx+a}}{3\,\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*x + 2*a)*sqrt(b*x + a)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

giac [A] time = 1.06, size = 20, normalized size = 0.62

$$-\frac{2\,(3\,bx+2\,a)}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*x + 2*a)/((b*x + a)^(3/2)*b^2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.66

$$-\frac{2\,(3bx+2a)}{3\,(bx+a)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(5/2),x)
```

```
[Out] -2/3/(b*x+a)^(3/2)*(3*b*x+2*a)/b^2
```

maxima [A] time = 1.34, size = 26, normalized size = 0.81

$$-\frac{2}{\sqrt{bx+a}\,b^2}+\frac{2\,a}{3\,(bx+a)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/(sqrt(b*x + a)*b^2) + 2/3*a/((b*x + a)^(3/2)*b^2)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.62

$$-\frac{4\,a+6\,b\,x}{3\,b^2\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(5/2),x)
```

```
[Out] -(4*a + 6*b*x)/(3*b^2*(a + b*x)^(3/2))
```

sympy [A] time = 1.13, size = 80, normalized size = 2.50

$$\begin{cases} -\frac{4a}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} - \frac{6bx}{3ab^2\sqrt{a+bx}+3b^3x\sqrt{a+bx}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-4*a/(3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)) - 6*b*x/(
3*a*b**2*sqrt(a + b*x) + 3*b**3*x*sqrt(a + b*x)), Ne(b, 0)), (x**2/(2*a**(5
/2)), True))
```

3.355 $\int \frac{1}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/2}}\,dx = -\frac{2}{3b(a+bx)^{3/2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-5/2),x]
[Out] -2/(3*b*(a + b*x)^(3/2))
```

fricas [B] time = 0.91, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{bx+a}}{3\left(b^3x^2+2\,ab^2x+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b)
```

giac [A] time = 1.04, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="giac")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2),x)
[Out] -2/3/b/(b*x+a)^(3/2)
```

maxima [A] time = 1.31, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(bx+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2),x, algorithm="maxima")
[Out] -2/3/((b*x + a)^(3/2)*b)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$-\frac{2}{3\,b\,(a+b\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(5/2),x)
[Out] -2/(3*b*(a + b*x)^(3/2))
```

sympy [A] time = 0.07, size = 14, normalized size = 0.88

$$-\frac{2}{3b\,(a+bx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2),x)
[Out] -2/(3*b*(a + b*x)**(3/2))
```

3.356 $\int \frac{1}{x(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{a+bx}}+\frac{2}{3a(a+bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{2}{a^2\sqrt{a+bx}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*(a + b*x)^(3/2)) + 2/(a^2*Sqrt[a + b*x]) - (2*ArcTanh[Sqrt[a + b*x]/
Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{5/2}}\,dx = \frac{2}{3a(a+bx)^{3/2}} + \frac{\int \frac{1}{x(a+bx)^{3/2}}\,dx}{a}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} + \frac{\int \frac{1}{x\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a^2 b}$$
$$= \frac{2}{3a(a+bx)^{3/2}} + \frac{2}{a^2\sqrt{a+bx}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 32, normalized size = 0.59

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*x)/a])/(3*a*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 49, normalized size = 0.91

$$\frac{2(3(a+bx)+a)}{3a^2(a+bx)^{3/2}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(5/2)),x]
```

```
[Out] (2*(a + 3*(a + b*x)))/(3*a^2*(a + b*x)^(3/2)) - (2*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]])/a^(5/2)
```

fricas [B] time = 1.02, size = 177, normalized size = 3.28

$$\left[\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(3\,abx+4\,a^2\right)\sqrt{bx+a}}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)}, \frac{2\left(3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(3\,abx+4\,a^2\right)\sqrt{bx+a}\right)}{3\left(a^3b^2x^2+2\,a^4bx+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a
) + 2*a)/x) + 2*(3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x +
 a^5), 2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(
-a)/a) + (3*a*b*x + 4*a^2)*sqrt(b*x + a))/(a^3*b^2*x^2 + 2*a^4*b*x + a^5)]
```

giac [A] time = 1.00, size = 45, normalized size = 0.83

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} + \frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) + 2/3*(3*b*x + 4*a)/((b*x +
 a)^(3/2)*a^2)
```

maple [A] time = 0.01, size = 43, normalized size = 0.80

$$\frac{2}{3\,(bx+a)^{\frac{3}{2}}\,a}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{\sqrt{bx+a}\;a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/a/(b*x+a)^(3/2)-2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(5/2)+2/(b*x+a)^(1/2
)/a^2
```

maxima [A] time = 2.91, size = 53, normalized size = 0.98

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2\,(3\,bx+4\,a)}{3\,(bx+a)^{\frac{3}{2}}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(5/2) + 2/3*(3*b
*x + 4*a)/((b*x + a)^(3/2)*a^2)
```

mupad [B] time = 0.05, size = 42, normalized size = 0.78

$$\frac{\frac{2\,(a+b\,x)}{a^2}+\frac{2}{3\,a}}{(a+b\,x)^{3/2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*(a + b*x))/a^2 + 2/(3*a))/(a + b*x)^(3/2) - (2*atanh((a + b*x)^(1/2)/a^
(1/2)))/a^(5/2)
```

sympy [B] time = 2.99, size = 697, normalized size = 12.91

$$\frac{8a^{7}\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{3a^{7}\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{14a^{6}bx\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{9a^{6}bx\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}-\frac{18a^{6}bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{6a^{5}b^{2}x^{2}\sqrt{1+\frac{bx}{a}}}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{9a^{5}b^{2}x^{2}\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}-\frac{18a^{5}b^{2}x^{2}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}+\frac{3a^{4}b^{3}x^{3}\log\left(\frac{bx}{a}\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}-\frac{6a^{4}b^{3}x^{3}\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{3a^{\frac{19}{2}}+9a^{\frac{17}{2}}bx+9a^{\frac{15}{2}}b^{2}x^{2}+3a^{\frac{13}{2}}b^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(5/2),x)
```

```
[Out] 8*a**7*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x*
*2 + 3*a**(13/2)*b**3*x**3) + 3*a**7*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*
b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**7*log(sqrt(1 +
b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(
13/2)*b**3*x**3) + 14*a**6*b*x*sqrt(1 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b
*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 9*a**6*b*x*log(b*x/a)
/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*
x**3) - 18*a**6*b*x*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) + 6*a**5*b**2*x**2*sqrt(1
```

```
 + b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13
/2)*b**3*x**3) + 9*a**5*b**2*x**2*log(b*x/a)/(3*a**(19/2) + 9*a**(17/2)*b*x
 + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 18*a**5*b**2*x**2*log(s
qrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b**2*x**2
+ 3*a**(13/2)*b**3*x**3) + 3*a**4*b**3*x**3*log(b*x/a)/(3*a**(19/2) + 9*a**
(17/2)*b*x + 9*a**(15/2)*b**2*x**2 + 3*a**(13/2)*b**3*x**3) - 6*a**4*b**3*x
**3*log(sqrt(1 + b*x/a) + 1)/(3*a**(19/2) + 9*a**(17/2)*b*x + 9*a**(15/2)*b
**2*x**2 + 3*a**(13/2)*b**3*x**3)
```

3.357 $\int \frac{1}{x^2(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=74

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{a+bx}}-\frac{5b}{3a^2(a+bx)^{3/2}}-\frac{1}{ax(a+bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{5\sqrt{a+bx}}{a^3x}+\frac{10}{3a^2x\sqrt{a+bx}}+\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2}{3ax(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x*(a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[a + b*x]) - (5*Sqrt[a + b*x])/
(a^3*x) + (5*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{5/2}}\,dx = \frac{2}{3ax(a+bx)^{3/2}} + \frac{5\int \frac{1}{x^2(a+bx)^{3/2}}\,dx}{3a}$$
$$= \frac{2}{3ax(a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{a+bx}} + \frac{5\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{a^2}$$
$$= \frac{2}{3ax(a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{a^3x} - \frac{(5b)\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a^3}$$
$$= \frac{2}{3ax(a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{a^3x} - \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{a^3}$$
$$= \frac{2}{3ax(a+bx)^{3/2}} + \frac{10}{3a^2x\sqrt{a+bx}} - \frac{5\sqrt{a+bx}}{a^3x} + \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.45

$$-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 + (b*x)/a])/(3*a^2*(a + b*x)^(3/2)
)
```

IntegrateAlgebraic [A] time = 0.07, size = 67, normalized size = 0.91

$$\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{2a^2+10a(a+bx)-15(a+bx)^2}{3a^3x(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(5/2)),x]
```

```
[Out] (2*a^2 + 10*a*(a + b*x) - 15*(a + b*x)^2)/(3*a^3*x*(a + b*x)^(3/2)) + (5*b*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 1.13, size = 221, normalized size = 2.99

$$\left[\frac{15\left(b^3x^3+2ab^2x^2+a^2bx\right)\sqrt{a}\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\left(15ab^2x^2+20a^2bx+3a^3\right)\sqrt{bx+a}}{6\left(a^4b^2x^3+2a^5bx^2+a^6x\right)}, -\frac{15\left(b^3x^3+2ab^2x^2+a^2bx\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15ab^2x^2+20a^2bx+3a^3\right)\sqrt{bx+a}}{3\left(a^4b^2x^3+2a^5bx^2+a^6x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2*b*x)*sqrt(a)*log((b*x + 2*sqrt(b*x +
a)*sqrt(a) + 2*a)/x) - 2*(15*a*b^2*x^2 + 20*a^2*b*x + 3*a^3)*sqrt(b*x + a))
/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x), -1/3*(15*(b^3*x^3 + 2*a*b^2*x^2 + a^2
*b*x)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*a*b^2*x^2 + 20*a^2*b*
x + 3*a^3)*sqrt(b*x + a))/(a^4*b^2*x^3 + 2*a^5*b*x^2 + a^6*x)]
```

giac [A] time = 1.03, size = 65, normalized size = 0.88

$$-\frac{5\,b\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}-\frac{2\,(6\,(bx+a)b+ab)}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{\sqrt{bx+a}}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3) - 2/3*(6*(b*x + a)*b + a
*b)/((b*x + a)^(3/2)*a^3) - sqrt(b*x + a)/(a^3*x)
```

maple [A] time = 0.01, size = 67, normalized size = 0.91

$$2\left(-\frac{1}{3\,(bx+a)^{\frac{3}{2}}\,a^2}-\frac{2}{\sqrt{bx+a}\,a^3}-\frac{-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}}+\frac{\sqrt{bx+a}}{2bx}}{a^3}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(5/2),x)
```

```
[Out] 2*b*(-1/3/a^2/(b*x+a)^(3/2)-2/(b*x+a)^(1/2)/a^3-1/a^3*(1/2*(b*x+a)^(1/2)/b/
x-5/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)))
```

maxima [A] time = 3.03, size = 89, normalized size = 1.20

$$-\frac{15\,(bx+a)^2b-10\,(bx+a)ab-2\,a^2b}{3\left((bx+a)^{\frac{5}{2}}a^3-(bx+a)^{\frac{3}{2}}a^4\right)}-\frac{5\,b\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{2\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(15*(b*x + a)^2*b - 10*(b*x + a)*a*b - 2*a^2*b)/((b*x + a)^(5/2)*a^3 -
 (b*x + a)^(3/2)*a^4) - 5/2*b*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a)
+ sqrt(a)))/a^(7/2)
```

mupad [B] time = 0.11, size = 73, normalized size = 0.99

$$\frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2\,b}{3\,a}+\frac{10\,b\,(a+b\,x)}{3\,a^2}-\frac{5\,b\,{(a+b\,x)}^2}{a^3}}{a\,{(a+b\,x)}^{3/2}-{(a+b\,x)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(5/2)),x)
```

```
[Out] (5*b*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(7/2) - ((2*b)/(3*a) + (10*b*(a + b*
x))/(3*a^2) - (5*b*(a + b*x)^2)/a^3)/(a*(a + b*x)^(3/2) - (a + b*x)^(5/2))
```

sympy [B] time = 5.60, size = 818, normalized size = 11.05

$$-\frac{6a^{17}\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{46a^{16}bx\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{16}bx\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{16}bx\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{70a^{15}b^2x^2\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{15}b^2x^2\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{15}b^2x^2\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{30a^{14}b^3x^3\sqrt{1+\frac{bx}{a}}}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{45a^{14}b^3x^3\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{90a^{14}b^3x^3\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}-\frac{15a^{13}b^4x^4\log\left(\frac{bx}{a}\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}+\frac{30a^{13}b^4x^4\log\left(\sqrt{1+\frac{bx}{a}}+1\right)}{6a^{\frac{39}{2}}x+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x^3+6a^{\frac{33}{2}}b^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**17*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2
)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 46*a**16*b*x*sqrt(1 + b*x/a)/(6*a**(
39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x
**4) - 15*a**16*b*x*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**16*b*x*log(sqrt(1 + b*x/
a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a
**(33/2)*b**3*x**4) - 70*a**15*b**2*x**2*sqrt(1 + b*x/a)/(6*a**(39/2)*x + 1
8*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*a
**15*b**2*x**2*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/
2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**15*b**2*x**2*log(sqrt(1 + b*x
/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*
a**(33/2)*b**3*x**4) - 30*a**14*b**3*x**3*sqrt(1 + b*x/a)/(6*a**(39/2)*x +
18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) - 45*
a**14*b**3*x**3*log(b*x/a)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35
/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 90*a**14*b**3*x**3*log(sqrt(1 + b*
x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6
*a**(33/2)*b**3*x**4) - 15*a**13*b**4*x**4*log(b*x/a)/(6*a**(39/2)*x + 18*a
**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4) + 30*a**1
3*b**4*x**4*log(sqrt(1 + b*x/a) + 1)/(6*a**(39/2)*x + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x**3 + 6*a**(33/2)*b**3*x**4)
```

3.358 $\int \frac{1}{x^3(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=106

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{a+bx}}+\frac{35b^2}{12a^3(a+bx)^{3/2}}+\frac{7b}{4a^2x(a+bx)^{3/2}}-\frac{1}{2ax^2(a+bx)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{2}{3ax^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^2*(a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[a + b*x]) - (35*Sqrt[a + b
*x])/(6*a^3*x^2) + (35*b*Sqrt[a + b*x])/(4*a^4*x) - (35*b^2*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(a+bx)^{5/2}}\,dx &= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{7\int \frac{1}{x^3(a+bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}+\frac{35\int \frac{1}{x^3\sqrt{a+bx}}\,dx}{3a^2}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}-\frac{(35b)\int \frac{1}{x^2\sqrt{a+bx}}\,dx}{4a^3}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{\left(35b^2\right)\int \frac{1}{x\sqrt{a+bx}}\,dx}{8a^4}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}+\frac{(35b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\right.}{4a^4}\\
&= \frac{2}{3ax^2(a+bx)^{3/2}}+\frac{14}{3a^2x^2\sqrt{a+bx}}-\frac{35\sqrt{a+bx}}{6a^3x^2}+\frac{35b\sqrt{a+bx}}{4a^4x}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.33

$$
\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}+1\right)}{3a^3(a+bx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 + (b*x)/a])/(3*a^3*(a + b*x)^(3/2
))
```

IntegrateAlgebraic [A] time = 0.11, size = 83, normalized size = 0.78

$$
\frac{8a^3+56a^2(a+bx)-175a(a+bx)^2+105(a+bx)^3}{12a^4x^2(a+bx)^{3/2}}-\frac{35b^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{9/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(5/2)),x]
```

```
[Out] (8*a^3 + 56*a^2*(a + b*x) - 175*a*(a + b*x)^2 + 105*(a + b*x)^3)/(12*a^4*x^
2*(a + b*x)^(3/2)) - (35*b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

fricas [A] time = 1.15, size = 255, normalized size = 2.41

$$
\left[\frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{bx+a}\,\sqrt{a}+2\,a}{x}\right)+2\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+a}}{24\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)},\ \frac{105\left(b^4x^4+2\,ab^3x^3+a^2b^2x^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\left(105\,ab^3x^3+140\,a^2b^2x^2+21\,a^3bx-6\,a^4\right)\sqrt{bx+a}}{12\left(a^5b^2x^4+2\,a^6bx^3+a^7x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*log((b*x - 2*sqrt(
b*x + a)*sqrt(a) + 2*a)/x) + 2*(105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*
x - 6*a^4)*sqrt(b*x + a))/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2), 1/12*(105*
(b^4*x^4 + 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a
```

```
)/a) + (105*a*b^3*x^3 + 140*a^2*b^2*x^2 + 21*a^3*b*x - 6*a^4)*sqrt(b*x + a)
)/(a^5*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)]
```

giac [A] time = 0.99, size = 93, normalized size = 0.88

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}\,a^4}+\frac{2\left(9\,(bx+a)b^2+ab^2\right)}{3\,(bx+a)^{\frac{3}{2}}a^4}+\frac{11\,(bx+a)^{\frac{3}{2}}b^2-13\,\sqrt{bx+a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4) + 2/3*(9*(b*x + a)*b
^2 + a*b^2)/((b*x + a)^(3/2)*a^4) + 1/4*(11*(b*x + a)^(3/2)*b^2 - 13*sqrt(b
*x + a)*a*b^2)/(a^4*b^2*x^2)
```

maple [A] time = 0.02, size = 80, normalized size = 0.75

$$2\left(\frac{1}{3\,(bx+a)^{\frac{3}{2}}\,a^3}+\frac{3}{\sqrt{bx+a}\;a^4}+\frac{-\frac{35\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8\sqrt{a}}+\frac{-\frac{13\,\sqrt{bx+a}\;a}{8}+\frac{11(bx+a)^{\frac{3}{2}}}{8}}{b^2x^2}}{a^4}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(5/2),x)
```

```
[Out] 2*b^2*(3/a^4/(b*x+a)^(1/2)+1/3/a^3/(b*x+a)^(3/2)+1/a^4*((11/8*(b*x+a)^(3/2)
-13/8*(b*x+a)^(1/2)*a)/x^2/b^2-35/8*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
)
```

maxima [A] time = 3.00, size = 123, normalized size = 1.16

$$\frac{105\,(bx+a)^3b^2-175\,(bx+a)^2ab^2+56\,(bx+a)a^2b^2+8\,a^3b^2}{12\left((bx+a)^{\frac{7}{2}}a^4-2\,(bx+a)^{\frac{5}{2}}a^5+(bx+a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{8\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x + a)^3*b^2 - 175*(b*x + a)^2*a*b^2 + 56*(b*x + a)*a^2*b^2 +
8*a^3*b^2)/((b*x + a)^(7/2)*a^4 - 2*(b*x + a)^(5/2)*a^5 + (b*x + a)^(3/2)*a
^6) + 35/8*b^2*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/a^(
9/2)
```

mupad [B] time = 0.12, size = 105, normalized size = 0.99

$$\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a+b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a+b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a+b\,x)}{3\,a^2}}{(a+b\,x)^{7/2}-2\,a\,(a+b\,x)^{5/2}+a^2\,(a+b\,x)^{3/2}}-\frac{35\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(5/2)),x)
```

```
[Out] ((2*b^2)/(3*a) - (175*b^2*(a + b*x)^2)/(12*a^3) + (35*b^2*(a + b*x)^3)/(4*a
^4) + (14*b^2*(a + b*x))/(3*a^2))/((a + b*x)^(7/2) - 2*a*(a + b*x)^(5/2) +
a^2*(a + b*x)^(3/2)) - (35*b^2*atanh((a + b*x)^(1/2)/a^(1/2)))/(4*a^(9/2))
```

sympy [B]   time = 8.57, size = 464, normalized size = 4.38

$$-\frac{6a^{\frac{89}{2}}b^{75}x^{75}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{21a^{\frac{87}{2}}b^{76}x^{76}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{140a^{\frac{85}{2}}b^{77}x^{77}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}+\frac{105a^{\frac{83}{2}}b^{78}x^{78}}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{42}b^{\frac{155}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}-\frac{105a^{41}b^{\frac{157}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{12a^{\frac{93}{2}}b^{\frac{151}{2}}x^{\frac{155}{2}}\sqrt{\frac{a}{bx}+1}+12a^{\frac{91}{2}}b^{\frac{153}{2}}x^{\frac{157}{2}}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**(89/2)*b**75*x**75/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) +
 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 21*a**(87/2)*
b**76*x**76/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(
91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/
(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(15
3/2)*x**(157/2)*sqrt(a/(b*x) + 1)) + 105*a**(83/2)*b**78*x**78/(12*a**(93/2
)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(a/(b*x) + 1)) - 105*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) + 1)*
asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a
/(b*x) + 1) + 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) + 1)) - 105*a
**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) + 1)*asinh(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) + 1) + 12*a**(91/2)*b**
(153/2)*x**(157/2)*sqrt(a/(b*x) + 1))
```

3.359 $\int \frac{1}{x\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{-a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[-a + b*x]),x]
[Out] (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.40, size = 58, normalized size = 2.32

$$\left[-\frac{\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)}{a}, \frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x)/a, 2*arctan(sqrt(b
*x - a)/sqrt(a))/sqrt(a)]
```

giac [A] time = 0.99, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

maple [A] time = 0.00, size = 20, normalized size = 0.80

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/2),x)
[Out] 2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 2.99, size = 19, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/2),x, algorithm="maxima")
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/sqrt(a)
```

mupad [B] time = 0.05, size = 19, normalized size = 0.76

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.23, size = 54, normalized size = 2.16

$$\begin{cases} \frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ -\frac{2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), Abs(a/(b*x)) > 1),
 (-2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a), True))
```

3.360 $\int \frac{1}{x^2\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{-a+bx}}\,dx&=\frac{\sqrt{-a+bx}}{ax}+\frac{b\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a}\\&=\frac{\sqrt{-a+bx}}{ax}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a}\\&=\frac{\sqrt{-a+bx}}{ax}+\frac{b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 1.20

$$\frac{b\sqrt{bx-a}\left(\frac{a}{bx}+\frac{\tanh^{-1}\left(\sqrt{1-\frac{bx}{a}}\right)}{\sqrt{1-\frac{bx}{a}}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-a + b*x]),x]
[Out] (b*Sqrt[-a + b*x]*(a/(b*x) + ArcTanh[Sqrt[1 - (b*x)/a]]/Sqrt[1 - (b*x)/a]))
/a^2
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\sqrt{bx-a}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[-a + b*x]),x]
[Out] Sqrt[-a + b*x]/(a*x) + (b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.91, size = 97, normalized size = 2.20

$$\left[-\frac{\sqrt{-a}\,bx\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\sqrt{bx-a}\,a}{2\,a^2x},\ \frac{\sqrt{a}\,bx\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="fricas")
[Out] [-1/2*(sqrt(-a)*b*x*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2*sqrt(
b*x - a)*a)/(a^2*x), (sqrt(a)*b*x*arctan(sqrt(b*x - a)/sqrt(a)) + sqrt(b*x
- a)*a)/(a^2*x)]
```

giac [A] time = 0.89, size = 43, normalized size = 0.98

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{bx-a}\,b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="giac")
[Out] (b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) + sqrt(b*x - a)*b/(a*x))/b
```

maple [A] time = 0.01, size = 37, normalized size = 0.84

$$\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{bx-a}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/2),x)
```

[Out] b*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)+(b*x-a)^(1/2)/a/x

maxima [A] time = 2.98, size = 46, normalized size = 1.05

$$\frac{\sqrt{bx-a}\,b}{(bx-a)a+a^2}+\frac{b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b*x - a)*b/((b*x - a)*a + a^2) + b*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/
2)
```

mupad [B] time = 0.04, size = 36, normalized size = 0.82

$$\frac{\sqrt{b\,x-a}}{a\,x}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/2)),x)
```

```
[Out] (b*x - a)^(1/2)/(a*x) + (b*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 2.46, size = 121, normalized size = 2.75

$$\begin{cases} \frac{i\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a\sqrt{x}}+\frac{ib\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{1}{\sqrt{b}x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{\sqrt{b}}{a\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I*sqrt(b)*sqrt(a/(b*x) - 1)/(a*sqrt(x)) + I*b*acosh(sqrt(a)/(sqr
t(b)*sqrt(x)))/a**(3/2), Abs(a/(b*x)) > 1), (-1/(sqrt(b)*x**(3/2)*sqrt(-a/(
b*x) + 1)) + sqrt(b)/(a*sqrt(x)*sqrt(-a/(b*x) + 1)) - b*asin(sqrt(a)/(sqrt(
b)*sqrt(x)))/a**(3/2), True))
```

3.361 $\int \frac{1}{x^3\sqrt{-a+bx}}\,dx$

Optimal. Leaf size=74

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3b\sqrt{bx-a}}{4a^2x}+\frac{\sqrt{bx-a}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] Sqrt[-a + b*x]/(2*a*x^2) + (3*b*Sqrt[-a + b*x])/(4*a^2*x) + (3*b^2*ArcTan[S
qrt[-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{-a+bx}}\,dx &= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{(3b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{\left(3b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{(3b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{4a^2}\\
&= \frac{\sqrt{-a+bx}}{2ax^2}+\frac{3b\sqrt{-a+bx}}{4a^2x}+\frac{3b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.49

$$\frac{2b^2\sqrt{bx-a}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (2*b^2*Sqrt[-a + b*x]*Hypergeometric2F1[1/2, 3, 3/2, 1 - (b*x)/a])/a^3
```

IntegrateAlgebraic [A] time = 0.07, size = 69, normalized size = 0.93

$$\frac{3b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{5/2}}+\frac{3(bx-a)^{3/2}+5a\sqrt{bx-a}}{4a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[-a + b*x]),x]
```

```
[Out] (5*a*Sqrt[-a + b*x] + 3*(-a + b*x)^(3/2))/(4*a^2*x^2) + (3*b^2*ArcTan[Sqrt[
-a + b*x]/Sqrt[a]])/(4*a^(5/2))
```

fricas [A] time = 1.36, size = 128, normalized size = 1.73

$$\left[-\frac{3\sqrt{-a}\,b^2x^2\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)-2\left(3abx+2a^2\right)\sqrt{bx-a}}{8a^3x^2},\ \frac{3\sqrt{a}\,b^2x^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx+2a^2\right)\sqrt{bx-a}}{4a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*sqrt(-a)*b^2*x^2*log((b*x - 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) - 2
*(3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2), 1/4*(3*sqrt(a)*b^2*x^2*arctan(
sqrt(b*x - a)/sqrt(a)) + (3*a*b*x + 2*a^2)*sqrt(b*x - a))/(a^3*x^2)]
```

giac [A] time = 0.97, size = 68, normalized size = 0.92

$$\frac{\frac{3b^3\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{3(bx-a)^{\frac{3}{2}}b^3+5\sqrt{bx-a}\,ab^3}{a^2b^2x^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + (3*(b*x - a)^(3/2)*b^3 +
 5*sqrt(b*x - a)*a*b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.01, size = 59, normalized size = 0.80

$$\frac{3b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{5}{2}}}+\frac{3\sqrt{bx-a}\,b}{4a^2x}+\frac{\sqrt{bx-a}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/2),x)
```

```
[Out] 3/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+1/2*(b*x-a)^(1/2)/a/x^2+3/4*b
*(b*x-a)^(1/2)/a^2/x
```

maxima [A] time = 2.96, size = 86, normalized size = 1.16

$$\frac{3\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{5}{2}}}+\frac{3\,(bx-a)^{\frac{3}{2}}b^2+5\,\sqrt{bx-a}\,ab^2}{4\left((bx-a)^2a^2+2\,(bx-a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 1/4*(3*(b*x - a)^(3/2)*b^2
+ 5*sqrt(b*x - a)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a^4)
```

mupad [B] time = 0.05, size = 57, normalized size = 0.77

$$\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{5/2}}+\frac{5\,\sqrt{b\,x-a}}{4\,a\,x^2}+\frac{3\,(b\,x-a)^{3/2}}{4\,a^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/2)),x)
```

```
[Out] (3*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(5/2)) + (5*(b*x - a)^(1/2))/(4*
a*x^2) + (3*(b*x - a)^(3/2))/(4*a^2*x^2)
```

sympy [A] time = 4.20, size = 216, normalized size = 2.92

$$\begin{cases}\frac{i}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{i\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{3ib^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{\frac{a}{bx}-1}}+\frac{3ib^2\operatorname{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{for}\ \left|\frac{a}{bx}\right|>1\\ -\frac{1}{2\sqrt{b}\,x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{\sqrt{b}}{4ax^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{3b^{\frac{3}{2}}}{4a^2\sqrt{x}\sqrt{-\frac{a}{bx}+1}}-\frac{3b^2\operatorname{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/2),x)
```

```
[Out] Piecewise((I/(2*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) + I*sqrt(b)/(4*a*x**(3/
2)*sqrt(a/(b*x) - 1)) - 3*I*b**(3/2)/(4*a**2*sqrt(x)*sqrt(a/(b*x) - 1)) + 3
*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), Abs(a/(b*x)) > 1), (
-1/(2*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) - sqrt(b)/(4*a*x**(3/2)*sqrt(-a/
(b*x) + 1)) + 3*b**(3/2)/(4*a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) - 3*b**2*asin(
sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(5/2)), True))
```

3.362 $\int \frac{1}{x(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(3/2)),x]
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(-a+bx)^{3/2}}\,dx &= -\frac{2}{a\sqrt{-a+bx}}-\frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a}\\
&= -\frac{2}{a\sqrt{-a+bx}}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{ab}\\
&= -\frac{2}{a\sqrt{-a+bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.79

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{bx}{a}\right)}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(3/2)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*x)/a])/(a*Sqrt[-a + b*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 42, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(3/2)),x]
[Out] -2/(a*Sqrt[-a + b*x]) - (2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 1.26, size = 124, normalized size = 2.95

$$\left[-\frac{(bx-a)\sqrt{-a}\,\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\sqrt{bx-a}\,a}{a^2bx-a^3},-\frac{2\left((bx-a)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\,a\right)}{a^2bx-a^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="fricas")
[Out] [-((b*x - a)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2*a)/x) + 2*sqr
t(b*x - a)*a)/(a^2*b*x - a^3), -2*((b*x - a)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + sqrt(b*x - a)*a)/(a^2*b*x - a^3)]
```

giac [A] time = 1.02, size = 34, normalized size = 0.81

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="giac")
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

maple [A] time = 0.01, size = 35, normalized size = 0.83

$$-\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(3/2),x)
[Out] -2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(3/2)-2/a/(b*x-a)^(1/2)
```

maxima [A] time = 2.97, size = 34, normalized size = 0.81

$$-\frac{2\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2}{\sqrt{bx-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(b*x - a)/sqrt(a))/a^(3/2) - 2/(sqrt(b*x - a)*a)
```

mupad [B] time = 0.10, size = 34, normalized size = 0.81

$$-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\,\sqrt{b\,x-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(3/2)),x)
```

```
[Out] - (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(3/2) - 2/(a*(b*x - a)^(1/2))
```

sympy [C] time = 2.20, size = 478, normalized size = 11.38

$$\begin{cases} \frac{2ia^3\sqrt{-1+\frac{bx}{a}}}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}-\frac{a^3\log\left(\frac{bx}{a}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}+\frac{2a^3\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}+\frac{2ia^3\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}+\frac{a^2bx\log\left(\frac{bx}{a}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}-\frac{2a^2bx\log\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx}-\frac{2ia^2bx\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{ia^{\frac{9}{2}}-ia^{\frac{7}{2}}bx} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{2a^3\sqrt{1-\frac{bx}{a}}}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}+\frac{a^3\log\left(\frac{bx}{a}\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}-\frac{2a^3\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}-\frac{i\pi a^3}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}-\frac{a^2bx\log\left(\frac{bx}{a}\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}+\frac{2a^2bx\log\left(\sqrt{1-\frac{bx}{a}}+1\right)}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx}+\frac{i\pi a^2bx}{-ia^{\frac{9}{2}}+ia^{\frac{7}{2}}bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((2*I*a**3*sqrt(-1 + b*x/a)/(I*a**(9/2) - I*a**(7/2)*b*x) - a**3*l
og(b*x/a)/(I*a**(9/2) - I*a**(7/2)*b*x) + 2*a**3*log(sqrt(b)*sqrt(x)/sqrt(a
))/(I*a**(9/2) - I*a**(7/2)*b*x) + 2*I*a**3*asin(sqrt(a)/(sqrt(b)*sqrt(x)))
/(I*a**(9/2) - I*a**(7/2)*b*x) + a**2*b*x*log(b*x/a)/(I*a**(9/2) - I*a**(7/
2)*b*x) - 2*a**2*b*x*log(sqrt(b)*sqrt(x)/sqrt(a))/(I*a**(9/2) - I*a**(7/2)*
b*x) - 2*I*a**2*b*x*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(I*a**(9/2) - I*a**(7/2
)*b*x), Abs(b*x/a) > 1), (2*a**3*sqrt(1 - b*x/a)/(-I*a**(9/2) + I*a**(7/2)*
b*x) + a**3*log(b*x/a)/(-I*a**(9/2) + I*a**(7/2)*b*x) - 2*a**3*log(sqrt(1 -
 b*x/a) + 1)/(-I*a**(9/2) + I*a**(7/2)*b*x) - I*pi*a**3/(-I*a**(9/2) + I*a*
*(7/2)*b*x) - a**2*b*x*log(b*x/a)/(-I*a**(9/2) + I*a**(7/2)*b*x) + 2*a**2*b
*x*log(sqrt(1 - b*x/a) + 1)/(-I*a**(9/2) + I*a**(7/2)*b*x) + I*pi*a**2*b*x/
(-I*a**(9/2) + I*a**(7/2)*b*x), True))
```

3.363 $\int \frac{1}{x^2(-a+bx)^{3/2}} \, dx$

Optimal. Leaf size=62

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3b}{a^2\sqrt{bx-a}}+\frac{1}{ax\sqrt{bx-a}}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{bx-a}}{a^2x}-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{ax\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x*Sqrt[-a + b*x]) - (3*Sqrt[-a + b*x])/(a^2*x) - (3*b*ArcTan[Sqrt[-a
+ b*x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx = -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{(3b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^2}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^2}$$
$$= -\frac{2}{ax\sqrt{-a+bx}} - \frac{3\sqrt{-a+bx}}{a^2x} - \frac{3b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.55

$$-\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{bx}{a}\right)}{a^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (b*x)/a])/(a^2*Sqrt[-a + b*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.95

$$-\frac{3b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{3(bx-a)+2a}{a^2x\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(3/2)),x]
```

```
[Out] -((2*a + 3*(-a + b*x))/(a^2*x*Sqrt[-a + b*x])) - (3*b*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.87, size = 164, normalized size = 2.65

$$\left[-\frac{3\left(b^2x^2-abx\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(3abx-a^2\right)\sqrt{bx-a}}{2\left(a^3bx^2-a^4x\right)}, -\frac{3\left(b^2x^2-abx\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3abx-a^2\right)\sqrt{bx-a}}{a^3bx^2-a^4x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^2*x^2 - a*b*x)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a) - 2
*a)/x) + 2*(3*a*b*x - a^2)*sqrt(b*x - a))/(a^3*b*x^2 - a^4*x), -(3*(b^2*x^2
 - a*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (3*a*b*x - a^2)*sqrt(b*x
- a))/(a^3*b*x^2 - a^4*x)]
```

giac [A] time = 1.01, size = 64, normalized size = 1.03

$$-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{3\,(bx-a)b+2\,ab}{\left((bx-a)^{\frac{3}{2}}+\sqrt{bx-a}\,a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) - (3*(b*x - a)*b + 2*a*b)/(((b*x
 - a)^(3/2) + sqrt(b*x - a)*a)*a^2)
```

maple [A] time = 0.01, size = 54, normalized size = 0.87

$$-\frac{3b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2b}{\sqrt{bx-a}\,a^{2}}-\frac{\sqrt{bx-a}}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(3/2),x)
```

```
[Out] -2*b/a^2/(b*x-a)^(1/2)-1/a^2*(b*x-a)^(1/2)/x-3*b*arctan((b*x-a)^(1/2)/a^(1/
2))/a^(5/2)
```

maxima [A] time = 3.02, size = 67, normalized size = 1.08

$$-\frac{3\,(bx-a)b+2\,ab}{(bx-a)^{\frac{3}{2}}a^{2}+\sqrt{bx-a}\,a^{3}}-\frac{3\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -(3*(b*x - a)*b + 2*a*b)/((b*x - a)^(3/2)*a^2 + sqrt(b*x - a)*a^3) - 3*b*ar
ctan(sqrt(b*x - a)/sqrt(a))/a^(5/2)
```

mupad [B] time = 0.06, size = 52, normalized size = 0.84

$$\frac{1}{a\,x\,\sqrt{b\,x-a}}-\frac{3\,b}{a^{2}\,\sqrt{b\,x-a}}-\frac{3\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(3/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(1/2)) - (3*b)/(a^2*(b*x - a)^(1/2)) - (3*b*atan((b*x - a)
^(1/2)/a^(1/2)))/a^(5/2)
```

sympy [A] time = 3.50, size = 156, normalized size = 2.52

$$\begin{cases} -\frac{i}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{3i\sqrt{b}}{a^{2}\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{3ib\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{1}{a\sqrt{b}\,x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{3\sqrt{b}}{a^{2}\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{3b\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(a*sqrt(b)*x**(3/2)*sqrt(a/(b*x) - 1)) + 3*I*sqrt(b)/(a**2*sq
rt(x)*sqrt(a/(b*x) - 1)) - 3*I*b*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/a**(5/2),
 Abs(a/(b*x)) > 1), (1/(a*sqrt(b)*x**(3/2)*sqrt(-a/(b*x) + 1)) - 3*sqrt(b)/
(a**2*sqrt(x)*sqrt(-a/(b*x) + 1)) + 3*b*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/a**
(5/2), True))
```

3.364 $\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx$

Optimal. Leaf size=95

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15b^2}{4a^3\sqrt{bx-a}}+\frac{5b}{4a^2x\sqrt{bx-a}}+\frac{1}{2ax^2\sqrt{bx-a}}$$

Rubi [A] time = 0.02, antiderivative size = 93, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{5\sqrt{bx-a}}{2a^2x^2}-\frac{15b\sqrt{bx-a}}{4a^3x}-\frac{2}{ax^2\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(3/2)),x]
```

```
[Out] -2/(a*x^2*Sqrt[-a + b*x]) - (5*Sqrt[-a + b*x])/(2*a^2*x^2) - (15*b*Sqrt[-a
+ b*x])/(4*a^3*x) - (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx &= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{a} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{(15b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^2} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{\left(15b^2\right)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{8a^3} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{(15b)\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{4a^3} \\
&= -\frac{2}{ax^2\sqrt{-a+bx}} - \frac{5\sqrt{-a+bx}}{2a^2x^2} - \frac{15b\sqrt{-a+bx}}{4a^3x} - \frac{15b^2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{4a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.38

$$
-\frac{2b^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};1-\frac{bx}{a}\right)}{a^3\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(3/2)),x]
[Out] (-2*b^2*Hypergeometric2F1[-1/2, 3, 1/2, 1 - (b*x)/a])/(a^3*Sqrt[-a + b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 79, normalized size = 0.83

$$
-\frac{15b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{7/2}} - \frac{8a^2+25a(bx-a)+15(bx-a)^2}{4a^3x^2\sqrt{bx-a}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(3/2)),x]
[Out] -1/4*(8*a^2 + 25*a*(-a + b*x) + 15*(-a + b*x)^2)/(a^3*x^2*Sqrt[-a + b*x]) -
 (15*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(7/2))
```

fricas [A] time = 0.89, size = 198, normalized size = 2.08

$$
\left[-\frac{15\left(b^3x^3-ab^2x^2\right)\sqrt{-a}\log\left(\frac{bx+2\sqrt{bx-a}\sqrt{-a}-2a}{x}\right)+2\left(15ab^2x^2-5a^2bx-2a^3\right)\sqrt{bx-a}}{8\left(a^4bx^3-a^5x^2\right)}, -\frac{15\left(b^3x^3-ab^2x^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(15ab^2x^2-5a^2bx-2a^3\right)\sqrt{bx-a}}{4\left(a^4bx^3-a^5x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="fricas")
[Out] [-1/8*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(-a)*log((b*x + 2*sqrt(b*x - a)*sqrt(-a
) - 2*a)/x) + 2*(15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^
3 - a^5*x^2), -1/4*(15*(b^3*x^3 - a*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/s
qrt(a)) + (15*a*b^2*x^2 - 5*a^2*b*x - 2*a^3)*sqrt(b*x - a))/(a^4*b*x^3 - a^
5*x^2)]
```

giac [A] time = 1.01, size = 81, normalized size = 0.85

$$-\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}-\frac{2\,b^2}{\sqrt{bx-a}\,a^3}-\frac{7\,(bx-a)^{\frac{3}{2}}b^2+9\,\sqrt{bx-a}\,ab^2}{4\,a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="giac")
```

```
[Out] -15/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) - 2*b^2/(sqrt(b*x - a)*a^3)
 - 1/4*(7*(b*x - a)^(3/2)*b^2 + 9*sqrt(b*x - a)*a*b^2)/(a^3*b^2*x^2)
```

maple [A] time = 0.01, size = 75, normalized size = 0.79

$$-\frac{15b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{7}{2}}}-\frac{2b^2}{\sqrt{bx-a}\,a^3}-\frac{9\sqrt{bx-a}}{4a^2x^2}-\frac{7\,(bx-a)^{\frac{3}{2}}}{4a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(3/2),x)
```

```
[Out] -2*b^2/a^3/(b*x-a)^(1/2)-7/4/a^3/x^2*(b*x-a)^(3/2)-9/4/a^2/x^2*(b*x-a)^(1/2
)-15/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

maxima [A] time = 2.93, size = 104, normalized size = 1.09

$$-\frac{15\,(bx-a)^2b^2+25\,(bx-a)ab^2+8\,a^2b^2}{4\left((bx-a)^{\frac{5}{2}}a^3+2\,(bx-a)^{\frac{3}{2}}a^4+\sqrt{bx-a}\,a^5\right)}-\frac{15\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(15*(b*x - a)^2*b^2 + 25*(b*x - a)*a*b^2 + 8*a^2*b^2)/((b*x - a)^(5/2)
*a^3 + 2*(b*x - a)^(3/2)*a^4 + sqrt(b*x - a)*a^5) - 15/4*b^2*arctan(sqrt(b*
x - a)/sqrt(a))/a^(7/2)
```

mupad [B] time = 0.13, size = 101, normalized size = 1.06

$$-\frac{\frac{2\,b^2}{a}+\frac{15\,b^2\,(a-b\,x)^2}{4\,a^3}-\frac{25\,b^2\,(a-b\,x)}{4\,a^2}}{2\,a\,(b\,x-a)^{3/2}+(b\,x-a)^{5/2}+a^2\,\sqrt{b\,x-a}}-\frac{15\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(3/2)),x)
```

```
[Out] - ((2*b^2)/a + (15*b^2*(a - b*x)^2)/(4*a^3) - (25*b^2*(a - b*x))/(4*a^2))/(
2*a*(b*x - a)^(3/2) + (b*x - a)^(5/2) + a^2*(b*x - a)^(1/2)) - (15*b^2*atan
((b*x - a)^(1/2)/a^(1/2)))/(4*a^(7/2))
```

sympy [A] time = 5.57, size = 226, normalized size = 2.38

$$\begin{cases} -\frac{i}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}-\frac{5i\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}+\frac{15ib^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{\frac{a}{bx}-1}}-\frac{15ib^2\,\mathrm{acosh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ \frac{1}{2a\sqrt{b}x^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}+\frac{5\sqrt{b}}{4a^2x^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}-\frac{15b^{\frac{3}{2}}}{4a^3\sqrt{x}\sqrt{-\frac{a}{bx}+1}}+\frac{15b^2\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{4a^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(3/2),x)
```

```
[Out] Piecewise((-I/(2*a*sqrt(b)*x**(5/2)*sqrt(a/(b*x) - 1)) - 5*I*sqrt(b)/(4*a**
2*x**(3/2)*sqrt(a/(b*x) - 1)) + 15*I*b**(3/2)/(4*a**3*sqrt(x)*sqrt(a/(b*x)
- 1)) - 15*I*b**2*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), Abs(a/(b*x
)) > 1), (1/(2*a*sqrt(b)*x**(5/2)*sqrt(-a/(b*x) + 1)) + 5*sqrt(b)/(4*a**2*x
**(3/2)*sqrt(-a/(b*x) + 1)) - 15*b**(3/2)/(4*a**3*sqrt(x)*sqrt(-a/(b*x) + 1
)) + 15*b**2*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(4*a**(7/2)), True))
```

3.365 $\int \frac{1}{x(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=60

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2}{a^2\sqrt{bx-a}}-\frac{2}{3a(bx-a)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{2}{a^2\sqrt{bx-a}}+\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*(-a + b*x)^(3/2)) + 2/(a^2*Sqrt[-a + b*x]) + (2*ArcTan[Sqrt[-a + b*
x]/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x(-a+bx)^{5/2}}\,dx = -\frac{2}{3a(-a+bx)^{3/2}} - \frac{\int \frac{1}{x(-a+bx)^{3/2}}\,dx}{a}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{\int \frac{1}{x\sqrt{-a+bx}}\,dx}{a^2}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{-a+bx}\right)}{a^2 b}$$
$$= -\frac{2}{3a(-a+bx)^{3/2}} + \frac{2}{a^2\sqrt{-a+bx}} + \frac{2\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.58

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*x)/a])/(3*a*(-a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.92

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2(a-3(bx-a))}{3a^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a - 3*(-a + b*x)))/(3*a^2*(-a + b*x)^(3/2)) + (2*ArcTan[Sqrt[-a + b*x]
/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.85, size = 182, normalized size = 3.03

$$\left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\left(3\,abx-4\,a^2\right)\sqrt{bx-a}}{3\left(a^3b^2x^2-2\,a^4bx+a^5\right)}, \frac{2\left(3\left(b^2x^2-2\,abx+a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(3\,abx-4\,a^2\right)\sqrt{bx-a}\right)}{3\left(a^3b^2x^2-2\,a^4bx+a^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a)*log((b*x - 2*sqrt(b*x - a)*sqrt
(-a) - 2*a)/x) - 2*(3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*
x + a^5), 2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqr
t(a)) + (3*a*b*x - 4*a^2)*sqrt(b*x - a))/(a^3*b^2*x^2 - 2*a^4*b*x + a^5)]
```

giac [A] time = 1.06, size = 42, normalized size = 0.70

$$\frac{2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2\,(3\,bx-4\,a)}{3\,(bx-a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.82

$$-\frac{2}{3 \left(b x -a \right)^{\frac{3}{2}} a}+\frac{2 \arctan \left(\frac{\sqrt{b x -a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2}{\sqrt{b x -a}\, a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(5/2),x)
```

```
[Out] -2/3/a/(b*x-a)^(3/2)+2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(5/2)+2/a^2/(b*x-a)^
(1/2)
```

maxima [A] time = 2.94, size = 42, normalized size = 0.70

$$\frac{2 \arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}+\frac{2 \, {\left(3 \, b x - 4 \, a\right)}}{3 \, {\left(b x - a\right)}^{\frac{3}{2}} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(b*x - a)/sqrt(a))/a^(5/2) + 2/3*(3*b*x - 4*a)/((b*x - a)^(3/2
)*a^2)
```

mupad [B] time = 0.09, size = 48, normalized size = 0.80

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2\,\left(a-b\,x\right)}{a^2}+\frac{2}{3\,a}}{{\left(b\,x-a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(5/2)),x)
```

```
[Out] (2*atan((b*x - a)^(1/2)/a^(1/2)))/a^(5/2) - ((2*(a - b*x))/a^2 + 2/(3*a))/(
b*x - a)^(3/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

3.366 $\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=81

$$\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{bx-a}}-\frac{5b}{3a^2(bx-a)^{3/2}}+\frac{1}{ax(bx-a)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{5\sqrt{bx-a}}{a^3x}+\frac{10}{3a^2x\sqrt{bx-a}}+\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2}{3ax(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x*(-a + b*x)^(3/2)) + 10/(3*a^2*x*Sqrt[-a + b*x]) + (5*Sqrt[-a + b*
x])/(a^3*x) + (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax(-a+bx)^{3/2}}-\frac{5\int \frac{1}{x^2(-a+bx)^{3/2}}\,dx}{3a}\\
&= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{a^2}\\
&= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{(5b)\int \frac{1}{x\sqrt{-a+bx}}\,dx}{2a^3}\\
&= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{-a+bx}\right)}{a^3}\\
&= -\frac{2}{3ax(-a+bx)^{3/2}}+\frac{10}{3a^2x\sqrt{-a+bx}}+\frac{5\sqrt{-a+bx}}{a^3x}+\frac{5b\tan^{-1}\left(\frac{\sqrt{-a+bx}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.44

$$
-\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^2(bx-a)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b*Hypergeometric2F1[-3/2, 2, -1/2, 1 - (b*x)/a])/(3*a^2*(-a + b*x)^(3/2
))
```

IntegrateAlgebraic [A] time = 0.07, size = 75, normalized size = 0.93

$$
\frac{5b\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2a^2-10a(bx-a)-15(bx-a)^2}{3a^3x(bx-a)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(5/2)),x]
```

```
[Out] -1/3*(2*a^2 - 10*a*(-a + b*x) - 15*(-a + b*x)^2)/(a^3*x*(-a + b*x)^(3/2)) +
 (5*b*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.83, size = 226, normalized size = 2.79

$$
\left[-\frac{15\left(b^3x^3-2\,ab^2x^2+a^2bx\right)\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{bx-a}\,\sqrt{-a}-2\,a}{x}\right)-2\left(15\,ab^2x^2-20\,a^2bx+3\,a^3\right)\sqrt{bx-a}}{6\left(a^4b^2x^3-2\,a^5bx^2+a^6x\right)},\ \frac{15\left(b^3x^3-2\,ab^2x^2+a^2bx\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(15\,ab^2x^2-20\,a^2bx+3\,a^3\right)\sqrt{bx-a}}{3\left(a^4b^2x^3-2\,a^5bx^2+a^6x\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^3*x^3 - 2*a*b^2*x^2 + a^2*b*x)*sqrt(-a)*log((b*x - 2*sqrt(b*x
- a)*sqrt(-a) - 2*a)/x) - 2*(15*a*b^2*x^2 - 20*a^2*b*x + 3*a^3)*sqrt(b*x -
a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x), 1/3*(15*(b^3*x^3 - 2*a*b^2*x^2 + a
^2*b*x)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a)) + (15*a*b^2*x^2 - 20*a^2*b*x
+ 3*a^3)*sqrt(b*x - a))/(a^4*b^2*x^3 - 2*a^5*b*x^2 + a^6*x)]
```

giac [A] time = 1.00, size = 66, normalized size = 0.81

$$\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{2\,(6\,(bx-a)b-ab)}{3\,(bx-a)^{\frac{3}{2}}a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2) + 2/3*(6*(b*x - a)*b - a*b)/((b*x
 - a)^(3/2)*a^3) + sqrt(b*x - a)/(a^3*x)
```

maple [A] time = 0.01, size = 68, normalized size = 0.84

$$-\frac{2b}{3\,(bx-a)^{\frac{3}{2}}\,a^{2}}+\frac{5b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}+\frac{4b}{\sqrt{bx-a}\;a^{3}}+\frac{\sqrt{bx-a}}{a^{3}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b/a^2/(b*x-a)^(3/2)+4*b/a^3/(b*x-a)^(1/2)+1/a^3*(b*x-a)^(1/2)/x+5*b*ar
ctan((b*x-a)^(1/2)/a^(1/2))/a^(7/2)
```

maxima [A] time = 2.93, size = 82, normalized size = 1.01

$$\frac{15\,(bx-a)^{2}b+10\,(bx-a)ab-2\,a^{2}b}{3\left((bx-a)^{\frac{5}{2}}a^{3}+(bx-a)^{\frac{3}{2}}a^{4}\right)}+\frac{5\,b\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(15*(b*x - a)^2*b + 10*(b*x - a)*a*b - 2*a^2*b)/((b*x - a)^(5/2)*a^3 +
(b*x - a)^(3/2)*a^4) + 5*b*arctan(sqrt(b*x - a)/sqrt(a))/a^(7/2)
```

mupad [B] time = 0.12, size = 70, normalized size = 0.86

$$\frac{1}{a\,x\,(b\,x-a)^{3/2}}-\frac{20\,b}{3\,a^{2}\,(b\,x-a)^{3/2}}+\frac{5\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5\,b^{2}\,x}{a^{3}\,(b\,x-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(5/2)),x)
```

```
[Out] 1/(a*x*(b*x - a)^(3/2)) - (20*b)/(3*a^2*(b*x - a)^(3/2)) + (5*b*atan((b*x -
 a)^(1/2)/a^(1/2)))/a^(7/2) + (5*b^2*x)/(a^3*(b*x - a)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(5/2),x)
```

```
[Out] Timed out
```

3.367 $\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b^2}{4a^4\sqrt{bx-a}}-\frac{35b^2}{12a^3(bx-a)^{3/2}}+\frac{7b}{4a^2x(bx-a)^{3/2}}+\frac{1}{2ax^2(bx-a)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 205}

$$\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35\sqrt{bx-a}}{6a^3x^2}+\frac{14}{3a^2x^2\sqrt{bx-a}}+\frac{35b\sqrt{bx-a}}{4a^4x}-\frac{2}{3ax^2(bx-a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] -2/(3*a*x^2*(-a + b*x)^(3/2)) + 14/(3*a^2*x^2*Sqrt[-a + b*x]) + (35*Sqrt[-a
 + b*x])/(6*a^3*x^2) + (35*b*Sqrt[-a + b*x])/(4*a^4*x) + (35*b^2*ArcTan[Sqr
t[-a + b*x]/Sqrt[a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(-a+bx)^{5/2}}\,dx &= -\frac{2}{3ax^2(-a+bx)^{3/2}} - \frac{7\int \frac{1}{x^3(-a+bx)^{3/2}}\,dx}{3a} \\
&= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\int \frac{1}{x^3\sqrt{-a+bx}}\,dx}{3a^2} \\
&= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{(35b)\int \frac{1}{x^2\sqrt{-a+bx}}\,dx}{4a^3} \\
&= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{\left(35b^2\right)\int \frac{1}{x\sqrt{\cdot}}}{8a^4} \\
&= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{(35b)\,\text{Subst}}{} \\
&= -\frac{2}{3ax^2(-a+bx)^{3/2}} + \frac{14}{3a^2x^2\sqrt{-a+bx}} + \frac{35\sqrt{-a+bx}}{6a^3x^2} + \frac{35b\sqrt{-a+bx}}{4a^4x} + \frac{35b^2\tan^{-1}\left(\right.}{4a^{9/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.33

$$
-\frac{2b^2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};1-\frac{bx}{a}\right)}{3a^3(bx-a)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] (-2*b^2*Hypergeometric2F1[-3/2, 3, -1/2, 1 - (b*x)/a])/(3*a^3*(-a + b*x)^(3
/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 93, normalized size = 0.80

$$
\frac{35b^2\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{9/2}} - \frac{8a^3-56a^2(bx-a)-175a(bx-a)^2-105(bx-a)^3}{12a^4x^2(bx-a)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(5/2)),x]
```

```
[Out] -1/12*(8*a^3 - 56*a^2*(-a + b*x) - 175*a*(-a + b*x)^2 - 105*(-a + b*x)^3)/(
a^4*x^2*(-a + b*x)^(3/2)) + (35*b^2*ArcTan[Sqrt[-a + b*x]/Sqrt[a]])/(4*a^(9
/2))
```

fricas [A] time = 0.96, size = 260, normalized size = 2.24

$$
\left[-\frac{105\left(b^4x^4-2\,ab^3x^3+a^2b^2x^2\right)\sqrt{-a}\log\left(\frac{bx-2\sqrt{bx-a}\sqrt{-a}-2\,a}{x}\right)-2\left(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4\right)\sqrt{bx-a}}{24\left(a^5b^2x^4-2\,a^6bx^3+a^7x^2\right)},\ \frac{105\left(b^4x^4-2\,ab^3x^3+a^2b^2x^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\left(105\,ab^3x^3-140\,a^2b^2x^2+21\,a^3bx+6\,a^4\right)\sqrt{bx-a}}{12\left(a^5b^2x^4-2\,a^6bx^3+a^7x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(105*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-a)*log((b*x - 2*sqr
t(b*x - a)*sqrt(-a) - 2*a)/x) - 2*(105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3
*b*x + 6*a^4)*sqrt(b*x - a))/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2), 1/12*(1
```

```
05*(b^4*x^4 - 2*a*b^3*x^3 + a^2*b^2*x^2)*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(
a)) + (105*a*b^3*x^3 - 140*a^2*b^2*x^2 + 21*a^3*b*x + 6*a^4)*sqrt(b*x - a))
/(a^5*b^2*x^4 - 2*a^6*b*x^3 + a^7*x^2)]
```

giac [A] time = 0.90, size = 97, normalized size = 0.84

$$\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}+\frac{2\left(9\,(bx-a)b^2-ab^2\right)}{3\,(bx-a)^{\frac{3}{2}}a^4}+\frac{11\,(bx-a)^{\frac{3}{2}}b^2+13\,\sqrt{bx-a}\,ab^2}{4\,a^4b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2) + 2/3*(9*(b*x - a)*b^2 - a*b
^2)/((b*x - a)^(3/2)*a^4) + 1/4*(11*(b*x - a)^(3/2)*b^2 + 13*sqrt(b*x - a)*
a*b^2)/(a^4*b^2*x^2)
```

maple [A] time = 0.02, size = 92, normalized size = 0.79

$$-\frac{2b^2}{3\,(bx-a)^{\frac{3}{2}}\,a^3}+\frac{35b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4a^{\frac{9}{2}}}+\frac{6b^2}{\sqrt{bx-a}\,a^4}+\frac{13\sqrt{bx-a}}{4a^3x^2}+\frac{11\,(bx-a)^{\frac{3}{2}}}{4a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(5/2),x)
```

```
[Out] -2/3*b^2/a^3/(b*x-a)^(3/2)+6*b^2/a^4/(b*x-a)^(1/2)+11/4/a^4/x^2*(b*x-a)^(3/
2)+13/4/a^3/x^2*(b*x-a)^(1/2)+35/4*b^2*arctan((b*x-a)^(1/2)/a^(1/2))/a^(9/2
)
```

maxima [A] time = 2.87, size = 121, normalized size = 1.04

$$\frac{105\,(bx-a)^3b^2+175\,(bx-a)^2ab^2+56\,(bx-a)a^2b^2-8\,a^3b^2}{12\left((bx-a)^{\frac{7}{2}}a^4+2\,(bx-a)^{\frac{5}{2}}a^5+(bx-a)^{\frac{3}{2}}a^6\right)}+\frac{35\,b^2\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)}{4\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(105*(b*x - a)^3*b^2 + 175*(b*x - a)^2*a*b^2 + 56*(b*x - a)*a^2*b^2 -
8*a^3*b^2)/((b*x - a)^(7/2)*a^4 + 2*(b*x - a)^(5/2)*a^5 + (b*x - a)^(3/2)*a
^6) + 35/4*b^2*arctan(sqrt(b*x - a)/sqrt(a))/a^(9/2)
```

mupad [B] time = 0.07, size = 117, normalized size = 1.01

$$\frac{35\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x-a}}{\sqrt{a}}\right)}{4\,a^{9/2}}-\frac{\frac{2\,b^2}{3\,a}-\frac{175\,b^2\,(a-b\,x)^2}{12\,a^3}+\frac{35\,b^2\,(a-b\,x)^3}{4\,a^4}+\frac{14\,b^2\,(a-b\,x)}{3\,a^2}}{2\,a\,(b\,x-a)^{5/2}+(b\,x-a)^{7/2}+a^2\,(b\,x-a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(5/2)),x)
```

```
[Out] (35*b^2*atan((b*x - a)^(1/2)/a^(1/2)))/(4*a^(9/2)) - ((2*b^2)/(3*a) - (175*
b^2*(a - b*x)^2)/(12*a^3) + (35*b^2*(a - b*x)^3)/(4*a^4) + (14*b^2*(a - b*x
))/(3*a^2))/(2*a*(b*x - a)^(5/2) + (b*x - a)^(7/2) + a^2*(b*x - a)^(3/2))
```

sympy [B] time = 11.22, size = 1108, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(5/2),x)
```

```
[Out] Piecewise((12*I*a**(89/2)*b**75*x**75/(24*a**(93/2)*b**(151/2)*x**(155/2)*s
qrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) +
42*I*a**(87/2)*b**76*x**76/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x)
 - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) - 280*I*a**(8
5/2)*b**77*x**77/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24
*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**(83/2)*b**78
*x**78/(24*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)
*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1)) + 210*I*a**42*b**(155/2)*x**(155/
2)*sqrt(a/(b*x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151
/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(
a/(b*x) - 1)) - 105*pi*a**42*b**(155/2)*x**(155/2)*sqrt(a/(b*x) - 1)/(24*a*
*(93/2)*b**(151/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x
**(157/2)*sqrt(a/(b*x) - 1)) - 210*I*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*
x) - 1)*acosh(sqrt(a)/(sqrt(b)*sqrt(x)))/(24*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(a/(b*x) - 1))
 + 105*pi*a**41*b**(157/2)*x**(157/2)*sqrt(a/(b*x) - 1)/(24*a**(93/2)*b**(1
51/2)*x**(155/2)*sqrt(a/(b*x) - 1) - 24*a**(91/2)*b**(153/2)*x**(157/2)*sqr
t(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-6*a**(89/2)*b**75*x**75/(12*a**(93/2)
*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157
/2)*sqrt(-a/(b*x) + 1)) - 21*a**(87/2)*b**76*x**76/(12*a**(93/2)*b**(151/2)
*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a
/(b*x) + 1)) + 140*a**(85/2)*b**77*x**77/(12*a**(93/2)*b**(151/2)*x**(155/2
)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1
)) - 105*a**(83/2)*b**78*x**78/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/
(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) - 105*a
**42*b**(155/2)*x**(155/2)*sqrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)
))/(12*a**(93/2)*b**(151/2)*x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b*
*(153/2)*x**(157/2)*sqrt(-a/(b*x) + 1)) + 105*a**41*b**(157/2)*x**(157/2)*s
qrt(-a/(b*x) + 1)*asin(sqrt(a)/(sqrt(b)*sqrt(x)))/(12*a**(93/2)*b**(151/2)*
x**(155/2)*sqrt(-a/(b*x) + 1) - 12*a**(91/2)*b**(153/2)*x**(157/2)*sqrt(-a/
(b*x) + 1)), True))
```

3.368 $\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}} dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {12, 74}

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}} dx &= \frac{1}{2} \int \frac{x^{-1+m}(2am+b(-1+2m)x)}{(a+bx)^{3/2}} dx \\ &= \frac{x^m}{\sqrt{a+bx}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)),x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}(2am+b(-1+2m)x)}{2(a+bx)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x)^(3/2)
),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^(-1 + m)*(2*a*m + b*(-1 + 2*m)*x))/(2*(a + b*x
)^(3/2)), x]
```

fricas [A] time = 0.91, size = 14, normalized size = 1.08

$$\frac{xx^{m-1}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="fri
cas")
```

```
[Out] x*x^(m - 1)/sqrt(b*x + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2m-1)x+2am)x^{m-1}}{2(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate(1/2*(b*(2*m - 1)*x + 2*a*m)*x^(m - 1)/(b*x + a)^(3/2), x)
```

maple [A] time = 0.01, size = 12, normalized size = 0.92

$$\frac{x^{m}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x)
```

```
[Out] x^m/(b*x+a)^(1/2)
```

maxima [A] time = 1.87, size = 11, normalized size = 0.85

$$\frac{x^{m}}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)^(3/2),x, algorithm="max
ima")
```

```
[Out] x^m/sqrt(b*x + a)
```

mupad [B] time = 0.41, size = 11, normalized size = 0.85

$$\frac{x^{m}}{\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(m - 1)*(2*a*m + b*x*(2*m - 1)))/(2*(a + b*x)^(3/2)),x)
```

```
[Out] x^m/(a + b*x)^(1/2)
```

sympy [C] time = 84.07, size = 78, normalized size = 6.00

$$\frac{mx^m\Gamma(m)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma(m+1)}+\frac{bxx^m(2m-1)\,\Gamma(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(-1+2*m)*x)/(b*x+a)**(3/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((3/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) + b*x*x**m*(2*m - 1)*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,)
, b*x*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

3.369 $\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx$

Optimal. Leaf size=13

$$\frac{x^m}{\sqrt{a+bx}}$$

Rubi [C] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 7.08, number of steps used = 5, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {67, 65}

$$\frac{x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};\frac{bx}{a}+1\right)}{\sqrt{a+bx}}-\frac{2mx^m\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(\frac{1}{2},1-m;\frac{3}{2};\frac{bx}{a}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[-(b*x^m)/(2*(a + b*x)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] (x^m*Hypergeometric2F1[-1/2, -m, 1/2, 1 + (b*x)/a])/((-((b*x)/a))^m*Sqrt[a
+ b*x]) - (2*m*x^m*Sqrt[a + b*x]*Hypergeometric2F1[1/2, 1 - m, 3/2, 1 + (b*
x)/a])/(a*(-((b*x)/a))^m)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[((-((b*c)
/d))^IntPart[m]*(b*x)^FracPart[m])/(-((d*x)/c))^FracPart[m], Int[(-((d*x)/c
))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
!IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]
```

Rubi steps

$$\begin{aligned}
\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx &= -\left(\frac{1}{2}b\int \frac{x^m}{(a+bx)^{3/2}}\,dx\right)+m\int \frac{x^{-1+m}}{\sqrt{a+bx}}\,dx\\
&= -\left(\frac{1}{2}\left(bx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^m}{(a+bx)^{3/2}}\,dx\right)-\frac{\left(bmx^m\left(-\frac{bx}{a}\right)^{-m}\right)\int \frac{\left(-\frac{bx}{a}\right)^{-1+m}}{\sqrt{a+bx}}\,d}{a}\\
&= \frac{x^m\left(-\frac{bx}{a}\right)^{-m}{}_2F_1\left(-\frac{1}{2},-m;\frac{1}{2};1+\frac{bx}{a}\right)}{\sqrt{a+bx}}-\frac{2mx^m\left(-\frac{bx}{a}\right)^{-m}\sqrt{a+bx}\,{}_2F_1\left(\frac{1}{2},1\right.}{a}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{x^m}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b*x],x]
```

```
[Out] x^m/Sqrt[a + b*x]
```

IntegrateAlgebraic [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \left(-\frac{bx^m}{2(a+bx)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx}}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqrt[a + b
*x],x]
```

```
[Out] Defer[IntegrateAlgebraic][-1/2*(b*x^m)/(a + b*x)^(3/2) + (m*x^(-1 + m))/Sqr
t[a + b*x], x]
```

fricas [A] time = 0.94, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="f
ricas")
```

```
[Out] x^m/sqrt(b*x + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{mx^{m-1}}{\sqrt{bx+a}} - \frac{bx^m}{2\,(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(m*x^(m - 1)/sqrt(b*x + a) - 1/2*b*x^m/(b*x + a)^(3/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int -\frac{b\,x^m}{2\,(bx+a)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{bx+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

```
[Out] int(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x)
```

maxima [A] time = 1.86, size = 11, normalized size = 0.85

$$\frac{x^m}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x^m/(b*x+a)^(3/2)+m*x^(-1+m)/(b*x+a)^(1/2),x, algorithm="m
axima")
```

```
[Out] x^m/sqrt(b*x + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{m\,x^{m-1}}{\sqrt{a+b\,x}}-\frac{b\,x^m}{2\,{\left(a+b\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)),x)
```

```
[Out] int((m*x^(m - 1))/(a + b*x)^(1/2) - (b*x^m)/(2*(a + b*x)^(3/2)), x)
```

sympy [C] time = 5.34, size = 73, normalized size = 5.62

$$\frac{mx^m\Gamma(m)_2F_1\left(\begin{matrix}\frac{1}{2},m\\m+1\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{\sqrt{a}\,\Gamma\,(m+1)}-\frac{bxx^m\Gamma\,(m+1)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},m+1\\m+2\end{matrix}\middle|\frac{bxe^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*b*x**m/(b*x+a)**(3/2)+m*x**(-1+m)/(b*x+a)**(1/2),x)
```

```
[Out] m*x**m*gamma(m)*hyper((1/2, m), (m + 1,), b*x*exp_polar(I*pi)/a)/(sqrt(a)*g
amma(m + 1)) - b*x*x**m*gamma(m + 1)*hyper((3/2, m + 1), (m + 2,), b*x*exp_
polar(I*pi)/a)/(2*a**(3/2)*gamma(m + 2))
```

3.370 $\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {7, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 && !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{\frac{1-n}{2}+\frac{1}{2}(-3+n)}}{\sqrt{a+bx}}\,dx &= \int \frac{1}{x\sqrt{a+bx}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^((1 - n)/2 + (-3 + n)/2)/Sqrt[a + b*x],x]
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

fricas [A] time = 1.11, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

giac [A] time = 0.85, size = 21, normalized size = 0.91

$$\frac{2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 2.99, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)
```

mupad [B] time = 0.00, size = 17, normalized size = 0.74

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{a+b\,x}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)
```

sympy [A] time = 1.09, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

3.371 $\int x^3 \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{13/3}}{13b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a + b*x)^(7/3))/(7*b^4) - (9*a*(
a + b*x)^(10/3))/(10*b^4) + (3*(a + b*x)^(13/3))/(13*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt[3]{a+bx}\,dx &= \int\left(-\frac{a^3\sqrt[3]{a+bx}}{b^3}+\frac{3a^2(a+bx)^{4/3}}{b^3}-\frac{3a(a+bx)^{7/3}}{b^3}+\frac{(a+bx)^{10/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{4/3}}{4b^4}+\frac{9a^2(a+bx)^{7/3}}{7b^4}-\frac{9a(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{13/3}}{13b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{4/3}\left(-81a^3+108a^2bx-126ab^2x^2+140b^3x^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-81*a^3 + 108*a^2*b*x - 126*a*b^2*x^2 + 140*b^3*x^3))/(
1820*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{4/3}\left(-455a^3+780a^2(a+bx)-546a(a+bx)^2+140(a+bx)^3\right)}{1820b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-455*a^3 + 780*a^2*(a + b*x) - 546*a*(a + b*x)^2 + 140*
(a + b*x)^3))/(1820*b^4)
```

fricas [A] time = 0.84, size = 53, normalized size = 0.74

$$\frac{3\left(140\,b^{4}x^{4}+14\,ab^{3}x^{3}-18\,a^{2}b^{2}x^{2}+27\,a^{3}bx-81\,a^{4}\right)(bx+a)^{\frac{1}{3}}}{1820\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(140*b^4*x^4 + 14*a*b^3*x^3 - 18*a^2*b^2*x^2 + 27*a^3*b*x - 81*a^4)*
(b*x + a)^(1/3)/b^4
```

giac [B] time = 0.87, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^{2}-140\,(bx+a)^{\frac{1}{3}}a^{3}\right)a}{b^{3}}+\frac{4\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^{2}-455\,(bx+a)^{\frac{4}{3}}a^{3}+455\,(bx+a)^{\frac{1}{3}}a^{4}\right)}{b^{3}}\right)}{1820\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/1820*(13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^3 + 4*(35*(b*x + a)^(13/3) - 182*(b*x
+ a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*(b*
x + a)^(1/3)*a^4)/b^3)/b
```

maple [A] time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{4}{3}}\left(-140b^{3}x^{3}+126a\,b^{2}x^{2}-108a^{2}bx+81a^{3}\right)}{1820b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(1/3),x)
```

```
[Out] -3/1820*(b*x+a)^(4/3)*(-140*b^3*x^3+126*a*b^2*x^2-108*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.39, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^{4}}-\frac{9\,(bx+a)^{\frac{10}{3}}a}{10\,b^{4}}+\frac{9\,(bx+a)^{\frac{7}{3}}a^{2}}{7\,b^{4}}-\frac{3\,(bx+a)^{\frac{4}{3}}a^{3}}{4\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^4 - 9/10*(b*x + a)^(10/3)*a/b^4 + 9/7*(b*x + a)^(7/
3)*a^2/b^4 - 3/4*(b*x + a)^(4/3)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{13/3}}{13\,b^{4}}-\frac{3\,a^{3}\,(a+b\,x)^{4/3}}{4\,b^{4}}+\frac{9\,a^{2}\,(a+b\,x)^{7/3}}{7\,b^{4}}-\frac{9\,a\,(a+b\,x)^{10/3}}{10\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(13/3))/(13*b^4) - (3*a^3*(a + b*x)^(4/3))/(4*b^4) + (9*a^2*(a
 + b*x)^(7/3))/(7*b^4) - (9*a*(a + b*x)^(10/3))/(10*b^4)
```

<u>**sympy**</u> **[B]** time = 2.83, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(73/3)*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 2
7300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 1092
0*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(73/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 1377*
a**(70/3)*b*x*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27
300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920
*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1458*a**(70/3)*b*x/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) - 3
213*a**(67/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b
**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x*
*4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 3645*a**(67/3)*b**2*x
**2/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a
**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14
*b**10*x**6) - 3927*a**(64/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1820*a**20*b**4
 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 273
00*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 4860*
a**(64/3)*b**3*x**3/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**
6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x
**5 + 1820*a**14*b**10*x**6) - 2163*a**(61/3)*b**4*x**4*(1 + b*x/a)**(1/3)/
(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17
*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**
10*x**6) + 3645*a**(61/3)*b**4*x**4/(1820*a**20*b**4 + 10920*a**19*b**5*x +
 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10
920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 1827*a**(58/3)*b**5*x**5*(1
+ b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x*
*2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5
+ 1820*a**14*b**10*x**6) + 1458*a**(58/3)*b**5*x**5/(1820*a**20*b**4 + 1092
0*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**1
6*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6573*a**(55/
3)*b**6*x**6*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 273
00*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*
a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 243*a**(55/3)*b**6*x**6/(1820*a*
*20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x*
*3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
 + 8787*a**(52/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**
19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**
8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6) + 6498*a**(49/3)*b*
*8*x**8*(1 + b*x/a)**(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a*
*18*b**6*x**2 + 36400*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15
*b**9*x**5 + 1820*a**14*b**10*x**6) + 2562*a**(46/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(1820*a**20*b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 3640
0*a**17*b**7*x**3 + 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a*
*14*b**10*x**6) + 420*a**(43/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(1820*a**20*
b**4 + 10920*a**19*b**5*x + 27300*a**18*b**6*x**2 + 36400*a**17*b**7*x**3 +
 27300*a**16*b**8*x**4 + 10920*a**15*b**9*x**5 + 1820*a**14*b**10*x**6)
```

3.372 $\int x^2 \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{4/3}}{4b^3}+\frac{3(a+bx)^{10/3}}{10b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(4/3))/(4*b^3) - (6*a*(a + b*x)^(7/3))/(7*b^3) + (3*(a + b
*x)^(10/3))/(10*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \sqrt[3]{a+bx}\,dx &= \int \left(\frac{a^2\sqrt[3]{a+bx}}{b^2}-\frac{2a(a+bx)^{4/3}}{b^2}+\frac{(a+bx)^{7/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{4/3}}{4b^3}-\frac{6a(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{10/3}}{10b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{4/3}\left(9a^2-12abx+14b^2x^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(9*a^2 - 12*a*b*x + 14*b^2*x^2))/(140*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{4/3}\left(35a^2-40a(a+bx)+14(a+bx)^2\right)}{140b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(35*a^2 - 40*a*(a + b*x) + 14*(a + b*x)^2))/(140*b^3)
```

fricas [A] time = 0.86, size = 42, normalized size = 0.79

$$\frac{3\left(14\,b^{3}x^{3}+2\,ab^{2}x^{2}-3\,a^{2}bx+9\,a^{3}\right)(bx+a)^{\frac{1}{3}}}{140\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/140*(14*b^3*x^3 + 2*a*b^2*x^2 - 3*a^2*b*x + 9*a^3)*(b*x + a)^(1/3)/b^3
```

giac [B] time = 0.83, size = 92, normalized size = 1.74

$$\frac{3\left(\frac{10\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^{2}\right)a}{b^{2}}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^{2}-140\,(bx+a)^{\frac{1}{3}}a^{3}}{b^{2}}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/140*(10*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a/b^2 + (14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)
*a^2 - 140*(b*x + a)^(1/3)*a^3)/b^2)/b
```

maple [A] time = 0.01, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{4}{3}}\left(14b^{2}x^{2}-12abx+9a^{2}\right)}{140b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/3),x)
[Out] 3/140*(b*x+a)^(4/3)*(14*b^2*x^2-12*a*b*x+9*a^2)/b^3
```

maxima [A] time = 1.30, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^{3}}-\frac{6\,(bx+a)^{\frac{7}{3}}a}{7\,b^{3}}+\frac{3\,(bx+a)^{\frac{4}{3}}a^{2}}{4\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/10*(b*x + a)^(10/3)/b^3 - 6/7*(b*x + a)^(7/3)*a/b^3 + 3/4*(b*x + a)^(4/3)
*a^2/b^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{42\,(a+b\,x)^{10/3}-120\,a\,(a+b\,x)^{7/3}+105\,a^{2}\,(a+b\,x)^{4/3}}{140\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(1/3),x)
[Out] (42*(a + b*x)^(10/3) - 120*a*(a + b*x)^(7/3) + 105*a^2*(a + b*x)^(4/3))/(14
0*b^3)
```

sympy [**B**] time = 1.86, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{27a^{\frac{34}{3}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{72a^{\frac{31}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{81a^{\frac{31}{3}}bx}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{60a^{\frac{28}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{81a^{\frac{28}{3}}b^2x^2}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}-\frac{27a^{\frac{25}{3}}b^3x^3}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{135a^{\frac{22}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{132a^{\frac{19}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}+\frac{42a^{\frac{16}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{140a^8b^3+420a^7b^4x+420a^6b^5x^2+140a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(34/3)*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) - 27*a**(34/3)/(140*a**8*b**3 + 420*a**7*b
**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 72*a**(31/3)*b*x*(1 + b*
x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**
5*b**6*x**3) - 81*a**(31/3)*b*x/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6
*b**5*x**2 + 140*a**5*b**6*x**3) + 60*a**(28/3)*b**2*x**2*(1 + b*x/a)**(1/3
)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**
3) - 81*a**(28/3)*b**2*x**2/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**
5*x**2 + 140*a**5*b**6*x**3) + 60*a**(25/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(1
40*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) -
 27*a**(25/3)*b**3*x**3/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x*
*2 + 140*a**5*b**6*x**3) + 135*a**(22/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(140*
a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 13
2*a**(19/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x +
 420*a**6*b**5*x**2 + 140*a**5*b**6*x**3) + 42*a**(16/3)*b**6*x**6*(1 + b*x
/a)**(1/3)/(140*a**8*b**3 + 420*a**7*b**4*x + 420*a**6*b**5*x**2 + 140*a**5
*b**6*x**3)
```

3.373 $\int x\sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{7/3}}{7b^2}-\frac{3a(a+bx)^{4/3}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*a*(a + b*x)^(4/3))/(4*b^2) + (3*(a + b*x)^(7/3))/(7*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{a+bx}\,dx&=\int\left(-\frac{a\sqrt[3]{a+bx}}{b}+\frac{(a+bx)^{4/3}}{b}\right)dx\\&=-\frac{3a(a+bx)^{4/3}}{4b^2}+\frac{3(a+bx)^{7/3}}{7b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{4/3}(4bx-3a)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*(-3*a + 4*b*x))/(28*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 35, normalized size = 1.03

$$-\frac{3\sqrt[3]{a+bx}\left(3a^2-abx-4b^2x^2\right)}{28b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(1/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3)*(3*a^2 - a*b*x - 4*b^2*x^2))/(28*b^2)
```

fricas [A] time = 0.91, size = 30, normalized size = 0.88

$$\frac{3\left(4\,b^2x^2+abx-3\,a^2\right)(bx+a)^{\frac{1}{3}}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^2*x^2 + a*b*x - 3*a^2)*(b*x + a)^(1/3)/b^2
```

giac [B] time = 1.04, size = 67, normalized size = 1.97

$$\frac{3\left(\frac{7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a}{b}+\frac{2\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)}{b}\right)}{28\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(7*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a/b + 2*(2*(b*x + a)^(7/3)
- 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{4}{3}}\,(-4bx+3a)}{28b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/3),x)
```

```
[Out] -3/28*(b*x+a)^(4/3)*(-4*b*x+3*a)/b^2
```

maxima [A] time = 1.29, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^2}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^2 - 3/4*(b*x + a)^(4/3)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{21\,a\,{\left(a+b\,x\right)}^{4/3}-12\,{\left(a+b\,x\right)}^{7/3}}{28\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(1/3),x)
```

```
[Out] -(21*a*(a + b*x)^(4/3) - 12*(a + b*x)^(7/3))/(28*b^2)
```

sympy [B] time = 1.20, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{13}{3}}}{28a^2b^2+28ab^3x}-\frac{6a^{\frac{10}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{9a^{\frac{10}{3}}bx}{28a^2b^2+28ab^3x}+\frac{15a^{\frac{7}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}+\frac{12a^{\frac{4}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{28a^2b^2+28ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(13/3)*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 9*a**(13/3)/
(28*a**2*b**2 + 28*a*b**3*x) - 6*a**(10/3)*b*x*(1 + b*x/a)**(1/3)/(28*a**2*
b**2 + 28*a*b**3*x) + 9*a**(10/3)*b*x/(28*a**2*b**2 + 28*a*b**3*x) + 15*a**
(7/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x) + 12*a**(4/
3)*b**3*x**3*(1 + b*x/a)**(1/3)/(28*a**2*b**2 + 28*a*b**3*x)
```

3.374 $\int \sqrt[3]{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{4/3}}{4b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt[3]{a+bx}\,dx = \frac{3(a+bx)^{4/3}}{4b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

fricas [A] time = 0.96, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b
```

giac [A] time = 0.89, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/4*(b*x + a)^(4/3)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3),x)
[Out] 3/4*(b*x+a)^(4/3)/b
```

maxima [A] time = 1.26, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x + a)^(4/3)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{4/3}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3),x)
[Out] (3*(a + b*x)^(4/3))/(4*b)
```

sympy [A] time = 0.07, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{4}{3}}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3),x)
[Out] 3*(a + b*x)**(4/3)/(4*b)
```

3.375 $\int \frac{\sqrt[3]{a+bx}}{x} \, dx$

Optimal. Leaf size=91

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Rubi [A] time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{a+bx}+\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}\sqrt[3]{a}\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(S
qrt[3]*a^(1/3))] - (a^(1/3)*Log[x])/2 + (3*a^(1/3)*Log[a^(1/3) - (a + b*x)^
(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{x}\,dx = 3\sqrt[3]{a+bx} + a\int \frac{1}{x(a+bx)^{2/3}}\,dx$$
$$= 3\sqrt[3]{a+bx} - \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \right.$$
$$= 3\sqrt[3]{a+bx} - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\right.$$
$$= 3\sqrt[3]{a+bx} - \sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

Mathematica [A] time = 0.05, size = 113, normalized size = 1.24

$$-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+3\sqrt[3]{a+bx}+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3)
)/Sqrt[3]] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[a^(2/3)
+ a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

IntegrateAlgebraic [A] time = 0.07, size = 116, normalized size = 1.27

$$-\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+3\sqrt[3]{a+bx}+\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x,x]
```

```
[Out] 3*(a + b*x)^(1/3) - Sqrt[3]*a^(1/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/
(Sqrt[3]*a^(1/3))] + a^(1/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(1/3)*Log[
a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

fricas [A] time = 1.08, size = 91, normalized size = 1.00

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

giac [A] time = 2.37, size = 87, normalized size = 0.96

$$-\sqrt{3}\,a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\,a^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3*(b*x + a)^(1/3)
```

maple [A] time = 0.01, size = 85, normalized size = 0.93

$$-\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{1}{3}} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{1}{3}} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x,x)
```

```
[Out] 3*(b*x+a)^(1/3)+a^(1/3)*ln((b*x+a)^(1/3)-a^(1/3))-1/2*a^(1/3)*ln((b*x+a)^(2
/3)+a^(1/3)*(b*x+a)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^
(1/3)*(b*x+a)^(1/3)+1))
```

maxima [A] time = 3.08, size = 86, normalized size = 0.95

$$-\sqrt{3}\, a^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)-\frac{1}{2}\, a^{\frac{1}{3}} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{1}{3}} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(1/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(1/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)
```

mupad [B] time = 0.12, size = 107, normalized size = 1.18

$$a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)+3\,(a+b\,x)^{1/3}+\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{a^{1/3}\,\ln\left(9\,a\,(a+b\,x)^{1/3}+\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x,x)
```

```
[Out] a^(1/3)*log(9*a*(a + b*x)^(1/3) - 9*a^(4/3)) + 3*(a + b*x)^(1/3) + (a^(1/3)
*log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1)
)/2 - (a^(1/3)*log(9*a*(a + b*x)^(1/3) + (9*a^(4/3)*(3^(1/2)*1i + 1))/2)*(3
^(1/2)*1i + 1))/2
```

sympy [C] time = 2.02, size = 180, normalized size = 1.98

$$\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}\,e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(3*gamma(
7/3)) + 4*a**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pol
ar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)
/(3*gamma(7/3)) + 4*b**(1/3)*(a/b + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

3.376 $\int \frac{\sqrt[3]{a+bx}}{x^2} dx$

Optimal. Leaf size=97

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}}-\frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)) - (b*Log[x])/(6*a^(2/3)) + (b*Log[a^(1/3) - (a +
b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx}}{x} + \frac{1}{3}b\int \frac{1}{x(a+bx)^{2/3}}\,dx$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a\ldots}\right.}{2\sqrt[3]{a}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt[3]{a+bx}}{x} - \frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{b\log(x)}{6a^{2/3}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.34

$$\frac{3b(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},2;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, 1 + (b*x)/a])/(4*a^2)
```

IntegrateAlgebraic [A] time = 0.16, size = 125, normalized size = 1.29

$$\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}} - \frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{2/3}} - \frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}} - \frac{\sqrt[3]{a+bx}}{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x^2,x]
```

```
[Out] -((a + b*x)^(1/3)/x) - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(2/3)) + (b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3)
) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(6*a^(2/3)
)
```

fricas [A] time = 0.76, size = 139, normalized size = 1.43

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}abx\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+6(bx+a)^{\frac{1}{3}}a^2}{6a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(a^2)^(1/6)*a*b*x*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2)^(1/
3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*b*x*log((b
*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2
/3)*b*x*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x + a)^(1/3)*a^2)/(a^2*
x)
```

giac [A] time = 2.55, size = 105, normalized size = 1.08

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{6(bx+a)^{\frac{1}{3}}b}{x}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(2/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x + a)^(1
/3)*b/x)/b
```

maple [A] time = 0.01, size = 92, normalized size = 0.95

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{2}{3}}}+\frac{b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^2,x)
```

```
[Out] -(b*x+a)^(1/3)/x+1/3*b/a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/6*b/a^(2/3)*ln(
a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-1/3*b/a^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.96, size = 93, normalized size = 0.96

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{2}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{2}{3}}}+\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{2}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(2/3) - 1/6*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2
/3) + 1/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - (b*x + a)^(1/3)/x
```

mupad [B] time = 0.07, size = 117, normalized size = 1.21

$$\frac{b\ln\left(3b(a+bx)^{1/3}-3a^{1/3}b\right)}{3a^{2/3}}-\frac{(a+bx)^{1/3}}{x}-\frac{\ln\left(\frac{3a^{1/3}(b-\sqrt{3}\,b\,1\mathrm{i})}{2}+3b(a+bx)^{1/3}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6a^{2/3}}-\frac{\ln\left(\frac{3a^{1/3}(b+\sqrt{3}\,b\,1\mathrm{i})}{2}+3b(a+bx)^{1/3}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^2,x)
```

```
[Out] (b*log(3*b*(a + b*x)^(1/3) - 3*a^(1/3)*b))/(3*a^(2/3)) - (a + b*x)^(1/3)/x
- (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)^(1/3))*(b - 3^(1/2)
*b*1i))/(6*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1/2)*b*1i))/2 + 3*b*(a + b*x)
^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(2/3))
```

sympy [C] time = 2.19, size = 643, normalized size = 6.63

$$\frac{4a^{\frac{7}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{4a^{\frac{7}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}-\frac{4a^{\frac{4}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{12a^2b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{4}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**2,x)
```

```
[Out] 4*a**(7/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*
gamma(7/3)) + 4*a**(7/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*
pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b
 + x)*exp(2*I*pi/3)*gamma(7/3)) + 4*a**(7/3)*b*exp(-2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2
*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(
4/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3
))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**
(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(
7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(4/3)*b**2*(a/b
+ x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a
**(1/3))*gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*e
xp(2*I*pi/3)*gamma(7/3)) + 12*a**2*b**(4/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*
gamma(4/3)/(9*a**3*exp(2*I*pi/3)*gamma(7/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi
/3)*gamma(7/3))
```

3.377 $\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx$

Optimal. Leaf size=127

$$\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}$$

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 57, 617, 204, 31}

$$\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -(a + b*x)^(1/3)/(2*x^2) - (b*(a + b*x)^(1/3))/(6*a*x) + (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) + (b^2*Log[
x])/(18*a^(5/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{a+bx}}{x^3}\,dx &= -\frac{\sqrt[3]{a+bx}}{2x^2}+\frac{1}{6}b\int \frac{1}{x^2(a+bx)^{2/3}}\,dx \\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}-\frac{b^2\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a} \\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{5/3}}+\frac{b^2\,\text{Subst}\left(\int \frac{}{a^{2/3}\ldots}\right.}{} \\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\ldots\right.}{3a^{5/3}} \\
&= -\frac{\sqrt[3]{a+bx}}{2x^2}-\frac{b\sqrt[3]{a+bx}}{6ax}+\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}+\frac{b^2\log(x)}{18a^{5/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{5/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.28

$$
-\frac{3b^2(a+bx)^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};\frac{bx}{a}+1\right)}{4a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, 1 + (b*x)/a])/(4*a^3
)
```

IntegrateAlgebraic [A] time = 0.22, size = 145, normalized size = 1.14

$$
-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{5/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{5/3}}+\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}\,(3a+bx)}{6ax^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(3*a + b*x))/(a*x^2) + (b^2*ArcTan[1/Sqrt[3] + (2*(a
+ b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)) - (b^2*Log[a^(1/3) -
(a + b*x)^(1/3)])/(9*a^(5/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)])/(18*a^(5/3))
```

fricas [A] time = 0.98, size = 187, normalized size = 1.47

$$
\frac{2\sqrt{3}\,ab^2x^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3\left(a^2bx+3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^3x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*a*b^2*x^2*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(
1/3)*a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) +
 (-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)
*(b*x + a)^(1/3)) - 2*(-a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (-a^2)^(
2/3)) - 3*(a^2*b*x + 3*a^3)*(b*x + a)^(1/3))/(a^3*x^2)
```

giac [A] time = 2.43, size = 128, normalized size = 1.01

$$\frac{\frac{2\,\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2\,b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3\left((bx+a)^{\frac{4}{3}}b^3+2\,(bx+a)^{\frac{1}{3}}ab^3\right)}{ab^2x^2}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="giac")
```

```
[Out] 1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(5/3) - 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*((b*x + a)^(
4/3)*b^3 + 2*(b*x + a)^(1/3)*a*b^3)/(a*b^2*x^2))/b
```

maple [A] time = 0.01, size = 113, normalized size = 0.89

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{5}{3}}}-\frac{b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{5}{3}}}+\frac{b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{3x^2}-\frac{(bx+a)^{\frac{4}{3}}}{6a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/x^3,x)
```

```
[Out] -1/6/x^2/a*(b*x+a)^(4/3)-1/3*(b*x+a)^(1/3)/x^2-1/9*b^2/a^(5/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))+1/18*b^2/a^(5/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2
/3))+1/9*b^2/a^(5/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1)
)
```

maxima [A] time = 2.99, size = 139, normalized size = 1.09

$$\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{5}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{4}{3}}b^2+2\,(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2a-2\,(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/x^3,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(5/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(5/3) - 1/6*((b*x + a)^(
4/3)*b^2 + 2*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3)
```

mupad [B] time = 0.23, size = 196, normalized size = 1.54

$$\frac{b^2\,\ln\left(\frac{b^2}{(-a)^{2/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)}{9\,(-a)^{5/3}}-\frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1\mathrm{i}}{2\,(-a)^{2/3}}+\frac{b^2\,(a+b\,x)^{1/3}}{a}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18\,(-a)^{5/3}}-\frac{\frac{b^2\,(a+b\,x)^{1/3}}{3}+\frac{b^2\,(a+b\,x)^{4/3}}{6\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}+\frac{b^2\,\ln\left(\frac{b^2\,(a+b\,x)^{1/3}}{a}-\frac{b^2\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{2/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/x^3,x)
```

```
[Out] (b^2*log(b^2/(-a)^(2/3) - (b^2*(a + b*x)^(1/3))/a))/(9*(-a)^(5/3)) - (log((
3^(1/2)*b^2*1i + b^2)/(2*(-a)^(2/3)) + (b^2*(a + b*x)^(1/3))/a)*(3^(1/2)*b^
2*1i + b^2))/(18*(-a)^(5/3)) - ((b^2*(a + b*x)^(1/3))/3 + (b^2*(a + b*x)^(4
/3))/(6*a))/((a + b*x)^2 - 2*a*(a + b*x) + a^2) + (b^2*log((b^2*(a + b*x)^(
1/3))/a - (b^2*((3^(1/2)*1i)/2 - 1/2))/(-a)^(2/3))*((3^(1/2)*1i)/2 - 1/2))/
(9*(-a)^(5/3))
```

sympy [C] time = 2.58, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/x**3,x)
```

```
[Out] -4*a**(16/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I
*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a*
*7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3)
+ 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(7/3)) - 4*a**(16/3)*b**2*exp(-2*I*pi/3)*log(1 - b
**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)
*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(
a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi
/3)*gamma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(
7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x
)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(7/3)) + 12*a**(13/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b
 + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3
)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*
(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*p
i/3)*gamma(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**
(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/
3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)*
*2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma
(7/3)) - 12*a**(10/3)*b**4*(a/b + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*e
xp_polar(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*e
xp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3
)) - 12*a**(10/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b +
x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*g
amma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/
b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3
)*gamma(7/3)) + 4*a**(7/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
) + 4*a**(7/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**
6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*
pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 4*
```

```
a**(7/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
*exp_polar(4*I*pi/3)/a**(1/3))*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3)
 - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2
*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7
/3)) - 12*a**5*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*
exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 8
1*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)*
*3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**4*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) - 81*a**6*b*(a/b + x)*exp(
2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(7/3) -
 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)) + 6*a**3*b**(13/3)*(a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(4/3)/(27*a**7*exp(2*I*pi/3)*gamma(7/3) -
81*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(7/3) + 81*a**5*b**2*(a/b + x)**2*ex
p(2*I*pi/3)*gamma(7/3) - 27*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(7/3)
)
```

3.378 $\int x^3(a+bx)^{2/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{14/3}}{14b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a + b*x)^(8/3))/(8*b^4) - (9*a*(
a + b*x)^(11/3))/(11*b^4) + (3*(a + b*x)^(14/3))/(14*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{2/3}\,dx &= \int \left(-\frac{a^3(a+bx)^{2/3}}{b^3}+\frac{3a^2(a+bx)^{5/3}}{b^3}-\frac{3a(a+bx)^{8/3}}{b^3}+\frac{(a+bx)^{11/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{5/3}}{5b^4}+\frac{9a^2(a+bx)^{8/3}}{8b^4}-\frac{9a(a+bx)^{11/3}}{11b^4}+\frac{3(a+bx)^{14/3}}{14b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{5/3}\left(-81a^3+135a^2bx-180ab^2x^2+220b^3x^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-81*a^3 + 135*a^2*b*x - 180*a*b^2*x^2 + 220*b^3*x^3))/(
3080*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{5/3}\left(-616a^3+1155a^2(a+bx)-840a(a+bx)^2+220(a+bx)^3\right)}{3080b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-616*a^3 + 1155*a^2*(a + b*x) - 840*a*(a + b*x)^2 + 220
*(a + b*x)^3))/(3080*b^4)
```

fricas [A] time = 0.82, size = 53, normalized size = 0.74

$$\frac{3\left(220\,b^{4}x^{4}+40\,ab^{3}x^{3}-45\,a^{2}b^{2}x^{2}+54\,a^{3}bx-81\,a^{4}\right)(bx+a)^{\frac{2}{3}}}{3080\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/3080*(220*b^4*x^4 + 40*a*b^3*x^3 - 45*a^2*b^2*x^2 + 54*a^3*b*x - 81*a^4)*
(b*x + a)^(2/3)/b^4
```

giac [B] time = 1.12, size = 117, normalized size = 1.62

$$\frac{3\left(\frac{7\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^{2}-220\,(bx+a)^{\frac{2}{3}}a^{3}\right)a}{b^{3}}+\frac{2\left(110\,(bx+a)^{\frac{14}{3}}-560\,(bx+a)^{\frac{11}{3}}a+1155\,(bx+a)^{\frac{8}{3}}a^{2}-1232\,(bx+a)^{\frac{5}{3}}a^{3}+770\,(bx+a)^{\frac{2}{3}}a^{4}\right)}{b^{3}}\right)}{3080\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/3080*(7*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3
)*a^2 - 220*(b*x + a)^(2/3)*a^3)*a/b^3 + 2*(110*(b*x + a)^(14/3) - 560*(b*x
 + a)^(11/3)*a + 1155*(b*x + a)^(8/3)*a^2 - 1232*(b*x + a)^(5/3)*a^3 + 770*
(b*x + a)^(2/3)*a^4)/b^3)/b
```

maple [A] time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{5}{3}}\left(-220b^{3}x^{3}+180a\,b^{2}x^{2}-135a^{2}bx+81a^{3}\right)}{3080b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(2/3),x)
```

```
[Out] -3/3080*(b*x+a)^(5/3)*(-220*b^3*x^3+180*a*b^2*x^2-135*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.36, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{14}{3}}}{14\,b^{4}}-\frac{9\,(bx+a)^{\frac{11}{3}}a}{11\,b^{4}}+\frac{9\,(bx+a)^{\frac{8}{3}}a^{2}}{8\,b^{4}}-\frac{3\,(bx+a)^{\frac{5}{3}}a^{3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/14*(b*x + a)^(14/3)/b^4 - 9/11*(b*x + a)^(11/3)*a/b^4 + 9/8*(b*x + a)^(8/
3)*a^2/b^4 - 3/5*(b*x + a)^(5/3)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{14/3}}{14\,b^{4}}-\frac{3\,a^{3}\,(a+b\,x)^{5/3}}{5\,b^{4}}+\frac{9\,a^{2}\,(a+b\,x)^{8/3}}{8\,b^{4}}-\frac{9\,a\,(a+b\,x)^{11/3}}{11\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(14/3))/(14*b^4) - (3*a^3*(a + b*x)^(5/3))/(5*b^4) + (9*a^2*(a
 + b*x)^(8/3))/(8*b^4) - (9*a*(a + b*x)^(11/3))/(11*b^4)
```

sympy [B] time = 3.07, size = 1742, normalized size = 24.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(74/3)*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 4
6200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 1848
0*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(74/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 1296*
a**(71/3)*b*x*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 1458*a**(71/3)*b*x/(3080*a**20*
b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 +
 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) - 2
808*a**(68/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b
**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x*
*4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 3645*a**(68/3)*b**2*x
**2/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a
**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14
*b**10*x**6) - 3120*a**(65/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(3080*a**20*b**4
 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 462
00*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4860*
a**(65/3)*b**3*x**3/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**
6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x
**5 + 3080*a**14*b**10*x**6) - 1050*a**(62/3)*b**4*x**4*(1 + b*x/a)**(2/3)/
(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17
*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**
10*x**6) + 3645*a**(62/3)*b**4*x**4/(3080*a**20*b**4 + 18480*a**19*b**5*x +
 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18
480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4032*a**(59/3)*b**5*x**5*(1
+ b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x*
*2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5
+ 3080*a**14*b**10*x**6) + 1458*a**(59/3)*b**5*x**5/(3080*a**20*b**4 + 1848
0*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**1
6*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 11004*a**(56
/3)*b**6*x**6*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46
200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480
*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 243*a**(56/3)*b**6*x**6/(3080*a
**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x
**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6
) + 14352*a**(53/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a
**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b
**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6) + 10485*a**(50/3)
*b**8*x**8*(1 + b*x/a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200
*a**18*b**6*x**2 + 61600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a*
*15*b**9*x**5 + 3080*a**14*b**10*x**6) + 4080*a**(47/3)*b**9*x**9*(1 + b*x/
a)**(2/3)/(3080*a**20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 6
1600*a**17*b**7*x**3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080
*a**14*b**10*x**6) + 660*a**(44/3)*b**10*x**10*(1 + b*x/a)**(2/3)/(3080*a**
20*b**4 + 18480*a**19*b**5*x + 46200*a**18*b**6*x**2 + 61600*a**17*b**7*x**
3 + 46200*a**16*b**8*x**4 + 18480*a**15*b**9*x**5 + 3080*a**14*b**10*x**6)
```

3.379 $\int x^2(a+bx)^{2/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{11/3}}{11b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(5/3))/(5*b^3) - (3*a*(a + b*x)^(8/3))/(4*b^3) + (3*(a + b
*x)^(11/3))/(11*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{2/3}\,dx&=\int\left(\frac{a^2(a+bx)^{2/3}}{b^2}-\frac{2a(a+bx)^{5/3}}{b^2}+\frac{(a+bx)^{8/3}}{b^2}\right)dx\\&=\frac{3a^2(a+bx)^{5/3}}{5b^3}-\frac{3a(a+bx)^{8/3}}{4b^3}+\frac{3(a+bx)^{11/3}}{11b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{5/3}\left(9a^2-15abx+20b^2x^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(9*a^2 - 15*a*b*x + 20*b^2*x^2))/(220*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{5/3}\left(44a^2-55a(a+bx)+20(a+bx)^2\right)}{220b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(44*a^2 - 55*a*(a + b*x) + 20*(a + b*x)^2))/(220*b^3)
```

fricas [**A**] time = 0.65, size = 42, normalized size = 0.79

$$\frac{3\left(20\,b^{3}x^{3}+5\,ab^{2}x^{2}-6\,a^{2}bx+9\,a^{3}\right)(bx+a)^{\frac{2}{3}}}{220\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^3*x^3 + 5*a*b^2*x^2 - 6*a^2*b*x + 9*a^3)*(b*x + a)^(2/3)/b^3
```

giac [**B**] time = 0.83, size = 92, normalized size = 1.74

$$\frac{3\left(\frac{11\left(5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^{2}\right)a}{b^{2}}+\frac{40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^{2}-220\,(bx+a)^{\frac{2}{3}}a^{3}}{b^{2}}\right)}{440\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/440*(11*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^
2)*a/b^2 + (40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/
3)*a^2 - 220*(b*x + a)^(2/3)*a^3)/b^2)/b
```

maple [**A**] time = 0.01, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{5}{3}}\left(20b^{2}x^{2}-15abx+9a^{2}\right)}{220b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(2/3),x)
```

```
[Out] 3/220*(b*x+a)^(5/3)*(20*b^2*x^2-15*a*b*x+9*a^2)/b^3
```

maxima [**A**] time = 1.35, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^{3}}-\frac{3\,(bx+a)^{\frac{8}{3}}a}{4\,b^{3}}+\frac{3\,(bx+a)^{\frac{5}{3}}a^{2}}{5\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^3 - 3/4*(b*x + a)^(8/3)*a/b^3 + 3/5*(b*x + a)^(5/3)
*a^2/b^3
```

mupad [**B**] time = 0.04, size = 37, normalized size = 0.70

$$\frac{60\,(a+b\,x)^{11/3}-165\,a\,(a+b\,x)^{8/3}+132\,a^{2}\,(a+b\,x)^{5/3}}{220\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(2/3),x)
```

```
[Out] (60*(a + b*x)^(11/3) - 165*a*(a + b*x)^(8/3) + 132*a^2*(a + b*x)^(5/3))/(22
0*b^3)
```

sympy [B]   time = 1.94, size = 666, normalized size = 12.57

$$\frac{27a^{\frac{35}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{35}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{63a^{\frac{32}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{32}{3}}bx}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{42a^{\frac{29}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{81a^{\frac{29}{3}}b^2x^2}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{78a^{\frac{26}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}-\frac{27a^{\frac{26}{3}}b^3x^3}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{207a^{\frac{23}{3}}b^4x^4\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{195a^{\frac{20}{3}}b^5x^5\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}+\frac{60a^{\frac{17}{3}}b^6x^6\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{220a^8b^3+660a^7b^4x+660a^6b^5x^2+220a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(35/3)*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) - 27*a**(35/3)/(220*a**8*b**3 + 660*a**7*b
**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 63*a**(32/3)*b*x*(1 + b*
x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**
5*b**6*x**3) - 81*a**(32/3)*b*x/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6
*b**5*x**2 + 220*a**5*b**6*x**3) + 42*a**(29/3)*b**2*x**2*(1 + b*x/a)**(2/3
)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**
3) - 81*a**(29/3)*b**2*x**2/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**
5*x**2 + 220*a**5*b**6*x**3) + 78*a**(26/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(2
20*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) -
 27*a**(26/3)*b**3*x**3/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x*
*2 + 220*a**5*b**6*x**3) + 207*a**(23/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(220*
a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 19
5*a**(20/3)*b**5*x**5*(1 + b*x/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x +
 660*a**6*b**5*x**2 + 220*a**5*b**6*x**3) + 60*a**(17/3)*b**6*x**6*(1 + b*x
/a)**(2/3)/(220*a**8*b**3 + 660*a**7*b**4*x + 660*a**6*b**5*x**2 + 220*a**5
*b**6*x**3)
```

3.380 $\int x(a+bx)^{2/3}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{8/3}}{8b^2}-\frac{3a(a+bx)^{5/3}}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*a*(a + b*x)^(5/3))/(5*b^2) + (3*(a + b*x)^(8/3))/(8*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{2/3}\,dx &= \int\left(-\frac{a(a+bx)^{2/3}}{b}+\frac{(a+bx)^{5/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{5/3}}{5b^2}+\frac{3(a+bx)^{8/3}}{8b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{5/3}(5bx-3a)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(-3*a + 5*b*x))/(40*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 35, normalized size = 1.03

$$-\frac{3(a+bx)^{2/3}\left(3a^2-2abx-5b^2x^2\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(2/3),x]
```

```
[Out] (-3*(a + b*x)^(2/3)*(3*a^2 - 2*a*b*x - 5*b^2*x^2))/(40*b^2)
```

fricas [A] time = 0.82, size = 31, normalized size = 0.91

$$\frac{3\left(5\,b^2x^2+2\,abx-3\,a^2\right)(bx+a)^{\frac{2}{3}}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 2*a*b*x - 3*a^2)*(b*x + a)^(2/3)/b^2
```

giac [B] time = 0.89, size = 68, normalized size = 2.00

$$\frac{3\left(\frac{4\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}\,a\right)a}{b}+\frac{5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}\,a+20\,(bx+a)^{\frac{2}{3}}\,a^2}{b}\right)}{40\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/40*(4*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)*a/b + (5*(b*x + a)^(8/3)
- 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{5}{3}}\,(-5bx+3a)}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(2/3),x)
```

```
[Out] -3/40*(b*x+a)^(5/3)*(-5*b*x+3*a)/b^2
```

maxima [A] time = 1.35, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^2}-\frac{3\,(bx+a)^{\frac{5}{3}}\,a}{5\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^2 - 3/5*(b*x + a)^(5/3)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{24\,a\,(a+b\,x)^{5/3}-15\,(a+b\,x)^{8/3}}{40\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(2/3),x)
```

```
[Out] -(24*a*(a + b*x)^(5/3) - 15*(a + b*x)^(8/3))/(40*b^2)
```

sympy [B] time = 1.28, size = 202, normalized size = 5.94

$$-\frac{9a^{\frac{14}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{14}{3}}}{40a^2b^2+40ab^3x}-\frac{3a^{\frac{11}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{9a^{\frac{11}{3}}bx}{40a^2b^2+40ab^3x}+\frac{21a^{\frac{8}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}+\frac{15a^{\frac{5}{3}}b^3x^3\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^2b^2+40ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(14/3)*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 9*a**(14/3)/
(40*a**2*b**2 + 40*a*b**3*x) - 3*a**(11/3)*b*x*(1 + b*x/a)**(2/3)/(40*a**2*
b**2 + 40*a*b**3*x) + 9*a**(11/3)*b*x/(40*a**2*b**2 + 40*a*b**3*x) + 21*a**
(8/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x) + 15*a**(5/
3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**2*b**2 + 40*a*b**3*x)
```

3.381 $\int (a+bx)^{2/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{5/3}}{5b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{2/3}\,dx = \frac{3(a+bx)^{5/3}}{5b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

fricas [A] time = 0.84, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b
```

giac [A] time = 1.14, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="giac")
[Out] 3/5*(b*x + a)^(5/3)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3),x)
[Out] 3/5*(b*x+a)^(5/3)/b
```

maxima [A] time = 1.29, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3),x, algorithm="maxima")
[Out] 3/5*(b*x + a)^(5/3)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,(a+b\,x)^{5/3}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3),x)
[Out] (3*(a + b*x)^(5/3))/(5*b)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{5}{3}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3),x)
[Out] 3*(a + b*x)**(5/3)/(5*b)
```

3.382 $\int \frac{(a+bx)^{2/3}}{x} dx$

Optimal. Leaf size=92

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 55, 617, 204, 31}

$$\frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{2/3}\log(x)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3)
)/(Sqrt[3]*a^(1/3))] - (a^(2/3)*Log[x])/2 + (3*a^(2/3)*Log[a^(1/3) - (a + b
*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x}\,dx = \frac{3}{2}(a+bx)^{2/3} + a\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$
$$= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) + \frac{1}{2}(3a)\,\text{Subst}\left(\int\right.$$
$$= \frac{3}{2}(a+bx)^{2/3} - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.$$
$$= \frac{3}{2}(a+bx)^{2/3} + \sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{2}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)$$

Mathematica [A] time = 0.04, size = 86, normalized size = 0.93

$$\frac{3}{2}\left(a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+(a+bx)^{2/3}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-\frac{1}{2}a^{2/3}\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x,x]
```

```
[Out] Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - (a^(2/3
)*Log[x])/2 + (3*((a + b*x)^(2/3) + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)])
)/2
```

IntegrateAlgebraic [A] time = 0.06, size = 117, normalized size = 1.27

$$a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{1}{2}a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)+\frac{3}{2}(a+bx)^{2/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x,x]
```

```
[Out] (3*(a + b*x)^(2/3))/2 + Sqrt[3]*a^(2/3)*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/
3))/(Sqrt[3]*a^(1/3))] + a^(2/3)*Log[a^(1/3) - (a + b*x)^(1/3)] - (a^(2/3)*
Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/2
```

fricas [A] time = 0.92, size = 110, normalized size = 1.20

$$\sqrt{3}\left(a^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\,a+2\sqrt{3}\left(a^2\right)^{\frac{1}{3}}(bx+a)^{\frac{1}{3}}}{3a}\right)-\frac{1}{2}\left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)+\left(a^2\right)^{\frac{1}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+\frac{3}{2}(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(a^2)^(1/3)*(b*x + a)
^(1/3))/a) - 1/2*(a^2)^(1/3)*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^
(2/3)*(b*x + a)^(1/3)) + (a^2)^(1/3)*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) +
 3/2*(b*x + a)^(2/3)
```

giac [A] time = 2.20, size = 86, normalized size = 0.93

$$\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)-\frac{1}{2}a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{2}(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="giac")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/2*(b*x + a)^(2/3)
```

maple [A] time = 0.00, size = 84, normalized size = 0.91

$$\sqrt{3}\, a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{2}{3}} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{2}{3}} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+\frac{3\,(bx+a)^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x,x)
```

```
[Out] 3/2*(b*x+a)^(2/3)+a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2*a^(2/3)*ln(a^(2/3)
+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.97, size = 85, normalized size = 0.92

$$\sqrt{3}\, a^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{1}{2}\, a^{\frac{2}{3}} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{2}{3}} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}\,(bx+a)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x,x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) -
 1/2*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^(
2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/2*(b*x + a)^(2/3)
```

mupad [B] time = 0.11, size = 117, normalized size = 1.27

$$\frac{3\,(a+b\,x)^{2/3}}{2}+a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-9\,a^{7/3}\right)+\frac{a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-\frac{9\,a^{7/3}\,(-1+\sqrt{3}\,1\mathrm{i})^2}{4}\right)\,(-1+\sqrt{3}\,1\mathrm{i})}{2}-\frac{a^{2/3}\,\ln\left(9\,a^2\,(a+b\,x)^{1/3}-\frac{9\,a^{7/3}\,(1+\sqrt{3}\,1\mathrm{i})^2}{4}\right)\,(1+\sqrt{3}\,1\mathrm{i})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x,x)
```

```
[Out] (3*(a + b*x)^(2/3))/2 + a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - 9*a^(7/3)) + (a
^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/
2)*1i - 1))/2 - (a^(2/3)*log(9*a^2*(a + b*x)^(1/3) - (9*a^(7/3)*(3^(1/2)*1i
 + 1)^2)/4)*(3^(1/2)*1i + 1))/2
```

sympy [C] time = 2.06, size = 182, normalized size = 1.98

$$\frac{5a^{\frac{2}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5a^{\frac{2}{3}}e^{-\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{3\Gamma\left(\frac{8}{3}\right)}+\frac{5b^{\frac{2}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x,x)
```

```
[Out] 5*a**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(3*gamma(
8/3)) + 5*a**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(5/3)/(3*gamma(8/3)) + 5*a**(2/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)
/(3*gamma(8/3)) + 5*b**(2/3)*(a/b + x)**(2/3)*gamma(5/3)/(2*gamma(8/3))
```

3.383 $\int \frac{(a+bx)^{2/3}}{x^2} dx$

Optimal. Leaf size=94

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 55, 617, 204, 31}

$$-\frac{(a+bx)^{2/3}}{x}-\frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a
^(1/3))])/(Sqrt[3]*a^(1/3)) - (b*Log[x])/(3*a^(1/3)) + (b*Log[a^(1/3) - (a
+ b*x)^(1/3)])/a^(1/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^2}\,dx = -\frac{(a+bx)^{2/3}}{x} + \frac{1}{3}(2b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$$

$$= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + b\,\mathrm{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right) - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x} \cdots\right.}{\sqrt[3]{\cdots}}$$

$$= -\frac{(a+bx)^{2/3}}{x} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}} - \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{(a+bx)^{2/3}}{x} + \frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}} + \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.35

$$\frac{3b(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] (3*b*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + (b*x)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.16, size = 125, normalized size = 1.33

$$-\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3\sqrt[3]{a}} - \frac{(a+bx)^{2/3}}{x} + \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3\sqrt[3]{a}} + \frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x^2,x]
```

```
[Out] -((a + b*x)^(2/3)/x) + (2*b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)) + (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(
1/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3*a^(
1/3))
```

fricas [A] time = 0.99, size = 252, normalized size = 2.68

$$\left[\frac{3\sqrt{\frac{1}{3}}abx\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(bx+a)^{\frac{2}{3}}a}{3ax}, \frac{6\sqrt{\frac{1}{3}}a^{\frac{2}{3}}bx\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2a^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3(bx+a)^{\frac{2}{3}}a}{3ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(1/3)*a*b*x*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^
(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3
*(b*x + a)^(2/3)*a)/(a*x), 1/3*(6*sqrt(1/3)*a^(2/3)*b*x*arctan(sqrt(1/3)*(2
*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b*x*log((b*x + a)^(2/3) + (b
```

```
*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x*log((b*x + a)^(1/3) - a^(1
/3)) - 3*(b*x + a)^(2/3)*a)/(a*x)]
```

giac [A] time = 2.29, size = 106, normalized size = 1.13

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}}+\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}-\frac{3(bx+a)^{\frac{2}{3}}b}{x}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(1/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x + a)^(2/
3)*b/x)/b
```

maple [A] time = 0.01, size = 92, normalized size = 0.98

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{1}{3}}}+\frac{2b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^2,x)
```

```
[Out] -(b*x+a)^(2/3)/x+2/3*b/a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/3*b/a^(1/3)*ln(
a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+2/3*b*3^(1/2)/a^(1/3)*arctan(1
/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.98, size = 93, normalized size = 0.99

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{1}{3}}}-\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{1}{3}}}+\frac{2b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^2,x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
1/3) - 1/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/
3) + 2/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(1/3) - (b*x + a)^(2/3)/x
```

mupad [B] time = 0.11, size = 127, normalized size = 1.35

$$\frac{2b\ln\left(4a^{1/3}b^2-4b^2(a+bx)^{1/3}\right)}{3a^{1/3}}-\frac{(a+bx)^{2/3}}{x}-\frac{\ln\left(a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2-4b^2(a+bx)^{1/3}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{3a^{1/3}}-\frac{\ln\left(a^{1/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2-4b^2(a+bx)^{1/3}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{3a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^2,x)
```

```
[Out] (2*b*log(4*a^(1/3)*b^2 - 4*b^2*(a + b*x)^(1/3)))/(3*a^(1/3)) - (a + b*x)^(2
/3)/x - (log(a^(1/3)*(b - 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*x)^(1/3))*(b - 3^(
```

```
1/2)*b*1i))/(3*a^(1/3)) - (log(a^(1/3)*(b + 3^(1/2)*b*1i)^2 - 4*b^2*(a + b*
x)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(1/3))
```

sympy [C] time = 2.24, size = 643, normalized size = 6.84

$$\frac{10a^{\frac{8}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{10a^{\frac{8}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}-\frac{10a^{\frac{5}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}+\frac{15a^2b^{\frac{5}{3}}\left(\frac{a}{b}+x\right)^{\frac{2}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{5}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{8}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**2,x)
```

```
[Out] 10*a**(8/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gam
ma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)
*gamma(8/3)) + 10*a**(8/3)*b*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3
) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*ex
p(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 10*
a**(5/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**
(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp
(2*I*pi/3)*gamma(8/3)) - 10*a**(5/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(9*a**3*
exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3)) - 1
0*a**(5/3)*b**2*(a/b + x)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(8/3)) + 15*a**2*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*
pi/3)*gamma(5/3)/(9*a**3*exp(2*I*pi/3)*gamma(8/3) - 9*a**2*b*(a/b + x)*exp(
2*I*pi/3)*gamma(8/3))
```

3.384 $\int \frac{(a+bx)^{2/3}}{x^3}\,dx$

Optimal. Leaf size=127

$$\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

Rubi [A] time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 55, 617, 204, 31}

$$\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -(a + b*x)^(2/3)/(2*x^2) - (b*(a + b*x)^(2/3))/(3*a*x) - (b^2*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) + (b^2*Log[
x])/(18*a^(4/3)) - (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{2/3}}{x^3}\,dx = -\frac{(a+bx)^{2/3}}{2x^2}+\frac{1}{3}b\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx$$

$$= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{4/3}}-\frac{b^2\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{3a^{4/3}}$$

$$= -\frac{(a+bx)^{2/3}}{2x^2}-\frac{b(a+bx)^{2/3}}{3ax}-\frac{b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}+\frac{b^2\log(x)}{18a^{4/3}}-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{4/3}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};\frac{bx}{a}+1\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, 1 + (b*x)/a])/(5*a^3
)
```

IntegrateAlgebraic [A] time = 0.25, size = 147, normalized size = 1.16

$$-\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{4/3}}+\frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{4/3}}-\frac{b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}(2(a+bx)+a)}{6ax^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(a + 2*(a + b*x)))/(a*x^2) - (b^2*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(4/3)) - (b^2*Log[a^(1
/3) - (a + b*x)^(1/3)])/(9*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a + b*x)^
(1/3) + (a + b*x)^(2/3)])/(18*a^(4/3))
```

fricas [A] time = 0.94, size = 350, normalized size = 2.76

$$\left[\frac{3\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(2abx+3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^2x^2},\ -\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(2abx+3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2
*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b^2*x^2*log((
b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b^
2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) - 3*(2*a*b*x + 3*a^2)*(b*x + a)^(2/
3))/(a^2*x^2), -1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-
a)^(2/3)*b^2*x^2*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(2*a*b*x + 3*a^2)*(b
*x + a)^(2/3))/(a^2*x^2)]
```

giac [A] time = 2.47, size = 129, normalized size = 1.02

$$-\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2(bx+a)^{\frac{5}{3}}b^3+(bx+a)^{\frac{2}{3}}ab^3\right)}{ab^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) - b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(4/3) + 2*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*(2*(b*x + a
)^(5/3)*b^3 + (b*x + a)^(2/3)*a*b^3)/(a*b^2*x^2))/b
```

maple [A] time = 0.01, size = 113, normalized size = 0.89

$$-\frac{\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{4}{3}}}-\frac{b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{6x^2}-\frac{(bx+a)^{\frac{5}{3}}}{3a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/x^3,x)
```

```
[Out] -1/3/x^2/a*(b*x+a)^(5/3)-1/6*(b*x+a)^(2/3)/x^2-1/9*b^2/a^(4/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))+1/18*b^2/a^(4/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2
/3))-1/9*b^2/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1)
)
```

maxima [A] time = 2.98, size = 139, normalized size = 1.09

$$-\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{4}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{4}{3}}}-\frac{2(bx+a)^{\frac{5}{3}}b^2+(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a-2(bx+a)a^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/x^3,x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(4/3) + 1/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(4/3) - 1/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(4/3) - 1/6*(2*(b*x + a
```

```
)^(5/3)*b^2 + (b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a - 2*(b*x + a)*a^2 + a^3
)
```

mupad [B] time = 0.33, size = 194, normalized size = 1.53

$$\frac{(-1)^{1/3}\,b^2\,\ln\left((a+b\,x)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{9\,a^{4/3}}-\frac{\frac{b^2\,(a+b\,x)^{2/3}}{6}+\frac{b^2\,(a+b\,x)^{5/3}}{3\,a}}{(a+b\,x)^2-2\,a\,(a+b\,x)+a^2}+\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9\,a^{4/3}}-\frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(a+b\,x)^{1/3}}{9\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2}{9\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/x^3,x)
```

```
[Out] ((-1)^(1/3)*b^2*log((a + b*x)^(1/3) - (-1)^(2/3)*a^(1/3)))/(9*a^(4/3)) - ((
b^2*(a + b*x)^(2/3))/6 + (b^2*(a + b*x)^(5/3))/(3*a))/((a + b*x)^2 - 2*a*(a
 + b*x) + a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^
(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(9
*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x)^(1/3))/(9*a^2) - ((-1)^(2/3)
*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(9*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(4
/3))
```

sympy [C] time = 2.65, size = 2266, normalized size = 17.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/x**3,x)
```

```
[Out] -10*a**(17/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3)
)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) -
 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 10*a**(17/3)*b**2*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(2*I
*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(
2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3
*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 30*a**(14/3)*b**3*(a/b + x)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a**7
*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) +
 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b +
x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(2*I*pi
/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2*I
*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**
5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*ex
p(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(54
*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8
/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a
/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 30*a**(11/3)*b**4*(a/b + x)**2*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
```

```
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(2*I*
pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(5/3)/(54*a**7*exp(2
*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a
**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*
exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(5/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(
a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) + 10*a**(8/3)*b**5*(a/b + x)**3*log(1
 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(5/3)/(54*a
**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3
) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b
 + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**5*b**(8/3)*(a/b + x)**(2/3)*exp(
2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b +
x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamm
a(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3)) - 15*a**4*b**(
11/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(5/3)/(54*a**7*exp(2*I*pi/3)*gamm
a(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(8/3) + 162*a**5*b**2*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)
*gamma(8/3)) + 30*a**3*b**(14/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(5/3)/
(54*a**7*exp(2*I*pi/3)*gamma(8/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamm
a(8/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(8/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(8/3))
```

3.385 $\int x^3(a+bx)^{4/3}\,dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{16/3}}{16b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a + b*x)^(10/3))/(10*b^4) - (9*a
*(a + b*x)^(13/3))/(13*b^4) + (3*(a + b*x)^(16/3))/(16*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^{4/3}\,dx&=\int\left(-\frac{a^3(a+bx)^{4/3}}{b^3}+\frac{3a^2(a+bx)^{7/3}}{b^3}-\frac{3a(a+bx)^{10/3}}{b^3}+\frac{(a+bx)^{13/3}}{b^3}\right)dx\\&=-\frac{3a^3(a+bx)^{7/3}}{7b^4}+\frac{9a^2(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{13/3}}{13b^4}+\frac{3(a+bx)^{16/3}}{16b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{7/3}\left(-81a^3+189a^2bx-315ab^2x^2+455b^3x^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-81*a^3 + 189*a^2*b*x - 315*a*b^2*x^2 + 455*b^3*x^3))/(
7280*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}\left(-1040a^3+2184a^2(a+bx)-1680a(a+bx)^2+455(a+bx)^3\right)}{7280b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-1040*a^3 + 2184*a^2*(a + b*x) - 1680*a*(a + b*x)^2 + 4
55*(a + b*x)^3))/(7280*b^4)
```

fricas [A] time = 0.92, size = 64, normalized size = 0.89

$$\frac{3\left(455\,b^{5}x^{5}+595\,ab^{4}x^{4}+14\,a^{2}b^{3}x^{3}-18\,a^{3}b^{2}x^{2}+27\,a^{4}bx-81\,a^{5}\right)(bx+a)^{\frac{1}{3}}}{7280\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^5*x^5 + 595*a*b^4*x^4 + 14*a^2*b^3*x^3 - 18*a^3*b^2*x^2 + 27*
a^4*b*x - 81*a^5)*(b*x + a)^(1/3)/b^4
```

giac [B] time = 1.22, size = 193, normalized size = 2.68

$$\frac{3\left(\frac{52\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^{2}-140\,(bx+a)^{\frac{1}{3}}a^{3}\right)a^{2}}{b^{3}}+\frac{32\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^{2}-455\,(bx+a)^{\frac{4}{3}}a^{3}+455\,(bx+a)^{\frac{1}{3}}a^{4}\right)a}{b^{3}}+\frac{5\left(91\,(bx+a)^{\frac{16}{3}}-560\,(bx+a)^{\frac{13}{3}}a+1456\,(bx+a)^{\frac{10}{3}}a^{2}-2080\,(bx+a)^{\frac{7}{3}}a^{3}+1820\,(bx+a)^{\frac{4}{3}}a^{4}-1456\,(bx+a)^{\frac{1}{3}}a^{5}\right)}{b^{3}}\right)}{7280\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/7280*(52*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3
)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a^2/b^3 + 32*(35*(b*x + a)^(13/3) - 182*(b
*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455*
(b*x + a)^(1/3)*a^4)*a/b^3 + 5*(91*(b*x + a)^(16/3) - 560*(b*x + a)^(13/3)*
a + 1456*(b*x + a)^(10/3)*a^2 - 2080*(b*x + a)^(7/3)*a^3 + 1820*(b*x + a)^(
4/3)*a^4 - 1456*(b*x + a)^(1/3)*a^5)/b^3)/b
```

maple [A] time = 0.01, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{7}{3}}\left(-455b^{3}x^{3}+315a\,b^{2}x^{2}-189a^{2}bx+81a^{3}\right)}{7280b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^(4/3),x)
```

```
[Out] -3/7280*(b*x+a)^(7/3)*(-455*b^3*x^3+315*a*b^2*x^2-189*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.36, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{16}{3}}}{16\,b^{4}}-\frac{9\,(bx+a)^{\frac{13}{3}}a}{13\,b^{4}}+\frac{9\,(bx+a)^{\frac{10}{3}}a^{2}}{10\,b^{4}}-\frac{3\,(bx+a)^{\frac{7}{3}}a^{3}}{7\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/16*(b*x + a)^(16/3)/b^4 - 9/13*(b*x + a)^(13/3)*a/b^4 + 9/10*(b*x + a)^(1
0/3)*a^2/b^4 - 3/7*(b*x + a)^(7/3)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.78

$$\frac{3\,(a+b\,x)^{16/3}}{16\,b^{4}}-\frac{3\,a^{3}\,(a+b\,x)^{7/3}}{7\,b^{4}}+\frac{9\,a^{2}\,(a+b\,x)^{10/3}}{10\,b^{4}}-\frac{9\,a\,(a+b\,x)^{13/3}}{13\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(16/3))/(16*b^4) - (3*a^3*(a + b*x)^(7/3))/(7*b^4) + (9*a^2*(a
 + b*x)^(10/3))/(10*b^4) - (9*a*(a + b*x)^(13/3))/(13*b^4)
```

<u>**sympy**</u> **[B]** time = 3.18, size = 1844, normalized size = 25.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**(4/3),x)
```

```
[Out] -243*a**(76/3)*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 1
09200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 4
3680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 243*a**(76/3)/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) -
 1377*a**(73/3)*b*x*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*
x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**
4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(73/3)*b*x/(72
80*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*
b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**
10*x**6) - 3213*a**(70/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 4
3680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 10920
0*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a
**(70/3)*b**2*x**2/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**
6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9
*x**5 + 7280*a**14*b**10*x**6) - 3927*a**(67/3)*b**3*x**3*(1 + b*x/a)**(1/3
)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a
**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**1
4*b**10*x**6) + 4860*a**(67/3)*b**3*x**3/(7280*a**20*b**4 + 43680*a**19*b**
5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x
**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) - 798*a**(64/3)*b**4*x
**4*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18
*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*
b**9*x**5 + 7280*a**14*b**10*x**6) + 3645*a**(64/3)*b**4*x**4/(7280*a**20*b
**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3
+ 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) +
 11382*a**(61/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**1
9*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b
**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1458*a**(61/3)*
b**5*x**5/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 +
145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7
280*a**14*b**10*x**6) + 35238*a**(58/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(7280*
a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**
7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*
x**6) + 243*a**(58/3)*b**6*x**6/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109
200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 436
80*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 56562*a**(55/3)*b**7*x**7*(1
+ b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x
**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x*
*5 + 7280*a**14*b**10*x**6) + 54273*a**(52/3)*b**8*x**8*(1 + b*x/a)**(1/3)/
(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**
17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*
b**10*x**6) + 31227*a**(49/3)*b**9*x**9*(1 + b*x/a)**(1/3)/(7280*a**20*b**4
 + 43680*a**19*b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 1
09200*a**16*b**8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 99
75*a**(46/3)*b**10*x**10*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*
b**5*x + 109200*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**
8*x**4 + 43680*a**15*b**9*x**5 + 7280*a**14*b**10*x**6) + 1365*a**(43/3)*b*
*11*x**11*(1 + b*x/a)**(1/3)/(7280*a**20*b**4 + 43680*a**19*b**5*x + 109200
*a**18*b**6*x**2 + 145600*a**17*b**7*x**3 + 109200*a**16*b**8*x**4 + 43680*
a**15*b**9*x**5 + 7280*a**14*b**10*x**6)
```

3.386 $\int x^2(a+bx)^{4/3}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{7/3}}{7b^3}+\frac{3(a+bx)^{13/3}}{13b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(7/3))/(7*b^3) - (3*a*(a + b*x)^(10/3))/(5*b^3) + (3*(a +
b*x)^(13/3))/(13*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^{4/3}\,dx &= \int\left(\frac{a^2(a+bx)^{4/3}}{b^2}-\frac{2a(a+bx)^{7/3}}{b^2}+\frac{(a+bx)^{10/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{10/3}}{5b^3}+\frac{3(a+bx)^{13/3}}{13b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{7/3}\left(9a^2-21abx+35b^2x^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(9*a^2 - 21*a*b*x + 35*b^2*x^2))/(455*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.74

$$\frac{3(a+bx)^{7/3}\left(65a^2-91a(a+bx)+35(a+bx)^2\right)}{455b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(65*a^2 - 91*a*(a + b*x) + 35*(a + b*x)^2))/(455*b^3)
```

fricas [A] time = 0.77, size = 53, normalized size = 1.00

$$\frac{3\left(35\,b^4x^4+49\,ab^3x^3+2\,a^2b^2x^2-3\,a^3bx+9\,a^4\right)(bx+a)^{\frac{1}{3}}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^4*x^4 + 49*a*b^3*x^3 + 2*a^2*b^2*x^2 - 3*a^3*b*x + 9*a^4)*(b*x
+ a)^(1/3)/b^3
```

giac [B] time = 0.98, size = 157, normalized size = 2.96

$$\frac{3\left(\frac{65\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a^2}{b^2}+\frac{13\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3\right)a}{b^2}+\frac{2\left(35\,(bx+a)^{\frac{13}{3}}-182\,(bx+a)^{\frac{10}{3}}a+390\,(bx+a)^{\frac{7}{3}}a^2-455\,(bx+a)^{\frac{4}{3}}a^3+455\,(bx+a)^{\frac{1}{3}}a^4\right)}{b^2}\right)}{910\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/910*(65*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2
)*a^2/b^2 + 13*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^
(4/3)*a^2 - 140*(b*x + a)^(1/3)*a^3)*a/b^2 + 2*(35*(b*x + a)^(13/3) - 182*(
b*x + a)^(10/3)*a + 390*(b*x + a)^(7/3)*a^2 - 455*(b*x + a)^(4/3)*a^3 + 455
*(b*x + a)^(1/3)*a^4)/b^2)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{7}{3}}\left(35b^2x^2-21abx+9a^2\right)}{455b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(4/3),x)
```

```
[Out] 3/455*(b*x+a)^(7/3)*(35*b^2*x^2-21*a*b*x+9*a^2)/b^3
```

maxima [A] time = 1.30, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{13}{3}}}{13\,b^3}-\frac{3\,(bx+a)^{\frac{10}{3}}a}{5\,b^3}+\frac{3\,(bx+a)^{\frac{7}{3}}a^2}{7\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/13*(b*x + a)^(13/3)/b^3 - 3/5*(b*x + a)^(10/3)*a/b^3 + 3/7*(b*x + a)^(7/3
)*a^2/b^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{105\,(a+b\,x)^{13/3}-273\,a\,(a+b\,x)^{10/3}+195\,a^2\,(a+b\,x)^{7/3}}{455\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^(4/3),x)
```

```
[Out] (105*(a + b*x)^(13/3) - 273*a*(a + b*x)^(10/3) + 195*a^2*(a + b*x)^(7/3))/(
455*b^3)
```

sympy [B] time = 2.16, size = 733, normalized size = 13.83

$$\frac{27a^{\frac{37}{3}}\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{37}{3}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{72a^{\frac{34}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{34}{3}}bx}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{60a^{\frac{31}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{81a^{\frac{31}{3}}b^2x^2}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{165a^{\frac{28}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}-\frac{27a^{\frac{28}{3}}b^3x^3}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{555a^{\frac{25}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{762a^{\frac{22}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{462a^{\frac{19}{3}}b^6x^6\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}+\frac{105a^{\frac{16}{3}}b^7x^7\sqrt[3]{1+\frac{bx}{a}}}{455a^8b^3+1365a^7b^4x+1365a^6b^5x^2+455a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(4/3),x)
```

```
[Out] 27*a**(37/3)*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a*
*6*b**5*x**2 + 455*a**5*b**6*x**3) - 27*a**(37/3)/(455*a**8*b**3 + 1365*a**
7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 72*a**(34/3)*b*x*(1
+ b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 4
55*a**5*b**6*x**3) - 81*a**(34/3)*b*x/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 60*a**(31/3)*b**2*x**2*(1 + b*x/
a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**
5*b**6*x**3) - 81*a**(31/3)*b**2*x**2/(455*a**8*b**3 + 1365*a**7*b**4*x + 1
365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 165*a**(28/3)*b**3*x**3*(1 + b*x
/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a*
*5*b**6*x**3) - 27*a**(28/3)*b**3*x**3/(455*a**8*b**3 + 1365*a**7*b**4*x +
1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 555*a**(25/3)*b**4*x**4*(1 + b*
x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a
**5*b**6*x**3) + 762*a**(22/3)*b**5*x**5*(1 + b*x/a)**(1/3)/(455*a**8*b**3
+ 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b**6*x**3) + 462*a**(19
/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a
**6*b**5*x**2 + 455*a**5*b**6*x**3) + 105*a**(16/3)*b**7*x**7*(1 + b*x/a)**
(1/3)/(455*a**8*b**3 + 1365*a**7*b**4*x + 1365*a**6*b**5*x**2 + 455*a**5*b*
*6*x**3)
```

3.387 $\int x(a+bx)^{4/3}\,dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{10/3}}{10b^2}-\frac{3a(a+bx)^{7/3}}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*a*(a + b*x)^(7/3))/(7*b^2) + (3*(a + b*x)^(10/3))/(10*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{4/3}\,dx &= \int\left(-\frac{a(a+bx)^{4/3}}{b}+\frac{(a+bx)^{7/3}}{b}\right)dx\\ &= -\frac{3a(a+bx)^{7/3}}{7b^2}+\frac{3(a+bx)^{10/3}}{10b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{7/3}(7bx-3a)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*(-3*a + 7*b*x))/(70*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{3(3a-7bx)(a+bx)^{7/3}}{70b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(a + b*x)^(4/3),x]
```

```
[Out] (-3*(3*a - 7*b*x)*(a + b*x)^(7/3))/(70*b^2)
```

fricas [A] time = 1.17, size = 41, normalized size = 1.21

$$\frac{3\left(7\,b^3x^3+11\,ab^2x^2+a^2bx-3\,a^3\right)(bx+a)^{\frac{1}{3}}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/70*(7*b^3*x^3 + 11*a*b^2*x^2 + a^2*b*x - 3*a^3)*(b*x + a)^(1/3)/b^2
```

giac [B] time = 1.02, size = 118, normalized size = 3.47

$$\frac{3\left(\frac{35\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a^2}{b}+\frac{20\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)a}{b}+\frac{14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^2-140\,(bx+a)^{\frac{1}{3}}a^3}{b}\right)}{140\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/140*(35*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a^2/b + 20*(2*(b*x + a)^(
7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)*a/b + (14*(b*x + a)^(1
0/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2 - 140*(b*x + a)^(1/3)
*a^3)/b)/b
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{7}{3}}\,(-7bx+3a)}{70b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(4/3),x)
```

```
[Out] -3/70*(b*x+a)^(7/3)*(-7*b*x+3*a)/b^2
```

maxima [A] time = 1.29, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^2}-\frac{3\,(bx+a)^{\frac{7}{3}}a}{7\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^2 - 3/7*(b*x + a)^(7/3)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{30\,a\,(a+b\,x)^{7/3}-21\,(a+b\,x)^{10/3}}{70\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^(4/3),x)
```

```
[Out] -(30*a*(a + b*x)^(7/3) - 21*(a + b*x)^(10/3))/(70*b^2)
```

sympy [A] time = 1.49, size = 80, normalized size = 2.35

$$\begin{cases} -\frac{9a^3\sqrt[3]{a+bx}}{70b^2}+\frac{3a^2x\sqrt[3]{a+bx}}{70b}+\frac{33ax^2\sqrt[3]{a+bx}}{70}+\frac{3bx^3\sqrt[3]{a+bx}}{10} & \text{for } b\neq 0 \\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((-9*a**3*(a + b*x)**(1/3)/(70*b**2) + 3*a**2*x*(a + b*x)**(1/3)/(
70*b) + 33*a*x**2*(a + b*x)**(1/3)/70 + 3*b*x**3*(a + b*x)**(1/3)/10, Ne(b,
 0)), (a**(4/3)*x**2/2, True))
```

3.388 $\int (a+bx)^{4/3}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{7/3}}{7b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{4/3}\,dx = \frac{3(a+bx)^{7/3}}{7b}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{7/3}}{7b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3),x]
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

fricas [B] time = 0.86, size = 28, normalized size = 1.75

$$\frac{3\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/7*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/3)/b
```

giac [B] time = 1.25, size = 58, normalized size = 3.62

$$\frac{3\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+28\,(bx+a)^{\frac{1}{3}}a^{2}+7\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)a\right)}{14\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 28*(b*x + a)^(1/3)*a^2 + 7*
((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)*a)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3),x)
```

```
[Out] 3/7*(b*x+a)^(7/3)/b
```

maxima [A] time = 1.33, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{7/3}}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(7/3))/(7*b)
```

sympy [A] time = 0.07, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{7}{3}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3),x)
```

```
[Out] 3*(a + b*x)**(7/3)/(7*b)
```

3.389 $\int \frac{(a+bx)^{4/3}}{x}\,dx$

Optimal. Leaf size=105

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Rubi [A] time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 57, 617, 204, 31}

$$\frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+3a\sqrt[3]{a+bx}+\frac{3}{4}(a+bx)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x,x]
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 - Sqrt[3]*a^(4/3)*ArcTan[(a^(1/
3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] - (a^(4/3)*Log[x])/2 + (3*a^(4/3
)*Log[a^(1/3) - (a + b*x)^(1/3)])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{x}\,dx = \frac{3}{4}(a+bx)^{4/3} + a\int \frac{\sqrt[3]{a+bx}}{x}\,dx$$
$$= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} + a^2\int \frac{1}{x(a+bx)^{2/3}}\,dx$$
$$= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \frac{1}{2}a^{4/3}\log(x) - \frac{1}{2}\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right) -$$
$$= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \frac{1}{2}a^{4/3}\log(x) + \frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) + \left(3a^{4/3}\right)\text{Subst}\Bigg(\int$$
$$= 3a\sqrt[3]{a+bx} + \frac{3}{4}(a+bx)^{4/3} - \sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right) - \frac{1}{2}a^{4/3}\log(x) + \frac{3}{2}a^{4/3}\log\left(\sqrt[3]{a}-\right.$$

Mathematica [A] time = 0.03, size = 130, normalized size = 1.24

$$\frac{1}{4}\left(4a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - 2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right) - 4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right) + 15a\sqrt[3]{a+bx} + 3bx\sqrt[3]{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x,x]
```

```
[Out] (15*a*(a + b*x)^(1/3) + 3*b*x*(a + b*x)^(1/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1
 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) - (a + b*x
)^(1/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3
)])/4
```

IntegrateAlgebraic [A] time = 0.06, size = 131, normalized size = 1.25

$$a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right) - \frac{1}{2}a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right) - \sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right) + \frac{3}{4}\left((a+bx)^{4/3}+4a\sqrt[3]{a+bx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x,x]
```

```
[Out] (3*(4*a*(a + b*x)^(1/3) + (a + b*x)^(4/3)))/4 - Sqrt[3]*a^(4/3)*ArcTan[1/Sq
rt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))] + a^(4/3)*Log[a^(1/3) - (a +
 b*x)^(1/3)] - (a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(
2/3)])/2
```

fricas [A] time = 0.66, size = 98, normalized size = 0.93

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right) - \frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{4}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{3}{4}\,(bx+5\,a)(bx+a)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="fricas")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) + sqrt(3)*a)
/a) - 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
+ a^(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + 5*a)*(b*x + a)^(1/3)
```

giac [A] time = 2.02, size = 97, normalized size = 0.92

$$-\sqrt{3}\,a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{2}\,a^{\frac{4}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + a^{\frac{4}{3}}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{3}{4}\,(bx+a)^{\frac{4}{3}} + 3\,(bx+a)^{\frac{1}{3}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log(abs((b*x + a)^(1/3) - a^(1/3))) + 3/4*(b*x + a)^(4/3) + 3*(b*x +
a)^(1/3)*a
```

maple [A] time = 0.01, size = 95, normalized size = 0.90

$$-\sqrt{3}\, a^{\frac{4}{3}} \arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)+a^{\frac{4}{3}} \ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)-\frac{a^{\frac{4}{3}} \ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2}+3\,(bx+a)^{\frac{1}{3}} a+\frac{3\,(bx+a)^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x,x)
```

```
[Out] 3/4*(b*x+a)^(4/3)+3*a*(b*x+a)^(1/3)+a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2*
a^(4/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-a^(4/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 3.03, size = 96, normalized size = 0.91

$$-\sqrt{3}\, a^{\frac{4}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\, a^{\frac{1}{3}}}\right)-\frac{1}{2}\, a^{\frac{4}{3}} \log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}} a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+a^{\frac{4}{3}} \log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx+a)^{\frac{4}{3}}+3\,(bx+a)^{\frac{1}{3}} a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))
- 1/2*a^(4/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + a^
(4/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3/4*(b*x + a)^(4/3) + 3*(b*x + a)^(1
/3)*a
```

mupad [B] time = 0.06, size = 123, normalized size = 1.17

$$3\,a\,{\left(a+b\,x\right)}^{1/3}+\frac{3\,{\left(a+b\,x\right)}^{4/3}}{4}+a^{4/3}\,\ln\left(9\,a^2\,{\left(a+b\,x\right)}^{1/3}-9\,a^{7/3}\right)+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}-\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}+9\,a^2\,{\left(a+b\,x\right)}^{1/3}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x,x)
```

```
[Out] 3*a*(a + b*x)^(1/3) + (3*(a + b*x)^(4/3))/4 + a^(4/3)*log(9*a^2*(a + b*x)^(
1/3) - 9*a^(7/3)) + (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i - 1))/2 - 9*a^2*(a
+ b*x)^(1/3))*(3^(1/2)*1i - 1))/2 - (a^(4/3)*log((9*a^(7/3)*(3^(1/2)*1i + 1
))/2 + 9*a^2*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/2
```

sympy [C] time = 2.39, size = 209, normalized size = 1.99

$$\frac{7a^{\frac{4}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{-\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a^{\frac{4}{3}}e^{\frac{2i\pi}{3}} \log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{7a\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{7}{3}\right)}{\Gamma\left(\frac{10}{3}\right)}+\frac{7b^{\frac{4}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}{4\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x,x)
```

```
[Out] 7*a**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(3*gamma(
10/3)) + 7*a**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
```

```
lar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(3*gamma(10/3)) + 7*a**(4/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/
3)/(3*gamma(10/3)) + 7*a*b**(1/3)*(a/b + x)**(1/3)*gamma(7/3)/gamma(10/3) +
 7*b**(4/3)*(a/b + x)**(4/3)*gamma(7/3)/(4*gamma(10/3))
```

3.390 $\int \frac{(a+bx)^{4/3}}{x^2} dx$

Optimal. Leaf size=107

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 57, 617, 204, 31}

$$-\frac{(a+bx)^{4/3}}{x}+4b\sqrt[3]{a+bx}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^2,x]
```

```
[Out] 4*b*(a + b*x)^(1/3) - (a + b*x)^(4/3)/x - (4*a^(1/3)*b*ArcTan[(a^(1/3) + 2*
(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] - (2*a^(1/3)*b*Log[x])/3 + 2*a
^(1/3)*b*Log[a^(1/3) - (a + b*x)^(1/3)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{4/3}}{x^2}\,dx &= -\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4b)\int \frac{\sqrt[3]{a+bx}}{x}\,dx\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}+\frac{1}{3}(4ab)\int \frac{1}{x(a+bx)^{2/3}}\,dx\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)-\left(2\sqrt[3]{a}\,b\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)-(\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+\left(4\sqrt[3]{a}\,b\right)\text{Subst}\Bigg(\\ &= 4b\sqrt[3]{a+bx}-\frac{(a+bx)^{4/3}}{x}-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{2}{3}\sqrt[3]{a}\,b\log(x)+2\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a}\right.\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.31

$$\frac{3b(a+bx)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};\frac{bx}{a}+1\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^2,x]
[Out] (3*b*(a + b*x)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 + (b*x)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.19, size = 135, normalized size = 1.26

$$-\frac{2}{3}\sqrt[3]{a}\,b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)+\frac{\sqrt[3]{a+bx}\,(3(a+bx)-4a)}{x}+\frac{4}{3}\sqrt[3]{a}\,b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)-\frac{4\sqrt[3]{a}\,b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x^2,x]
[Out] ((a + b*x)^(1/3)*(-4*a + 3*(a + b*x)))/x - (4*a^(1/3)*b*ArcTan[1/Sqrt[3] +
(2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/Sqrt[3] + (4*a^(1/3)*b*Log[a^(1/3)
- (a + b*x)^(1/3)])/3 - (2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)])/3
```

fricas [A] time = 0.72, size = 111, normalized size = 1.04

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}bx\arctan\left(\frac{2\sqrt{3}\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}\,a}{3\,a}\right)+2\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}bx\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-3\,(3\,bx-a)(bx+a)^{\frac{1}{3}}}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*a^(2/3) +
 sqrt(3)*a)/a) + 2*a^(1/3)*b*x*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3
) + a^(2/3)) - 4*a^(1/3)*b*x*log((b*x + a)^(1/3) - a^(1/3)) - 3*(3*b*x - a)
*(b*x + a)^(1/3))/x
```

giac [A] time = 2.35, size = 119, normalized size = 1.11

$$-\frac{4\,\sqrt{3}\,a^{\frac{1}{3}}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}b^{2}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}b^{2}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)-9\,(bx+a)^{\frac{1}{3}}b^{2}+\frac{3\,(bx+a)^{\frac{1}{3}}ab}{x}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/3*(4*sqrt(3)*a^(1/3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3)
)/a^(1/3)) + 2*a^(1/3)*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*b^2*log(abs((b*x + a)^(1/3) - a^(1/3))) - 9*(b*x + a)^
(1/3)*b^2 + 3*(b*x + a)^(1/3)*a*b/x)/b
```

maple [A] time = 0.01, size = 103, normalized size = 0.96

$$-\frac{4\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3}+\frac{4a^{\frac{1}{3}}b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3}-\frac{2a^{\frac{1}{3}}b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3}+3\,(bx+a)^{\frac{1}{3}}\,b-\frac{(bx+a)^{\frac{1}{3}}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^2,x)
```

```
[Out] 3*b*(b*x+a)^(1/3)-a*(b*x+a)^(1/3)/x+4/3*b*a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3)
)-2/3*b*a^(1/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-4/3*b*a^(1/
3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 3.03, size = 104, normalized size = 0.97

$$-\frac{4}{3}\,\sqrt{3}\,a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-\frac{2}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+\frac{4}{3}\,a^{\frac{1}{3}}b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\,(bx+a)^{\frac{1}{3}}b-\frac{(bx+a)^{\frac{1}{3}}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(
1/3)) - 2/3*a^(1/3)*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/
3)) + 4/3*a^(1/3)*b*log((b*x + a)^(1/3) - a^(1/3)) + 3*(b*x + a)^(1/3)*b -
(b*x + a)^(1/3)*a/x
```

mupad [B] time = 0.07, size = 131, normalized size = 1.22

$$3\,b\,(a+b\,x)^{1/3}+\frac{4\,a^{1/3}\,b\,\ln\left(12\,a^{4/3}\,b-12\,a\,b\,(a+b\,x)^{1/3}\right)}{3}-\frac{a\,(a+b\,x)^{1/3}}{x}+\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}-6\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}-\frac{2\,a^{1/3}\,b\,\ln\left(12\,a\,b\,(a+b\,x)^{1/3}+6\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^2,x)
```

```
[Out] 3*b*(a + b*x)^(1/3) + (4*a^(1/3)*b*log(12*a^(4/3)*b - 12*a*b*(a + b*x)^(1/3
)))/3 - (a*(a + b*x)^(1/3))/x + (2*a^(1/3)*b*log(12*a*b*(a + b*x)^(1/3) - 6
*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (2*a^(1/3)*b*log(12*a*b*
(a + b*x)^(1/3) + 6*a^(4/3)*b*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/3
```

sympy [C] time = 2.58, size = 719, normalized size = 6.72

$$\frac{28a^{\frac{10}{3}}be^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}b\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{28a^{\frac{10}{3}}be^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{28a^{\frac{7}{3}}b^2\left(\frac{a}{b}+x\right)e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}+\frac{84a^3b^{\frac{4}{3}}\sqrt[3]{\frac{a}{b}+x}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}-\frac{63a^2b^{\frac{7}{3}}\left(\frac{a}{b}+x\right)^{\frac{4}{3}}e^{\frac{2i\pi}{3}}\Gamma\left(\frac{7}{3}\right)}{9a^3e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)-9a^2b\left(\frac{a}{b}+x\right)e^{\frac{2i\pi}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**2,x)
```

```
[Out] 28*a**(10/3)*b*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*ga
mma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/
3)*gamma(10/3)) + 28*a**(10/3)*b*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_pola
r(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2
*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(10/3)*b*exp(-2*I*pi/3)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*
a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/
3)) - 28*a**(7/3)*b**2*(a/b + x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**
(1/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(7/3)*b**2*(a/b + x)*log(1 - b**(
1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(
2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(7/3)*b**2*(a/b + x)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) -
9*a**2*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) + 84*a**3*b**(4/3)*(a/b + x)*
*(1/3)*exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*
b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3)) - 63*a**2*b**(7/3)*(a/b + x)**(4/3)*
exp(2*I*pi/3)*gamma(7/3)/(9*a**3*exp(2*I*pi/3)*gamma(10/3) - 9*a**2*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3))
```

3.391 $\int \frac{(a+bx)^{4/3}}{x^3}\,dx$

Optimal. Leaf size=124

$$-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 57, 617, 204, 31}

$$-\frac{b^2\log(x)}{9a^{2/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}-\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}}-\frac{(a+bx)^{4/3}}{2x^2}-\frac{2b\sqrt[3]{a+bx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-2*b*(a + b*x)^(1/3))/(3*x) - (a + b*x)^(4/3)/(2*x^2) - (2*b^2*ArcTan[(a^(
1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) - (b^2*Lo
g[x])/(9*a^(2/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{x^3}\,dx = -\frac{(a+bx)^{4/3}}{2x^2} + \frac{1}{3}(2b)\int \frac{\sqrt[3]{a+bx}}{x^2}\,dx$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x} - \frac{(a+bx)^{4/3}}{2x^2} + \frac{1}{9}\left(2b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x} - \frac{(a+bx)^{4/3}}{2x^2} - \frac{b^2\log(x)}{9a^{2/3}} - \frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^{2/3}} - \frac{b^2\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x} - \frac{(a+bx)^{4/3}}{2x^2} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}} + \frac{\left(2b^2\right)\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a^{2/3}}$$
$$= -\frac{2b\sqrt[3]{a+bx}}{3x} - \frac{(a+bx)^{4/3}}{2x^2} - \frac{2b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{2/3}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.28

$$-\frac{3b^2(a+bx)^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};\frac{bx}{a}+1\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] (-3*b^2*(a + b*x)^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, 1 + (b*x)/a])/(7*a^
3)
```

IntegrateAlgebraic [A] time = 0.23, size = 146, normalized size = 1.18

$$\frac{2b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{2/3}} - \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{2/3}} - \frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}} - \frac{\sqrt[3]{a+bx}\,(7(a+bx)-4a)}{6x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/x^3,x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(-4*a + 7*(a + b*x)))/x^2 - (2*b^2*ArcTan[1/Sqrt[3] +
 (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)) + (2*b^2*Log[a
^(1/3) - (a + b*x)^(1/3)])/(9*a^(2/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a + b*
x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(2/3))
```

fricas [A] time = 0.98, size = 162, normalized size = 1.31

$$-\frac{4\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+2\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-4\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)+3\left(7a^2bx+3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^2
)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 2*(a^2)^(2/3)*b^2
*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) -
```

```
4*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2*b*x
+ 3*a^3)*(b*x + a)^(1/3))/(a^2*x^2)
```

giac [A] time = 1.94, size = 127, normalized size = 1.02

$$-\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}+\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}}-\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}+\frac{3\left(7(bx+a)^{\frac{4}{3}}b^3-4(bx+a)^{\frac{1}{3}}ab^3\right)}{b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="giac")
```

```
[Out] -1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(2/3) + 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3) + 3*(7*(b*x +
 a)^(4/3)*b^3 - 4*(b*x + a)^(1/3)*a*b^3)/(b^2*x^2))/b
```

maple [A] time = 0.01, size = 111, normalized size = 0.90

$$-\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{2}{3}}}+\frac{2b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2(bx+a)^{\frac{1}{3}}a}{3x^2}-\frac{7(bx+a)^{\frac{4}{3}}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/x^3,x)
```

```
[Out] -7/6*(b*x+a)^(4/3)/x^2+2/3/x^2*(b*x+a)^(1/3)*a+2/9*b^2/a^(2/3)*ln(-a^(1/3)+
(b*x+a)^(1/3))-1/9*b^2/a^(2/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/
3))-2/9*b^2/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 3.06, size = 136, normalized size = 1.10

$$-\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{2}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{2}{3}}}+\frac{2b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{2}{3}}}-\frac{7(bx+a)^{\frac{4}{3}}b^2-4(bx+a)^{\frac{1}{3}}ab^2}{6\left((bx+a)^2-2(bx+a)a+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/x^3,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(2/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(2/3) - 1/6*(7*(b*x + a)
^(4/3)*b^2 - 4*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2 - 2*(b*x + a)*a + a^2)
```

mupad [B] time = 0.12, size = 174, normalized size = 1.40

$$\frac{2b^2\ln\left(2b^2(a+bx)^{1/3}-2a^{1/3}b^2\right)}{9a^{2/3}}-\frac{\frac{7b^2(a+bx)^{4/3}}{6}-\frac{2ab^2(a+bx)^{1/3}}{3}}{(a+bx)^2-2a(a+bx)+a^2}-\frac{\ln\left(2b^2(a+bx)^{1/3}+a^{1/3}\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{9a^{2/3}}+\frac{b^2\ln\left(2b^2(a+bx)^{1/3}-9a^{1/3}b^2\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/x^3,x)
```

```
[Out] (2*b^2*log(2*b^2*(a + b*x)^(1/3) - 2*a^(1/3)*b^2))/(9*a^(2/3)) - ((7*b^2*(a
 + b*x)^(4/3))/6 - (2*a*b^2*(a + b*x)^(1/3))/3)/((a + b*x)^2 - 2*a*(a + b*x
) + a^2) - (log(2*b^2*(a + b*x)^(1/3) + a^(1/3)*(3^(1/2)*b^2*1i + b^2))*(3^
```

```
(1/2)*b^2*1i + b^2))/(9*a^(2/3)) + (b^2*log(2*b^2*(a + b*x)^(1/3) - 9*a^(1/
3)*b^2*((3^(1/2)*1i)/9 - 1/9))*((3^(1/2)*1i)/9 - 1/9))/a^(2/3)
```

sympy [C] time = 2.74, size = 2266, normalized size = 18.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/x**3,x)
```

```
[Out] 28*a**(19/3)*b**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))
*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2
*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2
*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)
/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*ga
mma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*
b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 28*a**(19/3)*b**2*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3
)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a
**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b +
 x)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(7/3)/(5
4*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma
(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**
3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*log
(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(7/3)/(54
*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(
10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3
*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 84*a**(16/3)*b**3*(a/b + x)*exp(
-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*
gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*
I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3)
 - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*
(a/b + x)**2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamm
a(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi
/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54
*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a/b
 + x)**2*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*ga
mma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*
pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) -
54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 84*a**(13/3)*b**4*(a
/b + x)**2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(
a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/
3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*
a**(10/3)*b**5*(a/b + x)**3*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)
/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b
+ x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*g
amma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a**(
10/3)*b**5*(a/b + x)**3*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/
b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)
*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 28*a*
*(10/3)*b**5*(a/b + x)**3*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(4*I*pi/3)/a**(1/3))*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3)
 - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)
**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gam
ma(10/3)) + 84*a**6*b**(7/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(7/3)/(54*
a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(1
```

```
0/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*
(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) - 231*a**5*b**(10/3)*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*I*pi/3)*gamma(10/3) - 162*a**6*b
*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*a**5*b**2*(a/b + x)**2*exp(2*I*p
i/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**3*exp(2*I*pi/3)*gamma(10/3)) + 1
47*a**4*b**(13/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(7/3)/(54*a**7*exp(2*
I*pi/3)*gamma(10/3) - 162*a**6*b*(a/b + x)*exp(2*I*pi/3)*gamma(10/3) + 162*
a**5*b**2*(a/b + x)**2*exp(2*I*pi/3)*gamma(10/3) - 54*a**4*b**3*(a/b + x)**
3*exp(2*I*pi/3)*gamma(10/3))
```

3.392 $\int \frac{x^3}{\sqrt[3]{a+bx}} dx$

Optimal. Leaf size=72

$$-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{3(a+bx)^{11/3}}{11b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a + b*x)^(5/3))/(5*b^4) - (9*a*(
a + b*x)^(8/3))/(8*b^4) + (3*(a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{a+bx}}\,dx &= \int \left(-\frac{a^3}{b^3\sqrt[3]{a+bx}}+\frac{3a^2(a+bx)^{2/3}}{b^3}-\frac{3a(a+bx)^{5/3}}{b^3}+\frac{(a+bx)^{8/3}}{b^3}\right)dx\\ &= -\frac{3a^3(a+bx)^{2/3}}{2b^4}+\frac{9a^2(a+bx)^{5/3}}{5b^4}-\frac{9a(a+bx)^{8/3}}{8b^4}+\frac{3(a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.64

$$\frac{3(a+bx)^{2/3}\left(-81a^3+54a^2bx-45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-81*a^3 + 54*a^2*b*x - 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 51, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}\left(-220a^3+264a^2(a+bx)-165a(a+bx)^2+40(a+bx)^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(-220*a^3 + 264*a^2*(a + b*x) - 165*a*(a + b*x)^2 + 40*(
a + b*x)^3))/(440*b^4)
```

fricas [A] time = 0.87, size = 42, normalized size = 0.58

$$\frac{3\left(40\,b^{3}x^{3}-45\,ab^{2}x^{2}+54\,a^{2}bx-81\,a^{3}\right)(bx+a)^{\frac{2}{3}}}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 - 45*a*b^2*x^2 + 54*a^2*b*x - 81*a^3)*(b*x + a)^(2/3)/b^4
```

giac [A] time = 0.90, size = 49, normalized size = 0.68

$$\frac{3\left(40\,(bx+a)^{\frac{11}{3}}-165\,(bx+a)^{\frac{8}{3}}a+264\,(bx+a)^{\frac{5}{3}}a^{2}-220\,(bx+a)^{\frac{2}{3}}a^{3}\right)}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x + a)^(11/3) - 165*(b*x + a)^(8/3)*a + 264*(b*x + a)^(5/3)*a^
2 - 220*(b*x + a)^(2/3)*a^3)/b^4
```

maple [A] time = 0.00, size = 43, normalized size = 0.60

$$-\frac{3\,(bx+a)^{\frac{2}{3}}\left(-40b^{3}x^{3}+45a\,b^{2}x^{2}-54a^{2}bx+81a^{3}\right)}{440b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(1/3),x)
```

```
[Out] -3/440*(b*x+a)^(2/3)*(-40*b^3*x^3+45*a*b^2*x^2-54*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.31, size = 56, normalized size = 0.78

$$\frac{3\,(bx+a)^{\frac{11}{3}}}{11\,b^{4}}-\frac{9\,(bx+a)^{\frac{8}{3}}a}{8\,b^{4}}+\frac{9\,(bx+a)^{\frac{5}{3}}a^{2}}{5\,b^{4}}-\frac{3\,(bx+a)^{\frac{2}{3}}a^{3}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x + a)^(11/3)/b^4 - 9/8*(b*x + a)^(8/3)*a/b^4 + 9/5*(b*x + a)^(5/3)
*a^2/b^4 - 3/2*(b*x + a)^(2/3)*a^3/b^4
```

mupad [B] time = 0.04, size = 56, normalized size = 0.78

$$\frac{3\,{\left(a+b\,x\right)}^{11/3}}{11\,b^4}-\frac{3\,a^3\,{\left(a+b\,x\right)}^{2/3}}{2\,b^4}+\frac{9\,a^2\,{\left(a+b\,x\right)}^{5/3}}{5\,b^4}-\frac{9\,a\,{\left(a+b\,x\right)}^{8/3}}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(1/3),x)
```

```
[Out] (3*(a + b*x)^(11/3))/(11*b^4) - (3*a^3*(a + b*x)^(2/3))/(2*b^4) + (9*a^2*(a
 + b*x)^(5/3))/(5*b^4) - (9*a*(a + b*x)^(8/3))/(8*b^4)
```

sympy [B] time = 2.78, size = 1640, normalized size = 22.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(1/3),x)
```

```
[Out] -243*a**(71/3)*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 660
0*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**1
5*b**9*x**5 + 440*a**14*b**10*x**6) + 243*a**(71/3)/(440*a**20*b**4 + 2640*
a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**
8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1296*a**(68/3)*b*x*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(68/3)*b*x/(440*a**20*b**4 + 2640*a**19*b**5
*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2
640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 2808*a**(65/3)*b**2*x**2*(1 +
 b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 +
 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a
**14*b**10*x**6) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b**4 + 2640*a**19*b*
*5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 +
 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 3120*a**(62/3)*b**3*x**3*(1
 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2
 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440
*a**14*b**10*x**6) + 4860*a**(62/3)*b**3*x**3/(440*a**20*b**4 + 2640*a**19*
b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4
 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) - 1710*a**(59/3)*b**4*x**4*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 72*a**(56/3)*b**5*x**5*
(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x*
*2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 4
40*a**14*b**10*x**6) + 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4 + 2640*a**1
9*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x*
*4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1104*a**(53/3)*b**6*x**
6*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*
x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 +
 440*a**14*b**10*x**6) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4 + 2640*a**
19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x
**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 1152*a**(50/3)*b**7*x*
*7*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6
*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5
+ 440*a**14*b**10*x**6) + 585*a**(47/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(440*a
**20*b**4 + 2640*a**19*b**5*x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3
 + 6600*a**16*b**8*x**4 + 2640*a**15*b**9*x**5 + 440*a**14*b**10*x**6) + 12
0*a**(44/3)*b**9*x**9*(1 + b*x/a)**(2/3)/(440*a**20*b**4 + 2640*a**19*b**5*
x + 6600*a**18*b**6*x**2 + 8800*a**17*b**7*x**3 + 6600*a**16*b**8*x**4 + 26
40*a**15*b**9*x**5 + 440*a**14*b**10*x**6)
```

3.393 $\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=53

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2(a+bx)^{2/3}}{2b^3}+\frac{3(a+bx)^{8/3}}{8b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(2/3))/(2*b^3) - (6*a*(a + b*x)^(5/3))/(5*b^3) + (3*(a + b
*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{a+bx}}-\frac{2a(a+bx)^{2/3}}{b^2}+\frac{(a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(a+bx)^{2/3}}{2b^3}-\frac{6a(a+bx)^{5/3}}{5b^3}+\frac{3(a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.66

$$\frac{3(a+bx)^{2/3}\left(9a^2-6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(9*a^2 - 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 45, normalized size = 0.85

$$\frac{3\left(20a^2(a+bx)^{2/3}+5(a+bx)^{8/3}-16a(a+bx)^{5/3}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(1/3),x]
```

```
[Out] (3*(20*a^2*(a + b*x)^(2/3) - 16*a*(a + b*x)^(5/3) + 5*(a + b*x)^(8/3)))/(40
*b^3)
```

fricas [A] time = 0.76, size = 31, normalized size = 0.58

$$\frac{3\left(5\,b^{2}x^{2}-6\,abx+9\,a^{2}\right)(bx+a)^{\frac{2}{3}}}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="fricas")
[Out] 3/40*(5*b^2*x^2 - 6*a*b*x + 9*a^2)*(b*x + a)^(2/3)/b^3
```

giac [A] time = 0.91, size = 37, normalized size = 0.70

$$\frac{3\left(5\,(bx+a)^{\frac{8}{3}}-16\,(bx+a)^{\frac{5}{3}}a+20\,(bx+a)^{\frac{2}{3}}a^{2}\right)}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/40*(5*(b*x + a)^(8/3) - 16*(b*x + a)^(5/3)*a + 20*(b*x + a)^(2/3)*a^2)/b^
3
```

maple [A] time = 0.00, size = 32, normalized size = 0.60

$$\frac{3\,(bx+a)^{\frac{2}{3}}\left(5b^{2}x^{2}-6abx+9a^{2}\right)}{40b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/3),x)
[Out] 3/40*(b*x+a)^(2/3)*(5*b^2*x^2-6*a*b*x+9*a^2)/b^3
```

maxima [A] time = 1.38, size = 41, normalized size = 0.77

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^{3}}-\frac{6\,(bx+a)^{\frac{5}{3}}a}{5\,b^{3}}+\frac{3\,(bx+a)^{\frac{2}{3}}a^{2}}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/8*(b*x + a)^(8/3)/b^3 - 6/5*(b*x + a)^(5/3)*a/b^3 + 3/2*(b*x + a)^(2/3)*a
^2/b^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{15\,(a+b\,x)^{8/3}-48\,a\,(a+b\,x)^{5/3}+60\,a^{2}\,(a+b\,x)^{2/3}}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(1/3),x)
[Out] (15*(a + b*x)^(8/3) - 48*a*(a + b*x)^(5/3) + 60*a^2*(a + b*x)^(2/3))/(40*b^
3)
```

sympy [B] time = 1.77, size = 600, normalized size = 11.32

$$\frac{27a^{\frac{32}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}-\frac{27a^{\frac{32}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}+\frac{63a^{\frac{29}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}-\frac{81a^{\frac{29}{3}}bx}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}+\frac{42a^{\frac{26}{3}}b^{2}x^{2}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}-\frac{81a^{\frac{26}{3}}b^{2}x^{2}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}+\frac{18a^{\frac{23}{3}}b^{3}x^{3}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}-\frac{27a^{\frac{23}{3}}b^{3}x^{3}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}+\frac{27a^{\frac{20}{3}}b^{4}x^{4}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}+\frac{15a^{\frac{17}{3}}b^{5}x^{5}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{40a^{8}b^{3}+120a^{7}b^{4}x+120a^{6}b^{5}x^{2}+40a^{5}b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/3),x)
```

```
[Out] 27*a**(32/3)*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*
b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(32/3)/(40*a**8*b**3 + 120*a**7*b**4
*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 + b*x/a)
**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6
*x**3) - 81*a**(29/3)*b*x/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x
**2 + 40*a**5*b**6*x**3) + 42*a**(26/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a*
*8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a*
*(26/3)*b**2*x**2/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40
*a**5*b**6*x**3) + 18*a**(23/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**8*b**3
+ 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*
b**3*x**3/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b*
*6*x**3) + 27*a**(20/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a*
*7*b**4*x + 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b**5*x**
5*(1 + b*x/a)**(2/3)/(40*a**8*b**3 + 120*a**7*b**4*x + 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3)
```

3.394 $\int \frac{x}{\sqrt[3]{a+bx}} dx$

Optimal. Leaf size=34

$$\frac{3(a+bx)^{5/3}}{5b^2} - \frac{3a(a+bx)^{2/3}}{2b^2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{5/3}}{5b^2} - \frac{3a(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(1/3),x]
[Out] (-3*a*(a + b*x)^(2/3))/(2*b^2) + (3*(a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{a+bx}} dx &= \int \left(-\frac{a}{b\sqrt[3]{a+bx}} + \frac{(a+bx)^{2/3}}{b}\right) dx\\ &= -\frac{3a(a+bx)^{2/3}}{2b^2} + \frac{3(a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.71

$$\frac{3(a+bx)^{2/3}(2bx-3a)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(1/3),x]
[Out] (3*(a + b*x)^(2/3)*(-3*a + 2*b*x))/(10*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.71

$$-\frac{3(3a-2bx)(a+bx)^{2/3}}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(1/3),x]
[Out] (-3*(3*a - 2*b*x)*(a + b*x)^(2/3))/(10*b^2)
```

fricas [A] time = 0.85, size = 20, normalized size = 0.59

$$\frac{3\,(2\,bx-3\,a)(bx+a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x - 3*a)*(b*x + a)^(2/3)/b^2
```

giac [A] time = 0.90, size = 25, normalized size = 0.74

$$\frac{3\left(2\,(bx+a)^{\frac{5}{3}}-5\,(bx+a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(2*(b*x + a)^(5/3) - 5*(b*x + a)^(2/3)*a)/b^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.62

$$-\frac{3\,(bx+a)^{\frac{2}{3}}\,(-2bx+3a)}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/3),x)
```

```
[Out] -3/10*(b*x+a)^(2/3)*(-2*b*x+3*a)/b^2
```

maxima [A] time = 1.30, size = 26, normalized size = 0.76

$$\frac{3\,(bx+a)^{\frac{5}{3}}}{5\,b^2}-\frac{3\,(bx+a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^2 - 3/2*(b*x + a)^(2/3)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.74

$$-\frac{15\,a\,(a+b\,x)^{2/3}-6\,(a+b\,x)^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(1/3),x)
```

```
[Out] -(15*a*(a + b*x)^(2/3) - 6*(a + b*x)^(5/3))/(10*b^2)
```

sympy [B] time = 1.16, size = 162, normalized size = 4.76

$$-\frac{9a^{\frac{11}{3}}\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{11}{3}}}{10a^2b^2+10ab^3x}-\frac{3a^{\frac{8}{3}}bx\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}+\frac{9a^{\frac{8}{3}}bx}{10a^2b^2+10ab^3x}+\frac{6a^{\frac{5}{3}}b^2x^2\left(1+\frac{bx}{a}\right)^{\frac{2}{3}}}{10a^2b^2+10ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/3),x)
```

```
[Out] -9*a**(11/3)*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x) + 9*a**(11/3)/
(10*a**2*b**2 + 10*a*b**3*x) - 3*a**(8/3)*b*x*(1 + b*x/a)**(2/3)/(10*a**2*b
**2 + 10*a*b**3*x) + 9*a**(8/3)*b*x/(10*a**2*b**2 + 10*a*b**3*x) + 6*a**(5/
3)*b**2*x**2*(1 + b*x/a)**(2/3)/(10*a**2*b**2 + 10*a*b**3*x)
```

3.395 $\int \frac{1}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=16

$$\frac{3(a+bx)^{2/3}}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}}\,dx = \frac{3(a+bx)^{2/3}}{2b}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{3(a+bx)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

fricas [A] time = 0.99, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b
```

giac [A] time = 0.97, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="giac")
[Out] 3/2*(b*x + a)^(2/3)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3),x)
[Out] 3/2*(b*x+a)^(2/3)/b
```

maxima [A] time = 1.32, size = 12, normalized size = 0.75

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3),x, algorithm="maxima")
[Out] 3/2*(b*x + a)^(2/3)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{3\,{\left(a+b\,x\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(1/3),x)
[Out] (3*(a + b*x)^(2/3))/(2*b)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{3\,(a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3),x)
[Out] 3*(a + b*x)**(2/3)/(2*b)
```

3.396 $\int \frac{1}{x\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=79

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3) -
 Log[x]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a+bx}}\,dx = -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a+bx}\right)-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.84

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)-\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - Log[x] + 3*L
og[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(1/3))
```

IntegrateAlgebraic [A] time = 0.05, size = 104, normalized size = 1.32

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2\sqrt[3]{a}}+\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{\sqrt[3]{a}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3)
 + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(1/3) - Log[a^(2/3) + a^(1/3)*(a + b*x)
^(1/3) + (a + b*x)^(2/3)]/(2*a^(1/3))
```

fricas [A] time = 0.87, size = 213, normalized size = 2.70

$$\left[\frac{\sqrt{3}\,a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx+\sqrt{3}\left(2\,(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a},\ \frac{2\sqrt{3}\,a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x + a)^(2/3)*a^
(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a
^(2/3) + 3*a)/x) - a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
a^(2/3)) + 2*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)))/a, 1/2*(2*sqrt(3)*a^(2
/3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*log
((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x
+ a)^(1/3) - a^(1/3)))/a]
```

giac [A] time = 2.37, size = 77, normalized size = 0.97

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(1/3)
```

maple [A] time = 0.00, size = 75, normalized size = 0.95

$$\frac{\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/3),x)
```

```
[Out] 1/a^(1/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(1/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a
^(1/3)+(b*x+a)^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a
^(1/3)+1))
```

maxima [A] time = 2.93, size = 76, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(1/3)
```

mupad [B] time = 0.09, size = 99, normalized size = 1.25

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{1/3}}+\frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}-\frac{\ln\left(9\,(a+b\,x)^{1/3}-\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(1/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(1/3) + (log(9*(a + b*x)^(1/3) - (9*a^
(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a^(1/3)) - (log(9*(a + b*
x)^(1/3) - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a^(1/3))
```

sympy [C] time = 1.88, size = 155, normalized size = 1.96

$$\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}+\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/3),x)
```

```
[Out] 2*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(
5/3)) + 2*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/
3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) + 2*exp(-2*I*pi/3)*log(1 -
b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(3*a**(1
/3)*gamma(5/3))
```

3.397 $\int \frac{1}{x^2 \sqrt[3]{a+bx}} \, dx$

Optimal. Leaf size=100

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {51, 55, 617, 204, 31}

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a} - \sqrt[3]{a+bx}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}} - \frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx &= -\frac{(a+bx)^{2/3}}{ax}-\frac{b\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a}\\ &= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{\ }\right.}{2a}\\ &= -\frac{(a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\ &= -\frac{(a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.33

$$\frac{3b(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 + (b*x)/a])/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.16, size = 128, normalized size = 1.28

$$-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{4/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}-\frac{(a+bx)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(1/3)),x]
```

```
[Out] -((a + b*x)^(2/3)/(a*x)) - (b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(4/3)) - (b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(
4/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(6*a^(
4/3))
```

fricas [A] time = 0.89, size = 306, normalized size = 3.06

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-6(bx+a)^{\frac{2}{3}}a}{6a^2x},\ -\frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+2(-a)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+6(bx+a)^{\frac{2}{3}}a}{6a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*sqrt(1/3)*(2*(b*x
 + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + (-a)^(2/3)*b*x*log((b*x + a)^
(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x + a)^(1/3) + (-a)^(1/3)) - 6*(b*x + a)^(2/3)*a)/(a^2*x), -1/6*(6*sqrt(1/3
)*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) - (-a)^(1/3
```

```
))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)*b*x*log((b*x + a)^(2/3) - (b*x + a)^(1
/3)*(-a)^(1/3) + (-a)^(2/3)) + 2*(-a)^(2/3)*b*x*log((b*x + a)^(1/3) + (-a)^
(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*x)]
```

giac [A] time = 2.45, size = 109, normalized size = 1.09

$$-\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{6(bx+a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] -1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(4/3) - b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x + a)^(2
/3)*b/(a*x))/b
```

maple [A] time = 0.01, size = 95, normalized size = 0.95

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{4}{3}}}-\frac{b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/3),x)
```

```
[Out] -(b*x+a)^(2/3)/a/x-1/3*b/a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))+1/6*b/a^(4/3)*l
n(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-1/3*b/a^(4/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 3.00, size = 106, normalized size = 1.06

$$-\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{4}{3}}}-\frac{(bx+a)^{\frac{2}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - (b*x + a)^(2/3)*b/((b*x + a)*a - a^2) + 1/6*b*log((b*x + a)^(2/3) +
 (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x + a)^(1/3) - a
^(1/3))/a^(4/3)
```

mupad [B] time = 0.14, size = 130, normalized size = 1.30

$$-\frac{(a+b\,x)^{2/3}}{a\,x}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}-\frac{b^2\,(a+b\,x)^{1/3}}{a^2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}-\frac{b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(1/3)),x)
```

```
[Out] (log((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b - 3^(
1/2)*b*1i))/(6*a^(4/3)) - (a + b*x)^(2/3)/(a*x) + (log((b + 3^(1/2)*b*1i)^2
/(4*a^(5/3)) - (b^2*(a + b*x)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3)) -
 (b*log((a + b*x)^(1/3) - a^(1/3)))/(3*a^(4/3))
```

sympy [C] time = 2.20, size = 831, normalized size = 8.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*p
i/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a
/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(a/b + x)**(4/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x
)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7/3)*
exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(9*a**
3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3)*b**(10/3)*(a/b + x)**(7
/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a*
*(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/
3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 2*a**(2/3
)*b**(10/3)*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*
I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) +
 6*a*b**3*(a/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(a/b + x)*
*(4/3)*exp(2*I*pi/3)*gamma(5/3) - 9*a**2*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3))
```

3.398 $\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=130

$$-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{b^2\log(x)}{9a^{7/3}}+\frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}+\frac{2b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}}+\frac{2b(a+bx)^{2/3}}{3a^2x}-\frac{(a+bx)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -(a + b*x)^(2/3)/(2*a*x^2) + (2*b*(a + b*x)^(2/3))/(3*a^2*x) + (2*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (
b^2*Log[x])/(9*a^(7/3)) + (b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \sqrt[3]{a+bx}}\,dx &= -\frac{(a+bx)^{2/3}}{2ax^2} - \frac{(2b)\int \frac{1}{x^2 \sqrt[3]{a+bx}}\,dx}{3a} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{\left(2b^2\right)\int \frac{1}{x \sqrt[3]{a+bx}}\,dx}{9a^2} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{b^2 \operatorname{Subst}\left(\int \right.}{} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} - \frac{b^2 \log(x)}{9a^{7/3}} + \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}} - \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{3a^{7/3}} \\
&= -\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2 \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{b^2 \log(x)}{9a^{7/3}} + \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.27

$$-\frac{3b^2(a+bx)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};\frac{bx}{a}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 + (b*x)/a])/(2*a^3
)
```

IntegrateAlgebraic [A] time = 0.15, size = 149, normalized size = 1.15

$$\frac{2b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{7/3}} - \frac{b^2 \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{7/3}} + \frac{2b^2 \tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} - \frac{(a+bx)^{2/3}(7a-4(a+bx))}{6a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(1/3)),x]
```

```
[Out] -1/6*((a + b*x)^(2/3)*(7*a - 4*(a + b*x)))/(a^2*x^2) + (2*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) + (2*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(7/3)) - (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(7/3))
```

fricas [A] time = 0.96, size = 296, normalized size = 2.28

$$\left[\frac{6\sqrt{\frac{1}{3}}\,ab^2x^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx-3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^3x^2}, \frac{12\sqrt{\frac{1}{3}}\,a^{\frac{2}{3}}b^2x^2\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-2a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+4a^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(4abx-3a^2\right)(bx+a)^{\frac{2}{3}}}{18a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt(-1/a^(2/3))*log((2*b*x + 3*sqrt(1/3)*(2*(
b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*
(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) +
```

```
(b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3)
 - a^(1/3)) + 3*(4*a*b*x - 3*a^2)*(b*x + a)^(2/3))/(a^3*x^2), 1/18*(12*sqrt
(1/3)*a^(2/3)*b^2*x^2*arctan(sqrt(1/3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
)) - 2*a^(2/3)*b^2*x^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2
/3)) + 4*a^(2/3)*b^2*x^2*log((b*x + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x - 3*a^
2)*(b*x + a)^(2/3))/(a^3*x^2)]
```

giac [A] time = 2.24, size = 130, normalized size = 1.00

$$\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}}-\frac{2b^3\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}}+\frac{4b^3\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}}+\frac{3\left(4(bx+a)^{\frac{5}{3}}b^3-7(bx+a)^{\frac{2}{3}}ab^3\right)}{a^2b^2x^2}}{18b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(7/3) - 2*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(7/3) + 4*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) + 3*(4*(b*x +
a)^(5/3)*b^3 - 7*(b*x + a)^(2/3)*a*b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.01, size = 117, normalized size = 0.90

$$\frac{2\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2(bx+a)^{\frac{2}{3}}b}{3a^2x}-\frac{(bx+a)^{\frac{2}{3}}}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(1/3),x)
```

```
[Out] -1/2*(b*x+a)^(2/3)/a/x^2+2/3*b*(b*x+a)^(2/3)/a^2/x+2/9*b^2/a^(7/3)*ln(-a^(1
/3)+(b*x+a)^(1/3))-1/9*b^2/a^(7/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)
^(2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)
+1))
```

maxima [A] time = 3.03, size = 142, normalized size = 1.09

$$\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{7}{3}}}-\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{4(bx+a)^{\frac{5}{3}}b^2-7(bx+a)^{\frac{2}{3}}ab^2}{6\left((bx+a)^2a^2-2(bx+a)a^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(7/3) - 1/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) + 2/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3) + 1/6*(4*(b*x + a)^
(5/3)*b^2 - 7*(b*x + a)^(2/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 + a
^4)
```

mupad [B] time = 0.23, size = 182, normalized size = 1.40

$$\frac{2b^2\ln\left((a+bx)^{1/3}-a^{1/3}\right)}{9a^{7/3}}-\frac{\frac{7b^2(a+bx)^{2/3}}{6a}-\frac{2b^2(a+bx)^{5/3}}{3a^2}}{(a+bx)^2-2a\,(a+bx)+a^2}-\frac{\ln\left(\frac{4b^4(a+bx)^{1/3}}{9a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)^2}{9a^{11/3}}\right)\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{9a^{7/3}}+\frac{b^2\ln\left(\frac{4b^4(a+bx)^{1/3}}{9a^4}-\frac{9b^4\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^2}{a^{11/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(1/3)),x)
```

```
[Out] (2*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(7/3)) - ((7*b^2*(a + b*x)^(2/3
))/(6*a) - (2*b^2*(a + b*x)^(5/3))/(3*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) - (log((4*b^4*(a + b*x)^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(9*a
^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*a^(7/3)) + (b^2*log((4*b^4*(a + b*x)^(
1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1i)/9 - 1/9)^2)/a^(11/3))*((3^(1/2)*1i)/9
- 1/9))/a^(7/3)
```

sympy [C] time = 2.59, size = 2730, normalized size = 21.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(1/3),x)
```

```
[Out] 4*a**(14/3)*b**(10/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I
*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5
/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**
4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b**(1
0/3)*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 4*a**(14/3)*b
**(10/3)*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 8
1*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(1
3/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(
a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*
a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)
*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**
(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(7/3)*
exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/
3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3)
- 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(
10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b +
 x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**(11/3)*b**(13/3)*(a/b + x)**(
7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*
b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3
)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b*
*(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b +
x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*ex
p(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(1
- b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a*
*7*b**(4/3)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a
/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*p
i/3)*gamma(5/3)) + 12*a**(8/3)*b**(16/3)*(a/b + x)**(10/3)*log(1 - b**(1/3)
*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3
)*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(
7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I
```

```
*pi/3)*gamma(5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 4*a**(5/3)*b**(19/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*log(1 - b**(1
/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3
)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3)
 - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/
3)*b**(19/3)*(a/b + x)**(13/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*
pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(
5/3) - 27*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a*
*(5/3)*b**(19/3)*(a/b + x)**(13/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma
(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*a
**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 21*a**4*b**4*(a
/b + x)**2*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamm
a(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27*
a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(
a/b + x)**3*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 27
*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 12*a**2*b**6*
(a/b + x)**4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) - 81*a**6*b**(7/3)*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3) + 81*a**5*b**(10/3)*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) - 2
7*a**4*b**(13/3)*(a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

3.399 $\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{9a^2(bx-a)^{5/3}}{5b^4}+\frac{3a^3(bx-a)^{2/3}}{2b^4}+\frac{3(bx-a)^{11/3}}{11b^4}+\frac{9a(bx-a)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^3*(-a + b*x)^(2/3))/(2*b^4) + (9*a^2*(-a + b*x)^(5/3))/(5*b^4) + (9*a*
(-a + b*x)^(8/3))/(8*b^4) + (3*(-a + b*x)^(11/3))/(11*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^3}{b^3\sqrt[3]{-a+bx}}+\frac{3a^2(-a+bx)^{2/3}}{b^3}+\frac{3a(-a+bx)^{5/3}}{b^3}+\frac{(-a+bx)^{8/3}}{b^3}\right)dx\\ &= \frac{3a^3(-a+bx)^{2/3}}{2b^4}+\frac{9a^2(-a+bx)^{5/3}}{5b^4}+\frac{9a(-a+bx)^{8/3}}{8b^4}+\frac{3(-a+bx)^{11/3}}{11b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.60

$$\frac{3(bx-a)^{2/3}\left(81a^3+54a^2bx+45ab^2x^2+40b^3x^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(81*a^3 + 54*a^2*b*x + 45*a*b^2*x^2 + 40*b^3*x^3))/(440
*b^4)
```

IntegrateAlgebraic [A] time = 0.02, size = 59, normalized size = 0.74

$$\frac{3(bx-a)^{2/3}\left(220a^3+264a^2(bx-a)+165a(bx-a)^2+40(bx-a)^3\right)}{440b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(220*a^3 + 264*a^2*(-a + b*x) + 165*a*(-a + b*x)^2 + 40
*(-a + b*x)^3))/(440*b^4)
```

fricas [A] time = 0.90, size = 44, normalized size = 0.55

$$\frac{3\left(40\,b^{3}x^{3}+45\,ab^{2}x^{2}+54\,a^{2}bx+81\,a^{3}\right)(bx-a)^{\frac{2}{3}}}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(40*b^3*x^3 + 45*a*b^2*x^2 + 54*a^2*b*x + 81*a^3)*(b*x - a)^(2/3)/b^4
```

giac [A] time = 1.07, size = 57, normalized size = 0.71

$$\frac{3\left(40\,(bx-a)^{\frac{11}{3}}+165\,(bx-a)^{\frac{8}{3}}a+264\,(bx-a)^{\frac{5}{3}}a^{2}+220\,(bx-a)^{\frac{2}{3}}a^{3}\right)}{440\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/440*(40*(b*x - a)^(11/3) + 165*(b*x - a)^(8/3)*a + 264*(b*x - a)^(5/3)*a^
2 + 220*(b*x - a)^(2/3)*a^3)/b^4
```

maple [A] time = 0.00, size = 45, normalized size = 0.56

$$\frac{3\left(40b^{3}x^{3}+45a\,b^{2}x^{2}+54a^{2}bx+81a^{3}\right)(bx-a)^{\frac{2}{3}}}{440b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x-a)^(1/3),x)
```

```
[Out] 3/440*(40*b^3*x^3+45*a*b^2*x^2+54*a^2*b*x+81*a^3)/b^4*(b*x-a)^(2/3)
```

maxima [A] time = 1.34, size = 64, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{11}{3}}}{11\,b^{4}}+\frac{9\,(bx-a)^{\frac{8}{3}}a}{8\,b^{4}}+\frac{9\,(bx-a)^{\frac{5}{3}}a^{2}}{5\,b^{4}}+\frac{3\,(bx-a)^{\frac{2}{3}}a^{3}}{2\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*(b*x - a)^(11/3)/b^4 + 9/8*(b*x - a)^(8/3)*a/b^4 + 9/5*(b*x - a)^(5/3)
*a^2/b^4 + 3/2*(b*x - a)^(2/3)*a^3/b^4
```

mupad [B] time = 0.05, size = 64, normalized size = 0.80

$$\frac{3\,(b\,x-a)^{11/3}}{11\,b^{4}}+\frac{9\,a\,(b\,x-a)^{8/3}}{8\,b^{4}}+\frac{3\,a^{3}\,(b\,x-a)^{2/3}}{2\,b^{4}}+\frac{9\,a^{2}\,(b\,x-a)^{5/3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x - a)^(1/3),x)
```

```
[Out] (3*(b*x - a)^(11/3))/(11*b^4) + (9*a*(b*x - a)^(8/3))/(8*b^4) + (3*a^3*(b*x
 - a)^(2/3))/(2*b^4) + (9*a^2*(b*x - a)^(5/3))/(5*b^4)
```

sympy [C] time = 2.98, size = 4974, normalized size = 62.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((243*a**(71/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*ex
p(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3
) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2
640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a
**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600
*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**1
6*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**1
0*x**6*exp(I*pi/3)) - 1296*a**(68/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(4
40*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6
*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4
*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(
I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5
*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*ex
p(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*p
i/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 2808*a**(65/3)*b**2*x**2*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
 + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a**20*b
**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(
I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/
3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) -
 3120*a**(62/3)*b**3*x**3*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*e
xp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/
3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) -
2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860
*a**(62/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*
pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3)
 + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 44
0*a**14*b**10*x**6*exp(I*pi/3)) + 1710*a**(59/3)*b**4*x**4*(-1 + b*x/a)**(2
/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
 + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 66
00*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**
14*b**10*x**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I
*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640
*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 72*a**(5
6/3)*b**5*x**5*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3)
- 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a
**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*
b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*
b**5*x**5/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 660
0*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**
16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**
10*x**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*x**6*(-1 + b*x/a)**(2/3)*exp(I*p
i/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**
18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b*
*8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x*
*6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440*a**20*b**4*exp(I*pi/3) - 264
0*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*
b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*
x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*
x**7*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**
19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*
x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*
exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*x**8*(
-1 + b*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
```

xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 120*a**(44/3)*b**9*x**9*(-1 + b
*x/a)**(2/3)*exp(I*pi/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*ex
p(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*p
i/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3)
+ 440*a**14*b**10*x**6*exp(I*pi/3)), Abs(b*x/a) > 1), (-243*a**(71/3)*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**4*exp(I*pi/3) - 2640
*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b
7*x3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x
5*exp(I*pi/3) + 440*a14*b**10*x**6*exp(I*pi/3)) + 1296*a**(68/3)*b*x*(1
- b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3
) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6
600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a*
*14*b**10*x**6*exp(I*pi/3)) - 1458*a**(68/3)*b*x/(440*a**20*b**4*exp(I*pi/3
) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800
*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**1
5*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 2808*a**(65/3
)*b**2*x**2*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**
5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*e
xp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*
pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*x**2/(440*a
20*b4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**
2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp
(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi
/3)) + 3120*a**(62/3)*b**3*x**3*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi
/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 88
00*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a*
*15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 4860*a**(62
/3)*b**3*x**3/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*x**4*(1 - b*x/a)**(2/3)/(440
*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x
2*exp(I*pi/3) - 8800*a17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*e
xp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*
pi/3)) + 3645*a**(59/3)*b**4*x**4/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 72*a**(56/3)*b**5*x**5*(1 -
b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) +
6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 6600
*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14
*b**10*x**6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*x**5/(440*a**20*b**4*exp(I*p
i/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8
800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a
15*b9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 1104*a**(5
3/3)*b**6*x**6*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 243*a**(53/3)*b**6*x**6/(440
*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x
2*exp(I*pi/3) - 8800*a17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*e
xp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*
pi/3)) - 1152*a**(50/3)*b**7*x**7*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*
pi/3) - 2640*a**19*b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) -
8800*a**17*b**7*x**3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*

```
a**15*b**9*x**5*exp(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) + 585*a**(4
7/3)*b**8*x**8*(1 - b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*
b**5*x*exp(I*pi/3) + 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**
3*exp(I*pi/3) + 6600*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp
(I*pi/3) + 440*a**14*b**10*x**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*x**9*(1 -
 b*x/a)**(2/3)/(440*a**20*b**4*exp(I*pi/3) - 2640*a**19*b**5*x*exp(I*pi/3)
+ 6600*a**18*b**6*x**2*exp(I*pi/3) - 8800*a**17*b**7*x**3*exp(I*pi/3) + 660
0*a**16*b**8*x**4*exp(I*pi/3) - 2640*a**15*b**9*x**5*exp(I*pi/3) + 440*a**1
4*b**10*x**6*exp(I*pi/3)), True))
```

3.400 $\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=59

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3a^2(bx-a)^{2/3}}{2b^3}+\frac{3(bx-a)^{8/3}}{8b^3}+\frac{6a(bx-a)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a^2*(-a + b*x)^(2/3))/(2*b^3) + (6*a*(-a + b*x)^(5/3))/(5*b^3) + (3*(-a
+ b*x)^(8/3))/(8*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt[3]{-a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt[3]{-a+bx}}+\frac{2a(-a+bx)^{2/3}}{b^2}+\frac{(-a+bx)^{5/3}}{b^2}\right)dx\\ &= \frac{3a^2(-a+bx)^{2/3}}{2b^3}+\frac{6a(-a+bx)^{5/3}}{5b^3}+\frac{3(-a+bx)^{8/3}}{8b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.63

$$\frac{3(bx-a)^{2/3}\left(9a^2+6abx+5b^2x^2\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(9*a^2 + 6*a*b*x + 5*b^2*x^2))/(40*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 51, normalized size = 0.86

$$\frac{3\left(20a^2(bx-a)^{2/3}+5(bx-a)^{8/3}+16a(bx-a)^{5/3}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(20*a^2*(-a + b*x)^(2/3) + 16*a*(-a + b*x)^(5/3) + 5*(-a + b*x)^(8/3)))/
(40*b^3)
```

fricas [A] time = 0.94, size = 33, normalized size = 0.56

$$\frac{3\left(5\,b^{2}x^{2}+6\,abx+9\,a^{2}\right)(bx-a)^{\frac{2}{3}}}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 6*a*b*x + 9*a^2)*(b*x - a)^(2/3)/b^3
```

giac [A] time = 1.07, size = 43, normalized size = 0.73

$$\frac{3\left(5\,(bx-a)^{\frac{8}{3}}+16\,(bx-a)^{\frac{5}{3}}a+20\,(bx-a)^{\frac{2}{3}}a^{2}\right)}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/40*(5*(b*x - a)^(8/3) + 16*(b*x - a)^(5/3)*a + 20*(b*x - a)^(2/3)*a^2)/b^
3
```

maple [A] time = 0.00, size = 34, normalized size = 0.58

$$\frac{3\left(5b^{2}x^{2}+6abx+9a^{2}\right)(bx-a)^{\frac{2}{3}}}{40b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x-a)^(1/3),x)
```

```
[Out] 3/40*(5*b^2*x^2+6*a*b*x+9*a^2)/b^3*(b*x-a)^(2/3)
```

maxima [A] time = 1.33, size = 47, normalized size = 0.80

$$\frac{3\,(bx-a)^{\frac{8}{3}}}{8\,b^{3}}+\frac{6\,(bx-a)^{\frac{5}{3}}a}{5\,b^{3}}+\frac{3\,(bx-a)^{\frac{2}{3}}a^{2}}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x - a)^(8/3)/b^3 + 6/5*(b*x - a)^(5/3)*a/b^3 + 3/2*(b*x - a)^(2/3)*a
^2/b^3
```

mupad [B] time = 0.04, size = 43, normalized size = 0.73

$$\frac{48\,a\,(b\,x-a)^{5/3}+15\,(b\,x-a)^{8/3}+60\,a^{2}\,(b\,x-a)^{2/3}}{40\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x - a)^(1/3),x)
```

```
[Out] (48*a*(b*x - a)^(5/3) + 15*(b*x - a)^(8/3) + 60*a^2*(b*x - a)^(2/3))/(40*b^
3)
```

sympy [C] time = 1.90, size = 1326, normalized size = 22.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4
*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(
-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) +
 63*a**(29/3)*b*x*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 12
0*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40
*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42
*a**(26/3)*b**2*x**2*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x -
 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*p
i/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x
**3) + 18*a**(23/3)*b**3*x**3*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7
*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23/3)*b**3*x**3*
exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a*
*5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 15*a**(17/3)*b
**5*x**5*(-1 + b*x/a)**(2/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b*
*5*x**2 + 40*a**5*b**6*x**3), Abs(b*x/a) > 1), (-27*a**(32/3)*(1 - b*x/a)**
(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 +
 40*a**5*b**6*x**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*
b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 63*a**(29/3)*b*x*(1 - b*
x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*
x**2 + 40*a**5*b**6*x**3) - 81*a**(29/3)*b*x*exp(2*I*pi/3)/(-40*a**8*b**3 +
 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 42*a**(26/3)*b
**2*x**2*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x
- 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) + 81*a**(26/3)*b**2*x**2*exp(2*I*
pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*
x**3) + 18*a**(23/3)*b**3*x**3*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b
**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3) - 27*a**(23
/3)*b**3*x**3*exp(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**
5*x**2 + 40*a**5*b**6*x**3) - 27*a**(20/3)*b**4*x**4*(1 - b*x/a)**(2/3)*exp
(2*I*pi/3)/(-40*a**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*
b**6*x**3) + 15*a**(17/3)*b**5*x**5*(1 - b*x/a)**(2/3)*exp(2*I*pi/3)/(-40*a
**8*b**3 + 120*a**7*b**4*x - 120*a**6*b**5*x**2 + 40*a**5*b**6*x**3), True)
)
```

3.401 $\int \frac{x}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=38

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3(bx-a)^{5/3}}{5b^2}+\frac{3a(bx-a)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*a*(-a + b*x)^(2/3))/(2*b^2) + (3*(-a + b*x)^(5/3))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt[3]{-a+bx}}\,dx&=\int \left(\frac{a}{b\sqrt[3]{-a+bx}}+\frac{(-a+bx)^{2/3}}{b}\right)dx\\&=\frac{3a(-a+bx)^{2/3}}{2b^2}+\frac{3(-a+bx)^{5/3}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(bx-a)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 26, normalized size = 0.68

$$\frac{3(bx-a)^{2/3}(3a+2bx)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(-a + b*x)^(1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*(3*a + 2*b*x))/(10*b^2)
```

fricas [A] time = 0.91, size = 22, normalized size = 0.58

$$\frac{3\,(2\,bx+3\,a)(bx-a)^{\frac{2}{3}}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 3*a)*(b*x - a)^(2/3)/b^2
```

giac [A] time = 0.98, size = 29, normalized size = 0.76

$$\frac{3\left(2\,(bx-a)^{\frac{5}{3}}+5\,(bx-a)^{\frac{2}{3}}a\right)}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(2*(b*x - a)^(5/3) + 5*(b*x - a)^(2/3)*a)/b^2
```

maple [A] time = 0.00, size = 23, normalized size = 0.61

$$\frac{3\,(2bx+3a)\,(bx-a)^{\frac{2}{3}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x-a)^(1/3),x)
```

```
[Out] 3/10*(2*b*x+3*a)/b^2*(b*x-a)^(2/3)
```

maxima [A] time = 1.34, size = 30, normalized size = 0.79

$$\frac{3\,(bx-a)^{\frac{5}{3}}}{5\,b^2}+\frac{3\,(bx-a)^{\frac{2}{3}}a}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x - a)^(5/3)/b^2 + 3/2*(b*x - a)^(2/3)*a/b^2
```

mupad [B] time = 0.03, size = 29, normalized size = 0.76

$$\frac{15\,a\,(b\,x-a)^{2/3}+6\,(b\,x-a)^{5/3}}{10\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x - a)^(1/3),x)
```

```
[Out] (15*a*(b*x - a)^(2/3) + 6*(b*x - a)^(5/3))/(10*b^2)
```

sympy [C] time = 1.26, size = 486, normalized size = 12.79

$$\begin{cases} -\frac{9a^{\frac{11}{3}}\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{3a^{\frac{8}{3}}bx\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{6a^{\frac{5}{3}}b^2x^2\left(-1+\frac{bx}{a}\right)^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ \frac{9a^{\frac{11}{3}}\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{9a^{\frac{11}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{3a^{\frac{8}{3}}bx\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}+\frac{9a^{\frac{8}{3}}bx}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}}-\frac{6a^{\frac{5}{3}}b^2x^2\left(1-\frac{bx}{a}\right)^{\frac{2}{3}}}{-10a^2b^2e^{\frac{i\pi}{3}}+10ab^3xe^{\frac{i\pi}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x-a)**(1/3),x)
```

```
[Out] Piecewise((-9*a**(11/3)*(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(
I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*pi/3)
 + 10*a*b**3*x*exp(I*pi/3)) + 3*a**(8/3)*b*x*(-1 + b*x/a)**(2/3)*exp(I*pi/3
)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-
10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 6*a**(5/3)*b**2*x**2*
(-1 + b*x/a)**(2/3)*exp(I*pi/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*ex
p(I*pi/3)), Abs(b*x/a) > 1), (9*a**(11/3)*(1 - b*x/a)**(2/3)/(-10*a**2*b**2
*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 9*a**(11/3)/(-10*a**2*b**2*exp(I*
pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 3*a**(8/3)*b*x*(1 - b*x/a)**(2/3)/(-10*a
**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) + 9*a**(8/3)*b*x/(-10*a**2*
b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)) - 6*a**(5/3)*b**2*x**2*(1 - b*x
/a)**(2/3)/(-10*a**2*b**2*exp(I*pi/3) + 10*a*b**3*x*exp(I*pi/3)), True))
```

3.402 $\int \frac{1}{\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=18

$$\frac{3(bx-a)^{2/3}}{2b}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32}

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{-a+bx}}\,dx=\frac{3(-a+bx)^{2/3}}{2b}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{3(bx-a)^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-a + b*x)^(-1/3),x]
```

```
[Out] (3*(-a + b*x)^(2/3))/(2*b)
```

fricas [A] time = 0.71, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x - a)^(2/3)/b
```

giac [A] time = 1.02, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="giac")
[Out] 3/2*(b*x - a)^(2/3)/b
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^(1/3),x)
[Out] 3/2*(b*x-a)^(2/3)/b
```

maxima [A] time = 1.32, size = 14, normalized size = 0.78

$$\frac{3\,(bx-a)^{\frac{2}{3}}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^(1/3),x, algorithm="maxima")
[Out] 3/2*(b*x - a)^(2/3)/b
```

mupad [B] time = 0.02, size = 14, normalized size = 0.78

$$\frac{3\,{\left(b\,x-a\right)}^{2/3}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x - a)^(1/3),x)
[Out] (3*(b*x - a)^(2/3))/(2*b)
```

sympy [A] time = 0.07, size = 12, normalized size = 0.67

$$\frac{3\,(-a+bx)^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**(1/3),x)
[Out] 3*(-a + b*x)**(2/3)/(2*b)
```

3.403 $\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{2\sqrt[3]{a}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}+\frac{\log(x)}{2\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1/3
)) + Log[x]/(2*a^(1/3)) - (3*Log[a^(1/3) + (-a + b*x)^(1/3)])/(2*a^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx = \frac{\log(x)}{2\sqrt[3]{a}} + \frac{3}{2}\mathrm{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{-a+bx}\right) - \frac{3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

$$= \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}} + \frac{3\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}} + \frac{\log(x)}{2\sqrt[3]{a}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2\sqrt[3]{a}}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.43

$$\frac{3(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b*x)/a])/(2*a)
```

IntegrateAlgebraic [A] time = 0.05, size = 113, normalized size = 1.38

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{2\sqrt[3]{a}} - \frac{\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{\sqrt[3]{a}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a + b*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(1
/3)) - Log[a^(1/3) + (-a + b*x)^(1/3)]/a^(1/3) + Log[a^(2/3) - a^(1/3)*(-a
+ b*x)^(1/3) + (-a + b*x)^(2/3)]/(2*a^(1/3))
```

fricas [A] time = 0.94, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{3}a\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+\sqrt{3}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2a}, \frac{2\sqrt{3}a\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)}{2a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*sqrt((-a)^(1/3)/a)*log((2*b*x + sqrt(3)*(2*(b*x - a)^(2/3)*
(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x
- a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a
)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)^(1/3) - (-a)^
(1/3)))/a, 1/2*(2*sqrt(3)*a*sqrt(-(-a)^(1/3)/a)*arctan(1/3*sqrt(3)*(2*(b*x
- a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*log((b*x - a)^(2
/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*log((b*x - a)
^(1/3) - (-a)^(1/3)))/a]
```

giac [A] time = 2.51, size = 112, normalized size = 1.37

$$-\frac{\sqrt{3}\left(-a\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx-a\right)^{\frac{1}{3}}+\left(-a\right)^{\frac{1}{3}}\right)}{3\left(-a\right)^{\frac{1}{3}}}\right)}{a}+\frac{\left(-a\right)^{\frac{2}{3}}\log\left(\left(bx-a\right)^{\frac{2}{3}}+\left(bx-a\right)^{\frac{1}{3}}\left(-a\right)^{\frac{1}{3}}+\left(-a\right)^{\frac{2}{3}}\right)}{2a}-\frac{\left(-a\right)^{\frac{2}{3}}\log\left(\left|\left(bx-a\right)^{\frac{1}{3}}-\left(-a\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/a + 1/2*(-a)^(2/3)*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3
) + (-a)^(2/3))/a - (-a)^(2/3)*log(abs((b*x - a)^(1/3) - (-a)^(1/3)))/a
```

maple [A] time = 0.01, size = 83, normalized size = 1.01

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{a^{\frac{1}{3}}}-\frac{\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}+\frac{\ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{2a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x-a)^(1/3),x)
```

```
[Out] -ln(a^(1/3)+(b*x-a)^(1/3))/a^(1/3)+1/2/a^(1/3)*ln((b*x-a)^(2/3)-a^(1/3)*(b*
x-a)^(1/3)+a^(2/3))+3^(1/2)/a^(1/3)*arctan(1/3*3^(1/2)*(2/a^(1/3)*(b*x-a)^(
1/3)-1))
```

maxima [A] time = 3.01, size = 86, normalized size = 1.05

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx-a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}+\frac{\log\left(\left(bx-a\right)^{\frac{2}{3}}-\left(bx-a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{1}{3}}}-\frac{\log\left(\left(bx-a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(1/3) +
 1/2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(1/3) - log
((b*x - a)^(1/3) + a^(1/3))/a^(1/3)
```

mupad [B] time = 0.09, size = 117, normalized size = 1.43

$$\frac{\ln\left(9\,{\left(b\,x-a\right)}^{1/3}-9\,{\left(-a\right)}^{1/3}\right)}{{\left(-a\right)}^{1/3}}+\frac{\ln\left(9\,{\left(b\,x-a\right)}^{1/3}-\frac{9\,{\left(-a\right)}^{1/3}\,{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}}-\frac{\ln\left(9\,{\left(b\,x-a\right)}^{1/3}-\frac{9\,{\left(-a\right)}^{1/3}\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2}{4}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,{\left(-a\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x - a)^(1/3)),x)
```

```
[Out] log(9*(b*x - a)^(1/3) - 9*(-a)^(1/3))/(-a)^(1/3) + (log(9*(b*x - a)^(1/3) -
 (9*(-a)^(1/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-a)^(1/3)) - (l
og(9*(b*x - a)^(1/3) - (9*(-a)^(1/3)*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1
))/(2*(-a)^(1/3))
```

sympy [C] time = 1.88, size = 160, normalized size = 1.95

$$-\frac{2e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}-\frac{2e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{-\frac{a}{b}+x}\,e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}\,\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x-a)**(1/3),x)
```

```
[Out] -2*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(
1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*log(1 - b**(1/3)*(-a/b + x)**(
1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*gamma(5/3)) - 2*exp(2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(3*a**(1/3)*gamma(5/3))
```

3.404 $\int \frac{1}{x^2 \sqrt[3]{-a+bx}} \, dx$

Optimal. Leaf size=103

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{2a^{4/3}} - \frac{b\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}} + \frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(4/3)) + (b*Log[x])/(6*a^(4/3)) - (b*Log[a^(1/3) + (-
a + b*x)^(1/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx &= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{3a}\\
&= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx,x,\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,x+x^2}\,dx,x\right.}{2a}\\
&= \frac{(-a+bx)^{2/3}}{ax}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\
&= \frac{(-a+bx)^{2/3}}{ax}-\frac{b\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{b\log(x)}{6a^{4/3}}-\frac{b\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{2a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.35

$$\frac{3b(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, 1 - (b*x)/a])/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.14, size = 136, normalized size = 1.32

$$-\frac{b\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{3a^{4/3}}+\frac{b\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{6a^{4/3}}-\frac{b\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}}+\frac{(bx-a)^{2/3}}{ax}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(a*x) - (b*ArcTan[1/Sqrt[3] - (2*(-a + b*x)^(1/3))/(Sqrt[3
]*a^(1/3))])/(Sqrt[3]*a^(4/3)) - (b*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(
4/3)) + (b*Log[a^(2/3) - a^(1/3)*(-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(6*a
^(4/3))
```

fricas [A] time = 1.06, size = 328, normalized size = 3.18

$$\left[\frac{3\sqrt{\frac{1}{3}}\,abx\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6(bx-a)^{\frac{2}{3}}a}{6a^2x},\ \frac{6\sqrt{\frac{1}{3}}\,abx\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-2(-a)^{\frac{2}{3}}bx\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+6(bx-a)^{\frac{2}{3}}a}{6a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*x*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2*(b*x
 - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(1/3)/
a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + (-a)^(2/3)*b*x*log((b*x - a)^
(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*
x - a)^(1/3) - (-a)^(1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x), 1/6*(6*sqrt(1/3)
*a*b*x*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3)
```

```
)*sqrt(-(-a)^(1/3)/a)) + (-a)^(2/3)*b*x*log((b*x - a)^(2/3) + (b*x - a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 2*(-a)^(2/3)*b*x*log((b*x - a)^(1/3) - (-a)^(
1/3)) + 6*(b*x - a)^(2/3)*a)/(a^2*x)]
```

giac [A] time = 2.43, size = 144, normalized size = 1.40

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{b^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a}-\frac{2(-a)^{\frac{2}{3}}b^2\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^2}+\frac{6(bx-a)^{\frac{2}{3}}b}{ax}}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/6*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a)
^(1/3))/((-a)^(1/3)*a) - b^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/
3) + (-a)^(2/3))/((-a)^(1/3)*a) - 2*(-a)^(2/3)*b^2*log(abs((b*x - a)^(1/3)
- (-a)^(1/3))))/a^2 + 6*(b*x - a)^(2/3)*b/(a*x))/b
```

maple [A] time = 0.01, size = 103, normalized size = 1.00

$$\frac{\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{3a^{\frac{4}{3}}}-\frac{b\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x-a)^(1/3),x)
```

```
[Out] (b*x-a)^(2/3)/a/x-1/3*b*ln(a^(1/3)+(b*x-a)^(1/3))/a^(4/3)+1/6*b/a^(4/3)*ln(
a^(2/3)-(b*x-a)^(1/3)*a^(1/3)+(b*x-a)^(2/3))+1/3*b/a^(4/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2*(b*x-a)^(1/3)/a^(1/3)-1))
```

maxima [A] time = 2.99, size = 116, normalized size = 1.13

$$\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{4}{3}}}+\frac{(bx-a)^{\frac{2}{3}}b}{(bx-a)a+a^2}+\frac{b\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6a^{\frac{4}{3}}}-\frac{b\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a^(
4/3) + (b*x - a)^(2/3)*b/((b*x - a)*a + a^2) + 1/6*b*log((b*x - a)^(2/3) -
(b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/3*b*log((b*x - a)^(1/3) + a^
(1/3))/a^(4/3)
```

mupad [B] time = 0.18, size = 133, normalized size = 1.29

$$\frac{(b\,x-a)^{2/3}}{a\,x}-\frac{b\,\ln\left((b\,x-a)^{1/3}+a^{1/3}\right)}{3\,a^{4/3}}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4\,a^{5/3}}+\frac{b^2\,(b\,x-a)^{1/3}}{a^2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b*x - a)^(1/3)),x)
```

```
[Out] (b*x - a)^(2/3)/(a*x) - (b*log((b*x - a)^(1/3) + a^(1/3)))/(3*a^(4/3)) + (l
og((b - 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x - a)^(1/3))/a^2)*(b - 3^(1/
2)*b*1i))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(4*a^(5/3)) + (b^2*(b*x -
 a)^(1/3))/a^2)*(b + 3^(1/2)*b*1i))/(6*a^(4/3))
```

sympy [C] time = 2.24, size = 838, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x-a)**(1/3),x)
```

```
[Out] -2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*e
xp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*ex
p(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**
(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)*
*(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(5/3)*b**(7/3)*(-a/b + x)**(4/3)*ex
p(-2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3
))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) +
 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b
**(10/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*p
i/3)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) -
2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b
 + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(
2*I*pi/3)*gamma(5/3)) - 2*a**(2/3)*b**(10/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2
/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 9*a**2*b*
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)) + 6*a*b**3*(-a/b + x)**2
*exp(2*I*pi/3)*gamma(2/3)/(9*a**3*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*
gamma(5/3) + 9*a**2*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3))
```

3.405 $\int \frac{1}{x^3 \sqrt[3]{-a+bx}} \, dx$

Optimal. Leaf size=136

$$\frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{3a^{7/3}} - \frac{2b^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2b(bx-a)^{2/3}}{3a^2x} + \frac{(bx-a)^{2/3}}{2ax^2}$$

Rubi [A] time = 0.04, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {51, 56, 617, 204, 31}

$$\frac{b^2 \log(x)}{9a^{7/3}} - \frac{b^2 \log\left(\sqrt[3]{bx-a} + \sqrt[3]{a}\right)}{3a^{7/3}} - \frac{2b^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{2b(bx-a)^{2/3}}{3a^2x} + \frac{(bx-a)^{2/3}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (-a + b*x)^(2/3)/(2*a*x^2) + (2*b*(-a + b*x)^(2/3))/(3*a^2*x) - (2*b^2*ArcT
an[(a^(1/3) - 2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) +
 (b^2*Log[x])/(9*a^(7/3)) - (b^2*Log[a^(1/3) + (-a + b*x)^(1/3)])/(3*a^(7/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt[3]{-a+bx}}\,dx = \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{(2b)\int \frac{1}{x^2\sqrt[3]{-a+bx}}\,dx}{3a}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{\left(2b^2\right)\int \frac{1}{x\sqrt[3]{-a+bx}}\,dx}{9a^2}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{b^2\operatorname{Subs}}{}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+bx}\right)}{3a^{7/3}} + \frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{}{-3}\right.}{}$$

$$= \frac{(-a+bx)^{2/3}}{2ax^2} + \frac{2b(-a+bx)^{2/3}}{3a^2x} - \frac{2b^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{b^2\log(x)}{9a^{7/3}} - \frac{b^2\log\left(\sqrt[3]{a}+\sqrt[3]{-a+\right.}{3a^{7/3}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.28

$$\frac{3b^2(bx-a)^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};1-\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] (3*b^2*(-a + b*x)^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, 1 - (b*x)/a])/(2*a^3
)
```

IntegrateAlgebraic [A] time = 0.13, size = 160, normalized size = 1.18

$$-\frac{2b^2\log\left(\sqrt[3]{bx-a}+\sqrt[3]{a}\right)}{9a^{7/3}} + \frac{b^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx-a}+(bx-a)^{2/3}\right)}{9a^{7/3}} - \frac{2b^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{bx-a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}} + \frac{(bx-a)^{2/3}(4(bx-a)+7a)}{6a^2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(-a + b*x)^(1/3)),x]
```

```
[Out] ((-a + b*x)^(2/3)*(7*a + 4*(-a + b*x)))/(6*a^2*x^2) - (2*b^2*ArcTan[1/Sqrt[
3] - (2*(-a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)) - (2*b^2*
Log[a^(1/3) + (-a + b*x)^(1/3)])/(9*a^(7/3)) + (b^2*Log[a^(2/3) - a^(1/3)*(
-a + b*x)^(1/3) + (-a + b*x)^(2/3)])/(9*a^(7/3))
```

fricas [A] time = 0.66, size = 374, normalized size = 2.75

$$\left[\frac{6\sqrt{\frac{1}{3}}ab^2x^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx-a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}-3a}{x}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\left(4abx+3a^2\right)(bx-a)^{\frac{2}{3}}}{18a^3x^2}, \frac{12\sqrt{\frac{1}{3}}ab^2x^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)+2(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4(-a)^{\frac{2}{3}}b^2x^2\log\left((bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)+3\left(4abx+3a^2\right)(bx-a)^{\frac{2}{3}}}{18a^3x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*a*b^2*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x + 3*sqrt(1/3)*(2
*(b*x - a)^(2/3)*(-a)^(2/3) + (b*x - a)^(1/3)*a + (-a)^(1/3)*a)*sqrt((-a)^(
1/3)/a) - 3*(b*x - a)^(1/3)*(-a)^(2/3) - 3*a)/x) + 2*(-a)^(2/3)*b^2*x^2*log
```

```
((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(-a)^(2/3)*
b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2)*(b*x - a)^(
2/3))/(a^3*x^2), 1/18*(12*sqrt(1/3)*a*b^2*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sq
rt(1/3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) + 2*(-a)^(2/3
)*b^2*x^2*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) -
4*(-a)^(2/3)*b^2*x^2*log((b*x - a)^(1/3) - (-a)^(1/3)) + 3*(4*a*b*x + 3*a^2
)*(b*x - a)^(2/3))/(a^3*x^2)]
```

giac [A] time = 2.24, size = 167, normalized size = 1.23

$$\frac{\frac{4\sqrt{3}\,b^{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)}{3\,(-a)^{\frac{1}{3}}}\right)}{(-a)^{\frac{1}{3}}a^{2}}-\frac{2\,b^{3}\log\left((bx-a)^{\frac{2}{3}}+(bx-a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)}{(-a)^{\frac{1}{3}}a^{2}}-\frac{4\,(-a)^{\frac{2}{3}}b^{3}\log\left(\left|(bx-a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right|\right)}{a^{3}}+\frac{3\left(4\,(bx-a)^{\frac{5}{3}}b^{3}+7\,(bx-a)^{\frac{2}{3}}ab^{3}\right)}{a^{2}b^{2}x^{2}}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="giac")
```

```
[Out] 1/18*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) + (-a)^(1/3))/(-a
)^(1/3))/((-a)^(1/3)*a^2) - 2*b^3*log((b*x - a)^(2/3) + (b*x - a)^(1/3)*(-a
)^(1/3) + (-a)^(2/3))/((-a)^(1/3)*a^2) - 4*(-a)^(2/3)*b^3*log(abs((b*x - a)
^(1/3) - (-a)^(1/3)))/a^3 + 3*(4*(b*x - a)^(5/3)*b^3 + 7*(b*x - a)^(2/3)*a*
b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.01, size = 128, normalized size = 0.94

$$\frac{2\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx-a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}-1\right)}{3}\right)}{9a^{\frac{7}{3}}}-\frac{2b^{2}\ln\left(a^{\frac{1}{3}}+(bx-a)^{\frac{1}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{b^{2}\ln\left(a^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx-a)^{\frac{2}{3}}\right)}{9a^{\frac{7}{3}}}+\frac{2\,(bx-a)^{\frac{2}{3}}b}{3a^{2}x}+\frac{(bx-a)^{\frac{2}{3}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x-a)^(1/3),x)
```

```
[Out] 1/2*(b*x-a)^(2/3)/a/x^2+2/3*b*(b*x-a)^(2/3)/a^2/x-2/9*b^2*ln(a^(1/3)+(b*x-a
)^(1/3))/a^(7/3)+1/9*b^2/a^(7/3)*ln(a^(2/3)-(b*x-a)^(1/3)*a^(1/3)+(b*x-a)^(
2/3))+2/9*b^2/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x-a)^(1/3)/a^(1/3)-1
))
```

maxima [A] time = 3.09, size = 159, normalized size = 1.17

$$\frac{2\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx-a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}}+\frac{b^{2}\log\left((bx-a)^{\frac{2}{3}}-(bx-a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9\,a^{\frac{7}{3}}}-\frac{2\,b^{2}\log\left((bx-a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}}+\frac{4\,(bx-a)^{\frac{5}{3}}b^{2}+7\,(bx-a)^{\frac{2}{3}}ab^{2}}{6\left((bx-a)^{2}a^{2}+2\,(bx-a)a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x-a)^(1/3),x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x - a)^(1/3) - a^(1/3))/a^(1/3))/a
^(7/3) + 1/9*b^2*log((b*x - a)^(2/3) - (b*x - a)^(1/3)*a^(1/3) + a^(2/3))/a
^(7/3) - 2/9*b^2*log((b*x - a)^(1/3) + a^(1/3))/a^(7/3) + 1/6*(4*(b*x - a)^
(5/3)*b^2 + 7*(b*x - a)^(2/3)*a*b^2)/((b*x - a)^2*a^2 + 2*(b*x - a)*a^3 + a
^4)
```

mupad [B] time = 0.22, size = 216, normalized size = 1.59

$$\frac{\frac{7\,b^{2}\,(b\,x-a)^{2/3}}{6\,a}+\frac{2\,b^{2}\,(b\,x-a)^{5/3}}{3\,a^{2}}}{(a-b\,x)^{2}-2\,a\,(a-b\,x)+a^{2}}-\frac{\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)^{2}}{9\,(-a)^{11/3}}\right)\left(b^{2}+\sqrt{3}\,b^{2}\,1\mathrm{i}\right)}{9\,(-a)^{7/3}}+\frac{2\,b^{2}\,\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{4\,b^{4}}{9\,(-a)^{11/3}}\right)}{9\,(-a)^{7/3}}+\frac{b^{2}\,\ln\left(\frac{4\,b^{4}\,(b\,x-a)^{1/3}}{9\,a^{4}}-\frac{9\,b^{4}\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)^{2}}{(-a)^{11/3}}\right)\left(-\frac{1}{9}+\frac{\sqrt{3}\,1\mathrm{i}}{9}\right)}{(-a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(b*x - a)^(1/3)),x)
```

```
[Out] ((7*b^2*(b*x - a)^(2/3))/(6*a) + (2*b^2*(b*x - a)^(5/3))/(3*a^2))/((a - b*x
)^2 - 2*a*(a - b*x) + a^2) - (log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (3^(1/2
)*b^2*1i + b^2)^2/(9*(-a)^(11/3)))*(3^(1/2)*b^2*1i + b^2))/(9*(-a)^(7/3)) +
 (2*b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (4*b^4)/(9*(-a)^(11/3))))/(9*
(-a)^(7/3)) + (b^2*log((4*b^4*(b*x - a)^(1/3))/(9*a^4) - (9*b^4*((3^(1/2)*1
i)/9 - 1/9)^2)/(-a)^(11/3))*((3^(1/2)*1i)/9 - 1/9))/(-a)^(7/3)
```

sympy [C] time = 2.62, size = 2744, normalized size = 20.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x-a)**(1/3),x)
```

```
[Out] -4*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)
*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)
*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3
)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3
) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(
14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)
**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**
(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I
*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamm
a(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) - 4
*a**(14/3)*b**(10/3)*(-a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(-a/
b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*log(1 - b**(1/3)*(-a/b +
x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp
(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
 - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/3)*(-
a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/
3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*
pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma
(5/3)) - 12*a**(11/3)*b**(13/3)*(-a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*log(1 - b**(1/
3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a**7*b**(4/
3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)
**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp
(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*
gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*log(1
- b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(27*a**7*
b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/
b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**(10/
3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*
pi/3)*gamma(5/3)) - 12*a**(8/3)*b**(16/3)*(-a/b + x)**(10/3)*exp(-2*I*pi/3)
*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(2/3
)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b*
```

```
*(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b
 + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/
3)*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*log(
1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(2/3)/(27*a
**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(7/3)*
(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b + x)**
(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)*exp(
2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(2*I*pi/
3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/
(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**6*b**(
7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/3)*(-a/b +
 x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b + x)**(13/3)
*exp(2*I*pi/3)*gamma(5/3)) - 4*a**(5/3)*b**(19/3)*(-a/b + x)**(13/3)*exp(-2
*I*pi/3)*log(1 - b**(1/3)*(-a/b + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*g
amma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3) + 81
*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b**(10/
3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-a/b +
x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 21*a**4*b**4*(-a/b + x)**2*exp(2*I*p
i/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(5/3
) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*a**5*b
**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13/3)*(-
a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 33*a**3*b**5*(-a/b + x)**3*exp
(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/3)*gam
ma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3) + 81*
a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*b**(13
/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3)) + 12*a**2*b**6*(-a/b + x)*
*4*exp(2*I*pi/3)*gamma(2/3)/(27*a**7*b**(4/3)*(-a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(5/3) + 81*a**6*b**(7/3)*(-a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(5/3)
 + 81*a**5*b**(10/3)*(-a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(5/3) + 27*a**4*
b**(13/3)*(-a/b + x)**(13/3)*exp(2*I*pi/3)*gamma(5/3))
```

3.406 $\int \frac{x^3}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=70

$$-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{3(a+bx)^{10/3}}{10b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (-3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4) + (3*(a + b*x)^(10/3))/(10*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{2/3}}+\frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{b^3}+\frac{(a+bx)^{7/3}}{b^3}\right)dx\\ &= -\frac{3a^3\sqrt[3]{a+bx}}{b^4}+\frac{9a^2(a+bx)^{4/3}}{4b^4}-\frac{9a(a+bx)^{7/3}}{7b^4}+\frac{3(a+bx)^{10/3}}{10b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\sqrt[3]{a+bx}\left(-81a^3+27a^2bx-18ab^2x^2+14b^3x^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-81*a^3 + 27*a^2*b*x - 18*a*b^2*x^2 + 14*b^3*x^3))/(140*b^4)
```

IntegrateAlgebraic [A] time = 0.03, size = 51, normalized size = 0.73

$$\frac{3\sqrt[3]{a+bx}\left(-140a^3+105a^2(a+bx)-60a(a+bx)^2+14(a+bx)^3\right)}{140b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(-140*a^3 + 105*a^2*(a + b*x) - 60*a*(a + b*x)^2 + 14*(a
 + b*x)^3))/(140*b^4)
```

fricas [A] time = 0.53, size = 42, normalized size = 0.60

$$\frac{3\left(14\,b^{3}x^{3}-18\,ab^{2}x^{2}+27\,a^{2}bx-81\,a^{3}\right)(bx+a)^{\frac{1}{3}}}{140\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 - 18*a*b^2*x^2 + 27*a^2*b*x - 81*a^3)*(b*x + a)^(1/3)/b^4
```

giac [A] time = 1.03, size = 49, normalized size = 0.70

$$\frac{3\left(14\,(bx+a)^{\frac{10}{3}}-60\,(bx+a)^{\frac{7}{3}}a+105\,(bx+a)^{\frac{4}{3}}a^{2}-140\,(bx+a)^{\frac{1}{3}}a^{3}\right)}{140\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/140*(14*(b*x + a)^(10/3) - 60*(b*x + a)^(7/3)*a + 105*(b*x + a)^(4/3)*a^2
 - 140*(b*x + a)^(1/3)*a^3)/b^4
```

maple [A] time = 0.01, size = 43, normalized size = 0.61

$$-\frac{3\,(bx+a)^{\frac{1}{3}}\left(-14b^{3}x^{3}+18a\,b^{2}x^{2}-27a^{2}bx+81a^{3}\right)}{140b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(2/3),x)
```

```
[Out] -3/140*(b*x+a)^(1/3)*(-14*b^3*x^3+18*a*b^2*x^2-27*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.34, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{10}{3}}}{10\,b^{4}}-\frac{9\,(bx+a)^{\frac{7}{3}}a}{7\,b^{4}}+\frac{9\,(bx+a)^{\frac{4}{3}}a^{2}}{4\,b^{4}}-\frac{3\,(bx+a)^{\frac{1}{3}}a^{3}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*(b*x + a)^(10/3)/b^4 - 9/7*(b*x + a)^(7/3)*a/b^4 + 9/4*(b*x + a)^(4/3)
*a^2/b^4 - 3*(b*x + a)^(1/3)*a^3/b^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,(a+b\,x)^{10/3}}{10\,b^{4}}-\frac{3\,a^{3}\,(a+b\,x)^{1/3}}{b^{4}}+\frac{9\,a^{2}\,(a+b\,x)^{4/3}}{4\,b^{4}}-\frac{9\,a\,(a+b\,x)^{7/3}}{7\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(2/3),x)
```

```
[Out] (3*(a + b*x)^(10/3))/(10*b^4) - (3*a^3*(a + b*x)^(1/3))/b^4 + (9*a^2*(a + b
*x)^(4/3))/(4*b^4) - (9*a*(a + b*x)^(7/3))/(7*b^4)
```

sympy [B] time = 2.79, size = 1640, normalized size = 23.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(2/3),x)
```

```
[Out] -243*a**(70/3)*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) + 243*a**(70/3)/(140*a**20*b**4 + 840*a**
19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x
**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) - 1377*a**(67/3)*b*x*(1 +
 b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 +
2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**
14*b**10*x**6) + 1458*a**(67/3)*b*x/(140*a**20*b**4 + 840*a**19*b**5*x + 21
00*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**1
5*b**9*x**5 + 140*a**14*b**10*x**6) - 3213*a**(64/3)*b**2*x**2*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6) + 3645*a**(64/3)*b**2*x**2/(140*a**20*b**4 + 840*a**19*b**5*x + 2100
*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*
b**9*x**5 + 140*a**14*b**10*x**6) - 3927*a**(61/3)*b**3*x**3*(1 + b*x/a)**(
1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17
*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x
**6) + 4860*a**(61/3)*b**3*x**3/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a
**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b*
*9*x**5 + 140*a**14*b**10*x**6) - 2583*a**(58/3)*b**4*x**4*(1 + b*x/a)**(1/
3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b
**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**
6) + 3645*a**(58/3)*b**4*x**4/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**
18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9
*x**5 + 140*a**14*b**10*x**6) - 693*a**(55/3)*b**5*x**5*(1 + b*x/a)**(1/3)/
(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7
*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6)
+ 1458*a**(55/3)*b**5*x**5/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*
b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x*
*5 + 140*a**14*b**10*x**6) + 273*a**(52/3)*b**6*x**6*(1 + b*x/a)**(1/3)/(14
0*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x*
*3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 2
43*a**(52/3)*b**6*x**6/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6
*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 +
 140*a**14*b**10*x**6) + 387*a**(49/3)*b**7*x**7*(1 + b*x/a)**(1/3)/(140*a*
*20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 +
 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10*x**6) + 198*a
**(46/3)*b**8*x**8*(1 + b*x/a)**(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x +
2100*a**18*b**6*x**2 + 2800*a**17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a*
*15*b**9*x**5 + 140*a**14*b**10*x**6) + 42*a**(43/3)*b**9*x**9*(1 + b*x/a)*
*(1/3)/(140*a**20*b**4 + 840*a**19*b**5*x + 2100*a**18*b**6*x**2 + 2800*a**
17*b**7*x**3 + 2100*a**16*b**8*x**4 + 840*a**15*b**9*x**5 + 140*a**14*b**10
*x**6)
```

3.407 $\int \frac{x^2}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=51

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3a^2\sqrt[3]{a+bx}}{b^3}+\frac{3(a+bx)^{7/3}}{7b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*a^2*(a + b*x)^(1/3))/b^3 - (3*a*(a + b*x)^(4/3))/(2*b^3) + (3*(a + b*x)^
(7/3))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{2/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{2/3}}-\frac{2a\sqrt[3]{a+bx}}{b^2}+\frac{(a+bx)^{4/3}}{b^2}\right)dx\\ &= \frac{3a^2\sqrt[3]{a+bx}}{b^3}-\frac{3a(a+bx)^{4/3}}{2b^3}+\frac{3(a+bx)^{7/3}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.69

$$\frac{3\sqrt[3]{a+bx}\left(9a^2-3abx+2b^2x^2\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3)*(9*a^2 - 3*a*b*x + 2*b^2*x^2))/(14*b^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 45, normalized size = 0.88

$$\frac{3\left(14a^2\sqrt[3]{a+bx}+2(a+bx)^{7/3}-7a(a+bx)^{4/3}\right)}{14b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(2/3),x]
```

```
[Out] (3*(14*a^2*(a + b*x)^(1/3) - 7*a*(a + b*x)^(4/3) + 2*(a + b*x)^(7/3)))/(14*
b^3)
```

fricas [A] time = 0.83, size = 31, normalized size = 0.61

$$\frac{3\left(2\,b^2x^2-3\,abx+9\,a^2\right)(bx+a)^{\frac{1}{3}}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 - 3*a*b*x + 9*a^2)*(b*x + a)^(1/3)/b^3
```

giac [A] time = 0.90, size = 37, normalized size = 0.73

$$\frac{3\left(2\,(bx+a)^{\frac{7}{3}}-7\,(bx+a)^{\frac{4}{3}}a+14\,(bx+a)^{\frac{1}{3}}a^2\right)}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/14*(2*(b*x + a)^(7/3) - 7*(b*x + a)^(4/3)*a + 14*(b*x + a)^(1/3)*a^2)/b^3
```

maple [A] time = 0.00, size = 32, normalized size = 0.63

$$\frac{3\,(bx+a)^{\frac{1}{3}}\left(2b^2x^2-3abx+9a^2\right)}{14b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(2/3),x)
```

```
[Out] 3/14*(b*x+a)^(1/3)*(2*b^2*x^2-3*a*b*x+9*a^2)/b^3
```

maxima [A] time = 1.31, size = 41, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{7}{3}}}{7\,b^3}-\frac{3\,(bx+a)^{\frac{4}{3}}a}{2\,b^3}+\frac{3\,(bx+a)^{\frac{1}{3}}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*(b*x + a)^(7/3)/b^3 - 3/2*(b*x + a)^(4/3)*a/b^3 + 3*(b*x + a)^(1/3)*a^2
/b^3
```

mupad [B] time = 0.04, size = 37, normalized size = 0.73

$$\frac{6\,(a+b\,x)^{7/3}-21\,a\,(a+b\,x)^{4/3}+42\,a^2\,(a+b\,x)^{1/3}}{14\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(2/3),x)
```

```
[Out] (6*(a + b*x)^(7/3) - 21*a*(a + b*x)^(4/3) + 42*a^2*(a + b*x)^(1/3))/(14*b^3
)
```

sympy [B] time = 1.82, size = 600, normalized size = 11.76

$$\frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{31}{3}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{72a^{\frac{28}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{28}{3}}bx}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{60a^{\frac{25}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{81a^{\frac{25}{3}}b^2x^2}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{18a^{\frac{22}{3}}b^3x^3\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}-\frac{27a^{\frac{22}{3}}b^3x^3}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{9a^{\frac{19}{3}}b^4x^4\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}+\frac{6a^{\frac{16}{3}}b^5x^5\sqrt[3]{1+\frac{bx}{a}}}{14a^8b^3+42a^7b^4x+42a^6b^5x^2+14a^5b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(2/3),x)
```

```
[Out] 27*a**(31/3)*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b*
*5*x**2 + 14*a**5*b**6*x**3) - 27*a**(31/3)/(14*a**8*b**3 + 42*a**7*b**4*x
+ 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 72*a**(28/3)*b*x*(1 + b*x/a)**(1
/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
 - 81*a**(28/3)*b*x/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14
*a**5*b**6*x**3) + 60*a**(25/3)*b**2*x**2*(1 + b*x/a)**(1/3)/(14*a**8*b**3
+ 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 81*a**(25/3)*b*
*2*x**2/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x
**3) + 18*a**(22/3)*b**3*x**3*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b*
*4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) - 27*a**(22/3)*b**3*x**3/(14*
a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3) + 9*a**
(19/3)*b**4*x**4*(1 + b*x/a)**(1/3)/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**
6*b**5*x**2 + 14*a**5*b**6*x**3) + 6*a**(16/3)*b**5*x**5*(1 + b*x/a)**(1/3)
/(14*a**8*b**3 + 42*a**7*b**4*x + 42*a**6*b**5*x**2 + 14*a**5*b**6*x**3)
```

3.408 $\int \frac{x}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3(a+bx)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(2/3),x]
[Out] (-3*a*(a + b*x)^(1/3))/b^2 + (3*(a + b*x)^(4/3))/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{2/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{2/3}}+\frac{\sqrt[3]{a+bx}}{b}\right)dx\\ &= -\frac{3a\sqrt[3]{a+bx}}{b^2}+\frac{3(a+bx)^{4/3}}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(bx-3a)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(2/3),x]
[Out] (3*(-3*a + b*x)*(a + b*x)^(1/3))/(4*b^2)
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 0.75

$$-\frac{3(3a-bx)\sqrt[3]{a+bx}}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(2/3),x]
[Out] (-3*(3*a - b*x)*(a + b*x)^(1/3))/(4*b^2)
```

fricas [A] time = 0.90, size = 19, normalized size = 0.59

$$\frac{3\,(bx+a)^{\frac{1}{3}}(bx-3\,a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(b*x + a)^(1/3)*(b*x - 3*a)/b^2
```

giac [A] time = 0.89, size = 23, normalized size = 0.72

$$\frac{3\left((bx+a)^{\frac{4}{3}}-4\,(bx+a)^{\frac{1}{3}}a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 3/4*((b*x + a)^(4/3) - 4*(b*x + a)^(1/3)*a)/b^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.66

$$-\frac{3\,(bx+a)^{\frac{1}{3}}\,(-bx+3a)}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(2/3),x)
```

```
[Out] -3/4*(b*x+a)^(1/3)*(-b*x+3*a)/b^2
```

maxima [A] time = 1.34, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{4}{3}}}{4\,b^2}-\frac{3\,(bx+a)^{\frac{1}{3}}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 3/4*(b*x + a)^(4/3)/b^2 - 3*(b*x + a)^(1/3)*a/b^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.78

$$-\frac{12\,a\,(a+b\,x)^{1/3}-3\,(a+b\,x)^{4/3}}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(2/3),x)
```

```
[Out] -(12*a*(a + b*x)^(1/3) - 3*(a + b*x)^(4/3))/(4*b^2)
```

sympy [B] time = 1.19, size = 162, normalized size = 5.06

$$-\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{10}{3}}}{4a^2b^2+4ab^3x}-\frac{6a^{\frac{7}{3}}bx\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}+\frac{9a^{\frac{7}{3}}bx}{4a^2b^2+4ab^3x}+\frac{3a^{\frac{4}{3}}b^2x^2\sqrt[3]{1+\frac{bx}{a}}}{4a^2b^2+4ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(2/3),x)
```

```
[Out] -9*a**(10/3)*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x) + 9*a**(10/3)/(4
*a**2*b**2 + 4*a*b**3*x) - 6*a**(7/3)*b*x*(1 + b*x/a)**(1/3)/(4*a**2*b**2 +
 4*a*b**3*x) + 9*a**(7/3)*b*x/(4*a**2*b**2 + 4*a*b**3*x) + 3*a**(4/3)*b**2*
x**2*(1 + b*x/a)**(1/3)/(4*a**2*b**2 + 4*a*b**3*x)
```

3.409 $\int \frac{1}{(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=14

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2/3),x]
```

```
[Out] (3*(a + b*x)^(1/3))/b
```

fricas [A] time = 0.89, size = 12, normalized size = 0.86

$$\frac{3\,(bx+a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="fricas")
[Out] 3*(b*x + a)^(1/3)/b
```

giac [A] time = 0.98, size = 12, normalized size = 0.86

$$\frac{3 (bx + a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="giac")
[Out] 3*(b*x + a)^(1/3)/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{3 (bx + a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3),x)
[Out] 3*(b*x+a)^(1/3)/b
```

maxima [A] time = 1.30, size = 12, normalized size = 0.86

$$\frac{3 (bx + a)^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3),x, algorithm="maxima")
[Out] 3*(b*x + a)^(1/3)/b
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{3 (a + b\, x)^{1/3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(2/3),x)
[Out] (3*(a + b*x)^(1/3))/b
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{3 \sqrt[3]{a + bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3),x)
[Out] 3*(a + b*x)**(1/3)/b
```

3.410 $\int \frac{1}{x(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=80

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}-\frac{\log(x)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/3)
) - Log[x]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x)^(1/3)])/(2*a^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x(a+bx)^{2/3}}\,dx = -\frac{\log(x)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{2a^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx, x, \sqrt[3]{a+bx}\right)}{2\sqrt[3]{a}}$$

$$= -\frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{\log(x)}{2a^{2/3}} + \frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{2/3}}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.16

$$-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)-2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{2a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) - (a + b*x)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*
x)^(2/3)])/a^(2/3)
```

IntegrateAlgebraic [A] time = 0.06, size = 105, normalized size = 1.31

$$\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{2/3}} - \frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/a^(2/
3)) + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(2/3) - Log[a^(2/3) + a^(1/3)*(a + b
*x)^(1/3) + (a + b*x)^(2/3)]/(2*a^(2/3))
```

fricas [B] time = 1.00, size = 115, normalized size = 1.44

$$-\frac{2\sqrt{3}\left(a^2\right)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}\left(a^2\right)^{\frac{1}{6}}\left(\left(a^2\right)^{\frac{1}{3}}a+2\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+\left(a^2\right)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(a^2\right)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*a
 + 2*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + (a^2)^(2/3)*log((b*x + a)^(2/3)*a
+ (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3)) - 2*(a^2)^(2/3)*log((b*x + a
)^(1/3)*a - (a^2)^(2/3)))/a^2
```

giac [A] time = 2.32, size = 78, normalized size = 0.98

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g(abs((b*x + a)^(1/3) - a^(1/3)))/a^(2/3)
```

maple [A] time = 0.00, size = 76, normalized size = 0.95

$$-\frac{\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{2}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(2/3),x)
```

```
[Out] 1/a^(2/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(2/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a
^(1/3)+(b*x+a)^(2/3))-1/a^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)
/a^(1/3)+1))
```

maxima [A] time = 2.87, size = 77, normalized size = 0.96

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{2}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(2/3)
- 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(2/3) + lo
g((b*x + a)^(1/3) - a^(1/3))/a^(2/3)
```

mupad [B] time = 0.17, size = 95, normalized size = 1.19

$$\frac{\ln\left(9\,(a+b\,x)^{1/3}-9\,a^{1/3}\right)}{a^{2/3}}+\frac{\ln\left(\frac{9\,a^{1/3}\,(-1+\sqrt{3}\,1\mathrm{i})}{2}-9\,(a+b\,x)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}-\frac{\ln\left(\frac{9\,a^{1/3}\,(1+\sqrt{3}\,1\mathrm{i})}{2}+9\,(a+b\,x)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(2/3)),x)
```

```
[Out] log(9*(a + b*x)^(1/3) - 9*a^(1/3))/a^(2/3) + (log((9*a^(1/3)*(3^(1/2)*1i -
1))/2 - 9*(a + b*x)^(1/3))*(3^(1/2)*1i - 1))/(2*a^(2/3)) - (log((9*a^(1/3)*
(3^(1/2)*1i + 1))/2 + 9*(a + b*x)^(1/3))*(3^(1/2)*1i + 1))/(2*a^(2/3))
```

sympy [C] time = 1.93, size = 150, normalized size = 1.88

$$\frac{\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\,\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(2/3),x)
```

```
[Out] log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/
3)) + exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/
a**(1/3))*gamma(1/3)/(3*a**(2/3)*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b**(1/
3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(3*a**(2/3)*ga
mma(4/3))
```

3.411 $\int \frac{1}{x^2(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=98

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) + (b*Log[x])/(3*a^(5/3)) - (b*Log[a^(1/3) -
 (a + b*x)^(1/3)])/a^(5/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{ax}-\frac{(2b)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{3a}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{a^{5/3}}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,\right.}{a^{4/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{5/3}}\\
&= -\frac{\sqrt[3]{a+bx}}{ax}+\frac{2b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{5/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.32

$$
\frac{3b\sqrt[3]{a+bx}\;{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};\frac{bx}{a}+1\right)}{a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] (3*b*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, 1 + (b*x)/a])/a^2
```

IntegrateAlgebraic [A] time = 0.15, size = 128, normalized size = 1.31

$$
-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{5/3}}+\frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{5/3}}+\frac{2b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\sqrt[3]{a+bx}}{ax}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(2/3)),x]
```

```
[Out] -((a + b*x)^(1/3)/(a*x)) + (2*b*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqr
t[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/(3
*a^(5/3)) + (b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(3
*a^(5/3))
```

fricas [B] time = 0.69, size = 166, normalized size = 1.69

$$
\frac{2\sqrt{3}\,abx\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\left(\sqrt{3}\left(-a^2\right)^{\frac{1}{3}}a-2\sqrt{3}\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{3a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{2}{3}}a-\left(-a^2\right)^{\frac{1}{3}}a+\left(-a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-2\left(-a^2\right)^{\frac{2}{3}}bx\log\left((bx+a)^{\frac{1}{3}}a-\left(-a^2\right)^{\frac{2}{3}}\right)-3(bx+a)^{\frac{1}{3}}a^2}{3a^3x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(3)*a*b*x*sqrt(-(-a^2)^(1/3))*arctan(-1/3*(sqrt(3)*(-a^2)^(1/3)*
a - 2*sqrt(3)*(-a^2)^(2/3)*(b*x + a)^(1/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^
2)^(2/3)*b*x*log((b*x + a)^(2/3)*a - (-a^2)^(1/3)*a + (-a^2)^(2/3)*(b*x + a
)^(1/3)) - 2*(-a^2)^(2/3)*b*x*log((b*x + a)^(1/3)*a - (-a^2)^(2/3)) - 3*(b*
x + a)^(1/3)*a^2)/(a^3*x)
```

giac [A] time = 2.37, size = 108, normalized size = 1.10

$$\frac{\frac{2\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\frac{2b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}}-\frac{3(bx+a)^{\frac{1}{3}}b}{ax}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] 1/3*(2*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(5/3) + b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(5/3) - 2*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x + a)^(1/
3)*b/(a*x))/b
```

maple [A] time = 0.01, size = 95, normalized size = 0.97

$$\frac{2\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{5}{3}}}-\frac{2b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{5}{3}}}+\frac{b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(2/3),x)
```

```
[Out] -(b*x+a)^(1/3)/a/x-2/3*b/a^(5/3)*ln(-a^(1/3)+(b*x+a)^(1/3))+1/3*b/a^(5/3)*l
n(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+2/3*b/a^(5/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.95, size = 106, normalized size = 1.08

$$\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{3a^{\frac{5}{3}}}-\frac{(bx+a)^{\frac{1}{3}}b}{(bx+a)a-a^2}+\frac{b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3a^{\frac{5}{3}}}-\frac{2b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3a^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(
5/3) - (b*x + a)^(1/3)*b/((b*x + a)*a - a^2) + 1/3*b*log((b*x + a)^(2/3) +
(b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 2/3*b*log((b*x + a)^(1/3) - a^
(1/3))/a^(5/3)
```

mupad [B] time = 0.13, size = 122, normalized size = 1.24

$$-\frac{(a+b\,x)^{1/3}}{a\,x}+\frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{5/3}}+\frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{a^{2/3}}+\frac{6\,b\,(a+b\,x)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{5/3}}-\frac{2\,b\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(2/3)),x)
```

```
[Out] (log((3*(b - 3^(1/2)*b*1i))/a^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b - 3^(1/2)
*b*1i))/(3*a^(5/3)) - (a + b*x)^(1/3)/(a*x) + (log((3*(b + 3^(1/2)*b*1i))/a
^(2/3) + (6*b*(a + b*x)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(3*a^(5/3)) - (2*b*lo
g((a + b*x)^(1/3) - a^(1/3)))/(3*a^(5/3))
```

sympy [C] time = 2.27, size = 830, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(2/3),x)
```

```
[Out] -2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*p
i/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)
) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*
exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*g
amma(4/3)) - 2*a**(4/3)*b**(5/3)*(a/b + x)**(2/3)*exp(-2*I*pi/3)*log(1 - b*
*(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(9*a**3*b*
*(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x
)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(9*a**3
*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b
+ x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b + x)**(5/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3
)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5
/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 2*a**(1/3)*b**(8/3)*(a/b +
 x)**(5/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*g
amma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3)) + 3*
a*b**2*(a/b + x)*exp(2*I*pi/3)*gamma(1/3)/(9*a**3*b**(2/3)*(a/b + x)**(2/3)
*exp(2*I*pi/3)*gamma(4/3) - 9*a**2*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*
gamma(4/3))
```

3.412 $\int \frac{1}{x^3(a+bx)^{2/3}}\,dx$

Optimal. Leaf size=130

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 57, 617, 204, 31}

$$-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{\sqrt[3]{a+bx}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -(a + b*x)^(1/3)/(2*a*x^2) + (5*b*(a + b*x)^(1/3))/(6*a^2*x) - (5*b^2*ArcTa
n[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) - (
5*b^2*Log[x])/(18*a^(8/3)) + (5*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(6*a^(8
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(a+bx)^{2/3}}\,dx &= -\frac{\sqrt[3]{a+bx}}{2ax^2}-\frac{(5b)\int \frac{1}{x^2(a+bx)^{2/3}}\,dx}{6a}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}+\frac{\left(5b^2\right)\int \frac{1}{x(a+bx)^{2/3}}\,dx}{9a^2}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}-\frac{\left(5b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{6a^{8/3}}-\frac{\left(5b^2\right)\mathrm{S}}{}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{6a^{8/3}}+\frac{\left(5b^2\right)\mathrm{Subst}\left(\int \frac{1}{-3-}\right.}{3}\\
&= -\frac{\sqrt[3]{a+bx}}{2ax^2}+\frac{5b\sqrt[3]{a+bx}}{6a^2x}-\frac{5b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{5b^2\log(x)}{18a^{8/3}}+\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{6a^{8/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.25

$$
-\frac{3b^2\sqrt[3]{a+bx}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};\frac{bx}{a}+1\right)}{a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] (-3*b^2*(a + b*x)^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, 1 + (b*x)/a])/a^3
```

IntegrateAlgebraic [A] time = 0.12, size = 149, normalized size = 1.15

$$
\frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{8/3}}-\frac{5b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{18a^{8/3}}-\frac{5b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}}-\frac{\sqrt[3]{a+bx}\,(8a-5(a+bx))}{6a^2x^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(2/3)),x]
```

```
[Out] -1/6*((a + b*x)^(1/3)*(8*a - 5*(a + b*x)))/(a^2*x^2) - (5*b^2*ArcTan[1/Sqrt
[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)) + (5*b^2*
Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(8/3)) - (5*b^2*Log[a^(2/3) + a^(1/3)*
(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(18*a^(8/3))
```

fricas [A] time = 0.82, size = 162, normalized size = 1.25

$$
-\frac{10\sqrt{3}\left(a^2\right)^{\frac{1}{6}}ab^2x^2\arctan\left(\frac{\left(a^2\right)^{\frac{1}{6}}\left(\sqrt{3}\left(a^2\right)^{\frac{1}{3}}a+2\sqrt{3}\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)}{3a^2}\right)+5\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{2}{3}}a+\left(a^2\right)^{\frac{1}{3}}a+\left(a^2\right)^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\right)-10\left(a^2\right)^{\frac{2}{3}}b^2x^2\log\left((bx+a)^{\frac{1}{3}}a-\left(a^2\right)^{\frac{2}{3}}\right)-3\left(5a^2bx-3a^3\right)(bx+a)^{\frac{1}{3}}}{18a^4x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/18*(10*sqrt(3)*(a^2)^(1/6)*a*b^2*x^2*arctan(1/3*(a^2)^(1/6)*(sqrt(3)*(a^
2)^(1/3)*a + 2*sqrt(3)*(a^2)^(2/3)*(b*x + a)^(1/3))/a^2) + 5*(a^2)^(2/3)*b^
2*x^2*log((b*x + a)^(2/3)*a + (a^2)^(1/3)*a + (a^2)^(2/3)*(b*x + a)^(1/3))
- 10*(a^2)^(2/3)*b^2*x^2*log((b*x + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^2*b*
x - 3*a^3)*(b*x + a)^(1/3))/(a^4*x^2)
```

giac [A] time = 2.05, size = 130, normalized size = 1.00

$$-\frac{\frac{10\,\sqrt{3}b^{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}}+\frac{5\,b^{3}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}}-\frac{10\,b^{3}\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}}-\frac{3\left(5\,(bx+a)^{\frac{4}{3}}b^{3}-8\,(bx+a)^{\frac{1}{3}}ab^{3}\right)}{a^{2}b^{2}x^{2}}}{18\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] -1/18*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(8/3) + 5*b^3*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(8/3) - 10*b^3*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(8/3) - 3*(5*(b*x
 + a)^(4/3)*b^3 - 8*(b*x + a)^(1/3)*a*b^3)/(a^2*b^2*x^2))/b
```

maple [A] time = 0.01, size = 117, normalized size = 0.90

$$-\frac{5\sqrt{3}\,b^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{8}{3}}}+\frac{5b^{2}\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{8}{3}}}-\frac{5b^{2}\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{18a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{1}{3}}b}{6a^{2}x}-\frac{(bx+a)^{\frac{1}{3}}}{2a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(2/3),x)
```

```
[Out] -1/2*(b*x+a)^(1/3)/a/x^2+5/6*b*(b*x+a)^(1/3)/a^2/x+5/9*b^2/a^(8/3)*ln(-a^(1
/3)+(b*x+a)^(1/3))-5/18*b^2/a^(8/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a
)^(2/3))-5/9*b^2/a^(8/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3
)+1))
```

maxima [A] time = 3.04, size = 142, normalized size = 1.09

$$-\frac{5\,\sqrt{3}b^{2}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{8}{3}}}-\frac{5\,b^{2}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{8}{3}}}+\frac{5\,b^{2}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{8}{3}}}+\frac{5\,(bx+a)^{\frac{4}{3}}b^{2}-8\,(bx+a)^{\frac{1}{3}}ab^{2}}{6\left((bx+a)^{2}a^{2}-2\,(bx+a)a^{3}+a^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -5/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(8/3) - 5/18*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(8/3) + 5/9*b^2*log((b*x + a)^(1/3) - a^(1/3))/a^(8/3) + 1/6*(5*(b*x + a
)^(4/3)*b^2 - 8*(b*x + a)^(1/3)*a*b^2)/((b*x + a)^2*a^2 - 2*(b*x + a)*a^3 +
 a^4)
```

mupad [B] time = 0.13, size = 175, normalized size = 1.35

$$\frac{5\,b^{2}\,\ln\left((a+b\,x)^{1/3}-a^{1/3}\right)}{9\,a^{8/3}}-\frac{\frac{4\,b^{2}\,(a+b\,x)^{1/3}}{3\,a}-\frac{5\,b^{2}\,(a+b\,x)^{4/3}}{6\,a^{2}}}{(a+b\,x)^{2}-2\,a\,(a+b\,x)+a^{2}}+\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,(a+b\,x)^{1/3}}{a^{2}}-\frac{5\,b^{2}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{a^{5/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9\,a^{8/3}}-\frac{5\,b^{2}\,\ln\left(\frac{5\,b^{2}\,(a+b\,x)^{1/3}}{a^{2}}+\frac{5\,b^{2}\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{a^{5/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{9\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(2/3)),x)
```

```
[Out] (5*b^2*log((a + b*x)^(1/3) - a^(1/3)))/(9*a^(8/3)) - ((4*b^2*(a + b*x)^(1/3
))/(3*a) - (5*b^2*(a + b*x)^(4/3))/(6*a^2))/((a + b*x)^2 - 2*a*(a + b*x) +
a^2) + (5*b^2*log((5*b^2*(a + b*x)^(1/3))/a^2 - (5*b^2*((3^(1/2)*1i)/2 - 1/
```

```
2))/a^(5/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(8/3)) - (5*b^2*log((5*b^2*(a + b
*x)^(1/3))/a^2 + (5*b^2*((3^(1/2)*1i)/2 + 1/2))/a^(5/3))*((3^(1/2)*1i)/2 +
1/2))/(9*a^(8/3))
```

sympy [C] time = 2.73, size = 2728, normalized size = 20.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(2/3),x)
```

```
[Out] 10*a**(13/3)*b**(8/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(
8/3)*(a/b + x)**(2/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)
/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamm
a(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*
a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)
*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 10*a**(13/3)*b**(8/3)*(a/b +
 x)**(2/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*p
i/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*
gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) +
162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(1
1/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(
a/b + x)**(5/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*g
amma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162
*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3
)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**
(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*
log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/
(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(
5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x
)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp
(2*I*pi/3)*gamma(4/3)) - 30*a**(10/3)*b**(11/3)*(a/b + x)**(5/3)*exp(-2*I*p
i/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(
1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6
*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/
b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3
)*exp(2*I*pi/3)*gamma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(2*
I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**
(2/3)*(a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b +
x)**(5/3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp
(2*I*pi/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*g
amma(4/3)) + 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*log(1 - b**(1/3)*(a/b +
 x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b
+ x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*e
xp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*
gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
+ 30*a**(7/3)*b**(14/3)*(a/b + x)**(8/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a
/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(
a/b + x)**(2/3)*exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/
3)*exp(2*I*pi/3)*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi
/3)*gamma(4/3) - 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/
3)) - 10*a**(4/3)*b**(17/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*
exp(2*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)
*gamma(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) -
 54*a**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3
```

```
)*b**(17/3)*(a/b + x)**(11/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I*pi
/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(4/3
) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**4*b
**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 10*a**(4/3)*b**(17/3
)*(a/b + x)**(11/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_po
lar(4*I*pi/3)/a**(1/3))*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 24*a**4*b**3*(a
/b + x)*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*I
*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma(
4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a**
4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) + 39*a**3*b**4*(a/b
 + x)**2*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2*
I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamma
(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a*
*4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3)) - 15*a**2*b**5*(a/
b + x)**3*exp(2*I*pi/3)*gamma(1/3)/(54*a**7*b**(2/3)*(a/b + x)**(2/3)*exp(2
*I*pi/3)*gamma(4/3) - 162*a**6*b**(5/3)*(a/b + x)**(5/3)*exp(2*I*pi/3)*gamm
a(4/3) + 162*a**5*b**(8/3)*(a/b + x)**(8/3)*exp(2*I*pi/3)*gamma(4/3) - 54*a
**4*b**(11/3)*(a/b + x)**(11/3)*exp(2*I*pi/3)*gamma(4/3))
```

3.413 $\int \frac{x^3}{(a+bx)^{4/3}} \, dx$

Optimal. Leaf size=70

$$\frac{3a^3}{b^4 \sqrt[3]{a+bx}} + \frac{9a^2(a+bx)^{2/3}}{2b^4} - \frac{9a(a+bx)^{5/3}}{5b^4} + \frac{3(a+bx)^{8/3}}{8b^4}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {43}

$$\frac{3a^3}{b^4 \sqrt[3]{a+bx}} + \frac{9a^2(a+bx)^{2/3}}{2b^4} - \frac{9a(a+bx)^{5/3}}{5b^4} + \frac{3(a+bx)^{8/3}}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*a^3)/(b^4*(a + b*x)^(1/3)) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) - (9*a*(a +
 b*x)^(5/3))/(5*b^4) + (3*(a + b*x)^(8/3))/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(a+bx)^{4/3}} \, dx &= \int \left(-\frac{a^3}{b^3(a+bx)^{4/3}} + \frac{3a^2}{b^3 \sqrt[3]{a+bx}} - \frac{3a(a+bx)^{2/3}}{b^3} + \frac{(a+bx)^{5/3}}{b^3} \right) dx \\ &= \frac{3a^3}{b^4 \sqrt[3]{a+bx}} + \frac{9a^2(a+bx)^{2/3}}{2b^4} - \frac{9a(a+bx)^{5/3}}{5b^4} + \frac{3(a+bx)^{8/3}}{8b^4} \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.66

$$\frac{3\left(81a^3 + 27a^2bx - 9ab^2x^2 + 5b^3x^3\right)}{40b^4 \sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(81*a^3 + 27*a^2*b*x - 9*a*b^2*x^2 + 5*b^3*x^3))/(40*b^4*(a + b*x)^(1/3)
)
```

IntegrateAlgebraic [A] time = 0.03, size = 51, normalized size = 0.73

$$\frac{3\left(40a^3 + 60a^2(a+bx) - 24a(a+bx)^2 + 5(a+bx)^3\right)}{40b^4 \sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*x)^(4/3),x]
```

```
[Out] (3*(40*a^3 + 60*a^2*(a + b*x) - 24*a*(a + b*x)^2 + 5*(a + b*x)^3))/(40*b^4*
(a + b*x)^(1/3))
```

fricas [A] time = 0.86, size = 52, normalized size = 0.74

$$\frac{3\left(5\,b^{3}x^{3}-9\,ab^{2}x^{2}+27\,a^{2}bx+81\,a^{3}\right)(bx+a)^{\frac{2}{3}}}{40\left(b^{5}x+ab^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^3*x^3 - 9*a*b^2*x^2 + 27*a^2*b*x + 81*a^3)*(b*x + a)^(2/3)/(b^5*x
 + a*b^4)
```

giac [A] time = 1.10, size = 62, normalized size = 0.89

$$\frac{3\,a^{3}}{(bx+a)^{\frac{1}{3}}b^{4}}+\frac{3\left(5\,(bx+a)^{\frac{8}{3}}b^{28}-24\,(bx+a)^{\frac{5}{3}}ab^{28}+60\,(bx+a)^{\frac{2}{3}}a^{2}b^{28}\right)}{40\,b^{32}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3*a^3/((b*x + a)^(1/3)*b^4) + 3/40*(5*(b*x + a)^(8/3)*b^28 - 24*(b*x + a)^(
5/3)*a*b^28 + 60*(b*x + a)^(2/3)*a^2*b^28)/b^32
```

maple [A] time = 0.00, size = 43, normalized size = 0.61

$$\frac{\frac{3}{8}b^{3}x^{3}-\frac{27}{40}a\,b^{2}x^{2}+\frac{81}{40}a^{2}bx+\frac{243}{40}a^{3}}{(bx+a)^{\frac{1}{3}}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3/40/(b*x+a)^(1/3)*(5*b^3*x^3-9*a*b^2*x^2+27*a^2*b*x+81*a^3)/b^4
```

maxima [A] time = 1.36, size = 56, normalized size = 0.80

$$\frac{3\,(bx+a)^{\frac{8}{3}}}{8\,b^{4}}-\frac{9\,(bx+a)^{\frac{5}{3}}a}{5\,b^{4}}+\frac{9\,(bx+a)^{\frac{2}{3}}a^{2}}{2\,b^{4}}+\frac{3\,a^{3}}{(bx+a)^{\frac{1}{3}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/8*(b*x + a)^(8/3)/b^4 - 9/5*(b*x + a)^(5/3)*a/b^4 + 9/2*(b*x + a)^(2/3)*a
^2/b^4 + 3*a^3/((b*x + a)^(1/3)*b^4)
```

mupad [B] time = 0.05, size = 56, normalized size = 0.80

$$\frac{3\,(a+b\,x)^{8/3}}{8\,b^{4}}+\frac{9\,a^{2}\,(a+b\,x)^{2/3}}{2\,b^{4}}+\frac{3\,a^{3}}{b^{4}\,(a+b\,x)^{1/3}}-\frac{9\,a\,(a+b\,x)^{5/3}}{5\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*x)^(4/3),x)
```

```
[Out] (3*(a + b*x)^(8/3))/(8*b^4) + (9*a^2*(a + b*x)^(2/3))/(2*b^4) + (3*a^3)/(b^
4*(a + b*x)^(1/3)) - (9*a*(a + b*x)^(5/3))/(5*b^4)
```

sympy [B] time = 2.89, size = 1538, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**(4/3),x)
```

```
[Out] 243*a**(68/3)*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a*
*18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*
x**5 + 40*a**14*b**10*x**6) - 243*a**(68/3)/(40*a**20*b**4 + 240*a**19*b**5
*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*
a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1296*a**(65/3)*b*x*(1 + b*x/a)**(2
/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**
7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) -
 1458*a**(65/3)*b*x/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2
 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**
14*b**10*x**6) + 2808*a**(62/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(40*a**20*b**4
 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16
*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(62/3)*b*
*2*x**2/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17
*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**
6) + 3120*a**(59/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19
*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 +
 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) - 4860*a**(59/3)*b**3*x**3/(40*
a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 1830*a*
*(56/3)*b**4*x**4*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 60
0*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b
**9*x**5 + 40*a**14*b**10*x**6) - 3645*a**(56/3)*b**4*x**4/(40*a**20*b**4 +
 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b
**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 528*a**(53/3)*b**5*
x**5*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*
x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40
*a**14*b**10*x**6) - 1458*a**(53/3)*b**5*x**5/(40*a**20*b**4 + 240*a**19*b*
*5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 24
0*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 96*a**(50/3)*b**6*x**6*(1 + b*x/
a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**
17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x
**6) - 243*a**(50/3)*b**6*x**6/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a**1
8*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9*x*
*5 + 40*a**14*b**10*x**6) + 48*a**(47/3)*b**7*x**7*(1 + b*x/a)**(2/3)/(40*a
**20*b**4 + 240*a**19*b**5*x + 600*a**18*b**6*x**2 + 800*a**17*b**7*x**3 +
600*a**16*b**8*x**4 + 240*a**15*b**9*x**5 + 40*a**14*b**10*x**6) + 15*a**(4
4/3)*b**8*x**8*(1 + b*x/a)**(2/3)/(40*a**20*b**4 + 240*a**19*b**5*x + 600*a
**18*b**6*x**2 + 800*a**17*b**7*x**3 + 600*a**16*b**8*x**4 + 240*a**15*b**9
*x**5 + 40*a**14*b**10*x**6)
```

3.414 $\int \frac{x^2}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (-3*a^2)/(b^3*(a + b*x)^(1/3)) - (3*a*(a + b*x)^(2/3))/b^3 + (3*(a + b*x)^(
5/3))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(a+bx)^{4/3}}\,dx &= \int \left(\frac{a^2}{b^2(a+bx)^{4/3}}-\frac{2a}{b^2\sqrt[3]{a+bx}}+\frac{(a+bx)^{2/3}}{b^2}\right)dx\\ &= -\frac{3a^2}{b^3\sqrt[3]{a+bx}}-\frac{3a(a+bx)^{2/3}}{b^3}+\frac{3(a+bx)^{5/3}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.69

$$\frac{3\left(-9a^2-3abx+b^2x^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (3*(-9*a^2 - 3*a*b*x + b^2*x^2))/(5*b^3*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.76

$$\frac{3\left(-5a^2-5a(a+bx)+(a+bx)^2\right)}{5b^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a + b*x)^(4/3),x]
```

```
[Out] (3*(-5*a^2 - 5*a*(a + b*x) + (a + b*x)^2))/(5*b^3*(a + b*x)^(1/3))
```

fricas [A] time = 0.83, size = 40, normalized size = 0.82

$$\frac{3\left(b^{2} x^{2} - 3\,abx - 9\,a^{2}\right)(bx + a)^{\frac{2}{3}}}{5\left(b^{4} x + ab^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(b^2*x^2 - 3*a*b*x - 9*a^2)*(b*x + a)^(2/3)/(b^4*x + a*b^3)
```

giac [A] time = 1.00, size = 46, normalized size = 0.94

$$-\frac{3\,a^{2}}{(bx + a)^{\frac{1}{3}} b^{3}} + \frac{3\left((bx + a)^{\frac{5}{3}} b^{12} - 5\,(bx + a)^{\frac{2}{3}} ab^{12}\right)}{5\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -3*a^2/((b*x + a)^(1/3)*b^3) + 3/5*((b*x + a)^(5/3)*b^12 - 5*(b*x + a)^(2/3
)*a*b^12)/b^15
```

maple [A] time = 0.01, size = 32, normalized size = 0.65

$$-\frac{3\left(-b^{2} x^{2} + 3abx + 9a^{2}\right)}{5\,(bx + a)^{\frac{1}{3}}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3/5/(b*x+a)^(1/3)*(-b^2*x^2+3*a*b*x+9*a^2)/b^3
```

maxima [A] time = 1.33, size = 41, normalized size = 0.84

$$\frac{3\,(bx + a)^{\frac{5}{3}}}{5\,b^{3}} - \frac{3\,(bx + a)^{\frac{2}{3}}\,a}{b^{3}} - \frac{3\,a^{2}}{(bx + a)^{\frac{1}{3}} b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/5*(b*x + a)^(5/3)/b^3 - 3*(b*x + a)^(2/3)*a/b^3 - 3*a^2/((b*x + a)^(1/3)*
b^3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$-\frac{15\,a\,\left(a + b\,x\right) - 3\,{\left(a + b\,x\right)}^{2} + 15\,a^{2}}{5\,b^{3}\,{\left(a + b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*x)^(4/3),x)
```

```
[Out] -(15*a*(a + b*x) - 3*(a + b*x)^2 + 15*a^2)/(5*b^3*(a + b*x)^(1/3))
```

sympy [B] time = 1.88, size = 534, normalized size = 10.90

$$-\frac{27a^{\frac{29}{3}}\left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} + \frac{27a^{\frac{29}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} - \frac{63a^{\frac{26}{3}}bx\left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} + \frac{81a^{\frac{26}{3}}bx}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} - \frac{42a^{\frac{23}{3}}b^{2}x^{2}\left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} + \frac{81a^{\frac{23}{3}}b^{2}x^{2}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} - \frac{3a^{\frac{20}{3}}b^{3}x^{3}\left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} + \frac{27a^{\frac{20}{3}}b^{3}x^{3}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}} + \frac{3a^{\frac{17}{3}}b^{4}x^{4}\left(1 + \frac{bx}{a}\right)^{\frac{2}{3}}}{5a^{8}b^{3} + 15a^{7}b^{4}x + 15a^{6}b^{5}x^{2} + 5a^{5}b^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(b*x+a)**(4/3),x)

[Out] -27*a**(29/3)*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b* *5*x**2 + 5*a**5*b**6*x**3) + 27*a**(29/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 63*a**(26/3)*b*x*(1 + b*x/a)**(2/3) /(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81 *a**(26/3)*b*x/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b **6*x**3) - 42*a**(23/3)*b**2*x**2*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a** 7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 81*a**(23/3)*b**2*x**2/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) - 3*a* *(20/3)*b**3*x**3*(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a** 6*b**5*x**2 + 5*a**5*b**6*x**3) + 27*a**(20/3)*b**3*x**3/(5*a**8*b**3 + 15* a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a**5*b**6*x**3) + 3*a**(17/3)*b**4*x**4 *(1 + b*x/a)**(2/3)/(5*a**8*b**3 + 15*a**7*b**4*x + 15*a**6*b**5*x**2 + 5*a **5*b**6*x**3)

3.415 $\int \frac{x}{(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=32

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*a)/(b^2*(a + b*x)^(1/3)) + (3*(a + b*x)^(2/3))/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+bx)^{4/3}}\,dx &= \int \left(-\frac{a}{b(a+bx)^{4/3}}+\frac{1}{b\sqrt[3]{a+bx}}\right)dx\\ &= \frac{3a}{b^2\sqrt[3]{a+bx}}+\frac{3(a+bx)^{2/3}}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.01, size = 23, normalized size = 0.72

$$\frac{3(3a+bx)}{2b^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(a + b*x)^(4/3),x]
```

```
[Out] (3*(3*a + b*x))/(2*b^2*(a + b*x)^(1/3))
```

fricas [A] time = 0.83, size = 29, normalized size = 0.91

$$\frac{3\,(bx+3\,a)(bx+a)^{\frac{2}{3}}}{2\left(b^3x+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/2*(b*x + 3*a)*(b*x + a)^(2/3)/(b^3*x + a*b^2)
```

giac [A] time = 0.93, size = 30, normalized size = 0.94

$$\frac{3\left(\frac{(bx+a)^{\frac{2}{3}}}{b}+\frac{2\,a}{(bx+a)^{\frac{1}{3}}\,b}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 3/2*((b*x + a)^(2/3)/b + 2*a/((b*x + a)^(1/3)*b))/b
```

maple [A] time = 0.00, size = 20, normalized size = 0.62

$$\frac{\frac{3bx}{2}+\frac{9a}{2}}{(bx+a)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(4/3),x)
```

```
[Out] 3/2/(b*x+a)^(1/3)*(b*x+3*a)/b^2
```

maxima [A] time = 1.30, size = 26, normalized size = 0.81

$$\frac{3\,(bx+a)^{\frac{2}{3}}}{2\,b^2}+\frac{3\,a}{(bx+a)^{\frac{1}{3}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 3/2*(b*x + a)^(2/3)/b^2 + 3*a/((b*x + a)^(1/3)*b^2)
```

mupad [B] time = 0.03, size = 20, normalized size = 0.62

$$\frac{9\,a+3\,b\,x}{2\,b^2\,{(a+b\,x)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*x)^(4/3),x)
```

```
[Out] (9*a + 3*b*x)/(2*b^2*(a + b*x)^(1/3))
```

sympy [A] time = 0.72, size = 41, normalized size = 1.28

$$\begin{cases} \frac{9a}{2b^2\sqrt[3]{a+bx}}+\frac{3x}{2b\sqrt[3]{a+bx}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(4/3),x)
```

```
[Out] Piecewise((9*a/(2*b**2*(a + b*x)**(1/3)) + 3*x/(2*b*(a + b*x)**(1/3)), Ne(b
, 0)), (x**2/(2*a**(4/3)), True))
```

3.416 $\int \frac{1}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=14

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4/3),x]
[Out] -3/(b*(a + b*x)^(1/3))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}} dx = -\frac{3}{b\sqrt[3]{a+bx}}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4/3),x]
[Out] -3/(b*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4/3),x]
[Out] -3/(b*(a + b*x)^(1/3))
```

fricas [A] time = 1.05, size = 20, normalized size = 1.43

$$-\frac{3\,(bx+a)^{\frac{2}{3}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)/(b^2*x + a*b)
```

giac [A] time = 0.76, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="giac")
[Out] -3/((b*x + a)^(1/3)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{3}{(bx+a)^{\frac{1}{3}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3),x)
[Out] -3/b/(b*x+a)^(1/3)
```

maxima [A] time = 1.35, size = 12, normalized size = 0.86

$$-\frac{3}{(bx+a)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3),x, algorithm="maxima")
[Out] -3/((b*x + a)^(1/3)*b)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{3}{b\,(a+b\,x)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^(4/3),x)
[Out] -3/(b*(a + b*x)^(1/3))
```

sympy [A] time = 0.07, size = 12, normalized size = 0.86

$$-\frac{3}{b\sqrt[3]{a+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3),x)
[Out] -3/(b*(a + b*x)**(1/3))
```

3.417 $\int \frac{1}{x(a+bx)^{4/3}} \, dx$

Optimal. Leaf size=93

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt
[3]*a^(1/3))])/a^(4/3) - Log[x]/(2*a^(4/3)) + (3*Log[a^(1/3) - (a + b*x)^(1
/3)])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(a+bx)^{4/3}}\,dx &= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{a}\\
&= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx,x,\sqrt[3]{a+bx}\right)}{2a^{4/3}}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^{2/3}+\sqrt[3]{a}\,x+x^2}\,dx,x,\sqrt[3]{a}\right.}{2a}\\
&= \frac{3}{a\sqrt[3]{a+bx}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}\right)}{a^{4/3}}\\
&= \frac{3}{a\sqrt[3]{a+bx}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{2a^{4/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.32

$$
\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};\frac{bx}{a}+1\right)}{a\sqrt[3]{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)^(4/3)),x]
[Out] (3*Hypergeometric2F1[-1/3, 1, 2/3, 1 + (b*x)/a])/(a*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.08, size = 118, normalized size = 1.27

$$
\frac{\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{4/3}}-\frac{\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{2a^{4/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{a^{4/3}}+\frac{3}{a\sqrt[3]{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b*x)^(4/3)),x]
[Out] 3/(a*(a + b*x)^(1/3)) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sq
rt[3]*a^(1/3))])/a^(4/3) + Log[a^(1/3) - (a + b*x)^(1/3)]/a^(4/3) - Log[a^(
2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]/(2*a^(4/3))
```

fricas [A] time = 0.95, size = 285, normalized size = 3.06

$$
\left[\frac{\sqrt{3}\left(abx+a^2\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+\sqrt{3}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+6(bx+a)^{\frac{2}{3}}a}{2\left(a^2bx+a^3\right)}, -\frac{(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2(bx+a)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}\left(abx+a^2\right)\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-6(bx+a)^{\frac{2}{3}}a}{2\left(a^2bx+a^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] [1/2*(sqrt(3)*(a*b*x + a^2)*sqrt(-1/a^(2/3))*log((2*b*x + sqrt(3)*(2*(b*x +
 a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x
+ a)^(1/3)*a^(2/3) + 3*a)/x) - (b*x + a)*a^(2/3)*log((b*x + a)^(2/3) + (b*x
 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x + a)*a^(2/3)*log((b*x + a)^(1/3) -
a^(1/3)) + 6*(b*x + a)^(2/3)*a)/(a^2*b*x + a^3), -1/2*((b*x + a)*a^(2/3)*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*(b*x + a)*a^(2/3
```

```
)*log((b*x + a)^(1/3) - a^(1/3)) - 2*sqrt(3)*(a*b*x + a^2)*arctan(1/3*sqrt(
3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 6*(b*x + a)^(2/3)*a)/(a
^2*b*x + a^3)]
```

giac [A] time = 2.38, size = 89, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
(abs((b*x + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

maple [A] time = 0.01, size = 87, normalized size = 0.94

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{a^{\frac{4}{3}}}+\frac{\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}-\frac{\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(4/3),x)
```

```
[Out] 3/a/(b*x+a)^(1/3)+1/a^(4/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-1/2/a^(4/3)*ln(a^(2/
3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+1/a^(4/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 3.03, size = 88, normalized size = 0.95

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{2\,a^{\frac{4}{3}}}+\frac{\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{a^{\frac{4}{3}}}+\frac{3}{(bx+a)^{\frac{1}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^(4/3) -
 1/2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) + log
((b*x + a)^(1/3) - a^(1/3))/a^(4/3) + 3/((b*x + a)^(1/3)*a)
```

mupad [B] time = 0.06, size = 114, normalized size = 1.23

$$\frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-9\,a^{4/3}\right)}{a^{4/3}}+\frac{3}{a\,(a+b\,x)^{1/3}}+\frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{4/3}}-\frac{\ln\left(9\,a\,(a+b\,x)^{1/3}-\frac{9\,a^{4/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{4}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*x)^(4/3)),x)
```

```
[Out] log(9*a*(a + b*x)^(1/3) - 9*a^(4/3))/a^(4/3) + 3/(a*(a + b*x)^(1/3)) + (log
(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/
```

(2*a^(4/3)) - (log(9*a*(a + b*x)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i + 1)^2)/4)*
(3^(1/2)*1i + 1))/(2*a^(4/3))

sympy [C] time = 2.21, size = 184, normalized size = 1.98

$$-\frac{\Gamma\left(-\frac{1}{3}\right)}{a\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{2i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}-\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{b}+x}\,e^{\frac{4i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(4/3),x)
```

```
[Out] -gamma(-1/3)/(a*b**(1/3)*(a/b + x)**(1/3)*gamma(2/3)) - log(1 - b**(1/3)*(a
/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3)) - exp(2*I*pi/3
)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/
3)/(3*a**(4/3)*gamma(2/3)) - exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/
3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(3*a**(4/3)*gamma(2/3))
```

3.418 $\int \frac{1}{x^2(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=113

$$\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4b}{a^2\sqrt[3]{a+bx}}-\frac{1}{ax\sqrt[3]{a+bx}}$$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{4(a+bx)^{2/3}}{a^2x}+\frac{2b\log(x)}{3a^{7/3}}-\frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}-\frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}+\frac{3}{ax\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x*(a + b*x)^(1/3)) - (4*(a + b*x)^(2/3))/(a^2*x) - (4*b*ArcTan[(a^(1/3
) + 2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b*Log[x])
/(3*a^(7/3)) - (2*b*Log[a^(1/3) - (a + b*x)^(1/3)])/a^(7/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^2(a+bx)^{4/3}}\,dx = \frac{3}{ax\sqrt[3]{a+bx}} + \frac{4\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{a}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{(4b)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{3a^2}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} + \frac{(2b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a+bx}\right)}{a^{7/3}} - \frac{(2b)\,\mathrm{Subst}}{}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}} + \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx\right.}{a^{7/3}}$$

$$= \frac{3}{ax\sqrt[3]{a+bx}} - \frac{4(a+bx)^{2/3}}{a^2x} - \frac{4b\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2b\log(x)}{3a^{7/3}} - \frac{2b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{a^{7/3}}$$

Mathematica [C] time = 0.01, size = 31, normalized size = 0.27

$$-\frac{3b\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};\frac{bx}{a}+1\right)}{a^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^(4/3)),x]
[Out] (-3*b*Hypergeometric2F1[-1/3, 2, 2/3, 1 + (b*x)/a])/(a^2*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.17, size = 138, normalized size = 1.22

$$-\frac{4b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{7/3}} + \frac{2b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{3a^{7/3}} - \frac{4b\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{3a-4(a+bx)}{a^2x\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(a + b*x)^(4/3)),x]
[Out] (3*a - 4*(a + b*x))/(a^2*x*(a + b*x)^(1/3)) - (4*b*ArcTan[1/Sqrt[3] + (2*(a
 + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) - (4*b*Log[a^(1/3) - (
a + b*x)^(1/3)])/(3*a^(7/3)) + (2*b*Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) +
 (a + b*x)^(2/3)])/(3*a^(7/3))
```

fricas [B] time = 0.83, size = 407, normalized size = 3.60

$$\left[\frac{6\sqrt{\frac{1}{3}}\left(ab^2x^2+a^2bx\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2bx-3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3(bx+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3a}{x}\right)+2\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)-4\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)-3\left(4abx+a^2\right)(bx+a)^{\frac{2}{3}}}{3\left(a^3bx^2+a^4x\right)}, -\frac{12\sqrt{\frac{1}{3}}\left(ab^2x^2+a^2bx\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(-a)^{\frac{1}{3}}+(-a)^{\frac{2}{3}}\right)+4\left(b^2x^2+abx\right)(-a)^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}+(-a)^{\frac{1}{3}}\right)+3\left(4abx+a^2\right)(bx+a)^{\frac{2}{3}}}{3\left(a^3bx^2+a^4x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="fricas")
[Out] [1/3*(6*sqrt(1/3)*(a*b^2*x^2 + a^2*b*x)*sqrt((-a)^(1/3)/a)*log((2*b*x - 3*s
qrt(1/3)*(2*(b*x + a)^(2/3)*(-a)^(2/3) - (b*x + a)^(1/3)*a + (-a)^(1/3)*a)*
sqrt((-a)^(1/3)/a) - 3*(b*x + a)^(1/3)*(-a)^(2/3) + 3*a)/x) + 2*(b^2*x^2 +
```

```
a*b*x)*(-a)^(2/3)*log((b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(
2/3)) - 4*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) -
3*(4*a*b*x + a^2)*(b*x + a)^(2/3))/(a^3*b*x^2 + a^4*x), -1/3*(12*sqrt(1/3)*
(a*b^2*x^2 + a^2*b*x)*sqrt(-(-a)^(1/3)/a)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - 2*(b^2*x^2 + a*b*x)*(-a)^(2/3)*log(
(b*x + a)^(2/3) - (b*x + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^2 + a
*b*x)*(-a)^(2/3)*log((b*x + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x + a^2)*(b*x
 + a)^(2/3))/(a^3*b*x^2 + a^4*x)]
```

giac [A] time = 2.40, size = 120, normalized size = 1.06

$$-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}+\frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,b\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx+a)b-3\,ab}{\left((bx+a)^{\frac{4}{3}}-(bx+a)^{\frac{1}{3}}a\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7
/3) - 4/3*b*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(7/3) - (4*(b*x + a)*b -
3*a*b)/(((b*x + a)^(4/3) - (b*x + a)^(1/3)*a)*a^2)
```

maple [A] time = 0.01, size = 108, normalized size = 0.96

$$-\frac{4\sqrt{3}\;b\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{3a^{\frac{7}{3}}}-\frac{4b\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{3a^{\frac{7}{3}}}+\frac{2b\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}\,a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{3a^{\frac{7}{3}}}-\frac{3b}{(bx+a)^{\frac{1}{3}}\,a^{2}}-\frac{(bx+a)^{\frac{2}{3}}}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(4/3),x)
```

```
[Out] -3*b/a^2/(b*x+a)^(1/3)-1/a^2*(b*x+a)^(2/3)/x-4/3*b/a^(7/3)*ln(-a^(1/3)+(b*x
+a)^(1/3))+2/3*b/a^(7/3)*ln(a^(2/3)+(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))-4/
3*b/a^(7/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.99, size = 122, normalized size = 1.08

$$-\frac{4\sqrt{3}\,b\arctan\left(\frac{\sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,(bx+a)b-3\,ab}{(bx+a)^{\frac{4}{3}}a^{2}-(bx+a)^{\frac{1}{3}}a^{3}}+\frac{2\,b\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{3\,a^{\frac{7}{3}}}-\frac{4\,b\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(7/3) - (4*(b*x + a)*b - 3*a*b)/((b*x + a)^(4/3)*a^2 - (b*x + a)^(1/3)*a^3)
 + 2/3*b*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(7/3) -
 4/3*b*log((b*x + a)^(1/3) - a^(1/3))/a^(7/3)
```

mupad [B] time = 0.07, size = 173, normalized size = 1.53

$$-\frac{\frac{3\,b}{a}-\frac{4\,b\,(a+b\,x)}{a^2}}{a\,(a+b\,x)^{1/3}-(a+b\,x)^{4/3}}+\frac{\ln\left(a^{7/3}\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\left(2\,b-\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}+\frac{\ln\left(a^{7/3}\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)\left(2\,b+\sqrt{3}\,b\,2\mathrm{i}\right)}{3\,a^{7/3}}-\frac{4\,b\,\ln\left(16\,a^{7/3}\,b^2-16\,a^2\,b^2\,(a+b\,x)^{1/3}\right)}{3\,a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*x)^(4/3)),x)
```

```
[Out] (log(a^(7/3)*(2*b - 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a + b*x)^(1/3))*(2*b - 3^
(1/2)*b*2i))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x))/a^2)/(a*(a + b*x)^(1/
3) - (a + b*x)^(4/3)) + (log(a^(7/3)*(2*b + 3^(1/2)*b*2i)^2 - 16*a^2*b^2*(a
 + b*x)^(1/3))*(2*b + 3^(1/2)*b*2i))/(3*a^(7/3)) - (4*b*log(16*a^(7/3)*b^2
- 16*a^2*b^2*(a + b*x)^(1/3)))/(3*a^(7/3))
```

sympy [C] time = 2.52, size = 857, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(4/3),x)
```

```
[Out] -9*a**(4/3)*b**(2/3)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*g
amma(2/3)) + 12*a**(1/3)*b**(5/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-9*a
**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)
**(4/3)*exp(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*
pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a
/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp
(2*I*pi/3)*gamma(2/3)) - 4*a*b*(a/b + x)**(1/3)*log(1 - b**(1/3)*(a/b + x)*
*(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)
*gamma(2/3)) + 4*b**2*(a/b + x)**(4/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b
+ x)**(1/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi
/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3)) +
4*b**2*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*ex
p_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/
3)) + 4*b**2*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-9*a**(10/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)
*gamma(2/3) + 9*a**(7/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3))
```

3.419 $\int \frac{1}{x^3(a+bx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{14b^2}{3a^3\sqrt[3]{a+bx}}+\frac{7b}{6a^2x\sqrt[3]{a+bx}}-\frac{1}{2ax^2\sqrt[3]{a+bx}}$$

Rubi [A] time = 0.06, antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 55, 617, 204, 31}

$$-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}}+\frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}+\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}-\frac{7(a+bx)^{2/3}}{2a^2x^2}+\frac{14b(a+bx)^{2/3}}{3a^3x}+\frac{3}{ax^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] 3/(a*x^2*(a + b*x)^(1/3)) - (7*(a + b*x)^(2/3))/(2*a^2*x^2) + (14*b*(a + b*
x)^(2/3))/(3*a^3*x) + (14*b^2*ArcTan[(a^(1/3) + 2*(a + b*x)^(1/3))/(Sqrt[3]
*a^(1/3))])/(3*Sqrt[3]*a^(10/3)) - (7*b^2*Log[x])/(9*a^(10/3)) + (7*b^2*Log
[a^(1/3) - (a + b*x)^(1/3)])/(3*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3(a+bx)^{4/3}}\,dx &= \frac{3}{ax^2\sqrt[3]{a+bx}} + \frac{7\int \frac{1}{x^3\sqrt[3]{a+bx}}\,dx}{a} \\
&= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} - \frac{(14b)\int \frac{1}{x^2\sqrt[3]{a+bx}}\,dx}{3a^2} \\
&= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{\left(14b^2\right)\int \frac{1}{x\sqrt[3]{a+bx}}\,dx}{9a^3} \\
&= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} - \frac{\left(7b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}-x}\,dx, x, \sqrt[3]{a \ldots}\right.}{3a^{10/3}} \\
&= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{3a^{10/3}} - \frac{(1\ldots}{\ldots} \\
&= \frac{3}{ax^2\sqrt[3]{a+bx}} - \frac{7(a+bx)^{2/3}}{2a^2x^2} + \frac{14b(a+bx)^{2/3}}{3a^3x} + \frac{14b^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a+bx}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} - \frac{7b^2\log(x)}{9a^{10/3}} + \frac{7b^2\,\mathrm{l}\ldots}{\ldots}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 33, normalized size = 0.22

$$\frac{3b^2\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};\frac{bx}{a}+1\right)}{a^3\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 3, 2/3, 1 + (b*x)/a])/(a^3*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.26, size = 161, normalized size = 1.08

$$\frac{14b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx}\right)}{9a^{10/3}} - \frac{7b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx}+(a+bx)^{2/3}\right)}{9a^{10/3}} + \frac{14b^2\tan^{-1}\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{a}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{18a^2-49a(a+bx)+28(a+bx)^2}{6a^3x^2\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*(a + b*x)^(4/3)),x]
```

```
[Out] (18*a^2 - 49*a*(a + b*x) + 28*(a + b*x)^2)/(6*a^3*x^2*(a + b*x)^(1/3)) + (1
4*b^2*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]
*a^(10/3)) + (14*b^2*Log[a^(1/3) - (a + b*x)^(1/3)])/(9*a^(10/3)) - (7*b^2*
Log[a^(2/3) + a^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)])/(9*a^(10/3))
```

fricas [A] time = 1.05, size = 407, normalized size = 2.73

$$\left[\frac{42\sqrt{\frac{1}{3}}\left(ab^3x^3+a^2b^2x^2\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2bx+3\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3(bx+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3a}{x}\right)-14\left(b^3x^3+ab^2x^2\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+28\left(b^3x^3+ab^2x^2\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+3\left(28ab^2x^2+7a^2bx-3a^3\right)(bx+a)^{\frac{2}{3}}}{18\left(a^4bx^3+a^5x^2\right)}, -\frac{14\left(b^3x^3+ab^2x^2\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\left(b^3x^3+ab^2x^2\right)a^{\frac{2}{3}}\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{84\sqrt{\frac{1}{3}}\left(ab^3x^3+a^2b^2x^2\right)\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}}-3\left(28ab^2x^2+7a^2bx-3a^3\right)(bx+a)^{\frac{2}{3}}}{18\left(a^4bx^3+a^5x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/18*(42*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*sqrt(-1/a^(2/3))*log((2*b*x +
 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*a^(2/3) - (b*x + a)^(1/3)*a - a^(4/3))*sqrt
(-1/a^(2/3)) - 3*(b*x + a)^(1/3)*a^(2/3) + 3*a)/x) - 14*(b^3*x^3 + a*b^2*x^
2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3)) + 28*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3)) + 3*(28*a*b^2*x^
2 + 7*a^2*b*x - 3*a^3)*(b*x + a)^(2/3))/(a^4*b*x^3 + a^5*x^2), -1/18*(14*(b
^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) +
 a^(2/3)) - 28*(b^3*x^3 + a*b^2*x^2)*a^(2/3)*log((b*x + a)^(1/3) - a^(1/3))
 - 84*sqrt(1/3)*(a*b^3*x^3 + a^2*b^2*x^2)*arctan(sqrt(1/3)*(2*(b*x + a)^(1/
3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(28*a*b^2*x^2 + 7*a^2*b*x - 3*a^3)*(b*x
+ a)^(2/3))/(a^4*b*x^3 + a^5*x^2)]
```

giac [A] time = 2.59, size = 140, normalized size = 0.94

$$\frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\log\left(\left|(bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9a^{\frac{10}{3}}}+\frac{3b^2}{(bx+a)^{\frac{1}{3}}a^3}+\frac{10(bx+a)^{\frac{5}{3}}b^2-13(bx+a)^{\frac{2}{3}}ab^2}{6a^3b^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) - 7/9*b^2*log((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 14/9*b^2*log(abs((b*x + a)^(1/3) - a^(1/3)))/a^(10/3) + 3*b^2/(
(b*x + a)^(1/3)*a^3) + 1/6*(10*(b*x + a)^(5/3)*b^2 - 13*(b*x + a)^(2/3)*a*b
^2)/(a^3*b^2*x^2)
```

maple [A] time = 0.01, size = 131, normalized size = 0.88

$$\frac{14\sqrt{3}\,b^2\arctan\left(\frac{\sqrt{3}\left(\frac{2(bx+a)^{\frac{1}{3}}}{a^{\frac{1}{3}}}+1\right)}{3}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\ln\left(-a^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}-\frac{7b^2\ln\left(a^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{3b^2}{(bx+a)^{\frac{1}{3}}\,a^3}-\frac{13(bx+a)^{\frac{2}{3}}}{6a^2x^2}+\frac{5(bx+a)^{\frac{5}{3}}}{3a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)^(4/3),x)
```

```
[Out] 3*b^2/a^3/(b*x+a)^(1/3)+5/3/a^3/x^2*(b*x+a)^(5/3)-13/6/a^2/x^2*(b*x+a)^(2/3
)+14/9*b^2/a^(10/3)*ln(-a^(1/3)+(b*x+a)^(1/3))-7/9*b^2/a^(10/3)*ln(a^(2/3)+
(b*x+a)^(1/3)*a^(1/3)+(b*x+a)^(2/3))+14/9*b^2/a^(10/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2*(b*x+a)^(1/3)/a^(1/3)+1))
```

maxima [A] time = 2.96, size = 158, normalized size = 1.06

$$\frac{14\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2(bx+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}}+\frac{28(bx+a)^2b^2-49(bx+a)ab^2+18a^2b^2}{6\left((bx+a)^{\frac{7}{3}}a^3-2(bx+a)^{\frac{4}{3}}a^4+(bx+a)^{\frac{1}{3}}a^5\right)}-\frac{7b^2\log\left((bx+a)^{\frac{2}{3}}+(bx+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{9a^{\frac{10}{3}}}+\frac{14b^2\log\left((bx+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] 14/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(10/3) + 1/6*(28*(b*x + a)^2*b^2 - 49*(b*x + a)*a*b^2 + 18*a^2*b^2)/((b*x
 + a)^(7/3)*a^3 - 2*(b*x + a)^(4/3)*a^4 + (b*x + a)^(1/3)*a^5) - 7/9*b^2*lo
g((b*x + a)^(2/3) + (b*x + a)^(1/3)*a^(1/3) + a^(2/3))/a^(10/3) + 14/9*b^2*
log((b*x + a)^(1/3) - a^(1/3))/a^(10/3)
```

mupad [B] time = 0.13, size = 221, normalized size = 1.48

$$\frac{\frac{3b^2}{a}+\frac{14b^2(a+bx)^2}{3a^3}-\frac{49b^2(a+bx)}{6a^2}}{(a+bx)^{7/3}-2a(a+bx)^{4/3}+a^2(a+bx)^{1/3}}+\frac{\ln\left(588a^3b^4(a+bx)^{1/3}-3a^{10/3}\left(-7b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\left(-7b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9a^{10/3}}-\frac{\ln\left(588a^3b^4(a+bx)^{1/3}-3a^{10/3}\left(7b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)^2\right)\left(7b^2+\sqrt{3}\,b^2\,7\mathrm{i}\right)}{9a^{10/3}}+\frac{14b^2\,\ln\left(588a^3b^4(a+bx)^{1/3}-588a^{10/3}b^4\right)}{9a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*x)^(4/3)),x)
```

```
[Out] ((3*b^2)/a + (14*b^2*(a + b*x)^2)/(3*a^3) - (49*b^2*(a + b*x))/(6*a^2))/((a
 + b*x)^(7/3) - 2*a*(a + b*x)^(4/3) + a^2*(a + b*x)^(1/3)) + (log(588*a^3*b
^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^(1/2)*b^2*7i - 7*b^2)^2)*(3^(1/2)*b^2*7i
 - 7*b^2))/(9*a^(10/3)) - (log(588*a^3*b^4*(a + b*x)^(1/3) - 3*a^(10/3)*(3^
(1/2)*b^2*7i + 7*b^2)^2)*(3^(1/2)*b^2*7i + 7*b^2))/(9*a^(10/3)) + (14*b^2*l
og(588*a^3*b^4*(a + b*x)^(1/3) - 588*a^(10/3)*b^4))/(9*a^(10/3))
```

sympy [C] time = 3.19, size = 2793, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)**(4/3),x)
```

```
[Out] 54*a**(13/3)*b**(5/3)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(
1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi
/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/
3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 201*a*
*(10/3)*b**(8/3)*(a/b + x)*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)*(a/b +
x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2
*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gam
ma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 2
31*a**(7/3)*b**(11/3)*(a/b + x)**2*exp(2*I*pi/3)*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) - 84*a**(4/3)*b**(14/3)*(a/b + x)**3*exp(2*I*pi/3)*gamma(-1/3)/(-54*a*
*(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*ex
p(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*
gamma(2/3)) + 28*a**4*b**2*(a/b + x)**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*
(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(
2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma
(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a
**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a
/b + x)**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2
*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3
)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) -
162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)
*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 28*a**4*b**2*(a/b + x)*
*(1/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamm
a(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(
19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
 + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)
*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*b**3*(a/b + x)**(4/3)*exp(2*I*pi/3)*lo
g(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b +
 x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(
2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*ga
mma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) -
84*a**3*b**3*(a/b + x)**(4/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1
/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/
3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3
)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3)
 + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 84*a**3*
b**3*(a/b + x)**(4/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(4*I*pi/3)
/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(
2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(
```

```
16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a
/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma(-1/3)/(-54*a
**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b +
 x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*e
xp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)
*gamma(2/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*exp(-2*I*pi/3)*log(1 - b**(1/3
)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)
*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/
3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*p
i/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2
/3)) + 84*a**2*b**4*(a/b + x)**(7/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)*exp_
polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2
*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(
2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a*
*(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b +
 x)**(10/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*(a/b + x)**(1/3)/a**(1/3))*gamma
(-1/3)/(-54*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(1
9/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b
+ x)**(7/3)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*
exp(2*I*pi/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*exp(-2*I*pi/3)*log(
1 - b**(1/3)*(a/b + x)**(1/3)*exp_polar(2*I*pi/3)/a**(1/3))*gamma(-1/3)/(-5
4*a**(22/3)*(a/b + x)**(1/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/
b + x)**(4/3)*exp(2*I*pi/3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3
)*exp(2*I*pi/3)*gamma(2/3) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi
/3)*gamma(2/3)) - 28*a*b**5*(a/b + x)**(10/3)*log(1 - b**(1/3)*(a/b + x)**(
1/3)*exp_polar(4*I*pi/3)/a**(1/3))*gamma(-1/3)/(-54*a**(22/3)*(a/b + x)**(1
/3)*exp(2*I*pi/3)*gamma(2/3) + 162*a**(19/3)*b*(a/b + x)**(4/3)*exp(2*I*pi/
3)*gamma(2/3) - 162*a**(16/3)*b**2*(a/b + x)**(7/3)*exp(2*I*pi/3)*gamma(2/3
) + 54*a**(13/3)*b**3*(a/b + x)**(10/3)*exp(2*I*pi/3)*gamma(2/3))
```

3.420 $\int \frac{1}{x \sqrt[3]{a^3+b^3x}} \, dx$

Optimal. Leaf size=71

$$\frac{3 \log \left(a-\sqrt[3]{a^3+b^3 x}\right)}{2 a}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{2 \sqrt[3]{a^3+b^3 x}+a}{\sqrt{3} a}\right)}{a}-\frac{\log (x)}{2 a}$$

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$\frac{3 \log \left(a-\sqrt[3]{a^3+b^3 x}\right)}{2 a}+\frac{\sqrt{3} \tan ^{-1}\left(\frac{2 \sqrt[3]{a^3+b^3 x}+a}{\sqrt{3} a}\right)}{a}-\frac{\log (x)}{2 a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3+b^3x}}\,dx = -\frac{\log(x)}{2a}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3+b^3x}\right)-\frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3+b^3x}\right)}{2a}$$
$$= -\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}-\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a}$$
$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a}$$

Mathematica [A] time = 0.02, size = 66, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)-\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 + b^3*x)^(1/3)])/(2*a)
```

IntegrateAlgebraic [A] time = 0.05, size = 102, normalized size = 1.44

$$\frac{\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a}-\frac{\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a + Log[a
 - (a^3 + b^3*x)^(1/3)]/a - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)
^(2/3)]/(2*a)
```

fricas [A] time = 0.97, size = 88, normalized size = 1.24

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) -
log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) + 2*log(-a + (b^3*x
+ a^3)^(1/3)))/a
```

giac [A] time = 1.03, size = 87, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(abs(-a + (b^3*x + a^3
)^(1/3)))/a
```

maple [A] time = 0.01, size = 87, normalized size = 1.23

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}-\frac{\ln\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln((b^3*x+a^3)^(1/3)-a)-1/2/a*ln((b^3*x+a^3)^(2/3)+(b^3*x+a^3)^(1/3)*a+
a^2)+arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

maxima [A] time = 3.09, size = 86, normalized size = 1.21

$$\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}-\frac{\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}+\frac{\log\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2 +
 (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a + log(-a + (b^3*x + a^3)^(1
/3))/a
```

mupad [B] time = 0.10, size = 105, normalized size = 1.48

$$\frac{\ln\left(9\left(a^{3}+x\,b^{3}\right)^{1/3}-9\,a\right)}{a}+\frac{\ln\left(9\left(a^{3}+x\,b^{3}\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}-\frac{\ln\left(9\left(a^{3}+x\,b^{3}\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(1/3)),x)
```

```
[Out] log(9*(b^3*x + a^3)^(1/3) - 9*a)/a + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(b^3*x + a^3)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

sympy [C] time = 2.13, size = 138, normalized size = 1.94

$$\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(1/3),x)
```

```
[Out] exp(I*pi/3)*log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamm
a(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(2*I
*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

3.421 $\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx$

Optimal. Leaf size=73

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {55, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a}-\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a - Log[x]/(2*a)
+ (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx = -\frac{\log(x)}{2a}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx,x,\sqrt[3]{a^3-b^3x}\right)-\frac{3\operatorname{Subst}\left(\int \frac{1}{a-x}\,dx,x,\sqrt[3]{a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}-\frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a}-\frac{\log(x)}{2a}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a}$$

Mathematica [A] time = 0.02, size = 68, normalized size = 0.93

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)-\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - Log[x] + 3*Log
[a - (a^3 - b^3*x)^(1/3)])/(2*a)
```

IntegrateAlgebraic [A] time = 0.05, size = 106, normalized size = 1.45

$$\frac{\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{a}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a}-\frac{\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a + Log[a
 - (a^3 - b^3*x)^(1/3)]/a - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)
^(2/3)]/(2*a)
```

fricas [A] time = 0.87, size = 92, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)+2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a) -
 log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) + 2*log(-a + (-b^
3*x + a^3)^(1/3)))/a
```

giac [A] time = 0.77, size = 91, normalized size = 1.25

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}-\frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a}+\frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(abs(-a + (-b^3*x +
 a^3)^(1/3)))/a
```

maple [A] time = 0.00, size = 91, normalized size = 1.25

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}+\frac{\ln\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}-\frac{\ln\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(1/3),x)
```

```
[Out] 1/a*ln((-b^3*x+a^3)^(1/3)-a)-1/2/a*ln((-b^3*x+a^3)^(2/3)+(-b^3*x+a^3)^(1/3)
*a+a^2)+arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))/a*3^(1/2))*3^(1/2)/a
```

maxima [A] time = 2.99, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a - 1/2*log(a^2
+ (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a + log(-a + (-b^3*x + a^3
)^(1/3))/a
```

mupad [B] time = 0.13, size = 108, normalized size = 1.48

$$\frac{\ln\left(9\left(a^{3}-b^{3}\,x\right)^{1/3}-9\,a\right)}{a}+\frac{\ln\left(9\left(a^{3}-b^{3}\,x\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}-\frac{\ln\left(9\left(a^{3}-b^{3}\,x\right)^{1/3}-\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(1/3)),x)
```

```
[Out] log(9*(a^3 - b^3*x)^(1/3) - 9*a)/a + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3^(
1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*a) - (log(9*(a^3 - b^3*x)^(1/3) - (
9*a*(3^(1/2)*1i + 1)^2)/4)*(3^(1/2)*1i + 1))/(2*a)
```

sympy [C] time = 1.89, size = 136, normalized size = 1.86

$$-\frac{e^{-\frac{2i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(1/3),x)
```

```
[Out] -exp(-2*I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*g
amma(-1/3)/(3*a*gamma(2/3)) + exp(-I*pi/3)*log(-a*exp_polar(I*pi)/(b*(-a**3
/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - log(-a*exp_polar(5*I
*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

3.422 $\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx$

Optimal. Leaf size=74

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3+b^3x}}\,dx = \frac{\log(x)}{2a}+\frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)-\frac{3\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3+\right.}{2a}$$

$$= \frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a}+\frac{\log(x)}{2a}-\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.55

$$\frac{3\left(b^3x-a^3\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};1-\frac{b^3x}{a^3}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 + b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 - (b^3*x)/a^3])/(2
*a^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 111, normalized size = 1.50

$$-\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a}+\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 + b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a) - L
og[a + (-a^3 + b^3*x)^(1/3)]/a + Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 +
 b^3*x)^(2/3)]/(2*a)
```

fricas [A] time = 0.83, size = 93, normalized size = 1.26

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) +
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) - 2*log(a + (b^3*x
- a^3)^(1/3)))/a
```

giac [A] time = 1.07, size = 95, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a}+\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a}-\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(abs(a + (b^3*x - a^3
)^(1/3)))/a
```

maple [A] time = 0.01, size = 97, normalized size = 1.31

$$\frac{\sqrt{3}\,\arctan\left(\frac{\left(-a+2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(a^{2}-\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(1/3),x)
```

```
[Out] 1/2/a*ln((b^3*x-a^3)^(2/3)-(b^3*x-a^3)^(1/3)*a+a^2)+1/a*3^(1/2)*arctan(1/3*
(2*(b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)-ln(a+(b^3*x-a^3)^(1/3))/a
```

maxima [A] time = 2.90, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}+\frac{\log\left(a^{2}-\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}-\frac{\log\left(a+\left(b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
- (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a - log(a + (b^3*x - a^3)^(1
/3))/a
```

mupad [B] time = 0.11, size = 112, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\,\left(b^{3}\,x-a^{3}\right)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1))/
(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(b^3*x - a^3)^(1/3))*(3^(1/2)*1
i - 1))/(2*a) - log(9*a + 9*(b^3*x - a^3)^(1/3))/a
```

sympy [C] time = 1.82, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{\log\left(-\frac{ae^{i\pi}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{5i\pi}{3}}}{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(1/3),x)
```

```
[Out] -exp(-I*pi/3)*log(-a*exp_polar(I*pi/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gam
ma(-1/3)/(3*a*gamma(2/3)) + log(-a*exp_polar(I*pi)/(b*(-a**3/b**3 + x)**(1/
3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(5*I*pi
/3)/(b*(-a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

3.423 $\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx$

Optimal. Leaf size=76

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a}+\frac{\log(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) + Log[x]/(2
*a) - (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\sqrt[3]{-a^3-b^3x}}\,dx = \frac{\log(x)}{2a} + \frac{3}{2}\operatorname{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3-b^3x}\right) - \frac{3\operatorname{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a}$$

$$= \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a} + \frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a} + \frac{\log(x)}{2a} - \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a}$$

Mathematica [C] time = 0.01, size = 41, normalized size = 0.54

$$\frac{3\left(-a^3-b^3x\right)^{2/3} {}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{xb^3}{a^3}+1\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] (3*(-a^3 - b^3*x)^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, 1 + (b^3*x)/a^3])/(2
*a^3)
```

IntegrateAlgebraic [A] time = 0.08, size = 115, normalized size = 1.51

$$-\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a} + \frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 - b^3*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a) - L
og[a + (-a^3 - b^3*x)^(1/3)]/a + Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 -
 b^3*x)^(2/3)]/(2*a)
```

fricas [A] time = 0.87, size = 97, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)-2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
+ log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) - 2*log(a + (-b^
3*x - a^3)^(1/3)))/a
```

giac [A] time = 1.00, size = 99, normalized size = 1.30

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a} + \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a} - \frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(abs(a + (-b^3*x -
 a^3)^(1/3)))/a
```

maple [A] time = 0.00, size = 101, normalized size = 1.33

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a}-\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}+\frac{\ln\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(1/3),x)
```

```
[Out] 1/2/a*ln((-b^3*x-a^3)^(2/3)-(-b^3*x-a^3)^(1/3)*a+a^2)+1/a*3^(1/2)*arctan(1/
3*(2*(-b^3*x-a^3)^(1/3)-a)*3^(1/2)/a)-ln(a+(-b^3*x-a^3)^(1/3))/a
```

maxima [A] time = 2.92, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\ \arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a}+\frac{\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2\,a}-\frac{\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(1/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a + 1/2*log(a^2
 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a - log(a + (-b^3*x - a^3
)^(1/3))/a
```

mupad [B] time = 0.07, size = 115, normalized size = 1.51

$$-\frac{\ln\left(9\,a+9\left(-a^{3}-x\,b^{3}\right)^{1/3}\right)}{a}-\frac{\ln\left(\frac{9\,a\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\left(-a^{3}-x\,b^{3}\right)^{1/3}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}+\frac{\ln\left(\frac{9\,a\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}+9\left(-a^{3}-x\,b^{3}\right)^{1/3}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(1/3)),x)
```

```
[Out] (log((9*a*(3^(1/2)*1i + 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/2)*1i + 1)
)/(2*a) - (log((9*a*(3^(1/2)*1i - 1)^2)/4 + 9*(- b^3*x - a^3)^(1/3))*(3^(1/
2)*1i - 1))/(2*a) - log(9*a + 9*(- b^3*x - a^3)^(1/3))/a
```

sympy [C] time = 1.83, size = 139, normalized size = 1.83

$$\frac{\log\left(-\frac{ae^{\frac{2i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(-\frac{ae^{\frac{4i\pi}{3}}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(-\frac{ae^{2i\pi}}{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}+1\right)\Gamma\left(-\frac{1}{3}\right)}{3a\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(1/3),x)
```

```
[Out] log(-a*exp_polar(2*I*pi/3)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a
*gamma(2/3)) - exp(I*pi/3)*log(-a*exp_polar(4*I*pi/3)/(b*(a**3/b**3 + x)**(
1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3)) + exp(2*I*pi/3)*log(-a*exp_polar(2*
I*pi)/(b*(a**3/b**3 + x)**(1/3)) + 1)*gamma(-1/3)/(3*a*gamma(2/3))
```

3.424 $\int \frac{1}{x(a^3+b^3x)^{2/3}}\,dx$

Optimal. Leaf size=72

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a^2}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 1.32

$$-\frac{-2\log\left(a-\sqrt[3]{a^3+b^3x}\right)+2\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}+a}{\sqrt{3}a}\right)+\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 + b^3*x)^(1/3)] + Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 + b^3*x)^(2/
3)])/a^2
```

IntegrateAlgebraic [A] time = 0.05, size = 103, normalized size = 1.43

$$\frac{\log\left(a-\sqrt[3]{a^3+b^3x}\right)}{a^2} - \frac{\log\left(a\sqrt[3]{a^3+b^3x}+\left(a^3+b^3x\right)^{2/3}+a^2\right)}{2a^2} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{2\sqrt[3]{a^3+b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
Log[a - (a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 + a*(a^3 + b^3*x)^(1/3) + (a^3 +
 b^3*x)^(2/3)]/(2*a^2)
```

fricas [A] time = 0.76, size = 86, normalized size = 1.19

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b^3*x + a^3)^(1/3))/a) +
 log(a^2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3)) - 2*log(-a + (b^3*x
 + a^3)^(1/3)))/a^2
```

giac [A] time = 1.00, size = 88, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2+\left(b^3x+a^3\right)^{\frac{1}{3}}a+\left(b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|-a+\left(b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (b^3*x
+ a^3)^(1/3)))/a^2
```

maple [A] time = 0.01, size = 88, normalized size = 1.22

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}-\frac{\ln\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x+a^3)^(2/3),x)
```

```
[Out] 1/a^2*ln(-a+(b^3*x+a^3)^(1/3))-1/2/a^2*ln(a^2+(b^3*x+a^3)^(1/3)*a+(b^3*x+a^
3)^(2/3))-arctan(1/3*(a+2*(b^3*x+a^3)^(1/3))*3^(1/2)/a)*3^(1/2)/a^2
```

maxima [A] time = 2.88, size = 87, normalized size = 1.21

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(a+2\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(-a+\left(b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 + (b^3*x + a^3)^(1/3)*a + (b^3*x + a^3)^(2/3))/a^2 + log(-a + (b^3*x + a^
3)^(1/3))/a^2
```

mupad [B] time = 0.14, size = 101, normalized size = 1.40

$$\frac{\ln\left(9\,a-9\,\left(a^{3}+x\,b^{3}\right)^{1/3}\right)}{a^{2}}+\frac{\ln\left(9\,\left(a^{3}+x\,b^{3}\right)^{1/3}-\frac{9\,a\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2}}-\frac{\ln\left(9\,\left(a^{3}+x\,b^{3}\right)^{1/3}+\frac{9\,a\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x + a^3)^(2/3)),x)
```

```
[Out] log(9*a - 9*(b^3*x + a^3)^(1/3))/a^2 + (log(9*(b^3*x + a^3)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x + a^3)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

sympy [C] time = 1.86, size = 134, normalized size = 1.86

$$\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}\,e^{\frac{2i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}\,e^{\frac{4i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(-2
*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/a)*gamma(1/3)
/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_p
olar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

3.425 $\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=74

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {57, 617, 204, 31}

$$\frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/(
2*a^2) + (3*Log[a - (a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3-b^3x}\right)}{2a^2} - \frac{3\,\text{Subst}\left(\int \frac{1}{a^2+ax+x^2}\,dx, x, \sqrt[3]{a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3-b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 1.34

$$-\frac{-2\log\left(a-\sqrt[3]{a^3-b^3x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}+a}{\sqrt{3}a}\right)+\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)] - 2*Log[a -
 (a^3 - b^3*x)^(1/3)] + Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 - b^3*x)^(2/
3)])/a^2
```

IntegrateAlgebraic [A] time = 0.05, size = 107, normalized size = 1.45

$$\frac{\log\left(a-\sqrt[3]{a^3-b^3x}\right)}{a^2} - \frac{\log\left(a\sqrt[3]{a^3-b^3x}+\left(a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2} - \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3x}}{\sqrt{3}a}+\frac{1}{\sqrt{3}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
Log[a - (a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 + a*(a^3 - b^3*x)^(1/3) + (a^3 -
 b^3*x)^(2/3)]/(2*a^2)
```

fricas [A] time = 0.96, size = 90, normalized size = 1.22

$$-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}\left(-b^3x+a^3\right)^{\frac{1}{3}}}{3a}\right)+\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)-2\log\left(-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(-b^3*x + a^3)^(1/3))/a)
+ log(a^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3)) - 2*log(-a + (-b
^3*x + a^3)^(1/3)))/a^2
```

giac [A] time = 1.09, size = 92, normalized size = 1.24

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3x+a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2+\left(-b^3x+a^3\right)^{\frac{1}{3}}a+\left(-b^3x+a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|-a+\left(-b^3x+a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="giac")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(abs(-a + (-b^
3*x + a^3)^(1/3)))/a^2
```

maple [A] time = 0.00, size = 92, normalized size = 1.24

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}-\frac{\ln\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x+a^3)^(2/3),x)
```

```
[Out] 1/a^2*ln(-a+(-b^3*x+a^3)^(1/3))-1/2/a^2*ln(a^2+(-b^3*x+a^3)^(1/3)*a+(-b^3*x
+a^3)^(2/3))-arctan(1/3*(a+2*(-b^3*x+a^3)^(1/3))*3^(1/2)/a)*3^(1/2)/a^2
```

maxima [A] time = 2.98, size = 91, normalized size = 1.23

$$-\frac{\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x+a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(-a+\left(-b^{3}x+a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x+a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*x + a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 + (-b^3*x + a^3)^(1/3)*a + (-b^3*x + a^3)^(2/3))/a^2 + log(-a + (-b^3*x
+ a^3)^(1/3))/a^2
```

mupad [B] time = 0.11, size = 104, normalized size = 1.41

$$\frac{\ln\left(9\,a-9\left(a^{3}-b^{3}\,x\right)^{1/3}\right)}{a^{2}}+\frac{\ln\left(9\left(a^{3}-b^{3}\,x\right)^{1/3}-\frac{9\,a\left(-1+\sqrt{3}\,1i\right)}{2}\right)\left(-1+\sqrt{3}\,1i\right)}{2\,a^{2}}-\frac{\ln\left(9\left(a^{3}-b^{3}\,x\right)^{1/3}+\frac{9\,a\left(1+\sqrt{3}\,1i\right)}{2}\right)\left(1+\sqrt{3}\,1i\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a^3 - b^3*x)^(2/3)),x)
```

```
[Out] log(9*a - 9*(a^3 - b^3*x)^(1/3))/a^2 + (log(9*(a^3 - b^3*x)^(1/3) - (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(a^3 - b^3*x)^(1/3) +
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

sympy [C] time = 1.92, size = 136, normalized size = 1.84

$$\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+xe^{\frac{i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+xe^{i\pi}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{e^{\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^{3}}{b^{3}}+xe^{\frac{5i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x+a**3)**(2/3),x)
```

```
[Out] log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(1/3)/(3*a**2*g
amma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + exp(2*I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

3.426 $\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=74

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 + b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3+b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx,x,\sqrt[3]{-a^3+b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}+\frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3+b^3x}}{a}}{\sqrt{3}}\right)}{a^2}-\frac{\log(x)}{2a^2}+\frac{3\log\left(a+\sqrt[3]{-a^3+b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.03, size = 108, normalized size = 1.46

$$\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a^2}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b^3x-a^3}-a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a^3 + b^3*x
)^(2/3)]/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.05, size = 110, normalized size = 1.49

$$\frac{\log\left(\sqrt[3]{b^3x-a^3}+a\right)}{a^2}-\frac{\log\left(-a\sqrt[3]{b^3x-a^3}+\left(b^3x-a^3\right)^{2/3}+a^2\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b^3x-a^3}}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 + b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 + b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
 Log[a + (-a^3 + b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 + b^3*x)^(1/3) + (-a
^3 + b^3*x)^(2/3)]/(2*a^2)
```

fricas [A] time = 0.89, size = 95, normalized size = 1.28

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right)-\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)+2\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(b^3*x - a^3)^(1/3))/a) -
 log(a^2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3)) + 2*log(a + (b^3*x
- a^3)^(1/3)))/a^2
```

giac [A] time = 0.99, size = 94, normalized size = 1.27

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(\left|a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (b^3*x -
 a^3)^(1/3)))/a^2
```

maple [A] time = 0.01, size = 96, normalized size = 1.30

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(-a+2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^2}+\frac{\ln\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}-\frac{\ln\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b^3*x-a^3)^(2/3),x)
```

```
[Out] -1/2/a^2*ln(a^2-(b^3*x-a^3)^(1/3)*a+(b^3*x-a^3)^(2/3))+1/a^2*3^(1/2)*arctan
(1/3*(-a+2*(b^3*x-a^3)^(1/3))*3^(1/2)/a)+ln(a+(b^3*x-a^3)^(1/3))/a^2
```

maxima [A] time = 2.95, size = 93, normalized size = 1.26

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2}-\frac{\log\left(a^2-\left(b^3x-a^3\right)^{\frac{1}{3}}a+\left(b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2}+\frac{\log\left(a+\left(b^3x-a^3\right)^{\frac{1}{3}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a^
2 - (b^3*x - a^3)^(1/3)*a + (b^3*x - a^3)^(2/3))/a^2 + log(a + (b^3*x - a^3
)^(1/3))/a^2
```

mupad [B] time = 0.16, size = 107, normalized size = 1.45

$$\frac{\ln\left(9a+9\left(b^3x-a^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}+\frac{9a\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}-\frac{\ln\left(9\left(b^3x-a^3\right)^{1/3}-\frac{9a\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(b^3*x - a^3)^(1/3))/a^2 + (log(9*(b^3*x - a^3)^(1/3) + (9*a*(3
^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(b^3*x - a^3)^(1/3) -
 (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

sympy [C] time = 1.98, size = 134, normalized size = 1.81

$$-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{i\pi}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}-\frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{-\frac{a^3}{b^3}+x}\,e^{\frac{5i\pi}{3}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^2\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b**3*x-a**3)**(2/3),x)
```

```
[Out] -exp(-I*pi/3)*log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi/3)/a)*gamma(
1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(-a**3/b**3 + x)**(1/3)*exp_polar(I*pi
)/a)*gamma(1/3)/(3*a**2*gamma(4/3)) - exp(I*pi/3)*log(1 - b*(-a**3/b**3 + x
)**(1/3)*exp_polar(5*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

3.427 $\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx$

Optimal. Leaf size=76

$$-\frac{\log(x)}{2a^2}+\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{2a^2}-\frac{\sqrt{3}\tan^{-1}\left(\frac{a-2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}-\frac{\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a - 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) - Log[x]/
(2*a^2) + (3*Log[a + (-a^3 - b^3*x)^(1/3)])/(2*a^2)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x\left(-a^3-b^3x\right)^{2/3}}\,dx = -\frac{\log(x)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{a^2-ax+x^2}\,dx, x, \sqrt[3]{-a^3-b^3x}\right)}{2a}$$

$$= -\frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}\right)}{a^2}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a^3-b^3x}}{a}}{\sqrt{3}}\right)}{a^2} - \frac{\log(x)}{2a^2} + \frac{3\log\left(a+\sqrt[3]{-a^3-b^3x}\right)}{2a^2}$$

Mathematica [A] time = 0.03, size = 112, normalized size = 1.47

$$\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a^2} - \frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2} + \frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{-a^3-b^3x}-a}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-a + 2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2 + Log[a + (
-a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a^3 - b^3*x
)^(2/3)]/(2*a^2)
```

IntegrateAlgebraic [A] time = 0.05, size = 114, normalized size = 1.50

$$\frac{\log\left(\sqrt[3]{-a^3-b^3x}+a\right)}{a^2} - \frac{\log\left(-a\sqrt[3]{-a^3-b^3x}+\left(-a^3-b^3x\right)^{2/3}+a^2\right)}{2a^2} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{-a^3-b^3x}}{\sqrt{3}a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(-a^3 - b^3*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*(-a^3 - b^3*x)^(1/3))/(Sqrt[3]*a)])/a^2) +
 Log[a + (-a^3 - b^3*x)^(1/3)]/a^2 - Log[a^2 - a*(-a^3 - b^3*x)^(1/3) + (-a
^3 - b^3*x)^(2/3)]/(2*a^2)
```

fricas [A] time = 1.15, size = 99, normalized size = 1.30

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\left(-b^3x-a^3\right)^{\frac{1}{3}}}{3a}\right) - \log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right) + 2\log\left(a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-b^3*x - a^3)^(1/3))/a)
- log(a^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3)) + 2*log(a + (-b^
3*x - a^3)^(1/3)))/a^2
```

giac [A] time = 1.07, size = 98, normalized size = 1.29

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^3x-a^3\right)^{\frac{1}{3}}\right)}{3a}\right)}{a^2} - \frac{\log\left(a^2-\left(-b^3x-a^3\right)^{\frac{1}{3}}a+\left(-b^3x-a^3\right)^{\frac{2}{3}}\right)}{2a^2} + \frac{\log\left(\left|a+\left(-b^3x-a^3\right)^{\frac{1}{3}}\right|\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="giac")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(abs(a + (-b^3
*x - a^3)^(1/3)))/a^2
```

maple [A] time = 0.00, size = 100, normalized size = 1.32

$$\frac{\sqrt{3}\ \arctan\left(\frac{\left(-a+2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a}\right)}{a^{2}}+\frac{\ln\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}-\frac{\ln\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-b^3*x-a^3)^(2/3),x)
```

```
[Out] -1/2/a^2*ln(a^2-(-b^3*x-a^3)^(1/3)*a+(-b^3*x-a^3)^(2/3))+1/a^2*3^(1/2)*arct
an(1/3*(-a+2*(-b^3*x-a^3)^(1/3))*3^(1/2)/a)+ln(a+(-b^3*x-a^3)^(1/3))/a^2
```

maxima [A] time = 2.89, size = 97, normalized size = 1.28

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(a-2\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^{2}}-\frac{\log\left(a^{2}-\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}a+\left(-b^{3}x-a^{3}\right)^{\frac{2}{3}}\right)}{2\,a^{2}}+\frac{\log\left(a+\left(-b^{3}x-a^{3}\right)^{\frac{1}{3}}\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b^3*x-a^3)^(2/3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*(-b^3*x - a^3)^(1/3))/a)/a^2 - 1/2*log(a
^2 - (-b^3*x - a^3)^(1/3)*a + (-b^3*x - a^3)^(2/3))/a^2 + log(a + (-b^3*x -
 a^3)^(1/3))/a^2
```

mupad [B] time = 0.16, size = 110, normalized size = 1.45

$$\frac{\ln\left(9\,a+9\,\left(-a^3-x\,b^3\right)^{1/3}\right)}{a^2}+\frac{\ln\left(9\,\left(-a^3-x\,b^3\right)^{1/3}+\frac{9\,a\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(9\,\left(-a^3-x\,b^3\right)^{1/3}-\frac{9\,a\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(- b^3*x - a^3)^(2/3)),x)
```

```
[Out] log(9*a + 9*(- b^3*x - a^3)^(1/3))/a^2 + (log(9*(- b^3*x - a^3)^(1/3) + (9*
a*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^2) - (log(9*(- b^3*x - a^3)^(
1/3) - (9*a*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(2*a^2)
```

sympy [C] time = 1.90, size = 133, normalized size = 1.75

$$\frac{e^{-\frac{2i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+x}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}-\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+xe^{\frac{2i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}+\frac{\log\left(1-\frac{b\sqrt[3]{\frac{a^{3}}{b^{3}}+xe^{\frac{4i\pi}{3}}}}{a}\right)\Gamma\left(\frac{1}{3}\right)}{3a^{2}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-b**3*x-a**3)**(2/3),x)
```

```
[Out] exp(-2*I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)/a)*gamma(1/3)/(3*a**2*gamma
(4/3)) - exp(-I*pi/3)*log(1 - b*(a**3/b**3 + x)**(1/3)*exp_polar(2*I*pi/3)/
a)*gamma(1/3)/(3*a**2*gamma(4/3)) + log(1 - b*(a**3/b**3 + x)**(1/3)*exp_po
lar(4*I*pi/3)/a)*gamma(1/3)/(3*a**2*gamma(4/3))
```

3.428 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{a x^{m+1}}{m+1}+\frac{b x^{m+2}}{m+2}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{a x^{m+1}}{m+1}+\frac{b x^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
```

```
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx&=\int \left(a x^m+b x^{1+m}\right)\,dx\\&=\frac{a x^{1+m}}{1+m}+\frac{b x^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
```

```
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^m(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x), x]
```

fricas [A] time = 0.96, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^2+(am+2a)x\right)x^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

giac [A] time = 1.01, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

maple [A] time = 0.00, size = 31, normalized size = 1.24

$$\frac{(bmx + am + bx + 2a)\,x^{m+1}}{(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
[Out] x^(1+m)*(b*m*x+a*m+b*x+2*a)/(2+m)/(1+m)
```

maxima [A] time = 1.34, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.31, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,\left(2\,a + a\,m + b\,x + b\,m\,x\right)}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

sympy [A] time = 0.30, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x} + b\log{(x)} & \text{for}\: m = -2 \\ a\log{(x)} + bx & \text{for}\: m = -1 \\ \frac{amxx^m}{m^2+3m+2} + \frac{2axx^m}{m^2+3m+2} + \frac{bmx^2x^m}{m^2+3m+2} + \frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

3.429 $\int x^{5/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x),x]
```

```
[Out] (2*a*x^(7/2))/7 + (2*b*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)\,dx &= \int \left(ax^{5/2}+bx^{7/2}\right)\,dx\\ &= \frac{2}{7}ax^{7/2}+\frac{2}{9}bx^{9/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{63}x^{7/2}(9a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x),x]
```

```
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{63}\left(9ax^{7/2}+7bx^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x),x]
```

```
[Out] (2*(9*a*x^(7/2) + 7*b*x^(9/2)))/63
```

fricas [A] time = 0.85, size = 18, normalized size = 0.86

$$\frac{2}{63}\left(7\,bx^4+9\,ax^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 2/63*(7*b*x^4 + 9*a*x^3)*sqrt(x)
```

giac [A] time = 0.83, size = 13, normalized size = 0.62

$$\frac{2}{9}bx^{\frac{9}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(7bx+9a)\,x^{\frac{7}{2}}}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a),x)
[Out] 2/63*x^(7/2)*(7*b*x+9*a)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.62

$$\frac{2}{9}bx^{\frac{9}{2}} + \frac{2}{7}ax^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 2/9*b*x^(9/2) + 2/7*a*x^(7/2)
```

mupad [B] time = 0.09, size = 13, normalized size = 0.62

$$\frac{2\,x^{7/2}\,\left(9\,a+7\,b\,x\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x),x)
[Out] (2*x^(7/2)*(9*a + 7*b*x))/63
```

sympy [A] time = 1.59, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a),x)
[Out] 2*a*x**(7/2)/7 + 2*b*x**(9/2)/9
```

3.430 $\int x^{3/2}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*a*x^(5/2))/5 + (2*b*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)\,dx &= \int \left(ax^{3/2}+bx^{5/2}\right)\,dx\\ &= \frac{2}{5}ax^{5/2}+\frac{2}{7}bx^{7/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{35}x^{5/2}(7a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{35}\left(7ax^{5/2}+5bx^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x),x]
```

```
[Out] (2*(7*a*x^(5/2) + 5*b*x^(7/2)))/35
```

fricas [A] time = 1.01, size = 18, normalized size = 0.86

$$\frac{2}{35}\left(5\,bx^{3}+7\,ax^{2}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 2/35*(5*b*x^3 + 7*a*x^2)*sqrt(x)
```

giac [A] time = 0.88, size = 13, normalized size = 0.62

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(5bx+7a)\,x^{\frac{5}{2}}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a),x)
[Out] 2/35*x^(5/2)*(5*b*x+7*a)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.62

$$\frac{2}{7}bx^{\frac{7}{2}} + \frac{2}{5}ax^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 2/7*b*x^(7/2) + 2/5*a*x^(5/2)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.62

$$\frac{2\,x^{5/2}\,\left(7\,a+5\,b\,x\right)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x),x)
[Out] (2*x^(5/2)*(7*a + 5*b*x))/35
```

sympy [A] time = 0.55, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{5}{2}}}{5} + \frac{2bx^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a),x)
[Out] 2*a*x**(5/2)/5 + 2*b*x**(7/2)/7
```

3.431 $\int \sqrt{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*a*x^(3/2))/3 + (2*b*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(a+bx)\,dx &= \int \left(a\sqrt{x}+bx^{3/2}\right)\,dx\\ &= \frac{2}{3}ax^{3/2}+\frac{2}{5}bx^{5/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{2}{15}x^{3/2}(5a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{2}{15}\left(5ax^{3/2}+3bx^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x),x]
```

```
[Out] (2*(5*a*x^(3/2) + 3*b*x^(5/2)))/15
```

fricas [A] time = 0.99, size = 16, normalized size = 0.76

$$\frac{2}{15}\left(3\,bx^2+5\,ax\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*x^2 + 5*a*x)*sqrt(x)
```

giac [A] time = 0.86, size = 13, normalized size = 0.62

$$\frac{2}{5}bx^{\frac{5}{2}} + \frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2\,(3bx+5a)\,x^{\frac{3}{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)*(3*b*x+5*a)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.62

$$\frac{2}{5}bx^{\frac{5}{2}} + \frac{2}{3}ax^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*b*x^(5/2) + 2/3*a*x^(3/2)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.62

$$\frac{2\,x^{3/2}\,\left(5\,a+3\,b\,x\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x),x)
```

```
[Out] (2*x^(3/2)*(5*a + 3*b*x))/15
```

sympy [A] time = 1.62, size = 19, normalized size = 0.90

$$\frac{2ax^{\frac{3}{2}}}{3} + \frac{2bx^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*x**(1/2),x)
```

```
[Out] 2*a*x**(3/2)/3 + 2*b*x**(5/2)/5
```

3.432 $\int \frac{a+bx}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2a\sqrt{x}+\frac{2}{3}bx^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[x],x]
[Out] 2*a*Sqrt[x] + (2*b*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{x}}\,dx &= \int \left(\frac{a}{\sqrt{x}}+b\sqrt{x}\right)dx\\ &= 2a\sqrt{x}+\frac{2}{3}bx^{3/2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{2}{3}\sqrt{x}\,(3a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[x],x]
[Out] (2*Sqrt[x]*(3*a + b*x))/3
```

IntegrateAlgebraic [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{2}{3}\left(3a\sqrt{x}+bx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[x],x]
[Out] (2*(3*a*Sqrt[x] + b*x^(3/2)))/3
```

fricas [A] time = 0.92, size = 12, normalized size = 0.63

$$\frac{2}{3}\,(bx+3\,a)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*x + 3*a)*sqrt(x)
```

giac [A] time = 1.08, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="giac")
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{2\,(bx+3a)\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/2),x)
[Out] 2/3*x^(1/2)*(b*x+3*a)
```

maxima [A] time = 1.32, size = 13, normalized size = 0.68

$$\frac{2}{3}\,bx^{\frac{3}{2}} + 2\,a\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/2),x, algorithm="maxima")
[Out] 2/3*b*x^(3/2) + 2*a*sqrt(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.63

$$\frac{2\,\sqrt{x}\,\left(3\,a+b\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/2),x)
[Out] (2*x^(1/2)*(3*a + b*x))/3
```

sympy [A] time = 0.16, size = 17, normalized size = 0.89

$$2a\sqrt{x} + \frac{2bx^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*b*x**(3/2)/3
```

3.433 $\int \frac{a+bx}{x^{3/2}}\,dx$

Optimal. Leaf size=17

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2b\sqrt{x}-\frac{2a}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + 2*b*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{3/2}}\,dx &= \int \left(\frac{a}{x^{3/2}}+\frac{b}{\sqrt{x}}\right)dx\\ &= -\frac{2a}{\sqrt{x}}+2b\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(bx-a)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(3/2),x]
```

```
[Out] (2*(-a + b*x))/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{2(bx-a)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(3/2),x]
```

```
[Out] (2*(-a + b*x))/Sqrt[x]
```

fricas [A] time = 0.55, size = 12, normalized size = 0.71

$$\frac{2\,(bx-a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*x - a)/sqrt(x)
```

giac [A] time = 0.92, size = 13, normalized size = 0.76

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

maple [A] time = 0.00, size = 12, normalized size = 0.71

$$-\frac{2\,(-bx + a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(3/2),x)
```

```
[Out] -2*(-b*x+a)/x^(1/2)
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$2\,b\sqrt{x} - \frac{2\,a}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x) - 2*a/sqrt(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$-\frac{2\,\left(a - b\,x\right)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(3/2),x)
```

```
[Out] -(2*(a - b*x))/x^(1/2)
```

sympy [A] time = 0.35, size = 15, normalized size = 0.88

$$-\frac{2a}{\sqrt{x}} + 2b\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(3/2),x)
```

```
[Out] -2*a/sqrt(x) + 2*b*sqrt(x)
```

3.434 $\int \frac{a+bx}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) - (2*b)/Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/2}}\,dx &= \int \left(\frac{a}{x^{5/2}}+\frac{b}{x^{3/2}}\right)dx\\ &= -\frac{2a}{3x^{3/2}}-\frac{2b}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/2),x]
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 0.79

$$-\frac{2(a+3bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(5/2),x]
[Out] (-2*(a + 3*b*x))/(3*x^(3/2))
```

fricas [A] time = 0.83, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

giac [A] time = 0.97, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="giac")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{2\,(3bx+a)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/2),x)
[Out] -2/3*(3*b*x+a)/x^(3/2)
```

maxima [A] time = 1.36, size = 11, normalized size = 0.58

$$-\frac{2\,(3\,bx+a)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*(3*b*x + a)/x^(3/2)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{2\,a+6\,b\,x}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/2),x)
[Out] -(2*a + 6*b*x)/(3*x^(3/2))
```

sympy [A] time = 0.56, size = 19, normalized size = 1.00

$$-\frac{2a}{3x^{\frac{3}{2}}}-\frac{2b}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/2),x)
[Out] -2*a/(3*x**(3/2)) - 2*b/sqrt(x)
```

3.435 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx&=\int\left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)dx\\&=\frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^2, x]
```

fricas [A] time = 0.78, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

giac [B] time = 1.10, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

maple [A] time = 0.00, size = 87, normalized size = 2.02

$$\frac{\left(b^2m^2x^2 + 2ab\,m^2x + 3b^2m\,x^2 + a^2m^2 + 8abmx + 2b^2x^2 + 5a^2m + 6abx + 6a^2\right)x^{m+1}}{(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] x^(m+1)*(b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*
a^2*m+6*a*b*x+6*a^2)/(3+m)/(m+2)/(m+1)
```

maxima [A] time = 1.36, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+3}}{m+3} + \frac{2\,abx^{m+2}}{m+2} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.42, size = 93, normalized size = 2.16

$$x^m\,\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

sympy [A] time = 0.53, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log{(x)} & \text{for}\: m = -3 \\ -\frac{a^2}{x} + 2ab\log{(x)} + b^2x & \text{for}\: m = -2 \\ a^2\log{(x)} + 2abx + \frac{b^2x^2}{2} & \text{for}\: m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.436 $\int x^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(7/2))/7 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(11/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^2\,dx &= \int \left(a^2x^{5/2}+2abx^{7/2}+b^2x^{9/2}\right)dx\\ &= \frac{2}{7}a^2x^{7/2}+\frac{4}{9}abx^{9/2}+\frac{2}{11}b^2x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{693}x^{7/2}\left(99a^2+154abx+63b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*(99*a^2 + 154*a*b*x + 63*b^2*x^2))/693
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{693}\left(99a^2x^{7/2}+154abx^{9/2}+63b^2x^{11/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^2,x]
```

```
[Out] (2*(99*a^2*x^(7/2) + 154*a*b*x^(9/2) + 63*b^2*x^(11/2)))/693
```

fricas [A] time = 0.71, size = 29, normalized size = 0.81

$$\frac{2}{693}\left(63\,b^2x^5+154\,abx^4+99\,a^2x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*x^5 + 154*a*b*x^4 + 99*a^2*x^3)*sqrt(x)
```

giac [A] time = 1.16, size = 24, normalized size = 0.67

$$\frac{2}{11} b^2 x^{\frac{11}{2}} + \frac{4}{9} abx^{\frac{9}{2}} + \frac{2}{7} a^2 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2 \left(63b^2x^2 + 154abx + 99a^2\right) x^{\frac{7}{2}}}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^2,x)
```

```
[Out] 2/693*x^(7/2)*(63*b^2*x^2+154*a*b*x+99*a^2)
```

maxima [A] time = 1.27, size = 24, normalized size = 0.67

$$\frac{2}{11} b^2 x^{\frac{11}{2}} + \frac{4}{9} abx^{\frac{9}{2}} + \frac{2}{7} a^2 x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2/11*b^2*x^(11/2) + 4/9*a*b*x^(9/2) + 2/7*a^2*x^(7/2)
```

mupad [B] time = 0.10, size = 24, normalized size = 0.67

$$\frac{2\,x^{7/2}\,\left(99\,a^2+154\,a\,b\,x+63\,b^2\,x^2\right)}{693}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^2,x)
```

```
[Out] (2*x^(7/2)*(99*a^2 + 63*b^2*x^2 + 154*a*b*x))/693
```

sympy [A] time = 2.60, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{7}{2}}}{7} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**2,x)
```

```
[Out] 2*a**2*x**(7/2)/7 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(11/2)/11
```

3.437 $\int x^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(5/2))/5 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(9/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^2\,dx &= \int\left(a^2x^{3/2}+2abx^{5/2}+b^2x^{7/2}\right)dx\\ &= \frac{2}{5}a^2x^{5/2}+\frac{4}{7}abx^{7/2}+\frac{2}{9}b^2x^{9/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{315}x^{5/2}\left(63a^2+90abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*(63*a^2 + 90*a*b*x + 35*b^2*x^2))/315
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{315}\left(63a^2x^{5/2}+90abx^{7/2}+35b^2x^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^2,x]
```

```
[Out] (2*(63*a^2*x^(5/2) + 90*a*b*x^(7/2) + 35*b^2*x^(9/2)))/315
```

fricas [A] time = 0.88, size = 29, normalized size = 0.81

$$\frac{2}{315}\left(35\,b^2x^4+90\,abx^3+63\,a^2x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 2/315*(35*b^2*x^4 + 90*a*b*x^3 + 63*a^2*x^2)*sqrt(x)
```

giac [A] time = 1.12, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(35b^2x^2+90abx+63a^2\right)x^{\frac{5}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^2,x)
[Out] 2/315*x^(5/2)*(35*b^2*x^2+90*a*b*x+63*a^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{2}{9}\,b^2x^{\frac{9}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{5}\,a^2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 2/9*b^2*x^(9/2) + 4/7*a*b*x^(7/2) + 2/5*a^2*x^(5/2)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{5/2}\,\left(63\,a^2+90\,a\,b\,x+35\,b^2\,x^2\right)}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^2,x)
[Out] (2*x^(5/2)*(63*a^2 + 35*b^2*x^2 + 90*a*b*x))/315
```

sympy [A] time = 1.03, size = 34, normalized size = 0.94

$$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**2,x)
[Out] 2*a**2*x**(5/2)/5 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(9/2)/9
```

3.438 $\int \sqrt{x}\,(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*a^2*x^(3/2))/3 + (4*a*b*x^(5/2))/5 + (2*b^2*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\,(a+bx)^2\,dx &= \int \left(a^2\sqrt{x}+2abx^{3/2}+b^2x^{5/2}\right)dx\\ &= \frac{2}{3}a^2x^{3/2}+\frac{4}{5}abx^{5/2}+\frac{2}{7}b^2x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{2}{105}x^{3/2}\left(35a^2+42abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*x^(3/2)*(35*a^2 + 42*a*b*x + 15*b^2*x^2))/105
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{2}{105}\left(35a^2x^{3/2}+42abx^{5/2}+15b^2x^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^2,x]
```

```
[Out] (2*(35*a^2*x^(3/2) + 42*a*b*x^(5/2) + 15*b^2*x^(7/2)))/105
```

fricas [A] time = 0.81, size = 27, normalized size = 0.75

$$\frac{2}{105}\left(15\,b^2x^3+42\,abx^2+35\,a^2x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="fricas")
[Out] 2/105*(15*b^2*x^3 + 42*a*b*x^2 + 35*a^2*x)*sqrt(x)
```

giac [A] time = 0.90, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="giac")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{2\left(15b^2x^2 + 42abx + 35a^2\right)x^{\frac{3}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*x^(1/2),x)
[Out] 2/105*x^(3/2)*(15*b^2*x^2+42*a*b*x+35*a^2)
```

maxima [A] time = 1.29, size = 24, normalized size = 0.67

$$\frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + \frac{2}{3}\,a^2x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^2*x^(7/2) + 4/5*a*b*x^(5/2) + 2/3*a^2*x^(3/2)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{2\,x^{3/2}\,\left(35\,a^2 + 42\,a\,b\,x + 15\,b^2\,x^2\right)}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^2,x)
[Out] (2*x^(3/2)*(35*a^2 + 15*b^2*x^2 + 42*a*b*x))/105
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*x**(1/2),x)
[Out] Timed out
```

3.439 $\int \frac{(a+bx)^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=34

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{x}}\,dx &= \int \left(\frac{a^2}{\sqrt{x}}+2ab\sqrt{x}+b^2x^{3/2}\right)dx\\ &= 2a^2\sqrt{x}+\frac{4}{3}abx^{3/2}+\frac{2}{5}b^2x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{2}{15}\sqrt{x}\left(15a^2+10abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 10*a*b*x + 3*b^2*x^2))/15
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{2}{15}\left(15a^2\sqrt{x}+10abx^{3/2}+3b^2x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[x],x]
```

```
[Out] (2*(15*a^2*Sqrt[x] + 10*a*b*x^(3/2) + 3*b^2*x^(5/2)))/15
```

fricas [A] time = 0.84, size = 24, normalized size = 0.71

$$\frac{2}{15}\left(3\,b^2x^2+10\,abx+15\,a^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + 10*a*b*x + 15*a^2)*sqrt(x)
```

giac [A] time = 0.92, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="giac")
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$\frac{2\left(3b^2x^2 + 10abx + 15a^2\right)\sqrt{x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/2),x)
[Out] 2/15*x^(1/2)*(3*b^2*x^2+10*a*b*x+15*a^2)
```

maxima [A] time = 1.38, size = 24, normalized size = 0.71

$$\frac{2}{5}\,b^2x^{\frac{5}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} + 2\,a^2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/2),x, algorithm="maxima")
[Out] 2/5*b^2*x^(5/2) + 4/3*a*b*x^(3/2) + 2*a^2*sqrt(x)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{2\,\sqrt{x}\,\left(15\,a^2 + 10\,a\,b\,x + 3\,b^2\,x^2\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/2),x)
[Out] (2*x^(1/2)*(15*a^2 + 3*b^2*x^2 + 10*a*b*x))/15
```

sympy [A] time = 0.26, size = 32, normalized size = 0.94

$$2a^2\sqrt{x} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/2),x)
[Out] 2*a**2*sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(5/2)/5
```

3.440 $\int \frac{(a+bx)^2}{x^{3/2}} \, dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (-2*a^2)/Sqrt[x] + 4*a*b*Sqrt[x] + (2*b^2*x^(3/2))/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{3/2}} \, dx &= \int \left(\frac{a^2}{x^{3/2}}+\frac{2ab}{\sqrt{x}}+b^2\sqrt{x}\right) dx \\ &= -\frac{2a^2}{\sqrt{x}}+4ab\sqrt{x}+\frac{2}{3}b^2x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\left(-3a^2+6abx+b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (2*(-3*a^2 + 6*a*b*x + b^2*x^2))/(3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2\left(-3a^2+6abx+b^2x^2\right)}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(3/2),x]
```

```
[Out] (2*(-3*a^2 + 6*a*b*x + b^2*x^2))/(3*Sqrt[x])
```

fricas [A] time = 0.89, size = 23, normalized size = 0.72

$$\frac{2\left(b^{2}x^{2}+6\,abx-3\,a^{2}\right)}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 + 6*a*b*x - 3*a^2)/sqrt(x)
```

giac [A] time = 1.02, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="giac")
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{2\left(-b^{2}x^{2}-6abx+3a^{2}\right)}{3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(3/2),x)
[Out] -2/3*(-b^2*x^2-6*a*b*x+3*a^2)/x^(1/2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.75

$$\frac{2}{3}\,b^{2}x^{\frac{3}{2}}+4\,ab\sqrt{x}-\frac{2\,a^{2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(3/2),x, algorithm="maxima")
[Out] 2/3*b^2*x^(3/2) + 4*a*b*sqrt(x) - 2*a^2/sqrt(x)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$\frac{-6\,a^{2}+12\,a\,b\,x+2\,b^{2}\,x^{2}}{3\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(3/2),x)
[Out] (2*b^2*x^2 - 6*a^2 + 12*a*b*x)/(3*x^(1/2))
```

sympy [A] time = 0.43, size = 31, normalized size = 0.97

$$-\frac{2a^{2}}{\sqrt{x}}+4ab\sqrt{x}+\frac{2b^{2}x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(3/2),x)
[Out] -2*a**2/sqrt(x) + 4*a*b*sqrt(x) + 2*b**2*x**(3/2)/3
```

3.441 $\int \frac{(a+bx)^2}{x^{5/2}} \, dx$

Optimal. Leaf size=32

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/2),x]
[Out] (-2*a^2)/(3*x^(3/2)) - (4*a*b)/Sqrt[x] + 2*b^2*Sqrt[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/2}} \, dx &= \int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{x^{3/2}}+\frac{b^2}{\sqrt{x}}\right) dx\\ &= -\frac{2a^2}{3x^{3/2}}-\frac{4ab}{\sqrt{x}}+2b^2\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.81

$$-\frac{2\left(a^2+6abx-3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/2),x]
[Out] (-2*(a^2 + 6*a*b*x - 3*b^2*x^2))/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{2\left(-a^2-6abx+3b^2x^2\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(5/2),x]
[Out] (2*(-a^2 - 6*a*b*x + 3*b^2*x^2))/(3*x^(3/2))
```

fricas [A] time = 0.81, size = 24, normalized size = 0.75

$$\frac{2\left(3\,b^2x^2-6\,abx-a^2\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(3*b^2*x^2 - 6*a*b*x - a^2)/x^(3/2)
```

giac [A] time = 1.00, size = 23, normalized size = 0.72

$$2\,b^{2}\sqrt{x} - \frac{2\,\left(6\,abx + a^{2}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="giac")
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

maple [A] time = 0.00, size = 23, normalized size = 0.72

$$-\frac{2\left(-3b^{2}x^{2}+6abx+a^{2}\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/2),x)
[Out] -2/3*(-3*b^2*x^2+6*a*b*x+a^2)/x^(3/2)
```

maxima [A] time = 1.29, size = 23, normalized size = 0.72

$$2\,b^{2}\sqrt{x} - \frac{2\,\left(6\,abx + a^{2}\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/2),x, algorithm="maxima")
[Out] 2*b^2*sqrt(x) - 2/3*(6*a*b*x + a^2)/x^(3/2)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$-\frac{2\,a^{2}+12\,a\,b\,x-6\,b^{2}\,x^{2}}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/2),x)
[Out] -(2*a^2 - 6*b^2*x^2 + 12*a*b*x)/(3*x^(3/2))
```

sympy [A] time = 0.59, size = 31, normalized size = 0.97

$$-\frac{2a^{2}}{3x^{\frac{3}{2}}} - \frac{4ab}{\sqrt{x}} + 2b^{2}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/2),x)
[Out] -2*a**2/(3*x**(3/2)) - 4*a*b/sqrt(x) + 2*b**2*sqrt(x)
```

3.442 $\int x^m(a+bx)^3\,dx$

Optimal. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^3\,dx &= \int\left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right)dx\\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx}{m+2}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^3, x]
```

fricas [B] time = 0.89, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b\right)x^2+\left(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3\right)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

giac [B] time = 1.03, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m + 42\,ab^2mx^3x^m + 6\,b^3x^4x^m + 9\,a^3m^2xx^m + 57\,a^2bmx^2x^m + 24\,ab^2x^3x^m + 26\,a^3mxx^m + 36\,a^2bx^2x^m + 24\,a^3xx^m}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

maple [B] time = 0.00, size = 170, normalized size = 2.79

$$\frac{\left(b^3m^3x^3 + 3a\,b^2m^3x^2 + 6b^3m^2x^3 + 3a^2b\,m^3x + 21a\,b^2m^2x^2 + 11b^3m\,x^3 + a^3m^3 + 24a^2b\,m^2x + 42a\,b^2m\,x^2 + 6b^3x^3 + 9a^3m^2 + 57a^2bmx + 24a\,b^2x^2 + 26a^3m + 36a^2bx + 24a^3\right)x^{m+1}}{(m+4)\,(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] x^(m+1)*(b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m
^2*x^2+11*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m
^2+57*a^2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(4+m)/(m+3)/(m+2)/
(m+1)
```

maxima [A] time = 1.36, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4} + \frac{3\,ab^2x^{m+3}}{m+3} + \frac{3\,a^2bx^{m+2}}{m+2} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.39, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

sympy [A] time = 0.88, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) & \text{for } m = -4 \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x & \text{for } m = -3 \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} & \text{for } m = -2 \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

3.443 $\int x^{5/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2}{3}a^2bx^{9/2}+\frac{2}{7}a^3x^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x^
(13/2))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/2}(a+bx)^3\,dx &= \int\left(a^3x^{5/2}+3a^2bx^{7/2}+3ab^2x^{9/2}+b^3x^{11/2}\right)dx\\ &= \frac{2}{7}a^3x^{7/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{13}b^3x^{13/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{7/2}\left(429a^3+1001a^2bx+819ab^2x^2+231b^3x^3\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*x^(7/2)*(429*a^3 + 1001*a^2*b*x + 819*a*b^2*x^2 + 231*b^3*x^3))/3003
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{2\left(429a^3x^{7/2}+1001a^2bx^{9/2}+819ab^2x^{11/2}+231b^3x^{13/2}\right)}{3003}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^3,x]
```

```
[Out] (2*(429*a^3*x^(7/2) + 1001*a^2*b*x^(9/2) + 819*a*b^2*x^(11/2) + 231*b^3*x^(
13/2)))/3003
```

fricas [A] time = 0.81, size = 40, normalized size = 0.78

$$\frac{2}{3003}\left(231\,b^{3}x^{6}+819\,ab^{2}x^{5}+1001\,a^{2}bx^{4}+429\,a^{3}x^{3}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*x^6 + 819*a*b^2*x^5 + 1001*a^2*b*x^4 + 429*a^3*x^3)*sqrt(x)
```

giac [A] time = 0.99, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^{3}x^{\frac{13}{2}}+\frac{6}{11}\,ab^{2}x^{\frac{11}{2}}+\frac{2}{3}\,a^{2}bx^{\frac{9}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(231b^{3}x^{3}+819a\,b^{2}x^{2}+1001a^{2}bx+429a^{3}\right)x^{\frac{7}{2}}}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^3,x)
```

```
[Out] 2/3003*x^(7/2)*(231*b^3*x^3+819*a*b^2*x^2+1001*a^2*b*x+429*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{13}\,b^{3}x^{\frac{13}{2}}+\frac{6}{11}\,ab^{2}x^{\frac{11}{2}}+\frac{2}{3}\,a^{2}bx^{\frac{9}{2}}+\frac{2}{7}\,a^{3}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/13*b^3*x^(13/2) + 6/11*a*b^2*x^(11/2) + 2/3*a^2*b*x^(9/2) + 2/7*a^3*x^(7/
2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^{3}\,x^{7/2}}{7}+\frac{2\,b^{3}\,x^{13/2}}{13}+\frac{2\,a^{2}\,b\,x^{9/2}}{3}+\frac{6\,a\,b^{2}\,x^{11/2}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(7/2))/7 + (2*b^3*x^(13/2))/13 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^
(11/2))/11
```

sympy [A] time = 3.88, size = 49, normalized size = 0.96

$$\frac{2a^{3}x^{\frac{7}{2}}}{7}+\frac{2a^{2}bx^{\frac{9}{2}}}{3}+\frac{6ab^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{13}{2}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(7/2)/7 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x*
*(13/2)/13
```

3.444 $\int x^{3/2}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{7}a^2bx^{7/2}+\frac{2}{5}a^3x^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(5/2))/5 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(1
1/2))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{3/2}(a+bx)^3\,dx&=\int\left(a^3x^{3/2}+3a^2bx^{5/2}+3ab^2x^{7/2}+b^3x^{9/2}\right)dx\\&=\frac{2}{5}a^3x^{5/2}+\frac{6}{7}a^2bx^{7/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{11}b^3x^{11/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{2x^{5/2}\left(231a^3+495a^2bx+385ab^2x^2+105b^3x^3\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*x^(5/2)*(231*a^3 + 495*a^2*b*x + 385*a*b^2*x^2 + 105*b^3*x^3))/1155
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{2\left(231a^3x^{5/2}+495a^2bx^{7/2}+385ab^2x^{9/2}+105b^3x^{11/2}\right)}{1155}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^3,x]
```

```
[Out] (2*(231*a^3*x^(5/2) + 495*a^2*b*x^(7/2) + 385*a*b^2*x^(9/2) + 105*b^3*x^(11
/2)))/1155
```

fricas [A] time = 0.88, size = 40, normalized size = 0.78

$$\frac{2}{1155}\left(105\,b^3x^5 + 385\,ab^2x^4 + 495\,a^2bx^3 + 231\,a^3x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*x^5 + 385*a*b^2*x^4 + 495*a^2*b*x^3 + 231*a^3*x^2)*sqrt(x)
```

giac [A] time = 0.99, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(105b^3x^3 + 385a\,b^2x^2 + 495a^2bx + 231a^3\right)x^{\frac{5}{2}}}{1155}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^3,x)
```

```
[Out] 2/1155*x^(5/2)*(105*b^3*x^3+385*a*b^2*x^2+495*a^2*b*x+231*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{7}\,a^2bx^{\frac{7}{2}} + \frac{2}{5}\,a^3x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 2/11*b^3*x^(11/2) + 2/3*a*b^2*x^(9/2) + 6/7*a^2*b*x^(7/2) + 2/5*a^3*x^(5/2)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{5/2}}{5} + \frac{2\,b^3\,x^{11/2}}{11} + \frac{6\,a^2\,b\,x^{7/2}}{7} + \frac{2\,a\,b^2\,x^{9/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(5/2))/5 + (2*b^3*x^(11/2))/11 + (6*a^2*b*x^(7/2))/7 + (2*a*b^2*x^
(9/2))/3
```

sympy [A] time = 1.70, size = 49, normalized size = 0.96

$$\frac{2a^3x^{\frac{5}{2}}}{5} + \frac{6a^2bx^{\frac{7}{2}}}{7} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**3,x)
```

```
[Out] 2*a**3*x**(5/2)/5 + 6*a**2*b*x**(7/2)/7 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(
11/2)/11
```

3.445 $\int \sqrt{x} (a + bx)^3 \, dx$

Optimal. Leaf size=51

$$\frac{2}{3}a^3x^{3/2} + \frac{6}{5}a^2bx^{5/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{9}b^3x^{9/2}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{6}{5}a^2bx^{5/2} + \frac{2}{3}a^3x^{3/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{9}b^3x^{9/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*a^3*x^(3/2))/3 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(9
/2))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \sqrt{x} (a + bx)^3 \, dx &= \int \left(a^3\sqrt{x} + 3a^2bx^{3/2} + 3ab^2x^{5/2} + b^3x^{7/2}\right) dx \\ &= \frac{2}{3}a^3x^{3/2} + \frac{6}{5}a^2bx^{5/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{9}b^3x^{9/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{2}{315}x^{3/2}\left(105a^3 + 189a^2bx + 135ab^2x^2 + 35b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*(105*a^3 + 189*a^2*b*x + 135*a*b^2*x^2 + 35*b^3*x^3))/315
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{2}{315}\left(105a^3x^{3/2} + 189a^2bx^{5/2} + 135ab^2x^{7/2} + 35b^3x^{9/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^3,x]
```

```
[Out] (2*(105*a^3*x^(3/2) + 189*a^2*b*x^(5/2) + 135*a*b^2*x^(7/2) + 35*b^3*x^(9/2
)))/315
```

fricas [A] time = 0.82, size = 38, normalized size = 0.75

$$\frac{2}{315}\left(35\,b^3x^4 + 135\,ab^2x^3 + 189\,a^2bx^2 + 105\,a^3x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*x^4 + 135*a*b^2*x^3 + 189*a^2*b*x^2 + 105*a^3*x)*sqrt(x)
```

giac [A] time = 1.08, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{2\left(35b^3x^3+135a\,b^2x^2+189a^2bx+105a^3\right)x^{\frac{3}{2}}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*x^(1/2),x)
```

```
[Out] 2/315*x^(3/2)*(35*b^3*x^3+135*a*b^2*x^2+189*a^2*b*x+105*a^3)
```

maxima [A] time = 1.26, size = 35, normalized size = 0.69

$$\frac{2}{9}\,b^3x^{\frac{9}{2}}+\frac{6}{7}\,ab^2x^{\frac{7}{2}}+\frac{6}{5}\,a^2bx^{\frac{5}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*b^3*x^(9/2) + 6/7*a*b^2*x^(7/2) + 6/5*a^2*b*x^(5/2) + 2/3*a^3*x^(3/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{9/2}}{9}+\frac{6\,a^2\,b\,x^{5/2}}{5}+\frac{6\,a\,b^2\,x^{7/2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^3,x)
```

```
[Out] (2*a^3*x^(3/2))/3 + (2*b^3*x^(9/2))/9 + (6*a^2*b*x^(5/2))/5 + (6*a*b^2*x^(7
/2))/7
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*x**(1/2),x)
```

```
[Out] Timed out
```

3.446 $\int \frac{(a+bx)^3}{\sqrt{x}}\,dx$

Optimal. Leaf size=47

$$2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$2a^2bx^{3/2}+2a^3\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[x],x]
[Out] 2*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{x}}\,dx &= \int \left(\frac{a^3}{\sqrt{x}}+3a^2b\sqrt{x}+3ab^2x^{3/2}+b^3x^{5/2}\right)dx\\ &= 2a^3\sqrt{x}+2a^2bx^{3/2}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{7}b^3x^{7/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.83

$$\frac{2}{35}\sqrt{x}\left(35a^3+35a^2bx+21ab^2x^2+5b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[x],x]
[Out] (2*Sqrt[x]*(35*a^3 + 35*a^2*b*x + 21*a*b^2*x^2 + 5*b^3*x^3))/35
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 1.00

$$\frac{2}{35}\left(35a^3\sqrt{x}+35a^2bx^{3/2}+21ab^2x^{5/2}+5b^3x^{7/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/Sqrt[x],x]
[Out] (2*(35*a^3*Sqrt[x] + 35*a^2*b*x^(3/2) + 21*a*b^2*x^(5/2) + 5*b^3*x^(7/2)))/
35
```

fricas [A] time = 0.77, size = 35, normalized size = 0.74

$$\frac{2}{35}\left(5\,b^3x^3+21\,ab^2x^2+35\,a^2bx+35\,a^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="fricas")
[Out] 2/35*(5*b^3*x^3 + 21*a*b^2*x^2 + 35*a^2*b*x + 35*a^3)*sqrt(x)
```

giac [A] time = 0.94, size = 35, normalized size = 0.74

$$\frac{2}{7}b^3x^{\frac{7}{2}} + \frac{6}{5}ab^2x^{\frac{5}{2}} + 2a^2bx^{\frac{3}{2}} + 2a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="giac")
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

maple [A] time = 0.00, size = 36, normalized size = 0.77

$$\frac{2\left(5b^3x^3 + 21ab^2x^2 + 35a^2bx + 35a^3\right)\sqrt{x}}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/2),x)
[Out] 2/35*x^(1/2)*(5*b^3*x^3+21*a*b^2*x^2+35*a^2*b*x+35*a^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.74

$$\frac{2}{7}b^3x^{\frac{7}{2}} + \frac{6}{5}ab^2x^{\frac{5}{2}} + 2a^2bx^{\frac{3}{2}} + 2a^3\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/2),x, algorithm="maxima")
[Out] 2/7*b^3*x^(7/2) + 6/5*a*b^2*x^(5/2) + 2*a^2*b*x^(3/2) + 2*a^3*sqrt(x)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$2a^3\sqrt{x} + \frac{2b^3x^{7/2}}{7} + 2a^2bx^{3/2} + \frac{6ab^2x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/2),x)
[Out] 2*a^3*x^(1/2) + (2*b^3*x^(7/2))/7 + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(5/2))/5
```

sympy [A] time = 0.44, size = 46, normalized size = 0.98

$$2a^3\sqrt{x} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/2),x)
[Out] 2*a**3*sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**(7/2)/
7
```

3.447 $\int \frac{(a+bx)^3}{x^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$6a^2b\sqrt{x}-\frac{2a^3}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (-2*a^3)/Sqrt[x] + 6*a^2*b*Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{3/2}}\,dx &= \int \left(\frac{a^3}{x^{3/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2\sqrt{x}+b^3x^{3/2}\right)dx\\ &= -\frac{2a^3}{\sqrt{x}}+6a^2b\sqrt{x}+2ab^2x^{3/2}+\frac{2}{5}b^3x^{5/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.84

$$\frac{2\left(-5a^3+15a^2bx+5ab^2x^2+b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (2*(-5*a^3 + 15*a^2*b*x + 5*a*b^2*x^2 + b^3*x^3))/(5*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 0.84

$$\frac{2\left(-5a^3+15a^2bx+5ab^2x^2+b^3x^3\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(3/2),x]
```

```
[Out] (2*(-5*a^3 + 15*a^2*b*x + 5*a*b^2*x^2 + b^3*x^3))/(5*Sqrt[x])
```

fricas [A] time = 0.80, size = 34, normalized size = 0.76

$$\frac{2\left(b^3x^3 + 5\,ab^2x^2 + 15\,a^2bx - 5\,a^3\right)}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="fricas")
[Out] 2/5*(b^3*x^3 + 5*a*b^2*x^2 + 15*a^2*b*x - 5*a^3)/sqrt(x)
```

giac [A] time = 1.03, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="giac")
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

maple [A] time = 0.00, size = 36, normalized size = 0.80

$$-\frac{2\left(-b^3x^3 - 5a\,b^2x^2 - 15a^2bx + 5a^3\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(3/2),x)
[Out] -2/5*(-b^3*x^3-5*a*b^2*x^2-15*a^2*b*x+5*a^3)/x^(1/2)
```

maxima [A] time = 1.36, size = 35, normalized size = 0.78

$$\frac{2}{5}\,b^3x^{\frac{5}{2}} + 2\,ab^2x^{\frac{3}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(3/2),x, algorithm="maxima")
[Out] 2/5*b^3*x^(5/2) + 2*a*b^2*x^(3/2) + 6*a^2*b*sqrt(x) - 2*a^3/sqrt(x)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.78

$$\frac{2\,b^3\,x^{5/2}}{5} - \frac{2\,a^3}{\sqrt{x}} + 6\,a^2\,b\,\sqrt{x} + 2\,a\,b^2\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(3/2),x)
[Out] (2*b^3*x^(5/2))/5 - (2*a^3)/x^(1/2) + 6*a^2*b*x^(1/2) + 2*a*b^2*x^(3/2)
```

sympy [A] time = 0.64, size = 44, normalized size = 0.98

$$-\frac{2a^3}{\sqrt{x}} + 6a^2b\sqrt{x} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(3/2),x)
[Out] -2*a**3/sqrt(x) + 6*a**2*b*sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(5/2)/5
```

3.448 $\int \frac{(a+bx)^3}{x^{5/2}} dx$

Optimal. Leaf size=47

$$-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{6a^2b}{\sqrt{x}}-\frac{2a^3}{3x^{3/2}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/2),x]
[Out] (-2*a^3)/(3*x^(3/2)) - (6*a^2*b)/Sqrt[x] + 6*a*b^2*Sqrt[x] + (2*b^3*x^(3/2)
)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/2}}\,dx&=\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{x^{3/2}}+\frac{3ab^2}{\sqrt{x}}+b^3\sqrt{x}\right)dx\\&=-\frac{2a^3}{3x^{3/2}}-\frac{6a^2b}{\sqrt{x}}+6ab^2\sqrt{x}+\frac{2}{3}b^3x^{3/2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.81

$$\frac{2\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/2),x]
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 38, normalized size = 0.81

$$\frac{2\left(-a^3-9a^2bx+9ab^2x^2+b^3x^3\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(5/2),x]
[Out] (2*(-a^3 - 9*a^2*b*x + 9*a*b^2*x^2 + b^3*x^3))/(3*x^(3/2))
```

fricas [A] time = 0.90, size = 34, normalized size = 0.72

$$\frac{2\left(b^3x^3 + 9\,ab^2x^2 - 9\,a^2bx - a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b^3*x^3 + 9*a*b^2*x^2 - 9*a^2*b*x - a^3)/x^(3/2)
```

giac [A] time = 1.07, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\left(9\,a^2bx + a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="giac")
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

maple [A] time = 0.00, size = 34, normalized size = 0.72

$$-\frac{2\left(-b^3x^3 - 9a\,b^2x^2 + 9a^2bx + a^3\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/2),x)
[Out] -2/3*(-b^3*x^3-9*a*b^2*x^2+9*a^2*b*x+a^3)/x^(3/2)
```

maxima [A] time = 1.37, size = 34, normalized size = 0.72

$$\frac{2}{3}\,b^3x^{\frac{3}{2}} + 6\,ab^2\sqrt{x} - \frac{2\left(9\,a^2bx + a^3\right)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/2),x, algorithm="maxima")
[Out] 2/3*b^3*x^(3/2) + 6*a*b^2*sqrt(x) - 2/3*(9*a^2*b*x + a^3)/x^(3/2)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.74

$$-\frac{2\,a^3 + 18\,a^2\,b\,x - 18\,a\,b^2\,x^2 - 2\,b^3\,x^3}{3\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/2),x)
[Out] -(2*a^3 - 2*b^3*x^3 - 18*a*b^2*x^2 + 18*a^2*b*x)/(3*x^(3/2))
```

sympy [A] time = 0.78, size = 46, normalized size = 0.98

$$-\frac{2a^3}{3x^{\frac{3}{2}}} - \frac{6a^2b}{\sqrt{x}} + 6ab^2\sqrt{x} + \frac{2b^3x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/2),x)
[Out] -2*a**3/(3*x**(3/2)) - 6*a**2*b/sqrt(x) + 6*a*b**2*sqrt(x) + 2*b**3*x**(3/2
)/3
```

3.449 $\int \frac{x^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 - (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{a+bx}\,dx = \frac{2x^{5/2}}{5b} - \frac{a\int \frac{x^{3/2}}{a+bx}\,dx}{b}$$
$$= -\frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{a+bx}\,dx}{b^2}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{a^3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{(2a^3)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} - \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\,(15a^2 - 5abx + 3b^2x^2)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 - 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTan[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.99

$$\frac{2\left(15a^2\sqrt{x} - 5abx^{3/2} + 3b^2x^{5/2}\right)}{15b^3} - \frac{2a^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x),x]
```

```
[Out] (2*(15*a^2*Sqrt[x] - 5*a*b*x^(3/2) + 3*b^2*x^(5/2)))/(15*b^3) - (2*a^(5/2)*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

fricas [A] time = 0.97, size = 132, normalized size = 1.94

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx - 2\,b\sqrt{x}\sqrt{-\frac{a}{b}} - a}{bx+a}\right) + 2\left(3\,b^2x^2 - 5\,abx + 15\,a^2\right)\sqrt{x}}{15\,b^3}, -\frac{2\left(15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right) - \left(3\,b^2x^2 - 5\,abx + 15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a))
+ 2*(3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3, -2/15*(15*a^2*sqrt(a/b)*ar
ctan(b*sqrt(x)*sqrt(a/b)/a) - (3*b^2*x^2 - 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

giac [A] time = 0.87, size = 59, normalized size = 0.87

$$-\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}} - 5\,ab^3x^{\frac{3}{2}} + 15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^4*x^(5/2) -
5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5b}-\frac{2a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^{3}}-\frac{2a\,x^{\frac{3}{2}}}{3b^{2}}+\frac{2a^{2}\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*x^(5/2)/b-2/3*a*x^(3/2)/b^2+2*a^2*x^(1/2)/b^3-2*a^3/b^3/(a*b)^(1/2)*arc
tan(x^(1/2)*b/(a*b)^(1/2))
```

maxima [A] time = 3.01, size = 54, normalized size = 0.79

$$-\frac{2\,a^{3}\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}+\frac{2\left(3\,b^{2}x^{\frac{5}{2}}-5\,abx^{\frac{3}{2}}+15\,a^{2}\sqrt{x}\right)}{15\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2/15*(3*b^2*x^(5/2) -
5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

mupad [B] time = 0.06, size = 48, normalized size = 0.71

$$\frac{2\,x^{5/2}}{5\,b}-\frac{2\,a\,x^{3/2}}{3\,b^{2}}+\frac{2\,a^{2}\,\sqrt{x}}{b^{3}}-\frac{2\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) - (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 - (2*a^(5/2
)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

sympy [A] time = 7.30, size = 121, normalized size = 1.78

$$\begin{cases}\frac{ia^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^{4}\sqrt{\frac{1}{b}}}-\frac{ia^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^{4}\sqrt{\frac{1}{b}}}+\frac{2a^{2}\sqrt{x}}{b^{3}}-\frac{2ax^{\frac{3}{2}}}{3b^{2}}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for } b\neq 0\\ \frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*a**(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b))
- I*a**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*s
qrt(x)/b**3 - 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (2*x**(7
/2)/(7*a), True))
```

3.450 $\int \frac{x^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x),x]
```

```
[Out] (-2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{a+bx}\,dx &= \frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{a+bx}\,dx}{b}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{a^2\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.92

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2\sqrt{x}\,(bx-3a)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(-3*a + b*x))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{2a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2\left(bx^{3/2}-3a\sqrt{x}\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x),x]
```

```
[Out] (2*(-3*a*Sqrt[x] + b*x^(3/2)))/(3*b^2) + (2*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

fricas [A] time = 0.70, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx+2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(bx-3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+(bx-3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*
(b*x - 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a)
+ (b*x - 3*a)*sqrt(x))/b^2]
```

giac [A] time = 1.24, size = 45, normalized size = 0.85

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}-3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b^2*x^(3/2) - 3*a*
b*sqrt(x))/b^3
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$\frac{2a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{2x^{\frac{3}{2}}}{3b}-\frac{2a\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*x^(3/2)/b-2*a*x^(1/2)/b^2+2*a^2/b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*
x^(1/2))
```

maxima [A] time = 2.94, size = 42, normalized size = 0.79

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}-3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2/3*(b*x^(3/2) - 3*a*sq
rt(x))/b^2
```

mupad [B] time = 0.05, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}-\frac{2\,a\,\sqrt{x}}{b^2}+\frac{2\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) - (2*a*x^(1/2))/b^2 + (2*a^(3/2)*atan((b^(1/2)*x^(1/2))/a
^(1/2)))/b^(5/2)
```

sympy [A] time = 1.93, size = 105, normalized size = 1.98

$$\begin{cases} -\frac{ia^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}+\frac{ia^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}-\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for } b\neq 0\\ \frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((-I*a**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b))
 + I*a**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - 2*a*sqr
t(x)/b**2 + 2*x**(3/2)/(3*b), Ne(b, 0)), (2*x**(5/2)/(5*a), True))
```

3.451 $\int \frac{\sqrt{x}}{a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 205}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{a+bx}\,dx &= \frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b}-\frac{(2a)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

fricas [A] time = 0.98, size = 85, normalized size = 2.12

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\sqrt{x}}{b},-\frac{2\left(\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="fricas")
[Out] [(sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x + a)) + 2*sqrt(x))
/b, -2*(sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) - sqrt(x))/b]
```

giac [A] time = 0.99, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="giac")
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a),x)
[Out] 2*x^(1/2)/b-2*a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.97, size = 31, normalized size = 0.78

$$-\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) + 2*sqrt(x)/b
```

mupad [B] time = 0.04, size = 28, normalized size = 0.70

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

sympy [A] time = 0.72, size = 92, normalized size = 2.30

$$\begin{cases} \frac{i\sqrt{a}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}}-\frac{i\sqrt{a}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{2\sqrt{x}}{b} & \text{for } b\neq 0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a),x)
```

```
[Out] Piecewise((I*sqrt(a)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) -
 I*sqrt(a)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/
b, Ne(b, 0)), (2*x**(3/2)/(3*a), True))
```

3.452 $\int \frac{1}{\sqrt{x}(a+bx)} dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}(a+bx)} dx &= 2\,\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{x}\right) \\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)),x]
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.91, size = 68, normalized size = 2.34

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqrt(x))/(b*x + a))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(b*sqrt(x)))/(a*b)]
```

giac [A] time = 0.95, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.01, size = 19, normalized size = 0.66

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/x^(1/2),x)
```

```
[Out] 2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.93, size = 18, normalized size = 0.62

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.04, size = 19, normalized size = 0.66

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)),x)
```

[Out] (2*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))

sympy [A] time = 1.29, size = 94, normalized size = 3.24

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ \frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
2*sqrt(x)/a, Eq(b, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*
sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), Tr
ue))
```

3.453 $\int \frac{1}{x^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=40

$$-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)),x]
```

```
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)}\,dx &= -\frac{2}{a\sqrt{x}}-\frac{b\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a}\\ &= -\frac{2}{a\sqrt{x}}-\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a}\\ &= -\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.62

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)),x]
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, -((b*x)/a)])/(a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.00

$$-\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)),x]
[Out] -2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.95, size = 93, normalized size = 2.32

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\sqrt{x}}{ax},\ \frac{2\left(x\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] [(x*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a)) - 2*sqrt(x
))/(a*x), 2*(x*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - sqrt(x))/(a*x)]
```

giac [A] time = 1.00, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="giac")
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a),x)
[Out] -2/a/x^(1/2)-2/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.88, size = 31, normalized size = 0.78

$$-\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) - 2/(a*sqrt(x))
```

mupad [B] time = 0.04, size = 28, normalized size = 0.70

$$-\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)),x)
```

```
[Out] - 2/(a*x^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

sympy [A] time = 2.77, size = 102, normalized size = 2.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for}\ b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{a\sqrt{x}}+\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(a*sqrt(x)), Eq(b, 0)),
(-2/(3*b*x**(3/2)), Eq(a, 0)), (-2/(a*sqrt(x)) + I*log(-I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(a**(3/2)*sqrt(1/b)) - I*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a*
*(3/2)*sqrt(1/b)), True))
```

3.454 $\int \frac{1}{x^{5/2}(a+bx)}\,dx$

Optimal. Leaf size=53

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)),x]
```

```
[Out] -2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)}\,dx &= -\frac{2}{3ax^{3/2}}-\frac{b\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\ &= -\frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.51

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\frac{bx}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)),x]
[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, -((b*x)/a)])/(3*a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 0.91

$$\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2(a-3bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)),x]
[Out] (-2*(a - 3*b*x))/(3*a^2*x^(3/2)) + (2*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

fricas [A] time = 0.89, size = 118, normalized size = 2.23

$$\left[\frac{3\,bx^2\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\,(3\,bx-a)\sqrt{x}}{3\,a^2x^2},-\frac{2\left(3\,bx^2\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx-a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="fricas")
[Out] [1/3*(3*b*x^2*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x + a))
+ 2*(3*b*x - a)*sqrt(x))/(a^2*x^2), -2/3*(3*b*x^2*sqrt(b/a)*arctan(a*sqrt(b
/a)/(b*sqrt(x))) - (3*b*x - a)*sqrt(x))/(a^2*x^2)]
```

giac [A] time = 1.16, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="giac")
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$\frac{2b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{2b}{a^2\sqrt{x}}-\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a),x)
```

```
[Out] -2/3/a/x^(3/2)+2*b/a^2/x^(1/2)+2/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b
*x^(1/2))
```

maxima [A] time = 3.02, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2\,(3\,bx-a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2/3*(3*b*x - a)/(a^2*x^
(3/2))
```

mupad [B] time = 0.10, size = 38, normalized size = 0.72

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{3\,a}-\frac{2\,b\,x}{a^2}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/(3*a) - (2*b*x)/a^
2)/x^(3/2)
```

sympy [A] time = 7.83, size = 121, normalized size = 2.28

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for } a=0\wedge b=0\\ -\frac{2}{3ax^{\frac{3}{2}}} & \text{for } b=0\\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for } a=0\\ -\frac{2}{3ax^{\frac{3}{2}}}+\frac{2b}{a^2\sqrt{x}}-\frac{ib\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{ib\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a*x**(3/2)), Eq(b, 0)
), (-2/(5*b*x**(5/2)), Eq(a, 0)), (-2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) -
 I*b*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) + I*b*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

3.455 $\int \frac{1}{x^{7/2}(a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}-\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)),x]
```

```
[Out] -2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) - (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)}\,dx &= -\frac{2}{5ax^{5/2}} - \frac{b\int \frac{1}{x^{5/2}(a+bx)}\,dx}{a} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} + \frac{b^2\int \frac{1}{x^{3/2}(a+bx)}\,dx}{a^2} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{b^3\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3} \\
&= -\frac{2}{5ax^{5/2}} + \frac{2b}{3a^2x^{3/2}} - \frac{2b^2}{a^3\sqrt{x}} - \frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.40

$$
-\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};-\frac{bx}{a}\right)}{5ax^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)),x]
[Out] (-2*Hypergeometric2F1[-5/2, 1, -3/2, -((b*x)/a)])/(5*a*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 0.90

$$
-\frac{2b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{2(3a^2-5abx+15b^2x^2)}{15a^3x^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x)),x]
[Out] (-2*(3*a^2 - 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTan[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 1.04, size = 144, normalized size = 2.12

$$
\left[\frac{15\,b^2x^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3}, \frac{2\left(15\,b^2x^3\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2-5\,abx+3\,a^2\right)\sqrt{x}\right)}{15\,a^3x^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="fricas")
[Out] [1/15*(15*b^2*x^3*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*x +
a)) - 2*(15*b^2*x^2 - 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3
*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 - 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

giac [A] time = 0.99, size = 52, normalized size = 0.76

$$
-\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3} - \frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^3}-\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a),x)
```

```
[Out] -2/5/a/x^(5/2)-2*b^2/a^3/x^(1/2)+2/3*b/a^2/x^(3/2)-2/a^3*b^3/(a*b)^(1/2)*ar
ctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.91, size = 52, normalized size = 0.76

$$-\frac{2\,b^3 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{2\left(15\,b^2x^2-5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*b^3*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/15*(15*b^2*x^2 - 5*a
*b*x + 3*a^2)/(a^3*x^(5/2))
```

mupad [B] time = 0.11, size = 49, normalized size = 0.72

$$-\frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}-\frac{2\,b\,x}{3\,a^2}}{x^{5/2}}-\frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)),x)
```

```
[Out] - (2/(5*a) + (2*b^2*x^2)/a^3 - (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atan((
b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

sympy [A] time = 24.82, size = 139, normalized size = 2.04

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ a = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}-\frac{2b^2}{a^3\sqrt{x}}+\frac{ib^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{ib^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2)
) - 2*b**2/(a**3*sqrt(x)) + I*b**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**
(7/2)*sqrt(1/b)) - I*b**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt
(1/b)), True))
```

3.456 $\int \frac{x^{5/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}-\frac{5a\sqrt{x}}{b^3}-\frac{x^{5/2}}{b(a+bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^2,x]
```

```
[Out] (-5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) - x^(5/2)/(b*(a + b*x)) + (5*a^(3/
2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^2}\,dx &= -\frac{x^{5/2}}{b(a+bx)}+\frac{5\int \frac{x^{3/2}}{a+bx}\,dx}{2b}\\
&= \frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}-\frac{(5a)\int \frac{\sqrt{x}}{a+bx}\,dx}{2b^2}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{\left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}-\frac{x^{5/2}}{b(a+bx)}+\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.39

$$\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^2,x]
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*x)/a)])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.08, size = 74, normalized size = 1.06

$$\frac{5a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{-15a^2\sqrt{x}-10abx^{3/2}+2b^2x^{5/2}}{3b^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^2,x]
[Out] (-15*a^2*Sqrt[x] - 10*a*b*x^(3/2) + 2*b^2*x^(5/2))/(3*b^3*(a + b*x)) + (5*a
^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

fricas [A] time = 0.92, size = 161, normalized size = 2.30

$$\left[\frac{15\left(abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{6\left(b^4x+ab^3\right)},\frac{15\left(abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(2b^2x^2-10abx-15a^2\right)\sqrt{x}}{3\left(b^4x+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] [1/6*(15*(a*b*x + a^2)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*sqrt(-a/b) - a)/(b
*x + a)) + 2*(2*b^2*x^2 - 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3), 1/3*
(15*(a*b*x + a^2)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (2*b^2*x^2 - 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x + a*b^3)]
```

giac [A] time = 0.93, size = 65, normalized size = 0.93

$$\frac{5a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}-\frac{a^2\sqrt{x}}{(bx+a)b^3}+\frac{2\left(b^4x^{\frac{3}{2}}-6ab^3\sqrt{x}\right)}{3b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) - a^2*sqrt(x)/((b*x + a)*
b^3) + 2/3*(b^4*x^(3/2) - 6*a*b^3*sqrt(x))/b^6
```

maple [A] time = 0.01, size = 61, normalized size = 0.87

$$\frac{5a^2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3}-\frac{a^2\sqrt{x}}{(bx+a)\, b^3}+\frac{2x^{\frac{3}{2}}}{3b^2}-\frac{4a\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^2,x)
```

```
[Out] 2/3*x^(3/2)/b^2-4*a*x^(1/2)/b^3-1/b^3*a^2*x^(1/2)/(b*x+a)+5/b^3*a^2/(a*b)^(
1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.96, size = 63, normalized size = 0.90

$$-\frac{a^2\sqrt{x}}{b^4x+ab^3}+\frac{5\,a^2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^3}+\frac{2\left(bx^{\frac{3}{2}}-6\,a\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x + a*b^3) + 5*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)
*b^3) + 2/3*(b*x^(3/2) - 6*a*sqrt(x))/b^3
```

mupad [B] time = 0.11, size = 58, normalized size = 0.83

$$\frac{2\,x^{3/2}}{3\,b^2}-\frac{4\,a\,\sqrt{x}}{b^3}-\frac{a^2\,\sqrt{x}}{x\,b^4+a\,b^3}+\frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) - (4*a*x^(1/2))/b^3 - (a^2*x^(1/2))/(a*b^3 + b^4*x) + (
5*a^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/b^(7/2)
```

sympy [A] time = 24.57, size = 479, normalized size = 6.84

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^2} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^2} & \text{for}\: a = 0 \\ -\frac{30ia^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{20ia^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{4i\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{15a^3\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{15a^3\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}+\frac{15a^2bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}}-\frac{15a^2bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{3}{2}}b^4\sqrt{\frac{1}{b}}+6i\sqrt{a}b^5x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-30*I*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(
6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 20*I*a**(3/2)
*b**2*x**(3/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*
sqrt(1/b)) + 4*I*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(6*I*a**(3/2)*b**4*sqrt(1/
b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(-I*sqrt(a)*sqrt(1/b) + sqr
```

```
t(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**
3*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqr
t(a)*b**5*x*sqrt(1/b)) + 15*a**2*b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6
*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*lo
g(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(3/2)*b**4*sqrt(1/b) + 6*I*sqrt(a)
*b**5*x*sqrt(1/b)), True))
```

3.457 $\int \frac{x^{3/2}}{(a+bx)^2} \, dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$-\frac{3\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}-\frac{x^{3/2}}{b(a+bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 - x^(3/2)/(b*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a+bx)^2}\,dx &= -\frac{x^{3/2}}{b(a+bx)}+\frac{3\int \frac{\sqrt{x}}{a+bx}\,dx}{2b}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b^2}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{(3a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b^2}\\ &= \frac{3\sqrt{x}}{b^2}-\frac{x^{3/2}}{b(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.47

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*x)/a)])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 58, normalized size = 1.02

$$\frac{3a\sqrt{x}+2bx^{3/2}}{b^2(a+bx)}-\frac{3\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*a*Sqrt[x] + 2*b*x^(3/2))/(b^2*(a + b*x)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/b^(5/2)
```

fricas [A] time = 0.83, size = 134, normalized size = 2.35

$$\left[\frac{3\,(bx+a)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\,(2\,bx+3\,a)\sqrt{x}}{2\left(b^3x+ab^2\right)},-\frac{3\,(bx+a)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-(2\,bx+3\,a)\sqrt{x}}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x + a)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-a/b) - a)/(b*x +
a)) + 2*(2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2), -(3*(b*x + a)*sqrt(a/b)*arc
tan(b*sqrt(x)*sqrt(a/b)/a) - (2*b*x + 3*a)*sqrt(x))/(b^3*x + a*b^2)]
```

giac [A] time = 0.96, size = 46, normalized size = 0.81

$$-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{a\sqrt{x}}{(bx+a)b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + a*sqrt(x)/((b*x + a)*b^2
) + 2*sqrt(x)/b^2
```

maple [A] time = 0.01, size = 47, normalized size = 0.82

$$-\frac{3a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{a\sqrt{x}}{\left(bx+a\right)b^{2}}+\frac{2\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*x^(1/2)/b^2+1/b^2*a*x^(1/2)/(b*x+a)-3/b^2*a/(a*b)^(1/2)*arctan(1/(a*b)^(1
/2)*b*x^(1/2))
```

maxima [A] time = 2.91, size = 49, normalized size = 0.86

$$\frac{a\sqrt{x}}{b^{3}x+ab^{2}}-\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\,\sqrt{x}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*sqrt(x)/(b^3*x + a*b^2) - 3*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
 + 2*sqrt(x)/b^2
```

mupad [B] time = 0.12, size = 46, normalized size = 0.81

$$\frac{2\,\sqrt{x}}{b^{2}}+\frac{a\,\sqrt{x}}{x\,b^{3}+a\,b^{2}}-\frac{3\,\sqrt{a}\,\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 + b^3*x) - (3*a^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/b^(5/2)
```

sympy [A] time = 9.18, size = 411, normalized size = 7.21

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^{2}} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^{2}} & \text{for}\: a = 0 \\ \frac{6ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{4i\sqrt{a}\,b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}-\frac{3a^{2}\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{3a^{2}\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(-i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(i\sqrt{a}\,\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^{3}\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^{4}x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(2*I*a**(
3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 4*I*sqrt(a)*b**2*x**(
3/2)*sqrt(1/b)/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b))
 - 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b)
+ 2*I*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a*b*x*log
(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**3*sqrt(1/b) + 2*I*sqrt(a)
*b**4*x*sqrt(1/b)) + 3*a*b*x*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/
2)*b**3*sqrt(1/b) + 2*I*sqrt(a)*b**4*x*sqrt(1/b)), True))
```

3.458 $\int \frac{\sqrt{x}}{(a+bx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = $\{47, 63, 205\}$

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(a+bx)^2}\,dx &= -\frac{\sqrt{x}}{b(a+bx)}+\frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{2b}\\
&= -\frac{\sqrt{x}}{b(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{b}\\
&= -\frac{\sqrt{x}}{b(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

IntegrateAlgebraic [A] time = 0.06, size = 46, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[x]/(b*(a + b*x))) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/
2))
```

fricas [A] time = 0.89, size = 115, normalized size = 2.50

$$\left[-\frac{2\,ab\sqrt{x}+\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\sqrt{-ab}\sqrt{x}}{bx+a}\right)}{2\left(ab^3x+a^2b^2\right)},\,-\frac{ab\sqrt{x}+\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{ab^3x+a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) + sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)))/(a*b^3*x + a^2*b^2), -(a*b*sqrt(x) + sqrt(a*b)*(b*x + a)*
arctan(sqrt(a*b)/(b*sqrt(x))))/(a*b^3*x + a^2*b^2)]
```

giac [A] time = 0.90, size = 36, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sqrt{x}}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b) - sqrt(x)/((b*x + a)*b)
```

maple [A] time = 0.01, size = 37, normalized size = 0.80

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{\sqrt{x}}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^2,x)
```

```
[Out] -x^(1/2)/b/(b*x+a)+1/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.94, size = 37, normalized size = 0.80

$$-\frac{\sqrt{x}}{b^2x+ab}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x + a*b) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b)
```

mupad [B] time = 0.04, size = 34, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^2,x)
```

```
[Out] atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2)) - x^(1/2)/(b*(a + b*x))
```

sympy [A] time = 4.45, size = 337, normalized size = 7.33

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for}\: b = 0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for}\: a = 0 \\ -\frac{2i\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}+\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}+\frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}-\frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2i\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a
**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + a*log(-I*sqrt(a)*s
qrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(
1/b)) - a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) +
 2*I*sqrt(a)*b**3*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2
*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(I*sqrt
(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(3/2)*b**2*sqrt(1/b) + 2*I*sqrt(a)*b**3*x*
sqrt(1/b)), True))
```

3.459 $\int \frac{1}{\sqrt{x}(a+bx)^2} dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^2),x]

[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)^2} dx &= \frac{\sqrt{x}}{a(a+bx)}+\frac{\int \frac{1}{\sqrt{x}(a+bx)} dx}{2a}\\ &= \frac{\sqrt{x}}{a(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{x}\right)}{a}\\ &= \frac{\sqrt{x}}{a(a+bx)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a^2+abx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a^2 + a*b*x) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.05, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^2),x]
[Out] Sqrt[x]/(a*(a + b*x)) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b])
```

fricas [A] time = 0.87, size = 116, normalized size = 2.58

$$\left[\frac{2\,ab\sqrt{x}-\sqrt{-ab}\,(bx+a)\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)}{2\left(a^2b^2x+a^3b\right)},\frac{ab\sqrt{x}-\sqrt{ab}\,(bx+a)\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)}{a^2b^2x+a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="fricas")
[Out] [1/2*(2*a*b*sqrt(x) - sqrt(-a*b)*(b*x + a)*log((b*x - a - 2*sqrt(-a*b)*sqrt
(x))/(b*x + a)))/(a^2*b^2*x + a^3*b), (a*b*sqrt(x) - sqrt(a*b)*(b*x + a)*ar
ctan(sqrt(a*b)/(b*sqrt(x))))/(a^2*b^2*x + a^3*b)]
```

giac [A] time = 0.89, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\sqrt{x}}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="giac")
[Out] arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a) + sqrt(x)/((b*x + a)*a)
```

maple [A] time = 0.01, size = 36, normalized size = 0.80

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\sqrt{x}}{(bx+a)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/x^(1/2),x)
[Out] x^(1/2)/a/(b*x+a)+1/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.92, size = 35, normalized size = 0.78

$$\frac{\sqrt{x}}{abx+a^2}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/(a*b*x + a^2) + arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 0.09, size = 33, normalized size = 0.73

$$\frac{\sqrt{x}}{a\,(a+b\,x)}+\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a + b*x)) + atan((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

sympy [A] time = 7.53, size = 328, normalized size = 7.29

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a=0 \wedge b=0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\ b=0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\ a=0 \\ \frac{2i\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}}+\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}}+\frac{bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}}-\frac{bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2ia^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (2*I*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(2*I*a**
(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b))
- a*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(
3/2)*b**2*x*sqrt(1/b)) + b*x*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(5
/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)) - b*x*log(I*sqrt(a)*sqrt(1
/b) + sqrt(x))/(2*I*a**(5/2)*b*sqrt(1/b) + 2*I*a**(3/2)*b**2*x*sqrt(1/b)),
True))
```

3.460 $\int \frac{1}{x^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a+bx)}+\frac{3\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{(3b)\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.45

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, -((b*x)/a)])/(a^2*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.07, size = 54, normalized size = 0.96

$$\frac{-2a-3bx}{a^2\sqrt{x}\,(a+bx)}-\frac{3\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a - 3*b*x)/(a^2*Sqrt[x]*(a + b*x)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.96, size = 147, normalized size = 2.62

$$\left[\frac{3\left(bx^2+ax\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\,(3\,bx+2\,a)\sqrt{x}}{2\left(a^2bx^2+a^3x\right)},\ \frac{3\left(bx^2+ax\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-(3\,bx+2\,a)\sqrt{x}}{a^2bx^2+a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 + a*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sqrt(-b/a) - a)/(b*
x + a)) - 2*(3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2 + a^3*x), (3*(b*x^2 + a*x)*sq
rt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (3*b*x + 2*a)*sqrt(x))/(a^2*b*x^2
 + a^3*x)]
```

giac [A] time = 1.01, size = 49, normalized size = 0.88

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}-\frac{3\,bx+2\,a}{\left(bx^{\frac{3}{2}}+a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) - (3*b*x + 2*a)/((b*x^(3/2
) + a*sqrt(x))*a^2)
```

maple [A] time = 0.01, size = 48, normalized size = 0.86

$$-\frac{3b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}-\frac{b\sqrt{x}}{(bx+a)\,a^2}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^2,x)
```

```
[Out] -2/a^2/x^(1/2)-1/a^2*b*x^(1/2)/(b*x+a)-3/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(
1/2)*b*x^(1/2))
```

maxima [A] time = 2.94, size = 51, normalized size = 0.91

$$-\frac{3\,bx+2\,a}{a^2bx^{\frac{3}{2}}+a^3\sqrt{x}}-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x + 2*a)/(a^2*b*x^(3/2) + a^3*sqrt(x)) - 3*b*arctan(b*sqrt(x)/sqrt(a*
b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.12, size = 48, normalized size = 0.86

$$-\frac{\frac{2}{a}+\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}+b\,x^{3/2}}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^2),x)
```

```
[Out] - (2/a + (3*b*x)/a^2)/(a*x^(1/2) + b*x^(3/2)) - (3*b^(1/2)*atan((b^(1/2)*x^
(1/2))/a^(1/2)))/a^(5/2)
```

sympy [A] time = 17.73, size = 434, normalized size = 7.75

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b^{2} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{a^{2} \sqrt{x}} & \text{for}\: b = 0 \\ - \frac{4 i a^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{6 i \sqrt{a} b x \sqrt{\frac{1}{b}}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{3 a \sqrt{x} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} + \frac{3 a \sqrt{x} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} - \frac{3 b x^{\frac{3}{2}} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} + \frac{3 b x^{\frac{3}{2}} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{2 i a^{\frac{7}{2}} \sqrt{x} \sqrt{\frac{1}{b}} + 2 i a^{\frac{5}{2}} b x^{\frac{3}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**2*x**(5/2)), Eq(a,
 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)), (-4*I*a**(3/2)*sqrt(1/b)/(2*I*a**(7/2)
*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 6*I*sqrt(a)*b*x*s
qrt(1/b)/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b
)) - 3*a*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*
sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3
/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(
7/2)*sqrt(x)*sqrt(1/b) + 2*I*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**(3/2)*
log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2*I*a**(7/2)*sqrt(x)*sqrt(1/b) + 2*I*a*
*(5/2)*b*x**(3/2)*sqrt(1/b)), True))
```

3.461 $\int \frac{1}{x^{5/2}(a+bx)^2}\,dx$

Optimal. Leaf size=69

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) + (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a + b*x)) + (5*b^(
3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^2}\,dx &= \frac{1}{ax^{3/2}(a+bx)}+\frac{5\int \frac{1}{x^{5/2}(a+bx)}\,dx}{2a}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a+bx)}-\frac{(5b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{2a^2}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{\left(5b^2\right)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{2a^3}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\
&= -\frac{5}{3a^2x^{3/2}}+\frac{5b}{a^3\sqrt{x}}+\frac{1}{ax^{3/2}(a+bx)}+\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.39

$$
-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{bx}{a}\right)}{3a^2x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, -((b*x)/a)])/(3*a^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 68, normalized size = 0.99

$$
\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{-2a^2+10abx+15b^2x^2}{3a^3x^{3/2}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^2),x]
```

```
[Out] (-2*a^2 + 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a + b*x)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 1.01, size = 184, normalized size = 2.67

$$
\left[\frac{15\left(b^2x^3+abx^2\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{-\frac{b}{a}}-a}{bx+a}\right)+2\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3+a^4x^2\right)},-\frac{15\left(b^2x^3+abx^2\right)\sqrt{\frac{b}{a}}\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15b^2x^2+10abx-2a^2\right)\sqrt{x}}{3\left(a^3bx^3+a^4x^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 + a*b*x^2)*sqrt(-b/a)*log((b*x + 2*a*sqrt(x)*sqrt(-b/a) -
 a)/(b*x + a)) + 2*(15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^
4*x^2), -1/3*(15*(b^2*x^3 + a*b*x^2)*sqrt(b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x
))) - (15*b^2*x^2 + 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 + a^4*x^2)]
```

giac [A] time = 0.96, size = 58, normalized size = 0.84

$$
\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{b^2\sqrt{x}}{(bx+a)a^3}+\frac{2\,(6\,bx-a)}{3\,a^3x^{\frac{3}{2}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) + b^2*sqrt(x)/((b*x + a)*
a^3) + 2/3*(6*b*x - a)/(a^3*x^(3/2))
```

maple [A] time = 0.02, size = 60, normalized size = 0.87

$$\frac{5b^2 \arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^3}+\frac{b^2\sqrt{x}}{(bx+a)\,a^3}+\frac{4b}{a^3\sqrt{x}}-\frac{2}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^2,x)
```

```
[Out] -2/3/a^2/x^(3/2)+4*b/a^3/x^(1/2)+1/a^3*b^2*x^(1/2)/(b*x+a)+5/a^3*b^2/(a*b)^
(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.88, size = 64, normalized size = 0.93

$$\frac{15\,b^2x^2+10\,abx-2\,a^2}{3\left(a^3bx^{\frac{5}{2}}+a^4x^{\frac{3}{2}}\right)}+\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(15*b^2*x^2 + 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) + a^4*x^(3/2)) + 5*b^2*a
rctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.15, size = 58, normalized size = 0.84

$$\frac{\frac{5\,b^2\,x^2}{a^3}-\frac{2}{3\,a}+\frac{10\,b\,x}{3\,a^2}}{a\,x^{3/2}+b\,x^{5/2}}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^2),x)
```

```
[Out] ((5*b^2*x^2)/a^3 - 2/(3*a) + (10*b*x)/(3*a^2))/(a*x^(3/2) + b*x^(5/2)) + (5
*b^(3/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

sympy [A] time = 50.52, size = 507, normalized size = 7.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{3a^2x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{7b^2x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ -\frac{4ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{20ia^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{30i\sqrt{a}\,b^2x^2\sqrt{\frac{1}{b}}}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15abx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15abx^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15b^2x^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15b^2x^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{6ia^{\frac{9}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}+6ia^{\frac{7}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(b,
 0)), (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (-4*I*a**(5/2)*sqrt(1/b)/(6*I*a**(9
/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 20*I*a**(3/2)
*b*x*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*s
qrt(1/b)) + 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*I*a**(9/2)*x**(3/2)*sqrt(1/
```

```
b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x**(3/2)*log(-I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/
2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**
(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b)
+ 6*I*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(
1/b) + sqrt(x))/(6*I*a**(9/2)*x**(3/2)*sqrt(1/b) + 6*I*a**(7/2)*b*x**(5/2)*
sqrt(1/b)), True))
```

3.462 $\int \frac{x^{7/2}}{(a+bx)^3} dx$

Optimal. Leaf size=95

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{35a\sqrt{x}}{4b^4}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 50, 63, 205}

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a+bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a + b*x)^2)
 - (7*x^(5/2))/(4*b^2*(a + b*x)) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a+bx)^2}+\frac{7\int \frac{x^{5/2}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35\int \frac{x^{3/2}}{a+bx}\,dx}{8b^2}\\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}-\frac{(35a)\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^3}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4}\\
&= -\frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a+bx)^2}-\frac{7x^{5/2}}{4b^2(a+bx)}+\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.28

$$\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};-\frac{bx}{a}\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, -((b*x)/a)])/(9*a^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 89, normalized size = 0.94

$$\frac{35a^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{-105a^3\sqrt{x}-175a^2bx^{3/2}-56ab^2x^{5/2}+8b^3x^{7/2}}{12b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(a + b*x)^3,x]
```

```
[Out] (-105*a^3*Sqrt[x] - 175*a^2*b*x^(3/2) - 56*a*b^2*x^(5/2) + 8*b^3*x^(7/2))/(
12*b^4*(a + b*x)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(
9/2))
```

fricas [A] time = 0.91, size = 227, normalized size = 2.39

$$\left[\frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx+2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{24\left(b^6x^2+2ab^5x+a^2b^4\right)}, \frac{105\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)+\left(8b^3x^3-56ab^2x^2-175a^2bx-105a^3\right)\sqrt{x}}{12\left(b^6x^2+2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(-a/b)*log((b*x + 2*b*sqrt(x)*
sqrt(-a/b) - a)/(b*x + a)) + 2*(8*b^3*x^3 - 56*a*b^2*x^2 - 175*a^2*b*x - 10
5*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 + 2*a
^2*b*x + a^3)*sqrt(a/b)*arctan(b*sqrt(x)*sqrt(a/b)/a) + (8*b^3*x^3 - 56*a*b
^2*x^2 - 175*a^2*b*x - 105*a^3)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

giac [A] time = 1.03, size = 77, normalized size = 0.81

$$\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^4}-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\,(bx+a)^2b^4}+\frac{2\left(b^6x^{\frac{3}{2}}-9\,ab^5\sqrt{x}\right)}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/4*(13*a^2*b*x^(3/2
) + 11*a^3*sqrt(x))/((b*x + a)^2*b^4) + 2/3*(b^6*x^(3/2) - 9*a*b^5*sqrt(x))
/b^9
```

maple [A] time = 0.02, size = 79, normalized size = 0.83

$$-\frac{13a^2x^{\frac{3}{2}}}{4\,(bx+a)^2\,b^3}-\frac{11a^3\sqrt{x}}{4\,(bx+a)^2\,b^4}+\frac{35a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,\,b^4}+\frac{2x^{\frac{3}{2}}}{3b^3}-\frac{6a\sqrt{x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x+a)^3,x)
```

```
[Out] 2/3*x^(3/2)/b^3-6*a*x^(1/2)/b^4-13/4/b^3*a^2/(b*x+a)^2*x^(3/2)-11/4/b^4*a^3
/(b*x+a)^2*x^(1/2)+35/4/b^4*a^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 3.07, size = 86, normalized size = 0.91

$$-\frac{13\,a^2bx^{\frac{3}{2}}+11\,a^3\sqrt{x}}{4\left(b^6x^2+2\,ab^5x+a^2b^4\right)}+\frac{35\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^4}+\frac{2\left(bx^{\frac{3}{2}}-9\,a\sqrt{x}\right)}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) + 11*a^3*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
35/4*a^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^4) + 2/3*(b*x^(3/2) - 9*a
*sqrt(x))/b^4
```

mupad [B] time = 0.12, size = 81, normalized size = 0.85

$$\frac{2\,x^{3/2}}{3\,b^3}-\frac{\frac{11\,a^3\,\sqrt{x}}{4}+\frac{13\,a^2\,b\,x^{3/2}}{4}}{a^2\,b^4+2\,a\,b^5\,x+b^6\,x^2}-\frac{6\,a\,\sqrt{x}}{b^4}+\frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a + b*x)^3,x)
```

```
[Out] (2*x^(3/2))/(3*b^3) - ((11*a^3*x^(1/2))/4 + (13*a^2*b*x^(3/2))/4)/(a^2*b^4
+ b^6*x^2 + 2*a*b^5*x) - (6*a*x^(1/2))/b^4 + (35*a^(3/2)*atan((b^(1/2)*x^(1
/2))/a^(1/2)))/(4*b^(9/2))
```

sympy [A] time = 135.24, size = 906, normalized size = 9.54

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{9}{2}}}{9a^3} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^3} & \text{for}\: a = 0 \\ -\frac{210ia^{\frac{7}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} - \frac{350ia^{\frac{5}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} - \frac{112ia^{\frac{3}{2}}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} + \frac{16i\sqrt{a}b^4x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} + \frac{105a^4\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} - \frac{105a^4\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} + \frac{210a^3bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} - \frac{210a^3bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} + \frac{105a^2b^2x^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} - \frac{105a^2b^2x^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24ia^{\frac{5}{2}}b^5\sqrt{\frac{1}{b}}+48ia^{\frac{3}{2}}b^6x\sqrt{\frac{1}{b}}+24i\sqrt{a}b^7x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**3), Eq(b,
0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (-210*I*a**(7/2)*b*sqrt(x)*sqrt(1/b)/
(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(
a)*b**7*x**2*sqrt(1/b)) - 350*I*a**(5/2)*b**2*x**(3/2)*sqrt(1/b)/(24*I*a**(
5/2)*b**5*sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x*
*2*sqrt(1/b)) - 112*I*a**(3/2)*b**3*x**(5/2)*sqrt(1/b)/(24*I*a**(5/2)*b**5*
sqrt(1/b) + 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/
b)) + 16*I*sqrt(a)*b**4*x**(7/2)*sqrt(1/b)/(24*I*a**(5/2)*b**5*sqrt(1/b) +
48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a*
*4*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I
*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**4*l
og(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(
3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 210*a**3*b*x*lo
g(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(
3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 210*a**3*b*x*lo
g(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I*a**(3
/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a**2*b**2*x*
*2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) + 48*I
*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**2*b
**2*x**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(5/2)*b**5*sqrt(1/b) +
 48*I*a**(3/2)*b**6*x*sqrt(1/b) + 24*I*sqrt(a)*b**7*x**2*sqrt(1/b)), True))
```

3.463 $\int \frac{x^{5/2}}{(a+bx)^3} dx$

Optimal. Leaf size=82

$$-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {47, 50, 63, 205}

$$-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a+bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a + b*x)^2) - (5*x^(3/2))/(4*b^2*(a +
b*x)) - (15*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{5/2}}{(a+bx)^3}\,dx &= -\frac{x^{5/2}}{2b(a+bx)^2}+\frac{5\int \frac{x^{3/2}}{(a+bx)^2}\,dx}{4b}\\ &= -\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}+\frac{15\int \frac{\sqrt{x}}{a+bx}\,dx}{8b^2}\\ &= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^3}\\ &= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{(15a)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^3}\\ &= \frac{15\sqrt{x}}{4b^3}-\frac{x^{5/2}}{2b(a+bx)^2}-\frac{5x^{3/2}}{4b^2(a+bx)}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.33

$$\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*x)/a)])/(7*a^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 76, normalized size = 0.93

$$\frac{15a^2\sqrt{x}+25abx^{3/2}+8b^2x^{5/2}}{4b^3(a+bx)^2}-\frac{15\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*a^2*Sqrt[x] + 25*a*b*x^(3/2) + 8*b^2*x^(5/2))/(4*b^3*(a + b*x)^2) - (15
*Sqrt[a]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

fricas [A] time = 0.59, size = 200, normalized size = 2.44

$$\left[\frac{15\left(b^2x^2+2abx+a^2\right)\sqrt{-\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{-\frac{a}{b}}-a}{bx+a}\right)+2\left(8b^2x^2+25abx+15a^2\right)\sqrt{x}}{8\left(b^5x^2+2ab^4x+a^2b^3\right)},-\frac{15\left(b^2x^2+2abx+a^2\right)\sqrt{\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{\frac{a}{b}}}{a}\right)-\left(8b^2x^2+25abx+15a^2\right)\sqrt{x}}{4\left(b^5x^2+2ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a/b)*log((b*x - 2*b*sqrt(x)*sqrt(-
a/b) - a)/(b*x + a)) + 2*(8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a/b)*arctan
(b*sqrt(x)*sqrt(a/b)/a) - (8*b^2*x^2 + 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
 + 2*a*b^4*x + a^2*b^3)]
```

giac [A] time = 0.95, size = 59, normalized size = 0.72

$$-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^3}+\frac{2\sqrt{x}}{b^3}+\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\,(bx+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3 + 1/4*(
9*a*b*x^(3/2) + 7*a^2*sqrt(x))/((b*x + a)^2*b^3)
```

maple [A] time = 0.02, size = 66, normalized size = 0.80

$$\frac{9a\,x^{\frac{3}{2}}}{4\,(bx+a)^2\,b^2}+\frac{7a^2\sqrt{x}}{4\,(bx+a)^2\,b^3}-\frac{15a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^3}+\frac{2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*x^(1/2)/b^3+9/4/b^2*a/(b*x+a)^2*x^(3/2)+7/4/b^3*a^2/(b*x+a)^2*x^(1/2)-15/
4/b^3*a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.94, size = 73, normalized size = 0.89

$$\frac{9\,abx^{\frac{3}{2}}+7\,a^2\sqrt{x}}{4\left(b^5x^2+2\,ab^4x+a^2b^3\right)}-\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*a*b*x^(3/2) + 7*a^2*sqrt(x))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3) - 15/4*
a*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^3) + 2*sqrt(x)/b^3
```

mupad [B] time = 0.14, size = 69, normalized size = 0.84

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}+\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3+2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 + (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

sympy [A] time = 53.29, size = 816, normalized size = 9.95

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a=0\wedge b=0\\ \frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\ b=0\\ \frac{2\sqrt{x}}{b^3} & \text{for}\ a=0\\ \frac{30ia^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}+\frac{50ia^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}+\frac{16i\sqrt{a}b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}-\frac{15a^3\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}+\frac{15a^3\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}-\frac{30a^2bx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}+\frac{30a^2bx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}-\frac{15ab^2x^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}}+\frac{15ab^2x^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^6x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**3), Eq(b, 0
)), (2*sqrt(x)/b**3, Eq(a, 0)), (30*I*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(8*I*a**
(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x*
*2*sqrt(1/b)) + 50*I*a**(3/2)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(5/2)*b**4*sq
rt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b))
 + 16*I*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I
*a**(3/2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a**3*log
(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/
```

```
2)*b**5*x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a**3*log(I*sqrt
(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*
x*sqrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 30*a**2*b*x*log(-I*sqrt(a)
*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*s
qrt(1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 30*a**2*b*x*log(I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(
1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a*b**2*x**2*log(-I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(
1/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a*b**2*x**2*log(I*sqrt(a)*sqrt
(1/b) + sqrt(x))/(8*I*a**(5/2)*b**4*sqrt(1/b) + 16*I*a**(3/2)*b**5*x*sqrt(1
/b) + 8*I*sqrt(a)*b**6*x**2*sqrt(1/b)), True))
```

3.464 $\int \frac{x^{3/2}}{(a+bx)^3} dx$

Optimal. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{3\sqrt{x}}{4b^2(a+bx)}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 205}

$$-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a + b*x)^2) - (3*Sqrt[x])/(4*b^2*(a + b*x)) + (3*ArcTan[(Sqr
t[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a+bx)^2}+\frac{3\int \frac{\sqrt{x}}{(a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{x^{3/2}}{2b(a+bx)^2}-\frac{3\sqrt{x}}{4b^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.84

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{\sqrt{x}\,(3a+5bx)}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(3*a + 5*b*x))/(b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 63, normalized size = 0.90

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{-3a\sqrt{x}\;-5bx^{3/2}}{4b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*a*Sqrt[x] - 5*b*x^(3/2))/(4*b^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

fricas [A] time = 1.01, size = 185, normalized size = 2.64

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)+2\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)+\left(5\,ab^2x+3\,a^2b\right)\sqrt{x}}{4\left(ab^5x^2+2\,a^2b^4x+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) + 2*(5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^
4*x + a^3*b^3), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) + (5*a*b^2*x + 3*a^2*b)*sqrt(x))/(a*b^5*x^2 + 2*a^2*b^4*x +
a^3*b^3)]
```

giac [A] time = 0.91, size = 47, normalized size = 0.67

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/4*(5*b*x^(3/2) + 3*a*sq
rt(x))/((b*x + a)^2*b^2)
```

maple [A] time = 0.01, size = 50, normalized size = 0.71

$$\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;b^2}+\frac{-\frac{5x^{\frac{3}{2}}}{4b}-\frac{3a\sqrt{x}}{4b^2}}{(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^3,x)
```

```
[Out] 2*(-5/8/b*x^(3/2)-3/8*a/b^2*x^(1/2))/(b*x+a)^2+3/4/b^2/(a*b)^(1/2)*arctan(1
/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.96, size = 61, normalized size = 0.87

$$-\frac{5\,bx^{\frac{3}{2}}+3\,a\sqrt{x}}{4\left(b^4x^2+2\,ab^3x+a^2b^2\right)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) + 3*a*sqrt(x))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 3/4*arct
an(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*b^2)
```

mupad [B] time = 0.13, size = 58, normalized size = 0.83

$$\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}-\frac{\frac{5\,x^{3/2}}{4\,b}+\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2)) - ((5*x^(3/2))/(4*b
) + (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 + 2*a*b*x)
```

sympy [A] time = 29.37, size = 726, normalized size = 10.37

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^3} & \text{for}\: b = 0 \\ -\frac{2}{b^3\sqrt{x}} & \text{for}\: a = 0 \\ -\frac{6ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{10i\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{6abx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{6abx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{3b^2x^2\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{3b^2x^2\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}+16ia^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8i\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b, 0
)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (-6*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*
a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5
*x**2*sqrt(1/b)) - 10*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(5/2)*b**3*
sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b
)) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b
) + 16*I*a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a
**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*
a**(3/2)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 6*a*b*x*log(
-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2
)*b**4*x*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x
*sqrt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*
sqrt(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sq
rt(1/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt
(1/b) + sqrt(x))/(8*I*a**(5/2)*b**3*sqrt(1/b) + 16*I*a**(3/2)*b**4*x*sqrt(1
/b) + 8*I*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

3.465 $\int \frac{\sqrt{x}}{(a+bx)^3} \, dx$

Optimal. Leaf size=73

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a+bx)}-\frac{\sqrt{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a + b*x)^2) + Sqrt[x]/(4*a*b*(a + b*x)) + ArcTan[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^3}\,dx = -\frac{\sqrt{x}}{2b(a+bx)^2} + \frac{\int \frac{1}{\sqrt{x}(a+bx)^2}\,dx}{4b}$$
$$= -\frac{\sqrt{x}}{2b(a+bx)^2} + \frac{\sqrt{x}}{4ab(a+bx)} + \frac{\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8ab}$$
$$= -\frac{\sqrt{x}}{2b(a+bx)^2} + \frac{\sqrt{x}}{4ab(a+bx)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sqrt{x}\right)}{4ab}$$
$$= -\frac{\sqrt{x}}{2b(a+bx)^2} + \frac{\sqrt{x}}{4ab(a+bx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.37

$$\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};-\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] (2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*x)/a)])/(3*a^3)
```

IntegrateAlgebraic [A] time = 0.11, size = 60, normalized size = 0.82

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\sqrt{x}(a-bx)}{4ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a - b*x))/(a*b*(a + b*x)^2) + ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]]/(4*a^(3/2)*b^(3/2))
```

fricas [A] time = 0.92, size = 186, normalized size = 2.55

$$\left[-\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(ab^2x-a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)},\ -\frac{\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(ab^2x-a^2b\right)\sqrt{x}}{4\left(a^2b^4x^2+2\,a^3b^3x+a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*sqr
t(x))/(b*x + a)) - 2*(a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x
+ a^4*b^2), -1/4*((b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b)/(b*s
qrt(x))) - (a*b^2*x - a^2*b)*sqrt(x))/(a^2*b^4*x^2 + 2*a^3*b^3*x + a^4*b^2)
]
```

giac [A] time = 1.07, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab} + \frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\,(bx+a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/4*(b*x^(3/2) - a*sqrt(x
))/((b*x + a)^2*a*b)
```

maple [A] time = 0.01, size = 52, normalized size = 0.71

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab}+\frac{\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^3,x)
```

```
[Out] 2*(1/8/a*x^(3/2)-1/8/b*x^(1/2))/(b*x+a)^2+1/4/b/a/(a*b)^(1/2)*arctan(1/(a*b
)^(1/2)*b*x^(1/2))
```

maxima [A] time = 3.01, size = 64, normalized size = 0.88

$$\frac{bx^{\frac{3}{2}}-a\sqrt{x}}{4\left(ab^{3}x^{2}+2\,a^{2}b^{2}x+a^{3}b\right)}+\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^(3/2) - a*sqrt(x))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/4*arctan(
b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.13, size = 56, normalized size = 0.77

$$\frac{\frac{x^{3/2}}{4\,a}-\frac{\sqrt{x}}{4\,b}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^3,x)
```

```
[Out] (x^(3/2)/(4*a) - x^(1/2)/(4*b))/(a^2 + b^2*x^2 + 2*a*b*x) + atan((b^(1/2)*x
^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2))
```

sympy [A] time = 15.27, size = 721, normalized size = 9.88

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{3}{2}}}{3a^{3}} & \text{for}\: b = 0 \\ -\frac{2}{3b^{3}x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ -\frac{2ia^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}+\frac{2i\sqrt{a}b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}+\frac{a^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}-\frac{a^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}+\frac{2abx\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}-\frac{2abx\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}+\frac{b^{2}x^{2}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}}-\frac{b^{2}x^{2}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{7}{2}}b^{2}\sqrt{\frac{1}{b}}+16ia^{\frac{5}{2}}b^{3}x\sqrt{\frac{1}{b}}+8ia^{\frac{3}{2}}b^{4}x^{2}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (-2*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(
8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)
*b**4*x**2*sqrt(1/b)) + 2*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(7/2)*b
**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqr
t(1/b)) + a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(
1/b) + 16*I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) -
```

```
 a**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*
I*a**(5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*l
og(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(
5/2)*b**3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - 2*a*b*x*log(I*s
qrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b*
*3*x*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x**2*log(-I*sqrt(
a)*sqrt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x
*sqrt(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)) - b**2*x**2*log(I*sqrt(a)*sq
rt(1/b) + sqrt(x))/(8*I*a**(7/2)*b**2*sqrt(1/b) + 16*I*a**(5/2)*b**3*x*sqrt
(1/b) + 8*I*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

3.466 $\int \frac{1}{\sqrt{x}(a+bx)^3} dx$

Optimal. Leaf size=70

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{\sqrt{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] Sqrt[x]/(2*a*(a + b*x)^2) + (3*Sqrt[x])/(4*a^2*(a + b*x)) + (3*ArcTan[(Sqrt
[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}(a+bx)^3} dx &= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\int \frac{1}{\sqrt{x}(a+bx)^2} dx}{4a}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\int \frac{1}{\sqrt{x}(a+bx)} dx}{8a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sqrt{x}\right)}{4a^2}\\
&= \frac{\sqrt{x}}{2a(a+bx)^2}+\frac{3\sqrt{x}}{4a^2(a+bx)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.36

$$\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*x)/a)])/a^3
```

IntegrateAlgebraic [A] time = 0.08, size = 63, normalized size = 0.90

$$\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}+\frac{5a\sqrt{x}+3bx^{3/2}}{4a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^3),x]
```

```
[Out] (5*a*Sqrt[x] + 3*b*x^(3/2))/(4*a^2*(a + b*x)^2) + (3*ArcTan[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

fricas [A] time = 0.98, size = 186, normalized size = 2.66

$$\left[-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{-ab}\,\log\left(\frac{bx-a-2\,\sqrt{-ab}\,\sqrt{x}}{bx+a}\right)-2\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)},-\frac{3\left(b^2x^2+2\,abx+a^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}}{b\sqrt{x}}\right)-\left(3\,ab^2x+5\,a^2b\right)\sqrt{x}}{4\left(a^3b^3x^2+2\,a^4b^2x+a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-a*b)*log((b*x - a - 2*sqrt(-a*b)*s
qrt(x))/(b*x + a)) - 2*(3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*
b^2*x + a^5*b), -1/4*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(a*b)*arctan(sqrt(a*b
)/(b*sqrt(x))) - (3*a*b^2*x + 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 + 2*a^4*b^2*x
+ a^5*b)]
```

giac [A] time = 0.86, size = 47, normalized size = 0.67

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^2}+\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\,(bx+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/4*(3*b*x^(3/2) + 5*a*sq
rt(x))/((b*x + a)^2*a^2)
```

maple [A] time = 0.01, size = 53, normalized size = 0.76

$$\frac{\sqrt{x}}{2\,(bx+a)^2\,a}+\frac{3\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,a^2}+\frac{3\sqrt{x}}{4\,(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/x^(1/2),x)
```

```
[Out] 1/2*x^(1/2)/a/(b*x+a)^2+3/4*x^(1/2)/a^2/(b*x+a)+3/4/a^2/(a*b)^(1/2)*arctan(
1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.96, size = 60, normalized size = 0.86

$$\frac{3\,bx^{\frac{3}{2}}+5\,a\sqrt{x}}{4\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}+\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) + 5*a*sqrt(x))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 3/4*arcta
n(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.13, size = 57, normalized size = 0.81

$$\frac{\frac{5\,\sqrt{x}}{4\,a}+\frac{3\,b\,x^{3/2}}{4\,a^{2}}}{a^{2}+2\,a\,b\,x+b^{2}\,x^{2}}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^3),x)
```

```
[Out] ((5*x^(1/2))/(4*a) + (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (3*
atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

sympy [A] time = 25.69, size = 712, normalized size = 10.17

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{5 b^{3} x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{2 \sqrt{x}}{a^{3}} & \text{for}\: b = 0 \\ \frac{10 i a^{\frac{3}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 i \sqrt{a} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 a^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 a^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{6 a b x \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{6 a b x \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} + \frac{3 b^{2} x^{2} \log{\left(- i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} - \frac{3 b^{2} x^{2} \log{\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{8 i a^{\frac{9}{2}} b \sqrt{\frac{1}{b}} + 16 i a^{\frac{7}{2}} b^{2} x \sqrt{\frac{1}{b}} + 8 i a^{\frac{5}{2}} b^{3} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b**3*x**(5/2)), Eq(a,
 0)), (2*sqrt(x)/a**3, Eq(b, 0)), (10*I*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*I*a
**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x*
*2*sqrt(1/b)) + 6*I*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*I*a**(9/2)*b*sqrt(1/
b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3
*a**2*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*
a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(
I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b*
*2*x*sqrt(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 6*a*b*x*log(-I*sqrt(a)
*sqrt(1/b) + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt
(1/b) + 8*I*a**(5/2)*b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(I*sqrt(a)*sqrt(1/b)
 + sqrt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*
I*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*log(-I*sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**
(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))
/(8*I*a**(9/2)*b*sqrt(1/b) + 16*I*a**(7/2)*b**2*x*sqrt(1/b) + 8*I*a**(5/2)*
b**3*x**2*sqrt(1/b)), True))
```

3.467 $\int \frac{1}{x^{3/2}(a+bx)^3}\,dx$

Optimal. Leaf size=82

$$-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

Rubi [A] time = 0.03, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}-\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^3),x]
```

```
[Out] -15/(4*a^3*Sqrt[x]) + 1/(2*a*Sqrt[x]*(a + b*x)^2) + 5/(4*a^2*Sqrt[x]*(a + b
*x)) - (15*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^3}\,dx &= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5\int \frac{1}{x^{3/2}(a+bx)^2}\,dx}{4a}\\ &= \frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}+\frac{15\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^2}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\int \frac{1}{\sqrt{x}\,(a+bx)}\,dx}{8a^3}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{(15b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^3}\\ &= -\frac{15}{4a^3\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx)^2}+\frac{5}{4a^2\sqrt{x}\,(a+bx)}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 25, normalized size = 0.30

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{bx}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^3),x]
[Out] (-2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*x)/a)])/(a^3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.12, size = 70, normalized size = 0.85

$$\frac{-8a^2-25abx-15b^2x^2}{4a^3\sqrt{x}\,(a+bx)^2}-\frac{15\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^3),x]
[Out] (-8*a^2 - 25*a*b*x - 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a + b*x)^2) - (15*Sqrt[b]*
ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

fricas [A] time = 0.72, size = 214, normalized size = 2.61

$$\left[\frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{-\frac{b}{a}}-a}{bx+a}\right)-2\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)},\ \frac{15\left(b^2x^3+2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{\frac{b}{a}}}{b\sqrt{x}}\right)-\left(15\,b^2x^2+25\,abx+8\,a^2\right)\sqrt{x}}{4\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="fricas")
[Out] [1/8*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(-b/a)*log((b*x - 2*a*sqrt(x)*sq
rt(-b/a) - a)/(b*x + a)) - 2*(15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*
b^2*x^3 + 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*sqrt(
b/a)*arctan(a*sqrt(b/a)/(b*sqrt(x))) - (15*b^2*x^2 + 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)]
```

giac [A] time = 1.14, size = 59, normalized size = 0.72

$$-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}-\frac{2}{a^{3}\sqrt{x}}-\frac{7\,b^{2}x^{\frac{3}{2}}+9\,ab\sqrt{x}}{4\,(bx+a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2/(a^3*sqrt(x)) - 1/4
*(7*b^2*x^(3/2) + 9*a*b*sqrt(x))/((b*x + a)^2*a^3)
```

maple [A] time = 0.02, size = 66, normalized size = 0.80

$$-\frac{7b^{2}x^{\frac{3}{2}}}{4\,(bx+a)^{2}\,a^{3}}-\frac{9b\sqrt{x}}{4\,(bx+a)^{2}\,a^{2}}-\frac{15b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\;a^{3}}-\frac{2}{a^{3}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^3,x)
```

```
[Out] -2/a^3/x^(1/2)-7/4/a^3*b^2/(b*x+a)^2*x^(3/2)-9/4/a^2*b/(b*x+a)^2*x^(1/2)-15
/4/a^3*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.99, size = 73, normalized size = 0.89

$$-\frac{15\,b^{2}x^{2}+25\,abx+8\,a^{2}}{4\left(a^{3}b^{2}x^{\frac{5}{2}}+2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}-\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(15*b^2*x^2 + 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) + 2*a^4*b*x^(3/2) + a
^5*sqrt(x)) - 15/4*b*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.15, size = 70, normalized size = 0.85

$$-\frac{\frac{2}{a}+\frac{15\,b^{2}\,x^{2}}{4\,a^{3}}+\frac{25\,b\,x}{4\,a^{2}}}{a^{2}\,\sqrt{x}+b^{2}\,x^{5/2}+2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^3),x)
```

```
[Out] - (2/a + (15*b^2*x^2)/(4*a^3) + (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2
) + 2*a*b*x^(3/2)) - (15*b^(1/2)*atan((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2
))
```

sympy [A] time = 54.35, size = 865, normalized size = 10.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{a^{3}\sqrt{x}} & \text{for } b=0 \\ -\frac{2}{7b^{3}x^{\frac{7}{2}}} & \text{for } a=0 \\ -\frac{16ia^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{50ia^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{30i\sqrt{a}b^{2}x^{2}\sqrt{\frac{1}{b}}}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15a^{2}\sqrt{x}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15a^{2}\sqrt{x}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{30abx^{\frac{3}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{30abx^{\frac{3}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}-\frac{15b^{2}x^{\frac{5}{2}}\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}+\frac{15b^{2}x^{\frac{5}{2}}\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8ia^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}+16ia^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8ia^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**3*sqrt(x)), Eq(b, 0)
), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-16*I*a**(5/2)*sqrt(1/b)/(8*I*a**(11/
2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b*
*2*x**(5/2)*sqrt(1/b)) - 50*I*a**(3/2)*b*x*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)
*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 30*I*sqrt(a)*b**2*x**2*sqrt(1/b)/(8*I*a**(11/2)*sqrt(x)*sqrt
(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqr
t(1/b)) - 15*a**2*sqrt(x)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2
)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**
2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/
(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I
*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/
b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*s
qrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*a*b*x**(3/2)*log(I*sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b) + 16*I*a**(9/2)
*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*b**2*x**
(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt(x)*sqrt(1/b)
+ 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)
) + 15*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(8*I*a**(11/2)*sqrt
(x)*sqrt(1/b) + 16*I*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*I*a**(7/2)*b**2*x**(
5/2)*sqrt(1/b)), True))
```

3.468 $\int \frac{1}{x^{5/2}(a+bx)^3} \, dx$

Optimal. Leaf size=95

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 205}

$$\frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{7}{4a^2x^{3/2}(a+bx)}+\frac{35b}{4a^4\sqrt{x}}-\frac{35}{12a^3x^{3/2}}+\frac{1}{2ax^{3/2}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^3),x]
```

```
[Out] -35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) + 1/(2*a*x^(3/2)*(a + b*x)^2)
 + 7/(4*a^2*x^(3/2)*(a + b*x)) + (35*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(a+bx)^3}\,dx = \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7\int \frac{1}{x^{5/2}(a+bx)^2}\,dx}{4a}$$

$$= \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7}{4a^2x^{3/2}(a+bx)} + \frac{35\int \frac{1}{x^{5/2}(a+bx)}\,dx}{8a^2}$$

$$= -\frac{35}{12a^3x^{3/2}} + \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7}{4a^2x^{3/2}(a+bx)} - \frac{(35b)\int \frac{1}{x^{3/2}(a+bx)}\,dx}{8a^3}$$

$$= -\frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} + \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7}{4a^2x^{3/2}(a+bx)} + \frac{\left(35b^2\right)\int \frac{1}{\sqrt{x}(a+bx)}\,dx}{8a^4}$$

$$= -\frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} + \frac{1}{2ax^{3/2}(a+bx)^2} + \frac{7}{4a^2x^{3/2}(a+bx)} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x

giac [A] time = 1.10, size = 71, normalized size = 0.75

$$\frac{35\, b^{2} \arctan\left(\frac{b \sqrt{x}}{\sqrt{ab}}\right)}{4\, \sqrt{ab}\, a^{4}} + \frac{2\, (9\, bx - a)}{3\, a^{4} x^{\frac{3}{2}}} + \frac{11\, b^{3} x^{\frac{3}{2}} + 13\, ab^{2} \sqrt{x}}{4\, (bx + a)^{2} a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt(a*b)*a^4) + 2/3*(9*b*x - a)/(a^4
*x^(3/2)) + 1/4*(11*b^3*x^(3/2) + 13*a*b^2*sqrt(x))/((b*x + a)^2*a^4)
```

maple [A] time = 0.02, size = 79, normalized size = 0.83

$$\frac{11 b^{3} x^{\frac{3}{2}}}{4\, (bx + a)^{2}\, a^{4}} + \frac{13 b^{2} \sqrt{x}}{4\, (bx + a)^{2}\, a^{3}} + \frac{35 b^{2} \arctan\left(\frac{b \sqrt{x}}{\sqrt{ab}}\right)}{4 \sqrt{ab}\ a^{4}} + \frac{6 b}{a^{4} \sqrt{x}} - \frac{2}{3 a^{3} x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^3,x)
```

```
[Out] -2/3/a^3/x^(3/2)+6*b/a^4/x^(1/2)+11/4/a^4*b^3/(b*x+a)^2*x^(3/2)+13/4/a^3*b^
2/(b*x+a)^2*x^(1/2)+35/4/a^4*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x^(1/2)
)
```

maxima [A] time = 2.98, size = 86, normalized size = 0.91

$$\frac{105\, b^{3} x^{3} + 175\, ab^{2} x^{2} + 56\, a^{2} bx - 8\, a^{3}}{12\, \left(a^{4} b^{2} x^{\frac{7}{2}} + 2\, a^{5} bx^{\frac{5}{2}} + a^{6} x^{\frac{3}{2}}\right)} + \frac{35\, b^{2} \arctan\left(\frac{b \sqrt{x}}{\sqrt{ab}}\right)}{4\, \sqrt{ab}\, a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 + 175*a*b^2*x^2 + 56*a^2*b*x - 8*a^3)/(a^4*b^2*x^(7/2) +
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/4*b^2*arctan(b*sqrt(x)/sqrt(a*b))/(sqrt
(a*b)*a^4)
```

mupad [B] time = 0.16, size = 80, normalized size = 0.84

$$\frac{\frac{175\, b^{2}\, x^{2}}{12\, a^{3}} - \frac{2}{3\, a} + \frac{35\, b^{3}\, x^{3}}{4\, a^{4}} + \frac{14\, b\, x}{3\, a^{2}}}{a^{2}\, x^{3/2} + b^{2}\, x^{7/2} + 2\, a\, b\, x^{5/2}} + \frac{35\, b^{3/2}\, \mathrm{atan}\left(\frac{\sqrt{b}\, \sqrt{x}}{\sqrt{a}}\right)}{4\, a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^3),x)
```

```
[Out] ((175*b^2*x^2)/(12*a^3) - 2/(3*a) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) + 2*a*b*x^(5/2)) + (35*b^(3/2)*atan((b^(1/2)*x
^(1/2))/a^(1/2)))/(4*a^(9/2))
```

sympy [A] time = 138.08, size = 962, normalized size = 10.13

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{3 a^{3} x^{\frac{3}{2}}} & \text{for}\ b = 0 \\ -\frac{2}{9 b^{3} x^{\frac{9}{2}}} & \text{for}\ a = 0 \\ -\frac{16 i a^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{D} + \frac{112 i a^{\frac{5}{2}} b x \sqrt{\frac{1}{b}}}{D} + \frac{350 i a^{\frac{3}{2}} b^{2} x^{2} \sqrt{\frac{1}{b}}}{D} + \frac{210 i \sqrt{a} b^{3} x^{3} \sqrt{\frac{1}{b}}}{D} + \frac{105 a^{2} b x^{\frac{3}{2}} \log\left(-i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} - \frac{105 a^{2} b x^{\frac{3}{2}} \log\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} + \frac{210 a b^{2} x^{\frac{5}{2}} \log\left(-i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} - \frac{210 a b^{2} x^{\frac{5}{2}} \log\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} + \frac{105 b^{3} x^{\frac{7}{2}} \log\left(-i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} - \frac{105 b^{3} x^{\frac{7}{2}} \log\left(i \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x}\right)}{D} & \text{otherwise} \end{cases}$$

where $D = 24 i a^{\frac{13}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} + 48 i a^{\frac{11}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}} + 24 i a^{\frac{9}{2}} b^{2} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}$ (each denominator printed in full)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**3*x**(3/2)), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (-16*I*a**(7/2)*sqrt(1/b)/(24*I*a**
(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(
9/2)*b**2*x**(7/2)*sqrt(1/b)) + 112*I*a**(5/2)*b*x*sqrt(1/b)/(24*I*a**(13/2
)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*
b**2*x**(7/2)*sqrt(1/b)) + 350*I*a**(3/2)*b**2*x**2*sqrt(1/b)/(24*I*a**(13/
2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)
*b**2*x**(7/2)*sqrt(1/b)) + 210*I*sqrt(a)*b**3*x**3*sqrt(1/b)/(24*I*a**(13/
2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)
*b**2*x**(7/2)*sqrt(1/b)) + 105*a**2*b*x**(3/2)*log(-I*sqrt(a)*sqrt(1/b) +
sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqr
t(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*a**2*b*x**(3/2)*log(I
*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**
(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 210*
a*b**2*x**(5/2)*log(-I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2
)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(
7/2)*sqrt(1/b)) - 210*a*b**2*x**(5/2)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(2
4*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24
*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*b**3*x**(7/2)*log(-I*sqrt(a)*sqr
t(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 48*I*a**(11/2)*b*x**
(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*b**3*x**(7/2
)*log(I*sqrt(a)*sqrt(1/b) + sqrt(x))/(24*I*a**(13/2)*x**(3/2)*sqrt(1/b) + 4
8*I*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*I*a**(9/2)*b**2*x**(7/2)*sqrt(1/b))
, True))
```

3.469 $\int \frac{x^{5/2}}{-a+bx}\,dx$

Optimal. Leaf size=68

$$-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 208}

$$\frac{2a^2\sqrt{x}}{b^3}-\frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{2ax^{3/2}}{3b^2}+\frac{2x^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*a^2*Sqrt[x])/b^3 + (2*a*x^(3/2))/(3*b^2) + (2*x^(5/2))/(5*b) - (2*a^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{-a+bx}\,dx = \frac{2x^{5/2}}{5b} + \frac{a\int \frac{x^{3/2}}{-a+bx}\,dx}{b}$$
$$= \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^2\int \frac{\sqrt{x}}{-a+bx}\,dx}{b^2}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{a^3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} + \frac{(2a^3)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= \frac{2a^2\sqrt{x}}{b^3} + \frac{2ax^{3/2}}{3b^2} + \frac{2x^{5/2}}{5b} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.90

$$\frac{2\sqrt{x}\left(15a^2+5abx+3b^2x^2\right)}{15b^3} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(15*a^2 + 5*a*b*x + 3*b^2*x^2))/(15*b^3) - (2*a^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.99

$$\frac{2\left(15a^2\sqrt{x}+5abx^{3/2}+3b^2x^{5/2}\right)}{15b^3} - \frac{2a^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x),x]
```

```
[Out] (2*(15*a^2*Sqrt[x] + 5*a*b*x^(3/2) + 3*b^2*x^(5/2)))/(15*b^3) - (2*a^(5/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

fricas [A] time = 1.08, size = 131, normalized size = 1.93

$$\left[\frac{15\,a^2\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}}{15\,b^3}, \frac{2\left(15\,a^2\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(3\,b^2x^2+5\,abx+15\,a^2\right)\sqrt{x}\right)}{15\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*a^2*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) +
2*(3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3, 2/15*(15*a^2*sqrt(-a/b)*arct
an(b*sqrt(x)*sqrt(-a/b)/a) + (3*b^2*x^2 + 5*a*b*x + 15*a^2)*sqrt(x))/b^3]
```

giac [A] time = 1.03, size = 61, normalized size = 0.90

$$\frac{2\,a^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^3} + \frac{2\left(3\,b^4x^{\frac{5}{2}}+5\,ab^3x^{\frac{3}{2}}+15\,a^2b^2\sqrt{x}\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2/15*(3*b^4*x^(5/2) +
 5*a*b^3*x^(3/2) + 15*a^2*b^2*sqrt(x))/b^5
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2a^3\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{\frac{2b^2x^{\frac{5}{2}}}{5}+\frac{2ab\,x^{\frac{3}{2}}}{3}+2a^2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a),x)
```

```
[Out] 2/b^3*(1/5*b^2*x^(5/2)+1/3*a*b*x^(3/2)+a^2*x^(1/2))-2*a^3/b^3/(a*b)^(1/2)*a
rctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.94, size = 70, normalized size = 1.03

$$\frac{a^3\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{2\left(3\,b^2x^{\frac{5}{2}}+5\,abx^{\frac{3}{2}}+15\,a^2\sqrt{x}\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) +
2/15*(3*b^2*x^(5/2) + 5*a*b*x^(3/2) + 15*a^2*sqrt(x))/b^3
```

mupad [B] time = 0.15, size = 51, normalized size = 0.75

$$\frac{2\,x^{5/2}}{5\,b}+\frac{2\,a\,x^{3/2}}{3\,b^2}+\frac{2\,a^2\,\sqrt{x}}{b^3}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x),x)
```

```
[Out] (2*x^(5/2))/(5*b) + (2*a*x^(3/2))/(3*b^2) + (2*a^2*x^(1/2))/b^3 + (a^(5/2)*
atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*2i)/b^(7/2)
```

sympy [A] time = 7.10, size = 116, normalized size = 1.71

$$\begin{cases}\frac{a^{\frac{5}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}-\frac{a^{\frac{5}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^4\sqrt{\frac{1}{b}}}+\frac{2a^2\sqrt{x}}{b^3}+\frac{2ax^{\frac{3}{2}}}{3b^2}+\frac{2x^{\frac{5}{2}}}{5b} & \text{for } b\neq 0\\ -\frac{2x^{\frac{7}{2}}}{7a} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) - a*
*(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**4*sqrt(1/b)) + 2*a**2*sqrt(x)/b
**3 + 2*a*x**(3/2)/(3*b**2) + 2*x**(5/2)/(5*b), Ne(b, 0)), (-2*x**(7/2)/(7*
a), True))
```

3.470 $\int \frac{x^{3/2}}{-a+bx} \, dx$

Optimal. Leaf size=53

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*a*Sqrt[x])/b^2 + (2*x^(3/2))/(3*b) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{-a+bx} \, dx &= \frac{2x^{3/2}}{3b}+\frac{a\int \frac{\sqrt{x}}{-a+bx} \, dx}{b}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{a^2\int \frac{1}{\sqrt{x}(-a+bx)} \, dx}{b^2}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}+\frac{\left(2a^2\right)\operatorname{Subst}\left(\int \frac{1}{-a+bx^2} \, dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2a\sqrt{x}}{b^2}+\frac{2x^{3/2}}{3b}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.92

$$\frac{2\sqrt{x}(3a+bx)}{3b^2}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x]*(3*a + b*x))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{2\left(3a\sqrt{x}+bx^{3/2}\right)}{3b^2}-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x),x]
```

```
[Out] (2*(3*a*Sqrt[x] + b*x^(3/2)))/(3*b^2) - (2*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/b^(5/2)
```

fricas [A] time = 0.67, size = 103, normalized size = 1.94

$$\left[\frac{3\,a\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(bx+3\,a)\sqrt{x}}{3\,b^2},\ \frac{2\left(3\,a\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+(bx+3\,a)\sqrt{x}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*a*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*(b
*x + 3*a)*sqrt(x))/b^2, 2/3*(3*a*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a)
+ (b*x + 3*a)*sqrt(x))/b^2]
```

giac [A] time = 0.94, size = 47, normalized size = 0.89

$$\frac{2\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2}+\frac{2\left(b^2x^{\frac{3}{2}}+3\,ab\sqrt{x}\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) + 2/3*(b^2*x^(3/2) + 3*
a*b*sqrt(x))/b^3
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$-\frac{2a^2\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b^2}+\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+2a\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a),x)
```

```
[Out] 2/b^2*(1/3*b*x^(3/2)+a*x^(1/2))-2*a^2/b^2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)
*b*x^(1/2))
```

maxima [A] time = 2.92, size = 58, normalized size = 1.09

$$\frac{a^2\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\left(bx^{\frac{3}{2}}+3\,a\sqrt{x}\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2) +
2/3*(b*x^(3/2) + 3*a*sqrt(x))/b^2
```

mupad [B] time = 0.11, size = 37, normalized size = 0.70

$$\frac{2\,x^{3/2}}{3\,b}+\frac{2\,a\,\sqrt{x}}{b^2}-\frac{2\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x),x)
```

```
[Out] (2*x^(3/2))/(3*b) + (2*a*x^(1/2))/b^2 - (2*a^(3/2)*atanh((b^(1/2)*x^(1/2))/
a^(1/2)))/b^(5/2)
```

sympy [A] time = 1.87, size = 100, normalized size = 1.89

$$\begin{cases} \frac{a^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}-\frac{a^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^3\sqrt{\frac{1}{b}}}+\frac{2a\sqrt{x}}{b^2}+\frac{2x^{\frac{3}{2}}}{3b} & \text{for } b\neq 0 \\ -\frac{2x^{\frac{5}{2}}}{5a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((a**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) - a*
*(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**3*sqrt(1/b)) + 2*a*sqrt(x)/b**2
 + 2*x**(3/2)/(3*b), Ne(b, 0)), (-2*x**(5/2)/(5*a), True))
```

3.471 $\int \frac{\sqrt{x}}{-a+bx}\,dx$

Optimal. Leaf size=40

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 63, 208}

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x),x]
```

```
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{-a+bx}\,dx &= \frac{2\sqrt{x}}{b}+\frac{a\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{b}\\ &= \frac{2\sqrt{x}}{b}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b}\\ &= \frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x),x]
[Out] (2*Sqrt[x])/b - (2*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(3/2)
```

fricas [A] time = 0.76, size = 83, normalized size = 2.08

$$\left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\,\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,\sqrt{x}}{b},\ \frac{2\left(\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\,\sqrt{-\frac{a}{b}}}{a}\right)+\sqrt{x}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="fricas")
[Out] [(sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)) + 2*sqrt(x))/b
, 2*(sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + sqrt(x))/b]
```

giac [A] time = 1.04, size = 33, normalized size = 0.82

$$\frac{2\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="giac")
[Out] 2*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) + 2*sqrt(x)/b
```

maple [A] time = 0.00, size = 32, normalized size = 0.80

$$-\frac{2a\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{2\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a),x)
[Out] 2/b*x^(1/2)-2*a/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 3.08, size = 47, normalized size = 1.18

$$\frac{a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{2\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b) + 2*sq
rt(x)/b
```

mupad [B] time = 0.11, size = 28, normalized size = 0.70

$$\frac{2\sqrt{x}}{b}-\frac{2\sqrt{a}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x),x)
```

```
[Out] (2*x^(1/2))/b - (2*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/b^(3/2)
```

sympy [A] time = 0.71, size = 87, normalized size = 2.18

$$\begin{cases} \frac{\sqrt{a}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}}-\frac{\sqrt{a}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{b^2\sqrt{\frac{1}{b}}}+\frac{2\sqrt{x}}{b} & \text{for}\ b\neq 0 \\ -\frac{2x^{\frac{3}{2}}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a),x)
```

```
[Out] Piecewise((sqrt(a)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) - sqr
t(a)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(b**2*sqrt(1/b)) + 2*sqrt(x)/b, Ne(b,
 0)), (-2*x**(3/2)/(3*a), True))
```

3.472 $\int \frac{1}{\sqrt{x}(-a+bx)}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(-a+bx)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.98, size = 67, normalized size = 2.31

$$\left[\frac{\sqrt{ab}\log\left(\frac{bx+a-2\sqrt{ab}\sqrt{x}}{bx-a}\right)}{ab}, \frac{2\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt(x))/(b*x - a))/(a*b), 2*sqrt(-a*
b)*arctan(sqrt(-a*b)/(b*sqrt(x)))/(a*b)]
```

giac [A] time = 1.00, size = 20, normalized size = 0.69

$$\frac{2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(b*sqrt(x)/sqrt(-a*b))/sqrt(-a*b)
```

maple [A] time = 0.00, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)/x^(1/2),x)
```

```
[Out] -2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 3.03, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x^(1/2),x, algorithm="maxima")
```

```
[Out] log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/sqrt(a*b)
```

mupad [B] time = 0.13, size = 19, normalized size = 0.66

$$-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)),x)
```

```
[Out] -(2*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(a^(1/2)*b^(1/2))
```

sympy [A] time = 1.25, size = 88, normalized size = 3.03

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a = 0 \\ -\frac{2\sqrt{x}}{a} & \text{for } b = 0 \\ \frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} - \frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)/x**(1/2),x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(b*sqrt(x)), Eq(a, 0)), (
-2*sqrt(x)/a, Eq(b, 0)), (log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt
(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(sqrt(a)*b*sqrt(1/b)), True))
```

3.473 $\int \frac{1}{x^{3/2}(-a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)),x]
```

```
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(-a+bx)}\,dx &= \frac{2}{a\sqrt{x}}+\frac{b\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{a}\\
&= \frac{2}{a\sqrt{x}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a}\\
&= \frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 24, normalized size = 0.60

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}\right)}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, (b*x)/a])/(a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.00

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)),x]
[Out] 2/(a*Sqrt[x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.70, size = 91, normalized size = 2.28

$$\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\sqrt{x}}{ax},\frac{2\left(x\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\sqrt{x}\right)}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="fricas")
[Out] [(x*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) + 2*sqrt(x))
/(a*x), 2*(x*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + sqrt(x))/(a*x)]
```

giac [A] time = 1.02, size = 33, normalized size = 0.82

$$\frac{2\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="giac")
[Out] 2*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) + 2/(a*sqrt(x))
```

maple [A] time = 0.01, size = 32, normalized size = 0.80

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a),x)
[Out] -2/a*b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/a/x^(1/2)
```

maxima [A] time = 2.87, size = 47, normalized size = 1.18

$$\frac{b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{2}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a) + 2/(a
*sqrt(x))
```

mupad [B] time = 0.06, size = 28, normalized size = 0.70

$$\frac{2}{a\sqrt{x}}-\frac{2\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)),x)
```

```
[Out] 2/(a*x^(1/2)) - (2*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(3/2)
```

sympy [A] time = 2.76, size = 94, normalized size = 2.35

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{a\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for}\: a = 0 \\ \frac{2}{a\sqrt{x}}+\frac{\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2/(a*sqrt(x)), Eq(b, 0)), (
-2/(3*b*x**(3/2)), Eq(a, 0)), (2/(a*sqrt(x)) + log(-sqrt(a)*sqrt(1/b) + sqr
t(x))/(a**(3/2)*sqrt(1/b)) - log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(3/2)*sqr
t(1/b)), True))
```

3.474 $\int \frac{1}{x^{5/2}(-a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] 2/(3*a*x^(3/2)) + (2*b)/(a^2*Sqrt[x]) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(-a+bx)}\,dx &= \frac{2}{3ax^{3/2}}+\frac{b\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{b^2\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a^2}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}+\frac{\left(2b^2\right)\operatorname{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= \frac{2}{3ax^{3/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.49

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{bx}{a}\right)}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*x)/a])/(3*a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 48, normalized size = 0.91

$$\frac{2(a+3bx)}{3a^2x^{3/2}}-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)),x]
```

```
[Out] (2*(a + 3*b*x))/(3*a^2*x^(3/2)) - (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]])/a^(5/2)
```

fricas [A] time = 1.06, size = 113, normalized size = 2.13

$$\left[\frac{3\,bx^2\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\,(3\,bx+a)\sqrt{x}}{3\,a^2x^2},\frac{2\left(3\,bx^2\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx+a)\sqrt{x}\right)}{3\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b*x^2*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)) +
2*(3*b*x + a)*sqrt(x))/(a^2*x^2), 2/3*(3*b*x^2*sqrt(-b/a)*arctan(a*sqrt(-b/
a)/(b*sqrt(x))) + (3*b*x + a)*sqrt(x))/(a^2*x^2)]
```

giac [A] time = 0.98, size = 41, normalized size = 0.77

$$\frac{2\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^2}+\frac{2\,(3\,bx+a)}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 2/3*(3*b*x + a)/(a^2*
x^(3/2))
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$-\frac{2b^2\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{2b}{a^2\sqrt{x}}+\frac{2}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a),x)
```

```
[Out] -2/a^2*b^2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/3/a/x^(3/2)+2/a^2
*b/x^(1/2)
```

maxima [A] time = 2.96, size = 55, normalized size = 1.04

$$\frac{b^2 \log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\, a^2} + \frac{2\,(3\,bx + a)}{3\,a^2 x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2) +
2/3*(3*b*x + a)/(a^2*x^(3/2))
```

mupad [B] time = 0.12, size = 37, normalized size = 0.70

$$\frac{\frac{2}{3\,a}+\frac{2\,b\,x}{a^2}}{x^{3/2}} - \frac{2\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)),x)
```

```
[Out] (2/(3*a) + (2*b*x)/a^2)/x^(3/2) - (2*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2
)))/a^(5/2)
```

sympy [A] time = 7.67, size = 112, normalized size = 2.11

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a = 0 \\ \frac{2}{3ax^{\frac{3}{2}}} + \frac{2b}{a^2\sqrt{x}} + \frac{b\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{b\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a*x**(3/2)), Eq(b, 0))
, (-2/(5*b*x**(5/2)), Eq(a, 0)), (2/(3*a*x**(3/2)) + 2*b/(a**2*sqrt(x)) + b
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)) - b*log(sqrt(a)*sqr
t(1/b) + sqrt(x))/(a**(5/2)*sqrt(1/b)), True))
```

3.475 $\int \frac{1}{x^{7/2}(-a+bx)}\,dx$

Optimal. Leaf size=68

$$-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(-a + b*x)),x]
```

```
[Out] 2/(5*a*x^(5/2)) + (2*b)/(3*a^2*x^(3/2)) + (2*b^2)/(a^3*Sqrt[x]) - (2*b^(5/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{7/2}(-a+bx)}\,dx &= \frac{2}{5ax^{5/2}}+\frac{b\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{a}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{b^2\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{a^2}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{a^3}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{(2b^3)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^3}\\
&= \frac{2}{5ax^{5/2}}+\frac{2b}{3a^2x^{3/2}}+\frac{2b^2}{a^3\sqrt{x}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\frac{bx}{a}\right)}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(-a + b*x)),x]
[Out] (2*Hypergeometric2F1[-5/2, 1, -3/2, (b*x)/a])/(5*a*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 61, normalized size = 0.90

$$\frac{2\left(3a^2+5abx+15b^2x^2\right)}{15a^3x^{5/2}}-\frac{2b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(-a + b*x)),x]
[Out] (2*(3*a^2 + 5*a*b*x + 15*b^2*x^2))/(15*a^3*x^(5/2)) - (2*b^(5/2)*ArcTanh[(S
qrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 0.90, size = 143, normalized size = 2.10

$$\left[\frac{15\,b^2x^3\sqrt{\frac{b}{a}}\log\left(\frac{bx-2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(15\,b^2x^2+5\,abx+3\,a^2\right)\sqrt{x}}{15\,a^3x^3},\ \frac{2\left(15\,b^2x^3\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^2x^2+5\,abx+3\,a^2\right)\sqrt{x}\right)}{15\,a^3x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="fricas")
[Out] [1/15*(15*b^2*x^3*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x - a)
) + 2*(15*b^2*x^2 + 5*a*b*x + 3*a^2)*sqrt(x))/(a^3*x^3), 2/15*(15*b^2*x^3*s
qrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 + 5*a*b*x + 3*a^2)
*sqrt(x))/(a^3*x^3)]
```

giac [A] time = 0.97, size = 54, normalized size = 0.79

$$\frac{2\,b^3\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^3}+\frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="giac")
```

```
[Out] 2*b^3*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/15*(15*b^2*x^2 + 5*
a*b*x + 3*a^2)/(a^3*x^(5/2))
```

maple [A] time = 0.01, size = 54, normalized size = 0.79

$$-\frac{2b^3 \operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\ a^3}+\frac{2b^2}{a^3\sqrt{x}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2}{5a\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x-a),x)
```

```
[Out] -2/a^3*b^3/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))+2/5/a/x^(5/2)+2/a^3
*b^2/x^(1/2)+2/3/a^2*b/x^(3/2)
```

maxima [A] time = 2.98, size = 68, normalized size = 1.00

$$\frac{b^3 \log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}+\frac{2\left(15\,b^2x^2+5\,abx+3\,a^2\right)}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x-a),x, algorithm="maxima")
```

```
[Out] b^3*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3) +
2/15*(15*b^2*x^2 + 5*a*b*x + 3*a^2)/(a^3*x^(5/2))
```

mupad [B] time = 0.13, size = 48, normalized size = 0.71

$$\frac{\frac{2}{5\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{2\,b\,x}{3\,a^2}}{x^{5/2}}-\frac{2\,b^{5/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(7/2)*(a - b*x)),x)
```

```
[Out] (2/(5*a) + (2*b^2*x^2)/a^3 + (2*b*x)/(3*a^2))/x^(5/2) - (2*b^(5/2)*atanh((b
^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2)
```

sympy [A] time = 24.44, size = 131, normalized size = 1.93

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ -\frac{2}{7bx^{\frac{7}{2}}} & \text{for}\ a = 0 \\ \frac{2}{5ax^{\frac{5}{2}}} & \text{for}\ b = 0 \\ \frac{2}{5ax^{\frac{5}{2}}}+\frac{2b}{3a^2x^{\frac{3}{2}}}+\frac{2b^2}{a^3\sqrt{x}}+\frac{b^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{b^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{a^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x-a),x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0)
), (2/(5*a*x**(5/2)), Eq(b, 0)), (2/(5*a*x**(5/2)) + 2*b/(3*a**2*x**(3/2))
+ 2*b**2/(a**3*sqrt(x)) + b**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*
sqrt(1/b)) - b**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(a**(7/2)*sqrt(1/b)), Tr
ue))
```

3.476 $\int \frac{x^{5/2}}{(-a+bx)^2} dx$

Optimal. Leaf size=70

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}+\frac{5a\sqrt{x}}{b^3}+\frac{x^{5/2}}{b(a-bx)}+\frac{5x^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (5*a*Sqrt[x])/b^3 + (5*x^(3/2))/(3*b^2) + x^(5/2)/(b*(a - b*x)) - (5*a^(3/2
)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(-a+bx)^2}\,dx &= \frac{x^{5/2}}{b(a-bx)}+\frac{5\int \frac{x^{3/2}}{-a+bx}\,dx}{2b} \\
&= \frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{(5a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b^2} \\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b^3} \\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b^3} \\
&= \frac{5a\sqrt{x}}{b^3}+\frac{5x^{3/2}}{3b^2}+\frac{x^{5/2}}{b(a-bx)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.37

$$
\frac{2x^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (2*x^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, (b*x)/a])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.07, size = 76, normalized size = 1.09

$$
\frac{-15a^2\sqrt{x}+10abx^{3/2}+2b^2x^{5/2}}{3b^3(bx-a)}-\frac{5a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x)^2,x]
```

```
[Out] (-15*a^2*Sqrt[x] + 10*a*b*x^(3/2) + 2*b^2*x^(5/2))/(3*b^3*(-a + b*x)) - (5*
a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/b^(7/2)
```

fricas [A] time = 0.70, size = 167, normalized size = 2.39

$$
\left[\frac{15\left(abx-a^2\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(2b^2x^2+10abx-15a^2\right)\sqrt{x}}{6\left(b^4x-ab^3\right)},\ \frac{15\left(abx-a^2\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(2b^2x^2+10abx-15a^2\right)\sqrt{x}}{3\left(b^4x-ab^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b*x - a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x
 - a)) + 2*(2*b^2*x^2 + 10*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3), 1/3*(1
5*(a*b*x - a^2)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (2*b^2*x^2 + 10
*a*b*x - 15*a^2)*sqrt(x))/(b^4*x - a*b^3)]
```

giac [A] time = 0.98, size = 69, normalized size = 0.99

$$
\frac{5\,a^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^3}-\frac{a^2\sqrt{x}}{(bx-a)b^3}+\frac{2\left(b^4x^{\frac{3}{2}}+6\,ab^3\sqrt{x}\right)}{3\,b^6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 5*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) - a^2*sqrt(x)/((b*x - a
)*b^3) + 2/3*(b^4*x^(3/2) + 6*a*b^3*sqrt(x))/b^6
```

maple [A] time = 0.01, size = 61, normalized size = 0.87

$$\frac{2\left(-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\sqrt{x}}{2(bx-a)}\right)a^{2}}{b^{3}}+\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+4a\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^2,x)
```

```
[Out] 2/b^3*(1/3*b*x^(3/2)+2*a*x^(1/2))+2/b^3*a^2*(-1/2*x^(1/2)/(b*x-a)-5/2/(a*b)
^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

maxima [A] time = 3.06, size = 81, normalized size = 1.16

$$-\frac{a^{2}\sqrt{x}}{b^{4}x-ab^{3}}+\frac{5\,a^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{2\left(bx^{\frac{3}{2}}+6\,a\sqrt{x}\right)}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a^2*sqrt(x)/(b^4*x - a*b^3) + 5/2*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(
x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2/3*(b*x^(3/2) + 6*a*sqrt(x))/b^3
```

mupad [B] time = 0.07, size = 61, normalized size = 0.87

$$\frac{2\,x^{3/2}}{3\,b^{2}}+\frac{4\,a\,\sqrt{x}}{b^{3}}+\frac{a^{2}\,\sqrt{x}}{a\,b^{3}-b^{4}\,x}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(3/2))/(3*b^2) + (4*a*x^(1/2))/b^3 + (a^2*x^(1/2))/(a*b^3 - b^4*x) + (
a^(3/2)*atan((b^(1/2)*x^(1/2)*1i)/a^(1/2))*5i)/b^(7/2)
```

sympy [A] time = 24.75, size = 444, normalized size = 6.34

$$\begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{7}{2}}}{7a^{2}} & \text{for}\: b = 0 \\ \frac{2x^{\frac{3}{2}}}{3b^{2}} & \text{for}\: a = 0 \\ -\frac{30a^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{20a^{\frac{3}{2}}b^{2}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{4\sqrt{a}b^{3}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}-\frac{15a^{3}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{15a^{3}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}+\frac{15a^{2}bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}}-\frac{15a^{2}bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-6a^{\frac{3}{2}}b^{4}\sqrt{\frac{1}{b}}+6\sqrt{a}b^{5}x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (2*x**(3/2)/(3*b**2), Eq(a, 0)), (-30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(-6
*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 20*a**(3/2)*b**2*x
**(3/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b))
 + 4*sqrt(a)*b**3*x**(5/2)*sqrt(1/b)/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a
```

```
)*b**5*x*sqrt(1/b)) - 15*a**3*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2
)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) + 15*a**3*log(sqrt(a)*sqrt(1
/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)) +
15*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-6*a**(3/2)*b**4*sqrt(1/b) +
 6*sqrt(a)*b**5*x*sqrt(1/b)) - 15*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))
/(-6*a**(3/2)*b**4*sqrt(1/b) + 6*sqrt(a)*b**5*x*sqrt(1/b)), True))
```

3.477 $\int \frac{x^{3/2}}{(-a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x^{3/2}}{b(a-bx)}+\frac{3\sqrt{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (3*Sqrt[x])/b^2 + x^(3/2)/(b*(a - b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[
x])/Sqrt[a]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(-a+bx)^2}\,dx = \frac{x^{3/2}}{b(a-bx)} + \frac{3\int \frac{\sqrt{x}}{-a+bx}\,dx}{2b}$$
$$= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} + \frac{(3a)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2b^2}$$
$$= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} + \frac{(3a)\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{3\sqrt{x}}{b^2} + \frac{x^{3/2}}{b(a-bx)} - \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.46

$$\frac{2x^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (2*x^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, (b*x)/a])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.06, size = 56, normalized size = 0.98

$$\frac{\sqrt{x}\,(2bx-3a)}{b^2(bx-a)} - \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x)^2,x]
```

```
[Out] (Sqrt[x]*(-3*a + 2*b*x))/(b^2*(-a + b*x)) - (3*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/b^(5/2)
```

fricas [A] time = 0.58, size = 138, normalized size = 2.42

$$\left[\frac{3\,(bx-a)\sqrt{\frac{a}{b}}\,\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\,(2\,bx-3\,a)\sqrt{x}}{2\,\left(b^3x-ab^2\right)}, \frac{3\,(bx-a)\sqrt{-\frac{a}{b}}\,\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+(2\,bx-3\,a)\sqrt{x}}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x - a)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/b) + a)/(b*x - a)
) + 2*(2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2), (3*(b*x - a)*sqrt(-a/b)*arcta
n(b*sqrt(x)*sqrt(-a/b)/a) + (2*b*x - 3*a)*sqrt(x))/(b^3*x - a*b^2)]
```

giac [A] time = 0.98, size = 51, normalized size = 0.89

$$\frac{3\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b^2} - \frac{a\sqrt{x}}{(bx-a)b^2} + \frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] 3*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - a*sqrt(x)/((b*x - a)*b^
2) + 2*sqrt(x)/b^2
```

maple [A] time = 0.01, size = 49, normalized size = 0.86

$$\frac{2\left(-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}-\frac{\sqrt{x}}{2(bx-a)}\right)a}{b^2}+\frac{2\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^2,x)
```

```
[Out] 2/b^2*x^(1/2)+2*a/b^2*(-1/2/(b*x-a)*x^(1/2)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)
^(1/2)*b*x^(1/2)))
```

maxima [A] time = 2.92, size = 68, normalized size = 1.19

$$-\frac{a\sqrt{x}}{b^3x-ab^2}+\frac{3\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{2\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -a*sqrt(x)/(b^3*x - a*b^2) + 3/2*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) +
 sqrt(a*b)))/(sqrt(a*b)*b^2) + 2*sqrt(x)/b^2
```

mupad [B] time = 0.11, size = 47, normalized size = 0.82

$$\frac{2\,\sqrt{x}}{b^2}+\frac{a\,\sqrt{x}}{a\,b^2-b^3\,x}-\frac{3\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^2,x)
```

```
[Out] (2*x^(1/2))/b^2 + (a*x^(1/2))/(a*b^2 - b^3*x) - (3*a^(1/2)*atanh((b^(1/2)*x
^(1/2))/a^(1/2)))/b^(5/2)
```

sympy [A] time = 9.13, size = 381, normalized size = 6.68

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2x^{\frac{5}{2}}}{5a^2} & \text{for}\: b = 0 \\ \frac{2\sqrt{x}}{b^2} & \text{for}\: a = 0 \\ -\frac{6a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{4\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}+\frac{3abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}}-\frac{3abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^3\sqrt{\frac{1}{b}}+2\sqrt{a}b^4x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b, 0
)), (2*sqrt(x)/b**2, Eq(a, 0)), (-6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(-2*a**(3/
2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 4*sqrt(a)*b**2*x**(3/2)*s
qrt(1/b)/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a**2
*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*
b**4*x*sqrt(1/b)) + 3*a**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b*
```

```
*3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) + 3*a*b*x*log(-sqrt(a)*sqrt(1/b)
 + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(a)*b**4*x*sqrt(1/b)) - 3*a
*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**3*sqrt(1/b) + 2*sqrt(
a)*b**4*x*sqrt(1/b)), True))
```

3.478 $\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 208}

$$\frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^2,x]
```

```
[Out] Sqrt[x]/(b*(a - b*x)) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(3/2)
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx &= \frac{\sqrt{x}}{b(a-bx)}+\frac{\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2b}\\
&= \frac{\sqrt{x}}{b(a-bx)}+\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{\sqrt{x}}{b(a-bx)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 1.30

$$\frac{\sqrt{a}\sqrt{b}\sqrt{x}+(bx-a)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^2,x]
[Out] (Sqrt[a]*Sqrt[b]*Sqrt[x] + (-a + b*x)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(
Sqrt[a]*b^(3/2)*(a - b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 1.04

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sqrt{x}}{b(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x)^2,x]
[Out] -(Sqrt[x]/(b*(-a + b*x))) - ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(Sqrt[a]*b^(
3/2))
```

fricas [A] time = 0.84, size = 123, normalized size = 2.62

$$\left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}\,(bx-a)\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)}{2\left(ab^3x-a^2b^2\right)},-\frac{ab\sqrt{x}-\sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{ab^3x-a^2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="fricas")
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a - 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a*b^3*x - a^2*b^2), -(a*b*sqrt(x) - sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a*b^3*x - a^2*b^2)]
```

giac [A] time = 1.06, size = 40, normalized size = 0.85

$$\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,b}-\frac{\sqrt{x}}{(bx-a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="giac")
[Out] arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b) - sqrt(x)/((b*x - a)*b)
```

maple [A] time = 0.01, size = 40, normalized size = 0.85

$$-\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}-\frac{\sqrt{x}}{(bx-a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^2,x)
[Out] -1/b*x^(1/2)/(b*x-a)-1/b/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.99, size = 56, normalized size = 1.19

$$-\frac{\sqrt{x}}{b^2x-ab}+\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(b^2*x - a*b) + 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*b)
```

mupad [B] time = 0.11, size = 35, normalized size = 0.74

$$\frac{\sqrt{x}}{b\,(a-b\,x)}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^2,x)
```

```
[Out] x^(1/2)/(b*(a - b*x)) - atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(1/2)*b^(3/2))
```

sympy [A] time = 4.43, size = 311, normalized size = 6.62

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a^2} & \text{for}\: b=0 \\ -\frac{2}{b^2\sqrt{x}} & \text{for}\: a=0 \\ -\frac{2\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}-\frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}+\frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}+\frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}}-\frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}+2\sqrt{a}\,b^3x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b, 0
)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(
3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - a*log(-sqrt(a)*sqrt(1/b
) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) + a*
log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b*
*3*x*sqrt(1/b)) + b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(3/2)*b**2*s
qrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)) - b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x
))/(-2*a**(3/2)*b**2*sqrt(1/b) + 2*sqrt(a)*b**3*x*sqrt(1/b)), True))
```

3.479 $\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx &= \frac{\sqrt{x}}{a(a-bx)}-\frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a}\\ &= \frac{\sqrt{x}}{a(a-bx)}-\frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a}\\ &= \frac{\sqrt{x}}{a(a-bx)}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a^2-abx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a^2 - a*b*x) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

IntegrateAlgebraic [A] time = 0.05, size = 46, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{\sqrt{x}}{a(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)^2),x]
```

```
[Out] Sqrt[x]/(a*(a - b*x)) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]/(a^(3/2)*Sqrt[b]
)
```

fricas [A] time = 0.58, size = 122, normalized size = 2.65

$$\left[-\frac{2\,ab\sqrt{x}-\sqrt{ab}\,(bx-a)\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)}{2\left(a^2b^2x-a^3b\right)},-\frac{ab\sqrt{x}+\sqrt{-ab}\,(bx-a)\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)}{a^2b^2x-a^3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sqrt(x) - sqrt(a*b)*(b*x - a)*log((b*x + a + 2*sqrt(a*b)*sqrt(
x))/(b*x - a)))/(a^2*b^2*x - a^3*b), -(a*b*sqrt(x) + sqrt(-a*b)*(b*x - a)*a
rctan(sqrt(-a*b)/(b*sqrt(x))))/(a^2*b^2*x - a^3*b)]
```

giac [A] time = 0.88, size = 41, normalized size = 0.89

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a}-\frac{\sqrt{x}}{(bx-a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a) - sqrt(x)/((b*x - a)*a)
```

maple [A] time = 0.01, size = 39, normalized size = 0.85

$$\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}\;a}-\frac{\sqrt{x}}{(bx-a)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^2/x^(1/2),x)
```

```
[Out] -x^(1/2)/a/(b*x-a)+1/a/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.97, size = 56, normalized size = 1.22

$$-\frac{\sqrt{x}}{abx - a^2} - \frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x)/(a*b*x - a^2) - 1/2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(
a*b)))/(sqrt(a*b)*a)
```

mupad [B] time = 0.05, size = 34, normalized size = 0.74

$$\frac{\sqrt{x}}{a\,(a-b\,x)} + \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^2),x)
```

```
[Out] x^(1/2)/(a*(a - b*x)) + atanh((b^(1/2)*x^(1/2))/a^(1/2))/(a^(3/2)*b^(1/2))
```

sympy [A] time = 7.41, size = 303, normalized size = 6.59

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{a^2} & \text{for}\ b = 0 \\ -\frac{2}{3b^2x^{\frac{3}{2}}} & \text{for}\ a = 0 \\ -\frac{2\sqrt{a}\,b\sqrt{x}\sqrt{\frac{1}{b}}}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{a\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{a\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} - \frac{bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} + \frac{bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{-2a^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+2a^{\frac{3}{2}}b^2x\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**2/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
(-2/(3*b**2*x**(3/2)), Eq(a, 0)), (-2*sqrt(a)*b*sqrt(x)*sqrt(1/b)/(-2*a**(5
/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) + a*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)) - a*log(s
qrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sq
rt(1/b)) - b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a**(5/2)*b*sqrt(1/b) +
 2*a**(3/2)*b**2*x*sqrt(1/b)) + b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(-2*a*
*(5/2)*b*sqrt(1/b) + 2*a**(3/2)*b**2*x*sqrt(1/b)), True))
```

3.480 $\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] -3/(a^2*Sqrt[x]) + 1/(a*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sq
rt[x])/Sqrt[a]])/a^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx &= \frac{1}{a\sqrt{x}\,(a-bx)}-\frac{3\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{2a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{a^2}\\
&= -\frac{3}{a^2\sqrt{x}}+\frac{1}{a\sqrt{x}\,(a-bx)}+\frac{3\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.42

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}\right)}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 2, 1/2, (b*x)/a])/(a^2*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.06, size = 55, normalized size = 0.96

$$\frac{3\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3bx-2a}{a^2\sqrt{x}\,(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a + 3*b*x)/(a^2*Sqrt[x]*(a - b*x)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x
])/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.85, size = 151, normalized size = 2.65

$$\left[\frac{3\left(bx^2-ax\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2\,a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\,(3\,bx-2\,a)\sqrt{x}}{2\left(a^2bx^2-a^3x\right)},-\frac{3\left(bx^2-ax\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+(3\,bx-2\,a)\sqrt{x}}{a^2bx^2-a^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*x^2 - a*x)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a)/(b*x
 - a)) - 2*(3*b*x - 2*a)*sqrt(x))/(a^2*b*x^2 - a^3*x), -(3*(b*x^2 - a*x)*sqr
t(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (3*b*x - 2*a)*sqrt(x))/(a^2*b*x^
2 - a^3*x)]
```

giac [A] time = 1.01, size = 52, normalized size = 0.91

$$-\frac{3\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^2}-\frac{3\,bx-2\,a}{\left(bx^{\frac{3}{2}}-a\sqrt{x}\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -3*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) - (3*b*x - 2*a)/((b*x^(3
/2) - a*sqrt(x))*a^2)
```

maple [A] time = 0.01, size = 49, normalized size = 0.86

$$-\frac{2\left(-\frac{3\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\sqrt{x}}{2bx-2a}\right)b}{a^2}-\frac{2}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^2,x)
```

```
[Out] -2/a^2*b*(1/2/(b*x-a)*x^(1/2)-3/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/
2)))-2/a^2/x^(1/2)
```

maxima [A] time = 3.00, size = 69, normalized size = 1.21

$$-\frac{3\,bx-2\,a}{a^2bx^{\frac{3}{2}}-a^3\sqrt{x}}-\frac{3\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -(3*b*x - 2*a)/(a^2*b*x^(3/2) - a^3*sqrt(x)) - 3/2*b*log((b*sqrt(x) - sqrt(
a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.07, size = 49, normalized size = 0.86

$$\frac{3\,\sqrt{b}\,\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{2}{a}-\frac{3\,b\,x}{a^2}}{a\,\sqrt{x}\,-b\,x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^2),x)
```

```
[Out] (3*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(5/2) - (2/a - (3*b*x)/a^2)/
(a*x^(1/2) - b*x^(3/2))
```

sympy [A] time = 17.54, size = 403, normalized size = 7.07

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2}{a^2\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{2}{5b^2x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{4a^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{6\sqrt{a}\,bx\sqrt{\frac{1}{b}}}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{3a\sqrt{x}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{3a\sqrt{x}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}+\frac{3bx^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}}-\frac{3bx^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{2a^{\frac{7}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-2a^{\frac{5}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0)
), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (-4*a**(3/2)*sqrt(1/b)/(2*a**(7/2)*sqr
t(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 6*sqrt(a)*b*x*sqrt(1/b)
/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*a*sqr
t(x)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a*
*(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*a*sqrt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))
/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)) + 3*b*x**
(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a
**(5/2)*b*x**(3/2)*sqrt(1/b)) - 3*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x
))/(2*a**(7/2)*sqrt(x)*sqrt(1/b) - 2*a**(5/2)*b*x**(3/2)*sqrt(1/b)), True))
```

3.481 $\int \frac{1}{x^{5/2}(-a+bx)^2} dx$

Optimal. Leaf size=70

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{5b}{a^3\sqrt{x}}-\frac{5}{3a^2x^{3/2}}+\frac{1}{ax^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] -5/(3*a^2*x^(3/2)) - (5*b)/(a^3*Sqrt[x]) + 1/(a*x^(3/2)*(a - b*x)) + (5*b^(
3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx = \frac{1}{ax^{3/2}(a-bx)} - \frac{5\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{2a}$$
$$= -\frac{5}{3a^2x^{3/2}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{(5b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{2a^2}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{\left(5b^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{2a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} - \frac{\left(5b^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{a^3}$$
$$= -\frac{5}{3a^2x^{3/2}} - \frac{5b}{a^3\sqrt{x}} + \frac{1}{ax^{3/2}(a-bx)} + \frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.37

$$-\frac{2\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{bx}{a}\right)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*Hypergeometric2F1[-3/2, 2, -1/2, (b*x)/a])/(3*a^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 69, normalized size = 0.99

$$\frac{5b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{-2a^2-10abx+15b^2x^2}{3a^3x^{3/2}(a-bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)^2),x]
```

```
[Out] (-2*a^2 - 10*a*b*x + 15*b^2*x^2)/(3*a^3*x^(3/2)*(a - b*x)) + (5*b^(3/2)*Arc
Tanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/a^(7/2)
```

fricas [A] time = 1.00, size = 187, normalized size = 2.67

$$\left[\frac{15\left(b^2x^3-abx^2\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx+2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)-2\left(15b^2x^2-10abx-2a^2\right)\sqrt{x}}{6\left(a^3bx^3-a^4x^2\right)}, -\frac{15\left(b^2x^3-abx^2\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15b^2x^2-10abx-2a^2\right)\sqrt{x}}{3\left(a^3bx^3-a^4x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(15*(b^2*x^3 - a*b*x^2)*sqrt(b/a)*log((b*x + 2*a*sqrt(x)*sqrt(b/a) + a
)/(b*x - a)) - 2*(15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*
x^2), -1/3*(15*(b^2*x^3 - a*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x
))) + (15*b^2*x^2 - 10*a*b*x - 2*a^2)*sqrt(x))/(a^3*b*x^3 - a^4*x^2)]
```

giac [A] time = 0.93, size = 61, normalized size = 0.87

$$-\frac{5\,b^2\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{\sqrt{-ab}\,a^3} - \frac{b^2\sqrt{x}}{(bx-a)a^3} - \frac{2\,(6\,bx+a)}{3\,a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="giac")
```

```
[Out] -5*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) - b^2*sqrt(x)/((b*x -
a)*a^3) - 2/3*(6*b*x + a)/(a^3*x^(3/2))
```

maple [A] time = 0.02, size = 60, normalized size = 0.86

$$-\frac{2\left(-\frac{5\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}}+\frac{\sqrt{x}}{2bx-2a}\right)b^{2}}{a^{3}}-\frac{4b}{a^{3}\sqrt{x}}-\frac{2}{3a^{2}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^2,x)
```

```
[Out] -2/a^3*b^2*(1/2/(b*x-a)*x^(1/2)-5/2/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(
1/2)))-2/3/a^2/x^(3/2)-4/a^3*b/x^(1/2)
```

maxima [A] time = 3.00, size = 82, normalized size = 1.17

$$-\frac{15\,b^{2}x^{2}-10\,abx-2\,a^{2}}{3\left(a^{3}bx^{\frac{5}{2}}-a^{4}x^{\frac{3}{2}}\right)}-\frac{5\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{2\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(15*b^2*x^2 - 10*a*b*x - 2*a^2)/(a^3*b*x^(5/2) - a^4*x^(3/2)) - 5/2*b^
2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

mupad [B] time = 0.14, size = 60, normalized size = 0.86

$$\frac{5\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{\frac{2}{3\,a}-\frac{5\,b^{2}\,x^{2}}{a^{3}}+\frac{10\,b\,x}{3\,a^{2}}}{a\,x^{3/2}-b\,x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^2),x)
```

```
[Out] (5*b^(3/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/a^(7/2) - (2/(3*a) - (5*b^2*x^
2)/a^3 + (10*b*x)/(3*a^2))/(a*x^(3/2) - b*x^(5/2))
```

sympy [A] time = 50.25, size = 471, normalized size = 6.73

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2}{7 b^{2} x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ - \frac{2}{3 a^{2} x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ - \frac{4 a^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{20 a^{\frac{3}{2}} b x \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{30 \sqrt{a} b^{2} x^{2} \sqrt{\frac{1}{b}}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 a b x^{\frac{3}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 a b x^{\frac{3}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} + \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} - \frac{15 b^{2} x^{\frac{5}{2}} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{6 a^{\frac{9}{2}} x^{\frac{3}{2}} \sqrt{\frac{1}{b}} - 6 a^{\frac{7}{2}} b x^{\frac{5}{2}} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**2,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b**2*x**(7/2)), Eq(a,
 0)), (-2/(3*a**2*x**(3/2)), Eq(b, 0)), (-4*a**(5/2)*sqrt(1/b)/(6*a**(9/2)*
x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) - 20*a**(3/2)*b*x*sqr
```

```
t(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) +
30*sqrt(a)*b**2*x**2*sqrt(1/b)/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*
b*x**(5/2)*sqrt(1/b)) - 15*a*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(
6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*a*b*x
**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6
*a**(7/2)*b*x**(5/2)*sqrt(1/b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) +
 sqrt(x))/(6*a**(9/2)*x**(3/2)*sqrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b))
 - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(6*a**(9/2)*x**(3/2)*s
qrt(1/b) - 6*a**(7/2)*b*x**(5/2)*sqrt(1/b)), True))
```

3.482 $\int \frac{x^{7/2}}{(-a+bx)^3} \, dx$

Optimal. Leaf size=97

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{35a\sqrt{x}}{4b^4}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35a\sqrt{x}}{4b^4}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{35x^{3/2}}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (35*a*Sqrt[x])/(4*b^4) + (35*x^(3/2))/(12*b^3) - x^(7/2)/(2*b*(a - b*x)^2)
+ (7*x^(5/2))/(4*b^2*(a - b*x)) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqr
t[a]])/(4*b^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{(-a+bx)^3}\,dx &= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7\int \frac{x^{5/2}}{(-a+bx)^2}\,dx}{4b} \\
&= -\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{35\int \frac{x^{3/2}}{-a+bx}\,dx}{8b^2} \\
&= \frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{(35a)\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^3} \\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^4} \\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}+\frac{\left(35a^2\right)\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^4} \\
&= \frac{35a\sqrt{x}}{4b^4}+\frac{35x^{3/2}}{12b^3}-\frac{x^{7/2}}{2b(a-bx)^2}+\frac{7x^{5/2}}{4b^2(a-bx)}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.27

$$-\frac{2x^{9/2}\,{}_2F_1\left(3,\frac{9}{2};\frac{11}{2};\frac{bx}{a}\right)}{9a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(9/2)*Hypergeometric2F1[3, 9/2, 11/2, (b*x)/a])/(9*a^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 91, normalized size = 0.94

$$\frac{105a^3\sqrt{x}-175a^2bx^{3/2}+56ab^2x^{5/2}+8b^3x^{7/2}}{12b^4(bx-a)^2}-\frac{35a^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(7/2)/(-a + b*x)^3,x]
```

```
[Out] (105*a^3*Sqrt[x] - 175*a^2*b*x^(3/2) + 56*a*b^2*x^(5/2) + 8*b^3*x^(7/2))/(1
2*b^4*(-a + b*x)^2) - (35*a^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^
(9/2))
```

fricas [A] time = 0.79, size = 227, normalized size = 2.34

$$\left[\frac{105\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8b^3x^3+56ab^2x^2-175a^2bx+105a^3\right)\sqrt{x}}{24\left(b^6x^2-2ab^5x+a^2b^4\right)},\ \frac{105\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(8b^3x^3+56ab^2x^2-175a^2bx+105a^3\right)\sqrt{x}}{12\left(b^6x^2-2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*s
qrt(a/b) + a)/(b*x - a)) + 2*(8*b^3*x^3 + 56*a*b^2*x^2 - 175*a^2*b*x + 105*
a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), 1/12*(105*(a*b^2*x^2 - 2*a^2
*b*x + a^3)*sqrt(-a/b)*arctan(b*sqrt(x)*sqrt(-a/b)/a) + (8*b^3*x^3 + 56*a*b
^2*x^2 - 175*a^2*b*x + 105*a^3)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4)]
```

giac [A] time = 1.03, size = 81, normalized size = 0.84

$$\frac{35\,a^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,b^{4}}-\frac{13\,a^{2}bx^{\frac{3}{2}}-11\,a^{3}\sqrt{x}}{4\,(bx-a)^{2}b^{4}}+\frac{2\left(b^{6}x^{\frac{3}{2}}+9\,ab^{5}\sqrt{x}\right)}{3\,b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*a^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^4) - 1/4*(13*a^2*b*x^(3
/2) - 11*a^3*sqrt(x))/((b*x - a)^2*b^4) + 2/3*(b^6*x^(3/2) + 9*a*b^5*sqrt(x
))/b^9
```

maple [A] time = 0.02, size = 70, normalized size = 0.72

$$\frac{2\left(-\frac{35\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{-\frac{13b\,x^{\frac{3}{2}}}{8}+\frac{11a\sqrt{x}}{8}}{(bx-a)^{2}}\right)a^{2}}{b^{4}}+\frac{\frac{2b\,x^{\frac{3}{2}}}{3}+6a\sqrt{x}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x-a)^3,x)
```

```
[Out] 2/b^4*(1/3*b*x^(3/2)+3*a*x^(1/2))+2/b^4*a^2*((-13/8*b*x^(3/2)+11/8*a*x^(1/2
))/(b*x-a)^2-35/8/(a*b)^(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

maxima [A] time = 2.99, size = 103, normalized size = 1.06

$$-\frac{13\,a^{2}bx^{\frac{3}{2}}-11\,a^{3}\sqrt{x}}{4\left(b^{6}x^{2}-2\,ab^{5}x+a^{2}b^{4}\right)}+\frac{35\,a^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{4}}+\frac{2\left(bx^{\frac{3}{2}}+9\,a\sqrt{x}\right)}{3\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(13*a^2*b*x^(3/2) - 11*a^3*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4) +
35/8*a^2*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^
4) + 2/3*(b*x^(3/2) + 9*a*sqrt(x))/b^4
```

mupad [B] time = 0.14, size = 83, normalized size = 0.86

$$\frac{\frac{11\,a^{3}\,\sqrt{x}}{4}-\frac{13\,a^{2}\,b\,x^{3/2}}{4}}{a^{2}\,b^{4}-2\,a\,b^{5}\,x+b^{6}\,x^{2}}+\frac{2\,x^{3/2}}{3\,b^{3}}+\frac{6\,a\,\sqrt{x}}{b^{4}}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{4\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(7/2)/(a - b*x)^3,x)
```

```
[Out] ((11*a^3*x^(1/2))/4 - (13*a^2*b*x^(3/2))/4)/(a^2*b^4 + b^6*x^2 - 2*a*b^5*x)
 + (2*x^(3/2))/(3*b^3) + (6*a*x^(1/2))/b^4 + (a^(3/2)*atan((b^(1/2)*x^(1/2)
*1i)/a^(1/2))*35i)/(4*b^(9/2))
```

sympy [A] time = 136.15, size = 840, normalized size = 8.66

$$\begin{cases} \tilde{\infty} x^{\frac{3}{2}} & \text{for}\: a = 0 \wedge b = 0 \\ - \frac{2 x^{\frac{9}{2}}}{9 a^{3}} & \text{for}\: b = 0 \\ \frac{2 x^{\frac{3}{2}}}{3 b^{3}} & \text{for}\: a = 0 \\ \frac{210 a^{\frac{7}{2}} b \sqrt{x} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{350 a^{\frac{5}{2}} b^{2} x^{\frac{3}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{112 a^{\frac{3}{2}} b^{3} x^{\frac{5}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{16 \sqrt{a} b^{4} x^{\frac{7}{2}} \sqrt{\frac{1}{b}}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{4} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{4} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{210 a^{3} b x \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{210 a^{3} b x \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} + \frac{105 a^{2} b^{2} x^{2} \log{\left(- \sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} - \frac{105 a^{2} b^{2} x^{2} \log{\left(\sqrt{a} \sqrt{\frac{1}{b}} + \sqrt{x} \right)}}{24 a^{\frac{5}{2}} b^{5} \sqrt{\frac{1}{b}} - 48 a^{\frac{3}{2}} b^{6} x \sqrt{\frac{1}{b}} + 24 \sqrt{a} b^{7} x^{2} \sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(9/2)/(9*a**3), Eq(b,
 0)), (2*x**(3/2)/(3*b**3), Eq(a, 0)), (210*a**(7/2)*b*sqrt(x)*sqrt(1/b)/(2
4*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*
x**2*sqrt(1/b)) - 350*a**(5/2)*b**2*x**(3/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sq
rt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) +
112*a**(3/2)*b**3*x**(5/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3
/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 16*sqrt(a)*b**4*x*
*(7/2)*sqrt(1/b)/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b)
 + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a**4*log(-sqrt(a)*sqrt(1/b) + sqrt
(x))/(24*a**(5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a
)*b**7*x**2*sqrt(1/b)) - 105*a**4*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(
5/2)*b**5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*s
qrt(1/b)) - 210*a**3*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**
5*sqrt(1/b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)
) + 210*a**3*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/
b) - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) + 105*a
**2*b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/b)
 - 48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)) - 105*a**
2*b**2*x**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(5/2)*b**5*sqrt(1/b) -
48*a**(3/2)*b**6*x*sqrt(1/b) + 24*sqrt(a)*b**7*x**2*sqrt(1/b)), True))
```

3.483 $\int \frac{x^{5/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=84

$$-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}+\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Rubi [A] time = 0.03, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 63, 208}

$$\frac{5x^{3/2}}{4b^2(a-bx)}-\frac{15\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}-\frac{x^{5/2}}{2b(a-bx)^2}+\frac{15\sqrt{x}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*Sqrt[x])/(4*b^3) - x^(5/2)/(2*b*(a - b*x)^2) + (5*x^(3/2))/(4*b^2*(a -
b*x)) - (15*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(-a+bx)^3}\,dx = -\frac{x^{5/2}}{2b(a-bx)^2} + \frac{5\int \frac{x^{3/2}}{(-a+bx)^2}\,dx}{4b}$$
$$= -\frac{x^{5/2}}{2b(a-bx)^2} + \frac{5x^{3/2}}{4b^2(a-bx)} + \frac{15\int \frac{\sqrt{x}}{-a+bx}\,dx}{8b^2}$$
$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b(a-bx)^2} + \frac{5x^{3/2}}{4b^2(a-bx)} + \frac{(15a)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^3}$$
$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b(a-bx)^2} + \frac{5x^{3/2}}{4b^2(a-bx)} + \frac{(15a)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4b^3}$$
$$= \frac{15\sqrt{x}}{4b^3} - \frac{x^{5/2}}{2b(a-bx)^2} + \frac{5x^{3/2}}{4b^2(a-bx)} - \frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.31

$$-\frac{2x^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, (b*x)/a])/(7*a^3)
```

IntegrateAlgebraic [A] time = 0.12, size = 78, normalized size = 0.93

$$\frac{15a^2\sqrt{x} - 25abx^{3/2} + 8b^2x^{5/2}}{4b^3(bx-a)^2} - \frac{15\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(-a + b*x)^3,x]
```

```
[Out] (15*a^2*Sqrt[x] - 25*a*b*x^(3/2) + 8*b^2*x^(5/2))/(4*b^3*(-a + b*x)^2) - (1
5*Sqrt[a]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(7/2))
```

fricas [A] time = 0.65, size = 199, normalized size = 2.37

$$\left[\frac{15\left(b^2x^2-2\,abx+a^2\right)\sqrt{\frac{a}{b}}\log\left(\frac{bx-2\,b\sqrt{x}\sqrt{\frac{a}{b}}+a}{bx-a}\right)+2\left(8\,b^2x^2-25\,abx+15\,a^2\right)\sqrt{x}}{8\left(b^5x^2-2\,ab^4x+a^2b^3\right)}, \frac{15\left(b^2x^2-2\,abx+a^2\right)\sqrt{-\frac{a}{b}}\arctan\left(\frac{b\sqrt{x}\sqrt{-\frac{a}{b}}}{a}\right)+\left(8\,b^2x^2-25\,abx+15\,a^2\right)\sqrt{x}}{4\left(b^5x^2-2\,ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a/b)*log((b*x - 2*b*sqrt(x)*sqrt(a/
b) + a)/(b*x - a)) + 2*(8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2 -
2*a*b^4*x + a^2*b^3), 1/4*(15*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a/b)*arctan(b
*sqrt(x)*sqrt(-a/b)/a) + (8*b^2*x^2 - 25*a*b*x + 15*a^2)*sqrt(x))/(b^5*x^2
- 2*a*b^4*x + a^2*b^3)]
```

giac [A] time = 0.95, size = 63, normalized size = 0.75

$$\frac{15\,a\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,b^3} + \frac{2\sqrt{x}}{b^3} - \frac{9\,abx^{\frac{3}{2}} - 7\,a^2\sqrt{x}}{4\,(bx-a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*a*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^3) + 2*sqrt(x)/b^3 - 1/4*
(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/((b*x - a)^2*b^3)
```

maple [A] time = 0.01, size = 58, normalized size = 0.69

$$\frac{2\left(-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{-\frac{9b\,x^{\frac{3}{2}}}{8}+\frac{7a\sqrt{x}}{8}}{(bx-a)^2}\right)a}{b^3}+\frac{2\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2/b^3*x^(1/2)+2/b^3*a*((-9/8*b*x^(3/2)+7/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^
(1/2)*arctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

maxima [A] time = 3.00, size = 90, normalized size = 1.07

$$-\frac{9\,abx^{\frac{3}{2}}-7\,a^2\sqrt{x}}{4\left(b^5x^2-2\,ab^4x+a^2b^3\right)}+\frac{15\,a\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{2\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(9*a*b*x^(3/2) - 7*a^2*sqrt(x))/(b^5*x^2 - 2*a*b^4*x + a^2*b^3) + 15/8
*a*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^3) + 2
*sqrt(x)/b^3
```

mupad [B] time = 0.06, size = 69, normalized size = 0.82

$$\frac{\frac{7\,a^2\,\sqrt{x}}{4}-\frac{9\,a\,b\,x^{3/2}}{4}}{a^2\,b^3-2\,a\,b^4\,x+b^5\,x^2}+\frac{2\,\sqrt{x}}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(5/2)/(a - b*x)^3,x)
```

```
[Out] ((7*a^2*x^(1/2))/4 - (9*a*b*x^(3/2))/4)/(a^2*b^3 + b^5*x^2 - 2*a*b^4*x) + (
2*x^(1/2))/b^3 - (15*a^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*b^(7/2))
```

sympy [A] time = 53.45, size = 756, normalized size = 9.00

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{2\sqrt{x}}{b^3} & \text{for}\ a = 0 \\ -\frac{2x^{\frac{7}{2}}}{7a^3} & \text{for}\ b = 0 \\ \frac{30a^{\frac{5}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} - \frac{50a^{\frac{3}{2}}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} + \frac{16\sqrt{a}\,b^3x^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} + \frac{15a^3\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} - \frac{15a^3\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} - \frac{30a^2bx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} + \frac{30a^2bx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} + \frac{15ab^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} - \frac{15ab^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^4\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^5x\sqrt{\frac{1}{b}}+8\sqrt{a}\,b^6x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/b**3, Eq(a, 0)), (
-2*x**(7/2)/(7*a**3), Eq(b, 0)), (30*a**(5/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(5/
2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt
(1/b)) - 50*a**(3/2)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 1
6*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 16*sqrt(a)*b
```

```
**3*x**(5/2)*sqrt(1/b)/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt
(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a**3*log(-sqrt(a)*sqrt(1/b) + s
qrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(
a)*b**6*x**2*sqrt(1/b)) - 15*a**3*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5
/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqr
t(1/b)) - 30*a**2*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sq
rt(1/b) - 16*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 3
0*a**2*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16
*a**(3/2)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) + 15*a*b**2*x**
2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2
)*b**5*x*sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)) - 15*a*b**2*x**2*log(sq
rt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**4*sqrt(1/b) - 16*a**(3/2)*b**5*x*
sqrt(1/b) + 8*sqrt(a)*b**6*x**2*sqrt(1/b)), True))
```

3.484 $\int \frac{x^{3/2}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 63, 208}

$$\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}-\frac{x^{3/2}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] -x^(3/2)/(2*b*(a - b*x)^2) + (3*Sqrt[x])/(4*b^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(-a+bx)^3}\,dx &= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\int \frac{\sqrt{x}}{(-a+bx)^2}\,dx}{4b}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8b^2}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}+\frac{3\,\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4b^2}\\
&= -\frac{x^{3/2}}{2b(a-bx)^2}+\frac{3\sqrt{x}}{4b^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.83

$$\frac{\sqrt{x}\,(3a-5bx)}{4b^2(a-bx)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (Sqrt[x]*(3*a - 5*b*x))/(4*b^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt[x])/
Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 65, normalized size = 0.90

$$\frac{3a\sqrt{x}\,-5bx^{3/2}}{4b^2(bx-a)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{a}\,b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(-a + b*x)^3,x]
```

```
[Out] (3*a*Sqrt[x] - 5*b*x^(3/2))/(4*b^2*(-a + b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(4*Sqrt[a]*b^(5/2))
```

fricas [A] time = 0.71, size = 186, normalized size = 2.58

$$\left[\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)-2\left(5\,ab^2x-3\,a^2b\right)\sqrt{x}}{8\left(ab^5x^2-2\,a^2b^4x+a^3b^3\right)},\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)-\left(5\,ab^2x-3\,a^2b\right)\sqrt{x}}{4\left(ab^5x^2-2\,a^2b^4x+a^3b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) - 2*(5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x
 + a^3*b^3), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) - (5*a*b^2*x - 3*a^2*b)*sqrt(x))/(a*b^5*x^2 - 2*a^2*b^4*x + a^
3*b^3)]
```

giac [A] time = 0.92, size = 51, normalized size = 0.71

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,b^2}-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\,(bx-a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*b^2) - 1/4*(5*b*x^(3/2) - 3*a*
sqrt(x))/((b*x - a)^2*b^2)
```

maple [A] time = 0.01, size = 52, normalized size = 0.72

$$-\frac{3\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,b^2}+\frac{-\frac{5x^{\frac{3}{2}}}{4b}+\frac{3a\sqrt{x}}{4b^2}}{(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-5/8/b*x^(3/2)+3/8*a/b^2*x^(1/2))/(b*x-a)^2-3/4/b^2/(a*b)^(1/2)*arctanh(
1/(a*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.99, size = 78, normalized size = 1.08

$$-\frac{5\,bx^{\frac{3}{2}}-3\,a\sqrt{x}}{4\left(b^4x^2-2\,ab^3x+a^2b^2\right)}+\frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(5*b*x^(3/2) - 3*a*sqrt(x))/(b^4*x^2 - 2*a*b^3*x + a^2*b^2) + 3/8*log(
(b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*b^2)
```

mupad [B] time = 0.14, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,x^{3/2}}{4\,b}-\frac{3\,a\,\sqrt{x}}{4\,b^2}}{a^2-2\,a\,b\,x+b^2\,x^2}-\frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(3/2)/(a - b*x)^3,x)
```

```
[Out] - ((5*x^(3/2))/(4*b) - (3*a*x^(1/2))/(4*b^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(1/2)*b^(5/2))
```

sympy [A] time = 29.25, size = 673, normalized size = 9.35

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{5}{2}}}{5a^3} & \text{for}\: b = 0 \\ -\frac{2}{b^3\sqrt{x}} & \text{for}\: a = 0 \\ \frac{6a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{10\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{6abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{6abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} + \frac{3b^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} - \frac{3b^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{5}{2}}b^3\sqrt{\frac{1}{b}}-16a^{\frac{3}{2}}b^4x\sqrt{\frac{1}{b}}+8\sqrt{a}b^5x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2*x**(5/2)/(5*a**3), Eq(b,
0)), (-2/(b**3*sqrt(x)), Eq(a, 0)), (6*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**(
5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sq
rt(1/b)) - 10*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) + 3*a**2*log(
-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4
*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b
) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*
sqrt(a)*b**5*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8
*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x*
*2*sqrt(1/b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*s
qrt(1/b) - 16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) +
3*b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) -
16*a**(3/2)*b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)) - 3*b**2*x**2
*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(5/2)*b**3*sqrt(1/b) - 16*a**(3/2)*
b**4*x*sqrt(1/b) + 8*sqrt(a)*b**5*x**2*sqrt(1/b)), True))
```

3.485 $\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx$

Optimal. Leaf size=75

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}+\frac{\sqrt{x}}{4ab(a-bx)}-\frac{\sqrt{x}}{2b(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -Sqrt[x]/(2*b*(a - b*x)^2) + Sqrt[x]/(4*a*b*(a - b*x)) + ArcTanh[(Sqrt[b]*S
qrt[x])/Sqrt[a]]/(4*a^(3/2)*b^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(-a+bx)^3}\,dx = -\frac{\sqrt{x}}{2b(a-bx)^2} + \frac{\int \frac{1}{\sqrt{x}(-a+bx)^2}\,dx}{4b}$$
$$= -\frac{\sqrt{x}}{2b(a-bx)^2} + \frac{\sqrt{x}}{4ab(a-bx)} - \frac{\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8ab}$$
$$= -\frac{\sqrt{x}}{2b(a-bx)^2} + \frac{\sqrt{x}}{4ab(a-bx)} - \frac{\text{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{x}\right)}{4ab}$$
$$= -\frac{\sqrt{x}}{2b(a-bx)^2} + \frac{\sqrt{x}}{4ab(a-bx)} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}}$$

Mathematica [C] time = 0.01, size = 26, normalized size = 0.35

$$-\frac{2x^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] (-2*x^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (b*x)/a])/(3*a^3)
```

IntegrateAlgebraic [A] time = 0.10, size = 60, normalized size = 0.80

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{3/2}b^{3/2}} - \frac{\sqrt{x}(a+bx)}{4ab(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(-a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[x]*(a + b*x))/(a*b*(a - b*x)^2) + ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt
[a]]/(4*a^(3/2)*b^(3/2))
```

fricas [A] time = 1.05, size = 183, normalized size = 2.44

$$\left[\frac{\left(b^2x^2 - 2\,abx + a^2\right)\sqrt{ab}\,\log\left(\frac{bx+a+2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right) - 2\left(ab^2x + a^2b\right)\sqrt{x}}{8\left(a^2b^4x^2 - 2\,a^3b^3x + a^4b^2\right)}, -\frac{\left(b^2x^2 - 2\,abx + a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right) + \left(ab^2x + a^2b\right)\sqrt{x}}{4\left(a^2b^4x^2 - 2\,a^3b^3x + a^4b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a + 2*sqrt(a*b)*sqrt(x
))/(b*x - a)) - 2*(a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a
^4*b^2), -1/4*((b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/(b*sq
rt(x))) + (a*b^2*x + a^2*b)*sqrt(x))/(a^2*b^4*x^2 - 2*a^3*b^3*x + a^4*b^2)]
```

giac [A] time = 0.93, size = 55, normalized size = 0.73

$$-\frac{\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\sqrt{-ab}\,ab} - \frac{bx^{\frac{3}{2}} + a\sqrt{x}}{4\,(bx-a)^2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] -1/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a*b) - 1/4*(b*x^(3/2) + a*sqr
t(x))/((b*x - a)^2*a*b)
```

maple [A] time = 0.01, size = 54, normalized size = 0.72

$$\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{4\sqrt{ab}\,ab}+\frac{-\frac{x^{\frac{3}{2}}}{4a}-\frac{\sqrt{x}}{4b}}{(bx-a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x-a)^3,x)
```

```
[Out] 2*(-1/8/a*x^(3/2)-1/8/b*x^(1/2))/(b*x-a)^2+1/4/b/a/(a*b)^(1/2)*arctanh(1/(a
*b)^(1/2)*b*x^(1/2))
```

maxima [A] time = 2.85, size = 80, normalized size = 1.07

$$-\frac{bx^{\frac{3}{2}}+a\sqrt{x}}{4\left(ab^3x^2-2\,a^2b^2x+a^3b\right)}-\frac{\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x^(3/2) + a*sqrt(x))/(a*b^3*x^2 - 2*a^2*b^2*x + a^3*b) - 1/8*log((b
*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.14, size = 57, normalized size = 0.76

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{3/2}\,b^{3/2}}-\frac{\frac{x^{3/2}}{4\,a}+\frac{\sqrt{x}}{4\,b}}{a^2-2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^(1/2)/(a - b*x)^3,x)
```

```
[Out] atanh((b^(1/2)*x^(1/2))/a^(1/2))/(4*a^(3/2)*b^(3/2)) - (x^(3/2)/(4*a) + x^(
1/2)/(4*b))/(a^2 + b^2*x^2 - 2*a*b*x)
```

sympy [A] time = 15.19, size = 668, normalized size = 8.91

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2x^{\frac{3}{2}}}{3a^3} & \text{for}\: b = 0 \\ -\frac{2}{3b^3x^{\frac{3}{2}}} & \text{for}\: a = 0 \\ -\frac{2a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{2\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{2abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{2abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} - \frac{b^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} + \frac{b^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{7}{2}}b^2\sqrt{\frac{1}{b}}-16a^{\frac{5}{2}}b^3x\sqrt{\frac{1}{b}}+8a^{\frac{3}{2}}b^4x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2*x**(3/2)/(3*a**3), Eq(b,
 0)), (-2/(3*b**3*x**(3/2)), Eq(a, 0)), (-2*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8
*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x
**2*sqrt(1/b)) - 2*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(7/2)*b**2*sqrt(1/
b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) - a**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*
b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + a**2*log(sqrt(a)*sqrt(
1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) +
 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + 2*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x)
```

```
)/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b*
*4*x**2*sqrt(1/b)) - 2*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b
**2*sqrt(1/b) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/
b)) - b**2*x**2*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b
) - 16*a**(5/2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)) + b**2*x
**2*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(7/2)*b**2*sqrt(1/b) - 16*a**(5/
2)*b**3*x*sqrt(1/b) + 8*a**(3/2)*b**4*x**2*sqrt(1/b)), True))
```

3.486 $\int \frac{1}{\sqrt{x}(-a+bx)^3} dx$

Optimal. Leaf size=72

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}-\frac{\sqrt{x}}{2a(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-a + b*x)^3),x]
```

```
[Out] -Sqrt[x]/(2*a*(a - b*x)^2) - (3*Sqrt[x])/(4*a^2*(a - b*x)) - (3*ArcTanh[(Sq
rt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}(-a+bx)^3} dx &= -\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\int \frac{1}{\sqrt{x}(-a+bx)^2} dx}{4a}\\
&= -\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\sqrt{x}}{4a^2(a-bx)}+\frac{3\int \frac{1}{\sqrt{x}(-a+bx)} dx}{8a^2}\\
&= -\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\sqrt{x}}{4a^2(a-bx)}+\frac{3\,\text{Subst}\left(\int \frac{1}{-a+bx^2} dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{\sqrt{x}}{2a(a-bx)^2}-\frac{3\sqrt{x}}{4a^2(a-bx)}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.33

$$-\frac{2\sqrt{x}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-a + b*x)^3),x]
[Out] (-2*Sqrt[x]*Hypergeometric2F1[1/2, 3, 3/2, (b*x)/a])/a^3
```

IntegrateAlgebraic [A] time = 0.09, size = 64, normalized size = 0.89

$$\frac{3bx^{3/2}-5a\sqrt{x}}{4a^2(a-bx)^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{5/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(-a + b*x)^3),x]
[Out] (-5*a*Sqrt[x] + 3*b*x^(3/2))/(4*a^2*(a - b*x)^2) - (3*ArcTanh[(Sqrt[b]*Sqrt
[x])/Sqrt[a]])/(4*a^(5/2)*Sqrt[b])
```

fricas [A] time = 1.00, size = 185, normalized size = 2.57

$$\left[\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{ab}\,\log\left(\frac{bx+a-2\,\sqrt{ab}\,\sqrt{x}}{bx-a}\right)+2\left(3\,ab^2x-5\,a^2b\right)\sqrt{x}}{8\left(a^3b^3x^2-2\,a^4b^2x+a^5b\right)},\ \frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-ab}\,\arctan\left(\frac{\sqrt{-ab}}{b\sqrt{x}}\right)+\left(3\,ab^2x-5\,a^2b\right)\sqrt{x}}{4\left(a^3b^3x^2-2\,a^4b^2x+a^5b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="fricas")
[Out] [1/8*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(a*b)*log((b*x + a - 2*sqrt(a*b)*sqrt
(x))/(b*x - a)) + 2*(3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2
*x + a^5*b), 1/4*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-a*b)*arctan(sqrt(-a*b)/
(b*sqrt(x))) + (3*a*b^2*x - 5*a^2*b)*sqrt(x))/(a^3*b^3*x^2 - 2*a^4*b^2*x +
a^5*b)]
```

giac [A] time = 1.15, size = 51, normalized size = 0.71

$$\frac{3\,\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^2}+\frac{3\,bx^{\frac{3}{2}}-5\,a\sqrt{x}}{4\,(bx-a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="giac")
[Out] 3/4*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^2) + 1/4*(3*b*x^(3/2) - 5*a*
sqrt(x))/((b*x - a)^2*a^2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.88

$$-\frac{\sqrt{x}}{2\,(bx-a)^2\,a}-\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}-\frac{\sqrt{x}}{2(bx-a)a}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-a)^3/x^(1/2),x)
```

```
[Out] -1/2*x^(1/2)/a/(b*x-a)^2-3/2/a*(-1/2/(b*x-a)/a*x^(1/2)+1/2/a/(a*b)^(1/2)*ar
ctanh(1/(a*b)^(1/2)*b*x^(1/2)))
```

maxima [A] time = 2.88, size = 77, normalized size = 1.07

$$\frac{3\,bx^{\frac{3}{2}} - 5\,a\sqrt{x}}{4\left(a^2b^2x^2 - 2\,a^3bx + a^4\right)} + \frac{3\,\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)^3/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*b*x^(3/2) - 5*a*sqrt(x))/(a^2*b^2*x^2 - 2*a^3*b*x + a^4) + 3/8*log((
b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

mupad [B] time = 0.13, size = 58, normalized size = 0.81

$$-\frac{\frac{5\,\sqrt{x}}{4\,a} - \frac{3\,b\,x^{3/2}}{4\,a^2}}{a^2 - 2\,a\,b\,x + b^2\,x^2} - \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(1/2)*(a - b*x)^3),x)
```

```
[Out] - ((5*x^(1/2))/(4*a) - (3*b*x^(3/2))/(4*a^2))/(a^2 + b^2*x^2 - 2*a*b*x) - (
3*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(5/2)*b^(1/2))
```

sympy [A] time = 25.67, size = 660, normalized size = 9.17

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{2\sqrt{x}}{a^3} & \text{for}\: b = 0 \\ -\frac{2}{5b^3x^{\frac{5}{2}}} & \text{for}\: a = 0 \\ -\frac{10a^{\frac{3}{2}}b\sqrt{x}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{6\sqrt{a}b^2x^{\frac{3}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{3a^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{3a^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{6abx\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{6abx\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} + \frac{3b^2x^2\log\left(-\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} - \frac{3b^2x^2\log\left(\sqrt{a}\sqrt{\frac{1}{b}} + \sqrt{x}\right)}{8a^{\frac{9}{2}}b\sqrt{\frac{1}{b}} - 16a^{\frac{7}{2}}b^2x\sqrt{\frac{1}{b}} + 8a^{\frac{5}{2}}b^3x^2\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-a)**3/x**(1/2),x)
```

```
[Out] Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2*sqrt(x)/a**3, Eq(b, 0)),
 (-2/(5*b**3*x**(5/2)), Eq(a, 0)), (-10*a**(3/2)*b*sqrt(x)*sqrt(1/b)/(8*a**
(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqr
t(1/b)) + 6*sqrt(a)*b**2*x**(3/2)*sqrt(1/b)/(8*a**(9/2)*b*sqrt(1/b) - 16*a*
*(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*a**2*log(-sqr
t(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt
(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*a**2*log(sqrt(a)*sqrt(1/b) + sq
rt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*
b**3*x**2*sqrt(1/b)) - 6*a*b*x*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2
)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/
b)) + 6*a*b*x*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16
*a**(7/2)*b**2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) + 3*b**2*x**2*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**
2*x*sqrt(1/b) + 8*a**(5/2)*b**3*x**2*sqrt(1/b)) - 3*b**2*x**2*log(sqrt(a)*s
qrt(1/b) + sqrt(x))/(8*a**(9/2)*b*sqrt(1/b) - 16*a**(7/2)*b**2*x*sqrt(1/b)
+ 8*a**(5/2)*b**3*x**2*sqrt(1/b)), True))
```

3.487 $\int \frac{1}{x^{3/2}(-a+bx)^3} dx$

Optimal. Leaf size=84

$$-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{15\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}+\frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] 15/(4*a^3*Sqrt[x]) - 1/(2*a*Sqrt[x]*(a - b*x)^2) - 5/(4*a^2*Sqrt[x]*(a - b*
x)) - (15*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(-a+bx)^3}\,dx &= -\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5\int \frac{1}{x^{3/2}(-a+bx)^2}\,dx}{4a}\\ &= -\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}+\frac{15\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^2}\\ &= \frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}+\frac{(15b)\int \frac{1}{\sqrt{x}\,(-a+bx)}\,dx}{8a^3}\\ &= \frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}+\frac{(15b)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx,x,\sqrt{x}\right)}{4a^3}\\ &= \frac{15}{4a^3\sqrt{x}}-\frac{1}{2a\sqrt{x}\,(a-bx)^2}-\frac{5}{4a^2\sqrt{x}\,(a-bx)}-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.29

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}\right)}{a^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 3, 1/2, (b*x)/a])/(a^3*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.11, size = 71, normalized size = 0.85

$$\frac{8a^2-25abx+15b^2x^2}{4a^3\sqrt{x}\,(a-bx)^2}-\frac{15\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^2 - 25*a*b*x + 15*b^2*x^2)/(4*a^3*Sqrt[x]*(a - b*x)^2) - (15*Sqrt[b]*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(7/2))
```

fricas [A] time = 0.97, size = 213, normalized size = 2.54

$$\left[\frac{15\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{\frac{b}{a}}\,\log\left(\frac{bx-2\,a\sqrt{x}\,\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(15\,b^2x^2-25\,abx+8\,a^2\right)\sqrt{x}}{8\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)},\ \frac{15\left(b^2x^3-2\,abx^2+a^2x\right)\sqrt{-\frac{b}{a}}\,\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(15\,b^2x^2-25\,abx+8\,a^2\right)\sqrt{x}}{4\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(b/a)*log((b*x - 2*a*sqrt(x)*sqr
t(b/a) + a)/(b*x - a)) + 2*(15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt(x))/(a^3*b^
2*x^3 - 2*a^4*b*x^2 + a^5*x), 1/4*(15*(b^2*x^3 - 2*a*b*x^2 + a^2*x)*sqrt(-b
/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) + (15*b^2*x^2 - 25*a*b*x + 8*a^2)*sqrt
(x))/(a^3*b^2*x^3 - 2*a^4*b*x^2 + a^5*x)]
```

giac [A] time = 1.05, size = 63, normalized size = 0.75

$$\frac{15\,b\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^{3}}+\frac{2}{a^{3}\sqrt{x}}+\frac{7\,b^{2}x^{\frac{3}{2}}-9\,ab\sqrt{x}}{4\,(bx-a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 15/4*b*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^3) + 2/(a^3*sqrt(x)) + 1/
4*(7*b^2*x^(3/2) - 9*a*b*sqrt(x))/((b*x - a)^2*a^3)
```

maple [A] time = 0.02, size = 58, normalized size = 0.69

$$\frac{2\left(-\frac{15\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{7bx^{\frac{3}{2}}}{8}-\frac{9a\sqrt{x}}{8}}{(bx-a)^{2}}\right)b}{a^{3}}+\frac{2}{a^{3}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x-a)^3,x)
```

```
[Out] 2/a^3*b*((7/8*b*x^(3/2)-9/8*a*x^(1/2))/(b*x-a)^2-15/8/(a*b)^(1/2)*arctanh(1
/(a*b)^(1/2)*b*x^(1/2)))+2/a^3/x^(1/2)
```

maxima [A] time = 2.93, size = 90, normalized size = 1.07

$$\frac{15\,b^{2}x^{2}-25\,abx+8\,a^{2}}{4\left(a^{3}b^{2}x^{\frac{5}{2}}-2\,a^{4}bx^{\frac{3}{2}}+a^{5}\sqrt{x}\right)}+\frac{15\,b\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(15*b^2*x^2 - 25*a*b*x + 8*a^2)/(a^3*b^2*x^(5/2) - 2*a^4*b*x^(3/2) + a^
5*sqrt(x)) + 15/8*b*log((b*sqrt(x) - sqrt(a*b))/(b*sqrt(x) + sqrt(a*b)))/(s
qrt(a*b)*a^3)
```

mupad [B] time = 0.15, size = 69, normalized size = 0.82

$$\frac{\frac{2}{a}+\frac{15\,b^{2}\,x^{2}}{4\,a^{3}}-\frac{25\,b\,x}{4\,a^{2}}}{a^{2}\,\sqrt{x}+b^{2}\,x^{5/2}-2\,a\,b\,x^{3/2}}-\frac{15\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(3/2)*(a - b*x)^3),x)
```

```
[Out] (2/a + (15*b^2*x^2)/(4*a^3) - (25*b*x)/(4*a^2))/(a^2*x^(1/2) + b^2*x^(5/2)
- 2*a*b*x^(3/2)) - (15*b^(1/2)*atanh((b^(1/2)*x^(1/2))/a^(1/2)))/(4*a^(7/2)
)
```

sympy [A] time = 54.56, size = 802, normalized size = 9.55

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{a^{3}\sqrt{x}} & \text{for}\: b = 0 \\ -\frac{2}{7b^{3}x^{\frac{7}{2}}} & \text{for}\: a = 0 \\ \frac{16a^{\frac{5}{2}}\sqrt{\frac{1}{b}}}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{50a^{\frac{3}{2}}bx\sqrt{\frac{1}{b}}}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{30\sqrt{a}b^{2}x^{2}\sqrt{\frac{1}{b}}}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{15a^{2}\sqrt{x}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{15a^{2}\sqrt{x}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{30abx^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{30abx^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} + \frac{15b^{2}x^{\frac{5}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} - \frac{15b^{2}x^{\frac{5}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{8a^{\frac{11}{2}}\sqrt{x}\sqrt{\frac{1}{b}}-16a^{\frac{9}{2}}bx^{\frac{3}{2}}\sqrt{\frac{1}{b}}+8a^{\frac{7}{2}}b^{2}x^{\frac{5}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2/(a**3*sqrt(x)), Eq(b, 0))
, (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (16*a**(5/2)*sqrt(1/b)/(8*a**(11/2)*sqr
t(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2
)*sqrt(1/b)) - 50*a**(3/2)*b*x*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 1
6*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 30*
sqrt(a)*b**2*x**2*sqrt(1/b)/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*
x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) + 15*a**2*sqrt(x)*
log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*a**(9
/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 15*a**2*sq
rt(x)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b) - 16*
a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) - 30*a*
b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt(1/b)
 - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/b)) +
 30*a*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(x)*sqrt
(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*sqrt(1/
b)) + 15*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)*sqrt(
x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**(5/2)*
sqrt(1/b)) - 15*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(8*a**(11/2)
*sqrt(x)*sqrt(1/b) - 16*a**(9/2)*b*x**(3/2)*sqrt(1/b) + 8*a**(7/2)*b**2*x**
(5/2)*sqrt(1/b)), True))
```

3.488 $\int \frac{1}{x^{5/2}(-a+bx)^3} dx$

Optimal. Leaf size=97

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}-\frac{7}{4a^2x^{3/2}(a-bx)}+\frac{35b}{4a^4\sqrt{x}}+\frac{35}{12a^3x^{3/2}}-\frac{1}{2ax^{3/2}(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] 35/(12*a^3*x^(3/2)) + (35*b)/(4*a^4*Sqrt[x]) - 1/(2*a*x^(3/2)*(a - b*x)^2)
- 7/(4*a^2*x^(3/2)*(a - b*x)) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[
a]])/(4*a^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x^{5/2}(-a+bx)^3}\,dx = -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7\int \frac{1}{x^{5/2}(-a+bx)^2}\,dx}{4a}$$
$$= -\frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{35\int \frac{1}{x^{5/2}(-a+bx)}\,dx}{8a^2}$$
$$= \frac{35}{12a^3x^{3/2}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{(35b)\int \frac{1}{x^{3/2}(-a+bx)}\,dx}{8a^3}$$
$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{(35b^2)\int \frac{1}{\sqrt{x}(-a+bx)}\,dx}{8a^4}$$
$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} + \frac{(35b^2)\,\mathrm{Subst}\left(\int \frac{1}{-a+bx^2}\,dx, x, \sqrt{\right.}}{4a^4}$$
$$= \frac{35}{12a^3x^{3/2}} + \frac{35b}{4a^4\sqrt{x}} - \frac{1}{2ax^{3/2}(a-bx)^2} - \frac{7}{4a^2x^{3/2}(a-bx)} - \frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Mathematica [C] time = 0.00, size = 26, normalized size = 0.27

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};\frac{bx}{a}\right)}{3a^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 3, -1/2, (b*x)/a])/(3*a^3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 82, normalized size = 0.85

$$\frac{8a^3+56a^2bx-175ab^2x^2+105b^3x^3}{12a^4x^{3/2}(a-bx)^2} - \frac{35b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(-a + b*x)^3),x]
```

```
[Out] (8*a^3 + 56*a^2*b*x - 175*a*b^2*x^2 + 105*b^3*x^3)/(12*a^4*x^(3/2)*(a - b*x
)^2) - (35*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*a^(9/2))
```

fricas [A] time = 1.01, size = 249, normalized size = 2.57

$$\left[\frac{105\left(b^3x^4-2ab^2x^3+a^2bx^2\right)\sqrt{\frac{b}{a}}\log\left(\frac{bx-2a\sqrt{x}\sqrt{\frac{b}{a}}+a}{bx-a}\right)+2\left(105b^3x^3-175ab^2x^2+56a^2bx+8a^3\right)\sqrt{x}}{24\left(a^4b^2x^4-2a^5bx^3+a^6x^2\right)}, \frac{105\left(b^3x^4-2ab^2x^3+a^2bx^2\right)\sqrt{-\frac{b}{a}}\arctan\left(\frac{a\sqrt{-\frac{b}{a}}}{b\sqrt{x}}\right)+\left(105b^3x^3-175ab^2x^2+56a^2bx+8a^3\right)\sqrt{x}}{12\left(a^4b^2x^4-2a^5bx^3+a^6x^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*x^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(b/a)*log((b*x - 2*a*sqr
t(x)*sqrt(b/a) + a)/(b*x - a)) + 2*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*
x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2), 1/12*(105*(b^3*x
^4 - 2*a*b^2*x^3 + a^2*b*x^2)*sqrt(-b/a)*arctan(a*sqrt(-b/a)/(b*sqrt(x))) +
 (105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)*sqrt(x))/(a^4*b^2*x^4 -
 2*a^5*b*x^3 + a^6*x^2)]
```

giac [A] time = 1.04, size = 73, normalized size = 0.75

$$\frac{35\,b^{2}\arctan\left(\frac{b\sqrt{x}}{\sqrt{-ab}}\right)}{4\,\sqrt{-ab}\,a^{4}}+\frac{2\,(9\,bx+a)}{3\,a^{4}x^{\frac{3}{2}}}+\frac{11\,b^{3}x^{\frac{3}{2}}-13\,ab^{2}\sqrt{x}}{4\,(bx-a)^{2}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="giac")
```

```
[Out] 35/4*b^2*arctan(b*sqrt(x)/sqrt(-a*b))/(sqrt(-a*b)*a^4) + 2/3*(9*b*x + a)/(a
^4*x^(3/2)) + 1/4*(11*b^3*x^(3/2) - 13*a*b^2*sqrt(x))/((b*x - a)^2*a^4)
```

maple [A] time = 0.02, size = 69, normalized size = 0.71

$$\frac{2\left(-\frac{35\,\mathrm{arctanh}\left(\frac{b\sqrt{x}}{\sqrt{ab}}\right)}{8\sqrt{ab}}+\frac{\frac{11b\,x^{\frac{3}{2}}}{8}-\frac{13a\sqrt{x}}{8}}{(bx-a)^{2}}\right)b^{2}}{a^{4}}+\frac{6b}{a^{4}\sqrt{x}}+\frac{2}{3a^{3}x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x-a)^3,x)
```

```
[Out] 2/a^4*b^2*((11/8*b*x^(3/2)-13/8*a*x^(1/2))/(b*x-a)^2-35/8/(a*b)^(1/2)*arcta
nh(1/(a*b)^(1/2)*b*x^(1/2)))+2/3/a^3/x^(3/2)+6/a^4*b/x^(1/2)
```

maxima [A] time = 2.95, size = 103, normalized size = 1.06

$$\frac{105\,b^{3}x^{3}-175\,ab^{2}x^{2}+56\,a^{2}bx+8\,a^{3}}{12\left(a^{4}b^{2}x^{\frac{7}{2}}-2\,a^{5}bx^{\frac{5}{2}}+a^{6}x^{\frac{3}{2}}\right)}+\frac{35\,b^{2}\log\left(\frac{b\sqrt{x}-\sqrt{ab}}{b\sqrt{x}+\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x-a)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(105*b^3*x^3 - 175*a*b^2*x^2 + 56*a^2*b*x + 8*a^3)/(a^4*b^2*x^(7/2) -
2*a^5*b*x^(5/2) + a^6*x^(3/2)) + 35/8*b^2*log((b*sqrt(x) - sqrt(a*b))/(b*sq
rt(x) + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

mupad [B] time = 0.17, size = 80, normalized size = 0.82

$$\frac{\frac{2}{3\,a}-\frac{175\,b^{2}\,x^{2}}{12\,a^{3}}+\frac{35\,b^{3}\,x^{3}}{4\,a^{4}}+\frac{14\,b\,x}{3\,a^{2}}}{a^{2}\,x^{3/2}+b^{2}\,x^{7/2}-2\,a\,b\,x^{5/2}}-\frac{35\,b^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\,a^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x^(5/2)*(a - b*x)^3),x)
```

```
[Out] (2/(3*a) - (175*b^2*x^2)/(12*a^3) + (35*b^3*x^3)/(4*a^4) + (14*b*x)/(3*a^2)
)/(a^2*x^(3/2) + b^2*x^(7/2) - 2*a*b*x^(5/2)) - (35*b^(3/2)*atanh((b^(1/2)*
x^(1/2))/a^(1/2)))/(4*a^(9/2))
```

sympy [A] time = 138.88, size = 892, normalized size = 9.20

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{2}{3a^{3}x^{\frac{3}{2}}} & \text{for}\: b = 0 \\ -\frac{2}{9b^{3}x^{\frac{9}{2}}} & \text{for}\: a = 0 \\ \frac{16a^{\frac{7}{2}}\sqrt{\frac{1}{b}}}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}+\frac{112a^{\frac{5}{2}}bx\sqrt{\frac{1}{b}}}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{350a^{\frac{3}{2}}b^{2}x^{2}\sqrt{\frac{1}{b}}}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}+\frac{210\sqrt{a}b^{3}x^{3}\sqrt{\frac{1}{b}}}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}+\frac{105a^{2}bx^{\frac{3}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{105a^{2}bx^{\frac{3}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{210ab^{2}x^{\frac{5}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}+\frac{210ab^{2}x^{\frac{5}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}+\frac{105b^{3}x^{\frac{7}{2}}\log\left(-\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}}-\frac{105b^{3}x^{\frac{7}{2}}\log\left(\sqrt{a}\sqrt{\frac{1}{b}}+\sqrt{x}\right)}{24a^{\frac{13}{2}}x^{\frac{3}{2}}\sqrt{\frac{1}{b}}-48a^{\frac{11}{2}}bx^{\frac{5}{2}}\sqrt{\frac{1}{b}}+24a^{\frac{9}{2}}b^{2}x^{\frac{7}{2}}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x-a)**3,x)
```

```
[Out] Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2/(3*a**3*x**(3/2)), Eq(b,
0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (16*a**(7/2)*sqrt(1/b)/(24*a**(13/2)
*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*
x**(7/2)*sqrt(1/b)) + 112*a**(5/2)*b*x*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqr
t(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt
(1/b)) - 350*a**(3/2)*b**2*x**2*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqrt(1/b)
- 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b))
+ 210*sqrt(a)*b**3*x**3*sqrt(1/b)/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**
(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*a*
*2*b*x**(3/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt
(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(
1/b)) - 105*a**2*b*x**(3/2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*
x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x
**(7/2)*sqrt(1/b)) - 210*a*b**2*x**(5/2)*log(-sqrt(a)*sqrt(1/b) + sqrt(x))/
(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a
**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 210*a*b**2*x**(5/2)*log(sqrt(a)*sqrt(1/b
) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*x**(5/2)*sqr
t(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) + 105*b**3*x**(7/2)*log(-sqrt
(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a**(11/2)*b*
x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)) - 105*b**3*x**(7/
2)*log(sqrt(a)*sqrt(1/b) + sqrt(x))/(24*a**(13/2)*x**(3/2)*sqrt(1/b) - 48*a
**(11/2)*b*x**(5/2)*sqrt(1/b) + 24*a**(9/2)*b**2*x**(7/2)*sqrt(1/b)), True)
)
```

3.489 $\int x^{5/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=122

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Rubi [A] time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3}-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}+\frac{ax^{5/2}\sqrt{a+bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a + b*x])/(96*
b^2) + (a*x^(5/2)*Sqrt[a + b*x])/(24*b) + (x^(7/2)*Sqrt[a + b*x])/4 - (5*a^
4*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a+bx} + \frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$

$$= \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} - \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2} + \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} + \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b^2}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2} + \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} - \frac{\left(5a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{128b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2} + \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} - \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+b}}\right.}{64b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2} + \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} - \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\right.}{64b^3}$$

$$= \frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a+bx}}{96b^2} + \frac{ax^{5/2}\sqrt{a+bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a+bx} - \frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{7/2}}$$

Mathematica [A] time = 0.18, size = 96, normalized size = 0.79

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^3-10a^2bx+8ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 - 10*a^2*b*x + 8*a*b^2*x^2 + 48*b^3
*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))
/(192*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 95, normalized size = 0.78

$$\frac{5a^4\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{64b^{7/2}} + \frac{\sqrt{a+bx}\left(15a^3\sqrt{x}-10a^2bx^{3/2}+8ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^3*Sqrt[x] - 10*a^2*b*x^(3/2) + 8*a*b^2*x^(5/2) + 48*b^
3*x^(7/2)))/(192*b^3) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64
*b^(7/2))
```

fricas [A] time = 0.95, size = 162, normalized size = 1.33

$$\left[\frac{15\,a^4\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{384\,b^4}, \frac{15\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(48\,b^4x^3+8\,ab^3x^2-10\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\sqrt{x}}{192\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x)
)/b^4, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
(48*b^4*x^3 + 8*a*b^3*x^2 - 10*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt(x))
/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.01, size = 120, normalized size = 0.98

$$\frac{(bx+a)^{\frac{3}{2}} x^{\frac{5}{2}}}{4b}-\frac{5\sqrt{(bx+a)\,x}\; a^{4} \ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{128\sqrt{bx+a}\; b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{bx+a}\; a^{3}\sqrt{x}}{64b^{3}}-\frac{5\,(bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24b^{2}}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a^{2}\sqrt{x}}{32b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(1/2),x)
[Out] 1/4/b*x^(5/2)*(b*x+a)^(3/2)-5/24*a/b^2*x^(3/2)*(b*x+a)^(3/2)+5/32*a^2/b^3*x
^(1/2)*(b*x+a)^(3/2)-5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^3-5/128*a^4/b^(7/2)*(
x*(b*x+a))^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(
1/2))
```

maxima [B] time = 2.95, size = 178, normalized size = 1.46

$$\frac{5\,a^{4}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^{4}b^{3}}{\sqrt{x}}+\frac{73\,(bx+a)^{\frac{3}{2}}a^{4}b^{2}}{x^{\frac{3}{2}}}-\frac{55\,(bx+a)^{\frac{5}{2}}a^{4}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}a^{4}}{x^{\frac{7}{2}}}}{192\left(b^{7}-\frac{4\,(bx+a)b^{6}}{x}+\frac{6\,(bx+a)^{2}b^{5}}{x^{2}}-\frac{4\,(bx+a)^{3}b^{4}}{x^{3}}+\frac{(bx+a)^{4}b^{3}}{x^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) + 73*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) - 55*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^7 - 4*(b*x + a)*b^6/x + 6*(b*x + a)^2*b^5/x^2 - 4*(b*x +
a)^3*b^4/x^3 + (b*x + a)^4*b^3/x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(1/2),x)
[Out] int(x^(5/2)*(a + b*x)^(1/2), x)
```

sympy [A] time = 11.70, size = 153, normalized size = 1.25

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}}+\frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 + b*x/a)) + 5*a**(5/2)*x**(3/2)/(192*b**
2*sqrt(1 + b*x/a)) - a**(3/2)*x**(5/2)/(96*b*sqrt(1 + b*x/a)) + 7*sqrt(a)*x
**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b*
*(7/2)) + b*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

3.490 $\int x^{3/2}\sqrt{a+bx}\,dx$

Optimal. Leaf size=98

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Rubi [A] time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b^2}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^2) + (a*x^(3/2)*Sqrt[a + b*x])/(12*b) + (
x^(5/2)*Sqrt[a + b*x])/3 + (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
8*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}\sqrt{a+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx \\
&= \frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}-\frac{a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b^2} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{8b^2} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^2} \\
&= -\frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b^2}+\frac{ax^{3/2}\sqrt{a+bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a+bx}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 85, normalized size = 0.87

$$
\frac{\sqrt{a+bx}\left(\frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}+\sqrt{b}\,\sqrt{x}\left(-3a^2+2abx+8b^2x^2\right)\right)}{24b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 82, normalized size = 0.84

$$
\frac{\sqrt{a+bx}\left(-3a^2\sqrt{x}+2abx^{3/2}+8b^2x^{5/2}\right)}{24b^2}-\frac{a^3\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{8b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^2*Sqrt[x] + 2*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^2) -
 (a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(5/2))
```

fricas [A] time = 0.54, size = 141, normalized size = 1.44

$$
\left[\frac{3\,a^3\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2+2\,ab^2x-3\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^3},-\frac{3\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(8\,b^3x^2+2\,ab^2x-3\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 2*a*b^2*x - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 2*a*b^2*x
 - 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.00, size = 102, normalized size = 1.04

$$\frac{\sqrt{(bx+a)\,x}\;a^3\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{16\sqrt{bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\;a^2\sqrt{x}}{8b^2}+\frac{(bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3b}-\frac{(bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+a)^(3/2)-1/4*a/b^2*x^(1/2)*(b*x+a)^(3/2)+1/8*a^2*x^(1/2)
*(b*x+a)^(1/2)/b^2+1/16*a^3/b^(5/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [**B**] time = 3.04, size = 146, normalized size = 1.49

$$-\frac{a^3\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{5}{2}}}-\frac{\frac{3\,\sqrt{bx+a}\,a^3b^2}{\sqrt{x}}+\frac{8\,(bx+a)^{\frac{3}{2}}\,a^3b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}\,a^3}{x^{\frac{5}{2}}}}{24\left(b^5-\frac{3\,(bx+a)b^4}{x}+\frac{3\,(bx+a)^2b^3}{x^2}-\frac{(bx+a)^3b^2}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(5/2) - 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) + 8*(b*x + a)^(3/2
)*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x + a)*b^4/x +
 3*(b*x + a)^2*b^3/x^2 - (b*x + a)^3*b^2/x^3)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a+b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(1/2), x)
```

sympy [**A**] time = 6.38, size = 122, normalized size = 1.24

$$-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}+\frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(1/2),x)
```

```
[Out] -a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 + b*x/a)) - a**(3/2)*x**(3/2)/(24*b*sqrt(1
 + b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*sqrt(1 + b*x/a)) + a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(5/2)) + b*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

3.491 $\int \sqrt{x}\sqrt{a+bx}\,dx$

Optimal. Leaf size=74

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{a\sqrt{x}\sqrt{a+bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a + b*x],x]
```

```
[Out] (a*Sqrt[x]*Sqrt[a + b*x])/(4*b) + (x^(3/2)*Sqrt[a + b*x])/2 - (a^2*ArcTanh[
(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{a+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a+bx}+\frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a+bx}}dx\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}dx}{8b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}dx,x,\sqrt{x}\right)}{4b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{1-bx^2}dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b}\\ &= \frac{a\sqrt{x}\sqrt{a+bx}}{4b}+\frac{1}{2}x^{3/2}\sqrt{a+bx}-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 0.97

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}(a+2bx)-\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a + b*x],x]
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(a + 2*b*x) - (a^(3/2)*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(4*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{a^2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{4b^{3/2}}+\frac{\sqrt{a+bx}\left(a\sqrt{x}+2bx^{3/2}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[a + b*x],x]
[Out] (Sqrt[a + b*x]*(a*Sqrt[x] + 2*b*x^(3/2)))/(4*b) + (a^2*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]])/(4*b^(3/2))
```

fricas [A] time = 0.91, size = 114, normalized size = 1.54

$$\left[\frac{a^2\sqrt{b}\log\left(2\,bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(2\,b^2x+ab\right)\sqrt{bx+a}\sqrt{x}}{8\,b^2},\frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(2\,b^2x+ab\right)\sqrt{bx+a}\sqrt{x}}{4\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="fricas")
[Out] [1/8*(a^2*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2*b
^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/4*(a^2*sqrt(-b)*arctan(sqrt(b*x +
 a)*sqrt(-b)/(b*sqrt(x))) + (2*b^2*x + a*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 81, normalized size = 1.09

$$\frac{\sqrt{bx+a}\,x^{\frac{3}{2}}}{2}-\frac{\sqrt{(bx+a)\,x}\;a^{2}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{bx+a}\;b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\;a\sqrt{x}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)+1/4*a*x^(1/2)*(b*x+a)^(1/2)/b-1/8*a^2/b^(3/2)*((b
*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/
2))
```

maxima [B] time = 2.96, size = 108, normalized size = 1.46

$$\frac{a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}+\frac{(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx+a)b^{2}}{x}+\frac{(bx+a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqr
t(x)))/b^(3/2) + 1/4*(sqrt(b*x + a)*a^2*b/sqrt(x) + (b*x + a)^(3/2)*a^2/x^(
3/2))/(b^3 - 2*(b*x + a)*b^2/x + (b*x + a)^2*b/x^2)
```

mupad [B] time = 0.15, size = 52, normalized size = 0.70

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{a}{4\,b}\right)\,\sqrt{a+b\,x}-\frac{a^2\,\ln\left(a+2\,b\,x+2\,\sqrt{b}\,\sqrt{x}\,\sqrt{a+b\,x}\right)}{8\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + a/(4*b))*(a + b*x)^(1/2) - (a^2*log(a + 2*b*x + 2*b^(1/2)*x^
(1/2)*(a + b*x)^(1/2)))/(8*b^(3/2))
```

sympy [A] time = 3.57, size = 97, normalized size = 1.31

$$\frac{a^{\frac{3}{2}}\sqrt{x}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3\sqrt{a}\,x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}-\frac{a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{3}{2}}}+\frac{bx^{\frac{5}{2}}}{2\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+a)**(1/2),x)
```

```
[Out] a**(3/2)*sqrt(x)/(4*b*sqrt(1 + b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 + b*x
/a)) - a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) + b*x**(5/2)/(2*sqr
t(a)*sqrt(1 + b*x/a))
```

3.492 $\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=44

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\sqrt{x}\sqrt{a+bx}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] + (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx = \sqrt{x}\sqrt{a+bx} + \frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$
$$= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x}\sqrt{a+bx} + a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \sqrt{x}\sqrt{a+bx} + \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Mathematica [A] time = 0.09, size = 62, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \sqrt{x}(a+bx)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*(a + b*x) + (a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/Sqrt[b])/Sqrt[a + b*x]
```

IntegrateAlgebraic [A] time = 0.06, size = 47, normalized size = 1.07

$$\sqrt{x}\sqrt{a+bx} - \frac{a\log\left(\sqrt{a+bx} - \sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a + b*x] - (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

fricas [A] time = 0.95, size = 93, normalized size = 2.11

$$\left[\frac{a\sqrt{b}\log\left(2bx + 2\sqrt{bx+a}\sqrt{b}\sqrt{x} + a\right) + 2\sqrt{bx+a}\,b\sqrt{x}}{2b}, -\frac{a\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right) - \sqrt{bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b, -(a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)
)) - sqrt(b*x + a)*b*sqrt(x))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 62, normalized size = 1.41

$$\frac{\sqrt{(bx+a)\,x}\;a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\sqrt{bx+a}\;\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)+1/2*a*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x
+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

maxima [B] time = 2.99, size = 70, normalized size = 1.59

$$-\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\sqrt{b}}-\frac{\sqrt{bx+a}\,a}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt
(x)))/sqrt(b) - sqrt(b*x + a)*a/((b - (b*x + a)/x)*sqrt(x))
```

mupad [B] time = 0.68, size = 41, normalized size = 0.93

$$\sqrt{x}\;\sqrt{a+b\,x}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\;\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a + b*x)^(1/2) + (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2) - a
^(1/2))))/b^(1/2)
```

sympy [A] time = 1.92, size = 42, normalized size = 0.95

$$\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a) + a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

3.493 $\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=45

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b
*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + b\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx \\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right) \\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + (2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right) \\ &= -\frac{2\sqrt{a+bx}}{\sqrt{x}} + 2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)\end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 1.42

$$\frac{2\left(\sqrt{a}\sqrt{b}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\frac{a+bx}{\sqrt{x}}\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (2*(-((a + b*x)/Sqrt[x]) + Sqrt[a]*Sqrt[b]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[
b]*Sqrt[x])/Sqrt[a]]))/Sqrt[a + b*x]
```

IntegrateAlgebraic [A] time = 0.07, size = 47, normalized size = 1.04

$$-\frac{2\sqrt{a+bx}}{\sqrt{x}} - 2\sqrt{b}\,\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*
x]]
```

fricas [A] time = 0.93, size = 89, normalized size = 1.98

$$\left[\frac{\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,\sqrt{bx+a}\,\sqrt{x}}{x}, -\frac{2\left(\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sqrt(b*x +
 a)*sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) +
 sqrt(b*x + a)*sqrt(x))/x]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.04, size = 61, normalized size = 1.36

$$\frac{\sqrt{(bx+a)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{bx+a}\,\sqrt{x}}-\frac{2\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/x^(1/2)+b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*
((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 3.00, size = 54, normalized size = 1.20

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/x^(3/2), x)
```

sympy [A] time = 1.56, size = 68, normalized size = 1.51

$$-\frac{2\sqrt{a}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{2b\sqrt{x}}{\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(a)/(sqrt(x)*sqrt(1 + b*x/a)) + 2*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt
(a)) - 2*b*sqrt(x)/(sqrt(a)*sqrt(1 + b*x/a))
```

3.494 $\int \frac{\sqrt{a+bx}}{x^{5/2}} \, dx$

Optimal. Leaf size=21

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{x^{5/2}} \, dx = -\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$-\frac{2(a+bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*a*x^(3/2))
```

fricas [A] time = 1.01, size = 15, normalized size = 0.71

$$-\frac{2\,{\left(bx+a\right)}^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

giac [B] time = 1.33, size = 33, normalized size = 1.57

$$-\frac{2\,(bx+a)^{\frac{3}{2}}b^4}{3\,((bx+a)b-ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] -2/3*(b*x + a)^(3/2)*b^4/(((b*x + a)*b - a*b)^(3/2)*a*abs(b))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(5/2),x)
[Out] -2/3*(b*x+a)^(3/2)/a/x^(3/2)
```

maxima [A] time = 1.36, size = 15, normalized size = 0.71

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*(b*x + a)^(3/2)/(a*x^(3/2))
```

mupad [B] time = 0.24, size = 21, normalized size = 1.00

$$-\frac{\left(\frac{2\,b\,x}{3\,a}+\frac{2}{3}\right)\,\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(5/2),x)
[Out] -(((2*b*x)/(3*a) + 2/3)*(a + b*x)^(1/2))/x^(3/2)
```

sympy [B] time = 1.46, size = 41, normalized size = 1.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(5/2),x)
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a)
```

3.495 $\int \frac{\sqrt{a+bx}}{x^{7/2}} dx$

Optimal. Leaf size=44

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}} - \frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {45, 37}

$$\frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}} - \frac{2(a+bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(5*a*x^(5/2)) + (4*b*(a + b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^{7/2}} dx &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}} - \frac{(2b)\int \frac{\sqrt{a+bx}}{x^{5/2}} dx}{5a}\\ &= -\frac{2(a+bx)^{3/2}}{5ax^{5/2}} + \frac{4b(a+bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.66

$$-\frac{2(3a-2bx)(a+bx)^{3/2}}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (-2*(3*a - 2*b*x)*(a + b*x)^(3/2))/(15*a^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 40, normalized size = 0.91

$$\frac{2\sqrt{a+bx}\left(-3a^2-abx+2b^2x^2\right)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(7/2),x]
```

```
[Out] (2*Sqrt[a + b*x]*(-3*a^2 - a*b*x + 2*b^2*x^2))/(15*a^2*x^(5/2))
```

fricas [A] time = 0.69, size = 34, normalized size = 0.77

$$\frac{2\left(2\,b^{2}x^{2}-abx-3\,a^{2}\right)\sqrt{bx+a}}{15\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b^2*x^2 - a*b*x - 3*a^2)*sqrt(b*x + a)/(a^2*x^(5/2))
```

giac [A] time = 1.10, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2\,(bx+a)b^{5}}{a^{2}}-\frac{5\,b^{5}}{a}\right)(bx+a)^{\frac{3}{2}}b}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(2*(b*x + a)*b^5/a^2 - 5*b^5/a)*(b*x + a)^(3/2)*b/(((b*x + a)*b - a*b)
^(5/2)*abs(b))
```

maple [A] time = 0.00, size = 24, normalized size = 0.55

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\,(-2bx+3a)}{15a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(7/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-2*b*x+3*a)/x^(5/2)/a^2
```

maxima [A] time = 1.29, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*(5*(b*x + a)^(3/2)*b/x^(3/2) - 3*(b*x + a)^(5/2)/x^(5/2))/a^2
```

mupad [B] time = 0.25, size = 32, normalized size = 0.73

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2\,b\,x}{15\,a}-\frac{4\,b^{2}\,x^{2}}{15\,a^{2}}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(7/2),x)
```

[Out] -((a + b*x)^(1/2)*((2*b*x)/(15*a) - (4*b^2*x^2)/(15*a^2) + 2/5))/x^(5/2)

sympy [A] time = 4.88, size = 65, normalized size = 1.48

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{5x^2}-\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{15ax}+\frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}+1}}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/2)/x**(7/2),x)

[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(5*x**2) - 2*b**(3/2)*sqrt(a/(b*x) + 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) + 1)/(15*a**2)

3.496 $\int \frac{\sqrt{a+bx}}{x^{9/2}} \, dx$

Optimal. Leaf size=68

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a+bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(7*a*x^(7/2)) + (8*b*(a + b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a + b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x^{9/2}} \, dx &= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}-\frac{(4b)\int \frac{\sqrt{a+bx}}{x^{7/2}} \, dx}{7a}\\
&= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{\sqrt{a+bx}}{x^{5/2}} \, dx}{35a^2}\\
&= -\frac{2(a+bx)^{3/2}}{7ax^{7/2}}+\frac{8b(a+bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a+bx)^{3/2}}{105a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.59

$$-\frac{2(a+bx)^{3/2}\left(15a^2-12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(15*a^2 - 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

IntegrateAlgebraic [A]   time = 0.09, size = 51, normalized size = 0.75

$$-\frac{2\sqrt{a+bx}\left(15a^3+3a^2bx-4ab^2x^2+8b^3x^3\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/x^(9/2),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(15*a^3 + 3*a^2*b*x - 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

fricas [A]   time = 0.76, size = 45, normalized size = 0.66

$$-\frac{2\left(8\,b^3x^3-4\,ab^2x^2+3\,a^2bx+15\,a^3\right)\sqrt{bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^3*x^3 - 4*a*b^2*x^2 + 3*a^2*b*x + 15*a^3)*sqrt(b*x + a)/(a^3*x^
(7/2))
```

giac [A]   time = 0.97, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx+a)b^7}{a^3}-\frac{7\,b^7}{a^2}\right)(bx+a)\right)(bx+a)^{\frac{3}{2}}b}{105\left((bx+a)b-ab\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] -2/105*(35*b^7/a + 4*(2*(b*x + a)*b^7/a^3 - 7*b^7/a^2)*(b*x + a))*(b*x + a)
^(3/2)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

maple [A]   time = 0.00, size = 35, normalized size = 0.51

$$-\frac{2\,(bx+a)^{\frac{3}{2}}\left(8b^2x^2-12abx+15a^2\right)}{105a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(b*x+a)^(3/2)*(8*b^2*x^2-12*a*b*x+15*a^2)/x^(7/2)/a^3
```

maxima [A]   time = 1.35, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{42\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(b*x + a)^(3/2)*b^2/x^(3/2) - 42*(b*x + a)^(5/2)*b/x^(5/2) + 15*
(b*x + a)^(7/2)/x^(7/2))/a^3
```

mupad [B] time = 0.26, size = 43, normalized size = 0.63

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{16\,b^3\,x^3}{105\,a^3}-\frac{8\,b^2\,x^2}{105\,a^2}+\frac{2\,b\,x}{35\,a}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/x^(9/2),x)
```

```
[Out] -((a + b*x)^(1/2)*((16*b^3*x^3)/(105*a^3) - (8*b^2*x^2)/(105*a^2) + (2*b*x)
/(35*a) + 2/7))/x^(7/2)
```

sympy [B] time = 13.77, size = 347, normalized size = 5.10

$$-\frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}-\frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}+1}}{105a^5b^4x^3+210a^4b^5x^4+105a^3b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] -30*a**5*b**(9/2)*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**
4 + 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) + 1)/(105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 34*a**3*b**(13/2)*x**
2*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**
6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 24*a*b**(17/2)*x**4*sqrt(a/(b*x) +
1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105*a**3*b**6*x**5) - 16*b**(
19/2)*x**5*sqrt(a/(b*x) + 1)/(105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 + 105
*a**3*b**6*x**5)
```

3.497 $\int x^{5/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=127

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Rubi [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2}-\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3}+\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}-\frac{ax^{5/2}\sqrt{a-bx}}{24b}+\frac{1}{4}x^{7/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^3) - (5*a^2*x^(3/2)*Sqrt[a - b*x])/(96
*b^2) - (a*x^(5/2)*Sqrt[a - b*x])/(24*b) + (x^(7/2)*Sqrt[a - b*x])/4 + (5*a
^4*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}\sqrt{a-bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{1}{8}a\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{48b}$$

$$= -\frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b^2}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{128b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-\cdots}}\cdots\right)}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{\left(5a^4\right)\text{Subst}\left(\int \frac{1}{1+b\cdots}\cdots\right)}{64b^3}$$

$$= -\frac{5a^3\sqrt{x}\sqrt{a-bx}}{64b^3} - \frac{5a^2x^{3/2}\sqrt{a-bx}}{96b^2} - \frac{ax^{5/2}\sqrt{a-bx}}{24b} + \frac{1}{4}x^{7/2}\sqrt{a-bx} + \frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{7/2}}$$

Mathematica [A] time = 0.14, size = 98, normalized size = 0.77

$$\frac{\sqrt{a-bx}\left(\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}\left(-15a^3-10a^2bx-8ab^2x^2+48b^3x^3\right)\right)}{192b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 - 10*a^2*b*x - 8*a*b^2*x^2 + 48*b^
3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(192*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 104, normalized size = 0.82

$$\frac{5a^4\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{64b^4} + \frac{\sqrt{a-bx}\left(-15a^3\sqrt{x}-10a^2bx^{3/2}-8ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^3*Sqrt[x] - 10*a^2*b*x^(3/2) - 8*a*b^2*x^(5/2) + 48*b
^3*x^(7/2)))/(192*b^3) + (5*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
 b*x]])/(64*b^4)
```

fricas [A] time = 0.95, size = 164, normalized size = 1.29

$$\left[-\frac{15a^4\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(48b^4x^3-8ab^3x^2-10a^2b^2x-15a^3b\right)\sqrt{-bx+a}\sqrt{x}}{384b^4}, -\frac{15a^4\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(48b^4x^3-8ab^3x^2-10a^2b^2x-15a^3b\right)\sqrt{-bx+a}\sqrt{x}}{192b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*s
qrt(x))/b^4, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))
) - (48*b^4*x^3 - 8*a*b^3*x^2 - 10*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a)*sqr
t(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 127, normalized size = 1.00

$$-\frac{(-bx+a)^{\frac{3}{2}} x^{\frac{5}{2}}}{4b}+\frac{5\sqrt{(-bx+a)\,x}\;a^{4}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{128\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{-bx+a}\;a^{3}\sqrt{x}}{64b^{3}}-\frac{5\,(-bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24b^{2}}-\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^{2}\sqrt{x}}{32b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+a)^(3/2)-5/24*a/b^2*x^(3/2)*(-b*x+a)^(3/2)-5/32*a^2/b^
3*x^(1/2)*(-b*x+a)^(3/2)+5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/128*a^4/b^(7
/2)*(x*(-b*x+a))^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/2*a/b)/(-
b*x^2+a*x)^(1/2))
```

maxima [A] time = 3.01, size = 170, normalized size = 1.34

$$-\frac{5\,a^{4}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{4}b^{3}}{\sqrt{x}}-\frac{73\,(-bx+a)^{\frac{3}{2}}\,a^{4}b^{2}}{x^{\frac{3}{2}}}-\frac{55\,(-bx+a)^{\frac{5}{2}}\,a^{4}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}\,a^{4}}{x^{\frac{7}{2}}}}{192\left(b^{7}-\frac{4\,(bx-a)b^{6}}{x}+\frac{6\,(bx-a)^{2}b^{5}}{x^{2}}-\frac{4\,(bx-a)^{3}b^{4}}{x^{3}}+\frac{(bx-a)^{4}b^{3}}{x^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) - 73*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 55*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^7 - 4*(b*x
- a)*b^6/x + 6*(b*x - a)^2*b^5/x^2 - 4*(b*x - a)^3*b^4/x^3 + (b*x - a)^4*b^
3/x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{a-b\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(1/2), x)
```

sympy [A] time = 11.65, size = 323, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} + \frac{ibx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{5}{2}}x^{\frac{3}{2}}}{192b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{3}{2}}x^{\frac{5}{2}}}{96b\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}\,x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}} + \frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{7}{2}}} - \frac{bx^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(5/2)*x
**(3/2)/(192*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(5/2)/(96*b*sqrt(-1 + b
*x/a)) - 7*I*sqrt(a)*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sqrt(b
)*sqrt(x)/sqrt(a))/(64*b**(7/2)) + I*b*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a)
), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b**3*sqrt(1 - b*x/a)) + 5*a**(
5/2)*x**(3/2)/(192*b**2*sqrt(1 - b*x/a)) + a**(3/2)*x**(5/2)/(96*b*sqrt(1 -
 b*x/a)) + 7*sqrt(a)*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(7/2)) - b*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True
))
```

3.498 $\int x^{3/2}\sqrt{a-bx}\,dx$

Optimal. Leaf size=102

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Rubi [A] time = 0.03, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2}+\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}-\frac{ax^{3/2}\sqrt{a-bx}}{12b}+\frac{1}{3}x^{5/2}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^2) - (a*x^(3/2)*Sqrt[a - b*x])/(12*b) + (
x^(5/2)*Sqrt[a - b*x])/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8
*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}\sqrt{a-bx}\,dx = \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{1}{6}a\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$

$$= -\frac{ax^{3/2}\sqrt{a-bx}}{12b} + \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b}$$

$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2} - \frac{ax^{3/2}\sqrt{a-bx}}{12b} + \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{16b^2}$$

$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2} - \frac{ax^{3/2}\sqrt{a-bx}}{12b} + \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{8b^2}$$

$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2} - \frac{ax^{3/2}\sqrt{a-bx}}{12b} + \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^2}$$

$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b^2} - \frac{ax^{3/2}\sqrt{a-bx}}{12b} + \frac{1}{3}x^{5/2}\sqrt{a-bx} + \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{5/2}}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.85

$$\frac{\sqrt{a-bx}\left(\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}\left(-3a^2-2abx+8b^2x^2\right)\right)}{24b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 - 2*a*b*x + 8*b^2*x^2) + (3*a^(5/2)
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 91, normalized size = 0.89

$$\frac{a^3\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{8b^3} + \frac{\sqrt{a-bx}\left(-3a^2\sqrt{x}-2abx^{3/2}+8b^2x^{5/2}\right)}{24b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^2*Sqrt[x] - 2*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^2) +
 (a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^3)
```

fricas [A] time = 1.02, size = 142, normalized size = 1.39

$$\left[-\frac{3a^3\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(8b^3x^2-2ab^2x-3a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48b^3}, -\frac{3a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(8b^3x^2-2ab^2x-3a^2b\right)\sqrt{-bx+a}\sqrt{x}}{24b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
- 2*(8*b^3*x^2 - 2*a*b^2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/24*(3
*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 2*a*b^
2*x - 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.01, size = 108, normalized size = 1.06

$$\frac{\sqrt{(-bx+a)\,x}\;a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{\sqrt{-bx+a}\;a^{2}\sqrt{x}}{8b^{2}}-\frac{(-bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3b}-\frac{(-bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+a)^(3/2)-1/4*a/b^2*x^(1/2)*(-b*x+a)^(3/2)+1/8*a^2*x^(1
/2)*(-b*x+a)^(1/2)/b^2+1/16*a^3/b^(5/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [**A**] time = 2.99, size = 135, normalized size = 1.32

$$-\frac{a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{8\,(-bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{5}-\frac{3\,(bx-a)b^{4}}{x}+\frac{3\,(bx-a)^{2}b^{3}}{x^{2}}-\frac{(bx-a)^{3}b^{2}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) - 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^5 - 3*(b*x - a)*b^4/x + 3*(b*x - a)^2*b^3/x^2 - (b*x -
 a)^3*b^2/x^3)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,\sqrt{a-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(1/2), x)
```

sympy [**A**] time = 6.33, size = 260, normalized size = 2.55

$$\begin{cases}\frac{ia^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{-1+\frac{bx}{a}}}-\frac{5i\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{ibx^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\-\frac{a^{\frac{5}{2}}\sqrt{x}}{8b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b\sqrt{1-\frac{bx}{a}}}+\frac{5\sqrt{a}\,x^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{bx^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b**2*sqrt(-1 + b*x/a)) - I*a**(3/2)*x**(3/
2)/(24*b*sqrt(-1 + b*x/a)) - 5*I*sqrt(a)*x**(5/2)/(12*sqrt(-1 + b*x/a)) - I
*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) + I*b*x**(7/2)/(3*sqrt(a)
*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b**2*sqrt(1 - b*
x/a)) + a**(3/2)*x**(3/2)/(24*b*sqrt(1 - b*x/a)) + 5*sqrt(a)*x**(5/2)/(12*s
qrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(5/2)) - b*x**(7
/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.499 $\int \sqrt{x}\,\sqrt{a-bx}\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\,\sqrt{a-bx}}{4b}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{a-bx}-\frac{a\sqrt{x}\,\sqrt{a-bx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] -(a*Sqrt[x]*Sqrt[a - b*x])/(4*b) + (x^(3/2)*Sqrt[a - b*x])/2 + (a^2*ArcTan[
(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\sqrt{a-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{a-bx} + \frac{1}{4}a\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx \\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a-bx} + \frac{a^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b} \\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a-bx} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{4b} \\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a-bx} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b} \\
&= -\frac{a\sqrt{x}\sqrt{a-bx}}{4b} + \frac{1}{2}x^{3/2}\sqrt{a-bx} + \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 75, normalized size = 0.97

$$
\frac{\sqrt{a-bx}\left(\frac{a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}(2bx-a)\right)}{4b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-a + 2*b*x) + (a^(3/2)*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(4*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 78, normalized size = 1.01

$$
\frac{a^2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^2} + \frac{\sqrt{a-bx}\left(2bx^{3/2}-a\sqrt{x}\right)}{4b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-(a*Sqrt[x]) + 2*b*x^(3/2)))/(4*b) + (a^2*Sqrt[-b]*Log[-(Sq
rt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b^2)
```

fricas [A] time = 0.90, size = 118, normalized size = 1.53

$$
\left[-\frac{a^2\sqrt{-b}\log\left(-2\,bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(2\,b^2x-ab\right)\sqrt{-bx+a}\sqrt{x}}{8\,b^2}, -\frac{a^2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(2\,b^2x-ab\right)\sqrt{-bx+a}\sqrt{x}}{4\,b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2
*(2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/4*(a^2*sqrt(b)*arctan(sqrt
(-b*x + a)/(sqrt(b)*sqrt(x))) - (2*b^2*x - a*b)*sqrt(-b*x + a)*sqrt(x))/b^2
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 86, normalized size = 1.12

$$\frac{\sqrt{-bx+a}\,x^{\frac{3}{2}}}{2}+\frac{\sqrt{(-bx+a)\,x}\,a^{2}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{8\sqrt{-bx+a}\,b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\,a\sqrt{x}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+a)^(1/2)-1/4*a*x^(1/2)*(-b*x+a)^(1/2)/b+1/8*a^2/b^(3/2)*(
(-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1
/2)*b^(1/2))
```

maxima [A] time = 2.97, size = 95, normalized size = 1.23

$$-\frac{a^{2}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}+\frac{\frac{\sqrt{-bx+a}\,a^{2}b}{\sqrt{x}}-\frac{(-bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{3}-\frac{2\,(bx-a)b^{2}}{x}+\frac{(bx-a)^{2}b}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/4*(sqrt(-b*x
+ a)*a^2*b/sqrt(x) - (-b*x + a)^(3/2)*a^2/x^(3/2))/(b^3 - 2*(b*x - a)*b^2/x
 + (b*x - a)^2*b/x^2)
```

mupad [B] time = 0.08, size = 58, normalized size = 0.75

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{a}{4\,b}\right)\,\sqrt{a-b\,x}-\frac{a^2\,\ln\left(a-2\,b\,x+2\,\sqrt{-b}\,\sqrt{x}\,\sqrt{a-b\,x}\right)}{8\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - a/(4*b))*(a - b*x)^(1/2) - (a^2*log(a - 2*b*x + 2*(-b)^(1/2)
*x^(1/2)*(a - b*x)^(1/2)))/(8*(-b)^(3/2))
```

sympy [A] time = 3.60, size = 207, normalized size = 2.69

$$\begin{cases} \frac{i a^{\frac{3}{2}} \sqrt{x}}{4 b \sqrt{-1 + \frac{b x}{a}}} - \frac{3 i \sqrt{a} x^{\frac{3}{2}}}{4 \sqrt{-1 + \frac{b x}{a}}} - \frac{i a^{2} \operatorname{acosh}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{4 b^{\frac{3}{2}}} + \frac{i b x^{\frac{5}{2}}}{2 \sqrt{a} \sqrt{-1 + \frac{b x}{a}}} & \text{for}\: \left|{\frac{b x}{a}}\right| > 1 \\- \frac{a^{\frac{3}{2}} \sqrt{x}}{4 b \sqrt{1 - \frac{b x}{a}}} + \frac{3 \sqrt{a} x^{\frac{3}{2}}}{4 \sqrt{1 - \frac{b x}{a}}} + \frac{a^{2} \operatorname{asin}{\left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}}{4 b^{\frac{3}{2}}} - \frac{b x^{\frac{5}{2}}}{2 \sqrt{a} \sqrt{1 - \frac{b x}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((I*a**(3/2)*sqrt(x)/(4*b*sqrt(-1 + b*x/a)) - 3*I*sqrt(a)*x**(3/2)
/(4*sqrt(-1 + b*x/a)) - I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2))
```

```
+ I*b*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(3/2)*sq
rt(x)/(4*b*sqrt(1 - b*x/a)) + 3*sqrt(a)*x**(3/2)/(4*sqrt(1 - b*x/a)) + a**2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(3/2)) - b*x**(5/2)/(2*sqrt(a)*sqrt(1
- b*x/a)), True))
```

3.500 $\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=46

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{a-bx}+\frac{1}{2}a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx \\
&= \sqrt{x}\sqrt{a-bx}+a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right) \\
&= \sqrt{x}\sqrt{a-bx}+a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right) \\
&= \sqrt{x}\sqrt{a-bx}+\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.41

$$\frac{\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}+\sqrt{x}(a-bx)}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/Sqrt[x],x]
[Out] (Sqrt[x]*(a - b*x) + (a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[a]])/Sqrt[b])/Sqrt[a - b*x]
```

IntegrateAlgebraic [A] time = 0.07, size = 55, normalized size = 1.20

$$\sqrt{x}\sqrt{a-bx}+\frac{a\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[a - b*x] + (a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]
])/b
```

fricas [A] time = 0.98, size = 94, normalized size = 2.04

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b},-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*s
qrt(-b*x + a)*b*sqrt(x))/b, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt
(x))) - sqrt(-b*x + a)*b*sqrt(x))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 66, normalized size = 1.43

$$\frac{\sqrt{(-bx+a)x}\, a \arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2\sqrt{-bx+a}\,\sqrt{b}\,\sqrt{x}} + \sqrt{-bx+a}\,\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)+1/2*a*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1
/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.90, size = 52, normalized size = 1.13

$$-\frac{a \arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}} + \frac{\sqrt{-bx+a}\,a}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + sqrt(-b*x + a)*a/((b
- (b*x - a)/x)*sqrt(x))
```

mupad [B] time = 0.59, size = 43, normalized size = 0.93

$$\sqrt{x}\,\sqrt{a-b\,x} + \frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(a - b*x)^(1/2) + (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^
(1/2))))/b^(1/2)
```

sympy [A] time = 1.96, size = 119, normalized size = 2.59

$$\begin{cases} -\frac{i\sqrt{a}\,\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}} - \frac{ia \operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} + \frac{ibx^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ \sqrt{a}\,\sqrt{x}\,\sqrt{1-\frac{bx}{a}} + \frac{a \operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)/sqrt(-1 + b*x/a) - I*a*acosh(sqrt(b)*sqrt(x)/
sqrt(a))/sqrt(b) + I*b*x**(3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (sqrt(a)*sqrt(x)*sqrt(1 - b*x/a) + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b
), True))
```

3.501 $\int \frac{\sqrt{a-bx}}{x^{3/2}} dx$

Optimal. Leaf size=47

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - b\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - (2b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= -\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 69, normalized size = 1.47

$$-\frac{2\left(\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+a-bx\right)}{\sqrt{x}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*(a - b*x + Sqrt[a]*Sqrt[b]*Sqrt[x]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[a]]))/(Sqrt[x]*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 53, normalized size = 1.13

$$-\frac{2\sqrt{a-bx}}{\sqrt{x}} - 2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[a - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a -
b*x]]
```

fricas [A] time = 0.75, size = 91, normalized size = 1.94

$$\left[\frac{\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\sqrt{-bx+a}\,\sqrt{x}}{x}, \frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+a}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) - 2*sqrt(-
b*x + a)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))
- sqrt(-b*x + a)*sqrt(x))/x]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="giac")
```

[Out] Timed out

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx+a}}{x^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(3/2),x)
```

```
[Out] int((-b*x+a)^(1/2)/x^(3/2),x)
```

maxima [A] time = 2.93, size = 35, normalized size = 0.74

$$2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right) - \frac{2\sqrt{-bx+a}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)/sqrt(
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a-b\,x}}{x^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(1/2)/x^(3/2), x)
```

sympy [A] time = 1.70, size = 148, normalized size = 3.15

$$\begin{cases} \frac{2i\sqrt{a}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}} + 2i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) - \frac{2ib\sqrt{x}}{\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ -\frac{2\sqrt{a}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}} - 2\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) + \frac{2b\sqrt{x}}{\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*sqrt(a)/(sqrt(x)*sqrt(-1 + b*x/a)) + 2*I*sqrt(b)*acosh(sqrt(
b)*sqrt(x)/sqrt(a)) - 2*I*b*sqrt(x)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a)
> 1), (-2*sqrt(a)/(sqrt(x)*sqrt(1 - b*x/a)) - 2*sqrt(b)*asin(sqrt(b)*sqrt(x
)/sqrt(a)) + 2*b*sqrt(x)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.502 $\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx = -\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 1.00

$$-\frac{2(a-bx)^{3/2}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(5/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(3*a*x^(3/2))
```

fricas [A] time = 1.02, size = 23, normalized size = 1.05

$$\frac{2\,(bx-a)\sqrt{-bx+a}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)/(a*x^(3/2))
```

giac [B] time = 1.40, size = 42, normalized size = 1.91

$$\frac{2\,(bx-a)\sqrt{-bx+a}\,b^4}{3\,((bx-a)b+ab)^{\frac{3}{2}}a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] 2/3*(b*x - a)*sqrt(-b*x + a)*b^4/(((b*x - a)*b + a*b)^(3/2)*a*abs(b))
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3a\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(5/2),x)
[Out] -2/3*(-b*x+a)^(3/2)/a/x^(3/2)
```

maxima [A] time = 1.31, size = 16, normalized size = 0.73

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -2/3*(-b*x + a)^(3/2)/(a*x^(3/2))
```

mupad [B] time = 0.24, size = 21, normalized size = 0.95

$$\frac{\left(\frac{2\,b\,x}{3\,a}-\frac{2}{3}\right)\,\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(5/2),x)
[Out] (((2*b*x)/(3*a) - 2/3)*(a - b*x)^(1/2))/x^(3/2)
```

sympy [B] time = 1.55, size = 88, normalized size = 4.00

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{2ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(5/2),x)
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 2*b**(3/2)*sqrt(a/(b*x) - 1
)/(3*a), Abs(a/(b*x)) > 1), (-2*I*sqrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 2*I*b*
*(3/2)*sqrt(-a/(b*x) + 1)/(3*a), True))
```

3.503 $\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}-\frac{2(a-bx)^{3/2}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(5*a*x^(5/2)) - (4*b*(a - b*x)^(3/2))/(15*a^2*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}+\frac{(2b)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{5a}\\ &= -\frac{2(a-bx)^{3/2}}{5ax^{5/2}}-\frac{4b(a-bx)^{3/2}}{15a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.65

$$-\frac{2(a-bx)^{3/2}(3a+2bx)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(7/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2)*(3*a + 2*b*x))/(15*a^2*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 40, normalized size = 0.87

$$\frac{2\sqrt{a-bx}\left(-3a^2+abx+2b^2x^2\right)}{15a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(7/2),x]
[Out] (2*Sqrt[a - b*x]*(-3*a^2 + a*b*x + 2*b^2*x^2))/(15*a^2*x^(5/2))
```

fricas [A] time = 0.86, size = 34, normalized size = 0.74

$$\frac{2\left(2\,b^{2}x^{2}+abx-3\,a^{2}\right)\sqrt{-bx+a}}{15\,a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="fricas")
[Out] 2/15*(2*b^2*x^2 + a*b*x - 3*a^2)*sqrt(-b*x + a)/(a^2*x^(5/2))
```

giac [A] time = 1.38, size = 61, normalized size = 1.33

$$\frac{2\left(\frac{2\,(bx-a)b^{5}}{a^{2}}+\frac{5\,b^{5}}{a}\right)(bx-a)\sqrt{-bx+a}\,b}{15\,((bx-a)b+ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="giac")
[Out] 2/15*(2*(b*x - a)*b^5/a^2 + 5*b^5/a)*(b*x - a)*sqrt(-b*x + a)*b/(((b*x - a)
*b + a*b)^(5/2)*abs(b))
```

maple [A] time = 0.00, size = 25, normalized size = 0.54

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}\,(2bx+3a)}{15a^{2}x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(7/2),x)
[Out] -2/15*(-b*x+a)^(3/2)*(2*b*x+3*a)/x^(5/2)/a^2
```

maxima [A] time = 1.34, size = 33, normalized size = 0.72

$$-\frac{2\left(\frac{5\,(-bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(7/2),x, algorithm="maxima")
[Out] -2/15*(5*(-b*x + a)^(3/2)*b/x^(3/2) + 3*(-b*x + a)^(5/2)/x^(5/2))/a^2
```

mupad [B] time = 0.25, size = 32, normalized size = 0.70

$$\frac{\sqrt{a-b\,x}\,\left(\frac{4\,b^{2}\,x^{2}}{15\,a^{2}}+\frac{2\,b\,x}{15\,a}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((4*b^2*x^2)/(15*a^2) + (2*b*x)/(15*a) - 2/5))/x^(5/2)
```

sympy [A] time = 5.01, size = 241, normalized size = 5.24

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{5x^2} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{15ax} + \frac{4b^{\frac{5}{2}}\sqrt{\frac{a}{bx}-1}}{15a^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{6ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{x(-15a^3bx+15a^2b^2x^2)} - \frac{8ia^2b^{\frac{5}{2}}\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} - \frac{2iab^{\frac{7}{2}}x\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} + \frac{4ib^{\frac{9}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-15a^3bx+15a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(5*x**2) + 2*b**(3/2)*sqrt(a/(b*x)
- 1)/(15*a*x) + 4*b**(5/2)*sqrt(a/(b*x) - 1)/(15*a**2), Abs(a/(b*x)) > 1),
(6*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1)/(x*(-15*a**3*b*x + 15*a**2*b**2*x**2)
) - 8*I*a**2*b**(5/2)*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2)
 - 2*I*a*b**(7/2)*x*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2) +
 4*I*b**(9/2)*x**2*sqrt(-a/(b*x) + 1)/(-15*a**3*b*x + 15*a**2*b**2*x**2), T
rue))
```

3.504 $\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{2(a-bx)^{3/2}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2))/(7*a*x^(7/2)) - (8*b*(a - b*x)^(3/2))/(35*a^2*x^(5/2))
 - (16*b^2*(a - b*x)^(3/2))/(105*a^3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a-bx}}{x^{9/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}+\frac{(4b)\int \frac{\sqrt{a-bx}}{x^{7/2}}\,dx}{7a}\\ &= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{\sqrt{a-bx}}{x^{5/2}}\,dx}{35a^2}\\ &= -\frac{2(a-bx)^{3/2}}{7ax^{7/2}}-\frac{8b(a-bx)^{3/2}}{35a^2x^{5/2}}-\frac{16b^2(a-bx)^{3/2}}{105a^3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.58

$$-\frac{2(a-bx)^{3/2}\left(15a^2+12abx+8b^2x^2\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (-2*(a - b*x)^(3/2)*(15*a^2 + 12*a*b*x + 8*b^2*x^2))/(105*a^3*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 52, normalized size = 0.73

$$\frac{2\sqrt{a-bx}\left(-15a^3+3a^2bx+4ab^2x^2+8b^3x^3\right)}{105a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a - b*x]/x^(9/2),x]
```

```
[Out] (2*Sqrt[a - b*x]*(-15*a^3 + 3*a^2*b*x + 4*a*b^2*x^2 + 8*b^3*x^3))/(105*a^3*
x^(7/2))
```

fricas [A] time = 0.96, size = 46, normalized size = 0.65

$$\frac{2\left(8\,b^3x^3+4\,ab^2x^2+3\,a^2bx-15\,a^3\right)\sqrt{-bx+a}}{105\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="fricas")
```

```
[Out] 2/105*(8*b^3*x^3 + 4*a*b^2*x^2 + 3*a^2*b*x - 15*a^3)*sqrt(-b*x + a)/(a^3*x^
(7/2))
```

giac [A] time = 1.34, size = 79, normalized size = 1.11

$$\frac{2\left(\frac{35\,b^7}{a}+4\left(\frac{2\,(bx-a)b^7}{a^3}+\frac{7\,b^7}{a^2}\right)(bx-a)\right)(bx-a)\sqrt{-bx+a}\,b}{105\,((bx-a)b+ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="giac")
```

```
[Out] 2/105*(35*b^7/a + 4*(2*(b*x - a)*b^7/a^3 + 7*b^7/a^2)*(b*x - a))*(b*x - a)*
sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^(7/2)*abs(b))
```

maple [A] time = 0.00, size = 36, normalized size = 0.51

$$-\frac{2\,(-bx+a)^{\frac{3}{2}}\left(8b^2x^2+12abx+15a^2\right)}{105a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(1/2)/x^(9/2),x)
```

```
[Out] -2/105*(-b*x+a)^(3/2)*(8*b^2*x^2+12*a*b*x+15*a^2)/x^(7/2)/a^3
```

maxima [A] time = 1.35, size = 49, normalized size = 0.69

$$-\frac{2\left(\frac{35\,(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{42\,(-bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(-bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{105\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(1/2)/x^(9/2),x, algorithm="maxima")
```

```
[Out] -2/105*(35*(-b*x + a)^(3/2)*b^2/x^(3/2) + 42*(-b*x + a)^(5/2)*b/x^(5/2) + 1
5*(-b*x + a)^(7/2)/x^(7/2))/a^3
```

mupad [B] time = 0.27, size = 43, normalized size = 0.61

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,b^2\,x^2}{105\,a^2}+\frac{16\,b^3\,x^3}{105\,a^3}+\frac{2\,b\,x}{35\,a}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*b^2*x^2)/(105*a^2) + (16*b^3*x^3)/(105*a^3) + (2*b*x)/
(35*a) - 2/7))/x^(7/2)
```

sympy [B] time = 26.81, size = 707, normalized size = 9.96

$$\begin{cases} \frac{30a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{66a^4b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{34a^3b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{6a^2b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{24ab^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{16b^{\frac{19}{2}}x^5\sqrt{\frac{a}{bx}-1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} & \text{for}\: \left|\frac{a}{bx}\right| > 1 \\ \frac{30ia^5b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{66ia^4b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{34ia^3b^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{6ia^2b^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} + \frac{24iab^{\frac{17}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} - \frac{16ib^{\frac{19}{2}}x^5\sqrt{-\frac{a}{bx}+1}}{-105a^5b^4x^3+210a^4b^5x^4-105a^3b^6x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((30*a**5*b**(9/2)*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a*
*4*b**5*x**4 - 105*a**3*b**6*x**5) - 66*a**4*b**(11/2)*x*sqrt(a/(b*x) - 1)/
(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 34*a**3*b
**(13/2)*x**2*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 -
 105*a**3*b**6*x**5) - 6*a**2*b**(15/2)*x**3*sqrt(a/(b*x) - 1)/(-105*a**5*b
**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 24*a*b**(17/2)*x**4*s
qrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*
x**5) - 16*b**(19/2)*x**5*sqrt(a/(b*x) - 1)/(-105*a**5*b**4*x**3 + 210*a**4
*b**5*x**4 - 105*a**3*b**6*x**5), Abs(a/(b*x)) > 1), (30*I*a**5*b**(9/2)*sq
rt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*
x**5) - 66*I*a**4*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210
*a**4*b**5*x**4 - 105*a**3*b**6*x**5) + 34*I*a**3*b**(13/2)*x**2*sqrt(-a/(b
*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) -
6*I*a**2*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*
b**5*x**4 - 105*a**3*b**6*x**5) + 24*I*a*b**(17/2)*x**4*sqrt(-a/(b*x) + 1)/
(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 105*a**3*b**6*x**5) - 16*I*b**(
19/2)*x**5*sqrt(-a/(b*x) + 1)/(-105*a**5*b**4*x**3 + 210*a**4*b**5*x**4 - 1
05*a**3*b**6*x**5), True))
```

3.505 $\int x^{5/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=108

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Rubi [A] time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{24b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(24*b^2) + (x
^(5/2)*Sqrt[2 + b*x])/(12*b) + (x^(7/2)*Sqrt[2 + b*x])/4 - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int x^{5/2}\sqrt{2+bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{2+bx} + \frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$$
$$= \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{12b}$$
$$= -\frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \cdot\right.}{4b^3}$$
$$= \frac{5\sqrt{x}\sqrt{2+bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{24b^2} + \frac{x^{5/2}\sqrt{2+bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2+bx} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{bx+2}\left(6b^3x^3+2b^2x^2-5bx+15\right)}{24b^3} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 85, normalized size = 0.79

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{7/2}} + \frac{\sqrt{bx+2}\left(6b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 6*b^3*x^(7/2)))/
(24*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(7/2))
```

fricas [A] time = 0.92, size = 140, normalized size = 1.30

$$\left[\frac{\left(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^4}, \frac{\left(6\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*
sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/24*((6*b^4*x^3
 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x
+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b
```

```
*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8
/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+4*abs(b)/b^2
*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sq
rt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln
(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.01, size = 108, normalized size = 1.00

$$\frac{(bx+2)^{\frac{3}{2}}x^{\frac{5}{2}}}{4b}-\frac{5\,(bx+2)^{\frac{3}{2}}x^{\frac{3}{2}}}{12b^2}+\frac{5\,(bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^3}-\frac{5\sqrt{bx+2}\,\sqrt{x}}{8b^3}-\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/4/b*x^(5/2)*(b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(b*x+2)^(3/2)+5/8/b^3*x^(1/2)*
(b*x+2)^(3/2)-5/8*x^(1/2)*(b*x+2)^(1/2)/b^3-5/8/b^(7/2)*(x*(b*x+2))^(1/2)/(
b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 3.03, size = 163, normalized size = 1.51

$$\frac{\frac{15\sqrt{bx+2}\,b^3}{\sqrt{x}}+\frac{73\,(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4\,(bx+2)b^6}{x}+\frac{6\,(bx+2)^2b^5}{x^2}-\frac{4\,(bx+2)^3b^4}{x^3}+\frac{(bx+2)^4b^3}{x^4}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) + 73*(b*x + 2)^(3/2)*b^2/x^(3/2) - 55*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x + 2)*b^
6/x + 6*(b*x + 2)^2*b^5/x^2 - 4*(b*x + 2)^3*b^4/x^3 + (b*x + 2)^4*b^3/x^4)
+ 5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(
x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,\sqrt{b\,x+2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(1/2), x)
```

sympy [A] time = 10.12, size = 117, normalized size = 1.08

$$\frac{bx^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{24b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b^3\sqrt{bx+2}}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(9/2)/(4*sqrt(b*x + 2)) + 7*x**(7/2)/(12*sqrt(b*x + 2)) - x**(5/2)/(24
*b*sqrt(b*x + 2)) + 5*x**(3/2)/(24*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(4*b**3*
sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2))
```

3.506 $\int x^{3/2}\sqrt{2+bx}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(6*b) + (x^(5/2)
*Sqrt[2 + b*x])/3 + ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\sqrt{2+bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{1}{3}\int\frac{x^{3/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}-\frac{\int\frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}\\
&= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\int\frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^2}\\
&= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2+bx}+\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.69

$$\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+bx-3\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 2*b^2*x^2))/(6*b^2) + ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]]/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 72, normalized size = 0.86

$$\frac{\sqrt{bx+2}\left(2b^2x^{5/2}+bx^{3/2}-3\sqrt{x}\right)}{6b^2}-\frac{\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^2) - Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]/b^(5/2)
```

fricas [A] time = 0.92, size = 121, normalized size = 1.44

$$\left[\frac{\left(2\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^{3}},\frac{\left(2\,b^{3}x^{2}+b^{2}x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x +
 sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 + b^2*x - 3*b)*sq
rt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))
)/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-
78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b
*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))
))+4*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(
b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
)
```

maple [A] time = 0.00, size = 93, normalized size = 1.11

$$\frac{(bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3b}-\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{2b^{2}}+\frac{\sqrt{bx+2}\,\sqrt{x}}{2b^{2}}+\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\;b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/3/b*x^(3/2)*(b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(b*x+2)^(3/2)+1/2*x^(1/2)*(b*x+
2)^(1/2)/b^2+1/2/b^(5/2)*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B]   time = 3.02, size = 134, normalized size = 1.60

$$-\frac{\frac{3\sqrt{bx+2}\,b^2}{\sqrt{x}}+\frac{8\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5-\frac{3\,(bx+2)b^4}{x}+\frac{3\,(bx+2)^2b^3}{x^2}-\frac{(bx+2)^3b^2}{x^3}\right)}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) + 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x +
2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x + 2)*b^4/x + 3*(b*x + 2)^2*b^3/x^2 - (b*x +
 2)^3*b^2/x^3) - 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt
(b*x + 2)/sqrt(x)))/b^(5/2)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{b\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(1/2), x)
```

sympy [A]   time = 5.22, size = 90, normalized size = 1.07

$$\frac{bx^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{5x^{\frac{5}{2}}}{6\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{6b\sqrt{bx+2}}-\frac{\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(7/2)/(3*sqrt(b*x + 2)) + 5*x**(5/2)/(6*sqrt(b*x + 2)) - x**(3/2)/(6*b
*sqrt(b*x + 2)) - sqrt(x)/(b**2*sqrt(b*x + 2)) + asinh(sqrt(2)*sqrt(b)*sqrt
(x)/2)/b**(5/2)
```

3.507 $\int \sqrt{x}\sqrt{2+bx}\,dx$

Optimal. Leaf size=64

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 - ArcSinh[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{2+bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx\\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b}\\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{\sqrt{x}\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.80

$$\frac{\sqrt{x}(bx+1)\sqrt{bx+2}}{2b}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(1 + b*x)*Sqrt[2 + b*x])/(2*b) - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.92

$$\frac{\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}}+\frac{\sqrt{bx+2}\left(bx^{3/2}+\sqrt{x}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(Sqrt[x] + b*x^(3/2)))/(2*b) + Log[-(Sqrt[b]*Sqrt[x]) + Sqrt
[2 + b*x]]/b^(3/2)
```

fricas [A] time = 1.14, size = 101, normalized size = 1.58

$$\left[\frac{(b^2x+b)\sqrt{bx+2}\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{2b^2},\frac{(b^2x+b)\sqrt{bx+2}\sqrt{x}+2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*s
qrt(b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x + b)*sqrt(b*x + 2)*sqrt(x) + 2*sqrt(-
b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [59.8656459
874,25.8388736797]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
```

```
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [33.9285577983,15.451549686]Warning, choosing root of [1,0,%%%{
-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%
{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3
]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{
-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]
%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%
%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,
[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%
}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-
20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,
4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%
}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{1
6,[0,0]%%%}] at parameters values [54.7579903365,81.9516051291]Warning, cho
osing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-
8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%
%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,
[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%
%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4
,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3
]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+
%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3
,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%
}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{
24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,
1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+
%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [18.4052062202,51.
6443148847]1/b*(2*b*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(
b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)
*sqrt(b*x+2))))+4*abs(b)/b^2*(1/2*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqr
t(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.00, size = 75, normalized size = 1.17

$$\frac{\sqrt{bx+2}\, x^{\frac{3}{2}}}{2}+\frac{\sqrt{bx+2}\, \sqrt{x}}{2b}-\frac{\sqrt{(bx+2)\, x}\, \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\, x^{2}+2x}\right)}{2\sqrt{bx+2}\, b^{\frac{3}{2}} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)+1/2*x^(1/2)*(b*x+2)^(1/2)/b-1/2/b^(3/2)*((b*x+2)*
x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.91, size = 98, normalized size = 1.53

$$\frac{\frac{\sqrt{bx+2}\,b}{\sqrt{x}} + \frac{(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3 - \frac{2\,(bx+2)b^2}{x} + \frac{(bx+2)^2 b}{x^2}} + \frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(b*x + 2)*b/sqrt(x) + (b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x + 2)*b^2/
x + (b*x + 2)^2*b/x^2) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b
) + sqrt(b*x + 2)/sqrt(x)))/b^(3/2)
```

mupad [B] time = 0.10, size = 46, normalized size = 0.72

$$\sqrt{x}\,\left(\frac{x}{2}+\frac{1}{2\,b}\right)\,\sqrt{b\,x+2}-\frac{\ln\left(b\,x+\sqrt{b}\,\sqrt{x}\,\sqrt{b\,x+2}\,+1\right)}{2\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 + 1/(2*b))*(b*x + 2)^(1/2) - log(b*x + b^(1/2)*x^(1/2)*(b*x +
2)^(1/2) + 1)/(2*b^(3/2))
```

sympy [A] time = 2.91, size = 71, normalized size = 1.11

$$\frac{bx^{\frac{5}{2}}}{2\sqrt{bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{bx+2}} + \frac{\sqrt{x}}{b\sqrt{bx+2}} - \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(b*x+2)**(1/2),x)
```

```
[Out] b*x**(5/2)/(2*sqrt(b*x + 2)) + 3*x**(3/2)/(2*sqrt(b*x + 2)) + sqrt(x)/(b*sq
rt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

3.508 $\int \frac{\sqrt{2+bx}}{\sqrt{x}} \, dx$

Optimal. Leaf size=40

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx &= \sqrt{x}\sqrt{2+bx}+\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx\\
&= \sqrt{x}\sqrt{2+bx}+2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \sqrt{x}\sqrt{2+bx}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$\sqrt{x}\sqrt{bx+2}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.05, size = 46, normalized size = 1.15

$$\sqrt{x}\sqrt{bx+2}-\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/Sqrt[x],x]
[Out] Sqrt[x]*Sqrt[2 + b*x] - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

fricas [A] time = 1.20, size = 86, normalized size = 2.15

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b},\frac{\sqrt{bx+2}\,b\sqrt{x}-2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b, (sqrt(b*x + 2)*b*sqrt(x) - 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-
b)/(b*sqrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
```

```
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(1/b*sqrt(b*x+2)*sqr
t(b*(b*x+2)-2*b)-2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))
)
```

maple [A] time = 0.00, size = 58, normalized size = 1.45

$$\sqrt{bx+2}\,\sqrt{x}+\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+2)^(1/2)+((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)*ln((b*x+1)/b^
(1/2)+(b*x^2+2*x)^(1/2))/b^(1/2)
```

maxima [B] time = 2.96, size = 68, normalized size = 1.70

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}-\frac{2\sqrt{bx+2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b) - 2*sqrt(b*x + 2)/((b - (b*x + 2)/x)*sqrt(x))
```

mupad [B] time = 0.62, size = 40, normalized size = 1.00

$$\sqrt{x}\,\sqrt{b\,x+2}-\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(b*x + 2)^(1/2) - (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(
1/2))))/b^(1/2)
```

sympy [A] time = 1.65, size = 37, normalized size = 0.92

$$\sqrt{x}\,\sqrt{bx+2}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] sqrt(x)*sqrt(b*x + 2) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

3.509 $\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx$

Optimal. Leaf size=41

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{3/2}}\,dx &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+b\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{2\sqrt{2+bx}}{\sqrt{x}}+2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$2\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] + 2*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 47, normalized size = 1.15

$$-\frac{2\sqrt{bx+2}}{\sqrt{x}}-2\sqrt{b}\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 + b*x])/Sqrt[x] - 2*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*
x]]
```

fricas [A] time = 1.02, size = 87, normalized size = 2.12

$$\left[\frac{\sqrt{b}x\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}\sqrt{x}}{x},-\frac{2\left(\sqrt{-b}x\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b*x + 2)*
sqrt(x))/x, -2*(sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + sqr
t(b*x + 2)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
```

```
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(-2*sqrt(b*x+2)*sqrt
(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)-2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b
)*sqrt(b*x+2))))
```

maple [A] time = 0.02, size = 59, normalized size = 1.44

$$\frac{\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}}-\frac{2\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] -2*(b*x+2)^(1/2)/x^(1/2)+b^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))*((b*
x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 2.95, size = 54, normalized size = 1.32

$$-\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x+2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(1/2)/x^(3/2), x)
```

sympy [A] time = 1.43, size = 48, normalized size = 1.17

$$-2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}-\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2\sqrt{b}\,\log\left(\sqrt{1+\frac{2}{bx}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(1 + 2/(b*x)) - sqrt(b)*log(1/(b*x)) + 2*sqrt(b)*log(sqrt(1
+ 2/(b*x)) + 1)
```

3.510 $\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(5/2),x]
[Out] -(2 + b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx = -\frac{(2+bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(5/2),x]
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$-\frac{(bx+2)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(5/2),x]
[Out] -1/3*(2 + b*x)^(3/2)/x^(3/2)
```

fricas [A] time = 0.78, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

giac [B] time = 1.15, size = 29, normalized size = 1.61

$$-\frac{(bx+2)^{\frac{3}{2}}b^4}{3\left((bx+2)b-2b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] -1/3*(b*x + 2)^(3/2)*b^4/(((b*x + 2)*b - 2*b)^(3/2)*abs(b))
```

maple [A] time = 0.00, size = 13, normalized size = 0.72

$$-\frac{(bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(5/2),x)
[Out] -1/3*(b*x+2)^(3/2)/x^(3/2)
```

maxima [A] time = 1.31, size = 12, normalized size = 0.67

$$-\frac{(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -1/3*(b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.21, size = 18, normalized size = 1.00

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}+\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(5/2),x)
[Out] -((b*x + 2)^(1/2)*((b*x)/3 + 2/3))/x^(3/2)
```

sympy [B] time = 1.45, size = 37, normalized size = 2.06

$$-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(5/2),x)
[Out] -b**(3/2)*sqrt(1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

3.511 $\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=38

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Rubi [A] time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(bx+2)^{3/2}}{15x^{3/2}}-\frac{(bx+2)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(7/2),x]
[Out] -(2 + b*x)^(3/2)/(5*x^(5/2)) + (b*(2 + b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2+bx}}{x^{7/2}}\,dx &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}-\frac{1}{5}b\int \frac{\sqrt{2+bx}}{x^{5/2}}\,dx\\ &= -\frac{(2+bx)^{3/2}}{5x^{5/2}}+\frac{b(2+bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.61

$$\frac{(bx-3)(bx+2)^{3/2}}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(7/2),x]
[Out] ((-3 + b*x)*(2 + b*x)^(3/2))/(15*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 31, normalized size = 0.82

$$\frac{\sqrt{bx+2}\left(b^2x^2-bx-6\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(7/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-6 - b*x + b^2*x^2))/(15*x^(5/2))
```

fricas [A] time = 1.04, size = 25, normalized size = 0.66

$$\frac{\left(b^{2} x^{2} - b x - 6\right) \sqrt{b x + 2}}{15 \, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 - b*x - 6)*sqrt(b*x + 2)/x^(5/2)
```

giac [A] time = 1.09, size = 42, normalized size = 1.11

$$\frac{\left((b x + 2) b^{5} - 5 \, b^{5}\right) (b x + 2)^{\frac{3}{2}} b}{15 \, \left((b x + 2) b - 2 \, b\right)^{\frac{5}{2}} |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2)^(3/2)*b/(((b*x + 2)*b - 2*b)^(5/2)*a
bs(b))
```

maple [A] time = 0.00, size = 18, normalized size = 0.47

$$\frac{(b x + 2)^{\frac{3}{2}} \, (b x - 3)}{15 x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] 1/15*(b*x+2)^(3/2)*(b*x-3)/x^(5/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.68

$$\frac{(b x + 2)^{\frac{3}{2}} b}{6 \, x^{\frac{3}{2}}} - \frac{(b x + 2)^{\frac{5}{2}}}{10 \, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(b*x + 2)^(5/2)/x^(5/2)
```

mupad [B] time = 0.22, size = 26, normalized size = 0.68

$$-\frac{\sqrt{b\,x+2}\,\left(-\frac{b^2\,x^2}{15}+\frac{b\,x}{15}+\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(7/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/15 - (b^2*x^2)/15 + 2/5))/x^(5/2)
```

sympy [A] time = 4.72, size = 56, normalized size = 1.47

$$\frac{b^{\frac{5}{2}}\sqrt{1+\frac{2}{bx}}}{15}-\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{15x}-\frac{2\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] b**(5/2)*sqrt(1 + 2/(b*x))/15 - b**(3/2)*sqrt(1 + 2/(b*x))/(15*x) - 2*sqrt(
b)*sqrt(1 + 2/(b*x))/(5*x**2)
```

3.512 $\int \frac{\sqrt{2+bx}}{x^{9/2}} dx$

Optimal. Leaf size=59

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2(bx+2)^{3/2}}{105x^{3/2}}+\frac{2b(bx+2)^{3/2}}{35x^{5/2}}-\frac{(bx+2)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -(2 + b*x)^(3/2)/(7*x^(7/2)) + (2*b*(2 + b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 + b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2+bx}}{x^{9/2}} dx &= -\frac{(2+bx)^{3/2}}{7x^{7/2}}-\frac{1}{7}(2b)\int \frac{\sqrt{2+bx}}{x^{7/2}} dx \\
&= -\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}+\frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2+bx}}{x^{5/2}} dx \\
&= -\frac{(2+bx)^{3/2}}{7x^{7/2}}+\frac{2b(2+bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2+bx)^{3/2}}{105x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.54

$$-\frac{(bx+2)^{3/2}\left(2b^2x^2-6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + b*x]/x^(9/2),x]
```

```
[Out] -1/105*((2 + b*x)^(3/2)*(15 - 6*b*x + 2*b^2*x^2))/x^(7/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 40, normalized size = 0.68

$$\frac{\sqrt{bx+2}\left(-2b^3x^3+2b^2x^2-3bx-30\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 + b*x]/x^(9/2),x]
[Out] (Sqrt[2 + b*x]*(-30 - 3*b*x + 2*b^2*x^2 - 2*b^3*x^3))/(105*x^(7/2))
```

fricas [A] time = 1.25, size = 34, normalized size = 0.58

$$-\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx+30\right)\sqrt{bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
[Out] -1/105*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x + 30)*sqrt(b*x + 2)/x^(7/2)
```

giac [A] time = 1.11, size = 55, normalized size = 0.93

$$-\frac{\left(35\,b^7+2\left((bx+2)b^7-7\,b^7\right)(bx+2)\right)(bx+2)^{\frac{3}{2}}b}{105\left((bx+2)b-2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
[Out] -1/105*(35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2)^(3/2)*b/(((
b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

maple [A] time = 0.00, size = 27, normalized size = 0.46

$$-\frac{(bx+2)^{\frac{3}{2}}\left(2b^2x^2-6bx+15\right)}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(1/2)/x^(9/2),x)
[Out] -1/105*(b*x+2)^(3/2)*(2*b^2*x^2-6*b*x+15)/x^(7/2)
```

maxima [A] time = 1.27, size = 41, normalized size = 0.69

$$-\frac{(bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
[Out] -1/12*(b*x + 2)^(3/2)*b^2/x^(3/2) + 1/10*(b*x + 2)^(5/2)*b/x^(5/2) - 1/28*(
b*x + 2)^(7/2)/x^(7/2)
```

mupad [B] time = 0.22, size = 34, normalized size = 0.58

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{105}-\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}+\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(1/2)/x^(9/2),x)
```

```
[Out] -((b*x + 2)^(1/2)*((b*x)/35 - (2*b^2*x^2)/105 + (2*b^3*x^3)/105 + 2/7))/x^(
7/2)
```

sympy [B] time = 13.80, size = 270, normalized size = 4.58

$$-\frac{2b^{\frac{19}{2}}x^5\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{6b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{3b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{34b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{132b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}-\frac{120b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{105b^6x^5+420b^5x^4+420b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] -2*b**(19/2)*x**5*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b*
*4*x**3) - 6*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 420*b**5*x**
4 + 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(105*b**6*x**5 + 42
0*b**5*x**4 + 420*b**4*x**3) - 34*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(105*b**
6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 132*b**(11/2)*x*sqrt(1 + 2/(b*x))
/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3) - 120*b**(9/2)*sqrt(1 + 2/
(b*x))/(105*b**6*x**5 + 420*b**5*x**4 + 420*b**4*x**3)
```

3.513 $\int x^{5/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=112

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Rubi [A] time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{24b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{4}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(24*b^2) - (
x^(5/2)*Sqrt[2 - b*x])/(12*b) + (x^(7/2)*Sqrt[2 - b*x])/4 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{5/2}\sqrt{2-bx}\,dx = \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{1}{4}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{12b}$$
$$= -\frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x,\right.}{4b^3}$$
$$= -\frac{5\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{24b^2} - \frac{x^{5/2}\sqrt{2-bx}}{12b} + \frac{1}{4}x^{7/2}\sqrt{2-bx} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.63

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(6b^3x^3-2b^2x^2-5bx-15\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 6*b^3*x^3))/(24*b^3) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 94, normalized size = 0.84

$$\frac{5\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^4} + \frac{\sqrt{2-bx}\left(6b^3x^{7/2}-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 6*b^3*x^(7/2)))
/(24*b^3) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^4)
```

fricas [A] time = 1.04, size = 141, normalized size = 1.26

$$\left[\frac{\left(6b^4x^3-2b^3x^2-5b^2x-15b\right)\sqrt{-bx+2}\sqrt{x}-15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{24b^4}, \frac{\left(6b^4x^3-2b^3x^2-5b^2x-15b\right)\sqrt{-bx+2}\sqrt{x}-30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{24b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 15
*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, 1/24*((6*b^
4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arc
tan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.13435
40126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,
%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[
2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}
+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,
[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(2*b*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt(-b*x+2
)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^11/1440
```

```
/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*
(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(
-b*x+2))))-4*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^
5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*
(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*
x+2)))))
```

maple [A] time = 0.01, size = 116, normalized size = 1.04

$$-\frac{(-bx+2)^{\frac{3}{2}}x^{\frac{5}{2}}}{4b}-\frac{5(-bx+2)^{\frac{3}{2}}x^{\frac{3}{2}}}{12b^2}-\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^3}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{8b^3}+\frac{5\sqrt{(-bx+2)x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-bx^2+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/4/b*x^(5/2)*(-b*x+2)^(3/2)-5/12/b^2*x^(3/2)*(-b*x+2)^(3/2)-5/8/b^3*x^(1/
2)*(-b*x+2)^(3/2)+5/8*x^(1/2)*(-b*x+2)^(1/2)/b^3+5/8/b^(7/2)*((-b*x+2)*x)^(
1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan(b^(1/2)*(x-1/b)/(-b*x^2+2*x)^(1/2))
```

maxima [A] time = 2.93, size = 147, normalized size = 1.31

$$\frac{\frac{15\sqrt{-bx+2}\,b^3}{\sqrt{x}}-\frac{73(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{55(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^7-\frac{4(bx-2)b^6}{x}+\frac{6(bx-2)^2b^5}{x^2}-\frac{4(bx-2)^3b^4}{x^3}+\frac{(bx-2)^4b^3}{x^4}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) - 73*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 55*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^7 - 4*(b*x - 2
)*b^6/x + 6*(b*x - 2)^2*b^5/x^2 - 4*(b*x - 2)^3*b^4/x^3 + (b*x - 2)^4*b^3/x
^4) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\sqrt{2-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(1/2), x)
```

sympy [A] time = 9.92, size = 252, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{7ix^{\frac{7}{2}}}{12\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{24b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{24b^2\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{5i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{bx^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{7x^{\frac{7}{2}}}{12\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{24b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{24b^2\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{5\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(9/2)/(4*sqrt(b*x - 2)) - 7*I*x**(7/2)/(12*sqrt(b*x - 2))
 - I*x**(5/2)/(24*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(24*b**2*sqrt(b*x - 2)) +
 5*I*sqrt(x)/(4*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(4*b**(7/2)), Abs(b*x)/2 > 1), (-b*x**(9/2)/(4*sqrt(-b*x + 2)) + 7*x**(7/2)
/(12*sqrt(-b*x + 2)) + x**(5/2)/(24*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(24*b**2
*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

3.514 $\int x^{3/2}\sqrt{2-bx}\,dx$

Optimal. Leaf size=87

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{1}{3}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(6*b) + (x^(5/2)
*Sqrt[2 - b*x])/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}\sqrt{2-bx}\,dx &= \frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{1}{3}\int\frac{x^{3/2}}{\sqrt{2-bx}}\,dx\\
&= -\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int\frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\int\frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\text{Subst}\left(\int\frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{6b}+\frac{1}{3}x^{5/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.69

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-bx-3\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-3 - b*x + 2*b^2*x^2))/(6*b^2) + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.11, size = 81, normalized size = 0.93

$$\frac{\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}+\frac{\sqrt{2-bx}\left(2b^2x^{5/2}-bx^{3/2}-3\sqrt{x}\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^2) + (Sqrt[-b
]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

fricas [A] time = 1.03, size = 125, normalized size = 1.44

$$\left[\frac{\left(2\,b^3x^2-b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}-3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^3},\frac{\left(2\,b^3x^2-b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}-6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x) - 3*sqrt(-b)*log(-b*
x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, 1/6*((2*b^3*x^2 - b^2*x - 3*
b)*sqrt(-b*x + 2)*sqrt(x) - 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x
))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.13435
40126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,
%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[
2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}
+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,
[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(2*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqr
t(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-
b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sq
rt(-b)*sqrt(-b*x+2))))+4*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8
)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)
+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.00, size = 100, normalized size = 1.15

$$-\frac{(-bx+2)^{\frac{3}{2}}x^{\frac{3}{2}}}{3b}-\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2b^{2}}+\frac{\sqrt{-bx+2}\,\sqrt{x}}{2b^{2}}+\frac{\sqrt{(-bx+2)\,x}\ \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\ b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(1/2),x)
```

```
[Out] -1/3/b*x^(3/2)*(-b*x+2)^(3/2)-1/2/b^2*x^(1/2)*(-b*x+2)^(3/2)+1/2*x^(1/2)*(-
b*x+2)^(1/2)/b^2+1/2/b^(5/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arct
an((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.98, size = 117, normalized size = 1.34

$$\frac{\frac{3\sqrt{-bx+2}\,b^2}{\sqrt{x}} - \frac{8(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}} - \frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^5 - \frac{3(bx-2)b^4}{x} + \frac{3(bx-2)^2b^3}{x^2} - \frac{(bx-2)^3b^2}{x^3}\right)} - \frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) - 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^5 - 3*(b*x - 2)*b^4/x + 3*(b*x - 2)^2*b^3/x^2 - (b*x
 - 2)^3*b^2/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\sqrt{2-b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(1/2), x)
```

sympy [A] time = 5.29, size = 196, normalized size = 2.25

$$\begin{cases} \frac{ibx^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{5ix^{\frac{5}{2}}}{6\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{6b\sqrt{bx-2}} + \frac{i\sqrt{x}}{b^2\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{5x^{\frac{5}{2}}}{6\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{6b\sqrt{-bx+2}} - \frac{\sqrt{x}}{b^2\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(7/2)/(3*sqrt(b*x - 2)) - 5*I*x**(5/2)/(6*sqrt(b*x - 2))
- I*x**(3/2)/(6*b*sqrt(b*x - 2)) + I*sqrt(x)/(b**2*sqrt(b*x - 2)) - I*acosh
(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-b*x**(7/2)/(3*sqrt
(-b*x + 2)) + 5*x**(5/2)/(6*sqrt(-b*x + 2)) + x**(3/2)/(6*b*sqrt(-b*x + 2))
 - sqrt(x)/(b**2*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
, True))
```

3.515 $\int \sqrt{x}\sqrt{2-bx}\,dx$

Optimal. Leaf size=65

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + ArcSin[(Sqrt[b
]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\sqrt{2-bx}\,dx &= \frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.78

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{2-bx}\,(bx-1)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-1 + b*x))/(2*b) + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]
]/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 70, normalized size = 1.08

$$\frac{\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}+\frac{\sqrt{2-bx}\,\left(bx^{3/2}-\sqrt{x}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-Sqrt[x] + b*x^(3/2)))/(2*b) + (Sqrt[-b]*Log[-(Sqrt[-b]*Sqr
t[x]) + Sqrt[2 - b*x]])/b^2
```

fricas [A] time = 1.20, size = 107, normalized size = 1.65

$$\left[\frac{\left(b^2x-b\right)\sqrt{-bx+2}\,\sqrt{x}-\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b^2},\ \frac{\left(b^2x-b\right)\sqrt{-bx+2}\,\sqrt{x}-2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x +
2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/2*((b^2*x - b)*sqrt(-b*x + 2)*sqrt(x) - 2*
sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-41.13435
40126,25.8388736797]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
```

```
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-67.0714422017,15.451549686]Warning, choosing root of [1,0,
%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[
2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}
+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,
[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}
+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-2
0,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2
]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}
+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[
3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}
+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{
4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,
0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+
%%%{16,[0,0]%%%}] at parameters values [-46.2420096635,81.9516051291]Warnin
g, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%
%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1
,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%
%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[
4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+
%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%
%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%
{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-82.594793
7798,51.6443148847]1/b*(-2*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)
-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*
x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-4*abs(b)/b^2*(1/2*sqrt(-b*x+2)*sqrt(-b*(
-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x
+2)))))
```

maple [A] time = 0.00, size = 81, normalized size = 1.25

$$\frac{\sqrt{-bx+2}\, x^{\frac{3}{2}}}{2}-\frac{\sqrt{-bx+2}\, \sqrt{x}}{2b}+\frac{\sqrt{(-bx+2)\, x}\, \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\, x^{2}+2x}}\right)}{2\sqrt{-bx+2}\, b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(-b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(-b*x+2)^(1/2)-1/2*x^(1/2)*(-b*x+2)^(1/2)/b+1/2/b^(3/2)*((-b*x+
2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2
))
```

maxima [A] time = 2.95, size = 81, normalized size = 1.25

$$\frac{\frac{\sqrt{-bx+2}\,b}{\sqrt{x}} - \frac{(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^3 - \frac{2\,(bx-2)b^2}{x} + \frac{(bx-2)^2 b}{x^2}} - \frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (sqrt(-b*x + 2)*b/sqrt(x) - (-b*x + 2)^(3/2)/x^(3/2))/(b^3 - 2*(b*x - 2)*b^
2/x + (b*x - 2)^2*b/x^2) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

mupad [B] time = 0.10, size = 53, normalized size = 0.82

$$\sqrt{x}\,\left(\frac{x}{2}-\frac{1}{2\,b}\right)\sqrt{2-b\,x}-\frac{\ln\left(\sqrt{-b}\,\sqrt{x}\,\sqrt{2-b\,x}-b\,x+1\right)}{2\,{\left(-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(1/2),x)
```

```
[Out] x^(1/2)*(x/2 - 1/(2*b))*(2 - b*x)^(1/2) - log((-b)^(1/2)*x^(1/2)*(2 - b*x)^
(1/2) - b*x + 1)/(2*(-b)^(3/2))
```

sympy [A] time = 2.94, size = 156, normalized size = 2.40

$$\begin{cases} \frac{ibx^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{3ix^{\frac{3}{2}}}{2\sqrt{bx-2}} + \frac{i\sqrt{x}}{b\sqrt{bx-2}} - \frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{bx^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{3x^{\frac{3}{2}}}{2\sqrt{-bx+2}} - \frac{\sqrt{x}}{b\sqrt{-bx+2}} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(5/2)/(2*sqrt(b*x - 2)) - 3*I*x**(3/2)/(2*sqrt(b*x - 2))
+ I*sqrt(x)/(b*sqrt(b*x - 2)) - I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
, Abs(b*x)/2 > 1), (-b*x**(5/2)/(2*sqrt(-b*x + 2)) + 3*x**(3/2)/(2*sqrt(-b*
x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(
3/2), True))
```

3.516 $\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx$

Optimal. Leaf size=41

$$\sqrt{x}\,\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\sqrt{x}\,\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx &= \sqrt{x}\,\sqrt{2-bx}+\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= \sqrt{x}\,\sqrt{2-bx}+2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \sqrt{x}\,\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\sqrt{x}\,\sqrt{2-bx}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.06, size = 55, normalized size = 1.34

$$\sqrt{x}\sqrt{2-bx}+\frac{2\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/Sqrt[x],x]
```

```
[Out] Sqrt[x]*Sqrt[2 - b*x] + (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]
])/b
```

fricas [A] time = 1.22, size = 89, normalized size = 2.17

$$\left[\frac{\sqrt{-bx+2}\,b\sqrt{x}-\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{b},\frac{\sqrt{-bx+2}\,b\sqrt{x}-2\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b*x + 2)*b*sqrt(x) - sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sq
rt(x) + 1))/b, (sqrt(-b*x + 2)*b*sqrt(x) - 2*sqrt(b)*arctan(sqrt(-b*x + 2)/
(sqrt(b)*sqrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
```

```
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(1/b*sqrt(-b*x+2
)*sqrt(-b*(-b*x+2)+2*b)+2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sq
rt(-b*x+2))))
```

maple [B] time = 0.00, size = 63, normalized size = 1.54

$$\sqrt{-bx+2}\,\sqrt{x}+\frac{\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(-b*x+2)^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)/b^(1/2)*ar
ctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.91, size = 49, normalized size = 1.20

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{2\,\sqrt{-bx+2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 2*sqrt(-b*x + 2)/((b
- (b*x - 2)/x)*sqrt(x))
```

mupad [B] time = 0.56, size = 42, normalized size = 1.02

$$\sqrt{x}\,\sqrt{2-b\,x}-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(1/2),x)
```

```
[Out] x^(1/2)*(2 - b*x)^(1/2) - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1
/2))))/b^(1/2)
```

sympy [A] time = 1.71, size = 121, normalized size = 2.95

$$\begin{cases} \frac{ibx^{\frac{3}{2}}}{\sqrt{bx-2}}-\frac{2i\sqrt{x}}{\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{bx^{\frac{3}{2}}}{\sqrt{-bx+2}}+\frac{2\sqrt{x}}{\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x**(3/2)/sqrt(b*x - 2) - 2*I*sqrt(x)/sqrt(b*x - 2) - 2*I*aco
sh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b*x**(3/2)/sqrt(-
b*x + 2) + 2*sqrt(x)/sqrt(-b*x + 2) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqr
t(b), True))
```

3.517 $\int \frac{\sqrt{2-bx}}{x^{3/2}} dx$

Optimal. Leaf size=42

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-bx}}{x^{3/2}} dx &= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-b\int \frac{1}{\sqrt{x}\sqrt{2-bx}} dx\\
&= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} dx, x, \sqrt{x}\right)\\
&= -\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{b}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.08, size = 53, normalized size = 1.26

$$-\frac{2\sqrt{2-bx}}{\sqrt{x}}-2\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(3/2),x]
```

```
[Out] (-2*Sqrt[2 - b*x])/Sqrt[x] - 2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 -
b*x]]
```

fricas [A] time = 1.24, size = 90, normalized size = 2.14

$$\left[\frac{\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-2\,\sqrt{-bx+2}\,\sqrt{x}}{x},\frac{2\left(\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\sqrt{-bx+2}\,\sqrt{x}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - 2*sqrt(-b*x
+ 2)*sqrt(x))/x, 2*(sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - sq
rt(-b*x + 2)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
```

```
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*sqrt(-b*x+2)
*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)+2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)
+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [B] time = 0.04, size = 90, normalized size = 2.14

$$-\frac{\sqrt{(-bx+2)\,x}\,\sqrt{b}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{2\,(bx-2)\,\sqrt{(-bx+2)\,x}}{\sqrt{-\,(bx-2)\,x}\,\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(3/2),x)
```

```
[Out] 2*(b*x-2)/(-x*(b*x-2))^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)-b^(1
/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^
(1/2)/x^(1/2)
```

maxima [A] time = 3.01, size = 35, normalized size = 0.83

$$2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + 2)/sqrt(
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{2-b\,x}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(1/2)/x^(3/2), x)
```

sympy [C] time = 1.58, size = 136, normalized size = 3.24

$$\begin{cases} -2\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}-i\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2i\sqrt{b}\,\log\left(\frac{1}{\sqrt{b}\,\sqrt{x}}\right)-2\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) & \text{for}\ \frac{2}{|bx|}>1 \\ -2i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}-i\sqrt{b}\,\log\left(\frac{1}{bx}\right)+2i\sqrt{b}\,\log\left(\sqrt{1-\frac{2}{bx}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(-1 + 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) + 2*I*sqr
t(b)*log(1/(sqrt(b)*sqrt(x))) - 2*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2),
2/Abs(b*x) > 1), (-2*I*sqrt(b)*sqrt(1 - 2/(b*x)) - I*sqrt(b)*log(1/(b*x)) +
 2*I*sqrt(b)*log(sqrt(1 - 2/(b*x)) + 1), True))
```

3.518 $\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(3*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx=-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$-\frac{(2-bx)^{3/2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] -1/3*(2 - b*x)^(3/2)/x^(3/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 24, normalized size = 1.26

$$\frac{\sqrt{2-bx}\,(bx-2)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-2 + b*x))/(3*x^(3/2))
```

fricas [A] time = 1.29, size = 18, normalized size = 0.95

$$\frac{(bx-2)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="fricas")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)/x^(3/2)
```

giac [B] time = 1.12, size = 35, normalized size = 1.84

$$\frac{(bx-2)\sqrt{-bx+2}\,b^4}{3\left((bx-2)b+2\,b\right)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="giac")
[Out] 1/3*(b*x - 2)*sqrt(-b*x + 2)*b^4/(((b*x - 2)*b + 2*b)^(3/2)*abs(b))
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(5/2),x)
[Out] -1/3*(-b*x+2)^(3/2)/x^(3/2)
```

maxima [A] time = 1.28, size = 13, normalized size = 0.68

$$-\frac{(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(5/2),x, algorithm="maxima")
[Out] -1/3*(-b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.22, size = 18, normalized size = 0.95

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}-\frac{2}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(5/2),x)
[Out] ((2 - b*x)^(1/2)*((b*x)/3 - 2/3))/x^(3/2)
```

sympy [B] time = 1.51, size = 82, normalized size = 4.32

$$\begin{cases} \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}-\frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|}>1 \\ \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}-\frac{2i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(5/2),x)
[Out] Piecewise((b**(3/2)*sqrt(-1 + 2/(b*x))/3 - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(3/2)*sqrt(1 - 2/(b*x))/3 - 2*I*sqrt(b)*sqrt(1 -
 2/(b*x))/(3*x), True))
```

3.519 $\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx$

Optimal. Leaf size=40

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Rubi [A] time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{b(2-bx)^{3/2}}{15x^{3/2}}-\frac{(2-bx)^{3/2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(5*x^(5/2)) - (b*(2 - b*x)^(3/2))/(15*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx&=-\frac{(2-bx)^{3/2}}{5x^{5/2}}+\frac{1}{5}b\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx\\&=-\frac{(2-bx)^{3/2}}{5x^{5/2}}-\frac{b(2-bx)^{3/2}}{15x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.60

$$-\frac{(2-bx)^{3/2}(bx+3)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] -1/15*((2 - b*x)^(3/2)*(3 + b*x))/x^(5/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 31, normalized size = 0.78

$$\frac{\sqrt{2-bx}\left(b^2x^2+bx-6\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(7/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-6 + b*x + b^2*x^2))/(15*x^(5/2))
```

fricas [A] time = 0.67, size = 25, normalized size = 0.62

$$\frac{\left(b^{2} x^{2} + b x - 6\right) \sqrt{-b x + 2}}{15 \, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(b^2*x^2 + b*x - 6)*sqrt(-b*x + 2)/x^(5/2)
```

giac [A] time = 0.85, size = 48, normalized size = 1.20

$$\frac{\left((bx-2)b^{5} + 5 \, b^{5}\right)(bx-2)\sqrt{-bx+2} \, b}{15 \, ((bx-2)b + 2 \, b)^{\frac{5}{2}} |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((b*x - 2)*b^5 + 5*b^5)*(b*x - 2)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b
)^(5/2)*abs(b))
```

maple [A] time = 0.00, size = 19, normalized size = 0.48

$$-\frac{(bx+3)(-bx+2)^{\frac{3}{2}}}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(7/2),x)
```

```
[Out] -1/15*(b*x+3)*(-b*x+2)^(3/2)/x^(5/2)
```

maxima [A] time = 1.33, size = 28, normalized size = 0.70

$$-\frac{(-bx+2)^{\frac{3}{2}} b}{6 \, x^{\frac{3}{2}}} - \frac{(-bx+2)^{\frac{5}{2}}}{10 \, x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(7/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-b*x + 2)^(3/2)*b/x^(3/2) - 1/10*(-b*x + 2)^(5/2)/x^(5/2)
```

mupad [B] time = 0.22, size = 26, normalized size = 0.65

$$\frac{\sqrt{2-b\,x}\,\left(\frac{b^2\,x^2}{15}+\frac{b\,x}{15}-\frac{2}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(7/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/15 + (b^2*x^2)/15 - 2/5))/x^(5/2)
```

sympy [A] time = 4.91, size = 194, normalized size = 4.85

$$\begin{cases} \frac{b^{\frac{5}{2}}\sqrt{-1+\frac{2}{bx}}}{15} + \frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{15x} - \frac{2\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{5x^2} & \text{for } \frac{2}{|bx|} > 1 \\ -\frac{ib^{\frac{9}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx} + \frac{ib^{\frac{7}{2}}x\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx} + \frac{8ib^{\frac{5}{2}}\sqrt{1-\frac{2}{bx}}}{-15b^2x^2+30bx} - \frac{12ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{x(-15b^2x^2+30bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(7/2),x)
```

```
[Out] Piecewise((b**(5/2)*sqrt(-1 + 2/(b*x))/15 + b**(3/2)*sqrt(-1 + 2/(b*x))/(15
*x) - 2*sqrt(b)*sqrt(-1 + 2/(b*x))/(5*x**2), 2/Abs(b*x) > 1), (-I*b**(9/2)*
x**2*sqrt(1 - 2/(b*x))/(-15*b**2*x**2 + 30*b*x) + I*b**(7/2)*x*sqrt(1 - 2/(
b*x))/(-15*b**2*x**2 + 30*b*x) + 8*I*b**(5/2)*sqrt(1 - 2/(b*x))/(-15*b**2*x
**2 + 30*b*x) - 12*I*b**(3/2)*sqrt(1 - 2/(b*x))/(x*(-15*b**2*x**2 + 30*b*x)
), True))
```

3.520 $\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{(2-bx)^{3/2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -(2 - b*x)^(3/2)/(7*x^(7/2)) - (2*b*(2 - b*x)^(3/2))/(35*x^(5/2)) - (2*b^2*
(2 - b*x)^(3/2))/(105*x^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{2-bx}}{x^{9/2}}\,dx &= -\frac{(2-bx)^{3/2}}{7x^{7/2}}+\frac{1}{7}(2b)\int \frac{\sqrt{2-bx}}{x^{7/2}}\,dx\\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}+\frac{1}{35}\left(2b^2\right)\int \frac{\sqrt{2-bx}}{x^{5/2}}\,dx\\ &= -\frac{(2-bx)^{3/2}}{7x^{7/2}}-\frac{2b(2-bx)^{3/2}}{35x^{5/2}}-\frac{2b^2(2-bx)^{3/2}}{105x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.53

$$-\frac{(2-bx)^{3/2}\left(2b^2x^2+6bx+15\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - b*x]/x^(9/2),x]
```

```
[Out] -1/105*((2 - b*x)^(3/2)*(15 + 6*b*x + 2*b^2*x^2))/x^(7/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 41, normalized size = 0.66

$$\frac{\sqrt{2-bx}\left(2b^3x^3+2b^2x^2+3bx-30\right)}{105x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[2 - b*x]/x^(9/2),x]
[Out] (Sqrt[2 - b*x]*(-30 + 3*b*x + 2*b^2*x^2 + 2*b^3*x^3))/(105*x^(7/2))
```

fricas [A] time = 0.90, size = 35, normalized size = 0.56

$$\frac{\left(2\,b^3x^3+2\,b^2x^2+3\,bx-30\right)\sqrt{-bx+2}}{105\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="fricas")
[Out] 1/105*(2*b^3*x^3 + 2*b^2*x^2 + 3*b*x - 30)*sqrt(-b*x + 2)/x^(7/2)
```

giac [A] time = 0.98, size = 61, normalized size = 0.98

$$\frac{\left(35\,b^7+2\left((bx-2)b^7+7\,b^7\right)(bx-2)\right)(bx-2)\sqrt{-bx+2}\,b}{105\left((bx-2)b+2\,b\right)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="giac")
[Out] 1/105*(35*b^7 + 2*((b*x - 2)*b^7 + 7*b^7)*(b*x - 2))*(b*x - 2)*sqrt(-b*x +
2)*b/(((b*x - 2)*b + 2*b)^(7/2)*abs(b))
```

maple [A] time = 0.00, size = 28, normalized size = 0.45

$$-\frac{\left(2b^2x^2+6bx+15\right)(-bx+2)^{\frac{3}{2}}}{105x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(1/2)/x^(9/2),x)
[Out] -1/105*(2*b^2*x^2+6*b*x+15)*(-b*x+2)^(3/2)/x^(7/2)
```

maxima [A] time = 1.31, size = 44, normalized size = 0.71

$$-\frac{(-bx+2)^{\frac{3}{2}}b^2}{12\,x^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{5}{2}}b}{10\,x^{\frac{5}{2}}}-\frac{(-bx+2)^{\frac{7}{2}}}{28\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(1/2)/x^(9/2),x, algorithm="maxima")
[Out] -1/12*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 1/10*(-b*x + 2)^(5/2)*b/x^(5/2) - 1/28
*(-b*x + 2)^(7/2)/x^(7/2)
```

mupad [B] time = 0.22, size = 34, normalized size = 0.55

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,b^3\,x^3}{105}+\frac{2\,b^2\,x^2}{105}+\frac{b\,x}{35}-\frac{2}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(1/2)/x^(9/2),x)
```

```
[Out] ((2 - b*x)^(1/2)*((b*x)/35 + (2*b^2*x^2)/105 + (2*b^3*x^3)/105 - 2/7))/x^(7
/2)
```

sympy [B] time = 24.60, size = 554, normalized size = 8.94

$$\begin{cases} -\frac{2b^{\frac{19}{2}}x^5\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{6b^{\frac{17}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} - \frac{3b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{34b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} - \frac{132b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{120b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} & \text{for } \frac{2}{|bx|} > 1 \\ -\frac{2ib^{\frac{19}{2}}x^5\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{6ib^{\frac{17}{2}}x^4\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} - \frac{3ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{34ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} - \frac{132ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} + \frac{120ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{-105b^6x^5+420b^5x^4-420b^4x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(1/2)/x**(9/2),x)
```

```
[Out] Piecewise((-2*b**(19/2)*x**5*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*
x**4 - 420*b**4*x**3) + 6*b**(17/2)*x**4*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5
 + 420*b**5*x**4 - 420*b**4*x**3) - 3*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(-1
05*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 34*b**(13/2)*x**2*sqrt(-1 +
 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) - 132*b**(11/2)*
x*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 120
*b**(9/2)*sqrt(-1 + 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**
3), 2/Abs(b*x) > 1), (-2*I*b**(19/2)*x**5*sqrt(1 - 2/(b*x))/(-105*b**6*x**5
 + 420*b**5*x**4 - 420*b**4*x**3) + 6*I*b**(17/2)*x**4*sqrt(1 - 2/(b*x))/(-
105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) - 3*I*b**(15/2)*x**3*sqrt(1
- 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) + 34*I*b**(13/2
)*x**2*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*b**4*x**3) -
 132*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x**4 - 420*
b**4*x**3) + 120*I*b**(9/2)*sqrt(1 - 2/(b*x))/(-105*b**6*x**5 + 420*b**5*x*
*4 - 420*b**4*x**3), True))
```

3.521 $\int x^{5/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=143

$$-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Rubi [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{7/2}}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a + b*x])/(64*b
^2) + (a^2*x^(5/2)*Sqrt[a + b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a + b*x])/40 +
 (x^(7/2)*(a + b*x)^(3/2))/5 - (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(a+bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{10}(3a)\int x^{5/2}\sqrt{a+bx}\,dx \\
&= \frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx \\
&= \frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}-\frac{a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{32b} \\
&= -\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}+\frac{\left(3a^4\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}}{128b^2} \\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2} \\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2} \\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2} \\
&= \frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{5/2}\sqrt{a+bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a+bx}+\frac{1}{5}x^{7/2}(a+bx)^{3/2}
\end{aligned}
$$

Mathematica [A] time = 0.21, size = 107, normalized size = 0.75

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^4-10a^3bx+8a^2b^2x^2+176ab^3x^3+128b^4x^4\right)-\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{640b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^4 - 10*a^3*b*x + 8*a^2*b^2*x^2 + 176*
a*b^3*x^3 + 128*b^4*x^4) - (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/
Sqrt[1 + (b*x)/a]))/(640*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 108, normalized size = 0.76

$$
\frac{3a^5\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{128b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^4\sqrt{x}-10a^3bx^{3/2}+8a^2b^2x^{5/2}+176ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) + 8*a^2*b^2*x^(5/2) + 176
*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^3) + (3*a^5*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[a + b*x]])/(128*b^(7/2))
```

fricas [A] time = 1.10, size = 184, normalized size = 1.29

$$
\left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b\right)\sqrt{bx+a}\,\sqrt{x}}{1280\,b^4},\ \frac{15\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(128\,b^5x^4+176\,ab^4x^3+8\,a^2b^3x^2-10\,a^3b^2x+15\,a^4b\right)\sqrt{bx+a}\,\sqrt{x}}{640\,b^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b^2*x + 15*a^4*b)*s
qrt(b*x + a)*sqrt(x))/b^4, 1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt
(-b)/(b*sqrt(x))) + (128*b^5*x^4 + 176*a*b^4*x^3 + 8*a^2*b^3*x^2 - 10*a^3*b
^2*x + 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.01, size = 138, normalized size = 0.97

$$-\frac{3\sqrt{(bx+a)\,x}\;a^5\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{256\sqrt{bx+a}\;b^{\frac{7}{2}}\sqrt{x}}-\frac{3\sqrt{bx+a}\;a^4\sqrt{x}}{128b^3}+\frac{(bx+a)^{\frac{5}{2}}x^{\frac{5}{2}}}{5b}-\frac{(bx+a)^{\frac{3}{2}}a^3\sqrt{x}}{64b^3}-\frac{(bx+a)^{\frac{5}{2}}a\,x^{\frac{3}{2}}}{8b^2}+\frac{(bx+a)^{\frac{5}{2}}a^2\sqrt{x}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(3/2),x)
[Out] 1/5/b*x^(5/2)*(b*x+a)^(5/2)-1/8*a/b^2*x^(3/2)*(b*x+a)^(5/2)+1/16*a^2/b^3*x^
(1/2)*(b*x+a)^(5/2)-1/64*a^3/b^3*(b*x+a)^(3/2)*x^(1/2)-3/128*a^4*x^(1/2)*(b
*x+a)^(1/2)/b^3-3/256*a^5/b^(7/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*l
n((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [B] time = 2.96, size = 212, normalized size = 1.48

$$\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{7}{2}}}+\frac{\frac{15\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}-\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^8-\frac{5\,(bx+a)b^7}{x}+\frac{10\,(bx+a)^2b^6}{x^2}-\frac{10\,(bx+a)^3b^5}{x^3}+\frac{5\,(bx+a)^4b^4}{x^4}-\frac{(bx+a)^5b^3}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(3/2),x, algorithm="maxima")
[Out] 3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(7/2) + 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^(
3/2)*a^5*b^3/x^(3/2) - 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^(
7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x + a)*b^7
/x + 10*(b*x + a)^2*b^6/x^2 - 10*(b*x + a)^3*b^5/x^3 + 5*(b*x + a)^4*b^4/x^
4 - (b*x + a)^5*b^3/x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(3/2),x)
[Out] int(x^(5/2)*(a + b*x)^(3/2), x)
```

sympy [A] time = 17.71, size = 178, normalized size = 1.24

$$\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^3\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1+\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}-\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^2x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(3/2),x)
```

```
[Out] 3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 + b*x/a)) + a**(7/2)*x**(3/2)/(128*b**2
*sqrt(1 + b*x/a)) - a**(5/2)*x**(5/2)/(320*b*sqrt(1 + b*x/a)) + 23*a**(3/2)
*x**(7/2)/(80*sqrt(1 + b*x/a)) + 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 + b*x/a))
 - 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(5
*sqrt(a)*sqrt(1 + b*x/a))
```

3.522 $\int x^{3/2}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=119

$$\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2}+\frac{3a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{5/2}}+\frac{a^2x^{3/2}\sqrt{a+bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a+bx}+\frac{1}{4}x^{5/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b^2) + (a^2*x^(3/2)*Sqrt[a + b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a + b*x])/8 + (x^(5/2)*(a + b*x)^(3/2))/4 + (3*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}(a+bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{8}(3a)\int x^{3/2}\sqrt{a+bx}\,dx$$
$$= \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{1}{16}a^2\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} - \frac{(3a^3)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{64b}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\int \frac{1}{\sqrt{x}\sqrt{a+}}}{128b^2}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\Big(\int}{}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{(3a^4)\,\mathrm{Subst}\Big(\int}{}$$
$$= -\frac{3a^3\sqrt{x}\sqrt{a+bx}}{64b^2} + \frac{a^2x^{3/2}\sqrt{a+bx}}{32b} + \frac{1}{8}ax^{5/2}\sqrt{a+bx} + \frac{1}{4}x^{5/2}(a+bx)^{3/2} + \frac{3a^4\tanh^{-1}\Big(\frac{\sqrt{}}{\sqrt{}}}{64b^{5/2}}$$

Mathematica [A] time = 0.13, size = 96, normalized size = 0.81

$$\frac{\sqrt{a+bx}\left(\frac{3a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}} + \sqrt{b}\sqrt{x}\left(-3a^3+2a^2bx+24ab^2x^2+16b^3x^3\right)\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^3 + 2*a^2*b*x + 24*a*b^2*x^2 + 16*b^3
*x^3) + (3*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/
(64*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 95, normalized size = 0.80

$$\frac{\sqrt{a+bx}\left(-3a^3\sqrt{x}+2a^2bx^{3/2}+24ab^2x^{5/2}+16b^3x^{7/2}\right)}{64b^2} - \frac{3a^4\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{64b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a^3*Sqrt[x] + 2*a^2*b*x^(3/2) + 24*a*b^2*x^(5/2) + 16*b^
3*x^(7/2)))/(64*b^2) - (3*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(64*
b^(5/2))
```

fricas [A] time = 0.70, size = 163, normalized size = 1.37

$$\left[\frac{3\,a^4\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{128\,b^3}, -\frac{3\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(16\,b^4x^3+24\,ab^3x^2+2\,a^2b^2x-3\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{64\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/128*(3*a^4*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(16*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/
```

```
b^3, -1/64*(3*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (16
*b^4*x^3 + 24*a*b^3*x^2 + 2*a^2*b^2*x - 3*a^3*b)*sqrt(b*x + a)*sqrt(x))/b^3
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.01, size = 120, normalized size = 1.01

$$\frac{3\sqrt{(bx+a)x}\,a^4\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{128\sqrt{bx+a}\,b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{bx+a}\,a^3\sqrt{x}}{64b^2}+\frac{(bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^2}+\frac{(bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4b}-\frac{(bx+a)^{\frac{5}{2}}\,a\sqrt{x}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(3/2),x)
[Out] 1/4/b*x^(3/2)*(b*x+a)^(5/2)-1/8*a/b^2*x^(1/2)*(b*x+a)^(5/2)+1/32*a^2/b^2*(b
*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b^2+3/128*a^4/b^(5/2)*((
b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

maxima [B] time = 2.96, size = 178, normalized size = 1.50

$$-\frac{3\,a^4\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{5}{2}}}-\frac{\frac{3\,\sqrt{bx+a}\,a^4b^3}{\sqrt{x}}-\frac{11\,(bx+a)^{\frac{3}{2}}\,a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(bx+a)^{\frac{5}{2}}\,a^4b}{x^{\frac{5}{2}}}+\frac{3\,(bx+a)^{\frac{7}{2}}\,a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx+a)b^5}{x}+\frac{6\,(bx+a)^2b^4}{x^2}-\frac{4\,(bx+a)^3b^3}{x^3}+\frac{(bx+a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(3/2),x, algorithm="maxima")
[Out] -3/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/64*(3*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 11*(b*x + a)^(3
/2)*a^4*b^2/x^(3/2) - 11*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 3*(b*x + a)^(7/2)*
a^4/x^(7/2))/(b^6 - 4*(b*x + a)*b^5/x + 6*(b*x + a)^2*b^4/x^2 - 4*(b*x + a)
^3*b^3/x^3 + (b*x + a)^4*b^2/x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(3/2),x)
[Out] int(x^(3/2)*(a + b*x)^(3/2), x)
```

sympy [A] time = 9.28, size = 153, normalized size = 1.29

$$-\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1+\frac{bx}{a}}}+\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1+\frac{bx}{a}}}+\frac{3a^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(3/2),x)
```

```
[Out] -3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 + b*x/a)) - a**(5/2)*x**(3/2)/(64*b*sqr
t(1 + b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 + b*x/a)) + 5*sqrt(a)*b*x**
(7/2)/(8*sqrt(1 + b*x/a)) + 3*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(5
/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

3.523 $\int \sqrt{x}(a+bx)^{3/2}\,dx$

Optimal. Leaf size=95

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}+\frac{a^2\sqrt{x}\sqrt{a+bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a+bx}+\frac{1}{3}x^{3/2}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b) + (a*x^(3/2)*Sqrt[a + b*x])/4 + (x^(3/2)*
(a + b*x)^(3/2))/3 - (a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8*b^(3
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{x}\,(a+bx)^{3/2}\,dx = \frac{1}{3}x^{3/2}(a+bx)^{3/2} + \frac{1}{2}a\int \sqrt{x}\,\sqrt{a+bx}\,dx$$
$$= \frac{1}{4}ax^{3/2}\sqrt{a+bx} + \frac{1}{3}x^{3/2}(a+bx)^{3/2} + \frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a+bx} + \frac{1}{3}x^{3/2}(a+bx)^{3/2} - \frac{a^3\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b}$$
$$= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a+bx} + \frac{1}{3}x^{3/2}(a+bx)^{3/2} - \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{8b}$$
$$= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a+bx} + \frac{1}{3}x^{3/2}(a+bx)^{3/2} - \frac{a^3\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b}$$
$$= \frac{a^2\sqrt{x}\,\sqrt{a+bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a+bx} + \frac{1}{3}x^{3/2}(a+bx)^{3/2} - \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{3/2}}$$

Mathematica [**A**] time = 0.11, size = 85, normalized size = 0.89

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\,\sqrt{x}\left(3a^2+14abx+8b^2x^2\right) - \frac{3a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(3*a^2 + 14*a*b*x + 8*b^2*x^2) - (3*a^(5/2)
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.11, size = 82, normalized size = 0.86

$$\frac{a^3\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{8b^{3/2}} + \frac{\sqrt{a+bx}\left(3a^2\sqrt{x}+14abx^{3/2}+8b^2x^{5/2}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(3*a^2*Sqrt[x] + 14*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b) + (
a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(3/2))
```

fricas [**A**] time = 0.72, size = 140, normalized size = 1.47

$$\left[\frac{3a^3\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8b^3x^2+14ab^2x+3a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48b^2}, \frac{3a^3\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(8b^3x^2+14ab^2x+3a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(
8*b^3*x^2 + 14*a*b^2*x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2, 1/24*(3*a^3*s
qrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 + 14*a*b^2*
x + 3*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^2]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 96, normalized size = 1.01

$$\frac{\sqrt{bx+a}\,a\,x^{\frac{3}{2}}}{4}-\frac{\sqrt{(bx+a)\,x}\,a^{3}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{bx+a}\,b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\,a^{2}\sqrt{x}}{8b}+\frac{(bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(b*x+a)^(3/2)+1/4*a*x^(3/2)*(b*x+a)^(1/2)+1/8*a^2*x^(1/2)*(b*x+
a)^(1/2)/b-1/16*a^3/b^(3/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x
+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [B] time = 3.02, size = 144, normalized size = 1.52

$$\frac{a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\frac{3\,\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{8\,(bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{4}-\frac{3\,(bx+a)b^{3}}{x}+\frac{3\,(bx+a)^{2}b^{2}}{x^{2}}-\frac{(bx+a)^{3}b}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(3/2) + 1/24*(3*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 8*(b*x + a)^(3/2)
*a^3*b/x^(3/2) - 3*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x + a)*b^3/x +
3*(b*x + a)^2*b^2/x^2 - (b*x + a)^3*b/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(3/2), x)
```

sympy [A] time = 5.59, size = 124, normalized size = 1.31

$$\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1+\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] a**(5/2)*sqrt(x)/(8*b*sqrt(1 + b*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 +
b*x/a)) + 11*sqrt(a)*b*x**(5/2)/(12*sqrt(1 + b*x/a)) - a**3*asinh(sqrt(b)*s
qrt(x)/sqrt(a))/(8*b**(3/2)) + b**2*x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

3.524 $\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=71

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a+bx}+\frac{1}{2}\sqrt{x}(a+bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a + b*x])/4 + (Sqrt[x]*(a + b*x)^(3/2))/2 + (3*a^2*ArcTan
h[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx = \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{4}(3a)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$
$$= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx} + \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx$$
$$= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx} + \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx} + \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2} + \frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= \frac{3}{4}a\sqrt{x}\,\sqrt{a+bx} + \frac{1}{2}\sqrt{x}\,(a+bx)^{3/2} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4\sqrt{b}}$$

Mathematica [A] time = 0.10, size = 69, normalized size = 0.97

$$\frac{1}{4}\sqrt{a+bx}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{bx}{a}+1}} + \sqrt{x}\,(5a+2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(5*a + 2*b*x) + (3*a^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/4
```

IntegrateAlgebraic [A] time = 0.09, size = 66, normalized size = 0.93

$$\frac{1}{4}\sqrt{a+bx}\left(5a\sqrt{x} + 2bx^{3/2}\right) - \frac{3a^2\log\left(\sqrt{a+bx} - \sqrt{b}\,\sqrt{x}\right)}{4\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(5*a*Sqrt[x] + 2*b*x^(3/2)))/4 - (3*a^2*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]])/(4*Sqrt[b])
```

fricas [A] time = 1.10, size = 119, normalized size = 1.68

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\left(2\,b^2x + 5\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{8\,b}, -\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) - \left(2\,b^2x + 5\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x + 5*a*b)*sqrt(b*x + a)*sqrt(x))/b
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 78, normalized size = 1.10

$$\frac{3\sqrt{(bx+a)\,x}\;a^{2}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{3\sqrt{bx+a}\;a\sqrt{x}}{4}+\frac{(bx+a)^{\frac{3}{2}}\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(b*x+a)^(1/2)+3/8*a^2*((b*x+a)*x)^(
1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2
)
```

maxima [B] time = 2.99, size = 107, normalized size = 1.51

$$-\frac{3\,a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{2}-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/sqrt(b) - 1/4*(3*sqrt(b*x + a)*a^2*b/sqrt(x) - 5*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x + a)^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(1/2), x)
```

sympy [A] time = 3.17, size = 75, normalized size = 1.06

$$\frac{5a^{\frac{3}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{4}+\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{2}+\frac{3a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 5*a**(3/2)*sqrt(x)*sqrt(1 + b*x/a)/4 + sqrt(a)*b*x**(3/2)*sqrt(1 + b*x/a)/2
 + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b))
```

3.525 $\int \frac{(a+bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=63

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{2(a+bx)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{a+bx}+3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[a + b*x] - (2*(a + b*x)^(3/2))/Sqrt[x] + 3*a*Sqrt[b]*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx = -\frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$
$$= 3b\sqrt{x}\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + \frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$
$$= 3b\sqrt{x}\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 3b\sqrt{x}\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + (3ab)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= 3b\sqrt{x}\sqrt{a+bx} - \frac{2(a+bx)^{3/2}}{\sqrt{x}} + 3a\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.73

$$-\frac{2a\sqrt{a+bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt[x
]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.13, size = 54, normalized size = 0.86

$$\frac{(bx-2a)\sqrt{a+bx}}{\sqrt{x}} - 3a\sqrt{b}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a + b*x)*Sqrt[a + b*x])/Sqrt[x] - 3*a*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) +
 Sqrt[a + b*x]]
```

fricas [A] time = 1.33, size = 109, normalized size = 1.73

$$\left[\frac{3\,a\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\,\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{2\,x}, -\frac{3\,a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+a}\,(bx-2\,a)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sq
rt(b*x + a)*(b*x - 2*a)*sqrt(x))/x, -(3*a*sqrt(-b)*x*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - sqrt(b*x + a)*(b*x - 2*a)*sqrt(x))/x]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 71, normalized size = 1.13

$$\frac{3\sqrt{(bx+a)\,x}\;a\sqrt{b}\;\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;(-bx+2a)}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] -(b*x+a)^(1/2)*(-b*x+2*a)/x^(1/2)+3/2*a*b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x
^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.99, size = 84, normalized size = 1.33

$$-\frac{3}{2}\,a\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*a*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 2*sqrt(b*x + a)*a/sqrt(x) - sqrt(b*x + a)*a*b/((b - (b*x +
a)/x)*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(3/2), x)
```

sympy [A] time = 2.72, size = 92, normalized size = 1.46

$$-\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1+\frac{bx}{a}}}+3a\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\frac{b^{2}x^{\frac{3}{2}}}{\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] -2*a**(3/2)/(sqrt(x)*sqrt(1 + b*x/a)) - sqrt(a)*b*sqrt(x)/sqrt(1 + b*x/a) +
 3*a*sqrt(b)*asinh(sqrt(b)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1
 + b*x/a))
```

3.526 $\int \frac{(a+bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=64

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{a+bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[a + b*x])/Sqrt[x] - (2*(a + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{x^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3x^{3/2}} + b\int \frac{\sqrt{a+bx}}{x^{3/2}}\,dx$$
$$= -\frac{2b\sqrt{a+bx}}{\sqrt{x}} - \frac{2(a+bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$$
$$= -\frac{2b\sqrt{a+bx}}{\sqrt{x}} - \frac{2(a+bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{2b\sqrt{a+bx}}{\sqrt{x}} - \frac{2(a+bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)$$
$$= -\frac{2b\sqrt{a+bx}}{\sqrt{x}} - \frac{2(a+bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.75

$$-\frac{2a\sqrt{a+bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/x^(5/2),x]
[Out] (-2*a*Sqrt[a + b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, -((b*x)/a)])/(3*x^(
3/2)*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.13, size = 55, normalized size = 0.86

$$-2b^{3/2}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)-\frac{2\sqrt{a+bx}\,(a+4bx)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/x^(5/2),x]
[Out] (-2*Sqrt[a + b*x]*(a + 4*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[a + b*x]]
```

fricas [A] time = 0.90, size = 109, normalized size = 1.70

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)-2\,(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+(4\,bx+a)\sqrt{bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
[Out] [1/3*(3*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*(4
*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*
x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b*x + a)*sqrt(b*x + a)*sqrt(x))/x^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 67, normalized size = 1.05

$$\frac{\sqrt{(bx+a)x}\, b^{\frac{3}{2}} \ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{bx+a}\,\sqrt{x}}-\frac{2\sqrt{bx+a}\,\left(4bx+a\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(4*b*x+a)/x^(3/2)+b^(3/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+
a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.93, size = 67, normalized size = 1.05

$$-b^{\frac{3}{2}} \log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x))) - 2*sqrt(b*x + a)*b/sqrt(x) - 2/3*(b*x + a)^(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/x^(5/2), x)
```

sympy [A] time = 3.04, size = 71, normalized size = 1.11

$$-\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-b^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] -2*a*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 8*b**(3/2)*sqrt(a/(b*x) + 1)/3 - b**
(3/2)*log(a/(b*x)) + 2*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1)
```

3.527 $\int x^{5/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=149

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Rubi [A] time = 0.05, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{7/2}}-\frac{a^2x^{5/2}\sqrt{a-bx}}{80b}+\frac{3}{40}ax^{7/2}\sqrt{a-bx}+\frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^3) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b^2) - (a^2*x^(5/2)*Sqrt[a - b*x])/(80*b) + (3*a*x^(7/2)*Sqrt[a - b*x])/40
+ (x^(7/2)*(a - b*x)^(3/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*
x]])/(128*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a-bx)^{3/2}\,dx = \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{10}(3a)\int x^{5/2}\sqrt{a-bx}\,dx$$
$$= \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{1}{80}\left(3a^2\right)\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{a^3\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{32b}$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2} + \frac{\left(3a^4\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,d}{128b^2}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^3} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b^2} - \frac{a^2x^{5/2}\sqrt{a-bx}}{80b} + \frac{3}{40}ax^{7/2}\sqrt{a-bx} + \frac{1}{5}x^{7/2}(a-bx)^{3/2}$$

Mathematica [A] time = 0.17, size = 110, normalized size = 0.74

$$\frac{\sqrt{a-bx}\left(\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} - \sqrt{b}\sqrt{x}\left(15a^4+10a^3bx+8a^2b^2x^2-176ab^3x^3+128b^4x^4\right)\right)}{640b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^4 + 10*a^3*b*x + 8*a^2*b^2*x^2 - 17
6*a*b^3*x^3 + 128*b^4*x^4)) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 117, normalized size = 0.79

$$\frac{3a^5\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{128b^4} + \frac{\sqrt{a-bx}\left(-15a^4\sqrt{x}-10a^3bx^{3/2}-8a^2b^2x^{5/2}+176ab^3x^{7/2}-128b^4x^{9/2}\right)}{640b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) - 8*a^2*b^2*x^(5/2) + 17
6*a*b^3*x^(7/2) - 128*b^4*x^(9/2)))/(640*b^3) + (3*a^5*Sqrt[-b]*Log[-(Sqrt[
-b]*Sqrt[x]) + Sqrt[a - b*x]])/(128*b^4)
```

fricas [A] time = 1.09, size = 185, normalized size = 1.24

$$\left[-\frac{15a^5\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(128b^5x^4-176ab^4x^3+8a^2b^3x^2+10a^3b^2x+15a^4b\right)\sqrt{-bx+a}\sqrt{x}}{1280b^4}, -\frac{15a^5\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(128b^5x^4-176ab^4x^3+8a^2b^3x^2+10a^3b^2x+15a^4b\right)\sqrt{-bx+a}\sqrt{x}}{640b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a^3*b^2*x + 15*a^4
*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) + (128*b^5*x^4 - 176*a*b^4*x^3 + 8*a^2*b^3*x^2 + 10*a
^3*b^2*x + 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 146, normalized size = 0.98

$$\frac{3\sqrt{(-bx+a)\,x}\;a^{5}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{256\sqrt{-bx+a}\;b^{\frac{7}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^{4}\sqrt{x}}{128b^{3}}-\frac{(-bx+a)^{\frac{5}{2}}\,x^{\frac{5}{2}}}{5b}+\frac{(-bx+a)^{\frac{3}{2}}\,a^{3}\sqrt{x}}{64b^{3}}-\frac{(-bx+a)^{\frac{5}{2}}\,a\,x^{\frac{3}{2}}}{8b^{2}}-\frac{(-bx+a)^{\frac{5}{2}}\,a^{2}\sqrt{x}}{16b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(3/2),x)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+a)^(5/2)-1/8*a/b^2*x^(3/2)*(-b*x+a)^(5/2)-1/16*a^2/b^3
*x^(1/2)*(-b*x+a)^(5/2)+1/64*a^3/b^3*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/
2)*(-b*x+a)^(1/2)/b^3+3/256*a^5/b^(7/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 3.12, size = 207, normalized size = 1.39

$$-\frac{3\,a^{5}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{5}b^{4}}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}a^{5}b^{3}}{x^{\frac{3}{2}}}-\frac{128\,(-bx+a)^{\frac{5}{2}}a^{5}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}a^{5}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}a^{5}}{x^{\frac{9}{2}}}}{640\left(b^{8}-\frac{5\,(bx-a)b^{7}}{x}+\frac{10\,(bx-a)^{2}b^{6}}{x^{2}}-\frac{10\,(bx-a)^{3}b^{5}}{x^{3}}+\frac{5\,(bx-a)^{4}b^{4}}{x^{4}}-\frac{(bx-a)^{5}b^{3}}{x^{5}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) - 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^8 - 5*(b*x - a)*b^7/x + 10*(b*x - a)^2*b^6/x^2
 - 10*(b*x - a)^3*b^5/x^3 + 5*(b*x - a)^4*b^4/x^4 - (b*x - a)^5*b^3/x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a-b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(3/2), x)
```

sympy [A] time = 17.69, size = 376, normalized size = 2.52

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{-1+\frac{bx}{a}}}-\frac{23ia^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}+\frac{19i\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^{5}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}-\frac{ib^{2}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{5}{2}}}{320b\sqrt{1-\frac{bx}{a}}}+\frac{23a^{\frac{3}{2}}x^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}-\frac{19\sqrt{a}\,bx^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^{5}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{7}{2}}}+\frac{b^{2}x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**3*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**(5/2)/(320*b*sqrt(-1 + b
*x/a)) - 23*I*a**(3/2)*x**(7/2)/(80*sqrt(-1 + b*x/a)) + 19*I*sqrt(a)*b*x**(
9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b
**(7/2)) - I*b**2*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1),
(-3*a**(9/2)*sqrt(x)/(128*b**3*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(128*b*
*2*sqrt(1 - b*x/a)) + a**(5/2)*x**(5/2)/(320*b*sqrt(1 - b*x/a)) + 23*a**(3/
2)*x**(7/2)/(80*sqrt(1 - b*x/a)) - 19*sqrt(a)*b*x**(9/2)/(40*sqrt(1 - b*x/a
)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(7/2)) + b**2*x**(11/2)/(
5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.528 $\int x^{3/2}(a-bx)^{3/2}\,dx$

Optimal. Leaf size=124

$$\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{5/2}}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (-3*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b^2) - (a^2*x^(3/2)*Sqrt[a - b*x])/(32*b
) + (a*x^(5/2)*Sqrt[a - b*x])/8 + (x^(5/2)*(a - b*x)^(3/2))/4 + (3*a^4*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(a-bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{8}(3a)\int x^{3/2}\sqrt{a-bx}\,dx \\
&= \frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{1}{16}a^2\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx \\
&= -\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{64b} \\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\int \frac{1}{\sqrt{x}\sqrt{a-b}}}{128b^2} \\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\left(\int\right.}{6} \\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{\left(3a^4\right)\text{Subst}\left(\int\right.}{6} \\
&= -\frac{3a^3\sqrt{x}\sqrt{a-bx}}{64b^2}-\frac{a^2x^{3/2}\sqrt{a-bx}}{32b}+\frac{1}{8}ax^{5/2}\sqrt{a-bx}+\frac{1}{4}x^{5/2}(a-bx)^{3/2}+\frac{3a^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{}}{\sqrt{a-b}}\right.}{64b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 99, normalized size = 0.80

$$
\frac{\sqrt{a-bx}\left(\frac{3a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\sqrt{x}\left(3a^3+2a^2bx-24ab^2x^2+16b^3x^3\right)\right)}{64b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(3*a^3 + 2*a^2*b*x - 24*a*b^2*x^2 + 16*b^
3*x^3)) + (3*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))
/(64*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 104, normalized size = 0.84

$$
\frac{3a^4\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{64b^3}+\frac{\sqrt{a-bx}\left(-3a^3\sqrt{x}-2a^2bx^{3/2}+24ab^2x^{5/2}-16b^3x^{7/2}\right)}{64b^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^3*Sqrt[x] - 2*a^2*b*x^(3/2) + 24*a*b^2*x^(5/2) - 16*b^
3*x^(7/2)))/(64*b^2) + (3*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b
*x]])/(64*b^3)
```

fricas [A] time = 1.12, size = 163, normalized size = 1.31

$$
\left[-\frac{3\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{128\,b^3},\;-\frac{3\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(16\,b^4x^3-24\,ab^3x^2+2\,a^2b^2x+3\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{64\,b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/128*(3*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqr
```

```
t(x))/b^3, -1/64*(3*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) +
(16*b^4*x^3 - 24*a*b^3*x^2 + 2*a^2*b^2*x + 3*a^3*b)*sqrt(-b*x + a)*sqrt(x))
/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.01, size = 127, normalized size = 1.02

$$\frac{3\sqrt{(-bx+a)\,x}\;a^4\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{128\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^3\sqrt{x}}{64b^2}+\frac{(-bx+a)^{\frac{3}{2}}\,a^2\sqrt{x}}{32b^2}-\frac{(-bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4b}-\frac{(-bx+a)^{\frac{5}{2}}\,a\sqrt{x}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(3/2),x)
[Out] -1/4/b*x^(3/2)*(-b*x+a)^(5/2)-1/8*a/b^2*x^(1/2)*(-b*x+a)^(5/2)+1/32*a^2/b^2
*(-b*x+a)^(3/2)*x^(1/2)+3/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/128*a^4/b^(5/
2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x
)^(1/2)*b^(1/2))
```

maxima [A] time = 2.97, size = 170, normalized size = 1.37

$$-\frac{3\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{5}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{11\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{64\left(b^6-\frac{4\,(bx-a)b^5}{x}+\frac{6\,(bx-a)^2b^4}{x^2}-\frac{4\,(bx-a)^3b^3}{x^3}+\frac{(bx-a)^4b^2}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(3/2),x, algorithm="maxima")
[Out] -3/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/64*(3*sqrt(-
b*x + a)*a^4*b^3/sqrt(x) + 11*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) - 11*(-b*x +
 a)^(5/2)*a^4*b/x^(5/2) - 3*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^6 - 4*(b*x - a
)*b^5/x + 6*(b*x - a)^2*b^4/x^2 - 4*(b*x - a)^3*b^3/x^3 + (b*x - a)^4*b^2/x
^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(3/2),x)
[Out] int(x^(3/2)*(a - b*x)^(3/2), x)
```

sympy [A] time = 9.06, size = 323, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{13ia^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{-1+\frac{bx}{a}}}+\frac{5i\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}-\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{7}{2}}\sqrt{x}}{64b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{5}{2}}x^{\frac{3}{2}}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{13a^{\frac{3}{2}}x^{\frac{5}{2}}}{32\sqrt{1-\frac{bx}{a}}}-\frac{5\sqrt{a}\,bx^{\frac{7}{2}}}{8\sqrt{1-\frac{bx}{a}}}+\frac{3a^4\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{5}{2}}}+\frac{b^2x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((3*I*a**(7/2)*sqrt(x)/(64*b**2*sqrt(-1 + b*x/a)) - I*a**(5/2)*x**
(3/2)/(64*b*sqrt(-1 + b*x/a)) - 13*I*a**(3/2)*x**(5/2)/(32*sqrt(-1 + b*x/a)
) + 5*I*sqrt(a)*b*x**(7/2)/(8*sqrt(-1 + b*x/a)) - 3*I*a**4*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(64*b**(5/2)) - I*b**2*x**(9/2)/(4*sqrt(a)*sqrt(-1 + b*x/a))
, Abs(b*x/a) > 1), (-3*a**(7/2)*sqrt(x)/(64*b**2*sqrt(1 - b*x/a)) + a**(5/2
)*x**(3/2)/(64*b*sqrt(1 - b*x/a)) + 13*a**(3/2)*x**(5/2)/(32*sqrt(1 - b*x/a
)) - 5*sqrt(a)*b*x**(7/2)/(8*sqrt(1 - b*x/a)) + 3*a**4*asin(sqrt(b)*sqrt(x)
/sqrt(a))/(64*b**(5/2)) + b**2*x**(9/2)/(4*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.529 $\int \sqrt{x}\,(a-bx)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}-\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b}+\frac{1}{4}ax^{3/2}\sqrt{a-bx}+\frac{1}{3}x^{3/2}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] -(a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b) + (a*x^(3/2)*Sqrt[a - b*x])/4 + (x^(3/2)
*(a - b*x)^(3/2))/3 + (a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*b^(3
/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{x}(a-bx)^{3/2}\,dx = \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{1}{2}a\int \sqrt{x}\sqrt{a-bx}\,dx$$
$$= \frac{1}{4}ax^{3/2}\sqrt{a-bx} + \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{1}{8}a^2\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a-bx} + \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{a^3\int \frac{1}{\sqrt{x}\sqrt{a-bx}}dx}{16b}$$
$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a-bx} + \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}dx, x, \sqrt{x}\right)}{8b}$$
$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a-bx} + \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{a^3\,\text{Subst}\left(\int \frac{1}{1+bx^2}dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b}$$
$$= -\frac{a^2\sqrt{x}\sqrt{a-bx}}{8b} + \frac{1}{4}ax^{3/2}\sqrt{a-bx} + \frac{1}{3}x^{3/2}(a-bx)^{3/2} + \frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{3/2}}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.88

$$\frac{\sqrt{a-bx}\left(\frac{3a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}\left(-3a^2+14abx-8b^2x^2\right)\right)}{24b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-3*a^2 + 14*a*b*x - 8*b^2*x^2) + (3*a^(5/2
)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 91, normalized size = 0.92

$$\frac{a^3\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{8b^2} + \frac{\sqrt{a-bx}\left(-3a^2\sqrt{x}+14abx^{3/2}-8b^2x^{5/2}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a - b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a^2*Sqrt[x] + 14*a*b*x^(3/2) - 8*b^2*x^(5/2)))/(24*b) +
(a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^2)
```

fricas [A] time = 1.16, size = 141, normalized size = 1.42

$$\left[-\frac{3a^3\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(8b^3x^2-14ab^2x+3a^2b\right)\sqrt{-bx+a}\sqrt{x}}{48b^2}, -\frac{3a^3\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(8b^3x^2-14ab^2x+3a^2b\right)\sqrt{-bx+a}\sqrt{x}}{24b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(8*b^3*x^2 - 14*a*b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2, -1/24*(
3*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 - 14*a*
b^2*x + 3*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 102, normalized size = 1.03

$$\frac{\sqrt{-bx+a}\,a\,x^{\frac{3}{2}}}{4}+\frac{\sqrt{(-bx+a)\,x}\,a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\,b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\,a^{2}\sqrt{x}}{8b}+\frac{(-bx+a)^{\frac{3}{2}}\,x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+a)^(3/2)+1/4*a*x^(3/2)*(-b*x+a)^(1/2)-1/8*a^2*x^(1/2)*(-b
*x+a)^(1/2)/b+1/16*a^3/b^(3/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*ar
ctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.99, size = 133, normalized size = 1.34

$$-\frac{a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{8\,(-bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}-\frac{3\,(-bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{4}-\frac{3\,(bx-a)b^{3}}{x}+\frac{3\,(bx-a)^{2}b^{2}}{x^{2}}-\frac{(bx-a)^{3}b}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/24*(3*sqrt(-b
*x + a)*a^3*b^2/sqrt(x) + 8*(-b*x + a)^(3/2)*a^3*b/x^(3/2) - 3*(-b*x + a)^(
5/2)*a^3/x^(5/2))/(b^4 - 3*(b*x - a)*b^3/x + 3*(b*x - a)^2*b^2/x^2 - (b*x -
 a)^3*b/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(a-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(a - b*x)^(3/2), x)
```

sympy [A] time = 5.54, size = 264, normalized size = 2.67

$$\begin{cases} \frac{ia^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{-1+\frac{bx}{a}}}-\frac{17ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}+\frac{11i\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}-\frac{ib^{2}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{a^{\frac{5}{2}}\sqrt{x}}{8b\sqrt{1-\frac{bx}{a}}}+\frac{17a^{\frac{3}{2}}x^{\frac{3}{2}}}{24\sqrt{1-\frac{bx}{a}}}-\frac{11\sqrt{a}\,bx^{\frac{5}{2}}}{12\sqrt{1-\frac{bx}{a}}}+\frac{a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{b^{2}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*a**(5/2)*sqrt(x)/(8*b*sqrt(-1 + b*x/a)) - 17*I*a**(3/2)*x**(3/
2)/(24*sqrt(-1 + b*x/a)) + 11*I*sqrt(a)*b*x**(5/2)/(12*sqrt(-1 + b*x/a)) -
```

```
I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) - I*b**2*x**(7/2)/(3*sqr
t(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-a**(5/2)*sqrt(x)/(8*b*sqrt(1 - b
*x/a)) + 17*a**(3/2)*x**(3/2)/(24*sqrt(1 - b*x/a)) - 11*sqrt(a)*b*x**(5/2)/
(12*sqrt(1 - b*x/a)) + a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(3/2)) + b*
*2*x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.530 $\int \frac{(a-bx)^{3/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=74

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}+\frac{3}{4}a\sqrt{x}\sqrt{a-bx}+\frac{1}{2}\sqrt{x}(a-bx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*a*Sqrt[x]*Sqrt[a - b*x])/4 + (Sqrt[x]*(a - b*x)^(3/2))/2 + (3*a^2*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}(3a)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{8}\left(3a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{1}{4}\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= \frac{3}{4}a\sqrt{x}\,\sqrt{a-bx}+\frac{1}{2}\sqrt{x}\,(a-bx)^{3/2}+\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{4\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 71, normalized size = 0.96

$$\frac{1}{4}\sqrt{a-bx}\left(\frac{3a^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{1-\frac{bx}{a}}}+\sqrt{x}\,(5a-2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(5*a - 2*b*x) + (3*a^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])
/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/4
```

IntegrateAlgebraic [A] time = 0.10, size = 75, normalized size = 1.01

$$\frac{3a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{4b}+\frac{1}{4}\sqrt{a-bx}\,\left(5a\sqrt{x}-2bx^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(5*a*Sqrt[x] - 2*b*x^(3/2)))/4 + (3*a^2*Sqrt[-b]*Log[-(Sqrt[
-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b)
```

fricas [A] time = 1.43, size = 119, normalized size = 1.61

$$\left[-\frac{3\,a^2\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(2\,b^2x-5\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,b},-\frac{3\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^2x-5\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/4*(3*a^2*sqrt(b)*arctan(
sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x - 5*a*b)*sqrt(-b*x + a)*sqrt(x
))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 83, normalized size = 1.12

$$\frac{3\sqrt{(-bx+a)\,x}\;a^2\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{8\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a\sqrt{x}}{4}+\frac{(-bx+a)^{\frac{3}{2}}\sqrt{x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+a)^(3/2)*x^(1/2)+3/4*a*x^(1/2)*(-b*x+a)^(1/2)+3/8*a^2*((-b*x+a)*x
)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2
)*b^(1/2))
```

maxima [A] time = 2.90, size = 93, normalized size = 1.26

$$-\frac{3\,a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,\sqrt{b}}+\frac{\frac{3\,\sqrt{-bx+a}\,a^2b}{\sqrt{x}}+\frac{5\,(-bx+a)^{\frac{3}{2}}a^2}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/4*(3*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 5*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^2 - 2*(b*x - a)*b
/x + (b*x - a)^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(1/2), x)
```

sympy [A] time = 3.21, size = 190, normalized size = 2.57

$$\begin{cases} -\frac{5ia^{\frac{3}{2}}\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{7i\sqrt{a}\,bx^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}}-\frac{ib^2x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1\\ \frac{5a^{\frac{3}{2}}\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}{4}-\frac{\sqrt{a}\,bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{2}+\frac{3a^2\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4\sqrt{b}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-5*I*a**(3/2)*sqrt(x)/(4*sqrt(-1 + b*x/a)) + 7*I*sqrt(a)*b*x**(3
/2)/(4*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(
b)) - I*b**2*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (5*a**
(3/2)*sqrt(x)*sqrt(1 - b*x/a)/4 - sqrt(a)*b*x**(3/2)*sqrt(1 - b*x/a)/2 + 3*
a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*sqrt(b)), True))
```

3.531 $\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{a-bx}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{2(a-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{a-bx}-3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[a - b*x] - (2*(a - b*x)^(3/2))/Sqrt[x] - 3*a*Sqrt[b]*ArcT
an[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx = -\frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3b)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$$
$$= -3b\sqrt{x}\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - \frac{1}{2}(3ab)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$$
$$= -3b\sqrt{x}\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= -3b\sqrt{x}\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - (3ab)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)$$
$$= -3b\sqrt{x}\sqrt{a-bx} - \frac{2(a-bx)^{3/2}}{\sqrt{x}} - 3a\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)$$

Mathematica [**C**] time = 0.01, size = 47, normalized size = 0.71

$$-\frac{2a\sqrt{a-bx}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]*S
qrt[1 - (b*x)/a])
```

IntegrateAlgebraic [**A**] time = 0.13, size = 61, normalized size = 0.92

$$\frac{(-2a-bx)\sqrt{a-bx}}{\sqrt{x}} - 3a\sqrt{-b}\,\log\left(\sqrt{a-bx} - \sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-2*a - b*x)*Sqrt[a - b*x])/Sqrt[x] - 3*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x])
 + Sqrt[a - b*x]]
```

fricas [**A**] time = 1.15, size = 109, normalized size = 1.65

$$\left[\frac{3\,a\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)-2\,(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{2\,x}, \frac{3\,a\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(bx+2\,a)\sqrt{-bx+a}\,\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*a*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) -
2*(b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x, (3*a*sqrt(b)*x*arctan(sqrt(-b*x +
a)/(sqrt(b)*sqrt(x))) - (b*x + 2*a)*sqrt(-b*x + a)*sqrt(x))/x]
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(3/2),x)
```

```
[Out] int((-b*x+a)^(3/2)/x^(3/2),x)
```

maxima [A] time = 2.85, size = 68, normalized size = 1.03

$$3\,a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}\,a}{\sqrt{x}}-\frac{\sqrt{-bx+a}\,ab}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 3*a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a)*a/s
qrt(x) - sqrt(-b*x + a)*a*b/((b - (b*x - a)/x)*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(3/2), x)
```

sympy [A] time = 2.88, size = 197, normalized size = 2.98

$$\begin{cases} \frac{2ia^{\frac{3}{2}}}{\sqrt{x}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,b\sqrt{x}}{\sqrt{-1+\frac{bx}{a}}}+3ia\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)-\frac{ib^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{3}{2}}}{\sqrt{x}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,b\sqrt{x}}{\sqrt{1-\frac{bx}{a}}}-3a\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((2*I*a**(3/2)/(sqrt(x)*sqrt(-1 + b*x/a)) - I*sqrt(a)*b*sqrt(x)/sq
rt(-1 + b*x/a) + 3*I*a*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a)) - I*b**2*x**(
3/2)/(sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(3/2)/(sqrt(x)*sqr
t(1 - b*x/a)) + sqrt(a)*b*sqrt(x)/sqrt(1 - b*x/a) - 3*a*sqrt(b)*asin(sqrt(b
)*sqrt(x)/sqrt(a)) + b**2*x**(3/2)/(sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.532 $\int \frac{(a-bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=67

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{a-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[a - b*x])/Sqrt[x] - (2*(a - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx)^{3/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{3/2}}{3x^{3/2}} - b\int \frac{\sqrt{a-bx}}{x^{3/2}}\,dx\\ &= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + b^2\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\ &= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + \left(2b^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= \frac{2b\sqrt{a-bx}}{\sqrt{x}} - \frac{2(a-bx)^{3/2}}{3x^{3/2}} + 2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.73

$$-\frac{2a\sqrt{a-bx}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a - b*x]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/a])/(3*x^(3/2
)*Sqrt[1 - (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.14, size = 62, normalized size = 0.93

$$2\sqrt{-b}\,b\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)-\frac{2(a-4bx)\sqrt{a-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*(a - 4*b*x)*Sqrt[a - b*x])/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[a - b*x]]
```

fricas [A] time = 1.62, size = 115, normalized size = 1.72

$$\left[\frac{3\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2},\,-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-(4\,bx-a)\sqrt{-bx+a}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
+ 2*(4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqr
t(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b*x - a)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(3/2)/x^(5/2),x)
```

```
[Out] int((-b*x+a)^(3/2)/x^(5/2),x)
```

maxima [A] time = 2.86, size = 49, normalized size = 0.73

$$-2\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{2\,\sqrt{-bx+a}\,b}{\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + a)*b/sq
rt(x) - 2/3*(-b*x + a)^(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(3/2)/x^(5/2), x)
```

sympy [C] time = 3.24, size = 187, normalized size = 2.79

$$\begin{cases} -\frac{2a\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-2ib^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)+2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right) & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{8ib^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+ib^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)-2ib^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 8*b**(3/2)*sqrt(a/(b*x) -
 1)/3 - 2*I*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + I*b**(3/2)*log(a/(b*x
)) + 2*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)), Abs(a/(b*x)) > 1), (-2*I*a*s
qrt(b)*sqrt(-a/(b*x) + 1)/(3*x) + 8*I*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + I*b**
(3/2)*log(a/(b*x)) - 2*I*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1), True))
```

3.533 $\int x^{5/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=126

$$-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Rubi [A] time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{x^{3/2}\sqrt{bx+2}}{8b^2}+\frac{3\sqrt{x}\sqrt{bx+2}}{8b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(bx+2)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 + b*x])/(8*b^2) + (x^(5
/2)*Sqrt[2 + b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 + b*x])/20 + (x^(7/2)*(2 + b*
x)^(3/2))/5 - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(2+bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{5}\int x^{5/2}\sqrt{2+bx}\,dx\\
&= \frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx\\
&= \frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{4b}\\
&= -\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b^2}\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3}{}\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3}{}\\
&= \frac{3\sqrt{x}\sqrt{2+bx}}{8b^3}-\frac{x^{3/2}\sqrt{2+bx}}{8b^2}+\frac{x^{5/2}\sqrt{2+bx}}{20b}+\frac{3}{20}x^{7/2}\sqrt{2+bx}+\frac{1}{5}x^{7/2}(2+bx)^{3/2}-\frac{3}{}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.62

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+22b^3x^3+2b^2x^2-5bx+15\right)}{40b^3}-\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 22*b^3*x^3 + 8*b^4*x^4))/(
40*b^3) - (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.75

$$\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{7/2}}+\frac{\sqrt{bx+2}\left(8b^4x^{9/2}+22b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{40b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 22*b^3*x^(7/2) +
 8*b^4*x^(9/2)))/(40*b^3) + (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*
b^(7/2))
```

fricas [A] time = 1.18, size = 156, normalized size = 1.24

$$\left[\frac{\left(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{40\,b^4},\ \frac{\left(8\,b^5x^4+22\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{40\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*
sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/4
0*((8*b^5*x^4 + 22*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt
(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
```

```
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,
3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16
,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]
%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%
{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2
,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%
}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%
{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,
0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)
/b^2*(2*((((5040*b^19/100800/b^23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/100800
/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+215460*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*
x+2)-469350*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+607950*b^19/100800/b^
23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-
2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x
+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/
b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+
2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+
8*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sq
rt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/
b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.01, size = 123, normalized size = 0.98

$$\frac{(bx+2)^{\frac{5}{2}} x^{\frac{5}{2}}}{5b}-\frac{(bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b^{2}}+\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{3}}-\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}-\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^{3}}-\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/5/b*x^(5/2)*(b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(b*x+2)^(5/2)+1/4/b^3*x^(1/2)*(
b*x+2)^(5/2)-1/8*(b*x+2)^(3/2)/b^3*x^(1/2)-3/8*(b*x+2)^(1/2)/b^3*x^(1/2)-3/
8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2
+2*x)^(1/2))
```

maxima [B] time = 2.99, size = 194, normalized size = 1.54

$$\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}} - \frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}} - \frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}} + \frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}} - \frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8 - \frac{5\,(bx+2)b^7}{x} + \frac{10\,(bx+2)^2b^6}{x^2} - \frac{10\,(bx+2)^3b^5}{x^3} + \frac{5\,(bx+2)^4b^4}{x^4} - \frac{(bx+2)^5b^3}{x^5}\right)} + \frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) - 128*(
b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(9
/2)/x^(9/2))/(b^8 - 5*(b*x + 2)*b^7/x + 10*(b*x + 2)^2*b^6/x^2 - 10*(b*x +
2)^3*b^5/x^3 + 5*(b*x + 2)^4*b^4/x^4 - (b*x + 2)^5*b^3/x^5) + 3/8*log(-(sqr
t(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(3/2), x)
```

sympy [A] time = 15.67, size = 136, normalized size = 1.08

$$\frac{b^2x^{\frac{11}{2}}}{5\sqrt{bx+2}} + \frac{19bx^{\frac{9}{2}}}{20\sqrt{bx+2}} + \frac{23x^{\frac{7}{2}}}{20\sqrt{bx+2}} - \frac{x^{\frac{5}{2}}}{40b\sqrt{bx+2}} + \frac{x^{\frac{3}{2}}}{8b^2\sqrt{bx+2}} + \frac{3\sqrt{x}}{4b^3\sqrt{bx+2}} - \frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(11/2)/(5*sqrt(b*x + 2)) + 19*b*x**(9/2)/(20*sqrt(b*x + 2)) + 23*x*
*(7/2)/(20*sqrt(b*x + 2)) - x**(5/2)/(40*b*sqrt(b*x + 2)) + x**(3/2)/(8*b**
2*sqrt(b*x + 2)) + 3*sqrt(x)/(4*b**3*sqrt(b*x + 2)) - 3*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(4*b**(7/2))
```

3.534 $\int x^{3/2}(2+bx)^{3/2}\,dx$

Optimal. Leaf size=105

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (3*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2+bx)^{3/2}\,dx &= \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{8b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,d\right.}{4b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.67

$$
\frac{\sqrt{b}\sqrt{x}\sqrt{bx+2}\left(2b^3x^3+6b^2x^2+bx-3\right)+6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*Sqrt[2 + b*x]*(-3 + b*x + 6*b^2*x^2 + 2*b^3*x^3) + 6*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 84, normalized size = 0.80

$$
\frac{\sqrt{bx+2}\left(2b^3x^{7/2}+6b^2x^{5/2}+bx^{3/2}-3\sqrt{x}\right)}{8b^2}-\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2) + 6*b^2*x^(5/2) + 2*b^3*x^(7/2)))/(8
*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(5/2))
```

fricas [A] time = 1.31, size = 137, normalized size = 1.30

$$
\left[\frac{\left(2\,b^4x^3+6\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{8\,b^3},\ \frac{\left(2\,b^4x^3+6\,b^3x^2+b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{8\,b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((2*b^4*x^3 + 6*b^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(
b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/8*((2*b^4*x^3 + 6*b
^3*x^2 + b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
```

```
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,
3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16
,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]
%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%
{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2
,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%
}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%
{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,
0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)
/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sq
rt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^1
1/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqr
t(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^
7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)
-sqrt(b)*sqrt(b*x+2))))+8*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)
*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-s
qrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.00, size = 108, normalized size = 1.03

$$\frac{(bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b}-\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^2}+\frac{(bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^2}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^2}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(3/2),x)
```

```
[Out] 1/4/b*x^(3/2)*(b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(b*x+2)^(5/2)+1/8*(b*x+2)^(3/2)
/b^2*x^(1/2)+3/8*(b*x+2)^(1/2)/b^2*x^(1/2)+3/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1
/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 3.04, size = 163, normalized size = 1.55

$$-\frac{\frac{3\sqrt{bx+2}\,b^3}{\sqrt{x}}-\frac{11\,(bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{11\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{3\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^6-\frac{4\,(bx+2)b^5}{x}+\frac{6\,(bx+2)^2b^4}{x^2}-\frac{4\,(bx+2)^3b^3}{x^3}+\frac{(bx+2)^4b^2}{x^4}\right)}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(3*sqrt(b*x + 2)*b^3/sqrt(x) - 11*(b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(b*
x + 2)^(5/2)*b/x^(5/2) + 3*(b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x + 2)*b^5/
x + 6*(b*x + 2)^2*b^4/x^2 - 4*(b*x + 2)^3*b^3/x^3 + (b*x + 2)^4*b^2/x^4) -
3/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} (b\,x+2)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(3/2), x)
```

sympy [A] time = 7.86, size = 117, normalized size = 1.11

$$\frac{b^2x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{5bx^{\frac{7}{2}}}{4\sqrt{bx+2}}+\frac{13x^{\frac{5}{2}}}{8\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(3/2),x)
```

```
[Out] b**2*x**(9/2)/(4*sqrt(b*x + 2)) + 5*b*x**(7/2)/(4*sqrt(b*x + 2)) + 13*x**(5
/2)/(8*sqrt(b*x + 2)) - x**(3/2)/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sq
rt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

3.535 $\int \sqrt{x}\,(2+bx)^{3/2}\,dx$

Optimal. Leaf size=82

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(bx+2)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{bx+2}+\frac{\sqrt{x}\,\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/(2*b) + (x^(3/2)*Sqrt[2 + b*x])/2 + (x^(3/2)*(2 + b
*x)^(3/2))/3 - ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(2+bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2+bx)^{3/2}+\int \sqrt{x}\,\sqrt{2+bx}\,dx\\
&= \frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx\\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b}\\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\
&= \frac{\sqrt{x}\,\sqrt{2+bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2+bx}+\frac{1}{3}x^{3/2}(2+bx)^{3/2}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.73

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+7bx+3\right)}{6b}-\frac{\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(3 + 7*b*x + 2*b^2*x^2))/(6*b) - ArcSinh[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 72, normalized size = 0.88

$$\frac{\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}}+\frac{\sqrt{bx+2}\left(2b^2x^{5/2}+7bx^{3/2}+3\sqrt{x}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 + b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(3*Sqrt[x] + 7*b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b) + Log[-(Sqr
t[b]*Sqrt[x]) + Sqrt[2 + b*x]]/b^(3/2)
```

fricas [A] time = 1.41, size = 124, normalized size = 1.51

$$\left[\frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}\,+3\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}\,+1\right)}{6\,b^2},\frac{\left(2\,b^3x^2+7\,b^2x+3\,b\right)\sqrt{bx+2}\,\sqrt{x}\,+6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 7*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x
 - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/6*((2*b^3*x^2 + 7*b^2*x + 3*b
)*sqrt(b*x + 2)*sqrt(x) + 6*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [82.7280518
371,8.05231268331]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [64.3995612673,28.4266860783]Warning, choosing root of [1,0,%%%
{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%
%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,
3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%
{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2
]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+
%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6
,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%
%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{
-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1
,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%
%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{
16,[0,0]%%%}] at parameters values [39.1803401988,96.7771189027]Warning, ch
oosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{
-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%
%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24
,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%
%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{
4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,
3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}
+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[
3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%
{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1
,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}
+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [95.5969694792,66
.1769613782]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%
}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}
+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,
[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%
%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{
16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0
]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%
%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[
3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%
}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{
-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,
3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters v
alues [39.9828299829,94.1262030317]Warning, choosing root of [1,0,%%%{-4,[1
,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%
}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1
,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}
+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[
2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+
%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16
```

```
,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]
%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%
{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2
,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%
}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%
{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,
0]%%%}] at parameters values [88.2886286299,17.6881634681]1/b*(2*b^2*abs(b)
/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2
)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b
)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*b*abs(b)/b^2/b*(2*(1/
8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b
)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+8*abs(b)/b^2*(1/2*sqrt(
b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)
*sqrt(b*x+2)))))
```

maple [A] time = 0.00, size = 87, normalized size = 1.06

$$\frac{(bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3}+\frac{\sqrt{bx+2}\, x^{\frac{3}{2}}}{2}+\frac{\sqrt{bx+2}\, \sqrt{x}}{2b}-\frac{\sqrt{(bx+2)\, x}\, \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\, x^{2}+2x}\right)}{2\sqrt{bx+2}\, b^{\frac{3}{2}} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(b*x+2)^(3/2)+1/2*(b*x+2)^(1/2)*x^(3/2)+1/2*(b*x+2)^(1/2)/b*x^(
1/2)-1/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)/x^(1/2)*ln((b*x+1)/b^(1/2)
+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.99, size = 132, normalized size = 1.61

$$\frac{\frac{3\, \sqrt{bx+2}\, b^{2}}{\sqrt{x}}-\frac{8\, (bx+2)^{\frac{3}{2}} b}{x^{\frac{3}{2}}}-\frac{3\, (bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{4}-\frac{3\, (bx+2) b^{3}}{x}+\frac{3\, (bx+2)^{2} b^{2}}{x^{2}}-\frac{(bx+2)^{3} b}{x^{3}}\right)}+\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\, b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(b*x + 2)*b^2/sqrt(x) - 8*(b*x + 2)^(3/2)*b/x^(3/2) - 3*(b*x + 2
)^(5/2)/x^(5/2))/(b^4 - 3*(b*x + 2)*b^3/x + 3*(b*x + 2)^2*b^2/x^2 - (b*x +
2)^3*b/x^3) + 1/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*
x + 2)/sqrt(x)))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\, (b\, x+2)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(3/2), x)
```

sympy [A] time = 4.81, size = 92, normalized size = 1.12

$$\frac{b^{2} x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{11 b x^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{17 x^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{\sqrt{x}}{b\sqrt{bx+2}}-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\, \sqrt{b}\, \sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] b**2*x**(7/2)/(3*sqrt(b*x + 2)) + 11*b*x**(5/2)/(6*sqrt(b*x + 2)) + 17*x**(
3/2)/(6*sqrt(b*x + 2)) + sqrt(x)/(b*sqrt(b*x + 2)) - asinh(sqrt(2)*sqrt(b)*
sqrt(x)/2)/b**(3/2)
```

3.536 $\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=61

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{1}{2}\sqrt{x}\,(bx+2)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 + b*x])/2 + (Sqrt[x]*(2 + b*x)^(3/2))/2 + (3*ArcSinh[(Sqr
t[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+3\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{1}{2}\sqrt{x}\,(2+bx)^{3/2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 0.79

$$\frac{1}{2}\sqrt{x}\sqrt{bx+2}(bx+5)+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(5 + b*x))/2 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
)/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.08, size = 59, normalized size = 0.97

$$\frac{1}{2}\sqrt{bx+2}\left(bx^{3/2}+5\sqrt{x}\right)-\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 + b*x]*(5*Sqrt[x] + b*x^(3/2)))/2 - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]])/Sqrt[b]
```

fricas [A] time = 1.51, size = 105, normalized size = 1.72

$$\left[\frac{\left(b^{2}x+5\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b},\frac{\left(b^{2}x+5\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b, 1/2*((b^2*x + 5*b)*sqrt(b*x + 2)*sqrt(x) - 6*sq
rt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*(1/4/b*sqrt(b*x+2
)*sqrt(b*x+2)+3/4/b)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-3/sqrt(b)*ln(abs(sqrt(
b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.00, size = 72, normalized size = 1.18

$$\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{2}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(b*x+2)^(3/2)*x^(1/2)+3/2*(b*x+2)^(1/2)*x^(1/2)+3/2*((b*x+2)*x)^(1/2)/(
b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.89, size = 98, normalized size = 1.61

$$-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\frac{3\,\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{5\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - (3*sqrt(b*x + 2)*b/sqrt(x) - 5*(b*x + 2)^(3/2)/x^(3/2))/(b^2
 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(1/2), x)
```

sympy [A] time = 2.82, size = 76, normalized size = 1.25

$$\frac{b^{2}x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{7bx^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{5\sqrt{x}}{\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] b**2*x**(5/2)/(2*sqrt(b*x + 2)) + 7*b*x**(3/2)/(2*sqrt(b*x + 2)) + 5*sqrt(x
)/sqrt(b*x + 2) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

3.537 $\int \frac{(2+bx)^{3/2}}{x^{3/2}} dx$

Optimal. Leaf size=58

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{bx+2}+6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{3/2}}{\sqrt{x}}+3b\sqrt{x}\sqrt{bx+2}+6\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] 3*b*Sqrt[x]*Sqrt[2 + b*x] - (2*(2 + b*x)^(3/2))/Sqrt[x] + 6*Sqrt[b]*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx = -\frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (3b)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + (6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)$$
$$= 3b\sqrt{x}\sqrt{2+bx} - \frac{2(2+bx)^{3/2}}{\sqrt{x}} + 6\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)$$

Mathematica [C] time = 0.00, size = 28, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, -1/2*(b*x)])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.10, size = 51, normalized size = 0.88

$$\frac{(bx-4)\sqrt{bx+2}}{\sqrt{x}} - 6\sqrt{b}\,\log\left(\sqrt{bx+2} - \sqrt{b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-4 + b*x)*Sqrt[2 + b*x])/Sqrt[x] - 6*Sqrt[b]*Log[-(Sqrt[b]*Sqrt[x]) + Sqr
t[2 + b*x]]
```

fricas [A] time = 1.23, size = 99, normalized size = 1.71

$$\left[\frac{3\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x}, -\frac{6\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\sqrt{bx+2}\,(bx-4)\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + sqrt(b*x + 2)*
(b*x - 4)*sqrt(x))/x, -(6*sqrt(-b)*x*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(
x))) - sqrt(b*x + 2)*(b*x - 4)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
```

```
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(2*(1/2*sqrt(b*x+2)*
sqrt(b*x+2)-3)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)-6/sqrt(b)*ln
(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.02, size = 72, normalized size = 1.24

$$\frac{3\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{\sqrt{bx+2}\,\sqrt{x}}+\frac{b^2x^2-2bx-8}{\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2-2*b*x-8)/(b*x+2)^(1/2)/x^(1/2)+3*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)*b
^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [A] time = 2.91, size = 81, normalized size = 1.40

$$-3\,\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{4\,\sqrt{bx+2}}{\sqrt{x}}-\frac{2\,\sqrt{bx+2}\,b}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -3*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 4*sqrt(b*x + 2)/sqrt(x) - 2*sqrt(b*x + 2)*b/((b - (b*x + 2)/x)*
sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(3/2), x)
```

sympy [A] time = 2.44, size = 73, normalized size = 1.26

$$6\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)+\frac{b^2x^{\frac{3}{2}}}{\sqrt{bx+2}}-\frac{2b\sqrt{x}}{\sqrt{bx+2}}-\frac{8}{\sqrt{x}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] 6*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)/sqrt(b*x + 2) -
2*b*sqrt(x)/sqrt(b*x + 2) - 8/(sqrt(x)*sqrt(b*x + 2))
```

3.538 $\int \frac{(2+bx)^{3/2}}{x^{5/2}} \, dx$

Optimal. Leaf size=60

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{3/2}}{3x^{3/2}}-\frac{2b\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-2*b*Sqrt[2 + b*x])/Sqrt[x] - (2*(2 + b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{3/2}}{x^{5/2}} \, dx &= -\frac{2(2+bx)^{3/2}}{3x^{3/2}}+b\int \frac{\sqrt{2+bx}}{x^{3/2}} \, dx\\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}}-\frac{2(2+bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\sqrt{2+bx}} \, dx\\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}}-\frac{2(2+bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} \, dx,x,\sqrt{x}\right)\\
&= -\frac{2b\sqrt{2+bx}}{\sqrt{x}}-\frac{2(2+bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.50

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, -1/2*(b*x)])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 55, normalized size = 0.92

$$-2b^{3/2}\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)-\frac{4\sqrt{bx+2}\,(2bx+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (-4*Sqrt[2 + b*x]*(1 + 2*b*x))/(3*x^(3/2)) - 2*b^(3/2)*Log[-(Sqrt[b]*Sqrt[x
]) + Sqrt[2 + b*x]]
```

fricas [A] time = 1.31, size = 108, normalized size = 1.80

$$\left[\frac{3\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)-4\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}}{3\,x^2},\,-\frac{2\left(3\,\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)+2\,(2\,bx+1)\sqrt{bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 4*(2*b*x
 + 1)*sqrt(b*x + 2)*sqrt(x))/x^2, -2/3*(3*sqrt(-b)*b*x^2*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + 2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
```

```
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*(-12*b^3/9*sqrt(b
*x+2)*sqrt(b*x+2)+18*b^3/9)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)
^2-2*b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.02, size = 73, normalized size = 1.22

$$\frac{\sqrt{(bx+2)\,x}\; b^{\frac{3}{2}} \ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;\sqrt{x}}-\frac{4\left(2b^{2}x^{2}+5bx+2\right)}{3\sqrt{bx+2}\; x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2+5*b*x+2)/x^(3/2)/(b*x+2)^(1/2)+b^(3/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 2.97, size = 67, normalized size = 1.12

$$-b^{\frac{3}{2}} \log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{2\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sq
rt(x))) - 2*sqrt(b*x + 2)*b/sqrt(x) - 2/3*(b*x + 2)^(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,x+2)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(3/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(3/2)/x^(5/2), x)
```

sympy [A] time = 2.81, size = 70, normalized size = 1.17

$$-\frac{8b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+2b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{4\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] -8*b**(3/2)*sqrt(1 + 2/(b*x))/3 - b**(3/2)*log(1/(b*x)) + 2*b**(3/2)*log(sq
rt(1 + 2/(b*x)) + 1) - 4*sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

3.539 $\int x^{5/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=131

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Rubi [A] time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{x^{3/2}\sqrt{2-bx}}{8b^2}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}}+\frac{1}{5}x^{7/2}(2-bx)^{3/2}+\frac{3}{20}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{20b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^3) - (x^(3/2)*Sqrt[2 - b*x])/(8*b^2) - (x^(
5/2)*Sqrt[2 - b*x])/(20*b) + (3*x^(7/2)*Sqrt[2 - b*x])/20 + (x^(7/2)*(2 - b
*x)^(3/2))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2-bx)^{3/2}\,dx &= \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{5}\int x^{5/2}\sqrt{2-bx}\,dx \\
&= \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{20}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{4b} \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b^2} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^3} - \frac{x^{3/2}\sqrt{2-bx}}{8b^2} - \frac{x^{5/2}\sqrt{2-bx}}{20b} + \frac{3}{20}x^{7/2}\sqrt{2-bx} + \frac{1}{5}x^{7/2}(2-bx)^{3/2} + \frac{3}{}
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.60

$$
\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{7/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4-22b^3x^3+2b^2x^2+5bx+15\right)}{40b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/40*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2 - 22*b^3*x^3 + 8*b^4*x
^4))/b^3 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 104, normalized size = 0.79

$$
\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^4} + \frac{\sqrt{2-bx}\left(-8b^4x^{9/2}+22b^3x^{7/2}-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{40b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 22*b^3*x^(7/2)
- 8*b^4*x^(9/2)))/(40*b^3) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 -
 b*x]])/(4*b^4)
```

fricas [A] time = 0.91, size = 157, normalized size = 1.20

$$
\left[-\frac{\left(8b^5x^4-22b^4x^3+2b^3x^2+5b^2x+15b\right)\sqrt{-bx+2}\sqrt{x}+15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{40b^4}, -\frac{\left(8b^5x^4-22b^4x^3+2b^3x^2+5b^2x+15b\right)\sqrt{-bx+2}\sqrt{x}+30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{40b^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2
)*sqrt(x) + 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
4, -1/40*((8*b^5*x^4 - 22*b^4*x^3 + 2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x +
 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
```

```
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(
-2*b^2*abs(b)/b^2*(2*((((5040*b^19/100800/b^23*sqrt(-b*x+2)*sqrt(-b*x+2)-51
660*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+215460*b^19/100800/b^23)*sq
rt(-b*x+2)*sqrt(-b*x+2)-469350*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+
607950*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-63/8/b^3/sqrt(-
b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*b*abs(b)/b^2*(2*
(((-90*b^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b
*x+2)*sqrt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11
/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqr
t(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-8*abs(b)/b^2*(2*((12*b^5/144/b^
7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b
^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-
b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.01, size = 132, normalized size = 1.01

$$-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{5}{2}}}{5b}-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b^{2}}-\frac{(-bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^{3}}+\frac{(-bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^{3}}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^{3}}+\frac{3\sqrt{(-bx+2)x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b x^{2}+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/5/b*x^(5/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^3*x^(1/2
)*(-b*x+2)^(5/2)+1/8*(-b*x+2)^(3/2)/b^3*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^3*x^(1
/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(7/2)/x^(1/2)*arctan((x-1/b)/(-
b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.94, size = 179, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^4}{\sqrt{x}}+\frac{70(-bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}-\frac{128(-bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}-\frac{70(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^8-\frac{5(bx-2)b^7}{x}+\frac{10(bx-2)^2b^6}{x^2}-\frac{10(bx-2)^3b^5}{x^3}+\frac{5(bx-2)^4b^4}{x^4}-\frac{(bx-2)^5b^3}{x^5}\right)}-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{4\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) - 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^8 - 5*(b*x - 2)*b^7/x + 10*(b*x - 2)^2*b^6/x^2 - 10*(b
*x - 2)^3*b^5/x^3 + 5*(b*x - 2)^4*b^4/x^4 - (b*x - 2)^5*b^3/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)*(2 - b*x)^(3/2), x)
```

sympy [A] time = 15.28, size = 291, normalized size = 2.22

$$\begin{cases}-\frac{ib^2x^{\frac{11}{2}}}{5\sqrt{bx-2}}+\frac{19ibx^{\frac{9}{2}}}{20\sqrt{bx-2}}-\frac{23ix^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{40b\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b^2\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^3\sqrt{bx-2}}-\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2}>1\\ \frac{b^2x^{\frac{11}{2}}}{5\sqrt{-bx+2}}-\frac{19bx^{\frac{9}{2}}}{20\sqrt{-bx+2}}+\frac{23x^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{40b\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b^2\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^3\sqrt{-bx+2}}+\frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{7}{2}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(11/2)/(5*sqrt(b*x - 2)) + 19*I*b*x**(9/2)/(20*sqrt(b
*x - 2)) - 23*I*x**(7/2)/(20*sqrt(b*x - 2)) - I*x**(5/2)/(40*b*sqrt(b*x - 2
)) - I*x**(3/2)/(8*b**2*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**3*sqrt(b*x - 2))
 - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(7/2)), Abs(b*x)/2 > 1), (b**
2*x**(11/2)/(5*sqrt(-b*x + 2)) - 19*b*x**(9/2)/(20*sqrt(-b*x + 2)) + 23*x**
(7/2)/(20*sqrt(-b*x + 2)) + x**(5/2)/(40*b*sqrt(-b*x + 2)) + x**(3/2)/(8*b*
*2*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**3*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqr
t(b)*sqrt(x)/2)/(4*b**(7/2)), True))
```

3.540 $\int x^{3/2}(2-bx)^{3/2}\,dx$

Optimal. Leaf size=109

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (3*ArcSin[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^{3/2}(2-bx)^{3/2}\,dx = \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx$$
$$= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$$
$$= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{8b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx\right.}{4b^2}$$
$$= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.64

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} - \frac{\sqrt{x}\sqrt{2-bx}\left(2b^3x^3-6b^2x^2+bx+3\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] -1/8*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x - 6*b^2*x^2 + 2*b^3*x^3))/b^2 + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 94, normalized size = 0.86

$$\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^3} + \frac{\sqrt{2-bx}\left(-2b^3x^{7/2}+6b^2x^{5/2}-bx^{3/2}-3\sqrt{x}\right)}{8b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2) + 6*b^2*x^(5/2) - 2*b^3*x^(7/2)))/(8
*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^3)
```

fricas [A] time = 1.12, size = 139, normalized size = 1.28

$$\left[-\frac{\left(2b^4x^3-6b^3x^2+b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+3\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{8b^3}, -\frac{\left(2b^4x^3-6b^3x^2+b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{8b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*b^4*x^3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqr
t(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/8*((2*b^4*x^
3 - 6*b^3*x^2 + b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt
(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
```

```
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(
-2*b^2*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^
11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*s
qrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^
2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*b*abs(b)
/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*
x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b
/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))+8*abs(b)/b^
2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*
b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.01, size = 116, normalized size = 1.06

$$-\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4b}-\frac{(-bx+2)^{\frac{5}{2}} \sqrt{x}}{4b^2}+\frac{(-bx+2)^{\frac{3}{2}} \sqrt{x}}{8b^2}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^2}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{8\sqrt{-bx+2}\;b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(3/2),x)
```

```
[Out] -1/4/b*x^(3/2)*(-b*x+2)^(5/2)-1/4/b^2*x^(1/2)*(-b*x+2)^(5/2)+1/8*(-b*x+2)^(
3/2)/b^2*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^2*x^(1/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.92, size = 147, normalized size = 1.35

$$\frac{\frac{3\sqrt{-bx+2}\,b^3}{\sqrt{x}}+\frac{11\,(-bx+2)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}-\frac{11\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{3\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{4\left(b^6-\frac{4\,(bx-2)b^5}{x}+\frac{6\,(bx-2)^2b^4}{x^2}-\frac{4\,(bx-2)^3b^3}{x^3}+\frac{(bx-2)^4b^2}{x^4}\right)}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*sqrt(-b*x + 2)*b^3/sqrt(x) + 11*(-b*x + 2)^(3/2)*b^2/x^(3/2) - 11*(-
b*x + 2)^(5/2)*b/x^(5/2) - 3*(-b*x + 2)^(7/2)/x^(7/2))/(b^6 - 4*(b*x - 2)*b
^5/x + 6*(b*x - 2)^2*b^4/x^2 - 4*(b*x - 2)^3*b^3/x^3 + (b*x - 2)^4*b^2/x^4)
 - 3/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2} (2 - b x)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(3/2), x)
```

sympy [A] time = 7.73, size = 252, normalized size = 2.31

$$\begin{cases} -\frac{ib^2x^{\frac{9}{2}}}{4\sqrt{bx-2}} + \frac{5ibx^{\frac{7}{2}}}{4\sqrt{bx-2}} - \frac{13ix^{\frac{5}{2}}}{8\sqrt{bx-2}} - \frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}} + \frac{3i\sqrt{x}}{4b^2\sqrt{bx-2}} - \frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ \frac{b^2x^{\frac{9}{2}}}{4\sqrt{-bx+2}} - \frac{5bx^{\frac{7}{2}}}{4\sqrt{-bx+2}} + \frac{13x^{\frac{5}{2}}}{8\sqrt{-bx+2}} + \frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}} - \frac{3\sqrt{x}}{4b^2\sqrt{-bx+2}} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(9/2)/(4*sqrt(b*x - 2)) + 5*I*b*x**(7/2)/(4*sqrt(b*x
- 2)) - 13*I*x**(5/2)/(8*sqrt(b*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) +
3*I*sqrt(x)/(4*b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(
4*b**(5/2)), Abs(b*x)/2 > 1), (b**2*x**(9/2)/(4*sqrt(-b*x + 2)) - 5*b*x**(7
/2)/(4*sqrt(-b*x + 2)) + 13*x**(5/2)/(8*sqrt(-b*x + 2)) + x**(3/2)/(8*b*sqr
t(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*s
qrt(x)/2)/(4*b**(5/2)), True))
```

3.541 $\int \sqrt{x}\,(2-bx)^{3/2}\,dx$

Optimal. Leaf size=84

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}x^{3/2}\sqrt{2-bx}-\frac{\sqrt{x}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 - b*x])/(2*b) + (x^(3/2)*Sqrt[2 - b*x])/2 + (x^(3/2)*(2 -
b*x)^(3/2))/3 + ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(2-bx)^{3/2}\,dx &= \frac{1}{3}x^{3/2}(2-bx)^{3/2}+\int \sqrt{x}\sqrt{2-bx}\,dx\\
&= \frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{2b}+\frac{1}{2}x^{3/2}\sqrt{2-bx}+\frac{1}{3}x^{3/2}(2-bx)^{3/2}+\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.71

$$\frac{\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-7bx+3\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(3 - 7*b*x + 2*b^2*x^2))/b + ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]]/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.12, size = 81, normalized size = 0.96

$$\frac{\sqrt{2-bx}\left(-2b^2x^{5/2}+7bx^{3/2}-3\sqrt{x}\right)}{6b}+\frac{\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] + 7*b*x^(3/2) - 2*b^2*x^(5/2)))/(6*b) + (Sqrt[-b
]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^2
```

fricas [A] time = 1.31, size = 125, normalized size = 1.49

$$\left[-\frac{\left(2\,b^3x^2-7\,b^2x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^2},-\frac{\left(2\,b^3x^2-7\,b^2x+3\,b\right)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 - 7*b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(
-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, -1/6*((2*b^3*x^2 - 7*b^2*
x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-18.27194
81629,8.05231268331]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
```

,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-36.6004387327,28.4266860783]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-61.8196598012,96.7771189027]Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-5.403030
52077,66.1769613782]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-61.0171700171,94.1262030317]Warning, choosing root of [1,0
,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,
[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%
}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4
,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%
}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-
20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,

```
2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%
}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,
[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%
}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%
{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1
,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}
+%%%{16,[0,0]%%%}] at parameters values [-12.7113713701,17.6881634681]1/b*(
-2*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/
b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+
2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))
)-8*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt
(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt
(-b*x+2))))-8*abs(b)/b^2*(1/2*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt
(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.00, size = 94, normalized size = 1.12

$$\frac{(-bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{3}+\frac{\sqrt{-bx+2}\, x^{\frac{3}{2}}}{2}-\frac{\sqrt{-bx+2}\, \sqrt{x}}{2b}+\frac{\sqrt{(-bx+2)\, x}\, \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\, x^2+2x}}\right)}{2\sqrt{-bx+2}\, b^{\frac{3}{2}} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)*x^(1/2),x)
```

```
[Out] 1/3*x^(3/2)*(-b*x+2)^(3/2)+1/2*(-b*x+2)^(1/2)*x^(3/2)-1/2*(-b*x+2)^(1/2)/b*
x^(1/2)+1/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(3/2)/x^(1/2)*arctan((x-1/b
)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 3.04, size = 115, normalized size = 1.37

$$\frac{\frac{3\sqrt{-bx+2}\, b^2}{\sqrt{x}}+\frac{8(-bx+2)^{\frac{3}{2}} b}{x^{\frac{3}{2}}}-\frac{3(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^4-\frac{3(bx-2)b^3}{x}+\frac{3(bx-2)^2 b^2}{x^2}-\frac{(bx-2)^3 b}{x^3}\right)}-\frac{\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sqrt(-b*x + 2)*b^2/sqrt(x) + 8*(-b*x + 2)^(3/2)*b/x^(3/2) - 3*(-b*x
+ 2)^(5/2)/x^(5/2))/(b^4 - 3*(b*x - 2)*b^3/x + 3*(b*x - 2)^2*b^2/x^2 - (b*x
 - 2)^3*b/x^3) - arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\, (2-b\, x)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(3/2), x)
```

sympy [A] time = 4.78, size = 199, normalized size = 2.37

$$\begin{cases} -\frac{ib^2x^{\frac{7}{2}}}{3\sqrt{bx-2}}+\frac{11ibx^{\frac{5}{2}}}{6\sqrt{bx-2}}-\frac{17ix^{\frac{3}{2}}}{6\sqrt{bx-2}}+\frac{i\sqrt{x}}{b\sqrt{bx-2}}-\frac{i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{b^2x^{\frac{7}{2}}}{3\sqrt{-bx+2}}-\frac{11bx^{\frac{5}{2}}}{6\sqrt{-bx+2}}+\frac{17x^{\frac{3}{2}}}{6\sqrt{-bx+2}}-\frac{\sqrt{x}}{b\sqrt{-bx+2}}+\frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)*x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(7/2)/(3*sqrt(b*x - 2)) + 11*I*b*x**(5/2)/(6*sqrt(b*x
 - 2)) - 17*I*x**(3/2)/(6*sqrt(b*x - 2)) + I*sqrt(x)/(b*sqrt(b*x - 2)) - I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (b**2*x**(7/2)/
(3*sqrt(-b*x + 2)) - 11*b*x**(5/2)/(6*sqrt(-b*x + 2)) + 17*x**(3/2)/(6*sqrt
(-b*x + 2)) - sqrt(x)/(b*sqrt(-b*x + 2)) + asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
b**(3/2), True))
```

3.542 $\int \frac{(2-bx)^{3/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}\,+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\sqrt{x}\,\sqrt{2-bx}\,+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (3*Sqrt[x]*Sqrt[2 - b*x])/2 + (Sqrt[x]*(2 - b*x)^(3/2))/2 + (3*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx &= \frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}\,+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}\,+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{3}{2}\sqrt{x}\,\sqrt{2-bx}\,+\frac{1}{2}\sqrt{x}\,(2-bx)^{3/2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.78

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}-\frac{1}{2}\sqrt{x}\sqrt{2-bx}\,(bx-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(-5 + b*x)) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[
2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.10, size = 69, normalized size = 1.10

$$\frac{1}{2}\sqrt{2-bx}\left(5\sqrt{x}-bx^{3/2}\right)+\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 - b*x]*(5*Sqrt[x] - b*x^(3/2)))/2 + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[2 - b*x]])/b
```

fricas [A] time = 1.47, size = 107, normalized size = 1.70

$$\left[-\frac{\left(b^{2}x-5\,b\right)\sqrt{-bx+2}\,\sqrt{x}+3\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{2\,b},-\frac{\left(b^{2}x-5\,b\right)\sqrt{-bx+2}\,\sqrt{x}+6\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, -1/2*((b^2*x - 5*b)*sqrt(-b*x + 2)*sqrt(x
) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*(1/4/b*sqrt(-
b*x+2)*sqrt(-b*x+2)+3/4/b)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+3/sqrt(-b)*ln
(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [A] time = 0.00, size = 78, normalized size = 1.24

$$\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{2}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{2}+\frac{3\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] 1/2*(-b*x+2)^(3/2)*x^(1/2)+3/2*(-b*x+2)^(1/2)*x^(1/2)+3/2*((-b*x+2)*x)^(1/2
)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.98, size = 79, normalized size = 1.25

$$-\frac{3\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{3\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{5(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{2}-\frac{2(bx-2)b}{x}+\frac{(bx-2)^{2}}{x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + (3*sqrt(-b*x + 2)*b/s
qrt(x) + 5*(-b*x + 2)^(3/2)/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)^2/x
^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(1/2), x)
```

sympy [A] time = 2.86, size = 167, normalized size = 2.65

$$\begin{cases} -\frac{ib^{2}x^{\frac{5}{2}}}{2\sqrt{bx-2}}+\frac{7ibx^{\frac{3}{2}}}{2\sqrt{bx-2}}-\frac{5i\sqrt{x}}{\sqrt{bx-2}}-\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{b^{2}x^{\frac{5}{2}}}{2\sqrt{-bx+2}}-\frac{7bx^{\frac{3}{2}}}{2\sqrt{-bx+2}}+\frac{5\sqrt{x}}{\sqrt{-bx+2}}+\frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((-I*b**2*x**(5/2)/(2*sqrt(b*x - 2)) + 7*I*b*x**(3/2)/(2*sqrt(b*x
- 2)) - 5*I*sqrt(x)/sqrt(b*x - 2) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sq
rt(b), Abs(b*x)/2 > 1), (b**2*x**(5/2)/(2*sqrt(-b*x + 2)) - 7*b*x**(3/2)/(2
*sqrt(-b*x + 2)) + 5*sqrt(x)/sqrt(-b*x + 2) + 3*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/sqrt(b), True))
```

3.543 $\int \frac{(2-bx)^{3/2}}{x^{3/2}} \, dx$

Optimal. Leaf size=60

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{2-bx}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-3b\sqrt{x}\,\sqrt{2-bx}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] -3*b*Sqrt[x]*Sqrt[2 - b*x] - (2*(2 - b*x)^(3/2))/Sqrt[x] - 6*Sqrt[b]*ArcSin
[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= -3b\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(3b)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\ &= -3b\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-(6b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\ &= -3b\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{3/2}}{\sqrt{x}}-6\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.47

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.12, size = 58, normalized size = 0.97

$$\frac{(-bx-4)\sqrt{2-bx}}{\sqrt{x}}-6\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/x^(3/2),x]
```

```
[Out] ((-4 - b*x)*Sqrt[2 - b*x])/Sqrt[x] - 6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + S
qrt[2 - b*x]]
```

fricas [A] time = 0.74, size = 101, normalized size = 1.68

$$\left[\frac{3\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)-(bx+4)\sqrt{-bx+2}\sqrt{x}}{x},\ \frac{6\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-(bx+4)\sqrt{-bx+2}\sqrt{x}}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [(3*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (b*x + 4)*
sqrt(-b*x + 2)*sqrt(x))/x, (6*sqrt(b)*x*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt
(x))) - (b*x + 4)*sqrt(-b*x + 2)*sqrt(x))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
```

```
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*(-1/2*sqrt(-
b*x+2)*sqrt(-b*x+2)+3)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)
+6/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [B] time = 0.02, size = 97, normalized size = 1.62

$$-\frac{3\sqrt{(-bx+2)\,x}\,\sqrt{b}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{\left(b^{2}x^{2}+2bx-8\right)\sqrt{(-bx+2)\,x}}{\sqrt{-\left(bx-2\right)x}\,\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(3/2),x)
```

```
[Out] (b^2*x^2+2*b*x-8)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1
/2)-3*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*
x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.99, size = 63, normalized size = 1.05

$$6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\frac{4\sqrt{-bx+2}}{\sqrt{x}}-\frac{2\sqrt{-bx+2}\,b}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - 4*sqrt(-b*x + 2)/sqrt(
x) - 2*sqrt(-b*x + 2)*b/((b - (b*x - 2)/x)*sqrt(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(3/2), x)
```

sympy [A] time = 2.49, size = 160, normalized size = 2.67

$$\begin{cases} 6i\sqrt{b}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) - \frac{ib^2x^{\frac{3}{2}}}{\sqrt{bx-2}} - \frac{2ib\sqrt{x}}{\sqrt{bx-2}} + \frac{8i}{\sqrt{x}\sqrt{bx-2}} & \text{for}\ \frac{|bx|}{2} > 1 \\ -6\sqrt{b}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right) + \frac{b^2x^{\frac{3}{2}}}{\sqrt{-bx+2}} + \frac{2b\sqrt{x}}{\sqrt{-bx+2}} - \frac{8}{\sqrt{x}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(3/2),x)
```

```
[Out] Piecewise((6*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) - I*b**2*x**(3/2)/s
qrt(b*x - 2) - 2*I*b*sqrt(x)/sqrt(b*x - 2) + 8*I/(sqrt(x)*sqrt(b*x - 2)), A
bs(b*x)/2 > 1), (-6*sqrt(b)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**2*x**(3/2)
/sqrt(-b*x + 2) + 2*b*sqrt(x)/sqrt(-b*x + 2) - 8/(sqrt(x)*sqrt(-b*x + 2)),
True))
```

3.544 $\int \frac{(2-bx)^{3/2}}{x^{5/2}} dx$

Optimal. Leaf size=62

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\frac{2b\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(3/2)/x^(5/2),x]
```

```
[Out] (2*b*Sqrt[2 - b*x])/Sqrt[x] - (2*(2 - b*x)^(3/2))/(3*x^(3/2)) + 2*b^(3/2)*A
rcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{3/2}}{x^{5/2}} dx &= -\frac{2(2-bx)^{3/2}}{3x^{3/2}}-b\int \frac{\sqrt{2-bx}}{x^{3/2}} dx\\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}}-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+b^2\int \frac{1}{\sqrt{x}\sqrt{2-bx}} dx\\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}}-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+\left(2b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} dx,x,\sqrt{x}\right)\\
&= \frac{2b\sqrt{2-bx}}{\sqrt{x}}-\frac{2(2-bx)^{3/2}}{3x^{3/2}}+2b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.48

$$-\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - b*x)^(3/2)/x^(5/2),x]

[Out] (-4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))

IntegrateAlgebraic [A] time = 0.13, size = 62, normalized size = 1.00

$$\frac{4\sqrt{2-bx}\,(2bx-1)}{3x^{3/2}}+2\sqrt{-b}\,b\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 - b*x)^(3/2)/x^(5/2),x]

[Out] (4*Sqrt[2 - b*x]*(-1 + 2*b*x))/(3*x^(3/2)) + 2*Sqrt[-b]*b*Log[-(Sqrt[-b]*Sq
rt[x]) + Sqrt[2 - b*x]]

fricas [A] time = 1.15, size = 111, normalized size = 1.79

$$\left[\frac{3\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+4\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}}{3\,x^2},-\frac{2\left(3\,b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\,(2\,bx-1)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="fricas")

[Out] [1/3*(3*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 4*
(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2, -2/3*(3*b^(3/2)*x^2*arctan(sqrt(-b
*x + 2)/(sqrt(b)*sqrt(x))) - 2*(2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x))/x^2]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%

```
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*(-12*b^3/9*sq
rt(-b*x+2)*sqrt(-b*x+2)+18*b^3/9)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-
b*x+2)+2*b)^2+2*b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*
x+2))))
```

maple [B] time = 0.02, size = 98, normalized size = 1.58

$$\frac{\sqrt{(-bx+2)x}\, b^{\frac{3}{2}} \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^2+2x}}\right)}{\sqrt{-bx+2}\,\sqrt{x}} - \frac{4\left(2b^2x^2-5bx+2\right)\sqrt{(-bx+2)x}}{3\sqrt{-(bx-2)x}\,\sqrt{-bx+2}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(3/2)/x^(5/2),x)
```

```
[Out] -4/3*(2*b^2*x^2-5*b*x+2)/x^(3/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)+b^(3/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))*((-b*x+2)*x)^
(1/2)/(-b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 3.00, size = 49, normalized size = 0.79

$$-2\,b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right) + \frac{2\sqrt{-bx+2}\,b}{\sqrt{x}} - \frac{2\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(3/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 2*sqrt(-b*x + 2)*b/sq
rt(x) - 2/3*(-b*x + 2)^(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(2-b\,x)^{3/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(3/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(3/2)/x^(5/2), x)
```

sympy [C] time = 2.92, size = 182, normalized size = 2.94

$$\begin{cases} \frac{8b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\,\sqrt{x}}\right) + 2b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) - \frac{4\sqrt{b}\,\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for}\ \frac{2}{|bx|} > 1 \\ \frac{8ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3} + ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right) - 2ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right) - \frac{4i\sqrt{b}\,\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(3/2)/x**(5/2),x)
```

```
[Out] Piecewise((8*b**(3/2)*sqrt(-1 + 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*
b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 2*b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)
/2) - 4*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x), 2/Abs(b*x) > 1), (8*I*b**(3/2)*sq
rt(1 - 2/(b*x))/3 + I*b**(3/2)*log(1/(b*x)) - 2*I*b**(3/2)*log(sqrt(1 - 2/(
b*x)) + 1) - 4*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

3.545 $\int x^{5/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=164

$$-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}-$$

Rubi [A] time = 0.06, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2}+\frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3}-\frac{5a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{512b^{7/2}}+\frac{a^3x^{5/2}\sqrt{a+bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a+bx}+\frac{1}{12}ax^{7/2}(a+bx)^{3/2}+\frac{1}{6}x^{7/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^5*Sqrt[x]*Sqrt[a + b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a + b*x])/(76
8*b^2) + (a^3*x^(5/2)*Sqrt[a + b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a + b*x])/
32 + (a*x^(7/2)*(a + b*x)^(3/2))/12 + (x^(7/2)*(a + b*x)^(5/2))/6 - (5*a^6*
ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int x^{5/2}(a+bx)^{5/2}\,dx = \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{12}(5a)\int x^{5/2}(a+bx)^{3/2}\,dx$$
$$= \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{8}a^2\int x^{5/2}\sqrt{a+bx}\,dx$$
$$= \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} + \frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$$
$$= \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}(a+bx)^{5/2} - \frac{(5a^4)\int}{38}$$
$$= -\frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a+bx)^{3/2} + \frac{1}{6}x^{7/2}($$
$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a$$
$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a$$
$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a$$
$$= \frac{5a^5\sqrt{x}\sqrt{a+bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a+bx}}{768b^2} + \frac{a^3x^{5/2}\sqrt{a+bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a+bx} + \frac{1}{12}ax^{7/2}(a$$

Mathematica [A] time = 0.24, size = 118, normalized size = 0.72

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\left(15a^5-10a^4bx+8a^3b^2x^2+432a^2b^3x^3+640ab^4x^4+256b^5x^5\right)-\frac{15a^{11/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^5 - 10*a^4*b*x + 8*a^3*b^2*x^2 + 432*
a^2*b^3*x^3 + 640*a*b^4*x^4 + 256*b^5*x^5) - (15*a^(11/2)*ArcSinh[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(1536*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 121, normalized size = 0.74

$$\frac{5a^6\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{512b^{7/2}}+\frac{\sqrt{a+bx}\left(15a^5\sqrt{x}-10a^4bx^{3/2}+8a^3b^2x^{5/2}+432a^2b^3x^{7/2}+640ab^4x^{9/2}+256b^5x^{11/2}\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^5*Sqrt[x] - 10*a^4*b*x^(3/2) + 8*a^3*b^2*x^(5/2) + 432
*a^2*b^3*x^(7/2) + 640*a*b^4*x^(9/2) + 256*b^5*x^(11/2)))/(1536*b^3) + (5*a
^6*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(512*b^(7/2))
```

fricas [A] time = 0.78, size = 206, normalized size = 1.26

$$\left[\frac{15\,a^6\sqrt{b}\log\left(2\,bx-2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(256\,b^6x^5+640\,ab^5x^4+432\,a^2b^4x^3+8\,a^3b^3x^2-10\,a^4b^2x+15\,a^5b\right)\sqrt{bx+a}\sqrt{x}}{3072\,b^4},\ \frac{15\,a^6\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(256\,b^6x^5+640\,ab^5x^4+432\,a^2b^4x^3+8\,a^3b^3x^2-10\,a^4b^2x+15\,a^5b\right)\sqrt{bx+a}\sqrt{x}}{1536\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3072*(15*a^6*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(256*b^6*x^5 + 640*a*b^5*x^4 + 432*a^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b
^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/1536*(15*a^6*sqrt(-b)*arctan
(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (256*b^6*x^5 + 640*a*b^5*x^4 + 432*a
^2*b^4*x^3 + 8*a^3*b^3*x^2 - 10*a^4*b^2*x + 15*a^5*b)*sqrt(b*x + a)*sqrt(x)
)/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 156, normalized size = 0.95

$$-\frac{5\sqrt{(bx+a)\,x}\,a^{6}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{1024\sqrt{bx+a}\;b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{bx+a}\;a^{5}\sqrt{x}}{512b^{3}}-\frac{5\,(bx+a)^{\frac{3}{2}}\,a^{4}\sqrt{x}}{768b^{3}}+\frac{(bx+a)^{\frac{7}{2}}\,x^{\frac{5}{2}}}{6b}-\frac{(bx+a)^{\frac{5}{2}}\,a^{3}\sqrt{x}}{192b^{3}}-\frac{(bx+a)^{\frac{7}{2}}\,a\,x^{\frac{3}{2}}}{12b^{2}}+\frac{(bx+a)^{\frac{7}{2}}\,a^{2}\sqrt{x}}{32b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+a)^(7/2)-1/12*a/b^2*x^(3/2)*(b*x+a)^(7/2)+1/32*a^2/b^3*x
^(1/2)*(b*x+a)^(7/2)-1/192*a^3/b^3*(b*x+a)^(5/2)*x^(1/2)-5/768*a^4/b^3*(b*x
+a)^(3/2)*x^(1/2)-5/512*a^5*x^(1/2)*(b*x+a)^(1/2)/b^3-5/1024*a^6/b^(7/2)*((
b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1
/2))
```

maxima [B] time = 2.99, size = 244, normalized size = 1.49

$$\frac{5\,a^{6}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{1024\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^{6}b^{5}}{\sqrt{x}}-\frac{85\,(bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}+\frac{198\,(bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}+\frac{15\,(bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx+a)b^{8}}{x}+\frac{15\,(bx+a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx+a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx+a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx+a)^{5}b^{4}}{x^{5}}+\frac{(bx+a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/1024*a^6*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(7/2) + 1/1536*(15*sqrt(b*x + a)*a^6*b^5/sqrt(x) - 85*(b*x + a)
^(3/2)*a^6*b^4/x^(3/2) + 198*(b*x + a)^(5/2)*a^6*b^3/x^(5/2) + 198*(b*x + a
)^(7/2)*a^6*b^2/x^(7/2) - 85*(b*x + a)^(9/2)*a^6*b/x^(9/2) + 15*(b*x + a)^(
11/2)*a^6/x^(11/2))/(b^9 - 6*(b*x + a)*b^8/x + 15*(b*x + a)^2*b^7/x^2 - 20*
(b*x + a)^3*b^6/x^3 + 15*(b*x + a)^4*b^5/x^4 - 6*(b*x + a)^5*b^4/x^5 + (b*x
 + a)^6*b^3/x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,{\left(a+b\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a + b*x)^(5/2), x)
```

sympy [A] time = 25.94, size = 207, normalized size = 1.26

$$\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1+\frac{bx}{a}}}+\frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1+\frac{bx}{a}}}-\frac{5a^6\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}}+\frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+a)**(5/2),x)
```

```
[Out] 5*a**(11/2)*sqrt(x)/(512*b**3*sqrt(1 + b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*
b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(5/2)/(768*b*sqrt(1 + b*x/a)) + 55*a**(
5/2)*x**(7/2)/(192*sqrt(1 + b*x/a)) + 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 + b
*x/a)) + 7*sqrt(a)*b**2*x**(11/2)/(12*sqrt(1 + b*x/a)) - 5*a**6*asinh(sqrt(
b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 + b*x
/a))
```

3.546 $\int x^{3/2}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=140

$$\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{3a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{128b^{5/2}}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a + b*x])/(128*b^2) + (a^3*x^(3/2)*Sqrt[a + b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a + b*x])/16 + (a*x^(5/2)*(a + b*x)^(3/2))/8 + (x^(5
/2)*(a + b*x)^(5/2))/5 + (3*a^5*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(
128*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(a+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{2}a\int x^{3/2}(a+bx)^{3/2}\,dx \\
&= \frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a+bx}\,dx \\
&= \frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}+\frac{1}{32}a^3\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx \\
&= \frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a+bx)^{5/2}-\frac{\left(3a^4\right)\int \frac{}{\sqrt{}}}{128} \\
&= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a \\
&= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a \\
&= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a \\
&= -\frac{3a^4\sqrt{x}\sqrt{a+bx}}{128b^2}+\frac{a^3x^{3/2}\sqrt{a+bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a+bx}+\frac{1}{8}ax^{5/2}(a+bx)^{3/2}+\frac{1}{5}x^{5/2}(a
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 107, normalized size = 0.76

$$
\frac{\sqrt{a+bx}\left(\frac{15a^{9/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}+\sqrt{b}\sqrt{x}\left(-15a^4+10a^3bx+248a^2b^2x^2+336ab^3x^3+128b^4x^4\right)\right)}{640b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 + 10*a^3*b*x + 248*a^2*b^2*x^2 + 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]
])/Sqrt[1 + (b*x)/a]))/(640*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 108, normalized size = 0.77

$$
\frac{\sqrt{a+bx}\left(-15a^4\sqrt{x}+10a^3bx^{3/2}+248a^2b^2x^{5/2}+336ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^2}-\frac{3a^5\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{128b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-15*a^4*Sqrt[x] + 10*a^3*b*x^(3/2) + 248*a^2*b^2*x^(5/2) +
336*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^2) - (3*a^5*Log[-(Sqrt[b]*Sqrt
[x]) + Sqrt[a + b*x]])/(128*b^(5/2))
```

fricas [A] time = 1.35, size = 185, normalized size = 1.32

$$
\left[\frac{15\,a^5\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx+a}\sqrt{x}}{1280\,b^3}, -\frac{15\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)-\left(128\,b^5x^4+336\,ab^4x^3+248\,a^2b^3x^2+10\,a^3b^2x-15\,a^4b\right)\sqrt{bx+a}\sqrt{x}}{640\,b^3}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/1280*(15*a^5*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) +
2*(128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*a^3*b^2*x - 15*a^4*b)
*sqrt(b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(-b)*arctan(sqrt(b*x + a)*s
qrt(-b)/(b*sqrt(x))) - (128*b^5*x^4 + 336*a*b^4*x^3 + 248*a^2*b^3*x^2 + 10*
a^3*b^2*x - 15*a^4*b)*sqrt(b*x + a)*sqrt(x))/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 138, normalized size = 0.99

$$\frac{3\sqrt{(bx+a)x}\,a^5\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{256\sqrt{bx+a}\,b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{bx+a}\,a^4\sqrt{x}}{128b^2}+\frac{(bx+a)^{\frac{3}{2}}a^3\sqrt{x}}{64b^2}+\frac{(bx+a)^{\frac{5}{2}}a^2\sqrt{x}}{80b^2}+\frac{(bx+a)^{\frac{7}{2}}x^{\frac{3}{2}}}{5b}-\frac{3\,(bx+a)^{\frac{7}{2}}a\sqrt{x}}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+a)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+a)^(7/2)-3/40*a/b^2*x^(1/2)*(b*x+a)^(7/2)+1/80*a^2/b^2*(
b*x+a)^(5/2)*x^(1/2)+1/64*a^3/b^2*(b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1/2)*(
b*x+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*
ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [B] time = 2.98, size = 212, normalized size = 1.51

$$-\frac{3\,a^5\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{256\,b^{\frac{5}{2}}}-\frac{\frac{15\,\sqrt{bx+a}\,a^5b^4}{\sqrt{x}}-\frac{70\,(bx+a)^{\frac{3}{2}}a^5b^3}{x^{\frac{3}{2}}}+\frac{128\,(bx+a)^{\frac{5}{2}}a^5b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+a)^{\frac{7}{2}}a^5b}{x^{\frac{7}{2}}}-\frac{15\,(bx+a)^{\frac{9}{2}}a^5}{x^{\frac{9}{2}}}}{640\left(b^7-\frac{5\,(bx+a)b^6}{x}+\frac{10\,(bx+a)^2b^5}{x^2}-\frac{10\,(bx+a)^3b^4}{x^3}+\frac{5\,(bx+a)^4b^3}{x^4}-\frac{(bx+a)^5b^2}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/256*a^5*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/
sqrt(x)))/b^(5/2) - 1/640*(15*sqrt(b*x + a)*a^5*b^4/sqrt(x) - 70*(b*x + a)^
(3/2)*a^5*b^3/x^(3/2) + 128*(b*x + a)^(5/2)*a^5*b^2/x^(5/2) + 70*(b*x + a)^
(7/2)*a^5*b/x^(7/2) - 15*(b*x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x + a)*b^
6/x + 10*(b*x + a)^2*b^5/x^2 - 10*(b*x + a)^3*b^4/x^3 + 5*(b*x + a)^4*b^3/x
^4 - (b*x + a)^5*b^2/x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a+b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a + b*x)^(5/2), x)
```

sympy [A] time = 16.41, size = 180, normalized size = 1.29

$$-\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1+\frac{bx}{a}}}-\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1+\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1+\frac{bx}{a}}}+\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1+\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^2x^{\frac{9}{2}}}{40\sqrt{1+\frac{bx}{a}}}+\frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+a)**(5/2),x)
```

```
[Out] -3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 + b*x/a)) - a**(7/2)*x**(3/2)/(128*b*s
qrt(1 + b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 + b*x/a)) + 73*a**(3/2)
*b*x**(7/2)/(80*sqrt(1 + b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1 + b*
x/a)) + 3*a**5*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) + b**3*x**(11/
2)/(5*sqrt(a)*sqrt(1 + b*x/a))
```

3.547 $\int \sqrt{x}(a+bx)^{5/2}\,dx$

Optimal. Leaf size=116

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$-\frac{5a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5a^3\sqrt{x}\sqrt{a+bx}}{64b}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (5*a^3*Sqrt[x]*Sqrt[a + b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a + b*x])/32 + (
5*a*x^(3/2)*(a + b*x)^(3/2))/24 + (x^(3/2)*(a + b*x)^(5/2))/4 - (5*a^4*ArcT
anh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(a+bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{8}(5a)\int \sqrt{x}\,(a+bx)^{3/2}\,dx\\
&= \frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a+bx}\,dx\\
&= \frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}+\frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\int}{}\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\mathrm{Su}}{}\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{\left(5a^4\right)\mathrm{Su}}{}\\
&= \frac{5a^3\sqrt{x}\,\sqrt{a+bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a+bx}+\frac{5}{24}ax^{3/2}(a+bx)^{3/2}+\frac{1}{4}x^{3/2}(a+bx)^{5/2}-\frac{5a^4\tan}{6}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 96, normalized size = 0.83

$$
\frac{\sqrt{a+bx}\left(\sqrt{b}\,\sqrt{x}\,\left(15a^3+118a^2bx+136ab^2x^2+48b^3x^3\right)-\frac{15a^{7/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{192b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^3 + 118*a^2*b*x + 136*a*b^2*x^2 + 48*
b^3*x^3) - (15*a^(7/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a
]))/(192*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.82

$$
\frac{5a^4\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{64b^{3/2}}+\frac{\sqrt{a+bx}\,\left(15a^3\sqrt{x}+118a^2bx^{3/2}+136ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^3*Sqrt[x] + 118*a^2*b*x^(3/2) + 136*a*b^2*x^(5/2) + 48
*b^3*x^(7/2)))/(192*b) + (5*a^4*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(6
4*b^(3/2))
```

fricas [A] time = 1.45, size = 162, normalized size = 1.40

$$
\left[\frac{15\,a^4\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{384\,b^2},\ \frac{15\,a^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(48\,b^4x^3+136\,ab^3x^2+118\,a^2b^2x+15\,a^3b\right)\sqrt{bx+a}\,\sqrt{x}}{192\,b^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(15*a^4*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sqrt
```

```
(x))/b^2, 1/192*(15*a^4*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))
 + (48*b^4*x^3 + 136*a*b^3*x^2 + 118*a^2*b^2*x + 15*a^3*b)*sqrt(b*x + a)*sq
rt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 111, normalized size = 0.96

$$\frac{5\sqrt{bx+a}\,a^{2}x^{\frac{3}{2}}}{32}-\frac{5\sqrt{(bx+a)\,x}\,a^{4}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{128\sqrt{bx+a}\,b^{\frac{3}{2}}\sqrt{x}}+\frac{5\sqrt{bx+a}\,a^{3}\sqrt{x}}{64b}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24}+\frac{(bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+a)^(5/2)+5/24*a*x^(3/2)*(b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(b*
x+a)^(1/2)+5/64*a^3*x^(1/2)*(b*x+a)^(1/2)/b-5/128*a^4/b^(3/2)*((b*x+a)*x)^(
1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [B] time = 2.98, size = 176, normalized size = 1.52

$$\frac{5\,a^{4}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{128\,b^{\frac{3}{2}}}+\frac{\frac{15\,\sqrt{bx+a}\,a^{4}b^{3}}{\sqrt{x}}-\frac{55\,(bx+a)^{\frac{3}{2}}\,a^{4}b^{2}}{x^{\frac{3}{2}}}+\frac{73\,(bx+a)^{\frac{5}{2}}\,a^{4}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+a)^{\frac{7}{2}}\,a^{4}}{x^{\frac{7}{2}}}}{192\left(b^{5}-\frac{4\,(bx+a)b^{4}}{x}+\frac{6\,(bx+a)^{2}b^{3}}{x^{2}}-\frac{4\,(bx+a)^{3}b^{2}}{x^{3}}+\frac{(bx+a)^{4}b}{x^{4}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 5/128*a^4*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/b^(3/2) + 1/192*(15*sqrt(b*x + a)*a^4*b^3/sqrt(x) - 55*(b*x + a)^(
3/2)*a^4*b^2/x^(3/2) + 73*(b*x + a)^(5/2)*a^4*b/x^(5/2) + 15*(b*x + a)^(7/2
)*a^4/x^(7/2))/(b^5 - 4*(b*x + a)*b^4/x + 6*(b*x + a)^2*b^3/x^2 - 4*(b*x +
a)^3*b^2/x^3 + (b*x + a)^4*b/x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(a+b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a + b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(a + b*x)^(5/2), x)
```

sympy [A] time = 9.86, size = 155, normalized size = 1.34

$$\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1+\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1+\frac{bx}{a}}}+\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1+\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^{2}x^{\frac{7}{2}}}{24\sqrt{1+\frac{bx}{a}}}-\frac{5a^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] 5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 + b*x/a)) + 133*a**(5/2)*x**(3/2)/(192*sqrt
(1 + b*x/a)) + 127*a**(3/2)*b*x**(5/2)/(96*sqrt(1 + b*x/a)) + 23*sqrt(a)*b*
*2*x**(7/2)/(24*sqrt(1 + b*x/a)) - 5*a**4*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(6
4*b**(3/2)) + b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 + b*x/a))
```

3.548 $\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=92

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a+bx}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}(a+bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/8 + (5*a*Sqrt[x]*(a + b*x)^(3/2))/12 + (Sqrt[
x]*(a + b*x)^(5/2))/3 + (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(8
*Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{1}{6}(5a)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx \\
&= \frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx\right. \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x\right. \\
&= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a+bx}+\frac{5}{12}a\sqrt{x}\,(a+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(a+bx)^{5/2}+\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8\sqrt{b}}
\end{aligned}
$$

Mathematica [A] time = 0.11, size = 80, normalized size = 0.87

$$\frac{1}{24}\sqrt{a+bx}\left(\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{\frac{bx}{a}+1}}+\sqrt{x}\left(33a^2+26abx+8b^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[x]*(33*a^2 + 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*x)/a])))/24
```

IntegrateAlgebraic [A] time = 0.11, size = 79, normalized size = 0.86

$$\frac{1}{24}\sqrt{a+bx}\left(33a^2\sqrt{x}+26abx^{3/2}+8b^2x^{5/2}\right)-\frac{5a^3\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{8\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a + b*x]*(33*a^2*Sqrt[x] + 26*a*b*x^(3/2) + 8*b^2*x^(5/2)))/24 - (5*a
^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*Sqrt[b])
```

fricas [A] time = 1.51, size = 141, normalized size = 1.53

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2+26\,ab^2x+33\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b},-\frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(8\,b^3x^2+26\,ab^2x+33\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 + 26*a*b^2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b, -1/24*(15*a^3
*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (8*b^3*x^2 + 26*a*b^
2*x + 33*a^2*b)*sqrt(b*x + a)*sqrt(x))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 93, normalized size = 1.01

$$\frac{5\sqrt{(bx+a)\,x}\;a^{3}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{5\sqrt{bx+a}\;a^{2}\sqrt{x}}{8}+\frac{5\,(bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{12}+\frac{(bx+a)^{\frac{5}{2}}\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*x^(1/2)+5/12*a*(b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(b*x
+a)^(1/2)+5/16*a^3*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)*ln((b*x+1/2*a)/b
^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)
```

maxima [B] time = 2.99, size = 141, normalized size = 1.53

$$-\frac{5\,a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}+\frac{33\,(bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{3}-\frac{3\,(bx+a)b^{2}}{x}+\frac{3\,(bx+a)^{2}b}{x^{2}}-\frac{(bx+a)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/s
qrt(x)))/sqrt(b) - 1/24*(15*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3
/2)*a^3*b/x^(3/2) + 33*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x + a)*b^2/
x + 3*(b*x + a)^2*b/x^2 - (b*x + a)^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(1/2), x)
```

sympy [A] time = 6.23, size = 102, normalized size = 1.11

$$\frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1+\frac{bx}{a}}}{8}+\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1+\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^{2}x^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}}{3}+\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 11*a**(5/2)*sqrt(x)*sqrt(1 + b*x/a)/8 + 13*a**(3/2)*b*x**(3/2)*sqrt(1 + b*x
/a)/12 + sqrt(a)*b**2*x**(5/2)*sqrt(1 + b*x/a)/3 + 5*a**3*asinh(sqrt(b)*sqr
t(x)/sqrt(a))/(8*sqrt(b))
```

3.549 $\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15}{4}a^2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}(a+bx)^{3/2}+\frac{15}{4}ab\sqrt{x}\sqrt{a+bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*a*b*Sqrt[x]*Sqrt[a + b*x])/4 + (5*b*Sqrt[x]*(a + b*x)^(3/2))/2 - (2*(a
+ b*x)^(5/2))/Sqrt[x] + (15*a^2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{x^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{\sqrt{x}} + (5b)\int \frac{(a+bx)^{3/2}}{\sqrt{x}}\,dx$$
$$= \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}(15ab)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{\right.$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{}}{\sqrt{a}}\right.$$
$$= \frac{15}{4}ab\sqrt{x}\,\sqrt{a+bx} + \frac{5}{2}b\sqrt{x}\,(a+bx)^{3/2} - \frac{2(a+bx)^{5/2}}{\sqrt{x}} + \frac{15}{4}a^2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.54

$$-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{a}\right)}{\sqrt{x}\,\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, -((b*x)/a)])/(Sqrt
[x]*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 0.82

$$\frac{\sqrt{a+bx}\,\left(-8a^2+9abx+2b^2x^2\right)}{4\sqrt{x}} - \frac{15}{4}a^2\sqrt{b}\,\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-8*a^2 + 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[b
]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/4
```

fricas [A] time = 1.39, size = 137, normalized size = 1.54

$$\left[\frac{15\,a^2\sqrt{b}\,x\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^2x^2+9\,abx-8\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{8\,x}, -\frac{15\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x^2+9\,abx-8\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{4\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(b)*x*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(2*b^2*x^2 + 9*a*b*x - 8*a^2)*sqrt(b*x + a)*sqrt(x))/x, -1/4*(15*a^2*sqrt(
-b)*x*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x^2 + 9*a*b*x - 8
*a^2)*sqrt(b*x + a)*sqrt(x))/x]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 84, normalized size = 0.94

$$\frac{15\sqrt{(bx+a)\,x}\;a^2\sqrt{b}\;\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;\left(-2b^2x^2-9abx+8a^2\right)}{4\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(3/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(-2*b^2*x^2-9*a*b*x+8*a^2)/x^(1/2)+15/8*a^2*b^(1/2)*ln((
b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/
2)
```

maxima [A] time = 2.98, size = 125, normalized size = 1.40

$$-\frac{15}{8}\,a^2\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{2\,\sqrt{bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\,\sqrt{bx+a}\,a^2b^2}{\sqrt{x}}-\frac{9\,(bx+a)^{\frac{3}{2}}\,a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx+a)b}{x}+\frac{(bx+a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/8*a^2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*
x + a)/sqrt(x))) - 2*sqrt(b*x + a)*a^2/sqrt(x) - 1/4*(7*sqrt(b*x + a)*a^2*b
^2/sqrt(x) - 9*(b*x + a)^(3/2)*a^2*b/x^(3/2))/(b^2 - 2*(b*x + a)*b/x + (b*x
 + a)^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(3/2), x)
```

sympy [A] time = 6.15, size = 126, normalized size = 1.42

$$-\frac{2a^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}+\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1+\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1+\frac{bx}{a}}}+\frac{15a^2\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(3/2),x)
```

```
[Out] -2*a**(5/2)/(sqrt(x)*sqrt(1 + b*x/a)) + a**(3/2)*b*sqrt(x)/(4*sqrt(1 + b*x/
a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sqrt(1 + b*x/a)) + 15*a**2*sqrt(b)*asinh(
sqrt(b)*sqrt(x)/sqrt(a))/4 + b**3*x**(5/2)/(2*sqrt(a)*sqrt(1 + b*x/a))
```

3.550 $\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=86

$$5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)+5b^2\sqrt{x}\sqrt{a+bx}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$5b^2\sqrt{x}\sqrt{a+bx}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)-\frac{2(a+bx)^{5/2}}{3x^{3/2}}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a + b*x] - (10*b*(a + b*x)^(3/2))/(3*Sqrt[x]) - (2*(a +
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*
x]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(a+bx)^{3/2}}{x^{3/2}}\,dx\\
&= -\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\
&= 5b^2\sqrt{x}\sqrt{a+bx}-\frac{10b(a+bx)^{3/2}}{3\sqrt{x}}-\frac{2(a+bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.58

$$
-\frac{2a^2\sqrt{a+bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{a}\right)}{3x^{3/2}\sqrt{\frac{bx}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a + b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, -((b*x)/a)])/(3*x
^(3/2)*Sqrt[1 + (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.16, size = 69, normalized size = 0.80

$$
\frac{\sqrt{a+bx}\left(-2a^2-14abx+3b^2x^2\right)}{3x^{3/2}}-5ab^{3/2}\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a + b*x]*(-2*a^2 - 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 5*a*b^(3/2)*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
```

fricas [A] time = 1.53, size = 138, normalized size = 1.60

$$
\left[\frac{15\,ab^{\frac{3}{2}}x^2\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{6\,x^2},-\frac{15\,a\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^2x^2-14\,abx-2\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\,x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*b^(3/2)*x^2*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2
*(3*b^2*x^2 - 14*a*b*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2, -1/3*(15*a*sqrt
(-b)*b*x^2*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 14*a*b
*x - 2*a^2)*sqrt(b*x + a)*sqrt(x))/x^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 82, normalized size = 0.95

$$\frac{5\sqrt{(bx+a)\,x}\;a\,b^{\frac{3}{2}}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\;\sqrt{x}}-\frac{\sqrt{bx+a}\;\left(-3b^{2}x^{2}+14abx+2a^{2}\right)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(b*x+a)^(1/2)*(-3*b^2*x^2+14*a*b*x+2*a^2)/x^(3/2)+5/2*a*b^(3/2)*ln((b*
x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.95, size = 100, normalized size = 1.16

$$-\frac{5}{2}\,ab^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)-\frac{4\sqrt{bx+a}\,ab}{\sqrt{x}}-\frac{\sqrt{bx+a}\,ab^{2}}{\left(b-\frac{bx+a}{x}\right)\sqrt{x}}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5/2*a*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x +
 a)/sqrt(x))) - 4*sqrt(b*x + a)*a*b/sqrt(x) - sqrt(b*x + a)*a*b^2/((b - (b*
x + a)/x)*sqrt(x)) - 2/3*(b*x + a)^(3/2)*a/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/x^(5/2), x)
```

sympy [A] time = 5.62, size = 99, normalized size = 1.15

$$-\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3x}-\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3}-\frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx}+1}+1\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] -2*a**2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*x) - 14*a*b**(3/2)*sqrt(a/(b*x) + 1)/3
 - 5*a*b**(3/2)*log(a/(b*x))/2 + 5*a*b**(3/2)*log(sqrt(a/(b*x) + 1) + 1) +
b**(5/2)*x*sqrt(a/(b*x) + 1)
```

3.551 $\int x^{5/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=171

$$\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+$$

Rubi [A] time = 0.06, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2}-\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3}+\frac{5a^6\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{512b^{7/2}}-\frac{a^3x^{5/2}\sqrt{a-bx}}{192b}+\frac{1}{32}a^2x^{7/2}\sqrt{a-bx}+\frac{1}{12}ax^{7/2}(a-bx)^{3/2}+\frac{1}{6}x^{7/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^5*Sqrt[x]*Sqrt[a - b*x])/(512*b^3) - (5*a^4*x^(3/2)*Sqrt[a - b*x])/(7
68*b^2) - (a^3*x^(5/2)*Sqrt[a - b*x])/(192*b) + (a^2*x^(7/2)*Sqrt[a - b*x])
/32 + (a*x^(7/2)*(a - b*x)^(3/2))/12 + (x^(7/2)*(a - b*x)^(5/2))/6 + (5*a^6
*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(512*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(a-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{12}(5a)\int x^{5/2}(a-bx)^{3/2}\,dx \\
&= \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{8}a^2\int x^{5/2}\sqrt{a-bx}\,dx \\
&= \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{1}{64}a^3\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx \\
&= -\frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-bx)^{5/2} + \frac{\left(5a^4\right)\int \frac{x^3}{\sqrt{a\cdot}}}{384b} \\
&= -\frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-bx)^{3/2} + \frac{1}{6}x^{7/2}(a-b \\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-b \\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-b \\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-b \\
&= -\frac{5a^5\sqrt{x}\sqrt{a-bx}}{512b^3} - \frac{5a^4x^{3/2}\sqrt{a-bx}}{768b^2} - \frac{a^3x^{5/2}\sqrt{a-bx}}{192b} + \frac{1}{32}a^2x^{7/2}\sqrt{a-bx} + \frac{1}{12}ax^{7/2}(a-b
\end{aligned}$$

Mathematica [A] time = 0.19, size = 120, normalized size = 0.70

$$\frac{\sqrt{a-bx}\left(\frac{15a^{11/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\sqrt{x}\left(-15a^5-10a^4bx-8a^3b^2x^2+432a^2b^3x^3-640ab^4x^4+256b^5x^5\right)\right)}{1536b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^5 - 10*a^4*b*x - 8*a^3*b^2*x^2 + 432
*a^2*b^3*x^3 - 640*a*b^4*x^4 + 256*b^5*x^5) + (15*a^(11/2)*ArcSin[(Sqrt[b]*
Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(1536*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.30, size = 130, normalized size = 0.76

$$\frac{5a^6\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{512b^4} + \frac{\sqrt{a-bx}\left(-15a^5\sqrt{x}-10a^4bx^{3/2}-8a^3b^2x^{5/2}+432a^2b^3x^{7/2}-640ab^4x^{9/2}+256b^5x^{11/2}\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^5*Sqrt[x] - 10*a^4*b*x^(3/2) - 8*a^3*b^2*x^(5/2) + 43
2*a^2*b^3*x^(7/2) - 640*a*b^4*x^(9/2) + 256*b^5*x^(11/2)))/(1536*b^3) + (5*
a^6*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(512*b^4)
```

fricas [A] time = 1.40, size = 208, normalized size = 1.22

$$\left[-\frac{15\,a^6\sqrt{-b}\log\left(-2\,bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b\right)\sqrt{-bx+a}\sqrt{x}}{3072\,b^4}, -\frac{15\,a^6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(256\,b^6x^5-640\,ab^5x^4+432\,a^2b^4x^3-8\,a^3b^3x^2-10\,a^4b^2x-15\,a^5b\right)\sqrt{-bx+a}\sqrt{x}}{1536\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3072*(15*a^6*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(256*b^6*x^5 - 640*a*b^5*x^4 + 432*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*
a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/1536*(15*a^6*sqrt(b)*
arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (256*b^6*x^5 - 640*a*b^5*x^4 + 4
32*a^2*b^4*x^3 - 8*a^3*b^3*x^2 - 10*a^4*b^2*x - 15*a^5*b)*sqrt(-b*x + a)*sq
rt(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 165, normalized size = 0.96

$$\frac{5\sqrt{(-bx+a)\,x}\,a^{6}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{1024\sqrt{-bx+a}\,b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{-bx+a}\,a^{5}\sqrt{x}}{512b^{3}}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a^{4}\sqrt{x}}{768b^{3}}-\frac{(-bx+a)^{\frac{7}{2}}\,x^{\frac{5}{2}}}{6b}+\frac{(-bx+a)^{\frac{5}{2}}\,a^{3}\sqrt{x}}{192b^{3}}-\frac{(-bx+a)^{\frac{7}{2}}\,a\,x^{\frac{3}{2}}}{12b^{2}}-\frac{(-bx+a)^{\frac{7}{2}}\,a^{2}\sqrt{x}}{32b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/6/b*x^(5/2)*(-b*x+a)^(7/2)-1/12*a/b^2*x^(3/2)*(-b*x+a)^(7/2)-1/32*a^2/b^
3*x^(1/2)*(-b*x+a)^(7/2)+1/192*a^3/b^3*(-b*x+a)^(5/2)*x^(1/2)+5/768*a^4/b^3
*(-b*x+a)^(3/2)*x^(1/2)+5/512*a^5*x^(1/2)*(-b*x+a)^(1/2)/b^3+5/1024*a^6/b^(
7/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a
*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.86, size = 242, normalized size = 1.42

$$-\frac{5\,a^{6}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{512\,b^{\frac{7}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{6}b^{5}}{\sqrt{x}}+\frac{85\,(-bx+a)^{\frac{3}{2}}a^{6}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(-bx+a)^{\frac{5}{2}}a^{6}b^{3}}{x^{\frac{5}{2}}}-\frac{198\,(-bx+a)^{\frac{7}{2}}a^{6}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(-bx+a)^{\frac{9}{2}}a^{6}b}{x^{\frac{9}{2}}}-\frac{15\,(-bx+a)^{\frac{11}{2}}a^{6}}{x^{\frac{11}{2}}}}{1536\left(b^{9}-\frac{6\,(bx-a)b^{8}}{x}+\frac{15\,(bx-a)^{2}b^{7}}{x^{2}}-\frac{20\,(bx-a)^{3}b^{6}}{x^{3}}+\frac{15\,(bx-a)^{4}b^{5}}{x^{4}}-\frac{6\,(bx-a)^{5}b^{4}}{x^{5}}+\frac{(bx-a)^{6}b^{3}}{x^{6}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -5/512*a^6*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) + 1/1536*(15*sq
rt(-b*x + a)*a^6*b^5/sqrt(x) + 85*(-b*x + a)^(3/2)*a^6*b^4/x^(3/2) + 198*(-
b*x + a)^(5/2)*a^6*b^3/x^(5/2) - 198*(-b*x + a)^(7/2)*a^6*b^2/x^(7/2) - 85*
(-b*x + a)^(9/2)*a^6*b/x^(9/2) - 15*(-b*x + a)^(11/2)*a^6/x^(11/2))/(b^9 -
6*(b*x - a)*b^8/x + 15*(b*x - a)^2*b^7/x^2 - 20*(b*x - a)^3*b^6/x^3 + 15*(b
*x - a)^4*b^5/x^4 - 6*(b*x - a)^5*b^4/x^5 + (b*x - a)^6*b^3/x^6)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)*(a - b*x)^(5/2), x)
```

sympy [A] time = 25.96, size = 435, normalized size = 2.54

$$\begin{cases} \frac{5ia^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{-1+\frac{bx}{a}}} - \frac{ia^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{-1+\frac{bx}{a}}} - \frac{55ia^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{-1+\frac{bx}{a}}} + \frac{67ia^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{-1+\frac{bx}{a}}} - \frac{7i\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{-1+\frac{bx}{a}}} - \frac{5ia^6\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} + \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ -\frac{5a^{\frac{11}{2}}\sqrt{x}}{512b^3\sqrt{1-\frac{bx}{a}}} + \frac{5a^{\frac{9}{2}}x^{\frac{3}{2}}}{1536b^2\sqrt{1-\frac{bx}{a}}} + \frac{a^{\frac{7}{2}}x^{\frac{5}{2}}}{768b\sqrt{1-\frac{bx}{a}}} + \frac{55a^{\frac{5}{2}}x^{\frac{7}{2}}}{192\sqrt{1-\frac{bx}{a}}} - \frac{67a^{\frac{3}{2}}bx^{\frac{9}{2}}}{96\sqrt{1-\frac{bx}{a}}} + \frac{7\sqrt{a}\,b^2x^{\frac{11}{2}}}{12\sqrt{1-\frac{bx}{a}}} + \frac{5a^6\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{512b^{\frac{7}{2}}} - \frac{b^3x^{\frac{13}{2}}}{6\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((5*I*a**(11/2)*sqrt(x)/(512*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(9/2)
*x**(3/2)/(1536*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x**(5/2)/(768*b*sqrt(-1
 + b*x/a)) - 55*I*a**(5/2)*x**(7/2)/(192*sqrt(-1 + b*x/a)) + 67*I*a**(3/2)*
b*x**(9/2)/(96*sqrt(-1 + b*x/a)) - 7*I*sqrt(a)*b**2*x**(11/2)/(12*sqrt(-1 +
 b*x/a)) - 5*I*a**6*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(512*b**(7/2)) + I*b**3*
x**(13/2)/(6*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(11/2)*sqrt
(x)/(512*b**3*sqrt(1 - b*x/a)) + 5*a**(9/2)*x**(3/2)/(1536*b**2*sqrt(1 - b*
x/a)) + a**(7/2)*x**(5/2)/(768*b*sqrt(1 - b*x/a)) + 55*a**(5/2)*x**(7/2)/(1
92*sqrt(1 - b*x/a)) - 67*a**(3/2)*b*x**(9/2)/(96*sqrt(1 - b*x/a)) + 7*sqrt(
a)*b**2*x**(11/2)/(12*sqrt(1 - b*x/a)) + 5*a**6*asin(sqrt(b)*sqrt(x)/sqrt(a
))/(512*b**(7/2)) - b**3*x**(13/2)/(6*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.552 $\int x^{3/2}(a-bx)^{5/2}\,dx$

Optimal. Leaf size=146

$$\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2}+\frac{3a^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{128b^{5/2}}-\frac{a^3x^{3/2}\sqrt{a-bx}}{64b}+\frac{1}{16}a^2x^{5/2}\sqrt{a-bx}+\frac{1}{8}ax^{5/2}(a-bx)^{3/2}+\frac{1}{5}x^{5/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (-3*a^4*Sqrt[x]*Sqrt[a - b*x])/(128*b^2) - (a^3*x^(3/2)*Sqrt[a - b*x])/(64*
b) + (a^2*x^(5/2)*Sqrt[a - b*x])/16 + (a*x^(5/2)*(a - b*x)^(3/2))/8 + (x^(5
/2)*(a - b*x)^(5/2))/5 + (3*a^5*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(1
28*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int x^{3/2}(a-bx)^{5/2}\,dx = \frac{1}{5}x^{5/2}(a-bx)^{5/2} + \frac{1}{2}a\int x^{3/2}(a-bx)^{3/2}\,dx$$
$$= \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx)^{5/2} + \frac{1}{16}\left(3a^2\right)\int x^{3/2}\sqrt{a-bx}\,dx$$
$$= \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx)^{5/2} + \frac{1}{32}a^3\int\frac{x^{3/2}}{\sqrt{a-bx}}\,dx$$
$$= -\frac{a^3x^{3/2}\sqrt{a-bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx)^{5/2} + \frac{\left(3a^4\right)\int\frac{\sqrt{x}}{\sqrt{a-b}}}{128b}$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx$$
$$= -\frac{3a^4\sqrt{x}\sqrt{a-bx}}{128b^2} - \frac{a^3x^{3/2}\sqrt{a-bx}}{64b} + \frac{1}{16}a^2x^{5/2}\sqrt{a-bx} + \frac{1}{8}ax^{5/2}(a-bx)^{3/2} + \frac{1}{5}x^{5/2}(a-bx$$

Mathematica [A] time = 0.15, size = 109, normalized size = 0.75

$$\frac{\sqrt{a-bx}\left(\frac{15a^{9/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}+\sqrt{b}\sqrt{x}\left(-15a^4-10a^3bx+248a^2b^2x^2-336ab^3x^3+128b^4x^4\right)\right)}{640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^4 - 10*a^3*b*x + 248*a^2*b^2*x^2 - 3
36*a*b^3*x^3 + 128*b^4*x^4) + (15*a^(9/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]
)/Sqrt[1 - (b*x)/a]))/(640*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.18, size = 117, normalized size = 0.80

$$\frac{3a^5\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{128b^3}+\frac{\sqrt{a-bx}\left(-15a^4\sqrt{x}-10a^3bx^{3/2}+248a^2b^2x^{5/2}-336ab^3x^{7/2}+128b^4x^{9/2}\right)}{640b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^4*Sqrt[x] - 10*a^3*b*x^(3/2) + 248*a^2*b^2*x^(5/2) -
336*a*b^3*x^(7/2) + 128*b^4*x^(9/2)))/(640*b^2) + (3*a^5*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(128*b^3)
```

fricas [A] time = 1.13, size = 186, normalized size = 1.27

$$\left[-\frac{15\,a^5\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2-10\,a^3b^2x-15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{1280\,b^3},\ -\frac{15\,a^5\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(128\,b^5x^4-336\,ab^4x^3+248\,a^2b^3x^2-10\,a^3b^2x-15\,a^4b\right)\sqrt{-bx+a}\,\sqrt{x}}{640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*a^5*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) - 2*(128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 - 10*a^3*b^2*x - 15*a
^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/640*(15*a^5*sqrt(b)*arctan(sqrt(-b*x
+ a)/(sqrt(b)*sqrt(x))) - (128*b^5*x^4 - 336*a*b^4*x^3 + 248*a^2*b^3*x^2 -
10*a^3*b^2*x - 15*a^4*b)*sqrt(-b*x + a)*sqrt(x))/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 146, normalized size = 1.00

$$\frac{3\sqrt{(-bx+a)\,x}\;a^5\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{256\sqrt{-bx+a}\;b^{\frac{5}{2}}\sqrt{x}}+\frac{3\sqrt{-bx+a}\;a^4\sqrt{x}}{128b^2}+\frac{(-bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}{64b^2}+\frac{(-bx+a)^{\frac{5}{2}}\,a^2\sqrt{x}}{80b^2}-\frac{(-bx+a)^{\frac{7}{2}}\,x^{\frac{3}{2}}}{5b}-\frac{3\,(-bx+a)^{\frac{7}{2}}\,a\sqrt{x}}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+a)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+a)^(7/2)-3/40*a/b^2*x^(1/2)*(-b*x+a)^(7/2)+1/80*a^2/b^
2*(-b*x+a)^(5/2)*x^(1/2)+1/64*a^3/b^2*(-b*x+a)^(3/2)*x^(1/2)+3/128*a^4*x^(1
/2)*(-b*x+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/
x^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 3.01, size = 207, normalized size = 1.42

$$-\frac{3\,a^5\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{128\,b^{\frac{5}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^5b^4}{\sqrt{x}}+\frac{70\,(-bx+a)^{\frac{3}{2}}\,a^5b^3}{x^{\frac{3}{2}}}+\frac{128\,(-bx+a)^{\frac{5}{2}}\,a^5b^2}{x^{\frac{5}{2}}}-\frac{70\,(-bx+a)^{\frac{7}{2}}\,a^5b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+a)^{\frac{9}{2}}\,a^5}{x^{\frac{9}{2}}}}{640\left(b^7-\frac{5\,(bx-a)b^6}{x}+\frac{10\,(bx-a)^2b^5}{x^2}-\frac{10\,(bx-a)^3b^4}{x^3}+\frac{5\,(bx-a)^4b^3}{x^4}-\frac{(bx-a)^5b^2}{x^5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/128*a^5*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) + 1/640*(15*sqr
t(-b*x + a)*a^5*b^4/sqrt(x) + 70*(-b*x + a)^(3/2)*a^5*b^3/x^(3/2) + 128*(-b
*x + a)^(5/2)*a^5*b^2/x^(5/2) - 70*(-b*x + a)^(7/2)*a^5*b/x^(7/2) - 15*(-b*
x + a)^(9/2)*a^5/x^(9/2))/(b^7 - 5*(b*x - a)*b^6/x + 10*(b*x - a)^2*b^5/x^2
 - 10*(b*x - a)^3*b^4/x^3 + 5*(b*x - a)^4*b^3/x^4 - (b*x - a)^5*b^2/x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(a - b*x)^(5/2), x)
```

sympy [A] time = 16.40, size = 379, normalized size = 2.60

$$\begin{cases} \frac{3ia^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{-1+\frac{bx}{a}}}-\frac{ia^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{-1+\frac{bx}{a}}}-\frac{129ia^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{-1+\frac{bx}{a}}}+\frac{73ia^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{-1+\frac{bx}{a}}}-\frac{29i\sqrt{a}\,b^2x^{\frac{9}{2}}}{40\sqrt{-1+\frac{bx}{a}}}-\frac{3ia^5\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{ib^3x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{3a^{\frac{9}{2}}\sqrt{x}}{128b^2\sqrt{1-\frac{bx}{a}}}+\frac{a^{\frac{7}{2}}x^{\frac{3}{2}}}{128b\sqrt{1-\frac{bx}{a}}}+\frac{129a^{\frac{5}{2}}x^{\frac{5}{2}}}{320\sqrt{1-\frac{bx}{a}}}-\frac{73a^{\frac{3}{2}}bx^{\frac{7}{2}}}{80\sqrt{1-\frac{bx}{a}}}+\frac{29\sqrt{a}\,b^2x^{\frac{9}{2}}}{40\sqrt{1-\frac{bx}{a}}}+\frac{3a^5\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}-\frac{b^3x^{\frac{11}{2}}}{5\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((3*I*a**(9/2)*sqrt(x)/(128*b**2*sqrt(-1 + b*x/a)) - I*a**(7/2)*x*
*(3/2)/(128*b*sqrt(-1 + b*x/a)) - 129*I*a**(5/2)*x**(5/2)/(320*sqrt(-1 + b*
x/a)) + 73*I*a**(3/2)*b*x**(7/2)/(80*sqrt(-1 + b*x/a)) - 29*I*sqrt(a)*b**2*
x**(9/2)/(40*sqrt(-1 + b*x/a)) - 3*I*a**5*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(1
28*b**(5/2)) + I*b**3*x**(11/2)/(5*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) >
1), (-3*a**(9/2)*sqrt(x)/(128*b**2*sqrt(1 - b*x/a)) + a**(7/2)*x**(3/2)/(12
8*b*sqrt(1 - b*x/a)) + 129*a**(5/2)*x**(5/2)/(320*sqrt(1 - b*x/a)) - 73*a**
(3/2)*b*x**(7/2)/(80*sqrt(1 - b*x/a)) + 29*sqrt(a)*b**2*x**(9/2)/(40*sqrt(1
 - b*x/a)) + 3*a**5*asin(sqrt(b)*sqrt(x)/sqrt(a))/(128*b**(5/2)) - b**3*x**
(11/2)/(5*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.553 $\int \sqrt{x}\,(a-bx)^{5/2}\,dx$

Optimal. Leaf size=121

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{5a^4\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{64b^{3/2}}+\frac{5}{32}a^2x^{3/2}\sqrt{a-bx}-\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b}+\frac{5}{24}ax^{3/2}(a-bx)^{3/2}+\frac{1}{4}x^{3/2}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (-5*a^3*Sqrt[x]*Sqrt[a - b*x])/(64*b) + (5*a^2*x^(3/2)*Sqrt[a - b*x])/32 +
(5*a*x^(3/2)*(a - b*x)^(3/2))/24 + (x^(3/2)*(a - b*x)^(5/2))/4 + (5*a^4*Arc
Tan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(64*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{x}\,(a-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{8}(5a)\int \sqrt{x}\,(a-bx)^{3/2}\,dx \\
&= \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{16}\left(5a^2\right)\int \sqrt{x}\,\sqrt{a-bx}\,dx \\
&= \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{1}{64}\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx \\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{\left(5a^4\right)\int \cdots}{12\cdots} \\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{\left(5a^4\right)\text{Sub}\cdots}{\cdots} \\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{\left(5a^4\right)\text{Sub}\cdots}{\cdots} \\
&= -\frac{5a^3\sqrt{x}\,\sqrt{a-bx}}{64b} + \frac{5}{32}a^2x^{3/2}\sqrt{a-bx} + \frac{5}{24}ax^{3/2}(a-bx)^{3/2} + \frac{1}{4}x^{3/2}(a-bx)^{5/2} + \frac{5a^4\tan^{-1}\cdots}{64b\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.13, size = 98, normalized size = 0.81

$$\frac{\sqrt{a-bx}\left(\frac{15a^{7/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}} + \sqrt{b}\,\sqrt{x}\left(-15a^3+118a^2bx-136ab^2x^2+48b^3x^3\right)\right)}{192b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[b]*Sqrt[x]*(-15*a^3 + 118*a^2*b*x - 136*a*b^2*x^2 + 48
*b^3*x^3) + (15*a^(7/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a
]))/(192*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.16, size = 104, normalized size = 0.86

$$\frac{5a^4\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{64b^2} + \frac{\sqrt{a-bx}\left(-15a^3\sqrt{x}+118a^2bx^{3/2}-136ab^2x^{5/2}+48b^3x^{7/2}\right)}{192b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^3*Sqrt[x] + 118*a^2*b*x^(3/2) - 136*a*b^2*x^(5/2) + 4
8*b^3*x^(7/2)))/(192*b) + (5*a^4*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a
- b*x]])/(64*b^2)
```

fricas [A] time = 1.25, size = 164, normalized size = 1.36

$$\left[-\frac{15\,a^4\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\left(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{384\,b^2},\ -\frac{15\,a^4\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(48\,b^4x^3-136\,ab^3x^2+118\,a^2b^2x-15\,a^3b\right)\sqrt{-bx+a}\,\sqrt{x}}{192\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/384*(15*a^4*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a
) - 2*(48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x + a
```

```
)*sqrt(x))/b^2, -1/192*(15*a^4*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(
x))) - (48*b^4*x^3 - 136*a*b^3*x^2 + 118*a^2*b^2*x - 15*a^3*b)*sqrt(-b*x +
a)*sqrt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.01, size = 118, normalized size = 0.98

$$\frac{5\sqrt{-bx+a}\,a^2x^{\frac{3}{2}}}{32}+\frac{5\sqrt{(-bx+a)\,x}\,a^4\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{128\sqrt{-bx+a}\,b^{\frac{3}{2}}\sqrt{x}}-\frac{5\sqrt{-bx+a}\,a^3\sqrt{x}}{64b}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a\,x^{\frac{3}{2}}}{24}+\frac{(-bx+a)^{\frac{5}{2}}\,x^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)*x^(1/2),x)
[Out] 1/4*x^(3/2)*(-b*x+a)^(5/2)+5/24*a*x^(3/2)*(-b*x+a)^(3/2)+5/32*a^2*x^(3/2)*(
-b*x+a)^(1/2)-5/64*a^3*x^(1/2)*(-b*x+a)^(1/2)/b+5/128*a^4/b^(3/2)*((-b*x+a)
*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1
/2))
```

maxima [A] time = 3.05, size = 168, normalized size = 1.39

$$-\frac{5\,a^4\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{64\,b^{\frac{3}{2}}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^4b^3}{\sqrt{x}}+\frac{55\,(-bx+a)^{\frac{3}{2}}a^4b^2}{x^{\frac{3}{2}}}+\frac{73\,(-bx+a)^{\frac{5}{2}}a^4b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+a)^{\frac{7}{2}}a^4}{x^{\frac{7}{2}}}}{192\left(b^5-\frac{4\,(bx-a)b^4}{x}+\frac{6\,(bx-a)^2b^3}{x^2}-\frac{4\,(bx-a)^3b^2}{x^3}+\frac{(bx-a)^4b}{x^4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)*x^(1/2),x, algorithm="maxima")
[Out] -5/64*a^4*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 1/192*(15*sqrt
(-b*x + a)*a^4*b^3/sqrt(x) + 55*(-b*x + a)^(3/2)*a^4*b^2/x^(3/2) + 73*(-b*x
 + a)^(5/2)*a^4*b/x^(5/2) - 15*(-b*x + a)^(7/2)*a^4/x^(7/2))/(b^5 - 4*(b*x
- a)*b^4/x + 6*(b*x - a)^2*b^3/x^2 - 4*(b*x - a)^3*b^2/x^3 + (b*x - a)^4*b/
x^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(a-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(a - b*x)^(5/2),x)
[Out] int(x^(1/2)*(a - b*x)^(5/2), x)
```

sympy [A] time = 9.81, size = 326, normalized size = 2.69

$$\begin{cases} \frac{5ia^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{-1+\frac{bx}{a}}}-\frac{133ia^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{-1+\frac{bx}{a}}}+\frac{127ia^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{-1+\frac{bx}{a}}}-\frac{23i\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^4\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}+\frac{ib^3x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{7}{2}}\sqrt{x}}{64b\sqrt{1-\frac{bx}{a}}}+\frac{133a^{\frac{5}{2}}x^{\frac{3}{2}}}{192\sqrt{1-\frac{bx}{a}}}-\frac{127a^{\frac{3}{2}}bx^{\frac{5}{2}}}{96\sqrt{1-\frac{bx}{a}}}+\frac{23\sqrt{a}\,b^2x^{\frac{7}{2}}}{24\sqrt{1-\frac{bx}{a}}}+\frac{5a^4\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{64b^{\frac{3}{2}}}-\frac{b^3x^{\frac{9}{2}}}{4\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(7/2)*sqrt(x)/(64*b*sqrt(-1 + b*x/a)) - 133*I*a**(5/2)*x*
*(3/2)/(192*sqrt(-1 + b*x/a)) + 127*I*a**(3/2)*b*x**(5/2)/(96*sqrt(-1 + b*x
/a)) - 23*I*sqrt(a)*b**2*x**(7/2)/(24*sqrt(-1 + b*x/a)) - 5*I*a**4*acosh(sq
rt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) + I*b**3*x**(9/2)/(4*sqrt(a)*sqrt(-1 +
 b*x/a)), Abs(b*x/a) > 1), (-5*a**(7/2)*sqrt(x)/(64*b*sqrt(1 - b*x/a)) + 13
3*a**(5/2)*x**(3/2)/(192*sqrt(1 - b*x/a)) - 127*a**(3/2)*b*x**(5/2)/(96*sqr
t(1 - b*x/a)) + 23*sqrt(a)*b**2*x**(7/2)/(24*sqrt(1 - b*x/a)) + 5*a**4*asin
(sqrt(b)*sqrt(x)/sqrt(a))/(64*b**(3/2)) - b**3*x**(9/2)/(4*sqrt(a)*sqrt(1 -
 b*x/a)), True))
```

3.554 $\int \frac{(a-bx)^{5/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=96

$$\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{5}{8}a^2\sqrt{x}\sqrt{a-bx}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}+\frac{5}{12}a\sqrt{x}(a-bx)^{3/2}+\frac{1}{3}\sqrt{x}(a-bx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a - b*x])/8 + (5*a*Sqrt[x]*(a - b*x)^(3/2))/12 + (Sqrt[
x]*(a - b*x)^(5/2))/3 + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(8*
Sqrt[b])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx)^{5/2}}{\sqrt{x}}\,dx = \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{1}{6}(5a)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx$$

$$= \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{1}{8}\left(5a^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx} + \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{1}{16}\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx} + \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \right.$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx} + \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{1}{8}\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \right.$$

$$= \frac{5}{8}a^2\sqrt{x}\,\sqrt{a-bx} + \frac{5}{12}a\sqrt{x}\,(a-bx)^{3/2} + \frac{1}{3}\sqrt{x}\,(a-bx)^{5/2} + \frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8\sqrt{b}}$$

Mathematica [A] time = 0.12, size = 82, normalized size = 0.85

$$\frac{1}{24}\sqrt{a-bx}\left(\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{1-\frac{bx}{a}}} + \sqrt{x}\left(33a^2-26abx+8b^2x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(Sqrt[x]*(33*a^2 - 26*a*b*x + 8*b^2*x^2) + (15*a^(5/2)*ArcSi
n[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 - (b*x)/a])))/24
```

IntegrateAlgebraic [A] time = 0.13, size = 88, normalized size = 0.92

$$\frac{5a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{8b} + \frac{1}{24}\sqrt{a-bx}\left(33a^2\sqrt{x}-26abx^{3/2}+8b^2x^{5/2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[a - b*x]*(33*a^2*Sqrt[x] - 26*a*b*x^(3/2) + 8*b^2*x^(5/2)))/24 + (5*a
^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b)
```

fricas [A] time = 0.69, size = 142, normalized size = 1.48

$$\left[-\frac{15\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)-2\,\left(8\,b^3x^2-26\,ab^2x+33\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b}, -\frac{15\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(8\,b^3x^2-26\,ab^2x+33\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 - 2*(8*b^3*x^2 - 26*a*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b, -1/24*(
15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (8*b^3*x^2 - 26*a
*b^2*x + 33*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 99, normalized size = 1.03

$$\frac{5\sqrt{(-bx+a)\,x}\;a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}+\frac{5\sqrt{-bx+a}\;a^{2}\sqrt{x}}{8}+\frac{5\,(-bx+a)^{\frac{3}{2}}\,a\sqrt{x}}{12}+\frac{(-bx+a)^{\frac{5}{2}}\,\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+a)^(5/2)*x^(1/2)+5/12*a*(-b*x+a)^(3/2)*x^(1/2)+5/8*a^2*x^(1/2)*(-
b*x+a)^(1/2)+5/16*a^3*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)/b^(1/2)*arc
tan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.93, size = 130, normalized size = 1.35

$$-\frac{5\,a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,\sqrt{b}}+\frac{\frac{15\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}+\frac{33\,(-bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{3}-\frac{3\,(bx-a)b^{2}}{x}+\frac{3\,(bx-a)^{2}b}{x^{2}}-\frac{(bx-a)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/24*(15*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 33*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^3 - 3*(b*x - a)*b^2/x + 3*(b*x - a)^2*b/x^2 - (b*x
- a)^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(1/2), x)
```

sympy [A] time = 6.23, size = 246, normalized size = 2.56

$$\begin{cases} -\frac{11ia^{\frac{5}{2}}\sqrt{x}}{8\sqrt{-1+\frac{bx}{a}}}+\frac{59ia^{\frac{3}{2}}bx^{\frac{3}{2}}}{24\sqrt{-1+\frac{bx}{a}}}-\frac{17i\sqrt{a}\,b^{2}x^{\frac{5}{2}}}{12\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{ib^{3}x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{11a^{\frac{5}{2}}\sqrt{x}\sqrt{1-\frac{bx}{a}}}{8}-\frac{13a^{\frac{3}{2}}bx^{\frac{3}{2}}\sqrt{1-\frac{bx}{a}}}{12}+\frac{\sqrt{a}\,b^{2}x^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}}{3}+\frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((-11*I*a**(5/2)*sqrt(x)/(8*sqrt(-1 + b*x/a)) + 59*I*a**(3/2)*b*x*
*(3/2)/(24*sqrt(-1 + b*x/a)) - 17*I*sqrt(a)*b**2*x**(5/2)/(12*sqrt(-1 + b*x
```

```
/a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)) + I*b**3*x**(7/2
)/(3*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (11*a**(5/2)*sqrt(x)*sqrt(
1 - b*x/a)/8 - 13*a**(3/2)*b*x**(3/2)*sqrt(1 - b*x/a)/12 + sqrt(a)*b**2*x**
(5/2)*sqrt(1 - b*x/a)/3 + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*sqrt(b)),
 True))
```

3.555 $\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}$$

Rubi [A] time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15}{4}a^2\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{15}{4}ab\sqrt{x}\sqrt{a-bx}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*a*b*Sqrt[x]*Sqrt[a - b*x])/4 - (5*b*Sqrt[x]*(a - b*x)^(3/2))/2 - (2*(a
 - b*x)^(5/2))/Sqrt[x] - (15*a^2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a -
b*x]])/4
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{\sqrt{x}}-(5b)\int \frac{(a-bx)^{3/2}}{\sqrt{x}}\,dx\\
&= -\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}(15ab)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{8}\left(15a^2b\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\cdot\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{1}{4}\left(15a^2b\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\cdot}{\sqrt{a}}\right.\\
&= -\frac{15}{4}ab\sqrt{x}\,\sqrt{a-bx}-\frac{5}{2}b\sqrt{x}\,(a-bx)^{3/2}-\frac{2(a-bx)^{5/2}}{\sqrt{x}}-\frac{15}{4}a^2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.53

$$
-\frac{2a^2\sqrt{a-bx}\;{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{a}\right)}{\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/a])/(Sqrt[x]
*Sqrt[1 - (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.16, size = 79, normalized size = 0.85

$$
\frac{\sqrt{a-bx}\,\left(-8a^2-9abx+2b^2x^2\right)}{4\sqrt{x}}-\frac{15}{4}a^2\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-8*a^2 - 9*a*b*x + 2*b^2*x^2))/(4*Sqrt[x]) - (15*a^2*Sqrt[-
b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/4
```

fricas [A] time = 1.30, size = 137, normalized size = 1.47

$$
\left[\frac{15\,a^2\sqrt{-b}\,x\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(2\,b^2x^2-9\,abx-8\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,x},\ \frac{15\,a^2\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^2x^2-9\,abx-8\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,x}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*a^2*sqrt(-b)*x*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(2*b^2*x^2 - 9*a*b*x - 8*a^2)*sqrt(-b*x + a)*sqrt(x))/x, 1/4*(15*a^2*s
qrt(b)*x*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x^2 - 9*a*b*x -
8*a^2)*sqrt(-b*x + a)*sqrt(x))/x]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{5}{2}}}{x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(3/2),x)
[Out] int((-b*x+a)^(5/2)/x^(3/2),x)
```

maxima [A] time = 2.99, size = 112, normalized size = 1.20

$$\frac{15}{4}\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{2\,\sqrt{-bx+a}\,a^2}{\sqrt{x}}-\frac{\frac{7\,\sqrt{-bx+a}\,a^2b^2}{\sqrt{x}}+\frac{9\,(-bx+a)^{\frac{3}{2}}\,a^2b}{x^{\frac{3}{2}}}}{4\left(b^2-\frac{2\,(bx-a)b}{x}+\frac{(bx-a)^2}{x^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(3/2),x, algorithm="maxima")
[Out] 15/4*a^2*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - 2*sqrt(-b*x + a
)*a^2/sqrt(x) - 1/4*(7*sqrt(-b*x + a)*a^2*b^2/sqrt(x) + 9*(-b*x + a)^(3/2)*
a^2*b/x^(3/2))/(b^2 - 2*(b*x - a)*b/x + (b*x - a)^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(3/2),x)
[Out] int((a - b*x)^(5/2)/x^(3/2), x)
```

sympy [A] time = 6.22, size = 267, normalized size = 2.87

$$\begin{cases} \frac{2ia^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{-1+\frac{bx}{a}}}+\frac{ia^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{-1+\frac{bx}{a}}}-\frac{11i\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{-1+\frac{bx}{a}}}+\frac{15ia^2\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}+\frac{ib^3x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{2a^{\frac{5}{2}}}{\sqrt{x}\,\sqrt{1-\frac{bx}{a}}}-\frac{a^{\frac{3}{2}}b\sqrt{x}}{4\sqrt{1-\frac{bx}{a}}}+\frac{11\sqrt{a}\,b^2x^{\frac{3}{2}}}{4\sqrt{1-\frac{bx}{a}}}-\frac{15a^2\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4}-\frac{b^3x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(3/2),x)
[Out] Piecewise((2*I*a**(5/2)/(sqrt(x)*sqrt(-1 + b*x/a)) + I*a**(3/2)*b*sqrt(x)/(
4*sqrt(-1 + b*x/a)) - 11*I*sqrt(a)*b**2*x**(3/2)/(4*sqrt(-1 + b*x/a)) + 15*
```

```
I*a**2*sqrt(b)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/4 + I*b**3*x**(5/2)/(2*sqrt(a
)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*a**(5/2)/(sqrt(x)*sqrt(1 - b*x/a)
) - a**(3/2)*b*sqrt(x)/(4*sqrt(1 - b*x/a)) + 11*sqrt(a)*b**2*x**(3/2)/(4*sq
rt(1 - b*x/a)) - 15*a**2*sqrt(b)*asin(sqrt(b)*sqrt(x)/sqrt(a))/4 - b**3*x**
(5/2)/(2*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.556 $\int \frac{(a-bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=90

$$5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)+5b^2\sqrt{x}\sqrt{a-bx}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$5b^2\sqrt{x}\sqrt{a-bx}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[a - b*x] + (10*b*(a - b*x)^(3/2))/(3*Sqrt[x]) - (2*(a -
b*x)^(5/2))/(3*x^(3/2)) + 5*a*b^(3/2)*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x
]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(a-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(a-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{a-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\frac{1}{2}\left(5ab^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+\left(5ab^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\
&= 5b^2\sqrt{x}\sqrt{a-bx}+\frac{10b(a-bx)^{3/2}}{3\sqrt{x}}-\frac{2(a-bx)^{5/2}}{3x^{3/2}}+5ab^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.57

$$-\frac{2a^2\sqrt{a-bx}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{a}\right)}{3x^{3/2}\sqrt{1-\frac{bx}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-2*a^2*Sqrt[a - b*x]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/a])/(3*x^(3
/2)*Sqrt[1 - (b*x)/a])
```

IntegrateAlgebraic [A] time = 0.19, size = 76, normalized size = 0.84

$$\frac{\sqrt{a-bx}\left(-2a^2+14abx+3b^2x^2\right)}{3x^{3/2}}+5a\sqrt{-b}\,b\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-2*a^2 + 14*a*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 5*a*Sqrt[-b]*
b*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]]
```

fricas [A] time = 1.33, size = 139, normalized size = 1.54

$$\left[\frac{15\,a\sqrt{-b}\,bx^2\log\left(-2\,bx-2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(3\,b^2x^2+14\,abx-2\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{6\,x^2},\ -\frac{15\,ab^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)-\left(3\,b^2x^2+14\,abx-2\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\,x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a*sqrt(-b)*b*x^2*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) +
a) + 2*(3*b^2*x^2 + 14*a*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2, -1/3*(15
*a*b^(3/2)*x^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 14*a
*b*x - 2*a^2)*sqrt(-b*x + a)*sqrt(x))/x^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(-bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+a)^(5/2)/x^(5/2),x)
```

```
[Out] int((-b*x+a)^(5/2)/x^(5/2),x)
```

maxima [A] time = 3.00, size = 84, normalized size = 0.93

$$-5\,ab^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\frac{4\,\sqrt{-bx+a}\,ab}{\sqrt{x}}+\frac{\sqrt{-bx+a}\,ab^{2}}{\left(b-\frac{bx-a}{x}\right)\sqrt{x}}-\frac{2\,(-bx+a)^{\frac{3}{2}}a}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*a*b^(3/2)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + 4*sqrt(-b*x + a)*a*
b/sqrt(x) + sqrt(-b*x + a)*a*b^2/((b - (b*x - a)/x)*sqrt(x)) - 2/3*(-b*x +
a)^(3/2)*a/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((a - b*x)^(5/2)/x^(5/2), x)
```

sympy [C] time = 5.85, size = 245, normalized size = 2.72

$$\begin{cases} -\frac{2a^{2}\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3x}+\frac{14ab^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3}-5iab^{\frac{3}{2}}\log\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}+5ab^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+b^{\frac{5}{2}}x\sqrt{\frac{a}{bx}-1} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2ia^{2}\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{3x}+\frac{14iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{3}+\frac{5iab^{\frac{3}{2}}\log\left(\frac{a}{bx}\right)}{2}-5iab^{\frac{3}{2}}\log\left(\sqrt{-\frac{a}{bx}+1}+1\right)+ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+a)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*x) + 14*a*b**(3/2)*sqrt(a/(
b*x) - 1)/3 - 5*I*a*b**(3/2)*log(sqrt(a)/(sqrt(b)*sqrt(x))) + 5*I*a*b**(3/2
)*log(a/(b*x))/2 + 5*a*b**(3/2)*asin(sqrt(b)*sqrt(x)/sqrt(a)) + b**(5/2)*x*
sqrt(a/(b*x) - 1), Abs(a/(b*x)) > 1), (-2*I*a**2*sqrt(b)*sqrt(-a/(b*x) + 1)
/(3*x) + 14*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/3 + 5*I*a*b**(3/2)*log(a/(b*x))
/2 - 5*I*a*b**(3/2)*log(sqrt(-a/(b*x) + 1) + 1) + I*b**(5/2)*x*sqrt(-a/(b*x
) + 1), True))
```

3.557 $\int x^{5/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=144

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

Rubi [A] time = 0.05, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{48b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{16b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(bx+2)^{5/2}+\frac{1}{6}x^{7/2}(bx+2)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{bx+2}+\frac{x^{5/2}\sqrt{bx+2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(48*b^2) + (
x^(5/2)*Sqrt[2 + b*x])/(24*b) + (x^(7/2)*Sqrt[2 + b*x])/8 + (x^(7/2)*(2 + b
*x)^(3/2))/6 + (x^(7/2)*(2 + b*x)^(5/2))/6 - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{5/2}(2+bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{5}{6}\int x^{5/2}(2+bx)^{3/2}\,dx \\
&= \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{2}\int x^{5/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} + \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} - \frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{24b} \\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}x^{7/2}(2+bx)^{5/2} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6} \\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2+bx}}{48b^2} + \frac{x^{5/2}\sqrt{2+bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2+bx} + \frac{1}{6}x^{7/2}(2+bx)^{3/2} + \frac{1}{6}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.60

$$\frac{\sqrt{x}\sqrt{bx+2}\left(8b^5x^5+40b^4x^4+54b^3x^3+2b^2x^2-5bx+15\right)}{48b^3} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2 + 54*b^3*x^3 + 40*b^4*x^4 +
8*b^5*x^5))/(48*b^3) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 105, normalized size = 0.73

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{8b^{7/2}} + \frac{\sqrt{bx+2}\left(8b^5x^{11/2}+40b^4x^{9/2}+54b^3x^{7/2}+2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{48b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2) + 54*b^3*x^(7/2) +
 40*b^4*x^(9/2) + 8*b^5*x^(11/2)))/(48*b^3) + (5*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[2 + b*x]])/(8*b^(7/2))
```

fricas [A] time = 1.29, size = 172, normalized size = 1.19

$$\left[\frac{\left(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{48\,b^4}, \frac{\left(8\,b^6x^5+40\,b^5x^4+54\,b^4x^3+2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x + 15*b)*s
qrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) +
```

```
1))/b^4, 1/48*((8*b^6*x^5 + 40*b^5*x^4 + 54*b^4*x^3 + 2*b^3*x^2 - 5*b^2*x
+ 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739
918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2
]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%
%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2
,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}
+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32
,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%
%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%
{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,
2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}
+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-
8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{-
4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2
]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{
8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]
%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-
64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%
%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%
%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[
4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}
+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2
0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4
]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}
+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16
,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo
sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8
,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%
}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[
0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%
}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,
[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]
%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%
%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,
```

```
4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2
4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1
]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%
%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0
204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+
%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+
%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((((113400*b^29/272160
```

```
0/b^34*sqrt(b*x+2)*sqrt(b*x+2)-1383480*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(
b*x+2)+7093170*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(b*x+2)-19737270*b^29/272
1600/b^34)*sqrt(b*x+2)*sqrt(b*x+2)+32304825*b^29/2721600/b^34)*sqrt(b*x+2)*
sqrt(b*x+2)-33722325*b^29/2721600/b^34)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-231
/16/b^4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+12*b^2*ab
s(b)/b^2*(2*((((5040*b^19/100800/b^23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/10
0800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+215460*b^19/100800/b^23)*sqrt(b*x+2)*sqr
t(b*x+2)-469350*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+607950*b^19/10080
0/b^23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x
+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqr
t(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/
1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*
(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2
))))+16*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b
^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b
)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.00, size = 138, normalized size = 0.96

$$\frac{(bx+2)^{\frac{7}{2}} x^{\frac{5}{2}}}{6b}-\frac{(bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{6b^{2}}+\frac{(bx+2)^{\frac{7}{2}} \sqrt{x}}{8b^{3}}-\frac{(bx+2)^{\frac{5}{2}} \sqrt{x}}{24b^{3}}-\frac{5\,(bx+2)^{\frac{3}{2}} \sqrt{x}}{48b^{3}}-\frac{5\sqrt{bx+2}\,\sqrt{x}}{16b^{3}}-\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{16\sqrt{bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/6/b*x^(5/2)*(b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(b*x+2)^(7/2)+1/8/b^3*x^(1/2)*(
b*x+2)^(7/2)-1/24*(b*x+2)^(5/2)/b^3*x^(1/2)-5/48*(b*x+2)^(3/2)/b^3*x^(1/2)-
5/16*(b*x+2)^(1/2)/b^3*x^(1/2)-5/16*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.98, size = 223, normalized size = 1.55

$$\frac{\frac{15\sqrt{bx+2}\,b^{5}}{\sqrt{x}}-\frac{85\,(bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{198\,(bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}+\frac{198\,(bx+2)^{\frac{7}{2}}b^{2}}{x^{\frac{7}{2}}}-\frac{85\,(bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}}+\frac{15\,(bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^{9}-\frac{6\,(bx+2)b^{8}}{x}+\frac{15\,(bx+2)^{2}b^{7}}{x^{2}}-\frac{20\,(bx+2)^{3}b^{6}}{x^{3}}+\frac{15\,(bx+2)^{4}b^{5}}{x^{4}}-\frac{6\,(bx+2)^{5}b^{4}}{x^{5}}+\frac{(bx+2)^{6}b^{3}}{x^{6}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*(15*sqrt(b*x + 2)*b^5/sqrt(x) - 85*(b*x + 2)^(3/2)*b^4/x^(3/2) + 198*(
b*x + 2)^(5/2)*b^3/x^(5/2) + 198*(b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(b*x + 2)
^(9/2)*b/x^(9/2) + 15*(b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x + 2)*b^8/x +
 15*(b*x + 2)^2*b^7/x^2 - 20*(b*x + 2)^3*b^6/x^3 + 15*(b*x + 2)^4*b^5/x^4 -
 6*(b*x + 2)^5*b^4/x^5 + (b*x + 2)^6*b^3/x^6) + 5/16*log(-(sqrt(b) - sqrt(b
*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(b\,x+2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)*(b*x + 2)^(5/2), x)
```

sympy [A] time = 22.96, size = 158, normalized size = 1.10

$$\frac{b^{3}x^{\frac{13}{2}}}{6\sqrt{bx+2}}+\frac{7b^{2}x^{\frac{11}{2}}}{6\sqrt{bx+2}}+\frac{67bx^{\frac{9}{2}}}{24\sqrt{bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{48b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^{2}\sqrt{bx+2}}+\frac{5\sqrt{x}}{8b^{3}\sqrt{bx+2}}-\frac{5\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(13/2)/(6*sqrt(b*x + 2)) + 7*b**2*x**(11/2)/(6*sqrt(b*x + 2)) + 67*
b*x**(9/2)/(24*sqrt(b*x + 2)) + 55*x**(7/2)/(24*sqrt(b*x + 2)) - x**(5/2)/(
48*b*sqrt(b*x + 2)) + 5*x**(3/2)/(48*b**2*sqrt(b*x + 2)) + 5*sqrt(x)/(8*b**
3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2))
```

3.558 $\int x^{3/2}(2+bx)^{5/2}\,dx$

Optimal. Leaf size=123

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{8b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(bx+2)^{5/2}+\frac{1}{4}x^{5/2}(bx+2)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{bx+2}+\frac{x^{3/2}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(8*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(8*b) + (x^(5/
2)*Sqrt[2 + b*x])/4 + (x^(5/2)*(2 + b*x)^(3/2))/4 + (x^(5/2)*(2 + b*x)^(5/2
))/5 + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^{3/2}(2+bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \int x^{3/2}(2+bx)^{3/2}\,dx \\
&= \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2+bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx \\
&= \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2} \\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{8b^2} + \frac{x^{3/2}\sqrt{2+bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2+bx} + \frac{1}{4}x^{5/2}(2+bx)^{3/2} + \frac{1}{5}x^{5/2}(2+bx)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 78, normalized size = 0.63

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} + \frac{\sqrt{x}\sqrt{bx+2}\left(8b^4x^4+42b^3x^3+62b^2x^2+5bx-15\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(-15 + 5*b*x + 62*b^2*x^2 + 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.77

$$\frac{\sqrt{bx+2}\left(8b^4x^{9/2}+42b^3x^{7/2}+62b^2x^{5/2}+5bx^{3/2}-15\sqrt{x}\right)}{40b^2} - \frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-15*Sqrt[x] + 5*b*x^(3/2) + 62*b^2*x^(5/2) + 42*b^3*x^(7/2)
 + 8*b^4*x^(9/2)))/(40*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(
4*b^(5/2))
```

fricas [A] time = 1.32, size = 155, normalized size = 1.26

$$\left[\frac{\left(8b^5x^4+42b^4x^3+62b^3x^2+5b^2x-15b\right)\sqrt{bx+2}\sqrt{x}+15\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{40b^3}, \frac{\left(8b^5x^4+42b^4x^3+62b^3x^2+5b^2x-15b\right)\sqrt{bx+2}\sqrt{x}-30\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{40b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)
*sqrt(x) + 15*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/
40*((8*b^5*x^4 + 42*b^4*x^3 + 62*b^3*x^2 + 5*b^2*x - 15*b)*sqrt(b*x + 2)*sq
rt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739
918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2
]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%
%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2
,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}
+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32
,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%
%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%
{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,
2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}
+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-
8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet
ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{-
4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2
]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{
8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]
%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-
64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%
%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%
%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[
4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}
+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2
0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4
]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}
+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16
,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo
sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8
,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%
}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[
0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%
}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,
[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]
%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%
%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,
4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}
+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2
4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1
```

```
]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%
%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0
204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+
%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+
%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*((((5040*b^19/100800/b^
23*sqrt(b*x+2)*sqrt(b*x+2)-51660*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)+
215460*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*x+2)-469350*b^19/100800/b^23)*s
qrt(b*x+2)*sqrt(b*x+2)+607950*b^19/100800/b^23)*sqrt(b*x+2)*sqrt(b*(b*x+2)-
```

```
2*b)+63/8/b^3/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+12*
b^2*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sqrt(b*x+2)*sqrt(b*x+2)-750*b^11/144
0/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2
)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(
abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b*abs(b)/b^2*(2*((12*b^5/
144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198
*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(
b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+16*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt
(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b
*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.00, size = 123, normalized size = 1.00

$$\frac{(bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{5b}-\frac{3\,(bx+2)^{\frac{7}{2}}\sqrt{x}}{20b^2}+\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{20b^2}+\frac{(bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^2}+\frac{3\sqrt{bx+2}\,\sqrt{x}}{8b^2}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x+2)^(5/2),x)
```

```
[Out] 1/5/b*x^(3/2)*(b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(b*x+2)^(7/2)+1/20*(b*x+2)^(5/
2)/b^2*x^(1/2)+1/8*(b*x+2)^(3/2)/b^2*x^(1/2)+3/8*(b*x+2)^(1/2)/b^2*x^(1/2)+
3/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))
```

maxima [B] time = 2.94, size = 194, normalized size = 1.58

$$-\frac{\frac{15\sqrt{bx+2}\,b^4}{\sqrt{x}}-\frac{70\,(bx+2)^{\frac{3}{2}}b^3}{x^{\frac{3}{2}}}+\frac{128\,(bx+2)^{\frac{5}{2}}b^2}{x^{\frac{5}{2}}}+\frac{70\,(bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^7-\frac{5\,(bx+2)b^6}{x}+\frac{10\,(bx+2)^2b^5}{x^2}-\frac{10\,(bx+2)^3b^4}{x^3}+\frac{5\,(bx+2)^4b^3}{x^4}-\frac{(bx+2)^5b^2}{x^5}\right)}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/20*(15*sqrt(b*x + 2)*b^4/sqrt(x) - 70*(b*x + 2)^(3/2)*b^3/x^(3/2) + 128*
(b*x + 2)^(5/2)*b^2/x^(5/2) + 70*(b*x + 2)^(7/2)*b/x^(7/2) - 15*(b*x + 2)^(
9/2)/x^(9/2))/(b^7 - 5*(b*x + 2)*b^6/x + 10*(b*x + 2)^2*b^5/x^2 - 10*(b*x +
 2)^3*b^4/x^3 + 5*(b*x + 2)^4*b^3/x^4 - (b*x + 2)^5*b^2/x^5) - 3/8*log(-(sq
rt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,{\left(b\,x+2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)*(b*x + 2)^(5/2), x)
```

sympy [A] time = 14.42, size = 138, normalized size = 1.12

$$\frac{b^3x^{\frac{11}{2}}}{5\sqrt{bx+2}}+\frac{29b^2x^{\frac{9}{2}}}{20\sqrt{bx+2}}+\frac{73bx^{\frac{7}{2}}}{20\sqrt{bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{8b\sqrt{bx+2}}-\frac{3\sqrt{x}}{4b^2\sqrt{bx+2}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(b*x+2)**(5/2),x)
```

```
[Out] b**3*x**(11/2)/(5*sqrt(b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(b*x + 2)) + 73
*b*x**(7/2)/(20*sqrt(b*x + 2)) + 129*x**(5/2)/(40*sqrt(b*x + 2)) - x**(3/2)
/(8*b*sqrt(b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*s
qrt(b)*sqrt(x)/2)/(4*b**(5/2))
```

3.559 $\int \sqrt{x}\,(2+bx)^{5/2}\,dx$

Optimal. Leaf size=102

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

Rubi [A] time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(bx+2)^{5/2}+\frac{5}{12}x^{3/2}(bx+2)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{bx+2}+\frac{5\sqrt{x}\sqrt{bx+2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 + b*x])/8 + (5*x^(3/2)*
(2 + b*x)^(3/2))/12 + (x^(3/2)*(2 + b*x)^(5/2))/4 - (5*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{x}\,(2+bx)^{5/2}\,dx = \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,(2+bx)^{3/2}\,dx$$
$$= \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,\sqrt{2+bx}\,dx$$
$$= \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,d}{8b}$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\,\mathrm{Subst}\left(\int \frac{}{\sqrt{2\cdots}}\right.}{4}$$
$$= \frac{5\sqrt{x}\,\sqrt{2+bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2+bx} + \frac{5}{12}x^{3/2}(2+bx)^{3/2} + \frac{1}{4}x^{3/2}(2+bx)^{5/2} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{\cdots}}{\sqrt{2}}\right.}{4b^{3/2}}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 0.69

$$\frac{\sqrt{x}\,\sqrt{bx+2}\,\left(6b^3x^3+34b^2x^2+59bx+15\right)}{24b} - \frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 + 59*b*x + 34*b^2*x^2 + 6*b^3*x^3))/(24*b) - (5*
ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 85, normalized size = 0.83

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{4b^{3/2}} + \frac{\sqrt{bx+2}\,\left(6b^3x^{7/2}+34b^2x^{5/2}+59bx^{3/2}+15\sqrt{x}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 + b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] + 59*b*x^(3/2) + 34*b^2*x^(5/2) + 6*b^3*x^(7/2))
)/(24*b) + (5*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/(4*b^(3/2))
```

fricas [A] time = 1.43, size = 140, normalized size = 1.37

$$\left[\frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{24\,b^2}, \frac{\left(6\,b^4x^3+34\,b^3x^2+59\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 1
5*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^2, 1/24*((6*b^4*x
^3 + 34*b^3*x^2 + 59*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arct
an(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%% }+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4, [1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+ %%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4, [3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%% }+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{ -4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4, 4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%% %{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%% %}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%% {48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0, 2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [83.4865739 918,53.112478131]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1, 0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+ %%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+% %%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2 ,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%} +%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32 ,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]% %%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%% {20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2, 2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%} +%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{- 8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at paramet ers values [38.6973876911,89.629912049]Warning, choosing root of [1,0,%%%{- 4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{ 8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3] %%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{- 64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]% %%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%% %{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%} +%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-2 0,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%} +%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16 ,[0,0]%%%}] at parameters values [6.82230772497,55.0343274642]Warning, choo sing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8 ,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%% }+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%% }+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4, [1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3] %%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+% %%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3, 4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%} +%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{2 4,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+% %%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [53.4880634798,16.0 204098616]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+ %%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[

```
2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%
%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3
,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}
+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16
,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%
%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%
{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,
1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+
%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-4
0,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]
%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters val
ues [46.2456374937,66.0382199469]Warning, choosing root of [1,0,%%%{-4,[1,1
]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+
%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0
]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%
%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,
2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%
%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[
0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%
%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-
20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3
]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+
%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1
,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]
%%%}] at parameters values [94.9264369817,51.8441526662]Warning, choosing r
oot of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]
%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{
28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%
%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{
4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%
%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%
%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,
[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}
+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6
,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0
]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+
%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32
,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [98.7121795234,4.66774101
928]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4
,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%
%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[
0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%
}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8
,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]
%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,
%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,
0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}
+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-4
0,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2
]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%
%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [9
0.2102860468,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((90*b^11/1440/b^14*sq
rt(b*x+2)*sqrt(b*x+2)-750*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)+2445*b^11
/1440/b^14)*sqrt(b*x+2)*sqrt(b*x+2)-4185*b^11/1440/b^14)*sqrt(b*x+2)*sqrt(b
*(b*x+2)-2*b)-35/8/b^2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+
2))))+12*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/
144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2
)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+24*b
*abs(b)/b^2/b*(2*(1/8*sqrt(b*x+2)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+
```

```
2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))+16*
abs(b)/b^2*(1/2*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))))
```

maple [A] time = 0.01, size = 99, normalized size = 0.97

$$\frac{(bx+2)^{\frac{5}{2}}x^{\frac{3}{2}}}{4}+\frac{5\,(bx+2)^{\frac{3}{2}}x^{\frac{3}{2}}}{12}+\frac{5\sqrt{bx+2}\,x^{\frac{3}{2}}}{8}+\frac{5\sqrt{bx+2}\,\sqrt{x}}{8b}-\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{8\sqrt{bx+2}\,b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x+2)^(5/2)+5/12*(b*x+2)^(3/2)*x^(3/2)+5/8*(b*x+2)^(1/2)*x^(3
/2)+5/8*(b*x+2)^(1/2)/b*x^(1/2)-5/8*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)
/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.99, size = 161, normalized size = 1.58

$$\frac{\frac{15\sqrt{bx+2}\,b^{3}}{\sqrt{x}}-\frac{55\,(bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}+\frac{73\,(bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}+\frac{15\,(bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^{5}-\frac{4\,(bx+2)b^{4}}{x}+\frac{6\,(bx+2)^{2}b^{3}}{x^{2}}-\frac{4\,(bx+2)^{3}b^{2}}{x^{3}}+\frac{(bx+2)^{4}b}{x^{4}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{8\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(b*x + 2)*b^3/sqrt(x) - 55*(b*x + 2)^(3/2)*b^2/x^(3/2) + 73*(b
*x + 2)^(5/2)*b/x^(5/2) + 15*(b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x + 2)*b^
4/x + 6*(b*x + 2)^2*b^3/x^2 - 4*(b*x + 2)^3*b^2/x^3 + (b*x + 2)^4*b/x^4) +
5/8*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)
))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,(b\,x+2)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(1/2)*(b*x + 2)^(5/2), x)
```

sympy [A] time = 8.61, size = 119, normalized size = 1.17

$$\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{bx+2}}+\frac{23b^{2}x^{\frac{7}{2}}}{12\sqrt{bx+2}}+\frac{127bx^{\frac{5}{2}}}{24\sqrt{bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{bx+2}}+\frac{5\sqrt{x}}{4b\sqrt{bx+2}}-\frac{5\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] b**3*x**(9/2)/(4*sqrt(b*x + 2)) + 23*b**2*x**(7/2)/(12*sqrt(b*x + 2)) + 127
*b*x**(5/2)/(24*sqrt(b*x + 2)) + 133*x**(3/2)/(24*sqrt(b*x + 2)) + 5*sqrt(x
)/(4*b*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2))
```

3.560 $\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx$

Optimal. Leaf size=79

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{1}{3}\sqrt{x}\,(bx+2)^{5/2}+\frac{5}{6}\sqrt{x}\,(bx+2)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{bx+2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*Sqrt[x]*(2 + b*x)^(3/2))/6 + (Sqrt[x]*(2 + 
 b*x)^(5/2))/3 + (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+bx)^{5/2}}{\sqrt{x}}\,dx &= \frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{3}\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\
&= \frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2+bx}+\frac{5}{6}\sqrt{x}\,(2+bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2+bx)^{5/2}+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.72

$$\frac{1}{6}\sqrt{x}\sqrt{bx+2}\left(2b^2x^2+13bx+33\right)+\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(33 + 13*b*x + 2*b^2*x^2))/6 + (5*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.10, size = 70, normalized size = 0.89

$$\frac{1}{6}\sqrt{bx+2}\left(2b^2x^{5/2}+13bx^{3/2}+33\sqrt{x}\right)-\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 + b*x]*(33*Sqrt[x] + 13*b*x^(3/2) + 2*b^2*x^(5/2)))/6 - (5*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

fricas [A] time = 1.24, size = 123, normalized size = 1.56

$$\left[\frac{\left(2\,b^{3}x^{2}+13\,b^{2}x+33\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b},\frac{\left(2\,b^{3}x^{2}+13\,b^{2}x+33\,b\right)\sqrt{bx+2}\,\sqrt{x}-30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 + 13*b^2*x + 33*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(
b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 + 13*b^2*x + 3
3*b)*sqrt(b*x + 2)*sqrt(x) - 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*((1/6/b*sqrt(b*x+
2)*sqrt(b*x+2)+5/12/b)*sqrt(b*x+2)*sqrt(b*x+2)+5/4/b)*sqrt(b*x+2)*sqrt(b*(b
*x+2)-2*b)-5/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.00, size = 84, normalized size = 1.06

$$\frac{(bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5\,(bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{bx+2}\,\sqrt{x}}{2}+\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(b*x+2)^(5/2)*x^(1/2)+5/6*(b*x+2)^(3/2)*x^(1/2)+5/2*(b*x+2)^(1/2)*x^(1/
2)+5/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(
b*x^2+2*x)^(1/2))
```

maxima [B] time = 3.03, size = 129, normalized size = 1.63

$$-\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,\sqrt{b}}-\frac{\frac{15\,\sqrt{bx+2}\,b^{2}}{\sqrt{x}}-\frac{40\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{3}-\frac{3\,(bx+2)b^{2}}{x}+\frac{3\,(bx+2)^{2}b}{x^{2}}-\frac{(bx+2)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x
)))/sqrt(b) - 1/3*(15*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3
/2) + 33*(b*x + 2)^(5/2)/x^(5/2))/(b^3 - 3*(b*x + 2)*b^2/x + 3*(b*x + 2)^2*
b/x^2 - (b*x + 2)^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(1/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(1/2), x)
```

sympy [A] time = 5.46, size = 97, normalized size = 1.23

$$\frac{b^{3}x^{\frac{7}{2}}}{3\sqrt{bx+2}}+\frac{17b^{2}x^{\frac{5}{2}}}{6\sqrt{bx+2}}+\frac{59bx^{\frac{3}{2}}}{6\sqrt{bx+2}}+\frac{11\sqrt{x}}{\sqrt{bx+2}}+\frac{5\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b**3*x**(7/2)/(3*sqrt(b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(b*x + 2)) + 59*b
*x**(3/2)/(6*sqrt(b*x + 2)) + 11*sqrt(x)/sqrt(b*x + 2) + 5*asinh(sqrt(2)*sq
rt(b)*sqrt(x)/2)/sqrt(b)
```

3.561 $\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{2(bx+2)^{5/2}}{\sqrt{x}}+\frac{5}{2}b\sqrt{x}\,(bx+2)^{3/2}+\frac{15}{2}b\sqrt{x}\,\sqrt{bx+2}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (15*b*Sqrt[x]*Sqrt[2 + b*x])/2 + (5*b*Sqrt[x]*(2 + b*x)^(3/2))/2 - (2*(2 + 
b*x)^(5/2))/Sqrt[x] + 15*Sqrt[b]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, 
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n 
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(5b)\int \frac{(2+bx)^{3/2}}{\sqrt{x}}\,dx\\ &= \frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+\frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+(15b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{15}{2}b\sqrt{x}\,\sqrt{2+bx}+\frac{5}{2}b\sqrt{x}\,(2+bx)^{3/2}-\frac{2(2+bx)^{5/2}}{\sqrt{x}}+15\sqrt{b}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.35

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};-\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, -1/2*(b*x)])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.12, size = 62, normalized size = 0.78

$$\frac{\sqrt{bx+2}\,\left(b^2x^2+9bx-16\right)}{2\sqrt{x}}-15\sqrt{b}\,\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-16 + 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[b]*Log[-(Sqrt
[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

fricas [A] time = 1.24, size = 116, normalized size = 1.47

$$\left[\frac{15\,\sqrt{b}\,x\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(b^2x^2+9\,bx-16\right)\sqrt{bx+2}\,\sqrt{x}}{2\,x},\,-\frac{30\,\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(b^2x^2+9\,bx-16\right)\sqrt{bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(b)*x*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x^2
+ 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))/x, -1/2*(30*sqrt(-b)*x*arctan(sqrt(b*x
 + 2)*sqrt(-b)/(b*sqrt(x))) - (b^2*x^2 + 9*b*x - 16)*sqrt(b*x + 2)*sqrt(x))
/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]b/abs(b)*b^2/b*(2*((1/4*sqrt(b*x+2)
*sqrt(b*x+2)+5/4)*sqrt(b*x+2)*sqrt(b*x+2)-15/2)*sqrt(b*x+2)*sqrt(b*(b*x+2)-
2*b)/(b*(b*x+2)-2*b)-15/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x
+2))))
```

maple [A] time = 0.02, size = 81, normalized size = 1.03

$$\frac{15\sqrt{(bx+2)\,x}\,\sqrt{b}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2\sqrt{bx+2}\,\sqrt{x}}+\frac{b^3x^3+11b^2x^2+2bx-32}{2\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] 1/2*(b^3*x^3+11*b^2*x^2+2*b*x-32)/(b*x+2)^(1/2)/x^(1/2)+15/2*((b*x+2)*x)^(1
/2)/(b*x+2)^(1/2)*b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.94, size = 113, normalized size = 1.43

$$-\frac{15}{2}\sqrt{b}\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{\frac{7\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{9\,(bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^2-\frac{2\,(bx+2)b}{x}+\frac{(bx+2)^2}{x^2}}-\frac{8\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] -15/2*sqrt(b)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x +
2)/sqrt(x))) - (7*sqrt(b*x + 2)*b^2/sqrt(x) - 9*(b*x + 2)^(3/2)*b/x^(3/2))/
(b^2 - 2*(b*x + 2)*b/x + (b*x + 2)^2/x^2) - 8*sqrt(b*x + 2)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(3/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(3/2), x)
```

sympy [A] time = 5.60, size = 94, normalized size = 1.19

$$15\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)+\frac{b^3x^{\frac{5}{2}}}{2\sqrt{bx+2}}+\frac{11b^2x^{\frac{3}{2}}}{2\sqrt{bx+2}}+\frac{b\sqrt{x}}{\sqrt{bx+2}}-\frac{16}{\sqrt{x}\,\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] 15*sqrt(b)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2) + b**3*x**(5/2)/(2*sqrt(b*x + 2
)) + 11*b**2*x**(3/2)/(2*sqrt(b*x + 2)) + b*sqrt(x)/sqrt(b*x + 2) - 16/(sqr
t(x)*sqrt(b*x + 2))
```

3.562 $\int \frac{(2+bx)^{5/2}}{x^{5/2}} dx$

Optimal. Leaf size=81

$$10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\sqrt{bx+2}-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$5b^2\sqrt{x}\sqrt{bx+2}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(bx+2)^{5/2}}{3x^{3/2}}-\frac{10b(bx+2)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 + b*x] - (10*b*(2 + b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 +
b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\frac{1}{3}(5b)\int \frac{(2+bx)^{3/2}}{x^{3/2}}\,dx \\
&= -\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2+bx}}{\sqrt{x}}\,dx \\
&= 5b^2\sqrt{x}\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx \\
&= 5b^2\sqrt{x}\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right) \\
&= 5b^2\sqrt{x}\sqrt{2+bx}-\frac{10b(2+bx)^{3/2}}{3\sqrt{x}}-\frac{2(2+bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.37

$$
-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};-\frac{bx}{2}\right)}{3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, -1/2*(b*x)])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 63, normalized size = 0.78

$$
\frac{\sqrt{bx+2}\left(3b^2x^2-28bx-8\right)}{3x^{3/2}}-10b^{3/2}\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 + b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[2 + b*x]*(-8 - 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) - 10*b^(3/2)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
```

fricas [A] time = 1.45, size = 123, normalized size = 1.52

$$
\left[\frac{15\,b^{\frac{3}{2}}x^2\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(3\,b^2x^2-28\,bx-8\right)\sqrt{bx+2}\sqrt{x}}{3\,x^2},-\frac{30\sqrt{-b}\,bx^2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)-\left(3\,b^2x^2-28\,bx-8\right)\sqrt{bx+2}\sqrt{x}}{3\,x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*b^(3/2)*x^2*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (3*b^2*
x^2 - 28*b*x - 8)*sqrt(b*x + 2)*sqrt(x))/x^2, -1/3*(30*sqrt(-b)*b*x^2*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (3*b^2*x^2 - 28*b*x - 8)*sqrt(b*x +
 2)*sqrt(x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]1/abs(b)*b^2/b*(2*((9*b^4/18/b*sqrt
(b*x+2)*sqrt(b*x+2)-120*b^4/18/b)*sqrt(b*x+2)*sqrt(b*x+2)+180*b^4/18/b)*sqr
t(b*x+2)*sqrt(b*(b*x+2)-2*b)/(b*(b*x+2)-2*b)^2-10*b^2/sqrt(b)*ln(abs(sqrt(b
*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.02, size = 82, normalized size = 1.01

$$\frac{5\sqrt{(bx+2)\,x}\;b^{\frac{3}{2}}\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;\sqrt{x}}+\frac{3b^{3}x^{3}-22b^{2}x^{2}-64bx-16}{3\sqrt{bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] 1/3*(3*b^3*x^3-22*b^2*x^2-64*b*x-16)/x^(3/2)/(b*x+2)^(1/2)+5*((b*x+2)*x)^(1
/2)/(b*x+2)^(1/2)*b^(3/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [A] time = 2.94, size = 96, normalized size = 1.19

$$-5\,b^{\frac{3}{2}}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)-\frac{8\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{2\sqrt{bx+2}\,b^{2}}{\left(b-\frac{bx+2}{x}\right)\sqrt{x}}-\frac{4\,(bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -5*b^(3/2)*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x))) - 8*sqrt(b*x + 2)*b/sqrt(x) - 2*sqrt(b*x + 2)*b^2/((b - (b*x + 2)
/x)*sqrt(x)) - 4/3*(b*x + 2)^(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x+2)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + 2)^(5/2)/x^(5/2),x)
```

```
[Out] int((b*x + 2)^(5/2)/x^(5/2), x)
```

sympy [A] time = 5.15, size = 88, normalized size = 1.09

$$b^{\frac{5}{2}}x\sqrt{1+\frac{2}{bx}}-\frac{28b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-5b^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)+10b^{\frac{3}{2}}\log\left(\sqrt{1+\frac{2}{bx}}+1\right)-\frac{8\sqrt{b}\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] b**(5/2)*x*sqrt(1 + 2/(b*x)) - 28*b**(3/2)*sqrt(1 + 2/(b*x))/3 - 5*b**(3/2)
*log(1/(b*x)) + 10*b**(3/2)*log(sqrt(1 + 2/(b*x)) + 1) - 8*sqrt(b)*sqrt(1 +
 2/(b*x))/(3*x)
```

3.563 $\int x^{5/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=150

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Rubi [A] time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{48b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}}+\frac{1}{6}x^{7/2}(2-bx)^{5/2}+\frac{1}{6}x^{7/2}(2-bx)^{3/2}+\frac{1}{8}x^{7/2}\sqrt{2-bx}-\frac{x^{5/2}\sqrt{2-bx}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(16*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(48*b^2) -
(x^(5/2)*Sqrt[2 - b*x])/(24*b) + (x^(7/2)*Sqrt[2 - b*x])/8 + (x^(7/2)*(2 -
b*x)^(3/2))/6 + (x^(7/2)*(2 - b*x)^(5/2))/6 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{5/2}(2-bx)^{5/2}\,dx &= \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{5}{6}\int x^{5/2}(2-bx)^{3/2}\,dx \\
&= \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{1}{2}\int x^{5/2}\sqrt{2-bx}\,dx \\
&= \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{1}{8}\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{24b} \\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/2}(2-bx)^{5/2} + \frac{5}{} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/} \\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{16b^3} - \frac{5x^{3/2}\sqrt{2-bx}}{48b^2} - \frac{x^{5/2}\sqrt{2-bx}}{24b} + \frac{1}{8}x^{7/2}\sqrt{2-bx} + \frac{1}{6}x^{7/2}(2-bx)^{3/2} + \frac{1}{6}x^{7/}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 87, normalized size = 0.58

$$
\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{8b^{7/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(8b^5x^5-40b^4x^4+54b^3x^3-2b^2x^2-5bx-15\right)}{48b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x - 2*b^2*x^2 + 54*b^3*x^3 - 40*b^4*x^4 +
 8*b^5*x^5))/(48*b^3) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(8*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 114, normalized size = 0.76

$$
\frac{5\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{8b^4} + \frac{\sqrt{2-bx}\left(8b^5x^{11/2}-40b^4x^{9/2}+54b^3x^{7/2}-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{48b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2) + 54*b^3*x^(7/2)
- 40*b^4*x^(9/2) + 8*b^5*x^(11/2)))/(48*b^3) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*S
qrt[x]) + Sqrt[2 - b*x]])/(8*b^4)
```

fricas [A] time = 1.28, size = 173, normalized size = 1.15

$$
\left[\frac{\left(8\,b^6x^5-40\,b^5x^4+54\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{48\,b^4},\ \frac{\left(8\,b^6x^5-40\,b^5x^4+54\,b^4x^3-2\,b^3x^2-5\,b^2x-15\,b\right)\sqrt{-bx+2}\,\sqrt{x}-30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{48\,b^4}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b^2*x - 15*b)*s
qrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt
```

```
(x) + 1))/b^4, 1/48*((8*b^6*x^5 - 40*b^5*x^4 + 54*b^4*x^3 - 2*b^3*x^2 - 5*b
^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.00, size = 148, normalized size = 0.99

$$-\frac{(-bx+2)^{\frac{7}{2}} x^{\frac{5}{2}}}{6b}-\frac{(-bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{6b^2}-\frac{(-bx+2)^{\frac{7}{2}} \sqrt{x}}{8b^3}+\frac{(-bx+2)^{\frac{5}{2}} \sqrt{x}}{24b^3}+\frac{5(-bx+2)^{\frac{3}{2}} \sqrt{x}}{48b^3}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{16b^3}+\frac{5\sqrt{(-bx+2)x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b x^2+2x}}\right)}{16\sqrt{-bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(-b*x+2)^(5/2),x)
[Out] -1/6/b*x^(5/2)*(-b*x+2)^(7/2)-1/6/b^2*x^(3/2)*(-b*x+2)^(7/2)-1/8/b^3*x^(1/2
)*(-b*x+2)^(7/2)+1/24*(-b*x+2)^(5/2)/b^3*x^(1/2)+5/48*(-b*x+2)^(3/2)/b^3*x^
(1/2)+5/16*(-b*x+2)^(1/2)/b^3*x^(1/2)+5/16*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2
)/b^(7/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.96, size = 209, normalized size = 1.39

$$\frac{\frac{15\sqrt{-bx+2}\,b^5}{\sqrt{x}}+\frac{85(-bx+2)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{198(-bx+2)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}-\frac{198(-bx+2)^{\frac{7}{2}}b^2}{x^{\frac{7}{2}}}-\frac{85(-bx+2)^{\frac{9}{2}}b}{x^{\frac{9}{2}}}-\frac{15(-bx+2)^{\frac{11}{2}}}{x^{\frac{11}{2}}}}{24\left(b^9-\frac{6(bx-2)b^8}{x}+\frac{15(bx-2)^2b^7}{x^2}-\frac{20(bx-2)^3b^6}{x^3}+\frac{15(bx-2)^4b^5}{x^4}-\frac{6(bx-2)^5b^4}{x^5}+\frac{(bx-2)^6b^3}{x^6}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
[Out] 1/24*(15*sqrt(-b*x + 2)*b^5/sqrt(x) + 85*(-b*x + 2)^(3/2)*b^4/x^(3/2) + 198
*(-b*x + 2)^(5/2)*b^3/x^(5/2) - 198*(-b*x + 2)^(7/2)*b^2/x^(7/2) - 85*(-b*x
 + 2)^(9/2)*b/x^(9/2) - 15*(-b*x + 2)^(11/2)/x^(11/2))/(b^9 - 6*(b*x - 2)*b
^8/x + 15*(b*x - 2)^2*b^7/x^2 - 20*(b*x - 2)^3*b^6/x^3 + 15*(b*x - 2)^4*b^5
/x^4 - 6*(b*x - 2)^5*b^4/x^5 + (b*x - 2)^6*b^3/x^6) - 5/8*arctan(sqrt(-b*x
+ 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{5/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)*(2 - b*x)^(5/2),x)
[Out] int(x^(5/2)*(2 - b*x)^(5/2), x)
```

sympy [A] time = 22.84, size = 337, normalized size = 2.25

$$\begin{cases} \frac{ib^3x^{\frac{13}{2}}}{6\sqrt{bx-2}}-\frac{7ib^2x^{\frac{11}{2}}}{6\sqrt{bx-2}}+\frac{67ibx^{\frac{9}{2}}}{24\sqrt{bx-2}}-\frac{55ix^{\frac{7}{2}}}{24\sqrt{bx-2}}-\frac{ix^{\frac{5}{2}}}{48b\sqrt{bx-2}}-\frac{5ix^{\frac{3}{2}}}{48b^2\sqrt{bx-2}}+\frac{5i\sqrt{x}}{8b^3\sqrt{bx-2}}-\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{b^3x^{\frac{13}{2}}}{6\sqrt{-bx+2}}+\frac{7b^2x^{\frac{11}{2}}}{6\sqrt{-bx+2}}-\frac{67bx^{\frac{9}{2}}}{24\sqrt{-bx+2}}+\frac{55x^{\frac{7}{2}}}{24\sqrt{-bx+2}}+\frac{x^{\frac{5}{2}}}{48b\sqrt{-bx+2}}+\frac{5x^{\frac{3}{2}}}{48b^2\sqrt{-bx+2}}-\frac{5\sqrt{x}}{8b^3\sqrt{-bx+2}}+\frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{8b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(13/2)/(6*sqrt(b*x - 2)) - 7*I*b**2*x**(11/2)/(6*sqrt(
b*x - 2)) + 67*I*b*x**(9/2)/(24*sqrt(b*x - 2)) - 55*I*x**(7/2)/(24*sqrt(b*x
 - 2)) - I*x**(5/2)/(48*b*sqrt(b*x - 2)) - 5*I*x**(3/2)/(48*b**2*sqrt(b*x -
 2)) + 5*I*sqrt(x)/(8*b**3*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/(8*b**(7/2)), Abs(b*x)/2 > 1), (-b**3*x**(13/2)/(6*sqrt(-b*x + 2)) +
7*b**2*x**(11/2)/(6*sqrt(-b*x + 2)) - 67*b*x**(9/2)/(24*sqrt(-b*x + 2)) + 5
5*x**(7/2)/(24*sqrt(-b*x + 2)) + x**(5/2)/(48*b*sqrt(-b*x + 2)) + 5*x**(3/2
)/(48*b**2*sqrt(-b*x + 2)) - 5*sqrt(x)/(8*b**3*sqrt(-b*x + 2)) + 5*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(8*b**(7/2)), True))
```

3.564 $\int x^{3/2}(2-bx)^{5/2}\,dx$

Optimal. Leaf size=128

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Rubi [A] time = 0.03, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}}+\frac{1}{5}x^{5/2}(2-bx)^{5/2}+\frac{1}{4}x^{5/2}(2-bx)^{3/2}+\frac{1}{4}x^{5/2}\sqrt{2-bx}-\frac{x^{3/2}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(8*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(8*b) + (x^(5/
2)*Sqrt[2 - b*x])/4 + (x^(5/2)*(2 - b*x)^(3/2))/4 + (x^(5/2)*(2 - b*x)^(5/2
))/5 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^{3/2}(2-bx)^{5/2}\,dx &= \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \int x^{3/2}(2-bx)^{3/2}\,dx \\
&= \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3}{4}\int x^{3/2}\sqrt{2-bx}\,dx \\
&= \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{1}{4}\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx \\
&= -\frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{8b} \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \cdots \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \cdots \\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{8b^2} - \frac{x^{3/2}\sqrt{2-bx}}{8b} + \frac{1}{4}x^{5/2}\sqrt{2-bx} + \frac{1}{4}x^{5/2}(2-bx)^{3/2} + \frac{1}{5}x^{5/2}(2-bx)^{5/2} + \cdots
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.62

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{5/2}} + \frac{\sqrt{x}\sqrt{2-bx}\left(8b^4x^4-42b^3x^3+62b^2x^2-5bx-15\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 - 5*b*x + 62*b^2*x^2 - 42*b^3*x^3 + 8*b^4*x^4))
/(40*b^2) + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(5/2))
```

IntegrateAlgebraic [A] time = 0.19, size = 104, normalized size = 0.81

$$\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^3} + \frac{\sqrt{2-bx}\left(8b^4x^{9/2}-42b^3x^{7/2}+62b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{40b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) + 62*b^2*x^(5/2) - 42*b^3*x^(7/2)
 + 8*b^4*x^(9/2)))/(40*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
- b*x]])/(4*b^3)
```

fricas [A] time = 1.32, size = 157, normalized size = 1.23

$$\left[\frac{\left(8b^5x^4-42b^4x^3+62b^3x^2-5b^2x-15b\right)\sqrt{-bx+2}\sqrt{x}-15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{40b^3}, \frac{\left(8b^5x^4-42b^4x^3+62b^3x^2-5b^2x-15b\right)\sqrt{-bx+2}\sqrt{x}-30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{40b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x + 2
)*sqrt(x) - 15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^
3, 1/40*((8*b^5*x^4 - 42*b^4*x^3 + 62*b^3*x^2 - 5*b^2*x - 15*b)*sqrt(-b*x +
 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-17.51342
60082,53.112478131]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[
0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%
}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20
,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]
%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%
%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[
4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}
+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46
,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]
%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%
%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at para
meters values [-62.3026123089,89.629912049]Warning, choosing root of [1,0,%
%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+
%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[
3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+
%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20
,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]
%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+
%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3
,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+
%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4
,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0
]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%
%%{16,[0,0]%%%}] at parameters values [-94.177692275,55.0343274642]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
```

48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-47.51193652
02,16.0204098616]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-54.7543625063,66.0382199469]Warning, choosing root of [1,0,%%
%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%
%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3
,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%
%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,
[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%
%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%
%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,
3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%
%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,
[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]
%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%
%{16,[0,0]%%%}] at parameters values [-6.07356301835,51.8441526662]Warning,
choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-2.287820476
57,4.66774101928]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-10.7897139532,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((((5040
*b^19/100800/b^23*sqrt(-b*x+2)*sqrt(-b*x+2)-51660*b^19/100800/b^23)*sqrt(-b
*x+2)*sqrt(-b*x+2)+215460*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)-46935
0*b^19/100800/b^23)*sqrt(-b*x+2)*sqrt(-b*x+2)+607950*b^19/100800/b^23)*sqrt

```
(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-63/8/b^3/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*
b)-sqrt(-b)*sqrt(-b*x+2))))-12*b^2*abs(b)/b^2*(2*(((-90*b^11/1440/b^14*sqrt
(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)-2445*b^
11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^14)*sqrt(-b*x+2)*s
qrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b
)*sqrt(-b*x+2))))+24*b*abs(b)/b^2*(2*((12*b^5/144/b^7*sqrt(-b*x+2)*sqrt(-b*
x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/144/b^7)*sqrt(-b*x+2
)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b
)*sqrt(-b*x+2))))+16*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2)*sqrt(-b*x+2)-5/8)*sq
rt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b
)-sqrt(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.01, size = 132, normalized size = 1.03

$$-\frac{(-bx+2)^{\frac{7}{2}} x^{\frac{3}{2}}}{5b}-\frac{3\,(-bx+2)^{\frac{7}{2}}\sqrt{x}}{20b^{2}}+\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{20b^{2}}+\frac{(-bx+2)^{\frac{3}{2}}\sqrt{x}}{8b^{2}}+\frac{3\sqrt{-bx+2}\,\sqrt{x}}{8b^{2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(-b*x+2)^(5/2),x)
```

```
[Out] -1/5/b*x^(3/2)*(-b*x+2)^(7/2)-3/20/b^2*x^(1/2)*(-b*x+2)^(7/2)+1/20*(-b*x+2)
^(5/2)/b^2*x^(1/2)+1/8*(-b*x+2)^(3/2)/b^2*x^(1/2)+3/8*(-b*x+2)^(1/2)/b^2*x^
(1/2)+3/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/
(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [B] time = 3.01, size = 179, normalized size = 1.40

$$\frac{\frac{15\sqrt{-bx+2}\,b^{4}}{\sqrt{x}}+\frac{70\,(-bx+2)^{\frac{3}{2}}b^{3}}{x^{\frac{3}{2}}}+\frac{128\,(-bx+2)^{\frac{5}{2}}b^{2}}{x^{\frac{5}{2}}}-\frac{70\,(-bx+2)^{\frac{7}{2}}b}{x^{\frac{7}{2}}}-\frac{15\,(-bx+2)^{\frac{9}{2}}}{x^{\frac{9}{2}}}}{20\left(b^{7}-\frac{5\,(bx-2)b^{6}}{x}+\frac{10\,(bx-2)^{2}b^{5}}{x^{2}}-\frac{10\,(bx-2)^{3}b^{4}}{x^{3}}+\frac{5\,(bx-2)^{4}b^{3}}{x^{4}}-\frac{(bx-2)^{5}b^{2}}{x^{5}}\right)}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/20*(15*sqrt(-b*x + 2)*b^4/sqrt(x) + 70*(-b*x + 2)^(3/2)*b^3/x^(3/2) + 128
*(-b*x + 2)^(5/2)*b^2/x^(5/2) - 70*(-b*x + 2)^(7/2)*b/x^(7/2) - 15*(-b*x +
2)^(9/2)/x^(9/2))/(b^7 - 5*(b*x - 2)*b^6/x + 10*(b*x - 2)^2*b^5/x^2 - 10*(b
*x - 2)^3*b^4/x^3 + 5*(b*x - 2)^4*b^3/x^4 - (b*x - 2)^5*b^2/x^5) - 3/4*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3/2}\,(2-b\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)*(2 - b*x)^(5/2), x)
```

sympy [A] time = 14.30, size = 294, normalized size = 2.30

$$\begin{cases} \frac{ib^{3}x^{\frac{11}{2}}}{5\sqrt{bx-2}}-\frac{29ib^{2}x^{\frac{9}{2}}}{20\sqrt{bx-2}}+\frac{73ibx^{\frac{7}{2}}}{20\sqrt{bx-2}}-\frac{129ix^{\frac{5}{2}}}{40\sqrt{bx-2}}-\frac{ix^{\frac{3}{2}}}{8b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{4b^{2}\sqrt{bx-2}}-\frac{3i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{b^{3}x^{\frac{11}{2}}}{5\sqrt{-bx+2}}+\frac{29b^{2}x^{\frac{9}{2}}}{20\sqrt{-bx+2}}-\frac{73bx^{\frac{7}{2}}}{20\sqrt{-bx+2}}+\frac{129x^{\frac{5}{2}}}{40\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{8b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{4b^{2}\sqrt{-bx+2}}+\frac{3\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*b**3*x**(11/2)/(5*sqrt(b*x - 2)) - 29*I*b**2*x**(9/2)/(20*sqrt
(b*x - 2)) + 73*I*b*x**(7/2)/(20*sqrt(b*x - 2)) - 129*I*x**(5/2)/(40*sqrt(b
*x - 2)) - I*x**(3/2)/(8*b*sqrt(b*x - 2)) + 3*I*sqrt(x)/(4*b**2*sqrt(b*x -
2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), Abs(b*x)/2 > 1), (
-b**3*x**(11/2)/(5*sqrt(-b*x + 2)) + 29*b**2*x**(9/2)/(20*sqrt(-b*x + 2)) -
 73*b*x**(7/2)/(20*sqrt(-b*x + 2)) + 129*x**(5/2)/(40*sqrt(-b*x + 2)) + x**
(3/2)/(8*b*sqrt(-b*x + 2)) - 3*sqrt(x)/(4*b**2*sqrt(-b*x + 2)) + 3*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/(4*b**(5/2)), True))
```

3.565 $\int \sqrt{x}\,(2-bx)^{5/2}\,dx$

Optimal. Leaf size=106

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

Rubi [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}+\frac{1}{4}x^{3/2}(2-bx)^{5/2}+\frac{5}{12}x^{3/2}(2-bx)^{3/2}+\frac{5}{8}x^{3/2}\sqrt{2-bx}-\frac{5\sqrt{x}\sqrt{2-bx}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(8*b) + (5*x^(3/2)*Sqrt[2 - b*x])/8 + (5*x^(3/2)
*(2 - b*x)^(3/2))/12 + (x^(3/2)*(2 - b*x)^(5/2))/4 + (5*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[2]])/(4*b^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}\,(2-bx)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,(2-bx)^{3/2}\,dx \\
&= \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{4}\int \sqrt{x}\,\sqrt{2-bx}\,dx \\
&= \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5}{8}\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{8b} \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx}}\right.}{4b} \\
&= -\frac{5\sqrt{x}\,\sqrt{2-bx}}{8b} + \frac{5}{8}x^{3/2}\sqrt{2-bx} + \frac{5}{12}x^{3/2}(2-bx)^{3/2} + \frac{1}{4}x^{3/2}(2-bx)^{5/2} + \frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 0.67

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{4b^{3/2}} + \frac{\sqrt{x}\,\sqrt{2-bx}\,\left(6b^3x^3 - 34b^2x^2 + 59bx - 15\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(-15 + 59*b*x - 34*b^2*x^2 + 6*b^3*x^3))/(24*b) + (5
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(4*b^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 94, normalized size = 0.89

$$\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)}{4b^2} + \frac{\sqrt{2-bx}\,\left(6b^3x^{7/2} - 34b^2x^{5/2} + 59bx^{3/2} - 15\sqrt{x}\right)}{24b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]*(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] + 59*b*x^(3/2) - 34*b^2*x^(5/2) + 6*b^3*x^(7/2)
))/(24*b) + (5*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/(4*b^2)
```

fricas [A] time = 1.44, size = 141, normalized size = 1.33

$$\left[\frac{\left(6\,b^4x^3 - 34\,b^3x^2 + 59\,b^2x - 15\,b\right)\sqrt{-bx+2}\,\sqrt{x} - 15\,\sqrt{-b}\,\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x} + 1\right)}{24\,b^2}, \frac{\left(6\,b^4x^3 - 34\,b^3x^2 + 59\,b^2x - 15\,b\right)\sqrt{-bx+2}\,\sqrt{x} - 30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{24\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*((6*b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) -
15*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^2, 1/24*((6*
b^4*x^3 - 34*b^3*x^2 + 59*b^2*x - 15*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)
*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-17.51342
60082,53.112478131]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,
0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+
%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[
0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%
}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20
,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]
%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%
%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[
4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}
+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46
,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]
%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%
%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at para
meters values [-62.3026123089,89.629912049]Warning, choosing root of [1,0,%
%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2
,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+
%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[
3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+
%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20
,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]
%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+
%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3
,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+
%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4
,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0
]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%
%%{16,[0,0]%%%}] at parameters values [-94.177692275,55.0343274642]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-47.51193652
02,16.0204098616]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
```

```
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-54.7543625063,66.0382199469]Warning, choosing root of [1,0,%%
%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,
2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%
%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3
,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%
%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,
[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%
%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%
%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,
3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%
%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,
[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]
%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%
%{16,[0,0]%%%}] at parameters values [-6.07356301835,51.8441526662]Warning,
 choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+%%%
{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[1,2
]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+%%
%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,[3
,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%}+
%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8,[3
,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%%
}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%%{-
48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,2]
%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-2.287820476
57,4.66774101928]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]
%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%
%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+
%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%
%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{
-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,
1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%
%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[
2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%
%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [-10.7897139532,38.2197840363]1/b*(2*b^3*abs(b)/b^2*(2*(((-90*b
^11/1440/b^14*sqrt(-b*x+2)*sqrt(-b*x+2)+750*b^11/1440/b^14)*sqrt(-b*x+2)*sq
rt(-b*x+2)-2445*b^11/1440/b^14)*sqrt(-b*x+2)*sqrt(-b*x+2)+4185*b^11/1440/b^
14)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-35/8/b^2/sqrt(-b)*ln(abs(sqrt(-b*(-b
*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-12*b^2*abs(b)/b^2*(2*((12*b^5/144/b^7*s
qrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+198*b^5/
144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqrt(-b*(
-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-24*b*abs(b)/b^2/b*(2*(1/8*sqrt(-b*x+2
```

```
)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*ln(ab
s(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))-16*abs(b)/b^2*(1/2*sqrt(-b
*x+2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqr
t(-b)*sqrt(-b*x+2)))))
```

maple [A] time = 0.00, size = 107, normalized size = 1.01

$$\frac{(-bx+2)^{\frac{5}{2}} x^{\frac{3}{2}}}{4}+\frac{5\,(-bx+2)^{\frac{3}{2}} x^{\frac{3}{2}}}{12}+\frac{5\sqrt{-bx+2}\; x^{\frac{3}{2}}}{8}-\frac{5\sqrt{-bx+2}\;\sqrt{x}}{8b}+\frac{5\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{8\sqrt{-bx+2}\; b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)*x^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(-b*x+2)^(5/2)+5/12*(-b*x+2)^(3/2)*x^(3/2)+5/8*(-b*x+2)^(1/2)*x
^(3/2)-5/8*(-b*x+2)^(1/2)/b*x^(1/2)+5/8*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b
^(3/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.89, size = 145, normalized size = 1.37

$$\frac{\frac{15\sqrt{-bx+2}\,b^{3}}{\sqrt{x}}+\frac{55\,(-bx+2)^{\frac{3}{2}}b^{2}}{x^{\frac{3}{2}}}+\frac{73\,(-bx+2)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{15\,(-bx+2)^{\frac{7}{2}}}{x^{\frac{7}{2}}}}{12\left(b^{5}-\frac{4\,(bx-2)b^{4}}{x}+\frac{6\,(bx-2)^{2}b^{3}}{x^{2}}-\frac{4\,(bx-2)^{3}b^{2}}{x^{3}}+\frac{(bx-2)^{4}b}{x^{4}}\right)}-\frac{5\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{4\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(15*sqrt(-b*x + 2)*b^3/sqrt(x) + 55*(-b*x + 2)^(3/2)*b^2/x^(3/2) + 73*
(-b*x + 2)^(5/2)*b/x^(5/2) - 15*(-b*x + 2)^(7/2)/x^(7/2))/(b^5 - 4*(b*x - 2
)*b^4/x + 6*(b*x - 2)^2*b^3/x^2 - 4*(b*x - 2)^3*b^2/x^3 + (b*x - 2)^4*b/x^4
) - 5/4*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x}\,\left(2-b\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(1/2)*(2 - b*x)^(5/2), x)
```

sympy [A] time = 8.59, size = 255, normalized size = 2.41

$$\begin{cases} \frac{ib^{3}x^{\frac{9}{2}}}{4\sqrt{bx-2}}-\frac{23ib^{2}x^{\frac{7}{2}}}{12\sqrt{bx-2}}+\frac{127ibx^{\frac{5}{2}}}{24\sqrt{bx-2}}-\frac{133ix^{\frac{3}{2}}}{24\sqrt{bx-2}}+\frac{5i\sqrt{x}}{4b\sqrt{bx-2}}-\frac{5i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{b^{3}x^{\frac{9}{2}}}{4\sqrt{-bx+2}}+\frac{23b^{2}x^{\frac{7}{2}}}{12\sqrt{-bx+2}}-\frac{127bx^{\frac{5}{2}}}{24\sqrt{-bx+2}}+\frac{133x^{\frac{3}{2}}}{24\sqrt{-bx+2}}-\frac{5\sqrt{x}}{4b\sqrt{-bx+2}}+\frac{5\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{4b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)*x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(9/2)/(4*sqrt(b*x - 2)) - 23*I*b**2*x**(7/2)/(12*sqrt(
b*x - 2)) + 127*I*b*x**(5/2)/(24*sqrt(b*x - 2)) - 133*I*x**(3/2)/(24*sqrt(b
*x - 2)) + 5*I*sqrt(x)/(4*b*sqrt(b*x - 2)) - 5*I*acosh(sqrt(2)*sqrt(b)*sqrt
```

```
(x)/2)/(4*b**(3/2)), Abs(b*x)/2 > 1), (-b**3*x**(9/2)/(4*sqrt(-b*x + 2)) +
23*b**2*x**(7/2)/(12*sqrt(-b*x + 2)) - 127*b*x**(5/2)/(24*sqrt(-b*x + 2)) +
 133*x**(3/2)/(24*sqrt(-b*x + 2)) - 5*sqrt(x)/(4*b*sqrt(-b*x + 2)) + 5*asin
(sqrt(2)*sqrt(b)*sqrt(x)/2)/(4*b**(3/2)), True))
```

3.566 $\int \frac{(2-bx)^{5/2}}{\sqrt{x}} \, dx$

Optimal. Leaf size=82

$$\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (5*Sqrt[x]*Sqrt[2 - b*x])/2 + (5*Sqrt[x]*(2 - b*x)^(3/2))/6 + (Sqrt[x]*(2 -
 b*x)^(5/2))/3 + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2-bx)^{5/2}}{\sqrt{x}} \, dx &= \frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{3}\int \frac{(2-bx)^{3/2}}{\sqrt{x}} \, dx\\
&= \frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{\sqrt{2-bx}}{\sqrt{x}} \, dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}} \, dx\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} \, dx,x,\sqrt{x}\right)\\
&= \frac{5}{2}\sqrt{x}\,\sqrt{2-bx}+\frac{5}{6}\sqrt{x}\,(2-bx)^{3/2}+\frac{1}{3}\sqrt{x}\,(2-bx)^{5/2}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.71

$$\frac{1}{6}\sqrt{x}\sqrt{2-bx}\left(2b^2x^2-13bx+33\right)+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 - b*x]*(33 - 13*b*x + 2*b^2*x^2))/6 + (5*ArcSin[(Sqrt[b]*Sq
rt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.14, size = 79, normalized size = 0.96

$$\frac{1}{6}\sqrt{2-bx}\left(2b^2x^{5/2}-13bx^{3/2}+33\sqrt{x}\right)+\frac{5\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/Sqrt[x],x]
```

```
[Out] (Sqrt[2 - b*x]*(33*Sqrt[x] - 13*b*x^(3/2) + 2*b^2*x^(5/2)))/6 + (5*Sqrt[-b]
*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b
```

fricas [A] time = 1.26, size = 125, normalized size = 1.52

$$\left[\frac{\left(2b^3x^2-13b^2x+33b\right)\sqrt{-bx+2}\sqrt{x}-15\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{6b},\frac{\left(2b^3x^2-13b^2x+33b\right)\sqrt{-bx+2}\sqrt{x}-30\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{6b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 13*b^2*x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b, 1/6*((2*b^3*x^2 - 13*b^2*
x + 33*b)*sqrt(-b*x + 2)*sqrt(x) - 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b
)*sqrt(x))))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*((1/6/b*sqrt(
-b*x+2)*sqrt(-b*x+2)+5/12/b)*sqrt(-b*x+2)*sqrt(-b*x+2)+5/4/b)*sqrt(-b*x+2)*
sqrt(-b*(-b*x+2)+2*b)+5/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt
(-b*x+2))))
```

maple [A] time = 0.00, size = 91, normalized size = 1.11

$$\frac{(-bx+2)^{\frac{5}{2}}\sqrt{x}}{3}+\frac{5(-bx+2)^{\frac{3}{2}}\sqrt{x}}{6}+\frac{5\sqrt{-bx+2}\,\sqrt{x}}{2}+\frac{5\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*(-b*x+2)^(5/2)*x^(1/2)+5/6*(-b*x+2)^(3/2)*x^(1/2)+5/2*(-b*x+2)^(1/2)*x^
(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/
(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.98, size = 112, normalized size = 1.37

$$-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}+\frac{\frac{15\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{40(-bx+2)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{33(-bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{3}-\frac{3(bx-2)b^{2}}{x}+\frac{3(bx-2)^{2}b}{x^{2}}-\frac{(bx-2)^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b) + 1/3*(15*sqrt(-b*x + 2
)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 33*(-b*x + 2)^(5/2)/x^(5/2)
)/(b^3 - 3*(b*x - 2)*b^2/x + 3*(b*x - 2)^2*b/x^2 - (b*x - 2)^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(1/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(1/2), x)
```

sympy [A] time = 5.52, size = 209, normalized size = 2.55

$$\begin{cases} \frac{ib^3x^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{17ib^2x^{\frac{5}{2}}}{6\sqrt{bx-2}} + \frac{59ibx^{\frac{3}{2}}}{6\sqrt{bx-2}} - \frac{11i\sqrt{x}}{\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for } \frac{|bx|}{2} > 1 \\ -\frac{b^3x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{17b^2x^{\frac{5}{2}}}{6\sqrt{-bx+2}} - \frac{59bx^{\frac{3}{2}}}{6\sqrt{-bx+2}} + \frac{11\sqrt{x}}{\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b**3*x**(7/2)/(3*sqrt(b*x - 2)) - 17*I*b**2*x**(5/2)/(6*sqrt(b
*x - 2)) + 59*I*b*x**(3/2)/(6*sqrt(b*x - 2)) - 11*I*sqrt(x)/sqrt(b*x - 2) -
 5*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1), (-b**3*x**(
7/2)/(3*sqrt(-b*x + 2)) + 17*b**2*x**(5/2)/(6*sqrt(-b*x + 2)) - 59*b*x**(3/
2)/(6*sqrt(-b*x + 2)) + 11*sqrt(x)/sqrt(-b*x + 2) + 5*asin(sqrt(2)*sqrt(b)*
sqrt(x)/2)/sqrt(b), True))
```

3.567 $\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}\,-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{2(2-bx)^{5/2}}{\sqrt{x}}-\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2}-\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx}\,-15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-15*b*Sqrt[x]*Sqrt[2 - b*x])/2 - (5*b*Sqrt[x]*(2 - b*x)^(3/2))/2 - (2*(2 - b*x)^(5/2))/Sqrt[x] - 15*Sqrt[b]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(2-bx)^{5/2}}{x^{3/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{\sqrt{x}} - (5b)\int \frac{(2-bx)^{3/2}}{\sqrt{x}}\,dx\\ &= -\frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - \frac{1}{2}(15b)\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - (15b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= -\frac{15}{2}b\sqrt{x}\,\sqrt{2-bx} - \frac{5}{2}b\sqrt{x}\,(2-bx)^{3/2} - \frac{2(2-bx)^{5/2}}{\sqrt{x}} - 15\sqrt{b}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.34

$$-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{bx}{2}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (b*x)/2])/Sqrt[x]
```

IntegrateAlgebraic [A] time = 0.15, size = 68, normalized size = 0.83

$$\frac{\sqrt{2-bx}\,\left(b^2x^2-9bx-16\right)}{2\sqrt{x}} - 15\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/x^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-16 - 9*b*x + b^2*x^2))/(2*Sqrt[x]) - 15*Sqrt[-b]*Log[-(Sqr
t[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

fricas [A] time = 1.32, size = 117, normalized size = 1.43

$$\left[\frac{15\sqrt{-b}\,x\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)+\left(b^2x^2-9\,bx-16\right)\sqrt{-bx+2}\,\sqrt{x}}{2\,x}, \frac{30\,\sqrt{b}\,x\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^2x^2-9\,bx-16\right)\sqrt{-bx+2}\,\sqrt{x}}{2\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*sqrt(-b)*x*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (b^2*
x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x))/x, 1/2*(30*sqrt(b)*x*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x^2 - 9*b*x - 16)*sqrt(-b*x + 2)*sqrt(x
))/x]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-b/abs(b)*b^2/b*(2*((-5/4-1/4*s
qrt(-b*x+2)*sqrt(-b*x+2))*sqrt(-b*x+2)*sqrt(-b*x+2)+15/2)*sqrt(-b*x+2)*sqrt
(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)+15/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b
)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [A] time = 0.02, size = 106, normalized size = 1.29

$$-\frac{15\sqrt{\left(-bx+2\right)x}\,\sqrt{b}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,\sqrt{x}}-\frac{\left(b^{3}x^{3}-11b^{2}x^{2}+2bx+32\right)\sqrt{\left(-bx+2\right)x}}{2\sqrt{-\left(bx-2\right)x}\,\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(3/2),x)
```

```
[Out] -1/2*(b^3*x^3-11*b^2*x^2+2*b*x+32)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-
b*x+2)^(1/2)/x^(1/2)-15/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(1/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.92, size = 96, normalized size = 1.17

$$15\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-\frac{\frac{7\,\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{9\,\left(-bx+2\right)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}}{b^{2}-\frac{2\,\left(bx-2\right)b}{x}+\frac{\left(bx-2\right)^{2}}{x^{2}}}-\frac{8\,\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(3/2),x, algorithm="maxima")
```

```
[Out] 15*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (7*sqrt(-b*x + 2)*b^2
/sqrt(x) + 9*(-b*x + 2)^(3/2)*b/x^(3/2))/(b^2 - 2*(b*x - 2)*b/x + (b*x - 2)
^2/x^2) - 8*sqrt(-b*x + 2)/sqrt(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(3/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(3/2), x)
```

sympy [A] time = 5.62, size = 202, normalized size = 2.46

$$\begin{cases} 15i\sqrt{b}\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) + \frac{ib^3x^{\frac{5}{2}}}{2\sqrt{bx-2}} - \frac{11ib^2x^{\frac{3}{2}}}{2\sqrt{bx-2}} + \frac{ib\sqrt{x}}{\sqrt{bx-2}} + \frac{16i}{\sqrt{x}\,\sqrt{bx-2}} & \text{for } \frac{|bx|}{2} > 1 \\ -15\sqrt{b}\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right) - \frac{b^3x^{\frac{5}{2}}}{2\sqrt{-bx+2}} + \frac{11b^2x^{\frac{3}{2}}}{2\sqrt{-bx+2}} - \frac{b\sqrt{x}}{\sqrt{-bx+2}} - \frac{16}{\sqrt{x}\,\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(3/2),x)
```

```
[Out] Piecewise((15*I*sqrt(b)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2) + I*b**3*x**(5/2)/
(2*sqrt(b*x - 2)) - 11*I*b**2*x**(3/2)/(2*sqrt(b*x - 2)) + I*b*sqrt(x)/sqrt
(b*x - 2) + 16*I/(sqrt(x)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-15*sqrt(b)*asi
n(sqrt(2)*sqrt(b)*sqrt(x)/2) - b**3*x**(5/2)/(2*sqrt(-b*x + 2)) + 11*b**2*x
**(3/2)/(2*sqrt(-b*x + 2)) - b*sqrt(x)/sqrt(-b*x + 2) - 16/(sqrt(x)*sqrt(-b
*x + 2)), True))
```

3.568 $\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx$

Optimal. Leaf size=84

$$10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)+5b^2\sqrt{x}\sqrt{2-bx}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$5b^2\sqrt{x}\sqrt{2-bx}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] 5*b^2*Sqrt[x]*Sqrt[2 - b*x] + (10*b*(2 - b*x)^(3/2))/(3*Sqrt[x]) - (2*(2 - b*x)^(5/2))/(3*x^(3/2)) + 10*b^(3/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2-bx)^{5/2}}{x^{5/2}}\,dx &= -\frac{2(2-bx)^{5/2}}{3x^{3/2}}-\frac{1}{3}(5b)\int \frac{(2-bx)^{3/2}}{x^{3/2}}\,dx\\
&= \frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{\sqrt{2-bx}}{\sqrt{x}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(5b^2\right)\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+\left(10b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)\\
&= 5b^2\sqrt{x}\sqrt{2-bx}+\frac{10b(2-bx)^{3/2}}{3\sqrt{x}}-\frac{2(2-bx)^{5/2}}{3x^{3/2}}+10b^{3/2}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.36

$$
-\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{bx}{2}\right)}{3x^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (-8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (b*x)/2])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 70, normalized size = 0.83

$$
\frac{\sqrt{2-bx}\left(3b^2x^2+28bx-8\right)}{3x^{3/2}}+10\sqrt{-b}\,b\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - b*x)^(5/2)/x^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-8 + 28*b*x + 3*b^2*x^2))/(3*x^(3/2)) + 10*Sqrt[-b]*b*Log[-
(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]]
```

fricas [A] time = 0.86, size = 126, normalized size = 1.50

$$
\left[\frac{15\sqrt{-b}\,bx^2\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+\left(3b^2x^2+28bx-8\right)\sqrt{-bx+2}\sqrt{x}}{3x^2},-\frac{30b^{\frac{3}{2}}x^2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\left(3b^2x^2+28bx-8\right)\sqrt{-bx+2}\sqrt{x}}{3x^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(15*sqrt(-b)*b*x^2*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + (
3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*x + 2)*sqrt(x))/x^2, -1/3*(30*b^(3/2)*x^2*a
rctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) - (3*b^2*x^2 + 28*b*x - 8)*sqrt(-b*
x + 2)*sqrt(x))/x^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]1/abs(b)*b^2/b*(2*((9*b^4/18/b*
sqrt(-b*x+2)*sqrt(-b*x+2)-120*b^4/18/b)*sqrt(-b*x+2)*sqrt(-b*x+2)+180*b^4/1
8/b)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)/(-b*(-b*x+2)+2*b)^2+10*b^2/sqrt(-b)
*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [A] time = 0.02, size = 107, normalized size = 1.27

$$\frac{5\sqrt{(-bx+2)\,x}\; b^{\frac{3}{2}} \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\;\sqrt{x}} - \frac{\left(3b^{3}x^{3}+22b^{2}x^{2}-64bx+16\right)\sqrt{(-bx+2)\,x}}{3\sqrt{-(bx-2)\,x}\;\sqrt{-bx+2}\;x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x+2)^(5/2)/x^(5/2),x)
```

```
[Out] -1/3*(3*b^3*x^3+22*b^2*x^2-64*b*x+16)/x^(3/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*
x)^(1/2)/(-b*x+2)^(1/2)+5*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)*b^(3/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.89, size = 79, normalized size = 0.94

$$-10\,b^{\frac{3}{2}} \arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right) + \frac{8\,\sqrt{-bx+2}\,b}{\sqrt{x}} + \frac{2\,\sqrt{-bx+2}\,b^{2}}{\left(b-\frac{bx-2}{x}\right)\sqrt{x}} - \frac{4\,(-bx+2)^{\frac{3}{2}}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)^(5/2)/x^(5/2),x, algorithm="maxima")
```

```
[Out] -10*b^(3/2)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + 8*sqrt(-b*x + 2)*b/s
qrt(x) + 2*sqrt(-b*x + 2)*b^2/((b - (b*x - 2)/x)*sqrt(x)) - 4/3*(-b*x + 2)^
(3/2)/x^(3/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(2-b\,x)^{5/2}}{x^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2 - b*x)^(5/2)/x^(5/2),x)
```

```
[Out] int((2 - b*x)^(5/2)/x^(5/2), x)
```

sympy [C] time = 5.35, size = 221, normalized size = 2.63

$$\begin{cases} b^{\frac{5}{2}}x\sqrt{-1+\frac{2}{bx}}+\frac{28b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\frac{1}{\sqrt{b}\sqrt{x}}\right)+10b^{\frac{3}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)-\frac{8\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|}>1 \\ ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}+\frac{28ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{3}+5ib^{\frac{3}{2}}\log\left(\frac{1}{bx}\right)-10ib^{\frac{3}{2}}\log\left(\sqrt{1-\frac{2}{bx}}+1\right)-\frac{8i\sqrt{b}\sqrt{1-\frac{2}{bx}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x+2)**(5/2)/x**(5/2),x)
```

```
[Out] Piecewise((b**(5/2)*x*sqrt(-1 + 2/(b*x)) + 28*b**(3/2)*sqrt(-1 + 2/(b*x))/3
 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(1/(sqrt(b)*sqrt(x))) + 10*
b**(3/2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2) - 8*sqrt(b)*sqrt(-1 + 2/(b*x))/(3*
x), 2/Abs(b*x) > 1), (I*b**(5/2)*x*sqrt(1 - 2/(b*x)) + 28*I*b**(3/2)*sqrt(1
 - 2/(b*x))/3 + 5*I*b**(3/2)*log(1/(b*x)) - 10*I*b**(3/2)*log(sqrt(1 - 2/(b
*x)) + 1) - 8*I*sqrt(b)*sqrt(1 - 2/(b*x))/(3*x), True))
```

3.569 $\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=101

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{5a^2\sqrt{x}\sqrt{a+bx}}{8b^3}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}-\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2}+\frac{x^{5/2}\sqrt{a+bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*a^2*Sqrt[x]*Sqrt[a + b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a + b*x])/(12*b^2
) + (x^(5/2)*Sqrt[a + b*x])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a
 + b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{a+bx}}\,dx = \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{(5a)\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= -\frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} + \frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{16b^3}$$

$$= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{8b^3}$$

$$= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^3}$$

$$= \frac{5a^2\sqrt{x}\,\sqrt{a+bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a+bx}}{12b^2} + \frac{x^{5/2}\sqrt{a+bx}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 0.84

$$\frac{\sqrt{a+bx}\left(\sqrt{b}\,\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15*a^(5/
2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 82, normalized size = 0.81

$$\frac{5a^3\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{8b^{7/2}} + \frac{\sqrt{a+bx}\left(15a^2\sqrt{x}-10abx^{3/2}+8b^2x^{5/2}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(15*a^2*Sqrt[x] - 10*a*b*x^(3/2) + 8*b^2*x^(5/2)))/(24*b^3)
+ (5*a^3*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(8*b^(7/2))
```

fricas [A] time = 1.33, size = 140, normalized size = 1.39

$$\left[\frac{15\,a^3\sqrt{b}\,\log\left(2\,bx-2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{48\,b^4}, \frac{15\,a^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)+\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*
(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4, 1/24*(15*a^
3*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (8*b^3*x^2 - 10*a*b
^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 102, normalized size = 1.01

$$\frac{\sqrt{bx+a}\,x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{bx+a}\,a\,x^{\frac{3}{2}}}{12b^{2}}-\frac{5\sqrt{(bx+a)\,x}\,a^{3}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{16\sqrt{bx+a}\,b^{\frac{7}{2}}\sqrt{x}}+\frac{5\sqrt{bx+a}\,a^{2}\sqrt{x}}{8b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(b*x+a)^(1/2)/b^2+5/8*a^2*x^(1/2
)*(b*x+a)^(1/2)/b^3-5/16*a^3/b^(7/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2
)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [A] time = 3.01, size = 146, normalized size = 1.45

$$\frac{5\,a^{3}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{16\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{bx+a}\,a^{3}b^{2}}{\sqrt{x}}-\frac{40\,(bx+a)^{\frac{3}{2}}a^{3}b}{x^{\frac{3}{2}}}+\frac{15\,(bx+a)^{\frac{5}{2}}a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{6}-\frac{3\,(bx+a)b^{5}}{x}+\frac{3\,(bx+a)^{2}b^{4}}{x^{2}}-\frac{(bx+a)^{3}b^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(7/2) - 1/24*(33*sqrt(b*x + a)*a^3*b^2/sqrt(x) - 40*(b*x + a)^(3/
2)*a^3*b/x^(3/2) + 15*(b*x + a)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x + a)*b^5/x
 + 3*(b*x + a)^2*b^4/x^2 - (b*x + a)^3*b^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(1/2), x)
```

sympy [A] time = 8.52, size = 128, normalized size = 1.27

$$\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1+\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1+\frac{bx}{a}}}-\frac{5a^{3}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*sqrt(x)/(8*b**3*sqrt(1 + b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*
sqrt(1 + b*x/a)) - sqrt(a)*x**(5/2)/(12*b*sqrt(1 + b*x/a)) - 5*a**3*asinh(s
qrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 + b*x/a))
```

3.570 $\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=77

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2}+\frac{x^{3/2}\sqrt{a+bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^2) + (x^(3/2)*Sqrt[a + b*x])/(2*b) + (3*a
^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx = \frac{x^{3/2}\sqrt{a+bx}}{2b} - \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{8b^2}$$

$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$

$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^2}$$

$$= -\frac{3a\sqrt{x}\sqrt{a+bx}}{4b^2} + \frac{x^{3/2}\sqrt{a+bx}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.05, size = 85, normalized size = 1.10

$$\frac{3a^{5/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 + (b*x)/a]
*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.08, size = 69, normalized size = 0.90

$$\frac{\sqrt{a+bx}\left(2bx^{3/2}-3a\sqrt{x}\right)}{4b^2}-\frac{3a^2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*(-3*a*Sqrt[x] + 2*b*x^(3/2)))/(4*b^2) - (3*a^2*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(5/2))
```

fricas [A] time = 1.15, size = 119, normalized size = 1.55

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{8\,b^3},\,-\frac{3\,a^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^2x-3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{4\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*(2
*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(-b)*arctan(sqr
t(b*x + a)*sqrt(-b)/(b*sqrt(x))) - (2*b^2*x - 3*a*b)*sqrt(b*x + a)*sqrt(x))
/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 84, normalized size = 1.09

$$\frac{\sqrt{bx+a}\,x^{\frac{3}{2}}}{2b}+\frac{3\sqrt{(bx+a)\,x}\,a^{2}\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{8\sqrt{bx+a}\,b^{\frac{5}{2}}\sqrt{x}}-\frac{3\sqrt{bx+a}\,a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(b*x+a)^(1/2)/b^2+3/8*a^2/b^(5/2)
*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)
^(1/2))
```

maxima [B] time = 2.87, size = 112, normalized size = 1.45

$$-\frac{3\,a^{2}\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\frac{5\,\sqrt{bx+a}\,a^{2}b}{\sqrt{x}}-\frac{3\,(bx+a)^{\frac{3}{2}}a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{4}-\frac{2\,(bx+a)b^{3}}{x}+\frac{(bx+a)^{2}b^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sq
rt(x)))/b^(5/2) + 1/4*(5*sqrt(b*x + a)*a^2*b/sqrt(x) - 3*(b*x + a)^(3/2)*a^
2/x^(3/2))/(b^4 - 2*(b*x + a)*b^3/x + (b*x + a)^2*b^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(1/2), x)
```

sympy [A] time = 4.30, size = 100, normalized size = 1.30

$$-\frac{3a^{\frac{3}{2}}\sqrt{x}}{4b^{2}\sqrt{1+\frac{bx}{a}}}-\frac{\sqrt{a}\,x^{\frac{3}{2}}}{4b\sqrt{1+\frac{bx}{a}}}+\frac{3a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*sqrt(x)/(4*b**2*sqrt(1 + b*x/a)) - sqrt(a)*x**(3/2)/(4*b*sqrt(1
 + b*x/a)) + 3*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/
(2*sqrt(a)*sqrt(1 + b*x/a))
```

3.571 $\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=48

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{x}\,\sqrt{a+bx}}{b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b - (a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^
(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}$$
$$= \frac{\sqrt{x}\sqrt{a+bx}}{b} - \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 1.42

$$\frac{\sqrt{b}\sqrt{x}(a+bx) - a^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(a + b*x) - a^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 1.02

$$\frac{a\log\left(\sqrt{a+bx} - \sqrt{b}\sqrt{x}\right)}{b^{3/2}} + \frac{\sqrt{x}\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[a + b*x])/b + (a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(
3/2)
```

fricas [A] time = 1.47, size = 91, normalized size = 1.90

$$\left[\frac{a\sqrt{b}\,\log\left(2\,bx - 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) + 2\,\sqrt{bx+a}\,b\sqrt{x}}{2\,b^2}, \frac{a\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) + \sqrt{bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) + 2*sqrt(b
*x + a)*b*sqrt(x))/b^2, (a*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x
))) + sqrt(b*x + a)*b*sqrt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.00, size = 65, normalized size = 1.35

$$-\frac{\sqrt{(bx+a)\,x}\;a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2\sqrt{bx+a}\;b^{\frac{3}{2}}\sqrt{x}}+\frac{\sqrt{bx+a}\;\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b-1/2*a/b^(3/2)*((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/
2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))
```

maxima [B] time = 2.91, size = 73, normalized size = 1.52

$$\frac{a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{3}{2}}}-\frac{\sqrt{bx+a}\,a}{\left(b^{2}-\frac{(bx+a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(
x)))/b^(3/2) - sqrt(b*x + a)*a/((b^2 - (b*x + a)*b/x)*sqrt(x))
```

mupad [B] time = 0.55, size = 44, normalized size = 0.92

$$\frac{\sqrt{x}\;\sqrt{a+b\,x}}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{b}\;\sqrt{x}}{\sqrt{a+b\,x}-\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (x^(1/2)*(a + b*x)^(1/2))/b - (2*a*atanh((b^(1/2)*x^(1/2))/((a + b*x)^(1/2)
 - a^(1/2))))/b^(3/2)
```

sympy [A] time = 2.19, size = 44, normalized size = 0.92

$$\frac{\sqrt{a}\,\sqrt{x}\,\sqrt{1+\frac{bx}{a}}}{b}-\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] sqrt(a)*sqrt(x)*sqrt(1 + b*x/a)/b - a*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/
2)
```

3.572 $\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=28

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*S
qrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.07

$$-\frac{2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/Sqrt[b]
```

fricas [A] time = 1.04, size = 57, normalized size = 2.04

$$\left[\frac{\log\left(2\,bx+2\,\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a)/sqrt(b), -2*sqrt(-b)*arct
an(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x)))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.00, size = 48, normalized size = 1.71

$$\frac{\sqrt{(bx+a)\,x}\,\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] ((b*x+a)*x)^(1/2)/x^(1/2)/(b*x+a)^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^
(1/2))/b^(1/2)
```

maxima [B] time = 2.95, size = 41, normalized size = 1.46

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
sqrt(b)
```

mupad [B] time = 0.03, size = 30, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}-\sqrt{a}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a + b*x)^(1/2) - a^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

sympy [A] time = 1.10, size = 22, normalized size = 0.79

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b)
```

3.573 $\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=19

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx = -\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{a+bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(a*Sqrt[x])
```

fricas [A] time = 0.95, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

giac [B] time = 2.05, size = 33, normalized size = 1.74

$$-\frac{2\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a*abs(b))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/a/x^(1/2)
```

maxima [A] time = 1.34, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(b*x + a)/(a*sqrt(x))
```

mupad [B] time = 0.35, size = 15, normalized size = 0.79

$$-\frac{2\sqrt{a+b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/(a*x^(1/2))
```

sympy [A] time = 0.91, size = 19, normalized size = 1.00

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/a
```

3.574 $\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=44

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a+bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(3*a*x^(3/2)) + (4*b*Sqrt[a + b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a+bx}}{3ax^{3/2}}+\frac{4b\sqrt{a+bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.61

$$-\frac{2(a-2bx)\sqrt{a+bx}}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[a + b*x])/(3*a^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 29, normalized size = 0.66

$$\frac{2\sqrt{a+bx}\,(2bx-a)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[a + b*x]),x]
[Out] (2*Sqrt[a + b*x]*(-a + 2*b*x))/(3*a^2*x^(3/2))
```

fricas [A] time = 1.15, size = 23, normalized size = 0.52

$$\frac{2\,(2\,bx-a)\sqrt{bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x - a)*sqrt(b*x + a)/(a^2*x^(3/2))
```

giac [A] time = 1.90, size = 50, normalized size = 1.14

$$\frac{2\left(\frac{2\,(bx+a)b^3}{a^2}-\frac{3\,b^3}{a}\right)\sqrt{bx+a}\,b}{3\,((bx+a)b-ab)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] 2/3*(2*(b*x + a)*b^3/a^2 - 3*b^3/a)*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(3
/2)*abs(b))
```

maple [A] time = 0.00, size = 22, normalized size = 0.50

$$-\frac{2\sqrt{bx+a}\;(-2bx+a)}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(1/2),x)
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*x+a)/x^(3/2)/a^2
```

maxima [A] time = 1.30, size = 31, normalized size = 0.70

$$\frac{2\left(\frac{3\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^2
```

mupad [B] time = 0.34, size = 25, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}-\frac{4\,b\,x}{3\,a^2}\right)\sqrt{a+b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) - (4*b*x)/(3*a^2))*(a + b*x)^(1/2))/x^(3/2)
```

sympy [A] time = 1.92, size = 42, normalized size = 0.95

$$-\frac{2\sqrt{b}\sqrt{\frac{a}{bx}+1}}{3ax}+\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] -2*sqrt(b)*sqrt(a/(b*x) + 1)/(3*a*x) + 4*b**(3/2)*sqrt(a/(b*x) + 1)/(3*a**2
)
```

3.575 $\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=68

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{2\sqrt{a+bx}}{5ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(5*a*x^(5/2)) + (8*b*Sqrt[a + b*x])/(15*a^2*x^(3/2)) - (
16*b^2*Sqrt[a + b*x])/(15*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}-\frac{(4b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a}\\ &= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}+\frac{\left(8b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{a+bx}}{5ax^{5/2}}+\frac{8b\sqrt{a+bx}}{15a^2x^{3/2}}-\frac{16b^2\sqrt{a+bx}}{15a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a + b*x]),x]
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 40, normalized size = 0.59

$$-\frac{2\sqrt{a+bx}\,(3a^2-4abx+8b^2x^2)}{15a^3x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[a + b*x]),x]
[Out] (-2*Sqrt[a + b*x]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(5/2))
```

fricas [A] time = 1.21, size = 34, normalized size = 0.50

$$-\frac{2\,(8\,b^2x^2-4\,abx+3\,a^2)\sqrt{bx+a}}{15\,a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x + a)/(a^3*x^(5/2))
```

giac [A] time = 1.67, size = 66, normalized size = 0.97

$$-\frac{2\left(\frac{15\,b^5}{a}+4\left(\frac{2\,(bx+a)b^5}{a^3}-\frac{5\,b^5}{a^2}\right)(bx+a)\right)\sqrt{bx+a}\,b}{15\,((bx+a)b-ab)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="giac")
[Out] -2/15*(15*b^5/a + 4*(2*(b*x + a)*b^5/a^3 - 5*b^5/a^2)*(b*x + a))*sqrt(b*x +
 a)*b/(((b*x + a)*b - a*b)^(5/2)*abs(b))
```

maple [A] time = 0.00, size = 35, normalized size = 0.51

$$-\frac{2\sqrt{bx+a}\,(8b^2x^2-4abx+3a^2)}{15a^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(1/2),x)
[Out] -2/15*(b*x+a)^(1/2)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(5/2)/a^3
```

maxima [A] time = 1.34, size = 46, normalized size = 0.68

$$-\frac{2\left(\frac{15\,\sqrt{bx+a}\,b^2}{\sqrt{x}}-\frac{10\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{3\,(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/15*(15*sqrt(b*x + a)*b^2/sqrt(x) - 10*(b*x + a)^(3/2)*b/x^(3/2) + 3*(b*x
 + a)^(5/2)/x^(5/2))/a^3
```

mupad [B] time = 0.35, size = 36, normalized size = 0.53

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a}+\frac{16\,b^2\,x^2}{15\,a^3}-\frac{8\,b\,x}{15\,a^2}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(5*a) + (16*b^2*x^2)/(15*a^3) - (8*b*x)/(15*a^2)))/x^(
5/2)
```

sympy [B] time = 6.25, size = 287, normalized size = 4.22

$$-\frac{6a^4b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{4a^3b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{6a^2b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{24ab^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}-\frac{16b^{\frac{17}{2}}x^4\sqrt{\frac{a}{bx}+1}}{15a^5b^4x^2+30a^4b^5x^3+15a^3b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(1/2),x)
```

```
[Out] -6*a**4*b**(9/2)*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 +
 15*a**3*b**6*x**4) - 4*a**3*b**(11/2)*x*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 6*a**2*b**(13/2)*x**2*sqrt(a/
(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4) - 24
*a*b**(15/2)*x**3*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x**2 + 30*a**4*b**5*x**3
+ 15*a**3*b**6*x**4) - 16*b**(17/2)*x**4*sqrt(a/(b*x) + 1)/(15*a**5*b**4*x*
*2 + 30*a**4*b**5*x**3 + 15*a**3*b**6*x**4)
```

3.576 $\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=92

$$\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Rubi [A] time = 0.02, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{2\sqrt{a+bx}}{7ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(7*a*x^(7/2)) + (12*b*Sqrt[a + b*x])/(35*a^2*x^(5/2)) -
(16*b^2*Sqrt[a + b*x])/(35*a^3*x^(3/2)) + (32*b^3*Sqrt[a + b*x])/(35*a^4*Sq
rt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{a+bx}}\,dx &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}}-\frac{(6b)\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{7a}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}+\frac{\left(24b^2\right)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{35a^2}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}-\frac{\left(16b^3\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{35a^3}\\ &= -\frac{2\sqrt{a+bx}}{7ax^{7/2}}+\frac{12b\sqrt{a+bx}}{35a^2x^{5/2}}-\frac{16b^2\sqrt{a+bx}}{35a^3x^{3/2}}+\frac{32b^3\sqrt{a+bx}}{35a^4\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.55

$$-\frac{2\sqrt{a+bx}\left(5a^3-6a^2bx+8ab^2x^2-16b^3x^3\right)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(5*a^3 - 6*a^2*b*x + 8*a*b^2*x^2 - 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 51, normalized size = 0.55

$$\frac{2\sqrt{a+bx}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(9/2)*Sqrt[a + b*x]),x]
```

```
[Out] (2*Sqrt[a + b*x]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35*a^4*x
^(7/2))
```

fricas [A] time = 1.21, size = 45, normalized size = 0.49

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx+a}}{35\,a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x + a)/(a^4*x^(7
/2))
```

giac [A] time = 1.41, size = 82, normalized size = 0.89

$$-\frac{2\left(\frac{35\,b^7}{a}-2\left(\frac{35\,b^7}{a^2}+4\left(\frac{2\,(bx+a)b^7}{a^4}-\frac{7\,b^7}{a^3}\right)(bx+a)\right)(bx+a)\right)\sqrt{bx+a}\,b}{35\,((bx+a)b-ab)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(35*b^7/a - 2*(35*b^7/a^2 + 4*(2*(b*x + a)*b^7/a^4 - 7*b^7/a^3)*(b*x
+ a))*(b*x + a))*sqrt(b*x + a)*b/(((b*x + a)*b - a*b)^(7/2)*abs(b))
```

maple [A] time = 0.00, size = 46, normalized size = 0.50

$$-\frac{2\sqrt{bx+a}\,\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35a^4x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+a)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)^(1/2)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(7/2)/a^4
```

maxima [A] time = 1.30, size = 61, normalized size = 0.66

$$\frac{2\left(\frac{35\,\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{35\,(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}+\frac{21\,(bx+a)^{\frac{5}{2}}b}{x^{\frac{5}{2}}}-\frac{5\,(bx+a)^{\frac{7}{2}}}{x^{\frac{7}{2}}}\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(35*sqrt(b*x + a)*b^3/sqrt(x) - 35*(b*x + a)^(3/2)*b^2/x^(3/2) + 21*(b
*x + a)^(5/2)*b/x^(5/2) - 5*(b*x + a)^(7/2)/x^(7/2))/a^4
```

mupad [B] time = 0.38, size = 47, normalized size = 0.51

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{7\,a}+\frac{16\,b^2\,x^2}{35\,a^3}-\frac{32\,b^3\,x^3}{35\,a^4}-\frac{12\,b\,x}{35\,a^2}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(7*a) + (16*b^2*x^2)/(35*a^3) - (32*b^3*x^3)/(35*a^4)
- (12*b*x)/(35*a^2)))/x^(7/2)
```

sympy [B] time = 16.14, size = 488, normalized size = 5.30

$$-\frac{10a^6b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{18a^5b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}-\frac{10a^4b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{10a^3b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{60a^2b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{80ab^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}+\frac{32b^{\frac{31}{2}}x^6\sqrt{\frac{a}{bx}+1}}{35a^7b^9x^3+105a^6b^{10}x^4+105a^5b^{11}x^5+35a^4b^{12}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+a)**(1/2),x)
```

```
[Out] -10*a**6*b**(19/2)*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x*
*4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) - 18*a**5*b**(21/2)*x*sqrt(a
/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5
+ 35*a**4*b**12*x**6) - 10*a**4*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(35*a**7*b
**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
+ 10*a**3*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b*
*10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6) + 60*a**2*b**(27/2)*x*
*4*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b*
*11*x**5 + 35*a**4*b**12*x**6) + 80*a*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(35*
a**7*b**9*x**3 + 105*a**6*b**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*
x**6) + 32*b**(31/2)*x**6*sqrt(a/(b*x) + 1)/(35*a**7*b**9*x**3 + 105*a**6*b
**10*x**4 + 105*a**5*b**11*x**5 + 35*a**4*b**12*x**6)
```

3.577 $\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=96

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{15a\sqrt{x}\sqrt{a+bx}}{4b^3}-\frac{2x^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[a + b*x]) - (15*a*Sqrt[x]*Sqrt[a + b*x])/(4*b^3) + (5*
x^(3/2)*Sqrt[a + b*x])/(2*b^2) + (15*a^2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a +
 b*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{4b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{8b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{a+bx}}-\frac{15a\sqrt{x}\,\sqrt{a+bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a+bx}}{2b^2}+\frac{15a^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.52

$$\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\ {}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, -((b*x)/a)])/
(7*a*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 82, normalized size = 0.85

$$\frac{-15a^2\sqrt{x}-5abx^{3/2}+2b^2x^{5/2}}{4b^3\sqrt{a+bx}}-\frac{15a^2\log\left(\sqrt{a+bx}-\sqrt{b}\,\sqrt{x}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-15*a^2*Sqrt[x] - 5*a*b*x^(3/2) + 2*b^2*x^(5/2))/(4*b^3*Sqrt[a + b*x]) - (
15*a^2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(4*b^(7/2))
```

fricas [A] time = 1.41, size = 175, normalized size = 1.82

$$\left[\frac{15\left(a^2bx+a^3\right)\sqrt{b}\log\left(2\,bx+2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x}+a\right)+2\left(2\,b^3x^2-5\,ab^2x-15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{8\left(b^5x+ab^4\right)},-\frac{15\left(a^2bx+a^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(2\,b^3x^2-5\,ab^2x-15\,a^2b\right)\sqrt{bx+a}\,\sqrt{x}}{4\left(b^5x+ab^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(15*(a^2*b*x + a^3)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x
) + a) + 2*(2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x
 + a*b^4), -1/4*(15*(a^2*b*x + a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/
```

```
(b*sqrt(x))) - (2*b^3*x^2 - 5*a*b^2*x - 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x + a*b^4)]
```

giac [A] time = 92.14, size = 131, normalized size = 1.36

$$\frac{\left(2\sqrt{(bx+a)b-ab}\sqrt{bx+a}\left(\frac{2(bx+a)}{b^3}-\frac{9a}{b^3}\right)-\frac{32a^3}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{3}{2}}}-\frac{15a^2\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{b^{\frac{5}{2}}}\right)|b|}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)*(2*(b*x + a)/b^3 - 9*a/b^3) -
32*a^3/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(3/2)
) - 15*a^2*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
)*abs(b)/b^2
```

maple [A] time = 0.04, size = 119, normalized size = 1.24

$$\frac{\left(\frac{15a^2\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{8b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^2 b}\;a^2}{\left(x+\frac{a}{b}\right)b^4}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\;\sqrt{x}}-\frac{(-2bx+7a)\sqrt{bx+a}\;\sqrt{x}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] -1/4*(-2*b*x+7*a)*(b*x+a)^(1/2)*x^(1/2)/b^3+(15/8/b^(7/2)*a^2*ln((b*x+1/2*a
)/b^(1/2)+(b*x^2+a*x)^(1/2))-2/b^4*a^2/(x+a/b)*(b*(x+a/b)^2-(x+a/b)*a)^(1/2
))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.92, size = 131, normalized size = 1.36

$$-\frac{8a^2b^2-\frac{25(bx+a)a^2b}{x}+\frac{15(bx+a)^2a^2}{x^2}}{4\left(\frac{\sqrt{bx+a}\,b^5}{\sqrt{x}}-\frac{2(bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{15a^2\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{8b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^2*b^2 - 25*(b*x + a)*a^2*b/x + 15*(b*x + a)^2*a^2/x^2)/(sqrt(b*x
+ a)*b^5/sqrt(x) - 2*(b*x + a)^(3/2)*b^4/x^(3/2) + (b*x + a)^(5/2)*b^3/x^(5
/2)) - 15/8*a^2*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
+ a)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(3/2), x)
```

sympy [A] time = 8.14, size = 105, normalized size = 1.09

$$-\frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^{3}\sqrt{1+\frac{bx}{a}}}-\frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^{2}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{2}\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 + b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**2*
sqrt(1 + b*x/a)) + 15*a**2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) + x*
*(5/2)/(2*sqrt(a)*b*sqrt(1 + b*x/a))
```

3.578 $\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[a + b*x]) + (3*Sqrt[x]*Sqrt[a + b*x])/b^2 - (3*a*ArcTa
nh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{(3a)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{x}\sqrt{a+bx}}{b^2}-\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.74

$$\frac{2x^{5/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{a}\right)}{5a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, -((b*x)/a)])/
(5*a*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.13, size = 61, normalized size = 0.90

$$\frac{3a\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}+\frac{3a\sqrt{x}+bx^{3/2}}{b^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*a*Sqrt[x] + b*x^(3/2))/(b^2*Sqrt[a + b*x]) + (3*a*Log[-(Sqrt[b]*Sqrt[x])
 + Sqrt[a + b*x]])/b^(5/2)
```

fricas [A] time = 1.39, size = 145, normalized size = 2.13

$$\left[\frac{3\left(abx+a^2\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}}{2\left(b^4x+ab^3\right)},\frac{3\left(abx+a^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2x+3ab\right)\sqrt{bx+a}\sqrt{x}}{b^4x+ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + a^2)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) +
 a) + 2*(b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3), (3*(a*b*x +
 a^2)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 3*a*b)
*sqrt(b*x + a)*sqrt(x))/(b^4*x + a*b^3)]
```

giac [B] time = 93.12, size = 115, normalized size = 1.69

$$\frac{\left(\frac{8\,a^2\sqrt{b}}{\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{3\,a\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}+\frac{2\,\sqrt{(bx+a)b-ab}\,\sqrt{bx+a}}{b}\right)|b|}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(8*a^2*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a
*b) + 3*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b)
+ 2*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b)*abs(b)/b^3
```

maple [B] time = 0.03, size = 106, normalized size = 1.56

$$\frac{\left(-\frac{3a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^2+ax}\right)}{2b^{\frac{5}{2}}}+\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^2b}\,a}{\left(x+\frac{a}{b}\right)b^3}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\,\sqrt{x}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x
)^(1/2))+2*a/b^3/(x+a/b)*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2))*((b*x+a)*x)^(1/2)/
(b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 3.02, size = 92, normalized size = 1.35

$$\frac{2\,ab-\frac{3\,(bx+a)a}{x}}{\frac{\sqrt{bx+a}\,b^3}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x + a)*a/x)/(sqrt(b*x + a)*b^3/sqrt(x) - (b*x + a)^(3/2)*b^2/
x^(3/2)) + 3/2*a*log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x
 + a)/sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(3/2), x)
```

sympy [A] time = 3.68, size = 71, normalized size = 1.04

$$\frac{3\sqrt{a}\,\sqrt{x}}{b^2\sqrt{1+\frac{bx}{a}}}-\frac{3a\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] 3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 + b*x/a)) - 3*a*asinh(sqrt(b)*sqrt(x)/sqrt(a
))/b**(5/2) + x**(3/2)/(sqrt(a)*b*sqrt(1 + b*x/a))
```

3.579 $\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) + (2*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx = -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b}$$
$$= -\frac{2\sqrt{x}}{b\sqrt{a+bx}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 1.33

$$\frac{2\left(\sqrt{a}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)-\sqrt{b}\sqrt{x}\right)}{b^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(3/2),x]
[Out] (2*(-(Sqrt[b]*Sqrt[x]) + Sqrt[a]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x]
)/Sqrt[a]]))/(b^(3/2)*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.08, size = 50, normalized size = 1.04

$$-\frac{2\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^(3/2),x]
[Out] (-2*Sqrt[x])/(b*Sqrt[a + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]
)/b^(3/2)
```

fricas [A] time = 1.33, size = 119, normalized size = 2.48

$$\left[\frac{(bx+a)\sqrt{b}\log\left(2\,bx+2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)-2\sqrt{bx+a}\,b\sqrt{x}}{b^3x+ab^2}, -\frac{2\left((bx+a)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+a}\,b\sqrt{x}\right)}{b^3x+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] [((b*x + a)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(b)*sqrt(x) + a) - 2*sq
rt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2), -2*((b*x + a)*sqrt(-b)*arctan(sqrt(
b*x + a)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + a)*b*sqrt(x))/(b^3*x + a*b^2)]
```

giac [B] time = 94.86, size = 85, normalized size = 1.77

$$-\frac{\left(\frac{4\,a\sqrt{b}}{\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab}+\frac{\log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2\right)}{\sqrt{b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(b)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b) +
 log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b))*abs(b)/b
^2
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x+a)^(3/2),x)
```

maxima [A] time = 2.98, size = 57, normalized size = 1.19

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + a)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a+b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a + b*x)^(3/2), x)
```

sympy [A] time = 1.78, size = 46, normalized size = 0.96

$$\frac{2\,\operatorname{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*sqrt(x)/(sqrt(a)*b*sqrt(1 + b
*x/a))
```

3.580 $\int \frac{1}{\sqrt{x}(a+bx)^{3/2}} dx$

Optimal. Leaf size=19

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a+bx)^{3/2}} dx = \frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^(3/2)),x]
[Out] (2*Sqrt[x])/(a*Sqrt[a + b*x])
```

fricas [A] time = 1.01, size = 22, normalized size = 1.16

$$\frac{2\sqrt{bx+a}\sqrt{x}}{abx+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(x)/(a*b*x + a^2)
```

giac [B] time = 1.11, size = 45, normalized size = 2.37

$$\frac{4 b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*abs(
b))
```

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a/(b*x+a)^(1/2)
```

maxima [A] time = 1.32, size = 15, normalized size = 0.79

$$\frac{2\sqrt{x}}{\sqrt{bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(b*x + a)*a)
```

mupad [B] time = 0.33, size = 22, normalized size = 1.16

$$\frac{2\sqrt{x}\sqrt{a+bx}}{a^2+b\,x\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a + b*x)^(1/2))/(a^2 + a*b*x)
```

sympy [A] time = 0.88, size = 17, normalized size = 0.89

$$\frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] 2/(a*sqrt(b)*sqrt(a/(b*x) + 1))
```

3.581 $\int \frac{1}{x^{3/2}(a+bx)^{3/2}} \, dx$

Optimal. Leaf size=39

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] 2/(a*Sqrt[x]*Sqrt[a + b*x]) - (4*Sqrt[a + b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^{3/2}} \, dx &= \frac{2}{a\sqrt{x}\sqrt{a+bx}} + \frac{2\int \frac{1}{x^{3/2}\sqrt{a+bx}} \, dx}{a} \\ &= \frac{2}{a\sqrt{x}\sqrt{a+bx}} - \frac{4\sqrt{a+bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.08, size = 25, normalized size = 0.64

$$-\frac{2(a+2bx)}{a^2\sqrt{x}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a + 2*b*x))/(a^2*Sqrt[x]*Sqrt[a + b*x])
```

fricas [A] time = 0.89, size = 34, normalized size = 0.87

$$-\frac{2\,(2\,bx+a)\sqrt{bx+a}\,\sqrt{x}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x + a)*sqrt(b*x + a)*sqrt(x)/(a^2*b*x^2 + a^3*x)
```

giac [B] time = 1.05, size = 82, normalized size = 2.10

$$-\frac{4\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a|b|}-\frac{2\,\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(5/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a*a
bs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^2*abs(b))
```

maple [A] time = 0.01, size = 22, normalized size = 0.56

$$-\frac{2\,(2bx+a)}{\sqrt{bx+a}\;a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2*(2*b*x+a)/(b*x+a)^(1/2)/x^(1/2)/a^2
```

maxima [A] time = 1.32, size = 32, normalized size = 0.82

$$-\frac{2\,b\sqrt{x}}{\sqrt{bx+a}\,a^2}-\frac{2\,\sqrt{bx+a}}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b*sqrt(x)/(sqrt(b*x + a)*a^2) - 2*sqrt(b*x + a)/(a^2*sqrt(x))
```

mupad [B] time = 0.39, size = 39, normalized size = 1.00

$$-\frac{2\,a\,\sqrt{a+b\,x}\;+4\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\;\left(a^3+b\,x\,a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(3/2)),x)
[Out] -(2*a*(a + b*x)^(1/2) + 4*b*x*(a + b*x)^(1/2))/(x^(1/2)*(a^3 + a^2*b*x))
```

sympy [A] time = 1.60, size = 41, normalized size = 1.05

$$-\frac{2}{a\sqrt{b}\,x\sqrt{\frac{a}{bx}+1}}-\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(3/2),x)
[Out] -2/(a*sqrt(b)*x*sqrt(a/(b*x) + 1)) - 4*sqrt(b)/(a**2*sqrt(a/(b*x) + 1))
```

3.582 $\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a + b*x]) - (8*Sqrt[a + b*x])/(3*a^2*x^(3/2)) + (16*b*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx &= \frac{2}{ax^{3/2}\sqrt{a+bx}}+\frac{4\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a}\\ &= \frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}-\frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}\\ &= \frac{2}{ax^{3/2}\sqrt{a+bx}}-\frac{8\sqrt{a+bx}}{3a^2x^{3/2}}+\frac{16b\sqrt{a+bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.60

$$-\frac{2\left(a^2-4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(3/2)),x]
[Out] (-2*(a^2 - 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 40, normalized size = 0.63

$$\frac{2(-a^2 + 4abx + 8b^2x^2)}{3a^3x^{3/2}\sqrt{a + bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^(3/2)),x]
[Out] (2*(-a^2 + 4*a*b*x + 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a + b*x])
```

fricas [A] time = 1.35, size = 49, normalized size = 0.78

$$\frac{2\left(8\,b^2x^2 + 4\,abx - a^2\right)\sqrt{bx + a}\,\sqrt{x}}{3\left(a^3bx^3 + a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
[Out] 2/3*(8*b^2*x^2 + 4*a*b*x - a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b*x^3 + a^4*x^2)
```

giac [B] time = 1.20, size = 98, normalized size = 1.56

$$\frac{2\sqrt{bx + a}\left(\frac{5\,(bx+a)b^2|b|}{a^3} - \frac{6\,b^2|b|}{a^2}\right)}{3\,((bx + a)b - ab)^{\frac{3}{2}}} + \frac{4\,b^{\frac{7}{2}}}{\left(\left(\sqrt{bx + a}\,\sqrt{b} - \sqrt{(bx + a)b - ab}\right)^2 + ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
[Out] 2/3*sqrt(b*x + a)*(5*(b*x + a)*b^2*abs(b)/a^3 - 6*b^2*abs(b)/a^2)/((b*x + a
)*b - a*b)^(3/2) + 4*b^(7/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b -
a*b))^2 + a*b)*a^2*abs(b))
```

maple [A] time = 0.00, size = 33, normalized size = 0.52

$$-\frac{2\left(-8b^2x^2 - 4abx + a^2\right)}{3\sqrt{bx + a}\;a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(3/2),x)
[Out] -2/3*(-8*b^2*x^2-4*a*b*x+a^2)/(b*x+a)^(1/2)/x^(3/2)/a^3
```

maxima [A] time = 1.31, size = 50, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{bx + a}\,a^3} + \frac{2\left(\frac{6\,\sqrt{bx+a}\,b}{\sqrt{x}} - \frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(b*x + a)*a^3) + 2/3*(6*sqrt(b*x + a)*b/sqrt(x) - (b*x +
 a)^(3/2)/x^(3/2))/a^3
```

mupad [B] time = 0.41, size = 46, normalized size = 0.73

$$\frac{\sqrt{a+b\,x}\,\left(\frac{8\,x}{3\,a^2}-\frac{2}{3\,a\,b}+\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}+\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(3/2)),x)
```

```
[Out] ((a + b*x)^(1/2)*((8*x)/(3*a^2) - 2/(3*a*b) + (16*b*x^2)/(3*a^3)))/(x^(5/2)
 + (a*x^(3/2))/b)
```

sympy [B] time = 3.98, size = 219, normalized size = 3.48

$$-\frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}+\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^5b^4x+6a^4b^5x^2+3a^3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**3*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a*
*3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**
4*b**5*x**2 + 3*a**3*b**6*x**3) + 24*a*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*
a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3) + 16*b**(15/2)*x**3*sqrt
(a/(b*x) + 1)/(3*a**5*b**4*x + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**3)
```

3.583 $\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{2}{ax^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(5/2)*Sqrt[a + b*x]) - (12*Sqrt[a + b*x])/(5*a^2*x^(5/2)) + (16*b*Sq
rt[a + b*x])/(5*a^3*x^(3/2)) - (32*b^2*Sqrt[a + b*x])/(5*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(a+bx)^{3/2}}\,dx &= \frac{2}{ax^{5/2}\sqrt{a+bx}}+\frac{6\int \frac{1}{x^{7/2}\sqrt{a+bx}}\,dx}{a}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}-\frac{(24b)\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{5a^2}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}+\frac{\left(16b^2\right)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{5a^3}\\
&= \frac{2}{ax^{5/2}\sqrt{a+bx}}-\frac{12\sqrt{a+bx}}{5a^2x^{5/2}}+\frac{16b\sqrt{a+bx}}{5a^3x^{3/2}}-\frac{32b^2\sqrt{a+bx}}{5a^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.56

$$-\frac{2\left(a^3-2a^2bx+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 49, normalized size = 0.56

$$-\frac{2\left(a^3-2a^2bx+8ab^2x^2+16b^3x^3\right)}{5a^4x^{5/2}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(a + b*x)^(3/2)),x]
```

```
[Out] (-2*(a^3 - 2*a^2*b*x + 8*a*b^2*x^2 + 16*b^3*x^3))/(5*a^4*x^(5/2)*Sqrt[a + b
*x])
```

fricas [A] time = 1.18, size = 58, normalized size = 0.67

$$-\frac{2\left(16\,b^3x^3+8\,ab^2x^2-2\,a^2bx+a^3\right)\sqrt{bx+a}\,\sqrt{x}}{5\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*x^3 + 8*a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b*x^4 + a^5*x^3)
```

giac [A] time = 1.21, size = 121, normalized size = 1.39

$$-\frac{4\,b^{\frac{9}{2}}}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)a^3|b|}-\frac{2\left(\frac{15\,b^6}{a^2|b|}+(bx+a)\left(\frac{11\,(bx+a)b^6}{a^4|b|}-\frac{25\,b^6}{a^3|b|}\right)\right)\sqrt{bx+a}}{5\left((bx+a)b-ab\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*b^(9/2)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*a^3
*abs(b)) - 2/5*(15*b^6/(a^2*abs(b)) + (b*x + a)*(11*(b*x + a)*b^6/(a^4*abs(
b)) - 25*b^6/(a^3*abs(b))))*sqrt(b*x + a)/((b*x + a)*b - a*b)^(5/2)
```

maple [A] time = 0.00, size = 44, normalized size = 0.51

$$-\frac{2\left(16b^3x^3+8a\,b^2x^2-2a^2bx+a^3\right)}{5\sqrt{bx+a}\;a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+a)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*x^3+8*a*b^2*x^2-2*a^2*b*x+a^3)/(b*x+a)^(1/2)/x^(5/2)/a^4
```

maxima [A] time = 1.31, size = 64, normalized size = 0.74

$$-\frac{2\,b^3\sqrt{x}}{\sqrt{bx+a}\,a^4}-\frac{2\left(\frac{15\,\sqrt{bx+a}\,b^2}{\sqrt{x}}-\frac{5\,(bx+a)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{(bx+a)^{\frac{5}{2}}}{x^{\frac{5}{2}}}\right)}{5\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*b^3*sqrt(x)/(sqrt(b*x + a)*a^4) - 2/5*(15*sqrt(b*x + a)*b^2/sqrt(x) - 5*
(b*x + a)^(3/2)*b/x^(3/2) + (b*x + a)^(5/2)/x^(5/2))/a^4
```

mupad [B] time = 0.43, size = 58, normalized size = 0.67

$$-\frac{\sqrt{a+b\,x}\,\left(\frac{2}{5\,a\,b}-\frac{4\,x}{5\,a^2}+\frac{16\,b\,x^2}{5\,a^3}+\frac{32\,b^2\,x^3}{5\,a^4}\right)}{x^{7/2}+\frac{a\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a + b*x)^(3/2)),x)
```

```
[Out] -((a + b*x)^(1/2)*(2/(5*a*b) - (4*x)/(5*a^2) + (16*b*x^2)/(5*a^3) + (32*b^2
*x^3)/(5*a^4)))/(x^(7/2) + (a*x^(5/2))/b)
```

sympy [B] time = 11.15, size = 348, normalized size = 4.00

$$-\frac{2a^5b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{10a^3b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{60a^2b^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{80ab^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}-\frac{32b^{\frac{29}{2}}x^5\sqrt{\frac{a}{bx}+1}}{5a^7b^9x^2+15a^6b^{10}x^3+15a^5b^{11}x^4+5a^4b^{12}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+a)**(3/2),x)
```

```
[Out] -2*a**5*b**(19/2)*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3
+ 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 10*a**3*b**(23/2)*x**2*sqrt(a/(
b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a
**4*b**12*x**5) - 60*a**2*b**(25/2)*x**3*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**
2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x**5) - 80*a*b**
(27/2)*x**4*sqrt(a/(b*x) + 1)/(5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a
**5*b**11*x**4 + 5*a**4*b**12*x**5) - 32*b**(29/2)*x**5*sqrt(a/(b*x) + 1)/(
5*a**7*b**9*x**2 + 15*a**6*b**10*x**3 + 15*a**5*b**11*x**4 + 5*a**4*b**12*x
**5)
```

3.584 $\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {47, 50, 63, 217, 206}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(a + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a + b*x]) + (
5*Sqrt[x]*Sqrt[a + b*x])/b^3 - (5*a*ArcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]
])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{(a+bx)^{5/2}}\,dx &= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}+\frac{5\int \frac{x^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a+bx}}\,dx}{b^2}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{(5a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{3b(a+bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a+bx}}+\frac{5\sqrt{x}\sqrt{a+bx}}{b^3}-\frac{5a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.55

$$\frac{2x^{7/2}\sqrt{\frac{bx}{a}+1}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{a}\right)}{7a^2\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, -((b*x)/a)])/
(7*a^2*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.15, size = 78, normalized size = 0.86

$$\frac{15a^2\sqrt{x}+20abx^{3/2}+3b^2x^{5/2}}{3b^3(a+bx)^{3/2}}+\frac{5a\log\left(\sqrt{a+bx}-\sqrt{b}\sqrt{x}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (15*a^2*Sqrt[x] + 20*a*b*x^(3/2) + 3*b^2*x^(5/2))/(3*b^3*(a + b*x)^(3/2)) +
 (5*a*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/b^(7/2)
```

fricas [A] time = 1.42, size = 214, normalized size = 2.35

$$\left[\frac{15\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{b}\log\left(2bx-2\sqrt{bx+a}\sqrt{b}\sqrt{x}+a\right)+2\left(3b^3x^2+20ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{6\left(b^6x^2+2ab^5x+a^2b^4\right)},\frac{15\left(ab^2x^2+2a^2bx+a^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-b}}{b\sqrt{x}}\right)+\left(3b^3x^2+20ab^2x+15a^2b\right)\sqrt{bx+a}\sqrt{x}}{3\left(b^6x^2+2ab^5x+a^2b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a*b^2*x^2 + 2*a^2*b*x + a^3)*sqrt(b)*log(2*b*x - 2*sqrt(b*x + a)*
sqrt(b)*sqrt(x) + a) + 2*(3*b^3*x^2 + 20*a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*
sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4), 1/3*(15*(a*b^2*x^2 + 2*a^2*b*x +
```

```
a^3)*sqrt(-b)*arctan(sqrt(b*x + a)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 20*
a*b^2*x + 15*a^2*b)*sqrt(b*x + a)*sqrt(x))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)]
```

giac [B] time = 92.46, size = 197, normalized size = 2.16

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{b^{\frac{5}{2}}}+\frac{6\,\sqrt{(bx+a)b-ab}\,\sqrt{bx+a}}{b^{3}}+\frac{8\left(9\,a^{2}\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+12\,a^{3}\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}b^{\frac{3}{2}}+7\,a^{4}b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}b^{2}}\right)|b|}{6\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/b^(5/2)
+ 6*sqrt((b*x + a)*b - a*b)*sqrt(b*x + a)/b^3 + 8*(9*a^2*(sqrt(b*x + a)*sqr
t(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 12*a^3*(sqrt(b*x + a)*sqrt(b) -
 sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 7*a^4*b^(5/2))/(((sqrt(b*x + a)*sqrt(
b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*b^2))*abs(b)/b^2
```

maple [B] time = 0.05, size = 147, normalized size = 1.62

$$\frac{\left(-\frac{5a\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}}+\sqrt{b\,x^{2}+ax}\right)}{2b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^{2}b}\;a^{2}}{3\left(x+\frac{a}{b}\right)^{2}b^{5}}+\frac{14\sqrt{-\left(x+\frac{a}{b}\right)a+\left(x+\frac{a}{b}\right)^{2}b}\;a}{3\left(x+\frac{a}{b}\right)b^{4}}\right)\sqrt{(bx+a)\,x}}{\sqrt{bx+a}\;\sqrt{x}}+\frac{\sqrt{bx+a}\;\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] x^(1/2)*(b*x+a)^(1/2)/b^3+(-5/2/b^(7/2)*a*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x
)^(1/2))+14/3/b^4*a/(x+a/b)*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2)-2/3/b^5*a^2/(x+a
/b)^2*(-(x+a/b)*a+(x+a/b)^2*b)^(1/2))*((b*x+a)*x)^(1/2)/(b*x+a)^(1/2)/x^(1/
2)
```

maxima [A] time = 2.94, size = 109, normalized size = 1.20

$$\frac{2\,ab^{2}+\frac{10\,(bx+a)ab}{x}-\frac{15\,(bx+a)^{2}a}{x^{2}}}{3\left(\frac{(bx+a)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}-\frac{(bx+a)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}+\frac{5\,a\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x + a)*a*b/x - 15*(b*x + a)^2*a/x^2)/((b*x + a)^(3/2)*
b^4/x^(3/2) - (b*x + a)^(5/2)*b^3/x^(5/2)) + 5/2*a*log(-(sqrt(b) - sqrt(b*x
 + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a + b*x)^(5/2), x)
```

sympy [B] time = 7.61, size = 396, normalized size = 4.35

$$-\frac{15a^{\frac{81}{2}}b^{22}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{15a^{\frac{79}{2}}b^{23}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{15a^{40}b^{\frac{45}{2}}x^{26}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{20a^{39}b^{\frac{47}{2}}x^{27}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{3a^{38}b^{\frac{49}{2}}x^{28}}{3a^{\frac{79}{2}}b^{\frac{51}{2}}x^{\frac{51}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{77}{2}}b^{\frac{53}{2}}x^{\frac{53}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -15*a**(81/2)*b**22*x**(51/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a)
)/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*
x**(53/2)*sqrt(1 + b*x/a)) - 15*a**(79/2)*b**23*x**(53/2)*sqrt(1 + b*x/a)*a
sinh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x
/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a)) + 15*a**40*b**(45/2)
*x**26/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(5
3/2)*x**(53/2)*sqrt(1 + b*x/a)) + 20*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**
(51/2)*x**(51/2)*sqrt(1 + b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 +
 b*x/a)) + 3*a**38*b**(49/2)*x**28/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1
+ b*x/a) + 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 + b*x/a))
```

3.585 $\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 63, 217, 206}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a+bx}}+\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(a + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a + b*x]) + (2*A
rcTanh[(Sqrt[b]*Sqrt[x])/Sqrt[a + b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a+bx)^{5/2}}\,dx = -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} + \frac{\int \frac{\sqrt{x}}{(a+bx)^{3/2}}\,dx}{b}$$

$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{\int \frac{1}{\sqrt{x}\,\sqrt{a+bx}}\,dx}{b^2}$$

$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$

$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a+bx}}\right)}{b^2}$$

$$= -\frac{2x^{3/2}}{3b(a+bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a+bx}} + \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a+bx}}\right)}{b^{5/2}}$$

Mathematica [**A**] time = 0.12, size = 80, normalized size = 1.16

$$\frac{6\sqrt{a}\,(a+bx)\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right) - 2\sqrt{b}\,\sqrt{x}\,(3a+4bx)}{3b^{5/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[b]*Sqrt[x]*(3*a + 4*b*x) + 6*Sqrt[a]*(a + b*x)*Sqrt[1 + (b*x)/a]*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(3*b^(5/2)*(a + b*x)^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.14, size = 64, normalized size = 0.93

$$-\frac{2\log\left(\sqrt{a+bx} - \sqrt{b}\,\sqrt{x}\right)}{b^{5/2}} - \frac{2\left(3a\sqrt{x} + 4bx^{3/2}\right)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(3*a*Sqrt[x] + 4*b*x^(3/2)))/(3*b^2*(a + b*x)^(3/2)) - (2*Log[-(Sqrt[b]
*Sqrt[x]) + Sqrt[a + b*x]])/b^(5/2)
```

fricas [**A**] time = 0.72, size = 186, normalized size = 2.70

$$\left[\frac{3\left(b^2x^2 + 2\,abx + a^2\right)\sqrt{b}\,\log\left(2\,bx + 2\,\sqrt{bx+a}\,\sqrt{b}\,\sqrt{x} + a\right) - 2\left(4\,b^2x + 3\,ab\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}, -\frac{2\left(3\left(b^2x^2 + 2\,abx + a^2\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-b}}{b\sqrt{x}}\right) + \left(4\,b^2x + 3\,ab\right)\sqrt{bx+a}\,\sqrt{x}\right)}{3\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b)*log(2*b*x + 2*sqrt(b*x + a)*sqrt(
b)*sqrt(x) + a) - 2*(4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b^5*x^2 + 2*a
*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(-b)*arctan(sqrt(b
*x + a)*sqrt(-b)/(b*sqrt(x))) + (4*b^2*x + 3*a*b)*sqrt(b*x + a)*sqrt(x))/(b
^5*x^2 + 2*a*b^4*x + a^2*b^3)]
```

giac [B] time = 105.59, size = 165, normalized size = 2.39

$$-\frac{\left(\frac{3 \log\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}\right)}{\sqrt{b}}+\frac{8\left(3 a\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+3 a^{2}\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2} b^{\frac{3}{2}}+2 a^{3} b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}}\right)|b|}{3 b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2)/sqrt(b) +
8*(3*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + 3*a^2*
(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(3/2) + 2*a^3*b^(5/2)
)/((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3)*abs(b)/b^3
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x+a)^(5/2),x)
```

maxima [A] time = 2.83, size = 69, normalized size = 1.00

$$-\frac{2\left(b+\frac{3(bx+a)}{x}\right)x^{\frac{3}{2}}}{3(bx+a)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+a}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+a}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + a)/sqrt(x))/(sqrt(b) + sqrt(b*x + a)/sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a+b\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a + b*x)^(5/2), x)
```

sympy [B] time = 4.03, size = 328, normalized size = 4.75

$$\frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}+\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}-\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] 6*a**(39/2)*b**11*x**(27/2)*sqrt(1 + b*x/a)*asinh(sqrt(b)*sqrt(x)/sqrt(a))/
(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x*
*(29/2)*sqrt(1 + b*x/a)) + 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 + b*x/a)*asin
h(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a)
 + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/a)) - 6*a**19*b**(23/2)*x**
14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)
*x**(29/2)*sqrt(1 + b*x/a)) - 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2
)*x**(27/2)*sqrt(1 + b*x/a) + 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 + b*x/
a))
```

3.586 $\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a+bx)^{5/2}} dx = \frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a + b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a + b*x)^(3/2))
```

fricas [B] time = 1.24, size = 33, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,x^{\frac{3}{2}}}{3\left(ab^2x^2+2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*x^(3/2)/(a*b^2*x^2 + 2*a^2*b*x + a^3)
```

giac [B] time = 1.63, size = 86, normalized size = 4.10

$$\frac{4\left(3\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{4}\sqrt{b}+a^{2}b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+a}\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^{2}+ab\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*sqrt(b) + a^2*b^
(5/2))*abs(b)/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^
3*b^2)
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{2x^{\frac{3}{2}}}{3\left(bx+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(b*x+a)^(3/2)
```

maxima [A] time = 1.33, size = 15, normalized size = 0.71

$$\frac{2\,x^{\frac{3}{2}}}{3\left(bx+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((b*x + a)^(3/2)*a)
```

mupad [B] time = 0.24, size = 36, normalized size = 1.71

$$\frac{2\,x^{3/2}\,\sqrt{a+b\,x}}{3\,\left(a^3+2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a + b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 + 2*a^2*b*x))
```

sympy [B] time = 1.43, size = 42, normalized size = 2.00

$$\frac{2x^{\frac{3}{2}}}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1+\frac{bx}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] 2*x**(3/2)/(3*a**(5/2)*sqrt(1 + b*x/a) + 3*a**(3/2)*b*x*sqrt(1 + b*x/a))
```

3.587 $\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}+\frac{2\sqrt{x}}{3a(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a + b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a+bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a+bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 29, normalized size = 0.67

$$\frac{2\sqrt{x}(3a+2bx)}{3a^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a + b*x)^(5/2)),x]
[Out] (2*Sqrt[x]*(3*a + 2*b*x))/(3*a^2*(a + b*x)^(3/2))
```

fricas [A] time = 1.49, size = 43, normalized size = 1.00

$$\frac{2\,(2\,bx+3\,a)\sqrt{bx+a}\,\sqrt{x}}{3\,\left(a^2b^2x^2+2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
[Out] 2/3*(2*b*x + 3*a)*sqrt(b*x + a)*sqrt(x)/(a^2*b^2*x^2 + 2*a^3*b*x + a^4)
```

giac [B] time = 1.50, size = 81, normalized size = 1.88

$$\frac{8\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
[Out] 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)*b^(5/2)/(
((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*abs(b))
```

maple [A] time = 0.00, size = 24, normalized size = 0.56

$$\frac{2\,(2bx+3a)\,\sqrt{x}}{3\,(bx+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/x^(1/2),x)
[Out] 2/3*x^(1/2)*(2*b*x+3*a)/(b*x+a)^(3/2)/a^2
```

maxima [A] time = 1.34, size = 27, normalized size = 0.63

$$-\frac{2\left(b-\frac{3\,(bx+a)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
[Out] -2/3*(b - 3*(b*x + a)/x)*x^(3/2)/((b*x + a)^(3/2)*a^2)
```

mupad [B] time = 0.40, size = 54, normalized size = 1.26

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a+b\,x}\,+4\,b\,x^{3/2}\,\sqrt{a+b\,x}}{3\,a^4+6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a + b*x)^(1/2) + 4*b*x^(3/2)*(a + b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 + 6*a^3*b*x)
```

sympy [B] time = 1.90, size = 92, normalized size = 2.14

$$\frac{6a}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}+\frac{4bx}{3a^3\sqrt{b}\sqrt{\frac{a}{bx}+1}+3a^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] 6*a/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x) + 1)
) + 4*b*x/(3*a**3*sqrt(b)*sqrt(a/(b*x) + 1) + 3*a**2*b**(3/2)*x*sqrt(a/(b*x
) + 1))
```

3.588 $\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a + b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a + b*x]) - (16*Sqr
t[a + b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a+bx)^{5/2}}\,dx &= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}+\frac{4\int \frac{1}{x^{3/2}(a+bx)^{3/2}}\,dx}{3a}\\ &= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}+\frac{8\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^2}\\ &= \frac{2}{3a\sqrt{x}\,(a+bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.62

$$-\frac{2\left(3a^2+12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 40, normalized size = 0.62

$$-\frac{2\left(3a^2+12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 + 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a + b*x)^(3/2))
```

fricas [A] time = 1.37, size = 58, normalized size = 0.91

$$-\frac{2\left(8\,b^2x^2+12\,abx+3\,a^2\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^3b^2x^3+2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 + 12*a*b*x + 3*a^2)*sqrt(b*x + a)*sqrt(x)/(a^3*b^2*x^3 + 2*
a^4*b*x^2 + a^5*x)
```

giac [B] time = 1.63, size = 159, normalized size = 2.48

$$-\frac{2\sqrt{bx+a}\,b^2}{\sqrt{(bx+a)b-ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{5}{2}}+12\,a\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{7}{2}}+5\,a^2b^{\frac{9}{2}}\right)}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2/(sqrt((b*x + a)*b - a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(b*
x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^4*b^(5/2) + 12*a*(sqrt(b*x + a)*s
qrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(7/2) + 5*a^2*b^(9/2))/(((sqrt(b*x +
a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 + a*b)^3*a^2*abs(b))
```

maple [A] time = 0.01, size = 35, normalized size = 0.55

$$-\frac{2\left(8b^2x^2+12abx+3a^2\right)}{3\,(bx+a)^{\frac{3}{2}}\,a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2+12*a*b*x+3*a^2)/(b*x+a)^(3/2)/x^(1/2)/a^3
```

maxima [A] time = 1.34, size = 46, normalized size = 0.72

$$\frac{2\left(b^2-\frac{6\,(bx+a)b}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}a^3}-\frac{2\sqrt{bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x + a)*b/x)*x^(3/2)/((b*x + a)^(3/2)*a^3) - 2*sqrt(b*x + a)
/(a^3*sqrt(x))
```

mupad [B] time = 0.42, size = 71, normalized size = 1.11

$$-\frac{6\,a^2\,\sqrt{a+b\,x}\,+16\,b^2\,x^2\,\sqrt{a+b\,x}\,+24\,a\,b\,x\,\sqrt{a+b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b+3\,x\,a^3\,b^2\right)+3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(6*a^2*(a + b*x)^(1/2) + 16*b^2*x^2*(a + b*x)^(1/2) + 24*a*b*x*(a + b*x)^(
1/2))/(x^(1/2)*(x*(6*a^4*b + 3*a^3*b^2*x) + 3*a^5))
```

sympy [B] time = 3.97, size = 153, normalized size = 2.39

$$-\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^5b^4+6a^4b^5x+3a^3b^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] -6*a**2*b**(9/2)*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x + 3*a**3*b*
*6*x**2) - 24*a*b**(11/2)*x*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**5*b**4 + 6*
a**4*b**5*x + 3*a**3*b**6*x**2)
```

3.589 $\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=84

$$\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a + b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a + b*x]) - (16*Sqrt[
a + b*x])/(3*a^3*x^(3/2)) + (32*b*Sqrt[a + b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(a+bx)^{5/2}}\,dx &= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{2\int \frac{1}{x^{5/2}(a+bx)^{3/2}}\,dx}{a}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{a+bx}}\,dx}{a^2}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}-\frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a+bx}}\,dx}{3a^3}\\
&= \frac{2}{3ax^{3/2}(a+bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a+bx}}-\frac{16\sqrt{a+bx}}{3a^3x^{3/2}}+\frac{32b\sqrt{a+bx}}{3a^4\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 0.58

$$-\frac{2\left(a^3-6a^2bx-24ab^2x^2-16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 - 6*a^2*b*x - 24*a*b^2*x^2 - 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 51, normalized size = 0.61

$$\frac{2\left(-a^3+6a^2bx+24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a + b*x)^(5/2)),x]
```

```
[Out] (2*(-a^3 + 6*a^2*b*x + 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a + b*x)
^(3/2))
```

fricas [A] time = 1.04, size = 71, normalized size = 0.85

$$\frac{2\left(16\,b^3x^3+24\,ab^2x^2+6\,a^2bx-a^3\right)\sqrt{bx+a}\,\sqrt{x}}{3\left(a^4b^2x^4+2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*x^3 + 24*a*b^2*x^2 + 6*a^2*b*x - a^3)*sqrt(b*x + a)*sqrt(x)/(a^
4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)
```

giac [B] time = 2.24, size = 175, normalized size = 2.08

$$\frac{2\sqrt{bx+a}\left(\frac{8\,(bx+a)b^2|b|}{a^4}-\frac{9\,b^2|b|}{a^3}\right)}{3\left((bx+a)b-ab\right)^{\frac{3}{2}}}+\frac{8\left(3\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^4b^{\frac{7}{2}}+9\,a\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2b^{\frac{9}{2}}+4\,a^2b^{\frac{11}{2}}\right)}{3\left(\left(\sqrt{bx+a}\,\sqrt{b}-\sqrt{(bx+a)b-ab}\right)^2+ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b*x + a)*b^2*abs(b)/a^4 - 9*b^2*abs(b)/a^3)/((b*x + a
)*b - a*b)^(3/2) + 8/3*(3*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))
^4*b^(7/2) + 9*a*(sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2*b^(9/2
) + 4*a^2*b^(11/2))/(((sqrt(b*x + a)*sqrt(b) - sqrt((b*x + a)*b - a*b))^2 +
 a*b)^3*a^3*abs(b))
```

maple [A] time = 0.00, size = 44, normalized size = 0.52

$$-\frac{2\left(-16b^3x^3-24a\,b^2x^2-6a^2bx+a^3\right)}{3\,(bx+a)^{\frac{3}{2}}\,a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*x^3-24*a*b^2*x^2-6*a^2*b*x+a^3)/(b*x+a)^(3/2)/x^(3/2)/a^4
```

maxima [A] time = 1.27, size = 64, normalized size = 0.76

$$\frac{2\left(\frac{9\,\sqrt{bx+a}\,b}{\sqrt{x}}-\frac{(bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}-\frac{2\left(b^3-\frac{9\,(bx+a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(9*sqrt(b*x + a)*b/sqrt(x) - (b*x + a)^(3/2)/x^(3/2))/a^4 - 2/3*(b^3 -
9*(b*x + a)*b^2/x)*x^(3/2)/((b*x + a)^(3/2)*a^4)
```

mupad [B] time = 0.47, size = 88, normalized size = 1.05

$$\frac{32\,b^3\,x^3\,\sqrt{a+b\,x}-2\,a^3\,\sqrt{a+b\,x}+12\,a^2\,b\,x\,\sqrt{a+b\,x}+48\,a\,b^2\,x^2\,\sqrt{a+b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b+3\,x\,a^4\,b^2\right)+3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] (32*b^3*x^3*(a + b*x)^(1/2) - 2*a^3*(a + b*x)^(1/2) + 12*a^2*b*x*(a + b*x)^
(1/2) + 48*a*b^2*x^2*(a + b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b + 3*a^4*b^2*x) +
 3*a^6))
```

sympy [B] time = 7.06, size = 337, normalized size = 4.01

$$-\frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}+\frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}+1}}{3a^7b^9x+9a^6b^{10}x^2+9a^5b^{11}x^3+3a^4b^{12}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+a)**(5/2),x)
```

```
[Out] -2*a**4*b**(19/2)*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*
a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(a/(b*x) + 1
)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**
4) + 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10
*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2)*x**3*sqrt(a
/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4
*b**12*x**4) + 32*b**(27/2)*x**4*sqrt(a/(b*x) + 1)/(3*a**7*b**9*x + 9*a**6*
b**10*x**2 + 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4)
```

3.590 $\int \frac{x^{5/2}}{\sqrt{a-bx}} dx$

Optimal. Leaf size=105

$$\frac{5a^3 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}} - \frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$-\frac{5a^2\sqrt{x}\sqrt{a-bx}}{8b^3} + \frac{5a^3 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}} - \frac{5ax^{3/2}\sqrt{a-bx}}{12b^2} - \frac{x^{5/2}\sqrt{a-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (-5*a^2*Sqrt[x]*Sqrt[a - b*x])/(8*b^3) - (5*a*x^(3/2)*Sqrt[a - b*x])/(12*b^
2) - (x^(5/2)*Sqrt[a - b*x])/(3*b) + (5*a^3*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a
 - b*x]])/(8*b^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{a-bx}}\,dx &= -\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{(5a)\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{6b}\\
&= -\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^2\right)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{8b^2}\\
&= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\int \frac{1}{\sqrt{x}\,\sqrt{a-bx}}\,dx}{16b^3}\\
&= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^3}\\
&= -\frac{5a^2\sqrt{x}\,\sqrt{a-bx}}{8b^3}-\frac{5ax^{3/2}\sqrt{a-bx}}{12b^2}-\frac{x^{5/2}\sqrt{a-bx}}{3b}+\frac{5a^3\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-bx}}\right)}{8b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 88, normalized size = 0.84

$$\frac{\sqrt{a-bx}\left(\frac{15a^{5/2}\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{1-\frac{bx}{a}}}-\sqrt{b}\,\sqrt{x}\left(15a^2+10abx+8b^2x^2\right)\right)}{24b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-(Sqrt[b]*Sqrt[x]*(15*a^2 + 10*a*b*x + 8*b^2*x^2)) + (15*a^
(5/2)*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 - (b*x)/a]))/(24*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 91, normalized size = 0.87

$$\frac{5a^3\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\,\sqrt{x}\right)}{8b^4}+\frac{\sqrt{a-bx}\left(-15a^2\sqrt{x}-10abx^{3/2}-8b^2x^{5/2}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^2*Sqrt[x] - 10*a*b*x^(3/2) - 8*b^2*x^(5/2)))/(24*b^3)
 + (5*a^3*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(8*b^4)
```

fricas [A] time = 1.35, size = 141, normalized size = 1.34

$$\left[-\frac{15\,a^3\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{48\,b^4},-\frac{15\,a^3\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(8\,b^3x^2+10\,ab^2x+15\,a^2b\right)\sqrt{-bx+a}\,\sqrt{x}}{24\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*a^3*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)
 + 2*(8*b^3*x^2 + 10*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4, -1/24
*(15*a^3*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (8*b^3*x^2 + 10
*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/b^4]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 108, normalized size = 1.03

$$-\frac{\sqrt{-bx+a}\,x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{-bx+a}\,a\,x^{\frac{3}{2}}}{12b^{2}}+\frac{5\sqrt{(-bx+a)\,x}\,a^{3}\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{16\sqrt{-bx+a}\,b^{\frac{7}{2}}\sqrt{x}}-\frac{5\sqrt{-bx+a}\,a^{2}\sqrt{x}}{8b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+a)^(1/2)/b-5/12*a*x^(3/2)*(-b*x+a)^(1/2)/b^2-5/8*a^2*x^(
1/2)*(-b*x+a)^(1/2)/b^3+5/16*a^3/b^(7/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a
)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.96, size = 135, normalized size = 1.29

$$-\frac{5\,a^{3}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{8\,b^{\frac{7}{2}}}-\frac{\frac{33\,\sqrt{-bx+a}\,a^{3}b^{2}}{\sqrt{x}}+\frac{40\,(-bx+a)^{\frac{3}{2}}\,a^{3}b}{x^{\frac{3}{2}}}+\frac{15\,(-bx+a)^{\frac{5}{2}}\,a^{3}}{x^{\frac{5}{2}}}}{24\left(b^{6}-\frac{3\,(bx-a)b^{5}}{x}+\frac{3\,(bx-a)^{2}b^{4}}{x^{2}}-\frac{(bx-a)^{3}b^{3}}{x^{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -5/8*a^3*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2) - 1/24*(33*sqrt(-
b*x + a)*a^3*b^2/sqrt(x) + 40*(-b*x + a)^(3/2)*a^3*b/x^(3/2) + 15*(-b*x + a
)^(5/2)*a^3/x^(5/2))/(b^6 - 3*(b*x - a)*b^5/x + 3*(b*x - a)^2*b^4/x^2 - (b*
x - a)^3*b^3/x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{a-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(1/2), x)
```

sympy [A] time = 8.43, size = 270, normalized size = 2.57

$$\begin{cases} \frac{5ia^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{-1+\frac{bx}{a}}}-\frac{i\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{-1+\frac{bx}{a}}}-\frac{5ia^{3}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}-\frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{5a^{\frac{5}{2}}\sqrt{x}}{8b^{3}\sqrt{1-\frac{bx}{a}}}+\frac{5a^{\frac{3}{2}}x^{\frac{3}{2}}}{24b^{2}\sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\,x^{\frac{5}{2}}}{12b\sqrt{1-\frac{bx}{a}}}+\frac{5a^{3}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{8b^{\frac{7}{2}}}+\frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((5*I*a**(5/2)*sqrt(x)/(8*b**3*sqrt(-1 + b*x/a)) - 5*I*a**(3/2)*x*
*(3/2)/(24*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(5/2)/(12*b*sqrt(-1 + b*x/
a)) - 5*I*a**3*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**(7/2)) - I*x**(7/2)/(3*
sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-5*a**(5/2)*sqrt(x)/(8*b**3*sq
rt(1 - b*x/a)) + 5*a**(3/2)*x**(3/2)/(24*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x*
*(5/2)/(12*b*sqrt(1 - b*x/a)) + 5*a**3*asin(sqrt(b)*sqrt(x)/sqrt(a))/(8*b**
(7/2)) + x**(7/2)/(3*sqrt(a)*sqrt(1 - b*x/a)), True))
```

3.591 $\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=80

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}-\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2}-\frac{x^{3/2}\sqrt{a-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (-3*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^2) - (x^(3/2)*Sqrt[a - b*x])/(2*b) + (3*a
^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/(4*b^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx = -\frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{(3a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{\left(3a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{\left(3a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{x}\sqrt{a-bx}}{4b^2} - \frac{x^{3/2}\sqrt{a-bx}}{2b} + \frac{3a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 1.08

$$\frac{3a^{5/2}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}\left(-3a^2+abx+2b^2x^2\right)}{4b^{5/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[b]*Sqrt[x]*(-3*a^2 + a*b*x + 2*b^2*x^2) + 3*a^(5/2)*Sqrt[1 - (b*x)/a]
*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 78, normalized size = 0.98

$$\frac{3a^2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^3}+\frac{\sqrt{a-bx}\left(-3a\sqrt{x}-2bx^{3/2}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[a - b*x],x]
```

```
[Out] (Sqrt[a - b*x]*(-3*a*Sqrt[x] - 2*b*x^(3/2)))/(4*b^2) + (3*a^2*Sqrt[-b]*Log[
-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4*b^3)
```

fricas [A] time = 0.77, size = 119, normalized size = 1.49

$$\left[-\frac{3\,a^2\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\left(2\,b^2x+3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{8\,b^3}, -\frac{3\,a^2\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\left(2\,b^2x+3\,ab\right)\sqrt{-bx+a}\,\sqrt{x}}{4\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(3*a^2*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*(2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt(x))/b^3, -1/4*(3*a^2*sqrt(b)*arcta
n(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (2*b^2*x + 3*a*b)*sqrt(-b*x + a)*sqrt
(x))/b^3]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 89, normalized size = 1.11

$$-\frac{\sqrt{-bx+a}\, x^{\frac{3}{2}}}{2b}+\frac{3\sqrt{(-bx+a)\, x}\, a^{2} \arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\, x^{2}+a x}}\right)}{8\sqrt{-bx+a}\, b^{\frac{5}{2}} \sqrt{x}}-\frac{3\sqrt{-bx+a}\, a\sqrt{x}}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+a)^(1/2)/b-3/4*a*x^(1/2)*(-b*x+a)^(1/2)/b^2+3/8*a^2/b^(5
/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*
x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.96, size = 98, normalized size = 1.22

$$-\frac{3\, a^{2} \arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\, \sqrt{x}}\right)}{4\, b^{\frac{5}{2}}}-\frac{\frac{5\, \sqrt{-bx+a}\, a^{2} b}{\sqrt{x}}+\frac{3\, (-bx+a)^{\frac{3}{2}} a^{2}}{x^{\frac{3}{2}}}}{4\left(b^{4}-\frac{2\,(bx-a) b^{3}}{x}+\frac{(bx-a)^{2} b^{2}}{x^{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2) - 1/4*(5*sqrt(-b*
x + a)*a^2*b/sqrt(x) + 3*(-b*x + a)^(3/2)*a^2/x^(3/2))/(b^4 - 2*(b*x - a)*b
^3/x + (b*x - a)^2*b^2/x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{a-b\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(1/2), x)
```

sympy [A] time = 4.30, size = 214, normalized size = 2.68

$$\begin{cases} \frac{3 i a^{\frac{3}{2}} \sqrt{x}}{4 b^{2} \sqrt{-1+\frac{bx}{a}}}-\frac{i \sqrt{a}\, x^{\frac{3}{2}}}{4 b \sqrt{-1+\frac{bx}{a}}}-\frac{3 i a^{2} \operatorname{acosh}\left(\frac{\sqrt{b}\, \sqrt{x}}{\sqrt{a}}\right)}{4 b^{\frac{5}{2}}}-\frac{i x^{\frac{5}{2}}}{2 \sqrt{a}\, \sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{3 a^{\frac{3}{2}} \sqrt{x}}{4 b^{2} \sqrt{1-\frac{bx}{a}}}+\frac{\sqrt{a}\, x^{\frac{3}{2}}}{4 b \sqrt{1-\frac{bx}{a}}}+\frac{3 a^{2} \operatorname{asin}\left(\frac{\sqrt{b}\, \sqrt{x}}{\sqrt{a}}\right)}{4 b^{\frac{5}{2}}}+\frac{x^{\frac{5}{2}}}{2 \sqrt{a}\, \sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((3*I*a**(3/2)*sqrt(x)/(4*b**2*sqrt(-1 + b*x/a)) - I*sqrt(a)*x**(3
/2)/(4*b*sqrt(-1 + b*x/a)) - 3*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**
```

```
(5/2)) - I*x**(5/2)/(2*sqrt(a)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-3*a**(
3/2)*sqrt(x)/(4*b**2*sqrt(1 - b*x/a)) + sqrt(a)*x**(3/2)/(4*b*sqrt(1 - b*x/
a)) + 3*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(5/2)) + x**(5/2)/(2*sqrt(
a)*sqrt(1 - b*x/a)), True))
```

3.592 $\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx$

Optimal. Leaf size=50

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {50, 63, 217, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/
b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx = -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}$$
$$= -\frac{\sqrt{x}\sqrt{a-bx}}{b} + \frac{a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.42

$$\frac{a^{3/2}\sqrt{1-\frac{bx}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\sqrt{x}\,(bx-a)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a - b*x],x]
[Out] (Sqrt[b]*Sqrt[x]*(-a + b*x) + a^(3/2)*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqr
t[x])/Sqrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 1.18

$$\frac{a\sqrt{-b}\,\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}-\frac{\sqrt{x}\sqrt{a-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[a - b*x],x]
[Out] -((Sqrt[x]*Sqrt[a - b*x])/b) + (a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a
 - b*x]])/b^2
```

fricas [A] time = 1.21, size = 93, normalized size = 1.86

$$\left[-\frac{a\sqrt{-b}\,\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}+a\right)+2\,\sqrt{-bx+a}\,b\sqrt{x}}{2\,b^2},\,-\frac{a\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)+\sqrt{-bx+a}\,b\sqrt{x}}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] [-1/2*(a*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) + 2*s
qrt(-b*x + a)*b*sqrt(x))/b^2, -(a*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sq
rt(x))) + sqrt(-b*x + a)*b*sqrt(x))/b^2]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 70, normalized size = 1.40

$$\frac{\sqrt{(-bx+a)\,x}\;a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+ax}}\right)}{2\sqrt{-bx+a}\;b^{\frac{3}{2}}\sqrt{x}}-\frac{\sqrt{-bx+a}\;\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b+1/2*a/b^(3/2)*((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)
^(1/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 3.00, size = 56, normalized size = 1.12

$$-\frac{a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{\sqrt{-bx+a}\,a}{\left(b^{2}-\frac{(bx-a)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) - sqrt(-b*x + a)*a/((b^
2 - (b*x - a)*b/x)*sqrt(x))
```

mupad [B] time = 0.52, size = 47, normalized size = 0.94

$$\frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a-b\,x}-\sqrt{a}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{a-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(1/2),x)
```

```
[Out] (2*a*atan((b^(1/2)*x^(1/2))/((a - b*x)^(1/2) - a^(1/2))))/b^(3/2) - (x^(1/2
)*(a - b*x)^(1/2))/b
```

sympy [A] time = 2.28, size = 121, normalized size = 2.42

$$\begin{cases} -\frac{i\sqrt{a}\,\sqrt{x}\,\sqrt{-1+\frac{bx}{a}}}{b}-\frac{ia\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ -\frac{\sqrt{a}\,\sqrt{x}}{b\sqrt{1-\frac{bx}{a}}}+\frac{a\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(a)*sqrt(x)*sqrt(-1 + b*x/a)/b - I*a*acosh(sqrt(b)*sqrt(x
)/sqrt(a))/b**(3/2), Abs(b*x/a) > 1), (-sqrt(a)*sqrt(x)/(b*sqrt(1 - b*x/a))
 + a*asin(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) + x**(3/2)/(sqrt(a)*sqrt(1 - b*
x/a)), True))
```

3.593 $\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a - b*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/Sqrt[b]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 1.79

$$\frac{2\sqrt{a}\sqrt{1-\frac{bx}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a - b*x]),x]
[Out] (2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sq
rt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 38, normalized size = 1.31

$$\frac{2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[a - b*x]),x]
[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b
```

fricas [A] time = 0.92, size = 57, normalized size = 1.97

$$\left[-\frac{\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\,\sqrt{-b}\,\sqrt{x}\,+a\right)}{b},-\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a)/b, -2*arctan
(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

maple [B] time = 0.01, size = 51, normalized size = 1.76

$$\frac{\sqrt{(-bx+a)\,x}\;\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{\sqrt{-bx+a}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+a)^(1/2),x)
[Out] ((-b*x+a)*x)^(1/2)/x^(1/2)/(-b*x+a)^(1/2)/b^(1/2)*arctan((x-1/2*a/b)/(-b*x^
2+a*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.88, size = 21, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

mupad [B] time = 0.03, size = 27, normalized size = 0.93

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{a-b\,x}-\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((a - b*x)^(1/2) - a^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

sympy [A] time = 1.15, size = 54, normalized size = 1.86

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{for}\ \left|\frac{bx}{a}\right| > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), Abs(b*x/a) > 1), (2
*asin(sqrt(b)*sqrt(x)/sqrt(a))/sqrt(b), True))
```

3.594 $\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx = -\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$-\frac{2\sqrt{a-bx}}{a\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(a*Sqrt[x])
```

fricas [A] time = 0.69, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

giac [B] time = 1.28, size = 35, normalized size = 1.75

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a*abs(b))
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(1/2),x)
```

```
[Out] -2*(-b*x+a)^(1/2)/a/x^(1/2)
```

maxima [A] time = 1.34, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{-bx+a}}{a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-b*x + a)/(a*sqrt(x))
```

mupad [B] time = 0.40, size = 16, normalized size = 0.80

$$-\frac{2\sqrt{a-b\,x}}{a\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -(2*(a - b*x)^(1/2))/(a*x^(1/2))
```

sympy [A] time = 0.97, size = 46, normalized size = 2.30

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{a} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ -\frac{2i\sqrt{b}\sqrt{-\frac{a}{bx}+1}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/a, Abs(a/(b*x)) > 1), (-2*I*sqrt(b)
*sqrt(-a/(b*x) + 1)/a, True))
```

3.595 $\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx$

Optimal. Leaf size=46

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}-\frac{2\sqrt{a-bx}}{3ax^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x])/(3*a*x^(3/2)) - (4*b*Sqrt[a - b*x])/(3*a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}+\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a}\\ &= -\frac{2\sqrt{a-bx}}{3ax^{3/2}}-\frac{4b\sqrt{a-bx}}{3a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a - b*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 28, normalized size = 0.61

$$-\frac{2\sqrt{a-bx}\,(a+2bx)}{3a^2x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[a - b*x]),x]
[Out] (-2*Sqrt[a - b*x]*(a + 2*b*x))/(3*a^2*x^(3/2))
```

fricas [A] time = 1.77, size = 22, normalized size = 0.48

$$-\frac{2\,(2\,bx+a)\sqrt{-bx+a}}{3\,a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/3*(2*b*x + a)*sqrt(-b*x + a)/(a^2*x^(3/2))
```

giac [A] time = 1.45, size = 54, normalized size = 1.17

$$-\frac{2\left(\frac{2\,(bx-a)b^3}{a^2}+\frac{3\,b^3}{a}\right)\sqrt{-bx+a}\,b}{3\,((bx-a)b+ab)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="giac")
[Out] -2/3*(2*(b*x - a)*b^3/a^2 + 3*b^3/a)*sqrt(-b*x + a)*b/(((b*x - a)*b + a*b)^
(3/2)*abs(b))
```

maple [A] time = 0.00, size = 23, normalized size = 0.50

$$-\frac{2\sqrt{-bx+a}\;(2bx+a)}{3a^2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(1/2),x)
[Out] -2/3*(-b*x+a)^(1/2)*(2*b*x+a)/x^(3/2)/a^2
```

maxima [A] time = 1.30, size = 32, normalized size = 0.70

$$-\frac{2\left(\frac{3\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(1/2),x, algorithm="maxima")
[Out] -2/3*(3*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^2
```

mupad [B] time = 0.35, size = 26, normalized size = 0.57

$$-\frac{\left(\frac{2}{3\,a}+\frac{4\,b\,x}{3\,a^2}\right)\sqrt{a-b\,x}}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(1/2)),x)
```

```
[Out] -((2/(3*a) + (4*b*x)/(3*a^2))*(a - b*x)^(1/2))/x^(3/2)
```

sympy [A] time = 2.06, size = 177, normalized size = 3.85

$$\begin{cases} -\frac{2\sqrt{b}\sqrt{\frac{a}{bx}-1}}{3ax} - \frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx}-1}}{3a^2} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^2b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} + \frac{2iab^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} - \frac{4ib^{\frac{7}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^3bx+3a^2b^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(1/2),x)
```

```
[Out] Piecewise((-2*sqrt(b)*sqrt(a/(b*x) - 1)/(3*a*x) - 4*b**(3/2)*sqrt(a/(b*x) -
 1)/(3*a**2), Abs(a/(b*x)) > 1), (2*I*a**2*b**(3/2)*sqrt(-a/(b*x) + 1)/(-3*
a**3*b*x + 3*a**2*b**2*x**2) + 2*I*a*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(-3*a**3
*b*x + 3*a**2*b**2*x**2) - 4*I*b**(7/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**3*b*
x + 3*a**2*b**2*x**2), True))
```

3.596 $\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{2x^{5/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[a - b*x]) + (15*a*Sqrt[x]*Sqrt[a - b*x])/(4*b^3) + (5*x
^(3/2)*Sqrt[a - b*x])/(2*b^2) - (15*a^2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b
*x]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{a-bx}}-\frac{5\int \frac{x^{3/2}}{\sqrt{a-bx}}\,dx}{b}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{(15a)\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{4b^2}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{8b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^3}\\
&= \frac{2x^{5/2}}{b\sqrt{a-bx}}+\frac{15a\sqrt{x}\sqrt{a-bx}}{4b^3}+\frac{5x^{3/2}\sqrt{a-bx}}{2b^2}-\frac{15a^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{4b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.51

$$
\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a\sqrt{a-bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/a])/(7*
a*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.17, size = 100, normalized size = 1.00

$$
-\frac{15a^2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{4b^4}-\frac{\sqrt{a-bx}\left(15a^2\sqrt{x}-5abx^{3/2}-2b^2x^{5/2}\right)}{4b^3(bx-a)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a - b*x)^(3/2),x]
```

```
[Out] -1/4*(Sqrt[a - b*x]*(15*a^2*Sqrt[x] - 5*a*b*x^(3/2) - 2*b^2*x^(5/2)))/(b^3*
(-a + b*x)) - (15*a^2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/(4
*b^4)
```

fricas [A] time = 1.37, size = 181, normalized size = 1.81

$$
\left[-\frac{15\left(a^2bx-a^3\right)\sqrt{-b}\log\left(-2\,bx-2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(2\,b^3x^2+5\,ab^2x-15\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{8\left(b^5x-ab^4\right)},\ \frac{15\left(a^2bx-a^3\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(2\,b^3x^2+5\,ab^2x-15\,a^2b\right)\sqrt{-bx+a}\sqrt{x}}{4\left(b^5x-ab^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(a^2*b*x - a^3)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*s
qrt(x) + a) - 2*(2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/
```

```
(b^5*x - a*b^4), 1/4*(15*(a^2*b*x - a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqr
t(b)*sqrt(x))) + (2*b^3*x^2 + 5*a*b^2*x - 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))
/(b^5*x - a*b^4)]
```

giac [B] time = 98.07, size = 154, normalized size = 1.54

$$\frac{\left(2\sqrt{(bx-a)b+ab}\sqrt{-bx+a}\left(\frac{2(bx-a)}{b^3}+\frac{9a}{b^3}\right)+\frac{32a^3}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}b}-\frac{15a^2\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}b^2}\right)|b|}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)*(2*(b*x - a)/b^3 + 9*a/b^3) +
 32*a^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt
(-b)*b) - 15*a^2*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)
/(sqrt(-b)*b^2))*abs(b)/b^2
```

maple [A] time = 0.04, size = 127, normalized size = 1.27

$$\frac{\left(-\frac{15a^2\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-bx^2+ax}}\right)}{8b^{\frac{7}{2}}}-\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2b}\,a^2}{\left(x-\frac{a}{b}\right)b^4}\right)\sqrt{(-bx+a)x}}{\sqrt{-bx+a}\,\sqrt{x}}+\frac{(2bx+7a)\sqrt{-bx+a}\,\sqrt{x}}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] 1/4*(2*b*x+7*a)/b^3*(-b*x+a)^(1/2)*x^(1/2)+(-15/8*a^2/b^(7/2)*arctan((x-1/2
*a/b)/(-b*x^2+a*x)^(1/2)*b^(1/2))-2*a^2/b^4/(x-a/b)*(-b*(x-a/b)^2-(x-a/b)*a
)^(1/2))*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.92, size = 118, normalized size = 1.18

$$\frac{8a^2b^2-\frac{25(bx-a)a^2b}{x}+\frac{15(bx-a)^2a^2}{x^2}}{4\left(\frac{\sqrt{-bx+a}\,b^5}{\sqrt{x}}+\frac{2(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}+\frac{15a^2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{4b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^2*b^2 - 25*(b*x - a)*a^2*b/x + 15*(b*x - a)^2*a^2/x^2)/(sqrt(-b*x
+ a)*b^5/sqrt(x) + 2*(-b*x + a)^(3/2)*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^
(5/2)) + 15/4*a^2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(3/2), x)
```

sympy [A] time = 8.03, size = 224, normalized size = 2.24

$$\begin{cases} -\frac{15ia^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{-1+\frac{bx}{a}}} + \frac{5i\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{-1+\frac{bx}{a}}} + \frac{15ia^2\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} + \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right| > 1 \\ \frac{15a^{\frac{3}{2}}\sqrt{x}}{4b^3\sqrt{1-\frac{bx}{a}}} - \frac{5\sqrt{a}\,x^{\frac{3}{2}}}{4b^2\sqrt{1-\frac{bx}{a}}} - \frac{15a^2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{4b^{\frac{7}{2}}} - \frac{x^{\frac{5}{2}}}{2\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-15*I*a**(3/2)*sqrt(x)/(4*b**3*sqrt(-1 + b*x/a)) + 5*I*sqrt(a)*x
**(3/2)/(4*b**2*sqrt(-1 + b*x/a)) + 15*I*a**2*acosh(sqrt(b)*sqrt(x)/sqrt(a)
)/(4*b**(7/2)) + I*x**(5/2)/(2*sqrt(a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1)
, (15*a**(3/2)*sqrt(x)/(4*b**3*sqrt(1 - b*x/a)) - 5*sqrt(a)*x**(3/2)/(4*b**
2*sqrt(1 - b*x/a)) - 15*a**2*asin(sqrt(b)*sqrt(x)/sqrt(a))/(4*b**(7/2)) - x
**(5/2)/(2*sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

3.597 $\int \frac{x^{3/2}}{(a-bx)^{3/2}} dx$

Optimal. Leaf size=71

$$-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$\frac{3\sqrt{x}\sqrt{a-bx}}{b^2}-\frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[a - b*x]) + (3*Sqrt[x]*Sqrt[a - b*x])/b^2 - (3*a*ArcTan
[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx = \frac{2x^{3/2}}{b\sqrt{a-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b}$$
$$= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\sqrt{a-bx}}{b^2} - \frac{(3a)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b^2}$$
$$= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\sqrt{a-bx}}{b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\sqrt{a-bx}}{b^2} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}$$
$$= \frac{2x^{3/2}}{b\sqrt{a-bx}} + \frac{3\sqrt{x}\sqrt{a-bx}}{b^2} - \frac{3a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.72

$$\frac{2x^{5/2}\sqrt{1-\frac{bx}{a}}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{a}\right)}{5a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/a])/(5*
a*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 81, normalized size = 1.14

$$-\frac{3a\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3} - \frac{\sqrt{a-bx}\left(3a\sqrt{x}-bx^{3/2}\right)}{b^2(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a - b*x)^(3/2),x]
```

```
[Out] -((Sqrt[a - b*x]*(3*a*Sqrt[x] - b*x^(3/2)))/(b^2*(-a + b*x))) - (3*a*Sqrt[-
b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^3
```

fricas [A] time = 1.30, size = 152, normalized size = 2.14

$$\left[-\frac{3\left(abx-a^2\right)\sqrt{-b}\log\left(-2\,bx-2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{2\left(b^4x-ab^3\right)}, \frac{3\left(abx-a^2\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{b^4x-ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(a*b*x - a^2)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt
(x) + a) - 2*(b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3), (3*(a
*b*x - a^2)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (b^2*x - 3*a
*b)*sqrt(-b*x + a)*sqrt(x))/(b^4*x - a*b^3)]
```

giac [B] time = 111.13, size = 130, normalized size = 1.83

$$-\frac{\left(\frac{8\,a^2\sqrt{-b}}{\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{3\,a\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}-\frac{2\,\sqrt{(bx-a)b+ab}\,\sqrt{-bx+a}}{b}\right)|b|}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(8*a^2*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2
 - a*b) + 3*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sq
rt(-b) - 2*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b)*abs(b)/b^3
```

maple [B] time = 0.03, size = 114, normalized size = 1.61

$$\frac{\left(-\frac{3a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2b^{\frac{5}{2}}}-\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2 b}\;a}{\left(x-\frac{a}{b}\right)b^3}\right)\sqrt{(-bx+a)\,x}}{\sqrt{-bx+a}\,\sqrt{x}}+\frac{\sqrt{-bx+a}\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] x^(1/2)*(-b*x+a)^(1/2)/b^2+(-3/2*a/b^(5/2)*arctan((x-1/2*a/b)/(-b*x^2+a*x)^
(1/2)*b^(1/2))-2*a/b^3/(x-a/b)*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2))*((-b*x+a)*x)
^(1/2)/(-b*x+a)^(1/2)/x^(1/2)
```

maxima [A] time = 2.95, size = 75, normalized size = 1.06

$$\frac{2\,ab-\frac{3\,(bx-a)a}{x}}{\frac{\sqrt{-bx+a}\,b^3}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}b^2}{x^{\frac{3}{2}}}}+\frac{3\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] (2*a*b - 3*(b*x - a)*a/x)/(sqrt(-b*x + a)*b^3/sqrt(x) + (-b*x + a)^(3/2)*b^
2/x^(3/2)) + 3*a*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(3/2), x)
```

sympy [A] time = 3.70, size = 155, normalized size = 2.18

$$\begin{cases} -\frac{3i\sqrt{a}\,\sqrt{x}}{b^2\sqrt{-1+\frac{bx}{a}}}+\frac{3ia\,\operatorname{acosh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}+\frac{ix^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for } \left|\frac{bx}{a}\right|>1 \\ \frac{3\sqrt{a}\,\sqrt{x}}{b^2\sqrt{1-\frac{bx}{a}}}-\frac{3a\,\operatorname{asin}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{5}{2}}}-\frac{x^{\frac{3}{2}}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-3*I*sqrt(a)*sqrt(x)/(b**2*sqrt(-1 + b*x/a)) + 3*I*a*acosh(sqrt(
b)*sqrt(x)/sqrt(a))/b**(5/2) + I*x**(3/2)/(sqrt(a)*b*sqrt(-1 + b*x/a)), Abs
(b*x/a) > 1), (3*sqrt(a)*sqrt(x)/(b**2*sqrt(1 - b*x/a)) - 3*a*asin(sqrt(b)*
sqrt(x)/sqrt(a))/b**(5/2) - x**(3/2)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

3.598 $\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=50

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$\frac{2\sqrt{x}}{b\sqrt{a-bx}}-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[a - b*x]) - (2*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b}$$
$$= \frac{2\sqrt{x}}{b\sqrt{a-bx}} - \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{3/2}}$$

Mathematica [A] time = 0.08, size = 66, normalized size = 1.32

$$\frac{2\sqrt{b}\sqrt{x} - 2\sqrt{a}\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b]*Sqrt[x] - 2*Sqrt[a]*Sqrt[1 - (b*x)/a]*ArcSin[(Sqrt[b]*Sqrt[x])/S
qrt[a]])/(b^(3/2)*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 68, normalized size = 1.36

$$-\frac{2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2} - \frac{2\sqrt{x}\sqrt{a-bx}}{b(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a - b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[a - b*x])/(b*(-a + b*x)) - (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[a - b*x]])/b^2
```

fricas [A] time = 1.21, size = 128, normalized size = 2.56

$$\left[-\frac{(bx-a)\sqrt{-b}\log\left(-2\,bx-2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\sqrt{-bx+a}\,b\sqrt{x}}{b^3x-ab^2}, \frac{2\left((bx-a)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+a}\,b\sqrt{x}\right)}{b^3x-ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - a)*sqrt(-b)*log(-2*b*x - 2*sqrt(-b*x + a)*sqrt(-b)*sqrt(x) + a) +
 2*sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2), 2*((b*x - a)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + a)*b*sqrt(x))/(b^3*x - a*b^2
)]
```

giac [B] time = 113.04, size = 98, normalized size = 1.96

$$-\frac{\left(\frac{4\,a\sqrt{-b}}{\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab}+\frac{\log\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(4*a*sqrt(-b)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
) + log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b))*ab
s(b)/b^2
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{(-bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

```
[Out] int(x^(1/2)/(-b*x+a)^(3/2),x)
```

maxima [A] time = 3.01, size = 38, normalized size = 0.76

$$\frac{2\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{-bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 a)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(a-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(a - b*x)^(3/2), x)
```

sympy [A] time = 1.89, size = 102, normalized size = 2.04

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{2i\sqrt{x}}{\sqrt{a}\,b\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1 \\ -\frac{2\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}+\frac{2\sqrt{x}}{\sqrt{a}\,b\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(b)*sqrt(x)/sqrt(a))/b**(3/2) - 2*I*sqrt(x)/(sqrt(
a)*b*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*asin(sqrt(b)*sqrt(x)/sqrt(a))/
b**(3/2) + 2*sqrt(x)/(sqrt(a)*b*sqrt(1 - b*x/a)), True))
```

3.599 $\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx = \frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 1.00

$$\frac{2\sqrt{x}}{a\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a - b*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[x])/(a*Sqrt[a - b*x])
```

fricas [A] time = 1.03, size = 25, normalized size = 1.25

$$-\frac{2\sqrt{-bx+a}\sqrt{x}}{abx-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-b*x + a)*sqrt(x)/(a*b*x - a^2)
```

giac [B] time = 1.37, size = 53, normalized size = 2.65

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b
)*abs(b))
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(3/2)/x^(1/2),x)
```

```
[Out] 2*x^(1/2)/a/(-b*x+a)^(1/2)
```

maxima [A] time = 1.30, size = 16, normalized size = 0.80

$$\frac{2\sqrt{x}}{\sqrt{-bx+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)/(sqrt(-b*x + a)*a)
```

mupad [B] time = 0.34, size = 24, normalized size = 1.20

$$\frac{2\sqrt{x}\sqrt{a-b\,x}}{a^2-a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(3/2)),x)
```

```
[Out] (2*x^(1/2)*(a - b*x)^(1/2))/(a^2 - a*b*x)
```

sympy [A] time = 0.94, size = 44, normalized size = 2.20

$$\begin{cases} \frac{2}{a\sqrt{b}\sqrt{\frac{a}{bx}-1}} & \text{for}\ \left|\frac{a}{bx}\right| > 1 \\ -\frac{2i}{a\sqrt{b}\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((2/(a*sqrt(b)*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (-2*I/(a*sqr
t(b)*sqrt(-a/(b*x) + 1)), True))
```

3.600 $\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*Sqrt[x]*Sqrt[a - b*x]) - (4*Sqrt[a - b*x])/(a^2*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx &= \frac{2}{a\sqrt{x}\,\sqrt{a-bx}}+\frac{2\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{a\sqrt{x}\,\sqrt{a-bx}}-\frac{4\sqrt{a-bx}}{a^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.63

$$-\frac{2(a-2bx)}{a^2\sqrt{x}\,\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a - 2*b*x))/(a^2*Sqrt[x]*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 37, normalized size = 0.90

$$-\frac{2\sqrt{a-bx}\,(2bx-a)}{a^2\sqrt{x}\,(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(-a + 2*b*x))/(a^2*Sqrt[x]*(-a + b*x))
```

fricas [A] time = 0.96, size = 38, normalized size = 0.93

$$-\frac{2\,(2\,bx-a)\sqrt{-bx+a}\,\sqrt{x}}{a^2bx^2-a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*x - a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b*x^2 - a^3*x)
```

giac [B] time = 1.44, size = 94, normalized size = 2.29

$$-\frac{4\,\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a|b|}-\frac{2\,\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)*a*abs(b)) - 2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^2*abs(b))
```

maple [A] time = 0.00, size = 23, normalized size = 0.56

$$-\frac{2\,(-2bx+a)}{\sqrt{-bx+a}\;a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2*(-2*b*x+a)/(-b*x+a)^(1/2)/x^(1/2)/a^2
```

maxima [A] time = 1.30, size = 34, normalized size = 0.83

$$\frac{2\,b\sqrt{x}}{\sqrt{-bx+a}\,a^2}-\frac{2\,\sqrt{-bx+a}}{a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b*sqrt(x)/(sqrt(-b*x + a)*a^2) - 2*sqrt(-b*x + a)/(a^2*sqrt(x))
```

mupad [B] time = 0.40, size = 42, normalized size = 1.02

$$-\frac{2\,a\,\sqrt{a-b\,x}\,-4\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(a^3-a^2\,b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(3/2)),x)
```

```
[Out] -(2*a*(a - b*x)^(1/2) - 4*b*x*(a - b*x)^(1/2))/(x^(1/2)*(a^3 - a^2*b*x))
```

sympy [A] time = 1.68, size = 112, normalized size = 2.73

$$\begin{cases} -\frac{2}{a\sqrt{b}x\sqrt{\frac{a}{bx}-1}}+\frac{4\sqrt{b}}{a^2\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{2iab^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}}{a^3b-a^2b^2x}+\frac{4ib^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}}{a^3b-a^2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((-2/(a*sqrt(b)*x*sqrt(a/(b*x) - 1)) + 4*sqrt(b)/(a**2*sqrt(a/(b*x
) - 1)), Abs(a/(b*x)) > 1), (-2*I*a*b**(3/2)*sqrt(-a/(b*x) + 1)/(a**3*b - a
**2*b**2*x) + 4*I*b**(5/2)*x*sqrt(-a/(b*x) + 1)/(a**3*b - a**2*b**2*x), Tru
e))
```

3.601 $\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{2}{ax^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] 2/(a*x^(3/2)*Sqrt[a - b*x]) - (8*Sqrt[a - b*x])/(3*a^2*x^(3/2)) - (16*b*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx &= \frac{2}{ax^{3/2}\sqrt{a-bx}}+\frac{4\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a}\\ &= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}+\frac{(8b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}\\ &= \frac{2}{ax^{3/2}\sqrt{a-bx}}-\frac{8\sqrt{a-bx}}{3a^2x^{3/2}}-\frac{16b\sqrt{a-bx}}{3a^3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.59

$$-\frac{2\left(a^2+4abx-8b^2x^2\right)}{3a^3x^{3/2}\sqrt{a-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*(a^2 + 4*a*b*x - 8*b^2*x^2))/(3*a^3*x^(3/2)*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.15, size = 50, normalized size = 0.76

$$-\frac{2\sqrt{a-bx}\left(-a^2-4abx+8b^2x^2\right)}{3a^3x^{3/2}(bx-a)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a - b*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(-a^2 - 4*a*b*x + 8*b^2*x^2))/(3*a^3*x^(3/2)*(-a + b*x))
```

fricas [A] time = 1.20, size = 51, normalized size = 0.77

$$-\frac{2\left(8\,b^2x^2-4\,abx-a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^3bx^3-a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 4*a*b*x - a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b*x^3 - a^4*x^
2)
```

giac [B] time = 1.48, size = 112, normalized size = 1.70

$$-\frac{2\sqrt{-bx+a}\left(\frac{5\,(bx-a)b^2|b|}{a^3}+\frac{6\,b^2|b|}{a^2}\right)}{3\left((bx-a)b+ab\right)^{\frac{3}{2}}}-\frac{4\sqrt{-b}\,b^3}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-b*x + a)*(5*(b*x - a)*b^2*abs(b)/a^3 + 6*b^2*abs(b)/a^2)/((b*x -
 a)*b + a*b)^(3/2) - 4*sqrt(-b)*b^3/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x
- a)*b + a*b))^2 - a*b)*a^2*abs(b))
```

maple [A] time = 0.00, size = 34, normalized size = 0.52

$$-\frac{2\left(-8b^2x^2+4abx+a^2\right)}{3\sqrt{-bx+a}\;a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(3/2),x)
```

```
[Out] -2/3*(-8*b^2*x^2+4*a*b*x+a^2)/(-b*x+a)^(1/2)/x^(3/2)/a^3
```

maxima [A] time = 1.35, size = 52, normalized size = 0.79

$$\frac{2\,b^2\sqrt{x}}{\sqrt{-bx+a}\,a^3}-\frac{2\left(\frac{6\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^2*sqrt(x)/(sqrt(-b*x + a)*a^3) - 2/3*(6*sqrt(-b*x + a)*b/sqrt(x) + (-b*
x + a)^(3/2)/x^(3/2))/a^3
```

mupad [B] time = 0.43, size = 48, normalized size = 0.73

$$\frac{\sqrt{a-b\,x}\,\left(\frac{8\,x}{3\,a^2}+\frac{2}{3\,a\,b}-\frac{16\,b\,x^2}{3\,a^3}\right)}{x^{5/2}-\frac{a\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(3/2)),x)
```

```
[Out] ((a - b*x)^(1/2)*((8*x)/(3*a^2) + 2/(3*a*b) - (16*b*x^2)/(3*a^3)))/(x^(5/2)
 - (a*x^(3/2))/b)
```

sympy [B] time = 4.64, size = 452, normalized size = 6.85

$$\begin{cases} \frac{2a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}+\frac{6a^2b^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}-\frac{24ab^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}+\frac{16b^{\frac{15}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} & \text{for}\ \left|\frac{a}{bx}\right|>1 \\ \frac{2ia^3b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}+\frac{6ia^2b^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}-\frac{24iab^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3}+\frac{16ib^{\frac{15}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^5b^4x+6a^4b^5x^2-3a^3b^6x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(3/2),x)
```

```
[Out] Piecewise((2*a**3*b**(9/2)*sqrt(a/(b*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*
x**2 - 3*a**3*b**6*x**3) + 6*a**2*b**(11/2)*x*sqrt(a/(b*x) - 1)/(-3*a**5*b*
*4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) - 24*a*b**(13/2)*x**2*sqrt(a/(b
*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) + 16*b**(15
/2)*x**3*sqrt(a/(b*x) - 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6
*x**3), Abs(a/(b*x)) > 1), (2*I*a**3*b**(9/2)*sqrt(-a/(b*x) + 1)/(-3*a**5*b
**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) + 6*I*a**2*b**(11/2)*x*sqrt(-a
/(b*x) + 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a**3*b**6*x**3) - 24*I*a
*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a**5*b**4*x + 6*a**4*b**5*x**2 - 3*a
**3*b**6*x**3) + 16*I*b**(15/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**5*b**4*x + 6
*a**4*b**5*x**2 - 3*a**3*b**6*x**3), True))
```

3.602 $\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=95

$$\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {47, 50, 63, 217, 203}

$$-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(a - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[a - b*x]) - (5
*Sqrt[x]*Sqrt[a - b*x])/b^3 + (5*a*ArcTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])
/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{5\int \frac{x^{3/2}}{(a-bx)^{3/2}}\,dx}{3b}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}+\frac{5\int \frac{\sqrt{x}}{\sqrt{a-bx}}\,dx}{b^2}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{2b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{(5a)\,\mathrm{Subst}\left(\int \frac{1}{1+bx^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^3}\\
&= \frac{2x^{5/2}}{3b(a-bx)^{3/2}}-\frac{10x^{3/2}}{3b^2\sqrt{a-bx}}-\frac{5\sqrt{x}\sqrt{a-bx}}{b^3}+\frac{5a\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.54

$$
\frac{2x^{7/2}\sqrt{1-\frac{bx}{a}}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{a}\right)}{7a^2\sqrt{a-bx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 - (b*x)/a]*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/a])/(7*
a^2*Sqrt[a - b*x])
```

IntegrateAlgebraic [A] time = 0.20, size = 96, normalized size = 1.01

$$
\frac{\sqrt{a-bx}\left(-15a^2\sqrt{x}+20abx^{3/2}-3b^2x^{5/2}\right)}{3b^3(bx-a)^2}+\frac{5a\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(a - b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - b*x]*(-15*a^2*Sqrt[x] + 20*a*b*x^(3/2) - 3*b^2*x^(5/2)))/(3*b^3*(
-a + b*x)^2) + (5*a*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^4
```

fricas [A] time = 1.35, size = 215, normalized size = 2.26

$$
\left[-\frac{15\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{-b}\log\left(-2bx+2\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)+2\left(3b^3x^2-20ab^2x+15a^2b\right)\sqrt{-bx+a}\sqrt{x}}{6\left(b^6x^2-2ab^5x+a^2b^4\right)},-\frac{15\left(ab^2x^2-2a^2bx+a^3\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)+\left(3b^3x^2-20ab^2x+15a^2b\right)\sqrt{-bx+a}\sqrt{x}}{3\left(b^6x^2-2ab^5x+a^2b^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b^2*x^2 - 2*a^2*b*x + a^3)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x +
 a)*sqrt(-b)*sqrt(x) + a) + 2*(3*b^3*x^2 - 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x
 + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*b^4), -1/3*(15*(a*b^2*x^2 - 2*a^2
```

```
*b*x + a^3)*sqrt(b)*arctan(sqrt(-b*x + a)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 -
 20*a*b^2*x + 15*a^2*b)*sqrt(-b*x + a)*sqrt(x))/(b^6*x^2 - 2*a*b^5*x + a^2*
b^4)]
```

giac [B] time = 112.52, size = 221, normalized size = 2.33

$$\frac{\left(\frac{15\,a\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\right)}{\sqrt{-b}\,b^2}-\frac{6\,\sqrt{(bx-a)b+ab}\,\sqrt{-bx+a}}{b^3}-\frac{8\left(9\,a^2\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4-12\,a^3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2b+7\,a^4b^2\right)}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3\sqrt{-b}\,b}\right)|b|}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(15*a*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/(sqrt(
-b)*b^2) - 6*sqrt((b*x - a)*b + a*b)*sqrt(-b*x + a)/b^3 - 8*(9*a^2*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4 - 12*a^3*(sqrt(-b*x + a)*sqrt
(-b) - sqrt((b*x - a)*b + a*b))^2*b + 7*a^4*b^2)/(((sqrt(-b*x + a)*sqrt(-b)
 - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

maple [B] time = 0.04, size = 160, normalized size = 1.68

$$\frac{\left(\frac{5a\arctan\left(\frac{\left(x-\frac{a}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^2+ax}}\right)}{2b^{\frac{7}{2}}}+\frac{2\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2b}\;a^2}{3\left(x-\frac{a}{b}\right)^2b^5}+\frac{14\sqrt{-\left(x-\frac{a}{b}\right)a-\left(x-\frac{a}{b}\right)^2b}\;a}{3\left(x-\frac{a}{b}\right)b^4}\right)\sqrt{(-bx+a)\,x}}{\sqrt{-bx+a}\,\sqrt{x}}-\frac{\sqrt{-bx+a}\,\sqrt{x}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -x^(1/2)*(-b*x+a)^(1/2)/b^3+(5/2/b^(7/2)*a*arctan((x-1/2*a/b)/(-b*x^2+a*x)^
(1/2)*b^(1/2))+2/3/b^5*a^2/(x-a/b)^2*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2)+14/3/b^
4*a/(x-a/b)*(-(x-a/b)*a-(x-a/b)^2*b)^(1/2))*((-b*x+a)*x)^(1/2)/(-b*x+a)^(1/
2)/x^(1/2)
```

maxima [A] time = 3.02, size = 94, normalized size = 0.99

$$\frac{2\,ab^2+\frac{10\,(bx-a)ab}{x}-\frac{15\,(bx-a)^2a}{x^2}}{3\left(\frac{(-bx+a)^{\frac{3}{2}}b^4}{x^{\frac{3}{2}}}+\frac{(-bx+a)^{\frac{5}{2}}b^3}{x^{\frac{5}{2}}}\right)}-\frac{5\,a\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a*b^2 + 10*(b*x - a)*a*b/x - 15*(b*x - a)^2*a/x^2)/((-b*x + a)^(3/2)
*b^4/x^(3/2) + (-b*x + a)^(5/2)*b^3/x^(5/2)) - 5*a*arctan(sqrt(-b*x + a)/(s
qrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(a-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(a - b*x)^(5/2), x)
```

sympy [B] time = 8.48, size = 971, normalized size = 10.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-30*I*a**(81/2)*b**22*x**(51/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*s
qrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(7
7/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 15*pi*a**(81/2)*b**22*x**(51/2
)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a*
*(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 30*I*a**(79/2)*b**23*x**(53
/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(6*a**(79/2)*b**(51/2)*
x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/
a)) - 15*pi*a**(79/2)*b**23*x**(53/2)*sqrt(-1 + b*x/a)/(6*a**(79/2)*b**(51/
2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b
*x/a)) + 30*I*a**40*b**(45/2)*x**26/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-
1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) - 40*I*a**39
*b**(47/2)*x**27/(6*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(
77/2)*b**(53/2)*x**(53/2)*sqrt(-1 + b*x/a)) + 6*I*a**38*b**(49/2)*x**28/(6*
a**(79/2)*b**(51/2)*x**(51/2)*sqrt(-1 + b*x/a) - 6*a**(77/2)*b**(53/2)*x**(
53/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (15*a**(81/2)*b**22*x**(51/2)*sqr
t(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(79/2)*b**(51/2)*x**(51/2)
*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 15*a*
*(79/2)*b**23*x**(53/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a*
*(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/
2)*sqrt(1 - b*x/a)) - 15*a**40*b**(45/2)*x**26/(3*a**(79/2)*b**(51/2)*x**(5
1/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) + 2
0*a**39*b**(47/2)*x**27/(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) -
3*a**(77/2)*b**(53/2)*x**(53/2)*sqrt(1 - b*x/a)) - 3*a**38*b**(49/2)*x**28/
(3*a**(79/2)*b**(51/2)*x**(51/2)*sqrt(1 - b*x/a) - 3*a**(77/2)*b**(53/2)*x*
*(53/2)*sqrt(1 - b*x/a)), True))
```

3.603 $\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 63, 217, 203}

$$-\frac{2\sqrt{x}}{b^2\sqrt{a-bx}}+\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(a - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[a - b*x]) + (2*Ar
cTan[(Sqrt[b]*Sqrt[x])/Sqrt[a - b*x]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(a-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{\int \frac{\sqrt{x}}{(a-bx)^{3/2}}\,dx}{b}\\ &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{a-bx}}\,dx}{b^2}\\ &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}\\ &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{a-bx}}\right)}{b^2}\\ &= \frac{2x^{3/2}}{3b(a-bx)^{3/2}} - \frac{2\sqrt{x}}{b^2\sqrt{a-bx}} + \frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a-bx}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 82, normalized size = 1.14

$$\frac{2\left(\sqrt{b}\sqrt{x}(4bx-3a)+3\sqrt{a}(a-bx)\sqrt{1-\frac{bx}{a}}\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{3b^{5/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (2*(Sqrt[b]*Sqrt[x]*(-3*a + 4*b*x) + 3*Sqrt[a]*(a - b*x)*Sqrt[1 - (b*x)/a]*
ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(3*b^(5/2)*(a - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 82, normalized size = 1.14

$$\frac{2\sqrt{-b}\log\left(\sqrt{a-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3} - \frac{2\sqrt{a-bx}\left(3a\sqrt{x}-4bx^{3/2}\right)}{3b^2(bx-a)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(a - b*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a - b*x]*(3*a*Sqrt[x] - 4*b*x^(3/2)))/(3*b^2*(-a + b*x)^2) + (2*Sq
rt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[a - b*x]])/b^3
```

fricas [A] time = 1.56, size = 188, normalized size = 2.61

$$\left[-\frac{3\left(b^2x^2-2\,abx+a^2\right)\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+a}\sqrt{-b}\sqrt{x}+a\right)-2\left(4\,b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}}{3\left(b^5x^2-2\,ab^4x+a^2b^3\right)}, -\frac{2\left(3\left(b^2x^2-2\,abx+a^2\right)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\sqrt{x}}\right)-\left(4\,b^2x-3\,ab\right)\sqrt{-bx+a}\sqrt{x}\right)}{3\left(b^5x^2-2\,ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + a)*s
qrt(-b)*sqrt(x) + a) - 2*(4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x))/(b^5*x^2
 - 2*a*b^4*x + a^2*b^3), -2/3*(3*(b^2*x^2 - 2*a*b*x + a^2)*sqrt(b)*arctan(s
qrt(-b*x + a)/(sqrt(b)*sqrt(x))) - (4*b^2*x - 3*a*b)*sqrt(-b*x + a)*sqrt(x)
)/(b^5*x^2 - 2*a*b^4*x + a^2*b^3)]
```

giac [B] time = 110.08, size = 194, normalized size = 2.69

$$\frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\right)}{\sqrt{-b}}+\frac{8\left(3\,a\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{4}\sqrt{-b}-3\,a^{2}\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}\sqrt{-b}\,b+2\,a^{3}\sqrt{-b}\,b^{2}\right)}{\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^{2}-ab\right)^{3}}\right)|b|}{3\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2)/sqrt(-b)
+ 8*(3*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) - 3
*a^2*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b + 2*a
^3*sqrt(-b)*b^2)/((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a
*b)^3)*abs(b)/b^3
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(-bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] int(x^(3/2)/(-b*x+a)^(5/2),x)
```

maxima [A] time = 2.94, size = 52, normalized size = 0.72

$$\frac{2\left(b+\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}b^{2}}-\frac{2\,\arctan\left(\frac{\sqrt{-bx+a}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + a)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a-b\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(a - b*x)^(5/2), x)
```

sympy [B] time = 4.50, size = 833, normalized size = 11.57

$$\begin{cases} -\frac{6ia^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{3\pi a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}\operatorname{acosh}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{3\pi a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}+\frac{6ia^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}}-\frac{8ia^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-1+\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{-1+\frac{bx}{a}}} & \text{for}\: \left|\frac{bx}{a}\right|>1 \\ \frac{6a^{\frac{39}{2}}b^{11}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{\frac{37}{2}}b^{12}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}\operatorname{asin}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}-\frac{6a^{19}b^{\frac{23}{2}}x^{14}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}}+\frac{8a^{18}b^{\frac{25}{2}}x^{15}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{1-\frac{bx}{a}}-3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{\frac{29}{2}}\sqrt{1-\frac{bx}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*I*a**(39/2)*b**11*x**(27/2)*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sq
rt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37
/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 3*pi*a**(39/2)*b**11*x**(27/2)*
sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(
37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) + 6*I*a**(37/2)*b**12*x**(29/2)
*sqrt(-1 + b*x/a)*acosh(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27/2)*x**
(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a))
 - 3*pi*a**(37/2)*b**12*x**(29/2)*sqrt(-1 + b*x/a)/(3*a**(39/2)*b**(27/2)*x
**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a
)) + 6*I*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b
*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)) - 8*I*a**18*b**(2
5/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(-1 + b*x/a) - 3*a**(37/2)*
b**(29/2)*x**(29/2)*sqrt(-1 + b*x/a)), Abs(b*x/a) > 1), (6*a**(39/2)*b**11*
x**(27/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sqrt(a))/(3*a**(39/2)*b**(27
/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*
x/a)) - 6*a**(37/2)*b**12*x**(29/2)*sqrt(1 - b*x/a)*asin(sqrt(b)*sqrt(x)/sq
rt(a))/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(2
9/2)*x**(29/2)*sqrt(1 - b*x/a)) - 6*a**19*b**(23/2)*x**14/(3*a**(39/2)*b**(
27/2)*x**(27/2)*sqrt(1 - b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 -
b*x/a)) + 8*a**18*b**(25/2)*x**15/(3*a**(39/2)*b**(27/2)*x**(27/2)*sqrt(1 -
 b*x/a) - 3*a**(37/2)*b**(29/2)*x**(29/2)*sqrt(1 - b*x/a)), True))
```

3.604 $\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a - b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(a-bx)^{5/2}}\,dx = \frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a - b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 22, normalized size = 1.00

$$\frac{2x^{3/2}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(a - b*x)^(5/2),x]
[Out] (2*x^(3/2))/(3*a*(a - b*x)^(3/2))
```

fricas [B] time = 1.11, size = 34, normalized size = 1.55

$$\frac{2\,\sqrt{-bx+a}\,x^{\frac{3}{2}}}{3\,\left(ab^2x^2-2\,a^2bx+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-b*x + a)*x^(3/2)/(a*b^2*x^2 - 2*a^2*b*x + a^3)
```

giac [B] time = 1.63, size = 102, normalized size = 4.64

$$\frac{4\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}+a^2\sqrt{-b}\,b^2\right)|b|}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b) + a^2
*sqrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))
^2 - a*b)^3*b^2)
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{2x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+a)^(5/2),x)
```

```
[Out] 2/3*x^(3/2)/a/(-b*x+a)^(3/2)
```

maxima [A] time = 1.36, size = 16, normalized size = 0.73

$$\frac{2\,x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)/((-b*x + a)^(3/2)*a)
```

mupad [B] time = 0.25, size = 37, normalized size = 1.68

$$\frac{2\,x^{3/2}\,\sqrt{a-b\,x}}{3\,\left(a^3-2\,a^2\,b\,x+a\,b^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a - b*x)^(5/2),x)
```

```
[Out] (2*x^(3/2)*(a - b*x)^(1/2))/(3*(a^3 + a*b^2*x^2 - 2*a^2*b*x))
```

sympy [B] time = 1.51, size = 95, normalized size = 4.32

$$\begin{cases}\frac{2ix^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{-1+\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{-1+\frac{bx}{a}}} & \text{for}\ \left|\frac{bx}{a}\right|>1\\ -\frac{2x^{\frac{3}{2}}}{-3a^{\frac{5}{2}}\sqrt{1-\frac{bx}{a}}+3a^{\frac{3}{2}}bx\sqrt{1-\frac{bx}{a}}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*I*x**(3/2)/(-3*a**(5/2)*sqrt(-1 + b*x/a) + 3*a**(3/2)*b*x*sqrt
(-1 + b*x/a)), Abs(b*x/a) > 1), (-2*x**(3/2)/(-3*a**(5/2)*sqrt(1 - b*x/a) +
 3*a**(3/2)*b*x*sqrt(1 - b*x/a)), True))
```

3.605 $\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=45

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}+\frac{2\sqrt{x}}{3a(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x])/(3*a*(a - b*x)^(3/2)) + (4*Sqrt[x])/(3*a^2*Sqrt[a - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(a-bx)^{5/2}}\,dx &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{2\int \frac{1}{\sqrt{x}(a-bx)^{3/2}}\,dx}{3a}\\ &= \frac{2\sqrt{x}}{3a(a-bx)^{3/2}}+\frac{4\sqrt{x}}{3a^2\sqrt{a-bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.67

$$\frac{2\sqrt{x}(3a-2bx)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[x]*(3*a - 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 30, normalized size = 0.67

$$-\frac{2\sqrt{x}\,(2bx-3a)}{3a^2(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*Sqrt[x]*(-3*a + 2*b*x))/(3*a^2*(a - b*x)^(3/2))
```

fricas [A] time = 1.25, size = 44, normalized size = 0.98

$$-\frac{2\,(2\,bx-3\,a)\sqrt{-bx+a}\,\sqrt{x}}{3\,\left(a^2b^2x^2-2\,a^3bx+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*b*x - 3*a)*sqrt(-b*x + a)*sqrt(x)/(a^2*b^2*x^2 - 2*a^3*b*x + a^4)
```

giac [B] time = 1.43, size = 96, normalized size = 2.13

$$\frac{8\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*abs(
b))
```

maple [A] time = 0.00, size = 25, normalized size = 0.56

$$\frac{2\,(-2bx+3a)\,\sqrt{x}}{3\,(-bx+a)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(5/2)/x^(1/2),x)
```

```
[Out] 2/3*x^(1/2)*(-2*b*x+3*a)/(-b*x+a)^(3/2)/a^2
```

maxima [A] time = 1.34, size = 30, normalized size = 0.67

$$\frac{2\left(b-\frac{3\,(bx-a)}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b - 3*(b*x - a)/x)*x^(3/2)/((-b*x + a)^(3/2)*a^2)
```

mupad [B] time = 0.41, size = 56, normalized size = 1.24

$$\frac{6\,a\,\sqrt{x}\,\sqrt{a-b\,x}\,-4\,b\,x^{3/2}\,\sqrt{a-b\,x}}{3\,a^4-6\,a^3\,b\,x+3\,a^2\,b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a*x^(1/2)*(a - b*x)^(1/2) - 4*b*x^(3/2)*(a - b*x)^(1/2))/(3*a^4 + 3*a^2*
b^2*x^2 - 6*a^3*b*x)
```

sympy [C] time = 2.00, size = 211, normalized size = 4.69

$$\begin{cases} \frac{6ia}{3ia^3\sqrt{b}\sqrt{\frac{a}{bx}-1}-3ia^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} - \frac{4ibx}{3ia^3\sqrt{b}\sqrt{\frac{a}{bx}-1}-3ia^2b^{\frac{3}{2}}x\sqrt{\frac{a}{bx}-1}} & \text{for } \left|\frac{a}{bx}\right| > 1 \\ \frac{6ab}{3ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}-3ia^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} - \frac{4b^2x}{3ia^3b^{\frac{3}{2}}\sqrt{-\frac{a}{bx}+1}-3ia^2b^{\frac{5}{2}}x\sqrt{-\frac{a}{bx}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((6*I*a/(3*I*a**3*sqrt(b)*sqrt(a/(b*x) - 1) - 3*I*a**2*b**(3/2)*x*
sqrt(a/(b*x) - 1)) - 4*I*b*x/(3*I*a**3*sqrt(b)*sqrt(a/(b*x) - 1) - 3*I*a**2
*b**(3/2)*x*sqrt(a/(b*x) - 1)), Abs(a/(b*x)) > 1), (6*a*b/(3*I*a**3*b**(3/2
)*sqrt(-a/(b*x) + 1) - 3*I*a**2*b**(5/2)*x*sqrt(-a/(b*x) + 1)) - 4*b**2*x/(
3*I*a**3*b**(3/2)*sqrt(-a/(b*x) + 1) - 3*I*a**2*b**(5/2)*x*sqrt(-a/(b*x) +
1)), True))
```

3.606 $\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*Sqrt[x]*(a - b*x)^(3/2)) + 8/(3*a^2*Sqrt[x]*Sqrt[a - b*x]) - (16*Sqr
t[a - b*x])/(3*a^3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{3/2}(a-bx)^{5/2}}\,dx &= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}+\frac{4\int \frac{1}{x^{3/2}(a-bx)^{3/2}}\,dx}{3a}\\
&= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}+\frac{8\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^2}\\
&= \frac{2}{3a\sqrt{x}\,(a-bx)^{3/2}}+\frac{8}{3a^2\sqrt{x}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 41, normalized size = 0.61

$$-\frac{2\left(3a^2-12abx+8b^2x^2\right)}{3a^3\sqrt{x}\,(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(3*a^2 - 12*a*b*x + 8*b^2*x^2))/(3*a^3*Sqrt[x]*(a - b*x)^(3/2))
```

fricas [A] time = 1.19, size = 59, normalized size = 0.88

$$-\frac{2\left(8\,b^2x^2-12\,abx+3\,a^2\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^3b^2x^3-2\,a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*x^2 - 12*a*b*x + 3*a^2)*sqrt(-b*x + a)*sqrt(x)/(a^3*b^2*x^3 - 2
*a^4*b*x^2 + a^5*x)
```

giac [B] time = 1.61, size = 189, normalized size = 2.82

$$-\frac{2\sqrt{-bx+a}\,b^2}{\sqrt{(bx-a)b+ab}\,a^3|b|}-\frac{4\left(3\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}\,b^2-12\,a\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}\,b^3+5\,a^2\sqrt{-b}\,b^4\right)}{3\left(\left(\sqrt{-bx+a}\,\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-b*x + a)*b^2/(sqrt((b*x - a)*b + a*b)*a^3*abs(b)) - 4/3*(3*(sqrt(-
b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^4*sqrt(-b)*b^2 - 12*a*(sqrt(-b
*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2*sqrt(-b)*b^3 + 5*a^2*sqrt(-b)
*b^4)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b))^2 - a*b)^3*a^2*
abs(b))
```

maple [A] time = 0.00, size = 36, normalized size = 0.54

$$-\frac{2\left(8b^2x^2-12abx+3a^2\right)}{3\left(-bx+a\right)^{\frac{3}{2}}a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(8*b^2*x^2-12*a*b*x+3*a^2)/(-b*x+a)^(3/2)/x^(1/2)/a^3
```

maxima [A] time = 1.29, size = 50, normalized size = 0.75

$$\frac{2\left(b^2-\frac{6\,(bx-a)b}{x}\right)x^{\frac{3}{2}}}{3\left(-bx+a\right)^{\frac{3}{2}}a^3}-\frac{2\sqrt{-bx+a}}{a^3\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2 - 6*(b*x - a)*b/x)*x^(3/2)/((-b*x + a)^(3/2)*a^3) - 2*sqrt(-b*x +
a)/(a^3*sqrt(x))
```

mupad [B] time = 0.44, size = 73, normalized size = 1.09

$$\frac{6\,a^2\,\sqrt{a-b\,x}\,+16\,b^2\,x^2\,\sqrt{a-b\,x}\,-24\,a\,b\,x\,\sqrt{a-b\,x}}{\sqrt{x}\,\left(x\,\left(6\,a^4\,b-3\,a^3\,b^2\,x\right)-3\,a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (6*a^2*(a - b*x)^(1/2) + 16*b^2*x^2*(a - b*x)^(1/2) - 24*a*b*x*(a - b*x)^(1
/2))/(x^(1/2)*(x*(6*a^4*b - 3*a^3*b^2*x) - 3*a^5))
```

sympy [B] time = 4.27, size = 314, normalized size = 4.69

$$\begin{cases} -\frac{6a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24ab^{\frac{11}{2}}x\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx}-1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{for } \left|\frac{a}{bx}\right|>1 \\ -\frac{6ia^2b^{\frac{9}{2}}\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}+\frac{24iab^{\frac{11}{2}}x\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2}-\frac{16ib^{\frac{13}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{3a^5b^4-6a^4b^5x+3a^3b^6x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((-6*a**2*b**(9/2)*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a**4*b**5*x
+ 3*a**3*b**6*x**2) + 24*a*b**(11/2)*x*sqrt(a/(b*x) - 1)/(3*a**5*b**4 - 6*a
**4*b**5*x + 3*a**3*b**6*x**2) - 16*b**(13/2)*x**2*sqrt(a/(b*x) - 1)/(3*a**
5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2), Abs(a/(b*x)) > 1), (-6*I*a**2*b
**(9/2)*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a**3*b**6*x**2)
 + 24*I*a*b**(11/2)*x*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a**4*b**5*x + 3*a
**3*b**6*x**2) - 16*I*b**(13/2)*x**2*sqrt(-a/(b*x) + 1)/(3*a**5*b**4 - 6*a*
*4*b**5*x + 3*a**3*b**6*x**2), True))
```

3.607 $\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}+\frac{2}{3ax^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] 2/(3*a*x^(3/2)*(a - b*x)^(3/2)) + 4/(a^2*x^(3/2)*Sqrt[a - b*x]) - (16*Sqrt[
a - b*x])/(3*a^3*x^(3/2)) - (32*b*Sqrt[a - b*x])/(3*a^4*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(a-bx)^{5/2}}\,dx &= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{2\int \frac{1}{x^{5/2}(a-bx)^{3/2}}\,dx}{a}\\ &= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}+\frac{8\int \frac{1}{x^{5/2}\sqrt{a-bx}}\,dx}{a^2}\\ &= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}+\frac{(16b)\int \frac{1}{x^{3/2}\sqrt{a-bx}}\,dx}{3a^3}\\ &= \frac{2}{3ax^{3/2}(a-bx)^{3/2}}+\frac{4}{a^2x^{3/2}\sqrt{a-bx}}-\frac{16\sqrt{a-bx}}{3a^3x^{3/2}}-\frac{32b\sqrt{a-bx}}{3a^4\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.57

$$-\frac{2\left(a^3+6a^2bx-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 50, normalized size = 0.57

$$-\frac{2\left(a^3+6a^2bx-24ab^2x^2+16b^3x^3\right)}{3a^4x^{3/2}(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(a - b*x)^(5/2)),x]
```

```
[Out] (-2*(a^3 + 6*a^2*b*x - 24*a*b^2*x^2 + 16*b^3*x^3))/(3*a^4*x^(3/2)*(a - b*x)
^(3/2))
```

fricas [A] time = 1.23, size = 70, normalized size = 0.80

$$-\frac{2\left(16\,b^3x^3-24\,ab^2x^2+6\,a^2bx+a^3\right)\sqrt{-bx+a}\,\sqrt{x}}{3\left(a^4b^2x^4-2\,a^5bx^3+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*b^3*x^3 - 24*a*b^2*x^2 + 6*a^2*b*x + a^3)*sqrt(-b*x + a)*sqrt(x)/(
a^4*b^2*x^4 - 2*a^5*b*x^3 + a^6*x^2)
```

giac [B] time = 1.70, size = 207, normalized size = 2.35

$$-\frac{2\sqrt{-bx+a}\left(\frac{8(bx-a)b^2|b|}{a^4}+\frac{9b^2|b|}{a^3}\right)}{3((bx-a)b+ab)^{\frac{3}{2}}}-\frac{8\left(3\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^4\sqrt{-b}\,b^3-9\,a\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2\sqrt{-b}\,b^4+4\,a^2\sqrt{-b}\,b^5\right)}{3\left(\left(\sqrt{-bx+a}\sqrt{-b}-\sqrt{(bx-a)b+ab}\right)^2-ab\right)^3a^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(-b*x + a)*(8*(b*x - a)*b^2*abs(b)/a^4 + 9*b^2*abs(b)/a^3)/((b*x -
 a)*b + a*b)^(3/2) - 8/3*(3*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a
*b))^4*sqrt(-b)*b^3 - 9*a*(sqrt(-b*x + a)*sqrt(-b) - sqrt((b*x - a)*b + a*b
))^2*sqrt(-b)*b^4 + 4*a^2*sqrt(-b)*b^5)/(((sqrt(-b*x + a)*sqrt(-b) - sqrt((
b*x - a)*b + a*b))^2 - a*b)^3*a^3*abs(b))
```

maple [A] time = 0.00, size = 45, normalized size = 0.51

$$-\frac{2\left(16b^3x^3-24a\,b^2x^2+6a^2bx+a^3\right)}{3\,(-bx+a)^{\frac{3}{2}}\,a^4x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+a)^(5/2),x)
```

```
[Out] -2/3*(16*b^3*x^3-24*a*b^2*x^2+6*a^2*b*x+a^3)/(-b*x+a)^(3/2)/x^(3/2)/a^4
```

maxima [A] time = 1.33, size = 68, normalized size = 0.77

$$-\frac{2\left(\frac{9\sqrt{-bx+a}\,b}{\sqrt{x}}+\frac{(-bx+a)^{\frac{3}{2}}}{x^{\frac{3}{2}}}\right)}{3\,a^4}+\frac{2\left(b^3-\frac{9\,(bx-a)b^2}{x}\right)x^{\frac{3}{2}}}{3\,(-bx+a)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(9*sqrt(-b*x + a)*b/sqrt(x) + (-b*x + a)^(3/2)/x^(3/2))/a^4 + 2/3*(b^3
 - 9*(b*x - a)*b^2/x)*x^(3/2)/((-b*x + a)^(3/2)*a^4)
```

mupad [B] time = 0.47, size = 92, normalized size = 1.05

$$\frac{2\,a^3\,\sqrt{a-b\,x}\,+32\,b^3\,x^3\,\sqrt{a-b\,x}\,+12\,a^2\,b\,x\,\sqrt{a-b\,x}\,-48\,a\,b^2\,x^2\,\sqrt{a-b\,x}}{x^{3/2}\,\left(x\,\left(6\,a^5\,b-3\,a^4\,b^2\,x\right)-3\,a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a - b*x)^(5/2)),x)
```

```
[Out] (2*a^3*(a - b*x)^(1/2) + 32*b^3*x^3*(a - b*x)^(1/2) + 12*a^2*b*x*(a - b*x)^
(1/2) - 48*a*b^2*x^2*(a - b*x)^(1/2))/(x^(3/2)*(x*(6*a^5*b - 3*a^4*b^2*x) -
 3*a^6))
```

sympy [B] time = 13.45, size = 688, normalized size = 7.82

$$\begin{cases} \frac{2a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10a^3b^{\frac{21}{2}}x\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60a^2b^{\frac{23}{2}}x^2\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80ab^{\frac{25}{2}}x^3\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32b^{\frac{27}{2}}x^4\sqrt{\frac{a}{bx}-1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{for}\: \left|\frac{a}{bx}\right| > 1 \\ \frac{2ia^4b^{\frac{19}{2}}\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{10ia^3b^{\frac{21}{2}}x\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{60ia^2b^{\frac{23}{2}}x^2\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} + \frac{80iab^{\frac{25}{2}}x^3\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} - \frac{32ib^{\frac{27}{2}}x^4\sqrt{-\frac{a}{bx}+1}}{-3a^7b^9x+9a^6b^{10}x^2-9a^5b^{11}x^3+3a^4b^{12}x^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+a)**(5/2),x)
```

```
[Out] Piecewise((2*a**4*b**(19/2)*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**1
0*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*a**3*b**(21/2)*x*sqrt(
a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a*
*4*b**12*x**4) - 60*a**2*b**(23/2)*x**2*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x +
 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80*a*b**(25/2
)*x**3*sqrt(a/(b*x) - 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11
*x**3 + 3*a**4*b**12*x**4) - 32*b**(27/2)*x**4*sqrt(a/(b*x) - 1)/(-3*a**7*b
**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4), Abs(a/(
b*x)) > 1), (2*I*a**4*b**(19/2)*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6
*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 10*I*a**3*b**(21/2)*
x*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**
3 + 3*a**4*b**12*x**4) - 60*I*a**2*b**(23/2)*x**2*sqrt(-a/(b*x) + 1)/(-3*a*
*7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) + 80
*I*a*b**(25/2)*x**3*sqrt(-a/(b*x) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2
- 9*a**5*b**11*x**3 + 3*a**4*b**12*x**4) - 32*I*b**(27/2)*x**4*sqrt(-a/(b*x
) + 1)/(-3*a**7*b**9*x + 9*a**6*b**10*x**2 - 9*a**5*b**11*x**3 + 3*a**4*b**
12*x**4), True))
```

3.608 $\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=88

$$-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{5x^{3/2}\sqrt{bx+2}}{6b^2}+\frac{5\sqrt{x}\sqrt{bx+2}}{2b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{x^{5/2}\sqrt{bx+2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 + b*x],x]

[Out] (5*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 + b*x])/(6*b^2) + (x^
(5/2)*Sqrt[2 + b*x])/(3*b) - (5*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{2+bx}}\,dx &= \frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^3}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= \frac{5\sqrt{x}\sqrt{2+bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2+bx}}{6b^2}+\frac{x^{5/2}\sqrt{2+bx}}{3b}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.68

$$\frac{\sqrt{x}\sqrt{bx+2}\left(2b^2x^2-5bx+15\right)}{6b^3}-\frac{5\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x]*(15 - 5*b*x + 2*b^2*x^2))/(6*b^3) - (5*ArcSinh[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 73, normalized size = 0.83

$$\frac{5\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{7/2}}+\frac{\sqrt{bx+2}\left(2b^2x^{5/2}-5bx^{3/2}+15\sqrt{x}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(15*Sqrt[x] - 5*b*x^(3/2) + 2*b^2*x^(5/2)))/(6*b^3) + (5*Log
[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

fricas [A] time = 1.15, size = 124, normalized size = 1.41

$$\left[\frac{\left(2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+15\,\sqrt{b}\,\log\left(bx-\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{6\,b^4},\ \frac{\left(2\,b^3x^2-5\,b^2x+15\,b\right)\sqrt{bx+2}\,\sqrt{x}+30\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*((2*b^3*x^2 - 5*b^2*x + 15*b)*sqrt(b*x + 2)*sqrt(x) + 15*sqrt(b)*log(b
*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1))/b^4, 1/6*((2*b^3*x^2 - 5*b^2*x + 1
5*b)*sqrt(b*x + 2)*sqrt(x) + 30*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*s
qrt(x))))/b^4]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b*(2*((12*b^5/144/b^7*
sqrt(b*x+2)*sqrt(b*x+2)-78*b^5/144/b^7)*sqrt(b*x+2)*sqrt(b*x+2)+198*b^5/144
/b^7)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)+5/2/b/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2
*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.00, size = 93, normalized size = 1.06

$$\frac{\sqrt{bx+2}\,x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{bx+2}\,x^{\frac{3}{2}}}{6b^2}+\frac{5\sqrt{bx+2}\,\sqrt{x}}{2b^3}-\frac{5\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2\sqrt{bx+2}\,b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/3*x^(5/2)*(b*x+2)^(1/2)/b-5/6*x^(3/2)*(b*x+2)^(1/2)/b^2+5/2*(b*x+2)^(1/2)
/b^3*x^(1/2)-5/2*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(7/2)/x^(1/2)*ln((b*x+1)
/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.97, size = 134, normalized size = 1.52

$$-\frac{\frac{33\,\sqrt{bx+2}\,b^2}{\sqrt{x}}-\frac{40\,(bx+2)^{\frac{3}{2}}\,b}{x^{\frac{3}{2}}}+\frac{15\,(bx+2)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^6-\frac{3\,(bx+2)b^5}{x}+\frac{3\,(bx+2)^2b^4}{x^2}-\frac{(bx+2)^3b^3}{x^3}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(b*x + 2)*b^2/sqrt(x) - 40*(b*x + 2)^(3/2)*b/x^(3/2) + 15*(b*x
 + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x + 2)*b^5/x + 3*(b*x + 2)^2*b^4/x^2 - (b*
x + 2)^3*b^3/x^3) + 5/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + s
qrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(1/2), x)
```

sympy [A] time = 7.40, size = 95, normalized size = 1.08

$$\frac{x^{\frac{7}{2}}}{3\sqrt{bx+2}}-\frac{x^{\frac{5}{2}}}{6b\sqrt{bx+2}}+\frac{5x^{\frac{3}{2}}}{6b^2\sqrt{bx+2}}+\frac{5\sqrt{x}}{b^3\sqrt{bx+2}}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(7/2)/(3*sqrt(b*x + 2)) - x**(5/2)/(6*b*sqrt(b*x + 2)) + 5*x**(3/2)/(6*b
**2*sqrt(b*x + 2)) + 5*sqrt(x)/(b**3*sqrt(b*x + 2)) - 5*asinh(sqrt(2)*sqrt(
b)*sqrt(x)/2)/b**(7/2)
```

3.609 $\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx$

Optimal. Leaf size=67

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$-\frac{3\sqrt{x}\sqrt{bx+2}}{2b^2}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{x^{3/2}\sqrt{bx+2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 + b*x])/(2*b^2) + (x^(3/2)*Sqrt[2 + b*x])/(2*b) + (3*Arc
Sinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx &= \frac{x^{3/2}\sqrt{2+bx}}{2b}-\frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{2b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2+bx}}{2b^2}+\frac{x^{3/2}\sqrt{2+bx}}{2b}+\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{3\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{\sqrt{x}\sqrt{bx+2}\,(bx-3)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*(-3 + b*x)*Sqrt[2 + b*x])/(2*b^2) + (3*ArcSinh[(Sqrt[b]*Sqrt[x])/S
qrt[2]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 62, normalized size = 0.93

$$\frac{\sqrt{bx+2}\,\left(bx^{3/2}-3\sqrt{x}\right)}{2b^2}-\frac{3\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[2 + b*x]*(-3*Sqrt[x] + b*x^(3/2)))/(2*b^2) - (3*Log[-(Sqrt[b]*Sqrt[x]
) + Sqrt[2 + b*x]])/b^(5/2)
```

fricas [A] time = 1.00, size = 105, normalized size = 1.57

$$\left[\frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}+3\,\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{2\,b^3},\frac{\left(b^2x-3\,b\right)\sqrt{bx+2}\,\sqrt{x}-6\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) + 3*sqrt(b)*log(b*x + sqrt(b*x +
2)*sqrt(b)*sqrt(x) + 1))/b^3, 1/2*((b^2*x - 3*b)*sqrt(b*x + 2)*sqrt(x) - 6*
sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
```

```
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b^2*(2*(1/8*sqrt(b*x+2
)*sqrt(b*x+2)-5/8)*sqrt(b*x+2)*sqrt(b*(b*x+2)-2*b)-6*b/4/sqrt(b)*ln(abs(sqr
t(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2))))
```

maple [A] time = 0.00, size = 78, normalized size = 1.16

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{2b}-\frac{3\sqrt{bx+2}\,\sqrt{x}}{2b^{2}}+\frac{3\sqrt{(bx+2)\,x}\,\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{2\sqrt{bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/2*x^(3/2)*(b*x+2)^(1/2)/b-3/2*(b*x+2)^(1/2)/b^2*x^(1/2)+3/2*((b*x+2)*x)^(
1/2)/(b*x+2)^(1/2)/b^(5/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.86, size = 102, normalized size = 1.52

$$\frac{\frac{5\sqrt{bx+2}\,b}{\sqrt{x}}-\frac{3\,(bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2\,(bx+2)b^{3}}{x}+\frac{(bx+2)^{2}b^{2}}{x^{2}}}-\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] (5*sqrt(b*x + 2)*b/sqrt(x) - 3*(b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x + 2)*
b^3/x + (b*x + 2)^2*b^2/x^2) - 3/2*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(
sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{b\,x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(1/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(1/2), x)
```

sympy [A] time = 3.67, size = 75, normalized size = 1.12

$$\frac{x^{\frac{5}{2}}}{2\sqrt{bx+2}}-\frac{x^{\frac{3}{2}}}{2b\sqrt{bx+2}}-\frac{3\sqrt{x}}{b^{2}\sqrt{bx+2}}+\frac{3\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(5/2)/(2*sqrt(b*x + 2)) - x**(3/2)/(2*b*sqrt(b*x + 2)) - 3*sqrt(x)/(b**2
*sqrt(b*x + 2)) + 3*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

3.610 $\int \frac{\sqrt{x}}{\sqrt{2+bx}} \, dx$

Optimal. Leaf size=43

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {50, 54, 215}

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{2+bx}} \, dx &= \frac{\sqrt{x}\sqrt{2+bx}}{b}-\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}} \, dx}{b}\\
&= \frac{\sqrt{x}\sqrt{2+bx}}{b}-\frac{2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}} \, dx,x,\sqrt{x}\right)}{b}\\
&= \frac{\sqrt{x}\sqrt{2+bx}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{\sqrt{x}\sqrt{bx+2}}{b}-\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b - (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 1.14

$$\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}}+\frac{\sqrt{x}\sqrt{bx+2}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[2 + b*x],x]
```

```
[Out] (Sqrt[x]*Sqrt[2 + b*x])/b + (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(
3/2)
```

fricas [A] time = 1.02, size = 87, normalized size = 2.02

$$\left[\frac{\sqrt{bx+2}\,b\sqrt{x}+\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)}{b^2}, \frac{\sqrt{bx+2}\,b\sqrt{x}+2\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(b*x + 2)*b*sqrt(x) + sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x)
 + 1))/b^2, (sqrt(b*x + 2)*b*sqrt(x) + 2*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt
(-b)/(b*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
```

```
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]2*abs(b)/b^2/b*(1/2*sqrt(b*x+2)*sqr
t(b*(b*x+2)-2*b)+2*b/2/sqrt(b)*ln(abs(sqrt(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+
2))))
```

maple [A] time = 0.00, size = 62, normalized size = 1.44

$$\frac{\sqrt{bx+2}\,\sqrt{x}}{b}-\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (b*x+2)^(1/2)/b*x^(1/2)-((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(3/2)/x^(1/2)*ln(
(b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
```

maxima [B] time = 2.92, size = 70, normalized size = 1.63

$$\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{bx+2}}{\left(b^{2}-\frac{(bx+2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b
^(3/2) - 2*sqrt(b*x + 2)/((b^2 - (b*x + 2)*b/x)*sqrt(x))
```

mupad [B] time = 0.59, size = 43, normalized size = 1.00

$$\frac{4\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{b\,x+2}}\right)}{b^{3/2}}+\frac{\sqrt{x}\,\sqrt{b\,x+2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(1/2),x)
```

```
[Out] (4*atanh((b^(1/2)*x^(1/2))/(2^(1/2) - (b*x + 2)^(1/2))))/b^(3/2) + (x^(1/2)
*(b*x + 2)^(1/2))/b
```

sympy [A] time = 1.93, size = 54, normalized size = 1.26

$$\frac{x^{\frac{3}{2}}}{\sqrt{bx+2}}+\frac{2\sqrt{x}}{b\sqrt{bx+2}}-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] x**(3/2)/sqrt(b*x + 2) + 2*sqrt(x)/(b*sqrt(b*x + 2)) - 2*asinh(sqrt(2)*sqrt
(b)*sqrt(x)/2)/b**(3/2)
```

3.611 $\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.25

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/Sqrt[b]
```

fricas [A] time = 1.28, size = 55, normalized size = 2.29

$$\left[\frac{\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)}{\sqrt{b}},\,-\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1)/sqrt(b), -2*sqrt(-b)*arctan(s
qrt(b*x + 2)*sqrt(-b)/(b*sqrt(x)))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{-4,[0,1]%%%
}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{4,
[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}+
%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+%%%{4,[3,1]%%%}+%%%{-4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%%
}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%}+%%%{16,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,[4,
4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%
%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%%{20,[3,1]%%%}+%%%{-8,[
3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]%%
%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%}+%%%{-40,[1,2]%%%}+%%%
{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0,
2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [85.3561567
818,61.7937478349]Warning, choosing root of [1,0,%%%{-4,[1,1]%%%}+%%%{-4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{4,[1,2]%%%}+%%%{28,[1,1]%%%}+%%%{8,[1,0]%%%}+%%%{6,[0,
2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{-4,[3,3]%%%}+%%%{4,[3,2]%%%}+
%%%{4,[3,1]%%%}+%%%{-4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[
2,1]%%%}+%%%{8,[2,0]%%%}+%%%{4,[1,3]%%%}+%%%{20,[1,2]%%%}+%%%{-128,[1,1]%%%
}+%%%{16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-3
2,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]
%%%}+%%%{1,[4,0]%%%}+%%%{-4,[3,4]%%%}+%%%{12,[3,3]%%%}+%%%{-20,[3,2]%%%}+%%
%{20,[3,1]%%%}+%%%{-8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2
,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{-4,[1,4]%%%}+%%%{20,[1,3]%%%
}+%%%{-40,[1,2]%%%}+%%%{48,[1,1]%%%}+%%%{-32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{
-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parame
ters values [71.707969239,78.6493344628]-2/abs(b)*b^2/b/sqrt(b)*ln(abs(sqrt
(b*(b*x+2)-2*b)-sqrt(b)*sqrt(b*x+2)))
```

maple [B] time = 0.00, size = 46, normalized size = 1.92

$$\frac{\sqrt{(bx+2)\,x}\;\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{\sqrt{bx+2}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/b^(1/2)/x^(1/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2
*x)^(1/2))
```

maxima [B] time = 2.96, size = 41, normalized size = 1.71

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
sqrt(b)
```

mupad [B] time = 0.04, size = 30, normalized size = 1.25

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{b\,x+2}}{\sqrt{-b}\,\sqrt{x}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (b*x + 2)^(1/2))/((-b)^(1/2)*x^(1/2))))/(-b)^(1/2)
```

sympy [A] time = 1.02, size = 24, normalized size = 1.00

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b)
```

3.612 $\int \frac{1}{x^{3/2}\sqrt{2+bx}} dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2+bx}} dx = -\frac{\sqrt{2+bx}}{\sqrt{x}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 16, normalized size = 1.00

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -(Sqrt[2 + b*x]/Sqrt[x])
```

fricas [A] time = 1.28, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

giac [B] time = 1.11, size = 29, normalized size = 1.81

$$-\frac{\sqrt{bx+2}\,b^2}{\sqrt{(bx+2)b-2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b))
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(1/2),x)
```

```
[Out] -(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 1.35, size = 12, normalized size = 0.75

$$-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(b*x + 2)/sqrt(x)
```

mupad [B] time = 0.33, size = 12, normalized size = 0.75

$$-\frac{\sqrt{b\,x+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(b*x + 2)^(1/2)/x^(1/2)
```

sympy [A] time = 0.88, size = 15, normalized size = 0.94

$$-\sqrt{b}\sqrt{1+\frac{2}{bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(1 + 2/(b*x))
```

3.613 $\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=38

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b\sqrt{bx+2}}{3\sqrt{x}}-\frac{\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(3*x^(3/2)) + (b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{3x^{3/2}}+\frac{b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.61

$$\frac{(bx-1)\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 + b*x]),x]
```

```
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 23, normalized size = 0.61

$$\frac{(bx-1)\sqrt{bx+2}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[2 + b*x]),x]
[Out] ((-1 + b*x)*Sqrt[2 + b*x])/(3*x^(3/2))
```

fricas [A] time = 1.11, size = 17, normalized size = 0.45

$$\frac{\sqrt{bx+2}\,(bx-1)}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*x + 2)*(b*x - 1)/x^(3/2)
```

giac [A] time = 1.09, size = 42, normalized size = 1.11

$$\frac{\left((bx+2)b^{3}-3\,b^{3}\right)\sqrt{bx+2}\,b}{3\,((bx+2)b-2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] 1/3*((b*x + 2)*b^3 - 3*b^3)*sqrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(3/2)*abs(
b))
```

maple [A] time = 0.00, size = 18, normalized size = 0.47

$$\frac{\sqrt{bx+2}\;(bx-1)}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(1/2),x)
[Out] 1/3*(b*x+2)^(1/2)*(b*x-1)/x^(3/2)
```

maxima [A] time = 1.32, size = 26, normalized size = 0.68

$$\frac{\sqrt{bx+2}\,b}{2\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/6*(b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.32, size = 17, normalized size = 0.45

$$\frac{\sqrt{b\,x+2}\,\left(\frac{b\,x}{3}-\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(1/2)),x)
[Out] ((b*x + 2)^(1/2)*((b*x)/3 - 1/3))/x^(3/2)
```

sympy [A] time = 1.87, size = 34, normalized size = 0.89

$$\frac{b^{\frac{3}{2}}\sqrt{1+\frac{2}{bx}}}{3}-\frac{\sqrt{b}\,\sqrt{1+\frac{2}{bx}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(1/2),x)
```

```
[Out] b**(3/2)*sqrt(1 + 2/(b*x))/3 - sqrt(b)*sqrt(1 + 2/(b*x))/(3*x)
```

3.614 $\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=59

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{15\sqrt{x}}+\frac{2b\sqrt{bx+2}}{15x^{3/2}}-\frac{\sqrt{bx+2}}{5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(5*x^(5/2)) + (2*b*Sqrt[2 + b*x])/(15*x^(3/2)) - (2*b^2*Sqrt
[2 + b*x])/(15*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(2b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}+\frac{1}{15}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= -\frac{\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{15x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{15\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.54

$$-\frac{\sqrt{bx+2}\left(2b^2x^2-2bx+3\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -1/15*(Sqrt[2 + b*x]*(3 - 2*b*x + 2*b^2*x^2))/x^(5/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 32, normalized size = 0.54

$$\frac{\sqrt{bx+2}\left(-2b^2x^2+2bx-3\right)}{15x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*Sqrt[2 + b*x]),x]
[Out] (Sqrt[2 + b*x]*(-3 + 2*b*x - 2*b^2*x^2))/(15*x^(5/2))
```

fricas [A] time = 0.87, size = 26, normalized size = 0.44

$$-\frac{\left(2\,b^2x^2-2\,bx+3\right)\sqrt{bx+2}}{15\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/15*(2*b^2*x^2 - 2*b*x + 3)*sqrt(b*x + 2)/x^(5/2)
```

giac [A] time = 1.06, size = 55, normalized size = 0.93

$$-\frac{\left(15\,b^5+2\left((bx+2)b^5-5\,b^5\right)(bx+2)\right)\sqrt{bx+2}\,b}{15\left((bx+2)b-2\,b\right)^{\frac{5}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -1/15*(15*b^5 + 2*((b*x + 2)*b^5 - 5*b^5)*(b*x + 2))*sqrt(b*x + 2)*b/(((b*x
 + 2)*b - 2*b)^(5/2)*abs(b))
```

maple [A] time = 0.01, size = 27, normalized size = 0.46

$$-\frac{\sqrt{bx+2}\,\left(2b^2x^2-2bx+3\right)}{15x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(1/2),x)
[Out] -1/15*(b*x+2)^(1/2)*(2*b^2*x^2-2*b*x+3)/x^(5/2)
```

maxima [A] time = 1.31, size = 41, normalized size = 0.69

$$-\frac{\sqrt{bx+2}\,b^2}{4\,\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{6\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{20\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] -1/4*sqrt(b*x + 2)*b^2/sqrt(x) + 1/6*(b*x + 2)^(3/2)*b/x^(3/2) - 1/20*(b*x
+ 2)^(5/2)/x^(5/2)
```

mupad [B] time = 0.32, size = 26, normalized size = 0.44

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^2\,x^2}{15}-\frac{2\,b\,x}{15}+\frac{1}{5}\right)}{x^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -((b*x + 2)^(1/2)*((2*b^2*x^2)/15 - (2*b*x)/15 + 1/5))/x^(5/2)
```

sympy [B] time = 6.10, size = 224, normalized size = 3.80

$$-\frac{2b^{\frac{17}{2}}x^4\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{6b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{3b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{4b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}-\frac{12b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{15b^6x^4+60b^5x^3+60b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*b**(17/2)*x**4*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 60*b**4*
x**2) - 6*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x**3 + 6
0*b**4*x**2) - 3*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b**5*x
**3 + 60*b**4*x**2) - 4*b**(11/2)*x*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*b*
*5*x**3 + 60*b**4*x**2) - 12*b**(9/2)*sqrt(1 + 2/(b*x))/(15*b**6*x**4 + 60*
b**5*x**3 + 60*b**4*x**2)
```

3.615 $\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=80

$$\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{35x^{3/2}}+\frac{2b^3\sqrt{bx+2}}{35\sqrt{x}}+\frac{3b\sqrt{bx+2}}{35x^{5/2}}-\frac{\sqrt{bx+2}}{7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] -Sqrt[2 + b*x]/(7*x^(7/2)) + (3*b*Sqrt[2 + b*x])/(35*x^(5/2)) - (2*b^2*Sqrt
[2 + b*x])/(35*x^(3/2)) + (2*b^3*Sqrt[2 + b*x])/(35*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{9/2}\sqrt{2+bx}}\,dx &= -\frac{\sqrt{2+bx}}{7x^{7/2}}-\frac{1}{7}(3b)\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}+\frac{1}{35}\left(6b^2\right)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}-\frac{1}{35}\left(2b^3\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= -\frac{\sqrt{2+bx}}{7x^{7/2}}+\frac{3b\sqrt{2+bx}}{35x^{5/2}}-\frac{2b^2\sqrt{2+bx}}{35x^{3/2}}+\frac{2b^3\sqrt{2+bx}}{35\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.50

$$\frac{\sqrt{bx+2}\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 40, normalized size = 0.50

$$\frac{\sqrt{bx+2}\,\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(9/2)*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*(-5 + 3*b*x - 2*b^2*x^2 + 2*b^3*x^3))/(35*x^(7/2))
```

fricas [A] time = 0.81, size = 34, normalized size = 0.42

$$\frac{\left(2\,b^3x^3-2\,b^2x^2+3\,bx-5\right)\sqrt{bx+2}}{35\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(2*b^3*x^3 - 2*b^2*x^2 + 3*b*x - 5)*sqrt(b*x + 2)/x^(7/2)
```

giac [A] time = 1.03, size = 68, normalized size = 0.85

$$-\frac{\left(35\,b^7-\left(35\,b^7+2\left((bx+2)b^7-7\,b^7\right)(bx+2)\right)(bx+2)\right)\sqrt{bx+2}\,b}{35\,((bx+2)b-2\,b)^{\frac{7}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*(35*b^7 - (35*b^7 + 2*((b*x + 2)*b^7 - 7*b^7)*(b*x + 2))*(b*x + 2))*s
qrt(b*x + 2)*b/(((b*x + 2)*b - 2*b)^(7/2)*abs(b))
```

maple [A] time = 0.01, size = 35, normalized size = 0.44

$$\frac{\sqrt{bx+2}\,\left(2b^3x^3-2b^2x^2+3bx-5\right)}{35x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x+2)^(1/2),x)
```

```
[Out] 1/35*(b*x+2)^(1/2)*(2*b^3*x^3-2*b^2*x^2+3*b*x-5)/x^(7/2)
```

maxima [A] time = 1.29, size = 56, normalized size = 0.70

$$\frac{\sqrt{bx+2}\,b^3}{8\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}b^2}{8\,x^{\frac{3}{2}}}+\frac{3\,(bx+2)^{\frac{5}{2}}b}{40\,x^{\frac{5}{2}}}-\frac{(bx+2)^{\frac{7}{2}}}{56\,x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x + 2)*b^3/sqrt(x) - 1/8*(b*x + 2)^(3/2)*b^2/x^(3/2) + 3/40*(b*x
 + 2)^(5/2)*b/x^(5/2) - 1/56*(b*x + 2)^(7/2)/x^(7/2)
```

mupad [B] time = 0.33, size = 33, normalized size = 0.41

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b^3\,x^3}{35}-\frac{2\,b^2\,x^2}{35}+\frac{3\,b\,x}{35}-\frac{1}{7}\right)}{x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((3*b*x)/35 - (2*b^2*x^2)/35 + (2*b^3*x^3)/35 - 1/7))/x^(7
/2)
```

sympy [B] time = 15.99, size = 374, normalized size = 4.68

$$\frac{2b^{\frac{31}{2}}x^6\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{10b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{15b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}+\frac{5b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{10b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{36b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}-\frac{40b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{35b^{12}x^6+210b^{11}x^5+420b^{10}x^4+280b^9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x+2)**(1/2),x)
```

```
[Out] 2*b**(31/2)*x**6*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b*
*10*x**4 + 280*b**9*x**3) + 10*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(35*b**12*x
**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) + 15*b**(27/2)*x**4*
sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b*
*9*x**3) + 5*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x*
*5 + 420*b**10*x**4 + 280*b**9*x**3) - 10*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/
(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4 + 280*b**9*x**3) - 36*b**(
21/2)*x*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**11*x**5 + 420*b**10*x**4
+ 280*b**9*x**3) - 40*b**(19/2)*sqrt(1 + 2/(b*x))/(35*b**12*x**6 + 210*b**1
1*x**5 + 420*b**10*x**4 + 280*b**9*x**3)
```

3.616 $\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=86

$$\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{15\sqrt{x}\,\sqrt{bx+2}}{2b^3}+\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Rubi [A] time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{5x^{3/2}\sqrt{bx+2}}{2b^2}-\frac{15\sqrt{x}\,\sqrt{bx+2}}{2b^3}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(5/2))/(b*Sqrt[2 + b*x]) - (15*Sqrt[x]*Sqrt[2 + b*x])/(2*b^3) + (5*x^
(3/2)*Sqrt[2 + b*x])/(2*b^2) + (15*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(2+bx)^{3/2}}\,dx &= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5\int \frac{x^{3/2}}{\sqrt{2+bx}}\,dx}{b}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}-\frac{15\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{2b^2}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\int \frac{1}{\sqrt{x}\,\sqrt{2+bx}}\,dx}{2b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{2x^{5/2}}{b\sqrt{2+bx}}-\frac{15\sqrt{x}\,\sqrt{2+bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2+bx}}{2b^2}+\frac{15\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.35

$$
\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{7\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, -1/2*(b*x)])/(7*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.12, size = 72, normalized size = 0.84

$$
\frac{b^2x^{5/2}-5bx^{3/2}-30\sqrt{x}}{2b^3\sqrt{bx+2}}-\frac{15\log\left(\sqrt{bx+2}-\sqrt{b}\,\sqrt{x}\right)}{b^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-30*Sqrt[x] - 5*b*x^(3/2) + b^2*x^(5/2))/(2*b^3*Sqrt[2 + b*x]) - (15*Log[-
(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

fricas [A] time = 1.25, size = 152, normalized size = 1.77

$$
\left[\frac{15\,(bx+2)\sqrt{b}\,\log\left(bx+\sqrt{bx+2}\,\sqrt{b}\,\sqrt{x}+1\right)+\left(b^3x^2-5\,b^2x-30\,b\right)\sqrt{bx+2}\,\sqrt{x}}{2\left(b^5x+2\,b^4\right)},-\frac{30\,(bx+2)\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-b}}{b\sqrt{x}}\right)-\left(b^3x^2-5\,b^2x-30\,b\right)\sqrt{bx+2}\,\sqrt{x}}{2\left(b^5x+2\,b^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(15*(b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (
b^3*x^2 - 5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4), -1/2*(30*
(b*x + 2)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) - (b^3*x^2 -
5*b^2*x - 30*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x + 2*b^4)]
```

giac [A] time = 11.12, size = 119, normalized size = 1.38

$$
\frac{\left(\sqrt{(bx+2)b-2\,b}\,\sqrt{bx+2}\left(\frac{bx+2}{b^3}-\frac{9}{b^3}\right)-\frac{15\,\log\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2\right)}{b^{\frac{5}{2}}}-\frac{64}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{3}{2}}}\right)|b|}{2\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)*((b*x + 2)/b^3 - 9/b^3) - 15*log
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) - 64/(((sqrt(
b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(3/2)))*abs(b)/b^2
```

maple [A] time = 0.03, size = 106, normalized size = 1.23

$$\frac{\left(\frac{15\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{2b^{\frac{7}{2}}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^2 b-2x-\frac{4}{b}}}{\left(x+\frac{2}{b}\right)b^4}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{(bx-7)\,\sqrt{bx+2}\,\sqrt{x}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(3/2),x)
```

```
[Out] 1/2*(b*x-7)*(b*x+2)^(1/2)*x^(1/2)/b^3+(15/2/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x
^2+2*x)^(1/2))-8/b^4/(x+2/b)*(b*(x+2/b)^2-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)
/(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 3.06, size = 119, normalized size = 1.38

$$-\frac{8\,b^2-\frac{25\,(bx+2)b}{x}+\frac{15\,(bx+2)^2}{x^2}}{\frac{\sqrt{bx+2}\,b^5}{\sqrt{x}}-\frac{2\,(bx+2)^{\frac{3}{2}}\,b^4}{x^{\frac{3}{2}}}+\frac{(bx+2)^{\frac{5}{2}}\,b^3}{x^{\frac{5}{2}}}}-\frac{15\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{2\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(8*b^2 - 25*(b*x + 2)*b/x + 15*(b*x + 2)^2/x^2)/(sqrt(b*x + 2)*b^5/sqrt(x)
 - 2*(b*x + 2)^(3/2)*b^4/x^(3/2) + (b*x + 2)^(5/2)*b^3/x^(5/2)) - 15/2*log(
-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/
2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(3/2), x)
```

sympy [A] time = 7.10, size = 80, normalized size = 0.93

$$\frac{x^{\frac{5}{2}}}{2b\sqrt{bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^2\sqrt{bx+2}}-\frac{15\sqrt{x}}{b^3\sqrt{bx+2}}+\frac{15\,\operatorname{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(5/2)/(2*b*sqrt(b*x + 2)) - 5*x**(3/2)/(2*b**2*sqrt(b*x + 2)) - 15*sqrt(
x)/(b**3*sqrt(b*x + 2)) + 15*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2)
```

3.617 $\int \frac{x^{3/2}}{(2+bx)^{3/2}} \, dx$

Optimal. Leaf size=63

$$-\frac{6 \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$\frac{3\sqrt{x}\sqrt{bx+2}}{b^2}-\frac{6 \sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*x^(3/2))/(b*Sqrt[2 + b*x]) + (3*Sqrt[x]*Sqrt[2 + b*x])/b^2 - (6*ArcSinh
[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx = -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^2}$$
$$= -\frac{2x^{3/2}}{b\sqrt{2+bx}} + \frac{3\sqrt{x}\sqrt{2+bx}}{b^2} - \frac{6\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.48

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(3/2),x]
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -1/2*(b*x)])/(5*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.10, size = 59, normalized size = 0.94

$$\frac{6\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}} + \frac{bx^{3/2}+6\sqrt{x}}{b^2\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 + b*x)^(3/2),x]
[Out] (6*Sqrt[x] + b*x^(3/2))/(b^2*Sqrt[2 + b*x]) + (6*Log[-(Sqrt[b]*Sqrt[x]) + S
qrt[2 + b*x]])/b^(5/2)
```

fricas [A] time = 1.33, size = 134, normalized size = 2.13

$$\left[\frac{3(bx+2)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(b^2x+6b\right)\sqrt{bx+2}\sqrt{x}}{b^4x+2b^3}, \frac{6(bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\left(b^2x+6b\right)\sqrt{bx+2}\sqrt{x}}{b^4x+2b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
[Out] [(3*(b*x + 2)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) + (b^2*x
 + 6*b)*sqrt(b*x + 2)*sqrt(x))/(b^4*x + 2*b^3), (6*(b*x + 2)*sqrt(-b)*arcta
n(sqrt(b*x + 2)*sqrt(-b)/(b*sqrt(x))) + (b^2*x + 6*b)*sqrt(b*x + 2)*sqrt(x)
)/(b^4*x + 2*b^3)]
```

giac [B] time = 10.16, size = 106, normalized size = 1.68

$$\frac{\left(\frac{3\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}} + \frac{\sqrt{(bx+2)b-2b}\sqrt{bx+2}}{b} + \frac{16\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] (3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + sqrt(
(b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b + 16*sqrt(b)/((sqrt(b*x + 2)*sqrt(b) - s
qrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^3
```

maple [B] time = 0.02, size = 100, normalized size = 1.59

$$\frac{\left(-\frac{3\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^2+2x}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{\left(x+\frac{2}{b}\right)^2 b-2x-\frac{4}{b}}}{\left(x+\frac{2}{b}\right)b^3}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{\sqrt{bx+2}\,\sqrt{x}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] (b*x+2)^(1/2)/b^2*x^(1/2)+(-3/b^(5/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
+4/b^3/(x+2/b)*((x+2/b)^2*b-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)
/x^(1/2)
```

maxima [A] time = 2.99, size = 90, normalized size = 1.43

$$\frac{2\left(2\,b-\frac{3\,(bx+2)}{x}\right)}{\frac{\sqrt{bx+2}\,b^3}{\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}\,b^2}{x^{\frac{3}{2}}}}+\frac{3\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x + 2)/x)/(sqrt(b*x + 2)*b^3/sqrt(x) - (b*x + 2)^(3/2)*b^2/x^
(3/2)) + 3*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/
sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(3/2), x)
```

sympy [A] time = 3.06, size = 58, normalized size = 0.92

$$\frac{x^{\frac{3}{2}}}{b\sqrt{bx+2}}+\frac{6\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{6\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] x**(3/2)/(b*sqrt(b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(b*x + 2)) - 6*asinh(sqrt(
2)*sqrt(b)*sqrt(x)/2)/b**(5/2)
```

3.618 $\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx &= -\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b}\\ &= -\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\ &= -\frac{2\sqrt{x}}{b\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) + (2*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(
3/2)
```

IntegrateAlgebraic [A] time = 0.08, size = 50, normalized size = 1.14

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{3/2}}-\frac{2\sqrt{x}}{b\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x])/(b*Sqrt[2 + b*x]) - (2*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]]
)/b^(3/2)
```

fricas [A] time = 1.29, size = 117, normalized size = 2.66

$$\left[\frac{(bx+2)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-2\sqrt{bx+2}\,b\sqrt{x}}{b^3x+2b^2},-\frac{2\left((bx+2)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{bx+2}\,b\sqrt{x}\right)}{b^3x+2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x + 2)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(x) + 1) - 2*sqrt(b
*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2), -2*((b*x + 2)*sqrt(-b)*arctan(sqrt(b*x
+ 2)*sqrt(-b)/(b*sqrt(x))) + sqrt(b*x + 2)*b*sqrt(x))/(b^3*x + 2*b^2)]
```

giac [B] time = 10.61, size = 82, normalized size = 1.86

$$-\frac{\left(\frac{\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}}+\frac{8\sqrt{b}}{\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) + 8*sqrt
(b)/((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b))*abs(b)/b^2
```

maple [A] time = 0.11, size = 48, normalized size = 1.09

$$\frac{-\frac{\sqrt{\pi}\sqrt{2}\sqrt{b}\sqrt{x}}{\sqrt{\frac{bx}{2}+1}}+2\sqrt{\pi}\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{\pi}\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(3/2),x)
```

```
[Out] 2/b^(3/2)/Pi^(1/2)*(-1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)/(1/2*b*x+1)^(1/2)
+Pi^(1/2)*arcsinh(1/2*b^(1/2)*x^(1/2)*2^(1/2)))
```

maxima [A] time = 2.90, size = 57, normalized size = 1.30

$$-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{3}{2}}}-\frac{2\sqrt{x}}{\sqrt{bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -log(-(sqrt(b) - sqrt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/
b^(3/2) - 2*sqrt(x)/(sqrt(b*x + 2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(b\,x+2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(3/2),x)
```

```
[Out] int(x^(1/2)/(b*x + 2)^(3/2), x)
```

sympy [A] time = 1.56, size = 41, normalized size = 0.93

$$-\frac{2\sqrt{x}}{b\sqrt{bx+2}}+\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*sqrt(x)/(b*sqrt(b*x + 2)) + 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2)
```

3.619 $\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=15

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2+bx}}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 + b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 + b*x]
```

fricas [A] time = 1.32, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

giac [B] time = 1.22, size = 44, normalized size = 2.93

$$\frac{4 b^{\frac{3}{2}}}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}+2b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(
b))
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(b*x+2)^(1/2)
```

maxima [A] time = 1.34, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(b*x + 2)
```

mupad [B] time = 0.31, size = 11, normalized size = 0.73

$$\frac{\sqrt{x}}{\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] x^(1/2)/(b*x + 2)^(1/2)
```

sympy [A] time = 0.86, size = 15, normalized size = 1.00

$$\frac{1}{\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] 1/(sqrt(b)*sqrt(1 + 2/(b*x)))
```

3.620 $\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\,\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 + b*x]) - Sqrt[2 + b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\,\sqrt{2+bx}}+\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{\sqrt{x}\,\sqrt{2+bx}}-\frac{\sqrt{2+bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{-bx-1}{\sqrt{x}\,\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 21, normalized size = 0.66

$$\frac{-bx-1}{\sqrt{x}\,\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 - b*x)/(Sqrt[x]*Sqrt[2 + b*x])
```

fricas [A] time = 1.00, size = 28, normalized size = 0.88

$$-\frac{\sqrt{bx+2}\,(bx+1)\sqrt{x}}{bx^2+2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(b*x + 2)*(b*x + 1)*sqrt(x)/(b*x^2 + 2*x)
```

giac [B] time = 1.12, size = 74, normalized size = 2.31

$$-\frac{\sqrt{bx+2}\,b^2}{2\,\sqrt{(bx+2)b-2\,b}\,|b|}-\frac{2\,b^{\frac{5}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 2*b^(5/2)/(((sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b))
```

maple [A] time = 0.00, size = 18, normalized size = 0.56

$$-\frac{bx+1}{\sqrt{bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(3/2),x)
```

```
[Out] -(b*x+1)/(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 1.34, size = 26, normalized size = 0.81

$$-\frac{b\sqrt{x}}{2\,\sqrt{bx+2}}-\frac{\sqrt{bx+2}}{2\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*b*sqrt(x)/sqrt(b*x + 2) - 1/2*sqrt(b*x + 2)/sqrt(x)
```

mupad [B] time = 0.35, size = 17, normalized size = 0.53

$$-\frac{b\,x+1}{\sqrt{x}\,\sqrt{b\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] -(b*x + 1)/(x^(1/2)*(b*x + 2)^(1/2))
```

sympy **[A]** time = 1.54, size = 34, normalized size = 1.06

$$-\frac{\sqrt{b}}{\sqrt{1+\frac{2}{bx}}}-\frac{1}{\sqrt{b}\,x\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(3/2),x)
```

```
[Out] -sqrt(b)/sqrt(1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(1 + 2/(b*x)))
```

3.621 $\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x])/(3*x^(3/2)) + (2*b*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2+bx}}+2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.60

$$\frac{2b^2x^2+2bx-1}{3x^{3/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 32, normalized size = 0.60

$$\frac{2b^2x^2+2bx-1}{3x^{3/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 + b*x)^(3/2)),x]
[Out] (-1 + 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 + b*x])
```

fricas [A] time = 0.64, size = 39, normalized size = 0.74

$$\frac{\left(2\,b^2x^2+2\,bx-1\right)\sqrt{bx+2}\,\sqrt{x}}{3\,\left(bx^3+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(2*b^2*x^2 + 2*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^3 + 2*x^2)
```

giac [B] time = 1.23, size = 86, normalized size = 1.62

$$\frac{b^{\frac{7}{2}}}{\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}+\frac{\left(5\,(bx+2)b^2|b|-12\,b^2|b|\right)\sqrt{bx+2}}{12\,((bx+2)b-2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="giac")
[Out] b^(7/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*abs(b)
) + 1/12*(5*(b*x + 2)*b^2*abs(b) - 12*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*
b - 2*b)^(3/2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.51

$$\frac{2b^2x^2+2bx-1}{3\sqrt{bx+2}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(3/2),x)
[Out] 1/3*(2*b^2*x^2+2*b*x-1)/(b*x+2)^(1/2)/x^(3/2)
```

maxima [A] time = 1.37, size = 41, normalized size = 0.77

$$\frac{b^2\sqrt{x}}{4\,\sqrt{bx+2}}+\frac{\sqrt{bx+2}\,b}{2\,\sqrt{x}}-\frac{(bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(3/2),x, algorithm="maxima")
[Out] 1/4*b^2*sqrt(x)/sqrt(b*x + 2) + 1/2*sqrt(b*x + 2)*b/sqrt(x) - 1/12*(b*x + 2
)^(3/2)/x^(3/2)
```

mupad [B] time = 0.38, size = 37, normalized size = 0.70

$$\frac{\sqrt{b\,x+2}\,\left(\frac{2\,x}{3}+\frac{2\,b\,x^2}{3}-\frac{1}{3\,b}\right)}{x^{5/2}+\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] ((b*x + 2)^(1/2)*((2*x)/3 + (2*b*x^2)/3 - 1/(3*b)))/(x^(5/2) + (2*x^(3/2))/
b)
```

sympy [B] time = 3.86, size = 170, normalized size = 3.21

$$\frac{2b^{\frac{15}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{6b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}+\frac{3b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}-\frac{2b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^3+12b^5x^2+12b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(3/2),x)
```

```
[Out] 2*b**(15/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
 + 6*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) + 3*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4
*x) - 2*b**(9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**3 + 12*b**5*x**2 + 12*b**4*x)
```

3.622 $\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx$

Optimal. Leaf size=74

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2\sqrt{bx+2}}{5\sqrt{x}}+\frac{2b\sqrt{bx+2}}{5x^{3/2}}-\frac{3\sqrt{bx+2}}{5x^{5/2}}+\frac{1}{x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] 1/(x^(5/2)*Sqrt[2 + b*x]) - (3*Sqrt[2 + b*x])/(5*x^(5/2)) + (2*b*Sqrt[2 + b
*x])/(5*x^(3/2)) - (2*b^2*Sqrt[2 + b*x])/(5*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}(2+bx)^{3/2}}\,dx &= \frac{1}{x^{5/2}\sqrt{2+bx}}+3\int \frac{1}{x^{7/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}-\frac{1}{5}(6b)\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}+\frac{1}{5}\left(2b^2\right)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{x^{5/2}\sqrt{2+bx}}-\frac{3\sqrt{2+bx}}{5x^{5/2}}+\frac{2b\sqrt{2+bx}}{5x^{3/2}}-\frac{2b^2\sqrt{2+bx}}{5\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.53

$$\frac{-2b^3x^3-2b^2x^2+bx-1}{5x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 39, normalized size = 0.53

$$\frac{-2b^3x^3-2b^2x^2+bx-1}{5x^{5/2}\sqrt{bx+2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(7/2)*(2 + b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x - 2*b^2*x^2 - 2*b^3*x^3)/(5*x^(5/2)*Sqrt[2 + b*x])
```

fricas [A] time = 1.22, size = 47, normalized size = 0.64

$$-\frac{\left(2\,b^3x^3+2\,b^2x^2-bx+1\right)\sqrt{bx+2}\,\sqrt{x}}{5\left(bx^4+2\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5*(2*b^3*x^3 + 2*b^2*x^2 - b*x + 1)*sqrt(b*x + 2)*sqrt(x)/(b*x^4 + 2*x^3
)
```

giac [B] time = 1.11, size = 107, normalized size = 1.45

$$-\frac{b^{\frac{9}{2}}}{2\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)|b|}-\frac{\left(\frac{60\,b^6}{|b|}+\left(\frac{11\,(bx+2)b^6}{|b|}-\frac{50\,b^6}{|b|}\right)(bx+2)\right)\sqrt{bx+2}}{40\,((bx+2)b-2\,b)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*b^(9/2)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*a
bs(b)) - 1/40*(60*b^6/abs(b) + (11*(b*x + 2)*b^6/abs(b) - 50*b^6/abs(b))*(b
*x + 2))*sqrt(b*x + 2)/((b*x + 2)*b - 2*b)^(5/2)
```

maple [A] time = 0.00, size = 35, normalized size = 0.47

$$-\frac{2b^3x^3+2b^2x^2-bx+1}{5\sqrt{bx+2}\;x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x+2)^(3/2),x)
```

```
[Out] -1/5*(2*b^3*x^3+2*b^2*x^2-b*x+1)/(b*x+2)^(1/2)/x^(5/2)
```

maxima [A] time = 1.30, size = 56, normalized size = 0.76

$$-\frac{b^3\sqrt{x}}{8\sqrt{bx+2}}-\frac{3\sqrt{bx+2}\,b^2}{8\sqrt{x}}+\frac{(bx+2)^{\frac{3}{2}}b}{8\,x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}}{40\,x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*b^3*sqrt(x)/sqrt(b*x + 2) - 3/8*sqrt(b*x + 2)*b^2/sqrt(x) + 1/8*(b*x +
 2)^(3/2)*b/x^(3/2) - 1/40*(b*x + 2)^(5/2)/x^(5/2)
```

mupad [B] time = 0.43, size = 46, normalized size = 0.62

$$-\frac{\sqrt{b\,x+2}\,\left(\frac{2\,b\,x^2}{5}-\frac{x}{5}+\frac{1}{5\,b}+\frac{2\,b^2\,x^3}{5}\right)}{x^{7/2}+\frac{2\,x^{5/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(b*x + 2)^(3/2)),x)
```

```
[Out] -((b*x + 2)^(1/2)*((2*b*x^2)/5 - x/5 + 1/(5*b) + (2*b^2*x^3)/5))/(x^(7/2) +
 (2*x^(5/2))/b)
```

sympy [B] time = 10.97, size = 269, normalized size = 3.64

$$-\frac{2b^{\frac{29}{2}}x^5\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{10b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{15b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{5b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}-\frac{4b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{5b^{12}x^5+30b^{11}x^4+60b^{10}x^3+40b^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x+2)**(3/2),x)
```

```
[Out] -2*b**(29/2)*x**5*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**1
0*x**3 + 40*b**9*x**2) - 10*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(5*b**12*x**5
+ 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) - 15*b**(25/2)*x**3*sqrt(1
+ 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10*x**3 + 40*b**9*x**2) -
5*b**(23/2)*x**2*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b**11*x**4 + 60*b**10
*x**3 + 40*b**9*x**2) - 4*b**(19/2)*sqrt(1 + 2/(b*x))/(5*b**12*x**5 + 30*b*
*11*x**4 + 60*b**10*x**3 + 40*b**9*x**2)
```

3.623 $\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=86

$$-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {47, 50, 54, 215}

$$-\frac{10x^{3/2}}{3b^2\sqrt{bx+2}}+\frac{5\sqrt{x}\sqrt{bx+2}}{b^3}-\frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{2x^{5/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(5/2))/(3*b*(2 + b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 + b*x]) + (
5*Sqrt[x]*Sqrt[2 + b*x])/b^3 - (10*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7
/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(2+bx)^{5/2}}\,dx = -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} + \frac{5\int \frac{x^{3/2}}{(2+bx)^{3/2}}\,dx}{3b}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2+bx}}\,dx}{b^2}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{5\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^3}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{10\,\text{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$
$$= -\frac{2x^{5/2}}{3b(2+bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2+bx}} + \frac{5\sqrt{x}\sqrt{2+bx}}{b^3} - \frac{10\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.35

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};-\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 + b*x)^(5/2),x]
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, -1/2*(b*x)])/(14*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 0.85

$$\frac{10\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{7/2}} + \frac{3b^2x^{5/2}+40bx^{3/2}+60\sqrt{x}}{3b^3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 + b*x)^(5/2),x]
[Out] (60*Sqrt[x] + 40*b*x^(3/2) + 3*b^2*x^(5/2))/(3*b^3*(2 + b*x)^(3/2)) + (10*L
og[-(Sqrt[b]*Sqrt[x]) + Sqrt[2 + b*x]])/b^(7/2)
```

fricas [A] time = 1.35, size = 186, normalized size = 2.16

$$\left[\frac{15\left(b^2x^2+4bx+4\right)\sqrt{b}\log\left(bx-\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)+\left(3b^3x^2+40b^2x+60b\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^6x^2+4b^5x+4b^4\right)}, \frac{30\left(b^2x^2+4bx+4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+\left(3b^3x^2+40b^2x+60b\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^6x^2+4b^5x+4b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
[Out] [1/3*(15*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x - sqrt(b*x + 2)*sqrt(b)*sqrt
(x) + 1) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt(x))/(b^6*x^2 +
4*b^5*x + 4*b^4), 1/3*(30*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x +
2)*sqrt(-b)/(b*sqrt(x))) + (3*b^3*x^2 + 40*b^2*x + 60*b)*sqrt(b*x + 2)*sqrt
(x))/(b^6*x^2 + 4*b^5*x + 4*b^4)]
```

giac [B] time = 10.82, size = 182, normalized size = 2.12

$$\frac{\left(\frac{15\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{b^{\frac{5}{2}}}+\frac{3\sqrt{(bx+2)b-2b}\sqrt{bx+2}}{b^3}+\frac{16\left(9\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4\sqrt{b}+24\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2b^{\frac{3}{2}}+28b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3b^2}\right)|b|}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/b^(5/2) +
3*sqrt((b*x + 2)*b - 2*b)*sqrt(b*x + 2)/b^3 + 16*(9*(sqrt(b*x + 2)*sqrt(b)
- sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 24*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*
x + 2)*b - 2*b))^2*b^(3/2) + 28*b^(5/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b
*x + 2)*b - 2*b))^2 + 2*b)^3*b^2))*abs(b)/b^2
```

maple [B] time = 0.04, size = 136, normalized size = 1.58

$$\frac{\left(-\frac{5\ln\left(\frac{bx+1}{\sqrt{b}}+\sqrt{b\,x^{2}+2x}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{3\left(x+\frac{2}{b}\right)b^{4}}-\frac{8\sqrt{\left(x+\frac{2}{b}\right)^{2}b-2x-\frac{4}{b}}}{3\left(x+\frac{2}{b}\right)^{2}b^{5}}\right)\sqrt{(bx+2)\,x}}{\sqrt{bx+2}\,\sqrt{x}}+\frac{\sqrt{bx+2}\,\sqrt{x}}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] (b*x+2)^(1/2)/b^3*x^(1/2)+(-5/b^(7/2)*ln((b*x+1)/b^(1/2)+(b*x^2+2*x)^(1/2))
+28/3/(x+2/b)*((x+2/b)^2*b-2*x-4/b)^(1/2)/b^4-8/3/b^5/(x+2/b)^2*((x+2/b)^2*
b-2*x-4/b)^(1/2))*((b*x+2)*x)^(1/2)/(b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 3.00, size = 105, normalized size = 1.22

$$\frac{2\left(2\,b^{2}+\frac{10\,(bx+2)b}{x}-\frac{15\,(bx+2)^{2}}{x^{2}}\right)}{3\left(\frac{(bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}-\frac{(bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}+\frac{5\,\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x + 2)*b/x - 15*(b*x + 2)^2/x^2)/((b*x + 2)^(3/2)*b^4/x^
(3/2) - (b*x + 2)^(5/2)*b^3/x^(5/2)) + 5*log(-(sqrt(b) - sqrt(b*x + 2)/sqrt
(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(5/2)/(b*x + 2)^(5/2), x)
```

sympy [B] time = 6.62, size = 308, normalized size = 3.58

$$\frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{30b^{10}x^{\frac{27}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{bx+2}\,\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx+2}+6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25
/2)*sqrt(b*x + 2)) + 40*b**(21/2)*x**14/(3*b**(27/2)*x**(27/2)*sqrt(b*x + 2
```

```
) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) + 60*b**(19/2)*x**13/(3*b**(27/2)*
x**(27/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 30*b**10*x
**(27/2)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27
/2)*sqrt(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2)) - 60*b**9*x**(25/2
)*sqrt(b*x + 2)*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqr
t(b*x + 2) + 6*b**(25/2)*x**(25/2)*sqrt(b*x + 2))
```

3.624 $\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {47, 54, 215}

$$-\frac{2\sqrt{x}}{b^2\sqrt{bx+2}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2x^{3/2}}{3b(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-2*x^(3/2))/(3*b*(2 + b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 + b*x]) + (2*A
rcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2+bx)^{5/2}}\,dx &= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}+\frac{\int \frac{\sqrt{x}}{(2+bx)^{3/2}}\,dx}{b}\\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2+bx}}\,dx}{b^2}\\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2+bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{2x^{3/2}}{3b(2+bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2+bx}}+\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 52, normalized size = 0.80

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{4\sqrt{x}(2bx+3)}{3b^2(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*Sqrt[x]*(3 + 2*b*x))/(3*b^2*(2 + b*x)^(3/2)) + (2*ArcSinh[(Sqrt[b]*Sqrt
[x])/Sqrt[2]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.12, size = 63, normalized size = 0.97

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{b}\sqrt{x}\right)}{b^{5/2}}-\frac{4\left(2bx^{3/2}+3\sqrt{x}\right)}{3b^2(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 + b*x)^(5/2),x]
```

```
[Out] (-4*(3*Sqrt[x] + 2*b*x^(3/2)))/(3*b^2*(2 + b*x)^(3/2)) - (2*Log[-(Sqrt[b]*S
qrt[x]) + Sqrt[2 + b*x]])/b^(5/2)
```

fricas [A] time = 1.29, size = 171, normalized size = 2.63

$$\left[\frac{3\left(b^2x^2+4bx+4\right)\sqrt{b}\log\left(bx+\sqrt{bx+2}\sqrt{b}\sqrt{x}+1\right)-4\left(2b^2x+3b\right)\sqrt{bx+2}\sqrt{x}}{3\left(b^5x^2+4b^4x+4b^3\right)},-\frac{2\left(3\left(b^2x^2+4bx+4\right)\sqrt{-b}\arctan\left(\frac{\sqrt{bx+2}\sqrt{-b}}{b\sqrt{x}}\right)+2\left(2b^2x+3b\right)\sqrt{bx+2}\sqrt{x}\right)}{3\left(b^5x^2+4b^4x+4b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(b)*log(b*x + sqrt(b*x + 2)*sqrt(b)*sqrt(
x) + 1) - 4*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x + 4*b
^3), -2/3*(3*(b^2*x^2 + 4*b*x + 4)*sqrt(-b)*arctan(sqrt(b*x + 2)*sqrt(-b)/(
b*sqrt(x))) + 2*(2*b^2*x + 3*b)*sqrt(b*x + 2)*sqrt(x))/(b^5*x^2 + 4*b^4*x +
 4*b^3)]
```

giac [B] time = 10.77, size = 154, normalized size = 2.37

$$-\frac{\left(\frac{3\log\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2\right)}{\sqrt{b}}+\frac{16\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^4\sqrt{b}+6\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2b^{\frac{3}{2}}+8b^{\frac{5}{2}}\right)}{\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^2+2b\right)^3}\right)|b|}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*log((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2)/sqrt(b) +
16*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 6*(sqrt
(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(3/2) + 8*b^(5/2))/((sqrt(
b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3)*abs(b)/b^3
```

maple [A] time = 0.04, size = 55, normalized size = 0.85

$$\frac{-\frac{\sqrt{\pi}\sqrt{2}(10bx+15)\sqrt{b}\sqrt{x}}{15\left(\frac{bx}{2}+1\right)^{\frac{3}{2}}}+2\sqrt{\pi}\operatorname{arcsinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{\pi}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] 4/3/b^(5/2)/Pi^(1/2)*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*b^(1/2)*(10*b*x+15)/(1
/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*arcsinh(1/2*2^(1/2)*b^(1/2)*x^(1/2)))
```

maxima [A] time = 2.97, size = 69, normalized size = 1.06

$$-\frac{2\left(b+\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{3\,(bx+2)^{\frac{3}{2}}b^{2}}-\frac{\log\left(-\frac{\sqrt{b}-\frac{\sqrt{bx+2}}{\sqrt{x}}}{\sqrt{b}+\frac{\sqrt{bx+2}}{\sqrt{x}}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b + 3*(b*x + 2)/x)*x^(3/2)/((b*x + 2)^(3/2)*b^2) - log(-(sqrt(b) - sq
rt(b*x + 2)/sqrt(x))/(sqrt(b) + sqrt(b*x + 2)/sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(b\,x+2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x + 2)^(5/2),x)
```

```
[Out] int(x^(3/2)/(b*x + 2)^(5/2), x)
```

sympy [B] time = 3.58, size = 257, normalized size = 3.95

$$-\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}-\frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{6b^{5}x^{\frac{15}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}+\frac{12b^{4}x^{\frac{13}{2}}\sqrt{bx+2}\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx+2}+6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -8*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x**(13
/2)*sqrt(b*x + 2)) - 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2)
+ 6*b**(13/2)*x**(13/2)*sqrt(b*x + 2)) + 6*b**5*x**(15/2)*sqrt(b*x + 2)*asi
nh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(
13/2)*x**(13/2)*sqrt(b*x + 2)) + 12*b**4*x**(13/2)*sqrt(b*x + 2)*asinh(sqrt
(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x + 2) + 6*b**(13/2)*x
**(13/2)*sqrt(b*x + 2))
```

3.625 $\int \frac{\sqrt{x}}{(2+bx)^{5/2}} \, dx$

Optimal. Leaf size=18

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 + b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2+bx)^{5/2}} \, dx = \frac{x^{3/2}}{3(2+bx)^{3/2}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 + b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{x^{3/2}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 + b*x)^(5/2),x]
[Out] x^(3/2)/(3*(2 + b*x)^(3/2))
```

fricas [B] time = 1.19, size = 27, normalized size = 1.50

$$\frac{\sqrt{bx+2}\,x^{\frac{3}{2}}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*x + 2)*x^(3/2)/(b^2*x^2 + 4*b*x + 4)
```

giac [B] time = 1.22, size = 82, normalized size = 4.56

$$\frac{4\left(3\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{4}\sqrt{b}+4b^{\frac{5}{2}}\right)|b|}{3\left(\left(\sqrt{bx+2}\sqrt{b}-\sqrt{(bx+2)b-2b}\right)^{2}+2b\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*sqrt(b) + 4*b^(5
/2))*abs(b)/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*
b^2)
```

maple [A] time = 0.00, size = 13, normalized size = 0.72

$$\frac{x^{\frac{3}{2}}}{3\left(bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(b*x+2)^(3/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.67

$$\frac{x^{\frac{3}{2}}}{3\left(bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(b*x + 2)^(3/2)
```

mupad [B] time = 0.25, size = 12, normalized size = 0.67

$$\frac{x^{3/2}}{3\,{\left(b\,x+2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x + 2)^(5/2),x)
```

```
[Out] x^(3/2)/(3*(b*x + 2)^(3/2))
```

sympy [A] time = 1.40, size = 27, normalized size = 1.50

$$\frac{x^{\frac{3}{2}}}{3bx\sqrt{bx+2}+6\sqrt{bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x+2)**(5/2),x)
```

```
[Out] x**(3/2)/(3*b*x*sqrt(b*x + 2) + 6*sqrt(b*x + 2))
```

3.626 $\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{bx+2}}+\frac{\sqrt{x}}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 + b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(2+bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}(2+bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2+bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2+bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}(bx+3)}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 23, normalized size = 0.62

$$\frac{\sqrt{x}(bx+3)}{3(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 + b*x)^(5/2)),x]
```

```
[Out] (Sqrt[x]*(3 + b*x))/(3*(2 + b*x)^(3/2))
```

fricas [A] time = 0.81, size = 32, normalized size = 0.86

$$\frac{(bx+3)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^2x^2+4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^2 + 4*b*x + 4)
```

giac [B] time = 1.22, size = 79, normalized size = 2.14

$$\frac{8\left(3\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)b^{\frac{5}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b}-\sqrt{(bx+2)b-2\,b}\right)^2+2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)*b^(5/2)/(
((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

maple [A] time = 0.00, size = 18, normalized size = 0.49

$$\frac{(bx+3)\,\sqrt{x}}{3\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] 1/3*x^(1/2)*(b*x+3)/(b*x+2)^(3/2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.65

$$-\frac{\left(b-\frac{3\,(bx+2)}{x}\right)x^{\frac{3}{2}}}{6\,(bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(b - 3*(b*x + 2)/x)*x^(3/2)/(b*x + 2)^(3/2)
```

mupad [B] time = 0.36, size = 42, normalized size = 1.14

$$\frac{3\,\sqrt{x}\,\sqrt{b\,x+2}+b\,x^{3/2}\,\sqrt{b\,x+2}}{3\,b^2\,x^2+12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(b*x + 2)^(1/2) + b*x^(3/2)*(b*x + 2)^(1/2))/(12*b*x + 3*b^2*x^2
 + 12)
```

sympy [B] time = 1.84, size = 75, normalized size = 2.03

$$\frac{bx}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}+\frac{3}{3b^{\frac{3}{2}}x\sqrt{1+\frac{2}{bx}}+6\sqrt{b}\sqrt{1+\frac{2}{bx}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] b*x/(3*b**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x))) + 3/(3*b
**(3/2)*x*sqrt(1 + 2/(b*x)) + 6*sqrt(b)*sqrt(1 + 2/(b*x)))
```

3.627 $\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=55

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{bx+2}}+\frac{1}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 + b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 + b*x]) - (2*Sqrt[2 + b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}}+\frac{2}{3}\int \frac{1}{x^{3/2}(2+bx)^{3/2}}\,dx\\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2+bx}}+\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\ &= \frac{1}{3\sqrt{x}\,(2+bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.58

$$\frac{-2b^2x^2-6bx-3}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 32, normalized size = 0.58

$$\frac{-2b^2x^2 - 6bx - 3}{3\sqrt{x}\,(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-3 - 6*b*x - 2*b^2*x^2)/(3*Sqrt[x]*(2 + b*x)^(3/2))
```

fricas [A] time = 1.47, size = 45, normalized size = 0.82

$$-\frac{\left(2\,b^2x^2 + 6\,bx + 3\right)\sqrt{bx+2}\,\sqrt{x}}{3\left(b^2x^3 + 4\,bx^2 + 4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 + 6*b*x + 3)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^3 + 4*b*x^2 + 4*x
)
```

giac [B] time = 1.36, size = 145, normalized size = 2.64

$$-\frac{\sqrt{bx+2}\,b^2}{4\,\sqrt{(bx+2)b-2\,b}\,|b|} - \frac{3\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^4 b^{\frac{5}{2}} + 24\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^2 b^{\frac{7}{2}} + 20\,b^{\frac{9}{2}}}{3\left(\left(\sqrt{bx+2}\,\sqrt{b} - \sqrt{(bx+2)b-2\,b}\right)^2 + 2\,b\right)^3 |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(b*x + 2)*b^2/(sqrt((b*x + 2)*b - 2*b)*abs(b)) - 1/3*(3*(sqrt(b*x
+ 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(5/2) + 24*(sqrt(b*x + 2)*sqrt(
b) - sqrt((b*x + 2)*b - 2*b))^2*b^(7/2) + 20*b^(9/2))/(((sqrt(b*x + 2)*sqrt
(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs(b))
```

maple [A] time = 0.00, size = 27, normalized size = 0.49

$$-\frac{2b^2x^2 + 6bx + 3}{3\,(bx+2)^{\frac{3}{2}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2+6*b*x+3)/(b*x+2)^(3/2)/x^(1/2)
```

maxima [A] time = 1.40, size = 40, normalized size = 0.73

$$\frac{\left(b^2 - \frac{6\,(bx+2)b}{x}\right)x^{\frac{3}{2}}}{12\,(bx+2)^{\frac{3}{2}}} - \frac{\sqrt{bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x + 2)*b/x)*x^(3/2)/(b*x + 2)^(3/2) - 1/4*sqrt(b*x + 2)/sq
rt(x)
```

mupad [B] time = 0.38, size = 57, normalized size = 1.04

$$-\frac{3\,\sqrt{b\,x+2}+6\,b\,x\,\sqrt{b\,x+2}+2\,b^2\,x^2\,\sqrt{b\,x+2}}{\sqrt{x}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] -(3*(b*x + 2)^(1/2) + 6*b*x*(b*x + 2)^(1/2) + 2*b^2*x^2*(b*x + 2)^(1/2))/(x
^(1/2)*(x*(12*b + 3*b^2*x) + 12))
```

sympy [B] time = 3.89, size = 117, normalized size = 2.13

$$-\frac{2b^{\frac{13}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{6b^{\frac{11}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{1+\frac{2}{bx}}}{3b^6x^2+12b^5x+12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x+2)**(5/2),x)
```

```
[Out] -2*b**(13/2)*x**2*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 6
*b**(11/2)*x*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4) - 3*b**(
9/2)*sqrt(1 + 2/(b*x))/(3*b**6*x**2 + 12*b**5*x + 12*b**4)
```

3.628 $\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx$

Optimal. Leaf size=71

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2\sqrt{bx+2}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{bx+2}}+\frac{1}{3x^{3/2}(bx+2)^{3/2}}+\frac{2b\sqrt{bx+2}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 + b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 + b*x]) - (2*Sqrt[2 + b*x
])/(3*x^(3/2)) + (2*b*Sqrt[2 + b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2+bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\int \frac{1}{x^{5/2}(2+bx)^{3/2}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}+2\int \frac{1}{x^{5/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}-\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2+bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2+bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2+bx}}-\frac{2\sqrt{2+bx}}{3x^{3/2}}+\frac{2b\sqrt{2+bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.56

$$\frac{2b^3x^3+6b^2x^2+3bx-1}{3x^{3/2}(bx+2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 40, normalized size = 0.56

$$\frac{2b^3x^3 + 6b^2x^2 + 3bx - 1}{3x^{3/2}(bx + 2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 + b*x)^(5/2)),x]
```

```
[Out] (-1 + 3*b*x + 6*b^2*x^2 + 2*b^3*x^3)/(3*x^(3/2)*(2 + b*x)^(3/2))
```

fricas [A] time = 1.05, size = 55, normalized size = 0.77

$$\frac{\left(2\,b^3x^3 + 6\,b^2x^2 + 3\,bx - 1\right)\sqrt{bx + 2}\,\sqrt{x}}{3\left(b^2x^4 + 4\,bx^3 + 4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^3*x^3 + 6*b^2*x^2 + 3*b*x - 1)*sqrt(b*x + 2)*sqrt(x)/(b^2*x^4 + 4*
b*x^3 + 4*x^2)
```

giac [B] time = 1.27, size = 158, normalized size = 2.23

$$\frac{\left(4\,(bx + 2)b^2|b| - 9\,b^2|b|\right)\sqrt{bx + 2}}{12\,((bx + 2)b - 2\,b)^{\frac{3}{2}}} + \frac{3\left(\sqrt{bx + 2}\,\sqrt{b} - \sqrt{(bx + 2)b - 2\,b}\right)^4 b^{\frac{7}{2}} + 18\left(\sqrt{bx + 2}\,\sqrt{b} - \sqrt{(bx + 2)b - 2\,b}\right)^2 b^{\frac{9}{2}} + 16\,b^{\frac{11}{2}}}{3\left(\left(\sqrt{bx + 2}\,\sqrt{b} - \sqrt{(bx + 2)b - 2\,b}\right)^2 + 2\,b\right)^3 |b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(4*(b*x + 2)*b^2*abs(b) - 9*b^2*abs(b))*sqrt(b*x + 2)/((b*x + 2)*b - 2
*b)^(3/2) + 1/3*(3*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^4*b^(7
/2) + 18*(sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2*b^(9/2) + 16*b
^(11/2))/(((sqrt(b*x + 2)*sqrt(b) - sqrt((b*x + 2)*b - 2*b))^2 + 2*b)^3*abs
(b))
```

maple [A] time = 0.00, size = 35, normalized size = 0.49

$$\frac{2b^3x^3 + 6b^2x^2 + 3bx - 1}{3\,(bx + 2)^{\frac{3}{2}}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x+2)^(5/2),x)
```

```
[Out] 1/3*(2*b^3*x^3+6*b^2*x^2+3*b*x-1)/(b*x+2)^(3/2)/x^(3/2)
```

maxima [A] time = 1.28, size = 55, normalized size = 0.77

$$\frac{3\,\sqrt{bx + 2}\,b}{8\,\sqrt{x}} - \frac{\left(b^3 - \frac{9\,(bx+2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(bx + 2)^{\frac{3}{2}}} - \frac{(bx + 2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(b*x + 2)*b/sqrt(x) - 1/24*(b^3 - 9*(b*x + 2)*b^2/x)*x^(3/2)/(b*x +
 2)^(3/2) - 1/24*(b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.42, size = 71, normalized size = 1.00

$$\frac{3\,b\,x\,\sqrt{b\,x+2}-\sqrt{b\,x+2}+6\,b^2\,x^2\,\sqrt{b\,x+2}+2\,b^3\,x^3\,\sqrt{b\,x+2}}{x^{3/2}\,\left(x\,\left(3\,x\,b^2+12\,b\right)+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(b*x + 2)^(5/2)),x)
```

```
[Out] (3*b*x*(b*x + 2)^(1/2) - (b*x + 2)^(1/2) + 6*b^2*x^2*(b*x + 2)^(1/2) + 2*b^
3*x^3*(b*x + 2)^(1/2))/(x^(3/2)*(x*(12*b + 3*b^2*x) + 12))
```

sympy [B] time = 6.85, size = 257, normalized size = 3.62

$$\frac{2b^{\frac{27}{2}}x^4\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{10b^{\frac{25}{2}}x^3\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{15b^{\frac{23}{2}}x^2\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}+\frac{5b^{\frac{21}{2}}x\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}-\frac{2b^{\frac{19}{2}}\sqrt{1+\frac{2}{bx}}}{3b^{12}x^4+18b^{11}x^3+36b^{10}x^2+24b^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x+2)**(5/2),x)
```

```
[Out] 2*b**(27/2)*x**4*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10
*x**2 + 24*b**9*x) + 10*b**(25/2)*x**3*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18
*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2*sqrt(1 + 2/(b*
x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b**9*x) + 5*b**(21/2
)*x*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b**10*x**2 + 24*b*
*9*x) - 2*b**(19/2)*sqrt(1 + 2/(b*x))/(3*b**12*x**4 + 18*b**11*x**3 + 36*b*
*10*x**2 + 24*b**9*x)
```

3.629 $\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=91

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{x^{5/2}\sqrt{2-bx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-5*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) - (5*x^(3/2)*Sqrt[2 - b*x])/(6*b^2) - (x
^(5/2)*Sqrt[2 - b*x])/(3*b) + (5*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{2-bx}}\,dx &= -\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{3b}\\
&= -\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^3}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^3}\\
&= -\frac{5\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{5x^{3/2}\sqrt{2-bx}}{6b^2}-\frac{x^{5/2}\sqrt{2-bx}}{3b}+\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}$$

Mathematica [**A**] time = 0.04, size = 61, normalized size = 0.67

$$\frac{5\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{\sqrt{x}\sqrt{2-bx}\left(2b^2x^2+5bx+15\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/6*(Sqrt[x]*Sqrt[2 - b*x]*(15 + 5*b*x + 2*b^2*x^2))/b^3 + (5*ArcSin[(Sqrt
[b]*Sqrt[x])/Sqrt[2]])/b^(7/2)
```

IntegrateAlgebraic [**A**] time = 0.11, size = 82, normalized size = 0.90

$$\frac{5\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^4}+\frac{\sqrt{2-bx}\left(-2b^2x^{5/2}-5bx^{3/2}-15\sqrt{x}\right)}{6b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-15*Sqrt[x] - 5*b*x^(3/2) - 2*b^2*x^(5/2)))/(6*b^3) + (5*Sq
rt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

fricas [**A**] time = 1.44, size = 125, normalized size = 1.37

$$\left[-\frac{\left(2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+15\,\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{6\,b^4},-\frac{\left(2\,b^3x^2+5\,b^2x+15\,b\right)\sqrt{-bx+2}\,\sqrt{x}+30\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{6\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*((2*b^3*x^2 + 5*b^2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 15*sqrt(-b)*lo
g(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^4, -1/6*((2*b^3*x^2 + 5*b^
2*x + 15*b)*sqrt(-b*x + 2)*sqrt(x) + 30*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt
(b)*sqrt(x))))/b^4]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
```

```
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-2*abs(b)/b^2/b*(2*((12*b^5/144
/b^7*sqrt(-b*x+2)*sqrt(-b*x+2)-78*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*x+2)+19
8*b^5/144/b^7)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)-5/2/b/sqrt(-b)*ln(abs(sqr
t(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [A] time = 0.00, size = 100, normalized size = 1.10

$$-\frac{\sqrt{-bx+2}\, x^{\frac{5}{2}}}{3b}-\frac{5\sqrt{-bx+2}\, x^{\frac{3}{2}}}{6b^{2}}-\frac{5\sqrt{-bx+2}\, \sqrt{x}}{2b^{3}}+\frac{5\sqrt{\left(-bx+2\right)x}\, \arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\, b^{\frac{7}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*x^(5/2)*(-b*x+2)^(1/2)/b-5/6*x^(3/2)*(-b*x+2)^(1/2)/b^2-5/2*(-b*x+2)^(
1/2)/b^3*x^(1/2)+5/2*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(7/2)/x^(1/2)*arct
an((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 3.03, size = 117, normalized size = 1.29

$$-\frac{\frac{33\sqrt{-bx+2}\,b^{2}}{\sqrt{x}}+\frac{40\left(-bx+2\right)^{\frac{3}{2}}b}{x^{\frac{3}{2}}}+\frac{15\left(-bx+2\right)^{\frac{5}{2}}}{x^{\frac{5}{2}}}}{3\left(b^{6}-\frac{3\left(bx-2\right)b^{5}}{x}+\frac{3\left(bx-2\right)^{2}b^{4}}{x^{2}}-\frac{\left(bx-2\right)^{3}b^{3}}{x^{3}}\right)}-\frac{5\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(33*sqrt(-b*x + 2)*b^2/sqrt(x) + 40*(-b*x + 2)^(3/2)*b/x^(3/2) + 15*(-
b*x + 2)^(5/2)/x^(5/2))/(b^6 - 3*(b*x - 2)*b^5/x + 3*(b*x - 2)^2*b^4/x^2 -
(b*x - 2)^3*b^3/x^3) - 5*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(1/2), x)
```

sympy [A] time = 7.51, size = 206, normalized size = 2.26

$$\begin{cases} -\frac{ix^{\frac{7}{2}}}{3\sqrt{bx-2}} - \frac{ix^{\frac{5}{2}}}{6b\sqrt{bx-2}} - \frac{5ix^{\frac{3}{2}}}{6b^2\sqrt{bx-2}} + \frac{5i\sqrt{x}}{b^3\sqrt{bx-2}} - \frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for } \frac{|bx|}{2} > 1 \\ \frac{x^{\frac{7}{2}}}{3\sqrt{-bx+2}} + \frac{x^{\frac{5}{2}}}{6b\sqrt{-bx+2}} + \frac{5x^{\frac{3}{2}}}{6b^2\sqrt{-bx+2}} - \frac{5\sqrt{x}}{b^3\sqrt{-bx+2}} + \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(7/2)/(3*sqrt(b*x - 2)) - I*x**(5/2)/(6*b*sqrt(b*x - 2)) -
 5*I*x**(3/2)/(6*b**2*sqrt(b*x - 2)) + 5*I*sqrt(x)/(b**3*sqrt(b*x - 2)) - 5
*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), Abs(b*x)/2 > 1), (x**(7/2)/(3
*sqrt(-b*x + 2)) + x**(5/2)/(6*b*sqrt(-b*x + 2)) + 5*x**(3/2)/(6*b**2*sqrt(
-b*x + 2)) - 5*sqrt(x)/(b**3*sqrt(-b*x + 2)) + 5*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), True))
```

3.630 $\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=69

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$-\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{x^{3/2}\sqrt{2-bx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (-3*Sqrt[x]*Sqrt[2 - b*x])/(2*b^2) - (x^(3/2)*Sqrt[2 - b*x])/(2*b) + (3*Arc
Sin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx &= -\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= -\frac{3\sqrt{x}\sqrt{2-bx}}{2b^2}-\frac{x^{3/2}\sqrt{2-bx}}{2b}+\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.75

$$\frac{3\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{\sqrt{x}\sqrt{2-bx}(bx+3)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] -1/2*(Sqrt[x]*Sqrt[2 - b*x]*(3 + b*x))/b^2 + (3*ArcSin[(Sqrt[b]*Sqrt[x])/Sq
rt[2]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 72, normalized size = 1.04

$$\frac{3\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}+\frac{\sqrt{2-bx}\left(-bx^{3/2}-3\sqrt{x}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/Sqrt[2 - b*x],x]
```

```
[Out] (Sqrt[2 - b*x]*(-3*Sqrt[x] - b*x^(3/2)))/(2*b^2) + (3*Sqrt[-b]*Log[-(Sqrt[-
b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

fricas [A] time = 1.34, size = 107, normalized size = 1.55

$$\left[-\frac{\left(b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+3\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)}{2b^3},-\frac{\left(b^2x+3b\right)\sqrt{-bx+2}\sqrt{x}+6\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{2b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt(x) + 3*sqrt(-b)*log(-b*x + sqrt(-b
*x + 2)*sqrt(-b)*sqrt(x) + 1))/b^3, -1/2*((b^2*x + 3*b)*sqrt(-b*x + 2)*sqrt
(x) + 6*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))))/b^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
```

```
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]2*abs(b)/b^2/b^2*(2*(1/8*sqrt(-
b*x+2)*sqrt(-b*x+2)-5/8)*sqrt(-b*x+2)*sqrt(-b*(-b*x+2)+2*b)+6*b/4/sqrt(-b)*
ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2))))
```

maple [A] time = 0.00, size = 84, normalized size = 1.22

$$-\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{2b}-\frac{3\sqrt{-bx+2}\,\sqrt{x}}{2b^{2}}+\frac{3\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2\sqrt{-bx+2}\,b^{\frac{5}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/2*x^(3/2)*(-b*x+2)^(1/2)/b-3/2*(-b*x+2)^(1/2)/b^2*x^(1/2)+3/2*((-b*x+2)*
x)^(1/2)/(-b*x+2)^(1/2)/b^(5/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b
^(1/2))
```

maxima [A] time = 2.93, size = 85, normalized size = 1.23

$$-\frac{\frac{5\sqrt{-bx+2}\,b}{\sqrt{x}}+\frac{3\,(-bx+2)^{\frac{3}{2}}}{x^{\frac{3}{2}}}}{b^{4}-\frac{2\,(bx-2)b^{3}}{x}+\frac{(bx-2)^{2}b^{2}}{x^{2}}}-\frac{3\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(5*sqrt(-b*x + 2)*b/sqrt(x) + 3*(-b*x + 2)^(3/2)/x^(3/2))/(b^4 - 2*(b*x -
2)*b^3/x + (b*x - 2)^2*b^2/x^2) - 3*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))
)/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\sqrt{2-b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(1/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(1/2), x)
```

sympy [A] time = 3.59, size = 163, normalized size = 2.36

$$\begin{cases} -\frac{i x^{\frac{5}{2}}}{2\sqrt{bx-2}}-\frac{i x^{\frac{3}{2}}}{2b\sqrt{bx-2}}+\frac{3i\sqrt{x}}{b^{2}\sqrt{bx-2}}-\frac{3i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{x^{\frac{5}{2}}}{2\sqrt{-bx+2}}+\frac{x^{\frac{3}{2}}}{2b\sqrt{-bx+2}}-\frac{3\sqrt{x}}{b^{2}\sqrt{-bx+2}}+\frac{3\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(5/2)/(2*sqrt(b*x - 2)) - I*x**(3/2)/(2*b*sqrt(b*x - 2)) +
 3*I*sqrt(x)/(b**2*sqrt(b*x - 2)) - 3*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b*
*(5/2), Abs(b*x)/2 > 1), (x**(5/2)/(2*sqrt(-b*x + 2)) + x**(3/2)/(2*b*sqrt(
-b*x + 2)) - 3*sqrt(x)/(b**2*sqrt(-b*x + 2)) + 3*asin(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(5/2), True))
```

3.631 $\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx$

Optimal. Leaf size=45

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {50, 54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx &= -\frac{\sqrt{x}\sqrt{2-bx}}{b}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{b}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b}\\
&= -\frac{\sqrt{x}\sqrt{2-bx}}{b}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}-\frac{\sqrt{x}\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/2
)
```

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 1.31

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\,\sqrt{x}\right)}{b^2}-\frac{\sqrt{x}\,\sqrt{2-bx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[2 - b*x],x]
```

```
[Out] -((Sqrt[x]*Sqrt[2 - b*x])/b) + (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2
 - b*x]])/b^2
```

fricas [A] time = 1.28, size = 90, normalized size = 2.00

$$\left[-\frac{\sqrt{-bx+2}\,b\sqrt{x}+\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b^2},\,-\frac{\sqrt{-bx+2}\,b\sqrt{x}+2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-(sqrt(-b*x + 2)*b*sqrt(x) + sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*s
qrt(x) + 1))/b^2, -(sqrt(-b*x + 2)*b*sqrt(x) + 2*sqrt(b)*arctan(sqrt(-b*x +
 2)/(sqrt(b)*sqrt(x))))/b^2]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
```

```
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]-2*abs(b)/b^2/b*(1/2*sqrt(-b*x+
2)*sqrt(-b*(-b*x+2)+2*b)-2*b/2/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+2)+2*b)-sqrt(-
b)*sqrt(-b*x+2))))
```

maple [A] time = 0.01, size = 67, normalized size = 1.49

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{b}+\frac{\sqrt{(-bx+2)\,x}\,\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\,b^{\frac{3}{2}}\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -(-b*x+2)^(1/2)/b*x^(1/2)+((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(3/2)/x^(1/2)
*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.93, size = 52, normalized size = 1.16

$$-\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}-\frac{2\,\sqrt{-bx+2}}{\left(b^{2}-\frac{(bx-2)b}{x}\right)\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) - 2*sqrt(-b*x + 2)/((b^
2 - (b*x - 2)*b/x)*sqrt(x))
```

mupad [B] time = 0.52, size = 46, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}-\sqrt{2-b\,x}}\right)}{b^{3/2}}-\frac{\sqrt{x}\,\sqrt{2-b\,x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(1/2),x)
```

```
[Out] - (4*atan((b^(1/2)*x^(1/2))/(2^(1/2) - (2 - b*x)^(1/2))))/b^(3/2) - (x^(1/2
)*(2 - b*x)^(1/2))/b
```

sympy [A] time = 1.97, size = 121, normalized size = 2.69

$$\begin{cases} -\frac{ix^{\frac{3}{2}}}{\sqrt{bx-2}}+\frac{2i\sqrt{x}}{b\sqrt{bx-2}}-\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{-bx+2}}-\frac{2\sqrt{x}}{b\sqrt{-bx+2}}+\frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*x**(3/2)/sqrt(b*x - 2) + 2*I*sqrt(x)/(b*sqrt(b*x - 2)) - 2*I*
acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), Abs(b*x)/2 > 1), (x**(3/2)/sqrt(
-b*x + 2) - 2*sqrt(x)/(b*sqrt(-b*x + 2)) + 2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2
)/b**(3/2), True))
```

3.632 $\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=24

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.06, size = 38, normalized size = 1.58

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b
```

fricas [A] time = 1.29, size = 56, normalized size = 2.33

$$\left[-\frac{\sqrt{-b}\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)}{b}, -\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan(sqr
t(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{8,[0,1]%%%}
+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4
,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{20,[2,1]%%%}+%%%{8,[2,0]%%
%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1]%%%}+%%%{-16,[1,0]%%%}+%
%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%%%{-32,[0,0]%%%},0,%%%{1,
[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}
+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%}+%%%{-20,[3,1]%%%}+%%%{8
,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{46,[2,2]%%%}+%%%{-40,[2,1]
%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3]%%%}+%%%{40,[1,2]%%%}+%%
%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-8,[0,3]%%%}+%%%{24,[0
,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at parameters values [-15.64384
32182,61.7937478349]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1
,0]%%%}+%%%{-4,[0,1]%%%}+%%%{-8,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}
+%%%{6,[2,0]%%%}+%%%{-4,[1,2]%%%}+%%%{-28,[1,1]%%%}+%%%{-8,[1,0]%%%}+%%%{6,
[0,2]%%%}+%%%{8,[0,1]%%%}+%%%{24,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%
%}+%%%{-4,[3,1]%%%}+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-64,[2,2]%%%}+%%%{2
0,[2,1]%%%}+%%%{8,[2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-20,[1,2]%%%}+%%%{128,[1,1
]%%%}+%%%{-16,[1,0]%%%}+%%%{-4,[0,3]%%%}+%%%{8,[0,2]%%%}+%%%{16,[0,1]%%%}+%
%%{-32,[0,0]%%%},0,%%%{1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,
[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{4,[3,4]%%%}+%%%{-12,[3,3]%%%}+%%%{20,[3,2]%%%
}+%%%{-20,[3,1]%%%}+%%%{8,[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-20,[2,3]%%%}+%%%{4
6,[2,2]%%%}+%%%{-40,[2,1]%%%}+%%%{24,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-20,[1,3
]%%%}+%%%{40,[1,2]%%%}+%%%{-48,[1,1]%%%}+%%%{32,[1,0]%%%}+%%%{1,[0,4]%%%}+%
%%{-8,[0,3]%%%}+%%%{24,[0,2]%%%}+%%%{-32,[0,1]%%%}+%%%{16,[0,0]%%%}] at par
ameters values [-29.292030761,78.6493344628]2/abs(b)*b^2/b/sqrt(-b)*ln(abs(
sqrt(-b*(-b*x+2)+2*b)-sqrt(-b)*sqrt(-b*x+2)))
```

maple [B] time = 0.00, size = 50, normalized size = 2.08

$$\frac{\sqrt{(-bx+2)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{\sqrt{-bx+2}\;\sqrt{b}\;\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/b^(1/2)/x^(1/2)*arctan((x-1/b)/(-b*x^2+2*
x)^(1/2)*b^(1/2))
```

maxima [A] time = 2.96, size = 21, normalized size = 0.88

$$-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

mupad [B] time = 0.03, size = 27, normalized size = 1.12

$$\frac{4\,\mathrm{atan}\left(\frac{\sqrt{2}-\sqrt{2-b\,x}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((2^(1/2) - (2 - b*x)^(1/2))/(b^(1/2)*x^(1/2))))/b^(1/2)
```

sympy [A] time = 1.08, size = 58, normalized size = 2.42

$$\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{for}\ \frac{|bx|}{2} > 1 \\ \frac{2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), Abs(b*x)/2 > 1),
(2*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/sqrt(b), True))
```

3.633 $\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx = -\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.00

$$-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -(Sqrt[2 - b*x]/Sqrt[x])
```

fricas [A] time = 1.13, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

giac [B] time = 1.28, size = 30, normalized size = 1.76

$$-\frac{\sqrt{-bx+2}\,b^2}{\sqrt{(bx-2)b+2\,b}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b))
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -(-b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.76

$$-\frac{\sqrt{-bx+2}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-b*x + 2)/sqrt(x)
```

mupad [B] time = 0.31, size = 13, normalized size = 0.76

$$-\frac{\sqrt{2-b\,x}}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(2 - b*x)^(1/2)/x^(1/2)
```

sympy [A] time = 0.93, size = 39, normalized size = 2.29

$$\begin{cases} -\sqrt{b}\,\sqrt{-1+\frac{2}{bx}} & \text{for}\ \frac{2}{|bx|}>1 \\ -i\sqrt{b}\,\sqrt{1-\frac{2}{bx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(b)*sqrt(-1 + 2/(b*x)), 2/Abs(b*x) > 1), (-I*sqrt(b)*sqrt(1
 - 2/(b*x)), True))
```

3.634 $\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx$

Optimal. Leaf size=40

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -Sqrt[2 - b*x]/(3*x^(3/2)) - (b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx &= -\frac{\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}b\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= -\frac{\sqrt{2-bx}}{3x^{3/2}}-\frac{b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.60

$$-\frac{\sqrt{2-bx}\,(bx+1)}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] -1/3*(Sqrt[2 - b*x]*(1 + b*x))/x^(3/2)
```

IntegrateAlgebraic [A] time = 0.09, size = 25, normalized size = 0.62

$$\frac{(-bx-1)\sqrt{2-bx}}{3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*Sqrt[2 - b*x]),x]
```

```
[Out] ((-1 - b*x)*Sqrt[2 - b*x])/(3*x^(3/2))
```

fricas [A] time = 0.87, size = 18, normalized size = 0.45

$$-\frac{(bx+1)\sqrt{-bx+2}}{3\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x + 1)*sqrt(-b*x + 2)/x^(3/2)
```

giac [A] time = 1.12, size = 43, normalized size = 1.08

$$-\frac{\left((bx-2)b^3+3\,b^3\right)\sqrt{-bx+2}\,b}{3\,((bx-2)b+2\,b)^{\frac{3}{2}}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*((b*x - 2)*b^3 + 3*b^3)*sqrt(-b*x + 2)*b/(((b*x - 2)*b + 2*b)^(3/2)*ab
s(b))
```

maple [A] time = 0.00, size = 19, normalized size = 0.48

$$-\frac{(bx+1)\,\sqrt{-bx+2}}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(1/2),x)
```

```
[Out] -1/3*(b*x+1)/x^(3/2)*(-b*x+2)^(1/2)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.70

$$-\frac{\sqrt{-bx+2}\,b}{2\,\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{6\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/6*(-b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.29, size = 19, normalized size = 0.48

$$-\frac{\sqrt{2-b\,x}\,\left(\frac{b\,x}{3}+\frac{1}{3}\right)}{x^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -((2 - b*x)^(1/2)*((b*x)/3 + 1/3))/x^(3/2)
```

sympy [A] time = 1.96, size = 139, normalized size = 3.48

$$\begin{cases} -\frac{b^{\frac{3}{2}}\sqrt{-1+\frac{2}{bx}}}{3} - \frac{\sqrt{b}\sqrt{-1+\frac{2}{bx}}}{3x} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{ib^{\frac{7}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx} - \frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx} - \frac{2ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^2x^2+6bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-b**(3/2)*sqrt(-1 + 2/(b*x))/3 - sqrt(b)*sqrt(-1 + 2/(b*x))/(3*x
), 2/Abs(b*x) > 1), (I*b**(7/2)*x**2*sqrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*
x) - I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*x) - 2*I*b**(3/2)*s
qrt(1 - 2/(b*x))/(-3*b**2*x**2 + 6*b*x), True))
```

3.635 $\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}+\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$\frac{5x^{3/2}\sqrt{2-bx}}{2b^2}+\frac{15\sqrt{x}\sqrt{2-bx}}{2b^3}-\frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(5/2))/(b*Sqrt[2 - b*x]) + (15*Sqrt[x]*Sqrt[2 - b*x])/(2*b^3) + (5*x^(
3/2)*Sqrt[2 - b*x])/(2*b^2) - (15*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{5/2}}{b\sqrt{2-bx}} - \frac{5\int \frac{x^{3/2}}{\sqrt{2-bx}}\,dx}{b} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{2b^2} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{2b^3} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3} \\
&= \frac{2x^{5/2}}{b\sqrt{2-bx}} + \frac{15\sqrt{x}\sqrt{2-bx}}{2b^3} + \frac{5x^{3/2}\sqrt{2-bx}}{2b^2} - \frac{15\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$
\frac{x^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{7\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(3/2),x]
[Out] (x^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (b*x)/2])/(7*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.17, size = 88, normalized size = 0.99

$$
\frac{\sqrt{2-bx}\left(b^2x^{5/2}+5bx^{3/2}-30\sqrt{x}\right)}{2b^3(bx-2)} - \frac{15\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 - b*x)^(3/2),x]
[Out] (Sqrt[2 - b*x]*(-30*Sqrt[x] + 5*b*x^(3/2) + b^2*x^(5/2)))/(2*b^3*(-2 + b*x)
) - (15*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

fricas [A] time = 1.23, size = 155, normalized size = 1.74

$$
\left[-\frac{15\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-\left(b^3x^2+5\,b^2x-30\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{2\left(b^5x-2\,b^4\right)}, \frac{30\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^3x^2+5\,b^2x-30\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{2\left(b^5x-2\,b^4\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
[Out] [-1/2*(15*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1
) - (b^3*x^2 + 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4), 1/2
*(30*(b*x - 2)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^3*x^2
+ 5*b^2*x - 30*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x - 2*b^4)]
```

giac [B] time = 10.85, size = 136, normalized size = 1.53

$$
\frac{\left(\sqrt{(bx-2)b+2\,b}\,\sqrt{-bx+2}\left(\frac{bx-2}{b^3}+\frac{9}{b^3}\right)-\frac{15\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}\,b^2}+\frac{64}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)\sqrt{-b}\,b}\right)|b|}{2\,b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)*((b*x - 2)/b^3 + 9/b^3) - 15*lo
g((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b)*b^2) + 6
4/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b)*b
))*abs(b)/b^2
```

maple [B] time = 0.03, size = 138, normalized size = 1.55

$$-\frac{\left(\frac{15\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{2b^{\frac{7}{2}}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{\left(x-\frac{2}{b}\right)b^{4}}\right)\sqrt{\left(-bx+2\right)x}}{\sqrt{-bx+2}\,\sqrt{x}}-\frac{\left(bx+7\right)\left(bx-2\right)\sqrt{\left(-bx+2\right)x}\,\sqrt{x}}{2\sqrt{-\left(bx-2\right)x}\,\sqrt{-bx+2}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/2*(b*x+7)*(b*x-2)*x^(1/2)/b^3/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*
x+2)^(1/2)-(15/2/b^(7/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+8/b^4/(
x-2/b)*(-b*(x-2/b)^2-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1
/2)
```

maxima [A] time = 2.99, size = 101, normalized size = 1.13

$$\frac{8\,b^{2}-\frac{25\,(bx-2)b}{x}+\frac{15\,(bx-2)^{2}}{x^{2}}}{\frac{\sqrt{-bx+2}\,b^{5}}{\sqrt{x}}+\frac{2\,(-bx+2)^{\frac{3}{2}}\,b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}\,b^{3}}{x^{\frac{5}{2}}}}+\frac{15\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] (8*b^2 - 25*(b*x - 2)*b/x + 15*(b*x - 2)^2/x^2)/(sqrt(-b*x + 2)*b^5/sqrt(x)
 + 2*(-b*x + 2)^(3/2)*b^4/x^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) + 15*arct
an(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(3/2), x)
```

sympy [A] time = 7.03, size = 173, normalized size = 1.94

$$\begin{cases} \frac{ix^{\frac{5}{2}}}{2b\sqrt{bx-2}}+\frac{5ix^{\frac{3}{2}}}{2b^{2}\sqrt{bx-2}}-\frac{15i\sqrt{x}}{b^{3}\sqrt{bx-2}}+\frac{15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{5}{2}}}{2b\sqrt{-bx+2}}-\frac{5x^{\frac{3}{2}}}{2b^{2}\sqrt{-bx+2}}+\frac{15\sqrt{x}}{b^{3}\sqrt{-bx+2}}-\frac{15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(5/2)/(2*b*sqrt(b*x - 2)) + 5*I*x**(3/2)/(2*b**2*sqrt(b*x -
 2)) - 15*I*sqrt(x)/(b**3*sqrt(b*x - 2)) + 15*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(7/2), Abs(b*x)/2 > 1), (-x**(5/2)/(2*b*sqrt(-b*x + 2)) - 5*x**(3/
2)/(2*b**2*sqrt(-b*x + 2)) + 15*sqrt(x)/(b**3*sqrt(-b*x + 2)) - 15*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(7/2), True))
```

3.636 $\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$\frac{3\sqrt{x}\sqrt{2-bx}}{b^2}-\frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{b\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (2*x^(3/2))/(b*Sqrt[2 - b*x]) + (3*Sqrt[x]*Sqrt[2 - b*x])/b^2 - (6*ArcSin[(
Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx &= \frac{2x^{3/2}}{b\sqrt{2-bx}} - \frac{3\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b} \\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{3\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2} \\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^2} \\ &= \frac{2x^{3/2}}{b\sqrt{2-bx}} + \frac{3\sqrt{x}\sqrt{2-bx}}{b^2} - \frac{6\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.46

$$\frac{x^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{bx}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (x^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (b*x)/2])/(5*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.15, size = 75, normalized size = 1.15

$$\frac{\sqrt{2-bx}\left(bx^{3/2}-6\sqrt{x}\right)}{b^2(bx-2)} - \frac{6\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 - b*x)^(3/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-6*Sqrt[x] + b*x^(3/2)))/(b^2*(-2 + b*x)) - (6*Sqrt[-b]*Log
[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

fricas [A] time = 1.30, size = 138, normalized size = 2.12

$$\left[-\frac{3\,(bx-2)\sqrt{-b}\,\log\left(-bx-\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}+1\right)-\left(b^2x-6\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3}, \frac{6\,(bx-2)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)+\left(b^2x-6\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{b^4x-2\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-(3*(b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) - (
b^2*x - 6*b)*sqrt(-b*x + 2)*sqrt(x))/(b^4*x - 2*b^3), (6*(b*x - 2)*sqrt(b)*
arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x))) + (b^2*x - 6*b)*sqrt(-b*x + 2)*sqr
t(x))/(b^4*x - 2*b^3)]
```

giac [B] time = 10.41, size = 119, normalized size = 1.83

$$-\frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}} - \frac{\sqrt{(bx-2)b+2\,b}\,\sqrt{-bx+2}}{b} + \frac{16\,\sqrt{-b}}{\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) - s
qrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b + 16*sqrt(-b)/((sqrt(-b*x + 2)*sqrt
(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(b)/b^3
```

maple [B] time = 0.03, size = 133, normalized size = 2.05

$$-\frac{\left(\frac{3\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{b^{\frac{5}{2}}}+\frac{4\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{\left(x-\frac{2}{b}\right)b^{3}}\right)\sqrt{(-bx+2)\,x}}{\sqrt{-bx+2}\,\sqrt{x}}-\frac{(bx-2)\sqrt{(-bx+2)\,x}\,\sqrt{x}}{\sqrt{-(bx-2)\,x}\,\sqrt{-bx+2}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -1/b^2*(b*x-2)*x^(1/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)
-(3/b^(5/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+4/b^3/(x-2/b)*(-(x-2
/b)^2*b-2*x+4/b)^(1/2))*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 3.00, size = 71, normalized size = 1.09

$$\frac{2\left(2\,b-\frac{3\,(bx-2)}{x}\right)}{\frac{\sqrt{-bx+2}\,b^{3}}{\sqrt{x}}+\frac{(-bx+2)^{\frac{3}{2}}\,b^{2}}{x^{\frac{3}{2}}}}+\frac{6\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(2*b - 3*(b*x - 2)/x)/(sqrt(-b*x + 2)*b^3/sqrt(x) + (-b*x + 2)^(3/2)*b^2/
x^(3/2)) + 6*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(3/2), x)
```

sympy [A] time = 3.20, size = 128, normalized size = 1.97

$$\begin{cases} \frac{i x^{\frac{3}{2}}}{b\sqrt{bx-2}}-\frac{6i\sqrt{x}}{b^{2}\sqrt{bx-2}}+\frac{6i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{for } \frac{|bx|}{2}>1 \\ -\frac{x^{\frac{3}{2}}}{b\sqrt{-bx+2}}+\frac{6\sqrt{x}}{b^{2}\sqrt{-bx+2}}-\frac{6\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(b*sqrt(b*x - 2)) - 6*I*sqrt(x)/(b**2*sqrt(b*x - 2))
+ 6*I*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b**(5/2), Abs(b*x)/2 > 1), (-x**(3/2
)/(b*sqrt(-b*x + 2)) + 6*sqrt(x)/(b**2*sqrt(-b*x + 2)) - 6*asin(sqrt(2)*sqr
t(b)*sqrt(x)/2)/b**(5/2), True))
```

3.637 $\int \frac{\sqrt{x}}{(2-bx)^{3/2}} \, dx$

Optimal. Leaf size=45

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{(2-bx)^{3/2}} \, dx &= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}} \, dx}{b} \\ &= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}} \, dx, x, \sqrt{x}\right)}{b} \\ &= \frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.00

$$\frac{2\sqrt{x}}{b\sqrt{2-bx}} - \frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (2*Sqrt[x])/(b*Sqrt[2 - b*x]) - (2*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(3/
2)
```

IntegrateAlgebraic [A] time = 0.12, size = 66, normalized size = 1.47

$$-\frac{2\sqrt{-b}\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^2}-\frac{2\sqrt{x}\sqrt{2-bx}}{b(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/(2 - b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[x]*Sqrt[2 - b*x])/(b*(-2 + b*x)) - (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt
[x]) + Sqrt[2 - b*x]])/b^2
```

fricas [A] time = 1.04, size = 122, normalized size = 2.71

$$\left[-\frac{(bx-2)\sqrt{-b}\log\left(-bx-\sqrt{-bx+2}\sqrt{-b}\sqrt{x}+1\right)+2\sqrt{-bx+2}\,b\sqrt{x}}{b^3x-2b^2},\ \frac{2\left((bx-2)\sqrt{b}\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)-\sqrt{-bx+2}\,b\sqrt{x}\right)}{b^3x-2b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((b*x - 2)*sqrt(-b)*log(-b*x - sqrt(-b*x + 2)*sqrt(-b)*sqrt(x) + 1) + 2*s
qrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2), 2*((b*x - 2)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) - sqrt(-b*x + 2)*b*sqrt(x))/(b^3*x - 2*b^2)]
```

giac [B] time = 10.06, size = 92, normalized size = 2.04

$$-\frac{\left(\frac{\log\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}}+\frac{8\sqrt{-b}}{\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^2-2b}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b) + 8*s
qrt(-b)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b))*abs(
b)/b^2
```

maple [A] time = 0.05, size = 67, normalized size = 1.49

$$-\frac{2\left(\frac{\sqrt{\pi}\sqrt{2}\,(-b)^{\frac{3}{2}}\sqrt{x}}{2\sqrt{-\frac{bx}{2}+1}\,b}-\frac{\sqrt{\pi}\,(-b)^{\frac{3}{2}}\arcsin\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}}\right)}{\sqrt{-b}\sqrt{\pi}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(3/2),x)
```

```
[Out] -2/(-b)^(1/2)/Pi^(1/2)/b*(1/2*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(3/2)/b/(-1/2*b
*x+1)^(1/2)-Pi^(1/2)*(-b)^(3/2)/b^(3/2)*arcsin(1/2*2^(1/2)*b^(1/2)*x^(1/2))
)
```

maxima [A] time = 2.98, size = 38, normalized size = 0.84

$$\frac{2\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{3}{2}}}+\frac{2\,\sqrt{x}}{\sqrt{-bx+2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(-b*x + 2)/(sqrt(b)*sqrt(x)))/b^(3/2) + 2*sqrt(x)/(sqrt(-b*x +
 2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{(2-b\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(3/2),x)
```

```
[Out] int(x^(1/2)/(2 - b*x)^(3/2), x)
```

sympy [A] time = 1.69, size = 92, normalized size = 2.04

$$\begin{cases} -\frac{2i\sqrt{x}}{b\sqrt{bx-2}}+\frac{2i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{for}\ \frac{|bx|}{2}>1 \\ \frac{2\sqrt{x}}{b\sqrt{-bx+2}}-\frac{2\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*I*sqrt(x)/(b*sqrt(b*x - 2)) + 2*I*acosh(sqrt(2)*sqrt(b)*sqrt(
x)/2)/b**(3/2), Abs(b*x)/2 > 1), (2*sqrt(x)/(b*sqrt(-b*x + 2)) - 2*asin(sqr
t(2)*sqrt(b)*sqrt(x)/2)/b**(3/2), True))
```

3.638 $\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx = \frac{\sqrt{x}}{\sqrt{2-bx}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

IntegrateAlgebraic [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{\sqrt{x}}{\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 - b*x)^(3/2)),x]
```

```
[Out] Sqrt[x]/Sqrt[2 - b*x]
```

fricas [A] time = 1.16, size = 20, normalized size = 1.25

$$-\frac{\sqrt{-bx+2}\,\sqrt{x}}{bx-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*x + 2)*sqrt(x)/(b*x - 2)
```

giac [B] time = 1.03, size = 50, normalized size = 3.12

$$-\frac{4\sqrt{-b}\,b}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(-b)*b/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b))
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(3/2)/x^(1/2),x)
```

```
[Out] x^(1/2)/(-b*x+2)^(1/2)
```

maxima [A] time = 1.27, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{-bx+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(3/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x)/sqrt(-b*x + 2)
```

mupad [B] time = 0.30, size = 12, normalized size = 0.75

$$\frac{\sqrt{x}}{\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] x^(1/2)/(2 - b*x)^(1/2)
```

sympy [A] time = 0.93, size = 39, normalized size = 2.44

$$\begin{cases} \frac{1}{\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for}\ \frac{2}{|bx|}>1 \\ -\frac{i}{\sqrt{b}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(3/2)/x**(1/2),x)
```

```
[Out] Piecewise((1/(sqrt(b)*sqrt(-1 + 2/(b*x))), 2/Abs(b*x) > 1), (-I/(sqrt(b)*sq
rt(1 - 2/(b*x))), True))
```

3.639 $\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(Sqrt[x]*Sqrt[2 - b*x]) - Sqrt[2 - b*x]/Sqrt[x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx &= \frac{1}{\sqrt{x}\sqrt{2-bx}}+\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{\sqrt{x}\sqrt{2-bx}}-\frac{\sqrt{2-bx}}{\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.62

$$\frac{bx-1}{\sqrt{x}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 + b*x)/(Sqrt[x]*Sqrt[2 - b*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 29, normalized size = 0.85

$$\frac{(1-bx)\sqrt{2-bx}}{\sqrt{x}\,(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] ((1 - b*x)*Sqrt[2 - b*x])/(Sqrt[x]*(-2 + b*x))
```

fricas [A] time = 1.13, size = 29, normalized size = 0.85

$$-\frac{(bx-1)\sqrt{-bx+2}\,\sqrt{x}}{bx^2-2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^2 - 2*x)
```

giac [B] time = 1.09, size = 83, normalized size = 2.44

$$-\frac{\sqrt{-bx+2}\,b^2}{2\,\sqrt{(bx-2)b+2\,b}\,|b|}-\frac{2\,\sqrt{-b}\,b^2}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 2*sqrt(-b)*b^2/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*abs(b))
```

maple [A] time = 0.00, size = 18, normalized size = 0.53

$$\frac{bx-1}{\sqrt{-bx+2}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(3/2),x)
```

```
[Out] (b*x-1)/x^(1/2)/(-b*x+2)^(1/2)
```

maxima [A] time = 1.26, size = 28, normalized size = 0.82

$$\frac{b\sqrt{x}}{2\,\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}}{2\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)/sqrt(x)
```

mupad [B] time = 0.32, size = 27, normalized size = 0.79

$$\frac{b\,\sqrt{x}}{\sqrt{2-b\,x}}-\frac{1}{\sqrt{x}\,\sqrt{2-b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] (b*x^(1/2))/(2 - b*x)^(1/2) - 1/(x^(1/2)*(2 - b*x)^(1/2))
```

sympy [A] time = 1.61, size = 90, normalized size = 2.65

$$\begin{cases} \frac{\sqrt{b}}{\sqrt{-1+\frac{2}{bx}}} - \frac{1}{\sqrt{b}\,x\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}}{-b^2x+2b} - \frac{ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}}{-b^2x+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((sqrt(b)/sqrt(-1 + 2/(b*x)) - 1/(sqrt(b)*x*sqrt(-1 + 2/(b*x))), 2
/Abs(b*x) > 1), (I*b**(5/2)*x*sqrt(1 - 2/(b*x))/(-b**2*x + 2*b) - I*b**(3/2
)*sqrt(1 - 2/(b*x))/(-b**2*x + 2*b), True))
```

3.640 $\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x])/(3*x^(3/2)) - (2*b*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx &= \frac{1}{x^{3/2}\sqrt{2-bx}}+2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{2b^2x^2-2bx-1}{3x^{3/2}\sqrt{2-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
```

```
[Out] (-1 - 2*b*x + 2*b^2*x^2)/(3*x^(3/2)*Sqrt[2 - b*x])
```

IntegrateAlgebraic [A] time = 0.13, size = 40, normalized size = 0.71

$$\frac{\sqrt{2-bx}\left(-2b^2x^2+2bx+1\right)}{3x^{3/2}(bx-2)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 - b*x)^(3/2)),x]
[Out] (Sqrt[2 - b*x]*(1 + 2*b*x - 2*b^2*x^2))/(3*x^(3/2)*(-2 + b*x))
```

fricas [A] time = 1.26, size = 40, normalized size = 0.71

$$-\frac{\left(2\,b^2x^2-2\,bx-1\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(bx^3-2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(2*b^2*x^2 - 2*b*x - 1)*sqrt(-b*x + 2)*sqrt(x)/(b*x^3 - 2*x^2)
```

giac [B] time = 1.17, size = 96, normalized size = 1.71

$$-\frac{\sqrt{-b}\,b^3}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)|b|}-\frac{\left(5\,(bx-2)b^2|b|+12\,b^2|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="giac")
[Out] -sqrt(-b)*b^3/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b
)*abs(b)) - 1/12*(5*(b*x - 2)*b^2*abs(b) + 12*b^2*abs(b))*sqrt(-b*x + 2)/((
b*x - 2)*b + 2*b)^(3/2)
```

maple [A] time = 0.01, size = 28, normalized size = 0.50

$$\frac{2b^2x^2-2bx-1}{3\sqrt{-bx+2}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(3/2),x)
[Out] 1/3*(2*b^2*x^2-2*b*x-1)/x^(3/2)/(-b*x+2)^(1/2)
```

maxima [A] time = 1.28, size = 44, normalized size = 0.79

$$\frac{b^2\sqrt{x}}{4\sqrt{-bx+2}}-\frac{\sqrt{-bx+2}\,b}{2\sqrt{x}}-\frac{(-bx+2)^{\frac{3}{2}}}{12\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(3/2),x, algorithm="maxima")
[Out] 1/4*b^2*sqrt(x)/sqrt(-b*x + 2) - 1/2*sqrt(-b*x + 2)*b/sqrt(x) - 1/12*(-b*x
+ 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.36, size = 38, normalized size = 0.68

$$\frac{\sqrt{2-b\,x}\,\left(\frac{2\,x}{3}-\frac{2\,b\,x^2}{3}+\frac{1}{3\,b}\right)}{x^{5/2}-\frac{2\,x^{3/2}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(3/2)),x)
```

```
[Out] ((2 - b*x)^(1/2)*((2*x)/3 - (2*b*x^2)/3 + 1/(3*b)))/(x^(5/2) - (2*x^(3/2))/
b)
```

sympy [B] time = 4.29, size = 354, normalized size = 6.32

$$\begin{cases} \frac{2b^{\frac{15}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} - \frac{6b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{3b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{2b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} & \text{for}\ \frac{2}{|bx|} > 1 \\ \frac{2ib^{\frac{15}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} - \frac{6ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{3ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} + \frac{2ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^6x^3+12b^5x^2-12b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(3/2),x)
```

```
[Out] Piecewise((2*b**(15/2)*x**3*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2
 - 12*b**4*x) - 6*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**5
*x**2 - 12*b**4*x) + 3*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b*
*5*x**2 - 12*b**4*x) + 2*b**(9/2)*sqrt(-1 + 2/(b*x))/(-3*b**6*x**3 + 12*b**
5*x**2 - 12*b**4*x), 2/Abs(b*x) > 1), (2*I*b**(15/2)*x**3*sqrt(1 - 2/(b*x))
/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) - 6*I*b**(13/2)*x**2*sqrt(1 - 2/
(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) + 3*I*b**(11/2)*x*sqrt(1 -
 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x) + 2*I*b**(9/2)*sqrt(1 -
 2/(b*x))/(-3*b**6*x**3 + 12*b**5*x**2 - 12*b**4*x), True))
```

3.641 $\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=89

$$\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {47, 50, 54, 216}

$$-\frac{10x^{3/2}}{3b^2\sqrt{2-bx}}-\frac{5\sqrt{x}\sqrt{2-bx}}{b^3}+\frac{10\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}+\frac{2x^{5/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(5/2))/(3*b*(2 - b*x)^(3/2)) - (10*x^(3/2))/(3*b^2*Sqrt[2 - b*x]) - (5
*Sqrt[x]*Sqrt[2 - b*x])/b^3 + (10*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(7/2
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^{5/2}}{(2-bx)^{5/2}}\,dx = \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{5\int \frac{x^{3/2}}{(2-bx)^{3/2}}\,dx}{3b}$$

$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} + \frac{5\int \frac{\sqrt{x}}{\sqrt{2-bx}}\,dx}{b^2}$$

$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{5\int \frac{1}{\sqrt{x}\,\sqrt{2-bx}}\,dx}{b^3}$$

$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx, x, \sqrt{x}\right)}{b^3}$$

$$= \frac{2x^{5/2}}{3b(2-bx)^{3/2}} - \frac{10x^{3/2}}{3b^2\sqrt{2-bx}} - \frac{5\sqrt{x}\,\sqrt{2-bx}}{b^3} + \frac{10\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.01, size = 30, normalized size = 0.34

$$\frac{x^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{bx}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (b*x)/2])/(14*Sqrt[2])
```

IntegrateAlgebraic [A] time = 0.19, size = 89, normalized size = 1.00

$$\frac{10\sqrt{-b}\,\log\left(\sqrt{2-bx} - \sqrt{-b}\,\sqrt{x}\right)}{b^4} + \frac{\sqrt{2-bx}\,\left(-3b^2x^{5/2} + 40bx^{3/2} - 60\sqrt{x}\right)}{3b^3(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (Sqrt[2 - b*x]*(-60*Sqrt[x] + 40*b*x^(3/2) - 3*b^2*x^(5/2)))/(3*b^3*(-2 + b
*x)^2) + (10*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^4
```

fricas [A] time = 1.14, size = 187, normalized size = 2.10

$$\left[-\frac{15\left(b^2x^2 - 4bx + 4\right)\sqrt{-b}\,\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x} + 1\right) + \left(3b^3x^2 - 40b^2x + 60b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^6x^2 - 4b^5x + 4b^4\right)}, -\frac{30\left(b^2x^2 - 4bx + 4\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right) + \left(3b^3x^2 - 40b^2x + 60b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^6x^2 - 4b^5x + 4b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(15*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)
*sqrt(x) + 1) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)*sqrt(x))/(b^6*
x^2 - 4*b^5*x + 4*b^4), -1/3*(30*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(
-b*x + 2)/(sqrt(b)*sqrt(x))) + (3*b^3*x^2 - 40*b^2*x + 60*b)*sqrt(-b*x + 2)
*sqrt(x))/(b^6*x^2 - 4*b^5*x + 4*b^4)]
```

giac [B] time = 10.72, size = 200, normalized size = 2.25

$$\frac{\left(\frac{15\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^2\right)}{\sqrt{-b}\,b^2} - \frac{3\sqrt{(bx-2)b+2b}\,\sqrt{-bx+2}}{b^3} - \frac{16\left(9\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^4 - 24\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^2 b + 28b^2\right)}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b} - \sqrt{(bx-2)b+2b}\right)^2 - 2b\right)^3\sqrt{-b}\,b}\right)|b|}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(15*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/(sqrt(-b
)*b^2) - 3*sqrt((b*x - 2)*b + 2*b)*sqrt(-b*x + 2)/b^3 - 16*(9*(sqrt(-b*x +
2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4 - 24*(sqrt(-b*x + 2)*sqrt(-b) - sq
rt((b*x - 2)*b + 2*b))^2*b + 28*b^2)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x
 - 2)*b + 2*b))^2 - 2*b)^3*sqrt(-b)*b))*abs(b)/b^2
```

maple [B] time = 0.04, size = 168, normalized size = 1.89

$$\frac{\left(\frac{5\arctan\left(\frac{\left(x-\frac{1}{b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+2x}}\right)}{b^{\frac{7}{2}}}+\frac{28\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{3\left(x-\frac{2}{b}\right)b^{4}}+\frac{8\sqrt{-\left(x-\frac{2}{b}\right)^{2}b-2x+\frac{4}{b}}}{3\left(x-\frac{2}{b}\right)^{2}b^{5}}\right)\sqrt{\left(-bx+2\right)x}}{\sqrt{-bx+2}\,\sqrt{x}}+\frac{\left(bx-2\right)\sqrt{\left(-bx+2\right)x}\,\sqrt{x}}{\sqrt{-\left(bx-2\right)x}\,\sqrt{-bx+2}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/b^3*(b*x-2)*x^(1/2)/(-(b*x-2)*x)^(1/2)*((-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)+
(5/b^(7/2)*arctan((x-1/b)/(-b*x^2+2*x)^(1/2)*b^(1/2))+8/3/b^5/(x-2/b)^2*(-(
x-2/b)^2*b-2*x+4/b)^(1/2)+28/3/(x-2/b)*(-(x-2/b)^2*b-2*x+4/b)^(1/2)/b^4)*((
-b*x+2)*x)^(1/2)/(-b*x+2)^(1/2)/x^(1/2)
```

maxima [A] time = 2.99, size = 86, normalized size = 0.97

$$\frac{2\left(2\,b^{2}+\frac{10\,(bx-2)b}{x}-\frac{15\,(bx-2)^{2}}{x^{2}}\right)}{3\left(\frac{(-bx+2)^{\frac{3}{2}}b^{4}}{x^{\frac{3}{2}}}+\frac{(-bx+2)^{\frac{5}{2}}b^{3}}{x^{\frac{5}{2}}}\right)}-\frac{10\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)}{b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(2*b^2 + 10*(b*x - 2)*b/x - 15*(b*x - 2)^2/x^2)/((-b*x + 2)^(3/2)*b^4/x
^(3/2) + (-b*x + 2)^(5/2)*b^3/x^(5/2)) - 10*arctan(sqrt(-b*x + 2)/(sqrt(b)*
sqrt(x)))/b^(7/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(2-b\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(5/2)/(2 - b*x)^(5/2), x)
```

sympy [B] time = 6.80, size = 753, normalized size = 8.46

$$\begin{cases} -\frac{3ib^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{40ib^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{60ib^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30ib^{10}x^{\frac{27}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{15\pi b^{10}x^{\frac{27}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}+\frac{60ib^{9}x^{\frac{25}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}}-\frac{30\pi b^{9}x^{\frac{25}{2}}\sqrt{bx-2}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{bx-2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{bx-2}} & \text{for}\: \frac{\left|bx\right|}{2} > 1 \\ \frac{3b^{\frac{23}{2}}x^{15}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{40b^{\frac{21}{2}}x^{14}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{60b^{\frac{19}{2}}x^{13}}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}+\frac{30b^{10}x^{\frac{27}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}}-\frac{60b^{9}x^{\frac{25}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{27}{2}}x^{\frac{27}{2}}\sqrt{-bx+2}-6b^{\frac{25}{2}}x^{\frac{25}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-3*I*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b*
*(25/2)*x**(25/2)*sqrt(b*x - 2)) + 40*I*b**(21/2)*x**14/(3*b**(27/2)*x**(27
/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 60*I*b**(19/2)*x
**13/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x
- 2)) - 30*I*b**10*x**(27/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)
/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)
) + 15*pi*b**10*x**(27/2)*sqrt(b*x - 2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2
) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) + 60*I*b**9*x**(25/2)*sqrt(b*x - 2
)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6
*b**(25/2)*x**(25/2)*sqrt(b*x - 2)) - 30*pi*b**9*x**(25/2)*sqrt(b*x - 2)/(3
*b**(27/2)*x**(27/2)*sqrt(b*x - 2) - 6*b**(25/2)*x**(25/2)*sqrt(b*x - 2)),
Abs(b*x)/2 > 1), (3*b**(23/2)*x**15/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) -
 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) - 40*b**(21/2)*x**14/(3*b**(27/2)*x*
*(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2)) + 60*b**(19/
2)*x**13/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt
(-b*x + 2)) + 30*b**10*x**(27/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x
)/2)/(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*
x + 2)) - 60*b**9*x**(25/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(b)*sqrt(x)/2)/
(3*b**(27/2)*x**(27/2)*sqrt(-b*x + 2) - 6*b**(25/2)*x**(25/2)*sqrt(-b*x + 2
)), True))
```

3.642 $\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {47, 54, 216}

$$-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{2x^{3/2}}{3b(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (2*x^(3/2))/(3*b*(2 - b*x)^(3/2)) - (2*Sqrt[x])/(b^2*Sqrt[2 - b*x]) + (2*Ar
cSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{(2-bx)^{5/2}}\,dx &= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{\int \frac{\sqrt{x}}{(2-bx)^{3/2}}\,dx}{b}\\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{\int \frac{1}{\sqrt{x}\sqrt{2-bx}}\,dx}{b^2}\\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2-bx^2}}\,dx,x,\sqrt{x}\right)}{b^2}\\
&= \frac{2x^{3/2}}{3b(2-bx)^{3/2}}-\frac{2\sqrt{x}}{b^2\sqrt{2-bx}}+\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 53, normalized size = 0.79

$$\frac{2\sin^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{b^{5/2}}+\frac{4\sqrt{x}\,(2bx-3)}{3b^2(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[x]*(-3 + 2*b*x))/(3*b^2*(2 - b*x)^(3/2)) + (2*ArcSin[(Sqrt[b]*Sqrt[
x])/Sqrt[2]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.17, size = 79, normalized size = 1.18

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{2-bx}-\sqrt{-b}\sqrt{x}\right)}{b^3}+\frac{4\sqrt{2-bx}\,\left(2bx^{3/2}-3\sqrt{x}\right)}{3b^2(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(3/2)/(2 - b*x)^(5/2),x]
```

```
[Out] (4*Sqrt[2 - b*x]*(-3*Sqrt[x] + 2*b*x^(3/2)))/(3*b^2*(-2 + b*x)^2) + (2*Sqrt
[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[2 - b*x]])/b^3
```

fricas [A] time = 1.39, size = 173, normalized size = 2.58

$$\left[-\frac{3\left(b^2x^2-4\,bx+4\right)\sqrt{-b}\,\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-b}\,\sqrt{x}\,+1\right)-4\left(2\,b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^5x^2-4\,b^4x+4\,b^3\right)},-\frac{2\left(3\left(b^2x^2-4\,bx+4\right)\sqrt{b}\,\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\,\sqrt{x}}\right)-2\left(2\,b^2x-3\,b\right)\sqrt{-bx+2}\,\sqrt{x}\right)}{3\left(b^5x^2-4\,b^4x+4\,b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(-b)*log(-b*x + sqrt(-b*x + 2)*sqrt(-b)*
sqrt(x) + 1) - 4*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4*x
 + 4*b^3), -2/3*(3*(b^2*x^2 - 4*b*x + 4)*sqrt(b)*arctan(sqrt(-b*x + 2)/(sqr
t(b)*sqrt(x))) - 2*(2*b^2*x - 3*b)*sqrt(-b*x + 2)*sqrt(x))/(b^5*x^2 - 4*b^4
*x + 4*b^3)]
```

giac [B] time = 10.63, size = 178, normalized size = 2.66

$$\frac{\left(\frac{3\,\log\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\right)}{\sqrt{-b}}+\frac{16\left(3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}-6\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b+8\,\sqrt{-b}\,b^2\right)}{\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3}\right)|b|}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*log((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2)/sqrt(-b)
+ 16*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) - 6*
(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b + 8*sqrt(-
b)*b^2)/((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3)*ab
s(b)/b^3
```

maple [A] time = 0.04, size = 73, normalized size = 1.09

$$-\frac{4\left(-\frac{\sqrt{\pi}\,\sqrt{2}\,(-b)^{\frac{5}{2}}(-10bx+15)\sqrt{x}}{20\left(-\frac{bx}{2}+1\right)^{\frac{3}{2}}b^2}+\frac{3\sqrt{\pi}\,(-b)^{\frac{5}{2}}\arcsin\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{2b^{\frac{5}{2}}}\right)}{3\,(-b)^{\frac{3}{2}}\sqrt{\pi}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -4/3/(-b)^(3/2)/Pi^(1/2)/b*(-1/20*Pi^(1/2)*x^(1/2)*2^(1/2)*(-b)^(5/2)*(-10*
b*x+15)/b^2/(-1/2*b*x+1)^(3/2)+3/2*Pi^(1/2)*(-b)^(5/2)/b^(5/2)*arcsin(1/2*2
^(1/2)*b^(1/2)*x^(1/2)))
```

maxima [A] time = 2.92, size = 50, normalized size = 0.75

$$\frac{2\left(b+\frac{3(bx-2)}{x}\right)x^{\frac{3}{2}}}{3(-bx+2)^{\frac{3}{2}}b^{2}}-\frac{2\arctan\left(\frac{\sqrt{-bx+2}}{\sqrt{b}\sqrt{x}}\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b + 3*(b*x - 2)/x)*x^(3/2)/((-b*x + 2)^(3/2)*b^2) - 2*arctan(sqrt(-b*x
 + 2)/(sqrt(b)*sqrt(x)))/b^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{(2-b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(2 - b*x)^(5/2),x)
```

```
[Out] int(x^(3/2)/(2 - b*x)^(5/2), x)
```

sympy [B] time = 3.70, size = 649, normalized size = 9.69

$$\begin{cases} \frac{8ib^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{12ib^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6ib^{5}x^{\frac{15}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{3\pi b^{5}x^{\frac{15}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}+\frac{12ib^{4}x^{\frac{13}{2}}\sqrt{bx-2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}}-\frac{6\pi b^{4}x^{\frac{13}{2}}\sqrt{bx-2}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{bx-2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{bx-2}} & \text{for}\: \frac{|bx|}{2}>1 \\ -\frac{8b^{\frac{11}{2}}x^{8}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{12b^{\frac{9}{2}}x^{7}}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}+\frac{6b^{5}x^{\frac{15}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}}-\frac{12b^{4}x^{\frac{13}{2}}\sqrt{-bx+2}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{3b^{\frac{15}{2}}x^{\frac{15}{2}}\sqrt{-bx+2}-6b^{\frac{13}{2}}x^{\frac{13}{2}}\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((8*I*b**(11/2)*x**8/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(
13/2)*x**(13/2)*sqrt(b*x - 2)) - 12*I*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*
sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) - 6*I*b**5*x**(15/2)*s
qrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(b
*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)) + 3*pi*b**5*x**(15/2)*sqrt(b
*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b
*x - 2)) + 12*I*b**4*x**(13/2)*sqrt(b*x - 2)*acosh(sqrt(2)*sqrt(b)*sqrt(x)/
2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2) - 6*b**(13/2)*x**(13/2)*sqrt(b*x -
2)) - 6*pi*b**4*x**(13/2)*sqrt(b*x - 2)/(3*b**(15/2)*x**(15/2)*sqrt(b*x - 2
) - 6*b**(13/2)*x**(13/2)*sqrt(b*x - 2)), Abs(b*x)/2 > 1), (-8*b**(11/2)*x*
*8/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)*sqrt(-b*x
+ 2)) + 12*b**(9/2)*x**7/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2
)*x**(13/2)*sqrt(-b*x + 2)) + 6*b**5*x**(15/2)*sqrt(-b*x + 2)*asin(sqrt(2)*
sqrt(b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(
13/2)*sqrt(-b*x + 2)) - 12*b**4*x**(13/2)*sqrt(-b*x + 2)*asin(sqrt(2)*sqrt(
b)*sqrt(x)/2)/(3*b**(15/2)*x**(15/2)*sqrt(-b*x + 2) - 6*b**(13/2)*x**(13/2)
*sqrt(-b*x + 2)), True))
```

3.643 $\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=19

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {37}

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(2 - b*x)^(5/2),x]

[Out] x^(3/2)/(3*(2 - b*x)^(3/2))

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(2-bx)^{5/2}}\,dx = \frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{x^{3/2}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(2 - b*x)^(5/2),x]

[Out] x^(3/2)/(3*(2 - b*x)^(3/2))

IntegrateAlgebraic [A] time = 0.11, size = 26, normalized size = 1.37

$$\frac{x^{3/2}\sqrt{2-bx}}{3(bx-2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(2 - b*x)^(5/2),x]

[Out] (x^(3/2)*Sqrt[2 - b*x])/(3*(-2 + b*x)^2)

fricas [B] time = 1.03, size = 28, normalized size = 1.47

$$\frac{\sqrt{-bx+2}\,x^{\frac{3}{2}}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(-b*x + 2)*x^(3/2)/(b^2*x^2 - 4*b*x + 4)
```

giac [B] time = 1.25, size = 95, normalized size = 5.00

$$\frac{4\left(3\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{4}\sqrt{-b}+4\sqrt{-b}b^{2}\right)|b|}{3\left(\left(\sqrt{-bx+2}\sqrt{-b}-\sqrt{(bx-2)b+2b}\right)^{2}-2b\right)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b) + 4*s
qrt(-b)*b^2)*abs(b)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2
 - 2*b)^3*b^2)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$\frac{x^{\frac{3}{2}}}{3\left(-bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-b*x+2)^(5/2),x)
```

```
[Out] 1/3*x^(3/2)/(-b*x+2)^(3/2)
```

maxima [A] time = 1.38, size = 13, normalized size = 0.68

$$\frac{x^{\frac{3}{2}}}{3\left(-bx+2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^(3/2)/(-b*x + 2)^(3/2)
```

mupad [B] time = 0.23, size = 13, normalized size = 0.68

$$\frac{x^{3/2}}{3\left(2-b\,x\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(2 - b*x)^(5/2),x)
```

```
[Out] x^(3/2)/(3*(2 - b*x)^(3/2))
```

sympy [B] time = 1.46, size = 65, normalized size = 3.42

$$\begin{cases} \frac{ix^{\frac{3}{2}}}{3bx\sqrt{bx-2}-6\sqrt{bx-2}} & \text{for}\: \frac{|bx|}{2} > 1 \\ -\frac{x^{\frac{3}{2}}}{3bx\sqrt{-bx+2}-6\sqrt{-bx+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((I*x**(3/2)/(3*b*x*sqrt(b*x - 2) - 6*sqrt(b*x - 2)), Abs(b*x)/2 >
 1), (-x**(3/2)/(3*b*x*sqrt(-b*x + 2) - 6*sqrt(-b*x + 2)), True))
```

3.644 $\int \frac{1}{\sqrt{x}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$\frac{\sqrt{x}}{3\sqrt{2-bx}}+\frac{\sqrt{x}}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] Sqrt[x]/(3*(2 - b*x)^(3/2)) + Sqrt[x]/(3*Sqrt[2 - b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(2-bx)^{5/2}}\,dx &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{x}(2-bx)^{3/2}}\,dx\\ &= \frac{\sqrt{x}}{3(2-bx)^{3/2}}+\frac{\sqrt{x}}{3\sqrt{2-bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.62

$$-\frac{\sqrt{x}(bx-3)}{3(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(Sqrt[x]*(-3 + b*x))/(2 - b*x)^(3/2)
```

IntegrateAlgebraic [A] time = 0.10, size = 31, normalized size = 0.79

$$-\frac{\sqrt{x}\sqrt{2-bx}(bx-3)}{3(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(Sqrt[x]*Sqrt[2 - b*x]*(-3 + b*x))/(-2 + b*x)^2
```

fricas [A] time = 1.31, size = 33, normalized size = 0.85

$$-\frac{(bx-3)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^2x^2-4\,bx+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(b*x - 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^2 - 4*b*x + 4)
```

giac [B] time = 1.10, size = 90, normalized size = 2.31

$$\frac{8\left(3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)\sqrt{-b}\,b^2}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)*sqrt(-b
)*b^2/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(
b))
```

maple [A] time = 0.00, size = 19, normalized size = 0.49

$$-\frac{(bx-3)\sqrt{x}}{3\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(5/2)/x^(1/2),x)
```

```
[Out] -1/3*x^(1/2)*(b*x-3)/(-b*x+2)^(3/2)
```

maxima [A] time = 1.30, size = 25, normalized size = 0.64

$$\frac{\left(b-\frac{3\,(bx-2)}{x}\right)x^{\frac{3}{2}}}{6\,(-bx+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(5/2)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b - 3*(b*x - 2)/x)*x^(3/2)/(-b*x + 2)^(3/2)
```

mupad [B] time = 0.36, size = 45, normalized size = 1.15

$$\frac{3\,\sqrt{x}\,\sqrt{2-b\,x}-b\,x^{3/2}\,\sqrt{2-b\,x}}{3\,b^2\,x^2-12\,b\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*x^(1/2)*(2 - b*x)^(1/2) - b*x^(3/2)*(2 - b*x)^(1/2))/(3*b^2*x^2 - 12*b*x
 + 12)
```

sympy [C] time = 1.91, size = 177, normalized size = 4.54

$$\begin{cases} \frac{ibx}{3ib^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6i\sqrt{b}\sqrt{-1+\frac{2}{bx}}} - \frac{3i}{3ib^{\frac{3}{2}}x\sqrt{-1+\frac{2}{bx}}-6i\sqrt{b}\sqrt{-1+\frac{2}{bx}}} & \text{for } \frac{2}{|bx|} > 1 \\ \frac{b^2x}{3ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}} - \frac{3b}{3ib^{\frac{5}{2}}x\sqrt{1-\frac{2}{bx}}-6ib^{\frac{3}{2}}\sqrt{1-\frac{2}{bx}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(5/2)/x**(1/2),x)
```

```
[Out] Piecewise((I*b*x/(3*I*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*I*sqrt(b)*sqrt(-1 +
 2/(b*x))) - 3*I/(3*I*b**(3/2)*x*sqrt(-1 + 2/(b*x)) - 6*I*sqrt(b)*sqrt(-1 +
 2/(b*x))), 2/Abs(b*x) > 1), (b**2*x/(3*I*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*
I*b**(3/2)*sqrt(1 - 2/(b*x))) - 3*b/(3*I*b**(5/2)*x*sqrt(1 - 2/(b*x)) - 6*I
*b**(3/2)*sqrt(1 - 2/(b*x))), True))
```

3.645 $\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3\sqrt{x}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*Sqrt[x]*(2 - b*x)^(3/2)) + 2/(3*Sqrt[x]*Sqrt[2 - b*x]) - (2*Sqrt[2 - b
*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3}\int \frac{1}{x^{3/2}(2-bx)^{3/2}}\,dx\\ &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}+\frac{2}{3}\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\ &= \frac{1}{3\sqrt{x}\,(2-bx)^{3/2}}+\frac{2}{3\sqrt{x}\,\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.57

$$-\frac{2b^2x^2-6bx+3}{3\sqrt{x}\,(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(3 - 6*b*x + 2*b^2*x^2)/(Sqrt[x]*(2 - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 40, normalized size = 0.69

$$\frac{\sqrt{2-bx}\left(-2b^2x^2+6bx-3\right)}{3\sqrt{x}\,(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(3/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] (Sqrt[2 - b*x]*(-3 + 6*b*x - 2*b^2*x^2))/(3*Sqrt[x]*(-2 + b*x)^2)
```

fricas [A] time = 1.18, size = 46, normalized size = 0.79

$$-\frac{\left(2\,b^2x^2-6\,bx+3\right)\sqrt{-bx+2}\,\sqrt{x}}{3\,\left(b^2x^3-4\,bx^2+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^2 - 6*b*x + 3)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^3 - 4*b*x^2 + 4*
x)
```

giac [B] time = 1.12, size = 170, normalized size = 2.93

$$-\frac{\sqrt{-bx+2}\,b^2}{4\,\sqrt{(bx-2)b+2\,b}\,|b|}-\frac{3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}\,b^2-24\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b^3+20\,\sqrt{-b}\,b^4}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(-b*x + 2)*b^2/(sqrt((b*x - 2)*b + 2*b)*abs(b)) - 1/3*(3*(sqrt(-b*
x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*sqrt(-b)*b^2 - 24*(sqrt(-b*x +
 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqrt(-b)*b^3 + 20*sqrt(-b)*b^4)/(
((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2 - 2*b)^3*abs(b))
```

maple [A] time = 0.00, size = 28, normalized size = 0.48

$$-\frac{2b^2x^2-6bx+3}{3\,(-bx+2)^{\frac{3}{2}}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^2*x^2-6*b*x+3)/x^(1/2)/(-b*x+2)^(3/2)
```

maxima [A] time = 1.35, size = 42, normalized size = 0.72

$$\frac{\left(b^2-\frac{6\,(bx-2)b}{x}\right)x^{\frac{3}{2}}}{12\,(-bx+2)^{\frac{3}{2}}}-\frac{\sqrt{-bx+2}}{4\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2 - 6*(b*x - 2)*b/x)*x^(3/2)/(-b*x + 2)^(3/2) - 1/4*sqrt(-b*x + 2)/
sqrt(x)
```

mupad [B] time = 0.37, size = 59, normalized size = 1.02

$$\frac{3\sqrt{2-b\,x}-6\,b\,x\,\sqrt{2-b\,x}+2\,b^2\,x^2\,\sqrt{2-b\,x}}{\sqrt{x}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] (3*(2 - b*x)^(1/2) - 6*b*x*(2 - b*x)^(1/2) + 2*b^2*x^2*(2 - b*x)^(1/2))/(x^
(1/2)*(x*(12*b - 3*b^2*x) - 12))
```

sympy [B] time = 4.00, size = 243, normalized size = 4.19

$$\begin{cases} -\frac{2b^{\frac{13}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4}+\frac{6b^{\frac{11}{2}}x\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4}-\frac{3b^{\frac{9}{2}}\sqrt{-1+\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{for } \frac{2}{|bx|}>1 \\ -\frac{2ib^{\frac{13}{2}}x^2\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4}+\frac{6ib^{\frac{11}{2}}x\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4}-\frac{3ib^{\frac{9}{2}}\sqrt{1-\frac{2}{bx}}}{3b^6x^2-12b^5x+12b^4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((-2*b**(13/2)*x**2*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x +
12*b**4) + 6*b**(11/2)*x*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b
**4) - 3*b**(9/2)*sqrt(-1 + 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12*b**4), 2
/Abs(b*x) > 1), (-2*I*b**(13/2)*x**2*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b*
*5*x + 12*b**4) + 6*I*b**(11/2)*x*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*
x + 12*b**4) - 3*I*b**(9/2)*sqrt(1 - 2/(b*x))/(3*b**6*x**2 - 12*b**5*x + 12
*b**4), True))
```

3.646 $\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Rubi [A] time = 0.01, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {45, 37}

$$-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+\frac{1}{3x^{3/2}(2-bx)^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] 1/(3*x^(3/2)*(2 - b*x)^(3/2)) + 1/(x^(3/2)*Sqrt[2 - b*x]) - (2*Sqrt[2 - b*x
])/(3*x^(3/2)) - (2*b*Sqrt[2 - b*x])/(3*Sqrt[x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}(2-bx)^{5/2}}\,dx &= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\int \frac{1}{x^{5/2}(2-bx)^{3/2}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}+2\int \frac{1}{x^{5/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}+\frac{1}{3}(2b)\int \frac{1}{x^{3/2}\sqrt{2-bx}}\,dx\\
&= \frac{1}{3x^{3/2}(2-bx)^{3/2}}+\frac{1}{x^{3/2}\sqrt{2-bx}}-\frac{2\sqrt{2-bx}}{3x^{3/2}}-\frac{2b\sqrt{2-bx}}{3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.55

$$-\frac{2b^3x^3-6b^2x^2+3bx+1}{3x^{3/2}(2-bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] -1/3*(1 + 3*b*x - 6*b^2*x^2 + 2*b^3*x^3)/(x^(3/2)*(2 - b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 48, normalized size = 0.64

$$\frac{\sqrt{2-bx}\left(-2b^3x^3+6b^2x^2-3bx-1\right)}{3x^{3/2}(bx-2)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/2)*(2 - b*x)^(5/2)),x]
```

```
[Out] (Sqrt[2 - b*x]*(-1 - 3*b*x + 6*b^2*x^2 - 2*b^3*x^3))/(3*x^(3/2)*(-2 + b*x)^
2)
```

fricas [A] time = 1.10, size = 56, normalized size = 0.75

$$-\frac{\left(2\,b^3x^3-6\,b^2x^2+3\,bx+1\right)\sqrt{-bx+2}\,\sqrt{x}}{3\left(b^2x^4-4\,bx^3+4\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^3*x^3 - 6*b^2*x^2 + 3*b*x + 1)*sqrt(-b*x + 2)*sqrt(x)/(b^2*x^4 -
4*b*x^3 + 4*x^2)
```

giac [B] time = 1.25, size = 183, normalized size = 2.44

$$-\frac{\left(4\,(bx-2)b^2|b|+9\,b^2|b|\right)\sqrt{-bx+2}}{12\,((bx-2)b+2\,b)^{\frac{3}{2}}}-\frac{3\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^4\sqrt{-b}\,b^3-18\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2\sqrt{-b}\,b^4+16\,\sqrt{-b}\,b^5}{3\left(\left(\sqrt{-bx+2}\,\sqrt{-b}-\sqrt{(bx-2)b+2\,b}\right)^2-2\,b\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(4*(b*x - 2)*b^2*abs(b) + 9*b^2*abs(b))*sqrt(-b*x + 2)/((b*x - 2)*b +
 2*b)^(3/2) - 1/3*(3*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^4*
sqrt(-b)*b^3 - 18*(sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b + 2*b))^2*sqr
t(-b)*b^4 + 16*sqrt(-b)*b^5)/(((sqrt(-b*x + 2)*sqrt(-b) - sqrt((b*x - 2)*b
+ 2*b))^2 - 2*b)^3*abs(b))
```

maple [A] time = 0.00, size = 36, normalized size = 0.48

$$-\frac{2b^3x^3-6b^2x^2+3bx+1}{3\,(-bx+2)^{\frac{3}{2}}\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(-b*x+2)^(5/2),x)
```

```
[Out] -1/3*(2*b^3*x^3-6*b^2*x^2+3*b*x+1)/x^(3/2)/(-b*x+2)^(3/2)
```

maxima [A] time = 1.35, size = 58, normalized size = 0.77

$$-\frac{3\,\sqrt{-bx+2}\,b}{8\,\sqrt{x}}+\frac{\left(b^3-\frac{9\,(bx-2)b^2}{x}\right)x^{\frac{3}{2}}}{24\,(-bx+2)^{\frac{3}{2}}}-\frac{(-bx+2)^{\frac{3}{2}}}{24\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(-b*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*sqrt(-b*x + 2)*b/sqrt(x) + 1/24*(b^3 - 9*(b*x - 2)*b^2/x)*x^(3/2)/(-b*
x + 2)^(3/2) - 1/24*(-b*x + 2)^(3/2)/x^(3/2)
```

mupad [B] time = 0.44, size = 73, normalized size = 0.97

$$\frac{\sqrt{2-b\,x}+3\,b\,x\,\sqrt{2-b\,x}-6\,b^2\,x^2\,\sqrt{2-b\,x}+2\,b^3\,x^3\,\sqrt{2-b\,x}}{x^{3/2}\,\left(x\,\left(12\,b-3\,b^2\,x\right)-12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(2 - b*x)^(5/2)),x)
```

```
[Out] ((2 - b*x)^(1/2) + 3*b*x*(2 - b*x)^(1/2) - 6*b^2*x^2*(2 - b*x)^(1/2) + 2*b^
3*x^3*(2 - b*x)^(1/2))/(x^(3/2)*(x*(12*b - 3*b^2*x) - 12))
```

sympy [B] time = 12.40, size = 529, normalized size = 7.05

$$\begin{cases} \frac{2b^{\frac{27}{2}}x^4\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{10b^{\frac{25}{2}}x^3\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} + \frac{15b^{\frac{23}{2}}x^2\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{5b^{\frac{21}{2}}x\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{2b^{\frac{19}{2}}\sqrt{-1+\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} & \text{for } \frac{2}{|bx|}>1 \\ \frac{2ib^{\frac{27}{2}}x^4\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{10ib^{\frac{25}{2}}x^3\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} + \frac{15ib^{\frac{23}{2}}x^2\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{5ib^{\frac{21}{2}}x\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} - \frac{2ib^{\frac{19}{2}}\sqrt{1-\frac{2}{bx}}}{-3b^{12}x^4+18b^{11}x^3-36b^{10}x^2+24b^9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(-b*x+2)**(5/2),x)
```

```
[Out] Piecewise((2*b**(27/2)*x**4*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x*
*3 - 36*b**10*x**2 + 24*b**9*x) - 10*b**(25/2)*x**3*sqrt(-1 + 2/(b*x))/(-3*
b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x) + 15*b**(23/2)*x**2
*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**
9*x) - 5*b**(21/2)*x*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36
*b**10*x**2 + 24*b**9*x) - 2*b**(19/2)*sqrt(-1 + 2/(b*x))/(-3*b**12*x**4 +
18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x), 2/Abs(b*x) > 1), (2*I*b**(27/2)
*x**4*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24
*b**9*x) - 10*I*b**(25/2)*x**3*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*
x**3 - 36*b**10*x**2 + 24*b**9*x) + 15*I*b**(23/2)*x**2*sqrt(1 - 2/(b*x))/(
-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**9*x) - 5*I*b**(21/2)*
x*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*b**10*x**2 + 24*b**
9*x) - 2*I*b**(19/2)*sqrt(1 - 2/(b*x))/(-3*b**12*x**4 + 18*b**11*x**3 - 36*
b**10*x**2 + 24*b**9*x), True))
```

3.647 $\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=27

$$-\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {50, 53, 619, 216}

$$-\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[1 - x],x]
```

```
[Out] -(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{x}+\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,1-2x\right)\\ &= -\sqrt{1-x}\,\sqrt{x}-\frac{1}{2}\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.93

$$-\sqrt{-((x-1)x)}-\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[1 - x],x]
[Out] -Sqrt[-((-1 + x)*x)] - ArcSin[Sqrt[1 - x]]
```

IntegrateAlgebraic [A] time = 0.08, size = 39, normalized size = 1.44

$$2\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-\sqrt{1-x}\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[x]) + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]
```

fricas [A] time = 1.52, size = 27, normalized size = 1.00

$$-\sqrt{x}\sqrt{-x+1}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x)*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))
```

giac [A] time = 1.17, size = 17, normalized size = 0.63

$$-\sqrt{x}\sqrt{-x+1}+\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x)*sqrt(-x + 1) + arcsin(sqrt(x))
```

maple [A] time = 0.01, size = 41, normalized size = 1.52

$$-\sqrt{-x+1}\,\sqrt{x}+\frac{\sqrt{(-x+1)\,x}\,\arcsin(2x-1)}{2\sqrt{-x+1}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1-x)^(1/2),x)
[Out] -(1-x)^(1/2)*x^(1/2)+1/2*(x*(1-x))^(1/2)/x^(1/2)/(1-x)^(1/2)*arcsin(-1+2*x)
```

maxima [A] time = 3.00, size = 37, normalized size = 1.37

$$\frac{\sqrt{-x+1}}{\sqrt{x}\left(\frac{x-1}{x}-1\right)}-\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
```

[Out] sqrt(-x + 1)/(sqrt(x)*((x - 1)/x - 1)) - arctan(sqrt(-x + 1)/sqrt(x))

mupad [B] time = 0.57, size = 31, normalized size = 1.15

$$2\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)-\sqrt{x}\,\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(1 - x)^(1/2),x)

[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - x^(1/2)*(1 - x)^(1/2)

sympy [A] time = 1.65, size = 54, normalized size = 2.00

$$\begin{cases} -i\sqrt{x}\,\sqrt{x-1}-i\,\mathrm{acosh}\left(\sqrt{x}\right) & \text{for } |x|>1 \\ \frac{x^{\frac{3}{2}}}{\sqrt{1-x}}-\frac{\sqrt{x}}{\sqrt{1-x}}+\mathrm{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(1-x)**(1/2),x)

[Out] Piecewise((-I*sqrt(x)*sqrt(x - 1) - I*acosh(sqrt(x)), Abs(x) > 1), (x**(3/2
)/sqrt(1 - x) - sqrt(x)/sqrt(1 - x) + asin(sqrt(x)), True))

3.648 $\int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] -ArcSin[1 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-x}\,\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{x-x^2}}\,dx \\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 1-2x\right) \\ &= -\sin^{-1}(1-2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] -2*ArcSin[Sqrt[1 - x]]
```

IntegrateAlgebraic [A] time = 0.04, size = 8, normalized size = 1.00

$$2\sin^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*Sqrt[x]),x]
[Out] 2*ArcSin[Sqrt[x]]
```

fricas [B] time = 1.26, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

giac [A] time = 1.09, size = 6, normalized size = 0.75

$$2\arcsin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*arcsin(sqrt(x))
```

maple [B] time = 0.00, size = 27, normalized size = 3.38

$$\frac{\sqrt{(-x+1)\,x}\ \arcsin(2x-1)}{\sqrt{-x+1}\ \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/x^(1/2),x)
[Out] ((-x+1)*x)^(1/2)/(-x+1)^(1/2)/x^(1/2)*arcsin(2*x-1)
```

maxima [B] time = 2.95, size = 14, normalized size = 1.75

$$-2\arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 1)/sqrt(x))
```

mupad [B] time = 0.05, size = 16, normalized size = 2.00

$$-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - x)^(1/2)),x)
[Out] -4*atan(((1 - x)^(1/2) - 1)/x^(1/2))
```

sympy [A] time = 0.97, size = 20, normalized size = 2.50

$$\begin{cases} -2i\,\mathrm{acosh}\left(\sqrt{x}\right) & \text{for}\ |x|>1 \\ 2\,\mathrm{asin}\left(\sqrt{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)), Abs(x) > 1), (2*asin(sqrt(x)), True))
```

3.649 $\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {54, 216}

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{1-bx}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^2}}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\sin^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*ArcSin[Sqrt[b]*Sqrt[x]])/Sqrt[b]
```

IntegrateAlgebraic [A] time = 0.05, size = 38, normalized size = 2.00

$$\frac{2\sqrt{-b}\,\log\left(\sqrt{1-bx}-\sqrt{-b}\sqrt{x}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[1 - b*x]),x]
```

```
[Out] (2*Sqrt[-b]*Log[-(Sqrt[-b]*Sqrt[x]) + Sqrt[1 - b*x]])/b
```

fricas [A] time = 1.16, size = 57, normalized size = 3.00

$$\left[-\frac{\sqrt{-b}\log\left(-2\,bx+2\,\sqrt{-bx+1}\,\sqrt{-b}\,\sqrt{x}\,+1\right)}{b},-\frac{2\,\arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-2*b*x + 2*sqrt(-b*x + 1)*sqrt(-b)*sqrt(x) + 1)/b, -2*arctan
(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,1]%%%}+
%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%%%{-4,[
1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0,1]%%%}
+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[2,0]%%%
}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{
-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{1,[4,4]
%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%%}+%%%{
4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,[3,0]%%
%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%}+%%%{6
,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[1,1]%%%
}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%%%{-4,[
0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-15.6438432182,61.793747834
9]Warning, choosing root of [1,0,%%%{4,[1,1]%%%}+%%%{4,[1,0]%%%}+%%%{-4,[0,
1]%%%}+%%%{-4,[0,0]%%%},0,%%%{6,[2,2]%%%}+%%%{4,[2,1]%%%}+%%%{6,[2,0]%%%}+%
%%{-4,[1,2]%%%}+%%%{-16,[1,1]%%%}+%%%{-4,[1,0]%%%}+%%%{6,[0,2]%%%}+%%%{4,[0
,1]%%%}+%%%{6,[0,0]%%%},0,%%%{4,[3,3]%%%}+%%%{-4,[3,2]%%%}+%%%{-4,[3,1]%%%}
+%%%{4,[3,0]%%%}+%%%{4,[2,3]%%%}+%%%{-52,[2,2]%%%}+%%%{12,[2,1]%%%}+%%%{4,[
2,0]%%%}+%%%{-4,[1,3]%%%}+%%%{-12,[1,2]%%%}+%%%{52,[1,1]%%%}+%%%{-4,[1,0]%%
%}+%%%{-4,[0,3]%%%}+%%%{4,[0,2]%%%}+%%%{4,[0,1]%%%}+%%%{-4,[0,0]%%%},0,%%%{
1,[4,4]%%%}+%%%{-4,[4,3]%%%}+%%%{6,[4,2]%%%}+%%%{-4,[4,1]%%%}+%%%{1,[4,0]%%
%}+%%%{4,[3,4]%%%}+%%%{-8,[3,3]%%%}+%%%{8,[3,2]%%%}+%%%{-8,[3,1]%%%}+%%%{4,
[3,0]%%%}+%%%{6,[2,4]%%%}+%%%{-8,[2,3]%%%}+%%%{20,[2,2]%%%}+%%%{-8,[2,1]%%%
}+%%%{6,[2,0]%%%}+%%%{4,[1,4]%%%}+%%%{-8,[1,3]%%%}+%%%{8,[1,2]%%%}+%%%{-8,[
1,1]%%%}+%%%{4,[1,0]%%%}+%%%{1,[0,4]%%%}+%%%{-4,[0,3]%%%}+%%%{6,[0,2]%%%}+%
%%{-4,[0,1]%%%}+%%%{1,[0,0]%%%}] at parameters values [-29.292030761,78.649
3344628]2/abs(b)*b^2/b/sqrt(-b)*ln(abs(sqrt(-b*(-b*x+1)+b)-sqrt(-b)*sqrt(-b
*x+1)))
```

maple [B] time = 0.01, size = 48, normalized size = 2.53

$$\frac{\sqrt{(-bx+1)\,x}\;\arctan\left(\frac{\left(x-\frac{1}{2b}\right)\sqrt{b}}{\sqrt{-b\,x^{2}+x}}\right)}{\sqrt{-bx+1}\,\sqrt{b}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-b*x+1)^(1/2),x)
```

```
[Out] (x*(-b*x+1))^(1/2)/x^(1/2)/(-b*x+1)^(1/2)/b^(1/2)*arctan(b^(1/2)*(x-1/2/b)/
(-b*x^2+x)^(1/2))
```

maxima [A] time = 2.89, size = 21, normalized size = 1.11

$$-\frac{2 \arctan\left(\frac{\sqrt{-bx+1}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-b*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*arctan(sqrt(-b*x + 1)/(sqrt(b)*sqrt(x)))/sqrt(b)
```

mupad [B] time = 0.13, size = 23, normalized size = 1.21

$$-\frac{4 \operatorname{atan}\left(\frac{\sqrt{1-b\,x}-1}{\sqrt{b}\,\sqrt{x}}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(1 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan(((1 - b*x)^(1/2) - 1)/(b^(1/2)*x^(1/2))))/b^(1/2)
```

sympy [A] time = 1.06, size = 42, normalized size = 2.21

$$\begin{cases} -\frac{2i \operatorname{acosh}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{for } |bx| > 1 \\ \frac{2 \operatorname{asin}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-b*x+1)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(b)*sqrt(x))/sqrt(b), Abs(b*x) > 1), (2*asin(sqrt
(b)*sqrt(x))/sqrt(b), True))
```

3.650 $\int x^{5/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x),x]
[Out] (3*a*x^(8/3))/8 + (3*b*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)\,dx&=\int\left(ax^{5/3}+bx^{8/3}\right)\,dx\\&=\frac{3}{8}ax^{8/3}+\frac{3}{11}bx^{11/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.81

$$\frac{3}{88}x^{8/3}(11a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x),x]
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{88}\left(11ax^{8/3}+8bx^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x),x]
[Out] (3*(11*a*x^(8/3) + 8*b*x^(11/3)))/88
```

fricas [A] time = 1.25, size = 18, normalized size = 0.86

$$\frac{3}{88}\left(8\,bx^{3}+11\,ax^{2}\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/88*(8*b*x^3 + 11*a*x^2)*x^(2/3)
```

giac [A] time = 0.98, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="giac")
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(8bx+11a)\,x^{\frac{8}{3}}}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a),x)
[Out] 3/88*x^(8/3)*(8*b*x+11*a)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{3}{11}\,bx^{\frac{11}{3}}+\frac{3}{8}\,ax^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/11*b*x^(11/3) + 3/8*a*x^(8/3)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.62

$$\frac{3\,x^{8/3}\,\left(11\,a+8\,b\,x\right)}{88}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x),x)
[Out] (3*x^(8/3)*(11*a + 8*b*x))/88
```

sympy [A] time = 2.01, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{8}{3}}}{8}+\frac{3bx^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a),x)
[Out] 3*a*x**(8/3)/8 + 3*b*x**(11/3)/11
```

3.651 $\int x^{4/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(7/3))/7 + (3*b*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)\,dx &= \int \left(ax^{4/3}+bx^{7/3}\right)\,dx\\ &= \frac{3}{7}ax^{7/3}+\frac{3}{10}bx^{10/3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{70}x^{7/3}(10a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{70}\left(10ax^{7/3}+7bx^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x),x]
```

```
[Out] (3*(10*a*x^(7/3) + 7*b*x^(10/3)))/70
```

fricas [A] time = 1.24, size = 18, normalized size = 0.86

$$\frac{3}{70}\left(7\,bx^{3}+10\,ax^{2}\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/70*(7*b*x^3 + 10*a*x^2)*x^(1/3)
```

giac [A] time = 1.04, size = 13, normalized size = 0.62

$$\frac{3}{10}bx^{\frac{10}{3}}+\frac{3}{7}ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="giac")
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\left(7bx+10a\right)x^{\frac{7}{3}}}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a),x)
[Out] 3/70*x^(7/3)*(7*b*x+10*a)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.62

$$\frac{3}{10}bx^{\frac{10}{3}}+\frac{3}{7}ax^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/10*b*x^(10/3) + 3/7*a*x^(7/3)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{7/3}\,\left(10\,a+7\,b\,x\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x),x)
[Out] (3*x^(7/3)*(10*a + 7*b*x))/70
```

sympy [A] time = 1.30, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{7}{3}}}{7}+\frac{3bx^{\frac{10}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a),x)
[Out] 3*a*x**(7/3)/7 + 3*b*x**(10/3)/10
```

3.652 $\int x^{2/3}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(5/3))/5 + (3*b*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)\,dx &= \int\left(ax^{2/3}+bx^{5/3}\right)\,dx\\ &= \frac{3}{5}ax^{5/3}+\frac{3}{8}bx^{8/3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{40}x^{5/3}(8a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{40}\left(8ax^{5/3}+5bx^{8/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x),x]
```

```
[Out] (3*(8*a*x^(5/3) + 5*b*x^(8/3)))/40
```

fricas [A] time = 1.24, size = 16, normalized size = 0.76

$$\frac{3}{40}\left(5\,bx^{2}+8\,ax\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/40*(5*b*x^2 + 8*a*x)*x^(2/3)
```

giac [A] time = 0.99, size = 13, normalized size = 0.62

$$\frac{3}{8} b x^{\frac{8}{3}} + \frac{3}{5} a x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="giac")
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3 (5 b x + 8 a) x^{\frac{5}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a),x)
[Out] 3/40*x^(5/3)*(5*b*x+8*a)
```

maxima [A] time = 1.35, size = 13, normalized size = 0.62

$$\frac{3}{8} b x^{\frac{8}{3}} + \frac{3}{5} a x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/8*b*x^(8/3) + 3/5*a*x^(5/3)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.62

$$\frac{3 x^{5/3} (8 a + 5 b x)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x),x)
[Out] (3*x^(5/3)*(8*a + 5*b*x))/40
```

sympy [A] time = 0.45, size = 19, normalized size = 0.90

$$\frac{3 a x^{\frac{5}{3}}}{5} + \frac{3 b x^{\frac{8}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a),x)
[Out] 3*a*x**(5/3)/5 + 3*b*x**(8/3)/8
```

3.653 $\int \sqrt[3]{x}(a+bx)\,dx$

Optimal. Leaf size=21

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*a*x^(4/3))/4 + (3*b*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)\,dx &= \int \left(a\sqrt[3]{x}+bx^{4/3}\right)\,dx\\ &= \frac{3}{4}ax^{4/3}+\frac{3}{7}bx^{7/3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{28}x^{4/3}(7a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{28}\left(7ax^{4/3}+4bx^{7/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x),x]
```

```
[Out] (3*(7*a*x^(4/3) + 4*b*x^(7/3)))/28
```

fricas [A] time = 1.17, size = 16, normalized size = 0.76

$$\frac{3}{28}\left(4\,bx^2+7\,ax\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="fricas")
[Out] 3/28*(4*b*x^2 + 7*a*x)*x^(1/3)
```

giac [A] time = 1.03, size = 13, normalized size = 0.62

$$\frac{3}{7}bx^{\frac{7}{3}} + \frac{3}{4}ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="giac")
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(4bx + 7a)\,x^{\frac{4}{3}}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a),x)
[Out] 3/28*x^(4/3)*(4*b*x+7*a)
```

maxima [A] time = 1.37, size = 13, normalized size = 0.62

$$\frac{3}{7}bx^{\frac{7}{3}} + \frac{3}{4}ax^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a),x, algorithm="maxima")
[Out] 3/7*b*x^(7/3) + 3/4*a*x^(4/3)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{4/3}\,\left(7\,a+4\,b\,x\right)}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x),x)
[Out] (3*x^(4/3)*(7*a + 4*b*x))/28
```

sympy [A] time = 1.52, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{4}{3}}}{4} + \frac{3bx^{\frac{7}{3}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a),x)
[Out] 3*a*x**(4/3)/4 + 3*b*x**(7/3)/7
```

3.654 $\int \frac{a+bx}{\sqrt[3]{x}} \, dx$

Optimal. Leaf size=21

$$\frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$\frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*a*x^(2/3))/2 + (3*b*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\sqrt[3]{x}} \, dx &= \int \left(\frac{a}{\sqrt[3]{x}} + bx^{2/3}\right) dx \\ &= \frac{3}{2}ax^{2/3} + \frac{3}{5}bx^{5/3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 0.81

$$\frac{3}{10}x^{2/3}(5a + 2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

IntegrateAlgebraic [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{3}{10}\left(5ax^{2/3} + 2bx^{5/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(1/3),x]
```

```
[Out] (3*(5*a*x^(2/3) + 2*b*x^(5/3)))/10
```

fricas [A] time = 1.30, size = 13, normalized size = 0.62

$$\frac{3}{10}\,(2\,bx + 5\,a)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(2*b*x + 5*a)*x^(2/3)
```

giac [A] time = 1.12, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}} + \frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{3\,(2bx+5a)\,x^{\frac{2}{3}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(1/3),x)
```

```
[Out] 3/10*x^(2/3)*(2*b*x+5*a)
```

maxima [A] time = 1.31, size = 13, normalized size = 0.62

$$\frac{3}{5}\,bx^{\frac{5}{3}} + \frac{3}{2}\,ax^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/5*b*x^(5/3) + 3/2*a*x^(2/3)
```

mupad [B] time = 0.02, size = 13, normalized size = 0.62

$$\frac{3\,x^{2/3}\,\left(5\,a+2\,b\,x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(5*a + 2*b*x))/10
```

sympy [A] time = 1.66, size = 19, normalized size = 0.90

$$\frac{3ax^{\frac{2}{3}}}{2} + \frac{3bx^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(1/3),x)
```

```
[Out] 3*a*x**(2/3)/2 + 3*b*x**(5/3)/5
```

3.655 $\int \frac{a+bx}{x^{2/3}}\,dx$

Optimal. Leaf size=19

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(2/3),x]
```

```
[Out] 3*a*x^(1/3) + (3*b*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{2/3}}\,dx &= \int \left(\frac{a}{x^{2/3}}+b\sqrt[3]{x}\right)dx\\ &= 3a\sqrt[3]{x}+\frac{3}{4}bx^{4/3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.84

$$\frac{3}{4}\sqrt[3]{x}\,(4a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

IntegrateAlgebraic [A] time = 0.01, size = 20, normalized size = 1.05

$$\frac{3}{4}\left(4a\sqrt[3]{x}+bx^{4/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(2/3),x]
```

```
[Out] (3*(4*a*x^(1/3) + b*x^(4/3)))/4
```

fricas [A] time = 1.13, size = 12, normalized size = 0.63

$$\frac{3}{4}\,(bx+4\,a)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="fricas")
[Out] 3/4*(b*x + 4*a)*x^(1/3)
```

giac [A] time = 1.12, size = 13, normalized size = 0.68

$$\frac{3}{4}\,bx^{\frac{4}{3}} + 3\,ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="giac")
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

maple [A] time = 0.00, size = 13, normalized size = 0.68

$$\frac{3\,(bx + 4a)\,x^{\frac{1}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(2/3),x)
[Out] 3/4*x^(1/3)*(b*x+4*a)
```

maxima [A] time = 1.28, size = 13, normalized size = 0.68

$$\frac{3}{4}\,bx^{\frac{4}{3}} + 3\,ax^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(2/3),x, algorithm="maxima")
[Out] 3/4*b*x^(4/3) + 3*a*x^(1/3)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.63

$$\frac{3\,x^{1/3}\,\left(4\,a + b\,x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(2/3),x)
[Out] (3*x^(1/3)*(4*a + b*x))/4
```

sympy [A] time = 1.49, size = 17, normalized size = 0.89

$$3a\sqrt[3]{x} + \frac{3bx^{\frac{4}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(2/3),x)
[Out] 3*a*x**(1/3) + 3*b*x**(4/3)/4
```

3.656 $\int \frac{a+bx}{x^{4/3}}\,dx$

Optimal. Leaf size=19

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3}{2}bx^{2/3}-\frac{3a}{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(4/3),x]
```

```
[Out] (-3*a)/x^(1/3) + (3*b*x^(2/3))/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{4/3}}\,dx &= \int \left(\frac{a}{x^{4/3}}+\frac{b}{\sqrt[3]{x}}\right)dx\\ &= -\frac{3a}{\sqrt[3]{x}}+\frac{3}{2}bx^{2/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{3(bx-2a)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(4/3),x]
```

```
[Out] (3*(-2*a + b*x))/(2*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 0.84

$$\frac{3(bx-2a)}{2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(4/3),x]
```

```
[Out] (3*(-2*a + b*x))/(2*x^(1/3))
```

fricas [A] time = 1.00, size = 12, normalized size = 0.63

$$\frac{3\,(bx-2\,a)}{2\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="fricas")
[Out] 3/2*(b*x - 2*a)/x^(1/3)
```

giac [A] time = 1.13, size = 13, normalized size = 0.68

$$\frac{3}{2}bx^{\frac{2}{3}} - \frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="giac")
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{3(-bx + 2a)}{2x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(4/3),x)
[Out] -3/2*(-b*x+2*a)/x^(1/3)
```

maxima [A] time = 1.34, size = 13, normalized size = 0.68

$$\frac{3}{2}bx^{\frac{2}{3}} - \frac{3a}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(4/3),x, algorithm="maxima")
[Out] 3/2*b*x^(2/3) - 3*a/x^(1/3)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{6a - 3bx}{2x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(4/3),x)
[Out] -(6*a - 3*b*x)/(2*x^(1/3))
```

sympy [A] time = 0.39, size = 17, normalized size = 0.89

$$-\frac{3a}{\sqrt[3]{x}} + \frac{3bx^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(4/3),x)
[Out] -3*a/x**(1/3) + 3*b*x**(2/3)/2
```

3.657 $\int \frac{a+bx}{x^{5/3}} \, dx$

Optimal. Leaf size=19

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {43}

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/x^(5/3),x]
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^{5/3}} \, dx &= \int \left(\frac{a}{x^{5/3}} + \frac{b}{x^{2/3}}\right) dx \\ &= -\frac{3a}{2x^{2/3}} + 3b\sqrt[3]{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$3b\sqrt[3]{x} - \frac{3a}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/x^(5/3),x]
[Out] (-3*a)/(2*x^(2/3)) + 3*b*x^(1/3)
```

IntegrateAlgebraic [A] time = 0.01, size = 17, normalized size = 0.89

$$\frac{3(2bx - a)}{2x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/x^(5/3),x]
[Out] (3*(-a + 2*b*x))/(2*x^(2/3))
```

fricas [A] time = 1.17, size = 13, normalized size = 0.68

$$\frac{3\,(2\,bx - a)}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/2*(2*b*x - a)/x^(2/3)
```

giac [A] time = 1.10, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="giac")
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{3\,(-2bx+a)}{2x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^(5/3),x)
```

```
[Out] -3/2*(-2*b*x+a)/x^(2/3)
```

maxima [A] time = 1.30, size = 13, normalized size = 0.68

$$3\,bx^{\frac{1}{3}} - \frac{3\,a}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3*b*x^(1/3) - 3/2*a/x^(2/3)
```

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$-\frac{3\,a-6\,b\,x}{2\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/x^(5/3),x)
```

```
[Out] -(3*a - 6*b*x)/(2*x^(2/3))
```

sympy [A] time = 0.45, size = 17, normalized size = 0.89

$$-\frac{3a}{2x^{\frac{2}{3}}} + 3b\sqrt[3]{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**(5/3),x)
```

```
[Out] -3*a/(2*x**(2/3)) + 3*b*x**(1/3)
```

3.658 $\int x^{5/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(8/3))/8 + (6*a*b*x^(11/3))/11 + (3*b^2*x^(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^2\,dx&=\int\left(a^2x^{5/3}+2abx^{8/3}+b^2x^{11/3}\right)dx\\&=\frac{3}{8}a^2x^{8/3}+\frac{6}{11}abx^{11/3}+\frac{3}{14}b^2x^{14/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

IntegrateAlgebraic [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{616}x^{8/3}\left(77a^2+112abx+44b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*(77*a^2 + 112*a*b*x + 44*b^2*x^2))/616
```

fricas [A] time = 1.30, size = 29, normalized size = 0.81

$$\frac{3}{616}\left(44\,b^2x^4+112\,abx^3+77\,a^2x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="fricas")
[Out] 3/616*(44*b^2*x^4 + 112*a*b*x^3 + 77*a^2*x^2)*x^(2/3)
```

giac [A] time = 1.04, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^{2}x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^{2}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="giac")
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(44b^{2}x^{2}+112abx+77a^{2}\right)x^{\frac{8}{3}}}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^2,x)
[Out] 3/616*x^(8/3)*(44*b^2*x^2+112*a*b*x+77*a^2)
```

maxima [A] time = 1.31, size = 24, normalized size = 0.67

$$\frac{3}{14}\,b^{2}x^{\frac{14}{3}}+\frac{6}{11}\,abx^{\frac{11}{3}}+\frac{3}{8}\,a^{2}x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^2,x, algorithm="maxima")
[Out] 3/14*b^2*x^(14/3) + 6/11*a*b*x^(11/3) + 3/8*a^2*x^(8/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{8/3}\,\left(77\,a^2+112\,a\,b\,x+44\,b^2\,x^2\right)}{616}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^2,x)
[Out] (3*x^(8/3)*(77*a^2 + 44*b^2*x^2 + 112*a*b*x))/616
```

sympy [A] time = 3.75, size = 34, normalized size = 0.94

$$\frac{3a^{2}x^{\frac{8}{3}}}{8}+\frac{6abx^{\frac{11}{3}}}{11}+\frac{3b^{2}x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**2,x)
[Out] 3*a**2*x**(8/3)/8 + 6*a*b*x**(11/3)/11 + 3*b**2*x**(14/3)/14
```

3.659 $\int x^{4/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(7/3))/7 + (3*a*b*x^(10/3))/5 + (3*b^2*x^(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^2\,dx&=\int\left(a^2x^{4/3}+2abx^{7/3}+b^2x^{10/3}\right)dx\\&=\frac{3}{7}a^2x^{7/3}+\frac{3}{5}abx^{10/3}+\frac{3}{13}b^2x^{13/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

IntegrateAlgebraic [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{455}x^{7/3}\left(65a^2+91abx+35b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(7/3)*(65*a^2 + 91*a*b*x + 35*b^2*x^2))/455
```

fricas [A] time = 1.02, size = 29, normalized size = 0.81

$$\frac{3}{455}\left(35\,b^2x^4+91\,abx^3+65\,a^2x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/455*(35*b^2*x^4 + 91*a*b*x^3 + 65*a^2*x^2)*x^(1/3)
```

giac [A] time = 1.06, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^{2}x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^{2}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(35b^{2}x^{2}+91abx+65a^{2}\right)x^{\frac{7}{3}}}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^2,x)
```

```
[Out] 3/455*x^(7/3)*(35*b^2*x^2+91*a*b*x+65*a^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{13}\,b^{2}x^{\frac{13}{3}}+\frac{3}{5}\,abx^{\frac{10}{3}}+\frac{3}{7}\,a^{2}x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/13*b^2*x^(13/3) + 3/5*a*b*x^(10/3) + 3/7*a^2*x^(7/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{7/3}\,\left(65\,a^2+91\,a\,b\,x+35\,b^2\,x^2\right)}{455}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(7/3)*(65*a^2 + 35*b^2*x^2 + 91*a*b*x))/455
```

sympy [A] time = 2.65, size = 34, normalized size = 0.94

$$\frac{3a^{2}x^{\frac{7}{3}}}{7}+\frac{3abx^{\frac{10}{3}}}{5}+\frac{3b^{2}x^{\frac{13}{3}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(7/3)/7 + 3*a*b*x**(10/3)/5 + 3*b**2*x**(13/3)/13
```

3.660 $\int x^{2/3}(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(5/3))/5 + (3*a*b*x^(8/3))/4 + (3*b^2*x^(11/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^2\,dx &= \int \left(a^2x^{2/3}+2abx^{5/3}+b^2x^{8/3}\right)dx\\ &= \frac{3}{5}a^2x^{5/3}+\frac{3}{4}abx^{8/3}+\frac{3}{11}b^2x^{11/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{220}x^{5/3}\left(44a^2+55abx+20b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*(44*a^2 + 55*a*b*x + 20*b^2*x^2))/220
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{220}\left(44a^2x^{5/3}+55abx^{8/3}+20b^2x^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x)^2,x]
```

```
[Out] (3*(44*a^2*x^(5/3) + 55*a*b*x^(8/3) + 20*b^2*x^(11/3)))/220
```

fricas [A] time = 1.10, size = 27, normalized size = 0.75

$$\frac{3}{220}\left(20\,b^2x^3+55\,abx^2+44\,a^2x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/220*(20*b^2*x^3 + 55*a*b*x^2 + 44*a^2*x)*x^(2/3)
```

giac [A] time = 1.05, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}} + \frac{3}{4}\,abx^{\frac{8}{3}} + \frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(20b^2x^2 + 55abx + 44a^2\right)x^{\frac{5}{3}}}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^2,x)
```

```
[Out] 3/220*x^(5/3)*(20*b^2*x^2+55*a*b*x+44*a^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{11}\,b^2x^{\frac{11}{3}} + \frac{3}{4}\,abx^{\frac{8}{3}} + \frac{3}{5}\,a^2x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/11*b^2*x^(11/3) + 3/4*a*b*x^(8/3) + 3/5*a^2*x^(5/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{5/3}\,\left(44\,a^2+55\,a\,b\,x+20\,b^2\,x^2\right)}{220}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(5/3)*(44*a^2 + 20*b^2*x^2 + 55*a*b*x))/220
```

sympy [A] time = 1.06, size = 34, normalized size = 0.94

$$\frac{3a^2x^{\frac{5}{3}}}{5} + \frac{3abx^{\frac{8}{3}}}{4} + \frac{3b^2x^{\frac{11}{3}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**2,x)
```

```
[Out] 3*a**2*x**(5/3)/5 + 3*a*b*x**(8/3)/4 + 3*b**2*x**(11/3)/11
```

3.661 $\int \sqrt[3]{x}\,(a+bx)^2\,dx$

Optimal. Leaf size=36

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*a^2*x^(4/3))/4 + (6*a*b*x^(7/3))/7 + (3*b^2*x^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}\,(a+bx)^2\,dx &= \int \left(a^2\sqrt[3]{x}+2abx^{4/3}+b^2x^{7/3}\right)dx\\ &= \frac{3}{4}a^2x^{4/3}+\frac{6}{7}abx^{7/3}+\frac{3}{10}b^2x^{10/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{140}x^{4/3}\left(35a^2+40abx+14b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*(35*a^2 + 40*a*b*x + 14*b^2*x^2))/140
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{140}\left(35a^2x^{4/3}+40abx^{7/3}+14b^2x^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x)^2,x]
```

```
[Out] (3*(35*a^2*x^(4/3) + 40*a*b*x^(7/3) + 14*b^2*x^(10/3)))/140
```

fricas [A] time = 1.15, size = 27, normalized size = 0.75

$$\frac{3}{140}\left(14\,b^2x^3+40\,abx^2+35\,a^2x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^2*x^3 + 40*a*b*x^2 + 35*a^2*x)*x^(1/3)
```

giac [A] time = 1.04, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}} + \frac{6}{7}\,abx^{\frac{7}{3}} + \frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.69

$$\frac{3\left(14b^2x^2 + 40abx + 35a^2\right)x^{\frac{4}{3}}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^2,x)
```

```
[Out] 3/140*x^(4/3)*(14*b^2*x^2+40*a*b*x+35*a^2)
```

maxima [A] time = 1.28, size = 24, normalized size = 0.67

$$\frac{3}{10}\,b^2x^{\frac{10}{3}} + \frac{6}{7}\,abx^{\frac{7}{3}} + \frac{3}{4}\,a^2x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/10*b^2*x^(10/3) + 6/7*a*b*x^(7/3) + 3/4*a^2*x^(4/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{4/3}\,\left(35\,a^2+40\,a\,b\,x+14\,b^2\,x^2\right)}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^2,x)
```

```
[Out] (3*x^(4/3)*(35*a^2 + 14*b^2*x^2 + 40*a*b*x))/140
```

sympy [C] time = 2.22, size = 2633, normalized size = 73.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**2,x)
```

```
[Out] Piecewise((27*a**(34/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(-140*a**
8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*
a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3
*exp(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7
*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I
*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 72*a**(31/3)*b*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2
*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(
a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3))
```

```
- 81*a**(31/3)*b*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 60*a**(28/3)*b**2*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp
(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)
*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)
) + 81*a**(28/3)*b**2*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*
a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp
(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(25/3)*
b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-140*a**8*b**(
4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b
**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2
*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3
) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x
)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 135*a
**(22/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(-140*
a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 4
20*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)
**3*exp(2*I*pi/3)) - 132*a**(19/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b +
x)**5*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(
a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 14
0*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 42*a**(16/3)*b**6*(-1 + b*(a
/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(-140*a**8*b**(4/3)*exp(2*I*pi
/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b +
 x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), Abs(
b*(a/b + x)/a) > 1), (-27*a**(34/3)*(1 - b*(a/b + x)/a)**(1/3)/(-140*a**8*b
**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**
6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*ex
p(2*I*pi/3)) + 27*a**(34/3)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b*
*(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi
/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 72*a**(31/3)*b*(1 -
b*(a/b + x)/a)**(1/3)*(a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**
7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*
I*pi/3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(31/3)*b*(
a/b + x)/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*ex
p(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(
13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(28/3)*b**2*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a
/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140
*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) + 81*a**(28/3)*b**2*(a/b + x)**
2/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*p
i/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(25/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(
a/b + x)**3/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)
*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b
**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 27*a**(25/3)*b**3*(a/b + x)**3/(-140
*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2*I*pi/3) -
420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/3)*(a/b + x
)**3*exp(2*I*pi/3)) - 135*a**(22/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b +
x)**4/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b + x)*exp(2
*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a**5*b**(13/
3)*(a/b + x)**3*exp(2*I*pi/3)) + 132*a**(19/3)*b**5*(1 - b*(a/b + x)/a)**(1
/3)*(a/b + x)**5/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**(7/3)*(a/b
 + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/3) + 140*a
**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)) - 42*a**(16/3)*b**6*(1 - b*(a/b +
 x)/a)**(1/3)*(a/b + x)**6/(-140*a**8*b**(4/3)*exp(2*I*pi/3) + 420*a**7*b**
(7/3)*(a/b + x)*exp(2*I*pi/3) - 420*a**6*b**(10/3)*(a/b + x)**2*exp(2*I*pi/
3) + 140*a**5*b**(13/3)*(a/b + x)**3*exp(2*I*pi/3)), True))
```

3.662 $\int \frac{(a+bx)^2}{\sqrt[3]{x}} dx$

Optimal. Leaf size=36

$$\frac{3}{2}a^2x^{2/3} + \frac{6}{5}abx^{5/3} + \frac{3}{8}b^2x^{8/3}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{2}a^2x^{2/3} + \frac{6}{5}abx^{5/3} + \frac{3}{8}b^2x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*a^2*x^(2/3))/2 + (6*a*b*x^(5/3))/5 + (3*b^2*x^(8/3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt[3]{x}} dx &= \int \left(\frac{a^2}{\sqrt[3]{x}} + 2abx^{2/3} + b^2x^{5/3}\right) dx \\ &= \frac{3}{2}a^2x^{2/3} + \frac{6}{5}abx^{5/3} + \frac{3}{8}b^2x^{8/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.78

$$\frac{3}{40}x^{2/3}\left(20a^2 + 16abx + 5b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(20*a^2 + 16*a*b*x + 5*b^2*x^2))/40
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 0.94

$$\frac{3}{40}\left(20a^2x^{2/3} + 16abx^{5/3} + 5b^2x^{8/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(1/3),x]
```

```
[Out] (3*(20*a^2*x^(2/3) + 16*a*b*x^(5/3) + 5*b^2*x^(8/3)))/40
```

fricas [A] time = 1.10, size = 24, normalized size = 0.67

$$\frac{3}{40}\left(5\,b^2x^2 + 16\,abx + 20\,a^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="fricas")
```

```
[Out] 3/40*(5*b^2*x^2 + 16*a*b*x + 20*a^2)*x^(2/3)
```

giac [A] time = 1.11, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="giac")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.69

$$\frac{3\left(5b^2x^2 + 16abx + 20a^2\right)x^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(1/3),x)
```

```
[Out] 3/40*x^(2/3)*(5*b^2*x^2+16*a*b*x+20*a^2)
```

maxima [A] time = 1.32, size = 24, normalized size = 0.67

$$\frac{3}{8}\,b^2x^{\frac{8}{3}} + \frac{6}{5}\,abx^{\frac{5}{3}} + \frac{3}{2}\,a^2x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(1/3),x, algorithm="maxima")
```

```
[Out] 3/8*b^2*x^(8/3) + 6/5*a*b*x^(5/3) + 3/2*a^2*x^(2/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.67

$$\frac{3\,x^{2/3}\,\left(20\,a^2+16\,a\,b\,x+5\,b^2\,x^2\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(1/3),x)
```

```
[Out] (3*x^(2/3)*(20*a^2 + 5*b^2*x^2 + 16*a*b*x))/40
```

sympy [C] time = 2.05, size = 1765, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(1/3),x)
```

```
[Out] Piecewise((-27*a**(32/3)*(-1 + b*(a/b + x)/a)**(2/3)/(-40*a**8*b**(2/3) + 1
20*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(1
1/3)*(a/b + x)**3) + 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a*
*7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*
(a/b + x)**3) + 63*a**(29/3)*b*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)
```

```
/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b
+ x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(-1 + b*(a/b
+ x)/a)**(2/3)*(a/b + x)**2/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 81*a
**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(
5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b +
x)**3) + 18*a**(23/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-40*a*
*8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
+ 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a/b + x)**3*exp(2*I*
pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*
(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(20/3)*b**4*(-1 + b*
(a/b + x)/a)**(2/3)*(a/b + x)**4/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 15*a**(17/3)*b**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5/(-40*a**8*b**(2
/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**
5*b**(11/3)*(a/b + x)**3), Abs(b*(a/b + x)/a) > 1), (-27*a**(32/3)*(1 - b*(
a/b + x)/a)**(2/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/
b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) +
 27*a**(32/3)*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x
) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) + 63*a
**(29/3)*b*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(-40*a**8*b**
(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a
**5*b**(11/3)*(a/b + x)**3) - 81*a**(29/3)*b*(a/b + x)*exp(2*I*pi/3)/(-40*a
**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2
 + 40*a**5*b**(11/3)*(a/b + x)**3) - 42*a**(26/3)*b**2*(1 - b*(a/b + x)/a)*
*(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a
/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3)
+ 81*a**(26/3)*b**2*(a/b + x)**2*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**
7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(
a/b + x)**3) + 18*a**(23/3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*ex
p(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**
(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(23/3)*b**3*(a
/b + x)**3*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) -
 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3) - 27*a**(
20/3)*b**4*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(-40*a**8*
b**(2/3) + 120*a**7*b**(5/3)*(a/b + x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 4
0*a**5*b**(11/3)*(a/b + x)**3) + 15*a**(17/3)*b**5*(1 - b*(a/b + x)/a)**(2/
3)*(a/b + x)**5*exp(2*I*pi/3)/(-40*a**8*b**(2/3) + 120*a**7*b**(5/3)*(a/b +
 x) - 120*a**6*b**(8/3)*(a/b + x)**2 + 40*a**5*b**(11/3)*(a/b + x)**3), Tru
e))
```

3.663 $\int \frac{(a+bx)^2}{x^{2/3}} \, dx$

Optimal. Leaf size=34

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(2/3),x]
```

```
[Out] 3*a^2*x^(1/3) + (3*a*b*x^(4/3))/2 + (3*b^2*x^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{2/3}} \, dx &= \int \left(\frac{a^2}{x^{2/3}}+2ab\sqrt[3]{x}+b^2x^{4/3}\right) \, dx\\ &= 3a^2\sqrt[3]{x}+\frac{3}{2}abx^{4/3}+\frac{3}{7}b^2x^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.82

$$\frac{3}{14}\sqrt[3]{x}\left(14a^2+7abx+2b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(14*a^2 + 7*a*b*x + 2*b^2*x^2))/14
```

IntegrateAlgebraic [A] time = 0.01, size = 34, normalized size = 1.00

$$\frac{3}{14}\left(14a^2\sqrt[3]{x}+7abx^{4/3}+2b^2x^{7/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(2/3),x]
```

```
[Out] (3*(14*a^2*x^(1/3) + 7*a*b*x^(4/3) + 2*b^2*x^(7/3)))/14
```

fricas [A] time = 1.33, size = 24, normalized size = 0.71

$$\frac{3}{14}\left(2\,b^2x^2+7\,abx+14\,a^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/14*(2*b^2*x^2 + 7*a*b*x + 14*a^2)*x^(1/3)
```

giac [A] time = 0.92, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

maple [A] time = 0.01, size = 25, normalized size = 0.74

$$\frac{3\left(2b^2x^2+7abx+14a^2\right)x^{\frac{1}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(2/3),x)
```

```
[Out] 3/14*x^(1/3)*(2*b^2*x^2+7*a*b*x+14*a^2)
```

maxima [A] time = 1.38, size = 24, normalized size = 0.71

$$\frac{3}{7}\,b^2x^{\frac{7}{3}}+\frac{3}{2}\,abx^{\frac{4}{3}}+3\,a^2x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^2*x^(7/3) + 3/2*a*b*x^(4/3) + 3*a^2*x^(1/3)
```

mupad [B] time = 0.03, size = 24, normalized size = 0.71

$$\frac{3\,x^{1/3}\,\left(14\,a^2+7\,a\,b\,x+2\,b^2\,x^2\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(2/3),x)
```

```
[Out] (3*x^(1/3)*(14*a^2 + 2*b^2*x^2 + 7*a*b*x))/14
```

sympy [C] time = 2.08, size = 1741, normalized size = 51.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(2/3),x)
```

```
[Out] Piecewise((-27*a**(31/3)*(-1 + b*(a/b + x)/a)**(1/3)/(-14*a**8*b**(1/3) + 4
2*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/
3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b*
*(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b +
 x)**3) + 72*a**(28/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)/(-14*a**8*b*
*(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a*
*5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)*b*(a/b + x)*exp(I*pi/3)/(-14*a**8
```

```
*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14
*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25/3)*b**2*(-1 + b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b
**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*
(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22
/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(-14*a**8*b**(1/3) + 42*a
**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*
(a/b + x)**3) - 27*a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/
3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b
**(10/3)*(a/b + x)**3) - 9*a**(19/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**4/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*
(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(-1 + b*(
a/b + x)/a)**(1/3)*(a/b + x)**5/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b
+ x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3), Abs
(b*(a/b + x)/a) > 1), (-27*a**(31/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)
/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b +
x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 27*a**(31/3)*exp(I*pi/3)/(-14*a**
8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 1
4*a**5*b**(10/3)*(a/b + x)**3) + 72*a**(28/3)*b*(1 - b*(a/b + x)/a)**(1/3)*
(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*
a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 81*a**(28/3)
*b*(a/b + x)*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) -
42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 60*a**(25
/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(-14*a**8*b**(
1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5
*b**(10/3)*(a/b + x)**3) + 81*a**(25/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(-14*
a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2
+ 14*a**5*b**(10/3)*(a/b + x)**3) + 18*a**(22/3)*b**3*(1 - b*(a/b + x)/a)**
(1/3)*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b +
 x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) - 27*
a**(22/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/
3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)*
*3) - 9*a**(19/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*pi/3)/
(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a/b + x) - 42*a**6*b**(7/3)*(a/b + x
)**2 + 14*a**5*b**(10/3)*(a/b + x)**3) + 6*a**(16/3)*b**5*(1 - b*(a/b + x)/
a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(-14*a**8*b**(1/3) + 42*a**7*b**(4/3)*(a
/b + x) - 42*a**6*b**(7/3)*(a/b + x)**2 + 14*a**5*b**(10/3)*(a/b + x)**3),
True))
```

3.664 $\int \frac{(a+bx)^2}{x^{4/3}}\,dx$

Optimal. Leaf size=32

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(4/3),x]
[Out] (-3*a^2)/x^(1/3) + 3*a*b*x^(2/3) + (3*b^2*x^(5/3))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{4/3}}\,dx &= \int \left(\frac{a^2}{x^{4/3}}+\frac{2ab}{\sqrt[3]{x}}+b^2x^{2/3}\right)dx\\ &= -\frac{3a^2}{\sqrt[3]{x}}+3abx^{2/3}+\frac{3}{5}b^2x^{5/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{3\left(-5a^2+5abx+b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(4/3),x]
[Out] (3*(-5*a^2 + 5*a*b*x + b^2*x^2))/(5*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.84

$$\frac{3\left(-5a^2+5abx+b^2x^2\right)}{5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(4/3),x]
[Out] (3*(-5*a^2 + 5*a*b*x + b^2*x^2))/(5*x^(1/3))
```

fricas [A] time = 1.29, size = 23, normalized size = 0.72

$$\frac{3\left(b^2x^2+5\,abx-5\,a^2\right)}{5\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="fricas")
[Out] 3/5*(b^2*x^2 + 5*a*b*x - 5*a^2)/x^(1/3)
```

giac [A] time = 1.17, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="giac")
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.78

$$-\frac{3\left(-b^2x^2-5abx+5a^2\right)}{5x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(4/3),x)
[Out] -3/5*(-b^2*x^2-5*a*b*x+5*a^2)/x^(1/3)
```

maxima [A] time = 1.36, size = 24, normalized size = 0.75

$$\frac{3}{5}\,b^2x^{\frac{5}{3}}+3\,abx^{\frac{2}{3}}-\frac{3\,a^2}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(4/3),x, algorithm="maxima")
[Out] 3/5*b^2*x^(5/3) + 3*a*b*x^(2/3) - 3*a^2/x^(1/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.75

$$\frac{-15\,a^2+15\,a\,b\,x+3\,b^2\,x^2}{5\,x^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(4/3),x)
[Out] (3*b^2*x^2 - 15*a^2 + 15*a*b*x)/(5*x^(1/3))
```

sympy [C] time = 2.09, size = 1826, normalized size = 57.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(4/3),x)
```

```
[Out] Piecewise((-27*a**(29/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(
-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
 x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/3)*b
**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b*
*2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 63*a*
*(26/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)*exp(I*pi/3)/(-5*a**8
*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*
exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)
*(a/b + x)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6
*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 42
*a**(23/3)*b**(7/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(I*pi/3)/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b*
*(7/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3)
- 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi
/3)) + 3*a**(20/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I
*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**
2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**
(20/3)*b**(10/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*ex
p(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**
3*exp(I*pi/3)) + 3*a**(17/3)*b**(13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x
)**4*exp(I*pi/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 1
5*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)
), Abs(b*(a/b + x)/a) > 1), (27*a**(29/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/
3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a
/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 27*a**(29/
3)*b**(1/3)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**
6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 6
3*a**(26/3)*b**(4/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)/(-5*a**8*exp(I*pi
/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/
3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 81*a**(26/3)*b**(4/3)*(a/b + x
)/(-5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/
b + x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) + 42*a**(23/3
)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(-5*a**8*exp(I*pi/3) + 1
5*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*
a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 81*a**(23/3)*b**(7/3)*(a/b + x)**2/(-
5*a**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b +
x)**2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(20/3)*b**
(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(-5*a**8*exp(I*pi/3) + 15*a*
*7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5
*b**3*(a/b + x)**3*exp(I*pi/3)) + 27*a**(20/3)*b**(10/3)*(a/b + x)**3/(-5*a
**8*exp(I*pi/3) + 15*a**7*b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)*
*2*exp(I*pi/3) + 5*a**5*b**3*(a/b + x)**3*exp(I*pi/3)) - 3*a**(17/3)*b**(13
/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(-5*a**8*exp(I*pi/3) + 15*a**7*
b*(a/b + x)*exp(I*pi/3) - 15*a**6*b**2*(a/b + x)**2*exp(I*pi/3) + 5*a**5*b*
*3*(a/b + x)**3*exp(I*pi/3)), True))
```

3.665 $\int \frac{(a+bx)^2}{x^{5/3}} \, dx$

Optimal. Leaf size=34

$$-\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/x^(5/3),x]
[Out] (-3*a^2)/(2*x^(2/3)) + 6*a*b*x^(1/3) + (3*b^2*x^(4/3))/4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^{5/3}} \, dx &= \int \left(\frac{a^2}{x^{5/3}} + \frac{2ab}{x^{2/3}} + b^2\sqrt[3]{x}\right) dx \\ &= -\frac{3a^2}{2x^{2/3}} + 6ab\sqrt[3]{x} + \frac{3}{4}b^2x^{4/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.79

$$\frac{3\left(-2a^2 + 8abx + b^2x^2\right)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/x^(5/3),x]
[Out] (3*(-2*a^2 + 8*a*b*x + b^2*x^2))/(4*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.79

$$\frac{3\left(-2a^2 + 8abx + b^2x^2\right)}{4x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/x^(5/3),x]
[Out] (3*(-2*a^2 + 8*a*b*x + b^2*x^2))/(4*x^(2/3))
```

fricas [A] time = 1.21, size = 23, normalized size = 0.68

$$\frac{3\left(b^2x^2 + 8\,abx - 2\,a^2\right)}{4\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="fricas")
[Out] 3/4*(b^2*x^2 + 8*a*b*x - 2*a^2)/x^(2/3)
```

giac [A] time = 1.22, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="giac")
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

maple [A] time = 0.00, size = 25, normalized size = 0.74

$$-\frac{3\left(-b^2x^2-8abx+2a^2\right)}{4x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^(5/3),x)
[Out] -3/4*(-b^2*x^2-8*a*b*x+2*a^2)/x^(2/3)
```

maxima [A] time = 1.35, size = 24, normalized size = 0.71

$$\frac{3}{4}\,b^2x^{\frac{4}{3}}+6\,abx^{\frac{1}{3}}-\frac{3\,a^2}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^(5/3),x, algorithm="maxima")
[Out] 3/4*b^2*x^(4/3) + 6*a*b*x^(1/3) - 3/2*a^2/x^(2/3)
```

mupad [B] time = 0.04, size = 24, normalized size = 0.71

$$\frac{-6\,a^2+24\,a\,b\,x+3\,b^2\,x^2}{4\,x^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/x^(5/3),x)
[Out] (3*b^2*x^2 - 6*a^2 + 24*a*b*x)/(4*x^(2/3))
```

sympy [C] time = 2.06, size = 1957, normalized size = 57.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**(5/3),x)
[Out] Piecewise((-27*a**(28/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)
/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*
(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 27*a
**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3
) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(
2*I*pi/3)) + 72*a**(25/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*ex
```

```
p(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12
*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi
/3)) + 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(
a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b
**3*(a/b + x)**3*exp(2*I*pi/3)) - 60*a**(22/3)*b**(8/3)*(-1 + b*(a/b + x)/a
)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b
 + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3
*(a/b + x)**3*exp(2*I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*
exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)*
*2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 12*a**(19/3)*b
**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(-4*a**8*ex
p(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2
*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**
(11/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi
/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*ex
p(2*I*pi/3)) + 3*a**(16/3)*b**(14/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*
*4*exp(2*I*pi/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
 - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2
*I*pi/3)), Abs(b*(a/b + x)/a) > 1), (27*a**(28/3)*b**(2/3)*(1 - b*(a/b + x)
/a)**(1/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*
a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/
3)) - 27*a**(28/3)*b**(2/3)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*ex
p(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
x)**3*exp(2*I*pi/3)) - 72*a**(25/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/
b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6
*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3))
+ 81*a**(25/3)*b**(5/3)*(a/b + x)/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b +
 x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(
a/b + x)**3*exp(2*I*pi/3)) + 60*a**(22/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/
3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3)
- 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*
I*pi/3)) - 81*a**(22/3)*b**(8/3)*(a/b + x)**2/(-4*a**8*exp(2*I*pi/3) + 12*a
**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4
*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 12*a**(19/3)*b**(11/3)*(1 - b*(a/b
 + x)/a)**(1/3)*(a/b + x)**3/(-4*a**8*exp(2*I*pi/3) + 12*a**7*b*(a/b + x)*e
xp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**5*b**3*(a/b +
 x)**3*exp(2*I*pi/3)) + 27*a**(19/3)*b**(11/3)*(a/b + x)**3/(-4*a**8*exp(2*
I*pi/3) + 12*a**7*b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp
(2*I*pi/3) + 4*a**5*b**3*(a/b + x)**3*exp(2*I*pi/3)) - 3*a**(16/3)*b**(14/3
)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(-4*a**8*exp(2*I*pi/3) + 12*a**7*
b*(a/b + x)*exp(2*I*pi/3) - 12*a**6*b**2*(a/b + x)**2*exp(2*I*pi/3) + 4*a**
5*b**3*(a/b + x)**3*exp(2*I*pi/3)), True))
```

3.666 $\int x^{5/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{11}a^2bx^{11/3}+\frac{3}{8}a^3x^{8/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(8/3))/8 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*x^(14/3))/14 + (3*b^3*
x^(17/3))/17
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{5/3}(a+bx)^3\,dx&=\int\left(a^3x^{5/3}+3a^2bx^{8/3}+3ab^2x^{11/3}+b^3x^{14/3}\right)dx\\&=\frac{3}{8}a^3x^{8/3}+\frac{9}{11}a^2bx^{11/3}+\frac{9}{14}ab^2x^{14/3}+\frac{3}{17}b^3x^{17/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{8/3}\left(1309a^3+2856a^2bx+2244ab^2x^2+616b^3x^3\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*(1309*a^3 + 2856*a^2*b*x + 2244*a*b^2*x^2 + 616*b^3*x^3))/10472
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(1309a^3x^{8/3}+2856a^2bx^{11/3}+2244ab^2x^{14/3}+616b^3x^{17/3}\right)}{10472}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)*(a + b*x)^3,x]
```

```
[Out] (3*(1309*a^3*x^(8/3) + 2856*a^2*b*x^(11/3) + 2244*a*b^2*x^(14/3) + 616*b^3*
x^(17/3)))/10472
```

fricas [A] time = 1.25, size = 40, normalized size = 0.78

$$\frac{3}{10472}\left(616\,b^3x^5+2244\,ab^2x^4+2856\,a^2bx^3+1309\,a^3x^2\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/10472*(616*b^3*x^5 + 2244*a*b^2*x^4 + 2856*a^2*b*x^3 + 1309*a^3*x^2)*x^(2
/3)
```

giac [A] time = 1.06, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(616b^3x^3+2244a\,b^2x^2+2856a^2bx+1309a^3\right)x^{\frac{8}{3}}}{10472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(b*x+a)^3,x)
```

```
[Out] 3/10472*x^(8/3)*(616*b^3*x^3+2244*a*b^2*x^2+2856*a^2*b*x+1309*a^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{3}{17}\,b^3x^{\frac{17}{3}}+\frac{9}{14}\,ab^2x^{\frac{14}{3}}+\frac{9}{11}\,a^2bx^{\frac{11}{3}}+\frac{3}{8}\,a^3x^{\frac{8}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/17*b^3*x^(17/3) + 9/14*a*b^2*x^(14/3) + 9/11*a^2*b*x^(11/3) + 3/8*a^3*x^(
8/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{8/3}}{8}+\frac{3\,b^3\,x^{17/3}}{17}+\frac{9\,a^2\,b\,x^{11/3}}{11}+\frac{9\,a\,b^2\,x^{14/3}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(8/3))/8 + (3*b^3*x^(17/3))/17 + (9*a^2*b*x^(11/3))/11 + (9*a*b^2*
x^(14/3))/14
```

sympy [A] time = 6.39, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{8}{3}}}{8}+\frac{9a^2bx^{\frac{11}{3}}}{11}+\frac{9ab^2x^{\frac{14}{3}}}{14}+\frac{3b^3x^{\frac{17}{3}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(8/3)/8 + 9*a**2*b*x**(11/3)/11 + 9*a*b**2*x**(14/3)/14 + 3*b**3*
x**(17/3)/17
```

3.667 $\int x^{4/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{10}a^2bx^{10/3}+\frac{3}{7}a^3x^{7/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(7/3))/7 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*x^(13/3))/13 + (3*b^3*
x^(16/3))/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{4/3}(a+bx)^3\,dx&=\int\left(a^3x^{4/3}+3a^2bx^{7/3}+3ab^2x^{10/3}+b^3x^{13/3}\right)dx\\&=\frac{3}{7}a^3x^{7/3}+\frac{9}{10}a^2bx^{10/3}+\frac{9}{13}ab^2x^{13/3}+\frac{3}{16}b^3x^{16/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{7/3}\left(1040a^3+2184a^2bx+1680ab^2x^2+455b^3x^3\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*(1040*a^3 + 2184*a^2*b*x + 1680*a*b^2*x^2 + 455*b^3*x^3))/7280
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(1040a^3x^{7/3}+2184a^2bx^{10/3}+1680ab^2x^{13/3}+455b^3x^{16/3}\right)}{7280}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)*(a + b*x)^3,x]
```

```
[Out] (3*(1040*a^3*x^(7/3) + 2184*a^2*b*x^(10/3) + 1680*a*b^2*x^(13/3) + 455*b^3*
x^(16/3)))/7280
```

fricas [A] time = 1.21, size = 40, normalized size = 0.78

$$\frac{3}{7280}\left(455\,b^3x^5 + 1680\,ab^2x^4 + 2184\,a^2bx^3 + 1040\,a^3x^2\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 3/7280*(455*b^3*x^5 + 1680*a*b^2*x^4 + 2184*a^2*b*x^3 + 1040*a^3*x^2)*x^(1/
3)
```

giac [A] time = 1.06, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(455b^3x^3 + 1680a\,b^2x^2 + 2184a^2bx + 1040a^3\right)x^{\frac{7}{3}}}{7280}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(b*x+a)^3,x)
```

```
[Out] 3/7280*x^(7/3)*(455*b^3*x^3+1680*a*b^2*x^2+2184*a^2*b*x+1040*a^3)
```

maxima [A] time = 1.29, size = 35, normalized size = 0.69

$$\frac{3}{16}\,b^3x^{\frac{16}{3}} + \frac{9}{13}\,ab^2x^{\frac{13}{3}} + \frac{9}{10}\,a^2bx^{\frac{10}{3}} + \frac{3}{7}\,a^3x^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 3/16*b^3*x^(16/3) + 9/13*a*b^2*x^(13/3) + 9/10*a^2*b*x^(10/3) + 3/7*a^3*x^(
7/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{7/3}}{7} + \frac{3\,b^3\,x^{16/3}}{16} + \frac{9\,a^2\,b\,x^{10/3}}{10} + \frac{9\,a\,b^2\,x^{13/3}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)*(a + b*x)^3,x)
```

```
[Out] (3*a^3*x^(7/3))/7 + (3*b^3*x^(16/3))/16 + (9*a^2*b*x^(10/3))/10 + (9*a*b^2*
x^(13/3))/13
```

sympy [A] time = 4.55, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{7}{3}}}{7} + \frac{9a^2bx^{\frac{10}{3}}}{10} + \frac{9ab^2x^{\frac{13}{3}}}{13} + \frac{3b^3x^{\frac{16}{3}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)*(b*x+a)**3,x)
```

```
[Out] 3*a**3*x**(7/3)/7 + 9*a**2*b*x**(10/3)/10 + 9*a*b**2*x**(13/3)/13 + 3*b**3*
x**(16/3)/16
```

3.668 $\int x^{2/3}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{8}a^2bx^{8/3}+\frac{3}{5}a^3x^{5/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(5/3))/5 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^(11/3))/11 + (3*b^3*x^
(14/3))/14
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^{2/3}(a+bx)^3\,dx&=\int\left(a^3x^{2/3}+3a^2bx^{5/3}+3ab^2x^{8/3}+b^3x^{11/3}\right)dx\\&=\frac{3}{5}a^3x^{5/3}+\frac{9}{8}a^2bx^{8/3}+\frac{9}{11}ab^2x^{11/3}+\frac{3}{14}b^3x^{14/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{5/3}\left(616a^3+1155a^2bx+840ab^2x^2+220b^3x^3\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*(616*a^3 + 1155*a^2*b*x + 840*a*b^2*x^2 + 220*b^3*x^3))/3080
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(616a^3x^{5/3}+1155a^2bx^{8/3}+840ab^2x^{11/3}+220b^3x^{14/3}\right)}{3080}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)*(a + b*x)^3,x]
```

```
[Out] (3*(616*a^3*x^(5/3) + 1155*a^2*b*x^(8/3) + 840*a*b^2*x^(11/3) + 220*b^3*x^(
14/3)))/3080
```

fricas [A] time = 1.40, size = 38, normalized size = 0.75

$$\frac{3}{3080}\left(220\,b^3x^4+840\,ab^2x^3+1155\,a^2bx^2+616\,a^3x\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="fricas")
[Out] 3/3080*(220*b^3*x^4 + 840*a*b^2*x^3 + 1155*a^2*b*x^2 + 616*a^3*x)*x^(2/3)
```

giac [A] time = 1.07, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="giac")
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

maple [A] time = 0.01, size = 36, normalized size = 0.71

$$\frac{3\left(220b^3x^3+840a\,b^2x^2+1155a^2bx+616a^3\right)x^{\frac{5}{3}}}{3080}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(b*x+a)^3,x)
[Out] 3/3080*x^(5/3)*(220*b^3*x^3+840*a*b^2*x^2+1155*a^2*b*x+616*a^3)
```

maxima [A] time = 1.34, size = 35, normalized size = 0.69

$$\frac{3}{14}\,b^3x^{\frac{14}{3}}+\frac{9}{11}\,ab^2x^{\frac{11}{3}}+\frac{9}{8}\,a^2bx^{\frac{8}{3}}+\frac{3}{5}\,a^3x^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)*(b*x+a)^3,x, algorithm="maxima")
[Out] 3/14*b^3*x^(14/3) + 9/11*a*b^2*x^(11/3) + 9/8*a^2*b*x^(8/3) + 3/5*a^3*x^(5/
3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{5/3}}{5}+\frac{3\,b^3\,x^{14/3}}{14}+\frac{9\,a^2\,b\,x^{8/3}}{8}+\frac{9\,a\,b^2\,x^{11/3}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)*(a + b*x)^3,x)
[Out] (3*a^3*x^(5/3))/5 + (3*b^3*x^(14/3))/14 + (9*a^2*b*x^(8/3))/8 + (9*a*b^2*x^
(11/3))/11
```

sympy [A] time = 2.19, size = 49, normalized size = 0.96

$$\frac{3a^3x^{\frac{5}{3}}}{5}+\frac{9a^2bx^{\frac{8}{3}}}{8}+\frac{9ab^2x^{\frac{11}{3}}}{11}+\frac{3b^3x^{\frac{14}{3}}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)*(b*x+a)**3,x)
[Out] 3*a**3*x**(5/3)/5 + 9*a**2*b*x**(8/3)/8 + 9*a*b**2*x**(11/3)/11 + 3*b**3*x*
*(14/3)/14
```

3.669 $\int \sqrt[3]{x}(a+bx)^3\,dx$

Optimal. Leaf size=51

$$\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{7}a^2bx^{7/3}+\frac{3}{4}a^3x^{4/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*a^3*x^(4/3))/4 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^(10/3))/10 + (3*b^3*x^
(13/3))/13
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x}(a+bx)^3\,dx&=\int \left(a^3\sqrt[3]{x}+3a^2bx^{4/3}+3ab^2x^{7/3}+b^3x^{10/3}\right)dx\\&=\frac{3}{4}a^3x^{4/3}+\frac{9}{7}a^2bx^{7/3}+\frac{9}{10}ab^2x^{10/3}+\frac{3}{13}b^3x^{13/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3x^{4/3}\left(455a^3+780a^2bx+546ab^2x^2+140b^3x^3\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*(455*a^3 + 780*a^2*b*x + 546*a*b^2*x^2 + 140*b^3*x^3))/1820
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{3\left(455a^3x^{4/3}+780a^2bx^{7/3}+546ab^2x^{10/3}+140b^3x^{13/3}\right)}{1820}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)*(a + b*x)^3,x]
```

```
[Out] (3*(455*a^3*x^(4/3) + 780*a^2*b*x^(7/3) + 546*a*b^2*x^(10/3) + 140*b^3*x^(1
3/3)))/1820
```

fricas [A] time = 1.18, size = 38, normalized size = 0.75

$$\frac{3}{1820}\left(140\,b^3x^4+546\,ab^2x^3+780\,a^2bx^2+455\,a^3x\right)x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="fricas")
[Out] 3/1820*(140*b^3*x^4 + 546*a*b^2*x^3 + 780*a^2*b*x^2 + 455*a^3*x)*x^(1/3)
```

giac [A] time = 1.16, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="giac")
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(140b^3x^3+546a\,b^2x^2+780a^2bx+455a^3\right)x^{\frac{4}{3}}}{1820}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(b*x+a)^3,x)
[Out] 3/1820*x^(4/3)*(140*b^3*x^3+546*a*b^2*x^2+780*a^2*b*x+455*a^3)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.69

$$\frac{3}{13}\,b^3x^{\frac{13}{3}}+\frac{9}{10}\,ab^2x^{\frac{10}{3}}+\frac{9}{7}\,a^2bx^{\frac{7}{3}}+\frac{3}{4}\,a^3x^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)*(b*x+a)^3,x, algorithm="maxima")
[Out] 3/13*b^3*x^(13/3) + 9/10*a*b^2*x^(10/3) + 9/7*a^2*b*x^(7/3) + 3/4*a^3*x^(4/
3)
```

mupad [B] time = 0.05, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{4/3}}{4}+\frac{3\,b^3\,x^{13/3}}{13}+\frac{9\,a^2\,b\,x^{7/3}}{7}+\frac{9\,a\,b^2\,x^{10/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)*(a + b*x)^3,x)
[Out] (3*a^3*x^(4/3))/4 + (3*b^3*x^(13/3))/13 + (9*a^2*b*x^(7/3))/7 + (9*a*b^2*x^
(10/3))/10
```

sympy [C] time = 3.22, size = 5012, normalized size = 98.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)*(b*x+a)**3,x)
```

```
[Out] Piecewise((-243*a**(73/3)*(-1 + b*(a/b + x)/a)**(1/3)/(1820*a**20*b**(4/3)
- 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 364
00*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 1092
0*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a
**(73/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x)
+ 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 +
1820*a**14*b**(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(-1 + b*(a/b + x)/a)*
*(1/3)*(a/b + x)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(18
20*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*
(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(
a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/
b + x)**6) - 3213*a**(67/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(
1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3
)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)
*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(
a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a**20*b**
(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2
 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4
- 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) +
 3927*a**(64/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(1820*a**20*b
**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)*
*2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**
4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6)
 - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a**20*b**(4/3) - 1092
0*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**
17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**1
5*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 2163*a**(61
/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(1820*a**20*b**(4/3) - 10
920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a
**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a*
*15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 3645*a**(
61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(
7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)
*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*
(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1827*a**(58/3)*b**5*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(1820*a**20*b**(4/3) - 10920*a**19*b*
*(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/
3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3
)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(
a/b + x)**5*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b +
x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**
3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5
 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 6573*a**(55/3)*b**6*(-1 + b*(a/b +
x)/a)**(1/3)*(a/b + x)**6/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
+ x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)
**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)*
*5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*e
xp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a*
*18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**
16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14
*b**(22/3)*(a/b + x)**6) - 8787*a**(52/3)*b**7*(-1 + b*(a/b + x)/a)**(1/3)*
(a/b + x)**7/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*
a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a
**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**
14*b**(22/3)*(a/b + x)**6) + 6498*a**(49/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3
```

```
)*(a/b + x)**8/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 2730
0*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300
*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a
**14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(-1 + b*(a/b + x)/a)**(1
/3)*(a/b + x)**9/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27
300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 273
00*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820
*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(-1 + b*(a/b + x)/a)**
(1/3)*(a/b + x)**10/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) +
 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 +
27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1
820*a**14*b**(22/3)*(a/b + x)**6), Abs(b*(a/b + x)/a) > 1), (-243*a**(73/3)
*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*
b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(1
3/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19
/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 243*a**(73/3)*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**
(22/3)*(a/b + x)**6) + 1377*a**(70/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x
)*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 1458*a**(70/3)*b*(a/b + x)*exp(I*pi/3)/(1820*
a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/
b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
 + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
 x)**6) - 3213*a**(67/3)*b**2*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I
*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*
b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b
**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**
(22/3)*(a/b + x)**6) + 3645*a**(67/3)*b**2*(a/b + x)**2*exp(I*pi/3)/(1820*a
**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
 + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b
+ x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b +
x)**6) + 3927*a**(64/3)*b**3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*
pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b
**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b*
*(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(
22/3)*(a/b + x)**6) - 4860*a**(64/3)*b**3*(a/b + x)**3*exp(I*pi/3)/(1820*a*
*20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b
+ x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
 x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x
)**6) - 2163*a**(61/3)*b**4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(I*p
i/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b*
*(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**
(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(2
2/3)*(a/b + x)**6) + 3645*a**(61/3)*b**4*(a/b + x)**4*exp(I*pi/3)/(1820*a**
20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
 x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b +
x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)
**6) - 1827*a**(58/3)*b**5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi
/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**
(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(
16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22
/3)*(a/b + x)**6) - 1458*a**(58/3)*b**5*(a/b + x)**5*exp(I*pi/3)/(1820*a**2
0*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b +
x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x
)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)*
```

```
*6) + 6573*a**(55/3)*b**6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/
3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(
10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(1
6/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/
3)*(a/b + x)**6) + 243*a**(55/3)*b**6*(a/b + x)**6*exp(I*pi/3)/(1820*a**20*
b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)
**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)*
*4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6
) - 8787*a**(52/3)*b**7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(I*pi/3)
/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300*a**18*b**(10
/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*a**16*b**(16/
3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)
*(a/b + x)**6) + 6498*a**(49/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**
8*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b + x) + 27300
*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x)**3 + 27300*
a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)**5 + 1820*a*
*14*b**(22/3)*(a/b + x)**6) - 2562*a**(46/3)*b**9*(1 - b*(a/b + x)/a)**(1/3
)*(a/b + x)**9*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19*b**(7/3)*(a/b
 + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(13/3)*(a/b + x
)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(19/3)*(a/b + x)
**5 + 1820*a**14*b**(22/3)*(a/b + x)**6) + 420*a**(43/3)*b**10*(1 - b*(a/b
+ x)/a)**(1/3)*(a/b + x)**10*exp(I*pi/3)/(1820*a**20*b**(4/3) - 10920*a**19
*b**(7/3)*(a/b + x) + 27300*a**18*b**(10/3)*(a/b + x)**2 - 36400*a**17*b**(
13/3)*(a/b + x)**3 + 27300*a**16*b**(16/3)*(a/b + x)**4 - 10920*a**15*b**(1
9/3)*(a/b + x)**5 + 1820*a**14*b**(22/3)*(a/b + x)**6), True))
```

3.670 $\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=51

$$\frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{5}a^2bx^{5/3}+\frac{3}{2}a^3x^{2/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*a^3*x^(2/3))/2 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^(8/3))/8 + (3*b^3*x^(1
1/3))/11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt[3]{x}}\,dx &= \int \left(\frac{a^3}{\sqrt[3]{x}}+3a^2bx^{2/3}+3ab^2x^{5/3}+b^3x^{8/3}\right)dx\\ &= \frac{3}{2}a^3x^{2/3}+\frac{9}{5}a^2bx^{5/3}+\frac{9}{8}ab^2x^{8/3}+\frac{3}{11}b^3x^{11/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.76

$$\frac{3}{440}x^{2/3}\left(220a^3+264a^2bx+165ab^2x^2+40b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*x^(2/3)*(220*a^3 + 264*a^2*b*x + 165*a*b^2*x^2 + 40*b^3*x^3))/440
```

IntegrateAlgebraic [A] time = 0.01, size = 47, normalized size = 0.92

$$\frac{3}{440}\left(220a^3x^{2/3}+264a^2bx^{5/3}+165ab^2x^{8/3}+40b^3x^{11/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(1/3),x]
```

```
[Out] (3*(220*a^3*x^(2/3) + 264*a^2*b*x^(5/3) + 165*a*b^2*x^(8/3) + 40*b^3*x^(11/
3)))/440
```

fricas [A] time = 0.78, size = 35, normalized size = 0.69

$$\frac{3}{440}\left(40\,b^3x^3 + 165\,ab^2x^2 + 264\,a^2bx + 220\,a^3\right)x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="fricas")
[Out] 3/440*(40*b^3*x^3 + 165*a*b^2*x^2 + 264*a^2*b*x + 220*a^3)*x^(2/3)
```

giac [A] time = 0.86, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}} + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="giac")
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.71

$$\frac{3\left(40b^3x^3 + 165a\,b^2x^2 + 264a^2bx + 220a^3\right)x^{\frac{2}{3}}}{440}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(1/3),x)
[Out] 3/440*x^(2/3)*(40*b^3*x^3+165*a*b^2*x^2+264*a^2*b*x+220*a^3)
```

maxima [A] time = 1.33, size = 35, normalized size = 0.69

$$\frac{3}{11}\,b^3x^{\frac{11}{3}} + \frac{9}{8}\,ab^2x^{\frac{8}{3}} + \frac{9}{5}\,a^2bx^{\frac{5}{3}} + \frac{3}{2}\,a^3x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(1/3),x, algorithm="maxima")
[Out] 3/11*b^3*x^(11/3) + 9/8*a*b^2*x^(8/3) + 9/5*a^2*b*x^(5/3) + 3/2*a^3*x^(2/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.69

$$\frac{3\,a^3\,x^{2/3}}{2} + \frac{3\,b^3\,x^{11/3}}{11} + \frac{9\,a^2\,b\,x^{5/3}}{5} + \frac{9\,a\,b^2\,x^{8/3}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(1/3),x)
[Out] (3*a^3*x^(2/3))/2 + (3*b^3*x^(11/3))/11 + (9*a^2*b*x^(5/3))/5 + (9*a*b^2*x^
(8/3))/8
```

sympy [C] time = 3.19, size = 6246, normalized size = 122.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(1/3),x)
[Out] Piecewise((243*a**(71/3)*(-1 + b*(a/b + x)/a)**(2/3)*exp(I*pi/3)/(440*a**20
*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a*
```

```
*18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*e
xp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**
(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi
/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1296*a**(68/3)*b*(-1 + b*(a/b + x)/a)
**(2/3)*(a/b + x)*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(
440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
+ 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
+ x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*
a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6
*exp(I*pi/3)) + 2808*a**(65/3)*b**2*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**
2*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b +
x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*
b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(
I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/
3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)) - 3120*a**(62/3)*b**3*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(I*pi
/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*p
i/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*
(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) -
2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b +
x)**6*exp(I*pi/3)) - 4860*a**(62/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*e
xp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3
)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3)
+ 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b
 + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1710
*a**(59/3)*b**4*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(I*pi/3)/(440*a
**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3)
 - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x
)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**
16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*e
xp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 72*a**(56/3)*b
**5*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**5*exp(I*pi/3)/(440*a**20*b**(2/3
)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(
8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/
3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(
a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1
458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**1
9*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*p
i/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)
*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) +
 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1104*a**(53/3)*b**6*(-1 +
b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi
/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
```

```
+ x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*
a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**
5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a**(53/
3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*
(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800
*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)*
*4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*
b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1152*a**(50/3)*b**7*(-1 + b*(a/b + x)
/a)**(2/3)*(a/b + x)**7*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*
a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp
(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(1
4/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/
3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 585*a**(47/3)*b**8*(-1
 + b*(a/b + x)/a)**(2/3)*(a/b + x)**8*exp(I*pi/3)/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 120*a**(
44/3)*b**9*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**9*exp(I*pi/3)/(440*a**20*
b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**
18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*ex
p(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(
17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/
3)), Abs(b*(a/b + x)/a) > 1), (-243*a**(71/3)*(1 - b*(a/b + x)/a)**(2/3)/(4
40*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) +
 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b +
 x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a
**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*
exp(I*pi/3)) + 243*a**(71/3)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1296*a**(68/3)*b*(1 - b*(a/b
 + x)/a)**(2/3)*(a/b + x)/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
 x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1458*a**(68/3)*b*(a/b + x)/(440*a
**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 660
0*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)*
*3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15
*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(
I*pi/3)) - 2808*a**(65/3)*b**2*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(440
*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6
600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x
)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**
15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*ex
p(I*pi/3)) + 3645*a**(65/3)*b**2*(a/b + x)**2/(440*a**20*b**(2/3)*exp(I*pi/
3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b +
 x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a
**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5
*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3120*a**(62/
3)*b**3*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*p
i/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b
 + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600
*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)*
*5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 4860*a**(6
2/3)*b**3*(a/b + x)**3/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3
)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 88
```

```
00*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x
)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**1
4*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1710*a**(59/3)*b**4*(1 - b*(a/b + x
)/a)**(2/3)*(a/b + x)**4/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5
/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b +
 x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a*
*14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 3645*a**(59/3)*b**4*(a/b + x)**4/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
 + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
 + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 72*a**(56/3)*b**5*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**5/
(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3)
 + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b
 + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640
*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**
6*exp(I*pi/3)) - 1458*a**(56/3)*b**5*(a/b + x)**5/(440*a**20*b**(2/3)*exp(I
*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a
/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 66
00*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x
)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 1104*a**
(53/3)*b**6*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(440*a**20*b**(2/3)*exp
(I*pi/3) - 2640*a**19*b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*
(a/b + x)**2*exp(I*pi/3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) +
6600*a**16*b**(14/3)*(a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b +
 x)**5*exp(I*pi/3) + 440*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 243*a*
*(53/3)*b**6*(a/b + x)**6/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b**(
5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3) -
 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/b
+ x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440*a
**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 1152*a**(50/3)*b**7*(1 - b*(a/b
+ x)/a)**(2/3)*(a/b + x)**7/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b*
*(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3)
 - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a/
b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 440
*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) + 585*a**(47/3)*b**8*(1 - b*(a/b
 + x)/a)**(2/3)*(a/b + x)**8/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*b
**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/3
) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(a
/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 44
0*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)) - 120*a**(44/3)*b**9*(1 - b*(a/
b + x)/a)**(2/3)*(a/b + x)**9/(440*a**20*b**(2/3)*exp(I*pi/3) - 2640*a**19*
b**(5/3)*(a/b + x)*exp(I*pi/3) + 6600*a**18*b**(8/3)*(a/b + x)**2*exp(I*pi/
3) - 8800*a**17*b**(11/3)*(a/b + x)**3*exp(I*pi/3) + 6600*a**16*b**(14/3)*(
a/b + x)**4*exp(I*pi/3) - 2640*a**15*b**(17/3)*(a/b + x)**5*exp(I*pi/3) + 4
40*a**14*b**(20/3)*(a/b + x)**6*exp(I*pi/3)), True))
```

3.671 $\int \frac{(a+bx)^3}{x^{2/3}}\,dx$

Optimal. Leaf size=49

$$3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{4}a^2bx^{4/3}+3a^3\sqrt[3]{x}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(2/3),x]
```

```
[Out] 3*a^3*x^(1/3) + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3))/7 + (3*b^3*x^(10/3)
)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{2/3}}\,dx &= \int \left(\frac{a^3}{x^{2/3}}+3a^2b\sqrt[3]{x}+3ab^2x^{4/3}+b^3x^{7/3}\right)dx\\ &= 3a^3\sqrt[3]{x}+\frac{9}{4}a^2bx^{4/3}+\frac{9}{7}ab^2x^{7/3}+\frac{3}{10}b^3x^{10/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{3}{140}\sqrt[3]{x}\left(140a^3+105a^2bx+60ab^2x^2+14b^3x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(2/3),x]
```

```
[Out] (3*x^(1/3)*(140*a^3 + 105*a^2*b*x + 60*a*b^2*x^2 + 14*b^3*x^3))/140
```

IntegrateAlgebraic [A] time = 0.02, size = 47, normalized size = 0.96

$$\frac{3}{140}\left(140a^3\sqrt[3]{x}+105a^2bx^{4/3}+60ab^2x^{7/3}+14b^3x^{10/3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(2/3),x]
```

```
[Out] (3*(140*a^3*x^(1/3) + 105*a^2*b*x^(4/3) + 60*a*b^2*x^(7/3) + 14*b^3*x^(10/3
)))/140
```

fricas [A] time = 1.30, size = 35, normalized size = 0.71

$$\frac{3}{140} \left(14 \, b^{3} x^{3} + 60 \, a b^{2} x^{2} + 105 \, a^{2} b x + 140 \, a^{3}\right) x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(14*b^3*x^3 + 60*a*b^2*x^2 + 105*a^2*b*x + 140*a^3)*x^(1/3)
```

giac [A] time = 0.94, size = 35, normalized size = 0.71

$$\frac{3}{10} \, b^{3} x^{\frac{10}{3}} + \frac{9}{7} \, a b^{2} x^{\frac{7}{3}} + \frac{9}{4} \, a^{2} b x^{\frac{4}{3}} + 3 \, a^{3} x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="giac")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

maple [A] time = 0.01, size = 36, normalized size = 0.73

$$\frac{3 \left(14 b^{3} x^{3}+60 a \,b^{2} x^{2}+105 a^{2} b x +140 a^{3}\right) x^{\frac{1}{3}}}{140}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(2/3),x)
```

```
[Out] 3/140*x^(1/3)*(14*b^3*x^3+60*a*b^2*x^2+105*a^2*b*x+140*a^3)
```

maxima [A] time = 1.29, size = 35, normalized size = 0.71

$$\frac{3}{10} \, b^{3} x^{\frac{10}{3}} + \frac{9}{7} \, a b^{2} x^{\frac{7}{3}} + \frac{9}{4} \, a^{2} b x^{\frac{4}{3}} + 3 \, a^{3} x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(2/3),x, algorithm="maxima")
```

```
[Out] 3/10*b^3*x^(10/3) + 9/7*a*b^2*x^(7/3) + 9/4*a^2*b*x^(4/3) + 3*a^3*x^(1/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$3\,a^3\,x^{1/3}+\frac{3\,b^3\,x^{10/3}}{10}+\frac{9\,a^2\,b\,x^{4/3}}{4}+\frac{9\,a\,b^2\,x^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(2/3),x)
```

```
[Out] 3*a^3*x^(1/3) + (3*b^3*x^(10/3))/10 + (9*a^2*b*x^(4/3))/4 + (9*a*b^2*x^(7/3
))/7
```

sympy [C] time = 3.17, size = 6667, normalized size = 136.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(2/3),x)
```

```
[Out] Piecewise((243*a**(70/3)*(-1 + b*(a/b + x)/a)**(1/3)*exp(2*I*pi/3)/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
```

00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1377*a**(67/3)*b*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3213*a**(64/3)*b**2*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 3927*a**(61/3)*b**3*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860*a**(61/3)*b**3*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 2583*a**(58/3)*b**4*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*a**(58/3)*b**4*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 693*a**(55/3)*b**5*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 1458*a**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 273*a**(52/3)*b**6*

(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6*exp(2*I*pi/3)/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) + 243*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
5*exp(2*I*pi/3) + 140*a14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 387*a
(49/3)*b7*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp(2*I*pi/3)/(140*a
20*b(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 8
40*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) - 198*a**(46/3)*b**8*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
+ x)**8*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3
)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)
- 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a
/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) +
140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 42*a**(43/3)*b**9*(-1 +
b*(a/b + x)/a)**(1/3)*(a/b + x)**9*exp(2*I*pi/3)/(140*a**20*b**(1/3)*exp(2*
I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*
(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3
) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(
a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3))
, Abs(b*(a/b + x)/a) > 1), (-243*a**(70/3)*(1 - b*(a/b + x)/a)**(1/3)/(140*
a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) +
2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b
+ x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) -
840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b +
x)**6*exp(2*I*pi/3)) + 243*a**(70/3)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 8
40*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**
2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a*
*16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5
*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 1377*a**
(67/3)*b*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*p
i/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/
b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) +
2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b
+ x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) -
1458*a**(67/3)*b*(a/b + x)/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b*
(4/3)(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*p
i/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/
3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi
/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 3213*a**(64/3)*b**2
(1 - b(a/b + x)/a)**(1/3)*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x
)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100
*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)
5*exp(2*I*pi/3) + 140*a14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3645*
a**(64/3)*b**2*(a/b + x)**2/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b
(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a18*b**(7/3)*(a/b + x)**2*exp(2*I*
pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13
/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*p
i/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 3927*a**(61/3)*b**
3*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3)
- 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210

```
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 4860
*a**(61/3)*b**3*(a/b + x)**3/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 2583*a**(58/3)*b*
*4*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 364
5*a**(58/3)*b**4*(a/b + x)**4/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 693*a**(55/3)*b*
*5*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 145
8*a**(55/3)*b**5*(a/b + x)**5/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19
*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*
I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(
13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I
*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 273*a**(52/3)*b*
*6*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3
) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
 x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 21
00*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b +
x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 243
*a**(52/3)*b**6*(a/b + x)**6/(140*a**20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*
b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b + x)**2*exp(2*I
*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 2100*a**16*b**(1
3/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*
pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) - 387*a**(49/3)*b**
7*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(140*a**20*b**(1/3)*exp(2*I*pi/3)
 - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**(7/3)*(a/b +
x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3) + 210
0*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(16/3)*(a/b + x
)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)) + 198*
a**(46/3)*b**8*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**8/(140*a**20*b**(1/3)*
exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 2100*a**18*b**
(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b + x)**3*exp(2*
I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840*a**15*b**(1
6/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)**6*exp(2*I*
pi/3)) - 42*a**(43/3)*b**9*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**9/(140*a**
20*b**(1/3)*exp(2*I*pi/3) - 840*a**19*b**(4/3)*(a/b + x)*exp(2*I*pi/3) + 21
00*a**18*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3) - 2800*a**17*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3) + 2100*a**16*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3) - 840
*a**15*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3) + 140*a**14*b**(19/3)*(a/b + x)
**6*exp(2*I*pi/3)), True))
```

3.672 $\int \frac{(a+bx)^3}{x^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{9}{2}a^2bx^{2/3}-\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (-3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3))/5 + (3*b^3*x^(8/
3))/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{4/3}}\,dx &= \int \left(\frac{a^3}{x^{4/3}}+\frac{3a^2b}{\sqrt[3]{x}}+3ab^2x^{2/3}+b^3x^{5/3}\right)dx\\ &= -\frac{3a^3}{\sqrt[3]{x}}+\frac{9}{2}a^2bx^{2/3}+\frac{9}{5}ab^2x^{5/3}+\frac{3}{8}b^3x^{8/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{3\left(-40a^3+60a^2bx+24ab^2x^2+5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (3*(-40*a^3 + 60*a^2*b*x + 24*a*b^2*x^2 + 5*b^3*x^3))/(40*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{3\left(-40a^3+60a^2bx+24ab^2x^2+5b^3x^3\right)}{40\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(4/3),x]
```

```
[Out] (3*(-40*a^3 + 60*a^2*b*x + 24*a*b^2*x^2 + 5*b^3*x^3))/(40*x^(1/3))
```

fricas [A] time = 0.98, size = 35, normalized size = 0.71

$$\frac{3\left(5\,b^3x^3+24\,ab^2x^2+60\,a^2bx-40\,a^3\right)}{40\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="fricas")
[Out] 3/40*(5*b^3*x^3 + 24*a*b^2*x^2 + 60*a^2*b*x - 40*a^3)/x^(1/3)
```

giac [A] time = 1.07, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="giac")
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.73

$$-\frac{3\left(-5b^3x^3-24a\,b^2x^2-60a^2bx+40a^3\right)}{40x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(4/3),x)
[Out] -3/40*(-5*b^3*x^3-24*a*b^2*x^2-60*a^2*b*x+40*a^3)/x^(1/3)
```

maxima [A] time = 1.32, size = 35, normalized size = 0.71

$$\frac{3}{8}\,b^3x^{\frac{8}{3}}+\frac{9}{5}\,ab^2x^{\frac{5}{3}}+\frac{9}{2}\,a^2bx^{\frac{2}{3}}-\frac{3\,a^3}{x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(4/3),x, algorithm="maxima")
[Out] 3/8*b^3*x^(8/3) + 9/5*a*b^2*x^(5/3) + 9/2*a^2*b*x^(2/3) - 3*a^3/x^(1/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{8/3}}{8}-\frac{3\,a^3}{x^{1/3}}+\frac{9\,a^2\,b\,x^{2/3}}{2}+\frac{9\,a\,b^2\,x^{5/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(4/3),x)
[Out] (3*b^3*x^(8/3))/8 - (3*a^3)/x^(1/3) + (9*a^2*b*x^(2/3))/2 + (9*a*b^2*x^(5/3
))/5
```

sympy [C] time = 3.26, size = 4004, normalized size = 81.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(4/3),x)
```

```
[Out] Piecewise((243*a**(68/3)*b**(1/3)*(-1 + b*(a/b + x)/a)**(2/3)/(40*a**20 - 2
40*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b +
x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**1
4*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 8
00*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(
a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b +
x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b +
x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**
15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3
)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**2/(40*a**20 - 240*a**19*b*(a/b + x
) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b
**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6
) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**1
9*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 +
 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*
(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b +
 x)**3/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 80
0*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a
/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b +
x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b
 + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*
a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(
13/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**4/(40*a**20 - 240*a**19*b*(a/b
 + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**
16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x
)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240
*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)
**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*
b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**5/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/
b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*
(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 -
240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b*
*(19/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a/b + x)**6/(40*a**20 - 240*a**19*b*(a
/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a
**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b +
 x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 24
0*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x
)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14
*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(-1 + b*(a/b + x)/a)**(2/3)*(a
/b + x)**7/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(-1 +
 b*(a/b + x)/a)**(2/3)*(a/b + x)**8/(40*a**20 - 240*a**19*b*(a/b + x) + 600
*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/
b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), Abs(
b*(a/b + x)/a) > 1), (243*a**(68/3)*b**(1/3)*(1 - b*(a/b + x)/a)**(2/3)*exp
(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2
- 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**
5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(68/3)*b**(1/3)*exp(2
*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 -
800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*
(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 1296*a**(65/3)*b**(4/3)*(1 - b
```

*(a/b + x)/a)**(2/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(65/3)*b**(4/3)*(a/b + x)*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 2808*a**(62/3)*b**(7/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3645*a**(62/3)*b**(7/3)*(a/b + x)**2*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3120*a**(59/3)*b**(10/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 4860*a**(59/3)*b**(10/3)*(a/b + x)**3*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1830*a**(56/3)*b**(13/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 3645*a**(56/3)*b**(13/3)*(a/b + x)**4*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 528*a**(53/3)*b**(16/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 1458*a**(53/3)*b**(16/3)*(a/b + x)**5*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 96*a**(50/3)*b**(19/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 243*a**(50/3)*b**(19/3)*(a/b + x)**6*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) - 48*a**(47/3)*b**(22/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**7*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6) + 15*a**(44/3)*b**(25/3)*(1 - b*(a/b + x)/a)**(2/3)*(a/b + x)**8*exp(2*I*pi/3)/(40*a**20 - 240*a**19*b*(a/b + x) + 600*a**18*b**2*(a/b + x)**2 - 800*a**17*b**3*(a/b + x)**3 + 600*a**16*b**4*(a/b + x)**4 - 240*a**15*b**5*(a/b + x)**5 + 40*a**14*b**6*(a/b + x)**6), True))

3.673 $\int \frac{(a+bx)^3}{x^{5/3}} dx$

Optimal. Leaf size=49

$$-\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$9a^2b\sqrt[3]{x}-\frac{3a^3}{2x^{2/3}}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (-3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3))/4 + (3*b^3*x^(7/
3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{x^{5/3}}\,dx &= \int \left(\frac{a^3}{x^{5/3}}+\frac{3a^2b}{x^{2/3}}+3ab^2\sqrt[3]{x}+b^3x^{4/3}\right)dx\\ &= -\frac{3a^3}{2x^{2/3}}+9a^2b\sqrt[3]{x}+\frac{9}{4}ab^2x^{4/3}+\frac{3}{7}b^3x^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.80

$$\frac{3\left(-14a^3+84a^2bx+21ab^2x^2+4b^3x^3\right)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (3*(-14*a^3 + 84*a^2*b*x + 21*a*b^2*x^2 + 4*b^3*x^3))/(28*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.02, size = 39, normalized size = 0.80

$$\frac{3\left(-14a^3+84a^2bx+21ab^2x^2+4b^3x^3\right)}{28x^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/x^(5/3),x]
```

```
[Out] (3*(-14*a^3 + 84*a^2*b*x + 21*a*b^2*x^2 + 4*b^3*x^3))/(28*x^(2/3))
```

fricas [A] time = 1.31, size = 35, normalized size = 0.71

$$\frac{3\left(4\,b^3x^3+21\,ab^2x^2+84\,a^2bx-14\,a^3\right)}{28\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="fricas")
```

```
[Out] 3/28*(4*b^3*x^3 + 21*a*b^2*x^2 + 84*a^2*b*x - 14*a^3)/x^(2/3)
```

giac [A] time = 1.17, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="giac")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

maple [A] time = 0.00, size = 36, normalized size = 0.73

$$-\frac{3\left(-4b^3x^3-21a\,b^2x^2-84a^2bx+14a^3\right)}{28x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/x^(5/3),x)
```

```
[Out] -3/28*(-4*b^3*x^3-21*a*b^2*x^2-84*a^2*b*x+14*a^3)/x^(2/3)
```

maxima [A] time = 1.36, size = 35, normalized size = 0.71

$$\frac{3}{7}\,b^3x^{\frac{7}{3}}+\frac{9}{4}\,ab^2x^{\frac{4}{3}}+9\,a^2bx^{\frac{1}{3}}-\frac{3\,a^3}{2\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/x^(5/3),x, algorithm="maxima")
```

```
[Out] 3/7*b^3*x^(7/3) + 9/4*a*b^2*x^(4/3) + 9*a^2*b*x^(1/3) - 3/2*a^3/x^(2/3)
```

mupad [B] time = 0.04, size = 35, normalized size = 0.71

$$\frac{3\,b^3\,x^{7/3}}{7}-\frac{3\,a^3}{2\,x^{2/3}}+9\,a^2\,b\,x^{1/3}+\frac{9\,a\,b^2\,x^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/x^(5/3),x)
```

```
[Out] (3*b^3*x^(7/3))/7 - (3*a^3)/(2*x^(2/3)) + 9*a^2*b*x^(1/3) + (9*a*b^2*x^(4/3
))/4
```

sympy [C] time = 3.24, size = 3964, normalized size = 80.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/x**(5/3),x)
```

```
[Out] Piecewise((243*a**(67/3)*b**(2/3)*(-1 + b*(a/b + x)/a)**(1/3)/(28*a**20 - 1
68*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b +
x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**1
4*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**20 - 168*a
**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**
3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b*
*6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b
 + x)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560
*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/
b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3)*b**(5/3)*(a/b + x)
*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**
2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b
**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a**(61/3)*b**(8/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**2/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a
/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a
**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
 x)**6) - 3927*a**(58/3)*b**(11/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**3
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(-
1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**4/(28*a**20 - 168*a**19*b*(a/b + x) +
420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*
(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) -
3645*a**(55/3)*b**(14/3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(
a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*
a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b
+ x)**6) - 903*a**(52/3)*b**(17/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**5
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*
exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2
 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b*
*5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(-1
 + b*(a/b + x)/a)**(1/3)*(a/b + x)**6/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(-1 + b*(a/b + x)/a)**(1/3)*(a/b + x)**7/(2
8*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b
**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**
5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(-1 + b*(a/b + x)/
a)**(1/3)*(a/b + x)**8/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(
a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 1
68*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6), Abs(b*(a/b + x)/a
) > 1), (243*a**(67/3)*b**(2/3)*(1 - b*(a/b + x)/a)**(1/3)*exp(I*pi/3)/(28*
a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**
3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5
+ 28*a**14*b**6*(a/b + x)**6) - 243*a**(67/3)*b**(2/3)*exp(I*pi/3)/(28*a**2
0 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a
/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28
*a**14*b**6*(a/b + x)**6) - 1377*a**(64/3)*b**(5/3)*(1 - b*(a/b + x)/a)**(1
```

```
/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**
2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4
- 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 1458*a**(64/3
)*b**(5/3)*(a/b + x)*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a*
*18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b +
 x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 3213*a
**(61/3)*b**(8/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**2*exp(I*pi/3)/(28*a
**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3
*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 +
 28*a**14*b**6*(a/b + x)**6) - 3645*a**(61/3)*b**(8/3)*(a/b + x)**2*exp(I*p
i/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*
a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b
 + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3927*a**(58/3)*b**(11/3)*(1 - b*(a
/b + x)/a)**(1/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x
) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b
**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6
) + 4860*a**(58/3)*b**(11/3)*(a/b + x)**3*exp(I*pi/3)/(28*a**20 - 168*a**19
*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 +
420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(
a/b + x)**6) + 2625*a**(55/3)*b**(14/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x
)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b +
x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**
15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 3645*a**(55/3)*b**(14/
3)*(a/b + x)**4*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b
**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**
4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 903*a**(52/
3)*b**(17/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**5*exp(I*pi/3)/(28*a**20
- 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b
 + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a
**14*b**6*(a/b + x)**6) + 1458*a**(52/3)*b**(17/3)*(a/b + x)**5*exp(I*pi/3)
/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**1
7*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x
)**5 + 28*a**14*b**6*(a/b + x)**6) + 147*a**(49/3)*b**(20/3)*(1 - b*(a/b +
x)/a)**(1/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 4
20*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(
a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) - 2
43*a**(49/3)*b**(20/3)*(a/b + x)**6*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/
b + x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a*
*16*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b +
x)**6) - 33*a**(46/3)*b**(23/3)*(1 - b*(a/b + x)/a)**(1/3)*(a/b + x)**7*exp
(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b + x) + 420*a**18*b**2*(a/b + x)**2 -
560*a**17*b**3*(a/b + x)**3 + 420*a**16*b**4*(a/b + x)**4 - 168*a**15*b**5*
(a/b + x)**5 + 28*a**14*b**6*(a/b + x)**6) + 12*a**(43/3)*b**(26/3)*(1 - b*
(a/b + x)/a)**(1/3)*(a/b + x)**8*exp(I*pi/3)/(28*a**20 - 168*a**19*b*(a/b +
 x) + 420*a**18*b**2*(a/b + x)**2 - 560*a**17*b**3*(a/b + x)**3 + 420*a**16
*b**4*(a/b + x)**4 - 168*a**15*b**5*(a/b + x)**5 + 28*a**14*b**6*(a/b + x)*
*6), True))
```

3.674 $\int \frac{x^{5/3}}{a+bx} \, dx$

Optimal. Leaf size=125

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Rubi [A] time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$-\frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}+\frac{a^{5/3}\log(a+bx)}{2b^{8/3}}-\frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^2}+\frac{3x^{5/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(2/3))/(2*b^2) + (3*x^(5/3))/(5*b) - (Sqrt[3]*a^(5/3)*ArcTan[(a^(1/
3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(8/3) - (3*a^(5/3)*Log[a^(1/3
) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) + (a^(5/3)*Log[a + b*x])/(2*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{a+bx}\,dx = \frac{3x^{5/3}}{5b} - \frac{a\int \frac{x^{2/3}}{a+bx}\,dx}{b}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{b^2}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} - \frac{(3a^{5/3})\,\text{Subst}\left(\int \right.}{2b^{8/}}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}} + \frac{(3a^{5/3})\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 -\right.}{b^{8/3}}$$
$$= -\frac{3ax^{2/3}}{2b^2} + \frac{3x^{5/3}}{5b} - \frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{8/3}} - \frac{3a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{2b^{8/3}} + \frac{a^{5/3}\log(a+bx)}{2b^{8/3}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.30

$$\frac{3x^{2/3}\left(5a\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right) - 5a + 2bx\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3)*(-5*a + 2*b*x + 5*a*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))
/(10*b^2)
```

IntegrateAlgebraic [A] time = 0.10, size = 150, normalized size = 1.20

$$\frac{a^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{8/3}} - \frac{a^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{b^{8/3}} - \frac{\sqrt{3}\,a^{5/3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{b^{8/3}} + \frac{3\left(2bx^{5/3}-5ax^{2/3}\right)}{10b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x),x]
```

```
[Out] (3*(-5*a*x^(2/3) + 2*b*x^(5/3)))/(10*b^2) - (Sqrt[3]*a^(5/3)*ArcTan[1/Sqrt[
3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]])/b^(8/3) - (a^(5/3)*Log[a^(1/3)
 + b^(1/3)*x^(1/3)])/b^(8/3) + (a^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/
3) + b^(2/3)*x^(2/3)])/(2*b^(8/3))
```

fricas [A] time = 0.85, size = 147, normalized size = 1.18

$$\frac{10\sqrt{3}\,a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}+\sqrt{3}\,a}{3a}\right) - 5a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}} + ax^{\frac{2}{3}} - a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) + 10a\left(-\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(-\frac{a^2}{b^2}\right)^{\frac{2}{3}} + ax^{\frac{1}{3}}\right) + 3\,(2bx-5a)x^{\frac{2}{3}}}{10b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/10*(10*sqrt(3)*a*(-a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a^2/b
^2)^(1/3) + sqrt(3)*a)/a) - 5*a*(-a^2/b^2)^(1/3)*log(-b*x^(1/3)*(-a^2/b^2)^
```

```
(2/3) + a*x^(2/3) - a*(-a^2/b^2)^(1/3)) + 10*a*(-a^2/b^2)^(1/3)*log(b*(-a^2
/b^2)^(2/3) + a*x^(1/3)) + 3*(2*b*x - 5*a)*x^(2/3))/b^2
```

giac [A] time = 1.04, size = 138, normalized size = 1.10

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^{2}}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{4}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{4}}+\frac{3\left(2b^{4}x^{\frac{5}{3}}-5ab^{3}x^{\frac{2}{3}}\right)}{10b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - sqrt(3)*(-a*b^2)^(2/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 + 1/2*
(-a*b^2)^(2/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4 + 3
/10*(2*b^4*x^(5/3) - 5*a*b^3*x^(2/3))/b^5
```

maple [A] time = 0.01, size = 122, normalized size = 0.98

$$\frac{3x^{\frac{5}{3}}}{5b}+\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{a^{2}\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{a^{2}\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{3a\,x^{\frac{2}{3}}}{2b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a),x)
```

```
[Out] 3/5*x^(5/3)/b-3/2*a*x^(2/3)/b^2-a^2/b^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))
+1/2*a^2/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+a^2/b^
3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 3.00, size = 130, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3\left(2bx^{\frac{5}{3}}-5ax^{\frac{2}{3}}\right)}{10b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(1/3)) + 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(1/3)) - a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/10*(2
*b*x^(5/3) - 5*a*x^(2/3))/b^2
```

mupad [B] time = 0.24, size = 151, normalized size = 1.21

$$\frac{3x^{5/3}}{5b}+\frac{(-a)^{5/3}\ln\left(\frac{9a^{4}x^{1/3}}{b^{3}}-\frac{9(-a)^{13/3}}{b^{10/3}}\right)}{b^{8/3}}-\frac{3ax^{2/3}}{2b^{2}}+\frac{(-a)^{5/3}\ln\left(\frac{9a^{4}x^{1/3}}{b^{3}}-\frac{9(-a)^{13/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}}{b^{10/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{8/3}}-\frac{(-a)^{5/3}\ln\left(\frac{9a^{4}x^{1/3}}{b^{3}}-\frac{9(-a)^{13/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}}{b^{10/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(5/3)/(a + b*x),x)

[Out] (3*x^(5/3))/(5*b) + ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3))/b ^(10/3)))/b^(8/3) - (3*a*x^(2/3))/(2*b^2) + ((-a)^(5/3)*log((9*a^4*x^(1/3)) /b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 - 1/2))/b^(8/3) - ((-a)^(5/3)*log((9*a^4*x^(1/3))/b^3 - (9*(-a)^(13/3)*((3^(1/2)*1i)/2 + 1/2)^2)/b^(10/3))*((3^(1/2)*1i)/2 + 1/2))/b^(8/3)

sympy [A] time = 47.12, size = 241, normalized size = 1.93

$$\begin{cases} \tilde{\infty} x^{\frac{5}{3}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3 x^{\frac{8}{3}}}{8 a} & \text{for}\: b = 0 \\ \frac{3 x^{\frac{5}{3}}}{5 b} & \text{for}\: a = 0 \\ - \frac{(-1)^{\frac{2}{3}} a^{\frac{5}{3}} \log\left(- \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{b^{4} \left(\frac{1}{b}\right)^{\frac{4}{3}}} + \frac{(-1)^{\frac{2}{3}} a^{\frac{5}{3}} \log\left(4 (-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4 \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{x} \sqrt[3]{\frac{1}{b}} + 4 x^{\frac{2}{3}}\right)}{2 b^{4} \left(\frac{1}{b}\right)^{\frac{4}{3}}} - \frac{(-1)^{\frac{2}{3}} \sqrt{3} a^{\frac{5}{3}} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2 (-1)^{\frac{2}{3}} \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{a} \sqrt[3]{\frac{1}{b}}}\right)}{b^{4} \left(\frac{1}{b}\right)^{\frac{4}{3}}} - \frac{3 a x^{\frac{2}{3}}}{2 b^{2}} + \frac{3 x^{\frac{5}{3}}}{5 b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(5/3)/(b*x+a),x)

[Out] Piecewise((zoo*x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(8/3)/(8*a), Eq(b, 0)) , (3*x**(5/3)/(5*b), Eq(a, 0)), (-(-1)**(2/3)*a**(5/3)*log(-(-1)**(1/3)*a** (1/3)*(1/b)**(1/3) + x**(1/3))/(b**4*(1/b)**(4/3)) + (-1)**(2/3)*a**(5/3)*l og(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1 /b)**(1/3) + 4*x**(2/3))/(2*b**4*(1/b)**(4/3)) - (-1)**(2/3)*sqrt(3)*a**(5/ 3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(b**4*(1/b)**(4/3)) - 3*a*x**(2/3)/(2*b**2) + 3*x**(5/3)/(5*b), True))

3.675 $\int \frac{x^{4/3}}{a+bx}\,dx$

Optimal. Leaf size=123

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Rubi [A] time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$\frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}}-\frac{a^{4/3}\log(a+bx)}{2b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}-\frac{3a\sqrt[3]{x}}{b^2}+\frac{3x^{4/3}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x),x]
```

```
[Out] (-3*a*x^(1/3))/b^2 + (3*x^(4/3))/(4*b) - (Sqrt[3]*a^(4/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (3*a^(4/3)*Log[a^(1/3) +
b^(1/3)*x^(1/3)])/(2*b^(7/3)) - (a^(4/3)*Log[a + b*x])/(2*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{a+bx}\,dx = \frac{3x^{4/3}}{4b} - \frac{a\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{b}$$
$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{a^2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b^2}$$
$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{5/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{8/3}} + \frac{\left(3a^{4/3}\right)\text{Subst}\left(\int \right.}{2b}$$
$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}} + \frac{\left(3a^{4/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{b^{7/3}}$$
$$= -\frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{4/3}}{4b} - \frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{7/3}} + \frac{3a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{7/3}} - \frac{a^{4/3}\log(a+bx)}{2b^{7/3}}$$

Mathematica [A] time = 0.06, size = 140, normalized size = 1.14

$$\frac{-2a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)+4a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)-4\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-12a\sqrt[3]{b}\sqrt[3]{x}+3b^{4/3}x^{4/3}}{4b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x),x]
```

```
[Out] (-12*a*b^(1/3)*x^(1/3) + 3*b^(4/3)*x^(4/3) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 -
(2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[a^(1/3) + b^(1/3)*x^(
1/3)] - 2*a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])
/(4*b^(7/3))
```

IntegrateAlgebraic [A] time = 0.09, size = 144, normalized size = 1.17

$$-\frac{a^{4/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2b^{7/3}}+\frac{a^{4/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}-\frac{\sqrt{3}\,a^{4/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{7/3}}+\frac{3\sqrt[3]{x}\,(bx-4a)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3)*(-4*a + b*x))/(4*b^2) - (Sqrt[3]*a^(4/3)*ArcTan[1/Sqrt[3] - (2*b
^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/b^(7/3) + (a^(4/3)*Log[a^(1/3) + b^(1/3
)*x^(1/3)])/b^(7/3) - (a^(4/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2
/3)*x^(2/3)])/(2*b^(7/3))
```

fricas [A] time = 1.42, size = 116, normalized size = 0.94

$$\frac{4\sqrt{3}\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}\,a}{3a}\right)-2a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)+4a\left(\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)+3(bx-4a)x^{\frac{1}{3}}}{4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(4*sqrt(3)*a*(a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a/b)^(2/3) -
sqrt(3)*a)/a) - 2*a*(a/b)^(1/3)*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(
2/3)) + 4*a*(a/b)^(1/3)*log(x^(1/3) + (a/b)^(1/3)) + 3*(b*x - 4*a)*x^(1/3))
/b^2
```

giac [A] time = 1.21, size = 136, normalized size = 1.11

$$-\frac{a\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b^{2}}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}a\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{3}}+\frac{3\left(b^{3}x^{\frac{4}{3}}-4ab^{2}x^{\frac{1}{3}}\right)}{4b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -a*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + sqrt(3)*(-a*b^2)^(1/
3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + 1/2*
(-a*b^2)^(1/3)*a*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3 + 3
/4*(b^3*x^(4/3) - 4*a*b^2*x^(1/3))/b^4
```

maple [A] time = 0.01, size = 121, normalized size = 0.98

$$\frac{3x^{\frac{4}{3}}}{4b}+\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{a^{2}\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{a^{2}\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{3a\,x^{\frac{1}{3}}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a),x)
```

```
[Out] 3/4*x^(4/3)/b-3*a*x^(1/3)/b^2+a^2/b^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1
/2*a^2/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+a^2/b^3/
(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 3.05, size = 128, normalized size = 1.04

$$\frac{\sqrt{3}\,a^{2}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{a^{2}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{a^{2}\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3\left(bx^{\frac{4}{3}}-4\,ax^{\frac{1}{3}}\right)}{4b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*a^2*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*
(a/b)^(2/3)) - 1/2*a^2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^
3*(a/b)^(2/3)) + a^2*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3/4*(b*
x^(4/3) - 4*a*x^(1/3))/b^2
```

mupad [B] time = 0.07, size = 126, normalized size = 1.02

$$\frac{3\,x^{4/3}}{4\,b}-\frac{3\,a\,x^{1/3}}{b^{2}}+\frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{1/3}}+9\,a^{2}\,x^{1/3}\right)}{b^{7/3}}+\frac{a^{4/3}\,\ln\left(9\,a^{2}\,x^{1/3}+\frac{9\,a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{7/3}}-\frac{a^{4/3}\,\ln\left(9\,a^{2}\,x^{1/3}-\frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{1/3}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x),x)
```

```
[Out] (3*x^(4/3))/(4*b) - (3*a*x^(1/3))/b^2 + (a^(4/3)*log((9*a^(7/3))/b^(1/3) +
9*a^2*x^(1/3)))/b^(7/3) + (a^(4/3)*log(9*a^2*x^(1/3) + (9*a^(7/3)*((3^(1/2)
*1i)/2 - 1/2))/b^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(7/3) - (a^(4/3)*log(9*a^
2*x^(1/3) - (9*a^(7/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1
/2))/b^(7/3)
```

sympy [A] time = 25.86, size = 240, normalized size = 1.95

$$\begin{cases} \tilde{\infty} x^{\frac{4}{3}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{7}{3}}}{7a} & \text{for } b = 0 \\ \frac{3x^{\frac{4}{3}}}{4b} & \text{for } a = 0 \\ -\frac{\sqrt[3]{-1}\, a^{\frac{4}{3}} \sqrt[3]{\frac{1}{b}} \log\left(-\sqrt[3]{-1}\, \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{b^2} + \frac{\sqrt[3]{-1}\, a^{\frac{4}{3}} \sqrt[3]{\frac{1}{b}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\, \sqrt[3]{a} \sqrt[3]{x} \sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2b^2} + \frac{\sqrt[3]{-1}\, \sqrt{3}\, a^{\frac{4}{3}} \sqrt[3]{\frac{1}{b}} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}} \sqrt{3} \sqrt[3]{x}}{3\sqrt[3]{a} \sqrt[3]{\frac{1}{b}}}\right)}{b^2} - \frac{3a\sqrt[3]{x}}{b^2} + \frac{3x^{\frac{4}{3}}}{4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(7/3)/(7*a), Eq(b, 0))
, (3*x**(4/3)/(4*b), Eq(a, 0)), (-(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(-(-
1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/b**2 + (-1)**(1/3)*a**(4/3)*(1/
b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*
x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(2*b**2) + (-1)**(1/3)*sqrt(3)*a**(4/3)
*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(
1/b)**(1/3)))/b**2 - 3*a*x**(1/3)/b**2 + 3*x**(4/3)/(4*b), True))
```

3.676 $\int \frac{x^{2/3}}{a+bx}\,dx$

Optimal. Leaf size=111

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Rubi [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 56, 617, 204, 31}

$$\frac{3a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}}-\frac{a^{2/3}\log(a+bx)}{2b^{5/3}}+\frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}}+\frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(
Sqrt[3]*a^(1/3))])/b^(5/3) + (3*a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*
b^(5/3)) - (a^(2/3)*Log[a + b*x])/(2*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{a+bx}\,dx = \frac{3x^{2/3}}{2b} - \frac{a\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{b}$$

$$= \frac{3x^{2/3}}{2b} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{(3a)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^2} + \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{3a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{5/3}}$$

$$= \frac{3x^{2/3}}{2b} + \frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{5/3}} + \frac{3a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{5/3}} - \frac{a^{2/3}\log(a+bx)}{2b^{5/3}}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.26

$$-\frac{3x^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)-1\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x),x]
```

```
[Out] (-3*x^(2/3)*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)]))/(2*b)
```

IntegrateAlgebraic [A] time = 0.08, size = 136, normalized size = 1.23

$$-\frac{a^{2/3}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{2b^{5/3}} + \frac{a^{2/3}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{b^{5/3}} + \frac{\sqrt{3}\,a^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{5/3}} + \frac{3x^{2/3}}{2b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x),x]
```

```
[Out] (3*x^(2/3))/(2*b) + (Sqrt[3]*a^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))
/(Sqrt[3]*a^(1/3))])/b^(5/3) + (a^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(
5/3) - (a^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(
2*b^(5/3))
```

fricas [A] time = 1.45, size = 128, normalized size = 1.15

$$-\frac{2\sqrt{3}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}} - \sqrt{3}\,a}{3a}\right) + \left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + ax^{\frac{2}{3}} + a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right) - 2\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}} + ax^{\frac{1}{3}}\right) - 3x^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(a^2/b^2)^(
1/3) - sqrt(3)*a)/a) + (a^2/b^2)^(1/3)*log(-b*x^(1/3)*(a^2/b^2)^(2/3) + a*x
^(2/3) + a*(a^2/b^2)^(1/3)) - 2*(a^2/b^2)^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x
^(1/3)) - 3*x^(2/3))/b
```

giac [A] time = 1.04, size = 118, normalized size = 1.06

$$\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}} \log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b} + \frac{3\,x^{\frac{2}{3}}}{2\,b} + \frac{\sqrt{3}\,\left(-ab^{2}\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{3}} - \frac{\left(-ab^{2}\right)^{\frac{2}{3}} \log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b + 3/2*x^(2/3)/b + sqrt(3)*(
-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b
^3 - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
b^3
```

maple [A] time = 0.01, size = 107, normalized size = 0.96

$$-\frac{\sqrt{3}\;a \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}} + \frac{a \ln\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}} - \frac{a \ln\left(x^{\frac{2}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}} b^{2}} + \frac{3x^{\frac{2}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a),x)
```

```
[Out] 3/2*x^(2/3)/b+a/b^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2*a/b^2/(a/b)^(1/
3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-a/b^2*3^(1/2)/(a/b)^(1/3)*ar
ctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.90, size = 114, normalized size = 1.03

$$-\frac{\sqrt{3}\,a \arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{3\,x^{\frac{2}{3}}}{2\,b} - \frac{a \log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{a \log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(1/3)) + 3/2*x^(2/3)/b - 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/
b)^(2/3))/(b^2*(a/b)^(1/3)) + a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(1/3)
)
```

mupad [B] time = 0.15, size = 130, normalized size = 1.17

$$\frac{3\,x^{2/3}}{2\,b} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}}{b^{4/3}} + \frac{9\,a^{2}\,x^{1/3}}{b}\right)}{b^{5/3}} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^{2}}{b^{4/3}} + \frac{9\,a^{2}\,x^{1/3}}{b}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{5/3}} - \frac{a^{2/3}\,\ln\left(\frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^{2}}{b^{4/3}} + \frac{9\,a^{2}\,x^{1/3}}{b}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x),x)
```

```
[Out] (3*x^(2/3))/(2*b) + (a^(2/3)*log((9*a^(7/3))/b^(4/3) + (9*a^2*x^(1/3))/b))/
b^(5/3) + (a^(2/3)*log((9*a^(7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(4/3) + (9*a^
2*x^(1/3))/b)*((3^(1/2)*1i)/2 - 1/2))/b^(5/3) - (a^(2/3)*log((9*a^(7/3)*((3
^(1/2)*1i)/2 + 1/2)^2)/b^(4/3) + (9*a^2*x^(1/3))/b)*((3^(1/2)*1i)/2 + 1/2))
/b^(5/3)
```

sympy [A] time = 9.08, size = 228, normalized size = 2.05

$$\begin{cases} \tilde{\infty}x^{\frac{2}{3}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{5}{3}}}{5a} & \text{for } b = 0 \\ \frac{3x^{\frac{2}{3}}}{2b} & \text{for } a = 0 \\ \frac{(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \log\left(-\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{b^2 \sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\,\sqrt[3]{a}\,\sqrt[3]{x}\,\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2b^2 \sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}} \sqrt{3}\, a^{\frac{2}{3}} \operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}} \sqrt{3}\,\sqrt[3]{x}}{3\sqrt[3]{a}\,\sqrt[3]{\frac{1}{b}}}\right)}{b^2 \sqrt[3]{\frac{1}{b}}} + \frac{3x^{\frac{2}{3}}}{2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a), Eq(b, 0))
, (3*x**(2/3)/(2*b), Eq(a, 0)), ((-1)**(2/3)*a**(2/3)*log(-(-1)**(1/3)*a**(
1/3)*(1/b)**(1/3) + x**(1/3))/(b**2*(1/b)**(1/3)) - (-1)**(2/3)*a**(2/3)*lo
g(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/
b)**(1/3) + 4*x**(2/3))/(2*b**2*(1/b)**(1/3)) + (-1)**(2/3)*sqrt(3)*a**(2/3
)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3))
)/(b**2*(1/b)**(1/3)) + 3*x**(2/3)/(2*b), True))
```

3.677 $\int \frac{\sqrt[3]{x}}{a+bx}\,dx$

Optimal. Leaf size=109

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$-\frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}}+\frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}}+\frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt
[3]*a^(1/3))])/b^(4/3) - (3*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(4
/3)) + (a^(1/3)*Log[a + b*x])/(2*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{a+bx}\,dx = \frac{3\sqrt[3]{x}}{b} - \frac{a\int \frac{1}{x^{2/3}(a+bx)}\,dx}{b}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^{5/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x\right.}{2b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}} - \frac{\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{b^{4/3}}$$

$$= \frac{3\sqrt[3]{x}}{b} + \frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{b^{4/3}} - \frac{3\sqrt[3]{a}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}\log(a+bx)}{2b^{4/3}}$$

Mathematica [A] time = 0.03, size = 126, normalized size = 1.16

$$\frac{\sqrt[3]{a}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right) - 2\sqrt[3]{a}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right) + 2\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 6\sqrt[3]{b}\sqrt[3]{x}}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x),x]
```

```
[Out] (6*b^(1/3)*x^(1/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1
/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)] + a^(1/3)*Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^(4/3))
```

IntegrateAlgebraic [A] time = 0.08, size = 135, normalized size = 1.24

$$\frac{\sqrt[3]{a}\log\left(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{2b^{4/3}} - \frac{\sqrt[3]{a}\log\left(\sqrt[3]{a} + \sqrt[3]{b}\sqrt[3]{x}\right)}{b^{4/3}} + \frac{\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{b^{4/3}} + \frac{3\sqrt[3]{x}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(a + b*x),x]
```

```
[Out] (3*x^(1/3))/b + (Sqrt[3]*a^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sq
rt[3]*a^(1/3))])/b^(4/3) - (a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/b^(4/3)
 + (a^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*b^
(4/3))
```

fricas [A] time = 1.28, size = 114, normalized size = 1.05

$$\frac{2\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}} - \sqrt{3}a}{3a}\right) - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) + 2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right) + 6x^{\frac{1}{3}}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(3)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b)^(2/3) -
sqrt(3)*a)/a) - (-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2
/3)) + 2*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^(1/3)) + 6*x^(1/3))/b
```

giac [A] time = 1.15, size = 119, normalized size = 1.09

$$\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x^{\frac{1}{3}} - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{b} - \frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}} + \frac{3\,x^{\frac{1}{3}}}{b} - \frac{\left(-ab^{2}\right)^{\frac{1}{3}} \log\left(x^{\frac{2}{3}} + x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] (-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b - sqrt(3)*(-a*b^2)^(1/3)*ar
ctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^2 + 3*x^(1/3)/b
 - 1/2*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^
2
```

maple [A] time = 0.01, size = 108, normalized size = 0.99

$$-\frac{\sqrt{3}\,a \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}} - \frac{a \ln\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}} + \frac{a \ln\left(x^{\frac{2}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}} x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}} b^{2}} + \frac{3x^{\frac{1}{3}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a),x)
```

```
[Out] 3*x^(1/3)/b-a/b^2/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2*a/b^2/(a/b)^(2/3)
*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-a/b^2/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.93, size = 115, normalized size = 1.06

$$-\frac{\sqrt{3}\,a \arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{3\,x^{\frac{1}{3}}}{b} + \frac{a \log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{a \log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^2*(
a/b)^(2/3)) + 3*x^(1/3)/b + 1/2*a*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)
^(2/3))/(b^2*(a/b)^(2/3)) - a*log(x^(1/3) + (a/b)^(1/3))/(b^2*(a/b)^(2/3))
```

mupad [B] time = 0.07, size = 126, normalized size = 1.16

$$\frac{3\,x^{1/3}}{b} + \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3} + 9\,a\,b\,x^{1/3}\right)}{b^{4/3}} + \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) + 9\,a\,b\,x^{1/3}\right)\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}} - \frac{(-a)^{1/3}\,\ln\left(9\,(-a)^{4/3}\,b^{2/3}\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right) - 9\,a\,b\,x^{1/3}\right)\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x),x)
```

```
[Out] (3*x^(1/3))/b + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3) + 9*a*b*x^(1/3)))/b^(4
/3) + ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2/3)*((3^(1/2)*1i)/2 - 1/2) + 9*a*b*x
^(1/3))*((3^(1/2)*1i)/2 - 1/2))/b^(4/3) - ((-a)^(1/3)*log(9*(-a)^(4/3)*b^(2
/3)*((3^(1/2)*1i)/2 + 1/2) - 9*a*b*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/b^(4/3)
```

sympy [A] time = 6.10, size = 219, normalized size = 2.01

$$\begin{cases} \tilde{\infty}\sqrt[3]{x} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a} & \text{for } b = 0 \\ \frac{3\sqrt[3]{x}}{b} & \text{for } a = 0 \\ \frac{\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{b} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2b} - \frac{\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{b} + \frac{3\sqrt[3]{x}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo*x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a), Eq(b, 0))
, (3*x**(1/3)/b, Eq(a, 0)), ((-1)**(1/3)*a**(1/3)*(1/b)**(1/3)*log(-(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/b - (-1)**(1/3)*a**(1/3)*(1/b)**(1/3
)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)
*(1/b)**(1/3) + 4*x**(2/3))/(2*b) - (-1)**(1/3)*sqrt(3)*a**(1/3)*(1/b)**(1/
3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/b + 3*x**(1/3)/b, True))
```

3.678 $\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {56, 617, 204, 31}

$$-\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}}+\frac{\log(a+bx)}{2\sqrt[3]{a}\,b^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1/3
)*b^(2/3))) - (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(1/3)*b^(2/3)) + Log[
a + b*x]/(2*a^(1/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx = \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2b} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{a}b^{2/3}} - \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{\log(a+bx)}{2\sqrt[3]{a}b^{2/3}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.27

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{bx}{a}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 1, 5/3, -((b*x)/a)])/(2*a)
```

IntegrateAlgebraic [A] time = 0.06, size = 126, normalized size = 1.26

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2\sqrt[3]{a}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}b^{2/3}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(1
/3)*b^(2/3))) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(1/3)*b^(2/3)) + Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(2*a^(1/3)*b^(2/3))
```

fricas [A] time = 1.30, size = 313, normalized size = 3.13

$$\left[\frac{\sqrt{3}ab\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+\sqrt{3}\left(abx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}a+2(-ab^2)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{(-ab^2)^{\frac{1}{3}}}{a}}-3(-ab^2)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{2ab^2}, \frac{2\sqrt{3}ab\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{3}\left(2bx^{\frac{1}{3}}+(-ab^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-ab^2)^{\frac{1}{3}}}{a}}}{3b}\right)+(-ab^2)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+(-ab^2)^{\frac{1}{3}}bx^{\frac{1}{3}}+(-ab^2)^{\frac{2}{3}}\right)-2(-ab^2)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-(-ab^2)^{\frac{1}{3}}\right)}{2ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt((-a*b^2)^(1/3)/a)*log((2*b^2*x - a*b + sqrt(3)*(a*b*
x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/
a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x + a)) + (-a*b^2)^(2/3)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/
3) - (-a*b^2)^(1/3)))/(a*b^2), 1/2*(2*sqrt(3)*a*b*sqrt(-(-a*b^2)^(1/3)/a)*a
rctan(1/3*sqrt(3)*(2*b*x^(1/3) + (-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b)
 + (-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/
3)) - 2*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^2)]
```

giac [A] time = 1.17, size = 118, normalized size = 1.18

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^{2}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a - sqrt(3)*(-a*b^2)^(2/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/2*(-
a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.00, size = 96, normalized size = 0.96

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}b}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a),x)
```

```
[Out] -1/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)
^(1/3)*x^(1/3)+(a/b)^(2/3))+3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/
b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.83, size = 103, normalized size = 1.03

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(1/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(1/3
)) - log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(1/3))
```

mupad [B] time = 0.11, size = 120, normalized size = 1.20

$$\frac{\ln\left(9\,b\,x^{1/3}-9\,(-a)^{1/3}\,b^{2/3}\right)}{(-a)^{1/3}\,b^{2/3}}+\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,(-a)^{1/3}\,b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}\,b^{2/3}}-\frac{\ln\left(9\,b\,x^{1/3}-\frac{9\,(-a)^{1/3}\,b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,(-a)^{1/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)),x)
```

```
[Out] log(9*b*x^(1/3) - 9*(-a)^(1/3)*b^(2/3))/((-a)^(1/3)*b^(2/3)) + (log(9*b*x^(
1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i - 1)^2)/4)*(3^(1/2)*1i - 1))/(2*(-
```

```
a)^(1/3)*b^(2/3)) - (log(9*b*x^(1/3) - (9*(-a)^(1/3)*b^(2/3)*(3^(1/2)*1i +
1)^2)/4)*(3^(1/2)*1i + 1))/(2*(-a)^(1/3)*b^(2/3))
```

sympy [A] time = 7.41, size = 212, normalized size = 2.12

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a} & \text{for } b = 0 \\ -\frac{3}{b\sqrt[3]{x}} & \text{for } a = 0 \\ -\frac{(-1)^{\frac{2}{3}} \log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}} \log\left(4(-1)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{\sqrt[3]{a}\, b\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a), Eq(b, 0))
, (-3/(b*x**(1/3)), Eq(a, 0)), (-(-1)**(2/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b
)**(1/3) + x**(1/3))/(a**(1/3)*b*(1/b)**(1/3)) + (-1)**(2/3)*log(4*(-1)**(2
/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) +
4*x**(2/3))/(2*a**(1/3)*b*(1/b)**(1/3)) - (-1)**(2/3)*sqrt(3)*atan(sqrt(3)/
3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(1/3)*b*(
1/b)**(1/3)), True))
```

3.679 $\int \frac{1}{x^{2/3}(a+bx)}\,dx$

Optimal. Leaf size=100

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {58, 617, 204, 31}

$$\frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2/3
)*b^(1/3))) + (3*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(2/3)*b^(1/3)) - Log[
a + b*x]/(2*a^(2/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{2/3}(a+bx)}\,dx = -\frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}}$$

$$= \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}\sqrt[3]{b}} + \frac{3\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}\sqrt[3]{b}} - \frac{\log(a+bx)}{2a^{2/3}\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 103, normalized size = 1.03

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{2a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(1/3))/a^(1/3))/Sqrt[3]] - 2*Log[a
^(1/3) + b^(1/3)*x^(1/3)] + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)
*x^(2/3)])/(a^(2/3)*b^(1/3))
```

IntegrateAlgebraic [A] time = 0.07, size = 125, normalized size = 1.25

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{2/3}\sqrt[3]{b}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(a^(2
/3)*b^(1/3))) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(2/3)*b^(1/3)) - Log[a^(2
/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(2*a^(2/3)*b^(1/3))
```

fricas [A] time = 1.49, size = 307, normalized size = 3.07

$$\left[\frac{\sqrt{3}\,ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx-a^2+\sqrt{3}\left(2\,abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{2\,a^2b}, \frac{2\sqrt{3}\,ab\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{3}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{3\,a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{2\,a^2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x - a^2 + sqrt(3)*(2*a*
b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b)
 - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x + a)) - (a^2*b)^(2/3)*log(a*b*x^(2/3) +
(a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3)
```

```
+ (a^2*b)^(2/3)))/(a^2*b), 1/2*(2*sqrt(3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(
-1/3*sqrt(3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)
/b)/a^2) - (a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*
x^(1/3)) + 2*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b)]
```

giac [A] time = 1.18, size = 117, normalized size = 1.17

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] -(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a + sqrt(3)*(-a*b^2)^(1/3)*a
rctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b) + 1/2*(-a*
b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b)
```

maple [A] time = 0.01, size = 95, normalized size = 0.95

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a),x)
```

```
[Out] 1/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^
(1/3)*x^(1/3)+(a/b)^(2/3))+1/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a
/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.96, size = 102, normalized size = 1.02

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b*(a/b)^
(2/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b*(a/b)^(2/3
)) + log(x^(1/3) + (a/b)^(1/3))/(b*(a/b)^(2/3))
```

mupad [B] time = 0.21, size = 110, normalized size = 1.10

$$\frac{\ln\left(9\,a^{1/3}\,b^{5/3}+9\,b^2\,x^{1/3}\right)}{a^{2/3}\,b^{1/3}}+\frac{\ln\left(9\,b^2\,x^{1/3}+\frac{9\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}-\frac{\ln\left(9\,b^2\,x^{1/3}-\frac{9\,a^{1/3}\,b^{5/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,a^{2/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)),x)
```

```
[Out] log(9*a^(1/3)*b^(5/3) + 9*b^2*x^(1/3))/(a^(2/3)*b^(1/3)) + (log(9*b^2*x^(1/
3) + (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(2*a^(2/3)*b
^(1/3)) - (log(9*b^2*x^(1/3) - (9*a^(1/3)*b^(5/3)*(3^(1/2)*1i + 1))/2)*(3^(
1/2)*1i + 1))/(2*a^(2/3)*b^(1/3))
```

sympy [A] time = 11.35, size = 212, normalized size = 2.12

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a} & \text{for } b = 0 \\ -\frac{3}{2bx^{\frac{2}{3}}} & \text{for } a = 0 \\ -\frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{2a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{2}{3}}b\left(\frac{1}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a, Eq(b, 0)), (-
3/(2*b*x**(2/3)), Eq(a, 0)), (-(-1)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3) + x**(1/3))/(a**(2/3)*b*(1/b)**(2/3)) + (-1)**(1/3)*log(4*(-1)**(2/3
)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*
x**(2/3))/(2*a**(2/3)*b*(1/b)**(2/3)) + (-1)**(1/3)*sqrt(3)*atan(sqrt(3)/3
- 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(a**(2/3)*b*(1/
b)**(2/3)), True))
```

3.680 $\int \frac{1}{x^{4/3}(a+bx)}\,dx$

Optimal. Leaf size=109

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Rubi [A] time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{3\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2a^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{4/3}}-\frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqr
t[3]*a^(1/3))])/a^(4/3) + (3*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(
4/3)) - (b^(1/3)*Log[a + b*x])/(2*a^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)}\,dx = -\frac{3}{a\sqrt[3]{x}} - \frac{b\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{a}$$

$$= -\frac{3}{a\sqrt[3]{x}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2a} + \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, \ldots\right.}{2a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\ldots}{\ldots}\right.}{a^{4/3}}$$

$$= -\frac{3}{a\sqrt[3]{x}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{4/3}} + \frac{3\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\sqrt[3]{b}\,\log(a+bx)}{2a^{4/3}}$$

Mathematica [C] time = 0.01, size = 25, normalized size = 0.23

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};-\frac{bx}{a}\right)}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, -((b*x)/a)])/(a*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.09, size = 134, normalized size = 1.23

$$-\frac{\sqrt[3]{b}\,\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{2a^{4/3}} + \frac{\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{4/3}} + \frac{\sqrt{3}\,\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{a^{4/3}} - \frac{3}{a\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)),x]
```

```
[Out] -3/(a*x^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(4/3) + (b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(4/3
) - (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2*a
^(4/3))
```

fricas [A] time = 1.24, size = 113, normalized size = 1.04

$$-\frac{2\sqrt{3}\,x\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}} - \frac{1}{3}\sqrt{3}\right) + x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}} + bx^{\frac{2}{3}} + a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right) - 2x\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}} + bx^{\frac{1}{3}}\right) + 6x^{\frac{2}{3}}}{2ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*x*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^(1/3) - 1/3*
sqrt(3)) + x*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(
1/3)) - 2*x*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 6*x^(2/3))/(a*x)
```

giac [A] time = 1.21, size = 125, normalized size = 1.15

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^{2}}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^{2}b}-\frac{3}{ax^{\frac{1}{3}}}-\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + sqrt(3)*(-a*b^2)^(2/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b) - 3/(
a*x^(1/3)) - 1/2*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)
^(2/3))/(a^2*b)
```

maple [A] time = 0.01, size = 104, normalized size = 0.95

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}a}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{3}}a}-\frac{3}{a\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a),x)
```

```
[Out] -3/a/x^(1/3)+1/a/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-1/2/a/(a/b)^(1/3)*ln(x
^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-1/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*
3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.96, size = 111, normalized size = 1.02

$$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\,a\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3}{ax^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(1/3)) - 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(1/
3)) + log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(1/3)) - 3/(a*x^(1/3))
```

mupad [B] time = 0.15, size = 124, normalized size = 1.14

$$\frac{b^{1/3}\,\ln\left(9\,a^{4/3}\,b^{8/3}+9\,a\,b^{3}\,x^{1/3}\right)}{a^{4/3}}-\frac{3}{a\,x^{1/3}}+\frac{b^{1/3}\,\ln\left(9\,a\,b^{3}\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}-\frac{b^{1/3}\,\ln\left(9\,a\,b^{3}\,x^{1/3}+9\,a^{4/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{a^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)),x)
```

```
[Out] (b^(1/3)*log(9*a^(4/3)*b^(8/3) + 9*a*b^3*x^(1/3)))/a^(4/3) - 3/(a*x^(1/3))
+ (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2)^2
```

```
)*((3^(1/2)*1i)/2 - 1/2))/a^(4/3) - (b^(1/3)*log(9*a*b^3*x^(1/3) + 9*a^(4/3
)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1/2)*1i)/2 + 1/2))/a^(4/3)
```

sympy [A] time = 25.29, size = 218, normalized size = 2.00

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{4bx^{\frac{4}{3}}} & \text{for } a = 0 \\ -\frac{3}{a\sqrt[3]{x}} & \text{for } b = 0 \\ -\frac{3}{a\sqrt[3]{x}} + \frac{(-1)^{\frac{2}{3}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} - \frac{(-1)^{\frac{2}{3}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} + \frac{(-1)^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (-3/(4*b*x**(4/3)), Eq(a, 0)
), (-3/(a*x**(1/3)), Eq(b, 0)), (-3/(a*x**(1/3)) + (-1)**(2/3)*log(-(-1)**(
1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(4/3)*(1/b)**(1/3)) - (-1)**(2/3
)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)
*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(4/3)*(1/b)**(1/3)) + (-1)**(2/3)*sqrt(3)
*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))
/(a**(4/3)*(1/b)**(1/3)), True))
```

3.681 $\int \frac{1}{x^{5/3}(a+bx)}\,dx$

Optimal. Leaf size=111

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Rubi [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}}+\frac{b^{2/3}\log(a+bx)}{2a^{5/3}}+\frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}-\frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)),x]
```

```
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(S
qrt[3]*a^(1/3))])/a^(5/3) - (3*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(2*a
^(5/3)) + (b^(2/3)*Log[a + b*x])/(2*a^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{5/3}(a+bx)}\,dx = -\frac{3}{2ax^{2/3}} - \frac{b\int \frac{1}{x^{2/3}(a+bx)}\,dx}{a}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} \cdots\right.}{2a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}} - \frac{\left(3b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\cdots}{\cdots}\right.}{a^{5/3}}$$

$$= -\frac{3}{2ax^{2/3}} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}} - \frac{3b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{5/3}} + \frac{b^{2/3}\log(a+bx)}{2a^{5/3}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.24

$$-\frac{3\,{}_2F_1\left(-\frac{2}{3},1;\frac{1}{3};-\frac{bx}{a}\right)}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)),x]
[Out] (-3*Hypergeometric2F1[-2/3, 1, 1/3, -((b*x)/a)])/(2*a*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.09, size = 137, normalized size = 1.23

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{2a^{5/3}} - \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}} + \frac{\sqrt{3}\,b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} - \frac{3}{2ax^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)),x]
[Out] -3/(2*a*x^(2/3)) + (Sqrt[3]*b^(2/3)*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/
(Sqrt[3]*a^(1/3))])/a^(5/3) - (b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/a^(5
/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(2
*a^(5/3))
```

fricas [A] time = 1.42, size = 147, normalized size = 1.32

$$\frac{2\sqrt{3}\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right) - x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right) + 2\,x\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right) - 3\,x^{\frac{1}{3}}}{2\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*sqrt(3)*x*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2
)^(2/3) - sqrt(3)*b)/b) - x*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*
(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 2*x*(-b^2/a^2)^(1/3)*log(b*x^(1/
3) - a*(-b^2/a^2)^(1/3)) - 3*x^(1/3))/(a*x)
```

giac [A] time = 1.14, size = 120, normalized size = 1.08

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2}-\frac{\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a^2}-\frac{3}{2ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="giac")
```

```
[Out] b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 - sqrt(3)*(-a*b^2)^(1/3
)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^2 - 1/2*(-a
*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a^2 - 3/2/(a
*x^(2/3))
```

maple [A] time = 0.01, size = 105, normalized size = 0.95

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{a}{b}\right)^{\frac{2}{3}}a}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{a}{b}\right)^{\frac{2}{3}}a}-\frac{3}{2a\,x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a),x)
```

```
[Out] -3/2/a/x^(2/3)-1/a/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+1/2/a/(a/b)^(2/3)*ln
(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-1/a/(a/b)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.99, size = 112, normalized size = 1.01

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{2a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{3}{2ax^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*(a/b)
^(2/3)) + 1/2*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*(a/b)^(2/
3)) - log(x^(1/3) + (a/b)^(1/3))/(a*(a/b)^(2/3)) - 3/2/(a*x^(2/3))
```

mupad [B] time = 0.07, size = 138, normalized size = 1.24

$$\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}-9\,a^2\,b^3\,x^{1/3}\right)}{(-a)^{5/3}}-\frac{3}{2\,a\,x^{2/3}}+\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-9\,a^2\,b^3\,x^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}-\frac{b^{2/3}\ln\left(9\,(-a)^{7/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+9\,a^2\,b^3\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{(-a)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)),x)
```

```
[Out] (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3) - 9*a^2*b^3*x^(1/3)))/(-a)^(5/3) - 3/(2*a
*x^(2/3)) + (b^(2/3)*log(9*(-a)^(7/3)*b^(8/3)*((3^(1/2)*1i)/2 - 1/2) - 9*a^
```

```
2*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(-a)^(5/3) - (b^(2/3)*log(9*(-a)^(7/
3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 9*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/
2))/(-a)^(5/3)
```

sympy [A] time = 34.96, size = 221, normalized size = 1.99

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} & \text{for } b = 0 \\ -\frac{3}{5bx^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{3}{2ax^{\frac{2}{3}}} + \frac{\sqrt[3]{-1}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}} + \sqrt[3]{x}\right)}{a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}} + 4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}} + 4x^{\frac{2}{3}}\right)}{2a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} - \frac{\sqrt[3]{-1}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}}{3} - \frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{a^{\frac{5}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a),x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (-3/(2*a*x**(2/3)), Eq(b, 0)
), (-3/(5*b*x**(5/3)), Eq(a, 0)), (-3/(2*a*x**(2/3)) + (-1)**(1/3)*log(-(-1
)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(a**(5/3)*(1/b)**(2/3)) - (-1)**
(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(
1/3)*(1/b)**(1/3) + 4*x**(2/3))/(2*a**(5/3)*(1/b)**(2/3)) - (-1)**(1/3)*sqr
t(3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/
3)))/(a**(5/3)*(1/b)**(2/3)), True))
```

3.682 $\int \frac{x^{5/3}}{(a+bx)^2} dx$

Optimal. Leaf size=129

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 56, 617, 204, 31}

$$\frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}}-\frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}+\frac{5a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}b^{8/3}}-\frac{x^{5/3}}{b(a+bx)}+\frac{5x^{2/3}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*x^(2/3))/(2*b^2) - x^(5/3)/(b*(a + b*x)) + (5*a^(2/3)*ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*Log[a
^(1/3) + b^(1/3)*x^(1/3)])/(2*b^(8/3)) - (5*a^(2/3)*Log[a + b*x])/(6*b^(8/3
))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^2}\,dx = -\frac{x^{5/3}}{b(a+bx)} + \frac{5\int \frac{x^{2/3}}{a+bx}\,dx}{3b}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{(5a)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b^2}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2b^3} + \frac{\left(5a^{2/3}\right)\text{Subst}}{}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}} - \frac{\left(5a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{b^{8/3}}$$
$$= \frac{5x^{2/3}}{2b^2} - \frac{x^{5/3}}{b(a+bx)} + \frac{5a^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2b^{8/3}} - \frac{5a^{2/3}\log(a+bx)}{6b^{8/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.21

$$\frac{3x^{8/3}\,{}_2F_1\left(2,\frac{8}{3};\frac{11}{3};-\frac{bx}{a}\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(8/3)*Hypergeometric2F1[2, 8/3, 11/3, -((b*x)/a)])/(8*a^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 159, normalized size = 1.23

$$-\frac{5a^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6b^{8/3}} + \frac{5a^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3b^{8/3}} + \frac{5a^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{8/3}} + \frac{5ax^{2/3}+3bx^{5/3}}{2b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x)^2,x]
```

```
[Out] (5*a*x^(2/3) + 3*b*x^(5/3))/(2*b^2*(a + b*x)) + (5*a^(2/3)*ArcTan[1/Sqrt[3]
 - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(8/3)) + (5*a^(2/3)*L
og[a^(1/3) + b^(1/3)*x^(1/3)])/(3*b^(8/3)) - (5*a^(2/3)*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(6*b^(8/3))
```

fricas [A] time = 1.35, size = 162, normalized size = 1.26

$$-\frac{10\sqrt{3}(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}-\sqrt{3}a}{3a}\right)+5(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(-bx^{\frac{1}{3}}\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{2}{3}}+a\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\right)-10(bx+a)\left(\frac{a^2}{b^2}\right)^{\frac{1}{3}}\log\left(b\left(\frac{a^2}{b^2}\right)^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)-3(3bx+5a)x^{\frac{2}{3}}}{6(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(10*sqrt(3)*(b*x + a)*(a^2/b^2)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*
(a^2/b^2)^(1/3) - sqrt(3)*a)/a) + 5*(b*x + a)*(a^2/b^2)^(1/3)*log(-b*x^(1/3
)*(a^2/b^2)^(2/3) + a*x^(2/3) + a*(a^2/b^2)^(1/3)) - 10*(b*x + a)*(a^2/b^2)
^(1/3)*log(b*(a^2/b^2)^(2/3) + a*x^(1/3)) - 3*(3*b*x + 5*a)*x^(2/3))/(b^3*x
 + a*b^2)
```

giac [A] time = 1.05, size = 135, normalized size = 1.05

$$\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3b^2}+\frac{ax^{\frac{2}{3}}}{(bx+a)b^2}+\frac{3x^{\frac{2}{3}}}{2b^2}+\frac{5\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^4}-\frac{5\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 + a*x^(2/3)/((b*x + a
)*b^2) + 3/2*x^(2/3)/b^2 + 5/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2
*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^4 - 5/6*(-a*b^2)^(2/3)*log(x^(2/3)
 + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^4
```

maple [A] time = 0.01, size = 123, normalized size = 0.95

$$\frac{a\,x^{\frac{2}{3}}}{(bx+a)\,b^2}-\frac{5\sqrt{3}\;a\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{5a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}-\frac{5a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}b^3}+\frac{3x^{\frac{2}{3}}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^2,x)
```

```
[Out] 3/2*x^(2/3)/b^2+a/b^2*x^(2/3)/(b*x+a)+5/3*a/b^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b
)^(1/3))-5/6*a/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-
5/3*a/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.96, size = 133, normalized size = 1.03

$$\frac{ax^{\frac{2}{3}}}{b^3x+ab^2}-\frac{5\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{3x^{\frac{2}{3}}}{2b^2}-\frac{5a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(2/3)/(b^3*x + a*b^2) - 5/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1/3)) + 3/2*x^(2/3)/b^2 - 5/6*a*log(x^
(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/3)) + 5/3*a*log(x^
(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

mupad [B] time = 0.26, size = 150, normalized size = 1.16

$$\frac{3\,x^{2/3}}{2\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)}{3\,b^{8/3}}+\frac{a\,x^{2/3}}{x\,b^3+a\,b^2}+\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}-\frac{5\,a^{2/3}\,\ln\left(\frac{25\,a^{7/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{b^{10/3}}+\frac{25\,a^2\,x^{1/3}}{b^3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(2/3))/(2*b^2) + (5*a^(2/3)*log((25*a^(7/3))/b^(10/3) + (25*a^2*x^(1/3
))/b^3))/(3*b^(8/3)) + (a*x^(2/3))/(a*b^2 + b^3*x) + (5*a^(2/3)*log((25*a^(
7/3)*((3^(1/2)*1i)/2 - 1/2)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1
i)/2 - 1/2))/(3*b^(8/3)) - (5*a^(2/3)*log((25*a^(7/3)*((3^(1/2)*1i)/2 + 1/2
)^2)/b^(10/3) + (25*a^2*x^(1/3))/b^3)*((3^(1/2)*1i)/2 + 1/2))/(3*b^(8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.683 $\int \frac{x^{4/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=125

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Rubi [A] time = 0.05, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 58, 617, 204, 31}

$$-\frac{2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}}+\frac{2\sqrt[3]{a}\log(a+bx)}{3b^{7/3}}+\frac{4\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}}-\frac{x^{4/3}}{b(a+bx)}+\frac{4\sqrt[3]{x}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^2,x]
```

```
[Out] (4*x^(1/3))/b^2 - x^(4/3)/(b*(a + b*x)) + (4*a^(1/3)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (2*a^(1/3)*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/b^(7/3) + (2*a^(1/3)*Log[a + b*x])/(3*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^2}\,dx = -\frac{x^{4/3}}{b(a+bx)} + \frac{4\int \frac{\sqrt[3]{x}}{a+bx}\,dx}{3b}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{(4a)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b^2}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{b^{8/3}} - \frac{\left(2\sqrt[3]{a}\right)\text{Sub}}{}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}} - \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.}{b^{7/3}}$$

$$= \frac{4\sqrt[3]{x}}{b^2} - \frac{x^{4/3}}{b(a+bx)} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,b^{7/3}} - \frac{2\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{b^{7/3}} + \frac{2\sqrt[3]{a}\,\log(a+bx)}{3b^{7/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.22

$$\frac{3x^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};-\frac{bx}{a}\right)}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^2,x]
[Out] (3*x^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, -((b*x)/a)])/(7*a^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 156, normalized size = 1.25

$$\frac{2\sqrt[3]{a}\,\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3b^{7/3}} - \frac{4\sqrt[3]{a}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3b^{7/3}} + \frac{4\sqrt[3]{a}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,b^{7/3}} + \frac{4a\sqrt[3]{x}+3bx^{4/3}}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x)^2,x]
[Out] (4*a*x^(1/3) + 3*b*x^(4/3))/(b^2*(a + b*x)) + (4*a^(1/3)*ArcTan[1/Sqrt[3] -
 (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*b^(7/3)) - (4*a^(1/3)*Log
[a^(1/3) + b^(1/3)*x^(1/3)])/(3*b^(7/3)) + (2*a^(1/3)*Log[a^(2/3) - a^(1/3)
*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(3*b^(7/3))
```

fricas [A] time = 1.40, size = 147, normalized size = 1.18

$$\frac{4\sqrt{3}(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}bx^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{2}{3}}-\sqrt{3}a}{3a}\right)-2(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)+4(bx+a)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)+3(3bx+4a)x^{\frac{1}{3}}}{3(b^3x+ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*x + a)*(-a/b)^(1/3)*arctan(1/3*(2*sqrt(3)*b*x^(1/3)*(-a/b
)^(2/3) - sqrt(3)*a)/a) - 2*(b*x + a)*(-a/b)^(1/3)*log(x^(2/3) + x^(1/3)*(-
a/b)^(1/3) + (-a/b)^(2/3)) + 4*(b*x + a)*(-a/b)^(1/3)*log(x^(1/3) - (-a/b)^
(1/3)) + 3*(3*b*x + 4*a)*x^(1/3))/(b^3*x + a*b^2)
```

giac [A] time = 1.07, size = 135, normalized size = 1.08

$$\frac{4\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3b^2}-\frac{4\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3}+\frac{ax^{\frac{1}{3}}}{(bx+a)b^2}+\frac{3x^{\frac{1}{3}}}{b^2}-\frac{2\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/b^2 - 4/3*sqrt(3)*(-a*b^2
)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/b^3 + a
*x^(1/3)/((b*x + a)*b^2) + 3*x^(1/3)/b^2 - 2/3*(-a*b^2)^(1/3)*log(x^(2/3) +
 x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/b^3
```

maple [A] time = 0.01, size = 123, normalized size = 0.98

$$\frac{a\,x^{\frac{1}{3}}}{(bx+a)\,b^2}-\frac{4\sqrt{3}\,a\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}-\frac{4a\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{2a\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^3}+\frac{3x^{\frac{1}{3}}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^2,x)
```

```
[Out] 3*x^(1/3)/b^2+a/b^2*x^(1/3)/(b*x+a)-4/3*a/b^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^
(1/3))+2/3*a/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-4/
3*a/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.93, size = 133, normalized size = 1.06

$$\frac{ax^{\frac{1}{3}}}{b^3x+ab^2}-\frac{4\sqrt{3}a\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{3x^{\frac{1}{3}}}{b^2}+\frac{2a\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{4a\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a*x^(1/3)/(b^3*x + a*b^2) - 4/3*sqrt(3)*a*arctan(1/3*sqrt(3)*(2*x^(1/3) - (
a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2/3)) + 3*x^(1/3)/b^2 + 2/3*a*log(x^(2
```

```
/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3)) - 4/3*a*log(x^(1
/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 0.15, size = 142, normalized size = 1.14

$$\frac{3\,x^{1/3}}{b^2}+\frac{a\,x^{1/3}}{x\,b^3+a\,b^2}+\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(\frac{12\,{\left(-a\right)}^{4/3}}{b^{1/3}}+12\,a\,x^{1/3}\right)}{3\,b^{7/3}}-\frac{4\,{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}-\frac{12\,{\left(-a\right)}^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{b^{1/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,b^{7/3}}+\frac{{\left(-a\right)}^{1/3}\,\ln\left(12\,a\,x^{1/3}+\frac{9\,{\left(-a\right)}^{4/3}\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{1/3}}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^2,x)
```

```
[Out] (3*x^(1/3))/b^2 + (a*x^(1/3))/(a*b^2 + b^3*x) + (4*(-a)^(1/3)*log((12*(-a)^
(4/3))/b^(1/3) + 12*a*x^(1/3)))/(3*b^(7/3)) - (4*(-a)^(1/3)*log(12*a*x^(1/3
) - (12*(-a)^(4/3)*((3^(1/2)*1i)/2 + 1/2))/b^(1/3))*((3^(1/2)*1i)/2 + 1/2))
/(3*b^(7/3)) + ((-a)^(1/3)*log(12*a*x^(1/3) + (9*(-a)^(4/3)*((3^(1/2)*2i)/3
 - 2/3))/b^(1/3))*((3^(1/2)*2i)/3 - 2/3))/b^(7/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.684 $\int \frac{x^{2/3}}{(a+bx)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Rubi [A] time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]
*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(
1/3)*b^(5/3)) + Log[a + b*x]/(3*a^(1/3)*b^(5/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{2/3}}{(a+bx)^2}\,dx &= -\frac{x^{2/3}}{b(a+bx)}+\frac{2\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3b}\\
&= -\frac{x^{2/3}}{b(a+bx)}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{b^2}-\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,x,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}\\
&= -\frac{x^{2/3}}{b(a+bx)}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{\sqrt[3]{a}\,b^{5/3}}\\
&= -\frac{x^{2/3}}{b(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{\sqrt[3]{a}\,b^{5/3}}+\frac{\log(a+bx)}{3\sqrt[3]{a}\,b^{5/3}}
\end{aligned}
$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.23

$$
\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};-\frac{bx}{a}\right)}{5a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, -((b*x)/a)])/(5*a^2)
```

IntegrateAlgebraic [A] time = 0.16, size = 145, normalized size = 1.26

$$
\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{5/3}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,\sqrt[3]{a}\,b^{5/3}}-\frac{x^{2/3}}{b(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(2/3)/(b*(a + b*x))) - (2*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*b^(5/3)) - (2*Log[a^(1/3) + b^(1/3)*x^(1/3)]
)/(3*a^(1/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(
2/3)]/(3*a^(1/3)*b^(5/3))
```

fricas [B] time = 1.32, size = 394, normalized size = 3.43

$$
\left[-\frac{3\,ab^2x^{\frac{2}{3}}-3\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)-\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)+2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{3\left(ab^4x+a^2b^3\right)},-\frac{3\,ab^2x^{\frac{2}{3}}-6\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)-\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)+2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)}{3\left(ab^4x+a^2b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/3*(3*a*b^2*x^(2/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/
a)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a
*b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*
x + a)) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/
```

```
3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^
(1/3)))/(a*b^4*x + a^2*b^3), -1/3*(3*a*b^2*x^(2/3) - 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) - (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) + 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a*b^4*x + a^2*b^3)]
```

giac [A] time = 1.20, size = 136, normalized size = 1.18

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}-\frac{x^{\frac{2}{3}}}{(bx+a)b}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) - x^(2/3)/((b*x +
a)*b) - 2/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(
1/3))/(-a/b)^(1/3))/(a*b^3) + 1/3*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^3)
```

maple [A] time = 0.01, size = 112, normalized size = 0.97

$$-\frac{x^{\frac{2}{3}}}{(bx+a)\,b}+\frac{2\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^2,x)
```

```
[Out] -x^(2/3)/b/(b*x+a)-2/3/b^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/3/b^2/(a/b
)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+2/3/b^2*3^(1/2)/(a/b)^(
1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.99, size = 120, normalized size = 1.04

$$-\frac{x^{\frac{2}{3}}}{b^{2}x+ab}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(2/3)/(b^2*x + a*b) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(1/3)) + 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(1/3)) - 2/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(1/3))
```

mupad [B] time = 0.24, size = 142, normalized size = 1.23

$$\frac{2\ln\left(\frac{4\,x^{1/3}}{b}-\frac{4\,(-a)^{1/3}}{b^{4/3}}\right)}{3\,(-a)^{1/3}\,b^{5/3}}-\frac{x^{2/3}}{b\,(a+b\,x)}+\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{b^{4/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}}-\frac{\ln\left(\frac{4\,x^{1/3}}{b}-\frac{(-a)^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{b^{4/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3\,(-a)^{1/3}\,b^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^2,x)
```

```
[Out] (2*log((4*x^(1/3))/b - (4*(-a)^(1/3))/b^(4/3)))/(3*(-a)^(1/3)*b^(5/3)) - x^
(2/3)/(b*(a + b*x)) + (log((4*x^(1/3))/b - ((-a)^(1/3)*(3^(1/2)*1i - 1)^2)/
b^(4/3))*(3^(1/2)*1i - 1))/(3*(-a)^(1/3)*b^(5/3)) - (log((4*x^(1/3))/b - ((
-a)^(1/3)*(3^(1/2)*1i + 1)^2)/b^(4/3))*(3^(1/2)*1i + 1))/(3*(-a)^(1/3)*b^(5
/3))
```

sympy [A] time = 106.98, size = 787, normalized size = 6.84

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt[3]{x}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3 x^{\frac{5}{3}}}{5 a^{2}} & \text{for}\: b = 0 \\ - \frac{3}{b^{2} \sqrt[3]{x}} & \text{for}\: a = 0 \\ - \frac{3 \sqrt[3]{-1} \sqrt[3]{a} b x^{\frac{2}{3}} \sqrt[3]{\frac{1}{b}}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 a \log{\left(- \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + \sqrt[3]{x} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} - \frac{a \log{\left(4 \left(-1\right)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4 \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{x} \sqrt[3]{\frac{1}{b}} + 4 x^{\frac{2}{3}} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 \sqrt{3} a \operatorname{atan}{\left(\frac{\sqrt{3}}{3} - \frac{2 \left(-1\right)^{\frac{2}{3}} \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{a} \sqrt[3]{\frac{1}{b}}} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 a \log{\left(2 \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 b x \log{\left(- \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{\frac{1}{b}} + \sqrt[3]{x} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} - \frac{b x \log{\left(4 \left(-1\right)^{\frac{2}{3}} a^{\frac{2}{3}} \left(\frac{1}{b}\right)^{\frac{2}{3}} + 4 \sqrt[3]{-1} \sqrt[3]{a} \sqrt[3]{x} \sqrt[3]{\frac{1}{b}} + 4 x^{\frac{2}{3}} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 \sqrt{3} b x \operatorname{atan}{\left(\frac{\sqrt{3}}{3} - \frac{2 \left(-1\right)^{\frac{2}{3}} \sqrt{3} \sqrt[3]{x}}{3 \sqrt[3]{a} \sqrt[3]{\frac{1}{b}}} \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} + \frac{2 b x \log{\left(2 \right)}}{3 \sqrt[3]{-1} a^{\frac{4}{3}} b^{2} \sqrt[3]{\frac{1}{b}} + 3 \sqrt[3]{-1} \sqrt[3]{a} b^{3} x \sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(1/3), Eq(a, 0) & Eq(b, 0)), (3*x**(5/3)/(5*a**2), Eq(b,
0)), (-3/(b**2*x**(1/3)), Eq(a, 0)), (-3*(-1)**(1/3)*a**(1/3)*b*x**(2/3)*(1
/b)**(1/3)/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/
3)*b**3*x*(1/b)**(1/3)) + 2*a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(
1/3))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b*
*3*x*(1/b)**(1/3)) - a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1
/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*(-1)**(1/3)*a**(4/3)*b*
*2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*sqrt(3)*a
*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))
/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*
(1/b)**(1/3)) + 2*a*log(2)/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-
1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*b*x*log(-(-1)**(1/3)*a**(1/3)*(
1/b)**(1/3) + x**(1/3))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)*
*(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) - b*x*log(4*(-1)**(2/3)*a**(2/3)*(1/b)
**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*(-1
)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**
(1/3)) + 2*sqrt(3)*b*x*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a
**(1/3)*(1/b)**(1/3)))/(3*(-1)**(1/3)*a**(4/3)*b**2*(1/b)**(1/3) + 3*(-1)**
(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)) + 2*b*x*log(2)/(3*(-1)**(1/3)*a**(4/3)*
b**2*(1/b)**(1/3) + 3*(-1)**(1/3)*a**(1/3)*b**3*x*(1/b)**(1/3)), True))
```

3.685 $\int \frac{\sqrt[3]{x}}{(a+bx)^2} \, dx$

Optimal. Leaf size=117

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Rubi [A] time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}-\frac{\log(a+bx)}{6a^{2/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}}-\frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(2/
3)*b^(4/3)) - Log[a + b*x]/(6*a^(2/3)*b^(4/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx = -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3b}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2\sqrt[3]{a}\,b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{2/3}b^{4/3}}$$

$$= -\frac{\sqrt[3]{x}}{b(a+bx)} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{2/3}b^{4/3}} - \frac{\log(a+bx)}{6a^{2/3}b^{4/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.23

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3}, 2; \frac{7}{3}; -\frac{bx}{a}\right)}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 2, 7/3, -((b*x)/a)])/(4*a^2)
```

IntegrateAlgebraic [A] time = 0.17, size = 145, normalized size = 1.24

$$-\frac{\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{6a^{2/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{2/3}b^{4/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}b^{4/3}} - \frac{\sqrt[3]{x}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(a + b*x)^2,x]
```

```
[Out] -(x^(1/3)/(b*(a + b*x))) - ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*
a^(1/3))]/(Sqrt[3]*a^(2/3)*b^(4/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(
2/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(6
*a^(2/3)*b^(4/3))
```

fricas [B] time = 1.25, size = 389, normalized size = 3.32

$$\left[-\frac{6a^2bx^{\frac{1}{3}} - 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx - a^2 + 3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}} - \left(a^2b\right)^{\frac{1}{3}}a + \left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}} - 3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right) + \left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}} + \left(a^2b\right)^{\frac{1}{3}}a - \left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right) - 2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}} + \left(a^2b\right)^{\frac{2}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}, -\frac{6a^2bx^{\frac{1}{3}} - 6\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a - 2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right) + \left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}} + \left(a^2b\right)^{\frac{1}{3}}a - \left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right) - 2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}} + \left(a^2b\right)^{\frac{2}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(6*a^2*b*x^(1/3) - 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/
b)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2
*b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
 + a)) + (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b
```

```
)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3
)))/(a^2*b^3*x + a^3*b^2), -1/6*(6*a^2*b*x^(1/3) - 6*sqrt(1/3)*(a*b^2*x + a
^2*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^
(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*(b*x + a)*log(a*b
*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) - 2*(a^2*b)^(2/3)*(b*x
+ a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^2*b^3*x + a^3*b^2)]
```

giac [A] time = 1.13, size = 136, normalized size = 1.16

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^{2}}-\frac{x^{\frac{1}{3}}}{(bx+a)b}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b) + 1/3*sqrt(3)*(-a*
b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b
^2) - x^(1/3)/((b*x + a)*b) + 1/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

maple [A] time = 0.01, size = 112, normalized size = 0.96

$$-\frac{x^{\frac{1}{3}}}{(bx+a)\,b}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}+\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^2,x)
```

```
[Out] -x^(1/3)/b/(b*x+a)+1/3/b^2/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/6/b^2/(a/b
)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/b^2/(a/b)^(2/3)*3^(
1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 3.03, size = 120, normalized size = 1.03

$$-\frac{x^{\frac{1}{3}}}{b^{2}x+ab}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -x^(1/3)/(b^2*x + a*b) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^
(1/3))/(a/b)^(1/3))/(b^2*(a/b)^(2/3)) - 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/
3) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) + 1/3*log(x^(1/3) + (a/b)^(1/3))/(b^2*(
a/b)^(2/3))
```

mupad [B] time = 0.06, size = 120, normalized size = 1.03

$$\frac{\ln\left(3\,b\,x^{1/3}+3\,a^{1/3}\,b^{2/3}\right)}{3\,a^{2/3}\,b^{4/3}}-\frac{x^{1/3}}{b\,\left(a+b\,x\right)}+\frac{\ln\left(3\,b\,x^{1/3}+\frac{3\,a^{1/3}\,b^{2/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}-\frac{\ln\left(3\,b\,x^{1/3}-\frac{3\,a^{1/3}\,b^{2/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/3)/(a + b*x)^2,x)

[Out] log(3*b*x^(1/3) + 3*a^(1/3)*b^(2/3))/(3*a^(2/3)*b^(4/3)) - x^(1/3)/(b*(a + b*x)) + (log(3*b*x^(1/3) + (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(2/3)*b^(4/3)) - (log(3*b*x^(1/3) - (3*a^(1/3)*b^(2/3)*(3^(1/2)*1i + 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(4/3))

sympy [A] time = 71.72, size = 607, normalized size = 5.19

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{2}{3}}} & \text{for}\: a = 0 \wedge b = 0 \\ \frac{3x^{\frac{4}{3}}}{4a^{2}} & \text{for}\: b = 0 \\ -\frac{3}{2b^{2}x^{\frac{2}{3}}} & \text{for}\: a = 0 \\ -\frac{2\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6a^{2}b+6ab^{2}x} + \frac{\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x} + \frac{2\sqrt[3]{-1}\sqrt{3}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}\,a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log(2)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6a^{2}b+6ab^{2}x} + \frac{\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6a^{2}b+6ab^{2}x} + \frac{2\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6a^{2}b+6ab^{2}x} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}\,bx\sqrt[3]{\frac{1}{b}}\log(2)}{6a^{2}b+6ab^{2}x} - \frac{6a\sqrt[3]{x}}{6a^{2}b+6ab^{2}x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/3)/(b*x+a)**2,x)

[Out] Piecewise((zoo/x**(2/3), Eq(a, 0) & Eq(b, 0)), (3*x**(4/3)/(4*a**2), Eq(b, 0)), (-3/(2*b**2*x**(2/3)), Eq(a, 0)), (-2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(6*a**2*b + 6*a*b**2*x) + (-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*(-1)**(1/3)*sqrt(3)*a**(4/3)*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a**2*b + 6*a*b**2*x) - 2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 2*(-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(6*a**2*b + 6*a*b**2*x) + (-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*a**2*b + 6*a*b**2*x) + 2*(-1)**(1/3)*sqrt(3)*a**(1/3)*b*x*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*a**2*b + 6*a*b**2*x) - 2*(-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(2)/(6*a**2*b + 6*a*b**2*x) - 6*a*x**(1/3)/(6*a**2*b + 6*a*b**2*x), True))

3.686 $\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx$

Optimal. Leaf size=116

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Rubi [A] time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}+\frac{\log(a+bx)}{6a^{4/3}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}b^{2/3}}+\frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/
3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(2*a^(4/3)*
b^(2/3)) + Log[a + b*x]/(6*a^(4/3)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}\,(a+bx)^2}\,dx = \frac{x^{2/3}}{a(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a}$$

$$= \frac{x^{2/3}}{a(a+bx)} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{2ab} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + x}\,dx, x, \sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{4/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{a(a+bx)} - \frac{\tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{2a^{4/3}b^{2/3}} + \frac{\log(a+bx)}{6a^{4/3}b^{2/3}}$$

Mathematica [**C**] time = 0.00, size = 27, normalized size = 0.23

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3}, 2; \frac{5}{3}; -\frac{bx}{a}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 2, 5/3, -((b*x)/a)])/(2*a^2)
```

IntegrateAlgebraic [**A**] time = 0.16, size = 144, normalized size = 1.24

$$\frac{\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{6a^{4/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{4/3}b^{2/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{4/3}b^{2/3}} + \frac{x^{2/3}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)^2),x]
```

```
[Out] x^(2/3)/(a*(a + b*x)) - ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(
1/3))]/(Sqrt[3]*a^(4/3)*b^(2/3)) - Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(4/3
)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(6*a^
(4/3)*b^(2/3))
```

fricas [**B**] time = 1.44, size = 396, normalized size = 3.41

$$\left[\frac{6ab^2x^{\frac{2}{3}} + 3\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x - ab + 3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}a + 2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}} - 3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right) + \left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}} + \left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{2}{3}}\right) - 2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}} - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}, \frac{6ab^2x^{\frac{2}{3}} + 6\sqrt{\frac{1}{3}}\left(ab^2x + a^2b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right) + \left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(b^2x^{\frac{2}{3}} + \left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}} + \left(-ab^2\right)^{\frac{2}{3}}\right) - 2\left(-ab^2\right)^{\frac{2}{3}}(bx+a)\log\left(bx^{\frac{1}{3}} - \left(-ab^2\right)^{\frac{1}{3}}\right)}{6\left(a^2b^3x + a^3b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*b^2*x^(2/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3
```

```
) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*log(b*x^(1/3) - (-a*b^2)^(
1/3)))/(a^2*b^3*x + a^3*b^2), 1/6*(6*a*b^2*x^(2/3) + 6*sqrt(1/3)*(a*b^2*x +
 a^2*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^2)^(1
/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (-a*b^2)^(2/3)*(b*x + a)*log(b^2*x^(2/3)
+ (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(-a*b^2)^(2/3)*(b*x + a)*l
og(b*x^(1/3) - (-a*b^2)^(1/3)))/(a^2*b^3*x + a^3*b^2)]
```

giac [A] time = 1.11, size = 132, normalized size = 1.14

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{2}}+\frac{x^{\frac{2}{3}}}{(bx+a)a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}b^{2}}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + x^(2/3)/((b*x + a)
*a) - 1/3*sqrt(3)*(-a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^2*b^2) + 1/6*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/
b)^(1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

maple [A] time = 0.01, size = 120, normalized size = 1.03

$$\frac{x^{\frac{2}{3}}}{(bx+a)\,a}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{1}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^2,x)
```

```
[Out] x^(2/3)/a/(b*x+a)-1/3/a/b/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))+1/6/a/b/(a/b)
^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+1/3/a*3^(1/2)/b/(a/b)^(1
/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.96, size = 127, normalized size = 1.09

$$\frac{x^{\frac{2}{3}}}{abx+a^{2}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(2/3)/(a*b*x + a^2) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(1/3)) + 1/6*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(1/3)) - 1/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(1/3))
```

mupad [B] time = 0.36, size = 144, normalized size = 1.24

$$\frac{x^{2/3}}{a\,\left(a+b\,x\right)}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{{\left(-1\right)}^{2/3}\,b^{2/3}}{a^{5/3}}+\frac{b\,x^{1/3}}{a^2}\right)}{3\,a^{4/3}\,b^{2/3}}-\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}^2}{a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{4/3}\,b^{2/3}}+\frac{{\left(-1\right)}^{1/3}\,\ln\left(\frac{b\,x^{1/3}}{a^2}+\frac{9\,{\left(-1\right)}^{2/3}\,b^{2/3}\,{\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}^2}{a^{5/3}}\right)\,\left(-\frac{1}{6}+\frac{\sqrt{3}\,1{}\mathrm{i}}{6}\right)}{a^{4/3}\,b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^2),x)
```

```
[Out] x^(2/3)/(a*(a + b*x)) + ((-1)^(1/3)*log(((-1)^(2/3)*b^(2/3))/a^(5/3) + (b*x
^(1/3))/a^2))/(3*a^(4/3)*b^(2/3)) - ((-1)^(1/3)*log((b*x^(1/3))/a^2 + ((-1)
^(2/3)*b^(2/3)*((3^(1/2)*1i)/2 + 1/2)^2)/a^(5/3))*((3^(1/2)*1i)/2 + 1/2))/(
3*a^(4/3)*b^(2/3)) + ((-1)^(1/3)*log((b*x^(1/3))/a^2 + (9*(-1)^(2/3)*b^(2/3
)*((3^(1/2)*1i)/6 - 1/6)^2)/a^(5/3))*((3^(1/2)*1i)/6 - 1/6))/(a^(4/3)*b^(2/
3))
```

sympy [A] time = 79.66, size = 774, normalized size = 6.67

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{4}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3x^{\frac{2}{3}}}{2a^2} & \text{for } b = 0 \\ -\frac{3}{4b^2x^{\frac{4}{3}}} & \text{for } a = 0 \\ \frac{6\sqrt[3]{-1}\sqrt[3]{a}bx^{\frac{2}{3}}\sqrt[3]{\frac{1}{b}}}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} - \frac{a\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}a\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2a\log(2)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} - \frac{bx\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2\sqrt{3}bx\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} + \frac{2bx\log(2)}{6\sqrt[3]{-1}a^{\frac{7}{3}}b\sqrt[3]{\frac{1}{b}}+6\sqrt[3]{-1}a^{\frac{4}{3}}b^2x\sqrt[3]{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(4/3), Eq(a, 0) & Eq(b, 0)), (3*x**(2/3)/(2*a**2), Eq(b,
0)), (-3/(4*b**2*x**(4/3)), Eq(a, 0)), (6*(-1)**(1/3)*a**(1/3)*b*x**(2/3)*(
1/b)**(1/3)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)
*b**2*x*(1/b)**(1/3)) + 2*a*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/
3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*
(1/b)**(1/3)) - a*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a
**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)
**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)) + 2*sqrt(3)*a*atan(sq
rt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(6*(-1)
**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3
)) + 2*a*log(2)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(
4/3)*b**2*x*(1/b)**(1/3)) + 2*b*x*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) +
x**(1/3))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b
**2*x*(1/b)**(1/3)) - b*x*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)*
*(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(6*(-1)**(1/3)*a**(7/3)
*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)) + 2*sqrt(3)*b
*x*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)
))/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)**(1/3)*a**(4/3)*b**2*x*(
1/b)**(1/3)) + 2*b*x*log(2)/(6*(-1)**(1/3)*a**(7/3)*b*(1/b)**(1/3) + 6*(-1)
**(1/3)*a**(4/3)*b**2*x*(1/b)**(1/3)), True))
```

3.687 $\int \frac{1}{x^{2/3}(a+bx)^2} dx$

Optimal. Leaf size=113

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + Log[a^(1/3) + b^(1/3)*x^(1/3)]/(a^(5/3
)*b^(1/3)) - Log[a + b*x]/(3*a^(5/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^2}\,dx &= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{a^{4/3}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}+x}\,dx,x,\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}+\frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}\\
&= \frac{\sqrt[3]{x}}{a(a+bx)}-\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{5/3}\sqrt[3]{b}}-\frac{\log(a+bx)}{3a^{5/3}\sqrt[3]{b}}
\end{aligned}$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.22

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},2;\frac{4}{3};-\frac{bx}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 2, 4/3, -((b*x)/a)])/a^2
```

IntegrateAlgebraic [A] time = 0.16, size = 144, normalized size = 1.27

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3a^{5/3}\sqrt[3]{b}}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{5/3}\sqrt[3]{b}}-\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)^2),x]
```

```
[Out] x^(1/3)/(a*(a + b*x)) - (2*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*
a^(1/3))])/(Sqrt[3]*a^(5/3)*b^(1/3)) + (2*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(
3*a^(5/3)*b^(1/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3
)]/(3*a^(5/3)*b^(1/3))
```

fricas [B] time = 1.19, size = 387, normalized size = 3.42

$$\left[\frac{3a^2bx^{\frac{1}{3}}+3\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-\left(a^2b\right)^{\frac{1}{3}}a+\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}-3\left(a^2b\right)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^3b^2x+a^4b\right)},\ \frac{3a^2bx^{\frac{1}{3}}+6\sqrt{\frac{1}{3}}\left(ab^2x+a^2b\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(a^2b\right)^{\frac{1}{3}}a-2\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{a^2}\right)-\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{2}{3}}+\left(a^2b\right)^{\frac{1}{3}}a-\left(a^2b\right)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+2\left(a^2b\right)^{\frac{2}{3}}(bx+a)\log\left(abx^{\frac{1}{3}}+\left(a^2b\right)^{\frac{2}{3}}\right)}{3\left(a^3b^2x+a^4b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^2*b*x^(1/3) + 3*sqrt(1/3)*(a*b^2*x + a^2*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)
```

```
^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(1/3) + (a^2*b)^(2/3)
))/(a^3*b^2*x + a^4*b), 1/3*(3*a^2*b*x^(1/3) + 6*sqrt(1/3)*(a*b^2*x + a^2*b
)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*b)^(2/3
)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(2/3)*(b*x + a)*log(a*b*x^(
2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(a^2*b)^(2/3)*(b*x + a)
*log(a*b*x^(1/3) + (a^2*b)^(2/3)))/(a^3*b^2*x + a^4*b)]
```

giac [A] time = 1.04, size = 132, normalized size = 1.17

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^2}+\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2b}+\frac{x^{\frac{1}{3}}}{(bx+a)a}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2/3*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^2 + 2/3*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b
) + x^(1/3)/((b*x + a)*a) + 1/3*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)
^(1/3) + (-a/b)^(2/3))/(a^2*b)
```

maple [A] time = 0.01, size = 120, normalized size = 1.06

$$\frac{x^{\frac{1}{3}}}{(bx+a)\,a}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}+\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^2,x)
```

```
[Out] x^(1/3)/a/(b*x+a)+2/3/a/b/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))-1/3/a/b/(a/b)
^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))+2/3/a/b/(a/b)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 3.01, size = 127, normalized size = 1.12

$$\frac{x^{\frac{1}{3}}}{abx+a^2}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] x^(1/3)/(a*b*x + a^2) + 2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(
1/3))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - 1/3*log(x^(2/3) - x^(1/3)*(a/b)^(1/3
) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2/3*log(x^(1/3) + (a/b)^(1/3))/(a*b*(a
/b)^(2/3))
```

mupad [B] time = 0.22, size = 134, normalized size = 1.19

$$\frac{2\ln\left(\frac{6\,b^{5/3}}{a^{2/3}}+\frac{6\,b^2\,x^{1/3}}{a}\right)}{3\,a^{5/3}\,b^{1/3}}+\frac{x^{1/3}}{a\,(a+b\,x)}+\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}+\frac{3\,b^{5/3}\,(-1+\sqrt{3}\,1\mathrm{i})}{a^{2/3}}\right)(-1+\sqrt{3}\,1\mathrm{i})}{3\,a^{5/3}\,b^{1/3}}-\frac{\ln\left(\frac{6\,b^2\,x^{1/3}}{a}-\frac{3\,b^{5/3}\,(1+\sqrt{3}\,1\mathrm{i})}{a^{2/3}}\right)(1+\sqrt{3}\,1\mathrm{i})}{3\,a^{5/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^2),x)
```

```
[Out] (2*log((6*b^(5/3))/a^(2/3) + (6*b^2*x^(1/3))/a))/(3*a^(5/3)*b^(1/3)) + x^(1
/3)/(a*(a + b*x)) + (log((6*b^2*x^(1/3))/a + (3*b^(5/3)*(3^(1/2)*1i - 1))/a
^(2/3))*(3^(1/2)*1i - 1))/(3*a^(5/3)*b^(1/3)) - (log((6*b^2*x^(1/3))/a - (3
*b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3^(1/2)*1i + 1))/(3*a^(5/3)*b^(1/3))
```

sympy [A] time = 115.98, size = 590, normalized size = 5.22

$$\begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{3}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{3\sqrt[3]{x}}{a^2} & \text{for } b = 0 \\ -\frac{3}{5b^2x^{\frac{5}{3}}} & \text{for } a = 0 \\ -\frac{2\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3a^3+3a^2bx} + \frac{\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3a^3+3a^2bx} + \frac{2\sqrt[3]{-1}\sqrt{3}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3a^3+3a^2bx} - \frac{2\sqrt[3]{-1}a^{\frac{4}{3}}\sqrt[3]{\frac{1}{b}}\log(2)}{3a^3+3a^2bx} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sqrt[3]{x}\right)}{3a^3+3a^2bx} + \frac{\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{x}\sqrt[3]{\frac{1}{b}}+4x^{\frac{2}{3}}\right)}{3a^3+3a^2bx} + \frac{2\sqrt[3]{-1}\sqrt{3}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sqrt{3}}{3}-\frac{2(-1)^{\frac{2}{3}}\sqrt{3}\sqrt[3]{x}}{3\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}}\right)}{3a^3+3a^2bx} - \frac{2\sqrt[3]{-1}\sqrt[3]{a}bx\sqrt[3]{\frac{1}{b}}\log(2)}{3a^3+3a^2bx} + \frac{3a\sqrt[3]{x}}{3a^3+3a^2bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo/x**(5/3), Eq(a, 0) & Eq(b, 0)), (3*x**(1/3)/a**2, Eq(b, 0)),
 (-3/(5*b**2*x**(5/3)), Eq(a, 0)), (-2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*lo
g(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x**(1/3))/(3*a**3 + 3*a**2*b*x) + (-
1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4
*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)**(1/3) + 4*x**(2/3))/(3*a**3 + 3*a**2*
b*x) + 2*(-1)**(1/3)*sqrt(3)*a**(4/3)*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)*
*(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/3)*(1/b)**(1/3)))/(3*a**3 + 3*a**2*b*x) -
2*(-1)**(1/3)*a**(4/3)*(1/b)**(1/3)*log(2)/(3*a**3 + 3*a**2*b*x) - 2*(-1)**
(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(-(-1)**(1/3)*a**(1/3)*(1/b)**(1/3) + x*
*(1/3))/(3*a**3 + 3*a**2*b*x) + (-1)**(1/3)*a**(1/3)*b*x*(1/b)**(1/3)*log(4
*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*x**(1/3)*(1/b)*
*(1/3) + 4*x**(2/3))/(3*a**3 + 3*a**2*b*x) + 2*(-1)**(1/3)*sqrt(3)*a**(1/3)
*b*x*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*x**(1/3)/(3*a**(1/
3)*(1/b)**(1/3)))/(3*a**3 + 3*a**2*b*x) - 2*(-1)**(1/3)*a**(1/3)*b*x*(1/b)*
*(1/3)*log(2)/(3*a**3 + 3*a**2*b*x) + 3*a*x**(1/3)/(3*a**3 + 3*a**2*b*x), T
rue))
```

3.688 $\int \frac{1}{x^{4/3}(a+bx)^2}\,dx$

Optimal. Leaf size=124

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}$$

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{2\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{a^{7/3}}-\frac{2\sqrt[3]{b}\log(a+bx)}{3a^{7/3}}+\frac{4\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}}-\frac{4}{a^2\sqrt[3]{x}}+\frac{1}{a\sqrt[3]{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] -4/(a^2*x^(1/3)) + 1/(a*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[(a^(1/3) - 2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (2*b^(1/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/a^(7/3) - (2*b^(1/3)*Log[a + b*x])/(3*a^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{x^{4/3}(a+bx)^2}\,dx = \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\int \frac{1}{x^{4/3}(a+bx)}\,dx}{3a}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{(4b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{3a^2}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{a^2} + \frac{\left(2\sqrt[3]{b}\right)}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a+bx)}{3a^{7/3}} - \frac{\left(4\sqrt[3]{b}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{4}{a^2\sqrt[3]{x}} + \frac{1}{a\sqrt[3]{x}\,(a+bx)} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{2\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{a^{7/3}} - \frac{2\sqrt[3]{b}\,\log(a}{3a^{7/3}}$$

Mathematica [C] time = 0.01, size = 25, normalized size = 0.20

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},2;\frac{2}{3};-\frac{bx}{a}\right)}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 2, 2/3, -((b*x)/a)])/(a^2*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.17, size = 152, normalized size = 1.23

$$-\frac{2\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{3a^{7/3}} + \frac{4\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{7/3}} + \frac{4\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{7/3}} + \frac{-3a-4bx}{a^2\sqrt[3]{x}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)^2),x]
```

```
[Out] (-3*a - 4*b*x)/(a^2*x^(1/3)*(a + b*x)) + (4*b^(1/3)*ArcTan[1/Sqrt[3] - (2*b
^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(7/3)) + (4*b^(1/3)*Log[a^(1
/3) + b^(1/3)*x^(1/3)])/(3*a^(7/3)) - (2*b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(3*a^(7/3))
```

fricas [A] time = 1.01, size = 156, normalized size = 1.26

$$-\frac{4\sqrt{3}\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+2\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-4\left(bx^2+ax\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\left(4bx+3a\right)x^{\frac{2}{3}}}{3\left(a^2bx^2+a^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*sqrt(3)*(b*x^2 + a*x)*(b/a)^(1/3)*arctan(2/3*sqrt(3)*x^(1/3)*(b/a)^
(1/3) - 1/3*sqrt(3)) + 2*(b*x^2 + a*x)*(b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/
```

```
3) + b*x^(2/3) + a*(b/a)^(1/3)) - 4*(b*x^2 + a*x)*(b/a)^(1/3)*log(a*(b/a)^(
2/3) + b*x^(1/3)) + 3*(4*b*x + 3*a)*x^(2/3))/(a^2*b*x^2 + a^3*x)
```

giac [A] time = 0.97, size = 145, normalized size = 1.17

$$\frac{4\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{3}}+\frac{4\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{3}b}-\frac{4\,bx+3\,a}{\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)a^{2}}-\frac{2\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4/3*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 4/3*sqrt(3)*(-a*b
^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*
b) - (4*b*x + 3*a)/((b*x^(4/3) + a*x^(1/3))*a^2) - 2/3*(-a*b^2)^(2/3)*log(x
^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b)
```

maple [A] time = 0.01, size = 121, normalized size = 0.98

$$-\frac{b\,x^{\frac{2}{3}}}{\left(bx+a\right)a^{2}}-\frac{4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}+\frac{4\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{2\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{2}}-\frac{3}{a^{2}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^2,x)
```

```
[Out] -3/a^2/x^(1/3)-1/a^2*b*x^(2/3)/(b*x+a)+4/3/a^2/(a/b)^(1/3)*ln(x^(1/3)+(a/b)
^(1/3))-2/3/a^2/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-4/3
/a^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 3.04, size = 132, normalized size = 1.06

$$-\frac{4\,bx+3\,a}{a^{2}bx^{\frac{4}{3}}+a^{3}x^{\frac{1}{3}}}-\frac{4\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{4\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b*x + 3*a)/(a^2*b*x^(4/3) + a^3*x^(1/3)) - 4/3*sqrt(3)*arctan(1/3*sqrt(
3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(1/3)) - 2/3*log(x^(2/
3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(1/3)) + 4/3*log(x^(1/3)
 + (a/b)^(1/3))/(a^2*(a/b)^(1/3))
```

mupad [B] time = 0.15, size = 151, normalized size = 1.22

$$\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}+16\,a^{2}\,b^{3}\,x^{1/3}\right)}{3\,a^{7/3}}-\frac{\frac{3}{a}+\frac{4\,b\,x}{a^{2}}}{a\,x^{1/3}+b\,x^{4/3}}-\frac{4\,b^{1/3}\,\ln\left(16\,a^{7/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^{2}+16\,a^{2}\,b^{3}\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{7/3}}+\frac{b^{1/3}\,\ln\left(9\,a^{7/3}\,b^{8/3}\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)^{2}+16\,a^{2}\,b^{3}\,x^{1/3}\right)\,\left(-\frac{2}{3}+\frac{\sqrt{3}\,2{}\mathrm{i}}{3}\right)}{a^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^2),x)
```

```
[Out] (4*b^(1/3)*log(16*a^(7/3)*b^(8/3) + 16*a^2*b^3*x^(1/3)))/(3*a^(7/3)) - (3/a
 + (4*b*x)/a^2)/(a*x^(1/3) + b*x^(4/3)) - (4*b^(1/3)*log(16*a^(7/3)*b^(8/3)
*((3^(1/2)*1i)/2 + 1/2)^2 + 16*a^2*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(3*
a^(7/3)) + (b^(1/3)*log(9*a^(7/3)*b^(8/3)*((3^(1/2)*2i)/3 - 2/3)^2 + 16*a^2
*b^3*x^(1/3))*((3^(1/2)*2i)/3 - 2/3))/a^(7/3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.689 $\int \frac{1}{x^{5/3}(a+bx)^2} dx$

Optimal. Leaf size=128

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Rubi [A] time = 0.05, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}}+\frac{5b^{2/3}\log(a+bx)}{6a^{8/3}}+\frac{5b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}}-\frac{5}{2a^2x^{2/3}}+\frac{1}{ax^{2/3}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] -5/(2*a^2*x^(2/3)) + 1/(a*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[(a^(1/3) -
 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(2*a^(8/3)) + (5*b^(2/3)*Log[a + b*x])/(6*a^(8/
3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/3}(a+bx)^2}\,dx &= \frac{1}{ax^{2/3}(a+bx)} + \frac{5\int \frac{1}{x^{5/3}(a+bx)}\,dx}{3a} \\
&= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{(5b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{3a^2} \\
&= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{x}\right)}{2a^{7/3}} - \\
&= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(a+bx)}{6a^{8/3}} - \frac{\left(5b^{2/3}\right)\text{Subst}\left(\right.}{} \\
&= -\frac{5}{2a^2x^{2/3}} + \frac{1}{ax^{2/3}(a+bx)} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{2a^{8/3}} + \frac{5b^{2/3}\log(}{6a^{8}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.21

$$
-\frac{3\,{}_2F_1\left(-\frac{2}{3},2;\frac{1}{3};-\frac{bx}{a}\right)}{2a^2x^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 2, 1/3, -((b*x)/a)])/(2*a^2*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.18, size = 155, normalized size = 1.21

$$
\frac{5b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{6a^{8/3}} - \frac{5b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{8/3}} + \frac{5b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{8/3}} + \frac{-3a-5bx}{2a^2x^{2/3}(a+bx)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)^2),x]
```

```
[Out] (-3*a - 5*b*x)/(2*a^2*x^(2/3)*(a + b*x)) + (5*b^(2/3)*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(8/3)) - (5*b^(2/3)*Log[a^
(1/3) + b^(1/3)*x^(1/3)])/(3*a^(8/3)) + (5*b^(2/3)*Log[a^(2/3) - a^(1/3)*b^
(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(6*a^(8/3))
```

fricas [B] time = 0.65, size = 189, normalized size = 1.48

$$
\frac{10\sqrt{3}\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}\,ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}\,b}{3\,b}\right)-5\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+10\left(bx^2+ax\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\left(5\,bx+3\,a\right)x^{\frac{1}{3}}}{6\left(a^2bx^2+a^3x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(10*sqrt(3)*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x^(1
/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 5*(b*x^2 + a*x)*(-b^2/a^2)^(1/3)*log
```

```
(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 10*(b
*x^2 + a*x)*(-b^2/a^2)^(1/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(5*b*x
 + 3*a)*x^(1/3))/(a^2*b*x^2 + a^3*x)
```

giac [A] time = 1.01, size = 137, normalized size = 1.07

$$\frac{5\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,a^{3}}-\frac{5\,\sqrt{3}\,\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{3}}-\frac{bx^{\frac{1}{3}}}{(bx+a)a^{2}}-\frac{5\,\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{3}}-\frac{3}{2\,a^{2}x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5/3*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 5/3*sqrt(3)*(-a*b
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^3 -
 b*x^(1/3)/((b*x + a)*a^2) - 5/6*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b
)^(1/3) + (-a/b)^(2/3))/a^3 - 3/2/(a^2*x^(2/3))
```

maple [A] time = 0.01, size = 121, normalized size = 0.95

$$-\frac{b\,x^{\frac{1}{3}}}{(bx+a)\,a^{2}}-\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}}-\frac{3}{2a^{2}x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^2,x)
```

```
[Out] -3/2/a^2/x^(2/3)-1/a^2*b*x^(1/3)/(b*x+a)-5/3/a^2/(a/b)^(2/3)*ln(x^(1/3)+(a/
b)^(1/3))+5/6/a^2/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-5
/3/a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.94, size = 132, normalized size = 1.03

$$-\frac{5\,bx+3\,a}{2\left(a^{2}bx^{\frac{5}{3}}+a^{3}x^{\frac{2}{3}}\right)}-\frac{5\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\,a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(5*b*x + 3*a)/(a^2*b*x^(5/3) + a^3*x^(2/3)) - 5/3*sqrt(3)*arctan(1/3*s
qrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*(a/b)^(2/3)) + 5/6*log(x
^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*(a/b)^(2/3)) - 5/3*log(x^(
1/3) + (a/b)^(1/3))/(a^2*(a/b)^(2/3))
```

mupad [B] time = 0.17, size = 166, normalized size = 1.30

$$\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}-15\,a^4\,b^3\,x^{1/3}\right)}{3\,a^{8/3}}-\frac{\frac{3}{2\,a}+\frac{5\,b\,x}{2\,a^2}}{a\,x^{2/3}+b\,x^{5/3}}+\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}-15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}-\frac{5\,{\left(-1\right)}^{1/3}\,b^{2/3}\,\ln\left(15\,a^4\,b^3\,x^{1/3}+15\,{\left(-1\right)}^{1/3}\,a^{13/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{3\,a^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^2),x)
```

```
[Out] (5*(-1)^(1/3)*b^(2/3)*log(15*(-1)^(1/3)*a^(13/3)*b^(8/3) - 15*a^4*b^3*x^(1/
3)))/(3*a^(8/3)) - (3/(2*a) + (5*b*x)/(2*a^2))/(a*x^(2/3) + b*x^(5/3)) + (5
*(-1)^(1/3)*b^(2/3)*log(15*a^4*b^3*x^(1/3) - 15*(-1)^(1/3)*a^(13/3)*b^(8/3)
*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(8/3)) - (5*(-1)^(1/3
)*b^(2/3)*log(15*a^4*b^3*x^(1/3) + 15*(-1)^(1/3)*a^(13/3)*b^(8/3)*((3^(1/2)
*1i)/2 + 1/2))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.690 $\int \frac{x^{5/3}}{(a+bx)^3} dx$

Optimal. Leaf size=140

$$-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 56, 617, 204, 31}

$$-\frac{5x^{2/3}}{6b^2(a+bx)}-\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6\sqrt[3]{a}b^{8/3}}+\frac{5\log(a+bx)}{18\sqrt[3]{a}b^{8/3}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\sqrt[3]{a}b^{8/3}}-\frac{x^{5/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] -x^(5/3)/(2*b*(a + b*x)^2) - (5*x^(2/3))/(6*b^2*(a + b*x)) - (5*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) -
 (5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(1/3)*b^(8/3)) + (5*Log[a + b*x])/
(18*a^(1/3)*b^(8/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{5/3}}{(a+bx)^3}\,dx = -\frac{x^{5/3}}{2b(a+bx)^2} + \frac{5\int \frac{x^{2/3}}{(a+bx)^2}\,dx}{6b}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} + \frac{5\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9b^2}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} + \frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6b^3} - \frac{5\,\mathrm{Subst}\left(\int \right.}{6}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} - \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}} + \frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}} + \frac{5\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, \right.}{3\sqrt[3]{a}\,b^{8/3}}$$

$$= -\frac{x^{5/3}}{2b(a+bx)^2} - \frac{5x^{2/3}}{6b^2(a+bx)} - \frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}} - \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{6\sqrt[3]{a}\,b^{8/3}} + \frac{5\log(a+bx)}{18\sqrt[3]{a}\,b^{8/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{8/3}\,{}_2F_1\left(\frac{8}{3},3;\frac{11}{3};-\frac{bx}{a}\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(8/3)*Hypergeometric2F1[8/3, 3, 11/3, -((b*x)/a)])/(8*a^3)
```

IntegrateAlgebraic [A] time = 0.26, size = 161, normalized size = 1.15

$$\frac{5\log\left(a^{2/3} - \sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x} + b^{2/3}x^{2/3}\right)}{18\sqrt[3]{a}\,b^{8/3}} - \frac{5\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{9\sqrt[3]{a}\,b^{8/3}} - \frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,\sqrt[3]{a}\,b^{8/3}} + \frac{-5ax^{2/3} - 8bx^{5/3}}{6b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(5/3)/(a + b*x)^3,x]
```

```
[Out] (-5*a*x^(2/3) - 8*b*x^(5/3))/(6*b^2*(a + b*x)^2) - (5*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1/3)*b^(8/3)) - (5*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(1/3)*b^(8/3)) + (5*Log[a^(2/3) - a^(1/3)*
b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(1/3)*b^(8/3))
```

fricas [B] time = 1.03, size = 506, normalized size = 3.61

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^2x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}a+2\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}-3\left(-ab^2\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)}, \frac{30\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^2\right)^{\frac{1}{3}}}{a}}}{b}\right)+5\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(b^2x^{\frac{2}{3}}+\left(-ab^2\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{2}{3}}\right)-10\left(b^2x^2+2abx+a^2\right)\left(-ab^2\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)-3\left(8ab^3x+5a^2b^2\right)x^{\frac{2}{3}}}{18\left(ab^6x^2+2a^2b^5x+a^3b^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a
)*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*
b^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
 + a)) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b
^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^
2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3
))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 5*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
10*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
 - 3*(8*a*b^3*x + 5*a^2*b^2)*x^(2/3))/(a*b^6*x^2 + 2*a^2*b^5*x + a^3*b^4)]
```

giac [A] time = 1.06, size = 146, normalized size = 1.04

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{2}}-\frac{5\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{4}}-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\,(bx+a)^{2}b^{2}}+\frac{5\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) - 5/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^4) - 1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/((b*x + a)^2*b^2) + 5/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4)
```

maple [A] time = 0.01, size = 124, normalized size = 0.89

$$\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}-\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}b^{3}}+\frac{-\frac{4x^{\frac{5}{3}}}{3b}-\frac{5a\,x^{\frac{2}{3}}}{6b^{2}}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-4/9/b*x^(5/3)-5/18*a/b^2*x^(2/3))/(b*x+a)^2-5/9/b^3/(a/b)^(1/3)*ln(x^(1
/3)+(a/b)^(1/3))+5/18/b^3/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^
(2/3))+5/9/b^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3
)-1))
```

maxima [A] time = 2.95, size = 143, normalized size = 1.02

$$-\frac{8\,bx^{\frac{5}{3}}+5\,ax^{\frac{2}{3}}}{6\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{5\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{5\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(8*b*x^(5/3) + 5*a*x^(2/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 5/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(1
```

```
/3)) + 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(1/
3)) - 5/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(1/3))
```

mupad [B] time = 0.17, size = 165, normalized size = 1.18

$$\frac{5\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{25(-a)^{1/3}}{9b^{10/3}}\right)}{9(-a)^{1/3}b^{8/3}}-\frac{\frac{4x^{5/3}}{3b}+\frac{5ax^{2/3}}{6b^2}}{a^2+2abx+b^2x^2}+\frac{\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{(-a)^{1/3}\left(-5+\sqrt{3}\,5\mathrm{i}\right)^2}{36b^{10/3}}\right)\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{18(-a)^{1/3}b^{8/3}}-\frac{\ln\left(\frac{25x^{1/3}}{9b^3}-\frac{(-a)^{1/3}\left(5+\sqrt{3}\,5\mathrm{i}\right)^2}{36b^{10/3}}\right)\left(5+\sqrt{3}\,5\mathrm{i}\right)}{18(-a)^{1/3}b^{8/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/3)/(a + b*x)^3,x)
```

```
[Out] (5*log((25*x^(1/3))/(9*b^3) - (25*(-a)^(1/3))/(9*b^(10/3))))/(9*(-a)^(1/3)*
b^(8/3)) - ((4*x^(5/3))/(3*b) + (5*a*x^(2/3))/(6*b^2))/(a^2 + b^2*x^2 + 2*a
*b*x) + (log((25*x^(1/3))/(9*b^3) - ((-a)^(1/3)*(3^(1/2)*5i - 5)^2)/(36*b^(
10/3)))*(3^(1/2)*5i - 5))/(18*(-a)^(1/3)*b^(8/3)) - (log((25*x^(1/3))/(9*b^
3) - ((-a)^(1/3)*(3^(1/2)*5i + 5)^2)/(36*b^(10/3)))*(3^(1/2)*5i + 5))/(18*(
-a)^(1/3)*b^(8/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.691 $\int \frac{x^{4/3}}{(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {47, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}}-\frac{\log(a+bx)}{9a^{2/3}b^{7/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}}-\frac{2\sqrt[3]{x}}{3b^2(a+bx)}-\frac{x^{4/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] -x^(4/3)/(2*b*(a + b*x)^2) - (2*x^(1/3))/(3*b^2*(a + b*x)) - (2*ArcTan[(a^(
1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) +
 Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(2/3)*b^(7/3)) - Log[a + b*x]/(9*a^(2/
3)*b^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{4/3}}{(a+bx)^3}\,dx = -\frac{x^{4/3}}{2b(a+bx)^2} + \frac{2\int \frac{\sqrt[3]{x}}{(a+bx)^2}\,dx}{3b}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2} - \frac{2\sqrt[3]{x}}{3b^2(a+bx)} + \frac{2\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9b^2}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2} - \frac{2\sqrt[3]{x}}{3b^2(a+bx)} - \frac{\log(a+bx)}{9a^{2/3}b^{7/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{3\sqrt[3]{a}\,b^{8/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} + \cdots\right.}{3a^{2/3}\cdots}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2} - \frac{2\sqrt[3]{x}}{3b^2(a+bx)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}} - \frac{\log(a+bx)}{9a^{2/3}b^{7/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1 - \cdots\right.}{3a^{2/3}b^{7/3}}$$

$$= -\frac{x^{4/3}}{2b(a+bx)^2} - \frac{2\sqrt[3]{x}}{3b^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{2/3}b^{7/3}} - \frac{\log(a+bx)}{9a^{2/3}b^{7/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{7/3}\,{}_2F_1\left(\frac{7}{3},3;\frac{10}{3};-\frac{bx}{a}\right)}{7a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(7/3)*Hypergeometric2F1[7/3, 3, 10/3, -((b*x)/a)])/(7*a^3)
```

IntegrateAlgebraic [A] time = 0.25, size = 161, normalized size = 1.15

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{2/3}b^{7/3}} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{2/3}b^{7/3}} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{2/3}b^{7/3}} + \frac{-4a\sqrt[3]{x}-7bx^{4/3}}{6b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(4/3)/(a + b*x)^3,x]
```

```
[Out] (-4*a*x^(1/3) - 7*b*x^(4/3))/(6*b^2*(a + b*x)^2) - (2*ArcTan[1/Sqrt[3] - (2
*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(2/3)*b^(7/3)) + (2*Log[
a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(2/3)*b^(7/3)) - Log[a^(2/3) - a^(1/3)*b^(
1/3)*x^(1/3) + b^(2/3)*x^(2/3)]/(9*a^(2/3)*b^(7/3))
```

fricas [B] time = 1.11, size = 503, normalized size = 3.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
```

```
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^
(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3
)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 4*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2
/3)) - 3*(7*a^2*b^2*x + 4*a^3*b)*x^(1/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*
b^3)]
```

giac [A] time = 1.13, size = 146, normalized size = 1.04

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,ab^{2}}+\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{3}}+\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,ab^{3}}-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\,(bx+a)^{2}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a*b^2) + 2/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
*b^3) + 1/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3
))/(a*b^3) - 1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/((b*x + a)^2*b^2)
```

maple [A] time = 0.01, size = 124, normalized size = 0.89

$$\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}-\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^{3}}+\frac{-\frac{7x^{\frac{4}{3}}}{6b}-\frac{2a\,x^{\frac{1}{3}}}{3b^{2}}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(b*x+a)^3,x)
```

```
[Out] 3*(-7/18/b*x^(4/3)-2/9*a/b^2*x^(1/3))/(b*x+a)^2+2/9/b^3/(a/b)^(2/3)*ln(x^(1
/3)+(a/b)^(1/3))-1/9/b^3/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+2/9/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)
-1))
```

maxima [A] time = 2.88, size = 143, normalized size = 1.02

$$-\frac{7\,bx^{\frac{4}{3}}+4\,ax^{\frac{1}{3}}}{6\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,b^{3}\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(7*b*x^(4/3) + 4*a*x^(1/3))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2) + 2/9*sqrt
(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(b^3*(a/b)^(2
```

```
/3)) - 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(b^3*(a/b)^(2/3
)) + 2/9*log(x^(1/3) + (a/b)^(1/3))/(b^3*(a/b)^(2/3))
```

mupad [B] time = 0.07, size = 139, normalized size = 0.99

$$\frac{2\,\ln\left(2\,x^{1/3}+\frac{2\,a^{1/3}}{b^{1/3}}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\frac{7\,x^{4/3}}{6\,b}+\frac{2\,a\,x^{1/3}}{3\,b^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{\ln\left(2\,x^{1/3}+\frac{a^{1/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}-\frac{\ln\left(2\,x^{1/3}-\frac{a^{1/3}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{b^{1/3}}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{2/3}\,b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(4/3)/(a + b*x)^3,x)
```

```
[Out] (2*log(2*x^(1/3) + (2*a^(1/3))/b^(1/3)))/(9*a^(2/3)*b^(7/3)) - ((7*x^(4/3))
/(6*b) + (2*a*x^(1/3))/(3*b^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (log(2*x^(1/3)
+ (a^(1/3)*(3^(1/2)*1i - 1))/b^(1/3))*(3^(1/2)*1i - 1))/(9*a^(2/3)*b^(7/3))
 - (log(2*x^(1/3) - (a^(1/3)*(3^(1/2)*1i + 1))/b^(1/3))*(3^(1/2)*1i + 1))/(
9*a^(2/3)*b^(7/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.692 $\int \frac{x^{2/3}}{(a+bx)^3} dx$

Optimal. Leaf size=143

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 56, 617, 204, 31}

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}}+\frac{\log(a+bx)}{18a^{4/3}b^{5/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}}+\frac{x^{2/3}}{3ab(a+bx)}-\frac{x^{2/3}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] -x^(2/3)/(2*b*(a + b*x)^2) + x^(2/3)/(3*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(4/3)*b^(5/3)) + Log[a + b*x]/(18*a^(4/3)*b^(5
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^{2/3}}{(a+bx)^3}\,dx = -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx}{3b}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9ab}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6ab^2} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{6a^{4/3}}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\right.}{3a^{4/3}b^{5/3}}$$

$$= -\frac{x^{2/3}}{2b(a+bx)^2} + \frac{x^{2/3}}{3ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\log(a+bx)}{18a^{4/3}b^{5/3}}$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.19

$$\frac{3x^{5/3}\,{}_2F_1\left(\frac{5}{3},3;\frac{8}{3};-\frac{bx}{a}\right)}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(5/3)*Hypergeometric2F1[5/3, 3, 8/3, -((b*x)/a)])/(5*a^3)
```

IntegrateAlgebraic [A] time = 0.24, size = 164, normalized size = 1.15

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{4/3}b^{5/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{4/3}b^{5/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{4/3}b^{5/3}} + \frac{2bx^{5/3}-ax^{2/3}}{6ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(2/3)/(a + b*x)^3,x]
```

```
[Out] (-(a*x^(2/3)) + 2*b*x^(5/3))/(6*a*b*(a + b*x)^2) - ArcTan[1/Sqrt[3] - (2*b^
(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(4/3)*b^(5/3)) - Log[a^(1/3)
 + b^(1/3)*x^(1/3)]/(9*a^(4/3)*b^(5/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(
1/3) + b^(2/3)*x^(2/3)]/(18*a^(4/3)*b^(5/3))
```

fricas [B] time = 0.99, size = 508, normalized size = 3.55

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2b^{2}x-ab+3\sqrt{\frac{1}{3}}\left(abx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}a+2\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{2}{3}}\right)\sqrt{\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}-3\left(-ab^{2}\right)^{\frac{2}{3}}x^{\frac{1}{3}}}{bx+a}\right)+\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)+3\left(2ab^{3}x-a^{2}b^{2}\right)x^{\frac{2}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)},\ \frac{6\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}}{a}}}{b}\right)+\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(b^{2}x^{\frac{2}{3}}+\left(-ab^{2}\right)^{\frac{1}{3}}bx^{\frac{1}{3}}+\left(-ab^{2}\right)^{\frac{2}{3}}\right)-2\left(b^{2}x^{2}+2abx+a^{2}\right)\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(bx^{\frac{1}{3}}-\left(-ab^{2}\right)^{\frac{1}{3}}\right)+3\left(2ab^{3}x-a^{2}b^{2}\right)x^{\frac{2}{3}}}{18\left(a^{2}b^{5}x^{2}+2a^{3}b^{4}x+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)
^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*a*b^3*x - a^2*b^2)*x^(2/3))/(a^
2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^
2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (-a*b^
2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + (b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(
2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 2*(b^2*
x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(2*
a*b^3*x - a^2*b^2)*x^(2/3))/(a^2*b^5*x^2 + 2*a^3*b^4*x + a^4*b^3)]
```

giac [A] time = 1.06, size = 149, normalized size = 1.04

$$-\frac{\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{2}b}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b^{3}}+\frac{2\,bx^{\frac{5}{3}}-ax^{\frac{2}{3}}}{6\,(bx+a)^{2}ab}+\frac{\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) - 1/9*sqrt(3)*(-
a*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^3) + 1/6*(2*b*x^(5/3) - a*x^(2/3))/((b*x + a)^2*a*b) + 1/18*(-a*b^2)^(
2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

maple [A] time = 0.01, size = 132, normalized size = 0.92

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}-\frac{\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{1}{3}}a\,b^{2}}+\frac{\frac{x^{\frac{5}{3}}}{3a}-\frac{x^{\frac{2}{3}}}{6b}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/9/a*x^(5/3)-1/18/b*x^(2/3))/(b*x+a)^2-1/9/b^2/a/(a/b)^(1/3)*ln(x^(1/3)
+(a/b)^(1/3))+1/18/b^2/a/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/9/b^2/a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/
3)-1))
```

maxima [A] time = 2.92, size = 153, normalized size = 1.07

$$\frac{2\,bx^{\frac{5}{3}} - ax^{\frac{2}{3}}}{6\left(ab^{3}x^{2} + 2\,a^{2}b^{2}x + a^{3}b\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x^(5/3) - a*x^(2/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
1/3)) + 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(1/3)) - 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(1/3))
```

mupad [B] time = 0.26, size = 172, normalized size = 1.20

$$\frac{\frac{x^{5/3}}{3\,a} - \frac{x^{2/3}}{6\,b}}{a^{2} + 2\,a\,b\,x + b^{2}\,x^{2}} + \frac{\ln\left(\frac{1}{9\,a^{5/3}\,(-b)^{4/3}} + \frac{x^{1/3}}{9\,a^{2}\,b}\right)}{9\,a^{4/3}\,(-b)^{5/3}} + \frac{\ln\left(\frac{x^{1/3}}{9\,a^{2}\,b} + \frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{36\,a^{5/3}\,(-b)^{4/3}}\right)\left(-1 + \sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,(-b)^{5/3}} - \frac{\ln\left(\frac{x^{1/3}}{9\,a^{2}\,b} + \frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)^{2}}{36\,a^{5/3}\,(-b)^{4/3}}\right)\left(1 + \sqrt{3}\,1\mathrm{i}\right)}{18\,a^{4/3}\,(-b)^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2/3)/(a + b*x)^3,x)
```

```
[Out] (x^(5/3)/(3*a) - x^(2/3)/(6*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(1/(9*a^(5/3
)*(-b)^(4/3)) + x^(1/3)/(9*a^2*b))/(9*a^(4/3)*(-b)^(5/3)) + (log(x^(1/3)/(9
*a^2*b) + (3^(1/2)*1i - 1)^2/(36*a^(5/3)*(-b)^(4/3)))*(3^(1/2)*1i - 1))/(18
*a^(4/3)*(-b)^(5/3)) - (log(x^(1/3)/(9*a^2*b) + (3^(1/2)*1i + 1)^2/(36*a^(5
/3)*(-b)^(4/3)))*(3^(1/2)*1i + 1))/(18*a^(4/3)*(-b)^(5/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.693 $\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx$

Optimal. Leaf size=143

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 51, 58, 617, 204, 31}

$$\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}}-\frac{\log(a+bx)}{18a^{5/3}b^{4/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}}+\frac{\sqrt[3]{x}}{6ab(a+bx)}-\frac{\sqrt[3]{x}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] -x^(1/3)/(2*b*(a + b*x)^2) + x^(1/3)/(6*a*b*(a + b*x)) - ArcTan[(a^(1/3) -
2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(
1/3) + b^(1/3)*x^(1/3)]/(6*a^(5/3)*b^(4/3)) - Log[a + b*x]/(18*a^(5/3)*b^(4
/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{x}}{(a+bx)^3}\,dx = -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6b}$$
$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9ab}$$
$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{6a^{4/3}b^{5/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{a}}{\sqrt[3]{b}} \cdots}\right)}{6a^{5/3}\cdots}$$
$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\cdots\right)}{3a^{5/3}b^{4/3}}$$
$$= -\frac{\sqrt[3]{x}}{2b(a+bx)^2} + \frac{\sqrt[3]{x}}{6ab(a+bx)} - \frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{5/3}b^{4/3}} - \frac{\log(a+bx)}{18a^{5/3}b^{4/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{4/3}\,{}_2F_1\left(\frac{4}{3},3;\frac{7}{3};-\frac{bx}{a}\right)}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] (3*x^(4/3)*Hypergeometric2F1[4/3, 3, 7/3, -((b*x)/a)])/(4*a^3)
```

IntegrateAlgebraic [A] time = 0.24, size = 163, normalized size = 1.14

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{5/3}b^{4/3}} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{5/3}b^{4/3}} - \frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}b^{4/3}} + \frac{bx^{4/3}-2a\sqrt[3]{x}}{6ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(1/3)/(a + b*x)^3,x]
```

```
[Out] (-2*a*x^(1/3) + b*x^(4/3))/(6*a*b*(a + b*x)^2) - ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))]/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + Log[a^(1/3) +
 b^(1/3)*x^(1/3)]/(9*a^(5/3)*b^(4/3)) - Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/
3) + b^(2/3)*x^(2/3)]/(18*a^(5/3)*b^(4/3))
```

fricas [B] time = 1.21, size = 501, normalized size = 3.50

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{-\frac{(a^{2}b)^{\frac13}}{b}}\log\left(\frac{2abx-a^{2}+3\sqrt{\frac13}\left(2abx^{\frac23}-(a^{2}b)^{\frac13}a+(a^{2}b)^{\frac23}x^{\frac13}\right)\sqrt{-\frac{(a^{2}b)^{\frac13}}{b}}-3(a^{2}b)^{\frac13}ax^{\frac13}}{bx+a}\right)-\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac23}\log\left(abx^{\frac23}+(a^{2}b)^{\frac13}a-(a^{2}b)^{\frac23}x^{\frac13}\right)+2\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac23}\log\left(abx^{\frac13}+(a^{2}b)^{\frac23}\right)+3\left(a^{2}b^{2}x-2a^{3}b\right)x^{\frac13}}{18\left(a^{3}b^{4}x^{2}+2a^{4}b^{3}x+a^{5}b^{2}\right)},\ \frac{6\sqrt{\frac13}\left(ab^{3}x^{2}+2a^{2}b^{2}x+a^{3}b\right)\sqrt{\frac{(a^{2}b)^{\frac13}}{b}}\arctan\left(-\frac{\sqrt{\frac13}\left((a^{2}b)^{\frac13}a-2(a^{2}b)^{\frac23}x^{\frac13}\right)\sqrt{\frac{(a^{2}b)^{\frac13}}{b}}}{a^{2}}\right)-\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac23}\log\left(abx^{\frac23}+(a^{2}b)^{\frac13}a-(a^{2}b)^{\frac23}x^{\frac13}\right)+2\left(b^{2}x^{2}+2abx+a^{2}\right)(a^{2}b)^{\frac23}\log\left(abx^{\frac13}+(a^{2}b)^{\frac23}\right)+3\left(a^{2}b^{2}x-2a^{3}b\right)x^{\frac13}}{18\left(a^{3}b^{4}x^{2}+2a^{4}b^{3}x+a^{5}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b)
*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*b
)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x +
 a)) - (b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1
/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*
log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^
4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x
+ a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*(a^2*
b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - (b^2*x^2 + 2*a*b*x + a^2)*(a
^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 2*
(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) +
3*(a^2*b^2*x - 2*a^3*b)*x^(1/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

giac [A] time = 1.11, size = 148, normalized size = 1.03

$$-\frac{\left(-\frac{a}{b}\right)^{\frac13}\log\left(\left|x^{\frac13}-\left(-\frac{a}{b}\right)^{\frac13}\right|\right)}{9\,a^{2}b}+\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac13}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac13}+\left(-\frac{a}{b}\right)^{\frac13}\right)}{3\left(-\frac{a}{b}\right)^{\frac13}}\right)}{9\,a^{2}b^{2}}+\frac{\left(-ab^{2}\right)^{\frac13}\log\left(x^{\frac23}+x^{\frac13}\left(-\frac{a}{b}\right)^{\frac13}+\left(-\frac{a}{b}\right)^{\frac23}\right)}{18\,a^{2}b^{2}}+\frac{bx^{\frac43}-2\,ax^{\frac13}}{6\,(bx+a)^{2}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/(a^2*b) + 1/9*sqrt(3)*(-
a*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a
^2*b^2) + 1/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(
2/3))/(a^2*b^2) + 1/6*(b*x^(4/3) - 2*a*x^(1/3))/((b*x + a)^2*a*b)
```

maple [A] time = 0.01, size = 132, normalized size = 0.92

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac13}}{\left(\frac{a}{b}\right)^{\frac13}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac23}a\,b^{2}}+\frac{\ln\left(x^{\frac13}+\left(\frac{a}{b}\right)^{\frac13}\right)}{9\left(\frac{a}{b}\right)^{\frac23}a\,b^{2}}-\frac{\ln\left(x^{\frac23}-\left(\frac{a}{b}\right)^{\frac13}x^{\frac13}+\left(\frac{a}{b}\right)^{\frac23}\right)}{18\left(\frac{a}{b}\right)^{\frac23}a\,b^{2}}+\frac{\frac{x^{\frac43}}{6a}-\frac{x^{\frac13}}{3b}}{(bx+a)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 3*(1/18/a*x^(4/3)-1/9/b*x^(1/3))/(b*x+a)^2+1/9/b^2/a/(a/b)^(2/3)*ln(x^(1/3)
+(a/b)^(1/3))-1/18/b^2/a/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(
2/3))+1/9/b^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x^(1/
3)-1))
```

maxima [A] time = 2.96, size = 152, normalized size = 1.06

$$\frac{bx^{\frac{4}{3}} - 2\,ax^{\frac{1}{3}}}{6\left(ab^3x^2 + 2\,a^2b^2x + a^3b\right)} + \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}} - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{\log\left(x^{\frac{2}{3}} - x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{\log\left(x^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^(4/3) - 2*a*x^(1/3))/(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b) + 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a*b^2*(a/b)^(
2/3)) - 1/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a*b^2*(a/b)^
(2/3)) + 1/9*log(x^(1/3) + (a/b)^(1/3))/(a*b^2*(a/b)^(2/3))
```

mupad [B] time = 0.24, size = 146, normalized size = 1.02

$$\frac{\frac{x^{4/3}}{6\,a} - \frac{x^{1/3}}{3\,b}}{a^2 + 2\,a\,b\,x + b^2\,x^2} + \frac{\ln\left(\frac{b^{2/3}}{a^{2/3}} + \frac{b\,x^{1/3}}{a}\right)}{9\,a^{5/3}\,b^{4/3}} + \frac{\ln\left(\frac{b\,x^{1/3}}{a} + \frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(-1 + \sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}} - \frac{\ln\left(\frac{b\,x^{1/3}}{a} - \frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,a^{2/3}}\right)\left(1 + \sqrt{3}\,1\mathrm{i}\right)}{18\,a^{5/3}\,b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/3)/(a + b*x)^3,x)
```

```
[Out] (x^(4/3)/(6*a) - x^(1/3)/(3*b))/(a^2 + b^2*x^2 + 2*a*b*x) + log(b^(2/3)/a^(
2/3) + (b*x^(1/3))/a)/(9*a^(5/3)*b^(4/3)) + (log((b*x^(1/3))/a + (b^(2/3)*(
3^(1/2)*1i - 1))/(2*a^(2/3)))*(3^(1/2)*1i - 1))/(18*a^(5/3)*b^(4/3)) - (log
((b*x^(1/3))/a - (b^(2/3)*(3^(1/2)*1i + 1))/(2*a^(2/3)))*(3^(1/2)*1i + 1))/
(18*a^(5/3)*b^(4/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.694 $\int \frac{1}{\sqrt[3]{x}\,(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = $\{51, 56, 617, 204, 31\}$

$$-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}}+\frac{\log(a+bx)}{9a^{7/3}b^{2/3}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}}+\frac{2x^{2/3}}{3a^2(a+bx)}+\frac{x^{2/3}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(1/3)*(a + b*x)^3),x]
```

```
[Out] x^(2/3)/(2*a*(a + b*x)^2) + (2*x^(2/3))/(3*a^2*(a + b*x)) - (2*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) -
Log[a^(1/3) + b^(1/3)*x^(1/3)]/(3*a^(7/3)*b^(2/3)) + Log[a + b*x]/(9*a^(7/3
)*b^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}(a+bx)^3}\,dx = \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)^2}\,dx}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{2\int \frac{1}{\sqrt[3]{x}(a+bx)}\,dx}{9a^2}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}x}{\sqrt[3]{b}} + x^2}\,dx, x, \sqrt[3]{x}\right)}{3a^2b} - \frac{\text{Subst}\left(\int \right.}{3a}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1\right.}{3a^{7/3}b^{2/3}}$$

$$= \frac{x^{2/3}}{2a(a+bx)^2} + \frac{2x^{2/3}}{3a^2(a+bx)} - \frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} - \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{7/3}b^{2/3}} + \frac{\log(a+bx)}{9a^{7/3}b^{2/3}}$$

Mathematica [C] time = 0.00, size = 27, normalized size = 0.19

$$\frac{3x^{2/3}\,{}_2F_1\left(\frac{2}{3},3;\frac{5}{3};-\frac{bx}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(1/3)*(a + b*x)^3),x]
[Out] (3*x^(2/3)*Hypergeometric2F1[2/3, 3, 5/3, -((b*x)/a)])/(2*a^3)
```

IntegrateAlgebraic [A] time = 0.16, size = 157, normalized size = 1.12

$$\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{7/3}b^{2/3}} - \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{7/3}b^{2/3}} - \frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{7/3}b^{2/3}} + \frac{x^{2/3}(7a+4bx)}{6a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(1/3)*(a + b*x)^3),x]
[Out] (x^(2/3)*(7*a + 4*b*x))/(6*a^2*(a + b*x)^2) - (2*ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(7/3)*b^(2/3)) - (2*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/(9*a^(7/3)*b^(2/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*
x^(1/3) + b^(2/3)*x^(2/3)]/(9*a^(7/3)*b^(2/3))
```

fricas [B] time = 0.89, size = 510, normalized size = 3.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="fricas")
[Out] [1/18*(6*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt((-a*b^2)^(1/3)/a)
*log((2*b^2*x - a*b + 3*sqrt(1/3)*(a*b*x^(1/3) + (-a*b^2)^(1/3)*a + 2*(-a*b
```

```
^2)^(2/3)*x^(2/3))*sqrt((-a*b^2)^(1/3)/a) - 3*(-a*b^2)^(2/3)*x^(1/3))/(b*x
+ a)) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^
2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) - 4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)
^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3)) + 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))
/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2), 1/18*(12*sqrt(1/3)*(a*b^3*x^2 + 2*a
^2*b^2*x + a^3*b)*sqrt(-(-a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*b*x^(1/3) + (
-a*b^2)^(1/3))*sqrt(-(-a*b^2)^(1/3)/a)/b) + 2*(b^2*x^2 + 2*a*b*x + a^2)*(-a
*b^2)^(2/3)*log(b^2*x^(2/3) + (-a*b^2)^(1/3)*b*x^(1/3) + (-a*b^2)^(2/3)) -
4*(b^2*x^2 + 2*a*b*x + a^2)*(-a*b^2)^(2/3)*log(b*x^(1/3) - (-a*b^2)^(1/3))
+ 3*(4*a*b^3*x + 7*a^2*b^2)*x^(2/3))/(a^3*b^4*x^2 + 2*a^4*b^3*x + a^5*b^2)]
```

giac [A] time = 1.06, size = 143, normalized size = 1.02

$$-\frac{2\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^3}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3b^2}+\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\,(bx+a)^2a^2}+\frac{\left(-ab^2\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -2/9*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 - 2/9*sqrt(3)*(-a*b^
2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
^2) + 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/((b*x + a)^2*a^2) + 1/9*(-a*b^2)^(2/3
)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^3*b^2)
```

maple [A] time = 0.01, size = 136, normalized size = 0.97

$$\frac{x^{\frac{2}{3}}}{2\,(bx+a)^2\,a}+\frac{2x^{\frac{2}{3}}}{3\,(bx+a)\,a^2}+\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}-\frac{2\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}+\frac{\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(2/3)/a/(b*x+a)^2+2/3*x^(2/3)/a^2/(b*x+a)-2/9/a^2/b/(a/b)^(1/3)*ln(x^
(1/3)+(a/b)^(1/3))+1/9/a^2/b/(a/b)^(1/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a/
b)^(2/3))+2/9/a^2*3^(1/2)/b/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x
^(1/3)-1))
```

maxima [A] time = 2.96, size = 151, normalized size = 1.08

$$\frac{4\,bx^{\frac{5}{3}}+7\,ax^{\frac{2}{3}}}{6\left(a^2b^2x^2+2\,a^3bx+a^4\right)}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^2b\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(4*b*x^(5/3) + 7*a*x^(2/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 2/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
1/3)) + 1/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^(
1/3)) - 2/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(1/3))
```

mupad [B] time = 0.19, size = 167, normalized size = 1.19

$$\frac{\frac{7x^{2/3}}{6a}+\frac{2bx^{5/3}}{3a^2}}{a^2+2abx+b^2x^2}+\frac{2\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{4b^{2/3}}{9(-a)^{11/3}}\right)}{9(-a)^{7/3}b^{2/3}}+\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}-\frac{\ln\left(\frac{4bx^{1/3}}{9a^4}-\frac{b^{2/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{9(-a)^{11/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{9(-a)^{7/3}b^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)*(a + b*x)^3),x)
```

```
[Out] ((7*x^(2/3))/(6*a) + (2*b*x^(5/3))/(3*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (2*
log((4*b*x^(1/3))/(9*a^4) - (4*b^(2/3))/(9*(-a)^(11/3))))/(9*(-a)^(7/3)*b^(
2/3)) + (log((4*b*x^(1/3))/(9*a^4) - (b^(2/3)*(3^(1/2)*1i - 1)^2)/(9*(-a)^(
11/3)))*(3^(1/2)*1i - 1))/(9*(-a)^(7/3)*b^(2/3)) - (log((4*b*x^(1/3))/(9*a^
4) - (b^(2/3)*(3^(1/2)*1i + 1)^2)/(9*(-a)^(11/3)))*(3^(1/2)*1i + 1))/(9*(-a
)^(7/3)*b^(2/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.695 $\int \frac{1}{x^{2/3}(a+bx)^3}\,dx$

Optimal. Leaf size=140

$$\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}}{2a(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] x^(1/3)/(2*a*(a + b*x)^2) + (5*x^(1/3))/(6*a^2*(a + b*x)) - (5*ArcTan[(a^(1
/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) +
(5*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(6*a^(8/3)*b^(1/3)) - (5*Log[a + b*x])/(
18*a^(8/3)*b^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{2/3}(a+bx)^3}\,dx &= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\int \frac{1}{x^{2/3}(a+bx)^2}\,dx}{6a}\\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^2}\\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}}-\frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}}+x^2}\,dx,x,\sqrt[3]{x}\right)}{6a^{7/3}b^{2/3}}+\frac{5\,\text{Subst} \cdots}{\cdots}\\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\cdots\right.}{3a^{8/3}\sqrt[3]{\cdots}}\\
&= \frac{\sqrt[3]{x}}{2a(a+bx)^2}+\frac{5\sqrt[3]{x}}{6a^2(a+bx)}-\frac{5\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{6a^{8/3}\sqrt[3]{b}}-\frac{5\log(a+bx)}{18a^{8/3}\sqrt[3]{b}}
\end{aligned}
$$

Mathematica [C] time = 0.00, size = 25, normalized size = 0.18

$$\frac{3\sqrt[3]{x}\,{}_2F_1\left(\frac{1}{3},3;\frac{4}{3};-\frac{bx}{a}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] (3*x^(1/3)*Hypergeometric2F1[1/3, 3, 4/3, -((b*x)/a)])/a^3
```

IntegrateAlgebraic [A] time = 0.15, size = 157, normalized size = 1.12

$$-\frac{5\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{18a^{8/3}\sqrt[3]{b}}+\frac{5\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{8/3}\sqrt[3]{b}}-\frac{5\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{8/3}\sqrt[3]{b}}+\frac{\sqrt[3]{x}(8a+5bx)}{6a^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(2/3)*(a + b*x)^3),x]
```

```
[Out] (x^(1/3)*(8*a + 5*b*x))/(6*a^2*(a + b*x)^2) - (5*ArcTan[1/Sqrt[3] - (2*b^(1
/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(8/3)*b^(1/3)) + (5*Log[a^(1/
3) + b^(1/3)*x^(1/3)])/(9*a^(8/3)*b^(1/3)) - (5*Log[a^(2/3) - a^(1/3)*b^(1/
3)*x^(1/3) + b^(2/3)*x^(2/3)])/(18*a^(8/3)*b^(1/3))
```

fricas [B] time = 1.37, size = 499, normalized size = 3.56

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx-a^2+3\sqrt{\frac{1}{3}}\left(2abx^{\frac{2}{3}}-(a^2b)^{\frac{1}{3}}a+(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}-3(a^2b)^{\frac{1}{3}}ax^{\frac{1}{3}}}{bx+a}\right)-5\left(b^2x^2+2abx+a^2\right)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\left(b^2x^2+2abx+a^2\right)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\left(5a^2b^2x+8a^3b\right)x^{\frac{1}{3}}}{18\left(a^4b^3x^2+2a^5b^2x+a^6b\right)}, \frac{30\sqrt{\frac{1}{3}}\left(ab^3x^2+2a^2b^2x+a^3b\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((a^2b)^{\frac{1}{3}}a-2(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)\sqrt{\frac{(a^2b)^{\frac{1}{3}}}{b}}}{a^2}\right)-5\left(b^2x^2+2abx+a^2\right)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{2}{3}}+(a^2b)^{\frac{1}{3}}a-(a^2b)^{\frac{2}{3}}x^{\frac{1}{3}}\right)+10\left(b^2x^2+2abx+a^2\right)(a^2b)^{\frac{2}{3}}\log\left(abx^{\frac{1}{3}}+(a^2b)^{\frac{2}{3}}\right)+3\left(5a^2b^2x+8a^3b\right)x^{\frac{1}{3}}}{18\left(a^4b^3x^2+2a^5b^2x+a^6b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b^2*x + a^3*b)*sqrt(-(a^2*b)^(1/3)/b
)*log((2*a*b*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^(2/3) - (a^2*b)^(1/3)*a + (a^2*
```

```
b)^(2/3)*x^(1/3))*sqrt(-(a^2*b)^(1/3)/b) - 3*(a^2*b)^(1/3)*a*x^(1/3))/(b*x
+ a)) - 5*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)
^(1/3)*a - (a^2*b)^(2/3)*x^(1/3)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2
/3)*log(a*b*x^(1/3) + (a^2*b)^(2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(
a^4*b^3*x^2 + 2*a^5*b^2*x + a^6*b), 1/18*(30*sqrt(1/3)*(a*b^3*x^2 + 2*a^2*b
^2*x + a^3*b)*sqrt((a^2*b)^(1/3)/b)*arctan(-sqrt(1/3)*((a^2*b)^(1/3)*a - 2*
(a^2*b)^(2/3)*x^(1/3))*sqrt((a^2*b)^(1/3)/b)/a^2) - 5*(b^2*x^2 + 2*a*b*x +
a^2)*(a^2*b)^(2/3)*log(a*b*x^(2/3) + (a^2*b)^(1/3)*a - (a^2*b)^(2/3)*x^(1/3
)) + 10*(b^2*x^2 + 2*a*b*x + a^2)*(a^2*b)^(2/3)*log(a*b*x^(1/3) + (a^2*b)^(
2/3)) + 3*(5*a^2*b^2*x + 8*a^3*b)*x^(1/3))/(a^4*b^3*x^2 + 2*a^5*b^2*x + a^6
*b)]
```

giac [A] time = 1.00, size = 143, normalized size = 1.02

$$-\frac{5\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{3}}+\frac{5\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}b}+\frac{5\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{3}b}+\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\,(bx+a)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -5/9*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^3 + 5/9*sqrt(3)*(-a*b^
2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^3*b
) + 5/18*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/
(a^3*b) + 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/((b*x + a)^2*a^2)
```

maple [A] time = 0.01, size = 136, normalized size = 0.97

$$\frac{x^{\frac{1}{3}}}{2\,(bx+a)^{2}\,a}+\frac{5x^{\frac{1}{3}}}{6\,(bx+a)\,a^{2}}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}+\frac{5\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}-\frac{5\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\left(\frac{a}{b}\right)^{\frac{2}{3}}a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(2/3)/(b*x+a)^3,x)
```

```
[Out] 1/2*x^(1/3)/a/(b*x+a)^2+5/6*x^(1/3)/a^2/(b*x+a)+5/9/a^2/b/(a/b)^(2/3)*ln(x^
(1/3)+(a/b)^(1/3))-5/18/a^2/b/(a/b)^(2/3)*ln(x^(2/3)-(a/b)^(1/3)*x^(1/3)+(a
/b)^(2/3))+5/9/a^2/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
x^(1/3)-1))
```

maxima [A] time = 3.00, size = 151, normalized size = 1.08

$$\frac{5\,bx^{\frac{4}{3}}+8\,ax^{\frac{1}{3}}}{6\left(a^{2}b^{2}x^{2}+2\,a^{3}bx+a^{4}\right)}+\frac{5\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{5\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{5\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{2}b\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(2/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(5*b*x^(4/3) + 8*a*x^(1/3))/(a^2*b^2*x^2 + 2*a^3*b*x + a^4) + 5/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/b)^(1/3))/(a^2*b*(a/b)^(
```

2/3)) - 5/18*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b)^(2/3))/(a^2*b*(a/b)^
(2/3)) + 5/9*log(x^(1/3) + (a/b)^(1/3))/(a^2*b*(a/b)^(2/3))

mupad [B] time = 0.24, size = 157, normalized size = 1.12

$$\frac{\frac{4\,x^{1/3}}{3\,a}+\frac{5\,b\,x^{4/3}}{6\,a^2}}{a^2+2\,a\,b\,x+b^2\,x^2}+\frac{5\,\ln\left(\frac{5\,b^{5/3}}{a^{5/3}}+\frac{5\,b^2\,x^{1/3}}{a^2}\right)}{9\,a^{8/3}\,b^{1/3}}+\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}+\frac{b^{5/3}\,\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(-5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}-\frac{\ln\left(\frac{5\,b^2\,x^{1/3}}{a^2}-\frac{b^{5/3}\,\left(5+\sqrt{3}\,5\mathrm{i}\right)}{2\,a^{5/3}}\right)\,\left(5+\sqrt{3}\,5\mathrm{i}\right)}{18\,a^{8/3}\,b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2/3)*(a + b*x)^3),x)
```

```
[Out] ((4*x^(1/3))/(3*a) + (5*b*x^(4/3))/(6*a^2))/(a^2 + b^2*x^2 + 2*a*b*x) + (5*
log((5*b^(5/3))/a^(5/3) + (5*b^2*x^(1/3))/a^2))/(9*a^(8/3)*b^(1/3)) + (log(
(5*b^2*x^(1/3))/a^2 + (b^(5/3)*(3^(1/2)*5i - 5))/(2*a^(5/3)))*(3^(1/2)*5i -
 5))/(18*a^(8/3)*b^(1/3)) - (log((5*b^2*x^(1/3))/a^2 - (b^(5/3)*(3^(1/2)*5i
 + 5))/(2*a^(5/3)))*(3^(1/2)*5i + 5))/(18*a^(8/3)*b^(1/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(2/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.696 $\int \frac{1}{x^{4/3}(a+bx)^3} dx$

Optimal. Leaf size=152

$$\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{1}{2a\sqrt[3]{x}(a+$$

Rubi [A] time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 56, 617, 204, 31}

$$\frac{7}{6a^2\sqrt[3]{x}(a+bx)}+\frac{7\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{10/3}}-\frac{7\sqrt[3]{b}\log(a+bx)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{14}{3a^3\sqrt[3]{x}}+\frac{1}{2a\sqrt[3]{x}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] -14/(3*a^3*x^(1/3)) + 1/(2*a*x^(1/3)*(a + b*x)^2) + 7/(6*a^2*x^(1/3)*(a + b
*x)) + (14*b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(10/3)) + (7*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(10
/3)) - (7*b^(1/3)*Log[a + b*x])/(9*a^(10/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 56

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q),
 x] + (Dist[3/(2*b), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)]
, x] - Dist[3/(2*b*q), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{4/3}(a+bx)^3}\,dx &= \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7\int \frac{1}{x^{4/3}(a+bx)^2}\,dx}{6a} \\
&= \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\int \frac{1}{x^{4/3}(a+bx)}\,dx}{9a^2} \\
&= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{(14b)\int \frac{1}{\sqrt[3]{x}\,(a+bx)}\,dx}{9a^3} \\
&= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} - \frac{7\sqrt[3]{b}\,\log(a+bx)}{9a^{10/3}} - \frac{7\,\mathrm{Subst}\left(\int \frac{1}{\frac{a^{2/3}}{b^{2/3}} - \frac{\sqrt[3]{a}\,x}{\sqrt[3]{b}} + x^2 \cdots}\right.}{3a^3} \\
&= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \sqrt[3]{b}\,\sqrt[3]{x}\right)}{3a^{10/3}} - \frac{7\sqrt[3]{b}\,\log(a + \cdots}{9a^{10/3}} \\
&= -\frac{14}{3a^3\sqrt[3]{x}} + \frac{1}{2a\sqrt[3]{x}\,(a+bx)^2} + \frac{7}{6a^2\sqrt[3]{x}\,(a+bx)} + \frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{10/3}} + \frac{7\sqrt[3]{b}\,\log\left(\sqrt[3]{a} + \cdots\right.}{3a^{10/3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 25, normalized size = 0.16

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},3;\frac{2}{3};-\frac{bx}{a}\right)}{a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 3, 2/3, -((b*x)/a)])/(a^3*x^(1/3))
```

IntegrateAlgebraic [A] time = 0.27, size = 168, normalized size = 1.11

$$-\frac{7\sqrt[3]{b}\,\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\,\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\sqrt[3]{x}\right)}{9a^{10/3}}+\frac{14\sqrt[3]{b}\,\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\,\sqrt[3]{x}}{\sqrt{3}\,\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{10/3}}+\frac{-18a^2-49abx-28b^2x^2}{6a^3\sqrt[3]{x}\,(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(4/3)*(a + b*x)^3),x]
```

```
[Out] (-18*a^2 - 49*a*b*x - 28*b^2*x^2)/(6*a^3*x^(1/3)*(a + b*x)^2) + (14*b^(1/3)
*ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(1
0/3)) + (14*b^(1/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(10/3)) - (7*b^(1/
3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(9*a^(10/3))
```

fricas [A] time = 1.29, size = 211, normalized size = 1.39

$$-\frac{28\sqrt{3}\left(b^2x^3+2abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}\,x^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)+14\left(b^2x^3+2abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(-ax^{\frac{1}{3}}\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{2}{3}}+a\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)-28\left(b^2x^3+2abx^2+a^2x\right)\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(a\left(\frac{b}{a}\right)^{\frac{2}{3}}+bx^{\frac{1}{3}}\right)+3\left(28b^2x^2+49abx+18a^2\right)x^{\frac{2}{3}}}{18\left(a^3b^2x^3+2a^4bx^2+a^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/18*(28*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*arctan(2/3*sqrt
(3)*x^(1/3)*(b/a)^(1/3) - 1/3*sqrt(3)) + 14*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(
b/a)^(1/3)*log(-a*x^(1/3)*(b/a)^(2/3) + b*x^(2/3) + a*(b/a)^(1/3)) - 28*(b^
2*x^3 + 2*a*b*x^2 + a^2*x)*(b/a)^(1/3)*log(a*(b/a)^(2/3) + b*x^(1/3)) + 3*(
28*b^2*x^2 + 49*a*b*x + 18*a^2)*x^(2/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x
)
```

giac [A] time = 1.16, size = 155, normalized size = 1.02

$$\frac{14\,b\left(-\frac{a}{b}\right)^{\frac{2}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^{4}}+\frac{14\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{4}b}-\frac{3}{a^{3}x^{\frac{1}{3}}}-\frac{7\left(-ab^{2}\right)^{\frac{2}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{4}b}-\frac{10\,b^{2}x^{\frac{5}{3}}+13\,abx^{\frac{2}{3}}}{6\,(bx+a)^{2}a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 14/9*b*(-a/b)^(2/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 + 14/9*sqrt(3)*(-a
*b^2)^(2/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
4*b) - 3/(a^3*x^(1/3)) - 7/9*(-a*b^2)^(2/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a^4*b) - 1/6*(10*b^2*x^(5/3) + 13*a*b*x^(2/3))/((b*x +
 a)^2*a^3)
```

maple [A] time = 0.02, size = 139, normalized size = 0.91

$$-\frac{5b^{2}x^{\frac{5}{3}}}{3\,(bx+a)^{2}\,a^{3}}-\frac{13b\,x^{\frac{2}{3}}}{6\,(bx+a)^{2}\,a^{2}}-\frac{14\sqrt{3}\;\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}+\frac{14\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{7\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{1}{3}}a^{3}}-\frac{3}{a^{3}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(4/3)/(b*x+a)^3,x)
```

```
[Out] -3/a^3/x^(1/3)-5/3*b^2/a^3/(b*x+a)^2*x^(5/3)-13/6*b/a^2/(b*x+a)^2*x^(2/3)+1
4/9/a^3/(a/b)^(1/3)*ln(x^(1/3)+(a/b)^(1/3))-7/9/a^3/(a/b)^(1/3)*ln(x^(2/3)-
(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-14/9/a^3*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^
(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.99, size = 154, normalized size = 1.01

$$-\frac{28\,b^{2}x^{2}+49\,abx+18\,a^{2}}{6\left(a^{3}b^{2}x^{\frac{7}{3}}+2\,a^{4}bx^{\frac{4}{3}}+a^{5}x^{\frac{1}{3}}\right)}-\frac{14\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{7\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}+\frac{14\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^{3}\left(\frac{a}{b}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(4/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(28*b^2*x^2 + 49*a*b*x + 18*a^2)/(a^3*b^2*x^(7/3) + 2*a^4*b*x^(4/3) +
a^5*x^(1/3)) - 14/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a
/b)^(1/3))/(a^3*(a/b)^(1/3)) - 7/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(1/3)) + 14/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(1/
3))
```

mupad [B] time = 0.09, size = 174, normalized size = 1.14

$$\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}+588\,a^{3}\,b^{3}\,x^{1/3}\right)}{9\,a^{10/3}}-\frac{\frac{3}{a}+\frac{14\,b^{2}\,x^{2}}{3\,a^{3}}+\frac{49\,b\,x}{6\,a^{2}}}{a^{2}\,x^{1/3}+b^{2}\,x^{7/3}+2\,a\,b\,x^{4/3}}+\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}+588\,a^{3}\,b^{3}\,x^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}-\frac{14\,b^{1/3}\,\ln\left(588\,a^{10/3}\,b^{8/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^{2}+588\,a^{3}\,b^{3}\,x^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(4/3)*(a + b*x)^3),x)
```

```
[Out] (14*b^(1/3)*log(588*a^(10/3)*b^(8/3) + 588*a^3*b^3*x^(1/3)))/(9*a^(10/3)) -
 (3/a + (14*b^2*x^2)/(3*a^3) + (49*b*x)/(6*a^2))/(a^2*x^(1/3) + b^2*x^(7/3)
 + 2*a*b*x^(4/3)) + (14*b^(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 -
1/2)^2 + 588*a^3*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (14*b^
(1/3)*log(588*a^(10/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2)^2 + 588*a^3*b^3*x^(1/
3))*((3^(1/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(4/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.697 $\int \frac{1}{x^{5/3}(a+bx)^3}\,dx$

Optimal. Leaf size=152

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{10}{3a^3x^{2/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{\quad}{2ax^{2/3}}$$

Rubi [A] time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {51, 58, 617, 204, 31}

$$-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] -10/(3*a^3*x^(2/3)) + 1/(2*a*x^(2/3)*(a + b*x)^2) + 4/(3*a^2*x^(2/3)*(a + b
*x)) + (20*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])
/(3*Sqrt[3]*a^(11/3)) - (10*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(3*a^(1
1/3)) + (10*b^(2/3)*Log[a + b*x])/(9*a^(11/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^{5/3}(a+bx)^3}\,dx &= \frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4\int \frac{1}{x^{5/3}(a+bx)^2}\,dx}{3a}\\
&= \frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20\int \frac{1}{x^{5/3}(a+bx)}\,dx}{9a^2}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{(20b)\int \frac{1}{x^{2/3}(a+bx)}\,dx}{9a^3}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{10b^{2/3}\log(a+bx)}{9a^{11/3}}-\frac{\left(10\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{}{\frac{a^{2/3}}{b^{2/3}}}\right.}{3a^1}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{3a^{11/3}}+\frac{10b^{2/3}\log(a}{9a^{11/3}}\\
&= -\frac{10}{3a^3x^{2/3}}+\frac{1}{2ax^{2/3}(a+bx)^2}+\frac{4}{3a^2x^{2/3}(a+bx)}+\frac{20b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{11/3}}-\frac{10b^{2/3}\log\left(\sqrt[3]{a}\right.}{3a^{11/3}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 27, normalized size = 0.18

$$
-\frac{3\,{}_2F_1\left(-\frac{2}{3},3;\frac{1}{3};-\frac{bx}{a}\right)}{2a^3x^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] (-3*Hypergeometric2F1[-2/3, 3, 1/3, -((b*x)/a)])/(2*a^3*x^(2/3))
```

IntegrateAlgebraic [A] time = 0.27, size = 168, normalized size = 1.11

$$
\frac{10b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sqrt[3]{x}+b^{2/3}x^{2/3}\right)}{9a^{11/3}}-\frac{20b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sqrt[3]{x}\right)}{9a^{11/3}}+\frac{20b^{2/3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{11/3}}+\frac{-9a^2-32abx-20b^2x^2}{6a^3x^{2/3}(a+bx)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^(5/3)*(a + b*x)^3),x]
```

```
[Out] (-9*a^2 - 32*a*b*x - 20*b^2*x^2)/(6*a^3*x^(2/3)*(a + b*x)^2) + (20*b^(2/3)*
ArcTan[1/Sqrt[3] - (2*b^(1/3)*x^(1/3))/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(11
/3)) - (20*b^(2/3)*Log[a^(1/3) + b^(1/3)*x^(1/3)])/(9*a^(11/3)) + (10*b^(2/
3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x^(1/3) + b^(2/3)*x^(2/3)])/(9*a^(11/3))
```

fricas [B] time = 0.67, size = 244, normalized size = 1.61

$$
\frac{40\sqrt{3}\left(b^2x^3+2abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-20\left(b^2x^3+2abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^{\frac{2}{3}}+abx^{\frac{1}{3}}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}\right)+40\left(b^2x^3+2abx^2+a^2x\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(bx^{\frac{1}{3}}-a\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)-3\left(20b^2x^2+32abx+9a^2\right)x^{\frac{1}{3}}}{18\left(a^3b^2x^3+2a^4bx^2+a^5x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*(40*sqrt(3)*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*arctan(1/3*
(2*sqrt(3)*a*x^(1/3)*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 20*(b^2*x^3 + 2*a*b
*x^2 + a^2*x)*(-b^2/a^2)^(1/3)*log(b^2*x^(2/3) + a*b*x^(1/3)*(-b^2/a^2)^(1/
3) + a^2*(-b^2/a^2)^(2/3)) + 40*(b^2*x^3 + 2*a*b*x^2 + a^2*x)*(-b^2/a^2)^(1
/3)*log(b*x^(1/3) - a*(-b^2/a^2)^(1/3)) - 3*(20*b^2*x^2 + 32*a*b*x + 9*a^2)
*x^(1/3))/(a^3*b^2*x^3 + 2*a^4*b*x^2 + a^5*x)
```

giac [A] time = 1.08, size = 150, normalized size = 0.99

$$\frac{20\,b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x^{\frac{1}{3}}-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^4}-\frac{20\,\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^4}-\frac{10\left(-ab^2\right)^{\frac{1}{3}}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^4}-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(bx^{\frac{4}{3}}+ax^{\frac{1}{3}}\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 20/9*b*(-a/b)^(1/3)*log(abs(x^(1/3) - (-a/b)^(1/3)))/a^4 - 20/9*sqrt(3)*(-a
*b^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x^(1/3) + (-a/b)^(1/3))/(-a/b)^(1/3))/a^4
 - 10/9*(-a*b^2)^(1/3)*log(x^(2/3) + x^(1/3)*(-a/b)^(1/3) + (-a/b)^(2/3))/a
^4 - 1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/((b*x^(4/3) + a*x^(1/3))^2*a^3)
```

maple [A] time = 0.02, size = 139, normalized size = 0.91

$$-\frac{11b^2x^{\frac{4}{3}}}{6\left(bx+a\right)^2a^3}-\frac{7b\,x^{\frac{1}{3}}}{3\left(bx+a\right)^2a^2}-\frac{20\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{\frac{1}{3}}}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{20\ln\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}+\frac{10\ln\left(x^{\frac{2}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}a^3}-\frac{3}{2a^3x^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/3)/(b*x+a)^3,x)
```

```
[Out] -3/2/a^3/x^(2/3)-11/6/a^3*b^2/(b*x+a)^2*x^(4/3)-7/3/a^2*b/(b*x+a)^2*x^(1/3)
-20/9/a^3/(a/b)^(2/3)*ln(x^(1/3)+(a/b)^(1/3))+10/9/a^3/(a/b)^(2/3)*ln(x^(2/
3)-(a/b)^(1/3)*x^(1/3)+(a/b)^(2/3))-20/9/a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*x^(1/3)-1))
```

maxima [A] time = 2.99, size = 154, normalized size = 1.01

$$-\frac{20\,b^2x^2+32\,abx+9\,a^2}{6\left(a^3b^2x^{\frac{8}{3}}+2\,a^4bx^{\frac{5}{3}}+a^5x^{\frac{2}{3}}\right)}-\frac{20\,\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2\,x^{\frac{1}{3}}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{10\,\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{20\,\log\left(x^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\,a^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/3)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(20*b^2*x^2 + 32*a*b*x + 9*a^2)/(a^3*b^2*x^(8/3) + 2*a^4*b*x^(5/3) + a
^5*x^(2/3)) - 20/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/3) - (a/b)^(1/3))/(a/
b)^(1/3))/(a^3*(a/b)^(2/3)) + 10/9*log(x^(2/3) - x^(1/3)*(a/b)^(1/3) + (a/b
)^(2/3))/(a^3*(a/b)^(2/3)) - 20/9*log(x^(1/3) + (a/b)^(1/3))/(a^3*(a/b)^(2/
3))
```

mupad [B] time = 0.17, size = 182, normalized size = 1.20

$$\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}-540\,a^6\,b^3\,x^{1/3}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{\frac{3}{2\,a}+\frac{10\,b^2\,x^2}{3\,a^3}+\frac{16\,b\,x}{3\,a^2}}{a^2\,x^{2/3}+b^2\,x^{8/3}+2\,a\,b\,x^{5/3}}+\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)-540\,a^6\,b^3\,x^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}-\frac{20\,b^{2/3}\,\ln\left(540\,{\left(-a\right)}^{19/3}\,b^{8/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)+540\,a^6\,b^3\,x^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,{\left(-a\right)}^{11/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/3)*(a + b*x)^3),x)
```

```
[Out] (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3) - 540*a^6*b^3*x^(1/3)))/(9*(-a)^(11
/3)) - (3/(2*a) + (10*b^2*x^2)/(3*a^3) + (16*b*x)/(3*a^2))/(a^2*x^(2/3) + b
^2*x^(8/3) + 2*a*b*x^(5/3)) + (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(
1/2)*1i)/2 - 1/2) - 540*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(9*(-a)^(1
1/3)) - (20*b^(2/3)*log(540*(-a)^(19/3)*b^(8/3)*((3^(1/2)*1i)/2 + 1/2) + 54
0*a^6*b^3*x^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(9*(-a)^(11/3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/3)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.698 $\int \frac{\sqrt[4]{1-x}}{1+x}\,dx$

Optimal. Leaf size=58

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 63, 212, 206, 203}

$$4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{1-x}}{1+x}\,dx &= 4\sqrt[4]{1-x}+2\int \frac{1}{(1-x)^{3/4}(1+x)}\,dx\\
&= 4\sqrt[4]{1-x}-8\,\text{Subst}\left(\int \frac{1}{2-x^4}\,dx,x,\sqrt[4]{1-x}\right)\\
&= 4\sqrt[4]{1-x}-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\sqrt[4]{1-x}\right)-\left(2\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx,x,\sqrt[4]{1-x}\right)\\
&= 4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$
4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

IntegrateAlgebraic [A] time = 0.08, size = 58, normalized size = 1.00

$$
4\sqrt[4]{1-x}-2\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)-2\sqrt[4]{2}\tanh^{-1}\left(\frac{\sqrt[4]{1-x}}{\sqrt[4]{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/4)/(1 + x),x]
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*ArcTan[(1 - x)^(1/4)/2^(1/4)] - 2*2^(1/4)*ArcTa
nh[(1 - x)^(1/4)/2^(1/4)]
```

fricas [A] time = 0.96, size = 82, normalized size = 1.41

$$
4\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}\sqrt{\sqrt{2}+\sqrt{-x+1}}-\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+4(-x+1)^{\frac{1}{4}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="fricas")
```

```
[Out] 4*2^(1/4)*arctan(1/2*2^(3/4)*sqrt(sqrt(2) + sqrt(-x + 1)) - 1/2*2^(3/4)*(-x
 + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x + 1)^(1/4)) + 2^(1/4)*log(-2^(1/4)
 + (-x + 1)^(1/4)) + 4*(-x + 1)^(1/4)
```

giac [A] time = 1.17, size = 64, normalized size = 1.10

$$
-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)-2^{\frac{1}{4}}\log\left(2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(\left|-2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}\right|\right)+4(-x+1)^{\frac{1}{4}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="giac")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) - 2^(1/4)*log(2^(1/4) + (-x +
 1)^(1/4)) + 2^(1/4)*log(abs(-2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

maple [A] time = 0.01, size = 62, normalized size = 1.07

$$-2\,2^{\frac{1}{4}}\arctan\left(\frac{(-x+1)^{\frac{1}{4}}\,2^{\frac{3}{4}}}{2}\right)-2^{\frac{1}{4}}\ln\left(\frac{(-x+1)^{\frac{1}{4}}+2^{\frac{1}{4}}}{(-x+1)^{\frac{1}{4}}-2^{\frac{1}{4}}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/4)/(1+x),x)
```

```
[Out] 4*(-x+1)^(1/4)-2*2^(1/4)*arctan(1/2*(-x+1)^(1/4)*2^(3/4))-2^(1/4)*ln(((-x+1
)^(1/4)+2^(1/4))/((-x+1)^(1/4)-2^(1/4)))
```

maxima [A] time = 2.91, size = 61, normalized size = 1.05

$$-2\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}(-x+1)^{\frac{1}{4}}\right)+2^{\frac{1}{4}}\log\left(-\frac{2^{\frac{1}{4}}-(-x+1)^{\frac{1}{4}}}{2^{\frac{1}{4}}+(-x+1)^{\frac{1}{4}}}\right)+4\,(-x+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/4)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*2^(1/4)*arctan(1/2*2^(3/4)*(-x + 1)^(1/4)) + 2^(1/4)*log(-(2^(1/4) - (-x
 + 1)^(1/4))/(2^(1/4) + (-x + 1)^(1/4))) + 4*(-x + 1)^(1/4)
```

mupad [B] time = 0.07, size = 46, normalized size = 0.79

$$4\,(1-x)^{1/4}-2\,2^{1/4}\,\mathrm{atanh}\left(\frac{2^{3/4}\,(1-x)^{1/4}}{2}\right)-2\,2^{1/4}\,\mathrm{atan}\left(\frac{2^{3/4}\,(1-x)^{1/4}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/4)/(x + 1),x)
```

```
[Out] 4*(1 - x)^(1/4) - 2*2^(1/4)*atanh((2^(3/4)*(1 - x)^(1/4))/2) - 2*2^(1/4)*at
an((2^(3/4)*(1 - x)^(1/4))/2)
```

sympy [C] time = 2.35, size = 243, normalized size = 4.19

$$\frac{5\sqrt[4]{-1}\,\sqrt[4]{x-1}\,\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\,(-1)^{\frac{3}{4}}\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{3i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}-\frac{5\sqrt[4]{-2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{5i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\,(-1)^{\frac{3}{4}}\sqrt[4]{2}\,e^{-\frac{i\pi}{4}}\log\left(-\frac{2^{\frac{3}{4}}\sqrt[4]{x-1}\,e^{\frac{7i\pi}{4}}}{2}+1\right)\Gamma\left(\frac{5}{4}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/4)/(1+x),x)
```

```
[Out] 5*(-1)**(1/4)*(x - 1)**(1/4)*gamma(5/4)/gamma(9/4) + 5*(-2)**(1/4)*exp(-I*p
i/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(I*pi/4)/2 + 1)*gamma(5/4)/(4*ga
mma(9/4)) - 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4
)*exp_polar(3*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4)) - 5*(-2)**(1/4)*exp(
-I*pi/4)*log(-2**(3/4)*(x - 1)**(1/4)*exp_polar(5*I*pi/4)/2 + 1)*gamma(5/4)
/(4*gamma(9/4)) + 5*(-1)**(3/4)*2**(1/4)*exp(-I*pi/4)*log(-2**(3/4)*(x - 1)
**(1/4)*exp_polar(7*I*pi/4)/2 + 1)*gamma(5/4)/(4*gamma(9/4))
```

3.699 $\int x^m(a+bx)^{10}\,dx$

Optimal. Leaf size=187

$$\frac{a^{10}x^{m+1}}{m+1}+\frac{10a^9bx^{m+2}}{m+2}+\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\cdots$$

Rubi [A] time = 0.08, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{45a^8b^2x^{m+3}}{m+3}+\frac{120a^7b^3x^{m+4}}{m+4}+\frac{210a^6b^4x^{m+5}}{m+5}+\frac{252a^5b^5x^{m+6}}{m+6}+\frac{210a^4b^6x^{m+7}}{m+7}+\frac{120a^3b^7x^{m+8}}{m+8}+\frac{45a^2b^8x^{m+9}}{m+9}+\frac{10a^9bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}+\frac{10ab^9x^{m+10}}{m+10}+\frac{b^{10}x^{m+11}}{m+11}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^10,x]
```

```
[Out] (a^10*x^(1 + m))/(1 + m) + (10*a^9*b*x^(2 + m))/(2 + m) + (45*a^8*b^2*x^(3
+ m))/(3 + m) + (120*a^7*b^3*x^(4 + m))/(4 + m) + (210*a^6*b^4*x^(5 + m))/(
5 + m) + (252*a^5*b^5*x^(6 + m))/(6 + m) + (210*a^4*b^6*x^(7 + m))/(7 + m)
+ (120*a^3*b^7*x^(8 + m))/(8 + m) + (45*a^2*b^8*x^(9 + m))/(9 + m) + (10*a*
b^9*x^(10 + m))/(10 + m) + (b^10*x^(11 + m))/(11 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^{10}\,dx &= \int \left(a^{10}x^m+10a^9bx^{1+m}+45a^8b^2x^{2+m}+120a^7b^3x^{3+m}+210a^6b^4x^{4+m}+252a^5b^5x^{5+m}+210\cdots\right.\\ &= \frac{a^{10}x^{1+m}}{1+m}+\frac{10a^9bx^{2+m}}{2+m}+\frac{45a^8b^2x^{3+m}}{3+m}+\frac{120a^7b^3x^{4+m}}{4+m}+\frac{210a^6b^4x^{5+m}}{5+m}+\frac{252a^5b^5x^{6+m}}{6+m}+\cdots\end{aligned}$$

Mathematica [A] time = 0.11, size = 166, normalized size = 0.89

$$x^{m+1}\left(\frac{a^{10}}{m+1}+\frac{10a^9bx}{m+2}+\frac{45a^8b^2x^2}{m+3}+\frac{120a^7b^3x^3}{m+4}+\frac{210a^6b^4x^4}{m+5}+\frac{252a^5b^5x^5}{m+6}+\frac{210a^4b^6x^6}{m+7}+\frac{120a^3b^7x^7}{m+8}+\frac{45a^2b^8x^8}{m+9}+\frac{10ab^9x^9}{m+10}+\frac{b^{10}x^{10}}{m+11}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^10,x]
```

```
[Out] x^(1 + m)*(a^10/(1 + m) + (10*a^9*b*x)/(2 + m) + (45*a^8*b^2*x^2)/(3 + m) +
 (120*a^7*b^3*x^3)/(4 + m) + (210*a^6*b^4*x^4)/(5 + m) + (252*a^5*b^5*x^5)/
(6 + m) + (210*a^4*b^6*x^6)/(7 + m) + (120*a^3*b^7*x^7)/(8 + m) + (45*a^2*b
^8*x^8)/(9 + m) + (10*a*b^9*x^9)/(10 + m) + (b^10*x^10)/(11 + m))
```

IntegrateAlgebraic [F] time = 0.18, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^10,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^10, x]
```

fricas [B] time = 1.17, size = 1277, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] ((b^10*m^10 + 55*b^10*m^9 + 1320*b^10*m^8 + 18150*b^10*m^7 + 157773*b^10*m^
6 + 902055*b^10*m^5 + 3416930*b^10*m^4 + 8409500*b^10*m^3 + 12753576*b^10*m
^2 + 10628640*b^10*m + 3628800*b^10)*x^11 + 10*(a*b^9*m^10 + 56*a*b^9*m^9 +
 1365*a*b^9*m^8 + 19020*a*b^9*m^7 + 167223*a*b^9*m^6 + 965328*a*b^9*m^5 + 3
686255*a*b^9*m^4 + 9133180*a*b^9*m^3 + 13926276*a*b^9*m^2 + 11655216*a*b^9*
m + 3991680*a*b^9)*x^10 + 45*(a^2*b^8*m^10 + 57*a^2*b^8*m^9 + 1412*a^2*b^8*
m^8 + 19962*a^2*b^8*m^7 + 177765*a^2*b^8*m^6 + 1037673*a^2*b^8*m^5 + 400047
8*a^2*b^8*m^4 + 9991428*a^2*b^8*m^3 + 15335224*a^2*b^8*m^2 + 12900960*a^2*b
^8*m + 4435200*a^2*b^8)*x^9 + 120*(a^3*b^7*m^10 + 58*a^3*b^7*m^9 + 1461*a^3
*b^7*m^8 + 20982*a^3*b^7*m^7 + 189567*a^3*b^7*m^6 + 1121022*a^3*b^7*m^5 + 4
371359*a^3*b^7*m^4 + 11024858*a^3*b^7*m^3 + 17059212*a^3*b^7*m^2 + 14444280
*a^3*b^7*m + 4989600*a^3*b^7)*x^8 + 210*(a^4*b^6*m^10 + 59*a^4*b^6*m^9 + 15
12*a^4*b^6*m^8 + 22086*a^4*b^6*m^7 + 202821*a^4*b^6*m^6 + 1217811*a^4*b^6*m
^5 + 4814858*a^4*b^6*m^4 + 12291724*a^4*b^6*m^3 + 19216008*a^4*b^6*m^2 + 16
405920*a^4*b^6*m + 5702400*a^4*b^6)*x^7 + 252*(a^5*b^5*m^10 + 60*a^5*b^5*m^
9 + 1565*a^5*b^5*m^8 + 23280*a^5*b^5*m^7 + 217743*a^5*b^5*m^6 + 1331100*a^5
*b^5*m^5 + 5352935*a^5*b^5*m^4 + 13878120*a^5*b^5*m^3 + 21989356*a^5*b^5*m^
2 + 18981840*a^5*b^5*m + 6652800*a^5*b^5)*x^6 + 210*(a^6*b^4*m^10 + 61*a^6*
b^4*m^9 + 1620*a^6*b^4*m^8 + 24570*a^6*b^4*m^7 + 234573*a^6*b^4*m^6 + 14646
93*a^6*b^4*m^5 + 6016070*a^6*b^4*m^4 + 15915380*a^6*b^4*m^3 + 25681176*a^6*
b^4*m^2 + 22512096*a^6*b^4*m + 7983360*a^6*b^4)*x^5 + 120*(a^7*b^3*m^10 + 6
2*a^7*b^3*m^9 + 1677*a^7*b^3*m^8 + 25962*a^7*b^3*m^7 + 253575*a^7*b^3*m^6 +
 1623258*a^7*b^3*m^5 + 6846503*a^7*b^3*m^4 + 18609718*a^7*b^3*m^3 + 3081920
4*a^7*b^3*m^2 + 27641160*a^7*b^3*m + 9979200*a^7*b^3)*x^4 + 45*(a^8*b^2*m^1
0 + 63*a^8*b^2*m^9 + 1736*a^8*b^2*m^8 + 27462*a^8*b^2*m^7 + 275037*a^8*b^2*
m^6 + 1812447*a^8*b^2*m^5 + 7902194*a^8*b^2*m^4 + 22289148*a^8*b^2*m^3 + 38
390632*a^8*b^2*m^2 + 35746080*a^8*b^2*m + 13305600*a^8*b^2)*x^3 + 10*(a^9*b
*m^10 + 64*a^9*b*m^9 + 1797*a^9*b*m^8 + 29076*a^9*b*m^7 + 299271*a^9*b*m^6
+ 2039016*a^9*b*m^5 + 9261503*a^9*b*m^4 + 27472724*a^9*b*m^3 + 50312628*a^9
*b*m^2 + 50292720*a^9*b*m + 19958400*a^9*b)*x^2 + (a^10*m^10 + 65*a^10*m^9
+ 1860*a^10*m^8 + 30810*a^10*m^7 + 326613*a^10*m^6 + 2310945*a^10*m^5 + 110
28590*a^10*m^4 + 34967140*a^10*m^3 + 70290936*a^10*m^2 + 80627040*a^10*m +
39916800*a^10)*x)*x^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 +
 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2
+ 120543840*m + 39916800)
```

giac [B] time = 1.20, size = 1925, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="giac")
```

```
[Out] (b^10*m^10*x^11*x^m + 10*a*b^9*m^10*x^10*x^m + 55*b^10*m^9*x^11*x^m + 45*a^
2*b^8*m^10*x^9*x^m + 560*a*b^9*m^9*x^10*x^m + 1320*b^10*m^8*x^11*x^m + 120*
a^3*b^7*m^10*x^8*x^m + 2565*a^2*b^8*m^9*x^9*x^m + 13650*a*b^9*m^8*x^10*x^m
+ 18150*b^10*m^7*x^11*x^m + 210*a^4*b^6*m^10*x^7*x^m + 6960*a^3*b^7*m^9*x^8
*x^m + 63540*a^2*b^8*m^8*x^9*x^m + 190200*a*b^9*m^7*x^10*x^m + 157773*b^10*
m^6*x^11*x^m + 252*a^5*b^5*m^10*x^6*x^m + 12390*a^4*b^6*m^9*x^7*x^m + 17532
0*a^3*b^7*m^8*x^8*x^m + 898290*a^2*b^8*m^7*x^9*x^m + 1672230*a*b^9*m^6*x^10
*x^m + 902055*b^10*m^5*x^11*x^m + 210*a^6*b^4*m^10*x^5*x^m + 15120*a^5*b^5*
m^9*x^6*x^m + 317520*a^4*b^6*m^8*x^7*x^m + 2517840*a^3*b^7*m^7*x^8*x^m + 79
```

```
99425*a^2*b^8*m^6*x^9*x^m + 9653280*a*b^9*m^5*x^10*x^m + 3416930*b^10*m^4*x
^11*x^m + 120*a^7*b^3*m^10*x^4*x^m + 12810*a^6*b^4*m^9*x^5*x^m + 394380*a^5
*b^5*m^8*x^6*x^m + 4638060*a^4*b^6*m^7*x^7*x^m + 22748040*a^3*b^7*m^6*x^8*x
^m + 46695285*a^2*b^8*m^5*x^9*x^m + 36862550*a*b^9*m^4*x^10*x^m + 8409500*b
^10*m^3*x^11*x^m + 45*a^8*b^2*m^10*x^3*x^m + 7440*a^7*b^3*m^9*x^4*x^m + 340
200*a^6*b^4*m^8*x^5*x^m + 5866560*a^5*b^5*m^7*x^6*x^m + 42592410*a^4*b^6*m^
6*x^7*x^m + 134522640*a^3*b^7*m^5*x^8*x^m + 180021510*a^2*b^8*m^4*x^9*x^m +
 91331800*a*b^9*m^3*x^10*x^m + 12753576*b^10*m^2*x^11*x^m + 10*a^9*b*m^10*x
^2*x^m + 2835*a^8*b^2*m^9*x^3*x^m + 201240*a^7*b^3*m^8*x^4*x^m + 5159700*a^
6*b^4*m^7*x^5*x^m + 54871236*a^5*b^5*m^6*x^6*x^m + 255740310*a^4*b^6*m^5*x^
7*x^m + 524563080*a^3*b^7*m^4*x^8*x^m + 449614260*a^2*b^8*m^3*x^9*x^m + 139
262760*a*b^9*m^2*x^10*x^m + 10628640*b^10*m*x^11*x^m + a^10*m^10*x*x^m + 64
0*a^9*b*m^9*x^2*x^m + 78120*a^8*b^2*m^8*x^3*x^m + 3115440*a^7*b^3*m^7*x^4*x
^m + 49260330*a^6*b^4*m^6*x^5*x^m + 335437200*a^5*b^5*m^5*x^6*x^m + 1011120
180*a^4*b^6*m^4*x^7*x^m + 1322982960*a^3*b^7*m^3*x^8*x^m + 690085080*a^2*b^
8*m^2*x^9*x^m + 116552160*a*b^9*m*x^10*x^m + 3628800*b^10*x^11*x^m + 65*a^1
0*m^9*x*x^m + 17970*a^9*b*m^8*x^2*x^m + 1235790*a^8*b^2*m^7*x^3*x^m + 30429
000*a^7*b^3*m^6*x^4*x^m + 307585530*a^6*b^4*m^5*x^5*x^m + 1348939620*a^5*b^
5*m^4*x^6*x^m + 2581262040*a^4*b^6*m^3*x^7*x^m + 2047105440*a^3*b^7*m^2*x^8
*x^m + 580543200*a^2*b^8*m*x^9*x^m + 39916800*a*b^9*x^10*x^m + 1860*a^10*m^
8*x*x^m + 290760*a^9*b*m^7*x^2*x^m + 12376665*a^8*b^2*m^6*x^3*x^m + 1947909
60*a^7*b^3*m^5*x^4*x^m + 1263374700*a^6*b^4*m^4*x^5*x^m + 3497286240*a^5*b^
5*m^3*x^6*x^m + 4035361680*a^4*b^6*m^2*x^7*x^m + 1733313600*a^3*b^7*m*x^8*x
^m + 199584000*a^2*b^8*x^9*x^m + 30810*a^10*m^7*x*x^m + 2992710*a^9*b*m^6*x
^2*x^m + 81560115*a^8*b^2*m^5*x^3*x^m + 821580360*a^7*b^3*m^4*x^4*x^m + 334
2229800*a^6*b^4*m^3*x^5*x^m + 5541317712*a^5*b^5*m^2*x^6*x^m + 3445243200*a
^4*b^6*m*x^7*x^m + 598752000*a^3*b^7*x^8*x^m + 326613*a^10*m^6*x*x^m + 2039
0160*a^9*b*m^5*x^2*x^m + 355598730*a^8*b^2*m^4*x^3*x^m + 2233166160*a^7*b^3
*m^3*x^4*x^m + 5393046960*a^6*b^4*m^2*x^5*x^m + 4783423680*a^5*b^5*m*x^6*x^
m + 1197504000*a^4*b^6*x^7*x^m + 2310945*a^10*m^5*x*x^m + 92615030*a^9*b*m^
4*x^2*x^m + 1003011660*a^8*b^2*m^3*x^3*x^m + 3698304480*a^7*b^3*m^2*x^4*x^m
 + 4727540160*a^6*b^4*m*x^5*x^m + 1676505600*a^5*b^5*x^6*x^m + 11028590*a^1
0*m^4*x*x^m + 274727240*a^9*b*m^3*x^2*x^m + 1727578440*a^8*b^2*m^2*x^3*x^m
+ 3316939200*a^7*b^3*m*x^4*x^m + 1676505600*a^6*b^4*x^5*x^m + 34967140*a^10
*m^3*x*x^m + 503126280*a^9*b*m^2*x^2*x^m + 1608573600*a^8*b^2*m*x^3*x^m + 1
197504000*a^7*b^3*x^4*x^m + 70290936*a^10*m^2*x*x^m + 502927200*a^9*b*m*x^2
*x^m + 598752000*a^8*b^2*x^3*x^m + 80627040*a^10*m*x*x^m + 199584000*a^9*b*
x^2*x^m + 39916800*a^10*x*x^m)/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357
423*m^7 + 2637558*m^6 + 13339535*m^5 + 45995730*m^4 + 105258076*m^3 + 15091
7976*m^2 + 120543840*m + 39916800)
```

maple [B] time = 0.01, size = 1535, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^10,x)
```

```
[Out] x^(m+1)*(b^10*m^10*x^10+10*a*b^9*m^10*x^9+55*b^10*m^9*x^10+45*a^2*b^8*m^10*
x^8+560*a*b^9*m^9*x^9+1320*b^10*m^8*x^10+120*a^3*b^7*m^10*x^7+2565*a^2*b^8*
m^9*x^8+13650*a*b^9*m^8*x^9+18150*b^10*m^7*x^10+210*a^4*b^6*m^10*x^6+6960*a
^3*b^7*m^9*x^7+63540*a^2*b^8*m^8*x^8+190200*a*b^9*m^7*x^9+157773*b^10*m^6*x
^10+252*a^5*b^5*m^10*x^5+12390*a^4*b^6*m^9*x^6+175320*a^3*b^7*m^8*x^7+89829
0*a^2*b^8*m^7*x^8+1672230*a*b^9*m^6*x^9+902055*b^10*m^5*x^10+210*a^6*b^4*m^
10*x^4+15120*a^5*b^5*m^9*x^5+317520*a^4*b^6*m^8*x^6+2517840*a^3*b^7*m^7*x^7
+7999425*a^2*b^8*m^6*x^8+9653280*a*b^9*m^5*x^9+3416930*b^10*m^4*x^10+120*a^
7*b^3*m^10*x^3+12810*a^6*b^4*m^9*x^4+394380*a^5*b^5*m^8*x^5+4638060*a^4*b^6
*m^7*x^6+22748040*a^3*b^7*m^6*x^7+46695285*a^2*b^8*m^5*x^8+36862550*a*b^9*m
^4*x^9+8409500*b^10*m^3*x^10+45*a^8*b^2*m^10*x^2+7440*a^7*b^3*m^9*x^3+34020
0*a^6*b^4*m^8*x^4+5866560*a^5*b^5*m^7*x^5+42592410*a^4*b^6*m^6*x^6+13452264
```

```
0*a^3*b^7*m^5*x^7+180021510*a^2*b^8*m^4*x^8+91331800*a*b^9*m^3*x^9+12753576
*b^10*m^2*x^10+10*a^9*b*m^10*x+2835*a^8*b^2*m^9*x^2+201240*a^7*b^3*m^8*x^3+
5159700*a^6*b^4*m^7*x^4+54871236*a^5*b^5*m^6*x^5+255740310*a^4*b^6*m^5*x^6+
524563080*a^3*b^7*m^4*x^7+449614260*a^2*b^8*m^3*x^8+139262760*a*b^9*m^2*x^9
+10628640*b^10*m*x^10+a^10*m^10+640*a^9*b*m^9*x+78120*a^8*b^2*m^8*x^2+31154
40*a^7*b^3*m^7*x^3+49260330*a^6*b^4*m^6*x^4+335437200*a^5*b^5*m^5*x^5+10111
20180*a^4*b^6*m^4*x^6+1322982960*a^3*b^7*m^3*x^7+690085080*a^2*b^8*m^2*x^8+
116552160*a*b^9*m*x^9+3628800*b^10*x^10+65*a^10*m^9+17970*a^9*b*m^8*x+12357
90*a^8*b^2*m^7*x^2+30429000*a^7*b^3*m^6*x^3+307585530*a^6*b^4*m^5*x^4+13489
39620*a^5*b^5*m^4*x^5+2581262040*a^4*b^6*m^3*x^6+2047105440*a^3*b^7*m^2*x^7
+580543200*a^2*b^8*m*x^8+39916800*a*b^9*x^9+1860*a^10*m^8+290760*a^9*b*m^7*
x+12376665*a^8*b^2*m^6*x^2+194790960*a^7*b^3*m^5*x^3+1263374700*a^6*b^4*m^4
*x^4+3497286240*a^5*b^5*m^3*x^5+4035361680*a^4*b^6*m^2*x^6+1733313600*a^3*b
^7*m*x^7+199584000*a^2*b^8*x^8+30810*a^10*m^7+2992710*a^9*b*m^6*x+81560115*
a^8*b^2*m^5*x^2+821580360*a^7*b^3*m^4*x^3+3342229800*a^6*b^4*m^3*x^4+554131
7712*a^5*b^5*m^2*x^5+3445243200*a^4*b^6*m*x^6+598752000*a^3*b^7*x^7+326613*
a^10*m^6+20390160*a^9*b*m^5*x+355598730*a^8*b^2*m^4*x^2+2233166160*a^7*b^3*
m^3*x^3+5393046960*a^6*b^4*m^2*x^4+4783423680*a^5*b^5*m*x^5+1197504000*a^4*
b^6*x^6+2310945*a^10*m^5+92615030*a^9*b*m^4*x+1003011660*a^8*b^2*m^3*x^2+36
98304480*a^7*b^3*m^2*x^3+4727540160*a^6*b^4*m*x^4+1676505600*a^5*b^5*x^5+11
028590*a^10*m^4+274727240*a^9*b*m^3*x+1727578440*a^8*b^2*m^2*x^2+3316939200
*a^7*b^3*m*x^3+1676505600*a^6*b^4*x^4+34967140*a^10*m^3+503126280*a^9*b*m^2
*x+1608573600*a^8*b^2*m*x^2+1197504000*a^7*b^3*x^3+70290936*a^10*m^2+502927
200*a^9*b*m*x+598752000*a^8*b^2*x^2+80627040*a^10*m+199584000*a^9*b*x+39916
800*a^10)/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)/(6+m)/(5+m)/(m+4)/(m+3)/(m+2)/(m+
1)
```

maxima [A] time = 1.39, size = 187, normalized size = 1.00

$$\frac{b^{10}x^{m+11}}{m+11}+\frac{10\,ab^9x^{m+10}}{m+10}+\frac{45\,a^2b^8x^{m+9}}{m+9}+\frac{120\,a^3b^7x^{m+8}}{m+8}+\frac{210\,a^4b^6x^{m+7}}{m+7}+\frac{252\,a^5b^5x^{m+6}}{m+6}+\frac{210\,a^6b^4x^{m+5}}{m+5}+\frac{120\,a^7b^3x^{m+4}}{m+4}+\frac{45\,a^8b^2x^{m+3}}{m+3}+\frac{10\,a^9bx^{m+2}}{m+2}+\frac{a^{10}x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] b^10*x^(m + 11)/(m + 11) + 10*a*b^9*x^(m + 10)/(m + 10) + 45*a^2*b^8*x^(m +
 9)/(m + 9) + 120*a^3*b^7*x^(m + 8)/(m + 8) + 210*a^4*b^6*x^(m + 7)/(m + 7)
 + 252*a^5*b^5*x^(m + 6)/(m + 6) + 210*a^6*b^4*x^(m + 5)/(m + 5) + 120*a^7*
b^3*x^(m + 4)/(m + 4) + 45*a^8*b^2*x^(m + 3)/(m + 3) + 10*a^9*b*x^(m + 2)/(
m + 2) + a^10*x^(m + 1)/(m + 1)
```

mupad [B] time = 1.37, size = 1274, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^10,x)
```

```
[Out] (a^10*x*x^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590*m^4 + 2310
945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39916800))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (b^10*x^m*x^11*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 +
 902055*m^5 + 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (45*a^2*b^8*x^m*x^9*(12900960*m + 15335224*m^2 + 9991428*m^3 + 40004
78*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 + 57*m^9 + m^10 +
4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 133
39535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^1
1 + 39916800) + (120*a^3*b^7*x^m*x^8*(14444280*m + 17059212*m^2 + 11024858*
```

```
m^3 + 4371359*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^
9 + m^10 + 4989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4599573
0*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66
*m^10 + m^11 + 39916800) + (210*a^4*b^6*x^m*x^7*(16405920*m + 19216008*m^2
+ 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821*m^6 + 22086*m^7 + 1512*
m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 150917976*m^2 + 105258076*m^
3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 19
25*m^9 + 66*m^10 + m^11 + 39916800) + (252*a^5*b^5*x^m*x^6*(18981840*m + 21
989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1331100*m^5 + 217743*m^6 + 23280*
m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/(120543840*m + 150917976*m^2 + 1
05258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 326
70*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (210*a^6*b^4*x^m*x^5*(2251
2096*m + 25681176*m^2 + 15915380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m
^6 + 24570*m^7 + 1620*m^8 + 61*m^9 + m^10 + 7983360))/(120543840*m + 150917
976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 35742
3*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (120*a^7*b^3*x^
m*x^4*(27641160*m + 30819204*m^2 + 18609718*m^3 + 6846503*m^4 + 1623258*m^5
 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9 + m^10 + 9979200))/(120543840
*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*
m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (45*
a^8*b^2*x^m*x^3*(35746080*m + 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1
812447*m^5 + 275037*m^6 + 27462*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))
/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916
800) + (10*a*b^9*x^m*x^10*(11655216*m + 13926276*m^2 + 9133180*m^3 + 368625
5*m^4 + 965328*m^5 + 167223*m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 39
91680))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339
535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
+ 39916800) + (10*a^9*b*x^m*x^2*(50292720*m + 50312628*m^2 + 27472724*m^3 +
 9261503*m^4 + 2039016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m
^10 + 19958400))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^
4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^1
0 + m^11 + 39916800)
```

sympy [A] time = 6.93, size = 9996, normalized size = 53.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**10,x)
```

```
[Out] Piecewise((-a**10/(10*x**10) - 10*a**9*b/(9*x**9) - 45*a**8*b**2/(8*x**8) -
 120*a**7*b**3/(7*x**7) - 35*a**6*b**4/x**6 - 252*a**5*b**5/(5*x**5) - 105*
a**4*b**6/(2*x**4) - 40*a**3*b**7/x**3 - 45*a**2*b**8/(2*x**2) - 10*a*b**9/
x + b**10*log(x), Eq(m, -11)), (-a**10/(9*x**9) - 5*a**9*b/(4*x**8) - 45*a*
*8*b**2/(7*x**7) - 20*a**7*b**3/x**6 - 42*a**6*b**4/x**5 - 63*a**5*b**5/x**
4 - 70*a**4*b**6/x**3 - 60*a**3*b**7/x**2 - 45*a**2*b**8/x + 10*a*b**9*log(
x) + b**10*x, Eq(m, -10)), (-a**10/(8*x**8) - 10*a**9*b/(7*x**7) - 15*a**8*
b**2/(2*x**6) - 24*a**7*b**3/x**5 - 105*a**6*b**4/(2*x**4) - 84*a**5*b**5/x
**3 - 105*a**4*b**6/x**2 - 120*a**3*b**7/x + 45*a**2*b**8*log(x) + 10*a*b**
9*x + b**10*x**2/2, Eq(m, -9)), (-a**10/(7*x**7) - 5*a**9*b/(3*x**6) - 9*a*
*8*b**2/x**5 - 30*a**7*b**3/x**4 - 70*a**6*b**4/x**3 - 126*a**5*b**5/x**2 -
 210*a**4*b**6/x + 120*a**3*b**7*log(x) + 45*a**2*b**8*x + 5*a*b**9*x**2 +
b**10*x**3/3, Eq(m, -8)), (-a**10/(6*x**6) - 2*a**9*b/x**5 - 45*a**8*b**2/(
4*x**4) - 40*a**7*b**3/x**3 - 105*a**6*b**4/x**2 - 252*a**5*b**5/x + 210*a*
*4*b**6*log(x) + 120*a**3*b**7*x + 45*a**2*b**8*x**2/2 + 10*a*b**9*x**3/3 +
 b**10*x**4/4, Eq(m, -7)), (-a**10/(5*x**5) - 5*a**9*b/(2*x**4) - 15*a**8*b
**2/x**3 - 60*a**7*b**3/x**2 - 210*a**6*b**4/x + 252*a**5*b**5*log(x) + 210
*a**4*b**6*x + 60*a**3*b**7*x**2 + 15*a**2*b**8*x**3 + 5*a*b**9*x**4/2 + b*
*10*x**5/5, Eq(m, -6)), (-a**10/(4*x**4) - 10*a**9*b/(3*x**3) - 45*a**8*b**
```

```
2/(2*x**2) - 120*a**7*b**3/x + 210*a**6*b**4*log(x) + 252*a**5*b**5*x + 105
*a**4*b**6*x**2 + 40*a**3*b**7*x**3 + 45*a**2*b**8*x**4/4 + 2*a*b**9*x**5 +
 b**10*x**6/6, Eq(m, -5)), (-a**10/(3*x**3) - 5*a**9*b/x**2 - 45*a**8*b**2/
x + 120*a**7*b**3*log(x) + 210*a**6*b**4*x + 126*a**5*b**5*x**2 + 70*a**4*b
**6*x**3 + 30*a**3*b**7*x**4 + 9*a**2*b**8*x**5 + 5*a*b**9*x**6/3 + b**10*x
**7/7, Eq(m, -4)), (-a**10/(2*x**2) - 10*a**9*b/x + 45*a**8*b**2*log(x) + 1
20*a**7*b**3*x + 105*a**6*b**4*x**2 + 84*a**5*b**5*x**3 + 105*a**4*b**6*x**
4/2 + 24*a**3*b**7*x**5 + 15*a**2*b**8*x**6/2 + 10*a*b**9*x**7/7 + b**10*x*
*8/8, Eq(m, -3)), (-a**10/x + 10*a**9*b*log(x) + 45*a**8*b**2*x + 60*a**7*b
**3*x**2 + 70*a**6*b**4*x**3 + 63*a**5*b**5*x**4 + 42*a**4*b**6*x**5 + 20*a
**3*b**7*x**6 + 45*a**2*b**8*x**7/7 + 5*a*b**9*x**8/4 + b**10*x**9/9, Eq(m,
 -2)), (a**10*log(x) + 10*a**9*b*x + 45*a**8*b**2*x**2/2 + 40*a**7*b**3*x**
3 + 105*a**6*b**4*x**4/2 + 252*a**5*b**5*x**5/5 + 35*a**4*b**6*x**6 + 120*a
**3*b**7*x**7/7 + 45*a**2*b**8*x**8/8 + 10*a*b**9*x**9/9 + b**10*x**10/10,
Eq(m, -1)), (a**10*m**10*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 65*a**10*m**9*x*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 1860*a**10*m**8*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m
**3 + 150917976*m**2 + 120543840*m + 39916800) + 30810*a**10*m**7*x*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 326613*a**10*m**6*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 2310945*a**10*m**5*x
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 11028590*a**10*m**4*x*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 34967140*a
**10*m**3*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 70290936*a**10*m**2*x*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 80627040*a**10*m*x*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35742
3*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15
0917976*m**2 + 120543840*m + 39916800) + 39916800*a**10*x*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 10*a**9*b*m**10*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 640*a**9*b*m**9*x**2*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 17970*a**9*b*m**8*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32
670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105
258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 290760*a**9*b*m**7
*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 2992710*a**9*b*m**6*x**2*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 20
390160*a**9*b*m**5*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3
57423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
+ 150917976*m**2 + 120543840*m + 39916800) + 92615030*a**9*b*m**4*x**2*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
```

13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 274727240*a**9*b*m**3*x**2*x**m/(m**11 + 66*m**10 + 1925*m
9 + 32670*m8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 503126280*
a**9*b*m**2*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 502927200*a**9*b*m*x**2*x**m/(m**11 +
66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 199584000*a**9*b*x**2*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45*a**8*b**2*m**10*x**3*
x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**
6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12054
3840*m + 39916800) + 2835*a**8*b**2*m**9*x**3*x**m/(m**11 + 66*m**10 + 1925
*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*
m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 78120*a*
*8*b**2*m**8*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*
m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1509
17976*m**2 + 120543840*m + 39916800) + 1235790*a**8*b**2*m**7*x**3*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
39916800) + 12376665*a**8*b**2*m**6*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 81560115*a**
8*b**2*m**5*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m
7 + 2637558*m6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 15091
7976*m**2 + 120543840*m + 39916800) + 355598730*a**8*b**2*m**4*x**3*x**m/(m
11 + 66*m10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 1003011660*a**8*b**2*m**3*x**3*x**m/(m**11 + 66*m**10 + 1925*
m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m
4 + 105258076*m3 + 150917976*m**2 + 120543840*m + 39916800) + 172757844
0*a**8*b**2*m**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 1608573600*a**8*b**2*m*x**3*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 598752000*a**8*b**2*x**3*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**7*b**
3*m**10*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7440*a**7*b**3*m**9*x**4*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 201240*a**7*b**3*m**8*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3115440*a**7*b**3*m**7
*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
120543840*m + 39916800) + 30429000*a**7*b**3*m**6*x**4*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 194790960*a**7*b**3*m**5*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 821580360*a**7*b**3*m*
*4*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2

```
 + 120543840*m + 39916800) + 2233166160*a**7*b**3*m**3*x**4*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 3698304480*a**7*b**3*m**2*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3316939200*a**7*b
**3*m*x**4*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 1197504000*a**7*b**3*x**4*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 210*a**6*b**4*m**10*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12810*a**6*b**4*m**9*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 340200*a**6*b**4*m**8*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 51597
00*a**6*b**4*m**7*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 35
7423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
 150917976*m**2 + 120543840*m + 39916800) + 49260330*a**6*b**4*m**6*x**5*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 307585530*a**6*b**4*m**5*x**5*x**m/(m**11 + 66*m**10 + 1
925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459957
30*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 12633
74700*a**6*b**4*m**4*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 3342229800*a**6*b**4*m**3*x*
*5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 5393046960*a**6*b**4*m**2*x**5*x**m/(m**11 + 66*m**
10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 4727540160*a**6*b**4*m*x**5*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505600*a**6*b**4*x**5
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 252*a**5*b**5*m**10*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 15120*a
**5*b**5*m**9*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 394380*a**5*b**5*m**8*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 5866560*a**5*b**5*m**7*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9
 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
+ 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 54871236*a**5
*b**5*m**6*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 335437200*a**5*b**5*m**5*x**6*x**m/(m*
*11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333
9535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m +
 39916800) + 1348939620*a**5*b**5*m**4*x**6*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3497286240
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*a**5*b**5*m**3*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 3574
23*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 1
50917976*m**2 + 120543840*m + 39916800) + 5541317712*a**5*b**5*m**2*x**6*x*
*m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6
+ 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205438
40*m + 39916800) + 4783423680*a**5*b**5*m*x**6*x**m/(m**11 + 66*m**10 + 192
5*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730
*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1676505
600*a**5*b**5*x**6*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423
*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150
917976*m**2 + 120543840*m + 39916800) + 210*a**4*b**6*m**10*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 12390*a**4*b**6*m**9*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 317520*a**4*b**6*
m**8*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 4638060*a**4*b**6*m**7*x**7*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 42592410*a**4*b**6*m**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3267
0*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10525
8076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 255740310*a**4*b**6*
m**5*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2
637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m*
*2 + 120543840*m + 39916800) + 1011120180*a**4*b**6*m**4*x**7*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 2581262040*a**4*b**6*m**3*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 4035361680*a**4
*b**6*m**2*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m*
*7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917
976*m**2 + 120543840*m + 39916800) + 3445243200*a**4*b**6*m*x**7*x**m/(m**1
1 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133395
35*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3
9916800) + 1197504000*a**4*b**6*x**7*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 120*a**3*b**7*m**
10*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 263
7558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2
 + 120543840*m + 39916800) + 6960*a**3*b**7*m**9*x**8*x**m/(m**11 + 66*m**1
0 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4
5995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
175320*a**3*b**7*m**8*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 2517840*a**3*b**7*m**7*x**8
*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m*
*6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 1205
43840*m + 39916800) + 22748040*a**3*b**7*m**6*x**8*x**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 134
522640*a**3*b**7*m**5*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 524563080*a**3*b**7*m**4*x*
*8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 1322982960*a**3*b**7*m**3*x**8*x**m/(m**11 + 66*m**
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10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) +
 2047105440*a**3*b**7*m**2*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*
m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 1052580
76*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1733313600*a**3*b**7*m
*x**8*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 26375
58*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 +
 120543840*m + 39916800) + 598752000*a**3*b**7*x**8*x**m/(m**11 + 66*m**10
+ 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 459
95730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 45
*a**2*b**8*m**10*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357
423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 +
150917976*m**2 + 120543840*m + 39916800) + 2565*a**2*b**8*m**9*x**9*x**m/(m
**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 133
39535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m
+ 39916800) + 63540*a**2*b**8*m**8*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 898290*a**2*b*
*8*m**7*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 7999425*a**2*b**8*m**6*x**9*x**m/(m**11 +
 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*
m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991
6800) + 46695285*a**2*b**8*m**5*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 3
2670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 10
5258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 180021510*a**2*b*
*8*m**4*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 449614260*a**2*b**8*m**3*x**9*x**m/(m**11
 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1333953
5*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39
916800) + 690085080*a**2*b**8*m**2*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 580543200*a**2
*b**8*m*x**9*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 199584000*a**2*b**8*x**9*x**m/(m**11 + 66
*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**
5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 3991680
0) + 10*a*b**9*m**10*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8
+ 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m*
*3 + 150917976*m**2 + 120543840*m + 39916800) + 560*a*b**9*m**9*x**10*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800) + 13650*a*b**9*m**8*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9
+ 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 190200*a*b**9*
m**7*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m
**2 + 120543840*m + 39916800) + 1672230*a*b**9*m**6*x**10*x**m/(m**11 + 66*
m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5
 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800
) + 9653280*a*b**9*m**5*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m*
*8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076
*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 36862550*a*b**9*m**4*x**
10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 91331800*a*b**9*m**3*x**10*x**m/(m**11 + 66*m**10 +
 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 4599
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5730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 139
262760*a*b**9*m**2*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3
 + 150917976*m**2 + 120543840*m + 39916800) + 116552160*a*b**9*m*x**10*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 39916800*a*b**9*x**10*x**m/(m**11 + 66*m**10 + 1925*m**9 +
 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 +
105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + b**10*m**10*x**
11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*
m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 12
0543840*m + 39916800) + 55*b**10*m**9*x**11*x**m/(m**11 + 66*m**10 + 1925*m
**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m*
*4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 1320*b**10
*m**8*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 +
 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*
m**2 + 120543840*m + 39916800) + 18150*b**10*m**7*x**11*x**m/(m**11 + 66*m*
*10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 +
 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800)
+ 157773*b**10*m**6*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 +
 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**
3 + 150917976*m**2 + 120543840*m + 39916800) + 902055*b**10*m**5*x**11*x**m
/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 +
13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840
*m + 39916800) + 3416930*b**10*m**4*x**11*x**m/(m**11 + 66*m**10 + 1925*m**
9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4
 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 39916800) + 8409500*b**1
0*m**3*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7
+ 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976
*m**2 + 120543840*m + 39916800) + 12753576*b**10*m**2*x**11*x**m/(m**11 + 6
6*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 13339535*m*
*5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*m + 399168
00) + 10628640*b**10*m*x**11*x**m/(m**11 + 66*m**10 + 1925*m**9 + 32670*m**
8 + 357423*m**7 + 2637558*m**6 + 13339535*m**5 + 45995730*m**4 + 105258076*
m**3 + 150917976*m**2 + 120543840*m + 39916800) + 3628800*b**10*x**11*x**m/
(m**11 + 66*m**10 + 1925*m**9 + 32670*m**8 + 357423*m**7 + 2637558*m**6 + 1
3339535*m**5 + 45995730*m**4 + 105258076*m**3 + 150917976*m**2 + 120543840*
m + 39916800), True))
```

3.700 $\int x^m(a+bx)^7\,dx$

Optimal. Leaf size=133

$$\frac{a^7x^{m+1}}{m+1}+\frac{7a^6bx^{m+2}}{m+2}+\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

Rubi [A] time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{21a^5b^2x^{m+3}}{m+3}+\frac{35a^4b^3x^{m+4}}{m+4}+\frac{35a^3b^4x^{m+5}}{m+5}+\frac{21a^2b^5x^{m+6}}{m+6}+\frac{7a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}+\frac{7ab^6x^{m+7}}{m+7}+\frac{b^7x^{m+8}}{m+8}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^7,x]
```

```
[Out] (a^7*x^(1 + m))/(1 + m) + (7*a^6*b*x^(2 + m))/(2 + m) + (21*a^5*b^2*x^(3 +
m))/(3 + m) + (35*a^4*b^3*x^(4 + m))/(4 + m) + (35*a^3*b^4*x^(5 + m))/(5 +
m) + (21*a^2*b^5*x^(6 + m))/(6 + m) + (7*a*b^6*x^(7 + m))/(7 + m) + (b^7*x^
(8 + m))/(8 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^m(a+bx)^7\,dx=\int\left(a^7x^m+7a^6bx^{1+m}+21a^5b^2x^{2+m}+35a^4b^3x^{3+m}+35a^3b^4x^{4+m}+21a^2b^5x^{5+m}+7ab^6x^{6+}\right.$$
$$=\frac{a^7x^{1+m}}{1+m}+\frac{7a^6bx^{2+m}}{2+m}+\frac{21a^5b^2x^{3+m}}{3+m}+\frac{35a^4b^3x^{4+m}}{4+m}+\frac{35a^3b^4x^{5+m}}{5+m}+\frac{21a^2b^5x^{6+m}}{6+m}+\frac{7ab^6x}{7+}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.89

$$x^{m+1}\left(\frac{a^7}{m+1}+\frac{7a^6bx}{m+2}+\frac{21a^5b^2x^2}{m+3}+\frac{35a^4b^3x^3}{m+4}+\frac{35a^3b^4x^4}{m+5}+\frac{21a^2b^5x^5}{m+6}+\frac{7ab^6x^6}{m+7}+\frac{b^7x^7}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^7,x]
```

```
[Out] x^(1 + m)*(a^7/(1 + m) + (7*a^6*b*x)/(2 + m) + (21*a^5*b^2*x^2)/(3 + m) + (
35*a^4*b^3*x^3)/(4 + m) + (35*a^3*b^4*x^4)/(5 + m) + (21*a^2*b^5*x^5)/(6 +
m) + (7*a*b^6*x^6)/(7 + m) + (b^7*x^7)/(8 + m))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^7,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^7, x]
```

fricas [B] time = 1.35, size = 665, normalized size = 5.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] ((b^7*m^7 + 28*b^7*m^6 + 322*b^7*m^5 + 1960*b^7*m^4 + 6769*b^7*m^3 + 13132*
b^7*m^2 + 13068*b^7*m + 5040*b^7)*x^8 + 7*(a*b^6*m^7 + 29*a*b^6*m^6 + 343*a
*b^6*m^5 + 2135*a*b^6*m^4 + 7504*a*b^6*m^3 + 14756*a*b^6*m^2 + 14832*a*b^6*
m + 5760*a*b^6)*x^7 + 21*(a^2*b^5*m^7 + 30*a^2*b^5*m^6 + 366*a^2*b^5*m^5 +
2340*a^2*b^5*m^4 + 8409*a^2*b^5*m^3 + 16830*a^2*b^5*m^2 + 17144*a^2*b^5*m +
 6720*a^2*b^5)*x^6 + 35*(a^3*b^4*m^7 + 31*a^3*b^4*m^6 + 391*a^3*b^4*m^5 + 2
581*a^3*b^4*m^4 + 9544*a^3*b^4*m^3 + 19564*a^3*b^4*m^2 + 20304*a^3*b^4*m +
8064*a^3*b^4)*x^5 + 35*(a^4*b^3*m^7 + 32*a^4*b^3*m^6 + 418*a^4*b^3*m^5 + 28
64*a^4*b^3*m^4 + 10993*a^4*b^3*m^3 + 23312*a^4*b^3*m^2 + 24876*a^4*b^3*m +
10080*a^4*b^3)*x^4 + 21*(a^5*b^2*m^7 + 33*a^5*b^2*m^6 + 447*a^5*b^2*m^5 + 3
195*a^5*b^2*m^4 + 12864*a^5*b^2*m^3 + 28692*a^5*b^2*m^2 + 32048*a^5*b^2*m +
 13440*a^5*b^2)*x^3 + 7*(a^6*b*m^7 + 34*a^6*b*m^6 + 478*a^6*b*m^5 + 3580*a^
6*b*m^4 + 15289*a^6*b*m^3 + 36706*a^6*b*m^2 + 44712*a^6*b*m + 20160*a^6*b)*
x^2 + (a^7*m^7 + 35*a^7*m^6 + 511*a^7*m^5 + 4025*a^7*m^4 + 18424*a^7*m^3 +
48860*a^7*m^2 + 69264*a^7*m + 40320*a^7)*x)*x^m/(m^8 + 36*m^7 + 546*m^6 + 4
536*m^5 + 22449*m^4 + 67284*m^3 + 118124*m^2 + 109584*m + 40320)
```

giac [B] time = 1.40, size = 992, normalized size = 7.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="giac")
```

```
[Out] (b^7*m^7*x^8*x^m + 7*a*b^6*m^7*x^7*x^m + 28*b^7*m^6*x^8*x^m + 21*a^2*b^5*m^
7*x^6*x^m + 203*a*b^6*m^6*x^7*x^m + 322*b^7*m^5*x^8*x^m + 35*a^3*b^4*m^7*x^
5*x^m + 630*a^2*b^5*m^6*x^6*x^m + 2401*a*b^6*m^5*x^7*x^m + 1960*b^7*m^4*x^8
*x^m + 35*a^4*b^3*m^7*x^4*x^m + 1085*a^3*b^4*m^6*x^5*x^m + 7686*a^2*b^5*m^5
*x^6*x^m + 14945*a*b^6*m^4*x^7*x^m + 6769*b^7*m^3*x^8*x^m + 21*a^5*b^2*m^7*
x^3*x^m + 1120*a^4*b^3*m^6*x^4*x^m + 13685*a^3*b^4*m^5*x^5*x^m + 49140*a^2*
b^5*m^4*x^6*x^m + 52528*a*b^6*m^3*x^7*x^m + 13132*b^7*m^2*x^8*x^m + 7*a^6*b
*m^7*x^2*x^m + 693*a^5*b^2*m^6*x^3*x^m + 14630*a^4*b^3*m^5*x^4*x^m + 90335*
a^3*b^4*m^4*x^5*x^m + 176589*a^2*b^5*m^3*x^6*x^m + 103292*a*b^6*m^2*x^7*x^m
 + 13068*b^7*m*x^8*x^m + a^7*m^7*x*x^m + 238*a^6*b*m^6*x^2*x^m + 9387*a^5*b
^2*m^5*x^3*x^m + 100240*a^4*b^3*m^4*x^4*x^m + 334040*a^3*b^4*m^3*x^5*x^m +
353430*a^2*b^5*m^2*x^6*x^m + 103824*a*b^6*m*x^7*x^m + 5040*b^7*x^8*x^m + 35
*a^7*m^6*x*x^m + 3346*a^6*b*m^5*x^2*x^m + 67095*a^5*b^2*m^4*x^3*x^m + 38475
5*a^4*b^3*m^3*x^4*x^m + 684740*a^3*b^4*m^2*x^5*x^m + 360024*a^2*b^5*m*x^6*x
^m + 40320*a*b^6*x^7*x^m + 511*a^7*m^5*x*x^m + 25060*a^6*b*m^4*x^2*x^m + 27
0144*a^5*b^2*m^3*x^3*x^m + 815920*a^4*b^3*m^2*x^4*x^m + 710640*a^3*b^4*m*x^
5*x^m + 141120*a^2*b^5*x^6*x^m + 4025*a^7*m^4*x*x^m + 107023*a^6*b*m^3*x^2*
x^m + 602532*a^5*b^2*m^2*x^3*x^m + 870660*a^4*b^3*m*x^4*x^m + 282240*a^3*b^
4*x^5*x^m + 18424*a^7*m^3*x*x^m + 256942*a^6*b*m^2*x^2*x^m + 673008*a^5*b^2
*m*x^3*x^m + 352800*a^4*b^3*x^4*x^m + 48860*a^7*m^2*x*x^m + 312984*a^6*b*m*
x^2*x^m + 282240*a^5*b^2*x^3*x^m + 69264*a^7*m*x*x^m + 141120*a^6*b*x^2*x^m
 + 40320*a^7*x*x^m)/(m^8 + 36*m^7 + 546*m^6 + 4536*m^5 + 22449*m^4 + 67284*
m^3 + 118124*m^2 + 109584*m + 40320)
```

maple [B] time = 0.01, size = 782, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^7,x)
```

```
[Out] x^(m+1)*(b^7*m^7*x^7+7*a*b^6*m^7*x^6+28*b^7*m^6*x^7+21*a^2*b^5*m^7*x^5+203*
a*b^6*m^6*x^6+322*b^7*m^5*x^7+35*a^3*b^4*m^7*x^4+630*a^2*b^5*m^6*x^5+2401*a
*b^6*m^5*x^6+1960*b^7*m^4*x^7+35*a^4*b^3*m^7*x^3+1085*a^3*b^4*m^6*x^4+7686*
a^2*b^5*m^5*x^5+14945*a*b^6*m^4*x^6+6769*b^7*m^3*x^7+21*a^5*b^2*m^7*x^2+112
0*a^4*b^3*m^6*x^3+13685*a^3*b^4*m^5*x^4+49140*a^2*b^5*m^4*x^5+52528*a*b^6*m
^3*x^6+13132*b^7*m^2*x^7+7*a^6*b*m^7*x+693*a^5*b^2*m^6*x^2+14630*a^4*b^3*m^
5*x^3+90335*a^3*b^4*m^4*x^4+176589*a^2*b^5*m^3*x^5+103292*a*b^6*m^2*x^6+130
68*b^7*m*x^7+a^7*m^7+238*a^6*b*m^6*x+9387*a^5*b^2*m^5*x^2+100240*a^4*b^3*m^
4*x^3+334040*a^3*b^4*m^3*x^4+353430*a^2*b^5*m^2*x^5+103824*a*b^6*m*x^6+5040
*b^7*x^7+35*a^7*m^6+3346*a^6*b*m^5*x+67095*a^5*b^2*m^4*x^2+384755*a^4*b^3*m
^3*x^3+684740*a^3*b^4*m^2*x^4+360024*a^2*b^5*m*x^5+40320*a*b^6*x^6+511*a^7*
m^5+25060*a^6*b*m^4*x+270144*a^5*b^2*m^3*x^2+815920*a^4*b^3*m^2*x^3+710640*
a^3*b^4*m*x^4+141120*a^2*b^5*x^5+4025*a^7*m^4+107023*a^6*b*m^3*x+602532*a^5
*b^2*m^2*x^2+870660*a^4*b^3*m*x^3+282240*a^3*b^4*x^4+18424*a^7*m^3+256942*a
^6*b*m^2*x+673008*a^5*b^2*m*x^2+352800*a^4*b^3*x^3+48860*a^7*m^2+312984*a^6
*b*m*x+282240*a^5*b^2*x^2+69264*a^7*m+141120*a^6*b*x+40320*a^7)/(m+8)/(m+7)
/(m+6)/(m+5)/(m+4)/(m+3)/(m+2)/(m+1)
```

maxima [A] time = 1.37, size = 133, normalized size = 1.00

$$\frac{b^7x^{m+8}}{m+8}+\frac{7\,ab^6x^{m+7}}{m+7}+\frac{21\,a^2b^5x^{m+6}}{m+6}+\frac{35\,a^3b^4x^{m+5}}{m+5}+\frac{35\,a^4b^3x^{m+4}}{m+4}+\frac{21\,a^5b^2x^{m+3}}{m+3}+\frac{7\,a^6bx^{m+2}}{m+2}+\frac{a^7x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] b^7*x^(m + 8)/(m + 8) + 7*a*b^6*x^(m + 7)/(m + 7) + 21*a^2*b^5*x^(m + 6)/(m
 + 6) + 35*a^3*b^4*x^(m + 5)/(m + 5) + 35*a^4*b^3*x^(m + 4)/(m + 4) + 21*a^
5*b^2*x^(m + 3)/(m + 3) + 7*a^6*b*x^(m + 2)/(m + 2) + a^7*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.78, size = 683, normalized size = 5.14

$$\frac{a^7\,x\,x^m\,\left(m^7+35\,m^6+511\,m^5+4025\,m^4+18424\,m^3+48860\,m^2+69264\,m+40320\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{b^7\,x^m\,x^8\,\left(m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^2\,b^5\,x^m\,x^6\,\left(m^7+30\,m^6+366\,m^5+2340\,m^4+8409\,m^3+16830\,m^2+17144\,m+6720\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^3\,b^4\,x^m\,x^5\,\left(m^7+31\,m^6+391\,m^5+2581\,m^4+9544\,m^3+19564\,m^2+20304\,m+8064\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{35\,a^4\,b^3\,x^m\,x^4\,\left(m^7+32\,m^6+418\,m^5+2864\,m^4+10993\,m^3+23312\,m^2+24876\,m+10080\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{21\,a^5\,b^2\,x^m\,x^3\,\left(m^7+33\,m^6+447\,m^5+3195\,m^4+12864\,m^3+28692\,m^2+32048\,m+13440\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a\,b^6\,x^m\,x^7\,\left(m^7+29\,m^6+343\,m^5+2135\,m^4+7504\,m^3+14756\,m^2+14832\,m+5760\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}+\frac{7\,a^6\,b\,x^m\,x^2\,\left(m^7+34\,m^6+478\,m^5+3580\,m^4+15289\,m^3+36706\,m^2+44712\,m+20160\right)}{m^8+36\,m^7+546\,m^6+4536\,m^5+22449\,m^4+67284\,m^3+118124\,m^2+109584\,m+40320}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^7,x)
```

```
[Out] (a^7*x*x^m*(69264*m + 48860*m^2 + 18424*m^3 + 4025*m^4 + 511*m^5 + 35*m^6 +
 m^7 + 40320))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (b^7*x^m*x^8*(13068*m + 13132*m^2 + 6769*
m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040))/(109584*m + 118124*m^2 + 6
7284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^2
*b^5*x^m*x^6*(17144*m + 16830*m^2 + 8409*m^3 + 2340*m^4 + 366*m^5 + 30*m^6
+ m^7 + 6720))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 +
546*m^6 + 36*m^7 + m^8 + 40320) + (35*a^3*b^4*x^m*x^5*(20304*m + 19564*m^2
+ 9544*m^3 + 2581*m^4 + 391*m^5 + 31*m^6 + m^7 + 8064))/(109584*m + 118124*
m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) +
(35*a^4*b^3*x^m*x^4*(24876*m + 23312*m^2 + 10993*m^3 + 2864*m^4 + 418*m^5 +
 32*m^6 + m^7 + 10080))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 45
36*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (21*a^5*b^2*x^m*x^3*(32048*m + 2
8692*m^2 + 12864*m^3 + 3195*m^4 + 447*m^5 + 33*m^6 + m^7 + 13440))/(109584*
m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8
+ 40320) + (7*a*b^6*x^m*x^7*(14832*m + 14756*m^2 + 7504*m^3 + 2135*m^4 + 34
3*m^5 + 29*m^6 + m^7 + 5760))/(109584*m + 118124*m^2 + 67284*m^3 + 22449*m^
4 + 4536*m^5 + 546*m^6 + 36*m^7 + m^8 + 40320) + (7*a^6*b*x^m*x^2*(44712*m
+ 36706*m^2 + 15289*m^3 + 3580*m^4 + 478*m^5 + 34*m^6 + m^7 + 20160))/(1095
```

```
84*m + 118124*m^2 + 67284*m^3 + 22449*m^4 + 4536*m^5 + 546*m^6 + 36*m^7 + m
^8 + 40320)
```

sympy [A] time = 3.34, size = 4257, normalized size = 32.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**7,x)
```

```
[Out] Piecewise((-a**7/(7*x**7) - 7*a**6*b/(6*x**6) - 21*a**5*b**2/(5*x**5) - 35*
a**4*b**3/(4*x**4) - 35*a**3*b**4/(3*x**3) - 21*a**2*b**5/(2*x**2) - 7*a*b*
*6/x + b**7*log(x), Eq(m, -8)), (-a**7/(6*x**6) - 7*a**6*b/(5*x**5) - 21*a*
*5*b**2/(4*x**4) - 35*a**4*b**3/(3*x**3) - 35*a**3*b**4/(2*x**2) - 21*a**2*
b**5/x + 7*a*b**6*log(x) + b**7*x, Eq(m, -7)), (-a**7/(5*x**5) - 7*a**6*b/(
4*x**4) - 7*a**5*b**2/x**3 - 35*a**4*b**3/(2*x**2) - 35*a**3*b**4/x + 21*a*
*2*b**5*log(x) + 7*a*b**6*x + b**7*x**2/2, Eq(m, -6)), (-a**7/(4*x**4) - 7*
a**6*b/(3*x**3) - 21*a**5*b**2/(2*x**2) - 35*a**4*b**3/x + 35*a**3*b**4*log
(x) + 21*a**2*b**5*x + 7*a*b**6*x**2/2 + b**7*x**3/3, Eq(m, -5)), (-a**7/(3
*x**3) - 7*a**6*b/(2*x**2) - 21*a**5*b**2/x + 35*a**4*b**3*log(x) + 35*a**3
*b**4*x + 21*a**2*b**5*x**2/2 + 7*a*b**6*x**3/3 + b**7*x**4/4, Eq(m, -4)),
(-a**7/(2*x**2) - 7*a**6*b/x + 21*a**5*b**2*log(x) + 35*a**4*b**3*x + 35*a*
*3*b**4*x**2/2 + 7*a**2*b**5*x**3 + 7*a*b**6*x**4/4 + b**7*x**5/5, Eq(m, -3
)), (-a**7/x + 7*a**6*b*log(x) + 21*a**5*b**2*x + 35*a**4*b**3*x**2/2 + 35*
a**3*b**4*x**3/3 + 21*a**2*b**5*x**4/4 + 7*a*b**6*x**5/5 + b**7*x**6/6, Eq(
m, -2)), (a**7*log(x) + 7*a**6*b*x + 21*a**5*b**2*x**2/2 + 35*a**4*b**3*x**
3/3 + 35*a**3*b**4*x**4/4 + 21*a**2*b**5*x**5/5 + 7*a*b**6*x**6/6 + b**7*x*
*7/7, Eq(m, -1)), (a**7*m**7*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**7*m**6*
x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 1
18124*m**2 + 109584*m + 40320) + 511*a**7*m**5*x*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 4025*a**7*m**4*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m
**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 18424*a**7*m**3*x*x**m
/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*
m**2 + 109584*m + 40320) + 48860*a**7*m**2*x*x**m/(m**8 + 36*m**7 + 546*m**
6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
 69264*a**7*m*x*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 +
67284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a**7*x*x**m/(m**8 + 36
*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1095
84*m + 40320) + 7*a**6*b*m**7*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 238*a**6*
b*m**6*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6728
4*m**3 + 118124*m**2 + 109584*m + 40320) + 3346*a**6*b*m**5*x**2*x**m/(m**8
 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
 109584*m + 40320) + 25060*a**6*b*m**4*x**2*x**m/(m**8 + 36*m**7 + 546*m**6
 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
107023*a**6*b*m**3*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 256942*a**6*b*m**2*x
**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 312984*a**6*b*m*x**2*x**m/(m**8 + 36*m**
7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m
 + 40320) + 141120*a**6*b*x**2*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5
+ 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**5*b**2*
m**7*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 693*a**5*b**2*m**6*x**3*x**m/(m**8
 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
 109584*m + 40320) + 9387*a**5*b**2*m**5*x**3*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 67095*a**5*b**2*m**4*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2
```

2449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 270144*a**5*b**2
*m**3*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 602532*a**5*b**2*m**2*x**3*x**m/(
m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m*
*2 + 109584*m + 40320) + 673008*a**5*b**2*m*x**3*x**m/(m**8 + 36*m**7 + 546
*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 4032
0) + 282240*a**5*b**2*x**3*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**4*b**3*m**7
*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
 + 118124*m**2 + 109584*m + 40320) + 1120*a**4*b**3*m**6*x**4*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 14630*a**4*b**3*m**5*x**4*x**m/(m**8 + 36*m**7 + 546*m**6
 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) +
100240*a**4*b**3*m**4*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22
449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 384755*a**4*b**3*
m**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*
m**3 + 118124*m**2 + 109584*m + 40320) + 815920*a**4*b**3*m**2*x**4*x**m/(m
8 + 36*m7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**
2 + 109584*m + 40320) + 870660*a**4*b**3*m*x**4*x**m/(m**8 + 36*m**7 + 546*
m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320
) + 352800*a**4*b**3*x**4*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 224
49*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 35*a**3*b**4*m**7*
x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3
+ 118124*m**2 + 109584*m + 40320) + 1085*a**3*b**4*m**6*x**5*x**m/(m**8 + 3
6*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109
584*m + 40320) + 13685*a**3*b**4*m**5*x**5*x**m/(m**8 + 36*m**7 + 546*m**6
+ 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 9
0335*a**3*b**4*m**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 334040*a**3*b**4*m*
*3*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320) + 684740*a**3*b**4*m**2*x**5*x**m/(m**
8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2
+ 109584*m + 40320) + 710640*a**3*b**4*m*x**5*x**m/(m**8 + 36*m**7 + 546*m*
*6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
+ 282240*a**3*b**4*x**5*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449
*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 21*a**2*b**5*m**7*x*
*6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
118124*m**2 + 109584*m + 40320) + 630*a**2*b**5*m**6*x**6*x**m/(m**8 + 36*m
7 + 546*m6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 7686*a**2*b**5*m**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 45
36*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 49140
*a**2*b**5*m**4*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m*
*4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 176589*a**2*b**5*m**3*x
6*xm/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 353430*a**2*b**5*m**2*x**6*x**m/(m**8 +
36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10
9584*m + 40320) + 360024*a**2*b**5*m*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 +
 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14
1120*a**2*b**5*x**6*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**
4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 7*a*b**6*m**7*x**7*x**m/
(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m
2 + 109584*m + 40320) + 203*a*b6*m**6*x**7*x**m/(m**8 + 36*m**7 + 546*m
6 + 4536*m5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320)
 + 2401*a*b**6*m**5*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 2244
9*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 14945*a*b**6*m**4*x
7*xm/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 +
 118124*m**2 + 109584*m + 40320) + 52528*a*b**6*m**3*x**7*x**m/(m**8 + 36*m
7 + 546*m6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584
*m + 40320) + 103292*a*b**6*m**2*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 453

```
6*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 103824
*a*b**6*m*x**7*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 6
7284*m**3 + 118124*m**2 + 109584*m + 40320) + 40320*a*b**6*x**7*x**m/(m**8
+ 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 +
109584*m + 40320) + b**7*m**7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 28*b**7*m
**6*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m
**3 + 118124*m**2 + 109584*m + 40320) + 322*b**7*m**5*x**8*x**m/(m**8 + 36*
m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 10958
4*m + 40320) + 1960*b**7*m**4*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m
**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 6769*b**7
*m**3*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284
*m**3 + 118124*m**2 + 109584*m + 40320) + 13132*b**7*m**2*x**8*x**m/(m**8 +
 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 1
09584*m + 40320) + 13068*b**7*m*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536
*m**5 + 22449*m**4 + 67284*m**3 + 118124*m**2 + 109584*m + 40320) + 5040*b*
*7*x**8*x**m/(m**8 + 36*m**7 + 546*m**6 + 4536*m**5 + 22449*m**4 + 67284*m*
*3 + 118124*m**2 + 109584*m + 40320), True))
```

3.701 $\int x^m (a+bx)^3 \, dx$

Optimal. Leaf size=61

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+2}}{m+2}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{3a^2bx^{m+2}}{m+2}+\frac{a^3x^{m+1}}{m+1}+\frac{3ab^2x^{m+3}}{m+3}+\frac{b^3x^{m+4}}{m+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^3,x]
```

```
[Out] (a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(2 + m))/(2 + m) + (3*a*b^2*x^(3 + m))
/(3 + m) + (b^3*x^(4 + m))/(4 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int x^m (a+bx)^3 \, dx &= \int \left(a^3x^m+3a^2bx^{1+m}+3ab^2x^{2+m}+b^3x^{3+m}\right) dx \\ &= \frac{a^3x^{1+m}}{1+m}+\frac{3a^2bx^{2+m}}{2+m}+\frac{3ab^2x^{3+m}}{3+m}+\frac{b^3x^{4+m}}{4+m} \end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.89

$$x^{m+1}\left(\frac{a^3}{m+1}+\frac{3a^2bx}{m+2}+\frac{3ab^2x^2}{m+3}+\frac{b^3x^3}{m+4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^3,x]
```

```
[Out] x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x)/(2 + m) + (3*a*b^2*x^2)/(3 + m) + (b^3
*x^3)/(4 + m))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m (a+bx)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^3, x]
```

fricas [B] time = 1.03, size = 157, normalized size = 2.57

$$\frac{\left(\left(b^3m^3+6\,b^3m^2+11\,b^3m+6\,b^3\right)x^4+3\left(ab^2m^3+7\,ab^2m^2+14\,ab^2m+8\,ab^2\right)x^3+3\left(a^2bm^3+8\,a^2bm^2+19\,a^2bm+12\,a^2b\right)x^2+\left(a^3m^3+9\,a^3m^2+26\,a^3m+24\,a^3\right)x\right)x^m}{m^4+10\,m^3+35\,m^2+50\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] ((b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)*x^4 + 3*(a*b^2*m^3 + 7*a*b^2*m^2
+ 14*a*b^2*m + 8*a*b^2)*x^3 + 3*(a^2*b*m^3 + 8*a^2*b*m^2 + 19*a^2*b*m + 12*
a^2*b)*x^2 + (a^3*m^3 + 9*a^3*m^2 + 26*a^3*m + 24*a^3)*x)*x^m/(m^4 + 10*m^3
 + 35*m^2 + 50*m + 24)
```

giac [B] time = 1.07, size = 224, normalized size = 3.67

$$\frac{b^3m^3x^4x^m + 3\,ab^2m^3x^3x^m + 6\,b^3m^2x^4x^m + 3\,a^2bm^3x^2x^m + 21\,ab^2m^2x^3x^m + 11\,b^3mx^4x^m + a^3m^3xx^m + 24\,a^2bm^2x^2x^m + 42\,ab^2mx^3x^m + 6\,b^3x^4x^m + 9\,a^3m^2xx^m + 57\,a^2bmx^2x^m + 24\,ab^2x^3x^m + 26\,a^3mxx^m + 36\,a^2bx^2x^m + 24\,a^3xx^m}{m^4 + 10\,m^3 + 35\,m^2 + 50\,m + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="giac")
```

```
[Out] (b^3*m^3*x^4*x^m + 3*a*b^2*m^3*x^3*x^m + 6*b^3*m^2*x^4*x^m + 3*a^2*b*m^3*x^
2*x^m + 21*a*b^2*m^2*x^3*x^m + 11*b^3*m*x^4*x^m + a^3*m^3*x*x^m + 24*a^2*b*
m^2*x^2*x^m + 42*a*b^2*m*x^3*x^m + 6*b^3*x^4*x^m + 9*a^3*m^2*x*x^m + 57*a^2
*b*m*x^2*x^m + 24*a*b^2*x^3*x^m + 26*a^3*m*x*x^m + 36*a^2*b*x^2*x^m + 24*a^
3*x*x^m)/(m^4 + 10*m^3 + 35*m^2 + 50*m + 24)
```

maple [B] time = 0.00, size = 170, normalized size = 2.79

$$\frac{\left(b^3m^3x^3 + 3a\,b^2m^3x^2 + 6b^3m^2x^3 + 3a^2b\,m^3x + 21a\,b^2m^2x^2 + 11b^3m\,x^3 + a^3m^3 + 24a^2b\,m^2x + 42a\,b^2m\,x^2 + 6b^3x^3 + 9a^3m^2 + 57a^2bmx + 24a\,b^2x^2 + 26a^3m + 36a^2bx + 24a^3\right)x^{m+1}}{(m+4)\,(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^3,x)
```

```
[Out] (b^3*m^3*x^3+3*a*b^2*m^3*x^2+6*b^3*m^2*x^3+3*a^2*b*m^3*x+21*a*b^2*m^2*x^2+1
1*b^3*m*x^3+a^3*m^3+24*a^2*b*m^2*x+42*a*b^2*m*x^2+6*b^3*x^3+9*a^3*m^2+57*a^
2*b*m*x+24*a*b^2*x^2+26*a^3*m+36*a^2*b*x+24*a^3)/(m+4)/(m+3)/(m+2)/(m+1)*x^
(m+1)
```

maxima [A] time = 1.30, size = 61, normalized size = 1.00

$$\frac{b^3x^{m+4}}{m+4} + \frac{3\,ab^2x^{m+3}}{m+3} + \frac{3\,a^2bx^{m+2}}{m+2} + \frac{a^3x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^3*x^(m + 4)/(m + 4) + 3*a*b^2*x^(m + 3)/(m + 3) + 3*a^2*b*x^(m + 2)/(m +
2) + a^3*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.44, size = 167, normalized size = 2.74

$$x^m\left(\frac{a^3\,x\,\left(m^3+9\,m^2+26\,m+24\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{b^3\,x^4\,\left(m^3+6\,m^2+11\,m+6\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a\,b^2\,x^3\,\left(m^3+7\,m^2+14\,m+8\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}+\frac{3\,a^2\,b\,x^2\,\left(m^3+8\,m^2+19\,m+12\right)}{m^4+10\,m^3+35\,m^2+50\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^3,x)
```

```
[Out] x^m*((a^3*x*(26*m + 9*m^2 + m^3 + 24))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (b^3*x^4*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24) +
(3*a*b^2*x^3*(14*m + 7*m^2 + m^3 + 8))/(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
+ (3*a^2*b*x^2*(19*m + 8*m^2 + m^3 + 12))/(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4))
```

sympy [A] time = 0.92, size = 663, normalized size = 10.87

$$\begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) & \text{for } m = -4 \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x & \text{for } m = -3 \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} & \text{for } m = -2 \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} & \text{for } m = -1 \\ \frac{a^3m^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{9a^3m^2xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{26a^3mxx^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^3xx^m}{m^4+10m^3+35m^2+50m+24} + \frac{3a^2bm^3x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24a^2bm^2x^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{57a^2bmx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{36a^2bx^2x^m}{m^4+10m^3+35m^2+50m+24} + \frac{3ab^2m^3x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{21ab^2m^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{42ab^2mx^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{24ab^2x^3x^m}{m^4+10m^3+35m^2+50m+24} + \frac{b^3m^3x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3m^2x^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{11b^3mx^4x^m}{m^4+10m^3+35m^2+50m+24} + \frac{6b^3x^4x^m}{m^4+10m^3+35m^2+50m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**3,x)
```

```
[Out] Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x), E
q(m, -4)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(m, -
3)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(m, -2)), (a*
*3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(m, -1)), (a**3*m
**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 9*a**3*m**2*x*x**m/(m**
4 + 10*m**3 + 35*m**2 + 50*m + 24) + 26*a**3*m*x*x**m/(m**4 + 10*m**3 + 35*
m**2 + 50*m + 24) + 24*a**3*x*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
 3*a**2*b*m**3*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 24*a**2*b
*m**2*x**2*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 57*a**2*b*m*x**2*x
**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 36*a**2*b*x**2*x**m/(m**4 + 10
*m**3 + 35*m**2 + 50*m + 24) + 3*a*b**2*m**3*x**3*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 21*a*b**2*m**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 5
0*m + 24) + 42*a*b**2*m*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) +
24*a*b**2*x**3*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + b**3*m**3*x**4
*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24) + 6*b**3*m**2*x**4*x**m/(m**4
+ 10*m**3 + 35*m**2 + 50*m + 24) + 11*b**3*m*x**4*x**m/(m**4 + 10*m**3 + 35
*m**2 + 50*m + 24) + 6*b**3*x**4*x**m/(m**4 + 10*m**3 + 35*m**2 + 50*m + 24
), True))
```

3.702 $\int x^m(a+bx)^2\,dx$

Optimal. Leaf size=43

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+2}}{m+2}+\frac{b^2x^{m+3}}{m+3}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x)^2,x]
```

```
[Out] (a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(2 + m))/(2 + m) + (b^2*x^(3 + m))/(3 +
m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)^2\,dx &= \int \left(a^2x^m+2abx^{1+m}+b^2x^{2+m}\right)dx\\ &= \frac{a^2x^{1+m}}{1+m}+\frac{2abx^{2+m}}{2+m}+\frac{b^2x^{3+m}}{3+m}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.88

$$x^{m+1}\left(\frac{a^2}{m+1}+\frac{2abx}{m+2}+\frac{b^2x^2}{m+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x)^2,x]
```

```
[Out] x^(1 + m)*(a^2/(1 + m) + (2*a*b*x)/(2 + m) + (b^2*x^2)/(3 + m))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x)^2, x]
```

fricas [A] time = 1.17, size = 85, normalized size = 1.98

$$\frac{\left(\left(b^2m^2+3\,b^2m+2\,b^2\right)x^3+2\left(abm^2+4\,abm+3\,ab\right)x^2+\left(a^2m^2+5\,a^2m+6\,a^2\right)x\right)x^m}{m^3+6\,m^2+11\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 3*b^2*m + 2*b^2)*x^3 + 2*(a*b*m^2 + 4*a*b*m + 3*a*b)*x^2 + (a^2
*m^2 + 5*a^2*m + 6*a^2)*x)*x^m/(m^3 + 6*m^2 + 11*m + 6)
```

giac [B] time = 1.04, size = 117, normalized size = 2.72

$$\frac{b^2m^2x^3x^m + 2\,abm^2x^2x^m + 3\,b^2mx^3x^m + a^2m^2xx^m + 8\,abmx^2x^m + 2\,b^2x^3x^m + 5\,a^2mxx^m + 6\,abx^2x^m + 6\,a^2xx^m}{m^3 + 6\,m^2 + 11\,m + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b^2*m^2*x^3*x^m + 2*a*b*m^2*x^2*x^m + 3*b^2*m*x^3*x^m + a^2*m^2*x*x^m + 8*
a*b*m*x^2*x^m + 2*b^2*x^3*x^m + 5*a^2*m*x*x^m + 6*a*b*x^2*x^m + 6*a^2*x*x^m
)/(m^3 + 6*m^2 + 11*m + 6)
```

maple [A] time = 0.00, size = 87, normalized size = 2.02

$$\frac{\left(b^2m^2x^2 + 2ab\,m^2x + 3b^2m\,x^2 + a^2m^2 + 8abmx + 2b^2x^2 + 5a^2m + 6abx + 6a^2\right)x^{m+1}}{(m+3)\,(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a)^2,x)
```

```
[Out] (b^2*m^2*x^2+2*a*b*m^2*x+3*b^2*m*x^2+a^2*m^2+8*a*b*m*x+2*b^2*x^2+5*a^2*m+6*
a*b*x+6*a^2)/(m+3)/(m+2)/(m+1)*x^(m+1)
```

maxima [A] time = 1.32, size = 43, normalized size = 1.00

$$\frac{b^2x^{m+3}}{m+3} + \frac{2\,abx^{m+2}}{m+2} + \frac{a^2x^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*x^(m + 3)/(m + 3) + 2*a*b*x^(m + 2)/(m + 2) + a^2*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.37, size = 93, normalized size = 2.16

$$x^m\,\left(\frac{a^2\,x\,\left(m^2+5\,m+6\right)}{m^3+6\,m^2+11\,m+6}+\frac{b^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{2\,a\,b\,x^2\,\left(m^2+4\,m+3\right)}{m^3+6\,m^2+11\,m+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x)^2,x)
```

```
[Out] x^m*((a^2*x*(5*m + m^2 + 6))/(11*m + 6*m^2 + m^3 + 6) + (b^2*x^3*(3*m + m^2
 + 2))/(11*m + 6*m^2 + m^3 + 6) + (2*a*b*x^2*(4*m + m^2 + 3))/(11*m + 6*m^2
 + m^3 + 6))
```

sympy [A] time = 0.55, size = 299, normalized size = 6.95

$$\begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2\log(x) & \text{for}\ m = -3 \\ -\frac{a^2}{x} + 2ab\log(x) + b^2x & \text{for}\ m = -2 \\ a^2\log(x) + 2abx + \frac{b^2x^2}{2} & \text{for}\ m = -1 \\ \frac{a^2m^2xx^m}{m^3+6m^2+11m+6} + \frac{5a^2mxx^m}{m^3+6m^2+11m+6} + \frac{6a^2xx^m}{m^3+6m^2+11m+6} + \frac{2abm^2x^2x^m}{m^3+6m^2+11m+6} + \frac{8abmx^2x^m}{m^3+6m^2+11m+6} + \frac{6abx^2x^m}{m^3+6m^2+11m+6} + \frac{b^2m^2x^3x^m}{m^3+6m^2+11m+6} + \frac{3b^2mx^3x^m}{m^3+6m^2+11m+6} + \frac{2b^2x^3x^m}{m^3+6m^2+11m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a)**2,x)
```

```
[Out] Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(m, -3)), (-a**2/x + 2
*a*b*log(x) + b**2*x, Eq(m, -2)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2, Eq(
m, -1)), (a**2*m**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 5*a**2*m*x*x**m/(m*
*3 + 6*m**2 + 11*m + 6) + 6*a**2*x*x**m/(m**3 + 6*m**2 + 11*m + 6) + 2*a*b*
m**2*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + 8*a*b*m*x**2*x**m/(m**3 + 6*m**
2 + 11*m + 6) + 6*a*b*x**2*x**m/(m**3 + 6*m**2 + 11*m + 6) + b**2*m**2*x**3
*x**m/(m**3 + 6*m**2 + 11*m + 6) + 3*b**2*m*x**3*x**m/(m**3 + 6*m**2 + 11*m
 + 6) + 2*b**2*x**3*x**m/(m**3 + 6*m**2 + 11*m + 6), True))
```

3.703 $\int x^m(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+2}}{m+2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a + b*x),x]
[Out] (a*x^(1 + m))/(1 + m) + (b*x^(2 + m))/(2 + m)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^m(a+bx)\,dx&=\int\left(ax^m+bx^{1+m}\right)\,dx\\&=\frac{ax^{1+m}}{1+m}+\frac{bx^{2+m}}{2+m}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.88

$$x^{m+1}\left(\frac{a}{m+1}+\frac{bx}{m+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a + b*x),x]
[Out] x^(1 + m)*(a/(1 + m) + (b*x)/(2 + m))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][x^m*(a + b*x), x]
```

fricas [A] time = 0.79, size = 33, normalized size = 1.32

$$\frac{\left((bm+b)x^2+(am+2a)x\right)x^m}{m^2+3m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

giac [A] time = 1.02, size = 43, normalized size = 1.72

$$\frac{bmx^2x^m + amxx^m + bx^2x^m + 2\,axx^m}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="giac")
[Out] (b*m*x^2*x^m + a*m*x*x^m + b*x^2*x^m + 2*a*x*x^m)/(m^2 + 3*m + 2)
```

maple [A] time = 0.00, size = 31, normalized size = 1.24

$$\frac{(bmx + am + bx + 2a)\,x^{m+1}}{(m+2)\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(b*x+a),x)
[Out] (b*m*x+a*m+b*x+2*a)/(m+2)/(m+1)*x^(m+1)
```

maxima [A] time = 1.35, size = 25, normalized size = 1.00

$$\frac{bx^{m+2}}{m+2} + \frac{ax^{m+1}}{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(b*x+a),x, algorithm="maxima")
[Out] b*x^(m + 2)/(m + 2) + a*x^(m + 1)/(m + 1)
```

mupad [B] time = 0.30, size = 30, normalized size = 1.20

$$\frac{x^{m+1}\,\left(2\,a + a\,m + b\,x + b\,m\,x\right)}{m^2 + 3\,m + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a + b*x),x)
[Out] (x^(m + 1)*(2*a + a*m + b*x + b*m*x))/(3*m + m^2 + 2)
```

sympy [A] time = 0.31, size = 87, normalized size = 3.48

$$\begin{cases} -\frac{a}{x} + b\log(x) & \text{for } m = -2 \\ a\log(x) + bx & \text{for } m = -1 \\ \frac{amxx^m}{m^2+3m+2} + \frac{2axx^m}{m^2+3m+2} + \frac{bmx^2x^m}{m^2+3m+2} + \frac{bx^2x^m}{m^2+3m+2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(b*x+a),x)
[Out] Piecewise((-a/x + b*log(x), Eq(m, -2)), (a*log(x) + b*x, Eq(m, -1)), (a*m*x
*x**m/(m**2 + 3*m + 2) + 2*a*x*x**m/(m**2 + 3*m + 2) + b*m*x**2*x**m/(m**2
+ 3*m + 2) + b*x**2*x**m/(m**2 + 3*m + 2), True))
```

3.704 $\int x^3(a+bx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{a^3(a+bx)^{n+1}}{b^4(n+1)}+\frac{3a^2(a+bx)^{n+2}}{b^4(n+2)}-\frac{3a(a+bx)^{n+3}}{b^4(n+3)}+\frac{(a+bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x)^n,x]
```

```
[Out] -((a^3*(a + b*x)^(1 + n))/(b^4*(1 + n))) + (3*a^2*(a + b*x)^(2 + n))/(b^4*(
2 + n)) - (3*a*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (a + b*x)^(4 + n)/(b^4*(4
 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3(a+bx)^n\,dx&=\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx\\&=-\frac{a^3(a+bx)^{1+n}}{b^4(1+n)}+\frac{3a^2(a+bx)^{2+n}}{b^4(2+n)}-\frac{3a(a+bx)^{3+n}}{b^4(3+n)}+\frac{(a+bx)^{4+n}}{b^4(4+n)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.81

$$\frac{(a+bx)^{n+1}\left(-\frac{a^3}{n+1}+\frac{3a^2(a+bx)}{n+2}-\frac{3a(a+bx)^2}{n+3}+\frac{(a+bx)^3}{n+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-(a^3/(1 + n)) + (3*a^2*(a + b*x))/(2 + n) - (3*a*(a +
b*x)^2)/(3 + n) + (a + b*x)^3/(4 + n)))/b^4
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^3(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^3*(a + b*x)^n, x]
```

fricas [A] time = 1.29, size = 143, normalized size = 1.72

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\,\left(a^2b^2n^2+a^2b^2n\right)x^2\right)(bx+a)^n}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*(b*
x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

giac [B] time = 1.21, size = 226, normalized size = 2.72

$$\frac{(bx+a)^nb^4n^3x^4+(bx+a)^nab^3n^3x^3+6\,(bx+a)^nb^4n^2x^4+3\,(bx+a)^nab^3n^2x^3+11\,(bx+a)^nb^4nx^4-3\,(bx+a)^na^2b^2n^2x^2+2\,(bx+a)^nab^3nx^3+6\,(bx+a)^nb^4x^4-3\,(bx+a)^na^2b^2nx^2+6\,(bx+a)^na^3bnx-6\,(bx+a)^na^4}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^4*n^3*x^4 + (b*x + a)^n*a*b^3*n^3*x^3 + 6*(b*x + a)^n*b^4*n^
2*x^4 + 3*(b*x + a)^n*a*b^3*n^2*x^3 + 11*(b*x + a)^n*b^4*n*x^4 - 3*(b*x + a
)^n*a^2*b^2*n^2*x^2 + 2*(b*x + a)^n*a*b^3*n*x^3 + 6*(b*x + a)^n*b^4*x^4 - 3
*(b*x + a)^n*a^2*b^2*n*x^2 + 6*(b*x + a)^n*a^3*b*n*x - 6*(b*x + a)^n*a^4)/(
b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)
```

maple [A] time = 0.01, size = 126, normalized size = 1.52

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9*a
*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/b^4/(n^4+10*n
^3+35*n^2+50*n+24)
```

maxima [A] time = 1.36, size = 101, normalized size = 1.22

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\,\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4)
```

mupad [B] time = 0.53, size = 176, normalized size = 2.12

$$(a+b\,x)^n\,\left(\frac{x^4\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^3\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^2\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x)^n,x)
```

```
[Out] (a + b*x)^n*((x^4*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 +
 24) - (6*a^4)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x)/(b^3
```

```
*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^3*(3*n + n^2 + 2))/(b*(50*n
+ 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^2*(n + 1))/(b^2*(50*n + 35*n^2
+ 10*n^3 + n^4 + 24)))
```

sympy [A] time = 2.33, size = 1318, normalized size = 15.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**4/4, Eq(b, 0)), (6*a**3*log(a/b + x)/(6*a**3*b**4 + 18*a
**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3/(6*a**3*b**4 + 18*a**2
*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*x*log(a/b + x)/(6*a**3*
b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*x/(6*a**3
*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*x**2*log
(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 1
8*a*b**2*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3)
 + 6*b**3*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2
+ 6*b**7*x**3), Eq(n, -4)), (-6*a**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - 9*a**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2
*b*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*x/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*x**2*log(a/b + x)/(2*a**2
*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*x**3/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2), Eq(n, -3)), (6*a**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a
**3/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) -
 3*a*b**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*x**3/(2*a*b**4 + 2*b**5*x), Eq(
n, -2)), (-a**3*log(a/b + x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3
*b), Eq(n, -1)), (-6*a**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*
n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**
4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*n**2*x**2*(a + b
*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*
a**2*b**2*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + a*b**3*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*n**2*x**3*(a + b*x)**n/(
b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*n
*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4) + b**4*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n
**2 + 50*b**4*n + 24*b**4) + 6*b**4*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*n*x**4*(a + b*x)*
*n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4
*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 2
4*b**4), True))
```

3.705 $\int x^2(a+bx)^n\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{n+1}}{b^3(n+1)}-\frac{2a(a+bx)^{n+2}}{b^3(n+2)}+\frac{(a+bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n,x]
```

```
[Out] (a^2*(a + b*x)^(1 + n))/(b^3*(1 + n)) - (2*a*(a + b*x)^(2 + n))/(b^3*(2 + n
)) + (a + b*x)^(3 + n)/(b^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^n\,dx&=\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx\\&=\frac{a^2(a+bx)^{1+n}}{b^3(1+n)}-\frac{2a(a+bx)^{2+n}}{b^3(2+n)}+\frac{(a+bx)^{3+n}}{b^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n,x]
```

```
[Out] ((a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(b^
3*(1 + n)*(2 + n)*(3 + n))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n, x]
```

fricas [A] time = 1.08, size = 96, normalized size = 1.60

$$-\frac{\left(2\,a^{2}bnx-\left(b^{3}n^{2}+3\,b^{3}n+2\,b^{3}\right)x^{3}-2\,a^{3}-\left(ab^{2}n^{2}+ab^{2}n\right)x^{2}\right)(bx+a)^{n}}{b^{3}n^{3}+6\,b^{3}n^{2}+11\,b^{3}n+6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

giac [B] time = 1.07, size = 140, normalized size = 2.33

$$\frac{(bx+a)^{n}b^{3}n^{2}x^{3}+(bx+a)^{n}ab^{2}n^{2}x^{2}+3\,(bx+a)^{n}b^{3}nx^{3}+(bx+a)^{n}ab^{2}nx^{2}+2\,(bx+a)^{n}b^{3}x^{3}-2\,(bx+a)^{n}a^{2}bnx+2\,(bx+a)^{n}a^{3}}{b^{3}n^{3}+6\,b^{3}n^{2}+11\,b^{3}n+6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^3*n^2*x^3 + (b*x + a)^n*a*b^2*n^2*x^2 + 3*(b*x + a)^n*b^3*n*
x^3 + (b*x + a)^n*a*b^2*n*x^2 + 2*(b*x + a)^n*b^3*x^3 - 2*(b*x + a)^n*a^2*b
*n*x + 2*(b*x + a)^n*a^3)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
```

maple [A] time = 0.01, size = 73, normalized size = 1.22

$$\frac{\left(b^{2}n^{2}x^{2}+3b^{2}n\,x^{2}-2abnx+2b^{2}x^{2}-2abx+2a^{2}\right)(bx+a)^{n+1}}{\left(n^{3}+6n^{2}+11n+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.37, size = 68, normalized size = 1.13

$$\frac{\left(\left(n^{2}+3\,n+2\right)b^{3}x^{3}+\left(n^{2}+n\right)ab^{2}x^{2}-2\,a^{2}bnx+2\,a^{3}\right)(bx+a)^{n}}{\left(n^{3}+6\,n^{2}+11\,n+6\right)b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 0.56, size = 192, normalized size = 3.20

$$\begin{cases} \frac{2\,a^2\,\ln(a+b\,x)+b^2\,x^2-2\,a\,b\,x}{2\,b^3} & \text{if } n=-1 \\ \frac{x}{b^2}-\frac{a^2}{b^3\,(a+b\,x)}-\frac{2\,a\,\ln(a+b\,x)}{b^3} & \text{if } n=-2 \\ \frac{\ln(a+b\,x)+\frac{2\,a}{a+b\,x}-\frac{a^2}{2\,(a+b\,x)^2}}{b^3} & \text{if } n=-3 \\ \frac{2\,(a+b\,x)^{n+1}\left(8\,a^2-8\,a\,b\,n\,x-8\,a\,b\,x+4\,b^2\,n^2\,x^2+12\,b^2\,n\,x^2+8\,b^2\,x^2\right)}{b^3\left(8\,n^3+48\,n^2+88\,n+48\right)} & \text{if } n\neq -1\land n\neq -2\land n\neq -3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3), n == -
2, x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*log(a + b*x))/b^3, n == -3, (log(a +
b*x) + (2*a)/(a + b*x) - a^2/(2*(a + b*x)^2))/b^3, n ~= -1 & n ~= -2 & n ~=
 -3, (2*(a + b*x)^(n + 1)*(8*a^2 + 8*b^2*x^2 + 12*b^2*n*x^2 - 8*a*b*x + 4*b
^2*n^2*x^2 - 8*a*b*n*x))/(b^3*(88*n + 48*n^2 + 8*n^3 + 48)))
```

sympy [**A**] time = 1.32, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^n x^3}{3} & \text{for}\: b = 0 \\ \frac{2a^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{3a^2}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{4abx}{2a^2b^3+4ab^4x+2b^5x^2} + \frac{2b^2x^2 \log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2} & \text{for}\: n = -3 \\ -\frac{2a^2 \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} - \frac{2a^2}{ab^3+b^4x} - \frac{2abx \log\left(\frac{a}{b}+x\right)}{ab^3+b^4x} + \frac{b^2x^2}{ab^3+b^4x} & \text{for}\: n = -2 \\ \frac{a^2 \log\left(\frac{a}{b}+x\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b} & \text{for}\: n = -1 \\ \frac{2a^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} - \frac{2a^2bnx(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2n^2x^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{ab^2nx^2(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{b^3n^2x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{3b^3nx^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} + \frac{2b^3x^3(a+bx)^n}{b^3n^3+6b^3n^2+11b^3n+6b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + 3*a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4
*a*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*x/(2*a
**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*log(a/b + x)/(2*a**2*b**
3 + 4*a*b**4*x + 2*b**5*x**2), Eq(n, -3)), (-2*a**2*log(a/b + x)/(a*b**3 +
b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*x)
 + b**2*x**2/(a*b**3 + b**4*x), Eq(n, -2)), (a**2*log(a/b + x)/b**3 - a*x/b
**2 + x**2/(2*b), Eq(n, -1)), (2*a**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) - 2*a**2*b*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2
 + 11*b**3*n + 6*b**3) + a*b**2*n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*
n**2 + 11*b**3*n + 6*b**3) + a*b**2*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3
*n**2 + 11*b**3*n + 6*b**3) + b**3*n**2*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3) + 3*b**3*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b
**3*n**2 + 11*b**3*n + 6*b**3) + 2*b**3*x**3*(a + b*x)**n/(b**3*n**3 + 6*b*
*3*n**2 + 11*b**3*n + 6*b**3), True))
```

3.706 $\int x(a+bx)^n\,dx$

Optimal. Leaf size=39

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{n+2}}{b^2(n+2)}-\frac{a(a+bx)^{n+1}}{b^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n,x]
[Out] -((a*(a + b*x)^(1 + n))/(b^2*(1 + n))) + (a + b*x)^(2 + n)/(b^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^n\,dx&=\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx\\&=-\frac{a(a+bx)^{1+n}}{b^2(1+n)}+\frac{(a+bx)^{2+n}}{b^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n,x]
[Out] ((a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n, x]
```

fricas [A] time = 1.14, size = 53, normalized size = 1.36

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)(bx+a)^n}{b^2n^2+3\,b^2n+2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*(b*x + a)^n/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

giac [A] time = 1.05, size = 76, normalized size = 1.95

$$\frac{(bx+a)^n b^2 nx^2 + (bx+a)^n abnx + (bx+a)^n b^2 x^2 - (bx+a)^n a^2}{b^2 n^2 + 3\,b^2 n + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^2*n*x^2 + (b*x + a)^n*a*b*n*x + (b*x + a)^n*b^2*x^2 - (b*x + a)^n*a^2)/(b^2*n^2 + 3*b^2*n + 2*b^2)
```

maple [A] time = 0.00, size = 36, normalized size = 0.92

$$-\frac{(-xnb - bx + a)\,(bx+a)^{n+1}}{\left(n^2+3n+2\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(-b*n*x-b*x+a)/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.31, size = 42, normalized size = 1.08

$$\frac{\left(b^2(n+1)x^2 + abnx - a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2)
```

mupad [B] time = 0.38, size = 94, normalized size = 2.41

$$\begin{cases} -\frac{a\,\ln(a+b\,x)-b\,x}{b^2} & \text{if } n=-1 \\ \frac{\ln(a+b\,x)+\frac{a}{a+b\,x}}{b^2} & \text{if } n=-2 \\ \frac{2\left(\frac{(a+b\,x)^{n+2}}{2\,n+4}-\frac{a\,(a+b\,x)^{n+1}}{2\,n+2}\right)}{b^2} & \text{if } n\neq -1 \wedge n\neq -2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x)^n,x)
```

```
[Out] piecewise(n == -1, -(a*log(a + b*x) - b*x)/b^2, n == -2, (log(a + b*x) + a/(a + b*x))/b^2, n ~= -1 & n ~= -2, (2*((a + b*x)^(n + 2)/(2*n + 4) - (a*(a + b*x)^(n + 1))/(2*n + 2)))/b^2)
```

sympy [A] time = 0.70, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^n x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } n = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } n = -1 \\ -\frac{a^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{abnx(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2nx^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} + \frac{b^2x^2(a+bx)^n}{b^2n^2+3b^2n+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n,x)
```

```
[Out] Piecewise((a**n*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n +
 2*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*n*x*
*2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x**2*(a + b*x)**n/(b
**2*n**2 + 3*b**2*n + 2*b**2), True))
```

3.707 $\int (a+bx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^n \, dx = \frac{(a+bx)^{1+n}}{b(1+n)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{n+1}}{bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n,x]
```

```
[Out] (a + b*x)^(1 + n)/(b + b*n)
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (a+bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n, x]
```

fricas [A] time = 1.23, size = 20, normalized size = 1.11

$$\frac{(bx+a)(bx+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^n/(b*n + b)
```

giac [A] time = 1.14, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="giac")
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{(bx+a)^{n+1}}{(n+1)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n,x)
[Out] (b*x+a)^(n+1)/b/(n+1)
```

maxima [A] time = 1.30, size = 18, normalized size = 1.00

$$\frac{(bx+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n,x, algorithm="maxima")
[Out] (b*x + a)^(n + 1)/(b*(n + 1))
```

mupad [B] time = 0.20, size = 18, normalized size = 1.00

$$\frac{(a+b\,x)^{n+1}}{b\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n,x)
[Out] (a + b*x)^(n + 1)/(b*(n + 1))
```

sympy [A] time = 0.07, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n,x)
[Out] Piecewise(((a + b*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(a + b*x), True))/b
```

3.708 $\int x^{-4+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=110

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Rubi [A] time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{2b^2x^{n-1}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}+\frac{2bx^{n-2}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{x^{n-3}(a+bx)^{1-n}}{a(3-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-3 + n)*(a + b*x)^(1 - n))/(a*(3 - n))) + (2*b*x^(-2 + n)*(a + b*x)^(
1 - n))/(a^2*(2 - n)*(3 - n)) - (2*b^2*x^(-1 + n)*(a + b*x)^(1 - n))/(a^3*(
1 - n)*(2 - n)*(3 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-4+n}(a+bx)^{-n}\,dx&=-\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}-\frac{(2b)\int x^{-3+n}(a+bx)^{-n}\,dx}{a(3-n)}\\&=-\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}+\frac{\left(2b^2\right)\int x^{-2+n}(a+bx)^{-n}\,dx}{a^2(2-n)(3-n)}\\&=-\frac{x^{-3+n}(a+bx)^{1-n}}{a(3-n)}+\frac{2bx^{-2+n}(a+bx)^{1-n}}{a^2(2-n)(3-n)}-\frac{2b^2x^{-1+n}(a+bx)^{1-n}}{a^3(1-n)(2-n)(3-n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 64, normalized size = 0.58

$$\frac{x^{n-3}(a+bx)^{1-n}\left(a^2\left(n^2-3n+2\right)+2ab(n-1)x+2b^2x^2\right)}{a^3(n-3)(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-3 + n)*(a + b*x)^(1 - n)*(a^2*(2 - 3*n + n^2) + 2*a*b*(-1 + n)*x + 2*b
^2*x^2))/(a^3*(-3 + n)*(-2 + n)*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-4+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-4 + n)/(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-4 + n)/(a + b*x)^n, x]
```

fricas [A] time = 0.99, size = 104, normalized size = 0.95

$$\frac{\left(2\,ab^2nx^3+2\,b^3x^4+\left(a^2bn^2-a^2bn\right)x^2+\left(a^3n^2-3\,a^3n+2\,a^3\right)x\right)x^{n-4}}{\left(a^3n^3-6\,a^3n^2+11\,a^3n-6\,a^3\right)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 + 2*b^3*x^4 + (a^2*b*n^2 - a^2*b*n)*x^2 + (a^3*n^2 - 3*a^3*n
 + 2*a^3)*x)*x^(n - 4)/((a^3*n^3 - 6*a^3*n^2 + 11*a^3*n - 6*a^3)*(b*x + a)^
n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

maple [A] time = 0.01, size = 77, normalized size = 0.70

$$\frac{(bx+a)\left(a^2n^2+2abnx+2b^2x^2-3a^2n-2abx+2a^2\right)x^{n-3}\,(bx+a)^{-n}}{(n-3)\,(n-2)\,(n-1)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4+n)/((b*x+a)^n),x)
```

```
[Out] x^(-3+n)*(b*x+a)*(a^2*n^2+2*a*b*n*x+2*b^2*x^2-3*a^2*n-2*a*b*x+2*a^2)/((b*x+
a)^n)/(-3+n)/(-2+n)/(-1+n)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-4}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 4)/(b*x + a)^n, x)
```

mupad [B] time = 0.52, size = 136, normalized size = 1.24

$$\frac{\frac{x\,x^{n-4}\,\left(n^2-3\,n+2\right)}{n^3-6\,n^2+11\,n-6}+\frac{2\,b^3\,x^{n-4}\,x^4}{a^3\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{2\,b^2\,n\,x^{n-4}\,x^3}{a^2\,\left(n^3-6\,n^2+11\,n-6\right)}+\frac{b\,n\,x^{n-4}\,x^2\,\left(n-1\right)}{a\,\left(n^3-6\,n^2+11\,n-6\right)}}{{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 4)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 4)*(n^2 - 3*n + 2))/(11*n - 6*n^2 + n^3 - 6) + (2*b^3*x^(n - 4)*
x^4)/(a^3*(11*n - 6*n^2 + n^3 - 6)) + (2*b^2*n*x^(n - 4)*x^3)/(a^2*(11*n -
6*n^2 + n^3 - 6)) + (b*n*x^(n - 4)*x^2*(n - 1))/(a*(11*n - 6*n^2 + n^3 - 6)
))/(a + b*x)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

3.709 $\int x^{-3+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=64

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{n-1}(a+bx)^{1-n}}{a^2(1-n)(2-n)}-\frac{x^{n-2}(a+bx)^{1-n}}{a(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] -((x^(-2 + n)*(a + b*x)^(1 - n))/(a*(2 - n))) + (b*x^(-1 + n)*(a + b*x)^(1
- n))/(a^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3+n}(a+bx)^{-n}\,dx&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}-\frac{b\int x^{-2+n}(a+bx)^{-n}\,dx}{a(2-n)}\\&=-\frac{x^{-2+n}(a+bx)^{1-n}}{a(2-n)}+\frac{bx^{-1+n}(a+bx)^{1-n}}{a^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.61

$$\frac{x^{n-2}(a+bx)^{1-n}(a(n-1)+bx)}{a^2(n-2)(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] (x^(-2 + n)*(a + b*x)^(1 - n)*(a*(-1 + n) + b*x))/(a^2*(-2 + n)*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-3+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 + n)/(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-3 + n)/(a + b*x)^n, x]
```

fricas [A] time = 1.23, size = 64, normalized size = 1.00

$$\frac{\left(abnx^2+b^2x^3+\left(a^2n-a^2\right)x\right)x^{n-3}}{\left(a^2n^2-3\,a^2n+2\,a^2\right)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="fricas")
```

```
[Out] (a*b*n*x^2 + b^2*x^3 + (a^2*n - a^2)*x)*x^(n - 3)/((a^2*n^2 - 3*a^2*n + 2*a
^2)*(b*x + a)^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-3}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.69

$$\frac{(an+bx-a)\,(bx+a)\,x^{n-2}\,(bx+a)^{-n}}{(n-2)\,(n-1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-3)/((b*x+a)^n),x)
```

```
[Out] x^(n-2)*(a*n+b*x-a)*(b*x+a)/((b*x+a)^n)/(n-2)/(n-1)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-3}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+n)/((b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate(x^(n - 3)/(b*x + a)^n, x)
```

mupad [B] time = 0.45, size = 80, normalized size = 1.25

$$\frac{\frac{x\,x^{n-3}\,(n-1)}{n^2-3\,n+2}+\frac{b^2\,x^{n-3}\,x^3}{a^2\,\left(n^2-3\,n+2\right)}+\frac{b\,n\,x^{n-3}\,x^2}{a\,\left(n^2-3\,n+2\right)}}{(a+b\,x)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 3)/(a + b*x)^n,x)
```

```
[Out] ((x*x^(n - 3)*(n - 1))/(n^2 - 3*n + 2) + (b^2*x^(n - 3)*x^3)/(a^2*(n^2 - 3*
n + 2)) + (b*n*x^(n - 3)*x^2)/(a*(n^2 - 3*n + 2)))/(a + b*x)^n
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+n)/((b*x+a)**n),x)
```

```
[Out] Timed out
```

3.710 $\int x^{-2+n}(a+bx)^{-n}\,dx$

Optimal. Leaf size=28

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{x^{n-1}(a+bx)^{1-n}}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + n)/(a + b*x)^n,x]
[Out] -((x^(-1 + n)*(a + b*x)^(1 - n))/(a*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-2+n}(a+bx)^{-n}\,dx=-\frac{x^{-1+n}(a+bx)^{1-n}}{a(1-n)}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.89

$$\frac{x^{n-1}(a+bx)^{1-n}}{a(n-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + n)/(a + b*x)^n,x]
[Out] (x^(-1 + n)*(a + b*x)^(1 - n))/(a*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int x^{-2+n}(a+bx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-2 + n)/(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-2 + n)/(a + b*x)^n, x]
```

fricas [A] time = 0.97, size = 33, normalized size = 1.18

$$\frac{\left(bx^2+ax\right)x^{n-2}}{(an-a)(bx+a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*x^(n - 2)/((a*n - a)*(b*x + a)^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="giac")
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

maple [A] time = 0.00, size = 29, normalized size = 1.04

$$\frac{(bx+a)\,x^{n-1}\,(bx+a)^{-n}}{(n-1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-2)/((b*x+a)^n),x)
[Out] (b*x+a)*x^(n-1)/a/(n-1)/((b*x+a)^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{n-2}}{(bx+a)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+n)/((b*x+a)^n),x, algorithm="maxima")
[Out] integrate(x^(n - 2)/(b*x + a)^n, x)
```

mupad [B] time = 0.35, size = 29, normalized size = 1.04

$$\frac{x^n\,\left(a+b\,x\right)}{a\,x\,\left(n-1\right)\,{\left(a+b\,x\right)}^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 2)/(a + b*x)^n,x)
[Out] (x^n*(a + b*x))/(a*x*(n - 1)*(a + b*x)^n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+n)/((b*x+a)**n),x)
[Out] Timed out
```

3.711 $\int x^{-1+n}(a+bx)^{-1-n}\,dx$

Optimal. Leaf size=19

$$\frac{x^n(a+bx)^{-n}}{an}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx=\frac{x^n(a+bx)^{-n}}{an}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{x^n(a+bx)^{-n}}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] x^n/(a*n*(a + b*x)^n)
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{-1+n}(a+bx)^{-1-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1 + n)*(a + b*x)^(-1 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(-1 + n)*(a + b*x)^(-1 - n), x]
```

fricas [A] time = 0.95, size = 32, normalized size = 1.68

$$\frac{\left(bx^2+ax\right)(bx+a)^{-n-1}x^{n-1}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^(-n - 1)*x^(n - 1)/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-1}x^{n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="giac")
[Out] integrate((b*x + a)^(-n - 1)*x^(n - 1), x)
```

maple [A] time = 0.00, size = 20, normalized size = 1.05

$$\frac{x^{n}\left(bx+a\right)^{-n}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n-1)*(b*x+a)^(-1-n),x)
[Out] (b*x+a)^(-n)*x^n/a/n
```

maxima [A] time = 1.30, size = 22, normalized size = 1.16

$$\frac{e^{\left(-n\log(bx+a)+n\log(x)\right)}}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b*x+a)^(-1-n),x, algorithm="maxima")
[Out] e^(-n*log(b*x + a) + n*log(x))/(a*n)
```

mupad [B] time = 0.50, size = 19, normalized size = 1.00

$$\frac{x^{n}}{a\,n\,{\left(a+b\,x\right)}^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)/(a + b*x)^(n + 1),x)
[Out] x^n/(a*n*(a + b*x)^n)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b*x+a)**(-1-n),x)
[Out] Timed out
```

3.712 $\int x^{-3-n}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{-3-n}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - n)*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int x^{-3-n}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3 - n)*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][x^(-3 - n)*(a + b*x)^n, x]
```

fricas [A] time = 1.15, size = 64, normalized size = 1.10

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)(bx+a)^n x^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

maple [A] time = 0.00, size = 41, normalized size = 0.71

$$-\frac{(an-bx+a)\,x^{-n-2}\,(bx+a)^{n+1}}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
[Out] -(b*x+a)^(n+1)*x^(-2-n)*(a*n-b*x+a)/(n+2)/(n+1)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

mupad [B] time = 0.50, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

sympy [A] time = 94.82, size = 323, normalized size = 5.57

$$\begin{cases} -\frac{b^n}{2x^2} & \text{for } a = 0 \\ \frac{a\log(x)}{a^3+a^2bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} + \frac{a}{a^3+a^2bx} + \frac{bx\log(x)}{a^3+a^2bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^3+a^2bx} & \text{for } n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^2} & \text{for } n = -1 \\ -\frac{a^2n(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{a^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} - \frac{abnx(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} + \frac{b^2x^2(a+bx)^n}{a^2n^2x^2x^n+3a^2nx^2x^n+2a^2x^2x^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-b**n/(2*x**2), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x) - a*log(a
/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3 + a**2*b
*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) - b*log(x)/
a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**2*n**2*x*
*2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)**n/(a**2*
n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*(a + b*x)
**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) + b**2*x*
*2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**
n), True))
```

3.713 $\int x^{2n-3(1+n)}(a+bx)^n\,dx$

Optimal. Leaf size=58

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{bx^{-n-1}(a+bx)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-n-2}(a+bx)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + b*x)^(1 + n))/(a*(2 + n))) + (b*x^(-1 - n)*(a + b*x)^(1
+ n))/(a^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int x^{2n-3(1+n)}(a+bx)^n\,dx&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-n}(a+bx)^n\,dx}{a(2+n)}\\&=-\frac{x^{-2-n}(a+bx)^{1+n}}{a(2+n)}+\frac{bx^{-1-n}(a+bx)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 0.69

$$-\frac{x^{-n-2}(an+a-bx)(a+bx)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] -((x^(-2 - n)*(a + a*n - b*x)*(a + b*x)^(1 + n))/(a^2*(1 + n)*(2 + n)))
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x^{2n-3(1+n)}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(2*n - 3*(1 + n))*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^(2*n - 3*(1 + n))*(a + b*x)^n, x]
```

fricas [A] time = 1.02, size = 64, normalized size = 1.10

$$-\frac{\left(abnx^2-b^2x^3+\left(a^2n+a^2\right)x\right)(bx+a)^n x^{-n-3}}{a^2n^2+3\,a^2n+2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x + a)^n*x^(-n - 3)/(a^2*n^2 +
3*a^2*n + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

maple [A] time = 0.00, size = 41, normalized size = 0.71

$$-\frac{(an-bx+a)\,x^{-n-2}\,(bx+a)^{n+1}}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-n)*(b*x+a)^n,x)
```

```
[Out] -(a*n-b*x+a)/(n+2)/(n+1)/a^2*x^(-n-2)*(b*x+a)^(n+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n x^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-n)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*x^(-n - 3), x)
```

mupad [B] time = 0.00, size = 86, normalized size = 1.48

$$-(a+b\,x)^n\left(\frac{x\,(n+1)}{x^{n+3}\,\left(n^2+3\,n+2\right)}-\frac{b^2\,x^3}{a^2\,x^{n+3}\,\left(n^2+3\,n+2\right)}+\frac{b\,n\,x^2}{a\,x^{n+3}\,\left(n^2+3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/x^(n + 3),x)
```

```
[Out] -(a + b*x)^n*((x*(n + 1))/(x^(n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(a^2*x^(n
 + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(n + 3)*(3*n + n^2 + 2)))
```

sympy [A] time = 94.49, size = 323, normalized size = 5.57

$$\begin{cases} -\frac{b^{n}}{2x^{2}} & \text{for } a = 0 \\ \frac{a\log(x)}{a^{3}+a^{2}bx} - \frac{a\log\left(\frac{a}{b}+x\right)}{a^{3}+a^{2}bx} + \frac{a}{a^{3}+a^{2}bx} + \frac{bx\log(x)}{a^{3}+a^{2}bx} - \frac{bx\log\left(\frac{a}{b}+x\right)}{a^{3}+a^{2}bx} & \text{for } n = -2 \\ -\frac{1}{ax} - \frac{b\log(x)}{a^{2}} + \frac{b\log\left(\frac{a}{b}+x\right)}{a^{2}} & \text{for } n = -1 \\ -\frac{a^{2}n(a+bx)^{n}}{a^{2}n^{2}x^{2}x^{n}+3a^{2}nx^{2}x^{n}+2a^{2}x^{2}x^{n}} - \frac{a^{2}(a+bx)^{n}}{a^{2}n^{2}x^{2}x^{n}+3a^{2}nx^{2}x^{n}+2a^{2}x^{2}x^{n}} - \frac{abnx(a+bx)^{n}}{a^{2}n^{2}x^{2}x^{n}+3a^{2}nx^{2}x^{n}+2a^{2}x^{2}x^{n}} + \frac{b^{2}x^{2}(a+bx)^{n}}{a^{2}n^{2}x^{2}x^{n}+3a^{2}nx^{2}x^{n}+2a^{2}x^{2}x^{n}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-n)*(b*x+a)**n,x)
```

```
[Out] Piecewise((-b**n/(2*x**2), Eq(a, 0)), (a*log(x)/(a**3 + a**2*b*x) - a*log(a
/b + x)/(a**3 + a**2*b*x) + a/(a**3 + a**2*b*x) + b*x*log(x)/(a**3 + a**2*b
*x) - b*x*log(a/b + x)/(a**3 + a**2*b*x), Eq(n, -2)), (-1/(a*x) - b*log(x)/
a**2 + b*log(a/b + x)/a**2, Eq(n, -1)), (-a**2*n*(a + b*x)**n/(a**2*n**2*x*
*2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a**2*(a + b*x)**n/(a**2*
n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) - a*b*n*x*(a + b*x)
**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**n) + b**2*x*
*2*(a + b*x)**n/(a**2*n**2*x**2*x**n + 3*a**2*n*x**2*x**n + 2*a**2*x**2*x**
n), True))
```

3.714 $\int x^3\sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*x^4*Sqrt[c*x^2])/5 + (b*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{cx^2}\,(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^4(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}ax^4\sqrt{cx^2}+\frac{1}{6}bx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{30}x^4\sqrt{cx^2}\,(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^4*Sqrt[c*x^2]*(6*a + 5*b*x))/30
```

fricas [A] time = 0.72, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^5 + 6\,ax^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(5*b*x^5 + 6*a*x^4)*sqrt(c*x^2)
```

giac [A] time = 0.89, size = 22, normalized size = 0.63

$$\frac{1}{30}\left(5\,bx^6\mathrm{sgn}(x) + 6\,ax^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*sqrt(c)
```

maple [A] time = 0.01, size = 21, normalized size = 0.60

$$\frac{(5bx + 6a)\sqrt{c\,x^2}\;x^4}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/30*x^4*(5*b*x+6*a)*(c*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 33, normalized size = 0.94

$$\frac{\left(cx^2\right)^{\frac{3}{2}}bx^3}{6\,c} + \frac{\left(cx^2\right)^{\frac{3}{2}}ax^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(3/2)*b*x^3/c + 1/5*(c*x^2)^(3/2)*a*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\sqrt{c\,x^2}\;(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x),x)
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x), x)
```

sympy [A] time = 0.43, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^4\sqrt{x^2}}{5} + \frac{b\sqrt{c}\,x^5\sqrt{x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)*(c*x**2)**(1/2),x)
[Out] a*sqrt(c)*x**4*sqrt(x**2)/5 + b*sqrt(c)*x**5*sqrt(x**2)/6
```

3.715 $\int x^2\sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (a*x^3*Sqrt[c*x^2])/4 + (b*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}\,(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^3(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^3+bx^4\right)dx}{x}\\&=\frac{1}{4}ax^3\sqrt{cx^2}+\frac{1}{5}bx^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{20}x^3\sqrt{cx^2}\,(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^3*Sqrt[c*x^2]*(5*a + 4*b*x))/20
```

fricas [A] time = 0.97, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^4+5\,ax^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/20*(4*b*x^4 + 5*a*x^3)*sqrt(c*x^2)
```

giac [A] time = 1.03, size = 22, normalized size = 0.63

$$\frac{1}{20}\left(4\,bx^5\mathrm{sgn}(x)+5\,ax^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(4bx+5a)\sqrt{c\,x^2}\;x^3}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/20*x^3*(4*b*x+5*a)*(c*x^2)^(1/2)
```

maxima [A] time = 1.37, size = 31, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{3}{2}}bx^2}{5\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}ax}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(3/2)*b*x^2/c + 1/4*(c*x^2)^(3/2)*a*x/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\,\sqrt{c\,x^2}\;\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x), x)
```

sympy [A] time = 0.34, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^3\sqrt{x^2}}{4}+\frac{b\sqrt{c}\,x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)*(c*x**2)**(1/2),x)
[Out] a*sqrt(c)*x**3*sqrt(x**2)/4 + b*sqrt(c)*x**4*sqrt(x**2)/5
```

3.716 $\int x\sqrt{cx^2}(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (a*x^2*Sqrt[c*x^2])/3 + (b*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(ax^2+bx^3\right)dx}{x}\\&=\frac{1}{3}ax^2\sqrt{cx^2}+\frac{1}{4}bx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{1}{12}x^2\sqrt{cx^2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

fricas [A] time = 0.97, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^{3}+4\,ax^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)
```

giac [A] time = 0.85, size = 22, normalized size = 0.63

$$\frac{1}{12}\left(3\,bx^{4}\mathrm{sgn}(x)+4\,ax^{3}\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(3bx+4a)\,\sqrt{c\,x^{2}}\;x^{2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/12*x^2*(3*b*x+4*a)*(c*x^2)^(1/2)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.80

$$\frac{\left(cx^{2}\right)^{\frac{3}{2}}bx}{4\,c}+\frac{\left(cx^{2}\right)^{\frac{3}{2}}a}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*b*x/c + 1/3*(c*x^2)^(3/2)*a/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\,\sqrt{c\,x^{2}}\;(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x),x)
[Out] int(x*(c*x^2)^(1/2)*(a + b*x), x)
```

sympy [A] time = 0.27, size = 36, normalized size = 1.03

$$\frac{a\sqrt{c}\,x^{2}\sqrt{x^{2}}}{3}+\frac{b\sqrt{c}\,x^{3}\sqrt{x^{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)*(c*x**2)**(1/2),x)
[Out] a*sqrt(c)*x**2*sqrt(x**2)/3 + b*sqrt(c)*x**3*sqrt(x**2)/4
```

3.717 $\int \sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=33

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x),x]
[Out] (a*x*Sqrt[c*x^2])/2 + (b*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(ax+bx^2\right)\,dx}{x}\\ &= \frac{1}{2}ax\sqrt{cx^2}+\frac{1}{3}bx^2\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.67

$$\frac{1}{6}x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

fricas [A] time = 0.95, size = 20, normalized size = 0.61

$$\frac{1}{6}\left(2\,bx^2+3\,ax\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)
```

giac [A] time = 1.05, size = 22, normalized size = 0.67

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x)+3\,ax^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*sqrt(c)
```

maple [A] time = 0.01, size = 19, normalized size = 0.58

$$\frac{(2bx+3a)\sqrt{c\,x^2}\;x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2),x)
[Out] 1/6*x*(2*b*x+3*a)*(c*x^2)^(1/2)
```

maxima [A] time = 1.32, size = 25, normalized size = 0.76

$$\frac{1}{2}\sqrt{cx^2}\,ax+\frac{\left(cx^2\right)^{\frac{3}{2}}b}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(c*x^2)*a*x + 1/3*(c*x^2)^(3/2)*b/c
```

mupad [B] time = 0.54, size = 20, normalized size = 0.61

$$\frac{\sqrt{c}\,\left(2\,b\,\sqrt{x^6}+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x),x)
[Out] (c^(1/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

sympy [A] time = 0.22, size = 34, normalized size = 1.03

$$\frac{a\sqrt{c}\,x\sqrt{x^2}}{2}+\frac{b\sqrt{c}\,x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2),x)
[Out] a*sqrt(c)*x*sqrt(x**2)/2 + b*sqrt(c)*x**2*sqrt(x**2)/3
```

3.718 $\int \frac{\sqrt{cx^2}(a+bx)}{x} dx$

Optimal. Leaf size=27

$$a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] a*Sqrt[c*x^2] + (b*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x} dx &= \frac{\sqrt{cx^2}\int (a+bx)\,dx}{x}\\ &= a\sqrt{cx^2} + \frac{1}{2}bx\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{cx^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.74

$$\frac{1}{2}\sqrt{cx^2}(2a+bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x,x]
```

```
[Out] (Sqrt[c*x^2]*(2*a + b*x))/2
```

fricas [A] time = 0.92, size = 16, normalized size = 0.59

$$\frac{1}{2}\sqrt{cx^2}(bx+2a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)
```

giac [A] time = 1.09, size = 17, normalized size = 0.63

$$\frac{1}{2}\left(bx^2 + 2\,ax\right)\sqrt{c}\,\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*sqrt(c)*sgn(x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.63

$$\frac{(bx + 2a)\sqrt{c\,x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/2*(b*x+2*a)*(c*x^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.19, size = 14, normalized size = 0.52

$$\frac{\sqrt{c}\,|x|\,(2\,a + b\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x,x)
```

```
[Out] (c^(1/2)*abs(x)*(2*a + b*x))/2
```

sympy [A] time = 0.23, size = 29, normalized size = 1.07

$$a\sqrt{c}\,\sqrt{x^2} + \frac{b\sqrt{c}\,x\sqrt{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x,x)
```

```
[Out] a*sqrt(c)*sqrt(x**2) + b*sqrt(c)*x*sqrt(x**2)/2
```

3.719 $\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=28

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}\log(x)}{x}+b\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] b*Sqrt[c*x^2] + (a*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b+\frac{a}{x}\right)dx}{x}\\ &= b\sqrt{cx^2}+\frac{a\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.71

$$\frac{cx(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 0.68

$$\sqrt{cx^2}\left(\frac{a\log(x)}{x}+b\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^2,x]
[Out] Sqrt[c*x^2]*(b + (a*Log[x])/x)
```

fricas [A] time = 1.27, size = 19, normalized size = 0.68

$$\frac{\sqrt{cx^2}\left(bx + a\log(x)\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a*log(x))/x
```

giac [A] time = 0.92, size = 17, normalized size = 0.61

$$\left(bx\mathrm{sgn}(x) + a\log\left(|x|\right)\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*sqrt(c)
```

maple [A] time = 0.02, size = 20, normalized size = 0.71

$$\frac{\sqrt{c\,x^2}\;\left(a\ln(x) + bx\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^2,x)
[Out] (c*x^2)^(1/2)/x*(a*ln(x)+b*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\;\left(a + b\,x\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^2,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,\left(a + bx\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**2, x)
```

3.720 $\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx$

Optimal. Leaf size=32

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{b\sqrt{cx^2}\log(x)}{x} - \frac{a\sqrt{cx^2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
[Out] -((a*Sqrt[c*x^2])/x^2) + (b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)}{x^3} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^2} dx}{x} \\
&= \frac{\sqrt{cx^2}\int \left(\frac{a}{x^2}+\frac{b}{x}\right) dx}{x} \\
&= -\frac{a\sqrt{cx^2}}{x^2} + \frac{b\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.62

$$\frac{c(bx\log(x)-a)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
[Out] (c*(-a + b*x*Log[x]))/Sqrt[c*x^2]
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.75

$$\sqrt{cx^2}\left(\frac{b\log(x)}{x} - \frac{a}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^3,x]
```

```
[Out] Sqrt[c*x^2]*(-(a/x^2) + (b*Log[x])/x)
```

fricas [A] time = 1.23, size = 20, normalized size = 0.62

$$\frac{\sqrt{cx^2}\left(bx\log(x) - a\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/x^2
```

giac [A] time = 1.10, size = 20, normalized size = 0.62

$$\left(b\log\left(|x|\right)\operatorname{sgn}(x) - \frac{a\operatorname{sgn}(x)}{x}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b*log(abs(x))*sgn(x) - a*sgn(x)/x)*sqrt(c)
```

maple [A] time = 0.01, size = 21, normalized size = 0.66

$$\frac{\sqrt{c\,x^2}\;\left(bx\ln(x) - a\right)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^3,x)
```

```
[Out] (c*x^2)^(1/2)*(b*ln(x)*x-a)/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}\;\left(a + b\,x\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^3,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,\left(a + bx\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)/x**3, x)
```

3.721 $\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx$

Optimal. Leaf size=26

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -(Sqrt[c*x^2]*(a + b*x)^2)/(2*a*x^3)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)}{x^4} dx &= \frac{\sqrt{cx^2}\int \frac{a+bx}{x^3} dx}{x}\\ &= -\frac{\sqrt{cx^2}(a+bx)^2}{2ax^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.85

$$-\frac{\sqrt{cx^2}(a+2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] -1/2*(Sqrt[c*x^2]*(a + 2*b*x))/x^3
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.92

$$\frac{\sqrt{cx^2}(-a-2bx)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x))/x^4,x]
```

```
[Out] (Sqrt[c*x^2]*(-a - 2*b*x))/(2*x^3)
```

fricas [A] time = 1.06, size = 18, normalized size = 0.69

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/x^3
```

giac [A] time = 0.97, size = 19, normalized size = 0.73

$$-\frac{\left(2\,bx\mathrm{sgn}(x)+a\mathrm{sgn}(x)\right)\sqrt{c}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*x*sgn(x) + a*sgn(x))*sqrt(c)/x^2
```

maple [A] time = 0.00, size = 19, normalized size = 0.73

$$-\frac{(2bx+a)\sqrt{c\,x^2}}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(c*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/2*(2*b*x+a)*(c*x^2)^(1/2)/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.14, size = 28, normalized size = 1.08

$$-\frac{a\,\sqrt{c}\,x^2+2\,b\,\sqrt{c}\,x^3}{2\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x))/x^4,x)
```

```
[Out] -(a*c^(1/2)*x^2 + 2*b*c^(1/2)*x^3)/(2*x*(x^2)^(3/2))
```

sympy [A] time = 0.51, size = 36, normalized size = 1.38

$$-\frac{a\sqrt{c}\,\sqrt{x^2}}{2x^3}-\frac{b\sqrt{c}\,\sqrt{x^2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(c*x**2)**(1/2)/x**4,x)
```

```
[Out] -a*sqrt(c)*sqrt(x**2)/(2*x**3) - b*sqrt(c)*sqrt(x**2)/x**2
```

3.722 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (a*c*x^6*Sqrt[c*x^2])/7 + (b*c*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^6+bx^7\right)dx}{x}\\&=\frac{1}{7}acx^6\sqrt{cx^2}+\frac{1}{8}bcx^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{56}x^4\left(cx^2\right)^{3/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(3/2)*(8*a + 7*b*x))/56
```

fricas [A] time = 0.91, size = 24, normalized size = 0.65

$$\frac{1}{56}\left(7\,bcx^7+8\,acx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/56*(7*b*c*x^7 + 8*a*c*x^6)*sqrt(c*x^2)
```

giac [A] time = 1.17, size = 22, normalized size = 0.59

$$\frac{1}{56}\left(7\,bx^8\mathrm{sgn}(x)+8\,ax^7\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/56*(7*b*x^8*sgn(x) + 8*a*x^7*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(7bx+8a)\left(c\,x^2\right)^{\frac{3}{2}}x^4}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a),x)
[Out] 1/56*x^4*(7*b*x+8*a)*(c*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 33, normalized size = 0.89

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^3}{8\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/8*(c*x^2)^(5/2)*b*x^3/c + 1/7*(c*x^2)^(5/2)*a*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^3\left(c\,x^2\right)^{3/2}\,(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x),x)
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x), x)
```

sympy [A] time = 1.16, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}+\frac{bc^{\frac{3}{2}}x^5\left(x^2\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**4*(x**2)**(3/2)/7 + b*c**(3/2)*x**5*(x**2)**(3/2)/8
```

3.723 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^5*Sqrt[c*x^2])/6 + (b*c*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\ &= \frac{1}{6}acx^5\sqrt{cx^2}+\frac{1}{7}bcx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{42}x^3\left(cx^2\right)^{3/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(7*a + 6*b*x))/42
```

fricas [A] time = 1.08, size = 24, normalized size = 0.65

$$\frac{1}{42}\left(6\,bcx^6+7\,acx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/42*(6*b*c*x^6 + 7*a*c*x^5)*sqrt(c*x^2)
```

giac [A] time = 0.93, size = 22, normalized size = 0.59

$$\frac{1}{42}\left(6\,bx^7\mathrm{sgn}(x)+7\,ax^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/42*(6*b*x^7*sgn(x) + 7*a*x^6*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(6bx+7a)\left(c\,x^2\right)^{\frac{3}{2}}x^3}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/42*x^3*(6*b*x+7*a)*(c*x^2)^(3/2)
```

maxima [A] time = 1.34, size = 31, normalized size = 0.84

$$\frac{\left(cx^2\right)^{\frac{5}{2}}bx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}ax}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/7*(c*x^2)^(5/2)*b*x^2/c + 1/6*(c*x^2)^(5/2)*a*x/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^2\left(c\,x^2\right)^{3/2}(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x), x)
```

sympy [A] time = 0.91, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}+\frac{bc^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**3*(x**2)**(3/2)/6 + b*c**(3/2)*x**4*(x**2)**(3/2)/7
```

3.724 $\int x\left(cx^2\right)^{3/2}(a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (a*c*x^4*Sqrt[c*x^2])/5 + (b*c*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(ax^4+bx^5\right)\,dx}{x}\\&=\frac{1}{5}acx^4\sqrt{cx^2}+\frac{1}{6}bcx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.65

$$\frac{1}{30}x^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

fricas [A] time = 0.86, size = 24, normalized size = 0.65

$$\frac{1}{30}\left(5\,bcx^{5}+6\,acx^{4}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/30*(5*b*c*x^5 + 6*a*c*x^4)*sqrt(c*x^2)
```

giac [A] time = 0.99, size = 22, normalized size = 0.59

$$\frac{1}{30}\left(5\,bx^{6}\mathrm{sgn}(x)+6\,ax^{5}\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
[Out] 1/30*(5*b*x^6*sgn(x) + 6*a*x^5*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.57

$$\frac{(5bx+6a)\left(c\,x^{2}\right)^{\frac{3}{2}}x^{2}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a),x)
[Out] 1/30*x^2*(5*b*x+6*a)*(c*x^2)^(3/2)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.76

$$\frac{\left(cx^{2}\right)^{\frac{5}{2}}bx}{6\,c}+\frac{\left(cx^{2}\right)^{\frac{5}{2}}a}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b*x/c + 1/5*(c*x^2)^(5/2)*a/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x\left(c\,x^{2}\right)^{3/2}\,(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x),x)
[Out] int(x*(c*x^2)^(3/2)*(a + b*x), x)
```

sympy [A] time = 0.73, size = 36, normalized size = 0.97

$$\frac{ac^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}{5}+\frac{bc^{\frac{3}{2}}x^{3}\left(x^{2}\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x**2*(x**2)**(3/2)/5 + b*c**(3/2)*x**3*(x**2)**(3/2)/6
```

3.725 $\int \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=37

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*x^3*Sqrt[c*x^2])/4 + (b*c*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2}(a+bx)\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)\,dx}{x}\\&=\frac{1}{4}acx^3\sqrt{cx^2}+\frac{1}{5}bcx^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.59

$$\frac{1}{20}x\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

fricas [A] time = 0.83, size = 24, normalized size = 0.65

$$\frac{1}{20}\left(4\,bcx^{4}+5\,acx^{3}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*c*x^4 + 5*a*c*x^3)*sqrt(c*x^2)
```

giac [A] time = 0.88, size = 22, normalized size = 0.59

$$\frac{1}{20}\left(4\,bx^{5}\mathrm{sgn}(x)+5\,ax^{4}\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] 1/20*(4*b*x^5*sgn(x) + 5*a*x^4*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 19, normalized size = 0.51

$$\frac{(4bx+5a)\left(c\,x^{2}\right)^{\frac{3}{2}}x}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] 1/20*x*(4*b*x+5*a)*(c*x^2)^(3/2)
```

maxima [A] time = 1.30, size = 25, normalized size = 0.68

$$\frac{1}{4}\left(cx^{2}\right)^{\frac{3}{2}}ax+\frac{\left(cx^{2}\right)^{\frac{5}{2}}b}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*a*x + 1/5*(c*x^2)^(5/2)*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \left(c\,x^{2}\right)^{3/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x), x)
```

sympy [A] time = 0.56, size = 34, normalized size = 0.92

$$\frac{ac^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}{4}+\frac{bc^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] a*c**(3/2)*x*(x**2)**(3/2)/4 + b*c**(3/2)*x**2*(x**2)**(3/2)/5
```

3.726 $\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx$

Optimal. Leaf size=37

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (a*c*x^2*Sqrt[c*x^2])/3 + (b*c*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\ &= \frac{1}{3}acx^2\sqrt{cx^2}+\frac{1}{4}bcx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.68

$$\frac{1}{12}cx^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 0.57

$$\frac{1}{12}\left(cx^2\right)^{3/2}(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x,x]
[Out] ((c*x^2)^(3/2)*(4*a + 3*b*x))/12
```

fricas [A] time = 1.09, size = 24, normalized size = 0.65

$$\frac{1}{12}\left(3\,bcx^{3}+4\,acx^{2}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="fricas")
[Out] 1/12*(3*b*c*x^3 + 4*a*c*x^2)*sqrt(c*x^2)
```

giac [A] time = 1.09, size = 22, normalized size = 0.59

$$\frac{1}{12}\left(3\,bx^{4}\mathrm{sgn}(x)+4\,ax^{3}\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="giac")
[Out] 1/12*(3*b*x^4*sgn(x) + 4*a*x^3*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 18, normalized size = 0.49

$$\frac{(3bx+4a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x,x)
[Out] 1/12*(3*b*x+4*a)*(c*x^2)^(3/2)
```

maxima [A] time = 1.34, size = 22, normalized size = 0.59

$$\frac{1}{4}\left(cx^{2}\right)^{\frac{3}{2}}bx+\frac{1}{3}\left(cx^{2}\right)^{\frac{3}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x,x, algorithm="maxima")
[Out] 1/4*(c*x^2)^(3/2)*b*x + 1/3*(c*x^2)^(3/2)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^{2}\right)^{3/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x, x)
```

sympy [A] time = 0.58, size = 31, normalized size = 0.84

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{3}+\frac{bc^{\frac{3}{2}}x\left(x^{2}\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/3 + b*c**(3/2)*x*(x**2)**(3/2)/4
```

3.727 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx$

Optimal. Leaf size=35

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 43}

$$\frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c*x*Sqrt[c*x^2])/2 + (b*c*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)}{x^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x}\\
&= \frac{1}{2}acx\sqrt{cx^2} + \frac{1}{3}bcx^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.66

$$\frac{1}{6}cx\sqrt{cx^2}(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.69

$$\frac{(cx^2)^{3/2}(3a+2bx)}{6x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^2,x]
[Out] ((c*x^2)^(3/2)*(3*a + 2*b*x))/(6*x)
```

fricas [A] time = 1.01, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bcx^2+3\,acx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="fricas")
[Out] 1/6*(2*b*c*x^2 + 3*a*c*x)*sqrt(c*x^2)
```

giac [A] time = 1.08, size = 22, normalized size = 0.63

$$\frac{1}{6}\left(2\,bx^3\mathrm{sgn}(x)+3\,ax^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/6*(2*b*x^3*sgn(x) + 3*a*x^2*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(2bx+3a)\left(c\,x^2\right)^{\frac{3}{2}}}{6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^2,x)
[Out] 1/6/x*(2*b*x+3*a)*(c*x^2)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.27, size = 20, normalized size = 0.57

$$\frac{c^{3/2}\left(2\,b\,\sqrt{x^6}\,+3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^2,x)
[Out] (c^(3/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

sympy [A] time = 0.57, size = 31, normalized size = 0.89

$$\frac{ac^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x}+\frac{bc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/(2*x) + b*c**(3/2)*(x**2)**(3/2)/3
```

3.728 $\int \frac{(cx^2)^{3/2}(a+bx)}{x^3} \, dx$

Optimal. Leaf size=29

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] a*c*Sqrt[c*x^2] + (b*c*x*Sqrt[c*x^2])/2
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^{3/2}(a+bx)}{x^3} \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)\, dx}{x} \\ &= ac\sqrt{cx^2} + \frac{1}{2}bcx\sqrt{cx^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.72

$$\frac{1}{2}c\sqrt{cx^2}\,(2a+bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(2*a + b*x))/2
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 0.79

$$\frac{(cx^2)^{3/2}\,(2a+bx)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a + b*x))/(2*x^2)
```

fricas [A] time = 1.04, size = 18, normalized size = 0.62

$$\frac{1}{2}\,(bcx + 2\,ac)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*c*x + 2*a*c)*sqrt(c*x^2)
```

giac [A] time = 0.99, size = 17, normalized size = 0.59

$$\frac{1}{2}\left(bx^{2}+2\,ax\right)c^{\frac{3}{2}}\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)*c^(3/2)*sgn(x)
```

maple [A] time = 0.00, size = 20, normalized size = 0.69

$$\frac{(bx+2a)\left(c\,x^{2}\right)^{\frac{3}{2}}}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^3,x)
```

```
[Out] 1/2/x^2*(b*x+2*a)*(c*x^2)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.22, size = 14, normalized size = 0.48

$$\frac{c^{3/2}\,|x|\,\left(2\,a+b\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^3,x)
```

```
[Out] (c^(3/2)*abs(x)*(2*a + b*x))/2
```

sympy [A] time = 0.74, size = 32, normalized size = 1.10

$$\frac{ac^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{x^{2}}+\frac{bc^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(3/2)*(x**2)**(3/2)/x**2 + b*c**(3/2)*(x**2)**(3/2)/(2*x)
```

3.729 $\int \frac{\left(cx^2\right)^{3/2}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=30

$$\frac{ac\sqrt{cx^2}\log(x)}{x}+bc\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}\log(x)}{x}+bc\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] b*c*Sqrt[c*x^2] + (a*c*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{3/2}(a+bx)}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{a+bx}{x}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(b+\frac{a}{x}\right)dx}{x}\\ &= bc\sqrt{cx^2}+\frac{ac\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.70

$$\frac{\left(cx^2\right)^{3/2}(a\log(x)+bx)}{x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x + a*Log[x]))/x^3
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 0.77

$$\left(cx^2\right)^{3/2}\left(\frac{a\log(x)}{x^3}+\frac{b}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x))/x^4,x]
[Out] (c*x^2)^(3/2)*(b/x^2 + (a*Log[x])/x^3)
```

fricas [A] time = 0.88, size = 21, normalized size = 0.70

$$\frac{\left(bcx + ac\log(x)\right)\sqrt{cx^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="fricas")
[Out] (b*c*x + a*c*log(x))*sqrt(c*x^2)/x
```

giac [A] time = 0.96, size = 17, normalized size = 0.57

$$\left(bx\mathrm{sgn}(x) + a\log\left(|x|\right)\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="giac")
[Out] (b*x*sgn(x) + a*log(abs(x))*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 20, normalized size = 0.67

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(a\ln(x) + bx\right)}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)/x^4,x)
[Out] (c*x^2)^(3/2)/x^3*(a*ln(x)+b*x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x))/x^4,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}\left(a + bx\right)}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)/x**4,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)/x**4, x)
```

3.730 $\int x^3 \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^8*Sqrt[c*x^2])/9 + (b*c^2*x^9*Sqrt[c*x^2])/10
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{5/2}(a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^8(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^8+bx^9\right)\,dx}{x}\\ &= \frac{1}{9}ac^2x^8\sqrt{cx^2}+\frac{1}{10}bc^2x^9\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{90}x^4\left(cx^2\right)^{5/2}(10a+9bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^4*(c*x^2)^(5/2)*(10*a + 9*b*x))/90
```

fricas [A] time = 1.22, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^9 + 10\,ac^2x^8\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/90*(9*b*c^2*x^9 + 10*a*c^2*x^8)*sqrt(c*x^2)
```

giac [A] time = 0.97, size = 28, normalized size = 0.68

$$\frac{1}{90}\left(9\,bc^2x^{10}\mathrm{sgn}(x) + 10\,ac^2x^9\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/90*(9*b*c^2*x^10*sgn(x) + 10*a*c^2*x^9*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(9bx + 10a)\left(c\,x^2\right)^{\frac{5}{2}}x^4}{90}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/90*x^4*(9*b*x+10*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.24, size = 33, normalized size = 0.80

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^3}{10\,c} + \frac{\left(cx^2\right)^{\frac{7}{2}}ax^2}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/10*(c*x^2)^(7/2)*b*x^3/c + 1/9*(c*x^2)^(7/2)*a*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\left(c\,x^2\right)^{5/2}\,(a + b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x^3*(c*x^2)^(5/2)*(a + b*x), x)
```

sympy [A] time = 2.54, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9} + \frac{bc^{\frac{5}{2}}x^5\left(x^2\right)^{\frac{5}{2}}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**4*(x**2)**(5/2)/9 + b*c**(5/2)*x**5*(x**2)**(5/2)/10
```

3.731 $\int x^2 \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^7*Sqrt[c*x^2])/8 + (b*c^2*x^8*Sqrt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(cx^2\right)^{5/2}(a+bx)\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^7(a+bx)\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(ax^7+bx^8\right)dx}{x}\\&=\frac{1}{8}ac^2x^7\sqrt{cx^2}+\frac{1}{9}bc^2x^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{72}x^3\left(cx^2\right)^{5/2}(9a+8bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^3*(c*x^2)^(5/2)*(9*a + 8*b*x))/72
```

fricas [A] time = 0.59, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^8 + 9\,ac^2x^7\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/72*(8*b*c^2*x^8 + 9*a*c^2*x^7)*sqrt(c*x^2)
```

giac [A] time = 0.80, size = 28, normalized size = 0.68

$$\frac{1}{72}\left(8\,bc^2x^9\text{sgn}(x) + 9\,ac^2x^8\text{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/72*(8*b*c^2*x^9*sgn(x) + 9*a*c^2*x^8*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(8bx + 9a)\left(c\,x^2\right)^{\frac{5}{2}}x^3}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/72*x^3*(8*b*x+9*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.30, size = 31, normalized size = 0.76

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx^2}{9\,c} + \frac{\left(cx^2\right)^{\frac{7}{2}}ax}{8\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/9*(c*x^2)^(7/2)*b*x^2/c + 1/8*(c*x^2)^(7/2)*a*x/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(c\,x^2\right)^{5/2}\,\left(a + b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x^2*(c*x^2)^(5/2)*(a + b*x), x)
```

sympy [A] time = 2.10, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{8} + \frac{bc^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**3*(x**2)**(5/2)/8 + b*c**(5/2)*x**4*(x**2)**(5/2)/9
```

3.732 $\int x \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^6*Sqrt[c*x^2])/7 + (b*c^2*x^7*Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x \left(cx^2\right)^{5/2} (a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^6+bx^7\right)\,dx}{x}\\ &= \frac{1}{7}ac^2x^6\sqrt{cx^2}+\frac{1}{8}bc^2x^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{1}{56}x^2\left(cx^2\right)^{5/2}(8a+7bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x^2*(c*x^2)^(5/2)*(8*a + 7*b*x))/56
```

fricas [A] time = 0.98, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^7+8\,ac^2x^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/56*(7*b*c^2*x^7 + 8*a*c^2*x^6)*sqrt(c*x^2)
```

giac [A] time = 1.03, size = 28, normalized size = 0.68

$$\frac{1}{56}\left(7\,bc^2x^8\mathrm{sgn}(x)+8\,ac^2x^7\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/56*(7*b*c^2*x^8*sgn(x) + 8*a*c^2*x^7*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(7bx+8a)\left(c\,x^2\right)^{\frac{5}{2}}x^2}{56}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/56*x^2*(7*b*x+8*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.35, size = 28, normalized size = 0.68

$$\frac{\left(cx^2\right)^{\frac{7}{2}}bx}{8\,c}+\frac{\left(cx^2\right)^{\frac{7}{2}}a}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/8*(c*x^2)^(7/2)*b*x/c + 1/7*(c*x^2)^(7/2)*a/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x),x)
[Out] int(x*(c*x^2)^(5/2)*(a + b*x), x)
```

sympy [A] time = 1.74, size = 36, normalized size = 0.88

$$\frac{ac^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}+\frac{bc^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x**2*(x**2)**(5/2)/7 + b*c**(5/2)*x**3*(x**2)**(5/2)/8
```

3.733 $\int (cx^2)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*x^5*Sqrt[c*x^2])/6 + (b*c^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (cx^2)^{5/2}(a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^5+bx^6\right)\,dx}{x}\\ &= \frac{1}{6}ac^2x^5\sqrt{cx^2}+\frac{1}{7}bc^2x^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.54

$$\frac{1}{42}x\left(cx^2\right)^{5/2}(7a+6bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(c*x^2)^(5/2)*(7*a + 6*b*x))/42
```

fricas [A] time = 0.99, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^{2}x^{6}+7\,ac^{2}x^{5}\right)\sqrt{cx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] 1/42*(6*b*c^2*x^6 + 7*a*c^2*x^5)*sqrt(c*x^2)
```

giac [A] time = 0.95, size = 28, normalized size = 0.68

$$\frac{1}{42}\left(6\,bc^{2}x^{7}\mathrm{sgn}(x)+7\,ac^{2}x^{6}\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] 1/42*(6*b*c^2*x^7*sgn(x) + 7*a*c^2*x^6*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 19, normalized size = 0.46

$$\frac{(6bx+7a)\left(c\,x^{2}\right)^{\frac{5}{2}}x}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a),x)
[Out] 1/42*x*(6*b*x+7*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.34, size = 25, normalized size = 0.61

$$\frac{1}{6}\left(cx^{2}\right)^{\frac{5}{2}}ax+\frac{\left(cx^{2}\right)^{\frac{7}{2}}b}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*a*x + 1/7*(c*x^2)^(7/2)*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(c\,x^2\right)}^{5/2}\,\left(a+b\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x),x)
[Out] int((c*x^2)^(5/2)*(a + b*x), x)
```

sympy [A] time = 1.41, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}{6}+\frac{bc^{\frac{5}{2}}x^{2}\left(x^{2}\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] a*c**(5/2)*x*(x**2)**(5/2)/6 + b*c**(5/2)*x**2*(x**2)**(5/2)/7
```

3.734 $\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx$

Optimal. Leaf size=41

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (a*c^2*x^4*Sqrt[c*x^2])/5 + (b*c^2*x^5*Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^4+bx^5\right)\,dx}{x}\\ &= \frac{1}{5}ac^2x^4\sqrt{cx^2}+\frac{1}{6}bc^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.61

$$\frac{1}{30}cx^2\left(cx^2\right)^{3/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x,x]
```

```
[Out] (c*x^2*(c*x^2)^(3/2)*(6*a + 5*b*x))/30
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{1}{30}\left(cx^2\right)^{5/2}(6a+5bx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x,x]
[Out] ((c*x^2)^(5/2)*(6*a + 5*b*x))/30
```

fricas [A] time = 1.21, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^5+6\,ac^2x^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="fricas")
[Out] 1/30*(5*b*c^2*x^5 + 6*a*c^2*x^4)*sqrt(c*x^2)
```

giac [A] time = 1.07, size = 28, normalized size = 0.68

$$\frac{1}{30}\left(5\,bc^2x^6\mathrm{sgn}(x)+6\,ac^2x^5\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="giac")
[Out] 1/30*(5*b*c^2*x^6*sgn(x) + 6*a*c^2*x^5*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$\frac{(5bx+6a)\left(c\,x^2\right)^{\frac{5}{2}}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x,x)
[Out] 1/30*(5*b*x+6*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.40, size = 22, normalized size = 0.54

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}bx+\frac{1}{5}\left(cx^2\right)^{\frac{5}{2}}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b*x + 1/5*(c*x^2)^(5/2)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x))/x, x)
```

sympy [A] time = 1.43, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{bc^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/5 + b*c**(5/2)*x*(x**2)**(5/2)/6
```

3.735 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx$

Optimal. Leaf size=41

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (a*c^2*x^3*Sqrt[c*x^2])/4 + (b*c^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^3+bx^4\right)dx}{x}\\ &= \frac{1}{4}ac^2x^3\sqrt{cx^2}+\frac{1}{5}bc^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.56

$$\frac{1}{20}cx\left(cx^2\right)^{3/2}(5a+4bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(5*a + 4*b*x))/20
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{\left(cx^2\right)^{5/2}(5a+4bx)}{20x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^2,x]
[Out] ((c*x^2)^(5/2)*(5*a + 4*b*x))/(20*x)
```

fricas [A] time = 0.79, size = 28, normalized size = 0.68

$$\frac{1}{20} \left(4 \, bc^2 x^4 + 5 \, ac^2 x^3\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="fricas")
[Out] 1/20*(4*b*c^2*x^4 + 5*a*c^2*x^3)*sqrt(c*x^2)
```

giac [A] time = 1.10, size = 28, normalized size = 0.68

$$\frac{1}{20} \left(4 \, bc^2 x^5 \text{sgn}(x) + 5 \, ac^2 x^4 \text{sgn}(x)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="giac")
[Out] 1/20*(4*b*c^2*x^5*sgn(x) + 5*a*c^2*x^4*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(4bx + 5a) \left(c \, x^2\right)^{\frac{5}{2}}}{20x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^2,x)
[Out] 1/20/x*(4*b*x+5*a)*(c*x^2)^(5/2)
```

maxima [A] time = 1.28, size = 24, normalized size = 0.59

$$\frac{1}{5} \left(cx^2\right)^{\frac{5}{2}} b + \frac{\left(cx^2\right)^{\frac{5}{2}} a}{4 \, x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^2,x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(5/2)*b + 1/4*(c*x^2)^(5/2)*a/x
```

mupad [B] time = 0.28, size = 25, normalized size = 0.61

$$\frac{c^{5/2} \left(4 \, b \, \sqrt{x^{10}} + 5 \, a \, x^3 \, \sqrt{x^2}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^2,x)
[Out] (c^(5/2)*(4*b*(x^10)^(1/2) + 5*a*x^3*(x^2)^(1/2)))/20
```

sympy [A] time = 1.53, size = 31, normalized size = 0.76

$$\frac{ac^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{4x}+\frac{bc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**2,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(4*x) + b*c**(5/2)*(x**2)**(5/2)/5
```

3.736 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (a*c^2*x^2*Sqrt[c*x^2])/3 + (b*c^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax^2+bx^3\right)\,dx}{x}\\ &= \frac{1}{3}ac^2x^2\sqrt{cx^2}+\frac{1}{4}bc^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.66

$$\frac{1}{12}c^2x^2\sqrt{cx^2}\,(4a+3bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/12
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{(cx^2)^{5/2}\,(4a+3bx)}{12x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^3,x]
[Out] ((c*x^2)^(5/2)*(4*a + 3*b*x))/(12*x^2)
```

fricas [A] time = 1.08, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^3+4\,ac^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="fricas")
[Out] 1/12*(3*b*c^2*x^3 + 4*a*c^2*x^2)*sqrt(c*x^2)
```

giac [A] time = 0.99, size = 28, normalized size = 0.68

$$\frac{1}{12}\left(3\,bc^2x^4\mathrm{sgn}(x)+4\,ac^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="giac")
[Out] 1/12*(3*b*c^2*x^4*sgn(x) + 4*a*c^2*x^3*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$\frac{(3bx+4a)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^3,x)
[Out] 1/12/x^2*(3*b*x+4*a)*(c*x^2)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.27, size = 25, normalized size = 0.61

$$\frac{c^{5/2}\left(4\,a\,\sqrt{x^6}+3\,b\,x^3\,\sqrt{x^2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^3,x)
[Out] (c^(5/2)*(4*a*(x^6)^(1/2) + 3*b*x^3*(x^2)^(1/2)))/12
```

sympy [A] time = 1.58, size = 34, normalized size = 0.83

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**3,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(3*x**2) + b*c**(5/2)*(x**2)**(5/2)/(4*x)
```

3.737 $\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx$

Optimal. Leaf size=39

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (a*c^2*x*Sqrt[c*x^2])/2 + (b*c^2*x^2*Sqrt[c*x^2])/3
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(ax+bx^2\right)\,dx}{x}\\
&= \frac{1}{2}ac^2x\sqrt{cx^2}+\frac{1}{3}bc^2x^2\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.64

$$\frac{1}{6}c^2x\sqrt{cx^2}\,(3a+2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(3*a + 2*b*x))/6
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.62

$$\frac{(cx^2)^{5/2}\,(3a+2bx)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x))/x^4,x]
[Out] ((c*x^2)^(5/2)*(3*a + 2*b*x))/(6*x^3)
```

fricas [A] time = 0.77, size = 26, normalized size = 0.67

$$\frac{1}{6}\left(2\,bc^2x^2 + 3\,ac^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="fricas")
[Out] 1/6*(2*b*c^2*x^2 + 3*a*c^2*x)*sqrt(c*x^2)
```

giac [A] time = 1.14, size = 28, normalized size = 0.72

$$\frac{1}{6}\left(2\,bc^2x^3\mathrm{sgn}(x) + 3\,ac^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="giac")
[Out] 1/6*(2*b*c^2*x^3*sgn(x) + 3*a*c^2*x^2*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.54

$$\frac{(2bx + 3a)\left(c\,x^2\right)^{\frac{5}{2}}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)/x^4,x)
[Out] 1/6/x^3*(2*b*x+3*a)*(c*x^2)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [B] time = 0.26, size = 20, normalized size = 0.51

$$\frac{c^{5/2}\left(2\,b\,\sqrt{x^6} + 3\,a\,x\,|x|\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x))/x^4,x)
[Out] (c^(5/2)*(2*b*(x^6)^(1/2) + 3*a*x*abs(x)))/6
```

sympy [A] time = 1.62, size = 36, normalized size = 0.92

$$\frac{ac^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x^3} + \frac{bc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)/x**4,x)
```

```
[Out] a*c**(5/2)*(x**2)**(5/2)/(2*x**3) + b*c**(5/2)*(x**2)**(5/2)/(3*x**2)
```

3.738 $\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^4)/(3*Sqrt[c*x^2]) + (b*x^5)/(4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(ax^2+bx^3\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{ax^4}{3\sqrt{cx^2}}+\frac{bx^5}{4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^4(4a+3bx)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(4*a + 3*b*x))/(12*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{x^2\sqrt{cx^2}\,(4a+3bx)}{12c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x^2*Sqrt[c*x^2]*(4*a + 3*b*x))/(12*c)
```

fricas [A] time = 1.26, size = 25, normalized size = 0.71

$$\frac{\left(3\,bx^3+4\,ax^2\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b*x^3 + 4*a*x^2)*sqrt(c*x^2)/c
```

giac [A] time = 1.30, size = 26, normalized size = 0.74

$$\frac{1}{12}\,\sqrt{cx^2}\left(\frac{3\,bx}{c}+\frac{4\,a}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(c*x^2)*(3*b*x/c + 4*a/c)*x^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(3bx+4a)\,x^4}{12\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(1/2),x)
[Out] 1/12*x^4*(3*b*x+4*a)/(c*x^2)^(1/2)
```

maxima [A] time = 1.40, size = 33, normalized size = 0.94

$$\frac{\sqrt{cx^2}\,bx^3}{4\,c}+\frac{\sqrt{cx^2}\,ax^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(c*x^2)*b*x^3/c + 1/3*sqrt(c*x^2)*a*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(1/2),x)
[Out] int((x^3*(a + b*x))/(c*x^2)^(1/2), x)
```

sympy [A] time = 0.61, size = 36, normalized size = 1.03

$$\frac{ax^4}{3\sqrt{c}\,\sqrt{x^2}}+\frac{bx^5}{4\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**4/(3*sqrt(c)*sqrt(x**2)) + b*x**5/(4*sqrt(c)*sqrt(x**2))
```

3.739 $\int \frac{x^2(a+bx)}{\sqrt{cx^2}} dx$

Optimal. Leaf size=35

$$\frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^3)/(2*Sqrt[c*x^2]) + (b*x^4)/(3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)}{\sqrt{cx^2}} dx &= \frac{x \int x(a+bx) dx}{\sqrt{cx^2}} \\
&= \frac{x \int \left(ax+bx^2\right) dx}{\sqrt{cx^2}} \\
&= \frac{ax^3}{2\sqrt{cx^2}} + \frac{bx^4}{3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.69

$$\frac{x^3(3a+2bx)}{6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^3*(3*a + 2*b*x))/(6*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.71

$$\frac{x\sqrt{cx^2}\,(3a+2bx)}{6c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x*Sqrt[c*x^2]*(3*a + 2*b*x))/(6*c)
```

fricas [A] time = 1.02, size = 23, normalized size = 0.66

$$\frac{\left(2\,bx^2 + 3\,ax\right)\sqrt{cx^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/6*(2*b*x^2 + 3*a*x)*sqrt(c*x^2)/c
```

giac [A] time = 1.16, size = 24, normalized size = 0.69

$$\frac{1}{6}\,\sqrt{cx^2}\left(\frac{2\,bx}{c} + \frac{3\,a}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(c*x^2)*(2*b*x/c + 3*a/c)*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{(2bx + 3a)\,x^3}{6\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(1/2),x)
[Out] 1/6*x^3*(2*b*x+3*a)/(c*x^2)^(1/2)
```

maxima [A] time = 1.30, size = 26, normalized size = 0.74

$$\frac{\sqrt{cx^2}\,bx^2}{3\,c} + \frac{ax^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(c*x^2)*b*x^2/c + 1/2*a*x^2/sqrt(c)
```

mupad [B] time = 0.25, size = 23, normalized size = 0.66

$$\frac{2\,b\,\sqrt{x^6} + 3\,a\,x\,\sqrt{x^2}}{6\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(1/2),x)
[Out] (2*b*(x^6)^(1/2) + 3*a*x*(x^2)^(1/2))/(6*c^(1/2))
```

sympy [A] time = 0.52, size = 36, normalized size = 1.03

$$\frac{ax^3}{2\sqrt{c}\,\sqrt{x^2}} + \frac{bx^4}{3\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] a*x**3/(2*sqrt(c)*sqrt(x**2)) + b*x**4/(3*sqrt(c)*sqrt(x**2))
```

3.740 $\int \frac{x(a+bx)}{\sqrt{cx^2}} dx$

Optimal. Leaf size=32

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {15}

$$\frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x^2)/Sqrt[c*x^2] + (b*x^3)/(2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)}{\sqrt{cx^2}} dx &= \frac{x\int (a+bx)\,dx}{\sqrt{cx^2}} \\ &= \frac{ax^2}{\sqrt{cx^2}} + \frac{bx^3}{2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.72

$$\frac{x^2(2a+bx)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (x^2*(2*a + b*x))/(2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 0.72

$$\frac{\sqrt{cx^2}\,(2a+bx)}{2c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (Sqrt[c*x^2]*(2*a + b*x))/(2*c)
```

fricas [A] time = 1.18, size = 19, normalized size = 0.59

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c
```

giac [A] time = 1.11, size = 22, normalized size = 0.69

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{bx}{c}+\frac{2a}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)
```

maple [A] time = 0.00, size = 20, normalized size = 0.62

$$\frac{(bx+2a)\,x^2}{2\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(1/2),x)
[Out] 1/2*x^2*(b*x+2*a)/(c*x^2)^(1/2)
```

maxima [A] time = 1.30, size = 22, normalized size = 0.69

$$\frac{bx^2}{2\sqrt{c}}+\frac{\sqrt{cx^2}\,a}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*b*x^2/sqrt(c) + sqrt(c*x^2)*a/c
```

mupad [B] time = 0.22, size = 19, normalized size = 0.59

$$\frac{2\,a\,|x|+b\,x\,\sqrt{x^2}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(1/2),x)
[Out] (2*a*abs(x) + b*x*(x^2)^(1/2))/(2*c^(1/2))
```

sympy [A] time = 0.46, size = 34, normalized size = 1.06

$$\frac{ax^2}{\sqrt{c}\,\sqrt{x^2}}+\frac{bx^3}{2\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(1/2),x)
[Out] a*x**2/(sqrt(c)*sqrt(x**2)) + b*x**3/(2*sqrt(c)*sqrt(x**2))
```

3.741 $\int \frac{a+bx}{\sqrt{cx^2}} dx$

Optimal. Leaf size=29

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$\frac{ax\log(x)}{\sqrt{cx^2}}+\frac{bx^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (b*x^2)/Sqrt[c*x^2] + (a*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right)dx}{\sqrt{cx^2}}\\ &= \frac{bx^2}{\sqrt{cx^2}}+\frac{ax\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.66

$$\frac{x(a\log(x)+bx)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] (x*(b*x + a*Log[x]))/Sqrt[c*x^2]
```

IntegrateAlgebraic [A] time = 0.02, size = 26, normalized size = 0.90

$$\sqrt{cx^2}\left(\frac{a\log(x)}{cx}+\frac{b}{c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[c*x^2],x]
```

```
[Out] Sqrt[c*x^2]*(b/c + (a*Log[x])/(c*x))
```

fricas [A] time = 1.02, size = 22, normalized size = 0.76

$$\frac{\sqrt{cx^2}\left(bx + a\log(x)\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c*x)
```

giac [A] time = 1.32, size = 35, normalized size = 1.21

$$-\frac{a\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2}\right|\right)}{\sqrt{c}} + \frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + sqrt(c*x^2)*b/c
```

maple [A] time = 0.00, size = 18, normalized size = 0.62

$$\frac{(a\ln(x) + bx)\,x}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/(c*x^2)^(1/2)*x*(a*ln(x)+b*x)
```

maxima [A] time = 1.32, size = 20, normalized size = 0.69

$$\frac{a\log(x)}{\sqrt{c}} + \frac{\sqrt{cx^2}\,b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(x)/sqrt(c) + sqrt(c*x^2)*b/c
```

mupad [B] time = 0.51, size = 17, normalized size = 0.59

$$\frac{b\,|x| + a\,\ln\left(c\,x\right)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(1/2),x)
```

```
[Out] (b*abs(x) + a*log(c*x)*sign(x))/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)/sqrt(c*x**2), x)
```

3.742 $\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=27

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{\sqrt{cx^2}}-\frac{a}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] -(a/Sqrt[c*x^2]) + (b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{\sqrt{cx^2}}+\frac{bx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{cx^2(bx\log(x)-a)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*Sqrt[c*x^2]),x]
```

```
[Out] (c*x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.11

$$\sqrt{cx^2}\left(\frac{b\log(x)}{cx}-\frac{a}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*Sqrt[c*x^2]),x]
[Out] Sqrt[c*x^2]*(-(a/(c*x^2)) + (b*Log[x])/(c*x))
```

fricas [A] time = 1.01, size = 23, normalized size = 0.85

$$\frac{\sqrt{cx^2}\left(bx\log(x) - a\right)}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c*x^2)
```

giac [B] time = 0.98, size = 47, normalized size = 1.74

$$-\frac{b\log\left(\left|-\sqrt{c}x + \sqrt{cx^2}\right|\right) - \frac{2a\sqrt{c}}{\sqrt{c}x - \sqrt{cx^2}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/sqrt(c)
```

maple [A] time = 0.00, size = 18, normalized size = 0.67

$$\frac{bx\ln(x) - a}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(1/2),x)
[Out] (b*x*ln(x)-a)/(c*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 17, normalized size = 0.63

$$\frac{b\log(x)}{\sqrt{c}} - \frac{a}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*log(x)/sqrt(c) - a/(sqrt(c)*x)
```

mupad [B] time = 1.22, size = 22, normalized size = 0.81

$$-\frac{\frac{a}{\sqrt{x^2}} - b\,\ln(c\,x)\,\mathrm{sign}(x)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(1/2)),x)
[Out] -(a/(x^2)^(1/2) - b*log(c*x)*sign(x))/c^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)/(x*sqrt(c*x**2)), x)
```

3.743 $\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^2/(2*a*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ax\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.88

$$\frac{cx(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (c*x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 1.04

$$\frac{\sqrt{cx^2}\,(-a-2bx)}{2cx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-a - 2*b*x))/(2*c*x^3)
```

fricas [A] time = 1.03, size = 21, normalized size = 0.81

$$-\frac{\sqrt{cx^2}\,(2\,bx + a)}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 19, normalized size = 0.73

$$-\frac{2bx + a}{2\sqrt{c\,x^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(1/2),x)
[Out] -1/2*(2*b*x+a)/x/(c*x^2)^(1/2)
```

maxima [A] time = 1.25, size = 19, normalized size = 0.73

$$-\frac{b}{\sqrt{c}\,x} - \frac{a}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -b/(sqrt(c)*x) - 1/2*a/(sqrt(c)*x^2)
```

mupad [B] time = 0.16, size = 25, normalized size = 0.96

$$-\frac{2\,b\,x^3 + a\,x^2}{2\,\sqrt{c}\,x\,{\left(x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(1/2)),x)
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(1/2)*x*(x^2)^(3/2))
```

sympy [A] time = 0.54, size = 31, normalized size = 1.19

$$-\frac{a}{2\sqrt{c}\,x\sqrt{x^2}} - \frac{b}{\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(1/2),x)
[Out] -a/(2*sqrt(c)*x*sqrt(x**2)) - b/(sqrt(c)*sqrt(x**2))
```

3.744 $\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -a/(3*x^2*Sqrt[c*x^2]) - b/(2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{a}{3x^2\sqrt{cx^2}}-\frac{b}{2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.63

$$\frac{c(-2a-3bx)}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-2*a - 3*b*x))/(6*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(-2a-3bx)}{6cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-2*a - 3*b*x))/(6*c*x^4)
```

fricas [A] time = 0.90, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\sqrt{cx^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)/(sqrt(c*x^2)*x^3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$-\frac{3bx+2a}{6\sqrt{c\,x^2}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(1/2),x)
[Out] -1/6*(3*b*x+2*a)/x^2/(c*x^2)^(1/2)
```

maxima [A] time = 1.28, size = 19, normalized size = 0.54

$$-\frac{b}{2\,\sqrt{c}\,x^2}-\frac{a}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*b/(sqrt(c)*x^2) - 1/3*a/(sqrt(c)*x^3)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.74

$$-\frac{2\,a\,\sqrt{x^2}\;+3\,b\,x\,\sqrt{x^2}}{6\,\sqrt{c}\;x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(1/2)),x)
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(1/2)*x^4)
```

sympy [A] time = 0.64, size = 36, normalized size = 1.03

$$-\frac{a}{3\sqrt{c}\,x^2\sqrt{x^2}}-\frac{b}{2\sqrt{c}\,x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a/(3*sqrt(c)*x**2*sqrt(x**2)) - b/(2*sqrt(c)*x*sqrt(x**2))
```

3.745 $\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=35

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a/(4*x^3*Sqrt[c*x^2]) - b/(3*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^4\sqrt{cx^2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a}{4x^3\sqrt{cx^2}}-\frac{b}{3x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.69

$$\frac{-3a-4bx}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a - 4*b*x)/(12*x^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 27, normalized size = 0.77

$$\frac{\sqrt{cx^2}\,(-3a-4bx)}{12cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-3*a - 4*b*x))/(12*c*x^5)
```

fricas [A] time = 1.10, size = 23, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.60

$$-\frac{4bx+3a}{12\sqrt{c\,x^2}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(1/2),x)
[Out] -1/12*(4*b*x+3*a)/x^3/(c*x^2)^(1/2)
```

maxima [A] time = 1.30, size = 19, normalized size = 0.54

$$-\frac{b}{3\,\sqrt{c}\,x^3}-\frac{a}{4\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/3*b/(sqrt(c)*x^3) - 1/4*a/(sqrt(c)*x^4)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.74

$$-\frac{3\,a\,\sqrt{x^2}\;+4\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\;x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(1/2)),x)
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(1/2)*x^5)
```

sympy [A] time = 0.81, size = 37, normalized size = 1.06

$$-\frac{a}{4\sqrt{c}\,x^3\sqrt{x^2}}-\frac{b}{3\sqrt{c}\,x^2\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a/(4*sqrt(c)*x**3*sqrt(x**2)) - b/(3*sqrt(c)*x**2*sqrt(x**2))
```

3.746 $\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=38

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {15}

$$\frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (a*x^2)/(c*Sqrt[c*x^2]) + (b*x^3)/(2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{(cx^2)^{3/2}}\,dx &= \frac{x\int (a+bx)\,dx}{c\sqrt{cx^2}}\\ &= \frac{ax^2}{c\sqrt{cx^2}}+\frac{bx^3}{2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 23, normalized size = 0.61

$$\frac{x^4(2a+bx)}{2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^4*(2*a + b*x))/(2*(c*x^2)^(3/2))
```

fricas [A] time = 1.05, size = 19, normalized size = 0.50

$$\frac{\sqrt{cx^2}\,(bx+2\,a)}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x + 2*a)/c^2
```

giac [A] time = 1.01, size = 25, normalized size = 0.66

$$\frac{\sqrt{cx^2}\left(\frac{bx}{c}+\frac{2\,a}{c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(b*x/c + 2*a/c)/c
```

maple [A] time = 0.00, size = 20, normalized size = 0.53

$$\frac{(bx+2a)\,x^4}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/2*x^4*(b*x+2*a)/(c*x^2)^(3/2)
```

maxima [A] time = 1.31, size = 32, normalized size = 0.84

$$\frac{bx^3}{2\,\sqrt{cx^2}\,c}+\frac{ax^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(c*x^2)*c) + a*x^2/(sqrt(c*x^2)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3\,\left(a+b\,x\right)}{{\left(c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(3/2), x)
```

sympy [A] time = 0.64, size = 34, normalized size = 0.89

$$\frac{ax^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{bx^5}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] a*x**4/(c**(3/2)*(x**2)**(3/2)) + b*x**5/(2*c**(3/2)*(x**2)**(3/2))
```

3.747 $\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx$

Optimal. Leaf size=35

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{ax\log(x)}{c\sqrt{cx^2}}+\frac{bx^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^2)/(c*Sqrt[c*x^2]) + (a*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{(cx^2)^{3/2}} \, dx &= \frac{x\int \frac{a+bx}{x} \, dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b+\frac{a}{x}\right) \, dx}{c\sqrt{cx^2}}\\ &= \frac{bx^2}{c\sqrt{cx^2}}+\frac{ax\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.60

$$\frac{x^3(a\log(x)+bx)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x + a*Log[x]))/(c*x^2)^(3/2)
```

IntegrateAlgebraic [A]   time = 0.02, size = 23, normalized size = 0.66

$$\frac{ax^3\log(x)+bx^4}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] (b*x^4 + a*x^3*Log[x])/(c*x^2)^(3/2)
```

fricas [A]   time = 1.11, size = 22, normalized size = 0.63

$$\frac{\sqrt{cx^2}\,(bx+a\log(x))}{c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a*log(x))/(c^2*x)
```

giac [A]   time = 0.99, size = 40, normalized size = 1.14

$$-\frac{\frac{a\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\frac{\sqrt{cx^2}\,b}{c}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(a*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*b/c)/c
```

maple [A]   time = 0.00, size = 20, normalized size = 0.57

$$\frac{(a\ln(x)+bx)\,x^3}{(c\,x^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(3/2),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*(a*ln(x)+b*x)
```

maxima [A]   time = 1.35, size = 23, normalized size = 0.66

$$\frac{bx^2}{\sqrt{cx^2}\,c}+\frac{a\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(c*x^2)*c) + a*log(x)/c^(3/2)
```

mupad [B]   time = 0.32, size = 30, normalized size = 0.86

$$\frac{b\,|x|}{c^{3/2}}+\frac{a\,\ln(x+|x|)}{c^{3/2}}-\frac{a\,x}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*abs(x))/c^(3/2) + (a*log(x + abs(x)))/c^(3/2) - (a*x)/(c^(3/2)*(x^2)^(1/
2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)}{\left(cx^2\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*x)/(c*x**2)**(3/2), x)
```

3.748 $\int \frac{x(a+bx)}{(cx^2)^{3/2}} \, dx$

Optimal. Leaf size=33

$$\frac{bx\log(x)}{c\sqrt{cx^2}} - \frac{a}{c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$\frac{bx\log(x)}{c\sqrt{cx^2}} - \frac{a}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] -(a/(c*Sqrt[c*x^2])) + (b*x*Log[x])/(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{(cx^2)^{3/2}} \, dx &= \frac{x \int \frac{a+bx}{x^2} \, dx}{c\sqrt{cx^2}} \\
&= \frac{x \int \left(\frac{a}{x^2} + \frac{b}{x}\right) dx}{c\sqrt{cx^2}} \\
&= -\frac{a}{c\sqrt{cx^2}} + \frac{bx\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.67

$$\frac{x^2(bx\log(x) - a)}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x^2*(-a + b*x*Log[x]))/(c*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.73

$$\frac{bx^3\log(x) - ax^2}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (-(a*x^2) + b*x^3*Log[x])/(c*x^2)^(3/2)
```

fricas [A] time = 1.07, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\left(bx\log(x) - a\right)}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^2*x^2)
```

giac [A] time = 1.22, size = 47, normalized size = 1.42

$$-\frac{b\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2}\right|\right) - \frac{2\,a\sqrt{c}}{\sqrt{c}\,x - \sqrt{cx^2}}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(3/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.64

$$\frac{(bx\ln(x) - a)\,x^2}{\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(3/2),x)
[Out] x^2*(b*x*ln(x)-a)/(c*x^2)^(3/2)
```

maxima [A] time = 1.31, size = 21, normalized size = 0.64

$$\frac{b\log(x)}{c^{\frac{3}{2}}} - \frac{a}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b*log(x)/c^(3/2) - a/(sqrt(c*x^2)*c)
```

mupad [B] time = 0.25, size = 28, normalized size = 0.85

$$-\frac{a + b\,x - b\,\ln\left(x + |x|\right)\,\sqrt{x^2}}{c^{3/2}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] -(a + b*x - b*log(x + abs(x))*(x^2)^(1/2))/(c^(3/2)*(x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+bx)}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)/(c*x**2)**(3/2), x)
```

3.749 $\int \frac{a+bx}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(3/2),x]
[Out] -(a + b*x)^2/(2*a*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2acx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.76

$$\frac{x(-a-2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(3/2),x]
[Out] (x*(-a - 2*b*x))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 20, normalized size = 0.69

$$-\frac{x(a+2bx)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c*x^2)^(3/2),x]
[Out] -1/2*(x*(a + 2*b*x))/(c*x^2)^(3/2)
```

fricas [A] time = 0.71, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 17, normalized size = 0.59

$$-\frac{(2bx+a)\,x}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(3/2),x)
[Out] -1/2*x*(2*b*x+a)/(c*x^2)^(3/2)
```

maxima [A] time = 1.35, size = 23, normalized size = 0.79

$$-\frac{b}{\sqrt{cx^2}\,c}-\frac{a}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -b/(sqrt(c*x^2)*c) - 1/2*a/(c^(3/2)*x^2)
```

mupad [B] time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{3/2}\,x\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(3/2),x)
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(3/2)*x*(x^2)^(3/2))
```

sympy [A] time = 0.54, size = 34, normalized size = 1.17

$$-\frac{ax}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{bx^2}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] -a*x/(2*c**(3/2)*(x**2)**(3/2)) - b*x**2/(c**(3/2)*(x**2)**(3/2))
```

3.750 $\int \frac{a+bx}{x(cx^2)^{3/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 43}

$$-\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(3/2)),x]

[Out] -a/(3*c*x^2*Sqrt[c*x^2]) - b/(2*c*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{3cx^2\sqrt{cx^2}}-\frac{b}{2cx\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.61

$$\frac{cx^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(3/2)),x]

[Out] (c*x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{-2a-3bx}{6\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*(c*x^2)^(3/2)),x]
[Out] (-2*a - 3*b*x)/(6*(c*x^2)^(3/2))
```

fricas [A] time = 1.76, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{3bx+2a}{6\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(3/2),x)
[Out] -1/6*(3*b*x+2*a)/(c*x^2)^(3/2)
```

maxima [A] time = 1.32, size = 19, normalized size = 0.46

$$-\frac{b}{2\,c^{\frac{3}{2}}x^2}-\frac{a}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b/(c^(3/2)*x^2) - 1/3*a/(c^(3/2)*x^3)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}\,+3\,b\,x\,\sqrt{x^2}}{6\,c^{3/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(3/2)),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(3/2)*x^4)
```

sympy [**A**] time = 0.63, size = 32, normalized size = 0.78

$$-\frac{a}{3c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}-\frac{bx}{2c^{\frac{3}{2}}\left(x^{2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(3/2),x)
```

```
[Out] -a/(3*c**(3/2)*(x**2)**(3/2)) - b*x/(2*c**(3/2)*(x**2)**(3/2))
```

3.751 $\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(4*c*x^3*Sqrt[c*x^2]) - b/(3*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{4cx^3\sqrt{cx^2}}-\frac{b}{3cx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^2*x^5)
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{-3a-4bx}{12x\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] (-3*a - 4*b*x)/(12*x*(c*x^2)^(3/2))
```

fricas [A] time = 1.07, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{4bx+3a}{12\left(c\,x^2\right)^{\frac{3}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(3/2),x)
```

```
[Out] -1/12*(4*b*x+3*a)/x/(c*x^2)^(3/2)
```

maxima [A] time = 1.34, size = 19, normalized size = 0.46

$$-\frac{b}{3\,c^{\frac{3}{2}}x^3}-\frac{a}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*b/(c^(3/2)*x^3) - 1/4*a/(c^(3/2)*x^4)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}\,+4\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(3/2)),x)
```

```
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(3/2)*x^5)
```

sympy [A] time = 0.77, size = 32, normalized size = 0.78

$$-\frac{a}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{b}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a/(4*c**(3/2)*x*(x**2)**(3/2)) - b/(3*c**(3/2)*(x**2)**(3/2))
```

3.752 $\int \frac{a+bx}{x^3(cx^2)^{3/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(5*c*x^4*Sqrt[c*x^2]) - b/(4*c*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^3\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a}{5cx^4\sqrt{cx^2}}-\frac{b}{4cx^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-4a-5bx)}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-4*a - 5*b*x))/(20*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{-4a-5bx}{20x^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*(c*x^2)^(3/2)),x]
[Out] (-4*a - 5*b*x)/(20*x^2*(c*x^2)^(3/2))
```

fricas [A] time = 0.96, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx+4\,a)}{20\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^2*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bx+a}{\left(cx^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{5bx+4a}{20\left(c\,x^2\right)^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(3/2),x)
[Out] -1/20*(5*b*x+4*a)/x^2/(c*x^2)^(3/2)
```

maxima [A] time = 1.28, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{3}{2}}x^4}-\frac{a}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*b/(c^(3/2)*x^4) - 1/5*a/(c^(3/2)*x^5)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2}\,+5\,b\,x\,\sqrt{x^2}}{20\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(3/2)),x)
```

```
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(3/2)*x^6)
```

sympy [**A**] time = 0.93, size = 36, normalized size = 0.88

$$-\frac{a}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{b}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a/(5*c**(3/2)*x**2*(x**2)**(3/2)) - b/(4*c**(3/2)*x*(x**2)**(3/2))
```

3.753 $\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -a/(6*c*x^5*Sqrt[c*x^2]) - b/(5*c*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\left(cx^2\right)^{3/2}} \, dx &= \frac{x\int \frac{a+bx}{x^7} \, dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right) dx}{c\sqrt{cx^2}}\\
&= -\frac{a}{6cx^5\sqrt{cx^2}}-\frac{b}{5cx^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*(c*x^2)^(3/2)),x]
[Out] (-5*a - 6*b*x)/(30*x^3*(c*x^2)^(3/2))
```

fricas [A] time = 1.28, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{6bx+5a}{30\left(c\,x^2\right)^{\frac{3}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(3/2),x)
[Out] -1/30*(6*b*x+5*a)/x^3/(c*x^2)^(3/2)
```

maxima [A] time = 1.35, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{3}{2}}x^5}-\frac{a}{6\,c^{\frac{3}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/5*b/(c^(3/2)*x^5) - 1/6*a/(c^(3/2)*x^6)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}\,+6\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(3/2)*x^7)
```

sympy [A] time = 1.16, size = 37, normalized size = 0.90

$$-\frac{a}{6c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}-\frac{b}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(3/2),x)
[Out] -a/(6*c**(3/2)*x**3*(x**2)**(3/2)) - b/(5*c**(3/2)*x**2*(x**2)**(3/2))
```

3.754 $\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{bx\log(x)}{c^2\sqrt{cx^2}}-\frac{a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a/(c^2*Sqrt[c*x^2])) + (b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^2}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^2}+\frac{b}{x}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{c^2\sqrt{cx^2}}+\frac{bx\log(x)}{c^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.67

$$\frac{bx\log(x)-a}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (-a + b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.73

$$\frac{bx^5\log(x)-ax^4}{(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x))/(c*x^2)^(5/2),x]
[Out] (-(a*x^4) + b*x^5*Log[x])/(c*x^2)^(5/2)
```

fricas [A] time = 0.79, size = 23, normalized size = 0.70

$$\frac{\sqrt{cx^2}\left(bx\log(x)-a\right)}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log(x) - a)/(c^3*x^2)
```

giac [A] time = 1.02, size = 47, normalized size = 1.42

$$-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{2\,a\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] -(b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - 2*a*sqrt(c)/(sqrt(c)*x - sqrt(c*x^
2)))/c^(5/2)
```

maple [A] time = 0.00, size = 21, normalized size = 0.64

$$\frac{(bx\ln(x)-a)\,x^4}{(c\,x^2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)/(c*x^2)^(5/2),x)
[Out] x^4*(b*x*ln(x)-a)/(c*x^2)^(5/2)
```

maxima [A] time = 1.43, size = 24, normalized size = 0.73

$$-\frac{ax^2}{(cx^2)^{\frac{3}{2}}\,c}+\frac{b\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -a*x^2/((c*x^2)^(3/2)*c) + b*log(x)/c^(5/2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{x^3\,(a+b\,x)}{(c\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x))/(c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a+bx)}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)/(c*x**2)**(5/2), x)
```

3.755 $\int \frac{x^2(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^2/(2*a*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^2}{2ac^2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{x^3(-a-2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-a - 2*b*x))/(2*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.76

$$-\frac{x^3(a+2bx)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -1/2*(x^3*(a + 2*b*x))/(c*x^2)^(5/2)
```

fricas [A] time = 1.05, size = 21, normalized size = 0.72

$$-\frac{\sqrt{cx^2}\,(2\,bx+a)}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(c*x^2)*(2*b*x + a)/(c^3*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 19, normalized size = 0.66

$$-\frac{(2bx+a)\,x^3}{2\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] -1/2*x^3*(2*b*x+a)/(c*x^2)^(5/2)
```

maxima [A] time = 1.34, size = 26, normalized size = 0.90

$$-\frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -b*x^2/((c*x^2)^(3/2)*c) - 1/2*a/(c^(5/2)*x^2)
```

mupad [B] time = 0.15, size = 25, normalized size = 0.86

$$-\frac{2\,b\,x^3+a\,x^2}{2\,c^{5/2}\,x\,\left(x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(a*x^2 + 2*b*x^3)/(2*c^(5/2)*x*(x^2)^(3/2))
```

sympy [A] time = 0.93, size = 36, normalized size = 1.24

$$-\frac{ax^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{bx^4}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**3/(2*c**(5/2)*(x**2)**(5/2)) - b*x**4/(c**(5/2)*(x**2)**(5/2))
```

3.756 $\int \frac{x(a+bx)}{(cx^2)^{5/2}} \, dx$

Optimal. Leaf size=41

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {15, 43}

$$-\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -a/(3*c^2*x^2*Sqrt[c*x^2]) - b/(2*c^2*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a+bx)}{(cx^2)^{5/2}} \, dx &= \frac{x\int \frac{a+bx}{x^4} \, dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^4}+\frac{b}{x^3}\right) \, dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{3c^2x^2\sqrt{cx^2}}-\frac{b}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.59

$$\frac{x^2(-2a-3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x^2*(-2*a - 3*b*x))/(6*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$-\frac{x^2(2a+3bx)}{6\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x))/(c*x^2)^(5/2),x]
[Out] -1/6*(x^2*(2*a + 3*b*x))/(c*x^2)^(5/2)
```

fricas [A] time = 1.09, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(3\,bx+2\,a)}{6\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/6*sqrt(c*x^2)*(3*b*x + 2*a)/(c^3*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)x}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)*x/(c*x^2)^(5/2), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{(3bx+2a)\,x^2}{6\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)/(c*x^2)^(5/2),x)
[Out] -1/6*x^2*(3*b*x+2*a)/(c*x^2)^(5/2)
```

maxima [A] time = 1.39, size = 23, normalized size = 0.56

$$-\frac{a}{3\left(cx^2\right)^{\frac{3}{2}}c}-\frac{b}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*a/((c*x^2)^(3/2)*c) - 1/2*b/(c^(5/2)*x^2)
```

mupad [B] time = 0.15, size = 26, normalized size = 0.63

$$-\frac{2\,a\,\sqrt{x^2}+3\,b\,x\,\sqrt{x^2}}{6\,c^{5/2}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -(2*a*(x^2)^(1/2) + 3*b*x*(x^2)^(1/2))/(6*c^(5/2)*x^4)
```

sympy [A] time = 0.92, size = 37, normalized size = 0.90

$$-\frac{ax^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{bx^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x**2/(3*c**(5/2)*(x**2)**(5/2)) - b*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

3.757 $\int \frac{a+bx}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {15, 43}

$$-\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -a/(4*c^2*x^3*Sqrt[c*x^2]) - b/(3*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(cx^2)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^5}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^5}+\frac{b}{x^4}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{4c^2x^3\sqrt{cx^2}}-\frac{b}{3c^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(3a+4bx)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c*x^2)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[c*x^2]*(3*a + 4*b*x))/(c^3*x^5)
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.54

$$-\frac{x(3a+4bx)}{12\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c*x^2)^(5/2),x]
[Out] -1/12*(x*(3*a + 4*b*x))/(c*x^2)^(5/2)
```

fricas [A] time = 1.00, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(4\,bx+3\,a)}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*sqrt(c*x^2)*(4*b*x + 3*a)/(c^3*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 19, normalized size = 0.46

$$-\frac{(4bx+3a)\,x}{12\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(c*x^2)^(5/2),x)
[Out] -1/12*x*(4*b*x+3*a)/(c*x^2)^(5/2)
```

maxima [A] time = 1.37, size = 23, normalized size = 0.56

$$-\frac{b}{3\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*b/((c*x^2)^(3/2)*c) - 1/4*a/(c^(5/2)*x^4)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{3\,a\,\sqrt{x^2}\,+4\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c*x^2)^(5/2),x)
[Out] -(3*a*(x^2)^(1/2) + 4*b*x*(x^2)^(1/2))/(12*c^(5/2)*x^5)
```

sympy [A] time = 0.92, size = 36, normalized size = 0.88

$$-\frac{ax}{4c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{bx^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] -a*x/(4*c**(5/2)*(x**2)**(5/2)) - b*x**2/(3*c**(5/2)*(x**2)**(5/2))
```

3.758 $\int \frac{a+bx}{x(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(5*c^2*x^4*Sqrt[c*x^2]) - b/(4*c^2*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^6}+\frac{b}{x^5}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{5c^2x^4\sqrt{cx^2}}-\frac{b}{4c^2x^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(4a+5bx)}{20c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/20*(Sqrt[c*x^2]*(4*a + 5*b*x))/(c^3*x^6)
```

IntegrateAlgebraic [A] time = 0.02, size = 21, normalized size = 0.51

$$\frac{-4a-5bx}{20\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x*(c*x^2)^(5/2)),x]
[Out] (-4*a - 5*b*x)/(20*(c*x^2)^(5/2))
```

fricas [A] time = 0.74, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(5\,bx + 4\,a)}{20\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/20*sqrt(c*x^2)*(5*b*x + 4*a)/(c^3*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx + a}{\left(cx^2\right)^{\frac{5}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x), x)
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{5bx + 4a}{20\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x/(c*x^2)^(5/2),x)
[Out] -1/20*(5*b*x+4*a)/(c*x^2)^(5/2)
```

maxima [A] time = 1.33, size = 19, normalized size = 0.46

$$-\frac{b}{4\,c^{\frac{5}{2}}x^4} - \frac{a}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4*b/(c^(5/2)*x^4) - 1/5*a/(c^(5/2)*x^5)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{4\,a\,\sqrt{x^2} + 5\,b\,x\,\sqrt{x^2}}{20\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x*(c*x^2)^(5/2)),x)
[Out] -(4*a*(x^2)^(1/2) + 5*b*x*(x^2)^(1/2))/(20*c^(5/2)*x^6)
```

sympy [A] time = 1.12, size = 32, normalized size = 0.78

$$-\frac{a}{5c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}-\frac{bx}{4c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x/(c*x**2)**(5/2),x)
[Out] -a/(5*c**(5/2)*(x**2)**(5/2)) - b*x/(4*c**(5/2)*(x**2)**(5/2))
```

3.759 $\int \frac{a+bx}{x^2(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(6*c^2*x^5*Sqrt[c*x^2]) - b/(5*c^2*x^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^7}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a}{x^7}+\frac{b}{x^6}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a}{6c^2x^5\sqrt{cx^2}}-\frac{b}{5c^2x^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.66

$$-\frac{\sqrt{cx^2}\,(5a+6bx)}{30c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(5*a + 6*b*x))/(c^3*x^7)
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.59

$$\frac{-5a-6bx}{30x\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^2*(c*x^2)^(5/2)),x]
[Out] (-5*a - 6*b*x)/(30*x*(c*x^2)^(5/2))
```

fricas [A] time = 1.03, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(6\,bx+5\,a)}{30\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/30*sqrt(c*x^2)*(6*b*x + 5*a)/(c^3*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{6bx+5a}{30\left(c\,x^2\right)^{\frac{5}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^2/(c*x^2)^(5/2),x)
[Out] -1/30*(6*b*x+5*a)/x/(c*x^2)^(5/2)
```

maxima [A] time = 1.32, size = 19, normalized size = 0.46

$$-\frac{b}{5\,c^{\frac{5}{2}}x^5}-\frac{a}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*b/(c^(5/2)*x^5) - 1/6*a/(c^(5/2)*x^6)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{5\,a\,\sqrt{x^2}\,+6\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^2*(c*x^2)^(5/2)),x)
[Out] -(5*a*(x^2)^(1/2) + 6*b*x*(x^2)^(1/2))/(30*c^(5/2)*x^7)
```

sympy [A] time = 1.34, size = 32, normalized size = 0.78

$$-\frac{a}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a/(6*c**(5/2)*x*(x**2)**(5/2)) - b/(5*c**(5/2)*(x**2)**(5/2))
```

3.760 $\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(7*c^2*x^6*Sqrt[c*x^2]) - b/(6*c^2*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^8}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^8}+\frac{b}{x^7}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{7c^2x^6\sqrt{cx^2}}-\frac{b}{6c^2x^5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.54

$$\frac{c(-6a-7bx)}{42\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] (c*(-6*a - 7*b*x))/(42*(c*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 0.59

$$\frac{-6a-7bx}{42x^2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^3*(c*x^2)^(5/2)),x]
[Out] (-6*a - 7*b*x)/(42*x^2*(c*x^2)^(5/2))
```

fricas [A] time = 1.10, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(7\,bx+6\,a)}{42\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/42*sqrt(c*x^2)*(7*b*x + 6*a)/(c^3*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{bx+a}{\left(cx^2\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)/((c*x^2)^(5/2)*x^3), x)
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{7bx+6a}{42\left(c\,x^2\right)^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^3/(c*x^2)^(5/2),x)
[Out] -1/42*(7*b*x+6*a)/x^2/(c*x^2)^(5/2)
```

maxima [A] time = 1.37, size = 19, normalized size = 0.46

$$-\frac{b}{6\,c^{\frac{5}{2}}x^6}-\frac{a}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6*b/(c^(5/2)*x^6) - 1/7*a/(c^(5/2)*x^7)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{6\,a\,\sqrt{x^2}\,+7\,b\,x\,\sqrt{x^2}}{42\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^3*(c*x^2)^(5/2)),x)
```

```
[Out] -(6*a*(x^2)^(1/2) + 7*b*x*(x^2)^(1/2))/(42*c^(5/2)*x^8)
```

sympy [A] time = 1.64, size = 36, normalized size = 0.88

$$-\frac{a}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a/(7*c**(5/2)*x**2*(x**2)**(5/2)) - b/(6*c**(5/2)*x*(x**2)**(5/2))
```

3.761 $\int \frac{a+bx}{x^4(cx^2)^{5/2}} dx$

Optimal. Leaf size=41

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a/(8*c^2*x^7*Sqrt[c*x^2]) - b/(7*c^2*x^6*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{a+bx}{x^9}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a}{x^9}+\frac{b}{x^8}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a}{8c^2x^7\sqrt{cx^2}}-\frac{b}{7c^2x^6\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.59

$$\frac{-7a-8bx}{56x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-7*a - 8*b*x)/(56*x^3*(c*x^2)^(5/2))
```

fricas [A] time = 0.83, size = 23, normalized size = 0.56

$$-\frac{\sqrt{cx^2}\,(8\,bx+7\,a)}{56\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/56*sqrt(c*x^2)*(8*b*x + 7*a)/(c^3*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 21, normalized size = 0.51

$$-\frac{8bx+7a}{56\left(c\,x^2\right)^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/x^4/(c*x^2)^(5/2),x)
```

```
[Out] -1/56*(8*b*x+7*a)/x^3/(c*x^2)^(5/2)
```

maxima [A] time = 1.34, size = 19, normalized size = 0.46

$$-\frac{b}{7\,c^{\frac{5}{2}}x^7}-\frac{a}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/7*b/(c^(5/2)*x^7) - 1/8*a/(c^(5/2)*x^8)
```

mupad [B] time = 0.16, size = 26, normalized size = 0.63

$$-\frac{7\,a\,\sqrt{x^2}+8\,b\,x\,\sqrt{x^2}}{56\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(7*a*(x^2)^(1/2) + 8*b*x*(x^2)^(1/2))/(56*c^(5/2)*x^9)
```

sympy [A] time = 1.97, size = 37, normalized size = 0.90

$$-\frac{a}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}-\frac{b}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/x**4/(c*x**2)**(5/2),x)
[Out] -a/(8*c**(5/2)*x**3*(x**2)**(5/2)) - b/(7*c**(5/2)*x**2*(x**2)**(5/2))
```

3.762 $\int x^3 \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=57

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^4*Sqrt[c*x^2])/5 + (a*b*x^5*Sqrt[c*x^2])/3 + (b^2*x^6*Sqrt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^4(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\
&= \frac{1}{5}a^2x^4\sqrt{cx^2} + \frac{1}{3}abx^5\sqrt{cx^2} + \frac{1}{7}b^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.61

$$\frac{1}{105}x^4\sqrt{cx^2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^4*Sqrt[c*x^2]*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

fricas [A] time = 0.86, size = 33, normalized size = 0.58

$$\frac{1}{105} \left(15 b^2 x^6 + 35 a b x^5 + 21 a^2 x^4\right) \sqrt{c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/105*(15*b^2*x^6 + 35*a*b*x^5 + 21*a^2*x^4)*sqrt(c*x^2)
```

giac [A] time = 1.08, size = 35, normalized size = 0.61

$$\frac{1}{105} \left(15 b^2 x^7 \operatorname{sgn}(x) + 35 a b x^6 \operatorname{sgn}(x) + 21 a^2 x^5 \operatorname{sgn}(x)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(15 b^2 x^2 + 35 a b x + 21 a^2\right) \sqrt{c x^2}\, x^4}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2*(c*x^2)^(1/2),x)
[Out] 1/105*x^4*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(1/2)
```

maxima [A] time = 1.39, size = 54, normalized size = 0.95

$$\frac{\left(c x^2\right)^{\frac{3}{2}} b^2 x^4}{7 c} + \frac{\left(c x^2\right)^{\frac{3}{2}} a b x^3}{3 c} + \frac{\left(c x^2\right)^{\frac{3}{2}} a^2 x^2}{5 c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/7*(c*x^2)^(3/2)*b^2*x^4/c + 1/3*(c*x^2)^(3/2)*a*b*x^3/c + 1/5*(c*x^2)^(3/
2)*a^2*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \sqrt{c x^2}\, (a + b x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2,x)
[Out] int(x^3*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.59, size = 60, normalized size = 1.05

$$\frac{a^2 \sqrt{c} x^4 \sqrt{x^2}}{5} + \frac{a b \sqrt{c} x^5 \sqrt{x^2}}{3} + \frac{b^2 \sqrt{c} x^6 \sqrt{x^2}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**4*sqrt(x**2)/5 + a*b*sqrt(c)*x**5*sqrt(x**2)/3 + b**2*sqrt(
c)*x**6*sqrt(x**2)/7
```

3.763 $\int x^2\sqrt{cx^2}\,(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (a^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*x^4*Sqrt[c*x^2])/5 + (b^2*x^5*Sqrt[c*x^2])
/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x^3(a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(a^2x^3+2abx^4+b^2x^5\right)dx}{x}\\ &= \frac{1}{4}a^2x^3\sqrt{cx^2}+\frac{2}{5}abx^4\sqrt{cx^2}+\frac{1}{6}b^2x^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.61

$$\frac{1}{60}x^3\sqrt{cx^2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^3*Sqrt[c*x^2]*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

fricas [A] time = 0.75, size = 33, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^5 + 24\,abx^4 + 15\,a^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*x^5 + 24*a*b*x^4 + 15*a^2*x^3)*sqrt(c*x^2)
```

giac [A] time = 1.04, size = 35, normalized size = 0.61

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x) + 24\,abx^5\mathrm{sgn}(x) + 15\,a^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(10b^2x^2 + 24abx + 15a^2\right)\sqrt{c\,x^2}\;x^3}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2*(c*x^2)^(1/2),x)
```

```
[Out] 1/60*x^3*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(1/2)
```

maxima [A] time = 1.35, size = 52, normalized size = 0.91

$$\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x^3}{6\,c} + \frac{2\left(cx^2\right)^{\frac{3}{2}}abx^2}{5\,c} + \frac{\left(cx^2\right)^{\frac{3}{2}}a^2x}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(3/2)*b^2*x^3/c + 2/5*(c*x^2)^(3/2)*a*b*x^2/c + 1/4*(c*x^2)^(3/
2)*a^2*x/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\,\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.46, size = 61, normalized size = 1.07

$$\frac{a^2\sqrt{c}\,x^3\sqrt{x^2}}{4} + \frac{2ab\sqrt{c}\,x^4\sqrt{x^2}}{5} + \frac{b^2\sqrt{c}\,x^5\sqrt{x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**3*sqrt(x**2)/4 + 2*a*b*sqrt(c)*x**4*sqrt(x**2)/5 + b**2*sqr
t(c)*x**5*sqrt(x**2)/6
```

3.764 $\int x\sqrt{cx^2}(a+bx)^2\,dx$

Optimal. Leaf size=57

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x^2*Sqrt[c*x^2])/3 + (a*b*x^3*Sqrt[c*x^2])/2 + (b^2*x^4*Sqrt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)^2\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^2\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(a^2x^2+2abx^3+b^2x^4\right)dx}{x}\\&=\frac{1}{3}a^2x^2\sqrt{cx^2}+\frac{1}{2}abx^3\sqrt{cx^2}+\frac{1}{5}b^2x^4\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

IntegrateAlgebraic [A] time = 0.02, size = 35, normalized size = 0.61

$$\frac{1}{30}x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

fricas [A] time = 1.11, size = 33, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)
```

giac [A] time = 1.11, size = 35, normalized size = 0.61

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\sqrt{c\,x^2}\;x^2}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2*(c*x^2)^(1/2),x)
[Out] 1/30*x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(1/2)
```

maxima [A] time = 1.31, size = 49, normalized size = 0.86

$$\frac{\left(cx^2\right)^{\frac{3}{2}}b^2x^2}{5\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}abx}{2\,c}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*(c*x^2)^(3/2)*b^2*x^2/c + 1/2*(c*x^2)^(3/2)*a*b*x/c + 1/3*(c*x^2)^(3/2)
*a^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^2,x)
[Out] int(x*(c*x^2)^(1/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.37, size = 60, normalized size = 1.05

$$\frac{a^2\sqrt{c}\,x^2\sqrt{x^2}}{3}+\frac{ab\sqrt{c}\,x^3\sqrt{x^2}}{2}+\frac{b^2\sqrt{c}\,x^4\sqrt{x^2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2*(c*x**2)**(1/2),x)
```

```
[Out] a**2*sqrt(c)*x**2*sqrt(x**2)/3 + a*b*sqrt(c)*x**3*sqrt(x**2)/2 + b**2*sqrt(
c)*x**4*sqrt(x**2)/5
```

3.765 $\int \sqrt{cx^2}\,(a+bx)^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*x*Sqrt[c*x^2])/2 + (2*a*b*x^2*Sqrt[c*x^2])/3 + (b^2*x^3*Sqrt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2x\sqrt{cx^2}+\frac{2}{3}abx^2\sqrt{cx^2}+\frac{1}{4}b^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.60

$$\frac{1}{12}x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x)^2,x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

fricas [A] time = 1.12, size = 31, normalized size = 0.56

$$\frac{1}{12}\left(3\,b^2x^3 + 8\,abx^2 + 6\,a^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)
```

giac [A] time = 0.95, size = 35, normalized size = 0.64

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x) + 8\,abx^3\mathrm{sgn}(x) + 6\,a^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*sqrt(c)
```

maple [A] time = 0.00, size = 30, normalized size = 0.55

$$\frac{\left(3b^2x^2 + 8abx + 6a^2\right)\sqrt{c\,x^2}\;x}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2),x)
[Out] 1/12*x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(1/2)
```

maxima [A] time = 1.35, size = 44, normalized size = 0.80

$$\frac{1}{2}\sqrt{cx^2}\,a^2x + \frac{\left(cx^2\right)^{\frac{3}{2}}b^2x}{4\,c} + \frac{2\left(cx^2\right)^{\frac{3}{2}}ab}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(c*x^2)*a^2*x + 1/4*(c*x^2)^(3/2)*b^2*x/c + 2/3*(c*x^2)^(3/2)*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{c\,x^2}\,\left(a + b\,x\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^2,x)
[Out] int((c*x^2)^(1/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.30, size = 60, normalized size = 1.09

$$\frac{a^2\sqrt{c}\,x\sqrt{x^2}}{2} + \frac{2ab\sqrt{c}\,x^2\sqrt{x^2}}{3} + \frac{b^2\sqrt{c}\,x^3\sqrt{x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2),x)
[Out] a**2*sqrt(c)*x*sqrt(x**2)/2 + 2*a*b*sqrt(c)*x**2*sqrt(x**2)/3 + b**2*sqrt(c
)*x**3*sqrt(x**2)/4
```

3.766 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x} dx$

Optimal. Leaf size=26

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x}\,dx &= \frac{\sqrt{cx^2}\,\int (a+bx)^2\,dx}{x}\\ &= \frac{\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{cx(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (c*x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 1.19

$$\frac{1}{3}\sqrt{cx^2}\,\left(3a^2+3abx+b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(3*a^2 + 3*a*b*x + b^2*x^2))/3
```

fricas [A] time = 1.08, size = 27, normalized size = 1.04

$$\frac{1}{3}\left(b^2x^2 + 3\,abx + 3\,a^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)
```

giac [A] time = 0.97, size = 29, normalized size = 1.12

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b} - \frac{a^3\mathrm{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*sqrt(c)
```

maple [A] time = 0.00, size = 28, normalized size = 1.08

$$\frac{\left(b^2x^2 + 3abx + 3a^2\right)\sqrt{c\,x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x,x)
```

```
[Out] 1/3*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x, x)
```

sympy [B] time = 0.30, size = 51, normalized size = 1.96

$$a^2\sqrt{c}\,\sqrt{x^2} + ab\sqrt{c}\,x\sqrt{x^2} + \frac{b^2\sqrt{c}\,x^2\sqrt{x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x,x)
```

```
[Out] a**2*sqrt(c)*sqrt(x**2) + a*b*sqrt(c)*x*sqrt(x**2) + b**2*sqrt(c)*x**2*sqrt
(x**2)/3
```

3.767 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\log(x)}{x}+2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] 2*a*b*Sqrt[c*x^2] + (b^2*x*Sqrt[c*x^2])/2 + (a^2*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\
&= 2ab\sqrt{cx^2}+\frac{1}{2}b^2x\sqrt{cx^2}+\frac{a^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{cx\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{a^2\log(x)}{x}+\frac{1}{2}\left(4ab+b^2x\right)\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^2,x]
[Out] Sqrt[c*x^2]*((4*a*b + b^2*x)/2 + (a^2*Log[x])/x)
```

fricas [A] time = 0.76, size = 32, normalized size = 0.65

$$\frac{\left(b^2x^2 + 4\,abx + 2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/x
```

giac [A] time = 1.08, size = 32, normalized size = 0.65

$$\frac{1}{2}\left(b^2x^2\mathrm{sgn}(x) + 4\,abx\mathrm{sgn}(x) + 2\,a^2\log(|x|)\,\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="giac")
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*sqrt(c)
```

maple [A] time = 0.01, size = 33, normalized size = 0.67

$$\frac{\sqrt{c\,x^2}\,\left(b^2x^2 + 2a^2\ln(x) + 4abx\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^2,x)
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a + b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a + bx)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**2, x)
```

3.768 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} \, dx$

Optimal. Leaf size=49

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}+b^2\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] b^2*Sqrt[c*x^2] - (a^2*Sqrt[c*x^2])/x^2 + (2*a*b*Sqrt[c*x^2]*Log[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^2}{x^3} \, dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^2} \, dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right) dx}{x}\\ &= b^2\sqrt{cx^2}-\frac{a^2\sqrt{cx^2}}{x^2}+\frac{2ab\sqrt{cx^2}\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.63

$$\frac{c\left(-a^2+2abx\log(x)+b^2x^2\right)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] (c*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/Sqrt[c*x^2]
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.76

$$\sqrt{cx^2}\left(\frac{b^2x^2-a^2}{x^2}+\frac{2ab\log(x)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^3,x]
[Out] Sqrt[c*x^2]*((-a^2 + b^2*x^2)/x^2 + (2*a*b*Log[x])/x)
```

fricas [A] time = 1.21, size = 31, normalized size = 0.63

$$\frac{\left(b^{2} x^{2}+2 \, a b x \log\left(x\right)-a^{2}\right) \sqrt{c x^{2}}}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/x^2
```

giac [A] time = 1.02, size = 31, normalized size = 0.63

$$\left(b^{2} x \mathrm{sgn}\left(x\right)+2 \, a b \log\left({\left| x \right|}\right) \mathrm{sgn}\left(x\right)-\frac{a^{2} \mathrm{sgn}\left(x\right)}{x}\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="giac")
[Out] (b^2*x*sgn(x) + 2*a*b*log(abs(x))*sgn(x) - a^2*sgn(x)/x)*sqrt(c)
```

maple [A] time = 0.01, size = 32, normalized size = 0.65

$$\frac{\sqrt{c \,x^{2}}\, \left(2 a b x \ln \left(x \right)+b^{2} x^{2}-a^{2}\right)}{x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^3,x)
[Out] (c*x^2)^(1/2)*(2*a*b*ln(x)*x+b^2*x^2-a^2)/x^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^3,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3,x)
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c x^{2}} \left(a + b x\right)^{2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**3, x)
```

3.769 $\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

Rubi [A] time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] -(a^2*Sqrt[c*x^2])/(2*x^3) - (2*a*b*Sqrt[c*x^2])/x^2 + (b^2*Sqrt[c*x^2]*Log
[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}(a+bx)^2}{x^4}\,dx &= \frac{\sqrt{cx^2}\int \frac{(a+bx)^2}{x^3}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{x}\\
&= -\frac{a^2\sqrt{cx^2}}{2x^3}-\frac{2ab\sqrt{cx^2}}{x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 38, normalized size = 0.70

$$\sqrt{cx^2}\left(\frac{-a^2-4abx}{2x^3}+\frac{b^2\log(x)}{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^2)/x^4,x]
[Out] Sqrt[c*x^2]*((-a^2 - 4*a*b*x)/(2*x^3) + (b^2*Log[x])/x)
```

fricas [A] time = 1.36, size = 33, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/x^3
```

giac [A] time = 0.99, size = 35, normalized size = 0.65

$$\frac{1}{2}\left(2\,b^2\log\left(|x|\right)\mathrm{sgn}(x)-\frac{4\,abx\mathrm{sgn}(x)+a^2\mathrm{sgn}(x)}{x^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] 1/2*(2*b^2*log(abs(x))*sgn(x) - (4*a*b*x*sgn(x) + a^2*sgn(x))/x^2)*sqrt(c)
```

maple [A] time = 0.01, size = 34, normalized size = 0.63

$$\frac{\sqrt{c\,x^2}\,\left(2b^2x^2\ln(x)-4abx-a^2\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(c*x^2)^(1/2)/x^4,x)
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*ln(x)*x^2-4*a*b*x-a^2)/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(c*x^2)^(1/2)/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(1/2)*(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}\,(a+bx)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(c*x**2)**(1/2)/x**4,x)
[Out] Integral(sqrt(c*x**2)*(a + b*x)**2/x**4, x)
```

3.770 $\int x^3 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = $\{15, 43\}$

$$\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^6*Sqrt[c*x^2])/7 + (a*b*c*x^7*Sqrt[c*x^2])/4 + (b^2*c*x^8*Sqrt[c*x
^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^6(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^6+2abx^7+b^2x^8\right)dx}{x}\\&=\frac{1}{7}a^2cx^6\sqrt{cx^2}+\frac{1}{4}abcx^7\sqrt{cx^2}+\frac{1}{9}b^2cx^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{252}x^4\left(cx^2\right)^{3/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^4*(c*x^2)^(3/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

fricas [A] time = 0.84, size = 36, normalized size = 0.60

$$\frac{1}{252}\left(28\,b^2cx^8 + 63\,abcx^7 + 36\,a^2cx^6\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/252*(28*b^2*c*x^8 + 63*a*b*c*x^7 + 36*a^2*c*x^6)*sqrt(c*x^2)
```

giac [A] time = 1.09, size = 35, normalized size = 0.58

$$\frac{1}{252}\left(28\,b^2x^9\text{sgn}(x) + 63\,abx^8\text{sgn}(x) + 36\,a^2x^7\text{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 1/252*(28*b^2*x^9*sgn(x) + 63*a*b*x^8*sgn(x) + 36*a^2*x^7*sgn(x))*c^(3/2)
```

maple [A] time = 0.01, size = 32, normalized size = 0.53

$$\frac{\left(28b^2x^2 + 63abx + 36a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^4}{252}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x)
[Out] 1/252*x^4*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(3/2)
```

maxima [A] time = 1.35, size = 54, normalized size = 0.90

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^4}{9\,c} + \frac{\left(cx^2\right)^{\frac{5}{2}}abx^3}{4\,c} + \frac{\left(cx^2\right)^{\frac{5}{2}}a^2x^2}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/9*(c*x^2)^(5/2)*b^2*x^4/c + 1/4*(c*x^2)^(5/2)*a*b*x^3/c + 1/7*(c*x^2)^(5/
2)*a^2*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^3\left(c\,x^2\right)^{3/2}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2,x)
[Out] int(x^3*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

sympy [A] time = 1.50, size = 60, normalized size = 1.00

$$\frac{a^2c^{\frac{3}{2}}x^4\left(x^2\right)^{\frac{3}{2}}}{7}+\frac{abc^{\frac{3}{2}}x^5\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{b^2c^{\frac{3}{2}}x^6\left(x^2\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**4*(x**2)**(3/2)/7 + a*b*c**(3/2)*x**5*(x**2)**(3/2)/4 + b*
*2*c**(3/2)*x**6*(x**2)**(3/2)/9
```

3.771 $\int x^2 \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^5*Sqrt[c*x^2])/6 + (2*a*b*c*x^6*Sqrt[c*x^2])/7 + (b^2*c*x^7*Sqrt[c
*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^5(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^5+2abx^6+b^2x^7\right)dx}{x}\\&=\frac{1}{6}a^2cx^5\sqrt{cx^2}+\frac{2}{7}abcx^6\sqrt{cx^2}+\frac{1}{8}b^2cx^7\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{168}x^3\left(cx^2\right)^{3/2}\left(28a^2+48abx+21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^3*(c*x^2)^(3/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

fricas [A] time = 1.14, size = 36, normalized size = 0.60

$$\frac{1}{168}\left(21\,b^2cx^7+48\,abcx^6+28\,a^2cx^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c*x^7 + 48*a*b*c*x^6 + 28*a^2*c*x^5)*sqrt(c*x^2)
```

giac [A] time = 1.08, size = 35, normalized size = 0.58

$$\frac{1}{168}\left(21\,b^2x^8\mathrm{sgn}(x)+48\,abx^7\mathrm{sgn}(x)+28\,a^2x^6\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*x^8*sgn(x) + 48*a*b*x^7*sgn(x) + 28*a^2*x^6*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.53

$$\frac{\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^3}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x^3*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 52, normalized size = 0.87

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^3}{8\,c}+\frac{2\left(cx^2\right)^{\frac{5}{2}}abx^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2x}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(c*x^2)^(5/2)*b^2*x^3/c + 2/7*(c*x^2)^(5/2)*a*b*x^2/c + 1/6*(c*x^2)^(5/
2)*a^2*x/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(c\,x^2\right)^{3/2}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int(x^2*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

sympy [A] time = 1.22, size = 61, normalized size = 1.02

$$\frac{a^{2}c^{\frac{3}{2}}x^{3}\left(x^{2}\right)^{\frac{3}{2}}}{6}+\frac{2abc^{\frac{3}{2}}x^{4}\left(x^{2}\right)^{\frac{3}{2}}}{7}+\frac{b^{2}c^{\frac{3}{2}}x^{5}\left(x^{2}\right)^{\frac{3}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**3*(x**2)**(3/2)/6 + 2*a*b*c**(3/2)*x**4*(x**2)**(3/2)/7 +
b**2*c**(3/2)*x**5*(x**2)**(3/2)/8
```

3.772 $\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx$

Optimal. Leaf size=60

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = $\{15, 43\}$

$$\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^4*Sqrt[c*x^2])/5 + (a*b*c*x^5*Sqrt[c*x^2])/3 + (b^2*c*x^6*Sqrt[c*x
^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{3/2}(a+bx)^2\,dx&=\frac{\left(c\sqrt{cx^2}\right)\int x^4(a+bx)^2\,dx}{x}\\&=\frac{\left(c\sqrt{cx^2}\right)\int\left(a^2x^4+2abx^5+b^2x^6\right)dx}{x}\\&=\frac{1}{5}a^2cx^4\sqrt{cx^2}+\frac{1}{3}abcx^5\sqrt{cx^2}+\frac{1}{7}b^2cx^6\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.58

$$\frac{1}{105}x^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x)^2,x]
[Out] (x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

fricas [A] time = 1.21, size = 36, normalized size = 0.60

$$\frac{1}{105}\left(15\,b^2cx^6+35\,abcx^5+21\,a^2cx^4\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
[Out] 1/105*(15*b^2*c*x^6 + 35*a*b*c*x^5 + 21*a^2*c*x^4)*sqrt(c*x^2)
```

giac [A] time = 1.18, size = 35, normalized size = 0.58

$$\frac{1}{105}\left(15\,b^2x^7\mathrm{sgn}(x)+35\,abx^6\mathrm{sgn}(x)+21\,a^2x^5\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
[Out] 1/105*(15*b^2*x^7*sgn(x) + 35*a*b*x^6*sgn(x) + 21*a^2*x^5*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.53

$$\frac{\left(15b^2x^2+35abx+21a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x^2}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^2,x)
[Out] 1/105*x^2*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(3/2)
```

maxima [A] time = 1.31, size = 49, normalized size = 0.82

$$\frac{\left(cx^2\right)^{\frac{5}{2}}b^2x^2}{7\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}abx}{3\,c}+\frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
[Out] 1/7*(c*x^2)^(5/2)*b^2*x^2/c + 1/3*(c*x^2)^(5/2)*a*b*x/c + 1/5*(c*x^2)^(5/2)
*a^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(c\,x^2\right)^{3/2}(a+b\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^2,x)
[Out] int(x*(c*x^2)^(3/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.97, size = 60, normalized size = 1.00

$$\frac{a^{2}c^{\frac{3}{2}}x^{2}\left(x^{2}\right)^{\frac{3}{2}}}{5}+\frac{abc^{\frac{3}{2}}x^{3}\left(x^{2}\right)^{\frac{3}{2}}}{3}+\frac{b^{2}c^{\frac{3}{2}}x^{4}\left(x^{2}\right)^{\frac{3}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x**2*(x**2)**(3/2)/5 + a*b*c**(3/2)*x**3*(x**2)**(3/2)/3 + b*
*2*c**(3/2)*x**4*(x**2)**(3/2)/7
```

3.773 $\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=60

$$\frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*x^3*Sqrt[c*x^2])/4 + (2*a*b*c*x^4*Sqrt[c*x^2])/5 + (b^2*c*x^5*Sqrt[c
*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^2 \, dx}{x} \\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^3 + 2abx^4 + b^2x^5\right) dx}{x} \\ &= \frac{1}{4}a^2cx^3\sqrt{cx^2} + \frac{2}{5}abcx^4\sqrt{cx^2} + \frac{1}{6}b^2cx^5\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2 + 24abx + 10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.55

$$\frac{1}{60}x\left(cx^2\right)^{3/2}\left(15a^2 + 24abx + 10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

fricas [A] time = 0.93, size = 36, normalized size = 0.60

$$\frac{1}{60}\left(10\,b^2cx^5 + 24\,abcx^4 + 15\,a^2cx^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^2*c*x^5 + 24*a*b*c*x^4 + 15*a^2*c*x^3)*sqrt(c*x^2)
```

giac [A] time = 1.13, size = 35, normalized size = 0.58

$$\frac{1}{60}\left(10\,b^2x^6\mathrm{sgn}(x) + 24\,abx^5\mathrm{sgn}(x) + 15\,a^2x^4\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^2*x^6*sgn(x) + 24*a*b*x^5*sgn(x) + 15*a^2*x^4*sgn(x))*c^(3/2)
```

maple [A] time = 0.01, size = 30, normalized size = 0.50

$$\frac{\left(10b^2x^2 + 24abx + 15a^2\right)\left(c\,x^2\right)^{\frac{3}{2}} x}{60}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] 1/60*x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(3/2)
```

maxima [A] time = 1.36, size = 44, normalized size = 0.73

$$\frac{1}{4}\left(cx^2\right)^{\frac{3}{2}} a^2x + \frac{\left(cx^2\right)^{\frac{5}{2}} b^2x}{6\,c} + \frac{2\left(cx^2\right)^{\frac{5}{2}} ab}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c*x^2)^(3/2)*a^2*x + 1/6*(c*x^2)^(5/2)*b^2*x/c + 2/5*(c*x^2)^(5/2)*a*b
/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)*(a + b*x)^2, x)
```

sympy [A] time = 0.77, size = 60, normalized size = 1.00

$$\frac{a^2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}+\frac{2abc^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}+\frac{b^2c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(3/2)*x*(x**2)**(3/2)/4 + 2*a*b*c**(3/2)*x**2*(x**2)**(3/2)/5 + b**
2*c**(3/2)*x**3*(x**2)**(3/2)/6
```

3.774 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x} dx$

Optimal. Leaf size=60

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c*x^2*Sqrt[c*x^2])/3 + (a*b*c*x^3*Sqrt[c*x^2])/2 + (b^2*c*x^4*Sqrt[c*x
^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^2}{x} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^2\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x^2+2abx^3+b^2x^4\right)dx}{x} \\
&= \frac{1}{3}a^2cx^2\sqrt{cx^2} + \frac{1}{2}abcx^3\sqrt{cx^2} + \frac{1}{5}b^2cx^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 36, normalized size = 0.60

$$\frac{1}{30}cx^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 0.53

$$\frac{1}{30}\left(cx^2\right)^{3/2}\left(10a^2+15abx+6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x,x]
[Out] ((c*x^2)^(3/2)*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

fricas [A] time = 1.07, size = 36, normalized size = 0.60

$$\frac{1}{30}\left(6\,b^2cx^4+15\,abcx^3+10\,a^2cx^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/30*(6*b^2*c*x^4 + 15*a*b*c*x^3 + 10*a^2*c*x^2)*sqrt(c*x^2)
```

giac [A] time = 0.95, size = 35, normalized size = 0.58

$$\frac{1}{30}\left(6\,b^2x^5\mathrm{sgn}(x)+15\,abx^4\mathrm{sgn}(x)+10\,a^2x^3\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/30*(6*b^2*x^5*sgn(x) + 15*a*b*x^4*sgn(x) + 10*a^2*x^3*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.48

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x,x)
[Out] 1/30*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(3/2)
```

maxima [A] time = 1.30, size = 40, normalized size = 0.67

$$\frac{1}{2}\left(cx^2\right)^{\frac{3}{2}}abx+\frac{1}{3}\left(cx^2\right)^{\frac{3}{2}}a^2+\frac{\left(cx^2\right)^{\frac{5}{2}}b^2}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/2*(c*x^2)^(3/2)*a*b*x + 1/3*(c*x^2)^(3/2)*a^2 + 1/5*(c*x^2)^(5/2)*b^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x, x)
```

sympy [A] time = 0.79, size = 54, normalized size = 0.90

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}+\frac{abc^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{2}+\frac{b^2c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/3 + a*b*c**(3/2)*x*(x**2)**(3/2)/2 + b**2*c**(3
/2)*x**2*(x**2)**(3/2)/5
```

3.775 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx$

Optimal. Leaf size=58

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c*x*Sqrt[c*x^2])/2 + (2*a*b*c*x^2*Sqrt[c*x^2])/3 + (b^2*c*x^3*Sqrt[c*x
^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2cx\sqrt{cx^2}+\frac{2}{3}abcx^2\sqrt{cx^2}+\frac{1}{4}b^2cx^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 0.59

$$\frac{1}{12}cx\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.60

$$\frac{(cx^2)^{3/2}\left(6a^2+8abx+3b^2x^2\right)}{12x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x]
[Out] ((c*x^2)^(3/2)*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*x)
```

fricas [A] time = 1.06, size = 34, normalized size = 0.59

$$\frac{1}{12}\left(3\,b^2cx^3+8\,abcx^2+6\,a^2cx\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] 1/12*(3*b^2*c*x^3 + 8*a*b*c*x^2 + 6*a^2*c*x)*sqrt(c*x^2)
```

giac [A] time = 1.03, size = 35, normalized size = 0.60

$$\frac{1}{12}\left(3\,b^2x^4\mathrm{sgn}(x)+8\,abx^3\mathrm{sgn}(x)+6\,a^2x^2\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="giac")
[Out] 1/12*(3*b^2*x^4*sgn(x) + 8*a*b*x^3*sgn(x) + 6*a^2*x^2*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.55

$$\frac{\left(3b^2x^2+8abx+6a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{12x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^2,x)
[Out] 1/12/x*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(3/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2,x)
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^2, x)
```

sympy [A] time = 0.80, size = 54, normalized size = 0.93

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x}+\frac{2abc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}+\frac{b^2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(3/2)*(x**2)**(3/2)/(2*x) + 2*a*b*c**(3/2)*(x**2)**(3/2)/3 + b**2*c
**(3/2)*x*(x**2)**(3/2)/4
```

3.776 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=27

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
[Out] (c*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.96

$$\frac{(cx^2)^{3/2}(a+bx)^3}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
[Out] ((c*x^2)^(3/2)*(a + b*x)^3)/(3*b*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 1.26

$$\frac{(cx^2)^{3/2}\left(3a^2+3abx+b^2x^2\right)}{3x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x]
```

[Out] ((c*x^2)^(3/2)*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*x^2)

fricas [A] time = 1.06, size = 30, normalized size = 1.11

$$\frac{1}{3}\left(b^2cx^2 + 3\,abcx + 3\,a^2c\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="fricas")

[Out] 1/3*(b^2*c*x^2 + 3*a*b*c*x + 3*a^2*c)*sqrt(c*x^2)

giac [A] time = 1.22, size = 29, normalized size = 1.07

$$\frac{1}{3}\left(\frac{(bx+a)^3\mathrm{sgn}(x)}{b} - \frac{a^3\mathrm{sgn}(x)}{b}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="giac")

[Out] 1/3*((b*x + a)^3*sgn(x)/b - a^3*sgn(x)/b)*c^(3/2)

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(b^2x^2 + 3abx + 3a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^3,x)

[Out] 1/3/x^2*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(3/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3,x)

[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^3, x)

sympy [B] time = 0.94, size = 51, normalized size = 1.89

$$\frac{a^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x^2} + \frac{abc^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{x} + \frac{b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**3,x)

[Out] a**2*c**(3/2)*(x**2)**(3/2)/x**2 + a*b*c**(3/2)*(x**2)**(3/2)/x + b**2*c**(3/2)*(x**2)**(3/2)/3

3.777 $\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx$

Optimal. Leaf size=52

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}+2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] 2*a*b*c*Sqrt[c*x^2] + (b^2*c*x*Sqrt[c*x^2])/2 + (a^2*c*Sqrt[c*x^2]*Log[x])/
x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\ &= 2abc\sqrt{cx^2}+\frac{1}{2}b^2cx\sqrt{cx^2}+\frac{a^2c\sqrt{cx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.65

$$\frac{(cx^2)^{3/2}\left(2a^2\log(x)+bx(4a+bx)\right)}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.71

$$(cx^2)^{3/2}\left(\frac{a^2\log(x)}{x^3}+\frac{4ab+b^2x}{2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x]
[Out] (c*x^2)^(3/2)*((4*a*b + b^2*x)/(2*x^2) + (a^2*Log[x])/x^3)
```

fricas [A] time = 1.13, size = 35, normalized size = 0.67

$$\frac{\left(b^2cx^2+4\,abcx+2\,a^2c\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] 1/2*(b^2*c*x^2 + 4*a*b*c*x + 2*a^2*c*log(x))*sqrt(c*x^2)/x
```

giac [A] time = 1.20, size = 32, normalized size = 0.62

$$\frac{1}{2}\left(b^2x^2\mathrm{sgn}(x)+4\,abx\mathrm{sgn}(x)+2\,a^2\log\left(|x|\right)\mathrm{sgn}(x)\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="giac")
[Out] 1/2*(b^2*x^2*sgn(x) + 4*a*b*x*sgn(x) + 2*a^2*log(abs(x))*sgn(x))*c^(3/2)
```

maple [A] time = 0.00, size = 33, normalized size = 0.63

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(b^2x^2+2a^2\ln(x)+4abx\right)}{2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^2/x^4,x)
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4,x)
```

```
[Out] int(((c*x^2)^(3/2)*(a + b*x)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}} (a + bx)^2}{x^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**2/x**4,x)
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**2/x**4, x)
```

3.778 $\int x\left(cx^2\right)^{5/2}(a+bx)^2\,dx$

Optimal. Leaf size=66

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^6*Sqrt[c*x^2])/7 + (a*b*c^2*x^7*Sqrt[c*x^2])/4 + (b^2*c^2*x^8*Sq
rt[c*x^2])/9
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\left(cx^2\right)^{5/2}(a+bx)^2\,dx&=\frac{\left(c^2\sqrt{cx^2}\right)\int x^6(a+bx)^2\,dx}{x}\\&=\frac{\left(c^2\sqrt{cx^2}\right)\int\left(a^2x^6+2abx^7+b^2x^8\right)dx}{x}\\&=\frac{1}{7}a^2c^2x^6\sqrt{cx^2}+\frac{1}{4}abc^2x^7\sqrt{cx^2}+\frac{1}{9}b^2c^2x^8\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{1}{252}x^2\left(cx^2\right)^{5/2}\left(36a^2+63abx+28b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x^2*(c*x^2)^(5/2)*(36*a^2 + 63*a*b*x + 28*b^2*x^2))/252
```

fricas [A] time = 0.94, size = 42, normalized size = 0.64

$$\frac{1}{252} \left(28 \, b^2 c^2 x^8 + 63 \, abc^2 x^7 + 36 \, a^2 c^2 x^6\right) \sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^2*c^2*x^8 + 63*a*b*c^2*x^7 + 36*a^2*c^2*x^6)*sqrt(c*x^2)
```

giac [A] time = 1.22, size = 44, normalized size = 0.67

$$\frac{1}{252} \left(28 \, b^2 c^2 x^9 \mathrm{sgn}(x) + 63 \, abc^2 x^8 \mathrm{sgn}(x) + 36 \, a^2 c^2 x^7 \mathrm{sgn}(x)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*b^2*c^2*x^9*sgn(x) + 63*a*b*c^2*x^8*sgn(x) + 36*a^2*c^2*x^7*sgn(x
))*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(28b^2 x^2 + 63abx + 36a^2\right) \left(c \, x^2\right)^{\frac{5}{2}} x^2}{252}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/252*x^2*(28*b^2*x^2+63*a*b*x+36*a^2)*(c*x^2)^(5/2)
```

maxima [A] time = 1.34, size = 49, normalized size = 0.74

$$\frac{\left(cx^2\right)^{\frac{7}{2}} b^2 x^2}{9 \, c} + \frac{\left(cx^2\right)^{\frac{7}{2}} abx}{4 \, c} + \frac{\left(cx^2\right)^{\frac{7}{2}} a^2}{7 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(c*x^2)^(7/2)*b^2*x^2/c + 1/4*(c*x^2)^(7/2)*a*b*x/c + 1/7*(c*x^2)^(7/2)
*a^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \left(c \, x^2\right)^{5/2} (a + b \, x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int(x*(c*x^2)^(5/2)*(a + b*x)^2, x)
```

sympy [A] time = 2.19, size = 60, normalized size = 0.91

$$\frac{a^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}+\frac{abc^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{4}+\frac{b^2c^{\frac{5}{2}}x^4\left(x^2\right)^{\frac{5}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x**2*(x**2)**(5/2)/7 + a*b*c**(5/2)*x**3*(x**2)**(5/2)/4 + b*
*2*c**(5/2)*x**4*(x**2)**(5/2)/9
```

3.779 $\int (cx^2)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=66

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2} + \frac{2}{7}abc^2x^6\sqrt{cx^2} + \frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{15, 43\}$

$$\frac{1}{6}a^2c^2x^5\sqrt{cx^2} + \frac{2}{7}abc^2x^6\sqrt{cx^2} + \frac{1}{8}b^2c^2x^7\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*x^5*Sqrt[c*x^2])/6 + (2*a*b*c^2*x^6*Sqrt[c*x^2])/7 + (b^2*c^2*x^7*
Sqrt[c*x^2])/8
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (cx^2)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^2 \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^5 + 2abx^6 + b^2x^7\right) dx}{x} \\
&= \frac{1}{6}a^2c^2x^5\sqrt{cx^2} + \frac{2}{7}abc^2x^6\sqrt{cx^2} + \frac{1}{8}b^2c^2x^7\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2 + 48abx + 21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.50

$$\frac{1}{168}x\left(cx^2\right)^{5/2}\left(28a^2 + 48abx + 21b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (x*(c*x^2)^(5/2)*(28*a^2 + 48*a*b*x + 21*b^2*x^2))/168
```

fricas [A] time = 1.02, size = 42, normalized size = 0.64

$$\frac{1}{168}\left(21\,b^2c^2x^7+48\,abc^2x^6+28\,a^2c^2x^5\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^2*c^2*x^7 + 48*a*b*c^2*x^6 + 28*a^2*c^2*x^5)*sqrt(c*x^2)
```

giac [A] time = 1.05, size = 44, normalized size = 0.67

$$\frac{1}{168}\left(21\,b^2c^2x^8\mathrm{sgn}(x)+48\,abc^2x^7\mathrm{sgn}(x)+28\,a^2c^2x^6\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^2*c^2*x^8*sgn(x) + 48*a*b*c^2*x^7*sgn(x) + 28*a^2*c^2*x^6*sgn(x
))*sqrt(c)
```

maple [A] time = 0.00, size = 30, normalized size = 0.45

$$\frac{\left(21b^2x^2+48abx+28a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}x}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] 1/168*x*(21*b^2*x^2+48*a*b*x+28*a^2)*(c*x^2)^(5/2)
```

maxima [A] time = 1.32, size = 44, normalized size = 0.67

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}a^2x+\frac{\left(cx^2\right)^{\frac{7}{2}}b^2x}{8\,c}+\frac{2\left(cx^2\right)^{\frac{7}{2}}ab}{7\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(c*x^2)^(5/2)*a^2*x + 1/8*(c*x^2)^(7/2)*b^2*x/c + 2/7*(c*x^2)^(7/2)*a*b
/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(c\,x^2\right)^{5/2}\,{\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(5/2)*(a + b*x)^2, x)
```

sympy [A] time = 1.80, size = 60, normalized size = 0.91

$$\frac{a^2c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{6}+\frac{2abc^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}+\frac{b^2c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] a**2*c**(5/2)*x*(x**2)**(5/2)/6 + 2*a*b*c**(5/2)*x**2*(x**2)**(5/2)/7 + b**
2*c**(5/2)*x**3*(x**2)**(5/2)/8
```

3.780 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x} \, dx$

Optimal. Leaf size=66

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2} + \frac{1}{3}abc^2x^5\sqrt{cx^2} + \frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{5}a^2c^2x^4\sqrt{cx^2} + \frac{1}{3}abc^2x^5\sqrt{cx^2} + \frac{1}{7}b^2c^2x^6\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (a^2*c^2*x^4*Sqrt[c*x^2])/5 + (a*b*c^2*x^5*Sqrt[c*x^2])/3 + (b^2*c^2*x^6*Sq
rt[c*x^2])/7
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x} \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^2 \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^4+2abx^5+b^2x^6\right) dx}{x} \\
&= \frac{1}{5}a^2c^2x^4\sqrt{cx^2} + \frac{1}{3}abc^2x^5\sqrt{cx^2} + \frac{1}{7}b^2c^2x^6\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 36, normalized size = 0.55

$$\frac{1}{105}cx^2\left(cx^2\right)^{3/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
```

```
[Out] (c*x^2*(c*x^2)^(3/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 0.48

$$\frac{1}{105}\left(cx^2\right)^{5/2}\left(21a^2+35abx+15b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x,x]
[Out] ((c*x^2)^(5/2)*(21*a^2 + 35*a*b*x + 15*b^2*x^2))/105
```

fricas [A] time = 0.82, size = 42, normalized size = 0.64

$$\frac{1}{105} \left(15 \, b^{2} c^{2} x^{6} + 35 \, a b c^{2} x^{5} + 21 \, a^{2} c^{2} x^{4}\right) \sqrt{c x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="fricas")
[Out] 1/105*(15*b^2*c^2*x^6 + 35*a*b*c^2*x^5 + 21*a^2*c^2*x^4)*sqrt(c*x^2)
```

giac [A] time = 0.96, size = 44, normalized size = 0.67

$$\frac{1}{105} \left(15 \, b^{2} c^{2} x^{7} \mathrm{sgn}\left(x\right) + 35 \, a b c^{2} x^{6} \mathrm{sgn}\left(x\right) + 21 \, a^{2} c^{2} x^{5} \mathrm{sgn}\left(x\right)\right) \sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="giac")
[Out] 1/105*(15*b^2*c^2*x^7*sgn(x) + 35*a*b*c^2*x^6*sgn(x) + 21*a^2*c^2*x^5*sgn(x
))*sqrt(c)
```

maple [A] time = 0.00, size = 29, normalized size = 0.44

$$\frac{\left(15 b^{2} x^{2}+35 a b x +21 a^{2}\right) \left(c \,x^{2}\right)^{\frac{5}{2}}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x,x)
[Out] 1/105*(15*b^2*x^2+35*a*b*x+21*a^2)*(c*x^2)^(5/2)
```

maxima [A] time = 1.28, size = 40, normalized size = 0.61

$$\frac{1}{3} \, \left(c x^{2}\right)^{\frac{5}{2}} a b x + \frac{1}{5} \, \left(c x^{2}\right)^{\frac{5}{2}} a^{2} + \frac{\left(c x^{2}\right)^{\frac{7}{2}} b^{2}}{7 \, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x,x, algorithm="maxima")
[Out] 1/3*(c*x^2)^(5/2)*a*b*x + 1/5*(c*x^2)^(5/2)*a^2 + 1/7*(c*x^2)^(7/2)*b^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(c\,x^2\right)}^{5/2}\,{\left(a+b\,x\right)}^2}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x, x)
```

sympy [A] time = 1.82, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}+\frac{abc^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}{3}+\frac{b^2c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/5 + a*b*c**(5/2)*x*(x**2)**(5/2)/3 + b**2*c**(5
/2)*x**2*(x**2)**(5/2)/7
```

3.781 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2} dx$

Optimal. Leaf size=66

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Rubi [A] time = 0.02, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (a^2*c^2*x^3*Sqrt[c*x^2])/4 + (2*a*b*c^2*x^4*Sqrt[c*x^2])/5 + (b^2*c^2*x^5*
Sqrt[c*x^2])/6
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^2} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^2 dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^3+2abx^4+b^2x^5\right) dx}{x} \\
&= \frac{1}{4}a^2c^2x^3\sqrt{cx^2}+\frac{2}{5}abc^2x^4\sqrt{cx^2}+\frac{1}{6}b^2c^2x^5\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.52

$$\frac{1}{60}cx\left(cx^2\right)^{3/2}\left(15a^2+24abx+10b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
```

```
[Out] (c*x*(c*x^2)^(3/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/60
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{\left(cx^2\right)^{5/2}\left(15a^2+24abx+10b^2x^2\right)}{60x}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x]
[Out] ((c*x^2)^(5/2)*(15*a^2 + 24*a*b*x + 10*b^2*x^2))/(60*x)
```

fricas [A] time = 1.11, size = 42, normalized size = 0.64

$$\frac{1}{60}\left(10\,b^2c^2x^5 + 24\,abc^2x^4 + 15\,a^2c^2x^3\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="fricas")
[Out] 1/60*(10*b^2*c^2*x^5 + 24*a*b*c^2*x^4 + 15*a^2*c^2*x^3)*sqrt(c*x^2)
```

giac [A] time = 0.99, size = 44, normalized size = 0.67

$$\frac{1}{60}\left(10\,b^2c^2x^6\mathrm{sgn}(x) + 24\,abc^2x^5\mathrm{sgn}(x) + 15\,a^2c^2x^4\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="giac")
[Out] 1/60*(10*b^2*c^2*x^6*sgn(x) + 24*a*b*c^2*x^5*sgn(x) + 15*a^2*c^2*x^4*sgn(x)
)*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(10b^2x^2 + 24abx + 15a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{60x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^2,x)
[Out] 1/60/x*(10*b^2*x^2+24*a*b*x+15*a^2)*(c*x^2)^(5/2)
```

maxima [A] time = 1.37, size = 40, normalized size = 0.61

$$\frac{1}{6}\left(cx^2\right)^{\frac{5}{2}}b^2x + \frac{2}{5}\left(cx^2\right)^{\frac{5}{2}}ab + \frac{\left(cx^2\right)^{\frac{5}{2}}a^2}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^2,x, algorithm="maxima")
[Out] 1/6*(c*x^2)^(5/2)*b^2*x + 2/5*(c*x^2)^(5/2)*a*b + 1/4*(c*x^2)^(5/2)*a^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^2, x)
```

sympy [A] time = 1.84, size = 54, normalized size = 0.82

$$\frac{a^{2}c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{4x}+\frac{2abc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{5}+\frac{b^{2}c^{\frac{5}{2}}x\left(x^{2}\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**2,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(4*x) + 2*a*b*c**(5/2)*(x**2)**(5/2)/5 + b**2*c
**(5/2)*x*(x**2)**(5/2)/6
```

3.782 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3} dx$

Optimal. Leaf size=66

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (a^2*c^2*x^2*Sqrt[c*x^2])/3 + (a*b*c^2*x^3*Sqrt[c*x^2])/2 + (b^2*c^2*x^4*Sq
rt[c*x^2])/5
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^3} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^2 dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x^2 + 2abx^3 + b^2x^4\right) dx}{x} \\
&= \frac{1}{3}a^2c^2x^2\sqrt{cx^2} + \frac{1}{2}abc^2x^3\sqrt{cx^2} + \frac{1}{5}b^2c^2x^4\sqrt{cx^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 0.58

$$\frac{1}{30}c^2x^2\sqrt{cx^2}\left(10a^2 + 15abx + 6b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] (c^2*x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/30
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{(cx^2)^{5/2}\left(10a^2 + 15abx + 6b^2x^2\right)}{30x^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x]
```

```
[Out] ((c*x^2)^(5/2)*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*x^2)
```

fricas [A] time = 1.19, size = 42, normalized size = 0.64

$$\frac{1}{30}\left(6\,b^2c^2x^4+15\,abc^2x^3+10\,a^2c^2x^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/30*(6*b^2*c^2*x^4 + 15*a*b*c^2*x^3 + 10*a^2*c^2*x^2)*sqrt(c*x^2)
```

giac [A] time = 0.94, size = 44, normalized size = 0.67

$$\frac{1}{30}\left(6\,b^2c^2x^5\mathrm{sgn}(x)+15\,abc^2x^4\mathrm{sgn}(x)+10\,a^2c^2x^3\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*c^2*x^5*sgn(x) + 15*a*b*c^2*x^4*sgn(x) + 10*a^2*c^2*x^3*sgn(x))
*sqrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$\frac{\left(6b^2x^2+15abx+10a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{30x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^3,x)
```

```
[Out] 1/30/x^2*(6*b^2*x^2+15*a*b*x+10*a^2)*(c*x^2)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3,x)
```

```
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^3, x)
```

sympy [A] time = 1.95, size = 54, normalized size = 0.82

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}+\frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{2x}+\frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**3,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(3*x**2) + a*b*c**(5/2)*(x**2)**(5/2)/(2*x) + b
**2*c**(5/2)*(x**2)**(5/2)/5
```

3.783 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx$

Optimal. Leaf size=64

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (a^2*c^2*x*Sqrt[c*x^2])/2 + (2*a*b*c^2*x^2*Sqrt[c*x^2])/3 + (b^2*c^2*x^3*Sq
rt[c*x^2])/4
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^2\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2x+2abx^2+b^2x^3\right)dx}{x}\\ &= \frac{1}{2}a^2c^2x\sqrt{cx^2}+\frac{2}{3}abc^2x^2\sqrt{cx^2}+\frac{1}{4}b^2c^2x^3\sqrt{cx^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 36, normalized size = 0.56

$$\frac{1}{12}c^2x\sqrt{cx^2}\left(6a^2+8abx+3b^2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
```

```
[Out] (c^2*x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/12
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.55

$$\frac{(cx^2)^{5/2}\left(6a^2+8abx+3b^2x^2\right)}{12x^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x]
[Out] ((c*x^2)^(5/2)*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*x^3)
```

fricas [A] time = 1.18, size = 40, normalized size = 0.62

$$\frac{1}{12}\left(3\,b^2c^2x^3+8\,abc^2x^2+6\,a^2c^2x\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="fricas")
[Out] 1/12*(3*b^2*c^2*x^3 + 8*a*b*c^2*x^2 + 6*a^2*c^2*x)*sqrt(c*x^2)
```

giac [A] time = 1.05, size = 44, normalized size = 0.69

$$\frac{1}{12}\left(3\,b^2c^2x^4\mathrm{sgn}(x)+8\,abc^2x^3\mathrm{sgn}(x)+6\,a^2c^2x^2\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="giac")
[Out] 1/12*(3*b^2*c^2*x^4*sgn(x) + 8*a*b*c^2*x^3*sgn(x) + 6*a^2*c^2*x^2*sgn(x))*s
qrt(c)
```

maple [A] time = 0.00, size = 32, normalized size = 0.50

$$\frac{\left(3b^2x^2+8abx+6a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{12x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^4,x)
[Out] 1/12/x^3*(3*b^2*x^2+8*a*b*x+6*a^2)*(c*x^2)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^4,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,\left(a+b\,x\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^4, x)
```

sympy [A] time = 2.01, size = 60, normalized size = 0.94

$$\frac{a^{2}c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{2x^{3}}+\frac{2abc^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{3x^{2}}+\frac{b^{2}c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**4,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/(2*x**3) + 2*a*b*c**(5/2)*(x**2)**(5/2)/(3*x**2
) + b**2*c**(5/2)*(x**2)**(5/2)/(4*x)
```

3.784 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx$

Optimal. Leaf size=29

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^3)/(3*b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^5}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^2\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^3}{3bx}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.90

$$\frac{(cx^2)^{5/2}(a+bx)^3}{3bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(a + b*x)^3)/(3*b*x^5)
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 1.17

$$\frac{(cx^2)^{5/2}\left(3a^2+3abx+b^2x^2\right)}{3x^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x]
```

```
[Out] ((c*x^2)^(5/2)*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*x^4)
```

fricas [A] time = 0.97, size = 36, normalized size = 1.24

$$\frac{1}{3}\left(b^2c^2x^2 + 3\,abc^2x + 3\,a^2c^2\right)\sqrt{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="fricas")
[Out] 1/3*(b^2*c^2*x^2 + 3*a*b*c^2*x + 3*a^2*c^2)*sqrt(c*x^2)
```

giac [A] time = 1.12, size = 41, normalized size = 1.41

$$\frac{1}{3}\left(b^2c^2x^3\mathrm{sgn}(x) + 3\,abc^2x^2\mathrm{sgn}(x) + 3\,a^2c^2x\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="giac")
[Out] 1/3*(b^2*c^2*x^3*sgn(x) + 3*a*b*c^2*x^2*sgn(x) + 3*a^2*c^2*x*sgn(x))*sqrt(c
)
```

maple [A] time = 0.00, size = 31, normalized size = 1.07

$$\frac{\left(b^2x^2 + 3abx + 3a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^5,x)
[Out] 1/3/x^4*(b^2*x^2+3*a*b*x+3*a^2)*(c*x^2)^(5/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^5,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(c\,x^2\right)^{5/2}\,{\left(a+b\,x\right)}^2}{x^5} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5,x)
[Out] int(((c*x^2)^(5/2)*(a + b*x)^2)/x^5, x)
```

sympy [B] time = 2.03, size = 56, normalized size = 1.93

$$\frac{a^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^4} + \frac{abc^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{x^3} + \frac{b^2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**5,x)
```

```
[Out] a**2*c**(5/2)*(x**2)**(5/2)/x**4 + a*b*c**(5/2)*(x**2)**(5/2)/x**3 + b**2*c
**(5/2)*(x**2)**(5/2)/(3*x**2)
```

3.785 $\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx$

Optimal. Leaf size=58

$$\frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}+2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}+2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] 2*a*b*c^2*Sqrt[c*x^2] + (b^2*c^2*x*Sqrt[c*x^2])/2 + (a^2*c^2*Sqrt[c*x^2]*Lo
g[x])/x
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^2}{x^6}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{(a+bx)^2}{x}\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{x}\\ &= 2abc^2\sqrt{cx^2}+\frac{1}{2}b^2c^2x\sqrt{cx^2}+\frac{a^2c^2\sqrt{cx^2}\,\log(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.60

$$\frac{c^3x\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
```

```
[Out] (c^3*x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.64

$$(cx^2)^{5/2}\left(\frac{a^2\log(x)}{x^5}+\frac{4ab+b^2x}{2x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x]
[Out] (c*x^2)^(5/2)*((4*a*b + b^2*x)/(2*x^4) + (a^2*Log[x])/x^5)
```

fricas [A] time = 1.09, size = 41, normalized size = 0.71

$$\frac{\left(b^2c^2x^2+4\,abc^2x+2\,a^2c^2\log(x)\right)\sqrt{cx^2}}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="fricas")
[Out] 1/2*(b^2*c^2*x^2 + 4*a*b*c^2*x + 2*a^2*c^2*log(x))*sqrt(c*x^2)/x
```

giac [A] time = 1.08, size = 41, normalized size = 0.71

$$\frac{1}{2}\left(b^2c^2x^2\mathrm{sgn}(x)+4\,abc^2x\mathrm{sgn}(x)+2\,a^2c^2\log\left(|x|\right)\mathrm{sgn}(x)\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="giac")
[Out] 1/2*(b^2*c^2*x^2*sgn(x) + 4*a*b*c^2*x*sgn(x) + 2*a^2*c^2*log(abs(x))*sgn(x)
)*sqrt(c)
```

maple [A] time = 0.01, size = 33, normalized size = 0.57

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(b^2x^2+2a^2\ln(x)+4abx\right)}{2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^2/x^6,x)
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/x^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^2/x^6,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}\,{\left(a+b\,x\right)}^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((c*x^2)^(5/2)*(a + b*x)^2)/x^6,x)
```

```
[Out] int((((c*x^2)^(5/2)*(a + b*x)^2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**2/x**6,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**2/x**6, x)
```

3.786 $\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}} \, dx$

Optimal. Leaf size=57

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^4)/(3*Sqrt[c*x^2]) + (a*b*x^5)/(2*Sqrt[c*x^2]) + (b^2*x^6)/(5*Sqrt[c
*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx)^2}{\sqrt{cx^2}} \, dx &= \frac{x\int x^2(a+bx)^2 \, dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(a^2x^2+2abx^3+b^2x^4\right) dx}{\sqrt{cx^2}}\\
&= \frac{a^2x^4}{3\sqrt{cx^2}}+\frac{abx^5}{2\sqrt{cx^2}}+\frac{b^2x^6}{5\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{x^4\left(10a^2+15abx+6b^2x^2\right)}{30\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x^4*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 0.67

$$\frac{x^2\sqrt{cx^2}\left(10a^2+15abx+6b^2x^2\right)}{30c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x^2*Sqrt[c*x^2]*(10*a^2 + 15*a*b*x + 6*b^2*x^2))/(30*c)
```

fricas [A] time = 1.24, size = 36, normalized size = 0.63

$$\frac{\left(6\,b^2x^4+15\,abx^3+10\,a^2x^2\right)\sqrt{cx^2}}{30\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/30*(6*b^2*x^4 + 15*a*b*x^3 + 10*a^2*x^2)*sqrt(c*x^2)/c
```

giac [A] time = 0.94, size = 41, normalized size = 0.72

$$\frac{1}{30}\sqrt{cx^2}\left(3\left(\frac{2\,b^2x}{c}+\frac{5\,ab}{c}\right)x+\frac{10\,a^2}{c}\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/30*sqrt(c*x^2)*(3*(2*b^2*x/c + 5*a*b/c)*x + 10*a^2/c)*x^2
```

maple [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(6b^2x^2+15abx+10a^2\right)x^4}{30\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(1/2),x)
[Out] 1/30*x^4*(6*b^2*x^2+15*a*b*x+10*a^2)/(c*x^2)^(1/2)
```

maxima [A] time = 1.34, size = 54, normalized size = 0.95

$$\frac{\sqrt{cx^2}\,b^2x^4}{5\,c}+\frac{\sqrt{cx^2}\,abx^3}{2\,c}+\frac{\sqrt{cx^2}\,a^2x^2}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/5*sqrt(c*x^2)*b^2*x^4/c + 1/2*sqrt(c*x^2)*a*b*x^3/c + 1/3*sqrt(c*x^2)*a^2
*x^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,(a+b\,x)^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2),x)
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

sympy [A] time = 0.79, size = 60, normalized size = 1.05

$$\frac{a^2x^4}{3\sqrt{c}\,\sqrt{x^2}}+\frac{abx^5}{2\sqrt{c}\,\sqrt{x^2}}+\frac{b^2x^6}{5\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**4/(3*sqrt(c)*sqrt(x**2)) + a*b*x**5/(2*sqrt(c)*sqrt(x**2)) + b**2*x
**6/(5*sqrt(c)*sqrt(x**2))
```

3.787 $\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} \, dx$

Optimal. Leaf size=57

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 43}

$$\frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x^3)/(2*Sqrt[c*x^2]) + (2*a*b*x^4)/(3*Sqrt[c*x^2]) + (b^2*x^5)/(4*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx)^2}{\sqrt{cx^2}} \, dx &= \frac{x \int x(a+bx)^2 \, dx}{\sqrt{cx^2}} \\
&= \frac{x \int \left(a^2x + 2abx^2 + b^2x^3\right) dx}{\sqrt{cx^2}} \\
&= \frac{a^2x^3}{2\sqrt{cx^2}} + \frac{2abx^4}{3\sqrt{cx^2}} + \frac{b^2x^5}{4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{x^3\left(6a^2 + 8abx + 3b^2x^2\right)}{12\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x^3*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 36, normalized size = 0.63

$$\frac{x\sqrt{cx^2}\left(6a^2 + 8abx + 3b^2x^2\right)}{12c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/Sqrt[c*x^2],x]
[Out] (x*Sqrt[c*x^2]*(6*a^2 + 8*a*b*x + 3*b^2*x^2))/(12*c)
```

fricas [A] time = 1.07, size = 34, normalized size = 0.60

$$\frac{\left(3\,b^2x^3 + 8\,abx^2 + 6\,a^2x\right)\sqrt{cx^2}}{12\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/12*(3*b^2*x^3 + 8*a*b*x^2 + 6*a^2*x)*sqrt(c*x^2)/c
```

giac [A] time = 1.25, size = 38, normalized size = 0.67

$$\frac{1}{12}\sqrt{cx^2}\left(\left(\frac{3\,b^2x}{c} + \frac{8\,ab}{c}\right)x + \frac{6\,a^2}{c}\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] 1/12*sqrt(c*x^2)*((3*b^2*x/c + 8*a*b/c)*x + 6*a^2/c)*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.56

$$\frac{\left(3b^2x^2 + 8abx + 6a^2\right)x^3}{12\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(1/2),x)
[Out] 1/12*x^3*(3*b^2*x^2+8*a*b*x+6*a^2)/(c*x^2)^(1/2)
```

maxima [A] time = 1.32, size = 47, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,b^2x^3}{4\,c} + \frac{2\,\sqrt{cx^2}\,abx^2}{3\,c} + \frac{a^2x^2}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(c*x^2)*b^2*x^3/c + 2/3*sqrt(c*x^2)*a*b*x^2/c + 1/2*a^2*x^2/sqrt(c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2),x)
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

sympy [A] time = 0.65, size = 61, normalized size = 1.07

$$\frac{a^2x^3}{2\sqrt{c}\,\sqrt{x^2}}+\frac{2abx^4}{3\sqrt{c}\,\sqrt{x^2}}+\frac{b^2x^5}{4\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**3/(2*sqrt(c)*sqrt(x**2)) + 2*a*b*x**4/(3*sqrt(c)*sqrt(x**2)) + b**2
*x**5/(4*sqrt(c)*sqrt(x**2))
```

3.788 $\int \frac{x(a+bx)^2}{\sqrt{cx^2}} dx$

Optimal. Leaf size=24

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\sqrt{cx^2}} dx &= \frac{x\int (a+bx)^2 dx}{\sqrt{cx^2}} \\ &= \frac{x(a+bx)^3}{3b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{x(a+bx)^3}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 1.42

$$\frac{\sqrt{cx^2}\left(3a^2+3abx+b^2x^2\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (Sqrt[c*x^2]*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*c)
```

fricas [A] time = 1.16, size = 30, normalized size = 1.25

$$\frac{\left(b^2x^2+3\,abx+3\,a^2\right)\sqrt{cx^2}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c
```

giac [A] time = 1.15, size = 36, normalized size = 1.50

$$\frac{1}{3}\sqrt{cx^2}\left(\left(\frac{b^2x}{c}+\frac{3\,ab}{c}\right)x+\frac{3\,a^2}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)
```

maple [A] time = 0.00, size = 31, normalized size = 1.29

$$\frac{\left(b^2x^2+3abx+3a^2\right)x^2}{3\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/3*x^2*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(1/2)
```

maxima [B] time = 1.37, size = 42, normalized size = 1.75

$$\frac{\sqrt{cx^2}\,b^2x^2}{3\,c}+\frac{abx^2}{\sqrt{c}}+\frac{\sqrt{cx^2}\,a^2}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*b^2*x^2/c + a*b*x^2/sqrt(c) + sqrt(c*x^2)*a^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x\,{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(1/2), x)
```

sympy [B] time = 0.54, size = 56, normalized size = 2.33

$$\frac{a^2x^2}{\sqrt{c}\,\sqrt{x^2}}+\frac{abx^3}{\sqrt{c}\,\sqrt{x^2}}+\frac{b^2x^4}{3\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] a**2*x**2/(sqrt(c)*sqrt(x**2)) + a*b*x**3/(sqrt(c)*sqrt(x**2)) + b**2*x**4/
(3*sqrt(c)*sqrt(x**2))
```

3.789 $\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=52

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2x\log(x)}{\sqrt{cx^2}}+\frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (2*a*b*x^2)/Sqrt[c*x^2] + (b^2*x^3)/(2*Sqrt[c*x^2]) + (a^2*x*Log[x])/Sqrt[c
*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)\,dx}{\sqrt{cx^2}}\\ &= \frac{2abx^2}{\sqrt{cx^2}}+\frac{b^2x^3}{2\sqrt{cx^2}}+\frac{a^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 32, normalized size = 0.62

$$\frac{x\left(2a^2\log(x)+bx(4a+bx)\right)}{2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c*x^2],x]
```

```
[Out] (x*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 0.77

$$\sqrt{cx^2}\left(\frac{a^2\log(x)}{cx}+\frac{4ab+b^2x}{2c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[c*x^2],x]
[Out] Sqrt[c*x^2]*((4*a*b + b^2*x)/(2*c) + (a^2*Log[x])/(c*x))
```

fricas [A] time = 0.99, size = 35, normalized size = 0.67

$$\frac{\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c*x)
```

giac [A] time = 1.12, size = 50, normalized size = 0.96

$$-\frac{a^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}+\frac{1}{2}\,\sqrt{cx^2}\left(\frac{b^2x}{c}+\frac{4\,ab}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] -a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) + 1/2*sqrt(c*x^2)*(b^2*x/c
+ 4*a*b/c)
```

maple [A] time = 0.00, size = 31, normalized size = 0.60

$$\frac{\left(b^2x^2+2a^2\ln(x)+4abx\right)x}{2\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(1/2),x)
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(1/2)
```

maxima [A] time = 1.35, size = 35, normalized size = 0.67

$$\frac{b^2x^2}{2\,\sqrt{c}}+\frac{a^2\log(x)}{\sqrt{c}}+\frac{2\,\sqrt{cx^2}\,ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*b^2*x^2/sqrt(c) + a^2*log(x)/sqrt(c) + 2*sqrt(c*x^2)*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{{\left(a+b\,x\right)}^2}{\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(1/2),x)
[Out] int((a + b*x)^2/(c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)**2/sqrt(c*x**2), x)
```

3.790 $\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
[Out] -(a^2/Sqrt[c*x^2]) + (b^2*x^2)/Sqrt[c*x^2] + (2*a*b*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{a^2}{\sqrt{cx^2}}+\frac{b^2x^2}{\sqrt{cx^2}}+\frac{2abx\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.72

$$\frac{cx^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
[Out] (c*x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 0.91

$$\sqrt{cx^2}\left(\frac{b^2x^2-a^2}{cx^2}+\frac{2ab\log(x)}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*Sqrt[c*x^2]),x]
[Out] Sqrt[c*x^2]*((-a^2 + b^2*x^2)/(c*x^2) + (2*a*b*Log[x])/(c*x))
```

fricas [A] time = 1.03, size = 34, normalized size = 0.72

$$\frac{\left(b^2x^2+2\,abx\log(x)-a^2\right)\sqrt{cx^2}}{cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c*x^2)
```

giac [A] time = 0.94, size = 65, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,b^2}{c}-\frac{2\left(ab\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{a^2\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c)
/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c)
```

maple [A] time = 0.00, size = 29, normalized size = 0.62

$$\frac{2abx\ln(x)+b^2x^2-a^2}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(1/2),x)
[Out] (2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(1/2)
```

maxima [A] time = 1.36, size = 35, normalized size = 0.74

$$\frac{2\,ab\log(x)}{\sqrt{c}}+\frac{\sqrt{cx^2}\,b^2}{c}-\frac{a^2}{\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] 2*a*b*log(x)/sqrt(c) + sqrt(c*x^2)*b^2/c - a^2/(sqrt(c)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\,x)^2}{x\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x*(c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(1/2),x)
```

```
[Out] Integral((a + b*x)**2/(x*sqrt(c*x**2)), x)
```

3.791 $\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{2x\sqrt{cx^2}}-\frac{2ab}{\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (-2*a*b)/Sqrt[c*x^2] - a^2/(2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/Sqrt[c*x^2]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{2ab}{\sqrt{cx^2}}-\frac{a^2}{2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.71

$$\frac{cx\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
```

```
[Out] (c*x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.90

$$\sqrt{cx^2}\left(\frac{-a^2-4abx}{2cx^3}+\frac{b^2\log(x)}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*Sqrt[c*x^2]),x]
[Out] Sqrt[c*x^2]*((-a^2 - 4*a*b*x)/(2*c*x^3) + (b^2*Log[x])/(c*x))
```

fricas [A] time = 1.09, size = 36, normalized size = 0.73

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 34, normalized size = 0.69

$$\frac{2b^2x^2\ln(x)-4abx-a^2}{2\sqrt{c\,x^2}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(1/2),x)
[Out] 1/2/x*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(1/2)
```

maxima [A] time = 1.29, size = 31, normalized size = 0.63

$$\frac{b^2\log(x)}{\sqrt{c}}-\frac{2\,ab}{\sqrt{c}\,x}-\frac{a^2}{2\,\sqrt{c}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b^2*log(x)/sqrt(c) - 2*a*b/(sqrt(c)*x) - 1/2*a^2/(sqrt(c)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\,x)^2}{x^2\,\sqrt{c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x)^2/(x^2*(c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{x^2\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(1/2),x)
[Out] Integral((a + b*x)**2/(x**2*sqrt(c*x**2)), x)
```

3.792 $\int \frac{(a+bx)^2}{x^3 \sqrt{cx^2}} \, dx$

Optimal. Leaf size=26

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] -(a + b*x)^3/(3*a*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\sqrt{cx^2}} \, dx &= \frac{x \int \frac{(a+bx)^2}{x^4} \, dx}{\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ax^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.27

$$\frac{c\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
```

```
[Out] (c*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 1.46

$$\frac{\sqrt{cx^2}\left(-a^2-3abx-3b^2x^2\right)}{3cx^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*c*x^4)
```

fricas [A] time = 1.07, size = 32, normalized size = 1.23

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^2}{\sqrt{cx^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/(sqrt(c*x^2)*x^3), x)
```

maple [A] time = 0.00, size = 30, normalized size = 1.15

$$-\frac{3b^2x^2+3abx+a^2}{3\sqrt{c\,x^2}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(1/2),x)
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/x^2/(c*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 33, normalized size = 1.27

$$-\frac{b^2}{\sqrt{c}\,x}-\frac{ab}{\sqrt{c}\,x^2}-\frac{a^2}{3\,\sqrt{c}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -b^2/(sqrt(c)*x) - a*b/(sqrt(c)*x^2) - 1/3*a^2/(sqrt(c)*x^3)
```

mupad [B] time = 0.18, size = 33, normalized size = 1.27

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,\sqrt{c}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(1/2)),x)
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(1/2)*(x^2)^(5/2))
```

sympy [B] time = 0.66, size = 53, normalized size = 2.04

$$-\frac{a^2}{3\sqrt{c}\,x^2\sqrt{x^2}}-\frac{ab}{\sqrt{c}\,x\sqrt{x^2}}-\frac{b^2}{\sqrt{c}\,\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(3*sqrt(c)*x**2*sqrt(x**2)) - a*b/(sqrt(c)*x*sqrt(x**2)) - b**2/(sqrt
(c)*sqrt(x**2))
```

3.793 $\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx$

Optimal. Leaf size=57

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] -a^2/(4*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*x^2*Sqrt[c*x^2]) - b^2/(2*x*Sqrt[c*x^
2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^4\sqrt{cx^2}}\,dx&=\frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{\sqrt{cx^2}}\\&=-\frac{a^2}{4x^3\sqrt{cx^2}}-\frac{2ab}{3x^2\sqrt{cx^2}}-\frac{b^2}{2x\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.61

$$\frac{-3a^2-8abx-6b^2x^2}{12x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
```

```
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(-3a^2-8abx-6b^2x^2\right)}{12cx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*Sqrt[c*x^2]),x]
[Out] (Sqrt[c*x^2]*(-3*a^2 - 8*a*b*x - 6*b^2*x^2))/(12*c*x^5)
```

fricas [A] time = 1.25, size = 34, normalized size = 0.60

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,cx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 32, normalized size = 0.56

$$-\frac{6b^2x^2+8abx+3a^2}{12\sqrt{c\,x^2}\;x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(1/2),x)
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x^3/(c*x^2)^(1/2)
```

maxima [A] time = 1.33, size = 33, normalized size = 0.58

$$-\frac{b^2}{2\,\sqrt{c}\,x^2}-\frac{2\,ab}{3\,\sqrt{c}\,x^3}-\frac{a^2}{4\,\sqrt{c}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -1/2*b^2/(sqrt(c)*x^2) - 2/3*a*b/(sqrt(c)*x^3) - 1/4*a^2/(sqrt(c)*x^4)
```

mupad [B] time = 0.19, size = 42, normalized size = 0.74

$$-\frac{3\,a^2\,\sqrt{x^2}\,+6\,b^2\,x^2\,\sqrt{x^2}\,+8\,a\,b\,x\,\sqrt{x^2}}{12\,\sqrt{c}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(1/2)),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(1
/2)*x^5)
```

sympy [A] time = 0.82, size = 61, normalized size = 1.07

$$-\frac{a^2}{4\sqrt{c}\,x^3\sqrt{x^2}}-\frac{2ab}{3\sqrt{c}\,x^2\sqrt{x^2}}-\frac{b^2}{2\sqrt{c}\,x\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(1/2),x)
```

```
[Out] -a**2/(4*sqrt(c)*x**3*sqrt(x**2)) - 2*a*b/(3*sqrt(c)*x**2*sqrt(x**2)) - b**
2/(2*sqrt(c)*x*sqrt(x**2))
```

3.794 $\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=27

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^3}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^3)/(3*b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{3/2}} \, dx &= \frac{x\int (a+bx)^2 \, dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^3}{3bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.96

$$\frac{x^3(a+bx)^3}{3b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^3)/(3*b*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 34, normalized size = 1.26

$$\frac{x^4\left(3a^2+3abx+b^2x^2\right)}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x^4*(3*a^2 + 3*a*b*x + b^2*x^2))/(3*(c*x^2)^(3/2))
```

fricas [A] time = 1.06, size = 30, normalized size = 1.11

$$\frac{\left(b^2x^2 + 3\,abx + 3\,a^2\right)\sqrt{cx^2}}{3\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(b^2*x^2 + 3*a*b*x + 3*a^2)*sqrt(c*x^2)/c^2
```

giac [A] time = 1.05, size = 39, normalized size = 1.44

$$\frac{\sqrt{cx^2}\left(\left(\frac{b^2x}{c} + \frac{3\,ab}{c}\right)x + \frac{3\,a^2}{c}\right)}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] 1/3*sqrt(c*x^2)*((b^2*x/c + 3*a*b/c)*x + 3*a^2/c)/c
```

maple [A] time = 0.00, size = 31, normalized size = 1.15

$$\frac{\left(b^2x^2 + 3abx + 3a^2\right)x^4}{3\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/3*x^4*(b^2*x^2+3*a*b*x+3*a^2)/(c*x^2)^(3/2)
```

maxima [B] time = 1.36, size = 52, normalized size = 1.93

$$\frac{b^2x^4}{3\sqrt{cx^2}\,c} + \frac{abx^3}{\sqrt{cx^2}\,c} + \frac{a^2x^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/3*b^2*x^4/(sqrt(c*x^2)*c) + a*b*x^3/(sqrt(c*x^2)*c) + a^2*x^2/(sqrt(c*x^2
)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3\,(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2),x)
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

sympy [B] time = 0.80, size = 56, normalized size = 2.07

$$\frac{a^2x^4}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{abx^5}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{b^2x^6}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] a**2*x**4/(c**(3/2)*(x**2)**(3/2)) + a*b*x**5/(c**(3/2)*(x**2)**(3/2)) + b*
*2*x**6/(3*c**(3/2)*(x**2)**(3/2))
```

3.795 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=61

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(x)}{c\sqrt{cx^2}}+\frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (2*a*b*x^2)/(c*Sqrt[c*x^2]) + (b^2*x^3)/(2*c*Sqrt[c*x^2]) + (a^2*x*Log[x])/
(c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(2ab+\frac{a^2}{x}+b^2x\right)dx}{c\sqrt{cx^2}}\\ &= \frac{2abx^2}{c\sqrt{cx^2}}+\frac{b^2x^3}{2c\sqrt{cx^2}}+\frac{a^2x\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.56

$$\frac{x^3\left(2a^2\log(x)+bx(4a+bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(b*x*(4*a + b*x) + 2*a^2*Log[x]))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.64

$$\frac{a^2x^3\log(x)+\frac{1}{2}\left(4abx^4+b^2x^5\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] ((4*a*b*x^4 + b^2*x^5)/2 + a^2*x^3*Log[x])/(c*x^2)^(3/2)
```

fricas [A] time = 1.18, size = 35, normalized size = 0.57

$$\frac{\left(b^2x^2+4\,abx+2\,a^2\log(x)\right)\sqrt{cx^2}}{2\,c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*sqrt(c*x^2)/(c^2*x)
```

giac [A] time = 1.10, size = 55, normalized size = 0.90

$$-\frac{\frac{2\,a^2\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{\sqrt{c}}-\sqrt{cx^2}\left(\frac{b^2x}{c}+\frac{4\,ab}{c}\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(2*a^2*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/sqrt(c) - sqrt(c*x^2)*(b^2*x
/c + 4*a*b/c))/c
```

maple [A] time = 0.00, size = 33, normalized size = 0.54

$$\frac{\left(b^2x^2+2a^2\ln(x)+4abx\right)x^3}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(x)+4*a*b*x)/(c*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 45, normalized size = 0.74

$$\frac{b^2x^3}{2\sqrt{cx^2}\,c}+\frac{2\,abx^2}{\sqrt{cx^2}\,c}+\frac{a^2\log(x)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] 1/2*b^2*x^3/(sqrt(c*x^2)*c) + 2*a*b*x^2/(sqrt(c*x^2)*c) + a^2*log(x)/c^(3/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^2\,(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2),x)
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(3/2),x)
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(3/2), x)
```

3.796 $\int \frac{x(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -(a^2/(c*Sqrt[c*x^2])) + (b^2*x^2)/(c*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(c*Sq
rt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a^2}{c\sqrt{cx^2}}+\frac{b^2x^2}{c\sqrt{cx^2}}+\frac{2abx\log(x)}{c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{x^2\left(-a^2+2abx\log(x)+b^2x^2\right)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] (x^2*(-a^2 + b^2*x^2 + 2*a*b*x*Log[x]))/(c*x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.62

$$\frac{-a^2x^2 + 2abx^3\log(x) + b^2x^4}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (-(a^2*x^2) + b^2*x^4 + 2*a*b*x^3*Log[x])/(c*x^2)^(3/2)
```

fricas [A] time = 1.20, size = 34, normalized size = 0.61

$$\frac{(b^2x^2 + 2\,abx\log(x) - a^2)\sqrt{cx^2}}{c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^2*x^2)
```

giac [A] time = 1.08, size = 69, normalized size = 1.23

$$\frac{\frac{\sqrt{cx^2}\,b^2}{c} - \frac{2\left(ab\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)-\frac{a^2\sqrt{c}}{\sqrt{c}\,x-\sqrt{cx^2}}\right)}{\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] (sqrt(c*x^2)*b^2/c - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(c
)/(sqrt(c)*x - sqrt(c*x^2)))/sqrt(c))/c
```

maple [A] time = 0.00, size = 32, normalized size = 0.57

$$\frac{(2abx\ln(x) + b^2x^2 - a^2)\,x^2}{(c\,x^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(3/2),x)
[Out] x^2*(2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(3/2)
```

maxima [A] time = 1.38, size = 42, normalized size = 0.75

$$\frac{b^2x^2}{\sqrt{cx^2}\,c} + \frac{2\,ab\log(x)}{c^{\frac{3}{2}}} - \frac{a^2}{\sqrt{cx^2}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b^2*x^2/(sqrt(c*x^2)*c) + 2*a*b*log(x)/c^(3/2) - a^2/(sqrt(c*x^2)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,(a+b\,x)^2}{(c\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*x)^2)/(c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x (a + bx)^2}{(cx^2)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral(x*(a + b*x)**2/(c*x**2)**(3/2), x)
```

3.797 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{2cx\sqrt{cx^2}}-\frac{2ab}{c\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (-2*a*b)/(c*Sqrt[c*x^2]) - a^2/(2*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c*Sqrt
[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{2ab}{c\sqrt{cx^2}}-\frac{a^2}{2cx\sqrt{cx^2}}+\frac{b^2x\log(x)}{c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 0.59

$$\frac{x\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(3/2),x]
```

```
[Out] (x*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 38, normalized size = 0.66

$$\frac{\frac{1}{2}\left(a^2(-x)-4abx^2\right)+b^2x^3\log(x)}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c*x^2)^(3/2),x]
[Out] ((-(a^2*x) - 4*a*b*x^2)/2 + b^2*x^3*Log[x])/(c*x^2)^(3/2)
```

fricas [A] time = 1.04, size = 36, normalized size = 0.62

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.55

$$\frac{\left(2b^2x^2\ln(x)-4abx-a^2\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(3/2),x)
[Out] 1/2*x*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(3/2)
```

maxima [A] time = 1.33, size = 35, normalized size = 0.60

$$\frac{b^2\log(x)}{c^{\frac{3}{2}}}-\frac{2\,ab}{\sqrt{cx^2}\,c}-\frac{a^2}{2\,c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b^2*log(x)/c^(3/2) - 2*a*b/(sqrt(c*x^2)*c) - 1/2*a^2/(c^(3/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\,x)^2}{\left(c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(3/2),x)
```

```
[Out] int((a + b*x)^2/(c*x^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Integral((a + b*x)**2/(c*x**2)**(3/2), x)
```

3.798 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] -(a + b*x)^3/(3*a*c*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3acx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.24

$$\frac{cx^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
```

```
[Out] (c*x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 1.10

$$\frac{-a^2-3abx-3b^2x^2}{3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*(c*x^2)^(3/2)),x]
[Out] (-a^2 - 3*a*b*x - 3*b^2*x^2)/(3*(c*x^2)^(3/2))
```

fricas [A] time = 0.80, size = 32, normalized size = 1.10

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^2*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$-\frac{3b^2x^2+3abx+a^2}{3\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(3/2),x)
[Out] -1/3*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(3/2)
```

maxima [A] time = 1.36, size = 37, normalized size = 1.28

$$-\frac{b^2}{\sqrt{cx^2}\,c}-\frac{ab}{c^{\frac{3}{2}}x^2}-\frac{a^2}{3\,c^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -b^2/(sqrt(c*x^2)*c) - a*b/(c^(3/2)*x^2) - 1/3*a^2/(c^(3/2)*x^3)
```

mupad [B] time = 0.19, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{3/2}\,{\left(x^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(3/2)),x)
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(3/2)*(x^2)^(5/2))
```

sympy [B] time = 0.67, size = 53, normalized size = 1.83

$$-\frac{a^2}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{abx}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2x^2}{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(3*c**(3/2)*(x**2)**(3/2)) - a*b*x/(c**(3/2)*(x**2)**(3/2)) - b**2*x*
*2/(c**(3/2)*(x**2)**(3/2))
```

3.799 $\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(4*c*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c*x^2*Sqrt[c*x^2]) - b^2/(2*c*x*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^2}{4cx^3\sqrt{cx^2}}-\frac{2ab}{3cx^2\sqrt{cx^2}}-\frac{b^2}{2cx\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
```

```
[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^2*x^5)
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-3a^2-8abx-6b^2x^2}{12x\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*(c*x^2)^(3/2)),x]
[Out] (-3*a^2 - 8*a*b*x - 6*b^2*x^2)/(12*x*(c*x^2)^(3/2))
```

fricas [A] time = 0.99, size = 34, normalized size = 0.52

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,c^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^2*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 32, normalized size = 0.48

$$-\frac{6b^2x^2+8abx+3a^2}{12\left(c\,x^2\right)^{\frac{3}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(3/2),x)
[Out] -1/12*(6*b^2*x^2+8*a*b*x+3*a^2)/x/(c*x^2)^(3/2)
```

maxima [A] time = 1.31, size = 33, normalized size = 0.50

$$-\frac{b^2}{2\,c^{\frac{3}{2}}x^2}-\frac{2\,ab}{3\,c^{\frac{3}{2}}x^3}-\frac{a^2}{4\,c^{\frac{3}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/2*b^2/(c^(3/2)*x^2) - 2/3*a*b/(c^(3/2)*x^3) - 1/4*a^2/(c^(3/2)*x^4)
```

mupad [B] time = 0.19, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}\,+6\,b^2\,x^2\,\sqrt{x^2}\,+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{3/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(3/2)),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(3
/2)*x^5)
```

sympy [**A**] time = 0.81, size = 56, normalized size = 0.85

$$-\frac{a^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{2ab}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2x}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(4*c**(3/2)*x*(x**2)**(3/2)) - 2*a*b/(3*c**(3/2)*(x**2)**(3/2)) - b**
2*x/(2*c**(3/2)*(x**2)**(3/2))
```

3.800 $\int \frac{(a+bx)^2}{x^3(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(5*c*x^4*Sqrt[c*x^2]) - (a*b)/(2*c*x^3*Sqrt[c*x^2]) - b^2/(3*c*x^2*Sqr
t[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{a^2}{5cx^4\sqrt{cx^2}}-\frac{ab}{2cx^3\sqrt{cx^2}}-\frac{b^2}{3cx^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{c\left(-6a^2-15abx-10b^2x^2\right)}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
```

```
[Out] (c*(-6*a^2 - 15*a*b*x - 10*b^2*x^2))/(30*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-6a^2-15abx-10b^2x^2}{30x^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*(c*x^2)^(3/2)),x]
[Out] (-6*a^2 - 15*a*b*x - 10*b^2*x^2)/(30*x^2*(c*x^2)^(3/2))
```

fricas [A] time = 1.28, size = 34, normalized size = 0.52

$$-\frac{\left(10\,b^2x^2+15\,abx+6\,a^2\right)\sqrt{cx^2}}{30\,c^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^2*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^2}{\left(cx^2\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(3/2)*x^3), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$-\frac{10b^2x^2+15abx+6a^2}{30\left(c\,x^2\right)^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(3/2),x)
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/x^2/(c*x^2)^(3/2)
```

maxima [A] time = 1.36, size = 33, normalized size = 0.50

$$-\frac{b^2}{3\,c^{\frac{3}{2}}x^3}-\frac{ab}{2\,c^{\frac{3}{2}}x^4}-\frac{a^2}{5\,c^{\frac{3}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/3*b^2/(c^(3/2)*x^3) - 1/2*a*b/(c^(3/2)*x^4) - 1/5*a^2/(c^(3/2)*x^5)
```

mupad [B] time = 0.20, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2}+10\,b^2\,x^2\,\sqrt{x^2}+15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{3/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(3/2)),x)
```

```
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(3/2)*x^6)
```

sympy [A] time = 0.98, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{ab}{2c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{3c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(5*c**(3/2)*x**2*(x**2)**(3/2)) - a*b/(2*c**(3/2)*x*(x**2)**(3/2)) -
b**2/(3*c**(3/2)*(x**2)**(3/2))
```

3.801 $\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] -a^2/(6*c*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c*x^4*Sqrt[c*x^2]) - b^2/(4*c*x^3*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{3/2}}\,dx&=\frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c\sqrt{cx^2}}\\&=\frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c\sqrt{cx^2}}\\&=-\frac{a^2}{6cx^5\sqrt{cx^2}}-\frac{2ab}{5cx^4\sqrt{cx^2}}-\frac{b^2}{4cx^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.53

$$\frac{-10a^2-24abx-15b^2x^2}{60x^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
```

```
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x^3 (cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*(c*x^2)^(3/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x^3*(c*x^2)^(3/2))
```

fricas [A] time = 1.15, size = 34, normalized size = 0.52

$$-\frac{\left(15\, b^2 x^2 + 24\, abx + 10\, a^2\right)\sqrt{cx^2}}{60\, c^2 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^2*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$-\frac{15b^2x^2 + 24abx + 10a^2}{60 \left(c\, x^2\right)^{\frac{3}{2}} x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(3/2),x)
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x^3/(c*x^2)^(3/2)
```

maxima [A] time = 1.29, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\, c^{\frac{3}{2}} x^4} - \frac{2\, ab}{5\, c^{\frac{3}{2}} x^5} - \frac{a^2}{6\, c^{\frac{3}{2}} x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(3/2)*x^4) - 2/5*a*b/(c^(3/2)*x^5) - 1/6*a^2/(c^(3/2)*x^6)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\, a^2\, \sqrt{x^2}\, + 15\, b^2\, x^2\, \sqrt{x^2}\, + 24\, a\, b\, x\, \sqrt{x^2}}{60\, c^{3/2}\, x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(3/2)),x)
```

```
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(3/2)*x^7)
```

sympy [**A**] time = 1.18, size = 61, normalized size = 0.92

$$-\frac{a^2}{6c^{\frac{3}{2}}x^3\left(x^2\right)^{\frac{3}{2}}}-\frac{2ab}{5c^{\frac{3}{2}}x^2\left(x^2\right)^{\frac{3}{2}}}-\frac{b^2}{4c^{\frac{3}{2}}x\left(x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(3/2),x)
```

```
[Out] -a**2/(6*c**(3/2)*x**3*(x**2)**(3/2)) - 2*a*b/(5*c**(3/2)*x**2*(x**2)**(3/2
)) - b**2/(4*c**(3/2)*x*(x**2)**(3/2))
```

3.802 $\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}} \, dx$

Optimal. Leaf size=56

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a^2/(c^2*Sqrt[c*x^2])) + (b^2*x^2)/(c^2*Sqrt[c*x^2]) + (2*a*b*x*Log[x])/(
c^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^2}{\left(cx^2\right)^{5/2}} \, dx &= \frac{x\int \frac{(a+bx)^2}{x^2} \, dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(b^2+\frac{a^2}{x^2}+\frac{2ab}{x}\right) dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{c^2\sqrt{cx^2}}+\frac{b^2x^2}{c^2\sqrt{cx^2}}+\frac{2abx\log(x)}{c^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.59

$$\frac{-a^2+2abx\log(x)+b^2x^2}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-a^2 + b^2*x^2 + 2*a*b*x*Log[x])/(c^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 35, normalized size = 0.62

$$\frac{-a^2x^4 + 2abx^5\log(x) + b^2x^6}{(cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] (-(a^2*x^4) + b^2*x^6 + 2*a*b*x^5*Log[x])/(c*x^2)^(5/2)
```

fricas [A] time = 0.97, size = 34, normalized size = 0.61

$$\frac{\left(b^2x^2 + 2\,abx\log(x) - a^2\right)\sqrt{cx^2}}{c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] (b^2*x^2 + 2*a*b*x*log(x) - a^2)*sqrt(c*x^2)/(c^3*x^2)
```

giac [A] time = 1.07, size = 65, normalized size = 1.16

$$\frac{\sqrt{cx^2}\,b^2}{c^3} - \frac{2\left(ab\log\left(\left|-\sqrt{c}\,x + \sqrt{cx^2}\right|\right) - \frac{a^2\sqrt{c}}{\sqrt{c}\,x - \sqrt{cx^2}}\right)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sqrt(c*x^2)*b^2/c^3 - 2*(a*b*log(abs(-sqrt(c)*x + sqrt(c*x^2))) - a^2*sqrt(
c)/(sqrt(c)*x - sqrt(c*x^2)))/c^(5/2)
```

maple [A] time = 0.00, size = 32, normalized size = 0.57

$$\frac{\left(2abx\ln(x) + b^2x^2 - a^2\right)x^4}{\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^2/(c*x^2)^(5/2),x)
[Out] x^4*(2*a*b*x*ln(x)+b^2*x^2-a^2)/(c*x^2)^(5/2)
```

maxima [A] time = 1.45, size = 45, normalized size = 0.80

$$\frac{b^2x^4}{(cx^2)^{\frac{3}{2}}\,c} - \frac{a^2x^2}{(cx^2)^{\frac{3}{2}}\,c} + \frac{2\,ab\log(x)}{c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] b^2*x^4/((c*x^2)^(3/2)*c) - a^2*x^2/((c*x^2)^(3/2)*c) + 2*a*b*log(x)/c^(5/2
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,(a+b\,x)^2}{\left(c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a+bx)^2}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*x)**2/(c*x**2)**(5/2), x)
```

3.803 $\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=58

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{2c^2x\sqrt{cx^2}}-\frac{2ab}{c^2\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (-2*a*b)/(c^2*Sqrt[c*x^2]) - a^2/(2*c^2*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(c^
2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^3}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^3}+\frac{2ab}{x^2}+\frac{b^2}{x}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{2ab}{c^2\sqrt{cx^2}}-\frac{a^2}{2c^2x\sqrt{cx^2}}+\frac{b^2x\log(x)}{c^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.62

$$\frac{x^3\left(2b^2x^2\log(x)-a(a+4bx)\right)}{2\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x^3*(-(a*(a + 4*b*x)) + 2*b^2*x^2*Log[x]))/(2*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 40, normalized size = 0.69

$$\frac{\frac{1}{2}\left(-a^2x^3-4abx^4\right)+b^2x^5\log(x)}{\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] ((-(a^2*x^3) - 4*a*b*x^4)/2 + b^2*x^5*Log[x])/(c*x^2)^(5/2)
```

fricas [A] time = 0.84, size = 36, normalized size = 0.62

$$\frac{\left(2\,b^2x^2\log(x)-4\,abx-a^2\right)\sqrt{cx^2}}{2\,c^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x) - 4*a*b*x - a^2)*sqrt(c*x^2)/(c^3*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 34, normalized size = 0.59

$$\frac{\left(2b^2x^2\ln(x)-4abx-a^2\right)x^3}{2\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^2/(c*x^2)^(5/2),x)
[Out] 1/2*x^3*(2*b^2*x^2*ln(x)-4*a*b*x-a^2)/(c*x^2)^(5/2)
```

maxima [A] time = 1.40, size = 38, normalized size = 0.66

$$-\frac{2\,abx^2}{\left(cx^2\right)^{\frac{3}{2}}c}+\frac{b^2\log(x)}{c^{\frac{5}{2}}}-\frac{a^2}{2\,c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -2*a*b*x^2/((c*x^2)^(3/2)*c) + b^2*log(x)/c^(5/2) - 1/2*a^2/(c^(5/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2\,(a+b\,x)^2}{\left(c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*x)^2)/(c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a+bx)^2}{(cx^2)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*x)**2/(c*x**2)**(5/2), x)
```

3.804 $\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 37}

$$-\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -(a + b*x)^3/(3*a*c^2*x^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^4}\,dx}{c^2\sqrt{cx^2}}\\ &= -\frac{(a+bx)^3}{3ac^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.21

$$\frac{x^2\left(-a^2-3abx-3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x^2*(-a^2 - 3*a*b*x - 3*b^2*x^2))/(3*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.14

$$-\frac{x^2\left(a^2+3abx+3b^2x^2\right)}{3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^2)/(c*x^2)^(5/2),x]
[Out] -1/3*(x^2*(a^2 + 3*a*b*x + 3*b^2*x^2))/(c*x^2)^(5/2)
```

fricas [A] time = 0.86, size = 32, normalized size = 1.10

$$-\frac{\left(3\,b^2x^2+3\,abx+a^2\right)\sqrt{cx^2}}{3\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*sqrt(c*x^2)/(c^3*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2x}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2*x/(c*x^2)^(5/2), x)
```

maple [A] time = 0.00, size = 30, normalized size = 1.03

$$-\frac{\left(3b^2x^2+3abx+a^2\right)x^2}{3\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^2/(c*x^2)^(5/2),x)
[Out] -1/3*x^2*(3*b^2*x^2+3*a*b*x+a^2)/(c*x^2)^(5/2)
```

maxima [A] time = 1.38, size = 44, normalized size = 1.52

$$-\frac{b^2x^2}{\left(cx^2\right)^{\frac{3}{2}}c}-\frac{a^2}{3\left(cx^2\right)^{\frac{3}{2}}c}-\frac{ab}{c^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -b^2*x^2/((c*x^2)^(3/2)*c) - 1/3*a^2/((c*x^2)^(3/2)*c) - a*b/(c^(5/2)*x^2)
```

mupad [B] time = 0.18, size = 33, normalized size = 1.14

$$-\frac{a^2\,x^2+3\,a\,b\,x^3+3\,b^2\,x^4}{3\,c^{5/2}\,\left(x^2\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^2)/(c*x^2)^(5/2),x)
[Out] -(a^2*x^2 + 3*b^2*x^4 + 3*a*b*x^3)/(3*c^(5/2)*(x^2)^(5/2))
```

sympy [B] time = 0.96, size = 58, normalized size = 2.00

$$-\frac{a^2x^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{abx^3}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x^4}{c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x**2/(3*c**(5/2)*(x**2)**(5/2)) - a*b*x**3/(c**(5/2)*(x**2)**(5/2)) -
 b**2*x**4/(c**(5/2)*(x**2)**(5/2))
```

3.805 $\int \frac{(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c*x^2)^(5/2),x]

[Out] -a^2/(4*c^2*x^3*Sqrt[c*x^2]) - (2*a*b)/(3*c^2*x^2*Sqrt[c*x^2]) - b^2/(2*c^2
*x*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^5}+\frac{2ab}{x^4}+\frac{b^2}{x^3}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^2}{4c^2x^3\sqrt{cx^2}}-\frac{2ab}{3c^2x^2\sqrt{cx^2}}-\frac{b^2}{2c^2x\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(3a^2+8abx+6b^2x^2\right)}{12c^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c*x^2)^(5/2),x]

[Out] -1/12*(Sqrt[c*x^2]*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c^3*x^5)
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.50

$$-\frac{x\left(3a^2+8abx+6b^2x^2\right)}{12\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c*x^2)^(5/2),x]
[Out] -1/12*(x*(3*a^2 + 8*a*b*x + 6*b^2*x^2))/(c*x^2)^(5/2)
```

fricas [A] time = 0.83, size = 34, normalized size = 0.52

$$-\frac{\left(6\,b^2x^2+8\,abx+3\,a^2\right)\sqrt{cx^2}}{12\,c^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/12*(6*b^2*x^2 + 8*a*b*x + 3*a^2)*sqrt(c*x^2)/(c^3*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 30, normalized size = 0.45

$$-\frac{\left(6b^2x^2+8abx+3a^2\right)x}{12\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(c*x^2)^(5/2),x)
[Out] -1/12*x*(6*b^2*x^2+8*a*b*x+3*a^2)/(c*x^2)^(5/2)
```

maxima [A] time = 1.36, size = 37, normalized size = 0.56

$$-\frac{2\,ab}{3\,\left(cx^2\right)^{\frac{3}{2}}c}-\frac{b^2}{2\,c^{\frac{5}{2}}x^2}-\frac{a^2}{4\,c^{\frac{5}{2}}x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -2/3*a*b/((c*x^2)^(3/2)*c) - 1/2*b^2/(c^(5/2)*x^2) - 1/4*a^2/(c^(5/2)*x^4)
```

mupad [B] time = 0.17, size = 42, normalized size = 0.64

$$-\frac{3\,a^2\,\sqrt{x^2}\,+6\,b^2\,x^2\,\sqrt{x^2}\,+8\,a\,b\,x\,\sqrt{x^2}}{12\,c^{5/2}\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c*x^2)^(5/2),x)
```

```
[Out] -(3*a^2*(x^2)^(1/2) + 6*b^2*x^2*(x^2)^(1/2) + 8*a*b*x*(x^2)^(1/2))/(12*c^(5
/2)*x^5)
```

sympy [A] time = 0.96, size = 61, normalized size = 0.92

$$-\frac{a^2x}{4c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{2abx^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x^3}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2*x/(4*c**(5/2)*(x**2)**(5/2)) - 2*a*b*x**2/(3*c**(5/2)*(x**2)**(5/2))
- b**2*x**3/(2*c**(5/2)*(x**2)**(5/2))
```

3.806 $\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(5*c^2*x^4*Sqrt[c*x^2]) - (a*b)/(2*c^2*x^3*Sqrt[c*x^2]) - b^2/(3*c^2*x
^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^6}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^5}+\frac{b^2}{x^4}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{5c^2x^4\sqrt{cx^2}}-\frac{ab}{2c^2x^3\sqrt{cx^2}}-\frac{b^2}{3c^2x^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(6a^2+15abx+10b^2x^2\right)}{30c^3x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
```

```
[Out] -1/30*(Sqrt[c*x^2]*(6*a^2 + 15*a*b*x + 10*b^2*x^2))/(c^3*x^6)
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 0.48

$$\frac{-6a^2 - 15abx - 10b^2x^2}{30\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x*(c*x^2)^(5/2)),x]
[Out] (-6*a^2 - 15*a*b*x - 10*b^2*x^2)/(30*(c*x^2)^(5/2))
```

fricas [A] time = 1.49, size = 34, normalized size = 0.52

$$-\frac{\left(10\,b^2x^2 + 15\,abx + 6\,a^2\right)\sqrt{cx^2}}{30\,c^3x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/30*(10*b^2*x^2 + 15*a*b*x + 6*a^2)*sqrt(c*x^2)/(c^3*x^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x), x)
```

maple [A] time = 0.01, size = 29, normalized size = 0.44

$$-\frac{10b^2x^2 + 15abx + 6a^2}{30\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x/(c*x^2)^(5/2),x)
[Out] -1/30*(10*b^2*x^2+15*a*b*x+6*a^2)/(c*x^2)^(5/2)
```

maxima [A] time = 1.31, size = 37, normalized size = 0.56

$$-\frac{b^2}{3\left(cx^2\right)^{\frac{3}{2}}c} - \frac{ab}{2\,c^{\frac{5}{2}}x^4} - \frac{a^2}{5\,c^{\frac{5}{2}}x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/3*b^2/((c*x^2)^(3/2)*c) - 1/2*a*b/(c^(5/2)*x^4) - 1/5*a^2/(c^(5/2)*x^5)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.64

$$-\frac{6\,a^2\,\sqrt{x^2} + 10\,b^2\,x^2\,\sqrt{x^2} + 15\,a\,b\,x\,\sqrt{x^2}}{30\,c^{5/2}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x*(c*x^2)^(5/2)),x)
```

```
[Out] -(6*a^2*(x^2)^(1/2) + 10*b^2*x^2*(x^2)^(1/2) + 15*a*b*x*(x^2)^(1/2))/(30*c^
(5/2)*x^6)
```

sympy [A] time = 1.15, size = 56, normalized size = 0.85

$$-\frac{a^2}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{abx}{2c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x^2}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(5*c**(5/2)*(x**2)**(5/2)) - a*b*x/(2*c**(5/2)*(x**2)**(5/2)) - b**2*
x**2/(3*c**(5/2)*(x**2)**(5/2))
```

3.807 $\int \frac{(a+bx)^2}{x^2(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(6*c^2*x^5*Sqrt[c*x^2]) - (2*a*b)/(5*c^2*x^4*Sqrt[c*x^2]) - b^2/(4*c^2
*x^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^2\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^7}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^7}+\frac{2ab}{x^6}+\frac{b^2}{x^5}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^2}{6c^2x^5\sqrt{cx^2}}-\frac{2ab}{5c^2x^4\sqrt{cx^2}}-\frac{b^2}{4c^2x^3\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.58

$$-\frac{\sqrt{cx^2}\left(10a^2+24abx+15b^2x^2\right)}{60c^3x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
```

```
[Out] -1/60*(Sqrt[c*x^2]*(10*a^2 + 24*a*b*x + 15*b^2*x^2))/(c^3*x^7)
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-10a^2 - 24abx - 15b^2x^2}{60x\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^2*(c*x^2)^(5/2)),x]
[Out] (-10*a^2 - 24*a*b*x - 15*b^2*x^2)/(60*x*(c*x^2)^(5/2))
```

fricas [A] time = 0.77, size = 34, normalized size = 0.52

$$-\frac{\left(15\,b^2x^2 + 24\,abx + 10\,a^2\right)\sqrt{cx^2}}{60\,c^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/60*(15*b^2*x^2 + 24*a*b*x + 10*a^2)*sqrt(c*x^2)/(c^3*x^7)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$-\frac{15b^2x^2 + 24abx + 10a^2}{60\left(c\,x^2\right)^{\frac{5}{2}}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^2/(c*x^2)^(5/2),x)
[Out] -1/60*(15*b^2*x^2+24*a*b*x+10*a^2)/x/(c*x^2)^(5/2)
```

maxima [A] time = 1.34, size = 33, normalized size = 0.50

$$-\frac{b^2}{4\,c^{\frac{5}{2}}x^4} - \frac{2\,ab}{5\,c^{\frac{5}{2}}x^5} - \frac{a^2}{6\,c^{\frac{5}{2}}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^2/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/4*b^2/(c^(5/2)*x^4) - 2/5*a*b/(c^(5/2)*x^5) - 1/6*a^2/(c^(5/2)*x^6)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.64

$$-\frac{10\,a^2\,\sqrt{x^2}\, + 15\,b^2\,x^2\,\sqrt{x^2}\, + 24\,a\,b\,x\,\sqrt{x^2}}{60\,c^{5/2}\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^2*(c*x^2)^(5/2)),x)
```

```
[Out] -(10*a^2*(x^2)^(1/2) + 15*b^2*x^2*(x^2)^(1/2) + 24*a*b*x*(x^2)^(1/2))/(60*c
^(5/2)*x^7)
```

sympy [A] time = 1.40, size = 56, normalized size = 0.85

$$-\frac{a^2}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{2ab}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2x}{4c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**2/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(6*c**(5/2)*x*(x**2)**(5/2)) - 2*a*b/(5*c**(5/2)*(x**2)**(5/2)) - b**
2*x/(4*c**(5/2)*(x**2)**(5/2))
```

3.808 $\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(7*c^2*x^6*Sqrt[c*x^2]) - (a*b)/(3*c^2*x^5*Sqrt[c*x^2]) - b^2/(5*c^2*x
^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^2}{x^3\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^8}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^7}+\frac{b^2}{x^6}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^2}{7c^2x^6\sqrt{cx^2}}-\frac{ab}{3c^2x^5\sqrt{cx^2}}-\frac{b^2}{5c^2x^4\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.50

$$\frac{c\left(-15a^2-35abx-21b^2x^2\right)}{105\left(cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
```

```
[Out] (c*(-15*a^2 - 35*a*b*x - 21*b^2*x^2))/(105*(c*x^2)^(7/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-15a^2 - 35abx - 21b^2x^2}{105x^2 \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^3*(c*x^2)^(5/2)),x]
[Out] (-15*a^2 - 35*a*b*x - 21*b^2*x^2)/(105*x^2*(c*x^2)^(5/2))
```

fricas [A] time = 0.74, size = 34, normalized size = 0.52

$$-\frac{\left(21\,b^2x^2 + 35\,abx + 15\,a^2\right)\sqrt{cx^2}}{105\,c^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/105*(21*b^2*x^2 + 35*a*b*x + 15*a^2)*sqrt(c*x^2)/(c^3*x^8)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2}{\left(cx^2\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2/((c*x^2)^(5/2)*x^3), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.48

$$-\frac{21b^2x^2 + 35abx + 15a^2}{105\left(c\,x^2\right)^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^3/(c*x^2)^(5/2),x)
[Out] -1/105*(21*b^2*x^2+35*a*b*x+15*a^2)/x^2/(c*x^2)^(5/2)
```

maxima [A] time = 1.37, size = 33, normalized size = 0.50

$$-\frac{b^2}{5\,c^{\frac{5}{2}}x^5} - \frac{ab}{3\,c^{\frac{5}{2}}x^6} - \frac{a^2}{7\,c^{\frac{5}{2}}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^3/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/5*b^2/(c^(5/2)*x^5) - 1/3*a*b/(c^(5/2)*x^6) - 1/7*a^2/(c^(5/2)*x^7)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.64

$$-\frac{15\,a^2\,\sqrt{x^2} + 21\,b^2\,x^2\,\sqrt{x^2} + 35\,a\,b\,x\,\sqrt{x^2}}{105\,c^{5/2}\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^3*(c*x^2)^(5/2)),x)
```

```
[Out] -(15*a^2*(x^2)^(1/2) + 21*b^2*x^2*(x^2)^(1/2) + 35*a*b*x*(x^2)^(1/2))/(105*
c^(5/2)*x^8)
```

sympy [A] time = 1.70, size = 56, normalized size = 0.85

$$-\frac{a^2}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{ab}{3c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2}{5c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**3/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(7*c**(5/2)*x**2*(x**2)**(5/2)) - a*b/(3*c**(5/2)*x*(x**2)**(5/2)) -
b**2/(5*c**(5/2)*(x**2)**(5/2))
```

3.809 $\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] -a^2/(8*c^2*x^7*Sqrt[c*x^2]) - (2*a*b)/(7*c^2*x^6*Sqrt[c*x^2]) - b^2/(6*c^2
*x^5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{x^4\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(a+bx)^2}{x^9}\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{x^9}+\frac{2ab}{x^8}+\frac{b^2}{x^7}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{a^2}{8c^2x^7\sqrt{cx^2}}-\frac{2ab}{7c^2x^6\sqrt{cx^2}}-\frac{b^2}{6c^2x^5\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
```

```
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 35, normalized size = 0.53

$$\frac{-21a^2-48abx-28b^2x^2}{168x^3\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(x^4*(c*x^2)^(5/2)),x]
[Out] (-21*a^2 - 48*a*b*x - 28*b^2*x^2)/(168*x^3*(c*x^2)^(5/2))
```

fricas [A] time = 1.20, size = 34, normalized size = 0.52

$$-\frac{\left(28\,b^2x^2+48\,abx+21\,a^2\right)\sqrt{cx^2}}{168\,c^3x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] -1/168*(28*b^2*x^2 + 48*a*b*x + 21*a^2)*sqrt(c*x^2)/(c^3*x^9)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 32, normalized size = 0.48

$$-\frac{28b^2x^2+48abx+21a^2}{168\left(c\,x^2\right)^{\frac{5}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/x^4/(c*x^2)^(5/2),x)
[Out] -1/168*(28*b^2*x^2+48*a*b*x+21*a^2)/x^3/(c*x^2)^(5/2)
```

maxima [A] time = 1.35, size = 33, normalized size = 0.50

$$-\frac{b^2}{6\,c^{\frac{5}{2}}x^6}-\frac{2\,ab}{7\,c^{\frac{5}{2}}x^7}-\frac{a^2}{8\,c^{\frac{5}{2}}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/x^4/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] -1/6*b^2/(c^(5/2)*x^6) - 2/7*a*b/(c^(5/2)*x^7) - 1/8*a^2/(c^(5/2)*x^8)
```

mupad [B] time = 0.18, size = 42, normalized size = 0.64

$$-\frac{21\,a^2\,\sqrt{x^2}\,+28\,b^2\,x^2\,\sqrt{x^2}\,+48\,a\,b\,x\,\sqrt{x^2}}{168\,c^{5/2}\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(x^4*(c*x^2)^(5/2)),x)
```

```
[Out] -(21*a^2*(x^2)^(1/2) + 28*b^2*x^2*(x^2)^(1/2) + 48*a*b*x*(x^2)^(1/2))/(168*
c^(5/2)*x^9)
```

sympy [**A**] time = 2.03, size = 61, normalized size = 0.92

$$-\frac{a^2}{8c^{\frac{5}{2}}x^3\left(x^2\right)^{\frac{5}{2}}}-\frac{2ab}{7c^{\frac{5}{2}}x^2\left(x^2\right)^{\frac{5}{2}}}-\frac{b^2}{6c^{\frac{5}{2}}x\left(x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/x**4/(c*x**2)**(5/2),x)
```

```
[Out] -a**2/(8*c**(5/2)*x**3*(x**2)**(5/2)) - 2*a*b/(7*c**(5/2)*x**2*(x**2)**(5/2
)) - b**2/(6*c**(5/2)*x*(x**2)**(5/2))
```

3.810 $\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=102

$$\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}+\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a^3*Sqrt[c*x^2])/b^4) + (a^2*x*Sqrt[c*x^2])/(2*b^3) - (a*x^2*Sqrt[c*x^2]
)/(3*b^2) + (x^3*Sqrt[c*x^2])/(4*b) + (a^4*Sqrt[c*x^2]*Log[a + b*x])/(b^5*x
)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^4}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\ &= -\frac{a^3\sqrt{cx^2}}{b^4}+\frac{a^2x\sqrt{cx^2}}{2b^3}-\frac{ax^2\sqrt{cx^2}}{3b^2}+\frac{x^3\sqrt{cx^2}}{4b}+\frac{a^4\sqrt{cx^2}\,\log(a+bx)}{b^5x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.62

$$\frac{cx\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^4*Log[a +
b*x]))/(12*b^5*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.06, size = 64, normalized size = 0.63

$$\sqrt{cx^2}\left(\frac{a^4\log(a+bx)}{b^5x}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4) + (a^
4*Log[a + b*x])/(b^5*x))
```

fricas [A] time = 0.97, size = 62, normalized size = 0.61

$$\frac{\left(3\,b^4x^4-4\,ab^3x^3+6\,a^2b^2x^2-12\,a^3bx+12\,a^4\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))*sqrt(c*x^2)/(b^5*x)
```

giac [A] time = 1.11, size = 81, normalized size = 0.79

$$\frac{1}{12}\sqrt{c}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3x^4\mathrm{sgn}(x)-4\,ab^2x^3\mathrm{sgn}(x)+6\,a^2bx^2\mathrm{sgn}(x)-12\,a^3x\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(c)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.62

$$\frac{\sqrt{c\,x^2}\,\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(1/2)*(3*b^4*x^4-4*x^3*a*b^3+6*x^2*a^2*b^2+12*a^4*ln(b*x+a)-12
*b*x*a^3)/x/b^5
```

maxima [A] time = 1.57, size = 128, normalized size = 1.25

$$\frac{(-1)^{\frac{2cx}{b}}a^4\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2acx}{b}}a^4\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}+\frac{\sqrt{cx^2}\,a^2x}{2\,b^3}+\frac{\left(cx^2\right)^{\frac{3}{2}}x}{4\,bc}-\frac{\sqrt{cx^2}\,a^3}{b^4}-\frac{\left(cx^2\right)^{\frac{3}{2}}a}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*sqrt(c)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/2*sqrt(c*x^2)*a^2*x/b^3 + 1/4*(c*x^2
)^(3/2)*x/(b*c) - sqrt(c*x^2)*a^3/b^4 - 1/3*(c*x^2)^(3/2)*a/(b^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x), x)
```

3.811 $\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=80

$$-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}+\frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}}{b^3}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (a^2*Sqrt[c*x^2])/b^3 - (a*x*Sqrt[c*x^2])/(2*b^2) + (x^2*Sqrt[c*x^2])/(3*b)
 - (a^3*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2\sqrt{cx^2}}{b^3}-\frac{ax\sqrt{cx^2}}{2b^2}+\frac{x^2\sqrt{cx^2}}{3b}-\frac{a^3\sqrt{cx^2}\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.65

$$\frac{cx\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[
c*x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 54, normalized size = 0.68

$$\sqrt{cx^2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3}-\frac{a^3\log(a+bx)}{b^4x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3) - (a^3*Log[a + b*x]])/(b^
4*x))
```

fricas [A] time = 1.07, size = 51, normalized size = 0.64

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*x)
```

giac [A] time = 0.94, size = 69, normalized size = 0.86

$$-\frac{1}{6}\sqrt{c}\left(\frac{6\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2x^3\mathrm{sgn}(x)-3\,abx^2\mathrm{sgn}(x)+6\,a^2x\mathrm{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(c)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

maple [A] time = 0.01, size = 52, normalized size = 0.65

$$-\frac{\sqrt{c\,x^2}\,\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)}{6b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(1/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x/b^4
```

maxima [A] time = 1.56, size = 110, normalized size = 1.38

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ax}{2\,b^2}+\frac{\sqrt{cx^2}\,a^2}{b^3}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*sqrt(c)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*x/b^2 + sqrt(c*x^2)
*a^2/b^3 + 1/3*(c*x^2)^(3/2)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{cx^2}}{a + bx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x), x)
```

3.812 $\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=58

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] -((a*Sqrt[c*x^2])/b^2) + (x*Sqrt[c*x^2])/(2*b) + (a^2*Sqrt[c*x^2]*Log[a + b
*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{a+bx}\,dx&=\frac{\sqrt{cx^2}\int \frac{x^2}{a+bx}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\&=-\frac{a\sqrt{cx^2}}{b^2}+\frac{x\sqrt{cx^2}}{2b}+\frac{a^2\sqrt{cx^2}\,\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{cx\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] (c*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 0.71

$$\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3x}+\frac{bx-2a}{2b^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c*x^2])/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2) + (a^2*Log[a + b*x])/(b^3*x))
```

fricas [A] time = 1.19, size = 39, normalized size = 0.67

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

giac [A] time = 1.02, size = 54, normalized size = 0.93

$$\frac{1}{2}\sqrt{c}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bx^2\mathrm{sgn}(x)-2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

maple [A] time = 0.01, size = 40, normalized size = 0.69

$$\frac{\sqrt{c\,x^2}\,\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x/b^3
```

maxima [A] time = 1.49, size = 91, normalized size = 1.57

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,x}{2\,b}-\frac{\sqrt{cx^2}\,a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*sqrt(c)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*x/b - sqrt(c*x^2)*a/b^
2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a),x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x), x)
```

3.813 $\int \frac{\sqrt{cx^2}}{a+bx}\,dx$

Optimal. Leaf size=38

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x),x]
[Out] Sqrt[c*x^2]/b - (a*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{a+bx}\,dx &= \frac{\sqrt{cx^2}\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{b}-\frac{a\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.74

$$\frac{cx(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x),x]
[Out] (c*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 0.76

$$\sqrt{cx^2}\left(\frac{1}{b}-\frac{a\log(a+bx)}{b^2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(a + b*x),x]
```

```
[Out] Sqrt[c*x^2]*(b^(-1) - (a*Log[a + b*x])/(b^2*x))
```

fricas [A] time = 1.04, size = 27, normalized size = 0.71

$$\frac{\sqrt{cx^2}\left(bx - a\log\left(bx + a\right)\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*x)
```

giac [A] time = 0.96, size = 37, normalized size = 0.97

$$\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} + \frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.76

$$-\frac{\sqrt{c\,x^2}\,\left(a\ln\left(bx + a\right) - bx\right)}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a),x)
```

```
[Out] -(c*x^2)^(1/2)*(a*ln(b*x+a)-b*x)/b^2/x
```

maxima [B] time = 1.45, size = 74, normalized size = 1.95

$$-\frac{(-1)^{\frac{2cx}{b}}a\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}a\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*sqrt(c)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{c\,x^2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(1/2)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(a + b*x), x)
```

3.814 $\int \frac{\sqrt{cx^2}}{x(a+bx)} dx$

Optimal. Leaf size=22

$$\frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)),x]
[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)} dx &= \frac{\sqrt{cx^2}\int \frac{1}{a+bx} dx}{x}\\ &= \frac{\sqrt{cx^2}\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 0.95

$$\frac{cx\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)),x]
[Out] (c*x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 1.00

$$\frac{\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x*(a + b*x)),x]
```

[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*x)

fricas [**A**] time = 1.17, size = 20, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="fricas")

[Out] sqrt(c*x^2)*log(b*x + a)/(b*x)

giac [**A**] time = 0.95, size = 28, normalized size = 1.27

$$\sqrt{c}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="giac")

[Out] sqrt(c)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)

maple [**A**] time = 0.00, size = 21, normalized size = 0.95

$$\frac{\sqrt{c\,x^2}\;\ln\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(1/2)/x/(b*x+a),x)

[Out] ln(b*x+a)*(c*x^2)^(1/2)/b/x

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2)^(1/2)/x/(b*x+a),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

mupad [**F**] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c\,x^2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2)^(1/2)/(x*(a + b*x)),x)

[Out] int((c*x^2)^(1/2)/(x*(a + b*x)), x)

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(1/2)/x/(b*x+a),x)

[Out] Integral(sqrt(c*x**2)/(x*(a + b*x)), x)

3.815 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*Log[x])/(a*x) - (Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^2(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \frac{1}{x}\,dx}{ax}-\frac{\left(b\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}\\ &= \frac{\sqrt{cx^2}\log(x)}{ax}-\frac{\sqrt{cx^2}\log(a+bx)}{ax}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.62

$$\frac{cx(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] (c*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 0.88

$$\sqrt{cx^2}\left(\frac{\log(x)}{ax}-\frac{\log\left(a^2+abx\right)}{ax}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^2*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*(Log[x]/(a*x) - Log[a^2 + a*b*x]/(a*x))
```

fricas [A] time = 1.42, size = 64, normalized size = 1.52

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x +
 a)*sqrt(-c)/(a*c*x))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 26, normalized size = 0.62

$$\frac{\sqrt{c\,x^2}\,\left(\ln(x)-\ln\left(bx+a\right)\right)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a),x)
```

```
[Out] (c*x^2)^(1/2)*(ln(x)-ln(b*x+a))/x/a
```

maxima [A] time = 1.38, size = 24, normalized size = 0.57

$$-\frac{\sqrt{c}\log\left(bx+a\right)}{a}+\frac{\sqrt{c}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -sqrt(c)*log(b*x + a)/a + sqrt(c)*log(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^2\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)), x)
```

3.816 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)} dx$

Optimal. Leaf size=61

$$-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -(Sqrt[c*x^2]/(a*x^2)) - (b*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*Sqrt[c*x^2]*Lo
g[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{x^3(a+bx)} dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)} dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right) dx}{x}\\
&= -\frac{\sqrt{cx^2}}{ax^2}-\frac{b\sqrt{cx^2}\log(x)}{a^2x}+\frac{b\sqrt{cx^2}\log(a+bx)}{a^2x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.52

$$-\frac{c(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] -((c*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.72

$$\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2x}+\frac{b\log(a+bx)}{a^2x}-\frac{1}{ax^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^3*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*(-(1/(a*x^2)) - (b*Log[x])/(a^2*x) + (b*Log[a + b*x])/(a^2*x))
```

fricas [A] time = 0.77, size = 31, normalized size = 0.51

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 33, normalized size = 0.54

$$-\frac{\sqrt{c\,x^2}\,\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\right)}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x*ln(x)-b*ln(b*x+a)*x+a)/x^2/a^2
```

maxima [A] time = 1.38, size = 37, normalized size = 0.61

$$\frac{b\sqrt{c}\,\log\left(bx+a\right)}{a^2}-\frac{b\sqrt{c}\,\log(x)}{a^2}-\frac{\sqrt{c}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*sqrt(c)*log(b*x + a)/a^2 - b*sqrt(c)*log(x)/a^2 - sqrt(c)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^3\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a),x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)), x)
```

3.817 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=84

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{b\sqrt{cx^2}}{a^2x^2}-\frac{\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a*x^3) + (b*Sqrt[c*x^2])/(a^2*x^2) + (b^2*Sqrt[c*x^2]*Log[x
])/(a^3*x) - (b^2*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{\sqrt{cx^2}}{2ax^3}+\frac{b\sqrt{cx^2}}{a^2x^2}+\frac{b^2\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2\sqrt{cx^2}\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] (Sqrt[c*x^2]*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x])
)/(2*a^3*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3x}-\frac{b^2\log(a+bx)}{a^3x}+\frac{2bx-a}{2a^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^4*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-a + 2*b*x)/(2*a^2*x^3) + (b^2*Log[x])/(a^3*x) - (b^2*Log[a +
 b*x])/(a^3*x))
```

fricas [A] time = 1.15, size = 44, normalized size = 0.52

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 51, normalized size = 0.61

$$\frac{\sqrt{c\,x^2}\,\left(2b^2x^2\ln(x)-2b^2x^2\ln\left(bx+a\right)+2abx-a^2\right)}{2a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(2*b^2*x^2*ln(x)-2*b^2*ln(b*x+a)*x^2+2*a*b*x-a^2)/a^3/x^3
```

maxima [A] time = 1.36, size = 52, normalized size = 0.62

$$-\frac{b^2\sqrt{c}\,\log\left(bx+a\right)}{a^3}+\frac{b^2\sqrt{c}\,\log(x)}{a^3}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*sqrt(c)*log(b*x + a)/a^3 + b^2*sqrt(c)*log(x)/a^3 + 1/2*(2*b*sqrt(c)*x
 - a*sqrt(c))/(a^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)),x)
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^4(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a),x)
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)), x)
```

3.818 $\int \frac{x(cx^2)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=107

$$\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Rubi [A] time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2])/b^4) + (a^2*c*x*Sqrt[c*x^2])/(2*b^3) - (a*c*x^2*Sqrt[
c*x^2])/(3*b^2) + (c*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c*Sqrt[c*x^2]*Log[a + b*
x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(cx^2\right)^{3/2}}{a+bx}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\
&= -\frac{a^3c\sqrt{cx^2}}{b^4}+\frac{a^2cx\sqrt{cx^2}}{2b^3}-\frac{acx^2\sqrt{cx^2}}{3b^2}+\frac{cx^3\sqrt{cx^2}}{4b}+\frac{a^4c\sqrt{cx^2}\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 64, normalized size = 0.60

$$\frac{\left(cx^2\right)^{3/2}\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*a^
4*Log[a + b*x]))/(12*b^5*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.63

$$(cx^2)^{3/2}\left(\frac{a^4\log(a+bx)}{b^5x^3}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c*x^2)^(3/2))/(a + b*x),x]
```

```
[Out] (c*x^2)^(3/2)*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4*x^2)
 + (a^4*Log[a + b*x])/(b^5*x^3))
```

fricas [A] time = 1.04, size = 67, normalized size = 0.63

$$\frac{\left(3\,b^4cx^4-4\,ab^3cx^3+6\,a^2b^2cx^2-12\,a^3bcx+12\,a^4c\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c*x^4 - 4*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 - 12*a^3*b*c*x + 12*a^4
*c*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

giac [A] time = 1.00, size = 81, normalized size = 0.76

$$\frac{1}{12}\,c^{\frac{3}{2}}\left(\frac{12\,a^4\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3x^4\mathrm{sgn}(x)-4\,ab^2x^3\mathrm{sgn}(x)+6\,a^2bx^2\mathrm{sgn}(x)-12\,a^3x\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*c^(3/2)*(12*a^4*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*log(abs(a))*sgn(
x)/b^5 + (3*b^3*x^4*sgn(x) - 4*a*b^2*x^3*sgn(x) + 6*a^2*b*x^2*sgn(x) - 12*a
^3*x*sgn(x))/b^4)
```

maple [A] time = 0.01, size = 63, normalized size = 0.59

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(3/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/x^3/b^5
```

maxima [A] time = 1.62, size = 124, normalized size = 1.16

$$\frac{(-1)^{\frac{2cx}{b}}\,a^4c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^4c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}+\frac{\left(cx^2\right)^{\frac{3}{2}}x}{4\,b}+\frac{\sqrt{cx^2}\,a^2cx}{2\,b^3}-\frac{\left(cx^2\right)^{\frac{3}{2}}a}{3\,b^2}-\frac{\sqrt{cx^2}\,a^3c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(3/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*x/b + 1/2*sqrt(c*x^2
)*a^2*c*x/b^3 - 1/3*(c*x^2)^(3/2)*a/b^2 - sqrt(c*x^2)*a^3*c/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c\,x^2\right)^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x),x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(cx^2\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x), x)
```

3.819 $\int \frac{(cx^2)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=84

$$-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}+\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}}{b^3}-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (a^2*c*Sqrt[c*x^2])/b^3 - (a*c*x*Sqrt[c*x^2])/(2*b^2) + (c*x^2*Sqrt[c*x^2])
/(3*b) - (a^3*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{a+bx}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\
&= \frac{a^2c\sqrt{cx^2}}{b^3}-\frac{acx\sqrt{cx^2}}{2b^2}+\frac{cx^2\sqrt{cx^2}}{3b}-\frac{a^3c\sqrt{cx^2}\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.63

$$\frac{(cx^2)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] ((c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6
*b^4*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 57, normalized size = 0.68

$$(cx^2)^{3/2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3x^2}-\frac{a^3\log(a+bx)}{b^4x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(a + b*x),x]
```

```
[Out] (c*x^2)^(3/2)*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*x^2) - (a^3*Log[a + b*x
])/(b^4*x^3))
```

fricas [A] time = 1.31, size = 55, normalized size = 0.65

$$\frac{\left(2\,b^3cx^3-3\,ab^2cx^2+6\,a^2bcx-6\,a^3c\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c*x^3 - 3*a*b^2*c*x^2 + 6*a^2*b*c*x - 6*a^3*c*log(b*x + a))*sqrt
(c*x^2)/(b^4*x)
```

giac [A] time = 1.15, size = 69, normalized size = 0.82

$$-\frac{1}{6}c^{\frac{3}{2}}\left(\frac{6\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2x^3\mathrm{sgn}(x)-3\,abx^2\mathrm{sgn}(x)+6\,a^2x\mathrm{sgn}(x)}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*c^(3/2)*(6*a^3*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*log(abs(a))*sgn(x)
/b^4 - (2*b^2*x^3*sgn(x) - 3*a*b*x^2*sgn(x) + 6*a^2*x*sgn(x))/b^3)
```

maple [A] time = 0.00, size = 52, normalized size = 0.62

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)}{6b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^3/b
^4
```

maxima [A] time = 1.54, size = 109, normalized size = 1.30

$$-\frac{(-1)^{\frac{2\,cx}{b}}\,a^3c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,acx}{2\,b^2}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,b}+\frac{\sqrt{cx^2}\,a^2c}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(3/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c*x/b^2 + 1/3*(c*x^
2)^(3/2)/b + sqrt(c*x^2)*a^2*c/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x), x)
```

3.820 $\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx$

Optimal. Leaf size=61

$$\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x} - \frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x} - \frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] -((a*c*Sqrt[c*x^2])/b^2) + (c*x*Sqrt[c*x^2])/(2*b) + (a^2*c*Sqrt[c*x^2]*Log
[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x(a+bx)} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{a+bx} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right) dx}{x} \\
&= -\frac{ac\sqrt{cx^2}}{b^2} + \frac{cx\sqrt{cx^2}}{2b} + \frac{a^2c\sqrt{cx^2}\log(a+bx)}{b^3x}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 0.69

$$\frac{c^2x\left(2a^2\log(a+bx) + bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)),x]
```

```
[Out] (c^2*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.72

$$(cx^2)^{3/2}\left(\frac{a^2\log(a+bx)}{b^3x^3}+\frac{bx-2a}{2b^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x*(a + b*x)),x]
[Out] (c*x^2)^(3/2)*((-2*a + b*x)/(2*b^2*x^2) + (a^2*Log[a + b*x])/(b^3*x^3))
```

fricas [A] time = 1.42, size = 42, normalized size = 0.69

$$\frac{\left(b^2cx^2-2\,abcx+2\,a^2c\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(b^2*c*x^2 - 2*a*b*c*x + 2*a^2*c*log(b*x + a))*sqrt(c*x^2)/(b^3*x)
```

giac [A] time = 1.12, size = 54, normalized size = 0.89

$$\frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{2\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bx^2\mathrm{sgn}(x)-2\,ax\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="giac")
[Out] 1/2*c^(3/2)*(2*a^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*log(abs(a))*sgn(x)/
b^3 + (b*x^2*sgn(x) - 2*a*x*sgn(x))/b^2)
```

maple [A] time = 0.00, size = 40, normalized size = 0.66

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a),x)
[Out] 1/2*(c*x^2)^(3/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/b^3/x^3
```

maxima [A] time = 1.48, size = 93, normalized size = 1.52

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,cx}{2\,b}-\frac{\sqrt{cx^2}\,ac}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a),x, algorithm="maxima")
[Out] (-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(3/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c*x/b - sqrt(c*x^2)*a*
c/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)), x)
```

3.821 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=40

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
[Out] (c*Sqrt[c*x^2])/b - (a*c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^2(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{a+bx}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{x}\\ &= \frac{c\sqrt{cx^2}}{b}-\frac{ac\sqrt{cx^2}\log(a+bx)}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.75

$$\frac{c^2x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
[Out] (c^2*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.82

$$(cx^2)^{3/2}\left(\frac{1}{bx^2}-\frac{a\log(a+bx)}{b^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^2*(a + b*x)),x]
[Out] (c*x^2)^(3/2)*(1/(b*x^2) - (a*Log[a + b*x])/(b^2*x^3))
```

fricas [A] time = 0.84, size = 29, normalized size = 0.72

$$\frac{\left(bcx - ac\log\left(bx + a\right)\right)\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="fricas")
[Out] (b*c*x - a*c*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

giac [A] time = 1.00, size = 37, normalized size = 0.92

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b} - \frac{a\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} + \frac{a\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="giac")
[Out] c^(3/2)*(x*sgn(x)/b - a*log(abs(b*x + a))*sgn(x)/b^2 + a*log(abs(a))*sgn(x)
/b^2)
```

maple [A] time = 0.00, size = 29, normalized size = 0.72

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(a\ln\left(bx + a\right) - bx\right)}{b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a),x)
[Out] -(c*x^2)^(3/2)*(a*ln(b*x+a)-b*x)/b^2/x^3
```

maxima [B] time = 1.48, size = 75, normalized size = 1.88

$$-\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}ac^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}c}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a),x, algorithm="maxima")
[Out] -(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(3/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^2\,\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)), x)
```

3.822 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{c\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{c\sqrt{cx^2}\,\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.96

$$\frac{(cx^2)^{3/2}\log(a+bx)}{bx^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^3*(a + b*x)),x]
[Out] ((c*x^2)^(3/2)*Log[a + b*x])/(b*x^3)
```

fricas [A] time = 1.05, size = 21, normalized size = 0.91

$$\frac{\sqrt{cx^2}\,c\log\left(bx+a\right)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*c*log(b*x + a)/(b*x)
```

giac [A] time = 1.14, size = 28, normalized size = 1.22

$$c^{\frac{3}{2}}\left(\frac{\log\left(|bx+a|\right)\operatorname{sgn}(x)}{b}-\frac{\log\left(|a|\right)\operatorname{sgn}(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="giac")
[Out] c^(3/2)*(log(abs(b*x + a))*sgn(x)/b - log(abs(a))*sgn(x)/b)
```

maple [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\ln\left(bx+a\right)}{b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a),x)
[Out] (c*x^2)^(3/2)/x^3*ln(b*x+a)/b
```

maxima [A] time = 1.36, size = 13, normalized size = 0.57

$$\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a),x, algorithm="maxima")
[Out] c^(3/2)*log(b*x + a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^3*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**3*(a + b*x)), x)
```

3.823 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{c\sqrt{cx^2}\log(x)}{ax}-\frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{c\sqrt{cx^2}\log(x)}{ax}-\frac{c\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*Sqrt[c*x^2]*Log[x])/(a*x) - (c*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^4(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x}\,dx}{ax}-\frac{\left(bc\sqrt{cx^2}\right)\int \frac{1}{a+bx}\,dx}{ax}\\ &= \frac{c\sqrt{cx^2}\log(x)}{ax}-\frac{c\sqrt{cx^2}\log(a+bx)}{ax}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{(cx^2)^{3/2}(\log(x)-\log(a+bx))}{ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(Log[x] - Log[a + b*x]))/(a*x^3)
```

IntegrateAlgebraic [A] time = 0.03, size = 37, normalized size = 0.84

$$(cx^2)^{3/2}\left(\frac{\log(x)}{ax^3}-\frac{\log\left(a^2+abx\right)}{ax^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*(Log[x]/(a*x^3) - Log[a^2 + a*b*x]/(a*x^3))
```

fricas [A] time = 0.79, size = 66, normalized size = 1.50

$$\left[\frac{\sqrt{cx^2}\,c\log\left(\frac{x}{bx+a}\right)}{ax}, \frac{2\sqrt{-c}\,c\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(\ln(x)-\ln\left(bx+a\right)\right)}{a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a),x)
```

```
[Out] (c*x^2)^(3/2)*(ln(x)-ln(b*x+a))/a/x^3
```

maxima [A] time = 1.38, size = 24, normalized size = 0.55

$$-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{3}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(3/2)*log(b*x + a)/a + c^(3/2)*log(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{4}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)), x)
```

3.824 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx$

Optimal. Leaf size=64

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]

[Out] -((c*Sqrt[c*x^2])/(a*x^2)) - (b*c*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c*Sqrt[c
*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^5(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{ax^2}-\frac{bc\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.53

$$-\frac{c^2(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]

[Out] -((c^2*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.69

$$\left(cx^2\right)^{3/2}\left(-\frac{b\log(x)}{a^2x^3}+\frac{b\log(a+bx)}{a^2x^3}-\frac{1}{ax^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^5*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*(-(1/(a*x^4)) - (b*Log[x])/(a^2*x^3) + (b*Log[a + b*x])/(a^2*
x^3))
```

fricas [A] time = 1.10, size = 33, normalized size = 0.52

$$\frac{\left(bcx\log\left(\frac{bx+a}{x}\right)-ac\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c*x*log((b*x + a)/x) - a*c)*sqrt(c*x^2)/(a^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 33, normalized size = 0.52

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\right)}{a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a),x)
```

```
[Out] -(c*x^2)^(3/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a)/x^4/a^2
```

maxima [A] time = 1.36, size = 37, normalized size = 0.58

$$\frac{bc^{\frac{3}{2}}\log\left(bx+a\right)}{a^2}-\frac{bc^{\frac{3}{2}}\log(x)}{a^2}-\frac{c^{\frac{3}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*log(b*x + a)/a^2 - b*c^(3/2)*log(x)/a^2 - c^(3/2)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c\,x^2\right)^{3/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{5}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)), x)
```

3.825 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}+\frac{bc\sqrt{cx^2}}{a^2x^2}-\frac{c\sqrt{cx^2}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a*x^3) + (b*c*Sqrt[c*x^2])/(a^2*x^2) + (b^2*c*Sqrt[c*x^
2]*Log[x])/(a^3*x) - (b^2*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^6(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{x}\\
&= -\frac{c\sqrt{cx^2}}{2ax^3}+\frac{bc\sqrt{cx^2}}{a^2x^2}+\frac{b^2c\sqrt{cx^2}\log(x)}{a^3x}-\frac{b^2c\sqrt{cx^2}\log(a+bx)}{a^3x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.60

$$\frac{(cx^2)^{3/2}\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(3/2)*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x
]))/(2*a^3*x^5)
```

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 0.66

$$\left(cx^2\right)^{3/2}\left(\frac{b^2\log(x)}{a^3x^3}-\frac{b^2\log(a+bx)}{a^3x^3}+\frac{2bx-a}{2a^2x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*((-a + 2*b*x)/(2*a^2*x^5) + (b^2*Log[x])/(a^3*x^3) - (b^2*Log
[a + b*x])/(a^3*x^3))
```

fricas [A] time = 0.91, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2cx^2\log\left(\frac{x}{bx+a}\right)+2\,abcx-a^2c\right)\sqrt{cx^2}}{2\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*c*x^2*log(x/(b*x + a)) + 2*a*b*c*x - a^2*c)*sqrt(c*x^2)/(a^3*x^3
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.00, size = 51, normalized size = 0.58

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx-a^2\right)}{2a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/x^5/a^3
```

maxima [A] time = 1.45, size = 52, normalized size = 0.59

$$-\frac{b^2c^{\frac{3}{2}}\log(bx+a)}{a^3}+\frac{b^2c^{\frac{3}{2}}\log(x)}{a^3}+\frac{2\,bc^{\frac{3}{2}}x-ac^{\frac{3}{2}}}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -b^2*c^(3/2)*log(b*x + a)/a^3 + b^2*c^(3/2)*log(x)/a^3 + 1/2*(2*b*c^(3/2)*x
 - a*c^(3/2))/(a^2*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{3/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x^6(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)), x)
```

3.826 $\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=112

$$-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Rubi [A] time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{c\sqrt{cx^2}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(3*a*x^4) + (b*c*Sqrt[c*x^2])/(2*a^2*x^3) - (b^2*c*Sqrt[c*
x^2])/(a^3*x^2) - (b^3*c*Sqrt[c*x^2]*Log[x])/(a^4*x) + (b^3*c*Sqrt[c*x^2]*L
og[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^7(a+bx)}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^4(a+bx)}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{x}\\
&= -\frac{c\sqrt{cx^2}}{3ax^4}+\frac{bc\sqrt{cx^2}}{2a^2x^3}-\frac{b^2c\sqrt{cx^2}}{a^3x^2}-\frac{b^3c\sqrt{cx^2}\log(x)}{a^4x}+\frac{b^3c\sqrt{cx^2}\log(a+bx)}{a^4x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.58

$$-\frac{(cx^2)^{3/2}\left(a\left(2a^2-3abx+6b^2x^2\right)-6b^3x^3\log(a+bx)+6b^3x^3\log(x)\right)}{6a^4x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] -1/6*((c*x^2)^(3/2)*(a*(2*a^2 - 3*a*b*x + 6*b^2*x^2) + 6*b^3*x^3*Log[x] - 6
*b^3*x^3*Log[a + b*x]))/(a^4*x^6)
```

IntegrateAlgebraic [A] time = 0.07, size = 69, normalized size = 0.62

$$(cx^2)^{3/2}\left(-\frac{b^3\log(x)}{a^4x^3}+\frac{b^3\log(a+bx)}{a^4x^3}+\frac{-2a^2+3abx-6b^2x^2}{6a^3x^6}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^7*(a + b*x)),x]
```

```
[Out] (c*x^2)^(3/2)*((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3*x^6) - (b^3*Log[x])/(a
^4*x^3) + (b^3*Log[a + b*x])/(a^4*x^3))
```

fricas [A] time = 1.01, size = 59, normalized size = 0.53

$$\frac{\left(6\,b^3cx^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2cx^2+3\,a^2bcx-2\,a^3c\right)\sqrt{cx^2}}{6\,a^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*c*x^3*log((b*x + a)/x) - 6*a*b^2*c*x^2 + 3*a^2*b*c*x - 2*a^3*c)*
sqrt(c*x^2)/(a^4*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 62, normalized size = 0.55

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(6b^3x^3\ln(x)-6b^3x^3\ln\left(bx+a\right)+6a\,b^2x^2-3a^2bx+2a^3\right)}{6a^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^7/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(3/2)*(6*b^3*ln(x)*x^3-6*b^3*ln(b*x+a)*x^3+6*a*b^2*x^2-3*a^2*b
*x+2*a^3)/x^6/a^4
```

maxima [A] time = 1.45, size = 66, normalized size = 0.59

$$\frac{b^3c^{\frac{3}{2}}\log\left(bx+a\right)}{a^4}-\frac{b^3c^{\frac{3}{2}}\log(x)}{a^4}-\frac{6\,b^2c^{\frac{3}{2}}x^2-3\,abc^{\frac{3}{2}}x+2\,a^2c^{\frac{3}{2}}}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b^3*c^(3/2)*log(b*x + a)/a^4 - b^3*c^(3/2)*log(x)/a^4 - 1/6*(6*b^2*c^(3/2)*
x^2 - 3*a*b*c^(3/2)*x + 2*a^2*c^(3/2))/(a^3*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^7\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^7*(a + b*x)),x)
[Out] int((c*x^2)^(3/2)/(x^7*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{x^{7}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**7/(b*x+a),x)
[Out] Integral((c*x**2)**(3/2)/(x**7*(a + b*x)), x)
```

3.827 $\int \frac{(cx^2)^{5/2}}{a+bx}\,dx$

Optimal. Leaf size=142

$$-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}+\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Rubi [A] time = 0.04, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2])/b^5 - (a^3*c^2*x*Sqrt[c*x^2])/(2*b^4) + (a^2*c^2*x^2*
Sqrt[c*x^2])/(3*b^3) - (a*c^2*x^3*Sqrt[c*x^2])/(4*b^2) + (c^2*x^4*Sqrt[c*x^
2])/(5*b) - (a^5*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^6*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{a+bx}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^5}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4}{b^5}-\frac{a^3x}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^3}{b^2}+\frac{x^4}{b}-\frac{a^5}{b^5(a+bx)}\right)dx}{x}\\
&= \frac{a^4c^2\sqrt{cx^2}}{b^5}-\frac{a^3c^2x\sqrt{cx^2}}{2b^4}+\frac{a^2c^2x^2\sqrt{cx^2}}{3b^3}-\frac{ac^2x^3\sqrt{cx^2}}{4b^2}+\frac{c^2x^4\sqrt{cx^2}}{5b}-\frac{a^5c^2\sqrt{cx^2}\log(a+bx)}{b^6x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 76, normalized size = 0.54

$$\frac{c^3x\left(bx\left(60a^4-30a^3bx+20a^2b^2x^2-15ab^3x^3+12b^4x^4\right)-60a^5\log(a+bx)\right)}{60b^6\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c^3*x*(b*x*(60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^4*x
^4) - 60*a^5*Log[a + b*x]))/(60*b^6*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.06, size = 79, normalized size = 0.56

$$\left(cx^2\right)^{5/2}\left(\frac{60a^4-30a^3bx+20a^2b^2x^2-15ab^3x^3+12b^4x^4}{60b^5x^4}-\frac{a^5\log(a+bx)}{b^6x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(a + b*x),x]
```

```
[Out] (c*x^2)^(5/2)*((60*a^4 - 30*a^3*b*x + 20*a^2*b^2*x^2 - 15*a*b^3*x^3 + 12*b^
4*x^4)/(60*b^5*x^4) - (a^5*Log[a + b*x])/(b^6*x^5))
```

fricas [A] time = 1.34, size = 91, normalized size = 0.64

$$\frac{\left(12\,b^5c^2x^5-15\,ab^4c^2x^4+20\,a^2b^3c^2x^3-30\,a^3b^2c^2x^2+60\,a^4bc^2x-60\,a^5c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{60\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*c^2*x^5 - 15*a*b^4*c^2*x^4 + 20*a^2*b^3*c^2*x^3 - 30*a^3*b^2*c
^2*x^2 + 60*a^4*b*c^2*x - 60*a^5*c^2*log(b*x + a))*sqrt(c*x^2)/(b^6*x)
```

giac [A] time = 1.18, size = 116, normalized size = 0.82

$$-\frac{1}{60}\left(\frac{60\,a^5c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^6}-\frac{60\,a^5c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^6}-\frac{12\,b^4c^2x^5\mathrm{sgn}(x)-15\,ab^3c^2x^4\mathrm{sgn}(x)+20\,a^2b^2c^2x^3\mathrm{sgn}(x)-30\,a^3bc^2x^2\mathrm{sgn}(x)+60\,a^4c^2x\mathrm{sgn}(x)}{b^5}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/60*(60*a^5*c^2*log(abs(b*x + a))*sgn(x)/b^6 - 60*a^5*c^2*log(abs(a))*sgn
(x)/b^6 - (12*b^4*c^2*x^5*sgn(x) - 15*a*b^3*c^2*x^4*sgn(x) + 20*a^2*b^2*c^2
*x^3*sgn(x) - 30*a^3*b*c^2*x^2*sgn(x) + 60*a^4*c^2*x*sgn(x))/b^5)*sqrt(c)
```

maple [A] time = 0.01, size = 74, normalized size = 0.52

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(-12b^5x^5+15a\,b^4x^4-20a^2b^3x^3+30a^3b^2x^2+60a^5\ln\left(bx+a\right)-60a^4bx\right)}{60b^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(b*x+a),x)
```

```
[Out] -1/60*(c*x^2)^(5/2)*(-12*b^5*x^5+15*a*b^4*x^4-20*a^2*b^3*x^3+30*a^3*b^2*x^2
+60*a^5*ln(b*x+a)-60*a^4*b*x)/x^5/b^6
```

maxima [A] time = 1.58, size = 146, normalized size = 1.03

$$-\frac{(-1)^{\frac{2cx}{b}}\,a^5c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^6}-\frac{(-1)^{\frac{2acx}{b}}\,a^5c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^6}-\frac{\left(cx^2\right)^{\frac{3}{2}}acx}{4\,b^2}-\frac{\sqrt{cx^2}\,a^3c^2x}{2\,b^4}+\frac{\left(cx^2\right)^{\frac{5}{2}}}{5\,b}+\frac{\left(cx^2\right)^{\frac{3}{2}}a^2c}{3\,b^3}+\frac{\sqrt{cx^2}\,a^4c^2}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^5*c^(5/2)*log(2*c*x/b)/b^6 - (-1)^(2*a*c*x/b)*a^5*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^6 - 1/4*(c*x^2)^(3/2)*a*c*x/b^2 - 1/2*sqr
t(c*x^2)*a^3*c^2*x/b^4 + 1/5*(c*x^2)^(5/2)/b + 1/3*(c*x^2)^(3/2)*a^2*c/b^3
+ sqrt(c*x^2)*a^4*c^2/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{a+b\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(a + b*x),x)
```

```
[Out] int((c*x^2)^(5/2)/(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{a+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(a + b*x), x)
```

3.828 $\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx$

Optimal. Leaf size=117

$$\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Rubi [A] time = 0.05, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}+\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2])/b^4) + (a^2*c^2*x*Sqrt[c*x^2])/(2*b^3) - (a*c^2*x^2
*Sqrt[c*x^2])/(3*b^2) + (c^2*x^3*Sqrt[c*x^2])/(4*b) + (a^4*c^2*Sqrt[c*x^2]*
Log[a + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^4}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3}{b^4}+\frac{a^2x}{b^3}-\frac{ax^2}{b^2}+\frac{x^3}{b}+\frac{a^4}{b^4(a+bx)}\right)dx}{x}\\
&= -\frac{a^3c^2\sqrt{cx^2}}{b^4}+\frac{a^2c^2x\sqrt{cx^2}}{2b^3}-\frac{ac^2x^2\sqrt{cx^2}}{3b^2}+\frac{c^2x^3\sqrt{cx^2}}{4b}+\frac{a^4c^2\sqrt{cx^2}\,\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 0.56

$$\frac{c\left(cx^2\right)^{3/2}\left(12a^4\log(a+bx)+bx\left(-12a^3+6a^2bx-4ab^2x^2+3b^3x^3\right)\right)}{12b^5x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3) + 12*
a^4*Log[a + b*x]))/(12*b^5*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.57

$$(cx^2)^{5/2}\left(\frac{a^4\log(a+bx)}{b^5x^5}+\frac{-12a^3+6a^2bx-4ab^2x^2+3b^3x^3}{12b^4x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*((-12*a^3 + 6*a^2*b*x - 4*a*b^2*x^2 + 3*b^3*x^3)/(12*b^4*x^4)
 + (a^4*Log[a + b*x])/(b^5*x^5))
```

fricas [A] time = 1.19, size = 77, normalized size = 0.66

$$\frac{\left(3\,b^4c^2x^4-4\,ab^3c^2x^3+6\,a^2b^2c^2x^2-12\,a^3bc^2x+12\,a^4c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{12\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*c^2*x^4 - 4*a*b^3*c^2*x^3 + 6*a^2*b^2*c^2*x^2 - 12*a^3*b*c^2*x
+ 12*a^4*c^2*log(b*x + a))*sqrt(c*x^2)/(b^5*x)
```

giac [A] time = 1.01, size = 99, normalized size = 0.85

$$\frac{1}{12}\left(\frac{12\,a^4c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}-\frac{12\,a^4c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,b^3c^2x^4\mathrm{sgn}(x)-4\,ab^2c^2x^3\mathrm{sgn}(x)+6\,a^2bc^2x^2\mathrm{sgn}(x)-12\,a^3c^2x\mathrm{sgn}(x)}{b^4}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/12*(12*a^4*c^2*log(abs(b*x + a))*sgn(x)/b^5 - 12*a^4*c^2*log(abs(a))*sgn(
x)/b^5 + (3*b^3*c^2*x^4*sgn(x) - 4*a*b^2*c^2*x^3*sgn(x) + 6*a^2*b*c^2*x^2*s
gn(x) - 12*a^3*c^2*x*sgn(x))/b^4)*sqrt(c)
```

maple [A] time = 0.01, size = 63, normalized size = 0.54

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(3b^4x^4-4a\,b^3x^3+6a^2b^2x^2+12a^4\ln\left(bx+a\right)-12a^3bx\right)}{12b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x/(b*x+a),x)
```

```
[Out] 1/12*(c*x^2)^(5/2)*(3*b^4*x^4-4*a*b^3*x^3+6*a^2*b^2*x^2+12*a^4*ln(b*x+a)-12
*a^3*b*x)/b^5/x^5
```

maxima [A] time = 1.60, size = 130, normalized size = 1.11

$$\frac{(-1)^{\frac{2cx}{b}}\,a^4c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^5}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^4c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}+\frac{\left(cx^2\right)^{\frac{3}{2}}cx}{4\,b}+\frac{\sqrt{cx^2}\,a^2c^2x}{2\,b^3}-\frac{\left(cx^2\right)^{\frac{3}{2}}ac}{3\,b^2}-\frac{\sqrt{cx^2}\,a^3c^2}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^4*c^(5/2)*log(2*c*x/b)/b^5 + (-1)^(2*a*c*x/b)*a^4*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^5 + 1/4*(c*x^2)^(3/2)*c*x/b + 1/2*sqrt(c*x
^2)*a^2*c^2*x/b^3 - 1/3*(c*x^2)^(3/2)*a*c/b^2 - sqrt(c*x^2)*a^3*c^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x*(a + b*x)), x)
```

3.829 $\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx$

Optimal. Leaf size=92

$$-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}+\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2])/b^3 - (a*c^2*x*Sqrt[c*x^2])/(2*b^2) + (c^2*x^2*Sqrt[c
*x^2])/(3*b) - (a^3*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^2(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^3}{a+bx}\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{x}\\ &= \frac{a^2c^2\sqrt{cx^2}}{b^3}-\frac{ac^2x\sqrt{cx^2}}{2b^2}+\frac{c^2x^2\sqrt{cx^2}}{3b}-\frac{a^3c^2\sqrt{cx^2}\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.59

$$\frac{c\left(cx^2\right)^{3/2}\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*(c*x^2)^(3/2)*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/
(6*b^4*x^3)
```

IntegrateAlgebraic [A] time = 0.05, size = 57, normalized size = 0.62

$$\left(cx^2\right)^{5/2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3x^4}-\frac{a^3\log(a+bx)}{b^4x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^2*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*x^4) - (a^3*Log[a + b*x
])/(b^4*x^5))
```

fricas [A] time = 0.87, size = 63, normalized size = 0.68

$$\frac{\left(2\,b^3c^2x^3-3\,ab^2c^2x^2+6\,a^2bc^2x-6\,a^3c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*c^2*x^3 - 3*a*b^2*c^2*x^2 + 6*a^2*b*c^2*x - 6*a^3*c^2*log(b*x +
a))*sqrt(c*x^2)/(b^4*x)
```

giac [A] time = 0.96, size = 84, normalized size = 0.91

$$-\frac{1}{6}\left(\frac{6\,a^3c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}-\frac{6\,a^3c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^4}-\frac{2\,b^2c^2x^3\mathrm{sgn}(x)-3\,abc^2x^2\mathrm{sgn}(x)+6\,a^2c^2x\mathrm{sgn}(x)}{b^3}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] -1/6*(6*a^3*c^2*log(abs(b*x + a))*sgn(x)/b^4 - 6*a^3*c^2*log(abs(a))*sgn(x)
/b^4 - (2*b^2*c^2*x^3*sgn(x) - 3*a*b*c^2*x^2*sgn(x) + 6*a^2*c^2*x*sgn(x))/b
^3)*sqrt(c)
```

maple [A] time = 0.00, size = 52, normalized size = 0.57

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)}{6b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^2/(b*x+a),x)
```

```
[Out] -1/6*(c*x^2)^(5/2)*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/x^5/b
^4
```

maxima [A] time = 1.56, size = 114, normalized size = 1.24

$$-\frac{(-1)^{\frac{2cx}{b}}a^3c^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^4}-\frac{(-1)^{\frac{2acx}{b}}a^3c^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^4}-\frac{\sqrt{cx^2}\,ac^2x}{2\,b^2}+\frac{\left(cx^2\right)^{\frac{3}{2}}c}{3\,b}+\frac{\sqrt{cx^2}\,a^2c^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a^3*c^(5/2)*log(2*c*x/b)/b^4 - (-1)^(2*a*c*x/b)*a^3*c^(5/2)
*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - 1/2*sqrt(c*x^2)*a*c^2*x/b^2 + 1/3*(c*
x^2)^(3/2)*c/b + sqrt(c*x^2)*a^2*c^2/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^2\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^2*(a + b*x)),x)
[Out] int((c*x^2)^(5/2)/(x^2*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**2/(b*x+a),x)
[Out] Integral((c*x**2)**(5/2)/(x**2*(a + b*x)), x)
```

3.830 $\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx$

Optimal. Leaf size=67

$$\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}-\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2])/b^2) + (c^2*x*Sqrt[c*x^2])/(2*b) + (a^2*c^2*Sqrt[c*x^
2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^3(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x^2}{a+bx}\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{x}\\
&= -\frac{ac^2\sqrt{cx^2}}{b^2}+\frac{c^2x\sqrt{cx^2}}{2b}+\frac{a^2c^2\sqrt{cx^2}\log(a+bx)}{b^3x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.63

$$\frac{c^3x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c^3*x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.66

$$\left(cx^2\right)^{5/2}\left(\frac{a^2\log(a+bx)}{b^3x^5}+\frac{bx-2a}{2b^2x^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^3*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*((-2*a + b*x)/(2*b^2*x^4) + (a^2*Log[a + b*x])/(b^3*x^5))
```

fricas [A] time = 1.23, size = 48, normalized size = 0.72

$$\frac{\left(b^2c^2x^2-2\,abc^2x+2\,a^2c^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 2*a*b*c^2*x + 2*a^2*c^2*log(b*x + a))*sqrt(c*x^2)/(b^3*x
)
```

giac [A] time = 1.13, size = 66, normalized size = 0.99

$$\frac{1}{2}\left(\frac{2\,a^2c^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}-\frac{2\,a^2c^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^3}+\frac{bc^2x^2\mathrm{sgn}(x)-2\,ac^2x\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*c^2*log(abs(b*x + a))*sgn(x)/b^3 - 2*a^2*c^2*log(abs(a))*sgn(x)/
b^3 + (b*c^2*x^2*sgn(x) - 2*a*c^2*x*sgn(x))/b^2)*sqrt(c)
```

maple [A] time = 0.01, size = 40, normalized size = 0.60

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)}{2b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^3/(b*x+a),x)
```

```
[Out] 1/2*(c*x^2)^(5/2)*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/x^5/b^3
```

maxima [A] time = 1.50, size = 97, normalized size = 1.45

$$\frac{(-1)^{\frac{2\,cx}{b}}\,a^2c^{\frac{5}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{5}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c^2x}{2\,b}-\frac{\sqrt{cx^2}\,ac^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*a^2*c^(5/2)*log(2*c*x/b)/b^3 + (-1)^(2*a*c*x/b)*a^2*c^(5/2)*
log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + 1/2*sqrt(c*x^2)*c^2*x/b - sqrt(c*x^2)*
a*c^2/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{5/2}}{x^3\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^3*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^3*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^3\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**3/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**3*(a + b*x)), x)
```

3.831 $\int \frac{(cx^2)^{5/2}}{x^4(a+bx)} dx$

Optimal. Leaf size=44

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2])/b - (a*c^2*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^4(a+bx)} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{x}{a+bx} dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx}{x} \\
&= \frac{c^2\sqrt{cx^2}}{b} - \frac{ac^2\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 0.68

$$\frac{c^3x(bx - a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c^3*x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.75

$$(cx^2)^{5/2}\left(\frac{1}{bx^4} - \frac{a\log(a+bx)}{b^2x^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^4*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*(1/(b*x^4) - (a*Log[a + b*x])/(b^2*x^5))
```

fricas [A] time = 0.68, size = 33, normalized size = 0.75

$$\frac{\left(bc^2x - ac^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x - a*c^2*log(b*x + a))*sqrt(c*x^2)/(b^2*x)
```

giac [A] time = 1.04, size = 46, normalized size = 1.05

$$\left(\frac{c^2x\mathrm{sgn}(x)}{b} - \frac{ac^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^2} + \frac{ac^2\log\left(|a|\right)\mathrm{sgn}(x)}{b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="giac")
```

```
[Out] (c^2*x*sgn(x)/b - a*c^2*log(abs(b*x + a))*sgn(x)/b^2 + a*c^2*log(abs(a))*sg
n(x)/b^2)*sqrt(c)
```

maple [A] time = 0.00, size = 29, normalized size = 0.66

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(a\ln\left(bx+a\right)-bx\right)}{b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^4/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(a*ln(b*x+a)-b*x)/x^5/b^2
```

maxima [A] time = 1.51, size = 77, normalized size = 1.75

$$-\frac{(-1)^{\frac{2cx}{b}}ac^{\frac{5}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} - \frac{(-1)^{\frac{2acx}{b}}ac^{\frac{5}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} + \frac{\sqrt{cx^2}\,c^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^4/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*c*x/b)*a*c^(5/2)*log(2*c*x/b)/b^2 - (-1)^(2*a*c*x/b)*a*c^(5/2)*log
(-2*a*c*x/(b*abs(b*x + a)))/b^2 + sqrt(c*x^2)*c^2/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^4\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^4*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^4*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**4/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**4*(a + b*x)), x)
```

3.832 $\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} dx$

Optimal. Leaf size=25

$$\frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[a + b*x])/(b*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^5(a+bx)} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{a+bx} dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\log(a+bx)}{bx}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
```

```
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 0.88

$$\frac{(cx^2)^{5/2}\log(a+bx)}{bx^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^5*(a + b*x)),x]
[Out] ((c*x^2)^(5/2)*Log[a + b*x])/(b*x^5)
```

fricas [A] time = 1.36, size = 23, normalized size = 0.92

$$\frac{\sqrt{cx^2}\,c^2\log(bx+a)}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*c^2*log(b*x + a)/(b*x)
```

giac [A] time = 1.10, size = 34, normalized size = 1.36

$$\left(\frac{c^2\log(|bx+a|)\operatorname{sgn}(x)}{b}-\frac{c^2\log(|a|)\operatorname{sgn}(x)}{b}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="giac")
[Out] (c^2*log(abs(b*x + a))*sgn(x)/b - c^2*log(abs(a))*sgn(x)/b)*sqrt(c)
```

maple [A] time = 0.00, size = 21, normalized size = 0.84

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}\ln(bx+a)}{b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^5/(b*x+a),x)
[Out] (c*x^2)^(5/2)/x^5*ln(b*x+a)/b
```

maxima [A] time = 1.36, size = 13, normalized size = 0.52

$$\frac{c^{\frac{5}{2}}\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^5/(b*x+a),x, algorithm="maxima")
[Out] c^(5/2)*log(b*x + a)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{\left(c\,x^2\right)^{5/2}}{x^5\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^5*(a + b*x)),x)
[Out] int((c*x^2)^(5/2)/(x^5*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{5}{2}}}{x^5\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**5/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**5*(a + b*x)), x)
```

3.833 $\int \frac{(cx^2)^{5/2}}{x^6(a+bx)} \, dx$

Optimal. Leaf size=48

$$\frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {15, 36, 29, 31}

$$\frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^2*Sqrt[c*x^2]*Log[x])/(a*x) - (c^2*Sqrt[c*x^2]*Log[a + b*x])/(a*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}}{x^6(a+bx)} \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)} \, dx}{x} \\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x} \, dx}{ax} - \frac{\left(bc^2\sqrt{cx^2}\right)\int \frac{1}{a+bx} \, dx}{ax} \\
&= \frac{c^2\sqrt{cx^2}\log(x)}{ax} - \frac{c^2\sqrt{cx^2}\log(a+bx)}{ax}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.58

$$\frac{c^3x(\log(x) - \log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c^3*x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 0.77

$$(cx^2)^{5/2}\left(\frac{\log(x)}{ax^5}-\frac{\log\left(a^2+abx\right)}{ax^5}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^6*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*(Log[x]/(a*x^5) - Log[a^2 + a*b*x]/(a*x^5))
```

fricas [A] time = 1.54, size = 70, normalized size = 1.46

$$\left[\frac{\sqrt{cx^2}\,c^2\log\left(\frac{x}{bx+a}\right)}{ax},\,\frac{2\sqrt{-c}\,c^2\arctan\left(\frac{\sqrt{cx^2}\,(2\,bx+a)\sqrt{-c}}{acx}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*c^2*log(x/(b*x + a))/(a*x), 2*sqrt(-c)*c^2*arctan(sqrt(c*x^2)*
(2*b*x + a)*sqrt(-c)/(a*c*x))/a]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 27, normalized size = 0.56

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(-\ln(x)+\ln\left(bx+a\right)\right)}{a\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^6/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(-ln(x)+ln(b*x+a))/x^5/a
```

maxima [A] time = 1.35, size = 24, normalized size = 0.50

$$-\frac{c^{\frac{5}{2}}\log\left(bx+a\right)}{a}+\frac{c^{\frac{5}{2}}\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^6/(b*x+a),x, algorithm="maxima")
```

```
[Out] -c^(5/2)*log(b*x + a)/a + c^(5/2)*log(x)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{5/2}}{x^6\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^6*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^6*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{6}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**6/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**6*(a + b*x)), x)
```

3.834 $\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx$

Optimal. Leaf size=70

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}-\frac{c^2\sqrt{cx^2}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^2*Sqrt[c*x^2])/(a*x^2)) - (b*c^2*Sqrt[c*x^2]*Log[x])/(a^2*x) + (b*c^2*
Sqrt[c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}}{x^7(a+bx)}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)}\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{x}\\ &= -\frac{c^2\sqrt{cx^2}}{ax^2}-\frac{bc^2\sqrt{cx^2}\log(x)}{a^2x}+\frac{bc^2\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.49

$$-\frac{c^3(-bx\log(a+bx)+a+bx\log(x))}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] -((c^3*(a + b*x*Log[x] - b*x*Log[a + b*x]))/(a^2*Sqrt[c*x^2]))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.63

$$(cx^2)^{5/2}\left(-\frac{b\log(x)}{a^2x^5}+\frac{b\log(a+bx)}{a^2x^5}-\frac{1}{ax^6}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)/(x^7*(a + b*x)),x]
```

```
[Out] (c*x^2)^(5/2)*(-(1/(a*x^6)) - (b*Log[x])/(a^2*x^5) + (b*Log[a + b*x])/(a^2*
x^5))
```

fricas [A] time = 0.94, size = 37, normalized size = 0.53

$$\frac{\left(bc^2x\log\left(\frac{bx+a}{x}\right)-ac^2\right)\sqrt{cx^2}}{a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*c^2*x*log((b*x + a)/x) - a*c^2)*sqrt(c*x^2)/(a^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 33, normalized size = 0.47

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\right)}{a^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/x^7/(b*x+a),x)
```

```
[Out] -(c*x^2)^(5/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a)/x^6/a^2
```

maxima [A] time = 1.39, size = 37, normalized size = 0.53

$$\frac{bc^{\frac{5}{2}}\log\left(bx+a\right)}{a^2}-\frac{bc^{\frac{5}{2}}\log(x)}{a^2}-\frac{c^{\frac{5}{2}}}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)/x^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(5/2)*log(b*x + a)/a^2 - b*c^(5/2)*log(x)/a^2 - c^(5/2)/(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{5/2}}{x^7\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)/(x^7*(a + b*x)),x)
```

```
[Out] int((c*x^2)^(5/2)/(x^7*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^7\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)/x**7/(b*x+a),x)
```

```
[Out] Integral((c*x**2)**(5/2)/(x**7*(a + b*x)), x)
```

3.835 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=83

$$-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}+\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*Sqrt[c*x^2]) + x^4/(3*b*Sqrt[c
*x^2]) - (a^3*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{x^3}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{a^2x^2}{b^3\sqrt{cx^2}}-\frac{ax^3}{2b^2\sqrt{cx^2}}+\frac{x^4}{3b\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.61

$$\frac{x\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*Sqrt[c*
x^2])
```

IntegrateAlgebraic [A] time = 0.05, size = 60, normalized size = 0.72

$$\sqrt{cx^2}\left(\frac{6a^2-3abx+2b^2x^2}{6b^3c}-\frac{a^3\log(a+bx)}{b^4cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((6*a^2 - 3*a*b*x + 2*b^2*x^2)/(6*b^3*c) - (a^3*Log[a + b*x]])/(
b^4*c*x))
```

fricas [A] time = 1.39, size = 54, normalized size = 0.65

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c*x)
```

giac [A] time = 1.15, size = 81, normalized size = 0.98

$$\frac{1}{6}\,\sqrt{cx^2}\left(x\left(\frac{2\,x}{bc}-\frac{3\,a}{b^2c}\right)+\frac{6\,a^2}{b^3c}\right)+\frac{a^3\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}-2\,ac\right|\right)}{b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + a^3*log(abs
(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^4*sqrt(c))
```

maple [A] time = 0.01, size = 50, normalized size = 0.60

$$-\frac{\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)x}{6\sqrt{c\,x^2}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6*x*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(1/2)/b^4
```

maxima [A] time = 1.52, size = 142, normalized size = 1.71

$$\frac{\sqrt{cx^2}\,x^2}{3\,bc}-\frac{7\,ax^2}{6\,b^2\sqrt{c}}-\frac{(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\,\sqrt{cx^2}\,ax}{3\,b^2c}-\frac{14\,a^2x}{3\,b^3\sqrt{c}}-\frac{a^3\log\left(bx\right)}{b^4\sqrt{c}}+\frac{17\,\sqrt{cx^2}\,a^2}{3\,b^3c}-\frac{7\,a^3}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c*x^2)*x^2/(b*c) - 7/6*a*x^2/(b^2*sqrt(c)) - (-1)^(2*a*c*x/b)*a^3*
log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^2*c)
- 14/3*a^2*x/(b^3*sqrt(c)) - a^3*log(b*x)/(b^4*sqrt(c)) + 17/3*sqrt(c*x^2)*
a^2/(b^3*c) - 7/2*a^3/(b^4*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x)
```

3.836 $\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$

Optimal. Leaf size=61

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}-\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*Sqrt[c*x^2])) + x^3/(2*b*Sqrt[c*x^2]) + (a^2*x*Log[a + b*x])
/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx &= \frac{x\int \frac{x^2}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{ax^2}{b^2\sqrt{cx^2}}+\frac{x^3}{2b\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 39, normalized size = 0.64

$$\frac{x\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.04, size = 47, normalized size = 0.77

$$\sqrt{cx^2}\left(\frac{a^2\log(a+bx)}{b^3cx}+\frac{bx-2a}{2b^2c}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-2*a + b*x)/(2*b^2*c) + (a^2*Log[a + b*x])/(b^3*c*x))
```

fricas [A] time = 1.35, size = 42, normalized size = 0.69

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c*x)
```

giac [A] time = 1.09, size = 67, normalized size = 1.10

$$\frac{1}{2}\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2\,a}{b^2c}\right)-\frac{a^2\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}\,-2\,ac\right|\right)}{b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - a^2*log(abs(-(sqrt(c)*x - sqrt(c*
x^2))*b*sqrt(c) - 2*a*c))/(b^3*sqrt(c))
```

maple [A] time = 0.00, size = 38, normalized size = 0.62

$$\frac{\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)x}{2\sqrt{c\,x^2}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(1/2)/b^3
```

maxima [A] time = 1.49, size = 100, normalized size = 1.64

$$\frac{x^2}{2\,b\sqrt{c}}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}+\frac{2\,ax}{b^2\sqrt{c}}+\frac{a^2\log\left(bx\right)}{b^3\sqrt{c}}-\frac{3\sqrt{cx^2}\,a}{b^2c}+\frac{3\,a^2}{2\,b^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/(b*sqrt(c)) + (-1)^(2*a*c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^3*sqrt(c)) + 2*a*x/(b^2*sqrt(c)) + a^2*log(b*x)/(b^3*sqrt(c)) - 3*sqrt(c*
x^2)*a/(b^2*c) + 3/2*a^2/(b^3*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^3}{\sqrt{c\,x^2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x)
```

3.837 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=39

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] x^2/(b*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{x^2}{b\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.69

$$\frac{x(bx-a\log(a+bx))}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(b*x - a*Log[a + b*x]))/(b^2*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 36, normalized size = 0.92

$$\sqrt{cx^2}\left(\frac{1}{bc}-\frac{a\log(a+bx)}{b^2cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] Sqrt[c*x^2]*(1/(b*c) - (a*Log[a + b*x])/(b^2*c*x))
```

fricas [A] time = 0.95, size = 30, normalized size = 0.77

$$\frac{\sqrt{cx^2}\left(bx - a\log\left(bx + a\right)\right)}{b^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c*x)
```

giac [A] time = 1.19, size = 51, normalized size = 1.31

$$\frac{a\log\left(\left|-\left(\sqrt{c}\,x - \sqrt{cx^2}\right)b\sqrt{c} - 2\,ac\right|\right)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^2*sqrt(c)) + sq
rt(c*x^2)/(b*c)
```

maple [A] time = 0.00, size = 27, normalized size = 0.69

$$-\frac{\left(a\ln\left(bx + a\right) - bx\right)x}{\sqrt{c\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)/(c*x^2)^(1/2),x)
[Out] -x*(a*ln(b*x+a)-b*x)/(c*x^2)^(1/2)/b^2
```

maxima [A] time = 1.47, size = 64, normalized size = 1.64

$$-\frac{\left(-1\right)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^2\sqrt{c}} - \frac{a\log\left(bx\right)}{b^2\sqrt{c}} + \frac{\sqrt{cx^2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^2*sqrt(c)) - a*log(b*
x)/(b^2*sqrt(c)) + sqrt(c*x^2)/(b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\sqrt{c\,x^2}\;\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)), x)
```

3.838 $\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=20

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{b\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x\log(a+bx)}{b\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)),x]
[Out] (x*Log[a + b*x])/(b*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.25

$$\frac{\sqrt{cx^2}\log(a+bx)}{bcx}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[c*x^2]*(a + b*x)),x]
```

[Out] (Sqrt[c*x^2]*Log[a + b*x])/(b*c*x)

fricas [A] time = 0.63, size = 23, normalized size = 1.15

$$\frac{\sqrt{cx^2}\log(bx+a)}{bcx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")

[Out] sqrt(c*x^2)*log(b*x + a)/(b*c*x)

giac [A] time = 0.90, size = 36, normalized size = 1.80

$$-\frac{\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}-2\,ac\right|\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")

[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b*sqrt(c))

maple [A] time = 0.00, size = 19, normalized size = 0.95

$$\frac{x\ln(bx+a)}{\sqrt{c\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x+a)/(c*x^2)^(1/2),x)

[Out] x*ln(b*x+a)/b/(c*x^2)^(1/2)

maxima [B] time = 1.46, size = 46, normalized size = 2.30

$$\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b\sqrt{c}}+\frac{\log(bx)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")

[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*sqrt(c)) + log(b*x)/(b*s
qrt(c))

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{x}{\sqrt{c\,x^2}\;(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((c*x^2)^(1/2)*(a + b*x)),x)

[Out] int(x/((c*x^2)^(1/2)*(a + b*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{cx^2}\;(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(c*x**2)*(a + b*x)), x)
```

3.839 $\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \frac{1}{x}\,dx}{a\sqrt{cx^2}}-\frac{(bx)\int \frac{1}{a+bx}\,dx}{a\sqrt{cx^2}}\\ &= \frac{x\log(x)}{a\sqrt{cx^2}}-\frac{x\log(a+bx)}{a\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 0.66

$$\frac{x(\log(x)-\log(a+bx))}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (x*(Log[x] - Log[a + b*x]))/(a*Sqrt[c*x^2])
```

IntegrateAlgebraic [A] time = 0.03, size = 43, normalized size = 1.13

$$\sqrt{cx^2}\left(\frac{\log(x)}{acx} - \frac{\log\left(a^2+abx\right)}{acx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*(Log[x]/(a*c*x) - Log[a^2 + a*b*x]/(a*c*x))
```

fricas [A] time = 0.94, size = 70, normalized size = 1.84

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{acx}, \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b*x
 + a)*sqrt(-c)/(a*c*x))/(a*c)]
```

giac [A] time = 0.97, size = 59, normalized size = 1.55

$$\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a\sqrt{c}} - \frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs(
-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c))
```

maple [A] time = 0.00, size = 24, normalized size = 0.63

$$\frac{(\ln(x)-\ln(bx+a))\,x}{\sqrt{c\,x^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] x*(ln(x)-ln(b*x+a))/(c*x^2)^(1/2)/a
```

maxima [A] time = 1.46, size = 35, normalized size = 0.92

$$-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*sqrt(c))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{c\,x^2}\,\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)), x)
```

3.840 $\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=54

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}-\frac{1}{a\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(1/(a*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*Sqrt[c*x^2]) + (b*x*Log[a + b*x])/
(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{1}{a\sqrt{cx^2}}-\frac{bx\log(x)}{a^2\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.67

$$\frac{cx^2(bx\log(a+bx)-a-bx\log(x))}{a^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 53, normalized size = 0.98

$$\sqrt{cx^2}\left(-\frac{b\log(x)}{a^2cx}+\frac{b\log(a+bx)}{a^2cx}-\frac{1}{acx^2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*(-(1/(a*c*x^2)) - (b*Log[x])/(a^2*c*x) + (b*Log[a + b*x])/(a^2*
c*x))
```

fricas [A] time = 0.77, size = 34, normalized size = 0.63

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c*x^2)
```

giac [A] time = 1.19, size = 91, normalized size = 1.69

$$-\sqrt{c}\left(\frac{b\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b-2\,a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}\,x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}\,x-\sqrt{cx^2}\right)a\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(c)*(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) -
b*log(abs(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))
*a*sqrt(c)))
```

maple [A] time = 0.01, size = 30, normalized size = 0.56

$$-\frac{bx\ln(x)-bx\ln\left(bx+a\right)+a}{\sqrt{c\,x^2}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x*ln(x)-b*x*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2
```

maxima [A] time = 1.37, size = 37, normalized size = 0.69

$$\frac{b\log\left(bx+a\right)}{a^2\sqrt{c}}-\frac{b\log(x)}{a^2\sqrt{c}}-\frac{1}{a\sqrt{c}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(a^2*sqrt(c)) - b*log(x)/(a^2*sqrt(c)) - 1/(a*sqrt(c)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x\,\sqrt{c\,x^2}\;\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)), x)
```

3.841 $\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}+\frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] b/(a^2*Sqrt[c*x^2]) - 1/(2*a*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*Sqrt[c*x^
2]) - (b^2*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{b}{a^2\sqrt{cx^2}}-\frac{1}{2ax\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.68

$$\frac{cx\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3
*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 67, normalized size = 0.87

$$\sqrt{cx^2}\left(\frac{b^2\log(x)}{a^3cx}-\frac{b^2\log(a+bx)}{a^3cx}+\frac{2bx-a}{2a^2cx^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-a + 2*b*x)/(2*a^2*c*x^3) + (b^2*Log[x])/(a^3*c*x) - (b^2*Log
[a + b*x])/(a^3*c*x))
```

fricas [A] time = 0.95, size = 47, normalized size = 0.61

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right)+2\,abx-a^2\right)\sqrt{cx^2}}{2\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [A] time = 0.00, size = 51, normalized size = 0.66

$$\frac{2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx-a^2}{2\sqrt{c\,x^2}\;a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/(c*x^2)^(1/2)/a^3
```

maxima [A] time = 1.38, size = 55, normalized size = 0.71

$$-\frac{b^2\log(bx+a)}{a^3\sqrt{c}}+\frac{b^2\log(x)}{a^3\sqrt{c}}+\frac{2\,b\sqrt{c}\,x-a\sqrt{c}}{2\,a^2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/(a^3*sqrt(c)) + b^2*log(x)/(a^3*sqrt(c)) + 1/2*(2*b*sqrt(
c)*x - a*sqrt(c))/(a^2*c*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{x^2\,\sqrt{c\,x^2}\;(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)),x)
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)/(c*x**2)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)), x)
```

3.842 $\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx$

Optimal. Leaf size=100

$$-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}-\frac{b^2}{a^3\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2}{a^3\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*Sqrt[c*x^2])) - 1/(3*a*x^2*Sqrt[c*x^2]) + b/(2*a^2*x*Sqrt[c*x^2]
) - (b^3*x*Log[x])/(a^4*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*Sqrt[c*x^2
])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{cx^2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{b^2}{a^3\sqrt{cx^2}}-\frac{1}{3ax^2\sqrt{cx^2}}+\frac{b}{2a^2x\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 63, normalized size = 0.63

$$\frac{c\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] (c*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log[a +
 b*x]))/(6*a^4*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 78, normalized size = 0.78

$$\sqrt{cx^2}\left(-\frac{b^3\log(x)}{a^4cx}+\frac{b^3\log(a+bx)}{a^4cx}+\frac{-2a^2+3abx-6b^2x^2}{6a^3cx^4}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[c*x^2]*(a + b*x)),x]
```

```
[Out] Sqrt[c*x^2]*((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3*c*x^4) - (b^3*Log[x])/(a
^4*c*x) + (b^3*Log[a + b*x])/(a^4*c*x))
```

fricas [A] time = 0.84, size = 58, normalized size = 0.58

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4cx^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{cx^2}\,(bx+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2)*(b*x + a)*x^3), x)
```

maple [A] time = 0.00, size = 62, normalized size = 0.62

$$-\frac{6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2-3a^2bx+2a^3}{6\sqrt{c\,x^2}\;a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x+a)/(c*x^2)^(1/2),x)
```

```
[Out] -1/6/x^2*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/
(c*x^2)^(1/2)/a^4
```

maxima [A] time = 1.40, size = 69, normalized size = 0.69

$$\frac{b^3\log(bx+a)}{a^4\sqrt{c}}-\frac{b^3\log(x)}{a^4\sqrt{c}}-\frac{6\,b^2\sqrt{c}\,x^2-3\,ab\sqrt{c}\,x+2\,a^2\sqrt{c}}{6\,a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/(a^4*sqrt(c)) - b^3*log(x)/(a^4*sqrt(c)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{c\,x^2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)),x)
```

```
[Out] int(1/(x^3*(c*x^2)^(1/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \sqrt{c x^{2}} \left(a + b x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(c*x**2)*(a + b*x)), x)
```

3.843 $\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=95

$$-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}+\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (a^2*x^2)/(b^3*c*Sqrt[c*x^2]) - (a*x^3)/(2*b^2*c*Sqrt[c*x^2]) + x^4/(3*b*c*
Sqrt[c*x^2]) - (a^3*x*Log[a + b*x])/(b^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x^3}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2}{b^3}-\frac{ax}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{a^2x^2}{b^3c\sqrt{cx^2}}-\frac{ax^3}{2b^2c\sqrt{cx^2}}+\frac{x^4}{3bc\sqrt{cx^2}}-\frac{a^3x\log(a+bx)}{b^4c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 53, normalized size = 0.56

$$\frac{x^3\left(bx\left(6a^2-3abx+2b^2x^2\right)-6a^3\log(a+bx)\right)}{6b^4\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(6*a^2 - 3*a*b*x + 2*b^2*x^2) - 6*a^3*Log[a + b*x]))/(6*b^4*(c*x^
2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 59, normalized size = 0.62

$$\frac{\frac{6a^2x^4-3abx^5+2b^2x^6}{6b^3}-\frac{a^3x^3\log(a+bx)}{b^4}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^6/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] ((6*a^2*x^4 - 3*a*b*x^5 + 2*b^2*x^6)/(6*b^3) - (a^3*x^3*Log[a + b*x])/b^4)/
(c*x^2)^(3/2)
```

fricas [A] time = 1.15, size = 54, normalized size = 0.57

$$\frac{\left(2\,b^3x^3-3\,ab^2x^2+6\,a^2bx-6\,a^3\log\left(bx+a\right)\right)\sqrt{cx^2}}{6\,b^4c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))*sqrt(c*x^2)/
(b^4*c^2*x)
```

giac [A] time = 1.18, size = 86, normalized size = 0.91

$$\frac{\sqrt{cx^2}\left(x\left(\frac{2\,x}{bc}-\frac{3\,a}{b^2c}\right)+\frac{6\,a^2}{b^3c}\right)+\frac{6\,a^3\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}-2\,ac\right|\right)}{b^4\sqrt{c}}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(c*x^2)*(x*(2*x/(b*c) - 3*a/(b^2*c)) + 6*a^2/(b^3*c)) + 6*a^3*log(
abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^4*sqrt(c)))/c
```

maple [A] time = 0.01, size = 52, normalized size = 0.55

$$-\frac{\left(-2b^3x^3+3a\,b^2x^2+6a^3\ln\left(bx+a\right)-6a^2bx\right)x^3}{6\left(c\,x^2\right)^{\frac{3}{2}}b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] -1/6*x^3*(-2*b^3*x^3+3*a*b^2*x^2+6*a^3*ln(b*x+a)-6*a^2*b*x)/(c*x^2)^(3/2)/b
^4
```

maxima [A] time = 1.79, size = 162, normalized size = 1.71

$$\frac{x^4}{3\sqrt{cx^2}\,bc}-\frac{ax^3}{2\sqrt{cx^2}\,b^2c}+\frac{a^2x^2}{\sqrt{cx^2}\,b^3c}-\frac{(-1)^{\frac{2acx}{b}}a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4c^{\frac{3}{2}}}+\frac{29\,a^3x}{6\sqrt{cx^2}\,b^4c}-\frac{a^3\log\left(bx\right)}{b^4c^{\frac{3}{2}}}-\frac{2\,a^4}{\sqrt{cx^2}\,b^5c}+\frac{2\,a^4}{b^5c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/(sqrt(c*x^2)*b*c) - 1/2*a*x^3/(sqrt(c*x^2)*b^2*c) + a^2*x^2/(sqrt(c
*x^2)*b^3*c) - (-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*c^(
3/2)) + 29/6*a^3*x/(sqrt(c*x^2)*b^4*c) - a^3*log(b*x)/(b^4*c^(3/2)) - 2*a^4
/(sqrt(c*x^2)*b^5*c) + 2*a^4/(b^5*c^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6}{\left(c\,x^2\right)^{3/2}\,(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^6/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.844 $\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=70

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}-\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -((a*x^2)/(b^2*c*Sqrt[c*x^2])) + x^3/(2*b*c*Sqrt[c*x^2]) + (a^2*x*Log[a + b
*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x^2}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{ax^2}{b^2c\sqrt{cx^2}}+\frac{x^3}{2bc\sqrt{cx^2}}+\frac{a^2x\log(a+bx)}{b^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 0.59

$$\frac{x^3\left(2a^2\log(a+bx)+bx(bx-2a)\right)}{2b^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x]))/(2*b^3*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 46, normalized size = 0.66

$$\frac{\frac{a^2x^3\log(a+bx)}{b^3}+\frac{bx^5-2ax^4}{2b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] ((-2*a*x^4 + b*x^5)/(2*b^2) + (a^2*x^3*Log[a + b*x])/b^3)/(c*x^2)^(3/2)
```

fricas [A] time = 0.83, size = 42, normalized size = 0.60

$$\frac{\left(b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,b^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))*sqrt(c*x^2)/(b^3*c^2*x)
```

giac [A] time = 0.96, size = 71, normalized size = 1.01

$$\frac{\sqrt{cx^2}\left(\frac{x}{bc}-\frac{2\,a}{b^2c}\right)-\frac{2\,a^2\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}-2\,ac\right|\right)}{b^3\sqrt{c}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(c*x^2)*(x/(b*c) - 2*a/(b^2*c)) - 2*a^2*log(abs(-(sqrt(c)*x - sqrt
(c*x^2))*b*sqrt(c) - 2*a*c))/(b^3*sqrt(c)))/c
```

maple [A] time = 0.00, size = 40, normalized size = 0.57

$$\frac{\left(b^2x^2+2a^2\ln\left(bx+a\right)-2abx\right)x^3}{2\left(c\,x^2\right)^{\frac{3}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/2*x^3*(b^2*x^2+2*a^2*ln(b*x+a)-2*a*b*x)/(c*x^2)^(3/2)/b^3
```

maxima [B] time = 1.66, size = 140, normalized size = 2.00

$$\frac{x^3}{2\sqrt{cx^2}\,bc}-\frac{ax^2}{\sqrt{cx^2}\,b^2c}+\frac{(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}-\frac{7\,a^2x}{2\sqrt{cx^2}\,b^3c}+\frac{a^2\log\left(bx\right)}{b^3c^{\frac{3}{2}}}+\frac{2\,a^3}{\sqrt{cx^2}\,b^4c}-\frac{2\,a^3}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^3/(sqrt(c*x^2)*b*c) - a*x^2/(sqrt(c*x^2)*b^2*c) + (-1)^(2*a*c*x/b)*a^
2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) - 7/2*a^2*x/(sqrt(c*x^2)*b^3
*c) + a^2*log(b*x)/(b^3*c^(3/2)) + 2*a^3/(sqrt(c*x^2)*b^4*c) - 2*a^3/(b^4*c
^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.845 $\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=45

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] x^2/(b*c*Sqrt[c*x^2]) - (a*x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{x}{a+bx}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{x^2}{bc\sqrt{cx^2}}-\frac{ax\log(a+bx)}{b^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.64

$$\frac{x^3(bx-a\log(a+bx))}{b^2\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(b*x - a*Log[a + b*x]))/(b^2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.73

$$\frac{\frac{x^4}{b}-\frac{ax^3\log(a+bx)}{b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^4/b - (a*x^3*Log[a + b*x])/b^2)/(c*x^2)^(3/2)
```

fricas [A] time = 1.11, size = 30, normalized size = 0.67

$$\frac{\sqrt{cx^2}\left(bx-a\log\left(bx+a\right)\right)}{b^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x - a*log(b*x + a))/(b^2*c^2*x)
```

giac [A] time = 1.09, size = 55, normalized size = 1.22

$$\frac{\frac{a\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b\sqrt{c}-2ac\right|\right)}{b^2\sqrt{c}}+\frac{\sqrt{cx^2}}{bc}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] (a*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b^2*sqrt(c)) + s
qrt(c*x^2)/(b*c))/c
```

maple [A] time = 0.00, size = 29, normalized size = 0.64

$$-\frac{\left(a\ln\left(bx+a\right)-bx\right)x^3}{\left(c\,x^2\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a),x)
[Out] -x^3*(a*ln(b*x+a)-b*x)/(c*x^2)^(3/2)/b^2
```

maxima [B] time = 1.60, size = 116, normalized size = 2.58

$$\frac{x^2}{\sqrt{cx^2}\,bc}-\frac{(-1)^{\frac{2acx}{b}}a\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^2}\,b^2c}-\frac{a\log\left(bx\right)}{b^2c^{\frac{3}{2}}}-\frac{2\,a^2}{\sqrt{cx^2}\,b^3c}+\frac{2\,a^2}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] x^2/(sqrt(c*x^2)*b*c) - (-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(
b^2*c^(3/2)) + 2*a*x/(sqrt(c*x^2)*b^2*c) - a*log(b*x)/(b^2*c^(3/2)) - 2*a^2
/(sqrt(c*x^2)*b^3*c) + 2*a^2/(b^3*c^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(c x^{2}\right)^{\frac{3}{2}} \left(a + b x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.846 $\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=23

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 31}

$$\frac{x\log(a+bx)}{bc\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[a + b*x])/(b*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{a+bx}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\log(a+bx)}{bc\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 0.96

$$\frac{x^3\log(a+bx)}{b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 22, normalized size = 0.96

$$\frac{x^3\log(a+bx)}{b\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x^3*Log[a + b*x])/(b*(c*x^2)^(3/2))
```

fricas [A] time = 1.09, size = 23, normalized size = 1.00

$$\frac{\sqrt{cx^2}\log(bx+a)}{bc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*log(b*x + a)/(b*c^2*x)
```

giac [A] time = 0.97, size = 36, normalized size = 1.57

$$-\frac{\log\left(\left|-\left(\sqrt{c}\,x-\sqrt{cx^2}\right)b\sqrt{c}-2\,ac\right|\right)}{bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b*sqrt(c) - 2*a*c))/(b*c^(3/2))
```

maple [A] time = 0.00, size = 21, normalized size = 0.91

$$\frac{x^3\ln(bx+a)}{\left(c\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a),x)
```

```
[Out] 1/(c*x^2)^(3/2)*x^3*ln(b*x+a)/b
```

maxima [B] time = 1.60, size = 74, normalized size = 3.22

$$\frac{(-1)^{\frac{2\,acx}{b}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{bc^{\frac{3}{2}}}+\frac{\log(bx)}{bc^{\frac{3}{2}}}+\frac{2\,a}{\sqrt{cx^2}\,b^2c}-\frac{2\,a}{b^2c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(b*c^(3/2)) + log(b*x)/(b*c
^(3/2)) + 2*a/(sqrt(c*x^2)*b^2*c) - 2*a/(b^2*c^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^3}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^3/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**3/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.847 $\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=44

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {15, 36, 29, 31}

$$\frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*Log[x])/(a*c*Sqrt[c*x^2]) - (x*Log[a + b*x])/(a*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \frac{1}{x}\,dx}{ac\sqrt{cx^2}}-\frac{(bx)\int \frac{1}{a+bx}\,dx}{ac\sqrt{cx^2}}\\ &= \frac{x\log(x)}{ac\sqrt{cx^2}}-\frac{x\log(a+bx)}{ac\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{x^3(\log(x)-\log(a+bx))}{a\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (x^3*(Log[x] - Log[a + b*x]))/(a*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 0.84

$$\frac{\frac{x^3\log(x)}{a}-\frac{x^3\log\left(a^2+abx\right)}{a}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] ((x^3*Log[x])/a - (x^3*Log[a^2 + a*b*x])/a)/(c*x^2)^(3/2)
```

fricas [A] time = 1.29, size = 70, normalized size = 1.59

$$\left[\frac{\sqrt{cx^2}\log\left(\frac{x}{bx+a}\right)}{ac^2x},\ \frac{2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2}(2bx+a)\sqrt{-c}}{acx}\right)}{ac^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] [sqrt(c*x^2)*log(x/(b*x + a))/(a*c^2*x), 2*sqrt(-c)*arctan(sqrt(c*x^2)*(2*b
*x + a)*sqrt(-c)/(a*c*x))/(a*c^2)]
```

giac [A] time = 1.05, size = 63, normalized size = 1.43

$$\frac{\frac{\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a\sqrt{c}}-\frac{\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a\sqrt{c}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] (log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a*sqrt(c)) - log(abs
(-sqrt(c)*x + sqrt(c*x^2)))/(a*sqrt(c)))/c
```

maple [A] time = 0.00, size = 26, normalized size = 0.59

$$\frac{(\ln(x)-\ln(bx+a))\,x^3}{\left(c\,x^2\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a),x)
[Out] x^3*(ln(x)-ln(b*x+a))/(c*x^2)^(3/2)/a
```

maxima [A] time = 1.43, size = 35, normalized size = 0.80

$$-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{ac^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] -(-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x + a)))/(a*c^(3/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.848 $\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx$

Optimal. Leaf size=63

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}-\frac{1}{ac\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)),x]

[Out] -(1/(a*c*Sqrt[c*x^2])) - (b*x*Log[x])/(a^2*c*Sqrt[c*x^2]) + (b*x*Log[a + b*
x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(cx^2)^{3/2}(a+bx)} dx &= \frac{x\int \frac{1}{x^2(a+bx)} dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right) dx}{c\sqrt{cx^2}}\\ &= -\frac{1}{ac\sqrt{cx^2}}-\frac{bx\log(x)}{a^2c\sqrt{cx^2}}+\frac{bx\log(a+bx)}{a^2c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.56

$$\frac{x^2(bx\log(a+bx)-a-bx\log(x))}{a^2(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)),x]

[Out] (x^2*(-a - b*x*Log[x] + b*x*Log[a + b*x]))/(a^2*(c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 0.70

$$\frac{-\frac{bx^3\log(x)}{a^2}+\frac{bx^3\log(a+bx)}{a^2}-\frac{x^2}{a}}{(cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] (-(x^2/a) - (b*x^3*Log[x])/a^2 + (b*x^3*Log[a + b*x])/a^2)/(c*x^2)^(3/2)
```

fricas [A] time = 0.95, size = 34, normalized size = 0.54

$$\frac{\sqrt{cx^2}\left(bx\log\left(\frac{bx+a}{x}\right)-a\right)}{a^2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x*log((b*x + a)/x) - a)/(a^2*c^2*x^2)
```

giac [A] time = 1.08, size = 91, normalized size = 1.44

$$-\frac{\frac{b\log\left(\left|-\left(\sqrt{c}x-\sqrt{cx^2}\right)b-2a\sqrt{c}\right|\right)}{a^2c}-\frac{b\log\left(\left|-\sqrt{c}x+\sqrt{cx^2}\right|\right)}{a^2c}-\frac{2}{\left(\sqrt{c}x-\sqrt{cx^2}\right)a\sqrt{c}}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] -(b*log(abs(-(sqrt(c)*x - sqrt(c*x^2))*b - 2*a*sqrt(c)))/(a^2*c) - b*log(ab
s(-sqrt(c)*x + sqrt(c*x^2)))/(a^2*c) - 2/((sqrt(c)*x - sqrt(c*x^2))*a*sqrt(
c)))/sqrt(c)
```

maple [A] time = 0.01, size = 33, normalized size = 0.52

$$-\frac{(bx\ln(x)-bx\ln(bx+a)+a)\,x^2}{(c\,x^2)^{\frac{3}{2}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a),x)
[Out] -x^2*(b*x*ln(x)-b*x*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2
```

maxima [A] time = 1.48, size = 51, normalized size = 0.81

$$\frac{(-1)^{\frac{2acx}{b}}\,b\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}-\frac{1}{\sqrt{cx^2}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] (-1)^(2*a*c*x/b)*b*log(-2*a*c*x/(b*abs(b*x + a)))/(a^2*c^(3/2)) - 1/(sqrt(c
*x^2)*a*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.849 $\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=89

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}+\frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] b/(a^2*c*Sqrt[c*x^2]) - 1/(2*a*c*x*Sqrt[c*x^2]) + (b^2*x*Log[x])/(a^3*c*Sqr
t[c*x^2]) - (b^2*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(cx^2)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{b}{a^2c\sqrt{cx^2}}-\frac{1}{2acx\sqrt{cx^2}}+\frac{b^2x\log(x)}{a^3c\sqrt{cx^2}}-\frac{b^2x\log(a+bx)}{a^3c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 0.57

$$\frac{x\left(-2b^2x^2\log(a+bx)-a(a-2bx)+2b^2x^2\log(x)\right)}{2a^3\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] (x*(-(a*(a - 2*b*x)) + 2*b^2*x^2*Log[x] - 2*b^2*x^2*Log[a + b*x]))/(2*a^3*(
c*x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 0.65

$$\frac{\frac{b^2x^3\log(x)}{a^3} - \frac{b^2x^3\log(a+bx)}{a^3} + \frac{2bx^2-ax}{2a^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x^2)^(3/2)*(a + b*x)),x]
[Out] ((-(a*x) + 2*b*x^2)/(2*a^2) + (b^2*x^3*Log[x])/a^3 - (b^2*x^3*Log[a + b*x])
/a^3)/(c*x^2)^(3/2)
```

fricas [A] time = 1.16, size = 47, normalized size = 0.53

$$\frac{\left(2\,b^2x^2\log\left(\frac{x}{bx+a}\right) + 2\,abx - a^2\right)\sqrt{cx^2}}{2\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")
[Out] 1/2*(2*b^2*x^2*log(x/(b*x + a)) + 2*a*b*x - a^2)*sqrt(c*x^2)/(a^3*c^2*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.01, size = 49, normalized size = 0.55

$$\frac{\left(2b^2x^2\ln(x) - 2b^2x^2\ln(bx+a) + 2abx - a^2\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a),x)
[Out] 1/2*x*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x-a^2)/(c*x^2)^(3/2)/a^3
```

maxima [A] time = 1.54, size = 65, normalized size = 0.73

$$-\frac{(-1)^{\frac{2acx}{b}}b^2\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}} + \frac{b}{\sqrt{cx^2}\,a^2c} - \frac{1}{2\,ac^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")
[Out] -(-1)^(2*a*c*x/b)*b^2*log(-2*a*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) + b/(sqr
t(c*x^2)*a^2*c) - 1/2/(a*c^(3/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)), x)
```

3.850 $\int \frac{1}{x(cx^2)^{3/2}(a+bx)}\,dx$

Optimal. Leaf size=115

$$-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}-\frac{b^2}{a^3c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

```
[Out] -(b^2/(a^3*c*Sqrt[c*x^2])) - 1/(3*a*c*x^2*Sqrt[c*x^2]) + b/(2*a^2*c*x*Sqrt[
c*x^2]) - (b^3*x*Log[x])/(a^4*c*Sqrt[c*x^2]) + (b^3*x*Log[a + b*x])/(a^4*c*
Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(cx^2\right)^{3/2}(a+bx)}\,dx &= \frac{x\int \frac{1}{x^4(a+bx)}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= -\frac{b^2}{a^3c\sqrt{cx^2}}-\frac{1}{3acx^2\sqrt{cx^2}}+\frac{b}{2a^2cx\sqrt{cx^2}}-\frac{b^3x\log(x)}{a^4c\sqrt{cx^2}}+\frac{b^3x\log(a+bx)}{a^4c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 66, normalized size = 0.57

$$\frac{cx^2\left(a\left(-2a^2+3abx-6b^2x^2\right)+6b^3x^3\log(a+bx)-6b^3x^3\log(x)\right)}{6a^4\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]
```

[Out] (c*x^2*(a*(-2*a^2 + 3*a*b*x - 6*b^2*x^2) - 6*b^3*x^3*Log[x] + 6*b^3*x^3*Log
[a + b*x]))/(6*a^4*(c*x^2)^(5/2))

IntegrateAlgebraic [A] time = 0.06, size = 66, normalized size = 0.57

$$\frac{-\frac{b^3x^3\log(x)}{a^4}+\frac{b^3x^3\log(a+bx)}{a^4}+\frac{-2a^2+3abx-6b^2x^2}{6a^3}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(c*x^2)^(3/2)*(a + b*x)),x]

[Out] ((-2*a^2 + 3*a*b*x - 6*b^2*x^2)/(6*a^3) - (b^3*x^3*Log[x])/a^4 + (b^3*x^3*L
og[a + b*x])/a^4)/(c*x^2)^(3/2)

fricas [A] time = 1.05, size = 58, normalized size = 0.50

$$\frac{\left(6\,b^3x^3\log\left(\frac{bx+a}{x}\right)-6\,ab^2x^2+3\,a^2bx-2\,a^3\right)\sqrt{cx^2}}{6\,a^4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="fricas")

[Out] 1/6*(6*b^3*x^3*log((b*x + a)/x) - 6*a*b^2*x^2 + 3*a^2*b*x - 2*a^3)*sqrt(c*x
^2)/(a^4*c^2*x^4)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(cx^2\right)^{\frac{3}{2}}(bx+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="giac")

[Out] integrate(1/((c*x^2)^(3/2)*(b*x + a)*x), x)

maple [A] time = 0.01, size = 59, normalized size = 0.51

$$-\frac{6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2-3a^2bx+2a^3}{6\left(c\,x^2\right)^{\frac{3}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(c*x^2)^(3/2)/(b*x+a),x)

[Out] -1/6*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2-3*a^2*b*x+2*a^3)/(c*x
^2)^(3/2)/a^4

maxima [A] time = 1.36, size = 69, normalized size = 0.60

$$\frac{b^3\log(bx+a)}{a^4c^{\frac{3}{2}}}-\frac{b^3\log(x)}{a^4c^{\frac{3}{2}}}-\frac{6\,b^2\sqrt{c}\,x^2-3\,ab\sqrt{c}\,x+2\,a^2\sqrt{c}}{6\,a^3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(c*x^2)^(3/2)/(b*x+a),x, algorithm="maxima")

```
[Out] b^3*log(b*x + a)/(a^4*c^(3/2)) - b^3*log(x)/(a^4*c^(3/2)) - 1/6*(6*b^2*sqrt
(c)*x^2 - 3*a*b*sqrt(c)*x + 2*a^2*sqrt(c))/(a^3*c^2*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(3/2)*(a + b*x)),x)
```

```
[Out] int(1/(x*(c*x^2)^(3/2)*(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(c*x**2)**(3/2)/(b*x+a),x)
```

```
[Out] Integral(1/(x*(c*x**2)**(3/2)*(a + b*x)), x)
```

3.851 $\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Rubi [A] time = 0.04, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2\sqrt{cx^2}}{b^4}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (3*a^2*Sqrt[c*x^2])/b^4 - (a*x*Sqrt[c*x^2])/b^3 + (x^2*Sqrt[c*x^2])/(3*b^2)
 - (a^4*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*Sqrt[c*x^2]*Log[a + b*x])/(
b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^4}{(a+bx)^2}\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}\\
&= \frac{3a^2\sqrt{cx^2}}{b^4}-\frac{ax\sqrt{cx^2}}{b^3}+\frac{x^2\sqrt{cx^2}}{3b^2}-\frac{a^4\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3\sqrt{cx^2}\log(a+bx)}{b^5x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.76

$$\frac{cx\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(
a + b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.08, size = 85, normalized size = 0.80

$$\sqrt{cx^2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5x(a+bx)}-\frac{4a^3\log(a+bx)}{b^5x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)/(
3*b^5*x*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*x))
```

fricas [A] time = 1.14, size = 83, normalized size = 0.78

$$\frac{\left(b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\left(a^3bx+a^4\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\left(b^6x^2+ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

giac [A] time = 1.00, size = 96, normalized size = 0.91

$$-\frac{1}{3}\sqrt{c}\left(\frac{12\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,a^4\mathrm{sgn}(x)}{(bx+a)b^5}-\frac{3\left(4\,a^3\log\left(|a|\right)+a^3\right)\mathrm{sgn}(x)}{b^5}-\frac{b^4x^3\mathrm{sgn}(x)-3\,ab^3x^2\mathrm{sgn}(x)+9\,a^2b^2x\mathrm{sgn}(x)}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*sqrt(c)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

maple [A] time = 0.01, size = 88, normalized size = 0.83

$$-\frac{\sqrt{c\,x^2}\,\left(-b^4x^4+2a\,b^3x^3+12a^3bx\ln\left(bx+a\right)-6a^2b^2x^2+12a^4\ln\left(bx+a\right)-9a^3bx+3a^4\right)}{3\left(bx+a\right)b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(1/2)*(-b^4*x^4+2*a*b^3*x^3+12*ln(b*x+a)*x*a^3*b-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x/b^5/(b*x+a)
```

maxima [A] time = 1.52, size = 135, normalized size = 1.27

$$\frac{\sqrt{cx^2}\,a^3}{b^5x+ab^4}-\frac{4\,(-1)^{\frac{2\,cx}{b}}\,a^3\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^5}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5}-\frac{\sqrt{cx^2}\,ax}{b^3}+\frac{3\,\sqrt{cx^2}\,a^2}{b^4}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*x + a*b^4) - 4*(-1)^(2*c*x/b)*a^3*sqrt(c)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
sqrt(c*x^2)*a*x/b^3 + 3*sqrt(c*x^2)*a^2/b^4 + 1/3*(c*x^2)^(3/2)/(b^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3 \sqrt{c\,x^2}}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^3*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**3*sqrt(c*x**2)/(a + b*x)**2, x)
```

3.852 $\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (-2*a*Sqrt[c*x^2])/b^3 + (x*Sqrt[c*x^2])/(2*b^2) + (a^3*Sqrt[c*x^2])/(b^4*x
*(a + b*x)) + (3*a^2*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^3}{(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{x}\\ &= -\frac{2a\sqrt{cx^2}}{b^3}+\frac{x\sqrt{cx^2}}{2b^2}+\frac{a^3\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2\sqrt{cx^2}\log(a+bx)}{b^4x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 0.82

$$\frac{cx\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b
*x]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 74, normalized size = 0.87

$$\sqrt{cx^2}\left(\frac{3a^2\log(a+bx)}{b^4x}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4x(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*x*(a + b*x)
) + (3*a^2*Log[a + b*x])/(b^4*x))
```

fricas [A] time = 0.85, size = 72, normalized size = 0.85

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

giac [A] time = 1.06, size = 80, normalized size = 0.94

$$\frac{1}{2}\sqrt{c}\left(\frac{6\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}+\frac{2\,a^3\mathrm{sgn}(x)}{(bx+a)b^4}-\frac{2\left(3\,a^2\log\left(|a|\right)+a^2\right)\mathrm{sgn}(x)}{b^4}+\frac{b^2x^2\mathrm{sgn}(x)-4\,abx\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b
^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sg
n(x))/b^4)
```

maple [A] time = 0.01, size = 76, normalized size = 0.89

$$\frac{\sqrt{c\,x^2}\,\left(b^3x^3+6a^2bx\ln\left(bx+a\right)-3a\,b^2x^2+6a^3\ln\left(bx+a\right)-4a^2bx+2a^3\right)}{2\left(bx+a\right)b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(b^3*x^3+6*ln(b*x+a)*x*a^2*b-3*a*b^2*x^2+6*a^3*ln(b*x+a)-
4*a^2*b*x+2*a^3)/x/b^4/(b*x+a)
```

maxima [A] time = 1.55, size = 118, normalized size = 1.39

$$-\frac{\sqrt{cx^2}\,a^2}{b^4x+ab^3}+\frac{3\,(-1)^{\frac{2\,cx}{b}}\,a^2\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}+\frac{\sqrt{cx^2}\,x}{2\,b^2}-\frac{2\,\sqrt{cx^2}\,a}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*x + a*b^3) + 3*(-1)^(2*c*x/b)*a^2*sqrt(c)*log(2*c*x/b
)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 +
 1/2*sqrt(c*x^2)*x/b^2 - 2*sqrt(c*x^2)*a/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x^2*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}\sqrt{cx^{2}}}{\left(a+bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x)
```

3.853 $\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]/b^2 - (a^2*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*Sqrt[c*x^2]*Lo
g[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{x^2}{(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right)dx}{x}\\ &= \frac{\sqrt{cx^2}}{b^2}-\frac{a^2\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2a\sqrt{cx^2}\log(a+bx)}{b^3x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.82

$$\frac{cx\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] (c*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2
]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 57, normalized size = 0.88

$$\sqrt{cx^2}\left(\frac{-a^2+abx+b^2x^2}{b^3x(a+bx)}-\frac{2a\log(a+bx)}{b^3x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[c*x^2])/(a + b*x)^2,x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + a*b*x + b^2*x^2)/(b^3*x*(a + b*x)) - (2*a*Log[a + b*x]
)/(b^3*x))
```

fricas [A] time = 1.02, size = 57, normalized size = 0.88

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*x^2
 + a*b^3*x)
```

giac [A] time = 0.92, size = 58, normalized size = 0.89

$$\sqrt{c}\left(\frac{x\mathrm{sgn}(x)}{b^2}-\frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}+\frac{\left(2\,a\log\left(|a|\right)+a\right)\mathrm{sgn}(x)}{b^3}-\frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] sqrt(c)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

maple [A] time = 0.01, size = 62, normalized size = 0.95

$$-\frac{\sqrt{c\,x^2}\,\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)}{\left(bx+a\right)b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*ln(b*x+a)*x*a*b-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x/b^3/
(b*x+a)
```

maxima [A] time = 1.51, size = 96, normalized size = 1.48

$$\frac{\sqrt{cx^2}\,a}{b^3x+ab^2}-\frac{2\,(-1)^{\frac{2\,cx}{b}}\,a\sqrt{c}\,\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\sqrt{c}\,\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*sqrt(c)*log(2*c*x/b)/b^3
 - 2*(-1)^(2*a*c*x/b)*a*sqrt(c)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt(c
*x^2)/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\sqrt{c\,x^2}}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(1/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(1/2))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
```

3.854 $\int \frac{\sqrt{cx^2}}{(a+bx)^2} dx$

Optimal. Leaf size=47

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (a*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{cx^2}}{(a+bx)^2} dx &= \frac{\sqrt{cx^2}\int \frac{x}{(a+bx)^2} dx}{x}\\
&= \frac{\sqrt{cx^2}\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx}{x}\\
&= \frac{a\sqrt{cx^2}}{b^2x(a+bx)} + \frac{\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 0.77

$$\frac{cx((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(a + b*x)^2,x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 0.83

$$\sqrt{cx^2}\left(\frac{a}{b^2x(a+bx)} + \frac{\log(a+bx)}{b^2x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(a + b*x)^2,x]
[Out] Sqrt[c*x^2]*(a/(b^2*x*(a + b*x)) + Log[a + b*x]/(b^2*x))
```

fricas [A] time = 0.67, size = 38, normalized size = 0.81

$$\frac{\sqrt{cx^2}\left((bx+a)\log(bx+a)+a\right)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*x^2 + a*b^2*x)
```

giac [A] time = 1.06, size = 46, normalized size = 0.98

$$-\sqrt{c}\left(\frac{\left(\log(|a|)+1\right)\operatorname{sgn}(x)}{b^2}-\frac{\log(|bx+a|)\operatorname{sgn}(x)}{b^2}-\frac{a\operatorname{sgn}(x)}{(bx+a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="giac")
[Out] -sqrt(c)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

maple [A] time = 0.01, size = 41, normalized size = 0.87

$$\frac{\sqrt{c\,x^2}\;\left(bx\ln(bx+a)+a\ln(bx+a)+a\right)}{(bx+a)\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(b*x+a)^2,x)
[Out] (c*x^2)^(1/2)*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/x/b^2/(b*x+a)
```

maxima [A] time = 1.47, size = 79, normalized size = 1.68

$$\frac{(-1)^{\frac{2cx}{b}}\sqrt{c}\log\left(\frac{2cx}{b}\right)}{b^2}+\frac{(-1)^{\frac{2acx}{b}}\sqrt{c}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2}-\frac{\sqrt{cx^2}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] (-1)^(2*c*x/b)*sqrt(c)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*sqrt(c)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)/(b^2*x + a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{{\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(a + b*x)^2,x)
[Out] int((c*x^2)^(1/2)/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(a + b*x)**2, x)
```

3.855 $\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx$

Optimal. Leaf size=24

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{(a+bx)^2}\,dx}{x}\\ &= -\frac{\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.96

$$-\frac{cx}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -((c*x)/(b*Sqrt[c*x^2]*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 1.00

$$-\frac{\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*x*(a + b*x)))
```

fricas [A] time = 1.25, size = 23, normalized size = 0.96

$$-\frac{\sqrt{cx^2}}{b^2x^2 + abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)/(b^2*x^2 + a*b*x)
```

giac [A] time = 1.02, size = 29, normalized size = 1.21

$$-\sqrt{c}\left(\frac{\operatorname{sgn}(x)}{(bx+a)b} - \frac{\operatorname{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
[Out] -sqrt(c)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

maple [A] time = 0.00, size = 23, normalized size = 0.96

$$-\frac{\sqrt{c\,x^2}}{(bx+a)\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x/(b*x+a)^2,x)
[Out] -(c*x^2)^(1/2)/b/x/(b*x+a)
```

maxima [A] time = 1.36, size = 16, normalized size = 0.67

$$-\frac{\sqrt{c}}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
[Out] -sqrt(c)/(b^2*x + a*b)
```

mupad [B] time = 0.16, size = 22, normalized size = 0.92

$$-\frac{\sqrt{c\,x^2}}{b\,x\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x*(a + b*x)^2),x)
[Out] -(c*x^2)^(1/2)/(b*x*(a + b*x))
```

sympy [A] time = 0.82, size = 39, normalized size = 1.62

$$\begin{cases} -\frac{\sqrt{c}\sqrt{x^2}}{abx+b^2x^2} & \text{for } b \neq 0 \\ \frac{\sqrt{c}\sqrt{x^2}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] Piecewise((-sqrt(c)*sqrt(x**2)/(a*b*x + b**2*x**2), Ne(b, 0)), (sqrt(c)*sqr
t(x**2)/a**2, True))
```

3.856 $\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx$

Optimal. Leaf size=65

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{\sqrt{cx^2}\log(x)}{a^2x}+\frac{\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]/(a*x*(a + b*x)) + (Sqrt[c*x^2]*Log[x])/(a^2*x) - (Sqrt[c*x^2]*L
og[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^2(a+bx)^2}\,dx&=\frac{\sqrt{cx^2}\int \frac{1}{x(a+bx)^2}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx}{x}\\&=\frac{\sqrt{cx^2}}{ax(a+bx)}+\frac{\sqrt{cx^2}\log(x)}{a^2x}-\frac{\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.69

$$\frac{cx(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] (c*x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a +
 b*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 48, normalized size = 0.74

$$\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2x}+\frac{\log(x)}{a^2x}+\frac{1}{ax(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^2*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*(1/(a*x*(a + b*x)) + Log[x]/(a^2*x) - Log[a + b*x]/(a^2*x))
```

fricas [A] time = 1.05, size = 42, normalized size = 0.65

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*x^2 + a^3*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 52, normalized size = 0.80

$$\frac{\sqrt{c\,x^2}\,\left(bx\ln(x)-bx\ln(bx+a)+a\ln(x)-a\ln(bx+a)+a\right)}{(bx+a)\,a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(1/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/x/a^2/(b*x+a)
```

maxima [A] time = 1.32, size = 38, normalized size = 0.58

$$\frac{\sqrt{c}}{abx+a^2}-\frac{\sqrt{c}\log(bx+a)}{a^2}+\frac{\sqrt{c}\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c)/(a*b*x + a^2) - sqrt(c)*log(b*x + a)/a^2 + sqrt(c)*log(x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x^2}}{x^2\,{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^2*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^2 (a+bx)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**2/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**2*(a + b*x)**2), x)
```

3.857 $\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}-\frac{\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(a^2*x^2)) - (b*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*Sqrt[c*
x^2]*Log[x])/(a^3*x) + (2*b*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^3(a+bx)^2}\,dx &= \frac{\sqrt{cx^2}\int \frac{1}{x^2(a+bx)^2}\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}\\ &= -\frac{\sqrt{cx^2}}{a^2x^2}-\frac{b\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2b\sqrt{cx^2}\,\log(x)}{a^3x}+\frac{2b\sqrt{cx^2}\,\log(a+bx)}{a^3x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.66

$$-\frac{c(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*x]
))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.07, size = 59, normalized size = 0.68

$$\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3x}+\frac{2b\log(a+bx)}{a^3x}+\frac{-a-2bx}{a^2x^2(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^3*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a - 2*b*x)/(a^2*x^2*(a + b*x)) - (2*b*Log[x])/(a^3*x) + (2*b
*Log[a + b*x])/(a^3*x))
```

fricas [A] time = 1.04, size = 60, normalized size = 0.69

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
x^3 + a^4*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 74, normalized size = 0.85

$$-\frac{\sqrt{c\,x^2}\,\left(2b^2x^2\ln(x)-2b^2x^2\ln\left(bx+a\right)+2abx\ln(x)-2abx\ln\left(bx+a\right)+2abx+a^2\right)}{\left(bx+a\right)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^3/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(1/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*l
n(b*x+a)+2*a*b*x+a^2)/x^2/a^3/(b*x+a)
```

maxima [A] time = 1.40, size = 58, normalized size = 0.67

$$-\frac{2\,b\sqrt{c}\,x+a\sqrt{c}}{a^2bx^2+a^3x}+\frac{2\,b\sqrt{c}\,\log\left(bx+a\right)}{a^3}-\frac{2\,b\sqrt{c}\,\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*sqrt(c)*x + a*sqrt(c))/(a^2*b*x^2 + a^3*x) + 2*b*sqrt(c)*log(b*x + a)
/a^3 - 2*b*sqrt(c)*log(x)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^3\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^3*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^3\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**3*(a + b*x)**2), x)
```

3.858 $\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx$

Optimal. Leaf size=112

$$\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Rubi [A] time = 0.04, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}+\frac{2b\sqrt{cx^2}}{a^3x^2}-\frac{\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] -Sqrt[c*x^2]/(2*a^2*x^3) + (2*b*Sqrt[c*x^2])/(a^3*x^2) + (b^2*Sqrt[c*x^2])/
(a^3*x*(a + b*x)) + (3*b^2*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*Sqrt[c*x^2]
*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}}{x^4(a+bx)^2}\,dx&=\frac{\sqrt{cx^2}\int \frac{1}{x^3(a+bx)^2}\,dx}{x}\\&=\frac{\sqrt{cx^2}\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{x}\\&=-\frac{\sqrt{cx^2}}{2a^2x^3}+\frac{2b\sqrt{cx^2}}{a^3x^2}+\frac{b^2\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2\sqrt{cx^2}\,\log(x)}{a^4x}-\frac{3b^2\sqrt{cx^2}\,\log(a+bx)}{a^4x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.73

$$\frac{\sqrt{cx^2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] (Sqrt[c*x^2]*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] -
 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^3*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.09, size = 77, normalized size = 0.69

$$\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4x}-\frac{3b^2\log(a+bx)}{a^4x}+\frac{-a^2+3abx+6b^2x^2}{2a^3x^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]/(x^4*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*x^3*(a + b*x)) + (3*b^2*Lo
g[x])/(a^4*x) - (3*b^2*Log[a + b*x])/(a^4*x))
```

fricas [A] time = 1.16, size = 77, normalized size = 0.69

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.02, size = 95, normalized size = 0.85

$$\frac{\sqrt{c\,x^2}\,\left(6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3\right)}{2\,(bx+a)\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/x^4/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(1/2)*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*ln(x)*x^2*a*b^2-6*
ln(b*x+a)*x^2*a*b^2+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^3/a^4/(b*x+a)
```

maxima [A] time = 1.42, size = 79, normalized size = 0.71

$$\frac{6\,b^2\sqrt{c}\,x^2+3\,ab\sqrt{c}\,x-a^2\sqrt{c}}{2\left(a^3bx^3+a^4x^2\right)}-\frac{3\,b^2\sqrt{c}\,\log\left(bx+a\right)}{a^4}+\frac{3\,b^2\sqrt{c}\,\log(x)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(1/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*sqrt(c)*x^2 + 3*a*b*sqrt(c)*x - a^2*sqrt(c))/(a^3*b*x^3 + a^4*x^
2) - 3*b^2*sqrt(c)*log(b*x + a)/a^4 + 3*b^2*sqrt(c)*log(x)/a^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^2}}{x^4\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(1/2)/(x^4*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}}{x^4\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(1/2)/x**4/(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c*x**2)/(x**4*(a + b*x)**2), x)
```

3.859 $\int \frac{x(cx^2)^{3/2}}{(a+bx)^2}\,dx$

Optimal. Leaf size=111

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

Rubi [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}+\frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (3*a^2*c*Sqrt[c*x^2])/b^4 - (a*c*x*Sqrt[c*x^2])/b^3 + (c*x^2*Sqrt[c*x^2])/(
3*b^2) - (a^4*c*Sqrt[c*x^2])/(b^5*x*(a + b*x)) - (4*a^3*c*Sqrt[c*x^2]*Log[a
 + b*x])/(b^5*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(cx^2\right)^{3/2}}{(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^4}{(a+bx)^2}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{x}\\ &= \frac{3a^2c\sqrt{cx^2}}{b^4}-\frac{acx\sqrt{cx^2}}{b^3}+\frac{cx^2\sqrt{cx^2}}{3b^2}-\frac{a^4c\sqrt{cx^2}}{b^5x(a+bx)}-\frac{4a^3c\sqrt{cx^2}\log(a+bx)}{b^5x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.74

$$\frac{\left(cx^2\right)^{3/2}\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4
- 12*a^3*(a + b*x)*Log[a + b*x]))/(3*b^5*x^3*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 85, normalized size = 0.77

$$\left(cx^2\right)^{3/2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5x^3(a+bx)}-\frac{4a^3\log(a+bx)}{b^5x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(c*x^2)^(3/2))/(a + b*x)^2,x]
```

```
[Out] (c*x^2)^(3/2)*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)
/(3*b^5*x^3*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*x^3))
```

fricas [A] time = 1.10, size = 91, normalized size = 0.82

$$\frac{\left(b^4cx^4-2\,ab^3cx^3+6\,a^2b^2cx^2+9\,a^3bcx-3\,a^4c-12\left(a^3bcx+a^4c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\left(b^6x^2+ab^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*c*x^4 - 2*a*b^3*c*x^3 + 6*a^2*b^2*c*x^2 + 9*a^3*b*c*x - 3*a^4*c -
12*(a^3*b*c*x + a^4*c)*log(b*x + a))*sqrt(c*x^2)/(b^6*x^2 + a*b^5*x)
```

giac [A] time = 1.17, size = 96, normalized size = 0.86

$$-\frac{1}{3}\,c^{\frac{3}{2}}\left(\frac{12\,a^3\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^5}+\frac{3\,a^4\mathrm{sgn}(x)}{(bx+a)b^5}-\frac{3\left(4\,a^3\log\left(|a|\right)+a^3\right)\mathrm{sgn}(x)}{b^5}-\frac{b^4x^3\mathrm{sgn}(x)-3\,ab^3x^2\mathrm{sgn}(x)+9\,a^2b^2x\mathrm{sgn}(x)}{b^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/3*c^(3/2)*(12*a^3*log(abs(b*x + a))*sgn(x)/b^5 + 3*a^4*sgn(x)/((b*x + a)
*b^5) - 3*(4*a^3*log(abs(a)) + a^3)*sgn(x)/b^5 - (b^4*x^3*sgn(x) - 3*a*b^3*
x^2*sgn(x) + 9*a^2*b^2*x*sgn(x))/b^6)
```

maple [A] time = 0.01, size = 88, normalized size = 0.79

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(-b^4x^4+2a\,b^3x^3+12a^3bx\ln\left(bx+a\right)-6a^2b^2x^2+12a^4\ln\left(bx+a\right)-9a^3bx+3a^4\right)}{3\left(bx+a\right)b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -1/3*(c*x^2)^(3/2)*(-b^4*x^4+2*a*b^3*x^3+12*a^3*b*x*ln(b*x+a)-6*a^2*b^2*x^2
+12*a^4*ln(b*x+a)-9*a^3*b*x+3*a^4)/x^3/b^5/(b*x+a)
```

maxima [A] time = 1.62, size = 132, normalized size = 1.19

$$\frac{\left(cx^2\right)^{\frac{3}{2}}a}{b^3x+ab^2}-\frac{4\left(-1\right)^{\frac{2cx}{b}}a^3c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^5}-\frac{4\left(-1\right)^{\frac{2acx}{b}}a^3c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^5}-\frac{2\sqrt{cx^2}\,acx}{b^3}+\frac{\left(cx^2\right)^{\frac{3}{2}}}{3\,b^2}+\frac{4\sqrt{cx^2}\,a^2c}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (c*x^2)^(3/2)*a/(b^3*x + a*b^2) - 4*(-1)^(2*c*x/b)*a^3*c^(3/2)*log(2*c*x/b)
/b^5 - 4*(-1)^(2*a*c*x/b)*a^3*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^5 -
2*sqrt(c*x^2)*a*c*x/b^3 + 1/3*(c*x^2)^(3/2)/b^2 + 4*sqrt(c*x^2)*a^2*c/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(c\,x^2\right)^{3/2}}{(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^(3/2))/(a + b*x)^2,x)
```

```
[Out] int((x*(c*x^2)^(3/2))/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(cx^2\right)^{\frac{3}{2}}}{(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)/(a + b*x)**2, x)
```

3.860 $\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=89

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Rubi [A] time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}-\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (-2*a*c*Sqrt[c*x^2])/b^3 + (c*x*Sqrt[c*x^2])/(2*b^2) + (a^3*c*Sqrt[c*x^2])/
(b^4*x*(a + b*x)) + (3*a^2*c*Sqrt[c*x^2]*Log[a + b*x])/(b^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^3}{(a+bx)^2} dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx}{x}\\
&= -\frac{2ac\sqrt{cx^2}}{b^3}+\frac{cx\sqrt{cx^2}}{2b^2}+\frac{a^3c\sqrt{cx^2}}{b^4x(a+bx)}+\frac{3a^2c\sqrt{cx^2}\log(a+bx)}{b^4x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 71, normalized size = 0.80

$$\frac{(cx^2)^{3/2}\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] ((c*x^2)^(3/2)*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)
*Log[a + b*x]))/(2*b^4*x^3*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 74, normalized size = 0.83

$$(cx^2)^{3/2}\left(\frac{3a^2\log(a+bx)}{b^4x^3}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4x^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (c*x^2)^(3/2)*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*x^3*(a + b*x)) + (3*a^2*Log[a + b*x])/(b^4*x^3))
```

fricas [A] time = 1.10, size = 79, normalized size = 0.89

$$\frac{\left(b^3cx^3-3\,ab^2cx^2-4\,a^2bcx+2\,a^3c+6\,\left(a^2bcx+a^3c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\,\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*c*x^3 - 3*a*b^2*c*x^2 - 4*a^2*b*c*x + 2*a^3*c + 6*(a^2*b*c*x + a^3*c)*log(b*x + a))*sqrt(c*x^2)/(b^5*x^2 + a*b^4*x)
```

giac [A] time = 0.95, size = 80, normalized size = 0.90

$$\frac{1}{2}\,c^{\frac{3}{2}}\left(\frac{6\,a^2\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^4}+\frac{2\,a^3\mathrm{sgn}(x)}{(bx+a)b^4}-\frac{2\left(3\,a^2\log\left(|a|\right)+a^2\right)\mathrm{sgn}(x)}{b^4}+\frac{b^2x^2\mathrm{sgn}(x)-4\,abx\mathrm{sgn}(x)}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*c^(3/2)*(6*a^2*log(abs(b*x + a))*sgn(x)/b^4 + 2*a^3*sgn(x)/((b*x + a)*b^4) - 2*(3*a^2*log(abs(a)) + a^2)*sgn(x)/b^4 + (b^2*x^2*sgn(x) - 4*a*b*x*sgn(x))/b^4)
```

maple [A] time = 0.00, size = 76, normalized size = 0.85

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(b^3x^3+6a^2bx\ln\left(bx+a\right)-3a\,b^2x^2+6a^3\ln\left(bx+a\right)-4a^2bx+2a^3\right)}{2\left(bx+a\right)b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(b^3*x^3+6*a^2*b*x*ln(b*x+a)-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*a^3)/x^3/b^4/(b*x+a)
```

maxima [A] time = 1.55, size = 115, normalized size = 1.29

$$\frac{3\,(-1)^{\frac{2cx}{b}}\,a^2c^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^4}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2c^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4}-\frac{\left(cx^2\right)^{\frac{3}{2}}}{b^2x+ab}+\frac{3\,\sqrt{cx^2}\,cx}{2\,b^2}-\frac{3\,\sqrt{cx^2}\,ac}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3*(-1)^(2*c*x/b)*a^2*c^(3/2)*log(2*c*x/b)/b^4 + 3*(-1)^(2*a*c*x/b)*a^2*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^4 - (c*x^2)^(3/2)/(b^2*x + a*b) + 3/2*sqrt(c*x^2)*c*x/b^2 - 3*sqrt(c*x^2)*a*c/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(a + b*x)^2,x)
```

```
[Out] int((c*x^2)^(3/2)/(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{3}{2}}}{\left(a + bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(a + b*x)**2, x)
```

3.861 $\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx$

Optimal. Leaf size=68

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}+\frac{c\sqrt{cx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/b^2 - (a^2*c*Sqrt[c*x^2])/(b^3*x*(a + b*x)) - (2*a*c*Sqrt[c
*x^2]*Log[a + b*x])/(b^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x^2}{(a+bx)^2} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right) dx}{x} \\
&= \frac{c\sqrt{cx^2}}{b^2}-\frac{a^2c\sqrt{cx^2}}{b^3x(a+bx)}-\frac{2ac\sqrt{cx^2}\log(a+bx)}{b^3x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 55, normalized size = 0.81

$$\frac{c^2x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x
^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 57, normalized size = 0.84

$$(cx^2)^{3/2}\left(\frac{-a^2+abx+b^2x^2}{b^3x^3(a+bx)}-\frac{2a\log(a+bx)}{b^3x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a^2 + a*b*x + b^2*x^2)/(b^3*x^3*(a + b*x)) - (2*a*Log[a +
b*x])/(b^3*x^3))
```

fricas [A] time = 1.26, size = 63, normalized size = 0.93

$$\frac{\left(b^2cx^2+abcx-a^2c-2\left(abcx+a^2c\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4x^2+ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x^2 + a*b*c*x - a^2*c - 2*(a*b*c*x + a^2*c)*log(b*x + a))*sqrt(c*x^2
)/(b^4*x^2 + a*b^3*x)
```

giac [A] time = 0.96, size = 58, normalized size = 0.85

$$c^{\frac{3}{2}}\left(\frac{x\mathrm{sgn}(x)}{b^2}-\frac{2\,a\log\left(|bx+a|\right)\mathrm{sgn}(x)}{b^3}+\frac{\left(2\,a\log\left(|a|\right)+a\right)\mathrm{sgn}(x)}{b^3}-\frac{a^2\mathrm{sgn}(x)}{(bx+a)b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c^(3/2)*(x*sgn(x)/b^2 - 2*a*log(abs(b*x + a))*sgn(x)/b^3 + (2*a*log(abs(a))
 + a)*sgn(x)/b^3 - a^2*sgn(x)/((b*x + a)*b^3))
```

maple [A] time = 0.01, size = 62, normalized size = 0.91

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)}{\left(bx+a\right)b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/x^3/b^
3/(b*x+a)
```

maxima [A] time = 1.42, size = 98, normalized size = 1.44

$$\frac{\sqrt{cx^2}\,ac}{b^3x+ab^2}-\frac{2\,(-1)^{\frac{2\,cx}{b}}\,ac^{\frac{3}{2}}\log\left(\frac{2\,cx}{b}\right)}{b^3}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,ac^{\frac{3}{2}}\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3}+\frac{\sqrt{cx^2}\,c}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a*c/(b^3*x + a*b^2) - 2*(-1)^(2*c*x/b)*a*c^(3/2)*log(2*c*x/b)/b
^3 - 2*(-1)^(2*a*c*x/b)*a*c^(3/2)*log(-2*a*c*x/(b*abs(b*x + a)))/b^3 + sqrt
(c*x^2)*c/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x^2)^{3/2}}{x\,(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{3}{2}}}{x\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x*(a + b*x)**2), x)
```

3.862 $\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx$

Optimal. Leaf size=49

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (a*c*Sqrt[c*x^2])/(b^2*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[a + b*x])/(b^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^2(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{x}{(a+bx)^2} dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx}{x} \\
&= \frac{ac\sqrt{cx^2}}{b^2x(a+bx)}+\frac{c\sqrt{cx^2}\log(a+bx)}{b^2x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 0.78

$$\frac{c^2x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (c^2*x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 0.80

$$(cx^2)^{3/2}\left(\frac{a}{b^2x^3(a+bx)}+\frac{\log(a+bx)}{b^2x^3}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^2*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*(a/(b^2*x^3*(a + b*x)) + Log[a + b*x]/(b^2*x^3))
```

fricas [A] time = 1.41, size = 43, normalized size = 0.88

$$\frac{\sqrt{cx^2}\left(ac + (bcx + ac)\log\left(bx + a\right)\right)}{b^3x^2 + ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(b*x + a))/(b^3*x^2 + a*b^2*x)
```

giac [A] time = 0.96, size = 46, normalized size = 0.94

$$-c^{\frac{3}{2}}\left(\frac{\left(\log\left(|a|\right) + 1\right)\mathrm{sgn}(x)}{b^2} - \frac{\log\left(|bx + a|\right)\mathrm{sgn}(x)}{b^2} - \frac{a\mathrm{sgn}(x)}{(bx + a)b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -c^(3/2)*((log(abs(a)) + 1)*sgn(x)/b^2 - log(abs(b*x + a))*sgn(x)/b^2 - a*s
gn(x)/((b*x + a)*b^2))
```

maple [A] time = 0.00, size = 41, normalized size = 0.84

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(bx\ln\left(bx + a\right) + a\ln\left(bx + a\right) + a\right)}{\left(bx + a\right)b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^2/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/x^3/b^2/(b*x+a)
```

maxima [A] time = 1.48, size = 80, normalized size = 1.63

$$\frac{(-1)^{\frac{2cx}{b}}c^{\frac{3}{2}}\log\left(\frac{2cx}{b}\right)}{b^2} + \frac{(-1)^{\frac{2acx}{b}}c^{\frac{3}{2}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2} - \frac{\sqrt{cx^2}\,c}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-1)^(2*c*x/b)*c^(3/2)*log(2*c*x/b)/b^2 + (-1)^(2*a*c*x/b)*c^(3/2)*log(-2*a
*c*x/(b*abs(b*x + a)))/b^2 - sqrt(c*x^2)*c/(b^2*x + a*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^2\,\left(a + b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^2*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**2/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x**2*(a + b*x)**2), x)
```

3.863 $\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} dx$

Optimal. Leaf size=25

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{c\sqrt{cx^2}}{bx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(b*x*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^3(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{(a+bx)^2} dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{bx(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
```

```
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.96

$$-\frac{(cx^2)^{3/2}}{bx^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^3*(a + b*x)^2),x]
[Out] -((c*x^2)^(3/2)/(b*x^3*(a + b*x)))
```

fricas [A] time = 1.09, size = 24, normalized size = 0.96

$$-\frac{\sqrt{cx^2}\,c}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)*c/(b^2*x^2 + a*b*x)
```

giac [A] time = 1.14, size = 29, normalized size = 1.16

$$-c^{\frac{3}{2}}\left(\frac{\mathrm{sgn}(x)}{(bx+a)b}-\frac{\mathrm{sgn}(x)}{ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="giac")
[Out] -c^(3/2)*(sgn(x)/((b*x + a)*b) - sgn(x)/(a*b))
```

maple [A] time = 0.00, size = 23, normalized size = 0.92

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}}{(bx+a)\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^3/(b*x+a)^2,x)
[Out] -1/(b*x+a)/b*(c*x^2)^(3/2)/x^3
```

maxima [A] time = 1.33, size = 16, normalized size = 0.64

$$-\frac{c^{\frac{3}{2}}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^3/(b*x+a)^2,x, algorithm="maxima")
[Out] -c^(3/2)/(b^2*x + a*b)
```

mupad [B] time = 0.15, size = 24, normalized size = 0.96

$$-\frac{c^{3/2}\,\sqrt{x^2}}{b^2\,x^2+a\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^3*(a + b*x)^2),x)
[Out] -(c^(3/2)*(x^2)^(1/2))/(b^2*x^2 + a*b*x)
```

sympy [A] time = 2.23, size = 44, normalized size = 1.76

$$\begin{cases} -\frac{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{abx^3+b^2x^4} & \text{for } b \neq 0 \\ \frac{c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{a^2x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**3/(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**(3/2)*(x**2)**(3/2)/(a*b*x**3 + b**2*x**4), Ne(b, 0)), (c**(
3/2)*(x**2)**(3/2)/(a**2*x**2), True))
```

3.864 $\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} dx$

Optimal. Leaf size=68

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}+\frac{c\sqrt{cx^2}}{ax(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*Sqrt[c*x^2])/(a*x*(a + b*x)) + (c*Sqrt[c*x^2]*Log[x])/(a^2*x) - (c*Sqrt[
c*x^2]*Log[a + b*x])/(a^2*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^4(a+bx)^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x(a+bx)^2} dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx}{x}\\ &= \frac{c\sqrt{cx^2}}{ax(a+bx)}+\frac{c\sqrt{cx^2}\log(x)}{a^2x}-\frac{c\sqrt{cx^2}\log(a+bx)}{a^2x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.68

$$\frac{(cx^2)^{3/2}(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2x^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*x^3*(a
 + b*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 48, normalized size = 0.71

$$\left(cx^2\right)^{3/2}\left(-\frac{\log(a+bx)}{a^2x^3}+\frac{\log(x)}{a^2x^3}+\frac{1}{ax^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^4*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*(1/(a*x^3*(a + b*x)) + Log[x]/(a^2*x^3) - Log[a + b*x]/(a^2*x
^3))
```

fricas [A] time = 1.12, size = 47, normalized size = 0.69

$$\frac{\sqrt{cx^2}\left(ac+(bcx+ac)\log\left(\frac{x}{bx+a}\right)\right)}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*c + (b*c*x + a*c)*log(x/(b*x + a)))/(a^2*b*x^2 + a^3*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 52, normalized size = 0.76

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(bx\ln(x)-bx\ln\left(bx+a\right)+a\ln(x)-a\ln\left(bx+a\right)+a\right)}{\left(bx+a\right)a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^4/(b*x+a)^2,x)
```

```
[Out] (c*x^2)^(3/2)*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/x^3/a^2/(b*x+
a)
```

maxima [A] time = 1.40, size = 38, normalized size = 0.56

$$\frac{c^{\frac{3}{2}}}{abx+a^2}-\frac{c^{\frac{3}{2}}\log\left(bx+a\right)}{a^2}+\frac{c^{\frac{3}{2}}\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^4/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] c^(3/2)/(a*b*x + a^2) - c^(3/2)*log(b*x + a)/a^2 + c^(3/2)*log(x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(c\,x^2\right)^{3/2}}{x^4\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2),x)
[Out] int((c*x^2)^(3/2)/(x^4*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^4\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**4/(b*x+a)**2,x)
[Out] Integral((c*x**2)**(3/2)/(x**4*(a + b*x)**2), x)
```

3.865 $\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}-\frac{c\sqrt{cx^2}}{a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c*Sqrt[c*x^2])/(a^2*x^2)) - (b*c*Sqrt[c*x^2])/(a^2*x*(a + b*x)) - (2*b*c
*Sqrt[c*x^2]*Log[x])/(a^3*x) + (2*b*c*Sqrt[c*x^2]*Log[a + b*x])/(a^3*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}}{x^5(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^2(a+bx)^2}\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{x}\\
&= -\frac{c\sqrt{cx^2}}{a^2x^2}-\frac{bc\sqrt{cx^2}}{a^2x(a+bx)}-\frac{2bc\sqrt{cx^2}\log(x)}{a^3x}+\frac{2bc\sqrt{cx^2}\log(a+bx)}{a^3x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.65

$$-\frac{c^2(a(a+2bx)+2bx\log(x)(a+bx)-2bx(a+bx)\log(a+bx))}{a^3\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] -((c^2*(a*(a + 2*b*x) + 2*b*x*(a + b*x)*Log[x] - 2*b*x*(a + b*x)*Log[a + b*
x]))/(a^3*Sqrt[c*x^2]*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.65

$$(cx^2)^{3/2}\left(-\frac{2b\log(x)}{a^3x^3}+\frac{2b\log(a+bx)}{a^3x^3}+\frac{-a-2bx}{a^2x^4(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^5*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a - 2*b*x)/(a^2*x^4*(a + b*x)) - (2*b*Log[x])/(a^3*x^3) +
(2*b*Log[a + b*x])/(a^3*x^3))
```

fricas [A] time = 1.16, size = 65, normalized size = 0.71

$$-\frac{\left(2\,abcx+a^2c-2\left(b^2cx^2+abcx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bx^3+a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*c*x + a^2*c - 2*(b^2*c*x^2 + a*b*c*x)*log((b*x + a)/x))*sqrt(c*x^2)
/(a^3*b*x^3 + a^4*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 74, normalized size = 0.81

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx\ln(x)-2abx\ln(bx+a)+2abx+a^2\right)}{(bx+a)\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^5/(b*x+a)^2,x)
```

```
[Out] -(c*x^2)^(3/2)*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*l
n(b*x+a)+2*a*b*x+a^2)/x^4/a^3/(b*x+a)
```

maxima [A] time = 1.43, size = 58, normalized size = 0.64

$$\frac{2\,bc^{\frac{3}{2}}\log(bx+a)}{a^3}-\frac{2\,bc^{\frac{3}{2}}\log(x)}{a^3}-\frac{2\,bc^{\frac{3}{2}}x+ac^{\frac{3}{2}}}{a^2bx^2+a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^5/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 2*b*c^(3/2)*log(b*x + a)/a^3 - 2*b*c^(3/2)*log(x)/a^3 - (2*b*c^(3/2)*x + a*
c^(3/2))/(a^2*b*x^2 + a^3*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^5\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^5*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^5\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**5/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**5*(a + b*x)**2), x)
```

3.866 $\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx$

Optimal. Leaf size=117

$$\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Rubi [A] time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}+\frac{2bc\sqrt{cx^2}}{a^3x^2}-\frac{c\sqrt{cx^2}}{2a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] -(c*Sqrt[c*x^2])/(2*a^2*x^3) + (2*b*c*Sqrt[c*x^2])/(a^3*x^2) + (b^2*c*Sqrt[
c*x^2])/(a^3*x*(a + b*x)) + (3*b^2*c*Sqrt[c*x^2]*Log[x])/(a^4*x) - (3*b^2*c
*Sqrt[c*x^2]*Log[a + b*x])/(a^4*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}}{x^6(a+bx)^2}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int \frac{1}{x^3(a+bx)^2}\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{x}\\ &= -\frac{c\sqrt{cx^2}}{2a^2x^3}+\frac{2bc\sqrt{cx^2}}{a^3x^2}+\frac{b^2c\sqrt{cx^2}}{a^3x(a+bx)}+\frac{3b^2c\sqrt{cx^2}\log(x)}{a^4x}-\frac{3b^2c\sqrt{cx^2}\log(a+bx)}{a^4x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 82, normalized size = 0.70

$$\frac{(cx^2)^{3/2}\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4x^5(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] ((c*x^2)^(3/2)*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x]
 - 6*b^2*x^2*(a + b*x)*Log[a + b*x]))/(2*a^4*x^5*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 77, normalized size = 0.66

$$\left(cx^2\right)^{3/2}\left(\frac{3b^2\log(x)}{a^4x^3}-\frac{3b^2\log(a+bx)}{a^4x^3}+\frac{-a^2+3abx+6b^2x^2}{2a^3x^5(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)/(x^6*(a + b*x)^2),x]
```

```
[Out] (c*x^2)^(3/2)*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*x^5*(a + b*x)) + (3*b^2*
Log[x])/(a^4*x^3) - (3*b^2*Log[a + b*x])/(a^4*x^3))
```

fricas [A] time = 0.93, size = 82, normalized size = 0.70

$$\frac{\left(6\,ab^2cx^2+3\,a^2bcx-a^3c+6\left(b^3cx^3+ab^2cx^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bx^4+a^5x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*c*x^2 + 3*a^2*b*c*x - a^3*c + 6*(b^3*c*x^3 + a*b^2*c*x^2)*log(
x/(b*x + a)))*sqrt(c*x^2)/(a^4*b*x^4 + a^5*x^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Sign error (%%%{a,0%%%}+%%%{b,1%%%})
```

maple [A] time = 0.01, size = 95, normalized size = 0.81

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3\right)}{2\,(bx+a)\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/x^6/(b*x+a)^2,x)
```

```
[Out] 1/2*(c*x^2)^(3/2)*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*
a*b^2*x^2*ln(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/x^5/a^4/(b*x+a)
```

maxima [A] time = 1.33, size = 79, normalized size = 0.68

$$-\frac{3\,b^2c^{\frac{3}{2}}\log(bx+a)}{a^4}+\frac{3\,b^2c^{\frac{3}{2}}\log(x)}{a^4}+\frac{6\,b^2c^{\frac{3}{2}}x^2+3\,abc^{\frac{3}{2}}x-a^2c^{\frac{3}{2}}}{2\left(a^3bx^3+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)/x^6/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*b^2*c^(3/2)*log(b*x + a)/a^4 + 3*b^2*c^(3/2)*log(x)/a^4 + 1/2*(6*b^2*c^(
3/2)*x^2 + 3*a*b*c^(3/2)*x - a^2*c^(3/2))/(a^3*b*x^3 + a^4*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\,x^2\right)^{3/2}}{x^6\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2),x)
```

```
[Out] int((c*x^2)^(3/2)/(x^6*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^6\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)/x**6/(b*x+a)**2,x)
```

```
[Out] Integral((c*x**2)**(3/2)/(x**6*(a + b*x)**2), x)
```

3.867 $\int \frac{x^5}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (3*a^2*x^2)/(b^4*Sqrt[c*x^2]) - (a*x^3)/(b^3*Sqrt[c*x^2]) + x^4/(3*b^2*Sqrt
[c*x^2]) - (a^4*x)/(b^5*Sqrt[c*x^2]*(a + b*x)) - (4*a^3*x*Log[a + b*x])/(b^
5*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{x^4}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{3a^2x^2}{b^4\sqrt{cx^2}}-\frac{ax^3}{b^3\sqrt{cx^2}}+\frac{x^4}{3b^2\sqrt{cx^2}}-\frac{a^4x}{b^5\sqrt{cx^2}\,(a+bx)}-\frac{4a^3x\log(a+bx)}{b^5\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 80, normalized size = 0.75

$$\frac{x\left(-3a^4+9a^3bx-12a^3(a+bx)\log(a+bx)+6a^2b^2x^2-2ab^3x^3+b^4x^4\right)}{3b^5\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4 - 12*a^3*(a
+ b*x)*Log[a + b*x]))/(3*b^5*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 91, normalized size = 0.85

$$\sqrt{cx^2}\left(\frac{-3a^4+9a^3bx+6a^2b^2x^2-2ab^3x^3+b^4x^4}{3b^5cx(a+bx)}-\frac{4a^3\log(a+bx)}{b^5cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-3*a^4 + 9*a^3*b*x + 6*a^2*b^2*x^2 - 2*a*b^3*x^3 + b^4*x^4)/(
3*b^5*c*x*(a + b*x)) - (4*a^3*Log[a + b*x])/(b^5*c*x))
```

fricas [A] time = 0.97, size = 85, normalized size = 0.79

$$\frac{\left(b^4x^4-2\,ab^3x^3+6\,a^2b^2x^2+9\,a^3bx-3\,a^4-12\,\left(a^3bx+a^4\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{3\,\left(b^6cx^2+ab^5cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))*sqrt(c*x^2)/(b^6*c*x^2 + a*b^5*c*x)
```

giac [A] time = 1.13, size = 155, normalized size = 1.45

$$\frac{\frac{(bx+a)^3\left(\frac{6\,a}{bx+a}-\frac{18\,a^2}{(bx+a)^2}-1\right)}{b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{12\,a^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{3\,a^4}{(bx+a)b^5\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{3\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)^3*(6*a/(b*x + a) - 18*a^2/(b*x + a)^2 - 1)/(b^5*sgn(-b/(b*x
+ a) + a*b/(b*x + a)^2)) - 12*a^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b
^5*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) + 3*a^4/((b*x + a)*b^5*sgn(-b/(b*x
+ a) + a*b/(b*x + a)^2)))/sqrt(c)
```

maple [A] time = 0.01, size = 86, normalized size = 0.80

$$-\frac{\left(-b^4x^4+2a\,b^3x^3+12a^3bx\ln\left(bx+a\right)-6a^2b^2x^2+12a^4\ln\left(bx+a\right)-9a^3bx+3a^4\right)x}{3\sqrt{c\,x^2}\,\left(bx+a\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -1/3*x*(-b^4*x^4+2*a*b^3*x^3+12*a^3*b*x*ln(b*x+a)-6*a^2*b^2*x^2+12*a^4*ln(b
*x+a)-9*a^3*b*x+3*a^4)/(c*x^2)^(1/2)/b^5/(b*x+a)
```

maxima [A] time = 1.56, size = 168, normalized size = 1.57

$$\frac{\sqrt{cx^2}\,a^3}{b^5cx+ab^4c}+\frac{\sqrt{cx^2}\,x^2}{3\,b^2c}-\frac{5\,ax^2}{3\,b^3\sqrt{c}}-\frac{4\,(-1)^{\frac{2\,acx}{b}}\,a^3\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^5\sqrt{c}}+\frac{2\,\sqrt{cx^2}\,ax}{3\,b^3c}-\frac{20\,a^2x}{3\,b^4\sqrt{c}}-\frac{4\,a^3\log\left(bx\right)}{b^5\sqrt{c}}+\frac{29\,\sqrt{cx^2}\,a^2}{3\,b^4c}-\frac{5\,a^3}{b^5\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a^3/(b^5*c*x + a*b^4*c) + 1/3*sqrt(c*x^2)*x^2/(b^2*c) - 5/3*a*x
^2/(b^3*sqrt(c)) - 4*(-1)^(2*a*c*x/b)*a^3*log(-2*a*c*x/(b*abs(b*x + a)))/(b
^5*sqrt(c)) + 2/3*sqrt(c*x^2)*a*x/(b^3*c) - 20/3*a^2*x/(b^4*sqrt(c)) - 4*a^
3*log(b*x)/(b^5*sqrt(c)) + 29/3*sqrt(c*x^2)*a^2/(b^4*c) - 5*a^3/(b^5*sqrt(c
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^5/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(c*x**2)*(a + b*x)**2), x)
```

3.868 $\int \frac{x^4}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=86

$$\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}-\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (-2*a*x^2)/(b^3*Sqrt[c*x^2]) + x^3/(2*b^2*Sqrt[c*x^2]) + (a^3*x)/(b^4*Sqrt[
c*x^2]*(a + b*x)) + (3*a^2*x*Log[a + b*x])/(b^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{x^3}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{2ax^2}{b^3\sqrt{cx^2}}+\frac{x^3}{2b^2\sqrt{cx^2}}+\frac{a^3x}{b^4\sqrt{cx^2}\,(a+bx)}+\frac{3a^2x\log(a+bx)}{b^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 0.80

$$\frac{x\left(2a^3-4a^2bx+6a^2(a+bx)\log(a+bx)-3ab^2x^2+b^3x^3\right)}{2b^4\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 + 6*a^2*(a + b*x)*Log[a + b*x
]))/(2*b^4*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 80, normalized size = 0.93

$$\sqrt{cx^2}\left(\frac{3a^2\log(a+bx)}{b^4cx}+\frac{2a^3-4a^2bx-3ab^2x^2+b^3x^3}{2b^4cx(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((2*a^3 - 4*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3)/(2*b^4*c*x*(a + b*
x)) + (3*a^2*Log[a + b*x])/(b^4*c*x))
```

fricas [A] time = 0.99, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3-3\,ab^2x^2-4\,a^2bx+2\,a^3+6\left(a^2bx+a^3\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{2\left(b^5cx^2+ab^4cx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))*sqrt(c*x^2)/(b^5*c*x^2 + a*b^4*c*x)
```

giac [A] time = 1.04, size = 143, normalized size = 1.66

$$\frac{\frac{(bx+a)^2\left(\frac{6\,a}{bx+a}-1\right)}{b^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{6\,a^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{2\,a^3}{(bx+a)b^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*((b*x + a)^2*(6*a/(b*x + a) - 1)/(b^4*sgn(-b/(b*x + a) + a*b/(b*x + a)^
2)) + 6*a^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^4*sgn(-b/(b*x + a) +
a*b/(b*x + a)^2)) - 2*a^3/((b*x + a)*b^4*sgn(-b/(b*x + a) + a*b/(b*x + a)^2
)))/sqrt(c)
```

maple [A] time = 0.00, size = 74, normalized size = 0.86

$$\frac{\left(b^3x^3+6a^2bx\ln\left(bx+a\right)-3a\,b^2x^2+6a^3\ln\left(bx+a\right)-4a^2bx+2a^3\right)x}{2\sqrt{c\,x^2}\,\left(bx+a\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2*x*(b^3*x^3+6*a^2*b*x*ln(b*x+a)-3*a*b^2*x^2+6*a^3*ln(b*x+a)-4*a^2*b*x+2*
a^3)/(c*x^2)^(1/2)/b^4/(b*x+a)
```

maxima [A] time = 1.55, size = 129, normalized size = 1.50

$$-\frac{\sqrt{cx^2}\,a^2}{b^4cx+ab^3c}+\frac{x^2}{2\,b^2\sqrt{c}}+\frac{3\,(-1)^{\frac{2\,acx}{b}}\,a^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^4\sqrt{c}}+\frac{2\,ax}{b^3\sqrt{c}}+\frac{3\,a^2\log\left(bx\right)}{b^4\sqrt{c}}-\frac{4\,\sqrt{cx^2}\,a}{b^3c}+\frac{3\,a^2}{2\,b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*a^2/(b^4*c*x + a*b^3*c) + 1/2*x^2/(b^2*sqrt(c)) + 3*(-1)^(2*a*
c*x/b)*a^2*log(-2*a*c*x/(b*abs(b*x + a)))/(b^4*sqrt(c)) + 2*a*x/(b^3*sqrt(c
)) + 3*a^2*log(b*x)/(b^4*sqrt(c)) - 4*sqrt(c*x^2)*a/(b^3*c) + 3/2*a^2/(b^4*
sqrt(c))
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{c\,x^2}\,\left(a+b\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^4/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\sqrt{cx^{2}} \left(a + bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x)
```

3.869 $\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=64

$$-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}+\frac{x^2}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*Sqrt[c*x^2]) - (a^2*x)/(b^3*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Log[a
+ b*x])/(b^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x\int \frac{x^2}{(a+bx)^2} dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right) dx}{\sqrt{cx^2}}\\ &= \frac{x^2}{b^2\sqrt{cx^2}}-\frac{a^2x}{b^3\sqrt{cx^2}(a+bx)}-\frac{2ax\log(a+bx)}{b^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.81

$$\frac{x\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*Sqrt[c*x^2]*
(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 63, normalized size = 0.98

$$\sqrt{cx^2}\left(\frac{-a^2+abx+b^2x^2}{b^3cx(a+bx)}-\frac{2a\log(a+bx)}{b^3cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + a*b*x + b^2*x^2)/(b^3*c*x*(a + b*x)) - (2*a*Log[a + b*
x])/(b^3*c*x))
```

fricas [A] time = 1.05, size = 59, normalized size = 0.92

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4cx^2+ab^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c*x
^2 + a*b^3*c*x)
```

giac [B] time = 1.13, size = 127, normalized size = 1.98

$$-\frac{\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{bx+a}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a^2}{(bx+a)b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^3*sgn(-b/(b*x + a) + a*b/(b
*x + a)^2)) + (b*x + a)/(b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - a^2/((b
*x + a)*b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

maple [A] time = 0.01, size = 60, normalized size = 0.94

$$-\frac{\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)x}{\sqrt{c\,x^2}\,\left(bx+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(1/2)/b^3/
(b*x+a)
```

maxima [A] time = 1.45, size = 88, normalized size = 1.38

$$\frac{\sqrt{cx^2}\,a}{b^3cx+ab^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3\sqrt{c}}-\frac{2\,a\log\left(bx\right)}{b^3\sqrt{c}}+\frac{\sqrt{cx^2}}{b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)*a/(b^3*c*x + a*b^2*c) - 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*ab
s(b*x + a)))/(b^3*sqrt(c)) - 2*a*log(b*x)/(b^3*sqrt(c)) + sqrt(c*x^2)/(b^2*
c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^3/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{c x^{2}} \left(a + b x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(c*x**2)*(a + b*x)**2), x)
```

3.870 $\int \frac{x^2}{\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=43

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (a*x)/(b^2*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= \frac{ax}{b^2\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.81

$$\frac{x((a+bx)\log(a+bx)+a)}{b^2\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] (x*(a + (a + b*x)*Log[a + b*x]))/(b^2*Sqrt[c*x^2]*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.04, size = 45, normalized size = 1.05

$$\sqrt{cx^2}\left(\frac{a}{b^2cx(a+bx)}+\frac{\log(a+bx)}{b^2cx}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(Sqrt[c*x^2]*(a + b*x)^2),x]
[Out] Sqrt[c*x^2]*(a/(b^2*c*x*(a + b*x)) + Log[a + b*x]/(b^2*c*x))
```

fricas [A] time = 1.20, size = 40, normalized size = 0.93

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(bx+a\right)+a\right)}{b^3cx^2+ab^2cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c*x^2 + a*b^2*c*x)
```

giac [B] time = 1.15, size = 89, normalized size = 2.07

$$\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a}{(bx+a)b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
[Out] (log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^2*sgn(-b/(b*x + a) + a*b/(b*x +
a)^2)) - a/((b*x + a)*b^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

maple [A] time = 0.00, size = 39, normalized size = 0.91

$$\frac{\left(bx\ln\left(bx+a\right)+a\ln\left(bx+a\right)+a\right)x}{\sqrt{c\,x^2}\,\left(bx+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
[Out] x*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/(c*x^2)^(1/2)/b^2/(b*x+a)
```

maxima [A] time = 1.41, size = 68, normalized size = 1.58

$$-\frac{\sqrt{cx^2}}{b^2cx+abc}+\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2\sqrt{c}}+\frac{\log\left(bx\right)}{b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(c*x^2)/(b^2*c*x + a*b*c) + (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b*x +
 a)))/(b^2*sqrt(c)) + log(b*x)/(b^2*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{c\,x^2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(x^2/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
[Out] Integral(x**2/(sqrt(c*x**2)*(a + b*x)**2), x)
```

3.871 $\int \frac{x}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=22

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{cx^2}\,(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= -\frac{x}{b\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{x}{b\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(x/(b*Sqrt[c*x^2]*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.03, size = 27, normalized size = 1.23

$$-\frac{\sqrt{cx^2}}{bcx(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(Sqrt[c*x^2]/(b*c*x*(a + b*x)))
```

fricas [A]   time = 1.08, size = 25, normalized size = 1.14

$$-\frac{\sqrt{cx^2}}{b^2cx^2 + abcx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c*x^2)/(b^2*c*x^2 + a*b*c*x)
```

giac [A]   time = 1.16, size = 38, normalized size = 1.73

$$\frac{1}{(bx+a)b\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a} + \frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/((b*x + a)*b*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

maple [A]   time = 0.00, size = 21, normalized size = 0.95

$$-\frac{x}{(bx+a)\sqrt{c\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -x/b/(b*x+a)/(c*x^2)^(1/2)
```

maxima [A]   time = 1.45, size = 21, normalized size = 0.95

$$\frac{\sqrt{cx^2}}{abcx + a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c*x^2)/(a*b*c*x + a^2*c)
```

mupad [B]   time = 0.16, size = 25, normalized size = 1.14

$$-\frac{\sqrt{c\,x^2}}{b\,c\,x\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] -(c*x^2)^(1/2)/(b*c*x*(a + b*x))
```

sympy [A]   time = 1.17, size = 85, normalized size = 3.86

$$\begin{cases} \frac{\tilde{\infty}}{\sqrt{c}\sqrt{x^2}} & \text{for}\ a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^2}{\sqrt{c}\sqrt{x^2}} & \text{for}\ a = -bx \\ \frac{x^2}{a^2\sqrt{c}\sqrt{x^2}} & \text{for}\ b = 0 \\ -\frac{x}{ab\sqrt{c}\sqrt{x^2} + b^2\sqrt{c}\,x\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((zoo/(sqrt(c)*sqrt(x**2)), Eq(a, 0) & Eq(b, 0)), (zoo*x**2/(sqrt(
c)*sqrt(x**2)), Eq(a, -b*x)), (x**2/(a**2*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (
-x/(a*b*sqrt(c)*sqrt(x**2) + b**2*sqrt(c)*x*sqrt(x**2)), True))
```

3.872 $\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx$

Optimal. Leaf size=59

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}+\frac{x\log(x)}{a^2\sqrt{cx^2}}+\frac{x}{a\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] x/(a*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*Sqrt[c*x^2]) - (x*Log[a + b*x
])/(a^2*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx^2}(a+bx)^2} dx &= \frac{x\int \frac{1}{x(a+bx)^2} dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx}{\sqrt{cx^2}}\\ &= \frac{x}{a\sqrt{cx^2}(a+bx)}+\frac{x\log(x)}{a^2\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 44, normalized size = 0.75

$$\frac{x(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\sqrt{cx^2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (x*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*Sqrt[c*x^2]*(a + b
*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 57, normalized size = 0.97

$$\sqrt{cx^2}\left(-\frac{\log(a+bx)}{a^2cx}+\frac{\log(x)}{a^2cx}+\frac{1}{acx(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*(1/(a*c*x*(a + b*x)) + Log[x]/(a^2*c*x) - Log[a + b*x]/(a^2*c*x
))
```

fricas [A] time = 0.99, size = 44, normalized size = 0.75

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bcx^2+a^3cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c*x^2 + a^3*c*x)
```

giac [A] time = 1.08, size = 86, normalized size = 1.46

$$-\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{1}{(bx+a)a\sqrt{c}\,\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-a/(b*x + a) + 1))/(a^2*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2
)) - 1/((b*x + a)*a*sqrt(c)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

maple [A] time = 0.01, size = 50, normalized size = 0.85

$$\frac{(bx\ln(x)-bx\ln(bx+a)+a\ln(x)-a\ln(bx+a)+a)\,x}{\sqrt{c\,x^2}\,\,(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(1/2)/a^2/(b*x+a)
```

maxima [A] time = 1.46, size = 61, normalized size = 1.03

$$-\frac{\sqrt{cx^2}\,b}{a^2bcx+a^3c}-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(c*x^2)*b/(a^2*b*c*x + a^3*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*x/(b*abs(b
*x + a)))/(a^2*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x^2}\,\,{(a+b\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x**2)*(a + b*x)**2), x)
```

3.873 $\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx$

Optimal. Leaf size=78

$$-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}-\frac{1}{a^2\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*Sqrt[c*x^2])) - (b*x)/(a^2*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log[x])
/(a^3*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{cx^2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{1}{a^2\sqrt{cx^2}}-\frac{bx}{a^2\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.77

$$\frac{cx^2(-a(a+2bx)-2bx\log(x)(a+bx)+2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a +
 b*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 68, normalized size = 0.87

$$\sqrt{cx^2}\left(-\frac{2b\log(x)}{a^3cx}+\frac{2b\log(a+bx)}{a^3cx}+\frac{-a-2bx}{a^2cx^2(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a - 2*b*x)/(a^2*c*x^2*(a + b*x)) - (2*b*Log[x])/(a^3*c*x) +
(2*b*Log[a + b*x])/(a^3*c*x))
```

fricas [A] time = 0.95, size = 62, normalized size = 0.79

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bcx^3+a^4cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c*x^3 + a^4*c*x^2)
```

giac [A] time = 1.14, size = 126, normalized size = 1.62

$$\frac{b\left(\frac{2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{1}{(bx+a)a^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{1}{a^3\left(\frac{a}{bx+a}-1\right)\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] b*(2*log(abs(-a/(b*x + a) + 1))/(a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) +
 1/((b*x + a)*a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - 1/(a^3*(a/(b*x + a
) - 1)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/sqrt(c)
```

maple [A] time = 0.01, size = 71, normalized size = 0.91

$$-\frac{2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx\ln(x)-2abx\ln(bx+a)+2abx+a^2}{\sqrt{c\,x^2}\,\left(bx+a\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] -(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*ln(b*x+a)+2*a*b
*x+a^2)/(c*x^2)^(1/2)/a^3/(b*x+a)
```

maxima [A] time = 1.43, size = 57, normalized size = 0.73

$$-\frac{2\,bx+a}{a^2b\sqrt{c}\,x^2+a^3\sqrt{c}\,x}+\frac{2\,b\log\left(bx+a\right)}{a^3\sqrt{c}}-\frac{2\,b\log(x)}{a^3\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*sqrt(c)*x^2 + a^3*sqrt(c)*x) + 2*b*log(b*x + a)/(a^3*sq
rt(c)) - 2*b*log(x)/(a^3*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(c*x**2)*(a + b*x)**2), x)
```

3.874 $\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$

Optimal. Leaf size=103

$$\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}+\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*Sqrt[c*x^2]) - 1/(2*a^2*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*Sqrt[c*x^2
]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*Sqrt[c*x^2]) - (3*b^2*x*Log[a + b*x])/
(a^4*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx&=\frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{\sqrt{cx^2}}\\&=\frac{x\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{\sqrt{cx^2}}\\&=\frac{2b}{a^3\sqrt{cx^2}}-\frac{1}{2a^2x\sqrt{cx^2}}+\frac{b^2x}{a^3\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 81, normalized size = 0.79

$$\frac{cx\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] (c*x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x
^2*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 86, normalized size = 0.83

$$\sqrt{cx^2}\left(\frac{3b^2\log(x)}{a^4cx}-\frac{3b^2\log(a+bx)}{a^4cx}+\frac{-a^2+3abx+6b^2x^2}{2a^3cx^3(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*Sqrt[c*x^2]*(a + b*x)^2),x]
```

```
[Out] Sqrt[c*x^2]*((-a^2 + 3*a*b*x + 6*b^2*x^2)/(2*a^3*c*x^3*(a + b*x)) + (3*b^2*
Log[x])/(a^4*c*x) - (3*b^2*Log[a + b*x])/(a^4*c*x))
```

fricas [A] time = 1.28, size = 79, normalized size = 0.77

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bcx^4+a^5cx^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c*x^4 + a^5*c*x^3)
```

giac [A] time = 1.22, size = 152, normalized size = 1.48

$$-\frac{\frac{6\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{2\,b^2}{(bx+a)a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{a^4\left(\frac{a}{bx+a}-1\right)^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(6*b^2*log(abs(-a/(b*x + a) + 1))/(a^4*sgn(-b/(b*x + a) + a*b/(b*x + a
)^2)) + 2*b^2/((b*x + a)*a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - (6*a*b^
2/(b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2*sgn(-b/(b*x + a) + a*b/(b*x +
 a)^2)))/sqrt(c)
```

maple [A] time = 0.01, size = 95, normalized size = 0.92

$$\frac{6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3}{2\sqrt{c\,x^2}\,\left(bx+a\right)a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] 1/2/x*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*a*b^2*x^2*ln
(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(1/2)/a^4/(b*x+a)
```

maxima [A] time = 1.41, size = 76, normalized size = 0.74

$$\frac{6\,b^2x^2+3\,abx-a^2}{2\left(a^3b\sqrt{c}\,x^3+a^4\sqrt{c}\,x^2\right)}-\frac{3\,b^2\log\left(bx+a\right)}{a^4\sqrt{c}}+\frac{3\,b^2\log(x)}{a^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*sqrt(c)*x^3 + a^4*sqrt(c)*x^2) - 3*b
^2*log(b*x + a)/(a^4*sqrt(c)) + 3*b^2*log(x)/(a^4*sqrt(c))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,\sqrt{c\,x^2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2),x)
```

```
[Out] int(1/(x^2*(c*x^2)^(1/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{cx^2}\,(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(c*x**2)*(a + b*x)**2), x)
```

3.875 $\int \frac{x^5}{\left(cx^2\right)^{3/2}(a+bx)^2} \, dx$

Optimal. Leaf size=73

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}+\frac{x^2}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x^2/(b^2*c*Sqrt[c*x^2]) - (a^2*x)/(b^3*c*Sqrt[c*x^2]*(a + b*x)) - (2*a*x*Lo
g[a + b*x])/(b^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(cx^2\right)^{3/2}(a+bx)^2} \, dx &= \frac{x\int \frac{x^2}{(a+bx)^2} \, dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{b^2}+\frac{a^2}{b^2(a+bx)^2}-\frac{2a}{b^2(a+bx)}\right) dx}{c\sqrt{cx^2}}\\
&= \frac{x^2}{b^2c\sqrt{cx^2}}-\frac{a^2x}{b^3c\sqrt{cx^2}\,(a+bx)}-\frac{2ax\log(a+bx)}{b^3c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 0.74

$$\frac{x^3\left(-a^2+abx-2a(a+bx)\log(a+bx)+b^2x^2\right)}{b^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(-a^2 + a*b*x + b^2*x^2 - 2*a*(a + b*x)*Log[a + b*x]))/(b^3*(c*x^2)^(3
/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.81

$$\frac{\frac{-a^2x^3+abx^4+b^2x^5}{b^3(a+bx)}-\frac{2ax^3\log(a+bx)}{b^3}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a^2*x^3) + a*b*x^4 + b^2*x^5)/(b^3*(a + b*x)) - (2*a*x^3*Log[a + b*x])/
b^3)/(c*x^2)^(3/2)
```

fricas [A] time = 1.00, size = 63, normalized size = 0.86

$$\frac{\left(b^2x^2+abx-a^2-2\left(abx+a^2\right)\log\left(bx+a\right)\right)\sqrt{cx^2}}{b^4c^2x^2+ab^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))*sqrt(c*x^2)/(b^4*c^2
*x^2 + a*b^3*c^2*x)
```

giac [A] time = 1.26, size = 127, normalized size = 1.74

$$-\frac{\frac{2\,a\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{bx+a}{b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a^2}{(bx+a)b^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^3*sgn(-b/(b*x + a) + a*b/(b
*x + a)^2)) + (b*x + a)/(b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - a^2/((b
*x + a)*b^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

maple [A] time = 0.00, size = 62, normalized size = 0.85

$$-\frac{\left(2abx\ln\left(bx+a\right)-b^2x^2+2a^2\ln\left(bx+a\right)-abx+a^2\right)x^3}{\left(c\,x^2\right)^{\frac{3}{2}}\left(bx+a\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^3*(2*a*b*x*ln(b*x+a)-b^2*x^2+2*a^2*ln(b*x+a)-a*b*x+a^2)/(c*x^2)^(3/2)/b^
3/(b*x+a)
```

maxima [B] time = 1.61, size = 149, normalized size = 2.04

$$\frac{a^3}{\sqrt{cx^2}\,b^5cx+\sqrt{cx^2}\,ab^4c}+\frac{x^2}{\sqrt{cx^2}\,b^2c}-\frac{2\,(-1)^{\frac{2\,acx}{b}}\,a\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{b^3c^{\frac{3}{2}}}+\frac{2\,ax}{\sqrt{cx^2}\,b^3c}-\frac{2\,a\log\left(bx\right)}{b^3c^{\frac{3}{2}}}-\frac{5\,a^2}{\sqrt{cx^2}\,b^4c}+\frac{4\,a^2}{b^4c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] a^3/(sqrt(c*x^2)*b^5*c*x + sqrt(c*x^2)*a*b^4*c) + x^2/(sqrt(c*x^2)*b^2*c) -
 2*(-1)^(2*a*c*x/b)*a*log(-2*a*c*x/(b*abs(b*x + a)))/(b^3*c^(3/2)) + 2*a*x/
(sqrt(c*x^2)*b^3*c) - 2*a*log(b*x)/(b^3*c^(3/2)) - 5*a^2/(sqrt(c*x^2)*b^4*c
) + 4*a^2/(b^4*c^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x^5/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}}{\left(cx^{2}\right)^{\frac{3}{2}} \left(a + bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x**5/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

3.876 $\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=49

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (a*x)/(b^2*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[a + b*x])/(b^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{x}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{ax}{b^2c\sqrt{cx^2}\,(a+bx)}+\frac{x\log(a+bx)}{b^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{x^3((a+bx)\log(a+bx)+a)}{b^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[a + b*x]))/(b^2*(c*x^2)^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 39, normalized size = 0.80

$$\frac{\frac{ax^3}{b^2(a+bx)}+\frac{x^3\log(a+bx)}{b^2}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^4/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] ((a*x^3)/(b^2*(a + b*x)) + (x^3*Log[a + b*x])/b^2)/(c*x^2)^(3/2)
```

fricas [A] time = 1.12, size = 44, normalized size = 0.90

$$\frac{\sqrt{cx^2}\left((bx+a)\log(bx+a)+a\right)}{b^3c^2x^2+ab^2c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] sqrt(c*x^2)*((b*x + a)*log(b*x + a) + a)/(b^3*c^2*x^2 + a*b^2*c^2*x)
```

giac [A] time = 1.03, size = 89, normalized size = 1.82

$$\frac{\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{a}{(bx+a)b^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
[Out] (log(abs(b*x + a)/((b*x + a)^2*abs(b)))/(b^2*sgn(-b/(b*x + a) + a*b/(b*x +
a)^2)) - a/((b*x + a)*b^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

maple [A] time = 0.00, size = 41, normalized size = 0.84

$$\frac{(bx\ln(bx+a)+a\ln(bx+a)+a)\,x^3}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x)
[Out] x^3*(b*x*ln(b*x+a)+a*ln(b*x+a)+a)/(c*x^2)^(3/2)/b^2/(b*x+a)
```

maxima [B] time = 1.56, size = 108, normalized size = 2.20

$$-\frac{a^2}{\sqrt{cx^2}\,b^4cx+\sqrt{cx^2}\,ab^3c}+\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{b^2c^{\frac{3}{2}}}+\frac{\log(bx)}{b^2c^{\frac{3}{2}}}+\frac{3a}{\sqrt{cx^2}\,b^3c}-\frac{2a}{b^3c^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] -a^2/(sqrt(c*x^2)*b^4*c*x + sqrt(c*x^2)*a*b^3*c) + (-1)^(2*a*c*x/b)*log(-2*
a*c*x/(b*abs(b*x + a)))/(b^2*c^(3/2)) + log(b*x)/(b^2*c^(3/2)) + 3*a/(sqrt(
c*x^2)*b^3*c) - 2*a/(b^3*c^(3/2)*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{{\left(c\,x^2\right)}^{3/2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] int(x^4/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral(x**4/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

3.877 $\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$-\frac{x}{bc\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x/(b*c*Sqrt[c*x^2]*(a + b*x)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= -\frac{x}{bc\sqrt{cx^2}\,(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

IntegrateAlgebraic [A] time = 0.03, size = 24, normalized size = 0.96

$$-\frac{x^3}{b\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/((c*x^2)^(3/2)*(a + b*x)^2),x]
[Out] -(x^3/(b*(c*x^2)^(3/2)*(a + b*x)))
```

fricas [A] time = 0.96, size = 29, normalized size = 1.16

$$-\frac{\sqrt{cx^2}}{b^2c^2x^2 + abc^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
[Out] -sqrt(c*x^2)/(b^2*c^2*x^2 + a*b*c^2*x)
```

giac [A] time = 1.16, size = 38, normalized size = 1.52

$$\frac{1}{(bx + a)bc^{\frac{3}{2}}\operatorname{sgn}\left(-\frac{b}{bx+a} + \frac{ab}{(bx+a)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
[Out] 1/((b*x + a)*b*c^(3/2)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
```

maple [A] time = 0.00, size = 23, normalized size = 0.92

$$-\frac{x^3}{(bx + a)\left(c\,x^2\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x)
[Out] -1/(b*x+a)/b*x^3/(c*x^2)^(3/2)
```

maxima [B] time = 1.48, size = 47, normalized size = 1.88

$$\frac{a}{\sqrt{cx^2}\,b^3cx + \sqrt{cx^2}\,ab^2c} - \frac{1}{\sqrt{cx^2}\,b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
[Out] a/(sqrt(c*x^2)*b^3*c*x + sqrt(c*x^2)*a*b^2*c) - 1/(sqrt(c*x^2)*b^2*c)
```

mupad [B] time = 0.17, size = 25, normalized size = 1.00

$$-\frac{\sqrt{c\,x^2}}{b\,c^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] -(c*x^2)^(1/2)/(b*c^2*x*(a + b*x))
```

sympy [A] time = 1.94, size = 90, normalized size = 3.60

$$\begin{cases} \frac{\tilde{\infty}x^2}{c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } a = 0 \wedge b = 0 \\ \frac{\tilde{\infty}x^4}{c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } a = -bx \\ \frac{x^4}{a^2c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ -\frac{x^3}{abc^{\frac{3}{2}}(x^2)^{\frac{3}{2}} + b^2c^{\frac{3}{2}}x(x^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x**2/(c**(3/2)*(x**2)**(3/2)), Eq(a, 0) & Eq(b, 0)), (zoo*x*
*4/(c**(3/2)*(x**2)**(3/2)), Eq(a, -b*x)), (x**4/(a**2*c**(3/2)*(x**2)**(3/
2)), Eq(b, 0)), (-x**3/(a*b*c**(3/2)*(x**2)**(3/2) + b**2*c**(3/2)*x*(x**2)
**(3/2)), True))
```

3.878 $\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)^2} dx$

Optimal. Leaf size=68

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 44}

$$-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}+\frac{x}{ac\sqrt{cx^2}\,(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] x/(a*c*Sqrt[c*x^2]*(a + b*x)) + (x*Log[x])/(a^2*c*Sqrt[c*x^2]) - (x*Log[a +
 b*x])/(a^2*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{x}{ac\sqrt{cx^2}\,(a+bx)}+\frac{x\log(x)}{a^2c\sqrt{cx^2}}-\frac{x\log(a+bx)}{a^2c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.68

$$\frac{x^3(\log(x)(a+bx)-(a+bx)\log(a+bx)+a)}{a^2\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3*(a + (a + b*x)*Log[x] - (a + b*x)*Log[a + b*x]))/(a^2*(c*x^2)^(3/2)*(a
 + b*x))
```

IntegrateAlgebraic [A] time = 0.05, size = 48, normalized size = 0.71

$$\frac{-\frac{x^3\log(a+bx)}{a^2}+\frac{x^3\log(x)}{a^2}+\frac{x^3}{a(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^3/(a*(a + b*x)) + (x^3*Log[x])/a^2 - (x^3*Log[a + b*x])/a^2)/(c*x^2)^(3/
2)
```

fricas [A] time = 1.02, size = 48, normalized size = 0.71

$$\frac{\sqrt{cx^2}\left((bx+a)\log\left(\frac{x}{bx+a}\right)+a\right)}{a^2bc^2x^2+a^3c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*((b*x + a)*log(x/(b*x + a)) + a)/(a^2*b*c^2*x^2 + a^3*c^2*x)
```

giac [A] time = 1.06, size = 83, normalized size = 1.22

$$-\frac{\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^2\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{1}{(bx+a)a\operatorname{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -(log(abs(-a/(b*x + a) + 1))/(a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) + 1/
((b*x + a)*a*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/c^(3/2)
```

maple [A] time = 0.01, size = 52, normalized size = 0.76

$$\frac{\left(bx\ln(x)-bx\ln(bx+a)+a\ln(x)-a\ln(bx+a)+a\right)x^3}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] x^3*(b*x*ln(x)-b*x*ln(b*x+a)+a*ln(x)-a*ln(b*x+a)+a)/(c*x^2)^(3/2)/a^2/(b*x+
a)
```

maxima [A] time = 1.52, size = 82, normalized size = 1.21

$$-\frac{1}{\sqrt{cx^2}\,b^2cx+\sqrt{cx^2}\,abc}-\frac{(-1)^{\frac{2acx}{b}}\log\left(-\frac{2acx}{b|bx+a|}\right)}{a^2c^{\frac{3}{2}}}+\frac{1}{\sqrt{cx^2}\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/(sqrt(c*x^2)*b^2*c*x + sqrt(c*x^2)*a*b*c) - (-1)^(2*a*c*x/b)*log(-2*a*c*
x/(b*abs(b*x + a)))/(a^2*c^(3/2)) + 1/(sqrt(c*x^2)*a*b*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2),x)
[Out] int(x^2/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(cx^2\right)^{\frac{3}{2}}\left(a+bx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**2)**(3/2)/(b*x+a)**2,x)
[Out] Integral(x**2/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

3.879 $\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{1}{a^2c\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 44}

$$-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}-\frac{1}{a^2c\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*c*Sqrt[c*x^2])) - (b*x)/(a^2*c*Sqrt[c*x^2]*(a + b*x)) - (2*b*x*Log
[x])/(a^3*c*Sqrt[c*x^2]) + (2*b*x*Log[a + b*x])/(a^3*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(cx^2)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^2(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{1}{a^2c\sqrt{cx^2}}-\frac{bx}{a^2c\sqrt{cx^2}\,(a+bx)}-\frac{2bx\log(x)}{a^3c\sqrt{cx^2}}+\frac{2bx\log(a+bx)}{a^3c\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 0.66

$$\frac{x^2(-a(a+2bx)-2bx\log(x)(a+bx)+2bx(a+bx)\log(a+bx))}{a^3\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x^2*(-(a*(a + 2*b*x)) - 2*b*x*(a + b*x)*Log[x] + 2*b*x*(a + b*x)*Log[a + b
*x]))/(a^3*(c*x^2)^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.06, size = 61, normalized size = 0.68

$$\frac{-\frac{2bx^3\log(x)}{a^3}+\frac{2bx^3\log(a+bx)}{a^3}+\frac{-ax^2-2bx^3}{a^2(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a*x^2) - 2*b*x^3)/(a^2*(a + b*x)) - (2*b*x^3*Log[x])/a^3 + (2*b*x^3*Log
[a + b*x])/a^3)/(c*x^2)^(3/2)
```

fricas [A] time = 1.21, size = 66, normalized size = 0.73

$$-\frac{\left(2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(\frac{bx+a}{x}\right)\right)\sqrt{cx^2}}{a^3bc^2x^3+a^4c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log((b*x + a)/x))*sqrt(c*x^2)/(a^3*b*
c^2*x^3 + a^4*c^2*x^2)
```

giac [A] time = 1.11, size = 137, normalized size = 1.52

$$\frac{\frac{2\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{b^2}{(bx+a)a^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{b^2}{a^3\left(\frac{a}{bx+a}-1\right)\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (2*b^2*log(abs(-a/(b*x + a) + 1))/(a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2))
 + b^2/((b*x + a)*a^2*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - b^2/(a^3*(a/(b
*x + a) - 1)*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)))/(b*c^(3/2))
```

maple [A] time = 0.00, size = 74, normalized size = 0.82

$$-\frac{\left(2b^2x^2\ln(x)-2b^2x^2\ln(bx+a)+2abx\ln(x)-2abx\ln(bx+a)+2abx+a^2\right)x^2}{\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] -x^2*(2*b^2*x^2*ln(x)-2*b^2*x^2*ln(b*x+a)+2*a*b*x*ln(x)-2*a*b*x*ln(b*x+a)+2
*a*b*x+a^2)/(c*x^2)^(3/2)/a^3/(b*x+a)
```

maxima [A] time = 1.41, size = 79, normalized size = 0.88

$$\frac{1}{\sqrt{cx^2}\,abcx+\sqrt{cx^2}\,a^2c}+\frac{2\,(-1)^{\frac{2\,acx}{b}}\,b\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^3c^{\frac{3}{2}}}-\frac{2}{\sqrt{cx^2}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/(sqrt(c*x^2)*a*b*c*x + sqrt(c*x^2)*a^2*c) + 2*(-1)^(2*a*c*x/b)*b*log(-2*a
*c*x/(b*abs(b*x + a)))/(a^3*c^(3/2)) - 2/(sqrt(c*x^2)*a^2*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(c\,x^2\right)^{3/2}\,{\left(a+b\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(x/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(cx^{2}\right)^{\frac{3}{2}} \left(a + bx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(x/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

3.880 $\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx$

Optimal. Leaf size=118

$$\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 44}

$$\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}+\frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (2*b)/(a^3*c*Sqrt[c*x^2]) - 1/(2*a^2*c*x*Sqrt[c*x^2]) + (b^2*x)/(a^3*c*Sqrt
[c*x^2]*(a + b*x)) + (3*b^2*x*Log[x])/(a^4*c*Sqrt[c*x^2]) - (3*b^2*x*Log[a
+ b*x])/(a^4*c*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(cx^2\right)^{3/2}(a+bx)^2}\,dx &= \frac{x\int \frac{1}{x^3(a+bx)^2}\,dx}{c\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{1}{a^2x^3}-\frac{2b}{a^3x^2}+\frac{3b^2}{a^4x}-\frac{b^3}{a^3(a+bx)^2}-\frac{3b^3}{a^4(a+bx)}\right)dx}{c\sqrt{cx^2}}\\ &= \frac{2b}{a^3c\sqrt{cx^2}}-\frac{1}{2a^2cx\sqrt{cx^2}}+\frac{b^2x}{a^3c\sqrt{cx^2}\,(a+bx)}+\frac{3b^2x\log(x)}{a^4c\sqrt{cx^2}}-\frac{3b^2x\log(a+bx)}{a^4c\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 80, normalized size = 0.68

$$\frac{x\left(a\left(-a^2+3abx+6b^2x^2\right)+6b^2x^2\log(x)(a+bx)-6b^2x^2(a+bx)\log(a+bx)\right)}{2a^4\left(cx^2\right)^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] (x*(a*(-a^2 + 3*a*b*x + 6*b^2*x^2) + 6*b^2*x^2*(a + b*x)*Log[x] - 6*b^2*x^2
*(a + b*x)*Log[a + b*x]))/(2*a^4*(c*x^2)^(3/2)*(a + b*x))
```

IntegrateAlgebraic [A] time = 0.07, size = 77, normalized size = 0.65

$$\frac{\frac{3b^2x^3\log(x)}{a^4}-\frac{3b^2x^3\log(a+bx)}{a^4}+\frac{-a^2x+3abx^2+6b^2x^3}{2a^3(a+bx)}}{\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((c*x^2)^(3/2)*(a + b*x)^2),x]
```

```
[Out] ((-(a^2*x) + 3*a*b*x^2 + 6*b^2*x^3)/(2*a^3*(a + b*x)) + (3*b^2*x^3*Log[x])/
a^4 - (3*b^2*x^3*Log[a + b*x])/a^4)/(c*x^2)^(3/2)
```

fricas [A] time = 0.98, size = 83, normalized size = 0.70

$$\frac{\left(6\,ab^2x^2+3\,a^2bx-a^3+6\left(b^3x^3+ab^2x^2\right)\log\left(\frac{x}{bx+a}\right)\right)\sqrt{cx^2}}{2\left(a^4bc^2x^4+a^5c^2x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 + 6*(b^3*x^3 + a*b^2*x^2)*log(x/(b*x + a
)))*sqrt(c*x^2)/(a^4*b*c^2*x^4 + a^5*c^2*x^3)
```

giac [A] time = 1.03, size = 152, normalized size = 1.29

$$-\frac{\frac{6\,b^2\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^4\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}+\frac{2\,b^2}{(bx+a)a^3\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}-\frac{\frac{6\,ab^2}{bx+a}-5\,b^2}{a^4\left(\frac{a}{bx+a}-1\right)^2\mathrm{sgn}\left(-\frac{b}{bx+a}+\frac{ab}{(bx+a)^2}\right)}}{2\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(6*b^2*log(abs(-a/(b*x + a) + 1))/(a^4*sgn(-b/(b*x + a) + a*b/(b*x + a
)^2)) + 2*b^2/((b*x + a)*a^3*sgn(-b/(b*x + a) + a*b/(b*x + a)^2)) - (6*a*b^
2/(b*x + a) - 5*b^2)/(a^4*(a/(b*x + a) - 1)^2*sgn(-b/(b*x + a) + a*b/(b*x +
 a)^2)))/c^(3/2)
```

maple [A] time = 0.01, size = 93, normalized size = 0.79

$$\frac{\left(6b^3x^3\ln(x)-6b^3x^3\ln(bx+a)+6a\,b^2x^2\ln(x)-6a\,b^2x^2\ln(bx+a)+6a\,b^2x^2+3a^2bx-a^3\right)x}{2\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 1/2*x*(6*b^3*x^3*ln(x)-6*b^3*x^3*ln(b*x+a)+6*a*b^2*x^2*ln(x)-6*a*b^2*x^2*ln
(b*x+a)+6*a*b^2*x^2+3*a^2*b*x-a^3)/(c*x^2)^(3/2)/a^4/(b*x+a)
```

maxima [A] time = 1.46, size = 98, normalized size = 0.83

$$-\frac{b}{\sqrt{cx^2}\,a^2bcx+\sqrt{cx^2}\,a^3c}-\frac{3\,(-1)^{\frac{2\,acx}{b}}\,b^2\log\left(-\frac{2\,acx}{b|bx+a|}\right)}{a^4c^{\frac{3}{2}}}+\frac{3\,b}{\sqrt{cx^2}\,a^3c}-\frac{1}{2\,a^2c^{\frac{3}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -b/(sqrt(c*x^2)*a^2*b*c*x + sqrt(c*x^2)*a^3*c) - 3*(-1)^(2*a*c*x/b)*b^2*log
(-2*a*c*x/(b*abs(b*x + a)))/(a^4*c^(3/2)) + 3*b/(sqrt(c*x^2)*a^3*c) - 1/2/(
a^2*c^(3/2)*x^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(c\,x^2\right)^{3/2}\,\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x^2)^(3/2)*(a + b*x)^2),x)
```

```
[Out] int(1/((c*x^2)^(3/2)*(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(cx^{2}\right)^{\frac{3}{2}}\left(a+bx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Integral(1/((c*x**2)**(3/2)*(a + b*x)**2), x)
```

3.881 $\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Rubi [A] time = 0.04, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a^3*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(
b^4*(3 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\,\int x^3(a+bx)^n\,dx}{x}\\
&= \frac{\sqrt{cx^2}\,\int\left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}\\
&= -\frac{a^3\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3a\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{\sqrt{cx^2}\,(a+bx)^{4+n}}{b^4(4+n)x}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 97, normalized size = 0.74

$$\frac{cx(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int x^2\sqrt{cx^2}\,(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x^2*Sqrt[c*x^2]*(a + b*x)^n, x]
```

fricas [A] time = 1.24, size = 153, normalized size = 1.17

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^
4)*x)
```

giac [B] time = 1.11, size = 300, normalized size = 2.29

$$\left(\frac{6\,a^4a^n\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^nb^4n^3x^4\mathrm{sgn}(x)+(bx+a)^nab^3n^3x^3\mathrm{sgn}(x)+6\,(bx+a)^nb^4n^2x^4\mathrm{sgn}(x)+3\,(bx+a)^nab^3n^2x^3\mathrm{sgn}(x)+11\,(bx+a)^nb^4nx^4\mathrm{sgn}(x)-3\,(bx+a)^na^2b^2n^2x^2\mathrm{sgn}(x)+2\,(bx+a)^nab^3nx^3\mathrm{sgn}(x)+6\,(bx+a)^nb^4x^4\mathrm{sgn}(x)-3\,(bx+a)^na^2b^2nx^2\mathrm{sgn}(x)+6\,(bx+a)^na^3bnx\mathrm{sgn}(x)-6\,(bx+a)^na^4\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*sqrt(c)
```

maple [A] time = 0.01, size = 136, normalized size = 1.04

$$-\frac{\sqrt{c\,x^2}\,\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] -(c*x^2)^(1/2)*(b*x+a)^(n+1)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 1.46, size = 116, normalized size = 0.89

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4\sqrt{c}\,x^4+\left(n^3+3\,n^2+2\,n\right)ab^3\sqrt{c}\,x^3-3\left(n^2+n\right)a^2b^2\sqrt{c}\,x^2+6\,a^3b\sqrt{c}\,nx-6\,a^4\sqrt{c}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*sqrt(c)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*sqrt(
c)*x^3 - 3*(n^2 + n)*a^2*b^2*sqrt(c)*x^2 + 6*a^3*b*sqrt(c)*n*x - 6*a^4*sqrt
(c))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

mupad [B] time = 0.35, size = 214, normalized size = 1.63

$$\frac{(a+bx)^n\left(\frac{x^4\sqrt{cx^2}\,(n^3+6n^2+11n+6)}{n^4+10n^3+35n^2+50n+24}-\frac{6a^4\sqrt{cx^2}}{b^4\,(n^4+10n^3+35n^2+50n+24)}+\frac{6a^3nx\sqrt{cx^2}}{b^3\,(n^4+10n^3+35n^2+50n+24)}+\frac{anx^3\sqrt{cx^2}\,(n^2+3n+2)}{b\,(n^4+10n^3+35n^2+50n+24)}-\frac{3a^2nx^2\sqrt{cx^2}\,(n+1)}{b^2\,(n^4+10n^3+35n^2+50n+24)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) - (6*a^4*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^3 +
n^4 + 24)) + (6*a^3*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 +
 24)) + (a*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3
+ n^4 + 24)) - (3*a^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 10
*n^3 + n^4 + 24))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n\sqrt{c}\,x^3\sqrt{x^2}}{4} & \text{for } b=0 \\ \int\frac{x^2\sqrt{cx^2}}{(a+bx)^4}\,dx & \text{for } n=-4 \\ \int\frac{x^2\sqrt{cx^2}}{(a+bx)^3}\,dx & \text{for } n=-3 \\ \int\frac{x^2\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for } n=-2 \\ \int\frac{x^2\sqrt{cx^2}}{a+bx}\,dx & \text{for } n=-1 \\ -\frac{6a^4\sqrt{c}(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{6a^3b\sqrt{c}\,nx(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}-\frac{3a^2b^2\sqrt{c}\,n^2x^2(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}-\frac{3a^2b^2\sqrt{c}\,nx^2(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{ab^3\sqrt{c}\,n^3x^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{3ab^3\sqrt{c}\,n^2x^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{2ab^3\sqrt{c}\,nx^3(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{b^4\sqrt{c}\,n^3x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{6b^4\sqrt{c}\,n^2x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{11b^4\sqrt{c}\,nx^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x}+\frac{6b^4\sqrt{c}\,x^4(a+bx)^n\sqrt{x^2}}{b^4n^4x+10b^4n^3x+35b^4n^2x+50b^4nx+24b^4x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x**3*sqrt(x**2)/4, Eq(b, 0)), (Integral(x**2*sqrt(c
*x**2)/(a + b*x)**4, x), Eq(n, -4)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)*
*3, x), Eq(n, -3)), (Integral(x**2*sqrt(c*x**2)/(a + b*x)**2, x), Eq(n, -2)
), (Integral(x**2*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (-6*a**4*sqrt(c)*
(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50
*b**4*n*x + 24*b**4*x) + 6*a**3*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**4
*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a*
*2*b**2*sqrt(c)*n**2*x**2*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n*
*3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) - 3*a**2*b**2*sqrt(c)*n*x*
*2*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x +
 50*b**4*n*x + 24*b**4*x) + a*b**3*sqrt(c)*n**3*x**3*(a + b*x)**n*sqrt(x**2
)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x)
 + 3*a*b**3*sqrt(c)*n**2*x**3*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**
4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 2*a*b**3*sqrt(c)*n*x
**3*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x
+ 50*b**4*n*x + 24*b**4*x) + b**4*sqrt(c)*n**3*x**4*(a + b*x)**n*sqrt(x**2)
/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x)
+ 6*b**4*sqrt(c)*n**2*x**4*(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n
**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x) + 11*b**4*sqrt(c)*n*x**4*
(a + b*x)**n*sqrt(x**2)/(b**4*n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50
*b**4*n*x + 24*b**4*x) + 6*b**4*sqrt(c)*x**4*(a + b*x)**n*sqrt(x**2)/(b**4*
n**4*x + 10*b**4*n**3*x + 35*b**4*n**2*x + 50*b**4*n*x + 24*b**4*x), True))
```

3.882 $\int x\sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=96

$$\frac{a^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 43}

$$\frac{a^2\sqrt{cx^2}(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2a\sqrt{cx^2}(a+bx)^{n+2}}{b^3(n+2)x}+\frac{\sqrt{cx^2}(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (a^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*Sqrt[c*x^2]*(a +
 b*x)^(2 + n))/(b^3*(2 + n)*x) + (Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3*(3 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{cx^2}(a+bx)^n\,dx&=\frac{\sqrt{cx^2}\int x^2(a+bx)^n\,dx}{x}\\&=\frac{\sqrt{cx^2}\int\left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\&=\frac{a^2\sqrt{cx^2}(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2a\sqrt{cx^2}(a+bx)^{2+n}}{b^3(2+n)x}+\frac{\sqrt{cx^2}(a+bx)^{3+n}}{b^3(3+n)x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.71

$$\frac{cx(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x\sqrt{cx^2}\,(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*Sqrt[c*x^2]*(a + b*x)^n, x]
```

fricas [A] time = 1.56, size = 106, normalized size = 1.10

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)
*x)
```

giac [B] time = 0.96, size = 200, normalized size = 2.08

$$-\left(\frac{2\,a^3a^n\mathrm{sgn}(x)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}-\frac{(bx+a)^nb^3n^2x^3\mathrm{sgn}(x)+(bx+a)^nab^2n^2x^2\mathrm{sgn}(x)+3\,(bx+a)^nb^3nx^3\mathrm{sgn}(x)+(bx+a)^nab^2nx^2\mathrm{sgn}(x)+2\,(bx+a)^nb^3x^3\mathrm{sgn}(x)-2\,(bx+a)^na^2bnx\mathrm{sgn}(x)+2\,(bx+a)^na^3\mathrm{sgn}(x)}{b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^3*a^n*sgn(x)/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3) - ((b*x + a)^n*
b^3*n^2*x^3*sgn(x) + (b*x + a)^n*a*b^2*n^2*x^2*sgn(x) + 3*(b*x + a)^n*b^3*n
*x^3*sgn(x) + (b*x + a)^n*a*b^2*n*x^2*sgn(x) + 2*(b*x + a)^n*b^3*x^3*sgn(x)
 - 2*(b*x + a)^n*a^2*b*n*x*sgn(x) + 2*(b*x + a)^n*a^3*sgn(x))/(b^3*n^3 + 6*
b^3*n^2 + 11*b^3*n + 6*b^3))*sqrt(c)
```

maple [A] time = 0.01, size = 83, normalized size = 0.86

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)\sqrt{c\,x^2}\;\,(bx+a)^{n+1}}{\left(n^3+6n^2+11n+6\right)b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(1/2)/x/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.41, size = 80, normalized size = 0.83

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 0.25, size = 142, normalized size = 1.48

$$\frac{(a+b\,x)^n\,\left(\frac{2\,a^3\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (x^3*(
c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*n*x*(c*x^2)
^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b
*(11*n + 6*n^2 + n^3 + 6))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n\sqrt{c}\,x^2\sqrt{x^2}}{3} & \text{for}\ b = 0 \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^3}\,dx & \text{for}\ n = -3 \\ \int \frac{x\sqrt{cx^2}}{(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{x\sqrt{cx^2}}{a+bx}\,dx & \text{for}\ n = -1 \\ \frac{2a^3\sqrt{c}\,(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} - \frac{2a^2b\sqrt{c}\,nx(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2\sqrt{c}\,n^2x^2(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{ab^2\sqrt{c}\,nx^2(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{b^3\sqrt{c}\,n^2x^3(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{3b^3\sqrt{c}\,nx^3(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} + \frac{2b^3\sqrt{c}\,x^3(a+bx)^n\sqrt{x^2}}{b^3n^3x+6b^3n^2x+11b^3nx+6b^3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x**2*sqrt(x**2)/3, Eq(b, 0)), (Integral(x*sqrt(c*x*
*2)/(a + b*x)**3, x), Eq(n, -3)), (Integral(x*sqrt(c*x**2)/(a + b*x)**2, x)
, Eq(n, -2)), (Integral(x*sqrt(c*x**2)/(a + b*x), x), Eq(n, -1)), (2*a**3*s
qrt(c)*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x +
 6*b**3*x) - 2*a**2*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*
b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b**2*sqrt(c)*n**2*x**2*(a + b*x)*
*n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + a*b*
*2*sqrt(c)*n*x**2*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11
*b**3*n*x + 6*b**3*x) + b**3*sqrt(c)*n**2*x**3*(a + b*x)**n*sqrt(x**2)/(b**
3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3*x) + 3*b**3*sqrt(c)*n*x**3*
(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**2*x + 11*b**3*n*x + 6*b**3
*x) + 2*b**3*sqrt(c)*x**3*(a + b*x)**n*sqrt(x**2)/(b**3*n**3*x + 6*b**3*n**
2*x + 11*b**3*n*x + 6*b**3*x), True))
```

3.883 $\int \sqrt{cx^2}(a+bx)^n\,dx$

Optimal. Leaf size=63

$$\frac{\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$\frac{\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x}-\frac{a\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] -((a*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (Sqrt[c*x^2]*(a + b*
x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{cx^2}(a+bx)^n\,dx &= \frac{\sqrt{cx^2}\int x(a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\ &= -\frac{a\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x}+\frac{\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.70

$$\frac{cx(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x^2]*(a + b*x)^n,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2]
)
```

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \sqrt{cx^2}\,(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[Sqrt[c*x^2]*(a + b*x)^n,x]
[Out] Defer[IntegrateAlgebraic][Sqrt[c*x^2]*(a + b*x)^n, x]
```

fricas [A] time = 0.79, size = 63, normalized size = 1.00

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2+3\,b^2n+2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n^2 + 3*b
^2*n + 2*b^2)*x)
```

giac [B] time = 1.14, size = 119, normalized size = 1.89

$$\left(\frac{a^2a^n\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}+\frac{(bx+a)^nb^2nx^2\mathrm{sgn}(x)+(bx+a)^nabnx\mathrm{sgn}(x)+(bx+a)^nb^2x^2\mathrm{sgn}(x)-(bx+a)^na^2\mathrm{sgn}(x)}{b^2n^2+3\,b^2n+2\,b^2}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="giac")
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*sqrt(c)
```

maple [A] time = 0.00, size = 46, normalized size = 0.73

$$-\frac{\sqrt{c\,x^2}\;(-xnb-bx+a)\,(bx+a)^{n+1}}{\left(n^2+3n+2\right)b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2),x)
[Out] -(c*x^2)^(1/2)*(b*x+a)^(n+1)*(-b*n*x-b*x+a)/x/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.45, size = 51, normalized size = 0.81

$$\frac{\left(b^2\sqrt{c}\,(n+1)x^2+ab\sqrt{c}\,nx-a^2\sqrt{c}\,\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2),x, algorithm="maxima")
[Out] (b^2*sqrt(c)*(n + 1)*x^2 + a*b*sqrt(c)*n*x - a^2*sqrt(c))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

mupad [B] time = 0.22, size = 85, normalized size = 1.35

$$\frac{(a+b\,x)^n\left(\frac{x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,\sqrt{c\,x^2}}{b^2\left(n^2+3\,n+2\right)}+\frac{a\,n\,x\,\sqrt{c\,x^2}}{b\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(1/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*(c*x^2)^(1
/2))/(b^2*(3*n + n^2 + 2)) + (a*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n \sqrt{c}\, x \sqrt{x^2}}{2} & \text{for } b = 0 \\ \int \frac{\sqrt{cx^2}}{(a+bx)^2}\, dx & \text{for } n = -2 \\ \int \frac{\sqrt{cx^2}}{a+bx}\, dx & \text{for } n = -1 \\ -\frac{a^2 \sqrt{c}\,(a+bx)^n \sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x} + \frac{ab\sqrt{c}\, nx(a+bx)^n \sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x} + \frac{b^2\sqrt{c}\, nx^2(a+bx)^n \sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x} + \frac{b^2\sqrt{c}\, x^2(a+bx)^n \sqrt{x^2}}{b^2n^2x+3b^2nx+2b^2x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*sqrt(c)*x*sqrt(x**2)/2, Eq(b, 0)), (Integral(sqrt(c*x**2)/(
a + b*x)**2, x), Eq(n, -2)), (Integral(sqrt(c*x**2)/(a + b*x), x), Eq(n, -1
)), (-a**2*sqrt(c)*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b**2*n*x + 2*b*
*2*x) + a*b*sqrt(c)*n*x*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b**2*n*x +
 2*b**2*x) + b**2*sqrt(c)*n*x**2*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*x + 3*b
**2*n*x + 2*b**2*x) + b**2*sqrt(c)*x**2*(a + b*x)**n*sqrt(x**2)/(b**2*n**2*
x + 3*b**2*n*x + 2*b**2*x), True))
```

3.884 $\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx$

Optimal. Leaf size=30

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{\sqrt{cx^2}(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx &= \frac{\sqrt{cx^2}\int (a+bx)^n\,dx}{x}\\ &= \frac{\sqrt{cx^2}(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.97

$$\frac{cx(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] (c*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx^2}(a+bx)^n}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[c*x^2]*(a + b*x)^n)/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][(Sqrt[c*x^2]*(a + b*x)^n)/x, x]
```

fricas [A] time = 0.99, size = 30, normalized size = 1.00

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bn+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*n + b)*x)
```

giac [A] time = 1.01, size = 42, normalized size = 1.40

$$-\sqrt{c}\left(\frac{a^{n+1}\mathrm{sgn}(x)}{bn+b}-\frac{(bx+a)^{n+1}\mathrm{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -sqrt(c)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

maple [A] time = 0.00, size = 29, normalized size = 0.97

$$\frac{\sqrt{c\,x^2}\;(bx+a)^{n+1}}{(n+1)\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(c*x^2)^(1/2)/x,x)
```

```
[Out] (b*x+a)^(n+1)*(c*x^2)^(1/2)/b/(n+1)/x
```

maxima [A] time = 1.40, size = 28, normalized size = 0.93

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(c*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*(n + 1))
```

mupad [B] time = 0.23, size = 31, normalized size = 1.03

$$\frac{\sqrt{c\,x^2}\;(a+b\,x)^n\;(a+b\,x)}{b\,x\;(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(1/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((c*x^2)^(1/2)*(a + b*x)^n*(a + b*x))/(b*x*(n + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\sqrt{c}\sqrt{x^2}}{a} & \text{for } b = 0 \wedge n = -1 \\ a^n\sqrt{c}\sqrt{x^2} & \text{for } b = 0 \\ \int \frac{\sqrt{cx^2}}{x(a+bx)}\,dx & \text{for } n = -1 \\ \frac{a\sqrt{c}\,(a+bx)^n\sqrt{x^2}}{bnx+bx} + \frac{b\sqrt{c}\,x(a+bx)^n\sqrt{x^2}}{bnx+bx} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(c*x**2)**(1/2)/x,x)
```

```
[Out] Piecewise((sqrt(c)*sqrt(x**2)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sqrt(c)*sqrt(
x**2), Eq(b, 0)), (Integral(sqrt(c*x**2)/(x*(a + b*x)), x), Eq(n, -1)), (a*
sqrt(c)*(a + b*x)**n*sqrt(x**2)/(b*n*x + b*x) + b*sqrt(c)*x*(a + b*x)**n*sq
rt(x**2)/(b*n*x + b*x), True))
```

3.885 $\int x \left(cx^2\right)^{3/2} (a+bx)^n \, dx$

Optimal. Leaf size=169

$$\frac{a^4 c \sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x} - \frac{4a^3 c \sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x} + \frac{6a^2 c \sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x} - \frac{4ac \sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x} + \frac{c \sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Rubi [A] time = 0.06, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {15, 43}

$$\frac{a^4 c \sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x} - \frac{4a^3 c \sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x} + \frac{6a^2 c \sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x} - \frac{4ac \sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x} + \frac{c \sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] (a^4*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c*Sqrt[c*x^2
]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c*Sqrt[c*x^2]*(a + b*x)^(3 +
n))/(b^5*(3 + n)*x) - (4*a*c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4 + n)*x)
 + (c*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x \left(cx^2\right)^{3/2} (a+bx)^n \, dx &= \frac{\left(c\sqrt{cx^2}\right) \int x^4 (a+bx)^n \, dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right) \int \left(\frac{a^4(a+bx)^n}{b^4} - \frac{4a^3(a+bx)^{1+n}}{b^4} + \frac{6a^2(a+bx)^{2+n}}{b^4} - \frac{4a(a+bx)^{3+n}}{b^4} + \frac{(a+bx)^{4+n}}{b^4}\right) dx}{x} \\
&= \frac{a^4 c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x} - \frac{4a^3 c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x} + \frac{6a^2 c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x} - \frac{4ac\sqrt{cx^2}\,(a+
\end{aligned}$$

Mathematica [A] time = 0.08, size = 132, normalized size = 0.78

$$\frac{\left(cx^2\right)^{3/2} (a+bx)^{n+1} \left(24a^4 - 24a^3 b(n+1)x + 12a^2 b^2 \left(n^2+3n+2\right) x^2 - 4ab^3 \left(n^3+6n^2+11n+6\right) x^3 + b^4 \left(n^4+10n^3+35n^2+50n+24\right) x^4\right)}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^2*
(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50*n
```

```
+ 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n
)*x^3)
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int x\left(cx^2\right)^{3/2}(a+bx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(c*x^2)^(3/2)*(a + b*x)^n, x]
```

fricas [A] time = 1.34, size = 233, normalized size = 1.38

$$-\frac{\left(24\,a^4bcnx-24\,a^5c-\left(b^5cn^4+10\,b^5cn^3+35\,b^5cn^2+50\,b^5cn+24\,b^5c\right)x^5-\left(ab^4cn^4+6\,ab^4cn^3+11\,ab^4cn^2+6\,ab^4cn\right)x^4+4\left(a^2b^3cn^3+3\,a^2b^3cn^2+2\,a^2b^3cn\right)x^3-12\left(a^3b^2cn^2+a^3b^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c*n*x - 24*a^5*c - (b^5*c*n^4 + 10*b^5*c*n^3 + 35*b^5*c*n^2 + 50
*b^5*c*n + 24*b^5*c)*x^5 - (a*b^4*c*n^4 + 6*a*b^4*c*n^3 + 11*a*b^4*c*n^2 +
6*a*b^4*c*n)*x^4 + 4*(a^2*b^3*c*n^3 + 3*a^2*b^3*c*n^2 + 2*a^2*b^3*c*n)*x^3
- 12*(a^3*b^2*c*n^2 + a^3*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^5*n^5 +
 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)*x)
```

giac [B] time = 1.20, size = 426, normalized size = 2.52

$$-\left(\frac{24\,a^5a^n\mathrm{sgn}(x)}{b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5}-\frac{(bx+a)^nb^5n^4x^5\mathrm{sgn}(x)+(bx+a)^nab^4n^4x^4\mathrm{sgn}(x)+10\,(bx+a)^nb^5n^3x^5\mathrm{sgn}(x)+6\,(bx+a)^nab^4n^3x^4\mathrm{sgn}(x)+35\,(bx+a)^nb^5n^2x^5\mathrm{sgn}(x)-4\,(bx+a)^na^2b^3n^3x^3\mathrm{sgn}(x)+11\,(bx+a)^nab^4n^2x^4\mathrm{sgn}(x)+50\,(bx+a)^nb^5nx^5\mathrm{sgn}(x)-12\,(bx+a)^na^2b^3n^2x^3\mathrm{sgn}(x)+6\,(bx+a)^nab^4nx^4\mathrm{sgn}(x)+24\,(bx+a)^nb^5x^5\mathrm{sgn}(x)+12\,(bx+a)^na^3b^2n^2x^2\mathrm{sgn}(x)-8\,(bx+a)^na^2b^3nx^3\mathrm{sgn}(x)+12\,(bx+a)^na^3b^2nx^2\mathrm{sgn}(x)-24\,(bx+a)^na^4bnx\mathrm{sgn}(x)+24\,(bx+a)^na^5\mathrm{sgn}(x)}{b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] -(24*a^5*a^n*sgn(x)/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*
b^5*n + 120*b^5) - ((b*x + a)^n*b^5*n^4*x^5*sgn(x) + (b*x + a)^n*a*b^4*n^4*
x^4*sgn(x) + 10*(b*x + a)^n*b^5*n^3*x^5*sgn(x) + 6*(b*x + a)^n*a*b^4*n^3*x^
4*sgn(x) + 35*(b*x + a)^n*b^5*n^2*x^5*sgn(x) - 4*(b*x + a)^n*a^2*b^3*n^3*x^
3*sgn(x) + 11*(b*x + a)^n*a*b^4*n^2*x^4*sgn(x) + 50*(b*x + a)^n*b^5*n*x^5*s
gn(x) - 12*(b*x + a)^n*a^2*b^3*n^2*x^3*sgn(x) + 6*(b*x + a)^n*a*b^4*n*x^4*s
gn(x) + 24*(b*x + a)^n*b^5*x^5*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n^2*x^2*sgn(
x) - 8*(b*x + a)^n*a^2*b^3*n*x^3*sgn(x) + 12*(b*x + a)^n*a^3*b^2*n*x^2*sgn(
x) - 24*(b*x + a)^n*a^4*b*n*x*sgn(x) + 24*(b*x + a)^n*a^5*sgn(x))/(b^5*n^5
+ 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5))*c^(3/2)
```

maple [A] time = 0.01, size = 199, normalized size = 1.18

$$\frac{\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44a\,b^3n\,x^3+24b^4x^4+36a^2b^2n\,x^2-24a\,b^3x^3-24a^3bnx+24a^2b^2x^2-24a^3bx+24a^4\right)\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)^{n+1}}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(3/2)/x^3/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

maxima [A] time = 1.44, size = 157, normalized size = 0.93

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{3}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{3}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{3}{2}}x^3+12\left(n^2+n\right)a^3b^2c^{\frac{3}{2}}x^2-24\,a^4bc^{\frac{3}{2}}nx+24\,a^5c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(3/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(3/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(3/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(3/2)*x^2 - 24*a^4*b*c^(3/2)*n*x + 24*a^5*c^(3/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 0.41, size = 307, normalized size = 1.82

$$\frac{(a+b\,x)^n\,\left(\frac{24\,a^5\,c\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{c\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}-\frac{24\,a^4\,c\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((24*a^5*c*(c*x^2)^(1/2))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5 + 120)) + (c*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24
))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) - (24*a^4*c*n*x*(c*x^2)^
(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c*n*x^4*(
c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^
4 + n^5 + 120)) + (12*a^3*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(274*n + 225*
n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c*n*x^3*(c*x^2)^(1/2)*(3*n + n
^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(cx^2\right)^{\frac{3}{2}} (a+bx)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral(x*(c*x**2)**(3/2)*(a + b*x)**n, x)
```

3.886 $\int \left(cx^2\right)^{3/2} (a+bx)^n\,dx$

Optimal. Leaf size=135

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {15, 43}

$$-\frac{a^3c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] -((a^3*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c*Sqrt[c*
x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c*Sqrt[c*x^2]*(a + b*x)^(3 +
 n))/(b^4*(3 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{3/2} (a+bx)^n\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}\\ &= -\frac{a^3c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c\sqrt{cx^2}\,(a+}{b^4(4+}\end{aligned}$$

Mathematica [A] time = 0.05, size = 98, normalized size = 0.73

$$\frac{\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 +
 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(
3 + n)*(4 + n)*x^3)
```

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{3/2} (a+bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^(3/2)*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x^2)^(3/2)*(a + b*x)^n, x]
```

fricas [A] time = 1.06, size = 164, normalized size = 1.21

$$\frac{\left(6\,a^3bcnx-6\,a^4c+\left(b^4cn^3+6\,b^4cn^2+11\,b^4cn+6\,b^4c\right)x^4+\left(ab^3cn^3+3\,ab^3cn^2+2\,ab^3cn\right)x^3-3\left(a^2b^2cn^2+a^2b^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c*n*x - 6*a^4*c + (b^4*c*n^3 + 6*b^4*c*n^2 + 11*b^4*c*n + 6*b^4*c)
*x^4 + (a*b^3*c*n^3 + 3*a*b^3*c*n^2 + 2*a*b^3*c*n)*x^3 - 3*(a^2*b^2*c*n^2 +
 a^2*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4 + 10*b^4*n^3 + 35*b^4*
n^2 + 50*b^4*n + 24*b^4)*x)
```

giac [B] time = 1.14, size = 300, normalized size = 2.22

$$\left(\frac{6\,a^4a^n\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}+\frac{(bx+a)^nb^4n^3x^4\mathrm{sgn}(x)+(bx+a)^nab^3n^3x^3\mathrm{sgn}(x)+6\,(bx+a)^nb^4n^2x^4\mathrm{sgn}(x)+3\,(bx+a)^nab^3n^2x^3\mathrm{sgn}(x)+11\,(bx+a)^nb^4nx^4\mathrm{sgn}(x)-3\,(bx+a)^na^2b^2n^2x^2\mathrm{sgn}(x)+2\,(bx+a)^nab^3nx^3\mathrm{sgn}(x)+6\,(bx+a)^nb^4x^4\mathrm{sgn}(x)-3\,(bx+a)^na^2b^2nx^2\mathrm{sgn}(x)+6\,(bx+a)^na^3bnx\mathrm{sgn}(x)-6\,(bx+a)^na^4\mathrm{sgn}(x)}{b^4n^4+10\,b^4n^3+35\,b^4n^2+50\,b^4n+24\,b^4}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (6*a^4*a^n*sgn(x)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4) +
 ((b*x + a)^n*b^4*n^3*x^4*sgn(x) + (b*x + a)^n*a*b^3*n^3*x^3*sgn(x) + 6*(b*
x + a)^n*b^4*n^2*x^4*sgn(x) + 3*(b*x + a)^n*a*b^3*n^2*x^3*sgn(x) + 11*(b*x
+ a)^n*b^4*n*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b^2*n^2*x^2*sgn(x) + 2*(b*x + a
)^n*a*b^3*n*x^3*sgn(x) + 6*(b*x + a)^n*b^4*x^4*sgn(x) - 3*(b*x + a)^n*a^2*b
^2*n*x^2*sgn(x) + 6*(b*x + a)^n*a^3*b*n*x*sgn(x) - 6*(b*x + a)^n*a^4*sgn(x)
)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4))*c^(3/2)
```

maple [A] time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)(bx+a)^{n+1}}{\left(n^4+10n^3+35n^2+50n+24\right)b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(3/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^3/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 1.44, size = 116, normalized size = 0.86

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4c^{\frac{3}{2}}x^4+\left(n^3+3\,n^2+2\,n\right)ab^3c^{\frac{3}{2}}x^3-3\left(n^2+n\right)a^2b^2c^{\frac{3}{2}}x^2+6\,a^3bc^{\frac{3}{2}}nx-6\,a^4c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(3/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(3/
2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(3/2)*x^2 + 6*a^3*b*c^(3/2)*n*x - 6*a^4*c^(3
/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)
```

mupad [B] time = 0.32, size = 219, normalized size = 1.62

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2
 + 10*n^3 + n^4 + 24) - (6*a^4*c*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^
3 + n^4 + 24)) + (6*a^3*c*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (3*a^2*c*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 1
0*n^3 + n^4 + 24)) + (a*c*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 3
5*n^2 + 10*n^3 + n^4 + 24))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^{2}\right)^{\frac{3}{2}} \left(a + bx\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n, x)
```

3.887 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=99

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^2*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c*Sqrt[c*x^2]*
(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c*Sqrt[c*x^2]*(a + b*x)^(3 + n))/(b^3
*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}\\ &= \frac{\left(c\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\ &= \frac{a^2c\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{3/2}(a+bx)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x, x]
```

fricas [A] time = 1.02, size = 113, normalized size = 1.14

$$-\frac{\left(2\,a^2bcnx-2\,a^3c-\left(b^3cn^2+3\,b^3cn+2\,b^3c\right)x^3-\left(ab^2cn^2+ab^2cn\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c*n*x - 2*a^3*c - (b^3*c*n^2 + 3*b^3*c*n + 2*b^3*c)*x^3 - (a*b^2*
c*n^2 + a*b^2*c*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6*b^3*n^2 + 11*
b^3*n + 6*b^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{3}{2}}(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(3/2)*(b*x + a)^n/x, x)
```

maple [A] time = 0.01, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}(bx+a)^{n+1}}{\left(n^3+6n^2+11n+6\right)b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(3/2)/x^3/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.43, size = 80, normalized size = 0.81

$$\frac{\left(\left(n^2+3\,n+2\right)b^3c^{\frac{3}{2}}x^3+\left(n^2+n\right)ab^2c^{\frac{3}{2}}x^2-2\,a^2bc^{\frac{3}{2}}nx+2\,a^3c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(3/2)*x^3 + (n^2 + n)*a*b^2*c^(3/2)*x^2 - 2*a^2*b*c^
(3/2)*n*x + 2*a^3*c^(3/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 0.26, size = 146, normalized size = 1.47

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,c\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,c\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6
) + (2*a^3*c*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*c*n*x*(
c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c*n*x^2*(c*x^2)^(1/2)*(n
+ 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(cx^2\right)^{\frac{3}{2}}\,(a+bx)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(3/2)*(a + b*x)**n/x, x)
```

3.888 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2} dx$

Optimal. Leaf size=65

$$\frac{c\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x} - \frac{ac\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{c\sqrt{cx^2}(a+bx)^{n+2}}{b^2(n+2)x} - \frac{ac\sqrt{cx^2}(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a*c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c*Sqrt[c*x^2]*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^2} dx &= \frac{\left(c\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x} \\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x} \\
&= -\frac{ac\sqrt{cx^2}(a+bx)^{1+n}}{b^2(1+n)x} + \frac{c\sqrt{cx^2}(a+bx)^{2+n}}{b^2(2+n)x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.71

$$\frac{c^2x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c^2*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{3/2}(a+bx)^n}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x^2, x]
```

fricas [A] time = 0.84, size = 68, normalized size = 1.05

$$\frac{\left(abcnx - a^2c + \left(b^2cn + b^2c\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2 + 3\,b^2n + 2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (a*b*c*n*x - a^2*c + (b^2*c*n + b^2*c)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*n
^2 + 3*b^2*n + 2*b^2)*x)
```

giac [A] time = 0.91, size = 119, normalized size = 1.83

$$\left(\frac{a^2a^n\mathrm{sgn}(x)}{b^2n^2 + 3\,b^2n + 2\,b^2} + \frac{(bx+a)^n b^2nx^2\mathrm{sgn}(x) + (bx+a)^n abnx\mathrm{sgn}(x) + (bx+a)^n b^2x^2\mathrm{sgn}(x) - (bx+a)^n a^2\mathrm{sgn}(x)}{b^2n^2 + 3\,b^2n + 2\,b^2}\right)c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] (a^2*a^n*sgn(x)/(b^2*n^2 + 3*b^2*n + 2*b^2) + ((b*x + a)^n*b^2*n*x^2*sgn(x)
 + (b*x + a)^n*a*b*n*x*sgn(x) + (b*x + a)^n*b^2*x^2*sgn(x) - (b*x + a)^n*a^
2*sgn(x))/(b^2*n^2 + 3*b^2*n + 2*b^2))*c^(3/2)
```

maple [A] time = 0.00, size = 46, normalized size = 0.71

$$-\frac{\left(c\,x^2\right)^{\frac{3}{2}}(-xnb - bx + a)\,(bx+a)^{n+1}}{\left(n^2 + 3n + 2\right)b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(3/2)*(-b*n*x-b*x+a)/x^3/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.43, size = 51, normalized size = 0.78

$$\frac{\left(b^2c^{\frac{3}{2}}(n+1)x^2 + abc^{\frac{3}{2}}nx - a^2c^{\frac{3}{2}}\right)(bx+a)^n}{\left(n^2 + 3\,n + 2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

```
[Out] (b^2*c^(3/2)*(n + 1)*x^2 + a*b*c^(3/2)*n*x - a^2*c^(3/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

mupad [B] time = 0.23, size = 88, normalized size = 1.35

$$\frac{(a+b\,x)^n\,\left(\frac{c\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c\,\sqrt{c\,x^2}}{b^2\,\left(n^2+3\,n+2\right)}+\frac{a\,c\,n\,x\,\sqrt{c\,x^2}}{b\,\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^2,x)
```

```
[Out] ((a + b*x)^n*((c*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c*(c*x^2
)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c*n*x*(c*x^2)^(1/2))/(b*(3*n + n^2 + 2)
)))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}{2x} & \text{for}\ b=0 \\ \int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{\left(cx^2\right)^{\frac{3}{2}}}{x^2(a+bx)}\,dx & \text{for}\ n=-1 \\ -\frac{a^2c^{\frac{3}{2}}(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{abc^{\frac{3}{2}}nx(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{b^2c^{\frac{3}{2}}nx^2(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{b^2n^2x^3+3b^2nx^3+2b^2x^3}+\frac{b^2c^{\frac{3}{2}}x^2(a+bx)^n\left(x^2\right)^{\frac{3}{2}}}{b^2n^2x^3+3b^2nx^3+2b^2x^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**2,x)
```

```
[Out] Piecewise((a**n*c**(3/2)*(x**2)**(3/2)/(2*x), Eq(b, 0)), (Integral((c*x**2)
**(3/2)/(x**2*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(3/2)/(x**
2*(a + b*x)), x), Eq(n, -1)), (-a**2*c**(3/2)*(a + b*x)**n*(x**2)**(3/2)/(b
**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + a*b*c**(3/2)*n*x*(a + b*x)**
n*(x**2)**(3/2)/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b**2*x**3) + b**2*c**(3
/2)*n*x**2*(a + b*x)**n*(x**2)**(3/2)/(b**2*n**2*x**3 + 3*b**2*n*x**3 + 2*b
**2*x**3) + b**2*c**(3/2)*x**2*(a + b*x)**n*(x**2)**(3/2)/(b**2*n**2*x**3 +
 3*b**2*n*x**3 + 2*b**2*x**3), True))
```

3.889 $\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3} dx$

Optimal. Leaf size=31

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3}\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{(cx^2)^{3/2}(a+bx)^{n+1}}{b(n+1)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] ((c*x^2)^(3/2)*(a + b*x)^(1 + n))/(b*(1 + n)*x^3)
```

IntegrateAlgebraic [F] time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{3/2}(a+bx)^n}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(3/2)*(a + b*x)^n)/x^3, x]
```

fricas [A] time = 1.01, size = 33, normalized size = 1.06

$$\frac{(bcx + ac)\sqrt{cx^2}\,(bx + a)^n}{(bn + b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] (b*c*x + a*c)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

giac [A] time = 1.09, size = 42, normalized size = 1.35

$$-c^{\frac{3}{2}}\left(\frac{a^{n+1}\mathrm{sgn}(x)}{bn + b} - \frac{(bx + a)^{n+1}\mathrm{sgn}(x)}{b(n+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] -c^(3/2)*(a^(n + 1)*sgn(x)/(b*n + b) - (b*x + a)^(n + 1)*sgn(x)/(b*(n + 1))
)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{\left(c\,x^2\right)^{\frac{3}{2}}(bx + a)^{n+1}}{(n+1)\,b\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(3/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(n+1)/b/(n+1)*(c*x^2)^(3/2)/x^3
```

maxima [A] time = 1.42, size = 28, normalized size = 0.90

$$\frac{\left(bc^{\frac{3}{2}}x + ac^{\frac{3}{2}}\right)(bx + a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(3/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] (b*c^(3/2)*x + a*c^(3/2))*(b*x + a)^n/(b*(n + 1))
```

mupad [B] time = 0.23, size = 45, normalized size = 1.45

$$\frac{\left(\frac{c\,x\,\sqrt{c\,x^2}}{n+1} + \frac{a\,c\,\sqrt{c\,x^2}}{b\,(n+1)}\right)(a + b\,x)^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(3/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] (((c*x*(c*x^2)^(1/2))/(n + 1) + (a*c*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b*x)^
n)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}}{ax^2} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}}{x^2} & \text{for } b = 0 \\ \int \frac{(cx^2)^{\frac{3}{2}}}{x^3(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ac^{\frac{3}{2}}(a+bx)^n(x^2)^{\frac{3}{2}}}{bnx^3+bx^3} + \frac{bc^{\frac{3}{2}}x(a+bx)^n(x^2)^{\frac{3}{2}}}{bnx^3+bx^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(3/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Piecewise((c**(3/2)*(x**2)**(3/2)/(a*x**2), Eq(b, 0) & Eq(n, -1)), (a**n*c*
*(3/2)*(x**2)**(3/2)/x**2, Eq(b, 0)), (Integral((c*x**2)**(3/2)/(x**3*(a +
b*x)), x), Eq(n, -1)), (a*c**(3/2)*(a + b*x)**n*(x**2)**(3/2)/(b*n*x**3 + b
*x**3) + b*c**(3/2)*x*(a + b*x)**n*(x**2)**(3/2)/(b*n*x**3 + b*x**3), True)
)
```

3.890 $\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$

Optimal. Leaf size=217

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}}{b^6(n}$$

Rubi [A] time = 0.07, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {15, 43}

$$-\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^6(n+1)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^6(n+2)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^6(n+3)x}+\frac{10a^2c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^6(n+4)x}-\frac{5ac^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^6(n+5)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+6}}{b^6(n+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] -((a^5*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^6*(1 + n)*x)) + (5*a^4*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^6*(2 + n)*x) - (10*a^3*c^2*Sqrt[c*x^2]*(a +
b*x)^(3 + n))/(b^6*(3 + n)*x) + (10*a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/
(b^6*(4 + n)*x) - (5*a*c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^6*(5 + n)*x) +
 (c^2*Sqrt[c*x^2]*(a + b*x)^(6 + n))/(b^6*(6 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(a+bx)^n \, dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^5(a+bx)^n}{b^5}+\frac{5a^4(a+bx)^{1+n}}{b^5}-\frac{10a^3(a+bx)^{2+n}}{b^5}+\frac{10a^2(a+bx)^{3+n}}{b^5}-\frac{5a(a+bx)^{4+n}}{b^5}+\frac{(a+}{}\right.}{x}\\ &= -\frac{a^5c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^6(1+n)x}+\frac{5a^4c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^6(2+n)x}-\frac{10a^3c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^6(3+n)x}+\frac{10a^2c}{}\end{aligned}$$

Mathematica [A] time = 0.11, size = 172, normalized size = 0.79

$$\frac{c^3x(a+bx)^{n+1}\left(-120a^5+120a^4b(n+1)x-60a^3b^2\left(n^2+3n+2\right)x^2+20a^2b^3\left(n^3+6n^2+11n+6\right)x^3-5ab^4\left(n^4+10n^3+35n^2+50n+24\right)x^4+b^5\left(n^5+15n^4+85n^3+225n^2+274n+120\right)x^5\right)}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-120*a^5 + 120*a^4*b*(1 + n)*x - 60*a^3*b^2*(2 +
3*n + n^2)*x^2 + 20*a^2*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 - 5*a*b^4*(24 + 50
```

```
*n + 35*n^2 + 10*n^3 + n^4)*x^4 + b^5*(120 + 274*n + 225*n^2 + 85*n^3 + 15*
n^4 + n^5)*x^5))/(b^6*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*Sqrt[
c*x^2])
```

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{5/2} (a+bx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^(5/2)*(a + b*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c*x^2)^(5/2)*(a + b*x)^n, x]
```

fricas [A] time = 0.89, size = 352, normalized size = 1.62

$$\frac{\left(120 \, a^{5} b c^{2} n x - 120 \, a^{6} c^{2} + \left(b^{6} c^{2} n^{5} + 15 \, b^{6} c^{2} n^{4} + 85 \, b^{6} c^{2} n^{3} + 225 \, b^{6} c^{2} n^{2} + 274 \, b^{6} c^{2} n + 120 \, b^{6} c^{2}\right)x^{6} + \left(a b^{5} c^{2} n^{5} + 10 \, a b^{5} c^{2} n^{4} + 35 \, a b^{5} c^{2} n^{3} + 50 \, a b^{5} c^{2} n^{2} + 24 \, a b^{5} c^{2} n\right)x^{5} - 5 \, \left(a^{2} b^{4} c^{2} n^{4} + 6 \, a^{2} b^{4} c^{2} n^{3} + 11 \, a^{2} b^{4} c^{2} n^{2} + 6 \, a^{2} b^{4} c^{2} n\right)x^{4} + 20 \, \left(a^{3} b^{3} c^{2} n^{3} + 3 \, a^{3} b^{3} c^{2} n^{2} + 2 \, a^{3} b^{3} c^{2} n\right)x^{3} - 60 \, \left(a^{4} b^{2} c^{2} n^{2} + a^{4} b^{2} c^{2} n\right)x^{2}\right)\sqrt{c x^{2}} (b x + a)^{n}}{\left(b^{6} n^{6} + 21 \, b^{6} n^{5} + 175 \, b^{6} n^{4} + 735 \, b^{6} n^{3} + 1624 \, b^{6} n^{2} + 1764 \, b^{6} n + 720 \, b^{6}\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] (120*a^5*b*c^2*n*x - 120*a^6*c^2 + (b^6*c^2*n^5 + 15*b^6*c^2*n^4 + 85*b^6*c
^2*n^3 + 225*b^6*c^2*n^2 + 274*b^6*c^2*n + 120*b^6*c^2)*x^6 + (a*b^5*c^2*n^
5 + 10*a*b^5*c^2*n^4 + 35*a*b^5*c^2*n^3 + 50*a*b^5*c^2*n^2 + 24*a*b^5*c^2*n
)*x^5 - 5*(a^2*b^4*c^2*n^4 + 6*a^2*b^4*c^2*n^3 + 11*a^2*b^4*c^2*n^2 + 6*a^2
*b^4*c^2*n)*x^4 + 20*(a^3*b^3*c^2*n^3 + 3*a^3*b^3*c^2*n^2 + 2*a^3*b^3*c^2*n
)*x^3 - 60*(a^4*b^2*c^2*n^2 + a^4*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/(
(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6
*n + 720*b^6)*x)
```

giac [B] time = 1.06, size = 640, normalized size = 2.95

$$\left(\frac{120 \, a^{6} a^{n} c^{2} \mathrm{sgn}\left(x\right)}{b^{6} n^{6} + 21 \, b^{6} n^{5} + 175 \, b^{6} n^{4} + 735 \, b^{6} n^{3} + 1624 \, b^{6} n^{2} + 1764 \, b^{6} n + 720 \, b^{6}} + \frac{(b x + a)^{n} b^{6} c^{2} n^{5} x^{6} \mathrm{sgn}\left(x\right) + (b x + a)^{n} a b^{5} c^{2} n^{5} x^{5} \mathrm{sgn}\left(x\right) + 15 \, (b x + a)^{n} b^{6} c^{2} n^{4} x^{6} \mathrm{sgn}\left(x\right) + 10 \, (b x + a)^{n} a b^{5} c^{2} n^{4} x^{5} \mathrm{sgn}\left(x\right) + 85 \, (b x + a)^{n} b^{6} c^{2} n^{3} x^{6} \mathrm{sgn}\left(x\right) - 5 \, (b x + a)^{n} a^{2} b^{4} c^{2} n^{4} x^{4} \mathrm{sgn}\left(x\right) + 35 \, (b x + a)^{n} a b^{5} c^{2} n^{3} x^{5} \mathrm{sgn}\left(x\right) + 225 \, (b x + a)^{n} b^{6} c^{2} n^{2} x^{6} \mathrm{sgn}\left(x\right) - 30 \, (b x + a)^{n} a^{2} b^{4} c^{2} n^{3} x^{4} \mathrm{sgn}\left(x\right) + 50 \, (b x + a)^{n} a b^{5} c^{2} n^{2} x^{5} \mathrm{sgn}\left(x\right) + 274 \, (b x + a)^{n} b^{6} c^{2} n x^{6} \mathrm{sgn}\left(x\right) + 20 \, (b x + a)^{n} a^{3} b^{3} c^{2} n^{3} x^{3} \mathrm{sgn}\left(x\right) - 55 \, (b x + a)^{n} a^{2} b^{4} c^{2} n^{2} x^{4} \mathrm{sgn}\left(x\right) + 24 \, (b x + a)^{n} a b^{5} c^{2} n x^{5} \mathrm{sgn}\left(x\right) + 120 \, (b x + a)^{n} b^{6} c^{2} x^{6} \mathrm{sgn}\left(x\right) + 60 \, (b x + a)^{n} a^{3} b^{3} c^{2} n^{2} x^{3} \mathrm{sgn}\left(x\right) - 30 \, (b x + a)^{n} a^{2} b^{4} c^{2} n x^{4} \mathrm{sgn}\left(x\right) - 60 \, (b x + a)^{n} a^{4} b^{2} c^{2} n^{2} x^{2} \mathrm{sgn}\left(x\right) + 40 \, (b x + a)^{n} a^{3} b^{3} c^{2} n x^{3} \mathrm{sgn}\left(x\right) - 60 \, (b x + a)^{n} a^{4} b^{2} c^{2} n x^{2} \mathrm{sgn}\left(x\right) + 120 \, (b x + a)^{n} a^{5} b c^{2} n x \mathrm{sgn}\left(x\right) - 120 \, (b x + a)^{n} a^{6} c^{2} \mathrm{sgn}\left(x\right)}{b^{6} n^{6} + 21 \, b^{6} n^{5} + 175 \, b^{6} n^{4} + 735 \, b^{6} n^{3} + 1624 \, b^{6} n^{2} + 1764 \, b^{6} n + 720 \, b^{6}}\right)\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="giac")
```

```
[Out] (120*a^6*a^n*c^2*sgn(x)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 +
 1624*b^6*n^2 + 1764*b^6*n + 720*b^6) + ((b*x + a)^n*b^6*c^2*n^5*x^6*sgn(x)
 + (b*x + a)^n*a*b^5*c^2*n^5*x^5*sgn(x) + 15*(b*x + a)^n*b^6*c^2*n^4*x^6*sg
n(x) + 10*(b*x + a)^n*a*b^5*c^2*n^4*x^5*sgn(x) + 85*(b*x + a)^n*b^6*c^2*n^3
*x^6*sgn(x) - 5*(b*x + a)^n*a^2*b^4*c^2*n^4*x^4*sgn(x) + 35*(b*x + a)^n*a*b
^5*c^2*n^3*x^5*sgn(x) + 225*(b*x + a)^n*b^6*c^2*n^2*x^6*sgn(x) - 30*(b*x +
a)^n*a^2*b^4*c^2*n^3*x^4*sgn(x) + 50*(b*x + a)^n*a*b^5*c^2*n^2*x^5*sgn(x) +
 274*(b*x + a)^n*b^6*c^2*n*x^6*sgn(x) + 20*(b*x + a)^n*a^3*b^3*c^2*n^3*x^3*
sgn(x) - 55*(b*x + a)^n*a^2*b^4*c^2*n^2*x^4*sgn(x) + 24*(b*x + a)^n*a*b^5*c
^2*n*x^5*sgn(x) + 120*(b*x + a)^n*b^6*c^2*x^6*sgn(x) + 60*(b*x + a)^n*a^3*b
^3*c^2*n^2*x^3*sgn(x) - 30*(b*x + a)^n*a^2*b^4*c^2*n*x^4*sgn(x) - 60*(b*x +
 a)^n*a^4*b^2*c^2*n^2*x^2*sgn(x) + 40*(b*x + a)^n*a^3*b^3*c^2*n*x^3*sgn(x)
- 60*(b*x + a)^n*a^4*b^2*c^2*n*x^2*sgn(x) + 120*(b*x + a)^n*a^5*b*c^2*n*x*s
gn(x) - 120*(b*x + a)^n*a^6*c^2*sgn(x))/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4
 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6))*sqrt(c)
```

maple [A] time = 0.01, size = 280, normalized size = 1.29

$$-\frac{\left(c x^{2}\right)^{\frac{5}{2}} \left(-b^{5} n^{5} x^{5}-15 b^{5} n^{4} x^{5}+5 a \,b^{4} n^{4} x^{4}-85 b^{5} n^{3} x^{5}+50 a \,b^{4} n^{3} x^{4}-225 b^{5} n^{2} x^{5}-20 a^{2} b^{3} n^{3} x^{3}+175 a \,b^{4} n^{2} x^{4}-274 b^{5} n \,x^{5}-120 a^{2} b^{3} n^{2} x^{3}+250 a \,b^{4} n \,x^{4}-120 b^{5} x^{5}+60 a^{3} b^{2} n^{2} x^{2}-220 a^{2} b^{3} n \,x^{3}+120 a \,b^{4} x^{4}+180 a^{3} b^{2} n \,x^{2}-120 a^{2} b^{3} x^{3}-120 a^{4} b n x +120 a^{3} b^{2} x^{2}-120 a^{4} b x +120 a^{5}\right) \left(b x +a \right)^{n +1}}{\left(n^{6}+21 n^{5}+175 n^{4}+735 n^{3}+1624 n^{2}+1764 n +720\right) b^{6} x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b^5*n^5*x^5-15*b^5*n^4*x^5+5*a*b^4*n^4*x^4-8
5*b^5*n^3*x^5+50*a*b^4*n^3*x^4-225*b^5*n^2*x^5-20*a^2*b^3*n^3*x^3+175*a*b^4
*n^2*x^4-274*b^5*n*x^5-120*a^2*b^3*n^2*x^3+250*a*b^4*n*x^4-120*b^5*x^5+60*a
^3*b^2*n^2*x^2-220*a^2*b^3*n*x^3+120*a*b^4*x^4+180*a^3*b^2*n*x^2-120*a^2*b^
3*x^3-120*a^4*b*n*x+120*a^3*b^2*x^2-120*a^4*b*x+120*a^5)/x^5/b^6/(n^6+21*n^
5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

maxima [A] time = 1.53, size = 203, normalized size = 0.94

$$\frac{\left(\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^6c^{\frac{5}{2}}x^6+\left(n^5+10\,n^4+35\,n^3+50\,n^2+24\,n\right)ab^5c^{\frac{5}{2}}x^5-5\left(n^4+6\,n^3+11\,n^2+6\,n\right)a^2b^4c^{\frac{5}{2}}x^4+20\left(n^3+3\,n^2+2\,n\right)a^3b^3c^{\frac{5}{2}}x^3-60\left(n^2+n\right)a^4b^2c^{\frac{5}{2}}x^2+120\,a^5bc^{\frac{5}{2}}nx-120\,a^6c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*c^(5/2)*x^6 + (n^5 + 1
0*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*c^(5/2)*x^5 - 5*(n^4 + 6*n^3 + 11*n^2
 + 6*n)*a^2*b^4*c^(5/2)*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*c^(5/2)*x^3 -
60*(n^2 + n)*a^4*b^2*c^(5/2)*x^2 + 120*a^5*b*c^(5/2)*n*x - 120*a^6*c^(5/2))
*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*
b^6)
```

mupad [B] time = 0.50, size = 424, normalized size = 1.95

$$\frac{(a+b\,x)^n\left(\frac{c^2\,x^6\,\sqrt{c\,x^2}\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}{n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720}-\frac{120\,a^6\,c^2\,\sqrt{c\,x^2}}{b^6\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{120\,a^5\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^5\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{5\,a^2\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b^2\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}-\frac{60\,a^4\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b^4\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{a\,c^2\,n\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{b\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}+\frac{20\,a^3\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^3\,\left(n^6+21\,n^5+175\,n^4+735\,n^3+1624\,n^2+1764\,n+720\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(a + b*x)^n,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^6*(c*x^2)^(1/2)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n
^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) - (
120*a^6*c^2*(c*x^2)^(1/2))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21
*n^5 + n^6 + 720)) + (120*a^5*c^2*n*x*(c*x^2)^(1/2))/(b^5*(1764*n + 1624*n^
2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (5*a^2*c^2*n*x^4*(c*x^2)^(1/
2)*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 +
21*n^5 + n^6 + 720)) - (60*a^4*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^4*(1764*
n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*c^2*n*x^5*(c*x
^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*
n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (20*a^3*c^2*n*x^3*(c*x^2)^(1/2)*(3*n
 + n^2 + 2))/(b^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 7
20))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^{\frac{5}{2}}(a+bx)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n, x)
```

3.891 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx$

Optimal. Leaf size=179

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Rubi [A] time = 0.05, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^5(n+1)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^5(n+2)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^5(n+3)x}-\frac{4ac^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^5(n+4)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+5}}{b^5(n+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (a^4*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^5*(1 + n)*x) - (4*a^3*c^2*Sqrt[c
*x^2]*(a + b*x)^(2 + n))/(b^5*(2 + n)*x) + (6*a^2*c^2*Sqrt[c*x^2]*(a + b*x)
^(3 + n))/(b^5*(3 + n)*x) - (4*a*c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^5*(4
 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(5 + n))/(b^5*(5 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^4(a+bx)^n\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^4(a+bx)^n}{b^4}-\frac{4a^3(a+bx)^{1+n}}{b^4}+\frac{6a^2(a+bx)^{2+n}}{b^4}-\frac{4a(a+bx)^{3+n}}{b^4}+\frac{(a+bx)^{4+n}}{b^4}\right)dx}{x}\\ &= \frac{a^4c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^5(1+n)x}-\frac{4a^3c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^5(2+n)x}+\frac{6a^2c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^5(3+n)x}-\frac{4ac^2\sqrt{cx^2}\,(}{b^5(4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 133, normalized size = 0.74

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(24a^4-24a^3b(n+1)x+12a^2b^2\left(n^2+3n+2\right)x^2-4ab^3\left(n^3+6n^2+11n+6\right)x^3+b^4\left(n^4+10n^3+35n^2+50n+24\right)x^4\right)}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(24*a^4 - 24*a^3*b*(1 + n)*x + 12*a^2*b^
2*(2 + 3*n + n^2)*x^2 - 4*a*b^3*(6 + 11*n + 6*n^2 + n^3)*x^3 + b^4*(24 + 50
```

```
*n + 35*n^2 + 10*n^3 + n^4)*x^4))/(b^5*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 +
 n)*x^3)
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x, x]
```

fricas [A] time = 1.15, size = 265, normalized size = 1.48

$$-\frac{\left(24\,a^4bc^2nx-24\,a^5c^2-\left(b^5c^2n^4+10\,b^5c^2n^3+35\,b^5c^2n^2+50\,b^5c^2n+24\,b^5c^2\right)x^5-\left(ab^4c^2n^4+6\,ab^4c^2n^3+11\,ab^4c^2n^2+6\,ab^4c^2n\right)x^4+4\left(a^2b^3c^2n^3+3\,a^2b^3c^2n^2+2\,a^2b^3c^2n\right)x^3-12\left(a^3b^2c^2n^2+a^3b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^5n^5+15\,b^5n^4+85\,b^5n^3+225\,b^5n^2+274\,b^5n+120\,b^5\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="fricas")
```

```
[Out] -(24*a^4*b*c^2*n*x - 24*a^5*c^2 - (b^5*c^2*n^4 + 10*b^5*c^2*n^3 + 35*b^5*c^
2*n^2 + 50*b^5*c^2*n + 24*b^5*c^2)*x^5 - (a*b^4*c^2*n^4 + 6*a*b^4*c^2*n^3 +
 11*a*b^4*c^2*n^2 + 6*a*b^4*c^2*n)*x^4 + 4*(a^2*b^3*c^2*n^3 + 3*a^2*b^3*c^2
*n^2 + 2*a^2*b^3*c^2*n)*x^3 - 12*(a^3*b^2*c^2*n^2 + a^3*b^2*c^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 27
4*b^5*n + 120*b^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x, x)
```

maple [A] time = 0.01, size = 199, normalized size = 1.11

$$\frac{\left(b^4n^4x^4+10b^4n^3x^4-4a\,b^3n^3x^3+35b^4n^2x^4-24a\,b^3n^2x^3+50b^4n\,x^4+12a^2b^2n^2x^2-44a\,b^3n\,x^3+24b^4x^4+36a^2b^2n\,x^2-24a\,b^3x^3-24a^3bnx+24a^2b^2x^2-24a^3bx+24a^4\right)\left(c\,x^2\right)^{\frac{5}{2}}(bx+a)^{n+1}}{\left(n^5+15n^4+85n^3+225n^2+274n+120\right)b^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x,x)
```

```
[Out] (b*x+a)^(n+1)*(b^4*n^4*x^4+10*b^4*n^3*x^4-4*a*b^3*n^3*x^3+35*b^4*n^2*x^4-24
*a*b^3*n^2*x^3+50*b^4*n*x^4+12*a^2*b^2*n^2*x^2-44*a*b^3*n*x^3+24*b^4*x^4+36
*a^2*b^2*n*x^2-24*a*b^3*x^3-24*a^3*b*n*x+24*a^2*b^2*x^2-24*a^3*b*x+24*a^4)*
(c*x^2)^(5/2)/x^5/b^5/(n^5+15*n^4+85*n^3+225*n^2+274*n+120)
```

maxima [A] time = 1.45, size = 157, normalized size = 0.88

$$\frac{\left(\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^5c^{\frac{5}{2}}x^5+\left(n^4+6\,n^3+11\,n^2+6\,n\right)ab^4c^{\frac{5}{2}}x^4-4\left(n^3+3\,n^2+2\,n\right)a^2b^3c^{\frac{5}{2}}x^3+12\left(n^2+n\right)a^3b^2c^{\frac{5}{2}}x^2-24\,a^4bc^{\frac{5}{2}}nx+24\,a^5c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x,x, algorithm="maxima")
```

```
[Out] ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^(5/2)*x^5 + (n^4 + 6*n^3 + 11*n^
2 + 6*n)*a*b^4*c^(5/2)*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^(5/2)*x^3 + 12
*(n^2 + n)*a^3*b^2*c^(5/2)*x^2 - 24*a^4*b*c^(5/2)*n*x + 24*a^5*c^(5/2))*(b*
x + a)^n/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)
```

mupad [B] time = 0.38, size = 319, normalized size = 1.78

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^5\,\sqrt{c\,x^2}\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}{n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120}+\frac{24\,a^5\,c^2\,\sqrt{c\,x^2}}{b^5\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{24\,a^4\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^4\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{a\,c^2\,n\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{b\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}+\frac{12\,a^3\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^3\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}-\frac{4\,a^2\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b^2\,\left(n^5+15\,n^4+85\,n^3+225\,n^2+274\,n+120\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^5*(c*x^2)^(1/2)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(
274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) + (24*a^5*c^2*(c*x^2)^(1/2))
/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (24*a^4*c^2*n*x*(c
*x^2)^(1/2))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*c^2
*n*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3
 + 15*n^4 + n^5 + 120)) + (12*a^3*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^3*(27
4*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*c^2*n*x^3*(c*x^2)^(1
/2)*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))
)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^{2}\right)^{\frac{5}{2}} \left(a + bx\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x, x)
```

3.892 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx$

Optimal. Leaf size=143

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Rubi [A] time = 0.04, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^4(n+1)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^4(n+2)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^4(n+3)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+4}}{b^4(n+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] -((a^3*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^4*(1 + n)*x)) + (3*a^2*c^2*Sqr
t[c*x^2]*(a + b*x)^(2 + n))/(b^4*(2 + n)*x) - (3*a*c^2*Sqrt[c*x^2]*(a + b*x
)^(3 + n))/(b^4*(3 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(4 + n))/(b^4*(4 +
n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^2}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^3(a+bx)^n\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{x}\\
&= -\frac{a^3c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^4(1+n)x}+\frac{3a^2c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^4(2+n)x}-\frac{3ac^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^4(3+n)x}+\frac{c^2\sqrt{cx^2}}{b^4(}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 99, normalized size = 0.69

$$\frac{c\left(cx^2\right)^{3/2}(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] (c*(c*x^2)^(3/2)*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2
 + 3*n + n^2)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)
*(3 + n)*(4 + n)*x^3)
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2} (a+bx)^n}{x^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^2, x]
```

fricas [A] time = 1.15, size = 186, normalized size = 1.30

$$\frac{\left(6\,a^3bc^2nx - 6\,a^4c^2 + \left(b^4c^2n^3 + 6\,b^4c^2n^2 + 11\,b^4c^2n + 6\,b^4c^2\right)x^4 + \left(ab^3c^2n^3 + 3\,ab^3c^2n^2 + 2\,ab^3c^2n\right)x^3 - 3\left(a^2b^2c^2n^2 + a^2b^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4n^4 + 10\,b^4n^3 + 35\,b^4n^2 + 50\,b^4n + 24\,b^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="fricas")
```

```
[Out] (6*a^3*b*c^2*n*x - 6*a^4*c^2 + (b^4*c^2*n^3 + 6*b^4*c^2*n^2 + 11*b^4*c^2*n
+ 6*b^4*c^2)*x^4 + (a*b^3*c^2*n^3 + 3*a*b^3*c^2*n^2 + 2*a*b^3*c^2*n)*x^3 -
3*(a^2*b^2*c^2*n^2 + a^2*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^4*n^4
+ 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}} (bx+a)^n}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^2, x)
```

maple [A] time = 0.01, size = 136, normalized size = 0.95

$$-\frac{(c\,x^2)^{\frac{5}{2}}\left(-b^3n^3x^3 - 6b^3n^2x^3 + 3a\,b^2n^2x^2 - 11b^3n\,x^3 + 9a\,b^2n\,x^2 - 6b^3x^3 - 6a^2bnx + 6a\,b^2x^2 - 6a^2bx + 6a^3\right)(bx+a)^{n+1}}{\left(n^4 + 10n^3 + 35n^2 + 50n + 24\right)b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^2,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11
*b^3*n*x^3+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)
/x^5/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 1.45, size = 116, normalized size = 0.81

$$\frac{\left(\left(n^3 + 6\,n^2 + 11\,n + 6\right)b^4c^{\frac{5}{2}}x^4 + \left(n^3 + 3\,n^2 + 2\,n\right)ab^3c^{\frac{5}{2}}x^3 - 3\left(n^2 + n\right)a^2b^2c^{\frac{5}{2}}x^2 + 6\,a^3bc^{\frac{5}{2}}nx - 6\,a^4c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^4 + 10\,n^3 + 35\,n^2 + 50\,n + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^2,x, algorithm="maxima")
```

[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*c^(5/2)*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^(5/2)*x^3 - 3*(n^2 + n)*a^2*b^2*c^(5/2)*x^2 + 6*a^3*b*c^(5/2)*n*x - 6*a^4*c^(5/2))*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)

mupad [B] time = 0.32, size = 229, normalized size = 1.60

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^4\,\sqrt{c\,x^2}\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,c^2\,\sqrt{c\,x^2}}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,c^2\,n\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,\left(n+1\right)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^2,x)

[Out] ((a + b*x)^n*((c^2*x^4*(c*x^2)^(1/2)*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4 + 24) - (6*a^4*c^2*(c*x^2)^(1/2))/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*c^2*n*x*(c*x^2)^(1/2))/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*c^2*n*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*c^2*n*x^2*(c*x^2)^(1/2)*(n + 1))/(b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}} (a+bx)^n}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**2,x)

[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**2, x)

3.893 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^3}\,dx$

Optimal. Leaf size=105

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Rubi [A] time = 0.03, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^3(n+1)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^3(n+2)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+3}}{b^3(n+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (a^2*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^3*(1 + n)*x) - (2*a*c^2*Sqrt[c*x
^2]*(a + b*x)^(2 + n))/(b^3*(2 + n)*x) + (c^2*Sqrt[c*x^2]*(a + b*x)^(3 + n)
)/(b^3*(3 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^3}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^2(a+bx)^n\,dx}{x}\\ &= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{x}\\ &= \frac{a^2c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^3(1+n)x}-\frac{2ac^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^3(2+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{3+n}}{b^3(3+n)x}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2
))/(b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^3, x]
```

fricas [A] time = 1.05, size = 127, normalized size = 1.21

$$-\frac{\left(2\,a^2bc^2nx-2\,a^3c^2-\left(b^3c^2n^2+3\,b^3c^2n+2\,b^3c^2\right)x^3-\left(ab^2c^2n^2+ab^2c^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3n^3+6\,b^3n^2+11\,b^3n+6\,b^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*c^2*n*x - 2*a^3*c^2 - (b^3*c^2*n^2 + 3*b^3*c^2*n + 2*b^3*c^2)*x^3
 - (a*b^2*c^2*n^2 + a*b^2*c^2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*n^3 + 6
*b^3*n^2 + 11*b^3*n + 6*b^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^3, x)
```

maple [A] time = 0.01, size = 83, normalized size = 0.79

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}(bx+a)^{n+1}}{\left(n^3+6n^2+11n+6\right)b^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^3,x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*(
c*x^2)^(5/2)/x^5/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.36, size = 80, normalized size = 0.76

$$\frac{\left(\left(n^2+3\,n+2\right)b^3c^{\frac{5}{2}}x^3+\left(n^2+n\right)ab^2c^{\frac{5}{2}}x^2-2\,a^2bc^{\frac{5}{2}}nx+2\,a^3c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*c^(5/2)*x^3 + (n^2 + n)*a*b^2*c^(5/2)*x^2 - 2*a^2*b*c^
(5/2)*n*x + 2*a^3*c^(5/2))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3)
```

mupad [B] time = 0.27, size = 154, normalized size = 1.47

$$\frac{(a+b\,x)^n\,\left(\frac{2\,a^3\,c^2\,\sqrt{c\,x^2}}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{c^2\,x^3\,\sqrt{c\,x^2}\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}-\frac{2\,a^2\,c^2\,n\,x\,\sqrt{c\,x^2}}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c^2\,n\,x^2\,\sqrt{c\,x^2}\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^3,x)
```

```
[Out] ((a + b*x)^n*((2*a^3*c^2*(c*x^2)^(1/2))/(b^3*(11*n + 6*n^2 + n^3 + 6)) + (c
^2*x^3*(c*x^2)^(1/2)*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) - (2*a^2*c^2
*n*x*(c*x^2)^(1/2))/(b^2*(11*n + 6*n^2 + n^3 + 6)) + (a*c^2*n*x^2*(c*x^2)^(
1/2)*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^n}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**3,x)
```

```
[Out] Integral((c*x**2)**(5/2)*(a + b*x)**n/x**3, x)
```

3.894 $\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^4}\,dx$

Optimal. Leaf size=69

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = $\{15, 43\}$

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+2}}{b^2(n+2)x}-\frac{ac^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b^2(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] -((a*c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b^2*(1 + n)*x)) + (c^2*Sqrt[c*x^2]
*(a + b*x)^(2 + n))/(b^2*(2 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(cx^2\right)^{5/2}(a+bx)^n}{x^4}\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x(a+bx)^n\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{x}\\
&= -\frac{ac^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b^2(1+n)x}+\frac{c^2\sqrt{cx^2}\,(a+bx)^{2+n}}{b^2(2+n)x}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.67

$$\frac{c^3x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x]
```

```
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^4, x]
```

fricas [A] time = 1.05, size = 76, normalized size = 1.10

$$\frac{\left(abc^2nx - a^2c^2 + \left(b^2c^2n + b^2c^2\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2n^2 + 3\,b^2n + 2\,b^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="fricas")
```

```
[Out] (a*b*c^2*n*x - a^2*c^2 + (b^2*c^2*n + b^2*c^2)*x^2)*sqrt(c*x^2)*(b*x + a)^n
/((b^2*n^2 + 3*b^2*n + 2*b^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{\frac{5}{2}}(bx+a)^n}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="giac")
```

```
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^4, x)
```

maple [A] time = 0.00, size = 46, normalized size = 0.67

$$-\frac{\left(c\,x^2\right)^{\frac{5}{2}}(-xnb - bx + a)\,(bx+a)^{n+1}}{\left(n^2+3n+2\right)b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^4,x)
```

```
[Out] -(b*x+a)^(n+1)*(c*x^2)^(5/2)*(-b*n*x-b*x+a)/x^5/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.45, size = 51, normalized size = 0.74

$$\frac{\left(b^2c^{\frac{5}{2}}(n+1)x^2 + abc^{\frac{5}{2}}nx - a^2c^{\frac{5}{2}}\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^4,x, algorithm="maxima")
```

```
[Out] (b^2*c^(5/2)*(n + 1)*x^2 + a*b*c^(5/2)*n*x - a^2*c^(5/2))*(b*x + a)^n/((n^2
 + 3*n + 2)*b^2)
```

mupad [B] time = 0.24, size = 94, normalized size = 1.36

$$\frac{(a+b\,x)^n\,\left(\frac{c^2\,x^2\,\sqrt{c\,x^2}\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,c^2\,\sqrt{c\,x^2}}{b^2\,\left(n^2+3\,n+2\right)}+\frac{a\,c^2\,n\,x\,\sqrt{c\,x^2}}{b\,\left(n^2+3\,n+2\right)}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^4,x)
```

```
[Out] ((a + b*x)^n*((c^2*x^2*(c*x^2)^(1/2)*(n + 1))/(3*n + n^2 + 2) - (a^2*c^2*(c
*x^2)^(1/2))/(b^2*(3*n + n^2 + 2)) + (a*c^2*n*x*(c*x^2)^(1/2))/(b*(3*n + n^
2 + 2))))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n c^{\frac{5}{2}} \left(x^2\right)^{\frac{5}{2}}}{2x^3} & \text{for } b = 0 \\ \int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4 (a+bx)^2}\, dx & \text{for } n = -2 \\ \int \frac{\left(cx^2\right)^{\frac{5}{2}}}{x^4 (a+bx)}\, dx & \text{for } n = -1 \\ -\frac{a^2 c^{\frac{5}{2}} (a+bx)^n \left(x^2\right)^{\frac{5}{2}}}{b^2 n^2 x^5 + 3b^2 n x^5 + 2b^2 x^5} + \frac{abc^{\frac{5}{2}} n x (a+bx)^n \left(x^2\right)^{\frac{5}{2}}}{b^2 n^2 x^5 + 3b^2 n x^5 + 2b^2 x^5} + \frac{b^2 c^{\frac{5}{2}} n x^2 (a+bx)^n \left(x^2\right)^{\frac{5}{2}}}{b^2 n^2 x^5 + 3b^2 n x^5 + 2b^2 x^5} + \frac{b^2 c^{\frac{5}{2}} x^2 (a+bx)^n \left(x^2\right)^{\frac{5}{2}}}{b^2 n^2 x^5 + 3b^2 n x^5 + 2b^2 x^5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**4,x)
```

```
[Out] Piecewise((a**n*c**(5/2)*(x**2)**(5/2)/(2*x**3), Eq(b, 0)), (Integral((c*x*
*2)**(5/2)/(x**4*(a + b*x)**2), x), Eq(n, -2)), (Integral((c*x**2)**(5/2)/(
x**4*(a + b*x)), x), Eq(n, -1)), (-a**2*c**(5/2)*(a + b*x)**n*(x**2)**(5/2)
/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + a*b*c**(5/2)*n*x*(a + b*x
)**n*(x**2)**(5/2)/(b**2*n**2*x**5 + 3*b**2*n*x**5 + 2*b**2*x**5) + b**2*c*
*(5/2)*n*x**2*(a + b*x)**n*(x**2)**(5/2)/(b**2*n**2*x**5 + 3*b**2*n*x**5 +
2*b**2*x**5) + b**2*c**(5/2)*x**2*(a + b*x)**n*(x**2)**(5/2)/(b**2*n**2*x**
5 + 3*b**2*n*x**5 + 2*b**2*x**5), True))
```

3.895 $\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5} dx$

Optimal. Leaf size=33

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{c^2\sqrt{cx^2}\,(a+bx)^{n+1}}{b(n+1)x}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^2*Sqrt[c*x^2]*(a + b*x)^(1 + n))/(b*(1 + n)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5} dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int (a+bx)^n\,dx}{x}\\ &= \frac{c^2\sqrt{cx^2}\,(a+bx)^{1+n}}{b(1+n)x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.94

$$\frac{c^3x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
```

```
[Out] (c^3*x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^{5/2}(a+bx)^n}{x^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^(5/2)*(a + b*x)^n)/x^5, x]
```

fricas [A] time = 0.84, size = 37, normalized size = 1.12

$$\frac{\left(bc^2x + ac^2\right)\sqrt{cx^2}\,(bx + a)^n}{(bn + b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="fricas")
[Out] (b*c^2*x + a*c^2)*sqrt(c*x^2)*(b*x + a)^n/((b*n + b)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^{\frac{5}{2}}(bx + a)^n}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="giac")
[Out] integrate((c*x^2)^(5/2)*(b*x + a)^n/x^5, x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.88

$$\frac{\left(c\,x^2\right)^{\frac{5}{2}}(bx + a)^{n+1}}{(n+1)\,b\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^(5/2)*(b*x+a)^n/x^5,x)
[Out] (b*x+a)^(n+1)/b/(n+1)*(c*x^2)^(5/2)/x^5
```

maxima [A] time = 1.39, size = 28, normalized size = 0.85

$$\frac{\left(bc^{\frac{5}{2}}x + ac^{\frac{5}{2}}\right)(bx + a)^n}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^(5/2)*(b*x+a)^n/x^5,x, algorithm="maxima")
[Out] (b*c^(5/2)*x + a*c^(5/2))*(b*x + a)^n/(b*(n + 1))
```

mupad [B] time = 0.23, size = 49, normalized size = 1.48

$$\frac{\left(\frac{c^2\,x\,\sqrt{c\,x^2}}{n+1} + \frac{a\,c^2\,\sqrt{c\,x^2}}{b\,(n+1)}\right)(a + b\,x)^n}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^(5/2)*(a + b*x)^n)/x^5,x)
[Out] (((c^2*x*(c*x^2)^(1/2))/(n + 1) + (a*c^2*(c*x^2)^(1/2))/(b*(n + 1)))*(a + b
*x)^n)/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{ax^{4}} & \text{for}\: b=0 \wedge n=-1 \\ \frac{a^{n}c^{\frac{5}{2}}\left(x^{2}\right)^{\frac{5}{2}}}{x^{4}} & \text{for}\: b=0 \\ \int \frac{\left(cx^{2}\right)^{\frac{5}{2}}}{x^{5}(a+bx)}\, dx & \text{for}\: n=-1 \\ \frac{ac^{\frac{5}{2}}(a+bx)^{n}\left(x^{2}\right)^{\frac{5}{2}}}{bnx^{5}+bx^{5}} + \frac{bc^{\frac{5}{2}}x(a+bx)^{n}\left(x^{2}\right)^{\frac{5}{2}}}{bnx^{5}+bx^{5}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**(5/2)*(b*x+a)**n/x**5,x)
```

```
[Out] Piecewise((c**(5/2)*(x**2)**(5/2)/(a*x**4), Eq(b, 0) & Eq(n, -1)), (a**n*c*
*(5/2)*(x**2)**(5/2)/x**4, Eq(b, 0)), (Integral((c*x**2)**(5/2)/(x**5*(a +
b*x)), x), Eq(n, -1)), (a*c**(5/2)*(a + b*x)**n*(x**2)**(5/2)/(b*n*x**5 + b
*x**5) + b*c**(5/2)*x*(a + b*x)**n*(x**2)**(5/2)/(b*n*x**5 + b*x**5), True)
)
```

3.896 $\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=123

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*x)
^(2 + n))/(b^4*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*(3 + n
)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{\sqrt{cx^2}}\\
&= -\frac{a^3x(a+bx)^{1+n}}{b^4(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4(4+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.78

$$\frac{x(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4 +
n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^4*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^4*(a + b*x)^n)/Sqrt[c*x^2], x]
```

fricas [A] time = 1.06, size = 158, normalized size = 1.28

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4cn^4+10\,b^4cn^3+35\,b^4cn^2+50\,b^4cn+24\,b^4c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c*n^4 + 10*b^4*c*n^3 + 35*b^4*c*n^2 + 50*b^4*c*n
 + 24*b^4*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^4}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^4/sqrt(c*x^2), x)
```

maple [A] time = 0.01, size = 134, normalized size = 1.09

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\,\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3+9
*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(1/
2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 1.45, size = 104, normalized size = 0.85

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*sqrt(c))
```

mupad [B] time = 0.37, size = 186, normalized size = 1.51

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}-\frac{6\,a^4\,x}{b^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 + 10*n^3 + n^4
+ 24) - (6*a^4*x)/(b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*n*x^2)
/(b^3*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 + 2))/(b*(
50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*(50*n + 35
*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**5/(4*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**4/(sqr
t(c*x**2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**4/(sqrt(c*x**2)*(a +
b*x)**3), x), Eq(n, -3)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)**2), x), E
q(n, -2)), (Integral(x**4/(sqrt(c*x**2)*(a + b*x)), x), Eq(n, -1)), (-6*a**
4*x*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(
x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24
*b**4*sqrt(c)*sqrt(x**2)) + 6*a**3*b*n*x**2*(a + b*x)**n/(b**4*sqrt(c)*n**4
*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x
**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) - 3*a**2*
b**2*n**2*x**3*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)
*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt
(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) - 3*a**2*b**2*n*x**3*(a + b*x)**n/(b**
4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(
c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x*
*2)) + a*b**3*n**3*x**4*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**
4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(
c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + 3*a*b**3*n**2*x**4*(a + b*x
)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b
**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c
)*sqrt(x**2)) + 2*a*b**3*n*x**4*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2)
+ 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b*
*4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + b**4*n**3*x**5*(a +
 b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) +
35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sq
rt(c)*sqrt(x**2)) + 6*b**4*n**2*x**5*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x
**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) +
50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)) + 11*b**4*n*x**5
*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**
2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) + 50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b*
*4*sqrt(c)*sqrt(x**2)) + 6*b**4*x**5*(a + b*x)**n/(b**4*sqrt(c)*n**4*sqrt(x
**2) + 10*b**4*sqrt(c)*n**3*sqrt(x**2) + 35*b**4*sqrt(c)*n**2*sqrt(x**2) +
50*b**4*sqrt(c)*n*sqrt(x**2) + 24*b**4*sqrt(c)*sqrt(x**2)), True))
```

3.897 $\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=90

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2 +
 n))/(b^3*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*(3 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{\sqrt{cx^2}}\\ &= \frac{a^2x(a+bx)^{1+n}}{b^3(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3(3+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 67, normalized size = 0.74

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^3*(a + b*x)^n)/Sqrt[c*x^2], x]
```

fricas [A] time = 1.33, size = 110, normalized size = 1.22

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3cn^3+6\,b^3cn^2+11\,b^3cn+6\,b^3c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c*n^3 + 6*b^3*c*n^2 + 11*b^3*c*n +
6*b^3*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^3}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^3/sqrt(c*x^2), x)
```

maple [A] time = 0.00, size = 81, normalized size = 0.90

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\;\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
/(c*x^2)^(1/2)/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.46, size = 83, normalized size = 0.92

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c)
```

mupad [B] time = 0.29, size = 121, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{n^3+6\,n^2+11\,n+6}+\frac{2\,a^3\,x}{b^3\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(11*n + 6*n^2 + n^3 + 6) + (2*a^3*x)/(b
^3*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*(11*n + 6*n^2 + n^3 + 6))
 + (a*n*x^3*(n + 1))/(b*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^4}{3\sqrt{c}\sqrt{x^2}} & \text{for}\ b=0 \\ \int \frac{x^3}{\sqrt{c x^2}\,(a+bx)^3}\,dx & \text{for}\ n=-3 \\ \int \frac{x^3}{\sqrt{c x^2}\,(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{x^3}{\sqrt{c x^2}\,(a+bx)}\,dx & \text{for}\ n=-1 \\ \frac{2a^3x(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} - \frac{2a^2bnx^2(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{ab^2n^2x^3(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{ab^2nx^3(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{b^3n^2x^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{3b^3nx^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} + \frac{2b^3x^4(a+bx)^n}{b^3\sqrt{c}n^3\sqrt{x^2}+6b^3\sqrt{c}n^2\sqrt{x^2}+11b^3\sqrt{c}n\sqrt{x^2}+6b^3\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**4/(3*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**3/(sqr
t(c*x**2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**3/(sqrt(c*x**2)*(a +
b*x)**2), x), Eq(n, -2)), (Integral(x**3/(sqrt(c*x**2)*(a + b*x)), x), Eq(n
, -1)), (2*a**3*x*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b**3*sqrt(
c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)*sqrt(x**
2)) - 2*a**2*b*n*x**2*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b**3*s
qrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)*sqrt
(x**2)) + a*b**2*n**2*x**3*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6*b
**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(c)
*sqrt(x**2)) + a*b**2*n*x**3*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2) + 6
*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sqrt(
c)*sqrt(x**2)) + b**3*n**2*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2)
+ 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*sq
rt(c)*sqrt(x**2)) + 3*b**3*n*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2
) + 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*
sqrt(c)*sqrt(x**2)) + 2*b**3*x**4*(a + b*x)**n/(b**3*sqrt(c)*n**3*sqrt(x**2
) + 6*b**3*sqrt(c)*n**2*sqrt(x**2) + 11*b**3*sqrt(c)*n*sqrt(x**2) + 6*b**3*
sqrt(c)*sqrt(x**2)), True))
```

3.898 $\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=59

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 + n)
)/(b^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{\sqrt{cx^2}}\\ &= -\frac{ax(a+bx)^{1+n}}{b^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2(2+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^2*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^2*(a + b*x)^n)/Sqrt[c*x^2], x]
```

fricas [A] time = 0.96, size = 66, normalized size = 1.12

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2cn^2+3\,b^2cn+2\,b^2c\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c*n^2 + 3
*b^2*c*n + 2*b^2*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^2}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^2/sqrt(c*x^2), x)
```

maple [A] time = 0.00, size = 44, normalized size = 0.75

$$-\frac{(-xnb-bx+a)\,x\,(bx+a)^{n+1}}{\sqrt{c\,x^2}\,\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n/(c*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x*(-b*n*x-b*x+a)/(c*x^2)^(1/2)/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.49, size = 45, normalized size = 0.76

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*sqrt(c))
```

mupad [B] time = 0.28, size = 71, normalized size = 1.20

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{n^2+3\,n+2}-\frac{a^2\,x}{b^2\,\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x)^n)/(c*x^2)^(1/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(3*n + n^2 + 2) - (a^2*x)/(b^2*(3*n + n^2 + 2))
 + (a*n*x^2)/(b*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^3}{2\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^2}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } n = -1 \\ -\frac{a^2x(a+bx)^n}{b^2\sqrt{c}\,n^2\sqrt{x^2}+3b^2\sqrt{c}\,n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{abnx^2(a+bx)^n}{b^2\sqrt{c}\,n^2\sqrt{x^2}+3b^2\sqrt{c}\,n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{b^2nx^3(a+bx)^n}{b^2\sqrt{c}\,n^2\sqrt{x^2}+3b^2\sqrt{c}\,n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} + \frac{b^2x^3(a+bx)^n}{b^2\sqrt{c}\,n^2\sqrt{x^2}+3b^2\sqrt{c}\,n\sqrt{x^2}+2b^2\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((a**n*x**3/(2*sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x**2/(sqr
t(c*x**2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**2/(sqrt(c*x**2)*(a +
b*x)), x), Eq(n, -1)), (-a**2*x*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2)
+ 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b**2*sqrt(c)*sqrt(x**2)) + a*b*n*x**2*(a
+ b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b
**2*sqrt(c)*sqrt(x**2)) + b**2*n*x**3*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(
x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2) + 2*b**2*sqrt(c)*sqrt(x**2)) + b**2*x**
3*(a + b*x)**n/(b**2*sqrt(c)*n**2*sqrt(x**2) + 3*b**2*sqrt(c)*n*sqrt(x**2)
+ 2*b**2*sqrt(c)*sqrt(x**2)), True))
```

3.899 $\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=28

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{b(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x(a+bx)^{n+1}}{b(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
[Out] (x*(a + b*x)^(1 + n))/(b*(1 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx)^n}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x*(a + b*x)^n)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][(x*(a + b*x)^n)/Sqrt[c*x^2], x]
```

fricas [A] time = 1.69, size = 33, normalized size = 1.18

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{(bcn+bc)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c*n + b*c)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x/sqrt(c*x^2), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.96

$$\frac{x\,(bx+a)^{n+1}}{(n+1)\,\sqrt{c\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n/(c*x^2)^(1/2),x)
[Out] x*(b*x+a)^(n+1)/b/(n+1)/(c*x^2)^(1/2)
```

maxima [A] time = 1.44, size = 31, normalized size = 1.11

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c*(n + 1))
```

mupad [B] time = 0.22, size = 36, normalized size = 1.29

$$\frac{\left(\frac{x^2}{n+1}+\frac{a\,x}{b\,(n+1)}\right)\,(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x)^n)/(c*x^2)^(1/2),x)
[Out] ((x^2/(n + 1) + (a*x)/(b*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^2}{a\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^2}{\sqrt{c}\sqrt{x^2}} & \text{for } b = 0 \\ \int \frac{x}{\sqrt{cx^2}\,(a+bx)}\,dx & \text{for } n = -1 \\ \frac{ax(a+bx)^n}{b\sqrt{c}\,n\sqrt{x^2}+b\sqrt{c}\sqrt{x^2}} + \frac{bx^2(a+bx)^n}{b\sqrt{c}\,n\sqrt{x^2}+b\sqrt{c}\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**n/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise((x**2/(a*sqrt(c)*sqrt(x**2)), Eq(b, 0) & Eq(n, -1)), (a**n*x**2/(
sqrt(c)*sqrt(x**2)), Eq(b, 0)), (Integral(x/(sqrt(c*x**2)*(a + b*x)), x), E
q(n, -1)), (a*x*(a + b*x)**n/(b*sqrt(c)*n*sqrt(x**2) + b*sqrt(c)*sqrt(x**2)
) + b*x**2*(a + b*x)**n/(b*sqrt(c)*n*sqrt(x**2) + b*sqrt(c)*sqrt(x**2)), Tr
ue))
```

3.900 $\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a + b*
x)^(2 + n))/(b^4*c*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^4*c*
(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c*(4 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^3x(a+bx)^{1+n}}{b^4c(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c(4+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 0.73

$$\frac{x^3(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4(n+1)(n+2)(n+3)(n+4)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2
)*x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*(1 + n)*(2 + n)*(3 + n)*(4
+ n)*(c*x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx)^n}{(cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^6*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

fricas [A] time = 0.99, size = 168, normalized size = 1.24

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4c^2n^4+10\,b^4c^2n^3+35\,b^4c^2n^2+50\,b^4c^2n+24\,b^4c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^2*n^4 + 10*b^4*c^2*n^3 + 35*b^4*c^2*n^2 + 50*b
^4*c^2*n + 24*b^4*c^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^6}{(cx^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(3/2), x)
```

maple [A] time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] -(b*x+a)^(n+1)*x^3*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
3/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)
```

maxima [A] time = 1.46, size = 104, normalized size = 0.77

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(3/2))
```

mupad [B] time = 0.40, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,c\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) - (6*a^4*x)/(b^4*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*a^3*
n*x^2)/(b^3*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n + n^2 +
2))/(b*c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1))/(b^2*
c*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**7/(4*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**6/
((c*x**2)**(3/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**6/((c*x**2)**(
3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*
x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(3/2)*(a + b*x)), x), Eq(
n, -1)), (-6*a**4*x**3*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*
b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50
*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 6*a**3*b
*n*x**4*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n
**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*
n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) - 3*a**2*b**2*n**2*x**5*(
a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2
)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)*
*(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) - 3*a**2*b**2*n*x**5*(a + b*x)**n/
(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 3
5*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*
b**4*c**(3/2)*(x**2)**(3/2)) + a*b**3*n**3*x**6*(a + b*x)**n/(b**4*c**(3/2)
*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2
)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*
(x**2)**(3/2)) + 3*a*b**3*n**2*x**6*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)
**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2
)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2
)) + 2*a*b**3*n*x**6*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b*
*4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b
**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + b**4*n**3*
x**7*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3
*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(
x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)) + 6*b**4*n**2*x**7*(a + b*x)
**n/(b**4*c**(3/2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2)
 + 35*b**4*c**(3/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) +
 24*b**4*c**(3/2)*(x**2)**(3/2)) + 11*b**4*n*x**7*(a + b*x)**n/(b**4*c**(3/
2)*n**4*(x**2)**(3/2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3
/2)*n**2*(x**2)**(3/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2
)*(x**2)**(3/2)) + 6*b**4*x**7*(a + b*x)**n/(b**4*c**(3/2)*n**4*(x**2)**(3/
2) + 10*b**4*c**(3/2)*n**3*(x**2)**(3/2) + 35*b**4*c**(3/2)*n**2*(x**2)**(3
/2) + 50*b**4*c**(3/2)*n*(x**2)**(3/2) + 24*b**4*c**(3/2)*(x**2)**(3/2)), T
rue))
```

3.901 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^(2
 + n))/(b^3*c*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c*(3 + n)*S
qrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{c\sqrt{cx^2}}\\
&= \frac{a^2x(a+bx)^{1+n}}{b^3c(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c(3+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 69, normalized size = 0.70

$$\frac{x^3(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3(n+1)(n+2)(n+3)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))
/(b^3*(1 + n)*(2 + n)*(3 + n)*(c*x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

fricas [A] time = 1.17, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3c^2n^3+6\,b^3c^2n^2+11\,b^3c^2n+6\,b^3c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^2*n^3 + 6*b^3*c^2*n^2 + 11*b^3*c^
2*n + 6*b^3*c^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^5}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(3/2), x)
```

maple [A] time = 0.00, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(3/2),x)
```

```
[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^3/(c*x^2)^(3/2)/b^3/(n^3+6*n^2+11*n+6)
```

maxima [A] time = 1.46, size = 83, normalized size = 0.84

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^2)
```

mupad [B] time = 0.31, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,c\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3*x
)/(b^3*c*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c*(11*n + 6*n^2 + n
^3 + 6)) + (a*n*x^3*(n + 1))/(b*c*(11*n + 6*n^2 + n^3 + 6))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^6}{3 c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}} & \text{for}\: b = 0 \\ \int \frac{x^5}{\left(c x^2\right)^{\frac{3}{2}} \left(a + b x\right)^3}\, dx & \text{for}\: n = -3 \\ \int \frac{x^5}{\left(c x^2\right)^{\frac{3}{2}} \left(a + b x\right)^2}\, dx & \text{for}\: n = -2 \\ \int \frac{x^5}{\left(c x^2\right)^{\frac{3}{2}} \left(a + b x\right)}\, dx & \text{for}\: n = -1 \\ \frac{2 a^3 x^3 (a+bx)^n}{D} - \frac{2 a^2 b n x^4 (a+bx)^n}{D} + \frac{a b^2 n^2 x^5 (a+bx)^n}{D} + \frac{a b^2 n x^5 (a+bx)^n}{D} + \frac{b^3 n^2 x^6 (a+bx)^n}{D} + \frac{3 b^3 n x^6 (a+bx)^n}{D} + \frac{2 b^3 x^6 (a+bx)^n}{D} & \text{otherwise} \end{cases}$$

$$D = b^3 c^{\frac{3}{2}} n^3 \left(x^2\right)^{\frac{3}{2}} + 6 b^3 c^{\frac{3}{2}} n^2 \left(x^2\right)^{\frac{3}{2}} + 11 b^3 c^{\frac{3}{2}} n \left(x^2\right)^{\frac{3}{2}} + 6 b^3 c^{\frac{3}{2}} \left(x^2\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**6/(3*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**5/
((c*x**2)**(3/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**5/((c*x**2)**(
3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**5/((c*x**2)**(3/2)*(a + b*
x)), x), Eq(n, -1)), (2*a**3*x**3*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**
(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/
2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) - 2*a**2*b*n*x**4*(a + b*x)**n/(b**3*c*
*(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c*
*(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + a*b**2*n**2*x**5*
(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2
)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)
) + a*b**2*n*x**5*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c
**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(
3/2)*(x**2)**(3/2)) + b**3*n**2*x**6*(a + b*x)**n/(b**3*c**(3/2)*n**3*(x**2
)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c**(3/2)*n*(x**2)**
(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + 3*b**3*n*x**6*(a + b*x)**n/(b**3*c
**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(3/2) + 11*b**3*c
**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)) + 2*b**3*x**6*(a +
 b*x)**n/(b**3*c**(3/2)*n**3*(x**2)**(3/2) + 6*b**3*c**(3/2)*n**2*(x**2)**(
3/2) + 11*b**3*c**(3/2)*n*(x**2)**(3/2) + 6*b**3*c**(3/2)*(x**2)**(3/2)), T
rue))
```

3.902 $\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2 +
n))/(b^2*c*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{c\sqrt{cx^2}}\\
&= \frac{x\int\left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{ax(a+bx)^{1+n}}{b^2c(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c(2+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.69

$$\frac{x^3(a+bx)^{n+1}(b(n+1)x-a)}{b^2(n+1)(n+2)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

[Out] (x^3*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*(1 + n)*(2 + n)*(c*x^2)^(3/2))

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+bx)^n}{(cx^2)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^4*(a + b*x)^n)/(c*x^2)^(3/2),x]

[Out] Defer[IntegrateAlgebraic][(x^4*(a + b*x)^n)/(c*x^2)^(3/2), x]

fricas [A] time = 1.59, size = 72, normalized size = 1.11

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2c^2n^2+3\,b^2c^2n+2\,b^2c^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")

[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^2*n^2 + 3*b^2*c^2*n + 2*b^2*c^2)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^4}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*x + a)^n*x^4/(c*x^2)^(3/2), x)

maple [A] time = 0.00, size = 46, normalized size = 0.71

$$-\frac{(-xnb-bx+a)\,x^3\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{3}{2}}\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(b*x+a)^n/(c*x^2)^(3/2),x)

[Out] -(b*x+a)^(n+1)*x^3*(-b*n*x-b*x+a)/(c*x^2)^(3/2)/b^2/(n^2+3*n+2)

maxima [A] time = 1.47, size = 45, normalized size = 0.69

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")

[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(3/2))

mupad [B] time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\,\left(\frac{x^3\,(n+1)}{c\,\left(n^2+3\,n+2\right)}-\frac{a^2\,x}{b^2\,c\,\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,c\,\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x)^n)/(c*x^2)^(3/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c*(3*n + n^2
 + 2)) + (a*n*x^2)/(b*c*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^5}{2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{for}\ b=0 \\ \int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)^2}\,dx & \text{for}\ n=-2 \\ \int \frac{x^4}{\left(cx^2\right)^{\frac{3}{2}}(a+bx)}\,dx & \text{for}\ n=-1 \\ -\frac{a^2x^3(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{abnx^4(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{b^2nx^5(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{b^2x^5(a+bx)^n}{b^2c^{\frac{3}{2}}n^2\left(x^2\right)^{\frac{3}{2}}+3b^2c^{\frac{3}{2}}n\left(x^2\right)^{\frac{3}{2}}+2b^2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((a**n*x**5/(2*c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**4/
((c*x**2)**(3/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**4/((c*x**2)**(
3/2)*(a + b*x)), x), Eq(n, -1)), (-a**2*x**3*(a + b*x)**n/(b**2*c**(3/2)*n*
*2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*(x**2)
**(3/2)) + a*b*n*x**4*(a + b*x)**n/(b**2*c**(3/2)*n**2*(x**2)**(3/2) + 3*b*
*2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*(x**2)**(3/2)) + b**2*n*x**5*
(a + b*x)**n/(b**2*c**(3/2)*n**2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**
(3/2) + 2*b**2*c**(3/2)*(x**2)**(3/2)) + b**2*x**5*(a + b*x)**n/(b**2*c**(3
/2)*n**2*(x**2)**(3/2) + 3*b**2*c**(3/2)*n*(x**2)**(3/2) + 2*b**2*c**(3/2)*
(x**2)**(3/2)), True))
```

3.903 $\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.97

$$\frac{x^3(a+bx)^{n+1}}{b(n+1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
```

```
[Out] (x^3*(a + b*x)^(1 + n))/(b*(1 + n)*(c*x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx)^n}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^3*(a + b*x)^n)/(c*x^2)^(3/2),x]
[Out] Defer[IntegrateAlgebraic][(x^3*(a + b*x)^n)/(c*x^2)^(3/2), x]
```

fricas [A] time = 0.91, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{\left(bc^2n+bc^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^2*n + b*c^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^3}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^3/(c*x^2)^(3/2), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{x^3\,(bx+a)^{n+1}}{(n+1)\left(c\,x^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x+a)^n/(c*x^2)^(3/2),x)
[Out] (b*x+a)^(n+1)/b/(n+1)*x^3/(c*x^2)^(3/2)
```

maxima [A] time = 1.44, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{bc^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x+a)^n/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^2*(n + 1))
```

mupad [B] time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c\,(n+1)}+\frac{a\,x}{b\,c\,(n+1)}\right)\,(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x)^n)/(c*x^2)^(3/2),x)
[Out] ((x^2/(c*(n + 1)) + (a*x)/(b*c*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^4}{ac^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^4}{c^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{for } b = 0 \\ \int \frac{x^3}{(cx^2)^{\frac{3}{2}}(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ax^3(a+bx)^n}{bc^{\frac{3}{2}}n(x^2)^{\frac{3}{2}} + bc^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} + \frac{bx^4(a+bx)^n}{bc^{\frac{3}{2}}n(x^2)^{\frac{3}{2}} + bc^{\frac{3}{2}}(x^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x+a)**n/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((x**4/(a*c**(3/2)*(x**2)**(3/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x*
*4/(c**(3/2)*(x**2)**(3/2)), Eq(b, 0)), (Integral(x**3/((c*x**2)**(3/2)*(a
+ b*x)), x), Eq(n, -1)), (a*x**3*(a + b*x)**n/(b*c**(3/2)*n*(x**2)**(3/2) +
 b*c**(3/2)*(x**2)**(3/2)) + b*x**4*(a + b*x)**n/(b*c**(3/2)*n*(x**2)**(3/2
) + b*c**(3/2)*(x**2)**(3/2)), True))
```

3.904 $\int \frac{x^8(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$-\frac{a^3x(a+bx)^{n+1}}{b^4c^2(n+1)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{n+2}}{b^4c^2(n+2)\sqrt{cx^2}}-\frac{3ax(a+bx)^{n+3}}{b^4c^2(n+3)\sqrt{cx^2}}+\frac{x(a+bx)^{n+4}}{b^4c^2(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^3*x*(a + b*x)^(1 + n))/(b^4*c^2*(1 + n)*Sqrt[c*x^2])) + (3*a^2*x*(a +
b*x)^(2 + n))/(b^4*c^2*(2 + n)*Sqrt[c*x^2]) - (3*a*x*(a + b*x)^(3 + n))/(b^
4*c^2*(3 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(4 + n))/(b^4*c^2*(4 + n)*Sqrt[c*
x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int x^3(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(-\frac{a^3(a+bx)^n}{b^3}+\frac{3a^2(a+bx)^{1+n}}{b^3}-\frac{3a(a+bx)^{2+n}}{b^3}+\frac{(a+bx)^{3+n}}{b^3}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^3x(a+bx)^{1+n}}{b^4c^2(1+n)\sqrt{cx^2}}+\frac{3a^2x(a+bx)^{2+n}}{b^4c^2(2+n)\sqrt{cx^2}}-\frac{3ax(a+bx)^{3+n}}{b^4c^2(3+n)\sqrt{cx^2}}+\frac{x(a+bx)^{4+n}}{b^4c^2(4+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 99, normalized size = 0.73

$$\frac{x(a+bx)^{n+1}\left(-6a^3+6a^2b(n+1)x-3ab^2\left(n^2+3n+2\right)x^2+b^3\left(n^3+6n^2+11n+6\right)x^3\right)}{b^4c^2(n+1)(n+2)(n+3)(n+4)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

[Out] (x*(a + b*x)^(1 + n)*(-6*a^3 + 6*a^2*b*(1 + n)*x - 3*a*b^2*(2 + 3*n + n^2)*
x^2 + b^3*(6 + 11*n + 6*n^2 + n^3)*x^3))/(b^4*c^2*(1 + n)*(2 + n)*(3 + n)*(
4 + n)*Sqrt[c*x^2])

IntegrateAlgebraic [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{x^8(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^8*(a + b*x)^n)/(c*x^2)^(5/2),x]

[Out] Defer[IntegrateAlgebraic][(x^8*(a + b*x)^n)/(c*x^2)^(5/2), x]

fricas [A] time = 0.95, size = 168, normalized size = 1.24

$$\frac{\left(6\,a^3bnx+\left(b^4n^3+6\,b^4n^2+11\,b^4n+6\,b^4\right)x^4-6\,a^4+\left(ab^3n^3+3\,ab^3n^2+2\,ab^3n\right)x^3-3\left(a^2b^2n^2+a^2b^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^4c^3n^4+10\,b^4c^3n^3+35\,b^4c^3n^2+50\,b^4c^3n+24\,b^4c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")

[Out] (6*a^3*b*n*x + (b^4*n^3 + 6*b^4*n^2 + 11*b^4*n + 6*b^4)*x^4 - 6*a^4 + (a*b^
3*n^3 + 3*a*b^3*n^2 + 2*a*b^3*n)*x^3 - 3*(a^2*b^2*n^2 + a^2*b^2*n)*x^2)*sqr
t(c*x^2)*(b*x + a)^n/((b^4*c^3*n^4 + 10*b^4*c^3*n^3 + 35*b^4*c^3*n^2 + 50*b
^4*c^3*n + 24*b^4*c^3)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^8}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")

[Out] integrate((b*x + a)^n*x^8/(c*x^2)^(5/2), x)

maple [A] time = 0.01, size = 136, normalized size = 1.01

$$-\frac{\left(-b^3n^3x^3-6b^3n^2x^3+3a\,b^2n^2x^2-11b^3n\,x^3+9a\,b^2n\,x^2-6b^3x^3-6a^2bnx+6a\,b^2x^2-6a^2bx+6a^3\right)x^5\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{5}{2}}\left(n^4+10n^3+35n^2+50n+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8*(b*x+a)^n/(c*x^2)^(5/2),x)

[Out] -(b*x+a)^(n+1)*x^5*(-b^3*n^3*x^3-6*b^3*n^2*x^3+3*a*b^2*n^2*x^2-11*b^3*n*x^3
+9*a*b^2*n*x^2-6*b^3*x^3-6*a^2*b*n*x+6*a*b^2*x^2-6*a^2*b*x+6*a^3)/(c*x^2)^(
5/2)/b^4/(n^4+10*n^3+35*n^2+50*n+24)

maxima [A] time = 1.47, size = 104, normalized size = 0.77

$$\frac{\left(\left(n^3+6\,n^2+11\,n+6\right)b^4x^4+\left(n^3+3\,n^2+2\,n\right)ab^3x^3-3\left(n^2+n\right)a^2b^2x^2+6\,a^3bnx-6\,a^4\right)(bx+a)^n}{\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)b^4c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")

```
[Out] ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2
+ n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n/((n^4 + 10*n^3 + 35*n^2
 + 50*n + 24)*b^4*c^(5/2))
```

mupad [B] time = 0.41, size = 201, normalized size = 1.49

$$\frac{(a+b\,x)^n\,\left(\frac{x^5\,\left(n^3+6\,n^2+11\,n+6\right)}{c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{6\,a^4\,x}{b^4\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{6\,a^3\,n\,x^2}{b^3\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{a\,n\,x^4\,\left(n^2+3\,n+2\right)}{b\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}-\frac{3\,a^2\,n\,x^3\,(n+1)}{b^2\,c^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^5*(11*n + 6*n^2 + n^3 + 6))/(c^2*(50*n + 35*n^2 + 10*n^3 +
 n^4 + 24)) - (6*a^4*x)/(b^4*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*
a^3*n*x^2)/(b^3*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*n*x^4*(3*n +
n^2 + 2))/(b*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (3*a^2*n*x^3*(n + 1
))/(b^2*c^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**9/(4*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**8/
((c*x**2)**(5/2)*(a + b*x)**4), x), Eq(n, -4)), (Integral(x**8/((c*x**2)**(
5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*
x)**2), x), Eq(n, -2)), (Integral(x**8/((c*x**2)**(5/2)*(a + b*x)), x), Eq(
n, -1)), (-6*a**4*x**5*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*
b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50
*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 6*a**3*b
*n*x**6*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n
**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*
n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) - 3*a**2*b**2*n**2*x**7*(
a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2
)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)*
*(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) - 3*a**2*b**2*n*x**7*(a + b*x)**n/
(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 3
5*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*
b**4*c**(5/2)*(x**2)**(5/2)) + a*b**3*n**3*x**8*(a + b*x)**n/(b**4*c**(5/2)
*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2
)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*
(x**2)**(5/2)) + 3*a*b**3*n**2*x**8*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)
**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2
)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2
)) + 2*a*b**3*n*x**8*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b*
*4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b
**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + b**4*n**3*
x**9*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3
*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(
x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)) + 6*b**4*n**2*x**9*(a + b*x)
**n/(b**4*c**(5/2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2)
 + 35*b**4*c**(5/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) +
 24*b**4*c**(5/2)*(x**2)**(5/2)) + 11*b**4*n*x**9*(a + b*x)**n/(b**4*c**(5/
2)*n**4*(x**2)**(5/2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5
/2)*n**2*(x**2)**(5/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2
)*(x**2)**(5/2)) + 6*b**4*x**9*(a + b*x)**n/(b**4*c**(5/2)*n**4*(x**2)**(5/
2) + 10*b**4*c**(5/2)*n**3*(x**2)**(5/2) + 35*b**4*c**(5/2)*n**2*(x**2)**(5
/2) + 50*b**4*c**(5/2)*n*(x**2)**(5/2) + 24*b**4*c**(5/2)*(x**2)**(5/2)), T
rue))
```

3.905 $\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{a^2x(a+bx)^{n+1}}{b^3c^2(n+1)\sqrt{cx^2}}-\frac{2ax(a+bx)^{n+2}}{b^3c^2(n+2)\sqrt{cx^2}}+\frac{x(a+bx)^{n+3}}{b^3c^2(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (a^2*x*(a + b*x)^(1 + n))/(b^3*c^2*(1 + n)*Sqrt[c*x^2]) - (2*a*x*(a + b*x)^
(2 + n))/(b^3*c^2*(2 + n)*Sqrt[c*x^2]) + (x*(a + b*x)^(3 + n))/(b^3*c^2*(3
+ n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int x^2(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{x\int \left(\frac{a^2(a+bx)^n}{b^2}-\frac{2a(a+bx)^{1+n}}{b^2}+\frac{(a+bx)^{2+n}}{b^2}\right)dx}{c^2\sqrt{cx^2}}\\
&= \frac{a^2x(a+bx)^{1+n}}{b^3c^2(1+n)\sqrt{cx^2}}-\frac{2ax(a+bx)^{2+n}}{b^3c^2(2+n)\sqrt{cx^2}}+\frac{x(a+bx)^{3+n}}{b^3c^2(3+n)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}\left(2a^2-2ab(n+1)x+b^2\left(n^2+3n+2\right)x^2\right)}{b^3c^2(n+1)(n+2)(n+3)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

[Out] (x*(a + b*x)^(1 + n)*(2*a^2 - 2*a*b*(1 + n)*x + b^2*(2 + 3*n + n^2)*x^2))/(
b^3*c^2*(1 + n)*(2 + n)*(3 + n)*Sqrt[c*x^2])

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{x^7(a+bx)^n}{(cx^2)^{5/2}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^7*(a + b*x)^n)/(c*x^2)^(5/2),x]

[Out] Defer[IntegrateAlgebraic][(x^7*(a + b*x)^n)/(c*x^2)^(5/2), x]

fricas [A] time = 1.87, size = 118, normalized size = 1.19

$$-\frac{\left(2\,a^2bnx-\left(b^3n^2+3\,b^3n+2\,b^3\right)x^3-2\,a^3-\left(ab^2n^2+ab^2n\right)x^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^3c^3n^3+6\,b^3c^3n^2+11\,b^3c^3n+6\,b^3c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")

[Out] -(2*a^2*b*n*x - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 - 2*a^3 - (a*b^2*n^2 + a*b^
2*n)*x^2)*sqrt(c*x^2)*(b*x + a)^n/((b^3*c^3*n^3 + 6*b^3*c^3*n^2 + 11*b^3*c^
3*n + 6*b^3*c^3)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^7}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")

[Out] integrate((b*x + a)^n*x^7/(c*x^2)^(5/2), x)

maple [A] time = 0.01, size = 83, normalized size = 0.84

$$\frac{\left(b^2n^2x^2+3b^2n\,x^2-2abnx+2b^2x^2-2abx+2a^2\right)x^5\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{5}{2}}\left(n^3+6n^2+11n+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(b*x+a)^n/(c*x^2)^(5/2),x)

[Out] (b*x+a)^(n+1)*(b^2*n^2*x^2+3*b^2*n*x^2-2*a*b*n*x+2*b^2*x^2-2*a*b*x+2*a^2)*x
^5/(c*x^2)^(5/2)/b^3/(n^3+6*n^2+11*n+6)

maxima [A] time = 1.45, size = 83, normalized size = 0.84

$$\frac{\left(\left(n^2+3\,n+2\right)b^3\sqrt{c}\,x^3+\left(n^2+n\right)ab^2\sqrt{c}\,x^2-2\,a^2b\sqrt{c}\,nx+2\,a^3\sqrt{c}\right)(bx+a)^n}{\left(n^3+6\,n^2+11\,n+6\right)b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")

```
[Out] ((n^2 + 3*n + 2)*b^3*sqrt(c)*x^3 + (n^2 + n)*a*b^2*sqrt(c)*x^2 - 2*a^2*b*sq
rt(c)*n*x + 2*a^3*sqrt(c))*(b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b^3*c^3)
```

mupad [B] time = 0.35, size = 133, normalized size = 1.34

$$\frac{(a+b\,x)^n\,\left(\frac{x^4\,\left(n^2+3\,n+2\right)}{c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,a^3\,x}{b^3\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,a^2\,n\,x^2}{b^2\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,n\,x^3\,(n+1)}{b\,c^2\,\left(n^3+6\,n^2+11\,n+6\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^4*(3*n + n^2 + 2))/(c^2*(11*n + 6*n^2 + n^3 + 6)) + (2*a^3
*x)/(b^3*c^2*(11*n + 6*n^2 + n^3 + 6)) - (2*a^2*n*x^2)/(b^2*c^2*(11*n + 6*n
^2 + n^3 + 6)) + (a*n*x^3*(n + 1))/(b*c^2*(11*n + 6*n^2 + n^3 + 6))))/(c*x^
2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^8}{3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{for}\ b = 0 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^3}\,dx & \text{for}\ n = -3 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for}\ n = -2 \\ \int \frac{x^7}{\left(cx^2\right)^{\frac{5}{2}}(a+bx)}\,dx & \text{for}\ n = -1 \\ \frac{2a^3x^5(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} - \frac{2a^2bnx^6(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{ab^2n^2x^7(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{ab^2nx^7(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{b^3n^2x^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{3b^3nx^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} + \frac{2b^3x^8(a+bx)^n}{b^3c^{\frac{5}{2}}n^3\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}n^2\left(x^2\right)^{\frac{5}{2}}+11b^3c^{\frac{5}{2}}n\left(x^2\right)^{\frac{5}{2}}+6b^3c^{\frac{5}{2}}\left(x^2\right)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**8/(3*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**7/
((c*x**2)**(5/2)*(a + b*x)**3), x), Eq(n, -3)), (Integral(x**7/((c*x**2)**(
5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**7/((c*x**2)**(5/2)*(a + b*
x)), x), Eq(n, -1)), (2*a**3*x**5*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**
(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/
2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) - 2*a**2*b*n*x**6*(a + b*x)**n/(b**3*c*
*(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c*
*(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + a*b**2*n**2*x**7*
(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2
)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)
) + a*b**2*n*x**7*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c
**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(
5/2)*(x**2)**(5/2)) + b**3*n**2*x**8*(a + b*x)**n/(b**3*c**(5/2)*n**3*(x**2
)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c**(5/2)*n*(x**2)**
(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + 3*b**3*n*x**8*(a + b*x)**n/(b**3*c
**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(5/2) + 11*b**3*c
**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)) + 2*b**3*x**8*(a +
 b*x)**n/(b**3*c**(5/2)*n**3*(x**2)**(5/2) + 6*b**3*c**(5/2)*n**2*(x**2)**(
5/2) + 11*b**3*c**(5/2)*n*(x**2)**(5/2) + 6*b**3*c**(5/2)*(x**2)**(5/2)), T
rue))
```

3.906 $\int \frac{x^6(a+bx)^n}{(cx^2)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 43}

$$\frac{x(a+bx)^{n+2}}{b^2c^2(n+2)\sqrt{cx^2}}-\frac{ax(a+bx)^{n+1}}{b^2c^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] -((a*x*(a + b*x)^(1 + n))/(b^2*c^2*(1 + n)*Sqrt[c*x^2])) + (x*(a + b*x)^(2
+ n))/(b^2*c^2*(2 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6(a+bx)^n}{(cx^2)^{5/2}}\,dx &= \frac{x\int x(a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x\int \left(-\frac{a(a+bx)^n}{b}+\frac{(a+bx)^{1+n}}{b}\right)dx}{c^2\sqrt{cx^2}}\\ &= -\frac{ax(a+bx)^{1+n}}{b^2c^2(1+n)\sqrt{cx^2}}+\frac{x(a+bx)^{2+n}}{b^2c^2(2+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.71

$$\frac{x(a+bx)^{n+1}(b(n+1)x-a)}{b^2c^2(n+1)(n+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n)*(-a + b*(1 + n)*x))/(b^2*c^2*(1 + n)*(2 + n)*Sqrt[c*x^
2])
```

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^6*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] Defer[IntegrateAlgebraic][(x^6*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

fricas [A] time = 0.92, size = 72, normalized size = 1.11

$$\frac{\left(abnx+\left(b^2n+b^2\right)x^2-a^2\right)\sqrt{cx^2}\,(bx+a)^n}{\left(b^2c^3n^2+3\,b^2c^3n+2\,b^2c^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] (a*b*n*x + (b^2*n + b^2)*x^2 - a^2)*sqrt(c*x^2)*(b*x + a)^n/((b^2*c^3*n^2 +
 3*b^2*c^3*n + 2*b^2*c^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^6}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^6/(c*x^2)^(5/2), x)
```

maple [A] time = 0.00, size = 46, normalized size = 0.71

$$-\frac{(-xnb-bx+a)\,x^5\,(bx+a)^{n+1}}{\left(c\,x^2\right)^{\frac{5}{2}}\left(n^2+3n+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] -(b*x+a)^(n+1)*x^5*(-b*n*x-b*x+a)/(c*x^2)^(5/2)/b^2/(n^2+3*n+2)
```

maxima [A] time = 1.41, size = 45, normalized size = 0.69

$$\frac{\left(b^2(n+1)x^2+abnx-a^2\right)(bx+a)^n}{\left(n^2+3\,n+2\right)b^2c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n/((n^2 + 3*n + 2)*b^2*c^(5/2))
```

mupad [B] time = 0.29, size = 80, normalized size = 1.23

$$\frac{(a+b\,x)^n\left(\frac{x^3\,(n+1)}{c^2\left(n^2+3\,n+2\right)}-\frac{a^2\,x}{b^2\,c^2\left(n^2+3\,n+2\right)}+\frac{a\,n\,x^2}{b\,c^2\left(n^2+3\,n+2\right)}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x)^n)/(c*x^2)^(5/2),x)
```

```
[Out] ((a + b*x)^n*((x^3*(n + 1))/(c^2*(3*n + n^2 + 2)) - (a^2*x)/(b^2*c^2*(3*n +
 n^2 + 2)) + (a*n*x^2)/(b*c^2*(3*n + n^2 + 2))))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^n x^7}{2c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)^2}\,dx & \text{for } n = -2 \\ \int \frac{x^6}{(cx^2)^{\frac{5}{2}}(a+bx)}\,dx & \text{for } n = -1 \\ -\frac{a^2x^5(a+bx)^n}{b^2c^{\frac{5}{2}}n^2(x^2)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} + \frac{abnx^6(a+bx)^n}{b^2c^{\frac{5}{2}}n^2(x^2)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} + \frac{b^2nx^7(a+bx)^n}{b^2c^{\frac{5}{2}}n^2(x^2)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} + \frac{b^2x^7(a+bx)^n}{b^2c^{\frac{5}{2}}n^2(x^2)^{\frac{5}{2}}+3b^2c^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+2b^2c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((a**n*x**7/(2*c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**6/
((c*x**2)**(5/2)*(a + b*x)**2), x), Eq(n, -2)), (Integral(x**6/((c*x**2)**(
5/2)*(a + b*x)), x), Eq(n, -1)), (-a**2*x**5*(a + b*x)**n/(b**2*c**(5/2)*n*
*2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*(x**2)
**(5/2)) + a*b*n*x**6*(a + b*x)**n/(b**2*c**(5/2)*n**2*(x**2)**(5/2) + 3*b*
*2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*(x**2)**(5/2)) + b**2*n*x**7*
(a + b*x)**n/(b**2*c**(5/2)*n**2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**
(5/2) + 2*b**2*c**(5/2)*(x**2)**(5/2)) + b**2*x**7*(a + b*x)**n/(b**2*c**(5
/2)*n**2*(x**2)**(5/2) + 3*b**2*c**(5/2)*n*(x**2)**(5/2) + 2*b**2*c**(5/2)*
(x**2)**(5/2)), True))
```

3.907 $\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=31

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 32}

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int (a+bx)^n\,dx}{c^2\sqrt{cx^2}}\\ &= \frac{x(a+bx)^{1+n}}{bc^2(1+n)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x(a+bx)^{n+1}}{bc^2(n+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*x)^(1 + n))/(b*c^2*(1 + n)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+bx)^n}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(x^5*(a + b*x)^n)/(c*x^2)^(5/2),x]
[Out] Defer[IntegrateAlgebraic][(x^5*(a + b*x)^n)/(c*x^2)^(5/2), x]
```

fricas [A] time = 1.49, size = 37, normalized size = 1.19

$$\frac{\sqrt{cx^2}\,(bx+a)(bx+a)^n}{\left(bc^3n+bc^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(b*x + a)*(b*x + a)^n/((b*c^3*n + b*c^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^n x^5}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="giac")
[Out] integrate((b*x + a)^n*x^5/(c*x^2)^(5/2), x)
```

maple [A] time = 0.00, size = 29, normalized size = 0.94

$$\frac{x^5\,(bx+a)^{n+1}}{(n+1)\left(c\,x^2\right)^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x+a)^n/(c*x^2)^(5/2),x)
[Out] (b*x+a)^(n+1)/b/(n+1)*x^5/(c*x^2)^(5/2)
```

maxima [A] time = 1.45, size = 31, normalized size = 1.00

$$\frac{\left(b\sqrt{c}\,x+a\sqrt{c}\right)(bx+a)^n}{bc^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x+a)^n/(c*x^2)^(5/2),x, algorithm="maxima")
[Out] (b*sqrt(c)*x + a*sqrt(c))*(b*x + a)^n/(b*c^3*(n + 1))
```

mupad [B] time = 0.23, size = 42, normalized size = 1.35

$$\frac{\left(\frac{x^2}{c^2\,(n+1)}+\frac{a\,x}{b\,c^2\,(n+1)}\right)\,(a+b\,x)^n}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x)^n)/(c*x^2)^(5/2),x)
[Out] ((x^2/(c^2*(n + 1)) + (a*x)/(b*c^2*(n + 1)))*(a + b*x)^n)/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{x^6}{ac^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{for } b = 0 \wedge n = -1 \\ \frac{a^n x^6}{c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{for } b = 0 \\ \int \frac{x^5}{(cx^2)^{\frac{5}{2}}(a+bx)}\, dx & \text{for } n = -1 \\ \frac{ax^5(a+bx)^n}{bc^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+bc^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} + \frac{bx^6(a+bx)^n}{bc^{\frac{5}{2}}n(x^2)^{\frac{5}{2}}+bc^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x+a)**n/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((x**6/(a*c**(5/2)*(x**2)**(5/2)), Eq(b, 0) & Eq(n, -1)), (a**n*x*
*6/(c**(5/2)*(x**2)**(5/2)), Eq(b, 0)), (Integral(x**5/((c*x**2)**(5/2)*(a
+ b*x)), x), Eq(n, -1)), (a*x**5*(a + b*x)**n/(b*c**(5/2)*n*(x**2)**(5/2) +
 b*c**(5/2)*(x**2)**(5/2)) + b*x**6*(a + b*x)**n/(b*c**(5/2)*n*(x**2)**(5/2
) + b*c**(5/2)*(x**2)**(5/2)), True))
```

3.908 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ac^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{bc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
```

```
[Out] (a*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (b*c^2*(d*x)^(7 + m)*Sq
rt[c*x^2])/(d^7*(7 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)\,dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)\,dx}{d^5x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a(dx)^{5+m}+\frac{b(dx)^{6+m}}{d}\right)dx}{d^5x}\\
&= \frac{ac^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{bc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.58

$$\frac{x\left(cx^2\right)^{5/2}(dx)^m(a(m+7)+b(m+6)x)}{(m+6)(m+7)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] (x*(d*x)^m*(c*x^2)^(5/2)*(a*(7 + m) + b*(6 + m)*x))/((6 + m)*(7 + m))
```

IntegrateAlgebraic [F] time = 0.46, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(5/2)*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(5/2)*(a + b*x), x]
```

fricas [A] time = 1.41, size = 58, normalized size = 0.89

$$\frac{\left(\left(bc^2m+6\,bc^2\right)x^6+\left(ac^2m+7\,ac^2\right)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="fricas")
[Out] ((b*c^2*m + 6*b*c^2)*x^6 + (a*c^2*m + 7*a*c^2)*x^5)*sqrt(c*x^2)*(d*x)^m/(m^
2 + 13*m + 42)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 40, normalized size = 0.62

$$\frac{(bmx+am+6bx+7a)\left(c\,x^2\right)^{\frac{5}{2}}x\,(dx)^m}{(m+7)\,(m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x)
[Out] x*(b*m*x+a*m+6*b*x+7*a)*(d*x)^m*(c*x^2)^(5/2)/(m+7)/(m+6)
```

maxima [A] time = 1.52, size = 39, normalized size = 0.60

$$\frac{bc^{\frac{5}{2}}d^mx^7x^m}{m+7}+\frac{ac^{\frac{5}{2}}d^mx^6x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a),x, algorithm="maxima")
[Out] b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a*c^(5/2)*d^m*x^6*x^m/(m + 6)
```

mupad [B] time = 0.27, size = 44, normalized size = 0.68

$$\frac{c^2\,x^5\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(7\,a+a\,m+6\,b\,x+b\,m\,x\right)}{m^2+13\,m+42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x),x)
```

```
[Out] (c^2*x^5*(d*x)^m*(c*x^2)^(1/2)*(7*a + a*m + 6*b*x + b*m*x))/(13*m + m^2 + 4
2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^2\right)^{\frac{5}{2}}}{x^7}\,dx+\int \frac{b\left(cx^2\right)^{\frac{5}{2}}}{x^6}\,dx}{d^7} & \text{for } m = -7 \\ \frac{\int \frac{a\left(cx^2\right)^{\frac{5}{2}}}{x^6}\,dx+\int \frac{b\left(cx^2\right)^{\frac{5}{2}}}{x^5}\,dx}{d^6} & \text{for } m = -6 \\ \frac{ac^{\frac{5}{2}}d^m mxx^m\left(x^2\right)^{\frac{5}{2}}}{m^2+13m+42} + \frac{7ac^{\frac{5}{2}}d^m xx^m\left(x^2\right)^{\frac{5}{2}}}{m^2+13m+42} + \frac{bc^{\frac{5}{2}}d^m mx^2x^m\left(x^2\right)^{\frac{5}{2}}}{m^2+13m+42} + \frac{6bc^{\frac{5}{2}}d^m x^2x^m\left(x^2\right)^{\frac{5}{2}}}{m^2+13m+42} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(5/2)/x**7, x) + Integral(b*(c*x**2)**(5/2
)/x**6, x))/d**7, Eq(m, -7)), ((Integral(a*(c*x**2)**(5/2)/x**6, x) + Integ
ral(b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a*c**(5/2)*d**m*m*x*x**m
*(x**2)**(5/2)/(m**2 + 13*m + 42) + 7*a*c**(5/2)*d**m*x*x**m*(x**2)**(5/2)/
(m**2 + 13*m + 42) + b*c**(5/2)*d**m*m*x**2*x**m*(x**2)**(5/2)/(m**2 + 13*m
 + 42) + 6*b*c**(5/2)*d**m*x**2*x**m*(x**2)**(5/2)/(m**2 + 13*m + 42), True
))
```

3.909 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ac\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{bc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (a*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (b*c*(d*x)^(5 + m)*Sqrt[c
*x^2])/(d^5*(5 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\,dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)\,dx}{d^3x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a(dx)^{3+m}+\frac{b(dx)^{4+m}}{d}\right)dx}{d^3x}\\
&= \frac{ac(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{bc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.62

$$\frac{x\left(cx^2\right)^{3/2}(dx)^m(a(m+5)+b(m+4)x)}{(m+4)(m+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^(3/2)*(a*(5 + m) + b*(4 + m)*x))/((4 + m)*(5 + m))
```

IntegrateAlgebraic [F] time = 0.39, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(3/2)*(a + b*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(3/2)*(a + b*x), x]
```

fricas [A] time = 1.49, size = 50, normalized size = 0.82

$$\frac{\left((bcm+4\,bc)x^4+(acm+5\,ac)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="fricas")
```

```
[Out] ((b*c*m + 4*b*c)*x^4 + (a*c*m + 5*a*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^2 + 9*m
+ 20)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 40, normalized size = 0.66

$$\frac{(bmx+am+4bx+5a)\left(c\,x^2\right)^{\frac{3}{2}} x\,(dx)^m}{(m+5)\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x)
```

```
[Out] x*(b*m*x+a*m+4*b*x+5*a)*(d*x)^m*(c*x^2)^(3/2)/(m+5)/(m+4)
```

maxima [A] time = 1.55, size = 39, normalized size = 0.64

$$\frac{bc^{\frac{3}{2}}d^m x^5 x^m}{m+5}+\frac{ac^{\frac{3}{2}}d^m x^4 x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a),x, algorithm="maxima")
```

```
[Out] b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a*c^(3/2)*d^m*x^4*x^m/(m + 4)
```

mupad [B] time = 0.24, size = 42, normalized size = 0.69

$$\frac{c\,x^3\,{\left(d\,x\right)}^m\,\sqrt{c\,x^2}\,\left(5\,a+a\,m+4\,b\,x+b\,m\,x\right)}{m^2+9\,m+20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x),x)
```

```
[Out] (c*x^3*(d*x)^m*(c*x^2)^(1/2)*(5*a + a*m + 4*b*x + b*m*x))/(9*m + m^2 + 20)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^5}\,dx + \int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx}{d^5} & \text{for } m = -5 \\ \frac{\int \frac{a\left(cx^2\right)^{\frac{3}{2}}}{x^4}\,dx + \int \frac{b\left(cx^2\right)^{\frac{3}{2}}}{x^3}\,dx}{d^4} & \text{for } m = -4 \\ \frac{ac^{\frac{3}{2}}d^m m x x^m\left(x^2\right)^{\frac{3}{2}}}{m^2+9m+20} + \frac{5ac^{\frac{3}{2}}d^m x x^m\left(x^2\right)^{\frac{3}{2}}}{m^2+9m+20} + \frac{bc^{\frac{3}{2}}d^m m x^2 x^m\left(x^2\right)^{\frac{3}{2}}}{m^2+9m+20} + \frac{4bc^{\frac{3}{2}}d^m x^2 x^m\left(x^2\right)^{\frac{3}{2}}}{m^2+9m+20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*(c*x**2)**(3/2)/x**5, x) + Integral(b*(c*x**2)**(3/2
)/x**4, x))/d**5, Eq(m, -5)), ((Integral(a*(c*x**2)**(3/2)/x**4, x) + Integ
ral(b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a*c**(3/2)*d**m*m*x*x**m
*(x**2)**(3/2)/(m**2 + 9*m + 20) + 5*a*c**(3/2)*d**m*x*x**m*(x**2)**(3/2)/(
m**2 + 9*m + 20) + b*c**(3/2)*d**m*m*x**2*x**m*(x**2)**(3/2)/(m**2 + 9*m +
20) + 4*b*c**(3/2)*d**m*x**2*x**m*(x**2)**(3/2)/(m**2 + 9*m + 20), True))
```

3.910 $\int (dx)^m \sqrt{cx^2}\,(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x}+\frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{a\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x}+\frac{b\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
```

```
[Out] (a*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (b*(d*x)^(3 + m)*Sqrt[c*x^2
])/(d^3*(3 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{cx^2}\,(a+bx)\,dx &= \frac{\sqrt{cx^2}\int x(dx)^m(a+bx)\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)\,dx}{dx}\\
&= \frac{\sqrt{cx^2}\int \left(a(dx)^{1+m}+\frac{b(dx)^{2+m}}{d}\right)dx}{dx}\\
&= \frac{a(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{b(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.64

$$\frac{x\sqrt{cx^2}\,(dx)^m(a(m+3)+b(m+2)x)}{(m+2)(m+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a*(3 + m) + b*(2 + m)*x))/((2 + m)*(3 + m))
```

IntegrateAlgebraic [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (dx)^m\sqrt{cx^2}\,(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[c*x^2]*(a + b*x),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[c*x^2]*(a + b*x), x]
```

fricas [A] time = 1.52, size = 44, normalized size = 0.75

$$\frac{\left((bm+2\,b)x^2+(am+3\,a)x\right)\sqrt{cx^2}\,\left(dx\right)^m}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="fricas")
[Out] ((b*m + 2*b)*x^2 + (a*m + 3*a)*x)*sqrt(c*x^2)*(d*x)^m/(m^2 + 5*m + 6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 40, normalized size = 0.68

$$\frac{(bmx+am+2bx+3a)\sqrt{c\,x^2}\;x\,(dx)^m}{(m+3)\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x)
[Out] x*(b*m*x+a*m+2*b*x+3*a)*(d*x)^m*(c*x^2)^(1/2)/(m+3)/(m+2)
```

maxima [A] time = 1.52, size = 39, normalized size = 0.66

$$\frac{b\sqrt{c}\,d^m x^3 x^m}{m+3}+\frac{a\sqrt{c}\,d^m x^2 x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a),x, algorithm="maxima")
[Out] b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a*sqrt(c)*d^m*x^2*x^m/(m + 2)
```

mupad [B] time = 0.21, size = 39, normalized size = 0.66

$$\frac{x\,(d\,x)^m\,\sqrt{c\,x^2}\,\;(3\,a+a\,m+2\,b\,x+b\,m\,x)}{m^2+5\,m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x),x)
```

```
[Out] (x*(d*x)^m*(c*x^2)^(1/2)*(3*a + a*m + 2*b*x + b*m*x))/(5*m + m^2 + 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a\sqrt{cx^2}}{x^3}\,dx + \int \frac{b\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for } m = -3 \\ \frac{\int \frac{a\sqrt{cx^2}}{x^2}\,dx + \int \frac{b\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for } m = -2 \\ \frac{a\sqrt{c}\,d^m m x x^m \sqrt{x^2}}{m^2+5m+6} + \frac{3a\sqrt{c}\,d^m x x^m \sqrt{x^2}}{m^2+5m+6} + \frac{b\sqrt{c}\,d^m m x^2 x^m \sqrt{x^2}}{m^2+5m+6} + \frac{2b\sqrt{c}\,d^m x^2 x^m \sqrt{x^2}}{m^2+5m+6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a),x)
```

```
[Out] Piecewise(((Integral(a*sqrt(c*x**2)/x**3, x) + Integral(b*sqrt(c*x**2)/x**2
, x))/d**3, Eq(m, -3)), ((Integral(a*sqrt(c*x**2)/x**2, x) + Integral(b*sqr
t(c*x**2)/x, x))/d**2, Eq(m, -2)), (a*sqrt(c)*d**m*m*x*x**m*sqrt(x**2)/(m**
2 + 5*m + 6) + 3*a*sqrt(c)*d**m*x*x**m*sqrt(x**2)/(m**2 + 5*m + 6) + b*sqrt
(c)*d**m*m*x**2*x**m*sqrt(x**2)/(m**2 + 5*m + 6) + 2*b*sqrt(c)*d**m*x**2*x*
*m*sqrt(x**2)/(m**2 + 5*m + 6), True))
```

3.911 $\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$\frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
```

```
[Out] (a*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int (dx)^{-1+m}(a+bx)\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int \left(a(dx)^{-1+m}+\frac{b(dx)^m}{d}\right)dx}{\sqrt{cx^2}}\\ &= \frac{ax(dx)^m}{m\sqrt{cx^2}}+\frac{bx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.69

$$\frac{x(dx)^m(am+a+bmx)}{m(m+1)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
[Out] (x*(d*x)^m*(a + a*m + b*m*x))/(m*(1 + m)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/Sqrt[c*x^2],x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/Sqrt[c*x^2], x]
```

fricas [A] time = 0.75, size = 36, normalized size = 0.75

$$\frac{(bmx+am+a)\sqrt{cx^2}\,(dx)^m}{\left(cm^2+cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="fricas")
[Out] (b*m*x + a*m + a)*sqrt(c*x^2)*(d*x)^m/((c*m^2 + c*m)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\,(dx)^m}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a)*(d*x)^m/sqrt(c*x^2), x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.67

$$\frac{(bmx+am+a)\,x\,(dx)^m}{(m+1)\sqrt{c\,x^2}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x)
[Out] x*(b*m*x+a*m+a)*(d*x)^m/(m+1)/m/(c*x^2)^(1/2)
```

maxima [A] time = 1.48, size = 32, normalized size = 0.67

$$\frac{bd^m x x^m}{\sqrt{c}\,(m+1)}+\frac{ad^m x^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(1/2),x, algorithm="maxima")
[Out] b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a*d^m*x^m/(sqrt(c)*m)
```

mupad [B] time = 0.21, size = 30, normalized size = 0.62

$$\frac{\left(\frac{a\,x}{m}+\frac{b\,x^2}{m+1}\right)\,(d\,x)^m}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(1/2),x)
```

```
[Out] (((a*x)/m + (b*x^2)/(m + 1))*(d*x)^m)/(c*x^2)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b}{\sqrt{cx^2}}\,dx + \int \frac{a}{x\sqrt{cx^2}}\,dx}{d} & \text{for } m = -1 \\ \int \frac{a+bx}{\sqrt{cx^2}}\,dx & \text{for } m = 0 \\ \frac{ad^m m x x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} + \frac{ad^m x x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} + \frac{bd^m m x^2 x^m}{\sqrt{c}\,m^2\sqrt{x^2} + \sqrt{c}\,m\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b/sqrt(c*x**2), x) + Integral(a/(x*sqrt(c*x**2)), x))/
d, Eq(m, -1)), (Integral((a + b*x)/sqrt(c*x**2), x), Eq(m, 0)), (a*d**m*m*x
*x**m/(sqrt(c)*m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)) + a*d**m*x*x**m/(sqr
t(c)*m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)) + b*d**m*m*x**2*x**m/(sqrt(c)*
m**2*sqrt(x**2) + sqrt(c)*m*sqrt(x**2)), True))
```

3.912 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$-\frac{ad^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
```

```
[Out] -((a*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (b*d*x*(d*x)^(-1 + m)
)/(c*(1 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{3/2}} \, dx &= \frac{x\int \frac{(dx)^m(a+bx)}{x^3} \, dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x\right)\int (dx)^{-3+m}(a+bx) \, dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x\right)\int \left(a(dx)^{-3+m}+\frac{b(dx)^{-2+m}}{d}\right) \, dx}{c\sqrt{cx^2}}\\
&= -\frac{ad^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{bdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.58

$$\frac{x(dx)^m(a(m-1)+b(m-2)x)}{(m-2)(m-1)\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a*(-1 + m) + b*(-2 + m)*x))/((-2 + m)*(-1 + m)*(c*x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/(c*x^2)^(3/2), x]
```

fricas [A] time = 1.06, size = 53, normalized size = 0.82

$$\frac{\sqrt{cx^2}\,(am+(bm-2\,b)x-a)\,(dx)^m}{\left(c^2m^2-3\,c^2m+2\,c^2\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] sqrt(c*x^2)*(a*m + (b*m - 2*b)*x - a)*(d*x)^m/((c^2*m^2 - 3*c^2*m + 2*c^2)*
x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\,(dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(3/2), x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.62

$$\frac{(bmx+am-2bx-a)\,x\,(dx)^m}{(m-1)\,(m-2)\left(c\,x^2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x)
[Out] x*(b*m*x+a*m-2*b*x-a)*(d*x)^m/(m-1)/(m-2)/(c*x^2)^(3/2)
```

maxima [A] time = 1.51, size = 39, normalized size = 0.60

$$\frac{bd^mx^m}{c^{\frac{3}{2}}(m-1)x}+\frac{ad^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(3/2),x, algorithm="maxima")
[Out] b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a*d^m*x^m/(c^(3/2)*(m - 2)*x^2)
```

mupad [B] time = 0.26, size = 48, normalized size = 0.74

$$\frac{b\,(d\,x)^m}{c\,\sqrt{c\,x^2}\,\,(m-1)}+\frac{a\,(d\,x)^m}{c\,x\,\sqrt{c\,x^2}\,\,(m-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(3/2),x)
```

```
[Out] (b*(d*x)^m)/(c*(c*x^2)^(1/2)*(m - 1)) + (a*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m -
 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d\left(\int \frac{ax}{\left(cx^2\right)^{\frac{3}{2}}}\,dx+\int \frac{bx^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx\right) & \text{for}\ m=1 \\ d^2\left(\int \frac{ax^2}{\left(cx^2\right)^{\frac{3}{2}}}\,dx+\int \frac{bx^3}{\left(cx^2\right)^{\frac{3}{2}}}\,dx\right) & \text{for}\ m=2 \\ \frac{ad^m mxx^m}{c^{\frac{3}{2}}m^2\left(x^2\right)^{\frac{3}{2}}-3c^{\frac{3}{2}}m\left(x^2\right)^{\frac{3}{2}}+2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{ad^m xx^m}{c^{\frac{3}{2}}m^2\left(x^2\right)^{\frac{3}{2}}-3c^{\frac{3}{2}}m\left(x^2\right)^{\frac{3}{2}}+2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}+\frac{bd^m mx^2x^m}{c^{\frac{3}{2}}m^2\left(x^2\right)^{\frac{3}{2}}-3c^{\frac{3}{2}}m\left(x^2\right)^{\frac{3}{2}}+2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}}-\frac{2bd^m x^2x^m}{c^{\frac{3}{2}}m^2\left(x^2\right)^{\frac{3}{2}}-3c^{\frac{3}{2}}m\left(x^2\right)^{\frac{3}{2}}+2c^{\frac{3}{2}}\left(x^2\right)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((d*(Integral(a*x/(c*x**2)**(3/2), x) + Integral(b*x**2/(c*x**2)**
(3/2), x)), Eq(m, 1)), (d**2*(Integral(a*x**2/(c*x**2)**(3/2), x) + Integra
l(b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a*d**m*m*x*x**m/(c**(3/2)*m**2*(
x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**2)**(3/2)) - a*d
**m*x*x**m/(c**(3/2)*m**2*(x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c*
*(3/2)*(x**2)**(3/2)) + b*d**m*m*x**2*x**m/(c**(3/2)*m**2*(x**2)**(3/2) - 3
*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**2)**(3/2)) - 2*b*d**m*x**2*x**m/
(c**(3/2)*m**2*(x**2)**(3/2) - 3*c**(3/2)*m*(x**2)**(3/2) + 2*c**(3/2)*(x**
2)**(3/2)), True))
```

3.913 $\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {15, 16, 43}

$$-\frac{ad^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] -((a*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (b*d^3*x*(d*x)^(-3
+ m))/(c^2*(3 - m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{\left(d^5x\right)\int (dx)^{-5+m}(a+bx)\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{\left(d^5x\right)\int \left(a(dx)^{-5+m}+\frac{b(dx)^{-4+m}}{d}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{ad^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{bd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 0.57

$$\frac{x(dx)^m(a(m-3)+b(m-4)x)}{(m-4)(m-3)\left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a*(-3 + m) + b*(-4 + m)*x))/((-4 + m)*(-3 + m)*(c*x^2)^(5/2))
```

IntegrateAlgebraic [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x))/(c*x^2)^(5/2), x]
```

fricas [A] time = 1.21, size = 53, normalized size = 0.79

$$\frac{\sqrt{cx^2}\,(am+(bm-4\,b)x-3\,a)\,(dx)^m}{\left(c^3m^2-7\,c^3m+12\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2)*(a*m + (b*m - 4*b)*x - 3*a)*(d*x)^m/((c^3*m^2 - 7*c^3*m + 12*c^
3)*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)\,(dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(d*x)^m/(c*x^2)^(5/2), x)
```

maple [A] time = 0.00, size = 40, normalized size = 0.60

$$\frac{(bmx+am-4bx-3a)\,x\,(dx)^m}{(m-3)\,(m-4)\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x)
```

```
[Out] x*(b*m*x+a*m-4*b*x-3*a)*(d*x)^m/(m-3)/(-4+m)/(c*x^2)^(5/2)
```

maxima [A] time = 1.52, size = 39, normalized size = 0.58

$$\frac{bd^mx^m}{c^{\frac{5}{2}}(m-3)x^3}+\frac{ad^mx^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

mupad [B] time = 0.28, size = 47, normalized size = 0.70

$$-\frac{(d\,x)^m\,\left(3\,a-a\,m+4\,b\,x-b\,m\,x\right)}{c^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2-7\,m+12\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x))/(c*x^2)^(5/2),x)
```

```
[Out] -((d*x)^m*(3*a - a*m + 4*b*x - b*m*x))/(c^2*x^3*(c*x^2)^(1/2)*(m^2 - 7*m +
12))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} d^3\left(\int \frac{ax^3}{(cx^2)^{\frac{5}{2}}}\,dx+\int \frac{bx^4}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m=3 \\ d^4\left(\int \frac{ax^4}{(cx^2)^{\frac{5}{2}}}\,dx+\int \frac{bx^5}{(cx^2)^{\frac{5}{2}}}\,dx\right) & \text{for } m=4 \\ \frac{ad^m mxx^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}}-7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}}+12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}}-\frac{3ad^m xx^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}}-7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}}+12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}}+\frac{bd^m mx^2x^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}}-7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}}+12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}}-\frac{4bd^m x^2x^m}{c^{\frac{5}{2}}m^2(x^2)^{\frac{5}{2}}-7c^{\frac{5}{2}}m(x^2)^{\frac{5}{2}}+12c^{\frac{5}{2}}(x^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**3*(Integral(a*x**3/(c*x**2)**(5/2), x) + Integral(b*x**4/(c*x
**2)**(5/2), x)), Eq(m, 3)), (d**4*(Integral(a*x**4/(c*x**2)**(5/2), x) + I
ntegral(b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a*d**m*m*x*x**m/(c**(5/2)*
m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c**(5/2)*(x**2)**(5/2)
) - 3*a*d**m*x*x**m/(c**(5/2)*m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/
2) + 12*c**(5/2)*(x**2)**(5/2)) + b*d**m*m*x**2*x**m/(c**(5/2)*m**2*(x**2)*
*(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c**(5/2)*(x**2)**(5/2)) - 4*b*d**m
*x**2*x**m/(c**(5/2)*m**2*(x**2)**(5/2) - 7*c**(5/2)*m*(x**2)**(5/2) + 12*c
**(5/2)*(x**2)**(5/2)), True))
```

3.914 $\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$

Optimal. Leaf size=103

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

Rubi [A] time = 0.05, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c^2\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}+\frac{2abc^2\sqrt{cx^2}\,(dx)^{m+7}}{d^7(m+7)x}+\frac{b^2c^2\sqrt{cx^2}\,(dx)^{m+8}}{d^8(m+8)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c^2*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 + m)*x) + (2*a*b*c^2*(d*x)^(7 +
 m)*Sqrt[c*x^2])/(d^7*(7 + m)*x) + (b^2*c^2*(d*x)^(8 + m)*Sqrt[c*x^2])/(d^8
*(8 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx &= \frac{\left(c^2\sqrt{cx^2}\right)\int x^5(dx)^m(a+bx)^2 \, dx}{x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int (dx)^{5+m}(a+bx)^2 \, dx}{d^5x}\\
&= \frac{\left(c^2\sqrt{cx^2}\right)\int \left(a^2(dx)^{5+m}+\frac{2ab(dx)^{6+m}}{d}+\frac{b^2(dx)^{7+m}}{d^2}\right) dx}{d^5x}\\
&= \frac{a^2c^2(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}+\frac{2abc^2(dx)^{7+m}\sqrt{cx^2}}{d^7(7+m)x}+\frac{b^2c^2(dx)^{8+m}\sqrt{cx^2}}{d^8(8+m)x}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.47

$$x\left(cx^2\right)^{5/2}(dx)^m\left(\frac{a^2}{m+6}+\frac{2abx}{m+7}+\frac{b^2x^2}{m+8}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(5/2)*(a^2/(6 + m) + (2*a*b*x)/(7 + m) + (b^2*x^2)/(8 + m
))
```

IntegrateAlgebraic [F] time = 0.85, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{5/2} (a+bx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2, x]
```

fricas [A] time = 1.28, size = 123, normalized size = 1.19

$$\frac{\left(\left(b^2c^2m^2+13\,b^2c^2m+42\,b^2c^2\right)x^7+2\left(abc^2m^2+14\,abc^2m+48\,abc^2\right)x^6+\left(a^2c^2m^2+15\,a^2c^2m+56\,a^2c^2\right)x^5\right)\sqrt{cx^2}\,(dx)^m}{m^3+21\,m^2+146\,m+336}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c^2*m^2 + 13*b^2*c^2*m + 42*b^2*c^2)*x^7 + 2*(a*b*c^2*m^2 + 14*a*b*c^
2*m + 48*a*b*c^2)*x^6 + (a^2*c^2*m^2 + 15*a^2*c^2*m + 56*a^2*c^2)*x^5)*sqrt
(c*x^2)*(d*x)^m/(m^3 + 21*m^2 + 146*m + 336)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 95, normalized size = 0.92

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+13b^2m\,x^2+a^2m^2+28abmx+42b^2x^2+15a^2m+96abx+56a^2\right)\left(c\,x^2\right)^{\frac{5}{2}}x\,(dx)^m}{(m+8)\,(m+7)\,(m+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+13*b^2*m*x^2+a^2*m^2+28*a*b*m*x+42*b^2*x^2+15*a^
2*m+96*a*b*x+56*a^2)*(d*x)^m*(c*x^2)^(5/2)/(m+8)/(m+7)/(m+6)
```

maxima [A] time = 1.60, size = 64, normalized size = 0.62

$$\frac{b^2c^{\frac{5}{2}}d^mx^8x^m}{m+8}+\frac{2\,abc^{\frac{5}{2}}d^mx^7x^m}{m+7}+\frac{a^2c^{\frac{5}{2}}d^mx^6x^m}{m+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(5/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(5/2)*d^m*x^8*x^m/(m + 8) + 2*a*b*c^(5/2)*d^m*x^7*x^m/(m + 7) + a^2*c
^(5/2)*d^m*x^6*x^m/(m + 6)
```

mupad [B] time = 0.31, size = 127, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,c^2\,x^5\,\sqrt{c\,x^2}\,\left(m^2+15\,m+56\right)}{m^3+21\,m^2+146\,m+336}+\frac{b^2\,c^2\,x^7\,\sqrt{c\,x^2}\,\left(m^2+13\,m+42\right)}{m^3+21\,m^2+146\,m+336}+\frac{2\,a\,b\,c^2\,x^6\,\sqrt{c\,x^2}\,\left(m^2+14\,m+48\right)}{m^3+21\,m^2+146\,m+336}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(5/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c^2*x^5*(c*x^2)^(1/2)*(15*m + m^2 + 56))/(146*m + 21*m^2 + m^
3 + 336) + (b^2*c^2*x^7*(c*x^2)^(1/2)*(13*m + m^2 + 42))/(146*m + 21*m^2 +
m^3 + 336) + (2*a*b*c^2*x^6*(c*x^2)^(1/2)*(14*m + m^2 + 48))/(146*m + 21*m^
2 + m^3 + 336))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{8}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{6}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{5}{2}}}{x^{7}}\, dx}{d^{8}} & \text{for}\: m = -8 \\ \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{7}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{5}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{5}{2}}}{x^{6}}\, dx}{d^{7}} & \text{for}\: m = -7 \\ \frac{\int \frac{a^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{6}}\, dx + \int \frac{b^{2} \left(c x^{2}\right)^{\frac{5}{2}}}{x^{4}}\, dx + \int \frac{2 a b \left(c x^{2}\right)^{\frac{5}{2}}}{x^{5}}\, dx}{d^{6}} & \text{for}\: m = -6 \\ \frac{a^{2} c^{\frac{5}{2}} d^{m} m^{2} x x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{15 a^{2} c^{\frac{5}{2}} d^{m} m x x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{56 a^{2} c^{\frac{5}{2}} d^{m} x x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{2 a b c^{\frac{5}{2}} d^{m} m^{2} x^{2} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{28 a b c^{\frac{5}{2}} d^{m} m x^{2} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{96 a b c^{\frac{5}{2}} d^{m} x^{2} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{b^{2} c^{\frac{5}{2}} d^{m} m^{2} x^{3} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{13 b^{2} c^{\frac{5}{2}} d^{m} m x^{3} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} + \frac{42 b^{2} c^{\frac{5}{2}} d^{m} x^{3} x^{m} \left(x^{2}\right)^{\frac{5}{2}}}{m^{3} + 21 m^{2} + 146 m + 336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(5/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(5/2)/x**8, x) + Integral(b**2*(c*x**2)
**(5/2)/x**6, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**7, x))/d**8, Eq(m, -8)
), ((Integral(a**2*(c*x**2)**(5/2)/x**7, x) + Integral(b**2*(c*x**2)**(5/2)
/x**5, x) + Integral(2*a*b*(c*x**2)**(5/2)/x**6, x))/d**7, Eq(m, -7)), ((In
tegral(a**2*(c*x**2)**(5/2)/x**6, x) + Integral(b**2*(c*x**2)**(5/2)/x**4,
x) + Integral(2*a*b*(c*x**2)**(5/2)/x**5, x))/d**6, Eq(m, -6)), (a**2*c**(5
/2)*d**m*m**2*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 15*a**2
*c**(5/2)*d**m*m*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 56*a
**2*c**(5/2)*d**m*x*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336) + 2*a
*b*c**(5/2)*d**m*m**2*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m + 336
) + 28*a*b*c**(5/2)*d**m*m*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146*m
+ 336) + 96*a*b*c**(5/2)*d**m*x**2*x**m*(x**2)**(5/2)/(m**3 + 21*m**2 + 146
*m + 336) + b**2*c**(5/2)*d**m*m**2*x**3*x**m*(x**2)**(5/2)/(m**3 + 21*m**2
 + 146*m + 336) + 13*b**2*c**(5/2)*d**m*m*x**3*x**m*(x**2)**(5/2)/(m**3 + 2
1*m**2 + 146*m + 336) + 42*b**2*c**(5/2)*d**m*x**3*x**m*(x**2)**(5/2)/(m**3
 + 21*m**2 + 146*m + 336), True))
```

3.915 $\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$

Optimal. Leaf size=97

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

Rubi [A] time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2c\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}+\frac{2abc\sqrt{cx^2}\,(dx)^{m+5}}{d^5(m+5)x}+\frac{b^2c\sqrt{cx^2}\,(dx)^{m+6}}{d^6(m+6)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] (a^2*c*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x) + (2*a*b*c*(d*x)^(5 + m)*
Sqrt[c*x^2])/(d^5*(5 + m)*x) + (b^2*c*(d*x)^(6 + m)*Sqrt[c*x^2])/(d^6*(6 +
m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx &= \frac{\left(c\sqrt{cx^2}\right)\int x^3(dx)^m(a+bx)^2\,dx}{x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int (dx)^{3+m}(a+bx)^2\,dx}{d^3x}\\
&= \frac{\left(c\sqrt{cx^2}\right)\int \left(a^2(dx)^{3+m}+\frac{2ab(dx)^{4+m}}{d}+\frac{b^2(dx)^{5+m}}{d^2}\right)dx}{d^3x}\\
&= \frac{a^2c(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}+\frac{2abc(dx)^{5+m}\sqrt{cx^2}}{d^5(5+m)x}+\frac{b^2c(dx)^{6+m}\sqrt{cx^2}}{d^6(6+m)x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 48, normalized size = 0.49

$$x\left(cx^2\right)^{3/2}(dx)^m\left(\frac{a^2}{m+4}+\frac{2abx}{m+5}+\frac{b^2x^2}{m+6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] x*(d*x)^m*(c*x^2)^(3/2)*(a^2/(4 + m) + (2*a*b*x)/(5 + m) + (b^2*x^2)/(6 + m
))
```

IntegrateAlgebraic [F] time = 0.62, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^{3/2} (a+bx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2, x]
```

fricas [A] time = 1.42, size = 105, normalized size = 1.08

$$\frac{\left(\left(b^2cm^2+9\,b^2cm+20\,b^2c\right)x^5+2\left(abcm^2+10\,abcm+24\,abc\right)x^4+\left(a^2cm^2+11\,a^2cm+30\,a^2c\right)x^3\right)\sqrt{cx^2}\,(dx)^m}{m^3+15\,m^2+74\,m+120}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*c*m^2 + 9*b^2*c*m + 20*b^2*c)*x^5 + 2*(a*b*c*m^2 + 10*a*b*c*m + 24*a*
b*c)*x^4 + (a^2*c*m^2 + 11*a^2*c*m + 30*a^2*c)*x^3)*sqrt(c*x^2)*(d*x)^m/(m^
3 + 15*m^2 + 74*m + 120)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 95, normalized size = 0.98

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+9b^2m\,x^2+a^2m^2+20abmx+20b^2x^2+11a^2m+48abx+30a^2\right)\left(c\,x^2\right)^{\frac{3}{2}}x\,(dx)^m}{(m+6)\,(m+5)\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+9*b^2*m*x^2+a^2*m^2+20*a*b*m*x+20*b^2*x^2+11*a^2
*m+48*a*b*x+30*a^2)*(d*x)^m*(c*x^2)^(3/2)/(m+6)/(m+5)/(m+4)
```

maxima [A] time = 1.55, size = 64, normalized size = 0.66

$$\frac{b^2c^{\frac{3}{2}}d^mx^6x^m}{m+6}+\frac{2\,abc^{\frac{3}{2}}d^mx^5x^m}{m+5}+\frac{a^2c^{\frac{3}{2}}d^mx^4x^m}{m+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(3/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*c^(3/2)*d^m*x^6*x^m/(m + 6) + 2*a*b*c^(3/2)*d^m*x^5*x^m/(m + 5) + a^2*c
^(3/2)*d^m*x^4*x^m/(m + 4)
```

mupad [B] time = 0.28, size = 121, normalized size = 1.25

$$(d\,x)^m\,\left(\frac{a^2\,c\,x^3\,\sqrt{c\,x^2}\,\left(m^2+11\,m+30\right)}{m^3+15\,m^2+74\,m+120}+\frac{b^2\,c\,x^5\,\sqrt{c\,x^2}\,\left(m^2+9\,m+20\right)}{m^3+15\,m^2+74\,m+120}+\frac{2\,a\,b\,c\,x^4\,\sqrt{c\,x^2}\,\left(m^2+10\,m+24\right)}{m^3+15\,m^2+74\,m+120}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(3/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*c*x^3*(c*x^2)^(1/2)*(11*m + m^2 + 30))/(74*m + 15*m^2 + m^3 +
 120) + (b^2*c*x^5*(c*x^2)^(1/2)*(9*m + m^2 + 20))/(74*m + 15*m^2 + m^3 + 1
20) + (2*a*b*c*x^4*(c*x^2)^(1/2)*(10*m + m^2 + 24))/(74*m + 15*m^2 + m^3 +
120))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2 (cx^2)^{\frac{3}{2}}}{x^6}\, dx + \int \frac{b^2 (cx^2)^{\frac{3}{2}}}{x^4}\, dx + \int \frac{2ab (cx^2)^{\frac{3}{2}}}{x^5}\, dx}{d^6} & \text{for}\: m = -6 \\ \frac{\int \frac{a^2 (cx^2)^{\frac{3}{2}}}{x^5}\, dx + \int \frac{b^2 (cx^2)^{\frac{3}{2}}}{x^3}\, dx + \int \frac{2ab (cx^2)^{\frac{3}{2}}}{x^4}\, dx}{d^5} & \text{for}\: m = -5 \\ \frac{\int \frac{a^2 (cx^2)^{\frac{3}{2}}}{x^4}\, dx + \int \frac{b^2 (cx^2)^{\frac{3}{2}}}{x^2}\, dx + \int \frac{2ab (cx^2)^{\frac{3}{2}}}{x^3}\, dx}{d^4} & \text{for}\: m = -4 \\ \frac{a^2 c^{\frac{3}{2}} d^m m^2 x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{11 a^2 c^{\frac{3}{2}} d^m m x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{30 a^2 c^{\frac{3}{2}} d^m x x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{2abc^{\frac{3}{2}} d^m m^2 x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{20abc^{\frac{3}{2}} d^m m x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{48abc^{\frac{3}{2}} d^m x^2 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{b^2 c^{\frac{3}{2}} d^m m^2 x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{9b^2 c^{\frac{3}{2}} d^m m x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} + \frac{20b^2 c^{\frac{3}{2}} d^m x^3 x^m (x^2)^{\frac{3}{2}}}{m^3+15m^2+74m+120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(3/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*(c*x**2)**(3/2)/x**6, x) + Integral(b**2*(c*x**2)
**(3/2)/x**4, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**5, x))/d**6, Eq(m, -6)
), ((Integral(a**2*(c*x**2)**(3/2)/x**5, x) + Integral(b**2*(c*x**2)**(3/2)
/x**3, x) + Integral(2*a*b*(c*x**2)**(3/2)/x**4, x))/d**5, Eq(m, -5)), ((In
tegral(a**2*(c*x**2)**(3/2)/x**4, x) + Integral(b**2*(c*x**2)**(3/2)/x**2,
x) + Integral(2*a*b*(c*x**2)**(3/2)/x**3, x))/d**4, Eq(m, -4)), (a**2*c**(3
/2)*d**m*m**2*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 11*a**2*
c**(3/2)*d**m*m*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 30*a**
2*c**(3/2)*d**m*x*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) + 2*a*b*
c**(3/2)*d**m*m**2*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120) +
20*a*b*c**(3/2)*d**m*m*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 120
) + 48*a*b*c**(3/2)*d**m*x**2*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*m + 1
20) + b**2*c**(3/2)*d**m*m**2*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 + 74*
m + 120) + 9*b**2*c**(3/2)*d**m*m*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**2 +
 74*m + 120) + 20*b**2*c**(3/2)*d**m*x**3*x**m*(x**2)**(3/2)/(m**3 + 15*m**
2 + 74*m + 120), True))
```

3.916 $\int (dx)^m \sqrt{cx^2} (a+bx)^2 \, dx$

Optimal. Leaf size=94

$$\frac{a^2\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x}+\frac{2ab\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}+\frac{b^2\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}$$

Rubi [A] time = 0.04, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2\sqrt{cx^2}\,(dx)^{m+2}}{d^2(m+2)x}+\frac{2ab\sqrt{cx^2}\,(dx)^{m+3}}{d^3(m+3)x}+\frac{b^2\sqrt{cx^2}\,(dx)^{m+4}}{d^4(m+4)x}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (a^2*(d*x)^(2 + m)*Sqrt[c*x^2])/(d^2*(2 + m)*x) + (2*a*b*(d*x)^(3 + m)*Sqrt
[c*x^2])/(d^3*(3 + m)*x) + (b^2*(d*x)^(4 + m)*Sqrt[c*x^2])/(d^4*(4 + m)*x)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (dx)^m\sqrt{cx^2}\,(a+bx)^2\,dx &= \frac{\sqrt{cx^2}\int x(dx)^m(a+bx)^2\,dx}{x}\\
&= \frac{\sqrt{cx^2}\int (dx)^{1+m}(a+bx)^2\,dx}{dx}\\
&= \frac{\sqrt{cx^2}\int \left(a^2(dx)^{1+m}+\frac{2ab(dx)^{2+m}}{d}+\frac{b^2(dx)^{3+m}}{d^2}\right)dx}{dx}\\
&= \frac{a^2(dx)^{2+m}\sqrt{cx^2}}{d^2(2+m)x}+\frac{2ab(dx)^{3+m}\sqrt{cx^2}}{d^3(3+m)x}+\frac{b^2(dx)^{4+m}\sqrt{cx^2}}{d^4(4+m)x}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 72, normalized size = 0.77

$$\frac{x\sqrt{cx^2}\,(dx)^m\left(a^2\left(m^2+7m+12\right)+2ab\left(m^2+6m+8\right)x+b^2\left(m^2+5m+6\right)x^2\right)}{(m+2)(m+3)(m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] (x*(d*x)^m*Sqrt[c*x^2]*(a^2*(12 + 7*m + m^2) + 2*a*b*(8 + 6*m + m^2)*x + b^
2*(6 + 5*m + m^2)*x^2))/((2 + m)*(3 + m)*(4 + m))
```

IntegrateAlgebraic [F] time = 0.55, size = 0, normalized size = 0.00

$$\int (dx)^m \sqrt{cx^2}\,(a+bx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(d*x)^m*Sqrt[c*x^2]*(a + b*x)^2, x]
```

fricas [A] time = 1.55, size = 94, normalized size = 1.00

$$\frac{\left(\left(b^2m^2+5\,b^2m+6\,b^2\right)x^3+2\left(abm^2+6\,abm+8\,ab\right)x^2+\left(a^2m^2+7\,a^2m+12\,a^2\right)x\right)\sqrt{cx^2}\,(dx)^m}{m^3+9\,m^2+26\,m+24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] ((b^2*m^2 + 5*b^2*m + 6*b^2)*x^3 + 2*(a*b*m^2 + 6*a*b*m + 8*a*b)*x^2 + (a^2
*m^2 + 7*a^2*m + 12*a^2)*x)*sqrt(c*x^2)*(d*x)^m/(m^3 + 9*m^2 + 26*m + 24)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limit
```

maple [A] time = 0.00, size = 95, normalized size = 1.01

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+5b^2m\,x^2+a^2m^2+12abmx+6b^2x^2+7a^2m+16abx+12a^2\right)\sqrt{c\,x^2}\;x\,(dx)^m}{(m+4)\,(m+3)\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+5*b^2*m*x^2+a^2*m^2+12*a*b*m*x+6*b^2*x^2+7*a^2*m
+16*a*b*x+12*a^2)*(d*x)^m*(c*x^2)^(1/2)/(m+4)/(m+3)/(m+2)
```

maxima [A] time = 1.52, size = 64, normalized size = 0.68

$$\frac{b^2\sqrt{c}\,d^mx^4x^m}{m+4}+\frac{2\,ab\sqrt{c}\,d^mx^3x^m}{m+3}+\frac{a^2\sqrt{c}\,d^mx^2x^m}{m+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^(1/2)*(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] b^2*sqrt(c)*d^m*x^4*x^m/(m + 4) + 2*a*b*sqrt(c)*d^m*x^3*x^m/(m + 3) + a^2*s
qrt(c)*d^m*x^2*x^m/(m + 2)
```

mupad [B] time = 0.26, size = 116, normalized size = 1.23

$$(d\,x)^m\,\left(\frac{a^2\,x\,\sqrt{c\,x^2}\,\left(m^2+7\,m+12\right)}{m^3+9\,m^2+26\,m+24}+\frac{b^2\,x^3\,\sqrt{c\,x^2}\,\left(m^2+5\,m+6\right)}{m^3+9\,m^2+26\,m+24}+\frac{2\,a\,b\,x^2\,\sqrt{c\,x^2}\,\left(m^2+6\,m+8\right)}{m^3+9\,m^2+26\,m+24}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^(1/2)*(a + b*x)^2,x)
```

```
[Out] (d*x)^m*((a^2*x*(c*x^2)^(1/2)*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24) +
 (b^2*x^3*(c*x^2)^(1/2)*(5*m + m^2 + 6))/(26*m + 9*m^2 + m^3 + 24) + (2*a*b
*x^2*(c*x^2)^(1/2)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{a^2\sqrt{cx^2}}{x^4}\,dx+\int \frac{b^2\sqrt{cx^2}}{x^2}\,dx+\int \frac{2ab\sqrt{cx^2}}{x^3}\,dx}{d^4} & \text{for } m=-4 \\ \frac{\int \frac{a^2\sqrt{cx^2}}{x^3}\,dx+\int \frac{b^2\sqrt{cx^2}}{x}\,dx+\int \frac{2ab\sqrt{cx^2}}{x^2}\,dx}{d^3} & \text{for } m=-3 \\ \frac{\int b^2\sqrt{cx^2}\,dx+\int \frac{a^2\sqrt{cx^2}}{x^2}\,dx+\int \frac{2ab\sqrt{cx^2}}{x}\,dx}{d^2} & \text{for } m=-2 \\ \frac{a^2\sqrt{c}d^m m^2 x x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{7a^2\sqrt{c}d^m m x x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{12a^2\sqrt{c}d^m x x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{2ab\sqrt{c}d^m m^2x^2x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{12ab\sqrt{c}d^m m x^2x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{16ab\sqrt{c}d^m x^2x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{b^2\sqrt{c}d^m m^2x^3x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{5b^2\sqrt{c}d^m m x^3x^m\sqrt{x^2}}{m^3+9m^2+26m+24}+\frac{6b^2\sqrt{c}d^m x^3x^m\sqrt{x^2}}{m^3+9m^2+26m+24} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**(1/2)*(b*x+a)**2,x)
```

```
[Out] Piecewise(((Integral(a**2*sqrt(c*x**2)/x**4, x) + Integral(b**2*sqrt(c*x**2
)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)/x**3, x))/d**4, Eq(m, -4)), ((Inte
gral(a**2*sqrt(c*x**2)/x**3, x) + Integral(b**2*sqrt(c*x**2)/x, x) + Integr
al(2*a*b*sqrt(c*x**2)/x**2, x))/d**3, Eq(m, -3)), ((Integral(b**2*sqrt(c*x*
*2), x) + Integral(a**2*sqrt(c*x**2)/x**2, x) + Integral(2*a*b*sqrt(c*x**2)
/x, x))/d**2, Eq(m, -2)), (a**2*sqrt(c)*d**m*m**2*x*x**m*sqrt(x**2)/(m**3 +
 9*m**2 + 26*m + 24) + 7*a**2*sqrt(c)*d**m*m*x*x**m*sqrt(x**2)/(m**3 + 9*m*
*2 + 26*m + 24) + 12*a**2*sqrt(c)*d**m*x*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 2
6*m + 24) + 2*a*b*sqrt(c)*d**m*m**2*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 2
6*m + 24) + 12*a*b*sqrt(c)*d**m*m*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*
m + 24) + 16*a*b*sqrt(c)*d**m*x**2*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m +
24) + b**2*sqrt(c)*d**m*m**2*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 2
4) + 5*b**2*sqrt(c)*d**m*m*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 24)
 + 6*b**2*sqrt(c)*d**m*x**3*x**m*sqrt(x**2)/(m**3 + 9*m**2 + 26*m + 24), Tr
ue))
```

3.917 $\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$\frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{m+1}}{d(m+1)\sqrt{cx^2}}+\frac{b^2x(dx)^{m+2}}{d^2(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (a^2*x*(d*x)^m)/(m*Sqrt[c*x^2]) + (2*a*b*x*(d*x)^(1 + m))/(d*(1 + m)*Sqrt[c
*x^2]) + (b^2*x*(d*x)^(2 + m))/(d^2*(2 + m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^2}{x}\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int (dx)^{-1+m}(a+bx)^2\,dx}{\sqrt{cx^2}}\\ &= \frac{(dx)\int \left(a^2(dx)^{-1+m}+\frac{2ab(dx)^m}{d}+\frac{b^2(dx)^{1+m}}{d^2}\right)dx}{\sqrt{cx^2}}\\ &= \frac{a^2x(dx)^m}{m\sqrt{cx^2}}+\frac{2abx(dx)^{1+m}}{d(1+m)\sqrt{cx^2}}+\frac{b^2x(dx)^{2+m}}{d^2(2+m)\sqrt{cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.77

$$\frac{x(dx)^m\left(a^2\left(m^2+3m+2\right)+2abm(m+2)x+b^2m(m+1)x^2\right)}{m(m+1)(m+2)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] (x*(d*x)^m*(a^2*(2 + 3*m + m^2) + 2*a*b*m*(2 + m)*x + b^2*m*(1 + m)*x^2))/(
m*(1 + m)*(2 + m)*Sqrt[c*x^2])
```

IntegrateAlgebraic [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\sqrt{cx^2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2],x]
```

```
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/Sqrt[c*x^2], x]
```

fricas [A] time = 1.18, size = 85, normalized size = 1.05

$$\frac{\left(a^2m^2+3\,a^2m+\left(b^2m^2+b^2m\right)x^2+2\,a^2+2\left(abm^2+2\,abm\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(cm^3+3\,cm^2+2\,cm\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 + 3*a^2*m + (b^2*m^2 + b^2*m)*x^2 + 2*a^2 + 2*(a*b*m^2 + 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c*m^3 + 3*c*m^2 + 2*c*m)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2\,(dx)^m}{\sqrt{cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/sqrt(c*x^2), x)
```

maple [A] time = 0.00, size = 79, normalized size = 0.98

$$\frac{\left(b^2m^2x^2+2ab\,m^2x+b^2m\,x^2+a^2m^2+4abmx+3a^2m+2a^2\right)x\,(dx)^m}{(m+2)\,(m+1)\,\sqrt{c\,x^2}\;m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x+b^2*m*x^2+a^2*m^2+4*a*b*m*x+3*a^2*m+2*a^2)*(d*x)
^m/(m+2)/(m+1)/m/(c*x^2)^(1/2)
```

maxima [A] time = 1.64, size = 57, normalized size = 0.70

$$\frac{b^2d^mx^2x^m}{\sqrt{c}\,(m+2)}+\frac{2\,abd^mxx^m}{\sqrt{c}\,(m+1)}+\frac{a^2d^mx^m}{\sqrt{c}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^2*x^m/(sqrt(c)*(m + 2)) + 2*a*b*d^m*x*x^m/(sqrt(c)*(m + 1)) + a^2
*d^m*x^m/(sqrt(c)*m)
```

mupad [B] time = 0.26, size = 62, normalized size = 0.77

$$\frac{(d\,x)^m\,\left(\frac{a^2\,x}{m}+\frac{b^2\,x^3\,(m+1)}{m^2+3\,m+2}+\frac{2\,a\,b\,x^2\,(m+2)}{m^2+3\,m+2}\right)}{\sqrt{c\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(1/2),x)
```

```
[Out] ((d*x)^m*((a^2*x)/m + (b^2*x^3*(m + 1))/(3*m + m^2 + 2) + (2*a*b*x^2*(m + 2
))/(3*m + m^2 + 2)))/(c*x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{\int \frac{b^2}{\sqrt{cx^2}}\,dx+\int \frac{a^2}{x^2\sqrt{cx^2}}\,dx+\int \frac{2ab}{x\sqrt{cx^2}}\,dx}{d^2} & \text{for}\ m = -2 \\ \frac{\int \frac{2ab}{\sqrt{cx^2}}\,dx+\int \frac{a^2}{x\sqrt{cx^2}}\,dx+\int \frac{b^2x}{\sqrt{cx^2}}\,dx}{d} & \text{for}\ m = -1 \\ \int \frac{(a+bx)^2}{\sqrt{cx^2}}\,dx & \text{for}\ m = 0 \\ \frac{a^2d^mm^2xx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{3a^2d^mmxx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{2a^2d^mxx^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{2abd^mm^2x^2x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{4abd^mmx^2x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{b^2d^mm^2x^3x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} + \frac{b^2d^mmx^3x^m}{\sqrt{c}m^3\sqrt{x^2}+3\sqrt{c}m^2\sqrt{x^2}+2\sqrt{c}m\sqrt{x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(1/2),x)
```

```
[Out] Piecewise(((Integral(b**2/sqrt(c*x**2), x) + Integral(a**2/(x**2*sqrt(c*x**
2)), x) + Integral(2*a*b/(x*sqrt(c*x**2)), x))/d**2, Eq(m, -2)), ((Integral
(2*a*b/sqrt(c*x**2), x) + Integral(a**2/(x*sqrt(c*x**2)), x) + Integral(b**
2*x/sqrt(c*x**2), x))/d, Eq(m, -1)), (Integral((a + b*x)**2/sqrt(c*x**2), x
), Eq(m, 0)), (a**2*d**m*m**2*x*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m
**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 3*a**2*d**m*m*x*x**m/(sqrt(c)*m*
*3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 2*a**
2*d**m*x*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt
(c)*m*sqrt(x**2)) + 2*a*b*d**m*m**2*x**2*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*
sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + 4*a*b*d**m*m*x**2*x**m/
(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**
2)) + b**2*d**m*m**2*x**3*x**m/(sqrt(c)*m**3*sqrt(x**2) + 3*sqrt(c)*m**2*sq
rt(x**2) + 2*sqrt(c)*m*sqrt(x**2)) + b**2*d**m*m*x**3*x**m/(sqrt(c)*m**3*sq
rt(x**2) + 3*sqrt(c)*m**2*sqrt(x**2) + 2*sqrt(c)*m*sqrt(x**2)), True))
```

3.918 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^2x(dx)^{m-2}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{m-1}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
```

```
[Out] -((a^2*d^2*x*(d*x)^(-2 + m))/(c*(2 - m)*Sqrt[c*x^2])) - (2*a*b*d*x*(d*x)^(-
1 + m))/(c*(1 - m)*Sqrt[c*x^2]) + (b^2*x*(d*x)^m)/(c*m*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^2}{x^3}\,dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x\right)\int (dx)^{-3+m}(a+bx)^2\,dx}{c\sqrt{cx^2}}\\
&= \frac{\left(d^3x\right)\int \left(a^2(dx)^{-3+m}+\frac{2ab(dx)^{-2+m}}{d}+\frac{b^2(dx)^{-1+m}}{d^2}\right)dx}{c\sqrt{cx^2}}\\
&= -\frac{a^2d^2x(dx)^{-2+m}}{c(2-m)\sqrt{cx^2}}-\frac{2abdx(dx)^{-1+m}}{c(1-m)\sqrt{cx^2}}+\frac{b^2x(dx)^m}{cm\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.67

$$\frac{x(dx)^m\left(a^2(m-1)m+2ab(m-2)mx+b^2\left(m^2-3m+2\right)x^2\right)}{(m-2)(m-1)m\left(cx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] (x*(d*x)^m*(a^2*(-1 + m)*m + 2*a*b*(-2 + m)*m*x + b^2*(2 - 3*m + m^2)*x^2))
/((-2 + m)*(-1 + m)*m*(c*x^2)^(3/2))
```

IntegrateAlgebraic [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x]
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2), x]
```

fricas [A] time = 1.31, size = 92, normalized size = 0.99

$$\frac{\left(a^2m^2-a^2m+\left(b^2m^2-3\,b^2m+2\,b^2\right)x^2+2\left(abm^2-2\,abm\right)x\right)\sqrt{cx^2}\,(dx)^m}{\left(c^2m^3-3\,c^2m^2+2\,c^2m\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="fricas")
[Out] (a^2*m^2 - a^2*m + (b^2*m^2 - 3*b^2*m + 2*b^2)*x^2 + 2*(a*b*m^2 - 2*a*b*m)*
x)*sqrt(c*x^2)*(d*x)^m/((c^2*m^3 - 3*c^2*m^2 + 2*c^2*m)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2\,(dx)^m}{\left(cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(3/2), x)
```

maple [A] time = 0.01, size = 83, normalized size = 0.89

$$\frac{\left(b^2m^2x^2+2ab\,m^2x-3b^2m\,x^2+a^2m^2-4abmx+2b^2x^2-a^2m\right)x\,(dx)^m}{(m-1)\,(m-2)\left(c\,x^2\right)^{\frac{3}{2}}\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x)
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-3*b^2*m*x^2+a^2*m^2-4*a*b*m*x+2*b^2*x^2-a^2*m)*(
d*x)^m/m/(m-1)/(m-2)/(c*x^2)^(3/2)
```

maxima [A] time = 1.58, size = 59, normalized size = 0.63

$$\frac{b^2d^mx^m}{c^{\frac{3}{2}}m}+\frac{2\,abd^mx^m}{c^{\frac{3}{2}}(m-1)x}+\frac{a^2d^mx^m}{c^{\frac{3}{2}}(m-2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(3/2)*m) + 2*a*b*d^m*x^m/(c^(3/2)*(m - 1)*x) + a^2*d^m*x^m/(
c^(3/2)*(m - 2)*x^2)
```

mupad [B] time = 0.32, size = 66, normalized size = 0.71

$$\frac{a^2 (d x)^m}{c x \sqrt{c x^2}\ (m-2)} + \frac{b (d x)^m\ (2 a m - b x + b m x)}{c m \sqrt{c x^2}\ (m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(3/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c*x*(c*x^2)^(1/2)*(m - 2)) + (b*(d*x)^m*(2*a*m - b*x + b*m*x
))/(c*m*(c*x^2)^(1/2)*(m - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \int \frac{(a+bx)^2}{(cx^2)^{\frac{3}{2}}}\, dx & \text{for } m = 0 \\ d \left(\int \frac{a^2 x}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2 x^3}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^2}{(cx^2)^{\frac{3}{2}}}\, dx \right) & \text{for } m = 1 \\ d^2 \left(\int \frac{a^2 x^2}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{b^2 x^4}{(cx^2)^{\frac{3}{2}}}\, dx + \int \frac{2abx^3}{(cx^2)^{\frac{3}{2}}}\, dx \right) & \text{for } m = 2 \\ \frac{a^2 d^m m^2 x x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} - \frac{a^2 d^m m x x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} + \frac{2abd^m m^2 x^2 x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} - \frac{4abd^m m x^2 x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} + \frac{b^2 d^m m^2 x^3 x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} - \frac{3b^2 d^m m x^3 x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} + \frac{2b^2 d^m x^3 x^m}{c^{\frac{3}{2}} m^3 (x^2)^{\frac{3}{2}} - 3c^{\frac{3}{2}} m^2 (x^2)^{\frac{3}{2}} + 2c^{\frac{3}{2}} m (x^2)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(3/2),x)
```

```
[Out] Piecewise((Integral((a + b*x)**2/(c*x**2)**(3/2), x), Eq(m, 0)), (d*(Integr
al(a**2*x/(c*x**2)**(3/2), x) + Integral(b**2*x**3/(c*x**2)**(3/2), x) + In
tegral(2*a*b*x**2/(c*x**2)**(3/2), x)), Eq(m, 1)), (d**2*(Integral(a**2*x**
2/(c*x**2)**(3/2), x) + Integral(b**2*x**4/(c*x**2)**(3/2), x) + Integral(2
*a*b*x**3/(c*x**2)**(3/2), x)), Eq(m, 2)), (a**2*d**m*m**2*x*x**m/(c**(3/2)
*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**
(3/2)) - a**2*d**m*m*x*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*
(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + 2*a*b*d**m*m**2*x**2*x**m/(c*
*(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x
**2)**(3/2)) - 4*a*b*d**m*m*x**2*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(
3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + b**2*d**m*m**2*x**3
*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(
3/2)*m*(x**2)**(3/2)) - 3*b**2*d**m*m*x**3*x**m/(c**(3/2)*m**3*(x**2)**(3/2
) - 3*c**(3/2)*m**2*(x**2)**(3/2) + 2*c**(3/2)*m*(x**2)**(3/2)) + 2*b**2*d*
*m*x**3*x**m/(c**(3/2)*m**3*(x**2)**(3/2) - 3*c**(3/2)*m**2*(x**2)**(3/2) +
 2*c**(3/2)*m*(x**2)**(3/2)), True))
```

3.919 $\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=105

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Rubi [A] time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {15, 16, 43}

$$-\frac{a^2d^4x(dx)^{m-4}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{m-3}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{m-2}}{c^2(2-m)\sqrt{cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] -((a^2*d^4*x*(d*x)^(-4 + m))/(c^2*(4 - m)*Sqrt[c*x^2])) - (2*a*b*d^3*x*(d*x
)^(-3 + m))/(c^2*(3 - m)*Sqrt[c*x^2]) - (b^2*d^2*x*(d*x)^(-2 + m))/(c^2*(2
- m)*Sqrt[c*x^2])
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx &= \frac{x\int \frac{(dx)^m(a+bx)^2}{x^5}\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{\left(d^5x\right)\int (dx)^{-5+m}(a+bx)^2\,dx}{c^2\sqrt{cx^2}}\\
&= \frac{\left(d^5x\right)\int \left(a^2(dx)^{-5+m}+\frac{2ab(dx)^{-4+m}}{d}+\frac{b^2(dx)^{-3+m}}{d^2}\right)dx}{c^2\sqrt{cx^2}}\\
&= -\frac{a^2d^4x(dx)^{-4+m}}{c^2(4-m)\sqrt{cx^2}}-\frac{2abd^3x(dx)^{-3+m}}{c^2(3-m)\sqrt{cx^2}}-\frac{b^2d^2x(dx)^{-2+m}}{c^2(2-m)\sqrt{cx^2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.69

$$\frac{x(dx)^m \left(a^2 \left(m^2-5m+6\right)+2ab \left(m^2-6m+8\right) x+b^2 \left(m^2-7m+12\right) x^2\right)}{(m-4)(m-3)(m-2) \left(cx^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] (x*(d*x)^m*(a^2*(6 - 5*m + m^2) + 2*a*b*(8 - 6*m + m^2)*x + b^2*(12 - 7*m +
 m^2)*x^2))/((-4 + m)*(-3 + m)*(-2 + m)*(c*x^2)^(5/2))
```

IntegrateAlgebraic [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m(a+bx)^2}{\left(cx^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x]
```

```
[Out] Defer[IntegrateAlgebraic][((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2), x]
```

fricas [A] time = 1.41, size = 106, normalized size = 1.01

$$\frac{\left(a^2m^2-5\,a^2m+\left(b^2m^2-7\,b^2m+12\,b^2\right)x^2+6\,a^2+2\left(abm^2-6\,abm+8\,ab\right)x\right)\sqrt{cx^2}\,\left(dx\right)^m}{\left(c^3m^3-9\,c^3m^2+26\,c^3m-24\,c^3\right)x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] (a^2*m^2 - 5*a^2*m + (b^2*m^2 - 7*b^2*m + 12*b^2)*x^2 + 6*a^2 + 2*(a*b*m^2
- 6*a*b*m + 8*a*b)*x)*sqrt(c*x^2)*(d*x)^m/((c^3*m^3 - 9*c^3*m^2 + 26*c^3*m
- 24*c^3)*x^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^2\,(dx)^m}{\left(cx^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^2*(d*x)^m/(c*x^2)^(5/2), x)
```

maple [A] time = 0.01, size = 95, normalized size = 0.90

$$\frac{\left(b^2m^2x^2+2ab\,m^2x-7b^2m\,x^2+a^2m^2-12abmx+12b^2x^2-5a^2m+16abx+6a^2\right)x\,(dx)^m}{(m-2)\,(m-3)\,(m-4)\left(c\,x^2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x)
```

```
[Out] x*(b^2*m^2*x^2+2*a*b*m^2*x-7*b^2*m*x^2+a^2*m^2-12*a*b*m*x+12*b^2*x^2-5*a^2*
m+16*a*b*x+6*a^2)*(d*x)^m/(m-2)/(m-3)/(m-4)/(c*x^2)^(5/2)
```

maxima [A] time = 1.57, size = 64, normalized size = 0.61

$$\frac{b^2 d^m x^m}{c^{\frac{5}{2}}(m-2)x^2} + \frac{2\,abd^m x^m}{c^{\frac{5}{2}}(m-3)x^3} + \frac{a^2 d^m x^m}{c^{\frac{5}{2}}(m-4)x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(b*x+a)^2/(c*x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] b^2*d^m*x^m/(c^(5/2)*(m - 2)*x^2) + 2*a*b*d^m*x^m/(c^(5/2)*(m - 3)*x^3) + a
^2*d^m*x^m/(c^(5/2)*(m - 4)*x^4)
```

mupad [B] time = 0.34, size = 82, normalized size = 0.78

$$\frac{a^2\,(d\,x)^m}{c^2\,x^3\,\sqrt{c\,x^2}\,\,(m-4)} + \frac{b^2\,(d\,x)^m}{c^2\,x\,\sqrt{c\,x^2}\,\,(m-2)} + \frac{2\,a\,b\,(d\,x)^m}{c^2\,x^2\,\sqrt{c\,x^2}\,\,(m-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(a + b*x)^2)/(c*x^2)^(5/2),x)
```

```
[Out] (a^2*(d*x)^m)/(c^2*x^3*(c*x^2)^(1/2)*(m - 4)) + (b^2*(d*x)^m)/(c^2*x*(c*x^2
)^(1/2)*(m - 2)) + (2*a*b*(d*x)^m)/(c^2*x^2*(c*x^2)^(1/2)*(m - 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(b*x+a)**2/(c*x**2)**(5/2),x)
```

```
[Out] Piecewise((d**2*(Integral(a**2*x**2/(c*x**2)**(5/2), x) + Integral(b**2*x**
4/(c*x**2)**(5/2), x) + Integral(2*a*b*x**3/(c*x**2)**(5/2), x)), Eq(m, 2))
, (d**3*(Integral(a**2*x**3/(c*x**2)**(5/2), x) + Integral(b**2*x**5/(c*x**
2)**(5/2), x) + Integral(2*a*b*x**4/(c*x**2)**(5/2), x)), Eq(m, 3)), (d**4*
(Integral(a**2*x**4/(c*x**2)**(5/2), x) + Integral(b**2*x**6/(c*x**2)**(5/2
), x) + Integral(2*a*b*x**5/(c*x**2)**(5/2), x)), Eq(m, 4)), (a**2*d**m*m**
2*x*x**m/(c**(5/2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*
c**(5/2)*m*(x**2)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) - 5*a**2*d**m*m*x*x**
m/(c**(5/2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2
)*m*(x**2)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) + 6*a**2*d**m*x*x**m/(c**(5/
2)*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2
)**(5/2) - 24*c**(5/2)*(x**2)**(5/2)) + 2*a*b*d**m*m**2*x**2*x**m/(c**(5/2)
*m**3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)*
*(5/2) - 24*c**(5/2)*(x**2)**(5/2)) - 12*a*b*d**m*m*x**2*x**m/(c**(5/2)*m**
3*(x**2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/
2) - 24*c**(5/2)*(x**2)**(5/2)) + 16*a*b*d**m*x**2*x**m/(c**(5/2)*m**3*(x**
2)**(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 2
4*c**(5/2)*(x**2)**(5/2)) + b**2*d**m*m**2*x**3*x**m/(c**(5/2)*m**3*(x**2)*
*(5/2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c
**(5/2)*(x**2)**(5/2)) - 7*b**2*d**m*m*x**3*x**m/(c**(5/2)*m**3*(x**2)**(5/
2) - 9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c**(5
/2)*(x**2)**(5/2)) + 12*b**2*d**m*x**3*x**m/(c**(5/2)*m**3*(x**2)**(5/2) -
9*c**(5/2)*m**2*(x**2)**(5/2) + 26*c**(5/2)*m*(x**2)**(5/2) - 24*c**(5/2)*(
x**2)**(5/2)), True))
```

3.920 $\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx$

Optimal. Leaf size=33

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(p+2)}}{2a(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p)/(2*a*(2 + p)*(a + b*x)^(2*(2 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{3+2p}(a+bx)^{-5-2p}\,dx\\ &= \frac{x^4 \left(cx^2\right)^p (a+bx)^{-2(2+p)}}{2a(2+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{x^4 \left(cx^2\right)^p (a+bx)^{-2p-4}}{a(2p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
```

```
[Out] (x^4*(c*x^2)^p*(a + b*x)^(-4 - 2*p))/(a*(4 + 2*p))
```

IntegrateAlgebraic [F] time = 0.09, size = 0, normalized size = 0.00

$$\int x^3 \left(cx^2\right)^p (a+bx)^{-5-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^3*(c*x^2)^p*(a + b*x)^(-5 - 2*p), x]
```

fricas [A] time = 1.22, size = 40, normalized size = 1.21

$$\frac{\left(bx^5 + ax^4\right)\left(cx^2\right)^p (bx + a)^{-2p-5}}{2\left(ap + 2a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="fricas")
[Out] 1/2*(b*x^5 + a*x^4)*(c*x^2)^p*(b*x + a)^(-2*p - 5)/(a*p + 2*a)
```

giac [B] time = 1.26, size = 74, normalized size = 2.24

$$\frac{\left(cx^2\right)^p bx^5 e^{\left(-2p\log(bx+a)-5\log(bx+a)\right)} + \left(cx^2\right)^p ax^4 e^{\left(-2p\log(bx+a)-5\log(bx+a)\right)}}{2\left(ap + 2a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="giac")
[Out] 1/2*((c*x^2)^p*b*x^5*e^(-2*p*log(b*x + a) - 5*log(b*x + a)) + (c*x^2)^p*a*x
^4*e^(-2*p*log(b*x + a) - 5*log(b*x + a)))/(a*p + 2*a)
```

maple [A] time = 0.00, size = 32, normalized size = 0.97

$$\frac{x^4\left(c\,x^2\right)^p (bx + a)^{-2p-4}}{2\left(p + 2\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x)
[Out] 1/2*(b*x+a)^(-4-2*p)*x^4/a/(2+p)*(c*x^2)^p
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx + a)^{-2p-5} x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^2)^p*(b*x+a)^(-5-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 5)*x^3, x)
```

mupad [B] time = 0.27, size = 33, normalized size = 1.00

$$\frac{x^4\left(c\,x^2\right)^p}{2\,a\,\left(p + 2\right)\,(a + b\,x)^{2p+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c*x^2)^p)/(a + b*x)^(2*p + 5),x)
[Out] (x^4*(c*x^2)^p)/(2*a*(p + 2)*(a + b*x)^(2*p + 4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**2)**p*(b*x+a)**(-5-2*p),x)
```

```
[Out] Timed out
```

3.921 $\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p} \, dx$

Optimal. Leaf size=32

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{-2p-3}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(-3 - 2*p))/(a*(3 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p} \, dx &= \left(x^{-2p} \left(cx^2\right)^p\right) \int x^{2+2p} (a+bx)^{-4-2p} \, dx \\ &= \frac{x^3 \left(cx^2\right)^p (a+bx)^{-3-2p}}{a(3+2p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.06

$$\frac{x^3 \left(cx^2\right)^p (a+bx)^{1-2(p+2)}}{a(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
```

```
[Out] (x^3*(c*x^2)^p*(a + b*x)^(1 - 2*(2 + p)))/(a*(3 + 2*p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x^2 \left(cx^2\right)^p (a+bx)^{-4-2p} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^2*(c*x^2)^p*(a + b*x)^(-4 - 2*p), x]
```

fricas [A] time = 1.23, size = 40, normalized size = 1.25

$$\frac{\left(bx^4+ax^3\right)\left(cx^2\right)^p(bx+a)^{-2p-4}}{2ap+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="fricas")
[Out] (b*x^4 + a*x^3)*(c*x^2)^p*(b*x + a)^(-2*p - 4)/(2*a*p + 3*a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2\right)^p(bx+a)^{-2p-4}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

maple [A] time = 0.00, size = 33, normalized size = 1.03

$$\frac{x^3\left(cx^2\right)^p(bx+a)^{-2p-3}}{\left(2p+3\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^2)^p*(b*x+a)^(-2*p-4),x)
[Out] x^3*(c*x^2)^p*(b*x+a)^(-3-2*p)/a/(3+2*p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2\right)^p(bx+a)^{-2p-4}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^2)^p*(b*x+a)^(-4-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 4)*x^2, x)
```

mupad [B] time = 0.24, size = 34, normalized size = 1.06

$$\frac{x^3\left(cx^2\right)^p}{a\left(2p+3\right)(a+bx)^{2p+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c*x^2)^p)/(a + b*x)^(2*p + 4),x)
[Out] (x^3*(c*x^2)^p)/(a*(2*p + 3)*(a + b*x)^(2*p + 3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**2)**p*(b*x+a)**(-4-2*p),x)
[Out] Timed out
```

3.922 $\int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx$

Optimal. Leaf size=33

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2(p+1)}}{2a(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p)/(2*a*(1 + p)*(a + b*x)^(2*(1 + p)))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{1+2p}(a+bx)^{-3-2p}\,dx\\ &= \frac{x^2\left(cx^2\right)^p (a+bx)^{-2(1+p)}}{2a(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 0.97

$$\frac{x^2 \left(cx^2\right)^p (a+bx)^{-2p-2}}{a(2p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
```

```
[Out] (x^2*(c*x^2)^p*(a + b*x)^(-2 - 2*p))/(a*(2 + 2*p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x \left(cx^2\right)^p (a+bx)^{-3-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(c*x^2)^p*(a + b*x)^(-3 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x*(c*x^2)^p*(a + b*x)^(-3 - 2*p), x]
```

fricas [A] time = 1.51, size = 38, normalized size = 1.15

$$\frac{\left(bx^3+ax^2\right)\left(cx^2\right)^p(bx+a)^{-2p-3}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="fricas")
[Out] 1/2*(b*x^3 + a*x^2)*(c*x^2)^p*(b*x + a)^(-2*p - 3)/(a*p + a)
```

giac [B] time = 1.07, size = 72, normalized size = 2.18

$$\frac{\left(cx^2\right)^p bx^3 e^{\left(-2p\log(bx+a)-3\log(bx+a)\right)}+\left(cx^2\right)^p ax^2 e^{\left(-2p\log(bx+a)-3\log(bx+a)\right)}}{2\left(ap+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="giac")
[Out] 1/2*((c*x^2)^p*b*x^3*e^(-2*p*log(b*x + a) - 3*log(b*x + a)) + (c*x^2)^p*a*x
^2*e^(-2*p*log(b*x + a) - 3*log(b*x + a)))/(a*p + a)
```

maple [A] time = 0.00, size = 32, normalized size = 0.97

$$\frac{x^2\left(c\,x^2\right)^p(bx+a)^{-2p-2}}{2\left(p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2)^p*(b*x+a)^(-2*p-3),x)
[Out] 1/2*(b*x+a)^(-2-2*p)*x^2/a/(1+p)*(c*x^2)^p
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^2\right)^p(bx+a)^{-2p-3}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2)^p*(b*x+a)^(-3-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 3)*x, x)
```

mupad [B] time = 0.22, size = 33, normalized size = 1.00

$$\frac{x^2\left(c\,x^2\right)^p}{2\,a\,\left(p+1\right)\,\left(a+b\,x\right)^{2\,p+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c*x^2)^p)/(a + b*x)^(2*p + 3),x)
[Out] (x^2*(c*x^2)^p)/(2*a*(p + 1)*(a + b*x)^(2*p + 2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2)**p*(b*x+a)**(-3-2*p),x)
```

```
[Out] Timed out
```

3.923 $\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx$

Optimal. Leaf size=30

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 37}

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{a(2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a*(1 + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(a+bx)^{-2-2p}\,dx\\ &= \frac{x\left(cx^2\right)^p (a+bx)^{-1-2p}}{a(1+2p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.93

$$\frac{x\left(cx^2\right)^p (a+bx)^{-2p-1}}{2ap+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
```

```
[Out] (x*(c*x^2)^p*(a + b*x)^(-1 - 2*p))/(a + 2*a*p)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (a+bx)^{-2-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^p*(a + b*x)^(-2 - 2*p),x]
[Out] Defer[IntegrateAlgebraic][(c*x^2)^p*(a + b*x)^(-2 - 2*p), x]
```

fricas [A] time = 1.37, size = 36, normalized size = 1.20

$$\frac{\left(bx^{2}+ax\right)\left(cx^{2}\right)^{p}(bx+a)^{-2\,p-2}}{2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(c*x^2)^p*(b*x + a)^(-2*p - 2)/(2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2}\right)^{p}(bx+a)^{-2\,p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

maple [A] time = 0.00, size = 31, normalized size = 1.03

$$\frac{x\left(c\,x^{2}\right)^{p}(bx+a)^{-2p-1}}{\left(2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p-2),x)
[Out] x*(c*x^2)^p*(b*x+a)^(-1-2*p)/a/(1+2*p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(cx^{2}\right)^{p}(bx+a)^{-2\,p-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-2-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 2), x)
```

mupad [B] time = 0.20, size = 32, normalized size = 1.07

$$\frac{x\left(c\,x^{2}\right)^{p}}{a\left(2\,p+1\right)(a+b\,x)^{2\,p+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(a + b*x)^(2*p + 2),x)
[Out] (x*(c*x^2)^p)/(a*(2*p + 1)*(a + b*x)^(2*p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{b^{-2p}c^{p}x^{-2p}\left(x^{2}\right)^{p}}{b^{2}x} & \text{for}\ a=0 \\ \frac{0^{-2p-2}c^{p}x\left(x^{2}\right)^{p}}{2p+1} & \text{for}\ a=-bx \\ \frac{c^{p}x\left(0^{\frac{1}{p}}\right)^{-2p-2}\left(x^{2}\right)^{p}}{2p+1} & \text{for}\ a=0^{\frac{1}{p}}-bx \\ \int\frac{1}{\sqrt{cx^{2}}\,(a+bx)}\,dx & \text{for}\ p=-\frac{1}{2} \\ \frac{a^{3}c^{p}x\left(x^{2}\right)^{p}}{2a^{5}p(a+bx)^{2p}+a^{5}(a+bx)^{2p}+8a^{4}bpx(a+bx)^{2p}+4a^{4}bx(a+bx)^{2p}+12a^{3}b^{2}px^{2}(a+bx)^{2p}+6a^{3}b^{2}x^{2}(a+bx)^{2p}+8a^{2}b^{3}px^{3}(a+bx)^{2p}+4a^{2}b^{3}x^{3}(a+bx)^{2p}+2ab^{4}px^{4}(a+bx)^{2p}+ab^{4}x^{4}(a+bx)^{2p}}+\frac{2a^{2}bc^{p}x^{2}\left(x^{2}\right)^{p}}{2a^{5}p(a+bx)^{2p}+a^{5}(a+bx)^{2p}+8a^{4}bpx(a+bx)^{2p}+4a^{4}bx(a+bx)^{2p}+12a^{3}b^{2}px^{2}(a+bx)^{2p}+6a^{3}b^{2}x^{2}(a+bx)^{2p}+8a^{2}b^{3}px^{3}(a+bx)^{2p}+4a^{2}b^{3}x^{3}(a+bx)^{2p}+2ab^{4}px^{4}(a+bx)^{2p}+ab^{4}x^{4}(a+bx)^{2p}}+\frac{ab^{2}c^{p}x^{3}\left(x^{2}\right)^{p}}{2a^{5}p(a+bx)^{2p}+a^{5}(a+bx)^{2p}+8a^{4}bpx(a+bx)^{2p}+4a^{4}bx(a+bx)^{2p}+12a^{3}b^{2}px^{2}(a+bx)^{2p}+6a^{3}b^{2}x^{2}(a+bx)^{2p}+8a^{2}b^{3}px^{3}(a+bx)^{2p}+4a^{2}b^{3}x^{3}(a+bx)^{2p}+2ab^{4}px^{4}(a+bx)^{2p}+ab^{4}x^{4}(a+bx)^{2p}}+\frac{bc^{p}x^{2}\left(x^{2}\right)^{p}}{2a^{3}p(a+bx)^{2p}+a^{3}(a+bx)^{2p}+4a^{2}bpx(a+bx)^{2p}+2a^{2}bx(a+bx)^{2p}+2ab^{2}px^{2}(a+bx)^{2p}+ab^{2}x^{2}(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-2-2*p),x)
```

```
[Out] Piecewise((-b**(-2*p)*c**p*x**(-2*p)*(x**2)**p/(b**2*x), Eq(a, 0)), (0**(-2
*p - 2)*c**p*x*(x**2)**p/(2*p + 1), Eq(a, -b*x)), (c**p*x*(0**(1/p))**(-2*p
 - 2)*(x**2)**p/(2*p + 1), Eq(a, 0**(1/p) - b*x)), (Integral(1/(sqrt(c*x**2
)*(a + b*x)), x), Eq(p, -1/2)), (a**3*c**p*x*(x**2)**p/(2*a**5*p*(a + b*x)*
*(2*p) + a**5*(a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x
*(a + b*x)**(2*p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2
*(a + b*x)**(2*p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*
(a + b*x)**(2*p) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)
**(2*p)) + 2*a**2*b*c**p*x**2*(x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(
a + b*x)**(2*p) + 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*
p) + 12*a**3*b**2*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*
p) + 8*a**2*b**3*p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p
) + 2*a*b**4*p*x**4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + a*b*
*2*c**p*x**3*(x**2)**p/(2*a**5*p*(a + b*x)**(2*p) + a**5*(a + b*x)**(2*p) +
 8*a**4*b*p*x*(a + b*x)**(2*p) + 4*a**4*b*x*(a + b*x)**(2*p) + 12*a**3*b**2
*p*x**2*(a + b*x)**(2*p) + 6*a**3*b**2*x**2*(a + b*x)**(2*p) + 8*a**2*b**3*
p*x**3*(a + b*x)**(2*p) + 4*a**2*b**3*x**3*(a + b*x)**(2*p) + 2*a*b**4*p*x*
*4*(a + b*x)**(2*p) + a*b**4*x**4*(a + b*x)**(2*p)) + b*c**p*x**2*(x**2)**p
/(2*a**3*p*(a + b*x)**(2*p) + a**3*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x
)**(2*p) + 2*a**2*b*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p) +
 a*b**2*x**2*(a + b*x)**(2*p)), True))
```

3.924 $\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx$

Optimal. Leaf size=26

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-1+2p}(a+bx)^{-1-2p} dx \\ &= \frac{(cx^2)^p (a+bx)^{-2p}}{2ap} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{(cx^2)^p (a+bx)^{-2p}}{2ap}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
```

```
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{-1-2p}}{x} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(-1 - 2*p))/x, x]
```

fricas [A] time = 1.18, size = 31, normalized size = 1.19

$$\frac{(bx+a)\left(cx^2\right)^p(bx+a)^{-2p-1}}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="fricas")
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p - 1)/(a*p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p(bx+a)^{-2p-1}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p - 1)/x, x)
```

maple [A] time = 0.00, size = 25, normalized size = 0.96

$$\frac{\left(c\,x^2\right)^p(bx+a)^{-2p}}{2ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p-1)/x,x)
[Out] 1/2*(b*x+a)^(-2*p)/a/p*(c*x^2)^p
```

maxima [A] time = 1.45, size = 27, normalized size = 1.04

$$\frac{c^p e^{\left(-2\,p\log(bx+a)+2\,p\log(x)\right)}}{2\,ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(-1-2*p)/x,x, algorithm="maxima")
[Out] 1/2*c^p*e^(-2*p*log(b*x + a) + 2*p*log(x))/(a*p)
```

mupad [B] time = 0.26, size = 26, normalized size = 1.00

$$\frac{\left(c\,x^2\right)^p}{2\,a\,p\,(a+b\,x)^{2\,p}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x*(a + b*x)^(2*p + 1)),x)
[Out] (c*x^2)^p/(2*a*p*(a + b*x)^(2*p))
```

sympy [A] time = 59.69, size = 264, normalized size = 10.15

$$\begin{cases} -\frac{b^{-2p}c^p x^{-2p}(x^2)^p}{bx} & \text{for } a = 0 \\ \frac{0^{-2p-1}c^p(x^2)^p}{2p} & \text{for } a = -bx \\ \frac{c^p\left(0^{\frac{1}{p}}\right)^{-2p-1}(x^2)^p}{2p} & \text{for } a = 0^{\frac{1}{p}} - bx \\ \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x\right)}{a} & \text{for } p = 0 \\ \frac{a^2c^p(x^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}} + \frac{abc^px(x^2)^p}{2a^3p(a+bx)^{2p}+4a^2bpx(a+bx)^{2p}+2ab^2px^2(a+bx)^{2p}} + \frac{bc^px(x^2)^p}{2a^2p(a+bx)^{2p}+2abpx(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(-1-2*p)/x,x)
```

```
[Out] Piecewise((-b**(-2*p)*c**p*x**(-2*p)*(x**2)**p/(b*x), Eq(a, 0)), (0**(-2*p
 - 1)*c**p*(x**2)**p/(2*p), Eq(a, -b*x)), (c**p*(0**(1/p))**(-2*p - 1)*(x**2
)**p/(2*p), Eq(a, 0**(1/p) - b*x)), (log(x)/a - log(a/b + x)/a, Eq(p, 0)),
(a**2*c**p*(x**2)**p/(2*a**3*p*(a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(
2*p) + 2*a*b**2*p*x**2*(a + b*x)**(2*p)) + a*b*c**p*x*(x**2)**p/(2*a**3*p*(
a + b*x)**(2*p) + 4*a**2*b*p*x*(a + b*x)**(2*p) + 2*a*b**2*p*x**2*(a + b*x)
**(2*p)) + b*c**p*x*(x**2)**p/(2*a**2*p*(a + b*x)**(2*p) + 2*a*b*p*x*(a + b
*x)**(2*p)), True))
```

3.925 $\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(1 - 2*p)*x))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-2+2p}(a+bx)^{-2p} dx\\ &= -\frac{(cx^2)^p (a+bx)^{1-2p}}{a(1-2p)x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{1-2p}}{a(2p-1)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*(-1 + 2*p)*x)
```

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{-2p}}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c*x^2)^p/(x^2*(a + b*x)^(2*p)),x]
[Out] Defer[IntegrateAlgebraic][(c*x^2)^p/(x^2*(a + b*x)^(2*p)), x]
```

fricas [A] time = 1.17, size = 37, normalized size = 1.12

$$\frac{(bx+a)\left(cx^2\right)^p}{\left(2\,ap-a\right)(bx+a)^{2\,p}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="fricas")
[Out] (b*x + a)*(c*x^2)^p/((2*a*p - a)*(b*x + a)^(2*p)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p}{(bx+a)^{2\,p}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="giac")
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

maple [A] time = 0.00, size = 38, normalized size = 1.15

$$\frac{(bx+a)\left(c\,x^2\right)^p(bx+a)^{-2p}}{\left(2p-1\right)ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/x^2/((b*x+a)^(2*p)),x)
[Out] (b*x+a)/x/a/(2*p-1)*(c*x^2)^p/((b*x+a)^(2*p))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p}{(bx+a)^{2\,p}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p/x^2/((b*x+a)^(2*p)),x, algorithm="maxima")
[Out] integrate((c*x^2)^p/((b*x + a)^(2*p)*x^2), x)
```

mupad [B] time = 0.24, size = 32, normalized size = 0.97

$$\frac{\left(c\,x^2\right)^p\,(a+b\,x)^{1-2\,p}}{a\,x\,\left(2\,p-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p/(x^2*(a + b*x)^(2*p)),x)
[Out] ((c*x^2)^p*(a + b*x)^(1 - 2*p))/(a*x*(2*p - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} -\frac{\sqrt{c}\sqrt{x^2}}{bx^2} & \text{for } a = 0 \wedge p = \frac{1}{2} \\ -\frac{b^{-2p}c^px^{-2p}\left(x^2\right)^p}{x} & \text{for } a = 0 \\ \int \frac{\sqrt{cx^2}}{x^2(a+bx)}\, dx & \text{for } p = \frac{1}{2} \\ \frac{ac^p\left(x^2\right)^p}{2apx(a+bx)^{2p} - ax(a+bx)^{2p}} + \frac{bc^px\left(x^2\right)^p}{2apx(a+bx)^{2p} - ax(a+bx)^{2p}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p/x**2/((b*x+a)**(2*p)),x)
```

```
[Out] Piecewise((-sqrt(c)*sqrt(x**2)/(b*x**2), Eq(a, 0) & Eq(p, 1/2)), (-b**(-2*p
)*c**p*x**(-2*p)*(x**2)**p/x, Eq(a, 0)), (Integral(sqrt(c*x**2)/(x**2*(a +
b*x)), x), Eq(p, 1/2)), (a*c**p*(x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(
a + b*x)**(2*p)) + b*c**p*x*(x**2)**p/(2*a*p*x*(a + b*x)**(2*p) - a*x*(a +
b*x)**(2*p)), True))
```

3.926 $\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} \, dx$

Optimal. Leaf size=35

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] -((c*x^2)^p*(a + b*x)^(2 - 2*p))/(2*a*(1 - p)*x^2)
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-3+2p}(a+bx)^{1-2p} \, dx \\ &= -\frac{(cx^2)^p (a+bx)^{2-2p}}{2a(1-p)x^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.91

$$\frac{(cx^2)^p (a+bx)^{2-2p}}{a(2p-2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(2 - 2*p))/(a*(-2 + 2*p)*x^2)
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{1-2p}}{x^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3, x]
```

fricas [A] time = 0.88, size = 37, normalized size = 1.06

$$\frac{(bx+a)\left(cx^2\right)^p(bx+a)^{-2p+1}}{2\left(ap-a\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="fricas")
[Out] 1/2*(b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 1)/((a*p - a)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p(bx+a)^{-2p+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

maple [A] time = 0.00, size = 32, normalized size = 0.91

$$\frac{\left(c\,x^2\right)^p(bx+a)^{-2p+2}}{2\left(p-1\right)a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x)
[Out] 1/2*(b*x+a)^(2-2*p)/x^2/a/(p-1)*(c*x^2)^p
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p(bx+a)^{-2p+1}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(1-2*p)/x^3,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 1)/x^3, x)
```

mupad [B] time = 0.25, size = 50, normalized size = 1.43

$$\frac{\left(\frac{\left(c\,x^2\right)^p}{2\,\left(p-1\right)}+\frac{b\,x\,\left(c\,x^2\right)^p}{2\,a\,\left(p-1\right)}\right)\,{\left(a+b\,x\right)}^{1-2\,p}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(1 - 2*p))/x^3,x)
[Out] (((c*x^2)^p/(2*(p - 1)) + (b*x*(c*x^2)^p)/(2*a*(p - 1)))*(a + b*x)^(1 - 2*p
))/x^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (a+bx)^{1-2p}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(1-2*p)/x**3,x)
```

```
[Out] Integral((c*x**2)**p*(a + b*x)**(1 - 2*p)/x**3, x)
```

3.927 $\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} \, dx$

Optimal. Leaf size=33

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {15, 37}

$$-\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] -(((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(3 - 2*p)*x^3))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned} \int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} \, dx &= \left(x^{-2p} (cx^2)^p\right) \int x^{-4+2p}(a+bx)^{2-2p} \, dx \\ &= -\frac{(cx^2)^p (a+bx)^{3-2p}}{a(3-2p)x^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.97

$$\frac{(cx^2)^p (a+bx)^{3-2p}}{a(2p-3)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
```

```
[Out] ((c*x^2)^p*(a + b*x)^(3 - 2*p))/(a*(-3 + 2*p)*x^3)
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(cx^2)^p (a+bx)^{2-2p}}{x^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x]
[Out] Defer[IntegrateAlgebraic][((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4, x]
```

fricas [A] time = 1.32, size = 37, normalized size = 1.12

$$\frac{(bx+a)\left(cx^2\right)^p (bx+a)^{-2p+2}}{\left(2\,ap-3\,a\right)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="fricas")
[Out] (b*x + a)*(c*x^2)^p*(b*x + a)^(-2*p + 2)/((2*a*p - 3*a)*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

maple [A] time = 0.00, size = 33, normalized size = 1.00

$$\frac{\left(c\,x^2\right)^p (bx+a)^{-2p+3}}{\left(2p-3\right)a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2)^p*(b*x+a)^(-2*p+2)/x^4,x)
[Out] (b*x+a)^(3-2*p)/x^3/a/(2*p-3)*(c*x^2)^p
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(cx^2\right)^p (bx+a)^{-2p+2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2)^p*(b*x+a)^(2-2*p)/x^4,x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-2*p + 2)/x^4, x)
```

mupad [B] time = 0.25, size = 51, normalized size = 1.55

$$\frac{\left(\frac{\left(c\,x^2\right)^p}{2\,p-3}+\frac{b\,x\,\left(c\,x^2\right)^p}{a\,\left(2\,p-3\right)}\right)\,{\left(a+b\,x\right)}^{2-2\,p}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x^2)^p*(a + b*x)^(2 - 2*p))/x^4,x)
[Out] (((c*x^2)^p/(2*p - 3) + (b*x*(c*x^2)^p)/(a*(2*p - 3)))*(a + b*x)^(2 - 2*p))
/x^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2)**p*(b*x+a)**(2-2*p)/x**4,x)
[Out] Timed out
```

3.928 $\int x^m (cx^2)^p (a+bx)^{-2-m-2p}\,dx$

Optimal. Leaf size=38

$$\frac{x^{m+1}(cx^2)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {15, 37}

$$\frac{x^{m+1}(cx^2)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]

[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int x^m (cx^2)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}(cx^2)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\ &= \frac{x^{1+m}(cx^2)^p (a+bx)^{-1-m-2p}}{a(1+m+2p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 38, normalized size = 1.00

$$\frac{x^{m+1}(cx^2)^p (a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]

[Out] (x^(1 + m)*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^m (cx^2)^p (a+bx)^{-2-m-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] Defer[IntegrateAlgebraic][x^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p), x]
```

fricas [A] time = 1.42, size = 49, normalized size = 1.29

$$\frac{\left(bx^2+ax\right)(bx+a)^{-m-2\,p-2}x^m e^{\left(p\log(c)+2\,p\log(x)\right)}}{am+2\,ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*x^m*e^(p*log(c) + 2*p*log(x))/(a*m +
 2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2\,p-2}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

maple [A] time = 0.00, size = 39, normalized size = 1.03

$$\frac{x^{m+1}\left(c\,x^2\right)^p (bx+a)^{-m-2p-1}}{\left(m+2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
[Out] x^(m+1)*(c*x^2)^p*(b*x+a)^(-1-m-2*p)/a/(1+m+2*p)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2\,p-2}x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*x^m, x)
```

mupad [B] time = 0.34, size = 50, normalized size = 1.32

$$\frac{x\,x^m\,\left(c\,x^2\right)^p}{a\,(a+b\,x)^m\,(a+b\,x)^{2\,p}\,(a+b\,x)\,\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
[Out] (x*x^m*(c*x^2)^p)/(a*(a + b*x)^m*(a + b*x)^(2*p)*(a + b*x)*(m + 2*p + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
[Out] Timed out
```

3.929 $\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$

Optimal. Leaf size=39

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {15, 20, 37}

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}
\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx &= \left(x^{-2p}\left(cx^2\right)^p\right)\int x^{2p}(dx)^m(a+bx)^{-2-m-2p}\,dx\\
&= \left(x^{-m-2p}(dx)^m\left(cx^2\right)^p\right)\int x^{m+2p}(a+bx)^{-2-m-2p}\,dx\\
&= \frac{x(dx)^m\left(cx^2\right)^p(a+bx)^{-1-m-2p}}{a(1+m+2p)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{x\left(cx^2\right)^p(dx)^m(a+bx)^{-m-2p-1}}{a(m+2p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
```

```
[Out] (x*(d*x)^m*(c*x^2)^p*(a + b*x)^(-1 - m - 2*p))/(a*(1 + m + 2*p))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx)^m \left(cx^2\right)^p (a+bx)^{-2-m-2p}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p),x]
[Out] Defer[IntegrateAlgebraic][(d*x)^m*(c*x^2)^p*(a + b*x)^(-2 - m - 2*p), x]
```

fricas [A] time = 1.36, size = 57, normalized size = 1.46

$$\frac{\left(bx^2+ax\right)(bx+a)^{-m-2p-2}(dx)^m e^{\left(2p\log(dx)+p\log\left(\frac{c}{d^2}\right)\right)}}{am+2ap+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="fricas")
[Out] (b*x^2 + a*x)*(b*x + a)^(-m - 2*p - 2)*(d*x)^m*e^(2*p*log(d*x) + p*log(c/d^
2))/(a*m + 2*a*p + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="giac")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

maple [A] time = 0.00, size = 40, normalized size = 1.03

$$\frac{x\left(c\,x^2\right)^p(dx)^m(bx+a)^{-m-2p-1}}{\left(m+2p+1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x)
[Out] x*(d*x)^m*(c*x^2)^p*(b*x+a)^(-m-2*p-1)/a/(m+2*p+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(cx^2\right)^p (bx+a)^{-m-2p-2}(dx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(c*x^2)^p*(b*x+a)^(-2-m-2*p),x, algorithm="maxima")
[Out] integrate((c*x^2)^p*(b*x + a)^(-m - 2*p - 2)*(d*x)^m, x)
```

mupad [B] time = 0.26, size = 39, normalized size = 1.00

$$\frac{x\,(d\,x)^m\left(c\,x^2\right)^p}{a\,(a+b\,x)^{m+2\,p+1}\left(m+2\,p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(c*x^2)^p)/(a + b*x)^(m + 2*p + 2),x)
[Out] (x*(d*x)^m*(c*x^2)^p)/(a*(a + b*x)^(m + 2*p + 1)*(m + 2*p + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(c*x**2)**p*(b*x+a)**(-2-m-2*p),x)
[Out] Timed out
```

3.930 $\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$\frac{b^2(a+bx)^3}{3d^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int (a+bx)^2\,dx}{d^3}\\ &= \frac{b^2(a+bx)^3}{3d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^2(a+bx)^3}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*(a + b*x)^3)/(3*d^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^5/((a*d)/b + d*x)^3, x]
```

fricas [B] time = 1.19, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

giac [B] time = 1.09, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx + a)^3\,b^2}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(a*d/b+d*x)^3,x)
[Out] 1/3*b^2*(b*x+a)^3/d^3
```

maxima [B] time = 1.37, size = 31, normalized size = 1.82

$$\frac{b^5x^3 + 3\,ab^4x^2 + 3\,a^2b^3x}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] 1/3*(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x)/d^3
```

mupad [B] time = 0.05, size = 27, normalized size = 1.59

$$\frac{b^3\,x\,\left(3\,a^2 + 3\,a\,b\,x + b^2\,x^2\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(d*x + (a*d)/b)^3,x)
[Out] (b^3*x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*d^3)
```

sympy [B] time = 0.12, size = 34, normalized size = 2.00

$$\frac{a^2b^3x}{d^3} + \frac{ab^4x^2}{d^3} + \frac{b^5x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(a*d/b+d*x)**3,x)
[Out] a**2*b**3*x/d**3 + a*b**4*x**2/d**3 + b**5*x**3/(3*d**3)
```

3.931 $\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=23

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] (a*b^3*x)/d^3 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int (a+bx)\, dx}{d^3}\\ &= \frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^3\left(ax+\frac{bx^2}{2}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] (b^3*(a*x + (b*x^2)/2))/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{\left(\frac{ad}{b}+dx\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^4/((a*d)/b + d*x)^3, x]
```

fricas [A] time = 1.25, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

giac [A] time = 1.19, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\left(\frac{1}{2}b\,x^2 + ax\right)b^3}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(a*d/b+d*x)^3,x)
[Out] b^3/d^3*(1/2*b*x^2+a*x)
```

maxima [A] time = 1.24, size = 20, normalized size = 0.87

$$\frac{b^4x^2 + 2\,ab^3x}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] 1/2*(b^4*x^2 + 2*a*b^3*x)/d^3
```

mupad [B] time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^3\,x\,\left(2\,a + b\,x\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(d*x + (a*d)/b)^3,x)
[Out] (b^3*x*(2*a + b*x))/(2*d^3)
```

sympy [A] time = 0.12, size = 20, normalized size = 0.87

$$\frac{ab^3x}{d^3} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(a*d/b+d*x)**3,x)
[Out] a*b**3*x/d**3 + b**4*x**2/(2*d**3)
```

3.932 $\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/((a*d)/b + d*x)^3,x]
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/((a*d)/b + d*x)^3,x]
[Out] (b^3*x)/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^3/((a*d)/b + d*x)^3, x]
```

fricas [A] time = 1.35, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] b^3*x/d^3
```

giac [A] time = 1.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] b^3*x/d^3
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(a*d/b+d*x)^3,x)
[Out] b^3*x/d^3
```

maxima [A] time = 1.27, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] b^3*x/d^3
```

mupad [B] time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(d*x + (a*d)/b)^3,x)
[Out] (b^3*x)/d^3
```

sympy [A] time = 0.11, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(a*d/b+d*x)**3,x)
[Out] b**3*x/d**3
```

3.933 $\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=13

$$\frac{b^2\log(a+bx)}{d^3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 31}

$$\frac{b^2\log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{a+bx}\,dx}{d^3}\\ &= \frac{b^2\log(a+bx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2\log(a+bx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] (b^2*Log[a + b*x])/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/((a*d)/b + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/((a*d)/b + d*x)^3, x]
```

fricas [A] time = 1.22, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] b^2*log(b*x + a)/d^3
```

giac [A] time = 1.07, size = 14, normalized size = 1.08

$$\frac{b^2 \log(|bx + a|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(b*x + a))/d^3
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\frac{b^2 \ln(bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] b^2*ln(b*x+a)/d^3
```

maxima [A] time = 1.31, size = 13, normalized size = 1.00

$$\frac{b^2 \log(bx + a)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/d^3
```

mupad [B] time = 0.05, size = 13, normalized size = 1.00

$$\frac{b^2 \ln(a + b\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(d*x + (a*d)/b)^3,x)
```

```
[Out] (b^2*log(a + b*x))/d^3
```

sympy [A] time = 0.11, size = 19, normalized size = 1.46

$$\frac{b^2 \log\left(ad^3 + bd^3x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] b**2*log(a*d**3 + b*d**3*x)/d**3
```

3.934 $\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=15

$$-\frac{b^2}{d^3(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((a*d)/b + d*x)^3,x]
```

```
[Out] -(b^2/(d^3*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{(a+bx)^2} dx}{d^3} \\ &= -\frac{b^2}{d^3(a+bx)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((a*d)/b + d*x)^3,x]
```

```
[Out] -(b^2/(d^3*(a + b*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{\left(\frac{ad}{b}+dx\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/((a*d)/b + d*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)/((a*d)/b + d*x)^3, x]
```

fricas [A] time = 1.34, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -b^2/(b*d^3*x + a*d^3)
```

giac [A] time = 0.99, size = 15, normalized size = 1.00

$$-\frac{b^2}{(bx + a)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] -b^2/((b*x + a)*d^3)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{b^2}{(bx + a)\, d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(a*d/b+d*x)^3,x)
[Out] -b^2/d^3/(b*x+a)
```

maxima [A] time = 1.34, size = 19, normalized size = 1.27

$$-\frac{b^2}{bd^3x + ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] -b^2/(b*d^3*x + a*d^3)
```

mupad [B] time = 0.04, size = 15, normalized size = 1.00

$$-\frac{b^2}{d^3\,\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(d*x + (a*d)/b)^3,x)
[Out] -b^2/(d^3*(a + b*x))
```

sympy [A] time = 0.18, size = 19, normalized size = 1.27

$$-\frac{b^3}{abd^3 + b^2d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(a*b*d**3 + b**2*d**3*x)
```

3.935 $\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=17

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] -b^2/(3*d^3*(a + b*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{(a+bx)^4} dx}{d^3}\\ &= -\frac{b^2}{3d^3(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{3d^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] -1/3*b^2/(d^3*(a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\left(\frac{ad}{b}+dx\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*((a*d)/b + d*x)^3),x]
[Out] IntegrateAlgebraic[1/((a + b*x)*((a*d)/b + d*x)^3), x]
```

fricas [B] time = 1.33, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="fricas")
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

giac [A] time = 1.19, size = 15, normalized size = 0.88

$$-\frac{b^2}{3\,(bx+a)^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="giac")
[Out] -1/3*b^2/((b*x + a)^3*d^3)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^2}{3\,(bx+a)^3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(a*d/b+d*x)^3,x)
[Out] -1/3*b^2/d^3/(b*x+a)^3
```

maxima [B] time = 1.31, size = 47, normalized size = 2.76

$$-\frac{b^2}{3\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)^3,x, algorithm="maxima")
[Out] -1/3*b^2/(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
```

mupad [B] time = 0.15, size = 49, normalized size = 2.88

$$-\frac{b^2}{3\,\left(a^3\,d^3+3\,a^2\,b\,d^3\,x+3\,a\,b^2\,d^3\,x^2+b^3\,d^3\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)),x)
[Out] -b^2/(3*(a^3*d^3 + b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x))
```

sympy [B] time = 0.29, size = 53, normalized size = 3.12

$$-\frac{b^3}{3a^3bd^3+9a^2b^2d^3x+9ab^3d^3x^2+3b^4d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(3*a**3*b*d**3 + 9*a**2*b**2*d**3*x + 9*a*b**3*d**3*x**2 + 3*b**4*d**
3*x**3)
```

3.936 $\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3} dx$

Optimal. Leaf size=17

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(4*d^3*(a + b*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3} dx &= \frac{b^3 \int \frac{1}{(a+bx)^5} dx}{d^3}\\ &= -\frac{b^2}{4d^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{4d^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/4*b^2/(d^3*(a + b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\left(\frac{ad}{b}+dx\right)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*((a*d)/b + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*((a*d)/b + d*x)^3), x]
```

fricas [B] time = 1.36, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

giac [A] time = 0.91, size = 15, normalized size = 0.88

$$-\frac{b^2}{4\,(bx+a)^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + a)^4*d^3)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^2}{4\,(bx+a)^4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(a*d/b+d*x)^3,x)
```

```
[Out] -1/4*b^2/d^3/(b*x+a)^4
```

maxima [B] time = 1.35, size = 61, normalized size = 3.59

$$-\frac{b^2}{4\left(b^4d^3x^4+4\,ab^3d^3x^3+6\,a^2b^2d^3x^2+4\,a^3bd^3x+a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2/(b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^3*b*d^3*x
 + a^4*d^3)
```

mupad [B] time = 0.06, size = 63, normalized size = 3.71

$$-\frac{b^2}{4\,\left(a^4\,d^3+4\,a^3\,b\,d^3\,x+6\,a^2\,b^2\,d^3\,x^2+4\,a\,b^3\,d^3\,x^3+b^4\,d^3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^2),x)
```

```
[Out] -b^2/(4*(a^4*d^3 + b^4*d^3*x^4 + 4*a*b^3*d^3*x^3 + 6*a^2*b^2*d^3*x^2 + 4*a^
3*b*d^3*x))
```

sympy [B] time = 0.36, size = 68, normalized size = 4.00

$$-\frac{b^3}{4a^4bd^3+16a^3b^2d^3x+24a^2b^3d^3x^2+16ab^4d^3x^3+4b^5d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(4*a**4*b*d**3 + 16*a**3*b**2*d**3*x + 24*a**2*b**3*d**3*x**2 + 16*a*
b**4*d**3*x**3 + 4*b**5*d**3*x**4)
```

3.937 $\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$

Optimal. Leaf size=17

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -b^2/(5*d^3*(a + b*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx &= \frac{b^3\int \frac{1}{(a+bx)^6}\,dx}{d^3}\\ &= -\frac{b^2}{5d^3(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{b^2}{5d^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] -1/5*b^2/(d^3*(a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3\left(\frac{ad}{b}+dx\right)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*((a*d)/b + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*((a*d)/b + d*x)^3), x]
```

fricas [B] time = 0.88, size = 75, normalized size = 4.41

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="fricas")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

giac [A] time = 0.96, size = 15, normalized size = 0.88

$$-\frac{b^2}{5\,(bx+a)^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="giac")
```

```
[Out] -1/5*b^2/((b*x + a)^5*d^3)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{b^2}{5\,(bx+a)^5\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(a*d/b+d*x)^3,x)
```

```
[Out] -1/5*b^2/d^3/(b*x+a)^5
```

maxima [B] time = 1.33, size = 75, normalized size = 4.41

$$-\frac{b^2}{5\left(b^5d^3x^5+5\,ab^4d^3x^4+10\,a^2b^3d^3x^3+10\,a^3b^2d^3x^2+5\,a^4bd^3x+a^5d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(a*d/b+d*x)^3,x, algorithm="maxima")
```

```
[Out] -1/5*b^2/(b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^2*b^3*d^3*x^3 + 10*a^3*b^2*d
^3*x^2 + 5*a^4*b*d^3*x + a^5*d^3)
```

mupad [B] time = 0.05, size = 77, normalized size = 4.53

$$-\frac{b^2}{5\,\left(a^5\,d^3+5\,a^4\,b\,d^3\,x+10\,a^3\,b^2\,d^3\,x^2+10\,a^2\,b^3\,d^3\,x^3+5\,a\,b^4\,d^3\,x^4+b^5\,d^3\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d*x + (a*d)/b)^3*(a + b*x)^3),x)
```

```
[Out] -b^2/(5*(a^5*d^3 + b^5*d^3*x^5 + 5*a*b^4*d^3*x^4 + 10*a^3*b^2*d^3*x^2 + 10*
a^2*b^3*d^3*x^3 + 5*a^4*b*d^3*x))
```

sympy [B] time = 0.42, size = 83, normalized size = 4.88

$$-\frac{b^3}{5a^5bd^3 + 25a^4b^2d^3x + 50a^3b^3d^3x^2 + 50a^2b^4d^3x^3 + 25ab^5d^3x^4 + 5b^6d^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(a*d/b+d*x)**3,x)
```

```
[Out] -b**3/(5*a**5*b*d**3 + 25*a**4*b**2*d**3*x + 50*a**3*b**3*d**3*x**2 + 50*a*
*2*b**4*d**3*x**3 + 25*a*b**5*d**3*x**4 + 5*b**6*d**3*x**5)
```

3.938 $\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3} dx$

Optimal. Leaf size=17

$$\frac{b^5(c+dx)^3}{3d^6}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3} dx &= \frac{b^5 \int (c+dx)^2 dx}{d^5} \\ &= \frac{b^5(c+dx)^3}{3d^6} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (b^5*(c + d*x)^3)/(3*d^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^5}{(c+dx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^5/(c + d*x)^3,x]
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^5/(c + d*x)^3, x]
```

fricas [B] time = 1.25, size = 35, normalized size = 2.06

$$\frac{b^5 d^2 x^3 + 3\, b^5 c d x^2 + 3\, b^5 c^2 x}{3\, d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="fricas")
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

giac [B] time = 1.01, size = 35, normalized size = 2.06

$$\frac{b^5 d^2 x^3 + 3\, b^5 c d x^2 + 3\, b^5 c^2 x}{3\, d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="giac")
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{(dx + c)^3\, b^5}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^5/(d*x+c)^3,x)
[Out] 1/3*b^5*(d*x+c)^3/d^6
```

maxima [B] time = 1.37, size = 35, normalized size = 2.06

$$\frac{b^5 d^2 x^3 + 3\, b^5 c d x^2 + 3\, b^5 c^2 x}{3\, d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^5/(d*x+c)^3,x, algorithm="maxima")
[Out] 1/3*(b^5*d^2*x^3 + 3*b^5*c*d*x^2 + 3*b^5*c^2*x)/d^5
```

mupad [B] time = 0.16, size = 27, normalized size = 1.59

$$\frac{b^5\, x\, \left(3\, c^2 + 3\, c\, d\, x + d^2\, x^2\right)}{3\, d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^5/(c + d*x)^3,x)
[Out] (b^5*x*(3*c^2 + d^2*x^2 + 3*c*d*x))/(3*d^5)
```

sympy [B] time = 0.13, size = 34, normalized size = 2.00

$$\frac{b^5 c^2 x}{d^5} + \frac{b^5 c x^2}{d^4} + \frac{b^5 x^3}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**5/(d*x+c)**3,x)
[Out] b**5*c**2*x/d**5 + b**5*c*x**2/d**4 + b**5*x**3/(3*d**3)
```

3.939 $\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx$

Optimal. Leaf size=23

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {21}

$$\frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*c*x)/d^4 + (b^4*x^2)/(2*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx &= \frac{b^4\int (c+dx)\,dx}{d^4}\\ &= \frac{b^4cx}{d^4}+\frac{b^4x^2}{2d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 0.83

$$\frac{b^4\left(cx+\frac{dx^2}{2}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (b^4*(c*x + (d*x^2)/2))/d^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^4}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^4/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^4/(c + d*x)^3, x]
```

fricas [A] time = 1.26, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="fricas")
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

giac [A] time = 1.04, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="giac")
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\left(\frac{1}{2}d\,x^2 + cx\right)b^4}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^4/(d*x+c)^3,x)
[Out] b^4/d^4*(c*x+1/2*d*x^2)
```

maxima [A] time = 1.32, size = 21, normalized size = 0.91

$$\frac{b^4dx^2 + 2\,b^4cx}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^4/(d*x+c)^3,x, algorithm="maxima")
[Out] 1/2*(b^4*d*x^2 + 2*b^4*c*x)/d^4
```

mupad [B] time = 0.03, size = 16, normalized size = 0.70

$$\frac{b^4\,x\,\left(2\,c + d\,x\right)}{2\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^4/(c + d*x)^3,x)
[Out] (b^4*x*(2*c + d*x))/(2*d^4)
```

sympy [A] time = 0.12, size = 20, normalized size = 0.87

$$\frac{b^4cx}{d^4} + \frac{b^4x^2}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**4/(d*x+c)**3,x)
[Out] b**4*c*x/d**4 + b**4*x**2/(2*d**3)
```

3.940 $\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$

Optimal. Leaf size=8

$$\frac{b^3x}{d^3}$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 8}

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx &= \frac{b^3\int 1\,dx}{d^3}\\ &= \frac{b^3x}{d^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^3}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^3/(c + d*x)^3,x]
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^3/(c + d*x)^3, x]
```

fricas [A] time = 1.19, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
[Out] b^3*x/d^3
```

giac [A] time = 1.05, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
[Out] b^3*x/d^3
```

maple [A] time = 0.00, size = 9, normalized size = 1.12

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^3/(d*x+c)^3,x)
[Out] b^3/d^3*x
```

maxima [A] time = 1.33, size = 8, normalized size = 1.00

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
[Out] b^3*x/d^3
```

mupad [B] time = 0.01, size = 8, normalized size = 1.00

$$\frac{b^3\,x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^3/(c + d*x)^3,x)
[Out] (b^3*x)/d^3
```

sympy [A] time = 0.10, size = 7, normalized size = 0.88

$$\frac{b^3x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**3/(d*x+c)**3,x)
[Out] b**3*x/d**3
```

3.941 $\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} dx$

Optimal. Leaf size=13

$$\frac{b^2 \log(c+dx)}{d^3}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 31}

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} dx &= \frac{b^2 \int \frac{1}{c+dx} dx}{d^2} \\
&= \frac{b^2 \log(c+dx)}{d^3}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{b^2 \log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (b^2*Log[c + d*x])/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{bc}{d}+bx\right)^2}{(c+dx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)^2/(c + d*x)^3,x]
[Out] IntegrateAlgebraic[((b*c)/d + b*x)^2/(c + d*x)^3, x]
```

fricas [A] time = 1.27, size = 13, normalized size = 1.00

$$\frac{b^2 \log (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
[Out] b^2*log(d*x + c)/d^3
```

giac [A] time = 0.89, size = 14, normalized size = 1.08

$$\frac{b^2 \log (|dx + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
[Out] b^2*log(abs(d*x + c))/d^3
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\frac{b^2 \ln (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)^2/(d*x+c)^3,x)
[Out] b^2*ln(d*x+c)/d^3
```

maxima [A] time = 1.37, size = 13, normalized size = 1.00

$$\frac{b^2 \log (dx + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
[Out] b^2*log(d*x + c)/d^3
```

mupad [B] time = 0.14, size = 13, normalized size = 1.00

$$\frac{b^2 \ln (c + d\,x)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)^2/(c + d*x)^3,x)
[Out] (b^2*log(c + d*x))/d^3
```

sympy [A] time = 0.10, size = 17, normalized size = 1.31

$$\frac{b^2 \log \left(cd^2 + d^3 x\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)**2/(d*x+c)**3,x)
[Out] b**2*log(c*d**2 + d**3*x)/d**3
```

3.942 $\int \frac{\frac{bc}{d}+bx}{(c+dx)^3} dx$

Optimal. Leaf size=13

$$-\frac{b}{d^2(c+dx)}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\frac{bc}{d}+bx}{(c+dx)^3} dx &= \frac{b\int \frac{1}{(c+dx)^2} dx}{d} \\ &= -\frac{b}{d^2(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{b}{d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*c)/d + b*x)/(c + d*x)^3,x]
```

```
[Out] -(b/(d^2*(c + d*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{bc}{d}+bx}{(c+dx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((b*c)/d + b*x)/(c + d*x)^3,x]
[Out] IntegrateAlgebraic[((b*c)/d + b*x)/(c + d*x)^3, x]
```

fricas [A] time = 1.20, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
[Out] -b/(d^3*x + c*d^2)
```

giac [A] time = 1.00, size = 13, normalized size = 1.00

$$-\frac{b}{(dx + c)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
[Out] -b/((d*x + c)*d^2)
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$-\frac{b}{(dx + c)\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*c/d+b*x)/(d*x+c)^3,x)
[Out] -b/d^2/(d*x+c)
```

maxima [A] time = 1.38, size = 16, normalized size = 1.23

$$-\frac{b}{d^3x + cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
[Out] -b/(d^3*x + c*d^2)
```

mupad [B] time = 0.04, size = 13, normalized size = 1.00

$$-\frac{b}{d^2\,\left(c + d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + (b*c)/d)/(c + d*x)^3,x)
[Out] -b/(d^2*(c + d*x))
```

sympy [A] time = 0.15, size = 12, normalized size = 0.92

$$-\frac{b}{cd^2 + d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*c/d+b*x)/(d*x+c)**3,x)
[Out] -b/(c*d**2 + d**3*x)
```

3.943 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{3b(c+dx)^3}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/(3*b*(c + d*x)^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx &= \frac{d\int \frac{1}{(c+dx)^4}\,dx}{b}\\ &= -\frac{1}{3b(c+dx)^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{3b(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
```

```
[Out] -1/3*1/(b*(c + d*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)*(c + d*x)^3),x]
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)*(c + d*x)^3), x]
```

fricas [B] time = 1.29, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="fricas")
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

giac [A] time = 1.14, size = 12, normalized size = 0.86

$$-\frac{1}{3\,(dx+c)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="giac")
[Out] -1/3/((d*x + c)^3*b)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{3\,(dx+c)^3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)/(d*x+c)^3,x)
[Out] -1/3/b/(d*x+c)^3
```

maxima [B] time = 1.34, size = 36, normalized size = 2.57

$$-\frac{1}{3\left(bd^3x^3 + 3\,bcd^2x^2 + 3\,bc^2dx + bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/3/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
```

mupad [B] time = 0.05, size = 38, normalized size = 2.71

$$-\frac{1}{3\,b\,c^3+9\,b\,c^2\,d\,x+9\,b\,c\,d^2\,x^2+3\,b\,d^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)*(c + d*x)^3),x)
[Out] -1/(3*b*c^3 + 3*b*d^3*x^3 + 9*b*c^2*d*x + 9*b*c*d^2*x^2)
```

sympy [B] time = 0.28, size = 44, normalized size = 3.14

$$-\frac{d}{3bc^3d + 9bc^2d^2x + 9bcd^3x^2 + 3bd^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)/(d*x+c)**3,x)
[Out] -d/(3*b*c**3*d + 9*b*c**2*d**2*x + 9*b*c*d**3*x**2 + 3*b*d**4*x**3)
```

3.944 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx$

Optimal. Leaf size=15

$$-\frac{d}{4b^2(c+dx)^4}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -d/(4*b^2*(c + d*x)^4)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx &= \frac{d^2 \int \frac{1}{(c+dx)^5} \, dx}{b^2}\\ &= -\frac{d}{4b^2(c+dx)^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$-\frac{d}{4b^2(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/4*d/(b^2*(c + d*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)^2 (c+dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)^2*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)^2*(c + d*x)^3), x]
```

fricas [B] time = 1.31, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^{2}d^{4}x^{4}+4\,b^{2}cd^{3}x^{3}+6\,b^{2}c^{2}d^{2}x^{2}+4\,b^{2}c^{3}dx+b^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

giac [A] time = 0.98, size = 20, normalized size = 1.33

$$-\frac{b^{2}}{4\left(bx+\frac{bc}{d}\right)^{4}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/4*b^2/((b*x + b*c/d)^4*d^3)
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$-\frac{d}{4\left(dx+c\right)^{4}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^2/(d*x+c)^3,x)
```

```
[Out] -1/4*d/b^2/(d*x+c)^4
```

maxima [B] time = 1.35, size = 59, normalized size = 3.93

$$-\frac{d}{4\left(b^{2}d^{4}x^{4}+4\,b^{2}cd^{3}x^{3}+6\,b^{2}c^{2}d^{2}x^{2}+4\,b^{2}c^{3}dx+b^{2}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*d/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +
 b^2*c^4)
```

mupad [B] time = 0.05, size = 61, normalized size = 4.07

$$-\frac{d}{4\,\left(b^2\,c^4+4\,b^2\,c^3\,d\,x+6\,b^2\,c^2\,d^2\,x^2+4\,b^2\,c\,d^3\,x^3+b^2\,d^4\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^2*(c + d*x)^3),x)
```

```
[Out] -d/(4*(b^2*c^4 + b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x))
```

sympy [B] time = 0.36, size = 68, normalized size = 4.53

$$-\frac{d^2}{4b^2c^4d+16b^2c^3d^2x+24b^2c^2d^3x^2+16b^2cd^4x^3+4b^2d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**2/(d*x+c)**3,x)
```

```
[Out] -d**2/(4*b**2*c**4*d + 16*b**2*c**3*d**2*x + 24*b**2*c**2*d**3*x**2 + 16*b*
*2*c*d**4*x**3 + 4*b**2*d**5*x**4)
```

3.945 $\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3} \, dx$

Optimal. Leaf size=17

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -d^2/(5*b^3*(c + d*x)^5)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3} \, dx &= \frac{d^3 \int \frac{1}{(c+dx)^6} \, dx}{b^3} \\ &= -\frac{d^2}{5b^3(c+dx)^5} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$-\frac{d^2}{5b^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -1/5*d^2/(b^3*(c + d*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{bc}{d}+bx\right)^3 (c+dx)^3} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(((b*c)/d + b*x)^3*(c + d*x)^3),x]
[Out] IntegrateAlgebraic[1/(((b*c)/d + b*x)^3*(c + d*x)^3), x]
```

fricas [B] time = 1.36, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5 + 5\,b^3cd^4x^4 + 10\,b^3c^2d^3x^3 + 10\,b^3c^3d^2x^2 + 5\,b^3c^4dx + b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="fricas")
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

giac [A] time = 0.92, size = 15, normalized size = 0.88

$$-\frac{d^2}{5\,(dx+c)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="giac")
[Out] -1/5*d^2/((d*x + c)^5*b^3)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{d^2}{5\,(dx+c)^5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*c/d+b*x)^3/(d*x+c)^3,x)
[Out] -1/5*d^2/b^3/(d*x+c)^5
```

maxima [B] time = 1.36, size = 75, normalized size = 4.41

$$-\frac{d^2}{5\left(b^3d^5x^5 + 5\,b^3cd^4x^4 + 10\,b^3c^2d^3x^3 + 10\,b^3c^3d^2x^2 + 5\,b^3c^4dx + b^3c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)^3/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/5*d^2/(b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^2*d^3*x^3 + 10*b^3*c^3*d
^2*x^2 + 5*b^3*c^4*d*x + b^3*c^5)
```

mupad [B] time = 0.17, size = 77, normalized size = 4.53

$$-\frac{d^2}{5\,\left(b^3\,c^5+5\,b^3\,c^4\,d\,x+10\,b^3\,c^3\,d^2\,x^2+10\,b^3\,c^2\,d^3\,x^3+5\,b^3\,c\,d^4\,x^4+b^3\,d^5\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + (b*c)/d)^3*(c + d*x)^3),x)
[Out] -d^2/(5*(b^3*c^5 + b^3*d^5*x^5 + 5*b^3*c*d^4*x^4 + 10*b^3*c^3*d^2*x^2 + 10*
b^3*c^2*d^3*x^3 + 5*b^3*c^4*d*x))
```

sympy [B] time = 0.42, size = 83, normalized size = 4.88

$$-\frac{d^3}{5b^3c^5d + 25b^3c^4d^2x + 50b^3c^3d^3x^2 + 50b^3c^2d^4x^3 + 25b^3cd^5x^4 + 5b^3d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*c/d+b*x)**3/(d*x+c)**3,x)
```

```
[Out] -d**3/(5*b**3*c**5*d + 25*b**3*c**4*d**2*x + 50*b**3*c**3*d**3*x**2 + 50*b*
*3*c**2*d**4*x**3 + 25*b**3*c*d**5*x**4 + 5*b**3*d**6*x**5)
```

3.946 $\int (a+bx)^5 (ac+bcx)^n \, dx$

Optimal. Leaf size=24

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(ac+bcx)^{n+6}}{bc^6(n+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] (a*c + b*c*x)^(6 + n)/(b*c^6*(6 + n))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int (a+bx)^5 (ac+bcx)^n \, dx &= \frac{\int (ac+bcx)^{5+n} \, dx}{c^5} \\ &= \frac{(ac+bcx)^{6+n}}{bc^6(6+n)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.04

$$\frac{(a+bx)^6 (c(a+bx))^n}{b(n+6)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] ((a + b*x)^6*(c*(a + b*x))^n)/(b*(6 + n))
```

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a+bx)^5 (ac+bcx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^5*(a*c + b*c*x)^n, x]
```

fricas [B] time = 1.00, size = 80, normalized size = 3.33

$$\frac{\left(b^{6} x^{6} + 6 \, a b^{5} x^{5} + 15 \, a^{2} b^{4} x^{4} + 20 \, a^{3} b^{3} x^{3} + 15 \, a^{4} b^{2} x^{2} + 6 \, a^{5} b x + a^{6}\right)(b c x + a c)^{n}}{b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] (b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*x^2 +
 6*a^5*b*x + a^6)*(b*c*x + a*c)^n/(b*n + 6*b)
```

giac [B] time = 1.13, size = 141, normalized size = 5.88

$$\frac{(b c x + a c)^{n} b^{6} x^{6} + 6 \,(b c x + a c)^{n} a b^{5} x^{5} + 15 \,(b c x + a c)^{n} a^{2} b^{4} x^{4} + 20 \,(b c x + a c)^{n} a^{3} b^{3} x^{3} + 15 \,(b c x + a c)^{n} a^{4} b^{2} x^{2} + 6 \,(b c x + a c)^{n} a^{5} b x + (b c x + a c)^{n} a^{6}}{b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((b*c*x + a*c)^n*b^6*x^6 + 6*(b*c*x + a*c)^n*a*b^5*x^5 + 15*(b*c*x + a*c)^n
*a^2*b^4*x^4 + 20*(b*c*x + a*c)^n*a^3*b^3*x^3 + 15*(b*c*x + a*c)^n*a^4*b^2*
x^2 + 6*(b*c*x + a*c)^n*a^5*b*x + (b*c*x + a*c)^n*a^6)/(b*n + 6*b)
```

maple [A] time = 0.00, size = 27, normalized size = 1.12

$$\frac{(b x + a)^{6} (b c x + a c)^{n}}{(n + 6) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^n,x)
```

```
[Out] (b*x+a)^6/b/(6+n)*(b*c*x+a*c)^n
```

maxima [B] time = 1.81, size = 649, normalized size = 27.04

$$\frac{5 \left(b^{2} c^{n} (n + 1) x^{2} + a b c^{n} n x - a^{2} c^{n}\right)(b x + a)^{n} a^{4}}{\left(n^{2} + 3 n + 2\right) b} + \frac{10 \left(\left(n^{2} + 3 n + 2\right) b^{3} c^{n} x^{3} + \left(n^{2} + n\right) a b^{2} c^{n} x^{2} - 2 a^{2} b c^{n} n x + 2 a^{3} c^{n}\right)(b x + a)^{n} a^{3}}{\left(n^{3} + 6 n^{2} + 11 n + 6\right) b} + \frac{(b c x + a c)^{n + 1} a^{5}}{b c (n + 1)} + \frac{10 \left(\left(n^{3} + 6 n^{2} + 11 n + 6\right) b^{4} c^{n} x^{4} + \left(n^{3} + 3 n^{2} + 2 n\right) a b^{3} c^{n} x^{3} - 3 \left(n^{2} + n\right) a^{2} b^{2} c^{n} x^{2} + 6 a^{3} b c^{n} n x - 6 a^{4} c^{n}\right)(b x + a)^{n} a^{2}}{\left(n^{4} + 10 n^{3} + 35 n^{2} + 50 n + 24\right) b} + \frac{5 \left(\left(n^{4} + 10 n^{3} + 35 n^{2} + 50 n + 24\right) b^{5} c^{n} x^{5} + \left(n^{4} + 6 n^{3} + 11 n^{2} + 6 n\right) a b^{4} c^{n} x^{4} - 4 \left(n^{3} + 3 n^{2} + 2 n\right) a^{2} b^{3} c^{n} x^{3} + 12 \left(n^{2} + n\right) a^{3} b^{2} c^{n} x^{2} - 24 a^{4} b c^{n} n x + 24 a^{5} c^{n}\right)(b x + a)^{n} a}{\left(n^{5} + 15 n^{4} + 85 n^{3} + 225 n^{2} + 274 n + 120\right) b} + \frac{\left(\left(n^{5} + 15 n^{4} + 85 n^{3} + 225 n^{2} + 274 n + 120\right) b^{6} c^{n} x^{6} + \left(n^{5} + 10 n^{4} + 35 n^{3} + 50 n^{2} + 24 n\right) a b^{5} c^{n} x^{5} - 5 \left(n^{4} + 6 n^{3} + 11 n^{2} + 6 n\right) a^{2} b^{4} c^{n} x^{4} + 20 \left(n^{3} + 3 n^{2} + 2 n\right) a^{3} b^{3} c^{n} x^{3} - 60 \left(n^{2} + n\right) a^{4} b^{2} c^{n} x^{2} + 120 a^{5} b c^{n} n x - 120 a^{6} c^{n}\right)(b x + a)^{n}}{\left(n^{6} + 21 n^{5} + 175 n^{4} + 735 n^{3} + 1624 n^{2} + 1764 n + 720\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 5*(b^2*c^n*(n + 1)*x^2 + a*b*c^n*n*x - a^2*c^n)*(b*x + a)^n*a^4/((n^2 + 3*n
 + 2)*b) + 10*((n^2 + 3*n + 2)*b^3*c^n*x^3 + (n^2 + n)*a*b^2*c^n*x^2 - 2*a^
2*b*c^n*n*x + 2*a^3*c^n)*(b*x + a)^n*a^3/((n^3 + 6*n^2 + 11*n + 6)*b) + (b*
c*x + a*c)^(n + 1)*a^5/(b*c*(n + 1)) + 10*((n^3 + 6*n^2 + 11*n + 6)*b^4*c^n
*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*c^n*x^3 - 3*(n^2 + n)*a^2*b^2*c^n*x^2 + 6*
a^3*b*c^n*n*x - 6*a^4*c^n)*(b*x + a)^n*a^2/((n^4 + 10*n^3 + 35*n^2 + 50*n +
 24)*b) + 5*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*c^n*x^5 + (n^4 + 6*n^3
 + 11*n^2 + 6*n)*a*b^4*c^n*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*c^n*x^3 + 12
*(n^2 + n)*a^3*b^2*c^n*x^2 - 24*a^4*b*c^n*n*x + 24*a^5*c^n)*(b*x + a)^n*a/(
(n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b) + ((n^5 + 15*n^4 + 85*n^
3 + 225*n^2 + 274*n + 120)*b^6*c^n*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 +
24*n)*a*b^5*c^n*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*c^n*x^4 + 20*(
n^3 + 3*n^2 + 2*n)*a^3*b^3*c^n*x^3 - 60*(n^2 + n)*a^4*b^2*c^n*x^2 + 120*a^5
*b*c^n*n*x - 120*a^6*c^n)*(b*x + a)^n/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 +
1624*n^2 + 1764*n + 720)*b)
```

mupad [B] time = 0.33, size = 107, normalized size = 4.46

$$(a\,c+b\,c\,x)^n\,\left(\frac{a^6}{b\,\left(n+6\right)}+\frac{b^5\,x^6}{n+6}+\frac{6\,a^5\,x}{n+6}+\frac{15\,a^4\,b\,x^2}{n+6}+\frac{6\,a\,b^4\,x^5}{n+6}+\frac{20\,a^3\,b^2\,x^3}{n+6}+\frac{15\,a^2\,b^3\,x^4}{n+6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^n*(a + b*x)^5,x)
```

```
[Out] (a*c + b*c*x)^n*(a^6/(b*(n + 6)) + (b^5*x^6)/(n + 6) + (6*a^5*x)/(n + 6) +
(15*a^4*b*x^2)/(n + 6) + (6*a*b^4*x^5)/(n + 6) + (20*a^3*b^2*x^3)/(n + 6) +
 (15*a^2*b^3*x^4)/(n + 6))
```

sympy [A] time = 2.29, size = 212, normalized size = 8.83

$$\begin{cases} \frac{x}{ac^6} & \text{for } b = 0 \wedge n = -6 \\ a^5 x (ac)^n & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + x\right)}{bc^6} & \text{for } n = -6 \\ \frac{a^6(ac+bcx)^n}{bn+6b} + \frac{6a^5bx(ac+bcx)^n}{bn+6b} + \frac{15a^4b^2x^2(ac+bcx)^n}{bn+6b} + \frac{20a^3b^3x^3(ac+bcx)^n}{bn+6b} + \frac{15a^2b^4x^4(ac+bcx)^n}{bn+6b} + \frac{6ab^5x^5(ac+bcx)^n}{bn+6b} + \frac{b^6x^6(ac+bcx)^n}{bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((x/(a*c**6), Eq(b, 0) & Eq(n, -6)), (a**5*x*(a*c)**n, Eq(b, 0)),
(log(a/b + x)/(b*c**6), Eq(n, -6)), (a**6*(a*c + b*c*x)**n/(b*n + 6*b) + 6*
a**5*b*x*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**4*b**2*x**2*(a*c + b*c*x)**n/
(b*n + 6*b) + 20*a**3*b**3*x**3*(a*c + b*c*x)**n/(b*n + 6*b) + 15*a**2*b**4
*x**4*(a*c + b*c*x)**n/(b*n + 6*b) + 6*a*b**5*x**5*(a*c + b*c*x)**n/(b*n +
6*b) + b**6*x**6*(a*c + b*c*x)**n/(b*n + 6*b), True))
```

3.947 $\int (a+bx)^5(ac+bcx)^3\,dx$

Optimal. Leaf size=17

$$\frac{c^3(a+bx)^9}{9b}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^3,x]
[Out] (c^3*(a + b*x)^9)/(9*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^3\,dx &= c^3\int (a+bx)^8\,dx\\ &= \frac{c^3(a+bx)^9}{9b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^3(a+bx)^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^3,x]
[Out] (c^3*(a + b*x)^9)/(9*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^3, x]
```

fricas [B] time = 1.25, size = 113, normalized size = 6.65

$$\frac{1}{9}x^9c^3b^8 + x^8c^3b^7a + 4x^7c^3b^6a^2 + \frac{28}{3}x^6c^3b^5a^3 + 14x^5c^3b^4a^4 + 14x^4c^3b^3a^5 + \frac{28}{3}x^3c^3b^2a^6 + 4x^2c^3ba^7 + xc^3a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^3*b^8 + x^8*c^3*b^7*a + 4*x^7*c^3*b^6*a^2 + 28/3*x^6*c^3*b^5*a^3
+ 14*x^5*c^3*b^4*a^4 + 14*x^4*c^3*b^3*a^5 + 28/3*x^3*c^3*b^2*a^6 + 4*x^2*c^
3*b*a^7 + x*c^3*a^8
```

giac [B] time = 0.96, size = 113, normalized size = 6.65

$$\frac{1}{9}b^8c^3x^9 + ab^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

maple [B] time = 0.00, size = 114, normalized size = 6.71

$$\frac{1}{9}b^8c^3x^9 + a\,b^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7b\,c^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^3,x)
```

```
[Out] 1/9*b^8*c^3*x^9+a*b^7*c^3*x^8+4*a^2*b^6*c^3*x^7+28/3*a^3*b^5*c^3*x^6+14*a^4
*b^4*c^3*x^5+14*a^5*b^3*c^3*x^4+28/3*a^6*c^3*b^2*x^3+4*a^7*c^3*b*x^2+a^8*c^
3*x
```

maxima [B] time = 1.31, size = 113, normalized size = 6.65

$$\frac{1}{9}b^8c^3x^9 + ab^7c^3x^8 + 4a^2b^6c^3x^7 + \frac{28}{3}a^3b^5c^3x^6 + 14a^4b^4c^3x^5 + 14a^5b^3c^3x^4 + \frac{28}{3}a^6b^2c^3x^3 + 4a^7bc^3x^2 + a^8c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^8*c^3*x^9 + a*b^7*c^3*x^8 + 4*a^2*b^6*c^3*x^7 + 28/3*a^3*b^5*c^3*x^6
+ 14*a^4*b^4*c^3*x^5 + 14*a^5*b^3*c^3*x^4 + 28/3*a^6*b^2*c^3*x^3 + 4*a^7*b*
c^3*x^2 + a^8*c^3*x
```

mupad [B] time = 0.05, size = 113, normalized size = 6.65

$$a^8\,c^3\,x + 4\,a^7\,b\,c^3\,x^2 + \frac{28\,a^6\,b^2\,c^3\,x^3}{3} + 14\,a^5\,b^3\,c^3\,x^4 + 14\,a^4\,b^4\,c^3\,x^5 + \frac{28\,a^3\,b^5\,c^3\,x^6}{3} + 4\,a^2\,b^6\,c^3\,x^7 + a\,b^7\,c^3\,x^8 + \frac{b^8\,c^3\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^3*(a + b*x)^5,x)
```

```
[Out] a^8*c^3*x + (b^8*c^3*x^9)/9 + 4*a^7*b*c^3*x^2 + a*b^7*c^3*x^8 + (28*a^6*b^2
*c^3*x^3)/3 + 14*a^5*b^3*c^3*x^4 + 14*a^4*b^4*c^3*x^5 + (28*a^3*b^5*c^3*x^6
)/3 + 4*a^2*b^6*c^3*x^7
```

sympy [B] time = 0.10, size = 124, normalized size = 7.29

$$a^8c^3x + 4a^7bc^3x^2 + \frac{28a^6b^2c^3x^3}{3} + 14a^5b^3c^3x^4 + 14a^4b^4c^3x^5 + \frac{28a^3b^5c^3x^6}{3} + 4a^2b^6c^3x^7 + ab^7c^3x^8 + \frac{b^8c^3x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**3,x)
```

```
[Out] a**8*c**3*x + 4*a**7*b*c**3*x**2 + 28*a**6*b**2*c**3*x**3/3 + 14*a**5*b**3*
c**3*x**4 + 14*a**4*b**4*c**3*x**5 + 28*a**3*b**5*c**3*x**6/3 + 4*a**2*b**6
*c**3*x**7 + a*b**7*c**3*x**8 + b**8*c**3*x**9/9
```

3.948 $\int (a+bx)^5(ac+bcx)^2\,dx$

Optimal. Leaf size=17

$$\frac{c^2(a+bx)^8}{8b}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^2\,dx &= c^2\int (a+bx)^7\,dx\\ &= \frac{c^2(a+bx)^8}{8b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{c^2(a+bx)^8}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] (c^2*(a + b*x)^8)/(8*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^2, x]
```

fricas [B] time = 0.98, size = 99, normalized size = 5.82

$$\frac{1}{8}x^8c^2b^7 + x^7c^2b^6a + \frac{7}{2}x^6c^2b^5a^2 + 7x^5c^2b^4a^3 + \frac{35}{4}x^4c^2b^3a^4 + 7x^3c^2b^2a^5 + \frac{7}{2}x^2c^2ba^6 + xc^2a^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c^2*b^7 + x^7*c^2*b^6*a + 7/2*x^6*c^2*b^5*a^2 + 7*x^5*c^2*b^4*a^3 +
 35/4*x^4*c^2*b^3*a^4 + 7*x^3*c^2*b^2*a^5 + 7/2*x^2*c^2*b*a^6 + x*c^2*a^7
```

giac [B] time = 0.99, size = 99, normalized size = 5.82

$$\frac{1}{8}b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}a^2b^5c^2x^6 + 7a^3b^4c^2x^5 + \frac{35}{4}a^4b^3c^2x^4 + 7a^5b^2c^2x^3 + \frac{7}{2}a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

maple [B] time = 0.00, size = 100, normalized size = 5.88

$$\frac{1}{8}b^7c^2x^8 + a\,b^6c^2x^7 + \frac{7}{2}a^2b^5c^2x^6 + 7a^3b^4c^2x^5 + \frac{35}{4}a^4b^3c^2x^4 + 7a^5b^2c^2x^3 + \frac{7}{2}a^6b\,c^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^2,x)
```

```
[Out] 1/8*b^7*c^2*x^8+a*b^6*c^2*x^7+7/2*a^2*b^5*c^2*x^6+7*a^3*b^4*c^2*x^5+35/4*a^
4*b^3*c^2*x^4+7*a^5*b^2*c^2*x^3+7/2*a^6*c^2*b*x^2+a^7*c^2*x
```

maxima [B] time = 1.36, size = 99, normalized size = 5.82

$$\frac{1}{8}b^7c^2x^8 + ab^6c^2x^7 + \frac{7}{2}a^2b^5c^2x^6 + 7a^3b^4c^2x^5 + \frac{35}{4}a^4b^3c^2x^4 + 7a^5b^2c^2x^3 + \frac{7}{2}a^6bc^2x^2 + a^7c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/8*b^7*c^2*x^8 + a*b^6*c^2*x^7 + 7/2*a^2*b^5*c^2*x^6 + 7*a^3*b^4*c^2*x^5 +
 35/4*a^4*b^3*c^2*x^4 + 7*a^5*b^2*c^2*x^3 + 7/2*a^6*b*c^2*x^2 + a^7*c^2*x
```

mupad [B] time = 0.04, size = 99, normalized size = 5.82

$$a^7\,c^2\,x + \frac{7\,a^6\,b\,c^2\,x^2}{2} + 7\,a^5\,b^2\,c^2\,x^3 + \frac{35\,a^4\,b^3\,c^2\,x^4}{4} + 7\,a^3\,b^4\,c^2\,x^5 + \frac{7\,a^2\,b^5\,c^2\,x^6}{2} + a\,b^6\,c^2\,x^7 + \frac{b^7\,c^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^2*(a + b*x)^5,x)
```

```
[Out] a^7*c^2*x + (b^7*c^2*x^8)/8 + (7*a^6*b*c^2*x^2)/2 + a*b^6*c^2*x^7 + 7*a^5*b
^2*c^2*x^3 + (35*a^4*b^3*c^2*x^4)/4 + 7*a^3*b^4*c^2*x^5 + (7*a^2*b^5*c^2*x^
6)/2
```

sympy [B] time = 0.10, size = 110, normalized size = 6.47

$$a^7c^2x + \frac{7a^6bc^2x^2}{2} + 7a^5b^2c^2x^3 + \frac{35a^4b^3c^2x^4}{4} + 7a^3b^4c^2x^5 + \frac{7a^2b^5c^2x^6}{2} + ab^6c^2x^7 + \frac{b^7c^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**2,x)
```

```
[Out] a**7*c**2*x + 7*a**6*b*c**2*x**2/2 + 7*a**5*b**2*c**2*x**3 + 35*a**4*b**3*c
**2*x**4/4 + 7*a**3*b**4*c**2*x**5 + 7*a**2*b**5*c**2*x**6/2 + a*b**6*c**2*
x**7 + b**7*c**2*x**8/8
```

3.949 $\int (a+bx)^5(ac+bcx)\,dx$

Optimal. Leaf size=15

$$\frac{c(a+bx)^7}{7b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{21, 32\}$

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)\,dx &= c\int (a+bx)^6\,dx\\ &= \frac{c(a+bx)^7}{7b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{c(a+bx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] (c*(a + b*x)^7)/(7*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(ac+bcx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x), x]
```

fricas [B] time = 1.27, size = 71, normalized size = 4.73

$$\frac{1}{7}x^7cb^6 + x^6cb^5a + 3x^5cb^4a^2 + 5x^4cb^3a^3 + 5x^3cb^2a^4 + 3x^2cba^5 + xca^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*b^6 + x^6*c*b^5*a + 3*x^5*c*b^4*a^2 + 5*x^4*c*b^3*a^3 + 5*x^3*c*b
^2*a^4 + 3*x^2*c*b*a^5 + x*c*a^6
```

giac [B] time = 1.09, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^6cx^7 + ab^5cx^6 + 3\,a^2b^4cx^5 + 5\,a^3b^3cx^4 + 5\,a^4b^2cx^3 + 3\,a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="giac")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

maple [B] time = 0.00, size = 72, normalized size = 4.80

$$\frac{1}{7}b^6c\,x^7 + a\,b^5c\,x^6 + 3a^2b^4c\,x^5 + 5a^3b^3c\,x^4 + 5a^4b^2c\,x^3 + 3a^5bc\,x^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c),x)
```

```
[Out] 1/7*b^6*c*x^7+a*b^5*c*x^6+3*a^2*b^4*c*x^5+5*a^3*b^3*c*x^4+5*a^4*b^2*c*x^3+3
*a^5*b*c*x^2+a^6*c*x
```

maxima [B] time = 1.35, size = 71, normalized size = 4.73

$$\frac{1}{7}\,b^6cx^7 + ab^5cx^6 + 3\,a^2b^4cx^5 + 5\,a^3b^3cx^4 + 5\,a^4b^2cx^3 + 3\,a^5bcx^2 + a^6cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] 1/7*b^6*c*x^7 + a*b^5*c*x^6 + 3*a^2*b^4*c*x^5 + 5*a^3*b^3*c*x^4 + 5*a^4*b^2
*c*x^3 + 3*a^5*b*c*x^2 + a^6*c*x
```

mupad [B] time = 0.03, size = 71, normalized size = 4.73

$$c\,a^6\,x + 3\,c\,a^5\,b\,x^2 + 5\,c\,a^4\,b^2\,x^3 + 5\,c\,a^3\,b^3\,x^4 + 3\,c\,a^2\,b^4\,x^5 + c\,a\,b^5\,x^6 + \frac{c\,b^6\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)*(a + b*x)^5,x)
```

```
[Out] (b^6*c*x^7)/7 + a^6*c*x + 5*a^4*b^2*c*x^3 + 5*a^3*b^3*c*x^4 + 3*a^2*b^4*c*x
^5 + 3*a^5*b*c*x^2 + a*b^5*c*x^6
```

sympy [B] time = 0.08, size = 78, normalized size = 5.20

$$a^6cx + 3a^5bcx^2 + 5a^4b^2cx^3 + 5a^3b^3cx^4 + 3a^2b^4cx^5 + ab^5cx^6 + \frac{b^6cx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c),x)
```

```
[Out] a**6*c*x + 3*a**5*b*c*x**2 + 5*a**4*b**2*c*x**3 + 5*a**3*b**3*c*x**4 + 3*a*
*2*b**4*c*x**5 + a*b**5*c*x**6 + b**6*c*x**7/7
```

3.950 $\int \frac{(a+bx)^5}{ac+bcx} \, dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^5}{5bc}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{ac+bcx} \, dx &= \frac{\int (a+bx)^4 \, dx}{c} \\ &= \frac{(a+bx)^5}{5bc} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^5}{5bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] (a + b*x)^5/(5*b*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{ac+bcx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x), x]
```

fricas [B] time = 1.10, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="fricas")
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

giac [B] time = 0.89, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="giac")
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx + a)^5}{5bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c),x)
[Out] 1/5*(b*x+a)^5/b/c
```

maxima [B] time = 1.35, size = 48, normalized size = 2.82

$$\frac{b^4x^5 + 5\,ab^3x^4 + 10\,a^2b^2x^3 + 10\,a^3bx^2 + 5\,a^4x}{5\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c),x, algorithm="maxima")
[Out] 1/5*(b^4*x^5 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3 + 10*a^3*b*x^2 + 5*a^4*x)/c
```

mupad [B] time = 0.03, size = 57, normalized size = 3.35

$$\frac{a^4\,x}{c} + \frac{b^4\,x^5}{5\,c} + \frac{2\,a^3\,b\,x^2}{c} + \frac{a\,b^3\,x^4}{c} + \frac{2\,a^2\,b^2\,x^3}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x),x)
[Out] (a^4*x)/c + (b^4*x^5)/(5*c) + (2*a^3*b*x^2)/c + (a*b^3*x^4)/c + (2*a^2*b^2*
x^3)/c
```

sympy [B] time = 0.10, size = 51, normalized size = 3.00

$$\frac{a^4x}{c} + \frac{2a^3bx^2}{c} + \frac{2a^2b^2x^3}{c} + \frac{ab^3x^4}{c} + \frac{b^4x^5}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c),x)
[Out] a**4*x/c + 2*a**3*b*x**2/c + 2*a**2*b**2*x**3/c + a*b**3*x**4/c + b**4*x**5
/(5*c)
```

3.951 $\int \frac{(a+bx)^5}{(ac+bcx)^2}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^4}{4bc^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^2}\,dx &= \frac{\int (a+bx)^3\,dx}{c^2}\\ &= \frac{(a+bx)^4}{4bc^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^4}{4bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] (a + b*x)^4/(4*b*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^2, x]
```

fricas [B] time = 1.18, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

giac [A] time = 0.98, size = 18, normalized size = 1.06

$$\frac{(bcx + ac)^4}{4\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="giac")
[Out] 1/4*(b*c*x + a*c)^4/(b*c^6)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx + a)^4}{4b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^2,x)
[Out] 1/4*(b*x+a)^4/b/c^2
```

maxima [B] time = 1.31, size = 37, normalized size = 2.18

$$\frac{b^3x^4 + 4\,ab^2x^3 + 6\,a^2bx^2 + 4\,a^3x}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^2,x, algorithm="maxima")
[Out] 1/4*(b^3*x^4 + 4*a*b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)/c^2
```

mupad [B] time = 0.05, size = 43, normalized size = 2.53

$$\frac{a^3\,x}{c^2} + \frac{b^3\,x^4}{4\,c^2} + \frac{3\,a^2\,b\,x^2}{2\,c^2} + \frac{a\,b^2\,x^3}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^2,x)
[Out] (a^3*x)/c^2 + (b^3*x^4)/(4*c^2) + (3*a^2*b*x^2)/(2*c^2) + (a*b^2*x^3)/c^2
```

sympy [B] time = 0.11, size = 46, normalized size = 2.71

$$\frac{a^3x}{c^2} + \frac{3a^2bx^2}{2c^2} + \frac{ab^2x^3}{c^2} + \frac{b^3x^4}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**2,x)
[Out] a**3*x/c**2 + 3*a**2*b*x**2/(2*c**2) + a*b**2*x**3/c**2 + b**3*x**4/(4*c**2
)
```

3.952 $\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx$

Optimal. Leaf size=17

$$\frac{(a+bx)^3}{3bc^3}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx &= \frac{\int (a+bx)^2\,dx}{c^3}\\ &= \frac{(a+bx)^3}{3bc^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{(a+bx)^3}{3bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] (a + b*x)^3/(3*b*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^3, x]
```

fricas [A] time = 0.71, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="fricas")
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

giac [A] time = 1.00, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="giac")
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$\frac{(bx+a)^3}{3b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^3,x)
[Out] 1/3*(b*x+a)^3/b/c^3
```

maxima [A] time = 1.31, size = 26, normalized size = 1.53

$$\frac{b^2x^3 + 3\,abx^2 + 3\,a^2x}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^3,x, algorithm="maxima")
[Out] 1/3*(b^2*x^3 + 3*a*b*x^2 + 3*a^2*x)/c^3
```

mupad [B] time = 0.04, size = 24, normalized size = 1.41

$$\frac{x\,\left(3\,a^2+3\,a\,b\,x+b^2\,x^2\right)}{3\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^3,x)
[Out] (x*(3*a^2 + b^2*x^2 + 3*a*b*x))/(3*c^3)
```

sympy [B] time = 0.11, size = 29, normalized size = 1.71

$$\frac{a^2x}{c^3} + \frac{abx^2}{c^3} + \frac{b^2x^3}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**3,x)
[Out] a**2*x/c**3 + a*b*x**2/c**3 + b**2*x**3/(3*c**3)
```

3.953 $\int \frac{(a+bx)^5}{(ac+bcx)^4} \, dx$

Optimal. Leaf size=18

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {21}

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x)/c^4 + (b*x^2)/(2*c^4)

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^4} \, dx &= \frac{\int (a+bx) \, dx}{c^4} \\ &= \frac{ax}{c^4} + \frac{bx^2}{2c^4} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.89

$$\frac{ax + \frac{bx^2}{2}}{c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] (a*x + (b*x^2)/2)/c^4

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^4} \, dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^4,x]

[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^4, x]

fricas [A] time = 1.36, size = 15, normalized size = 0.83

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

giac [A] time = 1.21, size = 15, normalized size = 0.83

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{\frac{1}{2}b\,x^2 + ax}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(1/2*b*x^2+a*x)
```

maxima [A] time = 1.38, size = 15, normalized size = 0.83

$$\frac{bx^2 + 2\,ax}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + 2*a*x)/c^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(2\,a + b\,x\right)}{2\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^4,x)
```

```
[Out] (x*(2*a + b*x))/(2*c^4)
```

sympy [A] time = 0.11, size = 15, normalized size = 0.83

$$\frac{ax}{c^4} + \frac{bx^2}{2c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**4,x)
```

```
[Out] a*x/c**4 + b*x**2/(2*c**4)
```

3.954 $\int \frac{(a+bx)^5}{(ac+bcx)^5}\,dx$

Optimal. Leaf size=5

$$\frac{x}{c^5}$$

Rubi [A] time = 0.00, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 8}

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] x/c^5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^5}\,dx &= \frac{\int 1\,dx}{c^5}\\ &= \frac{x}{c^5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] x/c^5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^5}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^5, x]
```

fricas [**A**] time = 1.22, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="fricas")
[Out] x/c^5
```

giac [**B**] time = 1.11, size = 15, normalized size = 3.00

$$\frac{bcx + ac}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="giac")
[Out] (b*c*x + a*c)/(b*c^6)
```

maple [**A**] time = 0.00, size = 6, normalized size = 1.20

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^5,x)
[Out] x/c^5
```

maxima [**A**] time = 1.40, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^5,x, algorithm="maxima")
[Out] x/c^5
```

mupad [**B**] time = 0.01, size = 5, normalized size = 1.00

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^5,x)
[Out] x/c^5
```

sympy [**A**] time = 0.11, size = 3, normalized size = 0.60

$$\frac{x}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**5,x)
[Out] x/c**5
```

3.955 $\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$

Optimal. Leaf size=13

$$\frac{\log(a+bx)}{bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 31}

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^6} dx &= \frac{\int \frac{1}{a+bx} dx}{c^6}\\ &= \frac{\log(a+bx)}{bc^6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{\log(a+bx)}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] Log[a + b*x]/(b*c^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^6, x]
```

fricas [A] time = 1.33, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="fricas")
[Out] log(b*x + a)/(b*c^6)
```

giac [A] time = 0.95, size = 14, normalized size = 1.08

$$\frac{\log\left(|bx+a|\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="giac")
[Out] log(abs(b*x + a))/(b*c^6)
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$\frac{\ln\left(bx+a\right)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^6,x)
[Out] ln(b*x+a)/b/c^6
```

maxima [A] time = 1.31, size = 13, normalized size = 1.00

$$\frac{\log\left(bx+a\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^6,x, algorithm="maxima")
[Out] log(b*x + a)/(b*c^6)
```

mupad [B] time = 0.04, size = 13, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^6,x)
[Out] log(a + b*x)/(b*c^6)
```

sympy [A] time = 0.12, size = 17, normalized size = 1.31

$$\frac{\log\left(ac^6+bc^6x\right)}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**6,x)
[Out] log(a*c**6 + b*c**6*x)/(b*c**6)
```

3.956 $\int \frac{(a+bx)^5}{(ac+bcx)^7} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{bc^7(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {21, 32}

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^7} \, dx &= \frac{\int \frac{1}{(a+bx)^2} \, dx}{c^7} \\ &= -\frac{1}{bc^7(a+bx)} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] -(1/(b*c^7*(a + b*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^7} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^7, x]
```

fricas [A] time = 1.22, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x + abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="fricas")
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

giac [A] time = 1.09, size = 15, normalized size = 1.00

$$-\frac{1}{(bx + a)bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="giac")
[Out] -1/((b*x + a)*b*c^7)
```

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$-\frac{1}{(bx + a)\, b\, c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^7,x)
[Out] -1/b/c^7/(b*x+a)
```

maxima [A] time = 1.33, size = 19, normalized size = 1.27

$$-\frac{1}{b^2c^7x + abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^7,x, algorithm="maxima")
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

mupad [B] time = 0.05, size = 19, normalized size = 1.27

$$-\frac{1}{x\, b^2\, c^7 + a\, b\, c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^7,x)
[Out] -1/(b^2*c^7*x + a*b*c^7)
```

sympy [A] time = 0.20, size = 17, normalized size = 1.13

$$-\frac{1}{abc^7 + b^2c^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**7,x)
[Out] -1/(a*b*c**7 + b**2*c**7*x)
```

3.957 $\int \frac{(a+bx)^5}{(ac+bcx)^8} dx$

Optimal. Leaf size=17

$$-\frac{1}{2bc^8(a+bx)^2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {21, 32}

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/(2*b*c^8*(a + b*x)^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^8} dx &= \frac{\int \frac{1}{(a+bx)^3} dx}{c^8} \\ &= -\frac{1}{2bc^8(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$-\frac{1}{2bc^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] -1/2*1/(b*c^8*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(ac+bcx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^8, x]
```

fricas [B] time = 1.36, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^{3}c^{8}x^{2}+2\,ab^{2}c^{8}x+a^{2}bc^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="fricas")
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

giac [A] time = 1.05, size = 15, normalized size = 0.88

$$-\frac{1}{2\,(bx+a)^{2}bc^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="giac")
[Out] -1/2/((b*x + a)^2*b*c^8)
```

maple [A] time = 0.00, size = 16, normalized size = 0.94

$$-\frac{1}{2\,(bx+a)^{2}\,b\,c^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^8,x)
[Out] -1/2/b/c^8/(b*x+a)^2
```

maxima [B] time = 1.33, size = 33, normalized size = 1.94

$$-\frac{1}{2\left(b^{3}c^{8}x^{2}+2\,ab^{2}c^{8}x+a^{2}bc^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^8,x, algorithm="maxima")
[Out] -1/2/(b^3*c^8*x^2 + 2*a*b^2*c^8*x + a^2*b*c^8)
```

mupad [B] time = 0.15, size = 35, normalized size = 2.06

$$-\frac{1}{2\,a^{2}\,b\,c^{8}+4\,a\,b^{2}\,c^{8}\,x+2\,b^{3}\,c^{8}\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^8,x)
[Out] -1/(2*a^2*b*c^8 + 2*b^3*c^8*x^2 + 4*a*b^2*c^8*x)
```

sympy [B] time = 0.26, size = 36, normalized size = 2.12

$$-\frac{1}{2a^{2}bc^{8}+4ab^{2}c^{8}x+2b^{3}c^{8}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**8,x)
[Out] -1/(2*a**2*b*c**8 + 4*a*b**2*c**8*x + 2*b**3*c**8*x**2)
```

3.958 $\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {23, 31}

$$\frac{\sqrt{3x+2}\,\log(3x+2)}{3\sqrt{-3x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] (Sqrt[2 + 3*x]*Log[2 + 3*x])/(3*Sqrt[-2 - 3*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx &= \frac{\sqrt{2+3x}\int \frac{1}{2+3x}\,dx}{\sqrt{-2-3x}}\\ &= \frac{\sqrt{2+3x}\,\log(2+3x)}{3\sqrt{-2-3x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

IntegrateAlgebraic [F] time = 1.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2-3x}\,\sqrt{2+3x}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]),x]
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[-2 - 3*x]*Sqrt[2 + 3*x]), x]
```

fricas [A] time = 1.27, size = 1, normalized size = 0.04

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 1.01, size = 11, normalized size = 0.39

$$-\frac{1}{3}i \log\left(|3x+2|\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
[Out] -1/3*I*log(abs(3*x + 2))*sgn(x)
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{\sqrt{3x+2}\,\ln\left(3x+2\right)}{3\sqrt{-3x-2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2-3*x)^(1/2)/(3*x+2)^(1/2),x)
[Out] 1/3*ln(3*x+2)*(3*x+2)^(1/2)/(-2-3*x)^(1/2)
```

maxima [C] time = 2.99, size = 6, normalized size = 0.21

$$\frac{1}{3}i \log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
[Out] 1/3*I*log(x + 2/3)
```

mupad [B] time = 0.22, size = 35, normalized size = 1.25

$$-\frac{4\,\mathrm{atan}\left(\frac{-\sqrt{-3\,x-2}+\sqrt{2}\,1{}\mathrm{i}}{\sqrt{2}-\sqrt{3\,x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- 3*x - 2)^(1/2)*(3*x + 2)^(1/2)),x)
[Out] -(4*atan((2^(1/2)*1i - (- 3*x - 2)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3
```

sympy [C] time = 1.46, size = 53, normalized size = 1.89

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{3}\right)}{3} & \text{for } \left|x+\frac{2}{3}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{3}}\right)}{3} & \text{for } \frac{1}{\left|x+\frac{2}{3}\right|}<1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| x+\frac{2}{3}\right)}{3} - \frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| x+\frac{2}{3}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/3)/3, Abs(x + 2/3) < 1), (I*log(1/(x + 2/3))/3, 1/A
bs(x + 2/3) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/3)/3 - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/3)/3, True))
```

3.959 $\int (a+bx)(ac-bcx)^3\,dx$

Optimal. Leaf size=38

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^3(a-bx)^5}{5b}-\frac{ac^3(a-bx)^4}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^3,x]
[Out] -(a*c^3*(a - b*x)^4)/(2*b) + (c^3*(a - b*x)^5)/(5*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^3\,dx &= \int \left(2a(ac-bcx)^3-\frac{(ac-bcx)^4}{c}\right)dx\\ &= -\frac{ac^3(a-bx)^4}{2b}+\frac{c^3(a-bx)^5}{5b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.05

$$c^3\left(a^4x-a^3bx^2+\frac{1}{2}ab^3x^4-\frac{1}{5}b^4x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^3,x]
[Out] c^3*(a^4*x - a^3*b*x^2 + (a*b^3*x^4)/2 - (b^4*x^5)/5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^3, x]
```

fricas [A] time = 1.29, size = 44, normalized size = 1.16

$$-\frac{1}{5}x^5c^3b^4+\frac{1}{2}x^4c^3b^3a-x^2c^3ba^3+xc^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="fricas")
[Out] -1/5*x^5*c^3*b^4 + 1/2*x^4*c^3*b^3*a - x^2*c^3*b*a^3 + x*c^3*a^4
```

giac [A] time = 1.00, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="giac")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

maple [A] time = 0.00, size = 45, normalized size = 1.18

$$-\frac{1}{5}b^4c^3x^5+\frac{1}{2}a\,b^3c^3x^4-a^3b\,c^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^3,x)
[Out] -1/5*b^4*c^3*x^5+1/2*a*b^3*c^3*x^4-a^3*c^3*b*x^2+a^4*c^3*x
```

maxima [A] time = 1.23, size = 44, normalized size = 1.16

$$-\frac{1}{5}\,b^4c^3x^5+\frac{1}{2}\,ab^3c^3x^4-a^3bc^3x^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^3,x, algorithm="maxima")
[Out] -1/5*b^4*c^3*x^5 + 1/2*a*b^3*c^3*x^4 - a^3*b*c^3*x^2 + a^4*c^3*x
```

mupad [B] time = 0.16, size = 44, normalized size = 1.16

$$a^4\,c^3\,x-a^3\,b\,c^3\,x^2+\frac{a\,b^3\,c^3\,x^4}{2}-\frac{b^4\,c^3\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x),x)
[Out] a^4*c^3*x - (b^4*c^3*x^5)/5 - a^3*b*c^3*x^2 + (a*b^3*c^3*x^4)/2
```

sympy [A] time = 0.08, size = 44, normalized size = 1.16

$$a^4c^3x-a^3bc^3x^2+\frac{ab^3c^3x^4}{2}-\frac{b^4c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**3,x)
[Out] a**4*c**3*x - a**3*b*c**3*x**2 + a*b**3*c**3*x**4/2 - b**4*c**3*x**5/5
```

3.960 $\int (a+bx)(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{c^2(a-bx)^4}{4b}-\frac{2ac^2(a-bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^2,x]
[Out] (-2*a*c^2*(a - b*x)^3)/(3*b) + (c^2*(a - b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^2\,dx &= \int \left(2a(ac-bcx)^2-\frac{(ac-bcx)^3}{c}\right)dx\\ &= -\frac{2ac^2(a-bx)^3}{3b}+\frac{c^2(a-bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.11

$$c^2\left(a^3x-\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3+\frac{b^3x^4}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^2,x]
[Out] c^2*(a^3*x - (a^2*b*x^2)/2 - (a*b^2*x^3)/3 + (b^3*x^4)/4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^2, x]
```

fricas [A] time = 0.73, size = 44, normalized size = 1.16

$$\frac{1}{4}x^4c^2b^3-\frac{1}{3}x^3c^2b^2a-\frac{1}{2}x^2c^2ba^2+xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c^2*b^3 - 1/3*x^3*c^2*b^2*a - 1/2*x^2*c^2*b*a^2 + x*c^2*a^3
```

giac [A] time = 1.04, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^{3}c^{2}x^{4}-\frac{1}{3}\,ab^{2}c^{2}x^{3}-\frac{1}{2}\,a^{2}bc^{2}x^{2}+a^{3}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

maple [A] time = 0.00, size = 45, normalized size = 1.18

$$\frac{1}{4}b^{3}c^{2}x^{4}-\frac{1}{3}a\,b^{2}c^{2}x^{3}-\frac{1}{2}a^{2}b\,c^{2}x^{2}+a^{3}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4*b^3*c^2*x^4-1/3*a*b^2*c^2*x^3-1/2*a^2*c^2*b*x^2+a^3*c^2*x
```

maxima [A] time = 1.31, size = 44, normalized size = 1.16

$$\frac{1}{4}\,b^{3}c^{2}x^{4}-\frac{1}{3}\,ab^{2}c^{2}x^{3}-\frac{1}{2}\,a^{2}bc^{2}x^{2}+a^{3}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*c^2*x^4 - 1/3*a*b^2*c^2*x^3 - 1/2*a^2*b*c^2*x^2 + a^3*c^2*x
```

mupad [B] time = 0.05, size = 44, normalized size = 1.16

$$a^3\,c^2\,x-\frac{a^2\,b\,c^2\,x^2}{2}-\frac{a\,b^2\,c^2\,x^3}{3}+\frac{b^3\,c^2\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x),x)
```

```
[Out] a^3*c^2*x + (b^3*c^2*x^4)/4 - (a^2*b*c^2*x^2)/2 - (a*b^2*c^2*x^3)/3
```

sympy [A] time = 0.07, size = 46, normalized size = 1.21

$$a^{3}c^{2}x-\frac{a^{2}bc^{2}x^{2}}{2}-\frac{ab^{2}c^{2}x^{3}}{3}+\frac{b^{3}c^{2}x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**2,x)
```

```
[Out] a**3*c**2*x - a**2*b*c**2*x**2/2 - a*b**2*c**2*x**3/3 + b**3*c**2*x**4/4
```

3.961 $\int (a+bx)(ac-bcx)\,dx$

Optimal. Leaf size=18

$$a^2cx-\frac{1}{3}b^2cx^3$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {41}

$$a^2cx-\frac{1}{3}b^2cx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x),x]
[Out] a^2*c*x - (b^2*c*x^3)/3
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)\,dx &= \int \left(a^2c-b^2cx^2\right)\,dx\\ &= a^2cx-\frac{1}{3}b^2cx^3\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$c\left(a^2x-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x),x]
[Out] c*(a^2*x - (b^2*x^3)/3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x), x]
```

fricas [A] time = 1.03, size = 16, normalized size = 0.89

$$-\frac{1}{3}x^3cb^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/3*x^3*c*b^2 + x*c*a^2
```

giac [**A**] time = 1.01, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="giac")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

maple [**A**] time = 0.00, size = 17, normalized size = 0.94

$$-\frac{1}{3}b^2c\,x^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c),x)
[Out] a^2*c*x-1/3*b^2*c*x^3
```

maxima [**A**] time = 1.35, size = 16, normalized size = 0.89

$$-\frac{1}{3}\,b^2cx^3+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/3*b^2*c*x^3 + a^2*c*x
```

mupad [**B**] time = 0.02, size = 18, normalized size = 1.00

$$\frac{c\,x\,\left(3\,a^2-b^2\,x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x),x)
[Out] (c*x*(3*a^2 - b^2*x^2))/3
```

sympy [**A**] time = 0.06, size = 15, normalized size = 0.83

$$a^2cx-\frac{b^2cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c),x)
[Out] a**2*c*x - b**2*c*x**3/3
```

3.962 $\int (a+bx)\,dx$

Optimal. Leaf size=12

$$ax+\frac{bx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

Rubi steps

$$\int (a+bx)\,dx=ax+\frac{bx^2}{2}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x,x]
[Out] a*x + (b*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x,x]
[Out] IntegrateAlgebraic[a + b*x, x]
```

fricas [A] time = 1.07, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="fricas")
[Out] 1/2*x^2*b + x*a
```

giac [A] time = 0.86, size = 10, normalized size = 0.83

$$\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2} b x^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x+a,x)
```

```
[Out] 1/2*b*x^2+a*x
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{2} bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b x^2}{2} + a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x,x)
```

```
[Out] a*x + (b*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax + \frac{bx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x+a,x)
```

```
[Out] a*x + b*x**2/2
```

3.963 $\int \frac{a+bx}{ac-bcx} \, dx$

Optimal. Leaf size=23

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{ac-bcx} \, dx &= \int \left(-\frac{1}{c}+\frac{2a}{c(a-bx)}\right) dx\\ &= -\frac{x}{c}-\frac{2a\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2a\log(a-bx)}{bc}-\frac{x}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] -(x/c) - (2*a*Log[a - b*x])/(b*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{ac-bcx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x), x]
```

fricas [A] time = 1.31, size = 23, normalized size = 1.00

$$-\frac{bx+2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

giac [A] time = 0.92, size = 25, normalized size = 1.09

$$-\frac{x}{c} - \frac{2\,a\log\left(|bx-a|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
[Out] -x/c - 2*a*log(abs(b*x - a))/(b*c)
```

maple [A] time = 0.00, size = 25, normalized size = 1.09

$$-\frac{2a\ln\left(bx-a\right)}{bc} - \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c),x)
[Out] -x/c-2/c*a/b*ln(b*x-a)
```

maxima [A] time = 1.31, size = 24, normalized size = 1.04

$$-\frac{x}{c} - \frac{2\,a\log\left(bx-a\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
[Out] -x/c - 2*a*log(b*x - a)/(b*c)
```

mupad [B] time = 0.05, size = 23, normalized size = 1.00

$$-\frac{b\,x+2\,a\,\ln\left(b\,x-a\right)}{b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x),x)
[Out] -(b*x + 2*a*log(b*x - a))/(b*c)
```

sympy [A] time = 0.14, size = 17, normalized size = 0.74

$$-\frac{2a\log\left(-a+bx\right)}{bc} - \frac{x}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c),x)
[Out] -2*a*log(-a + b*x)/(b*c) - x/c
```

3.964 $\int \frac{a+bx}{(ac-bcx)^2} dx$

Optimal. Leaf size=32

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] (2*a)/(b*c^2*(a - b*x)) + Log[a - b*x]/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^2}\,dx&=\int \left(\frac{2a}{c^2(a-bx)^2}-\frac{1}{c^2(a-bx)}\right)dx\\&=\frac{2a}{bc^2(a-bx)}+\frac{\log(a-bx)}{bc^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.88

$$\frac{\log(c(a-bx))+\frac{2a}{a-bx}}{bc^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] ((2*a)/(a - b*x) + Log[c*(a - b*x)])/(b*c^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^2, x]
```

fricas [A] time = 1.25, size = 39, normalized size = 1.22

$$\frac{(bx-a)\log(bx-a)-2\,a}{b^2c^2x-abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] ((b*x - a)*log(b*x - a) - 2*a)/(b^2*c^2*x - a*b*c^2)
```

giac [B] time = 1.11, size = 81, normalized size = 2.53

$$-\frac{\frac{a}{(bcx-ac)b}+\frac{\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc}}{c}-\frac{a}{(bcx-ac)bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -(a/((b*c*x - a*c)*b) + log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(b)*abs(c)
))/(b*c))/c - a/((b*c*x - a*c)*b*c)
```

maple [A] time = 0.01, size = 35, normalized size = 1.09

$$-\frac{2a}{(bx-a)\,b\,c^2}+\frac{\ln(bx-a)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^2,x)
```

```
[Out] -2/c^2*a/b/(b*x-a)+1/c^2/b*ln(b*x-a)
```

maxima [A] time = 1.31, size = 37, normalized size = 1.16

$$-\frac{2\,a}{b^2c^2x-abc^2}+\frac{\log(bx-a)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -2*a/(b^2*c^2*x - a*b*c^2) + log(b*x - a)/(b*c^2)
```

mupad [B] time = 0.05, size = 37, normalized size = 1.16

$$\frac{\ln(b\,x-a)}{b\,c^2}+\frac{2\,a}{b\,\left(a\,c^2-b\,c^2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^2,x)
```

```
[Out] log(b*x - a)/(b*c^2) + (2*a)/(b*(a*c^2 - b*c^2*x))
```

sympy [A] time = 0.19, size = 29, normalized size = 0.91

$$-\frac{2a}{-abc^2+b^2c^2x}+\frac{\log(-a+bx)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -2*a/(-a*b*c**2 + b**2*c**2*x) + log(-a + b*x)/(b*c**2)
```

3.965 $\int \frac{a+bx}{(ac-bcx)^3} dx$

Optimal. Leaf size=13

$$\frac{x}{c^3(a-bx)^2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {34}

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^3,x]
[Out] x/(c^3*(a - b*x)^2)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int \frac{a+bx}{(ac-bcx)^3} dx = \frac{x}{c^3(a-bx)^2}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{x}{c^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^3,x]
[Out] x/(c^3*(a - b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^3,x]
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^3, x]
```

fricas [B] time = 0.68, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2 - 2\,abc^3x + a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

giac [A] time = 0.89, size = 14, normalized size = 1.08

$$\frac{x}{(bx-a)^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
[Out] x/((b*x - a)^2*c^3)
```

maple [B] time = 0.00, size = 33, normalized size = 2.54

$$\frac{\frac{a}{(bx-a)^2b}+\frac{1}{(bx-a)b}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^3,x)
[Out] 1/c^3*(a/b/(b*x-a)^2+1/b/(b*x-a))
```

maxima [B] time = 1.31, size = 30, normalized size = 2.31

$$\frac{x}{b^2c^3x^2-2\,abc^3x+a^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
[Out] x/(b^2*c^3*x^2 - 2*a*b*c^3*x + a^2*c^3)
```

mupad [B] time = 0.15, size = 13, normalized size = 1.00

$$\frac{x}{c^3\,(a-b\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^3,x)
[Out] x/(c^3*(a - b*x)^2)
```

sympy [B] time = 0.24, size = 27, normalized size = 2.08

$$\frac{x}{a^2c^3-2abc^3x+b^2c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**3,x)
[Out] x/(a**2*c**3 - 2*a*b*c**3*x + b**2*c**3*x**2)
```

3.966 $\int \frac{a+bx}{(ac-bcx)^4} dx$

Optimal. Leaf size=38

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] (2*a)/(3*b*c^4*(a - b*x)^3) - 1/(2*b*c^4*(a - b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^4} dx &= \int \left(\frac{2a}{c^4(a-bx)^4} - \frac{1}{c^4(a-bx)^3}\right) dx \\ &= \frac{2a}{3bc^4(a-bx)^3} - \frac{1}{2bc^4(a-bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.66

$$-\frac{a+3bx}{6bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] -1/6*(a + 3*b*x)/(b*c^4*(-a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^4, x]
```

fricas [A] time = 1.24, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

giac [A] time = 0.98, size = 23, normalized size = 0.61

$$-\frac{3\,bx+a}{6\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*x + a)/((b*x - a)^3*b*c^4)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$\frac{-\frac{2a}{3(bx-a)^3b}-\frac{1}{2(bx-a)^2b}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-1/2/b/(b*x-a)^2-2/3*a/b/(b*x-a)^3)
```

maxima [A] time = 1.34, size = 54, normalized size = 1.42

$$-\frac{3\,bx+a}{6\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*x + a)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a^3*b*c
^4)
```

mupad [B] time = 0.05, size = 54, normalized size = 1.42

$$\frac{\frac{x}{2}+\frac{a}{6\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^4,x)
```

```
[Out] (x/2 + a/(6*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*x)
```

sympy [A] time = 0.31, size = 56, normalized size = 1.47

$$\frac{-a-3bx}{-6a^3bc^4+18a^2b^2c^4x-18ab^3c^4x^2+6b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a - 3*b*x)/(-6*a**3*b*c**4 + 18*a**2*b**2*c**4*x - 18*a*b**3*c**4*x**2 +
6*b**4*c**4*x**3)
```

3.967 $\int \frac{a+bx}{(ac-bcx)^5} dx$

Optimal. Leaf size=38

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^5,x]
[Out] a/(2*b*c^5*(a - b*x)^4) - 1/(3*b*c^5*(a - b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^5} dx &= \int \left(\frac{2a}{c^5(a-bx)^5}-\frac{1}{c^5(a-bx)^4}\right) dx\\ &= \frac{a}{2bc^5(a-bx)^4}-\frac{1}{3bc^5(a-bx)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.63

$$\frac{a+2bx}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^5,x]
[Out] (a + 2*b*x)/(6*b*c^5*(a - b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^5,x]
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^5, x]
```

fricas [A] time = 1.41, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

giac [A] time = 0.95, size = 40, normalized size = 1.05

$$\frac{a}{2\,(bcx-ac)^4bc}+\frac{1}{3\,(bcx-ac)^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/2*a/((b*c*x - a*c)^4*b*c) + 1/3/((b*c*x - a*c)^3*b*c^2)
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$\frac{\frac{a}{2(bx-a)^4b}+\frac{1}{3(bx-a)^3b}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(1/2*a/b/(b*x-a)^4+1/3/b/(b*x-a)^3)
```

maxima [A] time = 1.35, size = 67, normalized size = 1.76

$$\frac{2\,bx+a}{6\,\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*x + a)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*c^5*x^2 - 4*a^3*
b^2*c^5*x + a^4*b*c^5)
```

mupad [B] time = 0.17, size = 67, normalized size = 1.76

$$\frac{\frac{x}{3}+\frac{a}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^5,x)
```

```
[Out] (x/3 + a/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3 + 6*a^2*b^2*c^5*x^
2 - 4*a^3*b*c^5*x)
```

sympy [B] time = 0.39, size = 73, normalized size = 1.92

$$-\frac{-a-2bx}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a - 2*b*x)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*a**2*b**3*c**5*x**2
 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

3.968 $\int \frac{a+bx}{(ac-bcx)^6} dx$

Optimal. Leaf size=38

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(a*c - b*c*x)^6,x]
[Out] (2*a)/(5*b*c^6*(a - b*x)^5) - 1/(4*b*c^6*(a - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(ac-bcx)^6} dx &= \int \left(\frac{2a}{c^6(a-bx)^6}-\frac{1}{c^6(a-bx)^5}\right) dx\\ &= \frac{2a}{5bc^6(a-bx)^5}-\frac{1}{4bc^6(a-bx)^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{3a+5bx}{20bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(a*c - b*c*x)^6,x]
[Out] -1/20*(3*a + 5*b*x)/(b*c^6*(-a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(ac-bcx)^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^6,x]
[Out] IntegrateAlgebraic[(a + b*x)/(a*c - b*c*x)^6, x]
```

fricas [B] time = 1.21, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

giac [A] time = 1.04, size = 25, normalized size = 0.66

$$-\frac{5\,bx+3\,a}{20\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/20*(5*b*x + 3*a)/((b*x - a)^5*b*c^6)
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$\frac{-\frac{2a}{5(bx-a)^5b}-\frac{1}{4(bx-a)^4b}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/4/b/(b*x-a)^4-2/5*a/b/(b*x-a)^5)
```

maxima [B] time = 1.31, size = 84, normalized size = 2.21

$$-\frac{5\,bx+3\,a}{20\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(5*b*x + 3*a)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2*b^4*c^6*x^3 - 1
0*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

mupad [B] time = 0.08, size = 82, normalized size = 2.16

$$\frac{\frac{x}{4}+\frac{3\,a}{20\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(a*c - b*c*x)^6,x)
```

```
[Out] (x/4 + (3*a)/(20*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6*x^4 + 10*a^3*b^2*
c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

sympy [B] time = 0.46, size = 88, normalized size = 2.32

$$\frac{-3a-5bx}{-20a^5bc^6+100a^4b^2c^6x-200a^3b^3c^6x^2+200a^2b^4c^6x^3-100ab^5c^6x^4+20b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(-b*c*x+a*c)**6,x)
```

```
[Out] (-3*a - 5*b*x)/(-20*a**5*b*c**6 + 100*a**4*b**2*c**6*x - 200*a**3*b**3*c**6
*x**2 + 200*a**2*b**4*c**6*x**3 - 100*a*b**5*c**6*x**4 + 20*b**6*c**6*x**5)
```

3.969 $\int (a+bx)^2(ac-bcx)^3\,dx$

Optimal. Leaf size=57

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{a^2c^3(a-bx)^4}{b}-\frac{c^3(a-bx)^6}{6b}+\frac{4ac^3(a-bx)^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] -((a^2*c^3*(a - b*x)^4)/b) + (4*a*c^3*(a - b*x)^5)/(5*b) - (c^3*(a - b*x)^6
)/(6*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^3\,dx &= \int \left(4a^2(ac-bcx)^3-\frac{4a(ac-bcx)^4}{c}+\frac{(ac-bcx)^5}{c^2}\right)dx\\ &= -\frac{a^2c^3(a-bx)^4}{b}+\frac{4ac^3(a-bx)^5}{5b}-\frac{c^3(a-bx)^6}{6b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 68, normalized size = 1.19

$$c^3\left(a^5x-\frac{1}{2}a^4bx^2-\frac{2}{3}a^3b^2x^3+\frac{1}{2}a^2b^3x^4+\frac{1}{5}ab^4x^5-\frac{1}{6}b^5x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] c^3*(a^5*x - (a^4*b*x^2)/2 - (2*a^3*b^2*x^3)/3 + (a^2*b^3*x^4)/2 + (a*b^4*x
^5)/5 - (b^5*x^6)/6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^3, x]
```

fricas [A]   time = 1.14, size = 72, normalized size = 1.26

$$-\frac{1}{6}x^6c^3b^5 + \frac{1}{5}x^5c^3b^4a + \frac{1}{2}x^4c^3b^3a^2 - \frac{2}{3}x^3c^3b^2a^3 - \frac{1}{2}x^2c^3ba^4 + xc^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/6*x^6*c^3*b^5 + 1/5*x^5*c^3*b^4*a + 1/2*x^4*c^3*b^3*a^2 - 2/3*x^3*c^3*b^
2*a^3 - 1/2*x^2*c^3*b*a^4 + x*c^3*a^5
```

giac [A]   time = 1.04, size = 72, normalized size = 1.26

$$-\frac{1}{6}b^5c^3x^6 + \frac{1}{5}ab^4c^3x^5 + \frac{1}{2}a^2b^3c^3x^4 - \frac{2}{3}a^3b^2c^3x^3 - \frac{1}{2}a^4bc^3x^2 + a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

maple [A]   time = 0.00, size = 73, normalized size = 1.28

$$-\frac{1}{6}b^5c^3x^6 + \frac{1}{5}a\,b^4c^3x^5 + \frac{1}{2}a^2b^3c^3x^4 - \frac{2}{3}a^3b^2c^3x^3 - \frac{1}{2}a^4b\,c^3x^2 + a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^3,x)
```

```
[Out] -1/6*b^5*c^3*x^6+1/5*a*b^4*c^3*x^5+1/2*a^2*b^3*c^3*x^4-2/3*a^3*c^3*b^2*x^3-
1/2*a^4*c^3*b*x^2+a^5*c^3*x
```

maxima [A]   time = 1.29, size = 72, normalized size = 1.26

$$-\frac{1}{6}b^5c^3x^6 + \frac{1}{5}ab^4c^3x^5 + \frac{1}{2}a^2b^3c^3x^4 - \frac{2}{3}a^3b^2c^3x^3 - \frac{1}{2}a^4bc^3x^2 + a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/6*b^5*c^3*x^6 + 1/5*a*b^4*c^3*x^5 + 1/2*a^2*b^3*c^3*x^4 - 2/3*a^3*b^2*c^
3*x^3 - 1/2*a^4*b*c^3*x^2 + a^5*c^3*x
```

mupad [B]   time = 0.03, size = 72, normalized size = 1.26

$$a^5\,c^3\,x - \frac{a^4\,b\,c^3\,x^2}{2} - \frac{2\,a^3\,b^2\,c^3\,x^3}{3} + \frac{a^2\,b^3\,c^3\,x^4}{2} + \frac{a\,b^4\,c^3\,x^5}{5} - \frac{b^5\,c^3\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3*(a + b*x)^2,x)
```

```
[Out] a^5*c^3*x - (b^5*c^3*x^6)/6 - (a^4*b*c^3*x^2)/2 + (a*b^4*c^3*x^5)/5 - (2*a^
3*b^2*c^3*x^3)/3 + (a^2*b^3*c^3*x^4)/2
```

sympy [A]   time = 0.09, size = 78, normalized size = 1.37

$$a^5c^3x - \frac{a^4bc^3x^2}{2} - \frac{2a^3b^2c^3x^3}{3} + \frac{a^2b^3c^3x^4}{2} + \frac{ab^4c^3x^5}{5} - \frac{b^5c^3x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**3,x)
```

```
[Out] a**5*c**3*x - a**4*b*c**3*x**2/2 - 2*a**3*b**2*c**3*x**3/3 + a**2*b**3*c**3
*x**4/2 + a*b**4*c**3*x**5/5 - b**5*c**3*x**6/6
```

3.970 $\int (a+bx)^2(ac-bcx)^2\,dx$

Optimal. Leaf size=38

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {41, 194}

$$-\frac{2}{3}a^2b^2c^2x^3+a^4c^2x+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^2\,dx &= \int \left(a^2c-b^2cx^2\right)^2\,dx\\ &= \int \left(a^4c^2-2a^2b^2c^2x^2+b^4c^2x^4\right)dx\\ &= a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5\end{aligned}$$

Mathematica [A] time = 0.00, size = 38, normalized size = 1.00

$$a^4c^2x-\frac{2}{3}a^2b^2c^2x^3+\frac{1}{5}b^4c^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] a^4*c^2*x - (2*a^2*b^2*c^2*x^3)/3 + (b^4*c^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^2, x]
```

fricas [A] time = 0.49, size = 34, normalized size = 0.89

$$\frac{1}{5}x^5c^2b^4 - \frac{2}{3}x^3c^2b^2a^2 + xc^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/5*x^5*c^2*b^4 - 2/3*x^3*c^2*b^2*a^2 + x*c^2*a^4
```

giac [A] time = 1.10, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="giac")
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$\frac{1}{5}b^4c^2x^5 - \frac{2}{3}a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^2,x)
[Out] a^4*c^2*x-2/3*a^2*b^2*c^2*x^3+1/5*b^4*c^2*x^5
```

maxima [A] time = 1.34, size = 34, normalized size = 0.89

$$\frac{1}{5}\,b^4c^2x^5 - \frac{2}{3}\,a^2b^2c^2x^3 + a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] 1/5*b^4*c^2*x^5 - 2/3*a^2*b^2*c^2*x^3 + a^4*c^2*x
```

mupad [B] time = 0.04, size = 31, normalized size = 0.82

$$\frac{c^2\,x\,\left(15\,a^4-10\,a^2\,b^2\,x^2+3\,b^4\,x^4\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2*(a + b*x)^2,x)
[Out] (c^2*x*(15*a^4 + 3*b^4*x^4 - 10*a^2*b^2*x^2))/15
```

sympy [A] time = 0.08, size = 36, normalized size = 0.95

$$a^4c^2x - \frac{2a^2b^2c^2x^3}{3} + \frac{b^4c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**2,x)
[Out] a**4*c**2*x - 2*a**2*b**2*c**2*x**3/3 + b**4*c**2*x**5/5
```

3.971 $\int (a+bx)^2(ac-bcx)\,dx$

Optimal. Leaf size=32

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] (2*a*c*(a + b*x)^3)/(3*b) - (c*(a + b*x)^4)/(4*b)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)\,dx &= \int \left(2ac(a+bx)^2-c(a+bx)^3\right)\,dx\\ &= \frac{2ac(a+bx)^3}{3b}-\frac{c(a+bx)^4}{4b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 40, normalized size = 1.25

$$c\left(a^3x+\frac{1}{2}a^2bx^2-\frac{1}{3}ab^2x^3-\frac{1}{4}b^3x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] c*(a^3*x + (a^2*b*x^2)/2 - (a*b^2*x^3)/3 - (b^3*x^4)/4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x), x]
```

fricas [A] time = 1.13, size = 36, normalized size = 1.12

$$-\frac{1}{4}x^4cb^3-\frac{1}{3}x^3cb^2a+\frac{1}{2}x^2cba^2+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/4*x^4*c*b^3 - 1/3*x^3*c*b^2*a + 1/2*x^2*c*b*a^2 + x*c*a^3
```

giac [A] time = 1.02, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="giac")
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

maple [A] time = 0.00, size = 37, normalized size = 1.16

$$-\frac{1}{4}b^3c\,x^4-\frac{1}{3}a\,b^2c\,x^3+\frac{1}{2}a^2bc\,x^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c),x)
[Out] -1/4*b^3*c*x^4-1/3*a*b^2*c*x^3+1/2*a^2*b*c*x^2+a^3*c*x
```

maxima [A] time = 1.32, size = 36, normalized size = 1.12

$$-\frac{1}{4}\,b^3cx^4-\frac{1}{3}\,ab^2cx^3+\frac{1}{2}\,a^2bcx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c),x, algorithm="maxima")
[Out] -1/4*b^3*c*x^4 - 1/3*a*b^2*c*x^3 + 1/2*a^2*b*c*x^2 + a^3*c*x
```

mupad [B] time = 0.05, size = 36, normalized size = 1.12

$$c\,a^3\,x+\frac{c\,a^2\,b\,x^2}{2}-\frac{c\,a\,b^2\,x^3}{3}-\frac{c\,b^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)*(a + b*x)^2,x)
[Out] a^3*c*x - (b^3*c*x^4)/4 + (a^2*b*c*x^2)/2 - (a*b^2*c*x^3)/3
```

sympy [A] time = 0.07, size = 39, normalized size = 1.22

$$a^3cx+\frac{a^2bcx^2}{2}-\frac{ab^2cx^3}{3}-\frac{b^3cx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c),x)
[Out] a**3*c*x + a**2*b*c*x**2/2 - a*b**2*c*x**3/3 - b**3*c*x**4/4
```

3.972 $\int (a+bx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(a+bx)^3}{3b}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^2 \, dx = \frac{(a+bx)^3}{3b}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(a+bx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2,x]
[Out] (a + b*x)^3/(3*b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(a + b*x)^2, x]
```

fricas [A] time = 1.13, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3b^2 + x^2ba + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="fricas")
[Out] 1/3*x^3*b^2 + x^2*b*a + x*a^2
```

giac [A] time = 0.97, size = 12, normalized size = 0.86

$$\frac{(bx+a)^3}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="giac")
[Out] 1/3*(b*x + a)^3/b
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(bx+a)^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2,x)
[Out] 1/3*(b*x+a)^3/b
```

maxima [A] time = 1.27, size = 20, normalized size = 1.43

$$\frac{1}{3}\,b^2x^3+abx^2+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + a*b*x^2 + a^2*x
```

mupad [B] time = 0.03, size = 20, normalized size = 1.43

$$a^2\,x+a\,b\,x^2+\frac{b^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2,x)
[Out] a^2*x + (b^2*x^3)/3 + a*b*x^2
```

sympy [B] time = 0.06, size = 19, normalized size = 1.36

$$a^2x+abx^2+\frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2,x)
[Out] a**2*x + a*b*x**2 + b**2*x**3/3
```

3.973 $\int \frac{(a+bx)^2}{ac-bcx} dx$

Optimal. Leaf size=43

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{(a+bx)^2}{2bc}-\frac{2ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x),x]
[Out] (-2*a*x)/c - (a + b*x)^2/(2*b*c) - (4*a^2*Log[a - b*x])/(b*c)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{ac-bcx}\,dx &= \int \left(-\frac{2a}{c}-\frac{a+bx}{c}+\frac{4a^2}{ac-bcx}\right)dx\\ &= -\frac{2ax}{c}-\frac{(a+bx)^2}{2bc}-\frac{4a^2\log(a-bx)}{bc}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.86

$$-\frac{4a^2\log(a-bx)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x),x]
[Out] (-3*a*x)/c - (b*x^2)/(2*c) - (4*a^2*Log[a - b*x])/(b*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{ac-bcx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x),x]
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x), x]
```

fricas [A] time = 1.23, size = 34, normalized size = 0.79

$$-\frac{b^2x^2+6\,abx+8\,a^2\log\left(bx-a\right)}{2\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
[Out] -1/2*(b^2*x^2 + 6*a*b*x + 8*a^2*log(b*x - a))/(b*c)
```

giac [A] time = 0.87, size = 46, normalized size = 1.07

$$-\frac{4\,a^2\log\left(|bx-a|\right)}{bc}-\frac{b^3cx^2+6\,ab^2cx}{2\,b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
[Out] -4*a^2*log(abs(b*x - a))/(b*c) - 1/2*(b^3*c*x^2 + 6*a*b^2*c*x)/(b^2*c^2)
```

maple [A] time = 0.00, size = 37, normalized size = 0.86

$$-\frac{b\,x^2}{2c}-\frac{4a^2\ln\left(bx-a\right)}{bc}-\frac{3ax}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c),x)
[Out] -1/2/c*x^2*b-3*a*x/c-4/c*a^2/b*ln(b*x-a)
```

maxima [A] time = 1.29, size = 35, normalized size = 0.81

$$-\frac{4\,a^2\log\left(bx-a\right)}{bc}-\frac{bx^2+6\,ax}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
[Out] -4*a^2*log(b*x - a)/(b*c) - 1/2*(b*x^2 + 6*a*x)/c
```

mupad [B] time = 0.05, size = 34, normalized size = 0.79

$$-\frac{8\,a^2\,\ln\left(b\,x-a\right)+b^2\,x^2+6\,a\,b\,x}{2\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x),x)
[Out] -(8*a^2*log(b*x - a) + b^2*x^2 + 6*a*b*x)/(2*b*c)
```

sympy [A] time = 0.17, size = 31, normalized size = 0.72

$$-\frac{4a^2\log\left(-a+bx\right)}{bc}-\frac{3ax}{c}-\frac{bx^2}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c),x)
[Out] -4*a**2*log(-a + b*x)/(b*c) - 3*a*x/c - b*x**2/(2*c)
```

3.974 $\int \frac{(a+bx)^2}{(ac-bcx)^2} dx$

Optimal. Leaf size=41

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}+\frac{x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] x/c^2 + (4*a^2)/(b*c^2*(a - b*x)) + (4*a*Log[a - b*x])/(b*c^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^2} dx &= \int \left(\frac{1}{c^2}+\frac{4a^2}{c^2(a-bx)^2}-\frac{4a}{c^2(a-bx)}\right) dx\\ &= \frac{x}{c^2}+\frac{4a^2}{bc^2(a-bx)}+\frac{4a\log(a-bx)}{bc^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 35, normalized size = 0.85

$$\frac{\frac{4a^2}{b(a-bx)}+\frac{4a\log(a-bx)}{b}+x}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] (x + (4*a^2)/(b*(a - b*x)) + (4*a*Log[a - b*x])/b)/c^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^2, x]
```

fricas [A] time = 0.60, size = 57, normalized size = 1.39

$$\frac{b^2x^2-abx-4\,a^2+4\left(abx-a^2\right)\log\left(bx-a\right)}{b^2c^2x-abc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*x^2 - a*b*x - 4*a^2 + 4*(a*b*x - a^2)*log(b*x - a))/(b^2*c^2*x - a*b*c
^2)
```

giac [A] time = 1.08, size = 79, normalized size = 1.93

$$-\frac{4\,a^2}{(bcx-ac)bc}-\frac{4\,a\log\left(\frac{|bcx-ac|}{(bcx-ac)^2|b||c|}\right)}{bc^2}+\frac{bcx-ac}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -4*a^2/((b*c*x - a*c)*b*c) - 4*a*log(abs(b*c*x - a*c)/((b*c*x - a*c)^2*abs(
b)*abs(c)))/(b*c^2) + (b*c*x - a*c)/(b*c^3)
```

maple [A] time = 0.01, size = 44, normalized size = 1.07

$$-\frac{4a^2}{(bx-a)\,b\,c^2}+\frac{4a\ln(bx-a)}{b\,c^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] x/c^2-4/c^2*a^2/b/(b*x-a)+4/c^2*a/b*ln(b*x-a)
```

maxima [A] time = 1.38, size = 46, normalized size = 1.12

$$-\frac{4\,a^2}{b^2c^2x-abc^2}+\frac{x}{c^2}+\frac{4\,a\log(bx-a)}{bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2/(b^2*c^2*x - a*b*c^2) + x/c^2 + 4*a*log(b*x - a)/(b*c^2)
```

mupad [B] time = 0.15, size = 46, normalized size = 1.12

$$\frac{x}{c^2}+\frac{4\,a^2}{b\,\left(a\,c^2-b\,c^2\,x\right)}+\frac{4\,a\,\ln(b\,x-a)}{b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^2,x)
```

```
[Out] x/c^2 + (4*a^2)/(b*(a*c^2 - b*c^2*x)) + (4*a*log(b*x - a))/(b*c^2)
```

sympy [A] time = 0.20, size = 39, normalized size = 0.95

$$-\frac{4a^2}{-abc^2+b^2c^2x}+\frac{4a\log(-a+bx)}{bc^2}+\frac{x}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**2,x)
```

```
[Out] -4*a**2/(-a*b*c**2 + b**2*c**2*x) + 4*a*log(-a + b*x)/(b*c**2) + x/c**2
```

3.975 $\int \frac{(a+bx)^2}{(ac-bcx)^3} \, dx$

Optimal. Leaf size=52

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

Rubi [A] time = 0.03, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] (2*a^2)/(b*c^3*(a - b*x)^2) - (4*a)/(b*c^3*(a - b*x)) - Log[a - b*x]/(b*c^3
)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx &= \int \left(\frac{4a^2}{c^3(a-bx)^3}-\frac{4a}{c^3(a-bx)^2}+\frac{1}{c^3(a-bx)}\right)dx\\ &= \frac{2a^2}{bc^3(a-bx)^2}-\frac{4a}{bc^3(a-bx)}-\frac{\log(a-bx)}{bc^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.63

$$-\frac{\frac{2a(a-2bx)}{(a-bx)^2}+\log(a-bx)}{bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] -(((2*a*(a - 2*b*x))/(a - b*x)^2 + Log[a - b*x])/(b*c^3))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^3, x]
```

fricas [A] time = 1.03, size = 69, normalized size = 1.33

$$\frac{4\,abx - 2\,a^2 - \left(b^2x^2 - 2\,abx + a^2\right)\log\left(bx - a\right)}{b^3c^3x^2 - 2\,ab^2c^3x + a^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] (4*a*b*x - 2*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x - a))/(b^3*c^3*x^2 - 2
*a*b^2*c^3*x + a^2*b*c^3)
```

giac [A] time = 1.07, size = 46, normalized size = 0.88

$$-\frac{\log\left(|bx - a|\right)}{bc^3} + \frac{2\left(2\,abx - a^2\right)}{(bx - a)^2bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*x - a))/(b*c^3) + 2*(2*a*b*x - a^2)/((b*x - a)^2*b*c^3)
```

maple [A] time = 0.01, size = 56, normalized size = 1.08

$$\frac{2a^2}{(bx - a)^2\,b\,c^3} + \frac{4a}{(bx - a)\,b\,c^3} - \frac{\ln\left(bx - a\right)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 2/c^3*a^2/b/(b*x-a)^2+4/c^3*a/b/(b*x-a)-1/c^3/b*ln(b*x-a)
```

maxima [A] time = 1.33, size = 61, normalized size = 1.17

$$\frac{2\left(2\,abx - a^2\right)}{b^3c^3x^2 - 2\,ab^2c^3x + a^2bc^3} - \frac{\log\left(bx - a\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] 2*(2*a*b*x - a^2)/(b^3*c^3*x^2 - 2*a*b^2*c^3*x + a^2*b*c^3) - log(b*x - a)/
(b*c^3)
```

mupad [B] time = 0.17, size = 59, normalized size = 1.13

$$\frac{4\,a\,x - \frac{2\,a^2}{b}}{a^2\,c^3 - 2\,a\,b\,c^3\,x + b^2\,c^3\,x^2} - \frac{\ln\left(b\,x - a\right)}{b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^3,x)
```

```
[Out] (4*a*x - (2*a^2)/b)/(a^2*c^3 + b^2*c^3*x^2 - 2*a*b*c^3*x) - log(b*x - a)/(b
*c^3)
```

sympy [A] time = 0.31, size = 54, normalized size = 1.04

$$-\frac{2a^2 - 4abx}{a^2bc^3 - 2ab^2c^3x + b^3c^3x^2} - \frac{\log\left(-a + bx\right)}{bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(2*a**2 - 4*a*b*x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) - log(
-a + b*x)/(b*c**3)
```

3.976 $\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] (a + b*x)^3/(6*a*b*c^4*(a - b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} dx = \frac{(a+bx)^3}{6abc^4(a-bx)^3}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.11

$$-\frac{a^2+3b^2x^2}{3bc^4(bx-a)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] -1/3*(a^2 + 3*b^2*x^2)/(b*c^4*(-a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^4} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^4, x]
```

fricas [B] time = 1.30, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

giac [A] time = 0.98, size = 29, normalized size = 1.04

$$-\frac{3\,b^2x^2+a^2}{3\,(bx-a)^3bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/((b*x - a)^3*b*c^4)
```

maple [A] time = 0.00, size = 52, normalized size = 1.86

$$\frac{-\frac{4a^2}{3(bx-a)^3b}-\frac{2a}{(bx-a)^2b}-\frac{1}{(bx-a)b}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^4,x)
```

```
[Out] 1/c^4*(-2/(b*x-a)^2*a/b-4/3*a^2/b/(b*x-a)^3-1/(b*x-a)/b)
```

maxima [B] time = 1.33, size = 60, normalized size = 2.14

$$-\frac{3\,b^2x^2+a^2}{3\left(b^4c^4x^3-3\,ab^3c^4x^2+3\,a^2b^2c^4x-a^3bc^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*x^2 + a^2)/(b^4*c^4*x^3 - 3*a*b^3*c^4*x^2 + 3*a^2*b^2*c^4*x - a
^3*b*c^4)
```

mupad [B] time = 0.05, size = 58, normalized size = 2.07

$$\frac{b\,x^2+\frac{a^2}{3\,b}}{a^3\,c^4-3\,a^2\,b\,c^4\,x+3\,a\,b^2\,c^4\,x^2-b^3\,c^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^4,x)
```

```
[Out] (b*x^2 + a^2/(3*b))/(a^3*c^4 - b^3*c^4*x^3 + 3*a*b^2*c^4*x^2 - 3*a^2*b*c^4*
x)
```

sympy [B] time = 0.35, size = 61, normalized size = 2.18

$$\frac{-a^2-3b^2x^2}{-3a^3bc^4+9a^2b^2c^4x-9ab^3c^4x^2+3b^4c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**4,x)
```

```
[Out] (-a**2 - 3*b**2*x**2)/(-3*a**3*b*c**4 + 9*a**2*b**2*c**4*x - 9*a*b**3*c**4*
x**2 + 3*b**4*c**4*x**3)
```

3.977 $\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx$

Optimal. Leaf size=56

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^5,x]

[Out] a^2/(b*c^5*(a - b*x)^4) - (4*a)/(3*b*c^5*(a - b*x)^3) + 1/(2*b*c^5*(a - b*x
)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx &= \int \left(\frac{4a^2}{c^5(a-bx)^5}-\frac{4a}{c^5(a-bx)^4}+\frac{1}{c^5(a-bx)^3}\right) dx\\ &= \frac{a^2}{bc^5(a-bx)^4}-\frac{4a}{3bc^5(a-bx)^3}+\frac{1}{2bc^5(a-bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.62

$$\frac{a^2+2abx+3b^2x^2}{6bc^5(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^5,x]

[Out] (a^2 + 2*a*b*x + 3*b^2*x^2)/(6*b*c^5*(a - b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^5,x]

[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^5, x]
```

fricas [A] time = 1.16, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

giac [A] time = 1.07, size = 64, normalized size = 1.14

$$\frac{\frac{6\,a^2}{(bcx-ac)^4b}+\frac{8\,a}{(bcx-ac)^3bc}+\frac{3}{(bcx-ac)^2bc^2}}{6\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="giac")
```

```
[Out] 1/6*(6*a^2/((b*c*x - a*c)^4*b) + 8*a/((b*c*x - a*c)^3*b*c) + 3/((b*c*x - a*
c)^2*b*c^2))/c
```

maple [A] time = 0.00, size = 51, normalized size = 0.91

$$\frac{\frac{a^2}{(bx-a)^4b}+\frac{4a}{3(bx-a)^3b}+\frac{1}{2(bx-a)^2b}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^5,x)
```

```
[Out] 1/c^5*(a^2/b/(b*x-a)^4+1/2/(b*x-a)^2/b+4/3/(b*x-a)^3*a/b)
```

maxima [A] time = 1.38, size = 78, normalized size = 1.39

$$\frac{3\,b^2x^2+2\,abx+a^2}{6\left(b^5c^5x^4-4\,ab^4c^5x^3+6\,a^2b^3c^5x^2-4\,a^3b^2c^5x+a^4bc^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^5,x, algorithm="maxima")
```

```
[Out] 1/6*(3*b^2*x^2 + 2*a*b*x + a^2)/(b^5*c^5*x^4 - 4*a*b^4*c^5*x^3 + 6*a^2*b^3*
c^5*x^2 - 4*a^3*b^2*c^5*x + a^4*b*c^5)
```

mupad [B] time = 0.05, size = 76, normalized size = 1.36

$$\frac{\frac{a\,x}{3}+\frac{b\,x^2}{2}+\frac{a^2}{6\,b}}{a^4\,c^5-4\,a^3\,b\,c^5\,x+6\,a^2\,b^2\,c^5\,x^2-4\,a\,b^3\,c^5\,x^3+b^4\,c^5\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^5,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/2 + a^2/(6*b))/(a^4*c^5 + b^4*c^5*x^4 - 4*a*b^3*c^5*x^3
+ 6*a^2*b^2*c^5*x^2 - 4*a^3*b*c^5*x)
```

sympy [A] time = 0.44, size = 85, normalized size = 1.52

$$-\frac{-a^2-2abx-3b^2x^2}{6a^4bc^5-24a^3b^2c^5x+36a^2b^3c^5x^2-24ab^4c^5x^3+6b^5c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**5,x)
```

```
[Out] -(-a**2 - 2*a*b*x - 3*b**2*x**2)/(6*a**4*b*c**5 - 24*a**3*b**2*c**5*x + 36*
a**2*b**3*c**5*x**2 - 24*a*b**4*c**5*x**3 + 6*b**5*c**5*x**4)
```

3.978 $\int \frac{(a+bx)^2}{(ac-bcx)^6} dx$

Optimal. Leaf size=57

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] (4*a^2)/(5*b*c^6*(a - b*x)^5) - a/(b*c^6*(a - b*x)^4) + 1/(3*b*c^6*(a - b*x
)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx &= \int \left(\frac{4a^2}{c^6(a-bx)^6}-\frac{4a}{c^6(a-bx)^5}+\frac{1}{c^6(a-bx)^4}\right)dx\\ &= \frac{4a^2}{5bc^6(a-bx)^5}-\frac{a}{bc^6(a-bx)^4}+\frac{1}{3bc^6(a-bx)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.67

$$-\frac{2a^2+5abx+5b^2x^2}{15bc^6(bx-a)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] -1/15*(2*a^2 + 5*a*b*x + 5*b^2*x^2)/(b*c^6*(-a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^6, x]
```

fricas [A] time = 1.19, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

giac [A] time = 0.85, size = 36, normalized size = 0.63

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,(bx-a)^5bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="giac")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/((b*x - a)^5*b*c^6)
```

maple [A] time = 0.00, size = 52, normalized size = 0.91

$$\frac{-\frac{4a^2}{5(bx-a)^5b}-\frac{a}{(bx-a)^4b}-\frac{1}{3(bx-a)^3b}}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^6,x)
```

```
[Out] 1/c^6*(-1/(b*x-a)^4*a/b-1/3/(b*x-a)^3/b-4/5*a^2/b/(b*x-a)^5)
```

maxima [A] time = 1.39, size = 95, normalized size = 1.67

$$-\frac{5\,b^2x^2+5\,abx+2\,a^2}{15\,\left(b^6c^6x^5-5\,ab^5c^6x^4+10\,a^2b^4c^6x^3-10\,a^3b^3c^6x^2+5\,a^4b^2c^6x-a^5bc^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(5*b^2*x^2 + 5*a*b*x + 2*a^2)/(b^6*c^6*x^5 - 5*a*b^5*c^6*x^4 + 10*a^2
*b^4*c^6*x^3 - 10*a^3*b^3*c^6*x^2 + 5*a^4*b^2*c^6*x - a^5*b*c^6)
```

mupad [B] time = 0.19, size = 91, normalized size = 1.60

$$\frac{\frac{a\,x}{3}+\frac{b\,x^2}{3}+\frac{2\,a^2}{15\,b}}{a^5\,c^6-5\,a^4\,b\,c^6\,x+10\,a^3\,b^2\,c^6\,x^2-10\,a^2\,b^3\,c^6\,x^3+5\,a\,b^4\,c^6\,x^4-b^5\,c^6\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^6,x)
```

```
[Out] ((a*x)/3 + (b*x^2)/3 + (2*a^2)/(15*b))/(a^5*c^6 - b^5*c^6*x^5 + 5*a*b^4*c^6
*x^4 + 10*a^3*b^2*c^6*x^2 - 10*a^2*b^3*c^6*x^3 - 5*a^4*b*c^6*x)
```

sympy [B] time = 0.51, size = 100, normalized size = 1.75

$$\frac{-2a^2-5abx-5b^2x^2}{-15a^5bc^6+75a^4b^2c^6x-150a^3b^3c^6x^2+150a^2b^4c^6x^3-75ab^5c^6x^4+15b^6c^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**6,x)
```

```
[Out] (-2*a**2 - 5*a*b*x - 5*b**2*x**2)/(-15*a**5*b*c**6 + 75*a**4*b**2*c**6*x -
150*a**3*b**3*c**6*x**2 + 150*a**2*b**4*c**6*x**3 - 75*a*b**5*c**6*x**4 + 1
5*b**6*c**6*x**5)
```

3.979 $\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$

Optimal. Leaf size=59

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (2*a^2)/(3*b*c^7*(a - b*x)^6) - (4*a)/(5*b*c^7*(a - b*x)^5) + 1/(4*b*c^7*(a
 - b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(ac-bcx)^7}dx &= \int \left(\frac{4a^2}{c^7(a-bx)^7}-\frac{4a}{c^7(a-bx)^6}+\frac{1}{c^7(a-bx)^5}\right)dx\\ &= \frac{2a^2}{3bc^7(a-bx)^6}-\frac{4a}{5bc^7(a-bx)^5}+\frac{1}{4bc^7(a-bx)^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.63

$$\frac{7a^2+18abx+15b^2x^2}{60bc^7(a-bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] (7*a^2 + 18*a*b*x + 15*b^2*x^2)/(60*b*c^7*(a - b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(ac-bcx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(a*c - b*c*x)^7, x]
```

fricas [A] time = 0.99, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

giac [A] time = 0.93, size = 36, normalized size = 0.61

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,(bx-a)^6bc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="giac")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/((b*x - a)^6*b*c^7)
```

maple [A] time = 0.01, size = 52, normalized size = 0.88

$$\frac{\frac{2a^2}{3(bx-a)^6b}+\frac{4a}{5(bx-a)^5b}+\frac{1}{4(bx-a)^4b}}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(-b*c*x+a*c)^7,x)
```

```
[Out] 1/c^7*(1/4/(b*x-a)^4/b+2/3*a^2/b/(b*x-a)^6+4/5/(b*x-a)^5*a/b)
```

maxima [A] time = 1.42, size = 108, normalized size = 1.83

$$\frac{15\,b^2x^2+18\,abx+7\,a^2}{60\,\left(b^7c^7x^6-6\,ab^6c^7x^5+15\,a^2b^5c^7x^4-20\,a^3b^4c^7x^3+15\,a^4b^3c^7x^2-6\,a^5b^2c^7x+a^6bc^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(-b*c*x+a*c)^7,x, algorithm="maxima")
```

```
[Out] 1/60*(15*b^2*x^2 + 18*a*b*x + 7*a^2)/(b^7*c^7*x^6 - 6*a*b^6*c^7*x^5 + 15*a^
2*b^5*c^7*x^4 - 20*a^3*b^4*c^7*x^3 + 15*a^4*b^3*c^7*x^2 - 6*a^5*b^2*c^7*x +
 a^6*b*c^7)
```

mupad [B] time = 0.11, size = 104, normalized size = 1.76

$$\frac{\frac{3\,a\,x}{10}+\frac{b\,x^2}{4}+\frac{7\,a^2}{60\,b}}{a^6\,c^7-6\,a^5\,b\,c^7\,x+15\,a^4\,b^2\,c^7\,x^2-20\,a^3\,b^3\,c^7\,x^3+15\,a^2\,b^4\,c^7\,x^4-6\,a\,b^5\,c^7\,x^5+b^6\,c^7\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(a*c - b*c*x)^7,x)
```

```
[Out] ((3*a*x)/10 + (b*x^2)/4 + (7*a^2)/(60*b))/(a^6*c^7 + b^6*c^7*x^6 - 6*a*b^5*
c^7*x^5 + 15*a^4*b^2*c^7*x^2 - 20*a^3*b^3*c^7*x^3 + 15*a^2*b^4*c^7*x^4 - 6*
a^5*b*c^7*x)
```

sympy [B] time = 0.60, size = 117, normalized size = 1.98

$$-\frac{-7a^2-18abx-15b^2x^2}{60a^6bc^7-360a^5b^2c^7x+900a^4b^3c^7x^2-1200a^3b^4c^7x^3+900a^2b^5c^7x^4-360ab^6c^7x^5+60b^7c^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(-b*c*x+a*c)**7,x)
```

```
[Out] -(-7*a**2 - 18*a*b*x - 15*b**2*x**2)/(60*a**6*b*c**7 - 360*a**5*b**2*c**7*x
 + 900*a**4*b**3*c**7*x**2 - 1200*a**3*b**4*c**7*x**3 + 900*a**2*b**5*c**7*
x**4 - 360*a*b**6*c**7*x**5 + 60*b**7*c**7*x**6)
```

3.980 $\int \frac{(ac-bcx)^3}{a+bx}\,dx$

Optimal. Leaf size=61

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{8a^3c^3\log(a+bx)}{b}-4a^2c^3x+\frac{c^3(a-bx)^3}{3b}+\frac{ac^3(a-bx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] -4*a^2*c^3*x + (a*c^3*(a - b*x)^2)/b + (c^3*(a - b*x)^3)/(3*b) + (8*a^3*c^3
*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{a+bx}\,dx &= \int \left(-4a^2c^3+\frac{8a^3c^3}{a+bx}-2ac^2(ac-bcx)-c(ac-bcx)^2\right)dx\\ &= -4a^2c^3x+\frac{ac^3(a-bx)^2}{b}+\frac{c^3(a-bx)^3}{3b}+\frac{8a^3c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.69

$$c^3\left(\frac{8a^3\log(a+bx)}{b}-7a^2x+2abx^2-\frac{b^2x^3}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] c^3*(-7*a^2*x + 2*a*b*x^2 - (b^2*x^3)/3 + (8*a^3*Log[a + b*x])/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^3}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x), x]
```

fricas [A] time = 1.37, size = 52, normalized size = 0.85

$$-\frac{b^3c^3x^3 - 6\,ab^2c^3x^2 + 21\,a^2bc^3x - 24\,a^3c^3\log\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/3*(b^3*c^3*x^3 - 6*a*b^2*c^3*x^2 + 21*a^2*b*c^3*x - 24*a^3*c^3*log(b*x +
 a))/b
```

giac [A] time = 1.15, size = 59, normalized size = 0.97

$$\frac{8\,a^3c^3\log\left(|bx+a|\right)}{b} - \frac{b^5c^3x^3 - 6\,ab^4c^3x^2 + 21\,a^2b^3c^3x}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 8*a^3*c^3*log(abs(b*x + a))/b - 1/3*(b^5*c^3*x^3 - 6*a*b^4*c^3*x^2 + 21*a^2
*b^3*c^3*x)/b^3
```

maple [A] time = 0.00, size = 49, normalized size = 0.80

$$-\frac{b^2c^3x^3}{3} + 2ab\,c^3x^2 + \frac{8a^3c^3\ln\left(bx+a\right)}{b} - 7a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a),x)
```

```
[Out] -1/3*c^3*b^2*x^3+2*c^3*b*x^2*a-7*a^2*c^3*x+8*a^3*c^3*ln(b*x+a)/b
```

maxima [A] time = 1.34, size = 48, normalized size = 0.79

$$-\frac{1}{3}\,b^2c^3x^3 + 2\,abc^3x^2 - 7\,a^2c^3x + \frac{8\,a^3c^3\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/3*b^2*c^3*x^3 + 2*a*b*c^3*x^2 - 7*a^2*c^3*x + 8*a^3*c^3*log(b*x + a)/b
```

mupad [B] time = 0.05, size = 48, normalized size = 0.79

$$\frac{8\,a^3\,c^3\,\ln\left(a+b\,x\right)}{b} - \frac{b^2\,c^3\,x^3}{3} - 7\,a^2\,c^3\,x + 2\,a\,b\,c^3\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x),x)
```

```
[Out] (8*a^3*c^3*log(a + b*x))/b - (b^2*c^3*x^3)/3 - 7*a^2*c^3*x + 2*a*b*c^3*x^2
```

sympy [A] time = 0.18, size = 49, normalized size = 0.80

$$\frac{8a^3c^3\log\left(a+bx\right)}{b} - 7a^2c^3x + 2abc^3x^2 - \frac{b^2c^3x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a),x)
```

```
[Out] 8*a**3*c**3*log(a + b*x)/b - 7*a**2*c**3*x + 2*a*b*c**3*x**2 - b**2*c**3*x*
*3/3
```

3.981 $\int \frac{(ac-bcx)^2}{a+bx}\,dx$

Optimal. Leaf size=43

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$\frac{4a^2c^2\log(a+bx)}{b}+\frac{c^2(a-bx)^2}{2b}-2ac^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] -2*a*c^2*x + (c^2*(a - b*x)^2)/(2*b) + (4*a^2*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{a+bx}\,dx &= \int \left(-2ac^2+\frac{4a^2c^2}{a+bx}-c(ac-bcx)\right)dx\\ &= -2ac^2x+\frac{c^2(a-bx)^2}{2b}+\frac{4a^2c^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.72

$$c^2\left(\frac{4a^2\log(a+bx)}{b}-3ax+\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] c^2*(-3*a*x + (b*x^2)/2 + (4*a^2*Log[a + b*x])/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^2}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x), x]
```

fricas [A] time = 1.23, size = 38, normalized size = 0.88

$$\frac{b^2c^2x^2-6\,abc^2x+8\,a^2c^2\log(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2 - 6*a*b*c^2*x + 8*a^2*c^2*log(b*x + a))/b
```

giac [A] time = 0.87, size = 45, normalized size = 1.05

$$\frac{4\,a^2c^2\log\left(|bx+a|\right)}{b}+\frac{b^3c^2x^2-6\,ab^2c^2x}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 4*a^2*c^2*log(abs(b*x + a))/b + 1/2*(b^3*c^2*x^2 - 6*a*b^2*c^2*x)/b^2
```

maple [A] time = 0.00, size = 35, normalized size = 0.81

$$\frac{b\,c^2x^2}{2}+\frac{4a^2c^2\ln\left(bx+a\right)}{b}-3a\,c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a),x)
```

```
[Out] 1/2*c^2*x^2*b-3*a*c^2*x+4*a^2*c^2*ln(b*x+a)/b
```

maxima [A] time = 1.34, size = 34, normalized size = 0.79

$$\frac{1}{2}\,bc^2x^2-3\,ac^2x+\frac{4\,a^2c^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*b*c^2*x^2 - 3*a*c^2*x + 4*a^2*c^2*log(b*x + a)/b
```

mupad [B] time = 0.15, size = 32, normalized size = 0.74

$$\frac{c^2\,\left(8\,a^2\,\ln\left(a+b\,x\right)+b^2\,x^2-6\,a\,b\,x\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x),x)
```

```
[Out] (c^2*(8*a^2*log(a + b*x) + b^2*x^2 - 6*a*b*x))/(2*b)
```

sympy [A] time = 0.15, size = 34, normalized size = 0.79

$$\frac{4a^2c^2\log\left(a+bx\right)}{b}-3ac^2x+\frac{bc^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a),x)
```

```
[Out] 4*a**2*c**2*log(a + b*x)/b - 3*a*c**2*x + b*c**2*x**2/2
```

3.982 $\int \frac{ac-bcx}{a+bx} dx$

Optimal. Leaf size=18

$$\frac{2ac\log(a+bx)}{b}-cx$$

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{2ac\log(a+bx)}{b}-cx$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] -(c*x) + (2*a*c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{a+bx}\,dx &= \int \left(-c+\frac{2ac}{a+bx}\right)dx\\ &= -cx+\frac{2ac\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$c\left(\frac{2a\log(a+bx)}{b}-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] c*(-x + (2*a*Log[a + b*x])/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac-bcx}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x), x]
```

fricas [A] time = 1.35, size = 20, normalized size = 1.11

$$-\frac{bcx-2\,ac\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="fricas")
[Out] -(b*c*x - 2*a*c*log(b*x + a))/b
```

giac [A] time = 1.01, size = 19, normalized size = 1.06

$$-cx + \frac{2\,ac\log\left(|bx + a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="giac")
[Out] -c*x + 2*a*c*log(abs(b*x + a))/b
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{2ac\ln\left(bx + a\right)}{b} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a),x)
[Out] -c*x+2*a*c*ln(b*x+a)/b
```

maxima [A] time = 1.34, size = 18, normalized size = 1.00

$$-cx + \frac{2\,ac\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x, algorithm="maxima")
[Out] -c*x + 2*a*c*log(b*x + a)/b
```

mupad [B] time = 0.04, size = 18, normalized size = 1.00

$$\frac{2\,a\,c\,\ln\left(a+b\,x\right)}{b} - c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x),x)
[Out] (2*a*c*log(a + b*x))/b - c*x
```

sympy [A] time = 0.12, size = 15, normalized size = 0.83

$$\frac{2ac\log\left(a + bx\right)}{b} - cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a),x)
[Out] 2*a*c*log(a + b*x)/b - c*x
```

3.983 $\int \frac{1}{a+bx}\,dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(-1),x]

[Out] Log[a + b*x]/b

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+bx}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^(-1),x]

[Out] IntegrateAlgebraic[(a + b*x)^(-1), x]

fricas [A] time = 1.16, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a),x, algorithm="fricas")

[Out] log(b*x + a)/b

giac [**A**] time = 1.06, size = 11, normalized size = 1.10

$$\frac{\log(|bx + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

maple [**A**] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
[Out] 1/b*ln(b*x+a)
```

maxima [**A**] time = 1.33, size = 10, normalized size = 1.00

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

mupad [**B**] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x),x)
[Out] log(a + b*x)/b
```

sympy [**A**] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
[Out] log(a + b*x)/b
```

3.984 $\int \frac{1}{(a+bx)(ac-bcx)}\,dx$

Optimal. Leaf size=17

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {35, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

Rule 35

```
Int[1/(((a_) + (b_.)*(x_))*((c_) + (d_.)*(x_))), x_Symbol] :> Int[1/(a*c +
b*d*x^2), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)}\,dx &= \int \frac{1}{a^2c-b^2cx^2}\,dx\\ &= \frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{abc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b*c)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)), x]
```

fricas [A] time = 1.23, size = 28, normalized size = 1.65

$$\frac{\log\left(bx+a\right)-\log\left(bx-a\right)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="fricas")
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b*c)
```

giac [B] time = 1.20, size = 39, normalized size = 2.29

$$\frac{\log\left(|bx+a|\right)}{2\,abc}-\frac{\log\left(|bx-a|\right)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="giac")
[Out] 1/2*log(abs(b*x + a))/(a*b*c) - 1/2*log(abs(b*x - a))/(a*b*c)
```

maple [B] time = 0.01, size = 38, normalized size = 2.24

$$-\frac{\ln\left(bx-a\right)}{2abc}+\frac{\ln\left(bx+a\right)}{2abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c),x)
[Out] 1/2/c/b/a*ln(b*x+a)-1/2/c/b/a*ln(b*x-a)
```

maxima [B] time = 1.40, size = 37, normalized size = 2.18

$$\frac{\log\left(bx+a\right)}{2\,abc}-\frac{\log\left(bx-a\right)}{2\,abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x, algorithm="maxima")
[Out] 1/2*log(b*x + a)/(a*b*c) - 1/2*log(b*x - a)/(a*b*c)
```

mupad [B] time = 0.17, size = 17, normalized size = 1.00

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{a\,b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)),x)
[Out] atanh((b*x)/a)/(a*b*c)
```

sympy [B] time = 0.17, size = 22, normalized size = 1.29

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c),x)
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b*c)
```

3.985 $\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}+\frac{1}{2abc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] 1/(2*a*b*c^2*(a - b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx&=\int \left(\frac{1}{2ac^2(a-bx)^2}+\frac{1}{2ac^2\left(a^2-b^2x^2\right)}\right)dx\\&=\frac{1}{2abc^2(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{2ac^2}\\&=\frac{1}{2abc^2(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.26

$$\frac{(bx-a)\log(a-bx)+(a-bx)\log(a+bx)+2a}{4a^2bc^2(a-bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a + (-a + b*x)*Log[a - b*x] + (a - b*x)*Log[a + b*x])/(4*a^2*b*c^2*(a -
b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^2),x]
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^2), x]
```

fricas [A] time = 1.41, size = 60, normalized size = 1.43

$$\frac{(bx-a)\log(bx+a)-(bx-a)\log(bx-a)-2a}{4\left(a^2b^2c^2x-a^3bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="fricas")
[Out] 1/4*((b*x - a)*log(b*x + a) - (b*x - a)*log(b*x - a) - 2*a)/(a^2*b^2*c^2*x
 - a^3*b*c^2)
```

giac [A] time = 1.03, size = 53, normalized size = 1.26

$$-\frac{1}{2(bcx-ac)abc}+\frac{\log\left(\left|-\frac{2ac}{bcx-ac}-1\right|\right)}{4a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="giac")
[Out] -1/2/((b*c*x - a*c)*a*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^2*b*
c^2)
```

maple [A] time = 0.01, size = 58, normalized size = 1.38

$$-\frac{1}{2(bx-a)abc^2}-\frac{\ln(bx-a)}{4a^2bc^2}+\frac{\ln(bx+a)}{4a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^2,x)
[Out] 1/4/c^2/a^2/b*ln(b*x+a)-1/4/c^2/a^2/b*ln(b*x-a)-1/2/c^2/b/a/(b*x-a)
```

maxima [A] time = 1.26, size = 60, normalized size = 1.43

$$-\frac{1}{2\left(ab^2c^2x-a^2bc^2\right)}+\frac{\log(bx+a)}{4a^2bc^2}-\frac{\log(bx-a)}{4a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^2,x, algorithm="maxima")
[Out] -1/2/(a*b^2*c^2*x - a^2*b*c^2) + 1/4*log(b*x + a)/(a^2*b*c^2) - 1/4*log(b*x
 - a)/(a^2*b*c^2)
```

mupad [B] time = 0.07, size = 42, normalized size = 1.00

$$\frac{1}{2ab\left(ac^2-bc^2x\right)}+\frac{\operatorname{atanh}\left(\frac{bx}{a}\right)}{2a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^2*(a + b*x)),x)
```

```
[Out] 1/(2*a*b*(a*c^2 - b*c^2*x)) + atanh((b*x)/a)/(2*a^2*b*c^2)
```

sympy [A] time = 0.28, size = 48, normalized size = 1.14

$$-\frac{1}{-2a^2bc^2 + 2ab^2c^2x} + \frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4} + \frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**2,x)
```

```
[Out] -1/(-2*a**2*b*c**2 + 2*a*b**2*c**2*x) + (-log(-a/b + x)/4 + log(a/b + x)/4)
/(a**2*b*c**2)
```

3.986 $\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4a^2bc^3(a-bx)}+\frac{1}{4abc^3(a-bx)^2}$$

Rubi [A] time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}+\frac{1}{4abc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(4*a*b*c^3*(a - b*x)^2) + 1/(4*a^2*b*c^3*(a - b*x)) + ArcTanh[(b*x)/a]/(4
*a^3*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx &= \int \left(\frac{1}{2ac^3(a-bx)^3}+\frac{1}{4a^2c^3(a-bx)^2}+\frac{1}{4a^2c^3\left(a^2-b^2x^2\right)}\right)dx\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\int \frac{1}{a^2-b^2x^2}\,dx}{4a^2c^3}\\
&= \frac{1}{4abc^3(a-bx)^2}+\frac{1}{4a^2bc^3(a-bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{4a^3bc^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 1.03

$$\frac{2a(2a-bx)+(a-bx)^2(-\log(a-bx))+(a-bx)^2\log(a+bx)}{8a^3bc^3(a-bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] (2*a*(2*a - b*x) - (a - b*x)^2*Log[a - b*x] + (a - b*x)^2*Log[a + b*x])/(8*
a^3*b*c^3*(a - b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(a*c - b*c*x)^3), x]
```

fricas [A] time = 1.49, size = 98, normalized size = 1.56

$$-\frac{2\,abx-4\,a^2-\left(b^2x^2-2\,abx+a^2\right)\log\left(bx+a\right)+\left(b^2x^2-2\,abx+a^2\right)\log\left(bx-a\right)}{8\left(a^3b^3c^3x^2-2\,a^4b^2c^3x+a^5bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a*b*x - 4*a^2 - (b^2*x^2 - 2*a*b*x + a^2)*log(b*x + a) + (b^2*x^2 -
 2*a*b*x + a^2)*log(b*x - a))/(a^3*b^3*c^3*x^2 - 2*a^4*b^2*c^3*x + a^5*b*c^
3)
```

giac [A] time = 0.87, size = 69, normalized size = 1.10

$$\frac{\log\left(|bx+a|\right)}{8\,a^3bc^3}-\frac{\log\left(|bx-a|\right)}{8\,a^3bc^3}-\frac{abx-2\,a^2}{4\,(bx-a)^2a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] 1/8*log(abs(b*x + a))/(a^3*b*c^3) - 1/8*log(abs(b*x - a))/(a^3*b*c^3) - 1/4
*(a*b*x - 2*a^2)/((b*x - a)^2*a^3*b*c^3)
```

maple [A] time = 0.01, size = 78, normalized size = 1.24

$$\frac{1}{4\,(bx-a)^2\,ab\,c^3}-\frac{1}{4\,(bx-a)\,a^2b\,c^3}-\frac{\ln\left(bx-a\right)}{8a^3b\,c^3}+\frac{\ln\left(bx+a\right)}{8a^3b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(-b*c*x+a*c)^3,x)
```

```
[Out] 1/8/c^3/a^3/b*ln(b*x+a)-1/8/c^3/a^3/b*ln(b*x-a)-1/4/c^3/a^2/b/(b*x-a)+1/4/c
^3/b/a/(b*x-a)^2
```

maxima [A] time = 1.35, size = 82, normalized size = 1.30

$$-\frac{bx-2\,a}{4\left(a^2b^3c^3x^2-2\,a^3b^2c^3x+a^4bc^3\right)}+\frac{\log\left(bx+a\right)}{8\,a^3bc^3}-\frac{\log\left(bx-a\right)}{8\,a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b*x - 2*a)/(a^2*b^3*c^3*x^2 - 2*a^3*b^2*c^3*x + a^4*b*c^3) + 1/8*log(
b*x + a)/(a^3*b*c^3) - 1/8*log(b*x - a)/(a^3*b*c^3)
```

mupad [B] time = 0.08, size = 64, normalized size = 1.02

$$\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{4\,a^3\,b\,c^3}-\frac{\frac{x}{4\,a^2}-\frac{1}{2\,a\,b}}{a^2\,c^3-2\,a\,b\,c^3\,x+b^2\,c^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)),x)
```

```
[Out] atanh((b*x)/a)/(4*a^3*b*c^3) - (x/(4*a^2) - 1/(2*a*b))/(a^2*c^3 + b^2*c^3*x
^2 - 2*a*b*c^3*x)
```

sympy [A] time = 0.38, size = 71, normalized size = 1.13

$$-\frac{-2a+bx}{4a^4bc^3-8a^3b^2c^3x+4a^2b^3c^3x^2}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{8}-\frac{\log\left(\frac{a}{b}+x\right)}{8}}{a^3bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a + b*x)/(4*a**4*b*c**3 - 8*a**3*b**2*c**3*x + 4*a**2*b**3*c**3*x**2)
- (log(-a/b + x)/8 - log(a/b + x)/8)/(a**3*b*c**3)
```

3.987 $\int \frac{(ac-bcx)^3}{(a+bx)^2} dx$

Optimal. Leaf size=54

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}+5ac^3x-\frac{1}{2}bc^3x^2$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (8*a^3*c^3)/(b*(a + b*x)) - (12*a^2*c^3*Log[a +
 b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx &= \int \left(5ac^3-bc^3x+\frac{8a^3c^3}{(a+bx)^2}-\frac{12a^2c^3}{a+bx}\right)dx\\ &= 5ac^3x-\frac{1}{2}bc^3x^2-\frac{8a^3c^3}{b(a+bx)}-\frac{12a^2c^3\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.85

$$c^3\left(-\frac{8a^3}{b(a+bx)}-\frac{12a^2\log(a+bx)}{b}+5ax-\frac{bx^2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] c^3*(5*a*x - (b*x^2)/2 - (8*a^3)/(b*(a + b*x)) - (12*a^2*Log[a + b*x])/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^3}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a*c - b*c*x)^3/(a + b*x)^2, x]
```

fricas [A] time = 0.77, size = 79, normalized size = 1.46

$$-\frac{b^3c^3x^3 - 9\,ab^2c^3x^2 - 10\,a^2bc^3x + 16\,a^3c^3 + 24\left(a^2bc^3x + a^3c^3\right)\log\left(bx + a\right)}{2\left(b^2x + ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3*x^3 - 9*a*b^2*c^3*x^2 - 10*a^2*b*c^3*x + 16*a^3*c^3 + 24*(a^2
*b*c^3*x + a^3*c^3)*log(b*x + a))/(b^2*x + a*b)
```

giac [A] time = 1.12, size = 80, normalized size = 1.48

$$\frac{12\,a^2c^3\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{8\,a^3c^3}{(bx+a)b} + \frac{\left(\frac{12\,ac^3}{bx+a} - c^3\right)(bx+a)^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 12*a^2*c^3*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - 8*a^3*c^3/((b*x + a)*
b) + 1/2*(12*a*c^3/(b*x + a) - c^3)*(b*x + a)^2/b
```

maple [A] time = 0.01, size = 53, normalized size = 0.98

$$-\frac{b\,c^3x^2}{2} - \frac{8a^3c^3}{(bx+a)\,b} - \frac{12a^2c^3\ln\left(bx+a\right)}{b} + 5a\,c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^3/(b*x+a)^2,x)
```

```
[Out] 5*a*c^3*x-1/2*b*c^3*x^2-8*a^3*c^3/b/(b*x+a)-12*a^2*c^3*ln(b*x+a)/b
```

maxima [A] time = 1.29, size = 53, normalized size = 0.98

$$-\frac{1}{2}\,bc^3x^2 - \frac{8\,a^3c^3}{b^2x + ab} + 5\,ac^3x - \frac{12\,a^2c^3\log\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*c^3*x^2 - 8*a^3*c^3/(b^2*x + a*b) + 5*a*c^3*x - 12*a^2*c^3*log(b*x +
 a)/b
```

mupad [B] time = 0.05, size = 52, normalized size = 0.96

$$5\,a\,c^3\,x - \frac{b\,c^3\,x^2}{2} - \frac{12\,a^2\,c^3\,\ln\left(a + b\,x\right)}{b} - \frac{8\,a^3\,c^3}{b\,\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^3/(a + b*x)^2,x)
```

```
[Out] 5*a*c^3*x - (b*c^3*x^2)/2 - (12*a^2*c^3*log(a + b*x))/b - (8*a^3*c^3)/(b*(a
 + b*x))
```

sympy [A] time = 0.25, size = 51, normalized size = 0.94

$$-\frac{8a^3c^3}{ab + b^2x} - \frac{12a^2c^3\log\left(a + bx\right)}{b} + 5ac^3x - \frac{bc^3x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**3/(b*x+a)**2,x)
```

```
[Out] -8*a**3*c**3/(a*b + b**2*x) - 12*a**2*c**3*log(a + b*x)/b + 5*a*c**3*x - b*
c**3*x**2/2
```

3.988 $\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx$

Optimal. Leaf size=39

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}+c^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)^2/(a + b*x)^2,x]
[Out] c^2*x - (4*a^2*c^2)/(b*(a + b*x)) - (4*a*c^2*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx &= \int \left(c^2+\frac{4a^2c^2}{(a+bx)^2}-\frac{4ac^2}{a+bx}\right)dx\\ &= c^2x-\frac{4a^2c^2}{b(a+bx)}-\frac{4ac^2\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.85

$$c^2\left(-\frac{4a^2}{b(a+bx)}-\frac{4a\log(a+bx)}{b}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)^2/(a + b*x)^2,x]
[Out] c^2*(x - (4*a^2)/(b*(a + b*x)) - (4*a*Log[a + b*x])/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ac-bcx)^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(a*c - b*c*x)^2/(a + b*x)^2, x]
```

fricas [A] time = 1.25, size = 61, normalized size = 1.56

$$\frac{b^2c^2x^2 + abc^2x - 4\,a^2c^2 - 4\left(abc^2x + a^2c^2\right)\log\left(bx + a\right)}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*c^2*x^2 + a*b*c^2*x - 4*a^2*c^2 - 4*(a*b*c^2*x + a^2*c^2)*log(b*x + a)
)/(b^2*x + a*b)
```

giac [A] time = 1.18, size = 59, normalized size = 1.51

$$\frac{4\,ac^2\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} + \frac{(bx+a)c^2}{b} - \frac{4\,a^2c^2}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 4*a*c^2*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b + (b*x + a)*c^2/b - 4*a^2*
c^2/((b*x + a)*b)
```

maple [A] time = 0.01, size = 40, normalized size = 1.03

$$-\frac{4a^2c^2}{(bx+a)\,b} - \frac{4a\,c^2\ln\left(bx+a\right)}{b} + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)^2/(b*x+a)^2,x)
```

```
[Out] c^2*x-4*a^2*c^2/b/(b*x+a)-4*a*c^2*ln(b*x+a)/b
```

maxima [A] time = 1.36, size = 40, normalized size = 1.03

$$-\frac{4\,a^2c^2}{b^2x + ab} + c^2x - \frac{4\,ac^2\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -4*a^2*c^2/(b^2*x + a*b) + c^2*x - 4*a*c^2*log(b*x + a)/b
```

mupad [B] time = 0.17, size = 39, normalized size = 1.00

$$c^2\,x - \frac{4\,a\,c^2\,\ln\left(a+b\,x\right)}{b} - \frac{4\,a^2\,c^2}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^2/(a + b*x)^2,x)
```

```
[Out] c^2*x - (4*a*c^2*log(a + b*x))/b - (4*a^2*c^2)/(b*(a + b*x))
```

sympy [A] time = 0.19, size = 36, normalized size = 0.92

$$-\frac{4a^2c^2}{ab + b^2x} - \frac{4ac^2\log\left(a+bx\right)}{b} + c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)**2/(b*x+a)**2,x)
```

```
[Out] -4*a**2*c**2/(a*b + b**2*x) - 4*a*c**2*log(a + b*x)/b + c**2*x
```

3.989 $\int \frac{ac-bcx}{(a+bx)^2} \, dx$

Optimal. Leaf size=27

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*c - b*c*x)/(a + b*x)^2,x]
[Out] (-2*a*c)/(b*(a + b*x)) - (c*Log[a + b*x])/b
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{ac-bcx}{(a+bx)^2} \, dx &= \int \left(\frac{2ac}{(a+bx)^2}-\frac{c}{a+bx}\right) dx\\ &= -\frac{2ac}{b(a+bx)}-\frac{c\log(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$-\frac{c\left(\frac{2a}{a+bx}+\log(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c - b*c*x)/(a + b*x)^2,x]
[Out] -((c*((2*a)/(a + b*x) + Log[a + b*x]))/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ac-bcx}{(a+bx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(a*c - b*c*x)/(a + b*x)^2, x]
```

fricas [A] time = 1.19, size = 33, normalized size = 1.22

$$-\frac{2\,ac+(bcx+ac)\log{(bx+a)}}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*c + (b*c*x + a*c)*log(b*x + a))/(b^2*x + a*b)
```

giac [A] time = 1.02, size = 54, normalized size = 2.00

$$c\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b} - \frac{a}{(bx+a)b}\right) - \frac{ac}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] c*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b)) - a*c/((b*x
+ a)*b)
```

maple [A] time = 0.00, size = 28, normalized size = 1.04

$$-\frac{2ac}{(bx+a)\,b} - \frac{c\ln(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*c*x+a*c)/(b*x+a)^2,x)
```

```
[Out] -2*a*c/b/(b*x+a)-c*ln(b*x+a)/b
```

maxima [A] time = 1.34, size = 28, normalized size = 1.04

$$-\frac{2\,ac}{b^2x+ab} - \frac{c\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*a*c/(b^2*x + a*b) - c*log(b*x + a)/b
```

mupad [B] time = 0.04, size = 27, normalized size = 1.00

$$-\frac{c\,\ln(a+b\,x)}{b} - \frac{2\,a\,c}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)/(a + b*x)^2,x)
```

```
[Out] - (c*log(a + b*x))/b - (2*a*c)/(b*(a + b*x))
```

sympy [A] time = 0.17, size = 24, normalized size = 0.89

$$-\frac{2ac}{ab+b^2x} - \frac{c\log(a+bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*c*x+a*c)/(b*x+a)**2,x)
```

```
[Out] -2*a*c/(a*b + b**2*x) - c*log(a + b*x)/b
```

3.990 $\int \frac{1}{(a+bx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^(-2), x]
```

fricas [A] time = 1.28, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 1.11, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
[Out] -1/(b*x+a)/b
```

maxima [A] time = 1.37, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
[Out] -1/((b*x + a)*b)
```

mupad [B] time = 0.02, size = 12, normalized size = 1.00

$$-\frac{1}{b\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*x)^2,x)
[Out] -1/(b*(a + b*x))
```

sympy [A] time = 0.14, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
[Out] -1/(a*b + b**2*x)
```

3.991 $\int \frac{1}{(a+bx)^2(ac-bcx)} dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}-\frac{1}{2abc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] -1/(2*a*b*c*(a + b*x)) + ArcTanh[(b*x)/a]/(2*a^2*b*c)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2(ac-bcx)} dx &= \int \left(\frac{1}{2ac(a+bx)^2}+\frac{1}{2ac\left(a^2-b^2x^2\right)}\right) dx\\
&= -\frac{1}{2abc(a+bx)}+\frac{\int \frac{1}{a^2-b^2x^2} dx}{2ac}\\
&= -\frac{1}{2abc(a+bx)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^2bc}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.22

$$\frac{-(a+bx)\log(a-bx)+(a+bx)\log(a+bx)-2a}{4a^2bc(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] (-2*a - (a + b*x)*Log[a - b*x] + (a + b*x)*Log[a + b*x])/(4*a^2*b*c*(a + b*
x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)), x]
```

fricas [A] time = 1.01, size = 51, normalized size = 1.24

$$\frac{(bx+a)\log(bx+a)-(bx+a)\log(bx-a)-2a}{4\left(a^2b^2cx+a^3bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="fricas")
```

```
[Out] 1/4*((b*x + a)*log(b*x + a) - (b*x + a)*log(b*x - a) - 2*a)/(a^2*b^2*c*x +
a^3*b*c)
```

giac [A] time = 0.95, size = 44, normalized size = 1.07

$$-\frac{\log\left(\left|-\frac{2a}{bx+a}+1\right|\right)}{4a^2bc}-\frac{1}{2(bx+a)abc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(-2*a/(b*x + a) + 1))/(a^2*b*c) - 1/2/((b*x + a)*a*b*c)
```

maple [A] time = 0.01, size = 56, normalized size = 1.37

$$-\frac{1}{2(bx+a)abc}-\frac{\ln(bx-a)}{4a^2bc}+\frac{\ln(bx+a)}{4a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c),x)
```

```
[Out] 1/4/c/a^2/b*ln(b*x+a)-1/2/a/b/c/(b*x+a)-1/4/c/a^2/b*ln(b*x-a)
```

maxima [A] time = 1.38, size = 55, normalized size = 1.34

$$-\frac{1}{2\left(ab^2cx+a^2bc\right)}+\frac{\log(bx+a)}{4a^2bc}-\frac{\log(bx-a)}{4a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c),x, algorithm="maxima")
```

```
[Out] -1/2/(a*b^2*c*x + a^2*b*c) + 1/4*log(b*x + a)/(a^2*b*c) - 1/4*log(b*x - a)/
(a^2*b*c)
```

mupad [B] time = 0.18, size = 37, normalized size = 0.90

$$\frac{\operatorname{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^2\,b\,c}-\frac{1}{2\,a\,b\,\left(a\,c+b\,c\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)*(a + b*x)^2),x)
```

```
[Out] atanh((b*x)/a)/(2*a^2*b*c) - 1/(2*a*b*(a*c + b*c*x))
```

sympy [A] time = 0.28, size = 44, normalized size = 1.07

$$-\frac{1}{2a^2bc+2ab^2cx}-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{4}-\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c),x)
```

```
[Out] -1/(2*a**2*b*c + 2*a*b**2*c*x) - (log(-a/b + x)/4 - log(a/b + x)/4)/(a**2*b
*c)
```

3.992 $\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$

Optimal. Leaf size=46

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}+\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {41, 199, 208}

$$\frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] x/(2*a^2*c^2*(a^2 - b^2*x^2)) + ArcTanh[(b*x)/a]/(2*a^3*b*c^2)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx &= \int \frac{1}{\left(a^2c-b^2cx^2\right)^2}\,dx\\ &= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\int \frac{1}{a^2c-b^2cx^2}\,dx}{2a^2c}\\ &= \frac{x}{2a^2c^2\left(a^2-b^2x^2\right)}+\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{2a^3bc^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.61

$$\frac{\left(b^2x^2-a^2\right)\log(a-bx)+\left(a^2-b^2x^2\right)\log(a+bx)+2abx}{4a^3bc^2(a-bx)(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] (2*a*b*x + (-a^2 + b^2*x^2)*Log[a - b*x] + (a^2 - b^2*x^2)*Log[a + b*x])/(4
*a^3*b*c^2*(a - b*x)*(a + b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^2), x]
```

fricas [A] time = 1.15, size = 76, normalized size = 1.65

$$-\frac{2\,abx-\left(b^2x^2-a^2\right)\log\left(bx+a\right)+\left(b^2x^2-a^2\right)\log\left(bx-a\right)}{4\left(a^3b^3c^2x^2-a^5bc^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*b*x - (b^2*x^2 - a^2)*log(b*x + a) + (b^2*x^2 - a^2)*log(b*x - a)
)/(a^3*b^3*c^2*x^2 - a^5*b*c^2)
```

giac [A] time = 1.07, size = 83, normalized size = 1.80

$$-\frac{1}{4\,(bcx-ac)a^2bc}+\frac{\log\left(\left|-\frac{2\,ac}{bcx-ac}-1\right|\right)}{4\,a^3bc^2}+\frac{1}{8\,a^3b\left(\frac{2\,ac}{bcx-ac}+1\right)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="giac")
```

```
[Out] -1/4/((b*c*x - a*c)*a^2*b*c) + 1/4*log(abs(-2*a*c/(b*c*x - a*c) - 1))/(a^3*
b*c^2) + 1/8/(a^3*b*(2*a*c/(b*c*x - a*c) + 1)*c^2)
```

maple [A] time = 0.01, size = 76, normalized size = 1.65

$$-\frac{1}{4\left(bx+a\right)a^2b\,c^2}-\frac{1}{4\left(bx-a\right)a^2b\,c^2}-\frac{\ln\left(bx-a\right)}{4a^3b\,c^2}+\frac{\ln\left(bx+a\right)}{4a^3b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^2,x)
```

```
[Out] 1/4/c^2/a^3/b*ln(b*x+a)-1/4/c^2/a^2/b/(b*x+a)-1/4/c^2/a^3/b*ln(b*x-a)-1/4/c
^2/a^2/b/(b*x-a)
```

maxima [A] time = 1.32, size = 64, normalized size = 1.39

$$-\frac{x}{2\left(a^2b^2c^2x^2-a^4c^2\right)}+\frac{\log\left(bx+a\right)}{4\,a^3bc^2}-\frac{\log\left(bx-a\right)}{4\,a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^2,x, algorithm="maxima")
```

[Out] -1/2*x/(a^2*b^2*c^2*x^2 - a^4*c^2) + 1/4*log(b*x + a)/(a^3*b*c^2) - 1/4*log(b*x - a)/(a^3*b*c^2)

mupad [B] time = 0.18, size = 46, normalized size = 1.00

$$\frac{x}{2\,a^2\,\left(a^2\,c^2-b^2\,c^2\,x^2\right)}+\frac{\mathrm{atanh}\left(\frac{b\,x}{a}\right)}{2\,a^3\,b\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*c - b*c*x)^2*(a + b*x)^2),x)

[Out] x/(2*a^2*(a^2*c^2 - b^2*c^2*x^2)) + atanh((b*x)/a)/(2*a^3*b*c^2)

sympy [A] time = 0.27, size = 49, normalized size = 1.07

$$-\frac{x}{-2a^4c^2+2a^2b^2c^2x^2}+\frac{-\frac{\log\left(-\frac{a}{b}+x\right)}{4}+\frac{\log\left(\frac{a}{b}+x\right)}{4}}{a^3bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**2,x)

[Out] -x/(-2*a**4*c**2 + 2*a**2*b**2*c**2*x**2) + (-log(-a/b + x)/4 + log(a/b + x)/4)/(a**3*b*c**2)

3.993 $\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$

Optimal. Leaf size=83

$$\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {44, 208}

$$\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{1}{8a^2bc^3(a-bx)^2}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] 1/(8*a^2*b*c^3*(a - b*x)^2) + 1/(4*a^3*b*c^3*(a - b*x)) - 1/(8*a^3*b*c^3*(a
 + b*x)) + (3*ArcTanh[(b*x)/a])/(8*a^4*b*c^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx &= \int \left(\frac{1}{4a^2c^3(a-bx)^3}+\frac{1}{4a^3c^3(a-bx)^2}+\frac{1}{8a^3c^3(a+bx)^2}+\frac{3}{8a^3c^3\left(a^2-b^2x^2\right)}\right)dx\\ &= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\int \frac{1}{a^2-b^2x^2}\,dx}{8a^3c^3}\\ &= \frac{1}{8a^2bc^3(a-bx)^2}+\frac{1}{4a^3bc^3(a-bx)}-\frac{1}{8a^3bc^3(a+bx)}+\frac{3\tanh^{-1}\left(\frac{bx}{a}\right)}{8a^4bc^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 68, normalized size = 0.82

$$\frac{\frac{2a\left(2a^2+3abx-3b^2x^2\right)}{(a-bx)^2(a+bx)}-3\log(a-bx)+3\log(a+bx)}{16a^4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] ((2*a*(2*a^2 + 3*a*b*x - 3*b^2*x^2))/((a - b*x)^2*(a + b*x)) - 3*Log[a - b*
x] + 3*Log[a + b*x])/(16*a^4*b*c^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(ac-bcx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(a*c - b*c*x)^3), x]
```

fricas [A] time = 1.17, size = 146, normalized size = 1.76

$$-\frac{6\,ab^2x^2-6\,a^2bx-4\,a^3-3\left(b^3x^3-ab^2x^2-a^2bx+a^3\right)\log\left(bx+a\right)+3\left(b^3x^3-ab^2x^2-a^2bx+a^3\right)\log\left(bx-a\right)}{16\left(a^4b^4c^3x^3-a^5b^3c^3x^2-a^6b^2c^3x+a^7bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*a*b^2*x^2 - 6*a^2*b*x - 4*a^3 - 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x +
 a^3)*log(b*x + a) + 3*(b^3*x^3 - a*b^2*x^2 - a^2*b*x + a^3)*log(b*x - a))/
(a^4*b^4*c^3*x^3 - a^5*b^3*c^3*x^2 - a^6*b^2*c^3*x + a^7*b*c^3)
```

giac [A] time = 0.96, size = 81, normalized size = 0.98

$$-\frac{3\,\log\left(\left|-\frac{2\,a}{bx+a}+1\right|\right)}{16\,a^4bc^3}-\frac{1}{8\,(bx+a)a^3bc^3}+\frac{\frac{12\,a}{bx+a}-5}{32\,a^4bc^3\left(\frac{2\,a}{bx+a}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="giac")
```

```
[Out] -3/16*log(abs(-2*a/(b*x + a) + 1))/(a^4*b*c^3) - 1/8/((b*x + a)*a^3*b*c^3)
+ 1/32*(12*a/(b*x + a) - 5)/(a^4*b*c^3*(2*a/(b*x + a) - 1)^2)
```

maple [A] time = 0.01, size = 96, normalized size = 1.16

$$\frac{1}{8\,(bx-a)^2\,a^2b\,c^3}-\frac{1}{8\,(bx+a)\,a^3b\,c^3}-\frac{1}{4\,(bx-a)\,a^3b\,c^3}-\frac{3\ln\left(bx-a\right)}{16a^4b\,c^3}+\frac{3\ln\left(bx+a\right)}{16a^4b\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(-b*c*x+a*c)^3,x)
```

```
[Out] 3/16/c^3/a^4/b*ln(b*x+a)-1/8/a^3/b/c^3/(b*x+a)-3/16/c^3/a^4/b*ln(b*x-a)-1/4
/c^3/a^3/b/(b*x-a)+1/8/c^3/a^2/b/(b*x-a)^2
```

maxima [A] time = 1.35, size = 108, normalized size = 1.30

$$-\frac{3\,b^2x^2-3\,abx-2\,a^2}{8\left(a^3b^4c^3x^3-a^4b^3c^3x^2-a^5b^2c^3x+a^6bc^3\right)}+\frac{3\,\log\left(bx+a\right)}{16\,a^4bc^3}-\frac{3\,\log\left(bx-a\right)}{16\,a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(-b*c*x+a*c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*b^2*x^2 - 3*a*b*x - 2*a^2)/(a^3*b^4*c^3*x^3 - a^4*b^3*c^3*x^2 - a^5
*b^2*c^3*x + a^6*b*c^3) + 3/16*log(b*x + a)/(a^4*b*c^3) - 3/16*log(b*x - a)
/(a^4*b*c^3)
```

mupad [B] time = 0.10, size = 86, normalized size = 1.04

$$\frac{\frac{3x}{8a^2}+\frac{1}{4ab}-\frac{3bx^2}{8a^3}}{a^3c^3-a^2bc^3x-ab^2c^3x^2+b^3c^3x^3}+\frac{3\operatorname{atanh}\left(\frac{bx}{a}\right)}{8a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^3*(a + b*x)^2),x)
```

```
[Out] ((3*x)/(8*a^2) + 1/(4*a*b) - (3*b*x^2)/(8*a^3))/(a^3*c^3 + b^3*c^3*x^3 - a*
b^2*c^3*x^2 - a^2*b*c^3*x) + (3*atanh((b*x)/a))/(8*a^4*b*c^3)
```

sympy [A] time = 0.51, size = 104, normalized size = 1.25

$$-\frac{-2a^2-3abx+3b^2x^2}{8a^6bc^3-8a^5b^2c^3x-8a^4b^3c^3x^2+8a^3b^4c^3x^3}-\frac{\frac{3\log\left(-\frac{a}{b}+x\right)}{16}-\frac{3\log\left(\frac{a}{b}+x\right)}{16}}{a^4bc^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(-b*c*x+a*c)**3,x)
```

```
[Out] -(-2*a**2 - 3*a*b*x + 3*b**2*x**2)/(8*a**6*b*c**3 - 8*a**5*b**2*c**3*x - 8*
a**4*b**3*c**3*x**2 + 8*a**3*b**4*c**3*x**3) - (3*log(-a/b + x)/16 - 3*log(
a/b + x)/16)/(a**4*b*c**3)
```

3.994 $\int (1-x)^{9/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=108

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{21}{16}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{3/2}(1-x)^{9/2}+\frac{3}{10}(x+1)^{3/2}(1-x)^{7/2}+\frac{21}{40}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{8}(x+1)^{3/2}(1-x)^{3/2}+\frac{21}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{21}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (21*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/8 + (21
*(1 - x)^(5/2)*(1 + x)^(3/2))/40 + (3*(1 - x)^(7/2)*(1 + x)^(3/2))/10 + ((1
 - x)^(9/2)*(1 + x)^(3/2))/6 + (21*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{9/2}\sqrt{1+x}\,dx &= \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{3}{2}\int (1-x)^{7/2}\sqrt{1+x}\,dx \\
&= \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{10}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\
&= \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1+x)^{3/2} + \frac{21}{8}\int (1-x)^{3/} \\
&= \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+x)^{3/2} + \frac{1}{6}(1-x)^{9/2}(1- \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+ \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+ \\
&= \frac{21}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{8}(1-x)^{3/2}(1+x)^{3/2} + \frac{21}{40}(1-x)^{5/2}(1+x)^{3/2} + \frac{3}{10}(1-x)^{7/2}(1+
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 60, normalized size = 0.56

$$\frac{1}{240}\left(\sqrt{1-x^2}\left(40x^5-192x^4+350x^3-256x^2-75x+448\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(448 - 75*x - 256*x^2 + 350*x^3 - 192*x^4 + 40*x^5) - 630*Ar
cSin[Sqrt[1 - x]/Sqrt[2]])/240
```

IntegrateAlgebraic [A] time = 0.09, size = 128, normalized size = 1.19

$$\frac{\sqrt{x+1}\left(\frac{315(x+1)^5}{(1-x)^5}+\frac{1785(x+1)^4}{(1-x)^4}+\frac{4158(x+1)^3}{(1-x)^3}+\frac{5058(x+1)^2}{(1-x)^2}+\frac{3335(x+1)}{1-x}-315\right)}{120\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^6}+\frac{21}{8}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-315 + (3335*(1 + x))/(1 - x) + (5058*(1 + x)^2)/(1 - x)^2 +
(4158*(1 + x)^3)/(1 - x)^3 + (1785*(1 + x)^4)/(1 - x)^4 + (315*(1 + x)^5)/(
1 - x)^5))/(120*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^6) + (21*ArcTan[Sqrt[1 +
x]/Sqrt[1 - x]])/8
```

fricas [A] time = 0.76, size = 62, normalized size = 0.57

$$\frac{1}{240}\left(40\,x^5-192\,x^4+350\,x^3-256\,x^2-75\,x+448\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{21}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/240*(40*x^5 - 192*x^4 + 350*x^3 - 256*x^2 - 75*x + 448)*sqrt(x + 1)*sqrt(
-x + 1) - 21/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.27, size = 185, normalized size = 1.71

$$\frac{1}{240}\,((2\,((4\,(5\,x-26)(x+1)+321)(x+1)-451)(x+1)+745)(x+1)-405)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{40}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\,\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{21}{8}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*
(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10
)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x - 5)*(x
+ 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 21/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.01, size = 113, normalized size = 1.05

$$\frac{21\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{9}{2}}(x+1)^{\frac{3}{2}}}{6}+\frac{3(-x+1)^{\frac{7}{2}}(x+1)^{\frac{3}{2}}}{10}+\frac{21(-x+1)^{\frac{5}{2}}(x+1)^{\frac{3}{2}}}{40}+\frac{7(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{8}+\frac{21\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{16}-\frac{21\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(1+x)^(1/2),x)
```

```
[Out] 1/6*(-x+1)^(9/2)*(1+x)^(3/2)+3/10*(-x+1)^(7/2)*(1+x)^(3/2)+21/40*(-x+1)^(5/
2)*(1+x)^(3/2)+7/8*(-x+1)^(3/2)*(1+x)^(3/2)+21/16*(-x+1)^(1/2)*(1+x)^(3/2)-
21/16*(-x+1)^(1/2)*(1+x)^(1/2)+21/16*((1+x)*(-x+1))^(1/2)/(1+x)^(1/2)/(-x+1
)^(1/2)*arcsin(x)
```

maxima [A] time = 3.00, size = 68, normalized size = 0.63

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{3}{2}}x^3+\frac{4}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{13}{8}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{28}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{21}{16}\sqrt{-x^2+1}\,x+\frac{21}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(3/2)*x^3 + 4/5*(-x^2 + 1)^(3/2)*x^2 - 13/8*(-x^2 + 1)^(3/2
)*x + 28/15*(-x^2 + 1)^(3/2) + 21/16*sqrt(-x^2 + 1)*x + 21/16*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(1/2), x)
```

sympy [A] time = 48.59, size = 289, normalized size = 2.68

$$\begin{cases} -\frac{21i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}-\frac{59i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}-\frac{2947i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}+\frac{8171i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{1045i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{21i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{21\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}+\frac{59(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}-\frac{1151(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}+\frac{2947(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}-\frac{8171(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{1045(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{21\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-21*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt
(x - 1)) - 59*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) + 1151*I*(x + 1)**(9/2)/(1
20*sqrt(x - 1)) - 2947*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) + 8171*I*(x + 1)*
*(5/2)/(240*sqrt(x - 1)) - 1045*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 21*I*sq
rt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (21*asin(sqrt(2)*sqrt(x + 1)/
```

```
2)/8 - (x + 1)**(13/2)/(6*sqrt(1 - x)) + 59*(x + 1)**(11/2)/(30*sqrt(1 - x)
) - 1151*(x + 1)**(9/2)/(120*sqrt(1 - x)) + 2947*(x + 1)**(7/2)/(120*sqrt(1
 - x)) - 8171*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 1045*(x + 1)**(3/2)/(48*sq
rt(1 - x)) - 21*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

3.995 $\int (1-x)^{7/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=88

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(x+1)^{3/2}(1-x)^{7/2}+\frac{7}{20}(x+1)^{3/2}(1-x)^{5/2}+\frac{7}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{8}x\sqrt{x+1}\sqrt{1-x}+\frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (7*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + (7*(
1 - x)^(5/2)*(1 + x)^(3/2))/20 + ((1 - x)^(7/2)*(1 + x)^(3/2))/5 + (7*ArcSi
n[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{7/2}\sqrt{1+x}\,dx &= \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{5}\int (1-x)^{5/2}\sqrt{1+x}\,dx \\
&= \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx \\
&= \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x)^{3/2} + \frac{7}{4}\int \sqrt{1-x}\,\sqrt{} \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x \\
&= \frac{7}{8}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{12}(1-x)^{3/2}(1+x)^{3/2} + \frac{7}{20}(1-x)^{5/2}(1+x)^{3/2} + \frac{1}{5}(1-x)^{7/2}(1+x
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.64

$$
\frac{1}{120}\sqrt{1-x^2}\left(-24x^4+90x^3-112x^2+15x+136\right)-\frac{7}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(136 + 15*x - 112*x^2 + 90*x^3 - 24*x^4))/120 - (7*ArcSin[Sq
rt[1 - x]/Sqrt[2]])/4
```

IntegrateAlgebraic [A] time = 0.08, size = 114, normalized size = 1.30

$$
\frac{\sqrt{x+1}\left(\frac{105(x+1)^4}{(1-x)^4}+\frac{490(x+1)^3}{(1-x)^3}+\frac{896(x+1)^2}{(1-x)^2}+\frac{790(x+1)}{1-x}-105\right)}{60\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^5}+\frac{7}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-105 + (790*(1 + x))/(1 - x) + (896*(1 + x)^2)/(1 - x)^2 + (4
90*(1 + x)^3)/(1 - x)^3 + (105*(1 + x)^4)/(1 - x)^4))/(60*Sqrt[1 - x]*(1 +
(1 + x)/(1 - x))^5) + (7*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

fricas [A] time = 1.32, size = 57, normalized size = 0.65

$$
-\frac{1}{120}\left(24\,x^4-90\,x^3+112\,x^2-15\,x-136\right)\sqrt{x+1}\,\sqrt{-x+1}\,-\frac{7}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}\,-1}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/120*(24*x^4 - 90*x^3 + 112*x^2 - 15*x - 136)*sqrt(x + 1)*sqrt(-x + 1) -
7/4*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [A] time = 1.06, size = 115, normalized size = 1.31

$$
-\frac{1}{120}\left(\left(2\left(3\left(4\,x-17\right)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}\,+\frac{1}{12}\left(\left(2\left(3\,x-10\right)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}\,-\sqrt{x+1}\,(x-2)\sqrt{-x+1}\,+\sqrt{x+1}\,\sqrt{-x+1}\,+\frac{7}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 7/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 99, normalized size = 1.12

$$\frac{7\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}(x+1)^{\frac{3}{2}}}{5}+\frac{7(-x+1)^{\frac{5}{2}}(x+1)^{\frac{3}{2}}}{20}+\frac{7(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{12}+\frac{7\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{7\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(1/2),x)
```

```
[Out] 1/5*(-x+1)^(7/2)*(x+1)^(3/2)+7/20*(-x+1)^(5/2)*(x+1)^(3/2)+7/12*(-x+1)^(3/2
)*(x+1)^(3/2)+7/8*(-x+1)^(1/2)*(x+1)^(3/2)-7/8*(-x+1)^(1/2)*(x+1)^(1/2)+7/8
*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.97, size = 54, normalized size = 0.61

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{3}{2}}x^2-\frac{3}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{17}{15}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{7}{8}\sqrt{-x^2+1}\,x+\frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)*x^2 - 3/4*(-x^2 + 1)^(3/2)*x + 17/15*(-x^2 + 1)^(3/2)
+ 7/8*sqrt(-x^2 + 1)*x + 7/8*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(1/2), x)
```

sympy [A] time = 21.07, size = 253, normalized size = 2.88

$$\begin{cases}-\frac{7i\,\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{39i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{449i(x+1)^{\frac{7}{2}}}{60\sqrt{x-1}}+\frac{1657i(x+1)^{\frac{5}{2}}}{120\sqrt{x-1}}-\frac{263i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1\\ \frac{7\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{39(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{449(x+1)^{\frac{7}{2}}}{60\sqrt{1-x}}-\frac{1657(x+1)^{\frac{5}{2}}}{120\sqrt{1-x}}+\frac{263(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{7\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 39*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 449*I*(x + 1)**(7/2)/(60*s
qrt(x - 1)) + 1657*I*(x + 1)**(5/2)/(120*sqrt(x - 1)) - 263*I*(x + 1)**(3/2
)/(24*sqrt(x - 1)) + 7*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7
*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 39*(x +
1)**(9/2)/(20*sqrt(1 - x)) + 449*(x + 1)**(7/2)/(60*sqrt(1 - x)) - 1657*(x
+ 1)**(5/2)/(120*sqrt(1 - x)) + 263*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 7*sqr
t(x + 1)/(4*sqrt(1 - x)), True))
```

3.996 $\int (1-x)^{5/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=68

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\,\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{4}(x+1)^{3/2}(1-x)^{5/2}+\frac{5}{12}(x+1)^{3/2}(1-x)^{3/2}+\frac{5}{8}x\sqrt{x+1}\,\sqrt{1-x}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 + ((1
 - x)^(5/2)*(1 + x)^(3/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}\sqrt{1+x}\,dx &= \frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int (1-x)^{3/2}\sqrt{1+x}\,dx\\
&= \frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+}}\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{4}(1-x)^{5/2}(1+x)^{3/2}+\frac{5}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.74

$$\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3-16x^2+9x+16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 - x^2]*(16 + 9*x - 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]]
)/24
```

IntegrateAlgebraic [A] time = 0.08, size = 100, normalized size = 1.47

$$\frac{\sqrt{x+1}\left(\frac{15(x+1)^3}{(1-x)^3}+\frac{55(x+1)^2}{(1-x)^2}+\frac{73(x+1)}{1-x}-15\right)}{12\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^4}+\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(-15 + (73*(1 + x))/(1 - x) + (55*(1 + x)^2)/(1 - x)^2 + (15*(
1 + x)^3)/(1 - x)^3))/(12*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^4) + (5*ArcTan[
Sqrt[1 + x]/Sqrt[1 - x]])/4
```

fricas [A] time = 1.36, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6x^3-16x^2+9x+16\right)\sqrt{x+1}\sqrt{-x+1}-\frac{5}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 - 16*x^2 + 9*x + 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.29, size = 101, normalized size = 1.49

$$\frac{1}{24}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{5}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

maple [A] time = 0.01, size = 85, normalized size = 1.25

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}(x+1)^{\frac{3}{2}}}{4}+\frac{5(-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{12}+\frac{5\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(1/2),x)
```

```
[Out] 1/4*(-x+1)^(5/2)*(x+1)^(3/2)+5/12*(-x+1)^(3/2)*(x+1)^(3/2)+5/8*(-x+1)^(1/2)
*(x+1)^(3/2)-5/8*(-x+1)^(1/2)*(x+1)^(1/2)+5/8*((x+1)*(-x+1))^(1/2)/(x+1)^(1
/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.95, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}\,x+\frac{5}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x + 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(1/2), x)
```

sympy [A] time = 9.03, size = 218, normalized size = 3.21

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}+\frac{127i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{133i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}-\frac{127(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{133(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 23*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) + 127*I*(x + 1)**(5/2)/(24*sq
rt(x - 1)) - 133*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqr
t(x - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(
9/2)/(4*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(12*sqrt(1 - x)) - 127*(x + 1)**(5
/2)/(24*sqrt(1 - x)) + 133*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/
(4*sqrt(1 - x)), True))
```

3.997 $\int (1-x)^{3/2}\sqrt{1+x}\,dx$

Optimal. Leaf size=48

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*Sqrt[1 + x],x]

[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{3/2}\sqrt{1+x}\,dx &= \frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.92

$$\frac{1}{6}\left(-2x^2+3x+2\right)\sqrt{1-x^2}-\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*Sqrt[1 + x],x]
[Out] ((2 + 3*x - 2*x^2)*Sqrt[1 - x^2])/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 82, normalized size = 1.71

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)^2}{(1-x)^2}+\frac{8(x+1)}{1-x}-3\right)}{3\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^3}+\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*Sqrt[1 + x],x]
[Out] (Sqrt[1 + x]*(-3 + (8*(1 + x))/(1 - x) + (3*(1 + x)^2)/(1 - x)^2))/(3*Sqrt[
1 - x]*(1 + (1 + x)/(1 - x))^3) + ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

fricas [A] time = 1.36, size = 47, normalized size = 0.98

$$-\frac{1}{6}\left(2\,x^2-3\,x-2\right)\sqrt{x+1}\,\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="fricas")
[Out] -1/6*(2*x^2 - 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

giac [A] time = 1.02, size = 50, normalized size = 1.04

$$-\frac{1}{6}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="giac")
[Out] -1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x
 + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.00, size = 71, normalized size = 1.48

$$\frac{\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{3}{2}}}{3}+\frac{\sqrt{-x+1}\;(x+1)^{\frac{3}{2}}}{2}-\frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(1/2),x)
[Out] 1/3*(-x+1)^(3/2)*(x+1)^(3/2)+1/2*(-x+1)^(1/2)*(x+1)^(3/2)-1/2*(-x+1)^(1/2)*
(x+1)^(1/2)+1/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.90, size = 28, normalized size = 0.58

$$\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(1/2), x)
```

sympy [B] time = 4.49, size = 168, normalized size = 3.50

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{11i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{17i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{11(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{17(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) + 11*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 17*I*(x + 1)**(3/2)/(6*sqrt(x -
1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x +
1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) - 11*(x + 1)**(5/2)/(6*sqrt(1 - x))
+ 17*(x + 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

3.998 $\int \sqrt{1-x}\sqrt{1+x}\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}\sqrt{1-x}\sqrt{x+1}\,x+\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{2}\sqrt{1-x}\sqrt{x+1}\,x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 + ArcSin[x]/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x}\sqrt{1+x}\,dx &= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.71

$$\frac{1}{2}\left(\sqrt{1-x^2}\,x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*Sqrt[1 + x],x]
[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

IntegrateAlgebraic [B] time = 0.06, size = 73, normalized size = 2.61

$$\frac{\frac{\sqrt{1-x}}{\sqrt{x+1}}-\frac{(1-x)^{3/2}}{(x+1)^{3/2}}}{\left(\frac{1-x}{x+1}+1\right)^2}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*Sqrt[1 + x],x]
```

```
[Out] (-((1 - x)^(3/2)/(1 + x)^(3/2)) + Sqrt[1 - x]/Sqrt[1 + x])/(1 + (1 - x)/(1
+ x))^2 - ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 0.88, size = 38, normalized size = 1.36

$$\frac{1}{2}\sqrt{x+1}\,x\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x + 1)*x*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.04, size = 42, normalized size = 1.50

$$\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/
2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{2}+\frac{\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(1/2),x)
```

```
[Out] -1/2*(-x+1)^(3/2)*(x+1)^(1/2)+1/2*(-x+1)^(1/2)*(x+1)^(1/2)+1/2*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.98, size = 17, normalized size = 0.61

$$\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

mupad [B] time = 0.20, size = 37, normalized size = 1.32

$$\frac{x\sqrt{1-x}\,\sqrt{x+1}}{2}-\frac{\ln\left(x-\sqrt{1-x}\,\sqrt{x+1}\,1\mathrm{i}\right)\,1\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(1/2),x)
```

```
[Out] (x*(1 - x)^(1/2)*(x + 1)^(1/2))/2 - (log(x - (1 - x)^(1/2)*(x + 1)^(1/2)*1i
)*1i)/2
```

sympy [B] time = 2.73, size = 133, normalized size = 4.75

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{3i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{3(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x - 1
)) - 3*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1), Abs(x
+ 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x))
+ 3*(x + 1)**(3/2)/(2*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

3.999 $\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=21

$$\sin^{-1}(x)-\sqrt{1-x}\,\sqrt{x+1}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\sin^{-1}(x)-\sqrt{1-x}\,\sqrt{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx &= -\sqrt{1-x}\,\sqrt{1+x}+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x}\,\sqrt{1+x}+\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.52

$$-\sqrt{1-x^2}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [C] time = 0.09, size = 45, normalized size = 2.14

$$-\sqrt{1-x}\,\sqrt{x+1}+2i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/Sqrt[1 - x],x]
[Out] -(Sqrt[1 - x]*Sqrt[1 + x]) + (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

fricas [B] time = 1.11, size = 37, normalized size = 1.76

$$-\sqrt{x+1}\,\sqrt{-x+1}-2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [A] time = 1.01, size = 28, normalized size = 1.33

$$-\sqrt{x+1}\,\sqrt{-x+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.01, size = 42, normalized size = 2.00

$$\frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{-x+1}}-\sqrt{-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(1/2),x)
[Out] -(-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arc
sin(x)
```

maxima [A] time = 2.90, size = 14, normalized size = 0.67

$$-\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")
[Out] -sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.14, size = 14, normalized size = 0.67

$$\mathrm{asin}(x)-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(1/2),x)
```

```
[Out] asin(x) - (1 - x^2)^(1/2)
```

sympy [**B**] time = 1.84, size = 100, normalized size = 4.76

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} + \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} - \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
 + 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) - 2*sqrt(x + 1)/sqrt(1 - x), True))
```

3.1000 $\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx&=\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\&=\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\int \frac{1}{\sqrt{1-x^2}}\,dx\\&=\frac{2\sqrt{1+x}}{\sqrt{1-x}}-\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.57

$$2\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(3/2),x]
[Out] 2*(Sqrt[1 + x]/Sqrt[1 - x] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.70

$$\frac{2\sqrt{x+1}}{\sqrt{1-x}}-2\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(3/2),x]
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - 2*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

fricas [B] time = 1.27, size = 48, normalized size = 2.09

$$\frac{2\left((x-1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="fricas")
[Out] 2*((x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x - 1)
```

giac [A] time = 1.06, size = 33, normalized size = 1.43

$$-\frac{2\sqrt{x+1}\sqrt{-x+1}}{x-1}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="giac")
[Out] -2*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.03, size = 64, normalized size = 2.78

$$-\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{x+1}\sqrt{-x+1}}+\frac{2\sqrt{x+1}\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(3/2),x)
[Out] 2*(x+1)^(1/2)/(-(x+1)*(-1+x))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)-((x+1
)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.95, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x-1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(3/2),x, algorithm="maxima")
[Out] -2*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x+1}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(1/2)/(1 - x)^(3/2), x)
```

sympy [A] time = 1.62, size = 71, normalized size = 3.09

$$\begin{cases} 2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1),
Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x + 1)/2) + 2*sqrt(x + 1)/sqrt(1 -
 x), True))
```

3.1001 $\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx = \frac{(1+x)^{3/2}}{3(1-x)^{3/2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 20, normalized size = 1.00

$$\frac{(x+1)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(5/2),x]
[Out] (1 + x)^(3/2)/(3*(1 - x)^(3/2))
```

fricas [B] time = 1.25, size = 33, normalized size = 1.65

$$\frac{x^2 + (x+1)^{\frac{3}{2}}\sqrt{-x+1} - 2\,x + 1}{3\left(x^2 - 2\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + (x + 1)^(3/2)*sqrt(-x + 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

giac [A] time = 0.94, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3\,(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="giac")
[Out] 1/3*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^2
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{3}{2}}}{3\,(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(5/2),x)
[Out] 1/3*(x+1)^(3/2)/(-x+1)^(3/2)
```

maxima [B] time = 1.26, size = 38, normalized size = 1.90

$$\frac{2\sqrt{-x^2+1}}{3\left(x^2-2\,x+1\right)}+\frac{\sqrt{-x^2+1}}{3\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(5/2),x, algorithm="maxima")
[Out] 2/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.27, size = 34, normalized size = 1.70

$$\frac{\left(\frac{x\,\sqrt{x+1}}{3}+\frac{\sqrt{x+1}}{3}\right)\sqrt{1-x}}{x^2-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(5/2),x)
[Out] (((x*(x + 1)^(1/2))/3 + (x + 1)^(1/2)/3)*(1 - x)^(1/2))/(x^2 - 2*x + 1)
```

sympy [A] time = 1.67, size = 61, normalized size = 3.05

$$\begin{cases} \frac{i(x+1)^{\frac{3}{2}}}{3\sqrt{x-1}\,(x+1)-6\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{3}{2}}}{3\sqrt{1-x}\,(x+1)-6\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(5/2),x)
[Out] Piecewise((I*(x + 1)**(3/2)/(3*sqrt(x - 1)*(x + 1) - 6*sqrt(x - 1)), Abs(x
+ 1)/2 > 1), (-(x + 1)**(3/2)/(3*sqrt(1 - x)*(x + 1) - 6*sqrt(1 - x)), True
))
```

3.1002 $\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{3/2}}{15(1-x)^{3/2}}+\frac{(x+1)^{3/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(3/2)/(5*(1 - x)^(5/2)) + (1 + x)^(3/2)/(15*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{1}{5}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx\\ &= \frac{(1+x)^{3/2}}{5(1-x)^{5/2}}+\frac{(1+x)^{3/2}}{15(1-x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-4)(x+1)^{3/2}}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(7/2),x]
```

```
[Out] -1/15*((-4 + x)*(1 + x)^(3/2))/(1 - x)^(5/2)
```

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 0.83

$$\frac{(x+1)^{3/2}\left(\frac{3(x+1)}{1-x}+5\right)}{30(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(7/2),x]
[Out] ((1 + x)^(3/2)*(5 + (3*(1 + x))/(1 - x)))/(30*(1 - x)^(3/2))
```

fricas [A] time = 0.92, size = 53, normalized size = 1.29

$$\frac{4\,x^3-12\,x^2+\left(x^2-3\,x-4\right)\sqrt{x+1}\,\sqrt{-x+1}\,+12\,x-4}{15\,\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="fricas")
[Out] 1/15*(4*x^3 - 12*x^2 + (x^2 - 3*x - 4)*sqrt(x + 1)*sqrt(-x + 1) + 12*x - 4)
/(x^3 - 3*x^2 + 3*x - 1)
```

giac [A] time = 1.05, size = 22, normalized size = 0.54

$$\frac{(x+1)^{\frac{3}{2}}(x-4)\sqrt{-x+1}}{15\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="giac")
[Out] 1/15*(x + 1)^(3/2)*(x - 4)*sqrt(-x + 1)/(x - 1)^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+1)^{\frac{3}{2}}\,(x-4)}{15\,(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(7/2),x)
[Out] -1/15*(x+1)^(3/2)*(x-4)/(-x+1)^(5/2)
```

maxima [B] time = 1.40, size = 64, normalized size = 1.56

$$-\frac{2\,\sqrt{-x^2+1}}{5\,\left(x^3-3\,x^2+3\,x-1\right)}-\frac{\sqrt{-x^2+1}}{15\,\left(x^2-2\,x+1\right)}+\frac{\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(7/2),x, algorithm="maxima")
[Out] -2/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 1/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) + 1/15*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.24, size = 50, normalized size = 1.22

$$-\frac{\sqrt{1-x}\,\left(\frac{x\,\sqrt{x+1}}{5}+\frac{4\,\sqrt{x+1}}{15}-\frac{x^2\,\sqrt{x+1}}{15}\right)}{x^3-3\,x^2+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((x*(x + 1)^(1/2))/5 + (4*(x + 1)^(1/2))/15 - (x^2*(x + 1)^
(1/2))/15))/(3*x - 3*x^2 + x^3 - 1)
```

sympy [B] time = 6.55, size = 173, normalized size = 4.22

$$\begin{cases} \frac{i(x+1)^{\frac{5}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{15\sqrt{x-1}\,(x+1)^2-60\sqrt{x-1}\,(x+1)+60\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ -\frac{(x+1)^{\frac{5}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{15\sqrt{1-x}\,(x+1)^2-60\sqrt{1-x}\,(x+1)+60\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(5/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60*sqrt(x - 1)*(x
+ 1) + 60*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(15*sqrt(x - 1)*(x + 1)**2 - 60
*sqrt(x - 1)*(x + 1) + 60*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x + 1)**(5/2)
/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqrt(1 - x)) + 5*
(x + 1)**(3/2)/(15*sqrt(1 - x)*(x + 1)**2 - 60*sqrt(1 - x)*(x + 1) + 60*sqr
t(1 - x)), True))
```

3.1003 $\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{105(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(3/2)/(7*(1 - x)^(7/2)) + (2*(1 + x)^(3/2))/(35*(1 - x)^(5/2)) + (2
*(1 + x)^(3/2))/(105*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2}{7}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2}{35}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{7(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{35(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{(x+1)^{3/2}\left(2x^2-10x+23\right)}{105(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] ((1 + x)^(3/2)*(23 - 10*x + 2*x^2))/(105*(1 - x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 0.79

$$\frac{(x+1)^{3/2}\left(\frac{15(x+1)^2}{(1-x)^2}+\frac{42(x+1)}{1-x}+35\right)}{420(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(9/2),x]
```

```
[Out] ((1 + x)^(3/2)*(35 + (42*(1 + x))/(1 - x) + (15*(1 + x)^2)/(1 - x)^2))/(420
*(1 - x)^(3/2))
```

fricas [A] time = 1.17, size = 70, normalized size = 1.15

$$\frac{23\,x^4-92\,x^3+138\,x^2+\left(2\,x^3-8\,x^2+13\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}\,-92\,x+23}{105\,\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(23*x^4 - 92*x^3 + 138*x^2 + (2*x^3 - 8*x^2 + 13*x + 23)*sqrt(x + 1)*
sqrt(-x + 1) - 92*x + 23)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

giac [A] time = 1.22, size = 29, normalized size = 0.48

$$\frac{(2\,(x+1)(x-6)+35)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{105\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(2*(x + 1)*(x - 6) + 35)*(x + 1)^(3/2)*sqrt(-x + 1)/(x - 1)^4
```

maple [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{(x+1)^{\frac{3}{2}}\left(2x^2-10x+23\right)}{105\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(9/2),x)
```

```
[Out] 1/105*(x+1)^(3/2)*(2*x^2-10*x+23)/(-x+1)^(7/2)
```

maxima [B] time = 1.27, size = 95, normalized size = 1.56

$$\frac{2\,\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{\sqrt{-x^2+1}}{35\left(x^3-3\,x^2+3\,x-1\right)}-\frac{2\,\sqrt{-x^2+1}}{105\left(x^2-2\,x+1\right)}+\frac{2\,\sqrt{-x^2+1}}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) - 2/105*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/105*sqrt(-
x^2 + 1)/(x - 1)
```

mupad [B] time = 0.27, size = 64, normalized size = 1.05

$$\frac{\sqrt{1-x}\,\left(\frac{13\,x\,\sqrt{x+1}}{105}+\frac{23\,\sqrt{x+1}}{105}-\frac{8\,x^2\,\sqrt{x+1}}{105}+\frac{2\,x^3\,\sqrt{x+1}}{105}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((13*x*(x + 1)^(1/2))/105 + (23*(x + 1)^(1/2))/105 - (8*x^2*
(x + 1)^(1/2))/105 + (2*x^3*(x + 1)^(1/2))/105))/(6*x^2 - 4*x - 4*x^3 + x^4
 + 1)
```

sympy [B] time = 19.92, size = 568, normalized size = 9.31

$$\begin{cases} \frac{2i(x+1)^{\frac{9}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{18i(x+1)^{\frac{7}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} + \frac{63i(x+1)^{\frac{5}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} - \frac{70i(x+1)^{\frac{3}{2}}}{105\sqrt{x-1}(x+1)^4-840\sqrt{x-1}(x+1)^3+2520\sqrt{x-1}(x+1)^2-3360\sqrt{x-1}(x+1)+1680\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -\frac{2(x+1)^{\frac{9}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{18(x+1)^{\frac{7}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} - \frac{63(x+1)^{\frac{5}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} + \frac{70(x+1)^{\frac{3}{2}}}{105\sqrt{1-x}(x+1)^4-840\sqrt{1-x}(x+1)^3+2520\sqrt{1-x}(x+1)^2-3360\sqrt{1-x}(x+1)+1680\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(9/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(x - 1)
*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1) + 1680
*sqrt(x - 1)) - 18*I*(x + 1)**(7/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840*sqrt(
x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x + 1)
+ 1680*sqrt(x - 1)) + 63*I*(x + 1)**(5/2)/(105*sqrt(x - 1)*(x + 1)**4 - 840
*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x - 1)*(x
 + 1) + 1680*sqrt(x - 1)) - 70*I*(x + 1)**(3/2)/(105*sqrt(x - 1)*(x + 1)**4
 - 840*sqrt(x - 1)*(x + 1)**3 + 2520*sqrt(x - 1)*(x + 1)**2 - 3360*sqrt(x -
 1)*(x + 1) + 1680*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(9/2)/(105
*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)*(x
+ 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 18*(x + 1)**(7/2)/
(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1 - x)
*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) - 63*(x + 1)**(5
/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqrt(1
- x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)) + 70*(x + 1)
**(3/2)/(105*sqrt(1 - x)*(x + 1)**4 - 840*sqrt(1 - x)*(x + 1)**3 + 2520*sqr
t(1 - x)*(x + 1)**2 - 3360*sqrt(1 - x)*(x + 1) + 1680*sqrt(1 - x)), True))
```

3.1004 $\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{3/2}}{315(1-x)^{3/2}}+\frac{2(x+1)^{3/2}}{105(1-x)^{5/2}}+\frac{(x+1)^{3/2}}{21(1-x)^{7/2}}+\frac{(x+1)^{3/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(3/2)/(9*(1 - x)^(9/2)) + (1 + x)^(3/2)/(21*(1 - x)^(7/2)) + (2*(1
+ x)^(3/2))/(105*(1 - x)^(5/2)) + (2*(1 + x)^(3/2))/(315*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{1}{3}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2}{21}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2}{105}\int \frac{\sqrt{1+x}}{(1-x)^{5/2}}\,dx\\ &= \frac{(1+x)^{3/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{3/2}}{21(1-x)^{7/2}}+\frac{2(1+x)^{3/2}}{105(1-x)^{5/2}}+\frac{2(1+x)^{3/2}}{315(1-x)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.43

$$\frac{(x+1)^{3/2}\left(-2x^3+12x^2-33x+58\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(11/2),x]
[Out] ((1 + x)^(3/2)*(58 - 33*x + 12*x^2 - 2*x^3))/(315*(1 - x)^(9/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 77, normalized size = 0.95

$$\frac{\frac{35(x+1)^{9/2}}{(1-x)^{9/2}}+\frac{135(x+1)^{7/2}}{(1-x)^{7/2}}+\frac{189(x+1)^{5/2}}{(1-x)^{5/2}}+\frac{105(x+1)^{3/2}}{(1-x)^{3/2}}}{2520}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(11/2),x]
[Out] ((105*(1 + x)^(3/2))/(1 - x)^(3/2) + (189*(1 + x)^(5/2))/(1 - x)^(5/2) + (1
35*(1 + x)^(7/2))/(1 - x)^(7/2) + (35*(1 + x)^(9/2))/(1 - x)^(9/2))/2520
```

fricas [A] time = 1.31, size = 85, normalized size = 1.05

$$\frac{58\,x^{5}-290\,x^{4}+580\,x^{3}-580\,x^{2}+\left(2\,x^{4}-10\,x^{3}+21\,x^{2}-25\,x-58\right)\sqrt{x+1}\,\sqrt{-x+1}+290\,x-58}{315\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="fricas")
[Out] 1/315*(58*x^5 - 290*x^4 + 580*x^3 - 580*x^2 + (2*x^4 - 10*x^3 + 21*x^2 - 25
*x - 58)*sqrt(x + 1)*sqrt(-x + 1) + 290*x - 58)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

giac [A] time = 1.12, size = 35, normalized size = 0.43

$$\frac{((2\,(x+1)(x-8)+63)(x+1)-105)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{315\,(x-1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="giac")
[Out] 1/315*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1)^(3/2)*sqrt(-x + 1)/(
x - 1)^5
```

maple [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{3}{2}}\left(2x^{3}-12x^{2}+33x-58\right)}{315\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(11/2),x)
[Out] -1/315*(x+1)^(3/2)*(2*x^3-12*x^2+33*x-58)/(-x+1)^(9/2)
```

maxima [B] time = 1.35, size = 131, normalized size = 1.62

$$-\frac{2\sqrt{-x^{2}+1}}{9\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}-\frac{\sqrt{-x^{2}+1}}{63\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{105\left(x^{3}-3\,x^{2}+3\,x-1\right)}-\frac{2\sqrt{-x^{2}+1}}{315\left(x^{2}-2\,x+1\right)}+\frac{2\sqrt{-x^{2}+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(11/2),x, algorithm="maxima")
```

[Out] -2/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/315*sqrt(-x^2 + 1)/(x - 1)

mupad [B] time = 0.28, size = 80, normalized size = 0.99

$$-\frac{\sqrt{1-x}\,\left(\frac{5\,x\,\sqrt{x+1}}{63}+\frac{58\,\sqrt{x+1}}{315}-\frac{x^2\,\sqrt{x+1}}{15}+\frac{2\,x^3\,\sqrt{x+1}}{63}-\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(1/2)/(1 - x)^(11/2),x)

[Out] -((1 - x)^(1/2)*((5*x*(x + 1)^(1/2))/63 + (58*(x + 1)^(1/2))/315 - (x^2*(x + 1)^(1/2))/15 + (2*x^3*(x + 1)^(1/2))/63 - (2*x^4*(x + 1)^(1/2))/315))/(5*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)

sympy [B] time = 53.78, size = 1562, normalized size = 19.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**(1/2)/(1-x)**(11/2),x)

[Out] Piecewise((2*I*(x + 1)**(15/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 30*I*(x + 1)**(13/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 195*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 715*I*(x + 1)**(9/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 1530*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) - 1764*I*(x + 1)**(5/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)) + 840*I*(x + 1)**(3/2)/(315*sqrt(x - 1)*(x + 1)**7 - 4410*sqrt(x - 1)*(x + 1)**6 + 26460*sqrt(x - 1)*(x + 1)**5 - 88200*sqrt(x - 1)*(x + 1)**4 + 176400*sqrt(x - 1)*(x + 1)**3 - 211680*sqrt(x - 1)*(x + 1)**2 + 141120*sqrt(x - 1)*(x + 1) - 40320*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(15/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 30*(x + 1)**(13/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) - 195*(x + 1)**(11/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) + 715*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1

```
)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 21168
0*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x))
- 1530*(x + 1)**(7/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1
)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400
*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x
)*(x + 1) - 40320*sqrt(1 - x)) + 1764*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x +
1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*sqrt(1 - x)*(x + 1)**5 - 88200*
sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(x + 1)**3 - 211680*sqrt(1 - x)
*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40320*sqrt(1 - x)) - 840*(x + 1)
**(3/2)/(315*sqrt(1 - x)*(x + 1)**7 - 4410*sqrt(1 - x)*(x + 1)**6 + 26460*s
qrt(1 - x)*(x + 1)**5 - 88200*sqrt(1 - x)*(x + 1)**4 + 176400*sqrt(1 - x)*(
x + 1)**3 - 211680*sqrt(1 - x)*(x + 1)**2 + 141120*sqrt(1 - x)*(x + 1) - 40
320*sqrt(1 - x)), True))
```

3.1005 $\int \frac{\sqrt{1+x}}{(1-x)^{13/2}} \, dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{45, 37\}$

$$\frac{8(x+1)^{3/2}}{3465(1-x)^{3/2}}+\frac{8(x+1)^{3/2}}{1155(1-x)^{5/2}}+\frac{4(x+1)^{3/2}}{231(1-x)^{7/2}}+\frac{4(x+1)^{3/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{3/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(3/2)/(11*(1 - x)^(11/2)) + (4*(1 + x)^(3/2))/(99*(1 - x)^(9/2)) +
(4*(1 + x)^(3/2))/(231*(1 - x)^(7/2)) + (8*(1 + x)^(3/2))/(1155*(1 - x)^(5/
2)) + (8*(1 + x)^(3/2))/(3465*(1 - x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{(1-x)^{13/2}} \, dx &= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4}{11}\int \frac{\sqrt{1+x}}{(1-x)^{11/2}} \, dx\\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4}{33}\int \frac{\sqrt{1+x}}{(1-x)^{9/2}} \, dx\\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8}{231}\int \frac{\sqrt{1+x}}{(1-x)^{7/2}} \, dx\\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8\int \frac{\sqrt{1+x}}{(1-x)^{5/2}} \, dx}{1155}\\
&= \frac{(1+x)^{3/2}}{11(1-x)^{11/2}}+\frac{4(1+x)^{3/2}}{99(1-x)^{9/2}}+\frac{4(1+x)^{3/2}}{231(1-x)^{7/2}}+\frac{8(1+x)^{3/2}}{1155(1-x)^{5/2}}+\frac{8(1+x)^{3/2}}{3465(1-x)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.40

$$\frac{(x+1)^{3/2}\left(8x^4-56x^3+180x^2-364x+547\right)}{3465(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(3/2)*(547 - 364*x + 180*x^2 - 56*x^3 + 8*x^4))/(3465*(1 - x)^(11/
2))
```

IntegrateAlgebraic [A] time = 0.07, size = 95, normalized size = 0.94

$$\frac{\frac{315(x+1)^{11/2}}{(1-x)^{11/2}}+\frac{1540(x+1)^{9/2}}{(1-x)^{9/2}}+\frac{2970(x+1)^{7/2}}{(1-x)^{7/2}}+\frac{2772(x+1)^{5/2}}{(1-x)^{5/2}}+\frac{1155(x+1)^{3/2}}{(1-x)^{3/2}}}{55440}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 + x]/(1 - x)^(13/2),x]
```

```
[Out] ((1155*(1 + x)^(3/2))/(1 - x)^(3/2) + (2772*(1 + x)^(5/2))/(1 - x)^(5/2) +
(2970*(1 + x)^(7/2))/(1 - x)^(7/2) + (1540*(1 + x)^(9/2))/(1 - x)^(9/2) + (
315*(1 + x)^(11/2))/(1 - x)^(11/2))/55440
```

fricas [A] time = 0.99, size = 100, normalized size = 0.99

$$\frac{547\,x^{6}-3282\,x^{5}+8205\,x^{4}-10940\,x^{3}+8205\,x^{2}+\left(8\,x^{5}-48\,x^{4}+124\,x^{3}-184\,x^{2}+183\,x+547\right)\sqrt{x+1}\,\sqrt{-x+1}-3282\,x+547}{3465\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(547*x^6 - 3282*x^5 + 8205*x^4 - 10940*x^3 + 8205*x^2 + (8*x^5 - 48*
x^4 + 124*x^3 - 184*x^2 + 183*x + 547)*sqrt(x + 1)*sqrt(-x + 1) - 3282*x +
547)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

giac [A] time = 1.36, size = 42, normalized size = 0.42

$$\frac{(4\,((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)+1155)(x+1)^{\frac{3}{2}}\sqrt{-x+1}}{3465\,(x-1)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/3465*(4*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1) + 1155)*(x + 1)
^(3/2)*sqrt(-x + 1)/(x - 1)^6
```

maple [A] time = 0.00, size = 35, normalized size = 0.35

$$\frac{(x+1)^{\frac{3}{2}}\left(8x^{4}-56x^{3}+180x^{2}-364x+547\right)}{3465\,(-x+1)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(1/2)/(-x+1)^(13/2),x)
```

```
[Out] 1/3465*(x+1)^(3/2)*(8*x^4-56*x^3+180*x^2-364*x+547)/(-x+1)^(11/2)
```

maxima [B] time = 1.32, size = 172, normalized size = 1.70

$$\frac{2\,\sqrt{-x^{2}+1}}{11\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{99\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}-\frac{4\,\sqrt{-x^{2}+1}}{693\,\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}+\frac{4\,\sqrt{-x^{2}+1}}{1155\,\left(x^{3}-3\,x^{2}+3\,x-1\right)}-\frac{8\,\sqrt{-x^{2}+1}}{3465\,\left(x^{2}-2\,x+1\right)}+\frac{8\,\sqrt{-x^{2}+1}}{3465\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 2/11*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 1/
99*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 4/693*sqrt(-x
^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/1155*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) - 8/3465*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/3465*sqrt(-x^2 + 1
)/(x - 1)
```

mupad [B] time = 0.29, size = 94, normalized size = 0.93

$$\frac{\sqrt{1-x}\,\left(\frac{61\,x\,\sqrt{x+1}}{1155}+\frac{547\,\sqrt{x+1}}{3465}-\frac{184\,x^2\,\sqrt{x+1}}{3465}+\frac{124\,x^3\,\sqrt{x+1}}{3465}-\frac{16\,x^4\,\sqrt{x+1}}{1155}+\frac{8\,x^5\,\sqrt{x+1}}{3465}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(1/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((61*x*(x + 1)^(1/2))/1155 + (547*(x + 1)^(1/2))/3465 - (184
*x^2*(x + 1)^(1/2))/3465 + (124*x^3*(x + 1)^(1/2))/3465 - (16*x^4*(x + 1)^(
1/2))/1155 + (8*x^5*(x + 1)^(1/2))/3465))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 -
 6*x^5 + x^6 + 1)
```

sympy [B] time = 135.09, size = 3650, normalized size = 36.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((8*I*(x + 1)**(23/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 184*I*(x + 1)**(21/2)/(3465*s
qrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)
*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)
**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 -
146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574
400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x
- 1)) + 1932*I*(x + 1)**(19/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x
 - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x
+ 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6
 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 14
6361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760
*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 12236*I*(x + 1)**(17/2)/(3465
*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x -
1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x +
1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5
- 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 975
74400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(
x - 1)) + 52003*I*(x + 1)**(15/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqr
t(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*
(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)
**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 +
 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029
760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 155316*I*(x + 1)**(13/2)/(
3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(
x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(
x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)
**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 -
```

```
97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*s
qrt(x - 1)) + 329588*I*(x + 1)**(11/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7623
0*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x
- 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x
 + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)
**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 +
39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 488224*I*(x + 1)**(9/
2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 762300*s
qrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x -
1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x
+ 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)*
*3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70963
20*sqrt(x - 1)) + 479952*I*(x + 1)**(7/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 7
6230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt
(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)
*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x +
 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2
 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)) - 280896*I*(x + 1)**
(5/2)/(3465*sqrt(x - 1)*(x + 1)**11 - 76230*sqrt(x - 1)*(x + 1)**10 + 76230
0*sqrt(x - 1)*(x + 1)**9 - 4573800*sqrt(x - 1)*(x + 1)**8 + 18295200*sqrt(x
 - 1)*(x + 1)**7 - 51226560*sqrt(x - 1)*(x + 1)**6 + 102453120*sqrt(x - 1)*
(x + 1)**5 - 146361600*sqrt(x - 1)*(x + 1)**4 + 146361600*sqrt(x - 1)*(x +
1)**3 - 97574400*sqrt(x - 1)*(x + 1)**2 + 39029760*sqrt(x - 1)*(x + 1) - 70
96320*sqrt(x - 1)) + 73920*I*(x + 1)**(3/2)/(3465*sqrt(x - 1)*(x + 1)**11 -
 76230*sqrt(x - 1)*(x + 1)**10 + 762300*sqrt(x - 1)*(x + 1)**9 - 4573800*sq
rt(x - 1)*(x + 1)**8 + 18295200*sqrt(x - 1)*(x + 1)**7 - 51226560*sqrt(x -
1)*(x + 1)**6 + 102453120*sqrt(x - 1)*(x + 1)**5 - 146361600*sqrt(x - 1)*(x
 + 1)**4 + 146361600*sqrt(x - 1)*(x + 1)**3 - 97574400*sqrt(x - 1)*(x + 1)*
*2 + 39029760*sqrt(x - 1)*(x + 1) - 7096320*sqrt(x - 1)), Abs(x + 1)/2 > 1)
, (-8*(x + 1)**(23/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x
+ 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 +
 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453
120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*s
qrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 -
 x)*(x + 1) - 7096320*sqrt(1 - x)) + 184*(x + 1)**(21/2)/(3465*sqrt(1 - x)*
(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9
 - 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226
560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*s
qrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1
- x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 193
2*(x + 1)**(19/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)
**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182
95200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*
sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(
1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*
(x + 1) - 7096320*sqrt(1 - x)) + 12236*(x + 1)**(17/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 52003
*(x + 1)**(15/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)*
*10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1829
5200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*s
qrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(
x + 1) - 7096320*sqrt(1 - x)) + 155316*(x + 1)**(13/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
```

```
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 32958
8*(x + 1)**(11/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)
**10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182
95200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*
sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(
1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*
(x + 1) - 7096320*sqrt(1 - x)) + 488224*(x + 1)**(9/2)/(3465*sqrt(1 - x)*(x
 + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
 4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 5122656
0*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqr
t(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 -
x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 47995
2*(x + 1)**(7/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)*
*10 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 1829
5200*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*s
qrt(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1
 - x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(
x + 1) - 7096320*sqrt(1 - x)) + 280896*(x + 1)**(5/2)/(3465*sqrt(1 - x)*(x
+ 1)**11 - 76230*sqrt(1 - x)*(x + 1)**10 + 762300*sqrt(1 - x)*(x + 1)**9 -
4573800*sqrt(1 - x)*(x + 1)**8 + 18295200*sqrt(1 - x)*(x + 1)**7 - 51226560
*sqrt(1 - x)*(x + 1)**6 + 102453120*sqrt(1 - x)*(x + 1)**5 - 146361600*sqrt
(1 - x)*(x + 1)**4 + 146361600*sqrt(1 - x)*(x + 1)**3 - 97574400*sqrt(1 - x
)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x + 1) - 7096320*sqrt(1 - x)) - 73920*
(x + 1)**(3/2)/(3465*sqrt(1 - x)*(x + 1)**11 - 76230*sqrt(1 - x)*(x + 1)**1
0 + 762300*sqrt(1 - x)*(x + 1)**9 - 4573800*sqrt(1 - x)*(x + 1)**8 + 182952
00*sqrt(1 - x)*(x + 1)**7 - 51226560*sqrt(1 - x)*(x + 1)**6 + 102453120*sqr
t(1 - x)*(x + 1)**5 - 146361600*sqrt(1 - x)*(x + 1)**4 + 146361600*sqrt(1 -
 x)*(x + 1)**3 - 97574400*sqrt(1 - x)*(x + 1)**2 + 39029760*sqrt(1 - x)*(x
+ 1) - 7096320*sqrt(1 - x)), True))
```

3.1006 $\int (1-x)^{9/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=109

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{9}{16}\sin^{-1}$$

Rubi [A] time = 0.02, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(x+1)^{5/2}(1-x)^{9/2}+\frac{3}{14}(x+1)^{5/2}(1-x)^{7/2}+\frac{3}{10}(x+1)^{5/2}(1-x)^{5/2}+\frac{3}{8}x(x+1)^{3/2}(1-x)^{3/2}+\frac{9}{16}x\sqrt{x+1}\sqrt{1-x}+\frac{9}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (9*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (3*(1 - x)^(3/2)*x*(1 + x)^(3/2))/8 + (3
*(1 - x)^(5/2)*(1 + x)^(5/2))/10 + (3*(1 - x)^(7/2)*(1 + x)^(5/2))/14 + ((1
 - x)^(9/2)*(1 + x)^(5/2))/7 + (9*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{9/2}(1+x)^{3/2}\,dx &= \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{9}{7}\int (1-x)^{7/2}(1+x)^{3/2}\,dx \\
&= \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{5/2}(1+x)^{3/2}\,dx \\
&= \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+x)^{5/2} + \frac{3}{2}\int (1-x)^{3/2}(1 \\
&= \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+x)^{5/2} + \frac{1}{7}(1-x)^{9/2}(1+ \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+ \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+ \\
&= \frac{9}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{3}{8}(1-x)^{3/2}x(1+x)^{3/2} + \frac{3}{10}(1-x)^{5/2}(1+x)^{5/2} + \frac{3}{14}(1-x)^{7/2}(1+
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.61

$$
\frac{1}{560}\sqrt{1-x^2}\left(80x^6-280x^5+208x^4+350x^3-656x^2+245x+368\right)-\frac{9}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(368 + 245*x - 656*x^2 + 350*x^3 + 208*x^4 - 280*x^5 + 80*x^
6))/560 - (9*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

IntegrateAlgebraic [A] time = 0.16, size = 169, normalized size = 1.55

$$
\frac{-\frac{315(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{2100(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{8393(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{9216(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{5943(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{2100(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{280\left(\frac{1-x}{x+1}+1\right)^7}-\frac{9}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-315*(1 - x)^(13/2))/(1 + x)^(13/2) - (2100*(1 - x)^(11/2))/(1 + x)^(11/2
) + (8393*(1 - x)^(9/2))/(1 + x)^(9/2) + (9216*(1 - x)^(7/2))/(1 + x)^(7/2)
 + (5943*(1 - x)^(5/2))/(1 + x)^(5/2) + (2100*(1 - x)^(3/2))/(1 + x)^(3/2)
+ (315*Sqrt[1 - x])/Sqrt[1 + x])/(280*(1 + (1 - x)/(1 + x))^7) - (9*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]])/8
```

fricas [A] time = 1.16, size = 67, normalized size = 0.61

$$
\frac{1}{560}\left(80\,x^6-280\,x^5+208\,x^4+350\,x^3-656\,x^2+245\,x+368\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{9}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/560*(80*x^6 - 280*x^5 + 208*x^4 + 350*x^3 - 656*x^2 + 245*x + 368)*sqrt(x
 + 1)*sqrt(-x + 1) - 9/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.26, size = 237, normalized size = 2.17

$$
\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(\left(4\left(5\,x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{120}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{6}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{6}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{9}{8}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)
*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/120*((2*((4*(
5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(
x + 1)*sqrt(-x + 1) - 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)
*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/6*((2*(3*x - 10)*(x + 1) + 43)
*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(
x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 9/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 127, normalized size = 1.17

$$\frac{9\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{9}{2}}(x+1)^{\frac{5}{2}}}{7}+\frac{3(-x+1)^{\frac{7}{2}}(x+1)^{\frac{5}{2}}}{14}+\frac{3(-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}{10}+\frac{3(-x+1)^{\frac{3}{2}}(x+1)^{\frac{5}{2}}}{8}+\frac{3\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{8}-\frac{3\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{16}-\frac{9\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(x+1)^(3/2),x)
```

```
[Out] 1/7*(-x+1)^(9/2)*(x+1)^(5/2)+3/14*(-x+1)^(7/2)*(x+1)^(5/2)+3/10*(-x+1)^(5/2
)*(x+1)^(5/2)+3/8*(-x+1)^(3/2)*(x+1)^(5/2)+3/8*(-x+1)^(1/2)*(x+1)^(5/2)-3/1
6*(-x+1)^(1/2)*(x+1)^(3/2)-9/16*(-x+1)^(1/2)*(x+1)^(1/2)+9/16*((x+1)*(-x+1)
)^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.03, size = 66, normalized size = 0.61

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{5}{2}}x^2-\frac{1}{2}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{23}{35}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{3}{8}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{9}{16}\sqrt{-x^2+1}\,x+\frac{9}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(5/2)*x^2 - 1/2*(-x^2 + 1)^(5/2)*x + 23/35*(-x^2 + 1)^(5/2)
+ 3/8*(-x^2 + 1)^(3/2)*x + 9/16*sqrt(-x^2 + 1)*x + 9/16*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(3/2), x)
```

sympy [A] time = 75.20, size = 325, normalized size = 2.98

$$\begin{cases} -\frac{9i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}}-\frac{23i(x+1)^{\frac{13}{2}}}{14\sqrt{x-1}}+\frac{541i(x+1)^{\frac{11}{2}}}{70\sqrt{x-1}}-\frac{5249i(x+1)^{\frac{9}{2}}}{280\sqrt{x-1}}+\frac{6653i(x+1)^{\frac{7}{2}}}{280\sqrt{x-1}}-\frac{1027i(x+1)^{\frac{5}{2}}}{80\sqrt{x-1}}-\frac{3i(x+1)^{\frac{3}{2}}}{16\sqrt{x-1}}+\frac{9i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{9\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}}+\frac{23(x+1)^{\frac{13}{2}}}{14\sqrt{1-x}}-\frac{541(x+1)^{\frac{11}{2}}}{70\sqrt{1-x}}+\frac{5249(x+1)^{\frac{9}{2}}}{280\sqrt{1-x}}-\frac{6653(x+1)^{\frac{7}{2}}}{280\sqrt{1-x}}+\frac{1027(x+1)^{\frac{5}{2}}}{80\sqrt{1-x}}+\frac{3(x+1)^{\frac{3}{2}}}{16\sqrt{1-x}}-\frac{9\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-9*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(15/2)/(7*sqrt(
x - 1)) - 23*I*(x + 1)**(13/2)/(14*sqrt(x - 1)) + 541*I*(x + 1)**(11/2)/(70
*sqrt(x - 1)) - 5249*I*(x + 1)**(9/2)/(280*sqrt(x - 1)) + 6653*I*(x + 1)**(
```

```
7/2)/(280*sqrt(x - 1)) - 1027*I*(x + 1)**(5/2)/(80*sqrt(x - 1)) - 3*I*(x +
1)**(3/2)/(16*sqrt(x - 1)) + 9*I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2
> 1), (9*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(15/2)/(7*sqrt(1 - x)) +
23*(x + 1)**(13/2)/(14*sqrt(1 - x)) - 541*(x + 1)**(11/2)/(70*sqrt(1 - x))
+ 5249*(x + 1)**(9/2)/(280*sqrt(1 - x)) - 6653*(x + 1)**(7/2)/(280*sqrt(1 -
 x)) + 1027*(x + 1)**(5/2)/(80*sqrt(1 - x)) + 3*(x + 1)**(3/2)/(16*sqrt(1 -
 x)) - 9*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

3.1007 $\int (1-x)^{7/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=89

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{6}(x+1)^{5/2}(1-x)^{7/2}+\frac{7}{30}(x+1)^{5/2}(1-x)^{5/2}+\frac{7}{24}x(x+1)^{3/2}(1-x)^{3/2}+\frac{7}{16}x\sqrt{x+1}\,\sqrt{1-x}+\frac{7}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (7*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (7*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
7*(1 - x)^(5/2)*(1 + x)^(5/2))/30 + ((1 - x)^(7/2)*(1 + x)^(5/2))/6 + (7*Ar
cSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (1-x)^{7/2}(1+x)^{3/2}\,dx = \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{5/2}(1+x)^{3/2}\,dx$$
$$= \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx$$
$$= \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+x)^{5/2} + \frac{7}{8}\int \sqrt{1-x}\,\sqrt{}$$
$$= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+$$
$$= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+$$
$$= \frac{7}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{7}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{7}{30}(1-x)^{5/2}(1+x)^{5/2} + \frac{1}{6}(1-x)^{7/2}(1+$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.69

$$\frac{1}{240}\sqrt{1-x^2}\left(-40x^5+96x^4+10x^3-192x^2+135x+96\right)-\frac{7}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(96 + 135*x - 192*x^2 + 10*x^3 + 96*x^4 - 40*x^5))/240 - (7*
ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

IntegrateAlgebraic [A] time = 0.13, size = 151, normalized size = 1.70

$$\frac{-\frac{105(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{595(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{1686(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{1386(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{595(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{120\left(\frac{1-x}{x+1}+1\right)^6}-\frac{7}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-105*(1 - x)^(11/2))/(1 + x)^(11/2) - (595*(1 - x)^(9/2))/(1 + x)^(9/2) +
 (1686*(1 - x)^(7/2))/(1 + x)^(7/2) + (1386*(1 - x)^(5/2))/(1 + x)^(5/2) +
(595*(1 - x)^(3/2))/(1 + x)^(3/2) + (105*Sqrt[1 - x])/Sqrt[1 + x])/(120*(1
+ (1 - x)/(1 + x))^6) - (7*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

fricas [A] time = 1.15, size = 62, normalized size = 0.70

$$-\frac{1}{240}\left(40\,x^5-96\,x^4-10\,x^3+192\,x^2-135\,x-96\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{7}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/240*(40*x^5 - 96*x^4 - 10*x^3 + 192*x^2 - 135*x - 96)*sqrt(x + 1)*sqrt(-
x + 1) - 7/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.33, size = 185, normalized size = 2.08

$$-\frac{1}{240}\left(\left(2\left(\left(4\left(5\,x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{120}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{12}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\,\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{7}{8}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133
)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x -
10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(
x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 7/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 113, normalized size = 1.27

$$\frac{7\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}(x+1)^{\frac{5}{2}}}{6}+\frac{7(-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}}{30}+\frac{7(-x+1)^{\frac{3}{2}}(x+1)^{\frac{5}{2}}}{24}+\frac{7\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{24}-\frac{7\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{48}-\frac{7\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(3/2),x)
```

```
[Out] 1/6*(-x+1)^(7/2)*(x+1)^(5/2)+7/30*(-x+1)^(5/2)*(x+1)^(5/2)+7/24*(-x+1)^(3/2
)*(x+1)^(5/2)+7/24*(-x+1)^(1/2)*(x+1)^(5/2)-7/48*(-x+1)^(1/2)*(x+1)^(3/2)-7
/16*(-x+1)^(1/2)*(x+1)^(1/2)+7/16*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(
1/2)*arcsin(x)
```

maxima [A] time = 2.90, size = 52, normalized size = 0.58

$$-\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{2}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{7}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{7}{16}\sqrt{-x^2+1}\,x+\frac{7}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)*x + 2/5*(-x^2 + 1)^(5/2) + 7/24*(-x^2 + 1)^(3/2)*x +
7/16*sqrt(-x^2 + 1)*x + 7/16*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(3/2), x)
```

sympy [A] time = 32.99, size = 289, normalized size = 3.25

$$\begin{cases} -\frac{7i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}}+\frac{47i(x+1)^{\frac{11}{2}}}{30\sqrt{x-1}}-\frac{683i(x+1)^{\frac{9}{2}}}{120\sqrt{x-1}}+\frac{1151i(x+1)^{\frac{7}{2}}}{120\sqrt{x-1}}-\frac{1543i(x+1)^{\frac{5}{2}}}{240\sqrt{x-1}}-\frac{7i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{7i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{7\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}}-\frac{47(x+1)^{\frac{11}{2}}}{30\sqrt{1-x}}+\frac{683(x+1)^{\frac{9}{2}}}{120\sqrt{1-x}}-\frac{1151(x+1)^{\frac{7}{2}}}{120\sqrt{1-x}}+\frac{1543(x+1)^{\frac{5}{2}}}{240\sqrt{1-x}}+\frac{7(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{7\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-7*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) + 47*I*(x + 1)**(11/2)/(30*sqrt(x - 1)) - 683*I*(x + 1)**(9/2)/(120
*sqrt(x - 1)) + 1151*I*(x + 1)**(7/2)/(120*sqrt(x - 1)) - 1543*I*(x + 1)**(
5/2)/(240*sqrt(x - 1)) - 7*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 7*I*sqrt(x +
 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (7*asin(sqrt(2)*sqrt(x + 1)/2)/8 +
(x + 1)**(13/2)/(6*sqrt(1 - x)) - 47*(x + 1)**(11/2)/(30*sqrt(1 - x)) + 683
*(x + 1)**(9/2)/(120*sqrt(1 - x)) - 1151*(x + 1)**(7/2)/(120*sqrt(1 - x)) +
 1543*(x + 1)**(5/2)/(240*sqrt(1 - x)) + 7*(x + 1)**(3/2)/(48*sqrt(1 - x))
- 7*sqrt(x + 1)/(8*sqrt(1 - x)), True))
```

3.1008 $\int (1-x)^{5/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + ((1 - x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}(1+x)^{3/2}\,dx &= \frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.80

$$\frac{1}{40}\left(\sqrt{1-x^2}\left(8x^4-10x^3-16x^2+25x+8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(8 + 25*x - 16*x^2 - 10*x^3 + 8*x^4) - 30*ArcSin[Sqrt[1 - x]
/Sqrt[2]])/40
```

IntegrateAlgebraic [A] time = 0.11, size = 133, normalized size = 1.93

$$\frac{-\frac{15(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{70(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{128(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{70(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{20\left(\frac{1-x}{x+1}+1\right)^5}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-15*(1 - x)^(9/2))/(1 + x)^(9/2) - (70*(1 - x)^(7/2))/(1 + x)^(7/2) + (12
8*(1 - x)^(5/2))/(1 + x)^(5/2) + (70*(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqr
t[1 - x])/Sqrt[1 + x])/(20*(1 + (1 - x)/(1 + x))^5) - (3*ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]])/4
```

fricas [A] time = 1.22, size = 57, normalized size = 0.83

$$\frac{1}{40}\left(8\,x^4-10\,x^3-16\,x^2+25\,x+8\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^4 - 10*x^3 - 16*x^2 + 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*ar
ctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [A] time = 1.16, size = 91, normalized size = 1.32

$$\frac{1}{120}\,((2\,(3\,(4\,x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{3}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x
 + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) +
 sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 99, normalized size = 1.43

$$\frac{3\sqrt{(x+1)\,(-x+1)}\ \arcsin(x)}{8\sqrt{x+1}\ \sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}\,(x+1)^{\frac{5}{2}}}{5}+\frac{(-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+1}\ \,(x+1)^{\frac{5}{2}}}{4}-\frac{\sqrt{-x+1}\ \,(x+1)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\ \sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(3/2),x)
```

```
[Out] 1/5*(-x+1)^(5/2)*(x+1)^(5/2)+1/4*(-x+1)^(3/2)*(x+1)^(5/2)+1/4*(-x+1)^(1/2)*
(x+1)^(5/2)-1/8*(-x+1)^(1/2)*(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8*(
(x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.98, size = 40, normalized size = 0.58

$$\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*
arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(3/2), x)
```

sympy [B] time = 15.25, size = 250, normalized size = 3.62

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}-\frac{29i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}+\frac{73i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{129i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}+\frac{29(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}-\frac{73(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{129(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) - 29*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) + 73*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - 129*I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sq
rt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sq
rt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(11/2)/(5*sqrt(1 - x)) + 29*(x + 1)**(9/2
)/(20*sqrt(1 - x)) - 73*(x + 1)**(7/2)/(20*sqrt(1 - x)) + 129*(x + 1)**(5/2
)/(40*sqrt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt
(1 - x)), True))
```

3.1009 $\int (1-x)^{3/2}(1+x)^{3/2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 + (3*Ar
cSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{3/2}(1+x)^{3/2}\,dx &= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.59

$$\frac{1}{8}\left(x\sqrt{1-x^2}\left(5-2x^2\right)+3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] (x*(5 - 2*x^2)*Sqrt[1 - x^2] + 3*ArcSin[x])/8
```

IntegrateAlgebraic [B] time = 0.10, size = 115, normalized size = 2.35

$$\frac{-\frac{3(1-x)^{7/2}}{(x+1)^{7/2}}-\frac{11(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{11(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3\sqrt{1-x}}{\sqrt{x+1}}}{4\left(\frac{1-x}{x+1}+1\right)^4}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*(1 + x)^(3/2),x]
```

```
[Out] ((-3*(1 - x)^(7/2))/(1 + x)^(7/2) - (11*(1 - x)^(5/2))/(1 + x)^(5/2) + (11*
(1 - x)^(3/2))/(1 + x)^(3/2) + (3*Sqrt[1 - x])/Sqrt[1 + x])/(4*(1 + (1 - x)
/(1 + x))^4) - (3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

fricas [A] time = 1.19, size = 46, normalized size = 0.94

$$-\frac{1}{8}\left(2x^3-5x\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(2*x^3 - 5*x)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*arctan((sqrt(x + 1)*sqrt(
-x + 1) - 1)/x)
```

giac [B] time = 1.12, size = 101, normalized size = 2.06

$$-\frac{1}{24}\left(\left(2\left(3x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{6}\left(\left(2x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\sqrt{-x+1}+\frac{1}{2}\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) -
 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(
x + 1))
```

maple [B] time = 0.00, size = 85, normalized size = 1.73

$$\frac{3\sqrt{\left(x+1\right)\left(-x+1\right)}\,\arcsin\left(x\right)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\left(-x+1\right)^{\frac{3}{2}}\left(x+1\right)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+1}\,\left(x+1\right)^{\frac{5}{2}}}{4}-\frac{\sqrt{-x+1}\,\left(x+1\right)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(3/2),x)
```

```
[Out] 1/4*(-x+1)^(3/2)*(x+1)^(5/2)+1/4*(-x+1)^(1/2)*(x+1)^(5/2)-1/8*(-x+1)^(1/2)*
(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8*((x+1)*(-x+1))^(1/2)/(x+1)^(1/
2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.91, size = 29, normalized size = 0.59

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (1-x)^{3/2}(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(3/2), x)
```

sympy [B] time = 7.46, size = 214, normalized size = 4.37

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}} + \frac{5i(x+1)^{\frac{7}{2}}}{4\sqrt{x-1}} - \frac{13i(x+1)^{\frac{5}{2}}}{8\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}} - \frac{5(x+1)^{\frac{7}{2}}}{4\sqrt{1-x}} + \frac{13(x+1)^{\frac{5}{2}}}{8\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}} - \frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) + 5*I*(x + 1)**(7/2)/(4*sqrt(x - 1)) - 13*I*(x + 1)**(5/2)/(8*sqrt(x
 - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1))
, Abs(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2)/4 + (x + 1)**(9/2)/(4*s
qrt(1 - x)) - 5*(x + 1)**(7/2)/(4*sqrt(1 - x)) + 13*(x + 1)**(5/2)/(8*sqrt(
1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x)), T
rue))
```

3.1010 $\int \sqrt{1-x}\,(1+x)^{3/2}\,dx$

Optimal. Leaf size=48

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{3}(1-x)^{3/2}(x+1)^{3/2}+\frac{1}{2}\sqrt{1-x}\,x\sqrt{x+1}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*x*Sqrt[1 + x])/2 - ((1 - x)^(3/2)*(1 + x)^(3/2))/3 + ArcSin[x]
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x}\,(1+x)^{3/2}\,dx &= -\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{2}\sqrt{1-x}\,x\sqrt{1+x}-\frac{1}{3}(1-x)^{3/2}(1+x)^{3/2}+\frac{1}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.92

$$\frac{1}{6}\sqrt{1-x^2}\left(2x^2+3x-2\right)-\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(3/2),x]
[Out] (Sqrt[1 - x^2]*(-2 + 3*x + 2*x^2))/6 - ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 95, normalized size = 1.98

$$\frac{-\frac{3(1-x)^{5/2}}{(x+1)^{5/2}}-\frac{8(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^3}-\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*(1 + x)^(3/2),x]
[Out] ((-3*(1 - x)^(5/2))/(1 + x)^(5/2) - (8*(1 - x)^(3/2))/(1 + x)^(3/2) + (3*Sq
rt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 - x)/(1 + x))^3) - ArcTan[Sqrt[1 - x]/Sq
rt[1 + x]]
```

fricas [A] time = 1.28, size = 47, normalized size = 0.98

$$\frac{1}{6}\left(2x^2+3x-2\right)\sqrt{x+1}\sqrt{-x+1}-\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="fricas")
[Out] 1/6*(2*x^2 + 3*x - 2)*sqrt(x + 1)*sqrt(-x + 1) - arctan((sqrt(x + 1)*sqrt(-
x + 1) - 1)/x)
```

giac [A] time = 0.92, size = 66, normalized size = 1.38

$$\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="giac")
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.01, size = 71, normalized size = 1.48

$$\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{2\sqrt{x+1}\sqrt{-x+1}}+\frac{\sqrt{-x+1}(x+1)^{\frac{5}{2}}}{3}-\frac{\sqrt{-x+1}(x+1)^{\frac{3}{2}}}{6}-\frac{\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(3/2),x)
[Out] 1/3*(-x+1)^(1/2)*(x+1)^(5/2)-1/6*(-x+1)^(1/2)*(x+1)^(3/2)-1/2*(-x+1)^(1/2)*
(x+1)^(1/2)+1/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.97, size = 28, normalized size = 0.58

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{1}{2}\sqrt{-x^2+1}\,x+\frac{1}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2) + 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{1-x}\,(x+1)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(3/2), x)
```

sympy [B] time = 4.82, size = 165, normalized size = 3.44

$$\begin{cases} -i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{5i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{5(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(3/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x - 1
)) - 5*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - I*(x + 1)**(3/2)/(6*sqrt(x - 1))
+ I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (asin(sqrt(2)*sqrt(x + 1)/2
) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 5*(x + 1)**(5/2)/(6*sqrt(1 - x)) + (x
+ 1)**(3/2)/(6*sqrt(1 - x)) - sqrt(x + 1)/sqrt(1 - x), True))
```

3.1011 $\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=47

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{2}\sqrt{1-x}\,(x+1)^{3/2}-\frac{3}{2}\sqrt{1-x}\,\sqrt{x+1}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] (-3*Sqrt[1 - x]*Sqrt[1 + x])/2 - (Sqrt[1 - x]*(1 + x)^(3/2))/2 + (3*ArcSin[
x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx &= -\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{1}{2}\sqrt{1-x}\,(1+x)^{3/2}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 0.79

$$-\frac{1}{2}\sqrt{1-x^2}\,(x+4)-3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -1/2*((4 + x)*Sqrt[1 - x^2]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.06, size = 68, normalized size = 1.45

$$-\frac{\sqrt{1-x}\left(\frac{3(1-x)}{x+1}+5\right)}{\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^2}-3\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/Sqrt[1 - x],x]
```

```
[Out] -((Sqrt[1 - x]*(5 + (3*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x)
)^2)) - 3*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.24, size = 40, normalized size = 0.85

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

giac [A] time = 1.17, size = 31, normalized size = 0.66

$$-\frac{1}{2}(x+4)\sqrt{x+1}\sqrt{-x+1}+3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(x + 4)*sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 57, normalized size = 1.21

$$\frac{3\sqrt{(x+1)(-x+1)}\arcsin(x)}{2\sqrt{x+1}\sqrt{-x+1}}-\frac{\sqrt{-x+1}(x+1)^{\frac{3}{2}}}{2}-\frac{3\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(1/2),x)
```

```
[Out] -1/2*(-x+1)^(1/2)*(x+1)^(3/2)-3/2*(-x+1)^(1/2)*(x+1)^(1/2)+3/2*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.09, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x-2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x - 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(1/2), x)
```

sympy [A] time = 3.28, size = 136, normalized size = 2.89

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}} - \frac{i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}} + \frac{3i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}} + \frac{(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}} - \frac{3\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + 3*I*sqrt(x + 1)/sqrt(x - 1), Abs(
x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1 -
x)) + (x + 1)**(3/2)/(2*sqrt(1 - x)) - 3*sqrt(x + 1)/sqrt(1 - x), True))
```

3.1012 $\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}} \, dx$

Optimal. Leaf size=41

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}} + 3\sqrt{1-x}\sqrt{x+1} - 3\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{3/2}}{\sqrt{1-x}} + 3\sqrt{1-x}\sqrt{x+1} - 3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] 3*Sqrt[1 - x]*Sqrt[1 + x] + (2*(1 + x)^(3/2))/Sqrt[1 - x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx = \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx$$
$$= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= 3\sqrt{1-x}\,\sqrt{1+x} + \frac{2(1+x)^{3/2}}{\sqrt{1-x}} - 3\sin^{-1}(x)$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.85

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

IntegrateAlgebraic [C] time = 0.15, size = 59, normalized size = 1.44

$$\frac{\sqrt{1-x}\,\left((x+1)^{3/2}-6\sqrt{x+1}\right)}{x-1} - 6i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(3/2),x]
```

```
[Out] (Sqrt[1 - x]*(-6*Sqrt[1 + x] + (1 + x)^(3/2)))/(-1 + x) - (6*I)*Log[Sqrt[1
 - x] - I*Sqrt[1 + x]]
```

fricas [A] time = 0.73, size = 52, normalized size = 1.27

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1} + 6\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) + 5\,x - 5}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x + 1)*(x - 5)*sqrt(-x + 1) + 6*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x +
 1) - 1)/x) + 5*x - 5)/(x - 1)
```

giac [A] time = 1.19, size = 35, normalized size = 0.85

$$\frac{\sqrt{x+1}\,(x-5)\sqrt{-x+1}}{x-1} - 6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(x + 1)*(x - 5)*sqrt(-x + 1)/(x - 1) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1)
)
```

maple [B] time = 0.02, size = 72, normalized size = 1.76

$$-\frac{3\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}-\frac{\left(x^{2}-4x-5\right)\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(3/2),x)
```

```
[Out] -(x^2-4*x-5)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(
1/2)-3*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.99, size = 42, normalized size = 1.02

$$-\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{2}-2\,x+1}-\frac{6\,\sqrt{-x^{2}+1}}{x-1}-3\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(3/2)/(x^2 - 2*x + 1) - 6*sqrt(-x^2 + 1)/(x - 1) - 3*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(x+1)^{3/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(3/2), x)
```

sympy [A] time = 2.92, size = 100, normalized size = 2.44

$$\begin{cases} 6i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{6i\,\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -6\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{6\,\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
- 6*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (-6*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 6*sqrt(x + 1)/sqrt(1 - x), True))
```

3.1013 $\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}-\frac{2\sqrt{x+1}}{\sqrt{1-x}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 + x])/Sqrt[1 - x] + (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}-\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx\\ &= -\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{2\sqrt{1+x}}{\sqrt{1-x}}+\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.90

$$\frac{4\sqrt{2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (4*Sqrt[2]*Hypergeometric2F1[-3/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

IntegrateAlgebraic [A] time = 0.06, size = 55, normalized size = 1.34

$$-\frac{2\left(\frac{3(1-x)}{x+1}-1\right)(x+1)^{3/2}}{3(1-x)^{3/2}}-2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(5/2),x]
```

```
[Out] (-2*(1 + x)^(3/2)*(-1 + (3*(1 - x))/(1 + x)))/(3*(1 - x)^(3/2)) - 2*ArcTan[
Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [B] time = 1.31, size = 71, normalized size = 1.73

$$-\frac{2\left(2x^2-2(2x-1)\sqrt{x+1}\sqrt{-x+1}+3\left(x^2-2x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-4x+2\right)}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(2*x^2 - 2*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*(x^2 - 2*x + 1)*arct
an((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 4*x + 2)/(x^2 - 2*x + 1)
```

giac [A] time = 1.02, size = 38, normalized size = 0.93

$$\frac{4(2x-1)\sqrt{x+1}\sqrt{-x+1}}{3(x-1)^2}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 4/3*(2*x - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 2*arcsin(1/2*sqrt(2)*sqr
t(x + 1))
```

maple [B] time = 0.02, size = 76, normalized size = 1.85

$$\frac{\sqrt{(x+1)(-x+1)}\arcsin(x)}{\sqrt{x+1}\sqrt{-x+1}}-\frac{4\left(2x^2+x-1\right)\sqrt{(x+1)(-x+1)}}{3(x-1)\sqrt{-(x+1)(x-1)}\sqrt{-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(5/2),x)
```

```
[Out] -4/3*(2*x^2+x-1)/(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/
2)/(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [B] time = 2.97, size = 66, normalized size = 1.61

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3x^2+3x-1\right)}+\frac{2\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{7\sqrt{-x^2+1}}{3(x-1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2 + 3*x - 1) + 2/3*sqrt(-x^2 + 1)/(x^2 - 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x - 1) + arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(3/2)/(1 - x)^(5/2), x)
```

sympy [B] time = 3.70, size = 500, normalized size = 12.20

$$\begin{cases} \frac{6i\sqrt{x-1}\,(x+1)^{\frac{15}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{3\pi\sqrt{x-1}\,(x+1)^{\frac{15}{2}}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{12i\sqrt{x-1}\,(x+1)^{\frac{13}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} + \frac{6\pi\sqrt{x-1}\,(x+1)^{\frac{13}{2}}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} - \frac{8i(x+1)^{8}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} + \frac{12i(x+1)^{7}}{-3\sqrt{x-1}\,(x+1)^{\frac{15}{2}}+6\sqrt{x-1}\,(x+1)^{\frac{13}{2}}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ \frac{6\sqrt{1-x}\,(x+1)^{\frac{15}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} - \frac{12\sqrt{1-x}\,(x+1)^{\frac{13}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} - \frac{8(x+1)^{8}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} + \frac{12(x+1)^{7}}{3\sqrt{1-x}\,(x+1)^{\frac{15}{2}}-6\sqrt{1-x}\,(x+1)^{\frac{13}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((6*I*sqrt(x - 1)*(x + 1)**(15/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(-3
*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x + 1)**(13/2)) - 3*pi*sqrt(x
 - 1)*(x + 1)**(15/2)/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x +
1)**(13/2)) - 12*I*sqrt(x - 1)*(x + 1)**(13/2)*acosh(sqrt(2)*sqrt(x + 1)/2)
/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(x + 1)**(13/2)) + 6*pi*sq
rt(x - 1)*(x + 1)**(13/2)/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x - 1)*(
x + 1)**(13/2)) - 8*I*(x + 1)**8/(-3*sqrt(x - 1)*(x + 1)**(15/2) + 6*sqrt(x
 - 1)*(x + 1)**(13/2)) + 12*I*(x + 1)**7/(-3*sqrt(x - 1)*(x + 1)**(15/2) +
6*sqrt(x - 1)*(x + 1)**(13/2)), Abs(x + 1)/2 > 1), (6*sqrt(1 - x)*(x + 1)**
(15/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(
1 - x)*(x + 1)**(13/2)) - 12*sqrt(1 - x)*(x + 1)**(13/2)*asin(sqrt(2)*sqrt(
x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2)) -
 8*(x + 1)**8/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**(13/2
)) + 12*(x + 1)**7/(3*sqrt(1 - x)*(x + 1)**(15/2) - 6*sqrt(1 - x)*(x + 1)**
(13/2)), True))
```

3.1014 $\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {37}

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx = \frac{(1+x)^{5/2}}{5(1-x)^{5/2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 20, normalized size = 1.00

$$\frac{(x+1)^{5/2}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(7/2),x]
```

```
[Out] (1 + x)^(5/2)/(5*(1 - x)^(5/2))
```

fricas [B] time = 1.00, size = 52, normalized size = 2.60

$$\frac{x^3 - 3x^2 - (x^2 + 2x + 1)\sqrt{x+1}\sqrt{-x+1} + 3x - 1}{5(x^3 - 3x^2 + 3x - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(x^3 - 3*x^2 - (x^2 + 2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) + 3*x - 1)/(x^3
 - 3*x^2 + 3*x - 1)
```

giac [A] time = 1.06, size = 19, normalized size = 0.95

$$-\frac{(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{5\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^3
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{5}{2}}}{5\,(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(7/2),x)
```

```
[Out] 1/5*(x+1)^(5/2)/(-x+1)^(5/2)
```

maxima [B] time = 1.28, size = 94, normalized size = 4.70

$$\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1}+\frac{6\,\sqrt{-x^{2}+1}}{5\,\left(x^{3}-3\,x^{2}+3\,x-1\right)}+\frac{\sqrt{-x^{2}+1}}{5\,\left(x^{2}-2\,x+1\right)}-\frac{\sqrt{-x^{2}+1}}{5\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 6/5*sqrt(-x^2 + 1)/(x^3
- 3*x^2 + 3*x - 1) + 1/5*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/5*sqrt(-x^2 + 1
)/(x - 1)
```

mupad [B] time = 0.25, size = 50, normalized size = 2.50

$$-\frac{\sqrt{1-x}\,\left(\frac{2\,x\,\sqrt{x+1}}{5}+\frac{\sqrt{x+1}}{5}+\frac{x^{2}\,\sqrt{x+1}}{5}\right)}{x^{3}-3\,x^{2}+3\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(7/2),x)
```

```
[Out] -((1 - x)^(1/2)*((2*x*(x + 1)^(1/2))/5 + (x + 1)^(1/2)/5 + (x^2*(x + 1)^(1/
2))/5))/(3*x - 3*x^2 + x^3 - 1)
```

sympy [B] time = 6.26, size = 88, normalized size = 4.40

$$\begin{cases} -\frac{i(x+1)^{\frac{5}{2}}}{5\sqrt{x-1}\,(x+1)^{2}-20\sqrt{x-1}\,(x+1)+20\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{(x+1)^{\frac{5}{2}}}{5\sqrt{1-x}\,(x+1)^{2}-20\sqrt{1-x}\,(x+1)+20\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(5/2)/(5*sqrt(x - 1)*(x + 1)**2 - 20*sqrt(x - 1)*(x
+ 1) + 20*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(5/2)/(5*sqrt(1 - x)*(
x + 1)**2 - 20*sqrt(1 - x)*(x + 1) + 20*sqrt(1 - x)), True))
```

3.1015 $\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{5/2}}{35(1-x)^{5/2}}+\frac{(x+1)^{5/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(5/2)/(7*(1 - x)^(7/2)) + (1 + x)^(5/2)/(35*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{1}{7}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx\\ &= \frac{(1+x)^{5/2}}{7(1-x)^{7/2}}+\frac{(1+x)^{5/2}}{35(1-x)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-6)(x+1)^{5/2}}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(9/2),x]
```

```
[Out] -1/35*((-6 + x)*(1 + x)^(5/2))/(1 - x)^(7/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 34, normalized size = 0.83

$$\frac{(x+1)^{7/2}\left(\frac{7(1-x)}{x+1}+5\right)}{70(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(9/2),x]
[Out] ((1 + x)^(7/2)*(5 + (7*(1 - x))/(1 + x)))/(70*(1 - x)^(7/2))
```

fricas [B] time = 1.15, size = 69, normalized size = 1.68

$$\frac{6 x^{4} - 24 x^{3} + 36 x^{2} - \left(x^{3} - 4 x^{2} - 11 x - 6\right) \sqrt{x + 1} \sqrt{-x + 1} - 24 x + 6}{35 \left(x^{4} - 4 x^{3} + 6 x^{2} - 4 x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="fricas")
[Out] 1/35*(6*x^4 - 24*x^3 + 36*x^2 - (x^3 - 4*x^2 - 11*x - 6)*sqrt(x + 1)*sqrt(-
x + 1) - 24*x + 6)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

giac [A] time = 1.13, size = 22, normalized size = 0.54

$$-\frac{{\left(x + 1\right)}^{\frac{5}{2}} {\left(x - 6\right)} \sqrt{-x + 1}}{35 \, {\left(x - 1\right)}^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="giac")
[Out] -1/35*(x + 1)^(5/2)*(x - 6)*sqrt(-x + 1)/(x - 1)^4
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{\left(x + 1\right)^{\frac{5}{2}} \left(x - 6\right)}{35 \left(-x + 1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(9/2),x)
[Out] -1/35*(x+1)^(5/2)*(x-6)/(-x+1)^(7/2)
```

maxima [B] time = 1.38, size = 131, normalized size = 3.20

$$-\frac{{\left(-x^{2} + 1\right)}^{\frac{3}{2}}}{2 \, {\left(x^{5} - 5 \, x^{4} + 10 \, x^{3} - 10 \, x^{2} + 5 \, x - 1\right)}} - \frac{3 \, \sqrt{-x^{2} + 1}}{7 \, {\left(x^{4} - 4 \, x^{3} + 6 \, x^{2} - 4 \, x + 1\right)}} - \frac{3 \, \sqrt{-x^{2} + 1}}{70 \, {\left(x^{3} - 3 \, x^{2} + 3 \, x - 1\right)}} + \frac{\sqrt{-x^{2} + 1}}{35 \, {\left(x^{2} - 2 \, x + 1\right)}} - \frac{\sqrt{-x^{2} + 1}}{35 \, {\left(x - 1\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(9/2),x, algorithm="maxima")
[Out] -1/2*(-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 3/7*sqrt(
-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/70*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 1/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/35*sqrt(-x^2 + 1)/(x
 - 1)
```

mupad [B] time = 0.27, size = 64, normalized size = 1.56

$$\frac{\sqrt{1-x}\,\left(\frac{11\,x\,\sqrt{x+1}}{35}+\frac{6\,\sqrt{x+1}}{35}+\frac{4\,x^2\,\sqrt{x+1}}{35}-\frac{x^3\,\sqrt{x+1}}{35}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11*x*(x + 1)^(1/2))/35 + (6*(x + 1)^(1/2))/35 + (4*x^2*(x
+ 1)^(1/2))/35 - (x^3*(x + 1)^(1/2))/35))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

sympy [**B**] time = 19.08, size = 228, normalized size = 5.56

$$\begin{cases} -\frac{i(x+1)^{\frac{7}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-1}(x+1)-280\sqrt{x-1}} + \frac{7i(x+1)^{\frac{5}{2}}}{35\sqrt{x-1}(x+1)^3-210\sqrt{x-1}(x+1)^2+420\sqrt{x-1}(x+1)-280\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \frac{(x+1)^{\frac{7}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(x+1)-280\sqrt{1-x}} - \frac{7(x+1)^{\frac{5}{2}}}{35\sqrt{1-x}(x+1)^3-210\sqrt{1-x}(x+1)^2+420\sqrt{1-x}(x+1)-280\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((-I*(x + 1)**(7/2)/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(
x + 1)**2 + 420*sqrt(x - 1)*(x + 1) - 280*sqrt(x - 1)) + 7*I*(x + 1)**(5/2)
/(35*sqrt(x - 1)*(x + 1)**3 - 210*sqrt(x - 1)*(x + 1)**2 + 420*sqrt(x - 1)*
(x + 1) - 280*sqrt(x - 1)), Abs(x + 1)/2 > 1), ((x + 1)**(7/2)/(35*sqrt(1 -
 x)*(x + 1)**3 - 210*sqrt(1 - x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280
*sqrt(1 - x)) - 7*(x + 1)**(5/2)/(35*sqrt(1 - x)*(x + 1)**3 - 210*sqrt(1 -
x)*(x + 1)**2 + 420*sqrt(1 - x)*(x + 1) - 280*sqrt(1 - x)), True))
```

3.1016 $\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{315(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] (1 + x)^(5/2)/(9*(1 - x)^(9/2)) + (2*(1 + x)^(5/2))/(63*(1 - x)^(7/2)) + (2
*(1 + x)^(5/2))/(315*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2}{9}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2}{63}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}}\,dx\\ &= \frac{(1+x)^{5/2}}{9(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{63(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{315(1-x)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{(x+1)^{5/2}\left(2x^2-14x+47\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(5/2)*(47 - 14*x + 2*x^2))/(315*(1 - x)^(9/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 0.79

$$\frac{(x+1)^{9/2}\left(\frac{63(1-x)^2}{(x+1)^2}+\frac{90(1-x)}{x+1}+35\right)}{1260(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(9/2)*(35 + (63*(1 - x)^2)/(1 + x)^2 + (90*(1 - x))/(1 + x)))/(126
0*(1 - x)^(9/2))
```

fricas [A] time = 0.67, size = 86, normalized size = 1.41

$$\frac{47\,x^5-235\,x^4+470\,x^3-470\,x^2-\left(2\,x^4-10\,x^3+21\,x^2+80\,x+47\right)\sqrt{x+1}\,\sqrt{-x+1}+235\,x-47}{315\,\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/315*(47*x^5 - 235*x^4 + 470*x^3 - 470*x^2 - (2*x^4 - 10*x^3 + 21*x^2 + 80
*x + 47)*sqrt(x + 1)*sqrt(-x + 1) + 235*x - 47)/(x^5 - 5*x^4 + 10*x^3 - 10*
x^2 + 5*x - 1)
```

giac [A] time = 1.11, size = 29, normalized size = 0.48

$$-\frac{(2\,(x+1)(x-8)+63)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{315\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] -1/315*(2*(x + 1)*(x - 8) + 63)*(x + 1)^(5/2)*sqrt(-x + 1)/(x - 1)^5
```

maple [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{(x+1)^{\frac{5}{2}}\left(2x^2-14x+47\right)}{315\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(11/2),x)
```

```
[Out] 1/315*(x+1)^(5/2)*(2*x^2-14*x+47)/(-x+1)^(9/2)
```

maxima [B] time = 1.37, size = 172, normalized size = 2.82

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}+\frac{2\sqrt{-x^2+1}}{9\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{\sqrt{-x^2+1}}{63\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{\sqrt{-x^2+1}}{105\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\sqrt{-x^2+1}}{315\left(x^2-2\,x+1\right)}-\frac{2\sqrt{-x^2+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 2
/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/63*sqrt(-x^
2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/105*sqrt(-x^2 + 1)/(x^3 - 3*x^2
```

```
+ 3*x - 1) + 2/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/315*sqrt(-x^2 + 1)/(x
 - 1)
```

mupad [B] time = 0.32, size = 80, normalized size = 1.31

$$-\frac{\sqrt{1-x}\,\left(\frac{16\,x\,\sqrt{x+1}}{63}+\frac{47\,\sqrt{x+1}}{315}+\frac{x^2\,\sqrt{x+1}}{15}-\frac{2\,x^3\,\sqrt{x+1}}{63}+\frac{2\,x^4\,\sqrt{x+1}}{315}\right)}{x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((16*x*(x + 1)^(1/2))/63 + (47*(x + 1)^(1/2))/315 + (x^2*(x
 + 1)^(1/2))/15 - (2*x^3*(x + 1)^(1/2))/63 + (2*x^4*(x + 1)^(1/2))/315))/(5
*x - 10*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

sympy [B] time = 51.65, size = 677, normalized size = 11.10

$$\begin{cases} -\frac{2i(x+1)^{\frac{11}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}+\frac{22i(x+1)^{\frac{9}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}-\frac{99i(x+1)^{\frac{7}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}}+\frac{126i(x+1)^{\frac{5}{2}}}{315\sqrt{x-1}(x+1)^5-3150\sqrt{x-1}(x+1)^4+12600\sqrt{x-1}(x+1)^3-25200\sqrt{x-1}(x+1)^2+25200\sqrt{x-1}(x+1)-10080\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{2(x+1)^{\frac{11}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}-\frac{22(x+1)^{\frac{9}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}+\frac{99(x+1)^{\frac{7}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}}-\frac{126(x+1)^{\frac{5}{2}}}{315\sqrt{1-x}(x+1)^5-3150\sqrt{1-x}(x+1)^4+12600\sqrt{1-x}(x+1)^3-25200\sqrt{1-x}(x+1)^2+25200\sqrt{1-x}(x+1)-10080\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(11/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x -
 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**
2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 22*I*(x + 1)**(9/2)/(3
15*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)
*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10
080*sqrt(x - 1)) - 99*I*(x + 1)**(7/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*s
qrt(x - 1)*(x + 1)**4 + 12600*sqrt(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x
 + 1)**2 + 25200*sqrt(x - 1)*(x + 1) - 10080*sqrt(x - 1)) + 126*I*(x + 1)**
(5/2)/(315*sqrt(x - 1)*(x + 1)**5 - 3150*sqrt(x - 1)*(x + 1)**4 + 12600*sqr
t(x - 1)*(x + 1)**3 - 25200*sqrt(x - 1)*(x + 1)**2 + 25200*sqrt(x - 1)*(x +
 1) - 10080*sqrt(x - 1)), Abs(x + 1)/2 > 1), (2*(x + 1)**(11/2)/(315*sqrt(1
 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)*
*3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(
1 - x)) - 22*(x + 1)**(9/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*
(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 +
25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)) + 99*(x + 1)**(7/2)/(315*sqr
t(1 - x)*(x + 1)**5 - 3150*sqrt(1 - x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x +
1)**3 - 25200*sqrt(1 - x)*(x + 1)**2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sq
rt(1 - x)) - 126*(x + 1)**(5/2)/(315*sqrt(1 - x)*(x + 1)**5 - 3150*sqrt(1 -
 x)*(x + 1)**4 + 12600*sqrt(1 - x)*(x + 1)**3 - 25200*sqrt(1 - x)*(x + 1)**
2 + 25200*sqrt(1 - x)*(x + 1) - 10080*sqrt(1 - x)), True))
```

3.1017 $\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}} \, dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{5/2}}{1155(1-x)^{5/2}}+\frac{2(x+1)^{5/2}}{231(1-x)^{7/2}}+\frac{(x+1)^{5/2}}{33(1-x)^{9/2}}+\frac{(x+1)^{5/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(5/2)/(11*(1 - x)^(11/2)) + (1 + x)^(5/2)/(33*(1 - x)^(9/2)) + (2*(
1 + x)^(5/2))/(231*(1 - x)^(7/2)) + (2*(1 + x)^(5/2))/(1155*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}} \, dx &= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{3}{11}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}} \, dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2}{33}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} \, dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2}{231}\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} \, dx\\
&= \frac{(1+x)^{5/2}}{11(1-x)^{11/2}}+\frac{(1+x)^{5/2}}{33(1-x)^{9/2}}+\frac{2(1+x)^{5/2}}{231(1-x)^{7/2}}+\frac{2(1+x)^{5/2}}{1155(1-x)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.43

$$\frac{(x+1)^{5/2}\left(-2x^3+16x^2-61x+152\right)}{1155(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(5/2)*(152 - 61*x + 16*x^2 - 2*x^3))/(1155*(1 - x)^(11/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 62, normalized size = 0.77

$$\frac{(x+1)^{11/2}\left(\frac{231(1-x)^3}{(x+1)^3}+\frac{495(1-x)^2}{(x+1)^2}+\frac{385(1-x)}{x+1}+105\right)}{9240(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(11/2)*(105 + (231*(1 - x)^3)/(1 + x)^3 + (495*(1 - x)^2)/(1 + x)^
2 + (385*(1 - x))/(1 + x)))/(9240*(1 - x)^(11/2))
```

fricas [A] time = 1.53, size = 101, normalized size = 1.25

$$\frac{152\,x^6-912\,x^5+2280\,x^4-3040\,x^3+2280\,x^2-\left(2\,x^5-12\,x^4+31\,x^3-46\,x^2-243\,x-152\right)\sqrt{x+1}\,\sqrt{-x+1}-912\,x+152}{1155\,\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(152*x^6 - 912*x^5 + 2280*x^4 - 3040*x^3 + 2280*x^2 - (2*x^5 - 12*x^
4 + 31*x^3 - 46*x^2 - 243*x - 152)*sqrt(x + 1)*sqrt(-x + 1) - 912*x + 152)/
(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

giac [A] time = 1.17, size = 35, normalized size = 0.43

$$-\frac{((2\,(x+1)(x-10)+99)(x+1)-231)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{1155\,(x-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] -1/1155*((2*(x + 1)*(x - 10) + 99)*(x + 1) - 231)*(x + 1)^(5/2)*sqrt(-x + 1
)/(x - 1)^6
```

maple [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{5}{2}}\left(2x^3-16x^2+61x-152\right)}{1155\,(-x+1)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(13/2),x)
```

```
[Out] -1/1155*(x+1)^(5/2)*(2*x^3-16*x^2+61*x-152)/(-x+1)^(11/2)
```

maxima [B] time = 1.39, size = 218, normalized size = 2.69

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{4\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}-\frac{3\sqrt{-x^2+1}}{22\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}-\frac{\sqrt{-x^2+1}}{132\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{\sqrt{-x^2+1}}{231\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{\sqrt{-x^2+1}}{385\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\sqrt{-x^2+1}}{1155\left(x^2-2\,x+1\right)}-\frac{2\sqrt{-x^2+1}}{1155\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 3/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
```

```
+ 1) - 1/132*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 1/2
31*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 1/385*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/1155*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/1155*sqrt
(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.31, size = 94, normalized size = 1.16

$$\frac{\sqrt{1-x}\,\left(\frac{81\,x\,\sqrt{x+1}}{385}+\frac{152\,\sqrt{x+1}}{1155}+\frac{46\,x^2\,\sqrt{x+1}}{1155}-\frac{31\,x^3\,\sqrt{x+1}}{1155}+\frac{4\,x^4\,\sqrt{x+1}}{385}-\frac{2\,x^5\,\sqrt{x+1}}{1155}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((81*x*(x + 1)^(1/2))/385 + (152*(x + 1)^(1/2))/1155 + (46*x
^2*(x + 1)^(1/2))/1155 - (31*x^3*(x + 1)^(1/2))/1155 + (4*x^4*(x + 1)^(1/2)
)/385 - (2*x^5*(x + 1)^(1/2))/1155))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^
5 + x^6 + 1)
```

sympy [B] time = 132.96, size = 1753, normalized size = 21.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**(17/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x
 - 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x +
1)**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2
069760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x
 - 1)) + 34*I*(x + 1)**(15/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
 1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)
**5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206
9760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
 1)) - 255*I*(x + 1)**(13/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) + 1105*I*(x + 1)**(11/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x -
1)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)*
*5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069
760*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x -
1)) - 2750*I*(x + 1)**(9/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1
)*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**
5 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 20697
60*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1
)) + 3564*I*(x + 1)**(7/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)
*(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
 + 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 206976
0*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1)
) - 1848*I*(x + 1)**(5/2)/(1155*sqrt(x - 1)*(x + 1)**8 - 18480*sqrt(x - 1)*
(x + 1)**7 + 129360*sqrt(x - 1)*(x + 1)**6 - 517440*sqrt(x - 1)*(x + 1)**5
+ 1293600*sqrt(x - 1)*(x + 1)**4 - 2069760*sqrt(x - 1)*(x + 1)**3 + 2069760
*sqrt(x - 1)*(x + 1)**2 - 1182720*sqrt(x - 1)*(x + 1) + 295680*sqrt(x - 1))
, Abs(x + 1)/2 > 1), (2*(x + 1)**(17/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 1848
0*sqrt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 -
x)*(x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1
)**3 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 29568
0*sqrt(1 - x)) - 34*(x + 1)**(15/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sq
rt(1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(
x + 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3
```

```
 + 2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sq
rt(1 - x)) + 255*(x + 1)**(13/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(
1 - x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x +
 1)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 +
2069760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(
1 - x)) - 1105*(x + 1)**(11/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1
- x)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1
)**5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20
69760*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1
- x)) + 2750*(x + 1)**(9/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x
)*(x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**
5 + 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 20697
60*sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x
)) - 3564*(x + 1)**(7/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(
x + 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 +
 1293600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*
sqrt(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x))
+ 1848*(x + 1)**(5/2)/(1155*sqrt(1 - x)*(x + 1)**8 - 18480*sqrt(1 - x)*(x +
 1)**7 + 129360*sqrt(1 - x)*(x + 1)**6 - 517440*sqrt(1 - x)*(x + 1)**5 + 12
93600*sqrt(1 - x)*(x + 1)**4 - 2069760*sqrt(1 - x)*(x + 1)**3 + 2069760*sqr
t(1 - x)*(x + 1)**2 - 1182720*sqrt(1 - x)*(x + 1) + 295680*sqrt(1 - x)), Tr
ue))
```

3.1018 $\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}} dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8(x+1)^{5/2}}{15015(1-x)^{5/2}}+\frac{8(x+1)^{5/2}}{3003(1-x)^{7/2}}+\frac{4(x+1)^{5/2}}{429(1-x)^{9/2}}+\frac{4(x+1)^{5/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{5/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(5/2)/(13*(1 - x)^(13/2)) + (4*(1 + x)^(5/2))/(143*(1 - x)^(11/2))
+ (4*(1 + x)^(5/2))/(429*(1 - x)^(9/2)) + (8*(1 + x)^(5/2))/(3003*(1 - x)^(
7/2)) + (8*(1 + x)^(5/2))/(15015*(1 - x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{3/2}}{(1-x)^{15/2}} dx &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4}{13}\int \frac{(1+x)^{3/2}}{(1-x)^{13/2}} dx\\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{12}{143}\int \frac{(1+x)^{3/2}}{(1-x)^{11/2}} dx\\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{4(1+x)^{5/2}}{429(1-x)^{9/2}}+\frac{8}{429}\int \frac{(1+x)^{3/2}}{(1-x)^{9/2}} dx\\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{4(1+x)^{5/2}}{429(1-x)^{9/2}}+\frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}}+\frac{8\int \frac{(1+x)^{3/2}}{(1-x)^{7/2}} dx}{3003}\\ &= \frac{(1+x)^{5/2}}{13(1-x)^{13/2}}+\frac{4(1+x)^{5/2}}{143(1-x)^{11/2}}+\frac{4(1+x)^{5/2}}{429(1-x)^{9/2}}+\frac{8(1+x)^{5/2}}{3003(1-x)^{7/2}}+\frac{8(1+x)^{5/2}}{15015(1-x)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.40

$$\frac{(x+1)^{5/2}\left(8x^4-72x^3+308x^2-852x+1763\right)}{15015(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(5/2)*(1763 - 852*x + 308*x^2 - 72*x^3 + 8*x^4))/(15015*(1 - x)^(1
3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 76, normalized size = 0.75

$$\frac{(x+1)^{13/2}\left(\frac{3003(1-x)^4}{(x+1)^4}+\frac{8580(1-x)^3}{(x+1)^3}+\frac{10010(1-x)^2}{(x+1)^2}+\frac{5460(1-x)}{x+1}+1155\right)}{240240(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(3/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(13/2)*(1155 + (3003*(1 - x)^4)/(1 + x)^4 + (8580*(1 - x)^3)/(1 +
x)^3 + (10010*(1 - x)^2)/(1 + x)^2 + (5460*(1 - x))/(1 + x)))/(240240*(1 -
x)^(13/2))
```

fricas [A] time = 1.23, size = 116, normalized size = 1.15

$$\frac{1763\,x^7-12341\,x^6+37023\,x^5-61705\,x^4+61705\,x^3-37023\,x^2-\left(8\,x^6-56\,x^5+172\,x^4-308\,x^3+367\,x^2+2674\,x+1763\right)\sqrt{x+1}\,\sqrt{-x+1}+12341\,x-1763}{15015\,\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/15015*(1763*x^7 - 12341*x^6 + 37023*x^5 - 61705*x^4 + 61705*x^3 - 37023*x
^2 - (8*x^6 - 56*x^5 + 172*x^4 - 308*x^3 + 367*x^2 + 2674*x + 1763)*sqrt(x
+ 1)*sqrt(-x + 1) + 12341*x - 1763)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3
 - 21*x^2 + 7*x - 1)
```

giac [A] time = 1.23, size = 42, normalized size = 0.42

$$-\frac{(4\,((2\,(x+1)(x-12)+143)(x+1)-429)(x+1)+3003)(x+1)^{\frac{5}{2}}\sqrt{-x+1}}{15015\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] -1/15015*(4*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1) + 3003)*(x +
 1)^(5/2)*sqrt(-x + 1)/(x - 1)^7
```

maple [A] time = 0.00, size = 35, normalized size = 0.35

$$\frac{(x+1)^{\frac{5}{2}}\left(8x^4-72x^3+308x^2-852x+1763\right)}{15015\,(-x+1)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(3/2)/(-x+1)^(15/2),x)
```

```
[Out] 1/15015*(x+1)^(5/2)*(8*x^4-72*x^3+308*x^2-852*x+1763)/(-x+1)^(13/2)
```

maxima [B] time = 1.38, size = 269, normalized size = 2.66

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{5\left(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1\right)}+\frac{6\sqrt{-x^2+1}}{65\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}+\frac{3\sqrt{-x^2+1}}{715\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}-\frac{\sqrt{-x^2+1}}{429\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}+\frac{4\sqrt{-x^2+1}}{3003\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{4\sqrt{-x^2+1}}{5005\left(x^3-3\,x^2+3\,x-1\right)}+\frac{8\sqrt{-x^2+1}}{15015\left(x^2-2\,x+1\right)}-\frac{8\sqrt{-x^2+1}}{15015\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(3/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 6/65*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^
3 - 21*x^2 + 7*x - 1) + 3/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3
 + 15*x^2 - 6*x + 1) - 1/429*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
+ 5*x - 1) + 4/3003*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/5005
*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 8/15015*sqrt(-x^2 + 1)/(x^2 - 2*x
 + 1) - 8/15015*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.33, size = 110, normalized size = 1.09

$$-\frac{\sqrt{1-x}\,\left(\frac{382\,x\,\sqrt{x+1}}{2145}+\frac{1763\,\sqrt{x+1}}{15015}+\frac{367\,x^2\,\sqrt{x+1}}{15015}-\frac{4\,x^3\,\sqrt{x+1}}{195}+\frac{172\,x^4\,\sqrt{x+1}}{15015}-\frac{8\,x^5\,\sqrt{x+1}}{2145}+\frac{8\,x^6\,\sqrt{x+1}}{15015}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(3/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((382*x*(x + 1)^(1/2))/2145 + (1763*(x + 1)^(1/2))/15015 +
(367*x^2*(x + 1)^(1/2))/15015 - (4*x^3*(x + 1)^(1/2))/195 + (172*x^4*(x + 1
)^(1/2))/15015 - (8*x^5*(x + 1)^(1/2))/2145 + (8*x^6*(x + 1)^(1/2))/15015))
/(7*x - 21*x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

3.1019 $\int (1-x)^{11/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=130

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x$$

Rubi [A] time = 0.03, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{9}(x+1)^{7/2}(1-x)^{11/2}+\frac{11}{72}(x+1)^{7/2}(1-x)^{9/2}+\frac{11}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{11}{48}x(x+1)^{5/2}(1-x)^{5/2}+\frac{55}{192}x(x+1)^{3/2}(1-x)^{3/2}+\frac{55}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{55}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (55*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (55*(1 - x)^(3/2)*x*(1 + x)^(3/2))/192
 + (11*(1 - x)^(5/2)*x*(1 + x)^(5/2))/48 + (11*(1 - x)^(7/2)*(1 + x)^(7/2))
/56 + (11*(1 - x)^(9/2)*(1 + x)^(7/2))/72 + ((1 - x)^(11/2)*(1 + x)^(7/2))/
9 + (55*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (1-x)^{11/2}(1+x)^{5/2}\,dx &= \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{9}\int (1-x)^{9/2}(1+x)^{5/2}\,dx \\ &= \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx \\ &= \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x)^{11/2}(1+x)^{7/2} + \frac{11}{8}\int (1- \\ &= \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1-x)^{9/2}(1+x)^{7/2} + \frac{1}{9}(1-x) \\ &= \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1-x)^{7/2}(1+x)^{7/2} + \frac{11}{72}(1- \\ &= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1- \\ &= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1- \\ &= \frac{55}{128}\sqrt{1-x}\,x\sqrt{1+x} + \frac{55}{192}(1-x)^{3/2}x(1+x)^{3/2} + \frac{11}{48}(1-x)^{5/2}x(1+x)^{5/2} + \frac{11}{56}(1- \end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.58

$$\frac{\sqrt{1-x^2}\left(-896x^8+3024x^7-1024x^6-7224x^5+8448x^4+3066x^3-10240x^2+4599x+3712\right)-6930\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)}{8064}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(3712 + 4599*x - 10240*x^2 + 3066*x^3 + 8448*x^4 - 7224*x^5
- 1024*x^6 + 3024*x^7 - 896*x^8) - 6930*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8064
```

IntegrateAlgebraic [A] time = 0.19, size = 205, normalized size = 1.58

$$\frac{-\frac{3465(1-x)^{17/2}}{(x+1)^{17/2}}-\frac{30030(1-x)^{15/2}}{(x+1)^{15/2}}-\frac{115038(1-x)^{13/2}}{(x+1)^{13/2}}+\frac{334602(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{360448(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{255222(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{115038(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{30030(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{3465\sqrt{1-x}}{\sqrt{x+1}}}{4032\left(\frac{1-x}{x+1}+1\right)^9}-\frac{55}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(11/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-3465*(1 - x)^(17/2))/(1 + x)^(17/2) - (30030*(1 - x)^(15/2))/(1 + x)^(15
/2) - (115038*(1 - x)^(13/2))/(1 + x)^(13/2) + (334602*(1 - x)^(11/2))/(1 +
 x)^(11/2) + (360448*(1 - x)^(9/2))/(1 + x)^(9/2) + (255222*(1 - x)^(7/2))/
(1 + x)^(7/2) + (115038*(1 - x)^(5/2))/(1 + x)^(5/2) + (30030*(1 - x)^(3/2)
)/(1 + x)^(3/2) + (3465*Sqrt[1 - x])/Sqrt[1 + x])/(4032*(1 + (1 - x)/(1 + x
))^9) - (55*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/64
```

fricas [A] time = 0.77, size = 77, normalized size = 0.59

$$-\frac{1}{8064}\left(896\,x^8-3024\,x^7+1024\,x^6+7224\,x^5-8448\,x^4-3066\,x^3+10240\,x^2-4599\,x-3712\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{55}{64}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8064*(896*x^8 - 3024*x^7 + 1024*x^6 + 7224*x^5 - 8448*x^4 - 3066*x^3 + 1
0240*x^2 - 4599*x - 3712)*sqrt(x + 1)*sqrt(-x + 1) - 55/64*arctan((sqrt(x +
 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.40, size = 323, normalized size = 2.48

$$-\frac{1}{40320}\,((2\,((4\,(5\,(2\,(7\,(8\,x-65)(x+1)+2073)(x+1)-9833)(x+1)+75293)(x+1)-310203)(x+1)+216993)(x+1)-205275)(x+1)+69615)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{6720}\,((2\,((4\,(5\,(6\,(7\,x-50)(x+1)+1219)(x+1)-12463)(x+1)+64233)(x+1)-53963)(x+1)+59465)(x+1)-23205)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{840}\,((2\,((4\,(5\,(6\,x-37)(x+1)+661)(x+1)-4551)(x+1)+4781)(x+1)-6335)(x+1)+2835)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{40}\,((2\,((4\,(5\,x-26)(x+1)+321)(x+1)-451)(x+1)+745)(x+1)-405)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{4}\,((2\,(3\,x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\,((2\,x-5)(x+1)+9)\sqrt{x+1}\,\sqrt{-x+1}-\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{55}{64}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/40320*((2*((4*(5*(2*(7*(8*x - 65)*(x + 1) + 2073)*(x + 1) - 9833)*(x + 1
) + 75293)*(x + 1) - 310203)*(x + 1) + 216993)*(x + 1) - 205275)*(x + 1) +
69615)*sqrt(x + 1)*sqrt(-x + 1) + 1/6720*((2*((4*(5*(6*(7*x - 50)*(x + 1) +
 1219)*(x + 1) - 12463)*(x + 1) + 64233)*(x + 1) - 53963)*(x + 1) + 59465)*
(x + 1) - 23205)*sqrt(x + 1)*sqrt(-x + 1) + 1/840*((2*((4*(5*(6*x - 37)*(x
+ 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)
*sqrt(x + 1)*sqrt(-x + 1) - 1/40*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1)
- 451)*(x + 1) + 745)*(x + 1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/4*((2*(3*
x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x -
5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
 + sqrt(x + 1)*sqrt(-x + 1) + 55/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.01, size = 155, normalized size = 1.19

$$\frac{55\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{128\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{11}{2}}(x+1)^{\frac{7}{2}}}{9}+\frac{11\,(-x+1)^{\frac{9}{2}}(x+1)^{\frac{7}{2}}}{72}+\frac{11\,(-x+1)^{\frac{7}{2}}(x+1)^{\frac{7}{2}}}{56}+\frac{11\,(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{48}+\frac{11\,(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{48}+\frac{11\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{64}-\frac{11\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{192}-\frac{55\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{384}-\frac{55\sqrt{-x+1}\,\sqrt{x+1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(11/2)*(x+1)^(5/2),x)
```

```
[Out] 1/9*(-x+1)^(11/2)*(x+1)^(7/2)+11/72*(-x+1)^(9/2)*(x+1)^(7/2)+11/56*(-x+1)^(
7/2)*(x+1)^(7/2)+11/48*(-x+1)^(5/2)*(x+1)^(7/2)+11/48*(-x+1)^(3/2)*(x+1)^(7
/2)+11/64*(-x+1)^(1/2)*(x+1)^(7/2)-11/192*(-x+1)^(1/2)*(x+1)^(5/2)-55/384*(
-x+1)^(1/2)*(x+1)^(3/2)-55/128*(-x+1)^(1/2)*(x+1)^(1/2)+55/128*((x+1)*(-x+1
))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.01, size = 78, normalized size = 0.60

$$\frac{1}{9}\left(-x^2+1\right)^{\frac{7}{2}}x^2-\frac{3}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{29}{63}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{11}{48}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{55}{192}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{55}{128}\sqrt{-x^2+1}\,x+\frac{55}{128}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(11/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(-x^2 + 1)^(7/2)*x^2 - 3/8*(-x^2 + 1)^(7/2)*x + 29/63*(-x^2 + 1)^(7/2)
+ 11/48*(-x^2 + 1)^(5/2)*x + 55/192*(-x^2 + 1)^(3/2)*x + 55/128*sqrt(-x^2 +
 1)*x + 55/128*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{11/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(11/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(11/2)*(x + 1)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(11/2)*(1+x)**(5/2),x)
```

```
[Out] Timed out
```

3.1020 $\int (1-x)^{9/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=110

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{45}{128}$$

Rubi [A] time = 0.02, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{8}(x+1)^{7/2}(1-x)^{9/2}+\frac{9}{56}(x+1)^{7/2}(1-x)^{7/2}+\frac{3}{16}x(x+1)^{5/2}(1-x)^{5/2}+\frac{15}{64}x(x+1)^{3/2}(1-x)^{3/2}+\frac{45}{128}x\sqrt{x+1}\sqrt{1-x}+\frac{45}{128}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (45*Sqrt[1 - x]*x*Sqrt[1 + x])/128 + (15*(1 - x)^(3/2)*x*(1 + x)^(3/2))/64
+ (3*(1 - x)^(5/2)*x*(1 + x)^(5/2))/16 + (9*(1 - x)^(7/2)*(1 + x)^(7/2))/56
 + ((1 - x)^(9/2)*(1 + x)^(7/2))/8 + (45*ArcSin[x])/128
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{9/2}(1+x)^{5/2}\,dx &= \frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{7/2}(1+x)^{5/2}\,dx \\
&= \frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{9}{8}\int (1-x)^{5/2}(1+x)^{5/2}\,dx \\
&= \frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(1+x)^{7/2}+\frac{15}{16}\int (1-x)^{3/} \\
&= \frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(1+x)^{7/2}+\frac{1}{8}(1-x)^{9/2}(\\
&= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(\\
&= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(\\
&= \frac{45}{128}\sqrt{1-x}\,x\sqrt{1+x}+\frac{15}{64}(1-x)^{3/2}x(1+x)^{3/2}+\frac{3}{16}(1-x)^{5/2}x(1+x)^{5/2}+\frac{9}{56}(1-x)^{7/2}(
\end{aligned}$$

Mathematica [A] time = 0.06, size = 70, normalized size = 0.64

$$\frac{1}{896}\left(\sqrt{1-x^2}\left(112x^7-256x^6-168x^5+768x^4-210x^3-768x^2+581x+256\right)-630\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(256 + 581*x - 768*x^2 - 210*x^3 + 768*x^4 - 168*x^5 - 256*x
^6 + 112*x^7) - 630*ArcSin[Sqrt[1 - x]/Sqrt[2]])/896
```

IntegrateAlgebraic [A] time = 0.16, size = 187, normalized size = 1.70

$$\frac{-\frac{315(1-x)^{15/2}}{(x+1)^{15/2}}-\frac{2415(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{8043(1-x)^{11/2}}{(x+1)^{11/2}}+\frac{17609(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{15159(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{8043(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{2415(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{448\left(\frac{1-x}{x+1}+1\right)^8}-\frac{45}{64}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-315*(1 - x)^(15/2))/(1 + x)^(15/2) - (2415*(1 - x)^(13/2))/(1 + x)^(13/2
) - (8043*(1 - x)^(11/2))/(1 + x)^(11/2) + (17609*(1 - x)^(9/2))/(1 + x)^(9
/2) + (15159*(1 - x)^(7/2))/(1 + x)^(7/2) + (8043*(1 - x)^(5/2))/(1 + x)^(5
/2) + (2415*(1 - x)^(3/2))/(1 + x)^(3/2) + (315*Sqrt[1 - x])/Sqrt[1 + x])/(
448*(1 + (1 - x)/(1 + x))^8) - (45*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/64
```

fricas [A] time = 1.16, size = 72, normalized size = 0.65

$$\frac{1}{896}\left(112\,x^7-256\,x^6-168\,x^5+768\,x^4-210\,x^3-768\,x^2+581\,x+256\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{45}{64}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/896*(112*x^7 - 256*x^6 - 168*x^5 + 768*x^4 - 210*x^3 - 768*x^2 + 581*x +
256)*sqrt(x + 1)*sqrt(-x + 1) - 45/64*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)
/x)
```

giac [B] time = 1.49, size = 296, normalized size = 2.69

$$\frac{1}{13440}\left(\left(2\left(\left(4\left(5\left(6\left(7\,x-50\right)\left(x+1\right)+1219\right)\left(x+1\right)-12463\right)\left(x+1\right)+64233\right)\left(x+1\right)-53963\right)\left(x+1\right)+59465\right)\left(x+1\right)-23205\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{80}\left(\left(2\left(\left(4\left(5\,x-26\right)\left(x+1\right)+321\right)\left(x+1\right)-451\right)\left(x+1\right)+745\right)\left(x+1\right)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{8}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\,\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{45}{64}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/13440*((2*((4*(5*(6*(7*x - 50)*(x + 1) + 1219)*(x + 1) - 12463)*(x + 1) +
 64233)*(x + 1) - 53963)*(x + 1) + 59465)*(x + 1) - 23205)*sqrt(x + 1)*sqrt
(-x + 1) - 1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x +
 1) + 4781)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) - 1/80
*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x + 1) -
405)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4*x - 17)*(x + 1) + 133)*(x +
1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/8*((2*(3*x - 10)*(x +
 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*((2*x - 5)*(x + 1) +
 9)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(
x + 1)*sqrt(-x + 1) + 45/64*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 141, normalized size = 1.28

$$\frac{45\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{128\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{9}{2}}(x+1)^{\frac{7}{2}}}{8}+\frac{9(-x+1)^{\frac{7}{2}}(x+1)^{\frac{7}{2}}}{56}+\frac{3(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{16}+\frac{3(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{16}+\frac{9\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{64}-\frac{3\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{64}-\frac{15\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{128}-\frac{45\sqrt{-x+1}\,\sqrt{x+1}}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)*(x+1)^(5/2),x)
```

```
[Out] 1/8*(-x+1)^(9/2)*(x+1)^(7/2)+9/56*(-x+1)^(7/2)*(x+1)^(7/2)+3/16*(-x+1)^(5/2
)*(x+1)^(7/2)+3/16*(-x+1)^(3/2)*(x+1)^(7/2)+9/64*(-x+1)^(1/2)*(x+1)^(7/2)-3
/64*(-x+1)^(1/2)*(x+1)^(5/2)-15/128*(-x+1)^(1/2)*(x+1)^(3/2)-45/128*(-x+1)^
(1/2)*(x+1)^(1/2)+45/128*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcs
in(x)
```

maxima [A] time = 2.97, size = 64, normalized size = 0.58

$$-\frac{1}{8}\left(-x^2+1\right)^{\frac{7}{2}}x+\frac{2}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{3}{16}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{15}{64}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{45}{128}\sqrt{-x^2+1}\,x+\frac{45}{128}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(-x^2 + 1)^(7/2)*x + 2/7*(-x^2 + 1)^(7/2) + 3/16*(-x^2 + 1)^(5/2)*x +
15/64*(-x^2 + 1)^(3/2)*x + 45/128*sqrt(-x^2 + 1)*x + 45/128*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{9/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)*(x + 1)^(5/2), x)
```

sympy [A] time = 117.57, size = 360, normalized size = 3.27

$$\begin{cases} -\frac{45i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64}+\frac{i(x+1)^{\frac{17}{2}}}{8\sqrt{x-1}}-\frac{79i(x+1)^{\frac{15}{2}}}{56\sqrt{x-1}}+\frac{725i(x+1)^{\frac{13}{2}}}{112\sqrt{x-1}}-\frac{1699i(x+1)^{\frac{11}{2}}}{112\sqrt{x-1}}+\frac{8191i(x+1)^{\frac{9}{2}}}{448\sqrt{x-1}}-\frac{4099i(x+1)^{\frac{7}{2}}}{448\sqrt{x-1}}-\frac{3i(x+1)^{\frac{5}{2}}}{128\sqrt{x-1}}-\frac{15i(x+1)^{\frac{3}{2}}}{128\sqrt{x-1}}+\frac{45i\sqrt{x+1}}{64\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{45\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{64}-\frac{(x+1)^{\frac{17}{2}}}{8\sqrt{1-x}}+\frac{79(x+1)^{\frac{15}{2}}}{56\sqrt{1-x}}-\frac{725(x+1)^{\frac{13}{2}}}{112\sqrt{1-x}}+\frac{1699(x+1)^{\frac{11}{2}}}{112\sqrt{1-x}}-\frac{8191(x+1)^{\frac{9}{2}}}{448\sqrt{1-x}}+\frac{4099(x+1)^{\frac{7}{2}}}{448\sqrt{1-x}}+\frac{3(x+1)^{\frac{5}{2}}}{128\sqrt{1-x}}+\frac{15(x+1)^{\frac{3}{2}}}{128\sqrt{1-x}}-\frac{45\sqrt{x+1}}{64\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-45*I*acosh(sqrt(2)*sqrt(x + 1)/2)/64 + I*(x + 1)**(17/2)/(8*sqr
t(x - 1)) - 79*I*(x + 1)**(15/2)/(56*sqrt(x - 1)) + 725*I*(x + 1)**(13/2)/(
```

```
112*sqrt(x - 1)) - 1699*I*(x + 1)**(11/2)/(112*sqrt(x - 1)) + 8191*I*(x + 1
)**(9/2)/(448*sqrt(x - 1)) - 4099*I*(x + 1)**(7/2)/(448*sqrt(x - 1)) - 3*I*
(x + 1)**(5/2)/(128*sqrt(x - 1)) - 15*I*(x + 1)**(3/2)/(128*sqrt(x - 1)) +
45*I*sqrt(x + 1)/(64*sqrt(x - 1)), Abs(x + 1)/2 > 1), (45*asin(sqrt(2)*sqrt
(x + 1)/2)/64 - (x + 1)**(17/2)/(8*sqrt(1 - x)) + 79*(x + 1)**(15/2)/(56*sq
rt(1 - x)) - 725*(x + 1)**(13/2)/(112*sqrt(1 - x)) + 1699*(x + 1)**(11/2)/(
112*sqrt(1 - x)) - 8191*(x + 1)**(9/2)/(448*sqrt(1 - x)) + 4099*(x + 1)**(7
/2)/(448*sqrt(1 - x)) + 3*(x + 1)**(5/2)/(128*sqrt(1 - x)) + 15*(x + 1)**(3
/2)/(128*sqrt(1 - x)) - 45*sqrt(x + 1)/(64*sqrt(1 - x)), True))
```

3.1021 $\int (1-x)^{7/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=90

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$\frac{1}{7}(1-x)^{7/2}(x+1)^{7/2}+\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + ((1 - x)^(7/2)*(1 + x)^(7/2))/7 + (5*Arc
Sin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int (1-x)^{7/2}(1+x)^{5/2}\,dx &= \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \int (1-x)^{5/2}(1+x)^{5/2}\,dx \\
&= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx \\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+x)^{7/2} + \frac{5}{8}\int \sqrt{1-x}\,\surd \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+ \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+ \\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x} + \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2} + \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2} + \frac{1}{7}(1-x)^{7/2}(1+
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 66, normalized size = 0.73

$$\frac{1}{336}\sqrt{1-x^2}\left(-48x^6+56x^5+144x^4-182x^3-144x^2+231x+48\right)-\frac{5}{8}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(48 + 231*x - 144*x^2 - 182*x^3 + 144*x^4 + 56*x^5 - 48*x^6)
)/336 - (5*ArcSin[Sqrt[1 - x]/Sqrt[2]])/8
```

IntegrateAlgebraic [A] time = 0.14, size = 169, normalized size = 1.88

$$\frac{-\frac{105(1-x)^{13/2}}{(x+1)^{13/2}}-\frac{700(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{1981(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{3072(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{1981(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{700(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{168\left(\frac{1-x}{x+1}+1\right)^7}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-105*(1 - x)^(13/2))/(1 + x)^(13/2) - (700*(1 - x)^(11/2))/(1 + x)^(11/2)
 - (1981*(1 - x)^(9/2))/(1 + x)^(9/2) + (3072*(1 - x)^(7/2))/(1 + x)^(7/2)
+ (1981*(1 - x)^(5/2))/(1 + x)^(5/2) + (700*(1 - x)^(3/2))/(1 + x)^(3/2) +
(105*Sqrt[1 - x])/Sqrt[1 + x])/(168*(1 + (1 - x)/(1 + x))^7) - (5*ArcTan[Sq
rt[1 - x]/Sqrt[1 + x]])/8
```

fricas [A] time = 1.32, size = 67, normalized size = 0.74

$$-\frac{1}{336}\left(48\,x^6-56\,x^5-144\,x^4+182\,x^3+144\,x^2-231\,x-48\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/336*(48*x^6 - 56*x^5 - 144*x^4 + 182*x^3 + 144*x^2 - 231*x - 48)*sqrt(x
+ 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.14, size = 143, normalized size = 1.59

$$-\frac{1}{1680}\left(\left(2\left(\left(4\left(5\left(6\,x-37\right)\left(x+1\right)+661\right)\left(x+1\right)-4551\right)\left(x+1\right)+4781\right)\left(x+1\right)-6335\right)\left(x+1\right)+2835\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{40}\left(\left(2\left(3\left(4\,x-17\right)\left(x+1\right)+133\right)\left(x+1\right)-295\right)\left(x+1\right)+195\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{2}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{8}\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/1680*((2*((4*(5*(6*x - 37)*(x + 1) + 661)*(x + 1) - 4551)*(x + 1) + 4781
)*(x + 1) - 6335)*(x + 1) + 2835)*sqrt(x + 1)*sqrt(-x + 1) + 1/40*((2*(3*(4
*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x +
 1) - 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + sqrt(x + 1)*sq
rt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 127, normalized size = 1.41

$$\frac{5\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{16\sqrt{x+1}\ \sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}(x+1)^{\frac{7}{2}}}{7}+\frac{(-x+1)^{\frac{5}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{6}+\frac{\sqrt{-x+1}\ (x+1)^{\frac{7}{2}}}{8}-\frac{\sqrt{-x+1}\ (x+1)^{\frac{5}{2}}}{24}-\frac{5\sqrt{-x+1}\ (x+1)^{\frac{3}{2}}}{48}-\frac{5\sqrt{-x+1}\ \sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)*(x+1)^(5/2),x)
```

```
[Out] 1/7*(-x+1)^(7/2)*(x+1)^(7/2)+1/6*(-x+1)^(5/2)*(x+1)^(7/2)+1/6*(-x+1)^(3/2)*
(x+1)^(7/2)+1/8*(-x+1)^(1/2)*(x+1)^(7/2)-1/24*(-x+1)^(1/2)*(x+1)^(5/2)-5/48
*(-x+1)^(1/2)*(x+1)^(3/2)-5/16*(-x+1)^(1/2)*(x+1)^(1/2)+5/16*((x+1)*(-x+1))
^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.01, size = 52, normalized size = 0.58

$$\frac{1}{7}\left(-x^2+1\right)^{\frac{7}{2}}+\frac{1}{6}\left(-x^2+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{5}{16}\sqrt{-x^2+1}\,x+\frac{5}{16}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/7*(-x^2 + 1)^(7/2) + 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5
/16*sqrt(-x^2 + 1)*x + 5/16*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{7/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)*(x + 1)^(5/2), x)
```

sympy [A] time = 53.58, size = 321, normalized size = 3.57

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}-\frac{i(x+1)^{\frac{15}{2}}}{7\sqrt{x-1}}+\frac{55i(x+1)^{\frac{13}{2}}}{42\sqrt{x-1}}-\frac{193i(x+1)^{\frac{11}{2}}}{42\sqrt{x-1}}+\frac{1237i(x+1)^{\frac{9}{2}}}{168\sqrt{x-1}}-\frac{769i(x+1)^{\frac{7}{2}}}{168\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8}+\frac{(x+1)^{\frac{15}{2}}}{7\sqrt{1-x}}-\frac{55(x+1)^{\frac{13}{2}}}{42\sqrt{1-x}}+\frac{193(x+1)^{\frac{11}{2}}}{42\sqrt{1-x}}-\frac{1237(x+1)^{\frac{9}{2}}}{168\sqrt{1-x}}+\frac{769(x+1)^{\frac{7}{2}}}{168\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}}-\frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 - I*(x + 1)**(15/2)/(7*sqrt(
x - 1)) + 55*I*(x + 1)**(13/2)/(42*sqrt(x - 1)) - 193*I*(x + 1)**(11/2)/(42
*sqrt(x - 1)) + 1237*I*(x + 1)**(9/2)/(168*sqrt(x - 1)) - 769*I*(x + 1)**(7
/2)/(168*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqrt(x - 1)) - 5*I*(x + 1)**(3
/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x - 1)), Abs(x + 1)/2 > 1),
(5*asin(sqrt(2)*sqrt(x + 1)/2)/8 + (x + 1)**(15/2)/(7*sqrt(1 - x)) - 55*(x
+ 1)**(13/2)/(42*sqrt(1 - x)) + 193*(x + 1)**(11/2)/(42*sqrt(1 - x)) - 1237
*(x + 1)**(9/2)/(168*sqrt(1 - x)) + 769*(x + 1)**(7/2)/(168*sqrt(1 - x)) +
(x + 1)**(5/2)/(48*sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqr
t(x + 1)/(8*sqrt(1 - x)), True))
```

3.1022 $\int (1-x)^{5/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {38, 41, 216}

$$\frac{1}{6}(1-x)^{5/2}x(x+1)^{5/2}+\frac{5}{24}(1-x)^{3/2}x(x+1)^{3/2}+\frac{5}{16}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{16}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/16 + (5*(1 - x)^(3/2)*x*(1 + x)^(3/2))/24 + (
(1 - x)^(5/2)*x*(1 + x)^(5/2))/6 + (5*ArcSin[x])/16
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{5/2}(1+x)^{5/2}\,dx &= \frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{6}\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{8}\int \sqrt{1-x}\sqrt{1+x}\,dx\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\int \frac{1}{\sqrt{1-x}\,\sqrt{}}\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\int \frac{1}{\sqrt{1-x^2}}\\
&= \frac{5}{16}\sqrt{1-x}\,x\sqrt{1+x}+\frac{5}{24}(1-x)^{3/2}x(1+x)^{3/2}+\frac{1}{6}(1-x)^{5/2}x(1+x)^{5/2}+\frac{5}{16}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 0.49

$$\frac{1}{48}\left(x\sqrt{1-x^2}\left(8x^4-26x^2+33\right)+15\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] (x*Sqrt[1 - x^2]*(33 - 26*x^2 + 8*x^4) + 15*ArcSin[x])/48
```

IntegrateAlgebraic [B] time = 0.12, size = 151, normalized size = 2.16

$$\frac{-\frac{15(1-x)^{11/2}}{(x+1)^{11/2}}-\frac{85(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{198(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{198(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{85(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{24\left(\frac{1-x}{x+1}+1\right)^{6}}-\frac{5}{8}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-15*(1 - x)^(11/2))/(1 + x)^(11/2) - (85*(1 - x)^(9/2))/(1 + x)^(9/2) - (
198*(1 - x)^(7/2))/(1 + x)^(7/2) + (198*(1 - x)^(5/2))/(1 + x)^(5/2) + (85*
(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqrt[1 - x])/Sqrt[1 + x])/(24*(1 + (1 -
x)/(1 + x))^6) - (5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]])/8
```

fricas [A] time = 1.29, size = 51, normalized size = 0.73

$$\frac{1}{48}\left(8\,x^{5}-26\,x^{3}+33\,x\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{8}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 26*x^3 + 33*x)*sqrt(x + 1)*sqrt(-x + 1) - 5/8*arctan((sqrt(x
 + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.31, size = 185, normalized size = 2.64

$$\frac{1}{240}\,\left(\left(2\,\left(\left(4\,(5\,x-26)(x+1)+321\right)(x+1)-451\right)(x+1)+745\right)(x+1)-405\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{120}\,\left(\left(2\,\left(3\,(4\,x-17)(x+1)+133\right)(x+1)-295\right)(x+1)+195\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{12}\,\left(\left(2\,(3\,x-10)(x+1)+43\right)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{1}{3}\,\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\,\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{8}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*((2*((4*(5*x - 26)*(x + 1) + 321)*(x + 1) - 451)*(x + 1) + 745)*(x +
1) - 405)*sqrt(x + 1)*sqrt(-x + 1) + 1/120*((2*(3*(4*x - 17)*(x + 1) + 133)
*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 1
0)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) - 1/3*((2*x - 5)*(x
 + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)
+ sqrt(x + 1)*sqrt(-x + 1) + 5/8*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.00, size = 113, normalized size = 1.61

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{16\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}\,(x+1)^{\frac{7}{2}}}{6}+\frac{(-x+1)^{\frac{3}{2}}\,(x+1)^{\frac{7}{2}}}{6}+\frac{\sqrt{-x+1}\,\,(x+1)^{\frac{7}{2}}}{8}-\frac{\sqrt{-x+1}\,\,(x+1)^{\frac{5}{2}}}{24}-\frac{5\sqrt{-x+1}\,\,(x+1)^{\frac{3}{2}}}{48}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)*(x+1)^(5/2),x)
```

```
[Out] 1/6*(-x+1)^(5/2)*(x+1)^(7/2)+1/6*(-x+1)^(3/2)*(x+1)^(7/2)+1/8*(-x+1)^(1/2)*
(x+1)^(7/2)-1/24*(-x+1)^(1/2)*(x+1)^(5/2)-5/48*(-x+1)^(1/2)*(x+1)^(3/2)-5/1
6*(-x+1)^(1/2)*(x+1)^(1/2)+5/16*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/
2)*arcsin(x)
```

maxima [A] time = 3.10, size = 41, normalized size = 0.59

$$\frac{1}{6}\left(-x^{2}+1\right)^{\frac{5}{2}}x+\frac{5}{24}\left(-x^{2}+1\right)^{\frac{3}{2}}x+\frac{5}{16}\,\sqrt{-x^{2}+1}\,x+\frac{5}{16}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-x^2 + 1)^(5/2)*x + 5/24*(-x^2 + 1)^(3/2)*x + 5/16*sqrt(-x^2 + 1)*x +
5/16*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{5/2} (x+1)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)*(x + 1)^(5/2), x)
```

sympy [B] time = 25.76, size = 286, normalized size = 4.09

$$\begin{cases} -\frac{5i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} + \frac{i(x+1)^{\frac{13}{2}}}{6\sqrt{x-1}} - \frac{7i(x+1)^{\frac{11}{2}}}{6\sqrt{x-1}} + \frac{67i(x+1)^{\frac{9}{2}}}{24\sqrt{x-1}} - \frac{55i(x+1)^{\frac{7}{2}}}{24\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{48\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{48\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{8\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ \frac{5 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{8} - \frac{(x+1)^{\frac{13}{2}}}{6\sqrt{1-x}} + \frac{7(x+1)^{\frac{11}{2}}}{6\sqrt{1-x}} - \frac{67(x+1)^{\frac{9}{2}}}{24\sqrt{1-x}} + \frac{55(x+1)^{\frac{7}{2}}}{24\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{48\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{48\sqrt{1-x}} - \frac{5\sqrt{x+1}}{8\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/8 + I*(x + 1)**(13/2)/(6*sqrt(
x - 1)) - 7*I*(x + 1)**(11/2)/(6*sqrt(x - 1)) + 67*I*(x + 1)**(9/2)/(24*sqr
t(x - 1)) - 55*I*(x + 1)**(7/2)/(24*sqrt(x - 1)) - I*(x + 1)**(5/2)/(48*sqr
t(x - 1)) - 5*I*(x + 1)**(3/2)/(48*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(8*sqrt(x
 - 1)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/8 - (x + 1)**(13/
2)/(6*sqrt(1 - x)) + 7*(x + 1)**(11/2)/(6*sqrt(1 - x)) - 67*(x + 1)**(9/2)/
(24*sqrt(1 - x)) + 55*(x + 1)**(7/2)/(24*sqrt(1 - x)) + (x + 1)**(5/2)/(48*
sqrt(1 - x)) + 5*(x + 1)**(3/2)/(48*sqrt(1 - x)) - 5*sqrt(x + 1)/(8*sqrt(1
- x)), True))
```

3.1023 $\int (1-x)^{3/2}(1+x)^{5/2}\,dx$

Optimal. Leaf size=69

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{5}(1-x)^{5/2}(x+1)^{5/2}+\frac{1}{4}(1-x)^{3/2}x(x+1)^{3/2}+\frac{3}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{3}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (3*Sqrt[1 - x]*x*Sqrt[1 + x])/8 + ((1 - x)^(3/2)*x*(1 + x)^(3/2))/4 - ((1 - x)^(5/2)*(1 + x)^(5/2))/5 + (3*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (1-x)^{3/2}(1+x)^{5/2}\,dx &= -\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\int (1-x)^{3/2}(1+x)^{3/2}\,dx\\
&= \frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1}}\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\int \frac{1}{\sqrt{1-x^2}}\,d\\
&= \frac{3}{8}\sqrt{1-x}\,x\sqrt{1+x}+\frac{1}{4}(1-x)^{3/2}x(1+x)^{3/2}-\frac{1}{5}(1-x)^{5/2}(1+x)^{5/2}+\frac{3}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 55, normalized size = 0.80

$$\frac{1}{40}\left(\sqrt{1-x^2}\left(-8x^4-10x^3+16x^2+25x-8\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-8 + 25*x + 16*x^2 - 10*x^3 - 8*x^4) - 30*ArcSin[Sqrt[1 - x
]/Sqrt[2]])/40
```

IntegrateAlgebraic [A] time = 0.10, size = 133, normalized size = 1.93

$$\frac{-\frac{15(1-x)^{9/2}}{(x+1)^{9/2}}-\frac{70(1-x)^{7/2}}{(x+1)^{7/2}}-\frac{128(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{70(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{15\sqrt{1-x}}{\sqrt{x+1}}}{20\left(\frac{1-x}{x+1}+1\right)^5}-\frac{3}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)*(1 + x)^(5/2),x]
```

```
[Out] ((-15*(1 - x)^(9/2))/(1 + x)^(9/2) - (70*(1 - x)^(7/2))/(1 + x)^(7/2) - (12
8*(1 - x)^(5/2))/(1 + x)^(5/2) + (70*(1 - x)^(3/2))/(1 + x)^(3/2) + (15*Sqr
t[1 - x])/Sqrt[1 + x])/(20*(1 + (1 - x)/(1 + x))^5) - (3*ArcTan[Sqrt[1 - x]
/Sqrt[1 + x]])/4
```

fricas [A] time = 1.06, size = 57, normalized size = 0.83

$$-\frac{1}{40}\left(8x^4+10x^3-16x^2-25x+8\right)\sqrt{x+1}\sqrt{-x+1}-\frac{3}{4}\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/40*(8*x^4 + 10*x^3 - 16*x^2 - 25*x + 8)*sqrt(x + 1)*sqrt(-x + 1) - 3/4*a
rctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.18, size = 114, normalized size = 1.65

$$-\frac{1}{120}\left((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195\right)\sqrt{x+1}\sqrt{-x+1}-\frac{1}{12}\left((2(3x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{-x+1}+\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+\frac{3}{4}\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/120*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(
x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(
x + 1)*sqrt(-x + 1) + sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-
x + 1) + 3/4*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 99, normalized size = 1.43

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}(x+1)^{\frac{7}{2}}}{5}+\frac{3\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{20}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{20}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)*(x+1)^(5/2),x)
```

```
[Out] 1/5*(-x+1)^(3/2)*(x+1)^(7/2)+3/20*(-x+1)^(1/2)*(x+1)^(7/2)-1/20*(-x+1)^(1/2
)*(x+1)^(5/2)-1/8*(-x+1)^(1/2)*(x+1)^(3/2)-3/8*(-x+1)^(1/2)*(x+1)^(1/2)+3/8
*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.05, size = 40, normalized size = 0.58

$$-\frac{1}{5}\left(-x^2+1\right)^{\frac{5}{2}}+\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{-x^2+1}\,x+\frac{3}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2) + 1/4*(-x^2 + 1)^(3/2)*x + 3/8*sqrt(-x^2 + 1)*x + 3/8
*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (1-x)^{3/2}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)*(x + 1)^(5/2), x)
```

sympy [B] time = 16.53, size = 246, normalized size = 3.57

$$\begin{cases} -\frac{3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{11}{2}}}{5\sqrt{x-1}}+\frac{19i(x+1)^{\frac{9}{2}}}{20\sqrt{x-1}}-\frac{23i(x+1)^{\frac{7}{2}}}{20\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{40\sqrt{x-1}}-\frac{i(x+1)^{\frac{3}{2}}}{8\sqrt{x-1}}+\frac{3i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{(x+1)^{\frac{11}{2}}}{5\sqrt{1-x}}-\frac{19(x+1)^{\frac{9}{2}}}{20\sqrt{1-x}}+\frac{23(x+1)^{\frac{7}{2}}}{20\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{40\sqrt{1-x}}+\frac{(x+1)^{\frac{3}{2}}}{8\sqrt{1-x}}-\frac{3\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(11/2)/(5*sqrt(
x - 1)) + 19*I*(x + 1)**(9/2)/(20*sqrt(x - 1)) - 23*I*(x + 1)**(7/2)/(20*sq
rt(x - 1)) - I*(x + 1)**(5/2)/(40*sqrt(x - 1)) - I*(x + 1)**(3/2)/(8*sqrt(x
 - 1)) + 3*I*sqrt(x + 1)/(4*sqrt(x - 1)), Abs(x + 1)/2 > 1), (3*asin(sqrt(2
)*sqrt(x + 1)/2)/4 + (x + 1)**(11/2)/(5*sqrt(1 - x)) - 19*(x + 1)**(9/2)/(2
0*sqrt(1 - x)) + 23*(x + 1)**(7/2)/(20*sqrt(1 - x)) + (x + 1)**(5/2)/(40*sq
rt(1 - x)) + (x + 1)**(3/2)/(8*sqrt(1 - x)) - 3*sqrt(x + 1)/(4*sqrt(1 - x))
, True))
```

3.1024 $\int \sqrt{1-x}\,(1+x)^{5/2}\,dx$

Optimal. Leaf size=68

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {49, 38, 41, 216}

$$-\frac{1}{4}(1-x)^{3/2}(x+1)^{5/2}-\frac{5}{12}(1-x)^{3/2}(x+1)^{3/2}+\frac{5}{8}\sqrt{1-x}\,x\sqrt{x+1}+\frac{5}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[1 - x]*x*Sqrt[1 + x])/8 - (5*(1 - x)^(3/2)*(1 + x)^(3/2))/12 - ((1
 - x)^(3/2)*(1 + x)^(5/2))/4 + (5*ArcSin[x])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 49

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(2*c*n)/(m + n + 1
), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && E
qQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0] && IGtQ[n + 1/2, 0] && LtQ[m, n]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1-x}\,(1+x)^{5/2}\,dx &= -\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,(1+x)^{3/2}\,dx\\
&= -\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{4}\int \sqrt{1-x}\,\sqrt{1+x}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{8}\sqrt{1-x}\,x\sqrt{1+x}-\frac{5}{12}(1-x)^{3/2}(1+x)^{3/2}-\frac{1}{4}(1-x)^{3/2}(1+x)^{5/2}+\frac{5}{8}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 0.74

$$\frac{1}{24}\left(\sqrt{1-x^2}\left(6x^3+16x^2+9x-16\right)-30\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x^2]*(-16 + 9*x + 16*x^2 + 6*x^3) - 30*ArcSin[Sqrt[1 - x]/Sqrt[2]
])/24
```

IntegrateAlgebraic [A] time = 0.08, size = 100, normalized size = 1.47

$$-\frac{\sqrt{1-x}\left(\frac{15(1-x)^3}{(x+1)^3}+\frac{55(1-x)^2}{(x+1)^2}+\frac{73(1-x)}{x+1}-15\right)}{12\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^4}-\frac{5}{4}\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]*(1 + x)^(5/2),x]
```

```
[Out] -1/12*(Sqrt[1 - x]*(-15 + (15*(1 - x)^3)/(1 + x)^3 + (55*(1 - x)^2)/(1 + x)
^2 + (73*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^4) - (5*ArcT
an[Sqrt[1 - x]/Sqrt[1 + x]])/4
```

fricas [A] time = 1.13, size = 52, normalized size = 0.76

$$\frac{1}{24}\left(6\,x^3+16\,x^2+9\,x-16\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{5}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(6*x^3 + 16*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(-x + 1) - 5/4*arctan((sqr
t(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 1.05, size = 101, normalized size = 1.49

$$\frac{1}{24}\left(\left(2\left(3\,x-10\right)\left(x+1\right)+43\right)\left(x+1\right)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\left(\left(2\,x-5\right)\left(x+1\right)+9\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{3}{2}\,\sqrt{x+1}\left(x-2\right)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{5}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 3/2*sqrt(x + 1)*(x -
 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5/4*arcsin(1/2*sqrt(2)*sqrt(x
 + 1))
```

maple [A] time = 0.00, size = 85, normalized size = 1.25

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{\sqrt{-x+1}\,(x+1)^{\frac{7}{2}}}{4}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{12}-\frac{5\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{24}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)*(x+1)^(5/2),x)
```

```
[Out] 1/4*(-x+1)^(1/2)*(x+1)^(7/2)-1/12*(-x+1)^(1/2)*(x+1)^(5/2)-5/24*(-x+1)^(1/2
)*(x+1)^(3/2)-5/8*(-x+1)^(1/2)*(x+1)^(1/2)+5/8*((x+1)*(-x+1))^(1/2)/(x+1)^(
1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.06, size = 40, normalized size = 0.59

$$-\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x-\frac{2}{3}\left(-x^2+1\right)^{\frac{3}{2}}+\frac{5}{8}\sqrt{-x^2+1}\,x+\frac{5}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(3/2)*x - 2/3*(-x^2 + 1)^(3/2) + 5/8*sqrt(-x^2 + 1)*x + 5/8
*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1-x}\,(x+1)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)*(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(1/2)*(x + 1)^(5/2), x)
```

sympy [A] time = 9.89, size = 214, normalized size = 3.15

$$\begin{cases} -\frac{5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}+\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}-\frac{7i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}-\frac{5i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}+\frac{5i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{1-x}}+\frac{7(x+1)^{\frac{7}{2}}}{12\sqrt{1-x}}+\frac{(x+1)^{\frac{5}{2}}}{24\sqrt{1-x}}+\frac{5(x+1)^{\frac{3}{2}}}{24\sqrt{1-x}}-\frac{5\sqrt{x+1}}{4\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 + I*(x + 1)**(9/2)/(4*sqrt(x
 - 1)) - 7*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - I*(x + 1)**(5/2)/(24*sqrt(x
- 1)) - 5*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) + 5*I*sqrt(x + 1)/(4*sqrt(x - 1
)), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x + 1)/2)/4 - (x + 1)**(9/2)/(4
*sqrt(1 - x)) + 7*(x + 1)**(7/2)/(12*sqrt(1 - x)) + (x + 1)**(5/2)/(24*sqrt
(1 - x)) + 5*(x + 1)**(3/2)/(24*sqrt(1 - x)) - 5*sqrt(x + 1)/(4*sqrt(1 - x)
), True))
```

3.1025 $\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$-\frac{1}{3}\sqrt{1-x}\,(x+1)^{5/2}-\frac{5}{6}\sqrt{1-x}\,(x+1)^{3/2}-\frac{5}{2}\sqrt{1-x}\,\sqrt{x+1}\,+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/Sqrt[1 - x],x]
```

```
[Out] (-5*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*Sqrt[1 - x]*(1 + x)^(3/2))/6 - (Sqrt[1
- x]*(1 + x)^(5/2))/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{\sqrt{1-x}}\,dx &= -\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{3}\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}-\frac{5}{6}\sqrt{1-x}\,(1+x)^{3/2}-\frac{1}{3}\sqrt{1-x}\,(1+x)^{5/2}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.66

$$-\frac{1}{6}\sqrt{1-x^2}\,\left(2x^2+9x+22\right)-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/Sqrt[1 - x],x]
[Out] -1/6*(Sqrt[1 - x^2]*(22 + 9*x + 2*x^2)) - 5*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 84, normalized size = 1.25

$$-\frac{\sqrt{1-x}\left(\frac{15(1-x)^2}{(x+1)^2}+\frac{40(1-x)}{x+1}+33\right)}{3\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^3}-5\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/Sqrt[1 - x],x]
[Out] -1/3*(Sqrt[1 - x]*(33 + (15*(1 - x)^2)/(1 + x)^2 + (40*(1 - x))/(1 + x)))/(
Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^3) - 5*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.50, size = 47, normalized size = 0.70

$$-\frac{1}{6}\left(2x^2+9x+22\right)\sqrt{x+1}\sqrt{-x+1}-5\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="fricas")
[Out] -1/6*(2*x^2 + 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sq
rt(-x + 1) - 1)/x)
```

giac [A] time = 1.05, size = 39, normalized size = 0.58

$$-\frac{1}{6}\left((2x+7)(x+1)+15\right)\sqrt{x+1}\sqrt{-x+1}+5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="giac")
[Out] -1/6*((2*x + 7)*(x + 1) + 15)*sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(
2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 71, normalized size = 1.06

$$\frac{5\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}-\frac{\sqrt{-x+1}\,(x+1)^{\frac{5}{2}}}{3}-\frac{5\sqrt{-x+1}\,(x+1)^{\frac{3}{2}}}{6}-\frac{5\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(1/2),x)
[Out] -1/3*(-x+1)^(1/2)*(x+1)^(5/2)-5/6*(-x+1)^(1/2)*(x+1)^(3/2)-5/2*(-x+1)^(1/2)
*(x+1)^(1/2)+5/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.93, size = 42, normalized size = 0.63

$$-\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x-\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x - 11/3*sqrt(-x^2 + 1) + 5/2*
arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x+1)^{5/2}}{\sqrt{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(1/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(1/2), x)
```

sympy [A] time = 7.50, size = 172, normalized size = 2.57

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}} - \frac{i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}} - \frac{5i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}} + \frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}} + \frac{(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}} + \frac{5(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}} - \frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 5*I*(x + 1)**(3/2)/(6*sqrt(x - 1)
) + 5*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sqrt(x
+ 1)/2) + (x + 1)**(7/2)/(3*sqrt(1 - x)) + (x + 1)**(5/2)/(6*sqrt(1 - x)) +
 5*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 5*sqrt(x + 1)/sqrt(1 - x), True))
```

3.1026 $\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{\sqrt{1-x}}+\frac{5}{2}\sqrt{1-x}\,(x+1)^{3/2}+\frac{15}{2}\sqrt{1-x}\,\sqrt{x+1}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(3/2),x]

[Out] (15*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*Sqrt[1 - x]*(1 + x)^(3/2))/2 + (2*(1 +
x)^(5/2))/Sqrt[1 - x] - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{3/2}}\,dx &= \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - 5\int \frac{(1+x)^{3/2}}{\sqrt{1-x}}\,dx \\ &= \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx \\ &= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{15}{2}\sqrt{1-x}\,\sqrt{1+x} + \frac{5}{2}\sqrt{1-x}\,(1+x)^{3/2} + \frac{2(1+x)^{5/2}}{\sqrt{1-x}} - \frac{15}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.54

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{1-x}{2}\right)}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(3/2),x]
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -1/2, 1/2, (1 - x)/2])/Sqrt[1 - x]
```

IntegrateAlgebraic [A] time = 0.08, size = 81, normalized size = 1.25

$$\frac{\sqrt{x+1}\left(\frac{15(1-x)^2}{(x+1)^2}+\frac{25(1-x)}{x+1}+8\right)}{\sqrt{1-x}\left(\frac{1-x}{x+1}+1\right)^2}+15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(3/2),x]
[Out] (Sqrt[1 + x]*(8 + (15*(1 - x)^2)/(1 + x)^2 + (25*(1 - x))/(1 + x)))/(Sqrt[1
 - x]*(1 + (1 - x)/(1 + x))^2) + 15*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.11, size = 58, normalized size = 0.89

$$\frac{\left(x^2+7\,x-24\right)\sqrt{x+1}\,\sqrt{-x+1}+30\,(x-1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+24\,x-24}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="fricas")
[Out] 1/2*((x^2 + 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x - 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) + 24*x - 24)/(x - 1)
```

giac [A] time = 1.01, size = 42, normalized size = 0.65

$$\frac{((x+6)(x+1)-30)\sqrt{x+1}\,\sqrt{-x+1}}{2\,(x-1)} - 15\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((x + 6)*(x + 1) - 30)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 15*arcsin(1/2
*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 77, normalized size = 1.18

$$-\frac{15\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}-\frac{\left(x^{3}+8x^{2}-17x-24\right)\sqrt{(x+1)(-x+1)}}{2\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(3/2),x)
```

```
[Out] -1/2*(x^3+8*x^2-17*x-24)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(
1/2)/(x+1)^(1/2)-15/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

maxima [A] time = 2.97, size = 56, normalized size = 0.86

$$-\frac{x^{3}}{2\sqrt{-x^{2}+1}}-\frac{4x^{2}}{\sqrt{-x^{2}+1}}+\frac{17x}{2\sqrt{-x^{2}+1}}+\frac{12}{\sqrt{-x^{2}+1}}-\frac{15}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) - 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) + 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(3/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(3/2), x)
```

sympy [A] time = 7.76, size = 139, normalized size = 2.14

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{5i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{15i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{5(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{15\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 5*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 15*I*sqrt(x + 1)/sqrt(x - 1), A
bs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x + 1)/2) - (x + 1)**(5/2)/(2*sqrt
(1 - x)) - 5*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 15*sqrt(x + 1)/sqrt(1 - x), T
rue))
```

3.1027 $\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}} dx$

Optimal. Leaf size=63

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}} - \frac{10(x+1)^{3/2}}{3\sqrt{1-x}} - 5\sqrt{1-x}\sqrt{x+1} + 5\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$\frac{2(x+1)^{5/2}}{3(1-x)^{3/2}} - \frac{10(x+1)^{3/2}}{3\sqrt{1-x}} - 5\sqrt{1-x}\sqrt{x+1} + 5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] -5*Sqrt[1 - x]*Sqrt[1 + x] - (10*(1 + x)^(3/2))/(3*Sqrt[1 - x]) + (2*(1 + x
)^(5/2))/(3*(1 - x)^(3/2)) + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{5/2}}\,dx &= \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} - \frac{5}{3}\int \frac{(1+x)^{3/2}}{(1-x)^{3/2}}\,dx\\ &= -\frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx\\ &= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\ &= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -5\sqrt{1-x}\,\sqrt{1+x} - \frac{10(1+x)^{3/2}}{3\sqrt{1-x}} + \frac{2(1+x)^{5/2}}{3(1-x)^{3/2}} + 5\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{1-x}{2}\right)}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -3/2, -1/2, (1 - x)/2])/(3*(1 - x)^(3/2)
)
```

IntegrateAlgebraic [C] time = 0.18, size = 73, normalized size = 1.16

$$\frac{\sqrt{1-x}\,\left(-3(x+1)^{5/2}+40(x+1)^{3/2}-60\sqrt{x+1}\right)}{3(x-1)^2} + 10i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-60*Sqrt[1 + x] + 40*(1 + x)^(3/2) - 3*(1 + x)^(5/2)))/(3*(-1
 + x)^2) + (10*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

fricas [A] time = 0.71, size = 75, normalized size = 1.19

$$-\frac{23\,x^2+\left(3\,x^2-34\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}+30\left(x^2-2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)-46\,x+23}{3\left(x^2-2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(23*x^2 + (3*x^2 - 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x^2 - 2*x
 + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 46*x + 23)/(x^2 - 2*x + 1)
```

giac [A] time = 0.98, size = 44, normalized size = 0.70

$$-\frac{((3\,x-37)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{3\,(x-1)^2} + 10\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((3*x - 37)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 + 10*arcs
in(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 84, normalized size = 1.33

$$\frac{5\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}+\frac{\left(3x^3-31x^2-11x+23\right)\sqrt{(x+1)(-x+1)}}{3(x-1)\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(5/2),x)
```

```
[Out] 1/3*(3*x^3-31*x^2-11*x+23)/(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/
(-x+1)^(1/2)/(x+1)^(1/2)+5*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*ar
csin(x)
```

maxima [B] time = 2.97, size = 99, normalized size = 1.57

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{x^4-4x^3+6x^2-4x+1}-\frac{5\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3x^2+3x-1\right)}+\frac{10\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}+\frac{35\sqrt{-x^2+1}}{3(x-1)}+5\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -(-x^2 + 1)^(5/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x
^3 - 3*x^2 + 3*x - 1) + 10/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 35/3*sqrt(-x^
2 + 1)/(x - 1) + 5*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(5/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(5/2), x)
```

sympy [B] time = 7.47, size = 576, normalized size = 9.14

$$\begin{cases} \frac{30i\sqrt{x-1}(x+1)^{\frac{27}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{15\pi\sqrt{x-1}(x+1)^{\frac{27}{2}}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{60i\sqrt{x-1}(x+1)^{\frac{25}{2}}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{30\pi\sqrt{x-1}(x+1)^{\frac{25}{2}}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{3i(x+1)^{15}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}-\frac{40i(x+1)^{14}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}}+\frac{60i(x+1)^{13}}{-3\sqrt{x-1}(x+1)^{\frac{27}{2}}+6\sqrt{x-1}(x+1)^{\frac{25}{2}}} & \text{for}\ \frac{|x+1|}{2}>1 \\ \frac{30\sqrt{1-x}(x+1)^{\frac{27}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{60\sqrt{1-x}(x+1)^{\frac{25}{2}}\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{3(x+1)^{15}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}-\frac{40(x+1)^{14}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}}+\frac{60(x+1)^{13}}{3\sqrt{1-x}(x+1)^{\frac{27}{2}}-6\sqrt{1-x}(x+1)^{\frac{25}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(5/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(27/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(-
3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) - 15*pi*sqrt
(x - 1)*(x + 1)**(27/2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x
+ 1)**(25/2)) - 60*I*sqrt(x - 1)*(x + 1)**(25/2)*acosh(sqrt(2)*sqrt(x + 1)/
2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) + 30*pi
*sqrt(x - 1)*(x + 1)**(25/2)/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sqrt(x - 1
)*(x + 1)**(25/2)) + 3*I*(x + 1)**15/(-3*sqrt(x - 1)*(x + 1)**(27/2) + 6*sq
rt(x - 1)*(x + 1)**(25/2)) - 40*I*(x + 1)**14/(-3*sqrt(x - 1)*(x + 1)**(27/
2) + 6*sqrt(x - 1)*(x + 1)**(25/2)) + 60*I*(x + 1)**13/(-3*sqrt(x - 1)*(x +
 1)**(27/2) + 6*sqrt(x - 1)*(x + 1)**(25/2)), Abs(x + 1)/2 > 1), (30*sqrt(1
 - x)*(x + 1)**(27/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(
```

```
27/2) - 6*sqrt(1 - x)*(x + 1)**(25/2)) - 60*sqrt(1 - x)*(x + 1)**(25/2)*asi
n(sqrt(2)*sqrt(x + 1)/2)/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x)*(x
+ 1)**(25/2)) + 3*(x + 1)**15/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt(1 - x
)*(x + 1)**(25/2)) - 40*(x + 1)**14/(3*sqrt(1 - x)*(x + 1)**(27/2) - 6*sqrt
(1 - x)*(x + 1)**(25/2)) + 60*(x + 1)**13/(3*sqrt(1 - x)*(x + 1)**(27/2) -
6*sqrt(1 - x)*(x + 1)**(25/2)), True))
```

3.1028 $\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$\frac{2(x+1)^{5/2}}{5(1-x)^{5/2}}-\frac{2(x+1)^{3/2}}{3(1-x)^{3/2}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*Sqrt[1 + x])/Sqrt[1 - x] - (2*(1 + x)^(3/2))/(3*(1 - x)^(3/2)) + (2*(1 +
 x)^(5/2))/(5*(1 - x)^(5/2)) - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{7/2}}\,dx &= \frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\int \frac{(1+x)^{3/2}}{(1-x)^{5/2}}\,dx\\
&= -\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}+\int \frac{\sqrt{1+x}}{(1-x)^{3/2}}\,dx\\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}}-\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx\\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}}-\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{2\sqrt{1+x}}{\sqrt{1-x}}-\frac{2(1+x)^{3/2}}{3(1-x)^{3/2}}+\frac{2(1+x)^{5/2}}{5(1-x)^{5/2}}-\sin^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$\frac{8\sqrt{2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{1-x}{2}\right)}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (8*Sqrt[2]*Hypergeometric2F1[-5/2, -5/2, -3/2, (1 - x)/2])/(5*(1 - x)^(5/2)
)
```

IntegrateAlgebraic [A] time = 0.07, size = 69, normalized size = 1.10

$$\frac{2\left(\frac{15(1-x)^2}{(x+1)^2}-\frac{5(1-x)}{x+1}+3\right)(x+1)^{5/2}}{15(1-x)^{5/2}}+2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(7/2),x]
```

```
[Out] (2*(1 + x)^(5/2)*(3 + (15*(1 - x)^2)/(1 + x)^2 - (5*(1 - x))/(1 + x)))/(15*
(1 - x)^(5/2)) + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.18, size = 91, normalized size = 1.44

$$\frac{2\left(13\,x^3-39\,x^2-\left(23\,x^2-24\,x+13\right)\sqrt{x+1}\,\sqrt{-x+1}+15\left(x^3-3\,x^2+3\,x-1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+39\,x-13\right)}{15\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(13*x^3 - 39*x^2 - (23*x^2 - 24*x + 13)*sqrt(x + 1)*sqrt(-x + 1) + 15*
(x^3 - 3*x^2 + 3*x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 39*x - 1
3)/(x^3 - 3*x^2 + 3*x - 1)
```

giac [A] time = 0.98, size = 44, normalized size = 0.70

$$-\frac{2\,((23\,x-47)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{15\,(x-1)^3}-2\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="giac")
```

```
[Out] -2/15*((23*x - 47)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3 - 2*arc
sin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 84, normalized size = 1.33

$$-\frac{\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{\sqrt{x+1}\,\sqrt{-x+1}}+\frac{2\left(23x^3-x^2-11x+13\right)\sqrt{(x+1)\,(-x+1)}}{15\,(x-1)^2\,\sqrt{-(x+1)\,(x-1)}\,\sqrt{-x+1}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(7/2),x)
```

```
[Out] 2/15*(23*x^3-x^2-11*x+13)/(x-1)^2/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)
/(-x+1)^(1/2)/(x+1)^(1/2)-((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arc
sin(x)
```

maxima [B] time = 3.07, size = 160, normalized size = 2.54

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{5\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^4-4x^3+6x^2-4x+1}+\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3-3x^2+3x-1\right)}+\frac{6\sqrt{-x^2+1}}{5\left(x^3-3x^2+3x-1\right)}-\frac{7\sqrt{-x^2+1}}{15\left(x^2-2x+1\right)}-\frac{38\sqrt{-x^2+1}}{15\left(x-1\right)}-\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + (-x^2 + 1
)^(3/2)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/3*(-x^2 + 1)^(3/2)/(x^3 - 3*x^2
 + 3*x - 1) + 6/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 7/15*sqrt(-x^2 +
 1)/(x^2 - 2*x + 1) - 38/15*sqrt(-x^2 + 1)/(x - 1) - arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{5/2}}{(1-x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(7/2),x)
```

```
[Out] int((x + 1)^(5/2)/(1 - x)^(7/2), x)
```

sympy [B] time = 11.21, size = 1608, normalized size = 25.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(7/2),x)
```

```
[Out] Piecewise((30*I*sqrt(x - 1)*(x + 1)**(35/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(1
5*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x
 - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 15*pi*sqrt(x - 1
)*(x + 1)**(35/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)*
*(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)
) - 180*I*sqrt(x - 1)*(x + 1)**(33/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt
(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*
(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 90*pi*sqrt(x - 1)*(x +
 1)**(33/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2
) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 36
0*I*sqrt(x - 1)*(x + 1)**(31/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1
)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1
)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 180*pi*sqrt(x - 1)*(x + 1)**
(31/2)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 1
80*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 240*I*s
qrt(x - 1)*(x + 1)**(29/2)*acosh(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(x - 1)*(x
+ 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sqrt(x - 1)*(x + 1)**(3
1/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 120*pi*sqrt(x - 1)*(x + 1)**(29/2
)/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180*sq
rt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 46*I*(x + 1)
**18/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) + 180
*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 232*I*(x
+ 1)**17/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/2) +
 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) - 400*I
*(x + 1)**16/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**(33/
2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2)) + 2
40*I*(x + 1)**15/(15*sqrt(x - 1)*(x + 1)**(35/2) - 90*sqrt(x - 1)*(x + 1)**
(33/2) + 180*sqrt(x - 1)*(x + 1)**(31/2) - 120*sqrt(x - 1)*(x + 1)**(29/2))
, Abs(x + 1)/2 > 1), (-30*sqrt(1 - x)*(x + 1)**(35/2)*asin(sqrt(2)*sqrt(x +
 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 1
```

```
80*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 180*sqr
t(1 - x)*(x + 1)**(33/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1
)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2
) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 360*sqrt(1 - x)*(x + 1)**(31/2)*asin
(sqrt(2)*sqrt(x + 1)/2)/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x
 + 1)**(33/2) + 180*sqrt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**
(29/2)) + 240*sqrt(1 - x)*(x + 1)**(29/2)*asin(sqrt(2)*sqrt(x + 1)/2)/(15*s
qrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 46*(x + 1)**18/(15*
sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1 -
 x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 232*(x + 1)**17/(1
5*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt(1
 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) + 400*(x + 1)**16/
(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sqrt
(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)) - 240*(x + 1)**1
5/(15*sqrt(1 - x)*(x + 1)**(35/2) - 90*sqrt(1 - x)*(x + 1)**(33/2) + 180*sq
rt(1 - x)*(x + 1)**(31/2) - 120*sqrt(1 - x)*(x + 1)**(29/2)), True))
```

3.1029 $\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx$

Optimal. Leaf size=20

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}} dx = \frac{(1+x)^{7/2}}{7(1-x)^{7/2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 20, normalized size = 1.00

$$\frac{(x+1)^{7/2}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(9/2),x]
```

```
[Out] (1 + x)^(7/2)/(7*(1 - x)^(7/2))
```

fricas [B] time = 1.30, size = 66, normalized size = 3.30

$$\frac{x^4 - 4x^3 + 6x^2 + \left(x^3 + 3x^2 + 3x + 1\right)\sqrt{x+1}\sqrt{-x+1} - 4x + 1}{7\left(x^4 - 4x^3 + 6x^2 - 4x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/7*(x^4 - 4*x^3 + 6*x^2 + (x^3 + 3*x^2 + 3*x + 1)*sqrt(x + 1)*sqrt(-x + 1)
 - 4*x + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

giac [A] time = 1.11, size = 19, normalized size = 0.95

$$\frac{(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{7\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/7*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^4
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$\frac{(x+1)^{\frac{7}{2}}}{7\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(9/2),x)
```

```
[Out] 1/7*(x+1)^(7/2)/(-x+1)^(7/2)
```

maxima [B] time = 1.36, size = 171, normalized size = 8.55

$$\frac{(-x^2+1)^{\frac{5}{2}}}{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}+\frac{5\,(-x^2+1)^{\frac{3}{2}}}{2\,(x^5-5x^4+10x^3-10x^2+5x-1)}+\frac{15\sqrt{-x^2+1}}{7\,(x^4-4x^3+6x^2-4x+1)}+\frac{3\sqrt{-x^2+1}}{14\,(x^3-3x^2+3x-1)}-\frac{\sqrt{-x^2+1}}{7\,(x^2-2x+1)}+\frac{\sqrt{-x^2+1}}{7\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(9/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/2*(
-x^2 + 1)^(3/2)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 15/7*sqrt(-x^2
+ 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 3/14*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3
*x - 1) - 1/7*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/7*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.28, size = 64, normalized size = 3.20

$$\frac{\sqrt{1-x}\,\left(\frac{3\,x\,\sqrt{x+1}}{7}+\frac{\sqrt{x+1}}{7}+\frac{3\,x^2\,\sqrt{x+1}}{7}+\frac{x^3\,\sqrt{x+1}}{7}\right)}{x^4-4\,x^3+6\,x^2-4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(9/2),x)
```

```
[Out] ((1 - x)^(1/2)*((3*x*(x + 1)^(1/2))/7 + (x + 1)^(1/2)/7 + (3*x^2*(x + 1)^(1
/2))/7 + (x^3*(x + 1)^(1/2))/7))/(6*x^2 - 4*x - 4*x^3 + x^4 + 1)
```

sympy [B] time = 19.49, size = 116, normalized size = 5.80

$$\begin{cases} \frac{i(x+1)^{\frac{7}{2}}}{7\sqrt{x-1}\,(x+1)^3-42\sqrt{x-1}\,(x+1)^2+84\sqrt{x-1}\,(x+1)-56\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{(x+1)^{\frac{7}{2}}}{7\sqrt{1-x}\,(x+1)^3-42\sqrt{1-x}\,(x+1)^2+84\sqrt{1-x}\,(x+1)-56\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(9/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(7/2)/(7*sqrt(x - 1)*(x + 1)**3 - 42*sqrt(x - 1)*(x +
 1)**2 + 84*sqrt(x - 1)*(x + 1) - 56*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x
+ 1)**(7/2)/(7*sqrt(1 - x)*(x + 1)**3 - 42*sqrt(1 - x)*(x + 1)**2 + 84*sqrt
(1 - x)*(x + 1) - 56*sqrt(1 - x)), True))
```

3.1030 $\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx$

Optimal. Leaf size=41

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{(x+1)^{7/2}}{63(1-x)^{7/2}}+\frac{(x+1)^{7/2}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] (1 + x)^(7/2)/(9*(1 - x)^(9/2)) + (1 + x)^(7/2)/(63*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{1}{9}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{9(1-x)^{9/2}}+\frac{(1+x)^{7/2}}{63(1-x)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-8)(x+1)^{7/2}}{63(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(11/2),x]
[Out] -1/63*((-8 + x)*(1 + x)^(7/2))/(1 - x)^(9/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 34, normalized size = 0.83

$$\frac{(x+1)^{9/2}\left(\frac{9(1-x)}{x+1}+7\right)}{126(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(11/2),x]
```

```
[Out] ((1 + x)^(9/2)*(7 + (9*(1 - x))/(1 + x)))/(126*(1 - x)^(9/2))
```

fricas [B] time = 1.06, size = 83, normalized size = 2.02

$$\frac{8\,x^{5}-40\,x^{4}+80\,x^{3}-80\,x^{2}+\left(x^{4}-5\,x^{3}-21\,x^{2}-23\,x-8\right)\sqrt{x+1}\,\sqrt{-x+1}\,+40\,x-8}{63\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/63*(8*x^5 - 40*x^4 + 80*x^3 - 80*x^2 + (x^4 - 5*x^3 - 21*x^2 - 23*x - 8)*
sqrt(x + 1)*sqrt(-x + 1) + 40*x - 8)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x -
 1)
```

giac [A] time = 1.23, size = 22, normalized size = 0.54

$$\frac{(x+1)^{\frac{7}{2}}(x-8)\sqrt{-x+1}}{63\,(x-1)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="giac")
```

```
[Out] 1/63*(x + 1)^(7/2)*(x - 8)*sqrt(-x + 1)/(x - 1)^5
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+1)^{\frac{7}{2}}\,(x-8)}{63\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(11/2),x)
```

```
[Out] -1/63*(x+1)^(7/2)*(x-8)/(-x+1)^(9/2)
```

maxima [B] time = 1.40, size = 218, normalized size = 5.32

$$-\frac{(-x^{2}+1)^{\frac{5}{2}}}{2\left(x^{7}-7x^{6}+21x^{5}-35x^{4}+35x^{3}-21x^{2}+7x-1\right)}-\frac{5(-x^{2}+1)^{\frac{3}{2}}}{6\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}-\frac{5\sqrt{-x^{2}+1}}{9\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}-\frac{5\sqrt{-x^{2}+1}}{126\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}+\frac{\sqrt{-x^{2}+1}}{42\left(x^{3}-3x^{2}+3x-1\right)}-\frac{\sqrt{-x^{2}+1}}{63\left(x^{2}-2x+1\right)}+\frac{\sqrt{-x^{2}+1}}{63\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/2*(-x^2 + 1)^(5/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*
x - 1) - 5/6*(-x^2 + 1)^(3/2)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x
 + 1) - 5/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/12
6*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/42*sqrt(-x^2 + 1)/(x^3
 - 3*x^2 + 3*x - 1) - 1/63*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 1/63*sqrt(-x^2
+ 1)/(x - 1)
```

mupad [B] time = 0.30, size = 80, normalized size = 1.95

$$-\frac{\sqrt{1-x}\,\left(\frac{23\,x\,\sqrt{x+1}}{63}+\frac{8\,\sqrt{x+1}}{63}+\frac{x^{2}\,\sqrt{x+1}}{3}+\frac{5\,x^{3}\,\sqrt{x+1}}{63}-\frac{x^{4}\,\sqrt{x+1}}{63}\right)}{x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(11/2),x)
```

```
[Out] -((1 - x)^(1/2)*((23*x*(x + 1)^(1/2))/63 + (8*(x + 1)^(1/2))/63 + (x^2*(x +
 1)^(1/2))/3 + (5*x^3*(x + 1)^(1/2))/63 - (x^4*(x + 1)^(1/2))/63))/(5*x - 1
0*x^2 + 10*x^3 - 5*x^4 + x^5 - 1)
```

sympy [B] time = 53.14, size = 282, normalized size = 6.88

$$\begin{cases} \frac{i(x+1)^{\frac{9}{2}}}{63\sqrt{x-1}\,(x+1)^4-504\sqrt{x-1}\,(x+1)^3+1512\sqrt{x-1}\,(x+1)^2-2016\sqrt{x-1}\,(x+1)+1008\sqrt{x-1}} - \frac{9i(x+1)^{\frac{7}{2}}}{63\sqrt{x-1}\,(x+1)^4-504\sqrt{x-1}\,(x+1)^3+1512\sqrt{x-1}\,(x+1)^2-2016\sqrt{x-1}\,(x+1)+1008\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -\frac{(x+1)^{\frac{9}{2}}}{63\sqrt{1-x}\,(x+1)^4-504\sqrt{1-x}\,(x+1)^3+1512\sqrt{1-x}\,(x+1)^2-2016\sqrt{1-x}\,(x+1)+1008\sqrt{1-x}} + \frac{9(x+1)^{\frac{7}{2}}}{63\sqrt{1-x}\,(x+1)^4-504\sqrt{1-x}\,(x+1)^3+1512\sqrt{1-x}\,(x+1)^2-2016\sqrt{1-x}\,(x+1)+1008\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(11/2),x)
```

```
[Out] Piecewise((I*(x + 1)**(9/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1)*(x
 + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 1008*sq
rt(x - 1)) - 9*I*(x + 1)**(7/2)/(63*sqrt(x - 1)*(x + 1)**4 - 504*sqrt(x - 1
)*(x + 1)**3 + 1512*sqrt(x - 1)*(x + 1)**2 - 2016*sqrt(x - 1)*(x + 1) + 100
8*sqrt(x - 1)), Abs(x + 1)/2 > 1), (-(x + 1)**(9/2)/(63*sqrt(1 - x)*(x + 1)
**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqrt(
1 - x)*(x + 1) + 1008*sqrt(1 - x)) + 9*(x + 1)**(7/2)/(63*sqrt(1 - x)*(x +
1)**4 - 504*sqrt(1 - x)*(x + 1)**3 + 1512*sqrt(1 - x)*(x + 1)**2 - 2016*sqr
t(1 - x)*(x + 1) + 1008*sqrt(1 - x)), True))
```

3.1031 $\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} dx$

Optimal. Leaf size=61

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{693(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{99(1-x)^{9/2}}+\frac{(x+1)^{7/2}}{11(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] (1 + x)^(7/2)/(11*(1 - x)^(11/2)) + (2*(1 + x)^(7/2))/(99*(1 - x)^(9/2)) +
(2*(1 + x)^(7/2))/(693*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2}{11}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2}{99}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\ &= \frac{(1+x)^{7/2}}{11(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{99(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{693(1-x)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.49

$$\frac{(x+1)^{7/2}\left(2x^2-18x+79\right)}{693(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(7/2)*(79 - 18*x + 2*x^2))/(693*(1 - x)^(11/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 48, normalized size = 0.79

$$\frac{(x+1)^{11/2}\left(\frac{99(1-x)^2}{(x+1)^2}+\frac{154(1-x)}{x+1}+63\right)}{2772(1-x)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(13/2),x]
```

```
[Out] ((1 + x)^(11/2)*(63 + (99*(1 - x)^2)/(1 + x)^2 + (154*(1 - x))/(1 + x)))/(2
772*(1 - x)^(11/2))
```

fricas [B] time = 0.97, size = 100, normalized size = 1.64

$$\frac{79\,x^{6}-474\,x^{5}+1185\,x^{4}-1580\,x^{3}+1185\,x^{2}+\left(2\,x^{5}-12\,x^{4}+31\,x^{3}+185\,x^{2}+219\,x+79\right)\sqrt{x+1}\,\sqrt{-x+1}-474\,x+79}{693\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/693*(79*x^6 - 474*x^5 + 1185*x^4 - 1580*x^3 + 1185*x^2 + (2*x^5 - 12*x^4
+ 31*x^3 + 185*x^2 + 219*x + 79)*sqrt(x + 1)*sqrt(-x + 1) - 474*x + 79)/(x^
6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
```

giac [A] time = 1.27, size = 29, normalized size = 0.48

$$\frac{\left(2\,{\left(x+1\right)}{\left(x-10\right)}+99\right){\left(x+1\right)}^{\frac{7}{2}}\sqrt{-x+1}}{693\,{\left(x-1\right)}^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="giac")
```

```
[Out] 1/693*(2*(x + 1)*(x - 10) + 99)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^6
```

maple [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{\left(x+1\right)^{\frac{7}{2}}\left(2x^{2}-18x+79\right)}{693\left(-x+1\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(13/2),x)
```

```
[Out] 1/693*(x+1)^(7/2)*(2*x^2-18*x+79)/(-x+1)^(11/2)
```

maxima [B] time = 1.42, size = 269, normalized size = 4.41

$$\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{3\,\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}-8\,x+1\right)}+\frac{5\,\left(-x^{2}+1\right)^{\frac{3}{2}}}{12\,\left(x^{7}-7\,x^{6}+21\,x^{5}-35\,x^{4}+35\,x^{3}-21\,x^{2}+7\,x-1\right)}+\frac{5\,\sqrt{-x^{2}+1}}{22\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}+\frac{5\,\sqrt{-x^{2}+1}}{396\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}-\frac{5\,\sqrt{-x^{2}+1}}{693\,\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{231\,\left(x^{3}-3\,x^{2}+3\,x-1\right)}-\frac{2\,\sqrt{-x^{2}+1}}{693\,\left(x^{2}-2\,x+1\right)}+\frac{2\,\sqrt{-x^{2}+1}}{693\,\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(13/2),x, algorithm="maxima")
```

```
[Out] 1/3*(-x^2 + 1)^(5/2)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*
x^2 - 8*x + 1) + 5/12*(-x^2 + 1)^(3/2)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*
x^3 - 21*x^2 + 7*x - 1) + 5/22*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^
3 + 15*x^2 - 6*x + 1) + 5/396*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2
```

```
 + 5*x - 1) - 5/693*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 1/231*
sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/693*sqrt(-x^2 + 1)/(x^2 - 2*x +
1) + 2/693*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.31, size = 94, normalized size = 1.54

$$\frac{\sqrt{1-x}\,\left(\frac{73\,x\,\sqrt{x+1}}{231}+\frac{79\,\sqrt{x+1}}{693}+\frac{185\,x^2\,\sqrt{x+1}}{693}+\frac{31\,x^3\,\sqrt{x+1}}{693}-\frac{4\,x^4\,\sqrt{x+1}}{231}+\frac{2\,x^5\,\sqrt{x+1}}{693}\right)}{x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(13/2),x)
```

```
[Out] ((1 - x)^(1/2)*((73*x*(x + 1)^(1/2))/231 + (79*(x + 1)^(1/2))/693 + (185*x^
2*(x + 1)^(1/2))/693 + (31*x^3*(x + 1)^(1/2))/693 - (4*x^4*(x + 1)^(1/2))/2
31 + (2*x^5*(x + 1)^(1/2))/693))/(15*x^2 - 6*x - 20*x^3 + 15*x^4 - 6*x^5 +
x^6 + 1)
```

sympy [B] time = 133.94, size = 785, normalized size = 12.87

$$\begin{cases} \frac{2i(x+1)^{\frac{13}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{26i(x+1)^{\frac{11}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} + \frac{143i(x+1)^{\frac{9}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} - \frac{198i(x+1)^{\frac{7}{2}}}{693\sqrt{x-1}(x+1)^6-8316\sqrt{x-1}(x+1)^5+41580\sqrt{x-1}(x+1)^4-110880\sqrt{x-1}(x+1)^3+166320\sqrt{x-1}(x+1)^2-133056\sqrt{x-1}(x+1)+44352\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -\frac{2(x+1)^{\frac{13}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{26(x+1)^{\frac{11}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} - \frac{143(x+1)^{\frac{9}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} + \frac{198(x+1)^{\frac{7}{2}}}{693\sqrt{1-x}(x+1)^6-8316\sqrt{1-x}(x+1)^5+41580\sqrt{1-x}(x+1)^4-110880\sqrt{1-x}(x+1)^3+166320\sqrt{1-x}(x+1)^2-133056\sqrt{1-x}(x+1)+44352\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(13/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**(13/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)**
3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqrt
(x - 1)) - 26*I*(x + 1)**(11/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x -
 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)*
*3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sqr
t(x - 1)) + 143*I*(x + 1)**(9/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
- 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1)
**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*sq
rt(x - 1)) - 198*I*(x + 1)**(7/2)/(693*sqrt(x - 1)*(x + 1)**6 - 8316*sqrt(x
 - 1)*(x + 1)**5 + 41580*sqrt(x - 1)*(x + 1)**4 - 110880*sqrt(x - 1)*(x + 1
)**3 + 166320*sqrt(x - 1)*(x + 1)**2 - 133056*sqrt(x - 1)*(x + 1) + 44352*s
qrt(x - 1)), Abs(x + 1)/2 > 1), (-2*(x + 1)**(13/2)/(693*sqrt(1 - x)*(x + 1
)**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*
sqrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)
*(x + 1) + 44352*sqrt(1 - x)) + 26*(x + 1)**(11/2)/(693*sqrt(1 - x)*(x + 1)
**6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*s
qrt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*
(x + 1) + 44352*sqrt(1 - x)) - 143*(x + 1)**(9/2)/(693*sqrt(1 - x)*(x + 1)*
*6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sq
rt(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(
x + 1) + 44352*sqrt(1 - x)) + 198*(x + 1)**(7/2)/(693*sqrt(1 - x)*(x + 1)**
6 - 8316*sqrt(1 - x)*(x + 1)**5 + 41580*sqrt(1 - x)*(x + 1)**4 - 110880*sqr
t(1 - x)*(x + 1)**3 + 166320*sqrt(1 - x)*(x + 1)**2 - 133056*sqrt(1 - x)*(x
 + 1) + 44352*sqrt(1 - x)), True))
```

3.1032 $\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx$

Optimal. Leaf size=81

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2(x+1)^{7/2}}{3003(1-x)^{7/2}}+\frac{2(x+1)^{7/2}}{429(1-x)^{9/2}}+\frac{3(x+1)^{7/2}}{143(1-x)^{11/2}}+\frac{(x+1)^{7/2}}{13(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] (1 + x)^(7/2)/(13*(1 - x)^(13/2)) + (3*(1 + x)^(7/2))/(143*(1 - x)^(11/2))
+ (2*(1 + x)^(7/2))/(429*(1 - x)^(9/2)) + (2*(1 + x)^(7/2))/(3003*(1 - x)^(
7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx &= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3}{13}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{6}{143}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2}{429}\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{13(1-x)^{13/2}}+\frac{3(1+x)^{7/2}}{143(1-x)^{11/2}}+\frac{2(1+x)^{7/2}}{429(1-x)^{9/2}}+\frac{2(1+x)^{7/2}}{3003(1-x)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.43

$$\frac{(x+1)^{7/2}\left(-2x^3+20x^2-97x+310\right)}{3003(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(7/2)*(310 - 97*x + 20*x^2 - 2*x^3))/(3003*(1 - x)^(13/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 62, normalized size = 0.77

$$\frac{(x+1)^{13/2}\left(\frac{429(1-x)^3}{(x+1)^3}+\frac{1001(1-x)^2}{(x+1)^2}+\frac{819(1-x)}{x+1}+231\right)}{24024(1-x)^{13/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(15/2),x]
```

```
[Out] ((1 + x)^(13/2)*(231 + (429*(1 - x)^3)/(1 + x)^3 + (1001*(1 - x)^2)/(1 + x)
^2 + (819*(1 - x))/(1 + x)))/(24024*(1 - x)^(13/2))
```

fricas [B] time = 1.31, size = 115, normalized size = 1.42

$$\frac{310\,x^7-2170\,x^6+6510\,x^5-10850\,x^4+10850\,x^3-6510\,x^2+\left(2\,x^6-14\,x^5+43\,x^4-77\,x^3-659\,x^2-833\,x-310\right)\sqrt{x+1}\,\sqrt{-x+1}+2170\,x-310}{3003\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="fricas")
```

```
[Out] 1/3003*(310*x^7 - 2170*x^6 + 6510*x^5 - 10850*x^4 + 10850*x^3 - 6510*x^2 +
(2*x^6 - 14*x^5 + 43*x^4 - 77*x^3 - 659*x^2 - 833*x - 310)*sqrt(x + 1)*sqrt
(-x + 1) + 2170*x - 310)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 +
 7*x - 1)
```

giac [A] time = 0.99, size = 35, normalized size = 0.43

$$\frac{\left(\left(2\,(x+1)(x-12)+143\right)(x+1)-429\right)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{3003\,(x-1)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="giac")
```

```
[Out] 1/3003*((2*(x + 1)*(x - 12) + 143)*(x + 1) - 429)*(x + 1)^(7/2)*sqrt(-x + 1
)/(x - 1)^7
```

maple [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{(x+1)^{\frac{7}{2}}\left(2x^3-20x^2+97x-310\right)}{3003\,(-x+1)^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(15/2),x)
```

```
[Out] -1/3003*(x+1)^(7/2)*(2*x^3-20*x^2+97*x-310)/(-x+1)^(13/2)
```

maxima [B] time = 1.46, size = 325, normalized size = 4.01

$$-\frac{\left(-x^2+1\right)^{\frac{5}{2}}}{4\left(x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1\right)}-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{4\left(x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1\right)}-\frac{3\sqrt{-x^2+1}}{26\left(x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1\right)}-\frac{3\sqrt{-x^2+1}}{572\left(x^6-6\,x^5+15\,x^4-20\,x^3+15\,x^2-6\,x+1\right)}+\frac{5\sqrt{-x^2+1}}{1716\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}-\frac{5\sqrt{-x^2+1}}{3003\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}+\frac{\sqrt{-x^2+1}}{1001\left(x^3-3\,x^2+3\,x-1\right)}-\frac{2\sqrt{-x^2+1}}{3003\left(x^2-2\,x+1\right)}+\frac{2\sqrt{-x^2+1}}{3003\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 1)^(5/2)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 +
84*x^3 - 36*x^2 + 9*x - 1) - 1/4*(-x^2 + 1)^(3/2)/(x^8 - 8*x^7 + 28*x^6 - 5
6*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) - 3/26*sqrt(-x^2 + 1)/(x^7 - 7*
x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1) - 3/572*sqrt(-x^2 + 1)/(
x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1) + 5/1716*sqrt(-x^2 + 1)/(
x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 5/3003*sqrt(-x^2 + 1)/(x^4 - 4*x
^3 + 6*x^2 - 4*x + 1) + 1/1001*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 2/3
003*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 2/3003*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.31, size = 110, normalized size = 1.36

$$-\frac{\sqrt{1-x}\,\left(\frac{119\,x\,\sqrt{x+1}}{429}+\frac{310\,\sqrt{x+1}}{3003}+\frac{659\,x^2\,\sqrt{x+1}}{3003}+\frac{x^3\,\sqrt{x+1}}{39}-\frac{43\,x^4\,\sqrt{x+1}}{3003}+\frac{2\,x^5\,\sqrt{x+1}}{429}-\frac{2\,x^6\,\sqrt{x+1}}{3003}\right)}{x^7-7\,x^6+21\,x^5-35\,x^4+35\,x^3-21\,x^2+7\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(15/2),x)
```

```
[Out] -((1 - x)^(1/2)*((119*x*(x + 1)^(1/2))/429 + (310*(x + 1)^(1/2))/3003 + (65
9*x^2*(x + 1)^(1/2))/3003 + (x^3*(x + 1)^(1/2))/39 - (43*x^4*(x + 1)^(1/2))
/3003 + (2*x^5*(x + 1)^(1/2))/429 - (2*x^6*(x + 1)^(1/2))/3003))/(7*x - 21*
x^2 + 35*x^3 - 35*x^4 + 21*x^5 - 7*x^6 + x^7 - 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(15/2),x)
```

```
[Out] Timed out
```

3.1033 $\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx$

Optimal. Leaf size=101

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{45045(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{6435(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{715(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{195(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{15(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] (1 + x)^(7/2)/(15*(1 - x)^(15/2)) + (4*(1 + x)^(7/2))/(195*(1 - x)^(13/2))
+ (4*(1 + x)^(7/2))/(715*(1 - x)^(11/2)) + (8*(1 + x)^(7/2))/(6435*(1 - x)^
(9/2)) + (8*(1 + x)^(7/2))/(45045*(1 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}}\,dx &= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4}{15}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4}{65}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8}{715}\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}}\,dx\\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{9/2}}\,dx}{6435}\\
&= \frac{(1+x)^{7/2}}{15(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{195(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{715(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{6435(1-x)^{9/2}}+\frac{8(1+x)^{7/2}}{45045(1-x)^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.40

$$\frac{(x+1)^{7/2}\left(8x^4-88x^3+468x^2-1628x+4243\right)}{45045(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(7/2)*(4243 - 1628*x + 468*x^2 - 88*x^3 + 8*x^4))/(45045*(1 - x)^(
15/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 76, normalized size = 0.75

$$\frac{(x+1)^{15/2}\left(\frac{6435(1-x)^4}{(x+1)^4}+\frac{20020(1-x)^3}{(x+1)^3}+\frac{24570(1-x)^2}{(x+1)^2}+\frac{13860(1-x)}{x+1}+3003\right)}{720720(1-x)^{15/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(17/2),x]
```

```
[Out] ((1 + x)^(15/2)*(3003 + (6435*(1 - x)^4)/(1 + x)^4 + (20020*(1 - x)^3)/(1 +
 x)^3 + (24570*(1 - x)^2)/(1 + x)^2 + (13860*(1 - x))/(1 + x)))/(720720*(1
- x)^(15/2))
```

fricas [A] time = 1.36, size = 130, normalized size = 1.29

$$\frac{4243\,x^{8}-33944\,x^{7}+118804\,x^{6}-237608\,x^{5}+297010\,x^{4}-237608\,x^{3}+118804\,x^{2}+\left(8\,x^{7}-64\,x^{6}+228\,x^{5}-480\,x^{4}+675\,x^{3}+8313\,x^{2}+11101\,x+4243\right)\sqrt{x+1}\,\sqrt{-x+1}-33944\,x+4243}{45045\,\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}-8\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="fricas")
```

```
[Out] 1/45045*(4243*x^8 - 33944*x^7 + 118804*x^6 - 237608*x^5 + 297010*x^4 - 2376
08*x^3 + 118804*x^2 + (8*x^7 - 64*x^6 + 228*x^5 - 480*x^4 + 675*x^3 + 8313*
x^2 + 11101*x + 4243)*sqrt(x + 1)*sqrt(-x + 1) - 33944*x + 4243)/(x^8 - 8*x
^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1)
```

giac [A] time = 0.88, size = 42, normalized size = 0.42

$$\frac{{\left(4\,{\left({\left(2\,{\left(x+1\right)}{\left(x-14\right)}+195\right)}{\left(x+1\right)}-715\right)}{\left(x+1\right)}+6435\right)}{\left(x+1\right)}^{\frac{7}{2}}\sqrt{-x+1}}{45045\,{\left(x-1\right)}^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="giac")
```

```
[Out] 1/45045*(4*((2*(x + 1)*(x - 14) + 195)*(x + 1) - 715)*(x + 1) + 6435)*(x +
1)^(7/2)*sqrt(-x + 1)/(x - 1)^8
```

maple [A] time = 0.00, size = 35, normalized size = 0.35

$$\frac{\left(x+1\right)^{\frac{7}{2}}\left(8x^{4}-88x^{3}+468x^{2}-1628x+4243\right)}{45045\left(-x+1\right)^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(17/2),x)
```

```
[Out] 1/45045*(x+1)^(7/2)*(8*x^4-88*x^3+468*x^2-1628*x+4243)/(-x+1)^(15/2)
```

maxima [B] time = 1.39, size = 386, normalized size = 3.82

$$\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{5\,\left(x^{10}-10\,x^{9}+45\,x^{8}-120\,x^{7}+210\,x^{6}-252\,x^{5}+210\,x^{4}-120\,x^{3}+45\,x^{2}-10\,x+1\right)}+\frac{\left(-x^{2}+1\right)^{\frac{3}{2}}}{6\,\left(x^{9}-9\,x^{8}+36\,x^{7}-84\,x^{6}+126\,x^{5}-126\,x^{4}+84\,x^{3}-36\,x^{2}+9\,x-1\right)}+\frac{\sqrt{-x^{2}+1}}{15\,\left(x^{8}-8\,x^{7}+28\,x^{6}-56\,x^{5}+70\,x^{4}-56\,x^{3}+28\,x^{2}-8\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{390\,\left(x^{7}-7\,x^{6}+21\,x^{5}-35\,x^{4}+35\,x^{3}-21\,x^{2}+7\,x-1\right)}-\frac{\sqrt{-x^{2}+1}}{715\,\left(x^{6}-6\,x^{5}+15\,x^{4}-20\,x^{3}+15\,x^{2}-6\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{1287\,\left(x^{5}-5\,x^{4}+10\,x^{3}-10\,x^{2}+5\,x-1\right)}-\frac{4\,\sqrt{-x^{2}+1}}{9009\,\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)}+\frac{4\,\sqrt{-x^{2}+1}}{15015\,\left(x^{3}-3\,x^{2}+3\,x-1\right)}-\frac{8\,\sqrt{-x^{2}+1}}{45045\,\left(x^{2}-2\,x+1\right)}+\frac{8\,\sqrt{-x^{2}+1}}{45045\,\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(17/2),x, algorithm="maxima")
```

```
[Out] 1/5*(-x^2 + 1)^(5/2)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5
+ 210*x^4 - 120*x^3 + 45*x^2 - 10*x + 1) + 1/6*(-x^2 + 1)^(3/2)/(x^9 - 9*x^
8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) + 1/15
*sqrt(-x^2 + 1)/(x^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 -
 8*x + 1) + 1/390*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 -
21*x^2 + 7*x - 1) - 1/715*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 1
5*x^2 - 6*x + 1) + 1/1287*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5
*x - 1) - 4/9009*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/15015*s
qrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) - 8/45045*sqrt(-x^2 + 1)/(x^2 - 2*x +
 1) + 8/45045*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.35, size = 124, normalized size = 1.23

$$\frac{\sqrt{1-x}\,\left(\frac{11101\,x\,\sqrt{x+1}}{45045}+\frac{4243\,\sqrt{x+1}}{45045}+\frac{2771\,x^2\,\sqrt{x+1}}{15015}+\frac{15\,x^3\,\sqrt{x+1}}{1001}-\frac{32\,x^4\,\sqrt{x+1}}{3003}+\frac{76\,x^5\,\sqrt{x+1}}{15015}-\frac{64\,x^6\,\sqrt{x+1}}{45045}+\frac{8\,x^7\,\sqrt{x+1}}{45045}\right)}{x^8-8\,x^7+28\,x^6-56\,x^5+70\,x^4-56\,x^3+28\,x^2-8\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(17/2),x)
```

```
[Out] ((1 - x)^(1/2)*((11101*x*(x + 1)^(1/2))/45045 + (4243*(x + 1)^(1/2))/45045
+ (2771*x^2*(x + 1)^(1/2))/15015 + (15*x^3*(x + 1)^(1/2))/1001 - (32*x^4*(x
 + 1)^(1/2))/3003 + (76*x^5*(x + 1)^(1/2))/15015 - (64*x^6*(x + 1)^(1/2))/4
5045 + (8*x^7*(x + 1)^(1/2))/45045))/(28*x^2 - 8*x - 56*x^3 + 70*x^4 - 56*x
^5 + 28*x^6 - 8*x^7 + x^8 + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(17/2),x)
```

```
[Out] Timed out
```

3.1034 $\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}} \, dx$

Optimal. Leaf size=121

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Rubi [A] time = 0.02, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {45, 37}

$$\frac{8(x+1)^{7/2}}{153153(1-x)^{7/2}}+\frac{8(x+1)^{7/2}}{21879(1-x)^{9/2}}+\frac{4(x+1)^{7/2}}{2431(1-x)^{11/2}}+\frac{4(x+1)^{7/2}}{663(1-x)^{13/2}}+\frac{(x+1)^{7/2}}{51(1-x)^{15/2}}+\frac{(x+1)^{7/2}}{17(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] (1 + x)^(7/2)/(17*(1 - x)^(17/2)) + (1 + x)^(7/2)/(51*(1 - x)^(15/2)) + (4*
(1 + x)^(7/2))/(663*(1 - x)^(13/2)) + (4*(1 + x)^(7/2))/(2431*(1 - x)^(11/2
)) + (8*(1 + x)^(7/2))/(21879*(1 - x)^(9/2)) + (8*(1 + x)^(7/2))/(153153*(1
 - x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x)^{5/2}}{(1-x)^{19/2}} \, dx &= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{5}{17}\int \frac{(1+x)^{5/2}}{(1-x)^{17/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4}{51}\int \frac{(1+x)^{5/2}}{(1-x)^{15/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4}{221}\int \frac{(1+x)^{5/2}}{(1-x)^{13/2}} \, dx\\
&= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8\int \frac{(1+x)^{5/2}}{(1-x)^{11/2}} \, dx}{2431}\\
&= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\frac{8\int \frac{(1+x)}{(1-x)}}{2187}\\
&= \frac{(1+x)^{7/2}}{17(1-x)^{17/2}}+\frac{(1+x)^{7/2}}{51(1-x)^{15/2}}+\frac{4(1+x)^{7/2}}{663(1-x)^{13/2}}+\frac{4(1+x)^{7/2}}{2431(1-x)^{11/2}}+\frac{8(1+x)^{7/2}}{21879(1-x)^{9/2}}+\frac{8(1+}{153153(}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.37

$$\frac{(x+1)^{7/2}\left(-8x^5+96x^4-556x^3+2096x^2-5871x+13252\right)}{153153(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(7/2)*(13252 - 5871*x + 2096*x^2 - 556*x^3 + 96*x^4 - 8*x^5))/(153
153*(1 - x)^(17/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 90, normalized size = 0.74

$$\frac{(x+1)^{17/2}\left(\frac{21879(1-x)^5}{(x+1)^5}+\frac{85085(1-x)^4}{(x+1)^4}+\frac{139230(1-x)^3}{(x+1)^3}+\frac{117810(1-x)^2}{(x+1)^2}+\frac{51051(1-x)}{x+1}+9009\right)}{4900896(1-x)^{17/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x)^(5/2)/(1 - x)^(19/2),x]
```

```
[Out] ((1 + x)^(17/2)*(9009 + (21879*(1 - x)^5)/(1 + x)^5 + (85085*(1 - x)^4)/(1
+ x)^4 + (139230*(1 - x)^3)/(1 + x)^3 + (117810*(1 - x)^2)/(1 + x)^2 + (510
51*(1 - x))/(1 + x)))/(4900896*(1 - x)^(17/2))
```

fricas [A] time = 1.16, size = 145, normalized size = 1.20

$$\frac{13252\,x^{9}-119268\,x^{8}+477072\,x^{7}-1113168\,x^{6}+1669752\,x^{5}-1669752\,x^{4}+1113168\,x^{3}-477072\,x^{2}+\left(8\,x^{8}-72\,x^{7}+292\,x^{6}-708\,x^{5}+1155\,x^{4}-1371\,x^{3}-24239\,x^{2}-33885\,x-13252\right)\sqrt{x+1}\,\sqrt{-x+1}+119268\,x-13252}{153153\,\left(x^{9}-9\,x^{8}+36\,x^{7}-84\,x^{6}+126\,x^{5}-126\,x^{4}+84\,x^{3}-36\,x^{2}+9\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="fricas")
```

```
[Out] 1/153153*(13252*x^9 - 119268*x^8 + 477072*x^7 - 1113168*x^6 + 1669752*x^5 -
 1669752*x^4 + 1113168*x^3 - 477072*x^2 + (8*x^8 - 72*x^7 + 292*x^6 - 708*x
^5 + 1155*x^4 - 1371*x^3 - 24239*x^2 - 33885*x - 13252)*sqrt(x + 1)*sqrt(-x
 + 1) + 119268*x - 13252)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6 + 126*x^5 - 126*x^
4 + 84*x^3 - 36*x^2 + 9*x - 1)
```

giac [A] time = 0.86, size = 48, normalized size = 0.40

$$\frac{((4\,((2\,(x+1)(x-16)+255)(x+1)-1105)(x+1)+12155)(x+1)-21879)(x+1)^{\frac{7}{2}}\sqrt{-x+1}}{153153\,(x-1)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="giac")
```

```
[Out] 1/153153*((4*((2*(x + 1)*(x - 16) + 255)*(x + 1) - 1105)*(x + 1) + 12155)*(
x + 1) - 21879)*(x + 1)^(7/2)*sqrt(-x + 1)/(x - 1)^9
```

maple [A] time = 0.00, size = 40, normalized size = 0.33

$$-\frac{(x+1)^{\frac{7}{2}}\left(8x^5-96x^4+556x^3-2096x^2+5871x-13252\right)}{153153\,(-x+1)^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+1)^(5/2)/(-x+1)^(19/2),x)
```

[Out] -1/153153*(x+1)^(7/2)*(8*x^5-96*x^4+556*x^3-2096*x^2+5871*x-13252)/(-x+1)^(
17/2)

maxima [B]   time = 1.38, size = 452, normalized size = 3.74

$$-\frac{\left(-x^{2}+1\right)^{\frac{5}{2}}}{6\left(x^{11}-11x^{10}+55x^{9}-165x^{8}+330x^{7}-462x^{6}+462x^{5}-330x^{4}+165x^{3}-55x^{2}+11x-1\right)}-\frac{5\left(-x^{2}+1\right)^{\frac{3}{2}}}{42\left(x^{10}-10x^{9}+45x^{8}-120x^{7}+210x^{6}-252x^{5}+210x^{4}-120x^{3}+45x^{2}-10x+1\right)}-\frac{5\sqrt{-x^{2}+1}}{119\left(x^{9}-9x^{8}+36x^{7}-84x^{6}+126x^{5}-126x^{4}+84x^{3}-36x^{2}+9x-1\right)}-\frac{\sqrt{-x^{2}+1}}{714\left(x^{8}-8x^{7}+28x^{6}-56x^{5}+70x^{4}-56x^{3}+28x^{2}-8x+1\right)}+\frac{\sqrt{-x^{2}+1}}{1326\left(x^{7}-7x^{6}+21x^{5}-35x^{4}+35x^{3}-21x^{2}+7x-1\right)}-\frac{\sqrt{-x^{2}+1}}{2431\left(x^{6}-6x^{5}+15x^{4}-20x^{3}+15x^{2}-6x+1\right)}+\frac{5\sqrt{-x^{2}+1}}{21879\left(x^{5}-5x^{4}+10x^{3}-10x^{2}+5x-1\right)}-\frac{20\sqrt{-x^{2}+1}}{153153\left(x^{4}-4x^{3}+6x^{2}-4x+1\right)}+\frac{4\sqrt{-x^{2}+1}}{51051\left(x^{3}-3x^{2}+3x-1\right)}-\frac{8\sqrt{-x^{2}+1}}{153153\left(x^{2}-2x+1\right)}+\frac{8\sqrt{-x^{2}+1}}{153153\left(x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(5/2)/(1-x)^(19/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 1)^(5/2)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 + 330*x^7 - 462*x^
6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1) - 5/42*(-x^2 + 1)^(3/2
)/(x^10 - 10*x^9 + 45*x^8 - 120*x^7 + 210*x^6 - 252*x^5 + 210*x^4 - 120*x^3
 + 45*x^2 - 10*x + 1) - 5/119*sqrt(-x^2 + 1)/(x^9 - 9*x^8 + 36*x^7 - 84*x^6
 + 126*x^5 - 126*x^4 + 84*x^3 - 36*x^2 + 9*x - 1) - 1/714*sqrt(-x^2 + 1)/(x
^8 - 8*x^7 + 28*x^6 - 56*x^5 + 70*x^4 - 56*x^3 + 28*x^2 - 8*x + 1) + 1/1326
*sqrt(-x^2 + 1)/(x^7 - 7*x^6 + 21*x^5 - 35*x^4 + 35*x^3 - 21*x^2 + 7*x - 1)
 - 1/2431*sqrt(-x^2 + 1)/(x^6 - 6*x^5 + 15*x^4 - 20*x^3 + 15*x^2 - 6*x + 1)
 + 5/21879*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) - 20/15
3153*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) + 4/51051*sqrt(-x^2 + 1
)/(x^3 - 3*x^2 + 3*x - 1) - 8/153153*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) + 8/153
153*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B]   time = 0.37, size = 140, normalized size = 1.16

$$-\frac{\sqrt{1-x}\,\left(\frac{3765\,x\,\sqrt{x+1}}{17017}+\frac{13252\,\sqrt{x+1}}{153153}+\frac{24239\,x^2\,\sqrt{x+1}}{153153}+\frac{457\,x^3\,\sqrt{x+1}}{51051}-\frac{5\,x^4\,\sqrt{x+1}}{663}+\frac{236\,x^5\,\sqrt{x+1}}{51051}-\frac{292\,x^6\,\sqrt{x+1}}{153153}+\frac{8\,x^7\,\sqrt{x+1}}{17017}-\frac{8\,x^8\,\sqrt{x+1}}{153153}\right)}{x^9-9\,x^8+36\,x^7-84\,x^6+126\,x^5-126\,x^4+84\,x^3-36\,x^2+9\,x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + 1)^(5/2)/(1 - x)^(19/2),x)
```

```
[Out] -((1 - x)^(1/2)*((3765*x*(x + 1)^(1/2))/17017 + (13252*(x + 1)^(1/2))/15315
3 + (24239*x^2*(x + 1)^(1/2))/153153 + (457*x^3*(x + 1)^(1/2))/51051 - (5*x
^4*(x + 1)^(1/2))/663 + (236*x^5*(x + 1)^(1/2))/51051 - (292*x^6*(x + 1)^(1
/2))/153153 + (8*x^7*(x + 1)^(1/2))/17017 - (8*x^8*(x + 1)^(1/2))/153153))/
(9*x - 36*x^2 + 84*x^3 - 126*x^4 + 126*x^5 - 84*x^6 + 36*x^7 - 9*x^8 + x^9
- 1)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(5/2)/(1-x)**(19/2),x)
```

```
[Out] Timed out
```

3.1035 $\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx$

Optimal. Leaf size=64

$$-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {50, 41, 216}

$$-\frac{\sqrt{1-ax}\,(ax+1)^{3/2}}{2a}-\frac{3\sqrt{1-ax}\,\sqrt{ax+1}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
```

```
[Out] (-3*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(2*a) - (Sqrt[1 - a*x]*(1 + a*x)^(3/2))/(2
*a) + (3*ArcSin[a*x])/(2*a)
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)^{3/2}}{\sqrt{1-ax}}\,dx &= -\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{\sqrt{1+ax}}{\sqrt{1-ax}}\,dx\\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{1}{\sqrt{1-ax}\,\sqrt{1+ax}}\,dx\\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1-ax}\,\sqrt{1+ax}}{2a}-\frac{\sqrt{1-ax}\,(1+ax)^{3/2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.73

$$-\frac{\sqrt{1-a^2x^2}\,(ax+4)+6\sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
[Out] -1/2*((4 + a*x)*Sqrt[1 - a^2*x^2] + 6*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/a
```

IntegrateAlgebraic [A] time = 0.08, size = 86, normalized size = 1.34

$$-\frac{\sqrt{1-ax}\left(\frac{3(1-ax)}{ax+1}+5\right)}{a\sqrt{ax+1}\left(\frac{1-ax}{ax+1}+1\right)^2}-\frac{3\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)^(3/2)/Sqrt[1 - a*x],x]
[Out] -((Sqrt[1 - a*x]*(5 + (3*(1 - a*x))/(1 + a*x)))/(a*Sqrt[1 + a*x]*(1 + (1 -
a*x)/(1 + a*x))^2)) - (3*ArcTan[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/a
```

fricas [A] time = 1.27, size = 55, normalized size = 0.86

$$-\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1}+6\arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) + 6*arctan((sqrt(a*x + 1)*sqrt
(-a*x + 1) - 1)/(a*x)))/a
```

giac [A] time = 0.70, size = 42, normalized size = 0.66

$$-\frac{(ax+4)\sqrt{ax+1}\sqrt{-ax+1}-6\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{ax+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -1/2*((a*x + 4)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 6*arcsin(1/2*sqrt(2)*sqrt(a*
x + 1)))/a
```

maple [A] time = 0.01, size = 98, normalized size = 1.53

$$\frac{3\sqrt{(ax+1)(-ax+1)}\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{ax+1}\sqrt{-ax+1}\sqrt{a^2}}-\frac{(ax+1)^{\frac{3}{2}}\sqrt{-ax+1}}{2a}-\frac{3\sqrt{-ax+1}\sqrt{ax+1}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)^(3/2)/(-a*x+1)^(1/2),x)
[Out] -1/2*(a*x+1)^(3/2)*(-a*x+1)^(1/2)/a-3/2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a+3/2*
((a*x+1)*(-a*x+1))^(1/2)/(a*x+1)^(1/2)/(-a*x+1)^(1/2)/(a^2)^(1/2)*arctan((a
^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))
```

maxima [A] time = 2.99, size = 42, normalized size = 0.66

$$-\frac{1}{2}\sqrt{-a^2x^2+1}\,x+\frac{3\arcsin(ax)}{2a}-\frac{2\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)^(3/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a\,x+1)^{3/2}}{\sqrt{1-a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2),x)
```

```
[Out] int((a*x + 1)^(3/2)/(1 - a*x)^(1/2), x)
```

sympy [A] time = 33.75, size = 75, normalized size = 1.17

$$\begin{cases} \frac{2\left(\begin{cases} -\frac{ax\sqrt{-ax+1}\sqrt{ax+1}}{4} - \sqrt{-ax+1}\sqrt{ax+1} + \frac{3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{ax+1}}{2}\right)}{2} & \text{for}\: ax-1 \geq -2 \wedge ax-1 < 0 \end{cases}\right)}{a} & \text{for}\: a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/(-a*x+1)**(1/2),x)
```

```
[Out] Piecewise((2*Piecewise((-a*x*sqrt(-a*x + 1)*sqrt(a*x + 1)/4 - sqrt(-a*x + 1
)*sqrt(a*x + 1) + 3*asin(sqrt(2)*sqrt(a*x + 1)/2)/2, (a*x - 1 >= -2) & (a*x
 - 1 < 0)))/a, Ne(a, 0)), (x, True))
```

3.1036 $\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx$

Optimal. Leaf size=62

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {795, 665, 216}

$$-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}-\frac{3\sqrt{1-a^2x^2}}{2a}+\frac{3\sin^{-1}(ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2])/(2*a) - (1 - a^2*x^2)^(3/2)/(2*a*(1 - a*x)) + (3*Arc
Sin[a*x])/(2*a)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 665

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e
^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0]
 || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+ax)\sqrt{1-a^2x^2}}{1-ax}\,dx &= -\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{\sqrt{1-a^2x^2}}{1-ax}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3}{2}\int \frac{1}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3\sqrt{1-a^2x^2}}{2a}-\frac{\left(1-a^2x^2\right)^{3/2}}{2a(1-ax)}+\frac{3\sin^{-1}(ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 91, normalized size = 1.47

$$\frac{\sqrt{1-a^2x^2}\left(6\sqrt{ax+1}\,\sin^{-1}\left(\frac{\sqrt{ax+1}}{\sqrt{2}}\right)-\sqrt{1-ax}\,\left(a^2x^2+5ax+4\right)\right)}{2a\sqrt{1-ax}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-(Sqrt[1 - a*x]*(4 + 5*a*x + a^2*x^2)) + 6*Sqrt[1 + a*x
]*ArcSin[Sqrt[1 + a*x]/Sqrt[2]]))/(2*a*Sqrt[1 - a*x]*(1 + a*x))
```

IntegrateAlgebraic [A] time = 0.29, size = 72, normalized size = 1.16

$$\frac{\sqrt{1-a^2x^2}\,(-ax-4)}{2a}+\frac{3\sqrt{-a^2}\,\log\left(\sqrt{1-a^2x^2}-\sqrt{-a^2}\,x\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + a*x)*Sqrt[1 - a^2*x^2])/(1 - a*x),x]
```

```
[Out] ((-4 - a*x)*Sqrt[1 - a^2*x^2])/(2*a) + (3*Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) +
Sqrt[1 - a^2*x^2]])/(2*a^2)
```

fricas [A] time = 1.32, size = 48, normalized size = 0.77

$$-\frac{\sqrt{-a^2x^2+1}\,(ax+4)+6\,\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(-a^2*x^2 + 1)*(a*x + 4) + 6*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x
)))/a
```

giac [A] time = 0.70, size = 34, normalized size = 0.55

$$-\frac{1}{2}\,\sqrt{-a^2x^2+1}\left(x+\frac{4}{a}\right)+\frac{3\,\arcsin\left(ax\right)\mathrm{sgn}\left(a\right)}{2\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x + 4/a) + 3/2*arcsin(a*x)*sgn(a)/abs(a)
```

maple [B] time = 0.01, size = 118, normalized size = 1.90

$$-\frac{\sqrt{-a^2x^2+1}\,x}{2}+\frac{2\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}\right)}{\sqrt{a^2}}-\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2\sqrt{a^2}}-\frac{2\sqrt{-\left(x-\frac{1}{a}\right)^2a^2-2\left(x-\frac{1}{a}\right)a}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x)
```

```
[Out] -1/2*x*(-a^2*x^2+1)^(1/2)-1/2/(a^2)^(1/2)*arctan((a^2)^(1/2)/(-a^2*x^2+1)^(
1/2)*x)-2/a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)+2/(a^2)^(1/2)*arctan((a^2)^(
1/2)*x/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2))
```

maxima [A] time = 3.03, size = 42, normalized size = 0.68

$$-\frac{1}{2}\,\sqrt{-a^2x^2+1}\,x+\frac{3\,\arcsin\left(ax\right)}{2\,a}-\frac{2\,\sqrt{-a^2x^2+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a^2*x^2+1)^(1/2)/(-a*x+1),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*x + 3/2*arcsin(a*x)/a - 2*sqrt(-a^2*x^2 + 1)/a
```

mupad [B] time = 0.15, size = 55, normalized size = 0.89

$$\frac{\frac{3\,\mathrm{asinh}\left(x\,\sqrt{-a^2}\right)}{2}+\sqrt{1-a^2\,x^2}\,\left(\frac{2\,a}{\sqrt{-a^2}}-\frac{x\,\sqrt{-a^2}}{2}\right)}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((1 - a^2*x^2)^(1/2)*(a*x + 1))/(a*x - 1),x)
```

```
[Out] ((3*asinh(x*(-a^2)^(1/2)))/2 + (1 - a^2*x^2)^(1/2)*((2*a)/(-a^2)^(1/2) - (x
*(-a^2)^(1/2))/2))/(-a^2)^(1/2)
```

sympy [A] time = 7.08, size = 76, normalized size = 1.23

$$-\begin{cases} -\frac{-\sqrt{-a^2x^2+1}+\mathrm{asin}\,(ax)}{a} & \text{for}\ ax>-1\wedge ax<1 \end{cases}-\begin{cases} -\frac{-\frac{ax\sqrt{-a^2x^2+1}}{2}-\sqrt{-a^2x^2+1}+\frac{\mathrm{asin}\,(ax)}{2}}{a} & \text{for}\ ax>-1\wedge ax<1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)*(-a**2*x**2+1)**(1/2)/(-a*x+1),x)
```

```
[Out] -Piecewise((-(-sqrt(-a**2*x**2 + 1) + asin(a*x))/a, (a*x > -1) & (a*x < 1))
) - Piecewise((-(-a*x*sqrt(-a**2*x**2 + 1)/2 - sqrt(-a**2*x**2 + 1) + asin(
a*x)/2)/a, (a*x > -1) & (a*x < 1)))
```

3.1037 $\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{4}\sqrt{x+1}\,(1-x)^{7/2}+\frac{7}{12}\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{24}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{8}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (35*Sqrt[1 - x]*Sqrt[1 + x])/8 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/24 + (7*(1
- x)^(5/2)*Sqrt[1 + x])/12 + ((1 - x)^(7/2)*Sqrt[1 + x])/4 + (35*ArcSin[x])
/8
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{7/2}}{\sqrt{1+x}}\,dx &= \frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{7}{4}\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx\\
&= \frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{12}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\
&= \frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\int\\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\int\\
&= \frac{35}{8}\sqrt{1-x}\,\sqrt{1+x}+\frac{35}{24}(1-x)^{3/2}\sqrt{1+x}+\frac{7}{12}(1-x)^{5/2}\sqrt{1+x}+\frac{1}{4}(1-x)^{7/2}\sqrt{1+x}+\frac{35}{8}\text{si}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.70

$$\frac{\sqrt{x+1}\left(6x^4-38x^3+113x^2-241x+160\right)}{24\sqrt{1-x}}-\frac{35}{4}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(160 - 241*x + 113*x^2 - 38*x^3 + 6*x^4))/(24*Sqrt[1 - x]) - (
35*ArcSin[Sqrt[1 - x]/Sqrt[2]])/4
```

IntegrateAlgebraic [A] time = 0.07, size = 100, normalized size = 1.15

$$\frac{\sqrt{x+1}\left(\frac{105(x+1)^3}{(1-x)^3}+\frac{385(x+1)^2}{(1-x)^2}+\frac{511(x+1)}{1-x}+279\right)}{12\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^4}+\frac{35}{4}\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(279 + (511*(1 + x))/(1 - x) + (385*(1 + x)^2)/(1 - x)^2 + (10
5*(1 + x)^3)/(1 - x)^3))/(12*Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^4) + (35*Arc
Tan[Sqrt[1 + x]/Sqrt[1 - x]])/4
```

fricas [A] time = 1.29, size = 52, normalized size = 0.60

$$-\frac{1}{24}\left(6\,x^3-32\,x^2+81\,x-160\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{35}{4}\,\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*(6*x^3 - 32*x^2 + 81*x - 160)*sqrt(x + 1)*sqrt(-x + 1) - 35/4*arctan(
(sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [A] time = 0.76, size = 101, normalized size = 1.16

$$-\frac{1}{24}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{1}{2}\left((2\,x-5)(x+1)+9\right)\sqrt{x+1}\,\sqrt{-x+1}-\frac{3}{2}\,\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\,\sqrt{-x+1}+\frac{35}{4}\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) +
 1/2*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - 3/2*sqrt(x + 1)*(x
- 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 35/4*arcsin(1/2*sqrt(2)*sqrt
(x + 1))
```

maple [A] time = 0.01, size = 85, normalized size = 0.98

$$\frac{35\sqrt{(x+1)\,(-x+1)}\ \arcsin(x)}{8\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{7}{2}}\,\sqrt{x+1}}{4}+\frac{7\,(-x+1)^{\frac{5}{2}}\,\sqrt{x+1}}{12}+\frac{35\,(-x+1)^{\frac{3}{2}}\,\sqrt{x+1}}{24}+\frac{35\sqrt{-x+1}\,\sqrt{x+1}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(1/2),x)
```

```
[Out] 1/4*(-x+1)^(7/2)*(x+1)^(1/2)+7/12*(-x+1)^(5/2)*(x+1)^(1/2)+35/24*(-x+1)^(3/
2)*(x+1)^(1/2)+35/8*(-x+1)^(1/2)*(x+1)^(1/2)+35/8*((x+1)*(-x+1))^(1/2)/(x+1
)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.99, size = 56, normalized size = 0.64

$$-\frac{1}{4}\sqrt{-x^2+1}\,x^3+\frac{4}{3}\sqrt{-x^2+1}\,x^2-\frac{27}{8}\sqrt{-x^2+1}\,x+\frac{20}{3}\sqrt{-x^2+1}+\frac{35}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-x^2 + 1)*x^3 + 4/3*sqrt(-x^2 + 1)*x^2 - 27/8*sqrt(-x^2 + 1)*x +
20/3*sqrt(-x^2 + 1) + 35/8*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(1/2), x)
```

sympy [A] time = 14.68, size = 201, normalized size = 2.31

$$\begin{cases} -\frac{35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{x-1}}+\frac{31i(x+1)^{\frac{7}{2}}}{12\sqrt{x-1}}-\frac{263i(x+1)^{\frac{5}{2}}}{24\sqrt{x-1}}+\frac{605i(x+1)^{\frac{3}{2}}}{24\sqrt{x-1}}-\frac{93i\sqrt{x+1}}{4\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -\frac{\sqrt{1-x}\,(x+1)^{\frac{7}{2}}}{4}+\frac{25\sqrt{1-x}\,(x+1)^{\frac{5}{2}}}{12}-\frac{163\sqrt{1-x}\,(x+1)^{\frac{3}{2}}}{24}+\frac{93\sqrt{1-x}\,\sqrt{x+1}}{8}+\frac{35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-35*I*acosh(sqrt(2)*sqrt(x + 1)/2)/4 - I*(x + 1)**(9/2)/(4*sqrt(
x - 1)) + 31*I*(x + 1)**(7/2)/(12*sqrt(x - 1)) - 263*I*(x + 1)**(5/2)/(24*s
qrt(x - 1)) + 605*I*(x + 1)**(3/2)/(24*sqrt(x - 1)) - 93*I*sqrt(x + 1)/(4*s
qrt(x - 1)), Abs(x + 1)/2 > 1), (-sqrt(1 - x)*(x + 1)**(7/2)/4 + 25*sqrt(1
- x)*(x + 1)**(5/2)/12 - 163*sqrt(1 - x)*(x + 1)**(3/2)/24 + 93*sqrt(1 - x)
*sqrt(x + 1)/8 + 35*asin(sqrt(2)*sqrt(x + 1)/2)/4, True))
```

3.1038 $\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{5}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{3}\sqrt{x+1}\,(1-x)^{5/2}+\frac{5}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{5}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{5}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (5*Sqrt[1 - x]*Sqrt[1 + x])/2 + (5*(1 - x)^(3/2)*Sqrt[1 + x])/6 + ((1 - x)^
(5/2)*Sqrt[1 + x])/3 + (5*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx &= \frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx\\
&= \frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{5}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{5}{6}(1-x)^{3/2}\sqrt{1+x}+\frac{1}{3}(1-x)^{5/2}\sqrt{1+x}+\frac{5}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 0.81

$$\frac{\sqrt{x+1}\left(-2x^3+11x^2-31x+22\right)}{6\sqrt{1-x}}-5\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(22 - 31*x + 11*x^2 - 2*x^3))/(6*Sqrt[1 - x]) - 5*ArcSin[Sqrt[
1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [A] time = 0.07, size = 84, normalized size = 1.25

$$\frac{\sqrt{x+1}\left(\frac{15(x+1)^2}{(1-x)^2}+\frac{40(x+1)}{1-x}+33\right)}{3\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^3}+5\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(33 + (40*(1 + x))/(1 - x) + (15*(1 + x)^2)/(1 - x)^2))/(3*Sqr
t[1 - x]*(1 + (1 + x)/(1 - x))^3) + 5*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

fricas [A] time = 1.26, size = 47, normalized size = 0.70

$$\frac{1}{6}\left(2x^2-9x+22\right)\sqrt{x+1}\sqrt{-x+1}-5\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 - 9*x + 22)*sqrt(x + 1)*sqrt(-x + 1) - 5*arctan((sqrt(x + 1)*sqr
t(-x + 1) - 1)/x)
```

giac [A] time = 0.70, size = 69, normalized size = 1.03

$$\frac{1}{6}\left((2x-5)(x+1)+9\right)\sqrt{x+1}\sqrt{-x+1}-\sqrt{x+1}(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+5\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) - sqrt(x + 1)*(x - 2)*
sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 5*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 71, normalized size = 1.06

$$\frac{5\sqrt{(x+1)(-x+1)}\arcsin(x)}{2\sqrt{x+1}\sqrt{-x+1}}+\frac{(-x+1)^{\frac{5}{2}}\sqrt{x+1}}{3}+\frac{5(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{6}+\frac{5\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(1/2),x)
```

```
[Out] 1/3*(-x+1)^(5/2)*(x+1)^(1/2)+5/6*(-x+1)^(3/2)*(x+1)^(1/2)+5/2*(-x+1)^(1/2)*
(x+1)^(1/2)+5/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.90, size = 42, normalized size = 0.63

$$\frac{1}{3}\sqrt{-x^2+1}\,x^2-\frac{3}{2}\sqrt{-x^2+1}\,x+\frac{11}{3}\sqrt{-x^2+1}+\frac{5}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(-x^2 + 1)*x^2 - 3/2*sqrt(-x^2 + 1)*x + 11/3*sqrt(-x^2 + 1) + 5/2*a
rcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{5/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(1/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(1/2), x)
```

sympy [A] time = 5.64, size = 175, normalized size = 2.61

$$\begin{cases} -5i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{17i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{59i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}-\frac{11i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2}>1 \\ 5\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{17(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{59(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}+\frac{11\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-5*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) - 17*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 59*I*(x + 1)**(3/2)/(6*sqrt(x
- 1)) - 11*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (5*asin(sqrt(2)*sq
rt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 17*(x + 1)**(5/2)/(6*sqrt(1
 - x)) - 59*(x + 1)**(3/2)/(6*sqrt(1 - x)) + 11*sqrt(x + 1)/sqrt(1 - x), Tr
ue))
```

3.1039 $\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{3}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 41, 216}

$$\frac{1}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{3}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (3*Sqrt[1 - x]*Sqrt[1 + x])/2 + ((1 - x)^(3/2)*Sqrt[1 + x])/2 + (3*ArcSin[x
])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx &= \frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \frac{3}{2}\sqrt{1-x}\,\sqrt{1+x}+\frac{1}{2}(1-x)^{3/2}\sqrt{1+x}+\frac{3}{2}\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{\sqrt{x+1}\,\left(x^2-5x+4\right)}{2\sqrt{1-x}}-3\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(4 - 5*x + x^2))/(2*Sqrt[1 - x]) - 3*ArcSin[Sqrt[1 - x]/Sqrt[2
]]
```

IntegrateAlgebraic [A] time = 0.07, size = 67, normalized size = 1.43

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)}{1-x}+5\right)}{\sqrt{1-x}\left(\frac{x+1}{1-x}+1\right)^2}+3\tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/Sqrt[1 + x],x]
```

```
[Out] (Sqrt[1 + x]*(5 + (3*(1 + x))/(1 - x)))/(Sqrt[1 - x]*(1 + (1 + x)/(1 - x))^
2) + 3*ArcTan[Sqrt[1 + x]/Sqrt[1 - x]]
```

fricas [A] time = 1.40, size = 40, normalized size = 0.85

$$-\frac{1}{2}\sqrt{x+1}\,(x-4)\sqrt{-x+1}-3\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 4)*sqrt(-x + 1) - 3*arctan((sqrt(x + 1)*sqrt(-x + 1)
- 1)/x)
```

giac [A] time = 0.70, size = 44, normalized size = 0.94

$$-\frac{1}{2}\sqrt{x+1}\,(x-2)\sqrt{-x+1}+\sqrt{x+1}\sqrt{-x+1}+3\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + sqrt(x + 1)*sqrt(-x + 1) + 3*arcsin
(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.00, size = 57, normalized size = 1.21

$$\frac{3\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{2\sqrt{x+1}\,\sqrt{-x+1}}+\frac{(-x+1)^{\frac{3}{2}}\sqrt{x+1}}{2}+\frac{3\sqrt{-x+1}\,\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(1/2),x)
```

```
[Out] 1/2*(-x+1)^(3/2)*(x+1)^(1/2)+3/2*(-x+1)^(1/2)*(x+1)^(1/2)+3/2*((x+1)*(-x+1)
)^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.02, size = 28, normalized size = 0.60

$$-\frac{1}{2}\sqrt{-x^2+1}\,x+2\sqrt{-x^2+1}+\frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] -1/2*sqrt(-x^2 + 1)*x + 2*sqrt(-x^2 + 1) + 3/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(1/2),x)
[Out] int((1 - x)^(3/2)/(x + 1)^(1/2), x)
```

sympy [A] time = 2.59, size = 139, normalized size = 2.96

$$\begin{cases} -3i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}+\frac{7i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}-\frac{5i\sqrt{x+1}}{\sqrt{x-1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 3\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}-\frac{7(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}+\frac{5\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(1/2),x)
[Out] Piecewise((-3*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) + 7*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) - 5*I*sqrt(x + 1)/sqrt(x - 1), Ab
s(x + 1)/2 > 1), (3*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(5/2)/(2*sqrt(1
- x)) - 7*(x + 1)**(3/2)/(2*sqrt(1 - x)) + 5*sqrt(x + 1)/sqrt(1 - x), True)
)
```

3.1040 $\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{1-x}\,\sqrt{x+1}\,+\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {50, 41, 216}

$$\sqrt{1-x}\,\sqrt{x+1}\,+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/Sqrt[1 + x],x]
[Out] Sqrt[1 - x]*Sqrt[1 + x] + ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx &= \sqrt{1-x}\,\sqrt{1+x}\,+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= \sqrt{1-x}\,\sqrt{1+x}\,+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= \sqrt{1-x}\,\sqrt{1+x}\,+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.50

$$\sqrt{1-x^2}\,-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/Sqrt[1 + x],x]
[Out] Sqrt[1 - x^2] - 2*ArcSin[Sqrt[1 - x]/Sqrt[2]]
```

IntegrateAlgebraic [C] time = 0.09, size = 44, normalized size = 2.20

$$\sqrt{1-x}\,\sqrt{x+1}+2i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/Sqrt[1 + x],x]
[Out] Sqrt[1 - x]*Sqrt[1 + x] + (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

fricas [B] time = 1.28, size = 36, normalized size = 1.80

$$\sqrt{x+1}\,\sqrt{-x+1}-2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] sqrt(x + 1)*sqrt(-x + 1) - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [A] time = 0.65, size = 27, normalized size = 1.35

$$\sqrt{x+1}\,\sqrt{-x+1}+2\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.00, size = 41, normalized size = 2.05

$$\frac{\sqrt{(x+1)(-x+1)}\,\arcsin(x)}{\sqrt{x+1}\,\sqrt{-x+1}}+\sqrt{-x+1}\,\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(1/2),x)
[Out] (-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcs
in(x)
```

maxima [A] time = 3.10, size = 12, normalized size = 0.60

$$\sqrt{-x^2+1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] sqrt(-x^2 + 1) + arcsin(x)
```

mupad [B] time = 0.12, size = 12, normalized size = 0.60

$$\mathrm{asin}(x)+\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(1/2),x)
```

```
[Out] asin(x) + (1 - x^2)^(1/2)
```

sympy [B]   time = 1.55, size = 100, normalized size = 5.00

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) + \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}} - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} & \text{for } \frac{|x+1|}{2} > 1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right) - \frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}} + \frac{2\sqrt{x+1}}{\sqrt{1-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(3/2)/sqrt(x - 1)
 - 2*I*sqrt(x + 1)/sqrt(x - 1), Abs(x + 1)/2 > 1), (2*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(3/2)/sqrt(1 - x) + 2*sqrt(x + 1)/sqrt(1 - x), True))
```

3.1041 $\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {41, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx &= \int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] ArcSin[x]
```

IntegrateAlgebraic [B] time = 0.04, size = 20, normalized size = 10.00

$$-2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*Sqrt[1 + x]),x]
```

```
[Out] -2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [B] time = 0.83, size = 22, normalized size = 11.00

$$-2\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)
```

giac [B] time = 0.65, size = 13, normalized size = 6.50

$$2 \arcsin\left(\frac{1}{2} \sqrt{2} \sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.00, size = 27, normalized size = 13.50

$$\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] ((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.95, size = 2, normalized size = 1.00

$$\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)
```

mupad [B] time = 0.08, size = 22, normalized size = 11.00

$$-4\,\mathrm{atan}\left(\frac{\sqrt{1-x}\ -1}{\sqrt{x+1}\ -1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] -4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1))
```

sympy [B] time = 1.04, size = 41, normalized size = 20.50

$$\begin{cases} -2i\,\mathrm{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for}\ \frac{|x+1|}{2} > 1 \\ 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (2*asin(sq
rt(2)*sqrt(x + 1)/2), True))
```

3.1042 $\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx = \frac{\sqrt{1+x}}{\sqrt{1-x}}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/Sqrt[1 - x]
```

fricas [A] time = 1.15, size = 23, normalized size = 1.35

$$\frac{x-\sqrt{x+1}\sqrt{-x+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] (x - sqrt(x + 1)*sqrt(-x + 1) - 1)/(x - 1)
```

giac [A] time = 0.68, size = 19, normalized size = 1.12

$$-\frac{\sqrt{x+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1)/(x - 1)
```

maple [A] time = 0.00, size = 14, normalized size = 0.82

$$\frac{\sqrt{x+1}}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(3/2)/(x+1)^(1/2),x)
```

```
[Out] (x+1)^(1/2)/(-x+1)^(1/2)
```

maxima [A] time = 2.98, size = 16, normalized size = 0.94

$$-\frac{\sqrt{-x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.28, size = 13, normalized size = 0.76

$$\frac{\sqrt{x+1}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x + 1)^(1/2)/(1 - x)^(1/2)
```

sympy [A] time = 0.94, size = 29, normalized size = 1.71

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{i}{\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I/sqrt(1 - 2/(x + 1
)), True))
```

3.1043 $\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{\sqrt{x+1}}{3\sqrt{1-x}}+\frac{\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(3*(1 - x)^(3/2)) + Sqrt[1 + x]/(3*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{1}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{3(1-x)^{3/2}}+\frac{\sqrt{1+x}}{3\sqrt{1-x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{(x-2)\sqrt{x+1}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
```

```
[Out] -1/3*((-2 + x)*Sqrt[1 + x])/(1 - x)^(3/2)
```

IntegrateAlgebraic [A] time = 0.06, size = 33, normalized size = 0.80

$$\frac{\sqrt{x+1}\left(\frac{x+1}{1-x}+3\right)}{6\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*Sqrt[1 + x]),x]
[Out] (Sqrt[1 + x]*(3 + (1 + x)/(1 - x)))/(6*Sqrt[1 - x])
```

fricas [A] time = 1.20, size = 39, normalized size = 0.95

$$\frac{2x^2-\sqrt{x+1}(x-2)\sqrt{-x+1}-4x+2}{3\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] 1/3*(2*x^2 - sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 4*x + 2)/(x^2 - 2*x + 1)
```

giac [A] time = 0.64, size = 22, normalized size = 0.54

$$-\frac{\sqrt{x+1}(x-2)\sqrt{-x+1}}{3(x-1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(x + 1)*(x - 2)*sqrt(-x + 1)/(x - 1)^2
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{\sqrt{x+1}\,(x-2)}{3(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(1/2),x)
[Out] -1/3*(x+1)^(1/2)*(-2+x)/(-x+1)^(3/2)
```

maxima [A] time = 3.12, size = 38, normalized size = 0.93

$$\frac{\sqrt{-x^2+1}}{3\left(x^2-2x+1\right)}-\frac{\sqrt{-x^2+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.31, size = 43, normalized size = 1.05

$$\frac{x\sqrt{1-x}+2\sqrt{1-x}-x^2\sqrt{1-x}}{3(x-1)^2\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(1/2)),x)
```

```
[Out] (x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x + 1
)^(1/2))
```

sympy [C] time = 2.25, size = 139, normalized size = 3.39

$$\begin{cases} \frac{i(x+1)}{3i\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6i\sqrt{-1+\frac{2}{x+1}}} - \frac{3i}{3i\sqrt{-1+\frac{2}{x+1}}\,(x+1)-6i\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{x+1}{-3i\sqrt{1-\frac{2}{x+1}}\,(x+1)+6i\sqrt{1-\frac{2}{x+1}}} + \frac{3}{-3i\sqrt{1-\frac{2}{x+1}}\,(x+1)+6i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((I*(x + 1)/(3*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*I*sqrt(-1 + 2/(x
 + 1))) - 3*I/(3*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 6*I*sqrt(-1 + 2/(x + 1)))
, 2/Abs(x + 1) > 1), (-(x + 1)/(-3*I*sqrt(1 - 2/(x + 1))*(x + 1) + 6*I*sqrt
(1 - 2/(x + 1))) + 3/(-3*I*sqrt(1 - 2/(x + 1))*(x + 1) + 6*I*sqrt(1 - 2/(x
+ 1))), True))
```

3.1044 $\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=61

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{15\sqrt{1-x}}+\frac{2\sqrt{x+1}}{15(1-x)^{3/2}}+\frac{\sqrt{x+1}}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(5*(1 - x)^(5/2)) + (2*Sqrt[1 + x])/(15*(1 - x)^(3/2)) + (2*Sqr
t[1 + x])/(15*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2}{5}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2}{15}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{5(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{15(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{15\sqrt{1-x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.49

$$\frac{\sqrt{x+1}\left(2x^2-6x+7\right)}{15(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(7 - 6*x + 2*x^2))/(15*(1 - x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 48, normalized size = 0.79

$$\frac{\sqrt{x+1}\left(\frac{3(x+1)^2}{(1-x)^2}+\frac{10(x+1)}{1-x}+15\right)}{60\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(15 + (10*(1 + x))/(1 - x) + (3*(1 + x)^2)/(1 - x)^2))/(60*Sqr
t[1 - x])
```

fricas [A] time = 1.18, size = 56, normalized size = 0.92

$$\frac{7\,x^3-21\,x^2-\left(2\,x^2-6\,x+7\right)\sqrt{x+1}\,\sqrt{-x+1}+21\,x-7}{15\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(7*x^3 - 21*x^2 - (2*x^2 - 6*x + 7)*sqrt(x + 1)*sqrt(-x + 1) + 21*x -
7)/(x^3 - 3*x^2 + 3*x - 1)
```

giac [A] time = 0.69, size = 29, normalized size = 0.48

$$-\frac{(2\,(x+1)(x-4)+15)\sqrt{x+1}\,\sqrt{-x+1}}{15\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/15*(2*(x + 1)*(x - 4) + 15)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

maple [A] time = 0.00, size = 25, normalized size = 0.41

$$\frac{\sqrt{x+1}\,\left(2x^2-6x+7\right)}{15\,(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(1/2),x)
```

```
[Out] 1/15*(x+1)^(1/2)*(2*x^2-6*x+7)/(-x+1)^(5/2)
```

maxima [A] time = 3.03, size = 64, normalized size = 1.05

$$-\frac{\sqrt{-x^2+1}}{5\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\,\sqrt{-x^2+1}}{15\left(x^2-2\,x+1\right)}-\frac{2\,\sqrt{-x^2+1}}{15\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*sqrt(-x^2 + 1)/(x^3 - 3*x^2 + 3*x - 1) + 2/15*sqrt(-x^2 + 1)/(x^2 - 2*
x + 1) - 2/15*sqrt(-x^2 + 1)/(x - 1)
```

mupad [B] time = 0.32, size = 55, normalized size = 0.90

$$-\frac{x\sqrt{1-x}+7\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{15(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 7*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(15*(x - 1)^3*(x + 1)^(1/2))
```

sympy [C] time = 7.89, size = 332, normalized size = 5.44

$$\begin{cases} -\frac{2i(x+1)^2}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}}+\frac{10i(x+1)}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}}-\frac{15i}{-15i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+60i\sqrt{-1+\frac{2}{x+1}}(x+1)-60i\sqrt{-1+\frac{2}{x+1}}} & \text{for } \frac{2}{|x+1|}>1 \\ \frac{2(x+1)^2}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}}-\frac{10(x+1)}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}}+\frac{15}{15i\sqrt{1-\frac{2}{x+1}}(x+1)^2-60i\sqrt{1-\frac{2}{x+1}}(x+1)+60i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*(x + 1)**2/(-15*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60*I*sq
rt(-1 + 2/(x + 1))*(x + 1) - 60*I*sqrt(-1 + 2/(x + 1))) + 10*I*(x + 1)/(-15
*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 60
*I*sqrt(-1 + 2/(x + 1))) - 15*I/(-15*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 60
*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 60*I*sqrt(-1 + 2/(x + 1))), 2/Abs(x + 1)
> 1), (2*(x + 1)**2/(15*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/
(x + 1))*(x + 1) + 60*I*sqrt(1 - 2/(x + 1))) - 10*(x + 1)/(15*I*sqrt(1 - 2/
(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/(x + 1))*(x + 1) + 60*I*sqrt(1 - 2/(x
 + 1))) + 15/(15*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 60*I*sqrt(1 - 2/(x + 1)
)*(x + 1) + 60*I*sqrt(1 - 2/(x + 1))), True))
```

3.1045 $\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=81

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Rubi [A] time = 0.01, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2\sqrt{x+1}}{35\sqrt{1-x}}+\frac{2\sqrt{x+1}}{35(1-x)^{3/2}}+\frac{3\sqrt{x+1}}{35(1-x)^{5/2}}+\frac{\sqrt{x+1}}{7(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(7*(1 - x)^(7/2)) + (3*Sqrt[1 + x])/(35*(1 - x)^(5/2)) + (2*Sqr
t[1 + x])/(35*(1 - x)^(3/2)) + (2*Sqrt[1 + x])/(35*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3}{7}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{6}{35}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2}{35}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\ &= \frac{\sqrt{1+x}}{7(1-x)^{7/2}}+\frac{3\sqrt{1+x}}{35(1-x)^{5/2}}+\frac{2\sqrt{1+x}}{35(1-x)^{3/2}}+\frac{2\sqrt{1+x}}{35\sqrt{1-x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 0.43

$$\frac{\sqrt{x+1}\left(-2x^3+8x^2-13x+12\right)}{35(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(12 - 13*x + 8*x^2 - 2*x^3))/(35*(1 - x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 62, normalized size = 0.77

$$\frac{\sqrt{x+1}\left(\frac{5(x+1)^3}{(1-x)^3}+\frac{21(x+1)^2}{(1-x)^2}+\frac{35(x+1)}{1-x}+35\right)}{280\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(35 + (35*(1 + x))/(1 - x) + (21*(1 + x)^2)/(1 - x)^2 + (5*(1
+ x)^3)/(1 - x)^3))/(280*Sqrt[1 - x])
```

fricas [A] time = 1.28, size = 71, normalized size = 0.88

$$\frac{12\,x^4-48\,x^3+72\,x^2-\left(2\,x^3-8\,x^2+13\,x-12\right)\sqrt{x+1}\,\sqrt{-x+1}-48\,x+12}{35\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/35*(12*x^4 - 48*x^3 + 72*x^2 - (2*x^3 - 8*x^2 + 13*x - 12)*sqrt(x + 1)*sq
rt(-x + 1) - 48*x + 12)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)
```

giac [A] time = 0.66, size = 35, normalized size = 0.43

$$-\frac{((2\,(x+1)(x-6)+35)(x+1)-35)\sqrt{x+1}\,\sqrt{-x+1}}{35\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*(x + 1)*(x - 6) + 35)*(x + 1) - 35)*sqrt(x + 1)*sqrt(-x + 1)/(x -
 1)^4
```

maple [A] time = 0.00, size = 30, normalized size = 0.37

$$-\frac{\sqrt{x+1}\,\left(2x^3-8x^2+13x-12\right)}{35\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(9/2)/(x+1)^(1/2),x)
```

```
[Out] -1/35*(x+1)^(1/2)*(2*x^3-8*x^2+13*x-12)/(-x+1)^(7/2)
```

maxima [A] time = 3.03, size = 95, normalized size = 1.17

$$\frac{\sqrt{-x^2+1}}{7\left(x^4-4\,x^3+6\,x^2-4\,x+1\right)}-\frac{3\,\sqrt{-x^2+1}}{35\left(x^3-3\,x^2+3\,x-1\right)}+\frac{2\,\sqrt{-x^2+1}}{35\left(x^2-2\,x+1\right)}-\frac{2\,\sqrt{-x^2+1}}{35\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(-x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 3/35*sqrt(-x^2 + 1)/(x
^3 - 3*x^2 + 3*x - 1) + 2/35*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 2/35*sqrt(-x^
2 + 1)/(x - 1)
```

mupad [B] time = 0.34, size = 67, normalized size = 0.83

$$-\frac{x\sqrt{1-x}-12\sqrt{1-x}+5x^2\sqrt{1-x}-6x^3\sqrt{1-x}+2x^4\sqrt{1-x}}{35(x-1)^4\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(1/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) - 12*(1 - x)^(1/2) + 5*x^2*(1 - x)^(1/2) - 6*x^3*(1 - x)^
(1/2) + 2*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

sympy [C] time = 22.13, size = 595, normalized size = 7.35

$$\begin{cases} \frac{2i(x+1)^3}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} - \frac{14i(x+1)^2}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} + \frac{35i(x+1)}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} - \frac{35i}{35i\sqrt{-1+\frac{2}{x+1}}(x+1)^3-210i\sqrt{-1+\frac{2}{x+1}}(x+1)^2+420i\sqrt{-1+\frac{2}{x+1}}(x+1)-280i\sqrt{-1+\frac{2}{x+1}}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{2(x+1)^3}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} + \frac{14(x+1)^2}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} - \frac{35(x+1)}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} + \frac{35}{-35i\sqrt{1-\frac{2}{x+1}}(x+1)^3+210i\sqrt{1-\frac{2}{x+1}}(x+1)^2-420i\sqrt{1-\frac{2}{x+1}}(x+1)+280i\sqrt{1-\frac{2}{x+1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((2*I*(x + 1)**3/(35*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 210*I*sqr
t(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 280*I*s
qrt(-1 + 2/(x + 1))) - 14*I*(x + 1)**2/(35*I*sqrt(-1 + 2/(x + 1))*(x + 1)**
3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x + 1))*(x +
 1) - 280*I*sqrt(-1 + 2/(x + 1))) + 35*I*(x + 1)/(35*I*sqrt(-1 + 2/(x + 1))
*(x + 1)**3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(x
+ 1))*(x + 1) - 280*I*sqrt(-1 + 2/(x + 1))) - 35*I/(35*I*sqrt(-1 + 2/(x + 1
))*(x + 1)**3 - 210*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 420*I*sqrt(-1 + 2/(
x + 1))*(x + 1) - 280*I*sqrt(-1 + 2/(x + 1))), 2/Abs(x + 1) > 1), (-2*(x +
1)**3/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x + 1))*(x
+ 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x + 1))) +
14*(x + 1)**2/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x +
 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x +
 1))) - 35*(x + 1)/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2
/(x + 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2
/(x + 1))) + 35/(-35*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 210*I*sqrt(1 - 2/(x
 + 1))*(x + 1)**2 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1) + 280*I*sqrt(1 - 2/(x
 + 1))), True))
```

3.1046 $\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx$

Optimal. Leaf size=101

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{8\sqrt{x+1}}{315\sqrt{1-x}}+\frac{8\sqrt{x+1}}{315(1-x)^{3/2}}+\frac{4\sqrt{x+1}}{105(1-x)^{5/2}}+\frac{4\sqrt{x+1}}{63(1-x)^{7/2}}+\frac{\sqrt{x+1}}{9(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 + x]/(9*(1 - x)^(9/2)) + (4*Sqrt[1 + x])/(63*(1 - x)^(7/2)) + (4*Sqr
t[1 + x])/(105*(1 - x)^(5/2)) + (8*Sqrt[1 + x])/(315*(1 - x)^(3/2)) + (8*Sq
rt[1 + x])/(315*Sqrt[1 - x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{11/2}\sqrt{1+x}}\,dx &= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4}{9}\int \frac{1}{(1-x)^{9/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4}{21}\int \frac{1}{(1-x)^{7/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8}{105}\int \frac{1}{(1-x)^{5/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8}{315}\int \frac{1}{(1-x)^{3/2}\sqrt{1+x}}\,dx\\
&= \frac{\sqrt{1+x}}{9(1-x)^{9/2}}+\frac{4\sqrt{1+x}}{63(1-x)^{7/2}}+\frac{4\sqrt{1+x}}{105(1-x)^{5/2}}+\frac{8\sqrt{1+x}}{315(1-x)^{3/2}}+\frac{8\sqrt{1+x}}{315\sqrt{1-x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.40

$$\frac{\sqrt{x+1}\left(8x^4-40x^3+84x^2-100x+83\right)}{315(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(83 - 100*x + 84*x^2 - 40*x^3 + 8*x^4))/(315*(1 - x)^(9/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 76, normalized size = 0.75

$$\frac{\sqrt{x+1}\left(\frac{35(x+1)^4}{(1-x)^4}+\frac{180(x+1)^3}{(1-x)^3}+\frac{378(x+1)^2}{(1-x)^2}+\frac{420(x+1)}{1-x}+315\right)}{5040\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 + x]*(315 + (420*(1 + x))/(1 - x) + (378*(1 + x)^2)/(1 - x)^2 + (18
0*(1 + x)^3)/(1 - x)^3 + (35*(1 + x)^4)/(1 - x)^4))/(5040*Sqrt[1 - x])
```

fricas [A] time = 1.22, size = 86, normalized size = 0.85

$$\frac{83\,x^5-415\,x^4+830\,x^3-830\,x^2-\left(8\,x^4-40\,x^3+84\,x^2-100\,x+83\right)\sqrt{x+1}\,\sqrt{-x+1}+415\,x-83}{315\left(x^5-5\,x^4+10\,x^3-10\,x^2+5\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(83*x^5 - 415*x^4 + 830*x^3 - 830*x^2 - (8*x^4 - 40*x^3 + 84*x^2 - 10
0*x + 83)*sqrt(x + 1)*sqrt(-x + 1) + 415*x - 83)/(x^5 - 5*x^4 + 10*x^3 - 10
*x^2 + 5*x - 1)
```

giac [A] time = 0.67, size = 42, normalized size = 0.42

$$-\frac{(4\,((2\,(x+1)(x-8)+63)(x+1)-105)(x+1)+315)\sqrt{x+1}\,\sqrt{-x+1}}{315\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/315*(4*((2*(x + 1)*(x - 8) + 63)*(x + 1) - 105)*(x + 1) + 315)*sqrt(x +
1)*sqrt(-x + 1)/(x - 1)^5
```

maple [A] time = 0.00, size = 35, normalized size = 0.35

$$\frac{\sqrt{x+1}\;\left(8x^4-40x^3+84x^2-100x+83\right)}{315\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(1/2),x)
```

```
[Out] 1/315*(x+1)^(1/2)*(8*x^4-40*x^3+84*x^2-100*x+83)/(-x+1)^(9/2)
```

maxima [A] time = 3.08, size = 131, normalized size = 1.30

$$-\frac{\sqrt{-x^2+1}}{9\left(x^5-5x^4+10x^3-10x^2+5x-1\right)}+\frac{4\sqrt{-x^2+1}}{63\left(x^4-4x^3+6x^2-4x+1\right)}-\frac{4\sqrt{-x^2+1}}{105\left(x^3-3x^2+3x-1\right)}+\frac{8\sqrt{-x^2+1}}{315\left(x^2-2x+1\right)}-\frac{8\sqrt{-x^2+1}}{315\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(-x^2 + 1)/(x^5 - 5*x^4 + 10*x^3 - 10*x^2 + 5*x - 1) + 4/63*sqrt(-
x^2 + 1)/(x^4 - 4*x^3 + 6*x^2 - 4*x + 1) - 4/105*sqrt(-x^2 + 1)/(x^3 - 3*x^
2 + 3*x - 1) + 8/315*sqrt(-x^2 + 1)/(x^2 - 2*x + 1) - 8/315*sqrt(-x^2 + 1)/
(x - 1)
```

mupad [B] time = 0.36, size = 80, normalized size = 0.79

$$\frac{17\,x\,\sqrt{1-x}-83\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{315\,(x-1)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(1/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 83*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(315*(x - 1)^5*(x +
 1)^(1/2))
```

sympy [C] time = 58.39, size = 933, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-8*I*(x + 1)**4/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I
*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 +
 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) + 72*I
*(x + 1)**3/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x
 + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-
1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) - 252*I*(x + 1)**2/(-
315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x + 1))*(x + 1)
**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-1 + 2/(x + 1))
*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))) + 420*I*(x + 1)/(-315*I*sqrt(-1 + 2
/(x + 1))*(x + 1)**4 + 2520*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt
(-1 + 2/(x + 1))*(x + 1)**2 + 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I
*sqrt(-1 + 2/(x + 1))) - 315*I/(-315*I*sqrt(-1 + 2/(x + 1))*(x + 1)**4 + 25
20*I*sqrt(-1 + 2/(x + 1))*(x + 1)**3 - 7560*I*sqrt(-1 + 2/(x + 1))*(x + 1)*
*2 + 10080*I*sqrt(-1 + 2/(x + 1))*(x + 1) - 5040*I*sqrt(-1 + 2/(x + 1))), 2
/Abs(x + 1) > 1), (8*(x + 1)**4/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 252
0*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2
- 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) - 72*(x
 + 1)**3/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))
*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x
 + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) + 252*(x + 1)**2/(315*I*sqrt(1
 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*s
qrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*
I*sqrt(1 - 2/(x + 1))) - 420*(x + 1)/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4
- 2520*I*sqrt(1 - 2/(x + 1))*(x + 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2 - 10080*I*sqrt(1 - 2/(x + 1))*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))) +
315/(315*I*sqrt(1 - 2/(x + 1))*(x + 1)**4 - 2520*I*sqrt(1 - 2/(x + 1))*(x +
 1)**3 + 7560*I*sqrt(1 - 2/(x + 1))*(x + 1)**2 - 10080*I*sqrt(1 - 2/(x + 1)
)*(x + 1) + 5040*I*sqrt(1 - 2/(x + 1))), True))
```

3.1047 $\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=85

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{\sqrt{x+1}}-\frac{7}{3}\sqrt{x+1}\,(1-x)^{5/2}-\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}-\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/Sqrt[1 + x] - (35*Sqrt[1 - x]*Sqrt[1 + x])/2 - (35*(1 -
x)^(3/2)*Sqrt[1 + x])/6 - (7*(1 - x)^(5/2)*Sqrt[1 + x])/3 - (35*ArcSin[x])/
2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx &= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - 7\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{\sqrt{1+x}} - \frac{35}{2}\sqrt{1-x}\sqrt{1+x} - \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} - \frac{7}{3}(1-x)^{5/2}\sqrt{1+x} - \frac{35}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.44

$$
-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{9\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(3/2),x]
[Out] -1/9*((1 - x)^(9/2)*Hypergeometric2F1[3/2, 9/2, 11/2, (1 - x)/2])/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.10, size = 98, normalized size = 1.15

$$
35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) - \frac{\sqrt{1-x}\left(\frac{48(1-x)^3}{(x+1)^3}+\frac{231(1-x)^2}{(x+1)^2}+\frac{280(1-x)}{x+1}+105\right)}{3\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/(1 + x)^(3/2),x]
[Out] -1/3*(Sqrt[1 - x]*(105 + (48*(1 - x)^3)/(1 + x)^3 + (231*(1 - x)^2)/(1 + x)
^2 + (280*(1 - x))/(1 + x)))/(Sqrt[1 + x]*(1 + (1 - x)/(1 + x))^3) + 35*Arc
Tan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.04, size = 65, normalized size = 0.76

$$
-\frac{\left(2x^3-13x^2+55x+166\right)\sqrt{x+1}\sqrt{-x+1}-210(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+166x+166}{6(x+1)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
[Out] -1/6*((2*x^3 - 13*x^2 + 55*x + 166)*sqrt(x + 1)*sqrt(-x + 1) - 210*(x + 1)*
arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 166*x + 166)/(x + 1)
```

giac [A] time = 0.75, size = 81, normalized size = 0.95

$$
-\frac{1}{6}\left((2x-17)(x+1)+87\right)\sqrt{x+1}\sqrt{-x+1}+\frac{8\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}-\frac{8\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-35\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/6*((2*x - 17)*(x + 1) + 87)*sqrt(x + 1)*sqrt(-x + 1) + 8*(sqrt(2) - sqrt
(-x + 1))/sqrt(x + 1) - 8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 35*arcsin(
1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 84, normalized size = 0.99

$$-\frac{35\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}+\frac{\left(2x^4-15x^3+68x^2+111x-166\right)\sqrt{(x+1)(-x+1)}}{6\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(3/2),x)
```

```
[Out] 1/6*(2*x^4-15*x^3+68*x^2+111*x-166)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/
2)/(-x+1)^(1/2)/(x+1)^(1/2)-35/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1
/2)*arcsin(x)
```

maxima [A] time = 2.84, size = 70, normalized size = 0.82

$$\frac{x^4}{3\sqrt{-x^2+1}}-\frac{5\,x^3}{2\sqrt{-x^2+1}}+\frac{34\,x^2}{3\sqrt{-x^2+1}}+\frac{37\,x}{2\sqrt{-x^2+1}}-\frac{83}{3\sqrt{-x^2+1}}-\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^4/sqrt(-x^2 + 1) - 5/2*x^3/sqrt(-x^2 + 1) + 34/3*x^2/sqrt(-x^2 + 1) +
 37/2*x/sqrt(-x^2 + 1) - 83/3/sqrt(-x^2 + 1) - 35/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(3/2), x)
```

sympy [A] time = 17.48, size = 207, normalized size = 2.44

$$\begin{cases} 35i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}+\frac{23i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}-\frac{125i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{13i\sqrt{x+1}}{\sqrt{x-1}}+\frac{32i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}-\frac{23(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}+\frac{125(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{13\sqrt{x+1}}{\sqrt{1-x}}-\frac{32}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((35*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(7/2)/(3*sqrt(x -
 1)) + 23*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) - 125*I*(x + 1)**(3/2)/(6*sqrt(x
 - 1)) + 13*I*sqrt(x + 1)/sqrt(x - 1) + 32*I/(sqrt(x - 1)*sqrt(x + 1)), Abs
(x + 1)/2 > 1), (-35*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(7/2)/(3*sqrt(1
 - x)) - 23*(x + 1)**(5/2)/(6*sqrt(1 - x)) + 125*(x + 1)**(3/2)/(6*sqrt(1 -
 x)) - 13*sqrt(x + 1)/sqrt(1 - x) - 32/(sqrt(1 - x)*sqrt(x + 1)), True))
```

3.1048 $\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx$

Optimal. Leaf size=65

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{\sqrt{x+1}}-\frac{5}{2}\sqrt{x+1}\,(1-x)^{3/2}-\frac{15}{2}\sqrt{x+1}\,\sqrt{1-x}-\frac{15}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/Sqrt[1 + x] - (15*Sqrt[1 - x]*Sqrt[1 + x])/2 - (5*(1 - x
)^(3/2)*Sqrt[1 + x])/2 - (15*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx = -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - 5\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2(1-x)^{5/2}}{\sqrt{1+x}} - \frac{15}{2}\sqrt{1-x}\sqrt{1+x} - \frac{5}{2}(1-x)^{3/2}\sqrt{1+x} - \frac{15}{2}\sin^{-1}(x)$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.57

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{7\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] -1/7*((1 - x)^(7/2)*Hypergeometric2F1[3/2, 7/2, 9/2, (1 - x)/2])/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.09, size = 82, normalized size = 1.26

$$15\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right) - \frac{\sqrt{1-x}\left(\frac{8(1-x)^2}{(x+1)^2}+\frac{25(1-x)}{x+1}+15\right)}{\sqrt{x+1}\left(\frac{1-x}{x+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/(1 + x)^(3/2),x]
```

```
[Out] -((Sqrt[1 - x]*(15 + (8*(1 - x)^2)/(1 + x)^2 + (25*(1 - x))/(1 + x)))/(Sqrt
[1 + x]*(1 + (1 - x)/(1 + x))^2)) + 15*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.32, size = 58, normalized size = 0.89

$$\frac{\left(x^2-7x-24\right)\sqrt{x+1}\sqrt{-x+1} + 30(x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) - 24x - 24}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 7*x - 24)*sqrt(x + 1)*sqrt(-x + 1) + 30*(x + 1)*arctan((sqrt(x
+ 1)*sqrt(-x + 1) - 1)/x) - 24*x - 24)/(x + 1)
```

giac [A] time = 0.79, size = 73, normalized size = 1.12

$$\frac{1}{2}\sqrt{x+1}(x-8)\sqrt{-x+1} + \frac{4\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{4\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}} - 15\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x + 1)*(x - 8)*sqrt(-x + 1) + 4*(sqrt(2) - sqrt(-x + 1))/sqrt(x +
1) - 4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 15*arcsin(1/2*sqrt(2)*sqrt(x
+ 1))
```

maple [A] time = 0.02, size = 77, normalized size = 1.18

$$-\frac{15\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}-\frac{\left(x^3-8x^2-17x+24\right)\sqrt{(x+1)(-x+1)}}{2\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(3/2),x)
```

```
[Out] -1/2*(x^3-8*x^2-17*x+24)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(
1/2)/(x+1)^(1/2)-15/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

maxima [A] time = 2.99, size = 56, normalized size = 0.86

$$-\frac{x^3}{2\sqrt{-x^2+1}}+\frac{4x^2}{\sqrt{-x^2+1}}+\frac{17x}{2\sqrt{-x^2+1}}-\frac{12}{\sqrt{-x^2+1}}-\frac{15}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^3/sqrt(-x^2 + 1) + 4*x^2/sqrt(-x^2 + 1) + 17/2*x/sqrt(-x^2 + 1) - 12
/sqrt(-x^2 + 1) - 15/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{5/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(3/2), x)
```

sympy [A] time = 6.99, size = 168, normalized size = 2.58

$$\begin{cases} 15i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{x-1}}-\frac{11i(x+1)^{\frac{3}{2}}}{2\sqrt{x-1}}+\frac{i\sqrt{x+1}}{\sqrt{x-1}}+\frac{16i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -15\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{1-x}}+\frac{11(x+1)^{\frac{3}{2}}}{2\sqrt{1-x}}-\frac{\sqrt{x+1}}{\sqrt{1-x}}-\frac{16}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((15*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(5/2)/(2*sqrt(x -
 1)) - 11*I*(x + 1)**(3/2)/(2*sqrt(x - 1)) + I*sqrt(x + 1)/sqrt(x - 1) + 16
*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-15*asin(sqrt(2)*sqrt(x +
 1)/2) - (x + 1)**(5/2)/(2*sqrt(1 - x)) + 11*(x + 1)**(3/2)/(2*sqrt(1 - x))
 - sqrt(x + 1)/sqrt(1 - x) - 16/(sqrt(1 - x)*sqrt(x + 1)), True))
```

3.1049 $\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}} dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}} - 3\sqrt{x+1}\sqrt{1-x} - 3\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{3/2}}{\sqrt{x+1}} - 3\sqrt{x+1}\sqrt{1-x} - 3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/Sqrt[1 + x] - 3*Sqrt[1 - x]*Sqrt[1 + x] - 3*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx = -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\,\sqrt{1+x} - 3\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\,\sqrt{1+x} - 3\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{2(1-x)^{3/2}}{\sqrt{1+x}} - 3\sqrt{1-x}\,\sqrt{1+x} - 3\sin^{-1}(x)$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.90

$$-\frac{(1-x)^{5/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{1-x}{2}\right)}{5\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] -1/5*((1 - x)^(5/2)*Hypergeometric2F1[3/2, 5/2, 7/2, (1 - x)/2])/Sqrt[2]
```

IntegrateAlgebraic [C] time = 0.13, size = 49, normalized size = 1.20

$$\frac{(-x-5)\sqrt{1-x}}{\sqrt{x+1}} - 6i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/(1 + x)^(3/2),x]
```

```
[Out] ((-5 - x)*Sqrt[1 - x])/Sqrt[1 + x] - (6*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

fricas [A] time = 1.39, size = 53, normalized size = 1.29

$$-\frac{(x+5)\sqrt{x+1}\,\sqrt{-x+1} - 6\,(x+1)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right) + 5\,x + 5}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x + 5)*sqrt(x + 1)*sqrt(-x + 1) - 6*(x + 1)*arctan((sqrt(x + 1)*sqrt(-x
+ 1) - 1)/x) + 5*x + 5)/(x + 1)
```

giac [B] time = 0.73, size = 70, normalized size = 1.71

$$-\sqrt{x+1}\,\sqrt{-x+1} + \frac{2\left(\sqrt{2} - \sqrt{-x+1}\right)}{\sqrt{x+1}} - \frac{2\,\sqrt{x+1}}{\sqrt{2} - \sqrt{-x+1}} - 6\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(x + 1)*sqrt(-x + 1) + 2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 2*sqrt
(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 6*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.02, size = 71, normalized size = 1.73

$$-\frac{3\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}+\frac{\left(x^2+4x-5\right)\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] (x^2+4*x-5)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1
/2)-3*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.86, size = 41, normalized size = 1.00

$$\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{x^2+2\,x+1}-\frac{6\,\sqrt{-x^2+1}}{x+1}-3\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(3/2)/(x^2 + 2*x + 1) - 6*sqrt(-x^2 + 1)/(x + 1) - 3*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(1-x)^{3/2}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(3/2), x)
```

sympy [A] time = 2.48, size = 133, normalized size = 3.24

$$\begin{cases} 6i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)-\frac{i(x+1)^{\frac{3}{2}}}{\sqrt{x-1}}-\frac{2i\sqrt{x+1}}{\sqrt{x-1}}+\frac{8i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2}>1 \\ -6\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)+\frac{(x+1)^{\frac{3}{2}}}{\sqrt{1-x}}+\frac{2\sqrt{x+1}}{\sqrt{1-x}}-\frac{8}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((6*I*acosh(sqrt(2)*sqrt(x + 1)/2) - I*(x + 1)**(3/2)/sqrt(x - 1)
- 2*I*sqrt(x + 1)/sqrt(x - 1) + 8*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2
 > 1), (-6*asin(sqrt(2)*sqrt(x + 1)/2) + (x + 1)**(3/2)/sqrt(1 - x) + 2*sqr
t(x + 1)/sqrt(1 - x) - 8/(sqrt(1 - x)*sqrt(x + 1)), True))
```

3.1050 $\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2\sqrt{1-x}}{\sqrt{x+1}}-\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] - ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-x}}{(1+x)^{3/2}} \, dx &= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \, dx\\ &= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\int \frac{1}{\sqrt{1-x^2}} \, dx\\ &= -\frac{2\sqrt{1-x}}{\sqrt{1+x}}-\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 34, normalized size = 1.48

$$2\left(\frac{x-1}{\sqrt{1-x^2}}+\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] 2*((-1 + x)/Sqrt[1 - x^2] + ArcSin[Sqrt[1 - x]/Sqrt[2]])
```

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 1.70

$$2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)-\frac{2\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + x)^(3/2),x]
```

```
[Out] (-2*Sqrt[1 - x])/Sqrt[1 + x] + 2*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [B] time = 0.77, size = 50, normalized size = 2.17

$$\frac{2\left((x+1)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-x-\sqrt{x+1}\sqrt{-x+1}-1\right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*((x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - x - sqrt(x + 1)*sqrt(
-x + 1) - 1)/(x + 1)
```

giac [B] time = 0.69, size = 55, normalized size = 2.39

$$\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}}-\frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}-2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
 - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.02, size = 67, normalized size = 2.91

$$-\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}+\frac{2(x-1)\sqrt{(x+1)(-x+1)}}{\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}\ \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(3/2),x)
```

```
[Out] 2*(x-1)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)-
((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 2.86, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{-x^2+1}}{x+1}-\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + 1)/(x + 1) - arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x}}{(x+1)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(3/2),x)
```

```
[Out] int((1 - x)^(1/2)/(x + 1)^(3/2), x)
```

sympy [B] time = 1.54, size = 104, normalized size = 4.52

$$\begin{cases} 2i\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) - \frac{2i\sqrt{x+1}}{\sqrt{x-1}} + \frac{4i}{\sqrt{x-1}\sqrt{x+1}} & \text{for}\ \frac{|x+1|}{2} > 1 \\ -2\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) + \frac{2\sqrt{x+1}}{\sqrt{1-x}} - \frac{4}{\sqrt{1-x}\sqrt{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*I*acosh(sqrt(2)*sqrt(x + 1)/2) - 2*I*sqrt(x + 1)/sqrt(x - 1) +
 4*I/(sqrt(x - 1)*sqrt(x + 1)), Abs(x + 1)/2 > 1), (-2*asin(sqrt(2)*sqrt(x
+ 1)/2) + 2*sqrt(x + 1)/sqrt(1 - x) - 4/(sqrt(1 - x)*sqrt(x + 1)), True))
```

3.1051 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx = -\frac{\sqrt{1-x}}{\sqrt{1+x}}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 1.00

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*(1 + x)^(3/2)),x]
```

```
[Out] -(Sqrt[1 - x]/Sqrt[1 + x])
```

fricas [A] time = 1.02, size = 23, normalized size = 1.28

$$-\frac{x+\sqrt{x+1}\,\sqrt{-x+1}\,+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x + 1)*sqrt(-x + 1) + 1)/(x + 1)
```

giac [B] time = 0.65, size = 43, normalized size = 2.39

$$\frac{\sqrt{2}-\sqrt{-x+1}}{2\sqrt{x+1}}-\frac{\sqrt{x+1}}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)/(sqrt(2) - sqrt(
-x + 1))
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$-\frac{\sqrt{-x+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(3/2),x)
```

```
[Out] -(-x+1)^(1/2)/(x+1)^(1/2)
```

maxima [A] time = 2.94, size = 16, normalized size = 0.89

$$-\frac{\sqrt{-x^2+1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/(x + 1)
```

mupad [B] time = 0.36, size = 14, normalized size = 0.78

$$-\frac{\sqrt{1-x}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(1 - x)^(1/2)/(x + 1)^(1/2)
```

sympy [A] time = 1.20, size = 29, normalized size = 1.61

$$\begin{cases} -\sqrt{-1+\frac{2}{x+1}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -i\sqrt{1-\frac{2}{x+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1)), 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1)
), True))
```

3.1052 $\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=18

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/(Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx = \frac{x}{\sqrt{1-x}\,\sqrt{1+x}}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 0.72

$$\frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] x/Sqrt[1 - x^2]
```

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 1.89

$$\frac{\sqrt{x+1}\left(1-\frac{1-x}{x+1}\right)}{2\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*(1 + x)^(3/2)),x]
```

```
[Out] (Sqrt[1 + x]*(1 - (1 - x)/(1 + x)))/(2*Sqrt[1 - x])
```

fricas [A] time = 1.24, size = 22, normalized size = 1.22

$$-\frac{\sqrt{x+1}\,x\sqrt{-x+1}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(x + 1)*x*sqrt(-x + 1)/(x^2 - 1)
```

giac [B] time = 0.72, size = 62, normalized size = 3.44

$$\frac{\sqrt{2}-\sqrt{-x+1}}{4\sqrt{x+1}}-\frac{\sqrt{x+1}\sqrt{-x+1}}{2(x-1)}-\frac{\sqrt{x+1}}{4\left(\sqrt{2}-\sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/2*sqrt(x + 1)*sqrt(-x + 1)/(x
- 1) - 1/4*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{x}{\sqrt{-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(3/2)/(x+1)^(3/2),x)
```

```
[Out] x/(-x+1)^(1/2)/(x+1)^(1/2)
```

maxima [A] time = 1.34, size = 11, normalized size = 0.61

$$\frac{x}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(3/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 1)
```

mupad [B] time = 0.31, size = 14, normalized size = 0.78

$$\frac{x}{\sqrt{1-x}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(3/2)),x)
```

```
[Out] x/((1 - x)^(1/2)*(x + 1)^(1/2))
```

sympy [A] time = 1.86, size = 65, normalized size = 3.61

$$\begin{cases} \frac{1}{\sqrt{-1+\frac{2}{x+1}}}-\frac{1}{\sqrt{-1+\frac{2}{x+1}}(x+1)} & \text{for } \frac{2}{|x+1|}>1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}(x+1)}{x-1}+\frac{i\sqrt{1-\frac{2}{x+1}}}{x-1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((1/sqrt(-1 + 2/(x + 1)) - 1/(sqrt(-1 + 2/(x + 1))*(x + 1)), 2/Abs
(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)/(x - 1) + I*sqrt(1 - 2/(x + 1
))/(x - 1), True))
```

3.1053 $\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{1}{3(1-x)^{3/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(3*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx&=\frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\&=\frac{1}{3(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{3\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.71

$$\frac{2x^2-2x-1}{3(x-1)\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] (-1 - 2*x + 2*x^2)/(3*(-1 + x)*Sqrt[1 - x^2])
```

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 1.14

$$\frac{(x+1)^{3/2}\left(-\frac{3(1-x)^2}{(x+1)^2}+\frac{6(1-x)}{x+1}+1\right)}{12(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(3/2)*(1 - (3*(1 - x)^2)/(1 + x)^2 + (6*(1 - x))/(1 + x)))/(12*(1
 - x)^(3/2))
```

fricas [A] time = 1.22, size = 54, normalized size = 1.29

$$\frac{x^3 - x^2 - \left(2x^2 - 2x - 1\right)\sqrt{x+1}\,\sqrt{-x+1} - x + 1}{3\left(x^3 - x^2 - x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 - x^2 - (2*x^2 - 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x + 1)/(x^3 -
 x^2 - x + 1)
```

giac [B] time = 0.70, size = 67, normalized size = 1.60

$$\frac{\sqrt{2} - \sqrt{-x+1}}{8\sqrt{x+1}} - \frac{(5x-7)\sqrt{x+1}\,\sqrt{-x+1}}{12\,(x-1)^2} - \frac{\sqrt{x+1}}{8\left(\sqrt{2} - \sqrt{-x+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/12*(5*x - 7)*sqrt(x + 1)*sqrt(
-x + 1)/(x - 1)^2 - 1/8*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1))
```

maple [A] time = 0.00, size = 25, normalized size = 0.60

$$-\frac{2x^2 - 2x - 1}{3\sqrt{x+1}\,\left(-x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(3/2),x)
```

```
[Out] -1/3*(2*x^2-2*x-1)/(x+1)^(1/2)/(-x+1)^(3/2)
```

maxima [A] time = 1.42, size = 40, normalized size = 0.95

$$\frac{2x}{3\sqrt{-x^2+1}} - \frac{1}{3\left(\sqrt{-x^2+1}\,x - \sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

mupad [B] time = 0.32, size = 42, normalized size = 1.00

$$\frac{2x\sqrt{1-x} + \sqrt{1-x} - 2x^2\sqrt{1-x}}{3\,(x-1)^2\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(3/2)),x)
```

[Out] (2*x*(1 - x)^(1/2) + (1 - x)^(1/2) - 2*x^2*(1 - x)^(1/2))/(3*(x - 1)^2*(x + 1)^(1/2))

sympy [B] time = 5.28, size = 158, normalized size = 3.76

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2} + \frac{6\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2} - \frac{3\sqrt{-1+\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-12x+3(x+1)^2} + \frac{6i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-12x+3(x+1)^2} - \frac{3i\sqrt{1-\frac{2}{x+1}}}{-12x+3(x+1)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-12*x + 3*(x + 1)**2) + 6*sq
rt(-1 + 2/(x + 1))*(x + 1)/(-12*x + 3*(x + 1)**2) - 3*sqrt(-1 + 2/(x + 1))/
(-12*x + 3*(x + 1)**2), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(-12*x + 3*(x + 1)**2) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-12*x + 3*(x
 + 1)**2) - 3*I*sqrt(1 - 2/(x + 1))/(-12*x + 3*(x + 1)**2), True))
```

3.1054 $\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}} dx$

Optimal. Leaf size=62

$$\frac{2x}{5\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{2x}{5\sqrt{1-x}\,\sqrt{x+1}}+\frac{1}{5(1-x)^{3/2}\sqrt{x+1}}+\frac{1}{5(1-x)^{5/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*Sqrt[1 + x]) + 1/(5*(1 - x)^(3/2)*Sqrt[1 + x]) + (2*x)/(
5*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx&=\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{3}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}}\,dx\\&=\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2}{5}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\&=\frac{1}{5(1-x)^{5/2}\sqrt{1+x}}+\frac{1}{5(1-x)^{3/2}\sqrt{1+x}}+\frac{2x}{5\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.53

$$\frac{2x^3-4x^2+x+2}{5(x-1)^2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] (2 + x - 4*x^2 + 2*x^3)/(5*(-1 + x)^2*Sqrt[1 - x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 62, normalized size = 1.00

$$\frac{(x+1)^{5/2}\left(-\frac{5(1-x)^3}{(x+1)^3}+\frac{15(1-x)^2}{(x+1)^2}+\frac{5(1-x)}{x+1}+1\right)}{40(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(5/2)*(1 - (5*(1 - x)^3)/(1 + x)^3 + (15*(1 - x)^2)/(1 + x)^2 + (5
*(1 - x))/(1 + x)))/(40*(1 - x)^(5/2))
```

fricas [A] time = 1.04, size = 59, normalized size = 0.95

$$\frac{2\,x^4-4\,x^3-\left(2\,x^3-4\,x^2+x+2\right)\sqrt{x+1}\,\sqrt{-x+1}+4\,x-2}{5\left(x^4-2\,x^3+2\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*x^4 - 4*x^3 - (2*x^3 - 4*x^2 + x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x
 - 2)/(x^4 - 2*x^3 + 2*x - 1)
```

giac [A] time = 0.66, size = 73, normalized size = 1.18

$$\frac{\sqrt{2}-\sqrt{-x+1}}{16\,\sqrt{x+1}}-\frac{\sqrt{x+1}}{16\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((11\,x-39)(x+1)+60)\sqrt{x+1}\,\sqrt{-x+1}}{40\,(x-1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/16*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/40*((11*x - 39)*(x + 1) + 60)*sqrt(x + 1)*sqrt(-x + 1)/(x -
1)^3
```

maple [A] time = 0.00, size = 28, normalized size = 0.45

$$\frac{2x^3-4x^2+x+2}{5\sqrt{x+1}\,\left(-x+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(3/2),x)
```

```
[Out] 1/5*(2*x^3-4*x^2+x+2)/(x+1)^(1/2)/(-x+1)^(5/2)
```

maxima [A] time = 1.35, size = 79, normalized size = 1.27

$$\frac{2\,x}{5\,\sqrt{-x^2+1}}+\frac{1}{5\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{1}{5\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x/sqrt(-x^2 + 1) + 1/5/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(
-x^2 + 1)) - 1/5/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

mupad [B] time = 0.34, size = 55, normalized size = 0.89

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}-4x^2\sqrt{1-x}+2x^3\sqrt{1-x}}{5(x-1)^3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2) - 4*x^2*(1 - x)^(1/2) + 2*x^3*(1 - x)^(
1/2))/(5*(x - 1)^3*(x + 1)^(1/2))
```

sympy [B] time = 16.83, size = 282, normalized size = 4.55

$$\begin{cases} \frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{10\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-60x-5(x+1)^3+30(x+1)^2-20} + \frac{15\sqrt{-1+\frac{2}{x+1}}(x+1)}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{5\sqrt{-1+\frac{2}{x+1}}}{-60x-5(x+1)^3+30(x+1)^2-20} & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{10i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-60x-5(x+1)^3+30(x+1)^2-20} + \frac{15i\sqrt{1-\frac{2}{x+1}}(x+1)}{-60x-5(x+1)^3+30(x+1)^2-20} - \frac{5i\sqrt{1-\frac{2}{x+1}}}{-60x-5(x+1)^3+30(x+1)^2-20} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-60*x - 5*(x + 1)**3 + 30*(x
+ 1)**2 - 20) - 10*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-60*x - 5*(x + 1)**3 +
30*(x + 1)**2 - 20) + 15*sqrt(-1 + 2/(x + 1))*(x + 1)/(-60*x - 5*(x + 1)**3
 + 30*(x + 1)**2 - 20) - 5*sqrt(-1 + 2/(x + 1))/(-60*x - 5*(x + 1)**3 + 30*
(x + 1)**2 - 20), 2/Abs(x + 1) > 1), (2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-
60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) - 10*I*sqrt(1 - 2/(x + 1))*(x + 1
)**2/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) + 15*I*sqrt(1 - 2/(x + 1))
*(x + 1)/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20) - 5*I*sqrt(1 - 2/(x +
1))/(-60*x - 5*(x + 1)**3 + 30*(x + 1)**2 - 20), True))
```

3.1055 $\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} \, dx$

Optimal. Leaf size=82

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Rubi [A] time = 0.01, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{35\sqrt{1-x}\sqrt{x+1}}+\frac{4}{35(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{35(1-x)^{5/2}\sqrt{x+1}}+\frac{1}{7(1-x)^{7/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(35*(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(35*
(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(35*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}} \, dx &= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}} \, dx\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{12}{35}\int \frac{1}{(1-x)^{5/2}(1+x)^{3/2}} \, dx\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8}{35}\int \frac{1}{(1-x)^{3/2}(}\\
&= \frac{1}{7(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{35(1-x)^{3/2}\sqrt{1+x}}+\frac{8x}{35\sqrt{1-x}\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.49

$$\frac{8x^4-24x^3+20x^2+4x-13}{35(x-1)^3\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]
```

[Out] (-13 + 4*x + 20*x^2 - 24*x^3 + 8*x^4)/(35*(-1 + x)^3*Sqrt[1 - x^2])

IntegrateAlgebraic [A] time = 0.08, size = 76, normalized size = 0.93

$$\frac{(x+1)^{7/2}\left(-\frac{35(1-x)^4}{(x+1)^4}+\frac{140(1-x)^3}{(x+1)^3}+\frac{70(1-x)^2}{(x+1)^2}+\frac{28(1-x)}{x+1}+5\right)}{560(1-x)^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*(1 + x)^(3/2)),x]

[Out] ((1 + x)^(7/2)*(5 - (35*(1 - x)^4)/(1 + x)^4 + (140*(1 - x)^3)/(1 + x)^3 + (70*(1 - x)^2)/(1 + x)^2 + (28*(1 - x))/(1 + x)))/(560*(1 - x)^(7/2))

fricas [A] time = 1.27, size = 86, normalized size = 1.05

$$\frac{13\,x^5-39\,x^4+26\,x^3+26\,x^2-\left(8\,x^4-24\,x^3+20\,x^2+4\,x-13\right)\sqrt{x+1}\,\sqrt{-x+1}-39\,x+13}{35\left(x^5-3\,x^4+2\,x^3+2\,x^2-3\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="fricas")

[Out] 1/35*(13*x^5 - 39*x^4 + 26*x^3 + 26*x^2 - (8*x^4 - 24*x^3 + 20*x^2 + 4*x - 13)*sqrt(x + 1)*sqrt(-x + 1) - 39*x + 13)/(x^5 - 3*x^4 + 2*x^3 + 2*x^2 - 3*x + 1)

giac [A] time = 0.70, size = 79, normalized size = 0.96

$$\frac{\sqrt{2}-\sqrt{-x+1}}{32\,\sqrt{x+1}}-\frac{\sqrt{x+1}}{32\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{(((93\,x-523)(x+1)+1400)(x+1)-1120)\sqrt{x+1}\,\sqrt{-x+1}}{560\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="giac")

[Out] 1/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/32*sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)) - 1/560*(((93*x - 523)*(x + 1) + 1400)*(x + 1) - 1120)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4

maple [A] time = 0.00, size = 35, normalized size = 0.43

$$-\frac{8x^4-24x^3+20x^2+4x-13}{35\sqrt{x+1}\,\left(-x+1\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x+1)^(9/2)/(x+1)^(3/2),x)

[Out] -1/35*(8*x^4-24*x^3+20*x^2+4*x-13)/(x+1)^(1/2)/(-x+1)^(7/2)

maxima [B] time = 1.38, size = 134, normalized size = 1.63

$$\frac{8\,x}{35\,\sqrt{-x^2+1}}-\frac{1}{7\left(\sqrt{-x^2+1}\,x^3-3\,\sqrt{-x^2+1}\,x^2+3\,\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{35\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{35\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(9/2)/(1+x)^(3/2),x, algorithm="maxima")

[Out] 8/35*x/sqrt(-x^2 + 1) - 1/7/(sqrt(-x^2 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4/35/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/35/(sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1))

mupad [B] time = 0.36, size = 68, normalized size = 0.83

$$-\frac{4\,x\,\sqrt{1-x}-13\,\sqrt{1-x}+20\,x^2\,\sqrt{1-x}-24\,x^3\,\sqrt{1-x}+8\,x^4\,\sqrt{1-x}}{35\,{\left(x-1\right)}^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(3/2)),x)
```

```
[Out] -(4*x*(1 - x)^(1/2) - 13*(1 - x)^(1/2) + 20*x^2*(1 - x)^(1/2) - 24*x^3*(1 -
 x)^(1/2) + 8*x^4*(1 - x)^(1/2))/(35*(x - 1)^4*(x + 1)^(1/2))
```

sympy [B] time = 44.94, size = 423, normalized size = 5.16

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}\,(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56\sqrt{-1+\frac{2}{x+1}}\,(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35\sqrt{-1+\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}\,(x+1)^4}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{56i\sqrt{1-\frac{2}{x+1}}\,(x+1)^3}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}+\frac{140i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560}-\frac{35i\sqrt{1-\frac{2}{x+1}}}{-1120x+35(x+1)^4-280(x+1)^3+840(x+1)^2-560} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 28
0*(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(
-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*sqrt
(-1 + 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840
*(x + 1)**2 - 560) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1)
**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*sqrt(-1 + 2/(x + 1))/(-11
20*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), 2/Abs(x + 1)
 > 1), (-8*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-1120*x + 35*(x + 1)**4 - 280*
(x + 1)**3 + 840*(x + 1)**2 - 560) + 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-
1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 140*I*sqr
t(1 - 2/(x + 1))*(x + 1)**2/(-1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840
*(x + 1)**2 - 560) + 140*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-1120*x + 35*(x + 1
)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560) - 35*I*sqrt(1 - 2/(x + 1))/(-
1120*x + 35*(x + 1)**4 - 280*(x + 1)**3 + 840*(x + 1)**2 - 560), True))
```

3.1056 $\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx$

Optimal. Leaf size=102

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Rubi [A] time = 0.02, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 39}

$$\frac{8x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{4}{63(1-x)^{3/2}\sqrt{x+1}}+\frac{4}{63(1-x)^{5/2}\sqrt{x+1}}+\frac{5}{63(1-x)^{7/2}\sqrt{x+1}}+\frac{1}{9(1-x)^{9/2}\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*Sqrt[1 + x]) + 5/(63*(1 - x)^(7/2)*Sqrt[1 + x]) + 4/(63*
(1 - x)^(5/2)*Sqrt[1 + x]) + 4/(63*(1 - x)^(3/2)*Sqrt[1 + x]) + (8*x)/(63*S
qrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-x)^{11/2}(1+x)^{3/2}}\,dx &= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{9}\int \frac{1}{(1-x)^{9/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{20}{63}\int \frac{1}{(1-x)^{7/2}(1+x)^{3/2}}\,dx\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{21}\int \frac{1}{(1-x)^{5/2}(1-}\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}\\
&= \frac{1}{9(1-x)^{9/2}\sqrt{1+x}}+\frac{5}{63(1-x)^{7/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{5/2}\sqrt{1+x}}+\frac{4}{63(1-x)^{3/2}\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.44

$$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63(x-1)^4\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] (20 - 17*x - 16*x^2 + 44*x^3 - 32*x^4 + 8*x^5)/(63*(-1 + x)^4*Sqrt[1 - x^2]
)
```

IntegrateAlgebraic [A] time = 0.08, size = 90, normalized size = 0.88

$$\frac{(x+1)^{9/2}\left(-\frac{63(1-x)^5}{(x+1)^5}+\frac{315(1-x)^4}{(x+1)^4}+\frac{210(1-x)^3}{(x+1)^3}+\frac{126(1-x)^2}{(x+1)^2}+\frac{45(1-x)}{x+1}+7\right)}{2016(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*(1 + x)^(3/2)),x]
```

```
[Out] ((1 + x)^(9/2)*(7 - (63*(1 - x)^5)/(1 + x)^5 + (315*(1 - x)^4)/(1 + x)^4 +
(210*(1 - x)^3)/(1 + x)^3 + (126*(1 - x)^2)/(1 + x)^2 + (45*(1 - x))/(1 + x
)))/(2016*(1 - x)^(9/2))
```

fricas [A] time = 1.25, size = 91, normalized size = 0.89

$$\frac{20\,x^6-80\,x^5+100\,x^4-100\,x^2-\left(8\,x^5-32\,x^4+44\,x^3-16\,x^2-17\,x+20\right)\sqrt{x+1}\,\sqrt{-x+1}+80\,x-20}{63\left(x^6-4\,x^5+5\,x^4-5\,x^2+4\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/63*(20*x^6 - 80*x^5 + 100*x^4 - 100*x^2 - (8*x^5 - 32*x^4 + 44*x^3 - 16*x
^2 - 17*x + 20)*sqrt(x + 1)*sqrt(-x + 1) + 80*x - 20)/(x^6 - 4*x^5 + 5*x^4
- 5*x^2 + 4*x - 1)
```

giac [A] time = 0.68, size = 85, normalized size = 0.83

$$\frac{\sqrt{2}-\sqrt{-x+1}}{64\,\sqrt{x+1}}-\frac{\sqrt{x+1}}{64\left(\sqrt{2}-\sqrt{-x+1}\right)}-\frac{((((193\,x-1481)(x+1)+5544)(x+1)-8400)(x+1)+5040)\sqrt{x+1}\,\sqrt{-x+1}}{2016\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/64*sqrt(x + 1)/(sqrt(2) - sqr
t(-x + 1)) - 1/2016*((((193*x - 1481)*(x + 1) + 5544)*(x + 1) - 8400)*(x +
1) + 5040)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

maple [A] time = 0.00, size = 40, normalized size = 0.39

$$\frac{8x^5-32x^4+44x^3-16x^2-17x+20}{63\sqrt{x+1}\,\left(-x+1\right)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(3/2),x)
```

```
[Out] 1/63*(8*x^5-32*x^4+44*x^3-16*x^2-17*x+20)/(x+1)^(1/2)/(-x+1)^(9/2)
```

maxima [B] time = 1.37, size = 201, normalized size = 1.97

$$\frac{8\,x}{63\,\sqrt{-x^2+1}}+\frac{1}{9\left(\sqrt{-x^2+1}\,x^4-4\,\sqrt{-x^2+1}\,x^3+6\,\sqrt{-x^2+1}\,x^2-4\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{5}{63\left(\sqrt{-x^2+1}\,x^3-3\,\sqrt{-x^2+1}\,x^2+3\,\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}+\frac{4}{63\left(\sqrt{-x^2+1}\,x^2-2\,\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}-\frac{4}{63\left(\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/63*x/sqrt(-x^2 + 1) + 1/9/(sqrt(-x^2 + 1)*x^4 - 4*sqrt(-x^2 + 1)*x^3 + 6*
sqrt(-x^2 + 1)*x^2 - 4*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 5/63/(sqrt(-x^2
 + 1)*x^3 - 3*sqrt(-x^2 + 1)*x^2 + 3*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1)) + 4
/63/(sqrt(-x^2 + 1)*x^2 - 2*sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1)) - 4/63/(sqrt
(-x^2 + 1)*x - sqrt(-x^2 + 1))
```

mupad [B] time = 0.36, size = 80, normalized size = 0.78

$$\frac{17\,x\,\sqrt{1-x}-20\,\sqrt{1-x}+16\,x^2\,\sqrt{1-x}-44\,x^3\,\sqrt{1-x}+32\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{63\,{\left(x-1\right)}^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(3/2)),x)
```

```
[Out] (17*x*(1 - x)^(1/2) - 20*(1 - x)^(1/2) + 16*x^2*(1 - x)^(1/2) - 44*x^3*(1 -
 x)^(1/2) + 32*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/(63*(x - 1)^5*(x +
1)^(1/2))
```

sympy [B] time = 113.61, size = 592, normalized size = 5.80

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^5}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{72\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{252\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{420\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{315\sqrt{-1+\frac{2}{x+1}}(x+1)}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{63\sqrt{-1+\frac{2}{x+1}}}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} & \text{for}\ \frac{2}{|x+1|}>1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^5}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{72i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{252i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{420i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} + \frac{315i\sqrt{1-\frac{2}{x+1}}(x+1)}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} - \frac{63i\sqrt{1-\frac{2}{x+1}}}{-5040x-63(x+1)^5+630(x+1)^4-2520(x+1)^3+5040(x+1)^2-3024} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(3/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-5040*x - 63*(x + 1)**5 + 630
*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) - 72*sqrt(-1 + 2/(x
 + 1))*(x + 1)**4/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)*
*3 + 5040*(x + 1)**2 - 3024) + 252*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-5040*x
 - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 302
4) - 420*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-5040*x - 63*(x + 1)**5 + 630*(x
+ 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) + 315*sqrt(-1 + 2/(x +
1))*(x + 1)/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5
040*(x + 1)**2 - 3024) - 63*sqrt(-1 + 2/(x + 1))/(-5040*x - 63*(x + 1)**5 +
 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024), 2/Abs(x + 1) >
 1), (8*I*sqrt(1 - 2/(x + 1))*(x + 1)**5/(-5040*x - 63*(x + 1)**5 + 630*(x
+ 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) - 72*I*sqrt(1 - 2/(x +
1))*(x + 1)**4/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3
+ 5040*(x + 1)**2 - 3024) + 252*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-5040*x -
 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024)
 - 420*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(-5040*x - 63*(x + 1)**5 + 630*(x +
 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024) + 315*I*sqrt(1 - 2/(x +
1))*(x + 1)/(-5040*x - 63*(x + 1)**5 + 630*(x + 1)**4 - 2520*(x + 1)**3 + 5
040*(x + 1)**2 - 3024) - 63*I*sqrt(1 - 2/(x + 1))/(-5040*x - 63*(x + 1)**5
+ 630*(x + 1)**4 - 2520*(x + 1)**3 + 5040*(x + 1)**2 - 3024), True))
```

3.1057 $\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=103

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{9/2}}{3(x+1)^{3/2}}+\frac{6(1-x)^{7/2}}{\sqrt{x+1}}+7\sqrt{x+1}\,(1-x)^{5/2}+\frac{35}{2}\sqrt{x+1}\,(1-x)^{3/2}+\frac{105}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{105}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(9/2))/(3*(1 + x)^(3/2)) + (6*(1 - x)^(7/2))/Sqrt[1 + x] + (105
*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/2 + 7*(1 - x)^
(5/2)*Sqrt[1 + x] + (105*ArcSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{(1-x)^{9/2}}{(1+x)^{5/2}}\,dx = -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} - 3\int \frac{(1-x)^{7/2}}{(1+x)^{3/2}}\,dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 21\int \frac{(1-x)^{5/2}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + 7(1-x)^{5/2}\sqrt{1+x} + 35\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + \frac{105}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + -$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + -$$
$$= -\frac{2(1-x)^{9/2}}{3(1+x)^{3/2}} + \frac{6(1-x)^{7/2}}{\sqrt{1+x}} + \frac{105}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{2}(1-x)^{3/2}\sqrt{1+x} + 7(1-x)^{5/2}\sqrt{1+x} + -$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.36

$$-\frac{(1-x)^{11/2}\,{}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{1-x}{2}\right)}{22\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/22*((1 - x)^(11/2)*Hypergeometric2F1[5/2, 11/2, 13/2, (1 - x)/2])/Sqrt[2
]
```

IntegrateAlgebraic [A] time = 0.10, size = 131, normalized size = 1.27

$$\frac{-\frac{16(1-x)^{9/2}}{(x+1)^{9/2}}+\frac{144(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{693(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{840(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{315\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^3}-105\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(9/2)/(1 + x)^(5/2),x]
```

```
[Out] ((-16*(1 - x)^(9/2))/(1 + x)^(9/2) + (144*(1 - x)^(7/2))/(1 + x)^(7/2) + (6
93*(1 - x)^(5/2))/(1 + x)^(5/2) + (840*(1 - x)^(3/2))/(1 + x)^(3/2) + (315*
Sqrt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 - x)/(1 + x))^3) - 105*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

fricas [A] time = 1.50, size = 85, normalized size = 0.83

$$\frac{494\,x^2+\left(2\,x^4-17\,x^3+102\,x^2+679\,x+494\right)\sqrt{x+1}\,\sqrt{-x+1}-630\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+988\,x+494}{6\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(494*x^2 + (2*x^4 - 17*x^3 + 102*x^2 + 679*x + 494)*sqrt(x + 1)*sqrt(-x
 + 1) - 630*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 988*
x + 494)/(x^2 + 2*x + 1)
```

giac [A] time = 0.89, size = 127, normalized size = 1.23

$$\frac{1}{6}\,\left((2\,x-23)(x+1)+165\right)\sqrt{x+1}\,\sqrt{-x+1}+\frac{2\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{3\,(x+1)^{\frac{3}{2}}}-\frac{34\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{2\,(x+1)^{\frac{3}{2}}\left(\frac{51\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}+105\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x - 23)*(x + 1) + 165)*sqrt(x + 1)*sqrt(-x + 1) + 2/3*(sqrt(2) - sq
rt(-x + 1))^3/(x + 1)^(3/2) - 34*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 2/3
*(x + 1)^(3/2)*(51*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(
-x + 1))^3 + 105*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 89, normalized size = 0.86

$$\frac{105\sqrt{(x+1)\,(-x+1)}\;\arcsin(x)}{2\sqrt{x+1}\;\sqrt{-x+1}}-\frac{\left(2x^{5}-19x^{4}+119x^{3}+577x^{2}-185x-494\right)\sqrt{(x+1)\,(-x+1)}}{6\,(x+1)^{\frac{3}{2}}\,\sqrt{-(x+1)\,(x-1)}\;\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(9/2)/(x+1)^(5/2),x)
```

```
[Out] -1/6*(2*x^5-19*x^4+119*x^3+577*x^2-185*x-494)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1
/2)*((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)+105/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2
)/(-x+1)^(1/2)*arcsin(x)
```

maxima [A] time = 3.01, size = 125, normalized size = 1.21

$$\frac{x^{6}}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}-\frac{7\,x^{5}}{2\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{23\,x^{4}}{\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{35}{2}\,x\left(\frac{3\,x^{2}}{\left(-x^{2}+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^{2}+1\right)^{\frac{3}{2}}}\right)-\frac{143\,x}{6\,\sqrt{-x^{2}+1}}-\frac{127\,x^{2}}{\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{22\,x}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{247}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}+\frac{105}{2}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x^6/(-x^2 + 1)^(3/2) - 7/2*x^5/(-x^2 + 1)^(3/2) + 23*x^4/(-x^2 + 1)^(3/
2) + 35/2*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 143/6*x/sqrt(-x
^2 + 1) - 127*x^2/(-x^2 + 1)^(3/2) + 22/3*x/(-x^2 + 1)^(3/2) + 247/3/(-x^2
+ 1)^(3/2) + 105/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{9/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(9/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(9/2)/(x + 1)^(5/2), x)
```

sympy [A] time = 45.20, size = 250, normalized size = 2.43

$$\begin{cases} -105i\,\operatorname{acosh}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)+\frac{i(x+1)^{\frac{7}{2}}}{3\sqrt{x-1}}-\frac{29i(x+1)^{\frac{5}{2}}}{6\sqrt{x-1}}+\frac{215i(x+1)^{\frac{3}{2}}}{6\sqrt{x-1}}+\frac{43i\sqrt{x+1}}{3\sqrt{x-1}}-\frac{448i}{3\sqrt{x-1}\,\sqrt{x+1}}+\frac{64i}{3\sqrt{x-1}\,(x+1)^{\frac{3}{2}}} & \text{for}\ \frac{|x+1|}{2}>1 \\ 105\,\operatorname{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)-\frac{(x+1)^{\frac{7}{2}}}{3\sqrt{1-x}}+\frac{29(x+1)^{\frac{5}{2}}}{6\sqrt{1-x}}-\frac{215(x+1)^{\frac{3}{2}}}{6\sqrt{1-x}}-\frac{43\sqrt{x+1}}{3\sqrt{1-x}}+\frac{448}{3\sqrt{1-x}\,\sqrt{x+1}}-\frac{64}{3\sqrt{1-x}\,(x+1)^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-105*I*acosh(sqrt(2)*sqrt(x + 1)/2) + I*(x + 1)**(7/2)/(3*sqrt(x
 - 1)) - 29*I*(x + 1)**(5/2)/(6*sqrt(x - 1)) + 215*I*(x + 1)**(3/2)/(6*sqrt
(x - 1)) + 43*I*sqrt(x + 1)/(3*sqrt(x - 1)) - 448*I/(3*sqrt(x - 1)*sqrt(x +
 1)) + 64*I/(3*sqrt(x - 1)*(x + 1)**(3/2)), Abs(x + 1)/2 > 1), (105*asin(sq
rt(2)*sqrt(x + 1)/2) - (x + 1)**(7/2)/(3*sqrt(1 - x)) + 29*(x + 1)**(5/2)/(
6*sqrt(1 - x)) - 215*(x + 1)**(3/2)/(6*sqrt(1 - x)) - 43*sqrt(x + 1)/(3*sqr
t(1 - x)) + 448/(3*sqrt(1 - x)*sqrt(x + 1)) - 64/(3*sqrt(1 - x)*(x + 1)**(3
/2)), True))
```

3.1058 $\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{7/2}}{3(x+1)^{3/2}}+\frac{14(1-x)^{5/2}}{3\sqrt{x+1}}+\frac{35}{6}\sqrt{x+1}\,(1-x)^{3/2}+\frac{35}{2}\sqrt{x+1}\,\sqrt{1-x}+\frac{35}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(7/2))/(3*(1 + x)^(3/2)) + (14*(1 - x)^(5/2))/(3*Sqrt[1 + x]) +
 (35*Sqrt[1 - x]*Sqrt[1 + x])/2 + (35*(1 - x)^(3/2)*Sqrt[1 + x])/6 + (35*Ar
cSin[x])/2
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(1-x)^{7/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} - \frac{7}{3}\int \frac{(1-x)^{5/2}}{(1+x)^{3/2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{3}\int \frac{(1-x)^{3/2}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}} \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= -\frac{2(1-x)^{7/2}}{3(1+x)^{3/2}} + \frac{14(1-x)^{5/2}}{3\sqrt{1+x}} + \frac{35}{2}\sqrt{1-x}\sqrt{1+x} + \frac{35}{6}(1-x)^{3/2}\sqrt{1+x} + \frac{35}{2}\sin^{-1}(x)
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.43

$$
-\frac{(1-x)^{9/2}\,{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{1-x}{2}\right)}{18\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/18*((1 - x)^(9/2)*Hypergeometric2F1[5/2, 9/2, 11/2, (1 - x)/2])/Sqrt[2]
```

IntegrateAlgebraic [A] time = 0.09, size = 113, normalized size = 1.30

$$
\frac{-\frac{8(1-x)^{7/2}}{(x+1)^{7/2}}+\frac{56(1-x)^{5/2}}{(x+1)^{5/2}}+\frac{175(1-x)^{3/2}}{(x+1)^{3/2}}+\frac{105\sqrt{1-x}}{\sqrt{x+1}}}{3\left(\frac{1-x}{x+1}+1\right)^2}-35\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(7/2)/(1 + x)^(5/2),x]
```

```
[Out] ((-8*(1 - x)^(7/2))/(1 + x)^(7/2) + (56*(1 - x)^(5/2))/(1 + x)^(5/2) + (175
*(1 - x)^(3/2))/(1 + x)^(3/2) + (105*Sqrt[1 - x])/Sqrt[1 + x])/(3*(1 + (1 -
 x)/(1 + x))^2) - 35*ArcTan[Sqrt[1 - x]/Sqrt[1 + x]]
```

fricas [A] time = 1.27, size = 81, normalized size = 0.93

$$
\frac{164\,x^2-\left(3\,x^3-30\,x^2-229\,x-164\right)\sqrt{x+1}\,\sqrt{-x+1}\,-210\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+328\,x+164}{6\left(x^2+2\,x+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(164*x^2 - (3*x^3 - 30*x^2 - 229*x - 164)*sqrt(x + 1)*sqrt(-x + 1) - 21
0*(x^2 + 2*x + 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 328*x + 164)/(
x^2 + 2*x + 1)
```

giac [A] time = 0.80, size = 119, normalized size = 1.37

$$
-\frac{1}{2}\,\sqrt{x+1}\,(x-12)\sqrt{-x+1}\,+\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{3\,(x+1)^{\frac{3}{2}}}-\frac{13\left(\sqrt{2}-\sqrt{-x+1}\right)}{\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{39\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{3\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+35\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(x + 1)*(x - 12)*sqrt(-x + 1) + 1/3*(sqrt(2) - sqrt(-x + 1))^3/(x
+ 1)^(3/2) - 13*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/3*(x + 1)^(3/2)*(3
9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 35*a
rcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 84, normalized size = 0.97

$$\frac{35\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{2\sqrt{x+1}\ \sqrt{-x+1}}+\frac{\left(3x^4-33x^3-199x^2+65x+164\right)\sqrt{(x+1)(-x+1)}}{6\,(x+1)^{\frac{3}{2}}\ \sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(7/2)/(x+1)^(5/2),x)
```

```
[Out] 1/6*(3*x^4-33*x^3-199*x^2+65*x+164)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)
*(-x+1))^(1/2)/(-x+1)^(1/2)+35/2*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1
/2)*arcsin(x)
```

maxima [A] time = 3.00, size = 111, normalized size = 1.28

$$-\frac{x^5}{2\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{6\,x^4}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{6}\,x\left(\frac{3\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}-\frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)-\frac{61\,x}{6\sqrt{-x^2+1}}-\frac{44\,x^2}{\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{16\,x}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{82}{3\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{35}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/2*x^5/(-x^2 + 1)^(3/2) + 6*x^4/(-x^2 + 1)^(3/2) + 35/6*x*(3*x^2/(-x^2 +
1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 61/6*x/sqrt(-x^2 + 1) - 44*x^2/(-x^2 + 1)^
(3/2) + 16/3*x/(-x^2 + 1)^(3/2) + 82/3/(-x^2 + 1)^(3/2) + 35/2*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(1-x)^{7/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(7/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(7/2)/(x + 1)^(5/2), x)
```

sympy [C] time = 17.50, size = 207, normalized size = 2.38

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^2}{2}+\frac{13\sqrt{-1+\frac{2}{x+1}}\,(x+1)}{2}+\frac{80\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{16\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}+\frac{35i\log(x+1)}{2}+35\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}\,(x+1)^2}{2}+\frac{13i\sqrt{1-\frac{2}{x+1}}\,(x+1)}{2}+\frac{80i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{16i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+\frac{35i\log\left(\frac{1}{x+1}\right)}{2}-35i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))*(x + 1)**2/2 + 13*sqrt(-1 + 2/(x + 1))*(x
+ 1)/2 + 80*sqrt(-1 + 2/(x + 1))/3 - 16*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
35*I*log(1/(x + 1))/2 + 35*I*log(x + 1)/2 + 35*asin(sqrt(2)*sqrt(x + 1)/2),
 2/Abs(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))*(x + 1)**2/2 + 13*I*sqrt(1 - 2/
(x + 1))*(x + 1)/2 + 80*I*sqrt(1 - 2/(x + 1))/3 - 16*I*sqrt(1 - 2/(x + 1))/
(3*(x + 1)) + 35*I*log(1/(x + 1))/2 - 35*I*log(sqrt(1 - 2/(x + 1)) + 1), Tr
ue))
```

3.1059 $\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}} \, dx$

Optimal. Leaf size=63

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {47, 50, 41, 216}

$$-\frac{2(1-x)^{5/2}}{3(x+1)^{3/2}}+\frac{10(1-x)^{3/2}}{3\sqrt{x+1}}+5\sqrt{x+1}\sqrt{1-x}+5\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(5/2))/(3*(1 + x)^(3/2)) + (10*(1 - x)^(3/2))/(3*Sqrt[1 + x]) +
 5*Sqrt[1 - x]*Sqrt[1 + x] + 5*ArcSin[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)^{5/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} - \frac{5}{3}\int \frac{(1-x)^{3/2}}{(1+x)^{3/2}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\int \frac{\sqrt{1-x}}{\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{2(1-x)^{5/2}}{3(1+x)^{3/2}} + \frac{10(1-x)^{3/2}}{3\sqrt{1+x}} + 5\sqrt{1-x}\,\sqrt{1+x} + 5\sin^{-1}(x)\end{aligned}$$

Mathematica [C] time = 0.01, size = 37, normalized size = 0.59

$$-\frac{(1-x)^{7/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{1-x}{2}\right)}{14\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] -1/14*((1 - x)^(7/2)*Hypergeometric2F1[5/2, 7/2, 9/2, (1 - x)/2])/Sqrt[2]
```

IntegrateAlgebraic [C] time = 0.17, size = 61, normalized size = 0.97

$$\frac{\sqrt{1-x}\,\left(3(x+1)^2+28(x+1)-8\right)}{3(x+1)^{3/2}} + 10i\log\left(\sqrt{1-x}-i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(5/2)/(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[1 - x]*(-8 + 28*(1 + x) + 3*(1 + x)^2))/(3*(1 + x)^(3/2)) + (10*I)*Lo
g[Sqrt[1 - x] - I*Sqrt[1 + x]]
```

fricas [A] time = 1.26, size = 75, normalized size = 1.19

$$\frac{23\,x^2+\left(3\,x^2+34\,x+23\right)\sqrt{x+1}\,\sqrt{-x+1}-30\left(x^2+2\,x+1\right)\arctan\left(\frac{\sqrt{x+1}\,\sqrt{-x+1}-1}{x}\right)+46\,x+23}{3\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(23*x^2 + (3*x^2 + 34*x + 23)*sqrt(x + 1)*sqrt(-x + 1) - 30*(x^2 + 2*x
+ 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 46*x + 23)/(x^2 + 2*x + 1)
```

giac [B] time = 0.75, size = 115, normalized size = 1.83

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{6\,(x+1)^{\frac{3}{2}}} + \sqrt{x+1}\,\sqrt{-x+1} - \frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)}{2\sqrt{x+1}} + \frac{(x+1)^{\frac{3}{2}}\left(\frac{27\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{6\left(\sqrt{2}-\sqrt{-x+1}\right)^3} + 10\,\arcsin\left(\frac{1}{2}\sqrt{2}\,\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + sqrt(x + 1)*sqrt(-x + 1) - 9
/2*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) + 1/6*(x + 1)^(3/2)*(27*(sqrt(2) -
sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1))^3 + 10*arcsin(1/2*sqr
t(2)*sqrt(x + 1))
```

maple [A] time = 0.02, size = 79, normalized size = 1.25

$$\frac{5\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}-\frac{(3x^3+31x^2-11x-23)\sqrt{(x+1)(-x+1)}}{3(x+1)^{\frac{3}{2}}\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(5/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*(3*x^3+31*x^2-11*x-23)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))
^(1/2)/(-x+1)^(1/2)+5*((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(
x)
```

maxima [B] time = 2.97, size = 98, normalized size = 1.56

$$\frac{(-x^2+1)^{\frac{5}{2}}}{x^4+4x^3+6x^2+4x+1}-\frac{5(-x^2+1)^{\frac{3}{2}}}{3(x^3+3x^2+3x+1)}-\frac{10\sqrt{-x^2+1}}{3(x^2+2x+1)}+\frac{35\sqrt{-x^2+1}}{3(x+1)}+5\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] (-x^2 + 1)^(5/2)/(x^4 + 4*x^3 + 6*x^2 + 4*x + 1) - 5/3*(-x^2 + 1)^(3/2)/(x^
3 + 3*x^2 + 3*x + 1) - 10/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 35/3*sqrt(-x^2
 + 1)/(x + 1) + 5*arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(1-x)^{5/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(5/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(5/2)/(x + 1)^(5/2), x)
```

sympy [C] time = 6.46, size = 160, normalized size = 2.54

$$\begin{cases}\sqrt{-1+\frac{2}{x+1}}\,(x+1)+\frac{28\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{8\sqrt{-1+\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)+5i\log(x+1)+10\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|}>1\\ i\sqrt{1-\frac{2}{x+1}}\,(x+1)+\frac{28i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{8i\sqrt{1-\frac{2}{x+1}}}{3(x+1)}+5i\log\left(\frac{1}{x+1}\right)-10i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))*(x + 1) + 28*sqrt(-1 + 2/(x + 1))/3 - 8*sqr
t(-1 + 2/(x + 1))/(3*(x + 1)) + 5*I*log(1/(x + 1)) + 5*I*log(x + 1) + 10*as
in(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))*(x + 1
) + 28*I*sqrt(1 - 2/(x + 1))/3 - 8*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) + 5*I*
log(1/(x + 1)) - 10*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

3.1060 $\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 41, 216}

$$-\frac{2(1-x)^{3/2}}{3(x+1)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{x+1}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*(1 - x)^(3/2))/(3*(1 + x)^(3/2)) + (2*Sqrt[1 - x])/Sqrt[1 + x] + ArcSin
[x]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x)^{3/2}}{(1+x)^{5/2}}\,dx &= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}-\int \frac{\sqrt{1-x}}{(1+x)^{3/2}}\,dx\\
&= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{1+x}}+\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}}\,dx\\
&= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{1+x}}+\int \frac{1}{\sqrt{1-x^2}}\,dx\\
&= -\frac{2(1-x)^{3/2}}{3(1+x)^{3/2}}+\frac{2\sqrt{1-x}}{\sqrt{1+x}}+\sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 1.20

$$\frac{-8x^2+4x+4}{3\sqrt{1-x}\,(x+1)^{3/2}}-2\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (4 + 4*x - 8*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2)) - 2*ArcSin[Sqrt[1 - x]/Sqrt
[2]]
```

IntegrateAlgebraic [A] time = 0.06, size = 54, normalized size = 1.32

$$-\frac{2\sqrt{1-x}\left(\frac{1-x}{x+1}-3\right)}{3\sqrt{x+1}}-2\tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(3/2)/(1 + x)^(5/2),x]
```

```
[Out] (-2*Sqrt[1 - x]*(-3 + (1 - x)/(1 + x)))/(3*Sqrt[1 + x]) - 2*ArcTan[Sqrt[1 -
 x]/Sqrt[1 + x]]
```

fricas [B] time = 1.19, size = 71, normalized size = 1.73

$$\frac{2\left(2x^2+2(2x+1)\sqrt{x+1}\sqrt{-x+1}-3\left(x^2+2x+1\right)\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)+4x+2\right)}{3\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + 2*(2*x + 1)*sqrt(x + 1)*sqrt(-x + 1) - 3*(x^2 + 2*x + 1)*arcta
n((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x + 2)/(x^2 + 2*x + 1)
```

giac [B] time = 0.70, size = 102, normalized size = 2.49

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{12(x+1)^{\frac{3}{2}}}-\frac{5\left(\sqrt{2}-\sqrt{-x+1}\right)}{4\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}-1\right)}{12\left(\sqrt{2}-\sqrt{-x+1}\right)^3}+2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 5/4*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/12*(x + 1)^(3/2)*(15*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
 1)/(sqrt(2) - sqrt(-x + 1))^3 + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1))
```

maple [B] time = 0.02, size = 73, normalized size = 1.78

$$\frac{\sqrt{(x+1)(-x+1)}\ \arcsin(x)}{\sqrt{x+1}\ \sqrt{-x+1}}-\frac{4\left(2x^2-x-1\right)\sqrt{(x+1)(-x+1)}}{3(x+1)^{\frac{3}{2}}\sqrt{-(x+1)(x-1)}\ \sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(3/2)/(x+1)^(5/2),x)
```

```
[Out] -4/3*(2*x^2-x-1)/(x+1)^(3/2)/(-(x+1)*(x-1))^(1/2)*((x+1)*(-x+1))^(1/2)/(-x+
1)^(1/2)+((x+1)*(-x+1))^(1/2)/(x+1)^(1/2)/(-x+1)^(1/2)*arcsin(x)
```

maxima [B] time = 3.01, size = 66, normalized size = 1.61

$$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3\left(x^3+3x^2+3x+1\right)}-\frac{2\sqrt{-x^2+1}}{3\left(x^2+2x+1\right)}+\frac{7\sqrt{-x^2+1}}{3(x+1)}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)/(x^3 + 3*x^2 + 3*x + 1) - 2/3*sqrt(-x^2 + 1)/(x^2 + 2
*x + 1) + 7/3*sqrt(-x^2 + 1)/(x + 1) + arcsin(x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(1-x)^{3/2}}{(x+1)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(3/2)/(x + 1)^(5/2),x)
```

```
[Out] int((1 - x)^(3/2)/(x + 1)^(5/2), x)
```

sympy [C] time = 3.28, size = 126, normalized size = 3.07

$$\begin{cases} \frac{8\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{4\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) + i\log(x+1) + 2\,\mathrm{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{2}{|x+1|} > 1 \\ \frac{8i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{4i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} + i\log\left(\frac{1}{x+1}\right) - 2i\log\left(\sqrt{1-\frac{2}{x+1}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))/3 - 4*sqrt(-1 + 2/(x + 1))/(3*(x + 1)) +
I*log(1/(x + 1)) + I*log(x + 1) + 2*asin(sqrt(2)*sqrt(x + 1)/2), 2/Abs(x +
1) > 1), (8*I*sqrt(1 - 2/(x + 1))/3 - 4*I*sqrt(1 - 2/(x + 1))/(3*(x + 1)) +
 I*log(1/(x + 1)) - 2*I*log(sqrt(1 - 2/(x + 1)) + 1), True))
```

3.1061 $\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {37}

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x]/(1 + x)^(5/2),x]
[Out] -(1 - x)^(3/2)/(3*(1 + x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{1-x}}{(1+x)^{5/2}}\,dx = -\frac{(1-x)^{3/2}}{3(1+x)^{3/2}}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x]/(1 + x)^(5/2),x]
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 20, normalized size = 1.00

$$-\frac{(1-x)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + x)^(5/2),x]
[Out] -1/3*(1 - x)^(3/2)/(1 + x)^(3/2)
```

fricas [B] time = 1.30, size = 37, normalized size = 1.85

$$-\frac{x^2 - \sqrt{x+1}\,(x-1)\sqrt{-x+1} + 2\,x + 1}{3\left(x^2 + 2\,x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(x^2 - sqrt(x + 1)*(x - 1)*sqrt(-x + 1) + 2*x + 1)/(x^2 + 2*x + 1)
```

giac [B] time = 0.71, size = 89, normalized size = 4.45

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{24\,(x+1)^{\frac{3}{2}}}-\frac{\sqrt{2}-\sqrt{-x+1}}{8\,\sqrt{x+1}}+\frac{(x+1)^{\frac{3}{2}}\left(\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}-1\right)}{24\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
[Out] 1/24*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) - 1/8*(sqrt(2) - sqrt(-x + 1)
)/sqrt(x + 1) + 1/24*(x + 1)^(3/2)*(3*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) -
1)/(sqrt(2) - sqrt(-x + 1))^3
```

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$-\frac{(-x+1)^{\frac{3}{2}}}{3\,(x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/2)/(x+1)^(5/2),x)
[Out] -1/3*(-x+1)^(3/2)/(x+1)^(3/2)
```

maxima [B] time = 1.32, size = 38, normalized size = 1.90

$$-\frac{2\,\sqrt{-x^{2}+1}}{3\left(x^{2}+2\,x+1\right)}+\frac{\sqrt{-x^{2}+1}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
[Out] -2/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) + 1/3*sqrt(-x^2 + 1)/(x + 1)
```

mupad [B] time = 0.26, size = 32, normalized size = 1.60

$$\frac{x\,\sqrt{1-x}-\sqrt{1-x}}{(3\,x+3)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/2)/(x + 1)^(5/2),x)
[Out] (x*(1 - x)^(1/2) - (1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

sympy [A] time = 1.69, size = 65, normalized size = 3.25

$$\begin{cases} \frac{\sqrt{-1+\frac{2}{x+1}}}{3}-\frac{2\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{2}{|x+1|}>1 \\ \frac{i\sqrt{1-\frac{2}{x+1}}}{3}-\frac{2i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((sqrt(-1 + 2/(x + 1))/3 - 2*sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/A
bs(x + 1) > 1), (I*sqrt(1 - 2/(x + 1))/3 - 2*I*sqrt(1 - 2/(x + 1))/(3*(x +
1)), True))
```

3.1062 $\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx$

Optimal. Leaf size=41

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{\sqrt{1-x}}{3\sqrt{x+1}}-\frac{\sqrt{1-x}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -Sqrt[1 - x]/(3*(1 + x)^(3/2)) - Sqrt[1 - x]/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x}\,(1+x)^{5/2}}\,dx &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{1-x}\,(1+x)^{3/2}}\,dx\\ &= -\frac{\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.56

$$-\frac{\sqrt{1-x}\,(x+2)}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*(Sqrt[1 - x]*(2 + x))/(1 + x)^(3/2)
```

IntegrateAlgebraic [A] time = 0.05, size = 33, normalized size = 0.80

$$-\frac{\sqrt{1-x}\left(\frac{1-x}{x+1}+3\right)}{6\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - x]*(1 + x)^(5/2)),x]
[Out] -1/6*(Sqrt[1 - x]*(3 + (1 - x)/(1 + x)))/Sqrt[1 + x]
```

fricas [A] time = 1.35, size = 38, normalized size = 0.93

$$-\frac{2x^2+(x+2)\sqrt{x+1}\sqrt{-x+1}+4x+2}{3(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="fricas")
[Out] -1/3*(2*x^2 + (x + 2)*sqrt(x + 1)*sqrt(-x + 1) + 4*x + 2)/(x^2 + 2*x + 1)
```

giac [B] time = 0.67, size = 89, normalized size = 2.17

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{48(x+1)^{\frac{3}{2}}}+\frac{3\left(\sqrt{2}-\sqrt{-x+1}\right)}{16\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{9\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{48\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="giac")
[Out] 1/48*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 3/16*(sqrt(2) - sqrt(-x + 1
))/sqrt(x + 1) - 1/48*(x + 1)^(3/2)*(9*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) +
 1)/(sqrt(2) - sqrt(-x + 1))^3
```

maple [A] time = 0.00, size = 18, normalized size = 0.44

$$-\frac{(x+2)\sqrt{-x+1}}{3(x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(1/2)/(x+1)^(5/2),x)
[Out] -1/3*(2+x)/(x+1)^(3/2)*(-x+1)^(1/2)
```

maxima [A] time = 2.93, size = 38, normalized size = 0.93

$$-\frac{\sqrt{-x^2+1}}{3(x^2+2x+1)}-\frac{\sqrt{-x^2+1}}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(1/2)/(1+x)^(5/2),x, algorithm="maxima")
[Out] -1/3*sqrt(-x^2 + 1)/(x^2 + 2*x + 1) - 1/3*sqrt(-x^2 + 1)/(x + 1)
```

mupad [B] time = 0.31, size = 33, normalized size = 0.80

$$-\frac{x\sqrt{1-x}+2\sqrt{1-x}}{(3x+3)\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(1/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(x*(1 - x)^(1/2) + 2*(1 - x)^(1/2))/((3*x + 3)*(x + 1)^(1/2))
```

sympy [A] time = 2.35, size = 65, normalized size = 1.59

$$\begin{cases} -\frac{\sqrt{-1+\frac{2}{x+1}}}{3} - \frac{\sqrt{-1+\frac{2}{x+1}}}{3(x+1)} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{i\sqrt{1-\frac{2}{x+1}}}{3} - \frac{i\sqrt{1-\frac{2}{x+1}}}{3(x+1)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(1/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-sqrt(-1 + 2/(x + 1))/3 - sqrt(-1 + 2/(x + 1))/(3*(x + 1)), 2/Ab
s(x + 1) > 1), (-I*sqrt(1 - 2/(x + 1))/3 - I*sqrt(1 - 2/(x + 1))/(3*(x + 1)
), True))
```

3.1063 $\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}} dx$

Optimal. Leaf size=58

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{2\sqrt{1-x}}{3\sqrt{x+1}}-\frac{2\sqrt{1-x}}{3(x+1)^{3/2}}+\frac{1}{(x+1)^{3/2}\sqrt{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(Sqrt[1 - x]*(1 + x)^(3/2)) - (2*Sqrt[1 - x])/(3*(1 + x)^(3/2)) - (2*Sqrt
[1 - x])/(3*Sqrt[1 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{3/2}(1+x)^{5/2}} dx &= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}+2\int \frac{1}{\sqrt{1-x}(1+x)^{5/2}} dx\\ &= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{\sqrt{1-x}(1+x)^{3/2}} dx\\ &= \frac{1}{\sqrt{1-x}(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3(1+x)^{3/2}}-\frac{2\sqrt{1-x}}{3\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.52

$$\frac{2x^2+2x-1}{3\sqrt{1-x}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]
```

[Out] (-1 + 2*x + 2*x^2)/(3*Sqrt[1 - x]*(1 + x)^(3/2))

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 0.83

$$\frac{\sqrt{x+1}\left(-\frac{(1-x)^2}{(x+1)^2}-\frac{6(1-x)}{x+1}+3\right)}{12\sqrt{1-x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 - x)^(3/2)*(1 + x)^(5/2)),x]

[Out] (Sqrt[1 + x]*(3 - (1 - x)^2/(1 + x)^2 - (6*(1 - x))/(1 + x)))/(12*Sqrt[1 - x])

fricas [A] time = 1.20, size = 49, normalized size = 0.84

$$-\frac{x^3+x^2+\left(2x^2+2x-1\right)\sqrt{x+1}\sqrt{-x+1}-x-1}{3\left(x^3+x^2-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="fricas")

[Out] -1/3*(x^3 + x^2 + (2*x^2 + 2*x - 1)*sqrt(x + 1)*sqrt(-x + 1) - x - 1)/(x^3 + x^2 - x - 1)

giac [B] time = 0.68, size = 108, normalized size = 1.86

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{96\,(x+1)^{\frac{3}{2}}}+\frac{7\left(\sqrt{2}-\sqrt{-x+1}\right)}{32\sqrt{x+1}}-\frac{\sqrt{x+1}\sqrt{-x+1}}{4\,(x-1)}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{21\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{96\left(\sqrt{2}-\sqrt{-x+1}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="giac")

[Out] 1/96*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 7/32*(sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - 1/4*sqrt(x + 1)*sqrt(-x + 1)/(x - 1) - 1/96*(x + 1)^(3/2)*(21*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3

maple [A] time = 0.00, size = 25, normalized size = 0.43

$$\frac{2x^2+2x-1}{3\,(x+1)^{\frac{3}{2}}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x+1)^(3/2)/(x+1)^(5/2),x)

[Out] 1/3*(2*x^2+2*x-1)/(x+1)^(3/2)/(-x+1)^(1/2)

maxima [A] time = 1.35, size = 38, normalized size = 0.66

$$\frac{2x}{3\sqrt{-x^2+1}}-\frac{1}{3\left(\sqrt{-x^2+1}\,x+\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(3/2)/(1+x)^(5/2),x, algorithm="maxima")

```
[Out] 2/3*x/sqrt(-x^2 + 1) - 1/3/(sqrt(-x^2 + 1)*x + sqrt(-x^2 + 1))
```

mupad [B] time = 0.34, size = 48, normalized size = 0.83

$$-\frac{2\,x\,\sqrt{1-x}-\sqrt{1-x}+2\,x^2\,\sqrt{1-x}}{\left(3\,x^2-3\right)\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(3/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(2*x*(1 - x)^(1/2) - (1 - x)^(1/2) + 2*x^2*(1 - x)^(1/2))/((3*x^2 - 3)*(x
+ 1)^(1/2))
```

sympy [A] time = 5.40, size = 165, normalized size = 2.84

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6}+\frac{\sqrt{-1+\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-6x+3(x+1)^2-6}+\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)}{-6x+3(x+1)^2-6}+\frac{i\sqrt{1-\frac{2}{x+1}}}{-6x+3(x+1)^2-6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(3/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2
*sqrt(-1 + 2/(x + 1))*(x + 1)/(-6*x + 3*(x + 1)**2 - 6) + sqrt(-1 + 2/(x +
1))/(-6*x + 3*(x + 1)**2 - 6), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))
*(x + 1)**2/(-6*x + 3*(x + 1)**2 - 6) + 2*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-6
*x + 3*(x + 1)**2 - 6) + I*sqrt(1 - 2/(x + 1))/(-6*x + 3*(x + 1)**2 - 6), T
rue))
```

3.1064 $\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {40, 39}

$$\frac{2x}{3\sqrt{1-x}\sqrt{x+1}}+\frac{x}{3(1-x)^{3/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] x/(3*(1 - x)^(3/2)*(1 + x)^(3/2)) + (2*x)/(3*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{x}{3(1-x)^{3/2}(1+x)^{3/2}}+\frac{2x}{3\sqrt{1-x}\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.53

$$-\frac{x\left(2x^2-3\right)}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] -1/3*(x*(-3 + 2*x^2))/(1 - x^2)^(3/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 62, normalized size = 1.44

$$\frac{(x+1)^{3/2}\left(-\frac{(1-x)^3}{(x+1)^3}-\frac{9(1-x)^2}{(x+1)^2}+\frac{9(1-x)}{x+1}+1\right)}{24(1-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(5/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(3/2)*(1 - (1 - x)^3/(1 + x)^3 - (9*(1 - x)^2)/(1 + x)^2 + (9*(1 -
 x))/(1 + x)))/(24*(1 - x)^(3/2))
```

fricas [A] time = 1.52, size = 35, normalized size = 0.81

$$-\frac{\left(2\,x^{3}-3\,x\right)\sqrt{x+1}\,\sqrt{-x+1}}{3\left(x^{4}-2\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(x + 1)*sqrt(-x + 1)/(x^4 - 2*x^2 + 1)
```

giac [B] time = 0.68, size = 113, normalized size = 2.63

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{192\,(x+1)^{\frac{3}{2}}}+\frac{11\left(\sqrt{2}-\sqrt{-x+1}\right)}{64\,\sqrt{x+1}}-\frac{(4\,x-5)\sqrt{x+1}\,\sqrt{-x+1}}{12\,(x-1)^{2}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{192\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 11/64*(sqrt(2) - sqrt(-x +
 1))/sqrt(x + 1) - 1/12*(4*x - 5)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^2 - 1/19
2*(x + 1)^(3/2)*(33*(sqrt(2) - sqrt(-x + 1))^2/(x + 1) + 1)/(sqrt(2) - sqrt
(-x + 1))^3
```

maple [A] time = 0.00, size = 23, normalized size = 0.53

$$-\frac{\left(2x^{2}-3\right)x}{3\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(5/2)/(x+1)^(5/2),x)
```

```
[Out] -1/3*x*(2*x^2-3)/(x+1)^(3/2)/(-x+1)^(3/2)
```

maxima [A] time = 1.39, size = 25, normalized size = 0.58

$$\frac{2\,x}{3\,\sqrt{-x^{2}+1}}+\frac{x}{3\left(-x^{2}+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(5/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(-x^2 + 1) + 1/3*x/(-x^2 + 1)^(3/2)
```

mupad [B] time = 0.37, size = 41, normalized size = 0.95

$$\frac{3\,x\,\sqrt{1-x}-2\,x^{3}\,\sqrt{1-x}}{(3\,x+3)\,\left(x-1\right)^{2}\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(5/2)*(x + 1)^(5/2)),x)
```

```
[Out] (3*x*(1 - x)^(1/2) - 2*x^3*(1 - x)^(1/2))/((3*x + 3)*(x - 1)^2*(x + 1)^(1/2
))
```

sympy [B] time = 9.61, size = 279, normalized size = 6.49

$$\begin{cases} -\frac{2\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12} + \frac{6\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{3\sqrt{-1+\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{\sqrt{-1+\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{for } \frac{2}{|x+1|} > 1 \\ -\frac{2i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{12x+3(x+1)^3-12(x+1)^2+12} + \frac{6i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{3i\sqrt{1-\frac{2}{x+1}}(x+1)}{12x+3(x+1)^3-12(x+1)^2+12} - \frac{i\sqrt{1-\frac{2}{x+1}}}{12x+3(x+1)^3-12(x+1)^2+12} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(5/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-2*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(12*x + 3*(x + 1)**3 - 12*(x
+ 1)**2 + 12) + 6*sqrt(-1 + 2/(x + 1))*(x + 1)**2/(12*x + 3*(x + 1)**3 - 12
*(x + 1)**2 + 12) - 3*sqrt(-1 + 2/(x + 1))*(x + 1)/(12*x + 3*(x + 1)**3 - 1
2*(x + 1)**2 + 12) - sqrt(-1 + 2/(x + 1))/(12*x + 3*(x + 1)**3 - 12*(x + 1)
**2 + 12), 2/Abs(x + 1) > 1), (-2*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(12*x +
3*(x + 1)**3 - 12*(x + 1)**2 + 12) + 6*I*sqrt(1 - 2/(x + 1))*(x + 1)**2/(12
*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - 3*I*sqrt(1 - 2/(x + 1))*(x + 1)/(
12*x + 3*(x + 1)**3 - 12*(x + 1)**2 + 12) - I*sqrt(1 - 2/(x + 1))/(12*x + 3
*(x + 1)**3 - 12*(x + 1)**2 + 12), True))
```

3.1065 $\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=63

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{15\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{15(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{5(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(5*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*x)/(15*(1 - x)^(3/2)*(1 + x)^(3/2))
+ (8*x)/(15*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{5}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{15}\int \frac{1}{(1-x)^{3/2}(1+x)^{3/2}}\,dx\\ &= \frac{1}{5(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{15(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{15\sqrt{1-x}\,\sqrt{1+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.63

$$\frac{8x^4-8x^3-12x^2+12x+3}{15(1-x)^{5/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] (3 + 12*x - 12*x^2 - 8*x^3 + 8*x^4)/(15*(1 - x)^(5/2)*(1 + x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 76, normalized size = 1.21

$$\frac{(x+1)^{5/2}\left(-\frac{5(1-x)^4}{(x+1)^4}-\frac{60(1-x)^3}{(x+1)^3}+\frac{90(1-x)^2}{(x+1)^2}+\frac{20(1-x)}{x+1}+3\right)}{240(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(7/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(5/2)*(3 - (5*(1 - x)^4)/(1 + x)^4 - (60*(1 - x)^3)/(1 + x)^3 + (9
0*(1 - x)^2)/(1 + x)^2 + (20*(1 - x))/(1 + x)))/(240*(1 - x)^(5/2))
```

fricas [A] time = 1.27, size = 84, normalized size = 1.33

$$\frac{3\,x^{5}-3\,x^{4}-6\,x^{3}+6\,x^{2}-\left(8\,x^{4}-8\,x^{3}-12\,x^{2}+12\,x+3\right)\sqrt{x+1}\,\sqrt{-x+1}+3\,x-3}{15\,\left(x^{5}-x^{4}-2\,x^{3}+2\,x^{2}+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^5 - 3*x^4 - 6*x^3 + 6*x^2 - (8*x^4 - 8*x^3 - 12*x^2 + 12*x + 3)*s
qrt(x + 1)*sqrt(-x + 1) + 3*x - 3)/(x^5 - x^4 - 2*x^3 + 2*x^2 + x - 1)
```

giac [B] time = 0.71, size = 119, normalized size = 1.89

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}{384\,(x+1)^{\frac{3}{2}}}+\frac{15\left(\sqrt{2}-\sqrt{-x+1}\right)}{128\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{45\left(\sqrt{2}-\sqrt{-x+1}\right)^{2}}{x+1}+1\right)}{384\left(\sqrt{2}-\sqrt{-x+1}\right)^{3}}-\frac{((73\,x-247)(x+1)+360)\sqrt{x+1}\,\sqrt{-x+1}}{240\,(x-1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 15/128*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/384*(x + 1)^(3/2)*(45*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/240*((73*x - 247)*(x + 1) + 360)*sq
rt(x + 1)*sqrt(-x + 1)/(x - 1)^3
```

maple [A] time = 0.00, size = 35, normalized size = 0.56

$$\frac{8x^{4}-8x^{3}-12x^{2}+12x+3}{15\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(7/2)/(x+1)^(5/2),x)
```

```
[Out] 1/15*(8*x^4-8*x^3-12*x^2+12*x+3)/(x+1)^(3/2)/(-x+1)^(5/2)
```

maxima [A] time = 1.39, size = 52, normalized size = 0.83

$$\frac{8\,x}{15\,\sqrt{-x^{2}+1}}+\frac{4\,x}{15\left(-x^{2}+1\right)^{\frac{3}{2}}}-\frac{1}{5\left(\left(-x^{2}+1\right)^{\frac{3}{2}}x-\left(-x^{2}+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-x^2 + 1) + 4/15*x/(-x^2 + 1)^(3/2) - 1/5/((-x^2 + 1)^(3/2)*x -
 (-x^2 + 1)^(3/2))
```

mupad [B] time = 0.38, size = 75, normalized size = 1.19

$$-\frac{12\,x\,\sqrt{1-x}\,+3\,\sqrt{1-x}\,-12\,x^2\,\sqrt{1-x}\,-8\,x^3\,\sqrt{1-x}\,+8\,x^4\,\sqrt{1-x}}{\left(15\,x+15\right)\,{\left(x-1\right)}^3\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(7/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(12*x*(1 - x)^(1/2) + 3*(1 - x)^(1/2) - 12*x^2*(1 - x)^(1/2) - 8*x^3*(1 -
x)^(1/2) + 8*x^4*(1 - x)^(1/2))/((15*x + 15)*(x - 1)^3*(x + 1)^(1/2))
```

sympy [B] time = 27.61, size = 423, normalized size = 6.71

$$\begin{cases} -\frac{8\sqrt{-1+\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40\sqrt{-1+\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60\sqrt{-1+\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{20\sqrt{-1+\frac{2}{x+1}}(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{5\sqrt{-1+\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{for}\ \frac{2}{|x+1|} > 1 \\ -\frac{8i\sqrt{1-\frac{2}{x+1}}(x+1)^4}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{40i\sqrt{1-\frac{2}{x+1}}(x+1)^3}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} - \frac{60i\sqrt{1-\frac{2}{x+1}}(x+1)^2}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{20i\sqrt{1-\frac{2}{x+1}}(x+1)}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} + \frac{5i\sqrt{1-\frac{2}{x+1}}}{-120x+15(x+1)^4-90(x+1)^3+180(x+1)^2-120} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((-8*sqrt(-1 + 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 40*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-1
20*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*sqrt(-1 +
 2/(x + 1))*(x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1
)**2 - 120) + 20*sqrt(-1 + 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*
(x + 1)**3 + 180*(x + 1)**2 - 120) + 5*sqrt(-1 + 2/(x + 1))/(-120*x + 15*(x
 + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120), 2/Abs(x + 1) > 1), (-8*I*
sqrt(1 - 2/(x + 1))*(x + 1)**4/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 18
0*(x + 1)**2 - 120) + 40*I*sqrt(1 - 2/(x + 1))*(x + 1)**3/(-120*x + 15*(x +
 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) - 60*I*sqrt(1 - 2/(x + 1))*(
x + 1)**2/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 + 180*(x + 1)**2 - 120) +
 20*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-120*x + 15*(x + 1)**4 - 90*(x + 1)**3 +
 180*(x + 1)**2 - 120) + 5*I*sqrt(1 - 2/(x + 1))/(-120*x + 15*(x + 1)**4 -
90*(x + 1)**3 + 180*(x + 1)**2 - 120), True))
```

3.1066 $\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=83

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{8x}{21\sqrt{1-x}\,\sqrt{x+1}}+\frac{4x}{21(1-x)^{3/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{5/2}(x+1)^{3/2}}+\frac{1}{7(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(7*(1 - x)^(7/2)*(1 + x)^(3/2)) + 1/(7*(1 - x)^(5/2)*(1 + x)^(3/2)) + (4*
x)/(21*(1 - x)^(3/2)*(1 + x)^(3/2)) + (8*x)/(21*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{5}{7}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4}{7}\int \frac{1}{(1-x)^{5/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8}{21}\int \frac{}{(1-x}\\ &= \frac{1}{7(1-x)^{7/2}(1+x)^{3/2}}+\frac{1}{7(1-x)^{5/2}(1+x)^{3/2}}+\frac{4x}{21(1-x)^{3/2}(1+x)^{3/2}}+\frac{8x}{21\sqrt{1-x}\,\sqrt{1}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 0.54

$$\frac{-8x^5+16x^4+4x^3-24x^2+9x+6}{21(1-x)^{7/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] (6 + 9*x - 24*x^2 + 4*x^3 + 16*x^4 - 8*x^5)/(21*(1 - x)^(7/2)*(1 + x)^(3/2)
)
```

IntegrateAlgebraic [A] time = 0.08, size = 90, normalized size = 1.08

$$\frac{(x+1)^{7/2}\left(-\frac{7(1-x)^5}{(x+1)^5}-\frac{105(1-x)^4}{(x+1)^4}+\frac{210(1-x)^3}{(x+1)^3}+\frac{70(1-x)^2}{(x+1)^2}+\frac{21(1-x)}{x+1}+3\right)}{672(1-x)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(9/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(7/2)*(3 - (7*(1 - x)^5)/(1 + x)^5 - (105*(1 - x)^4)/(1 + x)^4 + (
210*(1 - x)^3)/(1 + x)^3 + (70*(1 - x)^2)/(1 + x)^2 + (21*(1 - x))/(1 + x))
)/(672*(1 - x)^(7/2))
```

fricas [A] time = 1.33, size = 101, normalized size = 1.22

$$\frac{6x^6-12x^5-6x^4+24x^3-6x^2-\left(8x^5-16x^4-4x^3+24x^2-9x-6\right)\sqrt{x+1}\sqrt{-x+1}-12x+6}{21\left(x^6-2x^5-x^4+4x^3-x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(6*x^6 - 12*x^5 - 6*x^4 + 24*x^3 - 6*x^2 - (8*x^5 - 16*x^4 - 4*x^3 + 2
4*x^2 - 9*x - 6)*sqrt(x + 1)*sqrt(-x + 1) - 12*x + 6)/(x^6 - 2*x^5 - x^4 +
4*x^3 - x^2 - 2*x + 1)
```

giac [B] time = 0.69, size = 125, normalized size = 1.51

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{768\,(x+1)^{\frac{3}{2}}}+\frac{19\left(\sqrt{2}-\sqrt{-x+1}\right)}{256\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{57\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{768\left(\sqrt{2}-\sqrt{-x+1}\right)^3}-\frac{(((79\,x-432)(x+1)+1120)(x+1)-840)\sqrt{x+1}\sqrt{-x+1}}{336\,(x-1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/768*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 19/256*(sqrt(2) - sqrt(-x
+ 1))/sqrt(x + 1) - 1/768*(x + 1)^(3/2)*(57*(sqrt(2) - sqrt(-x + 1))^2/(x +
 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/336*(((79*x - 432)*(x + 1) + 1120)*
(x + 1) - 840)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^4
```

maple [A] time = 0.00, size = 40, normalized size = 0.48

$$-\frac{8x^5-16x^4-4x^3+24x^2-9x-6}{21\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(9/2)/(x+1)^(5/2),x)
```

```
[Out] -1/21*(8*x^5-16*x^4-4*x^3+24*x^2-9*x-6)/(x+1)^(3/2)/(-x+1)^(7/2)
```

maxima [A] time = 1.34, size = 91, normalized size = 1.10

$$\frac{8\,x}{21\,\sqrt{-x^2+1}}+\frac{4\,x}{21\,\left(-x^2+1\right)^{\frac{3}{2}}}+\frac{1}{7\,\left(\left(-x^2+1\right)^{\frac{3}{2}}x^2-2\,\left(-x^2+1\right)^{\frac{3}{2}}x+\left(-x^2+1\right)^{\frac{3}{2}}\right)}-\frac{1}{7\,\left(\left(-x^2+1\right)^{\frac{3}{2}}x-\left(-x^2+1\right)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(9/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 8/21*x/sqrt(-x^2 + 1) + 4/21*x/(-x^2 + 1)^(3/2) + 1/7/((-x^2 + 1)^(3/2)*x^2
 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 1/7/((-x^2 + 1)^(3/2)*x - (-x
^2 + 1)^(3/2))
```

mupad [B] time = 0.41, size = 86, normalized size = 1.04

$$\frac{9\,x\,\sqrt{1-x}+6\,\sqrt{1-x}-24\,x^2\,\sqrt{1-x}+4\,x^3\,\sqrt{1-x}+16\,x^4\,\sqrt{1-x}-8\,x^5\,\sqrt{1-x}}{\left(21\,x+21\right)\,{\left(x-1\right)}^4\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(9/2)*(x + 1)^(5/2)),x)
```

```
[Out] (9*x*(1 - x)^(1/2) + 6*(1 - x)^(1/2) - 24*x^2*(1 - x)^(1/2) + 4*x^3*(1 - x)
^(1/2) + 16*x^4*(1 - x)^(1/2) - 8*x^5*(1 - x)^(1/2))/((21*x + 21)*(x - 1)^4
*(x + 1)^(1/2))
```

sympy [B] time = 71.01, size = 592, normalized size = 7.13

$$\begin{cases} \frac{8 \sqrt{-1 + \frac{2}{x + 1}} \left(x + 1\right)^{5}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} - \frac{56 \sqrt{-1 + \frac{2}{x + 1}} \left(x + 1\right)^{4}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{140 \sqrt{-1 + \frac{2}{x + 1}} \left(x + 1\right)^{3}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} - \frac{140 \sqrt{-1 + \frac{2}{x + 1}} \left(x + 1\right)^{2}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{35 \sqrt{-1 + \frac{2}{x + 1}} \left(x + 1\right)}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{7 \sqrt{-1 + \frac{2}{x + 1}}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} & \text{for}\: \frac{2}{\left|{x + 1}\right|} > 1 \\ \frac{8 i \sqrt{1 - \frac{2}{x + 1}} \left(x + 1\right)^{5}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} - \frac{56 i \sqrt{1 - \frac{2}{x + 1}} \left(x + 1\right)^{4}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{140 i \sqrt{1 - \frac{2}{x + 1}} \left(x + 1\right)^{3}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} - \frac{140 i \sqrt{1 - \frac{2}{x + 1}} \left(x + 1\right)^{2}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{35 i \sqrt{1 - \frac{2}{x + 1}} \left(x + 1\right)}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} + \frac{7 i \sqrt{1 - \frac{2}{x + 1}}}{- 336 x - 21 \left(x + 1\right)^{5} + 168 \left(x + 1\right)^{4} - 504 \left(x + 1\right)^{3} + 672 \left(x + 1\right)^{2} - 336} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(9/2)/(1+x)**(5/2),x)
```

```
[Out] Piecewise((8*sqrt(-1 + 2/(x + 1))*(x + 1)**5/(-336*x - 21*(x + 1)**5 + 168*
(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) - 56*sqrt(-1 + 2/(x + 1
))*(x + 1)**4/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 6
72*(x + 1)**2 - 336) + 140*sqrt(-1 + 2/(x + 1))*(x + 1)**3/(-336*x - 21*(x
+ 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) - 140*sqr
t(-1 + 2/(x + 1))*(x + 1)**2/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504
*(x + 1)**3 + 672*(x + 1)**2 - 336) + 35*sqrt(-1 + 2/(x + 1))*(x + 1)/(-336
*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336
) + 7*sqrt(-1 + 2/(x + 1))/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(
x + 1)**3 + 672*(x + 1)**2 - 336), 2/Abs(x + 1) > 1), (8*I*sqrt(1 - 2/(x +
1))*(x + 1)**5/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 +
672*(x + 1)**2 - 336) - 56*I*sqrt(1 - 2/(x + 1))*(x + 1)**4/(-336*x - 21*(x
 + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) + 140*I*
sqrt(1 - 2/(x + 1))*(x + 1)**3/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 5
04*(x + 1)**3 + 672*(x + 1)**2 - 336) - 140*I*sqrt(1 - 2/(x + 1))*(x + 1)**
2/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**
2 - 336) + 35*I*sqrt(1 - 2/(x + 1))*(x + 1)/(-336*x - 21*(x + 1)**5 + 168*(
x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**2 - 336) + 7*I*sqrt(1 - 2/(x + 1)
)/(-336*x - 21*(x + 1)**5 + 168*(x + 1)**4 - 504*(x + 1)**3 + 672*(x + 1)**
2 - 336), True))
```

3.1067 $\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{16x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Rubi [A] time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {45, 40, 39}

$$\frac{16x}{63\sqrt{1-x}\sqrt{x+1}}+\frac{8x}{63(1-x)^{3/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{5/2}(x+1)^{3/2}}+\frac{2}{21(1-x)^{7/2}(x+1)^{3/2}}+\frac{1}{9(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] 1/(9*(1 - x)^(9/2)*(1 + x)^(3/2)) + 2/(21*(1 - x)^(7/2)*(1 + x)^(3/2)) + 2/
(21*(1 - x)^(5/2)*(1 + x)^(3/2)) + (8*x)/(63*(1 - x)^(3/2)*(1 + x)^(3/2)) +
 (16*x)/(63*Sqrt[1 - x]*Sqrt[1 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-x)^{11/2}(1+x)^{5/2}}\,dx &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{3}\int \frac{1}{(1-x)^{9/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{10}{21}\int \frac{1}{(1-x)^{7/2}(1+x)^{5/2}}\,dx\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8}{21}\int \frac{}{(1-x)^{5}}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1}\\ &= \frac{1}{9(1-x)^{9/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{7/2}(1+x)^{3/2}}+\frac{2}{21(1-x)^{5/2}(1+x)^{3/2}}+\frac{8x}{63(1-x)^{3/2}(1}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 0.49

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63(1-x)^{9/2}(x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] (19 + 6*x - 66*x^2 + 56*x^3 + 24*x^4 - 48*x^5 + 16*x^6)/(63*(1 - x)^(9/2)*(
1 + x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 104, normalized size = 1.01

$$\frac{(x+1)^{9/2}\left(-\frac{21(1-x)^6}{(x+1)^6}-\frac{378(1-x)^5}{(x+1)^5}+\frac{945(1-x)^4}{(x+1)^4}+\frac{420(1-x)^3}{(x+1)^3}+\frac{189(1-x)^2}{(x+1)^2}+\frac{54(1-x)}{x+1}+7\right)}{4032(1-x)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - x)^(11/2)*(1 + x)^(5/2)),x]
```

```
[Out] ((1 + x)^(9/2)*(7 - (21*(1 - x)^6)/(1 + x)^6 - (378*(1 - x)^5)/(1 + x)^5 +
(945*(1 - x)^4)/(1 + x)^4 + (420*(1 - x)^3)/(1 + x)^3 + (189*(1 - x)^2)/(1
+ x)^2 + (54*(1 - x))/(1 + x)))/(4032*(1 - x)^(9/2))
```

fricas [A] time = 0.88, size = 114, normalized size = 1.11

$$\frac{19\,x^7-57\,x^6+19\,x^5+95\,x^4-95\,x^3-19\,x^2-\left(16\,x^6-48\,x^5+24\,x^4+56\,x^3-66\,x^2+6\,x+19\right)\sqrt{x+1}\,\sqrt{-x+1}+57\,x-19}{63\,\left(x^7-3\,x^6+x^5+5\,x^4-5\,x^3-x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/63*(19*x^7 - 57*x^6 + 19*x^5 + 95*x^4 - 95*x^3 - 19*x^2 - (16*x^6 - 48*x^
5 + 24*x^4 + 56*x^3 - 66*x^2 + 6*x + 19)*sqrt(x + 1)*sqrt(-x + 1) + 57*x -
19)/(x^7 - 3*x^6 + x^5 + 5*x^4 - 5*x^3 - x^2 + 3*x - 1)
```

giac [A] time = 0.74, size = 131, normalized size = 1.27

$$\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^3}{1536\,(x+1)^{\frac{3}{2}}}+\frac{23\left(\sqrt{2}-\sqrt{-x+1}\right)}{512\,\sqrt{x+1}}-\frac{(x+1)^{\frac{3}{2}}\left(\frac{69\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1}+1\right)}{1536\left(\sqrt{2}-\sqrt{-x+1}\right)^3}-\frac{((((667\,x-5021)(x+1)+18396)(x+1)-26880)(x+1)+15120)\sqrt{x+1}\,\sqrt{-x+1}}{4032\,(x-1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(sqrt(2) - sqrt(-x + 1))^3/(x + 1)^(3/2) + 23/512*(sqrt(2) - sqrt(-x
 + 1))/sqrt(x + 1) - 1/1536*(x + 1)^(3/2)*(69*(sqrt(2) - sqrt(-x + 1))^2/(x
 + 1) + 1)/(sqrt(2) - sqrt(-x + 1))^3 - 1/4032*((((667*x - 5021)*(x + 1) +
18396)*(x + 1) - 26880)*(x + 1) + 15120)*sqrt(x + 1)*sqrt(-x + 1)/(x - 1)^5
```

maple [A] time = 0.00, size = 45, normalized size = 0.44

$$\frac{16x^6-48x^5+24x^4+56x^3-66x^2+6x+19}{63\,(x+1)^{\frac{3}{2}}\,(-x+1)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+1)^(11/2)/(x+1)^(5/2),x)
```

```
[Out] 1/63*(16*x^6-48*x^5+24*x^4+56*x^3-66*x^2+6*x+19)/(x+1)^(3/2)/(-x+1)^(9/2)
```

maxima [A] time = 1.42, size = 146, normalized size = 1.42

$$\frac{16\,x}{63\,\sqrt{-x^2+1}}+\frac{8\,x}{63\,(-x^2+1)^{\frac{3}{2}}}-\frac{1}{9\left((-x^2+1)^{\frac{3}{2}}x^3-3\,(-x^2+1)^{\frac{3}{2}}x^2+3\,(-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}+\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x^2-2\,(-x^2+1)^{\frac{3}{2}}x+(-x^2+1)^{\frac{3}{2}}\right)}-\frac{2}{21\left((-x^2+1)^{\frac{3}{2}}x-(-x^2+1)^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(11/2)/(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 16/63*x/sqrt(-x^2 + 1) + 8/63*x/(-x^2 + 1)^(3/2) - 1/9/((-x^2 + 1)^(3/2)*x^
3 - 3*(-x^2 + 1)^(3/2)*x^2 + 3*(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2)) + 2/2
1/((-x^2 + 1)^(3/2)*x^2 - 2*(-x^2 + 1)^(3/2)*x + (-x^2 + 1)^(3/2)) - 2/21/(
(-x^2 + 1)^(3/2)*x - (-x^2 + 1)^(3/2))
```

mupad [B] time = 0.42, size = 99, normalized size = 0.96

$$-\frac{6\,x\,\sqrt{1-x}+19\,\sqrt{1-x}-66\,x^2\,\sqrt{1-x}+56\,x^3\,\sqrt{1-x}+24\,x^4\,\sqrt{1-x}-48\,x^5\,\sqrt{1-x}+16\,x^6\,\sqrt{1-x}}{(63\,x+63)\,(x-1)^5\,\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - x)^(11/2)*(x + 1)^(5/2)),x)
```

```
[Out] -(6*x*(1 - x)^(1/2) + 19*(1 - x)^(1/2) - 66*x^2*(1 - x)^(1/2) + 56*x^3*(1 -
 x)^(1/2) + 24*x^4*(1 - x)^(1/2) - 48*x^5*(1 - x)^(1/2) + 16*x^6*(1 - x)^(1
/2))/((63*x + 63)*(x - 1)^5*(x + 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(11/2)/(1+x)**(5/2),x)
```

```
[Out] Timed out
```

3.1068 $\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx$

Optimal. Leaf size=126

$$\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^2c^2x\sqrt{ax+a}\sqrt{c-cx}+\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{5}{24}acx(ax+a)^{3/2}(c-cx)^{3/2}+\frac{1}{6}x(ax+a)^{5/2}(c-cx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (5*a^2*c^2*x*Sqrt[a + a*x]*Sqrt[c - c*x])/16 + (5*a*c*x*(a + a*x)^(3/2)*(c
- c*x)^(3/2))/24 + (x*(a + a*x)^(5/2)*(c - c*x)^(5/2))/6 + (5*a^(5/2)*c^(5/
2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/8
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+ax)^{5/2}(c-cx)^{5/2}\,dx = \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{6}(5ac)\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$$

$$= \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} + \frac{1}{8}\left(5a^2c^2\right)\int \sqrt{a+ax}\,\sqrt{c-c}$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

$$= \frac{5}{16}a^2c^2x\sqrt{a+ax}\,\sqrt{c-cx} + \frac{5}{24}acx(a+ax)^{3/2}(c-cx)^{3/2} + \frac{1}{6}x(a+ax)^{5/2}(c-cx)^{5/2} +$$

Mathematica [A] time = 0.10, size = 114, normalized size = 0.90

$$\frac{c^{3/2}(a(x+1))^{5/2}\sqrt{c-cx}\left(\sqrt{c}\,x\sqrt{x+1}\left(8x^5-8x^4-26x^3+26x^2+33x-33\right)+30\sqrt{c-cx}\,\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{48(x-1)(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] (c^(3/2)*(a*(1 + x))^(5/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-33 + 33*x
 + 26*x^2 - 26*x^3 - 8*x^4 + 8*x^5) + 30*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]
/(Sqrt[2]*Sqrt[c]])))/(48*(-1 + x)*(1 + x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.38, size = 206, normalized size = 1.63

$$-\frac{5}{8}a^{5/2}c^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-cx}}{\sqrt{c}\,\sqrt{ax+a}}\right)-\frac{a^3c^3\sqrt{c-cx}\left(\frac{15a^5(c-cx)^5}{(ax+a)^5}+\frac{85a^4c(c-cx)^4}{(ax+a)^4}+\frac{198a^3c^2(c-cx)^3}{(ax+a)^3}-\frac{198a^2c^3(c-cx)^2}{(ax+a)^2}-\frac{85ac^4(c-cx)}{ax+a}-15c^5\right)}{24\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + a*x)^(5/2)*(c - c*x)^(5/2),x]
```

```
[Out] -1/24*(a^3*c^3*Sqrt[c - c*x]*(-15*c^5 - (85*a*c^4*(c - c*x))/(a + a*x) - (1
98*a^2*c^3*(c - c*x)^2)/(a + a*x)^2 + (198*a^3*c^2*(c - c*x)^3)/(a + a*x)^3
 + (85*a^4*c*(c - c*x)^4)/(a + a*x)^4 + (15*a^5*(c - c*x)^5)/(a + a*x)^5))/
(Sqrt[a + a*x]*(c + (a*(c - c*x))/(a + a*x))^6) - (5*a^(5/2)*c^(5/2)*ArcTan
[(Sqrt[a]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/8
```

fricas [A] time = 1.61, size = 201, normalized size = 1.60

$$\left[\frac{5}{32}\sqrt{-ac}\,a^2c^2\log\left(2acx^2+2\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)+\frac{1}{48}\left(8a^2c^2x^5-26a^2c^2x^3+33a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c},\ -\frac{5}{16}\sqrt{ac}\,a^2c^2\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)+\frac{1}{48}\left(8a^2c^2x^5-26a^2c^2x^3+33a^2c^2x\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [5/32*sqrt(-a*c)*a^2*c^2*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c
*x + c)*x - a*c) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2*x^3 + 33*a^2*c^2*x)*sqr
t(a*x + a)*sqrt(-c*x + c), -5/16*sqrt(a*c)*a^2*c^2*arctan(sqrt(a*c)*sqrt(a*
x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a*c)) + 1/48*(8*a^2*c^2*x^5 - 26*a^2*c^2
*x^3 + 33*a^2*c^2*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

giac [B] time = 1.57, size = 679, normalized size = 5.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/240*(150*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c +
2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((2*((a*x + a)*(4*(a
*x + a)*(5*(a*x + a)/a^5 - 31/a^4) + 321/a^3) - 451/a^2)*(a*x + a) + 745/a)
*(a*x + a) - 405)*sqrt(a*x + a))*c^2*abs(a) - 1/120*(90*a^2*c*log(abs(-sqrt
(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-
(a*x + a)*a*c + 2*a^2*c)*((2*(a*x + a)*(3*(a*x + a)*(4*(a*x + a)/a^4 - 21/a
^3) + 133/a^2) - 295/a)*(a*x + a) + 195)*sqrt(a*x + a))*c^2*abs(a) - 1/12*(
18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)
))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x + a)*(3*(
a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c^2*abs(a) + 1/3*(6*a^2
*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqr
t(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2*(a*x +
 a)/a^2 - 7/a) + 9))*c^2*abs(a) - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a
) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a
^2*c)*sqrt(a*x + a))*c^2*abs(a) + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x
 + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c +
 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c^2*abs(a)/a
```

maple [B] time = 0.01, size = 193, normalized size = 1.53

$$\frac{5\sqrt{(-cx+c)(ax+a)}\, a^{3} c^{3} \arctan\left(\frac{\sqrt{ac}\, x}{\sqrt{-ac\,x^{2}+ac}}\right)}{16\sqrt{-cx+c}\, \sqrt{ax+a}\, \sqrt{ac}}+\frac{5\sqrt{-cx+c}\, \sqrt{ax+a}\, a^{2} c^{2}}{16}+\frac{5\left(-cx+c\right)^{\frac{3}{2}} \sqrt{ax+a}\, a^{2} c}{48}+\frac{\left(-cx+c\right)^{\frac{5}{2}} \sqrt{ax+a}\, a^{2}}{24}-\frac{\sqrt{ax+a}\, \left(-cx+c\right)^{\frac{7}{2}} a^{2}}{8c}-\frac{\left(ax+a\right)^{\frac{3}{2}} \left(-cx+c\right)^{\frac{7}{2}} a}{6c}-\frac{\left(ax+a\right)^{\frac{5}{2}} \left(-cx+c\right)^{\frac{7}{2}}}{6c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(5/2)*(-c*x+c)^(5/2),x)
```

```
[Out] -1/6/c*(a*x+a)^(5/2)*(-c*x+c)^(7/2)-1/6*a/c*(a*x+a)^(3/2)*(-c*x+c)^(7/2)-1/
8*a^2/c*(a*x+a)^(1/2)*(-c*x+c)^(7/2)+1/24*a^2*(-c*x+c)^(5/2)*(a*x+a)^(1/2)+
5/48*a^2*c*(-c*x+c)^(3/2)*(a*x+a)^(1/2)+5/16*a^2*c^2*(-c*x+c)^(1/2)*(a*x+a)
^(1/2)+5/16*a^3*c^3*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(
a*c)^(1/2)*arctan((a*c)^(1/2)*x/(-a*c*x^2+a*c)^(1/2))
```

maxima [A] time = 3.02, size = 72, normalized size = 0.57

$$\frac{5\, a^{3} c^{3} \arcsin\left(x\right)}{16\, \sqrt{ac}}+\frac{5}{16}\, \sqrt{-acx^{2}+ac}\, a^{2} c^{2} x+\frac{5}{24}\, \left(-acx^{2}+ac\right)^{\frac{3}{2}} acx+\frac{1}{6}\, \left(-acx^{2}+ac\right)^{\frac{5}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(5/2)*(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^3*c^3*arcsin(x)/sqrt(a*c) + 5/16*sqrt(-a*c*x^2 + a*c)*a^2*c^2*x + 5/
24*(-a*c*x^2 + a*c)^(3/2)*a*c*x + 1/6*(-a*c*x^2 + a*c)^(5/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,x)^{5/2}\,(c-c\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(5/2)*(c - c*x)^(5/2),x)
```

```
[Out] int((a + a*x)^(5/2)*(c - c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(x+1))^{\frac{5}{2}}(-c(x-1))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(5/2)*(-c*x+c)**(5/2),x)
```

```
[Out] Integral((a*(x + 1))**(5/2)*(-c*(x - 1))**(5/2), x)
```

3.1069 $\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx$

Optimal. Leaf size=96

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)+\frac{3}{8}acx\sqrt{ax+a}\sqrt{c-cx}+\frac{1}{4}x(ax+a)^{3/2}(c-cx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (3*a*c*x*Sqrt[a + a*x]*Sqrt[c - c*x])/8 + (x*(a + a*x)^(3/2)*(c - c*x)^(3/2
))/4 + (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c -
c*x])])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+ax)^{3/2}(c-cx)^{3/2}\,dx &= \frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}(3ac)\int \sqrt{a+ax}\,\sqrt{c-cx}\,dx \\
&= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{8}\left(3a^2c^2\right)\int \frac{1}{\sqrt{a+ax}\,\sqrt{c-cx}}\,dx \\
&= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx \right. \\
&= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{1}{4}\left(3ac^2\right)\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx,x, \right. \\
&= \frac{3}{8}acx\sqrt{a+ax}\,\sqrt{c-cx}+\frac{1}{4}x(a+ax)^{3/2}(c-cx)^{3/2}+\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+ax}}{\sqrt{a}\,\sqrt{c-cx}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.08, size = 104, normalized size = 1.08

$$
\frac{\sqrt{c}\,(a(x+1))^{3/2}\sqrt{c-cx}\left(\sqrt{c}\,x\sqrt{x+1}\left(-2x^3+2x^2+5x-5\right)+6\sqrt{c-cx}\,\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\,\sqrt{c}}\right)\right)}{8(x-1)(x+1)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] (Sqrt[c]*(a*(1 + x))^(3/2)*Sqrt[c - c*x]*(Sqrt[c]*x*Sqrt[1 + x]*(-5 + 5*x +
 2*x^2 - 2*x^3) + 6*Sqrt[c - c*x]*ArcSin[Sqrt[c - c*x]/(Sqrt[2]*Sqrt[c])]))
/(8*(-1 + x)*(1 + x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 160, normalized size = 1.67

$$
-\frac{3}{4}a^{3/2}c^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-cx}}{\sqrt{c}\,\sqrt{ax+a}}\right)-\frac{a^2c^2\sqrt{c-cx}\left(\frac{3a^3(c-cx)^3}{(ax+a)^3}+\frac{11a^2c(c-cx)^2}{(ax+a)^2}-\frac{11ac^2(c-cx)}{ax+a}-3c^3\right)}{4\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^4}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + a*x)^(3/2)*(c - c*x)^(3/2),x]
```

```
[Out] -1/4*(a^2*c^2*Sqrt[c - c*x]*(-3*c^3 - (11*a*c^2*(c - c*x))/(a + a*x) + (11*
a^2*c*(c - c*x)^2)/(a + a*x)^2 + (3*a^3*(c - c*x)^3)/(a + a*x)^3))/(Sqrt[a
+ a*x]*(c + (a*(c - c*x))/(a + a*x))^4) - (3*a^(3/2)*c^(3/2)*ArcTan[(Sqrt[a
]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/4
```

fricas [A] time = 1.40, size = 155, normalized size = 1.61

$$
\left[\frac{3}{16}\sqrt{-ac}\,ac\log\left(2acx^2+2\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)-\frac{1}{8}\left(2acx^3-5acx\right)\sqrt{ax+a}\,\sqrt{-cx+c},\ -\frac{3}{8}\sqrt{ac}\,ac\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)-\frac{1}{8}\left(2acx^3-5acx\right)\sqrt{ax+a}\,\sqrt{-cx+c}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/16*sqrt(-a*c)*a*c*log(2*a*c*x^2 + 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - a*c) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c), -3/8
*sqrt(a*c)*a*c*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*x^2 - a
*c)) - 1/8*(2*a*c*x^3 - 5*a*c*x)*sqrt(a*x + a)*sqrt(-c*x + c)]
```

giac [B] time = 1.25, size = 403, normalized size = 4.20

$$
-\frac{\left(\frac{18a^2c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2a^2c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2a^2c}\left((ax+a)\left(2(ax+a)\left(\frac{3(ax+a)}{a^3}-\frac{13}{a^2}\right)+\frac{43}{a}\right)-39\right)\sqrt{ax+a}\right)c|a|}{24a}+\frac{\left(\frac{6a^2c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}\left((ax+a)\left(\frac{2(ax+a)}{a^2}-\frac{7}{a}\right)+9\right)\right)c|a|}{6a}-\frac{\left(\frac{2a^2c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2a^2c}\right|\right)}{\sqrt{-ac}}-\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}\right)c|a|}{a}+\frac{\left(\frac{2a^3c\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2a^2c}\right|\right)}{\sqrt{-ac}}+\sqrt{-(ax+a)ac+2a^2c}\,\sqrt{ax+a}\,(ax-2a)\right)c|a|}{2a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(18*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2
*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*(2*(a*x +
a)*(3*(a*x + a)/a^3 - 13/a^2) + 43/a) - 39)*sqrt(a*x + a))*c*abs(a)/a + 1/6
*(6*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(2
*(a*x + a)/a^2 - 7/a) + 9))*c*abs(a)/a - (2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(
a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*
c + 2*a^2*c)*sqrt(a*x + a))*c*abs(a)/a + 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*s
qrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a
)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*a))*c*abs(a)/a^2
```

maple [B] time = 0.00, size = 143, normalized size = 1.49

$$\frac{3\sqrt{(-cx+c)\,(ax+a)}\;a^{2}c^{2}\arctan\left(\frac{\sqrt{ac}\;x}{\sqrt{-ac\,x^{2}+ac}}\right)}{8\sqrt{-cx+c}\;\sqrt{ax+a}\;\sqrt{ac}}+\frac{3\sqrt{-cx+c}\;\sqrt{ax+a}\;ac}{8}+\frac{\sqrt{ax+a}\;(-cx+c)^{\frac{3}{2}}\,a}{8}-\frac{\sqrt{ax+a}\;(-cx+c)^{\frac{5}{2}}\,a}{4c}-\frac{(ax+a)^{\frac{3}{2}}\,(-cx+c)^{\frac{5}{2}}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(3/2)*(-c*x+c)^(3/2),x)
```

```
[Out] -1/4/c*(a*x+a)^(3/2)*(-c*x+c)^(5/2)-1/4*a/c*(a*x+a)^(1/2)*(-c*x+c)^(5/2)+1/
8*(a*x+a)^(1/2)*(-c*x+c)^(3/2)*a+3/8*a*c*(-c*x+c)^(1/2)*(a*x+a)^(1/2)+3/8*a
^2*c^2*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*ar
ctan((a*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

maxima [A] time = 3.09, size = 50, normalized size = 0.52

$$\frac{3\,a^{2}c^{2}\arcsin(x)}{8\,\sqrt{ac}}+\frac{3}{8}\,\sqrt{-acx^{2}+ac}\,acx+\frac{1}{4}\left(-acx^{2}+ac\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(3/2)*(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^2*c^2*arcsin(x)/sqrt(a*c) + 3/8*sqrt(-a*c*x^2 + a*c)*a*c*x + 1/4*(-a*
c*x^2 + a*c)^(3/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+a\,x)^{3/2}\,(c-c\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(3/2)*(c - c*x)^(3/2),x)
```

```
[Out] int((a + a*x)^(3/2)*(c - c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\,(x+1))^{\frac{3}{2}}\,(-c\,(x-1))^{\frac{3}{2}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(3/2)*(-c*x+c)**(3/2),x)
```

```
[Out] Integral((a*(x + 1))**(3/2)*(-c*(x - 1))**(3/2), x)
```

3.1070 $\int \sqrt{a+ax}\sqrt{c-cx}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {38, 63, 217, 203}

$$\frac{1}{2}x\sqrt{ax+a}\sqrt{c-cx}+\sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (x*Sqrt[a + a*x]*Sqrt[c - c*x])/2 + Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
+ a*x])/(Sqrt[a]*Sqrt[c - c*x])]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+ax}\sqrt{c-cx}\,dx = \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx} + \frac{1}{2}(ac)\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx} + c\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)$$
$$= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx} + c\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)$$
$$= \frac{1}{2}x\sqrt{a+ax}\sqrt{c-cx} + \sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)$$

Mathematica [A] time = 0.06, size = 69, normalized size = 1.03

$$\frac{\sqrt{a(x+1)}\left(x\sqrt{x+1}\sqrt{c-cx} - 2\sqrt{c}\sin^{-1}\left(\frac{\sqrt{c-cx}}{\sqrt{2}\sqrt{c}}\right)\right)}{2\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] (Sqrt[a*(1 + x)]*(x*Sqrt[1 + x]*Sqrt[c - c*x] - 2*Sqrt[c]*ArcSin[Sqrt[c - c
*x]/(Sqrt[2]*Sqrt[c])]))/(2*Sqrt[1 + x])
```

IntegrateAlgebraic [A] time = 0.19, size = 105, normalized size = 1.57

$$-\frac{ac\sqrt{c-cx}\left(\frac{a(c-cx)}{ax+a}-c\right)}{\sqrt{ax+a}\left(\frac{a(c-cx)}{ax+a}+c\right)^2} - \sqrt{a}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{c-cx}}{\sqrt{c}\sqrt{ax+a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + a*x]*Sqrt[c - c*x],x]
```

```
[Out] -((a*c*Sqrt[c - c*x]*(-c + (a*(c - c*x))/(a + a*x)))/(Sqrt[a + a*x]*(c + (a
*(c - c*x))/(a + a*x))^2)) - Sqrt[a]*Sqrt[c]*ArcTan[(Sqrt[a]*Sqrt[c - c*x])
/(Sqrt[c]*Sqrt[a + a*x])]
```

fricas [A] time = 1.60, size = 127, normalized size = 1.90

$$\left[\frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}\,x + \frac{1}{4}\sqrt{-ac}\log\left(2\,acx^2 + 2\sqrt{-ac}\sqrt{ax+a}\sqrt{-cx+c}\,x - ac\right), \frac{1}{2}\sqrt{ax+a}\sqrt{-cx+c}\,x - \frac{1}{2}\sqrt{ac}\arctan\left(\frac{\sqrt{ac}\sqrt{ax+a}\sqrt{-cx+c}\,x}{acx^2-ac}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a*x + a)*sqrt(-c*x + c)*x + 1/4*sqrt(-a*c)*log(2*a*c*x^2 + 2*sqrt
(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x - a*c), 1/2*sqrt(a*x + a)*sqrt(-c*x +
 c)*x - 1/2*sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/(a*c*
x^2 - a*c))]
```

giac [B] time = 0.90, size = 173, normalized size = 2.58

$$-\frac{\left(\frac{2\,a^2c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} - \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\right)|a|}{a^2} + \frac{\left(\frac{2\,a^3c\log\left(\left|-\sqrt{-ac}\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}} + \sqrt{-(ax+a)ac+2\,a^2c}\sqrt{ax+a}\,(ax-2\,a)\right)|a|}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -(2*a^2*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c
)))/sqrt(-a*c) - sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a))*abs(a)/a^2 +
 1/2*(2*a^3*c*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a
^2*c)))/sqrt(-a*c) + sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(a*x - 2*
a))*abs(a)/a^3
```

maple [A] time = 0.01, size = 98, normalized size = 1.46

$$\frac{\sqrt{\left(-cx+c\right)\left(ax+a\right)}\, ac \arctan\left(\frac{\sqrt{ac}\, x}{\sqrt{-ac\, x^{2}+ac}}\right)}{2\sqrt{-cx+c}\, \sqrt{ax+a}\, \sqrt{ac}}-\frac{\sqrt{ax+a}\, \left(-cx+c\right)^{\frac{3}{2}}}{2c}+\frac{\sqrt{ax+a}\, \sqrt{-cx+c}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+a)^(1/2)*(-c*x+c)^(1/2),x)
```

```
[Out] -1/2/c*(a*x+a)^(1/2)*(-c*x+c)^(3/2)+1/2*(a*x+a)^(1/2)*(-c*x+c)^(1/2)+1/2*a*
c*((-c*x+c)*(a*x+a))^(1/2)/(-c*x+c)^(1/2)/(a*x+a)^(1/2)/(a*c)^(1/2)*arctan(
(a*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

maxima [A] time = 3.08, size = 28, normalized size = 0.42

$$\frac{ac \arcsin\left(x\right)}{2\sqrt{ac}}+\frac{1}{2}\sqrt{-acx^{2}+ac}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)^(1/2)*(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*c*arcsin(x)/sqrt(a*c) + 1/2*sqrt(-a*c*x^2 + a*c)*x
```

mupad [B] time = 0.30, size = 59, normalized size = 0.88

$$\frac{x\sqrt{a+a\,x}\,\sqrt{c-c\,x}}{2}-\frac{\sqrt{a}\,\sqrt{-c}\,\ln\left(\sqrt{-c}\,\sqrt{a\,\left(x+1\right)}\,\sqrt{-c\,\left(x-1\right)}-\sqrt{a}\,c\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*x)^(1/2)*(c - c*x)^(1/2),x)
```

```
[Out] (x*(a + a*x)^(1/2)*(c - c*x)^(1/2))/2 - (a^(1/2)*(-c)^(1/2)*log((-c)^(1/2)*
(a*(x + 1))^(1/2)*(-c*(x - 1))^(1/2) - a^(1/2)*c*x))/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(x+1\right)}\,\sqrt{-c\left(x-1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+a)**(1/2)*(-c*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(x + 1))*sqrt(-c*(x - 1)), x)
```

3.1071 $\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ax+a}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + a*x])/(Sqrt[a]*Sqrt[c - c*x])])/(Sqrt[a]*Sqrt[c
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+ax}\sqrt{c-cx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2c-\frac{cx^2}{a}}}\,dx, x, \sqrt{a+ax}\right)}{a}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{cx^2}{a}}\,dx, x, \frac{\sqrt{a+ax}}{\sqrt{c-cx}}\right)}{a}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+ax}}{\sqrt{a}\sqrt{c-cx}}\right)}{\sqrt{a}\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.09

$$\frac{2\sqrt{x+1}\,\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{x+1}}{\sqrt{c-cx}}\right)}{\sqrt{c}\,\sqrt{a(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (2*Sqrt[1 + x]*ArcTan[(Sqrt[c]*Sqrt[1 + x])/Sqrt[c - c*x]])/(Sqrt[c]*Sqrt[a
*(1 + x)])
```

IntegrateAlgebraic [A] time = 0.08, size = 43, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{c-cx}}{\sqrt{c}\,\sqrt{ax+a}}\right)}{\sqrt{a}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + a*x]*Sqrt[c - c*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[a]*Sqrt[c - c*x])/(Sqrt[c]*Sqrt[a + a*x])])/(Sqrt[a]*Sqrt[
c])
```

fricas [A] time = 1.64, size = 101, normalized size = 2.35

$$\left[-\frac{\sqrt{-ac}\log\left(2\,acx^2-2\,\sqrt{-ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x-ac\right)}{2\,ac},\,-\frac{\sqrt{ac}\,\arctan\left(\frac{\sqrt{ac}\,\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{acx^2-ac}\right)}{ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*c)*log(2*a*c*x^2 - 2*sqrt(-a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*
x - a*c)/(a*c), -sqrt(a*c)*arctan(sqrt(a*c)*sqrt(a*x + a)*sqrt(-c*x + c)*x/
(a*c*x^2 - a*c))/(a*c)]
```

giac [A] time = 0.76, size = 49, normalized size = 1.14

$$-\frac{2\,a\log\left(\left|-\sqrt{-ac}\,\sqrt{ax+a}+\sqrt{-(ax+a)ac+2\,a^2c}\right|\right)}{\sqrt{-ac}\,|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(-sqrt(-a*c)*sqrt(a*x + a) + sqrt(-(a*x + a)*a*c + 2*a^2*c)))/(
sqrt(-a*c)*abs(a))
```

maple [A] time = 0.00, size = 57, normalized size = 1.33

$$\frac{\sqrt{(-cx+c)\,(ax+a)}\;\arctan\left(\frac{\sqrt{ac}\,x}{\sqrt{-ac\,x^2+ac}}\right)}{\sqrt{ax+a}\,\sqrt{-cx+c}\,\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x)
```

```
[Out] ((-c*x+c)*(a*x+a))^(1/2)/(a*x+a)^(1/2)/(-c*x+c)^(1/2)/(a*c)^(1/2)*arctan((a
*c)^(1/2)/(-a*c*x^2+a*c)^(1/2)*x)
```

maxima [A] time = 2.99, size = 8, normalized size = 0.19

$$\frac{\arcsin(x)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(1/2)/(-c*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(a*c)
```

mupad [B] time = 0.18, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{c-c\,x}-\sqrt{c}\right)}{\sqrt{a\,c}\,\left(\sqrt{a+a\,x}-\sqrt{a}\right)}\right)}{\sqrt{a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(1/2)*(c - c*x)^(1/2)),x)
```

```
[Out] -(4*atan((a*((c - c*x)^(1/2) - c^(1/2)))/((a*c)^(1/2)*((a + a*x)^(1/2) - a^
(1/2)))))/(a*c)^(1/2)
```

sympy [C] time = 3.95, size = 85, normalized size = 1.98

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{x^{2}}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^{2}}\right)}{4\pi^{\frac{3}{2}}\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(1/2)/(-c*x+c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
x**(-2))/(4*pi**(3/2)*sqrt(a)*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2,
 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/x**2)/(4*pi**(
3/2)*sqrt(a)*sqrt(c))
```

3.1072 $\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{ac\sqrt{ax+a}\,\sqrt{c-cx}}$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{ac\sqrt{ax+a}\,\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] x/(a*c*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx = \frac{x}{ac\sqrt{a+ax}\,\sqrt{c-cx}}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$\frac{x(x+1)}{c(a(x+1))^{3/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (x*(1 + x))/(c*(a*(1 + x))^(3/2)*Sqrt[c - c*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 47, normalized size = 1.74

$$\frac{\sqrt{ax+a}\left(c-\frac{a(c-cx)}{ax+a}\right)}{2a^2c^2\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x]
```

```
[Out] (Sqrt[a + a*x]*(c - (a*(c - c*x))/(a + a*x)))/(2*a^2*c^2*Sqrt[c - c*x])
```

fricas [A] time = 1.48, size = 39, normalized size = 1.44

$$-\frac{\sqrt{ax+a}\,\sqrt{-cx+c}\,x}{a^2c^2x^2-a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(a*x + a)*sqrt(-c*x + c)*x/(a^2*c^2*x^2 - a^2*c^2)
```

giac [B] time = 0.70, size = 116, normalized size = 4.30

$$-\frac{2\sqrt{-ac}\,a}{\left(2\,a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)c|a|}-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\,\sqrt{ax+a}}{2\left((ax+a)ac-2\,a^2c\right)c|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-a*c)*a/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)*c*abs(a)) - 1/2*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a
)/(((a*x + a)*a*c - 2*a^2*c)*c*abs(a))
```

maple [A] time = 0.00, size = 25, normalized size = 0.93

$$-\frac{(x+1)\,(x-1)\,x}{(ax+a)^{\frac{3}{2}}\,(-cx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x)
```

```
[Out] -(x+1)*(x-1)*x/(a*x+a)^(3/2)/(-c*x+c)^(3/2)
```

maxima [A] time = 1.31, size = 21, normalized size = 0.78

$$\frac{x}{\sqrt{-acx^2+ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(3/2)/(-c*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-a*c*x^2 + a*c)*a*c)
```

mupad [B] time = 0.39, size = 23, normalized size = 0.85

$$\frac{x}{a\,c\,\sqrt{a+a\,x}\,\sqrt{c-c\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(3/2)*(c - c*x)^(3/2)),x)
```

```
[Out] x/(a*c*(a + a*x)^(1/2)*(c - c*x)^(1/2))
```

sympy [C] time = 4.44, size = 82, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}a^{\frac{3}{2}}c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(3/2)/(-c*x+c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), x
**(-2))/(2*pi**(3/2)*a**(3/2)*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4,
 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp_polar(-2*I*pi)/x**2)/(2*pi**(3
/2)*a**(3/2)*c**(3/2))
```

3.1073 $\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{2x}{3a^2c^2\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{x}{3ac(ax+a)^{3/2}(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] x/(3*a*c*(a + a*x)^(3/2)*(c - c*x)^(3/2)) + (2*x)/(3*a^2*c^2*Sqrt[a + a*x]*
Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{3ac}\\ &= \frac{x}{3ac(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{2x}{3a^2c^2\sqrt{a+ax}\,\sqrt{c-cx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.69

$$\frac{x(x+1)\left(2x^2-3\right)}{3c^2(x-1)(a(x+1))^{5/2}\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] (x*(1 + x)*(-3 + 2*x^2))/(3*c^2*(-1 + x)*(a*(1 + x))^(5/2)*Sqrt[c - c*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 93, normalized size = 1.52

$$\frac{(ax+a)^{3/2}\left(-\frac{a^3(c-cx)^3}{(ax+a)^3}-\frac{9a^2c(c-cx)^2}{(ax+a)^2}+\frac{9ac^2(c-cx)}{ax+a}+c^3\right)}{24a^4c^4(c-cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x]
```

```
[Out] ((a + a*x)^(3/2)*(c^3 + (9*a*c^2*(c - c*x))/(a + a*x) - (9*a^2*c*(c - c*x)^
2)/(a + a*x)^2 - (a^3*(c - c*x)^3)/(a + a*x)^3))/(24*a^4*c^4*(c - c*x)^(3/2
))
```

fricas [A] time = 1.08, size = 57, normalized size = 0.93

$$-\frac{\left(2\,x^{3}-3\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{3\left(a^{3}c^{3}x^{4}-2\,a^{3}c^{3}x^{2}+a^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*x^3 - 3*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^3*c^3*x^4 - 2*a^3*c^3*x^
2 + a^3*c^3)
```

giac [B] time = 0.81, size = 237, normalized size = 3.89

$$-\frac{\sqrt{-(ax+a)ac+2\,a^{2}c}\,\sqrt{ax+a}\left(\frac{4\,(ax+a)|a|}{a^{2}c}-\frac{9\,|a|}{ac}\right)}{12\left((ax+a)ac-2\,a^{2}c\right)^{2}}-\frac{16\,\sqrt{-ac}\,a^{4}c^{2}-18\,\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^{2}c}\right)^{2}a^{2}c+3\,\sqrt{-ac}\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^{2}c}\right)^{4}}{3\left(2\,a^{2}c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^{2}c}\right)^{2}\right)^{3}c^{2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*(4*(a*x + a)*abs(a)/(a^2
*c) - 9*abs(a)/(a*c))/((a*x + a)*a*c - 2*a^2*c)^2 - 1/3*(16*sqrt(-a*c)*a^4*
c^2 - 18*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2
*c))^2*a^2*c + 3*sqrt(-a*c)*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^4)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c
 + 2*a^2*c))^2)^3*c^2*abs(a))
```

maple [A] time = 0.00, size = 32, normalized size = 0.52

$$\frac{(x+1)\,(x-1)\left(2x^{2}-3\right)x}{3\,(ax+a)^{\frac{5}{2}}\,(-cx+c)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x)
```

```
[Out] 1/3*(x+1)*(x-1)*x*(2*x^2-3)/(a*x+a)^(5/2)/(-c*x+c)^(5/2)
```

maxima [A] time = 1.35, size = 45, normalized size = 0.74

$$\frac{x}{3\left(-acx^{2}+ac\right)^{\frac{3}{2}}ac}+\frac{2\,x}{3\,\sqrt{-acx^{2}+ac}\,a^{2}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(5/2)/(-c*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-a*c*x^2 + a*c)^(3/2)*a*c) + 2/3*x/(sqrt(-a*c*x^2 + a*c)*a^2*c^2)
```

mupad [B] time = 0.41, size = 62, normalized size = 1.02

$$-\frac{3\,x\,\sqrt{c-c\,x}-2\,x^{3}\,\sqrt{c-c\,x}}{\sqrt{a+a\,x}\,(c-c\,x)^{2}\left(3\,a^{2}\,(c-c\,x)-6\,a^{2}\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(5/2)*(c - c*x)^(5/2)),x)
```

```
[Out] -(3*x*(c - c*x)^(1/2) - 2*x^3*(c - c*x)^(1/2))/((a + a*x)^(1/2)*(c - c*x)^2
*(3*a^2*(c - c*x) - 6*a^2*c))
```

sympy [C] time = 13.69, size = 82, normalized size = 1.34

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{1}{2},\frac{5}{2},3 \\ \frac{5}{4},\frac{7}{4},2,\frac{5}{2},3 & 0 \end{matrix} \middle| \frac{1}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},0,\frac{1}{2},\frac{3}{4},\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & -\frac{1}{2},0,2,0 \end{matrix} \middle| \frac{e^{-2i\pi}}{x^2}\right)}{3\pi^{\frac{3}{2}}a^{\frac{5}{2}}c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(5/2)/(-c*x+c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), x*
*(-2))/(3*pi**(3/2)*a**(5/2)*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4, 5/4,
1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), exp_polar(-2*I*pi)/x**2)/(3*pi**(3/
2)*a**(5/2)*c**(5/2))
```

3.1074 $\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx$

Optimal. Leaf size=91

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{8x}{15a^3c^3\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{4x}{15a^2c^2(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{x}{5ac(ax+a)^{5/2}(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] x/(5*a*c*(a + a*x)^(5/2)*(c - c*x)^(5/2)) + (4*x)/(15*a^2*c^2*(a + a*x)^(3/
2)*(c - c*x)^(3/2)) + (8*x)/(15*a^3*c^3*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{5ac}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8\int \frac{1}{(a+ax)^{3/2}(c-cx)^{3/2}}\,dx}{15a^2c^2}\\ &= \frac{x}{5ac(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{4x}{15a^2c^2(a+ax)^{3/2}(c-cx)^{3/2}}+\frac{8x}{15a^3c^3\sqrt{a+ax}\,\sqrt{c-cx}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 0.54

$$\frac{x\left(8x^4-20x^2+15\right)}{15a^3c^3\left(x^2-1\right)^2\sqrt{a(x+1)}\,\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] (x*(15 - 20*x^2 + 8*x^4))/(15*a^3*c^3*Sqrt[a*(1 + x)]*Sqrt[c - c*x]*(-1 + x
^2)^2)
```

IntegrateAlgebraic [A] time = 0.13, size = 141, normalized size = 1.55

$$\frac{(ax+a)^{5/2}\left(-\frac{3a^5(c-cx)^5}{(ax+a)^5}-\frac{25a^4c(c-cx)^4}{(ax+a)^4}-\frac{150a^3c^2(c-cx)^3}{(ax+a)^3}+\frac{150a^2c^3(c-cx)^2}{(ax+a)^2}+\frac{25ac^4(c-cx)}{ax+a}+3c^5\right)}{480a^6c^6(c-cx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x]
```

```
[Out] ((a + a*x)^(5/2)*(3*c^5 + (25*a*c^4*(c - c*x))/(a + a*x) + (150*a^2*c^3*(c
 - c*x)^2)/(a + a*x)^2 - (150*a^3*c^2*(c - c*x)^3)/(a + a*x)^3 - (25*a^4*c*(
c - c*x)^4)/(a + a*x)^4 - (3*a^5*(c - c*x)^5)/(a + a*x)^5))/(480*a^6*c^6*(c
 - c*x)^(5/2))
```

fricas [A] time = 1.12, size = 74, normalized size = 0.81

$$-\frac{\left(8\,x^5-20\,x^3+15\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{15\,\left(a^4c^4x^6-3\,a^4c^4x^4+3\,a^4c^4x^2-a^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*x^5 - 20*x^3 + 15*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^4*c^4*x^6 - 3
*a^4*c^4*x^4 + 3*a^4*c^4*x^2 - a^4*c^4)
```

giac [B] time = 1.06, size = 333, normalized size = 3.66

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\,\sqrt{ax+a}\left((ax+a)\left(\frac{64\,(ax+a)}{c|a|}-\frac{275\,a}{c|a|}\right)+\frac{300\,a^2}{c|a|}\right)}{240\,\left((ax+a)ac-2\,a^2c\right)^3}+\frac{1024\,a^8c^4-2200\,\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^6c^3+1660\,\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^4a^4c^2-450\,\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^6a^2c+45\,\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^8}{60\,\left(2\,a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)^5\sqrt{-ac}\,c^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-(a*x + a)*a*c + 2*a^2*c)*sqrt(a*x + a)*((a*x + a)*(64*(a*x + a
)/(c*abs(a)) - 275*a/(c*abs(a))) + 300*a^2/(c*abs(a)))/((a*x + a)*a*c - 2*a
^2*c)^3 + 1/60*(1024*a^8*c^4 - 2200*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
+ a)*a*c + 2*a^2*c))^2*a^6*c^3 + 1660*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*
x + a)*a*c + 2*a^2*c))^4*a^4*c^2 - 450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a
*x + a)*a*c + 2*a^2*c))^6*a^2*c + 45*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^8)/((2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x
 + a)*a*c + 2*a^2*c))^2)^5*sqrt(-a*c)*c^2*abs(a))
```

maple [A] time = 0.00, size = 37, normalized size = 0.41

$$-\frac{(x+1)\,(x-1)\left(8x^4-20x^2+15\right)x}{15\,(ax+a)^{\frac{7}{2}}\,(-cx+c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x)
```

```
[Out] -1/15*(x+1)*(x-1)*x*(8*x^4-20*x^2+15)/(a*x+a)^(7/2)/(-c*x+c)^(7/2)
```

maxima [A] time = 1.36, size = 67, normalized size = 0.74

$$\frac{x}{5\left(-acx^2+ac\right)^{\frac{5}{2}}ac}+\frac{4\,x}{15\left(-acx^2+ac\right)^{\frac{3}{2}}a^2c^2}+\frac{8\,x}{15\,\sqrt{-acx^2+ac}\,a^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(7/2)/(-c*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-a*c*x^2 + a*c)^(5/2)*a*c) + 4/15*x/((-a*c*x^2 + a*c)^(3/2)*a^2*c^2
) + 8/15*x/(sqrt(-a*c*x^2 + a*c)*a^3*c^3)
```

mupad [B] time = 0.44, size = 50, normalized size = 0.55

$$\frac{x\left(8x^4-20x^2+15\right)}{15a^3\sqrt{a+ax}\,(c-cx)^{5/2}\,(c+3cx-x(c-cx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(7/2)*(c - c*x)^(7/2)),x)
```

```
[Out] (x*(8*x^4 - 20*x^2 + 15))/(15*a^3*(a + a*x)^(1/2)*(c - c*x)^(5/2)*(c + 3*c*
x - x*(c - c*x)))
```

sympy [C] time = 55.15, size = 85, normalized size = 0.93

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{e^{-2i\pi}}{x^2}\right)}{15\pi^{\frac{3}{2}}a^{\frac{7}{2}}c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(7/2)/(-c*x+c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 x**(-2))/(15*pi**(3/2)*a**(7/2)*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/2, 5/4,
 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), exp_polar(-2*I*pi)/x**2)/(15*
pi**(3/2)*a**(7/2)*c**(7/2))
```

3.1075 $\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx$

Optimal. Leaf size=121

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Rubi [A] time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {40, 39}

$$\frac{16x}{35a^4c^4\sqrt{ax+a}\,\sqrt{c-cx}}+\frac{8x}{35a^3c^3(ax+a)^{3/2}(c-cx)^{3/2}}+\frac{6x}{35a^2c^2(ax+a)^{5/2}(c-cx)^{5/2}}+\frac{x}{7ac(ax+a)^{7/2}(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] x/(7*a*c*(a + a*x)^(7/2)*(c - c*x)^(7/2)) + (6*x)/(35*a^2*c^2*(a + a*x)^(5/
2)*(c - c*x)^(5/2)) + (8*x)/(35*a^3*c^3*(a + a*x)^(3/2)*(c - c*x)^(3/2)) +
(16*x)/(35*a^4*c^4*Sqrt[a + a*x]*Sqrt[c - c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+ax)^{9/2}(c-cx)^{9/2}}\,dx &= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6\int \frac{1}{(a+ax)^{7/2}(c-cx)^{7/2}}\,dx}{7ac}\\ &= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{24\int \frac{1}{(a+ax)^{5/2}(c-cx)^{5/2}}\,dx}{35a^2c^2}\\ &= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}\\ &= \frac{x}{7ac(a+ax)^{7/2}(c-cx)^{7/2}}+\frac{6x}{35a^2c^2(a+ax)^{5/2}(c-cx)^{5/2}}+\frac{8x}{35a^3c^3(a+ax)^{3/2}(c-cx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.45

$$\frac{x\left(16x^6-56x^4+70x^2-35\right)}{35a^4c^4\left(x^2-1\right)^3\sqrt{a(x+1)}\,\sqrt{c-cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] (x*(-35 + 70*x^2 - 56*x^4 + 16*x^6))/(35*a^4*c^4*Sqrt[a*(1 + x)]*Sqrt[c - c
*x]*(-1 + x^2)^3)
```

IntegrateAlgebraic [A] time = 0.14, size = 187, normalized size = 1.55

$$\frac{(ax+a)^{7/2}\left(-\frac{5a^7(c-cx)^7}{(ax+a)^7}-\frac{49a^6c(c-cx)^6}{(ax+a)^6}-\frac{245a^5c^2(c-cx)^5}{(ax+a)^5}-\frac{1225a^4c^3(c-cx)^4}{(ax+a)^4}+\frac{1225a^3c^4(c-cx)^3}{(ax+a)^3}+\frac{245a^2c^5(c-cx)^2}{(ax+a)^2}+\frac{49ac^6(c-cx)}{ax+a}+5c^7\right)}{4480a^8c^8(c-cx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x]
```

```
[Out] ((a + a*x)^(7/2)*(5*c^7 + (49*a*c^6*(c - c*x))/(a + a*x) + (245*a^2*c^5*(c
- c*x)^2)/(a + a*x)^2 + (1225*a^3*c^4*(c - c*x)^3)/(a + a*x)^3 - (1225*a^4*
c^3*(c - c*x)^4)/(a + a*x)^4 - (245*a^5*c^2*(c - c*x)^5)/(a + a*x)^5 - (49*
a^6*c*(c - c*x)^6)/(a + a*x)^6 - (5*a^7*(c - c*x)^7)/(a + a*x)^7))/(4480*a^
8*c^8*(c - c*x)^(7/2))
```

fricas [A] time = 1.26, size = 89, normalized size = 0.74

$$-\frac{\left(16\,x^7-56\,x^5+70\,x^3-35\,x\right)\sqrt{ax+a}\,\sqrt{-cx+c}}{35\left(a^5c^5x^8-4\,a^5c^5x^6+6\,a^5c^5x^4-4\,a^5c^5x^2+a^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(16*x^7 - 56*x^5 + 70*x^3 - 35*x)*sqrt(a*x + a)*sqrt(-c*x + c)/(a^5*c
^5*x^8 - 4*a^5*c^5*x^6 + 6*a^5*c^5*x^4 - 4*a^5*c^5*x^2 + a^5*c^5)
```

giac [B] time = 1.56, size = 437, normalized size = 3.61

$$-\frac{\sqrt{-(ax+a)ac+2\,a^2c}\left((ax+a)\left((ax+a)\left(\frac{256\,(ax+a)|a|}{a^2c}-\frac{1617\,|a|}{ac}\right)+\frac{3430\,|a|}{c}\right)-\frac{2450\,a|a|}{c}\right)\sqrt{ax+a}}{1120\left((ax+a)ac-2\,a^2c\right)^4}+\frac{16384\,a^{12}c^6-51744\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2a^{10}c^5+66416\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^4a^8c^4-43120\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^6a^6c^3+14280\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^8a^4c^2-2450\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^{10}a^2c+175\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^{12}}{280\left(2\,a^2c-\left(\sqrt{-ac}\,\sqrt{ax+a}-\sqrt{-(ax+a)ac+2\,a^2c}\right)^2\right)^7\sqrt{-ac}\,ac^3|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] -1/1120*sqrt(-(a*x + a)*a*c + 2*a^2*c)*((a*x + a)*((a*x + a)*(256*(a*x + a)
*abs(a)/(a^2*c) - 1617*abs(a)/(a*c)) + 3430*abs(a)/c) - 2450*a*abs(a)/c)*sq
rt(a*x + a)/((a*x + a)*a*c - 2*a^2*c)^4 + 1/280*(16384*a^12*c^6 - 51744*(sq
rt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2*a^10*c^5 + 66416
*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^4*a^8*c^4 - 43
120*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^6*a^6*c^3 +
 14280*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^8*a^4*c^
2 - 2450*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^10*a^2
*c + 175*(sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^12)/((
2*a^2*c - (sqrt(-a*c)*sqrt(a*x + a) - sqrt(-(a*x + a)*a*c + 2*a^2*c))^2)^7*
sqrt(-a*c)*a*c^3*abs(a))
```

maple [A] time = 0.00, size = 42, normalized size = 0.35

$$\frac{(x+1)\,(x-1)\left(16x^6-56x^4+70x^2-35\right)x}{35\,(ax+a)^{\frac{9}{2}}\,(-cx+c)^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x)
```

```
[Out] 1/35*(x+1)*(x-1)*x*(16*x^6-56*x^4+70*x^2-35)/(a*x+a)^(9/2)/(-c*x+c)^(9/2)
```

maxima [A] time = 1.40, size = 89, normalized size = 0.74

$$\frac{x}{7\left(-acx^2+ac\right)^{\frac{7}{2}}ac}+\frac{6\,x}{35\left(-acx^2+ac\right)^{\frac{5}{2}}a^2c^2}+\frac{8\,x}{35\left(-acx^2+ac\right)^{\frac{3}{2}}a^3c^3}+\frac{16\,x}{35\sqrt{-acx^2+ac}\,a^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)^(9/2)/(-c*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-a*c*x^2 + a*c)^(7/2)*a*c) + 6/35*x/((-a*c*x^2 + a*c)^(5/2)*a^2*c^2
) + 8/35*x/((-a*c*x^2 + a*c)^(3/2)*a^3*c^3) + 16/35*x/(sqrt(-a*c*x^2 + a*c)
*a^4*c^4)
```

mupad [B] time = 0.48, size = 66, normalized size = 0.55

$$-\frac{x\,\left(16\,x^6-56\,x^4+70\,x^2-35\right)}{35\,a^4\,\sqrt{a+a\,x}\,{\left(c-c\,x\right)}^{7/2}\,\left(c-x^2\,\left(c-c\,x\right)+7\,c\,x-4\,x\,\left(c-c\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*x)^(9/2)*(c - c*x)^(9/2)),x)
```

```
[Out] -(x*(70*x^2 - 56*x^4 + 16*x^6 - 35))/(35*a^4*(a + a*x)^(1/2)*(c - c*x)^(7/2
)*(c - x^2*(c - c*x) + 7*c*x - 4*x*(c - c*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+a)**(9/2)/(-c*x+c)**(9/2),x)
```

```
[Out] Timed out
```

3.1076 $\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx$

Optimal. Leaf size=135

$$\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {38, 63, 217, 203}

$$\frac{5}{16}a^4c^2x\sqrt{a+bx}\sqrt{ac-bcx}+\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{8b}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (5*a^4*c^2*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/16 + (5*a^2*c*x*(a + b*x)^(3/
2)*(a*c - b*c*x)^(3/2))/24 + (x*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2))/6 + (5
*a^6*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(8*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+bx)^{5/2}(ac-bcx)^{5/2}\,dx &= \frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{1}{6}\left(5a^2c\right)\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx \\
&= \frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-bcx)^{5/2}+\frac{1}{8}\left(5a^4c^2\right)\int \sqrt{a+b} \\
&= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac- \\
&= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac- \\
&= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac- \\
&= \frac{5}{16}a^4c^2x\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{5}{24}a^2cx(a+bx)^{3/2}(ac-bcx)^{3/2}+\frac{1}{6}x(a+bx)^{5/2}(ac-
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 120, normalized size = 0.89

$$
\frac{c^3\left(-30a^{13/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)+33a^6bx-59a^4b^3x^3+34a^2b^5x^5-8b^7x^7\right)}{48b\sqrt{a+bx}\,\sqrt{c(a-bx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (c^3*(33*a^6*b*x - 59*a^4*b^3*x^3 + 34*a^2*b^5*x^5 - 8*b^7*x^7 - 30*a^(13/2
)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))
/(48*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.33, size = 215, normalized size = 1.59

$$
\frac{a^6c^3\sqrt{ac-bcx}\left(\frac{85c^4(ac-bcx)}{a+bx}+\frac{198c^3(ac-bcx)^2}{(a+bx)^2}-\frac{198c^2(ac-bcx)^3}{(a+bx)^3}-\frac{85c(ac-bcx)^4}{(a+bx)^4}-\frac{15(ac-bcx)^5}{(a+bx)^5}+15c^5\right)}{24b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^6}-\frac{5a^6c^{5/2}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{8b}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2),x]
```

```
[Out] (a^6*c^3*Sqrt[a*c - b*c*x]*(15*c^5 + (85*c^4*(a*c - b*c*x))/(a + b*x) + (19
8*c^3*(a*c - b*c*x)^2)/(a + b*x)^2 - (198*c^2*(a*c - b*c*x)^3)/(a + b*x)^3
- (85*c*(a*c - b*c*x)^4)/(a + b*x)^4 - (15*(a*c - b*c*x)^5)/(a + b*x)^5))/(
24*b*Sqrt[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^6) - (5*a^6*c^(5/2)*ArcTan
[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(8*b)
```

fricas [A] time = 1.16, size = 232, normalized size = 1.72

$$
\left[\frac{15\,a^6\sqrt{-c}\,c^2\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\left(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{96\,b},-\frac{15\,a^6c^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\left(8\,b^5c^2x^5-26\,a^2b^3c^2x^3+33\,a^4bc^2x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{48\,b}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^6*sqrt(-c)*c^2*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x
+ a)*b*sqrt(-c)*x - a^2*c) + 2*(8*b^5*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4
*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a))/b, -1/48*(15*a^6*c^(5/2)*arctan
(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) - (8*b^5
```

```
*c^2*x^5 - 26*a^2*b^3*c^2*x^3 + 33*a^4*b*c^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x
 + a))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 243, normalized size = 1.80

$$\frac{5\sqrt{(bx+a)(-bcx+ac)}\, a^{6} c^{3} \arctan\left(\frac{\sqrt{b^{2} c}\, x}{\sqrt{-b^{2} c\, x^{2}+a^{2} c}}\right)}{16\sqrt{-bcx+ac}\, \sqrt{bx+a}\, \sqrt{b^{2} c}}+\frac{5\sqrt{-bcx+ac}\, \sqrt{bx+a}\, a^{5} c^{2}}{16b}+\frac{5\,(-bcx+ac)^{\frac{3}{2}} \sqrt{bx+a}\, a^{4} c}{48b}+\frac{(-bcx+ac)^{\frac{5}{2}} \sqrt{bx+a}\, a^{3}}{24b}-\frac{\sqrt{bx+a}\, (-bcx+ac)^{\frac{7}{2}} a^{2}}{8bc}-\frac{(bx+a)^{\frac{3}{2}} (-bcx+ac)^{\frac{7}{2}} a}{6bc}-\frac{(bx+a)^{\frac{5}{2}} (-bcx+ac)^{\frac{7}{2}}}{6bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x)
```

```
[Out] -1/6/b/c*(b*x+a)^(5/2)*(-b*c*x+a*c)^(7/2)-1/6*a/b/c*(b*x+a)^(3/2)*(-b*c*x+a
*c)^(7/2)-1/8*a^2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(7/2)+1/24*a^3/b*(-b*c*x+a
*c)^(5/2)*(b*x+a)^(1/2)+5/48*a^4*c/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+5/16*
a^5*c^2/b*(-b*c*x+a*c)^(1/2)*(b*x+a)^(1/2)+5/16*a^6*c^3*((b*x+a)*(-b*c*x+a*
c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/
2)*x/(-b^2*c*x^2+a^2*c)^(1/2))
```

maxima [A] time = 3.12, size = 89, normalized size = 0.66

$$\frac{5\, a^{6} c^{\frac{5}{2}} \arcsin\left(\frac{bx}{a}\right)}{16\, b}+\frac{5}{16}\, \sqrt{-b^{2} c x^{2}+a^{2} c}\, a^{4} c^{2} x+\frac{5}{24}\, \left(-b^{2} c x^{2}+a^{2} c\right)^{\frac{3}{2}} a^{2} c x+\frac{1}{6}\, \left(-b^{2} c x^{2}+a^{2} c\right)^{\frac{5}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*c^(5/2)*arcsin(b*x/a)/b + 5/16*sqrt(-b^2*c*x^2 + a^2*c)*a^4*c^2*x
+ 5/24*(-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c*x + 1/6*(-b^2*c*x^2 + a^2*c)^(5/2)*
x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,c-b\,c\,x\right)}^{5/2}\,{\left(a+b\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2),x)
```

```
[Out] int((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(- a + b x\right)\right)^{\frac{5}{2}} \left(a + b x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(-b*c*x+a*c)**(5/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(5/2)*(a + b*x)**(5/2), x)
```

3.1077 $\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx$

Optimal. Leaf size=102

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Rubi [A] time = 0.04, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{4b}+\frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx}+\frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (3*a^2*c*x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/8 + (x*(a + b*x)^(3/2)*(a*c - b
*c*x)^(3/2))/4 + (3*a^4*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a -
b*x)]])/(4*b)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(ac-bcx)^{3/2}\,dx = \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{4}\left(3a^2c\right)\int \sqrt{a+bx}\,\sqrt{ac-bcx}\,dx$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{1}{8}\left(3a^4c^2\right)\int \frac{\ldots}{\sqrt{a+bx}\ldots}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{2ac}\ldots}\right.}{4b\ldots}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{\left(3a^4c^2\right)\text{Subst}\left(\int \frac{1}{1+cx\ldots}\right.}{4b}$$
$$= \frac{3}{8}a^2cx\sqrt{a+bx}\,\sqrt{ac-bcx} + \frac{1}{4}x(a+bx)^{3/2}(ac-bcx)^{3/2} + \frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{c}\,\sqrt{a+b\ldots}}{\sqrt{c(a-bx\ldots}}\right.}{4b}$$

Mathematica [A] time = 0.13, size = 109, normalized size = 1.07

$$\frac{c^2\left(-6a^{9/2}\sqrt{a-bx}\,\sqrt{\frac{bx}{a}+1}\,\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\,\sqrt{a}}\right)+5a^4bx-7a^2b^3x^3+2b^5x^5\right)}{8b\sqrt{a+bx}\,\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (c^2*(5*a^4*b*x - 7*a^2*b^3*x^3 + 2*b^5*x^5 - 6*a^(9/2)*Sqrt[a - b*x]*Sqrt[
1 + (b*x)/a]*ArcSin[Sqrt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(8*b*Sqrt[c*(a - b*x
)]*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.24, size = 169, normalized size = 1.66

$$\frac{a^4c^2\sqrt{ac-bcx}\left(\frac{11c^2(ac-bcx)}{a+bx}-\frac{11c(ac-bcx)^2}{(a+bx)^2}-\frac{3(ac-bcx)^3}{(a+bx)^3}+3c^3\right)}{4b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^4}-\frac{3a^4c^{3/2}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\,\sqrt{a+bx}}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2),x]
```

```
[Out] (a^4*c^2*Sqrt[a*c - b*c*x]*(3*c^3 + (11*c^2*(a*c - b*c*x))/(a + b*x) - (11*
c*(a*c - b*c*x)^2)/(a + b*x)^2 - (3*(a*c - b*c*x)^3)/(a + b*x)^3))/(4*b*Sqr
t[a + b*x]*(c + (a*c - b*c*x)/(a + b*x))^4) - (3*a^4*c^(3/2)*ArcTan[Sqrt[a*
c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(4*b)
```

fricas [A] time = 1.43, size = 193, normalized size = 1.89

$$\left[\frac{3\,a^4\sqrt{-c}\,c\log\left(2\,b^2cx^2+2\,\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)-2\,\left(2\,b^3cx^3-5\,a^2bcx\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{16\,b}, -\frac{3\,a^4c^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-bcx+ac}\,\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)+\left(2\,b^3cx^3-5\,a^2bcx\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{8\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^4*sqrt(-c)*c*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a
)*b*sqrt(-c)*x - a^2*c) - 2*(2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c*x + a*c)*
sqrt(b*x + a))/b, -1/8*(3*a^4*c^(3/2)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x +
a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2*c)) + (2*b^3*c*x^3 - 5*a^2*b*c*x)*sqrt(-b*c
*x + a*c)*sqrt(b*x + a))/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 185, normalized size = 1.81

$$\frac{3\sqrt{(bx+a)(-bcx+ac)}\, a^4c^2 \arctan\left(\frac{\sqrt{b^2c}\, x}{\sqrt{-b^2c x^2+a^2c}}\right)}{8\sqrt{-bcx+ac}\, \sqrt{bx+a}\, \sqrt{b^2c}}+\frac{3\sqrt{-bcx+ac}\, \sqrt{bx+a}\, a^3c}{8b}+\frac{(-bcx+ac)^{\frac{3}{2}} \sqrt{bx+a}\, a^2}{8b}-\frac{\sqrt{bx+a}\, (-bcx+ac)^{\frac{5}{2}} a}{4bc}-\frac{(bx+a)^{\frac{3}{2}} (-bcx+ac)^{\frac{5}{2}}}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x)
```

```
[Out] -1/4/b/c*(b*x+a)^(3/2)*(-b*c*x+a*c)^(5/2)-1/4*a/b/c*(b*x+a)^(1/2)*(-b*c*x+a
*c)^(5/2)+1/8*a^2/b*(-b*c*x+a*c)^(3/2)*(b*x+a)^(1/2)+3/8*a^3*c/b*(-b*c*x+a*
c)^(1/2)*(b*x+a)^(1/2)+3/8*a^4*c^2*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c
)^(1/2)/(b*x+a)^(1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)
^(1/2)*x)
```

maxima [A] time = 3.02, size = 63, normalized size = 0.62

$$\frac{3\, a^4 c^{\frac{3}{2}} \arcsin\left(\frac{bx}{a}\right)}{8\, b}+\frac{3}{8}\sqrt{-b^2cx^2+a^2c}\, a^2cx+\frac{1}{4}\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*a^4*c^(3/2)*arcsin(b*x/a)/b + 3/8*sqrt(-b^2*c*x^2 + a^2*c)*a^2*c*x + 1/
4*(-b^2*c*x^2 + a^2*c)^(3/2)*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,c-b\,c\,x)^{3/2}\,(a+b\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2),x)
```

```
[Out] int((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-c\,(-a+bx))^{\frac{3}{2}}\,(a+bx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(-b*c*x+a*c)**(3/2),x)
```

```
[Out] Integral((-c*(-a + b*x))**(3/2)*(a + b*x)**(3/2), x)
```

3.1078 $\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx$

Optimal. Leaf size=68

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {38, 63, 217, 203}

$$\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}+\frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (x*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])/2 + (a^2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[a
 + b*x])/Sqrt[c*(a - b*x)]])/b
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{ac-bcx}\,dx = \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{1}{2}\left(a^2c\right)\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{\left(a^2c\right)\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$
$$= \frac{1}{2}x\sqrt{a+bx}\sqrt{ac-bcx} + \frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}$$

Mathematica [A] time = 0.12, size = 95, normalized size = 1.40

$$\frac{c\left(-2a^{5/2}\sqrt{a-bx}\sqrt{\frac{bx}{a}+1}\sin^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{2}\sqrt{a}}\right)+a^2bx-b^3x^3\right)}{2b\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (c*(a^2*b*x - b^3*x^3 - 2*a^(5/2)*Sqrt[a - b*x]*Sqrt[1 + (b*x)/a]*ArcSin[Sq
rt[a - b*x]/(Sqrt[2]*Sqrt[a])]))/(2*b*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.16, size = 114, normalized size = 1.68

$$\frac{a^2c\sqrt{ac-bcx}\left(c-\frac{ac-bcx}{a+bx}\right)}{b\sqrt{a+bx}\left(\frac{ac-bcx}{a+bx}+c\right)^2}-\frac{a^2\sqrt{c}\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[a*c - b*c*x],x]
```

```
[Out] (a^2*c*Sqrt[a*c - b*c*x]*(c - (a*c - b*c*x)/(a + b*x)))/(b*Sqrt[a + b*x]*(c
 + (a*c - b*c*x)/(a + b*x))^2) - (a^2*Sqrt[c]*ArcTan[Sqrt[a*c - b*c*x]/(Sqr
t[c]*Sqrt[a + b*x])])/b
```

fricas [A] time = 1.40, size = 159, normalized size = 2.34

$$\left[\frac{a^2\sqrt{-c}\log\left(2b^2cx^2+2\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{-c}\,x-a^2c\right)+2\sqrt{-bcx+ac}\sqrt{bx+a}\,bx}{4b}, -\frac{a^2\sqrt{c}\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)-\sqrt{-bcx+ac}\sqrt{bx+a}\,bx}{2b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(a^2*sqrt(-c)*log(2*b^2*c*x^2 + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*s
qrt(-c)*x - a^2*c) + 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b, -1/2*(a^2*s
qrt(c)*arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/(b^2*c*x^2 - a^2
*c)) - sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*x)/b]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 127, normalized size = 1.87

$$\frac{\sqrt{(bx+a)(-bcx+ac)}\, a^2 c \arctan\left(\frac{\sqrt{b^2c}\, x}{\sqrt{-b^2c\, x^2+a^2c}}\right)}{2\sqrt{-bcx+ac}\, \sqrt{bx+a}\, \sqrt{b^2c}}+\frac{\sqrt{-bcx+ac}\, \sqrt{bx+a}\, a}{2b}-\frac{\sqrt{bx+a}\, (-bcx+ac)^{\frac{3}{2}}}{2bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x)
```

```
[Out] -1/2/b/c*(b*x+a)^(1/2)*(-b*c*x+a*c)^(3/2)+1/2*a/b*(-b*c*x+a*c)^(1/2)*(b*x+a
)^(1/2)+1/2*a^2*c*((b*x+a)*(-b*c*x+a*c))^(1/2)/(-b*c*x+a*c)^(1/2)/(b*x+a)^(
1/2)/(b^2*c)^(1/2)*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2)*x)
```

maxima [A] time = 3.09, size = 39, normalized size = 0.57

$$\frac{a^2\sqrt{c}\, \arcsin\left(\frac{bx}{a}\right)}{2\, b}+\frac{1}{2}\, \sqrt{-b^2cx^2+a^2c}\, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a^2*sqrt(c)*arcsin(b*x/a)/b + 1/2*sqrt(-b^2*c*x^2 + a^2*c)*x
```

mupad [B] time = 0.20, size = 72, normalized size = 1.06

$$\frac{x\sqrt{a\, c-b\, c\, x}\, \sqrt{a+b\, x}}{2}-\frac{a^2\, \sqrt{b}\, c^2\, \ln\left(\sqrt{-b\, c}\, \sqrt{c\, (a-b\, x)}\, \sqrt{a+b\, x}\, -b^{3/2}\, c\, x\right)}{2\, {(-b\, c)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2),x)
```

```
[Out] (x*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))/2 - (a^2*b^(1/2)*c^2*log((-b*c)^(1/
2)*(c*(a - b*x))^(1/2)*(a + b*x)^(1/2) - b^(3/2)*c*x))/(2*(-b*c)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\,(-a+bx)}\, \sqrt{a+bx}\; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(-b*c*x+a*c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(-a + b*x))*sqrt(a + b*x), x)
```

3.1079 $\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {63, 217, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[c*(a - b*x)]])/(b*Sqrt[c])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{ac-bcx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2ac-cx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+cx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{c(a-bx)}}\right)}{b\sqrt{c}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.26

$$-\frac{2\sqrt{a-bx}\tan^{-1}\left(\frac{\sqrt{a-bx}}{\sqrt{a+bx}}\right)}{b\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (-2*Sqrt[a - b*x]*ArcTan[Sqrt[a - b*x]/Sqrt[a + b*x]])/(b*Sqrt[c*(a - b*x)]
)
```

IntegrateAlgebraic [A] time = 0.08, size = 39, normalized size = 1.03

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{ac-bcx}}{\sqrt{c}\sqrt{a+bx}}\right)}{b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[a*c - b*c*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[a*c - b*c*x]/(Sqrt[c]*Sqrt[a + b*x])])/(b*Sqrt[c])
```

fricas [A] time = 0.83, size = 108, normalized size = 2.84

$$\left[-\frac{\sqrt{-c}\log\left(2b^2cx^2 - 2\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{-c}\,x - a^2c\right)}{2bc}, -\frac{\arctan\left(\frac{\sqrt{-bcx+ac}\sqrt{bx+a}\,b\sqrt{c}\,x}{b^2cx^2-a^2c}\right)}{b\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-c)*log(2*b^2*c*x^2 - 2*sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(
-c)*x - a^2*c)/(b*c), -arctan(sqrt(-b*c*x + a*c)*sqrt(b*x + a)*b*sqrt(c)*x/
(b^2*c*x^2 - a^2*c))/(b*sqrt(c))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.00, size = 71, normalized size = 1.87

$$\frac{\sqrt{(bx+a)(-bcx+ac)}\,\arctan\left(\frac{\sqrt{b^2c}\,x}{\sqrt{-b^2c\,x^2+a^2c}}\right)}{\sqrt{bx+a}\,\sqrt{-bcx+ac}\,\sqrt{b^2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x)
```

```
[Out] ((b*x+a)*(-b*c*x+a*c))^(1/2)/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2)/(b^2*c)^(1/2)
*arctan((b^2*c)^(1/2)/(-b^2*c*x^2+a^2*c)^(1/2)*x)
```

maxima [A] time = 2.93, size = 14, normalized size = 0.37

$$\frac{\arcsin\left(\frac{bx}{a}\right)}{b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(-b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(b*x/a)/(b*sqrt(c))
```

mupad [B] time = 0.18, size = 53, normalized size = 1.39

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{a\,c-b\,c\,x}-\sqrt{a\,c}\right)}{\sqrt{b^2\,c}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((a*c - b*c*x)^(1/2) - (a*c)^(1/2)))/((b^2*c)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(b^2*c)^(1/2)
```

sympy [C] time = 4.69, size = 90, normalized size = 2.37

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(-b*c*x+a*c)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
a**2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(c)) + meijerg(((-1/2, -1/4, 0, 1/4, 1
/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(-2*I*pi)/(b**2*
x**2))/(4*pi**(3/2)*b*sqrt(c))
```

3.1080 $\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}} dx$

Optimal. Leaf size=30

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {39}

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}} dx = \frac{x}{a^2c\sqrt{a+bx}\sqrt{ac-bcx}}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.97

$$\frac{x}{a^2c\sqrt{a+bx}\sqrt{c(a-bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] x/(a^2*c*Sqrt[c*(a - b*x)]*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 55, normalized size = 1.83

$$\frac{\sqrt{a+bx}\left(c-\frac{ac-bcx}{a+bx}\right)}{2a^2bc^2\sqrt{ac-bcx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)),x]
```

```
[Out] (Sqrt[a + b*x]*(c - (a*c - b*c*x)/(a + b*x)))/(2*a^2*b*c^2*Sqrt[a*c - b*c*x
])
```

fricas [A] time = 0.96, size = 45, normalized size = 1.50

$$-\frac{\sqrt{-bcx+ac}\sqrt{bx+a}\,x}{a^2b^2c^2x^2-a^4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*c*x + a*c)*sqrt(b*x + a)*x/(a^2*b^2*c^2*x^2 - a^4*c^2)
```

giac [B] time = 1.86, size = 115, normalized size = 3.83

$$\frac{2\sqrt{-c}\,c}{\left(2\,ac^2 - \left(\sqrt{-bcx+ac}\,\sqrt{-c} - \sqrt{2\,ac^2 + (bcx-ac)c}\right)^2\right)ab|c|} - \frac{\sqrt{-bcx+ac}}{2\sqrt{2\,ac^2 + (bcx-ac)c}\,a^2b|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-c)*c/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c
*x - a*c)*c))^2)*a*b*abs(c)) - 1/2*sqrt(-b*c*x + a*c)/(sqrt(2*a*c^2 + (b*c*
x - a*c)*c)*a^2*b*abs(c))
```

maple [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{(-bx+a)\,x}{\sqrt{bx+a}\,\left(-bcx+ac\right)^{\frac{3}{2}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x)
```

```
[Out] 1/(b*x+a)^(1/2)*(-b*x+a)/a^2*x/(-b*c*x+a*c)^(3/2)
```

maxima [A] time = 1.40, size = 25, normalized size = 0.83

$$\frac{x}{\sqrt{-b^2cx^2 + a^2c}\,a^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(-b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-b^2*c*x^2 + a^2*c)*a^2*c)
```

mupad [B] time = 0.50, size = 26, normalized size = 0.87

$$\frac{x}{a^2\,c\,\sqrt{a\,c-b\,c\,x}\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(3/2)*(a + b*x)^(3/2)),x)
```

```
[Out] x/(a^2*c*(a*c - b*c*x)^(1/2)*(a + b*x)^(1/2))
```

sympy [C] time = 5.18, size = 94, normalized size = 3.13

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{matrix} \middle| {\frac{a^2}{b^2x^2}} \right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \end{matrix} \middle| {\frac{a^2e^{-2i\pi}}{b^2x^2}} \right)}{2\pi^{\frac{3}{2}}a^2bc^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(-b*c*x+a*c)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), a
**2/(b**2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2)) + meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), a**2*exp_polar(-2*I*pi)/(b*
*2*x**2))/(2*pi**(3/2)*a**2*b*c**(3/2))
```

3.1081 $\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx$

Optimal. Leaf size=67

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}+\frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] x/(3*a^2*c*(a + b*x)^(3/2)*(a*c - b*c*x)^(3/2)) + (2*x)/(3*a^4*c^2*Sqrt[a +
 b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{3a^2c}\\ &= \frac{x}{3a^2c(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{2x}{3a^4c^2\sqrt{a+bx}\,\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.69

$$\frac{3a^2x-2b^2x^3}{3a^4c(a+bx)^{3/2}(c(a-bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] (3*a^2*x - 2*b^2*x^3)/(3*a^4*c*(c*(a - b*x))^(3/2)*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 101, normalized size = 1.51

$$\frac{(a+bx)^{3/2}\left(\frac{9c^2(ac-bcx)}{a+bx}-\frac{9c(ac-bcx)^2}{(a+bx)^2}-\frac{(ac-bcx)^3}{(a+bx)^3}+c^3\right)}{24a^4bc^4(ac-bcx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)),x]
```

```
[Out] ((a + b*x)^(3/2)*(c^3 + (9*c^2*(a*c - b*c*x))/(a + b*x) - (9*c*(a*c - b*c*x
)^2)/(a + b*x)^2 - (a*c - b*c*x)^3/(a + b*x)^3))/(24*a^4*b*c^4*(a*c - b*c*x
)^(3/2))
```

fricas [A] time = 1.23, size = 72, normalized size = 1.07

$$-\frac{\left(2\,b^{2}x^{3}-3\,a^{2}x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{3\,\left(a^{4}b^{4}c^{3}x^{4}-2\,a^{6}b^{2}c^{3}x^{2}+a^{8}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*b^2*x^3 - 3*a^2*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^4*b^4*c^3*x^
4 - 2*a^6*b^2*c^3*x^2 + a^8*c^3)
```

giac [B] time = 2.38, size = 251, normalized size = 3.75

$$-\frac{\sqrt{-bcx+ac}\left(\frac{9\,|c|}{a^{3}bc}+\frac{4\,(bcx-ac)|c|}{a^{4}bc^{2}}\right)}{12\,\left(2\,ac^{2}+(bcx-ac)c\right)^{\frac{3}{2}}}+\frac{16\,a^{2}\sqrt{-c}\,c^{4}-18\,a\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^{2}+(bcx-ac)c}\right)^{2}\sqrt{-c}\,c^{2}+3\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^{2}+(bcx-ac)c}\right)^{4}\sqrt{-c}}{3\left(2\,ac^{2}-\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^{2}+(bcx-ac)c}\right)^{2}\right)^{3}a^{3}b|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(-b*c*x + a*c)*(9*abs(c)/(a^3*b*c) + 4*(b*c*x - a*c)*abs(c)/(a^4*
b*c^2))/(2*a*c^2 + (b*c*x - a*c)*c)^(3/2) + 1/3*(16*a^2*sqrt(-c)*c^4 - 18*a
*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*sqrt(-c)
*c^2 + 3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*
sqrt(-c))/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x
- a*c)*c))^2)^3*a^3*b*abs(c))
```

maple [A] time = 0.00, size = 45, normalized size = 0.67

$$\frac{(-bx+a)\left(-2b^{2}x^{2}+3a^{2}\right)x}{3\,(bx+a)^{\frac{3}{2}}\,(-bcx+ac)^{\frac{5}{2}}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x)
```

```
[Out] 1/3*(-b*x+a)*x*(-2*b^2*x^2+3*a^2)/(b*x+a)^(3/2)/a^4/(-b*c*x+a*c)^(5/2)
```

maxima [A] time = 1.43, size = 53, normalized size = 0.79

$$\frac{x}{3\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{2}c}+\frac{2\,x}{3\,\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{4}c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(-b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^2*c) + 2/3*x/(sqrt(-b^2*c*x^2 + a^2*c)*
a^4*c^2)
```

mupad [B] time = 0.58, size = 80, normalized size = 1.19

$$-\frac{3\,a^{2}\,x\,\sqrt{a\,c-b\,c\,x}\,-2\,b^{2}\,x^{3}\,\sqrt{a\,c-b\,c\,x}}{(a\,c-b\,c\,x)^{2}\,\left(3\,a^{4}\,(a\,c-b\,c\,x)-6\,a^{5}\,c\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(5/2)*(a + b*x)^(5/2)),x)
```

```
[Out] -(3*a^2*x*(a*c - b*c*x)^(1/2) - 2*b^2*x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*
x)^2*(3*a^4*(a*c - b*c*x) - 6*a^5*c)*(a + b*x)^(1/2))
```

sympy [C] time = 15.85, size = 94, normalized size = 1.40

$$\frac{iG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{1}{2}, \frac{5}{2}, 3 \\ \frac{5}{4}, \frac{7}{4}, 2, \frac{5}{2}, 3 & 0 \end{matrix} \middle| \frac{a^2}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}} + \frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, 0, \frac{1}{2}, \frac{3}{4}, \frac{5}{4}, 1 & \\ \frac{3}{4}, \frac{5}{4} & -\frac{1}{2}, 0, 2, 0 \end{matrix} \middle| \frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{3\pi^{\frac{3}{2}}a^4bc^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(-b*c*x+a*c)**(5/2),x)
```

```
[Out] I*meijerg(((5/4, 7/4, 1), (1/2, 5/2, 3)), ((5/4, 7/4, 2, 5/2, 3), (0,)), a*
*2/(b**2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2)) + meijerg(((-1/2, 0, 1/2, 3/4
, 5/4, 1), ()), ((3/4, 5/4), (-1/2, 0, 2, 0)), a**2*exp_polar(-2*I*pi)/(b**
2*x**2))/(3*pi**(3/2)*a**4*b*c**(5/2))
```

3.1082 $\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx$

Optimal. Leaf size=100

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] x/(5*a^2*c*(a + b*x)^(5/2)*(a*c - b*c*x)^(5/2)) + (4*x)/(15*a^4*c^2*(a + b*
x)^(3/2)*(a*c - b*c*x)^(3/2)) + (8*x)/(15*a^6*c^3*Sqrt[a + b*x]*Sqrt[a*c -
b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4\int \frac{1}{(a+bx)^{5/2}(ac-bcx)^{5/2}}\,dx}{5a^2c}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8\int \frac{1}{(a+bx)^{3/2}(ac-bcx)^{3/2}}\,dx}{15a^4c^2}\\ &= \frac{x}{5a^2c(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{4x}{15a^4c^2(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{8x}{15a^6c^3\sqrt{a+bx}\sqrt{ac-bcx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.57

$$\frac{15a^4x-20a^2b^2x^3+8b^4x^5}{15a^6c(a+bx)^{5/2}(c(a-bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] (15*a^4*x - 20*a^2*b^2*x^3 + 8*b^4*x^5)/(15*a^6*c*(c*(a - b*x))^(5/2)*(a +
b*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 149, normalized size = 1.49

$$\frac{(a+bx)^{5/2}\left(\frac{25c^4(ac-bcx)}{a+bx}+\frac{150c^3(ac-bcx)^2}{(a+bx)^2}-\frac{150c^2(ac-bcx)^3}{(a+bx)^3}-\frac{25c(ac-bcx)^4}{(a+bx)^4}-\frac{3(ac-bcx)^5}{(a+bx)^5}+3c^5\right)}{480a^6bc^6(ac-bcx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)),x]
```

```
[Out] ((a + b*x)^(5/2)*(3*c^5 + (25*c^4*(a*c - b*c*x))/(a + b*x) + (150*c^3*(a*c
- b*c*x)^2)/(a + b*x)^2 - (150*c^2*(a*c - b*c*x)^3)/(a + b*x)^3 - (25*c*(a*
c - b*c*x)^4)/(a + b*x)^4 - (3*(a*c - b*c*x)^5)/(a + b*x)^5))/(480*a^6*b*c^
6*(a*c - b*c*x)^(5/2))
```

fricas [A] time = 1.40, size = 98, normalized size = 0.98

$$-\frac{\left(8\,b^4x^5-20\,a^2b^2x^3+15\,a^4x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{15\left(a^6b^6c^4x^6-3\,a^8b^4c^4x^4+3\,a^{10}b^2c^4x^2-a^{12}c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(8*b^4*x^5 - 20*a^2*b^2*x^3 + 15*a^4*x)*sqrt(-b*c*x + a*c)*sqrt(b*x +
 a)/(a^6*b^6*c^4*x^6 - 3*a^8*b^4*c^4*x^4 + 3*a^10*b^2*c^4*x^2 - a^12*c^4)
```

giac [B] time = 2.57, size = 366, normalized size = 3.66

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left(\frac{275\,c}{a^5b|c|}+\frac{64\,(bcx-ac)}{a^6b|c|}\right)+\frac{300\,c^2}{a^4b|c|}\right)}{240\left(2\,ac^2+(bcx-ac)c\right)^{\frac{5}{2}}}-\frac{1024\,a^4c^8-2200\,a^3\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2c^6+1660\,a^2\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^4c^4-450\,a\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^6c^2+45\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^8}{60\left(2\,ac^2-\left(\sqrt{-bcx+ac}\,\sqrt{-c}-\sqrt{2\,ac^2+(bcx-ac)c}\right)^2\right)^5a^5b\sqrt{-c}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] -1/240*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*(275*c/(a^5*b*abs(c)) + 64*(b*c*x
- a*c)/(a^6*b*abs(c))) + 300*c^2/(a^4*b*abs(c)))/(2*a*c^2 + (b*c*x - a*c)*c
)^(5/2) - 1/60*(1024*a^4*c^8 - 2200*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt
(2*a*c^2 + (b*c*x - a*c)*c))^2*c^6 + 1660*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c)
- sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*c^4 - 450*a*(sqrt(-b*c*x + a*c)*sqrt(-
c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*c^2 + 45*(sqrt(-b*c*x + a*c)*sqrt(-
c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^8)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sq
rt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^5*a^5*b*sqrt(-c)*abs(c))
```

maple [A] time = 0.00, size = 56, normalized size = 0.56

$$\frac{(-bx+a)\left(8b^4x^4-20a^2b^2x^2+15a^4\right)x}{15\,(bx+a)^{\frac{5}{2}}\,(-bcx+ac)^{\frac{7}{2}}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x)
```

```
[Out] 1/15*(-b*x+a)*x*(8*b^4*x^4-20*a^2*b^2*x^2+15*a^4)/(b*x+a)^(5/2)/a^6/(-b*c*x
+a*c)^(7/2)
```

maxima [A] time = 1.32, size = 79, normalized size = 0.79

$$\frac{x}{5\left(-b^2cx^2+a^2c\right)^{\frac{5}{2}}a^2c}+\frac{4\,x}{15\left(-b^2cx^2+a^2c\right)^{\frac{3}{2}}a^4c^2}+\frac{8\,x}{15\sqrt{-b^2cx^2+a^2c}\,a^6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(-b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-b^2*c*x^2 + a^2*c)^(5/2)*a^2*c) + 4/15*x/((-b^2*c*x^2 + a^2*c)^(3/
2)*a^4*c^2) + 8/15*x/(sqrt(-b^2*c*x^2 + a^2*c)*a^6*c^3)
```

mupad [B] time = 0.65, size = 111, normalized size = 1.11

$$\frac{15\,a^4\,x\,\sqrt{a\,c-b\,c\,x}+8\,b^4\,x^5\,\sqrt{a\,c-b\,c\,x}-20\,a^2\,b^2\,x^3\,\sqrt{a\,c-b\,c\,x}}{{\left(a\,c-b\,c\,x\right)}^3\,\left(60\,a^8\,c-\left(a\,c-b\,c\,x\right)\,\left(45\,a^7+15\,b\,x\,a^6\right)\right)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(7/2)*(a + b*x)^(7/2)),x)
```

```
[Out] (15*a^4*x*(a*c - b*c*x)^(1/2) + 8*b^4*x^5*(a*c - b*c*x)^(1/2) - 20*a^2*b^2*
x^3*(a*c - b*c*x)^(1/2))/((a*c - b*c*x)^3*(60*a^8*c - (a*c - b*c*x)*(45*a^7
 + 15*a^6*b*x))*(a + b*x)^(1/2))
```

sympy [C] time = 59.50, size = 97, normalized size = 0.97

$$-\frac{2iG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{1}{2},\frac{7}{2},4\\ \frac{7}{4},\frac{9}{4},3,\frac{7}{2},4 & 0\end{matrix}\middle|\frac{a^2}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}+\frac{2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{2},\frac{5}{4},\frac{7}{4},1 & \\ \frac{5}{4},\frac{7}{4} & -\frac{1}{2},0,3,0\end{matrix}\middle|\frac{a^2e^{-2i\pi}}{b^2x^2}\right)}{15\pi^{\frac{3}{2}}a^6bc^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(-b*c*x+a*c)**(7/2),x)
```

```
[Out] -2*I*meijerg(((7/4, 9/4, 1), (1/2, 7/2, 4)), ((7/4, 9/4, 3, 7/2, 4), (0,)),
 a**2/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2)) + 2*meijerg(((-1/2, 0, 1/
2, 5/4, 7/4, 1), ()), ((5/4, 7/4), (-1/2, 0, 3, 0)), a**2*exp_polar(-2*I*pi
)/(b**2*x**2))/(15*pi**(3/2)*a**6*b*c**(7/2))
```

3.1083 $\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx$

Optimal. Leaf size=133

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-\ldots}$$

Rubi [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {40, 39}

$$\frac{16x}{35a^8c^4\sqrt{a+bx}\sqrt{ac-bcx}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}(ac-bcx)^{3/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

```
[Out] x/(7*a^2*c*(a + b*x)^(7/2)*(a*c - b*c*x)^(7/2)) + (6*x)/(35*a^4*c^2*(a + b*
x)^(5/2)*(a*c - b*c*x)^(5/2)) + (8*x)/(35*a^6*c^3*(a + b*x)^(3/2)*(a*c - b*
c*x)^(3/2)) + (16*x)/(35*a^8*c^4*Sqrt[a + b*x]*Sqrt[a*c - b*c*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{9/2}(ac-bcx)^{9/2}}\,dx &= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6\int \frac{1}{(a+bx)^{7/2}(ac-bcx)^{7/2}}\,dx}{7a^2c}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{24\int \frac{1}{(a+bx)^{5/2}(ac-bc\ldots}}{35a^4c^2}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}\ldots}\\
&= \frac{x}{7a^2c(a+bx)^{7/2}(ac-bcx)^{7/2}}+\frac{6x}{35a^4c^2(a+bx)^{5/2}(ac-bcx)^{5/2}}+\frac{8x}{35a^6c^3(a+bx)^{3/2}\ldots}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.57

$$\frac{x\left(35a^6-70a^4b^2x^2+56a^2b^4x^4-16b^6x^6\right)\sqrt{c(a-bx)}}{35a^8c^5(a-bx)^4(a+bx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]
```

[Out] (x*Sqrt[c*(a - b*x)]*(35*a^6 - 70*a^4*b^2*x^2 + 56*a^2*b^4*x^4 - 16*b^6*x^6))/(35*a^8*c^5*(a - b*x)^4*(a + b*x)^(7/2))

IntegrateAlgebraic [A] time = 0.15, size = 195, normalized size = 1.47

$$\frac{(a+bx)^{7/2}\left(\frac{49c^6(ac-bcx)}{a+bx}+\frac{245c^5(ac-bcx)^2}{(a+bx)^2}+\frac{1225c^4(ac-bcx)^3}{(a+bx)^3}-\frac{1225c^3(ac-bcx)^4}{(a+bx)^4}-\frac{245c^2(ac-bcx)^5}{(a+bx)^5}-\frac{49c(ac-bcx)^6}{(a+bx)^6}-\frac{5(ac-bcx)^7}{(a+bx)^7}+5c^7\right)}{4480a^8bc^8(ac-bcx)^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(a*c - b*c*x)^(9/2)),x]

[Out] ((a + b*x)^(7/2)*(5*c^7 + (49*c^6*(a*c - b*c*x))/(a + b*x) + (245*c^5*(a*c - b*c*x)^2)/(a + b*x)^2 + (1225*c^4*(a*c - b*c*x)^3)/(a + b*x)^3 - (1225*c^3*(a*c - b*c*x)^4)/(a + b*x)^4 - (245*c^2*(a*c - b*c*x)^5)/(a + b*x)^5 - (49*c*(a*c - b*c*x)^6)/(a + b*x)^6 - (5*(a*c - b*c*x)^7)/(a + b*x)^7))/(4480*a^8*b*c^8*(a*c - b*c*x)^(7/2))

fricas [A] time = 1.60, size = 122, normalized size = 0.92

$$-\frac{\left(16\,b^6x^7-56\,a^2b^4x^5+70\,a^4b^2x^3-35\,a^6x\right)\sqrt{-bcx+ac}\,\sqrt{bx+a}}{35\left(a^8b^8c^5x^8-4\,a^{10}b^6c^5x^6+6\,a^{12}b^4c^5x^4-4\,a^{14}b^2c^5x^2+a^{16}c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="fricas")

[Out] -1/35*(16*b^6*x^7 - 56*a^2*b^4*x^5 + 70*a^4*b^2*x^3 - 35*a^6*x)*sqrt(-b*c*x + a*c)*sqrt(b*x + a)/(a^8*b^8*c^5*x^8 - 4*a^10*b^6*c^5*x^6 + 6*a^12*b^4*c^5*x^4 - 4*a^14*b^2*c^5*x^2 + a^16*c^5)

giac [B] time = 3.30, size = 487, normalized size = 3.66

$$-\frac{\sqrt{-bcx+ac}\left((bcx-ac)\left((bcx-ac)\left(\frac{1617|c|}{a^7bc}+\frac{256(bcx-ac)|c|}{a^8bc^2}\right)+\frac{3430|c|}{a^6b}\right)+\frac{2450c|c|}{a^5b}\right)}{1120\left(2ac^2+(bcx-ac)c\right)^{\frac{7}{2}}}-\frac{16384a^6c^{12}-51744a^5\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^2c^{10}+66416a^4\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^4c^8-43120a^3\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^6c^6+14280a^2\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^8c^4-2450a\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^{10}c^2+175\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^{12}}{280\left(2ac^2-\left(\sqrt{-bcx+ac}\sqrt{-c}-\sqrt{2ac^2+(bcx-ac)c}\right)^2\right)^7a^7b\sqrt{-c}\,c|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="giac")

[Out] -1/1120*sqrt(-b*c*x + a*c)*((b*c*x - a*c)*((b*c*x - a*c)*(1617*abs(c)/(a^7*b*c) + 256*(b*c*x - a*c)*abs(c)/(a^8*b*c^2)) + 3430*abs(c)/(a^6*b)) + 2450*c*abs(c)/(a^5*b))/(2*a*c^2 + (b*c*x - a*c)*c)^(7/2) - 1/280*(16384*a^6*c^12 - 51744*a^5*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2*c^10 + 66416*a^4*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^4*c^8 - 43120*a^3*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^6*c^6 + 14280*a^2*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^8*c^4 - 2450*a*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^10*c^2 + 175*(sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^12)/((2*a*c^2 - (sqrt(-b*c*x + a*c)*sqrt(-c) - sqrt(2*a*c^2 + (b*c*x - a*c)*c))^2)^7*a^7*b*sqrt(-c)*c*abs(c))

maple [A] time = 0.00, size = 67, normalized size = 0.50

$$\frac{(-bx+a)\left(-16b^6x^6+56b^4x^4a^2-70b^2x^2a^4+35a^6\right)x}{35\,(bx+a)^{\frac{7}{2}}\,(-bcx+ac)^{\frac{9}{2}}\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x)

[Out] 1/35*(-b*x+a)*x*(-16*b^6*x^6+56*a^2*b^4*x^4-70*a^4*b^2*x^2+35*a^6)/(b*x+a)^(7/2)/a^8/(-b*c*x+a*c)^(9/2)

maxima [A] time = 1.29, size = 105, normalized size = 0.79

$$\frac{x}{7\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{7}{2}}a^{2}c}+\frac{6\,x}{35\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{5}{2}}a^{4}c^{2}}+\frac{8\,x}{35\left(-b^{2}cx^{2}+a^{2}c\right)^{\frac{3}{2}}a^{6}c^{3}}+\frac{16\,x}{35\sqrt{-b^{2}cx^{2}+a^{2}c}\,a^{8}c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(-b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/7*x/((-b^2*c*x^2 + a^2*c)^(7/2)*a^2*c) + 6/35*x/((-b^2*c*x^2 + a^2*c)^(5/
2)*a^4*c^2) + 8/35*x/((-b^2*c*x^2 + a^2*c)^(3/2)*a^6*c^3) + 16/35*x/(sqrt(-
b^2*c*x^2 + a^2*c)*a^8*c^4)
```

mupad [B] time = 0.71, size = 170, normalized size = 1.28

$$-\frac{35\,a^{6}\,x\,\sqrt{a\,c-b\,c\,x}-16\,b^{6}\,x^{7}\,\sqrt{a\,c-b\,c\,x}-70\,a^{4}\,b^{2}\,x^{3}\,\sqrt{a\,c-b\,c\,x}+56\,a^{2}\,b^{4}\,x^{5}\,\sqrt{a\,c-b\,c\,x}}{\left(\left(70\,a^{9}\,(a\,c-b\,c\,x)^{5}+35\,a^{8}\,(a\,c-b\,c\,x)^{5}\,(a+b\,x)\right)(a+b\,x)+(a\,c-b\,c\,x)^{4}\left(140\,a^{10}\,(a\,c-b\,c\,x)-280\,a^{11}\,c\right)\right)\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a*c - b*c*x)^(9/2)*(a + b*x)^(9/2)),x)
```

```
[Out] -(35*a^6*x*(a*c - b*c*x)^(1/2) - 16*b^6*x^7*(a*c - b*c*x)^(1/2) - 70*a^4*b^
2*x^3*(a*c - b*c*x)^(1/2) + 56*a^2*b^4*x^5*(a*c - b*c*x)^(1/2))/(((70*a^9*(
a*c - b*c*x)^5 + 35*a^8*(a*c - b*c*x)^5*(a + b*x))*(a + b*x) + (a*c - b*c*x
)^4*(140*a^10*(a*c - b*c*x) - 280*a^11*c))*(a + b*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(-b*c*x+a*c)**(9/2),x)
```

```
[Out] Timed out
```

3.1084 $\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx$

Optimal. Leaf size=100

$$6\sqrt{6}\,(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$6\sqrt{6}\,(1-2x)^{5/2}x(2x+1)^{5/2}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{45}{4}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (45*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 15*Sqrt[3/2]*(1 - 2*x)^(3/
2)*x*(1 + 2*x)^(3/2) + 6*Sqrt[6]*(1 - 2*x)^(5/2)*x*(1 + 2*x)^(5/2) + (45*Sq
rt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int (3-6x)^{5/2}(2+4x)^{5/2}\,dx=6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2}+5\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$$
$$=15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}\,(1-2x)^{5/2}x(1+2x)^{5/2}+\frac{45}{2}\int \sqrt{3-6x}\,\sqrt{2}$$
$$=\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}\,(1-2x)^{5/2}x(1-$$
$$=\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}\,(1-2x)^{5/2}x(1-$$
$$=\frac{45}{2}\sqrt{\frac{3}{2}}\,\sqrt{1-2x}\,x\sqrt{1+2x}+15\sqrt{\frac{3}{2}}\,(1-2x)^{3/2}x(1+2x)^{3/2}+6\sqrt{6}\,(1-2x)^{5/2}x(1-$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.44

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(128x^4-104x^2+33\right)+15\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2]*(33 - 104*x^2 + 128*x^4) + 15*ArcSin[2*x]
))/4
```

IntegrateAlgebraic [B] time = 0.97, size = 229, normalized size = 2.29

$$\frac{48\sqrt{6}\sqrt{1-2x}\,x\sqrt{2x+1}\left(128x^4-104x^2+33\right)\left(-352x^5-6160x^4-26224x^3-41096x^2-26158x-5741\right)+48\sqrt{3}\sqrt{1-2x}\,x\left(128x^4-104x^2+33\right)\left(64x^6+3712x^5+30160x^4+80768x^3+91052x^2+45112x+8119\right)}{-22528x^5-394240x^4-1678336x^3-2630144x^2+\sqrt{2}\sqrt{2x+1}\left(1024x^5+58880x^4+453120x^3+1065728x^2+923968x+259808\right)-1674112x-367424}+45\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 6*x)^(5/2)*(2 + 4*x)^(5/2),x]
```

```
[Out] (48*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(33 - 104*x^2 + 128*x^4)*(-5741 -
 26158*x - 41096*x^2 - 26224*x^3 - 6160*x^4 - 352*x^5) + 48*Sqrt[3]*Sqrt[1
- 2*x]*x*(33 - 104*x^2 + 128*x^4)*(8119 + 45112*x + 91052*x^2 + 80768*x^3 +
 30160*x^4 + 3712*x^5 + 64*x^6))/(-367424 - 1674112*x - 2630144*x^2 - 16783
36*x^3 - 394240*x^4 - 22528*x^5 + Sqrt[2]*Sqrt[1 + 2*x]*(259808 + 923968*x
+ 1065728*x^2 + 453120*x^3 + 58880*x^4 + 1024*x^5)) + 45*Sqrt[3/2]*ArcTan[(
-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

fricas [A] time = 1.21, size = 65, normalized size = 0.65

$$\frac{3}{4}\left(128\,x^{5}-104\,x^{3}+33\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{45}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] 3/4*(128*x^5 - 104*x^3 + 33*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 45/8*sqrt(3)*
sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

giac [B] time = 1.24, size = 227, normalized size = 2.27

$$\frac{3}{40}\,\sqrt{3}\,\sqrt{2}\left(\left(\left(2\left(\left(8\left(5\,x-13\right)\left(2\,x+1\right)+321\right)\left(2\,x+1\right)-451\right)\left(2\,x+1\right)+745\right)\left(2\,x+1\right)-405\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+2\left(\left(2\left(3\left(8\,x-17\right)\left(2\,x+1\right)+133\right)\left(2\,x+1\right)-295\right)\left(2\,x+1\right)+195\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-20\left(\left(4\left(3\,x-5\right)\left(2\,x+1\right)+43\right)\left(2\,x+1\right)-39\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-80\left(\left(4\,x-5\right)\left(2\,x+1\right)+9\right)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+240\,\sqrt{2\,x+1}\left(x-1\right)\sqrt{-2\,x+1}+240\,\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+150\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] 3/40*sqrt(3)*sqrt(2)*(((2*((8*(5*x - 13)*(2*x + 1) + 321)*(2*x + 1) - 451)*
(2*x + 1) + 745)*(2*x + 1) - 405)*sqrt(2*x + 1)*sqrt(-2*x + 1) + 2*((2*(3*(
8*x - 17)*(2*x + 1) + 133)*(2*x + 1) - 295)*(2*x + 1) + 195)*sqrt(2*x + 1)*
sqrt(-2*x + 1) - 20*((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*x
+ 1)*sqrt(-2*x + 1) - 80*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
+ 1) + 240*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + 240*sqrt(2*x + 1)*sqrt(-2
*x + 1) + 150*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

maple [A] time = 0.01, size = 134, normalized size = 1.34

$$\frac{45\sqrt{(4x+2)(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{8\sqrt{4x+2}\,\sqrt{-6x+3}}+\frac{(-6x+3)^{\frac{5}{2}}(4x+2)^{\frac{7}{2}}}{24}+\frac{(-6x+3)^{\frac{3}{2}}(4x+2)^{\frac{7}{2}}}{8}+\frac{9\sqrt{-6x+3}\,(4x+2)^{\frac{7}{2}}}{32}-\frac{3(4x+2)^{\frac{5}{2}}\sqrt{-6x+3}}{16}-\frac{15(4x+2)^{\frac{3}{2}}\sqrt{-6x+3}}{16}-\frac{45\sqrt{-6x+3}\,\sqrt{4x+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-6*x)^(5/2)*(2+4*x)^(5/2),x)
```

```
[Out] 1/24*(3-6*x)^(5/2)*(2+4*x)^(7/2)+1/8*(3-6*x)^(3/2)*(2+4*x)^(7/2)+9/32*(3-6*
x)^(1/2)*(2+4*x)^(7/2)-3/16*(2+4*x)^(5/2)*(3-6*x)^(1/2)-15/16*(2+4*x)^(3/2)
*(3-6*x)^(1/2)-45/8*(3-6*x)^(1/2)*(2+4*x)^(1/2)+45/8*((2+4*x)*(3-6*x))^(1/2
)/(2+4*x)^(1/2)/(3-6*x)^(1/2)*6^(1/2)*arcsin(2*x)
```

maxima [A] time = 2.86, size = 46, normalized size = 0.46

$$\frac{1}{6}\left(-24\,x^2+6\right)^{\frac{5}{2}}x+\frac{5}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x+\frac{45}{4}\,\sqrt{-24\,x^2+6}\,x+\frac{45}{8}\,\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(5/2)*(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-24*x^2 + 6)^(5/2)*x + 5/4*(-24*x^2 + 6)^(3/2)*x + 45/4*sqrt(-24*x^2 +
 6)*x + 45/8*sqrt(6)*arcsin(2*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (4\,x+2)^{5/2}\,(3-6\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2),x)
```

```
[Out] int((4*x + 2)^(5/2)*(3 - 6*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(5/2)*(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

3.1085 $\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx$

Optimal. Leaf size=74

$$3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Rubi [A] time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(2x+1)^{3/2}+\frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{2x+1}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (9*Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x])/2 + 3*Sqrt[3/2]*(1 - 2*x)^(3/2)
*x*(1 + 2*x)^(3/2) + (9*Sqrt[3/2]*ArcSin[2*x])/4
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int (3-6x)^{3/2}(2+4x)^{3/2}\,dx &= 3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{2}\int \sqrt{3-6x}\sqrt{2+4x}\,dx\\ &= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{27}{2}\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx\\ &= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{27}{2}\int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{9}{2}\sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\sqrt{\frac{3}{2}}(1-2x)^{3/2}x(1+2x)^{3/2}+\frac{9}{4}\sqrt{\frac{3}{2}}\sin^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 39, normalized size = 0.53

$$\frac{3}{4}\sqrt{\frac{3}{2}}\left(2x\sqrt{1-4x^2}\left(5-8x^2\right)+3\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (3*Sqrt[3/2]*(2*x*(5 - 8*x^2)*Sqrt[1 - 4*x^2] + 3*ArcSin[2*x]))/4
```

IntegrateAlgebraic [B] time = 0.84, size = 179, normalized size = 2.42

$$\frac{-12\sqrt{6}\sqrt{1-2x}\,x\sqrt{2x+1}\left(8x^2-5\right)\left(-56x^3-364x^2-490x-169\right)-12\sqrt{3}\sqrt{1-2x}\,x\left(8x^2-5\right)\left(16x^4+368x^3+1088x^2+932x+239\right)}{-896x^3-5824x^2+\sqrt{2}\sqrt{2x+1}\left(64x^3+1440x^2+3632x+1912\right)-7840x-2704}+9\sqrt{\frac{3}{2}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 6*x)^(3/2)*(2 + 4*x)^(3/2),x]
```

```
[Out] (-12*Sqrt[6]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x]*(-5 + 8*x^2)*(-169 - 490*x - 364
*x^2 - 56*x^3) - 12*Sqrt[3]*Sqrt[1 - 2*x]*x*(-5 + 8*x^2)*(239 + 932*x + 108
8*x^2 + 368*x^3 + 16*x^4))/(-2704 - 7840*x - 5824*x^2 - 896*x^3 + Sqrt[2]*S
qrt[1 + 2*x]*(1912 + 3632*x + 1440*x^2 + 64*x^3)) + 9*Sqrt[3/2]*ArcTan[(-Sq
rt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

fricas [A] time = 0.87, size = 60, normalized size = 0.81

$$-\frac{3}{4}\left(8\,x^3-5\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}-\frac{9}{8}\,\sqrt{3}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{12\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -3/4*(8*x^3 - 5*x)*sqrt(4*x + 2)*sqrt(-6*x + 3) - 9/8*sqrt(3)*sqrt(2)*arcta
n(1/12*sqrt(3)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

giac [B] time = 0.98, size = 125, normalized size = 1.69

$$-\frac{1}{8}\,\sqrt{3}\,\sqrt{2}\left(((4\,(3\,x-5)(2\,x+1)+43)(2\,x+1)-39)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}+4\,((4\,x-5)(2\,x+1)+9)\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-24\,\sqrt{2\,x+1}\,(x-1)\sqrt{-2\,x+1}-24\,\sqrt{2\,x+1}\,\sqrt{-2\,x+1}-18\,\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{2\,x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*sqrt(2)*(((4*(3*x - 5)*(2*x + 1) + 43)*(2*x + 1) - 39)*sqrt(2*
x + 1)*sqrt(-2*x + 1) + 4*((4*x - 5)*(2*x + 1) + 9)*sqrt(2*x + 1)*sqrt(-2*x
 + 1) - 24*sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) - 24*sqrt(2*x + 1)*sqrt(-2*
x + 1) - 18*arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

maple [B] time = 0.00, size = 102, normalized size = 1.38

$$\frac{9\sqrt{(4x+2)\,(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{8\sqrt{4x+2}\,\sqrt{-6x+3}}+\frac{(-6x+3)^{\frac{3}{2}}\,(4x+2)^{\frac{5}{2}}}{16}+\frac{3\,(4x+2)^{\frac{5}{2}}\,\sqrt{-6x+3}}{16}-\frac{3\,(4x+2)^{\frac{3}{2}}\,\sqrt{-6x+3}}{16}-\frac{9\sqrt{-6x+3}\,\sqrt{4x+2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x+3)^(3/2)*(4*x+2)^(3/2),x)
```

```
[Out] 1/16*(-6*x+3)^(3/2)*(4*x+2)^(5/2)+3/16*(4*x+2)^(5/2)*(-6*x+3)^(1/2)-3/16*(4
*x+2)^(3/2)*(-6*x+3)^(1/2)-9/8*(-6*x+3)^(1/2)*(4*x+2)^(1/2)+9/8*((4*x+2)*(-
6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2*x)
```

maxima [A] time = 2.86, size = 34, normalized size = 0.46

$$\frac{1}{4}\left(-24\,x^2+6\right)^{\frac{3}{2}}x+\frac{9}{4}\,\sqrt{-24\,x^2+6}\,x+\frac{9}{8}\,\sqrt{6}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(3/2)*(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-24*x^2 + 6)^(3/2)*x + 9/4*sqrt(-24*x^2 + 6)*x + 9/8*sqrt(6)*arcsin(2*
x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (4\,x+2)^{3/2}\,(3-6\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2),x)
```

```
[Out] int((4*x + 2)^(3/2)*(3 - 6*x)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(3/2)*(4*x+2)**(3/2),x)
```

```
[Out] Timed out
```

3.1086 $\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx$

Optimal. Leaf size=43

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\,\sqrt{2x+1}\,x+\frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {38, 41, 216}

$$\sqrt{\frac{3}{2}}\sqrt{1-2x}\,\sqrt{2x+1}\,x+\frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] Sqrt[3/2]*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + (Sqrt[3/2]*ArcSin[2*x])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-6x}\,\sqrt{2+4x}\,dx &= \sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\int \frac{1}{\sqrt{3-6x}\,\sqrt{2+4x}}\,dx\\ &= \sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+3\int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \sqrt{\frac{3}{2}}\sqrt{1-2x}\,x\sqrt{1+2x}+\frac{1}{2}\sqrt{\frac{3}{2}}\,\sin^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.70

$$\frac{1}{2}\sqrt{\frac{3}{2}}\left(2\sqrt{1-4x^2}\,x+\sin^{-1}(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (Sqrt[3/2]*(2*x*Sqrt[1 - 4*x^2] + ArcSin[2*x]))/2
```

IntegrateAlgebraic [B] time = 0.68, size = 122, normalized size = 2.84

$$\frac{2\sqrt{3}\sqrt{1-2x}\left(4x^2+16x+7\right)x+2\sqrt{6}\,(-6x-5)\sqrt{1-2x}\,\sqrt{2x+1}\,x}{-24x+\sqrt{2}\sqrt{2x+1}\,(4x+14)-20}+\sqrt{6}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - 6*x]*Sqrt[2 + 4*x],x]
```

```
[Out] (2*Sqrt[6]*(-5 - 6*x)*Sqrt[1 - 2*x]*x*Sqrt[1 + 2*x] + 2*Sqrt[3]*Sqrt[1 - 2*
x]*x*(7 + 16*x + 4*x^2))/(-20 - 24*x + Sqrt[2]*Sqrt[1 + 2*x]*(14 + 4*x)) +
Sqrt[6]*ArcTan[(-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

fricas [A] time = 1.13, size = 52, normalized size = 1.21

$$\frac{1}{2}\sqrt{4x+2}\,x\sqrt{-6x+3}-\frac{1}{4}\sqrt{3}\sqrt{2}\arctan\left(\frac{\sqrt{3}\sqrt{2}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*x + 2)*x*sqrt(-6*x + 3) - 1/4*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3
)*sqrt(2)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

giac [A] time = 1.07, size = 55, normalized size = 1.28

$$\frac{1}{2}\sqrt{3}\sqrt{2}\left(\sqrt{2x+1}\,(x-1)\sqrt{-2x+1}+\sqrt{2x+1}\sqrt{-2x+1}+\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{2x+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*sqrt(2)*(sqrt(2*x + 1)*(x - 1)*sqrt(-2*x + 1) + sqrt(2*x + 1)*s
qrt(-2*x + 1) + arcsin(1/2*sqrt(2)*sqrt(2*x + 1)))
```

maple [B] time = 0.00, size = 70, normalized size = 1.63

$$\frac{\sqrt{(4x+2)(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{4\sqrt{4x+2}\,\sqrt{-6x+3}}-\frac{\sqrt{4x+2}\,(-6x+3)^{\frac{3}{2}}}{12}+\frac{\sqrt{-6x+3}\,\sqrt{4x+2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-6*x+3)^(1/2)*(4*x+2)^(1/2),x)
```

```
[Out] -1/12*(4*x+2)^(1/2)*(-6*x+3)^(3/2)+1/4*(-6*x+3)^(1/2)*(4*x+2)^(1/2)+1/4*((4
*x+2)*(-6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2*x)
```

maxima [A] time = 3.07, size = 22, normalized size = 0.51

$$\frac{1}{2}\sqrt{-24x^2+6x}+\frac{1}{4}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)^(1/2)*(4*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-24*x^2 + 6)*x + 1/4*sqrt(6)*arcsin(2*x)
```

mupad [B] time = 0.26, size = 44, normalized size = 1.02

$$\frac{x\sqrt{4x+2}\,\sqrt{3-6x}}{2}-\frac{\sqrt{6}\,\ln\left(x-\frac{\sqrt{1-2x}\,\sqrt{2x+1}\,1\mathrm{i}}{2}\right)1\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x + 2)^(1/2)*(3 - 6*x)^(1/2),x)
```

```
[Out] (x*(4*x + 2)^(1/2)*(3 - 6*x)^(1/2))/2 - (6^(1/2)*log(x - ((1 - 2*x)^(1/2)*(
2*x + 1)^(1/2)*1i)/2)*1i)/4
```

sympy [B] time = 4.74, size = 187, normalized size = 4.35

$$\begin{cases} -\frac{\sqrt{6}i\operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{2}+\frac{\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{x-\frac{1}{2}}}-\frac{3\sqrt{6}i\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{x-\frac{1}{2}}}+\frac{\sqrt{6}i\sqrt{x+\frac{1}{2}}}{2\sqrt{x-\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{2}-\frac{\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{5}{2}}}{\sqrt{\frac{1}{2}-x}}+\frac{3\sqrt{6}\left(x+\frac{1}{2}\right)^{\frac{3}{2}}}{2\sqrt{\frac{1}{2}-x}}-\frac{\sqrt{6}\sqrt{x+\frac{1}{2}}}{2\sqrt{\frac{1}{2}-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-6*x)**(1/2)*(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/2 + sqrt(6)*I*(x + 1/2)**(5/2)/s
qrt(x - 1/2) - 3*sqrt(6)*I*(x + 1/2)**(3/2)/(2*sqrt(x - 1/2)) + sqrt(6)*I*s
qrt(x + 1/2)/(2*sqrt(x - 1/2)), Abs(x + 1/2) > 1), (sqrt(6)*asin(sqrt(x + 1
/2))/2 - sqrt(6)*(x + 1/2)**(5/2)/sqrt(1/2 - x) + 3*sqrt(6)*(x + 1/2)**(3/2
)/(2*sqrt(1/2 - x)) - sqrt(6)*sqrt(x + 1/2)/(2*sqrt(1/2 - x)), True))
```

3.1087 $\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx$

Optimal. Leaf size=13

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{41, 216\}$

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-6x}\sqrt{2+4x}}\,dx &= \int \frac{1}{\sqrt{6-24x^2}}\,dx\\ &= \frac{\sin^{-1}(2x)}{2\sqrt{6}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 13, normalized size = 1.00

$$\frac{\sin^{-1}(2x)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] ArcSin[2*x]/(2*Sqrt[6])
```

IntegrateAlgebraic [B] time = 0.59, size = 36, normalized size = 2.77

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\frac{\sqrt{2x+1}-\sqrt{2}}{\sqrt{1-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 6*x]*Sqrt[2 + 4*x]),x]
```

```
[Out] Sqrt[2/3]*ArcTan[(-Sqrt[2] + Sqrt[1 + 2*x])/Sqrt[1 - 2*x]]
```

fricas [B] time = 1.09, size = 28, normalized size = 2.15

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{4x+2}\sqrt{-6x+3}}{12x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="fricas")
[Out] -1/12*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(4*x + 2)*sqrt(-6*x + 3)/x)
```

giac [A] time = 0.88, size = 15, normalized size = 1.15

$$\frac{1}{6}\sqrt{6}\arcsin\left(\frac{1}{2}\sqrt{4x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="giac")
[Out] 1/6*sqrt(6)*arcsin(1/2*sqrt(4*x + 2))
```

maple [B] time = 0.00, size = 37, normalized size = 2.85

$$\frac{\sqrt{(4x+2)(-6x+3)}\,\sqrt{6}\,\arcsin(2x)}{12\sqrt{4x+2}\,\sqrt{-6x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(1/2)/(4*x+2)^(1/2),x)
[Out] 1/12*((4*x+2)*(-6*x+3))^(1/2)/(4*x+2)^(1/2)/(-6*x+3)^(1/2)*6^(1/2)*arcsin(2
*x)
```

maxima [A] time = 2.94, size = 9, normalized size = 0.69

$$\frac{1}{12}\sqrt{6}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(1/2)/(4*x+2)^(1/2),x, algorithm="maxima")
[Out] 1/12*sqrt(6)*arcsin(2*x)
```

mupad [B] time = 0.05, size = 40, normalized size = 3.08

$$-\frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{24}\,\left(\sqrt{3}-\sqrt{3-6x}\right)}{6\left(\sqrt{2}-\sqrt{4x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(1/2)*(3 - 6*x)^(1/2)),x)
[Out] -(6^(1/2)*atan((24^(1/2)*(3^(1/2) - (3 - 6*x)^(1/2)))/(6*(2^(1/2) - (4*x +
2)^(1/2)))))/3
```

sympy [A] time = 3.35, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{\sqrt{6}\, i \operatorname{acosh}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ \frac{\sqrt{6}\operatorname{asin}\left(\sqrt{x+\frac{1}{2}}\right)}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(1/2)/(4*x+2)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(6)*I*acosh(sqrt(x + 1/2))/6, Abs(x + 1/2) > 1), (sqrt(6)*a
sin(sqrt(x + 1/2))/6, True))
```

3.1088 $\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {39}

$$\frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}}\,dx = \frac{x}{6\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 0.57

$$\frac{x}{6\sqrt{6-24x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] x/(6*Sqrt[6 - 24*x^2])
```

IntegrateAlgebraic [B] time = 0.73, size = 80, normalized size = 2.86

$$\frac{x(2x+3)-2\sqrt{2}x\sqrt{2x+1}}{6\sqrt{3}\sqrt{1-2x}(-8x-4)+6\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}(2x+3)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(3/2)*(2 + 4*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[2]*x*Sqrt[1 + 2*x] + x*(3 + 2*x))/(6*Sqrt[3]*(-4 - 8*x)*Sqrt[1 - 2
*x] + 6*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x]*(3 + 2*x))
```

fricas [A] time = 1.39, size = 26, normalized size = 0.93

$$-\frac{\sqrt{4x+2}\,x\sqrt{-6x+3}}{36\left(4x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(4*x + 2)*x*sqrt(-6*x + 3)/(4*x^2 - 1)
```

giac [B] time = 1.06, size = 71, normalized size = 2.54

$$-\frac{\sqrt{6}\left(\sqrt{-4\,x+2}-2\right)}{288\,\sqrt{4\,x+2}}-\frac{\sqrt{6}\,\sqrt{4\,x+2}\,\sqrt{-4\,x+2}}{288\,(2\,x-1)}+\frac{\sqrt{6}\,\sqrt{4\,x+2}}{288\left(\sqrt{-4\,x+2}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/288*sqrt(6)*(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2) - 1/288*sqrt(6)*sqrt(4*x
+ 2)*sqrt(-4*x + 2)/(2*x - 1) + 1/288*sqrt(6)*sqrt(4*x + 2)/(sqrt(-4*x + 2)
 - 2)
```

maple [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{(2x-1)\,(2x+1)\,x}{(-6x+3)^{\frac{3}{2}}\,(4x+2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(3/2)/(4*x+2)^(3/2),x)
```

```
[Out] -(2*x-1)*(1+2*x)*x/(-6*x+3)^(3/2)/(4*x+2)^(3/2)
```

maxima [A] time = 1.36, size = 12, normalized size = 0.43

$$\frac{x}{6\,\sqrt{-24\,x^2+6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(3/2)/(4*x+2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(-24*x^2 + 6)
```

mupad [B] time = 0.46, size = 24, normalized size = 0.86

$$-\frac{x\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(36\,x-18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(3/2)*(3 - 6*x)^(3/2)),x)
```

```
[Out] -(x*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(36*x - 18))
```

sympy [B] time = 85.28, size = 156, normalized size = 5.57

$$\begin{cases} -\frac{2\sqrt{6}\,i\sqrt{x-\frac{1}{2}}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,i\sqrt{x-\frac{1}{2}}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{for}\ \left|x+\frac{1}{2}\right|>1 \\ -\frac{2\sqrt{6}\,\sqrt{\frac{1}{2}-x}\left(x+\frac{1}{2}\right)}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}}+\frac{\sqrt{6}\,\sqrt{\frac{1}{2}-x}}{144\left(x+\frac{1}{2}\right)^{\frac{3}{2}}-144\sqrt{x+\frac{1}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(3/2)/(4*x+2)**(3/2),x)
```

```
[Out] Piecewise((-2*sqrt(6)*I*sqrt(x - 1/2)*(x + 1/2)/(144*(x + 1/2)**(3/2) - 144
*sqrt(x + 1/2)) + sqrt(6)*I*sqrt(x - 1/2)/(144*(x + 1/2)**(3/2) - 144*sqrt(
x + 1/2)), Abs(x + 1/2) > 1), (-2*sqrt(6)*sqrt(1/2 - x)*(x + 1/2)/(144*(x +
 1/2)**(3/2) - 144*sqrt(x + 1/2)) + sqrt(6)*sqrt(1/2 - x)/(144*(x + 1/2)**(
3/2) - 144*sqrt(x + 1/2)), True))
```

3.1089 $\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}} \, dx$

Optimal. Leaf size=57

$$\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{108\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{2x+1}}+\frac{x}{108\sqrt{6}(1-2x)^{3/2}(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] x/(108*Sqrt[6]*(1 - 2*x)^(3/2)*(1 + 2*x)^(3/2)) + x/(54*Sqrt[6]*Sqrt[1 - 2*
x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}} \, dx &= \frac{x}{108\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{1}{9}\int \frac{1}{(3-6x)^{3/2}(2+4x)^{3/2}} \, dx\\ &= \frac{x}{108\sqrt{6}(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{54\sqrt{6}\sqrt{1-2x}\sqrt{1+2x}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.65

$$\frac{x\left(8x^2-3\right)}{108\sqrt{6-12x}(2x-1)(2x+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] (x*(-3 + 8*x^2))/(108*Sqrt[6 - 12*x]*(-1 + 2*x)*(1 + 2*x)^(3/2))
```

IntegrateAlgebraic [B] time = 0.85, size = 334, normalized size = 5.86

$$\frac{\left(\sqrt{2}\sqrt{2x+1}-2\right)^9\left(\frac{91\left(4x^2-4x+1\right)}{294912\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^4}+\frac{35\left(8x^3-12x^2+6x-1\right)}{36864\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^6}+\frac{91\left(16x^4-32x^3+24x^2-8x+1\right)}{73728\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^8}+\frac{5\left(32x^5-80x^4+80x^3-40x^2+10x-1\right)}{18432\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{10}}-\frac{64x^6-192x^5+240x^4-160x^3+60x^2-12x+1}{55296\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{12}}+\frac{5(2x-1)}{294912\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^2}-\frac{1}{3538944\sqrt{3}}\right)}{(1-2x)^{3/2}\left(-2x+\sqrt{2}\sqrt{2x+1}-1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(5/2)*(2 + 4*x)^(5/2)),x]
```

```
[Out] ((-2 + Sqrt[2]*Sqrt[1 + 2*x])^9*(-1/3538944*1/Sqrt[3] - (1 - 12*x + 60*x^2
- 160*x^3 + 240*x^4 - 192*x^5 + 64*x^6)/(55296*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1
 + 2*x])^12) + (5*(-1 + 10*x - 40*x^2 + 80*x^3 - 80*x^4 + 32*x^5))/(18432*S
qrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^10) + (91*(1 - 8*x + 24*x^2 - 32*x^3 +
16*x^4))/(73728*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^8) + (35*(-1 + 6*x - 1
2*x^2 + 8*x^3))/(36864*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^6) + (91*(1 - 4
*x + 4*x^2))/(294912*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^4) + (5*(-1 + 2*x
))/(294912*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^2)))/((1 - 2*x)^(3/2)*(-1 -
 2*x + Sqrt[2]*Sqrt[1 + 2*x])^3)
```

fricas [A] time = 1.31, size = 39, normalized size = 0.68

$$-\frac{\left(8\,x^{3}-3\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{648\,\left(16\,x^{4}-8\,x^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/648*(8*x^3 - 3*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(16*x^4 - 8*x^2 + 1)
```

giac [B] time = 1.02, size = 128, normalized size = 2.25

$$-\frac{1}{82944}\,\sqrt{6}\left(\frac{\left(\sqrt{-4\,x+2}-2\right)^{3}}{\left(4\,x+2\right)^{\frac{3}{2}}}+\frac{33\,\left(\sqrt{-4\,x+2}-2\right)}{\sqrt{4\,x+2}}\right)-\frac{\left(4\,\sqrt{6}\,\left(2\,x+1\right)-9\,\sqrt{6}\right)\sqrt{4\,x+2}\,\sqrt{-4\,x+2}}{10368\,\left(2\,x-1\right)^{2}}+\frac{\sqrt{6}\,\left(4\,x+2\right)^{\frac{3}{2}}\left(\frac{33\,\left(\sqrt{-4\,x+2}-2\right)^{2}}{2\,x+1}+2\right)}{165888\,\left(\sqrt{-4\,x+2}-2\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/82944*sqrt(6)*((sqrt(-4*x + 2) - 2)^3/(4*x + 2)^(3/2) + 33*(sqrt(-4*x +
2) - 2)/sqrt(4*x + 2)) - 1/10368*(4*sqrt(6)*(2*x + 1) - 9*sqrt(6))*sqrt(4*x
 + 2)*sqrt(-4*x + 2)/(2*x - 1)^2 + 1/165888*sqrt(6)*(4*x + 2)^(3/2)*(33*(sq
rt(-4*x + 2) - 2)^2/(2*x + 1) + 2)/(sqrt(-4*x + 2) - 2)^3
```

maple [A] time = 0.00, size = 35, normalized size = 0.61

$$\frac{(2x-1)\,(2x+1)\left(8x^{2}-3\right)x}{3\,(-6x+3)^{\frac{5}{2}}\,(4x+2)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(5/2)/(4*x+2)^(5/2),x)
```

```
[Out] 1/3*(2*x-1)*(2*x+1)*x*(8*x^2-3)/(-6*x+3)^(5/2)/(4*x+2)^(5/2)
```

maxima [A] time = 1.28, size = 25, normalized size = 0.44

$$\frac{x}{54\,\sqrt{-24\,x^{2}+6}}+\frac{x}{18\,\left(-24\,x^{2}+6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(5/2)/(4*x+2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/54*x/sqrt(-24*x^2 + 6) + 1/18*x/(-24*x^2 + 6)^(3/2)
```

mupad [**B**] time = 0.31, size = 49, normalized size = 0.86

$$-\frac{3\,x\,\sqrt{3-6\,x}-8\,x^3\,\sqrt{3-6\,x}}{\sqrt{4\,x+2}\,\left(-2592\,x^3+1296\,x^2+648\,x-324\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(5/2)*(3 - 6*x)^(5/2)),x)
```

```
[Out] -(3*x*(3 - 6*x)^(1/2) - 8*x^3*(3 - 6*x)^(1/2))/((4*x + 2)^(1/2)*(648*x + 12
96*x^2 - 2592*x^3 - 324))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(5/2)/(4*x+2)**(5/2),x)
```

```
[Out] Timed out
```

3.1090 $\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx$

Optimal. Leaf size=85

$$\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(2x+1)^{5/2}}$$

Rubi [A] time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {40, 39}

$$\frac{x}{405\sqrt{6}\,\sqrt{1-2x}\,\sqrt{2x+1}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(2x+1)^{3/2}}+\frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] x/(1080*Sqrt[6]*(1 - 2*x)^(5/2)*(1 + 2*x)^(5/2)) + x/(810*Sqrt[6]*(1 - 2*x)
^(3/2)*(1 + 2*x)^(3/2)) + x/(405*Sqrt[6]*Sqrt[1 - 2*x]*Sqrt[1 + 2*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-6x)^{7/2}(2+4x)^{7/2}}\,dx &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{2}{15}\int \frac{1}{(3-6x)^{5/2}(2+4x)^{5/2}}\,dx\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{2}{135}\int \frac{}{(3-6x)}\\ &= \frac{x}{1080\sqrt{6}\,(1-2x)^{5/2}(1+2x)^{5/2}}+\frac{x}{810\sqrt{6}\,(1-2x)^{3/2}(1+2x)^{3/2}}+\frac{x}{405\sqrt{6}\,\sqrt{1-2x}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.49

$$\frac{x\left(128x^4-80x^2+15\right)}{3240\sqrt{6-12x}\,(1-2x)^2(2x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] (x*(15 - 80*x^2 + 128*x^4))/(3240*Sqrt[6 - 12*x]*(1 - 2*x)^2*(1 + 2*x)^(5/2
))
```

IntegrateAlgebraic [B] time = 1.25, size = 616, normalized size = 7.25

$$\frac{\left(\sqrt{2}\sqrt{2x+1}-2\right)^{15}\left(-\frac{347(4x^2-4x+1)}{2717908992\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^4}-\frac{539(8x^3-12x^2+6x-1)}{169869312\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^6}-\frac{2101(16x^4-32x^3+24x^2-8x+1)}{113246208\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^8}-\frac{7469(32x^5-80x^4+80x^3-40x^2+10x-1)}{141557760\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{10}}-\frac{2101(64x^6-192x^5+240x^4-160x^3+60x^2-12x+1)}{28311552\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{12}}-\frac{539(128x^7-448x^6+672x^5-560x^4+280x^3-84x^2+14x-1)}{10616832\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{14}}-\frac{347(256x^8-1024x^7+1792x^6-1792x^5+1120x^4-448x^3+112x^2-16x+1)}{42467328\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{16}}+\frac{7(512x^9-2304x^8+4608x^7-5376x^6+4032x^5-2016x^4+672x^3-144x^2+18x-1)}{10616832\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{18}}-\frac{1024x^{10}-5120x^9+11520x^8-15360x^7+13440x^6-8064x^5+3360x^4-960x^3+180x^2-20x+1}{17694720\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^{20}}+\frac{7(2x-1)}{2717908992\sqrt{3}\left(\sqrt{2}\sqrt{2x+1}-2\right)^2}-\frac{1}{18119393280\sqrt{3}}\right)}{(1-2x)^{5/2}\left(-2x+\sqrt{2}\sqrt{2x+1}-1\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - 6*x)^(7/2)*(2 + 4*x)^(7/2)),x]
```

```
[Out] ((-2 + Sqrt[2]*Sqrt[1 + 2*x])^15*(-1/18119393280*1/Sqrt[3] - (1 - 20*x + 18
0*x^2 - 960*x^3 + 3360*x^4 - 8064*x^5 + 13440*x^6 - 15360*x^7 + 11520*x^8 -
 5120*x^9 + 1024*x^10)/(17694720*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^20) +
 (7*(-1 + 18*x - 144*x^2 + 672*x^3 - 2016*x^4 + 4032*x^5 - 5376*x^6 + 4608*
x^7 - 2304*x^8 + 512*x^9))/(10616832*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^1
8) - (347*(1 - 16*x + 112*x^2 - 448*x^3 + 1120*x^4 - 1792*x^5 + 1792*x^6 -
1024*x^7 + 256*x^8))/(42467328*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^16) - (
539*(-1 + 14*x - 84*x^2 + 280*x^3 - 560*x^4 + 672*x^5 - 448*x^6 + 128*x^7))
/(10616832*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^14) - (2101*(1 - 12*x + 60*
x^2 - 160*x^3 + 240*x^4 - 192*x^5 + 64*x^6))/(28311552*Sqrt[3]*(-2 + Sqrt[2
]*Sqrt[1 + 2*x])^12) - (7469*(-1 + 10*x - 40*x^2 + 80*x^3 - 80*x^4 + 32*x^5
))/(141557760*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^10) - (2101*(1 - 8*x + 2
4*x^2 - 32*x^3 + 16*x^4))/(113246208*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1 + 2*x])^8
) - (539*(-1 + 6*x - 12*x^2 + 8*x^3))/(169869312*Sqrt[3]*(-2 + Sqrt[2]*Sqrt
[1 + 2*x])^6) - (347*(1 - 4*x + 4*x^2))/(2717908992*Sqrt[3]*(-2 + Sqrt[2]*S
qrt[1 + 2*x])^4) + (7*(-1 + 2*x))/(2717908992*Sqrt[3]*(-2 + Sqrt[2]*Sqrt[1
+ 2*x])^2)))/((1 - 2*x)^(5/2)*(-1 - 2*x + Sqrt[2]*Sqrt[1 + 2*x])^5)
```

fricas [A] time = 1.26, size = 49, normalized size = 0.58

$$-\frac{\left(128\,x^{5}-80\,x^{3}+15\,x\right)\sqrt{4\,x+2}\,\sqrt{-6\,x+3}}{19440\,\left(64\,x^{6}-48\,x^{4}+12\,x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/19440*(128*x^5 - 80*x^3 + 15*x)*sqrt(4*x + 2)*sqrt(-6*x + 3)/(64*x^6 - 4
8*x^4 + 12*x^2 - 1)
```

giac [B] time = 1.02, size = 181, normalized size = 2.13

$$-\frac{1}{39813120}\sqrt{6}\left(\frac{3\left(\sqrt{-4\,x+2}-2\right)^{5}}{(4\,x+2)^{\frac{5}{2}}}+\frac{85\left(\sqrt{-4\,x+2}-2\right)^{3}}{(4\,x+2)^{\frac{3}{2}}}+\frac{2130\left(\sqrt{-4\,x+2}-2\right)}{\sqrt{4\,x+2}}\right)-\frac{\left((64\,\sqrt{6}\,(2\,x+1)-275\,\sqrt{6})(2\,x+1)+300\,\sqrt{6}\right)\sqrt{4\,x+2}\,\sqrt{-4\,x+2}}{1244160\,(2\,x-1)^{3}}+\frac{\sqrt{6}\left(\frac{1065\left(\sqrt{-4\,x+2}-2\right)^{4}}{(2\,x+1)^{2}}+\frac{85\left(\sqrt{-4\,x+2}-2\right)^{2}}{2\,x+1}+6\right)(4\,x+2)^{\frac{5}{2}}}{79626240\left(\sqrt{-4\,x+2}-2\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/39813120*sqrt(6)*(3*(sqrt(-4*x + 2) - 2)^5/(4*x + 2)^(5/2) + 85*(sqrt(-4
*x + 2) - 2)^3/(4*x + 2)^(3/2) + 2130*(sqrt(-4*x + 2) - 2)/sqrt(4*x + 2)) -
 1/1244160*((64*sqrt(6)*(2*x + 1) - 275*sqrt(6))*(2*x + 1) + 300*sqrt(6))*s
qrt(4*x + 2)*sqrt(-4*x + 2)/(2*x - 1)^3 + 1/79626240*sqrt(6)*(1065*(sqrt(-4
*x + 2) - 2)^4/(2*x + 1)^2 + 85*(sqrt(-4*x + 2) - 2)^2/(2*x + 1) + 6)*(4*x
+ 2)^(5/2)/(sqrt(-4*x + 2) - 2)^5
```

maple [A] time = 0.00, size = 40, normalized size = 0.47

$$-\frac{(2x-1)\,(2x+1)\left(128x^{4}-80x^{2}+15\right)x}{15\,(-6x+3)^{\frac{7}{2}}\,(4x+2)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-6*x+3)^(7/2)/(4*x+2)^(7/2),x)
```

```
[Out] -1/15*(2*x-1)*(2*x+1)*x*(128*x^4-80*x^2+15)/(-6*x+3)^(7/2)/(4*x+2)^(7/2)
```

maxima [A] time = 1.31, size = 37, normalized size = 0.44

$$\frac{x}{405\sqrt{-24\,x^2+6}}+\frac{x}{135\left(-24\,x^2+6\right)^{\frac{3}{2}}}+\frac{x}{30\left(-24\,x^2+6\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)^(7/2)/(4*x+2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/405*x/sqrt(-24*x^2 + 6) + 1/135*x/(-24*x^2 + 6)^(3/2) + 1/30*x/(-24*x^2 +
 6)^(5/2)
```

mupad [B] time = 0.45, size = 66, normalized size = 0.78

$$-\frac{15\,x\,\sqrt{3-6\,x}\,-80\,x^3\,\sqrt{3-6\,x}\,+128\,x^5\,\sqrt{3-6\,x}}{\left(\left(6\,x-3\right)\,\left(240\,x+360\right)+1440\right)\,\sqrt{4\,x+2}\,\left(6\,x-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((4*x + 2)^(7/2)*(3 - 6*x)^(7/2)),x)
```

```
[Out] -(15*x*(3 - 6*x)^(1/2) - 80*x^3*(3 - 6*x)^(1/2) + 128*x^5*(3 - 6*x)^(1/2))/
(((6*x - 3)*(240*x + 360) + 1440)*(4*x + 2)^(1/2)*(6*x - 3)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-6*x)**(7/2)/(4*x+2)**(7/2),x)
```

```
[Out] Timed out
```

3.1091 $\int (3-x)^{3/2}(-2+x)^{3/2}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

Rubi [A] time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{4}(x-2)^{3/2}(3-x)^{5/2}-\frac{1}{8}\sqrt{x-2}\,(3-x)^{5/2}+\frac{1}{32}\sqrt{x-2}\,(3-x)^{3/2}+\frac{3}{64}\sqrt{x-2}\,\sqrt{3-x}-\frac{3}{128}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] (3*Sqrt[3 - x]*Sqrt[-2 + x])/64 + ((3 - x)^(3/2)*Sqrt[-2 + x])/32 - ((3 - x
)^(5/2)*Sqrt[-2 + x])/8 - ((3 - x)^(5/2)*(-2 + x)^(3/2))/4 - (3*ArcSin[5 -
2*x])/128
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int (3-x)^{3/2}(-2+x)^{3/2}\,dx &= -\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{8}\int (3-x)^{3/2}\sqrt{-2+x}\,dx \\
&= -\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{1}{16}\int \frac{(3-x)^{3/2}}{\sqrt{-2+x}}\,dx \\
&= \frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-2+x)^{3/2}+\frac{3}{64}\int \frac{\sqrt{3}}{\sqrt{-}} \\
&= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(- \\
&= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(- \\
&= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(- \\
&= \frac{3}{64}\sqrt{3-x}\sqrt{-2+x}+\frac{1}{32}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}(3-x)^{5/2}\sqrt{-2+x}-\frac{1}{4}(3-x)^{5/2}(-
\end{aligned}
$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.88

$$\frac{\sqrt{-x^2+5x-6}\left(\sqrt{x-2}\left(-16x^4+168x^3-650x^2+1095x-675\right)+3\sqrt{3-x}\sin^{-1}\left(\sqrt{3-x}\right)\right)}{64(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(-675 + 1095*x - 650*x^2 + 168*x^3 - 16
*x^4) + 3*Sqrt[3 - x]*ArcSin[Sqrt[3 - x]]))/(64*(-3 + x)*Sqrt[-2 + x])
```

IntegrateAlgebraic [A] time = 0.09, size = 115, normalized size = 1.26

$$\frac{-\frac{3(3-x)^{7/2}}{(x-2)^{7/2}}-\frac{11(3-x)^{5/2}}{(x-2)^{5/2}}+\frac{11(3-x)^{3/2}}{(x-2)^{3/2}}+\frac{3\sqrt{3-x}}{\sqrt{x-2}}}{64\left(\frac{3-x}{x-2}+1\right)^4}-\frac{3}{64}\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - x)^(3/2)*(-2 + x)^(3/2),x]
```

```
[Out] ((-3*(3 - x)^(7/2))/(-2 + x)^(7/2) - (11*(3 - x)^(5/2))/(-2 + x)^(5/2) + (1
1*(3 - x)^(3/2))/(-2 + x)^(3/2) + (3*Sqrt[3 - x])/Sqrt[-2 + x])/(64*(1 + (3
 - x)/(-2 + x))^4) - (3*ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]])/64
```

fricas [A] time = 1.10, size = 62, normalized size = 0.68

$$-\frac{1}{64}\left(16\,x^3-120\,x^2+290\,x-225\right)\sqrt{x-2}\,\sqrt{-x+3}-\frac{3}{128}\,\arctan\left(\frac{(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^2-5\,x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 120*x^2 + 290*x - 225)*sqrt(x - 2)*sqrt(-x + 3) - 3/128*arc
tan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

giac [A] time = 0.88, size = 101, normalized size = 1.11

$$-\frac{1}{192}\,(2\,(4\,(6\,x+35)(x-2)+523)(x-2)+801)\sqrt{x-2}\,\sqrt{-x+3}+\frac{7}{24}\,(2\,(4\,x+15)(x-2)+69)\sqrt{x-2}\,\sqrt{-x+3}-4\,(2\,x+3)\sqrt{x-2}\,\sqrt{-x+3}+12\,\sqrt{x-2}\,\sqrt{-x+3}+\frac{3}{64}\,\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/192*(2*(4*(6*x + 35)*(x - 2) + 523)*(x - 2) + 801)*sqrt(x - 2)*sqrt(-x +
 3) + 7/24*(2*(4*x + 15)*(x - 2) + 69)*sqrt(x - 2)*sqrt(-x + 3) - 4*(2*x +
3)*sqrt(x - 2)*sqrt(-x + 3) + 12*sqrt(x - 2)*sqrt(-x + 3) + 3/64*arcsin(sqr
t(x - 2))
```

maple [A] time = 0.01, size = 89, normalized size = 0.98

$$\frac{3\sqrt{(x-2)(-x+3)}\ \arcsin(2x-5)}{128\sqrt{x-2}\ \sqrt{-x+3}}+\frac{(-x+3)^{\frac{3}{2}}(x-2)^{\frac{5}{2}}}{4}+\frac{\sqrt{-x+3}\ (x-2)^{\frac{5}{2}}}{8}-\frac{\sqrt{-x+3}\ (x-2)^{\frac{3}{2}}}{32}-\frac{3\sqrt{-x+3}\ \sqrt{x-2}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-x)^(3/2)*(x-2)^(3/2),x)
```

```
[Out] 1/4*(3-x)^(3/2)*(x-2)^(5/2)+1/8*(3-x)^(1/2)*(x-2)^(5/2)-1/32*(3-x)^(1/2)*(x
-2)^(3/2)-3/64*(3-x)^(1/2)*(x-2)^(1/2)+3/128*((x-2)*(3-x))^(1/2)/(x-2)^(1/2
)/(3-x)^(1/2)*arcsin(-5+2*x)
```

maxima [A] time = 2.97, size = 67, normalized size = 0.74

$$\frac{1}{4}\left(-x^2+5x-6\right)^{\frac{3}{2}}x-\frac{5}{8}\left(-x^2+5x-6\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^2+5x-6}\,x-\frac{15}{64}\sqrt{-x^2+5x-6}+\frac{3}{128}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(3/2)*(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + 5*x - 6)^(3/2)*x - 5/8*(-x^2 + 5*x - 6)^(3/2) + 3/32*sqrt(-x^2
+ 5*x - 6)*x - 15/64*sqrt(-x^2 + 5*x - 6) + 3/128*arcsin(2*x - 5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x-2)^{3/2}\,(3-x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(3/2)*(3 - x)^(3/2),x)
```

```
[Out] int((x - 2)^(3/2)*(3 - x)^(3/2), x)
```

sympy [A] time = 7.48, size = 199, normalized size = 2.19

$$\begin{cases} -\frac{3i\,\mathrm{acosh}\left(\sqrt{x-2}\right)}{64}-\frac{i(x-2)^{\frac{9}{2}}}{4\sqrt{x-3}}+\frac{5i(x-2)^{\frac{7}{2}}}{8\sqrt{x-3}}-\frac{13i(x-2)^{\frac{5}{2}}}{32\sqrt{x-3}}-\frac{i(x-2)^{\frac{3}{2}}}{64\sqrt{x-3}}+\frac{3i\sqrt{x-2}}{64\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{3\,\mathrm{asin}\left(\sqrt{x-2}\right)}{64}+\frac{(x-2)^{\frac{9}{2}}}{4\sqrt{3-x}}-\frac{5(x-2)^{\frac{7}{2}}}{8\sqrt{3-x}}+\frac{13(x-2)^{\frac{5}{2}}}{32\sqrt{3-x}}+\frac{(x-2)^{\frac{3}{2}}}{64\sqrt{3-x}}-\frac{3\sqrt{x-2}}{64\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(3/2)*(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-3*I*acosh(sqrt(x - 2))/64 - I*(x - 2)**(9/2)/(4*sqrt(x - 3)) +
5*I*(x - 2)**(7/2)/(8*sqrt(x - 3)) - 13*I*(x - 2)**(5/2)/(32*sqrt(x - 3)) -
 I*(x - 2)**(3/2)/(64*sqrt(x - 3)) + 3*I*sqrt(x - 2)/(64*sqrt(x - 3)), Abs(
x - 2) > 1), (3*asin(sqrt(x - 2))/64 + (x - 2)**(9/2)/(4*sqrt(3 - x)) - 5*(
x - 2)**(7/2)/(8*sqrt(3 - x)) + 13*(x - 2)**(5/2)/(32*sqrt(3 - x)) + (x - 2
)**(3/2)/(64*sqrt(3 - x)) - 3*sqrt(x - 2)/(64*sqrt(3 - x)), True))
```

3.1092 $\int \sqrt{3-x}\sqrt{-2+x}\,dx$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

Rubi [A] time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {50, 53, 619, 216}

$$-\frac{1}{2}\sqrt{x-2}\,(3-x)^{3/2}+\frac{1}{4}\sqrt{x-2}\,\sqrt{3-x}-\frac{1}{8}\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x]*Sqrt[-2 + x],x]
```

```
[Out] (Sqrt[3 - x]*Sqrt[-2 + x])/4 - ((3 - x)^(3/2)*Sqrt[-2 + x])/2 - ArcSin[5 -
2*x]/8
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x}\sqrt{-2+x}\,dx &= -\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{4}\int \frac{\sqrt{3-x}}{\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}+\frac{1}{8}\int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\ &= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx,x,5-2x\right)\\ &= \frac{1}{4}\sqrt{3-x}\sqrt{-2+x}-\frac{1}{2}(3-x)^{3/2}\sqrt{-2+x}-\frac{1}{8}\sin^{-1}(5-2x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.35

$$\frac{\sqrt{-x^2+5x-6}\left(\sqrt{x-2}\left(2x^2-11x+15\right)+\sqrt{3-x}\,\sin^{-1}\left(\sqrt{3-x}\right)\right)}{4(x-3)\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x]*Sqrt[-2 + x],x]
[Out] (Sqrt[-6 + 5*x - x^2]*(Sqrt[-2 + x]*(15 - 11*x + 2*x^2) + Sqrt[3 - x]*ArcSi
n[Sqrt[3 - x]]))/(4*(-3 + x)*Sqrt[-2 + x])
```

IntegrateAlgebraic [A] time = 0.06, size = 78, normalized size = 1.53

$$\frac{\frac{\sqrt{3-x}}{\sqrt{x-2}}-\frac{(3-x)^{3/2}}{(x-2)^{3/2}}}{4\left(\frac{3-x}{x-2}+1\right)^2}-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[3 - x]*Sqrt[-2 + x],x]
[Out] (-((3 - x)^(3/2)/(-2 + x)^(3/2)) + Sqrt[3 - x]/Sqrt[-2 + x])/(4*(1 + (3 - x
)/(-2 + x))^2) - ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]]/4
```

fricas [A] time = 1.29, size = 52, normalized size = 1.02

$$\frac{1}{4}\,(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}\,-\frac{1}{8}\,\arctan\left(\frac{(2\,x-5)\sqrt{x-2}\,\sqrt{-x+3}}{2\left(x^2-5\,x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="fricas")
[Out] 1/4*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3) - 1/8*arctan(1/2*(2*x - 5)*sqrt(x -
2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

giac [A] time = 1.02, size = 42, normalized size = 0.82

$$\frac{1}{4}\,(2\,x+3)\sqrt{x-2}\,\sqrt{-x+3}\,-2\,\sqrt{x-2}\,\sqrt{-x+3}\,+\frac{1}{4}\,\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 3)*sqrt(x - 2)*sqrt(-x + 3) - 2*sqrt(x - 2)*sqrt(-x + 3) + 1/4*a
rcsin(sqrt(x - 2))
```

maple [A] time = 0.01, size = 61, normalized size = 1.20

$$\frac{\sqrt{(x-2)(-x+3)}\ \arcsin(2x-5)}{8\sqrt{x-2}\ \sqrt{-x+3}} - \frac{(-x+3)^{\frac{3}{2}}\sqrt{x-2}}{2} + \frac{\sqrt{-x+3}\ \sqrt{x-2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+3)^(1/2)*(x-2)^(1/2),x)
```

```
[Out] -1/2*(-x+3)^(3/2)*(x-2)^(1/2)+1/4*(-x+3)^(1/2)*(x-2)^(1/2)+1/8*((x-2)*(-x+3
))^(1/2)/(x-2)^(1/2)/(-x+3)^(1/2)*arcsin(2*x-5)
```

maxima [A] time = 2.96, size = 38, normalized size = 0.75

$$\frac{1}{2}\sqrt{-x^2+5x-6}\,x - \frac{5}{4}\sqrt{-x^2+5x-6} + \frac{1}{8}\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)^(1/2)*(-2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 5*x - 6)*x - 5/4*sqrt(-x^2 + 5*x - 6) + 1/8*arcsin(2*x - 5)
```

mupad [B] time = 0.21, size = 41, normalized size = 0.80

$$\left(\frac{x}{2}-\frac{5}{4}\right)\sqrt{x-2}\ \sqrt{3-x} - \frac{\ln\left(x-\frac{5}{2}-\sqrt{x-2}\ \sqrt{3-x}\ 1\mathrm{i}\right)\,1\mathrm{i}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 2)^(1/2)*(3 - x)^(1/2),x)
```

```
[Out] (x/2 - 5/4)*(x - 2)^(1/2)*(3 - x)^(1/2) - (log(x - (x - 2)^(1/2)*(3 - x)^(1
/2)*1i - 5/2)*1i)/8
```

sympy [A] time = 3.01, size = 124, normalized size = 2.43

$$\begin{cases} -\frac{i\,\operatorname{acosh}\left(\sqrt{x-2}\right)}{4} + \frac{i(x-2)^{\frac{5}{2}}}{2\sqrt{x-3}} - \frac{3i(x-2)^{\frac{3}{2}}}{4\sqrt{x-3}} + \frac{i\sqrt{x-2}}{4\sqrt{x-3}} & \text{for } |x-2|>1 \\ \frac{\operatorname{asin}\left(\sqrt{x-2}\right)}{4} - \frac{(x-2)^{\frac{5}{2}}}{2\sqrt{3-x}} + \frac{3(x-2)^{\frac{3}{2}}}{4\sqrt{3-x}} - \frac{\sqrt{x-2}}{4\sqrt{3-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-x)**(1/2)*(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(sqrt(x - 2))/4 + I*(x - 2)**(5/2)/(2*sqrt(x - 3)) - 3*I
*(x - 2)**(3/2)/(4*sqrt(x - 3)) + I*sqrt(x - 2)/(4*sqrt(x - 3)), Abs(x - 2)
 > 1), (asin(sqrt(x - 2))/4 - (x - 2)**(5/2)/(2*sqrt(3 - x)) + 3*(x - 2)**(
3/2)/(4*sqrt(3 - x)) - sqrt(x - 2)/(4*sqrt(3 - x)), True))
```

3.1093 $\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(5-2x)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {53, 619, 216}

$$-\sin^{-1}(5-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] -ArcSin[5 - 2*x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-x}\sqrt{-2+x}}\,dx &= \int \frac{1}{\sqrt{-6+5x-x^2}}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, 5-2x\right)\\
&= -\sin^{-1}(5-2x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-2\sin^{-1}\left(\sqrt{3-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
```

```
[Out] -2*ArcSin[Sqrt[3 - x]]
```

IntegrateAlgebraic [B] time = 0.04, size = 20, normalized size = 2.50

$$-2\tan^{-1}\left(\frac{\sqrt{3-x}}{\sqrt{x-2}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - x]*Sqrt[-2 + x]),x]
[Out] -2*ArcTan[Sqrt[3 - x]/Sqrt[-2 + x]]
```

fricas [B] time = 1.35, size = 32, normalized size = 4.00

$$-\arctan\left(\frac{(2x-5)\sqrt{x-2}\sqrt{-x+3}}{2\left(x^2-5x+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6))
```

giac [A] time = 1.02, size = 8, normalized size = 1.00

$$2\arcsin\left(\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(sqrt(x - 2))
```

maple [B] time = 0.00, size = 31, normalized size = 3.88

$$\frac{\sqrt{(x-2)(-x+3)}\,\arcsin(2x-5)}{\sqrt{x-2}\,\sqrt{-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(1/2)/(x-2)^(1/2),x)
[Out] ((x-2)*(-x+3))^(1/2)/(x-2)^(1/2)/(-x+3)^(1/2)*arcsin(2*x-5)
```

maxima [A] time = 3.00, size = 6, normalized size = 0.75

$$\arcsin(2x-5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(1/2)/(-2+x)^(1/2),x, algorithm="maxima")
[Out] arcsin(2*x - 5)
```

mupad [B] time = 0.18, size = 31, normalized size = 3.88

$$-4\,\mathrm{atan}\left(\frac{\sqrt{x-2}-\sqrt{2}\,1\mathrm{i}}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(1/2)*(3 - x)^(1/2)),x)
[Out] -4*atan(((x - 2)^(1/2) - 2^(1/2)*1i)/(3^(1/2) - (3 - x)^(1/2)))
```

sympy [A] time = 1.61, size = 26, normalized size = 3.25

$$\begin{cases} -2i\,\mathrm{acosh}\left(\sqrt{x-2}\right) & \text{for } |x-2|>1 \\ 2\,\mathrm{asin}\left(\sqrt{x-2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(-2+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x - 2)), Abs(x - 2) > 1), (2*asin(sqrt(x - 2)),
True))
```

3.1094 $\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Rubi [A] time = 0.00, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$\frac{2}{\sqrt{3-x}\,\sqrt{x-2}}-\frac{4\sqrt{3-x}}{\sqrt{x-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] 2/(Sqrt[3 - x]*Sqrt[-2 + x]) - (4*Sqrt[3 - x])/Sqrt[-2 + x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x)^{3/2}(-2+x)^{3/2}}\,dx&=\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}+2\int \frac{1}{\sqrt{3-x}\,(-2+x)^{3/2}}\,dx\\&=\frac{2}{\sqrt{3-x}\,\sqrt{-2+x}}-\frac{4\sqrt{3-x}}{\sqrt{-2+x}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.57

$$\frac{2(2x-5)}{\sqrt{-x^2+5x-6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] (2*(-5 + 2*x))/Sqrt[-6 + 5*x - x^2]
```

IntegrateAlgebraic [A] time = 0.05, size = 31, normalized size = 0.84

$$-\frac{2\left(\frac{3-x}{x-2}-1\right)\sqrt{x-2}}{\sqrt{3-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(3/2)*(-2 + x)^(3/2)),x]
```

```
[Out] (-2*(-1 + (3 - x)/(-2 + x))*Sqrt[-2 + x])/Sqrt[3 - x]
```

fricas [A] time = 1.27, size = 29, normalized size = 0.78

$$-\frac{2 \, {\left(2 \, x - 5\right)} \sqrt{x - 2} \sqrt{-x + 3}}{x^{2} - 5 \, x + 6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*x - 5)*sqrt(x - 2)*sqrt(-x + 3)/(x^2 - 5*x + 6)
```

giac [A] time = 0.85, size = 53, normalized size = 1.43

$$-\frac{\sqrt{-x + 3} - 1}{\sqrt{x - 2}} - \frac{2 \, \sqrt{x - 2} \sqrt{-x + 3}}{x - 3} + \frac{\sqrt{x - 2}}{\sqrt{-x + 3} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-x + 3) - 1)/sqrt(x - 2) - 2*sqrt(x - 2)*sqrt(-x + 3)/(x - 3) + sqrt
(x - 2)/(sqrt(-x + 3) - 1)
```

maple [A] time = 0.00, size = 20, normalized size = 0.54

$$\frac{-10+4 x}{\sqrt{-x +3}\, \sqrt{x -2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(3/2)/(x-2)^(3/2),x)
```

```
[Out] 2*(2*x-5)/(x-2)^(1/2)/(-x+3)^(1/2)
```

maxima [A] time = 1.32, size = 30, normalized size = 0.81

$$\frac{4 \, x}{\sqrt{-x^{2} + 5 \, x - 6}} - \frac{10}{\sqrt{-x^{2} + 5 \, x - 6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(-2+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 4*x/sqrt(-x^2 + 5*x - 6) - 10/sqrt(-x^2 + 5*x - 6)
```

mupad [B] time = 0.25, size = 32, normalized size = 0.86

$$-\frac{4\,x\,\sqrt{3-x}-10\,\sqrt{3-x}}{\sqrt{x-2}\,\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(3/2)*(3 - x)^(3/2)),x)
```

```
[Out] -(4*x*(3 - x)^(1/2) - 10*(3 - x)^(1/2))/((x - 2)^(1/2)*(x - 3))
```

sympy [A] time = 2.30, size = 100, normalized size = 2.70

$$\begin{cases} -\frac{4i\sqrt{x-3}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} + \frac{2i\sqrt{x-3}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{for } |x-2|>1 \\ -\frac{4\sqrt{3-x}\,(x-2)}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} + \frac{2\sqrt{3-x}}{(x-2)^{\frac{3}{2}}-\sqrt{x-2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(-2+x)**(3/2),x)
```

```
[Out] Piecewise((-4*I*sqrt(x - 3)*(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*I*sq
rt(x - 3)/((x - 2)**(3/2) - sqrt(x - 2)), Abs(x - 2) > 1), (-4*sqrt(3 - x)*
(x - 2)/((x - 2)**(3/2) - sqrt(x - 2)) + 2*sqrt(3 - x)/((x - 2)**(3/2) - sq
rt(x - 2)), True))
```

3.1095 $\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {45, 37}

$$-\frac{32\sqrt{3-x}}{3\sqrt{x-2}}-\frac{16\sqrt{3-x}}{3(x-2)^{3/2}}+\frac{4}{(x-2)^{3/2}\sqrt{3-x}}+\frac{2}{3(x-2)^{3/2}(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] 2/(3*(3 - x)^(3/2)*(-2 + x)^(3/2)) + 4/(Sqrt[3 - x]*(-2 + x)^(3/2)) - (16*S
qrt[3 - x])/(3*(-2 + x)^(3/2)) - (32*Sqrt[3 - x])/(3*Sqrt[-2 + x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(3-x)^{5/2}(-2+x)^{5/2}}\,dx &= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+2\int \frac{1}{(3-x)^{3/2}(-2+x)^{5/2}}\,dx\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}}+8\int \frac{1}{\sqrt{3-x}\,(-2+x)^{5/2}}\,dx\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}+\frac{16}{3}\int \frac{1}{\sqrt{3-x}\,(-2+x}\\
&= \frac{2}{3(3-x)^{3/2}(-2+x)^{3/2}}+\frac{4}{\sqrt{3-x}\,(-2+x)^{3/2}}-\frac{16\sqrt{3-x}}{3(-2+x)^{3/2}}-\frac{32\sqrt{3-x}}{3\sqrt{-2+x}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.42

$$\frac{-32x^3+240x^2-588x+470}{3\left(-x^2+5x-6\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (470 - 588*x + 240*x^2 - 32*x^3)/(3*(-6 + 5*x - x^2)^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 61, normalized size = 0.77

$$-\frac{2\left(\frac{(3-x)^3}{(x-2)^3}+\frac{9(3-x)^2}{(x-2)^2}-\frac{9(3-x)}{x-2}-1\right)(x-2)^{3/2}}{3(3-x)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(5/2)*(-2 + x)^(5/2)),x]
```

```
[Out] (-2*(-1 + (3 - x)^3/(-2 + x)^3 + (9*(3 - x)^2)/(-2 + x)^2 - (9*(3 - x))/(-2
 + x))*(-2 + x)^(3/2))/(3*(3 - x)^(3/2))
```

fricas [A] time = 1.22, size = 49, normalized size = 0.62

$$-\frac{2\left(16\,x^{3}-120\,x^{2}+294\,x-235\right)\sqrt{x-2}\,\sqrt{-x+3}}{3\left(x^{4}-10\,x^{3}+37\,x^{2}-60\,x+36\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(16*x^3 - 120*x^2 + 294*x - 235)*sqrt(x - 2)*sqrt(-x + 3)/(x^4 - 10*x^
3 + 37*x^2 - 60*x + 36)
```

giac [A] time = 1.10, size = 97, normalized size = 1.23

$$-\frac{\left(\sqrt{-x+3}-1\right)^{3}}{12\,(x-2)^{\frac{3}{2}}}-\frac{11\left(\sqrt{-x+3}-1\right)}{4\,\sqrt{x-2}}-\frac{2\,(8\,x-25)\sqrt{x-2}\,\sqrt{-x+3}}{3\,(x-3)^{2}}+\frac{(x-2)^{\frac{3}{2}}\left(\frac{33\left(\sqrt{-x+3}-1\right)^{2}}{x-2}+1\right)}{12\left(\sqrt{-x+3}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(sqrt(-x + 3) - 1)^3/(x - 2)^(3/2) - 11/4*(sqrt(-x + 3) - 1)/sqrt(x -
 2) - 2/3*(8*x - 25)*sqrt(x - 2)*sqrt(-x + 3)/(x - 3)^2 + 1/12*(x - 2)^(3/2
)*(33*(sqrt(-x + 3) - 1)^2/(x - 2) + 1)/(sqrt(-x + 3) - 1)^3
```

maple [A] time = 0.00, size = 30, normalized size = 0.38

$$-\frac{2\left(16x^{3}-120x^{2}+294x-235\right)}{3\,(x-2)^{\frac{3}{2}}\,(-x+3)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(5/2)/(x-2)^(5/2),x)
```

```
[Out] -2/3*(16*x^3-120*x^2+294*x-235)/(x-2)^(3/2)/(-x+3)^(3/2)
```

maxima [A] time = 1.34, size = 59, normalized size = 0.75

$$\frac{32\,x}{3\,\sqrt{-x^{2}+5\,x-6}}-\frac{80}{3\,\sqrt{-x^{2}+5\,x-6}}+\frac{4\,x}{3\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}}-\frac{10}{3\left(-x^{2}+5\,x-6\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(5/2)/(-2+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 32/3*x/sqrt(-x^2 + 5*x - 6) - 80/3/sqrt(-x^2 + 5*x - 6) + 4/3*x/(-x^2 + 5*x
 - 6)^(3/2) - 10/3/(-x^2 + 5*x - 6)^(3/2)
```

mupad [B] time = 0.37, size = 69, normalized size = 0.87

$$-\frac{32\,{\left(x-2\right)}^3\,\sqrt{3-x}-48\,{\left(x-2\right)}^2\,\sqrt{3-x}+2\,\sqrt{3-x}+12\,\left(x-2\right)\,\sqrt{3-x}}{\left(3\,x-6\right)\,\sqrt{x-2}\,{\left(x-3\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)^(5/2)*(3 - x)^(5/2)),x)
```

```
[Out] -(32*(x - 2)^3*(3 - x)^(1/2) - 48*(x - 2)^2*(3 - x)^(1/2) + 2*(3 - x)^(1/2)
 + 12*(x - 2)*(3 - x)^(1/2))/((3*x - 6)*(x - 2)^(1/2)*(x - 3)^2)
```

sympy [B] time = 9.85, size = 282, normalized size = 3.57

$$\begin{cases} - \frac{32 \sqrt{-1 + \frac{1}{x - 2}} \left(x - 2\right)^{3}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} + \frac{48 \sqrt{-1 + \frac{1}{x - 2}} \left(x - 2\right)^{2}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} - \frac{12 \sqrt{-1 + \frac{1}{x - 2}} \left(x - 2\right)}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} - \frac{2 \sqrt{-1 + \frac{1}{x - 2}}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} & \text{for}\: \frac{1}{\left|{x - 2}\right|} > 1 \\- \frac{32 i \sqrt{1 - \frac{1}{x - 2}} \left(x - 2\right)^{3}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} + \frac{48 i \sqrt{1 - \frac{1}{x - 2}} \left(x - 2\right)^{2}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} - \frac{12 i \sqrt{1 - \frac{1}{x - 2}} \left(x - 2\right)}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} - \frac{2 i \sqrt{1 - \frac{1}{x - 2}}}{3 x + 3 \left(x - 2\right)^{3} - 6 \left(x - 2\right)^{2} - 6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(5/2)/(-2+x)**(5/2),x)
```

```
[Out] Piecewise((-32*sqrt(-1 + 1/(x - 2))*(x - 2)**3/(3*x + 3*(x - 2)**3 - 6*(x -
 2)**2 - 6) + 48*sqrt(-1 + 1/(x - 2))*(x - 2)**2/(3*x + 3*(x - 2)**3 - 6*(x
 - 2)**2 - 6) - 12*sqrt(-1 + 1/(x - 2))*(x - 2)/(3*x + 3*(x - 2)**3 - 6*(x
- 2)**2 - 6) - 2*sqrt(-1 + 1/(x - 2))/(3*x + 3*(x - 2)**3 - 6*(x - 2)**2 -
6), 1/Abs(x - 2) > 1), (-32*I*sqrt(1 - 1/(x - 2))*(x - 2)**3/(3*x + 3*(x -
2)**3 - 6*(x - 2)**2 - 6) + 48*I*sqrt(1 - 1/(x - 2))*(x - 2)**2/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 12*I*sqrt(1 - 1/(x - 2))*(x - 2)/(3*x + 3*(x
 - 2)**3 - 6*(x - 2)**2 - 6) - 2*I*sqrt(1 - 1/(x - 2))/(3*x + 3*(x - 2)**3
- 6*(x - 2)**2 - 6), True))
```

3.1096 $\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}} \, dx$

Optimal. Leaf size=21

$$\frac{x}{9\sqrt{3-x}\sqrt{x+3}}$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{9\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-x)^{3/2}(3+x)^{3/2}} \, dx = \frac{x}{9\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{9-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - x^2])
```

IntegrateAlgebraic [A] time = 0.06, size = 34, normalized size = 1.62

$$\frac{\sqrt{x+3}\left(1-\frac{3-x}{x+3}\right)}{18\sqrt{3-x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] (Sqrt[3 + x]*(1 - (3 - x)/(3 + x)))/(18*Sqrt[3 - x])
```

fricas [A] time = 1.56, size = 22, normalized size = 1.05

$$-\frac{\sqrt{x+3}\,x\sqrt{-x+3}}{9\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(x + 3)*x*sqrt(-x + 3)/(x^2 - 9)
```

giac [B] time = 0.90, size = 62, normalized size = 2.95

$$\frac{\sqrt{6}-\sqrt{-x+3}}{36\sqrt{x+3}}-\frac{\sqrt{x+3}\sqrt{-x+3}}{18(x-3)}-\frac{\sqrt{x+3}}{36\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/18*sqrt(x + 3)*sqrt(-x + 3)/(
x - 3) - 1/36*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3))
```

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{x}{9\sqrt{-x+3}\,\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x+3)^(3/2)/(3+x)^(3/2),x)
```

```
[Out] 1/9*x/(-x+3)^(1/2)/(3+x)^(1/2)
```

maxima [A] time = 1.37, size = 12, normalized size = 0.57

$$\frac{x}{9\sqrt{-x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-x^2 + 9)
```

mupad [B] time = 0.36, size = 22, normalized size = 1.05

$$-\frac{x\sqrt{3-x}}{(9x-27)\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - x)^(1/2))/((9*x - 27)*(x + 3)^(1/2))
```

sympy [A] time = 1.80, size = 73, normalized size = 3.48

$$\begin{cases} \frac{1}{9\sqrt{-1+\frac{6}{x+3}}}-\frac{1}{3\sqrt{-1+\frac{6}{x+3}}\,(x+3)} & \text{for}\ \frac{6}{|x+3|}>1 \\ \frac{i\sqrt{1-\frac{6}{x+3}}\,(x+3)}{27-9x}-\frac{3i\sqrt{1-\frac{6}{x+3}}}{27-9x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((1/(9*sqrt(-1 + 6/(x + 3))) - 1/(3*sqrt(-1 + 6/(x + 3))*(x + 3)),
 6/Abs(x + 3) > 1), (I*sqrt(1 - 6/(x + 3))*(x + 3)/(27 - 9*x) - 3*I*sqrt(1
- 6/(x + 3))/(27 - 9*x), True))
```

3.1097 $\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}} \, dx$

Optimal. Leaf size=24

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{bx+3}}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{9\sqrt{3-bx}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(3-bx)^{3/2}(3+bx)^{3/2}} \, dx = \frac{x}{9\sqrt{3-bx}\,\sqrt{3+bx}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{9-b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(9*Sqrt[9 - b^2*x^2])
```

IntegrateAlgebraic [A] time = 0.08, size = 43, normalized size = 1.79

$$\frac{\sqrt{bx+3}\left(1-\frac{3-bx}{bx+3}\right)}{18b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((3 - b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] (Sqrt[3 + b*x]*(1 - (3 - b*x)/(3 + b*x)))/(18*b*Sqrt[3 - b*x])
```

fricas [A] time = 1.53, size = 29, normalized size = 1.21

$$-\frac{\sqrt{bx+3}\,\sqrt{-bx+3}\,x}{9\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(b*x + 3)*sqrt(-b*x + 3)*x/(b^2*x^2 - 9)
```

giac [B] time = 1.10, size = 82, normalized size = 3.42

$$\frac{\sqrt{6}-\sqrt{-bx+3}}{36\sqrt{bx+3}\,b}-\frac{\sqrt{bx+3}\sqrt{-bx+3}}{18\,(bx-3)b}-\frac{\sqrt{bx+3}}{36\,b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/36*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/18*sqrt(b*x + 3)*sqrt
(-b*x + 3)/((b*x - 3)*b) - 1/36*sqrt(b*x + 3)/(b*(sqrt(6) - sqrt(-b*x + 3))
)
```

maple [A] time = 0.00, size = 19, normalized size = 0.79

$$\frac{x}{9\sqrt{-bx+3}\,\sqrt{bx+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] 1/9*x/(-b*x+3)^(1/2)/(b*x+3)^(1/2)
```

maxima [A] time = 1.37, size = 15, normalized size = 0.62

$$\frac{x}{9\sqrt{-b^2x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*x/sqrt(-b^2*x^2 + 9)
```

mupad [B] time = 0.46, size = 26, normalized size = 1.08

$$-\frac{x\sqrt{3-b\,x}}{\sqrt{b\,x+3}\,\left(9\,b\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - b*x)^(3/2)*(b*x + 3)^(3/2)),x)
```

```
[Out] -(x*(3 - b*x)^(1/2))/((b*x + 3)^(1/2)*(9*b*x - 27))
```

sympy [C] time = 5.16, size = 73, normalized size = 3.04

$$-\frac{iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{18\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 9
/(b**2*x**2))/(18*pi**(3/2)*b) + meijerg(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()),
 ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*x**2))/(18*pi**(
3/2)*b)
```

3.1098 $\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

Rubi [A] time = 0.00, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - x]*Sqrt[3 + x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx = \frac{x}{18\sqrt{2}\sqrt{3-x}\sqrt{3+x}}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.81

$$\frac{x}{18\sqrt{6-2x}\sqrt{x+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[6 - 2*x]*Sqrt[3 + x])
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{(6-2x)^{3/2}(3+x)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 2*x)^(3/2)*(3 + x)^(3/2)), x]
```

fricas [A] time = 1.29, size = 22, normalized size = 0.85

$$-\frac{\sqrt{x+3}\,x\sqrt{-2\,x+6}}{36\left(x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(x + 3)*x*sqrt(-2*x + 6)/(x^2 - 9)
```

giac [B] time = 1.04, size = 71, normalized size = 2.73

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-x+3}\right)}{144\sqrt{x+3}}-\frac{\sqrt{2}\sqrt{x+3}\sqrt{-x+3}}{72\,(x-3)}-\frac{\sqrt{2}\sqrt{x+3}}{144\left(\sqrt{6}-\sqrt{-x+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-x + 3))/sqrt(x + 3) - 1/72*sqrt(2)*sqrt(x +
3)*sqrt(-x + 3)/(x - 3) - 1/144*sqrt(2)*sqrt(x + 3)/(sqrt(6) - sqrt(-x + 3)
)
```

maple [A] time = 0.00, size = 19, normalized size = 0.73

$$-\frac{(x-3)\,x}{9\sqrt{x+3}\;(-2x+6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(6-2*x)^(3/2)/(x+3)^(3/2),x)
```

```
[Out] -1/9*(-3+x)/(x+3)^(1/2)*x/(6-2*x)^(3/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.46

$$\frac{x}{18\sqrt{-2\,x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)^(3/2)/(3+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*x^2 + 18)
```

mupad [B] time = 0.37, size = 22, normalized size = 0.85

$$-\frac{x\sqrt{6-2\,x}}{(36\,x-108)\;\sqrt{x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((6 - 2*x)^(3/2)*(x + 3)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*x)^(1/2))/((36*x - 108)*(x + 3)^(1/2))
```

sympy [A] time = 20.45, size = 90, normalized size = 3.46

$$\begin{cases} \frac{\sqrt{2}}{36\sqrt{-1+\frac{6}{x+3}}}-\frac{\sqrt{2}}{12\sqrt{-1+\frac{6}{x+3}}\,(x+3)} & \text{for}\ \frac{6}{|x+3|}>1 \\ \frac{\sqrt{2}\,i\sqrt{1-\frac{6}{x+3}}\,(x+3)}{108-36x}-\frac{3\sqrt{2}\,i\sqrt{1-\frac{6}{x+3}}}{108-36x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(6-2*x)**(3/2)/(3+x)**(3/2),x)
```

```
[Out] Piecewise((sqrt(2)/(36*sqrt(-1 + 6/(x + 3))) - sqrt(2)/(12*sqrt(-1 + 6/(x +
 3))*(x + 3)), 6/Abs(x + 3) > 1), (sqrt(2)*I*sqrt(1 - 6/(x + 3))*(x + 3)/(1
08 - 36*x) - 3*sqrt(2)*I*sqrt(1 - 6/(x + 3))/(108 - 36*x), True))
```

3.1099 $\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx$

Optimal. Leaf size=29

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {39}

$$\frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[2]*Sqrt[3 - b*x]*Sqrt[3 + b*x])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx = \frac{x}{18\sqrt{2}\sqrt{3-bx}\sqrt{3+bx}}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 0.66

$$\frac{x}{18\sqrt{18-2b^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] x/(18*Sqrt[18 - 2*b^2*x^2])
```

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{1}{(6-2bx)^{3/2}(3+bx)^{3/2}} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 2*b*x)^(3/2)*(3 + b*x)^(3/2)), x]
```

fricas [A] time = 1.21, size = 29, normalized size = 1.00

$$-\frac{\sqrt{bx+3}\sqrt{-2\,bx+6}\,x}{36\left(b^2x^2-9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/36*sqrt(b*x + 3)*sqrt(-2*b*x + 6)*x/(b^2*x^2 - 9)
```

giac [B] time = 1.14, size = 91, normalized size = 3.14

$$\frac{\sqrt{2}\left(\sqrt{6}-\sqrt{-bx+3}\right)}{144\sqrt{bx+3}\,b}-\frac{\sqrt{2}\sqrt{bx+3}\sqrt{-bx+3}}{72\,(bx-3)b}-\frac{\sqrt{2}\sqrt{bx+3}}{144\,b\left(\sqrt{6}-\sqrt{-bx+3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/144*sqrt(2)*(sqrt(6) - sqrt(-b*x + 3))/(sqrt(b*x + 3)*b) - 1/72*sqrt(2)*s
qrt(b*x + 3)*sqrt(-b*x + 3)/((b*x - 3)*b) - 1/144*sqrt(2)*sqrt(b*x + 3)/(b*
(sqrt(6) - sqrt(-b*x + 3)))
```

maple [A] time = 0.00, size = 24, normalized size = 0.83

$$-\frac{(bx-3)\,x}{9\sqrt{bx+3}\;(-2bx+6)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x)
```

```
[Out] -1/9*(b*x-3)/(b*x+3)^(1/2)*x/(-2*b*x+6)^(3/2)
```

maxima [A] time = 1.25, size = 15, normalized size = 0.52

$$\frac{x}{18\sqrt{-2\,b^2x^2+18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)^(3/2)/(b*x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/18*x/sqrt(-2*b^2*x^2 + 18)
```

mupad [B] time = 0.32, size = 26, normalized size = 0.90

$$-\frac{x\sqrt{6-2\,b\,x}}{\sqrt{b\,x+3}\;(36\,b\,x-108)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 3)^(3/2)*(6 - 2*b*x)^(3/2)),x)
```

```
[Out] -(x*(6 - 2*b*x)^(1/2))/((b*x + 3)^(1/2)*(36*b*x - 108))
```

sympy [C] time = 31.50, size = 83, normalized size = 2.86

$$-\frac{\sqrt{2}\,iG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{9}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}+\frac{\sqrt{2}\,G_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{9e^{-2i\pi}}{b^2x^2}\right)}{72\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*b*x+6)**(3/2)/(b*x+3)**(3/2),x)
```

```
[Out] -sqrt(2)*I*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2),
(0,)), 9/(b**2*x**2))/(72*pi**(3/2)*b) + sqrt(2)*meijerg(((-1/2, 0, 1/4, 1/
2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), 9*exp_polar(-2*I*pi)/(b**2*
x**2))/(72*pi**(3/2)*b)
```

3.1100 $\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx$

Optimal. Leaf size=39

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Rubi [A] time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx}\sqrt{-ad+bdx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-2ad+dx^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{-ad+bdx}}\right)}{b\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bdx-ad}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[-(a*d) + b*d*x]])/(b*Sqrt[d])
```

IntegrateAlgebraic [A] time = 0.08, size = 39, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bdx-ad}}{\sqrt{d}\sqrt{a+bx}}\right)}{b\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[-(a*d) + b*d*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-(a*d) + b*d*x]/(Sqrt[d]*Sqrt[a + b*x])])/(b*Sqrt[d])
```

fricas [A] time = 1.27, size = 108, normalized size = 2.77

$$\left[\frac{\log\left(2\,b^2dx^2+2\,\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{d}\,x-a^2d\right)}{2\,b\sqrt{d}},\,-\frac{\sqrt{-d}\,\arctan\left(\frac{\sqrt{bdx-ad}\,\sqrt{bx+a}\,b\sqrt{-d}\,x}{b^2dx^2-a^2d}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(2*b^2*d*x^2 + 2*sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(d)*x - a^2*
d)/(b*sqrt(d)), -sqrt(-d)*arctan(sqrt(b*d*x - a*d)*sqrt(b*x + a)*b*sqrt(-d)
*x/(b^2*d*x^2 - a^2*d))/(b*d)]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.01, size = 76, normalized size = 1.95

$$\frac{\sqrt{(bx+a)\,(bdx-ad)}\,\ln\left(\frac{b^2dx}{\sqrt{b^2d}}+\sqrt{b^2d\,x^2-a^2d}\right)}{\sqrt{bx+a}\,\sqrt{bdx-ad}\,\sqrt{b^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x)
```

```
[Out] ((b*x+a)*(b*d*x-a*d))^(1/2)/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2)*ln(b^2*d*x/(b^2
*d)^(1/2)+(b^2*d*x^2-a^2*d)^(1/2))/(b^2*d)^(1/2)
```

maxima [A] time = 1.43, size = 39, normalized size = 1.00

$$\frac{\log\left(2\,b^2dx+2\,\sqrt{b^2dx^2-a^2d}\,b\sqrt{d}\right)}{b\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*d*x-a*d)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*d*x + 2*sqrt(b^2*d*x^2 - a^2*d)*b*sqrt(d))/(b*sqrt(d))
```

mupad [B] time = 0.22, size = 56, normalized size = 1.44

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,d\,x-a\,d}-\sqrt{-a\,d}\right)}{\sqrt{-b^2\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*d*x - a*d)^(1/2)*(a + b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*d*x - a*d)^(1/2) - (-a*d)^(1/2)))/((-b^2*d)^(1/2)*((a + b*x
)^(1/2) - a^(1/2)))))/(-b^2*d)^(1/2)
```

sympy [C] time = 4.77, size = 88, normalized size = 2.26

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{a^{2}}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b \sqrt{d}} - \frac{i G_{6,6}^{2,6}\left(\begin{matrix} - \frac{1}{2}, - \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ - \frac{1}{4}, \frac{1}{4} & - \frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{a^{2} e^{2 i \pi}}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*d*x-a*d)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), a**
2/(b**2*x**2))/(4*pi**(3/2)*b*sqrt(d)) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), a**2*exp_polar(2*I*pi)/(b**2*x*
*2))/(4*pi**(3/2)*b*sqrt(d))
```

3.1101 $\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$

Optimal. Leaf size=241

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\,\mathrm{t}}{}$$

Rubi [A] time = 0.25, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}-\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\log\left(\frac{\sqrt{6-3ex}+\sqrt{3}\sqrt{ex+2}+\sqrt{6}\sqrt[4]{2-ex}\sqrt[4]{ex+2}}{\sqrt{ex+2}}\right)}{\sqrt{2}\sqrt[4]{3}e}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}\right)}{\sqrt[4]{3}e}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{ex+2}}+1\right)}{\sqrt[4]{3}e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)*e)
 - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*(2 - e*x)^(1/4))/(2 + e*x)^(1/4)])/(3^(1/4)
*e) - Log[(Sqrt[6 - 3*e*x] - Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt
[3]*Sqrt[2 + e*x])/Sqrt[2 + e*x]]/(Sqrt[2]*3^(1/4)*e) + Log[(Sqrt[6 - 3*e*x
] + Sqrt[6]*(2 - e*x)^(1/4)*(2 + e*x)^(1/4) + Sqrt[3]*Sqrt[2 + e*x])/Sqrt[2
 + e*x]]/(Sqrt[2]*3^(1/4)*e)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx &= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(4-\frac{x^4}{3}\right)^{3/4}}\,dx, x, \sqrt[4]{6-3ex}\right)}{3e}\\
&= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}\\
&= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}-x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{3}+x^2}{1+\frac{x^4}{3}}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{3e}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}-\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{3}+\sqrt{2}\sqrt[4]{3}x+x^2}\,dx, x, \frac{\sqrt[4]{6-3ex}}{\sqrt[4]{2+ex}}\right)}{e}\\
&= -\frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} + \frac{\log\left(\frac{\sqrt{2-ex}+\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+ex}+\sqrt{2+ex}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e} - \frac{\sqrt{2}\,\text{S}}{}\\
&= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{2-ex}}{\sqrt[4]{2+ex}}\right)}{\sqrt[4]{3}e} - \frac{\log\left(\frac{\sqrt{2-ex}-\sqrt{2}\sqrt[4]{2-ex}\sqrt[4]{2+e}}{\sqrt{2+ex}}\right)}{\sqrt{2}\sqrt[4]{3}e}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.17

$$
-\frac{\sqrt{2}(6-3ex)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{12}(6-3ex)\right)}{9e}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] -1/9*(Sqrt[2]*(6 - 3*e*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (6 - 3*e*x
)/12])/e
```

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{6-3ex}\,(2+ex)^{3/4}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)),x]
```

```
[Out] Defer[IntegrateAlgebraic][1/((6 - 3*e*x)^(1/4)*(2 + e*x)^(3/4)), x]
```

fricas [B] time = 1.41, size = 505, normalized size = 2.10

$$2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e^3\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(e^4x-2e^3\right)\sqrt{\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}+3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}-\sqrt{ex+2}\sqrt{-3ex+6}}{ex-2}}\left(\frac{1}{e^4}\right)^{\frac{3}{4}}+ex-2}{ex-2}\right)+2\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e^3\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-\sqrt{3}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(e^4x-2e^3\right)\sqrt{-\frac{\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}-3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}+\sqrt{ex+2}\sqrt{-3ex+6}}{ex-2}}\left(\frac{1}{e^4}\right)^{\frac{3}{4}}-ex+2}{ex-2}\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\log\left(\frac{3\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}+3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}-\sqrt{ex+2}\sqrt{-3ex+6}\right)}{ex-2}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\frac{1}{e^4}\right)^{\frac{1}{4}}\log\left(-\frac{3\left(\sqrt{2}\left(\frac{1}{3}\right)^{\frac{1}{4}}(ex+2)^{\frac{1}{4}}(-3ex+6)^{\frac{3}{4}}e\left(\frac{1}{e^4}\right)^{\frac{1}{4}}-3\sqrt{\frac{1}{3}}\left(e^3x-2e^2\right)\sqrt{\frac{1}{e^4}}+\sqrt{ex+2}\sqrt{-3ex+6}\right)}{ex-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)
^(1/4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*
(e^4*x - 2*e^3)*sqrt((sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4
)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) - sqrt(e*x +
2)*sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) + e*x - 2)/(e*x - 2)) + 2*sq
rt(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*arctan(-(sqrt(2)*(1/3)^(3/4)*(e*x + 2)^(1/
4)*(-3*e*x + 6)^(3/4)*e^3*(e^(-4))^(3/4) - sqrt(3)*sqrt(2)*(1/3)^(3/4)*(e^4
*x - 2*e^3)*sqrt(-(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/4)*e
*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x + 2)*
sqrt(-3*e*x + 6))/(e*x - 2))*(e^(-4))^(3/4) - e*x + 2)/(e*x - 2)) - 1/2*sqr
t(2)*(1/3)^(1/4)*(e^(-4))^(1/4)*log(3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*
(-3*e*x + 6)^(3/4)*e*(e^(-4))^(1/4) + 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-
4)) - sqrt(e*x + 2)*sqrt(-3*e*x + 6))/(e*x - 2)) + 1/2*sqrt(2)*(1/3)^(1/4)*
(e^(-4))^(1/4)*log(-3*(sqrt(2)*(1/3)^(1/4)*(e*x + 2)^(1/4)*(-3*e*x + 6)^(3/
4)*e*(e^(-4))^(1/4) - 3*sqrt(1/3)*(e^3*x - 2*e^2)*sqrt(e^(-4)) + sqrt(e*x +
 2)*sqrt(-3*e*x + 6))/(e*x - 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(-3ex+6)^{\frac{1}{4}}\,(ex+2)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

```
[Out] int(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex+2)^{\frac{3}{4}}(-3\,ex+6)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)^(1/4)/(e*x+2)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x + 2)^(3/4)*(-3*e*x + 6)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x+2)^{3/4}\,(6-3\,e\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)),x)
```

```
[Out] int(1/((e*x + 2)^(3/4)*(6 - 3*e*x)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3^{\frac{3}{4}}\int \frac{1}{\sqrt[4]{-ex+2}\,(ex+2)^{\frac{3}{4}}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*e*x+6)**(1/4)/(e*x+2)**(3/4),x)
```

```
[Out] 3**(3/4)*Integral(1/((-e*x + 2)**(1/4)*(e*x + 2)**(3/4)), x)/3
```

3.1102 $\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}} dx$

Optimal. Leaf size=256

$$-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+$$

Rubi [A] time = 0.17, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\sqrt[4]{a-iax}(a+iax)^{3/4}}{a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))/a - (I*ArcTan[1 - (Sqrt[2]*(a -
I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (I*ArcTan[1 + (Sqrt[2]*(a - I*a
*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - ((I/2)*Log[1 + Sqrt[a - I*a*x]/Sqr
t[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((
I/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx = -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}a\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx,x,\sqrt[4]{a-iax}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+2i\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\ldots$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}+\frac{1}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{1}{2}i\,\text{Subst}\left(\int \frac{\ldots}{1+\ldots}\right.$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\ldots$$

$$= -\frac{i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a}-\frac{i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{i\log\left(1+\frac{\sqrt{a-\ldots}}{\sqrt{a+\ldots}}\ldots\right.}{2\ldots}$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.27

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{5/4}\,{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] (((2*I)/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.47, size = 126, normalized size = 0.49

$$\frac{\sqrt[4]{-1}\,\sqrt[4]{x-i}\,\sqrt[4]{a-iax}\left(-(-1)^{3/4}(x-i)^{3/4}\sqrt[4]{x+i}+\sqrt[4]{-1}\,\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)+\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{\sqrt[4]{x+i}\,\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(1/4),x]
```

```
[Out] ((-1)^(1/4)*(-I + x)^(1/4)*(a - I*a*x)^(1/4)*(-((-1)^(3/4)*(-I + x)^(3/4)*(
I + x)^(1/4)) + (-1)^(1/4)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] + (-1)^(1/4
)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(1/4)*(a + I*a*x)^(1/4))
```

fricas [A] time = 1.49, size = 194, normalized size = 0.76

$$\frac{\sqrt{i}\,a\log\left(\frac{\sqrt{i}\,(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{i}\,a\log\left(-\frac{\sqrt{i}\,(ax-ia)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)+\sqrt{-i}\,a\log\left(\frac{\sqrt{-i}\,(ax-ia)+(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-\sqrt{-i}\,a\log\left(-\frac{\sqrt{-i}\,(ax-ia)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{x-i}\right)-2i\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(I)*a*log((sqrt(I)*(a*x - I*a) + (I*a*x + a)^(3/4)*(-I*a*x + a)^(1
/4))/(x - I)) - sqrt(I)*a*log(-(sqrt(I)*(a*x - I*a) - (I*a*x + a)^(3/4)*(-I
*a*x + a)^(1/4))/(x - I)) + sqrt(-I)*a*log((sqrt(-I)*(a*x - I*a) + (I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - sqrt(-I)*a*log(-(sqrt(-I)*(a*x - I
*a) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(x - I)) - 2*I*(I*a*x + a)^(3/4
)*(-I*a*x + a)^(1/4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

maple [C] time = 2.28, size = 477, normalized size = 1.86

$$\frac{i(x-i)(x+i)(-(ix-1)a)^{\frac{1}{4}}}{(ix-1)((ix+1)a)^{\frac{1}{4}}}-\frac{\left(-\frac{\mathrm{RootOf}(\_Z^2-i)\ln\left(\frac{-x^3+(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x^2\mathrm{RootOf}(\_Z^2-i)-2ix^2+2i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x\,\mathrm{RootOf}(\_Z^2-i)-i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+i(-x^4-2ix^3-2ix+1)^{\frac{3}{4}}\mathrm{RootOf}(\_Z^2-i)-(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^2-i)+\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)}{2}-\frac{i\,\mathrm{RootOf}(\_Z^2-i)\ln\left(\frac{-x^3+i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x^2\mathrm{RootOf}(\_Z^2-i)-2ix^2-2(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}x\,\mathrm{RootOf}(\_Z^2-i)+i\sqrt{-x^4-2ix^3-2ix+1}\,x+x+(-x^4-2ix^3-2ix+1)^{\frac{3}{4}}\mathrm{RootOf}(\_Z^2-i)-i(-x^4-2ix^3-2ix+1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^2-i)-\sqrt{-x^4-2ix^3-2ix+1}}{(ix-1)^2}\right)}{2}\right)(-(ix-1)a)^{\frac{1}{4}}\left(-(ix-1)^3(ix+1)\right)^{\frac{1}{4}}}{(ix-1)((ix+1)a)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] I*(x-I)*(x+I)*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(-1/2*RootOf(_Z^
2-I)*ln((RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)*x^2+I*RootOf(_Z^2-I)*(1
-2*I*x-2*I*x^3-x^4)^(3/4)-x^3+2*I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4
)*x-I*(1-2*I*x-2*I*x^3-x^4)^(1/2)*x-2*I*x^2-RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3
-x^4)^(1/4)+(1-2*I*x-2*I*x^3-x^4)^(1/2)+x)/(I*x-1)^2)-1/2*I*RootOf(_Z^2-I)*
ln((I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)*x^2-2*RootOf(_Z^2-I)*(1-2*
I*x-2*I*x^3-x^4)^(1/4)*x-x^3+RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(3/4)+I*(
1-2*I*x-2*I*x^3-x^4)^(1/2)*x-I*RootOf(_Z^2-I)*(1-2*I*x-2*I*x^3-x^4)^(1/4)-2
*I*x^2-(1-2*I*x-2*I*x^3-x^4)^(1/2)+x)/(I*x-1)^2))*(-(I*x-1)*a)^(1/4)/(I*x-1
)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4),x)
[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{\sqrt[4]{ia\,(x-i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(1/4),x)
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(1/4), x)
```

3.1103 $\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=233

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Rubi [A] time = 0.13, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {63, 240, 211, 1165, 628, 1162, 617, 204}

$$-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a -
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx = \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Sub}}{}$$

$$= -\frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{\left(i\sqrt{2}\right)\text{Subs}}{}$$

$$= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.02, size = 68, normalized size = 0.29

$$\frac{2i2^{3/4}\sqrt[4]{1+ix}\sqrt[4]{a-iax}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1}{2}-\frac{ix}{2}\right)}{a\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((2*I)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(1/4)*Hypergeometric2F1[1/4, 1/4
, 5/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.20, size = 83, normalized size = 0.36

$$\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
- (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
a
```

fricas [A] time = 1.50, size = 227, normalized size = 0.97

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-
I*a*x + a)^(1/4))/(2*x - 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-i,[1,1]%%%}+%%%{-
1,[1,0]%%%}] at parameters values [-27,-87]ext_reduce Error: Bad Argument T
ypeintegrate((-4*i)/a/4*i*a*(-4*i)/a*((i*a*x+a)^(1/4))^2/((-((i*a*x+a)^(1/4
))^4+2*a)^(1/4))^3/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-iax+a\right)^{\frac{3}{4}}\left(iax+a\right)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(i\,ax+a\right)^{\frac{1}{4}}\left(-i\,ax+a\right)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{3/4}\,(a + a\,x\,1\mathrm{i})^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)),x)
[Out] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x - i)}\,(-ia\,(x + i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(3/4)), x)
```

3.1104 $\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=33

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Rubi [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx = -\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.11, size = 33, normalized size = 1.00

$$-\frac{2i(a+iax)^{3/4}}{3a^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/3)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(3/4))
```

fricas [A] time = 1.71, size = 32, normalized size = 0.97

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{3\,a^3x+3i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(3*a^3*x + 3*I*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

maple [A] time = 0.05, size = 31, normalized size = 0.94

$$\frac{\frac{2x}{3}-\frac{2i}{3}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(1/4),x)
[Out] 2/3/a/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(x-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(7/4)), x)
```

mupad [B] time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(x-\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{3\,a^2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(1/4)),x)
[Out] -(2*(x - 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^2*(x*1i - 1)*(a*(x*1i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\left(x-i\right)}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(1/4),x)
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(7/4)), x)
```

3.1105 $\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=67

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}-\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/7)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(7/4)) - (((4*I)/21)*(a + I
*a*x)^(3/4))/(a^3*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}+\frac{2\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{7a}\\ &= -\frac{2i(a+iax)^{3/4}}{7a^2(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{21a^3(a-iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(5-2ix)(a+iax)^{3/4}}{21a^3(x+i)(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(5 - (2*I)*x)*(a + I*a*x)^(3/4))/(21*a^3*(I + x)*(a - I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 55, normalized size = 0.82

$$-\frac{i(a+iax)^{7/4}\left(3+\frac{7(a-iax)}{a+iax}\right)}{21a^3(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-1/21*I)*(a + I*a*x)^(7/4)*(3 + (7*(a - I*a*x))/(a + I*a*x)))/(a^3*(a - I
*a*x)^(7/4))
```

fricas [A] time = 1.57, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x+10i)}{21\left(a^4x^2+2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/21*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 10*I)/(a^4*x^2 + 2*I*a^4*x
 - a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

maple [A] time = 0.05, size = 44, normalized size = 0.66

$$\frac{\frac{4}{21}x^2+\frac{2}{7}ix+\frac{10}{21}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] 2/21/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x^2+5+3*I*x)/(x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(11/4)), x)
```

mupad [B] time = 0.67, size = 46, normalized size = 0.69

$$-\frac{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,\left(2\,x^2+x\,3\mathrm{i}+5\right)\,2\mathrm{i}}{21\,a^3\,{\left(-1+x\,1\mathrm{i}\right)}^2\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*3i + 2*x^2 + 5)*2i)/(21*a^3*(x*1i - 1)^2*(a*(x*1
i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{ia\,(x-i)}\,\left(-ia\,(x+i)\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(1/4)*(-I*a*(x + I))**(11/4)), x)
```

3.1106 $\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx$

Optimal. Leaf size=100

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/11)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(11/4)) - (((8*I)/77)*(a + I*a*x)^(3/4))/(a^3*(a - I*a*x)^(7/4)) - (((16*I)/231)*(a + I*a*x)^(3/4))/(a^4*(a - I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*Simplify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}+\frac{4\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{11a}\\ &= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}+\frac{8\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{77a^2}\\ &= -\frac{2i(a+iax)^{3/4}}{11a^2(a-iax)^{11/4}}-\frac{8i(a+iax)^{3/4}}{77a^3(a-iax)^{7/4}}-\frac{16i(a+iax)^{3/4}}{231a^4(a-iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.52

$$\frac{2\left(-8ix^2+28x+41i\right)(a+iax)^{3/4}}{231a^4(x+i)^2(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(41*I + 28*x - (8*I)*x^2))/(231*a^4*(I + x)^2*(a - I*a
*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 77, normalized size = 0.77

$$-\frac{i(a+iax)^{11/4}\left(\frac{77(a-iax)^2}{(a+iax)^2}+\frac{66(a-iax)}{a+iax}+21\right)}{462a^4(a-iax)^{11/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(15/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-1/462*I)*(a + I*a*x)^(11/4)*(21 + (77*(a - I*a*x)^2)/(a + I*a*x)^2 + (66
*(a - I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(11/4))
```

fricas [A] time = 1.42, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+28i\,x-41\right)}{231\,a^5x^3+693i\,a^5x^2-693\,a^5x-231i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 28*I*x - 41)/(231*a^5*x^3 +
 693*I*a^5*x^2 - 693*a^5*x - 231*I*a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

maple [A] time = 0.05, size = 50, normalized size = 0.50

$$\frac{\frac{16}{231}x^3+\frac{40}{231}ix^2-\frac{26}{231}x+\frac{82}{231}i}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(15/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] 2/231/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(20*I*x^2+8*x^3-13*x+41*I)/(
x+I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{15}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(15/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(15/4)), x)
```

mupad [B] time = 0.75, size = 51, normalized size = 0.51

$$\frac{(x-\mathrm{i})^4\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,\left(8\,x^2+x\,28\mathrm{i}-41\right)\,2\mathrm{i}}{231\,a^4\,\left(x^2+1\right)^3\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(15/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] ((x - 1i)^4*(-a*(x*1i - 1))^(1/4)*(x*28i + 8*x^2 - 41)*2i)/(231*a^4*(x^2 +
1)^3*(a*(x*1i + 1))^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(15/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

3.1107 $\int \frac{1}{(a-iax)^{19/4} \sqrt[4]{a+iax}} \, dx$

Optimal. Leaf size=133

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Rubi [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (((-2*I)/15)*(a + I*a*x)^(3/4))/(a^2*(a - I*a*x)^(15/4)) - (((4*I)/55)*(a +
 I*a*x)^(3/4))/(a^3*(a - I*a*x)^(11/4)) - (((16*I)/385)*(a + I*a*x)^(3/4))/
(a^4*(a - I*a*x)^(7/4)) - (((32*I)/1155)*(a + I*a*x)^(3/4))/(a^5*(a - I*a*x
)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{19/4}\sqrt[4]{a+iax}}\,dx &= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}+\frac{2\int \frac{1}{(a-iax)^{15/4}\sqrt[4]{a+iax}}\,dx}{5a}\\
&= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}+\frac{8\int \frac{1}{(a-iax)^{11/4}\sqrt[4]{a+iax}}\,dx}{55a^2}\\
&= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}+\frac{16\int \frac{1}{(a-iax)^{7/4}\sqrt[4]{a+iax}}\,dx}{385a^3}\\
&= -\frac{2i(a+iax)^{3/4}}{15a^2(a-iax)^{15/4}}-\frac{4i(a+iax)^{3/4}}{55a^3(a-iax)^{11/4}}-\frac{16i(a+iax)^{3/4}}{385a^4(a-iax)^{7/4}}-\frac{32i(a+iax)^{3/4}}{1155a^5(a-iax)^{3/4}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.43

$$\frac{2\left(-16ix^3+72x^2+138ix-159\right)(a+iax)^{3/4}}{1155a^5(x+i)^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] (2*(a + I*a*x)^(3/4)*(-159 + (138*I)*x + 72*x^2 - (16*I)*x^3))/(1155*a^5*(I
 + x)^3*(a - I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 99, normalized size = 0.74

$$-\frac{i(a+iax)^{15/4}\left(\frac{385(a-iax)^3}{(a+iax)^3}+\frac{495(a-iax)^2}{(a+iax)^2}+\frac{315(a-iax)}{a+iax}+77\right)}{4620a^5(a-iax)^{15/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(19/4)*(a + I*a*x)^(1/4)),x]
```

```
[Out] ((-1/4620*I)*(a + I*a*x)^(15/4)*(77 + (385*(a - I*a*x)^3)/(a + I*a*x)^3 + (
495*(a - I*a*x)^2)/(a + I*a*x)^2 + (315*(a - I*a*x))/(a + I*a*x)))/(a^5*(a
- I*a*x)^(15/4))
```

fricas [A] time = 1.47, size = 70, normalized size = 0.53

$$\frac{\left(32\,x^3+144i\,x^2-276\,x-318i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{1155\,a^6x^4+4620i\,a^6x^3-6930\,a^6x^2-4620i\,a^6x+1155\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="fricas")
```

```
[Out] (32*x^3 + 144*I*x^2 - 276*x - 318*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(
1155*a^6*x^4 + 4620*I*a^6*x^3 - 6930*a^6*x^2 - 4620*I*a^6*x + 1155*a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

maple [A] time = 0.06, size = 55, normalized size = 0.41

$$\frac{\frac{32}{1155}x^4+\frac{16}{165}ix^3-\frac{4}{35}x^2-\frac{2}{55}ix-\frac{106}{385}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)^3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(19/4)/(I*a*x+a)^(1/4),x)
```

```
[Out] 2/1155/a^4/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(56*I*x^3+16*x^4-21*I*x-159
-66*x^2)/(x+I)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{19}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(19/4)/(a+I*a*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(1/4)*(-I*a*x + a)^(19/4)), x)
```

mupad [B] time = 0.79, size = 57, normalized size = 0.43

$$-\frac{(x-\mathrm{i})^5\,(-a\,(-1+x\,1\mathrm{i}))^{1/4}\,\left(-16\,x^3-x^2\,72\mathrm{i}+138\,x+159\mathrm{i}\right)\,2\mathrm{i}}{1155\,a^5\,\left(x^2+1\right)^4\,(a\,(1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(19/4)*(a + a*x*1i)^(1/4)),x)
```

```
[Out] -((x - 1i)^5*(-a*(x*1i - 1))^(1/4)*(138*x - x^2*72i - 16*x^3 + 159i)*2i)/(1
155*a^5*(x^2 + 1)^4*(a*(x*1i + 1))^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(19/4)/(a+I*a*x)**(1/4),x)
```

```
[Out] Timed out
```

3.1108 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=256

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a+}}\right.}{\sqrt{2}}$$

Rubi [A] time = 0.16, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-I)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))/a - ((3*I)*ArcTan[1 - (Sqrt[2]*(
a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + ((3*I)*ArcTan[1 + (Sqrt[2]*
(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((3*I)/2)*Log[1 + Sqrt[a
- I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/
Sqrt[2] - (((3*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a
- I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx = -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{1}{2}(3a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{a-iax}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+6i\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-3i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+3i\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{}}{\sqrt[4]{}}\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)+\frac{3}{2}i\,\text{Subst}\left(\int \frac{}{1+\sqrt{}}\right.$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}+\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}-\frac{3i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}$$

$$= -\frac{i(a-iax)^{3/4}\sqrt[4]{a+iax}}{a}-\frac{3i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}+\frac{3i\log\left(1+\right.}{}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.27

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] (((2*I)/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.60, size = 128, normalized size = 0.50

$$\frac{(-1)^{3/4}(x-i)^{3/4}(a-iax)^{3/4}\left(-\sqrt[4]{-1}\,\sqrt[4]{x-i}\,(x+i)^{3/4}+3(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)-3(-1)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{3/4}(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(3/4)/(a + I*a*x)^(3/4),x]
```

```
[Out] ((-1)^(3/4)*(-I + x)^(3/4)*(a - I*a*x)^(3/4)*(-((-1)^(1/4)*(-I + x)^(1/4)*(
I + x)^(3/4)) + 3*(-1)^(3/4)*ArcTan[(I + x)^(1/4)/(-I + x)^(1/4)] - 3*(-1)^
(3/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)]))/((I + x)^(3/4)*(a + I*a*x)^(3
/4))
```

fricas [A] time = 1.48, size = 204, normalized size = 0.80

$$\frac{\sqrt{9i}\,a\log\left(\frac{\sqrt{9i}(ax+ia)+3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-\sqrt{9i}\,a\log\left(-\frac{\sqrt{9i}(ax+ia)-3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)+\sqrt{-9i}\,a\log\left(\frac{\sqrt{-9i}(ax+ia)+3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-\sqrt{-9i}\,a\log\left(-\frac{\sqrt{-9i}(ax+ia)-3(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{3x+3i}\right)-2i(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(9*I)*a*log((sqrt(9*I)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x +
 a)^(3/4))/(3*x + 3*I)) - sqrt(9*I)*a*log(-(sqrt(9*I)*(a*x + I*a) - 3*(I*a*
x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) + sqrt(-9*I)*a*log((sqrt(-9*I
)*(a*x + I*a) + 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(3*x + 3*I)) - sqrt
(-9*I)*a*log(-(sqrt(-9*I)*(a*x + I*a) - 3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3
/4))/(3*x + 3*I)) - 2*I*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/a
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

maple [C] time = 2.14, size = 464, normalized size = 1.81

$$-\frac{i(x-i)(x+i)a}{((ix+1)a)^{\frac{3}{4}}(-(ix-1)a)^{\frac{1}{4}}}+\frac{\left(-\frac{3\,\mathrm{RootOf}(\_Z^2+i)\ln\left(-\frac{x^3-(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}x^2\,\mathrm{RootOf}(\_Z^2+i)-2ix^2+2i(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}x\,\mathrm{RootOf}(\_Z^2+i)-i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+i(-x^4+2ix^3+2ix+1)^{\frac{3}{4}}\mathrm{RootOf}(\_Z^2+i)+(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^2+i)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}-\frac{3i\,\mathrm{RootOf}(\_Z^2+i)\ln\left(-\frac{x^3-i(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}x^2\,\mathrm{RootOf}(\_Z^2+i)-2ix^2-2(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}x\,\mathrm{RootOf}(\_Z^2+i)+i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+(-x^4+2ix^3+2ix+1)^{\frac{3}{4}}\mathrm{RootOf}(\_Z^2+i)+i(-x^4+2ix^3+2ix+1)^{\frac{1}{4}}\mathrm{RootOf}(\_Z^2+i)+\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}\right)\left(-(ix-1)(ix+1)^3\right)^{\frac{1}{4}}a}{((ix+1)a)^{\frac{3}{4}}(-(ix-1)a)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(3/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] -I*(x-I)*(x+I)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*a+(-3/2*RootOf(_Z^2+I)*
ln(-(-RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)*x^2+x^3+I*RootOf(_Z^2+I)*(
1+2*I*x+2*I*x^3-x^4)^(3/4)+2*I*RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)*x
-I*(1+2*I*x+2*I*x^3-x^4)^(1/2)*x-2*I*x^2+(1+2*I*x+2*I*x^3-x^4)^(1/4)*RootOf
(_Z^2+I)-(1+2*I*x+2*I*x^3-x^4)^(1/2)-x)/(I*x+1)^2)-3/2*I*RootOf(_Z^2+I)*ln(
-(-I*(1+2*I*x+2*I*x^3-x^4)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(1+2*I
*x+2*I*x^3-x^4)^(1/4)*x+x^3+I*(1+2*I*x+2*I*x^3-x^4)^(1/2)*x+RootOf(_Z^2+I)*
(1+2*I*x+2*I*x^3-x^4)^(3/4)+I*RootOf(_Z^2+I)*(1+2*I*x+2*I*x^3-x^4)^(1/4)-2*
I*x^2+(1+2*I*x+2*I*x^3-x^4)^(1/2)-x)/(I*x+1)^2))/((I*x+1)*a)^(3/4)*(-(I*x-1
)*(I*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{3}{4}}}{(i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{3/4}}{(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(3/4), x)
```

3.1109 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=233

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-I)*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
+ (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a +
 (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(
a + I*a*x)^(1/4)])/(Sqrt[2]*a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx &= \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}\\
&= \frac{(4i)\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}\\
&= -\frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}\\
&= \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\right.}{}\\
&= \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{\left(i\sqrt{2}\right)\text{Subst}\left(\int\right.}{}\\
&= -\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}}{\right.}}{\sqrt{2}a}\end{aligned}$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.30

$$\frac{2i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}-\frac{ix}{2}\right)}{3a(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((2*I)/3)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, 1/2 - (I/2)*x])/(a*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.15, size = 83, normalized size = 0.36

$$\frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}-\frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (-2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a
 + (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])
/a
```

fricas [A] time = 1.41, size = 227, normalized size = 0.97

$$\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\frac{1}{2}\sqrt{\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)+\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\frac{1}{2}\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{(a^2x+ia^2)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*(I*a*x + a)^(1/4)*
(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(4*I/a^2)*log(-((a^2*x + I*a^2)*
sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) + 1/2*
sqrt(-4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-
I*a*x + a)^(3/4))/(2*x + 2*I)) - 1/2*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*s
qrt(-4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate((-4*i)/a/4*i
*a*(-4*i)/a/(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(-iax+a)^{\frac{1}{4}}\,(iax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{1/4}\,(a+a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(1/4)), x)
```

3.1110 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}} dx$

Optimal. Leaf size=31

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Rubi [A] time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}} dx = -\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.06, size = 31, normalized size = 1.00

$$-\frac{2i\sqrt[4]{a+iax}}{a^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-2*I)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(1/4))
```

fricas [A] time = 1.42, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{a^3x+i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(a^3*x + I*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax + a)^{\frac{3}{4}}(-i\,ax + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

maple [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{2x - 2i}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(5/4)/(I*a*x+a)^(3/4),x)
[Out] 2/a/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(x-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax + a)^{\frac{3}{4}}(-i\,ax + a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{5/4}\,(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{3}{4}}\,(-ia\,(x+i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(3/4),x)
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(5/4)), x)
```

3.1111 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=67

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {45, 37}

$$-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}-\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/5)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(5/4)) - (((4*I)/5)*(a + I*
a*x)^(1/4))/(a^3*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}+\frac{2\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{5a}\\ &= -\frac{2i\sqrt[4]{a+iax}}{5a^2(a-iax)^{5/4}}-\frac{4i\sqrt[4]{a+iax}}{5a^3\sqrt[4]{a-iax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(3-2ix)\sqrt[4]{a+iax}}{5a^3(x+i)\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(3 - (2*I)*x)*(a + I*a*x)^(1/4))/(5*a^3*(I + x)*(a - I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.11, size = 54, normalized size = 0.81

$$-\frac{i\sqrt[4]{a+iax}\left(5+\frac{a+iax}{a-iax}\right)}{5a^3\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-1/5*I)*(a + I*a*x)^(1/4)*(5 + (a + I*a*x)/(a - I*a*x)))/(a^3*(a - I*a*x)
^(1/4))
```

fricas [A] time = 1.16, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4\,x+6i)}{5\left(a^4x^2+2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 1/5*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x + 6*I)/(a^4*x^2 + 2*I*a^4*x -
 a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

maple [A] time = 0.05, size = 44, normalized size = 0.66

$$\frac{\frac{4}{5}x^2+\frac{2}{5}ix+\frac{6}{5}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,(x+i)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(9/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] 2/5/a^2/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(2*x^2+3+I*x)/(x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(9/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{1}{{(a-a\,x\,1\mathrm{i})}^{9/4}\,{(a+a\,x\,1\mathrm{i})}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(3/4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{3}{4}}\left(-ia\left(x+i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(3/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(3/4)*(-I*a*(x + I))**(9/4)), x)
```

3.1112 $\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx$

Optimal. Leaf size=100

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (((-2*I)/9)*(a + I*a*x)^(1/4))/(a^2*(a - I*a*x)^(9/4)) - (((8*I)/45)*(a + I
*a*x)^(1/4))/(a^3*(a - I*a*x)^(5/4)) - (((16*I)/45)*(a + I*a*x)^(1/4))/(a^4
*(a - I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{13/4}(a+iax)^{3/4}}\,dx &= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}+\frac{4\int \frac{1}{(a-iax)^{9/4}(a+iax)^{3/4}}\,dx}{9a}\\
&= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{5/4}(a+iax)^{3/4}}\,dx}{45a^2}\\
&= -\frac{2i\sqrt[4]{a+iax}}{9a^2(a-iax)^{9/4}}-\frac{8i\sqrt[4]{a+iax}}{45a^3(a-iax)^{5/4}}-\frac{16i\sqrt[4]{a+iax}}{45a^4\sqrt[4]{a-iax}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 0.52

$$\frac{2\left(-8ix^2+20x+17i\right)\sqrt[4]{a+iax}}{45a^4(x+i)^2\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] (2*(a + I*a*x)^(1/4)*(17*I + 20*x - (8*I)*x^2))/(45*a^4*(I + x)^2*(a - I*a*
x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 77, normalized size = 0.77

$$-\frac{i\sqrt[4]{a+iax}\left(\frac{5(a+iax)^2}{(a-iax)^2}+\frac{18(a+iax)}{a-iax}+45\right)}{90a^4\sqrt[4]{a-iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(13/4)*(a + I*a*x)^(3/4)),x]
```

```
[Out] ((-1/90*I)*(a + I*a*x)^(1/4)*(45 + (18*(a + I*a*x))/(a - I*a*x) + (5*(a + I
*a*x)^2)/(a - I*a*x)^2))/(a^4*(a - I*a*x)^(1/4))
```

fricas [A] time = 1.40, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+20i\,x-17\right)}{45\,a^5x^3+135i\,a^5x^2-135\,a^5x-45i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 20*I*x - 17)/(45*a^5*x^3 +
135*I*a^5*x^2 - 135*a^5*x - 45*I*a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

maple [A] time = 0.06, size = 50, normalized size = 0.50

$$\frac{\frac{16}{45}x^3+\frac{8}{15}ix^2+\frac{2}{15}x+\frac{34}{45}i}{\left((ix+1)\,a\right)^{\frac{3}{4}}\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}\left(x+i\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(13/4)/(I*a*x+a)^(3/4),x)
```

```
[Out] 2/45/a^3/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(12*I*x^2+8*x^3+3*x+17*I)/(x+
I)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(13/4)/(a+I*a*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(3/4)*(-I*a*x + a)^(13/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a - a\,x\,1\mathrm{i})^{13/4}\,(a + a\,x\,1\mathrm{i})^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)),x)
[Out] int(1/((a - a*x*1i)^(13/4)*(a + a*x*1i)^(3/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(13/4)/(a+I*a*x)**(3/4),x)
[Out] Timed out
```

3.1113 $\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}} \, dx$

Optimal. Leaf size=291

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+ 7i$$

Rubi [A] time = 0.18, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {47, 50, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i\sqrt[4]{a+iax}\,(a-iax)^{3/4}}{3a}-\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(7/4))/(a*(a + I*a*x)^(3/4)) + (((7*I)/3)*(a - I*a*x
)^(3/4)*(a + I*a*x)^(1/4))/a + ((7*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4)
)/(a + I*a*x)^(1/4)])/Sqrt[2] - ((7*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((7*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a
+ I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] + (((7*I
)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/
(a + I*a*x)^(1/4)])/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a-iax)^{7/4}}{(a+iax)^{7/4}}\,dx &= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}-\frac{7}{3}\int \frac{(a-iax)^{3/4}}{(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}-\frac{1}{2}(7a)\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}-14i\,\text{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx,x,\sqrt[4]{a-iax}\right) \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}-14i\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}+7i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)-7i\,\text{Subs} \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}-\frac{7}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)- \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}-\frac{7i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}}+\frac{7i\log\left(1+\right.}{} \\
&= \frac{4i(a-iax)^{7/4}}{3a(a+iax)^{3/4}}+\frac{7i(a-iax)^{3/4}\sqrt[4]{a+iax}}{3a}+\frac{7i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{7i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+i}}\right.}{\sqrt{2}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.24

$$
\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{11/4}\,{}_2F_1\left(\frac{7}{4},\frac{11}{4};\frac{15}{4};\frac{1}{2}-\frac{ix}{2}\right)}{11a^2(a+iax)^{3/4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/11)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(11/4)*Hypergeometric2F1[7/4, 1
1/4, 15/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.90, size = 150, normalized size = 0.52

$$
-\frac{(-1)^{3/4}(x-i)^{7/4}(a-iax)^{7/4}\left(\frac{3\sqrt[4]{-1}\,(x+i)^{7/4}-14(-1)^{3/4}(x+i)^{3/4}}{3(x-i)^{3/4}}-7(-1)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)+7(-1)^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{7/4}(a+iax)^{7/4}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(7/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] -(((-1)^(3/4)*(-I + x)^(7/4)*(a - I*a*x)^(7/4)*((-14*(-1)^(3/4)*(I + x)^(3/
4) + 3*(-1)^(1/4)*(I + x)^(7/4))/(3*(-I + x)^(3/4)) - 7*(-1)^(3/4)*ArcTan[(
I + x)^(1/4)/(-I + x)^(1/4)] + 7*(-1)^(3/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^
(1/4)]))/((I + x)^(7/4)*(a + I*a*x)^(7/4)))
```

fricas [A] time = 1.43, size = 244, normalized size = 0.84

$$
-\frac{\sqrt{49i}\,(3ax-3ia)\log\left(\frac{\sqrt{49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)-\sqrt{49i}\,(3ax-3ia)\log\left(-\frac{\sqrt{49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)+\sqrt{-49i}\,(3ax-3ia)\log\left(\frac{\sqrt{-49i}\,(ax+ia)+7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)-\sqrt{-49i}\,(3ax-3ia)\log\left(-\frac{\sqrt{-49i}\,(ax+ia)-7\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{7x+7i}\right)+2\,(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}(-3ix-11)}{6ax-6ia}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -(sqrt(49*I)*(3*a*x - 3*I*a)*log((sqrt(49*I)*(a*x + I*a) + 7*(I*a*x + a)^(1
/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - sqrt(49*I)*(3*a*x - 3*I*a)*log(-(sqr
t(49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I))
+ sqrt(-49*I)*(3*a*x - 3*I*a)*log((sqrt(-49*I)*(a*x + I*a) + 7*(I*a*x + a)^
(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*I)) - sqrt(-49*I)*(3*a*x - 3*I*a)*log(-(
sqrt(-49*I)*(a*x + I*a) - 7*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(7*x + 7*
I)) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(-3*I*x - 11))/(6*a*x - 6*I*a)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, choosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1
,[0,4]%%%}] at parameters values [-64,-30]Warning, choosing root of [1,0,0,
0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters values [70,22]Warning, ch
oosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters val
ues [42,56]Warning, choosing root of [1,0,0,0,%%%{-2,[1,0]%%%}+%%%{1,[0,4]%
%%}] at parameters values [-9,-13]Warning, choosing root of [1,0,0,0,%%%{-2
,[1,0]%%%}+%%%{1,[0,4]%%%}] at parameters values [46,24]ext_reduce Error: B
ad Argument TypeEvaluation time: 0.61integrate(i/4*a/a^2*(16*((i*a*x+a)^(1/
4))^4*((-((i*a*x+a)^(1/4))^4+2*a)^(1/4))^3-32*a*((-((i*a*x+a)^(1/4))^4+2*a)
^(1/4))^3)/((i*a*x+a)^(1/4))^4/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

maple [**C**] time = 2.01, size = 469, normalized size = 1.61

$$\frac{i\left(3x^2-8ix+11\right)a}{3\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}+\frac{\left(-\frac{7\,\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(\frac{-x^3-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2-i\right)+2ix^2+2i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2-i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x+x-i\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)+\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}+\frac{7i\,\mathrm{RootOf}\left(\_Z^2-i\right)\ln\left(-\frac{x^3-i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2-i\right)-2ix^2-2\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2-i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x-x-\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2-i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)}{2}\right)\left(-(ix-1)(ix+1)^3\right)^{\frac{1}{4}}a}{\left((ix+1)a\right)^{\frac{3}{4}}\left(-(ix-1)a\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(7/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 1/3*I*(-8*I*x+3*x^2+11)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*a+(-7/2*RootOf
(_Z^2-I)*ln((-(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2-I)*x^2-I*RootOf(_Z^2
-I)*(-x^4+2*I*x^3+2*I*x+1)^(3/4)-x^3+2*I*RootOf(_Z^2-I)*(-x^4+2*I*x^3+2*I*x
+1)^(1/4)*x-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+2*I*x^2+RootOf(_Z^2-I)*(-x^4+2
*I*x^3+2*I*x+1)^(1/4)-(-x^4+2*I*x^3+2*I*x+1)^(1/2)+x)/(I*x+1)^2)+7/2*I*Root
Of(_Z^2-I)*ln(-(-I*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2-I)*x^2-2*RootOf
(_Z^2-I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x+x^3-RootOf(_Z^2-I)*(-x^4+2*I*x^3+2*
I*x+1)^(3/4)-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+I*RootOf(_Z^2-I)*(-x^4+2*I*x^
3+2*I*x+1)^(1/4)-2*I*x^2-(-x^4+2*I*x^3+2*I*x+1)^(1/2)-x)/(I*x+1)^2))/((I*x+
1)*a)^(3/4)*(-(I*x-1)*(I*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)*a
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{7}{4}}}{(i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(7/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(7/4)/(I*a*x + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - a\,x\,1\mathrm{i})^{7/4}}{(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(7/4)/(a + a*x*1i)^(7/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{7}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(7/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(7/4)/(I*a*(x - I))**(7/4), x)
```

3.1114 $\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=266

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}}{}$$

Rubi [A] time = 0.14, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 331, 297, 1162, 617, 204, 1165, 628}

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (I*Sqrt[2]*ArcTan[1 -
 (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 +
(Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a
*x]/Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt
[2]*a) + (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^
(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{7/4}}\,dx = \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{3/4}}\,dx$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\mathrm{Subst}\left(\int \frac{x^2}{\left(2a-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{(4i)\,\mathrm{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\mathrm{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i}{}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{\left(i\sqrt{2}\right)\mathrm{Su}}{}$$

$$= \frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.26

$$\frac{i\sqrt[4]{2}\,(1+ix)^{3/4}(a-iax)^{7/4}\,{}_2F_1\left(\frac{7}{4},\frac{7}{4};\frac{11}{4};\frac{1}{2}-\frac{ix}{2}\right)}{7a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] ((I/7)*2^(1/4)*(1 + I*x)^(3/4)*(a - I*a*x)^(7/4)*Hypergeometric2F1[7/4, 7/4
, 11/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 116, normalized size = 0.44

$$\frac{4i(a-iax)^{3/4}}{3a(a+iax)^{3/4}} + \frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a} - \frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a+iax}}{\sqrt[4]{a-iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(3/4)/(a + I*a*x)^(7/4),x]
```

```
[Out] (((4*I)/3)*(a - I*a*x)^(3/4))/(a*(a + I*a*x)^(3/4)) + (2*(-1)^(1/4)*ArcTanh
[((-1)^(1/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a - (2*(-1)^(3/4)*ArcTa
nh[((-1)^(3/4)*(a + I*a*x)^(1/4))/(a - I*a*x)^(1/4)])/a
```

fricas [A] time = 0.84, size = 307, normalized size = 1.15

$$-\frac{\left(3a^2x-3ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\left(3a^2x-3ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)+\left(3a^2x-3ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-\left(3a^2x-3ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x+ia^2\right)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{2x+2i}\right)-8(iax+a)^{\frac{1}{4}}(-iax+a)^{\frac{3}{4}}}{6a^2x-6ia^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] -((3*a^2*x - 3*I*a^2)*sqrt(4*I/a^2)*log(((a^2*x + I*a^2)*sqrt(4*I/a^2) + 2*
(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - (3*a^2*x - 3*I*a^2)*sq
rt(4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(1/4)*(-I*a
*x + a)^(3/4))/(2*x + 2*I)) + (3*a^2*x - 3*I*a^2)*sqrt(-4*I/a^2)*log(((a^2*
x + I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x +
2*I)) - (3*a^2*x - 3*I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x + I*a^2)*sqrt(-4*I/
a^2) - 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4))/(2*x + 2*I)) - 8*(I*a*x + a)
^(1/4)*(-I*a*x + a)^(3/4))/(6*a^2*x - 6*I*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax + a)^{\frac{3}{4}}}{(i\,ax + a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

maple [C] time = 1.88, size = 459, normalized size = 1.73

$$\frac{\frac{4x}{3}+\frac{4i}{3}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}}-\frac{\left(i\,\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(\frac{-x^3-i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)+2ix^2-2\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x+x+\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)-\mathrm{RootOf}\left(\_Z^2+i\right)\ln\left(-\frac{x^3-\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x^2\,\mathrm{RootOf}\left(\_Z^2+i\right)-2ix^2+2i\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^2+i\right)-i\sqrt{-x^4+2ix^3+2ix+1}\,x-x+i\left(-x^4+2ix^3+2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)+\left(-x^4+2ix^3+2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^2+i\right)-\sqrt{-x^4+2ix^3+2ix+1}}{(ix+1)^2}\right)\right)\left(-(ix-1)(ix+1)^3\right)^{\frac{1}{4}}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(3/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 4/3*(x+I)/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)-(-RootOf(_Z^2+I)*ln(-(-RootO
f(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x^2+x^3+I*RootOf(_Z^2+I)*(-x^4+2*I*x
^3+2*I*x+1)^(3/4)+2*I*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)*x-I*(-x^4
+2*I*x^3+2*I*x+1)^(1/2)*x-2*I*x^2+(-x^4+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2+
I)-(-x^4+2*I*x^3+2*I*x+1)^(1/2)-x)/(I*x+1)^2)+I*RootOf(_Z^2+I)*ln((-I*(-x^4
+2*I*x^3+2*I*x+1)^(1/4)*RootOf(_Z^2+I)*x^2-2*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2
*I*x+1)^(1/4)*x-x^3-I*(-x^4+2*I*x^3+2*I*x+1)^(1/2)*x+RootOf(_Z^2+I)*(-x^4+2
*I*x^3+2*I*x+1)^(3/4)+I*RootOf(_Z^2+I)*(-x^4+2*I*x^3+2*I*x+1)^(1/4)+2*I*x^2
-(-x^4+2*I*x^3+2*I*x+1)^(1/2)+x)/(I*x+1)^2))/((I*x+1)*a)^(3/4)*(-(I*x-1)*(I
*x+1)^3)^(1/4)/(-(I*x-1)*a)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax + a)^{\frac{3}{4}}}{(i\,ax + a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(3/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(3/4)/(I*a*x + a)^(7/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - a\,x\,1\mathrm{i})^{3/4}}{(a + a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4),x)
```

```
[Out] int((a - a*x*1i)^(3/4)/(a + a*x*1i)^(7/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia\,(x+i))^{\frac{3}{4}}}{(ia\,(x-i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(3/4)/(a+I*a*x)**(7/4),x)
[Out] Integral((-I*a*(x + I))**(3/4)/(I*a*(x - I))**(7/4), x)
```

3.1115 $\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Rubi [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx = \frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.09, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{3/4}}{3a^2(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(1/4)*(a + I*a*x)^(7/4)),x]
[Out] (((2*I)/3)*(a - I*a*x)^(3/4))/(a^2*(a + I*a*x)^(3/4))
```

fricas [A] time = 1.51, size = 32, normalized size = 0.97

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}}{3\,a^3x-3i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)/(3*a^3*x - 3*I*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

maple [A] time = 0.04, size = 31, normalized size = 0.94

$$\frac{\frac{2x}{3}+\frac{2i}{3}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(1/4)/(I*a*x+a)^(7/4),x)
[Out] 2/3/a/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(1/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{1/4}\,{\left(a+a\,x\,1{}\mathrm{i}\right)}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)),x)
[Out] int(1/((a - a*x*1i)^(1/4)*(a + a*x*1i)^(7/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,\sqrt[4]{-ia\,(x+i)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(1/4)/(a+I*a*x)**(7/4),x)
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(1/4)), x)
```

3.1116 $\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=65

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Rubi [A] time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}-\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I)/(a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((4*I)/3)*(a - I*a*x)^(
3/4))/(a^3*(a + I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{2\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{a}\\ &= -\frac{2i}{a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{4i(a-iax)^{3/4}}{3a^3(a+iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.58

$$\frac{4x-2i}{3a^2\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (-2*I + 4*x)/(3*a^2*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.10, size = 55, normalized size = 0.85

$$-\frac{i(a-iax)^{3/4}\left(-1+\frac{3(a+iax)}{a-iax}\right)}{3a^3(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(5/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/3*I)*(a - I*a*x)^(3/4)*(-1 + (3*(a + I*a*x))/(a - I*a*x)))/(a^3*(a + I
*a*x)^(3/4))
```

fricas [A] time = 1.10, size = 36, normalized size = 0.55

$$\frac{(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}(4\,x-2i)}{3\left(a^4x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(4*x - 2*I)/(a^4*x^2 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

maple [A] time = 0.05, size = 33, normalized size = 0.51

$$\frac{\frac{4x}{3}-\frac{2i}{3}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(5/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 2/3/a^2/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(2*x-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(5/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{(a-a\,x\,1\mathrm{i})}^{5/4}\,{(a+a\,x\,1\mathrm{i})}^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(5/4)*(a + a*x*1i)^(7/4)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{7}{4}}\left(-ia\left(x+i\right)\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(5/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(5/4)), x)
```

3.1117 $\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}-\frac{8i}{5a^3(a+iax)^{3/4}\sqrt[4]{a-iax}}-\frac{2i}{5a^2(a+iax)^{3/4}(a-iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-2*I)/5)/(a^2*(a - I*a*x)^(5/4)*(a + I*a*x)^(3/4)) - ((8*I)/5)/(a^3*(a -
I*a*x)^(1/4)*(a + I*a*x)^(3/4)) + (((16*I)/15)*(a - I*a*x)^(3/4))/(a^4*(a +
 I*a*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{9/4}(a+iax)^{7/4}}\,dx &= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}+\frac{4\int \frac{1}{(a-iax)^{5/4}(a+iax)^{7/4}}\,dx}{5a}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{8\int \frac{1}{\sqrt[4]{a-iax}\,(a+iax)^{7/4}}\,dx}{5a^2}\\ &= -\frac{2i}{5a^2(a-iax)^{5/4}(a+iax)^{3/4}}-\frac{8i}{5a^3\sqrt[4]{a-iax}\,(a+iax)^{3/4}}+\frac{16i(a-iax)^{3/4}}{15a^4(a+iax)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.50

$$\frac{2\left(8x^2+4ix+7\right)}{15a^3(x+i)\sqrt[4]{a-iax}\,(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] (2*(7 + (4*I)*x + 8*x^2))/(15*a^3*(I + x)*(a - I*a*x)^(1/4)*(a + I*a*x)^(3/
4))
```

IntegrateAlgebraic [A] time = 0.11, size = 77, normalized size = 0.77

$$-\frac{i(a-iax)^{3/4}\left(\frac{3(a+iax)^2}{(a-iax)^2}+\frac{30(a+iax)}{a-iax}-5\right)}{30a^4(a+iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(9/4)*(a + I*a*x)^(7/4)),x]
```

```
[Out] ((-1/30*I)*(a - I*a*x)^(3/4)*(-5 + (30*(a + I*a*x))/(a - I*a*x) + (3*(a + I
*a*x)^2)/(a - I*a*x)^2))/(a^4*(a + I*a*x)^(3/4))
```

fricas [A] time = 1.51, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{1}{4}}(-i\,ax+a)^{\frac{3}{4}}\left(8\,x^2+4i\,x+7\right)}{15\,a^5x^3+15i\,a^5x^2+15\,a^5x+15i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(1/4)*(-I*a*x + a)^(3/4)*(8*x^2 + 4*I*x + 7)/(15*a^5*x^3 + 15
*I*a^5*x^2 + 15*a^5*x + 15*I*a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{7}{4}}(-i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(7/4)*(-I*a*x + a)^(9/4)), x)
```

maple [A] time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{15}x^2+\frac{8}{15}ix+\frac{14}{15}}{((ix+1)\,a)^{\frac{3}{4}}\,(-(ix-1)\,a)^{\frac{1}{4}}\,(x+i)\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(9/4)/(I*a*x+a)^(7/4),x)
```

```
[Out] 2/15/a^3/((I*x+1)*a)^(3/4)/(-(I*x-1)*a)^(1/4)*(8*x^2+4*I*x+7)/(x+I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(9/4)/(a+I*a*x)^(7/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a-a\,x\,1\mathrm{i})^{9/4}\,(a+a\,x\,1\mathrm{i})^{7/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)),x)
```

```
[Out] int(1/((a - a*x*1i)^(9/4)*(a + a*x*1i)^(7/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia\,(x-i))^{\frac{7}{4}}\,(-ia\,(x+i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(9/4)/(a+I*a*x)**(7/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(7/4)*(-I*a*(x + I))**(9/4)), x)
```

3.1118 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=287

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\,t}{}$$

Rubi [A] time = 0.18, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {47, 50, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}}+\frac{5i(a+iax)^{3/4}\sqrt[4]{a-iax}}{a}+\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}-\frac{5i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{2\sqrt{2}}+\frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}-\frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(1/4)) + ((5*I)*(a - I*a*x)^(1/4)*
(a + I*a*x)^(3/4))/a + ((5*I)*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I
*a*x)^(1/4)])/Sqrt[2] - ((5*I)*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4))/(a +
I*a*x)^(1/4)])/Sqrt[2] + (((5*I)/2)*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x]
 - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/Sqrt[2] - (((5*I)/2)*Log
[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a
*x)^(1/4)])/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{5/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} - 5\int \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{1}{2}(5a)\int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right) \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 10i\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - 5i\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - 5i\,\text{Subst} \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} - \frac{5}{2}i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right) - \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{2\sqrt{2}} - \frac{5i\log\left(1+\right.}{} \\
&= \frac{4i(a-iax)^{5/4}}{a\sqrt[4]{a+iax}} + \frac{5i\sqrt[4]{a-iax}\,(a+iax)^{3/4}}{a} + \frac{5i\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}} - \frac{5i\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-}}{\sqrt[4]{a+ia}}\right.}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.24

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{9/4}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/9)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[5/4, 9/4
, 13/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.71, size = 146, normalized size = 0.51

$$-\frac{\sqrt[4]{-1}\,(x-i)^{5/4}(a-iax)^{5/4}\left(\frac{(-1)^{3/4}(x+i)^{5/4}+10\sqrt[4]{-1}\sqrt[4]{x+i}}{\sqrt[4]{x-i}}-5\sqrt[4]{-1}\tan^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)-5\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{x+i}}{\sqrt[4]{x-i}}\right)\right)}{(x+i)^{5/4}(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(5/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] -(((-1)^(1/4)*(-I + x)^(5/4)*(a - I*a*x)^(5/4)*((10*(-1)^(1/4)*(I + x)^(1/4
) + (-1)^(3/4)*(I + x)^(5/4))/(-I + x)^(1/4) - 5*(-1)^(1/4)*ArcTan[(I + x)^
(1/4)/(-I + x)^(1/4)] - 5*(-1)^(1/4)*ArcTanh[(I + x)^(1/4)/(-I + x)^(1/4)])
)/((I + x)^(5/4)*(a + I*a*x)^(5/4)))
```

fricas [A] time = 1.28, size = 240, normalized size = 0.84

$$-\frac{\sqrt{25i}\,(ax-ia)\log\left(\frac{\sqrt{25i}\,(ax-ia)+5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)-\sqrt{25i}\,(ax-ia)\log\left(-\frac{\sqrt{25i}\,(ax-ia)-5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)+\sqrt{-25i}\,(ax-ia)\log\left(\frac{\sqrt{-25i}\,(ax-ia)+5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)-\sqrt{-25i}\,(ax-ia)\log\left(-\frac{\sqrt{-25i}\,(ax-ia)-5\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{5x-5i}\right)+2\,(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(-ix-9)}{2ax-2ia}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] -(sqrt(25*I)*(a*x - I*a)*log((sqrt(25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4)*
(-I*a*x + a)^(1/4))/(5*x - 5*I)) - sqrt(25*I)*(a*x - I*a)*log(-(sqrt(25*I)*
(a*x - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + sqrt(-
25*I)*(a*x - I*a)*log((sqrt(-25*I)*(a*x - I*a) + 5*(I*a*x + a)^(3/4)*(-I*a*
x + a)^(1/4))/(5*x - 5*I)) - sqrt(-25*I)*(a*x - I*a)*log(-(sqrt(-25*I)*(a*x
 - I*a) - 5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(5*x - 5*I)) + 2*(I*a*x +
 a)^(3/4)*(-I*a*x + a)^(1/4)*(-I*x - 9))/(2*a*x - 2*I*a)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(i/4*a/a^2*(1
6*((i*a*x+a)^(1/4))^4*(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)-32*a*(-((i*a*x+a)^(1
/4))^4+2*a)^(1/4))/((i*a*x+a)^(1/4))^2/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

maple [C] time = 2.04, size = 481, normalized size = 1.68

$$-\frac{i\left(x^{2}-8ix+9\right)\left(-(ix-1)a\right)^{\frac{1}{4}}}{(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}}-\frac{\left(\frac{5i\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(\frac{-x^{3}+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}-i\right)-2ix^{2}-2\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x+x+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}-2ix^{3}-2ix+1}}{(ix-1)^{2}}\right)}{2}-\frac{5\,\mathrm{RootOf}\left(\_Z^{2}-i\right)\ln\left(-\frac{x^{3}+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+2ix^{2}+2i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}-i\right)+i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x-x+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}-i\right)-\sqrt{-x^{4}-2ix^{3}-2ix+1}}{(ix-1)^{2}}\right)}{2}\right)\left(-(ix-1)a\right)^{\frac{1}{4}}\left(-(ix-1)^{3}(ix+1)\right)^{\frac{1}{4}}}{(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(5/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] -I*(x^2+9-8*I*x)*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(-5/2*RootOf(
_Z^2-I)*ln(-(RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_Z^2-
I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)+x^3+2*I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+
1)^(1/4)*x+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x+2*I*x^2-RootOf(_Z^2-I)*(-x^4-2*
I*x^3-2*I*x+1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)-x)/(I*x-1)^2)+5/2*I*RootO
f(_Z^2-I)*ln((I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2-2*RootOf(_Z
^2-I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2-I)*(-x^4-2*I*x^3-2*I*x
+1)^(3/4)+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-I*RootOf(_Z^2-I)*(-x^4-2*I*x^3-2
*I*x+1)^(1/4)-2*I*x^2-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))*(-(I*x-1)
*a)^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{5/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(5/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(5/4), x)
```

3.1119 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=264

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}a}+\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}-\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) + (I*Sqrt[2]*ArcTan[1 - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a - (I*Sqrt[2]*ArcTan[1 + (Sqr
t[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (I*Log[1 + Sqrt[a - I*a*x]/
Sqrt[a + I*a*x] - (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*
a) - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx = \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Su}}{}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}a} - \frac{\left(i\sqrt{2}\right)\text{Subs}}{}$$

$$= \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.02, size = 70, normalized size = 0.27

$$\frac{i2^{3/4}\sqrt[4]{1+ix}\,(a-iax)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{1}{2}-\frac{ix}{2}\right)}{5a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((I/5)*2^(3/4)*(1 + I*x)^(1/4)*(a - I*a*x)^(5/4)*Hypergeometric2F1[5/4, 5/4
, 9/4, 1/2 - (I/2)*x])/(a^2*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 114, normalized size = 0.43

$$\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{2\sqrt[4]{-1}\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(5/4),x]
```

```
[Out] ((4*I)*(a - I*a*x)^(1/4))/(a*(a + I*a*x)^(1/4)) - (2*(-1)^(1/4)*ArcTanh[((-
1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a + (2*(-1)^(3/4)*ArcTanh[(
(-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/a
```

fricas [A] time = 0.93, size = 303, normalized size = 1.15

$$-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right) - \left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right) + \left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}+2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right) - \left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}-2\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2x-2i}\right) - 8\left(iax+a\right)^{\frac{3}{4}}\left(-iax+a\right)^{\frac{1}{4}}}{2a^2x-2ia^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")

[Out] -((a^2*x - I*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4*I/a^2) + 2*(I*a *x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (a^2*x - I*a^2)*sqrt(4*I/a ^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^ (1/4))/(2*x - 2*I)) + (a^2*x - I*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*s qrt(-4*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (a^2 *x - I*a^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - 8*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*a^2*x - 2*I*a^2)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:

maple [C] time = 2.01, size = 476, normalized size = 1.80

$$-\frac{4 (x+i) \left(-(i x-1) a\right)^{\frac{1}{4}}}{(i x-1) \left((i x+1) a\right)^{\frac{1}{4}} a}+\frac{\left(\operatorname{RootOf}\left(\_Z^{2}+i\right) \ln\left(\frac{-x^{3}-\left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} x^{2} \operatorname{RootOf}\left(\_Z^{2}+i\right)-2 i x^{2}-2 i \left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} x \operatorname{RootOf}\left(\_Z^{2}+i\right)+i \sqrt{-x^{4}-2 i x^{3}-2 i x+1}\, x+x+i \left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{3}{4}} \operatorname{RootOf}\left(\_Z^{2}+i\right)+\left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^{2}+i\right)-\sqrt{-x^{4}-2 i x^{3}-2 i x+1}}{(i x-1)^{2}}\right)+i \operatorname{RootOf}\left(\_Z^{2}+i\right) \ln\left(\frac{-x^{3}-i \left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} x^{2} \operatorname{RootOf}\left(\_Z^{2}+i\right)-2 i x^{2}+2 \left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} x \operatorname{RootOf}\left(\_Z^{2}+i\right)-i \sqrt{-x^{4}-2 i x^{3}-2 i x+1}\, x+x+\left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{3}{4}} \operatorname{RootOf}\left(\_Z^{2}+i\right)+i \left(-x^{4}-2 i x^{3}-2 i x+1\right)^{\frac{1}{4}} \operatorname{RootOf}\left(\_Z^{2}+i\right)+\sqrt{-x^{4}-2 i x^{3}-2 i x+1}}{(i x-1)^{2}}\right)\right) \left(-(i x-1) a\right)^{\frac{1}{4}} \left(-(i x-1)^{3} (i x+1)\right)^{\frac{1}{4}}}{(i x-1) \left((i x+1) a\right)^{\frac{1}{4}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(5/4),x)

[Out] -4*(x+I)/a*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)+(RootOf(_Z^2+I)*ln((-RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_Z^2+I)*(-x^4-2* I*x^3-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x+ I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x +1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2)+I*RootOf(_Z^2+I)*ln((- I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+2*RootOf(_Z^2+I)*(-x^4-2* I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)-I*(- x^4-2*I*x^3-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)- 2*I*x^2+(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))/a*(-(I*x-1)*a)^(1/4)/(I *x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{1}{4}}}{(i\,ax+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")

[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(5/4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-a\,x\,1\mathrm{i})^{1/4}}{(a+a\,x\,1\mathrm{i})^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4),x)

[Out] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(5/4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\,(x+i)}}{(ia\,(x-i))^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(5/4), x)
```

3.1120 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=31

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Rubi [A] time = 0.00, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx = \frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.07, size = 31, normalized size = 1.00

$$\frac{2i\sqrt[4]{a-iax}}{a^2\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((2*I)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(1/4))
```

fricas [A] time = 0.97, size = 31, normalized size = 1.00

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{a^3x-i\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)/(a^3*x - I*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

maple [A] time = 0.04, size = 31, normalized size = 1.00

$$\frac{2x+2i}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(5/4),x)
[Out] 2/a/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(3/4)), x)
```

mupad [B] time = 1.16, size = 27, normalized size = 0.87

$$\frac{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,2\mathrm{i}}{a^2\,\left(a\,\left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(5/4)),x)
[Out] ((-a*(x*1i - 1))^(1/4)*2i)/(a^2*(a*(x*1i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(5/4),x)
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(3/4)), x)
```

3.1121 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}-\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((4*I)/3)*(a - I*a*
x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{3a}\\ &= -\frac{2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{4i\sqrt[4]{a-iax}}{3a^3\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 0.57

$$\frac{4x+2i}{3a^2(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (2*I + 4*x)/(3*a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 55, normalized size = 0.82

$$\frac{i(a+iax)^{3/4}\left(-1+\frac{3(a-iax)}{a+iax}\right)}{3a^3(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((I/3)*(a + I*a*x)^(3/4)*(-1 + (3*(a - I*a*x))/(a + I*a*x)))/(a^3*(a - I*a*
x)^(3/4))
```

fricas [A] time = 1.50, size = 36, normalized size = 0.54

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x+2i)}{3\left(a^4x^2+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/3*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x + 2*I)/(a^4*x^2 + a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

maple [A] time = 0.05, size = 33, normalized size = 0.49

$$\frac{\frac{4x}{3}+\frac{2i}{3}}{(-(ix-1)\,a)^{\frac{3}{4}}\,((ix+1)\,a)^{\frac{1}{4}}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] 2/3/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(7/4)), x)
```

mupad [B] time = 0.60, size = 40, normalized size = 0.60

$$-\frac{2\,(2\,x+1\mathrm{i})\,{\left(-a\,(-1+x\,1\mathrm{i})\right)}^{1/4}}{3\,a^3\,(-1+x\,1\mathrm{i})\,{\left(a\,(1+x\,1\mathrm{i})\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -(2*(2*x + 1i)*(-a*(x*1i - 1))^(1/4))/(3*a^3*(x*1i - 1)*(a*(x*1i + 1))^(1/4
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{5}{4}}(-ia(x+i))^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(7/4)), x)
```

3.1122 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}-\frac{8i}{21a^3\sqrt[4]{a+iax}\,(a-iax)^{3/4}}-\frac{2i}{7a^2\sqrt[4]{a+iax}\,(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(1/4)) - ((8*I)/21)/(a^3*(a -
 I*a*x)^(3/4)*(a + I*a*x)^(1/4)) + (((16*I)/21)*(a - I*a*x)^(1/4))/(a^4*(a
+ I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{11/4}(a+iax)^{5/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}+\frac{4\int \frac{1}{(a-iax)^{7/4}(a+iax)^{5/4}}\,dx}{7a}\\ &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{21a^2}\\ &= -\frac{2i}{7a^2(a-iax)^{7/4}\sqrt[4]{a+iax}}-\frac{8i}{21a^3(a-iax)^{3/4}\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{21a^4\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.50

$$\frac{16x^2+24ix-2}{21a^3(x+i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] (-2 + (24*I)*x + 16*x^2)/(21*a^3*(I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1/4
))
```

IntegrateAlgebraic [A] time = 0.14, size = 77, normalized size = 0.77

$$\frac{i(a+iax)^{7/4}\left(\frac{21(a-iax)^2}{(a+iax)^2}-\frac{14(a-iax)}{a+iax}-3\right)}{42a^4(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(5/4)),x]
```

```
[Out] ((I/42)*(a + I*a*x)^(7/4)*(-3 + (21*(a - I*a*x)^2)/(a + I*a*x)^2 - (14*(a -
 I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(7/4))
```

fricas [A] time = 0.88, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2+12i\,x-1\right)}{21\,a^5x^3+21i\,a^5x^2+21\,a^5x+21i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 + 12*I*x - 1)/(21*a^5*x^3 + 2
1*I*a^5*x^2 + 21*a^5*x + 21*I*a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{5}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(5/4)*(-I*a*x + a)^(11/4)), x)
```

maple [A] time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{21}x^2+\frac{8}{7}ix-\frac{2}{21}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x+i\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(5/4),x)
```

```
[Out] 2/21/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(8*x^2+12*I*x-1)/(x+I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(5/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 0.76, size = 46, normalized size = 0.46

$$-\frac{(-a\ (-1+x\,1\mathrm{i}))^{1/4}\,\left(8\,x^2+x\,12\mathrm{i}-1\right)\,2\mathrm{i}}{21\,a^4\,(-1+x\,1\mathrm{i})^2\,(a\ (1+x\,1\mathrm{i}))^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(5/4)),x)
```

```
[Out] -((-a*(x*1i - 1))^(1/4)*(x*12i + 8*x^2 - 1)*2i)/(21*a^4*(x*1i - 1)^2*(a*(x*
1i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{5}{4}}\left(-ia\left(x+i\right)\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(5/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(5/4)*(-I*a*(x + I))**(11/4)), x)
```

3.1123 $\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=297

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\right.}{a}$$

Rubi [A] time = 0.14, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {47, 63, 240, 211, 1165, 628, 1162, 617, 204}

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}}-\frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}}-\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}+\frac{i\log\left(\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}+1\right)}{\sqrt{2}\,a}-\frac{i\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}+\frac{i\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(5/4))/(a*(a + I*a*x)^(5/4)) - ((4*I)*(a - I*a*x)^(1
/4))/(a*(a + I*a*x)^(1/4)) - (I*Sqrt[2]*ArcTan[1 - (Sqrt[2]*(a - I*a*x)^(1/
4))/(a + I*a*x)^(1/4)])/a + (I*Sqrt[2]*ArcTan[1 + (Sqrt[2]*(a - I*a*x)^(1/4
))/(a + I*a*x)^(1/4)])/a - (I*Log[1 + Sqrt[a - I*a*x]/Sqrt[a + I*a*x] - (Sq
rt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)])/(Sqrt[2]*a) + (I*Log[1 + Sqrt[
a - I*a*x]/Sqrt[a + I*a*x] + (Sqrt[2]*(a - I*a*x)^(1/4))/(a + I*a*x)^(1/4)]
)/(Sqrt[2]*a)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-iax)^{5/4}}{(a+iax)^{9/4}}\,dx &= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \int \frac{\sqrt[4]{a-iax}}{(a+iax)^{5/4}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \int \frac{1}{(a-iax)^{3/4}\sqrt[4]{a+iax}}\,dx \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{2a-x^4}}\,dx, x, \sqrt[4]{a-iax}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(4i)\,\text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{(2i)\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{(2i)\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx,\right.}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} + \frac{i\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\right.}{a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{\sqrt{2}\,a} + \frac{i\log\left(1+\frac{\sqrt{a-iax}}{\sqrt{a+iax}}+\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{a+}}\right.}{\sqrt{2}\,a} \\
&= \frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i\sqrt[4]{a-iax}}{a\sqrt[4]{a+iax}} - \frac{i\sqrt{2}\,\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} + \frac{i\sqrt{2}\,\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} -
\end{aligned}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.24

$$\frac{i\sqrt[4]{1+ix}\,(a-iax)^{9/4}\,{}_2F_1\left(\frac{9}{4},\frac{9}{4};\frac{13}{4};\frac{1}{2}-\frac{ix}{2}\right)}{9\sqrt[4]{2}\,a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] ((I/9)*(1 + I*x)^(1/4)*(a - I*a*x)^(9/4)*Hypergeometric2F1[9/4, 9/4, 13/4,
1/2 - (I/2)*x])/(2^(1/4)*a^3*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 137, normalized size = 0.46

$$\frac{4i\sqrt[4]{a-iax}\left(-5+\frac{a-iax}{a+iax}\right)}{5a\sqrt[4]{a+iax}} + \frac{2\sqrt[4]{-1}\,\tanh^{-1}\left(\frac{\sqrt[4]{-1}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a} - \frac{2(-1)^{3/4}\tanh^{-1}\left(\frac{(-1)^{3/4}\sqrt[4]{a-iax}}{\sqrt[4]{a+iax}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(5/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((4*I)/5)*(a - I*a*x)^(1/4)*(-5 + (a - I*a*x)/(a + I*a*x)))/(a*(a + I*a*x)
^(1/4)) + (2*(-1)^(1/4)*ArcTanh[((-1)^(1/4)*(a - I*a*x)^(1/4))/(a + I*a*x)^
(1/4)])/a - (2*(-1)^(3/4)*ArcTanh[((-1)^(3/4)*(a - I*a*x)^(1/4))/(a + I*a*x
)^(1/4)])/a
```

fricas [A] time = 1.49, size = 351, normalized size = 1.18

$$\frac{\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)+\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}+2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-\left(5a^2x^2-10ia^2x-5a^2\right)\sqrt{-\frac{4i}{a^2}}\log\left(-\frac{\left(a^2x-ia^2\right)\sqrt{-\frac{4i}{a^2}}-2(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}}{2x-2i}\right)-(iax+a)^{\frac{3}{4}}(-iax+a)^{\frac{1}{4}}(48x-32i)}{10a^2x^2-20ia^2x-10a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] ((5*a^2*x^2 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(4
*I/a^2) + 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2
 - 10*I*a^2*x - 5*a^2)*sqrt(4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(4*I/a^2) -
2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) + (5*a^2*x^2 - 10*I*a^
2*x - 5*a^2)*sqrt(-4*I/a^2)*log(((a^2*x - I*a^2)*sqrt(-4*I/a^2) + 2*(I*a*x
+ a)^(3/4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (5*a^2*x^2 - 10*I*a^2*x - 5*a
^2)*sqrt(-4*I/a^2)*log(-((a^2*x - I*a^2)*sqrt(-4*I/a^2) - 2*(I*a*x + a)^(3/
4)*(-I*a*x + a)^(1/4))/(2*x - 2*I)) - (I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*
(48*x - 32*I))/(10*a^2*x^2 - 20*I*a^2*x - 10*a^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:ext_reduce Error: Bad Argument Typeintegrate(i/4*a/a^2*(1
6*((i*a*x+a)^(1/4))^4*(-((i*a*x+a)^(1/4))^4+2*a)^(1/4)-32*a*(-((i*a*x+a)^(1
/4))^4+2*a)^(1/4))/((i*a*x+a)^(1/4))^6/4*i*a*((i*a*x+a)^(1/4))^-3,x)
```

maple [C] time = 0.06, size = 490, normalized size = 1.65

$$\frac{8\left(3x^{2}+ix+2\right)\left(-(ix-1)a\right)^{\frac{1}{4}}}{5(x-i)(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}a}-\frac{\left(\mathrm{RootOf}\left(\_Z^{2}+i\right)\ln\left(\frac{-x^{3}-\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}+i\right)-2ix^{2}-2i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}+i\right)+i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x+x+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}+i\right)+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+i\right)-\sqrt{-x^{4}-2ix^{3}-2ix+1}}{(ix-1)^{2}}\right)+i\,\mathrm{RootOf}\left(\_Z^{2}+i\right)\ln\left(\frac{-x^{3}-i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x^{2}\,\mathrm{RootOf}\left(\_Z^{2}+i\right)-2ix^{2}+2\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}x\,\mathrm{RootOf}\left(\_Z^{2}+i\right)-i\sqrt{-x^{4}-2ix^{3}-2ix+1}\,x+x+\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{3}{4}}\mathrm{RootOf}\left(\_Z^{2}+i\right)+i\left(-x^{4}-2ix^{3}-2ix+1\right)^{\frac{1}{4}}\mathrm{RootOf}\left(\_Z^{2}+i\right)+\sqrt{-x^{4}-2ix^{3}-2ix+1}}{(ix-1)^{2}}\right)\right)\left(-(ix-1)a\right)^{\frac{1}{4}}\left(-(ix-1)^{3}(ix+1)\right)^{\frac{1}{4}}}{(ix-1)\left((ix+1)a\right)^{\frac{1}{4}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(5/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 8/5*(3*x^2+2+I*x)/(x-I)/a*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)-(Roo
tOf(_Z^2+I)*ln((-RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+I*RootOf(_
Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(3/4)-x^3-2*I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*
I*x+1)^(1/4)*x+I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x-2*I*x^2+RootOf(_Z^2+I)*(-x^
4-2*I*x^3-2*I*x+1)^(1/4)-(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2)+I*RootO
f(_Z^2+I)*ln((-I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x^2+2*RootOf(_
Z^2+I)*(-x^4-2*I*x^3-2*I*x+1)^(1/4)*x-x^3+RootOf(_Z^2+I)*(-x^4-2*I*x^3-2*I*
x+1)^(3/4)-I*(-x^4-2*I*x^3-2*I*x+1)^(1/2)*x+I*RootOf(_Z^2+I)*(-x^4-2*I*x^3-
2*I*x+1)^(1/4)-2*I*x^2+(-x^4-2*I*x^3-2*I*x+1)^(1/2)+x)/(I*x-1)^2))/a*(-(I*x
-1)*a)^(1/4)/(I*x-1)*(-(I*x-1)^3*(I*x+1))^(1/4)/((I*x+1)*a)^(1/4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-i\,ax+a)^{\frac{5}{4}}}{(i\,ax+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(5/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(5/4)/(I*a*x + a)^(9/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a-a\,x\,1{}\mathrm{i}\right)}^{5/4}}{{\left(a+a\,x\,1{}\mathrm{i}\right)}^{9/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] int((a - a*x*1i)^(5/4)/(a + a*x*1i)^(9/4), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-ia(x+i))^{\frac{5}{4}}}{(ia(x-i))^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(5/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(5/4)/(I*a*(x - I))**(9/4), x)
```

3.1124 $\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=33

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Rubi [A] time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {37}

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[4]{a-iax}}{(a+iax)^{9/4}}\,dx = \frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

IntegrateAlgebraic [A] time = 0.11, size = 33, normalized size = 1.00

$$\frac{2i(a-iax)^{5/4}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - I*a*x)^(1/4)/(a + I*a*x)^(9/4),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(5/4))/(a^2*(a + I*a*x)^(5/4))
```

fricas [B] time = 1.43, size = 45, normalized size = 1.36

$$-\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(2\,x+2i)}{5\,a^3x^2-10i\,a^3x-5\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] -(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(2*x + 2*I)/(5*a^3*x^2 - 10*I*a^3*x -
 5*a^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:
```

maple [B] time = 0.04, size = 50, normalized size = 1.52

$$\frac{2\left(-\left(ix-1\right)a\right)^{\frac{1}{4}}\left(x^{2}+2ix-1\right)}{5\left(ix-1\right)\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x-i\right)a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-I*a*x+a)^(1/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/5/a^2*(-(I*x-1)*a)^(1/4)/(I*x-1)/((I*x+1)*a)^(1/4)*(2*I*x+x^2-1)/(x-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-i\,ax+a\right)^{\frac{1}{4}}}{\left(i\,ax+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)^(1/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((-I*a*x + a)^(1/4)/(I*a*x + a)^(9/4), x)
```

mupad [B] time = 0.55, size = 38, normalized size = 1.15

$$-\frac{2\,\left(-1+x\,1\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^2\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*x*1i)^(1/4)/(a + a*x*1i)^(9/4),x)
```

```
[Out] -(2*(x*1i - 1)*(-a*(x*1i - 1))^(1/4))/(5*a^2*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{-ia\left(x+i\right)}}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-I*a*x)**(1/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral((-I*a*(x + I))**(1/4)/(I*a*(x - I))**(9/4), x)
```

3.1125 $\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=67

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Rubi [A] time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}+\frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (((2*I)/5)*(a - I*a*x)^(1/4))/(a^2*(a + I*a*x)^(5/4)) + (((4*I)/5)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx &= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{2\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{5a}\\ &= \frac{2i\sqrt[4]{a-iax}}{5a^2(a+iax)^{5/4}}+\frac{4i\sqrt[4]{a-iax}}{5a^3\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.67

$$\frac{2(3+2ix)\sqrt[4]{a-iax}}{5a^3(x-i)\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(3 + (2*I)*x)*(a - I*a*x)^(1/4))/(5*a^3*(-I + x)*(a + I*a*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.14, size = 54, normalized size = 0.81

$$\frac{i\sqrt[4]{a-iax}\left(5+\frac{a-iax}{a+iax}\right)}{5a^3\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(3/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/5)*(a - I*a*x)^(1/4)*(5 + (a - I*a*x)/(a + I*a*x)))/(a^3*(a + I*a*x)^(1
/4))
```

fricas [A] time = 1.36, size = 44, normalized size = 0.66

$$\frac{(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}(4\,x-6i)}{5\left(a^4x^2-2i\,a^4x-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/5*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(4*x - 6*I)/(a^4*x^2 - 2*I*a^4*x -
 a^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

maple [A] time = 0.04, size = 44, normalized size = 0.66

$$\frac{\frac{4}{5}x^2-\frac{2}{5}ix+\frac{6}{5}}{\left(-(ix-1)\,a\right)^{\frac{3}{4}}\left((ix+1)\,a\right)^{\frac{1}{4}}(x-i)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(3/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/5/a^2/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(2*x^2+3-I*x)/(x-I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(3/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(3/4)), x)
```

mupad [B] time = 0.63, size = 38, normalized size = 0.57

$$\frac{2\,\left(3+x\,2\mathrm{i}\right)\,{\left(-a\,\left(-1+x\,1\mathrm{i}\right)\right)}^{1/4}}{5\,a^3\,\left(x-\mathrm{i}\right)\,{\left(a\,\left(1+x\,1\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(3/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(x*2i + 3)*(-a*(x*1i - 1))^(1/4))/(5*a^3*(x - 1i)*(a*(x*1i + 1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ia(x-i))^{\frac{9}{4}}(-ia(x+i))^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(3/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(3/4)), x)
```

3.1126 $\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=100

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Rubi [A] time = 0.02, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}-\frac{2i}{3a^2(a+iax)^{5/4}(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/3)/(a^2*(a - I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((8*I)/15)*(a - I*a
*x)^(1/4))/(a^3*(a + I*a*x)^(5/4)) + (((16*I)/15)*(a - I*a*x)^(1/4))/(a^4*(
a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{4\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,dx}{3a}\\ &= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{5/4}}\,dx}{15a^2}\\ &= -\frac{2i}{3a^2(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8i\sqrt[4]{a-iax}}{15a^3(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{15a^4\sqrt[4]{a+iax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 0.50

$$\frac{2\left(8x^2-4ix+7\right)}{15a^3(x-i)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(7 - (4*I)*x + 8*x^2))/(15*a^3*(-I + x)*(a - I*a*x)^(3/4)*(a + I*a*x)^(1
/4))
```

IntegrateAlgebraic [A] time = 0.14, size = 77, normalized size = 0.77

$$\frac{i(a+iax)^{3/4}\left(\frac{3(a-iax)^2}{(a+iax)^2}+\frac{30(a-iax)}{a+iax}-5\right)}{30a^4(a-iax)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(7/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/30)*(a + I*a*x)^(3/4)*(-5 + (3*(a - I*a*x)^2)/(a + I*a*x)^2 + (30*(a -
I*a*x))/(a + I*a*x)))/(a^4*(a - I*a*x)^(3/4))
```

fricas [A] time = 1.27, size = 58, normalized size = 0.58

$$\frac{2\,(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}\left(8\,x^2-4i\,x+7\right)}{15\,a^5x^3-15i\,a^5x^2+15\,a^5x-15i\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 2*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)*(8*x^2 - 4*I*x + 7)/(15*a^5*x^3 - 15
*I*a^5*x^2 + 15*a^5*x - 15*I*a^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(7/4)), x)
```

maple [A] time = 0.06, size = 44, normalized size = 0.44

$$\frac{\frac{16}{15}x^2-\frac{8}{15}ix+\frac{14}{15}}{\left(-\left(ix-1\right)a\right)^{\frac{3}{4}}\left(\left(ix+1\right)a\right)^{\frac{1}{4}}\left(x-i\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(7/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/15/a^3/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(8*x^2-4*I*x+7)/(x-I)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(7/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 0.53, size = 45, normalized size = 0.45

$$\frac{2\left(-a\ \left(-1+x\,1\mathrm{i}\right)\right)^{1/4}\,\left(x^2\,8\mathrm{i}+4\,x+7\mathrm{i}\right)}{15\,a^4\,\left(x^2+1\right)\,\left(a\ \left(1+x\,1\mathrm{i}\right)\right)^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(7/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(4*x + x^2*8i + 7i))/(15*a^4*(x^2 + 1)*(a*(x*1i +
1))^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ia\left(x-i\right)\right)^{\frac{9}{4}}\left(-ia\left(x+i\right)\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(7/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Integral(1/((I*a*(x - I))**(9/4)*(-I*a*(x + I))**(7/4)), x)
```

3.1127 $\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx$

Optimal. Leaf size=133

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Rubi [A] time = 0.03, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {45, 37}

$$\frac{32i\sqrt[4]{a-iax}}{35a^5\sqrt[4]{a+iax}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}-\frac{4i}{7a^3(a+iax)^{5/4}(a-iax)^{3/4}}-\frac{2i}{7a^2(a+iax)^{5/4}(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((-2*I)/7)/(a^2*(a - I*a*x)^(7/4)*(a + I*a*x)^(5/4)) - ((4*I)/7)/(a^3*(a -
I*a*x)^(3/4)*(a + I*a*x)^(5/4)) + (((16*I)/35)*(a - I*a*x)^(1/4))/(a^4*(a +
 I*a*x)^(5/4)) + (((32*I)/35)*(a - I*a*x)^(1/4))/(a^5*(a + I*a*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-iax)^{11/4}(a+iax)^{9/4}}\,dx &= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}+\frac{6\int \frac{1}{(a-iax)^{7/4}(a+iax)^{9/4}}\,dx}{7a}\\
&= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{8\int \frac{1}{(a-iax)^{3/4}(a+iax)^{9/4}}\,d
\end{aligned}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{16}{}$$

$$= -\frac{2i}{7a^2(a-iax)^{7/4}(a+iax)^{5/4}}-\frac{4i}{7a^3(a-iax)^{3/4}(a+iax)^{5/4}}+\frac{16i\sqrt[4]{a-iax}}{35a^4(a+iax)^{5/4}}+\frac{3}{35}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 0.43

$$\frac{2\left(16x^3+8ix^2+22x+9i\right)}{35a^4\left(x^2+1\right)(a-iax)^{3/4}\sqrt[4]{a+iax}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] (2*(9*I + 22*x + (8*I)*x^2 + 16*x^3))/(35*a^4*(a - I*a*x)^(3/4)*(a + I*a*x)
^(1/4)*(1 + x^2))
```

IntegrateAlgebraic [A] time = 0.14, size = 99, normalized size = 0.74

$$\frac{i(a+iax)^{7/4}\left(\frac{7(a-iax)^3}{(a+iax)^3}+\frac{105(a-iax)^2}{(a+iax)^2}-\frac{35(a-iax)}{a+iax}-5\right)}{140a^5(a-iax)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a - I*a*x)^(11/4)*(a + I*a*x)^(9/4)),x]
```

```
[Out] ((I/140)*(a + I*a*x)^(7/4)*(-5 + (7*(a - I*a*x)^3)/(a + I*a*x)^3 + (105*(a
- I*a*x)^2)/(a + I*a*x)^2 - (35*(a - I*a*x))/(a + I*a*x)))/(a^5*(a - I*a*x)
^(7/4))
```

fricas [A] time = 1.49, size = 54, normalized size = 0.41

$$\frac{\left(32\,x^3+16i\,x^2+44\,x+18i\right)(i\,ax+a)^{\frac{3}{4}}(-i\,ax+a)^{\frac{1}{4}}}{35\left(a^6x^4+2\,a^6x^2+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="fricas")
```

```
[Out] 1/35*(32*x^3 + 16*I*x^2 + 44*x + 18*I)*(I*a*x + a)^(3/4)*(-I*a*x + a)^(1/4)
/(a^6*x^4 + 2*a^6*x^2 + a^6)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="giac")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

maple [A] time = 0.06, size = 56, normalized size = 0.42

$$\frac{\frac{32}{35}x^3+\frac{16}{35}ix^2+\frac{44}{35}x+\frac{18}{35}i}{(-(ix-1)\,a)^{\frac{3}{4}}\,((ix+1)\,a)^{\frac{1}{4}}\,(x-i)\,(x+i)\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-I*a*x+a)^(11/4)/(I*a*x+a)^(9/4),x)
```

```
[Out] 2/35/a^4/(-(I*x-1)*a)^(3/4)/((I*x+1)*a)^(1/4)*(16*x^3+8*I*x^2+22*x+9*I)/(x-
I)/(x+I)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(i\,ax+a)^{\frac{9}{4}}(-i\,ax+a)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)^(11/4)/(a+I*a*x)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((I*a*x + a)^(9/4)*(-I*a*x + a)^(11/4)), x)
```

mupad [B] time = 0.69, size = 56, normalized size = 0.42

$$\frac{2\,{\left(-a\,\left(-1+x\,1{}\mathrm{i}\right)\right)}^{1/4}\,\left(x^4\,16{}\mathrm{i}+8\,x^3+x^2\,30{}\mathrm{i}+13\,x+9{}\mathrm{i}\right)}{35\,a^5\,{\left(x^2+1\right)}^2\,{\left(a\,\left(1+x\,1{}\mathrm{i}\right)\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - a*x*1i)^(11/4)*(a + a*x*1i)^(9/4)),x)
```

```
[Out] (2*(-a*(x*1i - 1))^(1/4)*(13*x + x^2*30i + 8*x^3 + x^4*16i + 9i))/(35*a^5*(
x^2 + 1)^2*(a*(x*1i + 1))^(1/4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-I*a*x)**(11/4)/(a+I*a*x)**(9/4),x)
```

```
[Out] Timed out
```

3.1128 $\int (a+bx)^2(ac-bcx)^n\,dx$

Optimal. Leaf size=83

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {43}

$$-\frac{4a^2(ac-bcx)^{n+1}}{bc(n+1)}+\frac{4a(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{(ac-bcx)^{n+3}}{bc^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] (-4*a^2*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (4*a*(a*c - b*c*x)^(2 + n))/
(b*c^2*(2 + n)) - (a*c - b*c*x)^(3 + n)/(b*c^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(ac-bcx)^n\,dx &= \int \left(4a^2(ac-bcx)^n-\frac{4a(ac-bcx)^{1+n}}{c}+\frac{(ac-bcx)^{2+n}}{c^2}\right)dx\\ &= -\frac{4a^2(ac-bcx)^{1+n}}{bc(1+n)}+\frac{4a(ac-bcx)^{2+n}}{bc^2(2+n)}-\frac{(ac-bcx)^{3+n}}{bc^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.93

$$\frac{(bx-a)\left(a^2\left(n^2+7n+14\right)+2ab\left(n^2+5n+4\right)x+b^2\left(n^2+3n+2\right)x^2\right)(c(a-bx))^n}{b(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a^2*(14 + 7*n + n^2) + 2*a*b*(4 + 5*n + n^2)*x
 + b^2*(2 + 3*n + n^2)*x^2))/(b*(1 + n)*(2 + n)*(3 + n))
```

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (a+bx)^2(ac-bcx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(a*c - b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^2*(a*c - b*c*x)^n, x]
```

fricas [A] time = 1.25, size = 128, normalized size = 1.54

$$-\frac{\left(a^{3} n^{2} + 7 \, a^{3} n - \left(b^{3} n^{2} + 3 \, b^{3} n + 2 \, b^{3}\right) x^{3} + 14 \, a^{3} - \left(a b^{2} n^{2} + 7 \, a b^{2} n + 6 \, a b^{2}\right) x^{2} + \left(a^{2} b n^{2} + 3 \, a^{2} b n - 6 \, a^{2} b\right) x\right) (-b c x + a c)^{n}}{b n^{3} + 6 \, b n^{2} + 11 \, b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^3*n^2 + 7*a^3*n - (b^3*n^2 + 3*b^3*n + 2*b^3)*x^3 + 14*a^3 - (a*b^2*n^2
 + 7*a*b^2*n + 6*a*b^2)*x^2 + (a^2*b*n^2 + 3*a^2*b*n - 6*a^2*b)*x)*(-b*c*x
+ a*c)^n/(b*n^3 + 6*b*n^2 + 11*b*n + 6*b)
```

giac [B] time = 1.15, size = 256, normalized size = 3.08

$$\frac{(-bcx + ac)^{n} b^{3} n^{2} x^{3} + (-bcx + ac)^{n} a b^{2} n^{2} x^{2} + 3 \, (-bcx + ac)^{n} b^{3} n x^{3} - (-bcx + ac)^{n} a^{2} b n^{2} x + 7 \, (-bcx + ac)^{n} a b^{2} n x^{2} + 2 \, (-bcx + ac)^{n} b^{3} x^{3} - (-bcx + ac)^{n} a^{3} n^{2} - 3 \, (-bcx + ac)^{n} a^{2} b n x + 6 \, (-bcx + ac)^{n} a b^{2} x^{2} - 7 \, (-bcx + ac)^{n} a^{3} n + 6 \, (-bcx + ac)^{n} a^{2} b x - 14 \, (-bcx + ac)^{n} a^{3}}{b n^{3} + 6 \, b n^{2} + 11 \, b n + 6 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^3*n^2*x^3 + (-b*c*x + a*c)^n*a*b^2*n^2*x^2 + 3*(-b*c*x
+ a*c)^n*b^3*n*x^3 - (-b*c*x + a*c)^n*a^2*b*n^2*x + 7*(-b*c*x + a*c)^n*a*b^
2*n*x^2 + 2*(-b*c*x + a*c)^n*b^3*x^3 - (-b*c*x + a*c)^n*a^3*n^2 - 3*(-b*c*x
 + a*c)^n*a^2*b*n*x + 6*(-b*c*x + a*c)^n*a*b^2*x^2 - 7*(-b*c*x + a*c)^n*a^3
*n + 6*(-b*c*x + a*c)^n*a^2*b*x - 14*(-b*c*x + a*c)^n*a^3)/(b*n^3 + 6*b*n^2
 + 11*b*n + 6*b)
```

maple [A] time = 0.01, size = 103, normalized size = 1.24

$$-\frac{(-bx + a)\left(b^{2} n^{2} x^{2} + 2 a b\, n^{2} x + 3 b^{2} n\, x^{2} + a^{2} n^{2} + 10 a b n x + 2 b^{2} x^{2} + 7 a^{2} n + 8 a b x + 14 a^{2}\right) (-bcx + ac)^{n}}{\left(n^{3} + 6 n^{2} + 11 n + 6\right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*x+a)*(b^2*n^2*x^2+2*a*b*n^2*x+3*b^2*n*x^2+a^2*n^2+10*a*b*n*x+2*b^2*x^2
+7*a^2*n+8*a*b*x+14*a^2)*(-b*c*x+a*c)^n/b/(n^3+6*n^2+11*n+6)
```

maxima [B] time = 1.53, size = 167, normalized size = 2.01

$$\frac{2 \left(b^{2} c^{n} (n+1) x^{2} - a b c^{n} n x - a^{2} c^{n}\right) (-bx + a)^{n} a}{\left(n^{2} + 3 \, n + 2\right) b} + \frac{\left(\left(n^{2} + 3 \, n + 2\right) b^{3} c^{n} x^{3} - \left(n^{2} + n\right) a b^{2} c^{n} x^{2} - 2 \, a^{2} b c^{n} n x - 2 \, a^{3} c^{n}\right) (-bx + a)^{n}}{\left(n^{3} + 6 \, n^{2} + 11 \, n + 6\right) b} - \frac{(-bcx + ac)^{n+1} a^{2}}{b c (n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] 2*(b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n*a/((n^2 + 3*n
+ 2)*b) + ((n^2 + 3*n + 2)*b^3*c^n*x^3 - (n^2 + n)*a*b^2*c^n*x^2 - 2*a^2*b*
c^n*n*x - 2*a^3*c^n)*(-b*x + a)^n/((n^3 + 6*n^2 + 11*n + 6)*b) - (-b*c*x +
a*c)^(n + 1)*a^2/(b*c*(n + 1))
```

mupad [B] time = 0.49, size = 133, normalized size = 1.60

$$-(a\,c - b\,c\,x)^{n} \left(\frac{a^{2}\,x\,\left(n^{2} + 3\,n - 6\right)}{n^{3} + 6\,n^{2} + 11\,n + 6} + \frac{a^{3}\,\left(n^{2} + 7\,n + 14\right)}{b\,\left(n^{3} + 6\,n^{2} + 11\,n + 6\right)} - \frac{b^{2}\,x^{3}\,\left(n^{2} + 3\,n + 2\right)}{n^{3} + 6\,n^{2} + 11\,n + 6} - \frac{a\,b\,x^{2}\,\left(n^{2} + 7\,n + 6\right)}{n^{3} + 6\,n^{2} + 11\,n + 6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x)^2,x)
```

```
[Out] -(a*c - b*c*x)^n*((a^2*x*(3*n + n^2 - 6))/(11*n + 6*n^2 + n^3 + 6) + (a^3*(
7*n + n^2 + 14))/(b*(11*n + 6*n^2 + n^3 + 6)) - (b^2*x^3*(3*n + n^2 + 2))/(
11*n + 6*n^2 + n^3 + 6) - (a*b*x^2*(7*n + n^2 + 6))/(11*n + 6*n^2 + n^3 + 6
))
```

sympy [A] time = 1.30, size = 819, normalized size = 9.87

$$\begin{cases} a^2x\,(ac)^n & \text{for } b = 0 \\ -\frac{a^2\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} - \frac{2a^2}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} + \frac{2abx\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} + \frac{4abx}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} - \frac{b^2x^2\log\left(-\frac{a}{b}+x\right)}{a^2bc^3-2ab^2c^3x+b^3c^3x^2} & \text{for } n = -3 \\ -\frac{4a^2\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{5a^2}{-abc^2+b^2c^2x} + \frac{4abx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} + \frac{b^2x^2}{-abc^2+b^2c^2x} & \text{for } n = -2 \\ -\frac{4a^2\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{3ax}{c} - \frac{bx^2}{2c} & \text{for } n = -1 \\ -\frac{a^3n^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{7a^3n(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{14a^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{a^2bn^2x(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} - \frac{3a^2bnx(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{6a^2bx(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{ab^2n^2x^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{7ab^2nx^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{6ab^2x^2(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{b^3n^2x^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{3b^3nx^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} + \frac{2b^3x^3(ac-bcx)^n}{bn^3+6bn^2+11bn+6b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a**2*x*(a*c)**n, Eq(b, 0)), (-a**2*log(-a/b + x)/(a**2*b*c**3 -
2*a*b**2*c**3*x + b**3*c**3*x**2) - 2*a**2/(a**2*b*c**3 - 2*a*b**2*c**3*x +
 b**3*c**3*x**2) + 2*a*b*x*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b
**3*c**3*x**2) + 4*a*b*x/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2) -
 b**2*x**2*log(-a/b + x)/(a**2*b*c**3 - 2*a*b**2*c**3*x + b**3*c**3*x**2),
Eq(n, -3)), (-4*a**2*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) - 5*a**2/(-a*b
*c**2 + b**2*c**2*x) + 4*a*b*x*log(-a/b + x)/(-a*b*c**2 + b**2*c**2*x) + b*
*2*x**2/(-a*b*c**2 + b**2*c**2*x), Eq(n, -2)), (-4*a**2*log(-a/b + x)/(b*c)
 - 3*a*x/c - b*x**2/(2*c), Eq(n, -1)), (-a**3*n**2*(a*c - b*c*x)**n/(b*n**3
 + 6*b*n**2 + 11*b*n + 6*b) - 7*a**3*n*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2
+ 11*b*n + 6*b) - 14*a**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*
b) - a**2*b*n**2*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) - 3*
a**2*b*n*x*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a**2*b*x
*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + a*b**2*n**2*x**2*(a*
c - b*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 7*a*b**2*n*x**2*(a*c - b
*c*x)**n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + 6*a*b**2*x**2*(a*c - b*c*x)**
n/(b*n**3 + 6*b*n**2 + 11*b*n + 6*b) + b**3*n**2*x**3*(a*c - b*c*x)**n/(b*n
**3 + 6*b*n**2 + 11*b*n + 6*b) + 3*b**3*n*x**3*(a*c - b*c*x)**n/(b*n**3 + 6
*b*n**2 + 11*b*n + 6*b) + 2*b**3*x**3*(a*c - b*c*x)**n/(b*n**3 + 6*b*n**2 +
 11*b*n + 6*b), True))
```

3.1129 $\int (a+bx)(ac-bcx)^n\,dx$

Optimal. Leaf size=53

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{(ac-bcx)^{n+2}}{bc^2(n+2)}-\frac{2a(ac-bcx)^{n+1}}{bc(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] (-2*a*(a*c - b*c*x)^(1 + n))/(b*c*(1 + n)) + (a*c - b*c*x)^(2 + n)/(b*c^2*(
2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(ac-bcx)^n\,dx &= \int \left(2a(ac-bcx)^n-\frac{(ac-bcx)^{1+n}}{c}\right)dx\\ &= -\frac{2a(ac-bcx)^{1+n}}{bc(1+n)}+\frac{(ac-bcx)^{2+n}}{bc^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.81

$$\frac{(bx-a)(a(n+3)+b(n+1)x)(c(a-bx))^n}{b(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] ((c*(a - b*x))^n*(-a + b*x)*(a*(3 + n) + b*(1 + n)*x))/(b*(1 + n)*(2 + n))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (a+bx)(ac-bcx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(a*c - b*c*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)*(a*c - b*c*x)^n, x]
```

fricas [A] time = 1.34, size = 58, normalized size = 1.09

$$-\frac{\left(a^2n-2\,abx-\left(b^2n+b^2\right)x^2+3\,a^2\right)(-bcx+ac)^n}{bn^2+3\,bn+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="fricas")
```

```
[Out] -(a^2*n - 2*a*b*x - (b^2*n + b^2)*x^2 + 3*a^2)*(-b*c*x + a*c)^n/(b*n^2 + 3*
b*n + 2*b)
```

giac [A] time = 1.05, size = 103, normalized size = 1.94

$$\frac{(-bcx+ac)^n b^2nx^2 + (-bcx+ac)^n b^2x^2 - (-bcx+ac)^n a^2n + 2\,(-bcx+ac)^n abx - 3\,(-bcx+ac)^n a^2}{bn^2 + 3\,bn + 2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="giac")
```

```
[Out] ((-b*c*x + a*c)^n*b^2*n*x^2 + (-b*c*x + a*c)^n*b^2*x^2 - (-b*c*x + a*c)^n*a
^2*n + 2*(-b*c*x + a*c)^n*a*b*x - 3*(-b*c*x + a*c)^n*a^2)/(b*n^2 + 3*b*n +
2*b)
```

maple [A] time = 0.00, size = 47, normalized size = 0.89

$$-\frac{(bnx+an+bx+3a)\,(-bx+a)\,(-bcx+ac)^n}{\left(n^2+3n+2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(-b*c*x+a*c)^n,x)
```

```
[Out] -(-b*c*x+a*c)^n*(b*n*x+a*n+b*x+3*a)*(-b*x+a)/b/(n^2+3*n+2)
```

maxima [A] time = 1.40, size = 81, normalized size = 1.53

$$\frac{\left(b^2c^n(n+1)x^2 - abc^n nx - a^2c^n\right)(-bx+a)^n}{\left(n^2+3\,n+2\right)b} - \frac{(-bcx+ac)^{n+1}a}{bc(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)^n,x, algorithm="maxima")
```

```
[Out] (b^2*c^n*(n + 1)*x^2 - a*b*c^n*n*x - a^2*c^n)*(-b*x + a)^n/((n^2 + 3*n + 2)
*b) - (-b*c*x + a*c)^(n + 1)*a/(b*c*(n + 1))
```

mupad [B] time = 0.32, size = 66, normalized size = 1.25

$$(a\,c-b\,c\,x)^n\,\left(\frac{2\,a\,x}{n^2+3\,n+2}-\frac{a^2\,(n+3)}{b\,\left(n^2+3\,n+2\right)}+\frac{b\,x^2\,(n+1)}{n^2+3\,n+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c - b*c*x)^n*(a + b*x),x)
```

```
[Out] (a*c - b*c*x)^n*((2*a*x)/(3*n + n^2 + 2) - (a^2*(n + 3))/(b*(3*n + n^2 + 2)
) + (b*x^2*(n + 1))/(3*n + n^2 + 2))
```

sympy [A] time = 0.70, size = 245, normalized size = 4.62

$$\begin{cases} ax\,(ac)^n & \text{for}\ b=0 \\ -\frac{a\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} - \frac{2a}{-abc^2+b^2c^2x} + \frac{bx\log\left(-\frac{a}{b}+x\right)}{-abc^2+b^2c^2x} & \text{for}\ n=-2 \\ -\frac{2a\log\left(-\frac{a}{b}+x\right)}{bc} - \frac{x}{c} & \text{for}\ n=-1 \\ -\frac{a^2n(ac-bcx)^n}{bn^2+3bn+2b} - \frac{3a^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{2abx(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2nx^2(ac-bcx)^n}{bn^2+3bn+2b} + \frac{b^2x^2(ac-bcx)^n}{bn^2+3bn+2b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(-b*c*x+a*c)**n,x)
```

```
[Out] Piecewise((a*x*(a*c)**n, Eq(b, 0)), (-a*log(-a/b + x)/(-a*b*c**2 + b**2*c**
2*x) - 2*a/(-a*b*c**2 + b**2*c**2*x) + b*x*log(-a/b + x)/(-a*b*c**2 + b**2*
c**2*x), Eq(n, -2)), (-2*a*log(-a/b + x)/(b*c) - x/c, Eq(n, -1)), (-a**2*n*
(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) - 3*a**2*(a*c - b*c*x)**n/(b*n**2 +
 3*b*n + 2*b) + 2*a*b*x*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*n*x*
*2*(a*c - b*c*x)**n/(b*n**2 + 3*b*n + 2*b) + b**2*x**2*(a*c - b*c*x)**n/(b*
n**2 + 3*b*n + 2*b), True))
```

3.1130 $\int (a+bx)^4(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^5(bc-ad)}{5b^2}+\frac{d(a+bx)^6}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^5)/(5*b^2) + (d*(a + b*x)^6)/(6*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^4}{b}+\frac{d(a+bx)^5}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^5}{5b^2}+\frac{d(a+bx)^6}{6b^2}\end{aligned}$$

Mathematica [B] time = 0.02, size = 84, normalized size = 2.21

$$\frac{1}{30}x\left(15a^4(2c+dx)+20a^3bx(3c+2dx)+15a^2b^2x^2(4c+3dx)+6ab^3x^3(5c+4dx)+b^4x^4(6c+5dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x),x]
```

```
[Out] (x*(15*a^4*(2*c + d*x) + 20*a^3*b*x*(3*c + 2*d*x) + 15*a^2*b^2*x^2*(4*c + 3
*d*x) + 6*a*b^3*x^3*(5*c + 4*d*x) + b^4*x^4*(6*c + 5*d*x)))/30
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x), x]
```

fricas [B] time = 1.20, size = 97, normalized size = 2.55

$$\frac{1}{6}x^6db^4+\frac{1}{5}x^5cb^4+\frac{4}{5}x^5db^3a+x^4cb^3a+\frac{3}{2}x^4db^2a^2+2x^3cb^2a^2+\frac{4}{3}x^3dba^3+2x^2cba^3+\frac{1}{2}x^2da^4+xca^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d*b^4 + 1/5*x^5*c*b^4 + 4/5*x^5*d*b^3*a + x^4*c*b^3*a + 3/2*x^4*d*b
^2*a^2 + 2*x^3*c*b^2*a^2 + 4/3*x^3*d*b*a^3 + 2*x^2*c*b*a^3 + 1/2*x^2*d*a^4
+ x*c*a^4
```

giac [B] time = 1.03, size = 97, normalized size = 2.55

$$\frac{1}{6}\,b^4dx^6+\frac{1}{5}\,b^4cx^5+\frac{4}{5}\,ab^3dx^5+ab^3cx^4+\frac{3}{2}\,a^2b^2dx^4+2\,a^2b^2cx^3+\frac{4}{3}\,a^3bdx^3+2\,a^3bcx^2+\frac{1}{2}\,a^4dx^2+a^4cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*b^4*d*x^6 + 1/5*b^4*c*x^5 + 4/5*a*b^3*d*x^5 + a*b^3*c*x^4 + 3/2*a^2*b^2
*d*x^4 + 2*a^2*b^2*c*x^3 + 4/3*a^3*b*d*x^3 + 2*a^3*b*c*x^2 + 1/2*a^4*d*x^2
+ a^4*c*x
```

maple [B] time = 0.00, size = 97, normalized size = 2.55

$$\frac{b^4d\,x^6}{6}+a^4cx+\frac{\left(4a\,b^3d+b^4c\right)x^5}{5}+\frac{\left(6a^2b^2d+4a\,b^3c\right)x^4}{4}+\frac{\left(4a^3bd+6a^2b^2c\right)x^3}{3}+\frac{\left(a^4d+4a^3bc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c),x)
```

```
[Out] 1/6*b^4*d*x^6+1/5*(4*a*b^3*d+b^4*c)*x^5+1/4*(6*a^2*b^2*d+4*a*b^3*c)*x^4+1/3
*(4*a^3*b*d+6*a^2*b^2*c)*x^3+1/2*(a^4*d+4*a^3*b*c)*x^2+a^4*c*x
```

maxima [B] time = 1.38, size = 96, normalized size = 2.53

$$\frac{1}{6}\,b^4dx^6+a^4cx+\frac{1}{5}\,\left(b^4c+4\,ab^3d\right)x^5+\frac{1}{2}\,\left(2\,ab^3c+3\,a^2b^2d\right)x^4+\frac{2}{3}\,\left(3\,a^2b^2c+2\,a^3bd\right)x^3+\frac{1}{2}\,\left(4\,a^3bc+a^4d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*b^4*d*x^6 + a^4*c*x + 1/5*(b^4*c + 4*a*b^3*d)*x^5 + 1/2*(2*a*b^3*c + 3*
a^2*b^2*d)*x^4 + 2/3*(3*a^2*b^2*c + 2*a^3*b*d)*x^3 + 1/2*(4*a^3*b*c + a^4*d
)*x^2
```

mupad [B] time = 0.19, size = 88, normalized size = 2.32

$$x^5\,\left(\frac{c\,b^4}{5}+\frac{4\,a\,d\,b^3}{5}\right)+x^2\,\left(\frac{d\,a^4}{2}+2\,b\,c\,a^3\right)+\frac{b^4\,d\,x^6}{6}+a^4\,c\,x+\frac{2\,a^2\,b\,x^3\,\left(2\,a\,d+3\,b\,c\right)}{3}+\frac{a\,b^2\,x^4\,\left(3\,a\,d+2\,b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x),x)
```

```
[Out] x^5*((b^4*c)/5 + (4*a*b^3*d)/5) + x^2*((a^4*d)/2 + 2*a^3*b*c) + (b^4*d*x^6)
/6 + a^4*c*x + (2*a^2*b*x^3*(2*a*d + 3*b*c))/3 + (a*b^2*x^4*(3*a*d + 2*b*c)
)/2
```

sympy [B] time = 0.08, size = 100, normalized size = 2.63

$$a^4cx+\frac{b^4dx^6}{6}+x^5\left(\frac{4ab^3d}{5}+\frac{b^4c}{5}\right)+x^4\left(\frac{3a^2b^2d}{2}+ab^3c\right)+x^3\left(\frac{4a^3bd}{3}+2a^2b^2c\right)+x^2\left(\frac{a^4d}{2}+2a^3bc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c),x)
```

```
[Out] a**4*c*x + b**4*d*x**6/6 + x**5*(4*a*b**3*d/5 + b**4*c/5) + x**4*(3*a**2*b*
*2*d/2 + a*b**3*c) + x**3*(4*a**3*b*d/3 + 2*a**2*b**2*c) + x**2*(a**4*d/2 +
 2*a**3*b*c)
```

3.1131 $\int (a+bx)^3(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^4(bc-ad)}{4b^2}+\frac{d(a+bx)^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^4)/(4*b^2) + (d*(a + b*x)^5)/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^3}{b}+\frac{d(a+bx)^4}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^4}{4b^2}+\frac{d(a+bx)^5}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.76

$$a^3cx+\frac{1}{2}a^2x^2(ad+3bc)+\frac{1}{4}b^2x^4(3ad+bc)+abx^3(ad+bc)+\frac{1}{5}b^3dx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x),x]
```

```
[Out] a^3*c*x + (a^2*(3*b*c + a*d)*x^2)/2 + a*b*(b*c + a*d)*x^3 + (b^2*(b*c + 3*a
*d)*x^4)/4 + (b^3*d*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x), x]
```

fricas [B] time = 1.41, size = 72, normalized size = 1.89

$$\frac{1}{5}x^5db^3+\frac{1}{4}x^4cb^3+\frac{3}{4}x^4db^2a+x^3cb^2a+x^3dba^2+\frac{3}{2}x^2cba^2+\frac{1}{2}x^2da^3+xca^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d*b^3 + 1/4*x^4*c*b^3 + 3/4*x^4*d*b^2*a + x^3*c*b^2*a + x^3*d*b*a^2
 + 3/2*x^2*c*b*a^2 + 1/2*x^2*d*a^3 + x*c*a^3
```

giac [B] time = 1.14, size = 72, normalized size = 1.89

$$\frac{1}{5}\,b^3dx^5+\frac{1}{4}\,b^3cx^4+\frac{3}{4}\,ab^2dx^4+ab^2cx^3+a^2bdx^3+\frac{3}{2}\,a^2bcx^2+\frac{1}{2}\,a^3dx^2+a^3cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/5*b^3*d*x^5 + 1/4*b^3*c*x^4 + 3/4*a*b^2*d*x^4 + a*b^2*c*x^3 + a^2*b*d*x^3
 + 3/2*a^2*b*c*x^2 + 1/2*a^3*d*x^2 + a^3*c*x
```

maple [B] time = 0.00, size = 73, normalized size = 1.92

$$\frac{b^3d\,x^5}{5}+a^3cx+\frac{\left(3a\,b^2d+b^3c\right)x^4}{4}+\frac{\left(3a^2bd+3a\,b^2c\right)x^3}{3}+\frac{\left(a^3d+3a^2bc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c),x)
```

```
[Out] 1/5*b^3*d*x^5+1/4*(3*a*b^2*d+b^3*c)*x^4+1/3*(3*a^2*b*d+3*a*b^2*c)*x^3+1/2*(
a^3*d+3*a^2*b*c)*x^2+a^3*c*x
```

maxima [B] time = 1.41, size = 69, normalized size = 1.82

$$\frac{1}{5}\,b^3dx^5+a^3cx+\frac{1}{4}\left(b^3c+3\,ab^2d\right)x^4+\left(ab^2c+a^2bd\right)x^3+\frac{1}{2}\left(3\,a^2bc+a^3d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/5*b^3*d*x^5 + a^3*c*x + 1/4*(b^3*c + 3*a*b^2*d)*x^4 + (a*b^2*c + a^2*b*d)
*x^3 + 1/2*(3*a^2*b*c + a^3*d)*x^2
```

mupad [B] time = 0.16, size = 65, normalized size = 1.71

$$x^4\,\left(\frac{c\,b^3}{4}+\frac{3\,a\,d\,b^2}{4}\right)+x^2\,\left(\frac{d\,a^3}{2}+\frac{3\,b\,c\,a^2}{2}\right)+\frac{b^3\,d\,x^5}{5}+a^3\,c\,x+a\,b\,x^3\,\left(a\,d+b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x),x)
```

```
[Out] x^4*((b^3*c)/4 + (3*a*b^2*d)/4) + x^2*((a^3*d)/2 + (3*a^2*b*c)/2) + (b^3*d*
x^5)/5 + a^3*c*x + a*b*x^3*(a*d + b*c)
```

sympy [B] time = 0.08, size = 73, normalized size = 1.92

$$a^3cx+\frac{b^3dx^5}{5}+x^4\left(\frac{3ab^2d}{4}+\frac{b^3c}{4}\right)+x^3\left(a^2bd+ab^2c\right)+x^2\left(\frac{a^3d}{2}+\frac{3a^2bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c),x)
```

```
[Out] a**3*c*x + b**3*d*x**5/5 + x**4*(3*a*b**2*d/4 + b**3*c/4) + x**3*(a**2*b*d
+ a*b**2*c) + x**2*(a**3*d/2 + 3*a**2*b*c/2)
```

3.1132 $\int (a+bx)^2(c+dx)\,dx$

Optimal. Leaf size=38

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(a+bx)^3(bc-ad)}{3b^2}+\frac{d(a+bx)^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x),x]
[Out] ((b*c - a*d)*(a + b*x)^3)/(3*b^2) + (d*(a + b*x)^4)/(4*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^2}{b}+\frac{d(a+bx)^3}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^3}{3b^2}+\frac{d(a+bx)^4}{4b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.21

$$\frac{1}{12}x\left(6a^2(2c+dx)+4abx(3c+2dx)+b^2x^2(4c+3dx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x),x]
[Out] (x*(6*a^2*(2*c + d*x) + 4*a*b*x*(3*c + 2*d*x) + b^2*x^2*(4*c + 3*d*x)))/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x),x]
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x), x]
```

fricas [A] time = 1.73, size = 49, normalized size = 1.29

$$\frac{1}{4}x^4db^2+\frac{1}{3}x^3cb^2+\frac{2}{3}x^3dba+x^2cba+\frac{1}{2}x^2da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d*b^2 + 1/3*x^3*c*b^2 + 2/3*x^3*d*b*a + x^2*c*b*a + 1/2*x^2*d*a^2 +
 x*c*a^2
```

giac [A] time = 0.99, size = 49, normalized size = 1.29

$$\frac{1}{4}\,b^2dx^4+\frac{1}{3}\,b^2cx^3+\frac{2}{3}\,abdx^3+abcx^2+\frac{1}{2}\,a^2dx^2+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*b^2*d*x^4 + 1/3*b^2*c*x^3 + 2/3*a*b*d*x^3 + a*b*c*x^2 + 1/2*a^2*d*x^2 +
 a^2*c*x
```

maple [A] time = 0.00, size = 49, normalized size = 1.29

$$\frac{b^2d\,x^4}{4}+a^2cx+\frac{\left(2abd+b^2c\right)x^3}{3}+\frac{\left(a^2d+2abc\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c),x)
```

```
[Out] 1/4*b^2*d*x^4+1/3*(2*a*b*d+b^2*c)*x^3+1/2*(a^2*d+2*a*b*c)*x^2+a^2*c*x
```

maxima [A] time = 1.30, size = 48, normalized size = 1.26

$$\frac{1}{4}\,b^2dx^4+a^2cx+\frac{1}{3}\left(b^2c+2\,abd\right)x^3+\frac{1}{2}\left(2\,abc+a^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*b^2*d*x^4 + a^2*c*x + 1/3*(b^2*c + 2*a*b*d)*x^3 + 1/2*(2*a*b*c + a^2*d)
*x^2
```

mupad [B] time = 0.05, size = 47, normalized size = 1.24

$$x^2\,\left(\frac{d\,a^2}{2}+b\,c\,a\right)+x^3\,\left(\frac{c\,b^2}{3}+\frac{2\,a\,d\,b}{3}\right)+\frac{b^2\,d\,x^4}{4}+a^2\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x),x)
```

```
[Out] x^2*((a^2*d)/2 + a*b*c) + x^3*((b^2*c)/3 + (2*a*b*d)/3) + (b^2*d*x^4)/4 + a
^2*c*x
```

sympy [A] time = 0.07, size = 49, normalized size = 1.29

$$a^2cx+\frac{b^2dx^4}{4}+x^3\left(\frac{2abd}{3}+\frac{b^2c}{3}\right)+x^2\left(\frac{a^2d}{2}+abc\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**4/4 + x**3*(2*a*b*d/3 + b**2*c/3) + x**2*(a**2*d/2 + a
*b*c)
```

3.1133 $\int (a+bx)(c+dx)\,dx$

Optimal. Leaf size=28

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)\,dx &= \int \left(ac+(bc+ad)x+bdx^2\right)\,dx\\ &= acx+\frac{1}{2}(bc+ad)x^2+\frac{1}{3}bdx^3\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$\frac{1}{2}x^2(ad+bc)+acx+\frac{1}{3}bdx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^2)/2 + (b*d*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x), x]
```

fricas [A] time = 1.56, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3db+\frac{1}{2}x^2cb+\frac{1}{2}x^2da+xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="fricas")
[Out] 1/3*x^3*d*b + 1/2*x^2*c*b + 1/2*x^2*d*a + x*c*a
```

giac [A] time = 0.99, size = 26, normalized size = 0.93

$$\frac{1}{3}\,bdx^3 + \frac{1}{2}\,bcx^2 + \frac{1}{2}\,adx^2 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="giac")
[Out] 1/3*b*d*x^3 + 1/2*b*c*x^2 + 1/2*a*d*x^2 + a*c*x
```

maple [A] time = 0.00, size = 25, normalized size = 0.89

$$\frac{bd\,x^3}{3} + acx + \frac{(ad + bc)\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c),x)
[Out] a*c*x+1/2*(a*d+b*c)*x^2+1/3*b*d*x^3
```

maxima [A] time = 1.35, size = 24, normalized size = 0.86

$$\frac{1}{3}\,bdx^3 + acx + \frac{1}{2}\,(bc + ad)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x, algorithm="maxima")
[Out] 1/3*b*d*x^3 + a*c*x + 1/2*(b*c + a*d)*x^2
```

mupad [B] time = 0.03, size = 25, normalized size = 0.89

$$\frac{b\,d\,x^3}{3} + \left(\frac{a\,d}{2} + \frac{b\,c}{2}\right)\,x^2 + a\,c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x),x)
[Out] x^2*((a*d)/2 + (b*c)/2) + a*c*x + (b*d*x^3)/3
```

sympy [A] time = 0.06, size = 26, normalized size = 0.93

$$acx + \frac{bdx^3}{3} + x^2\left(\frac{ad}{2} + \frac{bc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c),x)
[Out] a*c*x + b*d*x**3/3 + x**2*(a*d/2 + b*c/2)
```

3.1134 $\int (c+dx)\,dx$

Optimal. Leaf size=12

$$cx+\frac{dx^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[c + d*x,x]
[Out] c*x + (d*x^2)/2
```

Rubi steps

$$\int (c+dx)\,dx=cx+\frac{dx^2}{2}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$cx+\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[c + d*x,x]
[Out] c*x + (d*x^2)/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[c + d*x,x]
[Out] IntegrateAlgebraic[c + d*x, x]
```

fricas [A] time = 1.46, size = 10, normalized size = 0.83

$$\frac{1}{2}x^2d+xc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="fricas")
[Out] 1/2*x^2*d + x*c
```

giac [A] time = 0.90, size = 10, normalized size = 0.83

$$\frac{1}{2}dx^2+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x
```

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$\frac{1}{2}d\,x^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x+c,x)
[Out] 1/2*d*x^2+c*x
```

maxima [A] time = 1.35, size = 10, normalized size = 0.83

$$\frac{1}{2}\,dx^2 + cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x, algorithm="maxima")
[Out] 1/2*d*x^2 + c*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{d\,x^2}{2} + c\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c + d*x,x)
[Out] c*x + (d*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$cx + \frac{dx^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x+c,x)
[Out] c*x + d*x**2/2
```

3.1135 $\int \frac{c+dx}{a+bx} \, dx$

Optimal. Leaf size=25

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Rubi **[A]** time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{a+bx}\,dx&=\int\left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx\\&=\frac{dx}{b}+\frac{(bc-ad)\log(a+bx)}{b^2}\end{aligned}$$

Mathematica **[A]** time = 0.01, size = 25, normalized size = 1.00

$$\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2
```

IntegrateAlgebraic **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x), x]
```

fricas **[A]** time = 1.70, size = 24, normalized size = 0.96

$$\frac{bdx+(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="fricas")
[Out] (b*d*x + (b*c - a*d)*log(b*x + a))/b^2
```

giac [A] time = 0.96, size = 26, normalized size = 1.04

$$\frac{dx}{b}+\frac{(bc-ad)\log(|bx+a|)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="giac")
[Out] d*x/b + (b*c - a*d)*log(abs(b*x + a))/b^2
```

maple [A] time = 0.00, size = 32, normalized size = 1.28

$$-\frac{ad\ln(bx+a)}{b^2}+\frac{c\ln(bx+a)}{b}+\frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a),x)
[Out] d*x/b-1/b^2*ln(b*x+a)*a*d+1/b*c*ln(b*x+a)
```

maxima [A] time = 1.25, size = 25, normalized size = 1.00

$$\frac{dx}{b}+\frac{(bc-ad)\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x, algorithm="maxima")
[Out] d*x/b + (b*c - a*d)*log(b*x + a)/b^2
```

mupad [B] time = 0.05, size = 26, normalized size = 1.04

$$\frac{d\,x}{b}-\frac{\ln(a+b\,x)\,(a\,d-b\,c)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x),x)
[Out] (d*x)/b - (log(a + b*x)*(a*d - b*c))/b^2
```

sympy [A] time = 0.15, size = 20, normalized size = 0.80

$$\frac{dx}{b}-\frac{(ad-bc)\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a),x)
[Out] d*x/b - (a*d - b*c)*log(a + b*x)/b**2
```

3.1136 $\int \frac{c+dx}{(a+bx)^2} dx$

Optimal. Leaf size=32

$$\frac{d\log(a+bx)}{b^2} - \frac{bc-ad}{b^2(a+bx)}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{d\log(a+bx)}{b^2} - \frac{bc-ad}{b^2(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] -((b*c - a*d)/(b^2*(a + b*x))) + (d*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^2}\,dx &= \int \left(\frac{bc-ad}{b(a+bx)^2} + \frac{d}{b(a+bx)}\right) dx\\ &= -\frac{bc-ad}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.97

$$\frac{ad-bc}{b^2(a+bx)} + \frac{d\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d)/(b^2*(a + b*x)) + (d*Log[a + b*x])/b^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^2, x]
```

fricas [A] time = 1.68, size = 39, normalized size = 1.22

$$-\frac{bc-ad-(bdx+ad)\log(bx+a)}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b*c - a*d - (b*d*x + a*d)*log(b*x + a))/(b^3*x + a*b^2)
```

giac [A] time = 1.02, size = 57, normalized size = 1.78

$$-\frac{d\left(\frac{\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b}-\frac{a}{(bx+a)b}\right)}{b}-\frac{c}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -d*(log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b - a/((b*x + a)*b))/b - c/((b*x
 + a)*b)
```

maple [A] time = 0.00, size = 39, normalized size = 1.22

$$\frac{ad}{(bx+a)\,b^2}-\frac{c}{(bx+a)\,b}+\frac{d\ln(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^2,x)
```

```
[Out] d*ln(b*x+a)/b^2+1/b^2/(b*x+a)*a*d-1/b/(b*x+a)*c
```

maxima [A] time = 1.34, size = 35, normalized size = 1.09

$$-\frac{bc-ad}{b^3x+ab^2}+\frac{d\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)/(b^3*x + a*b^2) + d*log(b*x + a)/b^2
```

mupad [B] time = 0.17, size = 31, normalized size = 0.97

$$\frac{a\,d-b\,c}{b^2\,(a+b\,x)}+\frac{d\,\ln(a+b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^2,x)
```

```
[Out] (a*d - b*c)/(b^2*(a + b*x)) + (d*log(a + b*x))/b^2
```

sympy [A] time = 0.19, size = 27, normalized size = 0.84

$$\frac{ad-bc}{ab^2+b^3x}+\frac{d\log(a+bx)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**2,x)
```

```
[Out] (a*d - b*c)/(a*b**2 + b**3*x) + d*log(a + b*x)/b**2
```

3.1137 $\int \frac{c+dx}{(a+bx)^3} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$-\frac{(c+dx)^2}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^3,x]
[Out] -(c + d*x)^2/(2*(b*c - a*d)*(a + b*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{c+dx}{(a+bx)^3} dx = -\frac{(c+dx)^2}{2(bc-ad)(a+bx)^2}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^3,x]
[Out] -1/2*(a*d + b*(c + 2*d*x))/(b^2*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^3,x]
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^3, x]
```

fricas [A] time = 1.75, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

giac [A] time = 1.06, size = 24, normalized size = 0.86

$$-\frac{2\,bdx+bc+ad}{2\,(bx+a)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((b*x + a)^2*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 1.25

$$-\frac{d}{(bx+a)\,b^2}-\frac{-ad+bc}{2\,(bx+a)^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^3,x)
```

```
[Out] -1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a)
```

maxima [A] time = 1.36, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(b^4x^2+2\,ab^3x+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(b^4*x^2 + 2*a*b^3*x + a^2*b^2)
```

mupad [B] time = 0.16, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,b^2}+\frac{d\,x}{b}}{a^2+2\,a\,b\,x+b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*b^2) + (d*x)/b)/(a^2 + b^2*x^2 + 2*a*b*x)
```

sympy [A] time = 0.26, size = 39, normalized size = 1.39

$$\frac{-ad-bc-2bdx}{2a^2b^2+4ab^3x+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*a**2*b**2 + 4*a*b**3*x + 2*b**4*x**2)
```

3.1138 $\int \frac{c+dx}{(a+bx)^4} \, dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)/(3*b^2*(a + b*x)^3) - d/(2*b^2*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^4} \, dx &= \int \left(\frac{bc-ad}{b(a+bx)^4}+\frac{d}{b(a+bx)^3}\right) dx\\ &= -\frac{bc-ad}{3b^2(a+bx)^3}-\frac{d}{2b^2(a+bx)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{ad+2bc+3bdx}{6b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] -1/6*(2*b*c + a*d + 3*b*d*x)/(b^2*(a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^4} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^4, x]
```

fricas [A] time = 1.75, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^5x^3+3\,ab^4x^2+3\,a^2b^3x+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

giac [A] time = 0.75, size = 25, normalized size = 0.66

$$-\frac{3\,bdx+2\,bc+ad}{6\,(bx+a)^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/((b*x + a)^3*b^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$-\frac{d}{2\,(bx+a)^{2}\,b^{2}}-\frac{-ad+bc}{3\,(bx+a)^{3}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^4,x)
```

```
[Out] -1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2
```

maxima [A] time = 1.31, size = 50, normalized size = 1.32

$$-\frac{3\,bdx+2\,bc+ad}{6\left(b^{5}x^{3}+3\,ab^{4}x^{2}+3\,a^{2}b^{3}x+a^{3}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*b*d*x + 2*b*c + a*d)/(b^5*x^3 + 3*a*b^4*x^2 + 3*a^2*b^3*x + a^3*b^2
)
```

mupad [B] time = 0.17, size = 52, normalized size = 1.37

$$-\frac{\frac{a\,d+2\,b\,c}{6\,b^{2}}+\frac{d\,x}{2\,b}}{a^{3}+3\,a^{2}\,b\,x+3\,a\,b^{2}\,x^{2}+b^{3}\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^4,x)
```

```
[Out] -((a*d + 2*b*c)/(6*b^2) + (d*x)/(2*b))/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2
*b*x)
```

sympy [A] time = 0.34, size = 53, normalized size = 1.39

$$\frac{-ad-2bc-3bdx}{6a^{3}b^{2}+18a^{2}b^{3}x+18ab^{4}x^{2}+6b^{5}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**4,x)
```

```
[Out] (-a*d - 2*b*c - 3*b*d*x)/(6*a**3*b**2 + 18*a**2*b**3*x + 18*a*b**4*x**2 + 6
*b**5*x**3)
```

3.1139 $\int \frac{c+dx}{(a+bx)^5} dx$

Optimal. Leaf size=38

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)/(4*b^2*(a + b*x)^4) - d/(3*b^2*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx}{(a+bx)^5} dx &= \int \left(\frac{bc-ad}{b(a+bx)^5}+\frac{d}{b(a+bx)^4}\right) dx\\ &= -\frac{bc-ad}{4b^2(a+bx)^4}-\frac{d}{3b^2(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{ad+3bc+4bdx}{12b^2(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] -1/12*(3*b*c + a*d + 4*b*d*x)/(b^2*(a + b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{(a+bx)^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)/(a + b*x)^5, x]
```

fricas [A] time = 1.20, size = 61, normalized size = 1.61

$$-\frac{4\,bdx+3\,bc+ad}{12\left(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

giac [A] time = 0.92, size = 41, normalized size = 1.08

$$-\frac{c}{4\,(bx+a)^4 b}-\frac{d}{3\,(bx+a)^3 b^2}+\frac{ad}{4\,(bx+a)^4 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*c/((b*x + a)^4*b) - 1/3*d/((b*x + a)^3*b^2) + 1/4*a*d/((b*x + a)^4*b^2
)
```

maple [A] time = 0.00, size = 35, normalized size = 0.92

$$-\frac{d}{3\,(bx+a)^3\,b^2}-\frac{-ad+bc}{4\,(bx+a)^4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(b*x+a)^5,x)
```

```
[Out] -1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4
```

maxima [A] time = 1.39, size = 61, normalized size = 1.61

$$-\frac{4\,bdx+3\,bc+ad}{12\,\left(b^6x^4+4\,ab^5x^3+6\,a^2b^4x^2+4\,a^3b^3x+a^4b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(4*b*d*x + 3*b*c + a*d)/(b^6*x^4 + 4*a*b^5*x^3 + 6*a^2*b^4*x^2 + 4*a^
3*b^3*x + a^4*b^2)
```

mupad [B] time = 0.04, size = 63, normalized size = 1.66

$$-\frac{\frac{a\,d+3\,b\,c}{12\,b^2}+\frac{d\,x}{3\,b}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*x)^5,x)
```

```
[Out] -((a*d + 3*b*c)/(12*b^2) + (d*x)/(3*b))/(a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^
2*b^2*x^2 + 4*a^3*b*x)
```

sympy [B] time = 0.43, size = 65, normalized size = 1.71

$$\frac{-ad-3bc-4bdx}{12a^4b^2+48a^3b^3x+72a^2b^4x^2+48ab^5x^3+12b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(b*x+a)**5,x)
```

```
[Out] (-a*d - 3*b*c - 4*b*d*x)/(12*a**4*b**2 + 48*a**3*b**3*x + 72*a**2*b**4*x**2
 + 48*a*b**5*x**3 + 12*b**6*x**4)
```

3.1140 $\int (a+bx)^4 (c+dx)^2 \, dx$

Optimal. Leaf size=65

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^6(bc-ad)}{3b^3}+\frac{(a+bx)^5(bc-ad)^2}{5b^3}+\frac{d^2(a+bx)^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^5)/(5*b^3) + (d*(b*c - a*d)*(a + b*x)^6)/(3*b^3) +
 (d^2*(a + b*x)^7)/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^4}{b^2}+\frac{2d(bc-ad)(a+bx)^5}{b^2}+\frac{d^2(a+bx)^6}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^5}{5b^3}+\frac{d(bc-ad)(a+bx)^6}{3b^3}+\frac{d^2(a+bx)^7}{7b^3}\end{aligned}$$

Mathematica [B] time = 0.03, size = 148, normalized size = 2.28

$$a^4c^2x+a^3cx^2(ad+2bc)+\frac{1}{5}b^2x^5\left(6a^2d^2+8abcd+b^2c^2\right)+abx^4\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{3}a^2x^3\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{1}{3}b^3dx^6(2ad+bc)+\frac{1}{7}b^4d^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] a^4*c^2*x + a^3*c*(2*b*c + a*d)*x^2 + (a^2*(6*b^2*c^2 + 8*a*b*c*d + a^2*d^2
)*x^3)/3 + a*b*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^4 + (b^2*(b^2*c^2 + 8*a*b*
c*d + 6*a^2*d^2)*x^5)/5 + (b^3*d*(b*c + 2*a*d)*x^6)/3 + (b^4*d^2*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^2, x]
```

fricas [B] time = 1.11, size = 170, normalized size = 2.62

$$\frac{1}{7}x^7d^2b^4+\frac{1}{3}x^6dcb^4+\frac{2}{3}x^6d^2b^3a+\frac{1}{5}x^5c^2b^4+\frac{8}{5}x^5dcb^3a+\frac{6}{5}x^5d^2b^2a^2+x^4c^2b^3a+3x^4dcb^2a^2+x^4d^2ba^3+2x^3c^2b^2a^2+\frac{8}{3}x^3dcba^3+\frac{1}{3}x^3d^2a^4+2x^2c^2ba^3+x^2dca^4+xc^2a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^2*b^4 + 1/3*x^6*d*c*b^4 + 2/3*x^6*d^2*b^3*a + 1/5*x^5*c^2*b^4 + 8
/5*x^5*d*c*b^3*a + 6/5*x^5*d^2*b^2*a^2 + x^4*c^2*b^3*a + 3*x^4*d*c*b^2*a^2
+ x^4*d^2*b*a^3 + 2*x^3*c^2*b^2*a^2 + 8/3*x^3*d*c*b*a^3 + 1/3*x^3*d^2*a^4 +
 2*x^2*c^2*b*a^3 + x^2*d*c*a^4 + x*c^2*a^4
```

giac [B] time = 1.13, size = 170, normalized size = 2.62

$$\frac{1}{7}b^4d^2x^7+\frac{1}{3}b^4cdx^6+\frac{2}{3}ab^3d^2x^6+\frac{1}{5}b^4c^2x^5+\frac{8}{5}ab^3cdx^5+\frac{6}{5}a^2b^2d^2x^5+ab^3c^2x^4+3a^2b^2cdx^4+a^3bd^2x^4+2a^2b^2c^2x^3+\frac{8}{3}a^3bcdx^3+\frac{1}{3}a^4d^2x^3+2a^3bc^2x^2+a^4cdx^2+a^4c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^4*d^2*x^7 + 1/3*b^4*c*d*x^6 + 2/3*a*b^3*d^2*x^6 + 1/5*b^4*c^2*x^5 + 8
/5*a*b^3*c*d*x^5 + 6/5*a^2*b^2*d^2*x^5 + a*b^3*c^2*x^4 + 3*a^2*b^2*c*d*x^4
+ a^3*b*d^2*x^4 + 2*a^2*b^2*c^2*x^3 + 8/3*a^3*b*c*d*x^3 + 1/3*a^4*d^2*x^3 +
 2*a^3*b*c^2*x^2 + a^4*c*d*x^2 + a^4*c^2*x
```

maple [B] time = 0.00, size = 163, normalized size = 2.51

$$\frac{b^4d^2x^7}{7}+a^4c^2x+\frac{\left(4a\,b^3d^2+2b^4cd\right)x^6}{6}+\frac{\left(6a^2b^2d^2+8a\,b^3cd+b^4c^2\right)x^5}{5}+\frac{\left(4a^3b\,d^2+12a^2b^2cd+4a\,b^3c^2\right)x^4}{4}+\frac{\left(a^4d^2+8a^3bcd+6a^2b^2c^2\right)x^3}{3}+\frac{\left(2a^4cd+4a^3b\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^2,x)
```

```
[Out] 1/7*b^4*d^2*x^7+1/6*(4*a*b^3*d^2+2*b^4*c*d)*x^6+1/5*(6*a^2*b^2*d^2+8*a*b^3*
c*d+b^4*c^2)*x^5+1/4*(4*a^3*b*d^2+12*a^2*b^2*c*d+4*a*b^3*c^2)*x^4+1/3*(a^4*
d^2+8*a^3*b*c*d+6*a^2*b^2*c^2)*x^3+1/2*(2*a^4*c*d+4*a^3*b*c^2)*x^2+a^4*c^2*
x
```

maxima [B] time = 1.36, size = 156, normalized size = 2.40

$$\frac{1}{7}b^4d^2x^7+a^4c^2x+\frac{1}{3}\left(b^4cd+2\,ab^3d^2\right)x^6+\frac{1}{5}\left(b^4c^2+8\,ab^3cd+6\,a^2b^2d^2\right)x^5+\left(ab^3c^2+3\,a^2b^2cd+a^3bd^2\right)x^4+\frac{1}{3}\left(6\,a^2b^2c^2+8\,a^3bcd+a^4d^2\right)x^3+\left(2\,a^3bc^2+a^4cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^4*d^2*x^7 + a^4*c^2*x + 1/3*(b^4*c*d + 2*a*b^3*d^2)*x^6 + 1/5*(b^4*c^
2 + 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x^5 + (a*b^3*c^2 + 3*a^2*b^2*c*d + a^3*b*d
^2)*x^4 + 1/3*(6*a^2*b^2*c^2 + 8*a^3*b*c*d + a^4*d^2)*x^3 + (2*a^3*b*c^2 +
a^4*c*d)*x^2
```

mupad [B] time = 0.07, size = 144, normalized size = 2.22

$$x^3\left(\frac{a^4\,d^2}{3}+\frac{8\,a^3\,b\,c\,d}{3}+2\,a^2\,b^2\,c^2\right)+x^5\left(\frac{6\,a^2\,b^2\,d^2}{5}+\frac{8\,a\,b^3\,c\,d}{5}+\frac{b^4\,c^2}{5}\right)+a^4\,c^2\,x+\frac{b^4\,d^2\,x^7}{7}+a^3\,c\,x^2\,\left(a\,d+2\,b\,c\right)+\frac{b^3\,d\,x^6\,\left(2\,a\,d+b\,c\right)}{3}+a\,b\,x^4\,\left(a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^2,x)
```

```
[Out] x^3*((a^4*d^2)/3 + 2*a^2*b^2*c^2 + (8*a^3*b*c*d)/3) + x^5*((b^4*c^2)/5 + (6
*a^2*b^2*d^2)/5 + (8*a*b^3*c*d)/5) + a^4*c^2*x + (b^4*d^2*x^7)/7 + a^3*c*x^
```

```
2*(a*d + 2*b*c) + (b^3*d*x^6*(2*a*d + b*c))/3 + a*b*x^4*(a^2*d^2 + b^2*c^2
+ 3*a*b*c*d)
```

sympy [B] time = 0.10, size = 168, normalized size = 2.58

$$a^4c^2x + \frac{b^4d^2x^7}{7} + x^6\left(\frac{2ab^3d^2}{3} + \frac{b^4cd}{3}\right) + x^5\left(\frac{6a^2b^2d^2}{5} + \frac{8ab^3cd}{5} + \frac{b^4c^2}{5}\right) + x^4\left(a^3bd^2 + 3a^2b^2cd + ab^3c^2\right) + x^3\left(\frac{a^4d^2}{3} + \frac{8a^3bcd}{3} + 2a^2b^2c^2\right) + x^2\left(a^4cd + 2a^3bc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**2,x)
```

```
[Out] a**4*c**2*x + b**4*d**2*x**7/7 + x**6*(2*a*b**3*d**2/3 + b**4*c*d/3) + x**5
*(6*a**2*b**2*d**2/5 + 8*a*b**3*c*d/5 + b**4*c**2/5) + x**4*(a**3*b*d**2 +
3*a**2*b**2*c*d + a*b**3*c**2) + x**3*(a**4*d**2/3 + 8*a**3*b*c*d/3 + 2*a**
2*b**2*c**2) + x**2*(a**4*c*d + 2*a**3*b*c**2)
```

3.1141 $\int (a+bx)^3(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2d(a+bx)^5(bc-ad)}{5b^3}+\frac{(a+bx)^4(bc-ad)^2}{4b^3}+\frac{d^2(a+bx)^6}{6b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^4)/(4*b^3) + (2*d*(b*c - a*d)*(a + b*x)^5)/(5*b^3)
 + (d^2*(a + b*x)^6)/(6*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^3}{b^2}+\frac{2d(bc-ad)(a+bx)^4}{b^2}+\frac{d^2(a+bx)^5}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^4}{4b^3}+\frac{2d(bc-ad)(a+bx)^5}{5b^3}+\frac{d^2(a+bx)^6}{6b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 122, normalized size = 1.88

$$a^3c^2x+\frac{1}{4}bx^4\left(3a^2d^2+6abcd+b^2c^2\right)+\frac{1}{3}ax^3\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{2}a^2cx^2(2ad+3bc)+\frac{1}{5}b^2dx^5(3ad+2bc)+\frac{1}{6}b^3d^2x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] a^3*c^2*x + (a^2*c*(3*b*c + 2*a*d)*x^2)/2 + (a*(3*b^2*c^2 + 6*a*b*c*d + a^2
*d^2)*x^3)/3 + (b*(b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2)*x^4)/4 + (b^2*d*(2*b*c
+ 3*a*d)*x^5)/5 + (b^3*d^2*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^2, x]
```

fricas [B] time = 1.56, size = 130, normalized size = 2.00

$$\frac{1}{6}x^6d^2b^3+\frac{2}{5}x^5dcb^3+\frac{3}{5}x^5d^2b^2a+\frac{1}{4}x^4c^2b^3+\frac{3}{2}x^4dcb^2a+\frac{3}{4}x^4d^2ba^2+x^3c^2b^2a+2x^3dcba^2+\frac{1}{3}x^3d^2a^3+\frac{3}{2}x^2c^2ba^2+x^2dca^3+xc^2a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^2*b^3 + 2/5*x^5*d*c*b^3 + 3/5*x^5*d^2*b^2*a + 1/4*x^4*c^2*b^3 + 3
/2*x^4*d*c*b^2*a + 3/4*x^4*d^2*b*a^2 + x^3*c^2*b^2*a + 2*x^3*d*c*b*a^2 + 1/
3*x^3*d^2*a^3 + 3/2*x^2*c^2*b*a^2 + x^2*d*c*a^3 + x*c^2*a^3
```

giac [B] time = 1.13, size = 130, normalized size = 2.00

$$\frac{1}{6}b^3d^2x^6+\frac{2}{5}b^3cdx^5+\frac{3}{5}ab^2d^2x^5+\frac{1}{4}b^3c^2x^4+\frac{3}{2}ab^2cdx^4+\frac{3}{4}a^2bd^2x^4+ab^2c^2x^3+2a^2bcdx^3+\frac{1}{3}a^3d^2x^3+\frac{3}{2}a^2bc^2x^2+a^3cdx^2+a^3c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/6*b^3*d^2*x^6 + 2/5*b^3*c*d*x^5 + 3/5*a*b^2*d^2*x^5 + 1/4*b^3*c^2*x^4 + 3
/2*a*b^2*c*d*x^4 + 3/4*a^2*b*d^2*x^4 + a*b^2*c^2*x^3 + 2*a^2*b*c*d*x^3 + 1/
3*a^3*d^2*x^3 + 3/2*a^2*b*c^2*x^2 + a^3*c*d*x^2 + a^3*c^2*x
```

maple [B] time = 0.00, size = 125, normalized size = 1.92

$$\frac{b^3d^2x^6}{6}+a^3c^2x+\frac{\left(3ab^2d^2+2b^3cd\right)x^5}{5}+\frac{\left(3a^2bd^2+6ab^2cd+b^3c^2\right)x^4}{4}+\frac{\left(a^3d^2+6a^2bcd+3ab^2c^2\right)x^3}{3}+\frac{\left(2a^3cd+3a^2bc^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^2,x)
```

```
[Out] 1/6*b^3*d^2*x^6+1/5*(3*a*b^2*d^2+2*b^3*c*d)*x^5+1/4*(3*a^2*b*d^2+6*a*b^2*c*
d+b^3*c^2)*x^4+1/3*(a^3*d^2+6*a^2*b*c*d+3*a*b^2*c^2)*x^3+1/2*(2*a^3*c*d+3*a
^2*b*c^2)*x^2+a^3*c^2*x
```

maxima [B] time = 1.34, size = 124, normalized size = 1.91

$$\frac{1}{6}b^3d^2x^6+a^3c^2x+\frac{1}{5}\left(2b^3cd+3ab^2d^2\right)x^5+\frac{1}{4}\left(b^3c^2+6ab^2cd+3a^2bd^2\right)x^4+\frac{1}{3}\left(3ab^2c^2+6a^2bcd+a^3d^2\right)x^3+\frac{1}{2}\left(3a^2bc^2+2a^3cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/6*b^3*d^2*x^6 + a^3*c^2*x + 1/5*(2*b^3*c*d + 3*a*b^2*d^2)*x^5 + 1/4*(b^3*
c^2 + 6*a*b^2*c*d + 3*a^2*b*d^2)*x^4 + 1/3*(3*a*b^2*c^2 + 6*a^2*b*c*d + a^3
*d^2)*x^3 + 1/2*(3*a^2*b*c^2 + 2*a^3*c*d)*x^2
```

mupad [B] time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(\frac{a^3d^2}{3}+2a^2bcd+ab^2c^2\right)+x^4\left(\frac{3a^2bd^2}{4}+\frac{3ab^2cd}{2}+\frac{b^3c^2}{4}\right)+a^3c^2x+\frac{b^3d^2x^6}{6}+\frac{a^2cx^2(2ad+3bc)}{2}+\frac{b^2dx^5(3ad+2bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] x^3*((a^3*d^2)/3 + a*b^2*c^2 + 2*a^2*b*c*d) + x^4*((b^3*c^2)/4 + (3*a^2*b*d
^2)/4 + (3*a*b^2*c*d)/2) + a^3*c^2*x + (b^3*d^2*x^6)/6 + (a^2*c*x^2*(2*a*d
+ 3*b*c))/2 + (b^2*d*x^5*(3*a*d + 2*b*c))/5
```

sympy [B] time = 0.09, size = 133, normalized size = 2.05

$$a^3c^2x+\frac{b^3d^2x^6}{6}+x^5\left(\frac{3ab^2d^2}{5}+\frac{2b^3cd}{5}\right)+x^4\left(\frac{3a^2bd^2}{4}+\frac{3ab^2cd}{2}+\frac{b^3c^2}{4}\right)+x^3\left(\frac{a^3d^2}{3}+2a^2bcd+ab^2c^2\right)+x^2\left(a^3cd+\frac{3a^2bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**2,x)
```

```
[Out] a**3*c**2*x + b**3*d**2*x**6/6 + x**5*(3*a*b**2*d**2/5 + 2*b**3*c*d/5) + x*
*4*(3*a**2*b*d**2/4 + 3*a*b**2*c*d/2 + b**3*c**2/4) + x**3*(a**3*d**2/3 + 2
*a**2*b*c*d + a*b**2*c**2) + x**2*(a**3*c*d + 3*a**2*b*c**2/2)
```

3.1142 $\int (a+bx)^2(c+dx)^2\,dx$

Optimal. Leaf size=65

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d(a+bx)^4(bc-ad)}{2b^3}+\frac{(a+bx)^3(bc-ad)^2}{3b^3}+\frac{d^2(a+bx)^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^3)/(3*b^3) + (d*(b*c - a*d)*(a + b*x)^4)/(2*b^3) +
 (d^2*(a + b*x)^5)/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^2}{b^2}+\frac{2d(bc-ad)(a+bx)^3}{b^2}+\frac{d^2(a+bx)^4}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^3}{3b^3}+\frac{d(bc-ad)(a+bx)^4}{2b^3}+\frac{d^2(a+bx)^5}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 79, normalized size = 1.22

$$\frac{1}{3}x^3\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{2}bdx^4(ad+bc)+acx^2(ad+bc)+\frac{1}{5}b^2d^2x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] a^2*c^2*x + a*c*(b*c + a*d)*x^2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^3)/3 +
 (b*d*(b*c + a*d)*x^4)/2 + (b^2*d^2*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^2, x]
```

fricas [A] time = 1.33, size = 89, normalized size = 1.37

$$\frac{1}{5}x^5d^2b^2 + \frac{1}{2}x^4dcb^2 + \frac{1}{2}x^4d^2ba + \frac{1}{3}x^3c^2b^2 + \frac{4}{3}x^3dcba + \frac{1}{3}x^3d^2a^2 + x^2c^2ba + x^2dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^2*b^2 + 1/2*x^4*d*c*b^2 + 1/2*x^4*d^2*b*a + 1/3*x^3*c^2*b^2 + 4/3
*x^3*d*c*b*a + 1/3*x^3*d^2*a^2 + x^2*c^2*b*a + x^2*d*c*a^2 + x*c^2*a^2
```

giac [A] time = 0.80, size = 89, normalized size = 1.37

$$\frac{1}{5}\,b^2d^2x^5 + \frac{1}{2}\,b^2cdx^4 + \frac{1}{2}\,abd^2x^4 + \frac{1}{3}\,b^2c^2x^3 + \frac{4}{3}\,abcdx^3 + \frac{1}{3}\,a^2d^2x^3 + abc^2x^2 + a^2cdx^2 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*d^2*x^5 + 1/2*b^2*c*d*x^4 + 1/2*a*b*d^2*x^4 + 1/3*b^2*c^2*x^3 + 4/3
*a*b*c*d*x^3 + 1/3*a^2*d^2*x^3 + a*b*c^2*x^2 + a^2*c*d*x^2 + a^2*c^2*x
```

maple [A] time = 0.00, size = 87, normalized size = 1.34

$$\frac{b^2d^2x^5}{5} + a^2c^2x + \frac{\left(2ab\,d^2 + 2b^2cd\right)x^4}{4} + \frac{\left(a^2d^2 + 4abcd + b^2c^2\right)x^3}{3} + \frac{\left(2a^2cd + 2ab\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^2,x)
```

```
[Out] 1/5*b^2*d^2*x^5+1/4*(2*a*b*d^2+2*b^2*c*d)*x^4+1/3*(a^2*d^2+4*a*b*c*d+b^2*c^
2)*x^3+1/2*(2*a^2*c*d+2*a*b*c^2)*x^2+a^2*c^2*x
```

maxima [A] time = 1.34, size = 81, normalized size = 1.25

$$\frac{1}{5}\,b^2d^2x^5 + a^2c^2x + \frac{1}{2}\left(b^2cd + abd^2\right)x^4 + \frac{1}{3}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^3 + \left(abc^2 + a^2cd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*d^2*x^5 + a^2*c^2*x + 1/2*(b^2*c*d + a*b*d^2)*x^4 + 1/3*(b^2*c^2 +
4*a*b*c*d + a^2*d^2)*x^3 + (a*b*c^2 + a^2*c*d)*x^2
```

mupad [B] time = 0.17, size = 74, normalized size = 1.14

$$x^3\left(\frac{a^2\,d^2}{3} + \frac{4\,a\,b\,c\,d}{3} + \frac{b^2\,c^2}{3}\right) + a^2\,c^2\,x + \frac{b^2\,d^2\,x^5}{5} + a\,c\,x^2\,\left(a\,d + b\,c\right) + \frac{b\,d\,x^4\,\left(a\,d + b\,c\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x^3*((a^2*d^2)/3 + (b^2*c^2)/3 + (4*a*b*c*d)/3) + a^2*c^2*x + (b^2*d^2*x^5)
/5 + a*c*x^2*(a*d + b*c) + (b*d*x^4*(a*d + b*c))/2
```

sympy [A] time = 0.08, size = 87, normalized size = 1.34

$$a^2c^2x + \frac{b^2d^2x^5}{5} + x^4\left(\frac{abd^2}{2} + \frac{b^2cd}{2}\right) + x^3\left(\frac{a^2d^2}{3} + \frac{4abcd}{3} + \frac{b^2c^2}{3}\right) + x^2\left(a^2cd + abc^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**5/5 + x**4*(a*b*d**2/2 + b**2*c*d/2) + x**3*(a**
2*d**2/3 + 4*a*b*c*d/3 + b**2*c**2/3) + x**2*(a**2*c*d + a*b*c**2)
```

3.1143 $\int (a+bx)(c+dx)^2\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^4}{4d^2}-\frac{(c+dx)^3(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^3)/(3*d^2) + (b*(c + d*x)^4)/(4*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^2\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^2}{d}+\frac{b(c+dx)^3}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^3}{3d^2}+\frac{b(c+dx)^4}{4d^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.24

$$\frac{1}{12}x\left(4dx^2(ad+2bc)+6cx(2ad+bc)+12ac^2+3bd^2x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] (x*(12*a*c^2 + 6*c*(b*c + 2*a*d)*x + 4*d*(2*b*c + a*d)*x^2 + 3*b*d^2*x^3))/
12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^2, x]
```

fricas [A] time = 1.18, size = 49, normalized size = 1.29

$$\frac{1}{4}x^4d^2b+\frac{2}{3}x^3dcb+\frac{1}{3}x^3d^2a+\frac{1}{2}x^2c^2b+x^2dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*d^2*b + 2/3*x^3*d*c*b + 1/3*x^3*d^2*a + 1/2*x^2*c^2*b + x^2*d*c*a +
 x*c^2*a
```

giac [A] time = 1.03, size = 49, normalized size = 1.29

$$\frac{1}{4}bd^2x^4 + \frac{2}{3}bcdx^3 + \frac{1}{3}ad^2x^3 + \frac{1}{2}bc^2x^2 + acdx^2 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*d^2*x^4 + 2/3*b*c*d*x^3 + 1/3*a*d^2*x^3 + 1/2*b*c^2*x^2 + a*c*d*x^2 +
 a*c^2*x
```

maple [A] time = 0.00, size = 49, normalized size = 1.29

$$\frac{b\,d^2x^4}{4} + a\,c^2x + \frac{\left(a\,d^2 + 2bcd\right)x^3}{3} + \frac{\left(2acd + b\,c^2\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^2,x)
```

```
[Out] 1/4*b*d^2*x^4+1/3*(a*d^2+2*b*c*d)*x^3+1/2*(2*a*c*d+b*c^2)*x^2+a*c^2*x
```

maxima [A] time = 1.31, size = 48, normalized size = 1.26

$$\frac{1}{4}bd^2x^4 + ac^2x + \frac{1}{3}\left(2\,bcd + ad^2\right)x^3 + \frac{1}{2}\left(bc^2 + 2\,acd\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*d^2*x^4 + a*c^2*x + 1/3*(2*b*c*d + a*d^2)*x^3 + 1/2*(b*c^2 + 2*a*c*d)
*x^2
```

mupad [B] time = 0.04, size = 47, normalized size = 1.24

$$x^2\left(\frac{b\,c^2}{2} + a\,d\,c\right) + x^3\left(\frac{a\,d^2}{3} + \frac{2\,b\,c\,d}{3}\right) + \frac{b\,d^2\,x^4}{4} + a\,c^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^2,x)
```

```
[Out] x^2*((b*c^2)/2 + a*c*d) + x^3*((a*d^2)/3 + (2*b*c*d)/3) + (b*d^2*x^4)/4 + a
*c^2*x
```

sympy [A] time = 0.07, size = 49, normalized size = 1.29

$$ac^2x + \frac{bd^2x^4}{4} + x^3\left(\frac{ad^2}{3} + \frac{2bcd}{3}\right) + x^2\left(acd + \frac{bc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**4/4 + x**3*(a*d**2/3 + 2*b*c*d/3) + x**2*(a*c*d + b*c*
*2/2)
```

3.1144 $\int (c+dx)^2 \, dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^3}{3d}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2,x]
[Out] (c + d*x)^3/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^2 \, dx = \frac{(c+dx)^3}{3d}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2,x]
[Out] (c + d*x)^3/(3*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2,x]
[Out] IntegrateAlgebraic[(c + d*x)^2, x]
```

fricas [A] time = 1.25, size = 20, normalized size = 1.43

$$\frac{1}{3}x^3d^2 + x^2dc + xc^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="fricas")
[Out] 1/3*x^3*d^2 + x^2*d*c + x*c^2
```

giac [A] time = 0.96, size = 12, normalized size = 0.86

$$\frac{(dx+c)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(d*x + c)^3/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx+c)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2,x)
```

```
[Out] 1/3*(d*x+c)^3/d
```

maxima [A] time = 1.31, size = 20, normalized size = 1.43

$$\frac{1}{3}\,d^2x^3+cdx^2+c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x
```

mupad [B] time = 0.03, size = 20, normalized size = 1.43

$$c^2\,x+c\,d\,x^2+\frac{d^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2,x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + c*d*x^2
```

sympy [B] time = 0.06, size = 19, normalized size = 1.36

$$c^2x+cdx^2+\frac{d^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2,x)
```

```
[Out] c**2*x + c*d*x**2 + d**2*x**3/3
```

3.1145 $\int \frac{(c+dx)^2}{a+bx}\,dx$

Optimal. Leaf size=49

$$\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)}{b^2}+\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{(c+dx)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c - a*d)^2*Log[a + b*x])/b^
3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(c+dx)}{b}\right)dx\\ &= \frac{d(bc-ad)x}{b^2}+\frac{(c+dx)^2}{2b}+\frac{(bc-ad)^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.88

$$\frac{bdx(-2ad+4bc+bdx)+2(bc-ad)^2\log(a+bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x),x]
```

```
[Out] (b*d*x*(4*b*c - 2*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[a + b*x])/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x), x]
```

fricas [A] time = 0.84, size = 63, normalized size = 1.29

$$\frac{b^2d^2x^2 + 2\left(2\,b^2cd - abd^2\right)x + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*(2*b^2*c*d - a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(b*x + a))/b^3
```

giac [A] time = 1.20, size = 60, normalized size = 1.22

$$\frac{bd^2x^2 + 4\,bcdx - 2\,ad^2x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|bx + a|\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d^2*x^2 + 4*b*c*d*x - 2*a*d^2*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(b*x + a))/b^3
```

maple [A] time = 0.00, size = 74, normalized size = 1.51

$$\frac{d^2x^2}{2b} + \frac{a^2d^2\ln\left(bx + a\right)}{b^3} - \frac{2acd\ln\left(bx + a\right)}{b^2} - \frac{a\,d^2x}{b^2} + \frac{c^2\ln\left(bx + a\right)}{b} + \frac{2cdx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a),x)
```

```
[Out] 1/2*d^2/b*x^2-d^2/b^2*a*x+2*d/b*x*c+1/b^3*ln(b*x+a)*a^2*d^2-2/b^2*ln(b*x+a)
*a*c*d+1/b*ln(b*x+a)*c^2
```

maxima [A] time = 1.36, size = 61, normalized size = 1.24

$$\frac{bd^2x^2 + 2\left(2\,bcd - ad^2\right)x}{2\,b^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d^2*x^2 + 2*(2*b*c*d - a*d^2)*x)/b^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(b*x + a)/b^3
```

mupad [B] time = 0.19, size = 62, normalized size = 1.27

$$\frac{\ln\left(a + b\,x\right)\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{b^3} - x\,\left(\frac{a\,d^2}{b^2} - \frac{2\,c\,d}{b}\right) + \frac{d^2\,x^2}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x),x)
```

```
[Out] (log(a + b*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/b^3 - x*((a*d^2)/b^2 - (2*c*
d)/b) + (d^2*x^2)/(2*b)
```

sympy [A] time = 0.22, size = 44, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) + \frac{d^2x^2}{2b} + \frac{\left(ad - bc\right)^2\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a),x)
```

```
[Out] x*(-a*d**2/b**2 + 2*c*d/b) + d**2*x**2/(2*b) + (a*d - b*c)**2*log(a + b*x)/
b**3
```

3.1146 $\int \frac{(c+dx)^2}{(a+bx)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}+\frac{d^2x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (d^2*x)/b^2 - (b*c - a*d)^2/(b^3*(a + b*x)) + (2*d*(b*c - a*d)*Log[a + b*x]
)/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^2}\,dx &= \int \left(\frac{d^2}{b^2}+\frac{(bc-ad)^2}{b^2(a+bx)^2}+\frac{2d(bc-ad)}{b^2(a+bx)}\right)dx\\ &= \frac{d^2x}{b^2}-\frac{(bc-ad)^2}{b^3(a+bx)}+\frac{2d(bc-ad)\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{a+bx}+2d(bc-ad)\log(a+bx)+bd^2x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] (b*d^2*x - (b*c - a*d)^2/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x])/b^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^2, x]
```

fricas [A] time = 1.42, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + abd^2x - b^2c^2 + 2\,abcd - a^2d^2 + 2\left(abcd - a^2d^2 + \left(b^2cd - abd^2\right)x\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + a*b*d^2*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 + 2*(a*b*c*d - a^2
*d^2 + (b^2*c*d - a*b*d^2)*x)*log(b*x + a))/(b^4*x + a*b^3)
```

giac [A] time = 0.94, size = 98, normalized size = 1.92

$$\frac{(bx + a)d^2}{b^3} - \frac{2\left(bcd - ad^2\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^3} - \frac{\frac{b^3c^2}{bx+a} - \frac{2\,ab^2cd}{bx+a} + \frac{a^2bd^2}{bx+a}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (b*x + a)*d^2/b^3 - 2*(b*c*d - a*d^2)*log(abs(b*x + a)/((b*x + a)^2*abs(b))
)/b^3 - (b^3*c^2/(b*x + a) - 2*a*b^2*c*d/(b*x + a) + a^2*b*d^2/(b*x + a))/b
^4
```

maple [A] time = 0.01, size = 86, normalized size = 1.69

$$-\frac{a^2d^2}{(bx + a)\,b^3} + \frac{2acd}{(bx + a)\,b^2} - \frac{2a\,d^2\ln\left(bx + a\right)}{b^3} - \frac{c^2}{(bx + a)\,b} + \frac{2cd\ln\left(bx + a\right)}{b^2} + \frac{d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^2,x)
```

```
[Out] d^2*x/b^2-2/b^3*d^2*ln(b*x+a)*a+2/b^2*d*ln(b*x+a)*c-1/b^3/(b*x+a)*a^2*d^2+2
/b^2/(b*x+a)*a*c*d-1/b/(b*x+a)*c^2
```

maxima [A] time = 1.37, size = 67, normalized size = 1.31

$$\frac{d^2x}{b^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{b^4x + ab^3} + \frac{2\left(bcd - ad^2\right)\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d^2*x/b^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(b^4*x + a*b^3) + 2*(b*c*d - a*
d^2)*log(b*x + a)/b^3
```

mupad [B] time = 0.20, size = 71, normalized size = 1.39

$$\frac{d^2\,x}{b^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{b\,\left(x\,b^3 + a\,b^2\right)} - \frac{\ln\left(a + b\,x\right)\,\left(2\,a\,d^2 - 2\,b\,c\,d\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^2,x)
```

```
[Out] (d^2*x)/b^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(b*(a*b^2 + b^3*x)) - (log(a
+ b*x)*(2*a*d^2 - 2*b*c*d))/b^3
```

sympy [A] time = 0.34, size = 60, normalized size = 1.18

$$\frac{-a^2d^2 + 2abcd - b^2c^2}{ab^3 + b^4x} + \frac{d^2x}{b^2} - \frac{2d\,(ad - bc)\log{(a + bx)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**2,x)
```

```
[Out] (-a**2*d**2 + 2*a*b*c*d - b**2*c**2)/(a*b**3 + b**4*x) + d**2*x/b**2 - 2*d*
(a*d - b*c)*log(a + b*x)/b**3
```

3.1147 $\int \frac{(c+dx)^2}{(a+bx)^3}\,dx$

Optimal. Leaf size=59

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{b^3(a+bx)}-\frac{(bc-ad)^2}{2b^3(a+bx)^2}+\frac{d^2\log(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*b^3*(a + b*x)^2) - (2*d*(b*c - a*d))/(b^3*(a + b*x)) + (d
^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^3}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^3}+\frac{2d(bc-ad)}{b^2(a+bx)^2}+\frac{d^2}{b^2(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2b^3(a+bx)^2}-\frac{2d(bc-ad)}{b^3(a+bx)}+\frac{d^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.83

$$\frac{2d^2\log(a+bx)-\frac{(bc-ad)(3ad+b(c+4dx))}{(a+bx)^2}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] (-(((b*c - a*d)*(3*a*d + b*(c + 4*d*x)))/(a + b*x)^2) + 2*d^2*Log[a + b*x])
/(2*b^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^3, x]
```

fricas [A] time = 1.14, size = 99, normalized size = 1.68

$$-\frac{b^2c^2 + 2\,abcd - 3\,a^2d^2 + 4\left(b^2cd - abd^2\right)x - 2\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(bx + a\right)}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3
)
```

giac [A] time = 1.03, size = 68, normalized size = 1.15

$$\frac{d^2\log\left(|bx + a|\right)}{b^3} - \frac{4\left(bcd - ad^2\right)x + \frac{b^2c^2 + 2\,abcd - 3\,a^2d^2}{b}}{2\left(bx + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^2*log(abs(b*x + a))/b^3 - 1/2*(4*(b*c*d - a*d^2)*x + (b^2*c^2 + 2*a*b*c*d
 - 3*a^2*d^2)/b)/((b*x + a)^2*b^2)
```

maple [A] time = 0.01, size = 92, normalized size = 1.56

$$-\frac{a^2d^2}{2\left(bx + a\right)^2b^3} + \frac{acd}{\left(bx + a\right)^2b^2} - \frac{c^2}{2\left(bx + a\right)^2b} + \frac{2a\,d^2}{\left(bx + a\right)b^3} - \frac{2cd}{\left(bx + a\right)b^2} + \frac{d^2\ln\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^3,x)
```

```
[Out] -1/2/b^3/(b*x+a)^2*a^2*d^2+1/b^2/(b*x+a)^2*a*c*d-1/2/b/(b*x+a)^2*c^2+d^2*ln
(b*x+a)/b^3+2/b^3*d^2/(b*x+a)*a-2/b^2*d/(b*x+a)*c
```

maxima [A] time = 1.30, size = 79, normalized size = 1.34

$$-\frac{b^2c^2 + 2\,abcd - 3\,a^2d^2 + 4\left(b^2cd - abd^2\right)x}{2\left(b^5x^2 + 2\,ab^4x + a^2b^3\right)} + \frac{d^2\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(b^5*x^2 +
 2*a*b^4*x + a^2*b^3) + d^2*log(b*x + a)/b^3
```

mupad [B] time = 0.20, size = 77, normalized size = 1.31

$$\frac{d^2\,\ln\left(a + b\,x\right)}{b^3} - \frac{\frac{-3\,a^2\,d^2 + 2\,a\,b\,c\,d + b^2\,c^2}{2\,b^3} - \frac{2\,d\,x\,\left(a\,d - b\,c\right)}{b^2}}{a^2 + 2\,a\,b\,x + b^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^3,x)
```

```
[Out] (d^2*log(a + b*x))/b^3 - ((b^2*c^2 - 3*a^2*d^2 + 2*a*b*c*d)/(2*b^3) - (2*d*
x*(a*d - b*c))/b^2)/(a^2 + b^2*x^2 + 2*a*b*x)
```

sympy [A] time = 0.45, size = 80, normalized size = 1.36

$$\frac{3a^2d^2 - 2abcd - b^2c^2 + x\left(4abd^2 - 4b^2cd\right)}{2a^2b^3 + 4ab^4x + 2b^5x^2} + \frac{d^2\log\left(a + bx\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**3,x)
```

```
[Out] (3*a**2*d**2 - 2*a*b*c*d - b**2*c**2 + x*(4*a*b*d**2 - 4*b**2*c*d))/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) + d**2*log(a + b*x)/b**3
```

3.1148 $\int \frac{(c+dx)^2}{(a+bx)^4}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^3}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -(c + d*x)^3/(3*(b*c - a*d)*(a + b*x)^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+bx)^4}\,dx = -\frac{(c+dx)^3}{3(bc-ad)(a+bx)^3}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 1.89

$$-\frac{a^2d^2+abd(c+3dx)+b^2\left(c^2+3cdx+3d^2x^2\right)}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] -1/3*(a^2*d^2 + a*b*d*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))/(b^3*(
a + b*x)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^4, x]
```

fricas [B] time = 1.39, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

giac [B] time = 0.96, size = 59, normalized size = 2.11

$$-\frac{3\,b^2d^2x^2+3\,b^2cdx+3\,abd^2x+b^2c^2+abcd+a^2d^2}{3\,(bx+a)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + 3*b^2*c*d*x + 3*a*b*d^2*x + b^2*c^2 + a*b*c*d + a^2*d
^2)/((b*x + a)^3*b^3)
```

maple [B] time = 0.01, size = 70, normalized size = 2.50

$$-\frac{d^2}{(bx+a)\,b^3}+\frac{(ad-bc)\,d}{(bx+a)^2\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{3\,(bx+a)^3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^4,x)
```

```
[Out] -1/3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^3+d*(a*d-b*c)/b^3/(b*x+a)^2-d^
2/b^3/(b*x+a)
```

maxima [B] time = 1.37, size = 84, normalized size = 3.00

$$-\frac{3\,b^2d^2x^2+b^2c^2+abcd+a^2d^2+3\left(b^2cd+abd^2\right)x}{3\left(b^6x^3+3\,ab^5x^2+3\,a^2b^4x+a^3b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*d^2*x^2 + b^2*c^2 + a*b*c*d + a^2*d^2 + 3*(b^2*c*d + a*b*d^2)*x
)/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)
```

mupad [B] time = 0.04, size = 80, normalized size = 2.86

$$-\frac{\frac{a^2\,d^2+a\,b\,c\,d+b^2\,c^2}{3\,b^3}+\frac{d^2\,x^2}{b}+\frac{d\,x\,(a\,d+b\,c)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^4,x)
```

```
[Out] -((a^2*d^2 + b^2*c^2 + a*b*c*d)/(3*b^3) + (d^2*x^2)/b + (d*x*(a*d + b*c))/b
^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

sympy [B] time = 0.60, size = 88, normalized size = 3.14

$$\frac{-a^2d^2-abcd-b^2c^2-3b^2d^2x^2+x\left(-3abd^2-3b^2cd\right)}{3a^3b^3+9a^2b^4x+9ab^5x^2+3b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**4,x)
```

```
[Out] (-a**2*d**2 - a*b*c*d - b**2*c**2 - 3*b**2*d**2*x**2 + x*(-3*a*b*d**2 - 3*b
**2*c*d))/(3*a**3*b**3 + 9*a**2*b**4*x + 9*a*b**5*x**2 + 3*b**6*x**3)
```

3.1149 $\int \frac{(c+dx)^2}{(a+bx)^5} \, dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{d^2}{2b^3(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -(b*c - a*d)^2/(4*b^3*(a + b*x)^4) - (2*d*(b*c - a*d))/(3*b^3*(a + b*x)^3)
- d^2/(2*b^3*(a + b*x)^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+bx)^5} \, dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^5}+\frac{2d(bc-ad)}{b^2(a+bx)^4}+\frac{d^2}{b^2(a+bx)^3}\right) dx\\
&= -\frac{(bc-ad)^2}{4b^3(a+bx)^4}-\frac{2d(bc-ad)}{3b^3(a+bx)^3}-\frac{d^2}{2b^3(a+bx)^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 56, normalized size = 0.86

$$-\frac{a^2d^2+2abd(c+2dx)+b^2\left(3c^2+8cdx+6d^2x^2\right)}{12b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] -1/12*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d*x + 6*d^2*x^2))/(
b^3*(a + b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^5, x]
```

fricas [A] time = 1.49, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2 + 3\,b^2c^2 + 2\,abcd + a^2d^2 + 4\left(2\,b^2cd + abd^2\right)x}{12\left(b^7x^4 + 4\,ab^6x^3 + 6\,a^2b^5x^2 + 4\,a^3b^4x + a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

giac [A] time = 1.13, size = 96, normalized size = 1.48

$$-\frac{\frac{3\,c^2}{(bx+a)^4} + \frac{8\,cd}{(bx+a)^3b} - \frac{6\,acd}{(bx+a)^4b} + \frac{6\,d^2}{(bx+a)^2b^2} - \frac{8\,ad^2}{(bx+a)^3b^2} + \frac{3\,a^2d^2}{(bx+a)^4b^2}}{12\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*c^2/(b*x + a)^4 + 8*c*d/((b*x + a)^3*b) - 6*a*c*d/((b*x + a)^4*b)
+ 6*d^2/((b*x + a)^2*b^2) - 8*a*d^2/((b*x + a)^3*b^2) + 3*a^2*d^2/((b*x + a
)^4*b^2))/b
```

maple [A] time = 0.01, size = 71, normalized size = 1.09

$$-\frac{d^2}{2\left(bx+a\right)^2b^3} + \frac{2\left(ad-bc\right)d}{3\left(bx+a\right)^3b^3} - \frac{a^2d^2 - 2abcd + b^2c^2}{4\left(bx+a\right)^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^5,x)
```

```
[Out] 2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/2*d^2/b^3/(b*x+a)^2-1/4*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/(b*x+a)^4
```

maxima [A] time = 1.33, size = 98, normalized size = 1.51

$$-\frac{6\,b^2d^2x^2 + 3\,b^2c^2 + 2\,abcd + a^2d^2 + 4\left(2\,b^2cd + abd^2\right)x}{12\left(b^7x^4 + 4\,ab^6x^3 + 6\,a^2b^5x^2 + 4\,a^3b^4x + a^4b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(6*b^2*d^2*x^2 + 3*b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*(2*b^2*c*d + a*b
*d^2)*x)/(b^7*x^4 + 4*a*b^6*x^3 + 6*a^2*b^5*x^2 + 4*a^3*b^4*x + a^4*b^3)
```

mupad [B] time = 0.19, size = 39, normalized size = 0.60

$$\frac{{\left(c+d\,x\right)}^3\,\left(4\,a\,d-3\,b\,c+b\,d\,x\right)}{12\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^5,x)
```

```
[Out] ((c + d*x)^3*(4*a*d - 3*b*c + b*d*x))/(12*(a*d - b*c)^2*(a + b*x)^4)
```

sympy [A] time = 0.76, size = 104, normalized size = 1.60

$$\frac{-a^2d^2 - 2abcd - 3b^2c^2 - 6b^2d^2x^2 + x\left(-4abd^2 - 8b^2cd\right)}{12a^4b^3 + 48a^3b^4x + 72a^2b^5x^2 + 48ab^6x^3 + 12b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**5,x)
```

```
[Out] (-a**2*d**2 - 2*a*b*c*d - 3*b**2*c**2 - 6*b**2*d**2*x**2 + x*(-4*a*b*d**2 -
 8*b**2*c*d))/(12*a**4*b**3 + 48*a**3*b**4*x + 72*a**2*b**5*x**2 + 48*a*b**
6*x**3 + 12*b**7*x**4)
```

3.1150 $\int \frac{(c+dx)^2}{(a+bx)^6} dx$

Optimal. Leaf size=65

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d^2}{3b^3(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -(b*c - a*d)^2/(5*b^3*(a + b*x)^5) - (d*(b*c - a*d))/(2*b^3*(a + b*x)^4) -
d^2/(3*b^3*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^6} dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^6}+\frac{2d(bc-ad)}{b^2(a+bx)^5}+\frac{d^2}{b^2(a+bx)^4}\right) dx\\ &= -\frac{(bc-ad)^2}{5b^3(a+bx)^5}-\frac{d(bc-ad)}{2b^3(a+bx)^4}-\frac{d^2}{3b^3(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 57, normalized size = 0.88

$$-\frac{a^2d^2+abd(3c+5dx)+b^2\left(6c^2+15cdx+10d^2x^2\right)}{30b^3(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] -1/30*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))
/(b^3*(a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^6} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^6, x]
```

fricas [A] time = 1.34, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2 + 6\,b^2c^2 + 3\,abcd + a^2d^2 + 5\left(3\,b^2cd + abd^2\right)x}{30\left(b^8x^5 + 5\,ab^7x^4 + 10\,a^2b^6x^3 + 10\,a^3b^5x^2 + 5\,a^4b^4x + a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

giac [A] time = 0.87, size = 61, normalized size = 0.94

$$-\frac{10\,b^2d^2x^2 + 15\,b^2cdx + 5\,abd^2x + 6\,b^2c^2 + 3\,abcd + a^2d^2}{30\,(bx + a)^5b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 15*b^2*c*d*x + 5*a*b*d^2*x + 6*b^2*c^2 + 3*a*b*c*d
+ a^2*d^2)/((b*x + a)^5*b^3)
```

maple [A] time = 0.01, size = 71, normalized size = 1.09

$$-\frac{d^2}{3\,(bx + a)^3\,b^3} + \frac{(ad - bc)\,d}{2\,(bx + a)^4\,b^3} - \frac{a^2d^2 - 2abcd + b^2c^2}{5\,(bx + a)^5\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^6,x)
```

```
[Out] -1/3*d^2/b^3/(b*x+a)^3-1/5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5+1/2*d*
(a*d-b*c)/b^3/(b*x+a)^4
```

maxima [A] time = 1.35, size = 109, normalized size = 1.68

$$-\frac{10\,b^2d^2x^2 + 6\,b^2c^2 + 3\,abcd + a^2d^2 + 5\left(3\,b^2cd + abd^2\right)x}{30\left(b^8x^5 + 5\,ab^7x^4 + 10\,a^2b^6x^3 + 10\,a^3b^5x^2 + 5\,a^4b^4x + a^5b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/30*(10*b^2*d^2*x^2 + 6*b^2*c^2 + 3*a*b*c*d + a^2*d^2 + 5*(3*b^2*c*d + a*
b*d^2)*x)/(b^8*x^5 + 5*a*b^7*x^4 + 10*a^2*b^6*x^3 + 10*a^3*b^5*x^2 + 5*a^4*
b^4*x + a^5*b^3)
```

mupad [B] time = 0.20, size = 107, normalized size = 1.65

$$-\frac{\frac{a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2}{30\,b^3}+\frac{d^2\,x^2}{3\,b}+\frac{d\,x\,\left(a\,d+3\,b\,c\right)}{6\,b^2}}{a^5+5\,a^4\,b\,x+10\,a^3\,b^2\,x^2+10\,a^2\,b^3\,x^3+5\,a\,b^4\,x^4+b^5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^6,x)
```

```
[Out] -((a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d)/(30*b^3) + (d^2*x^2)/(3*b) + (d*x*(a*d
+ 3*b*c))/(6*b^2))/(a^5 + b^5*x^5 + 5*a*b^4*x^4 + 10*a^3*b^2*x^2 + 10*a^2*b
^3*x^3 + 5*a^4*b*x)
```

sympy [B] time = 0.96, size = 116, normalized size = 1.78

$$\frac{-a^2d^2 - 3abcd - 6b^2c^2 - 10b^2d^2x^2 + x\left(-5abd^2 - 15b^2cd\right)}{30a^5b^3 + 150a^4b^4x + 300a^3b^5x^2 + 300a^2b^6x^3 + 150ab^7x^4 + 30b^8x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**6,x)
```

```
[Out] (-a**2*d**2 - 3*a*b*c*d - 6*b**2*c**2 - 10*b**2*d**2*x**2 + x*(-5*a*b*d**2
- 15*b**2*c*d))/(30*a**5*b**3 + 150*a**4*b**4*x + 300*a**3*b**5*x**2 + 300*
a**2*b**6*x**3 + 150*a*b**7*x**4 + 30*b**8*x**5)
```

3.1151 $\int \frac{(c+dx)^2}{(a+bx)^7} \, dx$

Optimal. Leaf size=65

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{d^2}{4b^3(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^2/(6*b^3*(a + b*x)^6) - (2*d*(b*c - a*d))/(5*b^3*(a + b*x)^5)
- d^2/(4*b^3*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+bx)^7}\,dx &= \int \left(\frac{(bc-ad)^2}{b^2(a+bx)^7}+\frac{2d(bc-ad)}{b^2(a+bx)^6}+\frac{d^2}{b^2(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^2}{6b^3(a+bx)^6}-\frac{2d(bc-ad)}{5b^3(a+bx)^5}-\frac{d^2}{4b^3(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.89

$$-\frac{a^2d^2+2abd(2c+3dx)+b^2\left(10c^2+24cdx+15d^2x^2\right)}{60b^3(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^2*d^2 + 2*a*b*d*(2*c + 3*d*x) + b^2*(10*c^2 + 24*c*d*x + 15*d^2*x^
2))/(b^3*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^2/(a + b*x)^7, x]
```

fricas [B] time = 1.33, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

giac [A] time = 1.04, size = 61, normalized size = 0.94

$$-\frac{15\,b^2d^2x^2+24\,b^2cdx+6\,abd^2x+10\,b^2c^2+4\,abcd+a^2d^2}{60\,(bx+a)^6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 24*b^2*c*d*x + 6*a*b*d^2*x + 10*b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)/((b*x + a)^6*b^3)
```

maple [A] time = 0.00, size = 71, normalized size = 1.09

$$-\frac{d^2}{4\,(bx+a)^4\,b^3}+\frac{2\,(ad-bc)\,d}{5\,(bx+a)^5\,b^3}-\frac{a^2d^2-2abcd+b^2c^2}{6\,(bx+a)^6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(b*x+a)^7,x)
```

```
[Out] 2/5*d*(a*d-b*c)/b^3/(b*x+a)^5-1/4*d^2/b^3/(b*x+a)^4-1/6*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^3/(b*x+a)^6
```

maxima [B] time = 1.39, size = 120, normalized size = 1.85

$$-\frac{15\,b^2d^2x^2+10\,b^2c^2+4\,abcd+a^2d^2+6\left(4\,b^2cd+abd^2\right)x}{60\left(b^9x^6+6\,ab^8x^5+15\,a^2b^7x^4+20\,a^3b^6x^3+15\,a^4b^5x^2+6\,a^5b^4x+a^6b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(15*b^2*d^2*x^2 + 10*b^2*c^2 + 4*a*b*c*d + a^2*d^2 + 6*(4*b^2*c*d + a
*b*d^2)*x)/(b^9*x^6 + 6*a*b^8*x^5 + 15*a^2*b^7*x^4 + 20*a^3*b^6*x^3 + 15*a^
4*b^5*x^2 + 6*a^5*b^4*x + a^6*b^3)
```

mupad [B] time = 0.09, size = 118, normalized size = 1.82

$$-\frac{\frac{a^2\,d^2+4\,a\,b\,c\,d+10\,b^2\,c^2}{60\,b^3}+\frac{d^2\,x^2}{4\,b}+\frac{d\,x\,\left(a\,d+4\,b\,c\right)}{10\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*x)^7,x)
```

```
[Out] -((a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d)/(60*b^3) + (d^2*x^2)/(4*b) + (d*x*(a*d
 + 4*b*c))/(10*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3
*b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

sympy [B] time = 1.16, size = 128, normalized size = 1.97

$$\frac{-a^2d^2 - 4abcd - 10b^2c^2 - 15b^2d^2x^2 + x\left(-6abd^2 - 24b^2cd\right)}{60a^6b^3 + 360a^5b^4x + 900a^4b^5x^2 + 1200a^3b^6x^3 + 900a^2b^7x^4 + 360ab^8x^5 + 60b^9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(b*x+a)**7,x)
```

```
[Out] (-a**2*d**2 - 4*a*b*c*d - 10*b**2*c**2 - 15*b**2*d**2*x**2 + x*(-6*a*b*d**2
 - 24*b**2*c*d))/(60*a**6*b**3 + 360*a**5*b**4*x + 900*a**4*b**5*x**2 + 120
0*a**3*b**6*x**3 + 900*a**2*b**7*x**4 + 360*a*b**8*x**5 + 60*b**9*x**6)
```

3.1152 $\int (a+bx)^5(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Rubi [A] time = 0.16, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(a+bx)^8(bc-ad)}{8b^4}+\frac{3d(a+bx)^7(bc-ad)^2}{7b^4}+\frac{(a+bx)^6(bc-ad)^3}{6b^4}+\frac{d^3(a+bx)^9}{9b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^6)/(6*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^7)/(7*b^
4) + (3*d^2*(b*c - a*d)*(a + b*x)^8)/(8*b^4) + (d^3*(a + b*x)^9)/(9*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^5}{b^3}+\frac{3d(bc-ad)^2(a+bx)^6}{b^3}+\frac{3d^2(bc-ad)(a+bx)^7}{b^3}+\frac{d^3(a+b}{b^3}\right.\\ &= \frac{(bc-ad)^3(a+bx)^6}{6b^4}+\frac{3d(bc-ad)^2(a+bx)^7}{7b^4}+\frac{3d^2(bc-ad)(a+bx)^8}{8b^4}+\frac{d^3(a+bx)^9}{9b^4}\end{aligned}$$

Mathematica [B] time = 0.08, size = 235, normalized size = 2.55

$$\frac{1}{504}x\left(126a^5\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)+126a^4bx\left(10c^3+20c^2dx+15cd^2x^2+4d^3x^3\right)+84a^3b^2x^2\left(20c^3+45c^2dx+36cd^2x^2+10d^3x^3\right)+36a^2b^3x^3\left(35c^3+84c^2dx+70cd^2x^2+20d^3x^3\right)+9ab^4x^4\left(56c^3+140c^2dx+120cd^2x^2+35d^3x^3\right)+b^5x^5\left(84c^3+216c^2dx+189cd^2x^2+56d^3x^3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] (x*(126*a^5*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 126*a^4*b*x*(10*c
^3 + 20*c^2*d*x + 15*c*d^2*x^2 + 4*d^3*x^3) + 84*a^3*b^2*x^2*(20*c^3 + 45*c
^2*d*x + 36*c*d^2*x^2 + 10*d^3*x^3) + 36*a^2*b^3*x^3*(35*c^3 + 84*c^2*d*x +
 70*c*d^2*x^2 + 20*d^3*x^3) + 9*a*b^4*x^4*(56*c^3 + 140*c^2*d*x + 120*c*d^2
*x^2 + 35*d^3*x^3) + b^5*x^5*(84*c^3 + 216*c^2*d*x + 189*c*d^2*x^2 + 56*d^3
*x^3)))/504
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^3, x]
```

fricas [**B**] time = 1.14, size = 303, normalized size = 3.29

$$\frac{1}{9}x^9d^3b^5+\frac{3}{8}x^8d^2cb^5+\frac{5}{8}x^8d^3b^4a+\frac{3}{7}x^7dc^2b^5+\frac{15}{7}x^7d^2cb^4a+\frac{10}{7}x^7d^3b^3a^2+\frac{1}{6}x^6c^3b^5+\frac{5}{2}x^6dc^2b^4a+5x^6d^2cb^3a^2+\frac{5}{3}x^6d^3b^2a^3+x^5c^3b^4a+6x^5dc^2b^3a^2+6x^5d^2cb^2a^3+x^5d^3ba^4+\frac{5}{2}x^4c^3b^3a^2+\frac{15}{2}x^4dc^2b^2a^3+\frac{15}{4}x^4d^2cba^4+\frac{1}{4}x^4d^3a^5+\frac{10}{3}x^3c^3b^2a^3+5x^3dc^2ba^4+x^3d^2ca^5+\frac{5}{2}x^2c^3ba^4+\frac{3}{2}x^2dc^2a^5+xc^3a^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^3*b^5 + 3/8*x^8*d^2*c*b^5 + 5/8*x^8*d^3*b^4*a + 3/7*x^7*d*c^2*b^5
 + 15/7*x^7*d^2*c*b^4*a + 10/7*x^7*d^3*b^3*a^2 + 1/6*x^6*c^3*b^5 + 5/2*x^6*
d*c^2*b^4*a + 5*x^6*d^2*c*b^3*a^2 + 5/3*x^6*d^3*b^2*a^3 + x^5*c^3*b^4*a + 6
*x^5*d*c^2*b^3*a^2 + 6*x^5*d^2*c*b^2*a^3 + x^5*d^3*b*a^4 + 5/2*x^4*c^3*b^3*
a^2 + 15/2*x^4*d*c^2*b^2*a^3 + 15/4*x^4*d^2*c*b*a^4 + 1/4*x^4*d^3*a^5 + 10/
3*x^3*c^3*b^2*a^3 + 5*x^3*d*c^2*b*a^4 + x^3*d^2*c*a^5 + 5/2*x^2*c^3*b*a^4 +
 3/2*x^2*d*c^2*a^5 + x*c^3*a^5
```

giac [**B**] time = 1.01, size = 303, normalized size = 3.29

$$\frac{1}{9}b^5d^3x^9+\frac{3}{8}b^5cd^2x^8+\frac{5}{8}ab^4d^3x^8+\frac{3}{7}b^5c^2dx^7+\frac{15}{7}ab^4cd^2x^7+\frac{10}{7}a^2b^3d^3x^7+\frac{1}{6}b^5c^3x^6+\frac{5}{2}ab^4c^2dx^6+5a^2b^3cd^2x^6+\frac{5}{3}a^3b^2d^3x^6+ab^4c^3x^5+6a^2b^3c^2dx^5+6a^3b^2cd^2x^5+a^4bd^3x^5+\frac{5}{2}a^2b^3c^3x^4+\frac{15}{2}a^3b^2c^2dx^4+\frac{15}{4}a^4bcd^2x^4+\frac{1}{4}a^5d^3x^4+\frac{10}{3}a^3b^2c^3x^3+5a^4bc^2dx^3+a^5cd^2x^3+\frac{5}{2}a^4bc^3x^2+\frac{3}{2}a^5c^2dx^2+a^5c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/9*b^5*d^3*x^9 + 3/8*b^5*c*d^2*x^8 + 5/8*a*b^4*d^3*x^8 + 3/7*b^5*c^2*d*x^7
 + 15/7*a*b^4*c*d^2*x^7 + 10/7*a^2*b^3*d^3*x^7 + 1/6*b^5*c^3*x^6 + 5/2*a*b^
4*c^2*d*x^6 + 5*a^2*b^3*c*d^2*x^6 + 5/3*a^3*b^2*d^3*x^6 + a*b^4*c^3*x^5 + 6
*a^2*b^3*c^2*d*x^5 + 6*a^3*b^2*c*d^2*x^5 + a^4*b*d^3*x^5 + 5/2*a^2*b^3*c^3*
x^4 + 15/2*a^3*b^2*c^2*d*x^4 + 15/4*a^4*b*c*d^2*x^4 + 1/4*a^5*d^3*x^4 + 10/
3*a^3*b^2*c^3*x^3 + 5*a^4*b*c^2*d*x^3 + a^5*c*d^2*x^3 + 5/2*a^4*b*c^3*x^2 +
 3/2*a^5*c^2*d*x^2 + a^5*c^3*x
```

maple [**B**] time = 0.00, size = 281, normalized size = 3.05

$$\frac{b^5d^3x^9}{9}+a^5c^3x+\frac{\left(5ab^4d^3+3b^5cd^2\right)x^8}{8}+\frac{\left(10a^2b^3d^3+15ab^4cd^2+3b^5c^2d\right)x^7}{7}+\frac{\left(10a^3b^2d^3+30a^2b^3cd^2+15ab^4c^2d+b^5c^3\right)x^6}{6}+\frac{\left(5a^4bd^3+30a^3b^2cd^2+30a^2b^3c^2d+5ab^4c^3\right)x^5}{5}+\frac{\left(a^5d^3+15a^4bcd^2+30a^3b^2c^2d+10a^2b^3c^3\right)x^4}{4}+\frac{\left(3a^5cd^2+15a^4bc^2d+10a^3b^2c^3\right)x^3}{3}+\frac{\left(3a^5c^2d+5a^4bc^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^3,x)
```

```
[Out] 1/9*b^5*d^3*x^9+1/8*(5*a*b^4*d^3+3*b^5*c*d^2)*x^8+1/7*(10*a^2*b^3*d^3+15*a*
b^4*c*d^2+3*b^5*c^2*d)*x^7+1/6*(10*a^3*b^2*d^3+30*a^2*b^3*c*d^2+15*a*b^4*c^
2*d+b^5*c^3)*x^6+1/5*(5*a^4*b*d^3+30*a^3*b^2*c*d^2+30*a^2*b^3*c^2*d+5*a*b^4
*c^3)*x^5+1/4*(a^5*d^3+15*a^4*b*c*d^2+30*a^3*b^2*c^2*d+10*a^2*b^3*c^3)*x^4+
1/3*(3*a^5*c*d^2+15*a^4*b*c^2*d+10*a^3*b^2*c^3)*x^3+1/2*(3*a^5*c^2*d+5*a^4*
b*c^3)*x^2+a^5*c^3*x
```

maxima [**B**] time = 1.38, size = 277, normalized size = 3.01

$$\frac{1}{9}b^5d^3x^9+a^5c^3x+\frac{1}{8}\left(3b^5cd^2+5ab^4d^3\right)x^8+\frac{1}{7}\left(3b^5c^2d+15ab^4cd^2+10a^2b^3d^3\right)x^7+\frac{1}{6}\left(b^5c^3+15ab^4c^2d+30a^2b^3cd^2+10a^3b^2d^3\right)x^6+\left(ab^4c^3+6a^2b^3c^2d+6a^3b^2cd^2+a^4bd^3\right)x^5+\frac{1}{4}\left(10a^2b^3c^3+30a^3b^2c^2d+15a^4bcd^2+a^5d^3\right)x^4+\frac{1}{3}\left(10a^3b^2c^3+15a^4bc^2d+3a^5cd^2\right)x^3+\frac{1}{2}\left(5a^4bc^3+3a^5c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/9*b^5*d^3*x^9 + a^5*c^3*x + 1/8*(3*b^5*c*d^2 + 5*a*b^4*d^3)*x^8 + 1/7*(3*
b^5*c^2*d + 15*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^7 + 1/6*(b^5*c^3 + 15*a*b^4*
c^2*d + 30*a^2*b^3*c*d^2 + 10*a^3*b^2*d^3)*x^6 + (a*b^4*c^3 + 6*a^2*b^3*c^2
*d + 6*a^3*b^2*c*d^2 + a^4*b*d^3)*x^5 + 1/4*(10*a^2*b^3*c^3 + 30*a^3*b^2*c^
2*d + 15*a^4*b*c*d^2 + a^5*d^3)*x^4 + 1/3*(10*a^3*b^2*c^3 + 15*a^4*b*c^2*d
+ 3*a^5*c*d^2)*x^3 + 1/2*(5*a^4*b*c^3 + 3*a^5*c^2*d)*x^2
```

mupad [B] time = 0.24, size = 261, normalized size = 2.84

$$x^5\left(a^4\,b\,d^3+6\,a^3\,b^2\,c\,d^2+6\,a^2\,b^3\,c^2\,d+a\,b^4\,c^3\right)+x^4\left(\frac{a^5\,d^3}{4}+\frac{15\,a^4\,b\,c\,d^2}{4}+\frac{15\,a^3\,b^2\,c^2\,d}{2}+\frac{5\,a^2\,b^3\,c^3}{2}\right)+x^6\left(\frac{5\,a^3\,b^2\,d^3}{3}+5\,a^2\,b^3\,c\,d^2+\frac{5\,a\,b^4\,c^2\,d}{2}+\frac{b^5\,c^3}{6}\right)+a^5\,c^3\,x+\frac{b^5\,d^3\,x^9}{9}+\frac{a^4\,c^2\,x^2\,(3\,a\,d+5\,b\,c)}{2}+\frac{b^4\,d^2\,x^8\,(5\,a\,d+3\,b\,c)}{8}+\frac{a^3\,c\,x^3\,\left(3\,a^2\,d^2+15\,a\,b\,c\,d+10\,b^2\,c^2\right)}{3}+\frac{b^3\,d\,x^7\,\left(10\,a^2\,d^2+15\,a\,b\,c\,d+3\,b^2\,c^2\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^3,x)
```

```
[Out] x^5*(a*b^4*c^3 + a^4*b*d^3 + 6*a^2*b^3*c^2*d + 6*a^3*b^2*c*d^2) + x^4*((a^5
*d^3)/4 + (5*a^2*b^3*c^3)/2 + (15*a^3*b^2*c^2*d)/2 + (15*a^4*b*c*d^2)/4) +
x^6*((b^5*c^3)/6 + (5*a^3*b^2*d^3)/3 + 5*a^2*b^3*c*d^2 + (5*a*b^4*c^2*d)/2)
 + a^5*c^3*x + (b^5*d^3*x^9)/9 + (a^4*c^2*x^2*(3*a*d + 5*b*c))/2 + (b^4*d^2
*x^8*(5*a*d + 3*b*c))/8 + (a^3*c*x^3*(3*a^2*d^2 + 10*b^2*c^2 + 15*a*b*c*d))
/3 + (b^3*d*x^7*(10*a^2*d^2 + 3*b^2*c^2 + 15*a*b*c*d))/7
```

sympy [B] time = 0.12, size = 308, normalized size = 3.35

$$a^5c^3x+\frac{b^5d^3x^9}{9}+x^8\left(\frac{5ab^4d^3}{8}+\frac{3b^5cd^2}{8}\right)+x^7\left(\frac{10a^2b^3d^3}{7}+\frac{15ab^4cd^2}{7}+\frac{3b^5c^2d}{7}\right)+x^6\left(\frac{5a^3b^2d^3}{3}+5a^2b^3cd^2+\frac{5ab^4c^2d}{2}+\frac{b^5c^3}{6}\right)+x^5\left(a^4bd^3+6a^3b^2cd^2+6a^2b^3c^2d+ab^4c^3\right)+x^4\left(\frac{a^5d^3}{4}+\frac{15a^4bcd^2}{4}+\frac{15a^3b^2c^2d}{2}+\frac{5a^2b^3c^3}{2}\right)+x^3\left(a^5cd^2+5a^4bc^2d+\frac{10a^3b^2c^3}{3}\right)+x^2\left(\frac{3a^5c^2d}{2}+\frac{5a^4bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**3,x)
```

```
[Out] a**5*c**3*x + b**5*d**3*x**9/9 + x**8*(5*a*b**4*d**3/8 + 3*b**5*c*d**2/8) +
 x**7*(10*a**2*b**3*d**3/7 + 15*a*b**4*c*d**2/7 + 3*b**5*c**2*d/7) + x**6*(
5*a**3*b**2*d**3/3 + 5*a**2*b**3*c*d**2 + 5*a*b**4*c**2*d/2 + b**5*c**3/6)
+ x**5*(a**4*b*d**3 + 6*a**3*b**2*c*d**2 + 6*a**2*b**3*c**2*d + a*b**4*c**3
) + x**4*(a**5*d**3/4 + 15*a**4*b*c*d**2/4 + 15*a**3*b**2*c**2*d/2 + 5*a**2
*b**3*c**3/2) + x**3*(a**5*c*d**2 + 5*a**4*b*c**2*d + 10*a**3*b**2*c**3/3)
+ x**2*(3*a**5*c**2*d/2 + 5*a**4*b*c**3/2)
```

3.1153 $\int (a+bx)^4(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{43\}$

$$\frac{3d^2(a+bx)^7(bc-ad)}{7b^4}+\frac{d(a+bx)^6(bc-ad)^2}{2b^4}+\frac{(a+bx)^5(bc-ad)^3}{5b^4}+\frac{d^3(a+bx)^8}{8b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^5)/(5*b^4) + (d*(b*c - a*d)^2*(a + b*x)^6)/(2*b^4)
 + (3*d^2*(b*c - a*d)*(a + b*x)^7)/(7*b^4) + (d^3*(a + b*x)^8)/(8*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^4}{b^3}+\frac{3d(bc-ad)^2(a+bx)^5}{b^3}+\frac{3d^2(bc-ad)(a+bx)^6}{b^3}+\frac{d^3(a+bx)}{b^3}\right.\\ &= \frac{(bc-ad)^3(a+bx)^5}{5b^4}+\frac{d(bc-ad)^2(a+bx)^6}{2b^4}+\frac{3d^2(bc-ad)(a+bx)^7}{7b^4}+\frac{d^3(a+bx)^8}{8b^4}\end{aligned}$$

Mathematica [B] time = 0.03, size = 217, normalized size = 2.36

$$a^4c^3x+\frac{1}{2}a^3c^2x^2(3ad+4bc)+\frac{1}{2}b^2dx^6\left(2a^2d^2+4abcd+b^2c^2\right)+a^2cx^3\left(a^2d^2+4abcd+2b^2c^2\right)+\frac{1}{5}bx^5\left(4a^3d^3+18a^2bcd^2+12ab^2c^2d+b^3c^3\right)+\frac{1}{4}ax^4\left(a^3d^3+12a^2bcd^2+18ab^2c^2d+4b^3c^3\right)+\frac{1}{7}b^3d^2x^7(4ad+3bc)+\frac{1}{8}b^4d^3x^8$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] a^4*c^3*x + (a^3*c^2*(4*b*c + 3*a*d)*x^2)/2 + a^2*c*(2*b^2*c^2 + 4*a*b*c*d
+ a^2*d^2)*x^3 + (a*(4*b^3*c^3 + 18*a*b^2*c^2*d + 12*a^2*b*c*d^2 + a^3*d^3)
*x^4)/4 + (b*(b^3*c^3 + 12*a*b^2*c^2*d + 18*a^2*b*c*d^2 + 4*a^3*d^3)*x^5)/5
 + (b^2*d*(b^2*c^2 + 4*a*b*c*d + 2*a^2*d^2)*x^6)/2 + (b^3*d^2*(3*b*c + 4*a*
d)*x^7)/7 + (b^4*d^3*x^8)/8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^3, x]
```

fricas [B] time = 1.20, size = 245, normalized size = 2.66

$$\frac{1}{8}x^8d^3b^4+\frac{3}{7}x^7d^2cb^4+\frac{4}{7}x^7d^3b^3a+\frac{1}{2}x^6dc^2b^4+2x^6d^2cb^3a+x^6d^3b^2a^2+\frac{1}{5}x^5c^3b^4+\frac{12}{5}x^5dc^2b^3a+\frac{18}{5}x^5d^2cb^2a^2+\frac{4}{5}x^5d^3ba^3+x^4c^3b^3a+\frac{9}{2}x^4dc^2b^2a^2+3x^4d^2cba^3+\frac{1}{4}x^4d^3a^4+2x^3c^3b^2a^2+4x^3dc^2ba^3+x^3d^2ca^4+2x^2c^3ba^3+\frac{3}{2}x^2dc^2a^4+xc^3a^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^3*b^4 + 3/7*x^7*d^2*c*b^4 + 4/7*x^7*d^3*b^3*a + 1/2*x^6*d*c^2*b^4
 + 2*x^6*d^2*c*b^3*a + x^6*d^3*b^2*a^2 + 1/5*x^5*c^3*b^4 + 12/5*x^5*d*c^2*b
^3*a + 18/5*x^5*d^2*c*b^2*a^2 + 4/5*x^5*d^3*b*a^3 + x^4*c^3*b^3*a + 9/2*x^4
*d*c^2*b^2*a^2 + 3*x^4*d^2*c*b*a^3 + 1/4*x^4*d^3*a^4 + 2*x^3*c^3*b^2*a^2 +
4*x^3*d*c^2*b*a^3 + x^3*d^2*c*a^4 + 2*x^2*c^3*b*a^3 + 3/2*x^2*d*c^2*a^4 + x
*c^3*a^4
```

giac [B] time = 1.12, size = 245, normalized size = 2.66

$$\frac{1}{8}b^4d^3x^8+\frac{3}{7}b^4cd^2x^7+\frac{4}{7}ab^3d^3x^7+\frac{1}{2}b^4c^2dx^6+2ab^3cd^2x^6+a^2b^2d^3x^6+\frac{1}{5}b^4c^3x^5+\frac{12}{5}ab^3c^2dx^5+\frac{18}{5}a^2b^2cd^2x^5+\frac{4}{5}a^3bd^3x^5+ab^3c^3x^4+\frac{9}{2}a^2b^2c^2dx^4+3a^3bcd^2x^4+\frac{1}{4}a^4d^3x^4+2a^2b^2c^3x^3+4a^3bc^2dx^3+a^4cd^2x^3+2a^3bc^3x^2+\frac{3}{2}a^4c^2dx^2+a^4c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^4*d^3*x^8 + 3/7*b^4*c*d^2*x^7 + 4/7*a*b^3*d^3*x^7 + 1/2*b^4*c^2*d*x^6
 + 2*a*b^3*c*d^2*x^6 + a^2*b^2*d^3*x^6 + 1/5*b^4*c^3*x^5 + 12/5*a*b^3*c^2*d
*x^5 + 18/5*a^2*b^2*c*d^2*x^5 + 4/5*a^3*b*d^3*x^5 + a*b^3*c^3*x^4 + 9/2*a^2
*b^2*c^2*d*x^4 + 3*a^3*b*c*d^2*x^4 + 1/4*a^4*d^3*x^4 + 2*a^2*b^2*c^3*x^3 +
4*a^3*b*c^2*d*x^3 + a^4*c*d^2*x^3 + 2*a^3*b*c^3*x^2 + 3/2*a^4*c^2*d*x^2 + a
^4*c^3*x
```

maple [B] time = 0.00, size = 229, normalized size = 2.49

$$\frac{b^4d^3x^8}{8}+a^4c^3x+\frac{\left(4ab^3d^3+3b^4cd^2\right)x^7}{7}+\frac{\left(6a^2b^2d^3+12ab^3cd^2+3b^4c^2d\right)x^6}{6}+\frac{\left(4a^3bd^3+18a^2b^2cd^2+12ab^3c^2d+b^4c^3\right)x^5}{5}+\frac{\left(a^4d^3+12a^3bcd^2+18a^2b^2c^2d+4ab^3c^3\right)x^4}{4}+\frac{\left(3a^4cd^2+12a^3bc^2d+6a^2b^2c^3\right)x^3}{3}+\frac{\left(3a^4c^2d+4a^3bc^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^3,x)
```

```
[Out] 1/8*b^4*d^3*x^8+1/7*(4*a*b^3*d^3+3*b^4*c*d^2)*x^7+1/6*(6*a^2*b^2*d^3+12*a*b
^3*c*d^2+3*b^4*c^2*d)*x^6+1/5*(4*a^3*b*d^3+18*a^2*b^2*c*d^2+12*a*b^3*c^2*d+
b^4*c^3)*x^5+1/4*(a^4*d^3+12*a^3*b*c*d^2+18*a^2*b^2*c^2*d+4*a*b^3*c^3)*x^4+
1/3*(3*a^4*c*d^2+12*a^3*b*c^2*d+6*a^2*b^2*c^3)*x^3+1/2*(3*a^4*c^2*d+4*a^3*b
*c^3)*x^2+a^4*c^3*x
```

maxima [B] time = 1.32, size = 225, normalized size = 2.45

$$\frac{1}{8}b^4d^3x^8+a^4c^3x+\frac{1}{7}\left(3b^4cd^2+4ab^3d^3\right)x^7+\frac{1}{2}\left(b^4c^2d+4ab^3cd^2+2a^2b^2d^3\right)x^6+\frac{1}{5}\left(b^4c^3+12ab^3c^2d+18a^2b^2cd^2+4a^3bd^3\right)x^5+\frac{1}{4}\left(4ab^3c^3+18a^2b^2c^2d+12a^3bcd^2+a^4d^3\right)x^4+\left(2a^2b^2c^3+4a^3bc^2d+a^4cd^2\right)x^3+\frac{1}{2}\left(4a^3bc^3+3a^4c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b^4*d^3*x^8 + a^4*c^3*x + 1/7*(3*b^4*c*d^2 + 4*a*b^3*d^3)*x^7 + 1/2*(b^
4*c^2*d + 4*a*b^3*c*d^2 + 2*a^2*b^2*d^3)*x^6 + 1/5*(b^4*c^3 + 12*a*b^3*c^2*
d + 18*a^2*b^2*c*d^2 + 4*a^3*b*d^3)*x^5 + 1/4*(4*a*b^3*c^3 + 18*a^2*b^2*c^2
*d + 12*a^3*b*c*d^2 + a^4*d^3)*x^4 + (2*a^2*b^2*c^3 + 4*a^3*b*c^2*d + a^4*c
*d^2)*x^3 + 1/2*(4*a^3*b*c^3 + 3*a^4*c^2*d)*x^2
```

mupad [B] time = 0.21, size = 208, normalized size = 2.26

$$x^4\left(\frac{a^4d^3}{4}+3a^3bcd^2+\frac{9a^2b^2c^2d}{2}+ab^3c^3\right)+x^5\left(\frac{4a^3bd^3}{5}+\frac{18a^2b^2cd^2}{5}+\frac{12ab^3c^2d}{5}+\frac{b^4c^3}{5}\right)+a^4c^3x+\frac{b^4d^3x^8}{8}+\frac{a^3c^2x^2\left(3ad+4bc\right)}{2}+\frac{b^3d^2x^7\left(4ad+3bc\right)}{7}+a^2cx^3\left(a^2d^2+4abcd+2b^2c^2\right)+\frac{b^2dx^6\left(2a^2d^2+4abcd+b^2c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^3,x)
```

```
[Out] x^4*((a^4*d^3)/4 + a*b^3*c^3 + (9*a^2*b^2*c^2*d)/2 + 3*a^3*b*c*d^2) + x^5*(
(b^4*c^3)/5 + (4*a^3*b*d^3)/5 + (18*a^2*b^2*c*d^2)/5 + (12*a*b^3*c^2*d)/5)
+ a^4*c^3*x + (b^4*d^3*x^8)/8 + (a^3*c^2*x^2*(3*a*d + 4*b*c))/2 + (b^3*d^2*
x^7*(4*a*d + 3*b*c))/7 + a^2*c*x^3*(a^2*d^2 + 2*b^2*c^2 + 4*a*b*c*d) + (b^2
*d*x^6*(2*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/2
```

sympy [B]   time = 0.11, size = 243, normalized size = 2.64

$$a^4c^3x + \frac{b^4d^3x^8}{8} + x^7\left(\frac{4ab^3d^3}{7} + \frac{3b^4cd^2}{7}\right) + x^6\left(a^2b^2d^3 + 2ab^3cd^2 + \frac{b^4c^2d}{2}\right) + x^5\left(\frac{4a^3bd^3}{5} + \frac{18a^2b^2cd^2}{5} + \frac{12ab^3c^2d}{5} + \frac{b^4c^3}{5}\right) + x^4\left(\frac{a^4d^3}{4} + 3a^3bcd^2 + \frac{9a^2b^2c^2d}{2} + ab^3c^3\right) + x^3\left(a^4cd^2 + 4a^3bc^2d + 2a^2b^2c^3\right) + x^2\left(\frac{3a^4c^2d}{2} + 2a^3bc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**3,x)
```

```
[Out] a**4*c**3*x + b**4*d**3*x**8/8 + x**7*(4*a*b**3*d**3/7 + 3*b**4*c*d**2/7) +
 x**6*(a**2*b**2*d**3 + 2*a*b**3*c*d**2 + b**4*c**2*d/2) + x**5*(4*a**3*b*d
**3/5 + 18*a**2*b**2*c*d**2/5 + 12*a*b**3*c**2*d/5 + b**4*c**3/5) + x**4*(a
**4*d**3/4 + 3*a**3*b*c*d**2 + 9*a**2*b**2*c**2*d/2 + a*b**3*c**3) + x**3*(
a**4*c*d**2 + 4*a**3*b*c**2*d + 2*a**2*b**2*c**3) + x**2*(3*a**4*c**2*d/2 +
 2*a**3*b*c**3)
```

3.1154 $\int (a+bx)^3(c+dx)^3\,dx$

Optimal. Leaf size=92

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Rubi [A] time = 0.08, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2(a+bx)^6(bc-ad)}{2b^4}+\frac{3d(a+bx)^5(bc-ad)^2}{5b^4}+\frac{(a+bx)^4(bc-ad)^3}{4b^4}+\frac{d^3(a+bx)^7}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^4)/(4*b^4) + (3*d*(b*c - a*d)^2*(a + b*x)^5)/(5*b^
4) + (d^2*(b*c - a*d)*(a + b*x)^6)/(2*b^4) + (d^3*(a + b*x)^7)/(7*b^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^3}{b^3}+\frac{3d(bc-ad)^2(a+bx)^4}{b^3}+\frac{3d^2(bc-ad)(a+bx)^5}{b^3}+\frac{d^3(a+b}{b^3}\right.\\ &= \frac{(bc-ad)^3(a+bx)^4}{4b^4}+\frac{3d(bc-ad)^2(a+bx)^5}{5b^4}+\frac{d^2(bc-ad)(a+bx)^6}{2b^4}+\frac{d^3(a+bx)^7}{7b^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 161, normalized size = 1.75

$$a^3c^3x+\frac{3}{5}bdx^5\left(a^2d^2+3abcd+b^2c^2\right)+acx^3\left(a^2d^2+3abcd+b^2c^2\right)+\frac{3}{2}a^2c^2x^2(ad+bc)+\frac{1}{4}x^4\left(a^3d^3+9a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{1}{2}b^2d^2x^6(ad+bc)+\frac{1}{7}b^3d^3x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] a^3*c^3*x + (3*a^2*c^2*(b*c + a*d)*x^2)/2 + a*c*(b^2*c^2 + 3*a*b*c*d + a^2*
d^2)*x^3 + ((b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^4)/4 + (3
*b*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5)/5 + (b^2*d^2*(b*c + a*d)*x^6)/2 +
 (b^3*d^3*x^7)/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^3, x]
```

fricas [B] time = 1.09, size = 188, normalized size = 2.04

$$\frac{1}{7}x^7d^3b^3 + \frac{1}{2}x^6d^2cb^3 + \frac{1}{2}x^6d^3b^2a + \frac{3}{5}x^5dc^2b^3 + \frac{9}{5}x^5d^2cb^2a + \frac{3}{5}x^5d^3ba^2 + \frac{1}{4}x^4c^3b^3 + \frac{9}{4}x^4dc^2b^2a + \frac{9}{4}x^4d^2cba^2 + \frac{1}{4}x^4d^3a^3 + x^3c^3b^2a + 3x^3dc^2ba^2 + x^3d^2ca^3 + \frac{3}{2}x^2c^3ba^2 + \frac{3}{2}x^2dc^2a^3 + xc^3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d^3*b^3 + 1/2*x^6*d^2*c*b^3 + 1/2*x^6*d^3*b^2*a + 3/5*x^5*d*c^2*b^3
 + 9/5*x^5*d^2*c*b^2*a + 3/5*x^5*d^3*b*a^2 + 1/4*x^4*c^3*b^3 + 9/4*x^4*d*c^
2*b^2*a + 9/4*x^4*d^2*c*b*a^2 + 1/4*x^4*d^3*a^3 + x^3*c^3*b^2*a + 3*x^3*d*c
^2*b*a^2 + x^3*d^2*c*a^3 + 3/2*x^2*c^3*b*a^2 + 3/2*x^2*d*c^2*a^3 + x*c^3*a^
3
```

giac [B] time = 0.97, size = 188, normalized size = 2.04

$$\frac{1}{7}b^3d^3x^7 + \frac{1}{2}b^3cd^2x^6 + \frac{1}{2}ab^2d^3x^6 + \frac{3}{5}b^3c^2dx^5 + \frac{9}{5}ab^2cd^2x^5 + \frac{3}{5}a^2bd^3x^5 + \frac{1}{4}b^3c^3x^4 + \frac{9}{4}ab^2c^2dx^4 + \frac{9}{4}a^2bcd^2x^4 + \frac{1}{4}a^3d^3x^4 + ab^2c^3x^3 + 3a^2bc^2dx^3 + a^3cd^2x^3 + \frac{3}{2}a^2bc^3x^2 + \frac{3}{2}a^3c^2dx^2 + a^3c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/7*b^3*d^3*x^7 + 1/2*b^3*c*d^2*x^6 + 1/2*a*b^2*d^3*x^6 + 3/5*b^3*c^2*d*x^5
 + 9/5*a*b^2*c*d^2*x^5 + 3/5*a^2*b*d^3*x^5 + 1/4*b^3*c^3*x^4 + 9/4*a*b^2*c^
2*d*x^4 + 9/4*a^2*b*c*d^2*x^4 + 1/4*a^3*d^3*x^4 + a*b^2*c^3*x^3 + 3*a^2*b*c
^2*d*x^3 + a^3*c*d^2*x^3 + 3/2*a^2*b*c^3*x^2 + 3/2*a^3*c^2*d*x^2 + a^3*c^3*
x
```

maple [B] time = 0.00, size = 177, normalized size = 1.92

$$\frac{b^3d^3x^7}{7} + a^3c^3x + \frac{\left(3ab^2d^3 + 3b^3cd^2\right)x^6}{6} + \frac{\left(3a^2bd^3 + 9ab^2cd^2 + 3b^3c^2d\right)x^5}{5} + \frac{\left(a^3d^3 + 9a^2bcd^2 + 9ab^2c^2d + b^3c^3\right)x^4}{4} + \frac{\left(3a^3cd^2 + 9a^2bc^2d + 3ab^2c^3\right)x^3}{3} + \frac{\left(3a^3c^2d + 3a^2bc^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^3,x)
```

```
[Out] 1/7*b^3*d^3*x^7+1/6*(3*a*b^2*d^3+3*b^3*c*d^2)*x^6+1/5*(3*a^2*b*d^3+9*a*b^2*
c*d^2+3*b^3*c^2*d)*x^5+1/4*(a^3*d^3+9*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)*x^
4+1/3*(3*a^3*c*d^2+9*a^2*b*c^2*d+3*a*b^2*c^3)*x^3+1/2*(3*a^3*c^2*d+3*a^2*b*
c^3)*x^2+a^3*c^3*x
```

maxima [A] time = 1.34, size = 167, normalized size = 1.82

$$\frac{1}{7}b^3d^3x^7 + a^3c^3x + \frac{1}{2}\left(b^3cd^2 + ab^2d^3\right)x^6 + \frac{3}{5}\left(b^3c^2d + 3ab^2cd^2 + a^2bd^3\right)x^5 + \frac{1}{4}\left(b^3c^3 + 9ab^2c^2d + 9a^2bcd^2 + a^3d^3\right)x^4 + \left(ab^2c^3 + 3a^2bc^2d + a^3cd^2\right)x^3 + \frac{3}{2}\left(a^2bc^3 + a^3c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*d^3*x^7 + a^3*c^3*x + 1/2*(b^3*c*d^2 + a*b^2*d^3)*x^6 + 3/5*(b^3*c^
2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*x^5 + 1/4*(b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2
*b*c*d^2 + a^3*d^3)*x^4 + (a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*x^3 + 3/2
*(a^2*b*c^3 + a^3*c^2*d)*x^2
```

mupad [B] time = 0.06, size = 152, normalized size = 1.65

$$x^4\left(\frac{a^3d^3}{4} + \frac{9a^2bcd^2}{4} + \frac{9ab^2c^2d}{4} + \frac{b^3c^3}{4}\right) + a^3c^3x + \frac{b^3d^3x^7}{7} + acx^3\left(a^2d^2 + 3abcd + b^2c^2\right) + \frac{3bdx^5\left(a^2d^2 + 3abcd + b^2c^2\right)}{5} + \frac{3a^2c^2x^2\left(ad + bc\right)}{2} + \frac{b^2d^2x^6\left(ad + bc\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] x^4*((a^3*d^3)/4 + (b^3*c^3)/4 + (9*a*b^2*c^2*d)/4 + (9*a^2*b*c*d^2)/4) + a
^3*c^3*x + (b^3*d^3*x^7)/7 + a*c*x^3*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d) + (3*b
```

```
*d*x^5*(a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/5 + (3*a^2*c^2*x^2*(a*d + b*c))/2 +
 (b^2*d^2*x^6*(a*d + b*c))/2
```

sympy [B] time = 0.10, size = 190, normalized size = 2.07

$$a^3c^3x + \frac{b^3d^3x^7}{7} + x^6\left(\frac{ab^2d^3}{2} + \frac{b^3cd^2}{2}\right) + x^5\left(\frac{3a^2bd^3}{5} + \frac{9ab^2cd^2}{5} + \frac{3b^3c^2d}{5}\right) + x^4\left(\frac{a^3d^3}{4} + \frac{9a^2bcd^2}{4} + \frac{9ab^2c^2d}{4} + \frac{b^3c^3}{4}\right) + x^3\left(a^3cd^2 + 3a^2bc^2d + ab^2c^3\right) + x^2\left(\frac{3a^3c^2d}{2} + \frac{3a^2bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**3,x)
```

```
[Out] a**3*c**3*x + b**3*d**3*x**7/7 + x**6*(a*b**2*d**3/2 + b**3*c*d**2/2) + x**
5*(3*a**2*b*d**3/5 + 9*a*b**2*c*d**2/5 + 3*b**3*c**2*d/5) + x**4*(a**3*d**3
/4 + 9*a**2*b*c*d**2/4 + 9*a*b**2*c**2*d/4 + b**3*c**3/4) + x**3*(a**3*c*d*
*2 + 3*a**2*b*c**2*d + a*b**2*c**3) + x**2*(3*a**3*c**2*d/2 + 3*a**2*b*c**3
/2)
```

3.1155 $\int (a+bx)^2(c+dx)^3\,dx$

Optimal. Leaf size=65

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^5(bc-ad)}{5d^3}+\frac{(c+dx)^4(bc-ad)^2}{4d^3}+\frac{b^2(c+dx)^6}{6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^4)/(4*d^3) - (2*b*(b*c - a*d)*(c + d*x)^5)/(5*d^3)
 + (b^2*(c + d*x)^6)/(6*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^3}{d^2}-\frac{2b(bc-ad)(c+dx)^4}{d^2}+\frac{b^2(c+dx)^5}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^4}{4d^3}-\frac{2b(bc-ad)(c+dx)^5}{5d^3}+\frac{b^2(c+dx)^6}{6d^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 122, normalized size = 1.88

$$\frac{1}{4}dx^4\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{3}cx^3\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{2}ac^2x^2(3ad+2bc)+\frac{1}{5}bd^2x^5(2ad+3bc)+\frac{1}{6}b^2d^3x^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^2)/2 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^3)/3 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4)/4 + (b*d^2*(3*b*c
+ 2*a*d)*x^5)/5 + (b^2*d^3*x^6)/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^3, x]
```

fricas [B] time = 1.27, size = 130, normalized size = 2.00

$$\frac{1}{6}x^6d^3b^2 + \frac{3}{5}x^5d^2cb^2 + \frac{2}{5}x^5d^3ba + \frac{3}{4}x^4dc^2b^2 + \frac{3}{2}x^4d^2cba + \frac{1}{4}x^4d^3a^2 + \frac{1}{3}x^3c^3b^2 + 2x^3dc^2ba + x^3d^2ca^2 + x^2c^3ba + \frac{3}{2}x^2dc^2a^2 + xc^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*d^3*b^2 + 3/5*x^5*d^2*c*b^2 + 2/5*x^5*d^3*b*a + 3/4*x^4*d*c^2*b^2 +
 3/2*x^4*d^2*c*b*a + 1/4*x^4*d^3*a^2 + 1/3*x^3*c^3*b^2 + 2*x^3*d*c^2*b*a +
x^3*d^2*c*a^2 + x^2*c^3*b*a + 3/2*x^2*d*c^2*a^2 + x*c^3*a^2
```

giac [B] time = 1.04, size = 130, normalized size = 2.00

$$\frac{1}{6}b^2d^3x^6 + \frac{3}{5}b^2cd^2x^5 + \frac{2}{5}abd^3x^5 + \frac{3}{4}b^2c^2dx^4 + \frac{3}{2}abcd^2x^4 + \frac{1}{4}a^2d^3x^4 + \frac{1}{3}b^2c^3x^3 + 2\,abc^2dx^3 + a^2cd^2x^3 + abc^3x^2 + \frac{3}{2}a^2c^2dx^2 + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/6*b^2*d^3*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*a*b*d^3*x^5 + 3/4*b^2*c^2*d*x^4 +
 3/2*a*b*c*d^2*x^4 + 1/4*a^2*d^3*x^4 + 1/3*b^2*c^3*x^3 + 2*a*b*c^2*d*x^3 +
a^2*c*d^2*x^3 + a*b*c^3*x^2 + 3/2*a^2*c^2*d*x^2 + a^2*c^3*x
```

maple [B] time = 0.00, size = 125, normalized size = 1.92

$$\frac{b^2d^3x^6}{6} + a^2c^3x + \frac{\left(2ab\,d^3 + 3b^2c\,d^2\right)x^5}{5} + \frac{\left(a^2d^3 + 6abc\,d^2 + 3b^2c^2d\right)x^4}{4} + \frac{\left(3a^2c\,d^2 + 6ab\,c^2d + b^2c^3\right)x^3}{3} + \frac{\left(3a^2c^2d + 2ab\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^3,x)
```

```
[Out] 1/6*b^2*d^3*x^6+1/5*(2*a*b*d^3+3*b^2*c*d^2)*x^5+1/4*(a^2*d^3+6*a*b*c*d^2+3*
b^2*c^2*d)*x^4+1/3*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^3+1/2*(3*a^2*c^2*d+2
*a*b*c^3)*x^2+a^2*c^3*x
```

maxima [B] time = 1.34, size = 124, normalized size = 1.91

$$\frac{1}{6}b^2d^3x^6 + a^2c^3x + \frac{1}{5}\left(3\,b^2cd^2 + 2\,abd^3\right)x^5 + \frac{1}{4}\left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^4 + \frac{1}{3}\left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^3 + \frac{1}{2}\left(2\,abc^3 + 3\,a^2c^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*d^3*x^6 + a^2*c^3*x + 1/5*(3*b^2*c*d^2 + 2*a*b*d^3)*x^5 + 1/4*(3*b^
2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^4 + 1/3*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c
*d^2)*x^3 + 1/2*(2*a*b*c^3 + 3*a^2*c^2*d)*x^2
```

mupad [B] time = 0.05, size = 115, normalized size = 1.77

$$x^3\left(a^2\,c\,d^2 + 2\,a\,b\,c^2\,d + \frac{b^2\,c^3}{3}\right) + x^4\left(\frac{a^2\,d^3}{4} + \frac{3\,a\,b\,c\,d^2}{2} + \frac{3\,b^2\,c^2\,d}{4}\right) + a^2\,c^3\,x + \frac{b^2\,d^3\,x^6}{6} + \frac{a\,c^2\,x^2\,\left(3\,a\,d + 2\,b\,c\right)}{2} + \frac{b\,d^2\,x^5\,\left(2\,a\,d + 3\,b\,c\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^3*((b^2*c^3)/3 + a^2*c*d^2 + 2*a*b*c^2*d) + x^4*((a^2*d^3)/4 + (3*b^2*c^2
*d)/4 + (3*a*b*c*d^2)/2) + a^2*c^3*x + (b^2*d^3*x^6)/6 + (a*c^2*x^2*(3*a*d
+ 2*b*c))/2 + (b*d^2*x^5*(2*a*d + 3*b*c))/5
```

sympy [B] time = 0.09, size = 133, normalized size = 2.05

$$a^2c^3x + \frac{b^2d^3x^6}{6} + x^5\left(\frac{2abd^3}{5} + \frac{3b^2cd^2}{5}\right) + x^4\left(\frac{a^2d^3}{4} + \frac{3abcd^2}{2} + \frac{3b^2c^2d}{4}\right) + x^3\left(a^2cd^2 + 2abc^2d + \frac{b^2c^3}{3}\right) + x^2\left(\frac{3a^2c^2d}{2} + abc^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**6/6 + x**5*(2*a*b*d**3/5 + 3*b**2*c*d**2/5) + x*
*4*(a**2*d**3/4 + 3*a*b*c*d**2/2 + 3*b**2*c**2*d/4) + x**3*(a**2*c*d**2 + 2
*a*b*c**2*d + b**2*c**3/3) + x**2*(3*a**2*c**2*d/2 + a*b*c**3)
```

3.1156 $\int (a+bx)(c+dx)^3\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^5}{5d^2}-\frac{(c+dx)^4(bc-ad)}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^4)/(4*d^2) + (b*(c + d*x)^5)/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^3\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^3}{d}+\frac{b(c+dx)^4}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^4}{4d^2}+\frac{b(c+dx)^5}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 67, normalized size = 1.76

$$\frac{1}{2}c^2x^2(3ad+bc)+\frac{1}{4}d^2x^4(ad+3bc)+cdx^3(ad+bc)+ac^3x+\frac{1}{5}bd^3x^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^2)/2 + c*d*(b*c + a*d)*x^3 + (d^2*(3*b*c + a
*d)*x^4)/4 + (b*d^3*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^3, x]
```

fricas [B] time = 1.09, size = 72, normalized size = 1.89

$$\frac{1}{5}x^5d^3b+\frac{3}{4}x^4d^2cb+\frac{1}{4}x^4d^3a+x^3dc^2b+x^3d^2ca+\frac{1}{2}x^2c^3b+\frac{3}{2}x^2dc^2a+xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*d^3*b + 3/4*x^4*d^2*c*b + 1/4*x^4*d^3*a + x^3*d*c^2*b + x^3*d^2*c*a
 + 1/2*x^2*c^3*b + 3/2*x^2*d*c^2*a + x*c^3*a
```

giac [B] time = 0.88, size = 72, normalized size = 1.89

$$\frac{1}{5}\,bd^3x^5 + \frac{3}{4}\,bcd^2x^4 + \frac{1}{4}\,ad^3x^4 + bc^2dx^3 + acd^2x^3 + \frac{1}{2}\,bc^3x^2 + \frac{3}{2}\,ac^2dx^2 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/5*b*d^3*x^5 + 3/4*b*c*d^2*x^4 + 1/4*a*d^3*x^4 + b*c^2*d*x^3 + a*c*d^2*x^3
 + 1/2*b*c^3*x^2 + 3/2*a*c^2*d*x^2 + a*c^3*x
```

maple [B] time = 0.00, size = 73, normalized size = 1.92

$$\frac{b\,d^3x^5}{5} + a\,c^3x + \frac{\left(a\,d^3 + 3bc\,d^2\right)x^4}{4} + \frac{\left(3ac\,d^2 + 3b\,c^2d\right)x^3}{3} + \frac{\left(3a\,c^2d + b\,c^3\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^3,x)
```

```
[Out] 1/5*b*d^3*x^5+1/4*(a*d^3+3*b*c*d^2)*x^4+1/3*(3*a*c*d^2+3*b*c^2*d)*x^3+1/2*(
3*a*c^2*d+b*c^3)*x^2+a*c^3*x
```

maxima [B] time = 1.39, size = 69, normalized size = 1.82

$$\frac{1}{5}\,bd^3x^5 + ac^3x + \frac{1}{4}\left(3\,bcd^2 + ad^3\right)x^4 + \left(bc^2d + acd^2\right)x^3 + \frac{1}{2}\left(bc^3 + 3\,ac^2d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/5*b*d^3*x^5 + a*c^3*x + 1/4*(3*b*c*d^2 + a*d^3)*x^4 + (b*c^2*d + a*c*d^2)
*x^3 + 1/2*(b*c^3 + 3*a*c^2*d)*x^2
```

mupad [B] time = 0.03, size = 65, normalized size = 1.71

$$x^2\left(\frac{b\,c^3}{2} + \frac{3\,a\,d\,c^2}{2}\right) + x^4\left(\frac{a\,d^3}{4} + \frac{3\,b\,c\,d^2}{4}\right) + \frac{b\,d^3\,x^5}{5} + a\,c^3\,x + c\,d\,x^3\,\left(a\,d + b\,c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^3,x)
```

```
[Out] x^2*((b*c^3)/2 + (3*a*c^2*d)/2) + x^4*((a*d^3)/4 + (3*b*c*d^2)/4) + (b*d^3*
x^5)/5 + a*c^3*x + c*d*x^3*(a*d + b*c)
```

sympy [B] time = 0.08, size = 73, normalized size = 1.92

$$ac^3x + \frac{bd^3x^5}{5} + x^4\left(\frac{ad^3}{4} + \frac{3bcd^2}{4}\right) + x^3\left(acd^2 + bc^2d\right) + x^2\left(\frac{3ac^2d}{2} + \frac{bc^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**5/5 + x**4*(a*d**3/4 + 3*b*c*d**2/4) + x**3*(a*c*d**2
+ b*c**2*d) + x**2*(3*a*c**2*d/2 + b*c**3/2)
```

3.1157 $\int (c + dx)^3 \, dx$

Optimal. Leaf size=14

$$\frac{(c + dx)^4}{4d}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c + dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3,x]
[Out] (c + d*x)^4/(4*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c + dx)^3 \, dx = \frac{(c + dx)^4}{4d}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c + dx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3,x]
[Out] (c + d*x)^4/(4*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3,x]
[Out] IntegrateAlgebraic[(c + d*x)^3, x]
```

fricas [B] time = 1.18, size = 31, normalized size = 2.21

$$\frac{1}{4}x^4d^3 + x^3d^2c + \frac{3}{2}x^2dc^2 + xc^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="fricas")
[Out] 1/4*x^4*d^3 + x^3*d^2*c + 3/2*x^2*d*c^2 + x*c^3
```

giac [**A**] time = 1.00, size = 12, normalized size = 0.86

$$\frac{(dx+c)^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(d*x + c)^4/d
```

maple [**A**] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx+c)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3,x)
```

```
[Out] 1/4*(d*x+c)^4/d
```

maxima [**B**] time = 1.35, size = 31, normalized size = 2.21

$$\frac{1}{4}\,d^3x^4 + cd^2x^3 + \frac{3}{2}\,c^2dx^2 + c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x
```

mupad [**B**] time = 0.04, size = 31, normalized size = 2.21

$$c^3\,x + \frac{3\,c^2\,d\,x^2}{2} + c\,d^2\,x^3 + \frac{d^3\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3,x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3
```

sympy [**B**] time = 0.06, size = 32, normalized size = 2.29

$$c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3,x)
```

```
[Out] c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
```

3.1158 $\int \frac{(c+dx)^3}{a+bx}\,dx$

Optimal. Leaf size=73

$$\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{(c+dx)^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^2)/(2*b^2) + (c + d*x)^3/(
3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^2}{b^3}+\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)(c+dx)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx\\ &= \frac{d(bc-ad)^2x}{b^3}+\frac{(bc-ad)(c+dx)^2}{2b^2}+\frac{(c+dx)^3}{3b}+\frac{(bc-ad)^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.01

$$\frac{bdx\left(6a^2d^2-3abd(6c+dx)+b^2\left(18c^2+9cdx+2d^2x^2\right)\right)+6(bc-ad)^3\log(a+bx)}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x),x]
```

```
[Out] (b*d*x*(6*a^2*d^2 - 3*a*b*d*(6*c + d*x) + b^2*(18*c^2 + 9*c*d*x + 2*d^2*x^2
)) + 6*(b*c - a*d)^3*Log[a + b*x])/(6*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x), x]
```

fricas [A] time = 1.79, size = 116, normalized size = 1.59

$$\frac{2\,b^3d^3x^3+3\left(3\,b^3cd^2-ab^2d^3\right)x^2+6\left(3\,b^3c^2d-3\,ab^2cd^2+a^2bd^3\right)x+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{6\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 + 3*(3*b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(3*b^3*c^2*d - 3*a
*b^2*c*d^2 + a^2*b*d^3)*x + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(b*x + a))/b^4
```

giac [A] time = 1.03, size = 115, normalized size = 1.58

$$\frac{2\,b^2d^3x^3+9\,b^2cd^2x^2-3\,abd^3x^2+18\,b^2c^2dx-18\,abcd^2x+6\,a^2d^3x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|bx+a|\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 9*b^2*c*d^2*x^2 - 3*a*b*d^3*x^2 + 18*b^2*c^2*d*x - 18*
a*b*c*d^2*x + 6*a^2*d^3*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(b*x + a))/b^4
```

maple [A] time = 0.00, size = 133, normalized size = 1.82

$$\frac{d^3x^3}{3b}-\frac{a\,d^3x^2}{2b^2}+\frac{3c\,d^2x^2}{2b}-\frac{a^3d^3\ln\left(bx+a\right)}{b^4}+\frac{3a^2c\,d^2\ln\left(bx+a\right)}{b^3}+\frac{a^2d^3x}{b^3}-\frac{3a\,c^2d\ln\left(bx+a\right)}{b^2}-\frac{3ac\,d^2x}{b^2}+\frac{c^3\ln\left(bx+a\right)}{b}+\frac{3c^2dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a),x)
```

```
[Out] 1/3*d^3/b*x^3-1/2*d^3/b^2*x^2*a+3/2*d^2/b*x^2*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c
*x+3*d/b*c^2*x-1/b^4*ln(b*x+a)*a^3*d^3+3/b^3*ln(b*x+a)*a^2*c*d^2-3/b^2*ln(b
*x+a)*a*c^2*d+1/b*ln(b*x+a)*c^3
```

maxima [A] time = 1.33, size = 114, normalized size = 1.56

$$\frac{2\,b^2d^3x^3+3\left(3\,b^2cd^2-abd^3\right)x^2+6\left(3\,b^2c^2d-3\,abcd^2+a^2d^3\right)x}{6\,b^3}+\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*d^3*x^3 + 3*(3*b^2*c*d^2 - a*b*d^3)*x^2 + 6*(3*b^2*c^2*d - 3*a*b
*c*d^2 + a^2*d^3)*x)/b^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(b*x + a)/b^4
```

mupad [B] time = 0.20, size = 118, normalized size = 1.62

$$x\left(\frac{3\,c^2\,d}{b}+\frac{a\,\left(\frac{a\,d^3}{b^2}-\frac{3\,c\,d^2}{b}\right)}{b}\right)-x^2\,\left(\frac{a\,d^3}{2\,b^2}-\frac{3\,c\,d^2}{2\,b}\right)+\frac{d^3\,x^3}{3\,b}-\frac{\ln\left(a+b\,x\right)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b) - x^2*((a*d^3)/(2*b^2)
- (3*c*d^2)/(2*b)) + (d^3*x^3)/(3*b) - (log(a + b*x)*(a^3*d^3 - b^3*c^3 + 3
*a*b^2*c^2*d - 3*a^2*b*c*d^2))/b^4
```

sympy [A] time = 0.30, size = 83, normalized size = 1.14

$$x^2\left(-\frac{ad^3}{2b^2}+\frac{3cd^2}{2b}\right)+x\left(\frac{a^2d^3}{b^3}-\frac{3acd^2}{b^2}+\frac{3c^2d}{b}\right)+\frac{d^3x^3}{3b}-\frac{(ad-bc)^3\log(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a),x)
```

```
[Out] x**2*(-a*d**3/(2*b**2) + 3*c*d**2/(2*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + d**3*x**3/(3*b) - (a*d - b*c)**3*log(a + b*x)/b**4
```

3.1159 $\int \frac{(c+dx)^3}{(a+bx)^2} dx$

Optimal. Leaf size=75

$$-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^2x(3bc-2ad)}{b^3}+\frac{d^3x^2}{2b^2}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^2x(3bc-2ad)}{b^3}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}+\frac{d^3x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^2)/(2*b^2) - (b*c - a*d)^3/(b^4*(a + b
*x)) + (3*d*(b*c - a*d)^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^2}\,dx&=\int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x}{b^2}+\frac{(bc-ad)^3}{b^3(a+bx)^2}+\frac{3d(bc-ad)^2}{b^3(a+bx)}\right)dx\\&=\frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^2}{2b^2}-\frac{(bc-ad)^3}{b^4(a+bx)}+\frac{3d(bc-ad)^2\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.96

$$\frac{2bd^2x(3bc-2ad)-\frac{2(bc-ad)^3}{a+bx}+6d(bc-ad)^2\log(a+bx)+b^2d^3x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] (2*b*d^2*(3*b*c - 2*a*d)*x + b^2*d^3*x^2 - (2*(b*c - a*d)^3)/(a + b*x) + 6*
d*(b*c - a*d)^2*Log[a + b*x])/(2*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^2, x]
```

fricas [B] time = 1.42, size = 173, normalized size = 2.31

$$\frac{b^3d^3x^3-2\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+3\left(2\,b^3cd^2-ab^2d^3\right)x^2+2\left(3\,ab^2cd^2-2\,a^2bd^3\right)x+6\left(ab^2c^2d-2\,a^2bcd^2+a^3d^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x\right)\log\left(bx+a\right)}{2\left(b^5x+ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 +
3*(2*b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(3*a*b^2*c*d^2 - 2*a^2*b*d^3)*x + 6*(a*
b^2*c^2*d - 2*a^2*b*c*d^2 + a^3*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(b*x + a))/(b^5*x + a*b^4)
```

giac [B] time = 0.99, size = 167, normalized size = 2.23

$$\frac{\left(d^3+\frac{6\left(b^2cd^2-abd^3\right)}{(bx+a)b}\right)(bx+a)^2}{2\,b^4}-\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(\frac{|bx+a|}{(bx+a)^2|b|}\right)}{b^4}-\frac{\frac{b^5c^3}{bx+a}-\frac{3\,ab^4c^2d}{bx+a}+\frac{3\,a^2b^3cd^2}{bx+a}-\frac{a^3b^2d^3}{bx+a}}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(d^3 + 6*(b^2*c*d^2 - a*b*d^3)/((b*x + a)*b))*(b*x + a)^2/b^4 - 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^4 -
 (b^5*c^3/(b*x + a) - 3*a*b^4*c^2*d/(b*x + a) + 3*a^2*b^3*c*d^2/(b*x + a) -
 a^3*b^2*d^3/(b*x + a))/b^6
```

maple [B] time = 0.01, size = 149, normalized size = 1.99

$$\frac{d^3x^2}{2b^2}+\frac{a^3d^3}{(bx+a)\,b^4}-\frac{3a^2c\,d^2}{(bx+a)\,b^3}+\frac{3a^2d^3\ln(bx+a)}{b^4}+\frac{3a\,c^2d}{(bx+a)\,b^2}-\frac{6ac\,d^2\ln(bx+a)}{b^3}-\frac{2a\,d^3x}{b^3}-\frac{c^3}{(bx+a)\,b}+\frac{3c^2d\ln(bx+a)}{b^2}+\frac{3c\,d^2x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^2,x)
```

```
[Out] 1/2*d^3*x^2/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c+3/b^4*d^3*ln(b*x+a)*a^2-6/b^3*d
^2*ln(b*x+a)*a*c+3/b^2*d*ln(b*x+a)*c^2+1/b^4/(b*x+a)*a^3*d^3-3/b^3/(b*x+a)*
a^2*c*d^2+3/b^2/(b*x+a)*a*c^2*d-1/b/(b*x+a)*c^3
```

maxima [A] time = 1.32, size = 118, normalized size = 1.57

$$-\frac{b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3}{b^5x+ab^4}+\frac{bd^3x^2+2\left(3\,bcd^2-2\,ad^3\right)x}{2\,b^3}+\frac{3\left(b^2c^2d-2\,abcd^2+a^2d^3\right)\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(b^5*x + a*b^4) + 1/2*
(b*d^3*x^2 + 2*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(b*x + a)/b^4
```

mupad [B] time = 0.21, size = 123, normalized size = 1.64

$$\frac{\ln\left(a+b\,x\right)\,\left(3\,a^2\,d^3-6\,a\,b\,c\,d^2+3\,b^2\,c^2\,d\right)}{b^4}-x\,\left(\frac{2\,a\,d^3}{b^3}-\frac{3\,c\,d^2}{b^2}\right)+\frac{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}{b\,\left(x\,b^4+a\,b^3\right)}+\frac{d^3\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^2,x)
```

```
[Out] (log(a + b*x)*(3*a^2*d^3 + 3*b^2*c^2*d - 6*a*b*c*d^2))/b^4 - x*((2*a*d^3)/b
^3 - (3*c*d^2)/b^2) + (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
b*(a*b^3 + b^4*x)) + (d^3*x^2)/(2*b^2)
```

sympy [A] time = 0.50, size = 102, normalized size = 1.36

$$x\left(-\frac{2ad^3}{b^3}+\frac{3cd^2}{b^2}\right)+\frac{a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3}{ab^4+b^5x}+\frac{d^3x^2}{2b^2}+\frac{3d\,(ad-bc)^2\log\,(a+bx)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**2,x)
```

```
[Out] x*(-2*a*d**3/b**3 + 3*c*d**2/b**2) + (a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**
2*c**2*d - b**3*c**3)/(a*b**4 + b**5*x) + d**3*x**2/(2*b**2) + 3*d*(a*d - b
*c)**2*log(a + b*x)/b**4
```

3.1160 $\int \frac{(c+dx)^3}{(a+bx)^3} dx$

Optimal. Leaf size=78

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4} - \frac{3d(bc-ad)^2}{b^4(a+bx)} - \frac{(bc-ad)^3}{2b^4(a+bx)^2} + \frac{d^3x}{b^3}$$

Rubi [A] time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{3d^2(bc-ad)\log(a+bx)}{b^4} - \frac{3d(bc-ad)^2}{b^4(a+bx)} - \frac{(bc-ad)^3}{2b^4(a+bx)^2} + \frac{d^3x}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (d^3*x)/b^3 - (b*c - a*d)^3/(2*b^4*(a + b*x)^2) - (3*d*(b*c - a*d)^2)/(b^4*
(a + b*x)) + (3*d^2*(b*c - a*d)*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^3}{(a+bx)^3} dx &= \int \left(\frac{d^3}{b^3} + \frac{(bc-ad)^3}{b^3(a+bx)^3} + \frac{3d(bc-ad)^2}{b^3(a+bx)^2} + \frac{3d^2(bc-ad)}{b^3(a+bx)}\right) dx \\ &= \frac{d^3x}{b^3} - \frac{(bc-ad)^3}{2b^4(a+bx)^2} - \frac{3d(bc-ad)^2}{b^4(a+bx)} + \frac{3d^2(bc-ad)\log(a+bx)}{b^4} \end{aligned}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 1.46

$$\frac{-5a^3d^3 + a^2bd^2(9c-4dx) + ab^2d\left(-3c^2+12cdx+4d^2x^2\right) - 6d^2(a+bx)^2(ad-bc)\log(a+bx) - \left(b^3\left(c^3+6c^2dx-2d^3x^3\right)\right)}{2b^4(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] (-5*a^3*d^3 + a^2*b*d^2*(9*c - 4*d*x) + a*b^2*d*(-3*c^2 + 12*c*d*x + 4*d^2*
x^2) - b^3*(c^3 + 6*c^2*d*x - 2*d^3*x^3) - 6*d^2*(-(b*c) + a*d)*(a + b*x)^2
*Log[a + b*x])/(2*b^4*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^3, x]
```

fricas [B] time = 1.34, size = 188, normalized size = 2.41

$$\frac{2 b^3 d^3 x^3 + 4 a b^2 d^3 x^2 - b^3 c^3 - 3 a b^2 c^2 d + 9 a^2 b c d^2 - 5 a^3 d^3 - 2 \left(3 b^3 c^2 d - 6 a b^2 c d^2 + 2 a^2 b d^3\right) x + 6 \left(a^2 b c d^2 - a^3 d^3 + \left(b^3 c d^2 - a b^2 d^3\right) x^2 + 2 \left(a b^2 c d^2 - a^2 b d^3\right) x\right) \log\left(bx + a\right)}{2 \left(b^6 x^2 + 2 a b^5 x + a^2 b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*a*b^2*d^3*x^2 - b^3*c^3 - 3*a*b^2*c^2*d + 9*a^2*b*c*
d^2 - 5*a^3*d^3 - 2*(3*b^3*c^2*d - 6*a*b^2*c*d^2 + 2*a^2*b*d^3)*x + 6*(a^2*
b*c*d^2 - a^3*d^3 + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(a*b^2*c*d^2 - a^2*b*d^
3)*x)*log(b*x + a))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

giac [A] time = 0.95, size = 112, normalized size = 1.44

$$\frac{d^3 x}{b^3} + \frac{3 \left(b c d^2 - a d^3\right) \log\left(|bx + a|\right)}{b^4} - \frac{b^3 c^3 + 3 a b^2 c^2 d - 9 a^2 b c d^2 + 5 a^3 d^3 + 6 \left(b^3 c^2 d - 2 a b^2 c d^2 + a^2 b d^3\right) x}{2 \left(bx + a\right)^2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] d^3*x/b^3 + 3*(b*c*d^2 - a*d^3)*log(abs(b*x + a))/b^4 - 1/2*(b^3*c^3 + 3*a*
b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((b*x + a)^2*b^4)
```

maple [B] time = 0.01, size = 160, normalized size = 2.05

$$\frac{a^3 d^3}{2 \left(bx + a\right)^2 b^4} - \frac{3 a^2 c d^2}{2 \left(bx + a\right)^2 b^3} + \frac{3 a c^2 d}{2 \left(bx + a\right)^2 b^2} - \frac{c^3}{2 \left(bx + a\right)^2 b} - \frac{3 a^2 d^3}{\left(bx + a\right) b^4} + \frac{6 a c d^2}{\left(bx + a\right) b^3} - \frac{3 a d^3 \ln\left(bx + a\right)}{b^4} - \frac{3 c^2 d}{\left(bx + a\right) b^2} + \frac{3 c d^2 \ln\left(bx + a\right)}{b^3} + \frac{d^3 x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^3,x)
```

```
[Out] d^3*x/b^3+1/2/b^4/(b*x+a)^2*a^3*d^3-3/2/b^3/(b*x+a)^2*a^2*c*d^2+3/2/b^2/(b*
x+a)^2*a*c^2*d-1/2/b/(b*x+a)^2*c^3-3/b^4*d^3*ln(b*x+a)*a+3/b^3*d^2*ln(b*x+a
)*c-3/b^4*d^3/(b*x+a)*a^2+6/b^3*d^2/(b*x+a)*a*c-3/b^2*d/(b*x+a)*c^2
```

maxima [A] time = 1.37, size = 125, normalized size = 1.60

$$\frac{d^3 x}{b^3} - \frac{b^3 c^3 + 3 a b^2 c^2 d - 9 a^2 b c d^2 + 5 a^3 d^3 + 6 \left(b^3 c^2 d - 2 a b^2 c d^2 + a^2 b d^3\right) x}{2 \left(b^6 x^2 + 2 a b^5 x + a^2 b^4\right)} + \frac{3 \left(b c d^2 - a d^3\right) \log\left(bx + a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d^3*x/b^3 - 1/2*(b^3*c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2 + 5*a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^6*x^2 + 2*a*b^5*x + a^2*b^4) +
3*(b*c*d^2 - a*d^3)*log(b*x + a)/b^4
```

mupad [B] time = 0.82, size = 130, normalized size = 1.67

$$\frac{d^3\,x}{b^3} - \frac{\ln\left(a + b\,x\right)\,\left(3\,a\,d^3 - 3\,b\,c\,d^2\right)}{b^4} - \frac{\frac{5\,a^3\,d^3 - 9\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d + b^3\,c^3}{2\,b} + x\,\left(3\,a^2\,d^3 - 6\,a\,b\,c\,d^2 + 3\,b^2\,c^2\,d\right)}{a^2\,b^3 + 2\,a\,b^4\,x + b^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^3,x)
```

```
[Out] (d^3*x)/b^3 - (log(a + b*x)*(3*a*d^3 - 3*b*c*d^2))/b^4 - ((5*a^3*d^3 + b^3*
c^3 + 3*a*b^2*c^2*d - 9*a^2*b*c*d^2)/(2*b) + x*(3*a^2*d^3 + 3*b^2*c^2*d - 6
*a*b*c*d^2))/(a^2*b^3 + b^5*x^2 + 2*a*b^4*x)
```

sympy [A] time = 0.82, size = 128, normalized size = 1.64

$$\frac{-5a^3d^3 + 9a^2bcd^2 - 3ab^2c^2d - b^3c^3 + x\left(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d\right)}{2a^2b^4 + 4ab^5x + 2b^6x^2} + \frac{d^3x}{b^3} - \frac{3d^2\left(ad - bc\right)\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**3,x)
```

```
[Out] (-5*a**3*d**3 + 9*a**2*b*c*d**2 - 3*a*b**2*c**2*d - b**3*c**3 + x*(-6*a**2*
b*d**3 + 12*a*b**2*c*d**2 - 6*b**3*c**2*d))/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) + d**3*x/b**3 - 3*d**2*(a*d - b*c)*log(a + b*x)/b**4
```

3.1161 $\int \frac{(c+dx)^3}{(a+bx)^4} dx$

Optimal. Leaf size=86

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{b^4(a+bx)}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{(bc-ad)^3}{3b^4(a+bx)^3}+\frac{d^3\log(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] -(b*c - a*d)^3/(3*b^4*(a + b*x)^3) - (3*d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^2
) - (3*d^2*(b*c - a*d))/(b^4*(a + b*x)) + (d^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^4}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^4}+\frac{3d(bc-ad)^2}{b^3(a+bx)^3}+\frac{3d^2(bc-ad)}{b^3(a+bx)^2}+\frac{d^3}{b^3(a+bx)}\right)dx\\ &= -\frac{(bc-ad)^3}{3b^4(a+bx)^3}-\frac{3d(bc-ad)^2}{2b^4(a+bx)^2}-\frac{3d^2(bc-ad)}{b^4(a+bx)}+\frac{d^3\log(a+bx)}{b^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 80, normalized size = 0.93

$$\frac{6d^3\log(a+bx)-\frac{(bc-ad)\left(11a^2d^2+abd(5c+27dx)+b^2\left(2c^2+9cdx+18d^2x^2\right)\right)}{(a+bx)^3}}{6b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] (-(((b*c - a*d)*(11*a^2*d^2 + a*b*d*(5*c + 27*d*x) + b^2*(2*c^2 + 9*c*d*x +
 18*d^2*x^2)))/(a + b*x)^3) + 6*d^3*Log[a + b*x])/(6*b^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^4, x]
```

fricas [B] time = 1.51, size = 176, normalized size = 2.05

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x-6\left(b^3d^3x^3+3\,ab^2d^3x^2+3\,a^2bd^3x+a^3d^3\right)\log\left(bx+a\right)}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4)
```

giac [A] time = 0.94, size = 118, normalized size = 1.37

$$\frac{d^3\log\left(|bx+a|\right)}{b^4}-\frac{18\left(b^2cd^2-abd^3\right)x^2+9\left(b^2c^2d+2\,abcd^2-3\,a^2d^3\right)x+\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3}{b}}{6\left(bx+a\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] d^3*log(abs(b*x + a))/b^4 - 1/6*(18*(b^2*c*d^2 - a*b*d^3)*x^2 + 9*(b^2*c^2*
d + 2*a*b*c*d^2 - 3*a^2*d^3)*x + (2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2
 - 11*a^3*d^3)/b)/((b*x + a)^3*b^3)
```

maple [B] time = 0.01, size = 166, normalized size = 1.93

$$\frac{a^3d^3}{3\left(bx+a\right)^3b^4}-\frac{a^2c\,d^2}{\left(bx+a\right)^3b^3}+\frac{a\,c^2d}{\left(bx+a\right)^3b^2}-\frac{c^3}{3\left(bx+a\right)^3b}-\frac{3a^2d^3}{2\left(bx+a\right)^2b^4}+\frac{3ac\,d^2}{\left(bx+a\right)^2b^3}-\frac{3c^2d}{2\left(bx+a\right)^2b^2}+\frac{3a\,d^3}{\left(bx+a\right)b^4}-\frac{3c\,d^2}{\left(bx+a\right)b^3}+\frac{d^3\ln\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^4,x)
```

```
[Out] 1/3/b^4/(b*x+a)^3*a^3*d^3-1/b^3/(b*x+a)^3*a^2*c*d^2+1/b^2/(b*x+a)^3*a*c^2*d
-1/3/b/(b*x+a)^3*c^3-3/2*d^3/b^4/(b*x+a)^2*a^2+3*d^2/b^3/(b*x+a)^2*a*c-3/2*
d/b^2/(b*x+a)^2*c^2+d^3*ln(b*x+a)/b^4+3/b^4*d^3/(b*x+a)*a-3/b^3*d^2/(b*x+a)
*c
```

maxima [A] time = 1.35, size = 142, normalized size = 1.65

$$-\frac{2\,b^3c^3+3\,ab^2c^2d+6\,a^2bcd^2-11\,a^3d^3+18\left(b^3cd^2-ab^2d^3\right)x^2+9\left(b^3c^2d+2\,ab^2cd^2-3\,a^2bd^3\right)x}{6\left(b^7x^3+3\,ab^6x^2+3\,a^2b^5x+a^3b^4\right)}+\frac{d^3\log\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 11*a^3*d^3 + 18*(b^3*c*d^
2 - a*b^2*d^3)*x^2 + 9*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x)/(b^7*x^
3 + 3*a*b^6*x^2 + 3*a^2*b^5*x + a^3*b^4) + d^3*log(b*x + a)/b^4
```

mupad [B] time = 0.25, size = 138, normalized size = 1.60

$$\frac{d^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{\frac{-11\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{6\,b^4}+\frac{3\,x\,\left(-3\,a^2\,d^3+2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{2\,b^3}-\frac{3\,d^2\,x^2\,\left(a\,d-b\,c\right)}{b^2}}{a^3+3\,a^2\,b\,x+3\,a\,b^2\,x^2+b^3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^4,x)
```

```
[Out] (d^3*log(a + b*x))/b^4 - ((2*b^3*c^3 - 11*a^3*d^3 + 3*a*b^2*c^2*d + 6*a^2*b
*c*d^2)/(6*b^4) + (3*x*(b^2*c^2*d - 3*a^2*d^3 + 2*a*b*c*d^2))/(2*b^3) - (3*
d^2*x^2*(a*d - b*c))/b^2)/(a^3 + b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x)
```

sympy [A] time = 1.13, size = 148, normalized size = 1.72

$$\frac{11a^3d^3 - 6a^2bcd^2 - 3ab^2c^2d - 2b^3c^3 + x^2\left(18ab^2d^3 - 18b^3cd^2\right) + x\left(27a^2bd^3 - 18ab^2cd^2 - 9b^3c^2d\right)}{6a^3b^4 + 18a^2b^5x + 18ab^6x^2 + 6b^7x^3} + \frac{d^3\log\left(a + bx\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**4,x)
```

```
[Out] (11*a**3*d**3 - 6*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 2*b**3*c**3 + x**2*(18*
a*b**2*d**3 - 18*b**3*c*d**2) + x*(27*a**2*b*d**3 - 18*a*b**2*c*d**2 - 9*b*
*3*c**2*d))/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
 d**3*log(a + b*x)/b**4
```

3.1162 $\int \frac{(c+dx)^3}{(a+bx)^5} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^4}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -(c + d*x)^4/(4*(b*c - a*d)*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+bx)^5} dx = -\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

Mathematica [B] time = 0.03, size = 91, normalized size = 3.25

$$-\frac{a^3d^3 + a^2bd^2(c+4dx) + ab^2d\left(c^2+4cdx+6d^2x^2\right) + b^3\left(c^3+4c^2dx+6cd^2x^2+4d^3x^3\right)}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] -1/4*(a^3*d^3 + a^2*b*d^2*(c + 4*d*x) + a*b^2*d*(c^2 + 4*c*d*x + 6*d^2*x^2)
 + b^3*(c^3 + 4*c^2*d*x + 6*c*d^2*x^2 + 4*d^3*x^3))/(b^4*(a + b*x)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^5} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^5, x]
```

fricas [B] time = 1.54, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3 + b^3c^3 + ab^2c^2d + a^2bcd^2 + a^3d^3 + 6\left(b^3cd^2 + ab^2d^3\right)x^2 + 4\left(b^3c^2d + ab^2cd^2 + a^2bd^3\right)x}{4\left(b^8x^4 + 4\,ab^7x^3 + 6\,a^2b^6x^2 + 4\,a^3b^5x + a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

giac [B] time = 0.97, size = 159, normalized size = 5.68

$$-\frac{\frac{b^2c^3}{(bx+a)^4}+\frac{4\,bc^2d}{(bx+a)^3}-\frac{3\,abc^2d}{(bx+a)^4}+\frac{6\,cd^2}{(bx+a)^2}-\frac{8\,acd^2}{(bx+a)^3}+\frac{3\,a^2cd^2}{(bx+a)^4}+\frac{4\,d^3}{(bx+a)b}-\frac{6\,ad^3}{(bx+a)^2b}+\frac{4\,a^2d^3}{(bx+a)^3b}-\frac{a^3d^3}{(bx+a)^4b}}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c^3/(b*x + a)^4 + 4*b*c^2*d/(b*x + a)^3 - 3*a*b*c^2*d/(b*x + a)^4
 + 6*c*d^2/(b*x + a)^2 - 8*a*c*d^2/(b*x + a)^3 + 3*a^2*c*d^2/(b*x + a)^4 +
4*d^3/((b*x + a)*b) - 6*a*d^3/((b*x + a)^2*b) + 4*a^2*d^3/((b*x + a)^3*b) -
 a^3*d^3/((b*x + a)^4*b))/b^3
```

maple [B] time = 0.01, size = 122, normalized size = 4.36

$$-\frac{d^3}{(bx+a)\,b^4}+\frac{3\,(ad-bc)\,d^2}{2\,(bx+a)^2\,b^4}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)d}{(bx+a)^3\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4\,(bx+a)^4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^5,x)
```

```
[Out] -d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^3+3/2*d^2*(a*d-b*c)/b^4/(b*x+a)^
2-1/4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^4-d^3/b^4/
(b*x+a)
```

maxima [B] time = 1.39, size = 143, normalized size = 5.11

$$-\frac{4\,b^3d^3x^3+b^3c^3+ab^2c^2d+a^2bcd^2+a^3d^3+6\left(b^3cd^2+ab^2d^3\right)x^2+4\left(b^3c^2d+ab^2cd^2+a^2bd^3\right)x}{4\left(b^8x^4+4\,ab^7x^3+6\,a^2b^6x^2+4\,a^3b^5x+a^4b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2 + a^3*d^3 + 6*(b^
3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + a*b^2*c*d^2 + a^2*b*d^3)*x)/(b^8*
x^4 + 4*a*b^7*x^3 + 6*a^2*b^6*x^2 + 4*a^3*b^5*x + a^4*b^4)
```

mupad [B] time = 0.07, size = 135, normalized size = 4.82

$$-\frac{\frac{a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3}{4\,b^4}+\frac{d^3\,x^3}{b}+\frac{d\,x\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{b^3}+\frac{3\,d^2\,x^2\,(a\,d+b\,c)}{2\,b^2}}{a^4+4\,a^3\,b\,x+6\,a^2\,b^2\,x^2+4\,a\,b^3\,x^3+b^4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^5,x)
```

```
[Out] -((a^3*d^3 + b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2)/(4*b^4) + (d^3*x^3)/b + (
d*x*(a^2*d^2 + b^2*c^2 + a*b*c*d))/b^3 + (3*d^2*x^2*(a*d + b*c))/(2*b^2))/(
a^4 + b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x)
```

sympy [B] time = 1.50, size = 155, normalized size = 5.54

$$\frac{-a^3d^3 - a^2bcd^2 - ab^2c^2d - b^3c^3 - 4b^3d^3x^3 + x^2\left(-6ab^2d^3 - 6b^3cd^2\right) + x\left(-4a^2bd^3 - 4ab^2cd^2 - 4b^3c^2d\right)}{4a^4b^4 + 16a^3b^5x + 24a^2b^6x^2 + 16ab^7x^3 + 4b^8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**5,x)
```

```
[Out] (-a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d - b**3*c**3 - 4*b**3*d**3*x**3
+ x**2*(-6*a*b**2*d**3 - 6*b**3*c*d**2) + x*(-4*a**2*b*d**3 - 4*a*b**2*c*d*
*2 - 4*b**3*c**2*d))/(4*a**4*b**4 + 16*a**3*b**5*x + 24*a**2*b**6*x**2 + 16
*a*b**7*x**3 + 4*b**8*x**4)
```

3.1163 $\int \frac{(c+dx)^3}{(a+bx)^6} \, dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^4}{20(a+bx)^4(bc-ad)^2}-\frac{(c+dx)^4}{5(a+bx)^5(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -(c + d*x)^4/(5*(b*c - a*d)*(a + b*x)^5) + (d*(c + d*x)^4)/(20*(b*c - a*d)^
2*(a + b*x)^4)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^6} \, dx &= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5}-\frac{d\int \frac{(c+dx)^3}{(a+bx)^5} \, dx}{5(bc-ad)}\\ &= -\frac{(c+dx)^4}{5(bc-ad)(a+bx)^5}+\frac{d(c+dx)^4}{20(bc-ad)^2(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.67

$$-\frac{a^3d^3+a^2bd^2(2c+5dx)+ab^2d\left(3c^2+10cdx+10d^2x^2\right)+b^3\left(4c^3+15c^2dx+20cd^2x^2+10d^3x^3\right)}{20b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] -1/20*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d*(3*c^2 + 10*c*d*x + 10*d
^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2 + 10*d^3*x^3))/(b^4*(a + b
*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^6, x]
```

fricas [B] time = 1.65, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

giac [B] time = 1.00, size = 114, normalized size = 1.97

$$-\frac{10\,b^3d^3x^3+20\,b^3cd^2x^2+10\,ab^2d^3x^2+15\,b^3c^2dx+10\,ab^2cd^2x+5\,a^2bd^3x+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3}{20\,(bx+a)^5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 20*b^3*c*d^2*x^2 + 10*a*b^2*d^3*x^2 + 15*b^3*c^2*d*
x + 10*a*b^2*c*d^2*x + 5*a^2*b*d^3*x + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*
c*d^2 + a^3*d^3)/((b*x + a)^5*b^4)
```

maple [B] time = 0.01, size = 121, normalized size = 2.09

$$-\frac{d^3}{2\,(bx+a)^2\,b^4}+\frac{(ad-bc)\,d^2}{(bx+a)^3\,b^4}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)d}{4\,(bx+a)^4\,b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5\,(bx+a)^5\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^6,x)
```

```
[Out] d^2*(a*d-b*c)/b^4/(b*x+a)^3-1/5*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^5-1/2*d^3/b^4/(b*x+a)^2-3/4*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b
^4/(b*x+a)^4
```

maxima [B] time = 1.47, size = 160, normalized size = 2.76

$$-\frac{10\,b^3d^3x^3+4\,b^3c^3+3\,ab^2c^2d+2\,a^2bcd^2+a^3d^3+10\left(2\,b^3cd^2+ab^2d^3\right)x^2+5\left(3\,b^3c^2d+2\,ab^2cd^2+a^2bd^3\right)x}{20\left(b^9x^5+5\,ab^8x^4+10\,a^2b^7x^3+10\,a^3b^6x^2+5\,a^4b^5x+a^5b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(10*b^3*d^3*x^3 + 4*b^3*c^3 + 3*a*b^2*c^2*d + 2*a^2*b*c*d^2 + a^3*d^3
 + 10*(2*b^3*c*d^2 + a*b^2*d^3)*x^2 + 5*(3*b^3*c^2*d + 2*a*b^2*c*d^2 + a^2*
```

```
b*d^3)*x)/(b^9*x^5 + 5*a*b^8*x^4 + 10*a^2*b^7*x^3 + 10*a^3*b^6*x^2 + 5*a^4*
b^5*x + a^5*b^4)
```

mupad [B] time = 0.08, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^4\,\left(5\,a\,d-4\,b\,c+b\,d\,x\right)}{20\,\left(a\,d-b\,c\right)^2\,\left(a+b\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^6,x)
```

```
[Out] ((c + d*x)^4*(5*a*d - 4*b*c + b*d*x))/(20*(a*d - b*c)^2*(a + b*x)^5)
```

sympy [B] time = 1.96, size = 172, normalized size = 2.97

$$\frac{-a^3d^3-2a^2bcd^2-3ab^2c^2d-4b^3c^3-10b^3d^3x^3+x^2\left(-10ab^2d^3-20b^3cd^2\right)+x\left(-5a^2bd^3-10ab^2cd^2-15b^3c^2d\right)}{20a^5b^4+100a^4b^5x+200a^3b^6x^2+200a^2b^7x^3+100ab^8x^4+20b^9x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**6,x)
```

```
[Out] (-a**3*d**3 - 2*a**2*b*c*d**2 - 3*a*b**2*c**2*d - 4*b**3*c**3 - 10*b**3*d**
3*x**3 + x**2*(-10*a*b**2*d**3 - 20*b**3*c*d**2) + x*(-5*a**2*b*d**3 - 10*a
*b**2*c*d**2 - 15*b**3*c**2*d))/(20*a**5*b**4 + 100*a**4*b**5*x + 200*a**3*
b**6*x**2 + 200*a**2*b**7*x**3 + 100*a*b**8*x**4 + 20*b**9*x**5)
```

3.1164 $\int \frac{(c+dx)^3}{(a+bx)^7} dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{d^3}{3b^4(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -(b*c - a*d)^3/(6*b^4*(a + b*x)^6) - (3*d*(b*c - a*d)^2)/(5*b^4*(a + b*x)^5
) - (3*d^2*(b*c - a*d))/(4*b^4*(a + b*x)^4) - d^3/(3*b^4*(a + b*x)^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^7} dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^7}+\frac{3d(bc-ad)^2}{b^3(a+bx)^6}+\frac{3d^2(bc-ad)}{b^3(a+bx)^5}+\frac{d^3}{b^3(a+bx)^4}\right) dx\\ &= -\frac{(bc-ad)^3}{6b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{5b^4(a+bx)^5}-\frac{3d^2(bc-ad)}{4b^4(a+bx)^4}-\frac{d^3}{3b^4(a+bx)^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+3a^2bd^2(c+2dx)+3ab^2d\left(2c^2+6cdx+5d^2x^2\right)+b^3\left(10c^3+36c^2dx+45cd^2x^2+20d^3x^3\right)}{60b^4(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^7,x]
```

```
[Out] -1/60*(a^3*d^3 + 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*d*(2*c^2 + 6*c*d*x + 5*d
^2*x^2) + b^3*(10*c^3 + 36*c^2*d*x + 45*c*d^2*x^2 + 20*d^3*x^3))/(b^4*(a +
b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^7} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^7,x]
```

[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^7, x]

fricas [B] time = 1.49, size = 171, normalized size = 1.86

$$-\frac{20 b^3d^3x^3 + 10 b^3c^3 + 6 ab^2c^2d + 3 a^2bcd^2 + a^3d^3 + 15 \left(3 b^3cd^2 + ab^2d^3\right)x^2 + 6 \left(6 b^3c^2d + 3 ab^2cd^2 + a^2bd^3\right)x}{60 \left(b^{10}x^6 + 6 ab^9x^5 + 15 a^2b^8x^4 + 20 a^3b^7x^3 + 15 a^4b^6x^2 + 6 a^5b^5x + a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

giac [A] time = 0.98, size = 114, normalized size = 1.24

$$-\frac{20 b^3d^3x^3 + 45 b^3cd^2x^2 + 15 ab^2d^3x^2 + 36 b^3c^2dx + 18 ab^2cd^2x + 6 a^2bd^3x + 10 b^3c^3 + 6 ab^2c^2d + 3 a^2bcd^2 + a^3d^3}{60 (bx + a)^6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 45*b^3*c*d^2*x^2 + 15*a*b^2*d^3*x^2 + 36*b^3*c^2*d*
x + 18*a*b^2*c*d^2*x + 6*a^2*b*d^3*x + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b
*c*d^2 + a^3*d^3)/((b*x + a)^6*b^4)
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^3}{3 (bx + a)^3 b^4} + \frac{3 (ad - bc) d^2}{4 (bx + a)^4 b^4} - \frac{3 \left(a^2d^2 - 2abcd + b^2c^2\right) d}{5 (bx + a)^5 b^4} - \frac{-a^3d^3 + 3a^2bc d^2 - 3a b^2c^2d + b^3c^3}{6 (bx + a)^6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^7,x)
```

```
[Out] -1/3*d^3/b^4/(b*x+a)^3-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5+3/4*
d^2*(a*d-b*c)/b^4/(b*x+a)^4-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c
^3)/b^4/(b*x+a)^6
```

maxima [B] time = 1.44, size = 171, normalized size = 1.86

$$-\frac{20 b^3d^3x^3 + 10 b^3c^3 + 6 ab^2c^2d + 3 a^2bcd^2 + a^3d^3 + 15 \left(3 b^3cd^2 + ab^2d^3\right)x^2 + 6 \left(6 b^3c^2d + 3 ab^2cd^2 + a^2bd^3\right)x}{60 \left(b^{10}x^6 + 6 ab^9x^5 + 15 a^2b^8x^4 + 20 a^3b^7x^3 + 15 a^4b^6x^2 + 6 a^5b^5x + a^6b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/60*(20*b^3*d^3*x^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^
3 + 15*(3*b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(6*b^3*c^2*d + 3*a*b^2*c*d^2 + a^2
*b*d^3)*x)/(b^10*x^6 + 6*a*b^9*x^5 + 15*a^2*b^8*x^4 + 20*a^3*b^7*x^3 + 15*a
^4*b^6*x^2 + 6*a^5*b^5*x + a^6*b^4)
```

mupad [B] time = 0.22, size = 165, normalized size = 1.79

$$-\frac{\frac{a^3\,d^3+3\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d+10\,b^3\,c^3}{60\,b^4}+\frac{d^3\,x^3}{3\,b}+\frac{d\,x\,\left(a^2\,d^2+3\,a\,b\,c\,d+6\,b^2\,c^2\right)}{10\,b^3}+\frac{d^2\,x^2\,\left(a\,d+3\,b\,c\right)}{4\,b^2}}{a^6+6\,a^5\,b\,x+15\,a^4\,b^2\,x^2+20\,a^3\,b^3\,x^3+15\,a^2\,b^4\,x^4+6\,a\,b^5\,x^5+b^6\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^7,x)
```

```
[Out] -((a^3*d^3 + 10*b^3*c^3 + 6*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(60*b^4) + (d^3*x^
3)/(3*b) + (d*x*(a^2*d^2 + 6*b^2*c^2 + 3*a*b*c*d))/(10*b^3) + (d^2*x^2*(a*d
 + 3*b*c))/(4*b^2))/(a^6 + b^6*x^6 + 6*a*b^5*x^5 + 15*a^4*b^2*x^2 + 20*a^3*
b^3*x^3 + 15*a^2*b^4*x^4 + 6*a^5*b*x)
```

sympy [B] time = 2.54, size = 184, normalized size = 2.00

$$\frac{-a^3d^3-3a^2bcd^2-6ab^2c^2d-10b^3c^3-20b^3d^3x^3+x^2\left(-15ab^2d^3-45b^3cd^2\right)+x\left(-6a^2bd^3-18ab^2cd^2-36b^3c^2d\right)}{60a^6b^4+360a^5b^5x+900a^4b^6x^2+1200a^3b^7x^3+900a^2b^8x^4+360ab^9x^5+60b^{10}x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**7,x)
```

```
[Out] (-a**3*d**3 - 3*a**2*b*c*d**2 - 6*a*b**2*c**2*d - 10*b**3*c**3 - 20*b**3*d*
*3*x**3 + x**2*(-15*a*b**2*d**3 - 45*b**3*c*d**2) + x*(-6*a**2*b*d**3 - 18*
a*b**2*c*d**2 - 36*b**3*c**2*d))/(60*a**6*b**4 + 360*a**5*b**5*x + 900*a**4
*b**6*x**2 + 1200*a**3*b**7*x**3 + 900*a**2*b**8*x**4 + 360*a*b**9*x**5 + 6
0*b**10*x**6)
```

3.1165 $\int \frac{(c+dx)^3}{(a+bx)^8} dx$

Optimal. Leaf size=92

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d^3}{4b^4(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^3/(7*b^4*(a + b*x)^7) - (d*(b*c - a*d)^2)/(2*b^4*(a + b*x)^6)
- (3*d^2*(b*c - a*d))/(5*b^4*(a + b*x)^5) - d^3/(4*b^4*(a + b*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^8}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^8}+\frac{3d(bc-ad)^2}{b^3(a+bx)^7}+\frac{3d^2(bc-ad)}{b^3(a+bx)^6}+\frac{d^3}{b^3(a+bx)^5}\right)dx\\ &= -\frac{(bc-ad)^3}{7b^4(a+bx)^7}-\frac{d(bc-ad)^2}{2b^4(a+bx)^6}-\frac{3d^2(bc-ad)}{5b^4(a+bx)^5}-\frac{d^3}{4b^4(a+bx)^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(4c+7dx)+ab^2d\left(10c^2+28cdx+21d^2x^2\right)+b^3\left(20c^3+70c^2dx+84cd^2x^2+35d^3x^3\right)}{140b^4(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] -1/140*(a^3*d^3 + a^2*b*d^2*(4*c + 7*d*x) + a*b^2*d*(10*c^2 + 28*c*d*x + 21
*d^2*x^2) + b^3*(20*c^3 + 70*c^2*d*x + 84*c*d^2*x^2 + 35*d^3*x^3))/(b^4*(a
+ b*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^8, x]
```

fricas [B] time = 1.46, size = 182, normalized size = 1.98

$$-\frac{35\,b^{3}d^{3}x^{3}+20\,b^{3}c^{3}+10\,ab^{2}c^{2}d+4\,a^{2}bcd^{2}+a^{3}d^{3}+21\,\left(4\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+7\,\left(10\,b^{3}c^{2}d+4\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{140\,\left(b^{11}x^{7}+7\,ab^{10}x^{6}+21\,a^{2}b^{9}x^{5}+35\,a^{3}b^{8}x^{4}+35\,a^{4}b^{7}x^{3}+21\,a^{5}b^{6}x^{2}+7\,a^{6}b^{5}x+a^{7}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

giac [A] time = 0.95, size = 114, normalized size = 1.24

$$-\frac{35\,b^{3}d^{3}x^{3}+84\,b^{3}cd^{2}x^{2}+21\,ab^{2}d^{3}x^{2}+70\,b^{3}c^{2}dx+28\,ab^{2}cd^{2}x+7\,a^{2}bd^{3}x+20\,b^{3}c^{3}+10\,ab^{2}c^{2}d+4\,a^{2}bcd^{2}+a^{3}d^{3}}{140\,(bx+a)^{7}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 84*b^3*c*d^2*x^2 + 21*a*b^2*d^3*x^2 + 70*b^3*c^2*d
*x + 28*a*b^2*c*d^2*x + 7*a^2*b*d^3*x + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2
*b*c*d^2 + a^3*d^3)/((b*x + a)^7*b^4)
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^{3}}{4\,(bx+a)^{4}\,b^{4}}+\frac{3\,(ad-bc)\,d^{2}}{5\,(bx+a)^{5}\,b^{4}}-\frac{\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)d}{2\,(bx+a)^{6}\,b^{4}}-\frac{-a^{3}d^{3}+3a^{2}bc\,d^{2}-3a\,b^{2}c^{2}d+b^{3}c^{3}}{7\,(bx+a)^{7}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^8,x)
```

```
[Out] -1/7*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^7+3/5*d^2*(
a*d-b*c)/b^4/(b*x+a)^5-1/4*d^3/b^4/(b*x+a)^4-1/2*d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/b^4/(b*x+a)^6
```

maxima [B] time = 1.45, size = 182, normalized size = 1.98

$$-\frac{35\,b^{3}d^{3}x^{3}+20\,b^{3}c^{3}+10\,ab^{2}c^{2}d+4\,a^{2}bcd^{2}+a^{3}d^{3}+21\,\left(4\,b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+7\,\left(10\,b^{3}c^{2}d+4\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x}{140\,\left(b^{11}x^{7}+7\,ab^{10}x^{6}+21\,a^{2}b^{9}x^{5}+35\,a^{3}b^{8}x^{4}+35\,a^{4}b^{7}x^{3}+21\,a^{5}b^{6}x^{2}+7\,a^{6}b^{5}x+a^{7}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2 + a^3*
d^3 + 21*(4*b^3*c*d^2 + a*b^2*d^3)*x^2 + 7*(10*b^3*c^2*d + 4*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^11*x^7 + 7*a*b^10*x^6 + 21*a^2*b^9*x^5 + 35*a^3*b^8*x^4 +
35*a^4*b^7*x^3 + 21*a^5*b^6*x^2 + 7*a^6*b^5*x + a^7*b^4)
```

mupad [B] time = 0.11, size = 176, normalized size = 1.91

$$-\frac{\frac{a^{3}\,d^{3}+4\,a^{2}\,b\,c\,d^{2}+10\,a\,b^{2}\,c^{2}\,d+20\,b^{3}\,c^{3}}{140\,b^{4}}+\frac{d^{3}\,x^{3}}{4\,b}+\frac{d\,x\,\left(a^{2}\,d^{2}+4\,a\,b\,c\,d+10\,b^{2}\,c^{2}\right)}{20\,b^{3}}+\frac{3\,d^{2}\,x^{2}\,(a\,d+4\,b\,c)}{20\,b^{2}}}{a^{7}+7\,a^{6}\,b\,x+21\,a^{5}\,b^{2}\,x^{2}+35\,a^{4}\,b^{3}\,x^{3}+35\,a^{3}\,b^{4}\,x^{4}+21\,a^{2}\,b^{5}\,x^{5}+7\,a\,b^{6}\,x^{6}+b^{7}\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^8,x)
```

```
[Out] -((a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2)/(140*b^4) + (d^3*
x^3)/(4*b) + (d*x*(a^2*d^2 + 10*b^2*c^2 + 4*a*b*c*d))/(20*b^3) + (3*d^2*x^2
*(a*d + 4*b*c))/(20*b^2))/(a^7 + b^7*x^7 + 7*a*b^6*x^6 + 21*a^5*b^2*x^2 + 3
5*a^4*b^3*x^3 + 35*a^3*b^4*x^4 + 21*a^2*b^5*x^5 + 7*a^6*b*x)
```

sympy [**B**] time = 3.12, size = 196, normalized size = 2.13

$$\frac{-a^3d^3 - 4a^2bcd^2 - 10ab^2c^2d - 20b^3c^3 - 35b^3d^3x^3 + x^2\left(-21ab^2d^3 - 84b^3cd^2\right) + x\left(-7a^2bd^3 - 28ab^2cd^2 - 70b^3c^2d\right)}{140a^7b^4 + 980a^6b^5x + 2940a^5b^6x^2 + 4900a^4b^7x^3 + 4900a^3b^8x^4 + 2940a^2b^9x^5 + 980ab^{10}x^6 + 140b^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**8,x)
```

```
[Out] (-a**3*d**3 - 4*a**2*b*c*d**2 - 10*a*b**2*c**2*d - 20*b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-21*a*b**2*d**3 - 84*b**3*c*d**2) + x*(-7*a**2*b*d**3 - 28
*a*b**2*c*d**2 - 70*b**3*c**2*d))/(140*a**7*b**4 + 980*a**6*b**5*x + 2940*a
**5*b**6*x**2 + 4900*a**4*b**7*x**3 + 4900*a**3*b**8*x**4 + 2940*a**2*b**9*
x**5 + 980*a*b**10*x**6 + 140*b**11*x**7)
```

3.1166 $\int \frac{(c+dx)^3}{(a+bx)^9}\,dx$

Optimal. Leaf size=92

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{d^3}{5b^4(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -(b*c - a*d)^3/(8*b^4*(a + b*x)^8) - (3*d*(b*c - a*d)^2)/(7*b^4*(a + b*x)^7
) - (d^2*(b*c - a*d))/(2*b^4*(a + b*x)^6) - d^3/(5*b^4*(a + b*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+bx)^9}\,dx &= \int \left(\frac{(bc-ad)^3}{b^3(a+bx)^9}+\frac{3d(bc-ad)^2}{b^3(a+bx)^8}+\frac{3d^2(bc-ad)}{b^3(a+bx)^7}+\frac{d^3}{b^3(a+bx)^6}\right)dx\\ &= -\frac{(bc-ad)^3}{8b^4(a+bx)^8}-\frac{3d(bc-ad)^2}{7b^4(a+bx)^7}-\frac{d^2(bc-ad)}{2b^4(a+bx)^6}-\frac{d^3}{5b^4(a+bx)^5}\end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.05

$$-\frac{a^3d^3+a^2bd^2(5c+8dx)+ab^2d\left(15c^2+40cdx+28d^2x^2\right)+b^3\left(35c^3+120c^2dx+140cd^2x^2+56d^3x^3\right)}{280b^4(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] -1/280*(a^3*d^3 + a^2*b*d^2*(5*c + 8*d*x) + a*b^2*d*(15*c^2 + 40*c*d*x + 28
*d^2*x^2) + b^3*(35*c^3 + 120*c^2*d*x + 140*c*d^2*x^2 + 56*d^3*x^3))/(b^4*(
a + b*x)^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{(a+bx)^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^3/(a + b*x)^9, x]
```

fricas [B] time = 1.44, size = 193, normalized size = 2.10

$$-\frac{56\,b^{3}d^{3}x^{3} + 35\,b^{3}c^{3} + 15\,ab^{2}c^{2}d + 5\,a^{2}bcd^{2} + a^{3}d^{3} + 28\left(5\,b^{3}cd^{2} + ab^{2}d^{3}\right)x^{2} + 8\left(15\,b^{3}c^{2}d + 5\,ab^{2}cd^{2} + a^{2}bd^{3}\right)x}{280\left(b^{12}x^{8} + 8\,ab^{11}x^{7} + 28\,a^{2}b^{10}x^{6} + 56\,a^{3}b^{9}x^{5} + 70\,a^{4}b^{8}x^{4} + 56\,a^{5}b^{7}x^{3} + 28\,a^{6}b^{6}x^{2} + 8\,a^{7}b^{5}x + a^{8}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

giac [A] time = 0.86, size = 114, normalized size = 1.24

$$-\frac{56\,b^{3}d^{3}x^{3} + 140\,b^{3}cd^{2}x^{2} + 28\,ab^{2}d^{3}x^{2} + 120\,b^{3}c^{2}dx + 40\,ab^{2}cd^{2}x + 8\,a^{2}bd^{3}x + 35\,b^{3}c^{3} + 15\,ab^{2}c^{2}d + 5\,a^{2}bcd^{2} + a^{3}d^{3}}{280\,(bx + a)^{8}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 140*b^3*c*d^2*x^2 + 28*a*b^2*d^3*x^2 + 120*b^3*c^2
*d*x + 40*a*b^2*c*d^2*x + 8*a^2*b*d^3*x + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a
^2*b*c*d^2 + a^3*d^3)/((b*x + a)^8*b^4)
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$-\frac{d^{3}}{5\,(bx + a)^{5}\,b^{4}} + \frac{(ad - bc)\,d^{2}}{2\,(bx + a)^{6}\,b^{4}} - \frac{3\left(a^{2}d^{2} - 2abcd + b^{2}c^{2}\right)d}{7\,(bx + a)^{7}\,b^{4}} - \frac{-a^{3}d^{3} + 3a^{2}bc\,d^{2} - 3a\,b^{2}c^{2}d + b^{3}c^{3}}{8\,(bx + a)^{8}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(b*x+a)^9,x)
```

```
[Out] -1/8*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^8-3/7*d*(a^
2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^7-1/5*d^3/b^4/(b*x+a)^5+1/2*d^2*(a*d-b
*c)/b^4/(b*x+a)^6
```

maxima [B] time = 1.52, size = 193, normalized size = 2.10

$$-\frac{56\,b^{3}d^{3}x^{3} + 35\,b^{3}c^{3} + 15\,ab^{2}c^{2}d + 5\,a^{2}bcd^{2} + a^{3}d^{3} + 28\left(5\,b^{3}cd^{2} + ab^{2}d^{3}\right)x^{2} + 8\left(15\,b^{3}c^{2}d + 5\,ab^{2}cd^{2} + a^{2}bd^{3}\right)x}{280\left(b^{12}x^{8} + 8\,ab^{11}x^{7} + 28\,a^{2}b^{10}x^{6} + 56\,a^{3}b^{9}x^{5} + 70\,a^{4}b^{8}x^{4} + 56\,a^{5}b^{7}x^{3} + 28\,a^{6}b^{6}x^{2} + 8\,a^{7}b^{5}x + a^{8}b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/280*(56*b^3*d^3*x^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*
d^3 + 28*(5*b^3*c*d^2 + a*b^2*d^3)*x^2 + 8*(15*b^3*c^2*d + 5*a*b^2*c*d^2 +
a^2*b*d^3)*x)/(b^12*x^8 + 8*a*b^11*x^7 + 28*a^2*b^10*x^6 + 56*a^3*b^9*x^5 +
 70*a^4*b^8*x^4 + 56*a^5*b^7*x^3 + 28*a^6*b^6*x^2 + 8*a^7*b^5*x + a^8*b^4)
```

mupad [B] time = 0.23, size = 187, normalized size = 2.03

$$-\frac{\frac{a^3\,d^3+5\,a^2\,b\,c\,d^2+15\,a\,b^2\,c^2\,d+35\,b^3\,c^3}{280\,b^4}+\frac{d^3\,x^3}{5\,b}+\frac{d\,x\,\left(a^2\,d^2+5\,a\,b\,c\,d+15\,b^2\,c^2\right)}{35\,b^3}+\frac{d^2\,x^2\,\left(a\,d+5\,b\,c\right)}{10\,b^2}}{a^8+8\,a^7\,b\,x+28\,a^6\,b^2\,x^2+56\,a^5\,b^3\,x^3+70\,a^4\,b^4\,x^4+56\,a^3\,b^5\,x^5+28\,a^2\,b^6\,x^6+8\,a\,b^7\,x^7+b^8\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*x)^9,x)
```

```
[Out] -((a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2)/(280*b^4) + (d^3*
x^3)/(5*b) + (d*x*(a^2*d^2 + 15*b^2*c^2 + 5*a*b*c*d))/(35*b^3) + (d^2*x^2*(
a*d + 5*b*c))/(10*b^2))/(a^8 + b^8*x^8 + 8*a*b^7*x^7 + 28*a^6*b^2*x^2 + 56*
a^5*b^3*x^3 + 70*a^4*b^4*x^4 + 56*a^3*b^5*x^5 + 28*a^2*b^6*x^6 + 8*a^7*b*x)
```

sympy [B] time = 3.95, size = 207, normalized size = 2.25

$$\frac{-a^3d^3-5a^2bcd^2-15ab^2c^2d-35b^3c^3-56b^3d^3x^3+x^2\left(-28ab^2d^3-140b^3cd^2\right)+x\left(-8a^2bd^3-40ab^2cd^2-120b^3c^2d\right)}{280a^8b^4+2240a^7b^5x+7840a^6b^6x^2+15680a^5b^7x^3+19600a^4b^8x^4+15680a^3b^9x^5+7840a^2b^{10}x^6+2240ab^{11}x^7+280b^{12}x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(b*x+a)**9,x)
```

```
[Out] (-a**3*d**3 - 5*a**2*b*c*d**2 - 15*a*b**2*c**2*d - 35*b**3*c**3 - 56*b**3*d
**3*x**3 + x**2*(-28*a*b**2*d**3 - 140*b**3*c*d**2) + x*(-8*a**2*b*d**3 - 4
0*a*b**2*c*d**2 - 120*b**3*c**2*d))/(280*a**8*b**4 + 2240*a**7*b**5*x + 784
0*a**6*b**6*x**2 + 15680*a**5*b**7*x**3 + 19600*a**4*b**8*x**4 + 15680*a**3
*b**9*x**5 + 7840*a**2*b**10*x**6 + 2240*a*b**11*x**7 + 280*b**12*x**8)
```

3.1167 $\int (a+bx)^9(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx}{4}$$

Rubi [A] time = 0.68, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^{16}(bc-ad)}{16b^8}+\frac{7d^5(a+bx)^{15}(bc-ad)^2}{5b^8}+\frac{5d^4(a+bx)^{14}(bc-ad)^3}{2b^8}+\frac{35d^3(a+bx)^{13}(bc-ad)^4}{13b^8}+\frac{7d^2(a+bx)^{12}(bc-ad)^5}{4b^8}+\frac{7d(a+bx)^{11}(bc-ad)^6}{11b^8}+\frac{(a+bx)^{10}(bc-ad)^7}{10b^8}+\frac{d^7(a+bx)^{17}}{17b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^10)/(10*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^11)/(1
1*b^8) + (7*d^2*(b*c - a*d)^5*(a + b*x)^12)/(4*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^13)/(13*b^8) + (5*d^4*(b*c - a*d)^3*(a + b*x)^14)/(2*b^8) + (7*d
^5*(b*c - a*d)^2*(a + b*x)^15)/(5*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^16)/(
16*b^8) + (d^7*(a + b*x)^17)/(17*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^7\,dx=\int \left(\frac{(bc-ad)^7(a+bx)^9}{b^7}+\frac{7d(bc-ad)^6(a+bx)^{10}}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{b^7}+\frac{35d^3(b}{}\right.$$
$$=\frac{(bc-ad)^7(a+bx)^{10}}{10b^8}+\frac{7d(bc-ad)^6(a+bx)^{11}}{11b^8}+\frac{7d^2(bc-ad)^5(a+bx)^{12}}{4b^8}+\frac{35d^3(bc-}{1}$$

Mathematica [B] time = 0.15, size = 993, normalized size = 4.96

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] a^9*c^7*x + (a^8*c^6*(9*b*c + 7*a*d)*x^2)/2 + a^7*c^5*(12*b^2*c^2 + 21*a*b*
c*d + 7*a^2*d^2)*x^3 + (7*a^6*c^4*(12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^5*c^3*(18*b^4*c^4 + 84*a*b^3*c^3*d + 108*a^
2*b^2*c^2*d^2 + 45*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^4*c^2*(6*b^5*c^5
+ 42*a*b^4*c^4*d + 84*a^2*b^3*c^3*d^2 + 60*a^3*b^2*c^2*d^3 + 15*a^4*b*c*d^4
 + a^5*d^5)*x^6)/2 + a^3*c*(12*b^6*c^6 + 126*a*b^5*c^5*d + 378*a^2*b^4*c^4*
d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 27*a^5*b*c*d^5 + a^6*d^6)
*x^7 + (a^2*(36*b^7*c^7 + 588*a*b^6*c^6*d + 2646*a^2*b^5*c^5*d^2 + 4410*a^3
*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 756*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6
+ a^7*d^7)*x^8)/8 + a*b*(b^7*c^7 + 28*a*b^6*c^6*d + 196*a^2*b^5*c^5*d^2 + 4
90*a^3*b^4*c^4*d^3 + 490*a^4*b^3*c^3*d^4 + 196*a^5*b^2*c^2*d^5 + 28*a^6*b*c
*d^6 + a^7*d^7)*x^9 + (b^2*(b^7*c^7 + 63*a*b^6*c^6*d + 756*a^2*b^5*c^5*d^2
+ 2940*a^3*b^4*c^4*d^3 + 4410*a^4*b^3*c^3*d^4 + 2646*a^5*b^2*c^2*d^5 + 588*
```

```
a^6*b*c*d^6 + 36*a^7*d^7)*x^10)/10 + (7*b^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 1
80*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*
c*d^5 + 12*a^6*d^6)*x^11)/11 + (7*b^4*d^2*(b^5*c^5 + 15*a*b^4*c^4*d + 60*a^
2*b^3*c^3*d^2 + 84*a^3*b^2*c^2*d^3 + 42*a^4*b*c*d^4 + 6*a^5*d^5)*x^12)/4 +
(7*b^5*d^3*(5*b^4*c^4 + 45*a*b^3*c^3*d + 108*a^2*b^2*c^2*d^2 + 84*a^3*b*c*d
^3 + 18*a^4*d^4)*x^13)/13 + (b^6*d^4*(5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b
*c*d^2 + 12*a^3*d^3)*x^14)/2 + (b^7*d^5*(7*b^2*c^2 + 21*a*b*c*d + 12*a^2*d^
2)*x^15)/5 + (b^8*d^6*(7*b*c + 9*a*d)*x^16)/16 + (b^9*d^7*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^9(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^7, x]
```

fricas [B] time = 0.86, size = 1175, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^7*b^9 + 7/16*x^16*d^6*c*b^9 + 9/16*x^16*d^7*b^8*a + 7/5*x^15*d^
5*c^2*b^9 + 21/5*x^15*d^6*c*b^8*a + 12/5*x^15*d^7*b^7*a^2 + 5/2*x^14*d^4*c^
3*b^9 + 27/2*x^14*d^5*c^2*b^8*a + 18*x^14*d^6*c*b^7*a^2 + 6*x^14*d^7*b^6*a^
3 + 35/13*x^13*d^3*c^4*b^9 + 315/13*x^13*d^4*c^3*b^8*a + 756/13*x^13*d^5*c^
2*b^7*a^2 + 588/13*x^13*d^6*c*b^6*a^3 + 126/13*x^13*d^7*b^5*a^4 + 7/4*x^12*
d^2*c^5*b^9 + 105/4*x^12*d^3*c^4*b^8*a + 105*x^12*d^4*c^3*b^7*a^2 + 147*x^1
2*d^5*c^2*b^6*a^3 + 147/2*x^12*d^6*c*b^5*a^4 + 21/2*x^12*d^7*b^4*a^5 + 7/11
*x^11*d*c^6*b^9 + 189/11*x^11*d^2*c^5*b^8*a + 1260/11*x^11*d^3*c^4*b^7*a^2
+ 2940/11*x^11*d^4*c^3*b^6*a^3 + 2646/11*x^11*d^5*c^2*b^5*a^4 + 882/11*x^11
*d^6*c*b^4*a^5 + 84/11*x^11*d^7*b^3*a^6 + 1/10*x^10*c^7*b^9 + 63/10*x^10*d*
c^6*b^8*a + 378/5*x^10*d^2*c^5*b^7*a^2 + 294*x^10*d^3*c^4*b^6*a^3 + 441*x^1
0*d^4*c^3*b^5*a^4 + 1323/5*x^10*d^5*c^2*b^4*a^5 + 294/5*x^10*d^6*c*b^3*a^6
+ 18/5*x^10*d^7*b^2*a^7 + x^9*c^7*b^8*a + 28*x^9*d*c^6*b^7*a^2 + 196*x^9*d^
2*c^5*b^6*a^3 + 490*x^9*d^3*c^4*b^5*a^4 + 490*x^9*d^4*c^3*b^4*a^5 + 196*x^9
*d^5*c^2*b^3*a^6 + 28*x^9*d^6*c*b^2*a^7 + x^9*d^7*b*a^8 + 9/2*x^8*c^7*b^7*a
^2 + 147/2*x^8*d*c^6*b^6*a^3 + 1323/4*x^8*d^2*c^5*b^5*a^4 + 2205/4*x^8*d^3*
c^4*b^4*a^5 + 735/2*x^8*d^4*c^3*b^3*a^6 + 189/2*x^8*d^5*c^2*b^2*a^7 + 63/8*
x^8*d^6*c*b*a^8 + 1/8*x^8*d^7*a^9 + 12*x^7*c^7*b^6*a^3 + 126*x^7*d*c^6*b^5*
a^4 + 378*x^7*d^2*c^5*b^4*a^5 + 420*x^7*d^3*c^4*b^3*a^6 + 180*x^7*d^4*c^3*b
^2*a^7 + 27*x^7*d^5*c^2*b*a^8 + x^7*d^6*c*a^9 + 21*x^6*c^7*b^5*a^4 + 147*x^
6*d*c^6*b^4*a^5 + 294*x^6*d^2*c^5*b^3*a^6 + 210*x^6*d^3*c^4*b^2*a^7 + 105/2
*x^6*d^4*c^3*b*a^8 + 7/2*x^6*d^5*c^2*a^9 + 126/5*x^5*c^7*b^4*a^5 + 588/5*x^
5*d*c^6*b^3*a^6 + 756/5*x^5*d^2*c^5*b^2*a^7 + 63*x^5*d^3*c^4*b*a^8 + 7*x^5*
d^4*c^3*a^9 + 21*x^4*c^7*b^3*a^6 + 63*x^4*d*c^6*b^2*a^7 + 189/4*x^4*d^2*c^5
*b*a^8 + 35/4*x^4*d^3*c^4*a^9 + 12*x^3*c^7*b^2*a^7 + 21*x^3*d*c^6*b*a^8 + 7
*x^3*d^2*c^5*a^9 + 9/2*x^2*c^7*b*a^8 + 7/2*x^2*d*c^6*a^9 + x*c^7*a^9
```

giac [B] time = 1.05, size = 1175, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/17*b^9*d^7*x^17 + 7/16*b^9*c*d^6*x^16 + 9/16*a*b^8*d^7*x^16 + 7/5*b^9*c^2
*d^5*x^15 + 21/5*a*b^8*c*d^6*x^15 + 12/5*a^2*b^7*d^7*x^15 + 5/2*b^9*c^3*d^4
*x^14 + 27/2*a*b^8*c^2*d^5*x^14 + 18*a^2*b^7*c*d^6*x^14 + 6*a^3*b^6*d^7*x^1
4 + 35/13*b^9*c^4*d^3*x^13 + 315/13*a*b^8*c^3*d^4*x^13 + 756/13*a^2*b^7*c^2
*d^5*x^13 + 588/13*a^3*b^6*c*d^6*x^13 + 126/13*a^4*b^5*d^7*x^13 + 7/4*b^9*c
^5*d^2*x^12 + 105/4*a*b^8*c^4*d^3*x^12 + 105*a^2*b^7*c^3*d^4*x^12 + 147*a^3
*b^6*c^2*d^5*x^12 + 147/2*a^4*b^5*c*d^6*x^12 + 21/2*a^5*b^4*d^7*x^12 + 7/11
*b^9*c^6*d*x^11 + 189/11*a*b^8*c^5*d^2*x^11 + 1260/11*a^2*b^7*c^4*d^3*x^11
+ 2940/11*a^3*b^6*c^3*d^4*x^11 + 2646/11*a^4*b^5*c^2*d^5*x^11 + 882/11*a^5*
b^4*c*d^6*x^11 + 84/11*a^6*b^3*d^7*x^11 + 1/10*b^9*c^7*x^10 + 63/10*a*b^8*c
^6*d*x^10 + 378/5*a^2*b^7*c^5*d^2*x^10 + 294*a^3*b^6*c^4*d^3*x^10 + 441*a^4
*b^5*c^3*d^4*x^10 + 1323/5*a^5*b^4*c^2*d^5*x^10 + 294/5*a^6*b^3*c*d^6*x^10
+ 18/5*a^7*b^2*d^7*x^10 + a*b^8*c^7*x^9 + 28*a^2*b^7*c^6*d*x^9 + 196*a^3*b^
6*c^5*d^2*x^9 + 490*a^4*b^5*c^4*d^3*x^9 + 490*a^5*b^4*c^3*d^4*x^9 + 196*a^6
*b^3*c^2*d^5*x^9 + 28*a^7*b^2*c*d^6*x^9 + a^8*b*d^7*x^9 + 9/2*a^2*b^7*c^7*x
^8 + 147/2*a^3*b^6*c^6*d*x^8 + 1323/4*a^4*b^5*c^5*d^2*x^8 + 2205/4*a^5*b^4*
c^4*d^3*x^8 + 735/2*a^6*b^3*c^3*d^4*x^8 + 189/2*a^7*b^2*c^2*d^5*x^8 + 63/8*
a^8*b*c*d^6*x^8 + 1/8*a^9*d^7*x^8 + 12*a^3*b^6*c^7*x^7 + 126*a^4*b^5*c^6*d*
x^7 + 378*a^5*b^4*c^5*d^2*x^7 + 420*a^6*b^3*c^4*d^3*x^7 + 180*a^7*b^2*c^3*d
^4*x^7 + 27*a^8*b*c^2*d^5*x^7 + a^9*c*d^6*x^7 + 21*a^4*b^5*c^7*x^6 + 147*a^
5*b^4*c^6*d*x^6 + 294*a^6*b^3*c^5*d^2*x^6 + 210*a^7*b^2*c^4*d^3*x^6 + 105/2
*a^8*b*c^3*d^4*x^6 + 7/2*a^9*c^2*d^5*x^6 + 126/5*a^5*b^4*c^7*x^5 + 588/5*a^
6*b^3*c^6*d*x^5 + 756/5*a^7*b^2*c^5*d^2*x^5 + 63*a^8*b*c^4*d^3*x^5 + 7*a^9*
c^3*d^4*x^5 + 21*a^6*b^3*c^7*x^4 + 63*a^7*b^2*c^6*d*x^4 + 189/4*a^8*b*c^5*d
^2*x^4 + 35/4*a^9*c^4*d^3*x^4 + 12*a^7*b^2*c^7*x^3 + 21*a^8*b*c^6*d*x^3 + 7
*a^9*c^5*d^2*x^3 + 9/2*a^8*b*c^7*x^2 + 7/2*a^9*c^6*d*x^2 + a^9*c^7*x
```

maple [B] time = 0.00, size = 1033, normalized size = 5.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^7,x)
```

```
[Out] 1/17*b^9*d^7*x^17+1/16*(9*a*b^8*d^7+7*b^9*c*d^6)*x^16+1/15*(36*a^2*b^7*d^7+
63*a*b^8*c*d^6+21*b^9*c^2*d^5)*x^15+1/14*(84*a^3*b^6*d^7+252*a^2*b^7*c*d^6+
189*a*b^8*c^2*d^5+35*b^9*c^3*d^4)*x^14+1/13*(126*a^4*b^5*d^7+588*a^3*b^6*c*
d^6+756*a^2*b^7*c^2*d^5+315*a*b^8*c^3*d^4+35*b^9*c^4*d^3)*x^13+1/12*(126*a^
5*b^4*d^7+882*a^4*b^5*c*d^6+1764*a^3*b^6*c^2*d^5+1260*a^2*b^7*c^3*d^4+315*a
*b^8*c^4*d^3+21*b^9*c^5*d^2)*x^12+1/11*(84*a^6*b^3*d^7+882*a^5*b^4*c*d^6+26
46*a^4*b^5*c^2*d^5+2940*a^3*b^6*c^3*d^4+1260*a^2*b^7*c^4*d^3+189*a*b^8*c^5*
d^2+7*b^9*c^6*d)*x^11+1/10*(36*a^7*b^2*d^7+588*a^6*b^3*c*d^6+2646*a^5*b^4*c
^2*d^5+4410*a^4*b^5*c^3*d^4+2940*a^3*b^6*c^4*d^3+756*a^2*b^7*c^5*d^2+63*a*b
^8*c^6*d+b^9*c^7)*x^10+1/9*(9*a^8*b*d^7+252*a^7*b^2*c*d^6+1764*a^6*b^3*c^2*
d^5+4410*a^5*b^4*c^3*d^4+4410*a^4*b^5*c^4*d^3+1764*a^3*b^6*c^5*d^2+252*a^2*
b^7*c^6*d+9*a*b^8*c^7)*x^9+1/8*(a^9*d^7+63*a^8*b*c*d^6+756*a^7*b^2*c^2*d^5+
2940*a^6*b^3*c^3*d^4+4410*a^5*b^4*c^4*d^3+2646*a^4*b^5*c^5*d^2+588*a^3*b^6*
c^6*d+36*a^2*b^7*c^7)*x^8+1/7*(7*a^9*c*d^6+189*a^8*b*c^2*d^5+1260*a^7*b^2*c
^3*d^4+2940*a^6*b^3*c^4*d^3+2646*a^5*b^4*c^5*d^2+882*a^4*b^5*c^6*d+84*a^3*b
^6*c^7)*x^7+1/6*(21*a^9*c^2*d^5+315*a^8*b*c^3*d^4+1260*a^7*b^2*c^4*d^3+1764
*a^6*b^3*c^5*d^2+882*a^5*b^4*c^6*d+126*a^4*b^5*c^7)*x^6+1/5*(35*a^9*c^3*d^4
+315*a^8*b*c^4*d^3+756*a^7*b^2*c^5*d^2+588*a^6*b^3*c^6*d+126*a^5*b^4*c^7)*x
^5+1/4*(35*a^9*c^4*d^3+189*a^8*b*c^5*d^2+252*a^7*b^2*c^6*d+84*a^6*b^3*c^7)*
x^4+1/3*(21*a^9*c^5*d^2+63*a^8*b*c^6*d+36*a^7*b^2*c^7)*x^3+1/2*(7*a^9*c^6*d
+9*a^8*b*c^7)*x^2+a^9*c^7*x
```

maxima [B] time = 1.43, size = 1023, normalized size = 5.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/17*b^9*d^7*x^17 + a^9*c^7*x + 1/16*(7*b^9*c*d^6 + 9*a*b^8*d^7)*x^16 + 1/5
*(7*b^9*c^2*d^5 + 21*a*b^8*c*d^6 + 12*a^2*b^7*d^7)*x^15 + 1/2*(5*b^9*c^3*d^
4 + 27*a*b^8*c^2*d^5 + 36*a^2*b^7*c*d^6 + 12*a^3*b^6*d^7)*x^14 + 7/13*(5*b^
9*c^4*d^3 + 45*a*b^8*c^3*d^4 + 108*a^2*b^7*c^2*d^5 + 84*a^3*b^6*c*d^6 + 18*
a^4*b^5*d^7)*x^13 + 7/4*(b^9*c^5*d^2 + 15*a*b^8*c^4*d^3 + 60*a^2*b^7*c^3*d^
4 + 84*a^3*b^6*c^2*d^5 + 42*a^4*b^5*c*d^6 + 6*a^5*b^4*d^7)*x^12 + 7/11*(b^9
*c^6*d + 27*a*b^8*c^5*d^2 + 180*a^2*b^7*c^4*d^3 + 420*a^3*b^6*c^3*d^4 + 378
*a^4*b^5*c^2*d^5 + 126*a^5*b^4*c*d^6 + 12*a^6*b^3*d^7)*x^11 + 1/10*(b^9*c^7
 + 63*a*b^8*c^6*d + 756*a^2*b^7*c^5*d^2 + 2940*a^3*b^6*c^4*d^3 + 4410*a^4*b
^5*c^3*d^4 + 2646*a^5*b^4*c^2*d^5 + 588*a^6*b^3*c*d^6 + 36*a^7*b^2*d^7)*x^1
0 + (a*b^8*c^7 + 28*a^2*b^7*c^6*d + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d
^3 + 490*a^5*b^4*c^3*d^4 + 196*a^6*b^3*c^2*d^5 + 28*a^7*b^2*c*d^6 + a^8*b*d
^7)*x^9 + 1/8*(36*a^2*b^7*c^7 + 588*a^3*b^6*c^6*d + 2646*a^4*b^5*c^5*d^2 +
4410*a^5*b^4*c^4*d^3 + 2940*a^6*b^3*c^3*d^4 + 756*a^7*b^2*c^2*d^5 + 63*a^8*
b*c*d^6 + a^9*d^7)*x^8 + (12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4 + 27*a^8*b*c^2*d^5 + a^
9*c*d^6)*x^7 + 7/2*(6*a^4*b^5*c^7 + 42*a^5*b^4*c^6*d + 84*a^6*b^3*c^5*d^2 +
 60*a^7*b^2*c^4*d^3 + 15*a^8*b*c^3*d^4 + a^9*c^2*d^5)*x^6 + 7/5*(18*a^5*b^4
*c^7 + 84*a^6*b^3*c^6*d + 108*a^7*b^2*c^5*d^2 + 45*a^8*b*c^4*d^3 + 5*a^9*c^
3*d^4)*x^5 + 7/4*(12*a^6*b^3*c^7 + 36*a^7*b^2*c^6*d + 27*a^8*b*c^5*d^2 + 5*
a^9*c^4*d^3)*x^4 + (12*a^7*b^2*c^7 + 21*a^8*b*c^6*d + 7*a^9*c^5*d^2)*x^3 +
1/2*(9*a^8*b*c^7 + 7*a^9*c^6*d)*x^2
```

mupad [B] time = 0.55, size = 997, normalized size = 4.98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^7,x)
```

```
[Out] x^5*((126*a^5*b^4*c^7)/5 + 7*a^9*c^3*d^4 + (588*a^6*b^3*c^6*d)/5 + 63*a^8*b
*c^4*d^3 + (756*a^7*b^2*c^5*d^2)/5) + x^13*((126*a^4*b^5*d^7)/13 + (35*b^9*
c^4*d^3)/13 + (315*a*b^8*c^3*d^4)/13 + (588*a^3*b^6*c*d^6)/13 + (756*a^2*b^
7*c^2*d^5)/13) + x^8*((a^9*d^7)/8 + (9*a^2*b^7*c^7)/2 + (147*a^3*b^6*c^6*d)
/2 + (1323*a^4*b^5*c^5*d^2)/4 + (2205*a^5*b^4*c^4*d^3)/4 + (735*a^6*b^3*c^3
*d^4)/2 + (189*a^7*b^2*c^2*d^5)/2 + (63*a^8*b*c*d^6)/8) + x^10*((b^9*c^7)/1
0 + (18*a^7*b^2*d^7)/5 + (294*a^6*b^3*c*d^6)/5 + (378*a^2*b^7*c^5*d^2)/5 +
294*a^3*b^6*c^4*d^3 + 441*a^4*b^5*c^3*d^4 + (1323*a^5*b^4*c^2*d^5)/5 + (63*
a*b^8*c^6*d)/10) + x^6*(21*a^4*b^5*c^7 + (7*a^9*c^2*d^5)/2 + 147*a^5*b^4*c^
6*d + (105*a^8*b*c^3*d^4)/2 + 294*a^6*b^3*c^5*d^2 + 210*a^7*b^2*c^4*d^3) +
x^12*((21*a^5*b^4*d^7)/2 + (7*b^9*c^5*d^2)/4 + (105*a*b^8*c^4*d^3)/4 + (147
*a^4*b^5*c*d^6)/2 + 105*a^2*b^7*c^3*d^4 + 147*a^3*b^6*c^2*d^5) + x^7*(a^9*c
*d^6 + 12*a^3*b^6*c^7 + 126*a^4*b^5*c^6*d + 27*a^8*b*c^2*d^5 + 378*a^5*b^4*
c^5*d^2 + 420*a^6*b^3*c^4*d^3 + 180*a^7*b^2*c^3*d^4) + x^11*((7*b^9*c^6*d)/
11 + (84*a^6*b^3*d^7)/11 + (189*a*b^8*c^5*d^2)/11 + (882*a^5*b^4*c*d^6)/11
+ (1260*a^2*b^7*c^4*d^3)/11 + (2940*a^3*b^6*c^3*d^4)/11 + (2646*a^4*b^5*c^2
*d^5)/11) + x^9*(a*b^8*c^7 + a^8*b*d^7 + 28*a^2*b^7*c^6*d + 28*a^7*b^2*c*d^
6 + 196*a^3*b^6*c^5*d^2 + 490*a^4*b^5*c^4*d^3 + 490*a^5*b^4*c^3*d^4 + 196*a
^6*b^3*c^2*d^5) + a^9*c^7*x + (b^9*d^7*x^17)/17 + (7*a^6*c^4*x^4*(5*a^3*d^3
 + 12*b^3*c^3 + 36*a*b^2*c^2*d + 27*a^2*b*c*d^2))/4 + (b^6*d^4*x^14*(12*a^3
*d^3 + 5*b^3*c^3 + 27*a*b^2*c^2*d + 36*a^2*b*c*d^2))/2 + (a^8*c^6*x^2*(7*a*
d + 9*b*c))/2 + (b^8*d^6*x^16*(9*a*d + 7*b*c))/16 + a^7*c^5*x^3*(7*a^2*d^2
+ 12*b^2*c^2 + 21*a*b*c*d) + (b^7*d^5*x^15*(12*a^2*d^2 + 7*b^2*c^2 + 21*a*b
*c*d))/5
```

sympy [B] time = 0.23, size = 1163, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**7,x)
```

```
[Out] a**9*c**7*x + b**9*d**7*x**17/17 + x**16*(9*a*b**8*d**7/16 + 7*b**9*c*d**6/
16) + x**15*(12*a**2*b**7*d**7/5 + 21*a*b**8*c*d**6/5 + 7*b**9*c**2*d**5/5)
 + x**14*(6*a**3*b**6*d**7 + 18*a**2*b**7*c*d**6 + 27*a*b**8*c**2*d**5/2 +
5*b**9*c**3*d**4/2) + x**13*(126*a**4*b**5*d**7/13 + 588*a**3*b**6*c*d**6/1
3 + 756*a**2*b**7*c**2*d**5/13 + 315*a*b**8*c**3*d**4/13 + 35*b**9*c**4*d**
3/13) + x**12*(21*a**5*b**4*d**7/2 + 147*a**4*b**5*c*d**6/2 + 147*a**3*b**6
*c**2*d**5 + 105*a**2*b**7*c**3*d**4 + 105*a*b**8*c**4*d**3/4 + 7*b**9*c**5
*d**2/4) + x**11*(84*a**6*b**3*d**7/11 + 882*a**5*b**4*c*d**6/11 + 2646*a**
4*b**5*c**2*d**5/11 + 2940*a**3*b**6*c**3*d**4/11 + 1260*a**2*b**7*c**4*d**
3/11 + 189*a*b**8*c**5*d**2/11 + 7*b**9*c**6*d/11) + x**10*(18*a**7*b**2*d*
*7/5 + 294*a**6*b**3*c*d**6/5 + 1323*a**5*b**4*c**2*d**5/5 + 441*a**4*b**5*
c**3*d**4 + 294*a**3*b**6*c**4*d**3 + 378*a**2*b**7*c**5*d**2/5 + 63*a*b**8
*c**6*d/10 + b**9*c**7/10) + x**9*(a**8*b*d**7 + 28*a**7*b**2*c*d**6 + 196*
a**6*b**3*c**2*d**5 + 490*a**5*b**4*c**3*d**4 + 490*a**4*b**5*c**4*d**3 + 1
96*a**3*b**6*c**5*d**2 + 28*a**2*b**7*c**6*d + a*b**8*c**7) + x**8*(a**9*d*
*7/8 + 63*a**8*b*c*d**6/8 + 189*a**7*b**2*c**2*d**5/2 + 735*a**6*b**3*c**3*
d**4/2 + 2205*a**5*b**4*c**4*d**3/4 + 1323*a**4*b**5*c**5*d**2/4 + 147*a**3
*b**6*c**6*d/2 + 9*a**2*b**7*c**7/2) + x**7*(a**9*c*d**6 + 27*a**8*b*c**2*d
**5 + 180*a**7*b**2*c**3*d**4 + 420*a**6*b**3*c**4*d**3 + 378*a**5*b**4*c**
5*d**2 + 126*a**4*b**5*c**6*d + 12*a**3*b**6*c**7) + x**6*(7*a**9*c**2*d**5
/2 + 105*a**8*b*c**3*d**4/2 + 210*a**7*b**2*c**4*d**3 + 294*a**6*b**3*c**5*
d**2 + 147*a**5*b**4*c**6*d + 21*a**4*b**5*c**7) + x**5*(7*a**9*c**3*d**4 +
 63*a**8*b*c**4*d**3 + 756*a**7*b**2*c**5*d**2/5 + 588*a**6*b**3*c**6*d/5 +
 126*a**5*b**4*c**7/5) + x**4*(35*a**9*c**4*d**3/4 + 189*a**8*b*c**5*d**2/4
 + 63*a**7*b**2*c**6*d + 21*a**6*b**3*c**7) + x**3*(7*a**9*c**5*d**2 + 21*a
**8*b*c**6*d + 12*a**7*b**2*c**7) + x**2*(7*a**9*c**6*d/2 + 9*a**8*b*c**7/2
)
```

3.1168 $\int (a+bx)^8(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a}{}$$

Rubi [A] time = 0.57, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{7d^6(a+bx)^{15}(bc-ad)}{15b^8}+\frac{3d^5(a+bx)^{14}(bc-ad)^2}{2b^8}+\frac{35d^4(a+bx)^{13}(bc-ad)^3}{13b^8}+\frac{35d^3(a+bx)^{12}(bc-ad)^4}{12b^8}+\frac{21d^2(a+bx)^{11}(bc-ad)^5}{11b^8}+\frac{7d(a+bx)^{10}(bc-ad)^6}{10b^8}+\frac{(a+bx)^9(bc-ad)^7}{9b^8}+\frac{d^7(a+bx)^{16}}{16b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^9)/(9*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^10)/(10*
b^8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^11)/(11*b^8) + (35*d^3*(b*c - a*d)^4
*(a + b*x)^12)/(12*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^13)/(13*b^8) + (3
*d^5*(b*c - a*d)^2*(a + b*x)^14)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^15)
/(15*b^8) + (d^7*(a + b*x)^16)/(16*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^8(c+dx)^7\,dx &= \int \left(\frac{(bc-ad)^7(a+bx)^8}{b^7}+\frac{7d(bc-ad)^6(a+bx)^9}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{b^7}+\frac{35d^3}{}\right.\\ &= \frac{(bc-ad)^7(a+bx)^9}{9b^8}+\frac{7d(bc-ad)^6(a+bx)^{10}}{10b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{11}}{11b^8}+\frac{35d^3(bc}{}\end{aligned}$$

Mathematica [B] time = 0.11, size = 897, normalized size = 4.48

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] a^8*c^7*x + (a^7*c^6*(8*b*c + 7*a*d)*x^2)/2 + (7*a^6*c^5*(4*b^2*c^2 + 8*a*b
*c*d + 3*a^2*d^2)*x^3)/3 + (7*a^5*c^4*(8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*
b*c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^4*c^3*(10*b^4*c^4 + 56*a*b^3*c^3*d + 84*
a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^3*c^2*(8*b^5*c^
5 + 70*a*b^4*c^4*d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c
*d^4 + 3*a^5*d^5)*x^6)/6 + a^2*c*(4*b^6*c^6 + 56*a*b^5*c^5*d + 210*a^2*b^4*
c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 140*a^4*b^2*c^2*d^4 + 24*a^5*b*c*d^5 + a^6*
d^6)*x^7 + (a*(8*b^7*c^7 + 196*a*b^6*c^6*d + 1176*a^2*b^5*c^5*d^2 + 2450*a^
3*b^4*c^4*d^3 + 1960*a^4*b^3*c^3*d^4 + 588*a^5*b^2*c^2*d^5 + 56*a^6*b*c*d^6
 + a^7*d^7)*x^8)/8 + (b*(b^7*c^7 + 56*a*b^6*c^6*d + 588*a^2*b^5*c^5*d^2 + 1
960*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 + 1176*a^5*b^2*c^2*d^5 + 196*a^6
*b*c*d^6 + 8*a^7*d^7)*x^9)/9 + (7*b^2*d*(b^6*c^6 + 24*a*b^5*c^5*d + 140*a^2
*b^4*c^4*d^2 + 280*a^3*b^3*c^3*d^3 + 210*a^4*b^2*c^2*d^4 + 56*a^5*b*c*d^5 +
```

```
 4*a^6*d^6)*x^10)/10 + (7*b^3*d^2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3
*c^3*d^2 + 168*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^11)/11 + (7*
b^4*d^3*(5*b^4*c^4 + 40*a*b^3*c^3*d + 84*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 +
 10*a^4*d^4)*x^12)/12 + (7*b^5*d^4*(5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a^2*b*c
*d^2 + 8*a^3*d^3)*x^13)/13 + (b^6*d^5*(3*b^2*c^2 + 8*a*b*c*d + 4*a^2*d^2)*x
^14)/2 + (b^7*d^6*(7*b*c + 8*a*d)*x^15)/15 + (b^8*d^7*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^8(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^7, x]
```

fricas [B] time = 1.28, size = 1050, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^7*b^8 + 7/15*x^15*d^6*c*b^8 + 8/15*x^15*d^7*b^7*a + 3/2*x^14*d^
5*c^2*b^8 + 4*x^14*d^6*c*b^7*a + 2*x^14*d^7*b^6*a^2 + 35/13*x^13*d^4*c^3*b^
8 + 168/13*x^13*d^5*c^2*b^7*a + 196/13*x^13*d^6*c*b^6*a^2 + 56/13*x^13*d^7*
b^5*a^3 + 35/12*x^12*d^3*c^4*b^8 + 70/3*x^12*d^4*c^3*b^7*a + 49*x^12*d^5*c^
2*b^6*a^2 + 98/3*x^12*d^6*c*b^5*a^3 + 35/6*x^12*d^7*b^4*a^4 + 21/11*x^11*d^
2*c^5*b^8 + 280/11*x^11*d^3*c^4*b^7*a + 980/11*x^11*d^4*c^3*b^6*a^2 + 1176/
11*x^11*d^5*c^2*b^5*a^3 + 490/11*x^11*d^6*c*b^4*a^4 + 56/11*x^11*d^7*b^3*a^
5 + 7/10*x^10*d*c^6*b^8 + 84/5*x^10*d^2*c^5*b^7*a + 98*x^10*d^3*c^4*b^6*a^2
 + 196*x^10*d^4*c^3*b^5*a^3 + 147*x^10*d^5*c^2*b^4*a^4 + 196/5*x^10*d^6*c*b
^3*a^5 + 14/5*x^10*d^7*b^2*a^6 + 1/9*x^9*c^7*b^8 + 56/9*x^9*d*c^6*b^7*a + 1
96/3*x^9*d^2*c^5*b^6*a^2 + 1960/9*x^9*d^3*c^4*b^5*a^3 + 2450/9*x^9*d^4*c^3*
b^4*a^4 + 392/3*x^9*d^5*c^2*b^3*a^5 + 196/9*x^9*d^6*c*b^2*a^6 + 8/9*x^9*d^7
*b*a^7 + x^8*c^7*b^7*a + 49/2*x^8*d*c^6*b^6*a^2 + 147*x^8*d^2*c^5*b^5*a^3 +
 1225/4*x^8*d^3*c^4*b^4*a^4 + 245*x^8*d^4*c^3*b^3*a^5 + 147/2*x^8*d^5*c^2*b
^2*a^6 + 7*x^8*d^6*c*b*a^7 + 1/8*x^8*d^7*a^8 + 4*x^7*c^7*b^6*a^2 + 56*x^7*d
*c^6*b^5*a^3 + 210*x^7*d^2*c^5*b^4*a^4 + 280*x^7*d^3*c^4*b^3*a^5 + 140*x^7*
d^4*c^3*b^2*a^6 + 24*x^7*d^5*c^2*b*a^7 + x^7*d^6*c*a^8 + 28/3*x^6*c^7*b^5*a
^3 + 245/3*x^6*d*c^6*b^4*a^4 + 196*x^6*d^2*c^5*b^3*a^5 + 490/3*x^6*d^3*c^4*
b^2*a^6 + 140/3*x^6*d^4*c^3*b*a^7 + 7/2*x^6*d^5*c^2*a^8 + 14*x^5*c^7*b^4*a^
4 + 392/5*x^5*d*c^6*b^3*a^5 + 588/5*x^5*d^2*c^5*b^2*a^6 + 56*x^5*d^3*c^4*b*
a^7 + 7*x^5*d^4*c^3*a^8 + 14*x^4*c^7*b^3*a^5 + 49*x^4*d*c^6*b^2*a^6 + 42*x^
4*d^2*c^5*b*a^7 + 35/4*x^4*d^3*c^4*a^8 + 28/3*x^3*c^7*b^2*a^6 + 56/3*x^3*d*
c^6*b*a^7 + 7*x^3*d^2*c^5*a^8 + 4*x^2*c^7*b*a^7 + 7/2*x^2*d*c^6*a^8 + x*c^7
*a^8
```

giac [B] time = 1.01, size = 1050, normalized size = 5.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/16*b^8*d^7*x^16 + 7/15*b^8*c*d^6*x^15 + 8/15*a*b^7*d^7*x^15 + 3/2*b^8*c^2
*d^5*x^14 + 4*a*b^7*c*d^6*x^14 + 2*a^2*b^6*d^7*x^14 + 35/13*b^8*c^3*d^4*x^1
3 + 168/13*a*b^7*c^2*d^5*x^13 + 196/13*a^2*b^6*c*d^6*x^13 + 56/13*a^3*b^5*d
```

```
^7*x^13 + 35/12*b^8*c^4*d^3*x^12 + 70/3*a*b^7*c^3*d^4*x^12 + 49*a^2*b^6*c^2
*d^5*x^12 + 98/3*a^3*b^5*c*d^6*x^12 + 35/6*a^4*b^4*d^7*x^12 + 21/11*b^8*c^5
*d^2*x^11 + 280/11*a*b^7*c^4*d^3*x^11 + 980/11*a^2*b^6*c^3*d^4*x^11 + 1176/
11*a^3*b^5*c^2*d^5*x^11 + 490/11*a^4*b^4*c*d^6*x^11 + 56/11*a^5*b^3*d^7*x^1
1 + 7/10*b^8*c^6*d*x^10 + 84/5*a*b^7*c^5*d^2*x^10 + 98*a^2*b^6*c^4*d^3*x^10
 + 196*a^3*b^5*c^3*d^4*x^10 + 147*a^4*b^4*c^2*d^5*x^10 + 196/5*a^5*b^3*c*d^
6*x^10 + 14/5*a^6*b^2*d^7*x^10 + 1/9*b^8*c^7*x^9 + 56/9*a*b^7*c^6*d*x^9 + 1
96/3*a^2*b^6*c^5*d^2*x^9 + 1960/9*a^3*b^5*c^4*d^3*x^9 + 2450/9*a^4*b^4*c^3*
d^4*x^9 + 392/3*a^5*b^3*c^2*d^5*x^9 + 196/9*a^6*b^2*c*d^6*x^9 + 8/9*a^7*b*d
^7*x^9 + a*b^7*c^7*x^8 + 49/2*a^2*b^6*c^6*d*x^8 + 147*a^3*b^5*c^5*d^2*x^8 +
 1225/4*a^4*b^4*c^4*d^3*x^8 + 245*a^5*b^3*c^3*d^4*x^8 + 147/2*a^6*b^2*c^2*d
^5*x^8 + 7*a^7*b*c*d^6*x^8 + 1/8*a^8*d^7*x^8 + 4*a^2*b^6*c^7*x^7 + 56*a^3*b
^5*c^6*d*x^7 + 210*a^4*b^4*c^5*d^2*x^7 + 280*a^5*b^3*c^4*d^3*x^7 + 140*a^6*
b^2*c^3*d^4*x^7 + 24*a^7*b*c^2*d^5*x^7 + a^8*c*d^6*x^7 + 28/3*a^3*b^5*c^7*x
^6 + 245/3*a^4*b^4*c^6*d*x^6 + 196*a^5*b^3*c^5*d^2*x^6 + 490/3*a^6*b^2*c^4*
d^3*x^6 + 140/3*a^7*b*c^3*d^4*x^6 + 7/2*a^8*c^2*d^5*x^6 + 14*a^4*b^4*c^7*x^
5 + 392/5*a^5*b^3*c^6*d*x^5 + 588/5*a^6*b^2*c^5*d^2*x^5 + 56*a^7*b*c^4*d^3*
x^5 + 7*a^8*c^3*d^4*x^5 + 14*a^5*b^3*c^7*x^4 + 49*a^6*b^2*c^6*d*x^4 + 42*a^
7*b*c^5*d^2*x^4 + 35/4*a^8*c^4*d^3*x^4 + 28/3*a^6*b^2*c^7*x^3 + 56/3*a^7*b*
c^6*d*x^3 + 7*a^8*c^5*d^2*x^3 + 4*a^7*b*c^7*x^2 + 7/2*a^8*c^6*d*x^2 + a^8*c
^7*x
```

maple [B] time = 0.00, size = 925, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^7,x)
```

```
[Out] 1/16*b^8*d^7*x^16+1/15*(8*a*b^7*d^7+7*b^8*c*d^6)*x^15+1/14*(28*a^2*b^6*d^7+
56*a*b^7*c*d^6+21*b^8*c^2*d^5)*x^14+1/13*(56*a^3*b^5*d^7+196*a^2*b^6*c*d^6+
168*a*b^7*c^2*d^5+35*b^8*c^3*d^4)*x^13+1/12*(70*a^4*b^4*d^7+392*a^3*b^5*c*d
^6+588*a^2*b^6*c^2*d^5+280*a*b^7*c^3*d^4+35*b^8*c^4*d^3)*x^12+1/11*(56*a^5*
b^3*d^7+490*a^4*b^4*c*d^6+1176*a^3*b^5*c^2*d^5+980*a^2*b^6*c^3*d^4+280*a*b^
7*c^4*d^3+21*b^8*c^5*d^2)*x^11+1/10*(28*a^6*b^2*d^7+392*a^5*b^3*c*d^6+1470*
a^4*b^4*c^2*d^5+1960*a^3*b^5*c^3*d^4+980*a^2*b^6*c^4*d^3+168*a*b^7*c^5*d^2+
7*b^8*c^6*d)*x^10+1/9*(8*a^7*b*d^7+196*a^6*b^2*c*d^6+1176*a^5*b^3*c^2*d^5+2
450*a^4*b^4*c^3*d^4+1960*a^3*b^5*c^4*d^3+588*a^2*b^6*c^5*d^2+56*a*b^7*c^6*d
+b^8*c^7)*x^9+1/8*(a^8*d^7+56*a^7*b*c*d^6+588*a^6*b^2*c^2*d^5+1960*a^5*b^3*
c^3*d^4+2450*a^4*b^4*c^4*d^3+1176*a^3*b^5*c^5*d^2+196*a^2*b^6*c^6*d+8*a*b^7
*c^7)*x^8+1/7*(7*a^8*c*d^6+168*a^7*b*c^2*d^5+980*a^6*b^2*c^3*d^4+1960*a^5*b
^3*c^4*d^3+1470*a^4*b^4*c^5*d^2+392*a^3*b^5*c^6*d+28*a^2*b^6*c^7)*x^7+1/6*(
21*a^8*c^2*d^5+280*a^7*b*c^3*d^4+980*a^6*b^2*c^4*d^3+1176*a^5*b^3*c^5*d^2+4
90*a^4*b^4*c^6*d+56*a^3*b^5*c^7)*x^6+1/5*(35*a^8*c^3*d^4+280*a^7*b*c^4*d^3+
588*a^6*b^2*c^5*d^2+392*a^5*b^3*c^6*d+70*a^4*b^4*c^7)*x^5+1/4*(35*a^8*c^4*d
^3+168*a^7*b*c^5*d^2+196*a^6*b^2*c^6*d+56*a^5*b^3*c^7)*x^4+1/3*(21*a^8*c^5*
d^2+56*a^7*b*c^6*d+28*a^6*b^2*c^7)*x^3+1/2*(7*a^8*c^6*d+8*a^7*b*c^7)*x^2+a^
8*c^7*x
```

maxima [B] time = 1.40, size = 921, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/16*b^8*d^7*x^16 + a^8*c^7*x + 1/15*(7*b^8*c*d^6 + 8*a*b^7*d^7)*x^15 + 1/2
*(3*b^8*c^2*d^5 + 8*a*b^7*c*d^6 + 4*a^2*b^6*d^7)*x^14 + 7/13*(5*b^8*c^3*d^4
 + 24*a*b^7*c^2*d^5 + 28*a^2*b^6*c*d^6 + 8*a^3*b^5*d^7)*x^13 + 7/12*(5*b^8*
```

```
c^4*d^3 + 40*a*b^7*c^3*d^4 + 84*a^2*b^6*c^2*d^5 + 56*a^3*b^5*c*d^6 + 10*a^4
*b^4*d^7)*x^12 + 7/11*(3*b^8*c^5*d^2 + 40*a*b^7*c^4*d^3 + 140*a^2*b^6*c^3*d
^4 + 168*a^3*b^5*c^2*d^5 + 70*a^4*b^4*c*d^6 + 8*a^5*b^3*d^7)*x^11 + 7/10*(b
^8*c^6*d + 24*a*b^7*c^5*d^2 + 140*a^2*b^6*c^4*d^3 + 280*a^3*b^5*c^3*d^4 + 2
10*a^4*b^4*c^2*d^5 + 56*a^5*b^3*c*d^6 + 4*a^6*b^2*d^7)*x^10 + 1/9*(b^8*c^7
+ 56*a*b^7*c^6*d + 588*a^2*b^6*c^5*d^2 + 1960*a^3*b^5*c^4*d^3 + 2450*a^4*b^
4*c^3*d^4 + 1176*a^5*b^3*c^2*d^5 + 196*a^6*b^2*c*d^6 + 8*a^7*b*d^7)*x^9 + 1
/8*(8*a*b^7*c^7 + 196*a^2*b^6*c^6*d + 1176*a^3*b^5*c^5*d^2 + 2450*a^4*b^4*c
^4*d^3 + 1960*a^5*b^3*c^3*d^4 + 588*a^6*b^2*c^2*d^5 + 56*a^7*b*c*d^6 + a^8*
d^7)*x^8 + (4*a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 210*a^4*b^4*c^5*d^2 + 280*a^
5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4 + 24*a^7*b*c^2*d^5 + a^8*c*d^6)*x^7 + 7
/6*(8*a^3*b^5*c^7 + 70*a^4*b^4*c^6*d + 168*a^5*b^3*c^5*d^2 + 140*a^6*b^2*c^
4*d^3 + 40*a^7*b*c^3*d^4 + 3*a^8*c^2*d^5)*x^6 + 7/5*(10*a^4*b^4*c^7 + 56*a^
5*b^3*c^6*d + 84*a^6*b^2*c^5*d^2 + 40*a^7*b*c^4*d^3 + 5*a^8*c^3*d^4)*x^5 +
7/4*(8*a^5*b^3*c^7 + 28*a^6*b^2*c^6*d + 24*a^7*b*c^5*d^2 + 5*a^8*c^4*d^3)*x
^4 + 7/3*(4*a^6*b^2*c^7 + 8*a^7*b*c^6*d + 3*a^8*c^5*d^2)*x^3 + 1/2*(8*a^7*b
*c^7 + 7*a^8*c^6*d)*x^2
```

mupad [B] time = 0.36, size = 892, normalized size = 4.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^7,x)
```

```
[Out] x^8*((a^8*d^7)/8 + a*b^7*c^7 + (49*a^2*b^6*c^6*d)/2 + 147*a^3*b^5*c^5*d^2 +
 (1225*a^4*b^4*c^4*d^3)/4 + 245*a^5*b^3*c^3*d^4 + (147*a^6*b^2*c^2*d^5)/2 +
 7*a^7*b*c*d^6) + x^9*((b^8*c^7)/9 + (8*a^7*b*d^7)/9 + (196*a^6*b^2*c*d^6)/
9 + (196*a^2*b^6*c^5*d^2)/3 + (1960*a^3*b^5*c^4*d^3)/9 + (2450*a^4*b^4*c^3*
d^4)/9 + (392*a^5*b^3*c^2*d^5)/3 + (56*a*b^7*c^6*d)/9) + x^5*(14*a^4*b^4*c^
7 + 7*a^8*c^3*d^4 + (392*a^5*b^3*c^6*d)/5 + 56*a^7*b*c^4*d^3 + (588*a^6*b^2
*c^5*d^2)/5) + x^12*((35*a^4*b^4*d^7)/6 + (35*b^8*c^4*d^3)/12 + (70*a*b^7*c
^3*d^4)/3 + (98*a^3*b^5*c*d^6)/3 + 49*a^2*b^6*c^2*d^5) + x^6*((28*a^3*b^5*c
^7)/3 + (7*a^8*c^2*d^5)/2 + (245*a^4*b^4*c^6*d)/3 + (140*a^7*b*c^3*d^4)/3 +
 196*a^5*b^3*c^5*d^2 + (490*a^6*b^2*c^4*d^3)/3) + x^11*((56*a^5*b^3*d^7)/11
 + (21*b^8*c^5*d^2)/11 + (280*a*b^7*c^4*d^3)/11 + (490*a^4*b^4*c*d^6)/11 +
(980*a^2*b^6*c^3*d^4)/11 + (1176*a^3*b^5*c^2*d^5)/11) + x^7*(a^8*c*d^6 + 4*
a^2*b^6*c^7 + 56*a^3*b^5*c^6*d + 24*a^7*b*c^2*d^5 + 210*a^4*b^4*c^5*d^2 + 2
80*a^5*b^3*c^4*d^3 + 140*a^6*b^2*c^3*d^4) + x^10*((7*b^8*c^6*d)/10 + (14*a^
6*b^2*d^7)/5 + (84*a*b^7*c^5*d^2)/5 + (196*a^5*b^3*c*d^6)/5 + 98*a^2*b^6*c^
4*d^3 + 196*a^3*b^5*c^3*d^4 + 147*a^4*b^4*c^2*d^5) + a^8*c^7*x + (b^8*d^7*x
^16)/16 + (7*a^5*c^4*x^4*(5*a^3*d^3 + 8*b^3*c^3 + 28*a*b^2*c^2*d + 24*a^2*b
*c*d^2))/4 + (7*b^5*d^4*x^13*(8*a^3*d^3 + 5*b^3*c^3 + 24*a*b^2*c^2*d + 28*a
^2*b*c*d^2))/13 + (a^7*c^6*x^2*(7*a*d + 8*b*c))/2 + (b^7*d^6*x^15*(8*a*d +
7*b*c))/15 + (7*a^6*c^5*x^3*(3*a^2*d^2 + 4*b^2*c^2 + 8*a*b*c*d))/3 + (b^6*d
^5*x^14*(4*a^2*d^2 + 3*b^2*c^2 + 8*a*b*c*d))/2
```

sympy [B] time = 0.21, size = 1046, normalized size = 5.23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**7,x)
```

```
[Out] a**8*c**7*x + b**8*d**7*x**16/16 + x**15*(8*a*b**7*d**7/15 + 7*b**8*c*d**6/
15) + x**14*(2*a**2*b**6*d**7 + 4*a*b**7*c*d**6 + 3*b**8*c**2*d**5/2) + x**
13*(56*a**3*b**5*d**7/13 + 196*a**2*b**6*c*d**6/13 + 168*a*b**7*c**2*d**5/1
3 + 35*b**8*c**3*d**4/13) + x**12*(35*a**4*b**4*d**7/6 + 98*a**3*b**5*c*d**
6/3 + 49*a**2*b**6*c**2*d**5 + 70*a*b**7*c**3*d**4/3 + 35*b**8*c**4*d**3/12
```

```
) + x**11*(56*a**5*b**3*d**7/11 + 490*a**4*b**4*c*d**6/11 + 1176*a**3*b**5*
c**2*d**5/11 + 980*a**2*b**6*c**3*d**4/11 + 280*a*b**7*c**4*d**3/11 + 21*b*
*8*c**5*d**2/11) + x**10*(14*a**6*b**2*d**7/5 + 196*a**5*b**3*c*d**6/5 + 14
7*a**4*b**4*c**2*d**5 + 196*a**3*b**5*c**3*d**4 + 98*a**2*b**6*c**4*d**3 +
84*a*b**7*c**5*d**2/5 + 7*b**8*c**6*d/10) + x**9*(8*a**7*b*d**7/9 + 196*a**
6*b**2*c*d**6/9 + 392*a**5*b**3*c**2*d**5/3 + 2450*a**4*b**4*c**3*d**4/9 +
1960*a**3*b**5*c**4*d**3/9 + 196*a**2*b**6*c**5*d**2/3 + 56*a*b**7*c**6*d/9
 + b**8*c**7/9) + x**8*(a**8*d**7/8 + 7*a**7*b*c*d**6 + 147*a**6*b**2*c**2*
d**5/2 + 245*a**5*b**3*c**3*d**4 + 1225*a**4*b**4*c**4*d**3/4 + 147*a**3*b*
*5*c**5*d**2 + 49*a**2*b**6*c**6*d/2 + a*b**7*c**7) + x**7*(a**8*c*d**6 + 2
4*a**7*b*c**2*d**5 + 140*a**6*b**2*c**3*d**4 + 280*a**5*b**3*c**4*d**3 + 21
0*a**4*b**4*c**5*d**2 + 56*a**3*b**5*c**6*d + 4*a**2*b**6*c**7) + x**6*(7*a
**8*c**2*d**5/2 + 140*a**7*b*c**3*d**4/3 + 490*a**6*b**2*c**4*d**3/3 + 196*
a**5*b**3*c**5*d**2 + 245*a**4*b**4*c**6*d/3 + 28*a**3*b**5*c**7/3) + x**5*
(7*a**8*c**3*d**4 + 56*a**7*b*c**4*d**3 + 588*a**6*b**2*c**5*d**2/5 + 392*a
**5*b**3*c**6*d/5 + 14*a**4*b**4*c**7) + x**4*(35*a**8*c**4*d**3/4 + 42*a**
7*b*c**5*d**2 + 49*a**6*b**2*c**6*d + 14*a**5*b**3*c**7) + x**3*(7*a**8*c**
5*d**2 + 56*a**7*b*c**6*d/3 + 28*a**6*b**2*c**7/3) + x**2*(7*a**8*c**6*d/2
+ 4*a**7*b*c**7)
```

3.1169 $\int (a+bx)^7(c+dx)^7\,dx$

Optimal. Leaf size=200

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+$$

Rubi [A] time = 0.45, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{d^6(a+bx)^{14}(bc-ad)}{2b^8}+\frac{21d^5(a+bx)^{13}(bc-ad)^2}{13b^8}+\frac{35d^4(a+bx)^{12}(bc-ad)^3}{12b^8}+\frac{35d^3(a+bx)^{11}(bc-ad)^4}{11b^8}+\frac{21d^2(a+bx)^{10}(bc-ad)^5}{10b^8}+\frac{7d(a+bx)^9(bc-ad)^6}{9b^8}+\frac{(a+bx)^8(bc-ad)^7}{8b^8}+\frac{d^7(a+bx)^{15}}{15b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^7*(a + b*x)^8)/(8*b^8) + (7*d*(b*c - a*d)^6*(a + b*x)^9)/(9*b^
8) + (21*d^2*(b*c - a*d)^5*(a + b*x)^10)/(10*b^8) + (35*d^3*(b*c - a*d)^4*(
a + b*x)^11)/(11*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^12)/(12*b^8) + (21*
d^5*(b*c - a*d)^2*(a + b*x)^13)/(13*b^8) + (d^6*(b*c - a*d)*(a + b*x)^14)/(
2*b^8) + (d^7*(a + b*x)^15)/(15*b^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^7\,dx = \int \left(\frac{(bc-ad)^7(a+bx)^7}{b^7}+\frac{7d(bc-ad)^6(a+bx)^8}{b^7}+\frac{21d^2(bc-ad)^5(a+bx)^9}{b^7}+\frac{35d^3(bc$$

$$= \frac{(bc-ad)^7(a+bx)^8}{8b^8}+\frac{7d(bc-ad)^6(a+bx)^9}{9b^8}+\frac{21d^2(bc-ad)^5(a+bx)^{10}}{10b^8}+\frac{35d^3(bc-a$$

Mathematica [B] time = 0.09, size = 785, normalized size = 3.92

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^7,x]
```

```
[Out] a^7*c^7*x + (7*a^6*c^6*(b*c + a*d)*x^2)/2 + (7*a^5*c^5*(3*b^2*c^2 + 7*a*b*c
*d + 3*a^2*d^2)*x^3)/3 + (7*a^4*c^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*
c*d^2 + 5*a^3*d^3)*x^4)/4 + (7*a^3*c^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2
*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^5)/5 + (7*a^2*c^2*(3*b^5*c^5 +
 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^
4 + 3*a^5*d^5)*x^6)/6 + a*c*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2
 + 175*a^3*b^3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^
7 + ((b^7*c^7 + 49*a*b^6*c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3
 + 1225*a^4*b^3*c^3*d^4 + 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x
^8)/8 + (7*b*d*(b^6*c^6 + 21*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 + 175*a^3*b^
3*c^3*d^3 + 105*a^4*b^2*c^2*d^4 + 21*a^5*b*c*d^5 + a^6*d^6)*x^9)/9 + (7*b^2
*d^2*(3*b^5*c^5 + 35*a*b^4*c^4*d + 105*a^2*b^3*c^3*d^2 + 105*a^3*b^2*c^2*d^
3 + 35*a^4*b*c*d^4 + 3*a^5*d^5)*x^10)/10 + (7*b^3*d^3*(5*b^4*c^4 + 35*a*b^3
```

```
*c^3*d + 63*a^2*b^2*c^2*d^2 + 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^11)/11 + (7*b^4
*d^4*(5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2 + 5*a^3*d^3)*x^12)/12 + (
7*b^5*d^5*(3*b^2*c^2 + 7*a*b*c*d + 3*a^2*d^2)*x^13)/13 + (b^6*d^6*(b*c + a*
d)*x^14)/2 + (b^7*d^7*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^7(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^7,x]
[Out] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^7, x]
```

fricas [B] time = 1.25, size = 924, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="fricas")
[Out] 1/15*x^15*d^7*b^7 + 1/2*x^14*d^6*c*b^7 + 1/2*x^14*d^7*b^6*a + 21/13*x^13*d^
5*c^2*b^7 + 49/13*x^13*d^6*c*b^6*a + 21/13*x^13*d^7*b^5*a^2 + 35/12*x^12*d^
4*c^3*b^7 + 49/4*x^12*d^5*c^2*b^6*a + 49/4*x^12*d^6*c*b^5*a^2 + 35/12*x^12*
d^7*b^4*a^3 + 35/11*x^11*d^3*c^4*b^7 + 245/11*x^11*d^4*c^3*b^6*a + 441/11*x
^11*d^5*c^2*b^5*a^2 + 245/11*x^11*d^6*c*b^4*a^3 + 35/11*x^11*d^7*b^3*a^4 +
21/10*x^10*d^2*c^5*b^7 + 49/2*x^10*d^3*c^4*b^6*a + 147/2*x^10*d^4*c^3*b^5*a
^2 + 147/2*x^10*d^5*c^2*b^4*a^3 + 49/2*x^10*d^6*c*b^3*a^4 + 21/10*x^10*d^7*
b^2*a^5 + 7/9*x^9*d*c^6*b^7 + 49/3*x^9*d^2*c^5*b^6*a + 245/3*x^9*d^3*c^4*b^
5*a^2 + 1225/9*x^9*d^4*c^3*b^4*a^3 + 245/3*x^9*d^5*c^2*b^3*a^4 + 49/3*x^9*d
^6*c*b^2*a^5 + 7/9*x^9*d^7*b*a^6 + 1/8*x^8*c^7*b^7 + 49/8*x^8*d*c^6*b^6*a +
 441/8*x^8*d^2*c^5*b^5*a^2 + 1225/8*x^8*d^3*c^4*b^4*a^3 + 1225/8*x^8*d^4*c^
3*b^3*a^4 + 441/8*x^8*d^5*c^2*b^2*a^5 + 49/8*x^8*d^6*c*b*a^6 + 1/8*x^8*d^7*
a^7 + x^7*c^7*b^6*a + 21*x^7*d*c^6*b^5*a^2 + 105*x^7*d^2*c^5*b^4*a^3 + 175*
x^7*d^3*c^4*b^3*a^4 + 105*x^7*d^4*c^3*b^2*a^5 + 21*x^7*d^5*c^2*b*a^6 + x^7*
d^6*c*a^7 + 7/2*x^6*c^7*b^5*a^2 + 245/6*x^6*d*c^6*b^4*a^3 + 245/2*x^6*d^2*c
^5*b^3*a^4 + 245/2*x^6*d^3*c^4*b^2*a^5 + 245/6*x^6*d^4*c^3*b*a^6 + 7/2*x^6*
d^5*c^2*a^7 + 7*x^5*c^7*b^4*a^3 + 49*x^5*d*c^6*b^3*a^4 + 441/5*x^5*d^2*c^5*
b^2*a^5 + 49*x^5*d^3*c^4*b*a^6 + 7*x^5*d^4*c^3*a^7 + 35/4*x^4*c^7*b^3*a^4 +
 147/4*x^4*d*c^6*b^2*a^5 + 147/4*x^4*d^2*c^5*b*a^6 + 35/4*x^4*d^3*c^4*a^7 +
 7*x^3*c^7*b^2*a^5 + 49/3*x^3*d*c^6*b*a^6 + 7*x^3*d^2*c^5*a^7 + 7/2*x^2*c^7
*b*a^6 + 7/2*x^2*d*c^6*a^7 + x*c^7*a^7
```

giac [B] time = 1.01, size = 924, normalized size = 4.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="giac")
[Out] 1/15*b^7*d^7*x^15 + 1/2*b^7*c*d^6*x^14 + 1/2*a*b^6*d^7*x^14 + 21/13*b^7*c^2
*d^5*x^13 + 49/13*a*b^6*c*d^6*x^13 + 21/13*a^2*b^5*d^7*x^13 + 35/12*b^7*c^3
*d^4*x^12 + 49/4*a*b^6*c^2*d^5*x^12 + 49/4*a^2*b^5*c*d^6*x^12 + 35/12*a^3*b
^4*d^7*x^12 + 35/11*b^7*c^4*d^3*x^11 + 245/11*a*b^6*c^3*d^4*x^11 + 441/11*a
^2*b^5*c^2*d^5*x^11 + 245/11*a^3*b^4*c*d^6*x^11 + 35/11*a^4*b^3*d^7*x^11 +
21/10*b^7*c^5*d^2*x^10 + 49/2*a*b^6*c^4*d^3*x^10 + 147/2*a^2*b^5*c^3*d^4*x^
10 + 147/2*a^3*b^4*c^2*d^5*x^10 + 49/2*a^4*b^3*c*d^6*x^10 + 21/10*a^5*b^2*d
^7*x^10 + 7/9*b^7*c^6*d*x^9 + 49/3*a*b^6*c^5*d^2*x^9 + 245/3*a^2*b^5*c^4*d^
```

```
3*x^9 + 1225/9*a^3*b^4*c^3*d^4*x^9 + 245/3*a^4*b^3*c^2*d^5*x^9 + 49/3*a^5*b
^2*c*d^6*x^9 + 7/9*a^6*b*d^7*x^9 + 1/8*b^7*c^7*x^8 + 49/8*a*b^6*c^6*d*x^8 +
 441/8*a^2*b^5*c^5*d^2*x^8 + 1225/8*a^3*b^4*c^4*d^3*x^8 + 1225/8*a^4*b^3*c^
3*d^4*x^8 + 441/8*a^5*b^2*c^2*d^5*x^8 + 49/8*a^6*b*c*d^6*x^8 + 1/8*a^7*d^7*
x^8 + a*b^6*c^7*x^7 + 21*a^2*b^5*c^6*d*x^7 + 105*a^3*b^4*c^5*d^2*x^7 + 175*
a^4*b^3*c^4*d^3*x^7 + 105*a^5*b^2*c^3*d^4*x^7 + 21*a^6*b*c^2*d^5*x^7 + a^7*
c*d^6*x^7 + 7/2*a^2*b^5*c^7*x^6 + 245/6*a^3*b^4*c^6*d*x^6 + 245/2*a^4*b^3*c
^5*d^2*x^6 + 245/2*a^5*b^2*c^4*d^3*x^6 + 245/6*a^6*b*c^3*d^4*x^6 + 7/2*a^7*
c^2*d^5*x^6 + 7*a^3*b^4*c^7*x^5 + 49*a^4*b^3*c^6*d*x^5 + 441/5*a^5*b^2*c^5*
d^2*x^5 + 49*a^6*b*c^4*d^3*x^5 + 7*a^7*c^3*d^4*x^5 + 35/4*a^4*b^3*c^7*x^4 +
 147/4*a^5*b^2*c^6*d*x^4 + 147/4*a^6*b*c^5*d^2*x^4 + 35/4*a^7*c^4*d^3*x^4 +
 7*a^5*b^2*c^7*x^3 + 49/3*a^6*b*c^6*d*x^3 + 7*a^7*c^5*d^2*x^3 + 7/2*a^6*b*c
^7*x^2 + 7/2*a^7*c^6*d*x^2 + a^7*c^7*x
```

maple [B] time = 0.00, size = 817, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^7,x)
```

```
[Out] 1/15*b^7*d^7*x^15+1/14*(7*a*b^6*d^7+7*b^7*c*d^6)*x^14+1/13*(21*a^2*b^5*d^7+
49*a*b^6*c*d^6+21*b^7*c^2*d^5)*x^13+1/12*(35*a^3*b^4*d^7+147*a^2*b^5*c*d^6+
147*a*b^6*c^2*d^5+35*b^7*c^3*d^4)*x^12+1/11*(35*a^4*b^3*d^7+245*a^3*b^4*c*d
^6+441*a^2*b^5*c^2*d^5+245*a*b^6*c^3*d^4+35*b^7*c^4*d^3)*x^11+1/10*(21*a^5*
b^2*d^7+245*a^4*b^3*c*d^6+735*a^3*b^4*c^2*d^5+735*a^2*b^5*c^3*d^4+245*a*b^6
*c^4*d^3+21*b^7*c^5*d^2)*x^10+1/9*(7*a^6*b*d^7+147*a^5*b^2*c*d^6+735*a^4*b^
3*c^2*d^5+1225*a^3*b^4*c^3*d^4+735*a^2*b^5*c^4*d^3+147*a*b^6*c^5*d^2+7*b^7*
c^6*d)*x^9+1/8*(a^7*d^7+49*a^6*b*c*d^6+441*a^5*b^2*c^2*d^5+1225*a^4*b^3*c^3
*d^4+1225*a^3*b^4*c^4*d^3+441*a^2*b^5*c^5*d^2+49*a*b^6*c^6*d+b^7*c^7)*x^8+1
/7*(7*a^7*c*d^6+147*a^6*b*c^2*d^5+735*a^5*b^2*c^3*d^4+1225*a^4*b^3*c^4*d^3+
735*a^3*b^4*c^5*d^2+147*a^2*b^5*c^6*d+7*a*b^6*c^7)*x^7+1/6*(21*a^7*c^2*d^5+
245*a^6*b*c^3*d^4+735*a^5*b^2*c^4*d^3+735*a^4*b^3*c^5*d^2+245*a^3*b^4*c^6*d
+21*a^2*b^5*c^7)*x^6+1/5*(35*a^7*c^3*d^4+245*a^6*b*c^4*d^3+441*a^5*b^2*c^5*
d^2+245*a^4*b^3*c^6*d+35*a^3*b^4*c^7)*x^5+1/4*(35*a^7*c^4*d^3+147*a^6*b*c^5
*d^2+147*a^5*b^2*c^6*d+35*a^4*b^3*c^7)*x^4+1/3*(21*a^7*c^5*d^2+49*a^6*b*c^6
*d+21*a^5*b^2*c^7)*x^3+1/2*(7*a^7*c^6*d+7*a^6*b*c^7)*x^2+a^7*c^7*x
```

maxima [B] time = 1.33, size = 807, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/15*b^7*d^7*x^15 + a^7*c^7*x + 1/2*(b^7*c*d^6 + a*b^6*d^7)*x^14 + 7/13*(3*
b^7*c^2*d^5 + 7*a*b^6*c*d^6 + 3*a^2*b^5*d^7)*x^13 + 7/12*(5*b^7*c^3*d^4 + 2
1*a*b^6*c^2*d^5 + 21*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^12 + 7/11*(5*b^7*c^4*
d^3 + 35*a*b^6*c^3*d^4 + 63*a^2*b^5*c^2*d^5 + 35*a^3*b^4*c*d^6 + 5*a^4*b^3*
d^7)*x^11 + 7/10*(3*b^7*c^5*d^2 + 35*a*b^6*c^4*d^3 + 105*a^2*b^5*c^3*d^4 +
105*a^3*b^4*c^2*d^5 + 35*a^4*b^3*c*d^6 + 3*a^5*b^2*d^7)*x^10 + 7/9*(b^7*c^6
*d + 21*a*b^6*c^5*d^2 + 105*a^2*b^5*c^4*d^3 + 175*a^3*b^4*c^3*d^4 + 105*a^4
*b^3*c^2*d^5 + 21*a^5*b^2*c*d^6 + a^6*b*d^7)*x^9 + 1/8*(b^7*c^7 + 49*a*b^6*
c^6*d + 441*a^2*b^5*c^5*d^2 + 1225*a^3*b^4*c^4*d^3 + 1225*a^4*b^3*c^3*d^4 +
 441*a^5*b^2*c^2*d^5 + 49*a^6*b*c*d^6 + a^7*d^7)*x^8 + (a*b^6*c^7 + 21*a^2*
b^5*c^6*d + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 105*a^5*b^2*c^3*d^4
 + 21*a^6*b*c^2*d^5 + a^7*c*d^6)*x^7 + 7/6*(3*a^2*b^5*c^7 + 35*a^3*b^4*c^6*
d + 105*a^4*b^3*c^5*d^2 + 105*a^5*b^2*c^4*d^3 + 35*a^6*b*c^3*d^4 + 3*a^7*c^
2*d^5)*x^6 + 7/5*(5*a^3*b^4*c^7 + 35*a^4*b^3*c^6*d + 63*a^5*b^2*c^5*d^2 + 3
```

```
5*a^6*b*c^4*d^3 + 5*a^7*c^3*d^4)*x^5 + 7/4*(5*a^4*b^3*c^7 + 21*a^5*b^2*c^6*
d + 21*a^6*b*c^5*d^2 + 5*a^7*c^4*d^3)*x^4 + 7/3*(3*a^5*b^2*c^7 + 7*a^6*b*c^
6*d + 3*a^7*c^5*d^2)*x^3 + 7/2*(a^6*b*c^7 + a^7*c^6*d)*x^2
```

mupad [**B**] time = 0.40, size = 781, normalized size = 3.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^7,x)
```

```
[Out] x^8*((a^7*d^7)/8 + (b^7*c^7)/8 + (441*a^2*b^5*c^5*d^2)/8 + (1225*a^3*b^4*c^
4*d^3)/8 + (1225*a^4*b^3*c^3*d^4)/8 + (441*a^5*b^2*c^2*d^5)/8 + (49*a*b^6*c
^6*d)/8 + (49*a^6*b*c*d^6)/8) + x^5*(7*a^3*b^4*c^7 + 7*a^7*c^3*d^4 + 49*a^4
*b^3*c^6*d + 49*a^6*b*c^4*d^3 + (441*a^5*b^2*c^5*d^2)/5) + x^11*((35*a^4*b^
3*d^7)/11 + (35*b^7*c^4*d^3)/11 + (245*a*b^6*c^3*d^4)/11 + (245*a^3*b^4*c*d
^6)/11 + (441*a^2*b^5*c^2*d^5)/11) + x^7*(a*b^6*c^7 + a^7*c*d^6 + 21*a^2*b^
5*c^6*d + 21*a^6*b*c^2*d^5 + 105*a^3*b^4*c^5*d^2 + 175*a^4*b^3*c^4*d^3 + 10
5*a^5*b^2*c^3*d^4) + x^9*((7*a^6*b*d^7)/9 + (7*b^7*c^6*d)/9 + (49*a*b^6*c^5
*d^2)/3 + (49*a^5*b^2*c*d^6)/3 + (245*a^2*b^5*c^4*d^3)/3 + (1225*a^3*b^4*c^
3*d^4)/9 + (245*a^4*b^3*c^2*d^5)/3) + x^6*((7*a^2*b^5*c^7)/2 + (7*a^7*c^2*d
^5)/2 + (245*a^3*b^4*c^6*d)/6 + (245*a^6*b*c^3*d^4)/6 + (245*a^4*b^3*c^5*d^
2)/2 + (245*a^5*b^2*c^4*d^3)/2) + x^10*((21*a^5*b^2*d^7)/10 + (21*b^7*c^5*d
^2)/10 + (49*a*b^6*c^4*d^3)/2 + (49*a^4*b^3*c*d^6)/2 + (147*a^2*b^5*c^3*d^4
)/2 + (147*a^3*b^4*c^2*d^5)/2) + a^7*c^7*x + (b^7*d^7*x^15)/15 + (7*a^4*c^4
*x^4*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/4 + (7*b^4*
d^4*x^12*(5*a^3*d^3 + 5*b^3*c^3 + 21*a*b^2*c^2*d + 21*a^2*b*c*d^2))/12 + (7
*a^6*c^6*x^2*(a*d + b*c))/2 + (b^6*d^6*x^14*(a*d + b*c))/2 + (7*a^5*c^5*x^3
*(3*a^2*d^2 + 3*b^2*c^2 + 7*a*b*c*d))/3 + (7*b^5*d^5*x^13*(3*a^2*d^2 + 3*b^
2*c^2 + 7*a*b*c*d))/13
```

sympy [**B**] time = 0.19, size = 935, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**7,x)
```

```
[Out] a**7*c**7*x + b**7*d**7*x**15/15 + x**14*(a*b**6*d**7/2 + b**7*c*d**6/2) +
x**13*(21*a**2*b**5*d**7/13 + 49*a*b**6*c*d**6/13 + 21*b**7*c**2*d**5/13) +
 x**12*(35*a**3*b**4*d**7/12 + 49*a**2*b**5*c*d**6/4 + 49*a*b**6*c**2*d**5/
4 + 35*b**7*c**3*d**4/12) + x**11*(35*a**4*b**3*d**7/11 + 245*a**3*b**4*c*d
**6/11 + 441*a**2*b**5*c**2*d**5/11 + 245*a*b**6*c**3*d**4/11 + 35*b**7*c**
4*d**3/11) + x**10*(21*a**5*b**2*d**7/10 + 49*a**4*b**3*c*d**6/2 + 147*a**3
*b**4*c**2*d**5/2 + 147*a**2*b**5*c**3*d**4/2 + 49*a*b**6*c**4*d**3/2 + 21*
b**7*c**5*d**2/10) + x**9*(7*a**6*b*d**7/9 + 49*a**5*b**2*c*d**6/3 + 245*a*
*4*b**3*c**2*d**5/3 + 1225*a**3*b**4*c**3*d**4/9 + 245*a**2*b**5*c**4*d**3/
3 + 49*a*b**6*c**5*d**2/3 + 7*b**7*c**6*d/9) + x**8*(a**7*d**7/8 + 49*a**6*
b*c*d**6/8 + 441*a**5*b**2*c**2*d**5/8 + 1225*a**4*b**3*c**3*d**4/8 + 1225*
a**3*b**4*c**4*d**3/8 + 441*a**2*b**5*c**5*d**2/8 + 49*a*b**6*c**6*d/8 + b*
*7*c**7/8) + x**7*(a**7*c*d**6 + 21*a**6*b*c**2*d**5 + 105*a**5*b**2*c**3*d
**4 + 175*a**4*b**3*c**4*d**3 + 105*a**3*b**4*c**5*d**2 + 21*a**2*b**5*c**6
*d + a*b**6*c**7) + x**6*(7*a**7*c**2*d**5/2 + 245*a**6*b*c**3*d**4/6 + 245
*a**5*b**2*c**4*d**3/2 + 245*a**4*b**3*c**5*d**2/2 + 245*a**3*b**4*c**6*d/6
 + 7*a**2*b**5*c**7/2) + x**5*(7*a**7*c**3*d**4 + 49*a**6*b*c**4*d**3 + 441
*a**5*b**2*c**5*d**2/5 + 49*a**4*b**3*c**6*d + 7*a**3*b**4*c**7) + x**4*(35
*a**7*c**4*d**3/4 + 147*a**6*b*c**5*d**2/4 + 147*a**5*b**2*c**6*d/4 + 35*a*
*4*b**3*c**7/4) + x**3*(7*a**7*c**5*d**2 + 49*a**6*b*c**6*d/3 + 7*a**5*b**2
*c**7) + x**2*(7*a**7*c**6*d/2 + 7*a**6*b*c**7/2)
```

3.1170 $\int (a+bx)^6 (c+dx)^7 \, dx$

Optimal. Leaf size=173

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)}{3}$$

Rubi [A] time = 0.43, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{6b^5(c+dx)^{13}(bc-ad)}{13d^7}+\frac{5b^4(c+dx)^{12}(bc-ad)^2}{4d^7}-\frac{20b^3(c+dx)^{11}(bc-ad)^3}{11d^7}+\frac{3b^2(c+dx)^{10}(bc-ad)^4}{2d^7}-\frac{2b(c+dx)^9(bc-ad)^5}{3d^7}+\frac{(c+dx)^8(bc-ad)^6}{8d^7}+\frac{b^6(c+dx)^{14}}{14d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^8)/(8*d^7) - (2*b*(b*c - a*d)^5*(c + d*x)^9)/(3*d^
7) + (3*b^2*(b*c - a*d)^4*(c + d*x)^10)/(2*d^7) - (20*b^3*(b*c - a*d)^3*(c
+ d*x)^11)/(11*d^7) + (5*b^4*(b*c - a*d)^2*(c + d*x)^12)/(4*d^7) - (6*b^5*(
b*c - a*d)*(c + d*x)^13)/(13*d^7) + (b^6*(c + d*x)^14)/(14*d^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^6(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^6(c+dx)^7}{d^6}-\frac{6b(bc-ad)^5(c+dx)^8}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^9}{d^6}-\frac{20b^3(bc\cdots}{}\right.$$

$$=\frac{(bc-ad)^6(c+dx)^8}{8d^7}-\frac{2b(bc-ad)^5(c+dx)^9}{3d^7}+\frac{3b^2(bc-ad)^4(c+dx)^{10}}{2d^7}-\frac{20b^3(bc-ad)}{11d}$$

Mathematica [B] time = 0.08, size = 684, normalized size = 3.95

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] a^6*c^7*x + (a^5*c^6*(6*b*c + 7*a*d)*x^2)/2 + a^4*c^5*(5*b^2*c^2 + 14*a*b*c
*d + 7*a^2*d^2)*x^3 + (a^3*c^4*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + a^2*c^3*(3*b^4*c^4 + 28*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 42*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (a*c^2*(2*b^5*c^5 + 35*a*b^4*c
^4*d + 140*a^2*b^3*c^3*d^2 + 175*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d
^5)*x^6)/2 + (c*(b^6*c^6 + 42*a*b^5*c^5*d + 315*a^2*b^4*c^4*d^2 + 700*a^3*b
^3*c^3*d^3 + 525*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 7*a^6*d^6)*x^7)/7 + (d
*(7*b^6*c^6 + 126*a*b^5*c^5*d + 525*a^2*b^4*c^4*d^2 + 700*a^3*b^3*c^3*d^3 +
 315*a^4*b^2*c^2*d^4 + 42*a^5*b*c*d^5 + a^6*d^6)*x^8)/8 + (b*d^2*(7*b^5*c^5
 + 70*a*b^4*c^4*d + 175*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 35*a^4*b*c*
d^4 + 2*a^5*d^5)*x^9)/3 + (b^2*d^3*(7*b^4*c^4 + 42*a*b^3*c^3*d + 63*a^2*b^2
*c^2*d^2 + 28*a^3*b*c*d^3 + 3*a^4*d^4)*x^10)/2 + (b^3*d^4*(35*b^3*c^3 + 126
*a*b^2*c^2*d + 105*a^2*b*c*d^2 + 20*a^3*d^3)*x^11)/11 + (b^4*d^5*(7*b^2*c^2
```

```
 + 14*a*b*c*d + 5*a^2*d^2)*x^12)/4 + (b^5*d^6*(7*b*c + 6*a*d)*x^13)/13 + (b
^6*d^7*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^6(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^7, x]
```

fricas [B] time = 1.20, size = 798, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^7*b^6 + 7/13*x^13*d^6*c*b^6 + 6/13*x^13*d^7*b^5*a + 7/4*x^12*d^
5*c^2*b^6 + 7/2*x^12*d^6*c*b^5*a + 5/4*x^12*d^7*b^4*a^2 + 35/11*x^11*d^4*c^
3*b^6 + 126/11*x^11*d^5*c^2*b^5*a + 105/11*x^11*d^6*c*b^4*a^2 + 20/11*x^11*
d^7*b^3*a^3 + 7/2*x^10*d^3*c^4*b^6 + 21*x^10*d^4*c^3*b^5*a + 63/2*x^10*d^5*
c^2*b^4*a^2 + 14*x^10*d^6*c*b^3*a^3 + 3/2*x^10*d^7*b^2*a^4 + 7/3*x^9*d^2*c^
5*b^6 + 70/3*x^9*d^3*c^4*b^5*a + 175/3*x^9*d^4*c^3*b^4*a^2 + 140/3*x^9*d^5*
c^2*b^3*a^3 + 35/3*x^9*d^6*c*b^2*a^4 + 2/3*x^9*d^7*b*a^5 + 7/8*x^8*d*c^6*b^
6 + 63/4*x^8*d^2*c^5*b^5*a + 525/8*x^8*d^3*c^4*b^4*a^2 + 175/2*x^8*d^4*c^3*
b^3*a^3 + 315/8*x^8*d^5*c^2*b^2*a^4 + 21/4*x^8*d^6*c*b*a^5 + 1/8*x^8*d^7*a^
6 + 1/7*x^7*c^7*b^6 + 6*x^7*d*c^6*b^5*a + 45*x^7*d^2*c^5*b^4*a^2 + 100*x^7*
d^3*c^4*b^3*a^3 + 75*x^7*d^4*c^3*b^2*a^4 + 18*x^7*d^5*c^2*b*a^5 + x^7*d^6*c
*a^6 + x^6*c^7*b^5*a + 35/2*x^6*d*c^6*b^4*a^2 + 70*x^6*d^2*c^5*b^3*a^3 + 17
5/2*x^6*d^3*c^4*b^2*a^4 + 35*x^6*d^4*c^3*b*a^5 + 7/2*x^6*d^5*c^2*a^6 + 3*x^
5*c^7*b^4*a^2 + 28*x^5*d*c^6*b^3*a^3 + 63*x^5*d^2*c^5*b^2*a^4 + 42*x^5*d^3*
c^4*b*a^5 + 7*x^5*d^4*c^3*a^6 + 5*x^4*c^7*b^3*a^3 + 105/4*x^4*d*c^6*b^2*a^4
 + 63/2*x^4*d^2*c^5*b*a^5 + 35/4*x^4*d^3*c^4*a^6 + 5*x^3*c^7*b^2*a^4 + 14*x
^3*d*c^6*b*a^5 + 7*x^3*d^2*c^5*a^6 + 3*x^2*c^7*b*a^5 + 7/2*x^2*d*c^6*a^6 +
x*c^7*a^6
```

giac [B] time = 0.95, size = 798, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/14*b^6*d^7*x^14 + 7/13*b^6*c*d^6*x^13 + 6/13*a*b^5*d^7*x^13 + 7/4*b^6*c^2
*d^5*x^12 + 7/2*a*b^5*c*d^6*x^12 + 5/4*a^2*b^4*d^7*x^12 + 35/11*b^6*c^3*d^4
*x^11 + 126/11*a*b^5*c^2*d^5*x^11 + 105/11*a^2*b^4*c*d^6*x^11 + 20/11*a^3*b
^3*d^7*x^11 + 7/2*b^6*c^4*d^3*x^10 + 21*a*b^5*c^3*d^4*x^10 + 63/2*a^2*b^4*c
^2*d^5*x^10 + 14*a^3*b^3*c*d^6*x^10 + 3/2*a^4*b^2*d^7*x^10 + 7/3*b^6*c^5*d^
2*x^9 + 70/3*a*b^5*c^4*d^3*x^9 + 175/3*a^2*b^4*c^3*d^4*x^9 + 140/3*a^3*b^3*
c^2*d^5*x^9 + 35/3*a^4*b^2*c*d^6*x^9 + 2/3*a^5*b*d^7*x^9 + 7/8*b^6*c^6*d*x^
8 + 63/4*a*b^5*c^5*d^2*x^8 + 525/8*a^2*b^4*c^4*d^3*x^8 + 175/2*a^3*b^3*c^3*
d^4*x^8 + 315/8*a^4*b^2*c^2*d^5*x^8 + 21/4*a^5*b*c*d^6*x^8 + 1/8*a^6*d^7*x^
8 + 1/7*b^6*c^7*x^7 + 6*a*b^5*c^6*d*x^7 + 45*a^2*b^4*c^5*d^2*x^7 + 100*a^3*
b^3*c^4*d^3*x^7 + 75*a^4*b^2*c^3*d^4*x^7 + 18*a^5*b*c^2*d^5*x^7 + a^6*c*d^6
*x^7 + a*b^5*c^7*x^6 + 35/2*a^2*b^4*c^6*d*x^6 + 70*a^3*b^3*c^5*d^2*x^6 + 17
5/2*a^4*b^2*c^4*d^3*x^6 + 35*a^5*b*c^3*d^4*x^6 + 7/2*a^6*c^2*d^5*x^6 + 3*a^
```

```
2*b^4*c^7*x^5 + 28*a^3*b^3*c^6*d*x^5 + 63*a^4*b^2*c^5*d^2*x^5 + 42*a^5*b*c^
4*d^3*x^5 + 7*a^6*c^3*d^4*x^5 + 5*a^3*b^3*c^7*x^4 + 105/4*a^4*b^2*c^6*d*x^4
 + 63/2*a^5*b*c^5*d^2*x^4 + 35/4*a^6*c^4*d^3*x^4 + 5*a^4*b^2*c^7*x^3 + 14*a
^5*b*c^6*d*x^3 + 7*a^6*c^5*d^2*x^3 + 3*a^5*b*c^7*x^2 + 7/2*a^6*c^6*d*x^2 +
a^6*c^7*x
```

maple [B] time = 0.00, size = 709, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^7,x)
```

```
[Out] 1/14*b^6*d^7*x^14+1/13*(6*a*b^5*d^7+7*b^6*c*d^6)*x^13+1/12*(15*a^2*b^4*d^7+
42*a*b^5*c*d^6+21*b^6*c^2*d^5)*x^12+1/11*(20*a^3*b^3*d^7+105*a^2*b^4*c*d^6+
126*a*b^5*c^2*d^5+35*b^6*c^3*d^4)*x^11+1/10*(15*a^4*b^2*d^7+140*a^3*b^3*c*d
^6+315*a^2*b^4*c^2*d^5+210*a*b^5*c^3*d^4+35*b^6*c^4*d^3)*x^10+1/9*(6*a^5*b*
d^7+105*a^4*b^2*c*d^6+420*a^3*b^3*c^2*d^5+525*a^2*b^4*c^3*d^4+210*a*b^5*c^4
*d^3+21*b^6*c^5*d^2)*x^9+1/8*(a^6*d^7+42*a^5*b*c*d^6+315*a^4*b^2*c^2*d^5+70
0*a^3*b^3*c^3*d^4+525*a^2*b^4*c^4*d^3+126*a*b^5*c^5*d^2+7*b^6*c^6*d)*x^8+1/
7*(7*a^6*c*d^6+126*a^5*b*c^2*d^5+525*a^4*b^2*c^3*d^4+700*a^3*b^3*c^4*d^3+31
5*a^2*b^4*c^5*d^2+42*a*b^5*c^6*d+b^6*c^7)*x^7+1/6*(21*a^6*c^2*d^5+210*a^5*b
*c^3*d^4+525*a^4*b^2*c^4*d^3+420*a^3*b^3*c^5*d^2+105*a^2*b^4*c^6*d+6*a*b^5*
c^7)*x^6+1/5*(35*a^6*c^3*d^4+210*a^5*b*c^4*d^3+315*a^4*b^2*c^5*d^2+140*a^3*
b^3*c^6*d+15*a^2*b^4*c^7)*x^5+1/4*(35*a^6*c^4*d^3+126*a^5*b*c^5*d^2+105*a^4
*b^2*c^6*d+20*a^3*b^3*c^7)*x^4+1/3*(21*a^6*c^5*d^2+42*a^5*b*c^6*d+15*a^4*b^
2*c^7)*x^3+1/2*(7*a^6*c^6*d+6*a^5*b*c^7)*x^2+a^6*c^7*x
```

maxima [B] time = 1.32, size = 706, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*b^6*d^7*x^14 + a^6*c^7*x + 1/13*(7*b^6*c*d^6 + 6*a*b^5*d^7)*x^13 + 1/4
*(7*b^6*c^2*d^5 + 14*a*b^5*c*d^6 + 5*a^2*b^4*d^7)*x^12 + 1/11*(35*b^6*c^3*d
^4 + 126*a*b^5*c^2*d^5 + 105*a^2*b^4*c*d^6 + 20*a^3*b^3*d^7)*x^11 + 1/2*(7*
b^6*c^4*d^3 + 42*a*b^5*c^3*d^4 + 63*a^2*b^4*c^2*d^5 + 28*a^3*b^3*c*d^6 + 3*
a^4*b^2*d^7)*x^10 + 1/3*(7*b^6*c^5*d^2 + 70*a*b^5*c^4*d^3 + 175*a^2*b^4*c^3
*d^4 + 140*a^3*b^3*c^2*d^5 + 35*a^4*b^2*c*d^6 + 2*a^5*b*d^7)*x^9 + 1/8*(7*b
^6*c^6*d + 126*a*b^5*c^5*d^2 + 525*a^2*b^4*c^4*d^3 + 700*a^3*b^3*c^3*d^4 +
315*a^4*b^2*c^2*d^5 + 42*a^5*b*c*d^6 + a^6*d^7)*x^8 + 1/7*(b^6*c^7 + 42*a*b
^5*c^6*d + 315*a^2*b^4*c^5*d^2 + 700*a^3*b^3*c^4*d^3 + 525*a^4*b^2*c^3*d^4
+ 126*a^5*b*c^2*d^5 + 7*a^6*c*d^6)*x^7 + 1/2*(2*a*b^5*c^7 + 35*a^2*b^4*c^6*
d + 140*a^3*b^3*c^5*d^2 + 175*a^4*b^2*c^4*d^3 + 70*a^5*b*c^3*d^4 + 7*a^6*c^
2*d^5)*x^6 + (3*a^2*b^4*c^7 + 28*a^3*b^3*c^6*d + 63*a^4*b^2*c^5*d^2 + 42*a^
5*b*c^4*d^3 + 7*a^6*c^3*d^4)*x^5 + 1/4*(20*a^3*b^3*c^7 + 105*a^4*b^2*c^6*d
+ 126*a^5*b*c^5*d^2 + 35*a^6*c^4*d^3)*x^4 + (5*a^4*b^2*c^7 + 14*a^5*b*c^6*d
 + 7*a^6*c^5*d^2)*x^3 + 1/2*(6*a^5*b*c^7 + 7*a^6*c^6*d)*x^2
```

mupad [B] time = 0.26, size = 683, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^7,x)
```

```
[Out] x^5*(3*a^2*b^4*c^7 + 7*a^6*c^3*d^4 + 28*a^3*b^3*c^6*d + 42*a^5*b*c^4*d^3 +
63*a^4*b^2*c^5*d^2) + x^10*((3*a^4*b^2*d^7)/2 + (7*b^6*c^4*d^3)/2 + 21*a*b^
```

```
5*c^3*d^4 + 14*a^3*b^3*c*d^6 + (63*a^2*b^4*c^2*d^5)/2) + x^6*(a*b^5*c^7 + (
7*a^6*c^2*d^5)/2 + (35*a^2*b^4*c^6*d)/2 + 35*a^5*b*c^3*d^4 + 70*a^3*b^3*c^5
*d^2 + (175*a^4*b^2*c^4*d^3)/2) + x^9*((2*a^5*b*d^7)/3 + (7*b^6*c^5*d^2)/3
+ (70*a*b^5*c^4*d^3)/3 + (35*a^4*b^2*c*d^6)/3 + (175*a^2*b^4*c^3*d^4)/3 + (
140*a^3*b^3*c^2*d^5)/3) + x^7*((b^6*c^7)/7 + a^6*c*d^6 + 18*a^5*b*c^2*d^5 +
 45*a^2*b^4*c^5*d^2 + 100*a^3*b^3*c^4*d^3 + 75*a^4*b^2*c^3*d^4 + 6*a*b^5*c^
6*d) + x^8*((a^6*d^7)/8 + (7*b^6*c^6*d)/8 + (63*a*b^5*c^5*d^2)/4 + (525*a^2
*b^4*c^4*d^3)/8 + (175*a^3*b^3*c^3*d^4)/2 + (315*a^4*b^2*c^2*d^5)/8 + (21*a
^5*b*c*d^6)/4) + x^4*(5*a^3*b^3*c^7 + (35*a^6*c^4*d^3)/4 + (105*a^4*b^2*c^6
*d)/4 + (63*a^5*b*c^5*d^2)/2) + x^11*((20*a^3*b^3*d^7)/11 + (35*b^6*c^3*d^4
)/11 + (126*a*b^5*c^2*d^5)/11 + (105*a^2*b^4*c*d^6)/11) + a^6*c^7*x + (b^6*
d^7*x^14)/14 + (a^5*c^6*x^2*(7*a*d + 6*b*c))/2 + (b^5*d^6*x^13*(6*a*d + 7*b
*c))/13 + a^4*c^5*x^3*(7*a^2*d^2 + 5*b^2*c^2 + 14*a*b*c*d) + (b^4*d^5*x^12*
(5*a^2*d^2 + 7*b^2*c^2 + 14*a*b*c*d))/4
```

sympy [B] time = 0.18, size = 796, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**7,x)
```

```
[Out] a**6*c**7*x + b**6*d**7*x**14/14 + x**13*(6*a*b**5*d**7/13 + 7*b**6*c*d**6/
13) + x**12*(5*a**2*b**4*d**7/4 + 7*a*b**5*c*d**6/2 + 7*b**6*c**2*d**5/4) +
 x**11*(20*a**3*b**3*d**7/11 + 105*a**2*b**4*c*d**6/11 + 126*a*b**5*c**2*d*
*5/11 + 35*b**6*c**3*d**4/11) + x**10*(3*a**4*b**2*d**7/2 + 14*a**3*b**3*c*
d**6 + 63*a**2*b**4*c**2*d**5/2 + 21*a*b**5*c**3*d**4 + 7*b**6*c**4*d**3/2)
 + x**9*(2*a**5*b*d**7/3 + 35*a**4*b**2*c*d**6/3 + 140*a**3*b**3*c**2*d**5/
3 + 175*a**2*b**4*c**3*d**4/3 + 70*a*b**5*c**4*d**3/3 + 7*b**6*c**5*d**2/3)
 + x**8*(a**6*d**7/8 + 21*a**5*b*c*d**6/4 + 315*a**4*b**2*c**2*d**5/8 + 175
*a**3*b**3*c**3*d**4/2 + 525*a**2*b**4*c**4*d**3/8 + 63*a*b**5*c**5*d**2/4
+ 7*b**6*c**6*d/8) + x**7*(a**6*c*d**6 + 18*a**5*b*c**2*d**5 + 75*a**4*b**2
*c**3*d**4 + 100*a**3*b**3*c**4*d**3 + 45*a**2*b**4*c**5*d**2 + 6*a*b**5*c*
*6*d + b**6*c**7/7) + x**6*(7*a**6*c**2*d**5/2 + 35*a**5*b*c**3*d**4 + 175*
a**4*b**2*c**4*d**3/2 + 70*a**3*b**3*c**5*d**2 + 35*a**2*b**4*c**6*d/2 + a*
b**5*c**7) + x**5*(7*a**6*c**3*d**4 + 42*a**5*b*c**4*d**3 + 63*a**4*b**2*c*
*5*d**2 + 28*a**3*b**3*c**6*d + 3*a**2*b**4*c**7) + x**4*(35*a**6*c**4*d**3
/4 + 63*a**5*b*c**5*d**2/2 + 105*a**4*b**2*c**6*d/4 + 5*a**3*b**3*c**7) + x
**3*(7*a**6*c**5*d**2 + 14*a**5*b*c**6*d + 5*a**4*b**2*c**7) + x**2*(7*a**6
*c**6*d/2 + 3*a**5*b*c**7)
```

3.1171 $\int (a+bx)^5(c+dx)^7\,dx$

Optimal. Leaf size=144

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-}{8d^6}$$

Rubi [A] time = 0.36, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^{12}(bc-ad)}{12d^6}+\frac{10b^3(c+dx)^{11}(bc-ad)^2}{11d^6}-\frac{b^2(c+dx)^{10}(bc-ad)^3}{d^6}+\frac{5b(c+dx)^9(bc-ad)^4}{9d^6}-\frac{(c+dx)^8(bc-ad)^5}{8d^6}+\frac{b^5(c+dx)^{13}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^8)/(8*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^9)/(9*d
^6) - (b^2*(b*c - a*d)^3*(c + d*x)^10)/d^6 + (10*b^3*(b*c - a*d)^2*(c + d*x
)^11)/(11*d^6) - (5*b^4*(b*c - a*d)*(c + d*x)^12)/(12*d^6) + (b^5*(c + d*x)
^13)/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^5(c+dx)^7}{d^5}+\frac{5b(bc-ad)^4(c+dx)^8}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^9}{d^5}+\frac{10b^3(bc}{}\right.$$

$$= -\frac{(bc-ad)^5(c+dx)^8}{8d^6}+\frac{5b(bc-ad)^4(c+dx)^9}{9d^6}-\frac{b^2(bc-ad)^3(c+dx)^{10}}{d^6}+\frac{10b^3(bc-ad}{11d}$$

Mathematica [B] time = 0.08, size = 574, normalized size = 3.99

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] a^5*c^7*x + (a^4*c^6*(5*b*c + 7*a*d)*x^2)/2 + (a^3*c^5*(10*b^2*c^2 + 35*a*b
*c*d + 21*a^2*d^2)*x^3)/3 + (5*a^2*c^4*(2*b^3*c^3 + 14*a*b^2*c^2*d + 21*a^2
*b*c*d^2 + 7*a^3*d^3)*x^4)/4 + a*c^3*(b^4*c^4 + 14*a*b^3*c^3*d + 42*a^2*b^2
*c^2*d^2 + 35*a^3*b*c*d^3 + 7*a^4*d^4)*x^5 + (c^2*(b^5*c^5 + 35*a*b^4*c^4*d
 + 210*a^2*b^3*c^3*d^2 + 350*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5
)*x^6)/6 + c*d*(b^5*c^5 + 15*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 + 50*a^3*b^2*
c^2*d^3 + 15*a^4*b*c*d^4 + a^5*d^5)*x^7 + (d^2*(21*b^5*c^5 + 175*a*b^4*c^4*
d + 350*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 + a^5*d^5)*x
^8)/8 + (5*b*d^3*(7*b^4*c^4 + 35*a*b^3*c^3*d + 42*a^2*b^2*c^2*d^2 + 14*a^3*
b*c*d^3 + a^4*d^4)*x^9)/9 + (b^2*d^4*(7*b^3*c^3 + 21*a*b^2*c^2*d + 14*a^2*b
*c*d^2 + 2*a^3*d^3)*x^10)/2 + (b^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^
2)*x^11)/11 + (b^4*d^6*(7*b*c + 5*a*d)*x^12)/12 + (b^5*d^7*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^7, x]
```

fricas [B] time = 1.09, size = 670, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^7*b^5 + 7/12*x^12*d^6*c*b^5 + 5/12*x^12*d^7*b^4*a + 21/11*x^11*
d^5*c^2*b^5 + 35/11*x^11*d^6*c*b^4*a + 10/11*x^11*d^7*b^3*a^2 + 7/2*x^10*d^
4*c^3*b^5 + 21/2*x^10*d^5*c^2*b^4*a + 7*x^10*d^6*c*b^3*a^2 + x^10*d^7*b^2*a
^3 + 35/9*x^9*d^3*c^4*b^5 + 175/9*x^9*d^4*c^3*b^4*a + 70/3*x^9*d^5*c^2*b^3*
a^2 + 70/9*x^9*d^6*c*b^2*a^3 + 5/9*x^9*d^7*b*a^4 + 21/8*x^8*d^2*c^5*b^5 + 1
75/8*x^8*d^3*c^4*b^4*a + 175/4*x^8*d^4*c^3*b^3*a^2 + 105/4*x^8*d^5*c^2*b^2*
a^3 + 35/8*x^8*d^6*c*b*a^4 + 1/8*x^8*d^7*a^5 + x^7*d*c^6*b^5 + 15*x^7*d^2*c
^5*b^4*a + 50*x^7*d^3*c^4*b^3*a^2 + 50*x^7*d^4*c^3*b^2*a^3 + 15*x^7*d^5*c^2
*b*a^4 + x^7*d^6*c*a^5 + 1/6*x^6*c^7*b^5 + 35/6*x^6*d*c^6*b^4*a + 35*x^6*d^
2*c^5*b^3*a^2 + 175/3*x^6*d^3*c^4*b^2*a^3 + 175/6*x^6*d^4*c^3*b*a^4 + 7/2*x
^6*d^5*c^2*a^5 + x^5*c^7*b^4*a + 14*x^5*d*c^6*b^3*a^2 + 42*x^5*d^2*c^5*b^2*
a^3 + 35*x^5*d^3*c^4*b*a^4 + 7*x^5*d^4*c^3*a^5 + 5/2*x^4*c^7*b^3*a^2 + 35/2
*x^4*d*c^6*b^2*a^3 + 105/4*x^4*d^2*c^5*b*a^4 + 35/4*x^4*d^3*c^4*a^5 + 10/3*
x^3*c^7*b^2*a^3 + 35/3*x^3*d*c^6*b*a^4 + 7*x^3*d^2*c^5*a^5 + 5/2*x^2*c^7*b*
a^4 + 7/2*x^2*d*c^6*a^5 + x*c^7*a^5
```

giac [B] time = 1.28, size = 670, normalized size = 4.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/13*b^5*d^7*x^13 + 7/12*b^5*c*d^6*x^12 + 5/12*a*b^4*d^7*x^12 + 21/11*b^5*c
^2*d^5*x^11 + 35/11*a*b^4*c*d^6*x^11 + 10/11*a^2*b^3*d^7*x^11 + 7/2*b^5*c^3
*d^4*x^10 + 21/2*a*b^4*c^2*d^5*x^10 + 7*a^2*b^3*c*d^6*x^10 + a^3*b^2*d^7*x^
10 + 35/9*b^5*c^4*d^3*x^9 + 175/9*a*b^4*c^3*d^4*x^9 + 70/3*a^2*b^3*c^2*d^5*
x^9 + 70/9*a^3*b^2*c*d^6*x^9 + 5/9*a^4*b*d^7*x^9 + 21/8*b^5*c^5*d^2*x^8 + 1
75/8*a*b^4*c^4*d^3*x^8 + 175/4*a^2*b^3*c^3*d^4*x^8 + 105/4*a^3*b^2*c^2*d^5*
x^8 + 35/8*a^4*b*c*d^6*x^8 + 1/8*a^5*d^7*x^8 + b^5*c^6*d*x^7 + 15*a*b^4*c^5
*d^2*x^7 + 50*a^2*b^3*c^4*d^3*x^7 + 50*a^3*b^2*c^3*d^4*x^7 + 15*a^4*b*c^2*d
^5*x^7 + a^5*c*d^6*x^7 + 1/6*b^5*c^7*x^6 + 35/6*a*b^4*c^6*d*x^6 + 35*a^2*b^
3*c^5*d^2*x^6 + 175/3*a^3*b^2*c^4*d^3*x^6 + 175/6*a^4*b*c^3*d^4*x^6 + 7/2*a
^5*c^2*d^5*x^6 + a*b^4*c^7*x^5 + 14*a^2*b^3*c^6*d*x^5 + 42*a^3*b^2*c^5*d^2*
x^5 + 35*a^4*b*c^4*d^3*x^5 + 7*a^5*c^3*d^4*x^5 + 5/2*a^2*b^3*c^7*x^4 + 35/2
*a^3*b^2*c^6*d*x^4 + 105/4*a^4*b*c^5*d^2*x^4 + 35/4*a^5*c^4*d^3*x^4 + 10/3*
a^3*b^2*c^7*x^3 + 35/3*a^4*b*c^6*d*x^3 + 7*a^5*c^5*d^2*x^3 + 5/2*a^4*b*c^7*
x^2 + 7/2*a^5*c^6*d*x^2 + a^5*c^7*x
```

maple [B] time = 0.00, size = 601, normalized size = 4.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^7,x)
```

```
[Out] 1/13*b^5*d^7*x^13+1/12*(5*a*b^4*d^7+7*b^5*c*d^6)*x^12+1/11*(10*a^2*b^3*d^7+
35*a*b^4*c*d^6+21*b^5*c^2*d^5)*x^11+1/10*(10*a^3*b^2*d^7+70*a^2*b^3*c*d^6+1
05*a*b^4*c^2*d^5+35*b^5*c^3*d^4)*x^10+1/9*(5*a^4*b*d^7+70*a^3*b^2*c*d^6+210
*a^2*b^3*c^2*d^5+175*a*b^4*c^3*d^4+35*b^5*c^4*d^3)*x^9+1/8*(a^5*d^7+35*a^4*
b*c*d^6+210*a^3*b^2*c^2*d^5+350*a^2*b^3*c^3*d^4+175*a*b^4*c^4*d^3+21*b^5*c^
5*d^2)*x^8+1/7*(7*a^5*c*d^6+105*a^4*b*c^2*d^5+350*a^3*b^2*c^3*d^4+350*a^2*b
^3*c^4*d^3+105*a*b^4*c^5*d^2+7*b^5*c^6*d)*x^7+1/6*(21*a^5*c^2*d^5+175*a^4*b
*c^3*d^4+350*a^3*b^2*c^4*d^3+210*a^2*b^3*c^5*d^2+35*a*b^4*c^6*d+b^5*c^7)*x^
6+1/5*(35*a^5*c^3*d^4+175*a^4*b*c^4*d^3+210*a^3*b^2*c^5*d^2+70*a^2*b^3*c^6*
d+5*a*b^4*c^7)*x^5+1/4*(35*a^5*c^4*d^3+105*a^4*b*c^5*d^2+70*a^3*b^2*c^6*d+1
0*a^2*b^3*c^7)*x^4+1/3*(21*a^5*c^5*d^2+35*a^4*b*c^6*d+10*a^3*b^2*c^7)*x^3+1
/2*(7*a^5*c^6*d+5*a^4*b*c^7)*x^2+a^5*c^7*x
```

maxima [B] time = 1.38, size = 594, normalized size = 4.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/13*b^5*d^7*x^13 + a^5*c^7*x + 1/12*(7*b^5*c*d^6 + 5*a*b^4*d^7)*x^12 + 1/1
1*(21*b^5*c^2*d^5 + 35*a*b^4*c*d^6 + 10*a^2*b^3*d^7)*x^11 + 1/2*(7*b^5*c^3*
d^4 + 21*a*b^4*c^2*d^5 + 14*a^2*b^3*c*d^6 + 2*a^3*b^2*d^7)*x^10 + 5/9*(7*b^
5*c^4*d^3 + 35*a*b^4*c^3*d^4 + 42*a^2*b^3*c^2*d^5 + 14*a^3*b^2*c*d^6 + a^4*
b*d^7)*x^9 + 1/8*(21*b^5*c^5*d^2 + 175*a*b^4*c^4*d^3 + 350*a^2*b^3*c^3*d^4
+ 210*a^3*b^2*c^2*d^5 + 35*a^4*b*c*d^6 + a^5*d^7)*x^8 + (b^5*c^6*d + 15*a*b
^4*c^5*d^2 + 50*a^2*b^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4 + 15*a^4*b*c^2*d^5 + a
^5*c*d^6)*x^7 + 1/6*(b^5*c^7 + 35*a*b^4*c^6*d + 210*a^2*b^3*c^5*d^2 + 350*a
^3*b^2*c^4*d^3 + 175*a^4*b*c^3*d^4 + 21*a^5*c^2*d^5)*x^6 + (a*b^4*c^7 + 14*
a^2*b^3*c^6*d + 42*a^3*b^2*c^5*d^2 + 35*a^4*b*c^4*d^3 + 7*a^5*c^3*d^4)*x^5
+ 5/4*(2*a^2*b^3*c^7 + 14*a^3*b^2*c^6*d + 21*a^4*b*c^5*d^2 + 7*a^5*c^4*d^3)
*x^4 + 1/3*(10*a^3*b^2*c^7 + 35*a^4*b*c^6*d + 21*a^5*c^5*d^2)*x^3 + 1/2*(5*
a^4*b*c^7 + 7*a^5*c^6*d)*x^2
```

mupad [B] time = 0.21, size = 570, normalized size = 3.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^7,x)
```

```
[Out] x^7*(a^5*c*d^6 + b^5*c^6*d + 15*a*b^4*c^5*d^2 + 15*a^4*b*c^2*d^5 + 50*a^2*b
^3*c^4*d^3 + 50*a^3*b^2*c^3*d^4) + x^6*((b^5*c^7)/6 + (7*a^5*c^2*d^5)/2 + (
175*a^4*b*c^3*d^4)/6 + 35*a^2*b^3*c^5*d^2 + (175*a^3*b^2*c^4*d^3)/3 + (35*a
*b^4*c^6*d)/6) + x^8*((a^5*d^7)/8 + (21*b^5*c^5*d^2)/8 + (175*a*b^4*c^4*d^3
)/8 + (175*a^2*b^3*c^3*d^4)/4 + (105*a^3*b^2*c^2*d^5)/4 + (35*a^4*b*c*d^6)/
8) + x^5*(a*b^4*c^7 + 7*a^5*c^3*d^4 + 14*a^2*b^3*c^6*d + 35*a^4*b*c^4*d^3 +
 42*a^3*b^2*c^5*d^2) + x^9*((5*a^4*b*d^7)/9 + (35*b^5*c^4*d^3)/9 + (175*a*b
^4*c^3*d^4)/9 + (70*a^3*b^2*c*d^6)/9 + (70*a^2*b^3*c^2*d^5)/3) + a^5*c^7*x
+ (b^5*d^7*x^13)/13 + (5*a^2*c^4*x^4*(7*a^3*d^3 + 2*b^3*c^3 + 14*a*b^2*c^2*
d + 21*a^2*b*c*d^2))/4 + (b^2*d^4*x^10*(2*a^3*d^3 + 7*b^3*c^3 + 21*a*b^2*c^
2*d + 14*a^2*b*c*d^2))/2 + (a^4*c^6*x^2*(7*a*d + 5*b*c))/2 + (b^4*d^6*x^12*
(5*a*d + 7*b*c))/12 + (a^3*c^5*x^3*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*b*c*d))/
3 + (b^3*d^5*x^11*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/11
```

sympy [B] time = 0.16, size = 673, normalized size = 4.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**7,x)
```

```
[Out] a**5*c**7*x + b**5*d**7*x**13/13 + x**12*(5*a*b**4*d**7/12 + 7*b**5*c*d**6/
12) + x**11*(10*a**2*b**3*d**7/11 + 35*a*b**4*c*d**6/11 + 21*b**5*c**2*d**5
/11) + x**10*(a**3*b**2*d**7 + 7*a**2*b**3*c*d**6 + 21*a*b**4*c**2*d**5/2 +
 7*b**5*c**3*d**4/2) + x**9*(5*a**4*b*d**7/9 + 70*a**3*b**2*c*d**6/9 + 70*a
**2*b**3*c**2*d**5/3 + 175*a*b**4*c**3*d**4/9 + 35*b**5*c**4*d**3/9) + x**8
*(a**5*d**7/8 + 35*a**4*b*c*d**6/8 + 105*a**3*b**2*c**2*d**5/4 + 175*a**2*b
**3*c**3*d**4/4 + 175*a*b**4*c**4*d**3/8 + 21*b**5*c**5*d**2/8) + x**7*(a**
5*c*d**6 + 15*a**4*b*c**2*d**5 + 50*a**3*b**2*c**3*d**4 + 50*a**2*b**3*c**4
*d**3 + 15*a*b**4*c**5*d**2 + b**5*c**6*d) + x**6*(7*a**5*c**2*d**5/2 + 175
*a**4*b*c**3*d**4/6 + 175*a**3*b**2*c**4*d**3/3 + 35*a**2*b**3*c**5*d**2 +
35*a*b**4*c**6*d/6 + b**5*c**7/6) + x**5*(7*a**5*c**3*d**4 + 35*a**4*b*c**4
*d**3 + 42*a**3*b**2*c**5*d**2 + 14*a**2*b**3*c**6*d + a*b**4*c**7) + x**4*
(35*a**5*c**4*d**3/4 + 105*a**4*b*c**5*d**2/4 + 35*a**3*b**2*c**6*d/2 + 5*a
**2*b**3*c**7/2) + x**3*(7*a**5*c**5*d**2 + 35*a**4*b*c**6*d/3 + 10*a**3*b*
*2*c**7/3) + x**2*(7*a**5*c**6*d/2 + 5*a**4*b*c**7/2)
```

3.1172 $\int (a+bx)^4(c+dx)^7\,dx$

Optimal. Leaf size=119

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Rubi [A] time = 0.28, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^3(c+dx)^{11}(bc-ad)}{11d^5}+\frac{3b^2(c+dx)^{10}(bc-ad)^2}{5d^5}-\frac{4b(c+dx)^9(bc-ad)^3}{9d^5}+\frac{(c+dx)^8(bc-ad)^4}{8d^5}+\frac{b^4(c+dx)^{12}}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^8)/(8*d^5) - (4*b*(b*c - a*d)^3*(c + d*x)^9)/(9*d^
5) + (3*b^2*(b*c - a*d)^2*(c + d*x)^10)/(5*d^5) - (4*b^3*(b*c - a*d)*(c + d
*x)^11)/(11*d^5) + (b^4*(c + d*x)^12)/(12*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^7\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^7}{d^4}-\frac{4b(bc-ad)^3(c+dx)^8}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^9}{d^4}-\frac{4b^3(bc-\ldots}{}\right.$$

$$=\frac{(bc-ad)^4(c+dx)^8}{8d^5}-\frac{4b(bc-ad)^3(c+dx)^9}{9d^5}+\frac{3b^2(bc-ad)^2(c+dx)^{10}}{5d^5}-\frac{4b^3(bc-ad)(\ldots}{11d^5}$$

Mathematica [B] time = 0.05, size = 473, normalized size = 3.97

$$a^4c^7x+\frac{1}{2}a^3c^6x^2(7ad+4bc)+\frac{1}{10}b^2d^5x^{10}\left(6a^2d^2+28abcd+21b^2c^2\right)+\frac{1}{3}a^2c^5x^3\left(21a^2d^2+28abcd+6b^2c^2\right)+\frac{1}{9}bd^4x^9\left(4a^3d^3+42a^2bcd^2+84ab^2c^2d+35b^3c^3\right)+\frac{1}{4}ac^4x^4\left(35a^3d^3+84a^2bcd^2+42ab^2c^2d+4b^3c^3\right)+\frac{1}{8}d^3x^8\left(a^4d^4+28a^3bcd^3+126a^2b^2c^2d^2+140ab^3c^3d+35b^4c^4\right)+cd^2x^7\left(a^4d^4+12a^3bcd^3+30a^2b^2c^2d^2+20ab^3c^3d+3b^4c^4\right)+\frac{7}{6}c^2dx^6\left(3a^4d^4+20a^3bcd^3+30a^2b^2c^2d^2+12ab^3c^3d+b^4c^4\right)+\frac{1}{5}c^3x^5\left(35a^4d^4+140a^3bcd^3+126a^2b^2c^2d^2+28ab^3c^3d+b^4c^4\right)+\frac{1}{11}b^3d^6x^{11}(4ad+7bc)+\frac{1}{12}b^4d^7x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] a^4*c^7*x + (a^3*c^6*(4*b*c + 7*a*d)*x^2)/2 + (a^2*c^5*(6*b^2*c^2 + 28*a*b*
c*d + 21*a^2*d^2)*x^3)/3 + (a*c^4*(4*b^3*c^3 + 42*a*b^2*c^2*d + 84*a^2*b*c*
d^2 + 35*a^3*d^3)*x^4)/4 + (c^3*(b^4*c^4 + 28*a*b^3*c^3*d + 126*a^2*b^2*c^2
*d^2 + 140*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + (7*c^2*d*(b^4*c^4 + 12*a*b^3*
c^3*d + 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 + 3*a^4*d^4)*x^6)/6 + c*d^2*(3*
b^4*c^4 + 20*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)*x
^7 + (d^3*(35*b^4*c^4 + 140*a*b^3*c^3*d + 126*a^2*b^2*c^2*d^2 + 28*a^3*b*c*
d^3 + a^4*d^4)*x^8)/8 + (b*d^4*(35*b^3*c^3 + 84*a*b^2*c^2*d + 42*a^2*b*c*d^
2 + 4*a^3*d^3)*x^9)/9 + (b^2*d^5*(21*b^2*c^2 + 28*a*b*c*d + 6*a^2*d^2)*x^10
)/10 + (b^3*d^6*(7*b*c + 4*a*d)*x^11)/11 + (b^4*d^7*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^7, x]
```

fricas [B] time = 1.31, size = 546, normalized size = 4.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^7*b^4 + 7/11*x^11*d^6*c*b^4 + 4/11*x^11*d^7*b^3*a + 21/10*x^10*
d^5*c^2*b^4 + 14/5*x^10*d^6*c*b^3*a + 3/5*x^10*d^7*b^2*a^2 + 35/9*x^9*d^4*c
^3*b^4 + 28/3*x^9*d^5*c^2*b^3*a + 14/3*x^9*d^6*c*b^2*a^2 + 4/9*x^9*d^7*b*a^
3 + 35/8*x^8*d^3*c^4*b^4 + 35/2*x^8*d^4*c^3*b^3*a + 63/4*x^8*d^5*c^2*b^2*a^
2 + 7/2*x^8*d^6*c*b*a^3 + 1/8*x^8*d^7*a^4 + 3*x^7*d^2*c^5*b^4 + 20*x^7*d^3*
c^4*b^3*a + 30*x^7*d^4*c^3*b^2*a^2 + 12*x^7*d^5*c^2*b*a^3 + x^7*d^6*c*a^4 +
 7/6*x^6*d*c^6*b^4 + 14*x^6*d^2*c^5*b^3*a + 35*x^6*d^3*c^4*b^2*a^2 + 70/3*x
^6*d^4*c^3*b*a^3 + 7/2*x^6*d^5*c^2*a^4 + 1/5*x^5*c^7*b^4 + 28/5*x^5*d*c^6*b
^3*a + 126/5*x^5*d^2*c^5*b^2*a^2 + 28*x^5*d^3*c^4*b*a^3 + 7*x^5*d^4*c^3*a^4
 + x^4*c^7*b^3*a + 21/2*x^4*d*c^6*b^2*a^2 + 21*x^4*d^2*c^5*b*a^3 + 35/4*x^4
*d^3*c^4*a^4 + 2*x^3*c^7*b^2*a^2 + 28/3*x^3*d*c^6*b*a^3 + 7*x^3*d^2*c^5*a^4
 + 2*x^2*c^7*b*a^3 + 7/2*x^2*d*c^6*a^4 + x*c^7*a^4
```

giac [B] time = 1.21, size = 546, normalized size = 4.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/12*b^4*d^7*x^12 + 7/11*b^4*c*d^6*x^11 + 4/11*a*b^3*d^7*x^11 + 21/10*b^4*c
^2*d^5*x^10 + 14/5*a*b^3*c*d^6*x^10 + 3/5*a^2*b^2*d^7*x^10 + 35/9*b^4*c^3*d
^4*x^9 + 28/3*a*b^3*c^2*d^5*x^9 + 14/3*a^2*b^2*c*d^6*x^9 + 4/9*a^3*b*d^7*x^
9 + 35/8*b^4*c^4*d^3*x^8 + 35/2*a*b^3*c^3*d^4*x^8 + 63/4*a^2*b^2*c^2*d^5*x^
8 + 7/2*a^3*b*c*d^6*x^8 + 1/8*a^4*d^7*x^8 + 3*b^4*c^5*d^2*x^7 + 20*a*b^3*c^
4*d^3*x^7 + 30*a^2*b^2*c^3*d^4*x^7 + 12*a^3*b*c^2*d^5*x^7 + a^4*c*d^6*x^7 +
 7/6*b^4*c^6*d*x^6 + 14*a*b^3*c^5*d^2*x^6 + 35*a^2*b^2*c^4*d^3*x^6 + 70/3*a
^3*b*c^3*d^4*x^6 + 7/2*a^4*c^2*d^5*x^6 + 1/5*b^4*c^7*x^5 + 28/5*a*b^3*c^6*d
*x^5 + 126/5*a^2*b^2*c^5*d^2*x^5 + 28*a^3*b*c^4*d^3*x^5 + 7*a^4*c^3*d^4*x^5
 + a*b^3*c^7*x^4 + 21/2*a^2*b^2*c^6*d*x^4 + 21*a^3*b*c^5*d^2*x^4 + 35/4*a^4
*c^4*d^3*x^4 + 2*a^2*b^2*c^7*x^3 + 28/3*a^3*b*c^6*d*x^3 + 7*a^4*c^5*d^2*x^3
 + 2*a^3*b*c^7*x^2 + 7/2*a^4*c^6*d*x^2 + a^4*c^7*x
```

maple [B] time = 0.00, size = 493, normalized size = 4.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^7,x)
```

```
[Out] 1/12*b^4*d^7*x^12+1/11*(4*a*b^3*d^7+7*b^4*c*d^6)*x^11+1/10*(6*a^2*b^2*d^7+2
8*a*b^3*c*d^6+21*b^4*c^2*d^5)*x^10+1/9*(4*a^3*b*d^7+42*a^2*b^2*c*d^6+84*a*b
^3*c^2*d^5+35*b^4*c^3*d^4)*x^9+1/8*(a^4*d^7+28*a^3*b*c*d^6+126*a^2*b^2*c^2*
d^5+140*a*b^3*c^3*d^4+35*b^4*c^4*d^3)*x^8+1/7*(7*a^4*c*d^6+84*a^3*b*c^2*d^5
+210*a^2*b^2*c^3*d^4+140*a*b^3*c^4*d^3+21*b^4*c^5*d^2)*x^7+1/6*(21*a^4*c^2*
d^5+140*a^3*b*c^3*d^4+210*a^2*b^2*c^4*d^3+84*a*b^3*c^5*d^2+7*b^4*c^6*d)*x^6
+1/5*(35*a^4*c^3*d^4+140*a^3*b*c^4*d^3+126*a^2*b^2*c^5*d^2+28*a*b^3*c^6*d+b
```

```
^4*c^7)*x^5+1/4*(35*a^4*c^4*d^3+84*a^3*b*c^5*d^2+42*a^2*b^2*c^6*d+4*a*b^3*c
^7)*x^4+1/3*(21*a^4*c^5*d^2+28*a^3*b*c^6*d+6*a^2*b^2*c^7)*x^3+1/2*(7*a^4*c^
6*d+4*a^3*b*c^7)*x^2+a^4*c^7*x
```

maxima [B] time = 1.48, size = 489, normalized size = 4.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/12*b^4*d^7*x^12 + a^4*c^7*x + 1/11*(7*b^4*c*d^6 + 4*a*b^3*d^7)*x^11 + 1/1
0*(21*b^4*c^2*d^5 + 28*a*b^3*c*d^6 + 6*a^2*b^2*d^7)*x^10 + 1/9*(35*b^4*c^3*
d^4 + 84*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 + 4*a^3*b*d^7)*x^9 + 1/8*(35*b^4*
c^4*d^3 + 140*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 + 28*a^3*b*c*d^6 + a^4*d^
7)*x^8 + (3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 30*a^2*b^2*c^3*d^4 + 12*a^3*b*
c^2*d^5 + a^4*c*d^6)*x^7 + 7/6*(b^4*c^6*d + 12*a*b^3*c^5*d^2 + 30*a^2*b^2*c
^4*d^3 + 20*a^3*b*c^3*d^4 + 3*a^4*c^2*d^5)*x^6 + 1/5*(b^4*c^7 + 28*a*b^3*c^
6*d + 126*a^2*b^2*c^5*d^2 + 140*a^3*b*c^4*d^3 + 35*a^4*c^3*d^4)*x^5 + 1/4*(
4*a*b^3*c^7 + 42*a^2*b^2*c^6*d + 84*a^3*b*c^5*d^2 + 35*a^4*c^4*d^3)*x^4 + 1
/3*(6*a^2*b^2*c^7 + 28*a^3*b*c^6*d + 21*a^4*c^5*d^2)*x^3 + 1/2*(4*a^3*b*c^7
 + 7*a^4*c^6*d)*x^2
```

mupad [B] time = 0.31, size = 470, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^7,x)
```

```
[Out] x^5*((b^4*c^7)/5 + 7*a^4*c^3*d^4 + 28*a^3*b*c^4*d^3 + (126*a^2*b^2*c^5*d^2)
/5 + (28*a*b^3*c^6*d)/5) + x^8*((a^4*d^7)/8 + (35*b^4*c^4*d^3)/8 + (35*a*b^
3*c^3*d^4)/2 + (63*a^2*b^2*c^2*d^5)/4 + (7*a^3*b*c*d^6)/2) + x^4*(a*b^3*c^7
 + (35*a^4*c^4*d^3)/4 + (21*a^2*b^2*c^6*d)/2 + 21*a^3*b*c^5*d^2) + x^9*((4*
a^3*b*d^7)/9 + (35*b^4*c^3*d^4)/9 + (28*a*b^3*c^2*d^5)/3 + (14*a^2*b^2*c*d^
6)/3) + x^7*(a^4*c*d^6 + 3*b^4*c^5*d^2 + 20*a*b^3*c^4*d^3 + 12*a^3*b*c^2*d^
5 + 30*a^2*b^2*c^3*d^4) + x^6*((7*b^4*c^6*d)/6 + (7*a^4*c^2*d^5)/2 + 14*a*b
^3*c^5*d^2 + (70*a^3*b*c^3*d^4)/3 + 35*a^2*b^2*c^4*d^3) + a^4*c^7*x + (b^4*
d^7*x^12)/12 + (a^3*c^6*x^2*(7*a*d + 4*b*c))/2 + (b^3*d^6*x^11*(4*a*d + 7*b
*c))/11 + (a^2*c^5*x^3*(21*a^2*d^2 + 6*b^2*c^2 + 28*a*b*c*d))/3 + (b^2*d^5*
x^10*(6*a^2*d^2 + 21*b^2*c^2 + 28*a*b*c*d))/10
```

sympy [B] time = 0.15, size = 549, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**7,x)
```

```
[Out] a**4*c**7*x + b**4*d**7*x**12/12 + x**11*(4*a*b**3*d**7/11 + 7*b**4*c*d**6/
11) + x**10*(3*a**2*b**2*d**7/5 + 14*a*b**3*c*d**6/5 + 21*b**4*c**2*d**5/10
) + x**9*(4*a**3*b*d**7/9 + 14*a**2*b**2*c*d**6/3 + 28*a*b**3*c**2*d**5/3 +
 35*b**4*c**3*d**4/9) + x**8*(a**4*d**7/8 + 7*a**3*b*c*d**6/2 + 63*a**2*b**
2*c**2*d**5/4 + 35*a*b**3*c**3*d**4/2 + 35*b**4*c**4*d**3/8) + x**7*(a**4*c
*d**6 + 12*a**3*b*c**2*d**5 + 30*a**2*b**2*c**3*d**4 + 20*a*b**3*c**4*d**3
+ 3*b**4*c**5*d**2) + x**6*(7*a**4*c**2*d**5/2 + 70*a**3*b*c**3*d**4/3 + 35
*a**2*b**2*c**4*d**3 + 14*a*b**3*c**5*d**2 + 7*b**4*c**6*d/6) + x**5*(7*a**
4*c**3*d**4 + 28*a**3*b*c**4*d**3 + 126*a**2*b**2*c**5*d**2/5 + 28*a*b**3*c
**6*d/5 + b**4*c**7/5) + x**4*(35*a**4*c**4*d**3/4 + 21*a**3*b*c**5*d**2 +
21*a**2*b**2*c**6*d/2 + a*b**3*c**7) + x**3*(7*a**4*c**5*d**2 + 28*a**3*b*c
**6*d/3 + 2*a**2*b**2*c**7) + x**2*(7*a**4*c**6*d/2 + 2*a**3*b*c**7)
```

3.1173 $\int (a+bx)^3(c+dx)^7\,dx$

Optimal. Leaf size=92

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Rubi [A] time = 0.22, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{10}(bc-ad)}{10d^4}+\frac{b(c+dx)^9(bc-ad)^2}{3d^4}-\frac{(c+dx)^8(bc-ad)^3}{8d^4}+\frac{b^3(c+dx)^{11}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^8)/(8*d^4) + (b*(b*c - a*d)^2*(c + d*x)^9)/(3*d^4
) - (3*b^2*(b*c - a*d)*(c + d*x)^10)/(10*d^4) + (b^3*(c + d*x)^11)/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^7\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^7}{d^3}+\frac{3b(bc-ad)^2(c+dx)^8}{d^3}-\frac{3b^2(bc-ad)(c+dx)^9}{d^3}+\frac{b^3(c+}{d}\right.$$

$$=-\frac{(bc-ad)^3(c+dx)^8}{8d^4}+\frac{b(bc-ad)^2(c+dx)^9}{3d^4}-\frac{3b^2(bc-ad)(c+dx)^{10}}{10d^4}+\frac{b^3(c+dx)^1}{11d^4}$$

Mathematica [B] time = 0.04, size = 360, normalized size = 3.91

$$a^3c^7x+\frac{1}{3}bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)+ac^5x^3\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{1}{2}a^2c^6x^2(7ad+3bc)+cd^3x^7\left(a^3d^3+9a^2bcd^2+15ab^2c^2d+5b^3c^3\right)+\frac{7}{2}c^2d^2x^6\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)+\frac{7}{5}c^3dx^5\left(5a^3d^3+15a^2bcd^2+9ab^2c^2d+b^3c^3\right)+\frac{1}{8}d^4x^8\left(a^3d^3+21a^2bcd^2+63ab^2c^2d+35b^3c^3\right)+\frac{1}{4}c^4x^4\left(35a^3d^3+63a^2bcd^2+21ab^2c^2d+b^3c^3\right)+\frac{1}{10}b^2d^6x^{10}(3ad+7bc)+\frac{1}{11}b^3d^7x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] a^3*c^7*x + (a^2*c^6*(3*b*c + 7*a*d)*x^2)/2 + a*c^5*(b^2*c^2 + 7*a*b*c*d +
7*a^2*d^2)*x^3 + (c^4*(b^3*c^3 + 21*a*b^2*c^2*d + 63*a^2*b*c*d^2 + 35*a^3*d
^3)*x^4)/4 + (7*c^3*d*(b^3*c^3 + 9*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
)*x^5)/5 + (7*c^2*d^2*(b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2 + a^3*d^3)*x
^6)/2 + c*d^3*(5*b^3*c^3 + 15*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*x^7 +
(d^4*(35*b^3*c^3 + 63*a*b^2*c^2*d + 21*a^2*b*c*d^2 + a^3*d^3)*x^8)/8 + (b*d
^5*(7*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*x^9)/3 + (b^2*d^6*(7*b*c + 3*a*d)*x^10
)/10 + (b^3*d^7*x^11)/11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^7, x]
```

fricas [B] time = 1.25, size = 420, normalized size = 4.57

$$\frac{1}{11}x^{11}d^7b^3 + \frac{7}{10}x^{10}d^6cb^3 + \frac{3}{10}x^{10}d^7b^2a + \frac{7}{3}x^9d^5c^2b^3 + \frac{7}{3}x^9d^6cb^2a + \frac{1}{3}x^9d^7ba^2 + \frac{35}{8}x^8d^4c^3b^3 + \frac{63}{8}x^8d^5c^2b^2a + \frac{21}{8}x^8d^6cba^2 + \frac{1}{8}x^8d^7a^3 + 5x^7d^3c^4b^3 + 15x^7d^4c^3b^2a + 9x^7d^5c^2ba^2 + x^7d^6ca^3 + \frac{7}{2}x^6d^2c^5b^3 + \frac{35}{2}x^6d^3c^4b^2a + \frac{35}{2}x^6d^4c^3ba^2 + \frac{7}{2}x^6d^5c^2a^3 + \frac{7}{5}x^5dc^6b^3 + \frac{63}{5}x^5d^2c^5b^2a + 21x^5d^3c^4ba^2 + 7x^5d^4c^3a^3 + \frac{1}{4}x^4c^7b^3 + \frac{21}{4}x^4dc^6b^2a + \frac{63}{4}x^4d^2c^5ba^2 + \frac{35}{4}x^4d^3c^4a^3 + x^3c^7b^2a + 7x^3dc^6ba^2 + 7x^3d^2c^5a^3 + \frac{3}{2}x^2c^7ba^2 + \frac{7}{2}x^2dc^6a^3 + xc^7a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^7*b^3 + 7/10*x^10*d^6*c*b^3 + 3/10*x^10*d^7*b^2*a + 7/3*x^9*d^5
*c^2*b^3 + 7/3*x^9*d^6*c*b^2*a + 1/3*x^9*d^7*b*a^2 + 35/8*x^8*d^4*c^3*b^3 +
 63/8*x^8*d^5*c^2*b^2*a + 21/8*x^8*d^6*c*b*a^2 + 1/8*x^8*d^7*a^3 + 5*x^7*d^
3*c^4*b^3 + 15*x^7*d^4*c^3*b^2*a + 9*x^7*d^5*c^2*b*a^2 + x^7*d^6*c*a^3 + 7/
2*x^6*d^2*c^5*b^3 + 35/2*x^6*d^3*c^4*b^2*a + 35/2*x^6*d^4*c^3*b*a^2 + 7/2*x
^6*d^5*c^2*a^3 + 7/5*x^5*d*c^6*b^3 + 63/5*x^5*d^2*c^5*b^2*a + 21*x^5*d^3*c^
4*b*a^2 + 7*x^5*d^4*c^3*a^3 + 1/4*x^4*c^7*b^3 + 21/4*x^4*d*c^6*b^2*a + 63/4
*x^4*d^2*c^5*b*a^2 + 35/4*x^4*d^3*c^4*a^3 + x^3*c^7*b^2*a + 7*x^3*d*c^6*b*a
^2 + 7*x^3*d^2*c^5*a^3 + 3/2*x^2*c^7*b*a^2 + 7/2*x^2*d*c^6*a^3 + x*c^7*a^3
```

giac [B] time = 1.26, size = 420, normalized size = 4.57

$$\frac{1}{11}b^3d^7x^{11} + \frac{7}{10}b^3cd^6x^{10} + \frac{3}{10}ab^2d^7x^{10} + \frac{7}{3}b^3c^2d^5x^9 + \frac{7}{3}ab^2cd^6x^9 + \frac{1}{3}a^2bd^7x^9 + \frac{35}{8}b^3c^3d^4x^8 + \frac{63}{8}ab^2c^2d^5x^8 + \frac{21}{8}a^2bcd^6x^8 + \frac{1}{8}a^3d^7x^8 + 5b^3c^4d^3x^7 + 15ab^2c^3d^4x^7 + 9a^2bc^2d^5x^7 + a^3cd^6x^7 + \frac{7}{2}b^3c^5d^2x^6 + \frac{35}{2}ab^2c^4d^3x^6 + \frac{35}{2}a^2bc^3d^4x^6 + \frac{7}{2}a^3c^2d^5x^6 + \frac{7}{5}b^3c^6dx^5 + \frac{63}{5}ab^2c^5d^2x^5 + 21a^2bc^4d^3x^5 + 7a^3c^3d^4x^5 + \frac{1}{4}b^3c^7x^4 + \frac{21}{4}ab^2c^6dx^4 + \frac{63}{4}a^2bc^5d^2x^4 + \frac{35}{4}a^3c^4d^3x^4 + ab^2c^7x^3 + 7a^2bc^6dx^3 + 7a^3c^5d^2x^3 + \frac{3}{2}a^2bc^7x^2 + \frac{7}{2}a^3c^6dx^2 + a^3c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/11*b^3*d^7*x^11 + 7/10*b^3*c*d^6*x^10 + 3/10*a*b^2*d^7*x^10 + 7/3*b^3*c^2
*d^5*x^9 + 7/3*a*b^2*c*d^6*x^9 + 1/3*a^2*b*d^7*x^9 + 35/8*b^3*c^3*d^4*x^8 +
 63/8*a*b^2*c^2*d^5*x^8 + 21/8*a^2*b*c*d^6*x^8 + 1/8*a^3*d^7*x^8 + 5*b^3*c^
4*d^3*x^7 + 15*a*b^2*c^3*d^4*x^7 + 9*a^2*b*c^2*d^5*x^7 + a^3*c*d^6*x^7 + 7/
2*b^3*c^5*d^2*x^6 + 35/2*a*b^2*c^4*d^3*x^6 + 35/2*a^2*b*c^3*d^4*x^6 + 7/2*a
^3*c^2*d^5*x^6 + 7/5*b^3*c^6*d*x^5 + 63/5*a*b^2*c^5*d^2*x^5 + 21*a^2*b*c^4*
d^3*x^5 + 7*a^3*c^3*d^4*x^5 + 1/4*b^3*c^7*x^4 + 21/4*a*b^2*c^6*d*x^4 + 63/4
*a^2*b*c^5*d^2*x^4 + 35/4*a^3*c^4*d^3*x^4 + a*b^2*c^7*x^3 + 7*a^2*b*c^6*d*x
^3 + 7*a^3*c^5*d^2*x^3 + 3/2*a^2*b*c^7*x^2 + 7/2*a^3*c^6*d*x^2 + a^3*c^7*x
```

maple [B] time = 0.00, size = 385, normalized size = 4.18

$$\frac{b^3d^7x^{11}}{11} + a^3c^7x + \frac{\left(3a b^2d^7 + 7b^3c d^6\right)x^{10}}{10} + \frac{\left(3a^2b d^7 + 21a b^2c d^6 + 21b^3c^2d^5\right)x^9}{9} + \frac{\left(a^3d^7 + 21a^2bc d^6 + 63a b^2c^2d^5 + 35b^3c^3d^4\right)x^8}{8} + \frac{\left(7a^3c d^6 + 63a^2b c^2d^5 + 105a b^2c^3d^4 + 35b^3c^4d^3\right)x^7}{7} + \frac{\left(21a^3c^2d^5 + 105a^2b c^3d^4 + 105a b^2c^4d^3 + 21b^3c^5d^2\right)x^6}{6} + \frac{\left(35a^3c^3d^4 + 105a^2b c^4d^3 + 63a b^2c^5d^2 + 7b^3c^6d\right)x^5}{5} + \frac{\left(35a^3c^4d^3 + 63a^2b c^5d^2 + 21a b^2c^6d + b^3c^7\right)x^4}{4} + \frac{\left(21a^3c^5d^2 + 21a^2b c^6d + 3a b^2c^7\right)x^3}{3} + \frac{\left(7a^3c^6d + 3a^2b c^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^7,x)
```

```
[Out] 1/11*b^3*d^7*x^11+1/10*(3*a*b^2*d^7+7*b^3*c*d^6)*x^10+1/9*(3*a^2*b*d^7+21*a
*b^2*c*d^6+21*b^3*c^2*d^5)*x^9+1/8*(a^3*d^7+21*a^2*b*c*d^6+63*a*b^2*c^2*d^5
+35*b^3*c^3*d^4)*x^8+1/7*(7*a^3*c*d^6+63*a^2*b*c^2*d^5+105*a*b^2*c^3*d^4+35
*b^3*c^4*d^3)*x^7+1/6*(21*a^3*c^2*d^5+105*a^2*b*c^3*d^4+105*a*b^2*c^4*d^3+2
1*b^3*c^5*d^2)*x^6+1/5*(35*a^3*c^3*d^4+105*a^2*b*c^4*d^3+63*a*b^2*c^5*d^2+7
*b^3*c^6*d)*x^5+1/4*(35*a^3*c^4*d^3+63*a^2*b*c^5*d^2+21*a*b^2*c^6*d+b^3*c^7
)*x^4+1/3*(21*a^3*c^5*d^2+21*a^2*b*c^6*d+3*a*b^2*c^7)*x^3+1/2*(7*a^3*c^6*d+
3*a^2*b*c^7)*x^2+a^3*c^7*x
```

maxima [B] time = 1.37, size = 376, normalized size = 4.09

$$\frac{1}{11}b^3d^7x^{11} + a^3c^7x + \frac{1}{10}\left(7b^3cd^6 + 3ab^2d^7\right)x^{10} + \frac{1}{3}\left(7b^3c^2d^5 + 7ab^2cd^6 + a^2bd^7\right)x^9 + \frac{1}{8}\left(35b^3c^3d^4 + 63ab^2c^2d^5 + 21a^2bcd^6 + a^3d^7\right)x^8 + \left(5b^3c^4d^3 + 15ab^2c^3d^4 + 9a^2bc^2d^5 + a^3cd^6\right)x^7 + \frac{7}{2}\left(b^3c^5d^2 + 5ab^2c^4d^3 + 5a^2bc^3d^4 + a^3c^2d^5\right)x^6 + \frac{7}{5}\left(b^3c^6d + 9ab^2c^5d^2 + 15a^2bc^4d^3 + 5a^3c^3d^4\right)x^5 + \frac{1}{4}\left(b^3c^7 + 21ab^2c^6d + 63a^2bc^5d^2 + 35a^3c^4d^3\right)x^4 + \left(ab^2c^7 + 7a^2bc^6d + 7a^3c^5d^2\right)x^3 + \frac{1}{2}\left(3a^2bc^7 + 7a^3c^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/11*b^3*d^7*x^11 + a^3*c^7*x + 1/10*(7*b^3*c*d^6 + 3*a*b^2*d^7)*x^10 + 1/3
*(7*b^3*c^2*d^5 + 7*a*b^2*c*d^6 + a^2*b*d^7)*x^9 + 1/8*(35*b^3*c^3*d^4 + 63
*a*b^2*c^2*d^5 + 21*a^2*b*c*d^6 + a^3*d^7)*x^8 + (5*b^3*c^4*d^3 + 15*a*b^2*
c^3*d^4 + 9*a^2*b*c^2*d^5 + a^3*c*d^6)*x^7 + 7/2*(b^3*c^5*d^2 + 5*a*b^2*c^4
*d^3 + 5*a^2*b*c^3*d^4 + a^3*c^2*d^5)*x^6 + 7/5*(b^3*c^6*d + 9*a*b^2*c^5*d^
2 + 15*a^2*b*c^4*d^3 + 5*a^3*c^3*d^4)*x^5 + 1/4*(b^3*c^7 + 21*a*b^2*c^6*d +
 63*a^2*b*c^5*d^2 + 35*a^3*c^4*d^3)*x^4 + (a*b^2*c^7 + 7*a^2*b*c^6*d + 7*a^
3*c^5*d^2)*x^3 + 1/2*(3*a^2*b*c^7 + 7*a^3*c^6*d)*x^2
```

mupad [B] time = 0.27, size = 356, normalized size = 3.87

$$x^7\left(a^3cd^6+9a^2bc^2d^5+15ab^2c^3d^4+5b^3c^4d^3\right)+x^5\left(7a^3c^3d^4+21a^2bc^4d^3+\frac{63ab^2c^5d^2}{5}+\frac{7b^3c^6d}{5}\right)+x^4\left(\frac{35a^3c^4d^3}{4}+\frac{63a^2bc^5d^2}{4}+\frac{21ab^2c^6d}{4}+\frac{b^3c^7}{4}\right)+x^8\left(\frac{a^3d^7}{8}+\frac{21a^2bcd^6}{8}+\frac{63ab^2c^2d^5}{8}+\frac{35b^3c^3d^4}{8}\right)+a^3c^7x+\frac{b^3d^7x^{11}}{11}+\frac{7c^2d^2x^6\left(a^3d^3+5a^2bcd^2+5ab^2c^2d+b^3c^3\right)}{2}+\frac{a^2c^6x^2\left(7ad+3bc\right)}{2}+\frac{b^2d^6x^{10}\left(3ad+7bc\right)}{10}+ac^5x^3\left(7a^2d^2+7abcd+b^2c^2\right)+\frac{bd^5x^9\left(a^2d^2+7abcd+7b^2c^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^7,x)
```

```
[Out] x^7*(a^3*c*d^6 + 5*b^3*c^4*d^3 + 15*a*b^2*c^3*d^4 + 9*a^2*b*c^2*d^5) + x^5*
((7*b^3*c^6*d)/5 + 7*a^3*c^3*d^4 + (63*a*b^2*c^5*d^2)/5 + 21*a^2*b*c^4*d^3)
 + x^4*((b^3*c^7)/4 + (35*a^3*c^4*d^3)/4 + (63*a^2*b*c^5*d^2)/4 + (21*a*b^2
*c^6*d)/4) + x^8*((a^3*d^7)/8 + (35*b^3*c^3*d^4)/8 + (63*a*b^2*c^2*d^5)/8 +
 (21*a^2*b*c*d^6)/8) + a^3*c^7*x + (b^3*d^7*x^11)/11 + (7*c^2*d^2*x^6*(a^3*
d^3 + b^3*c^3 + 5*a*b^2*c^2*d + 5*a^2*b*c*d^2))/2 + (a^2*c^6*x^2*(7*a*d + 3
*b*c))/2 + (b^2*d^6*x^10*(3*a*d + 7*b*c))/10 + a*c^5*x^3*(7*a^2*d^2 + b^2*c
^2 + 7*a*b*c*d) + (b*d^5*x^9*(a^2*d^2 + 7*b^2*c^2 + 7*a*b*c*d))/3
```

sympy [B] time = 0.13, size = 427, normalized size = 4.64

$$a^3c^7x+\frac{b^3d^7x^{11}}{11}+x^{10}\left(\frac{3ab^2d^7}{10}+\frac{7b^3cd^6}{10}\right)+x^9\left(\frac{a^2bd^7}{3}+\frac{7ab^2cd^6}{3}+\frac{7b^3c^2d^5}{3}\right)+x^8\left(\frac{a^3d^7}{8}+\frac{21a^2bcd^6}{8}+\frac{63ab^2c^2d^5}{8}+\frac{35b^3c^3d^4}{8}\right)+x^7\left(a^3cd^6+9a^2bc^2d^5+15ab^2c^3d^4+5b^3c^4d^3\right)+x^6\left(\frac{7a^3c^2d^5}{2}+\frac{35a^2bc^3d^4}{2}+\frac{35ab^2c^4d^3}{2}+\frac{7b^3c^5d^2}{2}\right)+x^5\left(7a^3c^3d^4+21a^2bc^4d^3+\frac{63ab^2c^5d^2}{5}+\frac{7b^3c^6d}{5}\right)+x^4\left(\frac{35a^3c^4d^3}{4}+\frac{63a^2bc^5d^2}{4}+\frac{21ab^2c^6d}{4}+\frac{b^3c^7}{4}\right)+x^3\left(7a^3c^5d^2+7a^2bc^6d+ab^2c^7\right)+x^2\left(\frac{7a^3c^6d}{2}+\frac{3a^2bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**7,x)
```

```
[Out] a**3*c**7*x + b**3*d**7*x**11/11 + x**10*(3*a*b**2*d**7/10 + 7*b**3*c*d**6/
10) + x**9*(a**2*b*d**7/3 + 7*a*b**2*c*d**6/3 + 7*b**3*c**2*d**5/3) + x**8*
(a**3*d**7/8 + 21*a**2*b*c*d**6/8 + 63*a*b**2*c**2*d**5/8 + 35*b**3*c**3*d*
*4/8) + x**7*(a**3*c*d**6 + 9*a**2*b*c**2*d**5 + 15*a*b**2*c**3*d**4 + 5*b*
*3*c**4*d**3) + x**6*(7*a**3*c**2*d**5/2 + 35*a**2*b*c**3*d**4/2 + 35*a*b**
2*c**4*d**3/2 + 7*b**3*c**5*d**2/2) + x**5*(7*a**3*c**3*d**4 + 21*a**2*b*c*
*4*d**3 + 63*a*b**2*c**5*d**2/5 + 7*b**3*c**6*d/5) + x**4*(35*a**3*c**4*d**
3/4 + 63*a**2*b*c**5*d**2/4 + 21*a*b**2*c**6*d/4 + b**3*c**7/4) + x**3*(7*a
**3*c**5*d**2 + 7*a**2*b*c**6*d + a*b**2*c**7) + x**2*(7*a**3*c**6*d/2 + 3*
a**2*b*c**7/2)
```

3.1174 $\int (a+bx)^2(c+dx)^7\,dx$

Optimal. Leaf size=65

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Rubi [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b(c+dx)^9(bc-ad)}{9d^3}+\frac{(c+dx)^8(bc-ad)^2}{8d^3}+\frac{b^2(c+dx)^{10}}{10d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^8)/(8*d^3) - (2*b*(b*c - a*d)*(c + d*x)^9)/(9*d^3)
 + (b^2*(c + d*x)^10)/(10*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^7}{d^2}-\frac{2b(bc-ad)(c+dx)^8}{d^2}+\frac{b^2(c+dx)^9}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^8}{8d^3}-\frac{2b(bc-ad)(c+dx)^9}{9d^3}+\frac{b^2(c+dx)^{10}}{10d^3}\end{aligned}$$

Mathematica [B] time = 0.03, size = 261, normalized size = 4.02

$$\frac{1}{8}d^5x^8\left(a^2d^2+14abcd+21b^2c^2\right)+cd^4x^7\left(a^2d^2+6abcd+5b^2c^2\right)+\frac{7}{6}c^2d^3x^6\left(3a^2d^2+10abcd+5b^2c^2\right)+\frac{1}{3}c^5x^3\left(21a^2d^2+14abcd+b^2c^2\right)+\frac{7}{4}c^4dx^4\left(5a^2d^2+6abcd+b^2c^2\right)+\frac{7}{5}c^3d^2x^5\left(5a^2d^2+10abcd+3b^2c^2\right)+a^2c^7x+\frac{1}{2}ac^6x^2(7ad+2bc)+\frac{1}{9}bd^6x^9(2ad+7bc)+\frac{1}{10}b^2d^7x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] a^2*c^7*x + (a*c^6*(2*b*c + 7*a*d)*x^2)/2 + (c^5*(b^2*c^2 + 14*a*b*c*d + 21
*a^2*d^2)*x^3)/3 + (7*c^4*d*(b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*x^4)/4 + (7*c
^3*d^2*(3*b^2*c^2 + 10*a*b*c*d + 5*a^2*d^2)*x^5)/5 + (7*c^2*d^3*(5*b^2*c^2
+ 10*a*b*c*d + 3*a^2*d^2)*x^6)/6 + c*d^4*(5*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
x^7 + (d^5*(21*b^2*c^2 + 14*a*b*c*d + a^2*d^2)*x^8)/8 + (b*d^6*(7*b*c + 2*a
*d)*x^9)/9 + (b^2*d^7*x^10)/10
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^7, x]
```

fricas [B] time = 1.21, size = 294, normalized size = 4.52

$$\frac{1}{10}x^{10}d^7b^2+\frac{7}{9}x^9d^6cb^2+\frac{2}{9}x^9d^7ba+\frac{21}{8}x^8d^5c^2b^2+\frac{7}{4}x^8d^6cba+\frac{1}{8}x^8d^7a^2+5x^7d^4c^3b^2+6x^7d^5c^2ba+x^7d^6ca^2+\frac{35}{6}x^6d^3c^4b^2+\frac{35}{3}x^6d^4c^3ba+\frac{7}{2}x^6d^5c^2a^2+\frac{21}{5}x^5d^2c^5b^2+14x^5d^3c^4ba+7x^5d^4c^3a^2+\frac{7}{4}x^4dc^6b^2+\frac{21}{2}x^4d^2c^5ba+\frac{35}{4}x^4d^3c^4a^2+\frac{1}{3}x^3c^7b^2+\frac{14}{3}x^3dc^6ba+7x^3d^2c^5a^2+x^2c^7ba+\frac{7}{2}x^2dc^6a^2+xc^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^7*b^2 + 7/9*x^9*d^6*c*b^2 + 2/9*x^9*d^7*b*a + 21/8*x^8*d^5*c^2*
b^2 + 7/4*x^8*d^6*c*b*a + 1/8*x^8*d^7*a^2 + 5*x^7*d^4*c^3*b^2 + 6*x^7*d^5*c
^2*b*a + x^7*d^6*c*a^2 + 35/6*x^6*d^3*c^4*b^2 + 35/3*x^6*d^4*c^3*b*a + 7/2*
x^6*d^5*c^2*a^2 + 21/5*x^5*d^2*c^5*b^2 + 14*x^5*d^3*c^4*b*a + 7*x^5*d^4*c^3
*a^2 + 7/4*x^4*d*c^6*b^2 + 21/2*x^4*d^2*c^5*b*a + 35/4*x^4*d^3*c^4*a^2 + 1/
3*x^3*c^7*b^2 + 14/3*x^3*d*c^6*b*a + 7*x^3*d^2*c^5*a^2 + x^2*c^7*b*a + 7/2*
x^2*d*c^6*a^2 + x*c^7*a^2
```

giac [B] time = 1.24, size = 294, normalized size = 4.52

$$\frac{1}{10}b^2d^7x^{10}+\frac{7}{9}b^2cd^6x^9+\frac{2}{9}abd^7x^9+\frac{21}{8}b^2c^2d^5x^8+\frac{7}{4}abcd^6x^8+\frac{1}{8}a^2d^7x^8+5b^2c^3d^4x^7+6abc^2d^5x^7+a^2cd^6x^7+\frac{35}{6}b^2c^4d^3x^6+\frac{35}{3}abc^3d^4x^6+\frac{7}{2}a^2c^2d^5x^6+\frac{21}{5}b^2c^5d^2x^5+14abc^4d^3x^5+7a^2c^3d^4x^5+\frac{7}{4}b^2c^6dx^4+\frac{21}{2}abc^5d^2x^4+\frac{35}{4}a^2c^4d^3x^4+\frac{1}{3}b^2c^7x^3+\frac{14}{3}abc^6dx^3+7a^2c^5d^2x^3+abc^7x^2+\frac{7}{2}a^2c^6dx^2+a^2c^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/10*b^2*d^7*x^10 + 7/9*b^2*c*d^6*x^9 + 2/9*a*b*d^7*x^9 + 21/8*b^2*c^2*d^5*
x^8 + 7/4*a*b*c*d^6*x^8 + 1/8*a^2*d^7*x^8 + 5*b^2*c^3*d^4*x^7 + 6*a*b*c^2*d
^5*x^7 + a^2*c*d^6*x^7 + 35/6*b^2*c^4*d^3*x^6 + 35/3*a*b*c^3*d^4*x^6 + 7/2*
a^2*c^2*d^5*x^6 + 21/5*b^2*c^5*d^2*x^5 + 14*a*b*c^4*d^3*x^5 + 7*a^2*c^3*d^4
*x^5 + 7/4*b^2*c^6*d*x^4 + 21/2*a*b*c^5*d^2*x^4 + 35/4*a^2*c^4*d^3*x^4 + 1/
3*b^2*c^7*x^3 + 14/3*a*b*c^6*d*x^3 + 7*a^2*c^5*d^2*x^3 + a*b*c^7*x^2 + 7/2*
a^2*c^6*d*x^2 + a^2*c^7*x
```

maple [B] time = 0.00, size = 277, normalized size = 4.26

$$\frac{b^2d^7x^{10}}{10}+a^2c^7x+\frac{\left(2abd^7+7b^2cd^6\right)x^9}{9}+\frac{\left(a^2d^7+14abcd^6+21b^2c^2d^5\right)x^8}{8}+\frac{\left(7a^2cd^6+42abc^2d^5+35b^2c^3d^4\right)x^7}{7}+\frac{\left(21a^2c^2d^5+70abc^3d^4+35b^2c^4d^3\right)x^6}{6}+\frac{\left(35a^2c^3d^4+70abc^4d^3+21b^2c^5d^2\right)x^5}{5}+\frac{\left(35a^2c^4d^3+42abc^5d^2+7b^2c^6d\right)x^4}{4}+\frac{\left(21a^2c^5d^2+14abc^6d+b^2c^7\right)x^3}{3}+\frac{\left(7a^2c^6d+2abc^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^7,x)
```

```
[Out] 1/10*b^2*d^7*x^10+1/9*(2*a*b*d^7+7*b^2*c*d^6)*x^9+1/8*(a^2*d^7+14*a*b*c*d^6
+21*b^2*c^2*d^5)*x^8+1/7*(7*a^2*c*d^6+42*a*b*c^2*d^5+35*b^2*c^3*d^4)*x^7+1/
6*(21*a^2*c^2*d^5+70*a*b*c^3*d^4+35*b^2*c^4*d^3)*x^6+1/5*(35*a^2*c^3*d^4+70
*a*b*c^4*d^3+21*b^2*c^5*d^2)*x^5+1/4*(35*a^2*c^4*d^3+42*a*b*c^5*d^2+7*b^2*c
^6*d)*x^4+1/3*(21*a^2*c^5*d^2+14*a*b*c^6*d+b^2*c^7)*x^3+1/2*(7*a^2*c^6*d+2*
a*b*c^7)*x^2+a^2*c^7*x
```

maxima [B] time = 1.36, size = 273, normalized size = 4.20

$$\frac{1}{10}b^2d^7x^{10}+a^2c^7x+\frac{1}{9}\left(7b^2cd^6+2abd^7\right)x^9+\frac{1}{8}\left(21b^2c^2d^5+14abcd^6+a^2d^7\right)x^8+\left(5b^2c^3d^4+6abc^2d^5+a^2cd^6\right)x^7+\frac{7}{6}\left(5b^2c^4d^3+10abc^3d^4+3a^2c^2d^5\right)x^6+\frac{7}{5}\left(3b^2c^5d^2+10abc^4d^3+5a^2c^3d^4\right)x^5+\frac{7}{4}\left(b^2c^6d+6abc^5d^2+5a^2c^4d^3\right)x^4+\frac{1}{3}\left(b^2c^7+14abc^6d+21a^2c^5d^2\right)x^3+\frac{1}{2}\left(2abc^7+7a^2c^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d^7*x^10 + a^2*c^7*x + 1/9*(7*b^2*c*d^6 + 2*a*b*d^7)*x^9 + 1/8*(21
*b^2*c^2*d^5 + 14*a*b*c*d^6 + a^2*d^7)*x^8 + (5*b^2*c^3*d^4 + 6*a*b*c^2*d^5
 + a^2*c*d^6)*x^7 + 7/6*(5*b^2*c^4*d^3 + 10*a*b*c^3*d^4 + 3*a^2*c^2*d^5)*x^
6 + 7/5*(3*b^2*c^5*d^2 + 10*a*b*c^4*d^3 + 5*a^2*c^3*d^4)*x^5 + 7/4*(b^2*c^6
*d + 6*a*b*c^5*d^2 + 5*a^2*c^4*d^3)*x^4 + 1/3*(b^2*c^7 + 14*a*b*c^6*d + 21*
a^2*c^5*d^2)*x^3 + 1/2*(2*a*b*c^7 + 7*a^2*c^6*d)*x^2
```

mupad [B] time = 0.11, size = 249, normalized size = 3.83

$$x^3\left(7a^2c^5d^2+\frac{14abc^6d}{3}+\frac{b^2c^7}{3}\right)+x^8\left(\frac{a^2d^7}{8}+\frac{7abcd^6}{4}+\frac{21b^2c^2d^5}{8}\right)+a^2c^7x+\frac{b^2d^7x^{10}}{10}+\frac{ac^6x^2\,(7ad+2bc)}{2}+\frac{bd^6x^9\,(2ad+7bc)}{9}+\frac{7c^4dx^4\left(5a^2d^2+6abcd+b^2c^2\right)}{4}+cd^4x^7\left(a^2d^2+6abcd+5b^2c^2\right)+\frac{7c^3d^2x^5\left(5a^2d^2+10abcd+3b^2c^2\right)}{5}+\frac{7c^2d^3x^6\left(3a^2d^2+10abcd+5b^2c^2\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^7,x)
```

```
[Out] x^3*((b^2*c^7)/3 + 7*a^2*c^5*d^2 + (14*a*b*c^6*d)/3) + x^8*((a^2*d^7)/8 + (
21*b^2*c^2*d^5)/8 + (7*a*b*c*d^6)/4) + a^2*c^7*x + (b^2*d^7*x^10)/10 + (a*c
^6*x^2*(7*a*d + 2*b*c))/2 + (b*d^6*x^9*(2*a*d + 7*b*c))/9 + (7*c^4*d*x^4*(5
*a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/4 + c*d^4*x^7*(a^2*d^2 + 5*b^2*c^2 + 6*a*b
*c*d) + (7*c^3*d^2*x^5*(5*a^2*d^2 + 3*b^2*c^2 + 10*a*b*c*d))/5 + (7*c^2*d^3
*x^6*(3*a^2*d^2 + 5*b^2*c^2 + 10*a*b*c*d))/6
```

sympy [B] time = 0.12, size = 303, normalized size = 4.66

$$a^2c^7x+\frac{b^2d^7x^{10}}{10}+x^9\left(\frac{2abd^7}{9}+\frac{7b^2cd^6}{9}\right)+x^8\left(\frac{a^2d^7}{8}+\frac{7abcd^6}{4}+\frac{21b^2c^2d^5}{8}\right)+x^7\left(a^2cd^6+6abc^2d^5+5b^2c^3d^4\right)+x^6\left(\frac{7a^2c^2d^5}{2}+\frac{35abc^3d^4}{3}+\frac{35b^2c^4d^3}{6}\right)+x^5\left(7a^2c^3d^4+14abc^4d^3+\frac{21b^2c^5d^2}{5}\right)+x^4\left(\frac{35a^2c^4d^3}{4}+\frac{21abc^5d^2}{2}+\frac{7b^2c^6d}{4}\right)+x^3\left(7a^2c^5d^2+\frac{14abc^6d}{3}+\frac{b^2c^7}{3}\right)+x^2\left(\frac{7a^2c^6d}{2}+abc^7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**7,x)
```

```
[Out] a**2*c**7*x + b**2*d**7*x**10/10 + x**9*(2*a*b*d**7/9 + 7*b**2*c*d**6/9) +
x**8*(a**2*d**7/8 + 7*a*b*c*d**6/4 + 21*b**2*c**2*d**5/8) + x**7*(a**2*c*d*
*6 + 6*a*b*c**2*d**5 + 5*b**2*c**3*d**4) + x**6*(7*a**2*c**2*d**5/2 + 35*a*
b*c**3*d**4/3 + 35*b**2*c**4*d**3/6) + x**5*(7*a**2*c**3*d**4 + 14*a*b*c**4
*d**3 + 21*b**2*c**5*d**2/5) + x**4*(35*a**2*c**4*d**3/4 + 21*a*b*c**5*d**2
/2 + 7*b**2*c**6*d/4) + x**3*(7*a**2*c**5*d**2 + 14*a*b*c**6*d/3 + b**2*c**
7/3) + x**2*(7*a**2*c**6*d/2 + a*b*c**7)
```

3.1175 $\int (a+bx)(c+dx)^7\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^9}{9d^2}-\frac{(c+dx)^8(bc-ad)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^8)/(8*d^2) + (b*(c + d*x)^9)/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^7\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^7}{d}+\frac{b(c+dx)^8}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^8}{8d^2}+\frac{b(c+dx)^9}{9d^2}\end{aligned}$$

Mathematica [B] time = 0.02, size = 151, normalized size = 3.97

$$\frac{1}{2}c^6x^2(7ad+bc)+\frac{7}{3}c^5dx^3(3ad+bc)+\frac{7}{4}c^4d^2x^4(5ad+3bc)+7c^3d^3x^5(ad+bc)+\frac{7}{6}c^2d^4x^6(3ad+5bc)+\frac{1}{8}d^6x^8(ad+7bc)+cd^5x^7(ad+3bc)+ac^7x+\frac{1}{9}bd^7x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] a*c^7*x + (c^6*(b*c + 7*a*d)*x^2)/2 + (7*c^5*d*(b*c + 3*a*d)*x^3)/3 + (7*c^
4*d^2*(3*b*c + 5*a*d)*x^4)/4 + 7*c^3*d^3*(b*c + a*d)*x^5 + (7*c^2*d^4*(5*b*
c + 3*a*d)*x^6)/6 + c*d^5*(3*b*c + a*d)*x^7 + (d^6*(7*b*c + a*d)*x^8)/8 + (
b*d^7*x^9)/9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^7, x]
```

fricas [B] time = 1.57, size = 169, normalized size = 4.45

$$\frac{1}{9}x^9d^7b+\frac{7}{8}x^8d^6cb+\frac{1}{8}x^8d^7a+3x^7d^5c^2b+x^7d^6ca+\frac{35}{6}x^6d^4c^3b+\frac{7}{2}x^6d^5c^2a+7x^5d^3c^4b+7x^5d^4c^3a+\frac{21}{4}x^4d^2c^5b+\frac{35}{4}x^4d^3c^4a+\frac{7}{3}x^3dc^6b+7x^3d^2c^5a+\frac{1}{2}x^2c^7b+\frac{7}{2}x^2dc^6a+xc^7a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d^7*b + 7/8*x^8*d^6*c*b + 1/8*x^8*d^7*a + 3*x^7*d^5*c^2*b + x^7*d^6
*c*a + 35/6*x^6*d^4*c^3*b + 7/2*x^6*d^5*c^2*a + 7*x^5*d^3*c^4*b + 7*x^5*d^4
*c^3*a + 21/4*x^4*d^2*c^5*b + 35/4*x^4*d^3*c^4*a + 7/3*x^3*d*c^6*b + 7*x^3*
d^2*c^5*a + 1/2*x^2*c^7*b + 7/2*x^2*d*c^6*a + x*c^7*a
```

giac [B] time = 1.30, size = 169, normalized size = 4.45

$$\frac{1}{9} bd^7x^9 + \frac{7}{8} bcd^6x^8 + \frac{1}{8} ad^7x^8 + 3 bc^2d^5x^7 + acd^6x^7 + \frac{35}{6} bc^3d^4x^6 + \frac{7}{2} ac^2d^5x^6 + 7 bc^4d^3x^5 + 7 ac^3d^4x^5 + \frac{21}{4} bc^5d^2x^4 + \frac{35}{4} ac^4d^3x^4 + \frac{7}{3} bc^6dx^3 + 7 ac^5d^2x^3 + \frac{1}{2} bc^7x^2 + \frac{7}{2} ac^6dx^2 + ac^7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/9*b*d^7*x^9 + 7/8*b*c*d^6*x^8 + 1/8*a*d^7*x^8 + 3*b*c^2*d^5*x^7 + a*c*d^6
*x^7 + 35/6*b*c^3*d^4*x^6 + 7/2*a*c^2*d^5*x^6 + 7*b*c^4*d^3*x^5 + 7*a*c^3*d
^4*x^5 + 21/4*b*c^5*d^2*x^4 + 35/4*a*c^4*d^3*x^4 + 7/3*b*c^6*d*x^3 + 7*a*c^
5*d^2*x^3 + 1/2*b*c^7*x^2 + 7/2*a*c^6*d*x^2 + a*c^7*x
```

maple [B] time = 0.00, size = 169, normalized size = 4.45

$$\frac{b\,d^7x^9}{9} + a\,c^7x + \frac{\left(a\,d^7 + 7bc\,d^6\right)x^8}{8} + \frac{\left(7ac\,d^6 + 21b\,c^2d^5\right)x^7}{7} + \frac{\left(21a\,c^2d^5 + 35b\,c^3d^4\right)x^6}{6} + \frac{\left(35a\,c^3d^4 + 35b\,c^4d^3\right)x^5}{5} + \frac{\left(35a\,c^4d^3 + 21b\,c^5d^2\right)x^4}{4} + \frac{\left(21a\,c^5d^2 + 7b\,c^6d\right)x^3}{3} + \frac{\left(7a\,c^6d + b\,c^7\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^7,x)
```

```
[Out] 1/9*b*d^7*x^9+1/8*(a*d^7+7*b*c*d^6)*x^8+1/7*(7*a*c*d^6+21*b*c^2*d^5)*x^7+1/
6*(21*a*c^2*d^5+35*b*c^3*d^4)*x^6+1/5*(35*a*c^3*d^4+35*b*c^4*d^3)*x^5+1/4*(
35*a*c^4*d^3+21*b*c^5*d^2)*x^4+1/3*(21*a*c^5*d^2+7*b*c^6*d)*x^3+1/2*(7*a*c^
6*d+b*c^7)*x^2+a*c^7*x
```

maxima [B] time = 1.38, size = 163, normalized size = 4.29

$$\frac{1}{9} bd^7x^9 + ac^7x + \frac{1}{8} \left(7 bcd^6 + ad^7\right)x^8 + \left(3 bc^2d^5 + acd^6\right)x^7 + \frac{7}{6} \left(5 bc^3d^4 + 3 ac^2d^5\right)x^6 + 7 \left(bc^4d^3 + ac^3d^4\right)x^5 + \frac{7}{4} \left(3 bc^5d^2 + 5 ac^4d^3\right)x^4 + \frac{7}{3} \left(bc^6d + 3 ac^5d^2\right)x^3 + \frac{1}{2} \left(bc^7 + 7 ac^6d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^7*x^9 + a*c^7*x + 1/8*(7*b*c*d^6 + a*d^7)*x^8 + (3*b*c^2*d^5 + a*c*
d^6)*x^7 + 7/6*(5*b*c^3*d^4 + 3*a*c^2*d^5)*x^6 + 7*(b*c^4*d^3 + a*c^3*d^4)*
x^5 + 7/4*(3*b*c^5*d^2 + 5*a*c^4*d^3)*x^4 + 7/3*(b*c^6*d + 3*a*c^5*d^2)*x^3
 + 1/2*(b*c^7 + 7*a*c^6*d)*x^2
```

mupad [B] time = 0.08, size = 143, normalized size = 3.76

$$x^2\,\left(\frac{b\,c^7}{2}+\frac{7\,a\,d\,c^6}{2}\right)+x^8\,\left(\frac{a\,d^7}{8}+\frac{7\,b\,c\,d^6}{8}\right)+\frac{b\,d^7\,x^9}{9}+a\,c^7\,x+\frac{7\,c^5\,d\,x^3\,\left(3\,a\,d+b\,c\right)}{3}+c\,d^5\,x^7\,\left(a\,d+3\,b\,c\right)+7\,c^3\,d^3\,x^5\,\left(a\,d+b\,c\right)+\frac{7\,c^4\,d^2\,x^4\,\left(5\,a\,d+3\,b\,c\right)}{4}+\frac{7\,c^2\,d^4\,x^6\,\left(3\,a\,d+5\,b\,c\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^7,x)
```

```
[Out] x^2*((b*c^7)/2 + (7*a*c^6*d)/2) + x^8*((a*d^7)/8 + (7*b*c*d^6)/8) + (b*d^7*
x^9)/9 + a*c^7*x + (7*c^5*d*x^3*(3*a*d + b*c))/3 + c*d^5*x^7*(a*d + 3*b*c)
+ 7*c^3*d^3*x^5*(a*d + b*c) + (7*c^4*d^2*x^4*(5*a*d + 3*b*c))/4 + (7*c^2*d^
4*x^6*(3*a*d + 5*b*c))/6
```

sympy [B] time = 0.10, size = 178, normalized size = 4.68

$$ac^7x + \frac{bd^7x^9}{9} + x^8 \left(\frac{ad^7}{8} + \frac{7bcd^6}{8}\right) + x^7 \left(acd^6 + 3bc^2d^5\right) + x^6 \left(\frac{7ac^2d^5}{2} + \frac{35bc^3d^4}{6}\right) + x^5 \left(7ac^3d^4 + 7bc^4d^3\right) + x^4 \left(\frac{35ac^4d^3}{4} + \frac{21bc^5d^2}{4}\right) + x^3 \left(7ac^5d^2 + \frac{7bc^6d}{3}\right) + x^2 \left(\frac{7ac^6d}{2} + \frac{bc^7}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**7,x)
```

```
[Out] a*c**7*x + b*d**7*x**9/9 + x**8*(a*d**7/8 + 7*b*c*d**6/8) + x**7*(a*c*d**6
+ 3*b*c**2*d**5) + x**6*(7*a*c**2*d**5/2 + 35*b*c**3*d**4/6) + x**5*(7*a*c*
*3*d**4 + 7*b*c**4*d**3) + x**4*(35*a*c**4*d**3/4 + 21*b*c**5*d**2/4) + x**
3*(7*a*c**5*d**2 + 7*b*c**6*d/3) + x**2*(7*a*c**6*d/2 + b*c**7/2)
```

3.1176 $\int (c+dx)^7\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^8}{8d}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7,x]
[Out] (c + d*x)^8/(8*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^7\,dx=\frac{(c+dx)^8}{8d}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^8}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7,x]
[Out] (c + d*x)^8/(8*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^7\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7,x]
[Out] IntegrateAlgebraic[(c + d*x)^7, x]
```

fricas [B] time = 1.29, size = 75, normalized size = 5.36

$$\frac{1}{8}x^8d^7+x^7d^6c+\frac{7}{2}x^6d^5c^2+7x^5d^4c^3+\frac{35}{4}x^4d^3c^4+7x^3d^2c^5+\frac{7}{2}x^2dc^6+xc^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*d^7 + x^7*d^6*c + 7/2*x^6*d^5*c^2 + 7*x^5*d^4*c^3 + 35/4*x^4*d^3*c^
4 + 7*x^3*d^2*c^5 + 7/2*x^2*d*c^6 + x*c^7
```

giac [A] time = 1.30, size = 12, normalized size = 0.86

$$\frac{(dx+c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/8*(d*x + c)^8/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx+c)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7,x)
```

```
[Out] 1/8*(d*x+c)^8/d
```

maxima [A] time = 1.32, size = 12, normalized size = 0.86

$$\frac{(dx+c)^8}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/8*(d*x + c)^8/d
```

mupad [B] time = 0.06, size = 75, normalized size = 5.36

$$c^7\,x+\frac{7\,c^6\,d\,x^2}{2}+7\,c^5\,d^2\,x^3+\frac{35\,c^4\,d^3\,x^4}{4}+7\,c^3\,d^4\,x^5+\frac{7\,c^2\,d^5\,x^6}{2}+c\,d^6\,x^7+\frac{d^7\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7,x)
```

```
[Out] c^7*x + (d^7*x^8)/8 + (7*c^6*d*x^2)/2 + c*d^6*x^7 + 7*c^5*d^2*x^3 + (35*c^4
*d^3*x^4)/4 + 7*c^3*d^4*x^5 + (7*c^2*d^5*x^6)/2
```

sympy [B] time = 0.08, size = 83, normalized size = 5.93

$$c^7x+\frac{7c^6dx^2}{2}+7c^5d^2x^3+\frac{35c^4d^3x^4}{4}+7c^3d^4x^5+\frac{7c^2d^5x^6}{2}+cd^6x^7+\frac{d^7x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7,x)
```

```
[Out] c**7*x + 7*c**6*d*x**2/2 + 7*c**5*d**2*x**3 + 35*c**4*d**3*x**4/4 + 7*c**3*
d**4*x**5 + 7*c**2*d**5*x**6/2 + c*d**6*x**7 + d**7*x**8/8
```

3.1177 $\int \frac{(c+dx)^7}{a+bx} \, dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)^7 \log(a+bx)}{b^8}+\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(b}{5b^3}$$

Rubi [A] time = 0.07, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{dx(bc-ad)^6}{b^7}+\frac{(c+dx)^2(bc-ad)^5}{2b^6}+\frac{(c+dx)^3(bc-ad)^4}{3b^5}+\frac{(c+dx)^4(bc-ad)^3}{4b^4}+\frac{(c+dx)^5(bc-ad)^2}{5b^3}+\frac{(c+dx)^6(bc-ad)}{6b^2}+\frac{(bc-ad)^7\log(a+bx)}{b^8}+\frac{(c+dx)^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^6*x)/b^7 + ((b*c - a*d)^5*(c + d*x)^2)/(2*b^6) + ((b*c - a*d
)^4*(c + d*x)^3)/(3*b^5) + ((b*c - a*d)^3*(c + d*x)^4)/(4*b^4) + ((b*c - a*
d)^2*(c + d*x)^5)/(5*b^3) + ((b*c - a*d)*(c + d*x)^6)/(6*b^2) + (c + d*x)^7
/(7*b) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{a+bx}\,dx &= \int \left(\frac{d(bc-ad)^6}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)}+\frac{d(bc-ad)^5(c+dx)}{b^6}+\frac{d(bc-ad)^4(c+dx)^2}{b^5}+\frac{d(bc-ad)^3(c+a}{b^4}\right.\\ &= \frac{d(bc-ad)^6x}{b^7}+\frac{(bc-ad)^5(c+dx)^2}{2b^6}+\frac{(bc-ad)^4(c+dx)^3}{3b^5}+\frac{(bc-ad)^3(c+dx)^4}{4b^4}+\frac{(bc-ad)^2(c}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.15, size = 304, normalized size = 1.80

$$\frac{dx\left(420a^6d^6-210a^5bd^5(14c+dx)+70a^4b^2d^4\left(126c^2+21cdx+2d^2x^2\right)-35a^3b^3d^3\left(420c^3+126c^2dx+28cd^2x^2+3d^3x^3\right)+21a^2b^4d^2\left(700c^4+350c^3dx+140c^2d^2x^2+35cd^3x^3+4d^4x^4\right)-7ab^5d\left(1260c^5+1050c^4dx+700c^3d^2x^2+315c^2d^3x^3+84cd^4x^4+10d^5x^5\right)+b^6\left(2940c^6+4410c^5dx+4900c^4d^2x^2+3675c^3d^3x^3+1764c^2d^4x^4+490cd^5x^5+60d^6x^6\right)\right)}{420b^7}+\frac{(bc-ad)^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x),x]
```

```
[Out] (d*x*(420*a^6*d^6 - 210*a^5*b*d^5*(14*c + d*x) + 70*a^4*b^2*d^4*(126*c^2 +
21*c*d*x + 2*d^2*x^2) - 35*a^3*b^3*d^3*(420*c^3 + 126*c^2*d*x + 28*c*d^2*x^
2 + 3*d^3*x^3) + 21*a^2*b^4*d^2*(700*c^4 + 350*c^3*d*x + 140*c^2*d^2*x^2 +
35*c*d^3*x^3 + 4*d^4*x^4) - 7*a*b^5*d*(1260*c^5 + 1050*c^4*d*x + 700*c^3*d^
2*x^2 + 315*c^2*d^3*x^3 + 84*c*d^4*x^4 + 10*d^5*x^5) + b^6*(2940*c^6 + 4410
*c^5*d*x + 4900*c^4*d^2*x^2 + 3675*c^3*d^3*x^3 + 1764*c^2*d^4*x^4 + 490*c*d
^5*x^5 + 60*d^6*x^6)))/(420*b^7) + ((b*c - a*d)^7*Log[a + b*x])/b^8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x),x]
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x), x]
```

fricas [B] time = 1.17, size = 462, normalized size = 2.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="fricas")
[Out] 1/420*(60*b^7*d^7*x^7 + 70*(7*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(21*b^7*c^2*d
^5 - 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(35*b^7*c^3*d^4 - 21*a*b^6*c^2*
d^5 + 7*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(35*b^7*c^4*d^3 - 35*a*b^6*c
^3*d^4 + 21*a^2*b^5*c^2*d^5 - 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(21*
b^7*c^5*d^2 - 35*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 - 21*a^3*b^4*c^2*d^5 +
7*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 420*(7*b^7*c^6*d - 21*a*b^6*c^5*d^2 +
35*a^2*b^5*c^4*d^3 - 35*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 - 7*a^5*b^2*c*
d^6 + a^6*b*d^7)*x + 420*(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35
*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6
- a^7*d^7)*log(b*x + a))/b^8
```

giac [B] time = 1.30, size = 497, normalized size = 2.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="giac")
[Out] 1/420*(60*b^6*d^7*x^7 + 490*b^6*c*d^6*x^6 - 70*a*b^5*d^7*x^6 + 1764*b^6*c^2
*d^5*x^5 - 588*a*b^5*c*d^6*x^5 + 84*a^2*b^4*d^7*x^5 + 3675*b^6*c^3*d^4*x^4
- 2205*a*b^5*c^2*d^5*x^4 + 735*a^2*b^4*c*d^6*x^4 - 105*a^3*b^3*d^7*x^4 + 49
00*b^6*c^4*d^3*x^3 - 4900*a*b^5*c^3*d^4*x^3 + 2940*a^2*b^4*c^2*d^5*x^3 - 98
0*a^3*b^3*c*d^6*x^3 + 140*a^4*b^2*d^7*x^3 + 4410*b^6*c^5*d^2*x^2 - 7350*a*b
^5*c^4*d^3*x^2 + 7350*a^2*b^4*c^3*d^4*x^2 - 4410*a^3*b^3*c^2*d^5*x^2 + 1470
*a^4*b^2*c*d^6*x^2 - 210*a^5*b*d^7*x^2 + 2940*b^6*c^6*d*x - 8820*a*b^5*c^5*
d^2*x + 14700*a^2*b^4*c^4*d^3*x - 14700*a^3*b^3*c^3*d^4*x + 8820*a^4*b^2*c^
2*d^5*x - 2940*a^5*b*c*d^6*x + 420*a^6*d^7*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*
d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b
^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*log(abs(b*x + a))/b^8
```

maple [B] time = 0.01, size = 539, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a),x)
[Out] 7/6*d^6/b*x^6*c+1/5*d^7/b^3*x^5*a^2+7*d/b*c^6*x+d^7/b^7*a^6*x-1/b^8*ln(b*x+
a)*a^7*d^7+21/5*d^5/b*x^5*c^2-1/4*d^7/b^4*x^4*a^3+35/4*d^4/b*x^4*c^3+1/3*d^
7/b^5*x^3*a^4+35/3*d^3/b*x^3*c^4-1/2*d^7/b^6*x^2*a^5+21/2*d^2/b*x^2*c^5-1/6
*d^7/b^2*x^6*a+35*d^3/b^3*a^2*c^4*x-21*d^2/b^2*a*c^5*x-21/2*d^5/b^4*x^2*a^3
*c^2+35/2*d^4/b^3*x^2*a^2*c^3+7/4*d^6/b^3*x^4*a^2*c-21/4*d^5/b^2*x^4*a*c^2-
7/b^2*ln(b*x+a)*a*c^6*d-35/2*d^3/b^2*x^2*a*c^4+7/2*d^6/b^5*x^2*a^4*c+7*d^5/
b^3*x^3*a^2*c^2-35/3*d^4/b^2*x^3*a*c^3+7/b^7*ln(b*x+a)*a^6*c*d^6-21/b^6*ln(
b*x+a)*a^5*c^2*d^5+35/b^5*ln(b*x+a)*a^4*c^3*d^4-7/3*d^6/b^4*x^3*a^3*c-7/5*d
^6/b^2*x^5*a*c-7*d^6/b^6*a^5*c*x+21*d^5/b^5*a^4*c^2*x-35*d^4/b^4*a^3*c^3*x-
35/b^4*ln(b*x+a)*a^3*c^4*d^3+21/b^3*ln(b*x+a)*a^2*c^5*d^2+1/b*ln(b*x+a)*c^7
+1/7*d^7/b*x^7
```

maxima [B] time = 1.39, size = 460, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/420*(60*b^6*d^7*x^7 + 70*(7*b^6*c*d^6 - a*b^5*d^7)*x^6 + 84*(21*b^6*c^2*d
^5 - 7*a*b^5*c*d^6 + a^2*b^4*d^7)*x^5 + 105*(35*b^6*c^3*d^4 - 21*a*b^5*c^2*
d^5 + 7*a^2*b^4*c*d^6 - a^3*b^3*d^7)*x^4 + 140*(35*b^6*c^4*d^3 - 35*a*b^5*c
^3*d^4 + 21*a^2*b^4*c^2*d^5 - 7*a^3*b^3*c*d^6 + a^4*b^2*d^7)*x^3 + 210*(21*
b^6*c^5*d^2 - 35*a*b^5*c^4*d^3 + 35*a^2*b^4*c^3*d^4 - 21*a^3*b^3*c^2*d^5 +
7*a^4*b^2*c*d^6 - a^5*b*d^7)*x^2 + 420*(7*b^6*c^6*d - 21*a*b^5*c^5*d^2 + 35
*a^2*b^4*c^4*d^3 - 35*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - 7*a^5*b*c*d^6
+ a^6*d^7)*x)/b^7 + (b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*
b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7
*d^7)*log(b*x + a)/b^8
```

mupad [B] time = 0.22, size = 509, normalized size = 3.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x),x)
```

```
[Out] x*((7*c^6*d)/b - (a*((a*((a*((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^
6)/b))/b + (21*c^2*d^5)/b))/b))/b - (35*c^4*d^3)/b))/b + (21*c^5*d^2)/b))/b
) - x^6*((a*d^7)/(6*b^2) - (7*c*d^6)/(6*b)) + x^4*((35*c^3*d^4)/(4*b) - (a*
((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/(4*b)) + x^2*((a*((a*
((35*c^3*d^4)/b - (a*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/
b))/b - (35*c^4*d^3)/b))/(2*b) + (21*c^5*d^2)/(2*b)) + x^5*((a*((a*d^7)/b^2
 - (7*c*d^6)/b))/(5*b) + (21*c^2*d^5)/(5*b)) - x^3*((a*((35*c^3*d^4)/b - (a
*((a*((a*d^7)/b^2 - (7*c*d^6)/b))/b + (21*c^2*d^5)/b))/b))/(3*b) - (35*c^4*
d^3)/(3*b)) - (log(a + b*x)*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6))/b^8 + (d^7*x^7)/(7*b)
```

sympy [B] time = 0.80, size = 408, normalized size = 2.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a),x)
```

```
[Out] x**6*(-a*d**7/(6*b**2) + 7*c*d**6/(6*b)) + x**5*(a**2*d**7/(5*b**3) - 7*a*c
*d**6/(5*b**2) + 21*c**2*d**5/(5*b)) + x**4*(-a**3*d**7/(4*b**4) + 7*a**2*c
*d**6/(4*b**3) - 21*a*c**2*d**5/(4*b**2) + 35*c**3*d**4/(4*b)) + x**3*(a**4
*d**7/(3*b**5) - 7*a**3*c*d**6/(3*b**4) + 7*a**2*c**2*d**5/b**3 - 35*a*c**3
*d**4/(3*b**2) + 35*c**4*d**3/(3*b)) + x**2*(-a**5*d**7/(2*b**6) + 7*a**4*c
```

```
*d**6/(2*b**5) - 21*a**3*c**2*d**5/(2*b**4) + 35*a**2*c**3*d**4/(2*b**3) -
35*a*c**4*d**3/(2*b**2) + 21*c**5*d**2/(2*b)) + x*(a**6*d**7/b**7 - 7*a**5*
c*d**6/b**6 + 21*a**4*c**2*d**5/b**5 - 35*a**3*c**3*d**4/b**4 + 35*a**2*c**
4*d**3/b**3 - 21*a*c**5*d**2/b**2 + 7*c**6*d/b) + d**7*x**7/(7*b) - (a*d -
b*c)**7*log(a + b*x)/b**8
```

3.1178 $\int \frac{(c+dx)^7}{(a+bx)^2} \, dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}-\frac{(bc-ad)^7}{b^8(a+bx)}+$$

Rubi [A] time = 0.23, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^5(bc-ad)}{5b^8}+\frac{21d^5(a+bx)^4(bc-ad)^2}{4b^8}+\frac{35d^4(a+bx)^3(bc-ad)^3}{3b^8}+\frac{35d^3(a+bx)^2(bc-ad)^4}{2b^8}+\frac{21d^2x(bc-ad)^5}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{7d(bc-ad)^6\log(a+bx)}{b^8}+\frac{d^7(a+bx)^6}{6b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (21*d^2*(b*c - a*d)^5*x)/b^7 - (b*c - a*d)^7/(b^8*(a + b*x)) + (35*d^3*(b*c
 - a*d)^4*(a + b*x)^2)/(2*b^8) + (35*d^4*(b*c - a*d)^3*(a + b*x)^3)/(3*b^8)
 + (21*d^5*(b*c - a*d)^2*(a + b*x)^4)/(4*b^8) + (7*d^6*(b*c - a*d)*(a + b*x
)^5)/(5*b^8) + (d^7*(a + b*x)^6)/(6*b^8) + (7*d*(b*c - a*d)^6*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^2}\,dx &= \int \left(\frac{21d^2(bc-ad)^5}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^2}+\frac{7d(bc-ad)^6}{b^7(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)}{b^7}+\frac{35d^4(bc-ad)^3(a}{b^7}\right.\\ &= \frac{21d^2(bc-ad)^5x}{b^7}-\frac{(bc-ad)^7}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8}+\frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8}+\frac{21d^5(bc}{}\end{aligned}$$

Mathematica [B] time = 0.12, size = 388, normalized size = 2.07

$$\frac{60a^7d^7-60a^6bd^6(7c+6dx)+210a^5b^2d^5(6c^2+10cdx-d^2x^2)+70a^4b^3d^4(-30c^3-72c^2dx+18cd^2x^2+d^3x^3)-35a^3b^4d^3(-60c^4-180c^3dx+90c^2d^2x^2+12cd^3x^3+d^4x^4)+21a^2b^5d^2(-60c^5-200c^4dx+200c^3d^2x^2+50c^2d^3x^3+10cd^4x^4+d^5x^5)-7ab^6d(-60c^6-180c^5dx+450c^4d^2x^2+200c^3d^3x^3+75c^2d^4x^4+18cd^5x^5+2d^6x^6)+420d(a+bx)(bc-ad)^6\log(a+bx)+b^7(-60c^7+1260c^5d^2x^2+1050c^4d^3x^3+700c^3d^4x^4+315c^2d^5x^5+84cd^6x^6+10d^7x^7)}{60b^8(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] (60*a^7*d^7 - 60*a^6*b*d^6*(7*c + 6*d*x) + 210*a^5*b^2*d^5*(6*c^2 + 10*c*d*
x - d^2*x^2) + 70*a^4*b^3*d^4*(-30*c^3 - 72*c^2*d*x + 18*c*d^2*x^2 + d^3*x^
3) - 35*a^3*b^4*d^3*(-60*c^4 - 180*c^3*d*x + 90*c^2*d^2*x^2 + 12*c*d^3*x^3
+ d^4*x^4) + 21*a^2*b^5*d^2*(-60*c^5 - 200*c^4*d*x + 200*c^3*d^2*x^2 + 50*c
^2*d^3*x^3 + 10*c*d^4*x^4 + d^5*x^5) - 7*a*b^6*d*(-60*c^6 - 180*c^5*d*x + 4
50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 75*c^2*d^4*x^4 + 18*c*d^5*x^5 + 2*d^6*x^
6) + b^7*(-60*c^7 + 1260*c^5*d^2*x^2 + 1050*c^4*d^3*x^3 + 700*c^3*d^4*x^4 +
 315*c^2*d^5*x^5 + 84*c*d^6*x^6 + 10*d^7*x^7) + 420*d*(b*c - a*d)^6*(a + b*
x)*Log[a + b*x])/(60*b^8*(a + b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^2, x]
```

fricas [B] time = 1.58, size = 632, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^7*d^7*x^7 - 60*b^7*c^7 + 420*a*b^6*c^6*d - 1260*a^2*b^5*c^5*d^2
+ 2100*a^3*b^4*c^4*d^3 - 2100*a^4*b^3*c^3*d^4 + 1260*a^5*b^2*c^2*d^5 - 420*
a^6*b*c*d^6 + 60*a^7*d^7 + 14*(6*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(15*b^7*c^
2*d^5 - 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(20*b^7*c^3*d^4 - 15*a*b^6*c^
2*d^5 + 6*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 70*(15*b^7*c^4*d^3 - 20*a*b^6*
c^3*d^4 + 15*a^2*b^5*c^2*d^5 - 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 210*(6*
b^7*c^5*d^2 - 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 15*a^3*b^4*c^2*d^5 +
6*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 60*(21*a*b^6*c^5*d^2 - 70*a^2*b^5*c^4*
d^3 + 105*a^3*b^4*c^3*d^4 - 84*a^4*b^3*c^2*d^5 + 35*a^5*b^2*c*d^6 - 6*a^6*b
*d^7)*x + 420*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d -
 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2
*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^9*x + a*b^8)
```

giac [B] time = 1.28, size = 567, normalized size = 3.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/60*(10*d^7 + 84*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 315*(b^4*c^2*d^5 -
2*a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2) + 700*(b^6*c^3*d^4 - 3*a*b^5
*c^2*d^5 + 3*a^2*b^4*c*d^6 - a^3*b^3*d^7)/((b*x + a)^3*b^3) + 1050*(b^8*c^4
*d^3 - 4*a*b^7*c^3*d^4 + 6*a^2*b^6*c^2*d^5 - 4*a^3*b^5*c*d^6 + a^4*b^4*d^7)
/((b*x + a)^4*b^4) + 1260*(b^10*c^5*d^2 - 5*a*b^9*c^4*d^3 + 10*a^2*b^8*c^3*
d^4 - 10*a^3*b^7*c^2*d^5 + 5*a^4*b^6*c*d^6 - a^5*b^5*d^7)/((b*x + a)^5*b^5)
)*(b*x + a)^6/b^8 - 7*(b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 2
0*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*log(abs(b
*x + a)/((b*x + a)^2*abs(b)))/b^8 - (b^13*c^7/(b*x + a) - 7*a*b^12*c^6*d/(b
*x + a) + 21*a^2*b^11*c^5*d^2/(b*x + a) - 35*a^3*b^10*c^4*d^3/(b*x + a) + 3
5*a^4*b^9*c^3*d^4/(b*x + a) - 21*a^5*b^8*c^2*d^5/(b*x + a) + 7*a^6*b^7*c*d^
6/(b*x + a) - a^7*b^6*d^7/(b*x + a))/b^14
```

maple [B] time = 0.01, size = 571, normalized size = 3.05

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^2,x)
```

```
[Out] 35/3*d^4/b^2*x^3*c^3+5/2*d^7/b^6*x^2*a^4+35/2*d^3/b^2*x^2*c^4-6*d^7/b^7*a^5
*x+21*d^2/b^2*c^5*x+7/b^8*d^7*ln(b*x+a)*a^6+7/b^2*d*ln(b*x+a)*c^6+1/b^8/(b*
x+a)*a^7*d^7-2/5*d^7/b^3*x^5*a+7/5*d^6/b^2*x^5*c+3/4*d^7/b^4*x^4*a^2+21/4*d
^5/b^2*x^4*c^2-4/3*d^7/b^5*x^3*a^3-21/b^3/(b*x+a)*a^2*c^5*d^2+7/b^2/(b*x+a)
*a*c^6*d-14*d^5/b^3*x^3*a*c^2-14*d^6/b^5*x^2*a^3*c+63/2*d^5/b^4*x^2*a^2*c^2
-35*d^4/b^3*x^2*a*c^3+35*d^6/b^6*a^4*c*x-7/2*d^6/b^3*x^4*a*c+7*d^6/b^4*x^3*
```

```
a^2*c-70*d^3/b^3*a*c^4*x-42/b^7*d^6*ln(b*x+a)*a^5*c+105/b^6*d^5*ln(b*x+a)*a
^4*c^2-140/b^5*d^4*ln(b*x+a)*a^3*c^3+105/b^4*d^3*ln(b*x+a)*a^2*c^4-42/b^3*d
^2*ln(b*x+a)*a*c^5-7/b^7/(b*x+a)*a^6*c*d^6+21/b^6/(b*x+a)*a^5*c^2*d^5-35/b^
5/(b*x+a)*a^4*c^3*d^4+35/b^4/(b*x+a)*a^3*c^4*d^3-84*d^5/b^5*a^3*c^2*x+105*d
^4/b^4*a^2*c^3*x-1/b/(b*x+a)*c^7+1/6*d^7/b^2*x^6
```

maxima [**B**] time = 1.40, size = 467, normalized size = 2.50

$$-\frac{b^7c^7 - 7\,ab^6c^6d + 21\,a^2b^5c^5d^2 - 35\,a^3b^4c^4d^3 + 35\,a^4b^3c^3d^4 - 21\,a^5b^2c^2d^5 + 7\,a^6bcd^6 - a^7d^7}{b^9x + ab^8} + \frac{10\,b^5d^7x^6 + 12\left(7\,b^5cd^6 - 2\,ab^4d^7\right)x^5 + 15\left(21\,b^5c^2d^5 - 14\,ab^4cd^6 + 3\,a^2b^3d^7\right)x^4 + 20\left(35\,b^5c^3d^4 - 42\,ab^4c^2d^5 + 21\,a^2b^3cd^6 - 4\,a^3b^2d^7\right)x^3 + 30\left(35\,b^5c^4d^3 - 70\,ab^4c^3d^4 + 63\,a^2b^3c^2d^5 - 28\,a^3b^2cd^6 + 5\,a^4bd^7\right)x^2 + 60\left(21\,b^5c^5d^2 - 70\,ab^4c^4d^3 + 105\,a^2b^3c^3d^4 - 84\,a^3b^2c^2d^5 + 35\,a^4bcd^6 - 6\,a^5d^7\right)x}{60\,b^7} + \frac{7\left(b^6c^6d - 6\,ab^5c^5d^2 + 15\,a^2b^4c^4d^3 - 20\,a^3b^3c^3d^4 + 15\,a^4b^2c^2d^5 - 6\,a^5bcd^6 + a^6d^7\right)\log\left(bx + a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(b^9*x + a*b^
8) + 1/60*(10*b^5*d^7*x^6 + 12*(7*b^5*c*d^6 - 2*a*b^4*d^7)*x^5 + 15*(21*b^5
*c^2*d^5 - 14*a*b^4*c*d^6 + 3*a^2*b^3*d^7)*x^4 + 20*(35*b^5*c^3*d^4 - 42*a*
b^4*c^2*d^5 + 21*a^2*b^3*c*d^6 - 4*a^3*b^2*d^7)*x^3 + 30*(35*b^5*c^4*d^3 -
70*a*b^4*c^3*d^4 + 63*a^2*b^3*c^2*d^5 - 28*a^3*b^2*c*d^6 + 5*a^4*b*d^7)*x^2
 + 60*(21*b^5*c^5*d^2 - 70*a*b^4*c^4*d^3 + 105*a^2*b^3*c^3*d^4 - 84*a^3*b^2
*c^2*d^5 + 35*a^4*b*c*d^6 - 6*a^5*d^7)*x)/b^7 + 7*(b^6*c^6*d - 6*a*b^5*c^5*
d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*
b*c*d^6 + a^6*d^7)*log(b*x + a)/b^8
```

mupad [**B**] time = 0.24, size = 841, normalized size = 4.50

$$x^4\left(\frac{a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{2b}-\frac{a^2d^7}{4b^4}+\frac{21c^2d^5}{4b^2}\right)-x^2\left(\frac{a\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b}-\frac{35c^4d^3}{2b^2}+\frac{a^2\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{2b^2}\right)-x^5\left(\frac{2ad^7}{5b^3}-\frac{7cd^6}{5b^2}\right)+x\left(\frac{2a\left(\frac{2a\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b}-\frac{35c^4d^3}{b^2}+\frac{a^2\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b^2}\right)}{b}-\frac{a^2\left(\frac{35c^3d^4}{b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b^2}\right)}{b^2}+\frac{21c^5d^2}{b^2}\right)+x^3\left(\frac{35c^3d^4}{3b^2}-\frac{2a\left(\frac{2a\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{b}-\frac{a^2d^7}{b^4}+\frac{21c^2d^5}{b^2}\right)}{3b}+\frac{a^2\left(\frac{2ad^7}{b^3}-\frac{7cd^6}{b^2}\right)}{3b^2}\right)+\frac{a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7}{b\left(xb^8+ab^7\right)}+\frac{d^7x^6}{6b^2}+\frac{\ln\left(a+bx\right)\left(7a^6d^7-42a^5bcd^6+105a^4b^2c^2d^5-140a^3b^3c^3d^4+105a^2b^4c^4d^3-42ab^5c^5d^2+7b^6c^6d\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^2,x)
```

```
[Out] x^4*((a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(2*b) - (a^2*d^7)/(4*b^4) + (21*c^
2*d^5)/(4*b^2)) - x^2*((a*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3 - (
7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^7)/b
^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)/(2*b^2) + (a^2*((2*a*((2*a*d^7)
/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/(2*b^2)) - x^
5*((2*a*d^7)/(5*b^3) - (7*c*d^6)/(5*b^2)) + x*((2*a*((2*a*((35*c^3*d^4)/b^2
 - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*
d^5)/b^2))/b + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b^2))/b - (35*c^4*d^3)
/b^2 + (a^2*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*
c^2*d^5)/b^2))/b^2))/b - (a^2*((35*c^3*d^4)/b^2 - (2*a*((2*a*((2*a*d^7)/b^3
 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (21*c^2*d^5)/b^2))/b + (a^2*((2*a*d^
7)/b^3 - (7*c*d^6)/b^2))/b^2))/b^2 + (21*c^5*d^2)/b^2) + x^3*((35*c^3*d^4)/
(3*b^2) - (2*a*((2*a*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/b - (a^2*d^7)/b^4 + (
21*c^2*d^5)/b^2))/(3*b) + (a^2*((2*a*d^7)/b^3 - (7*c*d^6)/b^2))/(3*b^2)) +
(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c
^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)/(b*(a*b^7 + b^
8*x)) + (d^7*x^6)/(6*b^2) + (log(a + b*x)*(7*a^6*d^7 + 7*b^6*c^6*d - 42*a*b
^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4 + 105*a^4*b^2*c^2*d^
5 - 42*a^5*b*c*d^6))/b^8
```

sympy [**B**] time = 1.44, size = 428, normalized size = 2.29

$$x^5\left(-\frac{2ad^7}{5b^3}+\frac{7cd^6}{5b^2}\right)+x^4\left(\frac{3a^2d^7}{4b^4}-\frac{7acd^6}{2b^3}+\frac{21c^2d^5}{4b^2}\right)+x^3\left(-\frac{4a^3d^7}{3b^5}+\frac{7a^2cd^6}{b^4}-\frac{14ac^2d^5}{b^3}+\frac{35c^3d^4}{3b^2}\right)+x^2\left(\frac{5a^4d^7}{2b^6}-\frac{14a^3cd^6}{b^5}+\frac{63a^2c^2d^5}{2b^4}-\frac{35ac^3d^4}{b^3}+\frac{35c^4d^3}{2b^2}\right)+x\left(-\frac{6a^5d^7}{b^7}+\frac{35a^4cd^6}{b^6}-\frac{84a^3c^2d^5}{b^5}+\frac{105a^2c^3d^4}{b^4}-\frac{70ac^4d^3}{b^3}+\frac{21c^5d^2}{b^2}\right)+\frac{a^7d^7-7a^6bcd^6+21a^5b^2c^2d^5-35a^4b^3c^3d^4+35a^3b^4c^4d^3-21a^2b^5c^5d^2+7ab^6c^6d-b^7c^7}{ab^8+b^9x}+\frac{d^7x^6}{6b^2}+\frac{7d\left(ad-bc\right)^6\log\left(a+bx\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**2,x)
```

```
[Out] x**5*(-2*a*d**7/(5*b**3) + 7*c*d**6/(5*b**2)) + x**4*(3*a**2*d**7/(4*b**4)
- 7*a*c*d**6/(2*b**3) + 21*c**2*d**5/(4*b**2)) + x**3*(-4*a**3*d**7/(3*b**5
) + 7*a**2*c*d**6/b**4 - 14*a*c**2*d**5/b**3 + 35*c**3*d**4/(3*b**2)) + x**
2*(5*a**4*d**7/(2*b**6) - 14*a**3*c*d**6/b**5 + 63*a**2*c**2*d**5/(2*b**4)
- 35*a*c**3*d**4/b**3 + 35*c**4*d**3/(2*b**2)) + x*(-6*a**5*d**7/b**7 + 35*
a**4*c*d**6/b**6 - 84*a**3*c**2*d**5/b**5 + 105*a**2*c**3*d**4/b**4 - 70*a*
c**4*d**3/b**3 + 21*c**5*d**2/b**2) + (a**7*d**7 - 7*a**6*b*c*d**6 + 21*a**
5*b**2*c**2*d**5 - 35*a**4*b**3*c**3*d**4 + 35*a**3*b**4*c**4*d**3 - 21*a**
2*b**5*c**5*d**2 + 7*a*b**6*c**6*d - b**7*c**7)/(a*b**8 + b**9*x) + d**7*x*
*6/(6*b**2) + 7*d*(a*d - b*c)**6*log(a + b*x)/b**8
```

3.1179 $\int \frac{(c+dx)^7}{(a+bx)^3} \, dx$

Optimal. Leaf size=185

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-ad}{b^8(a+bx}$$

Rubi [A] time = 0.22, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7d^6(a+bx)^4(bc-ad)}{4b^8}+\frac{7d^5(a+bx)^3(bc-ad)^2}{b^8}+\frac{35d^4(a+bx)^2(bc-ad)^3}{2b^8}+\frac{35d^3x(bc-ad)^4}{b^7}+\frac{21d^2(bc-ad)^5\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{b^8(a+bx)}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}+\frac{d^7(a+bx)^5}{5b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (35*d^3*(b*c - a*d)^4*x)/b^7 - (b*c - a*d)^7/(2*b^8*(a + b*x)^2) - (7*d*(b*
c - a*d)^6)/(b^8*(a + b*x)) + (35*d^4*(b*c - a*d)^3*(a + b*x)^2)/(2*b^8) +
(7*d^5*(b*c - a*d)^2*(a + b*x)^3)/b^8 + (7*d^6*(b*c - a*d)*(a + b*x)^4)/(4*
b^8) + (d^7*(a + b*x)^5)/(5*b^8) + (21*d^2*(b*c - a*d)^5*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^3}\,dx=\int\left(\frac{35d^3(bc-ad)^4}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^3}+\frac{7d(bc-ad)^6}{b^7(a+bx)^2}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)}{b^7}+\right.$$
$$=\frac{35d^3(bc-ad)^4x}{b^7}-\frac{(bc-ad)^7}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{b^8(a+bx)}+\frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8}+\frac{7d^5(bc-ad)^2(a+}{b^8}$$

Mathematica [B] time = 0.13, size = 389, normalized size = 2.10

$$\frac{-130a^7d^7+10a^6bd^6(77c+16dx)+10a^5b^2d^5\left(-189c^2-56cdx+50d^2x^2\right)+70a^4b^3d^4\left(35c^3+6c^2dx-34cd^2x^2+2d^3x^3\right)-35a^3b^4d^3\left(50c^4-20c^3dx-126c^2d^2x^2+20cd^3x^3+d^4x^4\right)+7a^2b^5d^2\left(90c^5-200c^4dx-550c^3d^2x^2+200c^2d^3x^3+25cd^4x^4+2d^5x^5\right)-7ab^6d\left(10c^6-120c^5dx-200c^4d^2x^2+200c^3d^3x^3+50c^2d^4x^4+10cd^5x^5+d^6x^6\right)-420d^2(a+bx)^2(ad-bc)^5\log(a+bx)+b^7\left(-10c^7-140c^6dx+700c^4d^3x^3+350c^3d^4x^4+140c^2d^5x^5+35cd^6x^6+4d^7x^7\right)}{20b^8(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] (-130*a^7*d^7 + 10*a^6*b*d^6*(77*c + 16*d*x) + 10*a^5*b^2*d^5*(-189*c^2 - 5
6*c*d*x + 50*d^2*x^2) + 70*a^4*b^3*d^4*(35*c^3 + 6*c^2*d*x - 34*c*d^2*x^2 +
 2*d^3*x^3) - 35*a^3*b^4*d^3*(50*c^4 - 20*c^3*d*x - 126*c^2*d^2*x^2 + 20*c*
d^3*x^3 + d^4*x^4) + 7*a^2*b^5*d^2*(90*c^5 - 200*c^4*d*x - 550*c^3*d^2*x^2
+ 200*c^2*d^3*x^3 + 25*c*d^4*x^4 + 2*d^5*x^5) - 7*a*b^6*d*(10*c^6 - 120*c^5
*d*x - 200*c^4*d^2*x^2 + 200*c^3*d^3*x^3 + 50*c^2*d^4*x^4 + 10*c*d^5*x^5 +
d^6*x^6) + b^7*(-10*c^7 - 140*c^6*d*x + 700*c^4*d^3*x^3 + 350*c^3*d^4*x^4 +
 140*c^2*d^5*x^5 + 35*c*d^6*x^6 + 4*d^7*x^7) - 420*d^2*(-(b*c) + a*d)^5*(a
+ b*x)^2*Log[a + b*x])/(20*b^8*(a + b*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^3, x]
```

fricas [B] time = 1.20, size = 703, normalized size = 3.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^7*d^7*x^7 - 10*b^7*c^7 - 70*a*b^6*c^6*d + 630*a^2*b^5*c^5*d^2 - 1
750*a^3*b^4*c^4*d^3 + 2450*a^4*b^3*c^3*d^4 - 1890*a^5*b^2*c^2*d^5 + 770*a^6
*b*c*d^6 - 130*a^7*d^7 + 7*(5*b^7*c*d^6 - a*b^6*d^7)*x^6 + 14*(10*b^7*c^2*d
^5 - 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 35*(10*b^7*c^3*d^4 - 10*a*b^6*c^2*d
^5 + 5*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 140*(5*b^7*c^4*d^3 - 10*a*b^6*c^3
*d^4 + 10*a^2*b^5*c^2*d^5 - 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 10*(140*a*
b^6*c^4*d^3 - 385*a^2*b^5*c^3*d^4 + 441*a^3*b^4*c^2*d^5 - 238*a^4*b^3*c*d^6
 + 50*a^5*b^2*d^7)*x^2 - 20*(7*b^7*c^6*d - 42*a*b^6*c^5*d^2 + 70*a^2*b^5*c^
4*d^3 - 35*a^3*b^4*c^3*d^4 - 21*a^4*b^3*c^2*d^5 + 28*a^5*b^2*c*d^6 - 8*a^6*
b*d^7)*x + 420*(a^2*b^5*c^5*d^2 - 5*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 -
10*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 - a^7*d^7 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d
^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^
7)*x^2 + 2*(a*b^6*c^5*d^2 - 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 - 10*a^4
*b^3*c^2*d^5 + 5*a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^10*x^2 + 2*
a*b^9*x + a^2*b^8)
```

giac [B] time = 1.26, size = 477, normalized size = 2.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2*c^2*d^5
 + 5*a^4*b*c*d^6 - a^5*d^7)*log(abs(b*x + a))/b^8 - 1/2*(b^7*c^7 + 7*a*b^6*
c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 14*(b^7*c^6*d - 6*a*b^6*c
^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a
^5*b^2*c*d^6 + a^6*b*d^7)*x)/((b*x + a)^2*b^8) + 1/20*(4*b^12*d^7*x^5 + 35*
b^12*c*d^6*x^4 - 15*a*b^11*d^7*x^4 + 140*b^12*c^2*d^5*x^3 - 140*a*b^11*c*d^
6*x^3 + 40*a^2*b^10*d^7*x^3 + 350*b^12*c^3*d^4*x^2 - 630*a*b^11*c^2*d^5*x^2
 + 420*a^2*b^10*c*d^6*x^2 - 100*a^3*b^9*d^7*x^2 + 700*b^12*c^4*d^3*x - 2100
*a*b^11*c^3*d^4*x + 2520*a^2*b^10*c^2*d^5*x - 1400*a^3*b^9*c*d^6*x + 300*a^
4*b^8*d^7*x)/b^15
```

maple [B] time = 0.01, size = 599, normalized size = 3.24

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^3,x)
```

```
[Out] 2*d^7/b^5*x^3*a^2+7*d^5/b^3*x^3*c^2-5*d^7/b^6*x^2*a^3+35/2*d^4/b^3*x^2*c^3+
15*d^7/b^7*a^4*x+35*d^3/b^3*c^4*x+1/2/b^8/(b*x+a)^2*a^7*d^7-21/b^8*d^7*ln(b
*x+a)*a^5+21/b^3*d^2*ln(b*x+a)*c^5-7/b^8*d^7/(b*x+a)*a^6-7/b^2*d/(b*x+a)*c^
6-3/4*d^7/b^4*x^4*a+7/4*d^6/b^3*x^4*c+210/b^5*d^4*ln(b*x+a)*a^2*c^3-105/b^4
*d^3*ln(b*x+a)*a*c^4+42/b^7*d^6/(b*x+a)*a^5*c-105/b^6*d^5/(b*x+a)*a^4*c^2+1
```

```
40/b^5*d^4/(b*x+a)*a^3*c^3-105/b^4*d^3/(b*x+a)*a^2*c^4+42/b^3*d^2/(b*x+a)*a
*c^5-63/2*d^5/b^4*x^2*a*c^2-70*d^6/b^6*a^3*c*x+126*d^5/b^5*a^2*c^2*x-105*d^
4/b^4*a*c^3*x-7/2/b^7/(b*x+a)^2*a^6*c*d^6+21/2/b^6/(b*x+a)^2*a^5*c^2*d^5-35
/2/b^5/(b*x+a)^2*a^4*c^3*d^4+35/2/b^4/(b*x+a)^2*a^3*c^4*d^3-21/2/b^3/(b*x+a
)^2*a^2*c^5*d^2+7/2/b^2/(b*x+a)^2*a*c^6*d+105/b^7*d^6*ln(b*x+a)*a^4*c-210/b
^6*d^5*ln(b*x+a)*a^3*c^2-7*d^6/b^4*x^3*a*c+21*d^6/b^5*x^2*a^2*c-1/2/b/(b*x+
a)^2*c^7+1/5*d^7/b^3*x^5
```

maxima [B] time = 1.54, size = 473, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^7*c^7 + 7*a*b^6*c^6*d - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 -
245*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 77*a^6*b*c*d^6 + 13*a^7*d^7 + 1
4*(b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 +
15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^10*x^2 + 2*a*b^9*x
+ a^2*b^8) + 1/20*(4*b^4*d^7*x^5 + 5*(7*b^4*c*d^6 - 3*a*b^3*d^7)*x^4 + 20*(
7*b^4*c^2*d^5 - 7*a*b^3*c*d^6 + 2*a^2*b^2*d^7)*x^3 + 10*(35*b^4*c^3*d^4 - 6
3*a*b^3*c^2*d^5 + 42*a^2*b^2*c*d^6 - 10*a^3*b*d^7)*x^2 + 20*(35*b^4*c^4*d^3
 - 105*a*b^3*c^3*d^4 + 126*a^2*b^2*c^2*d^5 - 70*a^3*b*c*d^6 + 15*a^4*d^7)*x
)/b^7 + 21*(b^5*c^5*d^2 - 5*a*b^4*c^4*d^3 + 10*a^2*b^3*c^3*d^4 - 10*a^3*b^2
*c^2*d^5 + 5*a^4*b*c*d^6 - a^5*d^7)*log(b*x + a)/b^8
```

mupad [B] time = 0.27, size = 690, normalized size = 3.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^3,x)
```

```
[Out] x*((3*a*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)/b^5 +
(21*c^2*d^5)/b^3))/b + (a^3*d^7)/b^6 - (35*c^3*d^4)/b^3 - (3*a^2*((3*a*d^7)
/b^4 - (7*c*d^6)/b^3))/b^2))/b + (35*c^4*d^3)/b^3 + (a^3*((3*a*d^7)/b^4 - (
7*c*d^6)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a
^2*d^7)/b^5 + (21*c^2*d^5)/b^3))/b^2) - x^4*((3*a*d^7)/(4*b^4) - (7*c*d^6)/
(4*b^3)) - x^2*((3*a*((3*a*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/b - (3*a^2*d^7)
/b^5 + (21*c^2*d^5)/b^3))/(2*b) + (a^3*d^7)/(2*b^6) - (35*c^3*d^4)/(2*b^3)
- (3*a^2*((3*a*d^7)/b^4 - (7*c*d^6)/b^3))/(2*b^2)) + x^3*((a*((3*a*d^7)/b^4
 - (7*c*d^6)/b^3))/b - (a^2*d^7)/b^5 + (7*c^2*d^5)/b^3) - ((13*a^7*d^7 + b^
7*c^7 - 63*a^2*b^5*c^5*d^2 + 175*a^3*b^4*c^4*d^3 - 245*a^4*b^3*c^3*d^4 + 18
9*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 77*a^6*b*c*d^6)/(2*b) + x*(7*a^6*d^7 +
7*b^6*c^6*d - 42*a*b^5*c^5*d^2 + 105*a^2*b^4*c^4*d^3 - 140*a^3*b^3*c^3*d^4
+ 105*a^4*b^2*c^2*d^5 - 42*a^5*b*c*d^6))/(a^2*b^7 + b^9*x^2 + 2*a*b^8*x) +
(d^7*x^5)/(5*b^3) - (log(a + b*x)*(21*a^5*d^7 - 21*b^5*c^5*d^2 + 105*a*b^4*
c^4*d^3 - 210*a^2*b^3*c^3*d^4 + 210*a^3*b^2*c^2*d^5 - 105*a^4*b*c*d^6))/b^8
```

sympy [B] time = 2.95, size = 447, normalized size = 2.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**3,x)
```

```
[Out] x**4*(-3*a*d**7/(4*b**4) + 7*c*d**6/(4*b**3)) + x**3*(2*a**2*d**7/b**5 - 7*
a*c*d**6/b**4 + 7*c**2*d**5/b**3) + x**2*(-5*a**3*d**7/b**6 + 21*a**2*c*d**
```

```
6/b**5 - 63*a*c**2*d**5/(2*b**4) + 35*c**3*d**4/(2*b**3)) + x*(15*a**4*d**7
/b**7 - 70*a**3*c*d**6/b**6 + 126*a**2*c**2*d**5/b**5 - 105*a*c**3*d**4/b**
4 + 35*c**4*d**3/b**3) + (-13*a**7*d**7 + 77*a**6*b*c*d**6 - 189*a**5*b**2*
c**2*d**5 + 245*a**4*b**3*c**3*d**4 - 175*a**3*b**4*c**4*d**3 + 63*a**2*b**
5*c**5*d**2 - 7*a*b**6*c**6*d - b**7*c**7 + x*(-14*a**6*b*d**7 + 84*a**5*b*
*2*c*d**6 - 210*a**4*b**3*c**2*d**5 + 280*a**3*b**4*c**3*d**4 - 210*a**2*b*
*5*c**4*d**3 + 84*a*b**6*c**5*d**2 - 14*b**7*c**6*d))/(2*a**2*b**8 + 4*a*b*
*9*x + 2*b**10*x**2) + d**7*x**5/(5*b**3) - 21*d**2*(a*d - b*c)**5*log(a +
b*x)/b**8
```

3.1180 $\int \frac{(c+dx)^7}{(a+bx)^4} dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{(bc}{3b^8(}$$

Rubi [A] time = 0.21, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{7d^6(a+bx)^3(bc-ad)}{3b^8}+\frac{21d^5(a+bx)^2(bc-ad)^2}{2b^8}+\frac{35d^4x(bc-ad)^3}{b^7}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{35d^3(bc-ad)^4\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}+\frac{d^7(a+bx)^4}{4b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (35*d^4*(b*c - a*d)^3*x)/b^7 - (b*c - a*d)^7/(3*b^8*(a + b*x)^3) - (7*d*(b*
c - a*d)^6)/(2*b^8*(a + b*x)^2) - (21*d^2*(b*c - a*d)^5)/(b^8*(a + b*x)) +
(21*d^5*(b*c - a*d)^2*(a + b*x)^2)/(2*b^8) + (7*d^6*(b*c - a*d)*(a + b*x)^3
)/(3*b^8) + (d^7*(a + b*x)^4)/(4*b^8) + (35*d^3*(b*c - a*d)^4*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^4}\,dx &= \int \left(\frac{35d^4(bc-ad)^3}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^4}+\frac{7d(bc-ad)^6}{b^7(a+bx)^3}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^2}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)}+\frac{21d^5(b\right.\\ &= \frac{35d^4(bc-ad)^3x}{b^7}-\frac{(bc-ad)^7}{3b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{2b^8(a+bx)^2}-\frac{21d^2(bc-ad)^5}{b^8(a+bx)}+\frac{21d^5(bc-ad)^2(a+bx)^2}{2b^8}+\end{aligned}$$

Mathematica [A] time = 0.11, size = 199, normalized size = 1.06

$$\frac{6b^2d^5x^2\left(10a^2d^2-28abcd+21b^2c^2\right)+12bd^4x\left(-20a^3d^3+70a^2bcd^2-84ab^2c^2d+35b^3c^3\right)+4b^3d^6x^3(7bc-4ad)+420d^3(bc-ad)^4\log(a+bx)+\frac{252d^2(ad-bc)^5}{a+bx}-\frac{42d(bc-ad)^6}{(a+bx)^2}-\frac{4(bc-ad)^7}{(a+bx)^3}+3b^4d^7x^4}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^4,x]
```

```
[Out] (12*b*d^4*(35*b^3*c^3 - 84*a*b^2*c^2*d + 70*a^2*b*c*d^2 - 20*a^3*d^3)*x + 6
*b^2*d^5*(21*b^2*c^2 - 28*a*b*c*d + 10*a^2*d^2)*x^2 + 4*b^3*d^6*(7*b*c - 4*
a*d)*x^3 + 3*b^4*d^7*x^4 - (4*(b*c - a*d)^7)/(a + b*x)^3 - (42*d*(b*c - a*d
)^6)/(a + b*x)^2 + (252*d^2*(-(b*c) + a*d)^5)/(a + b*x) + 420*d^3*(b*c - a*
d)^4*Log[a + b*x])/(12*b^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^4,x]

[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^4, x]

fricas [B] time = 1.51, size = 739, normalized size = 3.95

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="fricas")

[Out] 1/12*(3*b^7*d^7*x^7 - 4*b^7*c^7 - 14*a*b^6*c^6*d - 84*a^2*b^5*c^5*d^2 + 770*a^3*b^4*c^4*d^3 - 1820*a^4*b^3*c^3*d^4 + 1974*a^5*b^2*c^2*d^5 - 1036*a^6*b*c*d^6 + 214*a^7*d^7 + 7*(4*b^7*c*d^6 - a*b^6*d^7)*x^6 + 21*(6*b^7*c^2*d^5 - 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 105*(4*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 + 4*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 2*(630*a*b^6*c^3*d^4 - 1323*a^2*b^5*c^2*d^5 + 1022*a^3*b^4*c*d^6 - 278*a^4*b^3*d^7)*x^3 - 6*(42*b^7*c^5*d^2 - 210*a*b^6*c^4*d^3 + 210*a^2*b^5*c^3*d^4 + 63*a^3*b^4*c^2*d^5 - 182*a^4*b^3*c*d^6 + 68*a^5*b^2*d^7)*x^2 - 6*(7*b^7*c^6*d + 42*a*b^6*c^5*d^2 - 315*a^2*b^5*c^4*d^3 + 630*a^3*b^4*c^3*d^4 - 567*a^4*b^3*c^2*d^5 + 238*a^5*b^2*c*d^6 - 37*a^6*b*d^7)*x + 420*(a^3*b^4*c^4*d^3 - 4*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 - 4*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 3*(a*b^6*c^4*d^3 - 4*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 - 4*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 3*(a^2*b^5*c^4*d^3 - 4*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 - 4*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b^8)

giac [B] time = 1.32, size = 470, normalized size = 2.51

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="giac")

[Out] 35*(b^4*c^4*d^3 - 4*a*b^3*c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(abs(b*x + a))/b^8 - 1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*b^2*c*d^6 - 11*a^6*b*d^7)*x)/((b*x + a)^3*b^8) + 1/12*(3*b^12*d^7*x^4 + 28*b^12*c*d^6*x^3 - 16*a*b^11*d^7*x^3 + 126*b^12*c^2*d^5*x^2 - 168*a*b^11*c*d^6*x^2 + 60*a^2*b^10*d^7*x^2 + 420*b^12*c^3*d^4*x - 1008*a*b^11*c^2*d^5*x + 840*a^2*b^10*c*d^6*x - 240*a^3*b^9*d^7*x)/b^16

maple [B] time = 0.01, size = 622, normalized size = 3.33

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^7/(b*x+a)^4,x)

[Out] 210/b^6*d^5/(b*x+a)*a^3*c^2-210/b^5*d^4/(b*x+a)*a^2*c^3+105/b^4*d^3/(b*x+a)*a*c^4-7/2/b^8*d^7/(b*x+a)^2*a^6-7/2/b^2*d/(b*x+a)^2*c^6+35/b^8*d^7*ln(b*x+a)*a^4+35/b^4*d^3*ln(b*x+a)*c^4+21/b^8*d^7/(b*x+a)*a^5-21/b^3*d^2/(b*x+a)*c^5-4/3*d^7/b^5*x^3*a+7/3*d^6/b^4*x^3*c+5*d^7/b^6*x^2*a^2+21/2*d^5/b^4*x^2*c^2-20*d^7/b^7*a^3*x+35*d^4/b^4*c^3*x+1/3/b^8/(b*x+a)^3*a^7*d^7+1/4*d^7/b^4*x^4-1/3/b/(b*x+a)^3*c^7-84*d^5/b^5*a*c^2*x-7/3/b^7/(b*x+a)^3*a^6*c*d^6+7/b^6/(b*x+a)^3*a^5*c^2*d^5-35/3/b^5/(b*x+a)^3*a^4*c^3*d^4+35/3/b^4/(b*x+a)^3*a^3*c^4*d^3-7/b^3/(b*x+a)^3*a^2*c^5*d^2+7/3/b^2/(b*x+a)^3*a*c^6*d+21/b^7*d^6

```
/(b*x+a)^2*a^5*c-105/2/b^6*d^5/(b*x+a)^2*a^4*c^2+70/b^5*d^4/(b*x+a)^2*a^3*c
^3-105/2/b^4*d^3/(b*x+a)^2*a^2*c^4+21/b^3*d^2/(b*x+a)^2*a*c^5-14*d^6/b^5*x^
2*a*c+70*d^6/b^6*a^2*c*x-140/b^7*d^6*ln(b*x+a)*a^3*c+210/b^6*d^5*ln(b*x+a)*
a^2*c^2-140/b^5*d^4*ln(b*x+a)*a*c^3-105/b^7*d^6/(b*x+a)*a^4*c
```

maxima [B] time = 1.63, size = 484, normalized size = 2.59

$$-\frac{2 b^{7} c^{7} + 7 a b^{6} c^{6} d + 42 a^{2} b^{5} c^{5} d^{2} - 385 a^{3} b^{4} c^{4} d^{3} + 910 a^{4} b^{3} c^{3} d^{4} - 987 a^{5} b^{2} c^{2} d^{5} + 518 a^{6} b c d^{6} - 107 a^{7} d^{7} + 126 \left(b^{7} c^{5} d^{2} - 5 a b^{6} c^{4} d^{3} + 10 a^{2} b^{5} c^{3} d^{4} - 10 a^{3} b^{4} c^{2} d^{5} + 5 a^{4} b^{3} c d^{6} - a^{5} b^{2} d^{7}\right) x^{2} + 21 \left(b^{7} c^{6} d + 6 a b^{6} c^{5} d^{2} - 45 a^{2} b^{5} c^{4} d^{3} + 100 a^{3} b^{4} c^{3} d^{4} - 105 a^{4} b^{3} c^{2} d^{5} + 54 a^{5} b^{2} c d^{6} - 11 a^{6} b d^{7}\right) x}{6 \left(b^{11} x^{3} + 3 a b^{10} x^{2} + 3 a^{2} b^{9} x + a^{3} b^{8}\right)} + \frac{3 b^{3} d^{7} x^{4} + 4 \left(7 b^{3} c d^{6} - 4 a b^{2} d^{7}\right) x^{3} + 6 \left(21 b^{3} c^{2} d^{5} - 28 a b^{2} c d^{6} + 10 a^{2} b d^{7}\right) x^{2} + 12 \left(35 b^{3} c^{3} d^{4} - 84 a b^{2} c^{2} d^{5} + 70 a^{2} b c d^{6} - 20 a^{3} d^{7}\right) x}{12 b^{7}} + \frac{35 \left(b^{4} c^{4} d^{3} - 4 a b^{3} c^{3} d^{4} + 6 a^{2} b^{2} c^{2} d^{5} - 4 a^{3} b c d^{6} + a^{4} d^{7}\right) \log\left(b x + a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b^7*c^7 + 7*a*b^6*c^6*d + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3
+ 910*a^4*b^3*c^3*d^4 - 987*a^5*b^2*c^2*d^5 + 518*a^6*b*c*d^6 - 107*a^7*d^7
 + 126*(b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2
*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^2 + 21*(b^7*c^6*d + 6*a*b^6*c^5*d^2
 - 45*a^2*b^5*c^4*d^3 + 100*a^3*b^4*c^3*d^4 - 105*a^4*b^3*c^2*d^5 + 54*a^5*
b^2*c*d^6 - 11*a^6*b*d^7)*x)/(b^11*x^3 + 3*a*b^10*x^2 + 3*a^2*b^9*x + a^3*b
^8) + 1/12*(3*b^3*d^7*x^4 + 4*(7*b^3*c*d^6 - 4*a*b^2*d^7)*x^3 + 6*(21*b^3*c
^2*d^5 - 28*a*b^2*c*d^6 + 10*a^2*b*d^7)*x^2 + 12*(35*b^3*c^3*d^4 - 84*a*b^2
*c^2*d^5 + 70*a^2*b*c*d^6 - 20*a^3*d^7)*x)/b^7 + 35*(b^4*c^4*d^3 - 4*a*b^3*
c^3*d^4 + 6*a^2*b^2*c^2*d^5 - 4*a^3*b*c*d^6 + a^4*d^7)*log(b*x + a)/b^8
```

mupad [B] time = 0.29, size = 559, normalized size = 2.99

$$x^2\left(\frac{2a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{3a^2d^7}{b^6}+\frac{21c^2d^5}{2b^4}\right)-x^3\left(\frac{4ad^7}{3b^5}-\frac{7cd^6}{3b^4}\right)-\frac{\frac{-107a^7d^7+518a^6bcd^6-987a^5b^2c^2d^5+910a^4b^3c^3d^4-385a^3b^4c^4d^3+42a^2b^5c^5d^2+7ab^6c^6d+2b^7c^7}{6b}+x\left(-\frac{77a^6d^7}{2}+189a^5bcd^6-\frac{735a^4b^2c^2d^5}{2}+350a^3b^3c^3d^4-\frac{315a^2b^4c^4d^3}{2}+21ab^5c^5d^2+\frac{7b^6c^6d}{2}\right)-x^2\left(21a^5bd^7-105a^4b^2cd^6+210a^3b^3c^2d^5-210a^2b^4c^3d^4+105ab^5c^4d^3-21b^6c^5d^2\right)}{a^3b^7+3a^2b^8x+3ab^9x^2+b^{10}x^3}-x\left(\frac{4a\left(\frac{4a\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b}-\frac{6a^2d^7}{b^6}+\frac{21c^2d^5}{b^4}\right)}{b}+\frac{4a^3d^7}{b^7}-\frac{35c^3d^4}{b^4}-\frac{6a^2\left(\frac{4ad^7}{b^5}-\frac{7cd^6}{b^4}\right)}{b^2}\right)+\frac{\ln(a+bx)\left(35a^4d^7-140a^3bcd^6+210a^2b^2c^2d^5-140ab^3c^3d^4+35b^4c^4d^3\right)}{b^8}+\frac{d^7x^4}{4b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^4,x)
```

```
[Out] x^2*((2*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (3*a^2*d^7)/b^6 + (21*c^2*d^
5)/(2*b^4)) - x^3*((4*a*d^7)/(3*b^5) - (7*c*d^6)/(3*b^4)) - ((2*b^7*c^7 - 1
07*a^7*d^7 + 42*a^2*b^5*c^5*d^2 - 385*a^3*b^4*c^4*d^3 + 910*a^4*b^3*c^3*d^4
 - 987*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d + 518*a^6*b*c*d^6)/(6*b) + x*((7*b^6
*c^6*d)/2 - (77*a^6*d^7)/2 + 21*a*b^5*c^5*d^2 - (315*a^2*b^4*c^4*d^3)/2 + 3
50*a^3*b^3*c^3*d^4 - (735*a^4*b^2*c^2*d^5)/2 + 189*a^5*b*c*d^6) - x^2*(21*a
^5*b*d^7 - 21*b^6*c^5*d^2 + 105*a*b^5*c^4*d^3 - 105*a^4*b^2*c*d^6 - 210*a^2
*b^4*c^3*d^4 + 210*a^3*b^3*c^2*d^5))/(a^3*b^7 + b^10*x^3 + 3*a^2*b^8*x + 3*
a*b^9*x^2) - x*((4*a*((4*a*((4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b - (6*a^2*d^7)
/b^6 + (21*c^2*d^5)/b^4))/b + (4*a^3*d^7)/b^7 - (35*c^3*d^4)/b^4 - (6*a^2*(
(4*a*d^7)/b^5 - (7*c*d^6)/b^4))/b^2) + (log(a + b*x)*(35*a^4*d^7 + 35*b^4*c
^4*d^3 - 140*a*b^3*c^3*d^4 + 210*a^2*b^2*c^2*d^5 - 140*a^3*b*c*d^6))/b^8 +
(d^7*x^4)/(4*b^4)
```

sympy [B] time = 6.12, size = 474, normalized size = 2.53

$$x^3\left(-\frac{4ad^7}{3b^5}+\frac{7cd^6}{3b^4}\right)+x^2\left(\frac{5a^2d^7}{b^6}-\frac{14acd^6}{b^5}+\frac{21c^2d^5}{2b^4}\right)+x\left(-\frac{20a^3d^7}{b^7}+\frac{70a^2cd^6}{b^6}-\frac{84ac^2d^5}{b^5}+\frac{35c^3d^4}{b^4}\right)+\frac{107a^7d^7-518a^6bcd^6+987a^5b^2c^2d^5-910a^4b^3c^3d^4+385a^3b^4c^4d^3-42a^2b^5c^5d^2-7ab^6c^6d-2b^7c^7+x^2\left(126a^5b^2d^7-630a^4b^3cd^6+1260a^3b^4c^2d^5-1260a^2b^5c^3d^4+630ab^6c^4d^3-126b^7c^5d^2\right)+x\left(231a^6bd^7-1134a^5b^2cd^6+2205a^4b^3c^2d^5-2100a^3b^4c^3d^4+945a^2b^5c^4d^3-126ab^6c^5d^2-21b^7c^6d\right)}{6a^3b^8+18a^2b^9x+18ab^{10}x^2+6b^{11}x^3}+\frac{d^7x^4}{4b^4}+\frac{35d^3(ad-bc)^4\log(a+bx)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**4,x)
```

```
[Out] x**3*(-4*a*d**7/(3*b**5) + 7*c*d**6/(3*b**4)) + x**2*(5*a**2*d**7/b**6 - 14
*a*c*d**6/b**5 + 21*c**2*d**5/(2*b**4)) + x*(-20*a**3*d**7/b**7 + 70*a**2*c
*d**6/b**6 - 84*a*c**2*d**5/b**5 + 35*c**3*d**4/b**4) + (107*a**7*d**7 - 51
8*a**6*b*c*d**6 + 987*a**5*b**2*c**2*d**5 - 910*a**4*b**3*c**3*d**4 + 385*a
**3*b**4*c**4*d**3 - 42*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 2*b**7*c**7
 + x**2*(126*a**5*b**2*d**7 - 630*a**4*b**3*c*d**6 + 1260*a**3*b**4*c**2*d*
*5 - 1260*a**2*b**5*c**3*d**4 + 630*a*b**6*c**4*d**3 - 126*b**7*c**5*d**2)
+ x*(231*a**6*b*d**7 - 1134*a**5*b**2*c*d**6 + 2205*a**4*b**3*c**2*d**5 - 2
```

```
100*a**3*b**4*c**3*d**4 + 945*a**2*b**5*c**4*d**3 - 126*a*b**6*c**5*d**2 -
21*b**7*c**6*d))/(6*a**3*b**8 + 18*a**2*b**9*x + 18*a*b**10*x**2 + 6*b**11*
x**3) + d**7*x**4/(4*b**4) + 35*d**3*(a*d - b*c)**4*log(a + b*x)/b**8
```

3.1181 $\int \frac{(c+dx)^7}{(a+bx)^5} \, dx$

Optimal. Leaf size=187

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}$$

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{7d^6(a+bx)^2(bc-ad)}{2b^8}+\frac{21d^5x(bc-ad)^2}{b^7}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}+\frac{35d^4(bc-ad)^3\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}+\frac{d^7(a+bx)^3}{3b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (21*d^5*(b*c - a*d)^2*x)/b^7 - (b*c - a*d)^7/(4*b^8*(a + b*x)^4) - (7*d*(b*
c - a*d)^6)/(3*b^8*(a + b*x)^3) - (21*d^2*(b*c - a*d)^5)/(2*b^8*(a + b*x)^2
) - (35*d^3*(b*c - a*d)^4)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*(a + b*x)^2
)/(2*b^8) + (d^7*(a + b*x)^3)/(3*b^8) + (35*d^4*(b*c - a*d)^3*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^5} \, dx = \int \left(\frac{21d^5(bc-ad)^2}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^5}+\frac{7d(bc-ad)^6}{b^7(a+bx)^4}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^3}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^2}+ \dots\right)$$

$$= \frac{21d^5(bc-ad)^2x}{b^7}-\frac{(bc-ad)^7}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{b^8(a+bx)}+ \dots$$

Mathematica [A] time = 0.11, size = 173, normalized size = 0.93

$$\frac{12bd^5x\left(15a^2d^2-35abcd+21b^2c^2\right)+6b^2d^6x^2(7bc-5ad)+420d^4(bc-ad)^3\log(a+bx)-\frac{420d^3(bc-ad)^4}{a+bx}+\frac{126d^2(ad-bc)^5}{(a+bx)^2}-\frac{28d(bc-ad)^6}{(a+bx)^3}-\frac{3(bc-ad)^7}{(a+bx)^4}+4b^3d^7x^3}{12b^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(21*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2)*x + 6*b^2*d^6*(7*b*c - 5*a
*d)*x^2 + 4*b^3*d^7*x^3 - (3*(b*c - a*d)^7)/(a + b*x)^4 - (28*d*(b*c - a*d)
^6)/(a + b*x)^3 + (126*d^2*(-(b*c) + a*d)^5)/(a + b*x)^2 - (420*d^3*(b*c -
a*d)^4)/(a + b*x) + 420*d^4*(b*c - a*d)^3*Log[a + b*x])/(12*b^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^5, x]
```

fricas [B] time = 1.58, size = 754, normalized size = 4.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^7*d^7*x^7 - 3*b^7*c^7 - 7*a*b^6*c^6*d - 21*a^2*b^5*c^5*d^2 - 105*
a^3*b^4*c^4*d^3 + 875*a^4*b^3*c^3*d^4 - 1617*a^5*b^2*c^2*d^5 + 1197*a^6*b*c
*d^6 - 319*a^7*d^7 + 14*(3*b^7*c*d^6 - a*b^6*d^7)*x^6 + 84*(3*b^7*c^2*d^5 -
 3*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 4*(252*a*b^6*c^2*d^5 - 357*a^2*b^5*c*d^
6 + 139*a^3*b^4*d^7)*x^4 - 4*(105*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4 + 252*a^2
*b^5*c^2*d^5 + 168*a^3*b^4*c*d^6 - 136*a^4*b^3*d^7)*x^3 - 6*(21*b^7*c^5*d^2
 + 105*a*b^6*c^4*d^3 - 630*a^2*b^5*c^3*d^4 + 882*a^3*b^4*c^2*d^5 - 462*a^4*
b^3*c*d^6 + 74*a^5*b^2*d^7)*x^2 - 4*(7*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 105*a
^2*b^5*c^4*d^3 - 770*a^3*b^4*c^3*d^4 + 1302*a^4*b^3*c^2*d^5 - 882*a^5*b^2*c
*d^6 + 214*a^6*b*d^7)*x + 420*(a^4*b^3*c^3*d^4 - 3*a^5*b^2*c^2*d^5 + 3*a^6*
b*c*d^6 - a^7*d^7 + (b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*
b^4*d^7)*x^4 + 4*(a*b^6*c^3*d^4 - 3*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 - a^4
*b^3*d^7)*x^3 + 6*(a^2*b^5*c^3*d^4 - 3*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*d^6 -
a^5*b^2*d^7)*x^2 + 4*(a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6
 - a^6*b*d^7)*x)*log(b*x + a))/(b^12*x^4 + 4*a*b^11*x^3 + 6*a^2*b^10*x^2 +
4*a^3*b^9*x + a^4*b^8)
```

giac [B] time = 1.29, size = 660, normalized size = 3.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/6*(2*d^7 + 21*(b^2*c*d^6 - a*b*d^7)/((b*x + a)*b) + 126*(b^4*c^2*d^5 - 2*
a*b^3*c*d^6 + a^2*b^2*d^7)/((b*x + a)^2*b^2))*(b*x + a)^3/b^8 - 35*(b^3*c^3
*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(abs(b*x + a)/((b*x +
a)^2*abs(b)))/b^8 - 1/12*(3*b^43*c^7/(b*x + a)^4 + 28*b^42*c^6*d/(b*x + a)^
3 - 21*a*b^42*c^6*d/(b*x + a)^4 + 126*b^41*c^5*d^2/(b*x + a)^2 - 168*a*b^41
*c^5*d^2/(b*x + a)^3 + 63*a^2*b^41*c^5*d^2/(b*x + a)^4 + 420*b^40*c^4*d^3/(
b*x + a) - 630*a*b^40*c^4*d^3/(b*x + a)^2 + 420*a^2*b^40*c^4*d^3/(b*x + a)^
3 - 105*a^3*b^40*c^4*d^3/(b*x + a)^4 - 1680*a*b^39*c^3*d^4/(b*x + a) + 1260
*a^2*b^39*c^3*d^4/(b*x + a)^2 - 560*a^3*b^39*c^3*d^4/(b*x + a)^3 + 105*a^4*
b^39*c^3*d^4/(b*x + a)^4 + 2520*a^2*b^38*c^2*d^5/(b*x + a) - 1260*a^3*b^38*
c^2*d^5/(b*x + a)^2 + 420*a^4*b^38*c^2*d^5/(b*x + a)^3 - 63*a^5*b^38*c^2*d^
5/(b*x + a)^4 - 1680*a^3*b^37*c*d^6/(b*x + a) + 630*a^4*b^37*c*d^6/(b*x + a
)^2 - 168*a^5*b^37*c*d^6/(b*x + a)^3 + 21*a^6*b^37*c*d^6/(b*x + a)^4 + 420*
a^4*b^36*d^7/(b*x + a) - 126*a^5*b^36*d^7/(b*x + a)^2 + 28*a^6*b^36*d^7/(b*
x + a)^3 - 3*a^7*b^36*d^7/(b*x + a)^4)/b^44
```

maple [B] time = 0.02, size = 641, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^5,x)
```

```
[Out] 21/2/b^8*d^7/(b*x+a)^2*a^5-21/2/b^3*d^2/(b*x+a)^2*c^5-35/b^8*d^7*ln(b*x+a)*
a^3+35/b^5*d^4*ln(b*x+a)*c^3+1/4/b^8/(b*x+a)^4*a^7*d^7-35/b^8*d^7/(b*x+a)*a
```

```
^4-35/b^4*d^3/(b*x+a)*c^4-5/2*d^7/b^6*x^2*a+7/2*d^6/b^5*x^2*c+15*d^7/b^7*a^
2*x+21*d^5/b^5*c^2*x-7/3/b^8*d^7/(b*x+a)^3*a^6-7/3/b^2*d/(b*x+a)^3*c^6+1/3*
d^7/b^5*x^3-1/4/b/(b*x+a)^4*c^7-35*d^6/b^6*a*c*x+14/b^7*d^6/(b*x+a)^3*a^5*c
-35/b^6*d^5/(b*x+a)^3*a^4*c^2+140/3/b^5*d^4/(b*x+a)^3*a^3*c^3-35/b^4*d^3/(b
*x+a)^3*a^2*c^4+105/b^7*d^6*ln(b*x+a)*a^2*c-105/b^6*d^5*ln(b*x+a)*a*c^2-7/4
/b^7/(b*x+a)^4*a^6*c*d^6+21/4/b^6/(b*x+a)^4*a^5*c^2*d^5-35/4/b^5/(b*x+a)^4*
a^4*c^3*d^4+35/4/b^4/(b*x+a)^4*a^3*c^4*d^3-21/4/b^3/(b*x+a)^4*a^2*c^5*d^2+7
/4/b^2/(b*x+a)^4*a*c^6*d+140/b^7*d^6/(b*x+a)*a^3*c-210/b^6*d^5/(b*x+a)*a^2*
c^2+140/b^5*d^4/(b*x+a)*a*c^3+14/b^3*d^2/(b*x+a)^3*a*c^5-105/2/b^7*d^6/(b*x
+a)^2*a^4*c+105/b^6*d^5/(b*x+a)^2*a^3*c^2-105/b^5*d^4/(b*x+a)^2*a^2*c^3+105
/2/b^4*d^3/(b*x+a)^2*a*c^4
```

maxima [B] time = 1.73, size = 494, normalized size = 2.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^7*c^7 + 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3
 - 875*a^4*b^3*c^3*d^4 + 1617*a^5*b^2*c^2*d^5 - 1197*a^6*b*c*d^6 + 319*a^7*
d^7 + 420*(b^7*c^4*d^3 - 4*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 4*a^3*b^4*c*
d^6 + a^4*b^3*d^7)*x^3 + 126*(b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 - 30*a^2*b^5*c^
3*d^4 + 50*a^3*b^4*c^2*d^5 - 35*a^4*b^3*c*d^6 + 9*a^5*b^2*d^7)*x^2 + 28*(b^
7*c^6*d + 3*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 110*a^3*b^4*c^3*d^4 + 195*
a^4*b^3*c^2*d^5 - 141*a^5*b^2*c*d^6 + 37*a^6*b*d^7)*x)/(b^12*x^4 + 4*a*b^11
*x^3 + 6*a^2*b^10*x^2 + 4*a^3*b^9*x + a^4*b^8) + 1/6*(2*b^2*d^7*x^3 + 3*(7*
b^2*c*d^6 - 5*a*b*d^7)*x^2 + 6*(21*b^2*c^2*d^5 - 35*a*b*c*d^6 + 15*a^2*d^7)
*x)/b^7 + 35*(b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*log(
b*x + a)/b^8
```

mupad [B] time = 0.77, size = 512, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^5,x)
```

```
[Out] x*((5*a*((5*a*d^7)/b^6 - (7*c*d^6)/b^5))/b - (10*a^2*d^7)/b^7 + (21*c^2*d^5
)/b^5) - x^2*((5*a*d^7)/(2*b^6) - (7*c*d^6)/(2*b^5)) - ((319*a^7*d^7 + 3*b^
7*c^7 + 21*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 - 875*a^4*b^3*c^3*d^4 + 16
17*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 1197*a^6*b*c*d^6)/(12*b) + x*((259*a^6
*d^7)/3 + (7*b^6*c^6*d)/3 + 7*a*b^5*c^5*d^2 + 35*a^2*b^4*c^4*d^3 - (770*a^3
*b^3*c^3*d^4)/3 + 455*a^4*b^2*c^2*d^5 - 329*a^5*b*c*d^6) + x^3*(35*a^4*b^2*
d^7 + 35*b^6*c^4*d^3 - 140*a*b^5*c^3*d^4 - 140*a^3*b^3*c*d^6 + 210*a^2*b^4*
c^2*d^5) + x^2*((189*a^5*b*d^7)/2 + (21*b^6*c^5*d^2)/2 + (105*a*b^5*c^4*d^3
)/2 - (735*a^4*b^2*c*d^6)/2 - 315*a^2*b^4*c^3*d^4 + 525*a^3*b^3*c^2*d^5))/(
a^4*b^7 + b^11*x^4 + 4*a^3*b^8*x + 4*a*b^10*x^3 + 6*a^2*b^9*x^2) - (log(a +
 b*x)*(35*a^3*d^7 - 35*b^3*c^3*d^4 + 105*a*b^2*c^2*d^5 - 105*a^2*b*c*d^6))/
b^8 + (d^7*x^3)/(3*b^5)
```

sympy [B] time = 22.44, size = 500, normalized size = 2.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**5,x)
```

```
[Out] x**2*(-5*a*d**7/(2*b**6) + 7*c*d**6/(2*b**5)) + x*(15*a**2*d**7/b**7 - 35*a
*c*d**6/b**6 + 21*c**2*d**5/b**5) + (-319*a**7*d**7 + 1197*a**6*b*c*d**6 -
```

```
1617*a**5*b**2*c**2*d**5 + 875*a**4*b**3*c**3*d**4 - 105*a**3*b**4*c**4*d**
3 - 21*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 3*b**7*c**7 + x**3*(-420*a**
4*b**3*d**7 + 1680*a**3*b**4*c*d**6 - 2520*a**2*b**5*c**2*d**5 + 1680*a*b**
6*c**3*d**4 - 420*b**7*c**4*d**3) + x**2*(-1134*a**5*b**2*d**7 + 4410*a**4*
b**3*c*d**6 - 6300*a**3*b**4*c**2*d**5 + 3780*a**2*b**5*c**3*d**4 - 630*a*b
**6*c**4*d**3 - 126*b**7*c**5*d**2) + x*(-1036*a**6*b*d**7 + 3948*a**5*b**2
*c*d**6 - 5460*a**4*b**3*c**2*d**5 + 3080*a**3*b**4*c**3*d**4 - 420*a**2*b*
*5*c**4*d**3 - 84*a*b**6*c**5*d**2 - 28*b**7*c**6*d))/(12*a**4*b**8 + 48*a*
*3*b**9*x + 72*a**2*b**10*x**2 + 48*a*b**11*x**3 + 12*b**12*x**4) + d**7*x*
*3/(3*b**5) - 35*d**4*(a*d - b*c)**3*log(a + b*x)/b**8
```

3.1182 $\int \frac{(c+dx)^7}{(a+bx)^6} dx$

Optimal. Leaf size=181

$$\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^6x(7b}{}$$

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^6x(7bc-6ad)}{b^7}-\frac{35d^4(bc-ad)^3}{b^8(a+bx)}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}+\frac{21d^5(bc-ad)^2\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}+\frac{d^7x^2}{2b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (d^6*(7*b*c - 6*a*d)*x)/b^7 + (d^7*x^2)/(2*b^6) - (b*c - a*d)^7/(5*b^8*(a +
 b*x)^5) - (7*d*(b*c - a*d)^6)/(4*b^8*(a + b*x)^4) - (7*d^2*(b*c - a*d)^5)/
(b^8*(a + b*x)^3) - (35*d^3*(b*c - a*d)^4)/(2*b^8*(a + b*x)^2) - (35*d^4*(b
*c - a*d)^3)/(b^8*(a + b*x)) + (21*d^5*(b*c - a*d)^2*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^6}\,dx &= \int \left(\frac{d^6(7bc-6ad)}{b^7}+\frac{d^7x}{b^6}+\frac{(bc-ad)^7}{b^7(a+bx)^6}+\frac{7d(bc-ad)^6}{b^7(a+bx)^5}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^4}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^3}+\right.\\ &= \frac{d^6(7bc-6ad)x}{b^7}+\frac{d^7x^2}{2b^6}-\frac{(bc-ad)^7}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{4b^8(a+bx)^4}-\frac{7d^2(bc-ad)^5}{b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{2b^8(a+bx)^2}-\frac{35d}{b}\end{aligned}$$

Mathematica [B] time = 0.15, size = 389, normalized size = 2.15

$$\frac{459a^7d^7+3a^6bd^6(625dx-406c)+a^5b^2d^5\left(959c^2-5250cdx+2700d^2x^2\right)+5a^4b^3d^4\left(-28c^3+875c^2dx-1680cd^2x^2+260d^3x^3\right)-5a^3b^4d^3\left(7c^4+140c^3dx-1540c^2d^2x^2+1120cd^3x^3+80d^4x^4\right)-a^2b^5d^2\left(14c^5+175c^4dx+1400c^3d^2x^2-6300c^2d^3x^3+700cd^4x^4+500d^5x^5\right)-7ab^6d\left(c^6+10c^5dx+50c^4d^2x^2+200c^3d^3x^3-300c^2d^4x^4-100cd^5x^5+10d^6x^6\right)+420d^5(a+bx)^5(bc-ad)^2\log(a+bx)-\left(b^7\left(4c^7+35c^6dx+140c^5d^2x^2+350c^4d^3x^3+700c^3d^4x^4-140cd^6x^6-10d^7x^7\right)\right)}{20b^8(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] (459*a^7*d^7 + 3*a^6*b*d^6*(-406*c + 625*d*x) + a^5*b^2*d^5*(959*c^2 - 5250
*c*d*x + 2700*d^2*x^2) + 5*a^4*b^3*d^4*(-28*c^3 + 875*c^2*d*x - 1680*c*d^2*
x^2 + 260*d^3*x^3) - 5*a^3*b^4*d^3*(7*c^4 + 140*c^3*d*x - 1540*c^2*d^2*x^2
+ 1120*c*d^3*x^3 + 80*d^4*x^4) - a^2*b^5*d^2*(14*c^5 + 175*c^4*d*x + 1400*c
^3*d^2*x^2 - 6300*c^2*d^3*x^3 + 700*c*d^4*x^4 + 500*d^5*x^5) - 7*a*b^6*d*(c
^6 + 10*c^5*d*x + 50*c^4*d^2*x^2 + 200*c^3*d^3*x^3 - 300*c^2*d^4*x^4 - 100*
c*d^5*x^5 + 10*d^6*x^6) - b^7*(4*c^7 + 35*c^6*d*x + 140*c^5*d^2*x^2 + 350*c
^4*d^3*x^3 + 700*c^3*d^4*x^4 - 140*c*d^6*x^6 - 10*d^7*x^7) + 420*d^5*(b*c -
 a*d)^2*(a + b*x)^5*Log[a + b*x])/(20*b^8*(a + b*x)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^6}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^6, x]
```

fricas [B] time = 1.40, size = 732, normalized size = 4.04

$$\frac{10\,b^{7}d^{7}x^{7}-4\,b^{7}c^{7}-7\,ab^{6}c^{6}d-14\,a^{2}b^{5}c^{5}d^{2}-35\,a^{3}b^{4}c^{4}d^{3}-140\,a^{4}b^{3}c^{3}d^{4}+959\,a^{5}b^{2}c^{2}d^{5}-1218\,a^{6}bcd^{6}+459\,a^{7}d^{7}+70\left(2\,b^{7}cd^{6}-ab^{6}d^{7}\right)x^{6}+100\left(7\,ab^{6}cd^{6}-5\,a^{2}b^{5}d^{7}\right)x^{5}-100\left(7\,b^{7}c^{3}d^{4}-21\,ab^{6}c^{2}d^{5}+7\,a^{2}b^{5}cd^{6}+4\,a^{3}b^{4}d^{7}\right)x^{4}-50\left(7\,b^{7}c^{4}d^{3}+28\,ab^{6}c^{3}d^{4}-126\,a^{2}b^{5}c^{2}d^{5}+112\,a^{3}b^{4}cd^{6}-26\,a^{4}b^{3}d^{7}\right)x^{3}-10\left(14\,b^{7}c^{5}d^{2}+35\,ab^{6}c^{4}d^{3}+140\,a^{2}b^{5}c^{3}d^{4}-770\,a^{3}b^{4}c^{2}d^{5}+840\,a^{4}b^{3}cd^{6}-270\,a^{5}b^{2}d^{7}\right)x^{2}-5\left(7\,b^{7}c^{6}d+14\,ab^{6}c^{5}d^{2}+35\,a^{2}b^{5}c^{4}d^{3}+140\,a^{3}b^{4}c^{3}d^{4}-875\,a^{4}b^{3}c^{2}d^{5}+1050\,a^{5}b^{2}cd^{6}-375\,a^{6}bd^{7}\right)x+420\left(a^{5}b^{2}c^{2}d^{5}-2\,a^{6}bcd^{6}+a^{7}d^{7}+\left(b^{7}c^{2}d^{5}-2\,ab^{6}cd^{6}+a^{2}b^{5}d^{7}\right)x^{5}+5\left(ab^{6}c^{2}d^{5}-2\,a^{2}b^{5}cd^{6}+a^{3}b^{4}d^{7}\right)x^{4}+10\left(a^{2}b^{5}c^{2}d^{5}-2\,a^{3}b^{4}cd^{6}+a^{4}b^{3}d^{7}\right)x^{3}+10\left(a^{3}b^{4}c^{2}d^{5}-2\,a^{4}b^{3}cd^{6}+a^{5}b^{2}d^{7}\right)x^{2}+5\left(a^{4}b^{3}c^{2}d^{5}-2\,a^{5}b^{2}cd^{6}+a^{6}bd^{7}\right)x\right)\log\left(bx+a\right)}{20\left(b^{13}x^{5}+5\,ab^{12}x^{4}+10\,a^{2}b^{11}x^{3}+10\,a^{3}b^{10}x^{2}+5\,a^{4}b^{9}x+a^{5}b^{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/20*(10*b^7*d^7*x^7 - 4*b^7*c^7 - 7*a*b^6*c^6*d - 14*a^2*b^5*c^5*d^2 - 35*
a^3*b^4*c^4*d^3 - 140*a^4*b^3*c^3*d^4 + 959*a^5*b^2*c^2*d^5 - 1218*a^6*b*c*
d^6 + 459*a^7*d^7 + 70*(2*b^7*c*d^6 - a*b^6*d^7)*x^6 + 100*(7*a*b^6*c*d^6 -
 5*a^2*b^5*d^7)*x^5 - 100*(7*b^7*c^3*d^4 - 21*a*b^6*c^2*d^5 + 7*a^2*b^5*c*d
^6 + 4*a^3*b^4*d^7)*x^4 - 50*(7*b^7*c^4*d^3 + 28*a*b^6*c^3*d^4 - 126*a^2*b^
5*c^2*d^5 + 112*a^3*b^4*c*d^6 - 26*a^4*b^3*d^7)*x^3 - 10*(14*b^7*c^5*d^2 +
35*a*b^6*c^4*d^3 + 140*a^2*b^5*c^3*d^4 - 770*a^3*b^4*c^2*d^5 + 840*a^4*b^3*
c*d^6 - 270*a^5*b^2*d^7)*x^2 - 5*(7*b^7*c^6*d + 14*a*b^6*c^5*d^2 + 35*a^2*b
^5*c^4*d^3 + 140*a^3*b^4*c^3*d^4 - 875*a^4*b^3*c^2*d^5 + 1050*a^5*b^2*c*d^6
 - 375*a^6*b*d^7)*x + 420*(a^5*b^2*c^2*d^5 - 2*a^6*b*c*d^6 + a^7*d^7 + (b^7
*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 5*(a*b^6*c^2*d^5 - 2*a^2*b^5*
c*d^6 + a^3*b^4*d^7)*x^4 + 10*(a^2*b^5*c^2*d^5 - 2*a^3*b^4*c*d^6 + a^4*b^3*
d^7)*x^3 + 10*(a^3*b^4*c^2*d^5 - 2*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 5*(a^
4*b^3*c^2*d^5 - 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)*log(b*x + a))/(b^13*x^5 + 5
*a*b^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8)
```

giac [B] time = 1.38, size = 463, normalized size = 2.56

$$\frac{21\left(b^{2}c^{2}d^{5}-2\,abcd^{6}+a^{2}d^{7}\right)\log\left(|bx+a|\right)}{b^{8}}+\frac{b^{6}d^{7}x^{2}+14\,b^{6}cd^{6}x-12\,ab^{5}d^{7}x}{2\,b^{12}}-\frac{4\,b^{7}c^{7}+7\,ab^{6}c^{6}d+14\,a^{2}b^{5}c^{5}d^{2}+35\,a^{3}b^{4}c^{4}d^{3}+140\,a^{4}b^{3}c^{3}d^{4}-959\,a^{5}b^{2}c^{2}d^{5}+1218\,a^{6}bcd^{6}-459\,a^{7}d^{7}+700\left(b^{7}c^{3}d^{4}-3\,ab^{6}c^{2}d^{5}+3\,a^{2}b^{5}cd^{6}-a^{3}b^{4}d^{7}\right)x^{4}+350\left(b^{7}c^{4}d^{3}+4\,ab^{6}c^{3}d^{4}-18\,a^{2}b^{5}c^{2}d^{5}+20\,a^{3}b^{4}cd^{6}-7\,a^{4}b^{3}d^{7}\right)x^{3}+70\left(2\,b^{7}c^{5}d^{2}+5\,ab^{6}c^{4}d^{3}+20\,a^{2}b^{5}c^{3}d^{4}-110\,a^{3}b^{4}c^{2}d^{5}+130\,a^{4}b^{3}cd^{6}-47\,a^{5}b^{2}d^{7}\right)x^{2}+35\left(b^{7}c^{6}d+2\,ab^{6}c^{5}d^{2}+5\,a^{2}b^{5}c^{4}d^{3}+20\,a^{3}b^{4}c^{3}d^{4}-125\,a^{4}b^{3}c^{2}d^{5}+154\,a^{5}b^{2}cd^{6}-57\,a^{6}bd^{7}\right)x}{20\left(bx+a\right)^{5}b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 21*(b^2*c^2*d^5 - 2*a*b*c*d^6 + a^2*d^7)*log(abs(b*x + a))/b^8 + 1/2*(b^6*d
^7*x^2 + 14*b^6*c*d^6*x - 12*a*b^5*d^7*x)/b^12 - 1/20*(4*b^7*c^7 + 7*a*b^6*
c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959
*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^7 + 700*(b^7*c^3*d^4 - 3*a*
b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4 + 350*(b^7*c^4*d^3 + 4*a*b
^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d^6 - 7*a^4*b^3*d^7)*x^3 + 7
0*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 - 110*a^3*b^4*c^2*d
^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 35*(b^7*c^6*d + 2*a*b^6*c^5*
d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 125*a^4*b^3*c^2*d^5 + 154*a^
5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/((b*x + a)^5*b^8)
```

maple [B] time = 0.01, size = 656, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^6,x)
```

```
[Out] -105/b^7*d^6/(b*x+a)*a^2*c+105/b^6*d^5/(b*x+a)*a*c^2+1/2*d^7*x^2/b^6-35/2/b
^4*d^3/(b*x+a)^2*c^4-6*d^7/b^7*a*x+7*d^6/b^6*x*c+35/b^8*d^7/(b*x+a)*a^3-35/
b^5*d^4/(b*x+a)*c^3+21/b^8*d^7*ln(b*x+a)*a^2+21/b^6*d^5*ln(b*x+a)*c^2-7/4/b
^8*d^7/(b*x+a)^4*a^6-7/4/b^2*d/(b*x+a)^4*c^6+7/b^8*d^7/(b*x+a)^3*a^5-7/b^3*
d^2/(b*x+a)^3*c^5+1/5/b^8/(b*x+a)^5*a^7*d^7-35/2/b^8*d^7/(b*x+a)^2*a^4+7/b^
4/(b*x+a)^5*a^3*c^4*d^3-21/5/b^3/(b*x+a)^5*a^2*c^5*d^2-1/5/b/(b*x+a)^5*c^7-
```

```
35/b^7*d^6/(b*x+a)^3*a^4*c+70/b^6*d^5/(b*x+a)^3*a^3*c^2-70/b^5*d^4/(b*x+a)^
3*a^2*c^3+35/b^4*d^3/(b*x+a)^3*a*c^4-7/5/b^7/(b*x+a)^5*a^6*c*d^6+21/5/b^6/(
b*x+a)^5*a^5*c^2*d^5-7/b^5/(b*x+a)^5*a^4*c^3*d^4-105/b^6*d^5/(b*x+a)^2*a^2*
c^2+70/b^5*d^4/(b*x+a)^2*a*c^3-42/b^7*d^6*ln(b*x+a)*a*c+21/2/b^7*d^6/(b*x+a
)^4*a^5*c+7/5/b^2/(b*x+a)^5*a*c^6*d+70/b^7*d^6/(b*x+a)^2*a^3*c-105/4/b^6*d^
5/(b*x+a)^4*a^4*c^2+35/b^5*d^4/(b*x+a)^4*a^3*c^3-105/4/b^4*d^3/(b*x+a)^4*a^
2*c^4+21/2/b^3*d^2/(b*x+a)^4*a*c^5
```

maxima [B] time = 1.82, size = 504, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/20*(4*b^7*c^7 + 7*a*b^6*c^6*d + 14*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3
+ 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 1218*a^6*b*c*d^6 - 459*a^7*d^
7 + 700*(b^7*c^3*d^4 - 3*a*b^6*c^2*d^5 + 3*a^2*b^5*c*d^6 - a^3*b^4*d^7)*x^4
 + 350*(b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 - 18*a^2*b^5*c^2*d^5 + 20*a^3*b^4*c*d
^6 - 7*a^4*b^3*d^7)*x^3 + 70*(2*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 20*a^2*b^5*
c^3*d^4 - 110*a^3*b^4*c^2*d^5 + 130*a^4*b^3*c*d^6 - 47*a^5*b^2*d^7)*x^2 + 3
5*(b^7*c^6*d + 2*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 - 1
25*a^4*b^3*c^2*d^5 + 154*a^5*b^2*c*d^6 - 57*a^6*b*d^7)*x)/(b^13*x^5 + 5*a*b
^12*x^4 + 10*a^2*b^11*x^3 + 10*a^3*b^10*x^2 + 5*a^4*b^9*x + a^5*b^8) + 1/2*
(b*d^7*x^2 + 2*(7*b*c*d^6 - 6*a*d^7)*x)/b^7 + 21*(b^2*c^2*d^5 - 2*a*b*c*d^6
 + a^2*d^7)*log(b*x + a)/b^8
```

mupad [B] time = 0.34, size = 508, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^6,x)
```

```
[Out] (log(a + b*x)*(21*a^2*d^7 + 21*b^2*c^2*d^5 - 42*a*b*c*d^6))/b^8 - x*((6*a*d
^7)/b^7 - (7*c*d^6)/b^6) - ((4*b^7*c^7 - 459*a^7*d^7 + 14*a^2*b^5*c^5*d^2 +
 35*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 - 959*a^5*b^2*c^2*d^5 + 7*a*b^6*c
^6*d + 1218*a^6*b*c*d^6)/(20*b) + x*((7*b^6*c^6*d)/4 - (399*a^6*d^7)/4 + (7
*a*b^5*c^5*d^2)/2 + (35*a^2*b^4*c^4*d^3)/4 + 35*a^3*b^3*c^3*d^4 - (875*a^4*
b^2*c^2*d^5)/4 + (539*a^5*b*c*d^6)/2) + x^3*((35*b^6*c^4*d^3)/2 - (245*a^4*
b^2*d^7)/2 + 70*a*b^5*c^3*d^4 + 350*a^3*b^3*c*d^6 - 315*a^2*b^4*c^2*d^5) +
x^2*(7*b^6*c^5*d^2 - (329*a^5*b*d^7)/2 + (35*a*b^5*c^4*d^3)/2 + 455*a^4*b^2
*c*d^6 + 70*a^2*b^4*c^3*d^4 - 385*a^3*b^3*c^2*d^5) - x^4*(35*a^3*b^3*d^7 -
35*b^6*c^3*d^4 + 105*a*b^5*c^2*d^5 - 105*a^2*b^4*c*d^6))/(a^5*b^7 + b^12*x^
5 + 5*a^4*b^8*x + 5*a*b^11*x^4 + 10*a^3*b^9*x^2 + 10*a^2*b^10*x^3) + (d^7*x
^2)/(2*b^6)
```

sympy [B] time = 97.19, size = 524, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**6,x)
```

```
[Out] x*(-6*a*d**7/b**7 + 7*c*d**6/b**6) + (459*a**7*d**7 - 1218*a**6*b*c*d**6 +
959*a**5*b**2*c**2*d**5 - 140*a**4*b**3*c**3*d**4 - 35*a**3*b**4*c**4*d**3
- 14*a**2*b**5*c**5*d**2 - 7*a*b**6*c**6*d - 4*b**7*c**7 + x**4*(700*a**3*b
**4*d**7 - 2100*a**2*b**5*c*d**6 + 2100*a*b**6*c**2*d**5 - 700*b**7*c**3*d*
*4) + x**3*(2450*a**4*b**3*d**7 - 7000*a**3*b**4*c*d**6 + 6300*a**2*b**5*c*
```

```
*2*d**5 - 1400*a*b**6*c**3*d**4 - 350*b**7*c**4*d**3) + x**2*(3290*a**5*b**
2*d**7 - 9100*a**4*b**3*c*d**6 + 7700*a**3*b**4*c**2*d**5 - 1400*a**2*b**5*
c**3*d**4 - 350*a*b**6*c**4*d**3 - 140*b**7*c**5*d**2) + x*(1995*a**6*b*d**
7 - 5390*a**5*b**2*c*d**6 + 4375*a**4*b**3*c**2*d**5 - 700*a**3*b**4*c**3*d
**4 - 175*a**2*b**5*c**4*d**3 - 70*a*b**6*c**5*d**2 - 35*b**7*c**6*d))/(20*
a**5*b**8 + 100*a**4*b**9*x + 200*a**3*b**10*x**2 + 200*a**2*b**11*x**3 + 1
00*a*b**12*x**4 + 20*b**13*x**5) + d**7*x**2/(2*b**6) + 21*d**5*(a*d - b*c)
**2*log(a + b*x)/b**8
```

3.1183 $\int \frac{(c+dx)^7}{(a+bx)^7}\,dx$

Optimal. Leaf size=186

$$\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc-}{6b^8(a}$$

Rubi [A] time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{21d^5(bc-ad)^2}{b^8(a+bx)}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}+\frac{7d^6(bc-ad)\log(a+bx)}{b^8}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}+\frac{d^7x}{b^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] (d^7*x)/b^7 - (b*c - a*d)^7/(6*b^8*(a + b*x)^6) - (7*d*(b*c - a*d)^6)/(5*b^
8*(a + b*x)^5) - (21*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^4) - (35*d^3*(b*c
- a*d)^4)/(3*b^8*(a + b*x)^3) - (35*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^2)
- (21*d^5*(b*c - a*d)^2)/(b^8*(a + b*x)) + (7*d^6*(b*c - a*d)*Log[a + b*x])
/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^7}\,dx &= \int \left(\frac{d^7}{b^7}+\frac{(bc-ad)^7}{b^7(a+bx)^7}+\frac{7d(bc-ad)^6}{b^7(a+bx)^6}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^5}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^4}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^3}+\frac{2}{}\right.\\ &= \frac{d^7x}{b^7}-\frac{(bc-ad)^7}{6b^8(a+bx)^6}-\frac{7d(bc-ad)^6}{5b^8(a+bx)^5}-\frac{21d^2(bc-ad)^5}{4b^8(a+bx)^4}-\frac{35d^3(bc-ad)^4}{3b^8(a+bx)^3}-\frac{35d^4(bc-ad)^3}{2b^8(a+bx)^2}-\frac{21d}{b}\end{aligned}$$

Mathematica [B] time = 0.20, size = 390, normalized size = 2.10

$$-\frac{669a^7d^7+3a^6bd^6(1198dx-343c)+3a^5b^2d^5\left(70c^2-1918cdx+2575d^2x^2\right)+5a^4b^3d^4\left(14c^3+252c^2dx-2625cd^2x^2+1640d^3x^3\right)+5a^3b^4d^3\left(7c^4+84c^3dx+630c^2d^2x^2-3080cd^3x^3+810d^4x^4\right)+3a^2b^5d^2\left(7c^5+70c^4dx+350c^3d^2x^2+1400c^2d^3x^3-3150cd^4x^4+120d^5x^5\right)+ab^6d\left(14c^6+126c^5dx+525c^4d^2x^2+1400c^3d^3x^3+3150c^2d^4x^4-2520cd^5x^5-360d^6x^6\right)+420d^6(a+bx)^6(ad-bc)\log(a+bx)+b^7\left(10c^7+84c^6dx+315c^5d^2x^2+700c^4d^3x^3+1050c^3d^4x^4+1260c^2d^5x^5-60d^7x^7\right)}{60b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] -1/60*(669*a^7*d^7 + 3*a^6*b*d^6*(-343*c + 1198*d*x) + 3*a^5*b^2*d^5*(70*c^
2 - 1918*c*d*x + 2575*d^2*x^2) + 5*a^4*b^3*d^4*(14*c^3 + 252*c^2*d*x - 2625
*c*d^2*x^2 + 1640*d^3*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 84*c^3*d*x + 630*c^2*d^
2*x^2 - 3080*c*d^3*x^3 + 810*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 70*c^4*d*x +
 350*c^3*d^2*x^2 + 1400*c^2*d^3*x^3 - 3150*c*d^4*x^4 + 120*d^5*x^5) + a*b^6
*d*(14*c^6 + 126*c^5*d*x + 525*c^4*d^2*x^2 + 1400*c^3*d^3*x^3 + 3150*c^2*d^
4*x^4 - 2520*c*d^5*x^5 - 360*d^6*x^6) + b^7*(10*c^7 + 84*c^6*d*x + 315*c^5*
d^2*x^2 + 700*c^4*d^3*x^3 + 1050*c^3*d^4*x^4 + 1260*c^2*d^5*x^5 - 60*d^7*x^
7) + 420*d^6*(-(b*c) + a*d)*(a + b*x)^6*Log[a + b*x])/(b^8*(a + b*x)^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^7, x]
```

fricas [B] time = 1.51, size = 692, normalized size = 3.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*b^7*d^7*x^7 + 360*a*b^6*d^7*x^6 - 10*b^7*c^7 - 14*a*b^6*c^6*d - 21
*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 - 70*a^4*b^3*c^3*d^4 - 210*a^5*b^2*c^
2*d^5 + 1029*a^6*b*c*d^6 - 669*a^7*d^7 - 180*(7*b^7*c^2*d^5 - 14*a*b^6*c*d^
6 + 2*a^2*b^5*d^7)*x^5 - 150*(7*b^7*c^3*d^4 + 21*a*b^6*c^2*d^5 - 63*a^2*b^5
*c*d^6 + 27*a^3*b^4*d^7)*x^4 - 100*(7*b^7*c^4*d^3 + 14*a*b^6*c^3*d^4 + 42*a
^2*b^5*c^2*d^5 - 154*a^3*b^4*c*d^6 + 82*a^4*b^3*d^7)*x^3 - 15*(21*b^7*c^5*d
^2 + 35*a*b^6*c^4*d^3 + 70*a^2*b^5*c^3*d^4 + 210*a^3*b^4*c^2*d^5 - 875*a^4*
b^3*c*d^6 + 515*a^5*b^2*d^7)*x^2 - 6*(14*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 35*
a^2*b^5*c^4*d^3 + 70*a^3*b^4*c^3*d^4 + 210*a^4*b^3*c^2*d^5 - 959*a^5*b^2*c*
d^6 + 599*a^6*b*d^7)*x + 420*(a^6*b*c*d^6 - a^7*d^7 + (b^7*c*d^6 - a*b^6*d^
7)*x^6 + 6*(a*b^6*c*d^6 - a^2*b^5*d^7)*x^5 + 15*(a^2*b^5*c*d^6 - a^3*b^4*d^
7)*x^4 + 20*(a^3*b^4*c*d^6 - a^4*b^3*d^7)*x^3 + 15*(a^4*b^3*c*d^6 - a^5*b^2
*d^7)*x^2 + 6*(a^5*b^2*c*d^6 - a^6*b*d^7)*x)*log(b*x + a))/(b^14*x^6 + 6*a*
b^13*x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*
x + a^6*b^8)
```

giac [B] time = 1.30, size = 459, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] d^7*x/b^7 + 7*(b*c*d^6 - a*d^7)*log(abs(b*x + a))/b^8 - 1/60*(10*b^7*c^7 +
14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d
^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6 + 669*a^7*d^7 + 1260*(b^7*c^2*d
^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5
 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 700*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^
4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 13*a^4*b^3*d^7)*x^3 + 105*(3*b^7
*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 - 125*
a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5
*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 - 137*a^5*b^2*c*
d^6 + 87*a^6*b*d^7)*x)/((b*x + a)^6*b^8)
```

maple [B] time = 0.01, size = 666, normalized size = 3.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^7,x)
```

```
[Out] 42/5/b^3*d^2/(b*x+a)^5*a*c^5-105/2/b^7*d^6/(b*x+a)^2*a^2*c+7/b^7*d^6*ln(b*x
+a)*c-7/5/b^2*d/(b*x+a)^5*c^6+35/2/b^8*d^7/(b*x+a)^2*a^3-35/2/b^5*d^4/(b*x+
a)^2*c^3-7/b^8*d^7*ln(b*x+a)*a+1/6/b^8/(b*x+a)^6*a^7*d^7+21/4/b^8*d^7/(b*x+
a)^4*a^5-21/4/b^3*d^2/(b*x+a)^4*c^5-21/b^8*d^7/(b*x+a)*a^2-21/b^6*d^5/(b*x+
a)*c^2-35/3/b^8*d^7/(b*x+a)^3*a^4+d^7*x/b^7-35/3/b^4*d^3/(b*x+a)^3*c^4-7/5/
b^8*d^7/(b*x+a)^5*a^6+42/b^7*d^6/(b*x+a)*a*c-7/6/b^7/(b*x+a)^6*a^6*c*d^6-1/
6/b/(b*x+a)^6*c^7+105/2/b^6*d^5/(b*x+a)^2*a*c^2+140/3/b^7*d^6/(b*x+a)^3*a^3
*c-70/b^6*d^5/(b*x+a)^3*a^2*c^2+140/3/b^5*d^4/(b*x+a)^3*a*c^3+35/6/b^4/(b*x
+a)^6*a^3*c^4*d^3-7/2/b^3/(b*x+a)^6*a^2*c^5*d^2+7/6/b^2/(b*x+a)^6*a*c^6*d+7
/2/b^6/(b*x+a)^6*a^5*c^2*d^5-35/6/b^5/(b*x+a)^6*a^4*c^3*d^4-105/4/b^7*d^6/(
b*x+a)^4*a^4*c+105/2/b^6*d^5/(b*x+a)^4*a^3*c^2-105/2/b^5*d^4/(b*x+a)^4*a^2*
c^3+105/4/b^4*d^3/(b*x+a)^4*a*c^4+42/5/b^7*d^6/(b*x+a)^5*a^5*c-21/b^6*d^5/(
b*x+a)^5*a^4*c^2+28/b^5*d^4/(b*x+a)^5*a^3*c^3-21/b^4*d^3/(b*x+a)^5*a^2*c^4
```

maxima [B]   time = 1.80, size = 516, normalized size = 2.77

$$\frac{d^7x}{b^7} - \frac{10\,b^7c^7 + 14\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 35\,a^3b^4c^4d^3 + 70\,a^4b^3c^3d^4 + 210\,a^5b^2c^2d^5 - 1029\,a^6bcd^6 + 669\,a^7d^7 + 1260\left(b^7c^2d^5 - 2\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 1050\left(b^7c^3d^4 + 3\,ab^6c^2d^5 - 9\,a^2b^5cd^6 + 5\,a^3b^4d^7\right)x^4 + 700\left(b^7c^4d^3 + 2\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 - 22\,a^3b^4cd^6 + 13\,a^4b^3d^7\right)x^3 + 105\left(3\,b^7c^5d^2 + 5\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 + 30\,a^3b^4c^2d^5 - 125\,a^4b^3cd^6 + 77\,a^5b^2d^7\right)x^2 + 42\left(2\,b^7c^6d + 3\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 + 10\,a^3b^4c^3d^4 + 30\,a^4b^3c^2d^5 - 137\,a^5b^2cd^6 + 87\,a^6bd^7\right)x}{60\left(b^{14}x^6 + 6\,ab^{13}x^5 + 15\,a^2b^{12}x^4 + 20\,a^3b^{11}x^3 + 15\,a^4b^{10}x^2 + 6\,a^5b^9x + a^6b^8\right)} + \frac{7\left(bcd^6 - ad^7\right)\log\left(bx + a\right)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] d^7*x/b^7 - 1/60*(10*b^7*c^7 + 14*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 + 35*a^3
*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 - 1029*a^6*b*c*d^6
+ 669*a^7*d^7 + 1260*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1050
*(b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 - 9*a^2*b^5*c*d^6 + 5*a^3*b^4*d^7)*x^4 + 70
0*(b^7*c^4*d^3 + 2*a*b^6*c^3*d^4 + 6*a^2*b^5*c^2*d^5 - 22*a^3*b^4*c*d^6 + 1
3*a^4*b^3*d^7)*x^3 + 105*(3*b^7*c^5*d^2 + 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*
d^4 + 30*a^3*b^4*c^2*d^5 - 125*a^4*b^3*c*d^6 + 77*a^5*b^2*d^7)*x^2 + 42*(2*
b^7*c^6*d + 3*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 10*a^3*b^4*c^3*d^4 + 30*a
^4*b^3*c^2*d^5 - 137*a^5*b^2*c*d^6 + 87*a^6*b*d^7)*x)/(b^14*x^6 + 6*a*b^13*
x^5 + 15*a^2*b^12*x^4 + 20*a^3*b^11*x^3 + 15*a^4*b^10*x^2 + 6*a^5*b^9*x + a
^6*b^8) + 7*(b*c*d^6 - a*d^7)*log(b*x + a)/b^8
```

mupad [B]   time = 0.37, size = 517, normalized size = 2.78

$$\frac{d^7\,x}{b^7}-\frac{\ln\left(a+b\,x\right)\,\left(7\,a\,d^7-7\,b\,c\,d^6\right)}{b^8}-\frac{\frac{669\,a^7\,d^7-1029\,a^6\,b\,c\,d^6+210\,a^5\,b^2\,c^2\,d^5+70\,a^4\,b^3\,c^3\,d^4+35\,a^3\,b^4\,c^4\,d^3+21\,a^2\,b^5\,c^5\,d^2+14\,a\,b^6\,c^6\,d+10\,b^7\,c^7}{60\,b}+x\,\left(\frac{609\,a^6\,d^7}{10}-\frac{959\,a^5\,b\,c\,d^6}{10}+21\,a^4\,b^2\,c^2\,d^5+7\,a^3\,b^3\,c^3\,d^4+\frac{7\,a^2\,b^4\,c^4\,d^3}{2}+\frac{21\,a\,b^5\,c^5\,d^2}{10}+\frac{7\,b^6\,c^6\,d}{5}\right)+x^3\,\left(\frac{455\,a^4\,b^2\,d^7}{3}-\frac{770\,a^3\,b^3\,c\,d^6}{3}+70\,a^2\,b^4\,c^2\,d^5+\frac{70\,a\,b^5\,c^3\,d^4}{3}+\frac{35\,b^6\,c^4\,d^3}{3}\right)+x^2\,\left(\frac{539\,a^5\,b\,d^7}{4}-\frac{875\,a^4\,b^2\,c\,d^6}{4}+\frac{105\,a^3\,b^3\,c^2\,d^5}{2}+\frac{35\,a^2\,b^4\,c^3\,d^4}{2}+\frac{35\,a\,b^5\,c^4\,d^3}{4}+\frac{21\,b^6\,c^5\,d^2}{4}\right)+x^5\,\left(21\,a^2\,b^4\,d^7-42\,a\,b^5\,c\,d^6+21\,b^6\,c^2\,d^5\right)+x^4\,\left(\frac{175\,a^3\,b^3\,d^7}{2}-\frac{315\,a^2\,b^4\,c\,d^6}{2}+\frac{105\,a\,b^5\,c^2\,d^5}{2}+\frac{35\,b^6\,c^3\,d^4}{2}\right)}{a^6\,b^7+6\,a^5\,b^8\,x+15\,a^4\,b^9\,x^2+20\,a^3\,b^{10}\,x^3+15\,a^2\,b^{11}\,x^4+6\,a\,b^{12}\,x^5+b^{13}\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^7,x)
```

```
[Out] (d^7*x)/b^7 - (log(a + b*x)*(7*a*d^7 - 7*b*c*d^6))/b^8 - ((669*a^7*d^7 + 10
*b^7*c^7 + 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 70*a^4*b^3*c^3*d^4 + 2
10*a^5*b^2*c^2*d^5 + 14*a*b^6*c^6*d - 1029*a^6*b*c*d^6)/(60*b) + x*((609*a^
6*d^7)/10 + (7*b^6*c^6*d)/5 + (21*a*b^5*c^5*d^2)/10 + (7*a^2*b^4*c^4*d^3)/2
 + 7*a^3*b^3*c^3*d^4 + 21*a^4*b^2*c^2*d^5 - (959*a^5*b*c*d^6)/10) + x^3*((4
55*a^4*b^2*d^7)/3 + (35*b^6*c^4*d^3)/3 + (70*a*b^5*c^3*d^4)/3 - (770*a^3*b^
3*c*d^6)/3 + 70*a^2*b^4*c^2*d^5) + x^2*((539*a^5*b*d^7)/4 + (21*b^6*c^5*d^2
)/4 + (35*a*b^5*c^4*d^3)/4 - (875*a^4*b^2*c*d^6)/4 + (35*a^2*b^4*c^3*d^4)/2
 + (105*a^3*b^3*c^2*d^5)/2) + x^5*(21*a^2*b^4*d^7 + 21*b^6*c^2*d^5 - 42*a*b
^5*c*d^6) + x^4*((175*a^3*b^3*d^7)/2 + (35*b^6*c^3*d^4)/2 + (105*a*b^5*c^2*
d^5)/2 - (315*a^2*b^4*c*d^6)/2))/(a^6*b^7 + b^13*x^6 + 6*a^5*b^8*x + 6*a*b^
12*x^5 + 15*a^4*b^9*x^2 + 20*a^3*b^10*x^3 + 15*a^2*b^11*x^4)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**7,x)
```

```
[Out] Timed out
```

3.1184 $\int \frac{(c+dx)^7}{(a+bx)^8} \, dx$

Optimal. Leaf size=194

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}$$

Rubi [A] time = 0.16, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{b^8(a+bx)}-\frac{21d^5(bc-ad)^2}{2b^8(a+bx)^2}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{(bc-ad)^7}{7b^8(a+bx)^7}+\frac{d^7\log(a+bx)}{b^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -(b*c - a*d)^7/(7*b^8*(a + b*x)^7) - (7*d*(b*c - a*d)^6)/(6*b^8*(a + b*x)^6
) - (21*d^2*(b*c - a*d)^5)/(5*b^8*(a + b*x)^5) - (35*d^3*(b*c - a*d)^4)/(4*
b^8*(a + b*x)^4) - (35*d^4*(b*c - a*d)^3)/(3*b^8*(a + b*x)^3) - (21*d^5*(b*
c - a*d)^2)/(2*b^8*(a + b*x)^2) - (7*d^6*(b*c - a*d))/(b^8*(a + b*x)) + (d^
7*Log[a + b*x])/b^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^8}\,dx = \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^8}+\frac{7d(bc-ad)^6}{b^7(a+bx)^7}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^6}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^5}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^4}+\frac{21d\ldots}{b^7\ldots}\right.$$
$$= -\frac{(bc-ad)^7}{7b^8(a+bx)^7}-\frac{7d(bc-ad)^6}{6b^8(a+bx)^6}-\frac{21d^2(bc-ad)^5}{5b^8(a+bx)^5}-\frac{35d^3(bc-ad)^4}{4b^8(a+bx)^4}-\frac{35d^4(bc-ad)^3}{3b^8(a+bx)^3}-\frac{21d^5(\ldots}{2b^8(\ldots}$$

Mathematica [A] time = 0.16, size = 308, normalized size = 1.59

$$\frac{d^7\log(a+bx)}{b^8}-\frac{(bc-ad)\left(1089a^6d^6+3a^5bd^5(223c+2401dx)+3a^4b^2d^4(153c^2+1421cdx+6713d^2x^2)+a^3b^3d^3(319c^3+2793c^2dx+11319cd^2x^2+30625d^3x^3)+a^2b^4d^2(214c^4+1813c^3dx+6909c^2d^2x^2+15925cd^3x^3+26950d^4x^4)+ab^5d(130c^5+1078c^4dx+3969c^3d^2x^2+8575c^2d^3x^3+12250cd^4x^4+13230d^5x^5)+b^6(60c^6+490c^5dx+1764c^4d^2x^2+3675c^3d^3x^3+4900c^2d^4x^4+4410cd^5x^5+2940d^6x^6)\right)}{420b^8(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] -1/420*((b*c - a*d)*(1089*a^6*d^6 + 3*a^5*b*d^5*(223*c + 2401*d*x) + 3*a^4*
b^2*d^4*(153*c^2 + 1421*c*d*x + 6713*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 2793
*c^2*d*x + 11319*c*d^2*x^2 + 30625*d^3*x^3) + a^2*b^4*d^2*(214*c^4 + 1813*c
^3*d*x + 6909*c^2*d^2*x^2 + 15925*c*d^3*x^3 + 26950*d^4*x^4) + a*b^5*d*(130
*c^5 + 1078*c^4*d*x + 3969*c^3*d^2*x^2 + 8575*c^2*d^3*x^3 + 12250*c*d^4*x^4
 + 13230*d^5*x^5) + b^6*(60*c^6 + 490*c^5*d*x + 1764*c^4*d^2*x^2 + 3675*c^3
*d^3*x^3 + 4900*c^2*d^4*x^4 + 4410*c*d^5*x^5 + 2940*d^6*x^6)))/(b^8*(a + b*
x)^7) + (d^7*Log[a + b*x])/b^8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^8, x]
```

fricas [B] time = 1.42, size = 624, normalized size = 3.22

$$-\frac{60 b^7c^7 + 70 ab^6c^6d + 84 a^2b^5c^5d^2 + 105 a^3b^4c^4d^3 + 140 a^4b^3c^3d^4 + 210 a^5b^2c^2d^5 + 420 a^6bcd^6 - 1089 a^7d^7 + 2940 \left(b^7cd^6 - ab^6d^7\right)x^6 + 4410 \left(b^7c^2d^5 + 2 ab^6cd^6 - 3 a^2b^5d^7\right)x^5 + 2450 \left(2 b^7c^3d^4 + 3 ab^6c^2d^5 + 6 a^2b^5cd^6 - 11 a^3b^4d^7\right)x^4 + 1225 \left(3 b^7c^4d^3 + 4 ab^6c^3d^4 + 6 a^2b^5c^2d^5 + 12 a^3b^4cd^6 - 25 a^4b^3d^7\right)x^3 + 147 \left(12 b^7c^5d^2 + 15 ab^6c^4d^3 + 20 a^2b^5c^3d^4 + 30 a^3b^4c^2d^5 + 60 a^4b^3cd^6 - 137 a^5b^2d^7\right)x^2 + 49 \left(10 b^7c^6d + 12 ab^6c^5d^2 + 15 a^2b^5c^4d^3 + 20 a^3b^4c^3d^4 + 30 a^4b^3c^2d^5 + 60 a^5b^2cd^6 - 147 a^6bd^7\right)x - 420 \left(b^7d^7x^7 + 7 ab^6d^7x^6 + 21 a^2b^5d^7x^5 + 35 a^3b^4d^7x^4 + 35 a^4b^3d^7x^3 + 21 a^5b^2d^7x^2 + 7 a^6bd^7x + a^7d^7\right)\log\left(bx + a\right)}{420 \left(b^{15}x^7 + 7 ab^{14}x^6 + 21 a^2b^{13}x^5 + 35 a^3b^{12}x^4 + 35 a^4b^{11}x^3 + 21 a^5b^{10}x^2 + 7 a^6b^9x + a^7b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x - 420*(b^7*d^7*x^7 + 7*a*b^6*d^7*x^6 + 21*a^2*b^5*d^7*x^5
 + 35*a^3*b^4*d^7*x^4 + 35*a^4*b^3*d^7*x^3 + 21*a^5*b^2*d^7*x^2 + 7*a^6*b*d
^7*x + a^7*d^7)*log(b*x + a))/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 +
35*a^3*b^12*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8
)
```

giac [B] time = 1.28, size = 466, normalized size = 2.40

$$\frac{d^7\log\left(|bx + a|\right)}{b^8} - \frac{2940 \left(b^6cd^6 - ab^5d^7\right)x^6 + 4410 \left(b^6c^2d^5 + 2 ab^5cd^6 - 3 a^2b^4d^7\right)x^5 + 2450 \left(2 b^6c^3d^4 + 3 ab^5c^2d^5 + 6 a^2b^4cd^6 - 11 a^3b^3d^7\right)x^4 + 1225 \left(3 b^6c^4d^3 + 4 ab^5c^3d^4 + 6 a^2b^4c^2d^5 + 12 a^3b^3cd^6 - 25 a^4b^2d^7\right)x^3 + 147 \left(12 b^6c^5d^2 + 15 ab^5c^4d^3 + 20 a^2b^4c^3d^4 + 30 a^3b^3c^2d^5 + 60 a^4b^2cd^6 - 137 a^5bd^7\right)x^2 + 49 \left(10 b^6c^6d + 12 ab^5c^5d^2 + 15 a^2b^4c^4d^3 + 20 a^3b^3c^3d^4 + 30 a^4b^2c^2d^5 + 60 a^5bcd^6 - 147 a^6d^7\right)x + \frac{60 b^7c^7 + 70 ab^6c^6d + 84 a^2b^5c^5d^2 + 105 a^3b^4c^4d^3 + 140 a^4b^3c^3d^4 + 210 a^5b^2c^2d^5 + 420 a^6bcd^6 - 1089 a^7d^7}{b}}{420 \left(bx + a\right)^7b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] d^7*log(abs(b*x + a))/b^8 - 1/420*(2940*(b^6*c*d^6 - a*b^5*d^7)*x^6 + 4410*
(b^6*c^2*d^5 + 2*a*b^5*c*d^6 - 3*a^2*b^4*d^7)*x^5 + 2450*(2*b^6*c^3*d^4 + 3
*a*b^5*c^2*d^5 + 6*a^2*b^4*c*d^6 - 11*a^3*b^3*d^7)*x^4 + 1225*(3*b^6*c^4*d^
3 + 4*a*b^5*c^3*d^4 + 6*a^2*b^4*c^2*d^5 + 12*a^3*b^3*c*d^6 - 25*a^4*b^2*d^7
)*x^3 + 147*(12*b^6*c^5*d^2 + 15*a*b^5*c^4*d^3 + 20*a^2*b^4*c^3*d^4 + 30*a^
3*b^3*c^2*d^5 + 60*a^4*b^2*c*d^6 - 137*a^5*b*d^7)*x^2 + 49*(10*b^6*c^6*d +
12*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 + 20*a^3*b^3*c^3*d^4 + 30*a^4*b^2*c^2
*d^5 + 60*a^5*b*c*d^6 - 147*a^6*d^7)*x + (60*b^7*c^7 + 70*a*b^6*c^6*d + 84*
a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c
^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^7*d^7)/b)/((b*x + a)^7*b^7)
```

maple [B] time = 0.01, size = 672, normalized size = 3.46

$$\frac{a^7d^7}{7 (bx+a)^7 b^8} - \frac{a^6cd^6}{(bx+a)^7 b^7} + \frac{3a^5c^2d^5}{(bx+a)^7 b^6} - \frac{5a^4c^3d^4}{(bx+a)^7 b^5} + \frac{5a^3c^4d^3}{(bx+a)^7 b^4} - \frac{3a^2c^5d^2}{(bx+a)^7 b^3} + \frac{ac^6d}{(bx+a)^7 b^2} - \frac{c^7}{7 (bx+a)^7 b} - \frac{7a^6d^7}{6 (bx+a)^6 b^8} + \frac{7a^5cd^6}{(bx+a)^6 b^7} - \frac{35a^4c^2d^5}{2 (bx+a)^6 b^6} + \frac{70a^3c^3d^4}{3 (bx+a)^6 b^5} - \frac{35a^2c^4d^3}{2 (bx+a)^6 b^4} + \frac{7ac^5d^2}{(bx+a)^6 b^3} - \frac{7c^6d}{6 (bx+a)^6 b^2} + \frac{21a^5d^7}{5 (bx+a)^5 b^8} - \frac{21a^4cd^6}{(bx+a)^5 b^7} + \frac{42a^3c^2d^5}{(bx+a)^5 b^6} - \frac{42a^2c^3d^4}{(bx+a)^5 b^5} + \frac{21ac^4d^3}{(bx+a)^5 b^4} - \frac{21c^5d^2}{5 (bx+a)^5 b^3} - \frac{35a^4d^7}{4 (bx+a)^4 b^8} + \frac{35a^3cd^6}{(bx+a)^4 b^7} - \frac{105a^2c^2d^5}{2 (bx+a)^4 b^6} + \frac{35ac^3d^4}{(bx+a)^4 b^5} - \frac{35c^4d^3}{4 (bx+a)^4 b^4} + \frac{35a^3d^7}{3 (bx+a)^3 b^8} - \frac{35a^2cd^6}{(bx+a)^3 b^7} + \frac{35ac^2d^5}{(bx+a)^3 b^6} - \frac{35c^3d^4}{3 (bx+a)^3 b^5} - \frac{21a^2d^7}{2 (bx+a)^2 b^8} + \frac{21acd^6}{(bx+a)^2 b^7} - \frac{21c^2d^5}{2 (bx+a)^2 b^6} + \frac{7ad^7}{(bx+a) b^8} - \frac{7cd^6}{(bx+a) b^7} + \frac{d^7 \ln(bx+a)}{b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^8,x)
```

```
[Out] 7/b^8*d^7/(b*x+a)*a-7/b^7*d^6/(b*x+a)*c+35/3*d^7/b^8/(b*x+a)^3*a^3-35/3*d^4
/b^5/(b*x+a)^3*c^3+1/7/b^8/(b*x+a)^7*a^7*d^7+21/5*d^7/b^8/(b*x+a)^5*a^5-21/
5*d^2/b^3/(b*x+a)^5*c^5-7/6*d^7/b^8/(b*x+a)^6*a^6-7/6*d/b^2/(b*x+a)^6*c^6-2
1/2*d^7/b^8/(b*x+a)^2*a^2-21/2*d^5/b^6/(b*x+a)^2*c^2-35/4*d^7/b^8/(b*x+a)^4
*a^4-35/4*d^3/b^4/(b*x+a)^4*c^4+d^7*ln(b*x+a)/b^8+42*d^5/b^6/(b*x+a)^5*a^3*
c^2-21*d^6/b^7/(b*x+a)^5*a^4*c-1/7/b/(b*x+a)^7*c^7-42*d^4/b^5/(b*x+a)^5*a^2
*c^3+1/b^2/(b*x+a)^7*a*c^6*d-35*d^6/b^7/(b*x+a)^3*a^2*c+35*d^5/b^6/(b*x+a)^
```

```
3*a*c^2-1/b^7/(b*x+a)^7*a^6*c*d^6+3/b^6/(b*x+a)^7*a^5*c^2*d^5-5/b^5/(b*x+a)
^7*a^4*c^3*d^4+5/b^4/(b*x+a)^7*a^3*c^4*d^3-3/b^3/(b*x+a)^7*a^2*c^5*d^2+21*d
^3/b^4/(b*x+a)^5*a*c^4+21*d^6/b^7/(b*x+a)^2*a*c+35*d^6/b^7/(b*x+a)^4*a^3*c-
105/2*d^5/b^6/(b*x+a)^4*a^2*c^2+35*d^4/b^5/(b*x+a)^4*a*c^3+7*d^6/b^7/(b*x+a
)^6*a^5*c-35/2*d^5/b^6/(b*x+a)^6*a^4*c^2+70/3*d^4/b^5/(b*x+a)^6*a^3*c^3-35/
2*d^3/b^4/(b*x+a)^6*a^2*c^4+7*d^2/b^3/(b*x+a)^6*a*c^5
```

maxima [B] time = 1.65, size = 534, normalized size = 2.75

$$-\frac{60 \, b^{7} c^{7} + 70 \, a b^{6} c^{6} d + 84 \, a^{2} b^{5} c^{5} d^{2} + 105 \, a^{3} b^{4} c^{4} d^{3} + 140 \, a^{4} b^{3} c^{3} d^{4} + 210 \, a^{5} b^{2} c^{2} d^{5} + 420 \, a^{6} b c d^{6} - 1089 \, a^{7} d^{7} + 2940 \, {\left(b^{7} c d^{6} - a b^{6} d^{7}\right)} x^{6} + 4410 \, {\left(b^{7} c^{2} d^{5} + 2 \, a b^{6} c d^{6} - 3 \, a^{2} b^{5} d^{7}\right)} x^{5} + 2450 \, {\left(2 \, b^{7} c^{3} d^{4} + 3 \, a b^{6} c^{2} d^{5} + 6 \, a^{2} b^{5} c d^{6} - 11 \, a^{3} b^{4} d^{7}\right)} x^{4} + 1225 \, {\left(3 \, b^{7} c^{4} d^{3} + 4 \, a b^{6} c^{3} d^{4} + 6 \, a^{2} b^{5} c^{2} d^{5} + 12 \, a^{3} b^{4} c d^{6} - 25 \, a^{4} b^{3} d^{7}\right)} x^{3} + 147 \, {\left(12 \, b^{7} c^{5} d^{2} + 15 \, a b^{6} c^{4} d^{3} + 20 \, a^{2} b^{5} c^{3} d^{4} + 30 \, a^{3} b^{4} c^{2} d^{5} + 60 \, a^{4} b^{3} c d^{6} - 137 \, a^{5} b^{2} d^{7}\right)} x^{2} + 49 \, {\left(10 \, b^{7} c^{6} d + 12 \, a b^{6} c^{5} d^{2} + 15 \, a^{2} b^{5} c^{4} d^{3} + 20 \, a^{3} b^{4} c^{3} d^{4} + 30 \, a^{4} b^{3} c^{2} d^{5} + 60 \, a^{5} b^{2} c d^{6} - 147 \, a^{6} b d^{7}\right)} x}{420 \, {\left(b^{15} x^{7} + 7 \, a b^{14} x^{6} + 21 \, a^{2} b^{13} x^{5} + 35 \, a^{3} b^{12} x^{4} + 35 \, a^{4} b^{11} x^{3} + 21 \, a^{5} b^{10} x^{2} + 7 \, a^{6} b^{9} x + a^{7} b^{8}\right)}} + \frac{d^{7} \log\left(b x + a\right)}{b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/420*(60*b^7*c^7 + 70*a*b^6*c^6*d + 84*a^2*b^5*c^5*d^2 + 105*a^3*b^4*c^4*
d^3 + 140*a^4*b^3*c^3*d^4 + 210*a^5*b^2*c^2*d^5 + 420*a^6*b*c*d^6 - 1089*a^
7*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(b^7*c^2*d^5 + 2*a*b^6*c*d^
6 - 3*a^2*b^5*d^7)*x^5 + 2450*(2*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 6*a^2*b^5*
c*d^6 - 11*a^3*b^4*d^7)*x^4 + 1225*(3*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 6*a^2
*b^5*c^2*d^5 + 12*a^3*b^4*c*d^6 - 25*a^4*b^3*d^7)*x^3 + 147*(12*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 30*a^3*b^4*c^2*d^5 + 60*a^4*b^3*
c*d^6 - 137*a^5*b^2*d^7)*x^2 + 49*(10*b^7*c^6*d + 12*a*b^6*c^5*d^2 + 15*a^2
*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 30*a^4*b^3*c^2*d^5 + 60*a^5*b^2*c*d^6 -
 147*a^6*b*d^7)*x)/(b^15*x^7 + 7*a*b^14*x^6 + 21*a^2*b^13*x^5 + 35*a^3*b^12
*x^4 + 35*a^4*b^11*x^3 + 21*a^5*b^10*x^2 + 7*a^6*b^9*x + a^7*b^8) + d^7*log
(b*x + a)/b^8
```

mupad [B] time = 0.35, size = 461, normalized size = 2.38

$$\frac{d^7\,\ln\left(a+b\,x\right)}{b^8}-\frac{x\,\left(-\frac{343\,a^6\,b\,d^7}{20}+7\,a^5\,b^2\,c\,d^6+\frac{7\,a^4\,b^3\,c^2\,d^5}{2}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{4}+\frac{7\,a\,b^6\,c^5\,d^2}{5}+\frac{7\,b^7\,c^6\,d}{6}\right)-x^6\,\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\,\left(-\frac{875\,a^4\,b^3\,d^7}{12}+35\,a^3\,b^4\,c\,d^6+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+\frac{35\,a\,b^6\,c^3\,d^4}{3}+\frac{35\,b^7\,c^4\,d^3}{4}\right)+x^5\,\left(-\frac{63\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6+\frac{21\,b^7\,c^2\,d^5}{2}\right)+x^2\,\left(-\frac{959\,a^5\,b^2\,d^7}{20}+21\,a^4\,b^3\,c\,d^6+\frac{21\,a^3\,b^4\,c^2\,d^5}{2}+7\,a^2\,b^5\,c^3\,d^4+\frac{21\,a\,b^6\,c^4\,d^3}{4}+\frac{21\,b^7\,c^5\,d^2}{5}\right)-\frac{363\,a^7\,d^7}{140}+\frac{b^7\,c^7}{7}+x^4\,\left(-\frac{385\,a^3\,b^4\,d^7}{6}+35\,a^2\,b^5\,c\,d^6+\frac{35\,a\,b^6\,c^2\,d^5}{2}+\frac{35\,b^7\,c^3\,d^4}{3}\right)+\frac{a^2\,b^5\,c^5\,d^2}{5}+\frac{a^3\,b^4\,c^4\,d^3}{4}+\frac{a^4\,b^3\,c^3\,d^4}{3}+\frac{a^5\,b^2\,c^2\,d^5}{2}+\frac{a\,b^6\,c^6\,d}{6}+a^6\,b\,c\,d^6}{b^8\,{\left(a+b\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^8,x)
```

```
[Out] (d^7*log(a + b*x))/b^8 - (x*((7*b^7*c^6*d)/6 - (343*a^6*b*d^7)/20 + (7*a*b^
6*c^5*d^2)/5 + 7*a^5*b^2*c*d^6 + (7*a^2*b^5*c^4*d^3)/4 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/2) - x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*b
^7*c^4*d^3)/4 - (875*a^4*b^3*d^7)/12 + (35*a*b^6*c^3*d^4)/3 + 35*a^3*b^4*c*
d^6 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*b^7*c^2*d^5)/2 - (63*a^2*b^5*d^7)/
2 + 21*a*b^6*c*d^6) + x^2*((21*b^7*c^5*d^2)/5 - (959*a^5*b^2*d^7)/20 + (21*
a*b^6*c^4*d^3)/4 + 21*a^4*b^3*c*d^6 + 7*a^2*b^5*c^3*d^4 + (21*a^3*b^4*c^2*d
^5)/2) - (363*a^7*d^7)/140 + (b^7*c^7)/7 + x^4*((35*b^7*c^3*d^4)/3 - (385*a
^3*b^4*d^7)/6 + (35*a*b^6*c^2*d^5)/2 + 35*a^2*b^5*c*d^6) + (a^2*b^5*c^5*d^2
)/5 + (a^3*b^4*c^4*d^3)/4 + (a^4*b^3*c^3*d^4)/3 + (a^5*b^2*c^2*d^5)/2 + (a*
b^6*c^6*d)/6 + a^6*b*c*d^6)/(b^8*(a + b*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**8,x)
```

```
[Out] Timed out
```

3.1185 $\int \frac{(c+dx)^7}{(a+bx)^9} dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^8}{8(a+bx)^8(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -(c + d*x)^8/(8*(b*c - a*d)*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^9} dx = -\frac{(c+dx)^8}{8(bc-ad)(a+bx)^8}$$

Mathematica [B] time = 0.13, size = 353, normalized size = 12.61

$$-\frac{a^7d^7+a^6bd^6(c+8dx)+a^5b^2d^5(c^2+8cdx+28d^2x^2)+a^4b^3d^4(c^3+8c^2dx+28cd^2x^2+56d^3x^3)+a^3b^4d^3(c^4+8c^3dx+28c^2d^2x^2+56cd^3x^3+70d^4x^4)+a^2b^5d^2(c^5+8c^4dx+28c^3d^2x^2+56c^2d^3x^3+70cd^4x^4+56d^5x^5)+ab^6d(c^6+8c^5dx+28c^4d^2x^2+56c^3d^3x^3+70c^2d^4x^4+56cd^5x^5+28d^6x^6)+b^7(c^7+8c^6dx+28c^5d^2x^2+56c^4d^3x^3+70c^3d^4x^4+56c^2d^5x^5+28cd^6x^6+8d^7x^7)}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] -1/8*(a^7*d^7 + a^6*b*d^6*(c + 8*d*x) + a^5*b^2*d^5*(c^2 + 8*c*d*x + 28*d^2
*x^2) + a^4*b^3*d^4*(c^3 + 8*c^2*d*x + 28*c*d^2*x^2 + 56*d^3*x^3) + a^3*b^4
*d^3*(c^4 + 8*c^3*d*x + 28*c^2*d^2*x^2 + 56*c*d^3*x^3 + 70*d^4*x^4) + a^2*b
^5*d^2*(c^5 + 8*c^4*d*x + 28*c^3*d^2*x^2 + 56*c^2*d^3*x^3 + 70*c*d^4*x^4 +
56*d^5*x^5) + a*b^6*d*(c^6 + 8*c^5*d*x + 28*c^4*d^2*x^2 + 56*c^3*d^3*x^3 +
70*c^2*d^4*x^4 + 56*c*d^5*x^5 + 28*d^6*x^6) + b^7*(c^7 + 8*c^6*d*x + 28*c^5
*d^2*x^2 + 56*c^4*d^3*x^3 + 70*c^3*d^4*x^4 + 56*c^2*d^5*x^5 + 28*c*d^6*x^6
+ 8*d^7*x^7))/(b^8*(a + b*x)^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^9} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^9, x]
```

fricas [B] time = 1.39, size = 509, normalized size = 18.18

$$-\frac{8 b^7 d^7 x^7 + b^7 c^7 + a b^6 c^6 d + a^2 b^5 c^5 d^2 + a^3 b^4 c^4 d^3 + a^4 b^3 c^3 d^4 + a^5 b^2 c^2 d^5 + a^6 b c d^6 + a^7 d^7 + 28 \left(b^7 c d^6 + a b^6 d^7\right) x^6 + 56 \left(b^7 c^2 d^5 + a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 70 \left(b^7 c^3 d^4 + a b^6 c^2 d^5 + a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 56 \left(b^7 c^4 d^3 + a b^6 c^3 d^4 + a^2 b^5 c^2 d^5 + a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 28 \left(b^7 c^5 d^2 + a b^6 c^4 d^3 + a^2 b^5 c^3 d^4 + a^3 b^4 c^2 d^5 + a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 8 \left(b^7 c^6 d + a b^6 c^5 d^2 + a^2 b^5 c^4 d^3 + a^3 b^4 c^3 d^4 + a^4 b^3 c^2 d^5 + a^5 b^2 c d^6 + a^6 b d^7\right) x}{8 \left(b^{16} x^8 + 8 a b^{15} x^7 + 28 a^2 b^{14} x^6 + 56 a^3 b^{13} x^5 + 70 a^4 b^{12} x^4 + 56 a^5 b^{11} x^3 + 28 a^6 b^{10} x^2 + 8 a^7 b^9 x + a^8 b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

giac [B] time = 1.29, size = 489, normalized size = 17.46

$$-\frac{8 b^7 d^7 x^7 + 28 b^7 c d^6 x^6 + 28 a b^6 d^7 x^6 + 56 b^7 c^2 d^5 x^5 + 56 a b^6 c d^6 x^5 + 56 a^2 b^5 d^7 x^5 + 70 b^7 c^3 d^4 x^4 + 70 a b^6 c^2 d^5 x^4 + 70 a^2 b^5 c d^6 x^4 + 70 a^3 b^4 d^7 x^4 + 56 b^7 c^4 d^3 x^3 + 56 a b^6 c^3 d^4 x^3 + 56 a^2 b^5 c^2 d^5 x^3 + 56 a^3 b^4 c d^6 x^3 + 56 a^4 b^3 d^7 x^3 + 28 b^7 c^5 d^2 x^2 + 28 a b^6 c^4 d^3 x^2 + 28 a^2 b^5 c^3 d^4 x^2 + 28 a^3 b^4 c^2 d^5 x^2 + 28 a^4 b^3 c d^6 x^2 + 28 a^5 b^2 d^7 x^2 + 8 b^7 c^6 d x + 8 a b^6 c^5 d^2 x + 8 a^2 b^5 c^4 d^3 x + 8 a^3 b^4 c^3 d^4 x + 8 a^4 b^3 c^2 d^5 x + 8 a^5 b^2 c d^6 x + 8 a^6 b d^7 x + b^7 c^7 + a b^6 c^6 d + a^2 b^5 c^5 d^2 + a^3 b^4 c^4 d^3 + a^4 b^3 c^3 d^4 + a^5 b^2 c^2 d^5 + a^6 b c d^6 + a^7 d^7}{8 (b x + a)^8 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + 28*b^7*c*d^6*x^6 + 28*a*b^6*d^7*x^6 + 56*b^7*c^2*d^5*
x^5 + 56*a*b^6*c*d^6*x^5 + 56*a^2*b^5*d^7*x^5 + 70*b^7*c^3*d^4*x^4 + 70*a*b
^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4 + 70*a^3*b^4*d^7*x^4 + 56*b^7*c^4*d^3
*x^3 + 56*a*b^6*c^3*d^4*x^3 + 56*a^2*b^5*c^2*d^5*x^3 + 56*a^3*b^4*c*d^6*x^3
 + 56*a^4*b^3*d^7*x^3 + 28*b^7*c^5*d^2*x^2 + 28*a*b^6*c^4*d^3*x^2 + 28*a^2*
b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 28*a^4*b^3*c*d^6*x^2 + 28*a^5*b^
2*d^7*x^2 + 8*b^7*c^6*d*x + 8*a*b^6*c^5*d^2*x + 8*a^2*b^5*c^4*d^3*x + 8*a^3
*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 8*a^5*b^2*c*d^6*x + 8*a^6*b*d^7*x +
b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4
 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^8*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 16.57

$$-\frac{d^7}{(bx+a)\, b^8}+\frac{7 (a d-b c) d^6}{2 (bx+a)^2 b^8}-\frac{7 \left(a^2 d^2-2 a b c d+b^2 c^2\right) d^5}{(bx+a)^3 b^8}+\frac{35 \left(a^3 d^3-3 a^2 b c d^2+3 a b^2 c^2 d-b^3 c^3\right) d^4}{4 (bx+a)^4 b^8}-\frac{7 \left(a^4 d^4-4 a^3 b c d^3+6 a^2 b^2 c^2 d^2-4 a b^3 c^3 d+b^4 c^4\right) d^3}{(bx+a)^5 b^8}+\frac{7 \left(a^5 d^5-5 a^4 b c d^4+10 a^3 b^2 c^2 d^3-10 a^2 b^3 c^3 d^2+5 a b^4 c^4 d-b^5 c^5\right) d^2}{2 (bx+a)^6 b^8}-\frac{\left(a^6 d^6-6 a^5 b c d^5+15 a^4 b^2 c^2 d^4-20 a^3 b^3 c^3 d^3+15 a^2 b^4 c^4 d^2-6 a b^5 c^5 d+b^6 c^6\right) d}{(bx+a)^7 b^8}-\frac{-a^7 d^7+7 a^6 b c d^6-21 a^5 b^2 c^2 d^5+35 a^4 c^3 d^4 b^3-35 a^3 b^4 c^4 d^3+21 a^2 c^5 d^2 b^5-7 a b^6 c^6 d+b^7 c^7}{8 (bx+a)^8 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^9,x)
```

```
[Out] -1/8*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b
^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^8-7*d^5*(a
^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^3-d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2
*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(
b*x+a)^7-7*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c
^4)/b^8/(b*x+a)^5+7/2*d^6*(a*d-b*c)/b^8/(b*x+a)^2+35/4*d^4*(a^3*d^3-3*a^2*b
*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^4-d^7/b^8/(b*x+a)+7/2*d^2*(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)/b^8/(b*x+a)^6
```

maxima [B] time = 1.65, size = 509, normalized size = 18.18

$$-\frac{8 b^7 d^7 x^7 + b^7 c^7 + a b^6 c^6 d + a^2 b^5 c^5 d^2 + a^3 b^4 c^4 d^3 + a^4 b^3 c^3 d^4 + a^5 b^2 c^2 d^5 + a^6 b c d^6 + a^7 d^7 + 28 \left(b^7 c d^6 + a b^6 d^7\right) x^6 + 56 \left(b^7 c^2 d^5 + a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 70 \left(b^7 c^3 d^4 + a b^6 c^2 d^5 + a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 56 \left(b^7 c^4 d^3 + a b^6 c^3 d^4 + a^2 b^5 c^2 d^5 + a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 28 \left(b^7 c^5 d^2 + a b^6 c^4 d^3 + a^2 b^5 c^3 d^4 + a^3 b^4 c^2 d^5 + a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 8 \left(b^7 c^6 d + a b^6 c^5 d^2 + a^2 b^5 c^4 d^3 + a^3 b^4 c^3 d^4 + a^4 b^3 c^2 d^5 + a^5 b^2 c d^6 + a^6 b d^7\right) x}{8 \left(b^{16} x^8 + 8 a b^{15} x^7 + 28 a^2 b^{14} x^6 + 56 a^3 b^{13} x^5 + 70 a^4 b^{12} x^4 + 56 a^5 b^{11} x^3 + 28 a^6 b^{10} x^2 + 8 a^7 b^9 x + a^8 b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/8*(8*b^7*d^7*x^7 + b^7*c^7 + a*b^6*c^6*d + a^2*b^5*c^5*d^2 + a^3*b^4*c^4
*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + a^7*d^7 + 28*(b^7*
c*d^6 + a*b^6*d^7)*x^6 + 56*(b^7*c^2*d^5 + a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 +
 70*(b^7*c^3*d^4 + a*b^6*c^2*d^5 + a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 56*(b
^7*c^4*d^3 + a*b^6*c^3*d^4 + a^2*b^5*c^2*d^5 + a^3*b^4*c*d^6 + a^4*b^3*d^7)
*x^3 + 28*(b^7*c^5*d^2 + a*b^6*c^4*d^3 + a^2*b^5*c^3*d^4 + a^3*b^4*c^2*d^5
+ a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 8*(b^7*c^6*d + a*b^6*c^5*d^2 + a^2*b^5
*c^4*d^3 + a^3*b^4*c^3*d^4 + a^4*b^3*c^2*d^5 + a^5*b^2*c*d^6 + a^6*b*d^7)*x
)/(b^16*x^8 + 8*a*b^15*x^7 + 28*a^2*b^14*x^6 + 56*a^3*b^13*x^5 + 70*a^4*b^1
2*x^4 + 56*a^5*b^11*x^3 + 28*a^6*b^10*x^2 + 8*a^7*b^9*x + a^8*b^8)
```

mupad [**B**] time = 0.17, size = 571, normalized size = 20.39

$$-\frac{a^7 d^7 + a^6 b c d^6 + 8 a^6 b d^7 x + a^5 b^2 c^2 d^5 + 8 a^5 b^2 c d^6 x + 28 a^5 b^2 d^7 x^2 + a^4 b^3 c^3 d^4 + 8 a^4 b^3 c^2 d^5 x + 28 a^4 b^3 c d^6 x^2 + 56 a^4 b^3 d^7 x^3 + a^3 b^4 c^4 d^3 + 8 a^3 b^4 c^3 d^4 x + 28 a^3 b^4 c^2 d^5 x^2 + 56 a^3 b^4 c d^6 x^3 + 70 a^3 b^4 d^7 x^4 + a^2 b^5 c^5 d^2 + 8 a^2 b^5 c^4 d^3 x + 28 a^2 b^5 c^3 d^4 x^2 + 56 a^2 b^5 c^2 d^5 x^3 + 70 a^2 b^5 c d^6 x^4 + 56 a^2 b^5 d^7 x^5 + a b^6 c^6 d + 8 a b^6 c^5 d^2 x + 28 a b^6 c^4 d^3 x^2 + 56 a b^6 c^3 d^4 x^3 + 70 a b^6 c^2 d^5 x^4 + 56 a b^6 c d^6 x^5 + 28 a b^6 d^7 x^6 + b^7 c^7 + 8 b^7 c^6 d x + 28 b^7 c^5 d^2 x^2 + 56 b^7 c^4 d^3 x^3 + 70 b^7 c^3 d^4 x^4 + 56 b^7 c^2 d^5 x^5 + 28 b^7 c d^6 x^6 + 8 b^7 d^7 x^7}{8 a^8 b^8 + 64 a^7 b^9 x + 224 a^6 b^{10} x^2 + 448 a^5 b^{11} x^3 + 560 a^4 b^{12} x^4 + 448 a^3 b^{13} x^5 + 224 a^2 b^{14} x^6 + 64 a b^{15} x^7 + 8 b^{16} x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^9,x)
```

```
[Out] -(a^7*d^7 + b^7*c^7 + 8*b^7*d^7*x^7 + 28*a*b^6*d^7*x^6 + 28*b^7*c*d^6*x^6 +
 a^2*b^5*c^5*d^2 + a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4 + a^5*b^2*c^2*d^5 + 28
*a^5*b^2*d^7*x^2 + 56*a^4*b^3*d^7*x^3 + 70*a^3*b^4*d^7*x^4 + 56*a^2*b^5*d^7
*x^5 + 28*b^7*c^5*d^2*x^2 + 56*b^7*c^4*d^3*x^3 + 70*b^7*c^3*d^4*x^4 + 56*b^
7*c^2*d^5*x^5 + a*b^6*c^6*d + a^6*b*c*d^6 + 8*a^6*b*d^7*x + 8*b^7*c^6*d*x +
 28*a^2*b^5*c^3*d^4*x^2 + 28*a^3*b^4*c^2*d^5*x^2 + 56*a^2*b^5*c^2*d^5*x^3 +
 8*a*b^6*c^5*d^2*x + 8*a^5*b^2*c*d^6*x + 56*a*b^6*c*d^6*x^5 + 8*a^2*b^5*c^4
*d^3*x + 8*a^3*b^4*c^3*d^4*x + 8*a^4*b^3*c^2*d^5*x + 28*a*b^6*c^4*d^3*x^2 +
 28*a^4*b^3*c*d^6*x^2 + 56*a*b^6*c^3*d^4*x^3 + 56*a^3*b^4*c*d^6*x^3 + 70*a*
b^6*c^2*d^5*x^4 + 70*a^2*b^5*c*d^6*x^4)/(8*a^8*b^8 + 8*b^16*x^8 + 64*a^7*b^
9*x + 64*a*b^15*x^7 + 224*a^6*b^10*x^2 + 448*a^5*b^11*x^3 + 560*a^4*b^12*x^
4 + 448*a^3*b^13*x^5 + 224*a^2*b^14*x^6)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**9,x)
```

```
[Out] Timed out
```

3.1186 $\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^8}{72(a+bx)^8(bc-ad)^2}-\frac{(c+dx)^8}{9(a+bx)^9(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -(c + d*x)^8/(9*(b*c - a*d)*(a + b*x)^9) + (d*(c + d*x)^8)/(72*(b*c - a*d)^
2*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx &= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}-\frac{d\int \frac{(c+dx)^7}{(a+bx)^9}\,dx}{9(bc-ad)}\\
&= -\frac{(c+dx)^8}{9(bc-ad)(a+bx)^9}+\frac{d(c+dx)^8}{72(bc-ad)^2(a+bx)^8}
\end{aligned}$$

Mathematica [B] time = 0.13, size = 367, normalized size = 6.33

$$-\frac{a^7d^7+a^6bd^6(2c+9dx)+3a^5b^2d^5\left(c^2+6cdx+12d^2x^2\right)+a^4b^3d^4\left(4c^3+27c^2dx+72cd^2x^2+84d^3x^3\right)+a^3b^4d^3\left(5c^4+36c^3dx+108c^2d^2x^2+168cd^3x^3+126d^4x^4\right)+3a^2b^5d^2\left(2c^5+15c^4dx+48c^3d^2x^2+84c^2d^3x^3+84cd^4x^4+42d^5x^5\right)+ab^6d\left(7c^6+54c^5dx+180c^4d^2x^2+336c^3d^3x^3+378c^2d^4x^4+252cd^5x^5+84d^6x^6\right)+b^7\left(8c^7+63c^6dx+216c^5d^2x^2+420c^4d^3x^3+504c^3d^4x^4+378c^2d^5x^5+168cd^6x^6+36d^7x^7\right)}{72b^8(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] -1/72*(a^7*d^7 + a^6*b*d^6*(2*c + 9*d*x) + 3*a^5*b^2*d^5*(c^2 + 6*c*d*x + 1
2*d^2*x^2) + a^4*b^3*d^4*(4*c^3 + 27*c^2*d*x + 72*c*d^2*x^2 + 84*d^3*x^3) +
 a^3*b^4*d^3*(5*c^4 + 36*c^3*d*x + 108*c^2*d^2*x^2 + 168*c*d^3*x^3 + 126*d^
```

```
4*x^4) + 3*a^2*b^5*d^2*(2*c^5 + 15*c^4*d*x + 48*c^3*d^2*x^2 + 84*c^2*d^3*x^
3 + 84*c*d^4*x^4 + 42*d^5*x^5) + a*b^6*d*(7*c^6 + 54*c^5*d*x + 180*c^4*d^2*
x^2 + 336*c^3*d^3*x^3 + 378*c^2*d^4*x^4 + 252*c*d^5*x^5 + 84*d^6*x^6) + b^7
*(8*c^7 + 63*c^6*d*x + 216*c^5*d^2*x^2 + 420*c^4*d^3*x^3 + 504*c^3*d^4*x^4
+ 378*c^2*d^5*x^5 + 168*c*d^6*x^6 + 36*d^7*x^7))/(b^8*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^10, x]
```

fricas [B] time = 1.41, size = 548, normalized size = 9.45

$$-\frac{36\,b^7d^7x^7 + 8\,b^7c^7 + 7\,ab^6c^6d + 6\,a^2b^5c^5d^2 + 5\,a^3b^4c^4d^3 + 4\,a^4b^3c^3d^4 + 3\,a^5b^2c^2d^5 + 2\,a^6bcd^6 + a^7d^7 + 84\left(2\,b^7cd^6 + ab^6d^7\right)x^6 + 126\left(3\,b^7c^2d^5 + 2\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 126\left(4\,b^7c^3d^4 + 3\,ab^6c^2d^5 + 2\,a^2b^5cd^6 + a^3b^4d^7\right)x^4 + 84\left(5\,b^7c^4d^3 + 4\,ab^6c^3d^4 + 3\,a^2b^5c^2d^5 + 2\,a^3b^4cd^6 + a^4b^3d^7\right)x^3 + 36\left(6\,b^7c^5d^2 + 5\,ab^6c^4d^3 + 4\,a^2b^5c^3d^4 + 3\,a^3b^4c^2d^5 + 2\,a^4b^3cd^6 + a^5b^2d^7\right)x^2 + 9\left(7\,b^7c^6d + 6\,ab^6c^5d^2 + 5\,a^2b^5c^4d^3 + 4\,a^3b^4c^3d^4 + 3\,a^4b^3c^2d^5 + 2\,a^5b^2cd^6 + a^6bd^7\right)x}{72\left(b^{17}x^9 + 9\,ab^{16}x^8 + 36\,a^2b^{15}x^7 + 84\,a^3b^{14}x^6 + 126\,a^4b^{13}x^5 + 126\,a^5b^{12}x^4 + 84\,a^6b^{11}x^3 + 36\,a^7b^{10}x^2 + 9\,a^8b^9x + a^9b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

giac [B] time = 1.27, size = 496, normalized size = 8.55

$$-\frac{36\,b^7d^7x^7 + 168\,b^7cd^6x^6 + 84\,ab^6d^7x^6 + 378\,b^7c^2d^5x^5 + 252\,ab^6cd^6x^5 + 126\,a^2b^5d^7x^5 + 504\,b^7c^3d^4x^4 + 378\,ab^6c^2d^5x^4 + 252\,a^2b^5cd^6x^4 + 126\,a^3b^4d^7x^4 + 420\,b^7c^4d^3x^3 + 336\,ab^6c^3d^4x^3 + 252\,a^2b^5c^2d^5x^3 + 168\,a^3b^4cd^6x^3 + 84\,a^4b^3d^7x^3 + 216\,b^7c^5d^2x^2 + 180\,ab^6c^4d^3x^2 + 144\,a^2b^5c^3d^4x^2 + 108\,a^3b^4c^2d^5x^2 + 72\,a^4b^3cd^6x^2 + 36\,a^5b^2d^7x^2 + 63\,b^7c^6dx + 54\,ab^6c^5d^2x + 45\,a^2b^5c^4d^3x + 36\,a^3b^4c^3d^4x + 27\,a^4b^3c^2d^5x + 18\,a^5b^2cd^6x + 9\,a^6bd^7x + 8\,b^7c^7 + 7\,ab^6c^6d + 6\,a^2b^5c^5d^2 + 5\,a^3b^4c^4d^3 + 4\,a^4b^3c^3d^4 + 3\,a^5b^2c^2d^5 + 2\,a^6bcd^6 + a^7d^7}{72\,(bx+a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 168*b^7*c*d^6*x^6 + 84*a*b^6*d^7*x^6 + 378*b^7*c^2*
d^5*x^5 + 252*a*b^6*c*d^6*x^5 + 126*a^2*b^5*d^7*x^5 + 504*b^7*c^3*d^4*x^4 +
 378*a*b^6*c^2*d^5*x^4 + 252*a^2*b^5*c*d^6*x^4 + 126*a^3*b^4*d^7*x^4 + 420*
b^7*c^4*d^3*x^3 + 336*a*b^6*c^3*d^4*x^3 + 252*a^2*b^5*c^2*d^5*x^3 + 168*a^3
*b^4*c*d^6*x^3 + 84*a^4*b^3*d^7*x^3 + 216*b^7*c^5*d^2*x^2 + 180*a*b^6*c^4*d
^3*x^2 + 144*a^2*b^5*c^3*d^4*x^2 + 108*a^3*b^4*c^2*d^5*x^2 + 72*a^4*b^3*c*d
^6*x^2 + 36*a^5*b^2*d^7*x^2 + 63*b^7*c^6*d*x + 54*a*b^6*c^5*d^2*x + 45*a^2*
b^5*c^4*d^3*x + 36*a^3*b^4*c^3*d^4*x + 27*a^4*b^3*c^2*d^5*x + 18*a^5*b^2*c*
d^6*x + 9*a^6*b*d^7*x + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7)/((b*x + a)^9*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 8.00

$$-\frac{d^7}{2\,(bx+a)^2 b^8}+\frac{7\,(ad-bc)\,d^6}{3\,(bx+a)^3 b^8}-\frac{21\left(a^2d^2-2abcd+b^2c^2\right)d^5}{4\,(bx+a)^4 b^8}+\frac{7\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)d^4}{(bx+a)^5 b^8}-\frac{35\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)d^3}{6\,(bx+a)^6 b^8}+\frac{3\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)d^2}{(bx+a)^7 b^8}-\frac{7\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)d}{8\,(bx+a)^8 b^8}-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4c^3d^4b^3-35a^3b^4c^4d^3+21a^2c^5d^2b^5-7a\,b^6c^6d+b^7c^7}{9\,(bx+a)^9 b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^10,x)
```

```
[Out] -7/8*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*
b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^8+7/3*d^6*(a*d-b*c)/b^8/(b*x
+a)^3+3*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*
a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^7-1/9*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c
^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6
*d+b^7*c^7)/b^8/(b*x+a)^9+7*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)/b^8/(b*x+a)^5-1/2*d^7/b^8/(b*x+a)^2-21/4*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b^8/(b*x+a)^4-35/6*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^
3*d+b^4*c^4)/b^8/(b*x+a)^6
```

maxima [B] time = 1.71, size = 548, normalized size = 9.45

$$-\frac{36\, b^{7} d^{7} x^{7} + 8\, b^{7} c^{7} + 7\, a b^{6} c^{6} d + 6\, a^{2} b^{5} c^{5} d^{2} + 5\, a^{3} b^{4} c^{4} d^{3} + 4\, a^{4} b^{3} c^{3} d^{4} + 3\, a^{5} b^{2} c^{2} d^{5} + 2\, a^{6} b c d^{6} + a^{7} d^{7} + 84\, {\left(2\, b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 126\, {\left(3\, b^{7} c^{2} d^{5} + 2\, a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 126\, {\left(4\, b^{7} c^{3} d^{4} + 3\, a b^{6} c^{2} d^{5} + 2\, a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 84\, {\left(5\, b^{7} c^{4} d^{3} + 4\, a b^{6} c^{3} d^{4} + 3\, a^{2} b^{5} c^{2} d^{5} + 2\, a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 36\, {\left(6\, b^{7} c^{5} d^{2} + 5\, a b^{6} c^{4} d^{3} + 4\, a^{2} b^{5} c^{3} d^{4} + 3\, a^{3} b^{4} c^{2} d^{5} + 2\, a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 9\, {\left(7\, b^{7} c^{6} d + 6\, a b^{6} c^{5} d^{2} + 5\, a^{2} b^{5} c^{4} d^{3} + 4\, a^{3} b^{4} c^{3} d^{4} + 3\, a^{4} b^{3} c^{2} d^{5} + 2\, a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{72\, {\left(b^{17} x^{9} + 9\, a b^{16} x^{8} + 36\, a^{2} b^{15} x^{7} + 84\, a^{3} b^{14} x^{6} + 126\, a^{4} b^{13} x^{5} + 126\, a^{5} b^{12} x^{4} + 84\, a^{6} b^{11} x^{3} + 36\, a^{7} b^{10} x^{2} + 9\, a^{8} b^{9} x + a^{9} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] -1/72*(36*b^7*d^7*x^7 + 8*b^7*c^7 + 7*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 + 5*a
^3*b^4*c^4*d^3 + 4*a^4*b^3*c^3*d^4 + 3*a^5*b^2*c^2*d^5 + 2*a^6*b*c*d^6 + a^
7*d^7 + 84*(2*b^7*c*d^6 + a*b^6*d^7)*x^6 + 126*(3*b^7*c^2*d^5 + 2*a*b^6*c*d
^6 + a^2*b^5*d^7)*x^5 + 126*(4*b^7*c^3*d^4 + 3*a*b^6*c^2*d^5 + 2*a^2*b^5*c*
d^6 + a^3*b^4*d^7)*x^4 + 84*(5*b^7*c^4*d^3 + 4*a*b^6*c^3*d^4 + 3*a^2*b^5*c^
2*d^5 + 2*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 36*(6*b^7*c^5*d^2 + 5*a*b^6*c^
4*d^3 + 4*a^2*b^5*c^3*d^4 + 3*a^3*b^4*c^2*d^5 + 2*a^4*b^3*c*d^6 + a^5*b^2*d
^7)*x^2 + 9*(7*b^7*c^6*d + 6*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 + 4*a^3*b^4*
c^3*d^4 + 3*a^4*b^3*c^2*d^5 + 2*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^17*x^9 + 9
*a*b^16*x^8 + 36*a^2*b^15*x^7 + 84*a^3*b^14*x^6 + 126*a^4*b^13*x^5 + 126*a^
5*b^12*x^4 + 84*a^6*b^11*x^3 + 36*a^7*b^10*x^2 + 9*a^8*b^9*x + a^9*b^8)
```

mupad [B] time = 0.15, size = 39, normalized size = 0.67

$$\frac{{\left(c+d\,x\right)}^8\,\left(9\,a\,d-8\,b\,c+b\,d\,x\right)}{72\,{\left(a\,d-b\,c\right)}^2\,{\left(a+b\,x\right)}^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^10,x)
```

```
[Out] ((c + d*x)^8*(9*a*d - 8*b*c + b*d*x))/(72*(a*d - b*c)^2*(a + b*x)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**10,x)
```

```
[Out] Timed out
```

3.1187 $\int \frac{(c+dx)^7}{(a+bx)^{11}} \, dx$

Optimal. Leaf size=89

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{45, 37\}$

$$-\frac{d^2(c+dx)^8}{360(a+bx)^8(bc-ad)^3}+\frac{d(c+dx)^8}{45(a+bx)^9(bc-ad)^2}-\frac{(c+dx)^8}{10(a+bx)^{10}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -(c + d*x)^8/(10*(b*c - a*d)*(a + b*x)^10) + (d*(c + d*x)^8)/(45*(b*c - a*d
)^2*(a + b*x)^9) - (d^2*(c + d*x)^8)/(360*(b*c - a*d)^3*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{11}} \, dx &= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}-\frac{d\int \frac{(c+dx)^7}{(a+bx)^{10}} \, dx}{5(bc-ad)}\\
&= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}+\frac{d^2\int \frac{(c+dx)^7}{(a+bx)^9} \, dx}{45(bc-ad)^2}\\
&= -\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}}+\frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9}-\frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}
\end{aligned}$$

Mathematica [B] time = 0.12, size = 371, normalized size = 4.17

$$-\frac{a^7d^7+a^6bd^6(3c+10dx)+3a^5b^2d^5\left(2c^2+10cdx+15d^2x^2\right)+5a^4b^3d^4\left(2c^3+12c^2dx+27cd^2x^2+24d^3x^3\right)+5a^3b^4d^3\left(3c^4+20c^3dx+54c^2d^2x^2+72cd^3x^3+42d^4x^4\right)+3a^2b^5d^2\left(7c^5+50c^4dx+150c^3d^2x^2+240c^2d^3x^3+210cd^4x^4+84d^5x^5\right)+ab^6d\left(28c^6+210c^5dx+675c^4d^2x^2+1200c^3d^3x^3+1260c^2d^4x^4+756cd^5x^5+210d^6x^6\right)+b^7\left(36c^7+280c^6dx+945c^5d^2x^2+1800c^4d^3x^3+2100c^3d^4x^4+1512c^2d^5x^5+630cd^6x^6+120d^7x^7\right)}{360b^8(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] -1/360*(a^7*d^7 + a^6*b*d^6*(3*c + 10*d*x) + 3*a^5*b^2*d^5*(2*c^2 + 10*c*d*
x + 15*d^2*x^2) + 5*a^4*b^3*d^4*(2*c^3 + 12*c^2*d*x + 27*c*d^2*x^2 + 24*d^3
*x^3) + 5*a^3*b^4*d^3*(3*c^4 + 20*c^3*d*x + 54*c^2*d^2*x^2 + 72*c*d^3*x^3 +
 42*d^4*x^4) + 3*a^2*b^5*d^2*(7*c^5 + 50*c^4*d*x + 150*c^3*d^2*x^2 + 240*c^
2*d^3*x^3 + 210*c*d^4*x^4 + 84*d^5*x^5) + a*b^6*d*(28*c^6 + 210*c^5*d*x + 6
75*c^4*d^2*x^2 + 1200*c^3*d^3*x^3 + 1260*c^2*d^4*x^4 + 756*c*d^5*x^5 + 210*
d^6*x^6) + b^7*(36*c^7 + 280*c^6*d*x + 945*c^5*d^2*x^2 + 1800*c^4*d^3*x^3 +
 2100*c^3*d^4*x^4 + 1512*c^2*d^5*x^5 + 630*c*d^6*x^6 + 120*d^7*x^7))/(b^8*(
a + b*x)^10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{11}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^11,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^11, x]
```

fricas [B] time = 1.50, size = 559, normalized size = 6.28

$$-\frac{120\,b^7d^7x^7 + 36\,b^7c^7 + 28\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 15\,a^3b^4c^4d^3 + 10\,a^4b^3c^3d^4 + 6\,a^5b^2c^2d^5 + 3\,a^6bcd^6 + a^7d^7 + 210\left(3\,b^7cd^6 + ab^6d^7\right)x^6 + 252\left(6\,b^7c^2d^5 + 3\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 210\left(10\,b^7c^3d^4 + 6\,ab^6c^2d^5 + 3\,a^2b^5cd^6 + a^3b^4d^7\right)x^4 + 120\left(15\,b^7c^4d^3 + 10\,ab^6c^3d^4 + 6\,a^2b^5c^2d^5 + 3\,a^3b^4cd^6 + a^4b^3d^7\right)x^3 + 45\left(21\,b^7c^5d^2 + 15\,ab^6c^4d^3 + 10\,a^2b^5c^3d^4 + 6\,a^3b^4c^2d^5 + 3\,a^4b^3cd^6 + a^5b^2d^7\right)x^2 + 10\left(28\,b^7c^6d + 21\,ab^6c^5d^2 + 15\,a^2b^5c^4d^3 + 10\,a^3b^4c^3d^4 + 6\,a^4b^3c^2d^5 + 3\,a^5b^2cd^6 + a^6bd^7\right)x}{360\left(b^{18}x^{10} + 10\,ab^{17}x^9 + 45\,a^2b^{16}x^8 + 120\,a^3b^{15}x^7 + 210\,a^4b^{14}x^6 + 252\,a^5b^{13}x^5 + 210\,a^6b^{12}x^4 + 120\,a^7b^{11}x^3 + 45\,a^8b^{10}x^2 + 10\,a^9b^9x + a^{10}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="fricas")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

giac [B] time = 1.31, size = 496, normalized size = 5.57

$$-\frac{120\,b^7d^7x^7 + 630\,b^7cd^6x^6 + 210\,ab^6d^7x^6 + 1512\,b^7c^2d^5x^5 + 756\,ab^6cd^6x^5 + 252\,a^2b^5d^7x^5 + 2100\,b^7c^3d^4x^4 + 1260\,ab^6c^2d^5x^4 + 630\,a^2b^5cd^6x^4 + 210\,a^3b^4d^7x^4 + 1800\,b^7c^4d^3x^3 + 1200\,ab^6c^3d^4x^3 + 720\,a^2b^5c^2d^5x^3 + 360\,a^3b^4cd^6x^3 + 120\,a^4b^3d^7x^3 + 945\,b^7c^5d^2x^2 + 675\,ab^6c^4d^3x^2 + 450\,a^2b^5c^3d^4x^2 + 270\,a^3b^4c^2d^5x^2 + 135\,a^4b^3cd^6x^2 + 45\,a^5b^2d^7x^2 + 280\,b^7c^6dx + 210\,ab^6c^5d^2x + 150\,a^2b^5c^4d^3x + 100\,a^3b^4c^3d^4x + 60\,a^4b^3c^2d^5x + 30\,a^5b^2cd^6x + 10\,a^6bd^7x + 36\,b^7c^7 + 28\,ab^6c^6d + 21\,a^2b^5c^5d^2 + 15\,a^3b^4c^4d^3 + 10\,a^4b^3c^3d^4 + 6\,a^5b^2c^2d^5 + 3\,a^6bcd^6 + a^7d^7}{360\,(bx+a)^{10}b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="giac")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 630*b^7*c*d^6*x^6 + 210*a*b^6*d^7*x^6 + 1512*b^7*
c^2*d^5*x^5 + 756*a*b^6*c*d^6*x^5 + 252*a^2*b^5*d^7*x^5 + 2100*b^7*c^3*d^4*
x^4 + 1260*a*b^6*c^2*d^5*x^4 + 630*a^2*b^5*c*d^6*x^4 + 210*a^3*b^4*d^7*x^4
+ 1800*b^7*c^4*d^3*x^3 + 1200*a*b^6*c^3*d^4*x^3 + 720*a^2*b^5*c^2*d^5*x^3 +
 360*a^3*b^4*c*d^6*x^3 + 120*a^4*b^3*d^7*x^3 + 945*b^7*c^5*d^2*x^2 + 675*a*
b^6*c^4*d^3*x^2 + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*c^2*d^5*x^2 + 135*a
^4*b^3*c*d^6*x^2 + 45*a^5*b^2*d^7*x^2 + 280*b^7*c^6*d*x + 210*a*b^6*c^5*d^2
*x + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x + 60*a^4*b^3*c^2*d^5*x +
 30*a^5*b^2*c*d^6*x + 10*a^6*b*d^7*x + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2
*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5
+ 3*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^10*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 5.21

$$-\frac{d^7}{3\left(bx+a\right)^3 b^8}+\frac{7\left(ad-bc\right)d^6}{4\left(bx+a\right)^4 b^8}-\frac{21\left(a^2d^2-2abcd+b^2c^2\right)d^5}{5\left(bx+a\right)^5 b^8}+\frac{35\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)d^4}{6\left(bx+a\right)^6 b^8}-\frac{5\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)d^3}{\left(bx+a\right)^7 b^8}+\frac{21\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)d^2}{8\left(bx+a\right)^8 b^8}-\frac{7\left(a^6d^6-6a^5bcd^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6ab^5c^5d+b^6c^6\right)d}{9\left(bx+a\right)^9 b^8}-\frac{-a^7d^7+7a^6bcd^6-21a^5b^2c^2d^5+35a^4b^3c^3d^4-35a^3b^4c^4d^3+21a^2b^5c^5d^2-7ab^6c^6d+b^7c^7}{10\left(bx+a\right)^{10} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^11,x)
```

```
[Out] 21/8*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^8-1/3*d^7/b^8/(b*x+a)^3-5*d^3*(a^4*d^4-4*a^3*
b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^7-7/9*d*(a^6*d
^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6
*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^9-21/5*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^
8/(b*x+a)^5+7/4*d^6*(a*d-b*c)/b^8/(b*x+a)^4+35/6*d^4*(a^3*d^3-3*a^2*b*c*d^2
+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^6-1/10*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b
^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6
*c^6*d+b^7*c^7)/b^8/(b*x+a)^10
```

maxima [B] time = 1.73, size = 559, normalized size = 6.28

$$-\frac{120b^7d^7x^7+36b^7c^7+28ab^6c^6d+21a^2b^5c^5d^2+15a^3b^4c^4d^3+10a^4b^3c^3d^4+6a^5b^2c^2d^5+3a^6bcd^6+a^7d^7+210\left(3b^7cd^6+ab^6d^7\right)x^6+252\left(6b^7c^2d^5+3ab^6cd^6+a^2b^5d^7\right)x^5+210\left(10b^7c^3d^4+6ab^6c^2d^5+3a^2b^5cd^6+a^3b^4d^7\right)x^4+120\left(15b^7c^4d^3+10ab^6c^3d^4+6a^2b^5c^2d^5+3a^3b^4cd^6+a^4b^3d^7\right)x^3+45\left(21b^7c^5d^2+15ab^6c^4d^3+10a^2b^5c^3d^4+6a^3b^4c^2d^5+3a^4b^3cd^6+a^5b^2d^7\right)x^2+10\left(28b^7c^6d+21ab^6c^5d^2+15a^2b^5c^4d^3+10a^3b^4c^3d^4+6a^4b^3c^2d^5+3a^5b^2cd^6+a^6bd^7\right)x}{360\left(b^{18}x^{10}+10ab^{17}x^9+45a^2b^{16}x^8+120a^3b^{15}x^7+210a^4b^{14}x^6+252a^5b^{13}x^5+210a^6b^{12}x^4+120a^7b^{11}x^3+45a^8b^{10}x^2+10a^9b^9x+a^{10}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/360*(120*b^7*d^7*x^7 + 36*b^7*c^7 + 28*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2
+ 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^5*b^2*c^2*d^5 + 3*a^6*b*c*d
^6 + a^7*d^7 + 210*(3*b^7*c*d^6 + a*b^6*d^7)*x^6 + 252*(6*b^7*c^2*d^5 + 3*a
*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 210*(10*b^7*c^3*d^4 + 6*a*b^6*c^2*d^5 + 3*a
^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 120*(15*b^7*c^4*d^3 + 10*a*b^6*c^3*d^4 +
6*a^2*b^5*c^2*d^5 + 3*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 45*(21*b^7*c^5*d^2
 + 15*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 + 6*a^3*b^4*c^2*d^5 + 3*a^4*b^3*c*
d^6 + a^5*b^2*d^7)*x^2 + 10*(28*b^7*c^6*d + 21*a*b^6*c^5*d^2 + 15*a^2*b^5*c
^4*d^3 + 10*a^3*b^4*c^3*d^4 + 6*a^4*b^3*c^2*d^5 + 3*a^5*b^2*c*d^6 + a^6*b*d
^7)*x)/(b^18*x^10 + 10*a*b^17*x^9 + 45*a^2*b^16*x^8 + 120*a^3*b^15*x^7 + 21
0*a^4*b^14*x^6 + 252*a^5*b^13*x^5 + 210*a^6*b^12*x^4 + 120*a^7*b^11*x^3 + 4
5*a^8*b^10*x^2 + 10*a^9*b^9*x + a^10*b^8)
```

mupad [B] time = 0.45, size = 600, normalized size = 6.74

$$-\frac{a^7d^7+3a^6bcd^6+10a^6bd^7x+6a^5b^2c^2d^5+30a^5b^2cd^6x+45a^5b^2d^7x^2+10a^4b^3c^3d^4+60a^4b^3c^2d^5x+135a^4b^3cd^6x^2+120a^4b^3d^7x^3+15a^3b^4c^4d^3+100a^3b^4c^3d^4x+270a^3b^4c^2d^5x^2+360a^3b^4cd^6x^3+210a^3b^4d^7x^4+21a^2b^5c^5d^2+150a^2b^5c^4d^3x+450a^2b^5c^3d^4x^2+720a^2b^5c^2d^5x^3+630a^2b^5cd^6x^4+252a^2b^5d^7x^5+28ab^6c^6d+210ab^6c^5d^2x+675ab^6c^4d^3x^2+1200ab^6c^3d^4x^3+1260ab^6c^2d^5x^4+756ab^6cd^6x^5+210ab^6d^7x^6+36b^7c^7+280b^7c^6dx+945b^7c^5d^2x^2+1800b^7c^4d^3x^3+2100b^7c^3d^4x^4+1512b^7c^2d^5x^5+630b^7cd^6x^6+120b^7d^7x^7}{360a^{10}b^8+3600a^9b^9x+16200a^8b^{10}x^2+43200a^7b^{11}x^3+75600a^6b^{12}x^4+90720a^5b^{13}x^5+75600a^4b^{14}x^6+43200a^3b^{15}x^7+16200a^2b^{16}x^8+3600ab^{17}x^9+360b^{18}x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^11,x)
```

```
[Out] -(a^7*d^7 + 36*b^7*c^7 + 120*b^7*d^7*x^7 + 210*a*b^6*d^7*x^6 + 630*b^7*c*d^
6*x^6 + 21*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 + 10*a^4*b^3*c^3*d^4 + 6*a^
5*b^2*c^2*d^5 + 45*a^5*b^2*d^7*x^2 + 120*a^4*b^3*d^7*x^3 + 210*a^3*b^4*d^7*
x^4 + 252*a^2*b^5*d^7*x^5 + 945*b^7*c^5*d^2*x^2 + 1800*b^7*c^4*d^3*x^3 + 21
00*b^7*c^3*d^4*x^4 + 1512*b^7*c^2*d^5*x^5 + 28*a*b^6*c^6*d + 3*a^6*b*c*d^6
+ 10*a^6*b*d^7*x + 280*b^7*c^6*d*x + 450*a^2*b^5*c^3*d^4*x^2 + 270*a^3*b^4*
c^2*d^5*x^2 + 720*a^2*b^5*c^2*d^5*x^3 + 210*a*b^6*c^5*d^2*x + 30*a^5*b^2*c*
d^6*x + 756*a*b^6*c*d^6*x^5 + 150*a^2*b^5*c^4*d^3*x + 100*a^3*b^4*c^3*d^4*x
 + 60*a^4*b^3*c^2*d^5*x + 675*a*b^6*c^4*d^3*x^2 + 135*a^4*b^3*c*d^6*x^2 + 1
200*a*b^6*c^3*d^4*x^3 + 360*a^3*b^4*c*d^6*x^3 + 1260*a*b^6*c^2*d^5*x^4 + 63
0*a^2*b^5*c*d^6*x^4)/(360*a^10*b^8 + 360*b^18*x^10 + 3600*a^9*b^9*x + 3600*
a*b^17*x^9 + 16200*a^8*b^10*x^2 + 43200*a^7*b^11*x^3 + 75600*a^6*b^12*x^4 +
 90720*a^5*b^13*x^5 + 75600*a^4*b^14*x^6 + 43200*a^3*b^15*x^7 + 16200*a^2*b
^16*x^8)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**11,x)
```

```
[Out] Timed out
```

3.1188 $\int \frac{(c+dx)^7}{(a+bx)^{12}} \, dx$

Optimal. Leaf size=120

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^3(c+dx)^8}{1320(a+bx)^8(bc-ad)^4}-\frac{d^2(c+dx)^8}{165(a+bx)^9(bc-ad)^3}+\frac{3d(c+dx)^8}{110(a+bx)^{10}(bc-ad)^2}-\frac{(c+dx)^8}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^8/(11*(b*c - a*d)*(a + b*x)^11) + (3*d*(c + d*x)^8)/(110*(b*c -
a*d)^2*(a + b*x)^10) - (d^2*(c + d*x)^8)/(165*(b*c - a*d)^3*(a + b*x)^9) +
(d^3*(c + d*x)^8)/(1320*(b*c - a*d)^4*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{12}} \, dx &= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}-\frac{(3d)\int \frac{(c+dx)^7}{(a+bx)^{11}} \, dx}{11(bc-ad)}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}+\frac{\left(3d^2\right)\int \frac{(c+dx)^7}{(a+bx)^{10}} \, dx}{55(bc-ad)^2}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}-\frac{d^3\int \frac{(c+dx)^7}{(a+bx)^9} \, dx}{165(bc-ad)^3}\\
&= -\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}}+\frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}}-\frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9}+\frac{d^3(c+dx}{1320(bc-ad)^4(}
\end{aligned}$$

Mathematica [B] time = 0.12, size = 369, normalized size = 3.08

$$-\frac{a^7d^7+a^6bd^6(4c+11dx)+a^5b^2d^5\left(10c^2+44cdx+55d^2x^2\right)+5a^4b^3d^4\left(4c^3+22c^2dx+44cd^2x^2+33d^3x^3\right)+5a^3b^4d^3\left(7c^4+44c^3dx+110c^2d^2x^2+132cd^3x^3+66d^4x^4\right)+a^2b^5d^2\left(56c^5+385c^4dx+1100c^3d^2x^2+1650c^2d^3x^3+1320cd^4x^4+462d^5x^5\right)+ab^6d\left(84c^6+616c^5dx+1925c^4d^2x^2+3300c^3d^3x^3+3300c^2d^4x^4+1848cd^5x^5+462d^6x^6\right)+b^7\left(120c^7+924c^6dx+3080c^5d^2x^2+5775c^4d^3x^3+6600c^3d^4x^4+4620c^2d^5x^5+1848cd^6x^6+330d^7x^7\right)}{1320b^8(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] -1/1320*(a^7*d^7 + a^6*b*d^6*(4*c + 11*d*x) + a^5*b^2*d^5*(10*c^2 + 44*c*d*
x + 55*d^2*x^2) + 5*a^4*b^3*d^4*(4*c^3 + 22*c^2*d*x + 44*c*d^2*x^2 + 33*d^3
*x^3) + 5*a^3*b^4*d^3*(7*c^4 + 44*c^3*d*x + 110*c^2*d^2*x^2 + 132*c*d^3*x^3
 + 66*d^4*x^4) + a^2*b^5*d^2*(56*c^5 + 385*c^4*d*x + 1100*c^3*d^2*x^2 + 165
0*c^2*d^3*x^3 + 1320*c*d^4*x^4 + 462*d^5*x^5) + a*b^6*d*(84*c^6 + 616*c^5*d
*x + 1925*c^4*d^2*x^2 + 3300*c^3*d^3*x^3 + 3300*c^2*d^4*x^4 + 1848*c*d^5*x^
5 + 462*d^6*x^6) + b^7*(120*c^7 + 924*c^6*d*x + 3080*c^5*d^2*x^2 + 5775*c^4
*d^3*x^3 + 6600*c^3*d^4*x^4 + 4620*c^2*d^5*x^5 + 1848*c*d^6*x^6 + 330*d^7*x
^7))/(b^8*(a + b*x)^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^12,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^12, x]
```

fricas [B] time = 0.71, size = 570, normalized size = 4.75

$$-\frac{330 b^7 d^7 x^7 + 120 b^7 c^7 + 84 a b^6 c^6 d + 56 a^2 b^5 c^5 d^2 + 35 a^3 b^4 c^4 d^3 + 20 a^4 b^3 c^3 d^4 + 10 a^5 b^2 c^2 d^5 + 4 a^6 b c d^6 + a^7 d^7 + 462 (4 b^7 c d^6 + a b^6 d^7) x^6 + 462 (10 b^7 c^2 d^5 + 4 a b^6 c d^6 + a^2 b^5 d^7) x^5 + 330 (20 b^7 c^3 d^4 + 10 a b^6 c^2 d^5 + 4 a^2 b^5 c d^6 + a^3 b^4 d^7) x^4 + 165 (35 b^7 c^4 d^3 + 20 a b^6 c^3 d^4 + 10 a^2 b^5 c^2 d^5 + 4 a^3 b^4 c d^6 + a^4 b^3 d^7) x^3 + 55 (56 b^7 c^5 d^2 + 35 a b^6 c^4 d^3 + 20 a^2 b^5 c^3 d^4 + 10 a^3 b^4 c^2 d^5 + 4 a^4 b^3 c d^6 + a^5 b^2 d^7) x^2 + 11 (84 b^7 c^6 d + 56 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 + 20 a^3 b^4 c^3 d^4 + 10 a^4 b^3 c^2 d^5 + 4 a^5 b^2 c d^6 + a^6 b d^7) x}{1320 (b^{19} x^{11} + 11 a b^{18} x^{10} + 55 a^2 b^{17} x^9 + 165 a^3 b^{16} x^8 + 330 a^4 b^{15} x^7 + 462 a^5 b^{14} x^6 + 462 a^6 b^{13} x^5 + 330 a^7 b^{12} x^4 + 165 a^8 b^{11} x^3 + 55 a^9 b^{10} x^2 + 11 a^{10} b^9 x + a^{11} b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

giac [B] time = 1.30, size = 496, normalized size = 4.13

$$-\frac{330 b^7 d^7 x^7 + 1848 b^7 c d^6 x^6 + 462 a b^6 d^7 x^6 + 4620 b^7 c^2 d^5 x^5 + 1848 a b^6 c d^6 x^5 + 462 a^2 b^5 d^7 x^5 + 6600 b^7 c^3 d^4 x^4 + 3300 a b^6 c^2 d^5 x^4 + 1320 a^2 b^5 c d^6 x^4 + 330 a^3 b^4 d^7 x^4 + 5775 b^7 c^4 d^3 x^3 + 3300 a b^6 c^3 d^4 x^3 + 1650 a^2 b^5 c^2 d^5 x^3 + 660 a^3 b^4 c d^6 x^3 + 165 a^4 b^3 d^7 x^3 + 3080 b^7 c^5 d^2 x^2 + 1925 a b^6 c^4 d^3 x^2 + 1100 a^2 b^5 c^3 d^4 x^2 + 550 a^3 b^4 c^2 d^5 x^2 + 220 a^4 b^3 c d^6 x^2 + 55 a^5 b^2 d^7 x^2 + 924 b^7 c^6 d x + 616 a b^6 c^5 d^2 x + 385 a^2 b^5 c^4 d^3 x + 220 a^3 b^4 c^3 d^4 x + 110 a^4 b^3 c^2 d^5 x + 44 a^5 b^2 c d^6 x + 11 a^6 b d^7 x + 120 b^7 c^7 + 84 a b^6 c^6 d + 56 a^2 b^5 c^5 d^2 + 35 a^3 b^4 c^4 d^3 + 20 a^4 b^3 c^3 d^4 + 10 a^5 b^2 c^2 d^5 + 4 a^6 b c d^6 + a^7 d^7}{1320 (bx + a)^{11} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 1848*b^7*c*d^6*x^6 + 462*a*b^6*d^7*x^6 + 4620*b^
7*c^2*d^5*x^5 + 1848*a*b^6*c*d^6*x^5 + 462*a^2*b^5*d^7*x^5 + 6600*b^7*c^3*d
^4*x^4 + 3300*a*b^6*c^2*d^5*x^4 + 1320*a^2*b^5*c*d^6*x^4 + 330*a^3*b^4*d^7*
x^4 + 5775*b^7*c^4*d^3*x^3 + 3300*a*b^6*c^3*d^4*x^3 + 1650*a^2*b^5*c^2*d^5*
x^3 + 660*a^3*b^4*c*d^6*x^3 + 165*a^4*b^3*d^7*x^3 + 3080*b^7*c^5*d^2*x^2 +
1925*a*b^6*c^4*d^3*x^2 + 1100*a^2*b^5*c^3*d^4*x^2 + 550*a^3*b^4*c^2*d^5*x^2
 + 220*a^4*b^3*c*d^6*x^2 + 55*a^5*b^2*d^7*x^2 + 924*b^7*c^6*d*x + 616*a*b^6
*c^5*d^2*x + 385*a^2*b^5*c^4*d^3*x + 220*a^3*b^4*c^3*d^4*x + 110*a^4*b^3*c^
2*d^5*x + 44*a^5*b^2*c*d^6*x + 11*a^6*b*d^7*x + 120*b^7*c^7 + 84*a*b^6*c^6*
```

```
d + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b
^2*c^2*d^5 + 4*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^11*b^8)
```

maple [B] time = 0.00, size = 464, normalized size = 3.87

$$-\frac{d^7}{4 (bx+a)^4 b^8} + \frac{7 (ad-bc) d^6}{5 (bx+a)^5 b^8} - \frac{7 \left(a^2d^2 - 2abcd + b^2c^2\right) d^5}{2 (bx+a)^6 b^8} + \frac{5 \left(a^3d^3 - 3a^2bc d^2 + 3a b^2c^2d - b^3c^3\right) d^4}{(bx+a)^7 b^8} - \frac{35 \left(a^4d^4 - 4a^3bc d^3 + 6a^2b^2c^2d^2 - 4a b^3c^3d + b^4c^4\right) d^3}{8 (bx+a)^8 b^8} + \frac{7 \left(a^5d^5 - 5a^4bc d^4 + 10a^3b^2c^2d^3 - 10a^2b^3c^3d^2 + 5a b^4c^4d - b^5c^5\right) d^2}{3 (bx+a)^9 b^8} - \frac{7 \left(a^6d^6 - 6a^5bc d^5 + 15a^4b^2c^2d^4 - 20a^3b^3c^3d^3 + 15a^2b^4c^4d^2 - 6a b^5c^5d + b^6c^6\right) d}{10 (bx+a)^{10} b^8} - \frac{-a^7d^7 + 7a^6bc d^6 - 21a^5b^2c^2d^5 + 35a^4c^3d^4b^3 - 35a^3b^4c^4d^3 + 21a^2c^5d^2b^5 - 7a b^6c^6d + b^7c^7}{11 (bx+a)^{11} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^12,x)
```

```
[Out] -35/8*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b
^8/(b*x+a)^8+5*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a
)^7+7/3*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*
a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^9+7/5*d^6*(a*d-b*c)/b^8/(b*x+a)^5-1/4*d^7/
b^8/(b*x+a)^4-1/11*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^
3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x
+a)^11-7/2*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^6-7/10*d*(a^6*d^6-6*
a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^
5*c^5*d+b^6*c^6)/b^8/(b*x+a)^10
```

maxima [B] time = 1.81, size = 570, normalized size = 4.75

$$-\frac{330 b^7 d^7 x^7 + 120 b^7 c^7 + 84 a b^6 c^6 d + 56 a^2 b^5 c^5 d^2 + 35 a^3 b^4 c^4 d^3 + 20 a^4 b^3 c^3 d^4 + 10 a^5 b^2 c^2 d^5 + 4 a^6 b c d^6 + a^7 d^7 + 462 \left(4 b^7 c d^6 + a b^6 d^7\right) x^6 + 462 \left(10 b^7 c^2 d^5 + 4 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 330 \left(20 b^7 c^3 d^4 + 10 a b^6 c^2 d^5 + 4 a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 165 \left(35 b^7 c^4 d^3 + 20 a b^6 c^3 d^4 + 10 a^2 b^5 c^2 d^5 + 4 a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 55 \left(56 b^7 c^5 d^2 + 35 a b^6 c^4 d^3 + 20 a^2 b^5 c^3 d^4 + 10 a^3 b^4 c^2 d^5 + 4 a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 11 \left(84 b^7 c^6 d + 56 a b^6 c^5 d^2 + 35 a^2 b^5 c^4 d^3 + 20 a^3 b^4 c^3 d^4 + 10 a^4 b^3 c^2 d^5 + 4 a^5 b^2 c d^6 + a^6 b d^7\right) x}{1320 \left(b^{19} x^{11} + 11 a b^{18} x^{10} + 55 a^2 b^{17} x^9 + 165 a^3 b^{16} x^8 + 330 a^4 b^{15} x^7 + 462 a^5 b^{14} x^6 + 462 a^6 b^{13} x^5 + 330 a^7 b^{12} x^4 + 165 a^8 b^{11} x^3 + 55 a^9 b^{10} x^2 + 11 a^{10} b^9 x + a^{11} b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(330*b^7*d^7*x^7 + 120*b^7*c^7 + 84*a*b^6*c^6*d + 56*a^2*b^5*c^5*d^
2 + 35*a^3*b^4*c^4*d^3 + 20*a^4*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 4*a^6*b*
c*d^6 + a^7*d^7 + 462*(4*b^7*c*d^6 + a*b^6*d^7)*x^6 + 462*(10*b^7*c^2*d^5 +
 4*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 330*(20*b^7*c^3*d^4 + 10*a*b^6*c^2*d^5
+ 4*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 165*(35*b^7*c^4*d^3 + 20*a*b^6*c^3*d
^4 + 10*a^2*b^5*c^2*d^5 + 4*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 55*(56*b^7*c
^5*d^2 + 35*a*b^6*c^4*d^3 + 20*a^2*b^5*c^3*d^4 + 10*a^3*b^4*c^2*d^5 + 4*a^4
*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 11*(84*b^7*c^6*d + 56*a*b^6*c^5*d^2 + 35*a^
2*b^5*c^4*d^3 + 20*a^3*b^4*c^3*d^4 + 10*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 +
 a^6*b*d^7)*x)/(b^19*x^11 + 11*a*b^18*x^10 + 55*a^2*b^17*x^9 + 165*a^3*b^16
*x^8 + 330*a^4*b^15*x^7 + 462*a^5*b^14*x^6 + 462*a^6*b^13*x^5 + 330*a^7*b^1
2*x^4 + 165*a^8*b^11*x^3 + 55*a^9*b^10*x^2 + 11*a^10*b^9*x + a^11*b^8)
```

mupad [B] time = 0.52, size = 548, normalized size = 4.57

$$-\frac{\frac{a^7 d^7 + 4 a^6 b c d^6 + 10 a^5 b^2 c^2 d^5 + 20 a^4 b^3 c^3 d^4 + 35 a^3 b^4 c^4 d^3 + 56 a^2 b^5 c^5 d^2 + 84 a b^6 c^6 d + 120 b^7 c^7}{1320 b^8} + \frac{d^7 x^7}{4 b} + \frac{d^2 x^2 \left(a^5 d^5 + 4 a^4 b c d^4 + 10 a^3 b^2 c^2 d^3 + 20 a^2 b^3 c^3 d^2 + 35 a b^4 c^4 d + 56 b^5 c^5\right)}{24 b^6} + \frac{d^4 x^4 \left(a^3 d^3 + 4 a^2 b c d^2 + 10 a b^2 c^2 d + 20 b^3 c^3\right)}{4 b^4} + \frac{7 d^6 x^6 (a d + 4 b c)}{20 b^2} + \frac{d^3 x^3 \left(a^4 d^4 + 4 a^3 b c d^3 + 10 a^2 b^2 c^2 d^2 + 20 a b^3 c^3 d + 35 b^4 c^4\right)}{8 b^5} + \frac{d x \left(a^6 d^6 + 4 a^5 b c d^5 + 10 a^4 b^2 c^2 d^4 + 20 a^3 b^3 c^3 d^3 + 35 a^2 b^4 c^4 d^2 + 56 a b^5 c^5 d + 84 b^6 c^6\right)}{120 b^7} + \frac{7 d^5 x^5 \left(a^2 d^2 + 4 a b c d + 10 b^2 c^2\right)}{20 b^3}}{a^{11} + 11 a^{10} b x + 55 a^9 b^2 x^2 + 165 a^8 b^3 x^3 + 330 a^7 b^4 x^4 + 462 a^6 b^5 x^5 + 462 a^5 b^6 x^6 + 330 a^4 b^7 x^7 + 165 a^3 b^8 x^8 + 55 a^2 b^9 x^9 + 11 a b^{10} x^{10} + b^{11} x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^12,x)
```

```
[Out] -((a^7*d^7 + 120*b^7*c^7 + 56*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 + 20*a^4
*b^3*c^3*d^4 + 10*a^5*b^2*c^2*d^5 + 84*a*b^6*c^6*d + 4*a^6*b*c*d^6)/(1320*b
^8) + (d^7*x^7)/(4*b) + (d^2*x^2*(a^5*d^5 + 56*b^5*c^5 + 20*a^2*b^3*c^3*d^2
 + 10*a^3*b^2*c^2*d^3 + 35*a*b^4*c^4*d + 4*a^4*b*c*d^4))/(24*b^6) + (d^4*x^
4*(a^3*d^3 + 20*b^3*c^3 + 10*a*b^2*c^2*d + 4*a^2*b*c*d^2))/(4*b^4) + (7*d^6
*x^6*(a*d + 4*b*c))/(20*b^2) + (d^3*x^3*(a^4*d^4 + 35*b^4*c^4 + 10*a^2*b^2*
c^2*d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3))/(8*b^5) + (d*x*(a^6*d^6 + 84*b^6
*c^6 + 35*a^2*b^4*c^4*d^2 + 20*a^3*b^3*c^3*d^3 + 10*a^4*b^2*c^2*d^4 + 56*a*
b^5*c^5*d + 4*a^5*b*c*d^5))/(120*b^7) + (7*d^5*x^5*(a^2*d^2 + 10*b^2*c^2 +
4*a*b*c*d))/(20*b^3))/(a^11 + b^11*x^11 + 11*a*b^10*x^10 + 55*a^9*b^2*x^2 +
 165*a^8*b^3*x^3 + 330*a^7*b^4*x^4 + 462*a^6*b^5*x^5 + 462*a^5*b^6*x^6 + 33
0*a^4*b^7*x^7 + 165*a^3*b^8*x^8 + 55*a^2*b^9*x^9 + 11*a^10*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**12,x)
```

```
[Out] Timed out
```

3.1189 $\int \frac{(c+dx)^7}{(a+bx)^{13}} \, dx$

Optimal. Leaf size=151

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^8}{3960(a+bx)^8(bc-ad)^5}+\frac{d^3(c+dx)^8}{495(a+bx)^9(bc-ad)^4}-\frac{d^2(c+dx)^8}{110(a+bx)^{10}(bc-ad)^3}+\frac{d(c+dx)^8}{33(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^8}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -(c + d*x)^8/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^8)/(33*(b*c - a*d
)^2*(a + b*x)^11) - (d^2*(c + d*x)^8)/(110*(b*c - a*d)^3*(a + b*x)^10) + (d
^3*(c + d*x)^8)/(495*(b*c - a*d)^4*(a + b*x)^9) - (d^4*(c + d*x)^8)/(3960*(
b*c - a*d)^5*(a + b*x)^8)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^7}{(a+bx)^{13}} \, dx &= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}-\frac{d\int \frac{(c+dx)^7}{(a+bx)^{12}} \, dx}{3(bc-ad)}\\
&= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}+\frac{d^2\int \frac{(c+dx)^7}{(a+bx)^{11}} \, dx}{11(bc-ad)^2}\\
&= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}-\frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}}-\frac{d^3\int \frac{(c+dx)^7}{(a+bx)^{10}} \, dx}{55(bc-ad)^3}\\
&= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}-\frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}}+\frac{d^3(c+dx)^8}{495(bc-ad)^4(a+bx)^9}\\
&= -\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}}-\frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}}+\frac{d^3(c+dx)^8}{495(bc-ad)^4(a+bx)^9}
\end{aligned}$$

Mathematica [B] time = 0.13, size = 371, normalized size = 2.46

$$-\frac{a^7d^7+a^6bd^6(5c+12dx)+3a^5b^2d^5(5c^2+20cdx+22d^2x^2)+5a^4b^3d^4(7c^3+36c^2dx+66cd^2x^2+44d^3x^3)+5a^3b^4d^3(14c^4+84c^3dx+198c^2d^2x^2+220cd^3x^3+99d^4x^4)+3a^2b^5d^2(42c^5+280c^4dx+770c^3d^2x^2+1100c^2d^3x^3+825cd^4x^4+264d^5x^5)+ab^6d(210c^6+1512c^5dx+4620c^4d^2x^2+7700c^3d^3x^3+7425c^2d^4x^4+3960cd^5x^5+924d^6x^6)+b^7(330c^7+2520c^6dx+8316c^5d^2x^2+15400c^4d^3x^3+17325c^3d^4x^4+11880c^2d^5x^5+4620cd^6x^6+792d^7x^7)}{3960b^8(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] -1/3960*(a^7*d^7 + a^6*b*d^6*(5*c + 12*d*x) + 3*a^5*b^2*d^5*(5*c^2 + 20*c*d
*x + 22*d^2*x^2) + 5*a^4*b^3*d^4*(7*c^3 + 36*c^2*d*x + 66*c*d^2*x^2 + 44*d^
3*x^3) + 5*a^3*b^4*d^3*(14*c^4 + 84*c^3*d*x + 198*c^2*d^2*x^2 + 220*c*d^3*x
^3 + 99*d^4*x^4) + 3*a^2*b^5*d^2*(42*c^5 + 280*c^4*d*x + 770*c^3*d^2*x^2 +
1100*c^2*d^3*x^3 + 825*c*d^4*x^4 + 264*d^5*x^5) + a*b^6*d*(210*c^6 + 1512*c
^5*d*x + 4620*c^4*d^2*x^2 + 7700*c^3*d^3*x^3 + 7425*c^2*d^4*x^4 + 3960*c*d^
5*x^5 + 924*d^6*x^6) + b^7*(330*c^7 + 2520*c^6*d*x + 8316*c^5*d^2*x^2 + 154
00*c^4*d^3*x^3 + 17325*c^3*d^4*x^4 + 11880*c^2*d^5*x^5 + 4620*c*d^6*x^6 + 7
92*d^7*x^7))/(b^8*(a + b*x)^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^13,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^13, x]
```

fricas [B] time = 1.42, size = 581, normalized size = 3.85

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924\,(5\,b^7cd^6+ab^6d^7)x^6+792\,(15\,b^7c^2d^5+5\,ab^6cd^6+a^2b^5d^7)x^5+495\,(35\,b^7c^3d^4+15\,ab^6c^2d^5+5\,a^2b^5cd^6+a^3b^4d^7)x^4+220\,(70\,b^7c^4d^3+35\,ab^6c^3d^4+15\,a^2b^5c^2d^5+5\,a^3b^4cd^6+a^4b^3d^7)x^3+66\,(126\,b^7c^5d^2+70\,ab^6c^4d^3+35\,a^2b^5c^3d^4+15\,a^3b^4c^2d^5+5\,a^4b^3cd^6+a^5b^2d^7)x^2+12\,(210\,b^7c^6d+126\,ab^6c^5d^2+70\,a^2b^5c^4d^3+35\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5+5\,a^5b^2cd^6+a^6bd^7)x}{3960\,(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^2b^{18}x^{10}+220\,a^3b^{17}x^9+495\,a^4b^{16}x^8+792\,a^5b^{15}x^7+924\,a^6b^{14}x^6+792\,a^7b^{13}x^5+495\,a^8b^{12}x^4+220\,a^9b^{11}x^3+66\,a^{10}b^{10}x^2+12\,a^{11}b^9x+a^{12}b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

giac [B] time = 1.26, size = 496, normalized size = 3.28

$$-\frac{792\,b^7d^7x^7+4620\,b^7cd^6x^6+924\,ab^6d^7x^6+11880\,b^7c^2d^5x^5+3960\,ab^6cd^6x^5+792\,a^2b^5d^7x^5+17325\,b^7c^3d^4x^4+7425\,ab^6c^2d^5x^4+2475\,a^2b^5cd^6x^4+495\,a^3b^4d^7x^4+15400\,b^7c^4d^3x^3+7700\,ab^6c^3d^4x^3+3300\,a^2b^5c^2d^5x^3+1100\,a^3b^4cd^6x^3+220\,a^4b^3d^7x^3+8316\,b^7c^5d^2x^2+\cdots+4620\,ab^6c^4d^3x^2+2310\,a^2b^5c^3d^4x^2+990\,a^3b^4c^2d^5x^2+330\,a^4b^3cd^6x^2+66\,a^5b^2d^7x^2+2520\,b^7c^6dx+1512\,ab^6c^5d^2x+840\,a^2b^5c^4d^3x+420\,a^3b^4c^3d^4x+180\,a^4b^3c^2d^5x+60\,a^5b^2cd^6x+12\,a^6bd^7x+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7}{3960\,(bx+a)^{12}b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 4620*b^7*c*d^6*x^6 + 924*a*b^6*d^7*x^6 + 11880*b
^7*c^2*d^5*x^5 + 3960*a*b^6*c*d^6*x^5 + 792*a^2*b^5*d^7*x^5 + 17325*b^7*c^3
*d^4*x^4 + 7425*a*b^6*c^2*d^5*x^4 + 2475*a^2*b^5*c*d^6*x^4 + 495*a^3*b^4*d^
7*x^4 + 15400*b^7*c^4*d^3*x^3 + 7700*a*b^6*c^3*d^4*x^3 + 3300*a^2*b^5*c^2*d
```

```
^5*x^3 + 1100*a^3*b^4*c*d^6*x^3 + 220*a^4*b^3*d^7*x^3 + 8316*b^7*c^5*d^2*x^
2 + 4620*a*b^6*c^4*d^3*x^2 + 2310*a^2*b^5*c^3*d^4*x^2 + 990*a^3*b^4*c^2*d^5
*x^2 + 330*a^4*b^3*c*d^6*x^2 + 66*a^5*b^2*d^7*x^2 + 2520*b^7*c^6*d*x + 1512
*a*b^6*c^5*d^2*x + 840*a^2*b^5*c^4*d^3*x + 420*a^3*b^4*c^3*d^4*x + 180*a^4*
b^3*c^2*d^5*x + 60*a^5*b^2*c*d^6*x + 12*a^6*b*d^7*x + 330*b^7*c^7 + 210*a*b
^6*c^6*d + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 +
15*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^12*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 3.07

$$-\frac{d^7}{5(bx+a)^5 b^8}+\frac{7(ad-bc)d^6}{6(bx+a)^6 b^8}-\frac{3\left(a^2d^2-2abcd+b^2c^2\right)d^5}{(bx+a)^7 b^8}+\frac{35\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)d^4}{8(bx+a)^8 b^8}-\frac{35\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)d^3}{9(bx+a)^9 b^8}+\frac{21\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)d^2}{10(bx+a)^{10} b^8}-\frac{7\left(a^6d^6-6a^5bc\,d^5+15a^4b^2c^2d^4-20a^3b^3c^3d^3+15a^2b^4c^4d^2-6a\,b^5c^5d+b^6c^6\right)d}{11(bx+a)^{11} b^8}-\frac{-a^7d^7+7a^6bc\,d^6-21a^5b^2c^2d^5+35a^4c^3d^4b^3-35a^3b^4c^4d^3+21a^2c^5d^2b^5-7a\,b^6c^6d+b^7c^7}{12(bx+a)^{12} b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^13,x)
```

```
[Out] 35/8*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^8-3*d^5*
(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^7-35/9*d^3*(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^9-1/5*d^7/b^8/(b*x+a)^5
-7/11*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2
*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^11-1/12*(-a^7*d^7+7*a^6*b*c
*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^
5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^12+7/6*d^6*(a*d-b*c)/b^8/(b*x+a)^6
+21/10*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a
*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^10
```

maxima [B] time = 1.76, size = 581, normalized size = 3.85

$$-\frac{792\,b^7d^7x^7+330\,b^7c^7+210\,ab^6c^6d+126\,a^2b^5c^5d^2+70\,a^3b^4c^4d^3+35\,a^4b^3c^3d^4+15\,a^5b^2c^2d^5+5\,a^6bcd^6+a^7d^7+924\left(5\,b^7cd^6+ab^6d^7\right)x^6+792\left(15\,b^7c^2d^5+5\,ab^6cd^6+a^2b^5d^7\right)x^5+495\left(35\,b^7c^3d^4+15\,ab^6c^2d^5+5\,a^2b^5cd^6+a^3b^4d^7\right)x^4+220\left(70\,b^7c^4d^3+35\,ab^6c^3d^4+15\,a^2b^5c^2d^5+5\,a^3b^4cd^6+a^4b^3d^7\right)x^3+66\left(126\,b^7c^5d^2+70\,ab^6c^4d^3+35\,a^2b^5c^3d^4+15\,a^3b^4c^2d^5+5\,a^4b^3cd^6+a^5b^2d^7\right)x^2+12\left(210\,b^7c^6d+126\,ab^6c^5d^2+70\,a^2b^5c^4d^3+35\,a^3b^4c^3d^4+15\,a^4b^3c^2d^5+5\,a^5b^2cd^6+a^6bd^7\right)x}{3960\left(b^{20}x^{12}+12\,ab^{19}x^{11}+66\,a^2b^{18}x^{10}+220\,a^3b^{17}x^9+495\,a^4b^{16}x^8+792\,a^5b^{15}x^7+924\,a^6b^{14}x^6+792\,a^7b^{13}x^5+495\,a^8b^{12}x^4+220\,a^9b^{11}x^3+66\,a^{10}b^{10}x^2+12\,a^{11}b^9x+a^{12}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/3960*(792*b^7*d^7*x^7 + 330*b^7*c^7 + 210*a*b^6*c^6*d + 126*a^2*b^5*c^5*
d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 5*a^6*
b*c*d^6 + a^7*d^7 + 924*(5*b^7*c*d^6 + a*b^6*d^7)*x^6 + 792*(15*b^7*c^2*d^5
 + 5*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 495*(35*b^7*c^3*d^4 + 15*a*b^6*c^2*d^
5 + 5*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 220*(70*b^7*c^4*d^3 + 35*a*b^6*c^3
*d^4 + 15*a^2*b^5*c^2*d^5 + 5*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 66*(126*b^
7*c^5*d^2 + 70*a*b^6*c^4*d^3 + 35*a^2*b^5*c^3*d^4 + 15*a^3*b^4*c^2*d^5 + 5*
a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 12*(210*b^7*c^6*d + 126*a*b^6*c^5*d^2 +
70*a^2*b^5*c^4*d^3 + 35*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 + 5*a^5*b^2*c*
d^6 + a^6*b*d^7)*x)/(b^20*x^12 + 12*a*b^19*x^11 + 66*a^2*b^18*x^10 + 220*a^
3*b^17*x^9 + 495*a^4*b^16*x^8 + 792*a^5*b^15*x^7 + 924*a^6*b^14*x^6 + 792*a
^7*b^13*x^5 + 495*a^8*b^12*x^4 + 220*a^9*b^11*x^3 + 66*a^10*b^10*x^2 + 12*a
^11*b^9*x + a^12*b^8)
```

mupad [B] time = 0.23, size = 559, normalized size = 3.70

$$-\frac{\frac{a^7d^7+5a^6bcd^6+15a^5b^2c^2d^5+35a^4b^3c^3d^4+70a^3b^4c^4d^3+126a^2b^5c^5d^2+210ab^6c^6d+330b^7c^7}{3960\,b^8}+\frac{d^7x^7}{5\,b}+\frac{d^2x^2\left(a^5d^5+5a^4bcd^4+15a^3b^2c^2d^3+35a^2b^3c^3d^2+70ab^4c^4d+126b^5c^5\right)}{60\,b^6}+\frac{d^4x^4\left(a^3d^3+5a^2bcd^2+15ab^2c^2d+35b^3c^3\right)}{8\,b^4}+\frac{7d^6x^6\left(ad+5bc\right)}{30\,b^2}+\frac{d^3x^3\left(a^4d^4+5a^3bcd^3+15a^2b^2c^2d^2+35ab^3c^3d+70b^4c^4\right)}{18\,b^5}+\frac{dx\left(a^6d^6+5a^5bcd^5+15a^4b^2c^2d^4+35a^3b^3c^3d^3+70a^2b^4c^4d^2+126ab^5c^5d+210b^6c^6\right)}{330\,b^7}+\frac{d^5x^5\left(a^2d^2+5abcd+15b^2c^2\right)}{5\,b^3}}{a^{12}+12\,a^{11}bx+66\,a^{10}b^2x^2+220\,a^9b^3x^3+495\,a^8b^4x^4+792\,a^7b^5x^5+924\,a^6b^6x^6+792\,a^5b^7x^7+495\,a^4b^8x^8+220\,a^3b^9x^9+66\,a^2b^{10}x^{10}+12\,ab^{11}x^{11}+b^{12}x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^13,x)
```

```
[Out] -((a^7*d^7 + 330*b^7*c^7 + 126*a^2*b^5*c^5*d^2 + 70*a^3*b^4*c^4*d^3 + 35*a^
4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 + 210*a*b^6*c^6*d + 5*a^6*b*c*d^6)/(3960
*b^8) + (d^7*x^7)/(5*b) + (d^2*x^2*(a^5*d^5 + 126*b^5*c^5 + 35*a^2*b^3*c^3*
d^2 + 15*a^3*b^2*c^2*d^3 + 70*a*b^4*c^4*d + 5*a^4*b*c*d^4))/(60*b^6) + (d^4
*x^4*(a^3*d^3 + 35*b^3*c^3 + 15*a*b^2*c^2*d + 5*a^2*b*c*d^2))/(8*b^4) + (7*
d^6*x^6*(a*d + 5*b*c))/(30*b^2) + (d^3*x^3*(a^4*d^4 + 70*b^4*c^4 + 15*a^2*b
```

```
^2*c^2*d^2 + 35*a*b^3*c^3*d + 5*a^3*b*c*d^3))/(18*b^5) + (d*x*(a^6*d^6 + 21
0*b^6*c^6 + 70*a^2*b^4*c^4*d^2 + 35*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 +
126*a*b^5*c^5*d + 5*a^5*b*c*d^5))/(330*b^7) + (d^5*x^5*(a^2*d^2 + 15*b^2*c^
2 + 5*a*b*c*d))/(5*b^3))/(a^12 + b^12*x^12 + 12*a*b^11*x^11 + 66*a^10*b^2*x
^2 + 220*a^9*b^3*x^3 + 495*a^8*b^4*x^4 + 792*a^7*b^5*x^5 + 924*a^6*b^6*x^6
+ 792*a^5*b^7*x^7 + 495*a^4*b^8*x^8 + 220*a^3*b^9*x^9 + 66*a^2*b^10*x^10 +
12*a^11*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**13,x)
```

```
[Out] Timed out
```

3.1190 $\int \frac{(c+dx)^7}{(a+bx)^{14}} \, dx$

Optimal. Leaf size=198

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-$$

Rubi [A] time = 0.15, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{43\}$

$$-\frac{d^6(bc-ad)}{b^8(a+bx)^7}-\frac{21d^5(bc-ad)^2}{8b^8(a+bx)^8}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{d^7}{6b^8(a+bx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -(b*c - a*d)^7/(13*b^8*(a + b*x)^13) - (7*d*(b*c - a*d)^6)/(12*b^8*(a + b*x
)^12) - (21*d^2*(b*c - a*d)^5)/(11*b^8*(a + b*x)^11) - (7*d^3*(b*c - a*d)^4
)/(2*b^8*(a + b*x)^10) - (35*d^4*(b*c - a*d)^3)/(9*b^8*(a + b*x)^9) - (21*d
^5*(b*c - a*d)^2)/(8*b^8*(a + b*x)^8) - (d^6*(b*c - a*d))/(b^8*(a + b*x)^7)
 - d^7/(6*b^8*(a + b*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{14}} \, dx = \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{14}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{13}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{12}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{11}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{10}}+\frac{21d^5}{b^7(}\right.$$

$$= -\frac{(bc-ad)^7}{13b^8(a+bx)^{13}}-\frac{7d(bc-ad)^6}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{11b^8(a+bx)^{11}}-\frac{7d^3(bc-ad)^4}{2b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{9b^8(a+bx)^9}-\frac{21d^5}{8b^8}$$

Mathematica [A] time = 0.13, size = 369, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(6c+13dx)+3a^5b^2d^5(7c^2+26cdx+26d^2x^2)+a^4b^3d^4(56c^3+273c^2dx+468cd^2x^2+286d^3x^3)+a^3b^4d^3(126c^4+728c^3dx+1638c^2d^2x^2+1716cd^3x^3+715d^4x^4)+3a^2b^5d^2(84c^5+546c^4dx+1456c^3d^2x^2+2002c^2d^3x^3+1430cd^4x^4+429d^5x^5)+ab^6d(462c^6+3276c^5dx+9828c^4d^2x^2+16016c^3d^3x^3+15015c^2d^4x^4+7722cd^5x^5+1716d^6x^6)+b^7(792c^7+6006c^6dx+19656c^5d^2x^2+36036c^4d^3x^3+40040c^3d^4x^4+27027c^2d^5x^5+10296cd^6x^6+1716d^7x^7)}{10296b^8(a+bx)^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] -1/10296*(a^7*d^7 + a^6*b*d^6*(6*c + 13*d*x) + 3*a^5*b^2*d^5*(7*c^2 + 26*c*
d*x + 26*d^2*x^2) + a^4*b^3*d^4*(56*c^3 + 273*c^2*d*x + 468*c*d^2*x^2 + 286
*d^3*x^3) + a^3*b^4*d^3*(126*c^4 + 728*c^3*d*x + 1638*c^2*d^2*x^2 + 1716*c*
d^3*x^3 + 715*d^4*x^4) + 3*a^2*b^5*d^2*(84*c^5 + 546*c^4*d*x + 1456*c^3*d^2
*x^2 + 2002*c^2*d^3*x^3 + 1430*c*d^4*x^4 + 429*d^5*x^5) + a*b^6*d*(462*c^6
+ 3276*c^5*d*x + 9828*c^4*d^2*x^2 + 16016*c^3*d^3*x^3 + 15015*c^2*d^4*x^4 +
 7722*c*d^5*x^5 + 1716*d^6*x^6) + b^7*(792*c^7 + 6006*c^6*d*x + 19656*c^5*d
^2*x^2 + 36036*c^4*d^3*x^3 + 40040*c^3*d^4*x^4 + 27027*c^2*d^5*x^5 + 10296*
c*d^6*x^6 + 1716*d^7*x^7))/(b^8*(a + b*x)^13)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{14}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^14,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^14, x]
```

fricas [B] time = 1.23, size = 592, normalized size = 2.99

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="fricas")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

giac [B] time = 1.24, size = 496, normalized size = 2.51

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 10296*b^7*c*d^6*x^6 + 1716*a*b^6*d^7*x^6 + 270
27*b^7*c^2*d^5*x^5 + 7722*a*b^6*c*d^6*x^5 + 1287*a^2*b^5*d^7*x^5 + 40040*b^
7*c^3*d^4*x^4 + 15015*a*b^6*c^2*d^5*x^4 + 4290*a^2*b^5*c*d^6*x^4 + 715*a^3*
b^4*d^7*x^4 + 36036*b^7*c^4*d^3*x^3 + 16016*a*b^6*c^3*d^4*x^3 + 6006*a^2*b^
5*c^2*d^5*x^3 + 1716*a^3*b^4*c*d^6*x^3 + 286*a^4*b^3*d^7*x^3 + 19656*b^7*c^
5*d^2*x^2 + 9828*a*b^6*c^4*d^3*x^2 + 4368*a^2*b^5*c^3*d^4*x^2 + 1638*a^3*b^
4*c^2*d^5*x^2 + 468*a^4*b^3*c*d^6*x^2 + 78*a^5*b^2*d^7*x^2 + 6006*b^7*c^6*d
*x + 3276*a*b^6*c^5*d^2*x + 1638*a^2*b^5*c^4*d^3*x + 728*a^3*b^4*c^3*d^4*x
+ 273*a^4*b^3*c^2*d^5*x + 78*a^5*b^2*c*d^6*x + 13*a^6*b*d^7*x + 792*b^7*c^7
 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3
*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^13*b^8)
```

maple [B] time = 0.01, size = 463, normalized size = 2.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^14,x)
```

```
[Out] -21/8*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^8-1/13*(-a^7*d^7+7*a^6*b*
c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c
```

```
^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^13+d^6*(a*d-b*c)/b^8/(b*x+a)^7+35
/9*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^9+21/11*d^
2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*
d-b^5*c^5)/b^8/(b*x+a)^11-7/12*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-
20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^12
-1/6*d^7/b^8/(b*x+a)^6-7/2*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a
*b^3*c^3*d+b^4*c^4)/b^8/(b*x+a)^10
```

maxima [B] time = 1.74, size = 592, normalized size = 2.99

$$-\frac{1716\,b^7d^7x^7 + 792\,b^7c^7 + 462\,ab^6c^6d + 252\,a^2b^5c^5d^2 + 126\,a^3b^4c^4d^3 + 56\,a^4b^3c^3d^4 + 21\,a^5b^2c^2d^5 + 6\,a^6bcd^6 + a^7d^7 + 1716\left(6\,b^7cd^6 + ab^6d^7\right)x^6 + 1287\left(21\,b^7c^2d^5 + 6\,ab^6cd^6 + a^2b^5d^7\right)x^5 + 715\left(56\,b^7c^3d^4 + 21\,ab^6c^2d^5 + 6\,a^2b^5cd^6 + a^3b^4d^7\right)x^4 + 286\left(126\,b^7c^4d^3 + 56\,ab^6c^3d^4 + 21\,a^2b^5c^2d^5 + 6\,a^3b^4cd^6 + a^4b^3d^7\right)x^3 + 78\left(252\,b^7c^5d^2 + 126\,ab^6c^4d^3 + 56\,a^2b^5c^3d^4 + 21\,a^3b^4c^2d^5 + 6\,a^4b^3cd^6 + a^5b^2d^7\right)x^2 + 13\left(462\,b^7c^6d + 252\,ab^6c^5d^2 + 126\,a^2b^5c^4d^3 + 56\,a^3b^4c^3d^4 + 21\,a^4b^3c^2d^5 + 6\,a^5b^2cd^6 + a^6bd^7\right)x}{10296\left(b^{21}x^{13} + 13\,ab^{20}x^{12} + 78\,a^2b^{19}x^{11} + 286\,a^3b^{18}x^{10} + 715\,a^4b^{17}x^9 + 1287\,a^5b^{16}x^8 + 1716\,a^6b^{15}x^7 + 1716\,a^7b^{14}x^6 + 1287\,a^8b^{13}x^5 + 715\,a^9b^{12}x^4 + 286\,a^{10}b^{11}x^3 + 78\,a^{11}b^{10}x^2 + 13\,a^{12}b^9x + a^{13}b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/10296*(1716*b^7*d^7*x^7 + 792*b^7*c^7 + 462*a*b^6*c^6*d + 252*a^2*b^5*c^
5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 6*a
^6*b*c*d^6 + a^7*d^7 + 1716*(6*b^7*c*d^6 + a*b^6*d^7)*x^6 + 1287*(21*b^7*c^
2*d^5 + 6*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 715*(56*b^7*c^3*d^4 + 21*a*b^6*c
^2*d^5 + 6*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 286*(126*b^7*c^4*d^3 + 56*a*b
^6*c^3*d^4 + 21*a^2*b^5*c^2*d^5 + 6*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 78*(
252*b^7*c^5*d^2 + 126*a*b^6*c^4*d^3 + 56*a^2*b^5*c^3*d^4 + 21*a^3*b^4*c^2*d
^5 + 6*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 13*(462*b^7*c^6*d + 252*a*b^6*c^5
*d^2 + 126*a^2*b^5*c^4*d^3 + 56*a^3*b^4*c^3*d^4 + 21*a^4*b^3*c^2*d^5 + 6*a^
5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^21*x^13 + 13*a*b^20*x^12 + 78*a^2*b^19*x^11
+ 286*a^3*b^18*x^10 + 715*a^4*b^17*x^9 + 1287*a^5*b^16*x^8 + 1716*a^6*b^15*
x^7 + 1716*a^7*b^14*x^6 + 1287*a^8*b^13*x^5 + 715*a^9*b^12*x^4 + 286*a^10*b
^11*x^3 + 78*a^11*b^10*x^2 + 13*a^12*b^9*x + a^13*b^8)
```

mupad [B] time = 0.40, size = 570, normalized size = 2.88

$$-\frac{\frac{a^7\,d^7+6\,a^6\,b\,c\,d^6+21\,a^5\,b^2\,c^2\,d^5+56\,a^4\,b^3\,c^3\,d^4+126\,a^3\,b^4\,c^4\,d^3+252\,a^2\,b^5\,c^5\,d^2+462\,a\,b^6\,c^6\,d+792\,b^7\,c^7}{10296\,b^8}+\frac{d^7\,x^7}{6\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+6\,a^4\,b\,c\,d^4+21\,a^3\,b^2\,c^2\,d^3+56\,a^2\,b^3\,c^3\,d^2+126\,a\,b^4\,c^4\,d+252\,b^5\,c^5\right)}{132\,b^6}+\frac{5\,d^4\,x^4\,\left(a^3\,d^3+6\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d+56\,b^3\,c^3\right)}{72\,b^4}+\frac{d^6\,x^6\,\left(a\,d+6\,b\,c\right)}{6\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+6\,a^3\,b\,c\,d^3+21\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d+126\,b^4\,c^4\right)}{36\,b^5}+\frac{d\,x\,\left(a^6\,d^6+6\,a^5\,b\,c\,d^5+21\,a^4\,b^2\,c^2\,d^4+56\,a^3\,b^3\,c^3\,d^3+126\,a^2\,b^4\,c^4\,d^2+252\,a\,b^5\,c^5\,d+462\,b^6\,c^6\right)}{792\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+6\,a\,b\,c\,d+21\,b^2\,c^2\right)}{8\,b^3}}{a^{13}+13\,a^{12}\,b\,x+78\,a^{11}\,b^2\,x^2+286\,a^{10}\,b^3\,x^3+715\,a^9\,b^4\,x^4+1287\,a^8\,b^5\,x^5+1716\,a^7\,b^6\,x^6+1716\,a^6\,b^7\,x^7+1287\,a^5\,b^8\,x^8+715\,a^4\,b^9\,x^9+286\,a^3\,b^{10}\,x^{10}+78\,a^2\,b^{11}\,x^{11}+13\,a\,b^{12}\,x^{12}+b^{13}\,x^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^14,x)
```

```
[Out] -((a^7*d^7 + 792*b^7*c^7 + 252*a^2*b^5*c^5*d^2 + 126*a^3*b^4*c^4*d^3 + 56*a
^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 462*a*b^6*c^6*d + 6*a^6*b*c*d^6)/(102
96*b^8) + (d^7*x^7)/(6*b) + (d^2*x^2*(a^5*d^5 + 252*b^5*c^5 + 56*a^2*b^3*c^
3*d^2 + 21*a^3*b^2*c^2*d^3 + 126*a*b^4*c^4*d + 6*a^4*b*c*d^4))/(132*b^6) +
(5*d^4*x^4*(a^3*d^3 + 56*b^3*c^3 + 21*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(72*b^4
) + (d^6*x^6*(a*d + 6*b*c))/(6*b^2) + (d^3*x^3*(a^4*d^4 + 126*b^4*c^4 + 21*
a^2*b^2*c^2*d^2 + 56*a*b^3*c^3*d + 6*a^3*b*c*d^3))/(36*b^5) + (d*x*(a^6*d^6
 + 462*b^6*c^6 + 126*a^2*b^4*c^4*d^2 + 56*a^3*b^3*c^3*d^3 + 21*a^4*b^2*c^2*
d^4 + 252*a*b^5*c^5*d + 6*a^5*b*c*d^5))/(792*b^7) + (d^5*x^5*(a^2*d^2 + 21*
b^2*c^2 + 6*a*b*c*d))/(8*b^3))/(a^13 + b^13*x^13 + 13*a*b^12*x^12 + 78*a^11
*b^2*x^2 + 286*a^10*b^3*x^3 + 715*a^9*b^4*x^4 + 1287*a^8*b^5*x^5 + 1716*a^7
*b^6*x^6 + 1716*a^6*b^7*x^7 + 1287*a^5*b^8*x^8 + 715*a^4*b^9*x^9 + 286*a^3*
b^10*x^10 + 78*a^2*b^11*x^11 + 13*a^12*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**14,x)
```

```
[Out] Timed out
```

3.1191 $\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx$

Optimal. Leaf size=200

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{d^7}{7b^8(a+bx)^7}$$

Rubi [A] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{8b^8(a+bx)^8}-\frac{7d^5(bc-ad)^2}{3b^8(a+bx)^9}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{d^7}{7b^8(a+bx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -(b*c - a*d)^7/(14*b^8*(a + b*x)^14) - (7*d*(b*c - a*d)^6)/(13*b^8*(a + b*x
)^13) - (7*d^2*(b*c - a*d)^5)/(4*b^8*(a + b*x)^12) - (35*d^3*(b*c - a*d)^4)
/(11*b^8*(a + b*x)^11) - (7*d^4*(b*c - a*d)^3)/(2*b^8*(a + b*x)^10) - (7*d^
5*(b*c - a*d)^2)/(3*b^8*(a + b*x)^9) - (7*d^6*(b*c - a*d))/(8*b^8*(a + b*x)
^8) - d^7/(7*b^8*(a + b*x)^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx = \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{15}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{14}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{13}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{12}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{11}}+\ldots$$

$$= -\frac{(bc-ad)^7}{14b^8(a+bx)^{14}}-\frac{7d(bc-ad)^6}{13b^8(a+bx)^{13}}-\frac{7d^2(bc-ad)^5}{4b^8(a+bx)^{12}}-\frac{35d^3(bc-ad)^4}{11b^8(a+bx)^{11}}-\frac{7d^4(bc-ad)^3}{2b^8(a+bx)^{10}}-\ldots$$

Mathematica [A] time = 0.13, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+7a^6bd^6(c+2dx)+7a^5b^2d^5(4c^2+14cdx+13d^2x^2)+7a^4b^3d^4(12c^3+56c^2dx+91cd^2x^2+52d^3x^3)+7a^3b^4d^3(30c^4+168c^3dx+364c^2d^2x^2+364cd^3x^3+143d^4x^4)+7a^2b^5d^2(66c^5+420c^4dx+1092c^3d^2x^2+1456c^2d^3x^3+1001cd^4x^4+286d^5x^5)+7abd^6(132c^6+924c^5dx+2730c^4d^2x^2+4368c^3d^3x^3+4004c^2d^4x^4+2002cd^5x^5+429d^6x^6)+b^7(1716c^7+12936c^6dx+42042c^5d^2x^2+76440c^4d^3x^3+84084c^3d^4x^4+56056c^2d^5x^5+21021cd^6x^6+3432d^7x^7)}{24024b^8(a+bx)^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] -1/24024*(a^7*d^7 + 7*a^6*b*d^6*(c + 2*d*x) + 7*a^5*b^2*d^5*(4*c^2 + 14*c*d
*x + 13*d^2*x^2) + 7*a^4*b^3*d^4*(12*c^3 + 56*c^2*d*x + 91*c*d^2*x^2 + 52*d
^3*x^3) + 7*a^3*b^4*d^3*(30*c^4 + 168*c^3*d*x + 364*c^2*d^2*x^2 + 364*c*d^3
*x^3 + 143*d^4*x^4) + 7*a^2*b^5*d^2*(66*c^5 + 420*c^4*d*x + 1092*c^3*d^2*x^
2 + 1456*c^2*d^3*x^3 + 1001*c*d^4*x^4 + 286*d^5*x^5) + 7*a*b^6*d*(132*c^6 +
 924*c^5*d*x + 2730*c^4*d^2*x^2 + 4368*c^3*d^3*x^3 + 4004*c^2*d^4*x^4 + 200
2*c*d^5*x^5 + 429*d^6*x^6) + b^7*(1716*c^7 + 12936*c^6*d*x + 42042*c^5*d^2*
x^2 + 76440*c^4*d^3*x^3 + 84084*c^3*d^4*x^4 + 56056*c^2*d^5*x^5 + 21021*c*d
^6*x^6 + 3432*d^7*x^7))/(b^8*(a + b*x)^14)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^15,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^15, x]
```

fricas [B] time = 1.35, size = 603, normalized size = 3.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

giac [B] time = 1.28, size = 496, normalized size = 2.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 21021*b^7*c*d^6*x^6 + 3003*a*b^6*d^7*x^6 + 560
56*b^7*c^2*d^5*x^5 + 14014*a*b^6*c*d^6*x^5 + 2002*a^2*b^5*d^7*x^5 + 84084*b
^7*c^3*d^4*x^4 + 28028*a*b^6*c^2*d^5*x^4 + 7007*a^2*b^5*c*d^6*x^4 + 1001*a^
3*b^4*d^7*x^4 + 76440*b^7*c^4*d^3*x^3 + 30576*a*b^6*c^3*d^4*x^3 + 10192*a^2
*b^5*c^2*d^5*x^3 + 2548*a^3*b^4*c*d^6*x^3 + 364*a^4*b^3*d^7*x^3 + 42042*b^7
*c^5*d^2*x^2 + 19110*a*b^6*c^4*d^3*x^2 + 7644*a^2*b^5*c^3*d^4*x^2 + 2548*a^
3*b^4*c^2*d^5*x^2 + 637*a^4*b^3*c*d^6*x^2 + 91*a^5*b^2*d^7*x^2 + 12936*b^7*
c^6*d*x + 6468*a*b^6*c^5*d^2*x + 2940*a^2*b^5*c^4*d^3*x + 1176*a^3*b^4*c^3*
d^4*x + 392*a^4*b^3*c^2*d^5*x + 98*a^5*b^2*c*d^6*x + 14*a^6*b*d^7*x + 1716*
b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 + a^7*d^7)/((b*x + a)^
14*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 2.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^15,x)
```

```
[Out] 7/8*d^6*(a*d-b*c)/b^8/(b*x+a)^8-7/13*d*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^
2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x
+a)^13-1/7*d^7/b^8/(b*x+a)^7-7/3*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a
)^9-35/11*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^
4)/b^8/(b*x+a)^11+7/4*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*
b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^12+7/2*d^4*(a^3*d^3-3*a^2*b*
c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(b*x+a)^10-1/14*(-a^7*d^7+7*a^6*b*c*d^6-21
*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7
*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^14
```

maxima [B] time = 1.83, size = 603, normalized size = 3.02

$$-\frac{3432\,b^{7} d^{7} x^{7} + 1716\,b^{7} c^{7} + 924\,a b^{6} c^{6} d + 462\,a^{2} b^{5} c^{5} d^{2} + 210\,a^{3} b^{4} c^{4} d^{3} + 84\,a^{4} b^{3} c^{3} d^{4} + 28\,a^{5} b^{2} c^{2} d^{5} + 7\,a^{6} b c d^{6} + a^{7} d^{7} + 3003\,{\left(7\,b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 2002\,{\left(28\,b^{7} c^{2} d^{5} + 7\,a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 1001\,{\left(84\,b^{7} c^{3} d^{4} + 28\,a b^{6} c^{2} d^{5} + 7\,a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 364\,{\left(210\,b^{7} c^{4} d^{3} + 84\,a b^{6} c^{3} d^{4} + 28\,a^{2} b^{5} c^{2} d^{5} + 7\,a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 91\,{\left(462\,b^{7} c^{5} d^{2} + 210\,a b^{6} c^{4} d^{3} + 84\,a^{2} b^{5} c^{3} d^{4} + 28\,a^{3} b^{4} c^{2} d^{5} + 7\,a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 14\,{\left(924\,b^{7} c^{6} d + 462\,a b^{6} c^{5} d^{2} + 210\,a^{2} b^{5} c^{4} d^{3} + 84\,a^{3} b^{4} c^{3} d^{4} + 28\,a^{4} b^{3} c^{2} d^{5} + 7\,a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{24024\,{\left(b^{22} x^{14} + 14\,a b^{21} x^{13} + 91\,a^{2} b^{20} x^{12} + 364\,a^{3} b^{19} x^{11} + 1001\,a^{4} b^{18} x^{10} + 2002\,a^{5} b^{17} x^{9} + 3003\,a^{6} b^{16} x^{8} + 3432\,a^{7} b^{15} x^{7} + 3003\,a^{8} b^{14} x^{6} + 2002\,a^{9} b^{13} x^{5} + 1001\,a^{10} b^{12} x^{4} + 364\,a^{11} b^{11} x^{3} + 91\,a^{12} b^{10} x^{2} + 14\,a^{13} b^{9} x + a^{14} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/24024*(3432*b^7*d^7*x^7 + 1716*b^7*c^7 + 924*a*b^6*c^6*d + 462*a^2*b^5*c
^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 7*
a^6*b*c*d^6 + a^7*d^7 + 3003*(7*b^7*c*d^6 + a*b^6*d^7)*x^6 + 2002*(28*b^7*c
^2*d^5 + 7*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1001*(84*b^7*c^3*d^4 + 28*a*b^6
*c^2*d^5 + 7*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 364*(210*b^7*c^4*d^3 + 84*a
*b^6*c^3*d^4 + 28*a^2*b^5*c^2*d^5 + 7*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3 + 91
*(462*b^7*c^5*d^2 + 210*a*b^6*c^4*d^3 + 84*a^2*b^5*c^3*d^4 + 28*a^3*b^4*c^2
*d^5 + 7*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 14*(924*b^7*c^6*d + 462*a*b^6*c
^5*d^2 + 210*a^2*b^5*c^4*d^3 + 84*a^3*b^4*c^3*d^4 + 28*a^4*b^3*c^2*d^5 + 7*
a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^22*x^14 + 14*a*b^21*x^13 + 91*a^2*b^20*x^1
2 + 364*a^3*b^19*x^11 + 1001*a^4*b^18*x^10 + 2002*a^5*b^17*x^9 + 3003*a^6*b
^16*x^8 + 3432*a^7*b^15*x^7 + 3003*a^8*b^14*x^6 + 2002*a^9*b^13*x^5 + 1001*
a^10*b^12*x^4 + 364*a^11*b^11*x^3 + 91*a^12*b^10*x^2 + 14*a^13*b^9*x + a^14
*b^8)
```

mupad [B] time = 1.24, size = 581, normalized size = 2.90

$$-\frac{\frac{a^7\,d^7+7\,a^6\,b\,c\,d^6+28\,a^5\,b^2\,c^2\,d^5+84\,a^4\,b^3\,c^3\,d^4+210\,a^3\,b^4\,c^4\,d^3+462\,a^2\,b^5\,c^5\,d^2+924\,a\,b^6\,c^6\,d+1716\,b^7\,c^7}{24024\,b^8}+\frac{d^7\,x^7}{7\,b}+\frac{d^2\,x^2\,\left(a^5\,d^5+7\,a^4\,b\,c\,d^4+28\,a^3\,b^2\,c^2\,d^3+84\,a^2\,b^3\,c^3\,d^2+210\,a\,b^4\,c^4\,d+462\,b^5\,c^5\right)}{264\,b^6}+\frac{d^4\,x^4\,\left(a^3\,d^3+7\,a^2\,b\,c\,d^2+28\,a\,b^2\,c^2\,d+84\,b^3\,c^3\right)}{24\,b^4}+\frac{d^6\,x^6\,\left(a\,d+7\,b\,c\right)}{8\,b^2}+\frac{d^3\,x^3\,\left(a^4\,d^4+7\,a^3\,b\,c\,d^3+28\,a^2\,b^2\,c^2\,d^2+84\,a\,b^3\,c^3\,d+210\,b^4\,c^4\right)}{66\,b^5}+\frac{d\,x\,\left(a^6\,d^6+7\,a^5\,b\,c\,d^5+28\,a^4\,b^2\,c^2\,d^4+84\,a^3\,b^3\,c^3\,d^3+210\,a^2\,b^4\,c^4\,d^2+462\,a\,b^5\,c^5\,d+924\,b^6\,c^6\right)}{1716\,b^7}+\frac{d^5\,x^5\,\left(a^2\,d^2+7\,a\,b\,c\,d+28\,b^2\,c^2\right)}{12\,b^3}}{a^{14}+14\,a^{13}\,b\,x+91\,a^{12}\,b^2\,x^2+364\,a^{11}\,b^3\,x^3+1001\,a^{10}\,b^4\,x^4+2002\,a^9\,b^5\,x^5+3003\,a^8\,b^6\,x^6+3432\,a^7\,b^7\,x^7+3003\,a^6\,b^8\,x^8+2002\,a^5\,b^9\,x^9+1001\,a^4\,b^{10}\,x^{10}+364\,a^3\,b^{11}\,x^{11}+91\,a^2\,b^{12}\,x^{12}+14\,a\,b^{13}\,x^{13}+b^{14}\,x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^15,x)
```

```
[Out] -((a^7*d^7 + 1716*b^7*c^7 + 462*a^2*b^5*c^5*d^2 + 210*a^3*b^4*c^4*d^3 + 84*
a^4*b^3*c^3*d^4 + 28*a^5*b^2*c^2*d^5 + 924*a*b^6*c^6*d + 7*a^6*b*c*d^6)/(24
024*b^8) + (d^7*x^7)/(7*b) + (d^2*x^2*(a^5*d^5 + 462*b^5*c^5 + 84*a^2*b^3*c
^3*d^2 + 28*a^3*b^2*c^2*d^3 + 210*a*b^4*c^4*d + 7*a^4*b*c*d^4))/(264*b^6) +
 (d^4*x^4*(a^3*d^3 + 84*b^3*c^3 + 28*a*b^2*c^2*d + 7*a^2*b*c*d^2))/(24*b^4)
 + (d^6*x^6*(a*d + 7*b*c))/(8*b^2) + (d^3*x^3*(a^4*d^4 + 210*b^4*c^4 + 28*a
^2*b^2*c^2*d^2 + 84*a*b^3*c^3*d + 7*a^3*b*c*d^3))/(66*b^5) + (d*x*(a^6*d^6
+ 924*b^6*c^6 + 210*a^2*b^4*c^4*d^2 + 84*a^3*b^3*c^3*d^3 + 28*a^4*b^2*c^2*d
^4 + 462*a*b^5*c^5*d + 7*a^5*b*c*d^5))/(1716*b^7) + (d^5*x^5*(a^2*d^2 + 28*
b^2*c^2 + 7*a*b*c*d))/(12*b^3))/(a^14 + b^14*x^14 + 14*a*b^13*x^13 + 91*a^1
2*b^2*x^2 + 364*a^11*b^3*x^3 + 1001*a^10*b^4*x^4 + 2002*a^9*b^5*x^5 + 3003*
a^8*b^6*x^6 + 3432*a^7*b^7*x^7 + 3003*a^6*b^8*x^8 + 2002*a^5*b^9*x^9 + 1001
*a^4*b^10*x^10 + 364*a^3*b^11*x^11 + 91*a^2*b^12*x^12 + 14*a^13*b*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**15,x)
```

```
[Out] Timed out
```

3.1192 $\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=200

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}$$

Rubi [A] time = 0.14, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{7d^6(bc-ad)}{9b^8(a+bx)^9}-\frac{21d^5(bc-ad)^2}{10b^8(a+bx)^{10}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d^7}{8b^8(a+bx)^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -(b*c - a*d)^7/(15*b^8*(a + b*x)^15) - (d*(b*c - a*d)^6)/(2*b^8*(a + b*x)^1
4) - (21*d^2*(b*c - a*d)^5)/(13*b^8*(a + b*x)^13) - (35*d^3*(b*c - a*d)^4)/
(12*b^8*(a + b*x)^12) - (35*d^4*(b*c - a*d)^3)/(11*b^8*(a + b*x)^11) - (21*
d^5*(b*c - a*d)^2)/(10*b^8*(a + b*x)^10) - (7*d^6*(b*c - a*d))/(9*b^8*(a +
b*x)^9) - d^7/(8*b^8*(a + b*x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx &= \int \left(\frac{(bc-ad)^7}{b^7(a+bx)^{16}}+\frac{7d(bc-ad)^6}{b^7(a+bx)^{15}}+\frac{21d^2(bc-ad)^5}{b^7(a+bx)^{14}}+\frac{35d^3(bc-ad)^4}{b^7(a+bx)^{13}}+\frac{35d^4(bc-ad)^3}{b^7(a+bx)^{12}}+\frac{21d^5}{b^7(}\right.\\ &= -\frac{(bc-ad)^7}{15b^8(a+bx)^{15}}-\frac{d(bc-ad)^6}{2b^8(a+bx)^{14}}-\frac{21d^2(bc-ad)^5}{13b^8(a+bx)^{13}}-\frac{35d^3(bc-ad)^4}{12b^8(a+bx)^{12}}-\frac{35d^4(bc-ad)^3}{11b^8(a+bx)^{11}}-\frac{21d^5}{10b^8}\end{aligned}$$

Mathematica [A] time = 0.13, size = 371, normalized size = 1.86

$$-\frac{a^7d^7+a^6bd^6(8c+15dx)+3a^5b^2d^5\left(12c^2+40cdx+35d^2x^2\right)+5a^4b^3d^4\left(24c^3+108c^2dx+168cd^2x^2+91d^3x^3\right)+5a^3b^4d^3\left(66c^4+360c^3dx+756c^2d^2x^2+728cd^3x^3+273d^4x^4\right)+3a^2b^5d^2\left(264c^5+1650c^4dx+4200c^3d^2x^2+5460c^2d^3x^3+3640cd^4x^4+1001d^5x^5\right)+ab^6d\left(1716c^6+11880c^5dx+34650c^4d^2x^2+54600c^3d^3x^3+49140c^2d^4x^4+24024cd^5x^5+5005d^6x^6\right)+b^7\left(3432c^7+25740c^6dx+83160c^5d^2x^2+150150c^4d^3x^3+163800c^3d^4x^4+108108c^2d^5x^5+40040cd^6x^6+6435d^7x^7\right)}{51480b^8(a+bx)^{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] -1/51480*(a^7*d^7 + a^6*b*d^6*(8*c + 15*d*x) + 3*a^5*b^2*d^5*(12*c^2 + 40*c
*d*x + 35*d^2*x^2) + 5*a^4*b^3*d^4*(24*c^3 + 108*c^2*d*x + 168*c*d^2*x^2 +
91*d^3*x^3) + 5*a^3*b^4*d^3*(66*c^4 + 360*c^3*d*x + 756*c^2*d^2*x^2 + 728*c
*d^3*x^3 + 273*d^4*x^4) + 3*a^2*b^5*d^2*(264*c^5 + 1650*c^4*d*x + 4200*c^3*
d^2*x^2 + 5460*c^2*d^3*x^3 + 3640*c*d^4*x^4 + 1001*d^5*x^5) + a*b^6*d*(1716
*c^6 + 11880*c^5*d*x + 34650*c^4*d^2*x^2 + 54600*c^3*d^3*x^3 + 49140*c^2*d^
4*x^4 + 24024*c*d^5*x^5 + 5005*d^6*x^6) + b^7*(3432*c^7 + 25740*c^6*d*x + 8
3160*c^5*d^2*x^2 + 150150*c^4*d^3*x^3 + 163800*c^3*d^4*x^4 + 108108*c^2*d^5
*x^5 + 40040*c*d^6*x^6 + 6435*d^7*x^7))/(b^8*(a + b*x)^15)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^7}{(a+bx)^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^16,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^7/(a + b*x)^16, x]
```

fricas [B] time = 1.10, size = 614, normalized size = 3.07

$$-\frac{6435 \, b^{7} d^{7} x^{7} + 3432 \, b^{7} c^{7} + 1716 \, a b^{6} c^{6} d + 792 \, a^{2} b^{5} c^{5} d^{2} + 330 \, a^{3} b^{4} c^{4} d^{3} + 120 \, a^{4} b^{3} c^{3} d^{4} + 36 \, a^{5} b^{2} c^{2} d^{5} + 8 \, a^{6} b c d^{6} + a^{7} d^{7} + 5005 \, {\left(8 \, b^{7} c d^{6} + a b^{6} d^{7}\right)} x^{6} + 3003 \, {\left(36 \, b^{7} c^{2} d^{5} + 8 \, a b^{6} c d^{6} + a^{2} b^{5} d^{7}\right)} x^{5} + 1365 \, {\left(120 \, b^{7} c^{3} d^{4} + 36 \, a b^{6} c^{2} d^{5} + 8 \, a^{2} b^{5} c d^{6} + a^{3} b^{4} d^{7}\right)} x^{4} + 455 \, {\left(330 \, b^{7} c^{4} d^{3} + 120 \, a b^{6} c^{3} d^{4} + 36 \, a^{2} b^{5} c^{2} d^{5} + 8 \, a^{3} b^{4} c d^{6} + a^{4} b^{3} d^{7}\right)} x^{3} + 105 \, {\left(792 \, b^{7} c^{5} d^{2} + 330 \, a b^{6} c^{4} d^{3} + 120 \, a^{2} b^{5} c^{3} d^{4} + 36 \, a^{3} b^{4} c^{2} d^{5} + 8 \, a^{4} b^{3} c d^{6} + a^{5} b^{2} d^{7}\right)} x^{2} + 15 \, {\left(1716 \, b^{7} c^{6} d + 792 \, a b^{6} c^{5} d^{2} + 330 \, a^{2} b^{5} c^{4} d^{3} + 120 \, a^{3} b^{4} c^{3} d^{4} + 36 \, a^{4} b^{3} c^{2} d^{5} + 8 \, a^{5} b^{2} c d^{6} + a^{6} b d^{7}\right)} x}{51480 \, {\left(b^{23} x^{15} + 15 \, a b^{22} x^{14} + 105 \, a^{2} b^{21} x^{13} + 455 \, a^{3} b^{20} x^{12} + 1365 \, a^{4} b^{19} x^{11} + 3003 \, a^{5} b^{18} x^{10} + 5005 \, a^{6} b^{17} x^{9} + 6435 \, a^{7} b^{16} x^{8} + 6435 \, a^{8} b^{15} x^{7} + 5005 \, a^{9} b^{14} x^{6} + 3003 \, a^{10} b^{13} x^{5} + 1365 \, a^{11} b^{12} x^{4} + 455 \, a^{12} b^{11} x^{3} + 105 \, a^{13} b^{10} x^{2} + 15 \, a^{14} b^{9} x + a^{15} b^{8}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

giac [B] time = 1.29, size = 496, normalized size = 2.48

$$-\frac{6435 \, b^{7} d^{7} x^{7} + 40040 \, b^{7} c d^{6} x^{6} + 5005 \, a b^{6} d^{7} x^{6} + 108108 \, b^{7} c^{2} d^{5} x^{5} + 24024 \, a b^{6} c d^{6} x^{5} + 3003 \, a^{2} b^{5} d^{7} x^{5} + 163800 \, b^{7} c^{3} d^{4} x^{4} + 49140 \, a b^{6} c^{2} d^{5} x^{4} + 10920 \, a^{2} b^{5} c d^{6} x^{4} + 1365 \, a^{3} b^{4} d^{7} x^{4} + 150150 \, b^{7} c^{4} d^{3} x^{3} + 54600 \, a b^{6} c^{3} d^{4} x^{3} + 16380 \, a^{2} b^{5} c^{2} d^{5} x^{3} + 3640 \, a^{3} b^{4} c d^{6} x^{3} + 455 \, a^{4} b^{3} d^{7} x^{3} + 83160 \, b^{7} c^{5} d^{2} x^{2} + 34650 \, a b^{6} c^{4} d^{3} x^{2} + 12600 \, a^{2} b^{5} c^{3} d^{4} x^{2} + 3780 \, a^{3} b^{4} c^{2} d^{5} x^{2} + 840 \, a^{4} b^{3} c d^{6} x^{2} + 105 \, a^{5} b^{2} d^{7} x^{2} + 25740 \, b^{7} c^{6} d x + 11880 \, a b^{6} c^{5} d^{2} x + 4950 \, a^{2} b^{5} c^{4} d^{3} x + 1800 \, a^{3} b^{4} c^{3} d^{4} x + 540 \, a^{4} b^{3} c^{2} d^{5} x + 120 \, a^{5} b^{2} c d^{6} x + 15 \, a^{6} b d^{7} x + 3432 \, b^{7} c^{7} + 1716 \, a b^{6} c^{6} d + 792 \, a^{2} b^{5} c^{5} d^{2} + 330 \, a^{3} b^{4} c^{4} d^{3} + 120 \, a^{4} b^{3} c^{3} d^{4} + 36 \, a^{5} b^{2} c^{2} d^{5} + 8 \, a^{6} b c d^{6} + a^{7} d^{7}}{51480 \, {\left(b x + a\right)}^{15} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 40040*b^7*c*d^6*x^6 + 5005*a*b^6*d^7*x^6 + 108
108*b^7*c^2*d^5*x^5 + 24024*a*b^6*c*d^6*x^5 + 3003*a^2*b^5*d^7*x^5 + 163800
*b^7*c^3*d^4*x^4 + 49140*a*b^6*c^2*d^5*x^4 + 10920*a^2*b^5*c*d^6*x^4 + 1365
*a^3*b^4*d^7*x^4 + 150150*b^7*c^4*d^3*x^3 + 54600*a*b^6*c^3*d^4*x^3 + 16380
*a^2*b^5*c^2*d^5*x^3 + 3640*a^3*b^4*c*d^6*x^3 + 455*a^4*b^3*d^7*x^3 + 83160
*b^7*c^5*d^2*x^2 + 34650*a*b^6*c^4*d^3*x^2 + 12600*a^2*b^5*c^3*d^4*x^2 + 37
80*a^3*b^4*c^2*d^5*x^2 + 840*a^4*b^3*c*d^6*x^2 + 105*a^5*b^2*d^7*x^2 + 2574
0*b^7*c^6*d*x + 11880*a*b^6*c^5*d^2*x + 4950*a^2*b^5*c^4*d^3*x + 1800*a^3*b
^4*c^3*d^4*x + 540*a^4*b^3*c^2*d^5*x + 120*a^5*b^2*c*d^6*x + 15*a^6*b*d^7*x
 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*
d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 8*a^6*b*c*d^6 + a^7*d^7)/(
(b*x + a)^15*b^8)
```

maple [B] time = 0.01, size = 464, normalized size = 2.32

$$-\frac{d^{7}}{8 \left(b x +a \right)^{8} b^{8}}+\frac{7 \left(a d -b c \right) d^{6}}{9 \left(b x +a \right)^{9} b^{8}}-\frac{21 \left(a^{2} d^{2}-2 a b c d +b^{2} c^{2}\right) d^{5}}{10 \left(b x +a \right)^{10} b^{8}}+\frac{35 \left(a^{3} d^{3}-3 a^{2} b c \,d^{2}+3 a \,b^{2} c^{2} d -b^{3} c^{3}\right) d^{4}}{11 \left(b x +a \right)^{11} b^{8}}-\frac{35 \left(a^{4} d^{4}-4 a^{3} b c \,d^{3}+6 a^{2} b^{2} c^{2} d^{2}-4 a \,b^{3} c^{3} d +b^{4} c^{4}\right) d^{3}}{12 \left(b x +a \right)^{12} b^{8}}+\frac{21 \left(a^{5} d^{5}-5 a^{4} b c \,d^{4}+10 a^{3} b^{2} c^{2} d^{3}-10 a^{2} b^{3} c^{3} d^{2}+5 a \,b^{4} c^{4} d -b^{5} c^{5}\right) d^{2}}{13 \left(b x +a \right)^{13} b^{8}}-\frac{\left(a^{6} d^{6}-6 a^{5} b c \,d^{5}+15 a^{4} b^{2} c^{2} d^{4}-20 a^{3} b^{3} c^{3} d^{3}+15 a^{2} b^{4} c^{4} d^{2}-6 a \,b^{5} c^{5} d +b^{6} c^{6}\right) d}{2 \left(b x +a \right)^{14} b^{8}}-\frac{-a^{7} d^{7}+7 a^{6} b c \,d^{6}-21 a^{5} b^{2} c^{2} d^{5}+35 a^{4} c^{3} d^{4} b^{3}-35 a^{3} b^{4} c^{4} d^{3}+21 a^{2} c^{5} d^{2} b^{5}-7 a \,b^{6} c^{6} d +b^{7} c^{7}}{15 \left(b x +a \right)^{15} b^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^7/(b*x+a)^16,x)
```

```
[Out] -1/8*d^7/b^8/(b*x+a)^8+21/13*d^2*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-
10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^8/(b*x+a)^13+7/9*d^6*(a*d-b*c)/
b^8/(b*x+a)^9+35/11*d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^8/(
b*x+a)^11-1/15*(-a^7*d^7+7*a^6*b*c*d^6-21*a^5*b^2*c^2*d^5+35*a^4*b^3*c^3*d^
4-35*a^3*b^4*c^4*d^3+21*a^2*b^5*c^5*d^2-7*a*b^6*c^6*d+b^7*c^7)/b^8/(b*x+a)^
15-35/12*d^3*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/b^8/(b*x+a)^12-21/10*d^5*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^8/(b*x+a)^10-1/2*d
*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^
4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^8/(b*x+a)^14
```

maxima [**B**] time = 1.91, size = 614, normalized size = 3.07

$$-\frac{6435 b^7 d^7 x^7 + 3432 b^7 c^7 + 1716 a b^6 c^6 d + 792 a^2 b^5 c^5 d^2 + 330 a^3 b^4 c^4 d^3 + 120 a^4 b^3 c^3 d^4 + 36 a^5 b^2 c^2 d^5 + 8 a^6 b c d^6 + a^7 d^7 + 5005 \left(8 b^7 c d^6 + a b^6 d^7\right) x^6 + 3003 \left(36 b^7 c^2 d^5 + 8 a b^6 c d^6 + a^2 b^5 d^7\right) x^5 + 1365 \left(120 b^7 c^3 d^4 + 36 a b^6 c^2 d^5 + 8 a^2 b^5 c d^6 + a^3 b^4 d^7\right) x^4 + 455 \left(330 b^7 c^4 d^3 + 120 a b^6 c^3 d^4 + 36 a^2 b^5 c^2 d^5 + 8 a^3 b^4 c d^6 + a^4 b^3 d^7\right) x^3 + 105 \left(792 b^7 c^5 d^2 + 330 a b^6 c^4 d^3 + 120 a^2 b^5 c^3 d^4 + 36 a^3 b^4 c^2 d^5 + 8 a^4 b^3 c d^6 + a^5 b^2 d^7\right) x^2 + 15 \left(1716 b^7 c^6 d + 792 a b^6 c^5 d^2 + 330 a^2 b^5 c^4 d^3 + 120 a^3 b^4 c^3 d^4 + 36 a^4 b^3 c^2 d^5 + 8 a^5 b^2 c d^6 + a^6 b d^7\right) x}{51480 \left(b^{23} x^{15} + 15 a b^{22} x^{14} + 105 a^2 b^{21} x^{13} + 455 a^3 b^{20} x^{12} + 1365 a^4 b^{19} x^{11} + 3003 a^5 b^{18} x^{10} + 5005 a^6 b^{17} x^9 + 6435 a^7 b^{16} x^8 + 6435 a^8 b^{15} x^7 + 5005 a^9 b^{14} x^6 + 3003 a^{10} b^{13} x^5 + 1365 a^{11} b^{12} x^4 + 455 a^{12} b^{11} x^3 + 105 a^{13} b^{10} x^2 + 15 a^{14} b^9 x + a^{15} b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^7/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/51480*(6435*b^7*d^7*x^7 + 3432*b^7*c^7 + 1716*a*b^6*c^6*d + 792*a^2*b^5*
c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 +
8*a^6*b*c*d^6 + a^7*d^7 + 5005*(8*b^7*c*d^6 + a*b^6*d^7)*x^6 + 3003*(36*b^7
*c^2*d^5 + 8*a*b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 1365*(120*b^7*c^3*d^4 + 36*a*
b^6*c^2*d^5 + 8*a^2*b^5*c*d^6 + a^3*b^4*d^7)*x^4 + 455*(330*b^7*c^4*d^3 + 1
20*a*b^6*c^3*d^4 + 36*a^2*b^5*c^2*d^5 + 8*a^3*b^4*c*d^6 + a^4*b^3*d^7)*x^3
+ 105*(792*b^7*c^5*d^2 + 330*a*b^6*c^4*d^3 + 120*a^2*b^5*c^3*d^4 + 36*a^3*b
^4*c^2*d^5 + 8*a^4*b^3*c*d^6 + a^5*b^2*d^7)*x^2 + 15*(1716*b^7*c^6*d + 792*
a*b^6*c^5*d^2 + 330*a^2*b^5*c^4*d^3 + 120*a^3*b^4*c^3*d^4 + 36*a^4*b^3*c^2*
d^5 + 8*a^5*b^2*c*d^6 + a^6*b*d^7)*x)/(b^23*x^15 + 15*a*b^22*x^14 + 105*a^2
*b^21*x^13 + 455*a^3*b^20*x^12 + 1365*a^4*b^19*x^11 + 3003*a^5*b^18*x^10 +
5005*a^6*b^17*x^9 + 6435*a^7*b^16*x^8 + 6435*a^8*b^15*x^7 + 5005*a^9*b^14*x
^6 + 3003*a^10*b^13*x^5 + 1365*a^11*b^12*x^4 + 455*a^12*b^11*x^3 + 105*a^13
*b^10*x^2 + 15*a^14*b^9*x + a^15*b^8)
```

mupad [**B**] time = 2.20, size = 592, normalized size = 2.96

$$-\frac{\frac{a^7 d^7+8 a^6 b c d^6+36 a^5 b^2 c^2 d^5+120 a^4 b^3 c^3 d^4+330 a^3 b^4 c^4 d^3+792 a^2 b^5 c^5 d^2+1716 a b^6 c^6 d+3432 b^7 c^7}{51480 b^8}+\frac{d^7 x^7}{8 b}+\frac{7 d^2 x^2 \left(a^5 d^5+8 a^4 b c d^4+36 a^3 b^2 c^2 d^3+120 a^2 b^3 c^3 d^2+330 a b^4 c^4 d+792 b^5 c^5\right)}{3432 b^6}+\frac{7 d^4 x^4 \left(a^3 d^3+8 a^2 b c d^2+36 a b^2 c^2 d+120 b^3 c^3\right)}{264 b^4}+\frac{7 d^6 x^6 (a d+8 b c)}{72 b^2}+\frac{7 d^3 x^3 \left(a^4 d^4+8 a^3 b c d^3+36 a^2 b^2 c^2 d^2+120 a b^3 c^3 d+330 b^4 c^4\right)}{792 b^5}+\frac{d x \left(a^6 d^6+8 a^5 b c d^5+36 a^4 b^2 c^2 d^4+120 a^3 b^3 c^3 d^3+330 a^2 b^4 c^4 d^2+792 a b^5 c^5 d+1716 b^6 c^6\right)}{3432 b^7}+\frac{7 d^5 x^5 \left(a^2 d^2+8 a b c d+36 b^2 c^2\right)}{120 b^3}}{a^{15}+15 a^{14} b x+105 a^{13} b^2 x^2+455 a^{12} b^3 x^3+1365 a^{11} b^4 x^4+3003 a^{10} b^5 x^5+5005 a^9 b^6 x^6+6435 a^8 b^7 x^7+6435 a^7 b^8 x^8+5005 a^6 b^9 x^9+3003 a^5 b^{10} x^{10}+1365 a^4 b^{11} x^{11}+455 a^3 b^{12} x^{12}+105 a^2 b^{13} x^{13}+15 a b^{14} x^{14}+b^{15} x^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^7/(a + b*x)^16,x)
```

```
[Out] -((a^7*d^7 + 3432*b^7*c^7 + 792*a^2*b^5*c^5*d^2 + 330*a^3*b^4*c^4*d^3 + 120
*a^4*b^3*c^3*d^4 + 36*a^5*b^2*c^2*d^5 + 1716*a*b^6*c^6*d + 8*a^6*b*c*d^6)/(
51480*b^8) + (d^7*x^7)/(8*b) + (7*d^2*x^2*(a^5*d^5 + 792*b^5*c^5 + 120*a^2*
b^3*c^3*d^2 + 36*a^3*b^2*c^2*d^3 + 330*a*b^4*c^4*d + 8*a^4*b*c*d^4))/(3432*
b^6) + (7*d^4*x^4*(a^3*d^3 + 120*b^3*c^3 + 36*a*b^2*c^2*d + 8*a^2*b*c*d^2))
/(264*b^4) + (7*d^6*x^6*(a*d + 8*b*c))/(72*b^2) + (7*d^3*x^3*(a^4*d^4 + 330
*b^4*c^4 + 36*a^2*b^2*c^2*d^2 + 120*a*b^3*c^3*d + 8*a^3*b*c*d^3))/(792*b^5)
 + (d*x*(a^6*d^6 + 1716*b^6*c^6 + 330*a^2*b^4*c^4*d^2 + 120*a^3*b^3*c^3*d^3
 + 36*a^4*b^2*c^2*d^4 + 792*a*b^5*c^5*d + 8*a^5*b*c*d^5))/(3432*b^7) + (7*d
^5*x^5*(a^2*d^2 + 36*b^2*c^2 + 8*a*b*c*d))/(120*b^3))/(a^15 + b^15*x^15 + 1
5*a*b^14*x^14 + 105*a^13*b^2*x^2 + 455*a^12*b^3*x^3 + 1365*a^11*b^4*x^4 + 3
003*a^10*b^5*x^5 + 5005*a^9*b^6*x^6 + 6435*a^8*b^7*x^7 + 6435*a^7*b^8*x^8 +
 5005*a^6*b^9*x^9 + 3003*a^5*b^10*x^10 + 1365*a^4*b^11*x^11 + 455*a^3*b^12*
x^12 + 105*a^2*b^13*x^13 + 15*a^14*b*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**7/(b*x+a)**16,x)
```

```
[Out] Timed out
```

3.1193 $\int (a+bx)^{12}(c+dx)^{10}\,dx$

Optimal. Leaf size=275

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+\cdots$$

Rubi [A] time = 1.47, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{5d^9(a+bx)^{22}(bc-ad)}{11b^{11}}+\frac{15d^8(a+bx)^{21}(bc-ad)^2}{7b^{11}}+\frac{6d^7(a+bx)^{20}(bc-ad)^3}{b^{11}}+\frac{210d^6(a+bx)^{19}(bc-ad)^4}{19b^{11}}+\frac{14d^5(a+bx)^{18}(bc-ad)^5}{b^{11}}+\frac{210d^4(a+bx)^{17}(bc-ad)^6}{17b^{11}}+\frac{15d^3(a+bx)^{16}(bc-ad)^7}{2b^{11}}+\frac{3d^2(a+bx)^{15}(bc-ad)^8}{b^{11}}+\frac{5d(a+bx)^{14}(bc-ad)^9}{7b^{11}}+\frac{(a+bx)^{13}(bc-ad)^{10}}{13b^{11}}+\frac{d^{10}(a+bx)^{23}}{23b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^13)/(13*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^14)/
(7*b^11) + (3*d^2*(b*c - a*d)^8*(a + b*x)^15)/b^11 + (15*d^3*(b*c - a*d)^7*
(a + b*x)^16)/(2*b^11) + (210*d^4*(b*c - a*d)^6*(a + b*x)^17)/(17*b^11) + (
14*d^5*(b*c - a*d)^5*(a + b*x)^18)/b^11 + (210*d^6*(b*c - a*d)^4*(a + b*x)^
19)/(19*b^11) + (6*d^7*(b*c - a*d)^3*(a + b*x)^20)/b^11 + (15*d^8*(b*c - a*
d)^2*(a + b*x)^21)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^22)/(11*b^11) +
(d^10*(a + b*x)^23)/(23*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{12}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{12}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{b^{10}}+\cdots\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}}+\frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}}+\frac{15d^3(bc\cdots}{}\end{aligned}$$

Mathematica [B] time = 0.29, size = 1817, normalized size = 6.61

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] a^12*c^10*x + a^11*c^9*(6*b*c + 5*a*d)*x^2 + a^10*c^8*(22*b^2*c^2 + 40*a*b*
c*d + 15*a^2*d^2)*x^3 + 5*a^9*c^7*(11*b^3*c^3 + 33*a*b^2*c^2*d + 27*a^2*b*c
*d^2 + 6*a^3*d^3)*x^4 + a^8*c^6*(99*b^4*c^4 + 440*a*b^3*c^3*d + 594*a^2*b^2
*c^2*d^2 + 288*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + 3*a^7*c^5*(44*b^5*c^5 + 275*
a*b^4*c^4*d + 550*a^2*b^3*c^3*d^2 + 440*a^3*b^2*c^2*d^3 + 140*a^4*b*c*d^4 +
 14*a^5*d^5)*x^6 + (3*a^6*c^4*(308*b^6*c^6 + 2640*a*b^5*c^5*d + 7425*a^2*b^
4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 4620*a^4*b^2*c^2*d^4 + 1008*a^5*b*c*d^5
+ 70*a^6*d^6)*x^7)/7 + 3*a^5*c^3*(33*b^7*c^7 + 385*a*b^6*c^6*d + 1485*a^2*b
^5*c^5*d^2 + 2475*a^3*b^4*c^4*d^3 + 1925*a^4*b^3*c^3*d^4 + 693*a^5*b^2*c^2*
d^5 + 105*a^6*b*c*d^6 + 5*a^7*d^7)*x^8 + 5*a^4*c^2*(11*b^8*c^8 + 176*a*b^7*
c^7*d + 924*a^2*b^6*c^6*d^2 + 2112*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 +
 1232*a^5*b^3*c^3*d^5 + 308*a^6*b^2*c^2*d^6 + 32*a^7*b*c*d^7 + a^8*d^8)*x^9
```

```
 + a^3*c*(22*b^9*c^9 + 495*a*b^8*c^8*d + 3564*a^2*b^7*c^7*d^2 + 11088*a^3*b
^6*c^6*d^3 + 16632*a^4*b^5*c^5*d^4 + 12474*a^5*b^4*c^4*d^5 + 4620*a^6*b^3*c
^3*d^6 + 792*a^7*b^2*c^2*d^7 + 54*a^8*b*c*d^8 + a^9*d^9)*x^10 + (a^2*(66*b^
10*c^10 + 2200*a*b^9*c^9*d + 22275*a^2*b^8*c^8*d^2 + 95040*a^3*b^7*c^7*d^3
+ 194040*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5 + 103950*a^6*b^4*c^4*d^6
+ 26400*a^7*b^3*c^3*d^7 + 2970*a^8*b^2*c^2*d^8 + 120*a^9*b*c*d^9 + a^10*d^1
0)*x^11)/11 + a*b*(b^10*c^10 + 55*a*b^9*c^9*d + 825*a^2*b^8*c^8*d^2 + 4950*
a^3*b^7*c^7*d^3 + 13860*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 + 13860*a^6
*b^4*c^4*d^6 + 4950*a^7*b^3*c^3*d^7 + 825*a^8*b^2*c^2*d^8 + 55*a^9*b*c*d^9
+ a^10*d^10)*x^12 + (b^2*(b^10*c^10 + 120*a*b^9*c^9*d + 2970*a^2*b^8*c^8*d^
2 + 26400*a^3*b^7*c^7*d^3 + 103950*a^4*b^6*c^6*d^4 + 199584*a^5*b^5*c^5*d^5
 + 194040*a^6*b^4*c^4*d^6 + 95040*a^7*b^3*c^3*d^7 + 22275*a^8*b^2*c^2*d^8 +
 2200*a^9*b*c*d^9 + 66*a^10*d^10)*x^13)/13 + (5*b^3*d*(b^9*c^9 + 54*a*b^8*c
^8*d + 792*a^2*b^7*c^7*d^2 + 4620*a^3*b^6*c^6*d^3 + 12474*a^4*b^5*c^5*d^4 +
 16632*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 3564*a^7*b^2*c^2*d^7 + 495
*a^8*b*c*d^8 + 22*a^9*d^9)*x^14)/7 + 3*b^4*d^2*(b^8*c^8 + 32*a*b^7*c^7*d +
308*a^2*b^6*c^6*d^2 + 1232*a^3*b^5*c^5*d^3 + 2310*a^4*b^4*c^4*d^4 + 2112*a^
5*b^3*c^3*d^5 + 924*a^6*b^2*c^2*d^6 + 176*a^7*b*c*d^7 + 11*a^8*d^8)*x^15 +
(3*b^5*d^3*(5*b^7*c^7 + 105*a*b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1925*a^3*b^
4*c^4*d^3 + 2475*a^4*b^3*c^3*d^4 + 1485*a^5*b^2*c^2*d^5 + 385*a^6*b*c*d^6 +
 33*a^7*d^7)*x^16)/2 + (3*b^6*d^4*(70*b^6*c^6 + 1008*a*b^5*c^5*d + 4620*a^2
*b^4*c^4*d^2 + 8800*a^3*b^3*c^3*d^3 + 7425*a^4*b^2*c^2*d^4 + 2640*a^5*b*c*d
^5 + 308*a^6*d^6)*x^17)/17 + b^7*d^5*(14*b^5*c^5 + 140*a*b^4*c^4*d + 440*a^
2*b^3*c^3*d^2 + 550*a^3*b^2*c^2*d^3 + 275*a^4*b*c*d^4 + 44*a^5*d^5)*x^18 +
(5*b^8*d^6*(42*b^4*c^4 + 288*a*b^3*c^3*d + 594*a^2*b^2*c^2*d^2 + 440*a^3*b*
c*d^3 + 99*a^4*d^4)*x^19)/19 + b^9*d^7*(6*b^3*c^3 + 27*a*b^2*c^2*d + 33*a^2
*b*c*d^2 + 11*a^3*d^3)*x^20 + (b^10*d^8*(15*b^2*c^2 + 40*a*b*c*d + 22*a^2*d
^2)*x^21)/7 + (b^11*d^9*(5*b*c + 6*a*d)*x^22)/11 + (b^12*d^10*x^23)/23
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{12}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^12*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^12*(c + d*x)^10, x]
```

fricas [B] time = 1.33, size = 2186, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/23*x^23*d^10*b^12 + 5/11*x^22*d^9*c*b^12 + 6/11*x^22*d^10*b^11*a + 15/7*x
^21*d^8*c^2*b^12 + 40/7*x^21*d^9*c*b^11*a + 22/7*x^21*d^10*b^10*a^2 + 6*x^2
0*d^7*c^3*b^12 + 27*x^20*d^8*c^2*b^11*a + 33*x^20*d^9*c*b^10*a^2 + 11*x^20*
d^10*b^9*a^3 + 210/19*x^19*d^6*c^4*b^12 + 1440/19*x^19*d^7*c^3*b^11*a + 297
0/19*x^19*d^8*c^2*b^10*a^2 + 2200/19*x^19*d^9*c*b^9*a^3 + 495/19*x^19*d^10*
b^8*a^4 + 14*x^18*d^5*c^5*b^12 + 140*x^18*d^6*c^4*b^11*a + 440*x^18*d^7*c^3
*b^10*a^2 + 550*x^18*d^8*c^2*b^9*a^3 + 275*x^18*d^9*c*b^8*a^4 + 44*x^18*d^1
0*b^7*a^5 + 210/17*x^17*d^4*c^6*b^12 + 3024/17*x^17*d^5*c^5*b^11*a + 13860/
17*x^17*d^6*c^4*b^10*a^2 + 26400/17*x^17*d^7*c^3*b^9*a^3 + 22275/17*x^17*d^
8*c^2*b^8*a^4 + 7920/17*x^17*d^9*c*b^7*a^5 + 924/17*x^17*d^10*b^6*a^6 + 15/
2*x^16*d^3*c^7*b^12 + 315/2*x^16*d^4*c^6*b^11*a + 2079/2*x^16*d^5*c^5*b^10*
a^2 + 5775/2*x^16*d^6*c^4*b^9*a^3 + 7425/2*x^16*d^7*c^3*b^8*a^4 + 4455/2*x^
16*d^8*c^2*b^7*a^5 + 1155/2*x^16*d^9*c*b^6*a^6 + 99/2*x^16*d^10*b^5*a^7 + 3
*x^15*d^2*c^8*b^12 + 96*x^15*d^3*c^7*b^11*a + 924*x^15*d^4*c^6*b^10*a^2 + 3
696*x^15*d^5*c^5*b^9*a^3 + 6930*x^15*d^6*c^4*b^8*a^4 + 6336*x^15*d^7*c^3*b^
```

```
7*a^5 + 2772*x^15*d^8*c^2*b^6*a^6 + 528*x^15*d^9*c*b^5*a^7 + 33*x^15*d^10*b
^4*a^8 + 5/7*x^14*d*c^9*b^12 + 270/7*x^14*d^2*c^8*b^11*a + 3960/7*x^14*d^3*
c^7*b^10*a^2 + 3300*x^14*d^4*c^6*b^9*a^3 + 8910*x^14*d^5*c^5*b^8*a^4 + 1188
0*x^14*d^6*c^4*b^7*a^5 + 7920*x^14*d^7*c^3*b^6*a^6 + 17820/7*x^14*d^8*c^2*b
^5*a^7 + 2475/7*x^14*d^9*c*b^4*a^8 + 110/7*x^14*d^10*b^3*a^9 + 1/13*x^13*c^
10*b^12 + 120/13*x^13*d*c^9*b^11*a + 2970/13*x^13*d^2*c^8*b^10*a^2 + 26400/
13*x^13*d^3*c^7*b^9*a^3 + 103950/13*x^13*d^4*c^6*b^8*a^4 + 199584/13*x^13*d
^5*c^5*b^7*a^5 + 194040/13*x^13*d^6*c^4*b^6*a^6 + 95040/13*x^13*d^7*c^3*b^5
*a^7 + 22275/13*x^13*d^8*c^2*b^4*a^8 + 2200/13*x^13*d^9*c*b^3*a^9 + 66/13*x
^13*d^10*b^2*a^10 + x^12*c^10*b^11*a + 55*x^12*d*c^9*b^10*a^2 + 825*x^12*d^
2*c^8*b^9*a^3 + 4950*x^12*d^3*c^7*b^8*a^4 + 13860*x^12*d^4*c^6*b^7*a^5 + 19
404*x^12*d^5*c^5*b^6*a^6 + 13860*x^12*d^6*c^4*b^5*a^7 + 4950*x^12*d^7*c^3*b
^4*a^8 + 825*x^12*d^8*c^2*b^3*a^9 + 55*x^12*d^9*c*b^2*a^10 + x^12*d^10*b*a^
11 + 6*x^11*c^10*b^10*a^2 + 200*x^11*d*c^9*b^9*a^3 + 2025*x^11*d^2*c^8*b^8*
a^4 + 8640*x^11*d^3*c^7*b^7*a^5 + 17640*x^11*d^4*c^6*b^6*a^6 + 18144*x^11*d
^5*c^5*b^5*a^7 + 9450*x^11*d^6*c^4*b^4*a^8 + 2400*x^11*d^7*c^3*b^3*a^9 + 27
0*x^11*d^8*c^2*b^2*a^10 + 120/11*x^11*d^9*c*b*a^11 + 1/11*x^11*d^10*a^12 +
22*x^10*c^10*b^9*a^3 + 495*x^10*d*c^9*b^8*a^4 + 3564*x^10*d^2*c^8*b^7*a^5 +
 11088*x^10*d^3*c^7*b^6*a^6 + 16632*x^10*d^4*c^6*b^5*a^7 + 12474*x^10*d^5*c
^5*b^4*a^8 + 4620*x^10*d^6*c^4*b^3*a^9 + 792*x^10*d^7*c^3*b^2*a^10 + 54*x^1
0*d^8*c^2*b*a^11 + x^10*d^9*c*a^12 + 55*x^9*c^10*b^8*a^4 + 880*x^9*d*c^9*b^
7*a^5 + 4620*x^9*d^2*c^8*b^6*a^6 + 10560*x^9*d^3*c^7*b^5*a^7 + 11550*x^9*d^
4*c^6*b^4*a^8 + 6160*x^9*d^5*c^5*b^3*a^9 + 1540*x^9*d^6*c^4*b^2*a^10 + 160*
x^9*d^7*c^3*b*a^11 + 5*x^9*d^8*c^2*a^12 + 99*x^8*c^10*b^7*a^5 + 1155*x^8*d*
c^9*b^6*a^6 + 4455*x^8*d^2*c^8*b^5*a^7 + 7425*x^8*d^3*c^7*b^4*a^8 + 5775*x^
8*d^4*c^6*b^3*a^9 + 2079*x^8*d^5*c^5*b^2*a^10 + 315*x^8*d^6*c^4*b*a^11 + 15
*x^8*d^7*c^3*a^12 + 132*x^7*c^10*b^6*a^6 + 7920/7*x^7*d*c^9*b^5*a^7 + 22275
/7*x^7*d^2*c^8*b^4*a^8 + 26400/7*x^7*d^3*c^7*b^3*a^9 + 1980*x^7*d^4*c^6*b^2
*a^10 + 432*x^7*d^5*c^5*b*a^11 + 30*x^7*d^6*c^4*a^12 + 132*x^6*c^10*b^5*a^7
 + 825*x^6*d*c^9*b^4*a^8 + 1650*x^6*d^2*c^8*b^3*a^9 + 1320*x^6*d^3*c^7*b^2*
a^10 + 420*x^6*d^4*c^6*b*a^11 + 42*x^6*d^5*c^5*a^12 + 99*x^5*c^10*b^4*a^8 +
 440*x^5*d*c^9*b^3*a^9 + 594*x^5*d^2*c^8*b^2*a^10 + 288*x^5*d^3*c^7*b*a^11
+ 42*x^5*d^4*c^6*a^12 + 55*x^4*c^10*b^3*a^9 + 165*x^4*d*c^9*b^2*a^10 + 135*
x^4*d^2*c^8*b*a^11 + 30*x^4*d^3*c^7*a^12 + 22*x^3*c^10*b^2*a^10 + 40*x^3*d*
c^9*b*a^11 + 15*x^3*d^2*c^8*a^12 + 6*x^2*c^10*b*a^11 + 5*x^2*d*c^9*a^12 + x
*c^10*a^12
```

giac [B] time = 1.31, size = 2186, normalized size = 7.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/23*b^12*d^10*x^23 + 5/11*b^12*c*d^9*x^22 + 6/11*a*b^11*d^10*x^22 + 15/7*b
^12*c^2*d^8*x^21 + 40/7*a*b^11*c*d^9*x^21 + 22/7*a^2*b^10*d^10*x^21 + 6*b^1
2*c^3*d^7*x^20 + 27*a*b^11*c^2*d^8*x^20 + 33*a^2*b^10*c*d^9*x^20 + 11*a^3*b
^9*d^10*x^20 + 210/19*b^12*c^4*d^6*x^19 + 1440/19*a*b^11*c^3*d^7*x^19 + 297
0/19*a^2*b^10*c^2*d^8*x^19 + 2200/19*a^3*b^9*c*d^9*x^19 + 495/19*a^4*b^8*d^
10*x^19 + 14*b^12*c^5*d^5*x^18 + 140*a*b^11*c^4*d^6*x^18 + 440*a^2*b^10*c^3
*d^7*x^18 + 550*a^3*b^9*c^2*d^8*x^18 + 275*a^4*b^8*c*d^9*x^18 + 44*a^5*b^7*
d^10*x^18 + 210/17*b^12*c^6*d^4*x^17 + 3024/17*a*b^11*c^5*d^5*x^17 + 13860/
17*a^2*b^10*c^4*d^6*x^17 + 26400/17*a^3*b^9*c^3*d^7*x^17 + 22275/17*a^4*b^8
*c^2*d^8*x^17 + 7920/17*a^5*b^7*c*d^9*x^17 + 924/17*a^6*b^6*d^10*x^17 + 15/
2*b^12*c^7*d^3*x^16 + 315/2*a*b^11*c^6*d^4*x^16 + 2079/2*a^2*b^10*c^5*d^5*x
^16 + 5775/2*a^3*b^9*c^4*d^6*x^16 + 7425/2*a^4*b^8*c^3*d^7*x^16 + 4455/2*a^
5*b^7*c^2*d^8*x^16 + 1155/2*a^6*b^6*c*d^9*x^16 + 99/2*a^7*b^5*d^10*x^16 + 3
*b^12*c^8*d^2*x^15 + 96*a*b^11*c^7*d^3*x^15 + 924*a^2*b^10*c^6*d^4*x^15 + 3
696*a^3*b^9*c^5*d^5*x^15 + 6930*a^4*b^8*c^4*d^6*x^15 + 6336*a^5*b^7*c^3*d^7
*x^15 + 2772*a^6*b^6*c^2*d^8*x^15 + 528*a^7*b^5*c*d^9*x^15 + 33*a^8*b^4*d^1
```

```
0*x^15 + 5/7*b^12*c^9*d*x^14 + 270/7*a*b^11*c^8*d^2*x^14 + 3960/7*a^2*b^10*
c^7*d^3*x^14 + 3300*a^3*b^9*c^6*d^4*x^14 + 8910*a^4*b^8*c^5*d^5*x^14 + 1188
0*a^5*b^7*c^4*d^6*x^14 + 7920*a^6*b^6*c^3*d^7*x^14 + 17820/7*a^7*b^5*c^2*d^
8*x^14 + 2475/7*a^8*b^4*c*d^9*x^14 + 110/7*a^9*b^3*d^10*x^14 + 1/13*b^12*c^
10*x^13 + 120/13*a*b^11*c^9*d*x^13 + 2970/13*a^2*b^10*c^8*d^2*x^13 + 26400/
13*a^3*b^9*c^7*d^3*x^13 + 103950/13*a^4*b^8*c^6*d^4*x^13 + 199584/13*a^5*b^
7*c^5*d^5*x^13 + 194040/13*a^6*b^6*c^4*d^6*x^13 + 95040/13*a^7*b^5*c^3*d^7*
x^13 + 22275/13*a^8*b^4*c^2*d^8*x^13 + 2200/13*a^9*b^3*c*d^9*x^13 + 66/13*a
^10*b^2*d^10*x^13 + a*b^11*c^10*x^12 + 55*a^2*b^10*c^9*d*x^12 + 825*a^3*b^9
*c^8*d^2*x^12 + 4950*a^4*b^8*c^7*d^3*x^12 + 13860*a^5*b^7*c^6*d^4*x^12 + 19
404*a^6*b^6*c^5*d^5*x^12 + 13860*a^7*b^5*c^4*d^6*x^12 + 4950*a^8*b^4*c^3*d^
7*x^12 + 825*a^9*b^3*c^2*d^8*x^12 + 55*a^10*b^2*c*d^9*x^12 + a^11*b*d^10*x^
12 + 6*a^2*b^10*c^10*x^11 + 200*a^3*b^9*c^9*d*x^11 + 2025*a^4*b^8*c^8*d^2*x
^11 + 8640*a^5*b^7*c^7*d^3*x^11 + 17640*a^6*b^6*c^6*d^4*x^11 + 18144*a^7*b^
5*c^5*d^5*x^11 + 9450*a^8*b^4*c^4*d^6*x^11 + 2400*a^9*b^3*c^3*d^7*x^11 + 27
0*a^10*b^2*c^2*d^8*x^11 + 120/11*a^11*b*c*d^9*x^11 + 1/11*a^12*d^10*x^11 +
22*a^3*b^9*c^10*x^10 + 495*a^4*b^8*c^9*d*x^10 + 3564*a^5*b^7*c^8*d^2*x^10 +
 11088*a^6*b^6*c^7*d^3*x^10 + 16632*a^7*b^5*c^6*d^4*x^10 + 12474*a^8*b^4*c^
5*d^5*x^10 + 4620*a^9*b^3*c^4*d^6*x^10 + 792*a^10*b^2*c^3*d^7*x^10 + 54*a^1
1*b*c^2*d^8*x^10 + a^12*c*d^9*x^10 + 55*a^4*b^8*c^10*x^9 + 880*a^5*b^7*c^9*
d*x^9 + 4620*a^6*b^6*c^8*d^2*x^9 + 10560*a^7*b^5*c^7*d^3*x^9 + 11550*a^8*b^
4*c^6*d^4*x^9 + 6160*a^9*b^3*c^5*d^5*x^9 + 1540*a^10*b^2*c^4*d^6*x^9 + 160*
a^11*b*c^3*d^7*x^9 + 5*a^12*c^2*d^8*x^9 + 99*a^5*b^7*c^10*x^8 + 1155*a^6*b^
6*c^9*d*x^8 + 4455*a^7*b^5*c^8*d^2*x^8 + 7425*a^8*b^4*c^7*d^3*x^8 + 5775*a^
9*b^3*c^6*d^4*x^8 + 2079*a^10*b^2*c^5*d^5*x^8 + 315*a^11*b*c^4*d^6*x^8 + 15
*a^12*c^3*d^7*x^8 + 132*a^6*b^6*c^10*x^7 + 7920/7*a^7*b^5*c^9*d*x^7 + 22275
/7*a^8*b^4*c^8*d^2*x^7 + 26400/7*a^9*b^3*c^7*d^3*x^7 + 1980*a^10*b^2*c^6*d^
4*x^7 + 432*a^11*b*c^5*d^5*x^7 + 30*a^12*c^4*d^6*x^7 + 132*a^7*b^5*c^10*x^6
 + 825*a^8*b^4*c^9*d*x^6 + 1650*a^9*b^3*c^8*d^2*x^6 + 1320*a^10*b^2*c^7*d^3
*x^6 + 420*a^11*b*c^6*d^4*x^6 + 42*a^12*c^5*d^5*x^6 + 99*a^8*b^4*c^10*x^5 +
 440*a^9*b^3*c^9*d*x^5 + 594*a^10*b^2*c^8*d^2*x^5 + 288*a^11*b*c^7*d^3*x^5
+ 42*a^12*c^6*d^4*x^5 + 55*a^9*b^3*c^10*x^4 + 165*a^10*b^2*c^9*d*x^4 + 135*
a^11*b*c^8*d^2*x^4 + 30*a^12*c^7*d^3*x^4 + 22*a^10*b^2*c^10*x^3 + 40*a^11*b
*c^9*d*x^3 + 15*a^12*c^8*d^2*x^3 + 6*a^11*b*c^10*x^2 + 5*a^12*c^9*d*x^2 + a
^12*c^10*x
```

maple [B] time = 0.00, size = 1891, normalized size = 6.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^12*(d*x+c)^10,x)
```

```
[Out] 1/23*b^12*d^10*x^23+1/22*(12*a*b^11*d^10+10*b^12*c*d^9)*x^22+1/21*(66*a^2*b
^10*d^10+120*a*b^11*c*d^9+45*b^12*c^2*d^8)*x^21+1/20*(220*a^3*b^9*d^10+660*
a^2*b^10*c*d^9+540*a*b^11*c^2*d^8+120*b^12*c^3*d^7)*x^20+1/19*(495*a^4*b^8*
d^10+2200*a^3*b^9*c*d^9+2970*a^2*b^10*c^2*d^8+1440*a*b^11*c^3*d^7+210*b^12*
c^4*d^6)*x^19+1/18*(792*a^5*b^7*d^10+4950*a^4*b^8*c*d^9+9900*a^3*b^9*c^2*d^
8+7920*a^2*b^10*c^3*d^7+2520*a*b^11*c^4*d^6+252*b^12*c^5*d^5)*x^18+1/17*(92
4*a^6*b^6*d^10+7920*a^5*b^7*c*d^9+22275*a^4*b^8*c^2*d^8+26400*a^3*b^9*c^3*d
^7+13860*a^2*b^10*c^4*d^6+3024*a*b^11*c^5*d^5+210*b^12*c^6*d^4)*x^17+1/16*(
792*a^7*b^5*d^10+9240*a^6*b^6*c*d^9+35640*a^5*b^7*c^2*d^8+59400*a^4*b^8*c^3
*d^7+46200*a^3*b^9*c^4*d^6+16632*a^2*b^10*c^5*d^5+2520*a*b^11*c^6*d^4+120*b
^12*c^7*d^3)*x^16+1/15*(495*a^8*b^4*d^10+7920*a^7*b^5*c*d^9+41580*a^6*b^6*c
^2*d^8+95040*a^5*b^7*c^3*d^7+103950*a^4*b^8*c^4*d^6+55440*a^3*b^9*c^5*d^5+1
3860*a^2*b^10*c^6*d^4+1440*a*b^11*c^7*d^3+45*b^12*c^8*d^2)*x^15+1/14*(220*a
^9*b^3*d^10+4950*a^8*b^4*c*d^9+35640*a^7*b^5*c^2*d^8+110880*a^6*b^6*c^3*d^7
+166320*a^5*b^7*c^4*d^6+124740*a^4*b^8*c^5*d^5+46200*a^3*b^9*c^6*d^4+7920*a
^2*b^10*c^7*d^3+540*a*b^11*c^8*d^2+10*b^12*c^9*d)*x^14+1/13*(66*a^10*b^2*d^
10+2200*a^9*b^3*c*d^9+22275*a^8*b^4*c^2*d^8+95040*a^7*b^5*c^3*d^7+194040*a^
```

```
6*b^6*c^4*d^6+199584*a^5*b^7*c^5*d^5+103950*a^4*b^8*c^6*d^4+26400*a^3*b^9*c
^7*d^3+2970*a^2*b^10*c^8*d^2+120*a*b^11*c^9*d+b^12*c^10)*x^13+1/12*(12*a^11
*b*d^10+660*a^10*b^2*c*d^9+9900*a^9*b^3*c^2*d^8+59400*a^8*b^4*c^3*d^7+16632
0*a^7*b^5*c^4*d^6+232848*a^6*b^6*c^5*d^5+166320*a^5*b^7*c^6*d^4+59400*a^4*b
^8*c^7*d^3+9900*a^3*b^9*c^8*d^2+660*a^2*b^10*c^9*d+12*a*b^11*c^10)*x^12+1/1
1*(a^12*d^10+120*a^11*b*c*d^9+2970*a^10*b^2*c^2*d^8+26400*a^9*b^3*c^3*d^7+1
03950*a^8*b^4*c^4*d^6+199584*a^7*b^5*c^5*d^5+194040*a^6*b^6*c^6*d^4+95040*a
^5*b^7*c^7*d^3+22275*a^4*b^8*c^8*d^2+2200*a^3*b^9*c^9*d+66*a^2*b^10*c^10)*x
^11+1/10*(10*a^12*c*d^9+540*a^11*b*c^2*d^8+7920*a^10*b^2*c^3*d^7+46200*a^9*
b^3*c^4*d^6+124740*a^8*b^4*c^5*d^5+166320*a^7*b^5*c^6*d^4+110880*a^6*b^6*c^
7*d^3+35640*a^5*b^7*c^8*d^2+4950*a^4*b^8*c^9*d+220*a^3*b^9*c^10)*x^10+1/9*(
45*a^12*c^2*d^8+1440*a^11*b*c^3*d^7+13860*a^10*b^2*c^4*d^6+55440*a^9*b^3*c^
5*d^5+103950*a^8*b^4*c^6*d^4+95040*a^7*b^5*c^7*d^3+41580*a^6*b^6*c^8*d^2+79
20*a^5*b^7*c^9*d+495*a^4*b^8*c^10)*x^9+1/8*(120*a^12*c^3*d^7+2520*a^11*b*c^
4*d^6+16632*a^10*b^2*c^5*d^5+46200*a^9*b^3*c^6*d^4+59400*a^8*b^4*c^7*d^3+35
640*a^7*b^5*c^8*d^2+9240*a^6*b^6*c^9*d+792*a^5*b^7*c^10)*x^8+1/7*(210*a^12*
c^4*d^6+3024*a^11*b*c^5*d^5+13860*a^10*b^2*c^6*d^4+26400*a^9*b^3*c^7*d^3+22
275*a^8*b^4*c^8*d^2+7920*a^7*b^5*c^9*d+924*a^6*b^6*c^10)*x^7+1/6*(252*a^12*
c^5*d^5+2520*a^11*b*c^6*d^4+7920*a^10*b^2*c^7*d^3+9900*a^9*b^3*c^8*d^2+4950
*a^8*b^4*c^9*d+792*a^7*b^5*c^10)*x^6+1/5*(210*a^12*c^6*d^4+1440*a^11*b*c^7*
d^3+2970*a^10*b^2*c^8*d^2+2200*a^9*b^3*c^9*d+495*a^8*b^4*c^10)*x^5+1/4*(120
*a^12*c^7*d^3+540*a^11*b*c^8*d^2+660*a^10*b^2*c^9*d+220*a^9*b^3*c^10)*x^4+1
/3*(45*a^12*c^8*d^2+120*a^11*b*c^9*d+66*a^10*b^2*c^10)*x^3+1/2*(10*a^12*c^9
*d+12*a^11*b*c^10)*x^2+a^12*c^10*x
```

maxima [B] time = 1.55, size = 1877, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^12*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/23*b^12*d^10*x^23 + a^12*c^10*x + 1/11*(5*b^12*c*d^9 + 6*a*b^11*d^10)*x^2
2 + 1/7*(15*b^12*c^2*d^8 + 40*a*b^11*c*d^9 + 22*a^2*b^10*d^10)*x^21 + (6*b^
12*c^3*d^7 + 27*a*b^11*c^2*d^8 + 33*a^2*b^10*c*d^9 + 11*a^3*b^9*d^10)*x^20
+ 5/19*(42*b^12*c^4*d^6 + 288*a*b^11*c^3*d^7 + 594*a^2*b^10*c^2*d^8 + 440*a
^3*b^9*c*d^9 + 99*a^4*b^8*d^10)*x^19 + (14*b^12*c^5*d^5 + 140*a*b^11*c^4*d^
6 + 440*a^2*b^10*c^3*d^7 + 550*a^3*b^9*c^2*d^8 + 275*a^4*b^8*c*d^9 + 44*a^5
*b^7*d^10)*x^18 + 3/17*(70*b^12*c^6*d^4 + 1008*a*b^11*c^5*d^5 + 4620*a^2*b^
10*c^4*d^6 + 8800*a^3*b^9*c^3*d^7 + 7425*a^4*b^8*c^2*d^8 + 2640*a^5*b^7*c*d
^9 + 308*a^6*b^6*d^10)*x^17 + 3/2*(5*b^12*c^7*d^3 + 105*a*b^11*c^6*d^4 + 69
3*a^2*b^10*c^5*d^5 + 1925*a^3*b^9*c^4*d^6 + 2475*a^4*b^8*c^3*d^7 + 1485*a^5
*b^7*c^2*d^8 + 385*a^6*b^6*c*d^9 + 33*a^7*b^5*d^10)*x^16 + 3*(b^12*c^8*d^2
+ 32*a*b^11*c^7*d^3 + 308*a^2*b^10*c^6*d^4 + 1232*a^3*b^9*c^5*d^5 + 2310*a^
4*b^8*c^4*d^6 + 2112*a^5*b^7*c^3*d^7 + 924*a^6*b^6*c^2*d^8 + 176*a^7*b^5*c*
d^9 + 11*a^8*b^4*d^10)*x^15 + 5/7*(b^12*c^9*d + 54*a*b^11*c^8*d^2 + 792*a^2
*b^10*c^7*d^3 + 4620*a^3*b^9*c^6*d^4 + 12474*a^4*b^8*c^5*d^5 + 16632*a^5*b^
7*c^4*d^6 + 11088*a^6*b^6*c^3*d^7 + 3564*a^7*b^5*c^2*d^8 + 495*a^8*b^4*c*d^
9 + 22*a^9*b^3*d^10)*x^14 + 1/13*(b^12*c^10 + 120*a*b^11*c^9*d + 2970*a^2*b
^10*c^8*d^2 + 26400*a^3*b^9*c^7*d^3 + 103950*a^4*b^8*c^6*d^4 + 199584*a^5*b
^7*c^5*d^5 + 194040*a^6*b^6*c^4*d^6 + 95040*a^7*b^5*c^3*d^7 + 22275*a^8*b^4
*c^2*d^8 + 2200*a^9*b^3*c*d^9 + 66*a^10*b^2*d^10)*x^13 + (a*b^11*c^10 + 55*
a^2*b^10*c^9*d + 825*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7
*c^6*d^4 + 19404*a^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3
*d^7 + 825*a^9*b^3*c^2*d^8 + 55*a^10*b^2*c*d^9 + a^11*b*d^10)*x^12 + 1/11*(
66*a^2*b^10*c^10 + 2200*a^3*b^9*c^9*d + 22275*a^4*b^8*c^8*d^2 + 95040*a^5*b
^7*c^7*d^3 + 194040*a^6*b^6*c^6*d^4 + 199584*a^7*b^5*c^5*d^5 + 103950*a^8*b
^4*c^4*d^6 + 26400*a^9*b^3*c^3*d^7 + 2970*a^10*b^2*c^2*d^8 + 120*a^11*b*c*d
^9 + a^12*d^10)*x^11 + (22*a^3*b^9*c^10 + 495*a^4*b^8*c^9*d + 3564*a^5*b^7*
c^8*d^2 + 11088*a^6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5
```

```
*d^5 + 4620*a^9*b^3*c^4*d^6 + 792*a^10*b^2*c^3*d^7 + 54*a^11*b*c^2*d^8 + a^
12*c*d^9)*x^10 + 5*(11*a^4*b^8*c^10 + 176*a^5*b^7*c^9*d + 924*a^6*b^6*c^8*d
^2 + 2112*a^7*b^5*c^7*d^3 + 2310*a^8*b^4*c^6*d^4 + 1232*a^9*b^3*c^5*d^5 + 3
08*a^10*b^2*c^4*d^6 + 32*a^11*b*c^3*d^7 + a^12*c^2*d^8)*x^9 + 3*(33*a^5*b^7
*c^10 + 385*a^6*b^6*c^9*d + 1485*a^7*b^5*c^8*d^2 + 2475*a^8*b^4*c^7*d^3 + 1
925*a^9*b^3*c^6*d^4 + 693*a^10*b^2*c^5*d^5 + 105*a^11*b*c^4*d^6 + 5*a^12*c^
3*d^7)*x^8 + 3/7*(308*a^6*b^6*c^10 + 2640*a^7*b^5*c^9*d + 7425*a^8*b^4*c^8*
d^2 + 8800*a^9*b^3*c^7*d^3 + 4620*a^10*b^2*c^6*d^4 + 1008*a^11*b*c^5*d^5 +
70*a^12*c^4*d^6)*x^7 + 3*(44*a^7*b^5*c^10 + 275*a^8*b^4*c^9*d + 550*a^9*b^3
*c^8*d^2 + 440*a^10*b^2*c^7*d^3 + 140*a^11*b*c^6*d^4 + 14*a^12*c^5*d^5)*x^6
 + (99*a^8*b^4*c^10 + 440*a^9*b^3*c^9*d + 594*a^10*b^2*c^8*d^2 + 288*a^11*b
*c^7*d^3 + 42*a^12*c^6*d^4)*x^5 + 5*(11*a^9*b^3*c^10 + 33*a^10*b^2*c^9*d +
27*a^11*b*c^8*d^2 + 6*a^12*c^7*d^3)*x^4 + (22*a^10*b^2*c^10 + 40*a^11*b*c^9
*d + 15*a^12*c^8*d^2)*x^3 + (6*a^11*b*c^10 + 5*a^12*c^9*d)*x^2
```

mupad [B] time = 0.98, size = 1847, normalized size = 6.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^12*(c + d*x)^10,x)
```

```
[Out] x^12*(a*b^11*c^10 + a^11*b*d^10 + 55*a^2*b^10*c^9*d + 55*a^10*b^2*c*d^9 + 8
25*a^3*b^9*c^8*d^2 + 4950*a^4*b^8*c^7*d^3 + 13860*a^5*b^7*c^6*d^4 + 19404*a
^6*b^6*c^5*d^5 + 13860*a^7*b^5*c^4*d^6 + 4950*a^8*b^4*c^3*d^7 + 825*a^9*b^3
*c^2*d^8) + x^7*(132*a^6*b^6*c^10 + 30*a^12*c^4*d^6 + (7920*a^7*b^5*c^9*d)/
7 + 432*a^11*b*c^5*d^5 + (22275*a^8*b^4*c^8*d^2)/7 + (26400*a^9*b^3*c^7*d^3
)/7 + 1980*a^10*b^2*c^6*d^4) + x^17*((924*a^6*b^6*d^10)/17 + (210*b^12*c^6*
d^4)/17 + (3024*a*b^11*c^5*d^5)/17 + (7920*a^5*b^7*c*d^9)/17 + (13860*a^2*b
^10*c^4*d^6)/17 + (26400*a^3*b^9*c^3*d^7)/17 + (22275*a^4*b^8*c^2*d^8)/17)
+ x^5*(99*a^8*b^4*c^10 + 42*a^12*c^6*d^4 + 440*a^9*b^3*c^9*d + 288*a^11*b*c
^7*d^3 + 594*a^10*b^2*c^8*d^2) + x^19*((495*a^4*b^8*d^10)/19 + (210*b^12*c^
4*d^6)/19 + (1440*a*b^11*c^3*d^7)/19 + (2200*a^3*b^9*c*d^9)/19 + (2970*a^2*
b^10*c^2*d^8)/19) + x^8*(99*a^5*b^7*c^10 + 15*a^12*c^3*d^7 + 1155*a^6*b^6*c
^9*d + 315*a^11*b*c^4*d^6 + 4455*a^7*b^5*c^8*d^2 + 7425*a^8*b^4*c^7*d^3 + 5
775*a^9*b^3*c^6*d^4 + 2079*a^10*b^2*c^5*d^5) + x^16*((99*a^7*b^5*d^10)/2 +
(15*b^12*c^7*d^3)/2 + (315*a*b^11*c^6*d^4)/2 + (1155*a^6*b^6*c*d^9)/2 + (20
79*a^2*b^10*c^5*d^5)/2 + (5775*a^3*b^9*c^4*d^6)/2 + (7425*a^4*b^8*c^3*d^7)/
2 + (4455*a^5*b^7*c^2*d^8)/2) + x^11*((a^12*d^10)/11 + 6*a^2*b^10*c^10 + 20
0*a^3*b^9*c^9*d + 2025*a^4*b^8*c^8*d^2 + 8640*a^5*b^7*c^7*d^3 + 17640*a^6*b
^6*c^6*d^4 + 18144*a^7*b^5*c^5*d^5 + 9450*a^8*b^4*c^4*d^6 + 2400*a^9*b^3*c^
3*d^7 + 270*a^10*b^2*c^2*d^8 + (120*a^11*b*c*d^9)/11) + x^13*((b^12*c^10)/1
3 + (66*a^10*b^2*d^10)/13 + (2200*a^9*b^3*c*d^9)/13 + (2970*a^2*b^10*c^8*d^
2)/13 + (26400*a^3*b^9*c^7*d^3)/13 + (103950*a^4*b^8*c^6*d^4)/13 + (199584*
a^5*b^7*c^5*d^5)/13 + (194040*a^6*b^6*c^4*d^6)/13 + (95040*a^7*b^5*c^3*d^7)
/13 + (22275*a^8*b^4*c^2*d^8)/13 + (120*a*b^11*c^9*d)/13) + x^6*(132*a^7*b^
5*c^10 + 42*a^12*c^5*d^5 + 825*a^8*b^4*c^9*d + 420*a^11*b*c^6*d^4 + 1650*a^
9*b^3*c^8*d^2 + 1320*a^10*b^2*c^7*d^3) + x^18*(44*a^5*b^7*d^10 + 14*b^12*c^
5*d^5 + 140*a*b^11*c^4*d^6 + 275*a^4*b^8*c*d^9 + 440*a^2*b^10*c^3*d^7 + 550
*a^3*b^9*c^2*d^8) + x^9*(55*a^4*b^8*c^10 + 5*a^12*c^2*d^8 + 880*a^5*b^7*c^9
*d + 160*a^11*b*c^3*d^7 + 4620*a^6*b^6*c^8*d^2 + 10560*a^7*b^5*c^7*d^3 + 11
550*a^8*b^4*c^6*d^4 + 6160*a^9*b^3*c^5*d^5 + 1540*a^10*b^2*c^4*d^6) + x^15*
(33*a^8*b^4*d^10 + 3*b^12*c^8*d^2 + 96*a*b^11*c^7*d^3 + 528*a^7*b^5*c*d^9 +
 924*a^2*b^10*c^6*d^4 + 3696*a^3*b^9*c^5*d^5 + 6930*a^4*b^8*c^4*d^6 + 6336*
a^5*b^7*c^3*d^7 + 2772*a^6*b^6*c^2*d^8) + x^10*(a^12*c*d^9 + 22*a^3*b^9*c^1
0 + 495*a^4*b^8*c^9*d + 54*a^11*b*c^2*d^8 + 3564*a^5*b^7*c^8*d^2 + 11088*a^
6*b^6*c^7*d^3 + 16632*a^7*b^5*c^6*d^4 + 12474*a^8*b^4*c^5*d^5 + 4620*a^9*b^
3*c^4*d^6 + 792*a^10*b^2*c^3*d^7) + x^14*((5*b^12*c^9*d)/7 + (110*a^9*b^3*d
^10)/7 + (270*a*b^11*c^8*d^2)/7 + (2475*a^8*b^4*c*d^9)/7 + (3960*a^2*b^10*c
^7*d^3)/7 + 3300*a^3*b^9*c^6*d^4 + 8910*a^4*b^8*c^5*d^5 + 11880*a^5*b^7*c^4
```

```
*d^6 + 7920*a^6*b^6*c^3*d^7 + (17820*a^7*b^5*c^2*d^8)/7) + a^12*c^10*x + (b
^12*d^10*x^23)/23 + 5*a^9*c^7*x^4*(6*a^3*d^3 + 11*b^3*c^3 + 33*a*b^2*c^2*d
+ 27*a^2*b*c*d^2) + b^9*d^7*x^20*(11*a^3*d^3 + 6*b^3*c^3 + 27*a*b^2*c^2*d +
 33*a^2*b*c*d^2) + a^11*c^9*x^2*(5*a*d + 6*b*c) + (b^11*d^9*x^22*(6*a*d + 5
*b*c))/11 + a^10*c^8*x^3*(15*a^2*d^2 + 22*b^2*c^2 + 40*a*b*c*d) + (b^10*d^8
*x^21*(22*a^2*d^2 + 15*b^2*c^2 + 40*a*b*c*d))/7
```

sympy [B] time = 0.37, size = 2088, normalized size = 7.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**12*(d*x+c)**10,x)
```

```
[Out] a**12*c**10*x + b**12*d**10*x**23/23 + x**22*(6*a*b**11*d**10/11 + 5*b**12*
c*d**9/11) + x**21*(22*a**2*b**10*d**10/7 + 40*a*b**11*c*d**9/7 + 15*b**12*
c**2*d**8/7) + x**20*(11*a**3*b**9*d**10 + 33*a**2*b**10*c*d**9 + 27*a*b**1
1*c**2*d**8 + 6*b**12*c**3*d**7) + x**19*(495*a**4*b**8*d**10/19 + 2200*a**
3*b**9*c*d**9/19 + 2970*a**2*b**10*c**2*d**8/19 + 1440*a*b**11*c**3*d**7/19
 + 210*b**12*c**4*d**6/19) + x**18*(44*a**5*b**7*d**10 + 275*a**4*b**8*c*d*
*9 + 550*a**3*b**9*c**2*d**8 + 440*a**2*b**10*c**3*d**7 + 140*a*b**11*c**4*
d**6 + 14*b**12*c**5*d**5) + x**17*(924*a**6*b**6*d**10/17 + 7920*a**5*b**7
*c*d**9/17 + 22275*a**4*b**8*c**2*d**8/17 + 26400*a**3*b**9*c**3*d**7/17 +
13860*a**2*b**10*c**4*d**6/17 + 3024*a*b**11*c**5*d**5/17 + 210*b**12*c**6*
d**4/17) + x**16*(99*a**7*b**5*d**10/2 + 1155*a**6*b**6*c*d**9/2 + 4455*a**
5*b**7*c**2*d**8/2 + 7425*a**4*b**8*c**3*d**7/2 + 5775*a**3*b**9*c**4*d**6/
2 + 2079*a**2*b**10*c**5*d**5/2 + 315*a*b**11*c**6*d**4/2 + 15*b**12*c**7*d
**3/2) + x**15*(33*a**8*b**4*d**10 + 528*a**7*b**5*c*d**9 + 2772*a**6*b**6*
c**2*d**8 + 6336*a**5*b**7*c**3*d**7 + 6930*a**4*b**8*c**4*d**6 + 3696*a**3
*b**9*c**5*d**5 + 924*a**2*b**10*c**6*d**4 + 96*a*b**11*c**7*d**3 + 3*b**12
*c**8*d**2) + x**14*(110*a**9*b**3*d**10/7 + 2475*a**8*b**4*c*d**9/7 + 1782
0*a**7*b**5*c**2*d**8/7 + 7920*a**6*b**6*c**3*d**7 + 11880*a**5*b**7*c**4*d
**6 + 8910*a**4*b**8*c**5*d**5 + 3300*a**3*b**9*c**6*d**4 + 3960*a**2*b**10
*c**7*d**3/7 + 270*a*b**11*c**8*d**2/7 + 5*b**12*c**9*d/7) + x**13*(66*a**1
0*b**2*d**10/13 + 2200*a**9*b**3*c*d**9/13 + 22275*a**8*b**4*c**2*d**8/13 +
 95040*a**7*b**5*c**3*d**7/13 + 194040*a**6*b**6*c**4*d**6/13 + 199584*a**5
*b**7*c**5*d**5/13 + 103950*a**4*b**8*c**6*d**4/13 + 26400*a**3*b**9*c**7*d
**3/13 + 2970*a**2*b**10*c**8*d**2/13 + 120*a*b**11*c**9*d/13 + b**12*c**10
/13) + x**12*(a**11*b*d**10 + 55*a**10*b**2*c*d**9 + 825*a**9*b**3*c**2*d**
8 + 4950*a**8*b**4*c**3*d**7 + 13860*a**7*b**5*c**4*d**6 + 19404*a**6*b**6*
c**5*d**5 + 13860*a**5*b**7*c**6*d**4 + 4950*a**4*b**8*c**7*d**3 + 825*a**3
*b**9*c**8*d**2 + 55*a**2*b**10*c**9*d + a*b**11*c**10) + x**11*(a**12*d**1
0/11 + 120*a**11*b*c*d**9/11 + 270*a**10*b**2*c**2*d**8 + 2400*a**9*b**3*c*
*3*d**7 + 9450*a**8*b**4*c**4*d**6 + 18144*a**7*b**5*c**5*d**5 + 17640*a**6
*b**6*c**6*d**4 + 8640*a**5*b**7*c**7*d**3 + 2025*a**4*b**8*c**8*d**2 + 200
*a**3*b**9*c**9*d + 6*a**2*b**10*c**10) + x**10*(a**12*c*d**9 + 54*a**11*b*
c**2*d**8 + 792*a**10*b**2*c**3*d**7 + 4620*a**9*b**3*c**4*d**6 + 12474*a**
8*b**4*c**5*d**5 + 16632*a**7*b**5*c**6*d**4 + 11088*a**6*b**6*c**7*d**3 +
3564*a**5*b**7*c**8*d**2 + 495*a**4*b**8*c**9*d + 22*a**3*b**9*c**10) + x**
9*(5*a**12*c**2*d**8 + 160*a**11*b*c**3*d**7 + 1540*a**10*b**2*c**4*d**6 +
6160*a**9*b**3*c**5*d**5 + 11550*a**8*b**4*c**6*d**4 + 10560*a**7*b**5*c**7
*d**3 + 4620*a**6*b**6*c**8*d**2 + 880*a**5*b**7*c**9*d + 55*a**4*b**8*c**1
0) + x**8*(15*a**12*c**3*d**7 + 315*a**11*b*c**4*d**6 + 2079*a**10*b**2*c**
5*d**5 + 5775*a**9*b**3*c**6*d**4 + 7425*a**8*b**4*c**7*d**3 + 4455*a**7*b*
*5*c**8*d**2 + 1155*a**6*b**6*c**9*d + 99*a**5*b**7*c**10) + x**7*(30*a**12
*c**4*d**6 + 432*a**11*b*c**5*d**5 + 1980*a**10*b**2*c**6*d**4 + 26400*a**9
*b**3*c**7*d**3/7 + 22275*a**8*b**4*c**8*d**2/7 + 7920*a**7*b**5*c**9*d/7 +
 132*a**6*b**6*c**10) + x**6*(42*a**12*c**5*d**5 + 420*a**11*b*c**6*d**4 +
1320*a**10*b**2*c**7*d**3 + 1650*a**9*b**3*c**8*d**2 + 825*a**8*b**4*c**9*d
 + 132*a**7*b**5*c**10) + x**5*(42*a**12*c**6*d**4 + 288*a**11*b*c**7*d**3
```

```
+ 594*a**10*b**2*c**8*d**2 + 440*a**9*b**3*c**9*d + 99*a**8*b**4*c**10) + x
**4*(30*a**12*c**7*d**3 + 135*a**11*b*c**8*d**2 + 165*a**10*b**2*c**9*d + 5
5*a**9*b**3*c**10) + x**3*(15*a**12*c**8*d**2 + 40*a**11*b*c**9*d + 22*a**1
0*b**2*c**10) + x**2*(5*a**12*c**9*d + 6*a**11*b*c**10)
```

3.1194 $\int (a+bx)^{11}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(\cdots}{}$$

Rubi [A] time = 1.28, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^{21}(bc-ad)}{21b^{11}}+\frac{9d^8(a+bx)^{20}(bc-ad)^2}{4b^{11}}+\frac{120d^7(a+bx)^{19}(bc-ad)^3}{19b^{11}}+\frac{35d^6(a+bx)^{18}(bc-ad)^4}{3b^{11}}+\frac{252d^5(a+bx)^{17}(bc-ad)^5}{17b^{11}}+\frac{105d^4(a+bx)^{16}(bc-ad)^6}{8b^{11}}+\frac{8d^3(a+bx)^{15}(bc-ad)^7}{b^{11}}+\frac{45d^2(a+bx)^{14}(bc-ad)^8}{14b^{11}}+\frac{10d(a+bx)^{13}(bc-ad)^9}{13b^{11}}+\frac{(a+bx)^{12}(bc-ad)^{10}}{12b^{11}}+\frac{d^{10}(a+bx)^{22}}{22b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^12)/(12*b^11) + (10*d*(b*c - a*d)^9*(a + b*x)^13)
/(13*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^14)/(14*b^11) + (8*d^3*(b*c -
a*d)^7*(a + b*x)^15)/b^11 + (105*d^4*(b*c - a*d)^6*(a + b*x)^16)/(8*b^11) +
 (252*d^5*(b*c - a*d)^5*(a + b*x)^17)/(17*b^11) + (35*d^6*(b*c - a*d)^4*(a
+ b*x)^18)/(3*b^11) + (120*d^7*(b*c - a*d)^3*(a + b*x)^19)/(19*b^11) + (9*d
^8*(b*c - a*d)^2*(a + b*x)^20)/(4*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^21)
/(21*b^11) + (d^10*(a + b*x)^22)/(22*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{11}(c+dx)^{10}\,dx &= \int \left(\frac{(bc-ad)^{10}(a+bx)^{11}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{12}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{b^{10}}+\cdots\right.\\ &= \frac{(bc-ad)^{10}(a+bx)^{12}}{12b^{11}}+\frac{10d(bc-ad)^9(a+bx)^{13}}{13b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{14}}{14b^{11}}+\frac{8d^3(b\cdots}{}\end{aligned}$$

Mathematica [B] time = 0.23, size = 1702, normalized size = 6.10

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] a^11*c^10*x + (a^10*c^9*(11*b*c + 10*a*d)*x^2)/2 + (5*a^9*c^8*(11*b^2*c^2 +
 22*a*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^8*c^7*(33*b^3*c^3 + 110*a*b^2*c^2*d
+ 99*a^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 3*a^7*c^6*(22*b^4*c^4 + 110*a*b^3*c
^3*d + 165*a^2*b^2*c^2*d^2 + 88*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + (a^6*c^5*(1
54*b^5*c^5 + 1100*a*b^4*c^4*d + 2475*a^2*b^3*c^3*d^2 + 2200*a^3*b^2*c^2*d^3
 + 770*a^4*b*c*d^4 + 84*a^5*d^5)*x^6)/2 + (6*a^5*c^4*(77*b^6*c^6 + 770*a*b^
5*c^5*d + 2475*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 1925*a^4*b^2*c^2*d^
4 + 462*a^5*b*c*d^5 + 35*a^6*d^6)*x^7)/7 + (15*a^4*c^3*(11*b^7*c^7 + 154*a*
b^6*c^6*d + 693*a^2*b^5*c^5*d^2 + 1320*a^3*b^4*c^4*d^3 + 1155*a^4*b^3*c^3*d
^4 + 462*a^5*b^2*c^2*d^5 + 77*a^6*b*c*d^6 + 4*a^7*d^7)*x^8)/4 + (5*a^3*c^2*
(11*b^8*c^8 + 220*a*b^7*c^7*d + 1386*a^2*b^6*c^6*d^2 + 3696*a^3*b^5*c^5*d^3
```

```
 + 4620*a^4*b^4*c^4*d^4 + 2772*a^5*b^3*c^3*d^5 + 770*a^6*b^2*c^2*d^6 + 88*a
^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 + (a^2*c*(11*b^9*c^9 + 330*a*b^8*c^8*d + 297
0*a^2*b^7*c^7*d^2 + 11088*a^3*b^6*c^6*d^3 + 19404*a^4*b^5*c^5*d^4 + 16632*a
^5*b^4*c^4*d^5 + 6930*a^6*b^3*c^3*d^6 + 1320*a^7*b^2*c^2*d^7 + 99*a^8*b*c*d
^8 + 2*a^9*d^9)*x^10)/2 + (a*(11*b^10*c^10 + 550*a*b^9*c^9*d + 7425*a^2*b^8
*c^8*d^2 + 39600*a^3*b^7*c^7*d^3 + 97020*a^4*b^6*c^6*d^4 + 116424*a^5*b^5*c
^5*d^5 + 69300*a^6*b^4*c^4*d^6 + 19800*a^7*b^3*c^3*d^7 + 2475*a^8*b^2*c^2*d
^8 + 110*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (b*(b^10*c^10 + 110*a*b^9*c^9*
d + 2475*a^2*b^8*c^8*d^2 + 19800*a^3*b^7*c^7*d^3 + 69300*a^4*b^6*c^6*d^4 +
116424*a^5*b^5*c^5*d^5 + 97020*a^6*b^4*c^4*d^6 + 39600*a^7*b^3*c^3*d^7 + 74
25*a^8*b^2*c^2*d^8 + 550*a^9*b*c*d^9 + 11*a^10*d^10)*x^12)/12 + (5*b^2*d*(2
*b^9*c^9 + 99*a*b^8*c^8*d + 1320*a^2*b^7*c^7*d^2 + 6930*a^3*b^6*c^6*d^3 + 1
6632*a^4*b^5*c^5*d^4 + 19404*a^5*b^4*c^4*d^5 + 11088*a^6*b^3*c^3*d^6 + 2970
*a^7*b^2*c^2*d^7 + 330*a^8*b*c*d^8 + 11*a^9*d^9)*x^13)/13 + (15*b^3*d^2*(3*
b^8*c^8 + 88*a*b^7*c^7*d + 770*a^2*b^6*c^6*d^2 + 2772*a^3*b^5*c^5*d^3 + 462
0*a^4*b^4*c^4*d^4 + 3696*a^5*b^3*c^3*d^5 + 1386*a^6*b^2*c^2*d^6 + 220*a^7*b
*c*d^7 + 11*a^8*d^8)*x^14)/14 + 2*b^4*d^3*(4*b^7*c^7 + 77*a*b^6*c^6*d + 462
*a^2*b^5*c^5*d^2 + 1155*a^3*b^4*c^4*d^3 + 1320*a^4*b^3*c^3*d^4 + 693*a^5*b^
2*c^2*d^5 + 154*a^6*b*c*d^6 + 11*a^7*d^7)*x^15 + (3*b^5*d^4*(35*b^6*c^6 + 4
62*a*b^5*c^5*d + 1925*a^2*b^4*c^4*d^2 + 3300*a^3*b^3*c^3*d^3 + 2475*a^4*b^2
*c^2*d^4 + 770*a^5*b*c*d^5 + 77*a^6*d^6)*x^16)/8 + (3*b^6*d^5*(84*b^5*c^5 +
 770*a*b^4*c^4*d + 2200*a^2*b^3*c^3*d^2 + 2475*a^3*b^2*c^2*d^3 + 1100*a^4*b
*c*d^4 + 154*a^5*d^5)*x^17)/17 + (5*b^7*d^6*(14*b^4*c^4 + 88*a*b^3*c^3*d +
165*a^2*b^2*c^2*d^2 + 110*a^3*b*c*d^3 + 22*a^4*d^4)*x^18)/6 + (5*b^8*d^7*(2
4*b^3*c^3 + 99*a*b^2*c^2*d + 110*a^2*b*c*d^2 + 33*a^3*d^3)*x^19)/19 + (b^9*
d^8*(9*b^2*c^2 + 22*a*b*c*d + 11*a^2*d^2)*x^20)/4 + (b^10*d^9*(10*b*c + 11*
a*d)*x^21)/21 + (b^11*d^10*x^22)/22
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{11}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^11*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^11*(c + d*x)^10, x]
```

fricas [B] time = 1.11, size = 2010, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/22*x^22*d^10*b^11 + 10/21*x^21*d^9*c*b^11 + 11/21*x^21*d^10*b^10*a + 9/4*
x^20*d^8*c^2*b^11 + 11/2*x^20*d^9*c*b^10*a + 11/4*x^20*d^10*b^9*a^2 + 120/1
9*x^19*d^7*c^3*b^11 + 495/19*x^19*d^8*c^2*b^10*a + 550/19*x^19*d^9*c*b^9*a^
2 + 165/19*x^19*d^10*b^8*a^3 + 35/3*x^18*d^6*c^4*b^11 + 220/3*x^18*d^7*c^3*
b^10*a + 275/2*x^18*d^8*c^2*b^9*a^2 + 275/3*x^18*d^9*c*b^8*a^3 + 55/3*x^18*
d^10*b^7*a^4 + 252/17*x^17*d^5*c^5*b^11 + 2310/17*x^17*d^6*c^4*b^10*a + 660
0/17*x^17*d^7*c^3*b^9*a^2 + 7425/17*x^17*d^8*c^2*b^8*a^3 + 3300/17*x^17*d^9
*c*b^7*a^4 + 462/17*x^17*d^10*b^6*a^5 + 105/8*x^16*d^4*c^6*b^11 + 693/4*x^1
6*d^5*c^5*b^10*a + 5775/8*x^16*d^6*c^4*b^9*a^2 + 2475/2*x^16*d^7*c^3*b^8*a^
3 + 7425/8*x^16*d^8*c^2*b^7*a^4 + 1155/4*x^16*d^9*c*b^6*a^5 + 231/8*x^16*d^
10*b^5*a^6 + 8*x^15*d^3*c^7*b^11 + 154*x^15*d^4*c^6*b^10*a + 924*x^15*d^5*c
^5*b^9*a^2 + 2310*x^15*d^6*c^4*b^8*a^3 + 2640*x^15*d^7*c^3*b^7*a^4 + 1386*x
^15*d^8*c^2*b^6*a^5 + 308*x^15*d^9*c*b^5*a^6 + 22*x^15*d^10*b^4*a^7 + 45/14
*x^14*d^2*c^8*b^11 + 660/7*x^14*d^3*c^7*b^10*a + 825*x^14*d^4*c^6*b^9*a^2 +
 2970*x^14*d^5*c^5*b^8*a^3 + 4950*x^14*d^6*c^4*b^7*a^4 + 3960*x^14*d^7*c^3*
b^6*a^5 + 1485*x^14*d^8*c^2*b^5*a^6 + 1650/7*x^14*d^9*c*b^4*a^7 + 165/14*x^
```

```
14*d^10*b^3*a^8 + 10/13*x^13*d*c^9*b^11 + 495/13*x^13*d^2*c^8*b^10*a + 6600
/13*x^13*d^3*c^7*b^9*a^2 + 34650/13*x^13*d^4*c^6*b^8*a^3 + 83160/13*x^13*d^
5*c^5*b^7*a^4 + 97020/13*x^13*d^6*c^4*b^6*a^5 + 55440/13*x^13*d^7*c^3*b^5*a
^6 + 14850/13*x^13*d^8*c^2*b^4*a^7 + 1650/13*x^13*d^9*c*b^3*a^8 + 55/13*x^1
3*d^10*b^2*a^9 + 1/12*x^12*c^10*b^11 + 55/6*x^12*d*c^9*b^10*a + 825/4*x^12*
d^2*c^8*b^9*a^2 + 1650*x^12*d^3*c^7*b^8*a^3 + 5775*x^12*d^4*c^6*b^7*a^4 + 9
702*x^12*d^5*c^5*b^6*a^5 + 8085*x^12*d^6*c^4*b^5*a^6 + 3300*x^12*d^7*c^3*b^
4*a^7 + 2475/4*x^12*d^8*c^2*b^3*a^8 + 275/6*x^12*d^9*c*b^2*a^9 + 11/12*x^12
*d^10*b*a^10 + x^11*c^10*b^10*a + 50*x^11*d*c^9*b^9*a^2 + 675*x^11*d^2*c^8*
b^8*a^3 + 3600*x^11*d^3*c^7*b^7*a^4 + 8820*x^11*d^4*c^6*b^6*a^5 + 10584*x^1
1*d^5*c^5*b^5*a^6 + 6300*x^11*d^6*c^4*b^4*a^7 + 1800*x^11*d^7*c^3*b^3*a^8 +
 225*x^11*d^8*c^2*b^2*a^9 + 10*x^11*d^9*c*b*a^10 + 1/11*x^11*d^10*a^11 + 11
/2*x^10*c^10*b^9*a^2 + 165*x^10*d*c^9*b^8*a^3 + 1485*x^10*d^2*c^8*b^7*a^4 +
 5544*x^10*d^3*c^7*b^6*a^5 + 9702*x^10*d^4*c^6*b^5*a^6 + 8316*x^10*d^5*c^5*
b^4*a^7 + 3465*x^10*d^6*c^4*b^3*a^8 + 660*x^10*d^7*c^3*b^2*a^9 + 99/2*x^10*
d^8*c^2*b*a^10 + x^10*d^9*c*a^11 + 55/3*x^9*c^10*b^8*a^3 + 1100/3*x^9*d*c^9
*b^7*a^4 + 2310*x^9*d^2*c^8*b^6*a^5 + 6160*x^9*d^3*c^7*b^5*a^6 + 7700*x^9*d
^4*c^6*b^4*a^7 + 4620*x^9*d^5*c^5*b^3*a^8 + 3850/3*x^9*d^6*c^4*b^2*a^9 + 44
0/3*x^9*d^7*c^3*b*a^10 + 5*x^9*d^8*c^2*a^11 + 165/4*x^8*c^10*b^7*a^4 + 1155
/2*x^8*d*c^9*b^6*a^5 + 10395/4*x^8*d^2*c^8*b^5*a^6 + 4950*x^8*d^3*c^7*b^4*a
^7 + 17325/4*x^8*d^4*c^6*b^3*a^8 + 3465/2*x^8*d^5*c^5*b^2*a^9 + 1155/4*x^8*
d^6*c^4*b*a^10 + 15*x^8*d^7*c^3*a^11 + 66*x^7*c^10*b^6*a^5 + 660*x^7*d*c^9*
b^5*a^6 + 14850/7*x^7*d^2*c^8*b^4*a^7 + 19800/7*x^7*d^3*c^7*b^3*a^8 + 1650*
x^7*d^4*c^6*b^2*a^9 + 396*x^7*d^5*c^5*b*a^10 + 30*x^7*d^6*c^4*a^11 + 77*x^6
*c^10*b^5*a^6 + 550*x^6*d*c^9*b^4*a^7 + 2475/2*x^6*d^2*c^8*b^3*a^8 + 1100*x
^6*d^3*c^7*b^2*a^9 + 385*x^6*d^4*c^6*b*a^10 + 42*x^6*d^5*c^5*a^11 + 66*x^5*
c^10*b^4*a^7 + 330*x^5*d*c^9*b^3*a^8 + 495*x^5*d^2*c^8*b^2*a^9 + 264*x^5*d^
3*c^7*b*a^10 + 42*x^5*d^4*c^6*a^11 + 165/4*x^4*c^10*b^3*a^8 + 275/2*x^4*d*c
^9*b^2*a^9 + 495/4*x^4*d^2*c^8*b*a^10 + 30*x^4*d^3*c^7*a^11 + 55/3*x^3*c^10
*b^2*a^9 + 110/3*x^3*d*c^9*b*a^10 + 15*x^3*d^2*c^8*a^11 + 11/2*x^2*c^10*b*a
^10 + 5*x^2*d*c^9*a^11 + x*c^10*a^11
```

giac [B] time = 1.35, size = 2010, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/22*b^11*d^10*x^22 + 10/21*b^11*c*d^9*x^21 + 11/21*a*b^10*d^10*x^21 + 9/4*
b^11*c^2*d^8*x^20 + 11/2*a*b^10*c*d^9*x^20 + 11/4*a^2*b^9*d^10*x^20 + 120/1
9*b^11*c^3*d^7*x^19 + 495/19*a*b^10*c^2*d^8*x^19 + 550/19*a^2*b^9*c*d^9*x^1
9 + 165/19*a^3*b^8*d^10*x^19 + 35/3*b^11*c^4*d^6*x^18 + 220/3*a*b^10*c^3*d^
7*x^18 + 275/2*a^2*b^9*c^2*d^8*x^18 + 275/3*a^3*b^8*c*d^9*x^18 + 55/3*a^4*b
^7*d^10*x^18 + 252/17*b^11*c^5*d^5*x^17 + 2310/17*a*b^10*c^4*d^6*x^17 + 660
0/17*a^2*b^9*c^3*d^7*x^17 + 7425/17*a^3*b^8*c^2*d^8*x^17 + 3300/17*a^4*b^7*
c*d^9*x^17 + 462/17*a^5*b^6*d^10*x^17 + 105/8*b^11*c^6*d^4*x^16 + 693/4*a*b
^10*c^5*d^5*x^16 + 5775/8*a^2*b^9*c^4*d^6*x^16 + 2475/2*a^3*b^8*c^3*d^7*x^1
6 + 7425/8*a^4*b^7*c^2*d^8*x^16 + 1155/4*a^5*b^6*c*d^9*x^16 + 231/8*a^6*b^5
*d^10*x^16 + 8*b^11*c^7*d^3*x^15 + 154*a*b^10*c^6*d^4*x^15 + 924*a^2*b^9*c^
5*d^5*x^15 + 2310*a^3*b^8*c^4*d^6*x^15 + 2640*a^4*b^7*c^3*d^7*x^15 + 1386*a
^5*b^6*c^2*d^8*x^15 + 308*a^6*b^5*c*d^9*x^15 + 22*a^7*b^4*d^10*x^15 + 45/14
*b^11*c^8*d^2*x^14 + 660/7*a*b^10*c^7*d^3*x^14 + 825*a^2*b^9*c^6*d^4*x^14 +
 2970*a^3*b^8*c^5*d^5*x^14 + 4950*a^4*b^7*c^4*d^6*x^14 + 3960*a^5*b^6*c^3*d
^7*x^14 + 1485*a^6*b^5*c^2*d^8*x^14 + 1650/7*a^7*b^4*c*d^9*x^14 + 165/14*a^
8*b^3*d^10*x^14 + 10/13*b^11*c^9*d*x^13 + 495/13*a*b^10*c^8*d^2*x^13 + 6600
/13*a^2*b^9*c^7*d^3*x^13 + 34650/13*a^3*b^8*c^6*d^4*x^13 + 83160/13*a^4*b^7
*c^5*d^5*x^13 + 97020/13*a^5*b^6*c^4*d^6*x^13 + 55440/13*a^6*b^5*c^3*d^7*x^
13 + 14850/13*a^7*b^4*c^2*d^8*x^13 + 1650/13*a^8*b^3*c*d^9*x^13 + 55/13*a^9
*b^2*d^10*x^13 + 1/12*b^11*c^10*x^12 + 55/6*a*b^10*c^9*d*x^12 + 825/4*a^2*b
```

```
^9*c^8*d^2*x^12 + 1650*a^3*b^8*c^7*d^3*x^12 + 5775*a^4*b^7*c^6*d^4*x^12 + 9
702*a^5*b^6*c^5*d^5*x^12 + 8085*a^6*b^5*c^4*d^6*x^12 + 3300*a^7*b^4*c^3*d^7
*x^12 + 2475/4*a^8*b^3*c^2*d^8*x^12 + 275/6*a^9*b^2*c*d^9*x^12 + 11/12*a^10
*b*d^10*x^12 + a*b^10*c^10*x^11 + 50*a^2*b^9*c^9*d*x^11 + 675*a^3*b^8*c^8*d
^2*x^11 + 3600*a^4*b^7*c^7*d^3*x^11 + 8820*a^5*b^6*c^6*d^4*x^11 + 10584*a^6
*b^5*c^5*d^5*x^11 + 6300*a^7*b^4*c^4*d^6*x^11 + 1800*a^8*b^3*c^3*d^7*x^11 +
 225*a^9*b^2*c^2*d^8*x^11 + 10*a^10*b*c*d^9*x^11 + 1/11*a^11*d^10*x^11 + 11
/2*a^2*b^9*c^10*x^10 + 165*a^3*b^8*c^9*d*x^10 + 1485*a^4*b^7*c^8*d^2*x^10 +
 5544*a^5*b^6*c^7*d^3*x^10 + 9702*a^6*b^5*c^6*d^4*x^10 + 8316*a^7*b^4*c^5*d
^5*x^10 + 3465*a^8*b^3*c^4*d^6*x^10 + 660*a^9*b^2*c^3*d^7*x^10 + 99/2*a^10*
b*c^2*d^8*x^10 + a^11*c*d^9*x^10 + 55/3*a^3*b^8*c^10*x^9 + 1100/3*a^4*b^7*c
^9*d*x^9 + 2310*a^5*b^6*c^8*d^2*x^9 + 6160*a^6*b^5*c^7*d^3*x^9 + 7700*a^7*b
^4*c^6*d^4*x^9 + 4620*a^8*b^3*c^5*d^5*x^9 + 3850/3*a^9*b^2*c^4*d^6*x^9 + 44
0/3*a^10*b*c^3*d^7*x^9 + 5*a^11*c^2*d^8*x^9 + 165/4*a^4*b^7*c^10*x^8 + 1155
/2*a^5*b^6*c^9*d*x^8 + 10395/4*a^6*b^5*c^8*d^2*x^8 + 4950*a^7*b^4*c^7*d^3*x
^8 + 17325/4*a^8*b^3*c^6*d^4*x^8 + 3465/2*a^9*b^2*c^5*d^5*x^8 + 1155/4*a^10
*b*c^4*d^6*x^8 + 15*a^11*c^3*d^7*x^8 + 66*a^5*b^6*c^10*x^7 + 660*a^6*b^5*c^
9*d*x^7 + 14850/7*a^7*b^4*c^8*d^2*x^7 + 19800/7*a^8*b^3*c^7*d^3*x^7 + 1650*
a^9*b^2*c^6*d^4*x^7 + 396*a^10*b*c^5*d^5*x^7 + 30*a^11*c^4*d^6*x^7 + 77*a^6
*b^5*c^10*x^6 + 550*a^7*b^4*c^9*d*x^6 + 2475/2*a^8*b^3*c^8*d^2*x^6 + 1100*a
^9*b^2*c^7*d^3*x^6 + 385*a^10*b*c^6*d^4*x^6 + 42*a^11*c^5*d^5*x^6 + 66*a^7*
b^4*c^10*x^5 + 330*a^8*b^3*c^9*d*x^5 + 495*a^9*b^2*c^8*d^2*x^5 + 264*a^10*b
*c^7*d^3*x^5 + 42*a^11*c^6*d^4*x^5 + 165/4*a^8*b^3*c^10*x^4 + 275/2*a^9*b^2
*c^9*d*x^4 + 495/4*a^10*b*c^8*d^2*x^4 + 30*a^11*c^7*d^3*x^4 + 55/3*a^9*b^2*
c^10*x^3 + 110/3*a^10*b*c^9*d*x^3 + 15*a^11*c^8*d^2*x^3 + 11/2*a^10*b*c^10*
x^2 + 5*a^11*c^9*d*x^2 + a^11*c^10*x
```

maple [B] time = 0.00, size = 1741, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^11*(d*x+c)^10,x)
```

```
[Out] 1/22*b^11*d^10*x^22+1/21*(11*a*b^10*d^10+10*b^11*c*d^9)*x^21+1/20*(55*a^2*b
^9*d^10+110*a*b^10*c*d^9+45*b^11*c^2*d^8)*x^20+1/19*(165*a^3*b^8*d^10+550*a
^2*b^9*c*d^9+495*a*b^10*c^2*d^8+120*b^11*c^3*d^7)*x^19+1/18*(330*a^4*b^7*d^
10+1650*a^3*b^8*c*d^9+2475*a^2*b^9*c^2*d^8+1320*a*b^10*c^3*d^7+210*b^11*c^4
*d^6)*x^18+1/17*(462*a^5*b^6*d^10+3300*a^4*b^7*c*d^9+7425*a^3*b^8*c^2*d^8+6
600*a^2*b^9*c^3*d^7+2310*a*b^10*c^4*d^6+252*b^11*c^5*d^5)*x^17+1/16*(462*a^
6*b^5*d^10+4620*a^5*b^6*c*d^9+14850*a^4*b^7*c^2*d^8+19800*a^3*b^8*c^3*d^7+1
1550*a^2*b^9*c^4*d^6+2772*a*b^10*c^5*d^5+210*b^11*c^6*d^4)*x^16+1/15*(330*a
^7*b^4*d^10+4620*a^6*b^5*c*d^9+20790*a^5*b^6*c^2*d^8+39600*a^4*b^7*c^3*d^7+
34650*a^3*b^8*c^4*d^6+13860*a^2*b^9*c^5*d^5+2310*a*b^10*c^6*d^4+120*b^11*c^
7*d^3)*x^15+1/14*(165*a^8*b^3*d^10+3300*a^7*b^4*c*d^9+20790*a^6*b^5*c^2*d^8
+55440*a^5*b^6*c^3*d^7+69300*a^4*b^7*c^4*d^6+41580*a^3*b^8*c^5*d^5+11550*a^
2*b^9*c^6*d^4+1320*a*b^10*c^7*d^3+45*b^11*c^8*d^2)*x^14+1/13*(55*a^9*b^2*d^
10+1650*a^8*b^3*c*d^9+14850*a^7*b^4*c^2*d^8+55440*a^6*b^5*c^3*d^7+97020*a^5
*b^6*c^4*d^6+83160*a^4*b^7*c^5*d^5+34650*a^3*b^8*c^6*d^4+6600*a^2*b^9*c^7*d
^3+495*a*b^10*c^8*d^2+10*b^11*c^9*d)*x^13+1/12*(11*a^10*b*d^10+550*a^9*b^2*
c*d^9+7425*a^8*b^3*c^2*d^8+39600*a^7*b^4*c^3*d^7+97020*a^6*b^5*c^4*d^6+1164
24*a^5*b^6*c^5*d^5+69300*a^4*b^7*c^6*d^4+19800*a^3*b^8*c^7*d^3+2475*a^2*b^9
*c^8*d^2+110*a*b^10*c^9*d+b^11*c^10)*x^12+1/11*(a^11*d^10+110*a^10*b*c*d^9+
2475*a^9*b^2*c^2*d^8+19800*a^8*b^3*c^3*d^7+69300*a^7*b^4*c^4*d^6+116424*a^6
*b^5*c^5*d^5+97020*a^5*b^6*c^6*d^4+39600*a^4*b^7*c^7*d^3+7425*a^3*b^8*c^8*d
^2+550*a^2*b^9*c^9*d+11*a*b^10*c^10)*x^11+1/10*(10*a^11*c*d^9+495*a^10*b*c^
2*d^8+6600*a^9*b^2*c^3*d^7+34650*a^8*b^3*c^4*d^6+83160*a^7*b^4*c^5*d^5+9702
0*a^6*b^5*c^6*d^4+55440*a^5*b^6*c^7*d^3+14850*a^4*b^7*c^8*d^2+1650*a^3*b^8*
c^9*d+55*a^2*b^9*c^10)*x^10+1/9*(45*a^11*c^2*d^8+1320*a^10*b*c^3*d^7+11550*
a^9*b^2*c^4*d^6+41580*a^8*b^3*c^5*d^5+69300*a^7*b^4*c^6*d^4+55440*a^6*b^5*c
```

```
^7*d^3+20790*a^5*b^6*c^8*d^2+3300*a^4*b^7*c^9*d+165*a^3*b^8*c^10)*x^9+1/8*(
120*a^11*c^3*d^7+2310*a^10*b*c^4*d^6+13860*a^9*b^2*c^5*d^5+34650*a^8*b^3*c^
6*d^4+39600*a^7*b^4*c^7*d^3+20790*a^6*b^5*c^8*d^2+4620*a^5*b^6*c^9*d+330*a^
4*b^7*c^10)*x^8+1/7*(210*a^11*c^4*d^6+2772*a^10*b*c^5*d^5+11550*a^9*b^2*c^6
*d^4+19800*a^8*b^3*c^7*d^3+14850*a^7*b^4*c^8*d^2+4620*a^6*b^5*c^9*d+462*a^5
*b^6*c^10)*x^7+1/6*(252*a^11*c^5*d^5+2310*a^10*b*c^6*d^4+6600*a^9*b^2*c^7*d
^3+7425*a^8*b^3*c^8*d^2+3300*a^7*b^4*c^9*d+462*a^6*b^5*c^10)*x^6+1/5*(210*a
^11*c^6*d^4+1320*a^10*b*c^7*d^3+2475*a^9*b^2*c^8*d^2+1650*a^8*b^3*c^9*d+330
*a^7*b^4*c^10)*x^5+1/4*(120*a^11*c^7*d^3+495*a^10*b*c^8*d^2+550*a^9*b^2*c^9
*d+165*a^8*b^3*c^10)*x^4+1/3*(45*a^11*c^8*d^2+110*a^10*b*c^9*d+55*a^9*b^2*c
^10)*x^3+1/2*(10*a^11*c^9*d+11*a^10*b*c^10)*x^2+a^11*c^10*x
```

maxima [B] time = 1.58, size = 1740, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^11*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/22*b^11*d^10*x^22 + a^11*c^10*x + 1/21*(10*b^11*c*d^9 + 11*a*b^10*d^10)*x
^21 + 1/4*(9*b^11*c^2*d^8 + 22*a*b^10*c*d^9 + 11*a^2*b^9*d^10)*x^20 + 5/19*
(24*b^11*c^3*d^7 + 99*a*b^10*c^2*d^8 + 110*a^2*b^9*c*d^9 + 33*a^3*b^8*d^10)
*x^19 + 5/6*(14*b^11*c^4*d^6 + 88*a*b^10*c^3*d^7 + 165*a^2*b^9*c^2*d^8 + 11
0*a^3*b^8*c*d^9 + 22*a^4*b^7*d^10)*x^18 + 3/17*(84*b^11*c^5*d^5 + 770*a*b^1
0*c^4*d^6 + 2200*a^2*b^9*c^3*d^7 + 2475*a^3*b^8*c^2*d^8 + 1100*a^4*b^7*c*d^
9 + 154*a^5*b^6*d^10)*x^17 + 3/8*(35*b^11*c^6*d^4 + 462*a*b^10*c^5*d^5 + 19
25*a^2*b^9*c^4*d^6 + 3300*a^3*b^8*c^3*d^7 + 2475*a^4*b^7*c^2*d^8 + 770*a^5*
b^6*c*d^9 + 77*a^6*b^5*d^10)*x^16 + 2*(4*b^11*c^7*d^3 + 77*a*b^10*c^6*d^4 +
 462*a^2*b^9*c^5*d^5 + 1155*a^3*b^8*c^4*d^6 + 1320*a^4*b^7*c^3*d^7 + 693*a^
5*b^6*c^2*d^8 + 154*a^6*b^5*c*d^9 + 11*a^7*b^4*d^10)*x^15 + 15/14*(3*b^11*c
^8*d^2 + 88*a*b^10*c^7*d^3 + 770*a^2*b^9*c^6*d^4 + 2772*a^3*b^8*c^5*d^5 + 4
620*a^4*b^7*c^4*d^6 + 3696*a^5*b^6*c^3*d^7 + 1386*a^6*b^5*c^2*d^8 + 220*a^7
*b^4*c*d^9 + 11*a^8*b^3*d^10)*x^14 + 5/13*(2*b^11*c^9*d + 99*a*b^10*c^8*d^2
 + 1320*a^2*b^9*c^7*d^3 + 6930*a^3*b^8*c^6*d^4 + 16632*a^4*b^7*c^5*d^5 + 19
404*a^5*b^6*c^4*d^6 + 11088*a^6*b^5*c^3*d^7 + 2970*a^7*b^4*c^2*d^8 + 330*a^
8*b^3*c*d^9 + 11*a^9*b^2*d^10)*x^13 + 1/12*(b^11*c^10 + 110*a*b^10*c^9*d +
2475*a^2*b^9*c^8*d^2 + 19800*a^3*b^8*c^7*d^3 + 69300*a^4*b^7*c^6*d^4 + 1164
24*a^5*b^6*c^5*d^5 + 97020*a^6*b^5*c^4*d^6 + 39600*a^7*b^4*c^3*d^7 + 7425*a
^8*b^3*c^2*d^8 + 550*a^9*b^2*c*d^9 + 11*a^10*b*d^10)*x^12 + 1/11*(11*a*b^10
*c^10 + 550*a^2*b^9*c^9*d + 7425*a^3*b^8*c^8*d^2 + 39600*a^4*b^7*c^7*d^3 +
97020*a^5*b^6*c^6*d^4 + 116424*a^6*b^5*c^5*d^5 + 69300*a^7*b^4*c^4*d^6 + 19
800*a^8*b^3*c^3*d^7 + 2475*a^9*b^2*c^2*d^8 + 110*a^10*b*c*d^9 + a^11*d^10)*
x^11 + 1/2*(11*a^2*b^9*c^10 + 330*a^3*b^8*c^9*d + 2970*a^4*b^7*c^8*d^2 + 11
088*a^5*b^6*c^7*d^3 + 19404*a^6*b^5*c^6*d^4 + 16632*a^7*b^4*c^5*d^5 + 6930*
a^8*b^3*c^4*d^6 + 1320*a^9*b^2*c^3*d^7 + 99*a^10*b*c^2*d^8 + 2*a^11*c*d^9)*
x^10 + 5/3*(11*a^3*b^8*c^10 + 220*a^4*b^7*c^9*d + 1386*a^5*b^6*c^8*d^2 + 36
96*a^6*b^5*c^7*d^3 + 4620*a^7*b^4*c^6*d^4 + 2772*a^8*b^3*c^5*d^5 + 770*a^9*
b^2*c^4*d^6 + 88*a^10*b*c^3*d^7 + 3*a^11*c^2*d^8)*x^9 + 15/4*(11*a^4*b^7*c^
10 + 154*a^5*b^6*c^9*d + 693*a^6*b^5*c^8*d^2 + 1320*a^7*b^4*c^7*d^3 + 1155*
a^8*b^3*c^6*d^4 + 462*a^9*b^2*c^5*d^5 + 77*a^10*b*c^4*d^6 + 4*a^11*c^3*d^7)
*x^8 + 6/7*(77*a^5*b^6*c^10 + 770*a^6*b^5*c^9*d + 2475*a^7*b^4*c^8*d^2 + 33
00*a^8*b^3*c^7*d^3 + 1925*a^9*b^2*c^6*d^4 + 462*a^10*b*c^5*d^5 + 35*a^11*c^
4*d^6)*x^7 + 1/2*(154*a^6*b^5*c^10 + 1100*a^7*b^4*c^9*d + 2475*a^8*b^3*c^8*
d^2 + 2200*a^9*b^2*c^7*d^3 + 770*a^10*b*c^6*d^4 + 84*a^11*c^5*d^5)*x^6 + 3*
(22*a^7*b^4*c^10 + 110*a^8*b^3*c^9*d + 165*a^9*b^2*c^8*d^2 + 88*a^10*b*c^7*
d^3 + 14*a^11*c^6*d^4)*x^5 + 5/4*(33*a^8*b^3*c^10 + 110*a^9*b^2*c^9*d + 99*
a^10*b*c^8*d^2 + 24*a^11*c^7*d^3)*x^4 + 5/3*(11*a^9*b^2*c^10 + 22*a^10*b*c^
9*d + 9*a^11*c^8*d^2)*x^3 + 1/2*(11*a^10*b*c^10 + 10*a^11*c^9*d)*x^2
```

mupad [B] time = 1.03, size = 1702, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^11*(c + d*x)^10,x)
```

```
[Out] x^7*(66*a^5*b^6*c^10 + 30*a^11*c^4*d^6 + 660*a^6*b^5*c^9*d + 396*a^10*b*c^5
*d^5 + (14850*a^7*b^4*c^8*d^2)/7 + (19800*a^8*b^3*c^7*d^3)/7 + 1650*a^9*b^2
*c^6*d^4) + x^16*((231*a^6*b^5*d^10)/8 + (105*b^11*c^6*d^4)/8 + (693*a*b^10
*c^5*d^5)/4 + (1155*a^5*b^6*c*d^9)/4 + (5775*a^2*b^9*c^4*d^6)/8 + (2475*a^3
*b^8*c^3*d^7)/2 + (7425*a^4*b^7*c^2*d^8)/8) + x^11*((a^11*d^10)/11 + a*b^10
*c^10 + 50*a^2*b^9*c^9*d + 675*a^3*b^8*c^8*d^2 + 3600*a^4*b^7*c^7*d^3 + 882
0*a^5*b^6*c^6*d^4 + 10584*a^6*b^5*c^5*d^5 + 6300*a^7*b^4*c^4*d^6 + 1800*a^8
*b^3*c^3*d^7 + 225*a^9*b^2*c^2*d^8 + 10*a^10*b*c*d^9) + x^12*((b^11*c^10)/1
2 + (11*a^10*b*d^10)/12 + (275*a^9*b^2*c*d^9)/6 + (825*a^2*b^9*c^8*d^2)/4 +
 1650*a^3*b^8*c^7*d^3 + 5775*a^4*b^7*c^6*d^4 + 9702*a^5*b^6*c^5*d^5 + 8085*
a^6*b^5*c^4*d^6 + 3300*a^7*b^4*c^3*d^7 + (2475*a^8*b^3*c^2*d^8)/4 + (55*a*b
^10*c^9*d)/6) + x^5*(66*a^7*b^4*c^10 + 42*a^11*c^6*d^4 + 330*a^8*b^3*c^9*d
+ 264*a^10*b*c^7*d^3 + 495*a^9*b^2*c^8*d^2) + x^18*((55*a^4*b^7*d^10)/3 + (
35*b^11*c^4*d^6)/3 + (220*a*b^10*c^3*d^7)/3 + (275*a^3*b^8*c*d^9)/3 + (275*
a^2*b^9*c^2*d^8)/2) + x^8*((165*a^4*b^7*c^10)/4 + 15*a^11*c^3*d^7 + (1155*a
^5*b^6*c^9*d)/2 + (1155*a^10*b*c^4*d^6)/4 + (10395*a^6*b^5*c^8*d^2)/4 + 495
0*a^7*b^4*c^7*d^3 + (17325*a^8*b^3*c^6*d^4)/4 + (3465*a^9*b^2*c^5*d^5)/2) +
 x^15*(22*a^7*b^4*d^10 + 8*b^11*c^7*d^3 + 154*a*b^10*c^6*d^4 + 308*a^6*b^5*
c*d^9 + 924*a^2*b^9*c^5*d^5 + 2310*a^3*b^8*c^4*d^6 + 2640*a^4*b^7*c^3*d^7 +
 1386*a^5*b^6*c^2*d^8) + x^6*(77*a^6*b^5*c^10 + 42*a^11*c^5*d^5 + 550*a^7*b
^4*c^9*d + 385*a^10*b*c^6*d^4 + (2475*a^8*b^3*c^8*d^2)/2 + 1100*a^9*b^2*c^7
*d^3) + x^17*((462*a^5*b^6*d^10)/17 + (252*b^11*c^5*d^5)/17 + (2310*a*b^10*
c^4*d^6)/17 + (3300*a^4*b^7*c*d^9)/17 + (6600*a^2*b^9*c^3*d^7)/17 + (7425*a
^3*b^8*c^2*d^8)/17) + x^9*((55*a^3*b^8*c^10)/3 + 5*a^11*c^2*d^8 + (1100*a^4
*b^7*c^9*d)/3 + (440*a^10*b*c^3*d^7)/3 + 2310*a^5*b^6*c^8*d^2 + 6160*a^6*b^
5*c^7*d^3 + 7700*a^7*b^4*c^6*d^4 + 4620*a^8*b^3*c^5*d^5 + (3850*a^9*b^2*c^4
*d^6)/3) + x^14*((165*a^8*b^3*d^10)/14 + (45*b^11*c^8*d^2)/14 + (660*a*b^10
*c^7*d^3)/7 + (1650*a^7*b^4*c*d^9)/7 + 825*a^2*b^9*c^6*d^4 + 2970*a^3*b^8*c
^5*d^5 + 4950*a^4*b^7*c^4*d^6 + 3960*a^5*b^6*c^3*d^7 + 1485*a^6*b^5*c^2*d^8
) + x^10*(a^11*c*d^9 + (11*a^2*b^9*c^10)/2 + 165*a^3*b^8*c^9*d + (99*a^10*b
*c^2*d^8)/2 + 1485*a^4*b^7*c^8*d^2 + 5544*a^5*b^6*c^7*d^3 + 9702*a^6*b^5*c^
6*d^4 + 8316*a^7*b^4*c^5*d^5 + 3465*a^8*b^3*c^4*d^6 + 660*a^9*b^2*c^3*d^7)
+ x^13*((10*b^11*c^9*d)/13 + (55*a^9*b^2*d^10)/13 + (495*a*b^10*c^8*d^2)/13
 + (1650*a^8*b^3*c*d^9)/13 + (6600*a^2*b^9*c^7*d^3)/13 + (34650*a^3*b^8*c^6
*d^4)/13 + (83160*a^4*b^7*c^5*d^5)/13 + (97020*a^5*b^6*c^4*d^6)/13 + (55440
*a^6*b^5*c^3*d^7)/13 + (14850*a^7*b^4*c^2*d^8)/13) + a^11*c^10*x + (b^11*d^
10*x^22)/22 + (5*a^8*c^7*x^4*(24*a^3*d^3 + 33*b^3*c^3 + 110*a*b^2*c^2*d + 9
9*a^2*b*c*d^2))/4 + (5*b^8*d^7*x^19*(33*a^3*d^3 + 24*b^3*c^3 + 99*a*b^2*c^2
*d + 110*a^2*b*c*d^2))/19 + (a^10*c^9*x^2*(10*a*d + 11*b*c))/2 + (b^10*d^9*
x^21*(11*a*d + 10*b*c))/21 + (5*a^9*c^8*x^3*(9*a^2*d^2 + 11*b^2*c^2 + 22*a*
b*c*d))/3 + (b^9*d^8*x^20*(11*a^2*d^2 + 9*b^2*c^2 + 22*a*b*c*d))/4
```

sympy [B] time = 0.34, size = 1965, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**11*(d*x+c)**10,x)
```

```
[Out] a**11*c**10*x + b**11*d**10*x**22/22 + x**21*(11*a*b**10*d**10/21 + 10*b**1
1*c*d**9/21) + x**20*(11*a**2*b**9*d**10/4 + 11*a*b**10*c*d**9/2 + 9*b**11*
c**2*d**8/4) + x**19*(165*a**3*b**8*d**10/19 + 550*a**2*b**9*c*d**9/19 + 49
5*a*b**10*c**2*d**8/19 + 120*b**11*c**3*d**7/19) + x**18*(55*a**4*b**7*d**1
0/3 + 275*a**3*b**8*c*d**9/3 + 275*a**2*b**9*c**2*d**8/2 + 220*a*b**10*c**3
*d**7/3 + 35*b**11*c**4*d**6/3) + x**17*(462*a**5*b**6*d**10/17 + 3300*a**4
```

```
*b**7*c*d**9/17 + 7425*a**3*b**8*c**2*d**8/17 + 6600*a**2*b**9*c**3*d**7/17
 + 2310*a*b**10*c**4*d**6/17 + 252*b**11*c**5*d**5/17) + x**16*(231*a**6*b*
*5*d**10/8 + 1155*a**5*b**6*c*d**9/4 + 7425*a**4*b**7*c**2*d**8/8 + 2475*a*
*3*b**8*c**3*d**7/2 + 5775*a**2*b**9*c**4*d**6/8 + 693*a*b**10*c**5*d**5/4
+ 105*b**11*c**6*d**4/8) + x**15*(22*a**7*b**4*d**10 + 308*a**6*b**5*c*d**9
 + 1386*a**5*b**6*c**2*d**8 + 2640*a**4*b**7*c**3*d**7 + 2310*a**3*b**8*c**
4*d**6 + 924*a**2*b**9*c**5*d**5 + 154*a*b**10*c**6*d**4 + 8*b**11*c**7*d**
3) + x**14*(165*a**8*b**3*d**10/14 + 1650*a**7*b**4*c*d**9/7 + 1485*a**6*b*
*5*c**2*d**8 + 3960*a**5*b**6*c**3*d**7 + 4950*a**4*b**7*c**4*d**6 + 2970*a
**3*b**8*c**5*d**5 + 825*a**2*b**9*c**6*d**4 + 660*a*b**10*c**7*d**3/7 + 45
*b**11*c**8*d**2/14) + x**13*(55*a**9*b**2*d**10/13 + 1650*a**8*b**3*c*d**9
/13 + 14850*a**7*b**4*c**2*d**8/13 + 55440*a**6*b**5*c**3*d**7/13 + 97020*a
**5*b**6*c**4*d**6/13 + 83160*a**4*b**7*c**5*d**5/13 + 34650*a**3*b**8*c**6
*d**4/13 + 6600*a**2*b**9*c**7*d**3/13 + 495*a*b**10*c**8*d**2/13 + 10*b**1
1*c**9*d/13) + x**12*(11*a**10*b*d**10/12 + 275*a**9*b**2*c*d**9/6 + 2475*a
**8*b**3*c**2*d**8/4 + 3300*a**7*b**4*c**3*d**7 + 8085*a**6*b**5*c**4*d**6
+ 9702*a**5*b**6*c**5*d**5 + 5775*a**4*b**7*c**6*d**4 + 1650*a**3*b**8*c**7
*d**3 + 825*a**2*b**9*c**8*d**2/4 + 55*a*b**10*c**9*d/6 + b**11*c**10/12) +
 x**11*(a**11*d**10/11 + 10*a**10*b*c*d**9 + 225*a**9*b**2*c**2*d**8 + 1800
*a**8*b**3*c**3*d**7 + 6300*a**7*b**4*c**4*d**6 + 10584*a**6*b**5*c**5*d**5
 + 8820*a**5*b**6*c**6*d**4 + 3600*a**4*b**7*c**7*d**3 + 675*a**3*b**8*c**8
*d**2 + 50*a**2*b**9*c**9*d + a*b**10*c**10) + x**10*(a**11*c*d**9 + 99*a**
10*b*c**2*d**8/2 + 660*a**9*b**2*c**3*d**7 + 3465*a**8*b**3*c**4*d**6 + 831
6*a**7*b**4*c**5*d**5 + 9702*a**6*b**5*c**6*d**4 + 5544*a**5*b**6*c**7*d**3
 + 1485*a**4*b**7*c**8*d**2 + 165*a**3*b**8*c**9*d + 11*a**2*b**9*c**10/2)
+ x**9*(5*a**11*c**2*d**8 + 440*a**10*b*c**3*d**7/3 + 3850*a**9*b**2*c**4*d
**6/3 + 4620*a**8*b**3*c**5*d**5 + 7700*a**7*b**4*c**6*d**4 + 6160*a**6*b**
5*c**7*d**3 + 2310*a**5*b**6*c**8*d**2 + 1100*a**4*b**7*c**9*d/3 + 55*a**3*
b**8*c**10/3) + x**8*(15*a**11*c**3*d**7 + 1155*a**10*b*c**4*d**6/4 + 3465*
a**9*b**2*c**5*d**5/2 + 17325*a**8*b**3*c**6*d**4/4 + 4950*a**7*b**4*c**7*d
**3 + 10395*a**6*b**5*c**8*d**2/4 + 1155*a**5*b**6*c**9*d/2 + 165*a**4*b**7
*c**10/4) + x**7*(30*a**11*c**4*d**6 + 396*a**10*b*c**5*d**5 + 1650*a**9*b*
*2*c**6*d**4 + 19800*a**8*b**3*c**7*d**3/7 + 14850*a**7*b**4*c**8*d**2/7 +
660*a**6*b**5*c**9*d + 66*a**5*b**6*c**10) + x**6*(42*a**11*c**5*d**5 + 385
*a**10*b*c**6*d**4 + 1100*a**9*b**2*c**7*d**3 + 2475*a**8*b**3*c**8*d**2/2
+ 550*a**7*b**4*c**9*d + 77*a**6*b**5*c**10) + x**5*(42*a**11*c**6*d**4 + 2
64*a**10*b*c**7*d**3 + 495*a**9*b**2*c**8*d**2 + 330*a**8*b**3*c**9*d + 66*
a**7*b**4*c**10) + x**4*(30*a**11*c**7*d**3 + 495*a**10*b*c**8*d**2/4 + 275
*a**9*b**2*c**9*d/2 + 165*a**8*b**3*c**10/4) + x**3*(15*a**11*c**8*d**2 + 1
10*a**10*b*c**9*d/3 + 55*a**9*b**2*c**10/3) + x**2*(5*a**11*c**9*d + 11*a**
10*b*c**10/2)
```

3.1195 $\int (a+bx)^{10}(c+dx)^{10}\,dx$

Optimal. Leaf size=279

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5}{}$$

Rubi [A] time = 1.11, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{d^9(a+bx)^{20}(bc-ad)}{2b^{11}}+\frac{45d^8(a+bx)^{19}(bc-ad)^2}{19b^{11}}+\frac{20d^7(a+bx)^{18}(bc-ad)^3}{3b^{11}}+\frac{210d^6(a+bx)^{17}(bc-ad)^4}{17b^{11}}+\frac{63d^5(a+bx)^{16}(bc-ad)^5}{4b^{11}}+\frac{14d^4(a+bx)^{15}(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^{14}(bc-ad)^7}{7b^{11}}+\frac{45d^2(a+bx)^{13}(bc-ad)^8}{13b^{11}}+\frac{5d(a+bx)^{12}(bc-ad)^9}{6b^{11}}+\frac{(a+bx)^{11}(bc-ad)^{10}}{11b^{11}}+\frac{d^{10}(a+bx)^{21}}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^10*(a + b*x)^11)/(11*b^11) + (5*d*(b*c - a*d)^9*(a + b*x)^12)/
(6*b^11) + (45*d^2*(b*c - a*d)^8*(a + b*x)^13)/(13*b^11) + (60*d^3*(b*c - a
*d)^7*(a + b*x)^14)/(7*b^11) + (14*d^4*(b*c - a*d)^6*(a + b*x)^15)/b^11 + (
63*d^5*(b*c - a*d)^5*(a + b*x)^16)/(4*b^11) + (210*d^6*(b*c - a*d)^4*(a + b
*x)^17)/(17*b^11) + (20*d^7*(b*c - a*d)^3*(a + b*x)^18)/(3*b^11) + (45*d^8*
(b*c - a*d)^2*(a + b*x)^19)/(19*b^11) + (d^9*(b*c - a*d)*(a + b*x)^20)/(2*b
^11) + (d^10*(a + b*x)^21)/(21*b^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^{10}(c+dx)^{10}\,dx = \int \left(\frac{(bc-ad)^{10}(a+bx)^{10}}{b^{10}}+\frac{10d(bc-ad)^9(a+bx)^{11}}{b^{10}}+\frac{45d^2(bc-ad)^8(a+bx)^{12}}{b^{10}}+\right.$$
$$=\frac{(bc-ad)^{10}(a+bx)^{11}}{11b^{11}}+\frac{5d(bc-ad)^9(a+bx)^{12}}{6b^{11}}+\frac{45d^2(bc-ad)^8(a+bx)^{13}}{13b^{11}}+\frac{60d}{}$$

Mathematica [B] time = 0.17, size = 1539, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] a^10*c^10*x + 5*a^9*c^9*(b*c + a*d)*x^2 + (5*a^8*c^8*(9*b^2*c^2 + 20*a*b*c*
d + 9*a^2*d^2)*x^3)/3 + (15*a^7*c^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*
c*d^2 + 4*a^3*d^3)*x^4)/2 + 3*a^6*c^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^
2*b^2*c^2*d^2 + 80*a^3*b*c*d^3 + 14*a^4*d^4)*x^5 + 2*a^5*c^5*(21*b^5*c^5 +
175*a*b^4*c^4*d + 450*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d
^4 + 21*a^5*d^5)*x^6 + (30*a^4*c^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 315*a^2*b^
4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c*d^5 + 7*
a^6*d^6)*x^7)/7 + (15*a^3*c^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5
*d^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35
*a^6*b*c*d^6 + 2*a^7*d^7)*x^8)/2 + (5*a^2*c^2*(3*b^8*c^8 + 80*a*b^7*c^7*d +
 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2016*a
```

```
^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*d^8)*x^9)/3 +
 a*c*(b^9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3
 + 5292*a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540
*a^7*b^2*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^10 + ((b^10*c^10 + 100*a*b^9
*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100*a^4*b^6*c^6*d
^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^7*b^3*c^3*d^7
+ 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11)/11 + (5*b*d*(b^
9*c^9 + 45*a*b^8*c^8*d + 540*a^2*b^7*c^7*d^2 + 2520*a^3*b^6*c^6*d^3 + 5292*
a^4*b^5*c^5*d^4 + 5292*a^5*b^4*c^4*d^5 + 2520*a^6*b^3*c^3*d^6 + 540*a^7*b^2
*c^2*d^7 + 45*a^8*b*c*d^8 + a^9*d^9)*x^12)/6 + (15*b^2*d^2*(3*b^8*c^8 + 80*
a*b^7*c^7*d + 630*a^2*b^6*c^6*d^2 + 2016*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4
*d^4 + 2016*a^5*b^3*c^3*d^5 + 630*a^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + 3*a^8*
d^8)*x^13)/13 + (30*b^3*d^3*(2*b^7*c^7 + 35*a*b^6*c^6*d + 189*a^2*b^5*c^5*d
^2 + 420*a^3*b^4*c^4*d^3 + 420*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 + 35*a
^6*b*c*d^6 + 2*a^7*d^7)*x^14)/7 + 2*b^4*d^4*(7*b^6*c^6 + 84*a*b^5*c^5*d + 3
15*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c^3*d^3 + 315*a^4*b^2*c^2*d^4 + 84*a^5*b*c
*d^5 + 7*a^6*d^6)*x^15 + (3*b^5*d^5*(21*b^5*c^5 + 175*a*b^4*c^4*d + 450*a^2
*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 175*a^4*b*c*d^4 + 21*a^5*d^5)*x^16)/4
+ (15*b^6*d^6*(14*b^4*c^4 + 80*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 80*a^3*b
*c*d^3 + 14*a^4*d^4)*x^17)/17 + (5*b^7*d^7*(4*b^3*c^3 + 15*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + 4*a^3*d^3)*x^18)/3 + (5*b^8*d^8*(9*b^2*c^2 + 20*a*b*c*d + 9*
a^2*d^2)*x^19)/19 + (b^9*d^9*(b*c + a*d)*x^20)/2 + (b^10*d^10*x^21)/21
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{10}(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^10*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^10*(c + d*x)^10, x]
```

fricas [B] time = 0.95, size = 1833, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^10*b^10 + 1/2*x^20*d^9*c*b^10 + 1/2*x^20*d^10*b^9*a + 45/19*x^1
9*d^8*c^2*b^10 + 100/19*x^19*d^9*c*b^9*a + 45/19*x^19*d^10*b^8*a^2 + 20/3*x
^18*d^7*c^3*b^10 + 25*x^18*d^8*c^2*b^9*a + 25*x^18*d^9*c*b^8*a^2 + 20/3*x^1
8*d^10*b^7*a^3 + 210/17*x^17*d^6*c^4*b^10 + 1200/17*x^17*d^7*c^3*b^9*a + 20
25/17*x^17*d^8*c^2*b^8*a^2 + 1200/17*x^17*d^9*c*b^7*a^3 + 210/17*x^17*d^10*
b^6*a^4 + 63/4*x^16*d^5*c^5*b^10 + 525/4*x^16*d^6*c^4*b^9*a + 675/2*x^16*d^
7*c^3*b^8*a^2 + 675/2*x^16*d^8*c^2*b^7*a^3 + 525/4*x^16*d^9*c*b^6*a^4 + 63/
4*x^16*d^10*b^5*a^5 + 14*x^15*d^4*c^6*b^10 + 168*x^15*d^5*c^5*b^9*a + 630*x
^15*d^6*c^4*b^8*a^2 + 960*x^15*d^7*c^3*b^7*a^3 + 630*x^15*d^8*c^2*b^6*a^4 +
 168*x^15*d^9*c*b^5*a^5 + 14*x^15*d^10*b^4*a^6 + 60/7*x^14*d^3*c^7*b^10 + 1
50*x^14*d^4*c^6*b^9*a + 810*x^14*d^5*c^5*b^8*a^2 + 1800*x^14*d^6*c^4*b^7*a^
3 + 1800*x^14*d^7*c^3*b^6*a^4 + 810*x^14*d^8*c^2*b^5*a^5 + 150*x^14*d^9*c*b
^4*a^6 + 60/7*x^14*d^10*b^3*a^7 + 45/13*x^13*d^2*c^8*b^10 + 1200/13*x^13*d^
3*c^7*b^9*a + 9450/13*x^13*d^4*c^6*b^8*a^2 + 30240/13*x^13*d^5*c^5*b^7*a^3
+ 44100/13*x^13*d^6*c^4*b^6*a^4 + 30240/13*x^13*d^7*c^3*b^5*a^5 + 9450/13*x
^13*d^8*c^2*b^4*a^6 + 1200/13*x^13*d^9*c*b^3*a^7 + 45/13*x^13*d^10*b^2*a^8
+ 5/6*x^12*d*c^9*b^10 + 75/2*x^12*d^2*c^8*b^9*a + 450*x^12*d^3*c^7*b^8*a^2
+ 2100*x^12*d^4*c^6*b^7*a^3 + 4410*x^12*d^5*c^5*b^6*a^4 + 4410*x^12*d^6*c^4
*b^5*a^5 + 2100*x^12*d^7*c^3*b^4*a^6 + 450*x^12*d^8*c^2*b^3*a^7 + 75/2*x^12
*d^9*c*b^2*a^8 + 5/6*x^12*d^10*b*a^9 + 1/11*x^11*c^10*b^10 + 100/11*x^11*d*
c^9*b^9*a + 2025/11*x^11*d^2*c^8*b^8*a^2 + 14400/11*x^11*d^3*c^7*b^7*a^3 +
```

44100/11*x^11*d^4*c^6*b^6*a^4 + 63504/11*x^11*d^5*c^5*b^5*a^5 + 44100/11*x^11*d^6*c^4*b^4*a^6 + 14400/11*x^11*d^7*c^3*b^3*a^7 + 2025/11*x^11*d^8*c^2*b^2*a^8 + 100/11*x^11*d^9*c*b*a^9 + 1/11*x^11*d^10*a^10 + x^10*c^10*b^9*a + 45*x^10*d*c^9*b^8*a^2 + 540*x^10*d^2*c^8*b^7*a^3 + 2520*x^10*d^3*c^7*b^6*a^4 + 5292*x^10*d^4*c^6*b^5*a^5 + 5292*x^10*d^5*c^5*b^4*a^6 + 2520*x^10*d^6*c^4*b^3*a^7 + 540*x^10*d^7*c^3*b^2*a^8 + 45*x^10*d^8*c^2*b*a^9 + x^10*d^9*c*a^10 + 5*x^9*c^10*b^8*a^2 + 400/3*x^9*d*c^9*b^7*a^3 + 1050*x^9*d^2*c^8*b^6*a^4 + 3360*x^9*d^3*c^7*b^5*a^5 + 4900*x^9*d^4*c^6*b^4*a^6 + 3360*x^9*d^5*c^5*b^3*a^7 + 1050*x^9*d^6*c^4*b^2*a^8 + 400/3*x^9*d^7*c^3*b*a^9 + 5*x^9*d^8*c^2*a^10 + 15*x^8*c^10*b^7*a^3 + 525/2*x^8*d*c^9*b^6*a^4 + 2835/2*x^8*d^2*c^8*b^5*a^5 + 3150*x^8*d^3*c^7*b^4*a^6 + 3150*x^8*d^4*c^6*b^3*a^7 + 2835/2*x^8*d^5*c^5*b^2*a^8 + 525/2*x^8*d^6*c^4*b*a^9 + 15*x^8*d^7*c^3*a^10 + 30*x^7*c^10*b^6*a^4 + 360*x^7*d*c^9*b^5*a^5 + 1350*x^7*d^2*c^8*b^4*a^6 + 14400/7*x^7*d^3*c^7*b^3*a^7 + 1350*x^7*d^4*c^6*b^2*a^8 + 360*x^7*d^5*c^5*b*a^9 + 30*x^7*d^6*c^4*a^10 + 42*x^6*c^10*b^5*a^5 + 350*x^6*d*c^9*b^4*a^6 + 900*x^6*d^2*c^8*b^3*a^7 + 900*x^6*d^3*c^7*b^2*a^8 + 350*x^6*d^4*c^6*b*a^9 + 42*x^6*d^5*c^5*a^10 + 42*x^5*c^10*b^4*a^6 + 240*x^5*d*c^9*b^3*a^7 + 405*x^5*d^2*c^8*b^2*a^8 + 240*x^5*d^3*c^7*b*a^9 + 42*x^5*d^4*c^6*a^10 + 30*x^4*c^10*b^3*a^7 + 225/2*x^4*d*c^9*b^2*a^8 + 225/2*x^4*d^2*c^8*b*a^9 + 30*x^4*d^3*c^7*a^10 + 15*x^3*c^10*b^2*a^8 + 100/3*x^3*d*c^9*b*a^9 + 15*x^3*d^2*c^8*a^10 + 5*x^2*c^10*b*a^9 + 5*x^2*d*c^9*a^10 + x*c^10*a^10

giac [B] time = 1.34, size = 1833, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/21*b^10*d^10*x^21 + 1/2*b^10*c*d^9*x^20 + 1/2*a*b^9*d^10*x^20 + 45/19*b^10*c^2*d^8*x^19 + 100/19*a*b^9*c*d^9*x^19 + 45/19*a^2*b^8*d^10*x^19 + 20/3*b^10*c^3*d^7*x^18 + 25*a*b^9*c^2*d^8*x^18 + 25*a^2*b^8*c*d^9*x^18 + 20/3*a^3*b^7*d^10*x^18 + 210/17*b^10*c^4*d^6*x^17 + 1200/17*a*b^9*c^3*d^7*x^17 + 2025/17*a^2*b^8*c^2*d^8*x^17 + 1200/17*a^3*b^7*c*d^9*x^17 + 210/17*a^4*b^6*d^10*x^17 + 63/4*b^10*c^5*d^5*x^16 + 525/4*a*b^9*c^4*d^6*x^16 + 675/2*a^2*b^8*c^3*d^7*x^16 + 675/2*a^3*b^7*c^2*d^8*x^16 + 525/4*a^4*b^6*c*d^9*x^16 + 63/4*a^5*b^5*d^10*x^16 + 14*b^10*c^6*d^4*x^15 + 168*a*b^9*c^5*d^5*x^15 + 630*a^2*b^8*c^4*d^6*x^15 + 960*a^3*b^7*c^3*d^7*x^15 + 630*a^4*b^6*c^2*d^8*x^15 + 168*a^5*b^5*c*d^9*x^15 + 14*a^6*b^4*d^10*x^15 + 60/7*b^10*c^7*d^3*x^14 + 150*a*b^9*c^6*d^4*x^14 + 810*a^2*b^8*c^5*d^5*x^14 + 1800*a^3*b^7*c^4*d^6*x^14 + 1800*a^4*b^6*c^3*d^7*x^14 + 810*a^5*b^5*c^2*d^8*x^14 + 150*a^6*b^4*c*d^9*x^14 + 60/7*a^7*b^3*d^10*x^14 + 45/13*b^10*c^8*d^2*x^13 + 1200/13*a*b^9*c^7*d^3*x^13 + 9450/13*a^2*b^8*c^6*d^4*x^13 + 30240/13*a^3*b^7*c^5*d^5*x^13 + 44100/13*a^4*b^6*c^4*d^6*x^13 + 30240/13*a^5*b^5*c^3*d^7*x^13 + 9450/13*a^6*b^4*c^2*d^8*x^13 + 1200/13*a^7*b^3*c*d^9*x^13 + 45/13*a^8*b^2*d^10*x^13 + 5/6*b^10*c^9*d*x^12 + 75/2*a*b^9*c^8*d^2*x^12 + 450*a^2*b^8*c^7*d^3*x^12 + 2100*a^3*b^7*c^6*d^4*x^12 + 4410*a^4*b^6*c^5*d^5*x^12 + 4410*a^5*b^5*c^4*d^6*x^12 + 2100*a^6*b^4*c^3*d^7*x^12 + 450*a^7*b^3*c^2*d^8*x^12 + 75/2*a^8*b^2*c*d^9*x^12 + 5/6*a^9*b*d^10*x^12 + 1/11*b^10*c^10*x^11 + 100/11*a*b^9*c^9*d*x^11 + 2025/11*a^2*b^8*c^8*d^2*x^11 + 14400/11*a^3*b^7*c^7*d^3*x^11 + 44100/11*a^4*b^6*c^6*d^4*x^11 + 63504/11*a^5*b^5*c^5*d^5*x^11 + 44100/11*a^6*b^4*c^4*d^6*x^11 + 14400/11*a^7*b^3*c^3*d^7*x^11 + 2025/11*a^8*b^2*c^2*d^8*x^11 + 100/11*a^9*b*c*d^9*x^11 + 1/11*a^10*d^10*x^11 + a*b^9*c^10*x^10 + 45*a^2*b^8*c^9*d*x^10 + 540*a^3*b^7*c^8*d^2*x^10 + 2520*a^4*b^6*c^7*d^3*x^10 + 5292*a^5*b^5*c^6*d^4*x^10 + 5292*a^6*b^4*c^5*d^5*x^10 + 2520*a^7*b^3*c^4*d^6*x^10 + 540*a^8*b^2*c^3*d^7*x^10 + 45*a^9*b*c^2*d^8*x^10 + a^10*c*d^9*x^10 + 5*a^2*b^8*c^10*x^9 + 400/3*a^3*b^7*c^9*d*x^9 + 1050*a^4*b^6*c^8*d^2*x^9 + 3360*a^5*b^5*c^7*d^3*x^9 + 4900*a^6*b^4*c^6*d^4*x^9 + 3360*a^7*b^3*c^5*d^5*x^9 + 1050*a^8*b^2*c^4*d^6*x^9 + 400/3*a^9*b*c^3*d^7*x^9 + 5*a^10*c^2*d^8*x^9 + 15*a^3*b^7*c^10*x^8 + 525/2*a^4*b^6*c^9*d*x^8 + 2835/2*a^5*b^5*c
```

```
^8*d^2*x^8 + 3150*a^6*b^4*c^7*d^3*x^8 + 3150*a^7*b^3*c^6*d^4*x^8 + 2835/2*a
^8*b^2*c^5*d^5*x^8 + 525/2*a^9*b*c^4*d^6*x^8 + 15*a^10*c^3*d^7*x^8 + 30*a^4
*b^6*c^10*x^7 + 360*a^5*b^5*c^9*d*x^7 + 1350*a^6*b^4*c^8*d^2*x^7 + 14400/7*
a^7*b^3*c^7*d^3*x^7 + 1350*a^8*b^2*c^6*d^4*x^7 + 360*a^9*b*c^5*d^5*x^7 + 30
*a^10*c^4*d^6*x^7 + 42*a^5*b^5*c^10*x^6 + 350*a^6*b^4*c^9*d*x^6 + 900*a^7*b
^3*c^8*d^2*x^6 + 900*a^8*b^2*c^7*d^3*x^6 + 350*a^9*b*c^6*d^4*x^6 + 42*a^10*
c^5*d^5*x^6 + 42*a^6*b^4*c^10*x^5 + 240*a^7*b^3*c^9*d*x^5 + 405*a^8*b^2*c^8
*d^2*x^5 + 240*a^9*b*c^7*d^3*x^5 + 42*a^10*c^6*d^4*x^5 + 30*a^7*b^3*c^10*x^
4 + 225/2*a^8*b^2*c^9*d*x^4 + 225/2*a^9*b*c^8*d^2*x^4 + 30*a^10*c^7*d^3*x^4
 + 15*a^8*b^2*c^10*x^3 + 100/3*a^9*b*c^9*d*x^3 + 15*a^10*c^8*d^2*x^3 + 5*a^
9*b*c^10*x^2 + 5*a^10*c^9*d*x^2 + a^10*c^10*x
```

maple [B] time = 0.00, size = 1591, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^10*(d*x+c)^10,x)
```

```
[Out] 1/21*b^10*d^10*x^21+1/20*(10*a*b^9*d^10+10*b^10*c*d^9)*x^20+1/19*(45*a^2*b^
8*d^10+100*a*b^9*c*d^9+45*b^10*c^2*d^8)*x^19+1/18*(120*a^3*b^7*d^10+450*a^2
*b^8*c*d^9+450*a*b^9*c^2*d^8+120*b^10*c^3*d^7)*x^18+1/17*(210*a^4*b^6*d^10+
1200*a^3*b^7*c*d^9+2025*a^2*b^8*c^2*d^8+1200*a*b^9*c^3*d^7+210*b^10*c^4*d^6
)*x^17+1/16*(252*a^5*b^5*d^10+2100*a^4*b^6*c*d^9+5400*a^3*b^7*c^2*d^8+5400*
a^2*b^8*c^3*d^7+2100*a*b^9*c^4*d^6+252*b^10*c^5*d^5)*x^16+1/15*(210*a^6*b^4
*d^10+2520*a^5*b^5*c*d^9+9450*a^4*b^6*c^2*d^8+14400*a^3*b^7*c^3*d^7+9450*a^
2*b^8*c^4*d^6+2520*a*b^9*c^5*d^5+210*b^10*c^6*d^4)*x^15+1/14*(120*a^7*b^3*d
^10+2100*a^6*b^4*c*d^9+11340*a^5*b^5*c^2*d^8+25200*a^4*b^6*c^3*d^7+25200*a^
3*b^7*c^4*d^6+11340*a^2*b^8*c^5*d^5+2100*a*b^9*c^6*d^4+120*b^10*c^7*d^3)*x^
14+1/13*(45*a^8*b^2*d^10+1200*a^7*b^3*c*d^9+9450*a^6*b^4*c^2*d^8+30240*a^5*
b^5*c^3*d^7+44100*a^4*b^6*c^4*d^6+30240*a^3*b^7*c^5*d^5+9450*a^2*b^8*c^6*d^
4+1200*a*b^9*c^7*d^3+45*b^10*c^8*d^2)*x^13+1/12*(10*a^9*b*d^10+450*a^8*b^2*
c*d^9+5400*a^7*b^3*c^2*d^8+25200*a^6*b^4*c^3*d^7+52920*a^5*b^5*c^4*d^6+5292
0*a^4*b^6*c^5*d^5+25200*a^3*b^7*c^6*d^4+5400*a^2*b^8*c^7*d^3+450*a*b^9*c^8*
d^2+10*b^10*c^9*d)*x^12+1/11*(a^10*d^10+100*a^9*b*c*d^9+2025*a^8*b^2*c^2*d^
8+14400*a^7*b^3*c^3*d^7+44100*a^6*b^4*c^4*d^6+63504*a^5*b^5*c^5*d^5+44100*a
^4*b^6*c^6*d^4+14400*a^3*b^7*c^7*d^3+2025*a^2*b^8*c^8*d^2+100*a*b^9*c^9*d+b
^10*c^10)*x^11+1/10*(10*a^10*c*d^9+450*a^9*b*c^2*d^8+5400*a^8*b^2*c^3*d^7+2
5200*a^7*b^3*c^4*d^6+52920*a^6*b^4*c^5*d^5+52920*a^5*b^5*c^6*d^4+25200*a^4*
b^6*c^7*d^3+5400*a^3*b^7*c^8*d^2+450*a^2*b^8*c^9*d+10*a*b^9*c^10)*x^10+1/9*
(45*a^10*c^2*d^8+1200*a^9*b*c^3*d^7+9450*a^8*b^2*c^4*d^6+30240*a^7*b^3*c^5*
d^5+44100*a^6*b^4*c^6*d^4+30240*a^5*b^5*c^7*d^3+9450*a^4*b^6*c^8*d^2+1200*a
^3*b^7*c^9*d+45*a^2*b^8*c^10)*x^9+1/8*(120*a^10*c^3*d^7+2100*a^9*b*c^4*d^6+
11340*a^8*b^2*c^5*d^5+25200*a^7*b^3*c^6*d^4+25200*a^6*b^4*c^7*d^3+11340*a^5
*b^5*c^8*d^2+2100*a^4*b^6*c^9*d+120*a^3*b^7*c^10)*x^8+1/7*(210*a^10*c^4*d^6
+2520*a^9*b*c^5*d^5+9450*a^8*b^2*c^6*d^4+14400*a^7*b^3*c^7*d^3+9450*a^6*b^4
*c^8*d^2+2520*a^5*b^5*c^9*d+210*a^4*b^6*c^10)*x^7+1/6*(252*a^10*c^5*d^5+210
0*a^9*b*c^6*d^4+5400*a^8*b^2*c^7*d^3+5400*a^7*b^3*c^8*d^2+2100*a^6*b^4*c^9*
d+252*a^5*b^5*c^10)*x^6+1/5*(210*a^10*c^6*d^4+1200*a^9*b*c^7*d^3+2025*a^8*b
^2*c^8*d^2+1200*a^7*b^3*c^9*d+210*a^6*b^4*c^10)*x^5+1/4*(120*a^10*c^7*d^3+4
50*a^9*b*c^8*d^2+450*a^8*b^2*c^9*d+120*a^7*b^3*c^10)*x^4+1/3*(45*a^10*c^8*d
^2+100*a^9*b*c^9*d+45*a^8*b^2*c^10)*x^3+1/2*(10*a^10*c^9*d+10*a^9*b*c^10)*x
^2+a^10*c^10*x
```

maxima [B] time = 1.56, size = 1581, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^10*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/21*b^10*d^10*x^21 + a^10*c^10*x + 1/2*(b^10*c*d^9 + a*b^9*d^10)*x^20 + 5/
19*(9*b^10*c^2*d^8 + 20*a*b^9*c*d^9 + 9*a^2*b^8*d^10)*x^19 + 5/3*(4*b^10*c^
3*d^7 + 15*a*b^9*c^2*d^8 + 15*a^2*b^8*c*d^9 + 4*a^3*b^7*d^10)*x^18 + 15/17*
(14*b^10*c^4*d^6 + 80*a*b^9*c^3*d^7 + 135*a^2*b^8*c^2*d^8 + 80*a^3*b^7*c*d^
9 + 14*a^4*b^6*d^10)*x^17 + 3/4*(21*b^10*c^5*d^5 + 175*a*b^9*c^4*d^6 + 450*
a^2*b^8*c^3*d^7 + 450*a^3*b^7*c^2*d^8 + 175*a^4*b^6*c*d^9 + 21*a^5*b^5*d^10
)*x^16 + 2*(7*b^10*c^6*d^4 + 84*a*b^9*c^5*d^5 + 315*a^2*b^8*c^4*d^6 + 480*a
^3*b^7*c^3*d^7 + 315*a^4*b^6*c^2*d^8 + 84*a^5*b^5*c*d^9 + 7*a^6*b^4*d^10)*x
^15 + 30/7*(2*b^10*c^7*d^3 + 35*a*b^9*c^6*d^4 + 189*a^2*b^8*c^5*d^5 + 420*a
^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 + 35*a^6*b^4*c*d
^9 + 2*a^7*b^3*d^10)*x^14 + 15/13*(3*b^10*c^8*d^2 + 80*a*b^9*c^7*d^3 + 630*
a^2*b^8*c^6*d^4 + 2016*a^3*b^7*c^5*d^5 + 2940*a^4*b^6*c^4*d^6 + 2016*a^5*b^
5*c^3*d^7 + 630*a^6*b^4*c^2*d^8 + 80*a^7*b^3*c*d^9 + 3*a^8*b^2*d^10)*x^13 +
 5/6*(b^10*c^9*d + 45*a*b^9*c^8*d^2 + 540*a^2*b^8*c^7*d^3 + 2520*a^3*b^7*c^
6*d^4 + 5292*a^4*b^6*c^5*d^5 + 5292*a^5*b^5*c^4*d^6 + 2520*a^6*b^4*c^3*d^7
+ 540*a^7*b^3*c^2*d^8 + 45*a^8*b^2*c*d^9 + a^9*b*d^10)*x^12 + 1/11*(b^10*c^
10 + 100*a*b^9*c^9*d + 2025*a^2*b^8*c^8*d^2 + 14400*a^3*b^7*c^7*d^3 + 44100
*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^5*d^5 + 44100*a^6*b^4*c^4*d^6 + 14400*a^
7*b^3*c^3*d^7 + 2025*a^8*b^2*c^2*d^8 + 100*a^9*b*c*d^9 + a^10*d^10)*x^11 +
(a*b^9*c^10 + 45*a^2*b^8*c^9*d + 540*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3
 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540
*a^8*b^2*c^3*d^7 + 45*a^9*b*c^2*d^8 + a^10*c*d^9)*x^10 + 5/3*(3*a^2*b^8*c^1
0 + 80*a^3*b^7*c^9*d + 630*a^4*b^6*c^8*d^2 + 2016*a^5*b^5*c^7*d^3 + 2940*a^
6*b^4*c^6*d^4 + 2016*a^7*b^3*c^5*d^5 + 630*a^8*b^2*c^4*d^6 + 80*a^9*b*c^3*d
^7 + 3*a^10*c^2*d^8)*x^9 + 15/2*(2*a^3*b^7*c^10 + 35*a^4*b^6*c^9*d + 189*a^
5*b^5*c^8*d^2 + 420*a^6*b^4*c^7*d^3 + 420*a^7*b^3*c^6*d^4 + 189*a^8*b^2*c^5
*d^5 + 35*a^9*b*c^4*d^6 + 2*a^10*c^3*d^7)*x^8 + 30/7*(7*a^4*b^6*c^10 + 84*a
^5*b^5*c^9*d + 315*a^6*b^4*c^8*d^2 + 480*a^7*b^3*c^7*d^3 + 315*a^8*b^2*c^6*
d^4 + 84*a^9*b*c^5*d^5 + 7*a^10*c^4*d^6)*x^7 + 2*(21*a^5*b^5*c^10 + 175*a^6
*b^4*c^9*d + 450*a^7*b^3*c^8*d^2 + 450*a^8*b^2*c^7*d^3 + 175*a^9*b*c^6*d^4
+ 21*a^10*c^5*d^5)*x^6 + 3*(14*a^6*b^4*c^10 + 80*a^7*b^3*c^9*d + 135*a^8*b^
2*c^8*d^2 + 80*a^9*b*c^7*d^3 + 14*a^10*c^6*d^4)*x^5 + 15/2*(4*a^7*b^3*c^10
+ 15*a^8*b^2*c^9*d + 15*a^9*b*c^8*d^2 + 4*a^10*c^7*d^3)*x^4 + 5/3*(9*a^8*b^
2*c^10 + 20*a^9*b*c^9*d + 9*a^10*c^8*d^2)*x^3 + 5*(a^9*b*c^10 + a^10*c^9*d)
*x^2
```

mupad [B] time = 0.69, size = 1549, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^10*(c + d*x)^10,x)
```

```
[Out] x^7*(30*a^4*b^6*c^10 + 30*a^10*c^4*d^6 + 360*a^5*b^5*c^9*d + 360*a^9*b*c^5*
d^5 + 1350*a^6*b^4*c^8*d^2 + (14400*a^7*b^3*c^7*d^3)/7 + 1350*a^8*b^2*c^6*d
^4) + x^15*(14*a^6*b^4*d^10 + 14*b^10*c^6*d^4 + 168*a*b^9*c^5*d^5 + 168*a^5
*b^5*c*d^9 + 630*a^2*b^8*c^4*d^6 + 960*a^3*b^7*c^3*d^7 + 630*a^4*b^6*c^2*d^
8) + x^5*(42*a^6*b^4*c^10 + 42*a^10*c^6*d^4 + 240*a^7*b^3*c^9*d + 240*a^9*b
*c^7*d^3 + 405*a^8*b^2*c^8*d^2) + x^17*((210*a^4*b^6*d^10)/17 + (210*b^10*c
^4*d^6)/17 + (1200*a*b^9*c^3*d^7)/17 + (1200*a^3*b^7*c*d^9)/17 + (2025*a^2*
b^8*c^2*d^8)/17) + x^11*((a^10*d^10)/11 + (b^10*c^10)/11 + (2025*a^2*b^8*c^
8*d^2)/11 + (14400*a^3*b^7*c^7*d^3)/11 + (44100*a^4*b^6*c^6*d^4)/11 + (6350
4*a^5*b^5*c^5*d^5)/11 + (44100*a^6*b^4*c^4*d^6)/11 + (14400*a^7*b^3*c^3*d^7
)/11 + (2025*a^8*b^2*c^2*d^8)/11 + (100*a*b^9*c^9*d)/11 + (100*a^9*b*c*d^9)
/11) + x^8*(15*a^3*b^7*c^10 + 15*a^10*c^3*d^7 + (525*a^4*b^6*c^9*d)/2 + (52
5*a^9*b*c^4*d^6)/2 + (2835*a^5*b^5*c^8*d^2)/2 + 3150*a^6*b^4*c^7*d^3 + 3150
*a^7*b^3*c^6*d^4 + (2835*a^8*b^2*c^5*d^5)/2) + x^14*((60*a^7*b^3*d^10)/7 +
(60*b^10*c^7*d^3)/7 + 150*a*b^9*c^6*d^4 + 150*a^6*b^4*c*d^9 + 810*a^2*b^8*c
^5*d^5 + 1800*a^3*b^7*c^4*d^6 + 1800*a^4*b^6*c^3*d^7 + 810*a^5*b^5*c^2*d^8)
 + x^10*(a*b^9*c^10 + a^10*c*d^9 + 45*a^2*b^8*c^9*d + 45*a^9*b*c^2*d^8 + 54
```

```
0*a^3*b^7*c^8*d^2 + 2520*a^4*b^6*c^7*d^3 + 5292*a^5*b^5*c^6*d^4 + 5292*a^6*
b^4*c^5*d^5 + 2520*a^7*b^3*c^4*d^6 + 540*a^8*b^2*c^3*d^7) + x^12*((5*a^9*b*
d^10)/6 + (5*b^10*c^9*d)/6 + (75*a*b^9*c^8*d^2)/2 + (75*a^8*b^2*c*d^9)/2 +
450*a^2*b^8*c^7*d^3 + 2100*a^3*b^7*c^6*d^4 + 4410*a^4*b^6*c^5*d^5 + 4410*a^
5*b^5*c^4*d^6 + 2100*a^6*b^4*c^3*d^7 + 450*a^7*b^3*c^2*d^8) + x^6*(42*a^5*b
^5*c^10 + 42*a^10*c^5*d^5 + 350*a^6*b^4*c^9*d + 350*a^9*b*c^6*d^4 + 900*a^7
*b^3*c^8*d^2 + 900*a^8*b^2*c^7*d^3) + x^16*((63*a^5*b^5*d^10)/4 + (63*b^10*
c^5*d^5)/4 + (525*a*b^9*c^4*d^6)/4 + (525*a^4*b^6*c*d^9)/4 + (675*a^2*b^8*c
^3*d^7)/2 + (675*a^3*b^7*c^2*d^8)/2) + x^9*(5*a^2*b^8*c^10 + 5*a^10*c^2*d^8
 + (400*a^3*b^7*c^9*d)/3 + (400*a^9*b*c^3*d^7)/3 + 1050*a^4*b^6*c^8*d^2 + 3
360*a^5*b^5*c^7*d^3 + 4900*a^6*b^4*c^6*d^4 + 3360*a^7*b^3*c^5*d^5 + 1050*a^
8*b^2*c^4*d^6) + x^13*((45*a^8*b^2*d^10)/13 + (45*b^10*c^8*d^2)/13 + (1200*
a*b^9*c^7*d^3)/13 + (1200*a^7*b^3*c*d^9)/13 + (9450*a^2*b^8*c^6*d^4)/13 + (
30240*a^3*b^7*c^5*d^5)/13 + (44100*a^4*b^6*c^4*d^6)/13 + (30240*a^5*b^5*c^3
*d^7)/13 + (9450*a^6*b^4*c^2*d^8)/13) + a^10*c^10*x + (b^10*d^10*x^21)/21 +
 (15*a^7*c^7*x^4*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d^2))
/2 + (5*b^7*d^7*x^18*(4*a^3*d^3 + 4*b^3*c^3 + 15*a*b^2*c^2*d + 15*a^2*b*c*d
^2))/3 + 5*a^9*c^9*x^2*(a*d + b*c) + (b^9*d^9*x^20*(a*d + b*c))/2 + (5*a^8*
c^8*x^3*(9*a^2*d^2 + 9*b^2*c^2 + 20*a*b*c*d))/3 + (5*b^8*d^8*x^19*(9*a^2*d^
2 + 9*b^2*c^2 + 20*a*b*c*d))/19
```

sympy [B] time = 0.31, size = 1775, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**10*(d*x+c)**10,x)
```

```
[Out] a**10*c**10*x + b**10*d**10*x**21/21 + x**20*(a*b**9*d**10/2 + b**10*c*d**9
/2) + x**19*(45*a**2*b**8*d**10/19 + 100*a*b**9*c*d**9/19 + 45*b**10*c**2*d
**8/19) + x**18*(20*a**3*b**7*d**10/3 + 25*a**2*b**8*c*d**9 + 25*a*b**9*c**
2*d**8 + 20*b**10*c**3*d**7/3) + x**17*(210*a**4*b**6*d**10/17 + 1200*a**3*
b**7*c*d**9/17 + 2025*a**2*b**8*c**2*d**8/17 + 1200*a*b**9*c**3*d**7/17 + 2
10*b**10*c**4*d**6/17) + x**16*(63*a**5*b**5*d**10/4 + 525*a**4*b**6*c*d**9
/4 + 675*a**3*b**7*c**2*d**8/2 + 675*a**2*b**8*c**3*d**7/2 + 525*a*b**9*c**
4*d**6/4 + 63*b**10*c**5*d**5/4) + x**15*(14*a**6*b**4*d**10 + 168*a**5*b**
5*c*d**9 + 630*a**4*b**6*c**2*d**8 + 960*a**3*b**7*c**3*d**7 + 630*a**2*b**
8*c**4*d**6 + 168*a*b**9*c**5*d**5 + 14*b**10*c**6*d**4) + x**14*(60*a**7*b
**3*d**10/7 + 150*a**6*b**4*c*d**9 + 810*a**5*b**5*c**2*d**8 + 1800*a**4*b*
*6*c**3*d**7 + 1800*a**3*b**7*c**4*d**6 + 810*a**2*b**8*c**5*d**5 + 150*a*b
**9*c**6*d**4 + 60*b**10*c**7*d**3/7) + x**13*(45*a**8*b**2*d**10/13 + 1200
*a**7*b**3*c*d**9/13 + 9450*a**6*b**4*c**2*d**8/13 + 30240*a**5*b**5*c**3*d
**7/13 + 44100*a**4*b**6*c**4*d**6/13 + 30240*a**3*b**7*c**5*d**5/13 + 9450
*a**2*b**8*c**6*d**4/13 + 1200*a*b**9*c**7*d**3/13 + 45*b**10*c**8*d**2/13)
 + x**12*(5*a**9*b*d**10/6 + 75*a**8*b**2*c*d**9/2 + 450*a**7*b**3*c**2*d**
8 + 2100*a**6*b**4*c**3*d**7 + 4410*a**5*b**5*c**4*d**6 + 4410*a**4*b**6*c*
*5*d**5 + 2100*a**3*b**7*c**6*d**4 + 450*a**2*b**8*c**7*d**3 + 75*a*b**9*c*
*8*d**2/2 + 5*b**10*c**9*d/6) + x**11*(a**10*d**10/11 + 100*a**9*b*c*d**9/1
1 + 2025*a**8*b**2*c**2*d**8/11 + 14400*a**7*b**3*c**3*d**7/11 + 44100*a**6
*b**4*c**4*d**6/11 + 63504*a**5*b**5*c**5*d**5/11 + 44100*a**4*b**6*c**6*d*
*4/11 + 14400*a**3*b**7*c**7*d**3/11 + 2025*a**2*b**8*c**8*d**2/11 + 100*a*
b**9*c**9*d/11 + b**10*c**10/11) + x**10*(a**10*c*d**9 + 45*a**9*b*c**2*d**
8 + 540*a**8*b**2*c**3*d**7 + 2520*a**7*b**3*c**4*d**6 + 5292*a**6*b**4*c**
5*d**5 + 5292*a**5*b**5*c**6*d**4 + 2520*a**4*b**6*c**7*d**3 + 540*a**3*b**
7*c**8*d**2 + 45*a**2*b**8*c**9*d + a*b**9*c**10) + x**9*(5*a**10*c**2*d**8
 + 400*a**9*b*c**3*d**7/3 + 1050*a**8*b**2*c**4*d**6 + 3360*a**7*b**3*c**5*
d**5 + 4900*a**6*b**4*c**6*d**4 + 3360*a**5*b**5*c**7*d**3 + 1050*a**4*b**6
*c**8*d**2 + 400*a**3*b**7*c**9*d/3 + 5*a**2*b**8*c**10) + x**8*(15*a**10*c
**3*d**7 + 525*a**9*b*c**4*d**6/2 + 2835*a**8*b**2*c**5*d**5/2 + 3150*a**7*
b**3*c**6*d**4 + 3150*a**6*b**4*c**7*d**3 + 2835*a**5*b**5*c**8*d**2/2 + 52
```

```
5*a**4*b**6*c**9*d/2 + 15*a**3*b**7*c**10) + x**7*(30*a**10*c**4*d**6 + 360
*a**9*b*c**5*d**5 + 1350*a**8*b**2*c**6*d**4 + 14400*a**7*b**3*c**7*d**3/7
+ 1350*a**6*b**4*c**8*d**2 + 360*a**5*b**5*c**9*d + 30*a**4*b**6*c**10) + x
**6*(42*a**10*c**5*d**5 + 350*a**9*b*c**6*d**4 + 900*a**8*b**2*c**7*d**3 +
900*a**7*b**3*c**8*d**2 + 350*a**6*b**4*c**9*d + 42*a**5*b**5*c**10) + x**5
*(42*a**10*c**6*d**4 + 240*a**9*b*c**7*d**3 + 405*a**8*b**2*c**8*d**2 + 240
*a**7*b**3*c**9*d + 42*a**6*b**4*c**10) + x**4*(30*a**10*c**7*d**3 + 225*a*
*9*b*c**8*d**2/2 + 225*a**8*b**2*c**9*d/2 + 30*a**7*b**3*c**10) + x**3*(15*
a**10*c**8*d**2 + 100*a**9*b*c**9*d/3 + 15*a**8*b**2*c**10) + x**2*(5*a**10
*c**9*d + 5*a**9*b*c**10)
```

3.1196 $\int (a+bx)^9 (c+dx)^{10}\, dx$

Optimal. Leaf size=250

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+$$

Rubi [A] time = 1.04, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{9b^8(c+dx)^{19}(bc-ad)}{19d^{10}}+\frac{2b^7(c+dx)^{18}(bc-ad)^2}{d^{10}}-\frac{84b^6(c+dx)^{17}(bc-ad)^3}{17d^{10}}+\frac{63b^5(c+dx)^{16}(bc-ad)^4}{8d^{10}}-\frac{42b^4(c+dx)^{15}(bc-ad)^5}{5d^{10}}+\frac{6b^3(c+dx)^{14}(bc-ad)^6}{d^{10}}-\frac{36b^2(c+dx)^{13}(bc-ad)^7}{13d^{10}}+\frac{3b(c+dx)^{12}(bc-ad)^8}{4d^{10}}-\frac{(c+dx)^{11}(bc-ad)^9}{11d^{10}}+\frac{b^9(c+dx)^{20}}{20d^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^9*(c + d*x)^11)/(11*d^10) + (3*b*(b*c - a*d)^8*(c + d*x)^12)/
(4*d^10) - (36*b^2*(b*c - a*d)^7*(c + d*x)^13)/(13*d^10) + (6*b^3*(b*c - a*
d)^6*(c + d*x)^14)/d^10 - (42*b^4*(b*c - a*d)^5*(c + d*x)^15)/(5*d^10) + (6
3*b^5*(b*c - a*d)^4*(c + d*x)^16)/(8*d^10) - (84*b^6*(b*c - a*d)^3*(c + d*x
)^17)/(17*d^10) + (2*b^7*(b*c - a*d)^2*(c + d*x)^18)/d^10 - (9*b^8*(b*c - a
*d)*(c + d*x)^19)/(19*d^10) + (b^9*(c + d*x)^20)/(20*d^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^9(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^9(c+dx)^{10}}{d^9}+\frac{9b(bc-ad)^8(c+dx)^{11}}{d^9}-\frac{36b^2(bc-ad)^7(c+dx)^{12}}{d^9}+\frac{84b\right.$$

$$= -\frac{(bc-ad)^9(c+dx)^{11}}{11d^{10}}+\frac{3b(bc-ad)^8(c+dx)^{12}}{4d^{10}}-\frac{36b^2(bc-ad)^7(c+dx)^{13}}{13d^{10}}+\frac{6b^3(bc-$$

Mathematica [B] time = 0.19, size = 1397, normalized size = 5.59

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] a^9*c^10*x + (a^8*c^9*(9*b*c + 10*a*d)*x^2)/2 + 3*a^7*c^8*(4*b^2*c^2 + 10*a
*b*c*d + 5*a^2*d^2)*x^3 + (3*a^6*c^7*(28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^
2*b*c*d^2 + 40*a^3*d^3)*x^4)/4 + (6*a^5*c^6*(21*b^4*c^4 + 140*a*b^3*c^3*d +
 270*a^2*b^2*c^2*d^2 + 180*a^3*b*c*d^3 + 35*a^4*d^4)*x^5)/5 + 3*a^4*c^5*(7*
b^5*c^5 + 70*a*b^4*c^4*d + 210*a^2*b^3*c^3*d^2 + 240*a^3*b^2*c^2*d^3 + 105*
a^4*b*c*d^4 + 14*a^5*d^5)*x^6 + 6*a^3*c^4*(2*b^6*c^6 + 30*a*b^5*c^5*d + 135
*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^4 + 54*a^5*b*c*d
^5 + 5*a^6*d^6)*x^7 + (3*a^2*c^3*(6*b^7*c^7 + 140*a*b^6*c^6*d + 945*a^2*b^5
*c^5*d^2 + 2520*a^3*b^4*c^4*d^3 + 2940*a^4*b^3*c^3*d^4 + 1512*a^5*b^2*c^2*d
^5 + 315*a^6*b*c*d^6 + 20*a^7*d^7)*x^8)/4 + a*c^2*(b^8*c^8 + 40*a*b^7*c^7*d
 + 420*a^2*b^6*c^6*d^2 + 1680*a^3*b^5*c^5*d^3 + 2940*a^4*b^4*c^4*d^4 + 2352
*a^5*b^3*c^3*d^5 + 840*a^6*b^2*c^2*d^6 + 120*a^7*b*c*d^7 + 5*a^8*d^8)*x^9 +
```

```
(c*(b^9*c^9 + 90*a*b^8*c^8*d + 1620*a^2*b^7*c^7*d^2 + 10080*a^3*b^6*c^6*d^
3 + 26460*a^4*b^5*c^5*d^4 + 31752*a^5*b^4*c^4*d^5 + 17640*a^6*b^3*c^3*d^6 +
 4320*a^7*b^2*c^2*d^7 + 405*a^8*b*c*d^8 + 10*a^9*d^9)*x^10)/10 + (d*(10*b^9
*c^9 + 405*a*b^8*c^8*d + 4320*a^2*b^7*c^7*d^2 + 17640*a^3*b^6*c^6*d^3 + 317
52*a^4*b^5*c^5*d^4 + 26460*a^5*b^4*c^4*d^5 + 10080*a^6*b^3*c^3*d^6 + 1620*a
^7*b^2*c^2*d^7 + 90*a^8*b*c*d^8 + a^9*d^9)*x^11)/11 + (3*b*d^2*(5*b^8*c^8 +
 120*a*b^7*c^7*d + 840*a^2*b^6*c^6*d^2 + 2352*a^3*b^5*c^5*d^3 + 2940*a^4*b^
4*c^4*d^4 + 1680*a^5*b^3*c^3*d^5 + 420*a^6*b^2*c^2*d^6 + 40*a^7*b*c*d^7 + a
^8*d^8)*x^12)/4 + (6*b^2*d^3*(20*b^7*c^7 + 315*a*b^6*c^6*d + 1512*a^2*b^5*c
^5*d^2 + 2940*a^3*b^4*c^4*d^3 + 2520*a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5
+ 140*a^6*b*c*d^6 + 6*a^7*d^7)*x^13)/13 + 3*b^3*d^4*(5*b^6*c^6 + 54*a*b^5*c
^5*d + 180*a^2*b^4*c^4*d^2 + 240*a^3*b^3*c^3*d^3 + 135*a^4*b^2*c^2*d^4 + 30
*a^5*b*c*d^5 + 2*a^6*d^6)*x^14 + (6*b^4*d^5*(14*b^5*c^5 + 105*a*b^4*c^4*d +
 240*a^2*b^3*c^3*d^2 + 210*a^3*b^2*c^2*d^3 + 70*a^4*b*c*d^4 + 7*a^5*d^5)*x^
15)/5 + (3*b^5*d^6*(35*b^4*c^4 + 180*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 14
0*a^3*b*c*d^3 + 21*a^4*d^4)*x^16)/8 + (3*b^6*d^7*(40*b^3*c^3 + 135*a*b^2*c^
2*d + 120*a^2*b*c*d^2 + 28*a^3*d^3)*x^17)/17 + (b^7*d^8*(5*b^2*c^2 + 10*a*b
*c*d + 4*a^2*d^2)*x^18)/2 + (b^8*d^9*(10*b*c + 9*a*d)*x^19)/19 + (b^9*d^10*
x^20)/20
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^9(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^9*(c + d*x)^10, x]
```

fricas [B] time = 1.17, size = 1656, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/20*x^20*d^10*b^9 + 10/19*x^19*d^9*c*b^9 + 9/19*x^19*d^10*b^8*a + 5/2*x^18
*d^8*c^2*b^9 + 5*x^18*d^9*c*b^8*a + 2*x^18*d^10*b^7*a^2 + 120/17*x^17*d^7*c
^3*b^9 + 405/17*x^17*d^8*c^2*b^8*a + 360/17*x^17*d^9*c*b^7*a^2 + 84/17*x^17
*d^10*b^6*a^3 + 105/8*x^16*d^6*c^4*b^9 + 135/2*x^16*d^7*c^3*b^8*a + 405/4*x
^16*d^8*c^2*b^7*a^2 + 105/2*x^16*d^9*c*b^6*a^3 + 63/8*x^16*d^10*b^5*a^4 + 8
4/5*x^15*d^5*c^5*b^9 + 126*x^15*d^6*c^4*b^8*a + 288*x^15*d^7*c^3*b^7*a^2 +
252*x^15*d^8*c^2*b^6*a^3 + 84*x^15*d^9*c*b^5*a^4 + 42/5*x^15*d^10*b^4*a^5 +
 15*x^14*d^4*c^6*b^9 + 162*x^14*d^5*c^5*b^8*a + 540*x^14*d^6*c^4*b^7*a^2 +
720*x^14*d^7*c^3*b^6*a^3 + 405*x^14*d^8*c^2*b^5*a^4 + 90*x^14*d^9*c*b^4*a^5
 + 6*x^14*d^10*b^3*a^6 + 120/13*x^13*d^3*c^7*b^9 + 1890/13*x^13*d^4*c^6*b^8
*a + 9072/13*x^13*d^5*c^5*b^7*a^2 + 17640/13*x^13*d^6*c^4*b^6*a^3 + 15120/1
3*x^13*d^7*c^3*b^5*a^4 + 5670/13*x^13*d^8*c^2*b^4*a^5 + 840/13*x^13*d^9*c*b
^3*a^6 + 36/13*x^13*d^10*b^2*a^7 + 15/4*x^12*d^2*c^8*b^9 + 90*x^12*d^3*c^7*
b^8*a + 630*x^12*d^4*c^6*b^7*a^2 + 1764*x^12*d^5*c^5*b^6*a^3 + 2205*x^12*d^
6*c^4*b^5*a^4 + 1260*x^12*d^7*c^3*b^4*a^5 + 315*x^12*d^8*c^2*b^3*a^6 + 30*x
^12*d^9*c*b^2*a^7 + 3/4*x^12*d^10*b*a^8 + 10/11*x^11*d*c^9*b^9 + 405/11*x^1
1*d^2*c^8*b^8*a + 4320/11*x^11*d^3*c^7*b^7*a^2 + 17640/11*x^11*d^4*c^6*b^6*
a^3 + 31752/11*x^11*d^5*c^5*b^5*a^4 + 26460/11*x^11*d^6*c^4*b^4*a^5 + 10080
/11*x^11*d^7*c^3*b^3*a^6 + 1620/11*x^11*d^8*c^2*b^2*a^7 + 90/11*x^11*d^9*c*
b*a^8 + 1/11*x^11*d^10*a^9 + 1/10*x^10*c^10*b^9 + 9*x^10*d*c^9*b^8*a + 162*
x^10*d^2*c^8*b^7*a^2 + 1008*x^10*d^3*c^7*b^6*a^3 + 2646*x^10*d^4*c^6*b^5*a^
4 + 15876/5*x^10*d^5*c^5*b^4*a^5 + 1764*x^10*d^6*c^4*b^3*a^6 + 432*x^10*d^7
*c^3*b^2*a^7 + 81/2*x^10*d^8*c^2*b*a^8 + x^10*d^9*c*a^9 + x^9*c^10*b^8*a +
40*x^9*d*c^9*b^7*a^2 + 420*x^9*d^2*c^8*b^6*a^3 + 1680*x^9*d^3*c^7*b^5*a^4 +
```

```
 2940*x^9*d^4*c^6*b^4*a^5 + 2352*x^9*d^5*c^5*b^3*a^6 + 840*x^9*d^6*c^4*b^2*
a^7 + 120*x^9*d^7*c^3*b*a^8 + 5*x^9*d^8*c^2*a^9 + 9/2*x^8*c^10*b^7*a^2 + 10
5*x^8*d*c^9*b^6*a^3 + 2835/4*x^8*d^2*c^8*b^5*a^4 + 1890*x^8*d^3*c^7*b^4*a^5
 + 2205*x^8*d^4*c^6*b^3*a^6 + 1134*x^8*d^5*c^5*b^2*a^7 + 945/4*x^8*d^6*c^4*
b*a^8 + 15*x^8*d^7*c^3*a^9 + 12*x^7*c^10*b^6*a^3 + 180*x^7*d*c^9*b^5*a^4 +
810*x^7*d^2*c^8*b^4*a^5 + 1440*x^7*d^3*c^7*b^3*a^6 + 1080*x^7*d^4*c^6*b^2*a
^7 + 324*x^7*d^5*c^5*b*a^8 + 30*x^7*d^6*c^4*a^9 + 21*x^6*c^10*b^5*a^4 + 210
*x^6*d*c^9*b^4*a^5 + 630*x^6*d^2*c^8*b^3*a^6 + 720*x^6*d^3*c^7*b^2*a^7 + 31
5*x^6*d^4*c^6*b*a^8 + 42*x^6*d^5*c^5*a^9 + 126/5*x^5*c^10*b^4*a^5 + 168*x^5
*d*c^9*b^3*a^6 + 324*x^5*d^2*c^8*b^2*a^7 + 216*x^5*d^3*c^7*b*a^8 + 42*x^5*d
^4*c^6*a^9 + 21*x^4*c^10*b^3*a^6 + 90*x^4*d*c^9*b^2*a^7 + 405/4*x^4*d^2*c^8
*b*a^8 + 30*x^4*d^3*c^7*a^9 + 12*x^3*c^10*b^2*a^7 + 30*x^3*d*c^9*b*a^8 + 15
*x^3*d^2*c^8*a^9 + 9/2*x^2*c^10*b*a^8 + 5*x^2*d*c^9*a^9 + x*c^10*a^9
```

giac [B] time = 1.30, size = 1656, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/20*b^9*d^10*x^20 + 10/19*b^9*c*d^9*x^19 + 9/19*a*b^8*d^10*x^19 + 5/2*b^9*
c^2*d^8*x^18 + 5*a*b^8*c*d^9*x^18 + 2*a^2*b^7*d^10*x^18 + 120/17*b^9*c^3*d^
7*x^17 + 405/17*a*b^8*c^2*d^8*x^17 + 360/17*a^2*b^7*c*d^9*x^17 + 84/17*a^3*
b^6*d^10*x^17 + 105/8*b^9*c^4*d^6*x^16 + 135/2*a*b^8*c^3*d^7*x^16 + 405/4*a
^2*b^7*c^2*d^8*x^16 + 105/2*a^3*b^6*c*d^9*x^16 + 63/8*a^4*b^5*d^10*x^16 + 8
4/5*b^9*c^5*d^5*x^15 + 126*a*b^8*c^4*d^6*x^15 + 288*a^2*b^7*c^3*d^7*x^15 +
252*a^3*b^6*c^2*d^8*x^15 + 84*a^4*b^5*c*d^9*x^15 + 42/5*a^5*b^4*d^10*x^15 +
 15*b^9*c^6*d^4*x^14 + 162*a*b^8*c^5*d^5*x^14 + 540*a^2*b^7*c^4*d^6*x^14 +
720*a^3*b^6*c^3*d^7*x^14 + 405*a^4*b^5*c^2*d^8*x^14 + 90*a^5*b^4*c*d^9*x^14
 + 6*a^6*b^3*d^10*x^14 + 120/13*b^9*c^7*d^3*x^13 + 1890/13*a*b^8*c^6*d^4*x^
13 + 9072/13*a^2*b^7*c^5*d^5*x^13 + 17640/13*a^3*b^6*c^4*d^6*x^13 + 15120/1
3*a^4*b^5*c^3*d^7*x^13 + 5670/13*a^5*b^4*c^2*d^8*x^13 + 840/13*a^6*b^3*c*d^
9*x^13 + 36/13*a^7*b^2*d^10*x^13 + 15/4*b^9*c^8*d^2*x^12 + 90*a*b^8*c^7*d^3
*x^12 + 630*a^2*b^7*c^6*d^4*x^12 + 1764*a^3*b^6*c^5*d^5*x^12 + 2205*a^4*b^5
*c^4*d^6*x^12 + 1260*a^5*b^4*c^3*d^7*x^12 + 315*a^6*b^3*c^2*d^8*x^12 + 30*a
^7*b^2*c*d^9*x^12 + 3/4*a^8*b*d^10*x^12 + 10/11*b^9*c^9*d*x^11 + 405/11*a*b
^8*c^8*d^2*x^11 + 4320/11*a^2*b^7*c^7*d^3*x^11 + 17640/11*a^3*b^6*c^6*d^4*x
^11 + 31752/11*a^4*b^5*c^5*d^5*x^11 + 26460/11*a^5*b^4*c^4*d^6*x^11 + 10080
/11*a^6*b^3*c^3*d^7*x^11 + 1620/11*a^7*b^2*c^2*d^8*x^11 + 90/11*a^8*b*c*d^9
*x^11 + 1/11*a^9*d^10*x^11 + 1/10*b^9*c^10*x^10 + 9*a*b^8*c^9*d*x^10 + 162*
a^2*b^7*c^8*d^2*x^10 + 1008*a^3*b^6*c^7*d^3*x^10 + 2646*a^4*b^5*c^6*d^4*x^1
0 + 15876/5*a^5*b^4*c^5*d^5*x^10 + 1764*a^6*b^3*c^4*d^6*x^10 + 432*a^7*b^2*
c^3*d^7*x^10 + 81/2*a^8*b*c^2*d^8*x^10 + a^9*c*d^9*x^10 + a*b^8*c^10*x^9 +
40*a^2*b^7*c^9*d*x^9 + 420*a^3*b^6*c^8*d^2*x^9 + 1680*a^4*b^5*c^7*d^3*x^9 +
 2940*a^5*b^4*c^6*d^4*x^9 + 2352*a^6*b^3*c^5*d^5*x^9 + 840*a^7*b^2*c^4*d^6*
x^9 + 120*a^8*b*c^3*d^7*x^9 + 5*a^9*c^2*d^8*x^9 + 9/2*a^2*b^7*c^10*x^8 + 10
5*a^3*b^6*c^9*d*x^8 + 2835/4*a^4*b^5*c^8*d^2*x^8 + 1890*a^5*b^4*c^7*d^3*x^8
 + 2205*a^6*b^3*c^6*d^4*x^8 + 1134*a^7*b^2*c^5*d^5*x^8 + 945/4*a^8*b*c^4*d^
6*x^8 + 15*a^9*c^3*d^7*x^8 + 12*a^3*b^6*c^10*x^7 + 180*a^4*b^5*c^9*d*x^7 +
810*a^5*b^4*c^8*d^2*x^7 + 1440*a^6*b^3*c^7*d^3*x^7 + 1080*a^7*b^2*c^6*d^4*x
^7 + 324*a^8*b*c^5*d^5*x^7 + 30*a^9*c^4*d^6*x^7 + 21*a^4*b^5*c^10*x^6 + 210
*a^5*b^4*c^9*d*x^6 + 630*a^6*b^3*c^8*d^2*x^6 + 720*a^7*b^2*c^7*d^3*x^6 + 31
5*a^8*b*c^6*d^4*x^6 + 42*a^9*c^5*d^5*x^6 + 126/5*a^5*b^4*c^10*x^5 + 168*a^6
*b^3*c^9*d*x^5 + 324*a^7*b^2*c^8*d^2*x^5 + 216*a^8*b*c^7*d^3*x^5 + 42*a^9*c
^6*d^4*x^5 + 21*a^6*b^3*c^10*x^4 + 90*a^7*b^2*c^9*d*x^4 + 405/4*a^8*b*c^8*d
^2*x^4 + 30*a^9*c^7*d^3*x^4 + 12*a^7*b^2*c^10*x^3 + 30*a^8*b*c^9*d*x^3 + 15
*a^9*c^8*d^2*x^3 + 9/2*a^8*b*c^10*x^2 + 5*a^9*c^9*d*x^2 + a^9*c^10*x
```

maple [B] time = 0.00, size = 1441, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9*(d*x+c)^10,x)
```

```
[Out] 1/20*b^9*d^10*x^20+1/19*(9*a*b^8*d^10+10*b^9*c*d^9)*x^19+1/18*(36*a^2*b^7*d
^10+90*a*b^8*c*d^9+45*b^9*c^2*d^8)*x^18+1/17*(84*a^3*b^6*d^10+360*a^2*b^7*c
*d^9+405*a*b^8*c^2*d^8+120*b^9*c^3*d^7)*x^17+1/16*(126*a^4*b^5*d^10+840*a^3
*b^6*c*d^9+1620*a^2*b^7*c^2*d^8+1080*a*b^8*c^3*d^7+210*b^9*c^4*d^6)*x^16+1/
15*(126*a^5*b^4*d^10+1260*a^4*b^5*c*d^9+3780*a^3*b^6*c^2*d^8+4320*a^2*b^7*c
^3*d^7+1890*a*b^8*c^4*d^6+252*b^9*c^5*d^5)*x^15+1/14*(84*a^6*b^3*d^10+1260*
a^5*b^4*c*d^9+5670*a^4*b^5*c^2*d^8+10080*a^3*b^6*c^3*d^7+7560*a^2*b^7*c^4*d
^6+2268*a*b^8*c^5*d^5+210*b^9*c^6*d^4)*x^14+1/13*(36*a^7*b^2*d^10+840*a^6*b
^3*c*d^9+5670*a^5*b^4*c^2*d^8+15120*a^4*b^5*c^3*d^7+17640*a^3*b^6*c^4*d^6+9
072*a^2*b^7*c^5*d^5+1890*a*b^8*c^6*d^4+120*b^9*c^7*d^3)*x^13+1/12*(9*a^8*b*
d^10+360*a^7*b^2*c*d^9+3780*a^6*b^3*c^2*d^8+15120*a^5*b^4*c^3*d^7+26460*a^4
*b^5*c^4*d^6+21168*a^3*b^6*c^5*d^5+7560*a^2*b^7*c^6*d^4+1080*a*b^8*c^7*d^3+
45*b^9*c^8*d^2)*x^12+1/11*(a^9*d^10+90*a^8*b*c*d^9+1620*a^7*b^2*c^2*d^8+100
80*a^6*b^3*c^3*d^7+26460*a^5*b^4*c^4*d^6+31752*a^4*b^5*c^5*d^5+17640*a^3*b^
6*c^6*d^4+4320*a^2*b^7*c^7*d^3+405*a*b^8*c^8*d^2+10*b^9*c^9*d)*x^11+1/10*(1
0*a^9*c*d^9+405*a^8*b*c^2*d^8+4320*a^7*b^2*c^3*d^7+17640*a^6*b^3*c^4*d^6+31
752*a^5*b^4*c^5*d^5+26460*a^4*b^5*c^6*d^4+10080*a^3*b^6*c^7*d^3+1620*a^2*b^
7*c^8*d^2+90*a*b^8*c^9*d+b^9*c^10)*x^10+1/9*(45*a^9*c^2*d^8+1080*a^8*b*c^3*
d^7+7560*a^7*b^2*c^4*d^6+21168*a^6*b^3*c^5*d^5+26460*a^5*b^4*c^6*d^4+15120*
a^4*b^5*c^7*d^3+3780*a^3*b^6*c^8*d^2+360*a^2*b^7*c^9*d+9*a*b^8*c^10)*x^9+1/
8*(120*a^9*c^3*d^7+1890*a^8*b*c^4*d^6+9072*a^7*b^2*c^5*d^5+17640*a^6*b^3*c^
6*d^4+15120*a^5*b^4*c^7*d^3+5670*a^4*b^5*c^8*d^2+840*a^3*b^6*c^9*d+36*a^2*b
^7*c^10)*x^8+1/7*(210*a^9*c^4*d^6+2268*a^8*b*c^5*d^5+7560*a^7*b^2*c^6*d^4+1
0080*a^6*b^3*c^7*d^3+5670*a^5*b^4*c^8*d^2+1260*a^4*b^5*c^9*d+84*a^3*b^6*c^1
0)*x^7+1/6*(252*a^9*c^5*d^5+1890*a^8*b*c^6*d^4+4320*a^7*b^2*c^7*d^3+3780*a^
6*b^3*c^8*d^2+1260*a^5*b^4*c^9*d+126*a^4*b^5*c^10)*x^6+1/5*(210*a^9*c^6*d^4
+1080*a^8*b*c^7*d^3+1620*a^7*b^2*c^8*d^2+840*a^6*b^3*c^9*d+126*a^5*b^4*c^10
)*x^5+1/4*(120*a^9*c^7*d^3+405*a^8*b*c^8*d^2+360*a^7*b^2*c^9*d+84*a^6*b^3*c
^10)*x^4+1/3*(45*a^9*c^8*d^2+90*a^8*b*c^9*d+36*a^7*b^2*c^10)*x^3+1/2*(10*a^
9*c^9*d+9*a^8*b*c^10)*x^2+a^9*c^10*x
```

maxima [B] time = 1.51, size = 1437, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/20*b^9*d^10*x^20 + a^9*c^10*x + 1/19*(10*b^9*c*d^9 + 9*a*b^8*d^10)*x^19 +
 1/2*(5*b^9*c^2*d^8 + 10*a*b^8*c*d^9 + 4*a^2*b^7*d^10)*x^18 + 3/17*(40*b^9*
c^3*d^7 + 135*a*b^8*c^2*d^8 + 120*a^2*b^7*c*d^9 + 28*a^3*b^6*d^10)*x^17 + 3
/8*(35*b^9*c^4*d^6 + 180*a*b^8*c^3*d^7 + 270*a^2*b^7*c^2*d^8 + 140*a^3*b^6*
c*d^9 + 21*a^4*b^5*d^10)*x^16 + 6/5*(14*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 + 2
40*a^2*b^7*c^3*d^7 + 210*a^3*b^6*c^2*d^8 + 70*a^4*b^5*c*d^9 + 7*a^5*b^4*d^1
0)*x^15 + 3*(5*b^9*c^6*d^4 + 54*a*b^8*c^5*d^5 + 180*a^2*b^7*c^4*d^6 + 240*a
^3*b^6*c^3*d^7 + 135*a^4*b^5*c^2*d^8 + 30*a^5*b^4*c*d^9 + 2*a^6*b^3*d^10)*x
^14 + 6/13*(20*b^9*c^7*d^3 + 315*a*b^8*c^6*d^4 + 1512*a^2*b^7*c^5*d^5 + 294
0*a^3*b^6*c^4*d^6 + 2520*a^4*b^5*c^3*d^7 + 945*a^5*b^4*c^2*d^8 + 140*a^6*b^
3*c*d^9 + 6*a^7*b^2*d^10)*x^13 + 3/4*(5*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 8
40*a^2*b^7*c^6*d^4 + 2352*a^3*b^6*c^5*d^5 + 2940*a^4*b^5*c^4*d^6 + 1680*a^5
*b^4*c^3*d^7 + 420*a^6*b^3*c^2*d^8 + 40*a^7*b^2*c*d^9 + a^8*b*d^10)*x^12 +
1/11*(10*b^9*c^9*d + 405*a*b^8*c^8*d^2 + 4320*a^2*b^7*c^7*d^3 + 17640*a^3*b
^6*c^6*d^4 + 31752*a^4*b^5*c^5*d^5 + 26460*a^5*b^4*c^4*d^6 + 10080*a^6*b^3*
c^3*d^7 + 1620*a^7*b^2*c^2*d^8 + 90*a^8*b*c*d^9 + a^9*d^10)*x^11 + 1/10*(b^
9*c^10 + 90*a*b^8*c^9*d + 1620*a^2*b^7*c^8*d^2 + 10080*a^3*b^6*c^7*d^3 + 26
460*a^4*b^5*c^6*d^4 + 31752*a^5*b^4*c^5*d^5 + 17640*a^6*b^3*c^4*d^6 + 4320*
```

```
a^7*b^2*c^3*d^7 + 405*a^8*b*c^2*d^8 + 10*a^9*c*d^9)*x^10 + (a*b^8*c^10 + 40
*a^2*b^7*c^9*d + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 2940*a^5*b^4*
c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6 + 120*a^8*b*c^3*d^7 +
5*a^9*c^2*d^8)*x^9 + 3/4*(6*a^2*b^7*c^10 + 140*a^3*b^6*c^9*d + 945*a^4*b^5*
c^8*d^2 + 2520*a^5*b^4*c^7*d^3 + 2940*a^6*b^3*c^6*d^4 + 1512*a^7*b^2*c^5*d^
5 + 315*a^8*b*c^4*d^6 + 20*a^9*c^3*d^7)*x^8 + 6*(2*a^3*b^6*c^10 + 30*a^4*b^
5*c^9*d + 135*a^5*b^4*c^8*d^2 + 240*a^6*b^3*c^7*d^3 + 180*a^7*b^2*c^6*d^4 +
 54*a^8*b*c^5*d^5 + 5*a^9*c^4*d^6)*x^7 + 3*(7*a^4*b^5*c^10 + 70*a^5*b^4*c^9
*d + 210*a^6*b^3*c^8*d^2 + 240*a^7*b^2*c^7*d^3 + 105*a^8*b*c^6*d^4 + 14*a^9
*c^5*d^5)*x^6 + 6/5*(21*a^5*b^4*c^10 + 140*a^6*b^3*c^9*d + 270*a^7*b^2*c^8*
d^2 + 180*a^8*b*c^7*d^3 + 35*a^9*c^6*d^4)*x^5 + 3/4*(28*a^6*b^3*c^10 + 120*
a^7*b^2*c^9*d + 135*a^8*b*c^8*d^2 + 40*a^9*c^7*d^3)*x^4 + 3*(4*a^7*b^2*c^10
 + 10*a^8*b*c^9*d + 5*a^9*c^8*d^2)*x^3 + 1/2*(9*a^8*b*c^10 + 10*a^9*c^9*d)*
x^2
```

mupad [B] time = 0.79, size = 1404, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9*(c + d*x)^10,x)
```

```
[Out] x^7*(12*a^3*b^6*c^10 + 30*a^9*c^4*d^6 + 180*a^4*b^5*c^9*d + 324*a^8*b*c^5*d
^5 + 810*a^5*b^4*c^8*d^2 + 1440*a^6*b^3*c^7*d^3 + 1080*a^7*b^2*c^6*d^4) + x
^14*(6*a^6*b^3*d^10 + 15*b^9*c^6*d^4 + 162*a*b^8*c^5*d^5 + 90*a^5*b^4*c*d^9
 + 540*a^2*b^7*c^4*d^6 + 720*a^3*b^6*c^3*d^7 + 405*a^4*b^5*c^2*d^8) + x^5*(
(126*a^5*b^4*c^10)/5 + 42*a^9*c^6*d^4 + 168*a^6*b^3*c^9*d + 216*a^8*b*c^7*d
^3 + 324*a^7*b^2*c^8*d^2) + x^16*((63*a^4*b^5*d^10)/8 + (105*b^9*c^4*d^6)/8
 + (135*a*b^8*c^3*d^7)/2 + (105*a^3*b^6*c*d^9)/2 + (405*a^2*b^7*c^2*d^8)/4)
 + x^8*((9*a^2*b^7*c^10)/2 + 15*a^9*c^3*d^7 + 105*a^3*b^6*c^9*d + (945*a^8*
b*c^4*d^6)/4 + (2835*a^4*b^5*c^8*d^2)/4 + 1890*a^5*b^4*c^7*d^3 + 2205*a^6*b
^3*c^6*d^4 + 1134*a^7*b^2*c^5*d^5) + x^13*((36*a^7*b^2*d^10)/13 + (120*b^9*
c^7*d^3)/13 + (1890*a*b^8*c^6*d^4)/13 + (840*a^6*b^3*c*d^9)/13 + (9072*a^2*
b^7*c^5*d^5)/13 + (17640*a^3*b^6*c^4*d^6)/13 + (15120*a^4*b^5*c^3*d^7)/13 +
 (5670*a^5*b^4*c^2*d^8)/13) + x^9*(a*b^8*c^10 + 5*a^9*c^2*d^8 + 40*a^2*b^7*
c^9*d + 120*a^8*b*c^3*d^7 + 420*a^3*b^6*c^8*d^2 + 1680*a^4*b^5*c^7*d^3 + 29
40*a^5*b^4*c^6*d^4 + 2352*a^6*b^3*c^5*d^5 + 840*a^7*b^2*c^4*d^6) + x^12*((3
*a^8*b*d^10)/4 + (15*b^9*c^8*d^2)/4 + 90*a*b^8*c^7*d^3 + 30*a^7*b^2*c*d^9 +
 630*a^2*b^7*c^6*d^4 + 1764*a^3*b^6*c^5*d^5 + 2205*a^4*b^5*c^4*d^6 + 1260*a
^5*b^4*c^3*d^7 + 315*a^6*b^3*c^2*d^8) + x^6*(21*a^4*b^5*c^10 + 42*a^9*c^5*d
^5 + 210*a^5*b^4*c^9*d + 315*a^8*b*c^6*d^4 + 630*a^6*b^3*c^8*d^2 + 720*a^7*
b^2*c^7*d^3) + x^15*((42*a^5*b^4*d^10)/5 + (84*b^9*c^5*d^5)/5 + 126*a*b^8*c
^4*d^6 + 84*a^4*b^5*c*d^9 + 288*a^2*b^7*c^3*d^7 + 252*a^3*b^6*c^2*d^8) + x^
10*((b^9*c^10)/10 + a^9*c*d^9 + (81*a^8*b*c^2*d^8)/2 + 162*a^2*b^7*c^8*d^2
+ 1008*a^3*b^6*c^7*d^3 + 2646*a^4*b^5*c^6*d^4 + (15876*a^5*b^4*c^5*d^5)/5 +
 1764*a^6*b^3*c^4*d^6 + 432*a^7*b^2*c^3*d^7 + 9*a*b^8*c^9*d) + x^11*((a^9*d
^10)/11 + (10*b^9*c^9*d)/11 + (405*a*b^8*c^8*d^2)/11 + (4320*a^2*b^7*c^7*d^
3)/11 + (17640*a^3*b^6*c^6*d^4)/11 + (31752*a^4*b^5*c^5*d^5)/11 + (26460*a^
5*b^4*c^4*d^6)/11 + (10080*a^6*b^3*c^3*d^7)/11 + (1620*a^7*b^2*c^2*d^8)/11
+ (90*a^8*b*c*d^9)/11) + a^9*c^10*x + (b^9*d^10*x^20)/20 + (3*a^6*c^7*x^4*(
40*a^3*d^3 + 28*b^3*c^3 + 120*a*b^2*c^2*d + 135*a^2*b*c*d^2))/4 + (3*b^6*d^
7*x^17*(28*a^3*d^3 + 40*b^3*c^3 + 135*a*b^2*c^2*d + 120*a^2*b*c*d^2))/17 +
(a^8*c^9*x^2*(10*a*d + 9*b*c))/2 + (b^8*d^9*x^19*(9*a*d + 10*b*c))/19 + 3*a
^7*c^8*x^3*(5*a^2*d^2 + 4*b^2*c^2 + 10*a*b*c*d) + (b^7*d^8*x^18*(4*a^2*d^2
+ 5*b^2*c^2 + 10*a*b*c*d))/2
```

sympy [B] time = 0.30, size = 1598, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9*(d*x+c)**10,x)
```

```
[Out] a**9*c**10*x + b**9*d**10*x**20/20 + x**19*(9*a*b**8*d**10/19 + 10*b**9*c*d
**9/19) + x**18*(2*a**2*b**7*d**10 + 5*a*b**8*c*d**9 + 5*b**9*c**2*d**8/2)
+ x**17*(84*a**3*b**6*d**10/17 + 360*a**2*b**7*c*d**9/17 + 405*a*b**8*c**2*
d**8/17 + 120*b**9*c**3*d**7/17) + x**16*(63*a**4*b**5*d**10/8 + 105*a**3*b
**6*c*d**9/2 + 405*a**2*b**7*c**2*d**8/4 + 135*a*b**8*c**3*d**7/2 + 105*b**
9*c**4*d**6/8) + x**15*(42*a**5*b**4*d**10/5 + 84*a**4*b**5*c*d**9 + 252*a*
*3*b**6*c**2*d**8 + 288*a**2*b**7*c**3*d**7 + 126*a*b**8*c**4*d**6 + 84*b**
9*c**5*d**5/5) + x**14*(6*a**6*b**3*d**10 + 90*a**5*b**4*c*d**9 + 405*a**4*
b**5*c**2*d**8 + 720*a**3*b**6*c**3*d**7 + 540*a**2*b**7*c**4*d**6 + 162*a*
b**8*c**5*d**5 + 15*b**9*c**6*d**4) + x**13*(36*a**7*b**2*d**10/13 + 840*a*
*6*b**3*c*d**9/13 + 5670*a**5*b**4*c**2*d**8/13 + 15120*a**4*b**5*c**3*d**7
/13 + 17640*a**3*b**6*c**4*d**6/13 + 9072*a**2*b**7*c**5*d**5/13 + 1890*a*b
**8*c**6*d**4/13 + 120*b**9*c**7*d**3/13) + x**12*(3*a**8*b*d**10/4 + 30*a*
*7*b**2*c*d**9 + 315*a**6*b**3*c**2*d**8 + 1260*a**5*b**4*c**3*d**7 + 2205*
a**4*b**5*c**4*d**6 + 1764*a**3*b**6*c**5*d**5 + 630*a**2*b**7*c**6*d**4 +
90*a*b**8*c**7*d**3 + 15*b**9*c**8*d**2/4) + x**11*(a**9*d**10/11 + 90*a**8
*b*c*d**9/11 + 1620*a**7*b**2*c**2*d**8/11 + 10080*a**6*b**3*c**3*d**7/11 +
 26460*a**5*b**4*c**4*d**6/11 + 31752*a**4*b**5*c**5*d**5/11 + 17640*a**3*b
**6*c**6*d**4/11 + 4320*a**2*b**7*c**7*d**3/11 + 405*a*b**8*c**8*d**2/11 +
10*b**9*c**9*d/11) + x**10*(a**9*c*d**9 + 81*a**8*b*c**2*d**8/2 + 432*a**7*
b**2*c**3*d**7 + 1764*a**6*b**3*c**4*d**6 + 15876*a**5*b**4*c**5*d**5/5 + 2
646*a**4*b**5*c**6*d**4 + 1008*a**3*b**6*c**7*d**3 + 162*a**2*b**7*c**8*d**
2 + 9*a*b**8*c**9*d + b**9*c**10/10) + x**9*(5*a**9*c**2*d**8 + 120*a**8*b*
c**3*d**7 + 840*a**7*b**2*c**4*d**6 + 2352*a**6*b**3*c**5*d**5 + 2940*a**5*
b**4*c**6*d**4 + 1680*a**4*b**5*c**7*d**3 + 420*a**3*b**6*c**8*d**2 + 40*a*
*2*b**7*c**9*d + a*b**8*c**10) + x**8*(15*a**9*c**3*d**7 + 945*a**8*b*c**4*
d**6/4 + 1134*a**7*b**2*c**5*d**5 + 2205*a**6*b**3*c**6*d**4 + 1890*a**5*b*
*4*c**7*d**3 + 2835*a**4*b**5*c**8*d**2/4 + 105*a**3*b**6*c**9*d + 9*a**2*b
**7*c**10/2) + x**7*(30*a**9*c**4*d**6 + 324*a**8*b*c**5*d**5 + 1080*a**7*b
**2*c**6*d**4 + 1440*a**6*b**3*c**7*d**3 + 810*a**5*b**4*c**8*d**2 + 180*a*
*4*b**5*c**9*d + 12*a**3*b**6*c**10) + x**6*(42*a**9*c**5*d**5 + 315*a**8*b
*c**6*d**4 + 720*a**7*b**2*c**7*d**3 + 630*a**6*b**3*c**8*d**2 + 210*a**5*b
**4*c**9*d + 21*a**4*b**5*c**10) + x**5*(42*a**9*c**6*d**4 + 216*a**8*b*c**
7*d**3 + 324*a**7*b**2*c**8*d**2 + 168*a**6*b**3*c**9*d + 126*a**5*b**4*c**
10/5) + x**4*(30*a**9*c**7*d**3 + 405*a**8*b*c**8*d**2/4 + 90*a**7*b**2*c**
9*d + 21*a**6*b**3*c**10) + x**3*(15*a**9*c**8*d**2 + 30*a**8*b*c**9*d + 12
*a**7*b**2*c**10) + x**2*(5*a**9*c**9*d + 9*a**8*b*c**10/2)
```

3.1197 $\int (a+bx)^8(c+dx)^{10}\,dx$

Optimal. Leaf size=225

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+\cdots}{}$$

Rubi [A] time = 0.90, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{4b^7(c+dx)^{18}(bc-ad)}{9d^9}+\frac{28b^6(c+dx)^{17}(bc-ad)^2}{17d^9}-\frac{7b^5(c+dx)^{16}(bc-ad)^3}{2d^9}+\frac{14b^4(c+dx)^{15}(bc-ad)^4}{3d^9}-\frac{4b^3(c+dx)^{14}(bc-ad)^5}{d^9}+\frac{28b^2(c+dx)^{13}(bc-ad)^6}{13d^9}-\frac{2b(c+dx)^{12}(bc-ad)^7}{3d^9}+\frac{(c+dx)^{11}(bc-ad)^8}{11d^9}+\frac{b^8(c+dx)^{19}}{19d^9}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^8*(c + d*x)^11)/(11*d^9) - (2*b*(b*c - a*d)^7*(c + d*x)^12)/(3
*d^9) + (28*b^2*(b*c - a*d)^6*(c + d*x)^13)/(13*d^9) - (4*b^3*(b*c - a*d)^5
*(c + d*x)^14)/d^9 + (14*b^4*(b*c - a*d)^4*(c + d*x)^15)/(3*d^9) - (7*b^5*(
b*c - a*d)^3*(c + d*x)^16)/(2*d^9) + (28*b^6*(b*c - a*d)^2*(c + d*x)^17)/(1
7*d^9) - (4*b^7*(b*c - a*d)*(c + d*x)^18)/(9*d^9) + (b^8*(c + d*x)^19)/(19*
d^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^8(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^8(c+dx)^{10}}{d^8}-\frac{8b(bc-ad)^7(c+dx)^{11}}{d^8}+\frac{28b^2(bc-ad)^6(c+dx)^{12}}{d^8}-\frac{56b\cdots}{}\right.$$

$$=\frac{(bc-ad)^8(c+dx)^{11}}{11d^9}-\frac{2b(bc-ad)^7(c+dx)^{12}}{3d^9}+\frac{28b^2(bc-ad)^6(c+dx)^{13}}{13d^9}-\frac{4b^3(bc-\cdots}{}$$

Mathematica [B] time = 0.16, size = 1241, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] a^8*c^10*x + a^7*c^9*(4*b*c + 5*a*d)*x^2 + (a^6*c^8*(28*b^2*c^2 + 80*a*b*c*
d + 45*a^2*d^2)*x^3)/3 + 2*a^5*c^7*(7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c
*d^2 + 15*a^3*d^3)*x^4 + 2*a^4*c^6*(7*b^4*c^4 + 56*a*b^3*c^3*d + 126*a^2*b^
2*c^2*d^2 + 96*a^3*b*c*d^3 + 21*a^4*d^4)*x^5 + (14*a^3*c^5*(2*b^5*c^5 + 25*
a*b^4*c^4*d + 90*a^2*b^3*c^3*d^2 + 120*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 + 9
*a^5*d^5)*x^6)/3 + 2*a^2*c^4*(2*b^6*c^6 + 40*a*b^5*c^5*d + 225*a^2*b^4*c^4*
d^2 + 480*a^3*b^3*c^3*d^3 + 420*a^4*b^2*c^2*d^4 + 144*a^5*b*c*d^5 + 15*a^6*
d^6)*x^7 + a*c^3*(b^7*c^7 + 35*a*b^6*c^6*d + 315*a^2*b^5*c^5*d^2 + 1050*a^3
*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*d^5 + 210*a^6*b*c*d^6
 + 15*a^7*d^7)*x^8 + (c^2*(b^8*c^8 + 80*a*b^7*c^7*d + 1260*a^2*b^6*c^6*d^2
+ 6720*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 14112*a^5*b^3*c^3*d^5 + 58
80*a^6*b^2*c^2*d^6 + 960*a^7*b*c*d^7 + 45*a^8*d^8)*x^9)/9 + c*d*(b^8*c^8 +
```

```
36*a*b^7*c^7*d + 336*a^2*b^6*c^6*d^2 + 1176*a^3*b^5*c^5*d^3 + 1764*a^4*b^4*
c^4*d^4 + 1176*a^5*b^3*c^3*d^5 + 336*a^6*b^2*c^2*d^6 + 36*a^7*b*c*d^7 + a^8
*d^8)*x^10 + (d^2*(45*b^8*c^8 + 960*a*b^7*c^7*d + 5880*a^2*b^6*c^6*d^2 + 14
112*a^3*b^5*c^5*d^3 + 14700*a^4*b^4*c^4*d^4 + 6720*a^5*b^3*c^3*d^5 + 1260*a
^6*b^2*c^2*d^6 + 80*a^7*b*c*d^7 + a^8*d^8)*x^11)/11 + (2*b*d^3*(15*b^7*c^7
+ 210*a*b^6*c^6*d + 882*a^2*b^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1050*a^4*b
^3*c^3*d^4 + 315*a^5*b^2*c^2*d^5 + 35*a^6*b*c*d^6 + a^7*d^7)*x^12)/3 + (14*
b^2*d^4*(15*b^6*c^6 + 144*a*b^5*c^5*d + 420*a^2*b^4*c^4*d^2 + 480*a^3*b^3*c
^3*d^3 + 225*a^4*b^2*c^2*d^4 + 40*a^5*b*c*d^5 + 2*a^6*d^6)*x^13)/13 + 2*b^3
*d^5*(9*b^5*c^5 + 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3
 + 25*a^4*b*c*d^4 + 2*a^5*d^5)*x^14 + (2*b^4*d^6*(21*b^4*c^4 + 96*a*b^3*c^3
*d + 126*a^2*b^2*c^2*d^2 + 56*a^3*b*c*d^3 + 7*a^4*d^4)*x^15)/3 + (b^5*d^7*(
15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2 + 7*a^3*d^3)*x^16)/2 + (b^6*d^
8*(45*b^2*c^2 + 80*a*b*c*d + 28*a^2*d^2)*x^17)/17 + (b^7*d^9*(5*b*c + 4*a*d
)*x^18)/9 + (b^8*d^10*x^19)/19
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^8(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8*(c + d*x)^10, x]
```

fricas [B] time = 1.17, size = 1478, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/19*x^19*d^10*b^8 + 5/9*x^18*d^9*c*b^8 + 4/9*x^18*d^10*b^7*a + 45/17*x^17*
d^8*c^2*b^8 + 80/17*x^17*d^9*c*b^7*a + 28/17*x^17*d^10*b^6*a^2 + 15/2*x^16*
d^7*c^3*b^8 + 45/2*x^16*d^8*c^2*b^7*a + 35/2*x^16*d^9*c*b^6*a^2 + 7/2*x^16*
d^10*b^5*a^3 + 14*x^15*d^6*c^4*b^8 + 64*x^15*d^7*c^3*b^7*a + 84*x^15*d^8*c^
2*b^6*a^2 + 112/3*x^15*d^9*c*b^5*a^3 + 14/3*x^15*d^10*b^4*a^4 + 18*x^14*d^5
*c^5*b^8 + 120*x^14*d^6*c^4*b^7*a + 240*x^14*d^7*c^3*b^6*a^2 + 180*x^14*d^8
*c^2*b^5*a^3 + 50*x^14*d^9*c*b^4*a^4 + 4*x^14*d^10*b^3*a^5 + 210/13*x^13*d^
4*c^6*b^8 + 2016/13*x^13*d^5*c^5*b^7*a + 5880/13*x^13*d^6*c^4*b^6*a^2 + 672
0/13*x^13*d^7*c^3*b^5*a^3 + 3150/13*x^13*d^8*c^2*b^4*a^4 + 560/13*x^13*d^9*
c*b^3*a^5 + 28/13*x^13*d^10*b^2*a^6 + 10*x^12*d^3*c^7*b^8 + 140*x^12*d^4*c^
6*b^7*a + 588*x^12*d^5*c^5*b^6*a^2 + 980*x^12*d^6*c^4*b^5*a^3 + 700*x^12*d^
7*c^3*b^4*a^4 + 210*x^12*d^8*c^2*b^3*a^5 + 70/3*x^12*d^9*c*b^2*a^6 + 2/3*x^
12*d^10*b*a^7 + 45/11*x^11*d^2*c^8*b^8 + 960/11*x^11*d^3*c^7*b^7*a + 5880/1
1*x^11*d^4*c^6*b^6*a^2 + 14112/11*x^11*d^5*c^5*b^5*a^3 + 14700/11*x^11*d^6*
c^4*b^4*a^4 + 6720/11*x^11*d^7*c^3*b^3*a^5 + 1260/11*x^11*d^8*c^2*b^2*a^6 +
 80/11*x^11*d^9*c*b*a^7 + 1/11*x^11*d^10*a^8 + x^10*d*c^9*b^8 + 36*x^10*d^2
*c^8*b^7*a + 336*x^10*d^3*c^7*b^6*a^2 + 1176*x^10*d^4*c^6*b^5*a^3 + 1764*x^
10*d^5*c^5*b^4*a^4 + 1176*x^10*d^6*c^4*b^3*a^5 + 336*x^10*d^7*c^3*b^2*a^6 +
 36*x^10*d^8*c^2*b*a^7 + x^10*d^9*c*a^8 + 1/9*x^9*c^10*b^8 + 80/9*x^9*d*c^9
*b^7*a + 140*x^9*d^2*c^8*b^6*a^2 + 2240/3*x^9*d^3*c^7*b^5*a^3 + 4900/3*x^9*
d^4*c^6*b^4*a^4 + 1568*x^9*d^5*c^5*b^3*a^5 + 1960/3*x^9*d^6*c^4*b^2*a^6 + 3
20/3*x^9*d^7*c^3*b*a^7 + 5*x^9*d^8*c^2*a^8 + x^8*c^10*b^7*a + 35*x^8*d*c^9*
b^6*a^2 + 315*x^8*d^2*c^8*b^5*a^3 + 1050*x^8*d^3*c^7*b^4*a^4 + 1470*x^8*d^4
*c^6*b^3*a^5 + 882*x^8*d^5*c^5*b^2*a^6 + 210*x^8*d^6*c^4*b*a^7 + 15*x^8*d^7
*c^3*a^8 + 4*x^7*c^10*b^6*a^2 + 80*x^7*d*c^9*b^5*a^3 + 450*x^7*d^2*c^8*b^4*
a^4 + 960*x^7*d^3*c^7*b^3*a^5 + 840*x^7*d^4*c^6*b^2*a^6 + 288*x^7*d^5*c^5*b
*a^7 + 30*x^7*d^6*c^4*a^8 + 28/3*x^6*c^10*b^5*a^3 + 350/3*x^6*d*c^9*b^4*a^4
 + 420*x^6*d^2*c^8*b^3*a^5 + 560*x^6*d^3*c^7*b^2*a^6 + 280*x^6*d^4*c^6*b*a^
```

```
7 + 42*x^6*d^5*c^5*a^8 + 14*x^5*c^10*b^4*a^4 + 112*x^5*d*c^9*b^3*a^5 + 252*
x^5*d^2*c^8*b^2*a^6 + 192*x^5*d^3*c^7*b*a^7 + 42*x^5*d^4*c^6*a^8 + 14*x^4*c
^10*b^3*a^5 + 70*x^4*d*c^9*b^2*a^6 + 90*x^4*d^2*c^8*b*a^7 + 30*x^4*d^3*c^7*
a^8 + 28/3*x^3*c^10*b^2*a^6 + 80/3*x^3*d*c^9*b*a^7 + 15*x^3*d^2*c^8*a^8 + 4
*x^2*c^10*b*a^7 + 5*x^2*d*c^9*a^8 + x*c^10*a^8
```

giac [B] time = 1.29, size = 1478, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/19*b^8*d^10*x^19 + 5/9*b^8*c*d^9*x^18 + 4/9*a*b^7*d^10*x^18 + 45/17*b^8*c
^2*d^8*x^17 + 80/17*a*b^7*c*d^9*x^17 + 28/17*a^2*b^6*d^10*x^17 + 15/2*b^8*c
^3*d^7*x^16 + 45/2*a*b^7*c^2*d^8*x^16 + 35/2*a^2*b^6*c*d^9*x^16 + 7/2*a^3*b
^5*d^10*x^16 + 14*b^8*c^4*d^6*x^15 + 64*a*b^7*c^3*d^7*x^15 + 84*a^2*b^6*c^2
*d^8*x^15 + 112/3*a^3*b^5*c*d^9*x^15 + 14/3*a^4*b^4*d^10*x^15 + 18*b^8*c^5*
d^5*x^14 + 120*a*b^7*c^4*d^6*x^14 + 240*a^2*b^6*c^3*d^7*x^14 + 180*a^3*b^5*
c^2*d^8*x^14 + 50*a^4*b^4*c*d^9*x^14 + 4*a^5*b^3*d^10*x^14 + 210/13*b^8*c^6
*d^4*x^13 + 2016/13*a*b^7*c^5*d^5*x^13 + 5880/13*a^2*b^6*c^4*d^6*x^13 + 672
0/13*a^3*b^5*c^3*d^7*x^13 + 3150/13*a^4*b^4*c^2*d^8*x^13 + 560/13*a^5*b^3*c
*d^9*x^13 + 28/13*a^6*b^2*d^10*x^13 + 10*b^8*c^7*d^3*x^12 + 140*a*b^7*c^6*d
^4*x^12 + 588*a^2*b^6*c^5*d^5*x^12 + 980*a^3*b^5*c^4*d^6*x^12 + 700*a^4*b^4
*c^3*d^7*x^12 + 210*a^5*b^3*c^2*d^8*x^12 + 70/3*a^6*b^2*c*d^9*x^12 + 2/3*a^
7*b*d^10*x^12 + 45/11*b^8*c^8*d^2*x^11 + 960/11*a*b^7*c^7*d^3*x^11 + 5880/1
1*a^2*b^6*c^6*d^4*x^11 + 14112/11*a^3*b^5*c^5*d^5*x^11 + 14700/11*a^4*b^4*c
^4*d^6*x^11 + 6720/11*a^5*b^3*c^3*d^7*x^11 + 1260/11*a^6*b^2*c^2*d^8*x^11 +
 80/11*a^7*b*c*d^9*x^11 + 1/11*a^8*d^10*x^11 + b^8*c^9*d*x^10 + 36*a*b^7*c^
8*d^2*x^10 + 336*a^2*b^6*c^7*d^3*x^10 + 1176*a^3*b^5*c^6*d^4*x^10 + 1764*a^
4*b^4*c^5*d^5*x^10 + 1176*a^5*b^3*c^4*d^6*x^10 + 336*a^6*b^2*c^3*d^7*x^10 +
 36*a^7*b*c^2*d^8*x^10 + a^8*c*d^9*x^10 + 1/9*b^8*c^10*x^9 + 80/9*a*b^7*c^9
*d*x^9 + 140*a^2*b^6*c^8*d^2*x^9 + 2240/3*a^3*b^5*c^7*d^3*x^9 + 4900/3*a^4*
b^4*c^6*d^4*x^9 + 1568*a^5*b^3*c^5*d^5*x^9 + 1960/3*a^6*b^2*c^4*d^6*x^9 + 3
20/3*a^7*b*c^3*d^7*x^9 + 5*a^8*c^2*d^8*x^9 + a*b^7*c^10*x^8 + 35*a^2*b^6*c^
9*d*x^8 + 315*a^3*b^5*c^8*d^2*x^8 + 1050*a^4*b^4*c^7*d^3*x^8 + 1470*a^5*b^3
*c^6*d^4*x^8 + 882*a^6*b^2*c^5*d^5*x^8 + 210*a^7*b*c^4*d^6*x^8 + 15*a^8*c^3
*d^7*x^8 + 4*a^2*b^6*c^10*x^7 + 80*a^3*b^5*c^9*d*x^7 + 450*a^4*b^4*c^8*d^2*
x^7 + 960*a^5*b^3*c^7*d^3*x^7 + 840*a^6*b^2*c^6*d^4*x^7 + 288*a^7*b*c^5*d^5
*x^7 + 30*a^8*c^4*d^6*x^7 + 28/3*a^3*b^5*c^10*x^6 + 350/3*a^4*b^4*c^9*d*x^6
 + 420*a^5*b^3*c^8*d^2*x^6 + 560*a^6*b^2*c^7*d^3*x^6 + 280*a^7*b*c^6*d^4*x^
6 + 42*a^8*c^5*d^5*x^6 + 14*a^4*b^4*c^10*x^5 + 112*a^5*b^3*c^9*d*x^5 + 252*
a^6*b^2*c^8*d^2*x^5 + 192*a^7*b*c^7*d^3*x^5 + 42*a^8*c^6*d^4*x^5 + 14*a^5*b
^3*c^10*x^4 + 70*a^6*b^2*c^9*d*x^4 + 90*a^7*b*c^8*d^2*x^4 + 30*a^8*c^7*d^3*
x^4 + 28/3*a^6*b^2*c^10*x^3 + 80/3*a^7*b*c^9*d*x^3 + 15*a^8*c^8*d^2*x^3 + 4
*a^7*b*c^10*x^2 + 5*a^8*c^9*d*x^2 + a^8*c^10*x
```

maple [B] time = 0.00, size = 1291, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8*(d*x+c)^10,x)
```

```
[Out] 1/19*b^8*d^10*x^19+1/18*(8*a*b^7*d^10+10*b^8*c*d^9)*x^18+1/17*(28*a^2*b^6*d
^10+80*a*b^7*c*d^9+45*b^8*c^2*d^8)*x^17+1/16*(56*a^3*b^5*d^10+280*a^2*b^6*c
*d^9+360*a*b^7*c^2*d^8+120*b^8*c^3*d^7)*x^16+1/15*(70*a^4*b^4*d^10+560*a^3*
b^5*c*d^9+1260*a^2*b^6*c^2*d^8+960*a*b^7*c^3*d^7+210*b^8*c^4*d^6)*x^15+1/14
*(56*a^5*b^3*d^10+700*a^4*b^4*c*d^9+2520*a^3*b^5*c^2*d^8+3360*a^2*b^6*c^3*d
^7+1680*a*b^7*c^4*d^6+252*b^8*c^5*d^5)*x^14+1/13*(28*a^6*b^2*d^10+560*a^5*b
```

```
^3*c*d^9+3150*a^4*b^4*c^2*d^8+6720*a^3*b^5*c^3*d^7+5880*a^2*b^6*c^4*d^6+201
6*a*b^7*c^5*d^5+210*b^8*c^6*d^4)*x^13+1/12*(8*a^7*b*d^10+280*a^6*b^2*c*d^9+
2520*a^5*b^3*c^2*d^8+8400*a^4*b^4*c^3*d^7+11760*a^3*b^5*c^4*d^6+7056*a^2*b^
6*c^5*d^5+1680*a*b^7*c^6*d^4+120*b^8*c^7*d^3)*x^12+1/11*(a^8*d^10+80*a^7*b*
c*d^9+1260*a^6*b^2*c^2*d^8+6720*a^5*b^3*c^3*d^7+14700*a^4*b^4*c^4*d^6+14112
*a^3*b^5*c^5*d^5+5880*a^2*b^6*c^6*d^4+960*a*b^7*c^7*d^3+45*b^8*c^8*d^2)*x^1
1+1/10*(10*a^8*c*d^9+360*a^7*b*c^2*d^8+3360*a^6*b^2*c^3*d^7+11760*a^5*b^3*c
^4*d^6+17640*a^4*b^4*c^5*d^5+11760*a^3*b^5*c^6*d^4+3360*a^2*b^6*c^7*d^3+360
*a*b^7*c^8*d^2+10*b^8*c^9*d)*x^10+1/9*(45*a^8*c^2*d^8+960*a^7*b*c^3*d^7+588
0*a^6*b^2*c^4*d^6+14112*a^5*b^3*c^5*d^5+14700*a^4*b^4*c^6*d^4+6720*a^3*b^5*
c^7*d^3+1260*a^2*b^6*c^8*d^2+80*a*b^7*c^9*d+b^8*c^10)*x^9+1/8*(120*a^8*c^3*
d^7+1680*a^7*b*c^4*d^6+7056*a^6*b^2*c^5*d^5+11760*a^5*b^3*c^6*d^4+8400*a^4*
b^4*c^7*d^3+2520*a^3*b^5*c^8*d^2+280*a^2*b^6*c^9*d+8*a*b^7*c^10)*x^8+1/7*(2
10*a^8*c^4*d^6+2016*a^7*b*c^5*d^5+5880*a^6*b^2*c^6*d^4+6720*a^5*b^3*c^7*d^3
+3150*a^4*b^4*c^8*d^2+560*a^3*b^5*c^9*d+28*a^2*b^6*c^10)*x^7+1/6*(252*a^8*c
^5*d^5+1680*a^7*b*c^6*d^4+3360*a^6*b^2*c^7*d^3+2520*a^5*b^3*c^8*d^2+700*a^4
*b^4*c^9*d+56*a^3*b^5*c^10)*x^6+1/5*(210*a^8*c^6*d^4+960*a^7*b*c^7*d^3+1260
*a^6*b^2*c^8*d^2+560*a^5*b^3*c^9*d+70*a^4*b^4*c^10)*x^5+1/4*(120*a^8*c^7*d^
3+360*a^7*b*c^8*d^2+280*a^6*b^2*c^9*d+56*a^5*b^3*c^10)*x^4+1/3*(45*a^8*c^8*
d^2+80*a^7*b*c^9*d+28*a^6*b^2*c^10)*x^3+1/2*(10*a^8*c^9*d+8*a^7*b*c^10)*x^2
+a^8*c^10*x
```

maxima [B] time = 1.51, size = 1283, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/19*b^8*d^10*x^19 + a^8*c^10*x + 1/9*(5*b^8*c*d^9 + 4*a*b^7*d^10)*x^18 + 1
/17*(45*b^8*c^2*d^8 + 80*a*b^7*c*d^9 + 28*a^2*b^6*d^10)*x^17 + 1/2*(15*b^8*
c^3*d^7 + 45*a*b^7*c^2*d^8 + 35*a^2*b^6*c*d^9 + 7*a^3*b^5*d^10)*x^16 + 2/3*
(21*b^8*c^4*d^6 + 96*a*b^7*c^3*d^7 + 126*a^2*b^6*c^2*d^8 + 56*a^3*b^5*c*d^9
 + 7*a^4*b^4*d^10)*x^15 + 2*(9*b^8*c^5*d^5 + 60*a*b^7*c^4*d^6 + 120*a^2*b^6
*c^3*d^7 + 90*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 + 2*a^5*b^3*d^10)*x^14 + 1
4/13*(15*b^8*c^6*d^4 + 144*a*b^7*c^5*d^5 + 420*a^2*b^6*c^4*d^6 + 480*a^3*b^
5*c^3*d^7 + 225*a^4*b^4*c^2*d^8 + 40*a^5*b^3*c*d^9 + 2*a^6*b^2*d^10)*x^13 +
 2/3*(15*b^8*c^7*d^3 + 210*a*b^7*c^6*d^4 + 882*a^2*b^6*c^5*d^5 + 1470*a^3*b
^5*c^4*d^6 + 1050*a^4*b^4*c^3*d^7 + 315*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9
+ a^7*b*d^10)*x^12 + 1/11*(45*b^8*c^8*d^2 + 960*a*b^7*c^7*d^3 + 5880*a^2*b^
6*c^6*d^4 + 14112*a^3*b^5*c^5*d^5 + 14700*a^4*b^4*c^4*d^6 + 6720*a^5*b^3*c^
3*d^7 + 1260*a^6*b^2*c^2*d^8 + 80*a^7*b*c*d^9 + a^8*d^10)*x^11 + (b^8*c^9*d
 + 36*a*b^7*c^8*d^2 + 336*a^2*b^6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4
*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*d^6 + 336*a^6*b^2*c^3*d^7 + 36*a^7*b*c^2*d^
8 + a^8*c*d^9)*x^10 + 1/9*(b^8*c^10 + 80*a*b^7*c^9*d + 1260*a^2*b^6*c^8*d^2
 + 6720*a^3*b^5*c^7*d^3 + 14700*a^4*b^4*c^6*d^4 + 14112*a^5*b^3*c^5*d^5 + 5
880*a^6*b^2*c^4*d^6 + 960*a^7*b*c^3*d^7 + 45*a^8*c^2*d^8)*x^9 + (a*b^7*c^10
 + 35*a^2*b^6*c^9*d + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3 + 1470*a^5
*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5 + 210*a^7*b*c^4*d^6 + 15*a^8*c^3*d^7)*x^
8 + 2*(2*a^2*b^6*c^10 + 40*a^3*b^5*c^9*d + 225*a^4*b^4*c^8*d^2 + 480*a^5*b^
3*c^7*d^3 + 420*a^6*b^2*c^6*d^4 + 144*a^7*b*c^5*d^5 + 15*a^8*c^4*d^6)*x^7 +
 14/3*(2*a^3*b^5*c^10 + 25*a^4*b^4*c^9*d + 90*a^5*b^3*c^8*d^2 + 120*a^6*b^2
*c^7*d^3 + 60*a^7*b*c^6*d^4 + 9*a^8*c^5*d^5)*x^6 + 2*(7*a^4*b^4*c^10 + 56*a
^5*b^3*c^9*d + 126*a^6*b^2*c^8*d^2 + 96*a^7*b*c^7*d^3 + 21*a^8*c^6*d^4)*x^5
 + 2*(7*a^5*b^3*c^10 + 35*a^6*b^2*c^9*d + 45*a^7*b*c^8*d^2 + 15*a^8*c^7*d^3
)*x^4 + 1/3*(28*a^6*b^2*c^10 + 80*a^7*b*c^9*d + 45*a^8*c^8*d^2)*x^3 + (4*a^
7*b*c^10 + 5*a^8*c^9*d)*x^2
```

mupad [B] time = 0.71, size = 1253, normalized size = 5.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8*(c + d*x)^10,x)
```

```
[Out] x^7*(4*a^2*b^6*c^10 + 30*a^8*c^4*d^6 + 80*a^3*b^5*c^9*d + 288*a^7*b*c^5*d^5
 + 450*a^4*b^4*c^8*d^2 + 960*a^5*b^3*c^7*d^3 + 840*a^6*b^2*c^6*d^4) + x^13*
((28*a^6*b^2*d^10)/13 + (210*b^8*c^6*d^4)/13 + (2016*a*b^7*c^5*d^5)/13 + (5
60*a^5*b^3*c*d^9)/13 + (5880*a^2*b^6*c^4*d^6)/13 + (6720*a^3*b^5*c^3*d^7)/1
3 + (3150*a^4*b^4*c^2*d^8)/13) + x^8*(a*b^7*c^10 + 15*a^8*c^3*d^7 + 35*a^2*
b^6*c^9*d + 210*a^7*b*c^4*d^6 + 315*a^3*b^5*c^8*d^2 + 1050*a^4*b^4*c^7*d^3
+ 1470*a^5*b^3*c^6*d^4 + 882*a^6*b^2*c^5*d^5) + x^12*((2*a^7*b*d^10)/3 + 10
*b^8*c^7*d^3 + 140*a*b^7*c^6*d^4 + (70*a^6*b^2*c*d^9)/3 + 588*a^2*b^6*c^5*d
^5 + 980*a^3*b^5*c^4*d^6 + 700*a^4*b^4*c^3*d^7 + 210*a^5*b^3*c^2*d^8) + x^1
0*(a^8*c*d^9 + b^8*c^9*d + 36*a*b^7*c^8*d^2 + 36*a^7*b*c^2*d^8 + 336*a^2*b^
6*c^7*d^3 + 1176*a^3*b^5*c^6*d^4 + 1764*a^4*b^4*c^5*d^5 + 1176*a^5*b^3*c^4*
d^6 + 336*a^6*b^2*c^3*d^7) + x^5*(14*a^4*b^4*c^10 + 42*a^8*c^6*d^4 + 112*a^
5*b^3*c^9*d + 192*a^7*b*c^7*d^3 + 252*a^6*b^2*c^8*d^2) + x^15*((14*a^4*b^4*
d^10)/3 + 14*b^8*c^4*d^6 + 64*a*b^7*c^3*d^7 + (112*a^3*b^5*c*d^9)/3 + 84*a^
2*b^6*c^2*d^8) + x^6*((28*a^3*b^5*c^10)/3 + 42*a^8*c^5*d^5 + (350*a^4*b^4*c
^9*d)/3 + 280*a^7*b*c^6*d^4 + 420*a^5*b^3*c^8*d^2 + 560*a^6*b^2*c^7*d^3) +
x^14*(4*a^5*b^3*d^10 + 18*b^8*c^5*d^5 + 120*a*b^7*c^4*d^6 + 50*a^4*b^4*c*d^
9 + 240*a^2*b^6*c^3*d^7 + 180*a^3*b^5*c^2*d^8) + x^9*((b^8*c^10)/9 + 5*a^8*
c^2*d^8 + (320*a^7*b*c^3*d^7)/3 + 140*a^2*b^6*c^8*d^2 + (2240*a^3*b^5*c^7*d
^3)/3 + (4900*a^4*b^4*c^6*d^4)/3 + 1568*a^5*b^3*c^5*d^5 + (1960*a^6*b^2*c^4
*d^6)/3 + (80*a*b^7*c^9*d)/9) + x^11*((a^8*d^10)/11 + (45*b^8*c^8*d^2)/11 +
 (960*a*b^7*c^7*d^3)/11 + (5880*a^2*b^6*c^6*d^4)/11 + (14112*a^3*b^5*c^5*d^
5)/11 + (14700*a^4*b^4*c^4*d^6)/11 + (6720*a^5*b^3*c^3*d^7)/11 + (1260*a^6*
b^2*c^2*d^8)/11 + (80*a^7*b*c*d^9)/11) + a^8*c^10*x + (b^8*d^10*x^19)/19 +
2*a^5*c^7*x^4*(15*a^3*d^3 + 7*b^3*c^3 + 35*a*b^2*c^2*d + 45*a^2*b*c*d^2) +
(b^5*d^7*x^16*(7*a^3*d^3 + 15*b^3*c^3 + 45*a*b^2*c^2*d + 35*a^2*b*c*d^2))/2
 + a^7*c^9*x^2*(5*a*d + 4*b*c) + (b^7*d^9*x^18*(4*a*d + 5*b*c))/9 + (a^6*c^
8*x^3*(45*a^2*d^2 + 28*b^2*c^2 + 80*a*b*c*d))/3 + (b^6*d^8*x^17*(28*a^2*d^2
 + 45*b^2*c^2 + 80*a*b*c*d))/17
```

sympy [B]   time = 0.27, size = 1428, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8*(d*x+c)**10,x)
```

```
[Out] a**8*c**10*x + b**8*d**10*x**19/19 + x**18*(4*a*b**7*d**10/9 + 5*b**8*c*d**
9/9) + x**17*(28*a**2*b**6*d**10/17 + 80*a*b**7*c*d**9/17 + 45*b**8*c**2*d*
*8/17) + x**16*(7*a**3*b**5*d**10/2 + 35*a**2*b**6*c*d**9/2 + 45*a*b**7*c**
2*d**8/2 + 15*b**8*c**3*d**7/2) + x**15*(14*a**4*b**4*d**10/3 + 112*a**3*b*
*5*c*d**9/3 + 84*a**2*b**6*c**2*d**8 + 64*a*b**7*c**3*d**7 + 14*b**8*c**4*d
**6) + x**14*(4*a**5*b**3*d**10 + 50*a**4*b**4*c*d**9 + 180*a**3*b**5*c**2*
d**8 + 240*a**2*b**6*c**3*d**7 + 120*a*b**7*c**4*d**6 + 18*b**8*c**5*d**5)
+ x**13*(28*a**6*b**2*d**10/13 + 560*a**5*b**3*c*d**9/13 + 3150*a**4*b**4*c
**2*d**8/13 + 6720*a**3*b**5*c**3*d**7/13 + 5880*a**2*b**6*c**4*d**6/13 + 2
016*a*b**7*c**5*d**5/13 + 210*b**8*c**6*d**4/13) + x**12*(2*a**7*b*d**10/3
+ 70*a**6*b**2*c*d**9/3 + 210*a**5*b**3*c**2*d**8 + 700*a**4*b**4*c**3*d**7
 + 980*a**3*b**5*c**4*d**6 + 588*a**2*b**6*c**5*d**5 + 140*a*b**7*c**6*d**4
 + 10*b**8*c**7*d**3) + x**11*(a**8*d**10/11 + 80*a**7*b*c*d**9/11 + 1260*a
**6*b**2*c**2*d**8/11 + 6720*a**5*b**3*c**3*d**7/11 + 14700*a**4*b**4*c**4*
d**6/11 + 14112*a**3*b**5*c**5*d**5/11 + 5880*a**2*b**6*c**6*d**4/11 + 960*
a*b**7*c**7*d**3/11 + 45*b**8*c**8*d**2/11) + x**10*(a**8*c*d**9 + 36*a**7*
b*c**2*d**8 + 336*a**6*b**2*c**3*d**7 + 1176*a**5*b**3*c**4*d**6 + 1764*a**
4*b**4*c**5*d**5 + 1176*a**3*b**5*c**6*d**4 + 336*a**2*b**6*c**7*d**3 + 36*
a*b**7*c**8*d**2 + b**8*c**9*d) + x**9*(5*a**8*c**2*d**8 + 320*a**7*b*c**3*
d**7/3 + 1960*a**6*b**2*c**4*d**6/3 + 1568*a**5*b**3*c**5*d**5 + 4900*a**4*
```

```
b**4*c**6*d**4/3 + 2240*a**3*b**5*c**7*d**3/3 + 140*a**2*b**6*c**8*d**2 + 8
0*a*b**7*c**9*d/9 + b**8*c**10/9) + x**8*(15*a**8*c**3*d**7 + 210*a**7*b*c*
*4*d**6 + 882*a**6*b**2*c**5*d**5 + 1470*a**5*b**3*c**6*d**4 + 1050*a**4*b*
*4*c**7*d**3 + 315*a**3*b**5*c**8*d**2 + 35*a**2*b**6*c**9*d + a*b**7*c**10
) + x**7*(30*a**8*c**4*d**6 + 288*a**7*b*c**5*d**5 + 840*a**6*b**2*c**6*d**
4 + 960*a**5*b**3*c**7*d**3 + 450*a**4*b**4*c**8*d**2 + 80*a**3*b**5*c**9*d
 + 4*a**2*b**6*c**10) + x**6*(42*a**8*c**5*d**5 + 280*a**7*b*c**6*d**4 + 56
0*a**6*b**2*c**7*d**3 + 420*a**5*b**3*c**8*d**2 + 350*a**4*b**4*c**9*d/3 +
28*a**3*b**5*c**10/3) + x**5*(42*a**8*c**6*d**4 + 192*a**7*b*c**7*d**3 + 25
2*a**6*b**2*c**8*d**2 + 112*a**5*b**3*c**9*d + 14*a**4*b**4*c**10) + x**4*(
30*a**8*c**7*d**3 + 90*a**7*b*c**8*d**2 + 70*a**6*b**2*c**9*d + 14*a**5*b**
3*c**10) + x**3*(15*a**8*c**8*d**2 + 80*a**7*b*c**9*d/3 + 28*a**6*b**2*c**1
0/3) + x**2*(5*a**8*c**9*d + 4*a**7*b*c**10)
```

3.1198 $\int (a+bx)^7(c+dx)^{10}\,dx$

Optimal. Leaf size=200

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+\cdots}{}$$

Rubi [A] time = 0.77, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{7b^6(c+dx)^{17}(bc-ad)}{17d^8}+\frac{21b^5(c+dx)^{16}(bc-ad)^2}{16d^8}-\frac{7b^4(c+dx)^{15}(bc-ad)^3}{3d^8}+\frac{5b^3(c+dx)^{14}(bc-ad)^4}{2d^8}-\frac{21b^2(c+dx)^{13}(bc-ad)^5}{13d^8}+\frac{7b(c+dx)^{12}(bc-ad)^6}{12d^8}-\frac{(c+dx)^{11}(bc-ad)^7}{11d^8}+\frac{b^7(c+dx)^{18}}{18d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^7*(c + d*x)^11)/(11*d^8) + (7*b*(b*c - a*d)^6*(c + d*x)^12)/(
12*d^8) - (21*b^2*(b*c - a*d)^5*(c + d*x)^13)/(13*d^8) + (5*b^3*(b*c - a*d)
^4*(c + d*x)^14)/(2*d^8) - (7*b^4*(b*c - a*d)^3*(c + d*x)^15)/(3*d^8) + (21
*b^5*(b*c - a*d)^2*(c + d*x)^16)/(16*d^8) - (7*b^6*(b*c - a*d)*(c + d*x)^17
)/(17*d^8) + (b^7*(c + d*x)^18)/(18*d^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^7(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^7(c+dx)^{10}}{d^7}+\frac{7b(bc-ad)^6(c+dx)^{11}}{d^7}-\frac{21b^2(bc-ad)^5(c+dx)^{12}}{d^7}+\frac{35b\cdots}{}\right.$$

$$=-\frac{(bc-ad)^7(c+dx)^{11}}{11d^8}+\frac{7b(bc-ad)^6(c+dx)^{12}}{12d^8}-\frac{21b^2(bc-ad)^5(c+dx)^{13}}{13d^8}+\frac{5b^3(bc-\cdots}{}$$

Mathematica [B] time = 0.14, size = 1105, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] a^7*c^10*x + (a^6*c^9*(7*b*c + 10*a*d)*x^2)/2 + (a^5*c^8*(21*b^2*c^2 + 70*a
*b*c*d + 45*a^2*d^2)*x^3)/3 + (5*a^4*c^7*(7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a
^2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + 7*a^3*c^6*(b^4*c^4 + 10*a*b^3*c^3*d + 27*
a^2*b^2*c^2*d^2 + 24*a^3*b*c*d^3 + 6*a^4*d^4)*x^5 + (7*a^2*c^5*(3*b^5*c^5 +
 50*a*b^4*c^4*d + 225*a^2*b^3*c^3*d^2 + 360*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d
^4 + 36*a^5*d^5)*x^6)/6 + a*c^4*(b^6*c^6 + 30*a*b^5*c^5*d + 225*a^2*b^4*c^4
*d^2 + 600*a^3*b^3*c^3*d^3 + 630*a^4*b^2*c^2*d^4 + 252*a^5*b*c*d^5 + 30*a^6
*d^6)*x^7 + (c^3*(b^7*c^7 + 70*a*b^6*c^6*d + 945*a^2*b^5*c^5*d^2 + 4200*a^3
*b^4*c^4*d^3 + 7350*a^4*b^3*c^3*d^4 + 5292*a^5*b^2*c^2*d^5 + 1470*a^6*b*c*d
^6 + 120*a^7*d^7)*x^8)/8 + (5*c^2*d*(2*b^7*c^7 + 63*a*b^6*c^6*d + 504*a^2*b
^5*c^5*d^2 + 1470*a^3*b^4*c^4*d^3 + 1764*a^4*b^3*c^3*d^4 + 882*a^5*b^2*c^2*
d^5 + 168*a^6*b*c*d^6 + 9*a^7*d^7)*x^9)/9 + (c*d^2*(9*b^7*c^7 + 168*a*b^6*c
^6*d + 882*a^2*b^5*c^5*d^2 + 1764*a^3*b^4*c^4*d^3 + 1470*a^4*b^3*c^3*d^4 +
```

```
504*a^5*b^2*c^2*d^5 + 63*a^6*b*c*d^6 + 2*a^7*d^7)*x^10)/2 + (d^3*(120*b^7*c
^7 + 1470*a*b^6*c^6*d + 5292*a^2*b^5*c^5*d^2 + 7350*a^3*b^4*c^4*d^3 + 4200*
a^4*b^3*c^3*d^4 + 945*a^5*b^2*c^2*d^5 + 70*a^6*b*c*d^6 + a^7*d^7)*x^11)/11
+ (7*b*d^4*(30*b^6*c^6 + 252*a*b^5*c^5*d + 630*a^2*b^4*c^4*d^2 + 600*a^3*b^
3*c^3*d^3 + 225*a^4*b^2*c^2*d^4 + 30*a^5*b*c*d^5 + a^6*d^6)*x^12)/12 + (7*b
^2*d^5*(36*b^5*c^5 + 210*a*b^4*c^4*d + 360*a^2*b^3*c^3*d^2 + 225*a^3*b^2*c^
2*d^3 + 50*a^4*b*c*d^4 + 3*a^5*d^5)*x^13)/13 + (5*b^3*d^6*(6*b^4*c^4 + 24*a
*b^3*c^3*d + 27*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + a^4*d^4)*x^14)/2 + (b^4*
d^7*(24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*b*c*d^2 + 7*a^3*d^3)*x^15)/3 + (b
^5*d^8*(45*b^2*c^2 + 70*a*b*c*d + 21*a^2*d^2)*x^16)/16 + (b^6*d^9*(10*b*c +
 7*a*d)*x^17)/17 + (b^7*d^10*x^18)/18
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^7(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7*(c + d*x)^10, x]
```

fricas [B] time = 1.08, size = 1302, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/18*x^18*d^10*b^7 + 10/17*x^17*d^9*c*b^7 + 7/17*x^17*d^10*b^6*a + 45/16*x^
16*d^8*c^2*b^7 + 35/8*x^16*d^9*c*b^6*a + 21/16*x^16*d^10*b^5*a^2 + 8*x^15*d
^7*c^3*b^7 + 21*x^15*d^8*c^2*b^6*a + 14*x^15*d^9*c*b^5*a^2 + 7/3*x^15*d^10*
b^4*a^3 + 15*x^14*d^6*c^4*b^7 + 60*x^14*d^7*c^3*b^6*a + 135/2*x^14*d^8*c^2*
b^5*a^2 + 25*x^14*d^9*c*b^4*a^3 + 5/2*x^14*d^10*b^3*a^4 + 252/13*x^13*d^5*c
^5*b^7 + 1470/13*x^13*d^6*c^4*b^6*a + 2520/13*x^13*d^7*c^3*b^5*a^2 + 1575/1
3*x^13*d^8*c^2*b^4*a^3 + 350/13*x^13*d^9*c*b^3*a^4 + 21/13*x^13*d^10*b^2*a^
5 + 35/2*x^12*d^4*c^6*b^7 + 147*x^12*d^5*c^5*b^6*a + 735/2*x^12*d^6*c^4*b^5
*a^2 + 350*x^12*d^7*c^3*b^4*a^3 + 525/4*x^12*d^8*c^2*b^3*a^4 + 35/2*x^12*d^
9*c*b^2*a^5 + 7/12*x^12*d^10*b*a^6 + 120/11*x^11*d^3*c^7*b^7 + 1470/11*x^11
*d^4*c^6*b^6*a + 5292/11*x^11*d^5*c^5*b^5*a^2 + 7350/11*x^11*d^6*c^4*b^4*a^
3 + 4200/11*x^11*d^7*c^3*b^3*a^4 + 945/11*x^11*d^8*c^2*b^2*a^5 + 70/11*x^11
*d^9*c*b*a^6 + 1/11*x^11*d^10*a^7 + 9/2*x^10*d^2*c^8*b^7 + 84*x^10*d^3*c^7*
b^6*a + 441*x^10*d^4*c^6*b^5*a^2 + 882*x^10*d^5*c^5*b^4*a^3 + 735*x^10*d^6*
c^4*b^3*a^4 + 252*x^10*d^7*c^3*b^2*a^5 + 63/2*x^10*d^8*c^2*b*a^6 + x^10*d^9
*c*a^7 + 10/9*x^9*d*c^9*b^7 + 35*x^9*d^2*c^8*b^6*a + 280*x^9*d^3*c^7*b^5*a^
2 + 2450/3*x^9*d^4*c^6*b^4*a^3 + 980*x^9*d^5*c^5*b^3*a^4 + 490*x^9*d^6*c^4*
b^2*a^5 + 280/3*x^9*d^7*c^3*b*a^6 + 5*x^9*d^8*c^2*a^7 + 1/8*x^8*c^10*b^7 +
35/4*x^8*d*c^9*b^6*a + 945/8*x^8*d^2*c^8*b^5*a^2 + 525*x^8*d^3*c^7*b^4*a^3
+ 3675/4*x^8*d^4*c^6*b^3*a^4 + 1323/2*x^8*d^5*c^5*b^2*a^5 + 735/4*x^8*d^6*c
^4*b*a^6 + 15*x^8*d^7*c^3*a^7 + x^7*c^10*b^6*a + 30*x^7*d*c^9*b^5*a^2 + 225
*x^7*d^2*c^8*b^4*a^3 + 600*x^7*d^3*c^7*b^3*a^4 + 630*x^7*d^4*c^6*b^2*a^5 +
252*x^7*d^5*c^5*b*a^6 + 30*x^7*d^6*c^4*a^7 + 7/2*x^6*c^10*b^5*a^2 + 175/3*x
^6*d*c^9*b^4*a^3 + 525/2*x^6*d^2*c^8*b^3*a^4 + 420*x^6*d^3*c^7*b^2*a^5 + 24
5*x^6*d^4*c^6*b*a^6 + 42*x^6*d^5*c^5*a^7 + 7*x^5*c^10*b^4*a^3 + 70*x^5*d*c^
9*b^3*a^4 + 189*x^5*d^2*c^8*b^2*a^5 + 168*x^5*d^3*c^7*b*a^6 + 42*x^5*d^4*c^
6*a^7 + 35/4*x^4*c^10*b^3*a^4 + 105/2*x^4*d*c^9*b^2*a^5 + 315/4*x^4*d^2*c^8
*b*a^6 + 30*x^4*d^3*c^7*a^7 + 7*x^3*c^10*b^2*a^5 + 70/3*x^3*d*c^9*b*a^6 + 1
5*x^3*d^2*c^8*a^7 + 7/2*x^2*c^10*b*a^6 + 5*x^2*d*c^9*a^7 + x*c^10*a^7
```

giac [B] time = 1.26, size = 1302, normalized size = 6.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/18*b^7*d^10*x^18 + 10/17*b^7*c*d^9*x^17 + 7/17*a*b^6*d^10*x^17 + 45/16*b^
7*c^2*d^8*x^16 + 35/8*a*b^6*c*d^9*x^16 + 21/16*a^2*b^5*d^10*x^16 + 8*b^7*c^
3*d^7*x^15 + 21*a*b^6*c^2*d^8*x^15 + 14*a^2*b^5*c*d^9*x^15 + 7/3*a^3*b^4*d^
10*x^15 + 15*b^7*c^4*d^6*x^14 + 60*a*b^6*c^3*d^7*x^14 + 135/2*a^2*b^5*c^2*d
^8*x^14 + 25*a^3*b^4*c*d^9*x^14 + 5/2*a^4*b^3*d^10*x^14 + 252/13*b^7*c^5*d^
5*x^13 + 1470/13*a*b^6*c^4*d^6*x^13 + 2520/13*a^2*b^5*c^3*d^7*x^13 + 1575/1
3*a^3*b^4*c^2*d^8*x^13 + 350/13*a^4*b^3*c*d^9*x^13 + 21/13*a^5*b^2*d^10*x^1
3 + 35/2*b^7*c^6*d^4*x^12 + 147*a*b^6*c^5*d^5*x^12 + 735/2*a^2*b^5*c^4*d^6*
x^12 + 350*a^3*b^4*c^3*d^7*x^12 + 525/4*a^4*b^3*c^2*d^8*x^12 + 35/2*a^5*b^2
*c*d^9*x^12 + 7/12*a^6*b*d^10*x^12 + 120/11*b^7*c^7*d^3*x^11 + 1470/11*a*b^
6*c^6*d^4*x^11 + 5292/11*a^2*b^5*c^5*d^5*x^11 + 7350/11*a^3*b^4*c^4*d^6*x^1
1 + 4200/11*a^4*b^3*c^3*d^7*x^11 + 945/11*a^5*b^2*c^2*d^8*x^11 + 70/11*a^6*
b*c*d^9*x^11 + 1/11*a^7*d^10*x^11 + 9/2*b^7*c^8*d^2*x^10 + 84*a*b^6*c^7*d^3
*x^10 + 441*a^2*b^5*c^6*d^4*x^10 + 882*a^3*b^4*c^5*d^5*x^10 + 735*a^4*b^3*c
^4*d^6*x^10 + 252*a^5*b^2*c^3*d^7*x^10 + 63/2*a^6*b*c^2*d^8*x^10 + a^7*c*d^
9*x^10 + 10/9*b^7*c^9*d*x^9 + 35*a*b^6*c^8*d^2*x^9 + 280*a^2*b^5*c^7*d^3*x^
9 + 2450/3*a^3*b^4*c^6*d^4*x^9 + 980*a^4*b^3*c^5*d^5*x^9 + 490*a^5*b^2*c^4*
d^6*x^9 + 280/3*a^6*b*c^3*d^7*x^9 + 5*a^7*c^2*d^8*x^9 + 1/8*b^7*c^10*x^8 +
35/4*a*b^6*c^9*d*x^8 + 945/8*a^2*b^5*c^8*d^2*x^8 + 525*a^3*b^4*c^7*d^3*x^8
+ 3675/4*a^4*b^3*c^6*d^4*x^8 + 1323/2*a^5*b^2*c^5*d^5*x^8 + 735/4*a^6*b*c^4
*d^6*x^8 + 15*a^7*c^3*d^7*x^8 + a*b^6*c^10*x^7 + 30*a^2*b^5*c^9*d*x^7 + 225
*a^3*b^4*c^8*d^2*x^7 + 600*a^4*b^3*c^7*d^3*x^7 + 630*a^5*b^2*c^6*d^4*x^7 +
252*a^6*b*c^5*d^5*x^7 + 30*a^7*c^4*d^6*x^7 + 7/2*a^2*b^5*c^10*x^6 + 175/3*a
^3*b^4*c^9*d*x^6 + 525/2*a^4*b^3*c^8*d^2*x^6 + 420*a^5*b^2*c^7*d^3*x^6 + 24
5*a^6*b*c^6*d^4*x^6 + 42*a^7*c^5*d^5*x^6 + 7*a^3*b^4*c^10*x^5 + 70*a^4*b^3*
c^9*d*x^5 + 189*a^5*b^2*c^8*d^2*x^5 + 168*a^6*b*c^7*d^3*x^5 + 42*a^7*c^6*d^
4*x^5 + 35/4*a^4*b^3*c^10*x^4 + 105/2*a^5*b^2*c^9*d*x^4 + 315/4*a^6*b*c^8*d
^2*x^4 + 30*a^7*c^7*d^3*x^4 + 7*a^5*b^2*c^10*x^3 + 70/3*a^6*b*c^9*d*x^3 + 1
5*a^7*c^8*d^2*x^3 + 7/2*a^6*b*c^10*x^2 + 5*a^7*c^9*d*x^2 + a^7*c^10*x
```

maple [B] time = 0.00, size = 1141, normalized size = 5.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7*(d*x+c)^10,x)
```

```
[Out] 1/18*b^7*d^10*x^18+1/17*(7*a*b^6*d^10+10*b^7*c*d^9)*x^17+1/16*(21*a^2*b^5*d
^10+70*a*b^6*c*d^9+45*b^7*c^2*d^8)*x^16+1/15*(35*a^3*b^4*d^10+210*a^2*b^5*c
*d^9+315*a*b^6*c^2*d^8+120*b^7*c^3*d^7)*x^15+1/14*(35*a^4*b^3*d^10+350*a^3*
b^4*c*d^9+945*a^2*b^5*c^2*d^8+840*a*b^6*c^3*d^7+210*b^7*c^4*d^6)*x^14+1/13*
(21*a^5*b^2*d^10+350*a^4*b^3*c*d^9+1575*a^3*b^4*c^2*d^8+2520*a^2*b^5*c^3*d^
7+1470*a*b^6*c^4*d^6+252*b^7*c^5*d^5)*x^13+1/12*(7*a^6*b*d^10+210*a^5*b^2*c
*d^9+1575*a^4*b^3*c^2*d^8+4200*a^3*b^4*c^3*d^7+4410*a^2*b^5*c^4*d^6+1764*a*
b^6*c^5*d^5+210*b^7*c^6*d^4)*x^12+1/11*(a^7*d^10+70*a^6*b*c*d^9+945*a^5*b^2
*c^2*d^8+4200*a^4*b^3*c^3*d^7+7350*a^3*b^4*c^4*d^6+5292*a^2*b^5*c^5*d^5+147
0*a*b^6*c^6*d^4+120*b^7*c^7*d^3)*x^11+1/10*(10*a^7*c*d^9+315*a^6*b*c^2*d^8+
2520*a^5*b^2*c^3*d^7+7350*a^4*b^3*c^4*d^6+8820*a^3*b^4*c^5*d^5+4410*a^2*b^5
*c^6*d^4+840*a*b^6*c^7*d^3+45*b^7*c^8*d^2)*x^10+1/9*(45*a^7*c^2*d^8+840*a^6
*b*c^3*d^7+4410*a^5*b^2*c^4*d^6+8820*a^4*b^3*c^5*d^5+7350*a^3*b^4*c^6*d^4+2
520*a^2*b^5*c^7*d^3+315*a*b^6*c^8*d^2+10*b^7*c^9*d)*x^9+1/8*(120*a^7*c^3*d^
7+1470*a^6*b*c^4*d^6+5292*a^5*b^2*c^5*d^5+7350*a^4*b^3*c^6*d^4+4200*a^3*b^4
*c^7*d^3+945*a^2*b^5*c^8*d^2+70*a*b^6*c^9*d+b^7*c^10)*x^8+1/7*(210*a^7*c^4*
d^6+1764*a^6*b*c^5*d^5+4410*a^5*b^2*c^6*d^4+4200*a^4*b^3*c^7*d^3+1575*a^3*b
^4*c^8*d^2+210*a^2*b^5*c^9*d+7*a*b^6*c^10)*x^7+1/6*(252*a^7*c^5*d^5+1470*a^
6*b*c^6*d^4+2520*a^5*b^2*c^7*d^3+1575*a^4*b^3*c^8*d^2+350*a^3*b^4*c^9*d+21*
```

```
a^2*b^5*c^10)*x^6+1/5*(210*a^7*c^6*d^4+840*a^6*b*c^7*d^3+945*a^5*b^2*c^8*d^
2+350*a^4*b^3*c^9*d+35*a^3*b^4*c^10)*x^5+1/4*(120*a^7*c^7*d^3+315*a^6*b*c^8
*d^2+210*a^5*b^2*c^9*d+35*a^4*b^3*c^10)*x^4+1/3*(45*a^7*c^8*d^2+70*a^6*b*c^
9*d+21*a^5*b^2*c^10)*x^3+1/2*(10*a^7*c^9*d+7*a^6*b*c^10)*x^2+a^7*c^10*x
```

maxima [B] time = 1.52, size = 1135, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/18*b^7*d^10*x^18 + a^7*c^10*x + 1/17*(10*b^7*c*d^9 + 7*a*b^6*d^10)*x^17 +
 1/16*(45*b^7*c^2*d^8 + 70*a*b^6*c*d^9 + 21*a^2*b^5*d^10)*x^16 + 1/3*(24*b^
7*c^3*d^7 + 63*a*b^6*c^2*d^8 + 42*a^2*b^5*c*d^9 + 7*a^3*b^4*d^10)*x^15 + 5/
2*(6*b^7*c^4*d^6 + 24*a*b^6*c^3*d^7 + 27*a^2*b^5*c^2*d^8 + 10*a^3*b^4*c*d^9
 + a^4*b^3*d^10)*x^14 + 7/13*(36*b^7*c^5*d^5 + 210*a*b^6*c^4*d^6 + 360*a^2*
b^5*c^3*d^7 + 225*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 + 3*a^5*b^2*d^10)*x^13
 + 7/12*(30*b^7*c^6*d^4 + 252*a*b^6*c^5*d^5 + 630*a^2*b^5*c^4*d^6 + 600*a^3
*b^4*c^3*d^7 + 225*a^4*b^3*c^2*d^8 + 30*a^5*b^2*c*d^9 + a^6*b*d^10)*x^12 +
1/11*(120*b^7*c^7*d^3 + 1470*a*b^6*c^6*d^4 + 5292*a^2*b^5*c^5*d^5 + 7350*a^
3*b^4*c^4*d^6 + 4200*a^4*b^3*c^3*d^7 + 945*a^5*b^2*c^2*d^8 + 70*a^6*b*c*d^9
 + a^7*d^10)*x^11 + 1/2*(9*b^7*c^8*d^2 + 168*a*b^6*c^7*d^3 + 882*a^2*b^5*c^
6*d^4 + 1764*a^3*b^4*c^5*d^5 + 1470*a^4*b^3*c^4*d^6 + 504*a^5*b^2*c^3*d^7 +
 63*a^6*b*c^2*d^8 + 2*a^7*c*d^9)*x^10 + 5/9*(2*b^7*c^9*d + 63*a*b^6*c^8*d^2
 + 504*a^2*b^5*c^7*d^3 + 1470*a^3*b^4*c^6*d^4 + 1764*a^4*b^3*c^5*d^5 + 882*
a^5*b^2*c^4*d^6 + 168*a^6*b*c^3*d^7 + 9*a^7*c^2*d^8)*x^9 + 1/8*(b^7*c^10 +
70*a*b^6*c^9*d + 945*a^2*b^5*c^8*d^2 + 4200*a^3*b^4*c^7*d^3 + 7350*a^4*b^3*
c^6*d^4 + 5292*a^5*b^2*c^5*d^5 + 1470*a^6*b*c^4*d^6 + 120*a^7*c^3*d^7)*x^8
+ (a*b^6*c^10 + 30*a^2*b^5*c^9*d + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*c^7*d^
3 + 630*a^5*b^2*c^6*d^4 + 252*a^6*b*c^5*d^5 + 30*a^7*c^4*d^6)*x^7 + 7/6*(3*
a^2*b^5*c^10 + 50*a^3*b^4*c^9*d + 225*a^4*b^3*c^8*d^2 + 360*a^5*b^2*c^7*d^3
 + 210*a^6*b*c^6*d^4 + 36*a^7*c^5*d^5)*x^6 + 7*(a^3*b^4*c^10 + 10*a^4*b^3*c
^9*d + 27*a^5*b^2*c^8*d^2 + 24*a^6*b*c^7*d^3 + 6*a^7*c^6*d^4)*x^5 + 5/4*(7*
a^4*b^3*c^10 + 42*a^5*b^2*c^9*d + 63*a^6*b*c^8*d^2 + 24*a^7*c^7*d^3)*x^4 +
1/3*(21*a^5*b^2*c^10 + 70*a^6*b*c^9*d + 45*a^7*c^8*d^2)*x^3 + 1/2*(7*a^6*b*
c^10 + 10*a^7*c^9*d)*x^2
```

mupad [B] time = 0.61, size = 1106, normalized size = 5.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7*(c + d*x)^10,x)
```

```
[Out] x^10*(a^7*c*d^9 + (9*b^7*c^8*d^2)/2 + 84*a*b^6*c^7*d^3 + (63*a^6*b*c^2*d^8)
/2 + 441*a^2*b^5*c^6*d^4 + 882*a^3*b^4*c^5*d^5 + 735*a^4*b^3*c^4*d^6 + 252*
a^5*b^2*c^3*d^7) + x^9*((10*b^7*c^9*d)/9 + 5*a^7*c^2*d^8 + 35*a*b^6*c^8*d^2
 + (280*a^6*b*c^3*d^7)/3 + 280*a^2*b^5*c^7*d^3 + (2450*a^3*b^4*c^6*d^4)/3 +
 980*a^4*b^3*c^5*d^5 + 490*a^5*b^2*c^4*d^6) + x^5*(7*a^3*b^4*c^10 + 42*a^7*
c^6*d^4 + 70*a^4*b^3*c^9*d + 168*a^6*b*c^7*d^3 + 189*a^5*b^2*c^8*d^2) + x^1
4*((5*a^4*b^3*d^10)/2 + 15*b^7*c^4*d^6 + 60*a*b^6*c^3*d^7 + 25*a^3*b^4*c*d^
9 + (135*a^2*b^5*c^2*d^8)/2) + x^8*((b^7*c^10)/8 + 15*a^7*c^3*d^7 + (735*a^
6*b*c^4*d^6)/4 + (945*a^2*b^5*c^8*d^2)/8 + 525*a^3*b^4*c^7*d^3 + (3675*a^4*
b^3*c^6*d^4)/4 + (1323*a^5*b^2*c^5*d^5)/2 + (35*a*b^6*c^9*d)/4) + x^11*((a^
7*d^10)/11 + (120*b^7*c^7*d^3)/11 + (1470*a*b^6*c^6*d^4)/11 + (5292*a^2*b^5
*c^5*d^5)/11 + (7350*a^3*b^4*c^4*d^6)/11 + (4200*a^4*b^3*c^3*d^7)/11 + (945
*a^5*b^2*c^2*d^8)/11 + (70*a^6*b*c*d^9)/11) + x^6*((7*a^2*b^5*c^10)/2 + 42*
a^7*c^5*d^5 + (175*a^3*b^4*c^9*d)/3 + 245*a^6*b*c^6*d^4 + (525*a^4*b^3*c^8*
d^2)/2 + 420*a^5*b^2*c^7*d^3) + x^13*((21*a^5*b^2*d^10)/13 + (252*b^7*c^5*d
```

```
^5)/13 + (1470*a*b^6*c^4*d^6)/13 + (350*a^4*b^3*c*d^9)/13 + (2520*a^2*b^5*c
^3*d^7)/13 + (1575*a^3*b^4*c^2*d^8)/13) + x^7*(a*b^6*c^10 + 30*a^7*c^4*d^6
+ 30*a^2*b^5*c^9*d + 252*a^6*b*c^5*d^5 + 225*a^3*b^4*c^8*d^2 + 600*a^4*b^3*
c^7*d^3 + 630*a^5*b^2*c^6*d^4) + x^12*((7*a^6*b*d^10)/12 + (35*b^7*c^6*d^4)
/2 + 147*a*b^6*c^5*d^5 + (35*a^5*b^2*c*d^9)/2 + (735*a^2*b^5*c^4*d^6)/2 + 3
50*a^3*b^4*c^3*d^7 + (525*a^4*b^3*c^2*d^8)/4) + a^7*c^10*x + (b^7*d^10*x^18
)/18 + (5*a^4*c^7*x^4*(24*a^3*d^3 + 7*b^3*c^3 + 42*a*b^2*c^2*d + 63*a^2*b*c
*d^2))/4 + (b^4*d^7*x^15*(7*a^3*d^3 + 24*b^3*c^3 + 63*a*b^2*c^2*d + 42*a^2*
b*c*d^2))/3 + (a^6*c^9*x^2*(10*a*d + 7*b*c))/2 + (b^6*d^9*x^17*(7*a*d + 10*
b*c))/17 + (a^5*c^8*x^3*(45*a^2*d^2 + 21*b^2*c^2 + 70*a*b*c*d))/3 + (b^5*d^
8*x^16*(21*a^2*d^2 + 45*b^2*c^2 + 70*a*b*c*d))/16
```

sympy [B] time = 0.25, size = 1280, normalized size = 6.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7*(d*x+c)**10,x)
```

```
[Out] a**7*c**10*x + b**7*d**10*x**18/18 + x**17*(7*a*b**6*d**10/17 + 10*b**7*c*d
**9/17) + x**16*(21*a**2*b**5*d**10/16 + 35*a*b**6*c*d**9/8 + 45*b**7*c**2*
d**8/16) + x**15*(7*a**3*b**4*d**10/3 + 14*a**2*b**5*c*d**9 + 21*a*b**6*c**
2*d**8 + 8*b**7*c**3*d**7) + x**14*(5*a**4*b**3*d**10/2 + 25*a**3*b**4*c*d*
*9 + 135*a**2*b**5*c**2*d**8/2 + 60*a*b**6*c**3*d**7 + 15*b**7*c**4*d**6) +
 x**13*(21*a**5*b**2*d**10/13 + 350*a**4*b**3*c*d**9/13 + 1575*a**3*b**4*c*
*2*d**8/13 + 2520*a**2*b**5*c**3*d**7/13 + 1470*a*b**6*c**4*d**6/13 + 252*b
**7*c**5*d**5/13) + x**12*(7*a**6*b*d**10/12 + 35*a**5*b**2*c*d**9/2 + 525*
a**4*b**3*c**2*d**8/4 + 350*a**3*b**4*c**3*d**7 + 735*a**2*b**5*c**4*d**6/2
 + 147*a*b**6*c**5*d**5 + 35*b**7*c**6*d**4/2) + x**11*(a**7*d**10/11 + 70*
a**6*b*c*d**9/11 + 945*a**5*b**2*c**2*d**8/11 + 4200*a**4*b**3*c**3*d**7/11
 + 7350*a**3*b**4*c**4*d**6/11 + 5292*a**2*b**5*c**5*d**5/11 + 1470*a*b**6*
c**6*d**4/11 + 120*b**7*c**7*d**3/11) + x**10*(a**7*c*d**9 + 63*a**6*b*c**2
*d**8/2 + 252*a**5*b**2*c**3*d**7 + 735*a**4*b**3*c**4*d**6 + 882*a**3*b**4
*c**5*d**5 + 441*a**2*b**5*c**6*d**4 + 84*a*b**6*c**7*d**3 + 9*b**7*c**8*d*
*2/2) + x**9*(5*a**7*c**2*d**8 + 280*a**6*b*c**3*d**7/3 + 490*a**5*b**2*c**
4*d**6 + 980*a**4*b**3*c**5*d**5 + 2450*a**3*b**4*c**6*d**4/3 + 280*a**2*b*
*5*c**7*d**3 + 35*a*b**6*c**8*d**2 + 10*b**7*c**9*d/9) + x**8*(15*a**7*c**3
*d**7 + 735*a**6*b*c**4*d**6/4 + 1323*a**5*b**2*c**5*d**5/2 + 3675*a**4*b**
3*c**6*d**4/4 + 525*a**3*b**4*c**7*d**3 + 945*a**2*b**5*c**8*d**2/8 + 35*a*
b**6*c**9*d/4 + b**7*c**10/8) + x**7*(30*a**7*c**4*d**6 + 252*a**6*b*c**5*d
**5 + 630*a**5*b**2*c**6*d**4 + 600*a**4*b**3*c**7*d**3 + 225*a**3*b**4*c**
8*d**2 + 30*a**2*b**5*c**9*d + a*b**6*c**10) + x**6*(42*a**7*c**5*d**5 + 24
5*a**6*b*c**6*d**4 + 420*a**5*b**2*c**7*d**3 + 525*a**4*b**3*c**8*d**2/2 +
175*a**3*b**4*c**9*d/3 + 7*a**2*b**5*c**10/2) + x**5*(42*a**7*c**6*d**4 + 1
68*a**6*b*c**7*d**3 + 189*a**5*b**2*c**8*d**2 + 70*a**4*b**3*c**9*d + 7*a**
3*b**4*c**10) + x**4*(30*a**7*c**7*d**3 + 315*a**6*b*c**8*d**2/4 + 105*a**5
*b**2*c**9*d/2 + 35*a**4*b**3*c**10/4) + x**3*(15*a**7*c**8*d**2 + 70*a**6*
b*c**9*d/3 + 7*a**5*b**2*c**10) + x**2*(5*a**7*c**9*d + 7*a**6*b*c**10/2)
```

3.1199 $\int (a+bx)^6(c+dx)^{10}\,dx$

Optimal. Leaf size=170

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+d}{}$$

Rubi [A] time = 0.67, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^5(c+dx)^{16}(bc-ad)}{8d^7}+\frac{b^4(c+dx)^{15}(bc-ad)^2}{d^7}-\frac{10b^3(c+dx)^{14}(bc-ad)^3}{7d^7}+\frac{15b^2(c+dx)^{13}(bc-ad)^4}{13d^7}-\frac{b(c+dx)^{12}(bc-ad)^5}{2d^7}+\frac{(c+dx)^{11}(bc-ad)^6}{11d^7}+\frac{b^6(c+dx)^{17}}{17d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^6*(c + d*x)^11)/(11*d^7) - (b*(b*c - a*d)^5*(c + d*x)^12)/(2*d
^7) + (15*b^2*(b*c - a*d)^4*(c + d*x)^13)/(13*d^7) - (10*b^3*(b*c - a*d)^3*
(c + d*x)^14)/(7*d^7) + (b^4*(b*c - a*d)^2*(c + d*x)^15)/d^7 - (3*b^5*(b*c
- a*d)*(c + d*x)^16)/(8*d^7) + (b^6*(c + d*x)^17)/(17*d^7)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^6(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^6(c+dx)^{10}}{d^6}-\frac{6b(bc-ad)^5(c+dx)^{11}}{d^6}+\frac{15b^2(bc-ad)^4(c+dx)^{12}}{d^6}-2\right.$$
$$=\frac{(bc-ad)^6(c+dx)^{11}}{11d^7}-\frac{b(bc-ad)^5(c+dx)^{12}}{2d^7}+\frac{15b^2(bc-ad)^4(c+dx)^{13}}{13d^7}-\frac{10b^3(bc}{}$$

Mathematica [B] time = 0.12, size = 939, normalized size = 5.52

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] a^6*c^10*x + a^5*c^9*(3*b*c + 5*a*d)*x^2 + 5*a^4*c^8*(b^2*c^2 + 4*a*b*c*d +
 3*a^2*d^2)*x^3 + (5*a^3*c^7*(2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b*c*d^2 +
 12*a^3*d^3)*x^4)/2 + a^2*c^6*(3*b^4*c^4 + 40*a*b^3*c^3*d + 135*a^2*b^2*c^2
*d^2 + 144*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + a*c^5*(b^5*c^5 + 25*a*b^4*c^4*d
+ 150*a^2*b^3*c^3*d^2 + 300*a^3*b^2*c^2*d^3 + 210*a^4*b*c*d^4 + 42*a^5*d^5)
*x^6 + (c^4*(b^6*c^6 + 60*a*b^5*c^5*d + 675*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*
c^3*d^3 + 3150*a^4*b^2*c^2*d^4 + 1512*a^5*b*c*d^5 + 210*a^6*d^6)*x^7)/7 + (
5*c^3*d*(b^6*c^6 + 27*a*b^5*c^5*d + 180*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d
^3 + 378*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 + 12*a^6*d^6)*x^8)/4 + 5*c^2*d^2
*(b^6*c^6 + 16*a*b^5*c^5*d + 70*a^2*b^4*c^4*d^2 + 112*a^3*b^3*c^3*d^3 + 70*
a^4*b^2*c^2*d^4 + 16*a^5*b*c*d^5 + a^6*d^6)*x^9 + c*d^3*(12*b^6*c^6 + 126*a
*b^5*c^5*d + 378*a^2*b^4*c^4*d^2 + 420*a^3*b^3*c^3*d^3 + 180*a^4*b^2*c^2*d^
4 + 27*a^5*b*c*d^5 + a^6*d^6)*x^10 + (d^4*(210*b^6*c^6 + 1512*a*b^5*c^5*d +
 3150*a^2*b^4*c^4*d^2 + 2400*a^3*b^3*c^3*d^3 + 675*a^4*b^2*c^2*d^4 + 60*a^5
```

```
*b*c*d^5 + a^6*d^6)*x^11)/11 + (b*d^5*(42*b^5*c^5 + 210*a*b^4*c^4*d + 300*a
^2*b^3*c^3*d^2 + 150*a^3*b^2*c^2*d^3 + 25*a^4*b*c*d^4 + a^5*d^5)*x^12)/2 +
(5*b^2*d^6*(42*b^4*c^4 + 144*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 + 40*a^3*b*c
*d^3 + 3*a^4*d^4)*x^13)/13 + (5*b^3*d^7*(12*b^3*c^3 + 27*a*b^2*c^2*d + 15*a
^2*b*c*d^2 + 2*a^3*d^3)*x^14)/7 + b^4*d^8*(3*b^2*c^2 + 4*a*b*c*d + a^2*d^2)
*x^15 + (b^5*d^9*(5*b*c + 3*a*d)*x^16)/8 + (b^6*d^10*x^17)/17
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx)^6 (c + dx)^{10} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6*(c + d*x)^10, x]
```

fricas [B] time = 1.06, size = 1124, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^10*b^6 + 5/8*x^16*d^9*c*b^6 + 3/8*x^16*d^10*b^5*a + 3*x^15*d^8*
c^2*b^6 + 4*x^15*d^9*c*b^5*a + x^15*d^10*b^4*a^2 + 60/7*x^14*d^7*c^3*b^6 +
135/7*x^14*d^8*c^2*b^5*a + 75/7*x^14*d^9*c*b^4*a^2 + 10/7*x^14*d^10*b^3*a^3
 + 210/13*x^13*d^6*c^4*b^6 + 720/13*x^13*d^7*c^3*b^5*a + 675/13*x^13*d^8*c^
2*b^4*a^2 + 200/13*x^13*d^9*c*b^3*a^3 + 15/13*x^13*d^10*b^2*a^4 + 21*x^12*d
^5*c^5*b^6 + 105*x^12*d^6*c^4*b^5*a + 150*x^12*d^7*c^3*b^4*a^2 + 75*x^12*d^
8*c^2*b^3*a^3 + 25/2*x^12*d^9*c*b^2*a^4 + 1/2*x^12*d^10*b*a^5 + 210/11*x^11
*d^4*c^6*b^6 + 1512/11*x^11*d^5*c^5*b^5*a + 3150/11*x^11*d^6*c^4*b^4*a^2 +
2400/11*x^11*d^7*c^3*b^3*a^3 + 675/11*x^11*d^8*c^2*b^2*a^4 + 60/11*x^11*d^9
*c*b*a^5 + 1/11*x^11*d^10*a^6 + 12*x^10*d^3*c^7*b^6 + 126*x^10*d^4*c^6*b^5*
a + 378*x^10*d^5*c^5*b^4*a^2 + 420*x^10*d^6*c^4*b^3*a^3 + 180*x^10*d^7*c^3*
b^2*a^4 + 27*x^10*d^8*c^2*b*a^5 + x^10*d^9*c*a^6 + 5*x^9*d^2*c^8*b^6 + 80*x
^9*d^3*c^7*b^5*a + 350*x^9*d^4*c^6*b^4*a^2 + 560*x^9*d^5*c^5*b^3*a^3 + 350*
x^9*d^6*c^4*b^2*a^4 + 80*x^9*d^7*c^3*b*a^5 + 5*x^9*d^8*c^2*a^6 + 5/4*x^8*d*
c^9*b^6 + 135/4*x^8*d^2*c^8*b^5*a + 225*x^8*d^3*c^7*b^4*a^2 + 525*x^8*d^4*c
^6*b^3*a^3 + 945/2*x^8*d^5*c^5*b^2*a^4 + 315/2*x^8*d^6*c^4*b*a^5 + 15*x^8*d
^7*c^3*a^6 + 1/7*x^7*c^10*b^6 + 60/7*x^7*d*c^9*b^5*a + 675/7*x^7*d^2*c^8*b^
4*a^2 + 2400/7*x^7*d^3*c^7*b^3*a^3 + 450*x^7*d^4*c^6*b^2*a^4 + 216*x^7*d^5*
c^5*b*a^5 + 30*x^7*d^6*c^4*a^6 + x^6*c^10*b^5*a + 25*x^6*d*c^9*b^4*a^2 + 15
0*x^6*d^2*c^8*b^3*a^3 + 300*x^6*d^3*c^7*b^2*a^4 + 210*x^6*d^4*c^6*b*a^5 + 4
2*x^6*d^5*c^5*a^6 + 3*x^5*c^10*b^4*a^2 + 40*x^5*d*c^9*b^3*a^3 + 135*x^5*d^2
*c^8*b^2*a^4 + 144*x^5*d^3*c^7*b*a^5 + 42*x^5*d^4*c^6*a^6 + 5*x^4*c^10*b^3*
a^3 + 75/2*x^4*d*c^9*b^2*a^4 + 135/2*x^4*d^2*c^8*b*a^5 + 30*x^4*d^3*c^7*a^6
 + 5*x^3*c^10*b^2*a^4 + 20*x^3*d*c^9*b*a^5 + 15*x^3*d^2*c^8*a^6 + 3*x^2*c^1
0*b*a^5 + 5*x^2*d*c^9*a^6 + x*c^10*a^6
```

giac [B] time = 1.30, size = 1124, normalized size = 6.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/17*b^6*d^10*x^17 + 5/8*b^6*c*d^9*x^16 + 3/8*a*b^5*d^10*x^16 + 3*b^6*c^2*d
^8*x^15 + 4*a*b^5*c*d^9*x^15 + a^2*b^4*d^10*x^15 + 60/7*b^6*c^3*d^7*x^14 +
```

```
135/7*a*b^5*c^2*d^8*x^14 + 75/7*a^2*b^4*c*d^9*x^14 + 10/7*a^3*b^3*d^10*x^14
 + 210/13*b^6*c^4*d^6*x^13 + 720/13*a*b^5*c^3*d^7*x^13 + 675/13*a^2*b^4*c^2
*d^8*x^13 + 200/13*a^3*b^3*c*d^9*x^13 + 15/13*a^4*b^2*d^10*x^13 + 21*b^6*c^
5*d^5*x^12 + 105*a*b^5*c^4*d^6*x^12 + 150*a^2*b^4*c^3*d^7*x^12 + 75*a^3*b^3
*c^2*d^8*x^12 + 25/2*a^4*b^2*c*d^9*x^12 + 1/2*a^5*b*d^10*x^12 + 210/11*b^6*
c^6*d^4*x^11 + 1512/11*a*b^5*c^5*d^5*x^11 + 3150/11*a^2*b^4*c^4*d^6*x^11 +
2400/11*a^3*b^3*c^3*d^7*x^11 + 675/11*a^4*b^2*c^2*d^8*x^11 + 60/11*a^5*b*c*
d^9*x^11 + 1/11*a^6*d^10*x^11 + 12*b^6*c^7*d^3*x^10 + 126*a*b^5*c^6*d^4*x^1
0 + 378*a^2*b^4*c^5*d^5*x^10 + 420*a^3*b^3*c^4*d^6*x^10 + 180*a^4*b^2*c^3*d
^7*x^10 + 27*a^5*b*c^2*d^8*x^10 + a^6*c*d^9*x^10 + 5*b^6*c^8*d^2*x^9 + 80*a
*b^5*c^7*d^3*x^9 + 350*a^2*b^4*c^6*d^4*x^9 + 560*a^3*b^3*c^5*d^5*x^9 + 350*
a^4*b^2*c^4*d^6*x^9 + 80*a^5*b*c^3*d^7*x^9 + 5*a^6*c^2*d^8*x^9 + 5/4*b^6*c^
9*d*x^8 + 135/4*a*b^5*c^8*d^2*x^8 + 225*a^2*b^4*c^7*d^3*x^8 + 525*a^3*b^3*c
^6*d^4*x^8 + 945/2*a^4*b^2*c^5*d^5*x^8 + 315/2*a^5*b*c^4*d^6*x^8 + 15*a^6*c
^3*d^7*x^8 + 1/7*b^6*c^10*x^7 + 60/7*a*b^5*c^9*d*x^7 + 675/7*a^2*b^4*c^8*d^
2*x^7 + 2400/7*a^3*b^3*c^7*d^3*x^7 + 450*a^4*b^2*c^6*d^4*x^7 + 216*a^5*b*c^
5*d^5*x^7 + 30*a^6*c^4*d^6*x^7 + a*b^5*c^10*x^6 + 25*a^2*b^4*c^9*d*x^6 + 15
0*a^3*b^3*c^8*d^2*x^6 + 300*a^4*b^2*c^7*d^3*x^6 + 210*a^5*b*c^6*d^4*x^6 + 4
2*a^6*c^5*d^5*x^6 + 3*a^2*b^4*c^10*x^5 + 40*a^3*b^3*c^9*d*x^5 + 135*a^4*b^2
*c^8*d^2*x^5 + 144*a^5*b*c^7*d^3*x^5 + 42*a^6*c^6*d^4*x^5 + 5*a^3*b^3*c^10*
x^4 + 75/2*a^4*b^2*c^9*d*x^4 + 135/2*a^5*b*c^8*d^2*x^4 + 30*a^6*c^7*d^3*x^4
 + 5*a^4*b^2*c^10*x^3 + 20*a^5*b*c^9*d*x^3 + 15*a^6*c^8*d^2*x^3 + 3*a^5*b*c
^10*x^2 + 5*a^6*c^9*d*x^2 + a^6*c^10*x
```

maple [B] time = 0.00, size = 991, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6*(d*x+c)^10,x)
```

```
[Out] 1/17*b^6*d^10*x^17+1/16*(6*a*b^5*d^10+10*b^6*c*d^9)*x^16+1/15*(15*a^2*b^4*d
^10+60*a*b^5*c*d^9+45*b^6*c^2*d^8)*x^15+1/14*(20*a^3*b^3*d^10+150*a^2*b^4*c
*d^9+270*a*b^5*c^2*d^8+120*b^6*c^3*d^7)*x^14+1/13*(15*a^4*b^2*d^10+200*a^3*
b^3*c*d^9+675*a^2*b^4*c^2*d^8+720*a*b^5*c^3*d^7+210*b^6*c^4*d^6)*x^13+1/12*
(6*a^5*b*d^10+150*a^4*b^2*c*d^9+900*a^3*b^3*c^2*d^8+1800*a^2*b^4*c^3*d^7+12
60*a*b^5*c^4*d^6+252*b^6*c^5*d^5)*x^12+1/11*(a^6*d^10+60*a^5*b*c*d^9+675*a^
4*b^2*c^2*d^8+2400*a^3*b^3*c^3*d^7+3150*a^2*b^4*c^4*d^6+1512*a*b^5*c^5*d^5+
210*b^6*c^6*d^4)*x^11+1/10*(10*a^6*c*d^9+270*a^5*b*c^2*d^8+1800*a^4*b^2*c^3
*d^7+4200*a^3*b^3*c^4*d^6+3780*a^2*b^4*c^5*d^5+1260*a*b^5*c^6*d^4+120*b^6*c
^7*d^3)*x^10+1/9*(45*a^6*c^2*d^8+720*a^5*b*c^3*d^7+3150*a^4*b^2*c^4*d^6+504
0*a^3*b^3*c^5*d^5+3150*a^2*b^4*c^6*d^4+720*a*b^5*c^7*d^3+45*b^6*c^8*d^2)*x^
9+1/8*(120*a^6*c^3*d^7+1260*a^5*b*c^4*d^6+3780*a^4*b^2*c^5*d^5+4200*a^3*b^3
*c^6*d^4+1800*a^2*b^4*c^7*d^3+270*a*b^5*c^8*d^2+10*b^6*c^9*d)*x^8+1/7*(210*
a^6*c^4*d^6+1512*a^5*b*c^5*d^5+3150*a^4*b^2*c^6*d^4+2400*a^3*b^3*c^7*d^3+67
5*a^2*b^4*c^8*d^2+60*a*b^5*c^9*d+b^6*c^10)*x^7+1/6*(252*a^6*c^5*d^5+1260*a^
5*b*c^6*d^4+1800*a^4*b^2*c^7*d^3+900*a^3*b^3*c^8*d^2+150*a^2*b^4*c^9*d+6*a*
b^5*c^10)*x^6+1/5*(210*a^6*c^6*d^4+720*a^5*b*c^7*d^3+675*a^4*b^2*c^8*d^2+20
0*a^3*b^3*c^9*d+15*a^2*b^4*c^10)*x^5+1/4*(120*a^6*c^7*d^3+270*a^5*b*c^8*d^2
+150*a^4*b^2*c^9*d+20*a^3*b^3*c^10)*x^4+1/3*(45*a^6*c^8*d^2+60*a^5*b*c^9*d+
15*a^4*b^2*c^10)*x^3+1/2*(10*a^6*c^9*d+6*a^5*b*c^10)*x^2+a^6*c^10*x
```

maxima [B] time = 1.44, size = 977, normalized size = 5.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/17*b^6*d^10*x^17 + a^6*c^10*x + 1/8*(5*b^6*c*d^9 + 3*a*b^5*d^10)*x^16 + (
3*b^6*c^2*d^8 + 4*a*b^5*c*d^9 + a^2*b^4*d^10)*x^15 + 5/7*(12*b^6*c^3*d^7 +
27*a*b^5*c^2*d^8 + 15*a^2*b^4*c*d^9 + 2*a^3*b^3*d^10)*x^14 + 5/13*(42*b^6*c
^4*d^6 + 144*a*b^5*c^3*d^7 + 135*a^2*b^4*c^2*d^8 + 40*a^3*b^3*c*d^9 + 3*a^4
*b^2*d^10)*x^13 + 1/2*(42*b^6*c^5*d^5 + 210*a*b^5*c^4*d^6 + 300*a^2*b^4*c^3
*d^7 + 150*a^3*b^3*c^2*d^8 + 25*a^4*b^2*c*d^9 + a^5*b*d^10)*x^12 + 1/11*(21
0*b^6*c^6*d^4 + 1512*a*b^5*c^5*d^5 + 3150*a^2*b^4*c^4*d^6 + 2400*a^3*b^3*c^
3*d^7 + 675*a^4*b^2*c^2*d^8 + 60*a^5*b*c*d^9 + a^6*d^10)*x^11 + (12*b^6*c^7
*d^3 + 126*a*b^5*c^6*d^4 + 378*a^2*b^4*c^5*d^5 + 420*a^3*b^3*c^4*d^6 + 180*
a^4*b^2*c^3*d^7 + 27*a^5*b*c^2*d^8 + a^6*c*d^9)*x^10 + 5*(b^6*c^8*d^2 + 16*
a*b^5*c^7*d^3 + 70*a^2*b^4*c^6*d^4 + 112*a^3*b^3*c^5*d^5 + 70*a^4*b^2*c^4*d
^6 + 16*a^5*b*c^3*d^7 + a^6*c^2*d^8)*x^9 + 5/4*(b^6*c^9*d + 27*a*b^5*c^8*d^
2 + 180*a^2*b^4*c^7*d^3 + 420*a^3*b^3*c^6*d^4 + 378*a^4*b^2*c^5*d^5 + 126*a
^5*b*c^4*d^6 + 12*a^6*c^3*d^7)*x^8 + 1/7*(b^6*c^10 + 60*a*b^5*c^9*d + 675*a
^2*b^4*c^8*d^2 + 2400*a^3*b^3*c^7*d^3 + 3150*a^4*b^2*c^6*d^4 + 1512*a^5*b*c
^5*d^5 + 210*a^6*c^4*d^6)*x^7 + (a*b^5*c^10 + 25*a^2*b^4*c^9*d + 150*a^3*b^
3*c^8*d^2 + 300*a^4*b^2*c^7*d^3 + 210*a^5*b*c^6*d^4 + 42*a^6*c^5*d^5)*x^6 +
 (3*a^2*b^4*c^10 + 40*a^3*b^3*c^9*d + 135*a^4*b^2*c^8*d^2 + 144*a^5*b*c^7*d
^3 + 42*a^6*c^6*d^4)*x^5 + 5/2*(2*a^3*b^3*c^10 + 15*a^4*b^2*c^9*d + 27*a^5*
b*c^8*d^2 + 12*a^6*c^7*d^3)*x^4 + 5*(a^4*b^2*c^10 + 4*a^5*b*c^9*d + 3*a^6*c
^8*d^2)*x^3 + (3*a^5*b*c^10 + 5*a^6*c^9*d)*x^2
```

mupad [B] time = 0.53, size = 953, normalized size = 5.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6*(c + d*x)^10,x)
```

```
[Out] x^7*((b^6*c^10)/7 + 30*a^6*c^4*d^6 + 216*a^5*b*c^5*d^5 + (675*a^2*b^4*c^8*d
^2)/7 + (2400*a^3*b^3*c^7*d^3)/7 + 450*a^4*b^2*c^6*d^4 + (60*a*b^5*c^9*d)/7
) + x^11*((a^6*d^10)/11 + (210*b^6*c^6*d^4)/11 + (1512*a*b^5*c^5*d^5)/11 +
(3150*a^2*b^4*c^4*d^6)/11 + (2400*a^3*b^3*c^3*d^7)/11 + (675*a^4*b^2*c^2*d^
8)/11 + (60*a^5*b*c*d^9)/11) + x^9*(5*a^6*c^2*d^8 + 5*b^6*c^8*d^2 + 80*a*b^
5*c^7*d^3 + 80*a^5*b*c^3*d^7 + 350*a^2*b^4*c^6*d^4 + 560*a^3*b^3*c^5*d^5 +
350*a^4*b^2*c^4*d^6) + x^5*(3*a^2*b^4*c^10 + 42*a^6*c^6*d^4 + 40*a^3*b^3*c^
9*d + 144*a^5*b*c^7*d^3 + 135*a^4*b^2*c^8*d^2) + x^13*((15*a^4*b^2*d^10)/13
 + (210*b^6*c^4*d^6)/13 + (720*a*b^5*c^3*d^7)/13 + (200*a^3*b^3*c*d^9)/13 +
 (675*a^2*b^4*c^2*d^8)/13) + x^6*(a*b^5*c^10 + 42*a^6*c^5*d^5 + 25*a^2*b^4*
c^9*d + 210*a^5*b*c^6*d^4 + 150*a^3*b^3*c^8*d^2 + 300*a^4*b^2*c^7*d^3) + x^
12*((a^5*b*d^10)/2 + 21*b^6*c^5*d^5 + 105*a*b^5*c^4*d^6 + (25*a^4*b^2*c*d^9
)/2 + 150*a^2*b^4*c^3*d^7 + 75*a^3*b^3*c^2*d^8) + x^10*(a^6*c*d^9 + 12*b^6*
c^7*d^3 + 126*a*b^5*c^6*d^4 + 27*a^5*b*c^2*d^8 + 378*a^2*b^4*c^5*d^5 + 420*
a^3*b^3*c^4*d^6 + 180*a^4*b^2*c^3*d^7) + x^8*((5*b^6*c^9*d)/4 + 15*a^6*c^3*
d^7 + (135*a*b^5*c^8*d^2)/4 + (315*a^5*b*c^4*d^6)/2 + 225*a^2*b^4*c^7*d^3 +
 525*a^3*b^3*c^6*d^4 + (945*a^4*b^2*c^5*d^5)/2) + a^6*c^10*x + (b^6*d^10*x^
17)/17 + (5*a^3*c^7*x^4*(12*a^3*d^3 + 2*b^3*c^3 + 15*a*b^2*c^2*d + 27*a^2*b
*c*d^2))/2 + (5*b^3*d^7*x^14*(2*a^3*d^3 + 12*b^3*c^3 + 27*a*b^2*c^2*d + 15*
a^2*b*c*d^2))/7 + a^5*c^9*x^2*(5*a*d + 3*b*c) + (b^5*d^9*x^16*(3*a*d + 5*b*
c))/8 + 5*a^4*c^8*x^3*(3*a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + b^4*d^8*x^15*(a^2
*d^2 + 3*b^2*c^2 + 4*a*b*c*d)
```

sympy [B] time = 0.23, size = 1088, normalized size = 6.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6*(d*x+c)**10,x)
```

```
[Out] a**6*c**10*x + b**6*d**10*x**17/17 + x**16*(3*a*b**5*d**10/8 + 5*b**6*c*d**
9/8) + x**15*(a**2*b**4*d**10 + 4*a*b**5*c*d**9 + 3*b**6*c**2*d**8) + x**14
*(10*a**3*b**3*d**10/7 + 75*a**2*b**4*c*d**9/7 + 135*a*b**5*c**2*d**8/7 + 6
0*b**6*c**3*d**7/7) + x**13*(15*a**4*b**2*d**10/13 + 200*a**3*b**3*c*d**9/1
3 + 675*a**2*b**4*c**2*d**8/13 + 720*a*b**5*c**3*d**7/13 + 210*b**6*c**4*d*
*6/13) + x**12*(a**5*b*d**10/2 + 25*a**4*b**2*c*d**9/2 + 75*a**3*b**3*c**2*
d**8 + 150*a**2*b**4*c**3*d**7 + 105*a*b**5*c**4*d**6 + 21*b**6*c**5*d**5)
+ x**11*(a**6*d**10/11 + 60*a**5*b*c*d**9/11 + 675*a**4*b**2*c**2*d**8/11 +
 2400*a**3*b**3*c**3*d**7/11 + 3150*a**2*b**4*c**4*d**6/11 + 1512*a*b**5*c*
*5*d**5/11 + 210*b**6*c**6*d**4/11) + x**10*(a**6*c*d**9 + 27*a**5*b*c**2*d
**8 + 180*a**4*b**2*c**3*d**7 + 420*a**3*b**3*c**4*d**6 + 378*a**2*b**4*c**
5*d**5 + 126*a*b**5*c**6*d**4 + 12*b**6*c**7*d**3) + x**9*(5*a**6*c**2*d**8
 + 80*a**5*b*c**3*d**7 + 350*a**4*b**2*c**4*d**6 + 560*a**3*b**3*c**5*d**5
+ 350*a**2*b**4*c**6*d**4 + 80*a*b**5*c**7*d**3 + 5*b**6*c**8*d**2) + x**8*
(15*a**6*c**3*d**7 + 315*a**5*b*c**4*d**6/2 + 945*a**4*b**2*c**5*d**5/2 + 5
25*a**3*b**3*c**6*d**4 + 225*a**2*b**4*c**7*d**3 + 135*a*b**5*c**8*d**2/4 +
 5*b**6*c**9*d/4) + x**7*(30*a**6*c**4*d**6 + 216*a**5*b*c**5*d**5 + 450*a*
*4*b**2*c**6*d**4 + 2400*a**3*b**3*c**7*d**3/7 + 675*a**2*b**4*c**8*d**2/7
+ 60*a*b**5*c**9*d/7 + b**6*c**10/7) + x**6*(42*a**6*c**5*d**5 + 210*a**5*b
*c**6*d**4 + 300*a**4*b**2*c**7*d**3 + 150*a**3*b**3*c**8*d**2 + 25*a**2*b*
*4*c**9*d + a*b**5*c**10) + x**5*(42*a**6*c**6*d**4 + 144*a**5*b*c**7*d**3
+ 135*a**4*b**2*c**8*d**2 + 40*a**3*b**3*c**9*d + 3*a**2*b**4*c**10) + x**4
*(30*a**6*c**7*d**3 + 135*a**5*b*c**8*d**2/2 + 75*a**4*b**2*c**9*d/2 + 5*a*
*3*b**3*c**10) + x**3*(15*a**6*c**8*d**2 + 20*a**5*b*c**9*d + 5*a**4*b**2*c
**10) + x**2*(5*a**6*c**9*d + 3*a**5*b*c**10)
```

3.1200 $\int (a+bx)^5 (c+dx)^{10}\, dx$

Optimal. Leaf size=146

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc}{11d^6}$$

Rubi [A] time = 0.53, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^4(c+dx)^{15}(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^{14}(bc-ad)^2}{7d^6}-\frac{10b^2(c+dx)^{13}(bc-ad)^3}{13d^6}+\frac{5b(c+dx)^{12}(bc-ad)^4}{12d^6}-\frac{(c+dx)^{11}(bc-ad)^5}{11d^6}+\frac{b^5(c+dx)^{16}}{16d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^5*(c + d*x)^11)/(11*d^6) + (5*b*(b*c - a*d)^4*(c + d*x)^12)/(
12*d^6) - (10*b^2*(b*c - a*d)^3*(c + d*x)^13)/(13*d^6) + (5*b^3*(b*c - a*d)
^2*(c + d*x)^14)/(7*d^6) - (b^4*(b*c - a*d)*(c + d*x)^15)/(3*d^6) + (b^5*(c
 + d*x)^16)/(16*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{10}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{11}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{12}}{d^5}+\frac{10b\right.$$

$$=-\frac{(bc-ad)^5(c+dx)^{11}}{11d^6}+\frac{5b(bc-ad)^4(c+dx)^{12}}{12d^6}-\frac{10b^2(bc-ad)^3(c+dx)^{13}}{13d^6}+\frac{5b^3(bc-}{}$$

Mathematica [B] time = 0.09, size = 811, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] a^5*c^10*x + (5*a^4*c^9*(b*c + 2*a*d)*x^2)/2 + (5*a^3*c^8*(2*b^2*c^2 + 10*a
*b*c*d + 9*a^2*d^2)*x^3)/3 + (5*a^2*c^7*(2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^
2*b*c*d^2 + 24*a^3*d^3)*x^4)/4 + a*c^6*(b^4*c^4 + 20*a*b^3*c^3*d + 90*a^2*b
^2*c^2*d^2 + 120*a^3*b*c*d^3 + 42*a^4*d^4)*x^5 + (c^5*(b^5*c^5 + 50*a*b^4*c
^4*d + 450*a^2*b^3*c^3*d^2 + 1200*a^3*b^2*c^2*d^3 + 1050*a^4*b*c*d^4 + 252*
a^5*d^5)*x^6)/6 + (5*c^4*d*(2*b^5*c^5 + 45*a*b^4*c^4*d + 240*a^2*b^3*c^3*d^
2 + 420*a^3*b^2*c^2*d^3 + 252*a^4*b*c*d^4 + 42*a^5*d^5)*x^7)/7 + (15*c^3*d^
2*(3*b^5*c^5 + 40*a*b^4*c^4*d + 140*a^2*b^3*c^3*d^2 + 168*a^3*b^2*c^2*d^3 +
 70*a^4*b*c*d^4 + 8*a^5*d^5)*x^8)/8 + (5*c^2*d^3*(8*b^5*c^5 + 70*a*b^4*c^4*
d + 168*a^2*b^3*c^3*d^2 + 140*a^3*b^2*c^2*d^3 + 40*a^4*b*c*d^4 + 3*a^5*d^5)
*x^9)/3 + (c*d^4*(42*b^5*c^5 + 252*a*b^4*c^4*d + 420*a^2*b^3*c^3*d^2 + 240*
a^3*b^2*c^2*d^3 + 45*a^4*b*c*d^4 + 2*a^5*d^5)*x^10)/2 + (d^5*(252*b^5*c^5 +
 1050*a*b^4*c^4*d + 1200*a^2*b^3*c^3*d^2 + 450*a^3*b^2*c^2*d^3 + 50*a^4*b*c
```

```
*d^4 + a^5*d^5)*x^11)/11 + (5*b*d^6*(42*b^4*c^4 + 120*a*b^3*c^3*d + 90*a^2*
b^2*c^2*d^2 + 20*a^3*b*c*d^3 + a^4*d^4)*x^12)/12 + (5*b^2*d^7*(24*b^3*c^3 +
 45*a*b^2*c^2*d + 20*a^2*b*c*d^2 + 2*a^3*d^3)*x^13)/13 + (5*b^3*d^8*(9*b^2*
c^2 + 10*a*b*c*d + 2*a^2*d^2)*x^14)/14 + (b^4*d^9*(2*b*c + a*d)*x^15)/3 + (
b^5*d^10*x^16)/16
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^5(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^10, x]
```

fricas [B] time = 0.78, size = 948, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^10*b^5 + 2/3*x^15*d^9*c*b^5 + 1/3*x^15*d^10*b^4*a + 45/14*x^14*
d^8*c^2*b^5 + 25/7*x^14*d^9*c*b^4*a + 5/7*x^14*d^10*b^3*a^2 + 120/13*x^13*d
^7*c^3*b^5 + 225/13*x^13*d^8*c^2*b^4*a + 100/13*x^13*d^9*c*b^3*a^2 + 10/13*
x^13*d^10*b^2*a^3 + 35/2*x^12*d^6*c^4*b^5 + 50*x^12*d^7*c^3*b^4*a + 75/2*x^
12*d^8*c^2*b^3*a^2 + 25/3*x^12*d^9*c*b^2*a^3 + 5/12*x^12*d^10*b*a^4 + 252/1
1*x^11*d^5*c^5*b^5 + 1050/11*x^11*d^6*c^4*b^4*a + 1200/11*x^11*d^7*c^3*b^3*
a^2 + 450/11*x^11*d^8*c^2*b^2*a^3 + 50/11*x^11*d^9*c*b*a^4 + 1/11*x^11*d^10
*a^5 + 21*x^10*d^4*c^6*b^5 + 126*x^10*d^5*c^5*b^4*a + 210*x^10*d^6*c^4*b^3*
a^2 + 120*x^10*d^7*c^3*b^2*a^3 + 45/2*x^10*d^8*c^2*b*a^4 + x^10*d^9*c*a^5 +
 40/3*x^9*d^3*c^7*b^5 + 350/3*x^9*d^4*c^6*b^4*a + 280*x^9*d^5*c^5*b^3*a^2 +
 700/3*x^9*d^6*c^4*b^2*a^3 + 200/3*x^9*d^7*c^3*b*a^4 + 5*x^9*d^8*c^2*a^5 +
45/8*x^8*d^2*c^8*b^5 + 75*x^8*d^3*c^7*b^4*a + 525/2*x^8*d^4*c^6*b^3*a^2 + 3
15*x^8*d^5*c^5*b^2*a^3 + 525/4*x^8*d^6*c^4*b*a^4 + 15*x^8*d^7*c^3*a^5 + 10/
7*x^7*d*c^9*b^5 + 225/7*x^7*d^2*c^8*b^4*a + 1200/7*x^7*d^3*c^7*b^3*a^2 + 30
0*x^7*d^4*c^6*b^2*a^3 + 180*x^7*d^5*c^5*b*a^4 + 30*x^7*d^6*c^4*a^5 + 1/6*x^
6*c^10*b^5 + 25/3*x^6*d*c^9*b^4*a + 75*x^6*d^2*c^8*b^3*a^2 + 200*x^6*d^3*c^
7*b^2*a^3 + 175*x^6*d^4*c^6*b*a^4 + 42*x^6*d^5*c^5*a^5 + x^5*c^10*b^4*a + 2
0*x^5*d*c^9*b^3*a^2 + 90*x^5*d^2*c^8*b^2*a^3 + 120*x^5*d^3*c^7*b*a^4 + 42*x
^5*d^4*c^6*a^5 + 5/2*x^4*c^10*b^3*a^2 + 25*x^4*d*c^9*b^2*a^3 + 225/4*x^4*d^
2*c^8*b*a^4 + 30*x^4*d^3*c^7*a^5 + 10/3*x^3*c^10*b^2*a^3 + 50/3*x^3*d*c^9*b
*a^4 + 15*x^3*d^2*c^8*a^5 + 5/2*x^2*c^10*b*a^4 + 5*x^2*d*c^9*a^5 + x*c^10*a
^5
```

giac [B] time = 1.32, size = 948, normalized size = 6.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/16*b^5*d^10*x^16 + 2/3*b^5*c*d^9*x^15 + 1/3*a*b^4*d^10*x^15 + 45/14*b^5*c
^2*d^8*x^14 + 25/7*a*b^4*c*d^9*x^14 + 5/7*a^2*b^3*d^10*x^14 + 120/13*b^5*c^
3*d^7*x^13 + 225/13*a*b^4*c^2*d^8*x^13 + 100/13*a^2*b^3*c*d^9*x^13 + 10/13*
a^3*b^2*d^10*x^13 + 35/2*b^5*c^4*d^6*x^12 + 50*a*b^4*c^3*d^7*x^12 + 75/2*a^
2*b^3*c^2*d^8*x^12 + 25/3*a^3*b^2*c*d^9*x^12 + 5/12*a^4*b*d^10*x^12 + 252/1
1*b^5*c^5*d^5*x^11 + 1050/11*a*b^4*c^4*d^6*x^11 + 1200/11*a^2*b^3*c^3*d^7*x
```

```
^11 + 450/11*a^3*b^2*c^2*d^8*x^11 + 50/11*a^4*b*c*d^9*x^11 + 1/11*a^5*d^10*
x^11 + 21*b^5*c^6*d^4*x^10 + 126*a*b^4*c^5*d^5*x^10 + 210*a^2*b^3*c^4*d^6*x
^10 + 120*a^3*b^2*c^3*d^7*x^10 + 45/2*a^4*b*c^2*d^8*x^10 + a^5*c*d^9*x^10 +
 40/3*b^5*c^7*d^3*x^9 + 350/3*a*b^4*c^6*d^4*x^9 + 280*a^2*b^3*c^5*d^5*x^9 +
 700/3*a^3*b^2*c^4*d^6*x^9 + 200/3*a^4*b*c^3*d^7*x^9 + 5*a^5*c^2*d^8*x^9 +
45/8*b^5*c^8*d^2*x^8 + 75*a*b^4*c^7*d^3*x^8 + 525/2*a^2*b^3*c^6*d^4*x^8 + 3
15*a^3*b^2*c^5*d^5*x^8 + 525/4*a^4*b*c^4*d^6*x^8 + 15*a^5*c^3*d^7*x^8 + 10/
7*b^5*c^9*d*x^7 + 225/7*a*b^4*c^8*d^2*x^7 + 1200/7*a^2*b^3*c^7*d^3*x^7 + 30
0*a^3*b^2*c^6*d^4*x^7 + 180*a^4*b*c^5*d^5*x^7 + 30*a^5*c^4*d^6*x^7 + 1/6*b^
5*c^10*x^6 + 25/3*a*b^4*c^9*d*x^6 + 75*a^2*b^3*c^8*d^2*x^6 + 200*a^3*b^2*c^
7*d^3*x^6 + 175*a^4*b*c^6*d^4*x^6 + 42*a^5*c^5*d^5*x^6 + a*b^4*c^10*x^5 + 2
0*a^2*b^3*c^9*d*x^5 + 90*a^3*b^2*c^8*d^2*x^5 + 120*a^4*b*c^7*d^3*x^5 + 42*a
^5*c^6*d^4*x^5 + 5/2*a^2*b^3*c^10*x^4 + 25*a^3*b^2*c^9*d*x^4 + 225/4*a^4*b*
c^8*d^2*x^4 + 30*a^5*c^7*d^3*x^4 + 10/3*a^3*b^2*c^10*x^3 + 50/3*a^4*b*c^9*d
*x^3 + 15*a^5*c^8*d^2*x^3 + 5/2*a^4*b*c^10*x^2 + 5*a^5*c^9*d*x^2 + a^5*c^10
*x
```

maple [B] time = 0.00, size = 841, normalized size = 5.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^10,x)
```

```
[Out] 1/16*b^5*d^10*x^16+1/15*(5*a*b^4*d^10+10*b^5*c*d^9)*x^15+1/14*(10*a^2*b^3*d
^10+50*a*b^4*c*d^9+45*b^5*c^2*d^8)*x^14+1/13*(10*a^3*b^2*d^10+100*a^2*b^3*c
*d^9+225*a*b^4*c^2*d^8+120*b^5*c^3*d^7)*x^13+1/12*(5*a^4*b*d^10+100*a^3*b^2
*c*d^9+450*a^2*b^3*c^2*d^8+600*a*b^4*c^3*d^7+210*b^5*c^4*d^6)*x^12+1/11*(a^
5*d^10+50*a^4*b*c*d^9+450*a^3*b^2*c^2*d^8+1200*a^2*b^3*c^3*d^7+1050*a*b^4*c
^4*d^6+252*b^5*c^5*d^5)*x^11+1/10*(10*a^5*c*d^9+225*a^4*b*c^2*d^8+1200*a^3*
b^2*c^3*d^7+2100*a^2*b^3*c^4*d^6+1260*a*b^4*c^5*d^5+210*b^5*c^6*d^4)*x^10+1
/9*(45*a^5*c^2*d^8+600*a^4*b*c^3*d^7+2100*a^3*b^2*c^4*d^6+2520*a^2*b^3*c^5*
d^5+1050*a*b^4*c^6*d^4+120*b^5*c^7*d^3)*x^9+1/8*(120*a^5*c^3*d^7+1050*a^4*b
*c^4*d^6+2520*a^3*b^2*c^5*d^5+2100*a^2*b^3*c^6*d^4+600*a*b^4*c^7*d^3+45*b^5
*c^8*d^2)*x^8+1/7*(210*a^5*c^4*d^6+1260*a^4*b*c^5*d^5+2100*a^3*b^2*c^6*d^4+
1200*a^2*b^3*c^7*d^3+225*a*b^4*c^8*d^2+10*b^5*c^9*d)*x^7+1/6*(252*a^5*c^5*d
^5+1050*a^4*b*c^6*d^4+1200*a^3*b^2*c^7*d^3+450*a^2*b^3*c^8*d^2+50*a*b^4*c^9
*d+b^5*c^10)*x^6+1/5*(210*a^5*c^6*d^4+600*a^4*b*c^7*d^3+450*a^3*b^2*c^8*d^2
+100*a^2*b^3*c^9*d+5*a*b^4*c^10)*x^5+1/4*(120*a^5*c^7*d^3+225*a^4*b*c^8*d^2
+100*a^3*b^2*c^9*d+10*a^2*b^3*c^10)*x^4+1/3*(45*a^5*c^8*d^2+50*a^4*b*c^9*d+
10*a^3*b^2*c^10)*x^3+1/2*(10*a^5*c^9*d+5*a^4*b*c^10)*x^2+a^5*c^10*x
```

maxima [B] time = 1.46, size = 835, normalized size = 5.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/16*b^5*d^10*x^16 + a^5*c^10*x + 1/3*(2*b^5*c*d^9 + a*b^4*d^10)*x^15 + 5/1
4*(9*b^5*c^2*d^8 + 10*a*b^4*c*d^9 + 2*a^2*b^3*d^10)*x^14 + 5/13*(24*b^5*c^3
*d^7 + 45*a*b^4*c^2*d^8 + 20*a^2*b^3*c*d^9 + 2*a^3*b^2*d^10)*x^13 + 5/12*(4
2*b^5*c^4*d^6 + 120*a*b^4*c^3*d^7 + 90*a^2*b^3*c^2*d^8 + 20*a^3*b^2*c*d^9 +
 a^4*b*d^10)*x^12 + 1/11*(252*b^5*c^5*d^5 + 1050*a*b^4*c^4*d^6 + 1200*a^2*b
^3*c^3*d^7 + 450*a^3*b^2*c^2*d^8 + 50*a^4*b*c*d^9 + a^5*d^10)*x^11 + 1/2*(4
2*b^5*c^6*d^4 + 252*a*b^4*c^5*d^5 + 420*a^2*b^3*c^4*d^6 + 240*a^3*b^2*c^3*d
^7 + 45*a^4*b*c^2*d^8 + 2*a^5*c*d^9)*x^10 + 5/3*(8*b^5*c^7*d^3 + 70*a*b^4*c
^6*d^4 + 168*a^2*b^3*c^5*d^5 + 140*a^3*b^2*c^4*d^6 + 40*a^4*b*c^3*d^7 + 3*a
^5*c^2*d^8)*x^9 + 15/8*(3*b^5*c^8*d^2 + 40*a*b^4*c^7*d^3 + 140*a^2*b^3*c^6*
```

```
d^4 + 168*a^3*b^2*c^5*d^5 + 70*a^4*b*c^4*d^6 + 8*a^5*c^3*d^7)*x^8 + 5/7*(2*
b^5*c^9*d + 45*a*b^4*c^8*d^2 + 240*a^2*b^3*c^7*d^3 + 420*a^3*b^2*c^6*d^4 +
252*a^4*b*c^5*d^5 + 42*a^5*c^4*d^6)*x^7 + 1/6*(b^5*c^10 + 50*a*b^4*c^9*d +
450*a^2*b^3*c^8*d^2 + 1200*a^3*b^2*c^7*d^3 + 1050*a^4*b*c^6*d^4 + 252*a^5*c
^5*d^5)*x^6 + (a*b^4*c^10 + 20*a^2*b^3*c^9*d + 90*a^3*b^2*c^8*d^2 + 120*a^4
*b*c^7*d^3 + 42*a^5*c^6*d^4)*x^5 + 5/4*(2*a^2*b^3*c^10 + 20*a^3*b^2*c^9*d +
 45*a^4*b*c^8*d^2 + 24*a^5*c^7*d^3)*x^4 + 5/3*(2*a^3*b^2*c^10 + 10*a^4*b*c^
9*d + 9*a^5*c^8*d^2)*x^3 + 5/2*(a^4*b*c^10 + 2*a^5*c^9*d)*x^2
```

mupad [B] time = 0.34, size = 806, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^10,x)
```

```
[Out] x^10*(a^5*c*d^9 + 21*b^5*c^6*d^4 + 126*a*b^4*c^5*d^5 + (45*a^4*b*c^2*d^8)/2
 + 210*a^2*b^3*c^4*d^6 + 120*a^3*b^2*c^3*d^7) + x^7*((10*b^5*c^9*d)/7 + 30*
a^5*c^4*d^6 + (225*a*b^4*c^8*d^2)/7 + 180*a^4*b*c^5*d^5 + (1200*a^2*b^3*c^7
*d^3)/7 + 300*a^3*b^2*c^6*d^4) + x^6*((b^5*c^10)/6 + 42*a^5*c^5*d^5 + 175*a
^4*b*c^6*d^4 + 75*a^2*b^3*c^8*d^2 + 200*a^3*b^2*c^7*d^3 + (25*a*b^4*c^9*d)/
3) + x^11*((a^5*d^10)/11 + (252*b^5*c^5*d^5)/11 + (1050*a*b^4*c^4*d^6)/11 +
 (1200*a^2*b^3*c^3*d^7)/11 + (450*a^3*b^2*c^2*d^8)/11 + (50*a^4*b*c*d^9)/11
) + x^8*(15*a^5*c^3*d^7 + (45*b^5*c^8*d^2)/8 + 75*a*b^4*c^7*d^3 + (525*a^4*
b*c^4*d^6)/4 + (525*a^2*b^3*c^6*d^4)/2 + 315*a^3*b^2*c^5*d^5) + x^9*(5*a^5*
c^2*d^8 + (40*b^5*c^7*d^3)/3 + (350*a*b^4*c^6*d^4)/3 + (200*a^4*b*c^3*d^7)/
3 + 280*a^2*b^3*c^5*d^5 + (700*a^3*b^2*c^4*d^6)/3) + x^5*(a*b^4*c^10 + 42*a
^5*c^6*d^4 + 20*a^2*b^3*c^9*d + 120*a^4*b*c^7*d^3 + 90*a^3*b^2*c^8*d^2) + x
^12*((5*a^4*b*d^10)/12 + (35*b^5*c^4*d^6)/2 + 50*a*b^4*c^3*d^7 + (25*a^3*b^
2*c*d^9)/3 + (75*a^2*b^3*c^2*d^8)/2) + a^5*c^10*x + (b^5*d^10*x^16)/16 + (5
*a^2*c^7*x^4*(24*a^3*d^3 + 2*b^3*c^3 + 20*a*b^2*c^2*d + 45*a^2*b*c*d^2))/4
+ (5*b^2*d^7*x^13*(2*a^3*d^3 + 24*b^3*c^3 + 45*a*b^2*c^2*d + 20*a^2*b*c*d^2
))/13 + (5*a^4*c^9*x^2*(2*a*d + b*c))/2 + (b^4*d^9*x^15*(a*d + 2*b*c))/3 +
(5*a^3*c^8*x^3*(9*a^2*d^2 + 2*b^2*c^2 + 10*a*b*c*d))/3 + (5*b^3*d^8*x^14*(2
*a^2*d^2 + 9*b^2*c^2 + 10*a*b*c*d))/14
```

sympy [B] time = 0.21, size = 940, normalized size = 6.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**10,x)
```

```
[Out] a**5*c**10*x + b**5*d**10*x**16/16 + x**15*(a*b**4*d**10/3 + 2*b**5*c*d**9/
3) + x**14*(5*a**2*b**3*d**10/7 + 25*a*b**4*c*d**9/7 + 45*b**5*c**2*d**8/14
) + x**13*(10*a**3*b**2*d**10/13 + 100*a**2*b**3*c*d**9/13 + 225*a*b**4*c**
2*d**8/13 + 120*b**5*c**3*d**7/13) + x**12*(5*a**4*b*d**10/12 + 25*a**3*b**
2*c*d**9/3 + 75*a**2*b**3*c**2*d**8/2 + 50*a*b**4*c**3*d**7 + 35*b**5*c**4*
d**6/2) + x**11*(a**5*d**10/11 + 50*a**4*b*c*d**9/11 + 450*a**3*b**2*c**2*d
**8/11 + 1200*a**2*b**3*c**3*d**7/11 + 1050*a*b**4*c**4*d**6/11 + 252*b**5*
c**5*d**5/11) + x**10*(a**5*c*d**9 + 45*a**4*b*c**2*d**8/2 + 120*a**3*b**2*
c**3*d**7 + 210*a**2*b**3*c**4*d**6 + 126*a*b**4*c**5*d**5 + 21*b**5*c**6*d
**4) + x**9*(5*a**5*c**2*d**8 + 200*a**4*b*c**3*d**7/3 + 700*a**3*b**2*c**4
*d**6/3 + 280*a**2*b**3*c**5*d**5 + 350*a*b**4*c**6*d**4/3 + 40*b**5*c**7*d
**3/3) + x**8*(15*a**5*c**3*d**7 + 525*a**4*b*c**4*d**6/4 + 315*a**3*b**2*c
**5*d**5 + 525*a**2*b**3*c**6*d**4/2 + 75*a*b**4*c**7*d**3 + 45*b**5*c**8*d
**2/8) + x**7*(30*a**5*c**4*d**6 + 180*a**4*b*c**5*d**5 + 300*a**3*b**2*c**
6*d**4 + 1200*a**2*b**3*c**7*d**3/7 + 225*a*b**4*c**8*d**2/7 + 10*b**5*c**9
*d/7) + x**6*(42*a**5*c**5*d**5 + 175*a**4*b*c**6*d**4 + 200*a**3*b**2*c**7
```

```
*d**3 + 75*a**2*b**3*c**8*d**2 + 25*a*b**4*c**9*d/3 + b**5*c**10/6) + x**5*
(42*a**5*c**6*d**4 + 120*a**4*b*c**7*d**3 + 90*a**3*b**2*c**8*d**2 + 20*a**
2*b**3*c**9*d + a*b**4*c**10) + x**4*(30*a**5*c**7*d**3 + 225*a**4*b*c**8*d
**2/4 + 25*a**3*b**2*c**9*d + 5*a**2*b**3*c**10/2) + x**3*(15*a**5*c**8*d**
2 + 50*a**4*b*c**9*d/3 + 10*a**3*b**2*c**10/3) + x**2*(5*a**5*c**9*d + 5*a*
*4*b*c**10/2)
```

3.1201 $\int (a+bx)^4(c+dx)^{10}\,dx$

Optimal. Leaf size=119

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Rubi [A] time = 0.44, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{2b^3(c+dx)^{14}(bc-ad)}{7d^5}+\frac{6b^2(c+dx)^{13}(bc-ad)^2}{13d^5}-\frac{b(c+dx)^{12}(bc-ad)^3}{3d^5}+\frac{(c+dx)^{11}(bc-ad)^4}{11d^5}+\frac{b^4(c+dx)^{15}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^4*(c + d*x)^11)/(11*d^5) - (b*(b*c - a*d)^3*(c + d*x)^12)/(3*d
^5) + (6*b^2*(b*c - a*d)^2*(c + d*x)^13)/(13*d^5) - (2*b^3*(b*c - a*d)*(c +
 d*x)^14)/(7*d^5) + (b^4*(c + d*x)^15)/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4(c+dx)^{10}\,dx = \int \left(\frac{(-bc+ad)^4(c+dx)^{10}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{11}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{12}}{d^4}-\frac{4b\ldots}{\ldots}\right.$$

$$=\frac{(bc-ad)^4(c+dx)^{11}}{11d^5}-\frac{b(bc-ad)^3(c+dx)^{12}}{3d^5}+\frac{6b^2(bc-ad)^2(c+dx)^{13}}{13d^5}-\frac{2b^3(bc-\ldots}{\ldots}$$

Mathematica [B] time = 0.08, size = 660, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] a^4*c^10*x + a^3*c^9*(2*b*c + 5*a*d)*x^2 + (a^2*c^8*(6*b^2*c^2 + 40*a*b*c*d
 + 45*a^2*d^2)*x^3)/3 + a*c^7*(b^3*c^3 + 15*a*b^2*c^2*d + 45*a^2*b*c*d^2 +
30*a^3*d^3)*x^4 + (c^6*(b^4*c^4 + 40*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 48
0*a^3*b*c*d^3 + 210*a^4*d^4)*x^5)/5 + (c^5*d*(5*b^4*c^4 + 90*a*b^3*c^3*d +
360*a^2*b^2*c^2*d^2 + 420*a^3*b*c*d^3 + 126*a^4*d^4)*x^6)/3 + (3*c^4*d^2*(1
5*b^4*c^4 + 160*a*b^3*c^3*d + 420*a^2*b^2*c^2*d^2 + 336*a^3*b*c*d^3 + 70*a^
4*d^4)*x^7)/7 + 3*c^3*d^3*(5*b^4*c^4 + 35*a*b^3*c^3*d + 63*a^2*b^2*c^2*d^2
+ 35*a^3*b*c*d^3 + 5*a^4*d^4)*x^8 + (c^2*d^4*(70*b^4*c^4 + 336*a*b^3*c^3*d
+ 420*a^2*b^2*c^2*d^2 + 160*a^3*b*c*d^3 + 15*a^4*d^4)*x^9)/3 + (c*d^5*(126*
b^4*c^4 + 420*a*b^3*c^3*d + 360*a^2*b^2*c^2*d^2 + 90*a^3*b*c*d^3 + 5*a^4*d^
4)*x^10)/5 + (d^6*(210*b^4*c^4 + 480*a*b^3*c^3*d + 270*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 + a^4*d^4)*x^11)/11 + (b*d^7*(30*b^3*c^3 + 45*a*b^2*c^2*d + 15
*a^2*b*c*d^2 + a^3*d^3)*x^12)/3 + (b^2*d^8*(45*b^2*c^2 + 40*a*b*c*d + 6*a^2
*d^2)*x^13)/13 + (b^3*d^9*(5*b*c + 2*a*d)*x^14)/7 + (b^4*d^10*x^15)/15
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^4(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^10, x]
```

fricas [B] time = 1.10, size = 771, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/15*x^15*d^10*b^4 + 5/7*x^14*d^9*c*b^4 + 2/7*x^14*d^10*b^3*a + 45/13*x^13*
d^8*c^2*b^4 + 40/13*x^13*d^9*c*b^3*a + 6/13*x^13*d^10*b^2*a^2 + 10*x^12*d^7
*c^3*b^4 + 15*x^12*d^8*c^2*b^3*a + 5*x^12*d^9*c*b^2*a^2 + 1/3*x^12*d^10*b*a
^3 + 210/11*x^11*d^6*c^4*b^4 + 480/11*x^11*d^7*c^3*b^3*a + 270/11*x^11*d^8*
c^2*b^2*a^2 + 40/11*x^11*d^9*c*b*a^3 + 1/11*x^11*d^10*a^4 + 126/5*x^10*d^5*
c^5*b^4 + 84*x^10*d^6*c^4*b^3*a + 72*x^10*d^7*c^3*b^2*a^2 + 18*x^10*d^8*c^2
*b*a^3 + x^10*d^9*c*a^4 + 70/3*x^9*d^4*c^6*b^4 + 112*x^9*d^5*c^5*b^3*a + 14
0*x^9*d^6*c^4*b^2*a^2 + 160/3*x^9*d^7*c^3*b*a^3 + 5*x^9*d^8*c^2*a^4 + 15*x^
8*d^3*c^7*b^4 + 105*x^8*d^4*c^6*b^3*a + 189*x^8*d^5*c^5*b^2*a^2 + 105*x^8*d
^6*c^4*b*a^3 + 15*x^8*d^7*c^3*a^4 + 45/7*x^7*d^2*c^8*b^4 + 480/7*x^7*d^3*c^
7*b^3*a + 180*x^7*d^4*c^6*b^2*a^2 + 144*x^7*d^5*c^5*b*a^3 + 30*x^7*d^6*c^4*
a^4 + 5/3*x^6*d*c^9*b^4 + 30*x^6*d^2*c^8*b^3*a + 120*x^6*d^3*c^7*b^2*a^2 +
140*x^6*d^4*c^6*b*a^3 + 42*x^6*d^5*c^5*a^4 + 1/5*x^5*c^10*b^4 + 8*x^5*d*c^9
*b^3*a + 54*x^5*d^2*c^8*b^2*a^2 + 96*x^5*d^3*c^7*b*a^3 + 42*x^5*d^4*c^6*a^4
 + x^4*c^10*b^3*a + 15*x^4*d*c^9*b^2*a^2 + 45*x^4*d^2*c^8*b*a^3 + 30*x^4*d^
3*c^7*a^4 + 2*x^3*c^10*b^2*a^2 + 40/3*x^3*d*c^9*b*a^3 + 15*x^3*d^2*c^8*a^4
+ 2*x^2*c^10*b*a^3 + 5*x^2*d*c^9*a^4 + x*c^10*a^4
```

giac [B] time = 1.27, size = 771, normalized size = 6.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/15*b^4*d^10*x^15 + 5/7*b^4*c*d^9*x^14 + 2/7*a*b^3*d^10*x^14 + 45/13*b^4*c
^2*d^8*x^13 + 40/13*a*b^3*c*d^9*x^13 + 6/13*a^2*b^2*d^10*x^13 + 10*b^4*c^3*
d^7*x^12 + 15*a*b^3*c^2*d^8*x^12 + 5*a^2*b^2*c*d^9*x^12 + 1/3*a^3*b*d^10*x^
12 + 210/11*b^4*c^4*d^6*x^11 + 480/11*a*b^3*c^3*d^7*x^11 + 270/11*a^2*b^2*c
^2*d^8*x^11 + 40/11*a^3*b*c*d^9*x^11 + 1/11*a^4*d^10*x^11 + 126/5*b^4*c^5*d
^5*x^10 + 84*a*b^3*c^4*d^6*x^10 + 72*a^2*b^2*c^3*d^7*x^10 + 18*a^3*b*c^2*d^
8*x^10 + a^4*c*d^9*x^10 + 70/3*b^4*c^6*d^4*x^9 + 112*a*b^3*c^5*d^5*x^9 + 14
0*a^2*b^2*c^4*d^6*x^9 + 160/3*a^3*b*c^3*d^7*x^9 + 5*a^4*c^2*d^8*x^9 + 15*b^
4*c^7*d^3*x^8 + 105*a*b^3*c^6*d^4*x^8 + 189*a^2*b^2*c^5*d^5*x^8 + 105*a^3*b
*c^4*d^6*x^8 + 15*a^4*c^3*d^7*x^8 + 45/7*b^4*c^8*d^2*x^7 + 480/7*a*b^3*c^7*
d^3*x^7 + 180*a^2*b^2*c^6*d^4*x^7 + 144*a^3*b*c^5*d^5*x^7 + 30*a^4*c^4*d^6*
x^7 + 5/3*b^4*c^9*d*x^6 + 30*a*b^3*c^8*d^2*x^6 + 120*a^2*b^2*c^7*d^3*x^6 +
140*a^3*b*c^6*d^4*x^6 + 42*a^4*c^5*d^5*x^6 + 1/5*b^4*c^10*x^5 + 8*a*b^3*c^9
*d*x^5 + 54*a^2*b^2*c^8*d^2*x^5 + 96*a^3*b*c^7*d^3*x^5 + 42*a^4*c^6*d^4*x^5
 + a*b^3*c^10*x^4 + 15*a^2*b^2*c^9*d*x^4 + 45*a^3*b*c^8*d^2*x^4 + 30*a^4*c^
7*d^3*x^4 + 2*a^2*b^2*c^10*x^3 + 40/3*a^3*b*c^9*d*x^3 + 15*a^4*c^8*d^2*x^3
+ 2*a^3*b*c^10*x^2 + 5*a^4*c^9*d*x^2 + a^4*c^10*x
```

maple [B] time = 0.00, size = 691, normalized size = 5.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^10,x)
```

```
[Out] 1/15*b^4*d^10*x^15+1/14*(4*a*b^3*d^10+10*b^4*c*d^9)*x^14+1/13*(6*a^2*b^2*d^
10+40*a*b^3*c*d^9+45*b^4*c^2*d^8)*x^13+1/12*(4*a^3*b*d^10+60*a^2*b^2*c*d^9+
180*a*b^3*c^2*d^8+120*b^4*c^3*d^7)*x^12+1/11*(a^4*d^10+40*a^3*b*c*d^9+270*a
^2*b^2*c^2*d^8+480*a*b^3*c^3*d^7+210*b^4*c^4*d^6)*x^11+1/10*(10*a^4*c*d^9+1
80*a^3*b*c^2*d^8+720*a^2*b^2*c^3*d^7+840*a*b^3*c^4*d^6+252*b^4*c^5*d^5)*x^1
0+1/9*(45*a^4*c^2*d^8+480*a^3*b*c^3*d^7+1260*a^2*b^2*c^4*d^6+1008*a*b^3*c^5
*d^5+210*b^4*c^6*d^4)*x^9+1/8*(120*a^4*c^3*d^7+840*a^3*b*c^4*d^6+1512*a^2*b
^2*c^5*d^5+840*a*b^3*c^6*d^4+120*b^4*c^7*d^3)*x^8+1/7*(210*a^4*c^4*d^6+1008
*a^3*b*c^5*d^5+1260*a^2*b^2*c^6*d^4+480*a*b^3*c^7*d^3+45*b^4*c^8*d^2)*x^7+1
/6*(252*a^4*c^5*d^5+840*a^3*b*c^6*d^4+720*a^2*b^2*c^7*d^3+180*a*b^3*c^8*d^2
+10*b^4*c^9*d)*x^6+1/5*(210*a^4*c^6*d^4+480*a^3*b*c^7*d^3+270*a^2*b^2*c^8*d
^2+40*a*b^3*c^9*d+b^4*c^10)*x^5+1/4*(120*a^4*c^7*d^3+180*a^3*b*c^8*d^2+60*a
^2*b^2*c^9*d+4*a*b^3*c^10)*x^4+1/3*(45*a^4*c^8*d^2+40*a^3*b*c^9*d+6*a^2*b^2
*c^10)*x^3+1/2*(10*a^4*c^9*d+4*a^3*b*c^10)*x^2+a^4*c^10*x
```

maxima [B] time = 1.42, size = 686, normalized size = 5.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/15*b^4*d^10*x^15 + a^4*c^10*x + 1/7*(5*b^4*c*d^9 + 2*a*b^3*d^10)*x^14 + 1
/13*(45*b^4*c^2*d^8 + 40*a*b^3*c*d^9 + 6*a^2*b^2*d^10)*x^13 + 1/3*(30*b^4*c
^3*d^7 + 45*a*b^3*c^2*d^8 + 15*a^2*b^2*c*d^9 + a^3*b*d^10)*x^12 + 1/11*(210
*b^4*c^4*d^6 + 480*a*b^3*c^3*d^7 + 270*a^2*b^2*c^2*d^8 + 40*a^3*b*c*d^9 + a
^4*d^10)*x^11 + 1/5*(126*b^4*c^5*d^5 + 420*a*b^3*c^4*d^6 + 360*a^2*b^2*c^3*
d^7 + 90*a^3*b*c^2*d^8 + 5*a^4*c*d^9)*x^10 + 1/3*(70*b^4*c^6*d^4 + 336*a*b^
3*c^5*d^5 + 420*a^2*b^2*c^4*d^6 + 160*a^3*b*c^3*d^7 + 15*a^4*c^2*d^8)*x^9 +
 3*(5*b^4*c^7*d^3 + 35*a*b^3*c^6*d^4 + 63*a^2*b^2*c^5*d^5 + 35*a^3*b*c^4*d^
6 + 5*a^4*c^3*d^7)*x^8 + 3/7*(15*b^4*c^8*d^2 + 160*a*b^3*c^7*d^3 + 420*a^2*
b^2*c^6*d^4 + 336*a^3*b*c^5*d^5 + 70*a^4*c^4*d^6)*x^7 + 1/3*(5*b^4*c^9*d +
90*a*b^3*c^8*d^2 + 360*a^2*b^2*c^7*d^3 + 420*a^3*b*c^6*d^4 + 126*a^4*c^5*d^
5)*x^6 + 1/5*(b^4*c^10 + 40*a*b^3*c^9*d + 270*a^2*b^2*c^8*d^2 + 480*a^3*b*c
^7*d^3 + 210*a^4*c^6*d^4)*x^5 + (a*b^3*c^10 + 15*a^2*b^2*c^9*d + 45*a^3*b*c
^8*d^2 + 30*a^4*c^7*d^3)*x^4 + 1/3*(6*a^2*b^2*c^10 + 40*a^3*b*c^9*d + 45*a^
4*c^8*d^2)*x^3 + (2*a^3*b*c^10 + 5*a^4*c^9*d)*x^2
```

mupad [B] time = 0.43, size = 664, normalized size = 5.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^10,x)
```

```
[Out] x^5*((b^4*c^10)/5 + 42*a^4*c^6*d^4 + 96*a^3*b*c^7*d^3 + 54*a^2*b^2*c^8*d^2
+ 8*a*b^3*c^9*d) + x^11*((a^4*d^10)/11 + (210*b^4*c^4*d^6)/11 + (480*a*b^3*
c^3*d^7)/11 + (270*a^2*b^2*c^2*d^8)/11 + (40*a^3*b*c*d^9)/11) + x^8*(15*a^4
*c^3*d^7 + 15*b^4*c^7*d^3 + 105*a*b^3*c^6*d^4 + 105*a^3*b*c^4*d^6 + 189*a^2
*b^2*c^5*d^5) + x^9*(5*a^4*c^2*d^8 + (70*b^4*c^6*d^4)/3 + 112*a*b^3*c^5*d^5
 + (160*a^3*b*c^3*d^7)/3 + 140*a^2*b^2*c^4*d^6) + x^7*(30*a^4*c^4*d^6 + (45
*b^4*c^8*d^2)/7 + (480*a*b^3*c^7*d^3)/7 + 144*a^3*b*c^5*d^5 + 180*a^2*b^2*c
```

```
^6*d^4) + x^4*(a*b^3*c^10 + 30*a^4*c^7*d^3 + 15*a^2*b^2*c^9*d + 45*a^3*b*c^
8*d^2) + x^12*((a^3*b*d^10)/3 + 10*b^4*c^3*d^7 + 15*a*b^3*c^2*d^8 + 5*a^2*b
^2*c*d^9) + x^10*(a^4*c*d^9 + (126*b^4*c^5*d^5)/5 + 84*a*b^3*c^4*d^6 + 18*a
^3*b*c^2*d^8 + 72*a^2*b^2*c^3*d^7) + x^6*((5*b^4*c^9*d)/3 + 42*a^4*c^5*d^5
+ 30*a*b^3*c^8*d^2 + 140*a^3*b*c^6*d^4 + 120*a^2*b^2*c^7*d^3) + a^4*c^10*x
+ (b^4*d^10*x^15)/15 + a^3*c^9*x^2*(5*a*d + 2*b*c) + (b^3*d^9*x^14*(2*a*d +
 5*b*c))/7 + (a^2*c^8*x^3*(45*a^2*d^2 + 6*b^2*c^2 + 40*a*b*c*d))/3 + (b^2*d
^8*x^13*(6*a^2*d^2 + 45*b^2*c^2 + 40*a*b*c*d))/13
```

sympy [B] time = 0.18, size = 748, normalized size = 6.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**10,x)
```

```
[Out] a**4*c**10*x + b**4*d**10*x**15/15 + x**14*(2*a*b**3*d**10/7 + 5*b**4*c*d**
9/7) + x**13*(6*a**2*b**2*d**10/13 + 40*a*b**3*c*d**9/13 + 45*b**4*c**2*d**
8/13) + x**12*(a**3*b*d**10/3 + 5*a**2*b**2*c*d**9 + 15*a*b**3*c**2*d**8 +
10*b**4*c**3*d**7) + x**11*(a**4*d**10/11 + 40*a**3*b*c*d**9/11 + 270*a**2*
b**2*c**2*d**8/11 + 480*a*b**3*c**3*d**7/11 + 210*b**4*c**4*d**6/11) + x**1
0*(a**4*c*d**9 + 18*a**3*b*c**2*d**8 + 72*a**2*b**2*c**3*d**7 + 84*a*b**3*c
**4*d**6 + 126*b**4*c**5*d**5/5) + x**9*(5*a**4*c**2*d**8 + 160*a**3*b*c**3
*d**7/3 + 140*a**2*b**2*c**4*d**6 + 112*a*b**3*c**5*d**5 + 70*b**4*c**6*d**
4/3) + x**8*(15*a**4*c**3*d**7 + 105*a**3*b*c**4*d**6 + 189*a**2*b**2*c**5*
d**5 + 105*a*b**3*c**6*d**4 + 15*b**4*c**7*d**3) + x**7*(30*a**4*c**4*d**6
+ 144*a**3*b*c**5*d**5 + 180*a**2*b**2*c**6*d**4 + 480*a*b**3*c**7*d**3/7 +
 45*b**4*c**8*d**2/7) + x**6*(42*a**4*c**5*d**5 + 140*a**3*b*c**6*d**4 + 12
0*a**2*b**2*c**7*d**3 + 30*a*b**3*c**8*d**2 + 5*b**4*c**9*d/3) + x**5*(42*a
**4*c**6*d**4 + 96*a**3*b*c**7*d**3 + 54*a**2*b**2*c**8*d**2 + 8*a*b**3*c**
9*d + b**4*c**10/5) + x**4*(30*a**4*c**7*d**3 + 45*a**3*b*c**8*d**2 + 15*a*
*2*b**2*c**9*d + a*b**3*c**10) + x**3*(15*a**4*c**8*d**2 + 40*a**3*b*c**9*d
/3 + 2*a**2*b**2*c**10) + x**2*(5*a**4*c**9*d + 2*a**3*b*c**10)
```

3.1202 $\int (a+bx)^3(c+dx)^{10}\,dx$

Optimal. Leaf size=92

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Rubi [A] time = 0.35, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(c+dx)^{13}(bc-ad)}{13d^4}+\frac{b(c+dx)^{12}(bc-ad)^2}{4d^4}-\frac{(c+dx)^{11}(bc-ad)^3}{11d^4}+\frac{b^3(c+dx)^{14}}{14d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)^3*(c + d*x)^11)/(11*d^4) + (b*(b*c - a*d)^2*(c + d*x)^12)/(4*
d^4) - (3*b^2*(b*c - a*d)*(c + d*x)^13)/(13*d^4) + (b^3*(c + d*x)^14)/(14*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{10}\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^{10}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{11}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{12}}{d^3}+\frac{b^3(\ldots}{\ldots}\right.$$
$$=-\frac{(bc-ad)^3(c+dx)^{11}}{11d^4}+\frac{b(bc-ad)^2(c+dx)^{12}}{4d^4}-\frac{3b^2(bc-ad)(c+dx)^{13}}{13d^4}+\frac{b^3(c+d\ldots}{14d^4}$$

Mathematica [B] time = 0.07, size = 511, normalized size = 5.55

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] a^3*c^10*x + (a^2*c^9*(3*b*c + 10*a*d)*x^2)/2 + a*c^8*(b^2*c^2 + 10*a*b*c*d
 + 15*a^2*d^2)*x^3 + (c^7*(b^3*c^3 + 30*a*b^2*c^2*d + 135*a^2*b*c*d^2 + 120
*a^3*d^3)*x^4)/4 + c^6*d*(2*b^3*c^3 + 27*a*b^2*c^2*d + 72*a^2*b*c*d^2 + 42*
a^3*d^3)*x^5 + (3*c^5*d^2*(5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2 + 28
*a^3*d^3)*x^6)/2 + (6*c^4*d^3*(20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d
^2 + 35*a^3*d^3)*x^7)/7 + (3*c^3*d^4*(35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^
2*b*c*d^2 + 20*a^3*d^3)*x^8)/4 + c^2*d^5*(28*b^3*c^3 + 70*a*b^2*c^2*d + 40*
a^2*b*c*d^2 + 5*a^3*d^3)*x^9 + (c*d^6*(42*b^3*c^3 + 72*a*b^2*c^2*d + 27*a^2
*b*c*d^2 + 2*a^3*d^3)*x^10)/2 + (d^7*(120*b^3*c^3 + 135*a*b^2*c^2*d + 30*a^
2*b*c*d^2 + a^3*d^3)*x^11)/11 + (b*d^8*(15*b^2*c^2 + 10*a*b*c*d + a^2*d^2)*
x^12)/4 + (b^2*d^9*(10*b*c + 3*a*d)*x^13)/13 + (b^3*d^10*x^14)/14
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^10, x]
```

fricas [B] time = 1.05, size = 594, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/14*x^14*d^10*b^3 + 10/13*x^13*d^9*c*b^3 + 3/13*x^13*d^10*b^2*a + 15/4*x^1
2*d^8*c^2*b^3 + 5/2*x^12*d^9*c*b^2*a + 1/4*x^12*d^10*b*a^2 + 120/11*x^11*d^
7*c^3*b^3 + 135/11*x^11*d^8*c^2*b^2*a + 30/11*x^11*d^9*c*b*a^2 + 1/11*x^11*
d^10*a^3 + 21*x^10*d^6*c^4*b^3 + 36*x^10*d^7*c^3*b^2*a + 27/2*x^10*d^8*c^2*
b*a^2 + x^10*d^9*c*a^3 + 28*x^9*d^5*c^5*b^3 + 70*x^9*d^6*c^4*b^2*a + 40*x^9
*d^7*c^3*b*a^2 + 5*x^9*d^8*c^2*a^3 + 105/4*x^8*d^4*c^6*b^3 + 189/2*x^8*d^5*
c^5*b^2*a + 315/4*x^8*d^6*c^4*b*a^2 + 15*x^8*d^7*c^3*a^3 + 120/7*x^7*d^3*c^
7*b^3 + 90*x^7*d^4*c^6*b^2*a + 108*x^7*d^5*c^5*b*a^2 + 30*x^7*d^6*c^4*a^3 +
 15/2*x^6*d^2*c^8*b^3 + 60*x^6*d^3*c^7*b^2*a + 105*x^6*d^4*c^6*b*a^2 + 42*x
^6*d^5*c^5*a^3 + 2*x^5*d*c^9*b^3 + 27*x^5*d^2*c^8*b^2*a + 72*x^5*d^3*c^7*b*
a^2 + 42*x^5*d^4*c^6*a^3 + 1/4*x^4*c^10*b^3 + 15/2*x^4*d*c^9*b^2*a + 135/4*
x^4*d^2*c^8*b*a^2 + 30*x^4*d^3*c^7*a^3 + x^3*c^10*b^2*a + 10*x^3*d*c^9*b*a^
2 + 15*x^3*d^2*c^8*a^3 + 3/2*x^2*c^10*b*a^2 + 5*x^2*d*c^9*a^3 + x*c^10*a^3
```

giac [B] time = 1.28, size = 594, normalized size = 6.46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/14*b^3*d^10*x^14 + 10/13*b^3*c*d^9*x^13 + 3/13*a*b^2*d^10*x^13 + 15/4*b^3
*c^2*d^8*x^12 + 5/2*a*b^2*c*d^9*x^12 + 1/4*a^2*b*d^10*x^12 + 120/11*b^3*c^3
*d^7*x^11 + 135/11*a*b^2*c^2*d^8*x^11 + 30/11*a^2*b*c*d^9*x^11 + 1/11*a^3*d
^10*x^11 + 21*b^3*c^4*d^6*x^10 + 36*a*b^2*c^3*d^7*x^10 + 27/2*a^2*b*c^2*d^8
*x^10 + a^3*c*d^9*x^10 + 28*b^3*c^5*d^5*x^9 + 70*a*b^2*c^4*d^6*x^9 + 40*a^2
*b*c^3*d^7*x^9 + 5*a^3*c^2*d^8*x^9 + 105/4*b^3*c^6*d^4*x^8 + 189/2*a*b^2*c^
5*d^5*x^8 + 315/4*a^2*b*c^4*d^6*x^8 + 15*a^3*c^3*d^7*x^8 + 120/7*b^3*c^7*d^
3*x^7 + 90*a*b^2*c^6*d^4*x^7 + 108*a^2*b*c^5*d^5*x^7 + 30*a^3*c^4*d^6*x^7 +
 15/2*b^3*c^8*d^2*x^6 + 60*a*b^2*c^7*d^3*x^6 + 105*a^2*b*c^6*d^4*x^6 + 42*a
^3*c^5*d^5*x^6 + 2*b^3*c^9*d*x^5 + 27*a*b^2*c^8*d^2*x^5 + 72*a^2*b*c^7*d^3*
x^5 + 42*a^3*c^6*d^4*x^5 + 1/4*b^3*c^10*x^4 + 15/2*a*b^2*c^9*d*x^4 + 135/4*
a^2*b*c^8*d^2*x^4 + 30*a^3*c^7*d^3*x^4 + a*b^2*c^10*x^3 + 10*a^2*b*c^9*d*x^
3 + 15*a^3*c^8*d^2*x^3 + 3/2*a^2*b*c^10*x^2 + 5*a^3*c^9*d*x^2 + a^3*c^10*x
```

maple [B] time = 0.00, size = 541, normalized size = 5.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^10,x)
```

```
[Out] 1/14*b^3*d^10*x^14+1/13*(3*a*b^2*d^10+10*b^3*c*d^9)*x^13+1/12*(3*a^2*b*d^10
+30*a*b^2*c*d^9+45*b^3*c^2*d^8)*x^12+1/11*(a^3*d^10+30*a^2*b*c*d^9+135*a*b^
2*c^2*d^8+120*b^3*c^3*d^7)*x^11+1/10*(10*a^3*c*d^9+135*a^2*b*c^2*d^8+360*a*
b^2*c^3*d^7+210*b^3*c^4*d^6)*x^10+1/9*(45*a^3*c^2*d^8+360*a^2*b*c^3*d^7+630
*a*b^2*c^4*d^6+252*b^3*c^5*d^5)*x^9+1/8*(120*a^3*c^3*d^7+630*a^2*b*c^4*d^6+
```

```
756*a*b^2*c^5*d^5+210*b^3*c^6*d^4)*x^8+1/7*(210*a^3*c^4*d^6+756*a^2*b*c^5*d
^5+630*a*b^2*c^6*d^4+120*b^3*c^7*d^3)*x^7+1/6*(252*a^3*c^5*d^5+630*a^2*b*c^
6*d^4+360*a*b^2*c^7*d^3+45*b^3*c^8*d^2)*x^6+1/5*(210*a^3*c^6*d^4+360*a^2*b*
c^7*d^3+135*a*b^2*c^8*d^2+10*b^3*c^9*d)*x^5+1/4*(120*a^3*c^7*d^3+135*a^2*b*
c^8*d^2+30*a*b^2*c^9*d+b^3*c^10)*x^4+1/3*(45*a^3*c^8*d^2+30*a^2*b*c^9*d+3*a
*b^2*c^10)*x^3+1/2*(10*a^3*c^9*d+3*a^2*b*c^10)*x^2+a^3*c^10*x
```

maxima [B] time = 1.32, size = 535, normalized size = 5.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/14*b^3*d^10*x^14 + a^3*c^10*x + 1/13*(10*b^3*c*d^9 + 3*a*b^2*d^10)*x^13 +
 1/4*(15*b^3*c^2*d^8 + 10*a*b^2*c*d^9 + a^2*b*d^10)*x^12 + 1/11*(120*b^3*c^
3*d^7 + 135*a*b^2*c^2*d^8 + 30*a^2*b*c*d^9 + a^3*d^10)*x^11 + 1/2*(42*b^3*c
^4*d^6 + 72*a*b^2*c^3*d^7 + 27*a^2*b*c^2*d^8 + 2*a^3*c*d^9)*x^10 + (28*b^3*
c^5*d^5 + 70*a*b^2*c^4*d^6 + 40*a^2*b*c^3*d^7 + 5*a^3*c^2*d^8)*x^9 + 3/4*(3
5*b^3*c^6*d^4 + 126*a*b^2*c^5*d^5 + 105*a^2*b*c^4*d^6 + 20*a^3*c^3*d^7)*x^8
 + 6/7*(20*b^3*c^7*d^3 + 105*a*b^2*c^6*d^4 + 126*a^2*b*c^5*d^5 + 35*a^3*c^4
*d^6)*x^7 + 3/2*(5*b^3*c^8*d^2 + 40*a*b^2*c^7*d^3 + 70*a^2*b*c^6*d^4 + 28*a
^3*c^5*d^5)*x^6 + (2*b^3*c^9*d + 27*a*b^2*c^8*d^2 + 72*a^2*b*c^7*d^3 + 42*a
^3*c^6*d^4)*x^5 + 1/4*(b^3*c^10 + 30*a*b^2*c^9*d + 135*a^2*b*c^8*d^2 + 120*
a^3*c^7*d^3)*x^4 + (a*b^2*c^10 + 10*a^2*b*c^9*d + 15*a^3*c^8*d^2)*x^3 + 1/2
*(3*a^2*b*c^10 + 10*a^3*c^9*d)*x^2
```

mupad [B] time = 0.23, size = 495, normalized size = 5.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^10,x)
```

```
[Out] x^4*((b^3*c^10)/4 + 30*a^3*c^7*d^3 + (135*a^2*b*c^8*d^2)/4 + (15*a*b^2*c^9*
d)/2) + x^11*((a^3*d^10)/11 + (120*b^3*c^3*d^7)/11 + (135*a*b^2*c^2*d^8)/11
 + (30*a^2*b*c*d^9)/11) + a^3*c^10*x + (b^3*d^10*x^14)/14 + (3*c^5*d^2*x^6*
(28*a^3*d^3 + 5*b^3*c^3 + 40*a*b^2*c^2*d + 70*a^2*b*c*d^2))/2 + c^2*d^5*x^9
*(5*a^3*d^3 + 28*b^3*c^3 + 70*a*b^2*c^2*d + 40*a^2*b*c*d^2) + (6*c^4*d^3*x^
7*(35*a^3*d^3 + 20*b^3*c^3 + 105*a*b^2*c^2*d + 126*a^2*b*c*d^2))/7 + (3*c^3
*d^4*x^8*(20*a^3*d^3 + 35*b^3*c^3 + 126*a*b^2*c^2*d + 105*a^2*b*c*d^2))/4 +
 (a^2*c^9*x^2*(10*a*d + 3*b*c))/2 + (b^2*d^9*x^13*(3*a*d + 10*b*c))/13 + a*
c^8*x^3*(15*a^2*d^2 + b^2*c^2 + 10*a*b*c*d) + (b*d^8*x^12*(a^2*d^2 + 15*b^2
*c^2 + 10*a*b*c*d))/4 + c^6*d*x^5*(42*a^3*d^3 + 2*b^3*c^3 + 27*a*b^2*c^2*d
+ 72*a^2*b*c*d^2) + (c*d^6*x^10*(2*a^3*d^3 + 42*b^3*c^3 + 72*a*b^2*c^2*d +
27*a^2*b*c*d^2))/2
```

sympy [B] time = 0.16, size = 586, normalized size = 6.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**10,x)
```

```
[Out] a**3*c**10*x + b**3*d**10*x**14/14 + x**13*(3*a*b**2*d**10/13 + 10*b**3*c*d
**9/13) + x**12*(a**2*b*d**10/4 + 5*a*b**2*c*d**9/2 + 15*b**3*c**2*d**8/4)
+ x**11*(a**3*d**10/11 + 30*a**2*b*c*d**9/11 + 135*a*b**2*c**2*d**8/11 + 12
0*b**3*c**3*d**7/11) + x**10*(a**3*c*d**9 + 27*a**2*b*c**2*d**8/2 + 36*a*b*
*2*c**3*d**7 + 21*b**3*c**4*d**6) + x**9*(5*a**3*c**2*d**8 + 40*a**2*b*c**3
```

```
*d**7 + 70*a*b**2*c**4*d**6 + 28*b**3*c**5*d**5) + x**8*(15*a**3*c**3*d**7
+ 315*a**2*b*c**4*d**6/4 + 189*a*b**2*c**5*d**5/2 + 105*b**3*c**6*d**4/4) +
 x**7*(30*a**3*c**4*d**6 + 108*a**2*b*c**5*d**5 + 90*a*b**2*c**6*d**4 + 120
*b**3*c**7*d**3/7) + x**6*(42*a**3*c**5*d**5 + 105*a**2*b*c**6*d**4 + 60*a*
b**2*c**7*d**3 + 15*b**3*c**8*d**2/2) + x**5*(42*a**3*c**6*d**4 + 72*a**2*b
*c**7*d**3 + 27*a*b**2*c**8*d**2 + 2*b**3*c**9*d) + x**4*(30*a**3*c**7*d**3
 + 135*a**2*b*c**8*d**2/4 + 15*a*b**2*c**9*d/2 + b**3*c**10/4) + x**3*(15*a
**3*c**8*d**2 + 10*a**2*b*c**9*d + a*b**2*c**10) + x**2*(5*a**3*c**9*d + 3*
a**2*b*c**10/2)
```

3.1203 $\int (a+bx)^2(c+dx)^{10}\,dx$

Optimal. Leaf size=65

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Rubi [A] time = 0.25, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b(c+dx)^{12}(bc-ad)}{6d^3}+\frac{(c+dx)^{11}(bc-ad)^2}{11d^3}+\frac{b^2(c+dx)^{13}}{13d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^11)/(11*d^3) - (b*(b*c - a*d)*(c + d*x)^12)/(6*d^3
) + (b^2*(c + d*x)^13)/(13*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{10}}{d^2}-\frac{2b(bc-ad)(c+dx)^{11}}{d^2}+\frac{b^2(c+dx)^{12}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{11}}{11d^3}-\frac{b(bc-ad)(c+dx)^{12}}{6d^3}+\frac{b^2(c+dx)^{13}}{13d^3}\end{aligned}$$

Mathematica [B] time = 0.05, size = 358, normalized size = 5.51

$$\frac{1}{11}d^8x^{11}\left(a^2d^2+20abcd+45b^2c^2\right)+cd^7x^{10}\left(a^2d^2+9abcd+12b^2c^2\right)+\frac{5}{3}c^2d^6x^9\left(3a^2d^2+16abcd+14b^2c^2\right)+\frac{1}{3}c^8x^3\left(45a^2d^2+20abcd+b^2c^2\right)+\frac{5}{2}c^7dx^4\left(12a^2d^2+9abcd+b^2c^2\right)+3c^6d^2x^5\left(14a^2d^2+16abcd+3b^2c^2\right)+2c^5d^3x^6\left(21a^2d^2+35abcd+10b^2c^2\right)+6c^4d^4x^7\left(5a^2d^2+12abcd+5b^2c^2\right)+\frac{3}{2}c^3d^5x^8\left(10a^2d^2+35abcd+21b^2c^2\right)+a^2c^{10}x+ac^9x^2(5ad+bc)+\frac{1}{6}bd^9x^{12}(ad+5bc)+\frac{1}{13}b^2d^{10}x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] a^2*c^10*x + a*c^9*(b*c + 5*a*d)*x^2 + (c^8*(b^2*c^2 + 20*a*b*c*d + 45*a^2*
d^2)*x^3)/3 + (5*c^7*d*(b^2*c^2 + 9*a*b*c*d + 12*a^2*d^2)*x^4)/2 + 3*c^6*d^
2*(3*b^2*c^2 + 16*a*b*c*d + 14*a^2*d^2)*x^5 + 2*c^5*d^3*(10*b^2*c^2 + 35*a*
b*c*d + 21*a^2*d^2)*x^6 + 6*c^4*d^4*(5*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*x^
7 + (3*c^3*d^5*(21*b^2*c^2 + 35*a*b*c*d + 10*a^2*d^2)*x^8)/2 + (5*c^2*d^6*(
14*b^2*c^2 + 16*a*b*c*d + 3*a^2*d^2)*x^9)/3 + c*d^7*(12*b^2*c^2 + 9*a*b*c*d
 + a^2*d^2)*x^10 + (d^8*(45*b^2*c^2 + 20*a*b*c*d + a^2*d^2)*x^11)/11 + (b*d
^9*(5*b*c + a*d)*x^12)/6 + (b^2*d^10*x^13)/13
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^10, x]
```

fricas [B] time = 0.94, size = 417, normalized size = 6.42

$$\frac{1}{13}x^{13}d^{10}b^2+\frac{5}{6}x^{12}d^9cb^2+\frac{1}{6}x^{12}d^{10}ba+\frac{45}{11}x^{11}d^8c^2b^2+\frac{20}{11}x^{11}d^9cba+\frac{1}{11}x^{11}d^{10}a^2+12x^{10}d^7c^3b^2+9x^{10}d^8c^2ba+x^{10}d^9ca^2+\frac{70}{3}x^9d^6c^4b^2+\frac{80}{3}x^9d^7c^3ba+5x^9d^8c^2a^2+\frac{63}{2}x^8d^5c^5b^2+\frac{105}{2}x^8d^6c^4ba+15x^8d^7c^3a^2+30x^7d^4c^6b^2+72x^7d^5c^5ba+30x^7d^6c^4a^2+20x^6d^3c^7b^2+70x^6d^4c^6ba+42x^6d^5c^5a^2+9x^5d^2c^8b^2+48x^5d^3c^7ba+42x^5d^4c^6a^2+\frac{5}{2}x^4dc^9b^2+\frac{45}{2}x^4d^2c^8ba+30x^4d^3c^7a^2+\frac{1}{3}x^3c^{10}b^2+\frac{20}{3}x^3dc^9ba+15x^3d^2c^8a^2+x^2c^{10}ba+5x^2dc^9a^2+xc^{10}a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^10*b^2 + 5/6*x^12*d^9*c*b^2 + 1/6*x^12*d^10*b*a + 45/11*x^11*d^
8*c^2*b^2 + 20/11*x^11*d^9*c*b*a + 1/11*x^11*d^10*a^2 + 12*x^10*d^7*c^3*b^2
 + 9*x^10*d^8*c^2*b*a + x^10*d^9*c*a^2 + 70/3*x^9*d^6*c^4*b^2 + 80/3*x^9*d^
7*c^3*b*a + 5*x^9*d^8*c^2*a^2 + 63/2*x^8*d^5*c^5*b^2 + 105/2*x^8*d^6*c^4*b*
a + 15*x^8*d^7*c^3*a^2 + 30*x^7*d^4*c^6*b^2 + 72*x^7*d^5*c^5*b*a + 30*x^7*d
^6*c^4*a^2 + 20*x^6*d^3*c^7*b^2 + 70*x^6*d^4*c^6*b*a + 42*x^6*d^5*c^5*a^2 +
 9*x^5*d^2*c^8*b^2 + 48*x^5*d^3*c^7*b*a + 42*x^5*d^4*c^6*a^2 + 5/2*x^4*d*c^
9*b^2 + 45/2*x^4*d^2*c^8*b*a + 30*x^4*d^3*c^7*a^2 + 1/3*x^3*c^10*b^2 + 20/3
*x^3*d*c^9*b*a + 15*x^3*d^2*c^8*a^2 + x^2*c^10*b*a + 5*x^2*d*c^9*a^2 + x*c^
10*a^2
```

giac [B] time = 1.26, size = 417, normalized size = 6.42

$$\frac{1}{13}b^2d^{10}x^{13}+\frac{5}{6}b^2cd^9x^{12}+\frac{1}{6}abd^{10}x^{12}+\frac{45}{11}b^2c^2d^8x^{11}+\frac{20}{11}abcd^9x^{11}+\frac{1}{11}a^2d^{10}x^{11}+12b^2c^3d^7x^{10}+9abc^2d^8x^{10}+a^2cd^9x^{10}+\frac{70}{3}b^2c^4d^6x^9+\frac{80}{3}abc^3d^7x^9+5a^2c^2d^8x^9+\frac{63}{2}b^2c^5d^5x^8+\frac{105}{2}abc^4d^6x^8+15a^2c^3d^7x^8+30b^2c^6d^4x^7+72abc^5d^5x^7+30a^2c^4d^6x^7+20b^2c^7d^3x^6+70abc^6d^4x^6+42a^2c^5d^5x^6+9b^2c^8d^2x^5+48abc^7d^3x^5+42a^2c^6d^4x^5+\frac{5}{2}b^2c^9dx^4+\frac{45}{2}abc^8d^2x^4+30a^2c^7d^3x^4+\frac{1}{3}b^2c^{10}x^3+\frac{20}{3}abc^9dx^3+15a^2c^8d^2x^3+abc^{10}x^2+5a^2c^9dx^2+a^2c^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^10*x^13 + 5/6*b^2*c*d^9*x^12 + 1/6*a*b*d^10*x^12 + 45/11*b^2*c^2
*d^8*x^11 + 20/11*a*b*c*d^9*x^11 + 1/11*a^2*d^10*x^11 + 12*b^2*c^3*d^7*x^10
 + 9*a*b*c^2*d^8*x^10 + a^2*c*d^9*x^10 + 70/3*b^2*c^4*d^6*x^9 + 80/3*a*b*c^
3*d^7*x^9 + 5*a^2*c^2*d^8*x^9 + 63/2*b^2*c^5*d^5*x^8 + 105/2*a*b*c^4*d^6*x^
8 + 15*a^2*c^3*d^7*x^8 + 30*b^2*c^6*d^4*x^7 + 72*a*b*c^5*d^5*x^7 + 30*a^2*c
^4*d^6*x^7 + 20*b^2*c^7*d^3*x^6 + 70*a*b*c^6*d^4*x^6 + 42*a^2*c^5*d^5*x^6 +
 9*b^2*c^8*d^2*x^5 + 48*a*b*c^7*d^3*x^5 + 42*a^2*c^6*d^4*x^5 + 5/2*b^2*c^9*
d*x^4 + 45/2*a*b*c^8*d^2*x^4 + 30*a^2*c^7*d^3*x^4 + 1/3*b^2*c^10*x^3 + 20/3
*a*b*c^9*d*x^3 + 15*a^2*c^8*d^2*x^3 + a*b*c^10*x^2 + 5*a^2*c^9*d*x^2 + a^2*
c^10*x
```

maple [B] time = 0.00, size = 391, normalized size = 6.02

$$\frac{b^2d^{10}x^{13}}{13}+a^2c^{10}x+\frac{\left(2abd^{10}+10b^2cd^9\right)x^{12}}{12}+\frac{\left(a^2d^{10}+20abcd^9+45b^2c^2d^8\right)x^{11}}{11}+\frac{\left(10a^2cd^9+90abc^2d^8+120b^2c^3d^7\right)x^{10}}{10}+\frac{\left(45a^2c^2d^8+240abc^3d^7+210b^2c^4d^6\right)x^9}{9}+\frac{\left(120a^2c^3d^7+420abc^4d^6+252b^2c^5d^5\right)x^8}{8}+\frac{\left(210a^2c^4d^6+504abc^5d^5+210b^2c^6d^4\right)x^7}{7}+\frac{\left(252a^2c^5d^5+420abc^6d^4+120b^2c^7d^3\right)x^6}{6}+\frac{\left(210a^2c^6d^4+240abc^7d^3+45b^2c^8d^2\right)x^5}{5}+\frac{\left(120a^2c^7d^3+90abc^8d^2+10b^2c^9d\right)x^4}{4}+\frac{\left(45a^2c^8d^2+20abc^9d+b^2c^{10}\right)x^3}{3}+\frac{\left(10a^2c^9d+2abc^{10}\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^10,x)
```

```
[Out] 1/13*b^2*d^10*x^13+1/12*(2*a*b*d^10+10*b^2*c*d^9)*x^12+1/11*(a^2*d^10+20*a*
b*c*d^9+45*b^2*c^2*d^8)*x^11+1/10*(10*a^2*c*d^9+90*a*b*c^2*d^8+120*b^2*c^3*
d^7)*x^10+1/9*(45*a^2*c^2*d^8+240*a*b*c^3*d^7+210*b^2*c^4*d^6)*x^9+1/8*(120
*a^2*c^3*d^7+420*a*b*c^4*d^6+252*b^2*c^5*d^5)*x^8+1/7*(210*a^2*c^4*d^6+504*
a*b*c^5*d^5+210*b^2*c^6*d^4)*x^7+1/6*(252*a^2*c^5*d^5+420*a*b*c^6*d^4+120*b
^2*c^7*d^3)*x^6+1/5*(210*a^2*c^6*d^4+240*a*b*c^7*d^3+45*b^2*c^8*d^2)*x^5+1/
4*(120*a^2*c^7*d^3+90*a*b*c^8*d^2+10*b^2*c^9*d)*x^4+1/3*(45*a^2*c^8*d^2+20*
a*b*c^9*d+b^2*c^10)*x^3+1/2*(10*a^2*c^9*d+2*a*b*c^10)*x^2+a^2*c^10*x
```

maxima [B] time = 1.33, size = 384, normalized size = 5.91

$$\frac{1}{13}b^2d^{10}x^{13}+a^2c^{10}x+\frac{1}{6}\left(5b^2cd^9+abd^{10}\right)x^{12}+\frac{1}{11}\left(45b^2c^2d^8+20abcd^9+a^2d^{10}\right)x^{11}+\left(12b^2c^3d^7+9abc^2d^8+a^2cd^9\right)x^{10}+\frac{5}{3}\left(14b^2c^4d^6+16abc^3d^7+3a^2c^2d^8\right)x^9+\frac{3}{2}\left(21b^2c^5d^5+35abc^4d^6+10a^2c^3d^7\right)x^8+6\left(5b^2c^6d^4+12abc^5d^5+5a^2c^4d^6\right)x^7+2\left(10b^2c^7d^3+35abc^6d^4+21a^2c^5d^5\right)x^6+3\left(3b^2c^8d^2+16abc^7d^3+14a^2c^6d^4\right)x^5+\frac{5}{2}\left(b^2c^9d+9abc^8d^2+12a^2c^7d^3\right)x^4+\frac{1}{3}\left(b^2c^{10}+20abc^9d+45a^2c^8d^2\right)x^3+\left(abc^{10}+5a^2c^9d\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^10*x^13 + a^2*c^10*x + 1/6*(5*b^2*c*d^9 + a*b*d^10)*x^12 + 1/11*
(45*b^2*c^2*d^8 + 20*a*b*c*d^9 + a^2*d^10)*x^11 + (12*b^2*c^3*d^7 + 9*a*b*c
^2*d^8 + a^2*c*d^9)*x^10 + 5/3*(14*b^2*c^4*d^6 + 16*a*b*c^3*d^7 + 3*a^2*c^2
*d^8)*x^9 + 3/2*(21*b^2*c^5*d^5 + 35*a*b*c^4*d^6 + 10*a^2*c^3*d^7)*x^8 + 6*
(5*b^2*c^6*d^4 + 12*a*b*c^5*d^5 + 5*a^2*c^4*d^6)*x^7 + 2*(10*b^2*c^7*d^3 +
35*a*b*c^6*d^4 + 21*a^2*c^5*d^5)*x^6 + 3*(3*b^2*c^8*d^2 + 16*a*b*c^7*d^3 +
14*a^2*c^6*d^4)*x^5 + 5/2*(b^2*c^9*d + 9*a*b*c^8*d^2 + 12*a^2*c^7*d^3)*x^4
+ 1/3*(b^2*c^10 + 20*a*b*c^9*d + 45*a^2*c^8*d^2)*x^3 + (a*b*c^10 + 5*a^2*c^
9*d)*x^2
```

mupad [**B**] time = 0.32, size = 348, normalized size = 5.35

$$x^3\left(15a^2c^8d^2+\frac{20abc^9d}{3}+\frac{b^2c^{10}}{3}\right)+x^{11}\left(\frac{a^2d^{10}}{11}+\frac{20abcd^9}{11}+\frac{45b^2c^2d^8}{11}\right)+a^2c^{10}x+\frac{b^2d^{10}x^{13}}{13}+ac^9x^2(5ad+bc)+\frac{bd^9x^{12}(ad+5bc)}{6}+\frac{5c^7dx^4\left(12a^2d^2+9abcd+b^2c^2\right)}{2}+cd^7x^{10}\left(a^2d^2+9abcd+12b^2c^2\right)+6c^4d^4x^7\left(5a^2d^2+12abcd+5b^2c^2\right)+3c^6d^2x^5\left(14a^2d^2+16abcd+3b^2c^2\right)+\frac{5c^2d^6x^9\left(3a^2d^2+16abcd+14b^2c^2\right)}{3}+2c^5d^3x^6\left(21a^2d^2+35abcd+10b^2c^2\right)+\frac{3c^3d^5x^8\left(10a^2d^2+35abcd+21b^2c^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^10,x)
```

```
[Out] x^3*((b^2*c^10)/3 + 15*a^2*c^8*d^2 + (20*a*b*c^9*d)/3) + x^11*((a^2*d^10)/1
1 + (45*b^2*c^2*d^8)/11 + (20*a*b*c*d^9)/11) + a^2*c^10*x + (b^2*d^10*x^13)
/13 + a*c^9*x^2*(5*a*d + b*c) + (b*d^9*x^12*(a*d + 5*b*c))/6 + (5*c^7*d*x^4
*(12*a^2*d^2 + b^2*c^2 + 9*a*b*c*d))/2 + c*d^7*x^10*(a^2*d^2 + 12*b^2*c^2 +
 9*a*b*c*d) + 6*c^4*d^4*x^7*(5*a^2*d^2 + 5*b^2*c^2 + 12*a*b*c*d) + 3*c^6*d^
2*x^5*(14*a^2*d^2 + 3*b^2*c^2 + 16*a*b*c*d) + (5*c^2*d^6*x^9*(3*a^2*d^2 + 1
4*b^2*c^2 + 16*a*b*c*d))/3 + 2*c^5*d^3*x^6*(21*a^2*d^2 + 10*b^2*c^2 + 35*a*
b*c*d) + (3*c^3*d^5*x^8*(10*a^2*d^2 + 21*b^2*c^2 + 35*a*b*c*d))/2
```

sympy [**B**] time = 0.14, size = 415, normalized size = 6.38

$$a^2c^{10}x+\frac{b^2d^{10}x^{13}}{13}+x^{12}\left(\frac{abd^{10}}{6}+\frac{5b^2cd^9}{6}\right)+x^{11}\left(\frac{a^2d^{10}}{11}+\frac{20abcd^9}{11}+\frac{45b^2c^2d^8}{11}\right)+x^{10}\left(a^2cd^9+9abc^2d^8+12b^2c^3d^7\right)+x^9\left(5a^2c^2d^8+\frac{80abc^3d^7}{3}+\frac{70b^2c^4d^6}{3}\right)+x^8\left(15a^2c^3d^7+\frac{105abc^4d^6}{2}+\frac{63b^2c^5d^5}{2}\right)+x^7\left(30a^2c^4d^6+72abc^5d^5+30b^2c^6d^4\right)+x^6\left(42a^2c^5d^5+70abc^6d^4+20b^2c^7d^3\right)+x^5\left(42a^2c^6d^4+48abc^7d^3+9b^2c^8d^2\right)+x^4\left(30a^2c^7d^3+\frac{45abc^8d^2}{2}+\frac{5b^2c^9d}{2}\right)+x^3\left(15a^2c^8d^2+\frac{20abc^9d}{3}+\frac{b^2c^{10}}{3}\right)+x^2\left(5a^2c^9d+abc^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**10,x)
```

```
[Out] a**2*c**10*x + b**2*d**10*x**13/13 + x**12*(a*b*d**10/6 + 5*b**2*c*d**9/6)
+ x**11*(a**2*d**10/11 + 20*a*b*c*d**9/11 + 45*b**2*c**2*d**8/11) + x**10*(
a**2*c*d**9 + 9*a*b*c**2*d**8 + 12*b**2*c**3*d**7) + x**9*(5*a**2*c**2*d**8
 + 80*a*b*c**3*d**7/3 + 70*b**2*c**4*d**6/3) + x**8*(15*a**2*c**3*d**7 + 10
5*a*b*c**4*d**6/2 + 63*b**2*c**5*d**5/2) + x**7*(30*a**2*c**4*d**6 + 72*a*b
*c**5*d**5 + 30*b**2*c**6*d**4) + x**6*(42*a**2*c**5*d**5 + 70*a*b*c**6*d**
4 + 20*b**2*c**7*d**3) + x**5*(42*a**2*c**6*d**4 + 48*a*b*c**7*d**3 + 9*b**
2*c**8*d**2) + x**4*(30*a**2*c**7*d**3 + 45*a*b*c**8*d**2/2 + 5*b**2*c**9*d
/2) + x**3*(15*a**2*c**8*d**2 + 20*a*b*c**9*d/3 + b**2*c**10/3) + x**2*(5*a
**2*c**9*d + a*b*c**10)
```

3.1204 $\int (a+bx)(c+dx)^{10}\,dx$

Optimal. Leaf size=38

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{12}}{12d^2}-\frac{(c+dx)^{11}(bc-ad)}{11d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] -((b*c - a*d)*(c + d*x)^11)/(11*d^2) + (b*(c + d*x)^12)/(12*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{10}\,dx &= \int \left(\frac{(-bc+ad)(c+dx)^{10}}{d}+\frac{b(c+dx)^{11}}{d}\right)dx\\ &= -\frac{(bc-ad)(c+dx)^{11}}{11d^2}+\frac{b(c+dx)^{12}}{12d^2}\end{aligned}$$

Mathematica [B] time = 0.03, size = 220, normalized size = 5.79

$$\frac{1}{2}c^9x^2(10ad+bc)+\frac{5}{3}c^8dx^3(9ad+2bc)+\frac{15}{4}c^7d^2x^4(8ad+3bc)+6c^6d^3x^5(7ad+4bc)+7c^5d^4x^6(6ad+5bc)+6c^4d^5x^7(5ad+6bc)+\frac{15}{4}c^3d^6x^8(4ad+7bc)+\frac{5}{3}c^2d^7x^9(3ad+8bc)+\frac{1}{11}d^9x^{11}(ad+10bc)+\frac{1}{2}cd^8x^{10}(2ad+9bc)+ac^{10}x+\frac{1}{12}bd^{10}x^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] a*c^10*x + (c^9*(b*c + 10*a*d)*x^2)/2 + (5*c^8*d*(2*b*c + 9*a*d)*x^3)/3 + (
15*c^7*d^2*(3*b*c + 8*a*d)*x^4)/4 + 6*c^6*d^3*(4*b*c + 7*a*d)*x^5 + 7*c^5*d
^4*(5*b*c + 6*a*d)*x^6 + 6*c^4*d^5*(6*b*c + 5*a*d)*x^7 + (15*c^3*d^6*(7*b*c
 + 4*a*d)*x^8)/4 + (5*c^2*d^7*(8*b*c + 3*a*d)*x^9)/3 + (c*d^8*(9*b*c + 2*a*
d)*x^10)/2 + (d^9*(10*b*c + a*d)*x^11)/11 + (b*d^10*x^12)/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)^10, x]
```

fricas [B] time = 1.16, size = 241, normalized size = 6.34

$$\frac{1}{12}x^{12}d^{10}b+\frac{10}{11}x^{11}d^9cb+\frac{1}{11}x^{11}d^{10}a+\frac{9}{2}x^{10}d^8c^2b+x^{10}d^9ca+\frac{40}{3}x^9d^7c^3b+5x^9d^8c^2a+\frac{105}{4}x^8d^6c^4b+15x^8d^7c^3a+36x^7d^5c^5b+30x^7d^6c^4a+35x^6d^4c^6b+42x^6d^5c^5a+24x^5d^3c^7b+42x^5d^4c^6a+\frac{45}{4}x^4d^2c^8b+30x^4d^3c^7a+\frac{10}{3}x^3dc^9b+15x^3d^2c^8a+\frac{1}{2}x^2c^{10}b+5x^2dc^9a+xc^{10}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*d^10*b + 10/11*x^11*d^9*c*b + 1/11*x^11*d^10*a + 9/2*x^10*d^8*c^2
*b + x^10*d^9*c*a + 40/3*x^9*d^7*c^3*b + 5*x^9*d^8*c^2*a + 105/4*x^8*d^6*c^
4*b + 15*x^8*d^7*c^3*a + 36*x^7*d^5*c^5*b + 30*x^7*d^6*c^4*a + 35*x^6*d^4*c
^6*b + 42*x^6*d^5*c^5*a + 24*x^5*d^3*c^7*b + 42*x^5*d^4*c^6*a + 45/4*x^4*d^
2*c^8*b + 30*x^4*d^3*c^7*a + 10/3*x^3*d*c^9*b + 15*x^3*d^2*c^8*a + 1/2*x^2*
c^10*b + 5*x^2*d*c^9*a + x*c^10*a
```

giac [B] time = 1.26, size = 241, normalized size = 6.34

$$\frac{1}{12}bd^{10}x^{12} + \frac{10}{11}bcd^{9}x^{11} + \frac{1}{11}ad^{10}x^{11} + \frac{9}{2}bc^{2}d^{8}x^{10} + acd^{9}x^{10} + \frac{40}{3}bc^{3}d^{7}x^{9} + 5\,ac^{2}d^{8}x^{9} + \frac{105}{4}bc^{4}d^{6}x^{8} + 15\,ac^{3}d^{7}x^{8} + 36\,bc^{5}d^{5}x^{7} + 30\,ac^{4}d^{6}x^{7} + 35\,bc^{6}d^{4}x^{6} + 42\,ac^{5}d^{5}x^{6} + 24\,bc^{7}d^{3}x^{5} + 42\,ac^{6}d^{4}x^{5} + \frac{45}{4}bc^{8}d^{2}x^{4} + 30\,ac^{7}d^{3}x^{4} + \frac{10}{3}bc^{9}dx^{3} + 15\,ac^{8}d^{2}x^{3} + \frac{1}{2}bc^{10}x^{2} + 5\,ac^{9}dx^{2} + ac^{10}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/12*b*d^10*x^12 + 10/11*b*c*d^9*x^11 + 1/11*a*d^10*x^11 + 9/2*b*c^2*d^8*x^
10 + a*c*d^9*x^10 + 40/3*b*c^3*d^7*x^9 + 5*a*c^2*d^8*x^9 + 105/4*b*c^4*d^6*
x^8 + 15*a*c^3*d^7*x^8 + 36*b*c^5*d^5*x^7 + 30*a*c^4*d^6*x^7 + 35*b*c^6*d^4
*x^6 + 42*a*c^5*d^5*x^6 + 24*b*c^7*d^3*x^5 + 42*a*c^6*d^4*x^5 + 45/4*b*c^8*
d^2*x^4 + 30*a*c^7*d^3*x^4 + 10/3*b*c^9*d*x^3 + 15*a*c^8*d^2*x^3 + 1/2*b*c^
10*x^2 + 5*a*c^9*d*x^2 + a*c^10*x
```

maple [B] time = 0.00, size = 241, normalized size = 6.34

$$\frac{b\,d^{10}x^{12}}{12} + a\,c^{10}x + \frac{\left(a\,d^{10} + 10 b c\,d^{9}\right)x^{11}}{11} + \frac{\left(10 a c\,d^{9} + 45 b\,c^{2}d^{8}\right)x^{10}}{10} + \frac{\left(45 a\,c^{2}d^{8} + 120 b\,c^{3}d^{7}\right)x^{9}}{9} + \frac{\left(120 a\,c^{3}d^{7} + 210 b\,c^{4}d^{6}\right)x^{8}}{8} + \frac{\left(210 a\,c^{4}d^{6} + 252 b\,c^{5}d^{5}\right)x^{7}}{7} + \frac{\left(252 a\,c^{5}d^{5} + 210 b\,c^{6}d^{4}\right)x^{6}}{6} + \frac{\left(210 a\,c^{6}d^{4} + 120 b\,c^{7}d^{3}\right)x^{5}}{5} + \frac{\left(120 a\,c^{7}d^{3} + 45 b\,c^{8}d^{2}\right)x^{4}}{4} + \frac{\left(45 a\,c^{8}d^{2} + 10 b\,c^{9}d\right)x^{3}}{3} + \frac{\left(10 a\,c^{9}d + b\,c^{10}\right)x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^10,x)
```

```
[Out] 1/12*b*d^10*x^12+1/11*(a*d^10+10*b*c*d^9)*x^11+1/10*(10*a*c*d^9+45*b*c^2*d^
8)*x^10+1/9*(45*a*c^2*d^8+120*b*c^3*d^7)*x^9+1/8*(120*a*c^3*d^7+210*b*c^4*d
^6)*x^8+1/7*(210*a*c^4*d^6+252*b*c^5*d^5)*x^7+1/6*(252*a*c^5*d^5+210*b*c^6*
d^4)*x^6+1/5*(210*a*c^6*d^4+120*b*c^7*d^3)*x^5+1/4*(120*a*c^7*d^3+45*b*c^8*
d^2)*x^4+1/3*(45*a*c^8*d^2+10*b*c^9*d)*x^3+1/2*(10*a*c^9*d+b*c^10)*x^2+a*c^
10*x
```

maxima [B] time = 1.43, size = 240, normalized size = 6.32

$$\frac{1}{12}bd^{10}x^{12} + ac^{10}x + \frac{1}{11}\left(10\,bcd^{9} + ad^{10}\right)x^{11} + \frac{1}{2}\left(9\,bc^{2}d^{8} + 2\,acd^{9}\right)x^{10} + \frac{5}{3}\left(8\,bc^{3}d^{7} + 3\,ac^{2}d^{8}\right)x^{9} + \frac{15}{4}\left(7\,bc^{4}d^{6} + 4\,ac^{3}d^{7}\right)x^{8} + 6\left(6\,bc^{5}d^{5} + 5\,ac^{4}d^{6}\right)x^{7} + 7\left(5\,bc^{6}d^{4} + 6\,ac^{5}d^{5}\right)x^{6} + 6\left(4\,bc^{7}d^{3} + 7\,ac^{6}d^{4}\right)x^{5} + \frac{15}{4}\left(3\,bc^{8}d^{2} + 8\,ac^{7}d^{3}\right)x^{4} + \frac{5}{3}\left(2\,bc^{9}d + 9\,ac^{8}d^{2}\right)x^{3} + \frac{1}{2}\left(bc^{10} + 10\,ac^{9}d\right)x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/12*b*d^10*x^12 + a*c^10*x + 1/11*(10*b*c*d^9 + a*d^10)*x^11 + 1/2*(9*b*c^
2*d^8 + 2*a*c*d^9)*x^10 + 5/3*(8*b*c^3*d^7 + 3*a*c^2*d^8)*x^9 + 15/4*(7*b*c
^4*d^6 + 4*a*c^3*d^7)*x^8 + 6*(6*b*c^5*d^5 + 5*a*c^4*d^6)*x^7 + 7*(5*b*c^6*
d^4 + 6*a*c^5*d^5)*x^6 + 6*(4*b*c^7*d^3 + 7*a*c^6*d^4)*x^5 + 15/4*(3*b*c^8*
d^2 + 8*a*c^7*d^3)*x^4 + 5/3*(2*b*c^9*d + 9*a*c^8*d^2)*x^3 + 1/2*(b*c^10 +
10*a*c^9*d)*x^2
```

mupad [B] time = 0.13, size = 208, normalized size = 5.47

$$x^2\left(\frac{b\,c^{10}}{2}+5\,a\,d\,c^9\right)+x^{11}\left(\frac{a\,d^{10}}{11}+\frac{10\,b\,c\,d^9}{11}\right)+\frac{b\,d^{10}\,x^{12}}{12}+a\,c^{10}\,x+\frac{5\,c^8\,d\,x^3\,\left(9\,a\,d+2\,b\,c\right)}{3}+\frac{c\,d^8\,x^{10}\,\left(2\,a\,d+9\,b\,c\right)}{2}+\frac{15\,c^7\,d^2\,x^4\,\left(8\,a\,d+3\,b\,c\right)}{4}+6\,c^6\,d^3\,x^5\,\left(7\,a\,d+4\,b\,c\right)+7\,c^5\,d^4\,x^6\,\left(6\,a\,d+5\,b\,c\right)+6\,c^4\,d^5\,x^7\,\left(5\,a\,d+6\,b\,c\right)+\frac{15\,c^3\,d^6\,x^8\,\left(4\,a\,d+7\,b\,c\right)}{4}+\frac{5\,c^2\,d^7\,x^9\,\left(3\,a\,d+8\,b\,c\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^10,x)
```

```
[Out] x^2*((b*c^10)/2 + 5*a*c^9*d) + x^11*((a*d^10)/11 + (10*b*c*d^9)/11) + (b*d^
10*x^12)/12 + a*c^10*x + (5*c^8*d*x^3*(9*a*d + 2*b*c))/3 + (c*d^8*x^10*(2*a
*d + 9*b*c))/2 + (15*c^7*d^2*x^4*(8*a*d + 3*b*c))/4 + 6*c^6*d^3*x^5*(7*a*d
+ 4*b*c) + 7*c^5*d^4*x^6*(6*a*d + 5*b*c) + 6*c^4*d^5*x^7*(5*a*d + 6*b*c) +
(15*c^3*d^6*x^8*(4*a*d + 7*b*c))/4 + (5*c^2*d^7*x^9*(3*a*d + 8*b*c))/3
```

sympy [B] time = 0.12, size = 248, normalized size = 6.53

$$ac^{10}x + \frac{bd^{10}x^{12}}{12} + x^{11}\left(\frac{ad^{10}}{11} + \frac{10bcd^9}{11}\right) + x^{10}\left(acd^9 + \frac{9bc^2d^8}{2}\right) + x^9\left(5ac^2d^8 + \frac{40bc^3d^7}{3}\right) + x^8\left(15ac^3d^7 + \frac{105bc^4d^6}{4}\right) + x^7\left(30ac^4d^6 + 36bc^5d^5\right) + x^6\left(42ac^5d^5 + 35bc^6d^4\right) + x^5\left(42ac^6d^4 + 24bc^7d^3\right) + x^4\left(30ac^7d^3 + \frac{45bc^8d^2}{4}\right) + x^3\left(15ac^8d^2 + \frac{10bc^9d}{3}\right) + x^2\left(5ac^9d + \frac{bc^{10}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**10,x)
```

```
[Out] a*c**10*x + b*d**10*x**12/12 + x**11*(a*d**10/11 + 10*b*c*d**9/11) + x**10*
(a*c*d**9 + 9*b*c**2*d**8/2) + x**9*(5*a*c**2*d**8 + 40*b*c**3*d**7/3) + x*
*8*(15*a*c**3*d**7 + 105*b*c**4*d**6/4) + x**7*(30*a*c**4*d**6 + 36*b*c**5*
d**5) + x**6*(42*a*c**5*d**5 + 35*b*c**6*d**4) + x**5*(42*a*c**6*d**4 + 24*
b*c**7*d**3) + x**4*(30*a*c**7*d**3 + 45*b*c**8*d**2/4) + x**3*(15*a*c**8*d
**2 + 10*b*c**9*d/3) + x**2*(5*a*c**9*d + b*c**10/2)
```

3.1205 $\int (c+dx)^{10}\,dx$

Optimal. Leaf size=14

$$\frac{(c+dx)^{11}}{11d}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{10}\,dx = \frac{(c+dx)^{11}}{11d}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{(c+dx)^{11}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10,x]
```

```
[Out] (c + d*x)^11/(11*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{10}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10, x]
```

fricas [B] time = 0.98, size = 108, normalized size = 7.71

$$\frac{1}{11}x^{11}d^{10} + x^{10}d^{9}c + 5x^{9}d^{8}c^{2} + 15x^{8}d^{7}c^{3} + 30x^{7}d^{6}c^{4} + 42x^{6}d^{5}c^{5} + 42x^{5}d^{4}c^{6} + 30x^{4}d^{3}c^{7} + 15x^{3}d^{2}c^{8} + 5x^{2}dc^{9} + xc^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*d^10 + x^10*d^9*c + 5*x^9*d^8*c^2 + 15*x^8*d^7*c^3 + 30*x^7*d^6*c
^4 + 42*x^6*d^5*c^5 + 42*x^5*d^4*c^6 + 30*x^4*d^3*c^7 + 15*x^3*d^2*c^8 + 5*
x^2*d*c^9 + x*c^10
```

giac [A] time = 1.28, size = 12, normalized size = 0.86

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="giac")
```

```
[Out] 1/11*(d*x + c)^11/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{(dx+c)^{11}}{11d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10,x)
```

```
[Out] 1/11*(d*x+c)^11/d
```

maxima [A] time = 1.37, size = 12, normalized size = 0.86

$$\frac{(dx+c)^{11}}{11\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10,x, algorithm="maxima")
```

```
[Out] 1/11*(d*x + c)^11/d
```

mupad [B] time = 0.08, size = 108, normalized size = 7.71

$$c^{10}\,x+5\,c^9\,d\,x^2+15\,c^8\,d^2\,x^3+30\,c^7\,d^3\,x^4+42\,c^6\,d^4\,x^5+42\,c^5\,d^5\,x^6+30\,c^4\,d^6\,x^7+15\,c^3\,d^7\,x^8+5\,c^2\,d^8\,x^9+c\,d^9\,x^{10}+\frac{d^{10}\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10,x)
```

```
[Out] c^10*x + (d^10*x^11)/11 + 5*c^9*d*x^2 + c*d^9*x^10 + 15*c^8*d^2*x^3 + 30*c^
7*d^3*x^4 + 42*c^6*d^4*x^5 + 42*c^5*d^5*x^6 + 30*c^4*d^6*x^7 + 15*c^3*d^7*x
^8 + 5*c^2*d^8*x^9
```

sympy [B] time = 0.09, size = 114, normalized size = 8.14

$$c^{10}x+5c^9dx^2+15c^8d^2x^3+30c^7d^3x^4+42c^6d^4x^5+42c^5d^5x^6+30c^4d^6x^7+15c^3d^7x^8+5c^2d^8x^9+cd^9x^{10}+\frac{d^{10}x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10,x)
```

```
[Out] c**10*x + 5*c**9*d*x**2 + 15*c**8*d**2*x**3 + 30*c**7*d**3*x**4 + 42*c**6*d
**4*x**5 + 42*c**5*d**5*x**6 + 30*c**4*d**6*x**7 + 15*c**3*d**7*x**8 + 5*c*
*2*d**8*x**9 + c*d**9*x**10 + d**10*x**11/11
```

3.1206 $\int \frac{(c+dx)^{10}}{a+bx} dx$

Optimal. Leaf size=241

$$\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx \ldots}{}$$

Rubi [A] time = 0.10, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{dx(bc-ad)^9}{b^{10}}+\frac{(c+dx)^2(bc-ad)^8}{2b^9}+\frac{(c+dx)^3(bc-ad)^7}{3b^8}+\frac{(c+dx)^4(bc-ad)^6}{4b^7}+\frac{(c+dx)^5(bc-ad)^5}{5b^6}+\frac{(c+dx)^6(bc-ad)^4}{6b^5}+\frac{(c+dx)^7(bc-ad)^3}{7b^4}+\frac{(c+dx)^8(bc-ad)^2}{8b^3}+\frac{(c+dx)^9(bc-ad)}{9b^2}+\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}+\frac{(c+dx)^{10}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*(b*c - a*d)^9*x)/b^10 + ((b*c - a*d)^8*(c + d*x)^2)/(2*b^9) + ((b*c - a*
d)^7*(c + d*x)^3)/(3*b^8) + ((b*c - a*d)^6*(c + d*x)^4)/(4*b^7) + ((b*c - a
*d)^5*(c + d*x)^5)/(5*b^6) + ((b*c - a*d)^4*(c + d*x)^6)/(6*b^5) + ((b*c -
a*d)^3*(c + d*x)^7)/(7*b^4) + ((b*c - a*d)^2*(c + d*x)^8)/(8*b^3) + ((b*c -
 a*d)*(c + d*x)^9)/(9*b^2) + (c + d*x)^10/(10*b) + ((b*c - a*d)^10*Log[a +
b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{a+bx} dx = \int \left(\frac{d(bc-ad)^9}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)}+\frac{d(bc-ad)^8(c+dx)}{b^9}+\frac{d(bc-ad)^7(c+dx)^2}{b^8}+\frac{d(bc-ad)^6(\ldots}{b^7} \ldots\right.$$
$$= \frac{d(bc-ad)^9x}{b^{10}}+\frac{(bc-ad)^8(c+dx)^2}{2b^9}+\frac{(bc-ad)^7(c+dx)^3}{3b^8}+\frac{(bc-ad)^6(c+dx)^4}{4b^7}+\frac{(bc-ad \ldots}{5 \ldots}$$

Mathematica [B] time = 0.34, size = 591, normalized size = 2.45

$$\frac{dx\left(-2520a^9d^9+1260a^8bd^8(20c+dx)-840a^7b^2d^7\left(135c^2+15cdx+d^2x^2\right)+210a^6b^3d^6\left(1440c^3+270c^2dx+40cd^2x^2+3d^3x^3\right)-252a^5b^4d^5\left(2100c^4+600c^3dx+150c^2d^2x^2+25cd^3x^3+2d^4x^4\right)+210a^4b^5d^4\left(3024c^5+1260c^4dx+480c^3d^2x^2+135c^2d^3x^3+24cd^4x^4+2d^5x^5\right)-120a^3b^6d^3\left(4410c^6+2646c^5dx+1470c^4d^2x^2+630c^3d^3x^3+189c^2d^4x^4+35cd^5x^5+3d^6x^6\right)+45a^2b^7d^2\left(6720c^7+5880c^6dx+4704c^5d^2x^2+2940c^4d^3x^3+1344c^3d^4x^4+420c^2d^5x^5+80cd^6x^6+7d^7x^7\right)-10ab^8d\left(11340c^8+15120c^7dx+17640c^6d^2x^2+15876c^5d^3x^3+10584c^4d^4x^4+5040c^3d^5x^5+1620c^2d^6x^6+315cd^7x^7+28d^8x^8\right)+b^9\left(25200c^9+56700c^8dx+100800c^7d^2x^2+132300c^6d^3x^3+127008c^5d^4x^4+88200c^4d^5x^5+43200c^3d^6x^6+14175c^2d^7x^7+2800cd^8x^8+252d^9x^9\right)\right)}{2520b^{10}}+\frac{(bc-ad)^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x),x]
```

```
[Out] (d*x*(-2520*a^9*d^9 + 1260*a^8*b*d^8*(20*c + d*x) - 840*a^7*b^2*d^7*(135*c^
2 + 15*c*d*x + d^2*x^2) + 210*a^6*b^3*d^6*(1440*c^3 + 270*c^2*d*x + 40*c*d^
2*x^2 + 3*d^3*x^3) - 252*a^5*b^4*d^5*(2100*c^4 + 600*c^3*d*x + 150*c^2*d^2*
x^2 + 25*c*d^3*x^3 + 2*d^4*x^4) + 210*a^4*b^5*d^4*(3024*c^5 + 1260*c^4*d*x
+ 480*c^3*d^2*x^2 + 135*c^2*d^3*x^3 + 24*c*d^4*x^4 + 2*d^5*x^5) - 120*a^3*b
^6*d^3*(4410*c^6 + 2646*c^5*d*x + 1470*c^4*d^2*x^2 + 630*c^3*d^3*x^3 + 189*
c^2*d^4*x^4 + 35*c*d^5*x^5 + 3*d^6*x^6) + 45*a^2*b^7*d^2*(6720*c^7 + 5880*c
^6*d*x + 4704*c^5*d^2*x^2 + 2940*c^4*d^3*x^3 + 1344*c^3*d^4*x^4 + 420*c^2*d
^5*x^5 + 80*c*d^6*x^6 + 7*d^7*x^7) - 10*a*b^8*d*(11340*c^8 + 15120*c^7*d*x
+ 17640*c^6*d^2*x^2 + 15876*c^5*d^3*x^3 + 10584*c^4*d^4*x^4 + 5040*c^3*d^5*
x^5 + 1620*c^2*d^6*x^6 + 315*c*d^7*x^7 + 28*d^8*x^8) + b^9*(25200*c^9 + 567
```

```
00*c^8*d*x + 100800*c^7*d^2*x^2 + 132300*c^6*d^3*x^3 + 127008*c^5*d^4*x^4 +
 88200*c^4*d^5*x^5 + 43200*c^3*d^6*x^6 + 14175*c^2*d^7*x^7 + 2800*c*d^8*x^8
 + 252*d^9*x^9)))/(2520*b^10) + ((b*c - a*d)^10*Log[a + b*x])/b^11
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{a+bx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x), x]
```

fricas [B] time = 1.28, size = 868, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2520*(252*b^10*d^10*x^10 + 280*(10*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(45
*b^10*c^2*d^8 - 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(120*b^10*c^3*d^7
- 45*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(210*b^10*c
^4*d^6 - 120*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 - 10*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 504*(252*b^10*c^5*d^5 - 210*a*b^9*c^4*d^6 + 120*a^2*b^8*c^3*d
^7 - 45*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(210*b
^10*c^6*d^4 - 252*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 - 120*a^3*b^7*c^3*d^7
 + 45*a^4*b^6*c^2*d^8 - 10*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(120*b^1
0*c^7*d^3 - 210*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 - 210*a^3*b^7*c^4*d^6 +
 120*a^4*b^6*c^3*d^7 - 45*a^5*b^5*c^2*d^8 + 10*a^6*b^4*c*d^9 - a^7*b^3*d^10
)*x^3 + 1260*(45*b^10*c^8*d^2 - 120*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 - 2
52*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 120*a^5*b^5*c^3*d^7 + 45*a^6*b^4
*c^2*d^8 - 10*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2520*(10*b^10*c^9*d - 45*
a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d^3 - 210*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5
*d^5 - 210*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 - 45*a^7*b^3*c^2*d^8 + 10*
a^8*b^2*c*d^9 - a^9*b*d^10)*x + 2520*(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b
^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^
5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9
*b*c*d^9 + a^10*d^10)*log(b*x + a))/b^11
```

giac [B] time = 1.35, size = 961, normalized size = 3.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 2800*b^9*c*d^9*x^9 - 280*a*b^8*d^10*x^9 + 14175
*b^9*c^2*d^8*x^8 - 3150*a*b^8*c*d^9*x^8 + 315*a^2*b^7*d^10*x^8 + 43200*b^9*
c^3*d^7*x^7 - 16200*a*b^8*c^2*d^8*x^7 + 3600*a^2*b^7*c*d^9*x^7 - 360*a^3*b^
6*d^10*x^7 + 88200*b^9*c^4*d^6*x^6 - 50400*a*b^8*c^3*d^7*x^6 + 18900*a^2*b^
7*c^2*d^8*x^6 - 4200*a^3*b^6*c*d^9*x^6 + 420*a^4*b^5*d^10*x^6 + 127008*b^9*
c^5*d^5*x^5 - 105840*a*b^8*c^4*d^6*x^5 + 60480*a^2*b^7*c^3*d^7*x^5 - 22680*
a^3*b^6*c^2*d^8*x^5 + 5040*a^4*b^5*c*d^9*x^5 - 504*a^5*b^4*d^10*x^5 + 13230
0*b^9*c^6*d^4*x^4 - 158760*a*b^8*c^5*d^5*x^4 + 132300*a^2*b^7*c^4*d^6*x^4 -
 75600*a^3*b^6*c^3*d^7*x^4 + 28350*a^4*b^5*c^2*d^8*x^4 - 6300*a^5*b^4*c*d^9
*x^4 + 630*a^6*b^3*d^10*x^4 + 100800*b^9*c^7*d^3*x^3 - 176400*a*b^8*c^6*d^4
*x^3 + 211680*a^2*b^7*c^5*d^5*x^3 - 176400*a^3*b^6*c^4*d^6*x^3 + 100800*a^4
```

```
*b^5*c^3*d^7*x^3 - 37800*a^5*b^4*c^2*d^8*x^3 + 8400*a^6*b^3*c*d^9*x^3 - 840
*a^7*b^2*d^10*x^3 + 56700*b^9*c^8*d^2*x^2 - 151200*a*b^8*c^7*d^3*x^2 + 2646
00*a^2*b^7*c^6*d^4*x^2 - 317520*a^3*b^6*c^5*d^5*x^2 + 264600*a^4*b^5*c^4*d^
6*x^2 - 151200*a^5*b^4*c^3*d^7*x^2 + 56700*a^6*b^3*c^2*d^8*x^2 - 12600*a^7*
b^2*c*d^9*x^2 + 1260*a^8*b*d^10*x^2 + 25200*b^9*c^9*d*x - 113400*a*b^8*c^8*
d^2*x + 302400*a^2*b^7*c^7*d^3*x - 529200*a^3*b^6*c^6*d^4*x + 635040*a^4*b^
5*c^5*d^5*x - 529200*a^5*b^4*c^4*d^6*x + 302400*a^6*b^3*c^3*d^7*x - 113400*
a^7*b^2*c^2*d^8*x + 25200*a^8*b*c*d^9*x - 2520*a^9*d^10*x)/b^10 + (b^10*c^1
0 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6
*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7
+ 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*log(abs(b*x + a))/b^11
```

maple [B] time = 0.01, size = 1022, normalized size = 4.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a),x)
```

```
[Out] 105/2*d^4/b*x^4*c^6-1/3*d^10/b^8*x^3*a^7+40*d^3/b*x^3*c^7+1/2*d^10/b^9*x^2*
a^8+45/2*d^2/b*x^2*c^8+1/8*d^10/b^3*x^8*a^2+45/8*d^8/b*x^8*c^2-1/7*d^10/b^4
*x^7*a^3+120/7*d^7/b*x^7*c^3+1/6*d^10/b^5*x^6*a^4+35*d^6/b*x^6*c^4-1/5*d^10
/b^6*x^5*a^5+252/5*d^5/b*x^5*c^5+1/4*d^10/b^7*x^4*a^6-1/9*d^10/b^2*x^9*a+10
/9*d^9/b*x^9*c+1/b^11*ln(b*x+a)*a^10*d^10+10*d/b*c^9*x-d^10/b^10*a^9*x-42*d
^6/b^2*x^5*a*c^4-5/2*d^9/b^6*x^4*a^5*c+10/3*d^9/b^7*x^3*a^6*c-15*d^8/b^6*x^
3*a^5*c^2+24*d^7/b^3*x^5*a^2*c^3+40*d^7/b^5*x^3*a^4*c^3-30*d^7/b^4*x^4*a^3*
c^3+105/2*d^6/b^3*x^4*a^2*c^4-63*d^5/b^2*x^4*a*c^5-5/4*d^9/b^2*x^8*a*c+2*d^
9/b^5*x^5*a^4*c-9*d^8/b^4*x^5*a^3*c^2-5/3*d^9/b^4*x^6*a^3*c+15/2*d^8/b^3*x^
6*a^2*c^2-20*d^7/b^2*x^6*a*c^3+10/7*d^9/b^3*x^7*a^2*c-45/7*d^8/b^2*x^7*a*c^
2-120/b^8*ln(b*x+a)*a^7*c^3*d^7+210/b^7*ln(b*x+a)*a^6*c^4*d^6-252/b^6*ln(b*
x+a)*a^5*c^5*d^5+210/b^5*ln(b*x+a)*a^4*c^6*d^4-120/b^4*ln(b*x+a)*a^3*c^7*d^
3+45/b^3*ln(b*x+a)*a^2*c^8*d^2-10/b^2*ln(b*x+a)*a*c^9*d-10/b^10*ln(b*x+a)*a
^9*c*d^9+45/b^9*ln(b*x+a)*a^8*c^2*d^8+1/10*d^10/b*x^10+1/b*ln(b*x+a)*c^10+4
5/4*d^8/b^5*x^4*a^4*c^2+120*d^3/b^3*a^2*c^7*x-45*d^2/b^2*a*c^8*x-70*d^4/b^2
*x^3*a*c^6-5*d^9/b^8*x^2*a^7*c+45/2*d^8/b^7*x^2*a^6*c^2-60*d^7/b^6*x^2*a^5*
c^3+105*d^6/b^5*x^2*a^4*c^4-126*d^5/b^4*x^2*a^3*c^5+105*d^4/b^3*x^2*a^2*c^6
-60*d^3/b^2*x^2*a*c^7+10*d^9/b^9*a^8*c*x-45*d^8/b^8*a^7*c^2*x+120*d^7/b^7*a
^6*c^3*x-210*d^6/b^6*a^5*c^4*x-70*d^6/b^4*x^3*a^3*c^4+84*d^5/b^3*x^3*a^2*c^
5+252*d^5/b^5*a^4*c^5*x-210*d^4/b^4*a^3*c^6*x
```

maxima [B] time = 1.52, size = 866, normalized size = 3.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2520*(252*b^9*d^10*x^10 + 280*(10*b^9*c*d^9 - a*b^8*d^10)*x^9 + 315*(45*b
^9*c^2*d^8 - 10*a*b^8*c*d^9 + a^2*b^7*d^10)*x^8 + 360*(120*b^9*c^3*d^7 - 45
*a*b^8*c^2*d^8 + 10*a^2*b^7*c*d^9 - a^3*b^6*d^10)*x^7 + 420*(210*b^9*c^4*d^
6 - 120*a*b^8*c^3*d^7 + 45*a^2*b^7*c^2*d^8 - 10*a^3*b^6*c*d^9 + a^4*b^5*d^1
0)*x^6 + 504*(252*b^9*c^5*d^5 - 210*a*b^8*c^4*d^6 + 120*a^2*b^7*c^3*d^7 - 4
5*a^3*b^6*c^2*d^8 + 10*a^4*b^5*c*d^9 - a^5*b^4*d^10)*x^5 + 630*(210*b^9*c^6
*d^4 - 252*a*b^8*c^5*d^5 + 210*a^2*b^7*c^4*d^6 - 120*a^3*b^6*c^3*d^7 + 45*a
^4*b^5*c^2*d^8 - 10*a^5*b^4*c*d^9 + a^6*b^3*d^10)*x^4 + 840*(120*b^9*c^7*d^
3 - 210*a*b^8*c^6*d^4 + 252*a^2*b^7*c^5*d^5 - 210*a^3*b^6*c^4*d^6 + 120*a^4
*b^5*c^3*d^7 - 45*a^5*b^4*c^2*d^8 + 10*a^6*b^3*c*d^9 - a^7*b^2*d^10)*x^3 +
1260*(45*b^9*c^8*d^2 - 120*a*b^8*c^7*d^3 + 210*a^2*b^7*c^6*d^4 - 252*a^3*b^
6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 - 120*a^5*b^4*c^3*d^7 + 45*a^6*b^3*c^2*d^8
```

```
- 10*a^7*b^2*c*d^9 + a^8*b*d^10)*x^2 + 2520*(10*b^9*c^9*d - 45*a*b^8*c^8*d^
2 + 120*a^2*b^7*c^7*d^3 - 210*a^3*b^6*c^6*d^4 + 252*a^4*b^5*c^5*d^5 - 210*a
^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - 45*a^7*b^2*c^2*d^8 + 10*a^8*b*c*d^9
- a^9*d^10)*x)/b^10 + (b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 12
0*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4
*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10
*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.13, size = 979, normalized size = 4.06

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x),x)
```

```
[Out] x^7*((120*c^3*d^7)/(7*b) - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^
2*d^8)/b))/(7*b)) - x^9*((a*d^10)/(9*b^2) - (10*c*d^9)/(9*b)) + x^5*((a*((a
*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/
```

```
b))/b))/b - (210*c^4*d^6)/b))/(5*b) + (252*c^5*d^5)/(5*b)) + x^3*((a*((a*((
a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*
d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/
b))/(3*b) + (40*c^7*d^3)/b) + x*((a*((a*((a*((a*((a*((a*((120*c^3*d^7)/b -
(a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4
*d^6)/b))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))
/b - (45*c^8*d^2)/b))/b + (10*c^9*d)/b) + x^8*((a*((a*d^10)/b^2 - (10*c*d^9
)/b))/(8*b) + (45*c^2*d^8)/(8*b)) - x^6*((a*((120*c^3*d^7)/b - (a*((a*((a*d
^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/(6*b) - (35*c^4*d^6)/b)
- x^4*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*((a*d^10)/b^2 - (10*c*d^9)/b))/
b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b))/b + (252*c^5*d^5)/b))/(4*b)
- (105*c^6*d^4)/(2*b)) - x^2*((a*((a*((a*((a*((120*c^3*d^7)/b - (a*((a*
((a*d^10)/b^2 - (10*c*d^9)/b))/b + (45*c^2*d^8)/b))/b))/b - (210*c^4*d^6)/b
))/b + (252*c^5*d^5)/b))/b - (210*c^6*d^4)/b))/b + (120*c^7*d^3)/b))/(2*b)
- (45*c^8*d^2)/(2*b)) + (d^10*x^10)/(10*b) + (log(a + b*x)*(a^10*d^10 + b^1
0*c^10 + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 2
52*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2
*c^2*d^8 - 10*a*b^9*c^9*d - 10*a^9*b*c*d^9))/b^11
```

sympy [B] time = 1.42, size = 799, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a),x)
```

```
[Out] x**9*(-a*d**10/(9*b**2) + 10*c*d**9/(9*b)) + x**8*(a**2*d**10/(8*b**3) - 5*
a*c*d**9/(4*b**2) + 45*c**2*d**8/(8*b)) + x**7*(-a**3*d**10/(7*b**4) + 10*a
**2*c*d**9/(7*b**3) - 45*a*c**2*d**8/(7*b**2) + 120*c**3*d**7/(7*b)) + x**6
*(a**4*d**10/(6*b**5) - 5*a**3*c*d**9/(3*b**4) + 15*a**2*c**2*d**8/(2*b**3)
 - 20*a*c**3*d**7/b**2 + 35*c**4*d**6/b) + x**5*(-a**5*d**10/(5*b**6) + 2*a
**4*c*d**9/b**5 - 9*a**3*c**2*d**8/b**4 + 24*a**2*c**3*d**7/b**3 - 42*a*c**
4*d**6/b**2 + 252*c**5*d**5/(5*b)) + x**4*(a**6*d**10/(4*b**7) - 5*a**5*c*d
**9/(2*b**6) + 45*a**4*c**2*d**8/(4*b**5) - 30*a**3*c**3*d**7/b**4 + 105*a*
*2*c**4*d**6/(2*b**3) - 63*a*c**5*d**5/b**2 + 105*c**6*d**4/(2*b)) + x**3*(
-a**7*d**10/(3*b**8) + 10*a**6*c*d**9/(3*b**7) - 15*a**5*c**2*d**8/b**6 + 4
0*a**4*c**3*d**7/b**5 - 70*a**3*c**4*d**6/b**4 + 84*a**2*c**5*d**5/b**3 - 7
0*a*c**6*d**4/b**2 + 40*c**7*d**3/b) + x**2*(a**8*d**10/(2*b**9) - 5*a**7*c
*d**9/b**8 + 45*a**6*c**2*d**8/(2*b**7) - 60*a**5*c**3*d**7/b**6 + 105*a**4
*c**4*d**6/b**5 - 126*a**3*c**5*d**5/b**4 + 105*a**2*c**6*d**4/b**3 - 60*a*
c**7*d**3/b**2 + 45*c**8*d**2/(2*b)) + x*(-a**9*d**10/b**10 + 10*a**8*c*d**
9/b**9 - 45*a**7*c**2*d**8/b**8 + 120*a**6*c**3*d**7/b**7 - 210*a**5*c**4*d
**6/b**6 + 252*a**4*c**5*d**5/b**5 - 210*a**3*c**6*d**4/b**4 + 120*a**2*c**
7*d**3/b**3 - 45*a*c**8*d**2/b**2 + 10*c**9*d/b) + d**10*x**10/(10*b) + (a*
d - b*c)**10*log(a + b*x)/b**11
```

3.1207 $\int \frac{(c+dx)^{10}}{(a+bx)^2} \, dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx}{b}$$

Rubi [A] time = 0.47, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{43\}$

$$\frac{5d^9(a+bx)^8(bc-ad)}{4b^{11}}+\frac{45d^8(a+bx)^7(bc-ad)^2}{7b^{11}}+\frac{20d^7(a+bx)^6(bc-ad)^3}{b^{11}}+\frac{42d^6(a+bx)^5(bc-ad)^4}{b^{11}}+\frac{63d^5(a+bx)^4(bc-ad)^5}{b^{11}}+\frac{70d^4(a+bx)^3(bc-ad)^6}{b^{11}}+\frac{60d^3(a+bx)^2(bc-ad)^7}{b^{11}}+\frac{45d^2x(bc-ad)^8}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{10d(bc-ad)^9\log(a+bx)}{b^{11}}+\frac{d^{10}(a+bx)^9}{9b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (45*d^2*(b*c - a*d)^8*x)/b^10 - (b*c - a*d)^10/(b^11*(a + b*x)) + (60*d^3*(
b*c - a*d)^7*(a + b*x)^2)/b^11 + (70*d^4*(b*c - a*d)^6*(a + b*x)^3)/b^11 +
(63*d^5*(b*c - a*d)^5*(a + b*x)^4)/b^11 + (42*d^6*(b*c - a*d)^4*(a + b*x)^5
)/b^11 + (20*d^7*(b*c - a*d)^3*(a + b*x)^6)/b^11 + (45*d^8*(b*c - a*d)^2*(a
 + b*x)^7)/(7*b^11) + (5*d^9*(b*c - a*d)*(a + b*x)^8)/(4*b^11) + (d^10*(a +
 b*x)^9)/(9*b^11) + (10*d*(b*c - a*d)^9*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx &= \int \left(\frac{45d^2(bc-ad)^8}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^2}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)}+\frac{120d^3(bc-ad)^7(a+bx)}{b^{10}}+\frac{210d^4(bc-a}{b}\right.\\ &= \frac{45d^2(bc-ad)^8x}{b^{10}}-\frac{(bc-ad)^{10}}{b^{11}(a+bx)}+\frac{60d^3(bc-ad)^7(a+bx)^2}{b^{11}}+\frac{70d^4(bc-ad)^6(a+bx)^3}{b^{11}}+\frac{63d^5(}{}\end{aligned}$$

Mathematica [B] time = 0.24, size = 708, normalized size = 2.74

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] (-252*a^10*d^10 + 252*a^9*b*d^9*(10*c + 9*d*x) + 1260*a^8*b^2*d^8*(-9*c^2 -
 16*c*d*x + d^2*x^2) - 420*a^7*b^3*d^7*(-72*c^3 - 189*c^2*d*x + 27*c*d^2*x^
2 + d^3*x^3) + 210*a^6*b^4*d^6*(-252*c^4 - 864*c^3*d*x + 216*c^2*d^2*x^2 +
18*c*d^3*x^3 + d^4*x^4) - 126*a^5*b^5*d^5*(-504*c^5 - 2100*c^4*d*x + 840*c^
3*d^2*x^2 + 120*c^2*d^3*x^3 + 15*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(-12
60*c^6 - 6048*c^5*d*x + 3780*c^4*d^2*x^2 + 840*c^3*d^3*x^3 + 180*c^2*d^4*x^
4 + 27*c*d^5*x^5 + 2*d^6*x^6) - 12*a^3*b^7*d^3*(-2520*c^7 - 13230*c^6*d*x +
 13230*c^5*d^2*x^2 + 4410*c^4*d^3*x^3 + 1470*c^3*d^4*x^4 + 378*c^2*d^5*x^5
+ 63*c*d^6*x^6 + 5*d^7*x^7) + 9*a^2*b^8*d^2*(-1260*c^8 - 6720*c^7*d*x + 117
60*c^6*d^2*x^2 + 5880*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 1176*c^3*d^5*x^5 + 3
36*c^2*d^6*x^6 + 60*c*d^7*x^7 + 5*d^8*x^8) - a*b^9*d*(-2520*c^9 - 11340*c^8
```

```
*d*x + 45360*c^7*d^2*x^2 + 35280*c^6*d^3*x^3 + 26460*c^5*d^4*x^4 + 15876*c^
4*d^5*x^5 + 7056*c^3*d^6*x^6 + 2160*c^2*d^7*x^7 + 405*c*d^8*x^8 + 35*d^9*x^
9) + b^10*(-252*c^10 + 11340*c^8*d^2*x^2 + 15120*c^7*d^3*x^3 + 17640*c^6*d^
4*x^4 + 15876*c^5*d^5*x^5 + 10584*c^4*d^6*x^6 + 5040*c^3*d^7*x^7 + 1620*c^2
*d^8*x^8 + 315*c*d^9*x^9 + 28*d^10*x^10) - 2520*d*(-(b*c) + a*d)^9*(a + b*x
)*Log[a + b*x])/(252*b^11*(a + b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^2, x]
```

fricas [B] time = 1.31, size = 1124, normalized size = 4.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/252*(28*b^10*d^10*x^10 - 252*b^10*c^10 + 2520*a*b^9*c^9*d - 11340*a^2*b^8
*c^8*d^2 + 30240*a^3*b^7*c^7*d^3 - 52920*a^4*b^6*c^6*d^4 + 63504*a^5*b^5*c^
5*d^5 - 52920*a^6*b^4*c^4*d^6 + 30240*a^7*b^3*c^3*d^7 - 11340*a^8*b^2*c^2*d
^8 + 2520*a^9*b*c*d^9 - 252*a^10*d^10 + 35*(9*b^10*c*d^9 - a*b^9*d^10)*x^9
+ 45*(36*b^10*c^2*d^8 - 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(84*b^10*c^3
*d^7 - 36*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(126*b^1
0*c^4*d^6 - 84*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 - 9*a^3*b^7*c*d^9 + a^4*b
^6*d^10)*x^6 + 126*(126*b^10*c^5*d^5 - 126*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 - 36*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(84*b^1
0*c^6*d^4 - 126*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 - 84*a^3*b^7*c^3*d^7 +
36*a^4*b^6*c^2*d^8 - 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 420*(36*b^10*c^7
*d^3 - 84*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 - 126*a^3*b^7*c^4*d^6 + 84*a^
4*b^6*c^3*d^7 - 36*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 +
1260*(9*b^10*c^8*d^2 - 36*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 - 126*a^3*b^7*
c^5*d^5 + 126*a^4*b^6*c^4*d^6 - 84*a^5*b^5*c^3*d^7 + 36*a^6*b^4*c^2*d^8 - 9
*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 252*(45*a*b^9*c^8*d^2 - 240*a^2*b^8*c^
7*d^3 + 630*a^3*b^7*c^6*d^4 - 1008*a^4*b^6*c^5*d^5 + 1050*a^5*b^5*c^4*d^6 -
 720*a^6*b^4*c^3*d^7 + 315*a^7*b^3*c^2*d^8 - 80*a^8*b^2*c*d^9 + 9*a^9*b*d^1
0)*x + 2520*(a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 - 84*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 - 126*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^
7 - 36*a^8*b^2*c^2*d^8 + 9*a^9*b*c*d^9 - a^10*d^10 + (b^10*c^9*d - 9*a*b^9*
c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 1
26*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*
d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^12*x + a*b^11)
```

giac [B] time = 1.27, size = 1012, normalized size = 3.92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/252*(28*d^10 + 315*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 1620*(b^4*c^2*d
^8 - 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 5040*(b^6*c^3*d^7 -
3*a*b^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 10584
```

```
*(b^8*c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4
*b^4*d^10)/((b*x + a)^4*b^4) + 15876*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a
^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x
 + a)^5*b^5) + 17640*(b^12*c^6*d^4 - 6*a*b^11*c^5*d^5 + 15*a^2*b^10*c^4*d^6
 - 20*a^3*b^9*c^3*d^7 + 15*a^4*b^8*c^2*d^8 - 6*a^5*b^7*c*d^9 + a^6*b^6*d^10
)/((b*x + a)^6*b^6) + 15120*(b^14*c^7*d^3 - 7*a*b^13*c^6*d^4 + 21*a^2*b^12*
c^5*d^5 - 35*a^3*b^11*c^4*d^6 + 35*a^4*b^10*c^3*d^7 - 21*a^5*b^9*c^2*d^8 +
7*a^6*b^8*c*d^9 - a^7*b^7*d^10)/((b*x + a)^7*b^7) + 11340*(b^16*c^8*d^2 - 8
*a*b^15*c^7*d^3 + 28*a^2*b^14*c^6*d^4 - 56*a^3*b^13*c^5*d^5 + 70*a^4*b^12*c
^4*d^6 - 56*a^5*b^11*c^3*d^7 + 28*a^6*b^10*c^2*d^8 - 8*a^7*b^9*c*d^9 + a^8*
b^8*d^10)/((b*x + a)^8*b^8))*(b*x + a)^9/b^11 - 10*(b^9*c^9*d - 9*a*b^8*c^8
*d^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*
a^5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 -
 a^9*d^10)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - (b^19*c^10/(b*x +
a) - 10*a*b^18*c^9*d/(b*x + a) + 45*a^2*b^17*c^8*d^2/(b*x + a) - 120*a^3*b^
16*c^7*d^3/(b*x + a) + 210*a^4*b^15*c^6*d^4/(b*x + a) - 252*a^5*b^14*c^5*d^
5/(b*x + a) + 210*a^6*b^13*c^4*d^6/(b*x + a) - 120*a^7*b^12*c^3*d^7/(b*x +
a) + 45*a^8*b^11*c^2*d^8/(b*x + a) - 10*a^9*b^10*c*d^9/(b*x + a) + a^10*b^9
*d^10/(b*x + a))/b^20
```

maple [B] time = 0.02, size = 1066, normalized size = 4.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^2,x)
```

```
[Out] 45*d^2/b^2*c^8*x+9*d^10/b^10*a^8*x-10/b^11*d^10*ln(b*x+a)*a^9+10/b^2*d*ln(b
*x+a)*c^9-1/b^11/(b*x+a)*a^10*d^10-4*d^10/b^9*x^2*a^7+60*d^3/b^2*x^2*c^7+3/
7*d^10/b^4*x^7*a^2+45/7*d^8/b^2*x^7*c^2-2/3*d^10/b^5*x^6*a^3+20*d^7/b^2*x^6
*c^3+42*d^6/b^2*x^5*c^4-3/2*d^10/b^7*x^4*a^5+63*d^5/b^2*x^4*c^5+7/3*d^10/b^
8*x^3*a^6+70*d^4/b^2*x^3*c^6-1/4*d^10/b^3*x^8*a+5/4*d^9/b^2*x^8*c+d^10/b^6*
x^5*a^4+1/9*d^10/b^2*x^9-1/b/(b*x+a)*c^10+10/b^2/(b*x+a)*a*c^9*d-1008*d^5/b
^5*a^3*c^5*x+630*d^4/b^4*a^2*c^6*x-240*d^3/b^3*a*c^7*x+5*d^9/b^4*x^6*a^2*c-
15*d^8/b^3*x^6*a*c^2-8*d^9/b^5*x^5*a^3*c+27*d^8/b^4*x^5*a^2*c^2-48*d^7/b^3*
x^5*a*c^3+25/2*d^9/b^6*x^4*a^4*c+300*d^7/b^6*x^2*a^4*c^3-420*d^6/b^5*x^2*a^
3*c^4+378*d^5/b^4*x^2*a^2*c^5-210*d^4/b^3*x^2*a*c^6-80*d^9/b^9*a^7*c*x+315*
d^8/b^8*a^6*c^2*x-720*d^7/b^7*a^5*c^3*x+1050*d^6/b^6*a^4*c^4*x+35*d^9/b^8*x
^2*a^6*c-135*d^8/b^7*x^2*a^5*c^2-90/b^3*d^2*ln(b*x+a)*a*c^8+10/b^10/(b*x+a)
*a^9*c*d^9-45/b^9/(b*x+a)*a^8*c^2*d^8+120/b^8/(b*x+a)*a^7*c^3*d^7-210/b^7/(
b*x+a)*a^6*c^4*d^6+252/b^6/(b*x+a)*a^5*c^5*d^5-210/b^5/(b*x+a)*a^4*c^6*d^4+
120/b^4/(b*x+a)*a^3*c^7*d^3-45/b^3/(b*x+a)*a^2*c^8*d^2-840/b^5*d^4*ln(b*x+a
)*a^3*c^6+360/b^4*d^3*ln(b*x+a)*a^2*c^7-45*d^8/b^5*x^4*a^3*c^2+90*d^7/b^4*x
^4*a^2*c^3-105*d^6/b^3*x^4*a*c^4-20*d^9/b^7*x^3*a^5*c+75*d^8/b^6*x^3*a^4*c^
2-160*d^7/b^5*x^3*a^3*c^3+210*d^6/b^4*x^3*a^2*c^4-168*d^5/b^3*x^3*a*c^5-20/
7*d^9/b^3*x^7*a*c+90/b^10*d^9*ln(b*x+a)*a^8*c-360/b^9*d^8*ln(b*x+a)*a^7*c^2
+840/b^8*d^7*ln(b*x+a)*a^6*c^3-1260/b^7*d^6*ln(b*x+a)*a^5*c^4+1260/b^6*d^5*
ln(b*x+a)*a^4*c^5
```

maxima [B] time = 1.39, size = 874, normalized size = 3.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 2
10*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^
3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)/(b^12*x + a*b^
```

```
11) + 1/252*(28*b^8*d^10*x^9 + 63*(5*b^8*c*d^9 - a*b^7*d^10)*x^8 + 36*(45*b
^8*c^2*d^8 - 20*a*b^7*c*d^9 + 3*a^2*b^6*d^10)*x^7 + 84*(60*b^8*c^3*d^7 - 45
*a*b^7*c^2*d^8 + 15*a^2*b^6*c*d^9 - 2*a^3*b^5*d^10)*x^6 + 252*(42*b^8*c^4*d
^6 - 48*a*b^7*c^3*d^7 + 27*a^2*b^6*c^2*d^8 - 8*a^3*b^5*c*d^9 + a^4*b^4*d^10
)*x^5 + 126*(126*b^8*c^5*d^5 - 210*a*b^7*c^4*d^6 + 180*a^2*b^6*c^3*d^7 - 90
*a^3*b^5*c^2*d^8 + 25*a^4*b^4*c*d^9 - 3*a^5*b^3*d^10)*x^4 + 84*(210*b^8*c^6
*d^4 - 504*a*b^7*c^5*d^5 + 630*a^2*b^6*c^4*d^6 - 480*a^3*b^5*c^3*d^7 + 225*
a^4*b^4*c^2*d^8 - 60*a^5*b^3*c*d^9 + 7*a^6*b^2*d^10)*x^3 + 252*(60*b^8*c^7*
d^3 - 210*a*b^7*c^6*d^4 + 378*a^2*b^6*c^5*d^5 - 420*a^3*b^5*c^4*d^6 + 300*a
^4*b^4*c^3*d^7 - 135*a^5*b^3*c^2*d^8 + 35*a^6*b^2*c*d^9 - 4*a^7*b*d^10)*x^2
 + 252*(45*b^8*c^8*d^2 - 240*a*b^7*c^7*d^3 + 630*a^2*b^6*c^6*d^4 - 1008*a^3
*b^5*c^5*d^5 + 1050*a^4*b^4*c^4*d^6 - 720*a^5*b^3*c^3*d^7 + 315*a^6*b^2*c^2
*d^8 - 80*a^7*b*c*d^9 + 9*a^8*d^10)*x)/b^10 + 10*(b^9*c^9*d - 9*a*b^8*c^8*d
^2 + 36*a^2*b^7*c^7*d^3 - 84*a^3*b^6*c^6*d^4 + 126*a^4*b^5*c^5*d^5 - 126*a^
5*b^4*c^4*d^6 + 84*a^6*b^3*c^3*d^7 - 36*a^7*b^2*c^2*d^8 + 9*a^8*b*c*d^9 - a
^9*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.35, size = 3475, normalized size = 13.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^2,x)
```

```
[Out] x^7*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/(7*b) - (a^2*d^10)/(7*b^4) + (
45*c^2*d^8)/(7*b^2)) - x^5*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10
)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*(
(2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/(5*b) - (42*c^4*d^6)/b^2 + (a^2*((2
*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2
))/(5*b^2)) - x^8*((a*d^10)/(4*b^3) - (5*c*d^9)/(4*b^2)) + x^3*((70*c^6*d^4
)/b^2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 -
(10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^1
0)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^
10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b
- (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b
 - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9
)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/(3*b) + (a^2*((2*a*((120*c^3*d^7)/b
^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45
*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*
c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)
/b^4 + (45*c^2*d^8)/b^2))/b^2))/(3*b^2)) - x^2*((a*((210*c^6*d^4)/b^2 - (2*
a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/
b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (1
0*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (
10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((12
0*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^1
0)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2
))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a
*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))
/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 +
(a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*
d^8)/b^2))/b^2))/b^2))/b - (60*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^
7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 +
 (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (
210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d
^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*
a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2)
)/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2
))/(2*b^2)) + x^6*((20*c^3*d^7)/b^2 - (a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)
/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/(3*b) + (a^2*((2*a*d^10)/b^3
```

```
 - (10*c*d^9)/b^2))/(6*b^2)) + x*((45*c^8*d^2)/b^2 - (a^2*((210*c^6*d^4)/b^
2 - (2*a*((2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*
c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b
^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/
b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a
^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (
a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^
2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*
a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8
)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)
/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (
45*c^2*d^8)/b^2))/b^2))/b^2))/b^2 + (2*a*((2*a*((210*c^6*d^4)/b^2 - (2*a*((
2*a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2)
)/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*
d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c
*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^
3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b
^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b
^2 + (252*c^5*d^5)/b^2))/b + (a^2*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2
*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b +
 (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2
*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)
/b^2))/b^2))/b^2))/b - (120*c^7*d^3)/b^2 + (a^2*((2*a*((2*a*((120*c^3*d^7)/
b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (4
5*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210
*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10
)/b^4 + (45*c^2*d^8)/b^2))/b^2))/b - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*(
(2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b
 + (a^2*((2*a*d^10)/b^3 - (10*c*d^9)/b^2))/b^2))/b^2 + (252*c^5*d^5)/b^2))/
b^2))/b) + x^4*((a*((2*a*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 -
(10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^1
0)/b^3 - (10*c*d^9)/b^2))/b^2))/b - (210*c^4*d^6)/b^2 + (a^2*((2*a*((2*a*d^
10)/b^3 - (10*c*d^9)/b^2))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b^2))/(2
*b) - (a^2*((120*c^3*d^7)/b^2 - (2*a*((2*a*((2*a*d^10)/b^3 - (10*c*d^9)/b^2
))/b - (a^2*d^10)/b^4 + (45*c^2*d^8)/b^2))/b + (a^2*((2*a*d^10)/b^3 - (10*c
*d^9)/b^2))/b^2))/(4*b^2) + (63*c^5*d^5)/b^2) - (log(a + b*x)*(10*a^9*d^10
- 10*b^9*c^9*d + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d
^4 - 1260*a^4*b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 36
0*a^7*b^2*c^2*d^8 - 90*a^8*b*c*d^9))/b^11 - (a^10*d^10 + b^10*c^10 + 45*a^2
*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*
d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a
*b^9*c^9*d - 10*a^9*b*c*d^9)/(b*(a*b^10 + b^11*x)) + (d^10*x^9)/(9*b^2)
```

sympy [B] time = 2.65, size = 816, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**2,x)
```

```
[Out] x**8*(-a*d**10/(4*b**3) + 5*c*d**9/(4*b**2)) + x**7*(3*a**2*d**10/(7*b**4)
- 20*a*c*d**9/(7*b**3) + 45*c**2*d**8/(7*b**2)) + x**6*(-2*a**3*d**10/(3*b*
*5) + 5*a**2*c*d**9/b**4 - 15*a*c**2*d**8/b**3 + 20*c**3*d**7/b**2) + x**5*
(a**4*d**10/b**6 - 8*a**3*c*d**9/b**5 + 27*a**2*c**2*d**8/b**4 - 48*a*c**3*
d**7/b**3 + 42*c**4*d**6/b**2) + x**4*(-3*a**5*d**10/(2*b**7) + 25*a**4*c*d
**9/(2*b**6) - 45*a**3*c**2*d**8/b**5 + 90*a**2*c**3*d**7/b**4 - 105*a*c**4
*d**6/b**3 + 63*c**5*d**5/b**2) + x**3*(7*a**6*d**10/(3*b**8) - 20*a**5*c*d
**9/b**7 + 75*a**4*c**2*d**8/b**6 - 160*a**3*c**3*d**7/b**5 + 210*a**2*c**4
*d**6/b**4 - 168*a*c**5*d**5/b**3 + 70*c**6*d**4/b**2) + x**2*(-4*a**7*d**1
0/b**9 + 35*a**6*c*d**9/b**8 - 135*a**5*c**2*d**8/b**7 + 300*a**4*c**3*d**7
```

```
/b**6 - 420*a**3*c**4*d**6/b**5 + 378*a**2*c**5*d**5/b**4 - 210*a*c**6*d**4
/b**3 + 60*c**7*d**3/b**2) + x*(9*a**8*d**10/b**10 - 80*a**7*c*d**9/b**9 +
315*a**6*c**2*d**8/b**8 - 720*a**5*c**3*d**7/b**7 + 1050*a**4*c**4*d**6/b**
6 - 1008*a**3*c**5*d**5/b**5 + 630*a**2*c**6*d**4/b**4 - 240*a*c**7*d**3/b*
*3 + 45*c**8*d**2/b**2) + (-a**10*d**10 + 10*a**9*b*c*d**9 - 45*a**8*b**2*c
**2*d**8 + 120*a**7*b**3*c**3*d**7 - 210*a**6*b**4*c**4*d**6 + 252*a**5*b**
5*c**5*d**5 - 210*a**4*b**6*c**6*d**4 + 120*a**3*b**7*c**7*d**3 - 45*a**2*b
**8*c**8*d**2 + 10*a*b**9*c**9*d - b**10*c**10)/(a*b**11 + b**12*x) + d**10
*x**9/(9*b**2) - 10*d*(a*d - b*c)**9*log(a + b*x)/b**11
```

3.1208 $\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx$

Optimal. Leaf size=262

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+\dots}{}$$

Rubi [A] time = 0.44, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^7(bc-ad)}{7b^{11}}+\frac{15d^8(a+bx)^6(bc-ad)^2}{2b^{11}}+\frac{24d^7(a+bx)^5(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^4(bc-ad)^4}{2b^{11}}+\frac{84d^5(a+bx)^3(bc-ad)^5}{b^{11}}+\frac{105d^4(a+bx)^2(bc-ad)^6}{b^{11}}+\frac{120d^3x(bc-ad)^7}{b^{10}}+\frac{45d^2(bc-ad)^8\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}+\frac{d^{10}(a+bx)^8}{8b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (120*d^3*(b*c - a*d)^7*x)/b^10 - (b*c - a*d)^10/(2*b^11*(a + b*x)^2) - (10*
d*(b*c - a*d)^9)/(b^11*(a + b*x)) + (105*d^4*(b*c - a*d)^6*(a + b*x)^2)/b^1
1 + (84*d^5*(b*c - a*d)^5*(a + b*x)^3)/b^11 + (105*d^6*(b*c - a*d)^4*(a + b
*x)^4)/(2*b^11) + (24*d^7*(b*c - a*d)^3*(a + b*x)^5)/b^11 + (15*d^8*(b*c -
a*d)^2*(a + b*x)^6)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^7)/(7*b^11) +
(d^10*(a + b*x)^8)/(8*b^11) + (45*d^2*(b*c - a*d)^8*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^3}\,dx = \int \left(\frac{120d^3(bc-ad)^7}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^3}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^2}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)}+\frac{210d^4(bc-ad)^6(a+\dots}{b^{10}}\right.$$

$$=\frac{120d^3(bc-ad)^7x}{b^{10}}-\frac{(bc-ad)^{10}}{2b^{11}(a+bx)^2}-\frac{10d(bc-ad)^9}{b^{11}(a+bx)}+\frac{105d^4(bc-ad)^6(a+bx)^2}{b^{11}}+\frac{84d^5(bc-a\dots}{b^{\dots}}$$

Mathematica [B] time = 0.24, size = 708, normalized size = 2.70

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] (532*a^10*d^10 - 56*a^9*b*d^9*(85*c + 26*d*x) + 28*a^8*b^2*d^8*(675*c^2 + 3
80*c*d*x - 116*d^2*x^2) - 280*a^7*b^3*d^7*(156*c^3 + 117*c^2*d*x - 91*c*d^2
*x^2 + 3*d^3*x^3) + 210*a^6*b^4*d^6*(308*c^4 + 256*c^3*d*x - 414*c^2*d^2*x^
2 + 32*c*d^3*x^3 + d^4*x^4) - 84*a^5*b^5*d^5*(756*c^5 + 560*c^4*d*x - 2000*
c^3*d^2*x^2 + 280*c^2*d^3*x^3 + 20*c*d^4*x^4 + d^5*x^5) + 42*a^4*b^6*d^4*(9
80*c^6 + 336*c^5*d*x - 4760*c^4*d^2*x^2 + 1120*c^3*d^3*x^3 + 140*c^2*d^4*x^
4 + 16*c*d^5*x^5 + d^6*x^6) - 24*a^3*b^7*d^3*(700*c^7 - 490*c^6*d*x - 6174*
c^5*d^2*x^2 + 2450*c^4*d^3*x^3 + 490*c^3*d^4*x^4 + 98*c^2*d^5*x^5 + 14*c*d^
6*x^6 + d^7*x^7) + 3*a^2*b^8*d^2*(1260*c^8 - 4480*c^7*d*x - 21560*c^6*d^2*x
^2 + 15680*c^5*d^3*x^3 + 4900*c^4*d^4*x^4 + 1568*c^3*d^5*x^5 + 392*c^2*d^6*
x^6 + 64*c*d^7*x^7 + 5*d^8*x^8) - 2*a*b^9*d*(140*c^9 - 2520*c^8*d*x - 6720*
```

```
c^7*d^2*x^2 + 11760*c^6*d^3*x^3 + 5880*c^5*d^4*x^4 + 2940*c^4*d^5*x^5 + 117
6*c^3*d^6*x^6 + 336*c^2*d^7*x^7 + 60*c*d^8*x^8 + 5*d^9*x^9) + b^10*(-28*c^1
0 - 560*c^9*d*x + 6720*c^7*d^3*x^3 + 5880*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 +
2940*c^4*d^6*x^6 + 1344*c^3*d^7*x^7 + 420*c^2*d^8*x^8 + 80*c*d^9*x^9 + 7*d^
10*x^10) + 2520*d^2*(b*c - a*d)^8*(a + b*x)^2*Log[a + b*x])/(56*b^11*(a + b
*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^{10}}{(a + bx)^3}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^3, x]
```

fricas [B] time = 1.26, size = 1233, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/56*(7*b^10*d^10*x^10 - 28*b^10*c^10 - 280*a*b^9*c^9*d + 3780*a^2*b^8*c^8*
d^2 - 16800*a^3*b^7*c^7*d^3 + 41160*a^4*b^6*c^6*d^4 - 63504*a^5*b^5*c^5*d^5
 + 64680*a^6*b^4*c^4*d^6 - 43680*a^7*b^3*c^3*d^7 + 18900*a^8*b^2*c^2*d^8 -
4760*a^9*b*c*d^9 + 532*a^10*d^10 + 10*(8*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*
(28*b^10*c^2*d^8 - 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(56*b^10*c^3*d^7
- 28*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(70*b^10*c^4*
d^6 - 56*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 - 8*a^3*b^7*c*d^9 + a^4*b^6*d^1
0)*x^6 + 84*(56*b^10*c^5*d^5 - 70*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 - 28*a
^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 210*(28*b^10*c^6*d^4
 - 56*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 - 56*a^3*b^7*c^3*d^7 + 28*a^4*b^6*
c^2*d^8 - 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 840*(8*b^10*c^7*d^3 - 28*a*
b^9*c^6*d^4 + 56*a^2*b^8*c^5*d^5 - 70*a^3*b^7*c^4*d^6 + 56*a^4*b^6*c^3*d^7
- 28*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 28*(480*a*b^9*
c^7*d^3 - 2310*a^2*b^8*c^6*d^4 + 5292*a^3*b^7*c^5*d^5 - 7140*a^4*b^6*c^4*d^
6 + 6000*a^5*b^5*c^3*d^7 - 3105*a^6*b^4*c^2*d^8 + 910*a^7*b^3*c*d^9 - 116*a
^8*b^2*d^10)*x^2 - 56*(10*b^10*c^9*d - 90*a*b^9*c^8*d^2 + 240*a^2*b^8*c^7*d
^3 - 210*a^3*b^7*c^6*d^4 - 252*a^4*b^6*c^5*d^5 + 840*a^5*b^5*c^4*d^6 - 960*
a^6*b^4*c^3*d^7 + 585*a^7*b^3*c^2*d^8 - 190*a^8*b^2*c*d^9 + 26*a^9*b*d^10)*
x + 2520*(a^2*b^8*c^8*d^2 - 8*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 - 56*a^5
*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 - 56*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8
 - 8*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8
*c^6*d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 2
8*a^6*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 2*(a*b^9*c^8*d^2
- 8*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 - 56*a^4*b^6*c^5*d^5 + 70*a^5*b^5*
c^4*d^6 - 56*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 - 8*a^8*b^2*c*d^9 + a^9*b
*d^10)*x)*log(b*x + a))/(b^13*x^2 + 2*a*b^12*x + a^2*b^11)
```

giac [B] time = 1.25, size = 924, normalized size = 3.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a^3*b^5*c^5*d^5
 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d^8 - 8*a^7*b*c
```

```
*d^9 + a^8*d^10)*log(abs(b*x + a))/b^11 - 1/2*(b^10*c^10 + 10*a*b^9*c^9*d -
 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^
5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c
^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 + 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2
+ 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b
^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^
9*b*d^10)*x)/((b*x + a)^2*b^11) + 1/56*(7*b^21*d^10*x^8 + 80*b^21*c*d^9*x^7
 - 24*a*b^20*d^10*x^7 + 420*b^21*c^2*d^8*x^6 - 280*a*b^20*c*d^9*x^6 + 56*a^
2*b^19*d^10*x^6 + 1344*b^21*c^3*d^7*x^5 - 1512*a*b^20*c^2*d^8*x^5 + 672*a^2
*b^19*c*d^9*x^5 - 112*a^3*b^18*d^10*x^5 + 2940*b^21*c^4*d^6*x^4 - 5040*a*b^
20*c^3*d^7*x^4 + 3780*a^2*b^19*c^2*d^8*x^4 - 1400*a^3*b^18*c*d^9*x^4 + 210*
a^4*b^17*d^10*x^4 + 4704*b^21*c^5*d^5*x^3 - 11760*a*b^20*c^4*d^6*x^3 + 1344
0*a^2*b^19*c^3*d^7*x^3 - 8400*a^3*b^18*c^2*d^8*x^3 + 2800*a^4*b^17*c*d^9*x^
3 - 392*a^5*b^16*d^10*x^3 + 5880*b^21*c^6*d^4*x^2 - 21168*a*b^20*c^5*d^5*x^
2 + 35280*a^2*b^19*c^4*d^6*x^2 - 33600*a^3*b^18*c^3*d^7*x^2 + 18900*a^4*b^1
7*c^2*d^8*x^2 - 5880*a^5*b^16*c*d^9*x^2 + 784*a^6*b^15*d^10*x^2 + 6720*b^21
*c^7*d^3*x - 35280*a*b^20*c^6*d^4*x + 84672*a^2*b^19*c^5*d^5*x - 117600*a^3
*b^18*c^4*d^6*x + 100800*a^4*b^17*c^3*d^7*x - 52920*a^5*b^16*c^2*d^8*x + 15
680*a^6*b^15*c*d^9*x - 2016*a^7*b^14*d^10*x)/b^24
```

maple [B] time = 0.02, size = 1105, normalized size = 4.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^3,x)
```

```
[Out] -1/2/b^11/(b*x+a)^2*a^10*d^10+45/b^11*d^10*ln(b*x+a)*a^8+45/b^3*d^2*ln(b*x+
a)*c^8+10/b^11*d^10/(b*x+a)*a^9-10/b^2*d/(b*x+a)*c^9+105/2*d^6/b^3*x^4*c^4-
7*d^10/b^8*x^3*a^5+84*d^5/b^3*x^3*c^5+14*d^10/b^9*x^2*a^6+105*d^4/b^3*x^2*c
^6-3/7*d^10/b^4*x^7*a+10/7*d^9/b^3*x^7*c+15/2*d^8/b^3*x^6*c^2-2*d^10/b^6*x^
5*a^3+24*d^7/b^3*x^5*c^3+15/4*d^10/b^7*x^4*a^4+d^10/b^5*x^6*a^2-36*d^10/b^1
0*a^7*x+120*d^3/b^3*c^7*x-105/b^5/(b*x+a)^2*a^4*c^6*d^4+60/b^4/(b*x+a)^2*a^
3*c^7*d^3-45/2/b^3/(b*x+a)^2*a^2*c^8*d^2+5/b^2/(b*x+a)^2*a*c^9*d-360/b^10*d
^9*ln(b*x+a)*a^7*c+1/8*d^10/b^3*x^8-1/2/b/(b*x+a)^2*c^10+1260/b^9*d^8*ln(b*
x+a)*a^6*c^2-2520/b^8*d^7*ln(b*x+a)*a^5*c^3+3150/b^7*d^6*ln(b*x+a)*a^4*c^4-
2520/b^6*d^5*ln(b*x+a)*a^3*c^5+1260/b^5*d^4*ln(b*x+a)*a^2*c^6-360/b^4*d^3*l
n(b*x+a)*a*c^7-90/b^10*d^9/(b*x+a)*a^8*c+360/b^9*d^8/(b*x+a)*a^7*c^2-840/b^
8*d^7/(b*x+a)*a^6*c^3+1260/b^7*d^6/(b*x+a)*a^5*c^4-1260/b^6*d^5/(b*x+a)*a^4
*c^5+840/b^5*d^4/(b*x+a)*a^3*c^6-360/b^4*d^3/(b*x+a)*a^2*c^7+90/b^3*d^2/(b*
x+a)*a*c^8+675/2*d^8/b^7*x^2*a^4*c^2-600*d^7/b^6*x^2*a^3*c^3+630*d^6/b^5*x^
2*a^2*c^4-378*d^5/b^4*x^2*a*c^5+280*d^9/b^9*a^6*c*x-945*d^8/b^8*a^5*c^2*x+1
800*d^7/b^7*a^4*c^3*x-2100*d^6/b^6*a^3*c^4*x+1512*d^5/b^5*a^2*c^5*x-630*d^4
/b^4*a*c^6*x-27*d^8/b^4*x^5*a*c^2-25*d^9/b^6*x^4*a^3*c+135/2*d^8/b^5*x^4*a^
2*c^2-90*d^7/b^4*x^4*a*c^3+50*d^9/b^7*x^3*a^4*c-150*d^8/b^6*x^3*a^3*c^2+240
*d^7/b^5*x^3*a^2*c^3-210*d^6/b^4*x^3*a*c^4-105*d^9/b^8*x^2*a^5*c-5*d^9/b^4*
x^6*a*c+12*d^9/b^5*x^5*a^2*c+5/b^10/(b*x+a)^2*a^9*c*d^9-45/2/b^9/(b*x+a)^2*
a^8*c^2*d^8+60/b^8/(b*x+a)^2*a^7*c^3*d^7-105/b^7/(b*x+a)^2*a^6*c^4*d^6+126/
b^6/(b*x+a)^2*a^5*c^5*d^5
```

maxima [B] time = 1.62, size = 881, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(b^10*c^10 + 10*a*b^9*c^9*d - 135*a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^
3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 15
60*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^8 + 170*a^9*b*c*d^9 - 19*a^10*d^10 +
 20*(b^10*c^9*d - 9*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 - 84*a^3*b^7*c^6*d^4
```

```
 + 126*a^4*b^6*c^5*d^5 - 126*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 - 36*a^7*
b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 - a^9*b*d^10)*x)/(b^13*x^2 + 2*a*b^12*x + a^2
*b^11) + 1/56*(7*b^7*d^10*x^8 + 8*(10*b^7*c*d^9 - 3*a*b^6*d^10)*x^7 + 28*(1
5*b^7*c^2*d^8 - 10*a*b^6*c*d^9 + 2*a^2*b^5*d^10)*x^6 + 56*(24*b^7*c^3*d^7 -
 27*a*b^6*c^2*d^8 + 12*a^2*b^5*c*d^9 - 2*a^3*b^4*d^10)*x^5 + 70*(42*b^7*c^4
*d^6 - 72*a*b^6*c^3*d^7 + 54*a^2*b^5*c^2*d^8 - 20*a^3*b^4*c*d^9 + 3*a^4*b^3
*d^10)*x^4 + 56*(84*b^7*c^5*d^5 - 210*a*b^6*c^4*d^6 + 240*a^2*b^5*c^3*d^7 -
 150*a^3*b^4*c^2*d^8 + 50*a^4*b^3*c*d^9 - 7*a^5*b^2*d^10)*x^3 + 28*(210*b^7
*c^6*d^4 - 756*a*b^6*c^5*d^5 + 1260*a^2*b^5*c^4*d^6 - 1200*a^3*b^4*c^3*d^7
+ 675*a^4*b^3*c^2*d^8 - 210*a^5*b^2*c*d^9 + 28*a^6*b*d^10)*x^2 + 56*(120*b^
7*c^7*d^3 - 630*a*b^6*c^6*d^4 + 1512*a^2*b^5*c^5*d^5 - 2100*a^3*b^4*c^4*d^6
 + 1800*a^4*b^3*c^3*d^7 - 945*a^5*b^2*c^2*d^8 + 280*a^6*b*c*d^9 - 36*a^7*d^
10)*x)/b^10 + 45*(b^8*c^8*d^2 - 8*a*b^7*c^7*d^3 + 28*a^2*b^6*c^6*d^4 - 56*a
^3*b^5*c^5*d^5 + 70*a^4*b^4*c^4*d^6 - 56*a^5*b^3*c^3*d^7 + 28*a^6*b^2*c^2*d
^8 - 8*a^7*b*c*d^9 + a^8*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.38, size = 3299, normalized size = 12.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^3,x)
```

```
[Out] x^3*((84*c^5*d^5)/b^3 - (a*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b
^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^
3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d
^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2)
)/b + (a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/
b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((
3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(3*b^3)) - x^7*((
3*a*d^10)/(7*b^4) - (10*c*d^9)/(7*b^3)) - ((b^10*c^10 - 19*a^10*d^10 - 135*
a^2*b^8*c^8*d^2 + 600*a^3*b^7*c^7*d^3 - 1470*a^4*b^6*c^6*d^4 + 2268*a^5*b^5
*c^5*d^5 - 2310*a^6*b^4*c^4*d^6 + 1560*a^7*b^3*c^3*d^7 - 675*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 170*a^9*b*c*d^9)/(2*b) - x*(10*a^9*d^10 - 10*b^9*c^9*d
 + 90*a*b^8*c^8*d^2 - 360*a^2*b^7*c^7*d^3 + 840*a^3*b^6*c^6*d^4 - 1260*a^4*
b^5*c^5*d^5 + 1260*a^5*b^4*c^4*d^6 - 840*a^6*b^3*c^3*d^7 + 360*a^7*b^2*c^2*
d^8 - 90*a^8*b*c*d^9))/(a^2*b^10 + b^12*x^2 + 2*a*b^11*x) - x^5*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/(5*b) + (a^3*d^10)/(5*b^6) - (24*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
 (10*c*d^9)/b^3))/(5*b^2)) + x^6*((a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/(2*
b) - (a^2*d^10)/(2*b^5) + (15*c^2*d^8)/(2*b^3)) + x^4*((3*a*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
/b^3))/b^2))/(4*b) + (105*c^4*d^6)/(2*b^3) + (a^3*((3*a*d^10)/b^4 - (10*c*d
^9)/b^3))/(4*b^3) - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*
a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/(4*b^2)) - x^2*((3*a^2*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/(2*b^2) + (3*a*((252*c^5*d^5)/b^3 - (3*a*(
(3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
(45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^
10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b
^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))
/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3
*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b
 + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)
```

/b^3))/b^2))/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^3))/(2*b) - (105*c^6*d^4)/b^3 - (a^3*((3*
a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d
^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 -
 (10*c*d^9)/b^3))/b^2))/(2*b^3)) + x*((3*a*((3*a^2*((3*a*((3*a*((3*a*((3*a*
d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b +
(a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^
3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b
^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 +
 (45*c^2*d^8)/b^3))/b^2))/b^2 + (3*a*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*
((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8
)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2
*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^
4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^1
0)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2)
)/b^2 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5
+ (45*c^2*d^8)/b^3))/b^3))/b - (210*c^6*d^4)/b^3 - (a^3*((3*a*((3*a*((3*a*d
^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (
a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3
))/b^2))/b^3))/b - (a^3*((3*a*((3*a*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3)
)/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^6 - (120*c^3*d
^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b + (210*c^4*d^6)
/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^3 - (3*a^2*((3*a*((3*a*d^1
0)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b
^3 + (120*c^7*d^3)/b^3 - (3*a^2*((252*c^5*d^5)/b^3 - (3*a*((3*a*((3*a*((3*a
*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3
))/b + (a^3*d^10)/b^6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*
d^9)/b^3))/b^2))/b + (210*c^4*d^6)/b^3 + (a^3*((3*a*d^10)/b^4 - (10*c*d^9)/
b^3))/b^3 - (3*a^2*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10
)/b^5 + (45*c^2*d^8)/b^3))/b^2))/b + (3*a^2*((3*a*((3*a*((3*a*d^10)/b^4 - (
10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45*c^2*d^8)/b^3))/b + (a^3*d^10)/b^
6 - (120*c^3*d^7)/b^3 - (3*a^2*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b^2))/b^2
 - (a^3*((3*a*((3*a*d^10)/b^4 - (10*c*d^9)/b^3))/b - (3*a^2*d^10)/b^5 + (45
*c^2*d^8)/b^3))/b^3))/b^2) + (log(a + b*x)*(45*a^8*d^10 + 45*b^8*c^8*d^2 -
360*a*b^7*c^7*d^3 + 1260*a^2*b^6*c^6*d^4 - 2520*a^3*b^5*c^5*d^5 + 3150*a^4*
b^4*c^4*d^6 - 2520*a^5*b^3*c^3*d^7 + 1260*a^6*b^2*c^2*d^8 - 360*a^7*b*c*d^9
))/b^11 + (d^10*x^8)/(8*b^3)

sympy [B] time = 5.67, size = 843, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**10/(b*x+a)**3,x)

[Out] x**7*(-3*a*d**10/(7*b**4) + 10*c*d**9/(7*b**3)) + x**6*(a**2*d**10/b**5 - 5
*a*c*d**9/b**4 + 15*c**2*d**8/(2*b**3)) + x**5*(-2*a**3*d**10/b**6 + 12*a**
2*c*d**9/b**5 - 27*a*c**2*d**8/b**4 + 24*c**3*d**7/b**3) + x**4*(15*a**4*d*
*10/(4*b**7) - 25*a**3*c*d**9/b**6 + 135*a**2*c**2*d**8/(2*b**5) - 90*a*c**
3*d**7/b**4 + 105*c**4*d**6/(2*b**3)) + x**3*(-7*a**5*d**10/b**8 + 50*a**4*
c*d**9/b**7 - 150*a**3*c**2*d**8/b**6 + 240*a**2*c**3*d**7/b**5 - 210*a*c**
4*d**6/b**4 + 84*c**5*d**5/b**3) + x**2*(14*a**6*d**10/b**9 - 105*a**5*c*d*
*9/b**8 + 675*a**4*c**2*d**8/(2*b**7) - 600*a**3*c**3*d**7/b**6 + 630*a**2*
c**4*d**6/b**5 - 378*a*c**5*d**5/b**4 + 105*c**6*d**4/b**3) + x*(-36*a**7*d
10/b10 + 280*a**6*c*d**9/b**9 - 945*a**5*c**2*d**8/b**8 + 1800*a**4*c**
3*d**7/b**7 - 2100*a**3*c**4*d**6/b**6 + 1512*a**2*c**5*d**5/b**5 - 630*a*c
6*d4/b**4 + 120*c**7*d**3/b**3) + (19*a**10*d**10 - 170*a**9*b*c*d**9 +
 675*a**8*b**2*c**2*d**8 - 1560*a**7*b**3*c**3*d**7 + 2310*a**6*b**4*c**4*d

```
**6 - 2268*a**5*b**5*c**5*d**5 + 1470*a**4*b**6*c**6*d**4 - 600*a**3*b**7*c
**7*d**3 + 135*a**2*b**8*c**8*d**2 - 10*a*b**9*c**9*d - b**10*c**10 + x*(20
*a**9*b*d**10 - 180*a**8*b**2*c*d**9 + 720*a**7*b**3*c**2*d**8 - 1680*a**6*
b**4*c**3*d**7 + 2520*a**5*b**5*c**4*d**6 - 2520*a**4*b**6*c**5*d**5 + 1680
*a**3*b**7*c**6*d**4 - 720*a**2*b**8*c**7*d**3 + 180*a*b**9*c**8*d**2 - 20*
b**10*c**9*d))/(2*a**2*b**11 + 4*a*b**12*x + 2*b**13*x**2) + d**10*x**8/(8*
b**3) + 45*d**2*(a*d - b*c)**8*log(a + b*x)/b**11
```

3.1209 $\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx)^2(bc-ad)^5}{b^{11}}$$

Rubi [A] time = 0.44, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^6(bc-ad)}{3b^{11}}+\frac{9d^8(a+bx)^5(bc-ad)^2}{b^{11}}+\frac{30d^7(a+bx)^4(bc-ad)^3}{b^{11}}+\frac{70d^6(a+bx)^3(bc-ad)^4}{b^{11}}+\frac{126d^5(a+bx)^2(bc-ad)^5}{b^{11}}+\frac{210d^4x(bc-ad)^6}{b^{10}}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{120d^3(bc-ad)^7\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}+\frac{d^{10}(a+bx)^7}{7b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (210*d^4*(b*c - a*d)^6*x)/b^10 - (b*c - a*d)^10/(3*b^11*(a + b*x)^3) - (5*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^2) - (45*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
) + (126*d^5*(b*c - a*d)^5*(a + b*x)^2)/b^11 + (70*d^6*(b*c - a*d)^4*(a + b
*x)^3)/b^11 + (30*d^7*(b*c - a*d)^3*(a + b*x)^4)/b^11 + (9*d^8*(b*c - a*d)^
2*(a + b*x)^5)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^6)/(3*b^11) + (d^10*(a +
 b*x)^7)/(7*b^11) + (120*d^3*(b*c - a*d)^7*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx &= \int \left(\frac{210d^4(bc-ad)^6}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^4}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^3}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^2}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)}+\ldots\right)\\ &= \frac{210d^4(bc-ad)^6x}{b^{10}}-\frac{(bc-ad)^{10}}{3b^{11}(a+bx)^3}-\frac{5d(bc-ad)^9}{b^{11}(a+bx)^2}-\frac{45d^2(bc-ad)^8}{b^{11}(a+bx)}+\frac{126d^5(bc-ad)^5(a+bx)^2}{b^{11}}\end{aligned}$$

Mathematica [A] time = 0.18, size = 427, normalized size = 1.66

$$\frac{21b^5d^8x^5\left(2a^2d^2-8abcd+9b^2c^2\right)+105b^4d^7x^4\left(-a^3d^3+5a^2bcd^2-9ab^2c^2d+6b^3c^3\right)+35b^3d^6x^3\left(7a^4d^4-40a^3bcd^3+90a^2b^2c^2d^2-96ab^3c^3d+42b^4c^4\right)+21b^2d^5x^2\left(-28a^5d^5+175a^4bcd^4-450a^3b^2c^2d^3+600a^2b^3c^3d^2-420ab^4c^4d+126b^5c^5\right)+21bd^4x\left(84a^6d^6-560a^5bcd^5+1575a^4b^2c^2d^4-2400a^3b^3c^3d^3+2100a^2b^4c^4d^2-1008ab^5c^5d+210b^6c^6\right)+7b^6d^9x^6(5bc-2ad)+2520d^3(bc-ad)^7\log(a+bx)-\frac{945d^2(bc-ad)^8}{a+bx}+\frac{105d(ad-bc)^9}{(a+bx)^2}-\frac{7(bc-ad)^{10}}{(a+bx)^3}+3b^7d^{10}x^7}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] (21*b*d^4*(210*b^6*c^6 - 1008*a*b^5*c^5*d + 2100*a^2*b^4*c^4*d^2 - 2400*a^3
*b^3*c^3*d^3 + 1575*a^4*b^2*c^2*d^4 - 560*a^5*b*c*d^5 + 84*a^6*d^6)*x + 21*
b^2*d^5*(126*b^5*c^5 - 420*a*b^4*c^4*d + 600*a^2*b^3*c^3*d^2 - 450*a^3*b^2*
c^2*d^3 + 175*a^4*b*c*d^4 - 28*a^5*d^5)*x^2 + 35*b^3*d^6*(42*b^4*c^4 - 96*a
*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 40*a^3*b*c*d^3 + 7*a^4*d^4)*x^3 + 105*b^4
*d^7*(6*b^3*c^3 - 9*a*b^2*c^2*d + 5*a^2*b*c*d^2 - a^3*d^3)*x^4 + 21*b^5*d^8
*(9*b^2*c^2 - 8*a*b*c*d + 2*a^2*d^2)*x^5 + 7*b^6*d^9*(5*b*c - 2*a*d)*x^6 +
3*b^7*d^10*x^7 - (7*(b*c - a*d)^10)/(a + b*x)^3 + (105*d*(-(b*c) + a*d)^9)/
(a + b*x)^2 - (945*d^2*(b*c - a*d)^8)/(a + b*x) + 2520*d^3*(b*c - a*d)^7*Lo
g[a + b*x])/(21*b^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^4}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^4,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^4, x]
```

fricas [B] time = 1.25, size = 1316, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/21*(3*b^10*d^10*x^10 - 7*b^10*c^10 - 35*a*b^9*c^9*d - 315*a^2*b^8*c^8*d^2
 + 4620*a^3*b^7*c^7*d^3 - 19110*a^4*b^6*c^6*d^4 + 41454*a^5*b^5*c^5*d^5 - 5
4390*a^6*b^4*c^4*d^6 + 44940*a^7*b^3*c^3*d^7 - 22995*a^8*b^2*c^2*d^8 + 6685
*a^9*b*c*d^9 - 847*a^10*d^10 + 5*(7*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(21*b^
10*c^2*d^8 - 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 18*(35*b^10*c^3*d^7 - 21*a
*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 42*(35*b^10*c^4*d^6 -
35*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 - 7*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 126*(21*b^10*c^5*d^5 - 35*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 - 21*a^3*b^
7*c^2*d^8 + 7*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 630*(7*b^10*c^6*d^4 - 21*
a*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 - 35*a^3*b^7*c^3*d^7 + 21*a^4*b^6*c^2*d^
8 - 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 7*(1890*a*b^9*c^6*d^4 - 7938*a^2*
b^8*c^5*d^5 + 15330*a^3*b^7*c^4*d^6 - 16680*a^4*b^6*c^3*d^7 + 10575*a^5*b^5
*c^2*d^8 - 3665*a^6*b^4*c*d^9 + 539*a^7*b^3*d^10)*x^3 - 21*(45*b^10*c^8*d^2
 - 360*a*b^9*c^7*d^3 + 630*a^2*b^8*c^6*d^4 + 378*a^3*b^7*c^5*d^5 - 2730*a^4
*b^6*c^4*d^6 + 4080*a^5*b^5*c^3*d^7 - 3015*a^6*b^4*c^2*d^8 + 1145*a^7*b^3*c
*d^9 - 179*a^8*b^2*d^10)*x^2 - 21*(5*b^10*c^9*d + 45*a*b^9*c^8*d^2 - 540*a^
2*b^8*c^7*d^3 + 1890*a^3*b^7*c^6*d^4 - 3402*a^4*b^6*c^5*d^5 + 3570*a^5*b^5*
c^4*d^6 - 2220*a^6*b^4*c^3*d^7 + 765*a^7*b^3*c^2*d^8 - 115*a^8*b^2*c*d^9 +
a^9*b*d^10)*x + 2520*(a^3*b^7*c^7*d^3 - 7*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*
d^5 - 35*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 - 21*a^8*b^2*c^2*d^8 + 7*a^9*
b*c*d^9 - a^10*d^10 + (b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5
- 35*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*
c*d^9 - a^7*b^3*d^10)*x^3 + 3*(a*b^9*c^7*d^3 - 7*a^2*b^8*c^6*d^4 + 21*a^3*b
^7*c^5*d^5 - 35*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 - 21*a^6*b^4*c^2*d^8 +
 7*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 3*(a^2*b^8*c^7*d^3 - 7*a^3*b^7*c^6*d
^4 + 21*a^4*b^6*c^5*d^5 - 35*a^5*b^5*c^4*d^6 + 35*a^6*b^4*c^3*d^7 - 21*a^7*
b^3*c^2*d^8 + 7*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^14*x^3 + 3*
a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11)
```

giac [B] time = 1.26, size = 907, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^
6 + 35*a^4*b^3*c^3*d^7 - 21*a^5*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log
(abs(b*x + a))/b^11 - 1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a
^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*
d^9 + 121*a^10*d^10 + 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*
d^4 - 56*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6
```

```
*b^4*c^2*d^8 - 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b
^9*c^8*d^2 - 108*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^
5 + 1134*a^5*b^5*c^4*d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*
a^8*b^2*c*d^9 + 17*a^9*b*d^10)*x)/((b*x + a)^3*b^11) + 1/21*(3*b^24*d^10*x^
7 + 35*b^24*c*d^9*x^6 - 14*a*b^23*d^10*x^6 + 189*b^24*c^2*d^8*x^5 - 168*a*b
^23*c*d^9*x^5 + 42*a^2*b^22*d^10*x^5 + 630*b^24*c^3*d^7*x^4 - 945*a*b^23*c^
2*d^8*x^4 + 525*a^2*b^22*c*d^9*x^4 - 105*a^3*b^21*d^10*x^4 + 1470*b^24*c^4*
d^6*x^3 - 3360*a*b^23*c^3*d^7*x^3 + 3150*a^2*b^22*c^2*d^8*x^3 - 1400*a^3*b^
21*c*d^9*x^3 + 245*a^4*b^20*d^10*x^3 + 2646*b^24*c^5*d^5*x^2 - 8820*a*b^23*
c^4*d^6*x^2 + 12600*a^2*b^22*c^3*d^7*x^2 - 9450*a^3*b^21*c^2*d^8*x^2 + 3675
*a^4*b^20*c*d^9*x^2 - 588*a^5*b^19*d^10*x^2 + 4410*b^24*c^6*d^4*x - 21168*a
*b^23*c^5*d^5*x + 44100*a^2*b^22*c^4*d^6*x - 50400*a^3*b^21*c^3*d^7*x + 330
75*a^4*b^20*c^2*d^8*x - 11760*a^5*b^19*c*d^9*x + 1764*a^6*b^18*d^10*x)/b^28
```

maple [B] time = 0.02, size = 1141, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^4,x)
```

```
[Out] -1/3/b^11/(b*x+a)^3*a^10*d^10+5/b^11*d^10/(b*x+a)^2*a^9-5/b^2*d/(b*x+a)^2*c
^9-120/b^11*d^10*ln(b*x+a)*a^7+120/b^4*d^3*ln(b*x+a)*c^7-45/b^11*d^10/(b*x+
a)*a^8-45/b^3*d^2/(b*x+a)*c^8+9*d^8/b^4*x^5*c^2-5*d^10/b^7*x^4*a^3+30*d^7/b
^4*x^4*c^3+35/3*d^10/b^8*x^3*a^4+70*d^6/b^4*x^3*c^4-28*d^10/b^9*x^2*a^5+126
*d^5/b^4*x^2*c^5-2/3*d^10/b^5*x^6*a+5/3*d^9/b^4*x^6*c+2*d^10/b^6*x^5*a^2+21
0*d^4/b^4*c^6*x+84*d^10/b^10*a^6*x+1/7*d^10/b^4*x^7-1/3/b/(b*x+a)^3*c^10-70
/b^5/(b*x+a)^3*a^4*c^6*d^4+40/b^4/(b*x+a)^3*a^3*c^7*d^3-15/b^3/(b*x+a)^3*a^
2*c^8*d^2+10/3/b^2/(b*x+a)^3*a*c^9*d-45/b^10*d^9/(b*x+a)^2*a^8*c+180/b^9*d^
8/(b*x+a)^2*a^7*c^2-420/b^8*d^7/(b*x+a)^2*a^6*c^3+630/b^7*d^6/(b*x+a)^2*a^5
*c^4-630/b^6*d^5/(b*x+a)^2*a^4*c^5+420/b^5*d^4/(b*x+a)^2*a^3*c^6-180/b^4*d^
3/(b*x+a)^2*a^2*c^7+45/b^3*d^2/(b*x+a)^2*a*c^8+840/b^10*d^9*ln(b*x+a)*a^6*c
-2520/b^9*d^8*ln(b*x+a)*a^5*c^2+4200/b^8*d^7*ln(b*x+a)*a^4*c^3-4200/b^7*d^6
*ln(b*x+a)*a^3*c^4+2520/b^6*d^5*ln(b*x+a)*a^2*c^5-840/b^5*d^4*ln(b*x+a)*a*c
^6+360/b^10*d^9/(b*x+a)*a^7*c-1260/b^9*d^8/(b*x+a)*a^6*c^2+2520/b^8*d^7/(b*
x+a)*a^5*c^3-3150/b^7*d^6/(b*x+a)*a^4*c^4+2520/b^6*d^5/(b*x+a)*a^3*c^5-1260
/b^5*d^4/(b*x+a)*a^2*c^6+360/b^4*d^3/(b*x+a)*a*c^7-200/3*d^9/b^7*x^3*a^3*c-
8*d^9/b^5*x^5*a*c-560*d^9/b^9*a^5*c*x+1575*d^8/b^8*a^4*c^2*x-2400*d^7/b^7*a
^3*c^3*x+2100*d^6/b^6*a^2*c^4*x-1008*d^5/b^5*a*c^5*x-450*d^8/b^7*x^2*a^3*c^
2+600*d^7/b^6*x^2*a^2*c^3-420*d^6/b^5*x^2*a*c^4+175*d^9/b^8*x^2*a^4*c+150*d
^8/b^6*x^3*a^2*c^2-160*d^7/b^5*x^3*a*c^3+10/3/b^10/(b*x+a)^3*a^9*c*d^9-15/b
^9/(b*x+a)^3*a^8*c^2*d^8+40/b^8/(b*x+a)^3*a^7*c^3*d^7+25*d^9/b^6*x^4*a^2*c-
45*d^8/b^5*x^4*a*c^2-70/b^7/(b*x+a)^3*a^6*c^4*d^6+84/b^6/(b*x+a)^3*a^5*c^5*
d^5
```

maxima [B] time = 1.73, size = 891, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(b^10*c^10 + 5*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3
+ 2730*a^4*b^6*c^6*d^4 - 5922*a^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420
*a^7*b^3*c^3*d^7 + 3285*a^8*b^2*c^2*d^8 - 955*a^9*b*c*d^9 + 121*a^10*d^10 +
 135*(b^10*c^8*d^2 - 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 56*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 - 56*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c^2*d^8 - 8*a^7*
b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(b^10*c^9*d + 9*a*b^9*c^8*d^2 - 108*a^2*
b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 882*a^4*b^6*c^5*d^5 + 1134*a^5*b^5*c^4*
d^6 - 924*a^6*b^4*c^3*d^7 + 468*a^7*b^3*c^2*d^8 - 135*a^8*b^2*c*d^9 + 17*a^
9*b*d^10)*x)/(b^14*x^3 + 3*a*b^13*x^2 + 3*a^2*b^12*x + a^3*b^11) + 1/21*(3*
```

```
b^6*d^10*x^7 + 7*(5*b^6*c*d^9 - 2*a*b^5*d^10)*x^6 + 21*(9*b^6*c^2*d^8 - 8*a
*b^5*c*d^9 + 2*a^2*b^4*d^10)*x^5 + 105*(6*b^6*c^3*d^7 - 9*a*b^5*c^2*d^8 + 5
*a^2*b^4*c*d^9 - a^3*b^3*d^10)*x^4 + 35*(42*b^6*c^4*d^6 - 96*a*b^5*c^3*d^7
+ 90*a^2*b^4*c^2*d^8 - 40*a^3*b^3*c*d^9 + 7*a^4*b^2*d^10)*x^3 + 21*(126*b^6
*c^5*d^5 - 420*a*b^5*c^4*d^6 + 600*a^2*b^4*c^3*d^7 - 450*a^3*b^3*c^2*d^8 +
175*a^4*b^2*c*d^9 - 28*a^5*b*d^10)*x^2 + 21*(210*b^6*c^6*d^4 - 1008*a*b^5*c
^5*d^5 + 2100*a^2*b^4*c^4*d^6 - 2400*a^3*b^3*c^3*d^7 + 1575*a^4*b^2*c^2*d^8
 - 560*a^5*b*c*d^9 + 84*a^6*d^10)*x)/b^10 + 120*(b^7*c^7*d^3 - 7*a*b^6*c^6*
d^4 + 21*a^2*b^5*c^5*d^5 - 35*a^3*b^4*c^4*d^6 + 35*a^4*b^3*c^3*d^7 - 21*a^5
*b^2*c^2*d^8 + 7*a^6*b*c*d^9 - a^7*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.39, size = 2219, normalized size = 8.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^4,x)
```

```
[Out] x^3*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b
^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*(
(4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/(3*b) - (a^4*d^10)/(3*b^8) + (70*c^
4*d^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(3*b^3) - (2*a^2*((4
*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b
^4))/b^2) - x^6*((2*a*d^10)/(3*b^5) - (5*c*d^9)/(3*b^4)) - x^4*((a*((4*a*((
4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/
b + (a^3*d^10)/b^7 - (30*c^3*d^7)/b^4 - (3*a^2*((4*a*d^10)/b^5 - (10*c*d^9)
/b^4))/(2*b^2)) + x^5*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(5*b) - (6*a
^2*d^10)/(5*b^6) + (9*c^2*d^8)/b^4) - x*((4*a*((252*c^5*d^5)/b^4 - (4*a*((4
*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (4
5*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^
10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (
4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5
 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b + (a^
4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^4 + (6*a^2*((4*a*((4*a*((4*a*d^10)/b
^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*
d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
^2))/b^2 - (4*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)
/b^6 + (45*c^2*d^8)/b^4))/b^3))/b - (210*c^6*d^4)/b^4 + (6*a^2*((4*a*((4*a*
((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8
)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 -
 (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d^6)/b^4 + (4*a^3*((4
*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*a*d^10)/b^5 - (10*c*
d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^2))/b^2 - (4*a^3*((4
*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*
d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^
5 - (10*c*d^9)/b^4))/b^2))/b^3 + (a^4*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^
4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^4) + x^2*((126*c^5*d^5)/b^4
 - (2*a*((4*a*((4*a*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^1
0)/b^6 + (45*c^2*d^8)/b^4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a
^2*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^2))/b - (a^4*d^10)/b^8 + (210*c^4*d
^6)/b^4 + (4*a^3*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b^3 - (6*a^2*((4*a*((4*
a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^
2))/b + (a^4*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/(2*b^4) + (3*a^2*((4*a*((4*
a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^
4))/b + (4*a^3*d^10)/b^7 - (120*c^3*d^7)/b^4 - (6*a^2*((4*a*d^10)/b^5 - (10
*c*d^9)/b^4))/b^2))/b^2 - (2*a^3*((4*a*((4*a*d^10)/b^5 - (10*c*d^9)/b^4))/b
 - (6*a^2*d^10)/b^6 + (45*c^2*d^8)/b^4))/b^3) - ((121*a^10*d^10 + b^10*c^10
 + 45*a^2*b^8*c^8*d^2 - 660*a^3*b^7*c^7*d^3 + 2730*a^4*b^6*c^6*d^4 - 5922*a
^5*b^5*c^5*d^5 + 7770*a^6*b^4*c^4*d^6 - 6420*a^7*b^3*c^3*d^7 + 3285*a^8*b^2
*c^2*d^8 + 5*a*b^9*c^9*d - 955*a^9*b*c*d^9)/(3*b) + x*(85*a^9*d^10 + 5*b^9*
```

```
c^9*d + 45*a*b^8*c^8*d^2 - 540*a^2*b^7*c^7*d^3 + 2100*a^3*b^6*c^6*d^4 - 441
0*a^4*b^5*c^5*d^5 + 5670*a^5*b^4*c^4*d^6 - 4620*a^6*b^3*c^3*d^7 + 2340*a^7*
b^2*c^2*d^8 - 675*a^8*b*c*d^9) + x^2*(45*a^8*b*d^10 + 45*b^9*c^8*d^2 - 360*
a*b^8*c^7*d^3 - 360*a^7*b^2*c*d^9 + 1260*a^2*b^7*c^6*d^4 - 2520*a^3*b^6*c^5
*d^5 + 3150*a^4*b^5*c^4*d^6 - 2520*a^5*b^4*c^3*d^7 + 1260*a^6*b^3*c^2*d^8))
/(a^3*b^10 + b^13*x^3 + 3*a^2*b^11*x + 3*a*b^12*x^2) + (d^10*x^7)/(7*b^4) -
 (log(a + b*x)*(120*a^7*d^10 - 120*b^7*c^7*d^3 + 840*a*b^6*c^6*d^4 - 2520*a
^2*b^5*c^5*d^5 + 4200*a^3*b^4*c^4*d^6 - 4200*a^4*b^3*c^3*d^7 + 2520*a^5*b^2
*c^2*d^8 - 840*a^6*b*c*d^9))/b^11
```

sympy [B] time = 32.53, size = 867, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**4,x)
```

```
[Out] x**6*(-2*a*d**10/(3*b**5) + 5*c*d**9/(3*b**4)) + x**5*(2*a**2*d**10/b**6 -
8*a*c*d**9/b**5 + 9*c**2*d**8/b**4) + x**4*(-5*a**3*d**10/b**7 + 25*a**2*c*
d**9/b**6 - 45*a*c**2*d**8/b**5 + 30*c**3*d**7/b**4) + x**3*(35*a**4*d**10/
(3*b**8) - 200*a**3*c*d**9/(3*b**7) + 150*a**2*c**2*d**8/b**6 - 160*a*c**3*
d**7/b**5 + 70*c**4*d**6/b**4) + x**2*(-28*a**5*d**10/b**9 + 175*a**4*c*d**
9/b**8 - 450*a**3*c**2*d**8/b**7 + 600*a**2*c**3*d**7/b**6 - 420*a*c**4*d**
6/b**5 + 126*c**5*d**5/b**4) + x*(84*a**6*d**10/b**10 - 560*a**5*c*d**9/b**
9 + 1575*a**4*c**2*d**8/b**8 - 2400*a**3*c**3*d**7/b**7 + 2100*a**2*c**4*d*
*6/b**6 - 1008*a*c**5*d**5/b**5 + 210*c**6*d**4/b**4) + (-121*a**10*d**10 +
 955*a**9*b*c*d**9 - 3285*a**8*b**2*c**2*d**8 + 6420*a**7*b**3*c**3*d**7 -
7770*a**6*b**4*c**4*d**6 + 5922*a**5*b**5*c**5*d**5 - 2730*a**4*b**6*c**6*d
**4 + 660*a**3*b**7*c**7*d**3 - 45*a**2*b**8*c**8*d**2 - 5*a*b**9*c**9*d -
b**10*c**10 + x**2*(-135*a**8*b**2*d**10 + 1080*a**7*b**3*c*d**9 - 3780*a**
6*b**4*c**2*d**8 + 7560*a**5*b**5*c**3*d**7 - 9450*a**4*b**6*c**4*d**6 + 75
60*a**3*b**7*c**5*d**5 - 3780*a**2*b**8*c**6*d**4 + 1080*a*b**9*c**7*d**3 -
 135*b**10*c**8*d**2) + x*(-255*a**9*b*d**10 + 2025*a**8*b**2*c*d**9 - 7020
*a**7*b**3*c**2*d**8 + 13860*a**6*b**4*c**3*d**7 - 17010*a**5*b**5*c**4*d**
6 + 13230*a**4*b**6*c**5*d**5 - 6300*a**3*b**7*c**6*d**4 + 1620*a**2*b**8*c
**7*d**3 - 135*a*b**9*c**8*d**2 - 15*b**10*c**9*d))/(3*a**3*b**11 + 9*a**2*
b**12*x + 9*a*b**13*x**2 + 3*b**14*x**3) + d**10*x**7/(7*b**4) - 120*d**3*(
a*d - b*c)**7*log(a + b*x)/b**11
```

3.1210 $\int \frac{(c+dx)^{10}}{(a+bx)^5} \, dx$

Optimal. Leaf size=262

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}}+\frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}}+\frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}}+\frac{210d^4(b\ldots}{}$$

Rubi [A] time = 0.42, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{2d^9(a+bx)^5(bc-ad)}{b^{11}}+\frac{45d^8(a+bx)^4(bc-ad)^2}{4b^{11}}+\frac{40d^7(a+bx)^3(bc-ad)^3}{b^{11}}+\frac{105d^6(a+bx)^2(bc-ad)^4}{b^{11}}+\frac{252d^5x(bc-ad)^5}{b^{10}}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}+\frac{210d^4(bc-ad)^6\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}+\frac{d^{10}(a+bx)^6}{6b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (252*d^5*(b*c - a*d)^5*x)/b^10 - (b*c - a*d)^10/(4*b^11*(a + b*x)^4) - (10*
d*(b*c - a*d)^9)/(3*b^11*(a + b*x)^3) - (45*d^2*(b*c - a*d)^8)/(2*b^11*(a +
 b*x)^2) - (120*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)) + (105*d^6*(b*c - a*d)^
4*(a + b*x)^2)/b^11 + (40*d^7*(b*c - a*d)^3*(a + b*x)^3)/b^11 + (45*d^8*(b*
c - a*d)^2*(a + b*x)^4)/(4*b^11) + (2*d^9*(b*c - a*d)*(a + b*x)^5)/b^11 + (
d^10*(a + b*x)^6)/(6*b^11) + (210*d^4*(b*c - a*d)^6*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^5} \, dx &= \int \left(\frac{252d^5(bc-ad)^5}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^5}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^4}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^3}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^2}+\ldots\right.\\ &= \frac{252d^5(bc-ad)^5x}{b^{10}}-\frac{(bc-ad)^{10}}{4b^{11}(a+bx)^4}-\frac{10d(bc-ad)^9}{3b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{2b^{11}(a+bx)^2}-\frac{120d^3(bc-ad)^7}{b^{11}(a+bx)}+\ldots\end{aligned}$$

Mathematica [A] time = 0.20, size = 359, normalized size = 1.37

$$\frac{15b^4d^8x^4\left(3a^2d^2-10abcd+9b^2c^2\right)+20b^3d^7x^3\left(-7a^3d^3+30a^2bcd^2-45ab^2c^2d+24b^3c^3\right)+30b^2d^6x^2\left(14a^4d^4-70a^3bcd^3+135a^2b^2c^2d^2-120ab^3c^3d+42b^4c^4\right)+12bd^5x\left(-126a^5d^5+700a^4bcd^4-1575a^3b^2c^2d^3+1800a^2b^3c^3d^2-1050ab^4c^4d+252b^5c^5\right)+12b^5d^9x^5(2bc-ad)+2520d^4(bc-ad)^6\log(a+bx)+\frac{1440d^3(ad-bc)^7}{a+bx}-\frac{270d^2(bc-ad)^8}{(a+bx)^2}+\frac{40d(ad-bc)^9}{(a+bx)^3}-\frac{3(bc-ad)^{10}}{(a+bx)^4}+2b^6d^{10}x^6}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] (12*b*d^5*(252*b^5*c^5 - 1050*a*b^4*c^4*d + 1800*a^2*b^3*c^3*d^2 - 1575*a^3
*b^2*c^2*d^3 + 700*a^4*b*c*d^4 - 126*a^5*d^5)*x + 30*b^2*d^6*(42*b^4*c^4 -
120*a*b^3*c^3*d + 135*a^2*b^2*c^2*d^2 - 70*a^3*b*c*d^3 + 14*a^4*d^4)*x^2 +
20*b^3*d^7*(24*b^3*c^3 - 45*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 7*a^3*d^3)*x^3 +
 15*b^4*d^8*(9*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 12*b^5*d^9*(2*b*c -
a*d)*x^5 + 2*b^6*d^10*x^6 - (3*(b*c - a*d)^10)/(a + b*x)^4 + (40*d*(-(b*c)
+ a*d)^9)/(a + b*x)^3 - (270*d^2*(b*c - a*d)^8)/(a + b*x)^2 + (1440*d^3*(-(
b*c) + a*d)^7)/(a + b*x) + 2520*d^4*(b*c - a*d)^6*Log[a + b*x])/(12*b^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^5} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^5,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^5, x]
```

fricas [B] time = 1.25, size = 1365, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^10*d^10*x^10 - 3*b^10*c^10 - 10*a*b^9*c^9*d - 45*a^2*b^8*c^8*d^2
- 360*a^3*b^7*c^7*d^3 + 5250*a^4*b^6*c^6*d^4 - 19404*a^5*b^5*c^5*d^5 + 3591
0*a^6*b^4*c^4*d^6 - 38280*a^7*b^3*c^3*d^7 + 23985*a^8*b^2*c^2*d^8 - 8250*a^
9*b*c*d^9 + 1207*a^10*d^10 + 4*(6*b^10*c*d^9 - a*b^9*d^10)*x^9 + 9*(15*b^10
*c^2*d^8 - 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 24*(20*b^10*c^3*d^7 - 15*a*b
^9*c^2*d^8 + 6*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 84*(15*b^10*c^4*d^6 - 20
*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 - 6*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 504*(6*b^10*c^5*d^5 - 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 15*a^3*b^7*c
^2*d^8 + 6*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + (12096*a*b^9*c^5*d^5 - 42840
*a^2*b^8*c^4*d^6 + 66720*a^3*b^7*c^3*d^7 - 54765*a^4*b^6*c^2*d^8 + 23250*a^
5*b^5*c*d^9 - 4043*a^6*b^4*d^10)*x^4 - 4*(360*b^10*c^7*d^3 - 2520*a*b^9*c^6
*d^4 + 3024*a^2*b^8*c^5*d^5 + 5040*a^3*b^7*c^4*d^6 - 16320*a^4*b^6*c^3*d^7
+ 16965*a^5*b^5*c^2*d^8 - 8130*a^6*b^4*c*d^9 + 1523*a^7*b^3*d^10)*x^3 - 6*(
45*b^10*c^8*d^2 + 360*a*b^9*c^7*d^3 - 3780*a^2*b^8*c^6*d^4 + 10584*a^3*b^7*
c^5*d^5 - 13860*a^4*b^6*c^4*d^6 + 8880*a^5*b^5*c^3*d^7 - 1935*a^6*b^4*c^2*d
^8 - 570*a^7*b^3*c*d^9 + 263*a^8*b^2*d^10)*x^2 - 4*(10*b^10*c^9*d + 45*a*b^
9*c^8*d^2 + 360*a^2*b^8*c^7*d^3 - 4620*a^3*b^7*c^6*d^4 + 15624*a^4*b^6*c^5*
d^5 - 26460*a^5*b^5*c^4*d^6 + 25680*a^6*b^4*c^3*d^7 - 14535*a^7*b^3*c^2*d^8
 + 4470*a^8*b^2*c*d^9 - 577*a^9*b*d^10)*x + 2520*(a^4*b^6*c^6*d^4 - 6*a^5*b
^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 - 20*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 -
 6*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c
^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^
4*d^10)*x^4 + 4*(a*b^9*c^6*d^4 - 6*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 - 2
0*a^4*b^6*c^3*d^7 + 15*a^5*b^5*c^2*d^8 - 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^
3 + 6*(a^2*b^8*c^6*d^4 - 6*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 - 20*a^5*b^
5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 - 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 4*(a
^3*b^7*c^6*d^4 - 6*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 - 20*a^6*b^4*c^3*d^
7 + 15*a^7*b^3*c^2*d^8 - 6*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^
15*x^4 + 4*a*b^14*x^3 + 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11)
```

giac [B] time = 1.38, size = 1168, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/12*(2*d^10 + 24*(b^2*c*d^9 - a*b*d^10)/((b*x + a)*b) + 135*(b^4*c^2*d^8 -
 2*a*b^3*c*d^9 + a^2*b^2*d^10)/((b*x + a)^2*b^2) + 480*(b^6*c^3*d^7 - 3*a*b
^5*c^2*d^8 + 3*a^2*b^4*c*d^9 - a^3*b^3*d^10)/((b*x + a)^3*b^3) + 1260*(b^8*
c^4*d^6 - 4*a*b^7*c^3*d^7 + 6*a^2*b^6*c^2*d^8 - 4*a^3*b^5*c*d^9 + a^4*b^4*d
^10)/((b*x + a)^4*b^4) + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*
c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)/((b*x + a)^5
*b^5))*(b*x + a)^6/b^11 - 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4*c
^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^10
)*log(abs(b*x + a)/((b*x + a)^2*abs(b)))/b^11 - 1/12*(3*b^67*c^10/(b*x + a)
^4 + 40*b^66*c^9*d/(b*x + a)^3 - 30*a*b^66*c^9*d/(b*x + a)^4 + 270*b^65*c^8
*d^2/(b*x + a)^2 - 360*a*b^65*c^8*d^2/(b*x + a)^3 + 135*a^2*b^65*c^8*d^2/(b
*x + a)^4 + 1440*b^64*c^7*d^3/(b*x + a) - 2160*a*b^64*c^7*d^3/(b*x + a)^2 +
```

```
1440*a^2*b^64*c^7*d^3/(b*x + a)^3 - 360*a^3*b^64*c^7*d^3/(b*x + a)^4 - 100
80*a*b^63*c^6*d^4/(b*x + a) + 7560*a^2*b^63*c^6*d^4/(b*x + a)^2 - 3360*a^3*
b^63*c^6*d^4/(b*x + a)^3 + 630*a^4*b^63*c^6*d^4/(b*x + a)^4 + 30240*a^2*b^6
2*c^5*d^5/(b*x + a) - 15120*a^3*b^62*c^5*d^5/(b*x + a)^2 + 5040*a^4*b^62*c^
5*d^5/(b*x + a)^3 - 756*a^5*b^62*c^5*d^5/(b*x + a)^4 - 50400*a^3*b^61*c^4*d
^6/(b*x + a) + 18900*a^4*b^61*c^4*d^6/(b*x + a)^2 - 5040*a^5*b^61*c^4*d^6/(
b*x + a)^3 + 630*a^6*b^61*c^4*d^6/(b*x + a)^4 + 50400*a^4*b^60*c^3*d^7/(b*x
 + a) - 15120*a^5*b^60*c^3*d^7/(b*x + a)^2 + 3360*a^6*b^60*c^3*d^7/(b*x + a
)^3 - 360*a^7*b^60*c^3*d^7/(b*x + a)^4 - 30240*a^5*b^59*c^2*d^8/(b*x + a) +
 7560*a^6*b^59*c^2*d^8/(b*x + a)^2 - 1440*a^7*b^59*c^2*d^8/(b*x + a)^3 + 13
5*a^8*b^59*c^2*d^8/(b*x + a)^4 + 10080*a^6*b^58*c*d^9/(b*x + a) - 2160*a^7*
b^58*c*d^9/(b*x + a)^2 + 360*a^8*b^58*c*d^9/(b*x + a)^3 - 30*a^9*b^58*c*d^9
/(b*x + a)^4 - 1440*a^7*b^57*d^10/(b*x + a) + 270*a^8*b^57*d^10/(b*x + a)^2
 - 40*a^9*b^57*d^10/(b*x + a)^3 + 3*a^10*b^57*d^10/(b*x + a)^4)/b^68
```

maple [B] time = 0.02, size = 1172, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^5,x)
```

```
[Out] 10/3/b^11*d^10/(b*x+a)^3*a^9-10/3/b^2*d/(b*x+a)^3*c^9-45/2/b^11*d^10/(b*x+a
)^2*a^8-45/2/b^3*d^2/(b*x+a)^2*c^8+210/b^11*d^10*ln(b*x+a)*a^6+210/b^5*d^4*
ln(b*x+a)*c^6-1/4/b^11/(b*x+a)^4*a^10*d^10+120/b^11*d^10/(b*x+a)*a^7-120/b^
4*d^3/(b*x+a)*c^7-d^10/b^6*x^5*a+2*d^9/b^5*x^5*c+15/4*d^10/b^7*x^4*a^2+45/4
*d^8/b^5*x^4*c^2-35/3*d^10/b^8*x^3*a^3+40*d^7/b^5*x^3*c^3+35*d^10/b^9*x^2*a
^4+105*d^6/b^5*x^2*c^4-126*d^10/b^10*a^5*x+252*d^5/b^5*c^5*x+1/6*d^10/b^5*x
^6-1/4/b/(b*x+a)^4*c^10-25/2*d^9/b^6*x^4*a*c+50*d^9/b^7*x^3*a^2*c-1260/b^10
*d^9*ln(b*x+a)*a^5*c+3150/b^9*d^8*ln(b*x+a)*a^4*c^2-4200/b^8*d^7*ln(b*x+a)*
a^3*c^3+3150/b^7*d^6*ln(b*x+a)*a^2*c^4-1260/b^6*d^5*ln(b*x+a)*a*c^5+5/2/b^1
0/(b*x+a)^4*a^9*c*d^9-45/4/b^9/(b*x+a)^4*a^8*c^2*d^8+30/b^8/(b*x+a)^4*a^7*c
^3*d^7-105/2/b^7/(b*x+a)^4*a^6*c^4*d^6+63/b^6/(b*x+a)^4*a^5*c^5*d^5-105/2/b
^5/(b*x+a)^4*a^4*c^6*d^4+30/b^4/(b*x+a)^4*a^3*c^7*d^3-45/4/b^3/(b*x+a)^4*a^
2*c^8*d^2+5/2/b^2/(b*x+a)^4*a*c^9*d-840/b^10*d^9/(b*x+a)*a^6*c+2520/b^9*d^8
/(b*x+a)*a^5*c^2-4200/b^8*d^7/(b*x+a)*a^4*c^3+4200/b^7*d^6/(b*x+a)*a^3*c^4-
2520/b^6*d^5/(b*x+a)*a^2*c^5+840/b^5*d^4/(b*x+a)*a*c^6+700*d^9/b^9*a^4*c*x-
1575*d^8/b^8*a^3*c^2*x+1800*d^7/b^7*a^2*c^3*x-1050*d^6/b^6*a*c^4*x-30/b^10*
d^9/(b*x+a)^3*a^8*c+120/b^9*d^8/(b*x+a)^3*a^7*c^2-280/b^8*d^7/(b*x+a)^3*a^6
*c^3+420/b^7*d^6/(b*x+a)^3*a^5*c^4-420/b^6*d^5/(b*x+a)^3*a^4*c^5+280/b^5*d^
4/(b*x+a)^3*a^3*c^6-120/b^4*d^3/(b*x+a)^3*a^2*c^7+30/b^3*d^2/(b*x+a)^3*a*c^
8+180/b^10*d^9/(b*x+a)^2*a^7*c-630/b^9*d^8/(b*x+a)^2*a^6*c^2+1260/b^8*d^7/(
b*x+a)^2*a^5*c^3-1575/b^7*d^6/(b*x+a)^2*a^4*c^4+1260/b^6*d^5/(b*x+a)^2*a^3*
c^5-630/b^5*d^4/(b*x+a)^2*a^2*c^6+180/b^4*d^3/(b*x+a)^2*a*c^7-75*d^8/b^6*x^
3*a*c^2-175*d^9/b^8*x^2*a^3*c+675/2*d^8/b^7*x^2*a^2*c^2-300*d^7/b^6*x^2*a*c
^3
```

maxima [B] time = 1.93, size = 903, normalized size = 3.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*b^10*c^10 + 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*
d^3 - 5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6
+ 38280*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 8250*a^9*b*c*d^9 - 1207*a
^10*d^10 + 1440*(b^10*c^7*d^3 - 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 - 35*a
^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*d^7 - 21*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9
- a^7*b^3*d^10)*x^3 + 270*(b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 - 84*a^2*b^8*c^6*
d^4 + 280*a^3*b^7*c^5*d^5 - 490*a^4*b^6*c^4*d^6 + 504*a^5*b^5*c^3*d^7 - 308
```

```
*a^6*b^4*c^2*d^8 + 104*a^7*b^3*c*d^9 - 15*a^8*b^2*d^10)*x^2 + 20*(2*b^10*c^
9*d + 9*a*b^9*c^8*d^2 + 72*a^2*b^8*c^7*d^3 - 924*a^3*b^7*c^6*d^4 + 3276*a^4
*b^6*c^5*d^5 - 5922*a^5*b^5*c^4*d^6 + 6216*a^6*b^4*c^3*d^7 - 3852*a^7*b^3*c
^2*d^8 + 1314*a^8*b^2*c*d^9 - 191*a^9*b*d^10)*x)/(b^15*x^4 + 4*a*b^14*x^3 +
 6*a^2*b^13*x^2 + 4*a^3*b^12*x + a^4*b^11) + 1/12*(2*b^5*d^10*x^6 + 12*(2*b
^5*c*d^9 - a*b^4*d^10)*x^5 + 15*(9*b^5*c^2*d^8 - 10*a*b^4*c*d^9 + 3*a^2*b^3
*d^10)*x^4 + 20*(24*b^5*c^3*d^7 - 45*a*b^4*c^2*d^8 + 30*a^2*b^3*c*d^9 - 7*a
^3*b^2*d^10)*x^3 + 30*(42*b^5*c^4*d^6 - 120*a*b^4*c^3*d^7 + 135*a^2*b^3*c^2
*d^8 - 70*a^3*b^2*c*d^9 + 14*a^4*b*d^10)*x^2 + 12*(252*b^5*c^5*d^5 - 1050*a
*b^4*c^4*d^6 + 1800*a^2*b^3*c^3*d^7 - 1575*a^3*b^2*c^2*d^8 + 700*a^4*b*c*d^
9 - 126*a^5*d^10)*x)/b^10 + 210*(b^6*c^6*d^4 - 6*a*b^5*c^5*d^5 + 15*a^2*b^4
*c^4*d^6 - 20*a^3*b^3*c^3*d^7 + 15*a^4*b^2*c^2*d^8 - 6*a^5*b*c*d^9 + a^6*d^
10)*log(b*x + a)/b^11
```

mupad [**B**] time = 0.38, size = 1494, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^5,x)
```

```
[Out] x^2*((5*a*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/
b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^
2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/(2*b) - (5*a^4*d^10)/(2*b^9) + (
105*c^4*d^6)/b^5 + (5*a^3*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (5*a^2*(
(5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8
)/b^5))/b^2) - x^5*((a*d^10)/b^6 - (2*c*d^9)/b^5) - x^3*((5*a*((5*a*((5*a*d
^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/(3*b
) + (10*a^3*d^10)/(3*b^8) - (40*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 - (1
0*c*d^9)/b^5))/(3*b^2)) + x^4*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/(4*b
) - (5*a^2*d^10)/(2*b^7) + (45*c^2*d^8)/(4*b^5)) - x*((5*a*((5*a*((5*a*((5*
a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b
^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a^2*((5*a*d^10)/b^6 -
(10*c*d^9)/b^5))/b^2))/b - (5*a^4*d^10)/b^9 + (210*c^4*d^6)/b^5 + (10*a^3*(
(5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^3 - (10*a^2*((5*a*((5*a*d^10)/b^6 - (10
*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^2))/b + (a^5*d^1
0)/b^10 - (252*c^5*d^5)/b^5 - (5*a^4*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^4
 - (10*a^2*((5*a*((5*a*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)
/b^7 + (45*c^2*d^8)/b^5))/b + (10*a^3*d^10)/b^8 - (120*c^3*d^7)/b^5 - (10*a
^2*((5*a*d^10)/b^6 - (10*c*d^9)/b^5))/b^2))/b^2 + (10*a^3*((5*a*((5*a*d^10)
/b^6 - (10*c*d^9)/b^5))/b - (10*a^2*d^10)/b^7 + (45*c^2*d^8)/b^5))/b^3) - (
(3*b^10*c^10 - 1207*a^10*d^10 + 45*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 -
5250*a^4*b^6*c^6*d^4 + 19404*a^5*b^5*c^5*d^5 - 35910*a^6*b^4*c^4*d^6 + 3828
0*a^7*b^3*c^3*d^7 - 23985*a^8*b^2*c^2*d^8 + 10*a*b^9*c^9*d + 8250*a^9*b*c*d
^9)/(12*b) + x*((10*b^9*c^9*d)/3 - (955*a^9*d^10)/3 + 15*a*b^8*c^8*d^2 + 12
0*a^2*b^7*c^7*d^3 - 1540*a^3*b^6*c^6*d^4 + 5460*a^4*b^5*c^5*d^5 - 9870*a^5*
b^4*c^4*d^6 + 10360*a^6*b^3*c^3*d^7 - 6420*a^7*b^2*c^2*d^8 + 2190*a^8*b*c*d
^9) - x^3*(120*a^7*b^2*d^10 - 120*b^9*c^7*d^3 + 840*a*b^8*c^6*d^4 - 840*a^6
*b^3*c*d^9 - 2520*a^2*b^7*c^5*d^5 + 4200*a^3*b^6*c^4*d^6 - 4200*a^4*b^5*c^3
*d^7 + 2520*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)/2 - (675*a^8*b*d^10)/2
 + 180*a*b^8*c^7*d^3 + 2340*a^7*b^2*c*d^9 - 1890*a^2*b^7*c^6*d^4 + 6300*a^3
*b^6*c^5*d^5 - 11025*a^4*b^5*c^4*d^6 + 11340*a^5*b^4*c^3*d^7 - 6930*a^6*b^3
*c^2*d^8))/(a^4*b^10 + b^14*x^4 + 4*a^3*b^11*x + 4*a*b^13*x^3 + 6*a^2*b^12*
x^2) + (log(a + b*x)*(210*a^6*d^10 + 210*b^6*c^6*d^4 - 1260*a*b^5*c^5*d^5 +
 3150*a^2*b^4*c^4*d^6 - 4200*a^3*b^3*c^3*d^7 + 3150*a^4*b^2*c^2*d^8 - 1260*
a^5*b*c*d^9))/b^11 + (d^10*x^6)/(6*b^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.1211 $\int \frac{(c+dx)^{10}}{(a+bx)^6} \, dx$

Optimal. Leaf size=260

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{210d^4(b}{b^{11}(a}$$

Rubi [A] time = 0.42, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$\frac{5d^9(a+bx)^4(bc-ad)}{2b^{11}}+\frac{15d^8(a+bx)^3(bc-ad)^2}{b^{11}}+\frac{60d^7(a+bx)^2(bc-ad)^3}{b^{11}}+\frac{210d^6x(bc-ad)^4}{b^{10}}-\frac{210d^4(bc-ad)^6}{b^{11}(a+bx)}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}+\frac{252d^5(bc-ad)^5\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}+\frac{d^{10}(a+bx)^5}{5b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (210*d^6*(b*c - a*d)^4*x)/b^10 - (b*c - a*d)^10/(5*b^11*(a + b*x)^5) - (5*d
*(b*c - a*d)^9)/(2*b^11*(a + b*x)^4) - (15*d^2*(b*c - a*d)^8)/(b^11*(a + b*
x)^3) - (60*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^2) - (210*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)) + (60*d^7*(b*c - a*d)^3*(a + b*x)^2)/b^11 + (15*d^8*(b*c
- a*d)^2*(a + b*x)^3)/b^11 + (5*d^9*(b*c - a*d)*(a + b*x)^4)/(2*b^11) + (d^
10*(a + b*x)^5)/(5*b^11) + (252*d^5*(b*c - a*d)^5*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^6} \, dx &= \int \left(\frac{210d^6(bc-ad)^4}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^6}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^5}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^4}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^3}+\frac{2}{} \right.\\ &= \frac{210d^6(bc-ad)^4x}{b^{10}}-\frac{(bc-ad)^{10}}{5b^{11}(a+bx)^5}-\frac{5d(bc-ad)^9}{2b^{11}(a+bx)^4}-\frac{15d^2(bc-ad)^8}{b^{11}(a+bx)^3}-\frac{60d^3(bc-ad)^7}{b^{11}(a+bx)^2}-\frac{210d}{b^{11}}\end{aligned}$$

Mathematica [A] time = 0.21, size = 305, normalized size = 1.17

$$\frac{10b^3d^8x^3\left(7a^2d^2-20abcd+15b^2c^2\right)+10b^2d^7x^2\left(-28a^3d^3+105a^2bcd^2-135ab^2c^2d+60b^3c^3\right)+10bd^6x\left(126a^4d^4-560a^3bcd^3+945a^2b^2c^2d^2-720ab^3c^3d+210b^4c^4\right)+5b^4d^9x^4(5bc-3ad)+2520d^5(bc-ad)^5\log(a+bx)-\frac{2100d^4(bc-ad)^6}{a+bx}+\frac{600d^3(ad-bc)^7}{(a+bx)^2}-\frac{150d^2(bc-ad)^8}{(a+bx)^3}+\frac{25d(ad-bc)^9}{(a+bx)^4}-\frac{2(bc-ad)^{10}}{(a+bx)^5}+2b^5d^{10}x^5}{10b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] (10*b*d^6*(210*b^4*c^4 - 720*a*b^3*c^3*d + 945*a^2*b^2*c^2*d^2 - 560*a^3*b*
c*d^3 + 126*a^4*d^4)*x + 10*b^2*d^7*(60*b^3*c^3 - 135*a*b^2*c^2*d + 105*a^2
*b*c*d^2 - 28*a^3*d^3)*x^2 + 10*b^3*d^8*(15*b^2*c^2 - 20*a*b*c*d + 7*a^2*d^
2)*x^3 + 5*b^4*d^9*(5*b*c - 3*a*d)*x^4 + 2*b^5*d^10*x^5 - (2*(b*c - a*d)^10
)/(a + b*x)^5 + (25*d*(-(b*c) + a*d)^9)/(a + b*x)^4 - (150*d^2*(b*c - a*d)^
8)/(a + b*x)^3 + (600*d^3*(-(b*c) + a*d)^7)/(a + b*x)^2 - (2100*d^4*(b*c -
a*d)^6)/(a + b*x) + 2520*d^5*(b*c - a*d)^5*Log[a + b*x])/(10*b^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^6} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^6,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^6, x]
```

fricas [B] time = 1.27, size = 1395, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/10*(2*b^10*d^10*x^10 - 2*b^10*c^10 - 5*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2 -
 60*a^3*b^7*c^7*d^3 - 420*a^4*b^6*c^6*d^4 + 5754*a^5*b^5*c^5*d^5 - 18270*a^
6*b^4*c^4*d^6 + 27540*a^7*b^3*c^3*d^7 - 22290*a^8*b^2*c^2*d^8 + 9395*a^9*b*
c*d^9 - 1627*a^10*d^10 + 5*(5*b^10*c*d^9 - a*b^9*d^10)*x^9 + 15*(10*b^10*c^
2*d^8 - 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 60*(10*b^10*c^3*d^7 - 10*a*b^9*
c^2*d^8 + 5*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 420*(5*b^10*c^4*d^6 - 10*a*
b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 - 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + (1
0500*a*b^9*c^4*d^6 - 30000*a^2*b^8*c^3*d^7 + 35250*a^3*b^7*c^2*d^8 - 19375*
a^4*b^6*c*d^9 + 4127*a^5*b^5*d^10)*x^5 - 5*(420*b^10*c^6*d^4 - 2520*a*b^9*c
^5*d^5 + 2100*a^2*b^8*c^4*d^6 + 4800*a^3*b^7*c^3*d^7 - 10050*a^4*b^6*c^2*d^
8 + 6775*a^5*b^5*c*d^9 - 1607*a^6*b^4*d^10)*x^4 - 10*(60*b^10*c^7*d^3 + 420
*a*b^9*c^6*d^4 - 3780*a^2*b^8*c^5*d^5 + 8400*a^3*b^7*c^4*d^6 - 7800*a^4*b^6
*c^3*d^7 + 2550*a^5*b^5*c^2*d^8 + 475*a^6*b^4*c*d^9 - 347*a^7*b^3*d^10)*x^3
 - 10*(15*b^10*c^8*d^2 + 60*a*b^9*c^7*d^3 + 420*a^2*b^8*c^6*d^4 - 4620*a^3*
b^7*c^5*d^5 + 12600*a^4*b^6*c^4*d^6 - 16200*a^5*b^5*c^3*d^7 + 10950*a^6*b^4
*c^2*d^8 - 3725*a^7*b^3*c*d^9 + 493*a^8*b^2*d^10)*x^2 - 5*(5*b^10*c^9*d + 1
5*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 - 5250*a^4*b^6*c
^5*d^5 + 15750*a^5*b^5*c^4*d^6 - 22500*a^6*b^4*c^3*d^7 + 17250*a^7*b^3*c^2*
d^8 - 6875*a^8*b^2*c*d^9 + 1123*a^9*b*d^10)*x + 2520*(a^5*b^5*c^5*d^5 - 5*a
^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 - 10*a^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 -
a^10*d^10 + (b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b
^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 5*(a*b^9*c^5*d^5 - 5*a^2
*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 - 10*a^4*b^6*c^2*d^8 + 5*a^5*b^5*c*d^9 -
a^6*b^4*d^10)*x^4 + 10*(a^2*b^8*c^5*d^5 - 5*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^
3*d^7 - 10*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 10*(a^3*
b^7*c^5*d^5 - 5*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 - 10*a^6*b^4*c^2*d^8 +
 5*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 5*(a^4*b^6*c^5*d^5 - 5*a^5*b^5*c^4*d
^6 + 10*a^6*b^4*c^3*d^7 - 10*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 - a^9*b*d^10
)*x)*log(b*x + a))/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b^14*x^3 + 10*a^3*b^13
*x^2 + 5*a^4*b^12*x + a^5*b^11)
```

giac [B] time = 1.34, size = 883, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 252*(b^5*c^5*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^
8 + 5*a^4*b*c*d^9 - a^5*d^10)*log(abs(b*x + a))/b^11 - 1/10*(2*b^10*c^10 +
5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d
^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 +
 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*d^10 + 2100*(b^10*c^6
*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b^7*c^3*d^7 + 15*a^4*b
^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600*(b^10*c^7*d^3 + 7*a*
b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6 - 245*a^4*b^6*c^3*d^
7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^3*d^10)*x^3 + 150*(b^
10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 308*a^3*b^7*c^5*d^5 + 9
```

```
10*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b^4*c^2*d^8 - 428*a^7*
b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*a*b^9*c^8*d^2 + 12*a^
2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*d^5 + 3234*a^5*b^5*c^
4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 - 1599*a^8*b^2*c*d^9 +
275*a^9*b*d^10)*x)/((b*x + a)^5*b^11) + 1/10*(2*b^24*d^10*x^5 + 25*b^24*c*d
^9*x^4 - 15*a*b^23*d^10*x^4 + 150*b^24*c^2*d^8*x^3 - 200*a*b^23*c*d^9*x^3 +
 70*a^2*b^22*d^10*x^3 + 600*b^24*c^3*d^7*x^2 - 1350*a*b^23*c^2*d^8*x^2 + 10
50*a^2*b^22*c*d^9*x^2 - 280*a^3*b^21*d^10*x^2 + 2100*b^24*c^4*d^6*x - 7200*
a*b^23*c^3*d^7*x + 9450*a^2*b^22*c^2*d^8*x - 5600*a^3*b^21*c*d^9*x + 1260*a
^4*b^20*d^10*x)/b^30
```

maple [B] time = 0.02, size = 1199, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^6,x)
```

```
[Out] -252/b^11*d^10*ln(b*x+a)*a^5+252/b^6*d^5*ln(b*x+a)*c^5+5/2/b^11*d^10/(b*x+a
)^4*a^9-5/2/b^2*d/(b*x+a)^4*c^9-210/b^11*d^10/(b*x+a)*a^6-210/b^5*d^4/(b*x+
a)*c^6-3/2*d^10/b^7*x^4*a+5/2*d^9/b^6*x^4*c+7*d^10/b^8*x^3*a^2+15*d^8/b^6*x
^3*c^2-28*d^10/b^9*x^2*a^3+60*d^7/b^6*x^2*c^3+126*d^10/b^10*a^4*x+210*d^6/b
^6*c^4*x-15/b^11*d^10/(b*x+a)^3*a^8-15/b^3*d^2/(b*x+a)^3*c^8-1/5/b^11/(b*x+
a)^5*a^10*d^10+60/b^11*d^10/(b*x+a)^2*a^7-60/b^4*d^3/(b*x+a)^2*c^7+1260/b^1
0*d^9*ln(b*x+a)*a^4*c-2520/b^9*d^8*ln(b*x+a)*a^3*c^2-1260/b^6*d^5/(b*x+a)^2
*a^2*c^5+420/b^5*d^4/(b*x+a)^2*a*c^6-135*d^8/b^7*x^2*a*c^2-560*d^9/b^9*a^3*
c*x+945*d^8/b^8*a^2*c^2*x-720*d^7/b^7*a*c^3*x+120/b^10*d^9/(b*x+a)^3*a^7*c-
420/b^9*d^8/(b*x+a)^3*a^6*c^2+840/b^8*d^7/(b*x+a)^3*a^5*c^3-1050/b^7*d^6/(b
*x+a)^3*a^4*c^4+840/b^6*d^5/(b*x+a)^3*a^3*c^5-420/b^5*d^4/(b*x+a)^3*a^2*c^6
+120/b^4*d^3/(b*x+a)^3*a*c^7+2/b^10/(b*x+a)^5*a^9*c*d^9-9/b^9/(b*x+a)^5*a^8
*c^2*d^8+24/b^8/(b*x+a)^5*a^7*c^3*d^7-42/b^7/(b*x+a)^5*a^6*c^4*d^6+252/5/b^
6/(b*x+a)^5*a^5*c^5*d^5-42/b^5/(b*x+a)^5*a^4*c^6*d^4+24/b^4/(b*x+a)^5*a^3*c
^7*d^3-9/b^3/(b*x+a)^5*a^2*c^8*d^2+2520/b^8*d^7*ln(b*x+a)*a^2*c^3-1260/b^7*
d^6*ln(b*x+a)*a*c^4-20*d^9/b^7*x^3*a*c+105*d^9/b^8*x^2*a^2*c-45/2/b^10*d^9/
(b*x+a)^4*a^8*c+90/b^9*d^8/(b*x+a)^4*a^7*c^2-210/b^8*d^7/(b*x+a)^4*a^6*c^3+
315/b^7*d^6/(b*x+a)^4*a^5*c^4-315/b^6*d^5/(b*x+a)^4*a^4*c^5+210/b^5*d^4/(b*
x+a)^4*a^3*c^6-90/b^4*d^3/(b*x+a)^4*a^2*c^7+45/2/b^3*d^2/(b*x+a)^4*a*c^8+12
60/b^10*d^9/(b*x+a)*a^5*c-3150/b^9*d^8/(b*x+a)*a^4*c^2+4200/b^8*d^7/(b*x+a)
*a^3*c^3-3150/b^7*d^6/(b*x+a)*a^2*c^4+1260/b^6*d^5/(b*x+a)*a*c^5+2/b^2/(b*x
+a)^5*a*c^9*d-420/b^10*d^9/(b*x+a)^2*a^6*c+1260/b^9*d^8/(b*x+a)^2*a^5*c^2-2
100/b^8*d^7/(b*x+a)^2*a^4*c^3+2100/b^7*d^6/(b*x+a)^2*a^3*c^4+1/5*d^10/b^6*x
^5-1/5/b/(b*x+a)^5*c^10
```

maxima [B] time = 2.25, size = 912, normalized size = 3.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/10*(2*b^10*c^10 + 5*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^
3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 + 18270*a^6*b^4*c^4*d^6 - 27
540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 - 9395*a^9*b*c*d^9 + 1627*a^10*
d^10 + 2100*(b^10*c^6*d^4 - 6*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 20*a^3*b
^7*c^3*d^7 + 15*a^4*b^6*c^2*d^8 - 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 600
*(b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 - 63*a^2*b^8*c^5*d^5 + 175*a^3*b^7*c^4*d^6
 - 245*a^4*b^6*c^3*d^7 + 189*a^5*b^5*c^2*d^8 - 77*a^6*b^4*c*d^9 + 13*a^7*b^
3*d^10)*x^3 + 150*(b^10*c^8*d^2 + 4*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 - 30
8*a^3*b^7*c^5*d^5 + 910*a^4*b^6*c^4*d^6 - 1316*a^5*b^5*c^3*d^7 + 1036*a^6*b
^4*c^2*d^8 - 428*a^7*b^3*c*d^9 + 73*a^8*b^2*d^10)*x^2 + 25*(b^10*c^9*d + 3*
a*b^9*c^8*d^2 + 12*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 - 1050*a^4*b^6*c^5*
```

```
d^5 + 3234*a^5*b^5*c^4*d^6 - 4788*a^6*b^4*c^3*d^7 + 3828*a^7*b^3*c^2*d^8 -
1599*a^8*b^2*c*d^9 + 275*a^9*b*d^10)*x)/(b^16*x^5 + 5*a*b^15*x^4 + 10*a^2*b
^14*x^3 + 10*a^3*b^13*x^2 + 5*a^4*b^12*x + a^5*b^11) + 1/10*(2*b^4*d^10*x^5
 + 5*(5*b^4*c*d^9 - 3*a*b^3*d^10)*x^4 + 10*(15*b^4*c^2*d^8 - 20*a*b^3*c*d^9
 + 7*a^2*b^2*d^10)*x^3 + 10*(60*b^4*c^3*d^7 - 135*a*b^3*c^2*d^8 + 105*a^2*b
^2*c*d^9 - 28*a^3*b*d^10)*x^2 + 10*(210*b^4*c^4*d^6 - 720*a*b^3*c^3*d^7 + 9
45*a^2*b^2*c^2*d^8 - 560*a^3*b*c*d^9 + 126*a^4*d^10)*x)/b^10 + 252*(b^5*c^5
*d^5 - 5*a*b^4*c^4*d^6 + 10*a^2*b^3*c^3*d^7 - 10*a^3*b^2*c^2*d^8 + 5*a^4*b*
c*d^9 - a^5*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.40, size = 1141, normalized size = 4.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^6,x)
```

```
[Out] x^3*((2*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (5*a^2*d^10)/b^8 + (15*c^2
*d^8)/b^6) - x^2*((3*a*((6*a*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2
*d^10)/b^8 + (45*c^2*d^8)/b^6))/b + (10*a^3*d^10)/b^9 - (60*c^3*d^7)/b^6 -
(15*a^2*((6*a*d^10)/b^7 - (10*c*d^9)/b^6))/(2*b^2)) - x^4*((3*a*d^10)/(2*b^
7) - (5*c*d^9)/(2*b^6)) - (x^4*(210*a^6*b^3*d^10 + 210*b^9*c^6*d^4 - 1260*a
*b^8*c^5*d^5 - 1260*a^5*b^4*c*d^9 + 3150*a^2*b^7*c^4*d^6 - 4200*a^3*b^6*c^3
*d^7 + 3150*a^4*b^5*c^2*d^8) + (1627*a^10*d^10 + 2*b^10*c^10 + 15*a^2*b^8*c
^8*d^2 + 60*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 - 5754*a^5*b^5*c^5*d^5 +
18270*a^6*b^4*c^4*d^6 - 27540*a^7*b^3*c^3*d^7 + 22290*a^8*b^2*c^2*d^8 + 5*a
*b^9*c^9*d - 9395*a^9*b*c*d^9)/(10*b) + x*((1375*a^9*d^10)/2 + (5*b^9*c^9*d
)/2 + (15*a*b^8*c^8*d^2)/2 + 30*a^2*b^7*c^7*d^3 + 210*a^3*b^6*c^6*d^4 - 262
5*a^4*b^5*c^5*d^5 + 8085*a^5*b^4*c^4*d^6 - 11970*a^6*b^3*c^3*d^7 + 9570*a^7
*b^2*c^2*d^8 - (7995*a^8*b*c*d^9)/2) + x^3*(780*a^7*b^2*d^10 + 60*b^9*c^7*d
^3 + 420*a*b^8*c^6*d^4 - 4620*a^6*b^3*c*d^9 - 3780*a^2*b^7*c^5*d^5 + 10500*
a^3*b^6*c^4*d^6 - 14700*a^4*b^5*c^3*d^7 + 11340*a^5*b^4*c^2*d^8) + x^2*(109
5*a^8*b*d^10 + 15*b^9*c^8*d^2 + 60*a*b^8*c^7*d^3 - 6420*a^7*b^2*c*d^9 + 420
*a^2*b^7*c^6*d^4 - 4620*a^3*b^6*c^5*d^5 + 13650*a^4*b^5*c^4*d^6 - 19740*a^5
*b^4*c^3*d^7 + 15540*a^6*b^3*c^2*d^8))/(a^5*b^10 + b^15*x^5 + 5*a^4*b^11*x
+ 5*a*b^14*x^4 + 10*a^3*b^12*x^2 + 10*a^2*b^13*x^3) + x*((6*a*((6*a*((6*a*(
(6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6)
)/b + (20*a^3*d^10)/b^9 - (120*c^3*d^7)/b^6 - (15*a^2*((6*a*d^10)/b^7 - (10
*c*d^9)/b^6))/b^2))/b - (15*a^4*d^10)/b^10 + (210*c^4*d^6)/b^6 + (20*a^3*((
6*a*d^10)/b^7 - (10*c*d^9)/b^6))/b^3 - (15*a^2*((6*a*((6*a*d^10)/b^7 - (10*
c*d^9)/b^6))/b - (15*a^2*d^10)/b^8 + (45*c^2*d^8)/b^6))/b^2) + (d^10*x^5)/(
5*b^6) - (log(a + b*x)*(252*a^5*d^10 - 252*b^5*c^5*d^5 + 1260*a*b^4*c^4*d^6
 - 2520*a^2*b^3*c^3*d^7 + 2520*a^3*b^2*c^2*d^8 - 1260*a^4*b*c*d^9))/b^11
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**6,x)
```

```
[Out] Timed out
```

3.1212 $\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx$

Optimal. Leaf size=262

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)}{b^{11}(a+bx)^2}$$

Rubi [A] time = 0.39, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{10d^9(a+bx)^3(bc-ad)}{3b^{11}}+\frac{45d^8(a+bx)^2(bc-ad)^2}{2b^{11}}+\frac{120d^7x(bc-ad)^3}{b^{10}}-\frac{252d^5(bc-ad)^5}{b^{11}(a+bx)}-\frac{105d^4(bc-ad)^6}{b^{11}(a+bx)^2}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}+\frac{210d^6(bc-ad)^4\log(a+bx)}{b^{11}}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}+\frac{d^{10}(a+bx)^4}{4b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (120*d^7*(b*c - a*d)^3*x)/b^10 - (b*c - a*d)^10/(6*b^11*(a + b*x)^6) - (2*d
*(b*c - a*d)^9)/(b^11*(a + b*x)^5) - (45*d^2*(b*c - a*d)^8)/(4*b^11*(a + b*
x)^4) - (40*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^3) - (105*d^4*(b*c - a*d)^6)
/(b^11*(a + b*x)^2) - (252*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)) + (45*d^8*(b
*c - a*d)^2*(a + b*x)^2)/(2*b^11) + (10*d^9*(b*c - a*d)*(a + b*x)^3)/(3*b^1
1) + (d^10*(a + b*x)^4)/(4*b^11) + (210*d^6*(b*c - a*d)^4*Log[a + b*x])/b^1
1
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx = \int \left(\frac{120d^7(bc-ad)^3}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^7}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^6}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^5}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^4}+\frac{2}{}\right.$$

$$=\frac{120d^7(bc-ad)^3x}{b^{10}}-\frac{(bc-ad)^{10}}{6b^{11}(a+bx)^6}-\frac{2d(bc-ad)^9}{b^{11}(a+bx)^5}-\frac{45d^2(bc-ad)^8}{4b^{11}(a+bx)^4}-\frac{40d^3(bc-ad)^7}{b^{11}(a+bx)^3}-\frac{105d^4}{b^{11}(}$$

Mathematica [A] time = 0.22, size = 265, normalized size = 1.01

$$\frac{6b^2d^8x^2\left(28a^2d^2-70abcd+45b^2c^2\right)+12bd^7x\left(-84a^3d^3+280a^2bcd^2-315ab^2c^2d+120b^3c^3\right)+4b^3d^9x^3(10bc-7ad)+2520d^6(bc-ad)^4\log(a+bx)+\frac{3024d^5(ad-bc)^5}{a+bx}-\frac{1260d^4(bc-ad)^6}{(a+bx)^2}+\frac{480d^3(ad-bc)^7}{(a+bx)^3}-\frac{135d^2(bc-ad)^8}{(a+bx)^4}+\frac{24d(ad-bc)^9}{(a+bx)^5}-\frac{2(bc-ad)^{10}}{(a+bx)^6}+3b^4d^{10}x^4}{12b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] (12*b*d^7*(120*b^3*c^3 - 315*a*b^2*c^2*d + 280*a^2*b*c*d^2 - 84*a^3*d^3)*x
 + 6*b^2*d^8*(45*b^2*c^2 - 70*a*b*c*d + 28*a^2*d^2)*x^2 + 4*b^3*d^9*(10*b*c
 - 7*a*d)*x^3 + 3*b^4*d^10*x^4 - (2*(b*c - a*d)^10)/(a + b*x)^6 + (24*d*(-(b
*c) + a*d)^9)/(a + b*x)^5 - (135*d^2*(b*c - a*d)^8)/(a + b*x)^4 + (480*d^3*
(-(b*c) + a*d)^7)/(a + b*x)^3 - (1260*d^4*(b*c - a*d)^6)/(a + b*x)^2 + (302
4*d^5*(-(b*c) + a*d)^5)/(a + b*x) + 2520*d^6*(b*c - a*d)^4*Log[a + b*x])/(1
2*b^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^7}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^7,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^7, x]
```

fricas [B] time = 1.23, size = 1386, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^10*d^10*x^10 - 2*b^10*c^10 - 4*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
24*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 504*a^5*b^5*c^5*d^5 + 6174*a^6*b^
4*c^4*d^6 - 16056*a^7*b^3*c^3*d^7 + 18414*a^8*b^2*c^2*d^8 - 10036*a^9*b*c*d
^9 + 2131*a^10*d^10 + 10*(4*b^10*c*d^9 - a*b^9*d^10)*x^9 + 45*(6*b^10*c^2*d
^8 - 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 360*(4*b^10*c^3*d^7 - 6*a*b^9*c^2*
d^8 + 4*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + (8640*a*b^9*c^3*d^7 - 18630*a^2
*b^8*c^2*d^8 + 14660*a^3*b^7*c*d^9 - 4043*a^4*b^6*d^10)*x^6 - 6*(504*b^10*c
^5*d^5 - 2520*a*b^9*c^4*d^6 + 1440*a^2*b^8*c^3*d^7 + 3510*a^3*b^7*c^2*d^8 -
 4580*a^4*b^6*c*d^9 + 1523*a^5*b^5*d^10)*x^5 - 15*(84*b^10*c^6*d^4 + 504*a*
b^9*c^5*d^5 - 3780*a^2*b^8*c^4*d^6 + 6480*a^3*b^7*c^3*d^7 - 4050*a^4*b^6*c^
2*d^8 + 460*a^5*b^5*c*d^9 + 263*a^6*b^4*d^10)*x^4 - 20*(24*b^10*c^7*d^3 + 8
4*a*b^9*c^6*d^4 + 504*a^2*b^8*c^5*d^5 - 4620*a^3*b^7*c^4*d^6 + 9840*a^4*b^6
*c^3*d^7 - 9090*a^5*b^5*c^2*d^8 + 3820*a^6*b^4*c*d^9 - 577*a^7*b^3*d^10)*x^
3 - 15*(9*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 504*a^3*b^
7*c^5*d^5 - 5250*a^4*b^6*c^4*d^6 + 12360*a^5*b^5*c^3*d^7 - 12870*a^6*b^4*c^
2*d^8 + 6340*a^7*b^3*c*d^9 - 1207*a^8*b^2*d^10)*x^2 - 6*(4*b^10*c^9*d + 9*a
*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^
5 - 5754*a^5*b^5*c^4*d^6 + 14376*a^6*b^4*c^3*d^7 - 15894*a^7*b^3*c^2*d^8 +
8356*a^8*b^2*c*d^9 - 1711*a^9*b*d^10)*x + 2520*(a^6*b^4*c^4*d^6 - 4*a^7*b^3
*c^3*d^7 + 6*a^8*b^2*c^2*d^8 - 4*a^9*b*c*d^9 + a^10*d^10 + (b^10*c^4*d^6 -
4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 +
 6*(a*b^9*c^4*d^6 - 4*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 - 4*a^4*b^6*c*d^9
 + a^5*b^5*d^10)*x^5 + 15*(a^2*b^8*c^4*d^6 - 4*a^3*b^7*c^3*d^7 + 6*a^4*b^6*
c^2*d^8 - 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 20*(a^3*b^7*c^4*d^6 - 4*a^4
*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 - 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 15
*(a^4*b^6*c^4*d^6 - 4*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 - 4*a^7*b^3*c*d^9
 + a^8*b^2*d^10)*x^2 + 6*(a^5*b^5*c^4*d^6 - 4*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c
^2*d^8 - 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)*log(b*x + a))/(b^17*x^6 + 6*a*b^1
6*x^5 + 15*a^2*b^15*x^4 + 20*a^3*b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x
+ a^6*b^11)
```

giac [B] time = 1.29, size = 878, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="giac")
```

```
[Out] 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^
4*d^10)*log(abs(b*x + a))/b^11 - 1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*
b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5
 - 6174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 1
0036*a^9*b*c*d^9 - 2131*a^10*d^10 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 +
10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x
^5 + 1260*(b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^
7*c^3*d^7 - 105*a^4*b^6*c^2*d^8 + 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 +
 240*(2*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c
^4*d^6 + 910*a^4*b^6*c^3*d^7 - 987*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 10
```

```
7*a^7*b^3*d^10)*x^3 + 45*(3*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6
*d^4 + 168*a^3*b^7*c^5*d^5 - 1750*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 -
4788*a^6*b^4*c^2*d^8 + 2552*a^7*b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^
10*c^9*d + 9*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*
a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*
b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9 - 1879*a^9*b*d^10)*x)/((b*x + a)^6*b^11) +
 1/12*(3*b^21*d^10*x^4 + 40*b^21*c*d^9*x^3 - 28*a*b^20*d^10*x^3 + 270*b^21*
c^2*d^8*x^2 - 420*a*b^20*c*d^9*x^2 + 168*a^2*b^19*d^10*x^2 + 1440*b^21*c^3*
d^7*x - 3780*a*b^20*c^2*d^8*x + 3360*a^2*b^19*c*d^9*x - 1008*a^3*b^18*d^10*
x)/b^28
```

maple [B] time = 0.02, size = 1222, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^7,x)
```

```
[Out] 252/b^11*d^10/(b*x+a)*a^5-252/b^6*d^5/(b*x+a)*c^5-1/6/b^11/(b*x+a)^6*a^10*d
^10+2/b^11*d^10/(b*x+a)^5*a^9-2/b^2*d/(b*x+a)^5*c^9-105/b^11*d^10/(b*x+a)^2
*a^6-105/b^5*d^4/(b*x+a)^2*c^6+210/b^11*d^10*ln(b*x+a)*a^4+210/b^7*d^6*ln(b
*x+a)*c^4-45/4/b^11*d^10/(b*x+a)^4*a^8-45/4/b^3*d^2/(b*x+a)^4*c^8+45/2*d^8/
b^7*x^2*c^2-84*d^10/b^10*a^3*x+120*d^7/b^7*c^3*x+40/b^11*d^10/(b*x+a)^3*a^7
-40/b^4*d^3/(b*x+a)^3*c^7-7/3*d^10/b^8*x^3*a+10/3*d^9/b^7*x^3*c+14*d^10/b^9
*x^2*a^2-315*d^8/b^8*a*c^2*x-280/b^10*d^9/(b*x+a)^3*a^6*c+2100/b^8*d^7/(b*x
+a)^2*a^3*c^3-1575/b^7*d^6/(b*x+a)^2*a^2*c^4+72/b^9*d^8/(b*x+a)^5*a^7*c^2-1
68/b^8*d^7/(b*x+a)^5*a^6*c^3+252/b^7*d^6/(b*x+a)^5*a^5*c^4-252/b^6*d^5/(b*x
+a)^5*a^4*c^5+168/b^5*d^4/(b*x+a)^5*a^3*c^6-72/b^4*d^3/(b*x+a)^5*a^2*c^7+18
/b^3*d^2/(b*x+a)^5*a*c^8+840/b^9*d^8/(b*x+a)^3*a^5*c^2-1400/b^8*d^7/(b*x+a)
^3*a^4*c^3+1400/b^7*d^6/(b*x+a)^3*a^3*c^4-840/b^6*d^5/(b*x+a)^3*a^2*c^5+280
/b^5*d^4/(b*x+a)^3*a*c^6+630/b^6*d^5/(b*x+a)^2*a*c^5-35*d^9/b^8*x^2*a*c+280
*d^9/b^9*a^2*c*x-18/b^10*d^9/(b*x+a)^5*a^8*c+1260/b^7*d^6/(b*x+a)*a*c^4+5/3
/b^10/(b*x+a)^6*a^9*c*d^9-15/2/b^9/(b*x+a)^6*a^8*c^2*d^8+20/b^8/(b*x+a)^6*a
^7*c^3*d^7-35/b^7/(b*x+a)^6*a^6*c^4*d^6+42/b^6/(b*x+a)^6*a^5*c^5*d^5-35/b^5
/(b*x+a)^6*a^4*c^6*d^4+20/b^4/(b*x+a)^6*a^3*c^7*d^3-15/2/b^3/(b*x+a)^6*a^2*
c^8*d^2+5/3/b^2/(b*x+a)^6*a*c^9*d+90/b^10*d^9/(b*x+a)^4*a^7*c-315/b^9*d^8/(
b*x+a)^4*a^6*c^2+630/b^8*d^7/(b*x+a)^4*a^5*c^3-1575/2/b^7*d^6/(b*x+a)^4*a^4
*c^4+630/b^6*d^5/(b*x+a)^4*a^3*c^5-315/b^5*d^4/(b*x+a)^4*a^2*c^6+90/b^4*d^3
/(b*x+a)^4*a*c^7-1260/b^10*d^9/(b*x+a)*a^4*c+2520/b^9*d^8/(b*x+a)*a^3*c^2-2
520/b^8*d^7/(b*x+a)*a^2*c^3+1260/b^9*d^8*ln(b*x+a)*a^2*c^2-840/b^8*d^7*ln(b
*x+a)*a*c^3-840/b^10*d^9*ln(b*x+a)*a^3*c+630/b^10*d^9/(b*x+a)^2*a^5*c-1575/
b^9*d^8/(b*x+a)^2*a^4*c^2+1/4*d^10/b^7*x^4-1/6/b/(b*x+a)^6*c^10
```

maxima [B] time = 2.45, size = 925, normalized size = 3.53

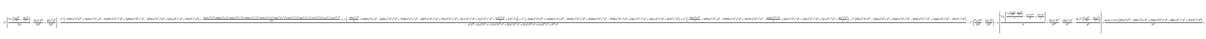
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^7,x, algorithm="maxima")
```

```
[Out] -1/12*(2*b^10*c^10 + 4*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3
 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6174*a^6*b^4*c^4*d^6 + 16056*
a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 10036*a^9*b*c*d^9 - 2131*a^10*d^1
0 + 3024*(b^10*c^5*d^5 - 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 - 10*a^3*b^7*
c^2*d^8 + 5*a^4*b^6*c*d^9 - a^5*b^5*d^10)*x^5 + 1260*(b^10*c^6*d^4 + 6*a*b^
9*c^5*d^5 - 45*a^2*b^8*c^4*d^6 + 100*a^3*b^7*c^3*d^7 - 105*a^4*b^6*c^2*d^8
+ 54*a^5*b^5*c*d^9 - 11*a^6*b^4*d^10)*x^4 + 240*(2*b^10*c^7*d^3 + 7*a*b^9*c
^6*d^4 + 42*a^2*b^8*c^5*d^5 - 385*a^3*b^7*c^4*d^6 + 910*a^4*b^6*c^3*d^7 - 9
87*a^5*b^5*c^2*d^8 + 518*a^6*b^4*c*d^9 - 107*a^7*b^3*d^10)*x^3 + 45*(3*b^10
*c^8*d^2 + 8*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c^5*d^5 - 175
0*a^4*b^6*c^4*d^6 + 4312*a^5*b^5*c^3*d^7 - 4788*a^6*b^4*c^2*d^8 + 2552*a^7*
```

```
b^3*c*d^9 - 533*a^8*b^2*d^10)*x^2 + 6*(4*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 24*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5 - 5754*a^5*b^5*c
^4*d^6 + 14616*a^6*b^4*c^3*d^7 - 16524*a^7*b^3*c^2*d^8 + 8916*a^8*b^2*c*d^9
 - 1879*a^9*b*d^10)*x)/(b^17*x^6 + 6*a*b^16*x^5 + 15*a^2*b^15*x^4 + 20*a^3*
b^14*x^3 + 15*a^4*b^13*x^2 + 6*a^5*b^12*x + a^6*b^11) + 1/12*(3*b^3*d^10*x^
4 + 4*(10*b^3*c*d^9 - 7*a*b^2*d^10)*x^3 + 6*(45*b^3*c^2*d^8 - 70*a*b^2*c*d^
9 + 28*a^2*b*d^10)*x^2 + 12*(120*b^3*c^3*d^7 - 315*a*b^2*c^2*d^8 + 280*a^2*
b*c*d^9 - 84*a^3*d^10)*x)/b^10 + 210*(b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2
*b^2*c^2*d^8 - 4*a^3*b*c*d^9 + a^4*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.42, size = 997, normalized size = 3.81



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^7,x)
```

```
[Out] x^2*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/(2*b) - (21*a^2*d^10)/(2*b^9)
+ (45*c^2*d^8)/(2*b^7)) - (x^4*(105*b^9*c^6*d^4 - 1155*a^6*b^3*d^10 + 630*a
*b^8*c^5*d^5 + 5670*a^5*b^4*c*d^9 - 4725*a^2*b^7*c^4*d^6 + 10500*a^3*b^6*c^
3*d^7 - 11025*a^4*b^5*c^2*d^8) + (2*b^10*c^10 - 2131*a^10*d^10 + 9*a^2*b^8*
c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 - 6
174*a^6*b^4*c^4*d^6 + 16056*a^7*b^3*c^3*d^7 - 18414*a^8*b^2*c^2*d^8 + 4*a*b
^9*c^9*d + 10036*a^9*b*c*d^9)/(12*b) + x*(2*b^9*c^9*d - (1879*a^9*d^10)/2 +
 (9*a*b^8*c^8*d^2)/2 + 12*a^2*b^7*c^7*d^3 + 42*a^3*b^6*c^6*d^4 + 252*a^4*b^
5*c^5*d^5 - 2877*a^5*b^4*c^4*d^6 + 7308*a^6*b^3*c^3*d^7 - 8262*a^7*b^2*c^2*
d^8 + 4458*a^8*b*c*d^9) + x^3*(40*b^9*c^7*d^3 - 2140*a^7*b^2*d^10 + 140*a*b
^8*c^6*d^4 + 10360*a^6*b^3*c*d^9 + 840*a^2*b^7*c^5*d^5 - 7700*a^3*b^6*c^4*d
^6 + 18200*a^4*b^5*c^3*d^7 - 19740*a^5*b^4*c^2*d^8) + x^2*((45*b^9*c^8*d^2)
/4 - (7995*a^8*b*d^10)/4 + 30*a*b^8*c^7*d^3 + 9570*a^7*b^2*c*d^9 + 105*a^2*
b^7*c^6*d^4 + 630*a^3*b^6*c^5*d^5 - (13125*a^4*b^5*c^4*d^6)/2 + 16170*a^5*b
^4*c^3*d^7 - 17955*a^6*b^3*c^2*d^8) - x^5*(252*a^5*b^4*d^10 - 252*b^9*c^5*d
^5 + 1260*a*b^8*c^4*d^6 - 1260*a^4*b^5*c*d^9 - 2520*a^2*b^7*c^3*d^7 + 2520*
a^3*b^6*c^2*d^8))/(a^6*b^10 + b^16*x^6 + 6*a^5*b^11*x + 6*a*b^15*x^5 + 15*a
^4*b^12*x^2 + 20*a^3*b^13*x^3 + 15*a^2*b^14*x^4) - x^3*((7*a*d^10)/(3*b^8)
- (10*c*d^9)/(3*b^7)) - x*((7*a*((7*a*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b
- (21*a^2*d^10)/b^9 + (45*c^2*d^8)/b^7))/b + (35*a^3*d^10)/b^10 - (120*c^3*
d^7)/b^7 - (21*a^2*((7*a*d^10)/b^8 - (10*c*d^9)/b^7))/b^2) + (log(a + b*x)*
(210*a^4*d^10 + 210*b^4*c^4*d^6 - 840*a*b^3*c^3*d^7 + 1260*a^2*b^2*c^2*d^8
- 840*a^3*b*c*d^9))/b^11 + (d^10*x^4)/(4*b^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**7,x)
```

```
[Out] Timed out
```

3.1213 $\int \frac{(c+dx)^{10}}{(a+bx)^8} dx$

Optimal. Leaf size=258

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc}{b^{11}(a}$$

Rubi [A] time = 0.36, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{5d^9(a+bx)^2(bc-ad)}{b^{11}}+\frac{45d^8x(bc-ad)^2}{b^{10}}-\frac{210d^6(bc-ad)^4}{b^{11}(a+bx)}-\frac{126d^5(bc-ad)^5}{b^{11}(a+bx)^2}-\frac{70d^4(bc-ad)^6}{b^{11}(a+bx)^3}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}+\frac{120d^7(bc-ad)^3\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}+\frac{d^{10}(a+bx)^3}{3b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (45*d^8*(b*c - a*d)^2*x)/b^10 - (b*c - a*d)^10/(7*b^11*(a + b*x)^7) - (5*d*
(b*c - a*d)^9)/(3*b^11*(a + b*x)^6) - (9*d^2*(b*c - a*d)^8)/(b^11*(a + b*x)
^5) - (30*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^4) - (70*d^4*(b*c - a*d)^6)/(b
^11*(a + b*x)^3) - (126*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^2) - (210*d^6*(b
*c - a*d)^4)/(b^11*(a + b*x)) + (5*d^9*(b*c - a*d)*(a + b*x)^2)/b^11 + (d^1
0*(a + b*x)^3)/(3*b^11) + (120*d^7*(b*c - a*d)^3*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^8} dx = \int \left(\frac{45d^8(bc-ad)^2}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^8}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^7}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^6}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^5}+\frac{21}{b}\right.$$
$$= \frac{45d^8(bc-ad)^2x}{b^{10}}-\frac{(bc-ad)^{10}}{7b^{11}(a+bx)^7}-\frac{5d(bc-ad)^9}{3b^{11}(a+bx)^6}-\frac{9d^2(bc-ad)^8}{b^{11}(a+bx)^5}-\frac{30d^3(bc-ad)^7}{b^{11}(a+bx)^4}-\frac{70d^4(bc}{b^{11}(a}$$

Mathematica [A] time = 0.25, size = 239, normalized size = 0.93

$$\frac{21bd^8x\left(36a^2d^2-80abcd+45b^2c^2\right)+21b^2d^9x^2(5bc-4ad)+2520d^7(bc-ad)^3\log(a+bx)-\frac{4410d^6(bc-ad)^4}{a+bx}+\frac{2646d^5(ad-bc)^5}{(a+bx)^2}-\frac{1470d^4(bc-ad)^6}{(a+bx)^3}+\frac{630d^3(ad-bc)^7}{(a+bx)^4}-\frac{189d^2(bc-ad)^8}{(a+bx)^5}+\frac{35d(ad-bc)^9}{(a+bx)^6}-\frac{3(bc-ad)^{10}}{(a+bx)^7}+7b^3d^{10}x^3}{21b^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] (21*b*d^8*(45*b^2*c^2 - 80*a*b*c*d + 36*a^2*d^2)*x + 21*b^2*d^9*(5*b*c - 4*
a*d)*x^2 + 7*b^3*d^10*x^3 - (3*(b*c - a*d)^10)/(a + b*x)^7 + (35*d*(-(b*c)
+ a*d)^9)/(a + b*x)^6 - (189*d^2*(b*c - a*d)^8)/(a + b*x)^5 + (630*d^3*(-(b
*c) + a*d)^7)/(a + b*x)^4 - (1470*d^4*(b*c - a*d)^6)/(a + b*x)^3 + (2646*d^
5*(-(b*c) + a*d)^5)/(a + b*x)^2 - (4410*d^6*(b*c - a*d)^4)/(a + b*x) + 2520
*d^7*(b*c - a*d)^3*Log[a + b*x])/(21*b^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^8, x]
```

fricas [B] time = 1.40, size = 1362, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/21*(7*b^10*d^10*x^10 - 3*b^10*c^10 - 5*a*b^9*c^9*d - 9*a^2*b^8*c^8*d^2 -
18*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 126*a^5*b^5*c^5*d^5 - 630*a^6*b^4
*c^4*d^6 + 6534*a^7*b^3*c^3*d^7 - 12987*a^8*b^2*c^2*d^8 + 10047*a^9*b*c*d^9
 - 2761*a^10*d^10 + 35*(3*b^10*c*d^9 - a*b^9*d^10)*x^9 + 315*(3*b^10*c^2*d^
8 - 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 49*(135*a*b^9*c^2*d^8 - 195*a^2*b^8
*c*d^9 + 77*a^3*b^7*d^10)*x^7 - 49*(90*b^10*c^4*d^6 - 360*a*b^9*c^3*d^7 + 1
35*a^2*b^8*c^2*d^8 + 285*a^3*b^7*c*d^9 - 179*a^4*b^6*d^10)*x^6 - 147*(18*b^
10*c^5*d^5 + 90*a*b^9*c^4*d^6 - 540*a^2*b^8*c^3*d^7 + 675*a^3*b^7*c^2*d^8 -
 255*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 - 245*(6*b^10*c^6*d^4 + 18*a*b^9*c^5
*d^5 + 90*a^2*b^8*c^4*d^6 - 660*a^3*b^7*c^3*d^7 + 1035*a^4*b^6*c^2*d^8 - 61
5*a^5*b^5*c*d^9 + 121*a^6*b^4*d^10)*x^4 - 35*(18*b^10*c^7*d^3 + 42*a*b^9*c^
6*d^4 + 126*a^2*b^8*c^5*d^5 + 630*a^3*b^7*c^4*d^6 - 5250*a^4*b^6*c^3*d^7 +
9135*a^5*b^5*c^2*d^8 - 6195*a^6*b^4*c*d^9 + 1477*a^7*b^3*d^10)*x^3 - 21*(9*
b^10*c^8*d^2 + 18*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
+ 630*a^4*b^6*c^4*d^6 - 5754*a^5*b^5*c^3*d^7 + 10647*a^6*b^4*c^2*d^8 - 7707
*a^7*b^3*c*d^9 + 1981*a^8*b^2*d^10)*x^2 - 7*(5*b^10*c^9*d + 9*a*b^9*c^8*d^2
 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 630*a^5*
b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 11907*a^7*b^3*c^2*d^8 - 8967*a^8*b^2*c
*d^9 + 2401*a^9*b*d^10)*x + 2520*(a^7*b^3*c^3*d^7 - 3*a^8*b^2*c^2*d^8 + 3*a
^9*b*c*d^9 - a^10*d^10 + (b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9
- a^3*b^7*d^10)*x^7 + 7*(a*b^9*c^3*d^7 - 3*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^
9 - a^4*b^6*d^10)*x^6 + 21*(a^2*b^8*c^3*d^7 - 3*a^3*b^7*c^2*d^8 + 3*a^4*b^6
*c*d^9 - a^5*b^5*d^10)*x^5 + 35*(a^3*b^7*c^3*d^7 - 3*a^4*b^6*c^2*d^8 + 3*a^
5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 35*(a^4*b^6*c^3*d^7 - 3*a^5*b^5*c^2*d^8 +
 3*a^6*b^4*c*d^9 - a^7*b^3*d^10)*x^3 + 21*(a^5*b^5*c^3*d^7 - 3*a^6*b^4*c^2*
d^8 + 3*a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 7*(a^6*b^4*c^3*d^7 - 3*a^7*b^3*
c^2*d^8 + 3*a^8*b^2*c*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^18*x^7 + 7*a*b^
17*x^6 + 21*a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*
x^2 + 7*a^6*b^12*x + a^7*b^11)
```

giac [B] time = 1.26, size = 872, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 120*(b^3*c^3*d^7 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(abs(b*x
+ a))/b^11 - 1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3
*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d
^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 276
1*a^10*d^10 + 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 -
30*a^2*b^8*c^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10
)*x^5 + 1470*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3
*b^7*c^3*d^7 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x
^4 + 210*(3*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b
^7*c^4*d^6 - 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^
```

9 + 319*a^7*b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^
7 + 3654*a^6*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(
5*b^10*c^9*d + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^
7*b^3*c^2*d^8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/((b*x + a)^7*b^11)
 + 1/3*(b^16*d^10*x^3 + 15*b^16*c*d^9*x^2 - 12*a*b^15*d^10*x^2 + 135*b^16*c
^2*d^8*x - 240*a*b^15*c*d^9*x + 108*a^2*b^14*d^10*x)/b^24

maple [B] time = 0.02, size = 1241, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^8,x)

[Out] -30/b^4*d^3/(b*x+a)^4*c^7-210/b^11*d^10/(b*x+a)*a^4-210/b^7*d^6/(b*x+a)*c^4
+5/3/b^11*d^10/(b*x+a)^6*a^9-5/3/b^2*d/(b*x+a)^6*c^9+120/b^8*d^7*ln(b*x+a)*
c^3+30/b^11*d^10/(b*x+a)^4*a^7-9/b^11*d^10/(b*x+a)^5*a^8-9/b^3*d^2/(b*x+a)^
5*c^8+126/b^11*d^10/(b*x+a)^2*a^5-126/b^6*d^5/(b*x+a)^2*c^5-120/b^11*d^10*l
n(b*x+a)*a^3-70/b^5*d^4/(b*x+a)^3*c^6-1/7/b^11/(b*x+a)^7*a^10*d^10-4*d^10/b
^9*x^2*a+5*d^9/b^8*x^2*c+36*d^10/b^10*a^2*x+45*d^8/b^8*c^2*x-70/b^11*d^10/(
b*x+a)^3*a^6+360/b^10*d^9*ln(b*x+a)*a^2*c-360/b^9*d^8*ln(b*x+a)*a*c^2-210/b
^10*d^9/(b*x+a)^4*a^6*c+630/b^9*d^8/(b*x+a)^4*a^5*c^2-1050/b^8*d^7/(b*x+a)^
4*a^4*c^3+1050/b^7*d^6/(b*x+a)^4*a^3*c^4-630/b^6*d^5/(b*x+a)^4*a^2*c^5+420/
b^6*d^5/(b*x+a)^3*a*c^5+10/7/b^10/(b*x+a)^7*a^9*c*d^9-45/7/b^9/(b*x+a)^7*a^
8*c^2*d^8+120/7/b^8/(b*x+a)^7*a^7*c^3*d^7-30/b^7/(b*x+a)^7*a^6*c^4*d^6+36/b
^6/(b*x+a)^7*a^5*c^5*d^5-30/b^5/(b*x+a)^7*a^4*c^6*d^4+120/7/b^4/(b*x+a)^7*a
^3*c^7*d^3-45/7/b^3/(b*x+a)^7*a^2*c^8*d^2+10/7/b^2/(b*x+a)^7*a*c^9*d+72/b^1
0*d^9/(b*x+a)^5*a^7*c-252/b^9*d^8/(b*x+a)^5*a^6*c^2+504/b^8*d^7/(b*x+a)^5*a
^5*c^3-630/b^7*d^6/(b*x+a)^5*a^4*c^4+504/b^6*d^5/(b*x+a)^5*a^3*c^5-252/b^5*
d^4/(b*x+a)^5*a^2*c^6+72/b^4*d^3/(b*x+a)^5*a*c^7-630/b^10*d^9/(b*x+a)^2*a^4
*c+1260/b^9*d^8/(b*x+a)^2*a^3*c^2-1260/b^8*d^7/(b*x+a)^2*a^2*c^3+630/b^7*d^
6/(b*x+a)^2*a*c^4-80*d^9/b^9*a*c*x+140/b^5*d^4/(b*x+a)^6*a^3*c^6-60/b^4*d^3
/(b*x+a)^6*a^2*c^7+15/b^3*d^2/(b*x+a)^6*a*c^8-15/b^10*d^9/(b*x+a)^6*a^8*c+6
0/b^9*d^8/(b*x+a)^6*a^7*c^2+420/b^10*d^9/(b*x+a)^3*a^5*c-1050/b^9*d^8/(b*x+
a)^3*a^4*c^2+1400/b^8*d^7/(b*x+a)^3*a^3*c^3-1050/b^7*d^6/(b*x+a)^3*a^2*c^4+
210/b^7*d^6/(b*x+a)^6*a^5*c^4-210/b^6*d^5/(b*x+a)^6*a^4*c^5+210/b^5*d^4/(b*
x+a)^4*a*c^6+840/b^10*d^9/(b*x+a)*a^3*c-1260/b^9*d^8/(b*x+a)*a^2*c^2+840/b^
8*d^7/(b*x+a)*a*c^3-140/b^8*d^7/(b*x+a)^6*a^6*c^3+1/3*d^10/b^8*x^3-1/7/b/(b
*x+a)^7*c^10

maxima [B] time = 2.44, size = 934, normalized size = 3.62

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^8,x, algorithm="maxima")

[Out] -1/21*(3*b^10*c^10 + 5*a*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3
 + 42*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^
7*b^3*c^3*d^7 + 12987*a^8*b^2*c^2*d^8 - 10047*a^9*b*c*d^9 + 2761*a^10*d^10
+ 4410*(b^10*c^4*d^6 - 4*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 4*a^3*b^7*c*d^
9 + a^4*b^6*d^10)*x^6 + 2646*(b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 - 30*a^2*b^8*c
^3*d^7 + 50*a^3*b^7*c^2*d^8 - 35*a^4*b^6*c*d^9 + 9*a^5*b^5*d^10)*x^5 + 1470
*(b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 - 110*a^3*b^7*c^3*d^7
 + 195*a^4*b^6*c^2*d^8 - 141*a^5*b^5*c*d^9 + 37*a^6*b^4*d^10)*x^4 + 210*(3*
b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 -
 875*a^4*b^6*c^3*d^7 + 1617*a^5*b^5*c^2*d^8 - 1197*a^6*b^4*c*d^9 + 319*a^7*
b^3*d^10)*x^3 + 63*(3*b^10*c^8*d^2 + 6*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 +
 42*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 - 1918*a^5*b^5*c^3*d^7 + 3654*a^6

```
*b^4*c^2*d^8 - 2754*a^7*b^3*c*d^9 + 743*a^8*b^2*d^10)*x^2 + 7*(5*b^10*c^9*d
 + 9*a*b^9*c^8*d^2 + 18*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 126*a^4*b^6*
c^5*d^5 + 630*a^5*b^5*c^4*d^6 - 6174*a^6*b^4*c^3*d^7 + 12042*a^7*b^3*c^2*d^
8 - 9207*a^8*b^2*c*d^9 + 2509*a^9*b*d^10)*x)/(b^18*x^7 + 7*a*b^17*x^6 + 21*
a^2*b^16*x^5 + 35*a^3*b^15*x^4 + 35*a^4*b^14*x^3 + 21*a^5*b^13*x^2 + 7*a^6*
b^12*x + a^7*b^11) + 1/3*(b^2*d^10*x^3 + 3*(5*b^2*c*d^9 - 4*a*b*d^10)*x^2 +
 3*(45*b^2*c^2*d^8 - 80*a*b*c*d^9 + 36*a^2*d^10)*x)/b^10 + 120*(b^3*c^3*d^7
 - 3*a*b^2*c^2*d^8 + 3*a^2*b*c*d^9 - a^3*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.43, size = 950, normalized size = 3.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^8,x)
```

```
[Out] x*((8*a*((8*a*d^10)/b^9 - (10*c*d^9)/b^8))/b - (28*a^2*d^10)/b^10 + (45*c^2
*d^8)/b^8) - (x^4*(2590*a^6*b^3*d^10 + 70*b^9*c^6*d^4 + 210*a*b^8*c^5*d^5 -
 9870*a^5*b^4*c*d^9 + 1050*a^2*b^7*c^4*d^6 - 7700*a^3*b^6*c^3*d^7 + 13650*a
^4*b^5*c^2*d^8) + x^6*(210*a^4*b^5*d^10 + 210*b^9*c^4*d^6 - 840*a*b^8*c^3*d
^7 - 840*a^3*b^6*c*d^9 + 1260*a^2*b^7*c^2*d^8) + (2761*a^10*d^10 + 3*b^10*c
^10 + 9*a^2*b^8*c^8*d^2 + 18*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 126*a^5
*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 - 6534*a^7*b^3*c^3*d^7 + 12987*a^8*b^2*c
^2*d^8 + 5*a*b^9*c^9*d - 10047*a^9*b*c*d^9)/(21*b) + x*((2509*a^9*d^10)/3 +
 (5*b^9*c^9*d)/3 + 3*a*b^8*c^8*d^2 + 6*a^2*b^7*c^7*d^3 + 14*a^3*b^6*c^6*d^4
 + 42*a^4*b^5*c^5*d^5 + 210*a^5*b^4*c^4*d^6 - 2058*a^6*b^3*c^3*d^7 + 4014*a
^7*b^2*c^2*d^8 - 3069*a^8*b*c*d^9) + x^3*(3190*a^7*b^2*d^10 + 30*b^9*c^7*d^
3 + 70*a*b^8*c^6*d^4 - 11970*a^6*b^3*c*d^9 + 210*a^2*b^7*c^5*d^5 + 1050*a^3
*b^6*c^4*d^6 - 8750*a^4*b^5*c^3*d^7 + 16170*a^5*b^4*c^2*d^8) + x^2*(2229*a^
8*b*d^10 + 9*b^9*c^8*d^2 + 18*a*b^8*c^7*d^3 - 8262*a^7*b^2*c*d^9 + 42*a^2*b
^7*c^6*d^4 + 126*a^3*b^6*c^5*d^5 + 630*a^4*b^5*c^4*d^6 - 5754*a^5*b^4*c^3*d
^7 + 10962*a^6*b^3*c^2*d^8) + x^5*(1134*a^5*b^4*d^10 + 126*b^9*c^5*d^5 + 63
0*a*b^8*c^4*d^6 - 4410*a^4*b^5*c*d^9 - 3780*a^2*b^7*c^3*d^7 + 6300*a^3*b^6*
c^2*d^8))/(a^7*b^10 + b^17*x^7 + 7*a^6*b^11*x + 7*a*b^16*x^6 + 21*a^5*b^12*
x^2 + 35*a^4*b^13*x^3 + 35*a^3*b^14*x^4 + 21*a^2*b^15*x^5) - x^2*((4*a*d^10
)/b^9 - (5*c*d^9)/b^8) - (log(a + b*x)*(120*a^3*d^10 - 120*b^3*c^3*d^7 + 36
0*a*b^2*c^2*d^8 - 360*a^2*b*c*d^9))/b^11 + (d^10*x^3)/(3*b^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**8,x)
```

```
[Out] Timed out
```

3.1214 $\int \frac{(c+dx)^{10}}{(a+bx)^9} \, dx$

Optimal. Leaf size=258

$$\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}$$

Rubi [A] time = 0.34, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{d^9x(10bc-9ad)}{b^{10}}-\frac{120d^7(bc-ad)^3}{b^{11}(a+bx)}-\frac{105d^6(bc-ad)^4}{b^{11}(a+bx)^2}-\frac{84d^5(bc-ad)^5}{b^{11}(a+bx)^3}-\frac{105d^4(bc-ad)^6}{2b^{11}(a+bx)^4}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}+\frac{45d^8(bc-ad)^2\log(a+bx)}{b^{11}}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}+\frac{d^{10}x^2}{2b^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (d^9*(10*b*c - 9*a*d)*x)/b^10 + (d^10*x^2)/(2*b^9) - (b*c - a*d)^10/(8*b^11
*(a + b*x)^8) - (10*d*(b*c - a*d)^9)/(7*b^11*(a + b*x)^7) - (15*d^2*(b*c -
a*d)^8)/(2*b^11*(a + b*x)^6) - (24*d^3*(b*c - a*d)^7)/(b^11*(a + b*x)^5) -
(105*d^4*(b*c - a*d)^6)/(2*b^11*(a + b*x)^4) - (84*d^5*(b*c - a*d)^5)/(b^11
*(a + b*x)^3) - (105*d^6*(b*c - a*d)^4)/(b^11*(a + b*x)^2) - (120*d^7*(b*c
- a*d)^3)/(b^11*(a + b*x)) + (45*d^8*(b*c - a*d)^2*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx &= \int \left(\frac{d^9(10bc-9ad)}{b^{10}}+\frac{d^{10}x}{b^9}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^9}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^8}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^7}+\frac{120d^3(bc-a}{b^{10}(a+bx)^6}\right.\\ &= \frac{d^9(10bc-9ad)x}{b^{10}}+\frac{d^{10}x^2}{2b^9}-\frac{(bc-ad)^{10}}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{7b^{11}(a+bx)^7}-\frac{15d^2(bc-ad)^8}{2b^{11}(a+bx)^6}-\frac{24d^3(bc-ad)^7}{b^{11}(a+bx)^5}\end{aligned}$$

Mathematica [B] time = 0.32, size = 712, normalized size = 2.76

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] (3601*a^10*d^10 + 2*a^9*b*d^9*(-4609*c + 13144*d*x) + a^8*b^2*d^8*(6849*c^2
 - 68704*c*d*x + 81928*d^2*x^2) + 8*a^7*b^3*d^7*(-105*c^3 + 6534*c^2*d*x -
27538*c*d^2*x^2 + 17542*d^3*x^3) + 14*a^6*b^4*d^6*(-15*c^4 - 480*c^3*d*x +
12348*c^2*d^2*x^2 - 28112*c*d^3*x^3 + 10010*d^4*x^4) - 28*a^5*b^5*d^5*(3*c^
5 + 60*c^4*d*x + 840*c^3*d^2*x^2 - 11508*c^2*d^3*x^3 + 15050*c*d^4*x^4 - 27
44*d^5*x^5) - 14*a^4*b^6*d^4*(3*c^6 + 48*c^5*d*x + 420*c^4*d^2*x^2 + 3360*c
^3*d^3*x^3 - 26250*c^2*d^4*x^4 + 19040*c*d^5*x^5 - 1064*d^6*x^6) - 8*a^3*b^
7*d^3*(3*c^7 + 42*c^6*d*x + 294*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 7350*c^3*d
^4*x^4 - 32340*c^2*d^5*x^5 + 10780*c*d^6*x^6 + 728*d^7*x^7) - a^2*b^8*d^2*(
15*c^8 + 192*c^7*d*x + 1176*c^6*d^2*x^2 + 4704*c^5*d^3*x^3 + 14700*c^4*d^4*
x^4 + 47040*c^3*d^5*x^5 - 105840*c^2*d^6*x^6 + 4480*c*d^7*x^7 + 3248*d^8*x^
```

```
8) - 2*a*b^9*d*(5*c^9 + 60*c^8*d*x + 336*c^7*d^2*x^2 + 1176*c^6*d^3*x^3 + 2
940*c^5*d^4*x^4 + 5880*c^4*d^5*x^5 + 11760*c^3*d^6*x^6 - 10080*c^2*d^7*x^7
- 2240*c*d^8*x^8 + 140*d^9*x^9) - b^10*(7*c^10 + 80*c^9*d*x + 420*c^8*d^2*x
^2 + 1344*c^7*d^3*x^3 + 2940*c^6*d^4*x^4 + 4704*c^5*d^5*x^5 + 5880*c^4*d^6*
x^6 + 6720*c^3*d^7*x^7 - 560*c*d^9*x^9 - 28*d^10*x^10) + 2520*d^8*(b*c - a*
d)^2*(a + b*x)^8*Log[a + b*x])/(56*b^11*(a + b*x)^8)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^9}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^9,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^9, x]
```

fricas [B] time = 1.39, size = 1296, normalized size = 5.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="fricas")
```

```
[Out] 1/56*(28*b^10*d^10*x^10 - 7*b^10*c^10 - 10*a*b^9*c^9*d - 15*a^2*b^8*c^8*d^2
 - 24*a^3*b^7*c^7*d^3 - 42*a^4*b^6*c^6*d^4 - 84*a^5*b^5*c^5*d^5 - 210*a^6*b
^4*c^4*d^6 - 840*a^7*b^3*c^3*d^7 + 6849*a^8*b^2*c^2*d^8 - 9218*a^9*b*c*d^9
+ 3601*a^10*d^10 + 280*(2*b^10*c*d^9 - a*b^9*d^10)*x^9 + 112*(40*a*b^9*c*d^
9 - 29*a^2*b^8*d^10)*x^8 - 448*(15*b^10*c^3*d^7 - 45*a*b^9*c^2*d^8 + 10*a^2
*b^8*c*d^9 + 13*a^3*b^7*d^10)*x^7 - 392*(15*b^10*c^4*d^6 + 60*a*b^9*c^3*d^7
 - 270*a^2*b^8*c^2*d^8 + 220*a^3*b^7*c*d^9 - 38*a^4*b^6*d^10)*x^6 - 784*(6*
b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 60*a^2*b^8*c^3*d^7 - 330*a^3*b^7*c^2*d^8
+ 340*a^4*b^6*c*d^9 - 98*a^5*b^5*d^10)*x^5 - 980*(3*b^10*c^6*d^4 + 6*a*b^9*
c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 60*a^3*b^7*c^3*d^7 - 375*a^4*b^6*c^2*d^8 + 4
30*a^5*b^5*c*d^9 - 143*a^6*b^4*d^10)*x^4 - 112*(12*b^10*c^7*d^3 + 21*a*b^9*
c^6*d^4 + 42*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 420*a^4*b^6*c^3*d^7 -
2877*a^5*b^5*c^2*d^8 + 3514*a^6*b^4*c*d^9 - 1253*a^7*b^3*d^10)*x^3 - 28*(15
*b^10*c^8*d^2 + 24*a*b^9*c^7*d^3 + 42*a^2*b^8*c^6*d^4 + 84*a^3*b^7*c^5*d^5
+ 210*a^4*b^6*c^4*d^6 + 840*a^5*b^5*c^3*d^7 - 6174*a^6*b^4*c^2*d^8 + 7868*a
^7*b^3*c*d^9 - 2926*a^8*b^2*d^10)*x^2 - 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^2
 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*b
^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8588*a^8*b^2*c*d^
9 - 3286*a^9*b*d^10)*x + 2520*(a^8*b^2*c^2*d^8 - 2*a^9*b*c*d^9 + a^10*d^10
+ (b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 8*(a*b^9*c^2*d^8 - 2*
a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 28*(a^2*b^8*c^2*d^8 - 2*a^3*b^7*c*d^9 +
 a^4*b^6*d^10)*x^6 + 56*(a^3*b^7*c^2*d^8 - 2*a^4*b^6*c*d^9 + a^5*b^5*d^10)*
x^5 + 70*(a^4*b^6*c^2*d^8 - 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 56*(a^5*b
^5*c^2*d^8 - 2*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 28*(a^6*b^4*c^2*d^8 - 2*
a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 8*(a^7*b^3*c^2*d^8 - 2*a^8*b^2*c*d^9 +
a^9*b*d^10)*x)*log(b*x + a))/(b^19*x^8 + 8*a*b^18*x^7 + 28*a^2*b^17*x^6 + 5
6*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*a^6*b^13*x^2 + 8*a^
7*b^12*x + a^8*b^11)
```

giac [B] time = 1.29, size = 871, normalized size = 3.38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="giac")
```

```
[Out] 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(abs(b*x + a))/b^11 + 1/2*(b^9
*d^10*x^2 + 20*b^9*c*d^9*x - 18*a*b^8*d^10*x)/b^18 - 1/56*(7*b^10*c^10 + 10
*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4
 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^
8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 + 6720*(b^10*c^3*d^7 - 3*
a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7 + 5880*(b^10*c^4*d^6 +
4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*d^9 - 7*a^4*b^6*d^10)*x
^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 - 110*a^3*
b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x^5 + 2940*(b^10*c^6*d^4
 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 - 125*a^4*b^6*c
^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 + 336*(4*b^10*c^7*d^3 + 7
*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*
d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459*a^7*b^3*d^10)*x^3 + 84
*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*d^4 + 28*a^3*b^7*c^5*d^
5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*a^6*b^4*c^2*d^8 + 2676*
a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*c^9*d + 15*a*b^9*c^8*d^
2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4*b^6*c^5*d^5 + 210*a^5*
b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2*d^8 + 8658*a^8*b^2*c*d
^9 - 3349*a^9*b*d^10)*x)/((b*x + a)^8*b^11)
```

maple [B] time = 0.02, size = 1256, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^9,x)
```

```
[Out] 1/2*d^10*x^2/b^9-9*d^10/b^10*a*x+10*d^9/b^9*x*c-15/2/b^11*d^10/(b*x+a)^6*a^
8-15/2/b^3*d^2/(b*x+a)^6*c^8-105/b^7*d^6/(b*x+a)^2*c^4+45/b^11*d^10*ln(b*x+
a)*a^2+45/b^9*d^8*ln(b*x+a)*c^2-105/2/b^11*d^10/(b*x+a)^4*a^6-105/2/b^5*d^4
/(b*x+a)^4*c^6+120/b^11*d^10/(b*x+a)*a^3-120/b^8*d^7/(b*x+a)*c^3-1/8/b^11/(
b*x+a)^8*a^10*d^10+84/b^11*d^10/(b*x+a)^3*a^5-84/b^6*d^5/(b*x+a)^3*c^5+10/7
/b^11*d^10/(b*x+a)^7*a^9-10/7/b^2*d/(b*x+a)^7*c^9+24/b^11*d^10/(b*x+a)^5*a^
7-24/b^4*d^3/(b*x+a)^5*c^7-105/b^11*d^10/(b*x+a)^2*a^4-1/8/b/(b*x+a)^8*c^10
-360/7/b^4*d^3/(b*x+a)^7*a^2*c^7+90/7/b^3*d^2/(b*x+a)^7*a*c^8-168/b^10*d^9/
(b*x+a)^5*a^6*c+504/b^9*d^8/(b*x+a)^5*a^5*c^2+180/b^7*d^6/(b*x+a)^7*a^5*c^4
-180/b^6*d^5/(b*x+a)^7*a^4*c^5+120/b^5*d^4/(b*x+a)^7*a^3*c^6-840/b^8*d^7/(b
*x+a)^5*a^4*c^3+840/b^7*d^6/(b*x+a)^5*a^3*c^4-504/b^6*d^5/(b*x+a)^5*a^2*c^5
-210/b^9*d^8/(b*x+a)^6*a^6*c^2+420/b^8*d^7/(b*x+a)^6*a^5*c^3-525/b^7*d^6/(b
*x+a)^6*a^4*c^4+420/b^6*d^5/(b*x+a)^6*a^3*c^5-210/b^5*d^4/(b*x+a)^6*a^2*c^6
+60/b^4*d^3/(b*x+a)^6*a*c^7+168/b^5*d^4/(b*x+a)^5*a*c^6+315/b^10*d^9/(b*x+a
)^4*a^5*c-1575/2/b^9*d^8/(b*x+a)^4*a^4*c^2+1050/b^8*d^7/(b*x+a)^4*a^3*c^3-1
575/2/b^7*d^6/(b*x+a)^4*a^2*c^4+315/b^6*d^5/(b*x+a)^4*a*c^5-360/b^10*d^9/(b
*x+a)*a^2*c+360/b^9*d^8/(b*x+a)*a*c^2+60/b^10*d^9/(b*x+a)^6*a^7*c+420/b^10*
d^9/(b*x+a)^2*a^3*c-630/b^9*d^8/(b*x+a)^2*a^2*c^2+420/b^8*d^7/(b*x+a)^2*a*c
^3-90/b^10*d^9*ln(b*x+a)*a*c+5/4/b^10/(b*x+a)^8*a^9*c*d^9-45/8/b^9/(b*x+a)^
8*a^8*c^2*d^8+15/b^8/(b*x+a)^8*a^7*c^3*d^7-105/4/b^7/(b*x+a)^8*a^6*c^4*d^6+
63/2/b^6/(b*x+a)^8*a^5*c^5*d^5-105/4/b^5/(b*x+a)^8*a^4*c^6*d^4+15/b^4/(b*x+
a)^8*a^3*c^7*d^3-45/8/b^3/(b*x+a)^8*a^2*c^8*d^2+5/4/b^2/(b*x+a)^8*a*c^9*d-4
20/b^10*d^9/(b*x+a)^3*a^4*c+840/b^9*d^8/(b*x+a)^3*a^3*c^2-840/b^8*d^7/(b*x+
a)^3*a^2*c^3+420/b^7*d^6/(b*x+a)^3*a*c^4-90/7/b^10*d^9/(b*x+a)^7*a^8*c+360/
7/b^9*d^8/(b*x+a)^7*a^7*c^2-120/b^8*d^7/(b*x+a)^7*a^6*c^3
```

maxima [B] time = 2.58, size = 945, normalized size = 3.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^9,x, algorithm="maxima")
```

```
[Out] -1/56*(7*b^10*c^10 + 10*a*b^9*c^9*d + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d
^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^
7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^8 + 9218*a^9*b*c*d^9 - 3601*a^10*d^10 +
6720*(b^10*c^3*d^7 - 3*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 - a^3*b^7*d^10)*x^7
+ 5880*(b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 - 18*a^2*b^8*c^2*d^8 + 20*a^3*b^7*c*
d^9 - 7*a^4*b^6*d^10)*x^6 + 2352*(2*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 20*a^2
*b^8*c^3*d^7 - 110*a^3*b^7*c^2*d^8 + 130*a^4*b^6*c*d^9 - 47*a^5*b^5*d^10)*x
^5 + 2940*(b^10*c^6*d^4 + 2*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 20*a^3*b^7*
c^3*d^7 - 125*a^4*b^6*c^2*d^8 + 154*a^5*b^5*c*d^9 - 57*a^6*b^4*d^10)*x^4 +
336*(4*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 14*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4
*d^6 + 140*a^4*b^6*c^3*d^7 - 959*a^5*b^5*c^2*d^8 + 1218*a^6*b^4*c*d^9 - 459
*a^7*b^3*d^10)*x^3 + 84*(5*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 14*a^2*b^8*c^6*
d^4 + 28*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 - 2058*
a^6*b^4*c^2*d^8 + 2676*a^7*b^3*c*d^9 - 1023*a^8*b^2*d^10)*x^2 + 8*(10*b^10*
c^9*d + 15*a*b^9*c^8*d^2 + 24*a^2*b^8*c^7*d^3 + 42*a^3*b^7*c^6*d^4 + 84*a^4
*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 - 6534*a^7*b^3*c^2
*d^8 + 8658*a^8*b^2*c*d^9 - 3349*a^9*b*d^10)*x)/(b^19*x^8 + 8*a*b^18*x^7 +
28*a^2*b^17*x^6 + 56*a^3*b^16*x^5 + 70*a^4*b^15*x^4 + 56*a^5*b^14*x^3 + 28*
a^6*b^13*x^2 + 8*a^7*b^12*x + a^8*b^11) + 1/2*(b*d^10*x^2 + 2*(10*b*c*d^9 -
 9*a*d^10)*x)/b^10 + 45*(b^2*c^2*d^8 - 2*a*b*c*d^9 + a^2*d^10)*log(b*x + a)
/b^11
```

mupad [B] time = 0.26, size = 946, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^9,x)
```

```
[Out] (log(a + b*x)*(45*a^2*d^10 + 45*b^2*c^2*d^8 - 90*a*b*c*d^9))/b^11 - (x^4*((
105*b^9*c^6*d^4)/2 - (5985*a^6*b^3*d^10)/2 + 105*a*b^8*c^5*d^5 + 8085*a^5*b
^4*c*d^9 + (525*a^2*b^7*c^4*d^6)/2 + 1050*a^3*b^6*c^3*d^7 - (13125*a^4*b^5*
c^2*d^8)/2) + x^6*(105*b^9*c^4*d^6 - 735*a^4*b^5*d^10 + 420*a*b^8*c^3*d^7 +
 2100*a^3*b^6*c*d^9 - 1890*a^2*b^7*c^2*d^8) + (7*b^10*c^10 - 3601*a^10*d^10
 + 15*a^2*b^8*c^8*d^2 + 24*a^3*b^7*c^7*d^3 + 42*a^4*b^6*c^6*d^4 + 84*a^5*b^
5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 - 6849*a^8*b^2*c^2*d^
8 + 10*a*b^9*c^9*d + 9218*a^9*b*c*d^9)/(56*b) + x*((10*b^9*c^9*d)/7 - (3349
*a^9*d^10)/7 + (15*a*b^8*c^8*d^2)/7 + (24*a^2*b^7*c^7*d^3)/7 + 6*a^3*b^6*c^
6*d^4 + 12*a^4*b^5*c^5*d^5 + 30*a^5*b^4*c^4*d^6 + 120*a^6*b^3*c^3*d^7 - (65
34*a^7*b^2*c^2*d^8)/7 + (8658*a^8*b*c*d^9)/7) + x^3*(24*b^9*c^7*d^3 - 2754*
a^7*b^2*d^10 + 42*a*b^8*c^6*d^4 + 7308*a^6*b^3*c*d^9 + 84*a^2*b^7*c^5*d^5 +
 210*a^3*b^6*c^4*d^6 + 840*a^4*b^5*c^3*d^7 - 5754*a^5*b^4*c^2*d^8) + x^2*((
15*b^9*c^8*d^2)/2 - (3069*a^8*b*d^10)/2 + 12*a*b^8*c^7*d^3 + 4014*a^7*b^2*c
*d^9 + 21*a^2*b^7*c^6*d^4 + 42*a^3*b^6*c^5*d^5 + 105*a^4*b^5*c^4*d^6 + 420*
a^5*b^4*c^3*d^7 - 3087*a^6*b^3*c^2*d^8) + x^5*(84*b^9*c^5*d^5 - 1974*a^5*b^
4*d^10 + 210*a*b^8*c^4*d^6 + 5460*a^4*b^5*c*d^9 + 840*a^2*b^7*c^3*d^7 - 462
0*a^3*b^6*c^2*d^8) - x^7*(120*a^3*b^6*d^10 - 120*b^9*c^3*d^7 + 360*a*b^8*c^
2*d^8 - 360*a^2*b^7*c*d^9))/(a^8*b^10 + b^18*x^8 + 8*a^7*b^11*x + 8*a*b^17*
x^7 + 28*a^6*b^12*x^2 + 56*a^5*b^13*x^3 + 70*a^4*b^14*x^4 + 56*a^3*b^15*x^5
 + 28*a^2*b^16*x^6) - x*((9*a*d^10)/b^10 - (10*c*d^9)/b^9) + (d^10*x^2)/(2*
b^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**9,x)
```

```
[Out] Timed out
```

3.1215 $\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx$

Optimal. Leaf size=257

$$\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20}{b}$$

Rubi [A] time = 0.31, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{45d^8(bc-ad)^2}{b^{11}(a+bx)}-\frac{60d^7(bc-ad)^3}{b^{11}(a+bx)^2}-\frac{70d^6(bc-ad)^4}{b^{11}(a+bx)^3}-\frac{63d^5(bc-ad)^5}{b^{11}(a+bx)^4}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}+\frac{10d^9(bc-ad)\log(a+bx)}{b^{11}}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}+\frac{d^{10}x}{b^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] (d^10*x)/b^10 - (b*c - a*d)^10/(9*b^11*(a + b*x)^9) - (5*d*(b*c - a*d)^9)/(
4*b^11*(a + b*x)^8) - (45*d^2*(b*c - a*d)^8)/(7*b^11*(a + b*x)^7) - (20*d^3
*(b*c - a*d)^7)/(b^11*(a + b*x)^6) - (42*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)
^5) - (63*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^4) - (70*d^6*(b*c - a*d)^4)/(b
^11*(a + b*x)^3) - (60*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^2) - (45*d^8*(b*c
 - a*d)^2)/(b^11*(a + b*x)) + (10*d^9*(b*c - a*d)*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx &= \int \left(\frac{d^{10}}{b^{10}}+\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{10}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^9}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^8}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^7}+\frac{210d^4(bc-a}{b^{10}(a+bx)}\right.\\ &= \frac{d^{10}x}{b^{10}}-\frac{(bc-ad)^{10}}{9b^{11}(a+bx)^9}-\frac{5d(bc-ad)^9}{4b^{11}(a+bx)^8}-\frac{45d^2(bc-ad)^8}{7b^{11}(a+bx)^7}-\frac{20d^3(bc-ad)^7}{b^{11}(a+bx)^6}-\frac{42d^4(bc-ad)^6}{b^{11}(a+bx)^5}-\end{aligned}$$

Mathematica [B] time = 0.42, size = 708, normalized size = 2.75

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] -1/252*(4861*a^10*d^10 + a^9*b*d^9*(-7129*c + 41229*d*x) + 9*a^8*b^2*d^8*(1
40*c^2 - 6849*c*d*x + 17064*d^2*x^2) + 12*a^7*b^3*d^7*(35*c^3 + 945*c^2*d*x
 - 19602*c*d^2*x^2 + 27342*d^3*x^3) + 42*a^6*b^4*d^6*(5*c^4 + 90*c^3*d*x +
1080*c^2*d^2*x^2 - 12348*c*d^3*x^3 + 10458*d^4*x^4) + 126*a^5*b^5*d^5*(c^5
+ 15*c^4*d*x + 120*c^3*d^2*x^2 + 840*c^2*d^3*x^3 - 5754*c*d^4*x^4 + 2982*d^
5*x^5) + 42*a^4*b^6*d^4*(2*c^6 + 27*c^5*d*x + 180*c^4*d^2*x^2 + 840*c^3*d^3
*x^3 + 3780*c^2*d^4*x^4 - 15750*c*d^5*x^5 + 4704*d^6*x^6) + 12*a^3*b^7*d^3*
(5*c^7 + 63*c^6*d*x + 378*c^5*d^2*x^2 + 1470*c^4*d^3*x^3 + 4410*c^3*d^4*x^4
 + 13230*c^2*d^5*x^5 - 32340*c*d^6*x^6 + 4536*d^7*x^7) + 9*a^2*b^8*d^2*(5*c
^8 + 60*c^7*d*x + 336*c^6*d^2*x^2 + 1176*c^5*d^3*x^3 + 2940*c^4*d^4*x^4 + 5
880*c^3*d^5*x^5 + 11760*c^2*d^6*x^6 - 15120*c*d^7*x^7 + 252*d^8*x^8) + a*b^
```

```
9*d*(35*c^9 + 405*c^8*d*x + 2160*c^7*d^2*x^2 + 7056*c^6*d^3*x^3 + 15876*c^5
*d^4*x^4 + 26460*c^4*d^5*x^5 + 35280*c^3*d^6*x^6 + 45360*c^2*d^7*x^7 - 2268
0*c*d^8*x^8 - 2268*d^9*x^9) + b^10*(28*c^10 + 315*c^9*d*x + 1620*c^8*d^2*x^
2 + 5040*c^7*d^3*x^3 + 10584*c^6*d^4*x^4 + 15876*c^5*d^5*x^5 + 17640*c^4*d^
6*x^6 + 15120*c^3*d^7*x^7 + 11340*c^2*d^8*x^8 - 252*d^10*x^10) + 2520*d^9*(
-(b*c) + a*d)*(a + b*x)^9*Log[a + b*x])/(b^11*(a + b*x)^9)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^10,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^10, x]
```

fricas [B] time = 1.23, size = 1216, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="fricas")
```

```
[Out] 1/252*(252*b^10*d^10*x^10 + 2268*a*b^9*d^10*x^9 - 28*b^10*c^10 - 35*a*b^9*c
^9*d - 45*a^2*b^8*c^8*d^2 - 60*a^3*b^7*c^7*d^3 - 84*a^4*b^6*c^6*d^4 - 126*a
^5*b^5*c^5*d^5 - 210*a^6*b^4*c^4*d^6 - 420*a^7*b^3*c^3*d^7 - 1260*a^8*b^2*c
^2*d^8 + 7129*a^9*b*c*d^9 - 4861*a^10*d^10 - 2268*(5*b^10*c^2*d^8 - 10*a*b^
9*c*d^9 + a^2*b^8*d^10)*x^8 - 3024*(5*b^10*c^3*d^7 + 15*a*b^9*c^2*d^8 - 45*
a^2*b^8*c*d^9 + 18*a^3*b^7*d^10)*x^7 - 3528*(5*b^10*c^4*d^6 + 10*a*b^9*c^3*
d^7 + 30*a^2*b^8*c^2*d^8 - 110*a^3*b^7*c*d^9 + 56*a^4*b^6*d^10)*x^6 - 5292*
(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8
 - 125*a^4*b^6*c*d^9 + 71*a^5*b^5*d^10)*x^5 - 5292*(2*b^10*c^6*d^4 + 3*a*b^
9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 1
37*a^5*b^5*c*d^9 + 83*a^6*b^4*d^10)*x^4 - 504*(10*b^10*c^7*d^3 + 14*a*b^9*c
^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210
*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 651*a^7*b^3*d^10)*x^3 - 108*(15*b^1
0*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70
*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3
*c*d^9 + 1422*a^8*b^2*d^10)*x^2 - 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*
a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^
4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4
581*a^9*b*d^10)*x + 2520*(a^9*b*c*d^9 - a^10*d^10 + (b^10*c*d^9 - a*b^9*d^1
0)*x^9 + 9*(a*b^9*c*d^9 - a^2*b^8*d^10)*x^8 + 36*(a^2*b^8*c*d^9 - a^3*b^7*d
^10)*x^7 + 84*(a^3*b^7*c*d^9 - a^4*b^6*d^10)*x^6 + 126*(a^4*b^6*c*d^9 - a^5
*b^5*d^10)*x^5 + 126*(a^5*b^5*c*d^9 - a^6*b^4*d^10)*x^4 + 84*(a^6*b^4*c*d^9
 - a^7*b^3*d^10)*x^3 + 36*(a^7*b^3*c*d^9 - a^8*b^2*d^10)*x^2 + 9*(a^8*b^2*c
*d^9 - a^9*b*d^10)*x)*log(b*x + a))/(b^20*x^9 + 9*a*b^19*x^8 + 36*a^2*b^18*
x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*x^4 + 84*a^6*b^14*x
^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11)
```

giac [B] time = 1.25, size = 867, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="giac")
```

```
[Out] d^10*x/b^10 + 10*(b*c*d^9 - a*d^10)*log(abs(b*x + a))/b^11 - 1/252*(28*b^10
*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 60*a^3*b^7*c^7*d^3 + 84*a^4*b
```

```
^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 420*a^7*b^3*c^3*d^
7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4861*a^10*d^10 + 11340*(b^10*
c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 15120*(b^10*c^3*d^7 + 3*a*b^9
*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^7 + 17640*(b^10*c^4*d^6 + 2*
a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*c*d^9 + 13*a^4*b^6*d^10)*x^6
 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 30*a^3*b^7
*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)*x^5 + 5292*(2*b^10*c^6*d^4
+ 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2
*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4 + 504*(10*b^10*c^7*d^3 + 14
*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 70*a^4*b^6*c^3*d
^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 + 669*a^7*b^3*d^10)*x^3 + 108
*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 42*a^3*b^7*c^5*
d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 - 2178
*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35*b^10*c^9*d + 45*a*b^9*c^8*d
^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 210*a^
5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 - 6849*a^8*b^2*c
*d^9 + 4609*a^9*b*d^10)*x)/((b*x + a)^9*b^11)
```

maple [B] time = 0.02, size = 1266, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^10,x)
```

```
[Out] 60/b^11*d^10/(b*x+a)^2*a^3-60/b^8*d^7/(b*x+a)^2*c^3-10/b^11*d^10*ln(b*x+a)*
a+10/b^10*d^9*ln(b*x+a)*c+63/b^11*d^10/(b*x+a)^4*a^5-63/b^6*d^5/(b*x+a)^4*c
^5-45/b^11*d^10/(b*x+a)*a^2-45/b^9*d^8/(b*x+a)*c^2+20/b^11*d^10/(b*x+a)^6*a
^7-20/b^4*d^3/(b*x+a)^6*c^7-45/7/b^11*d^10/(b*x+a)^7*a^8-45/7/b^3*d^2/(b*x+
a)^7*c^8-1/9/b^11/(b*x+a)^9*a^10*d^10+5/4/b^11*d^10/(b*x+a)^8*a^9-5/4/b^2*d
/(b*x+a)^8*c^9-70/b^11*d^10/(b*x+a)^3*a^4-70/b^7*d^6/(b*x+a)^3*c^4-42/b^11*
d^10/(b*x+a)^5*a^6-42/b^5*d^4/(b*x+a)^5*c^6+d^10*x/b^10-1/9/b/(b*x+a)^9*c^1
0-140/b^10*d^9/(b*x+a)^6*a^6*c+420/b^9*d^8/(b*x+a)^6*a^5*c^2+360/7/b^4*d^3/
(b*x+a)^7*a*c^7+10/9/b^10/(b*x+a)^9*a^9*c*d^9-5/b^9/(b*x+a)^9*a^8*c^2*d^8+4
0/3/b^8/(b*x+a)^9*a^7*c^3*d^7+700/b^7*d^6/(b*x+a)^6*a^3*c^4-420/b^6*d^5/(b*
x+a)^6*a^2*c^5+140/b^5*d^4/(b*x+a)^6*a*c^6-70/3/b^7/(b*x+a)^9*a^6*c^4*d^6+2
8/b^6/(b*x+a)^9*a^5*c^5*d^5-70/3/b^5/(b*x+a)^9*a^4*c^6*d^4+40/3/b^4/(b*x+a)
^9*a^3*c^7*d^3-5/b^3/(b*x+a)^9*a^2*c^8*d^2+10/9/b^2/(b*x+a)^9*a*c^9*d+252/b
^10*d^9/(b*x+a)^5*a^5*c-630/b^9*d^8/(b*x+a)^5*a^4*c^2+840/b^8*d^7/(b*x+a)^5
*a^3*c^3-630/b^7*d^6/(b*x+a)^5*a^2*c^4+252/b^6*d^5/(b*x+a)^5*a*c^5-180/b^10
*d^9/(b*x+a)^2*a^2*c+180/b^9*d^8/(b*x+a)^2*a*c^2-315/b^10*d^9/(b*x+a)^4*a^4
*c+630/b^9*d^8/(b*x+a)^4*a^3*c^2-630/b^8*d^7/(b*x+a)^4*a^2*c^3+315/b^7*d^6/
(b*x+a)^4*a*c^4+90/b^10*d^9/(b*x+a)*a*c+280/b^8*d^7/(b*x+a)^3*a*c^3+360/7/b
^10*d^9/(b*x+a)^7*a^7*c-180/b^9*d^8/(b*x+a)^7*a^6*c^2+360/b^8*d^7/(b*x+a)^7
*a^5*c^3-450/b^7*d^6/(b*x+a)^7*a^4*c^4+360/b^6*d^5/(b*x+a)^7*a^3*c^5-180/b^
5*d^4/(b*x+a)^7*a^2*c^6-700/b^8*d^7/(b*x+a)^6*a^4*c^3-45/4/b^10*d^9/(b*x+a)
^8*a^8*c+45/b^9*d^8/(b*x+a)^8*a^7*c^2-105/b^8*d^7/(b*x+a)^8*a^6*c^3+315/2/b
^7*d^6/(b*x+a)^8*a^5*c^4-315/2/b^6*d^5/(b*x+a)^8*a^4*c^5+105/b^5*d^4/(b*x+a
)^8*a^3*c^6-45/b^4*d^3/(b*x+a)^8*a^2*c^7+45/4/b^3*d^2/(b*x+a)^8*a*c^8+280/b
^10*d^9/(b*x+a)^3*a^3*c-420/b^9*d^8/(b*x+a)^3*a^2*c^2
```

maxima [B] time = 2.36, size = 957, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^10,x, algorithm="maxima")
```

```
[Out] d^10*x/b^10 - 1/252*(28*b^10*c^10 + 35*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 6
0*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*
c^4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 - 7129*a^9*b*c*d^9 + 4
```

```
861*a^10*d^10 + 11340*(b^10*c^2*d^8 - 2*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1
5120*(b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 - 9*a^2*b^8*c*d^9 + 5*a^3*b^7*d^10)*x^
7 + 17640*(b^10*c^4*d^6 + 2*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 - 22*a^3*b^7*
c*d^9 + 13*a^4*b^6*d^10)*x^6 + 5292*(3*b^10*c^5*d^5 + 5*a*b^9*c^4*d^6 + 10*
a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d^8 - 125*a^4*b^6*c*d^9 + 77*a^5*b^5*d^10)
*x^5 + 5292*(2*b^10*c^6*d^4 + 3*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6 + 10*a^3*
b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^8 - 137*a^5*b^5*c*d^9 + 87*a^6*b^4*d^10)*x^4
 + 504*(10*b^10*c^7*d^3 + 14*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 35*a^3*b^
7*c^4*d^6 + 70*a^4*b^6*c^3*d^7 + 210*a^5*b^5*c^2*d^8 - 1029*a^6*b^4*c*d^9 +
 669*a^7*b^3*d^10)*x^3 + 108*(15*b^10*c^8*d^2 + 20*a*b^9*c^7*d^3 + 28*a^2*b
^8*c^6*d^4 + 42*a^3*b^7*c^5*d^5 + 70*a^4*b^6*c^4*d^6 + 140*a^5*b^5*c^3*d^7
+ 420*a^6*b^4*c^2*d^8 - 2178*a^7*b^3*c*d^9 + 1443*a^8*b^2*d^10)*x^2 + 9*(35
*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 60*a^2*b^8*c^7*d^3 + 84*a^3*b^7*c^6*d^4 +
126*a^4*b^6*c^5*d^5 + 210*a^5*b^5*c^4*d^6 + 420*a^6*b^4*c^3*d^7 + 1260*a^7*
b^3*c^2*d^8 - 6849*a^8*b^2*c*d^9 + 4609*a^9*b*d^10)*x)/(b^20*x^9 + 9*a*b^19
*x^8 + 36*a^2*b^18*x^7 + 84*a^3*b^17*x^6 + 126*a^4*b^16*x^5 + 126*a^5*b^15*
x^4 + 84*a^6*b^14*x^3 + 36*a^7*b^13*x^2 + 9*a^8*b^12*x + a^9*b^11) + 10*(b*
c*d^9 - a*d^10)*log(b*x + a)/b^11
```

mupad [B] time = 0.50, size = 955, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^10,x)
```

```
[Out] (d^10*x)/b^10 - (log(a + b*x)*(10*a*d^10 - 10*b*c*d^9))/b^11 - (x^4*(1827*a
^6*b^3*d^10 + 42*b^9*c^6*d^4 + 63*a*b^8*c^5*d^5 - 2877*a^5*b^4*c*d^9 + 105*
a^2*b^7*c^4*d^6 + 210*a^3*b^6*c^3*d^7 + 630*a^4*b^5*c^2*d^8) + x^6*(910*a^4
*b^5*d^10 + 70*b^9*c^4*d^6 + 140*a*b^8*c^3*d^7 - 1540*a^3*b^6*c*d^9 + 420*a
^2*b^7*c^2*d^8) + (4861*a^10*d^10 + 28*b^10*c^10 + 45*a^2*b^8*c^8*d^2 + 60*
a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^
4*d^6 + 420*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 35*a*b^9*c^9*d - 7129*
a^9*b*c*d^9)/(252*b) + x*((4609*a^9*d^10)/28 + (5*b^9*c^9*d)/4 + (45*a*b^8*
c^8*d^2)/28 + (15*a^2*b^7*c^7*d^3)/7 + 3*a^3*b^6*c^6*d^4 + (9*a^4*b^5*c^5*d
^5)/2 + (15*a^5*b^4*c^4*d^6)/2 + 15*a^6*b^3*c^3*d^7 + 45*a^7*b^2*c^2*d^8 -
(6849*a^8*b*c*d^9)/28) + x^8*(45*a^2*b^7*d^10 + 45*b^9*c^2*d^8 - 90*a*b^8*c
*d^9) + x^3*(1338*a^7*b^2*d^10 + 20*b^9*c^7*d^3 + 28*a*b^8*c^6*d^4 - 2058*a
^6*b^3*c*d^9 + 42*a^2*b^7*c^5*d^5 + 70*a^3*b^6*c^4*d^6 + 140*a^4*b^5*c^3*d^
7 + 420*a^5*b^4*c^2*d^8) + x^2*((4329*a^8*b*d^10)/7 + (45*b^9*c^8*d^2)/7 +
(60*a*b^8*c^7*d^3)/7 - (6534*a^7*b^2*c*d^9)/7 + 12*a^2*b^7*c^6*d^4 + 18*a^3
*b^6*c^5*d^5 + 30*a^4*b^5*c^4*d^6 + 60*a^5*b^4*c^3*d^7 + 180*a^6*b^3*c^2*d^
8) + x^5*(1617*a^5*b^4*d^10 + 63*b^9*c^5*d^5 + 105*a*b^8*c^4*d^6 - 2625*a^4
*b^5*c*d^9 + 210*a^2*b^7*c^3*d^7 + 630*a^3*b^6*c^2*d^8) + x^7*(300*a^3*b^6*
d^10 + 60*b^9*c^3*d^7 + 180*a*b^8*c^2*d^8 - 540*a^2*b^7*c*d^9))/(a^9*b^10 +
 b^19*x^9 + 9*a^8*b^11*x + 9*a*b^18*x^8 + 36*a^7*b^12*x^2 + 84*a^6*b^13*x^3
 + 126*a^5*b^14*x^4 + 126*a^4*b^15*x^5 + 84*a^3*b^16*x^6 + 36*a^2*b^17*x^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**10,x)
```

```
[Out] Timed out
```

3.1216 $\int \frac{(c+dx)^{10}}{(a+bx)^{11}} \, dx$

Optimal. Leaf size=271

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-120d^3(bc$$

Rubi [A] time = 0.29, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{b^{11}(a+bx)}-\frac{45d^8(bc-ad)^2}{2b^{11}(a+bx)^2}-\frac{40d^7(bc-ad)^3}{b^{11}(a+bx)^3}-\frac{105d^6(bc-ad)^4}{2b^{11}(a+bx)^4}-\frac{252d^5(bc-ad)^5}{5b^{11}(a+bx)^5}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}+\frac{d^{10}\log(a+bx)}{b^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -(b*c - a*d)^10/(10*b^11*(a + b*x)^10) - (10*d*(b*c - a*d)^9)/(9*b^11*(a +
b*x)^9) - (45*d^2*(b*c - a*d)^8)/(8*b^11*(a + b*x)^8) - (120*d^3*(b*c - a*d
)^7)/(7*b^11*(a + b*x)^7) - (35*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^6) - (25
2*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^5) - (105*d^6*(b*c - a*d)^4)/(2*b^11
*(a + b*x)^4) - (40*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^3) - (45*d^8*(b*c -
a*d)^2)/(2*b^11*(a + b*x)^2) - (10*d^9*(b*c - a*d))/(b^11*(a + b*x)) + (d^1
0*Log[a + b*x])/b^11
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{11}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{10}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^9}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^8}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^7}+\right.\\ &= -\frac{(bc-ad)^{10}}{10b^{11}(a+bx)^{10}}-\frac{10d(bc-ad)^9}{9b^{11}(a+bx)^9}-\frac{45d^2(bc-ad)^8}{8b^{11}(a+bx)^8}-\frac{120d^3(bc-ad)^7}{7b^{11}(a+bx)^7}-\frac{35d^4(bc-ad)^6}{b^{11}(a+bx)^6}-\end{aligned}$$

Mathematica [B] time = 0.36, size = 591, normalized size = 2.18

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^11,x]
```

```
[Out] -1/2520*((b*c - a*d)*(7381*a^9*d^9 + a^8*b*d^8*(4861*c + 71290*d*x) + a^7*b
^2*d^7*(3601*c^2 + 46090*c*d*x + 308205*d^2*x^2) + a^6*b^3*d^6*(2761*c^3 +
33490*c^2*d*x + 194805*c*d^2*x^2 + 784080*d^3*x^3) + a^5*b^4*d^5*(2131*c^4
+ 25090*c^3*d*x + 138105*c^2*d^2*x^2 + 481680*c*d^3*x^3 + 1296540*d^4*x^4)
+ a^4*b^5*d^4*(1627*c^5 + 18790*c^4*d*x + 100305*c^3*d^2*x^2 + 330480*c^2*d
^3*x^3 + 767340*c*d^4*x^4 + 1450008*d^5*x^5) + a^3*b^6*d^3*(1207*c^6 + 1375
0*c^5*d*x + 71955*c^4*d^2*x^2 + 229680*c^3*d^3*x^3 + 502740*c^2*d^4*x^4 + 8
14968*c*d^5*x^5 + 1102500*d^6*x^6) + a^2*b^7*d^2*(847*c^7 + 9550*c^6*d*x +
49275*c^5*d^2*x^2 + 154080*c^4*d^3*x^3 + 326340*c^3*d^4*x^4 + 497448*c^2*d^
5*x^5 + 573300*c*d^6*x^6 + 554400*d^7*x^7) + a*b^8*d*(532*c^8 + 5950*c^7*d*
```

x + 30375*c^6*d^2*x^2 + 93600*c^5*d^3*x^3 + 194040*c^4*d^4*x^4 + 285768*c^3
*d^5*x^5 + 308700*c^2*d^6*x^6 + 252000*c*d^7*x^7 + 170100*d^8*x^8) + b^9*(2
52*c^9 + 2800*c^8*d*x + 14175*c^7*d^2*x^2 + 43200*c^6*d^3*x^3 + 88200*c^5*d
^4*x^4 + 127008*c^4*d^5*x^5 + 132300*c^3*d^6*x^6 + 100800*c^2*d^7*x^7 + 567
00*c*d^8*x^8 + 25200*d^9*x^9)))/(b^11*(a + b*x)^10) + (d^10*Log[a + b*x])/b
^11

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{11}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^11,x]

[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^11, x]

fricas [B] time = 1.22, size = 1107, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="fricas")

[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x - 2520*(b^10*d^10*x^10 + 10*a*b^9*d^10*x^
9 + 45*a^2*b^8*d^10*x^8 + 120*a^3*b^7*d^10*x^7 + 210*a^4*b^6*d^10*x^6 + 252
*a^5*b^5*d^10*x^5 + 210*a^6*b^4*d^10*x^4 + 120*a^7*b^3*d^10*x^3 + 45*a^8*b^
2*d^10*x^2 + 10*a^9*b*d^10*x + a^10*d^10)*log(b*x + a))/(b^21*x^10 + 10*a*b
^20*x^9 + 45*a^2*b^19*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b
^16*x^5 + 210*a^6*b^15*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^
12*x + a^10*b^11)

giac [B] time = 1.36, size = 874, normalized size = 3.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="giac")

[Out] d^10*log(abs(b*x + a))/b^11 - 1/2520*(25200*(b^9*c*d^9 - a*b^8*d^10)*x^9 +
56700*(b^9*c^2*d^8 + 2*a*b^8*c*d^9 - 3*a^2*b^7*d^10)*x^8 + 50400*(2*b^9*c^3

```
*d^7 + 3*a*b^8*c^2*d^8 + 6*a^2*b^7*c*d^9 - 11*a^3*b^6*d^10)*x^7 + 44100*(3*
b^9*c^4*d^6 + 4*a*b^8*c^3*d^7 + 6*a^2*b^7*c^2*d^8 + 12*a^3*b^6*c*d^9 - 25*a
^4*b^5*d^10)*x^6 + 10584*(12*b^9*c^5*d^5 + 15*a*b^8*c^4*d^6 + 20*a^2*b^7*c^
3*d^7 + 30*a^3*b^6*c^2*d^8 + 60*a^4*b^5*c*d^9 - 137*a^5*b^4*d^10)*x^5 + 882
0*(10*b^9*c^6*d^4 + 12*a*b^8*c^5*d^5 + 15*a^2*b^7*c^4*d^6 + 20*a^3*b^6*c^3*
d^7 + 30*a^4*b^5*c^2*d^8 + 60*a^5*b^4*c*d^9 - 147*a^6*b^3*d^10)*x^4 + 720*(
60*b^9*c^7*d^3 + 70*a*b^8*c^6*d^4 + 84*a^2*b^7*c^5*d^5 + 105*a^3*b^6*c^4*d^
6 + 140*a^4*b^5*c^3*d^7 + 210*a^5*b^4*c^2*d^8 + 420*a^6*b^3*c*d^9 - 1089*a^
7*b^2*d^10)*x^3 + 135*(105*b^9*c^8*d^2 + 120*a*b^8*c^7*d^3 + 140*a^2*b^7*c^
6*d^4 + 168*a^3*b^6*c^5*d^5 + 210*a^4*b^5*c^4*d^6 + 280*a^5*b^4*c^3*d^7 + 4
20*a^6*b^3*c^2*d^8 + 840*a^7*b^2*c*d^9 - 2283*a^8*b*d^10)*x^2 + 10*(280*b^9
*c^9*d + 315*a*b^8*c^8*d^2 + 360*a^2*b^7*c^7*d^3 + 420*a^3*b^6*c^6*d^4 + 50
4*a^4*b^5*c^5*d^5 + 630*a^5*b^4*c^4*d^6 + 840*a^6*b^3*c^3*d^7 + 1260*a^7*b^
2*c^2*d^8 + 2520*a^8*b*c*d^9 - 7129*a^9*d^10)*x + (252*b^10*c^10 + 280*a*b^
9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 +
 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6 + 840*a^7*b^3*c^3*d^7 + 1260*a^8
*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^10*d^10)/b)/((b*x + a)^10*b^10)
```

maple [B] time = 0.01, size = 1271, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^11,x)
```

```
[Out] -35*d^10/b^11/(b*x+a)^6*a^6-35*d^4/b^5/(b*x+a)^6*c^6-1/10/b^11/(b*x+a)^10*a
^10*d^10-120/7*d^3/b^4/(b*x+a)^7*c^7+10/9*d^10/b^11/(b*x+a)^9*a^9-10/9*d/b^
2/(b*x+a)^9*c^9+252/5*d^10/b^11/(b*x+a)^5*a^5-252/5*d^5/b^6/(b*x+a)^5*c^5-4
5/2*d^10/b^11/(b*x+a)^2*a^2-45/2*d^8/b^9/(b*x+a)^2*c^2-105/2*d^10/b^11/(b*x
+a)^4*a^4-105/2*d^6/b^7/(b*x+a)^4*c^4+10/b^11*d^10/(b*x+a)*a-10/b^10*d^9/(b
*x+a)*c-45/8*d^10/b^11/(b*x+a)^8*a^8-45/8*d^2/b^3/(b*x+a)^8*c^8+40*d^10/b^1
1/(b*x+a)^3*a^3-40*d^7/b^8/(b*x+a)^3*c^3+120/7*d^10/b^11/(b*x+a)^7*a^7-315/
2*d^4/b^5/(b*x+a)^8*a^2*c^6+45*d^3/b^4/(b*x+a)^8*a*c^7-120*d^9/b^10/(b*x+a)
^3*a^2*c+120*d^8/b^9/(b*x+a)^3*a*c^2-120*d^9/b^10/(b*x+a)^7*a^6*c+360*d^8/b
^9/(b*x+a)^7*a^5*c^2-600*d^7/b^8/(b*x+a)^7*a^4*c^3+600*d^6/b^7/(b*x+a)^7*a^
3*c^4-360*d^5/b^6/(b*x+a)^7*a^2*c^5+120*d^4/b^5/(b*x+a)^7*a*c^6-10*d^9/b^10
/(b*x+a)^9*a^8*c+40*d^8/b^9/(b*x+a)^9*a^7*c^2-280/3*d^7/b^8/(b*x+a)^9*a^6*c
^3+140*d^6/b^7/(b*x+a)^9*a^5*c^4-140*d^5/b^6/(b*x+a)^9*a^4*c^5+280/3*d^4/b^
5/(b*x+a)^9*a^3*c^6-40*d^3/b^4/(b*x+a)^9*a^2*c^7+10*d^2/b^3/(b*x+a)^9*a*c^8
-252*d^9/b^10/(b*x+a)^5*a^4*c+504*d^8/b^9/(b*x+a)^5*a^3*c^2-504*d^7/b^8/(b*
x+a)^5*a^2*c^3+252*d^6/b^7/(b*x+a)^5*a*c^4+45*d^9/b^10/(b*x+a)^2*a*c+210*d^
9/b^10/(b*x+a)^4*a^3*c-315*d^8/b^9/(b*x+a)^4*a^2*c^2+210*d^7/b^8/(b*x+a)^4*
a*c^3+210*d^9/b^10/(b*x+a)^6*a^5*c-525*d^8/b^9/(b*x+a)^6*a^4*c^2+700*d^7/b^
8/(b*x+a)^6*a^3*c^3-525*d^6/b^7/(b*x+a)^6*a^2*c^4+210*d^5/b^6/(b*x+a)^6*a*c
^5+1/b^10/(b*x+a)^10*a^9*c*d^9-9/2/b^9/(b*x+a)^10*a^8*c^2*d^8+12/b^8/(b*x+a
)^10*a^7*c^3*d^7-21/b^7/(b*x+a)^10*a^6*c^4*d^6+126/5/b^6/(b*x+a)^10*a^5*c^5
*d^5-21/b^5/(b*x+a)^10*a^4*c^6*d^4+12/b^4/(b*x+a)^10*a^3*c^7*d^3-9/2/b^3/(b
*x+a)^10*a^2*c^8*d^2+1/b^2/(b*x+a)^10*a*c^9*d+45*d^9/b^10/(b*x+a)^8*a^7*c-3
15/2*d^8/b^9/(b*x+a)^8*a^6*c^2+315*d^5/b^6/(b*x+a)^8*a^3*c^5+d^10*ln(b*x+a)
/b^11-1/10/b/(b*x+a)^10*c^10+315*d^7/b^8/(b*x+a)^8*a^5*c^3-1575/4*d^6/b^7/(
b*x+a)^8*a^4*c^4
```

maxima [B] time = 2.01, size = 975, normalized size = 3.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^11,x, algorithm="maxima")
```

```
[Out] -1/2520*(252*b^10*c^10 + 280*a*b^9*c^9*d + 315*a^2*b^8*c^8*d^2 + 360*a^3*b^
7*c^7*d^3 + 420*a^4*b^6*c^6*d^4 + 504*a^5*b^5*c^5*d^5 + 630*a^6*b^4*c^4*d^6
```

```
 + 840*a^7*b^3*c^3*d^7 + 1260*a^8*b^2*c^2*d^8 + 2520*a^9*b*c*d^9 - 7381*a^1
0*d^10 + 25200*(b^10*c*d^9 - a*b^9*d^10)*x^9 + 56700*(b^10*c^2*d^8 + 2*a*b^
9*c*d^9 - 3*a^2*b^8*d^10)*x^8 + 50400*(2*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 6
*a^2*b^8*c*d^9 - 11*a^3*b^7*d^10)*x^7 + 44100*(3*b^10*c^4*d^6 + 4*a*b^9*c^3
*d^7 + 6*a^2*b^8*c^2*d^8 + 12*a^3*b^7*c*d^9 - 25*a^4*b^6*d^10)*x^6 + 10584*
(12*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 30*a^3*b^7*c^2*d
^8 + 60*a^4*b^6*c*d^9 - 137*a^5*b^5*d^10)*x^5 + 8820*(10*b^10*c^6*d^4 + 12*
a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 30*a^4*b^6*c^2*d^
8 + 60*a^5*b^5*c*d^9 - 147*a^6*b^4*d^10)*x^4 + 720*(60*b^10*c^7*d^3 + 70*a*
b^9*c^6*d^4 + 84*a^2*b^8*c^5*d^5 + 105*a^3*b^7*c^4*d^6 + 140*a^4*b^6*c^3*d^
7 + 210*a^5*b^5*c^2*d^8 + 420*a^6*b^4*c*d^9 - 1089*a^7*b^3*d^10)*x^3 + 135*
(105*b^10*c^8*d^2 + 120*a*b^9*c^7*d^3 + 140*a^2*b^8*c^6*d^4 + 168*a^3*b^7*c
^5*d^5 + 210*a^4*b^6*c^4*d^6 + 280*a^5*b^5*c^3*d^7 + 420*a^6*b^4*c^2*d^8 +
840*a^7*b^3*c*d^9 - 2283*a^8*b^2*d^10)*x^2 + 10*(280*b^10*c^9*d + 315*a*b^9
*c^8*d^2 + 360*a^2*b^8*c^7*d^3 + 420*a^3*b^7*c^6*d^4 + 504*a^4*b^6*c^5*d^5
+ 630*a^5*b^5*c^4*d^6 + 840*a^6*b^4*c^3*d^7 + 1260*a^7*b^3*c^2*d^8 + 2520*a
^8*b^2*c*d^9 - 7129*a^9*b*d^10)*x)/(b^21*x^10 + 10*a*b^20*x^9 + 45*a^2*b^19
*x^8 + 120*a^3*b^18*x^7 + 210*a^4*b^17*x^6 + 252*a^5*b^16*x^5 + 210*a^6*b^1
5*x^4 + 120*a^7*b^14*x^3 + 45*a^8*b^13*x^2 + 10*a^9*b^12*x + a^10*b^11) + d
^10*log(b*x + a)/b^11
```

mupad [B] time = 0.56, size = 866, normalized size = 3.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^11,x)
```

```
[Out] (d^10*log(a + b*x))/b^11 - (x^4*(35*b^10*c^6*d^4 - (1029*a^6*b^4*d^10)/2 +
42*a*b^9*c^5*d^5 + 210*a^5*b^5*c*d^9 + (105*a^2*b^8*c^4*d^6)/2 + 70*a^3*b^7
*c^3*d^7 + 105*a^4*b^6*c^2*d^8) - x^9*(10*a*b^9*d^10 - 10*b^10*c*d^9) + x*(
(10*b^10*c^9*d)/9 - (7129*a^9*b*d^10)/252 + (5*a*b^9*c^8*d^2)/4 + 10*a^8*b^
2*c*d^9 + (10*a^2*b^8*c^7*d^3)/7 + (5*a^3*b^7*c^6*d^4)/3 + 2*a^4*b^6*c^5*d^
5 + (5*a^5*b^5*c^4*d^6)/2 + (10*a^6*b^4*c^3*d^7)/3 + 5*a^7*b^3*c^2*d^8) + x
^6*((105*b^10*c^4*d^6)/2 - (875*a^4*b^6*d^10)/2 + 70*a*b^9*c^3*d^7 + 210*a^
3*b^7*c*d^9 + 105*a^2*b^8*c^2*d^8) + x^8*((45*b^10*c^2*d^8)/2 - (135*a^2*b^
8*d^10)/2 + 45*a*b^9*c*d^9) + x^3*((120*b^10*c^7*d^3)/7 - (2178*a^7*b^3*d^1
0)/7 + 20*a*b^9*c^6*d^4 + 120*a^6*b^4*c*d^9 + 24*a^2*b^8*c^5*d^5 + 30*a^3*b
^7*c^4*d^6 + 40*a^4*b^6*c^3*d^7 + 60*a^5*b^5*c^2*d^8) + x^5*((252*b^10*c^5*
d^5)/5 - (2877*a^5*b^5*d^10)/5 + 63*a*b^9*c^4*d^6 + 252*a^4*b^6*c*d^9 + 84*
a^2*b^8*c^3*d^7 + 126*a^3*b^7*c^2*d^8) - (7381*a^10*d^10)/2520 + (b^10*c^10
)/10 + x^7*(40*b^10*c^3*d^7 - 220*a^3*b^7*d^10 + 60*a*b^9*c^2*d^8 + 120*a^2
*b^8*c*d^9) + x^2*((45*b^10*c^8*d^2)/8 - (6849*a^8*b^2*d^10)/56 + (45*a*b^9
*c^7*d^3)/7 + 45*a^7*b^3*c*d^9 + (15*a^2*b^8*c^6*d^4)/2 + 9*a^3*b^7*c^5*d^5
 + (45*a^4*b^6*c^4*d^6)/4 + 15*a^5*b^5*c^3*d^7 + (45*a^6*b^4*c^2*d^8)/2) +
(a^2*b^8*c^8*d^2)/8 + (a^3*b^7*c^7*d^3)/7 + (a^4*b^6*c^6*d^4)/6 + (a^5*b^5*
c^5*d^5)/5 + (a^6*b^4*c^4*d^6)/4 + (a^7*b^3*c^3*d^7)/3 + (a^8*b^2*c^2*d^8)/
2 + (a*b^9*c^9*d)/9 + a^9*b*c*d^9)/(b^11*(a + b*x)^10)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**11,x)
```

```
[Out] Timed out
```

3.1217 $\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx$

Optimal. Leaf size=28

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$-\frac{(c+dx)^{11}}{11(a+bx)^{11}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -(c + d*x)^11/(11*(b*c - a*d)*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx = -\frac{(c+dx)^{11}}{11(bc-ad)(a+bx)^{11}}$$

Mathematica [B] time = 0.28, size = 665, normalized size = 23.75

$$-\frac{a^{10}d^{10}+a^9bd^9(c+11dx)+a^8b^2d^8(c^2+11cdx+55d^2x^2)+a^7b^3d^7(c^3+11c^2dx+55cd^2x^2+165d^3x^3)+a^6b^4d^6(c^4+11c^3dx+55c^2d^2x^2+165cd^3x^3+330d^4x^4)+a^5b^5d^5(c^5+11c^4dx+55c^3d^2x^2+165c^2d^3x^3+330cd^4x^4+462d^5x^5)+a^4b^6d^4(c^6+11c^5dx+55c^4d^2x^2+165c^3d^3x^3+330c^2d^4x^4+462cd^5x^5+462d^6x^6)+a^3b^7d^3(c^7+11c^6dx+55c^5d^2x^2+165c^4d^3x^3+330c^3d^4x^4+462c^2d^5x^5+462cd^6x^6+330d^7x^7)+a^2b^8d^2(c^8+11c^7dx+55c^6d^2x^2+165c^5d^3x^3+330c^4d^4x^4+462c^3d^5x^5+462c^2d^6x^6+330cd^7x^7+165d^8x^8)+ab^9d(c^9+11c^8dx+55c^7d^2x^2+165c^6d^3x^3+330c^5d^4x^4+462c^4d^5x^5+462c^3d^6x^6+330c^2d^7x^7+165cd^8x^8+55d^9x^9)+b^{10}(c^{10}+11c^9dx+55c^8d^2x^2+165c^7d^3x^3+330c^6d^4x^4+462c^5d^5x^5+462c^4d^6x^6+330c^3d^7x^7+165c^2d^8x^8+55cd^9x^9+11d^{10}x^{10})}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] -1/11*(a^10*d^10 + a^9*b*d^9*(c + 11*d*x) + a^8*b^2*d^8*(c^2 + 11*c*d*x + 5
5*d^2*x^2) + a^7*b^3*d^7*(c^3 + 11*c^2*d*x + 55*c*d^2*x^2 + 165*d^3*x^3) +
a^6*b^4*d^6*(c^4 + 11*c^3*d*x + 55*c^2*d^2*x^2 + 165*c*d^3*x^3 + 330*d^4*x^
4) + a^5*b^5*d^5*(c^5 + 11*c^4*d*x + 55*c^3*d^2*x^2 + 165*c^2*d^3*x^3 + 330
*c*d^4*x^4 + 462*d^5*x^5) + a^4*b^6*d^4*(c^6 + 11*c^5*d*x + 55*c^4*d^2*x^2
+ 165*c^3*d^3*x^3 + 330*c^2*d^4*x^4 + 462*c*d^5*x^5 + 462*d^6*x^6) + a^3*b^
7*d^3*(c^7 + 11*c^6*d*x + 55*c^5*d^2*x^2 + 165*c^4*d^3*x^3 + 330*c^3*d^4*x^
4 + 462*c^2*d^5*x^5 + 462*c*d^6*x^6 + 330*d^7*x^7) + a^2*b^8*d^2*(c^8 + 11*
c^7*d*x + 55*c^6*d^2*x^2 + 165*c^5*d^3*x^3 + 330*c^4*d^4*x^4 + 462*c^3*d^5*
x^5 + 462*c^2*d^6*x^6 + 330*c*d^7*x^7 + 165*d^8*x^8) + a*b^9*d*(c^9 + 11*c^
8*d*x + 55*c^7*d^2*x^2 + 165*c^6*d^3*x^3 + 330*c^5*d^4*x^4 + 462*c^4*d^5*x^
5 + 462*c^3*d^6*x^6 + 330*c^2*d^7*x^7 + 165*c*d^8*x^8 + 55*d^9*x^9) + b^10*
(c^10 + 11*c^9*d*x + 55*c^8*d^2*x^2 + 165*c^7*d^3*x^3 + 330*c^6*d^4*x^4 + 4
62*c^5*d^5*x^5 + 462*c^4*d^6*x^6 + 330*c^3*d^7*x^7 + 165*c^2*d^8*x^8 + 55*c
*d^9*x^9 + 11*d^10*x^10))/(b^11*(a + b*x)^11)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^12,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^12, x]
```

fricas [B] time = 1.35, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="fricas")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

giac [B] time = 1.36, size = 951, normalized size = 33.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="giac")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + 55*b^10*c*d^9*x^9 + 55*a*b^9*d^10*x^9 + 165*b^10
*c^2*d^8*x^8 + 165*a*b^9*c*d^9*x^8 + 165*a^2*b^8*d^10*x^8 + 330*b^10*c^3*d^
7*x^7 + 330*a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7 + 330*a^3*b^7*d^10*x^
7 + 462*b^10*c^4*d^6*x^6 + 462*a*b^9*c^3*d^7*x^6 + 462*a^2*b^8*c^2*d^8*x^6
+ 462*a^3*b^7*c*d^9*x^6 + 462*a^4*b^6*d^10*x^6 + 462*b^10*c^5*d^5*x^5 + 462
*a*b^9*c^4*d^6*x^5 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 46
2*a^4*b^6*c*d^9*x^5 + 462*a^5*b^5*d^10*x^5 + 330*b^10*c^6*d^4*x^4 + 330*a*b
^9*c^5*d^5*x^4 + 330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^
4*b^6*c^2*d^8*x^4 + 330*a^5*b^5*c*d^9*x^4 + 330*a^6*b^4*d^10*x^4 + 165*b^10
*c^7*d^3*x^3 + 165*a*b^9*c^6*d^4*x^3 + 165*a^2*b^8*c^5*d^5*x^3 + 165*a^3*b^
7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 + 165*a^6
*b^4*c*d^9*x^3 + 165*a^7*b^3*d^10*x^3 + 55*b^10*c^8*d^2*x^2 + 55*a*b^9*c^7*
d^3*x^2 + 55*a^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*
d^6*x^2 + 55*a^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 55*a^7*b^3*c*d^
9*x^2 + 55*a^8*b^2*d^10*x^2 + 11*b^10*c^9*d*x + 11*a*b^9*c^8*d^2*x + 11*a^2
*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*c
^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 11*a^8*b^2*c*d^9*x
 + 11*a^9*b*d^10*x + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*b^7*c^
7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d
^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^11*b^11)
```

maple [B] time = 0.01, size = 866, normalized size = 30.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^12,x)
```

```
[Out] 15*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*
b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^8-15*d^8
*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^3-30*d^4*(a^6*d^6-6*a^5*b*c*d^5+1
5*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c
^6)/b^11/(b*x+a)^7-5*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b
^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7
*c^7*d+b^8*c^8)/b^11/(b*x+a)^9-42*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*
d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^5+5*d^9*(a*d-b*c)/b^11/(b*x+a)^2+30
*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^4-1/11*(a^1
0*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^
4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^
8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^11-d^10/b^11/(b*x+a)+42*d^
5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*
d-b^5*c^5)/b^11/(b*x+a)^6+d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^
6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36
*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^10
```

maxima [B] time = 2.13, size = 920, normalized size = 32.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^12,x, algorithm="maxima")
```

```
[Out] -1/11*(11*b^10*d^10*x^10 + b^10*c^10 + a*b^9*c^9*d + a^2*b^8*c^8*d^2 + a^3*
b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3
*c^3*d^7 + a^8*b^2*c^2*d^8 + a^9*b*c*d^9 + a^10*d^10 + 55*(b^10*c*d^9 + a*b
^9*d^10)*x^9 + 165*(b^10*c^2*d^8 + a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 330*(b
^10*c^3*d^7 + a*b^9*c^2*d^8 + a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 462*(b^10
*c^4*d^6 + a*b^9*c^3*d^7 + a^2*b^8*c^2*d^8 + a^3*b^7*c*d^9 + a^4*b^6*d^10)*
x^6 + 462*(b^10*c^5*d^5 + a*b^9*c^4*d^6 + a^2*b^8*c^3*d^7 + a^3*b^7*c^2*d^8
 + a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 330*(b^10*c^6*d^4 + a*b^9*c^5*d^5 +
a^2*b^8*c^4*d^6 + a^3*b^7*c^3*d^7 + a^4*b^6*c^2*d^8 + a^5*b^5*c*d^9 + a^6*b
^4*d^10)*x^4 + 165*(b^10*c^7*d^3 + a*b^9*c^6*d^4 + a^2*b^8*c^5*d^5 + a^3*b^
7*c^4*d^6 + a^4*b^6*c^3*d^7 + a^5*b^5*c^2*d^8 + a^6*b^4*c*d^9 + a^7*b^3*d^1
0)*x^3 + 55*(b^10*c^8*d^2 + a*b^9*c^7*d^3 + a^2*b^8*c^6*d^4 + a^3*b^7*c^5*d
^5 + a^4*b^6*c^4*d^6 + a^5*b^5*c^3*d^7 + a^6*b^4*c^2*d^8 + a^7*b^3*c*d^9 +
a^8*b^2*d^10)*x^2 + 11*(b^10*c^9*d + a*b^9*c^8*d^2 + a^2*b^8*c^7*d^3 + a^3*
b^7*c^6*d^4 + a^4*b^6*c^5*d^5 + a^5*b^5*c^4*d^6 + a^6*b^4*c^3*d^7 + a^7*b^3
*c^2*d^8 + a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^22*x^11 + 11*a*b^21*x^10 + 55*
a^2*b^20*x^9 + 165*a^3*b^19*x^8 + 330*a^4*b^18*x^7 + 462*a^5*b^17*x^6 + 462
*a^6*b^16*x^5 + 330*a^7*b^15*x^4 + 165*a^8*b^14*x^3 + 55*a^9*b^13*x^2 + 11*
a^10*b^12*x + a^11*b^11)
```

mupad [B] time = 0.46, size = 1066, normalized size = 38.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^12,x)
```

```
[Out] -(a^10*d^10 + b^10*c^10 + 11*b^10*d^10*x^10 + 55*a*b^9*d^10*x^9 + 55*b^10*c
*d^9*x^9 + a^2*b^8*c^8*d^2 + a^3*b^7*c^7*d^3 + a^4*b^6*c^6*d^4 + a^5*b^5*c^
5*d^5 + a^6*b^4*c^4*d^6 + a^7*b^3*c^3*d^7 + a^8*b^2*c^2*d^8 + 55*a^8*b^2*d^
10*x^2 + 165*a^7*b^3*d^10*x^3 + 330*a^6*b^4*d^10*x^4 + 462*a^5*b^5*d^10*x^5
 + 462*a^4*b^6*d^10*x^6 + 330*a^3*b^7*d^10*x^7 + 165*a^2*b^8*d^10*x^8 + 55*
b^10*c^8*d^2*x^2 + 165*b^10*c^7*d^3*x^3 + 330*b^10*c^6*d^4*x^4 + 462*b^10*c
^5*d^5*x^5 + 462*b^10*c^4*d^6*x^6 + 330*b^10*c^3*d^7*x^7 + 165*b^10*c^2*d^8
*x^8 + a*b^9*c^9*d + a^9*b*c*d^9 + 11*a^9*b*d^10*x + 11*b^10*c^9*d*x + 55*a
^2*b^8*c^6*d^4*x^2 + 55*a^3*b^7*c^5*d^5*x^2 + 55*a^4*b^6*c^4*d^6*x^2 + 55*a
^5*b^5*c^3*d^7*x^2 + 55*a^6*b^4*c^2*d^8*x^2 + 165*a^2*b^8*c^5*d^5*x^3 + 165
*a^3*b^7*c^4*d^6*x^3 + 165*a^4*b^6*c^3*d^7*x^3 + 165*a^5*b^5*c^2*d^8*x^3 +
330*a^2*b^8*c^4*d^6*x^4 + 330*a^3*b^7*c^3*d^7*x^4 + 330*a^4*b^6*c^2*d^8*x^4
 + 462*a^2*b^8*c^3*d^7*x^5 + 462*a^3*b^7*c^2*d^8*x^5 + 462*a^2*b^8*c^2*d^8*
x^6 + 11*a*b^9*c^8*d^2*x + 11*a^8*b^2*c*d^9*x + 165*a*b^9*c*d^9*x^8 + 11*a^
2*b^8*c^7*d^3*x + 11*a^3*b^7*c^6*d^4*x + 11*a^4*b^6*c^5*d^5*x + 11*a^5*b^5*
c^4*d^6*x + 11*a^6*b^4*c^3*d^7*x + 11*a^7*b^3*c^2*d^8*x + 55*a*b^9*c^7*d^3*
x^2 + 55*a^7*b^3*c*d^9*x^2 + 165*a*b^9*c^6*d^4*x^3 + 165*a^6*b^4*c*d^9*x^3
+ 330*a*b^9*c^5*d^5*x^4 + 330*a^5*b^5*c*d^9*x^4 + 462*a*b^9*c^4*d^6*x^5 + 4
62*a^4*b^6*c*d^9*x^5 + 462*a*b^9*c^3*d^7*x^6 + 462*a^3*b^7*c*d^9*x^6 + 330*
a*b^9*c^2*d^8*x^7 + 330*a^2*b^8*c*d^9*x^7)/(11*a^11*b^11 + 11*b^22*x^11 + 1
21*a^10*b^12*x + 121*a*b^21*x^10 + 605*a^9*b^13*x^2 + 1815*a^8*b^14*x^3 + 3
630*a^7*b^15*x^4 + 5082*a^6*b^16*x^5 + 5082*a^5*b^17*x^6 + 3630*a^4*b^18*x^
7 + 1815*a^3*b^19*x^8 + 605*a^2*b^20*x^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**12,x)
```

```
[Out] Timed out
```

3.1218 $\int \frac{(c+dx)^{10}}{(a+bx)^{13}} \, dx$

Optimal. Leaf size=58

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d(c+dx)^{11}}{132(a+bx)^{11}(bc-ad)^2}-\frac{(c+dx)^{11}}{12(a+bx)^{12}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -(c + d*x)^11/(12*(b*c - a*d)*(a + b*x)^12) + (d*(c + d*x)^11)/(132*(b*c -
a*d)^2*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned} \int \frac{(c+dx)^{10}}{(a+bx)^{13}} \, dx &= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}-\frac{d\int \frac{(c+dx)^{10}}{(a+bx)^{12}} \, dx}{12(bc-ad)}\\ &= -\frac{(c+dx)^{11}}{12(bc-ad)(a+bx)^{12}}+\frac{d(c+dx)^{11}}{132(bc-ad)^2(a+bx)^{11}} \end{aligned}$$

Mathematica [B] time = 0.28, size = 684, normalized size = 11.79

$$-\frac{a^{10}d^{10}+2a^9bd^9(c+6dx)+3a^8b^2d^8\left(c^2+8cdx+22d^2x^2\right)+4a^7b^3d^7\left(c^3+9c^2dx+33cd^2x^2+55d^3x^3\right)+a^6b^4d^6\left(5c^4+48c^3dx+198c^2d^2x^2+440cd^3x^3+495d^4x^4\right)+6a^5b^5d^5\left(c^5+10c^4dx+44c^3d^2x^2+110c^2d^3x^3+165cd^4x^4+132d^5x^5\right)+a^4b^6d^4\left(7c^6+72c^5dx+330c^4d^2x^2+880c^3d^3x^3+1485c^2d^4x^4+1584cd^5x^5+924d^6x^6\right)+4a^3b^7d^3\left(2c^7+21c^6dx+99c^5d^2x^2+275c^4d^3x^3+495c^3d^4x^4+594c^2d^5x^5+462cd^6x^6+198d^7x^7\right)+3a^2b^8d^2\left(3c^8+32c^7dx+154c^6d^2x^2+440c^5d^3x^3+825c^4d^4x^4+1056c^3d^5x^5+924c^2d^6x^6+528cd^7x^7+165d^8x^8\right)+2ab^9d\left(5c^9+54c^8dx+264c^7d^2x^2+770c^6d^3x^3+1485c^5d^4x^4+1980c^4d^5x^5+1848c^3d^6x^6+1188c^2d^7x^7+495cd^8x^8+110d^9x^9\right)+b^{10}\left(11c^{10}+120c^9dx+594c^8d^2x^2+1760c^7d^3x^3+3465c^6d^4x^4+4752c^5d^5x^5+4620c^4d^6x^6+3168c^3d^7x^7+1485c^2d^8x^8+440cd^9x^9+66d^{10}x^{10}\right)}{132b^{11}(a+bx)^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] -1/132*(a^10*d^10 + 2*a^9*b*d^9*(c + 6*d*x) + 3*a^8*b^2*d^8*(c^2 + 8*c*d*x
+ 22*d^2*x^2) + 4*a^7*b^3*d^7*(c^3 + 9*c^2*d*x + 33*c*d^2*x^2 + 55*d^3*x^3)
 + a^6*b^4*d^6*(5*c^4 + 48*c^3*d*x + 198*c^2*d^2*x^2 + 440*c*d^3*x^3 + 495*
```

```
d^4*x^4) + 6*a^5*b^5*d^5*(c^5 + 10*c^4*d*x + 44*c^3*d^2*x^2 + 110*c^2*d^3*x
^3 + 165*c*d^4*x^4 + 132*d^5*x^5) + a^4*b^6*d^4*(7*c^6 + 72*c^5*d*x + 330*c
^4*d^2*x^2 + 880*c^3*d^3*x^3 + 1485*c^2*d^4*x^4 + 1584*c*d^5*x^5 + 924*d^6*
x^6) + 4*a^3*b^7*d^3*(2*c^7 + 21*c^6*d*x + 99*c^5*d^2*x^2 + 275*c^4*d^3*x^3
 + 495*c^3*d^4*x^4 + 594*c^2*d^5*x^5 + 462*c*d^6*x^6 + 198*d^7*x^7) + 3*a^2
*b^8*d^2*(3*c^8 + 32*c^7*d*x + 154*c^6*d^2*x^2 + 440*c^5*d^3*x^3 + 825*c^4*
d^4*x^4 + 1056*c^3*d^5*x^5 + 924*c^2*d^6*x^6 + 528*c*d^7*x^7 + 165*d^8*x^8)
 + 2*a*b^9*d*(5*c^9 + 54*c^8*d*x + 264*c^7*d^2*x^2 + 770*c^6*d^3*x^3 + 1485
*c^5*d^4*x^4 + 1980*c^4*d^5*x^5 + 1848*c^3*d^6*x^6 + 1188*c^2*d^7*x^7 + 495
*c*d^8*x^8 + 110*d^9*x^9) + b^10*(11*c^10 + 120*c^9*d*x + 594*c^8*d^2*x^2 +
 1760*c^7*d^3*x^3 + 3465*c^6*d^4*x^4 + 4752*c^5*d^5*x^5 + 4620*c^4*d^6*x^6
+ 3168*c^3*d^7*x^7 + 1485*c^2*d^8*x^8 + 440*c*d^9*x^9 + 66*d^10*x^10))/(b^1
1*(a + b*x)^12)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^13,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^13, x]
```

fricas [B] time = 1.29, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="fricas")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

giac [B] time = 1.32, size = 961, normalized size = 16.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="giac")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 440*b^10*c*d^9*x^9 + 220*a*b^9*d^10*x^9 + 1485*
b^10*c^2*d^8*x^8 + 990*a*b^9*c*d^9*x^8 + 495*a^2*b^8*d^10*x^8 + 3168*b^10*c
^3*d^7*x^7 + 2376*a*b^9*c^2*d^8*x^7 + 1584*a^2*b^8*c*d^9*x^7 + 792*a^3*b^7*
d^10*x^7 + 4620*b^10*c^4*d^6*x^6 + 3696*a*b^9*c^3*d^7*x^6 + 2772*a^2*b^8*c^
2*d^8*x^6 + 1848*a^3*b^7*c*d^9*x^6 + 924*a^4*b^6*d^10*x^6 + 4752*b^10*c^5*d
^5*x^5 + 3960*a*b^9*c^4*d^6*x^5 + 3168*a^2*b^8*c^3*d^7*x^5 + 2376*a^3*b^7*c
^2*d^8*x^5 + 1584*a^4*b^6*c*d^9*x^5 + 792*a^5*b^5*d^10*x^5 + 3465*b^10*c^6*
d^4*x^4 + 2970*a*b^9*c^5*d^5*x^4 + 2475*a^2*b^8*c^4*d^6*x^4 + 1980*a^3*b^7*
c^3*d^7*x^4 + 1485*a^4*b^6*c^2*d^8*x^4 + 990*a^5*b^5*c*d^9*x^4 + 495*a^6*b^
4*d^10*x^4 + 1760*b^10*c^7*d^3*x^3 + 1540*a*b^9*c^6*d^4*x^3 + 1320*a^2*b^8*
c^5*d^5*x^3 + 1100*a^3*b^7*c^4*d^6*x^3 + 880*a^4*b^6*c^3*d^7*x^3 + 660*a^5*
b^5*c^2*d^8*x^3 + 440*a^6*b^4*c*d^9*x^3 + 220*a^7*b^3*d^10*x^3 + 594*b^10*c
^8*d^2*x^2 + 528*a*b^9*c^7*d^3*x^2 + 462*a^2*b^8*c^6*d^4*x^2 + 396*a^3*b^7*
c^5*d^5*x^2 + 330*a^4*b^6*c^4*d^6*x^2 + 264*a^5*b^5*c^3*d^7*x^2 + 198*a^6*b
^4*c^2*d^8*x^2 + 132*a^7*b^3*c*d^9*x^2 + 66*a^8*b^2*d^10*x^2 + 120*b^10*c^9
*d*x + 108*a*b^9*c^8*d^2*x + 96*a^2*b^8*c^7*d^3*x + 84*a^3*b^7*c^6*d^4*x +
72*a^4*b^6*c^5*d^5*x + 60*a^5*b^5*c^4*d^6*x + 48*a^6*b^4*c^3*d^7*x + 36*a^7
*b^3*c^2*d^8*x + 24*a^8*b^2*c*d^9*x + 12*a^9*b*d^10*x + 11*b^10*c^10 + 10*a
*b^9*c^9*d + 9*a^2*b^8*c^8*d^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*
a^5*b^5*c^5*d^5 + 5*a^6*b^4*c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8
 + 2*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^12*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 14.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^13,x)
```

```
[Out] -105/4*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*
a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^8+10/3*d^9*(a*d-b*c)/b^
11/(b*x+a)^3+36*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^
3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^7+40/3*d^3*(a^7*d^7-7*a^6*b*c*d^6
+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^
2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^9+24*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^5-1/2*d^10/b^11/(b*x+a)^2-45/4*d^8*(a^2*d^2-
2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^4+10/11*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2
*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*
b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^11-1/12*
(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^
4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^
2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^12-35*d^6*(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^6-9/2*d^2*
(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4
*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x
+a)^10
```

maxima [B] time = 2.17, size = 986, normalized size = 17.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^13,x, algorithm="maxima")
```

```
[Out] -1/132*(66*b^10*d^10*x^10 + 11*b^10*c^10 + 10*a*b^9*c^9*d + 9*a^2*b^8*c^8*d
^2 + 8*a^3*b^7*c^7*d^3 + 7*a^4*b^6*c^6*d^4 + 6*a^5*b^5*c^5*d^5 + 5*a^6*b^4*
c^4*d^6 + 4*a^7*b^3*c^3*d^7 + 3*a^8*b^2*c^2*d^8 + 2*a^9*b*c*d^9 + a^10*d^10
 + 220*(2*b^10*c*d^9 + a*b^9*d^10)*x^9 + 495*(3*b^10*c^2*d^8 + 2*a*b^9*c*d^
9 + a^2*b^8*d^10)*x^8 + 792*(4*b^10*c^3*d^7 + 3*a*b^9*c^2*d^8 + 2*a^2*b^8*c
```

```
*d^9 + a^3*b^7*d^10)*x^7 + 924*(5*b^10*c^4*d^6 + 4*a*b^9*c^3*d^7 + 3*a^2*b^
8*c^2*d^8 + 2*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 792*(6*b^10*c^5*d^5 + 5*a
*b^9*c^4*d^6 + 4*a^2*b^8*c^3*d^7 + 3*a^3*b^7*c^2*d^8 + 2*a^4*b^6*c*d^9 + a^
5*b^5*d^10)*x^5 + 495*(7*b^10*c^6*d^4 + 6*a*b^9*c^5*d^5 + 5*a^2*b^8*c^4*d^6
 + 4*a^3*b^7*c^3*d^7 + 3*a^4*b^6*c^2*d^8 + 2*a^5*b^5*c*d^9 + a^6*b^4*d^10)*
x^4 + 220*(8*b^10*c^7*d^3 + 7*a*b^9*c^6*d^4 + 6*a^2*b^8*c^5*d^5 + 5*a^3*b^7
*c^4*d^6 + 4*a^4*b^6*c^3*d^7 + 3*a^5*b^5*c^2*d^8 + 2*a^6*b^4*c*d^9 + a^7*b^
3*d^10)*x^3 + 66*(9*b^10*c^8*d^2 + 8*a*b^9*c^7*d^3 + 7*a^2*b^8*c^6*d^4 + 6*
a^3*b^7*c^5*d^5 + 5*a^4*b^6*c^4*d^6 + 4*a^5*b^5*c^3*d^7 + 3*a^6*b^4*c^2*d^8
 + 2*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 12*(10*b^10*c^9*d + 9*a*b^9*c^8*d^
2 + 8*a^2*b^8*c^7*d^3 + 7*a^3*b^7*c^6*d^4 + 6*a^4*b^6*c^5*d^5 + 5*a^5*b^5*c
^4*d^6 + 4*a^6*b^4*c^3*d^7 + 3*a^7*b^3*c^2*d^8 + 2*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^23*x^12 + 12*a*b^22*x^11 + 66*a^2*b^21*x^10 + 220*a^3*b^20*x^9 +
495*a^4*b^19*x^8 + 792*a^5*b^18*x^7 + 924*a^6*b^17*x^6 + 792*a^7*b^16*x^5 +
 495*a^8*b^15*x^4 + 220*a^9*b^14*x^3 + 66*a^10*b^13*x^2 + 12*a^11*b^12*x +
a^12*b^11)
```

mupad [B] time = 0.39, size = 39, normalized size = 0.67

$$\frac{(c+d\,x)^{11}\,\left(12\,a\,d-11\,b\,c+b\,d\,x\right)}{132\,(a\,d-b\,c)^2\,(a+b\,x)^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^13,x)
[Out] ((c + d*x)^11*(12*a*d - 11*b*c + b*d*x))/(132*(a*d - b*c)^2*(a + b*x)^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**13,x)
[Out] Timed out
```

3.1219 $\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx$

Optimal. Leaf size=89

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^2(c+dx)^{11}}{858(a+bx)^{11}(bc-ad)^3}+\frac{d(c+dx)^{11}}{78(a+bx)^{12}(bc-ad)^2}-\frac{(c+dx)^{11}}{13(a+bx)^{13}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^14,x]
```

```
[Out] -(c + d*x)^11/(13*(b*c - a*d)*(a + b*x)^13) + (d*(c + d*x)^11)/(78*(b*c - a
*d)^2*(a + b*x)^12) - (d^2*(c + d*x)^11)/(858*(b*c - a*d)^3*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx &= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}-\frac{(2d)\int \frac{(c+dx)^{10}}{(a+bx)^{13}}\,dx}{13(bc-ad)}\\
&= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}+\frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{12}}\,dx}{78(bc-ad)^2}\\
&= -\frac{(c+dx)^{11}}{13(bc-ad)(a+bx)^{13}}+\frac{d(c+dx)^{11}}{78(bc-ad)^2(a+bx)^{12}}-\frac{d^2(c+dx)^{11}}{858(bc-ad)^3(a+bx)^{11}}
\end{aligned}$$

Mathematica [B] time = 0.29, size = 690, normalized size = 7.75

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^14,x]
```

[Out] -1/858*(a^10*d^10 + a^9*b*d^9*(3*c + 13*d*x) + 3*a^8*b^2*d^8*(2*c^2 + 13*c*d*x + 26*d^2*x^2) + 2*a^7*b^3*d^7*(5*c^3 + 39*c^2*d*x + 117*c*d^2*x^2 + 143*d^3*x^3) + a^6*b^4*d^6*(15*c^4 + 130*c^3*d*x + 468*c^2*d^2*x^2 + 858*c*d^3*x^3 + 715*d^4*x^4) + 3*a^5*b^5*d^5*(7*c^5 + 65*c^4*d*x + 260*c^3*d^2*x^2 + 572*c^2*d^3*x^3 + 715*c*d^4*x^4 + 429*d^5*x^5) + a^4*b^6*d^4*(28*c^6 + 273*c^5*d*x + 1170*c^4*d^2*x^2 + 2860*c^3*d^3*x^3 + 4290*c^2*d^4*x^4 + 3861*c*d^5*x^5 + 1716*d^6*x^6) + 2*a^3*b^7*d^3*(18*c^7 + 182*c^6*d*x + 819*c^5*d^2*x^2 + 2145*c^4*d^3*x^3 + 3575*c^3*d^4*x^4 + 3861*c^2*d^5*x^5 + 2574*c*d^6*x^6 + 858*d^7*x^7) + 3*a^2*b^8*d^2*(15*c^8 + 156*c^7*d*x + 728*c^6*d^2*x^2 + 2002*c^5*d^3*x^3 + 3575*c^4*d^4*x^4 + 4290*c^3*d^5*x^5 + 3432*c^2*d^6*x^6 + 1716*c*d^7*x^7 + 429*d^8*x^8) + a*b^9*d*(55*c^9 + 585*c^8*d*x + 2808*c^7*d^2*x^2 + 8008*c^6*d^3*x^3 + 15015*c^5*d^4*x^4 + 19305*c^4*d^5*x^5 + 17160*c^3*d^6*x^6 + 10296*c^2*d^7*x^7 + 3861*c*d^8*x^8 + 715*d^9*x^9) + b^10*(66*c^10 + 715*c^9*d*x + 3510*c^8*d^2*x^2 + 10296*c^7*d^3*x^3 + 20020*c^6*d^4*x^4 + 27027*c^5*d^5*x^5 + 25740*c^4*d^6*x^6 + 17160*c^3*d^7*x^7 + 7722*c^2*d^8*x^8 + 2145*c*d^9*x^9 + 286*d^10*x^10))/(b^11*(a + b*x)^13)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^14,x]

[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^14, x]

fricas [B] time = 1.26, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="fricas")

[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^10*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5 + 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8 + 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 + 78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)

giac [B] time = 1.28, size = 961, normalized size = 10.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="giac")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 2145*b^10*c*d^9*x^9 + 715*a*b^9*d^10*x^9 + 772
2*b^10*c^2*d^8*x^8 + 3861*a*b^9*c*d^9*x^8 + 1287*a^2*b^8*d^10*x^8 + 17160*b
^10*c^3*d^7*x^7 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7 + 1716*a
^3*b^7*d^10*x^7 + 25740*b^10*c^4*d^6*x^6 + 17160*a*b^9*c^3*d^7*x^6 + 10296*
a^2*b^8*c^2*d^8*x^6 + 5148*a^3*b^7*c*d^9*x^6 + 1716*a^4*b^6*d^10*x^6 + 2702
7*b^10*c^5*d^5*x^5 + 19305*a*b^9*c^4*d^6*x^5 + 12870*a^2*b^8*c^3*d^7*x^5 +
7722*a^3*b^7*c^2*d^8*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 1287*a^5*b^5*d^10*x^5 +
 20020*b^10*c^6*d^4*x^4 + 15015*a*b^9*c^5*d^5*x^4 + 10725*a^2*b^8*c^4*d^6*x
^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 2145*a^5*b^5*c*d
^9*x^4 + 715*a^6*b^4*d^10*x^4 + 10296*b^10*c^7*d^3*x^3 + 8008*a*b^9*c^6*d^4
*x^3 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^6*x^3 + 2860*a^4*b^6*c
^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 858*a^6*b^4*c*d^9*x^3 + 286*a^7*b^3
*d^10*x^3 + 3510*b^10*c^8*d^2*x^2 + 2808*a*b^9*c^7*d^3*x^2 + 2184*a^2*b^8*c
^6*d^4*x^2 + 1638*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*
b^5*c^3*d^7*x^2 + 468*a^6*b^4*c^2*d^8*x^2 + 234*a^7*b^3*c*d^9*x^2 + 78*a^8*
b^2*d^10*x^2 + 715*b^10*c^9*d*x + 585*a*b^9*c^8*d^2*x + 468*a^2*b^8*c^7*d^3
*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^5*c^4*d^6*x
+ 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 39*a^8*b^2*c*d^9*x + 13*a^
9*b*d^10*x + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^
7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 +
10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^10*d^10)/((b*x +
 a)^13*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 9.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^14,x)
```

```
[Out] 63/2*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b
^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^8-1/13*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*
c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4
*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^1
0)/b^11/(b*x+a)^13-1/3*d^10/b^11/(b*x+a)^3-30*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*
a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^7-70/3*d^4*(a^6*d^6-6*a
^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5
*c^5*d+b^6*c^6)/b^11/(b*x+a)^9-9*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+
a)^5+5/2*d^9*(a*d-b*c)/b^11/(b*x+a)^4-45/11*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a
^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*
a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^11+5/6*d*(a^9*d^9-9*a^8
*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*
b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^
11/(b*x+a)^12+20*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*
x+a)^6+12*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+
35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^1
0
```

maxima [B] time = 2.21, size = 997, normalized size = 11.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^14,x, algorithm="maxima")
```

```
[Out] -1/858*(286*b^10*d^10*x^10 + 66*b^10*c^10 + 55*a*b^9*c^9*d + 45*a^2*b^8*c^8
*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^
```

```
6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*b^2*c^2*d^8 + 3*a^9*b*c*d^9 + a^
10*d^10 + 715*(3*b^10*c*d^9 + a*b^9*d^10)*x^9 + 1287*(6*b^10*c^2*d^8 + 3*a*
b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 1716*(10*b^10*c^3*d^7 + 6*a*b^9*c^2*d^8 + 3
*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 1716*(15*b^10*c^4*d^6 + 10*a*b^9*c^3*d
^7 + 6*a^2*b^8*c^2*d^8 + 3*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 1287*(21*b^1
0*c^5*d^5 + 15*a*b^9*c^4*d^6 + 10*a^2*b^8*c^3*d^7 + 6*a^3*b^7*c^2*d^8 + 3*a
^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 715*(28*b^10*c^6*d^4 + 21*a*b^9*c^5*d^5
+ 15*a^2*b^8*c^4*d^6 + 10*a^3*b^7*c^3*d^7 + 6*a^4*b^6*c^2*d^8 + 3*a^5*b^5*c
*d^9 + a^6*b^4*d^10)*x^4 + 286*(36*b^10*c^7*d^3 + 28*a*b^9*c^6*d^4 + 21*a^2
*b^8*c^5*d^5 + 15*a^3*b^7*c^4*d^6 + 10*a^4*b^6*c^3*d^7 + 6*a^5*b^5*c^2*d^8
+ 3*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 78*(45*b^10*c^8*d^2 + 36*a*b^9*c^7*
d^3 + 28*a^2*b^8*c^6*d^4 + 21*a^3*b^7*c^5*d^5 + 15*a^4*b^6*c^4*d^6 + 10*a^5
*b^5*c^3*d^7 + 6*a^6*b^4*c^2*d^8 + 3*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 13
*(55*b^10*c^9*d + 45*a*b^9*c^8*d^2 + 36*a^2*b^8*c^7*d^3 + 28*a^3*b^7*c^6*d^
4 + 21*a^4*b^6*c^5*d^5 + 15*a^5*b^5*c^4*d^6 + 10*a^6*b^4*c^3*d^7 + 6*a^7*b^
3*c^2*d^8 + 3*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^24*x^13 + 13*a*b^23*x^12 +
78*a^2*b^22*x^11 + 286*a^3*b^21*x^10 + 715*a^4*b^20*x^9 + 1287*a^5*b^19*x^8
 + 1716*a^6*b^18*x^7 + 1716*a^7*b^17*x^6 + 1287*a^8*b^16*x^5 + 715*a^9*b^15
*x^4 + 286*a^10*b^14*x^3 + 78*a^11*b^13*x^2 + 13*a^12*b^12*x + a^13*b^11)
```

mupad [B] time = 0.48, size = 1098, normalized size = 12.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^14,x)
```

```
[Out] -(a^10*d^10 + 66*b^10*c^10 + 286*b^10*d^10*x^10 + 715*a*b^9*d^10*x^9 + 2145
*b^10*c*d^9*x^9 + 45*a^2*b^8*c^8*d^2 + 36*a^3*b^7*c^7*d^3 + 28*a^4*b^6*c^6*
d^4 + 21*a^5*b^5*c^5*d^5 + 15*a^6*b^4*c^4*d^6 + 10*a^7*b^3*c^3*d^7 + 6*a^8*
b^2*c^2*d^8 + 78*a^8*b^2*d^10*x^2 + 286*a^7*b^3*d^10*x^3 + 715*a^6*b^4*d^10
*x^4 + 1287*a^5*b^5*d^10*x^5 + 1716*a^4*b^6*d^10*x^6 + 1716*a^3*b^7*d^10*x^
7 + 1287*a^2*b^8*d^10*x^8 + 3510*b^10*c^8*d^2*x^2 + 10296*b^10*c^7*d^3*x^3
+ 20020*b^10*c^6*d^4*x^4 + 27027*b^10*c^5*d^5*x^5 + 25740*b^10*c^4*d^6*x^6
+ 17160*b^10*c^3*d^7*x^7 + 7722*b^10*c^2*d^8*x^8 + 55*a*b^9*c^9*d + 3*a^9*b
*c*d^9 + 13*a^9*b*d^10*x + 715*b^10*c^9*d*x + 2184*a^2*b^8*c^6*d^4*x^2 + 16
38*a^3*b^7*c^5*d^5*x^2 + 1170*a^4*b^6*c^4*d^6*x^2 + 780*a^5*b^5*c^3*d^7*x^2
 + 468*a^6*b^4*c^2*d^8*x^2 + 6006*a^2*b^8*c^5*d^5*x^3 + 4290*a^3*b^7*c^4*d^
6*x^3 + 2860*a^4*b^6*c^3*d^7*x^3 + 1716*a^5*b^5*c^2*d^8*x^3 + 10725*a^2*b^8
*c^4*d^6*x^4 + 7150*a^3*b^7*c^3*d^7*x^4 + 4290*a^4*b^6*c^2*d^8*x^4 + 12870*
a^2*b^8*c^3*d^7*x^5 + 7722*a^3*b^7*c^2*d^8*x^5 + 10296*a^2*b^8*c^2*d^8*x^6
+ 585*a*b^9*c^8*d^2*x + 39*a^8*b^2*c*d^9*x + 3861*a*b^9*c*d^9*x^8 + 468*a^2
*b^8*c^7*d^3*x + 364*a^3*b^7*c^6*d^4*x + 273*a^4*b^6*c^5*d^5*x + 195*a^5*b^
5*c^4*d^6*x + 130*a^6*b^4*c^3*d^7*x + 78*a^7*b^3*c^2*d^8*x + 2808*a*b^9*c^7
*d^3*x^2 + 234*a^7*b^3*c*d^9*x^2 + 8008*a*b^9*c^6*d^4*x^3 + 858*a^6*b^4*c*d
^9*x^3 + 15015*a*b^9*c^5*d^5*x^4 + 2145*a^5*b^5*c*d^9*x^4 + 19305*a*b^9*c^4
*d^6*x^5 + 3861*a^4*b^6*c*d^9*x^5 + 17160*a*b^9*c^3*d^7*x^6 + 5148*a^3*b^7*
c*d^9*x^6 + 10296*a*b^9*c^2*d^8*x^7 + 5148*a^2*b^8*c*d^9*x^7)/(858*a^13*b^1
1 + 858*b^24*x^13 + 11154*a^12*b^12*x + 11154*a*b^23*x^12 + 66924*a^11*b^13
*x^2 + 245388*a^10*b^14*x^3 + 613470*a^9*b^15*x^4 + 1104246*a^8*b^16*x^5 +
1472328*a^7*b^17*x^6 + 1472328*a^6*b^18*x^7 + 1104246*a^5*b^19*x^8 + 613470
*a^4*b^20*x^9 + 245388*a^3*b^21*x^10 + 66924*a^2*b^22*x^11)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**14,x)
```

```
[Out] Timed out
```

3.1220 $\int \frac{(c+dx)^{10}}{(a+bx)^{15}} \, dx$

Optimal. Leaf size=120

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {45, 37}

$$\frac{d^3(c+dx)^{11}}{4004(a+bx)^{11}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{364(a+bx)^{12}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{182(a+bx)^{13}(bc-ad)^2}-\frac{(c+dx)^{11}}{14(a+bx)^{14}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -(c + d*x)^11/(14*(b*c - a*d)*(a + b*x)^14) + (3*d*(c + d*x)^11)/(182*(b*c
- a*d)^2*(a + b*x)^13) - (d^2*(c + d*x)^11)/(364*(b*c - a*d)^3*(a + b*x)^12
) + (d^3*(c + d*x)^11)/(4004*(b*c - a*d)^4*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{10}}{(a+bx)^{15}} \, dx &= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}-\frac{(3d)\int \frac{(c+dx)^{10}}{(a+bx)^{14}} \, dx}{14(bc-ad)}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}+\frac{\left(3d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{13}} \, dx}{91(bc-ad)^2}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}-\frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{12}} \, dx}{364(bc-ad)^3}\\
&= -\frac{(c+dx)^{11}}{14(bc-ad)(a+bx)^{14}}+\frac{3d(c+dx)^{11}}{182(bc-ad)^2(a+bx)^{13}}-\frac{d^2(c+dx)^{11}}{364(bc-ad)^3(a+bx)^{12}}+\frac{d^3(c+dx)^{11}}{4004(bc-ad)^4(a+bx)^{11}}
\end{aligned}$$

Mathematica [B] time = 0.29, size = 692, normalized size = 5.77

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] -1/4004*(a^10*d^10 + 2*a^9*b*d^9*(2*c + 7*d*x) + a^8*b^2*d^8*(10*c^2 + 56*c
*d*x + 91*d^2*x^2) + 4*a^7*b^3*d^7*(5*c^3 + 35*c^2*d*x + 91*c*d^2*x^2 + 91*
d^3*x^3) + 7*a^6*b^4*d^6*(5*c^4 + 40*c^3*d*x + 130*c^2*d^2*x^2 + 208*c*d^3*
x^3 + 143*d^4*x^4) + 14*a^5*b^5*d^5*(4*c^5 + 35*c^4*d*x + 130*c^3*d^2*x^2 +
 260*c^2*d^3*x^3 + 286*c*d^4*x^4 + 143*d^5*x^5) + 7*a^4*b^6*d^4*(12*c^6 + 1
12*c^5*d*x + 455*c^4*d^2*x^2 + 1040*c^3*d^3*x^3 + 1430*c^2*d^4*x^4 + 1144*c
*d^5*x^5 + 429*d^6*x^6) + 4*a^3*b^7*d^3*(30*c^7 + 294*c^6*d*x + 1274*c^5*d^
2*x^2 + 3185*c^4*d^3*x^3 + 5005*c^3*d^4*x^4 + 5005*c^2*d^5*x^5 + 3003*c*d^6
*x^6 + 858*d^7*x^7) + a^2*b^8*d^2*(165*c^8 + 1680*c^7*d*x + 7644*c^6*d^2*x^
2 + 20384*c^5*d^3*x^3 + 35035*c^4*d^4*x^4 + 40040*c^3*d^5*x^5 + 30030*c^2*d
^6*x^6 + 13728*c*d^7*x^7 + 3003*d^8*x^8) + 2*a*b^9*d*(110*c^9 + 1155*c^8*d*
x + 5460*c^7*d^2*x^2 + 15288*c^6*d^3*x^3 + 28028*c^5*d^4*x^4 + 35035*c^4*d^
5*x^5 + 30030*c^3*d^6*x^6 + 17160*c^2*d^7*x^7 + 6006*c*d^8*x^8 + 1001*d^9*x
^9) + b^10*(286*c^10 + 3080*c^9*d*x + 15015*c^8*d^2*x^2 + 43680*c^7*d^3*x^3
 + 84084*c^6*d^4*x^4 + 112112*c^5*d^5*x^5 + 105105*c^4*d^6*x^6 + 68640*c^3*
d^7*x^7 + 30030*c^2*d^8*x^8 + 8008*c*d^9*x^9 + 1001*d^10*x^10))/(b^11*(a +
b*x)^14)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^15,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^15, x]
```

fricas [B] time = 1.32, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="fricas")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
```

```
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

giac [B] time = 1.39, size = 961, normalized size = 8.01

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="giac")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 8008*b^10*c*d^9*x^9 + 2002*a*b^9*d^10*x^9 +
30030*b^10*c^2*d^8*x^8 + 12012*a*b^9*c*d^9*x^8 + 3003*a^2*b^8*d^10*x^8 + 68
640*b^10*c^3*d^7*x^7 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2*b^8*c*d^9*x^7 +
3432*a^3*b^7*d^10*x^7 + 105105*b^10*c^4*d^6*x^6 + 60060*a*b^9*c^3*d^7*x^6 +
 30030*a^2*b^8*c^2*d^8*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 3003*a^4*b^6*d^10*x^
6 + 112112*b^10*c^5*d^5*x^5 + 70070*a*b^9*c^4*d^6*x^5 + 40040*a^2*b^8*c^3*d
^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 2002*a^5*b^5*
d^10*x^5 + 84084*b^10*c^6*d^4*x^4 + 56056*a*b^9*c^5*d^5*x^4 + 35035*a^2*b^8
*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^2*d^8*x^4 + 4004
*a^5*b^5*c*d^9*x^4 + 1001*a^6*b^4*d^10*x^4 + 43680*b^10*c^7*d^3*x^3 + 30576
*a*b^9*c^6*d^4*x^3 + 20384*a^2*b^8*c^5*d^5*x^3 + 12740*a^3*b^7*c^4*d^6*x^3
+ 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3 + 1456*a^6*b^4*c*d^9*
x^3 + 364*a^7*b^3*d^10*x^3 + 15015*b^10*c^8*d^2*x^2 + 10920*a*b^9*c^7*d^3*x
^2 + 7644*a^2*b^8*c^6*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4
*d^6*x^2 + 1820*a^5*b^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 364*a^7*b^3
*c*d^9*x^2 + 91*a^8*b^2*d^10*x^2 + 3080*b^10*c^9*d*x + 2310*a*b^9*c^8*d^2*x
 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6*c^5*d^5*x
+ 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^2*d^8*x + 5
6*a^8*b^2*c*d^9*x + 14*a^9*b*d^10*x + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165
*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^
5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^
9*b*c*d^9 + a^10*d^10)/((b*x + a)^14*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^15,x)
```

```
[Out] -105/4*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/
b^11/(b*x+a)^8+10/13*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3
*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*
b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^13+120/7*d^7*(a^3*d^3-3*a^2
*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^7+28*d^5*(a^5*d^5-5*a^4*b*c*d^
4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)
^9+2*d^9*(a*d-b*c)/b^11/(b*x+a)^5-1/4*d^10/b^11/(b*x+a)^4+120/11*d^3*(a^7*d
^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-2
1*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^11-15/4*d^2*(a^8*d^8-
8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a
^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^12-15
/2*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^6-21*d^4*(a^6*d^6-6*a^5*b*c
*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d
+b^6*c^6)/b^11/(b*x+a)^10-1/14*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8
-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^
6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11
/(b*x+a)^14
```

maxima [B] time = 2.16, size = 1008, normalized size = 8.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^15,x, algorithm="maxima")
```

```
[Out] -1/4004*(1001*b^10*d^10*x^10 + 286*b^10*c^10 + 220*a*b^9*c^9*d + 165*a^2*b^
8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6*c^6*d^4 + 56*a^5*b^5*c^5*d^5 +
 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 10*a^8*b^2*c^2*d^8 + 4*a^9*b*c*d
^9 + a^10*d^10 + 2002*(4*b^10*c*d^9 + a*b^9*d^10)*x^9 + 3003*(10*b^10*c^2*d
^8 + 4*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 3432*(20*b^10*c^3*d^7 + 10*a*b^9*c
^2*d^8 + 4*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 3003*(35*b^10*c^4*d^6 + 20*a
*b^9*c^3*d^7 + 10*a^2*b^8*c^2*d^8 + 4*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 2
002*(56*b^10*c^5*d^5 + 35*a*b^9*c^4*d^6 + 20*a^2*b^8*c^3*d^7 + 10*a^3*b^7*c
^2*d^8 + 4*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1001*(84*b^10*c^6*d^4 + 56*a
*b^9*c^5*d^5 + 35*a^2*b^8*c^4*d^6 + 20*a^3*b^7*c^3*d^7 + 10*a^4*b^6*c^2*d^8
 + 4*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 364*(120*b^10*c^7*d^3 + 84*a*b^9*c
^6*d^4 + 56*a^2*b^8*c^5*d^5 + 35*a^3*b^7*c^4*d^6 + 20*a^4*b^6*c^3*d^7 + 10*
a^5*b^5*c^2*d^8 + 4*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 91*(165*b^10*c^8*d^
2 + 120*a*b^9*c^7*d^3 + 84*a^2*b^8*c^6*d^4 + 56*a^3*b^7*c^5*d^5 + 35*a^4*b^
6*c^4*d^6 + 20*a^5*b^5*c^3*d^7 + 10*a^6*b^4*c^2*d^8 + 4*a^7*b^3*c*d^9 + a^8
*b^2*d^10)*x^2 + 14*(220*b^10*c^9*d + 165*a*b^9*c^8*d^2 + 120*a^2*b^8*c^7*d
^3 + 84*a^3*b^7*c^6*d^4 + 56*a^4*b^6*c^5*d^5 + 35*a^5*b^5*c^4*d^6 + 20*a^6*
b^4*c^3*d^7 + 10*a^7*b^3*c^2*d^8 + 4*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^25*x
^14 + 14*a*b^24*x^13 + 91*a^2*b^23*x^12 + 364*a^3*b^22*x^11 + 1001*a^4*b^21
*x^10 + 2002*a^5*b^20*x^9 + 3003*a^6*b^19*x^8 + 3432*a^7*b^18*x^7 + 3003*a^
8*b^17*x^6 + 2002*a^9*b^16*x^5 + 1001*a^10*b^15*x^4 + 364*a^11*b^14*x^3 + 9
1*a^12*b^13*x^2 + 14*a^13*b^12*x + a^14*b^11)
```

mupad [B] time = 1.30, size = 1109, normalized size = 9.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^15,x)
```

```
[Out] -(a^10*d^10 + 286*b^10*c^10 + 1001*b^10*d^10*x^10 + 2002*a*b^9*d^10*x^9 + 8
008*b^10*c*d^9*x^9 + 165*a^2*b^8*c^8*d^2 + 120*a^3*b^7*c^7*d^3 + 84*a^4*b^6
*c^6*d^4 + 56*a^5*b^5*c^5*d^5 + 35*a^6*b^4*c^4*d^6 + 20*a^7*b^3*c^3*d^7 + 1
0*a^8*b^2*c^2*d^8 + 91*a^8*b^2*d^10*x^2 + 364*a^7*b^3*d^10*x^3 + 1001*a^6*b
^4*d^10*x^4 + 2002*a^5*b^5*d^10*x^5 + 3003*a^4*b^6*d^10*x^6 + 3432*a^3*b^7*
d^10*x^7 + 3003*a^2*b^8*d^10*x^8 + 15015*b^10*c^8*d^2*x^2 + 43680*b^10*c^7*
d^3*x^3 + 84084*b^10*c^6*d^4*x^4 + 112112*b^10*c^5*d^5*x^5 + 105105*b^10*c^
4*d^6*x^6 + 68640*b^10*c^3*d^7*x^7 + 30030*b^10*c^2*d^8*x^8 + 220*a*b^9*c^9
*d + 4*a^9*b*c*d^9 + 14*a^9*b*d^10*x + 3080*b^10*c^9*d*x + 7644*a^2*b^8*c^6
*d^4*x^2 + 5096*a^3*b^7*c^5*d^5*x^2 + 3185*a^4*b^6*c^4*d^6*x^2 + 1820*a^5*b
^5*c^3*d^7*x^2 + 910*a^6*b^4*c^2*d^8*x^2 + 20384*a^2*b^8*c^5*d^5*x^3 + 1274
0*a^3*b^7*c^4*d^6*x^3 + 7280*a^4*b^6*c^3*d^7*x^3 + 3640*a^5*b^5*c^2*d^8*x^3
 + 35035*a^2*b^8*c^4*d^6*x^4 + 20020*a^3*b^7*c^3*d^7*x^4 + 10010*a^4*b^6*c^
2*d^8*x^4 + 40040*a^2*b^8*c^3*d^7*x^5 + 20020*a^3*b^7*c^2*d^8*x^5 + 30030*a
^2*b^8*c^2*d^8*x^6 + 2310*a*b^9*c^8*d^2*x + 56*a^8*b^2*c*d^9*x + 12012*a*b^
9*c*d^9*x^8 + 1680*a^2*b^8*c^7*d^3*x + 1176*a^3*b^7*c^6*d^4*x + 784*a^4*b^6
*c^5*d^5*x + 490*a^5*b^5*c^4*d^6*x + 280*a^6*b^4*c^3*d^7*x + 140*a^7*b^3*c^
2*d^8*x + 10920*a*b^9*c^7*d^3*x^2 + 364*a^7*b^3*c*d^9*x^2 + 30576*a*b^9*c^6
*d^4*x^3 + 1456*a^6*b^4*c*d^9*x^3 + 56056*a*b^9*c^5*d^5*x^4 + 4004*a^5*b^5*
c*d^9*x^4 + 70070*a*b^9*c^4*d^6*x^5 + 8008*a^4*b^6*c*d^9*x^5 + 60060*a*b^9*
c^3*d^7*x^6 + 12012*a^3*b^7*c*d^9*x^6 + 34320*a*b^9*c^2*d^8*x^7 + 13728*a^2
*b^8*c*d^9*x^7)/(4004*a^14*b^11 + 4004*b^25*x^14 + 56056*a^13*b^12*x + 5605
6*a*b^24*x^13 + 364364*a^12*b^13*x^2 + 1457456*a^11*b^14*x^3 + 4008004*a^10
*b^15*x^4 + 8016008*a^9*b^16*x^5 + 12024012*a^8*b^17*x^6 + 13741728*a^7*b^1
8*x^7 + 12024012*a^6*b^19*x^8 + 8016008*a^5*b^20*x^9 + 4008004*a^4*b^21*x^1
0 + 1457456*a^3*b^22*x^11 + 364364*a^2*b^23*x^12)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**15,x)
[Out] Timed out
```

3.1221 $\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$

Optimal. Leaf size=151

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-15$$

Rubi [A] time = 0.04, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {45, 37}

$$-\frac{d^4(c+dx)^{11}}{15015(a+bx)^{11}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{1365(a+bx)^{12}(bc-ad)^4}-\frac{2d^2(c+dx)^{11}}{455(a+bx)^{13}(bc-ad)^3}+\frac{2d(c+dx)^{11}}{105(a+bx)^{14}(bc-ad)^2}-\frac{(c+dx)^{11}}{15(a+bx)^{15}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -(c + d*x)^11/(15*(b*c - a*d)*(a + b*x)^15) + (2*d*(c + d*x)^11)/(105*(b*c
- a*d)^2*(a + b*x)^14) - (2*d^2*(c + d*x)^11)/(455*(b*c - a*d)^3*(a + b*x)^
13) + (d^3*(c + d*x)^11)/(1365*(b*c - a*d)^4*(a + b*x)^12) - (d^4*(c + d*x)
^11)/(15015*(b*c - a*d)^5*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx = -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}-\frac{(4d)\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{15(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}+\frac{\left(2d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{35(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}-\frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}}-\frac{\left(4d^3\right)\int \frac{(c+}{(a+}}{455(bc-}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}-\frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}}+\frac{d^3(c}{1365(bc-}$$

$$= -\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}}+\frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}}-\frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}}+\frac{d^3(c}{1365(bc-}$$

Mathematica [B] time = 0.29, size = 690, normalized size = 4.57

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] -1/15015*(a^10*d^10 + 5*a^9*b*d^9*(c + 3*d*x) + 15*a^8*b^2*d^8*(c^2 + 5*c*d
*x + 7*d^2*x^2) + 5*a^7*b^3*d^7*(7*c^3 + 45*c^2*d*x + 105*c*d^2*x^2 + 91*d^
3*x^3) + 35*a^6*b^4*d^6*(2*c^4 + 15*c^3*d*x + 45*c^2*d^2*x^2 + 65*c*d^3*x^3
 + 39*d^4*x^4) + 21*a^5*b^5*d^5*(6*c^5 + 50*c^4*d*x + 175*c^3*d^2*x^2 + 325
*c^2*d^3*x^3 + 325*c*d^4*x^4 + 143*d^5*x^5) + 35*a^4*b^6*d^4*(6*c^6 + 54*c^
5*d*x + 210*c^4*d^2*x^2 + 455*c^3*d^3*x^3 + 585*c^2*d^4*x^4 + 429*c*d^5*x^5
 + 143*d^6*x^6) + 5*a^3*b^7*d^3*(66*c^7 + 630*c^6*d*x + 2646*c^5*d^2*x^2 +
6370*c^4*d^3*x^3 + 9555*c^3*d^4*x^4 + 9009*c^2*d^5*x^5 + 5005*c*d^6*x^6 + 1
287*d^7*x^7) + 15*a^2*b^8*d^2*(33*c^8 + 330*c^7*d*x + 1470*c^6*d^2*x^2 + 38
22*c^5*d^3*x^3 + 6370*c^4*d^4*x^4 + 7007*c^3*d^5*x^5 + 5005*c^2*d^6*x^6 + 2
145*c*d^7*x^7 + 429*d^8*x^8) + 5*a*b^9*d*(143*c^9 + 1485*c^8*d*x + 6930*c^7
*d^2*x^2 + 19110*c^6*d^3*x^3 + 34398*c^5*d^4*x^4 + 42042*c^4*d^5*x^5 + 3503
5*c^3*d^6*x^6 + 19305*c^2*d^7*x^7 + 6435*c*d^8*x^8 + 1001*d^9*x^9) + b^10*(
1001*c^10 + 10725*c^9*d*x + 51975*c^8*d^2*x^2 + 150150*c^7*d^3*x^3 + 286650
*c^6*d^4*x^4 + 378378*c^5*d^5*x^5 + 350350*c^4*d^6*x^6 + 225225*c^3*d^7*x^7
 + 96525*c^2*d^8*x^8 + 25025*c*d^9*x^9 + 3003*d^10*x^10))/(b^11*(a + b*x)^1
5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^16,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^16, x]
```

fricas [B] time = 1.12, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="fricas")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
```

```
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

giac [B] time = 1.32, size = 961, normalized size = 6.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="giac")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 25025*b^10*c*d^9*x^9 + 5005*a*b^9*d^10*x^9
+ 96525*b^10*c^2*d^8*x^8 + 32175*a*b^9*c*d^9*x^8 + 6435*a^2*b^8*d^10*x^8 +
225225*b^10*c^3*d^7*x^7 + 96525*a*b^9*c^2*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7
 + 6435*a^3*b^7*d^10*x^7 + 350350*b^10*c^4*d^6*x^6 + 175175*a*b^9*c^3*d^7*x
^6 + 75075*a^2*b^8*c^2*d^8*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 5005*a^4*b^6*d^1
0*x^6 + 378378*b^10*c^5*d^5*x^5 + 210210*a*b^9*c^4*d^6*x^5 + 105105*a^2*b^8
*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8*x^5 + 15015*a^4*b^6*c*d^9*x^5 + 3003*a
^5*b^5*d^10*x^5 + 286650*b^10*c^6*d^4*x^4 + 171990*a*b^9*c^5*d^5*x^4 + 9555
0*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 20475*a^4*b^6*c^2*d^8*x
^4 + 6825*a^5*b^5*c*d^9*x^4 + 1365*a^6*b^4*d^10*x^4 + 150150*b^10*c^7*d^3*x
^3 + 95550*a*b^9*c^6*d^4*x^3 + 57330*a^2*b^8*c^5*d^5*x^3 + 31850*a^3*b^7*c^
4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^5*c^2*d^8*x^3 + 2275*a^6
*b^4*c*d^9*x^3 + 455*a^7*b^3*d^10*x^3 + 51975*b^10*c^8*d^2*x^2 + 34650*a*b^
9*c^7*d^3*x^2 + 22050*a^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 735
0*a^4*b^6*c^4*d^6*x^2 + 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2
 + 525*a^7*b^3*c*d^9*x^2 + 105*a^8*b^2*d^10*x^2 + 10725*b^10*c^9*d*x + 7425
*a*b^9*c^8*d^2*x + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x + 1890*a
^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x + 225*a^7
*b^3*c^2*d^8*x + 75*a^8*b^2*c*d^9*x + 15*a^9*b*d^10*x + 1001*b^10*c^10 + 71
5*a*b^9*c^9*d + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6
*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a
^8*b^2*c^2*d^8 + 5*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^15*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^16,x)
```

```
[Out] 15*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^8-45/13*d
^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*
c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(
b*x+a)^13-45/7*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^7-70/3*d^6*(a^4
*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^9-
1/5*d^10/b^11/(b*x+a)^5-210/11*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^
4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)
^11-1/15*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+2
10*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*
d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^15+10*d^3*(a^
7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^
3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^12+5/3*d^9*(a*d-b*
c)/b^11/(b*x+a)^6+126/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^
2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^10+5/7*d*(a^9*d^9-9*a^8*b
*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^
```

```
5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11
/(b*x+a)^14
```

maxima [B] time = 2.26, size = 1019, normalized size = 6.75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^16,x, algorithm="maxima")
```

```
[Out] -1/15015*(3003*b^10*d^10*x^10 + 1001*b^10*c^10 + 715*a*b^9*c^9*d + 495*a^2*
b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 + 126*a^5*b^5*c^5*d
^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7 + 15*a^8*b^2*c^2*d^8 + 5*a^9*b
*c*d^9 + a^10*d^10 + 5005*(5*b^10*c*d^9 + a*b^9*d^10)*x^9 + 6435*(15*b^10*c
^2*d^8 + 5*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 6435*(35*b^10*c^3*d^7 + 15*a*b
^9*c^2*d^8 + 5*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 5005*(70*b^10*c^4*d^6 +
35*a*b^9*c^3*d^7 + 15*a^2*b^8*c^2*d^8 + 5*a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6
 + 3003*(126*b^10*c^5*d^5 + 70*a*b^9*c^4*d^6 + 35*a^2*b^8*c^3*d^7 + 15*a^3*
b^7*c^2*d^8 + 5*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1365*(210*b^10*c^6*d^4
+ 126*a*b^9*c^5*d^5 + 70*a^2*b^8*c^4*d^6 + 35*a^3*b^7*c^3*d^7 + 15*a^4*b^6*
c^2*d^8 + 5*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 455*(330*b^10*c^7*d^3 + 210
*a*b^9*c^6*d^4 + 126*a^2*b^8*c^5*d^5 + 70*a^3*b^7*c^4*d^6 + 35*a^4*b^6*c^3*
d^7 + 15*a^5*b^5*c^2*d^8 + 5*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 105*(495*b
^10*c^8*d^2 + 330*a*b^9*c^7*d^3 + 210*a^2*b^8*c^6*d^4 + 126*a^3*b^7*c^5*d^5
 + 70*a^4*b^6*c^4*d^6 + 35*a^5*b^5*c^3*d^7 + 15*a^6*b^4*c^2*d^8 + 5*a^7*b^3
*c*d^9 + a^8*b^2*d^10)*x^2 + 15*(715*b^10*c^9*d + 495*a*b^9*c^8*d^2 + 330*a
^2*b^8*c^7*d^3 + 210*a^3*b^7*c^6*d^4 + 126*a^4*b^6*c^5*d^5 + 70*a^5*b^5*c^4
*d^6 + 35*a^6*b^4*c^3*d^7 + 15*a^7*b^3*c^2*d^8 + 5*a^8*b^2*c*d^9 + a^9*b*d^
10)*x)/(b^26*x^15 + 15*a*b^25*x^14 + 105*a^2*b^24*x^13 + 455*a^3*b^23*x^12
+ 1365*a^4*b^22*x^11 + 3003*a^5*b^21*x^10 + 5005*a^6*b^20*x^9 + 6435*a^7*b^
19*x^8 + 6435*a^8*b^18*x^7 + 5005*a^9*b^17*x^6 + 3003*a^10*b^16*x^5 + 1365*
a^11*b^15*x^4 + 455*a^12*b^14*x^3 + 105*a^13*b^13*x^2 + 15*a^14*b^12*x + a^
15*b^11)
```

mupad [B] time = 2.28, size = 1120, normalized size = 7.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^16,x)
```

```
[Out] -(a^10*d^10 + 1001*b^10*c^10 + 3003*b^10*d^10*x^10 + 5005*a*b^9*d^10*x^9 +
25025*b^10*c*d^9*x^9 + 495*a^2*b^8*c^8*d^2 + 330*a^3*b^7*c^7*d^3 + 210*a^4*
b^6*c^6*d^4 + 126*a^5*b^5*c^5*d^5 + 70*a^6*b^4*c^4*d^6 + 35*a^7*b^3*c^3*d^7
 + 15*a^8*b^2*c^2*d^8 + 105*a^8*b^2*d^10*x^2 + 455*a^7*b^3*d^10*x^3 + 1365*
a^6*b^4*d^10*x^4 + 3003*a^5*b^5*d^10*x^5 + 5005*a^4*b^6*d^10*x^6 + 6435*a^3
*b^7*d^10*x^7 + 6435*a^2*b^8*d^10*x^8 + 51975*b^10*c^8*d^2*x^2 + 150150*b^1
0*c^7*d^3*x^3 + 286650*b^10*c^6*d^4*x^4 + 378378*b^10*c^5*d^5*x^5 + 350350*
b^10*c^4*d^6*x^6 + 225225*b^10*c^3*d^7*x^7 + 96525*b^10*c^2*d^8*x^8 + 715*a
*b^9*c^9*d + 5*a^9*b*c*d^9 + 15*a^9*b*d^10*x + 10725*b^10*c^9*d*x + 22050*a
^2*b^8*c^6*d^4*x^2 + 13230*a^3*b^7*c^5*d^5*x^2 + 7350*a^4*b^6*c^4*d^6*x^2 +
 3675*a^5*b^5*c^3*d^7*x^2 + 1575*a^6*b^4*c^2*d^8*x^2 + 57330*a^2*b^8*c^5*d^
5*x^3 + 31850*a^3*b^7*c^4*d^6*x^3 + 15925*a^4*b^6*c^3*d^7*x^3 + 6825*a^5*b^
5*c^2*d^8*x^3 + 95550*a^2*b^8*c^4*d^6*x^4 + 47775*a^3*b^7*c^3*d^7*x^4 + 204
75*a^4*b^6*c^2*d^8*x^4 + 105105*a^2*b^8*c^3*d^7*x^5 + 45045*a^3*b^7*c^2*d^8
*x^5 + 75075*a^2*b^8*c^2*d^8*x^6 + 7425*a*b^9*c^8*d^2*x + 75*a^8*b^2*c*d^9*
x + 32175*a*b^9*c*d^9*x^8 + 4950*a^2*b^8*c^7*d^3*x + 3150*a^3*b^7*c^6*d^4*x
 + 1890*a^4*b^6*c^5*d^5*x + 1050*a^5*b^5*c^4*d^6*x + 525*a^6*b^4*c^3*d^7*x
+ 225*a^7*b^3*c^2*d^8*x + 34650*a*b^9*c^7*d^3*x^2 + 525*a^7*b^3*c*d^9*x^2 +
```

```
 95550*a*b^9*c^6*d^4*x^3 + 2275*a^6*b^4*c*d^9*x^3 + 171990*a*b^9*c^5*d^5*x^
4 + 6825*a^5*b^5*c*d^9*x^4 + 210210*a*b^9*c^4*d^6*x^5 + 15015*a^4*b^6*c*d^9
*x^5 + 175175*a*b^9*c^3*d^7*x^6 + 25025*a^3*b^7*c*d^9*x^6 + 96525*a*b^9*c^2
*d^8*x^7 + 32175*a^2*b^8*c*d^9*x^7)/(15015*a^15*b^11 + 15015*b^26*x^15 + 22
5225*a^14*b^12*x + 225225*a*b^25*x^14 + 1576575*a^13*b^13*x^2 + 6831825*a^1
2*b^14*x^3 + 20495475*a^11*b^15*x^4 + 45090045*a^10*b^16*x^5 + 75150075*a^9
*b^17*x^6 + 96621525*a^8*b^18*x^7 + 96621525*a^7*b^19*x^8 + 75150075*a^6*b^
20*x^9 + 45090045*a^5*b^21*x^10 + 20495475*a^4*b^22*x^11 + 6831825*a^3*b^23
*x^12 + 1576575*a^2*b^24*x^13)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**16,x)
[Out] Timed out
```

3.1222 $\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$

Optimal. Leaf size=182

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\ldots$$

Rubi [A] time = 0.06, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^5(c+dx)^{11}}{48048(a+bx)^{11}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{4368(a+bx)^{12}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{728(a+bx)^{13}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{168(a+bx)^{14}(bc-ad)^3}+\frac{d(c+dx)^{11}}{48(a+bx)^{15}(bc-ad)^2}-\frac{(c+dx)^{11}}{16(a+bx)^{16}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -(c + d*x)^11/(16*(b*c - a*d)*(a + b*x)^16) + (d*(c + d*x)^11)/(48*(b*c - a
*d)^2*(a + b*x)^15) - (d^2*(c + d*x)^11)/(168*(b*c - a*d)^3*(a + b*x)^14) +
 (d^3*(c + d*x)^11)/(728*(b*c - a*d)^4*(a + b*x)^13) - (d^4*(c + d*x)^11)/(
4368*(b*c - a*d)^5*(a + b*x)^12) + (d^5*(c + d*x)^11)/(48048*(b*c - a*d)^6*
(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx &= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}-\frac{(5d)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{16(bc-ad)}\\
&= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}+\frac{d^2\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{12(bc-ad)^2}\\
&= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}-\frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{14}}\,dx}{56(bc-ad)^3}\\
&= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}+\ldots\\
&= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}+\ldots\\
&= -\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}}+\frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}}-\frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}}+\ldots
\end{aligned}
$$

Mathematica [B] time = 0.28, size = 694, normalized size = 3.81

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] -1/48048*(a^10*d^10 + 2*a^9*b*d^9*(3*c + 8*d*x) + 3*a^8*b^2*d^8*(7*c^2 + 32
*c*d*x + 40*d^2*x^2) + 8*a^7*b^3*d^7*(7*c^3 + 42*c^2*d*x + 90*c*d^2*x^2 + 7
0*d^3*x^3) + 14*a^6*b^4*d^6*(9*c^4 + 64*c^3*d*x + 180*c^2*d^2*x^2 + 240*c*d
^3*x^3 + 130*d^4*x^4) + 84*a^5*b^5*d^5*(3*c^5 + 24*c^4*d*x + 80*c^3*d^2*x^2
 + 140*c^2*d^3*x^3 + 130*c*d^4*x^4 + 52*d^5*x^5) + 14*a^4*b^6*d^4*(33*c^6 +
 288*c^5*d*x + 1080*c^4*d^2*x^2 + 2240*c^3*d^3*x^3 + 2730*c^2*d^4*x^4 + 187
2*c*d^5*x^5 + 572*d^6*x^6) + 8*a^3*b^7*d^3*(99*c^7 + 924*c^6*d*x + 3780*c^5
*d^2*x^2 + 8820*c^4*d^3*x^3 + 12740*c^3*d^4*x^4 + 11466*c^2*d^5*x^5 + 6006*
c*d^6*x^6 + 1430*d^7*x^7) + 3*a^2*b^8*d^2*(429*c^8 + 4224*c^7*d*x + 18480*c
^6*d^2*x^2 + 47040*c^5*d^3*x^3 + 76440*c^4*d^4*x^4 + 81536*c^3*d^5*x^5 + 56
056*c^2*d^6*x^6 + 22880*c*d^7*x^7 + 4290*d^8*x^8) + 2*a*b^9*d*(1001*c^9 + 1
0296*c^8*d*x + 47520*c^7*d^2*x^2 + 129360*c^6*d^3*x^3 + 229320*c^5*d^4*x^4
+ 275184*c^4*d^5*x^5 + 224224*c^3*d^6*x^6 + 120120*c^2*d^7*x^7 + 38610*c*d^
8*x^8 + 5720*d^9*x^9) + b^10*(3003*c^10 + 32032*c^9*d*x + 154440*c^8*d^2*x^
2 + 443520*c^7*d^3*x^3 + 840840*c^6*d^4*x^4 + 1100736*c^5*d^5*x^5 + 1009008
*c^4*d^6*x^6 + 640640*c^3*d^7*x^7 + 270270*c^2*d^8*x^8 + 68640*c*d^9*x^9 +
8008*d^10*x^10))/(b^11*(a + b*x)^16)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^17,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^17, x]
```

fricas [B] time = 1.26, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="fricas")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
```

```
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

giac [B] time = 1.31, size = 961, normalized size = 5.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="giac")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 68640*b^10*c*d^9*x^9 + 11440*a*b^9*d^10*x^9
 + 270270*b^10*c^2*d^8*x^8 + 77220*a*b^9*c*d^9*x^8 + 12870*a^2*b^8*d^10*x^8
 + 640640*b^10*c^3*d^7*x^7 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9
*x^7 + 11440*a^3*b^7*d^10*x^7 + 1009008*b^10*c^4*d^6*x^6 + 448448*a*b^9*c^3
*d^7*x^6 + 168168*a^2*b^8*c^2*d^8*x^6 + 48048*a^3*b^7*c*d^9*x^6 + 8008*a^4*
b^6*d^10*x^6 + 1100736*b^10*c^5*d^5*x^5 + 550368*a*b^9*c^4*d^6*x^5 + 244608
*a^2*b^8*c^3*d^7*x^5 + 91728*a^3*b^7*c^2*d^8*x^5 + 26208*a^4*b^6*c*d^9*x^5
+ 4368*a^5*b^5*d^10*x^5 + 840840*b^10*c^6*d^4*x^4 + 458640*a*b^9*c^5*d^5*x^
4 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^3*d^7*x^4 + 38220*a^4*b^6
*c^2*d^8*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 1820*a^6*b^4*d^10*x^4 + 443520*b^1
0*c^7*d^3*x^3 + 258720*a*b^9*c^6*d^4*x^3 + 141120*a^2*b^8*c^5*d^5*x^3 + 705
60*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 + 11760*a^5*b^5*c^2*d^8*
x^3 + 3360*a^6*b^4*c*d^9*x^3 + 560*a^7*b^3*d^10*x^3 + 154440*b^10*c^8*d^2*x
^2 + 95040*a*b^9*c^7*d^3*x^2 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^
5*d^5*x^2 + 15120*a^4*b^6*c^4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6
*b^4*c^2*d^8*x^2 + 720*a^7*b^3*c*d^9*x^2 + 120*a^8*b^2*d^10*x^2 + 32032*b^1
0*c^9*d*x + 20592*a*b^9*c^8*d^2*x + 12672*a^2*b^8*c^7*d^3*x + 7392*a^3*b^7*
c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*a^6*b^4*c
^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 96*a^8*b^2*c*d^9*x + 16*a^9*b*d^10*x + 3
003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d
^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a
^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)
^16*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 4.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^17,x)
```

```
[Out] -45/8*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^8+120/13*d^3*(a^7*d^7-7*
a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2
*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^13+10/7*d^9*(a*d-b*c)/b^11
/(b*x+a)^7+40/3*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x
+a)^9+252/11*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d
^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^11+2/3*d*(a^9*d^9-9*a^8*b*c*d^8+36*a
^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+8
4*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^15
-35/2*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a
^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^12-1/6*d^10/b^11/(b*x+a)
^6-21*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b
^11/(b*x+a)^10-45/14*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b
^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7
*c^7*d+b^8*c^8)/b^11/(b*x+a)^14-1/16*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c
```

```
^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*
b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10
)/b^11/(b*x+a)^16
```

maxima [B] time = 2.27, size = 1030, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^17,x, algorithm="maxima")
```

```
[Out] -1/48048*(8008*b^10*d^10*x^10 + 3003*b^10*c^10 + 2002*a*b^9*c^9*d + 1287*a^
2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^4*b^6*c^6*d^4 + 252*a^5*b^5*c^5
*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*d^7 + 21*a^8*b^2*c^2*d^8 + 6*a^
9*b*c*d^9 + a^10*d^10 + 11440*(6*b^10*c*d^9 + a*b^9*d^10)*x^9 + 12870*(21*b
^10*c^2*d^8 + 6*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 11440*(56*b^10*c^3*d^7 +
21*a*b^9*c^2*d^8 + 6*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 8008*(126*b^10*c^4
*d^6 + 56*a*b^9*c^3*d^7 + 21*a^2*b^8*c^2*d^8 + 6*a^3*b^7*c*d^9 + a^4*b^6*d^
10)*x^6 + 4368*(252*b^10*c^5*d^5 + 126*a*b^9*c^4*d^6 + 56*a^2*b^8*c^3*d^7 +
 21*a^3*b^7*c^2*d^8 + 6*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 1820*(462*b^10*
c^6*d^4 + 252*a*b^9*c^5*d^5 + 126*a^2*b^8*c^4*d^6 + 56*a^3*b^7*c^3*d^7 + 21
*a^4*b^6*c^2*d^8 + 6*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 560*(792*b^10*c^7*
d^3 + 462*a*b^9*c^6*d^4 + 252*a^2*b^8*c^5*d^5 + 126*a^3*b^7*c^4*d^6 + 56*a^
4*b^6*c^3*d^7 + 21*a^5*b^5*c^2*d^8 + 6*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 +
120*(1287*b^10*c^8*d^2 + 792*a*b^9*c^7*d^3 + 462*a^2*b^8*c^6*d^4 + 252*a^3*
b^7*c^5*d^5 + 126*a^4*b^6*c^4*d^6 + 56*a^5*b^5*c^3*d^7 + 21*a^6*b^4*c^2*d^8
 + 6*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 16*(2002*b^10*c^9*d + 1287*a*b^9*c
^8*d^2 + 792*a^2*b^8*c^7*d^3 + 462*a^3*b^7*c^6*d^4 + 252*a^4*b^6*c^5*d^5 +
126*a^5*b^5*c^4*d^6 + 56*a^6*b^4*c^3*d^7 + 21*a^7*b^3*c^2*d^8 + 6*a^8*b^2*c
*d^9 + a^9*b*d^10)*x)/(b^27*x^16 + 16*a*b^26*x^15 + 120*a^2*b^25*x^14 + 560
*a^3*b^24*x^13 + 1820*a^4*b^23*x^12 + 4368*a^5*b^22*x^11 + 8008*a^6*b^21*x^
10 + 11440*a^7*b^20*x^9 + 12870*a^8*b^19*x^8 + 11440*a^9*b^18*x^7 + 8008*a^
10*b^17*x^6 + 4368*a^11*b^16*x^5 + 1820*a^12*b^15*x^4 + 560*a^13*b^14*x^3 +
 120*a^14*b^13*x^2 + 16*a^15*b^12*x + a^16*b^11)
```

mupad [B] time = 0.58, size = 1131, normalized size = 6.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^17,x)
```

```
[Out] -(a^10*d^10 + 3003*b^10*c^10 + 8008*b^10*d^10*x^10 + 11440*a*b^9*d^10*x^9 +
 68640*b^10*c*d^9*x^9 + 1287*a^2*b^8*c^8*d^2 + 792*a^3*b^7*c^7*d^3 + 462*a^
4*b^6*c^6*d^4 + 252*a^5*b^5*c^5*d^5 + 126*a^6*b^4*c^4*d^6 + 56*a^7*b^3*c^3*
d^7 + 21*a^8*b^2*c^2*d^8 + 120*a^8*b^2*d^10*x^2 + 560*a^7*b^3*d^10*x^3 + 18
20*a^6*b^4*d^10*x^4 + 4368*a^5*b^5*d^10*x^5 + 8008*a^4*b^6*d^10*x^6 + 11440
*a^3*b^7*d^10*x^7 + 12870*a^2*b^8*d^10*x^8 + 154440*b^10*c^8*d^2*x^2 + 4435
20*b^10*c^7*d^3*x^3 + 840840*b^10*c^6*d^4*x^4 + 1100736*b^10*c^5*d^5*x^5 +
1009008*b^10*c^4*d^6*x^6 + 640640*b^10*c^3*d^7*x^7 + 270270*b^10*c^2*d^8*x^
8 + 2002*a*b^9*c^9*d + 6*a^9*b*c*d^9 + 16*a^9*b*d^10*x + 32032*b^10*c^9*d*x
 + 55440*a^2*b^8*c^6*d^4*x^2 + 30240*a^3*b^7*c^5*d^5*x^2 + 15120*a^4*b^6*c^
4*d^6*x^2 + 6720*a^5*b^5*c^3*d^7*x^2 + 2520*a^6*b^4*c^2*d^8*x^2 + 141120*a^
2*b^8*c^5*d^5*x^3 + 70560*a^3*b^7*c^4*d^6*x^3 + 31360*a^4*b^6*c^3*d^7*x^3 +
 11760*a^5*b^5*c^2*d^8*x^3 + 229320*a^2*b^8*c^4*d^6*x^4 + 101920*a^3*b^7*c^
3*d^7*x^4 + 38220*a^4*b^6*c^2*d^8*x^4 + 244608*a^2*b^8*c^3*d^7*x^5 + 91728*
a^3*b^7*c^2*d^8*x^5 + 168168*a^2*b^8*c^2*d^8*x^6 + 20592*a*b^9*c^8*d^2*x +
96*a^8*b^2*c*d^9*x + 77220*a*b^9*c*d^9*x^8 + 12672*a^2*b^8*c^7*d^3*x + 7392
*a^3*b^7*c^6*d^4*x + 4032*a^4*b^6*c^5*d^5*x + 2016*a^5*b^5*c^4*d^6*x + 896*
```

```
a^6*b^4*c^3*d^7*x + 336*a^7*b^3*c^2*d^8*x + 95040*a*b^9*c^7*d^3*x^2 + 720*a
^7*b^3*c*d^9*x^2 + 258720*a*b^9*c^6*d^4*x^3 + 3360*a^6*b^4*c*d^9*x^3 + 4586
40*a*b^9*c^5*d^5*x^4 + 10920*a^5*b^5*c*d^9*x^4 + 550368*a*b^9*c^4*d^6*x^5 +
 26208*a^4*b^6*c*d^9*x^5 + 448448*a*b^9*c^3*d^7*x^6 + 48048*a^3*b^7*c*d^9*x
^6 + 240240*a*b^9*c^2*d^8*x^7 + 68640*a^2*b^8*c*d^9*x^7)/(48048*a^16*b^11 +
 48048*b^27*x^16 + 768768*a^15*b^12*x + 768768*a*b^26*x^15 + 5765760*a^14*b
^13*x^2 + 26906880*a^13*b^14*x^3 + 87447360*a^12*b^15*x^4 + 209873664*a^11*
b^16*x^5 + 384768384*a^10*b^17*x^6 + 549669120*a^9*b^18*x^7 + 618377760*a^8
*b^19*x^8 + 549669120*a^7*b^20*x^9 + 384768384*a^6*b^21*x^10 + 209873664*a^
5*b^22*x^11 + 87447360*a^4*b^23*x^12 + 26906880*a^3*b^24*x^13 + 5765760*a^2
*b^25*x^14)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**17,x)
```

```
[Out] Timed out
```

3.1223 $\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx$

Optimal. Leaf size=213

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}$$

Rubi [A] time = 0.08, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$-\frac{d^6(c+dx)^{11}}{136136(a+bx)^{11}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{12376(a+bx)^{12}(bc-ad)^6}-\frac{3d^4(c+dx)^{11}}{6188(a+bx)^{13}(bc-ad)^5}+\frac{d^3(c+dx)^{11}}{476(a+bx)^{14}(bc-ad)^4}-\frac{d^2(c+dx)^{11}}{136(a+bx)^{15}(bc-ad)^3}+\frac{3d(c+dx)^{11}}{136(a+bx)^{16}(bc-ad)^2}-\frac{(c+dx)^{11}}{17(a+bx)^{17}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -(c + d*x)^11/(17*(b*c - a*d)*(a + b*x)^17) + (3*d*(c + d*x)^11)/(136*(b*c
- a*d)^2*(a + b*x)^16) - (d^2*(c + d*x)^11)/(136*(b*c - a*d)^3*(a + b*x)^15
) + (d^3*(c + d*x)^11)/(476*(b*c - a*d)^4*(a + b*x)^14) - (3*d^4*(c + d*x)^
11)/(6188*(b*c - a*d)^5*(a + b*x)^13) + (d^5*(c + d*x)^11)/(12376*(b*c - a*
d)^6*(a + b*x)^12) - (d^6*(c + d*x)^11)/(136136*(b*c - a*d)^7*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx = -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} - \frac{(6d)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{17(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} + \frac{\left(15d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{16}}\,dx}{136(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} - \frac{d^3\int \frac{(c+dx)^{10}}{(a+bx)^{15}}\,dx}{34(bc-ad)^3}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

$$= -\frac{(c+dx)^{11}}{17(bc-ad)(a+bx)^{17}} + \frac{3d(c+dx)^{11}}{136(bc-ad)^2(a+bx)^{16}} - \frac{d^2(c+dx)^{11}}{136(bc-ad)^3(a+bx)^{15}} + \frac{d^3(c+d}{476(bc-ad)^4(}$$

Mathematica [B] time = 0.32, size = 690, normalized size = 3.24

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] -1/136136*(a^10*d^10 + a^9*b*d^9*(7*c + 17*d*x) + a^8*b^2*d^8*(28*c^2 + 119
*c*d*x + 136*d^2*x^2) + 4*a^7*b^3*d^7*(21*c^3 + 119*c^2*d*x + 238*c*d^2*x^2
 + 170*d^3*x^3) + 14*a^6*b^4*d^6*(15*c^4 + 102*c^3*d*x + 272*c^2*d^2*x^2 +
340*c*d^3*x^3 + 170*d^4*x^4) + 14*a^5*b^5*d^5*(33*c^5 + 255*c^4*d*x + 816*c
^3*d^2*x^2 + 1360*c^2*d^3*x^3 + 1190*c*d^4*x^4 + 442*d^5*x^5) + 14*a^4*b^6*
d^4*(66*c^6 + 561*c^5*d*x + 2040*c^4*d^2*x^2 + 4080*c^3*d^3*x^3 + 4760*c^2*
d^4*x^4 + 3094*c*d^5*x^5 + 884*d^6*x^6) + 4*a^3*b^7*d^3*(429*c^7 + 3927*c^6
*d*x + 15708*c^5*d^2*x^2 + 35700*c^4*d^3*x^3 + 49980*c^3*d^4*x^4 + 43316*c^
2*d^5*x^5 + 21658*c*d^6*x^6 + 4862*d^7*x^7) + a^2*b^8*d^2*(3003*c^8 + 29172
*c^7*d*x + 125664*c^6*d^2*x^2 + 314160*c^5*d^3*x^3 + 499800*c^4*d^4*x^4 + 5
19792*c^3*d^5*x^5 + 346528*c^2*d^6*x^6 + 136136*c*d^7*x^7 + 24310*d^8*x^8)
+ a*b^9*d*(5005*c^9 + 51051*c^8*d*x + 233376*c^7*d^2*x^2 + 628320*c^6*d^3*x
^3 + 1099560*c^5*d^4*x^4 + 1299480*c^4*d^5*x^5 + 1039584*c^3*d^6*x^6 + 5445
44*c^2*d^7*x^7 + 170170*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(8008*c^10 + 8508
5*c^9*d*x + 408408*c^8*d^2*x^2 + 1166880*c^7*d^3*x^3 + 2199120*c^6*d^4*x^4
+ 2858856*c^5*d^5*x^5 + 2598960*c^4*d^6*x^6 + 1633632*c^3*d^7*x^7 + 680680*
c^2*d^8*x^8 + 170170*c*d^9*x^9 + 19448*d^10*x^10))/(b^11*(a + b*x)^17)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^18,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^18, x]
```

fricas [B] time = 1.31, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="fricas")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

giac [B] time = 1.32, size = 961, normalized size = 4.51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="giac")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 170170*b^10*c*d^9*x^9 + 24310*a*b^9*d^10*
x^9 + 680680*b^10*c^2*d^8*x^8 + 170170*a*b^9*c*d^9*x^8 + 24310*a^2*b^8*d^10
*x^8 + 1633632*b^10*c^3*d^7*x^7 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8
*c*d^9*x^7 + 19448*a^3*b^7*d^10*x^7 + 2598960*b^10*c^4*d^6*x^6 + 1039584*a*
b^9*c^3*d^7*x^6 + 346528*a^2*b^8*c^2*d^8*x^6 + 86632*a^3*b^7*c*d^9*x^6 + 12
376*a^4*b^6*d^10*x^6 + 2858856*b^10*c^5*d^5*x^5 + 1299480*a*b^9*c^4*d^6*x^5
 + 519792*a^2*b^8*c^3*d^7*x^5 + 173264*a^3*b^7*c^2*d^8*x^5 + 43316*a^4*b^6*
c*d^9*x^5 + 6188*a^5*b^5*d^10*x^5 + 2199120*b^10*c^6*d^4*x^4 + 1099560*a*b^
9*c^5*d^5*x^4 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*a^3*b^7*c^3*d^7*x^4 + 6
6640*a^4*b^6*c^2*d^8*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 2380*a^6*b^4*d^10*x^4
+ 1166880*b^10*c^7*d^3*x^3 + 628320*a*b^9*c^6*d^4*x^3 + 314160*a^2*b^8*c^5*
d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3*d^7*x^3 + 19040*a^
5*b^5*c^2*d^8*x^3 + 4760*a^6*b^4*c*d^9*x^3 + 680*a^7*b^3*d^10*x^3 + 408408*
b^10*c^8*d^2*x^2 + 233376*a*b^9*c^7*d^3*x^2 + 125664*a^2*b^8*c^6*d^4*x^2 +
62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d
^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 + 952*a^7*b^3*c*d^9*x^2 + 136*a^8*b^2*d^1
0*x^2 + 85085*b^10*c^9*d*x + 51051*a*b^9*c^8*d^2*x + 29172*a^2*b^8*c^7*d^3*
x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^4*d^6
*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 119*a^8*b^2*c*d^9*x +
 17*a^9*b*d^10*x + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*a^2*b^8*c^8*d^2
```

```
 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a
^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7*a^9*b*c*d^9 +
a^10*d^10)/((b*x + a)^17*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 4.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^18,x)
```

```
[Out] 5/4*d^9*(a*d-b*c)/b^11/(b*x+a)^8-210/13*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^1
1/(b*x+a)^13-1/7*d^10/b^11/(b*x+a)^7-5*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11
/(b*x+a)^9-1/17*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^
3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b
^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^17-210
/11*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^1
1/(b*x+a)^11-3*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3
*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d
+b^8*c^8)/b^11/(b*x+a)^15+21*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-
10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^12+12*d^7*(a^3*d^3-3
*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^10+60/7*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^14+5/8*d*(a^9*d^9-9*a^8*b*c*d
^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^
5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*
x+a)^16
```

maxima [B] time = 2.29, size = 1041, normalized size = 4.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^18,x, algorithm="maxima")
```

```
[Out] -1/136136*(19448*b^10*d^10*x^10 + 8008*b^10*c^10 + 5005*a*b^9*c^9*d + 3003*
a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924*a^4*b^6*c^6*d^4 + 462*a^5*b^5*
c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c^3*d^7 + 28*a^8*b^2*c^2*d^8 + 7
*a^9*b*c*d^9 + a^10*d^10 + 24310*(7*b^10*c*d^9 + a*b^9*d^10)*x^9 + 24310*(2
8*b^10*c^2*d^8 + 7*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 19448*(84*b^10*c^3*d^7
 + 28*a*b^9*c^2*d^8 + 7*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 12376*(210*b^10
*c^4*d^6 + 84*a*b^9*c^3*d^7 + 28*a^2*b^8*c^2*d^8 + 7*a^3*b^7*c*d^9 + a^4*b^
6*d^10)*x^6 + 6188*(462*b^10*c^5*d^5 + 210*a*b^9*c^4*d^6 + 84*a^2*b^8*c^3*d
^7 + 28*a^3*b^7*c^2*d^8 + 7*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 2380*(924*b
^10*c^6*d^4 + 462*a*b^9*c^5*d^5 + 210*a^2*b^8*c^4*d^6 + 84*a^3*b^7*c^3*d^7
+ 28*a^4*b^6*c^2*d^8 + 7*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 680*(1716*b^10
*c^7*d^3 + 924*a*b^9*c^6*d^4 + 462*a^2*b^8*c^5*d^5 + 210*a^3*b^7*c^4*d^6 +
84*a^4*b^6*c^3*d^7 + 28*a^5*b^5*c^2*d^8 + 7*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x
^3 + 136*(3003*b^10*c^8*d^2 + 1716*a*b^9*c^7*d^3 + 924*a^2*b^8*c^6*d^4 + 46
2*a^3*b^7*c^5*d^5 + 210*a^4*b^6*c^4*d^6 + 84*a^5*b^5*c^3*d^7 + 28*a^6*b^4*c
^2*d^8 + 7*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 17*(5005*b^10*c^9*d + 3003*a
*b^9*c^8*d^2 + 1716*a^2*b^8*c^7*d^3 + 924*a^3*b^7*c^6*d^4 + 462*a^4*b^6*c^5
*d^5 + 210*a^5*b^5*c^4*d^6 + 84*a^6*b^4*c^3*d^7 + 28*a^7*b^3*c^2*d^8 + 7*a^
8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^28*x^17 + 17*a*b^27*x^16 + 136*a^2*b^26*x^1
5 + 680*a^3*b^25*x^14 + 2380*a^4*b^24*x^13 + 6188*a^5*b^23*x^12 + 12376*a^6
*b^22*x^11 + 19448*a^7*b^21*x^10 + 24310*a^8*b^20*x^9 + 24310*a^9*b^19*x^8
+ 19448*a^10*b^18*x^7 + 12376*a^11*b^17*x^6 + 6188*a^12*b^16*x^5 + 2380*a^1
3*b^15*x^4 + 680*a^14*b^14*x^3 + 136*a^15*b^13*x^2 + 17*a^16*b^12*x + a^17*
b^11)
```

mupad [B] time = 0.66, size = 1142, normalized size = 5.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^18,x)
```

```
[Out] -(a^10*d^10 + 8008*b^10*c^10 + 19448*b^10*d^10*x^10 + 24310*a*b^9*d^10*x^9
+ 170170*b^10*c*d^9*x^9 + 3003*a^2*b^8*c^8*d^2 + 1716*a^3*b^7*c^7*d^3 + 924
*a^4*b^6*c^6*d^4 + 462*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 + 84*a^7*b^3*c
^3*d^7 + 28*a^8*b^2*c^2*d^8 + 136*a^8*b^2*d^10*x^2 + 680*a^7*b^3*d^10*x^3 +
 2380*a^6*b^4*d^10*x^4 + 6188*a^5*b^5*d^10*x^5 + 12376*a^4*b^6*d^10*x^6 + 1
9448*a^3*b^7*d^10*x^7 + 24310*a^2*b^8*d^10*x^8 + 408408*b^10*c^8*d^2*x^2 +
1166880*b^10*c^7*d^3*x^3 + 2199120*b^10*c^6*d^4*x^4 + 2858856*b^10*c^5*d^5*
x^5 + 2598960*b^10*c^4*d^6*x^6 + 1633632*b^10*c^3*d^7*x^7 + 680680*b^10*c^2
*d^8*x^8 + 5005*a*b^9*c^9*d + 7*a^9*b*c*d^9 + 17*a^9*b*d^10*x + 85085*b^10*
c^9*d*x + 125664*a^2*b^8*c^6*d^4*x^2 + 62832*a^3*b^7*c^5*d^5*x^2 + 28560*a^
4*b^6*c^4*d^6*x^2 + 11424*a^5*b^5*c^3*d^7*x^2 + 3808*a^6*b^4*c^2*d^8*x^2 +
314160*a^2*b^8*c^5*d^5*x^3 + 142800*a^3*b^7*c^4*d^6*x^3 + 57120*a^4*b^6*c^3
*d^7*x^3 + 19040*a^5*b^5*c^2*d^8*x^3 + 499800*a^2*b^8*c^4*d^6*x^4 + 199920*
a^3*b^7*c^3*d^7*x^4 + 66640*a^4*b^6*c^2*d^8*x^4 + 519792*a^2*b^8*c^3*d^7*x^
5 + 173264*a^3*b^7*c^2*d^8*x^5 + 346528*a^2*b^8*c^2*d^8*x^6 + 51051*a*b^9*c
^8*d^2*x + 119*a^8*b^2*c*d^9*x + 170170*a*b^9*c*d^9*x^8 + 29172*a^2*b^8*c^7
*d^3*x + 15708*a^3*b^7*c^6*d^4*x + 7854*a^4*b^6*c^5*d^5*x + 3570*a^5*b^5*c^
4*d^6*x + 1428*a^6*b^4*c^3*d^7*x + 476*a^7*b^3*c^2*d^8*x + 233376*a*b^9*c^7
*d^3*x^2 + 952*a^7*b^3*c*d^9*x^2 + 628320*a*b^9*c^6*d^4*x^3 + 4760*a^6*b^4*
c*d^9*x^3 + 1099560*a*b^9*c^5*d^5*x^4 + 16660*a^5*b^5*c*d^9*x^4 + 1299480*a
*b^9*c^4*d^6*x^5 + 43316*a^4*b^6*c*d^9*x^5 + 1039584*a*b^9*c^3*d^7*x^6 + 86
632*a^3*b^7*c*d^9*x^6 + 544544*a*b^9*c^2*d^8*x^7 + 136136*a^2*b^8*c*d^9*x^7
)/(136136*a^17*b^11 + 136136*b^28*x^17 + 2314312*a^16*b^12*x + 2314312*a*b^
27*x^16 + 18514496*a^15*b^13*x^2 + 92572480*a^14*b^14*x^3 + 324003680*a^13*
b^15*x^4 + 842409568*a^12*b^16*x^5 + 1684819136*a^11*b^17*x^6 + 2647572928*
a^10*b^18*x^7 + 3309466160*a^9*b^19*x^8 + 3309466160*a^8*b^20*x^9 + 2647572
928*a^7*b^21*x^10 + 1684819136*a^6*b^22*x^11 + 842409568*a^5*b^23*x^12 + 32
4003680*a^4*b^24*x^13 + 92572480*a^3*b^25*x^14 + 18514496*a^2*b^26*x^15)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**18,x)
```

```
[Out] Timed out
```

3.1224 $\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$

Optimal. Leaf size=244

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+2$$

Rubi [A] time = 0.10, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{d^7(c+dx)^{11}}{350064(a+bx)^{11}(bc-ad)^8}-\frac{d^6(c+dx)^{11}}{31824(a+bx)^{12}(bc-ad)^7}+\frac{d^5(c+dx)^{11}}{5304(a+bx)^{13}(bc-ad)^6}-\frac{d^4(c+dx)^{11}}{1224(a+bx)^{14}(bc-ad)^5}+\frac{7d^3(c+dx)^{11}}{2448(a+bx)^{15}(bc-ad)^4}-\frac{7d^2(c+dx)^{11}}{816(a+bx)^{16}(bc-ad)^3}+\frac{7d(c+dx)^{11}}{306(a+bx)^{17}(bc-ad)^2}-\frac{(c+dx)^{11}}{18(a+bx)^{18}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -(c + d*x)^11/(18*(b*c - a*d)*(a + b*x)^18) + (7*d*(c + d*x)^11)/(306*(b*c
- a*d)^2*(a + b*x)^17) - (7*d^2*(c + d*x)^11)/(816*(b*c - a*d)^3*(a + b*x)^
16) + (7*d^3*(c + d*x)^11)/(2448*(b*c - a*d)^4*(a + b*x)^15) - (d^4*(c + d*
x)^11)/(1224*(b*c - a*d)^5*(a + b*x)^14) + (d^5*(c + d*x)^11)/(5304*(b*c -
a*d)^6*(a + b*x)^13) - (d^6*(c + d*x)^11)/(31824*(b*c - a*d)^7*(a + b*x)^12
) + (d^7*(c + d*x)^11)/(350064*(b*c - a*d)^8*(a + b*x)^11)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx = -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} - \frac{(7d)\int \frac{(c+dx)^{10}}{(a+bx)^{18}}\,dx}{18(bc-ad)}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} + \frac{\left(7d^2\right)\int \frac{(c+dx)^{10}}{(a+bx)^{17}}\,dx}{51(bc-ad)^2}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} - \frac{\left(35d^3\right)\int \frac{(c}{(a}}{816(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c}{2448(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c}{2448(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c}{2448(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c}{2448(bc-}$$

$$= -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c}{2448(bc-}$$

Mathematica [B] time = 0.28, size = 694, normalized size = 2.84

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] -1/350064*(a^10*d^10 + 2*a^9*b*d^9*(4*c + 9*d*x) + 9*a^8*b^2*d^8*(4*c^2 + 1
6*c*d*x + 17*d^2*x^2) + 24*a^7*b^3*d^7*(5*c^3 + 27*c^2*d*x + 51*c*d^2*x^2 +
 34*d^3*x^3) + 6*a^6*b^4*d^6*(55*c^4 + 360*c^3*d*x + 918*c^2*d^2*x^2 + 1088
*c*d^3*x^3 + 510*d^4*x^4) + 36*a^5*b^5*d^5*(22*c^5 + 165*c^4*d*x + 510*c^3*
d^2*x^2 + 816*c^2*d^3*x^3 + 680*c*d^4*x^4 + 238*d^5*x^5) + 6*a^4*b^6*d^4*(2
86*c^6 + 2376*c^5*d*x + 8415*c^4*d^2*x^2 + 16320*c^3*d^3*x^3 + 18360*c^2*d^
4*x^4 + 11424*c*d^5*x^5 + 3094*d^6*x^6) + 24*a^3*b^7*d^3*(143*c^7 + 1287*c^
6*d*x + 5049*c^5*d^2*x^2 + 11220*c^4*d^3*x^3 + 15300*c^3*d^4*x^4 + 12852*c^
2*d^5*x^5 + 6188*c*d^6*x^6 + 1326*d^7*x^7) + 9*a^2*b^8*d^2*(715*c^8 + 6864*
c^7*d*x + 29172*c^6*d^2*x^2 + 71808*c^5*d^3*x^3 + 112200*c^4*d^4*x^4 + 1142
40*c^3*d^5*x^5 + 74256*c^2*d^6*x^6 + 28288*c*d^7*x^7 + 4862*d^8*x^8) + 2*a*
b^9*d*(5720*c^9 + 57915*c^8*d*x + 262548*c^7*d^2*x^2 + 700128*c^6*d^3*x^3 +
 1211760*c^5*d^4*x^4 + 1413720*c^4*d^5*x^5 + 1113840*c^3*d^6*x^6 + 572832*c
^2*d^7*x^7 + 175032*c*d^8*x^8 + 24310*d^9*x^9) + b^10*(19448*c^10 + 205920*
c^9*d*x + 984555*c^8*d^2*x^2 + 2800512*c^7*d^3*x^3 + 5250960*c^6*d^4*x^4 +
6785856*c^5*d^5*x^5 + 6126120*c^4*d^6*x^6 + 3818880*c^3*d^7*x^7 + 1575288*c
^2*d^8*x^8 + 388960*c*d^9*x^9 + 43758*d^10*x^10))/(b^11*(a + b*x)^18)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^19,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^19, x]
```

fricas [B] time = 1.19, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="fricas")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

giac [B] time = 1.40, size = 961, normalized size = 3.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="giac")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 388960*b^10*c*d^9*x^9 + 48620*a*b^9*d^10*
x^9 + 1575288*b^10*c^2*d^8*x^8 + 350064*a*b^9*c*d^9*x^8 + 43758*a^2*b^8*d^1
0*x^8 + 3818880*b^10*c^3*d^7*x^7 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a^2*b
^8*c*d^9*x^7 + 31824*a^3*b^7*d^10*x^7 + 6126120*b^10*c^4*d^6*x^6 + 2227680*
a*b^9*c^3*d^7*x^6 + 668304*a^2*b^8*c^2*d^8*x^6 + 148512*a^3*b^7*c*d^9*x^6 +
 18564*a^4*b^6*d^10*x^6 + 6785856*b^10*c^5*d^5*x^5 + 2827440*a*b^9*c^4*d^6*
x^5 + 1028160*a^2*b^8*c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 68544*a^4*
b^6*c*d^9*x^5 + 8568*a^5*b^5*d^10*x^5 + 5250960*b^10*c^6*d^4*x^4 + 2423520*
a*b^9*c^5*d^5*x^4 + 1009800*a^2*b^8*c^4*d^6*x^4 + 367200*a^3*b^7*c^3*d^7*x^
4 + 110160*a^4*b^6*c^2*d^8*x^4 + 24480*a^5*b^5*c*d^9*x^4 + 3060*a^6*b^4*d^1
0*x^4 + 2800512*b^10*c^7*d^3*x^3 + 1400256*a*b^9*c^6*d^4*x^3 + 646272*a^2*b
^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*b^6*c^3*d^7*x^3 + 2
9376*a^5*b^5*c^2*d^8*x^3 + 6528*a^6*b^4*c*d^9*x^3 + 816*a^7*b^3*d^10*x^3 +
984555*b^10*c^8*d^2*x^2 + 525096*a*b^9*c^7*d^3*x^2 + 262548*a^2*b^8*c^6*d^4
*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 50490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b
^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 153*a^
8*b^2*d^10*x^2 + 205920*b^10*c^9*d*x + 115830*a*b^9*c^8*d^2*x + 61776*a^2*b
^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 5940*a^5
```

```
*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 144*a^8*b
^2*c*d^9*x + 18*a^9*b*d^10*x + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 6435*a
^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b^5*
c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 +
8*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^18*b^11)
```

maple [B] time = 0.01, size = 867, normalized size = 3.55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^19,x)
```

```
[Out] -1/8*d^10/b^11/(b*x+a)^8+252/13*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d
^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^13+10/9*d^9*(a*d-
b*c)/b^11/(b*x+a)^9+10/17*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^
6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36
*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^17+120/11*d^7*(a^3*d^3
-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^11+8*d^3*(a^7*d^7-7*a^6*
b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5
*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^15-35/2*d^6*(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^12-1/18*(a^10*d^
10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^
6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^
8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^18-9/2*d^8*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/b^11/(b*x+a)^10-15*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-2
0*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^14
-45/16*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*
a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8
)/b^11/(b*x+a)^16
```

maxima [B] time = 2.54, size = 1052, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^19,x, algorithm="maxima")
```

```
[Out] -1/350064*(43758*b^10*d^10*x^10 + 19448*b^10*c^10 + 11440*a*b^9*c^9*d + 643
5*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 1716*a^4*b^6*c^6*d^4 + 792*a^5*b
^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^3*c^3*d^7 + 36*a^8*b^2*c^2*d^8
 + 8*a^9*b*c*d^9 + a^10*d^10 + 48620*(8*b^10*c*d^9 + a*b^9*d^10)*x^9 + 4375
8*(36*b^10*c^2*d^8 + 8*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 31824*(120*b^10*c^
3*d^7 + 36*a*b^9*c^2*d^8 + 8*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 18564*(330
*b^10*c^4*d^6 + 120*a*b^9*c^3*d^7 + 36*a^2*b^8*c^2*d^8 + 8*a^3*b^7*c*d^9 +
a^4*b^6*d^10)*x^6 + 8568*(792*b^10*c^5*d^5 + 330*a*b^9*c^4*d^6 + 120*a^2*b^
8*c^3*d^7 + 36*a^3*b^7*c^2*d^8 + 8*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 + 3060
*(1716*b^10*c^6*d^4 + 792*a*b^9*c^5*d^5 + 330*a^2*b^8*c^4*d^6 + 120*a^3*b^7
*c^3*d^7 + 36*a^4*b^6*c^2*d^8 + 8*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 + 816*(
3432*b^10*c^7*d^3 + 1716*a*b^9*c^6*d^4 + 792*a^2*b^8*c^5*d^5 + 330*a^3*b^7*
c^4*d^6 + 120*a^4*b^6*c^3*d^7 + 36*a^5*b^5*c^2*d^8 + 8*a^6*b^4*c*d^9 + a^7*
b^3*d^10)*x^3 + 153*(6435*b^10*c^8*d^2 + 3432*a*b^9*c^7*d^3 + 1716*a^2*b^8*
c^6*d^4 + 792*a^3*b^7*c^5*d^5 + 330*a^4*b^6*c^4*d^6 + 120*a^5*b^5*c^3*d^7 +
 36*a^6*b^4*c^2*d^8 + 8*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 18*(11440*b^10*
c^9*d + 6435*a*b^9*c^8*d^2 + 3432*a^2*b^8*c^7*d^3 + 1716*a^3*b^7*c^6*d^4 +
792*a^4*b^6*c^5*d^5 + 330*a^5*b^5*c^4*d^6 + 120*a^6*b^4*c^3*d^7 + 36*a^7*b^
3*c^2*d^8 + 8*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^29*x^18 + 18*a*b^28*x^17 +
153*a^2*b^27*x^16 + 816*a^3*b^26*x^15 + 3060*a^4*b^25*x^14 + 8568*a^5*b^24*
x^13 + 18564*a^6*b^23*x^12 + 31824*a^7*b^22*x^11 + 43758*a^8*b^21*x^10 + 48
```

```
620*a^9*b^20*x^9 + 43758*a^10*b^19*x^8 + 31824*a^11*b^18*x^7 + 18564*a^12*b
^17*x^6 + 8568*a^13*b^16*x^5 + 3060*a^14*b^15*x^4 + 816*a^15*b^14*x^3 + 153
*a^16*b^13*x^2 + 18*a^17*b^12*x + a^18*b^11)
```

mupad [B] time = 12.02, size = 1153, normalized size = 4.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^19,x)
```

```
[Out] -(a^10*d^10 + 19448*b^10*c^10 + 43758*b^10*d^10*x^10 + 48620*a*b^9*d^10*x^9
 + 388960*b^10*c*d^9*x^9 + 6435*a^2*b^8*c^8*d^2 + 3432*a^3*b^7*c^7*d^3 + 17
16*a^4*b^6*c^6*d^4 + 792*a^5*b^5*c^5*d^5 + 330*a^6*b^4*c^4*d^6 + 120*a^7*b^
3*c^3*d^7 + 36*a^8*b^2*c^2*d^8 + 153*a^8*b^2*d^10*x^2 + 816*a^7*b^3*d^10*x^
3 + 3060*a^6*b^4*d^10*x^4 + 8568*a^5*b^5*d^10*x^5 + 18564*a^4*b^6*d^10*x^6
+ 31824*a^3*b^7*d^10*x^7 + 43758*a^2*b^8*d^10*x^8 + 984555*b^10*c^8*d^2*x^2
 + 2800512*b^10*c^7*d^3*x^3 + 5250960*b^10*c^6*d^4*x^4 + 6785856*b^10*c^5*d
^5*x^5 + 6126120*b^10*c^4*d^6*x^6 + 3818880*b^10*c^3*d^7*x^7 + 1575288*b^10
*c^2*d^8*x^8 + 11440*a*b^9*c^9*d + 8*a^9*b*c*d^9 + 18*a^9*b*d^10*x + 205920
*b^10*c^9*d*x + 262548*a^2*b^8*c^6*d^4*x^2 + 121176*a^3*b^7*c^5*d^5*x^2 + 5
0490*a^4*b^6*c^4*d^6*x^2 + 18360*a^5*b^5*c^3*d^7*x^2 + 5508*a^6*b^4*c^2*d^8
*x^2 + 646272*a^2*b^8*c^5*d^5*x^3 + 269280*a^3*b^7*c^4*d^6*x^3 + 97920*a^4*
b^6*c^3*d^7*x^3 + 29376*a^5*b^5*c^2*d^8*x^3 + 1009800*a^2*b^8*c^4*d^6*x^4 +
 367200*a^3*b^7*c^3*d^7*x^4 + 110160*a^4*b^6*c^2*d^8*x^4 + 1028160*a^2*b^8*
c^3*d^7*x^5 + 308448*a^3*b^7*c^2*d^8*x^5 + 668304*a^2*b^8*c^2*d^8*x^6 + 115
830*a*b^9*c^8*d^2*x + 144*a^8*b^2*c*d^9*x + 350064*a*b^9*c*d^9*x^8 + 61776*
a^2*b^8*c^7*d^3*x + 30888*a^3*b^7*c^6*d^4*x + 14256*a^4*b^6*c^5*d^5*x + 594
0*a^5*b^5*c^4*d^6*x + 2160*a^6*b^4*c^3*d^7*x + 648*a^7*b^3*c^2*d^8*x + 5250
96*a*b^9*c^7*d^3*x^2 + 1224*a^7*b^3*c*d^9*x^2 + 1400256*a*b^9*c^6*d^4*x^3 +
 6528*a^6*b^4*c*d^9*x^3 + 2423520*a*b^9*c^5*d^5*x^4 + 24480*a^5*b^5*c*d^9*x
^4 + 2827440*a*b^9*c^4*d^6*x^5 + 68544*a^4*b^6*c*d^9*x^5 + 2227680*a*b^9*c^
3*d^7*x^6 + 148512*a^3*b^7*c*d^9*x^6 + 1145664*a*b^9*c^2*d^8*x^7 + 254592*a
^2*b^8*c*d^9*x^7)/(350064*a^18*b^11 + 350064*b^29*x^18 + 6301152*a^17*b^12*
x + 6301152*a*b^28*x^17 + 53559792*a^16*b^13*x^2 + 285652224*a^15*b^14*x^3
+ 1071195840*a^14*b^15*x^4 + 2999348352*a^13*b^16*x^5 + 6498588096*a^12*b^1
7*x^6 + 11140436736*a^11*b^18*x^7 + 15318100512*a^10*b^19*x^8 + 17020111680
*a^9*b^20*x^9 + 15318100512*a^8*b^21*x^10 + 11140436736*a^7*b^22*x^11 + 649
8588096*a^6*b^23*x^12 + 2999348352*a^5*b^24*x^13 + 1071195840*a^4*b^25*x^14
 + 285652224*a^3*b^26*x^15 + 53559792*a^2*b^27*x^16)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**19,x)
```

```
[Out] Timed out
```

3.1225 $\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$

Optimal. Leaf size=273

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc}{2b^{11}(a+}$$

Rubi [A] time = 0.28, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}}-\frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}}-\frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}}-\frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}}-\frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{d^{10}}{9b^{11}(a+bx)^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -(b*c - a*d)^10/(19*b^11*(a + b*x)^19) - (5*d*(b*c - a*d)^9)/(9*b^11*(a + b
*x)^18) - (45*d^2*(b*c - a*d)^8)/(17*b^11*(a + b*x)^17) - (15*d^3*(b*c - a*
d)^7)/(2*b^11*(a + b*x)^16) - (14*d^4*(b*c - a*d)^6)/(b^11*(a + b*x)^15) -
(18*d^5*(b*c - a*d)^5)/(b^11*(a + b*x)^14) - (210*d^6*(b*c - a*d)^4)/(13*b^
11*(a + b*x)^13) - (10*d^7*(b*c - a*d)^3)/(b^11*(a + b*x)^12) - (45*d^8*(b*
c - a*d)^2)/(11*b^11*(a + b*x)^11) - (d^9*(b*c - a*d))/(b^11*(a + b*x)^10)
- d^10/(9*b^11*(a + b*x)^9)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx = \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{20}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{19}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{18}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{17}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{16}}\right.$$

$$= -\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}}-\frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}}-\frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}}-\frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}}-$$

Mathematica [B] time = 0.28, size = 692, normalized size = 2.53

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] -1/831402*(a^10*d^10 + a^9*b*d^9*(9*c + 19*d*x) + 9*a^8*b^2*d^8*(5*c^2 + 19
*c*d*x + 19*d^2*x^2) + 3*a^7*b^3*d^7*(55*c^3 + 285*c^2*d*x + 513*c*d^2*x^2
+ 323*d^3*x^3) + 3*a^6*b^4*d^6*(165*c^4 + 1045*c^3*d*x + 2565*c^2*d^2*x^2 +
 2907*c*d^3*x^3 + 1292*d^4*x^4) + 9*a^5*b^5*d^5*(143*c^5 + 1045*c^4*d*x + 3
135*c^3*d^2*x^2 + 4845*c^2*d^3*x^3 + 3876*c*d^4*x^4 + 1292*d^5*x^5) + 3*a^4
*b^6*d^4*(1001*c^6 + 8151*c^5*d*x + 28215*c^4*d^2*x^2 + 53295*c^3*d^3*x^3 +
 58140*c^2*d^4*x^4 + 34884*c*d^5*x^5 + 9044*d^6*x^6) + 3*a^3*b^7*d^3*(2145*
c^7 + 19019*c^6*d*x + 73359*c^5*d^2*x^2 + 159885*c^4*d^3*x^3 + 213180*c^3*d
^4*x^4 + 174420*c^2*d^5*x^5 + 81396*c*d^6*x^6 + 16796*d^7*x^7) + 9*a^2*b^8*
d^2*(1430*c^8 + 13585*c^7*d*x + 57057*c^6*d^2*x^2 + 138567*c^5*d^3*x^3 + 21
```

```
3180*c^4*d^4*x^4 + 213180*c^3*d^5*x^5 + 135660*c^2*d^6*x^6 + 50388*c*d^7*x^
7 + 8398*d^8*x^8) + a*b^9*d*(24310*c^9 + 244530*c^8*d*x + 1100385*c^7*d^2*x
^2 + 2909907*c^6*d^3*x^3 + 4988412*c^5*d^4*x^4 + 5755860*c^4*d^5*x^5 + 4476
780*c^3*d^6*x^6 + 2267460*c^2*d^7*x^7 + 680238*c*d^8*x^8 + 92378*d^9*x^9) +
 b^10*(43758*c^10 + 461890*c^9*d*x + 2200770*c^8*d^2*x^2 + 6235515*c^7*d^3*
x^3 + 11639628*c^6*d^4*x^4 + 14965236*c^5*d^5*x^5 + 13430340*c^4*d^6*x^6 +
8314020*c^3*d^7*x^7 + 3401190*c^2*d^8*x^8 + 831402*c*d^9*x^9 + 92378*d^10*x
^10))/(b^11*(a + b*x)^19)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^20,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^20, x]
```

fricas [B] time = 1.30, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="fricas")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

giac [B] time = 1.31, size = 961, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="giac")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 831402*b^10*c*d^9*x^9 + 92378*a*b^9*d^10*
x^9 + 3401190*b^10*c^2*d^8*x^8 + 680238*a*b^9*c*d^9*x^8 + 75582*a^2*b^8*d^1
0*x^8 + 8314020*b^10*c^3*d^7*x^7 + 2267460*a*b^9*c^2*d^8*x^7 + 453492*a^2*b
^8*c*d^9*x^7 + 50388*a^3*b^7*d^10*x^7 + 13430340*b^10*c^4*d^6*x^6 + 4476780
*a*b^9*c^3*d^7*x^6 + 1220940*a^2*b^8*c^2*d^8*x^6 + 244188*a^3*b^7*c*d^9*x^6
 + 27132*a^4*b^6*d^10*x^6 + 14965236*b^10*c^5*d^5*x^5 + 5755860*a*b^9*c^4*d
^6*x^5 + 1918620*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 104652*
a^4*b^6*c*d^9*x^5 + 11628*a^5*b^5*d^10*x^5 + 11639628*b^10*c^6*d^4*x^4 + 49
88412*a*b^9*c^5*d^5*x^4 + 1918620*a^2*b^8*c^4*d^6*x^4 + 639540*a^3*b^7*c^3*
d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 34884*a^5*b^5*c*d^9*x^4 + 3876*a^6*b
^4*d^10*x^4 + 6235515*b^10*c^7*d^3*x^3 + 2909907*a*b^9*c^6*d^4*x^3 + 124710
3*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 159885*a^4*b^6*c^3*d^7
*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 969*a^7*b^3*d^1
0*x^3 + 2200770*b^10*c^8*d^2*x^2 + 1100385*a*b^9*c^7*d^3*x^2 + 513513*a^2*b
^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 2
8215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*c^2*d^8*x^2 + 1539*a^7*b^3*c*d^9*x^
2 + 171*a^8*b^2*d^10*x^2 + 461890*b^10*c^9*d*x + 244530*a*b^9*c^8*d^2*x + 1
22265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*d^5*x
 + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d^8*x
+ 171*a^8*b^2*c*d^9*x + 19*a^9*b*d^10*x + 43758*b^10*c^10 + 24310*a*b^9*c^9
*d + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 +
1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b
^2*c^2*d^8 + 9*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^19*b^11)
```

maple [B] time = 0.01, size = 866, normalized size = 3.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^20,x)
```

```
[Out] -210/13*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
/b^11/(b*x+a)^13-1/9*d^10/b^11/(b*x+a)^9-45/17*d^2*(a^8*d^8-8*a^7*b*c*d^7+2
8*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+
28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^17-45/11*d^8*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^11-14*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b
^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^1
1/(b*x+a)^15+10*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x
+a)^12+5/9*d*(a^9*d^9-9*a^8*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+1
26*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^
2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x+a)^18-1/19*(a^10*d^10-10*a^9*b*c*d^9+45*
a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5
+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+
b^10*c^10)/b^11/(b*x+a)^19+d^9*(a*d-b*c)/b^11/(b*x+a)^10+18*d^5*(a^5*d^5-5*
a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^
11/(b*x+a)^14+15/2*d^3*(a^7*d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3
*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/
(b*x+a)^16
```

maxima [B] time = 2.45, size = 1063, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^20,x, algorithm="maxima")
```

```
[Out] -1/831402*(92378*b^10*d^10*x^10 + 43758*b^10*c^10 + 24310*a*b^9*c^9*d + 128
70*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3003*a^4*b^6*c^6*d^4 + 1287*a^5
*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d
```

```
^8 + 9*a^9*b*c*d^9 + a^10*d^10 + 92378*(9*b^10*c*d^9 + a*b^9*d^10)*x^9 + 75
582*(45*b^10*c^2*d^8 + 9*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 50388*(165*b^10*
c^3*d^7 + 45*a*b^9*c^2*d^8 + 9*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 + 27132*(4
95*b^10*c^4*d^6 + 165*a*b^9*c^3*d^7 + 45*a^2*b^8*c^2*d^8 + 9*a^3*b^7*c*d^9
+ a^4*b^6*d^10)*x^6 + 11628*(1287*b^10*c^5*d^5 + 495*a*b^9*c^4*d^6 + 165*a^
2*b^8*c^3*d^7 + 45*a^3*b^7*c^2*d^8 + 9*a^4*b^6*c*d^9 + a^5*b^5*d^10)*x^5 +
3876*(3003*b^10*c^6*d^4 + 1287*a*b^9*c^5*d^5 + 495*a^2*b^8*c^4*d^6 + 165*a^
3*b^7*c^3*d^7 + 45*a^4*b^6*c^2*d^8 + 9*a^5*b^5*c*d^9 + a^6*b^4*d^10)*x^4 +
969*(6435*b^10*c^7*d^3 + 3003*a*b^9*c^6*d^4 + 1287*a^2*b^8*c^5*d^5 + 495*a^
3*b^7*c^4*d^6 + 165*a^4*b^6*c^3*d^7 + 45*a^5*b^5*c^2*d^8 + 9*a^6*b^4*c*d^9
+ a^7*b^3*d^10)*x^3 + 171*(12870*b^10*c^8*d^2 + 6435*a*b^9*c^7*d^3 + 3003*a
^2*b^8*c^6*d^4 + 1287*a^3*b^7*c^5*d^5 + 495*a^4*b^6*c^4*d^6 + 165*a^5*b^5*c
^3*d^7 + 45*a^6*b^4*c^2*d^8 + 9*a^7*b^3*c*d^9 + a^8*b^2*d^10)*x^2 + 19*(243
10*b^10*c^9*d + 12870*a*b^9*c^8*d^2 + 6435*a^2*b^8*c^7*d^3 + 3003*a^3*b^7*c
^6*d^4 + 1287*a^4*b^6*c^5*d^5 + 495*a^5*b^5*c^4*d^6 + 165*a^6*b^4*c^3*d^7 +
 45*a^7*b^3*c^2*d^8 + 9*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(b^30*x^19 + 19*a*b^
29*x^18 + 171*a^2*b^28*x^17 + 969*a^3*b^27*x^16 + 3876*a^4*b^26*x^15 + 1162
8*a^5*b^25*x^14 + 27132*a^6*b^24*x^13 + 50388*a^7*b^23*x^12 + 75582*a^8*b^2
2*x^11 + 92378*a^9*b^21*x^10 + 92378*a^10*b^20*x^9 + 75582*a^11*b^19*x^8 +
50388*a^12*b^18*x^7 + 27132*a^13*b^17*x^6 + 11628*a^14*b^16*x^5 + 3876*a^15
*b^15*x^4 + 969*a^16*b^14*x^3 + 171*a^17*b^13*x^2 + 19*a^18*b^12*x + a^19*b
^11)
```

mupad [B] time = 25.72, size = 1164, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^20,x)
```

```
[Out] -(a^10*d^10 + 43758*b^10*c^10 + 92378*b^10*d^10*x^10 + 92378*a*b^9*d^10*x^9
 + 831402*b^10*c*d^9*x^9 + 12870*a^2*b^8*c^8*d^2 + 6435*a^3*b^7*c^7*d^3 + 3
003*a^4*b^6*c^6*d^4 + 1287*a^5*b^5*c^5*d^5 + 495*a^6*b^4*c^4*d^6 + 165*a^7*
b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 + 171*a^8*b^2*d^10*x^2 + 969*a^7*b^3*d^10*
x^3 + 3876*a^6*b^4*d^10*x^4 + 11628*a^5*b^5*d^10*x^5 + 27132*a^4*b^6*d^10*x
^6 + 50388*a^3*b^7*d^10*x^7 + 75582*a^2*b^8*d^10*x^8 + 2200770*b^10*c^8*d^2
*x^2 + 6235515*b^10*c^7*d^3*x^3 + 11639628*b^10*c^6*d^4*x^4 + 14965236*b^10
*c^5*d^5*x^5 + 13430340*b^10*c^4*d^6*x^6 + 8314020*b^10*c^3*d^7*x^7 + 34011
90*b^10*c^2*d^8*x^8 + 24310*a*b^9*c^9*d + 9*a^9*b*c*d^9 + 19*a^9*b*d^10*x +
 461890*b^10*c^9*d*x + 513513*a^2*b^8*c^6*d^4*x^2 + 220077*a^3*b^7*c^5*d^5*
x^2 + 84645*a^4*b^6*c^4*d^6*x^2 + 28215*a^5*b^5*c^3*d^7*x^2 + 7695*a^6*b^4*
c^2*d^8*x^2 + 1247103*a^2*b^8*c^5*d^5*x^3 + 479655*a^3*b^7*c^4*d^6*x^3 + 15
9885*a^4*b^6*c^3*d^7*x^3 + 43605*a^5*b^5*c^2*d^8*x^3 + 1918620*a^2*b^8*c^4*
d^6*x^4 + 639540*a^3*b^7*c^3*d^7*x^4 + 174420*a^4*b^6*c^2*d^8*x^4 + 1918620
*a^2*b^8*c^3*d^7*x^5 + 523260*a^3*b^7*c^2*d^8*x^5 + 1220940*a^2*b^8*c^2*d^8
*x^6 + 244530*a*b^9*c^8*d^2*x + 171*a^8*b^2*c*d^9*x + 680238*a*b^9*c*d^9*x^
8 + 122265*a^2*b^8*c^7*d^3*x + 57057*a^3*b^7*c^6*d^4*x + 24453*a^4*b^6*c^5*
d^5*x + 9405*a^5*b^5*c^4*d^6*x + 3135*a^6*b^4*c^3*d^7*x + 855*a^7*b^3*c^2*d
^8*x + 1100385*a*b^9*c^7*d^3*x^2 + 1539*a^7*b^3*c*d^9*x^2 + 2909907*a*b^9*c
^6*d^4*x^3 + 8721*a^6*b^4*c*d^9*x^3 + 4988412*a*b^9*c^5*d^5*x^4 + 34884*a^5
*b^5*c*d^9*x^4 + 5755860*a*b^9*c^4*d^6*x^5 + 104652*a^4*b^6*c*d^9*x^5 + 447
6780*a*b^9*c^3*d^7*x^6 + 244188*a^3*b^7*c*d^9*x^6 + 2267460*a*b^9*c^2*d^8*x
^7 + 453492*a^2*b^8*c*d^9*x^7)/(831402*a^19*b^11 + 831402*b^30*x^19 + 15796
638*a^18*b^12*x + 15796638*a*b^29*x^18 + 142169742*a^17*b^13*x^2 + 80562853
8*a^16*b^14*x^3 + 3222514152*a^15*b^15*x^4 + 9667542456*a^14*b^16*x^5 + 225
57599064*a^13*b^17*x^6 + 41892683976*a^12*b^18*x^7 + 62839025964*a^11*b^19*
x^8 + 76803253956*a^10*b^20*x^9 + 76803253956*a^9*b^21*x^10 + 62839025964*a
^8*b^22*x^11 + 41892683976*a^7*b^23*x^12 + 22557599064*a^6*b^24*x^13 + 9667
542456*a^5*b^25*x^14 + 3222514152*a^4*b^26*x^15 + 805628538*a^3*b^27*x^16 +
 142169742*a^2*b^28*x^17)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**20,x)
```

```
[Out] Timed out
```

3.1226 $\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx$

Optimal. Leaf size=279

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc\ldots}{17b^{11}(a\ldots}$$

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{10d^9(bc-ad)}{11b^{11}(a+bx)^{11}}-\frac{15d^8(bc-ad)^2}{4b^{11}(a+bx)^{12}}-\frac{120d^7(bc-ad)^3}{13b^{11}(a+bx)^{13}}-\frac{15d^6(bc-ad)^4}{b^{11}(a+bx)^{14}}-\frac{84d^5(bc-ad)^5}{5b^{11}(a+bx)^{15}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{d^{10}}{10b^{11}(a+bx)^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -(b*c - a*d)^10/(20*b^11*(a + b*x)^20) - (10*d*(b*c - a*d)^9)/(19*b^11*(a +
 b*x)^19) - (5*d^2*(b*c - a*d)^8)/(2*b^11*(a + b*x)^18) - (120*d^3*(b*c - a
*d)^7)/(17*b^11*(a + b*x)^17) - (105*d^4*(b*c - a*d)^6)/(8*b^11*(a + b*x)^1
6) - (84*d^5*(b*c - a*d)^5)/(5*b^11*(a + b*x)^15) - (15*d^6*(b*c - a*d)^4)/
(b^11*(a + b*x)^14) - (120*d^7*(b*c - a*d)^3)/(13*b^11*(a + b*x)^13) - (15*
d^8*(b*c - a*d)^2)/(4*b^11*(a + b*x)^12) - (10*d^9*(b*c - a*d))/(11*b^11*(a
 + b*x)^11) - d^10/(10*b^11*(a + b*x)^10)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{21}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{20}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{19}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{18}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{17}}+\ldots\right.\\ &= -\frac{(bc-ad)^{10}}{20b^{11}(a+bx)^{20}}-\frac{10d(bc-ad)^9}{19b^{11}(a+bx)^{19}}-\frac{5d^2(bc-ad)^8}{2b^{11}(a+bx)^{18}}-\frac{120d^3(bc-ad)^7}{17b^{11}(a+bx)^{17}}-\frac{105d^4(bc-ad)^6}{8b^{11}(a+bx)^{16}}-\ldots\end{aligned}$$

Mathematica [B] time = 0.29, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] -1/1847560*(a^10*d^10 + 10*a^9*b*d^9*(c + 2*d*x) + 5*a^8*b^2*d^8*(11*c^2 +
40*c*d*x + 38*d^2*x^2) + 20*a^7*b^3*d^7*(11*c^3 + 55*c^2*d*x + 95*c*d^2*x^2
 + 57*d^3*x^3) + 5*a^6*b^4*d^6*(143*c^4 + 880*c^3*d*x + 2090*c^2*d^2*x^2 +
2280*c*d^3*x^3 + 969*d^4*x^4) + 2*a^5*b^5*d^5*(1001*c^5 + 7150*c^4*d*x + 20
900*c^3*d^2*x^2 + 31350*c^2*d^3*x^3 + 24225*c*d^4*x^4 + 7752*d^5*x^5) + 5*a
^4*b^6*d^4*(1001*c^6 + 8008*c^5*d*x + 27170*c^4*d^2*x^2 + 50160*c^3*d^3*x^3
 + 53295*c^2*d^4*x^4 + 31008*c*d^5*x^5 + 7752*d^6*x^6) + 20*a^3*b^7*d^3*(57
2*c^7 + 5005*c^6*d*x + 19019*c^5*d^2*x^2 + 40755*c^4*d^3*x^3 + 53295*c^3*d^
4*x^4 + 42636*c^2*d^5*x^5 + 19380*c*d^6*x^6 + 3876*d^7*x^7) + 5*a^2*b^8*d^2
*(4862*c^8 + 45760*c^7*d*x + 190190*c^6*d^2*x^2 + 456456*c^5*d^3*x^3 + 6928
```

```
35*c^4*d^4*x^4 + 682176*c^3*d^5*x^5 + 426360*c^2*d^6*x^6 + 155040*c*d^7*x^7
 + 25194*d^8*x^8) + 10*a*b^9*d*(4862*c^9 + 48620*c^8*d*x + 217360*c^7*d^2*x
^2 + 570570*c^6*d^3*x^3 + 969969*c^5*d^4*x^4 + 1108536*c^4*d^5*x^5 + 852720
*c^3*d^6*x^6 + 426360*c^2*d^7*x^7 + 125970*c*d^8*x^8 + 16796*d^9*x^9) + b^1
0*(92378*c^10 + 972400*c^9*d*x + 4618900*c^8*d^2*x^2 + 13041600*c^7*d^3*x^3
 + 24249225*c^6*d^4*x^4 + 31039008*c^5*d^5*x^5 + 27713400*c^4*d^6*x^6 + 170
54400*c^3*d^7*x^7 + 6928350*c^2*d^8*x^8 + 1679600*c*d^9*x^9 + 184756*d^10*x
^10))/(b^11*(a + b*x)^20)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{21}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^21,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^21, x]
```

fricas [B] time = 1.23, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="fricas")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

giac [B] time = 1.31, size = 961, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="giac")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 1679600*b^10*c*d^9*x^9 + 167960*a*b^9*d
^10*x^9 + 6928350*b^10*c^2*d^8*x^8 + 1259700*a*b^9*c*d^9*x^8 + 125970*a^2*b
^8*d^10*x^8 + 17054400*b^10*c^3*d^7*x^7 + 4263600*a*b^9*c^2*d^8*x^7 + 77520
0*a^2*b^8*c*d^9*x^7 + 77520*a^3*b^7*d^10*x^7 + 27713400*b^10*c^4*d^6*x^6 +
8527200*a*b^9*c^3*d^7*x^6 + 2131800*a^2*b^8*c^2*d^8*x^6 + 387600*a^3*b^7*c*
d^9*x^6 + 38760*a^4*b^6*d^10*x^6 + 31039008*b^10*c^5*d^5*x^5 + 11085360*a*b
^9*c^4*d^6*x^5 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 +
 155040*a^4*b^6*c*d^9*x^5 + 15504*a^5*b^5*d^10*x^5 + 24249225*b^10*c^6*d^4*
x^4 + 9699690*a*b^9*c^5*d^5*x^4 + 3464175*a^2*b^8*c^4*d^6*x^4 + 1065900*a^3
*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 4
845*a^6*b^4*d^10*x^4 + 13041600*b^10*c^7*d^3*x^3 + 5705700*a*b^9*c^6*d^4*x^
3 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d^6*x^3 + 250800*a^4*b
^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 1140
*a^7*b^3*d^10*x^3 + 4618900*b^10*c^8*d^2*x^2 + 2173600*a*b^9*c^7*d^3*x^2 +
950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7*c^5*d^5*x^2 + 135850*a^4*b^6*c^
4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 10450*a^6*b^4*c^2*d^8*x^2 + 1900*a^
7*b^3*c*d^9*x^2 + 190*a^8*b^2*d^10*x^2 + 972400*b^10*c^9*d*x + 486200*a*b^9
*c^8*d^2*x + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 40040*a^
4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 1100*a
^7*b^3*c^2*d^8*x + 200*a^8*b^2*c*d^9*x + 20*a^9*b*d^10*x + 92378*b^10*c^10
+ 48620*a*b^9*c^9*d + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*
a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*
c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10)/((b*x + a)^20*b^
11)
```

maple [**B**] time = 0.01, size = 867, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^10/(b*x+a)^21,x)
```

```
[Out] 120/13*d^7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^13-1/
20*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3*c^3*d^7+210*a^6
*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^3*b^7*c^7*d^3+45
*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^20+120/17*d^3*(a^7*
d^7-7*a^6*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-
21*a^2*b^5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^17+10/11*d^9*(a*d-b*
c)/b^11/(b*x+a)^11+84/5*d^5*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^
2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/b^11/(b*x+a)^15-15/4*d^8*(a^2*d^2-2*a*
b*c*d+b^2*c^2)/b^11/(b*x+a)^12-5/2*d^2*(a^8*d^8-8*a^7*b*c*d^7+28*a^6*b^2*c^
2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*c^5*d^3+28*a^2*b^6*c
^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^18+10/19*d*(a^9*d^9-9*a^8*b*c*d^
8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*b^5*c^5
*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^11/(b*x
+a)^19-1/10*d^10/b^11/(b*x+a)^10-15*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^
2*d^2-4*a*b^3*c^3*d+b^4*c^4)/b^11/(b*x+a)^14-105/8*d^4*(a^6*d^6-6*a^5*b*c*d
^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b
^6*c^6)/b^11/(b*x+a)^16
```

maxima [**B**] time = 2.51, size = 1074, normalized size = 3.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^21,x, algorithm="maxima")
```

```
[Out] -1/1847560*(184756*b^10*d^10*x^10 + 92378*b^10*c^10 + 48620*a*b^9*c^9*d + 2
4310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3 + 5005*a^4*b^6*c^6*d^4 + 2002*
```

```
a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^
2*d^8 + 10*a^9*b*c*d^9 + a^10*d^10 + 167960*(10*b^10*c*d^9 + a*b^9*d^10)*x^
9 + 125970*(55*b^10*c^2*d^8 + 10*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 77520*(2
20*b^10*c^3*d^7 + 55*a*b^9*c^2*d^8 + 10*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^7 +
 38760*(715*b^10*c^4*d^6 + 220*a*b^9*c^3*d^7 + 55*a^2*b^8*c^2*d^8 + 10*a^3*
b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 15504*(2002*b^10*c^5*d^5 + 715*a*b^9*c^4*d^
6 + 220*a^2*b^8*c^3*d^7 + 55*a^3*b^7*c^2*d^8 + 10*a^4*b^6*c*d^9 + a^5*b^5*d
^10)*x^5 + 4845*(5005*b^10*c^6*d^4 + 2002*a*b^9*c^5*d^5 + 715*a^2*b^8*c^4*d
^6 + 220*a^3*b^7*c^3*d^7 + 55*a^4*b^6*c^2*d^8 + 10*a^5*b^5*c*d^9 + a^6*b^4*
d^10)*x^4 + 1140*(11440*b^10*c^7*d^3 + 5005*a*b^9*c^6*d^4 + 2002*a^2*b^8*c^
5*d^5 + 715*a^3*b^7*c^4*d^6 + 220*a^4*b^6*c^3*d^7 + 55*a^5*b^5*c^2*d^8 + 10
*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 190*(24310*b^10*c^8*d^2 + 11440*a*b^9*
c^7*d^3 + 5005*a^2*b^8*c^6*d^4 + 2002*a^3*b^7*c^5*d^5 + 715*a^4*b^6*c^4*d^6
 + 220*a^5*b^5*c^3*d^7 + 55*a^6*b^4*c^2*d^8 + 10*a^7*b^3*c*d^9 + a^8*b^2*d^
10)*x^2 + 20*(48620*b^10*c^9*d + 24310*a*b^9*c^8*d^2 + 11440*a^2*b^8*c^7*d^
3 + 5005*a^3*b^7*c^6*d^4 + 2002*a^4*b^6*c^5*d^5 + 715*a^5*b^5*c^4*d^6 + 220
*a^6*b^4*c^3*d^7 + 55*a^7*b^3*c^2*d^8 + 10*a^8*b^2*c*d^9 + a^9*b*d^10)*x)/(
b^31*x^20 + 20*a*b^30*x^19 + 190*a^2*b^29*x^18 + 1140*a^3*b^28*x^17 + 4845*
a^4*b^27*x^16 + 15504*a^5*b^26*x^15 + 38760*a^6*b^25*x^14 + 77520*a^7*b^24*
x^13 + 125970*a^8*b^23*x^12 + 167960*a^9*b^22*x^11 + 184756*a^10*b^21*x^10
+ 167960*a^11*b^20*x^9 + 125970*a^12*b^19*x^8 + 77520*a^13*b^18*x^7 + 38760
*a^14*b^17*x^6 + 15504*a^15*b^16*x^5 + 4845*a^16*b^15*x^4 + 1140*a^17*b^14*
x^3 + 190*a^18*b^13*x^2 + 20*a^19*b^12*x + a^20*b^11)
```

mupad [B] time = 0.80, size = 1175, normalized size = 4.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^21,x)
```

```
[Out] -(a^10*d^10 + 92378*b^10*c^10 + 184756*b^10*d^10*x^10 + 167960*a*b^9*d^10*x
^9 + 1679600*b^10*c*d^9*x^9 + 24310*a^2*b^8*c^8*d^2 + 11440*a^3*b^7*c^7*d^3
 + 5005*a^4*b^6*c^6*d^4 + 2002*a^5*b^5*c^5*d^5 + 715*a^6*b^4*c^4*d^6 + 220*
a^7*b^3*c^3*d^7 + 55*a^8*b^2*c^2*d^8 + 190*a^8*b^2*d^10*x^2 + 1140*a^7*b^3*
d^10*x^3 + 4845*a^6*b^4*d^10*x^4 + 15504*a^5*b^5*d^10*x^5 + 38760*a^4*b^6*d
^10*x^6 + 77520*a^3*b^7*d^10*x^7 + 125970*a^2*b^8*d^10*x^8 + 4618900*b^10*c
^8*d^2*x^2 + 13041600*b^10*c^7*d^3*x^3 + 24249225*b^10*c^6*d^4*x^4 + 310390
08*b^10*c^5*d^5*x^5 + 27713400*b^10*c^4*d^6*x^6 + 17054400*b^10*c^3*d^7*x^7
 + 6928350*b^10*c^2*d^8*x^8 + 48620*a*b^9*c^9*d + 10*a^9*b*c*d^9 + 20*a^9*b
*d^10*x + 972400*b^10*c^9*d*x + 950950*a^2*b^8*c^6*d^4*x^2 + 380380*a^3*b^7
*c^5*d^5*x^2 + 135850*a^4*b^6*c^4*d^6*x^2 + 41800*a^5*b^5*c^3*d^7*x^2 + 104
50*a^6*b^4*c^2*d^8*x^2 + 2282280*a^2*b^8*c^5*d^5*x^3 + 815100*a^3*b^7*c^4*d
^6*x^3 + 250800*a^4*b^6*c^3*d^7*x^3 + 62700*a^5*b^5*c^2*d^8*x^3 + 3464175*a
^2*b^8*c^4*d^6*x^4 + 1065900*a^3*b^7*c^3*d^7*x^4 + 266475*a^4*b^6*c^2*d^8*x
^4 + 3410880*a^2*b^8*c^3*d^7*x^5 + 852720*a^3*b^7*c^2*d^8*x^5 + 2131800*a^2
*b^8*c^2*d^8*x^6 + 486200*a*b^9*c^8*d^2*x + 200*a^8*b^2*c*d^9*x + 1259700*a
*b^9*c*d^9*x^8 + 228800*a^2*b^8*c^7*d^3*x + 100100*a^3*b^7*c^6*d^4*x + 4004
0*a^4*b^6*c^5*d^5*x + 14300*a^5*b^5*c^4*d^6*x + 4400*a^6*b^4*c^3*d^7*x + 11
00*a^7*b^3*c^2*d^8*x + 2173600*a*b^9*c^7*d^3*x^2 + 1900*a^7*b^3*c*d^9*x^2 +
 5705700*a*b^9*c^6*d^4*x^3 + 11400*a^6*b^4*c*d^9*x^3 + 9699690*a*b^9*c^5*d^
5*x^4 + 48450*a^5*b^5*c*d^9*x^4 + 11085360*a*b^9*c^4*d^6*x^5 + 155040*a^4*b
^6*c*d^9*x^5 + 8527200*a*b^9*c^3*d^7*x^6 + 387600*a^3*b^7*c*d^9*x^6 + 42636
00*a*b^9*c^2*d^8*x^7 + 775200*a^2*b^8*c*d^9*x^7)/(1847560*a^20*b^11 + 18475
60*b^31*x^20 + 36951200*a^19*b^12*x + 36951200*a*b^30*x^19 + 351036400*a^18
*b^13*x^2 + 2106218400*a^17*b^14*x^3 + 8951428200*a^16*b^15*x^4 + 286445702
40*a^15*b^16*x^5 + 71611425600*a^14*b^17*x^6 + 143222851200*a^13*b^18*x^7 +
 232737133200*a^12*b^19*x^8 + 310316177600*a^11*b^20*x^9 + 341347795360*a^1
0*b^21*x^10 + 310316177600*a^9*b^22*x^11 + 232737133200*a^8*b^23*x^12 + 143
222851200*a^7*b^24*x^13 + 71611425600*a^6*b^25*x^14 + 28644570240*a^5*b^26*
```

```
x^15 + 8951428200*a^4*b^27*x^16 + 2106218400*a^3*b^28*x^17 + 351036400*a^2*
b^29*x^18)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**21,x)
```

```
[Out] Timed out
```

3.1227 $\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$

Optimal. Leaf size=279

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(b}{3b^{11}(a}$$

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{5d^9(bc-ad)}{6b^{11}(a+bx)^{12}}-\frac{45d^8(bc-ad)^2}{13b^{11}(a+bx)^{13}}-\frac{60d^7(bc-ad)^3}{7b^{11}(a+bx)^{14}}-\frac{14d^6(bc-ad)^4}{b^{11}(a+bx)^{15}}-\frac{63d^5(bc-ad)^5}{4b^{11}(a+bx)^{16}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d^{10}}{11b^{11}(a+bx)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -(b*c - a*d)^10/(21*b^11*(a + b*x)^21) - (d*(b*c - a*d)^9)/(2*b^11*(a + b*x
)^20) - (45*d^2*(b*c - a*d)^8)/(19*b^11*(a + b*x)^19) - (20*d^3*(b*c - a*d)
^7)/(3*b^11*(a + b*x)^18) - (210*d^4*(b*c - a*d)^6)/(17*b^11*(a + b*x)^17)
- (63*d^5*(b*c - a*d)^5)/(4*b^11*(a + b*x)^16) - (14*d^6*(b*c - a*d)^4)/(b^
11*(a + b*x)^15) - (60*d^7*(b*c - a*d)^3)/(7*b^11*(a + b*x)^14) - (45*d^8*(
b*c - a*d)^2)/(13*b^11*(a + b*x)^13) - (5*d^9*(b*c - a*d))/(6*b^11*(a + b*x
)^12) - d^10/(11*b^11*(a + b*x)^11)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx &= \int \left(\frac{(bc-ad)^{10}}{b^{10}(a+bx)^{22}}+\frac{10d(bc-ad)^9}{b^{10}(a+bx)^{21}}+\frac{45d^2(bc-ad)^8}{b^{10}(a+bx)^{20}}+\frac{120d^3(bc-ad)^7}{b^{10}(a+bx)^{19}}+\frac{210d^4(bc-ad)^6}{b^{10}(a+bx)^{18}}\right.\\ &= -\frac{(bc-ad)^{10}}{21b^{11}(a+bx)^{21}}-\frac{d(bc-ad)^9}{2b^{11}(a+bx)^{20}}-\frac{45d^2(bc-ad)^8}{19b^{11}(a+bx)^{19}}-\frac{20d^3(bc-ad)^7}{3b^{11}(a+bx)^{18}}-\frac{210d^4(bc-ad)^6}{17b^{11}(a+bx)^{17}}\end{aligned}$$

Mathematica [B] time = 0.30, size = 692, normalized size = 2.48

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] -1/3879876*(a^10*d^10 + a^9*b*d^9*(11*c + 21*d*x) + 3*a^8*b^2*d^8*(22*c^2 +
 77*c*d*x + 70*d^2*x^2) + 2*a^7*b^3*d^7*(143*c^3 + 693*c^2*d*x + 1155*c*d^2
*x^2 + 665*d^3*x^3) + 7*a^6*b^4*d^6*(143*c^4 + 858*c^3*d*x + 1980*c^2*d^2*x
^2 + 2090*c*d^3*x^3 + 855*d^4*x^4) + 21*a^5*b^5*d^5*(143*c^5 + 1001*c^4*d*x
 + 2860*c^3*d^2*x^2 + 4180*c^2*d^3*x^3 + 3135*c*d^4*x^4 + 969*d^5*x^5) + 7*
a^4*b^6*d^4*(1144*c^6 + 9009*c^5*d*x + 30030*c^4*d^2*x^2 + 54340*c^3*d^3*x^
3 + 56430*c^2*d^4*x^4 + 31977*c*d^5*x^5 + 7752*d^6*x^6) + 2*a^3*b^7*d^3*(97
24*c^7 + 84084*c^6*d*x + 315315*c^5*d^2*x^2 + 665665*c^4*d^3*x^3 + 855855*c
^3*d^4*x^4 + 671517*c^2*d^5*x^5 + 298452*c*d^6*x^6 + 58140*d^7*x^7) + 3*a^2
*b^8*d^2*(14586*c^8 + 136136*c^7*d*x + 560560*c^6*d^2*x^2 + 1331330*c^5*d^3
```

```
*x^3 + 1996995*c^4*d^4*x^4 + 1939938*c^3*d^5*x^5 + 1193808*c^2*d^6*x^6 + 42
6360*c*d^7*x^7 + 67830*d^8*x^8) + a*b^9*d*(92378*c^9 + 918918*c^8*d*x + 408
4080*c^7*d^2*x^2 + 10650640*c^6*d^3*x^3 + 17972955*c^5*d^4*x^4 + 20369349*c
^4*d^5*x^5 + 15519504*c^3*d^6*x^6 + 7674480*c^2*d^7*x^7 + 2238390*c*d^8*x^8
 + 293930*d^9*x^9) + b^10*(184756*c^10 + 1939938*c^9*d*x + 9189180*c^8*d^2*
x^2 + 25865840*c^7*d^3*x^3 + 47927880*c^6*d^4*x^4 + 61108047*c^5*d^5*x^5 +
54318264*c^4*d^6*x^6 + 33256080*c^3*d^7*x^7 + 13430340*c^2*d^8*x^8 + 323323
0*c*d^9*x^9 + 352716*d^10*x^10))/(b^11*(a + b*x)^21)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{10}}{(a+bx)^{22}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^22,x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^10/(a + b*x)^22, x]
```

fricas [B] time = 1.29, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="fricas")
```

```
[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

giac [B] time = 1.30, size = 961, normalized size = 3.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="giac")
```

[Out] -1/3879876*(352716*b^10*d^10*x^10 + 3233230*b^10*c*d^9*x^9 + 293930*a*b^9*d
^10*x^9 + 13430340*b^10*c^2*d^8*x^8 + 2238390*a*b^9*c*d^9*x^8 + 203490*a^2*
b^8*d^10*x^8 + 33256080*b^10*c^3*d^7*x^7 + 7674480*a*b^9*c^2*d^8*x^7 + 1279
080*a^2*b^8*c*d^9*x^7 + 116280*a^3*b^7*d^10*x^7 + 54318264*b^10*c^4*d^6*x^6
 + 15519504*a*b^9*c^3*d^7*x^6 + 3581424*a^2*b^8*c^2*d^8*x^6 + 596904*a^3*b^
7*c*d^9*x^6 + 54264*a^4*b^6*d^10*x^6 + 61108047*b^10*c^5*d^5*x^5 + 20369349
*a*b^9*c^4*d^6*x^5 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*
x^5 + 223839*a^4*b^6*c*d^9*x^5 + 20349*a^5*b^5*d^10*x^5 + 47927880*b^10*c^6
*d^4*x^4 + 17972955*a*b^9*c^5*d^5*x^4 + 5990985*a^2*b^8*c^4*d^6*x^4 + 17117
10*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^2*d^8*x^4 + 65835*a^5*b^5*c*d^9*x
^4 + 5985*a^6*b^4*d^10*x^4 + 25865840*b^10*c^7*d^3*x^3 + 10650640*a*b^9*c^6
*d^4*x^3 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^7*c^4*d^6*x^3 + 3803
80*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 14630*a^6*b^4*c*d^9*x^
3 + 1330*a^7*b^3*d^10*x^3 + 9189180*b^10*c^8*d^2*x^2 + 4084080*a*b^9*c^7*d^
3*x^2 + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a^3*b^7*c^5*d^5*x^2 + 210210*a
^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2 + 13860*a^6*b^4*c^2*d^8*x^2
+ 2310*a^7*b^3*c*d^9*x^2 + 210*a^8*b^2*d^10*x^2 + 1939938*b^10*c^9*d*x + 91
8918*a*b^9*c^8*d^2*x + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*x
+ 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^7*
x + 1386*a^7*b^3*c^2*d^8*x + 231*a^8*b^2*c*d^9*x + 21*a^9*b*d^10*x + 184756
*b^10*c^10 + 92378*a*b^9*c^9*d + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*
d^3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 +
286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10)/((b*
x + a)^21*b^11)

maple [B] time = 0.01, size = 867, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^10/(b*x+a)^22,x)

[Out] -45/13*d^8*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^11/(b*x+a)^13+1/2*d*(a^9*d^9-9*a^8
*b*c*d^8+36*a^7*b^2*c^2*d^7-84*a^6*b^3*c^3*d^6+126*a^5*b^4*c^4*d^5-126*a^4*
b^5*c^5*d^4+84*a^3*b^6*c^6*d^3-36*a^2*b^7*c^7*d^2+9*a*b^8*c^8*d-b^9*c^9)/b^
11/(b*x+a)^20-1/21*(a^10*d^10-10*a^9*b*c*d^9+45*a^8*b^2*c^2*d^8-120*a^7*b^3
*c^3*d^7+210*a^6*b^4*c^4*d^6-252*a^5*b^5*c^5*d^5+210*a^4*b^6*c^6*d^4-120*a^
3*b^7*c^7*d^3+45*a^2*b^8*c^8*d^2-10*a*b^9*c^9*d+b^10*c^10)/b^11/(b*x+a)^21-
210/17*d^4*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*
a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/b^11/(b*x+a)^17-1/11*d^10/b^11/(b*x+
a)^11-14*d^6*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4
)/b^11/(b*x+a)^15+5/6*d^9*(a*d-b*c)/b^11/(b*x+a)^12+20/3*d^3*(a^7*d^7-7*a^6
*b*c*d^6+21*a^5*b^2*c^2*d^5-35*a^4*b^3*c^3*d^4+35*a^3*b^4*c^4*d^3-21*a^2*b^
5*c^5*d^2+7*a*b^6*c^6*d-b^7*c^7)/b^11/(b*x+a)^18-45/19*d^2*(a^8*d^8-8*a^7*b
*c*d^7+28*a^6*b^2*c^2*d^6-56*a^5*b^3*c^3*d^5+70*a^4*b^4*c^4*d^4-56*a^3*b^5*
c^5*d^3+28*a^2*b^6*c^6*d^2-8*a*b^7*c^7*d+b^8*c^8)/b^11/(b*x+a)^19+60/7*d^7*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/b^11/(b*x+a)^14+63/4*d^5*(a^5
*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*
c^5)/b^11/(b*x+a)^16

maxima [B] time = 2.46, size = 1085, normalized size = 3.89

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^10/(b*x+a)^22,x, algorithm="maxima")

[Out] -1/3879876*(352716*b^10*d^10*x^10 + 184756*b^10*c^10 + 92378*a*b^9*c^9*d +
43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^3 + 8008*a^4*b^6*c^6*d^4 + 3003

```
*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 286*a^7*b^3*c^3*d^7 + 66*a^8*b^2*
c^2*d^8 + 11*a^9*b*c*d^9 + a^10*d^10 + 293930*(11*b^10*c*d^9 + a*b^9*d^10)*
x^9 + 203490*(66*b^10*c^2*d^8 + 11*a*b^9*c*d^9 + a^2*b^8*d^10)*x^8 + 116280
*(286*b^10*c^3*d^7 + 66*a*b^9*c^2*d^8 + 11*a^2*b^8*c*d^9 + a^3*b^7*d^10)*x^
7 + 54264*(1001*b^10*c^4*d^6 + 286*a*b^9*c^3*d^7 + 66*a^2*b^8*c^2*d^8 + 11*
a^3*b^7*c*d^9 + a^4*b^6*d^10)*x^6 + 20349*(3003*b^10*c^5*d^5 + 1001*a*b^9*c
^4*d^6 + 286*a^2*b^8*c^3*d^7 + 66*a^3*b^7*c^2*d^8 + 11*a^4*b^6*c*d^9 + a^5*
b^5*d^10)*x^5 + 5985*(8008*b^10*c^6*d^4 + 3003*a*b^9*c^5*d^5 + 1001*a^2*b^8
*c^4*d^6 + 286*a^3*b^7*c^3*d^7 + 66*a^4*b^6*c^2*d^8 + 11*a^5*b^5*c*d^9 + a^
6*b^4*d^10)*x^4 + 1330*(19448*b^10*c^7*d^3 + 8008*a*b^9*c^6*d^4 + 3003*a^2*
b^8*c^5*d^5 + 1001*a^3*b^7*c^4*d^6 + 286*a^4*b^6*c^3*d^7 + 66*a^5*b^5*c^2*d
^8 + 11*a^6*b^4*c*d^9 + a^7*b^3*d^10)*x^3 + 210*(43758*b^10*c^8*d^2 + 19448
*a*b^9*c^7*d^3 + 8008*a^2*b^8*c^6*d^4 + 3003*a^3*b^7*c^5*d^5 + 1001*a^4*b^6
*c^4*d^6 + 286*a^5*b^5*c^3*d^7 + 66*a^6*b^4*c^2*d^8 + 11*a^7*b^3*c*d^9 + a^
8*b^2*d^10)*x^2 + 21*(92378*b^10*c^9*d + 43758*a*b^9*c^8*d^2 + 19448*a^2*b^
8*c^7*d^3 + 8008*a^3*b^7*c^6*d^4 + 3003*a^4*b^6*c^5*d^5 + 1001*a^5*b^5*c^4*
d^6 + 286*a^6*b^4*c^3*d^7 + 66*a^7*b^3*c^2*d^8 + 11*a^8*b^2*c*d^9 + a^9*b*d
^10)*x)/(b^32*x^21 + 21*a*b^31*x^20 + 210*a^2*b^30*x^19 + 1330*a^3*b^29*x^1
8 + 5985*a^4*b^28*x^17 + 20349*a^5*b^27*x^16 + 54264*a^6*b^26*x^15 + 116280
*a^7*b^25*x^14 + 203490*a^8*b^24*x^13 + 293930*a^9*b^23*x^12 + 352716*a^10*
b^22*x^11 + 352716*a^11*b^21*x^10 + 293930*a^12*b^20*x^9 + 203490*a^13*b^19
*x^8 + 116280*a^14*b^18*x^7 + 54264*a^15*b^17*x^6 + 20349*a^16*b^16*x^5 + 5
985*a^17*b^15*x^4 + 1330*a^18*b^14*x^3 + 210*a^19*b^13*x^2 + 21*a^20*b^12*x
 + a^21*b^11)
```

mupad [B] time = 1.04, size = 1186, normalized size = 4.25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^10/(a + b*x)^22,x)
```

```
[Out] -(a^10*d^10 + 184756*b^10*c^10 + 352716*b^10*d^10*x^10 + 293930*a*b^9*d^10*
x^9 + 3233230*b^10*c*d^9*x^9 + 43758*a^2*b^8*c^8*d^2 + 19448*a^3*b^7*c^7*d^
3 + 8008*a^4*b^6*c^6*d^4 + 3003*a^5*b^5*c^5*d^5 + 1001*a^6*b^4*c^4*d^6 + 28
6*a^7*b^3*c^3*d^7 + 66*a^8*b^2*c^2*d^8 + 210*a^8*b^2*d^10*x^2 + 1330*a^7*b^
3*d^10*x^3 + 5985*a^6*b^4*d^10*x^4 + 20349*a^5*b^5*d^10*x^5 + 54264*a^4*b^6
*d^10*x^6 + 116280*a^3*b^7*d^10*x^7 + 203490*a^2*b^8*d^10*x^8 + 9189180*b^1
0*c^8*d^2*x^2 + 25865840*b^10*c^7*d^3*x^3 + 47927880*b^10*c^6*d^4*x^4 + 611
08047*b^10*c^5*d^5*x^5 + 54318264*b^10*c^4*d^6*x^6 + 33256080*b^10*c^3*d^7*
x^7 + 13430340*b^10*c^2*d^8*x^8 + 92378*a*b^9*c^9*d + 11*a^9*b*c*d^9 + 21*a
^9*b*d^10*x + 1939938*b^10*c^9*d*x + 1681680*a^2*b^8*c^6*d^4*x^2 + 630630*a
^3*b^7*c^5*d^5*x^2 + 210210*a^4*b^6*c^4*d^6*x^2 + 60060*a^5*b^5*c^3*d^7*x^2
 + 13860*a^6*b^4*c^2*d^8*x^2 + 3993990*a^2*b^8*c^5*d^5*x^3 + 1331330*a^3*b^
7*c^4*d^6*x^3 + 380380*a^4*b^6*c^3*d^7*x^3 + 87780*a^5*b^5*c^2*d^8*x^3 + 59
90985*a^2*b^8*c^4*d^6*x^4 + 1711710*a^3*b^7*c^3*d^7*x^4 + 395010*a^4*b^6*c^
2*d^8*x^4 + 5819814*a^2*b^8*c^3*d^7*x^5 + 1343034*a^3*b^7*c^2*d^8*x^5 + 358
1424*a^2*b^8*c^2*d^8*x^6 + 918918*a*b^9*c^8*d^2*x + 231*a^8*b^2*c*d^9*x + 2
238390*a*b^9*c*d^9*x^8 + 408408*a^2*b^8*c^7*d^3*x + 168168*a^3*b^7*c^6*d^4*
x + 63063*a^4*b^6*c^5*d^5*x + 21021*a^5*b^5*c^4*d^6*x + 6006*a^6*b^4*c^3*d^
7*x + 1386*a^7*b^3*c^2*d^8*x + 4084080*a*b^9*c^7*d^3*x^2 + 2310*a^7*b^3*c*d
^9*x^2 + 10650640*a*b^9*c^6*d^4*x^3 + 14630*a^6*b^4*c*d^9*x^3 + 17972955*a*
b^9*c^5*d^5*x^4 + 65835*a^5*b^5*c*d^9*x^4 + 20369349*a*b^9*c^4*d^6*x^5 + 22
3839*a^4*b^6*c*d^9*x^5 + 15519504*a*b^9*c^3*d^7*x^6 + 596904*a^3*b^7*c*d^9*
x^6 + 7674480*a*b^9*c^2*d^8*x^7 + 1279080*a^2*b^8*c*d^9*x^7)/(3879876*a^21*
b^11 + 3879876*b^32*x^21 + 81477396*a^20*b^12*x + 81477396*a*b^31*x^20 + 81
4773960*a^19*b^13*x^2 + 5160235080*a^18*b^14*x^3 + 23221057860*a^17*b^15*x^
4 + 78951596724*a^16*b^16*x^5 + 210537591264*a^15*b^17*x^6 + 451151981280*a
^14*b^18*x^7 + 789515967240*a^13*b^19*x^8 + 1140411952680*a^12*b^20*x^9 + 1
368494343216*a^11*b^21*x^10 + 1368494343216*a^10*b^22*x^11 + 1140411952680*
```

```
a^9*b^23*x^12 + 789515967240*a^8*b^24*x^13 + 451151981280*a^7*b^25*x^14 + 2
10537591264*a^6*b^26*x^15 + 78951596724*a^5*b^27*x^16 + 23221057860*a^4*b^2
8*x^17 + 5160235080*a^3*b^29*x^18 + 814773960*a^2*b^30*x^19)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**10/(b*x+a)**22,x)
```

```
[Out] Timed out
```

3.1228 $\int \frac{(a+bx)^5}{c+dx} \, dx$

Optimal. Leaf size=122

$$-\frac{(bc-ad)^5 \log(c+dx)}{d^6}+\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}+\frac{(a+bx)^5}{5d}$$

Rubi [A] time = 0.05, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^4}{d^5}-\frac{(a+bx)^2(bc-ad)^3}{2d^4}+\frac{(a+bx)^3(bc-ad)^2}{3d^3}-\frac{(a+bx)^4(bc-ad)}{4d^2}-\frac{(bc-ad)^5 \log(c+dx)}{d^6}+\frac{(a+bx)^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^4*x)/d^5 - ((b*c - a*d)^3*(a + b*x)^2)/(2*d^4) + ((b*c - a*d
)^2*(a + b*x)^3)/(3*d^3) - ((b*c - a*d)*(a + b*x)^4)/(4*d^2) + (a + b*x)^5/
(5*d) - ((b*c - a*d)^5*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{c+dx} \, dx = \int \left(\frac{b(bc-ad)^4}{d^5}-\frac{b(bc-ad)^3(a+bx)}{d^4}+\frac{b(bc-ad)^2(a+bx)^2}{d^3}-\frac{b(bc-ad)(a+bx)^3}{d^2}+\frac{b(a+bx}{d}\right.$$

$$=\frac{b(bc-ad)^4x}{d^5}-\frac{(bc-ad)^3(a+bx)^2}{2d^4}+\frac{(bc-ad)^2(a+bx)^3}{3d^3}-\frac{(bc-ad)(a+bx)^4}{4d^2}+\frac{(a+bx)^5}{5d}-($$

Mathematica [A] time = 0.07, size = 167, normalized size = 1.37

$$\frac{bdx\left(300a^4d^4+300a^3bd^3(dx-2c)+100a^2b^2d^2\left(6c^2-3cdx+2d^2x^2\right)+25ab^3d\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)+b^4\left(60c^4-30c^3dx+20c^2d^2x^2-15cd^3x^3+12d^4x^4\right)\right)-60(bc-ad)^5\log(c+dx)}{60d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x),x]
```

```
[Out] (b*d*x*(300*a^4*d^4 + 300*a^3*b*d^3*(-2*c + d*x) + 100*a^2*b^2*d^2*(6*c^2 -
 3*c*d*x + 2*d^2*x^2) + 25*a*b^3*d*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d
^3*x^3) + b^4*(60*c^4 - 30*c^3*d*x + 20*c^2*d^2*x^2 - 15*c*d^3*x^3 + 12*d^4
*x^4)) - 60*(b*c - a*d)^5*Log[c + d*x])/(60*d^6)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{c+dx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x), x]
```

fricas [B] time = 1.09, size = 259, normalized size = 2.12

$$\frac{12 b^5 d^5 x^5 - 15 \left(b^5 c d^4 - 5 a b^4 d^5\right) x^4 + 20 \left(b^5 c^2 d^3 - 5 a b^4 c d^4 + 10 a^2 b^3 d^5\right) x^3 - 30 \left(b^5 c^3 d^2 - 5 a b^4 c^2 d^3 + 10 a^2 b^3 c d^4 - 10 a^3 b^2 d^5\right) x^2 + 60 \left(b^5 c^4 d - 5 a b^4 c^3 d^2 + 10 a^2 b^3 c^2 d^3 - 10 a^3 b^2 c d^4 + 5 a^4 b d^5\right) x - 60 \left(b^5 c^5 - 5 a b^4 c^4 d + 10 a^2 b^3 c^3 d^2 - 10 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 - a^5 d^5\right) \log\left(d x + c\right)}{60 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b^5*d^5*x^5 - 15*(b^5*c*d^4 - 5*a*b^4*d^5)*x^4 + 20*(b^5*c^2*d^3 -
 5*a*b^4*c*d^4 + 10*a^2*b^3*d^5)*x^3 - 30*(b^5*c^3*d^2 - 5*a*b^4*c^2*d^3 +
10*a^2*b^3*c*d^4 - 10*a^3*b^2*d^5)*x^2 + 60*(b^5*c^4*d - 5*a*b^4*c^3*d^2 +
10*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x - 60*(b^5*c^5 - 5*a*
b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d
^5)*log(d*x + c))/d^6
```

giac [B] time = 1.26, size = 273, normalized size = 2.24

$$\frac{12 b^5 d^4 x^5 - 15 b^5 c d^3 x^4 + 75 a b^4 d^4 x^4 + 20 b^5 c^2 d^2 x^3 - 100 a b^4 c d^3 x^3 + 200 a^2 b^3 d^4 x^3 - 30 b^5 c^3 d x^2 + 150 a b^4 c^2 d^2 x^2 - 300 a^2 b^3 c d^3 x^2 + 300 a^3 b^2 d^4 x^2 + 60 b^5 c^4 x - 300 a b^4 c^3 d x + 600 a^2 b^3 c^2 d^2 x - 600 a^3 b^2 c d^3 x + 300 a^4 b d^4 x}{60 d^5} - \frac{\left(b^5 c^5 - 5 a b^4 c^4 d + 10 a^2 b^3 c^3 d^2 - 10 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 - a^5 d^5\right) \log\left({\left| d x + c \right|}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*b^5*c*d^3*x^4 + 75*a*b^4*d^4*x^4 + 20*b^5*c^2*d^2
*x^3 - 100*a*b^4*c*d^3*x^3 + 200*a^2*b^3*d^4*x^3 - 30*b^5*c^3*d*x^2 + 150*a
*b^4*c^2*d^2*x^2 - 300*a^2*b^3*c*d^3*x^2 + 300*a^3*b^2*d^4*x^2 + 60*b^5*c^4
*x - 300*a*b^4*c^3*d*x + 600*a^2*b^3*c^2*d^2*x - 600*a^3*b^2*c*d^3*x + 300*
a^4*b*d^4*x)/d^5 - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b
^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*log(abs(d*x + c))/d^6
```

maple [B] time = 0.00, size = 302, normalized size = 2.48

$$\frac{b^5 x^5}{5 d} + \frac{5 a b^4 x^4}{4 d} - \frac{b^5 c x^4}{4 d^2} + \frac{10 a^2 b^3 x^3}{3 d} - \frac{5 a b^4 c x^3}{3 d^2} + \frac{b^5 c^2 x^3}{3 d^3} + \frac{5 a^3 b^2 x^2}{d} - \frac{5 a^2 b^3 c x^2}{d^2} + \frac{5 a b^4 c^2 x^2}{2 d^3} - \frac{b^5 c^3 x^2}{2 d^4} + \frac{a^5 \ln\left(d x + c\right)}{d} - \frac{5 a^4 b c \ln\left(d x + c\right)}{d^2} + \frac{5 a^4 b x}{d} + \frac{10 a^3 b^2 c^2 \ln\left(d x + c\right)}{d^3} - \frac{10 a^3 b^2 c x}{d^2} - \frac{10 a^2 b^3 c^3 \ln\left(d x + c\right)}{d^4} + \frac{10 a^2 b^3 c^2 x}{d^3} + \frac{5 a b^4 c^4 \ln\left(d x + c\right)}{d^5} - \frac{5 a b^4 c^3 x}{d^4} - \frac{b^5 c^5 \ln\left(d x + c\right)}{d^6} + \frac{b^5 c^4 x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c),x)
```

```
[Out] 1/5*b^5/d*x^5+5/4*b^4/d*x^4*a-1/4*b^5/d^2*x^4*c+10/3*b^3/d*x^3*a^2-5/3*b^4/
d^2*x^3*a*c+1/3*b^5/d^3*x^3*c^2+5*b^2/d*x^2*a^3-5*b^3/d^2*x^2*a^2*c+5/2*b^4
/d^3*x^2*a*c^2-1/2*b^5/d^4*x^2*c^3+5*b/d*a^4*x-10*b^2/d^2*a^3*c*x+10*b^3/d^
3*a^2*c^2*x-5*b^4/d^4*a*c^3*x+b^5/d^5*c^4*x+1/d*ln(d*x+c)*a^5-5/d^2*ln(d*x+
c)*a^4*b*c+10/d^3*ln(d*x+c)*a^3*b^2*c^2-10/d^4*ln(d*x+c)*a^2*b^3*c^3+5/d^5*
ln(d*x+c)*a*b^4*c^4-1/d^6*ln(d*x+c)*b^5*c^5
```

maxima [B] time = 1.35, size = 258, normalized size = 2.11

$$\frac{12 b^5 d^4 x^5 - 15 \left(b^5 c d^3 - 5 a b^4 d^4\right) x^4 + 20 \left(b^5 c^2 d^2 - 5 a b^4 c d^3 + 10 a^2 b^3 d^4\right) x^3 - 30 \left(b^5 c^3 d - 5 a b^4 c^2 d^2 + 10 a^2 b^3 c d^3 - 10 a^3 b^2 d^4\right) x^2 + 60 \left(b^5 c^4 - 5 a b^4 c^3 d + 10 a^2 b^3 c^2 d^2 - 10 a^3 b^2 c d^3 + 5 a^4 b d^4\right) x}{60 d^5} - \frac{\left(b^5 c^5 - 5 a b^4 c^4 d + 10 a^2 b^3 c^3 d^2 - 10 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 - a^5 d^5\right) \log\left(d x + c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/60*(12*b^5*d^4*x^5 - 15*(b^5*c*d^3 - 5*a*b^4*d^4)*x^4 + 20*(b^5*c^2*d^2 -
 5*a*b^4*c*d^3 + 10*a^2*b^3*d^4)*x^3 - 30*(b^5*c^3*d - 5*a*b^4*c^2*d^2 + 10
*a^2*b^3*c*d^3 - 10*a^3*b^2*d^4)*x^2 + 60*(b^5*c^4 - 5*a*b^4*c^3*d + 10*a^2
*b^3*c^2*d^2 - 10*a^3*b^2*c*d^3 + 5*a^4*b*d^4)*x)/d^5 - (b^5*c^5 - 5*a*b^4*
c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
log(d*x + c)/d^6
```

mupad [B] time = 0.07, size = 280, normalized size = 2.30

$$x\left(\frac{5\,a^4\,b}{d}-\frac{c\left(\frac{10\,a^3\,b^2}{d}+\frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{d}\right)}{d}\right)+x^4\left(\frac{5\,a\,b^4}{4\,d}-\frac{b^5\,c}{4\,d^2}\right)+x^2\left(\frac{5\,a^3\,b^2}{d}+\frac{c\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{d}-\frac{10\,a^2\,b^3}{d}\right)}{2\,d}\right)-x^3\left(\frac{c\left(\frac{5\,a\,b^4}{d}-\frac{b^5\,c}{d^2}\right)}{3\,d}-\frac{10\,a^2\,b^3}{3\,d}\right)+\frac{b^5\,x^5}{5\,d}+\frac{\ln\left(c+d\,x\right)\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x),x)
```

```
[Out] x*((5*a^4*b)/d - (c*((10*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d
 - (10*a^2*b^3)/d))/d))/d) + x^4*((5*a*b^4)/(4*d) - (b^5*c)/(4*d^2)) + x^2*
((5*a^3*b^2)/d + (c*((c*((5*a*b^4)/d - (b^5*c)/d^2))/d - (10*a^2*b^3)/d))/(
2*d)) - x^3*((c*((5*a*b^4)/d - (b^5*c)/d^2))/(3*d) - (10*a^2*b^3)/(3*d)) +
(b^5*x^5)/(5*d) + (log(c + d*x)*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 1
0*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/d^6
```

sympy [B] time = 0.50, size = 209, normalized size = 1.71

$$\frac{b^5x^5}{5d}+x^4\left(\frac{5ab^4}{4d}-\frac{b^5c}{4d^2}\right)+x^3\left(\frac{10a^2b^3}{3d}-\frac{5ab^4c}{3d^2}+\frac{b^5c^2}{3d^3}\right)+x^2\left(\frac{5a^3b^2}{d}-\frac{5a^2b^3c}{d^2}+\frac{5ab^4c^2}{2d^3}-\frac{b^5c^3}{2d^4}\right)+x\left(\frac{5a^4b}{d}-\frac{10a^3b^2c}{d^2}+\frac{10a^2b^3c^2}{d^3}-\frac{5ab^4c^3}{d^4}+\frac{b^5c^4}{d^5}\right)+\frac{(ad-bc)^5\log(c+dx)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c),x)
```

```
[Out] b**5*x**5/(5*d) + x**4*(5*a*b**4/(4*d) - b**5*c/(4*d**2)) + x**3*(10*a**2*b
**3/(3*d) - 5*a*b**4*c/(3*d**2) + b**5*c**2/(3*d**3)) + x**2*(5*a**3*b**2/d
 - 5*a**2*b**3*c/d**2 + 5*a*b**4*c**2/(2*d**3) - b**5*c**3/(2*d**4)) + x*(5
*a**4*b/d - 10*a**3*b**2*c/d**2 + 10*a**2*b**3*c**2/d**3 - 5*a*b**4*c**3/d*
*4 + b**5*c**4/d**5) + (a*d - b*c)**5*log(c + d*x)/d**6
```

3.1229 $\int \frac{(a+bx)^4}{c+dx}\,dx$

Optimal. Leaf size=98

$$\frac{(bc-ad)^4\log(c+dx)}{d^5}-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(a+bx)^4}{4d}$$

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)^3}{d^4}+\frac{(a+bx)^2(bc-ad)^2}{2d^3}-\frac{(a+bx)^3(bc-ad)}{3d^2}+\frac{(bc-ad)^4\log(c+dx)}{d^5}+\frac{(a+bx)^4}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)^3*x)/d^4) + ((b*c - a*d)^2*(a + b*x)^2)/(2*d^3) - ((b*c -
a*d)*(a + b*x)^3)/(3*d^2) + (a + b*x)^4/(4*d) + ((b*c - a*d)^4*Log[c + d*x]
)/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)^3}{d^4}+\frac{b(bc-ad)^2(a+bx)}{d^3}-\frac{b(bc-ad)(a+bx)^2}{d^2}+\frac{b(a+bx)^3}{d}+\frac{(-bc+ad)^4}{d^4(c+dx)}\right)\\ &= -\frac{b(bc-ad)^3x}{d^4}+\frac{(bc-ad)^2(a+bx)^2}{2d^3}-\frac{(bc-ad)(a+bx)^3}{3d^2}+\frac{(a+bx)^4}{4d}+\frac{(bc-ad)^4\log(c+\end{aligned}$$

Mathematica [A] time = 0.04, size = 115, normalized size = 1.17

$$\frac{bdx\left(48a^3d^3+36a^2bd^2(dx-2c)+8ab^2d\left(6c^2-3cdx+2d^2x^2\right)+b^3\left(-12c^3+6c^2dx-4cd^2x^2+3d^3x^3\right)\right)+12(bc-ad)^4\log(c+dx)}{12d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x),x]
```

```
[Out] (b*d*x*(48*a^3*d^3 + 36*a^2*b*d^2*(-2*c + d*x) + 8*a*b^2*d*(6*c^2 - 3*c*d*x
 + 2*d^2*x^2) + b^3*(-12*c^3 + 6*c^2*d*x - 4*c*d^2*x^2 + 3*d^3*x^3)) + 12*(
b*c - a*d)^4*Log[c + d*x])/(12*d^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x), x]
```

fricas [A] time = 1.19, size = 179, normalized size = 1.83

$$\frac{3\,b^4d^4x^4-4\left(b^4cd^3-4\,ab^3d^4\right)x^3+6\left(b^4c^2d^2-4\,ab^3cd^3+6\,a^2b^2d^4\right)x^2-12\left(b^4c^3d-4\,ab^3c^2d^2+6\,a^2b^2cd^3-4\,a^3bd^4\right)x+12\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(dx+c\right)}{12\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*d^4*x^4 - 4*(b^4*c*d^3 - 4*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 4*
a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 - 12*(b^4*c^3*d - 4*a*b^3*c^2*d^2 + 6*a^2*
b^2*c*d^3 - 4*a^3*b*d^4)*x + 12*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c))/d^5
```

giac [A] time = 1.23, size = 184, normalized size = 1.88

$$\frac{3\,b^4d^3x^4-4\,b^4cd^2x^3+16\,ab^3d^3x^3+6\,b^4c^2dx^2-24\,ab^3cd^2x^2+36\,a^2b^2d^3x^2-12\,b^4c^3x+48\,ab^3c^2dx-72\,a^2b^2cd^2x+48\,a^3bd^3x}{12\,d^4}+\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(|dx+c|\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*b^4*c*d^2*x^3 + 16*a*b^3*d^3*x^3 + 6*b^4*c^2*d*x^2
- 24*a*b^3*c*d^2*x^2 + 36*a^2*b^2*d^3*x^2 - 12*b^4*c^3*x + 48*a*b^3*c^2*d*x
 - 72*a^2*b^2*c*d^2*x + 48*a^3*b*d^3*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(d*x + c))/d^5
```

maple [B] time = 0.00, size = 209, normalized size = 2.13

$$\frac{b^4x^4}{4d}+\frac{4a\,b^3x^3}{3d}-\frac{b^4c\,x^3}{3d^2}+\frac{3a^2b^2x^2}{d}-\frac{2a\,b^3c\,x^2}{d^2}+\frac{b^4c^2x^2}{2d^3}+\frac{a^4\ln\left(dx+c\right)}{d}-\frac{4a^3bc\ln\left(dx+c\right)}{d^2}+\frac{4a^3bx}{d}+\frac{6a^2b^2c^2\ln\left(dx+c\right)}{d^3}-\frac{6a^2b^2cx}{d^2}-\frac{4a\,b^3c^3\ln\left(dx+c\right)}{d^4}+\frac{4a\,b^3c^2x}{d^3}+\frac{b^4c^4\ln\left(dx+c\right)}{d^5}-\frac{b^4c^3x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c),x)
```

```
[Out] 1/4*b^4/d*x^4+4/3*b^3/d*x^3*a-1/3*b^4/d^2*x^3*c+3*b^2/d*x^2*a^2-2*b^3/d^2*x
^2*a*c+1/2*b^4/d^3*x^2*c^2+4*b/d*a^3*x-6*b^2/d^2*a^2*c*x+4*b^3/d^3*a*c^2*x-
b^4/d^4*c^3*x+1/d*ln(d*x+c)*a^4-4/d^2*ln(d*x+c)*a^3*b*c+6/d^3*ln(d*x+c)*a^2
*b^2*c^2-4/d^4*ln(d*x+c)*a*b^3*c^3+1/d^5*ln(d*x+c)*b^4*c^4
```

maxima [A] time = 1.41, size = 177, normalized size = 1.81

$$\frac{3\,b^4d^3x^4-4\left(b^4cd^2-4\,ab^3d^3\right)x^3+6\left(b^4c^2d-4\,ab^3cd^2+6\,a^2b^2d^3\right)x^2-12\left(b^4c^3-4\,ab^3c^2d+6\,a^2b^2cd^2-4\,a^3bd^3\right)x}{12\,d^4}+\frac{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\log\left(dx+c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/12*(3*b^4*d^3*x^4 - 4*(b^4*c*d^2 - 4*a*b^3*d^3)*x^3 + 6*(b^4*c^2*d - 4*a*
b^3*c*d^2 + 6*a^2*b^2*d^3)*x^2 - 12*(b^4*c^3 - 4*a*b^3*c^2*d + 6*a^2*b^2*c*
d^2 - 4*a^3*b*d^3)*x)/d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*log(d*x + c)/d^5
```

mupad [B] time = 0.22, size = 189, normalized size = 1.93

$$x^3\left(\frac{4\,a\,b^3}{3\,d}-\frac{b^4\,c}{3\,d^2}\right)+x\left(\frac{4\,a^3\,b}{d}+\frac{c\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{d}-\frac{6\,a^2\,b^2}{d}\right)}{d}\right)-x^2\left(\frac{c\left(\frac{4\,a\,b^3}{d}-\frac{b^4\,c}{d^2}\right)}{2\,d}-\frac{3\,a^2\,b^2}{d}\right)+\frac{\ln\left(c+d\,x\right)\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{b^4\,x^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x),x)
```

```
[Out] x^3*((4*a*b^3)/(3*d) - (b^4*c)/(3*d^2)) + x*((4*a^3*b)/d + (c*((c*((4*a*b^3
)/d - (b^4*c)/d^2))/d - (6*a^2*b^2)/d))/d) - x^2*((c*((4*a*b^3)/d - (b^4*c)
/d^2))/(2*d) - (3*a^2*b^2)/d) + (log(c + d*x)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^
2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/d^5 + (b^4*x^4)/(4*d)
```

sympy [A] time = 0.39, size = 136, normalized size = 1.39

$$\frac{b^4x^4}{4d} + x^3\left(\frac{4ab^3}{3d} - \frac{b^4c}{3d^2}\right) + x^2\left(\frac{3a^2b^2}{d} - \frac{2ab^3c}{d^2} + \frac{b^4c^2}{2d^3}\right) + x\left(\frac{4a^3b}{d} - \frac{6a^2b^2c}{d^2} + \frac{4ab^3c^2}{d^3} - \frac{b^4c^3}{d^4}\right) + \frac{(ad-bc)^4\log(c+dx)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c),x)
```

```
[Out] b**4*x**4/(4*d) + x**3*(4*a*b**3/(3*d) - b**4*c/(3*d**2)) + x**2*(3*a**2*b*
*2/d - 2*a*b**3*c/d**2 + b**4*c**2/(2*d**3)) + x*(4*a**3*b/d - 6*a**2*b**2*
c/d**2 + 4*a*b**3*c**2/d**3 - b**4*c**3/d**4) + (a*d - b*c)**4*log(c + d*x)
/d**5
```

3.1230 $\int \frac{(a+bx)^3}{c+dx} \, dx$

Optimal. Leaf size=74

$$-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}$$

Rubi [A] time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{(a+bx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)^2)/(2*d^2) + (a + b*x)^3/(
3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{c+dx}\,dx &= \int \left(\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+bx)}{d^2}+\frac{b(a+bx)^2}{d}+\frac{(-bc+ad)^3}{d^3(c+dx)}\right)dx\\ &= \frac{b(bc-ad)^2x}{d^3}-\frac{(bc-ad)(a+bx)^2}{2d^2}+\frac{(a+bx)^3}{3d}-\frac{(bc-ad)^3\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 74, normalized size = 1.00

$$\frac{bdx\left(18a^2d^2+9abd(dx-2c)+b^2\left(6c^2-3cdx+2d^2x^2\right)\right)-6(bc-ad)^3\log(c+dx)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x),x]
```

```
[Out] (b*d*x*(18*a^2*d^2 + 9*a*b*d*(-2*c + d*x) + b^2*(6*c^2 - 3*c*d*x + 2*d^2*x^
2)) - 6*(b*c - a*d)^3*Log[c + d*x])/(6*d^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x), x]
```

fricas [A] time = 0.67, size = 115, normalized size = 1.55

$$\frac{2\,b^3d^3x^3-3\left(b^3cd^2-3\,ab^2d^3\right)x^2+6\left(b^3c^2d-3\,ab^2cd^2+3\,a^2bd^3\right)x-6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(dx+c\right)}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*d^3*x^3 - 3*(b^3*c*d^2 - 3*a*b^2*d^3)*x^2 + 6*(b^3*c^2*d - 3*a*b
^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(d*x + c))/d^4
```

giac [A] time = 1.20, size = 116, normalized size = 1.57

$$\frac{2\,b^3d^2x^3-3\,b^3cdx^2+9\,ab^2d^2x^2+6\,b^3c^2x-18\,ab^2cdx+18\,a^2bd^2x}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(|dx+c|\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*b^3*c*d*x^2 + 9*a*b^2*d^2*x^2 + 6*b^3*c^2*x - 18*a*b
^2*c*d*x + 18*a^2*b*d^2*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*log(abs(d*x + c))/d^4
```

maple [A] time = 0.00, size = 133, normalized size = 1.80

$$\frac{b^3x^3}{3d}+\frac{3a\,b^2x^2}{2d}-\frac{b^3c\,x^2}{2d^2}+\frac{a^3\ln\left(dx+c\right)}{d}-\frac{3a^2bc\ln\left(dx+c\right)}{d^2}+\frac{3a^2bx}{d}+\frac{3a\,b^2c^2\ln\left(dx+c\right)}{d^3}-\frac{3a\,b^2cx}{d^2}-\frac{b^3c^3\ln\left(dx+c\right)}{d^4}+\frac{b^3c^2x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/3*b^3/d*x^3+3/2*b^2/d*x^2*a-1/2*b^3/d^2*x^2*c+3*b/d*a^2*x-3*b^2/d^2*a*c*x
+b^3/d^3*c^2*x+1/d*ln(d*x+c)*a^3-3/d^2*ln(d*x+c)*a^2*b*c+3/d^3*ln(d*x+c)*a*
b^2*c^2-1/d^4*ln(d*x+c)*b^3*c^3
```

maxima [A] time = 1.30, size = 114, normalized size = 1.54

$$\frac{2\,b^3d^2x^3-3\left(b^3cd-3\,ab^2d^2\right)x^2+6\left(b^3c^2-3\,ab^2cd+3\,a^2bd^2\right)x}{6\,d^3}-\frac{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\log\left(dx+c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^3*d^2*x^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*x^2 + 6*(b^3*c^2 - 3*a*b^2*c
*d + 3*a^2*b*d^2)*x)/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(d*x + c)/d^4
```

mupad [B] time = 0.07, size = 118, normalized size = 1.59

$$x^2\left(\frac{3\,a\,b^2}{2\,d}-\frac{b^3\,c}{2\,d^2}\right)+x\left(\frac{3\,a^2\,b}{d}-\frac{c\left(\frac{3\,a\,b^2}{d}-\frac{b^3\,c}{d^2}\right)}{d}\right)+\frac{\ln\left(c+d\,x\right)\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{d^4}+\frac{b^3\,x^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x),x)
```

```
[Out] x^2*((3*a*b^2)/(2*d) - (b^3*c)/(2*d^2)) + x*((3*a^2*b)/d - (c*((3*a*b^2)/d
- (b^3*c)/d^2))/d) + (log(c + d*x)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2))/d^4 + (b^3*x^3)/(3*d)
```

sympy [A] time = 0.30, size = 83, normalized size = 1.12

$$\frac{b^3x^3}{3d} + x^2\left(\frac{3ab^2}{2d} - \frac{b^3c}{2d^2}\right) + x\left(\frac{3a^2b}{d} - \frac{3ab^2c}{d^2} + \frac{b^3c^2}{d^3}\right) + \frac{(ad-bc)^3\log(c+dx)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c),x)
```

```
[Out] b**3*x**3/(3*d) + x**2*(3*a*b**2/(2*d) - b**3*c/(2*d**2)) + x*(3*a**2*b/d -
 3*a*b**2*c/d**2 + b**3*c**2/d**3) + (a*d - b*c)**3*log(c + d*x)/d**4
```

3.1231 $\int \frac{(a+bx)^2}{c+dx} dx$

Optimal. Leaf size=50

$$\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}$$

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{bx(bc-ad)}{d^2}+\frac{(bc-ad)^2\log(c+dx)}{d^3}+\frac{(a+bx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x),x]
```

```
[Out] -((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + ((b*c - a*d)^2*Log[c + d*x])
/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{c+dx}\,dx &= \int \left(-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}+\frac{(-bc+ad)^2}{d^2(c+dx)}\right)dx\\ &= -\frac{b(bc-ad)x}{d^2}+\frac{(a+bx)^2}{2d}+\frac{(bc-ad)^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 0.86

$$\frac{bdx(4ad-2bc+bdx)+2(bc-ad)^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x),x]
```

```
[Out] (b*d*x*(-2*b*c + 4*a*d + b*d*x) + 2*(b*c - a*d)^2*Log[c + d*x])/(2*d^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x), x]
```

fricas [A] time = 1.48, size = 62, normalized size = 1.24

$$\frac{b^2d^2x^2 - 2\left(b^2cd - 2\,abd^2\right)x + 2\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx + c\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 - 2*(b^2*c*d - 2*a*b*d^2)*x + 2*(b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*log(d*x + c))/d^3
```

giac [A] time = 1.21, size = 60, normalized size = 1.20

$$\frac{b^2dx^2 - 2\,b^2cx + 4\,abdx}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(|dx + c|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*b^2*c*x + 4*a*b*d*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(abs(d*x + c))/d^3
```

maple [A] time = 0.00, size = 74, normalized size = 1.48

$$\frac{b^2x^2}{2d} + \frac{a^2\ln\left(dx + c\right)}{d} - \frac{2abc\ln\left(dx + c\right)}{d^2} + \frac{2abx}{d} + \frac{b^2c^2\ln\left(dx + c\right)}{d^3} - \frac{b^2cx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/2*b^2/d*x^2+2*b/d*a*x-b^2/d^2*x*c+1/d*ln(d*x+c)*a^2-2/d^2*ln(d*x+c)*a*b*c
+1/d^3*ln(d*x+c)*b^2*c^2
```

maxima [A] time = 1.35, size = 60, normalized size = 1.20

$$\frac{b^2dx^2 - 2\left(b^2c - 2\,abd\right)x}{2\,d^2} + \frac{\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*d*x^2 - 2*(b^2*c - 2*a*b*d)*x)/d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^
2)*log(d*x + c)/d^3
```

mupad [B] time = 0.22, size = 62, normalized size = 1.24

$$\frac{\ln\left(c + d\,x\right)\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{d^3} - x\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{b^2\,x^2}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x),x)
```

```
[Out] (log(c + d*x)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - x*((b^2*c)/d^2 - (2*a*
b)/d) + (b^2*x^2)/(2*d)
```

sympy [A] time = 0.22, size = 44, normalized size = 0.88

$$\frac{b^2x^2}{2d} + x\left(\frac{2ab}{d} - \frac{b^2c}{d^2}\right) + \frac{(ad - bc)^2\log\left(c + dx\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c),x)
```

```
[Out] b**2*x**2/(2*d) + x*(2*a*b/d - b**2*c/d**2) + (a*d - b*c)**2*log(c + d*x)/d
**3
```

3.1232 $\int \frac{a+bx}{c+dx}\,dx$

Optimal. Leaf size=26

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{c+dx}\,dx &= \int \left(\frac{b}{d}+\frac{-bc+ad}{d(c+dx)}\right)dx\\ &= \frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 0.96

$$\frac{(ad-bc)\log(c+dx)}{d^2}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x),x]
```

```
[Out] (b*x)/d + ((-(b*c) + a*d)*Log[c + d*x])/d^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x), x]
```

fricas [A] time = 0.93, size = 25, normalized size = 0.96

$$\frac{bdx-(bc-ad)\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] (b*d*x - (b*c - a*d)*log(d*x + c))/d^2
```

giac [A] time = 1.20, size = 27, normalized size = 1.04

$$\frac{bx}{d} - \frac{(bc - ad)\log(|dx + c|)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="giac")
[Out] b*x/d - (b*c - a*d)*log(abs(d*x + c))/d^2
```

maple [A] time = 0.00, size = 32, normalized size = 1.23

$$\frac{a\ln(dx + c)}{d} - \frac{bc\ln(dx + c)}{d^2} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c),x)
[Out] b*x/d+1/d*ln(d*x+c)*a-1/d^2*ln(d*x+c)*b*c
```

maxima [A] time = 1.32, size = 26, normalized size = 1.00

$$\frac{bx}{d} - \frac{(bc - ad)\log(dx + c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] b*x/d - (b*c - a*d)*log(d*x + c)/d^2
```

mupad [B] time = 0.20, size = 25, normalized size = 0.96

$$\frac{\ln(c + d\,x)\,(a\,d - b\,c)}{d^2} + \frac{b\,x}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x),x)
[Out] (log(c + d*x)*(a*d - b*c))/d^2 + (b*x)/d
```

sympy [A] time = 0.15, size = 20, normalized size = 0.77

$$\frac{bx}{d} + \frac{(ad - bc)\log(c + dx)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c),x)
[Out] b*x/d + (a*d - b*c)*log(c + d*x)/d**2
```

3.1233 $\int \frac{1}{c+dx}\,dx$

Optimal. Leaf size=10

$$\frac{\log(c+dx)}{d}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{c+dx}\,dx = \frac{\log(c+dx)}{d}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-1),x]
```

```
[Out] Log[c + d*x]/d
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-1),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-1), x]
```

fricas [A] time = 0.90, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="fricas")
```

```
[Out] log(d*x + c)/d
```

giac [A] time = 1.26, size = 11, normalized size = 1.10

$$\frac{\log(|dx+c|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="giac")
[Out] log(abs(d*x + c))/d
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c),x)
[Out] ln(d*x+c)/d
```

maxima [A] time = 1.30, size = 10, normalized size = 1.00

$$\frac{\log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x, algorithm="maxima")
[Out] log(d*x + c)/d
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(c+d\,x)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x),x)
[Out] log(c + d*x)/d
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c),x)
[Out] log(c + d*x)/d
```

3.1234 $\int \frac{1}{(a+bx)(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {36, 31}

$$\frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] Log[a + b*x]/(b*c - a*d) - Log[c + d*x]/(b*c - a*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)}\,dx &= \frac{b\int \frac{1}{a+bx}\,dx}{bc-ad}-\frac{d\int \frac{1}{c+dx}\,dx}{bc-ad}\\ &= \frac{\log(a+bx)}{bc-ad}-\frac{\log(c+dx)}{bc-ad}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.72

$$\frac{\log(a+bx)-\log(c+dx)}{bc-ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] (Log[a + b*x] - Log[c + d*x])/(b*c - a*d)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)), x]
```

fricas [A] time = 1.33, size = 26, normalized size = 0.72

$$\frac{\log(bx+a)-\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] (log(b*x + a) - log(d*x + c))/(b*c - a*d)
```

giac [A] time = 1.24, size = 46, normalized size = 1.28

$$\frac{b\log(|bx+a|)}{b^2c-abd}-\frac{d\log(|dx+c|)}{bcd-ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/(b^2*c - a*b*d) - d*log(abs(d*x + c))/(b*c*d - a*d^2)
```

maple [A] time = 0.01, size = 37, normalized size = 1.03

$$-\frac{\ln(bx+a)}{ad-bc}+\frac{\ln(dx+c)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c),x)
[Out] 1/(a*d-b*c)*ln(d*x+c)-1/(a*d-b*c)*ln(b*x+a)
```

maxima [A] time = 1.36, size = 36, normalized size = 1.00

$$\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)
```

mupad [B] time = 0.26, size = 25, normalized size = 0.69

$$\frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{a\,d-b\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)),x)
[Out] log((c + d*x)/(a + b*x))/(a*d - b*c)
```

sympy [B] time = 0.33, size = 128, normalized size = 3.56

$$\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c),x)
[Out] log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c**2/(
a*d - b*c) + b*c)/(2*b*d))/(a*d - b*c) - log(x + (a**2*d**2/(a*d - b*c) - 2
*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*d))/(a*d - b
*c)
```

3.1235 $\int \frac{1}{(a+bx)^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right)dx\\ &= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)), x]
```

fricas [**A**] time = 1.27, size = 93, normalized size = 1.63

$$-\frac{bc - ad + (bdx + ad)\log(bx + a) - (bdx + ad)\log(dx + c)}{ab^2c^2 - 2\,a^2bcd + a^3d^2 + \left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

giac [**A**] time = 1.32, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^3c^2 - 2\,ab^2cd + a^2bd^2} - \frac{b}{\left(b^2c - abd\right)(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

maple [**A**] time = 0.01, size = 57, normalized size = 1.00

$$-\frac{d\ln(bx + a)}{(ad - bc)^2} + \frac{d\ln(dx + c)}{(ad - bc)^2} + \frac{1}{(ad - bc)(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
```

```
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

maxima [**A**] time = 1.36, size = 92, normalized size = 1.61

$$-\frac{d\log(bx + a)}{b^2c^2 - 2\,abcd + a^2d^2} + \frac{d\log(dx + c)}{b^2c^2 - 2\,abcd + a^2d^2} - \frac{1}{abc - a^2d + \left(b^2c - abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

mupad [**B**] time = 0.14, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d - b\,c)\,(a + b\,x)} - \frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d - b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

sympy [**B**] time = 0.68, size = 233, normalized size = 4.09

$$\frac{d\log\left(x + \frac{-\frac{a^3d^4}{(ad-bc)^2} + \frac{3a^2bcd^3}{(ad-bc)^2} - \frac{3ab^2c^2d^2}{(ad-bc)^2} + ad^2 + \frac{b^3c^3d}{(ad-bc)^2} + bcd}{2bd^2}\right)}{(ad - bc)^2} - \frac{d\log\left(x + \frac{\frac{a^3d^4}{(ad-bc)^2} - \frac{3a^2bcd^3}{(ad-bc)^2} + \frac{3ab^2c^2d^2}{(ad-bc)^2} + ad^2 - \frac{b^3c^3d}{(ad-bc)^2} + bcd}{2bd^2}\right)}{(ad - bc)^2} + \frac{1}{a^2d - abc + x\left(abd - b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

3.1236 $\int \frac{1}{(a+bx)^3(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}+\frac{d}{(a+bx)(bc-ad)^2}-\frac{1}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] -1/(2*(b*c - a*d)*(a + b*x)^2) + d/((b*c - a*d)^2*(a + b*x)) + (d^2*Log[a +
 b*x])/(b*c - a*d)^3 - (d^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)}\,dx &= \int \left(\frac{b}{(bc-ad)(a+bx)^3}-\frac{bd}{(bc-ad)^2(a+bx)^2}+\frac{bd^2}{(bc-ad)^3(a+bx)}-\frac{d^3}{(bc-ad)^3(c+dx)}\right)dx\\ &= -\frac{1}{2(bc-ad)(a+bx)^2}+\frac{d}{(bc-ad)^2(a+bx)}+\frac{d^2\log(a+bx)}{(bc-ad)^3}-\frac{d^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 67, normalized size = 0.82

$$\frac{\frac{(bc-ad)(3ad-bc+2bdx)}{(a+bx)^2}+2d^2\log(a+bx)-2d^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] (((b*c - a*d)*(-(b*c) + 3*a*d + 2*b*d*x))/(a + b*x)^2 + 2*d^2*Log[a + b*x]
- 2*d^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)), x]
```

fricas [B] time = 1.36, size = 242, normalized size = 2.95

$$-\frac{b^2c^2 - 4\,abcd + 3\,a^2d^2 - 2\,\left(b^2cd - abd^2\right)x - 2\,\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(bx + a\right) + 2\,\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\log\left(dx + c\right)}{2\,\left(a^2b^3c^3 - 3\,a^3b^2c^2d + 3\,a^4bcd^2 - a^5d^3 + \left(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3\right)x^2 + 2\,\left(ab^4c^3 - 3\,a^2b^3c^2d + 3\,a^3b^2cd^2 - a^4bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x - 2*(b^2*d^
2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*log(d*x + c))/(a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
^5*d^3 + (b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^2 + 2*
(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*x)
```

giac [B] time = 1.29, size = 165, normalized size = 2.01

$$\frac{bd^2\log\left(|bx + a|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{d^3\log\left(|dx + c|\right)}{b^3c^3d - 3\,ab^2c^2d^2 + 3\,a^2bcd^3 - a^3d^4} - \frac{b^2c^2 - 4\,abcd + 3\,a^2d^2 - 2\,\left(b^2cd - abd^2\right)x}{2\,(bc - ad)^3(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] b*d^2*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*
d^3) - d^3*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) - 1/2*(b^2*c^2 - 4*a*b*c*d + 3*a^2*d^2 - 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(b*x + a)^2)
```

maple [A] time = 0.01, size = 81, normalized size = 0.99

$$-\frac{d^2\ln\left(bx + a\right)}{\left(ad - bc\right)^3} + \frac{d^2\ln\left(dx + c\right)}{\left(ad - bc\right)^3} + \frac{d}{\left(ad - bc\right)^2\left(bx + a\right)} + \frac{1}{2\left(ad - bc\right)\left(bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c),x)
```

```
[Out] d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(b*x+a)-d^2
/(a*d-b*c)^3*ln(b*x+a)
```

maxima [B] time = 1.41, size = 202, normalized size = 2.46

$$\frac{d^2\log\left(bx + a\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} - \frac{d^2\log\left(dx + c\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} + \frac{2\,bdx - bc + 3\,ad}{2\,\left(a^2b^2c^2 - 2\,a^3bcd + a^4d^2 + \left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)x^2 + 2\,\left(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] d^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - d^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x - b*c + 3*a*d)/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b
^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

mupad [B] time = 0.16, size = 182, normalized size = 2.22

$$\frac{\frac{3\,a\,d-b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{a^2+2\,a\,b\,x+b^2\,x^2}-\frac{2\,d^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)),x)
```

```
[Out] ((3*a*d - b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x)/(a^2*d^2 + b^2
*c^2 - 2*a*b*c*d))/(a^2 + b^2*x^2 + 2*a*b*x) - (2*d^2*atanh((a^3*d^3 + b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

sympy [B] time = 1.06, size = 381, normalized size = 4.65

$$\frac{d^2\log\left(x+\frac{-\frac{a^4d^6}{(ad-bc)^3}+\frac{4a^3bcd^5}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4}{(ad-bc)^3}+\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3-\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}-\frac{d^2\log\left(x+\frac{\frac{a^4d^6}{(ad-bc)^3}-\frac{4a^3bcd^5}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4}{(ad-bc)^3}-\frac{4ab^3c^3d^3}{(ad-bc)^3}+ad^3+\frac{b^4c^4d^2}{(ad-bc)^3}+bcd^2}{2bd^3}\right)}{(ad-bc)^3}+\frac{3ad-bc+2bdx}{2a^4d^2-4a^3bcd+2a^2b^2c^2+x^2\left(2a^2b^2d^2-4ab^3cd+2b^4c^2\right)+x\left(4a^3bd^2-8a^2b^2cd+4ab^3c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c),x)
```

```
[Out] d**2*log(x + (-a**4*d**6/(a*d - b*c)**3 + 4*a**3*b*c*d**5/(a*d - b*c)**3 -
6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3/(a*d - b*c)**3 +
a*d**3 - b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b*c)*
*3 - d**2*log(x + (a**4*d**6/(a*d - b*c)**3 - 4*a**3*b*c*d**5/(a*d - b*c)**
3 + 6*a**2*b**2*c**2*d**4/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3/(a*d - b*c)**
3 + a*d**3 + b**4*c**4*d**2/(a*d - b*c)**3 + b*c*d**2)/(2*b*d**3))/(a*d - b
*c)**3 + (3*a*d - b*c + 2*b*d*x)/(2*a**4*d**2 - 4*a**3*b*c*d + 2*a**2*b**2*
c**2 + x**2*(2*a**2*b**2*d**2 - 4*a*b**3*c*d + 2*b**4*c**2) + x*(4*a**3*b*d
**2 - 8*a**2*b**2*c*d + 4*a*b**3*c**2))
```

3.1237 $\int \frac{(a+bx)^5}{(c+dx)^2} \, dx$

Optimal. Leaf size=130

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+}{4d^6}$$

Rubi [A] time = 0.14, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^3(bc-ad)}{3d^6}+\frac{5b^3(c+dx)^2(bc-ad)^2}{d^6}-\frac{10b^2x(bc-ad)^3}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b(bc-ad)^4\log(c+dx)}{d^6}+\frac{b^5(c+dx)^4}{4d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (-10*b^2*(b*c - a*d)^3*x)/d^5 + (b*c - a*d)^5/(d^6*(c + d*x)) + (5*b^3*(b*c
 - a*d)^2*(c + d*x)^2)/d^6 - (5*b^4*(b*c - a*d)*(c + d*x)^3)/(3*d^6) + (b^5
*(c + d*x)^4)/(4*d^6) + (5*b*(b*c - a*d)^4*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^2}\,dx &= \int \left(-\frac{10b^2(bc-ad)^3}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^2}+\frac{5b(bc-ad)^4}{d^5(c+dx)}+\frac{10b^3(bc-ad)^2(c+dx)}{d^5}-\frac{5b^4(bc-ad)(c+}{d^5}\right.\\ &= -\frac{10b^2(bc-ad)^3x}{d^5}+\frac{(bc-ad)^5}{d^6(c+dx)}+\frac{5b^3(bc-ad)^2(c+dx)^2}{d^6}-\frac{5b^4(bc-ad)(c+dx)^3}{3d^6}+\frac{b^5(c+dx)}{4d^6}\end{aligned}$$

Mathematica [A] time = 0.08, size = 228, normalized size = 1.75

$$\frac{-12a^5d^5+60a^4bcd^4+120a^3b^2d^3\left(-c^2+cdx+d^2x^2\right)+60a^2b^3d^2\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)+20ab^4d\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)+60b(c+dx)(bc-ad)^4\log(c+dx)+b^5\left(12c^5-48c^4dx-30c^3d^2x^2+10c^2d^3x^3-5cd^4x^4+3d^5x^5\right)}{12d^6(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] (60*a^4*b*c*d^4 - 12*a^5*d^5 + 120*a^3*b^2*d^3*(-c^2 + c*d*x + d^2*x^2) + 6
0*a^2*b^3*d^2*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + 20*a*b^4*d*(-3*
c^4 + 9*c^3*d*x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) + b^5*(12*c^5 - 48
*c^4*d*x - 30*c^3*d^2*x^2 + 10*c^2*d^3*x^3 - 5*c*d^4*x^4 + 3*d^5*x^5) + 60*
b*(b*c - a*d)^4*(c + d*x)*Log[c + d*x])/(12*d^6*(c + d*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^2, x]
```

fricas [**B**] time = 1.21, size = 373, normalized size = 2.87

$$\frac{3\,b^{5}d^{5}x^{5} + 12\,b^{5}c^{5} - 60\,ab^{4}c^{4}d + 120\,a^{2}b^{3}c^{3}d^{2} - 120\,a^{3}b^{2}c^{2}d^{3} + 60\,a^{4}bcd^{4} - 12\,a^{5}d^{5} - 5\left(b^{5}cd^{4} - 4\,ab^{4}d^{5}\right)x^{4} + 10\left(b^{5}c^{2}d^{3} - 4\,ab^{4}cd^{4} + 6\,a^{2}b^{3}d^{5}\right)x^{3} - 30\left(b^{5}c^{3}d^{2} - 4\,ab^{4}c^{2}d^{3} + 6\,a^{2}b^{3}cd^{4} - 4\,a^{3}b^{2}d^{5}\right)x^{2} - 12\left(4\,b^{5}c^{4}d - 15\,ab^{4}c^{3}d^{2} + 20\,a^{2}b^{3}c^{2}d^{3} - 10\,a^{3}b^{2}cd^{4}\right)x + 60\left(b^{5}c^{5} - 4\,ab^{4}c^{4}d + 6\,a^{2}b^{3}c^{3}d^{2} - 4\,a^{3}b^{2}c^{2}d^{3} + a^{4}bcd^{4} + \left(b^{5}c^{4}d - 4\,ab^{4}c^{3}d^{2} + 6\,a^{2}b^{3}c^{2}d^{3} - 4\,a^{3}b^{2}cd^{4} + a^{4}bd^{5}\right)x\right)\log\left(dx + c\right)}{12\left(d^{7}x + cd^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^5*d^5*x^5 + 12*b^5*c^5 - 60*a*b^4*c^4*d + 120*a^2*b^3*c^3*d^2 - 1
20*a^3*b^2*c^2*d^3 + 60*a^4*b*c*d^4 - 12*a^5*d^5 - 5*(b^5*c*d^4 - 4*a*b^4*d
^5)*x^4 + 10*(b^5*c^2*d^3 - 4*a*b^4*c*d^4 + 6*a^2*b^3*d^5)*x^3 - 30*(b^5*c^
3*d^2 - 4*a*b^4*c^2*d^3 + 6*a^2*b^3*c*d^4 - 4*a^3*b^2*d^5)*x^2 - 12*(4*b^5*
c^4*d - 15*a*b^4*c^3*d^2 + 20*a^2*b^3*c^2*d^3 - 10*a^3*b^2*c*d^4)*x + 60*(b
^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*c*d^
4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^
4*b*d^5)*x)*log(d*x + c))/(d^7*x + c*d^6)
```

giac [**B**] time = 1.27, size = 339, normalized size = 2.61

$$\frac{\left(3\,b^{5} - \frac{20\left(b^{5}cd - ab^{4}d^{2}\right)}{(dx+c)d} + \frac{60\left(b^{5}c^{2}d^{2} - 2\,ab^{4}cd^{3} + a^{2}b^{3}d^{4}\right)}{(dx+c)^{2}d^{2}} - \frac{120\left(b^{5}c^{3}d^{3} - 3\,ab^{4}c^{2}d^{4} + 3\,a^{2}b^{3}cd^{5} - a^{3}b^{2}d^{6}\right)}{(dx+c)^{3}d^{3}}\right)(dx + c)^{4}}{12\,d^{6}} - \frac{5\left(b^{5}c^{4} - 4\,ab^{4}c^{3}d + 6\,a^{2}b^{3}c^{2}d^{2} - 4\,a^{3}b^{2}cd^{3} + a^{4}bd^{4}\right)\log\left(\frac{|dx+c|}{(dx+c)^{2}|d|}\right)}{d^{6}} + \frac{\frac{b^{5}c^{5}d^{4}}{dx+c} - \frac{5\,ab^{4}c^{4}d^{5}}{dx+c} + \frac{10\,a^{2}b^{3}c^{3}d^{6}}{dx+c} - \frac{10\,a^{3}b^{2}c^{2}d^{7}}{dx+c} + \frac{5\,a^{4}bcd^{8}}{dx+c} - \frac{a^{5}d^{9}}{dx+c}}{d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5 - 20*(b^5*c*d - a*b^4*d^2)/((d*x + c)*d) + 60*(b^5*c^2*d^2 - 2*
a*b^4*c*d^3 + a^2*b^3*d^4)/((d*x + c)^2*d^2) - 120*(b^5*c^3*d^3 - 3*a*b^4*c
^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^2*d^6)/((d*x + c)^3*d^3))*(d*x + c)^4/d^6
- 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*
d^4)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^6 + (b^5*c^5*d^4/(d*x + c) -
5*a*b^4*c^4*d^5/(d*x + c) + 10*a^2*b^3*c^3*d^6/(d*x + c) - 10*a^3*b^2*c^2*d
^7/(d*x + c) + 5*a^4*b*c*d^8/(d*x + c) - a^5*d^9/(d*x + c))/d^10
```

maple [**B**] time = 0.01, size = 326, normalized size = 2.51

$$\frac{b^{5}x^{4}}{4d^{2}} + \frac{5a\,b^{4}x^{3}}{3d^{2}} - \frac{2b^{5}c\,x^{3}}{3d^{3}} + \frac{5a^{2}b^{3}x^{2}}{d^{2}} - \frac{5a\,b^{4}c\,x^{2}}{d^{3}} + \frac{3b^{5}c^{2}x^{2}}{2d^{4}} - \frac{a^{5}}{(dx+c)\,d} + \frac{5a^{4}bc}{(dx+c)\,d^{2}} + \frac{5a^{4}b\ln(dx+c)}{d^{2}} - \frac{10a^{3}b^{2}c^{2}}{(dx+c)\,d^{3}} - \frac{20a^{3}b^{2}c\ln(dx+c)}{d^{3}} + \frac{10a^{3}b^{2}x}{d^{2}} + \frac{10a^{2}b^{3}c^{3}}{(dx+c)\,d^{4}} + \frac{30a^{2}b^{3}c^{2}\ln(dx+c)}{d^{4}} - \frac{20a^{2}b^{3}cx}{d^{3}} - \frac{5a\,b^{4}c^{4}}{(dx+c)\,d^{5}} - \frac{20a\,b^{4}c^{3}\ln(dx+c)}{d^{5}} + \frac{15a\,b^{4}c^{2}x}{d^{4}} + \frac{b^{5}c^{5}}{(dx+c)\,d^{6}} + \frac{5b^{5}c^{4}\ln(dx+c)}{d^{6}} - \frac{4b^{5}c^{3}x}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^2,x)
```

```
[Out] 1/4*b^5/d^2*x^4+5/3*b^4/d^2*x^3*a-2/3*b^5/d^3*x^3*c+5*b^3/d^2*x^2*a^2-5*b^4
/d^3*x^2*a*c+3/2*b^5/d^4*x^2*c^2+10*b^2/d^2*a^3*x-20*b^3/d^3*a^2*c*x+15*b^4
/d^4*a*c^2*x-4*b^5/d^5*c^3*x-1/d/(d*x+c)*a^5+5/d^2/(d*x+c)*a^4*b*c-10/d^3/(
d*x+c)*a^3*b^2*c^2+10/d^4/(d*x+c)*a^2*b^3*c^3-5/d^5/(d*x+c)*a*b^4*c^4+1/d^6
/(d*x+c)*b^5*c^5+5*b/d^2*ln(d*x+c)*a^4-20*b^2/d^3*ln(d*x+c)*a^3*c+30*b^3/d^
4*ln(d*x+c)*a^2*c^2-20*b^4/d^5*ln(d*x+c)*a*c^3+5*b^5/d^6*ln(d*x+c)*c^4
```

maxima [**B**] time = 1.40, size = 264, normalized size = 2.03

$$\frac{b^{5}c^{5} - 5\,ab^{4}c^{4}d + 10\,a^{2}b^{3}c^{3}d^{2} - 10\,a^{3}b^{2}c^{2}d^{3} + 5\,a^{4}bcd^{4} - a^{5}d^{5}}{d^{7}x + cd^{6}} + \frac{3\,b^{5}d^{3}x^{4} - 4\left(2\,b^{5}cd^{2} - 5\,ab^{4}d^{3}\right)x^{3} + 6\left(3\,b^{5}c^{2}d - 10\,ab^{4}cd^{2} + 10\,a^{2}b^{3}d^{3}\right)x^{2} - 12\left(4\,b^{5}c^{3} - 15\,ab^{4}c^{2}d + 20\,a^{2}b^{3}cd^{2} - 10\,a^{3}b^{2}d^{3}\right)x}{12\,d^{5}} + \frac{5\left(b^{5}c^{4} - 4\,ab^{4}c^{3}d + 6\,a^{2}b^{3}c^{2}d^{2} - 4\,a^{3}b^{2}cd^{3} + a^{4}bd^{4}\right)\log\left(dx + c\right)}{d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*
b*c*d^4 - a^5*d^5)/(d^7*x + c*d^6) + 1/12*(3*b^5*d^3*x^4 - 4*(2*b^5*c*d^2 -
 5*a*b^4*d^3)*x^3 + 6*(3*b^5*c^2*d - 10*a*b^4*c*d^2 + 10*a^2*b^3*d^3)*x^2 -
 12*(4*b^5*c^3 - 15*a*b^4*c^2*d + 20*a^2*b^3*c*d^2 - 10*a^3*b^2*d^3)*x)/d^5
```

```
 + 5*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b
*d^4)*log(d*x + c)/d^6
```

mupad [B] time = 0.25, size = 327, normalized size = 2.52

$$x^3\left(\frac{5ab^4}{3d^2}-\frac{2b^5c}{3d^3}\right)+x\left(\frac{2c\left(\frac{2c\left(\frac{5ab^4}{d^2}-\frac{2b^5c}{d^3}\right)}{d}-\frac{10a^2b^3}{d^2}+\frac{b^5c^2}{d^4}\right)}{d}+\frac{10a^3b^2}{d^2}-\frac{c^2\left(\frac{5ab^4}{d^2}-\frac{2b^5c}{d^3}\right)}{d^2}\right)-x^2\left(\frac{c\left(\frac{5ab^4}{d^2}-\frac{2b^5c}{d^3}\right)}{d}-\frac{5a^2b^3}{d^2}+\frac{b^5c^2}{2d^4}\right)+\frac{\ln(c+dx)\left(5a^4bd^4-20a^3b^2cd^3+30a^2b^3c^2d^2-20ab^4c^3d+5b^5c^4\right)}{d^6}-\frac{a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5}{d\left(xd^6+cd^5\right)}+\frac{b^5x^4}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^2,x)
```

```
[Out] x^3*((5*a*b^4)/(3*d^2) - (2*b^5*c)/(3*d^3)) + x*((2*c*((2*c*((5*a*b^4)/d^2
- (2*b^5*c)/d^3))/d - (10*a^2*b^3)/d^2 + (b^5*c^2)/d^4))/d + (10*a^3*b^2)/d
^2 - (c^2*((5*a*b^4)/d^2 - (2*b^5*c)/d^3))/d^2) - x^2*((c*((5*a*b^4)/d^2 -
(2*b^5*c)/d^3))/d - (5*a^2*b^3)/d^2 + (b^5*c^2)/(2*d^4)) + (log(c + d*x)*(5
*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c
^3*d))/d^6 - (a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 +
 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)/(d*(c*d^5 + d^6*x)) + (b^5*x^4)/(4*d^2)
```

sympy [A] time = 0.89, size = 231, normalized size = 1.78

$$\frac{b^5x^4}{4d^2}+\frac{5b(ad-bc)^4\log(c+dx)}{d^6}+x^3\left(\frac{5ab^4}{3d^2}-\frac{2b^5c}{3d^3}\right)+x^2\left(\frac{5a^2b^3}{d^2}-\frac{5ab^4c}{d^3}+\frac{3b^5c^2}{2d^4}\right)+x\left(\frac{10a^3b^2}{d^2}-\frac{20a^2b^3c}{d^3}+\frac{15ab^4c^2}{d^4}-\frac{4b^5c^3}{d^5}\right)+\frac{-a^5d^5+5a^4bcd^4-10a^3b^2c^2d^3+10a^2b^3c^3d^2-5ab^4c^4d+b^5c^5}{cd^6+d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**2,x)
```

```
[Out] b**5*x**4/(4*d**2) + 5*b*(a*d - b*c)**4*log(c + d*x)/d**6 + x**3*(5*a*b**4/
(3*d**2) - 2*b**5*c/(3*d**3)) + x**2*(5*a**2*b**3/d**2 - 5*a*b**4*c/d**3 +
3*b**5*c**2/(2*d**4)) + x*(10*a**3*b**2/d**2 - 20*a**2*b**3*c/d**3 + 15*a*b
**4*c**2/d**4 - 4*b**5*c**3/d**5) + (-a**5*d**5 + 5*a**4*b*c*d**4 - 10*a**3
*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d + b**5*c**5)/(c*
d**6 + d**7*x)
```

3.1238 $\int \frac{(a+bx)^4}{(c+dx)^2} dx$

Optimal. Leaf size=104

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Rubi [A] time = 0.10, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^3(c+dx)^2(bc-ad)}{d^5}+\frac{6b^2x(bc-ad)^2}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{4b(bc-ad)^3\log(c+dx)}{d^5}+\frac{b^4(c+dx)^3}{3d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (6*b^2*(b*c - a*d)^2*x)/d^4 - (b*c - a*d)^4/(d^5*(c + d*x)) - (2*b^3*(b*c -
 a*d)*(c + d*x)^2)/d^5 + (b^4*(c + d*x)^3)/(3*d^5) - (4*b*(b*c - a*d)^3*Log
[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^2}\,dx &= \int \left(\frac{6b^2(bc-ad)^2}{d^4}+\frac{(-bc+ad)^4}{d^4(c+dx)^2}-\frac{4b(bc-ad)^3}{d^4(c+dx)}-\frac{4b^3(bc-ad)(c+dx)}{d^4}+\frac{b^4(c+dx)^2}{d^4}\right)dx\\ &= \frac{6b^2(bc-ad)^2x}{d^4}-\frac{(bc-ad)^4}{d^5(c+dx)}-\frac{2b^3(bc-ad)(c+dx)^2}{d^5}+\frac{b^4(c+dx)^3}{3d^5}-\frac{4b(bc-ad)^3\log(c+d}{d^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 165, normalized size = 1.59

$$\frac{-3a^4d^4+12a^3bcd^3+18a^2b^2d^2\left(-c^2+cdx+d^2x^2\right)+6ab^3d\left(2c^3-4c^2dx-3cd^2x^2+d^3x^3\right)-12b(c+dx)(bc-ad)^3\log(c+dx)+b^4\left(-3c^4+9c^3dx+6c^2d^2x^2-2cd^3x^3+d^4x^4\right)}{3d^5(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] (12*a^3*b*c*d^3 - 3*a^4*d^4 + 18*a^2*b^2*d^2*(-c^2 + c*d*x + d^2*x^2) + 6*a
*b^3*d*(2*c^3 - 4*c^2*d*x - 3*c*d^2*x^2 + d^3*x^3) + b^4*(-3*c^4 + 9*c^3*d*
x + 6*c^2*d^2*x^2 - 2*c*d^3*x^3 + d^4*x^4) - 12*b*(b*c - a*d)^3*(c + d*x)*L
og[c + d*x])/(3*d^5*(c + d*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^2, x]
```

fricas [B] time = 0.90, size = 267, normalized size = 2.57

$$\frac{b^4d^4x^4 - 3\,b^4c^4 + 12\,ab^3c^3d - 18\,a^2b^2c^2d^2 + 12\,a^3bcd^3 - 3\,a^4d^4 - 2\left(b^4cd^3 - 3\,ab^3d^4\right)x^3 + 6\left(b^4c^2d^2 - 3\,ab^3cd^3 + 3\,a^2b^2d^4\right)x^2 + 3\left(3\,b^4c^3d - 8\,ab^3c^2d^2 + 6\,a^2b^2cd^3\right)x - 12\left(b^4c^4 - 3\,ab^3c^3d + 3\,a^2b^2c^2d^2 - a^3bcd^3 + \left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)x\right)\log\left(dx + c\right)}{3\left(d^6x + cd^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*d^4*x^4 - 3*b^4*c^4 + 12*a*b^3*c^3*d - 18*a^2*b^2*c^2*d^2 + 12*a^3
*b*c*d^3 - 3*a^4*d^4 - 2*(b^4*c*d^3 - 3*a*b^3*d^4)*x^3 + 6*(b^4*c^2*d^2 - 3
*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*x^2 + 3*(3*b^4*c^3*d - 8*a*b^3*c^2*d^2 + 6*a^
2*b^2*c*d^3)*x - 12*(b^4*c^4 - 3*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 - a^3*b*c*
d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)*log(d*
x + c))/(d^6*x + c*d^5)
```

giac [B] time = 1.27, size = 245, normalized size = 2.36

$$\frac{\left(b^4 - \frac{6\left(b^4cd - ab^3d^2\right)}{(dx+c)d} + \frac{18\left(b^4c^2d^2 - 2\,ab^3cd^3 + a^2b^2d^4\right)}{(dx+c)^2d^2}\right)(dx+c)^3}{3\,d^5} + \frac{4\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^5} - \frac{\frac{b^4c^4d^3}{dx+c} - \frac{4\,ab^3c^3d^4}{dx+c} + \frac{6\,a^2b^2c^2d^5}{dx+c} - \frac{4\,a^3bcd^6}{dx+c} + \frac{a^4d^7}{dx+c}}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^4 - 6*(b^4*c*d - a*b^3*d^2)/((d*x + c)*d) + 18*(b^4*c^2*d^2 - 2*a*b^
3*c*d^3 + a^2*b^2*d^4)/((d*x + c)^2*d^2))*(d*x + c)^3/d^5 + 4*(b^4*c^3 - 3*
a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*log(abs(d*x + c)/((d*x + c)^2*ab
s(d)))/d^5 - (b^4*c^4*d^3/(d*x + c) - 4*a*b^3*c^3*d^4/(d*x + c) + 6*a^2*b^2
*c^2*d^5/(d*x + c) - 4*a^3*b*c*d^6/(d*x + c) + a^4*d^7/(d*x + c))/d^8
```

maple [B] time = 0.01, size = 230, normalized size = 2.21

$$\frac{b^4x^3}{3d^2} + \frac{2a\,b^3x^2}{d^2} - \frac{b^4c\,x^2}{d^3} - \frac{a^4}{(dx+c)\,d} + \frac{4a^3bc}{(dx+c)\,d^2} + \frac{4a^3b\ln(dx+c)}{d^2} - \frac{6a^2b^2c^2}{(dx+c)\,d^3} - \frac{12a^2b^2c\ln(dx+c)}{d^3} + \frac{6a^2b^2x}{d^2} + \frac{4a\,b^3c^3}{(dx+c)\,d^4} + \frac{12a\,b^3c^2\ln(dx+c)}{d^4} - \frac{8a\,b^3cx}{d^3} - \frac{b^4c^4}{(dx+c)\,d^5} - \frac{4b^4c^3\ln(dx+c)}{d^5} + \frac{3b^4c^2x}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^2,x)
```

```
[Out] 1/3*b^4/d^2*x^3+2*b^3/d^2*x^2*a-b^4/d^3*x^2*c+6*b^2/d^2*a^2*x-8*b^3/d^3*a*c
*x+3*b^4/d^4*c^2*x-1/d/(d*x+c)*a^4+4/d^2/(d*x+c)*a^3*b*c-6/d^3/(d*x+c)*a^2*
b^2*c^2+4/d^4/(d*x+c)*a*b^3*c^3-1/d^5/(d*x+c)*b^4*c^4+4*b/d^2*ln(d*x+c)*a^3
-12*b^2/d^3*ln(d*x+c)*a^2*c+12*b^3/d^4*ln(d*x+c)*a*c^2-4*b^4/d^5*ln(d*x+c)*
c^3
```

maxima [A] time = 1.36, size = 183, normalized size = 1.76

$$-\frac{b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4}{d^6x + cd^5} + \frac{b^4d^2x^3 - 3\left(b^4cd - 2\,ab^3d^2\right)x^2 + 3\left(3\,b^4c^2 - 8\,ab^3cd + 6\,a^2b^2d^2\right)x}{3\,d^4} - \frac{4\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)\log\left(dx + c\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(d
^6*x + c*d^5) + 1/3*(b^4*d^2*x^3 - 3*(b^4*c*d - 2*a*b^3*d^2)*x^2 + 3*(3*b^4
*c^2 - 8*a*b^3*c*d + 6*a^2*b^2*d^2)*x)/d^4 - 4*(b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*log(d*x + c)/d^5
```

mupad [B] time = 0.07, size = 203, normalized size = 1.95

$$x^2\left(\frac{2\,a\,b^3}{d^2} - \frac{b^4\,c}{d^3}\right) - x\left(\frac{2\,c\left(\frac{4\,a\,b^3}{d^2} - \frac{2\,b^4\,c}{d^3}\right)}{d} - \frac{6\,a^2\,b^2}{d^2} + \frac{b^4\,c^2}{d^4}\right) + \frac{b^4\,x^3}{3\,d^2} - \frac{\ln\left(c + d\,x\right)\left(-4\,a^3\,b\,d^3 + 12\,a^2\,b^2\,c\,d^2 - 12\,a\,b^3\,c^2\,d + 4\,b^4\,c^3\right)}{d^5} - \frac{a^4\,d^4 - 4\,a^3\,b\,c\,d^3 + 6\,a^2\,b^2\,c^2\,d^2 - 4\,a\,b^3\,c^3\,d + b^4\,c^4}{d\left(x\,d^5 + c\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^2,x)
```

```
[Out] x^2*((2*a*b^3)/d^2 - (b^4*c)/d^3) - x*((2*c*((4*a*b^3)/d^2 - (2*b^4*c)/d^3)
)/d - (6*a^2*b^2)/d^2 + (b^4*c^2)/d^4) + (b^4*x^3)/(3*d^2) - (log(c + d*x)*
(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/d^5 - (a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)/(d*(c*d^4
 + d^5*x))
```

sympy [A] time = 0.68, size = 155, normalized size = 1.49

$$\frac{b^4x^3}{3d^2} + \frac{4b(ad-bc)^3\log(c+dx)}{d^5} + x^2\left(\frac{2ab^3}{d^2} - \frac{b^4c}{d^3}\right) + x\left(\frac{6a^2b^2}{d^2} - \frac{8ab^3c}{d^3} + \frac{3b^4c^2}{d^4}\right) + \frac{-a^4d^4 + 4a^3bcd^3 - 6a^2b^2c^2d^2 + 4ab^3c^3d - b^4c^4}{cd^5 + d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**2,x)
```

```
[Out] b**4*x**3/(3*d**2) + 4*b*(a*d - b*c)**3*log(c + d*x)/d**5 + x**2*(2*a*b**3/
d**2 - b**4*c/d**3) + x*(6*a**2*b**2/d**2 - 8*a*b**3*c/d**3 + 3*b**4*c**2/d
**4) + (-a**4*d**4 + 4*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 + 4*a*b**3*c**
3*d - b**4*c**4)/(c*d**5 + d**6*x)
```

3.1239 $\int \frac{(a+bx)^3}{(c+dx)^2} dx$

Optimal. Leaf size=75

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^2x(2bc-3ad)}{d^3}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b*c - a*d)^3/(d^4*(c
+ d*x)) + (3*b*(b*c - a*d)^2*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^2}\,dx &= \int \left(-\frac{b^2(2bc-3ad)}{d^3}+\frac{b^3x}{d^2}+\frac{(-bc+ad)^3}{d^3(c+dx)^2}+\frac{3b(bc-ad)^2}{d^3(c+dx)}\right)dx\\ &= -\frac{b^2(2bc-3ad)x}{d^3}+\frac{b^3x^2}{2d^2}+\frac{(bc-ad)^3}{d^4(c+dx)}+\frac{3b(bc-ad)^2\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 1.52

$$\frac{3\left(a^2bd^2-2ab^2cd+b^3c^2\right)\log(c+dx)}{d^4}+\frac{-a^3d^3+3a^2bcd^2-3ab^2c^2d+b^3c^3}{d^4(c+dx)}-\frac{b^2x(2bc-3ad)}{d^3}+\frac{b^3x^2}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] -((b^2*(2*b*c - 3*a*d)*x)/d^3) + (b^3*x^2)/(2*d^2) + (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^4*(c + d*x)) + (3*(b^3*c^2 - 2*a*b^2*c*d +
 a^2*b*d^2)*Log[c + d*x])/d^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^2, x]
```

fricas [B] time = 1.82, size = 172, normalized size = 2.29

$$\frac{b^3d^3x^3 + 2\,b^3c^3 - 6\,ab^2c^2d + 6\,a^2bcd^2 - 2\,a^3d^3 - 3\left(b^3cd^2 - 2\,ab^2d^3\right)x^2 - 2\left(2\,b^3c^2d - 3\,ab^2cd^2\right)x + 6\left(b^3c^3 - 2\,ab^2c^2d + a^2bcd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x\right)\log\left(dx + c\right)}{2\left(d^5x + cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 -
3*(b^3*c*d^2 - 2*a*b^2*d^3)*x^2 - 2*(2*b^3*c^2*d - 3*a*b^2*c*d^2)*x + 6*(b^
3*c^3 - 2*a*b^2*c^2*d + a^2*b*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^
3)*x)*log(d*x + c))/(d^5*x + c*d^4)
```

giac [B] time = 1.26, size = 166, normalized size = 2.21

$$\frac{\left(b^3 - \frac{6\left(b^3cd - ab^2d^2\right)}{(dx+c)d}\right)(dx+c)^2}{2\,d^4} - \frac{3\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^4} + \frac{\frac{b^3c^3d^2}{dx+c} - \frac{3\,ab^2c^2d^3}{dx+c} + \frac{3\,a^2bcd^4}{dx+c} - \frac{a^3d^5}{dx+c}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3 - 6*(b^3*c*d - a*b^2*d^2)/((d*x + c)*d))*(d*x + c)^2/d^4 - 3*(b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d^4 +
 (b^3*c^3*d^2/(d*x + c) - 3*a*b^2*c^2*d^3/(d*x + c) + 3*a^2*b*c*d^4/(d*x +
c) - a^3*d^5/(d*x + c))/d^6
```

maple [B] time = 0.01, size = 149, normalized size = 1.99

$$\frac{b^3x^2}{2d^2} - \frac{a^3}{(dx+c)\,d} + \frac{3a^2bc}{(dx+c)\,d^2} + \frac{3a^2b\ln(dx+c)}{d^2} - \frac{3a\,b^2c^2}{(dx+c)\,d^3} - \frac{6a\,b^2c\ln(dx+c)}{d^3} + \frac{3a\,b^2x}{d^2} + \frac{b^3c^3}{(dx+c)\,d^4} + \frac{3b^3c^2\ln(dx+c)}{d^4} - \frac{2b^3cx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/2*b^3*x^2/d^2+3*b^2/d^2*a*x-2*b^3/d^3*x*c-1/d/(d*x+c)*a^3+3/d^2/(d*x+c)*a
^2*b*c-3/d^3/(d*x+c)*a*b^2*c^2+1/d^4/(d*x+c)*b^3*c^3+3*b/d^2*ln(d*x+c)*a^2-
6*b^2/d^3*ln(d*x+c)*a*c+3*b^3/d^4*ln(d*x+c)*c^2
```

maxima [A] time = 1.36, size = 117, normalized size = 1.56

$$\frac{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3}{d^5x + cd^4} + \frac{b^3dx^2 - 2\left(2\,b^3c - 3\,ab^2d\right)x}{2\,d^3} + \frac{3\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)\log\left(dx + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(d^5*x + c*d^4) + 1/2*(
b^3*d*x^2 - 2*(2*b^3*c - 3*a*b^2*d)*x)/d^3 + 3*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*log(d*x + c)/d^4
```

mupad [B] time = 0.08, size = 123, normalized size = 1.64

$$x\left(\frac{3\,a\,b^2}{d^2} - \frac{2\,b^3\,c}{d^3}\right) + \frac{\ln\left(c + d\,x\right)\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{d^4} - \frac{a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3}{d\,\left(x\,d^4 + c\,d^3\right)} + \frac{b^3\,x^2}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] x*((3*a*b^2)/d^2 - (2*b^3*c)/d^3) + (log(c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d))/d^4 - (a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)/(
d*(c*d^3 + d^4*x)) + (b^3*x^2)/(2*d^2)
```

sympy [A] time = 0.51, size = 102, normalized size = 1.36

$$\frac{b^3x^2}{2d^2} + \frac{3b\left(ad-bc\right)^2\log\left(c+dx\right)}{d^4} + x\left(\frac{3ab^2}{d^2} - \frac{2b^3c}{d^3}\right) + \frac{-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3}{cd^4 + d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] b**3*x**2/(2*d**2) + 3*b*(a*d - b*c)**2*log(c + d*x)/d**4 + x*(3*a*b**2/d**
2 - 2*b**3*c/d**3) + (-a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3
*c**3)/(c*d**4 + d**5*x)
```

3.1240 $\int \frac{(a+bx)^2}{(c+dx)^2} dx$

Optimal. Leaf size=51

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}+\frac{b^2x}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*x)/d^2 - (b*c - a*d)^2/(d^3*(c + d*x)) - (2*b*(b*c - a*d)*Log[c + d*x]
)/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^2}\,dx &= \int \left(\frac{b^2}{d^2}+\frac{(-bc+ad)^2}{d^2(c+dx)^2}-\frac{2b(bc-ad)}{d^2(c+dx)}\right)dx\\ &= \frac{b^2x}{d^2}-\frac{(bc-ad)^2}{d^3(c+dx)}-\frac{2b(bc-ad)\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.92

$$\frac{-\frac{(bc-ad)^2}{c+dx}+2b(ad-bc)\log(c+dx)+b^2dx}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] (b^2*d*x - (b*c - a*d)^2/(c + d*x) + 2*b*(-(b*c) + a*d)*Log[c + d*x])/d^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^2, x]
```

fricas [A] time = 1.16, size = 92, normalized size = 1.80

$$\frac{b^2d^2x^2 + b^2cdx - b^2c^2 + 2\,abcd - a^2d^2 - 2\left(b^2c^2 - abcd + \left(b^2cd - abd^2\right)x\right)\log\left(dx + c\right)}{d^4x + cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + b^2*c*d*x - b^2*c^2 + 2*a*b*c*d - a^2*d^2 - 2*(b^2*c^2 - a*b
*c*d + (b^2*c*d - a*b*d^2)*x)*log(d*x + c))/(d^4*x + c*d^3)
```

giac [A] time = 1.25, size = 98, normalized size = 1.92

$$\frac{(dx + c)b^2}{d^3} + \frac{2\left(b^2c - abd\right)\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d^3} - \frac{\frac{b^2c^2d}{dx+c} - \frac{2\,abcd^2}{dx+c} + \frac{a^2d^3}{dx+c}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d^3 + 2*(b^2*c - a*b*d)*log(abs(d*x + c)/((d*x + c)^2*abs(d))
)/d^3 - (b^2*c^2*d/(d*x + c) - 2*a*b*c*d^2/(d*x + c) + a^2*d^3/(d*x + c))/d
^4
```

maple [A] time = 0.01, size = 86, normalized size = 1.69

$$-\frac{a^2}{(dx + c)\,d} + \frac{2abc}{(dx + c)\,d^2} + \frac{2ab\ln\left(dx + c\right)}{d^2} - \frac{b^2c^2}{(dx + c)\,d^3} - \frac{2b^2c\ln\left(dx + c\right)}{d^3} + \frac{b^2x}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] b^2*x/d^2-1/d/(d*x+c)*a^2+2/d^2/(d*x+c)*a*b*c-1/d^3/(d*x+c)*b^2*c^2+2*b/d^2
*ln(d*x+c)*a-2*b^2/d^3*ln(d*x+c)*c
```

maxima [A] time = 1.35, size = 67, normalized size = 1.31

$$\frac{b^2x}{d^2} - \frac{b^2c^2 - 2\,abcd + a^2d^2}{d^4x + cd^3} - \frac{2\left(b^2c - abd\right)\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b^2*x/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(d^4*x + c*d^3) - 2*(b^2*c - a*
b*d)*log(d*x + c)/d^3
```

mupad [B] time = 0.24, size = 71, normalized size = 1.39

$$\frac{b^2\,x}{d^2} - \frac{a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2}{d\,\left(x\,d^3 + c\,d^2\right)} - \frac{\ln\left(c + d\,x\right)\,\left(2\,b^2\,c - 2\,a\,b\,d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] (b^2*x)/d^2 - (a^2*d^2 + b^2*c^2 - 2*a*b*c*d)/(d*(c*d^2 + d^3*x)) - (log(c
+ d*x)*(2*b^2*c - 2*a*b*d))/d^3
```

sympy [A] time = 0.34, size = 60, normalized size = 1.18

$$\frac{b^2x}{d^2} + \frac{2b\,(ad-bc)\log{(c+dx)}}{d^3} + \frac{-a^2d^2+2abcd-b^2c^2}{cd^3+d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] b**2*x/d**2 + 2*b*(a*d - b*c)*log(c + d*x)/d**3 + (-a**2*d**2 + 2*a*b*c*d -
 b**2*c**2)/(c*d**3 + d**4*x)
```

3.1241 $\int \frac{a+bx}{(c+dx)^2} \, dx$

Optimal. Leaf size=31

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx}{(c+dx)^2} \, dx &= \int \left(\frac{-bc+ad}{d(c+dx)^2}+\frac{b}{d(c+dx)}\right) dx \\
&= \frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{bc-ad}{d^2(c+dx)}+\frac{b\log(c+dx)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] (b*c - a*d)/(d^2*(c + d*x)) + (b*Log[c + d*x])/d^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^2,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^2, x]
```

fricas [A] time = 1.31, size = 37, normalized size = 1.19

$$\frac{bc-ad+(bdx+bc)\log(dx+c)}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(d*x + c))/(d^3*x + c*d^2)
```

giac [A] time = 1.26, size = 57, normalized size = 1.84

$$-\frac{b\left(\frac{\log\left(\frac{|dx+c|}{(dx+c)^2|d|}\right)}{d}-\frac{c}{(dx+c)d}\right)}{d}-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -b*(log(abs(d*x + c)/((d*x + c)^2*abs(d)))/d - c/((d*x + c)*d))/d - a/((d*x
 + c)*d)
```

maple [A] time = 0.01, size = 39, normalized size = 1.26

$$-\frac{a}{(dx+c)\,d}+\frac{bc}{(dx+c)\,d^2}+\frac{b\ln(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/d/(d*x+c)*a+1/d^2/(d*x+c)*b*c+b*ln(d*x+c)/d^2
```

maxima [A] time = 1.34, size = 34, normalized size = 1.10

$$\frac{bc-ad}{d^3x+cd^2}+\frac{b\log(dx+c)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (b*c - a*d)/(d^3*x + c*d^2) + b*log(d*x + c)/d^2
```

mupad [B] time = 0.04, size = 32, normalized size = 1.03

$$\frac{b\,\ln(c+d\,x)}{d^2}-\frac{a\,d-b\,c}{d^2\,(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^2,x)
```

```
[Out] (b*log(c + d*x))/d^2 - (a*d - b*c)/(d^2*(c + d*x))
```

sympy [A] time = 0.19, size = 27, normalized size = 0.87

$$\frac{b\log(c+dx)}{d^2}+\frac{-ad+bc}{cd^2+d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**2,x)
```

```
[Out] b*log(c + d*x)/d**2 + (-a*d + b*c)/(c*d**2 + d**3*x)
```

3.1242 $\int \frac{1}{(c+dx)^2} dx$

Optimal. Leaf size=12

$$-\frac{1}{d(c+dx)}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2} dx = -\frac{1}{d(c+dx)}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-2),x]
```

```
[Out] -(1/(d*(c + d*x)))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-2),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-2), x]
```

fricas [A] time = 1.15, size = 13, normalized size = 1.08

$$-\frac{1}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/(d^2*x + c*d)
```

giac [A] time = 1.23, size = 12, normalized size = 1.00

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="giac")
[Out] -1/((d*x + c)*d)
```

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$-\frac{1}{(dx+c)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2,x)
[Out] -1/d/(d*x+c)
```

maxima [A] time = 1.28, size = 12, normalized size = 1.00

$$-\frac{1}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2,x, algorithm="maxima")
[Out] -1/((d*x + c)*d)
```

mupad [B] time = 0.19, size = 12, normalized size = 1.00

$$-\frac{1}{d\,\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^2,x)
[Out] -1/(d*(c + d*x))
```

sympy [A] time = 0.13, size = 10, normalized size = 0.83

$$-\frac{1}{cd+d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2,x)
[Out] -1/(c*d + d**2*x)
```

3.1243 $\int \frac{1}{(a+bx)(c+dx)^2} \, dx$

Optimal. Leaf size=56

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{1}{(c+dx)(bc-ad)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] 1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Log[c + d*x
])/(b*c - a*d)^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^2} \, dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)(c+dx)^2}-\frac{bd}{(bc-ad)^2(c+dx)}\right) \, dx\\ &= \frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.95

$$\frac{b(c+dx)\log(a+bx)-ad-b(c+dx)\log(c+dx)+bc}{(c+dx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] (b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])/((b*c - a
*d)^2*(c + d*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^2), x]
```

fricas [A] time = 1.21, size = 92, normalized size = 1.64

$$\frac{bc - ad + (bdx + bc)\log(bx + a) - (bdx + bc)\log(dx + c)}{b^2c^3 - 2\,abc^2d + a^2cd^2 + \left(b^2c^2d - 2\,abcd^2 + a^2d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (b*c - a*d + (b*d*x + b*c)*log(b*x + a) - (b*d*x + b*c)*log(d*x + c))/(b^2*
c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)
```

giac [A] time = 1.34, size = 77, normalized size = 1.38

$$\frac{bd\log\left(\left|b - \frac{bc}{dx+c} + \frac{ad}{dx+c}\right|\right)}{b^2c^2d - 2\,abcd^2 + a^2d^3} + \frac{d}{\left(bcd - ad^2\right)(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] b*d*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3) + d/((b*c*d - a*d^2)*(d*x + c))
```

maple [A] time = 0.01, size = 58, normalized size = 1.04

$$\frac{b\ln(bx + a)}{(ad - bc)^2} - \frac{b\ln(dx + c)}{(ad - bc)^2} - \frac{1}{(ad - bc)(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^2,x)
```

```
[Out] -1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x+a)
```

maxima [A] time = 1.32, size = 90, normalized size = 1.61

$$\frac{b\log(bx + a)}{b^2c^2 - 2\,abcd + a^2d^2} - \frac{b\log(dx + c)}{b^2c^2 - 2\,abcd + a^2d^2} + \frac{1}{bc^2 - acd + \left(bcd - ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - b*log(d*x + c)/(b^2*c^2 -
2*a*b*c*d + a^2*d^2) + 1/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

mupad [B] time = 0.29, size = 47, normalized size = 0.84

$$-\frac{1}{(a\,d - b\,c)\,(c + d\,x)} - \frac{b\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{(a\,d - b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^2),x)
```

```
[Out] - 1/((a*d - b*c)*(c + d*x)) - (b*log((c + d*x)/(a + b*x)))/(a*d - b*c)^2
```

sympy [B] time = 0.68, size = 233, normalized size = 4.16

$$-\frac{b\log\left(x + \frac{-\frac{a^3bd^3}{(ad-bc)^2} + \frac{3a^2b^2cd^2}{(ad-bc)^2} - \frac{3ab^3c^2d}{(ad-bc)^2} + abd + \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{(ad - bc)^2} + \frac{b\log\left(x + \frac{\frac{a^3bd^3}{(ad-bc)^2} - \frac{3a^2b^2cd^2}{(ad-bc)^2} + \frac{3ab^3c^2d}{(ad-bc)^2} + abd - \frac{b^4c^3}{(ad-bc)^2} + b^2c}{2b^2d}\right)}{(ad - bc)^2} - \frac{1}{acd - bc^2 + x\left(ad^2 - bcd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**2,x)
```

```
[Out] -b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2
 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2
*c)/(2*b**2*d))/(a*d - b*c)**2 + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*
a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d -
b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(a*d - b*c)**2 - 1/(a*c*d -
b*c**2 + x*(a*d**2 - b*c*d))
```

3.1244 $\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$

Optimal. Leaf size=81

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Rubi [A] time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b}{(a+bx)(bc-ad)^2}-\frac{d}{(c+dx)(bc-ad)^2}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] -(b/((b*c - a*d)^2*(a + b*x))) - d/((b*c - a*d)^2*(c + d*x)) - (2*b*d*Log[a
 + b*x])/(b*c - a*d)^3 + (2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^2}-\frac{2b^2d}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^2}+\frac{2bd^2}{(bc-ad)^3(c \right.\\ &= -\frac{b}{(bc-ad)^2(a+bx)}-\frac{d}{(bc-ad)^2(c+dx)}-\frac{2bd\log(a+bx)}{(bc-ad)^3}+\frac{2bd\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.81

$$\frac{\frac{b(ad-bc)}{a+bx}+\frac{d(ad-bc)}{c+dx}-2bd\log(a+bx)+2bd\log(c+dx)}{(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] ((b*(-(b*c) + a*d))/(a + b*x) + (d*(-(b*c) + a*d))/(c + d*x) - 2*b*d*Log[a
+ b*x] + 2*b*d*Log[c + d*x])/(b*c - a*d)^3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^2), x]
```

fricas [B] time = 1.24, size = 241, normalized size = 2.98

$$-\frac{b^2c^2 - a^2d^2 + 2\left(b^2cd - abd^2\right)x + 2\left(b^2d^2x^2 + abcd + \left(b^2cd + abd^2\right)x\right)\log\left(bx + a\right) - 2\left(b^2d^2x^2 + abcd + \left(b^2cd + abd^2\right)x\right)\log\left(dx + c\right)}{ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3 + \left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)x^2 + \left(b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*c^2 - a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2*x^2 + a*b*c*d +
(b^2*c*d + a*b*d^2)*x)*log(b*x + a) - 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d +
 a*b*d^2)*x)*log(d*x + c))/(a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 -
 a^4*c*d^3 + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*x)
```

giac [A] time = 1.21, size = 153, normalized size = 1.89

$$\frac{2\,b^2d\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3} - \frac{b^3}{\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)(bx + a)} + \frac{bd^2}{(bc - ad)^3\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*b^2*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^4*c^3 - 3*a*b^3*c^2*
d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - b^3/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*(b*x + a)) + b*d^2/((b*c - a*d)^3*(b*c/(b*x + a) - a*d/(b*x + a) + d))
```

maple [A] time = 0.01, size = 82, normalized size = 1.01

$$\frac{2bd\ln\left(bx + a\right)}{\left(ad - bc\right)^3} - \frac{2bd\ln\left(dx + c\right)}{\left(ad - bc\right)^3} - \frac{b}{\left(ad - bc\right)^2\left(bx + a\right)} - \frac{d}{\left(ad - bc\right)^2\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] -d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^2/(b*x+a)+2*
d/(a*d-b*c)^3*b*ln(b*x+a)
```

maxima [B] time = 1.43, size = 208, normalized size = 2.57

$$-\frac{2\,bd\log\left(bx + a\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} + \frac{2\,bd\log\left(dx + c\right)}{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3} - \frac{2\,bdx + bc + ad}{ab^2c^3 - 2\,a^2bc^2d + a^3cd^2 + \left(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3\right)x^2 + \left(b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -2*b*d*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 2
*b*d*log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - (2*
b*d*x + b*c + a*d)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*
a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d
^3)*x)
```

mupad [B] time = 0.33, size = 74, normalized size = 0.91

$$\frac{1}{\left(a\,d - b\,c\right)\,\left(a + b\,x\right)\,\left(c + d\,x\right)} - \frac{2\,d}{{\left(a\,d - b\,c\right)}^2\,\left(c + d\,x\right)} - \frac{2\,b\,d\,\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{{\left(a\,d - b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)*(c + d*x)) - (2*d)/((a*d - b*c)^2*(c + d*x)) - (2*
b*d*log((c + d*x)/(a + b*x)))/(a*d - b*c)^3
```

sympy [B] time = 1.11, size = 406, normalized size = 5.01

$$-\frac{2bd\log\left(x+\frac{-\frac{2a^4bd^5}{(ad-bc)^3}+\frac{8a^3b^2cd^4}{(ad-bc)^3}-\frac{12a^2b^3c^2d^3}{(ad-bc)^3}+\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2-\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{2bd\log\left(x+\frac{\frac{2a^4bd^5}{(ad-bc)^3}-\frac{8a^3b^2cd^4}{(ad-bc)^3}+\frac{12a^2b^3c^2d^3}{(ad-bc)^3}-\frac{8ab^4c^3d^2}{(ad-bc)^3}+2abd^2+\frac{2b^5c^4d}{(ad-bc)^3}+2b^2cd}{4b^2d^2}\right)}{(ad-bc)^3}+\frac{-ad-bc-2bdx}{a^3cd^2-2a^2bc^2d+ab^2c^3+x^2\left(a^2bd^3-2ab^2cd^2+b^3c^2d\right)+x\left(a^3d^3-a^2bcd^2-ab^2c^2d+b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] -2*b*d*log(x + (-2*a**4*b*d**5/(a*d - b*c)**3 + 8*a**3*b**2*c*d**4/(a*d - b
*c)**3 - 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2/(a*d -
b*c)**3 + 2*a*b*d**2 - 2*b**5*c**4*d/(a*d - b*c)**3 + 2*b**2*c*d)/(4*b**2*d
**2))/(a*d - b*c)**3 + 2*b*d*log(x + (2*a**4*b*d**5/(a*d - b*c)**3 - 8*a**3
*b**2*c*d**4/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*a*b
**4*c**3*d**2/(a*d - b*c)**3 + 2*a*b*d**2 + 2*b**5*c**4*d/(a*d - b*c)**3 +
2*b**2*c*d)/(4*b**2*d**2))/(a*d - b*c)**3 + (-a*d - b*c - 2*b*d*x)/(a**3*c*
d**2 - 2*a**2*b*c**2*d + a*b**2*c**3 + x**2*(a**2*b*d**3 - 2*a*b**2*c*d**2
+ b**3*c**2*d) + x*(a**3*d**3 - a**2*b*c*d**2 - a*b**2*c**2*d + b**3*c**3))
```

3.1245 $\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$

Optimal. Leaf size=109

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{d^2}{(c+dx)(bc-ad)^3}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}-\frac{3bd^2\log(c+dx)}{(bc-ad)^4}+\frac{2bd}{(a+bx)(bc-ad)^3}-\frac{b}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] -b/(2*(b*c - a*d)^2*(a + b*x)^2) + (2*b*d)/((b*c - a*d)^3*(a + b*x)) + d^2/
((b*c - a*d)^3*(c + d*x)) + (3*b*d^2*Log[a + b*x])/(b*c - a*d)^4 - (3*b*d^2
*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx &= \int \left(\frac{b^2}{(bc-ad)^2(a+bx)^3}-\frac{2b^2d}{(bc-ad)^3(a+bx)^2}+\frac{3b^2d^2}{(bc-ad)^4(a+bx)}-\frac{d^3}{(bc-ad)^3(c+d\right.\\ &= -\frac{b}{2(bc-ad)^2(a+bx)^2}+\frac{2bd}{(bc-ad)^3(a+bx)}+\frac{d^2}{(bc-ad)^3(c+dx)}+\frac{3bd^2\log(a+bx)}{(bc-ad)^4}\end{aligned}$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.90

$$\frac{\frac{2d^2(bc-ad)}{c+dx}+\frac{4bd(bc-ad)}{a+bx}-\frac{b(bc-ad)^2}{(a+bx)^2}+6bd^2\log(a+bx)-6bd^2\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] (-((b*(b*c - a*d)^2)/(a + b*x)^2) + (4*b*d*(b*c - a*d))/(a + b*x) + (2*d^2*
(b*c - a*d))/(c + d*x) + 6*b*d^2*Log[a + b*x] - 6*b*d^2*Log[c + d*x])/(2*(b
*c - a*d)^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^2}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^2),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^2), x]
```

fricas [B] time = 1.23, size = 494, normalized size = 4.53

$$-\frac{b^{3} c^{3} - 6 \, a b^{2} c^{2} d + 3 \, a^{2} b c d^{2} + 2 \, a^{3} d^{3} - 6 \, {\left(b^{3} c d^{2} - a b^{2} d^{3}\right)} x^{2} - 3 \, {\left(b^{3} c^{2} d + 2 \, a b^{2} c d^{2} - 3 \, a^{2} b d^{3}\right)} x - 6 \, {\left(b^{3} d^{3} x^{3} + a^{2} b c d^{2} + {\left(b^{3} c d^{2} + 2 \, a b^{2} d^{3}\right)} x^{2} + {\left(2 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x\right)} \log\left(b x + a\right) + 6 \, {\left(b^{3} d^{3} x^{3} + a^{2} b c d^{2} + {\left(b^{3} c d^{2} + 2 \, a b^{2} d^{3}\right)} x^{2} + {\left(2 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x\right)} \log\left(d x + c\right)}{2 \, {\left(a^{2} b^{4} c^{5} - 4 \, a^{3} b^{3} c^{4} d + 6 \, a^{4} b^{2} c^{3} d^{2} - 4 \, a^{5} b c^{2} d^{3} + a^{6} c d^{4} + {\left(b^{6} c^{4} d - 4 \, a b^{5} c^{3} d^{2} + 6 \, a^{2} b^{4} c^{2} d^{3} - 4 \, a^{3} b^{3} c d^{4} + a^{4} b^{2} d^{5}\right)} x^{3} + {\left(b^{6} c^{5} - 2 \, a b^{5} c^{4} d - 2 \, a^{2} b^{4} c^{3} d^{2} + 8 \, a^{3} b^{3} c^{2} d^{3} - 7 \, a^{4} b^{2} c d^{4} + 2 \, a^{5} b d^{5}\right)} x^{2} + {\left(2 \, a b^{5} c^{5} - 7 \, a^{2} b^{4} c^{4} d + 8 \, a^{3} b^{3} c^{3} d^{2} - 2 \, a^{4} b^{2} c^{2} d^{3} - 2 \, a^{5} b c d^{4} + a^{6} d^{5}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*c^3 - 6*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 2*a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b
*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2
*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(d*x + c))/(a^2*b^4*c^5 - 4*a
^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4 + (b^6*c^4*d
 - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^4*b^2*d^5)*x^3
 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4
*b^2*c*d^4 + 2*a^5*b*d^5)*x^2 + (2*a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*
c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^4 + a^6*d^5)*x)
```

giac [B] time = 1.35, size = 216, normalized size = 1.98

$$\frac{3 \, b d^{3} \log\left({\left| b - \frac{b c}{d x + c} + \frac{a d}{d x + c} \right|}\right)}{b^{4} c^{4} d - 4 \, a b^{3} c^{3} d^{2} + 6 \, a^{2} b^{2} c^{2} d^{3} - 4 \, a^{3} b c d^{4} + a^{4} d^{5}} + \frac{d^{5}}{{\left(b^{3} c^{3} d^{3} - 3 \, a b^{2} c^{2} d^{4} + 3 \, a^{2} b c d^{5} - a^{3} d^{6}\right)} {\left(d x + c\right)}} + \frac{5 \, b^{3} d^{2} - \frac{6 \, {\left(b^{3} c d^{3} - a b^{2} d^{4}\right)}}{{\left(d x + c\right)} d}}{2 \, {\left(b c - a d\right)}^{4} {\left(b - \frac{b c}{d x + c} + \frac{a d}{d x + c}\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 3*b*d^3*log(abs(b - b*c/(d*x + c) + a*d/(d*x + c)))/(b^4*c^4*d - 4*a*b^3*c^
3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5) + d^5/((b^3*c^3*d^3 -
3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 - a^3*d^6)*(d*x + c)) + 1/2*(5*b^3*d^2 - 6*
(b^3*c*d^3 - a*b^2*d^4)/((d*x + c)*d))/((b*c - a*d)^4*(b - b*c/(d*x + c) +
a*d/(d*x + c))^2)
```

maple [A] time = 0.01, size = 109, normalized size = 1.00

$$\frac{3 b \,d^{2} \ln \left(b x +a \right)}{\left(a d -b c \right)^{4}}-\frac{3 b \,d^{2} \ln \left(d x +c \right)}{\left(a d -b c \right)^{4}}-\frac{2 b d}{\left(a d -b c \right)^{3} \left(b x +a \right)}-\frac{d^{2}}{\left(a d -b c \right)^{3} \left(d x +c \right)}-\frac{b}{2 \left(a d -b c \right)^{2} \left(b x +a \right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] -d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a*d-b*c)^2/(b
*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a)
```

maxima [B] time = 1.55, size = 386, normalized size = 3.54

$$\frac{3 \, b d^{2} \log\left(b x + a\right)}{b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}} - \frac{3 \, b d^{2} \log\left(d x + c\right)}{b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}} + \frac{6 \, b^{2} d^{2} x^{2} - b^{2} c^{2} + 5 \, a b c d + 2 \, a^{2} d^{2} + 3 \, {\left(b^{2} c d + 3 \, a b d^{2}\right)} x}{2 \, {\left(a^{2} b^{3} c^{4} - 3 \, a^{3} b^{2} c^{3} d + 3 \, a^{4} b c^{2} d^{2} - a^{5} c d^{3} + {\left(b^{5} c^{3} d - 3 \, a b^{4} c^{2} d^{2} + 3 \, a^{2} b^{3} c d^{3} - a^{3} b^{2} d^{4}\right)} x^{3} + {\left(b^{5} c^{4} - a b^{4} c^{3} d - 3 \, a^{2} b^{3} c^{2} d^{2} + 5 \, a^{3} b^{2} c d^{3} - 2 \, a^{4} b d^{4}\right)} x^{2} + {\left(2 \, a b^{4} c^{4} - 5 \, a^{2} b^{3} c^{3} d + 3 \, a^{3} b^{2} c^{2} d^{2} + a^{4} b c d^{3} - a^{5} d^{4}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 3*b*d^2*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4) - 3*b*d^2*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b
^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) + 1/2*(6*b^2*d^2*x^2 - b^2*c^2 + 5*a*
b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^2)*x)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3
*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*
```

```
a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b
^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

mupad [B] time = 0.40, size = 330, normalized size = 3.03

$$\frac{6\,b\,d^2\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{\frac{2\,a^2\,d^2+5\,a\,b\,c\,d-b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,d\,x\,\left(c\,b^2+3\,a\,d\,b\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(d\,a^2+2\,b\,c\,a\right)+a^2\,c+x^2\,\left(c\,b^2+2\,a\,d\,b\right)+b^2\,d\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] (6*b*d^2*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c*d^3)/(a*d - b
*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*d
- b*c)^4))/(a*d - b*c)^4 - ((2*a^2*d^2 - b^2*c^2 + 5*a*b*c*d)/(2*(a^3*d^3 -
 b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*d*x*(b^2*c + 3*a*b*d))/(2*(
a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(x*(a^2*d + 2*a*b*c) + a^2*
c + x^2*(b^2*c + 2*a*b*d) + b^2*d*x^3)
```

sympy [B] time = 1.72, size = 634, normalized size = 5.82

$$-\frac{3bd^2\log\left(x+\frac{-\frac{3a^5bd^7}{(ad-bc)^4}+\frac{15a^4b^2cd^6}{(ad-bc)^4}-\frac{30a^3b^3c^2d^5}{(ad-bc)^4}+\frac{30a^2b^4c^3d^4}{(ad-bc)^4}-\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3+\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{3bd^2\log\left(x+\frac{\frac{3a^5bd^7}{(ad-bc)^4}-\frac{15a^4b^2cd^6}{(ad-bc)^4}+\frac{30a^3b^3c^2d^5}{(ad-bc)^4}-\frac{30a^2b^4c^3d^4}{(ad-bc)^4}+\frac{15ab^5c^4d^3}{(ad-bc)^4}+3abd^3-\frac{3b^6c^5d^2}{(ad-bc)^4}+3b^2cd^2}{6b^2d^3}\right)}{(ad-bc)^4}+\frac{-2a^2d^2-5abcd+b^2c^2-6b^2d^2x^2+x\left(-9abd^2-3b^2cd\right)}{2a^5cd^3-6a^4bc^2d^2+6a^3b^2c^3d-2a^2b^3c^4+x^3\left(2a^3b^2d^4-6a^2b^3cd^3+6ab^4c^2d^2-2b^5c^3d\right)+x^2\left(4a^4bd^4-10a^3b^2cd^3+6a^2b^3c^2d^2+2ab^4c^3d-2b^5c^4\right)+x\left(2a^5d^4-2a^4bcd^3-6a^3b^2c^2d^2+10a^2b^3c^3d-4ab^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] -3*b*d**2*log(x + (-3*a**5*b*d**7/(a*d - b*c)**4 + 15*a**4*b**2*c*d**6/(a*d
 - b*c)**4 - 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 + 30*a**2*b**4*c**3*d**4
/(a*d - b*c)**4 - 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 + 3*b**6*
c**5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + 3
*b*d**2*log(x + (3*a**5*b*d**7/(a*d - b*c)**4 - 15*a**4*b**2*c*d**6/(a*d -
b*c)**4 + 30*a**3*b**3*c**2*d**5/(a*d - b*c)**4 - 30*a**2*b**4*c**3*d**4/(a
*d - b*c)**4 + 15*a*b**5*c**4*d**3/(a*d - b*c)**4 + 3*a*b*d**3 - 3*b**6*c**
5*d**2/(a*d - b*c)**4 + 3*b**2*c*d**2)/(6*b**2*d**3))/(a*d - b*c)**4 + (-2*
a**2*d**2 - 5*a*b*c*d + b**2*c**2 - 6*b**2*d**2*x**2 + x*(-9*a*b*d**2 - 3*b
**2*c*d))/(2*a**5*c*d**3 - 6*a**4*b*c**2*d**2 + 6*a**3*b**2*c**3*d - 2*a**2
*b**3*c**4 + x**3*(2*a**3*b**2*d**4 - 6*a**2*b**3*c*d**3 + 6*a*b**4*c**2*d*
*2 - 2*b**5*c**3*d) + x**2*(4*a**4*b*d**4 - 10*a**3*b**2*c*d**3 + 6*a**2*b*
*3*c**2*d**2 + 2*a*b**4*c**3*d - 2*b**5*c**4) + x*(2*a**5*d**4 - 2*a**4*b*c
*d**3 - 6*a**3*b**2*c**2*d**2 + 10*a**2*b**3*c**3*d - 4*a*b**4*c**4))
```

3.1246 $\int \frac{(a+bx)^6}{(c+dx)^3} \, dx$

Optimal. Leaf size=158

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}$$

Rubi [A] time = 0.20, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{2b^5(c+dx)^3(bc-ad)}{d^7}+\frac{15b^4(c+dx)^2(bc-ad)^2}{2d^7}-\frac{20b^3x(bc-ad)^3}{d^6}+\frac{15b^2(bc-ad)^4\log(c+dx)}{d^7}+\frac{6b(bc-ad)^5}{d^7(c+dx)}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{b^6(c+dx)^4}{4d^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^3,x]

[Out] (-20*b^3*(b*c - a*d)^3*x)/d^6 - (b*c - a*d)^6/(2*d^7*(c + d*x)^2) + (6*b*(b
*c - a*d)^5)/(d^7*(c + d*x)) + (15*b^4*(b*c - a*d)^2*(c + d*x)^2)/(2*d^7) -
 (2*b^5*(b*c - a*d)*(c + d*x)^3)/d^7 + (b^6*(c + d*x)^4)/(4*d^7) + (15*b^2*
(b*c - a*d)^4*Log[c + d*x])/d^7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^6}{(c+dx)^3}\,dx &= \int \left(-\frac{20b^3(bc-ad)^3}{d^6}+\frac{(-bc+ad)^6}{d^6(c+dx)^3}-\frac{6b(bc-ad)^5}{d^6(c+dx)^2}+\frac{15b^2(bc-ad)^4}{d^6(c+dx)}+\frac{15b^4(bc-ad)^2(c+ \\ &= -\frac{20b^3(bc-ad)^3x}{d^6}-\frac{(bc-ad)^6}{2d^7(c+dx)^2}+\frac{6b(bc-ad)^5}{d^7(c+dx)}+\frac{15b^4(bc-ad)^2(c+dx)^2}{2d^7}-\frac{2b^5(bc-ad)(}{d^7}\end{aligned}$$

Mathematica [A] time = 0.11, size = 303, normalized size = 1.92

$$\frac{-2a^6d^6-12a^5bd^5(c+2dx)+30a^4b^2cd^4(3c+4dx)+40a^3b^3d^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+30a^2b^4d^2\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)+4ab^5d\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)+60b^2(c+dx)^2(bc-ad)^4\log(c+dx)+b^6\left(22c^6-16c^5dx-68c^4d^2x^2-20c^3d^3x^3+5c^2d^4x^4-2cd^5x^5+d^6x^6\right)}{4d^7(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^3,x]

[Out] (-2*a^6*d^6 - 12*a^5*b*d^5*(c + 2*d*x) + 30*a^4*b^2*c*d^4*(3*c + 4*d*x) + 4
0*a^3*b^3*d^3*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 30*a^2*b^4*d
^2*(7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 4*a*b^5*d
*(-27*c^5 + 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d
^5*x^5) + b^6*(22*c^6 - 16*c^5*d*x - 68*c^4*d^2*x^2 - 20*c^3*d^3*x^3 + 5*c^
2*d^4*x^4 - 2*c*d^5*x^5 + d^6*x^6) + 60*b^2*(b*c - a*d)^4*(c + d*x)^2*Log[c
 + d*x])/(4*d^7*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^3, x]
```

fricas [B] time = 1.26, size = 548, normalized size = 3.47

$$\frac{b^6d^6x^6 + 22\,b^6c^6 - 108\,ab^5c^5d + 210\,a^2b^4c^4d^2 - 200\,a^3b^3c^3d^3 + 90\,a^4b^2c^2d^4 - 12\,a^5bcd^5 - 2\,a^6d^6 - 2\left(b^6cd^5 - 4\,ab^5d^6\right)x^5 + 5\left(b^6c^2d^4 - 4\,ab^5cd^5 + 6\,a^2b^4d^6\right)x^4 - 20\left(b^6c^3d^3 - 4\,ab^5c^2d^4 + 6\,a^2b^4cd^5 - 4\,a^3b^3d^6\right)x^3 - 2\left(34\,b^6c^4d^2 - 126\,ab^5c^3d^3 + 165\,a^2b^4c^2d^4 - 80\,a^3b^3cd^5\right)x^2 - 4\left(4\,b^6c^5d - 6\,ab^5c^4d^2 - 15\,a^2b^4c^3d^3 + 40\,a^3b^3c^2d^4 - 30\,a^4b^2cd^5 + 6\,a^5bd^6\right)x + 60\left(b^6c^6 - 4\,ab^5c^5d + 6\,a^2b^4c^4d^2 - 4\,a^3b^3c^3d^3 + a^4b^2c^2d^4 + \left(b^6c^4d^2 - 4\,ab^5c^3d^3 + 6\,a^2b^4c^2d^4 - 4\,a^3b^3cd^5 + a^4b^2d^6\right)x^2 + 2\left(b^6c^5d - 4\,ab^5c^4d^2 + 6\,a^2b^4c^3d^3 - 4\,a^3b^3c^2d^4 + a^4b^2cd^5\right)x\right)\log\left(dx + c\right)}{4\left(d^9x^2 + 2\,cd^8x + c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^6*d^6*x^6 + 22*b^6*c^6 - 108*a*b^5*c^5*d + 210*a^2*b^4*c^4*d^2 - 200
*a^3*b^3*c^3*d^3 + 90*a^4*b^2*c^2*d^4 - 12*a^5*b*c*d^5 - 2*a^6*d^6 - 2*(b^6
*c*d^5 - 4*a*b^5*d^6)*x^5 + 5*(b^6*c^2*d^4 - 4*a*b^5*c*d^5 + 6*a^2*b^4*d^6)
*x^4 - 20*(b^6*c^3*d^3 - 4*a*b^5*c^2*d^4 + 6*a^2*b^4*c*d^5 - 4*a^3*b^3*d^6)
*x^3 - 2*(34*b^6*c^4*d^2 - 126*a*b^5*c^3*d^3 + 165*a^2*b^4*c^2*d^4 - 80*a^3
*b^3*c*d^5)*x^2 - 4*(4*b^6*c^5*d - 6*a*b^5*c^4*d^2 - 15*a^2*b^4*c^3*d^3 + 4
0*a^3*b^3*c^2*d^4 - 30*a^4*b^2*c*d^5 + 6*a^5*b*d^6)*x + 60*(b^6*c^6 - 4*a*b
^5*c^5*d + 6*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + (b^6*c
^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^
6)*x^2 + 2*(b^6*c^5*d - 4*a*b^5*c^4*d^2 + 6*a^2*b^4*c^3*d^3 - 4*a^3*b^3*c^2
*d^4 + a^4*b^2*c*d^5)*x)*log(d*x + c))/(d^9*x^2 + 2*c*d^8*x + c^2*d^7)
```

giac [B] time = 1.28, size = 362, normalized size = 2.29

$$\frac{15\left(b^6c^4 - 4\,ab^5c^3d + 6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4\right)\log\left(|dx + c|\right)}{d^7} + \frac{11\,b^6c^6 - 54\,ab^5c^5d + 105\,a^2b^4c^4d^2 - 100\,a^3b^3c^3d^3 + 45\,a^4b^2c^2d^4 - 6\,a^5bcd^5 - a^6d^6 + 12\left(b^6c^5d - 5\,ab^5c^4d^2 + 10\,a^2b^4c^3d^3 - 10\,a^3b^3c^2d^4 + 5\,a^4b^2cd^5 - a^5bd^6\right)x}{2\left(dx + c\right)^2d^7} + \frac{b^6d^9x^4 - 4\,b^6cd^8x^3 + 8\,ab^5d^9x^3 + 12\,b^6c^2d^7x^2 - 36\,ab^5cd^8x^2 + 30\,a^2b^4d^9x^2 - 40\,b^6c^3d^6x + 144\,ab^5c^2d^7x - 180\,a^2b^4cd^8x + 80\,a^3b^3d^9x}{4\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2
*d^4)*log(abs(d*x + c))/d^7 + 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^
4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*
d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2
*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/((d*x + c)^2*d^7) + 1/4*(b^6*d^9*x^4
 - 4*b^6*c*d^8*x^3 + 8*a*b^5*d^9*x^3 + 12*b^6*c^2*d^7*x^2 - 36*a*b^5*c*d^8*
x^2 + 30*a^2*b^4*d^9*x^2 - 40*b^6*c^3*d^6*x + 144*a*b^5*c^2*d^7*x - 180*a^2
*b^4*c*d^8*x + 80*a^3*b^3*d^9*x)/d^12
```

maple [B] time = 0.01, size = 464, normalized size = 2.94

$$\frac{b^6x^4}{4d^3} + \frac{2ab^5x^3}{d^3} - \frac{b^6cx^3}{d^4} - \frac{a^6}{2\left(dx+c\right)^2d} + \frac{3a^5bc}{\left(dx+c\right)^2d^2} - \frac{15a^4b^2c^2}{2\left(dx+c\right)^2d^3} + \frac{10a^3b^3c^3}{\left(dx+c\right)^2d^4} - \frac{15a^2b^4c^4}{2\left(dx+c\right)^2d^5} + \frac{15a^2b^4x^2}{2d^3} + \frac{3ab^5c^5}{\left(dx+c\right)^2d^6} - \frac{9ab^5cx^2}{d^4} - \frac{b^6c^6}{2\left(dx+c\right)^2d^7} + \frac{3b^6c^2x^2}{d^5} - \frac{6a^5b}{\left(dx+c\right)d^2} + \frac{30a^4b^2c}{\left(dx+c\right)d^3} + \frac{15a^4b^2\ln\left(dx+c\right)}{d^3} - \frac{60a^3b^3c^2}{\left(dx+c\right)d^4} - \frac{60a^3b^3c\ln\left(dx+c\right)}{d^4} + \frac{20a^3b^3x}{d^3} + \frac{60a^2b^4c^3}{\left(dx+c\right)d^5} + \frac{90a^2b^4c^2\ln\left(dx+c\right)}{d^5} - \frac{45a^2b^4cx}{d^4} - \frac{30ab^5c^4}{\left(dx+c\right)d^6} - \frac{60ab^5c^3\ln\left(dx+c\right)}{d^6} + \frac{36ab^5c^2x}{d^5} + \frac{6b^6c^5}{\left(dx+c\right)d^7} + \frac{15b^6c^4\ln\left(dx+c\right)}{d^7} - \frac{10b^6c^3x}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^3,x)
```

```
[Out] -1/2/d^7/(d*x+c)^2*b^6*c^6-6*b/d^2/(d*x+c)*a^5+6*b^6/d^7/(d*x+c)*c^5+15*b^2
/d^3*ln(d*x+c)*a^4+15*b^6/d^7*ln(d*x+c)*c^4+20*b^3/d^3*a^3*x-10*b^6/d^6*c^3
*x+3*b^6/d^5*x^2*c^2+15/2*b^4/d^3*x^2*a^2-b^6/d^4*x^3*c+2*b^5/d^3*x^3*a+3/d
^2/(d*x+c)^2*a^5*b*c-15/2/d^3/(d*x+c)^2*a^4*b^2*c^2+10/d^4/(d*x+c)^2*a^3*b^
3*c^3-15/2/d^5/(d*x+c)^2*a^2*b^4*c^4+3/d^6/(d*x+c)^2*a*b^5*c^5+36*b^5/d^5*a
*c^2*x+30*b^2/d^3/(d*x+c)*a^4*c-60*b^3/d^4/(d*x+c)*a^3*c^2+60*b^4/d^5/(d*x+
c)*a^2*c^3-30*b^5/d^6/(d*x+c)*a*c^4-9*b^5/d^4*x^2*a*c-45*b^4/d^4*a^2*c*x+1/
4*b^6/d^3*x^4-1/2/d/(d*x+c)^2*a^6-60*b^5/d^6*ln(d*x+c)*a*c^3-60*b^3/d^4*ln(
d*x+c)*a^3*c+90*b^4/d^5*ln(d*x+c)*a^2*c^2
```

maxima [B] time = 1.47, size = 364, normalized size = 2.30

$$\frac{11\,b^6c^6 - 54\,ab^5c^5d + 105\,a^2b^4c^4d^2 - 100\,a^3b^3c^3d^3 + 45\,a^4b^2c^2d^4 - 6\,a^5bcd^5 - a^6d^6 + 12\left(b^6c^5d - 5\,ab^5c^4d^2 + 10\,a^2b^4c^3d^3 - 10\,a^3b^3c^2d^4 + 5\,a^4b^2cd^5 - a^5bd^6\right)x}{2\left(d^9x^2 + 2\,cd^8x + c^2d^7\right)} + \frac{b^6d^3x^4 - 4\left(b^6cd^2 - 2\,ab^5d^3\right)x^3 + 6\left(2\,b^6c^2d - 6\,ab^5cd^2 + 5\,a^2b^4d^3\right)x^2 - 4\left(10\,b^6c^3 - 36\,ab^5c^2d + 45\,a^2b^4cd^2 - 20\,a^3b^3d^3\right)x}{4\,d^6} + \frac{15\left(b^6c^4 - 4\,ab^5c^3d + 6\,a^2b^4c^2d^2 - 4\,a^3b^3cd^3 + a^4b^2d^4\right)\log\left(dx + c\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^6/(d*x+c)^3,x, algorithm="maxima")

[Out] 1/2*(11*b^6*c^6 - 54*a*b^5*c^5*d + 105*a^2*b^4*c^4*d^2 - 100*a^3*b^3*c^3*d^3 + 45*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 - a^6*d^6 + 12*(b^6*c^5*d - 5*a*b^5*c^4*d^2 + 10*a^2*b^4*c^3*d^3 - 10*a^3*b^3*c^2*d^4 + 5*a^4*b^2*c*d^5 - a^5*b*d^6)*x)/(d^9*x^2 + 2*c*d^8*x + c^2*d^7) + 1/4*(b^6*d^3*x^4 - 4*(b^6*c*d^2 - 2*a*b^5*d^3)*x^3 + 6*(2*b^6*c^2*d - 6*a*b^5*c*d^2 + 5*a^2*b^4*d^3)*x^2 - 4*(10*b^6*c^3 - 36*a*b^5*c^2*d + 45*a^2*b^4*c*d^2 - 20*a^3*b^3*d^3)*x)/d^6 + 15*(b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*log(d*x + c)/d^7

mupad [**B**] time = 0.27, size = 441, normalized size = 2.79

$$x^3\left(\frac{2\,a\,b^5}{d^3}-\frac{b^6\,c}{d^4}\right)-\frac{\frac{a^6\,d^6+6\,a^5\,b\,c\,d^5-45\,a^4\,b^2\,c^2\,d^4+100\,a^3\,b^3\,c^3\,d^3-105\,a^2\,b^4\,c^4\,d^2+54\,a\,b^5\,c^5\,d-11\,b^6\,c^6}{2\,d}-x\,\left(-6\,a^5\,b\,d^5+30\,a^4\,b^2\,c\,d^4-60\,a^3\,b^3\,c^2\,d^3+60\,a^2\,b^4\,c^3\,d^2-30\,a\,b^5\,c^4\,d+6\,b^6\,c^5\right)}{c^2\,d^6+2\,c\,d^7\,x+d^8\,x^2}-x^2\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{2\,d}-\frac{15\,a^2\,b^4}{2\,d^3}+\frac{3\,b^6\,c^2}{2\,d^5}\right)+x\,\left(\frac{3\,c\,\left(\frac{3\,c\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d}-\frac{15\,a^2\,b^4}{d^3}+\frac{3\,b^6\,c^2}{d^5}\right)}{d}+\frac{20\,a^3\,b^3}{d^3}-\frac{b^6\,c^3}{d^6}-\frac{3\,c^2\,\left(\frac{6\,a\,b^5}{d^3}-\frac{3\,b^6\,c}{d^4}\right)}{d^2}\right)+\frac{\ln\left(c+d\,x\right)\,\left(15\,a^4\,b^2\,d^4-60\,a^3\,b^3\,c\,d^3+90\,a^2\,b^4\,c^2\,d^2-60\,a\,b^5\,c^3\,d+15\,b^6\,c^4\right)}{d^7}+\frac{b^6\,x^4}{4\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^6/(c + d*x)^3,x)

[Out] x^3*((2*a*b^5)/d^3 - (b^6*c)/d^4) - ((a^6*d^6 - 11*b^6*c^6 - 105*a^2*b^4*c^4*d^2 + 100*a^3*b^3*c^3*d^3 - 45*a^4*b^2*c^2*d^4 + 54*a*b^5*c^5*d + 6*a^5*b*c*d^5)/(2*d) - x*(6*b^6*c^5 - 6*a^5*b*d^5 + 30*a^4*b^2*c*d^4 + 60*a^2*b^4*c^3*d^2 - 60*a^3*b^3*c^2*d^3 - 30*a*b^5*c^4*d))/(c^2*d^6 + d^8*x^2 + 2*c*d^7*x) - x^2*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/(2*d) - (15*a^2*b^4)/(2*d^3) + (3*b^6*c^2)/(2*d^5)) + x*((3*c*((3*c*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/d - (15*a^2*b^4)/d^3 + (3*b^6*c^2)/d^5))/d + (20*a^3*b^3)/d^3 - (b^6*c^3)/d^6 - (3*c^2*((6*a*b^5)/d^3 - (3*b^6*c)/d^4))/d^2) + (log(c + d*x)*(15*b^6*c^4 + 15*a^4*b^2*d^4 - 60*a^3*b^3*c*d^3 + 90*a^2*b^4*c^2*d^2 - 60*a*b^5*c^3*d))/d^7 + (b^6*x^4)/(4*d^3)

sympy [**B**] time = 2.15, size = 340, normalized size = 2.15

$$\frac{b^6x^4}{4d^3}+\frac{15b^2\left(ad-bc\right)^4\log\left(c+dx\right)}{d^7}+x^3\left(\frac{2ab^5}{d^3}-\frac{b^6c}{d^4}\right)+x^2\left(\frac{15a^2b^4}{2d^3}-\frac{9ab^5c}{d^4}+\frac{3b^6c^2}{d^5}\right)+x\left(\frac{20a^3b^3}{d^3}-\frac{45a^2b^4c}{d^4}+\frac{36ab^5c^2}{d^5}-\frac{10b^6c^3}{d^6}\right)+\frac{-a^6d^6-6a^5bcd^5+45a^4b^2c^2d^4-100a^3b^3c^3d^3+105a^2b^4c^4d^2-54ab^5c^5d+11b^6c^6+x\left(-12a^5bd^6+60a^4b^2cd^5-120a^3b^3c^2d^4+120a^2b^4c^3d^3-60ab^5c^4d^2+12b^6c^5d\right)}{2c^2d^7+4cd^8x+2d^9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**6/(d*x+c)**3,x)

[Out] b**6*x**4/(4*d**3) + 15*b**2*(a*d - b*c)**4*log(c + d*x)/d**7 + x**3*(2*a*b**5/d**3 - b**6*c/d**4) + x**2*(15*a**2*b**4/(2*d**3) - 9*a*b**5*c/d**4 + 3*b**6*c**2/d**5) + x*(20*a**3*b**3/d**3 - 45*a**2*b**4*c/d**4 + 36*a*b**5*c**2/d**5 - 10*b**6*c**3/d**6) + (-a**6*d**6 - 6*a**5*b*c*d**5 + 45*a**4*b**2*c**2*d**4 - 100*a**3*b**3*c**3*d**3 + 105*a**2*b**4*c**4*d**2 - 54*a*b**5*c**5*d + 11*b**6*c**6 + x*(-12*a**5*b*d**6 + 60*a**4*b**2*c*d**5 - 120*a**3*b**3*c**2*d**4 + 120*a**2*b**4*c**3*d**3 - 60*a*b**5*c**4*d**2 + 12*b**6*c**5*d))/(2*c**2*d**7 + 4*c*d**8*x + 2*d**9*x**2)

3.1247 $\int \frac{(a+bx)^5}{(c+dx)^3} \, dx$

Optimal. Leaf size=133

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Rubi [A] time = 0.12, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{5b^4(c+dx)^2(bc-ad)}{2d^6}+\frac{10b^3x(bc-ad)^2}{d^5}-\frac{10b^2(bc-ad)^3\log(c+dx)}{d^6}-\frac{5b(bc-ad)^4}{d^6(c+dx)}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}+\frac{b^5(c+dx)^3}{3d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (10*b^3*(b*c - a*d)^2*x)/d^5 + (b*c - a*d)^5/(2*d^6*(c + d*x)^2) - (5*b*(b*
c - a*d)^4)/(d^6*(c + d*x)) - (5*b^4*(b*c - a*d)*(c + d*x)^2)/(2*d^6) + (b^
5*(c + d*x)^3)/(3*d^6) - (10*b^2*(b*c - a*d)^3*Log[c + d*x])/d^6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^3}\,dx &= \int \left(\frac{10b^3(bc-ad)^2}{d^5}+\frac{(-bc+ad)^5}{d^5(c+dx)^3}+\frac{5b(bc-ad)^4}{d^5(c+dx)^2}-\frac{10b^2(bc-ad)^3}{d^5(c+dx)}-\frac{5b^4(bc-ad)(c+dx)}{d^5}+ b \ldots\right)\\ &= \frac{10b^3(bc-ad)^2x}{d^5}+\frac{(bc-ad)^5}{2d^6(c+dx)^2}-\frac{5b(bc-ad)^4}{d^6(c+dx)}-\frac{5b^4(bc-ad)(c+dx)^2}{2d^6}+\frac{b^5(c+dx)^3}{3d^6}-10b^2(\ldots\end{aligned}$$

Mathematica [A] time = 0.07, size = 230, normalized size = 1.73

$$\frac{-3a^5d^5-15a^4bd^4(c+2dx)+30a^3b^2cd^3(3c+4dx)+30a^2b^3d^2\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+15ab^4d\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)-60b^2(c+dx)^2(bc-ad)^3\log(c+dx)+b^5\left(-27c^5+6c^4dx+63c^3d^2x^2+20c^2d^3x^3-5cd^4x^4+2d^5x^5\right)}{6d^6(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] (-3*a^5*d^5 - 15*a^4*b*d^4*(c + 2*d*x) + 30*a^3*b^2*c*d^3*(3*c + 4*d*x) + 3
0*a^2*b^3*d^2*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + 15*a*b^4*d*(
7*c^4 + 2*c^3*d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + b^5*(-27*c^5
+ 6*c^4*d*x + 63*c^3*d^2*x^2 + 20*c^2*d^3*x^3 - 5*c*d^4*x^4 + 2*d^5*x^5) -
60*b^2*(b*c - a*d)^3*(c + d*x)^2*Log[c + d*x])/(6*d^6*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^3, x]
```

fricas [B] time = 1.20, size = 416, normalized size = 3.13

$$\frac{2 b^5 d^5 x^5 - 27 b^5 c^5 + 105 a b^4 c^4 d - 150 a^2 b^3 c^3 d^2 + 90 a^3 b^2 c^2 d^3 - 15 a^4 b c d^4 - 3 a^5 d^5 - 5 \left(b^5 c d^4 - 3 a b^4 d^5\right) x^4 + 20 \left(b^5 c^2 d^3 - 3 a b^4 c d^4 + 3 a^2 b^3 d^5\right) x^3 + 3 \left(21 b^5 c^3 d^2 - 55 a b^4 c^2 d^3 + 40 a^2 b^3 c d^4\right) x^2 + 6 \left(b^5 c^4 d + 5 a b^4 c^3 d^2 - 20 a^2 b^3 c^2 d^3 + 20 a^3 b^2 c d^4 - 5 a^4 b d^5\right) x - 60 \left(b^5 c^5 - 3 a b^4 c^4 d + 3 a^2 b^3 c^3 d^2 - a^3 b^2 c^2 d^3 + \left(b^5 c^3 d^2 - 3 a b^4 c^2 d^3 + 3 a^2 b^3 c d^4 - a^3 b^2 d^5\right) x^2 + 2 \left(b^5 c^4 d - 3 a b^4 c^3 d^2 + 3 a^2 b^3 c^2 d^3 - a^3 b^2 c d^4\right) x\right) \log\left(dx + c\right)}{6 \left(d^8 x^2 + 2 c d^7 x + c^2 d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^5*d^5*x^5 - 27*b^5*c^5 + 105*a*b^4*c^4*d - 150*a^2*b^3*c^3*d^2 + 9
0*a^3*b^2*c^2*d^3 - 15*a^4*b*c*d^4 - 3*a^5*d^5 - 5*(b^5*c*d^4 - 3*a*b^4*d^5
)*x^4 + 20*(b^5*c^2*d^3 - 3*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 3*(21*b^5*c^
3*d^2 - 55*a*b^4*c^2*d^3 + 40*a^2*b^3*c*d^4)*x^2 + 6*(b^5*c^4*d + 5*a*b^4*c
^3*d^2 - 20*a^2*b^3*c^2*d^3 + 20*a^3*b^2*c*d^4 - 5*a^4*b*d^5)*x - 60*(b^5*c
^5 - 3*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 - a^3*b^2*c^2*d^3 + (b^5*c^3*d^2 - 3
*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 - a^3*b^2*d^5)*x^2 + 2*(b^5*c^4*d - 3*a*b^
4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 - a^3*b^2*c*d^4)*x)*log(d*x + c))/(d^8*x^2 +
2*c*d^7*x + c^2*d^6)
```

giac [B] time = 1.29, size = 264, normalized size = 1.98

$$-\frac{10 \left(b^5 c^3 - 3 a b^4 c^2 d + 3 a^2 b^3 c d^2 - a^3 b^2 d^3\right) \log\left(|dx + c|\right)}{d^6} - \frac{9 b^5 c^5 - 35 a b^4 c^4 d + 50 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 + a^5 d^5 + 10 \left(b^5 c^4 d - 4 a b^4 c^3 d^2 + 6 a^2 b^3 c^2 d^3 - 4 a^3 b^2 c d^4 + a^4 b d^5\right) x}{2 \left(dx + c\right)^2 d^6} + \frac{2 b^5 d^6 x^3 - 9 b^5 c d^5 x^2 + 15 a b^4 d^6 x^2 + 36 b^5 c^2 d^4 x - 90 a b^4 c d^5 x + 60 a^2 b^3 d^6 x}{6 d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(d*x +
 c))/d^6 - 1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a
^2*b^3*c^2*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/((d*x + c)^2*d^6) + 1/6*(2
*b^5*d^6*x^3 - 9*b^5*c*d^5*x^2 + 15*a*b^4*d^6*x^2 + 36*b^5*c^2*d^4*x - 90*a
*b^4*c*d^5*x + 60*a^2*b^3*d^6*x)/d^9
```

maple [B] time = 0.01, size = 346, normalized size = 2.60

$$\frac{b^5 x^3}{3 d^3} - \frac{a^5}{2 \left(dx + c\right)^2 d} + \frac{5 a^4 b c}{2 \left(dx + c\right)^2 d^2} - \frac{5 a^3 b^2 c^2}{\left(dx + c\right)^2 d^3} + \frac{5 a^2 b^3 c^3}{\left(dx + c\right)^2 d^4} - \frac{5 a b^4 c^4}{2 \left(dx + c\right)^2 d^5} + \frac{5 a b^4 x^2}{2 d^3} + \frac{b^5 c^5}{2 \left(dx + c\right)^2 d^6} - \frac{3 b^5 c x^2}{2 d^4} - \frac{5 a^4 b}{\left(dx + c\right) d^2} + \frac{20 a^3 b^2 c}{\left(dx + c\right) d^3} + \frac{10 a^3 b^2 \ln\left(dx + c\right)}{d^3} - \frac{30 a^2 b^3 c^2}{\left(dx + c\right) d^4} - \frac{30 a^2 b^3 c \ln\left(dx + c\right)}{d^4} + \frac{10 a^2 b^3 x}{d^3} + \frac{20 a b^4 c^3}{\left(dx + c\right) d^5} + \frac{30 a b^4 c^2 \ln\left(dx + c\right)}{d^5} - \frac{15 a b^4 c x}{d^4} - \frac{5 b^5 c^4}{\left(dx + c\right) d^6} - \frac{10 b^5 c^3 \ln\left(dx + c\right)}{d^6} + \frac{6 b^5 c^2 x}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^3,x)
```

```
[Out] 1/3*b^5/d^3*x^3+5/2*b^4/d^3*x^2*a-3/2*b^5/d^4*x^2*c+10*b^3/d^3*a^2*x-15*b^4
/d^4*a*c*x+6*b^5/d^5*c^2*x-5*b/d^2/(d*x+c)*a^4+20*b^2/d^3/(d*x+c)*a^3*c-30*
b^3/d^4/(d*x+c)*a^2*c^2+20*b^4/d^5/(d*x+c)*a*c^3-5*b^5/d^6/(d*x+c)*c^4+10*b
^2/d^3*ln(d*x+c)*a^3-30*b^3/d^4*ln(d*x+c)*a^2*c+30*b^4/d^5*ln(d*x+c)*a*c^2-
10*b^5/d^6*ln(d*x+c)*c^3-1/2/d/(d*x+c)^2*a^5+5/2/d^2/(d*x+c)^2*a^4*b*c-5/d^
3/(d*x+c)^2*a^3*b^2*c^2+5/d^4/(d*x+c)^2*a^2*b^3*c^3-5/2/d^5/(d*x+c)^2*a*b^4
*c^4+1/2/d^6/(d*x+c)^2*b^5*c^5
```

maxima [B] time = 1.48, size = 271, normalized size = 2.04

$$-\frac{9 b^5 c^5 - 35 a b^4 c^4 d + 50 a^2 b^3 c^3 d^2 - 30 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 + a^5 d^5 + 10 \left(b^5 c^4 d - 4 a b^4 c^3 d^2 + 6 a^2 b^3 c^2 d^3 - 4 a^3 b^2 c d^4 + a^4 b d^5\right) x}{2 \left(d^8 x^2 + 2 c d^7 x + c^2 d^6\right)} + \frac{2 b^5 d^2 x^3 - 3 \left(3 b^5 c d - 5 a b^4 d^2\right) x^2 + 6 \left(6 b^5 c^2 - 15 a b^4 c d + 10 a^2 b^3 d^2\right) x}{6 d^5} - \frac{10 \left(b^5 c^3 - 3 a b^4 c^2 d + 3 a^2 b^3 c d^2 - a^3 b^2 d^3\right) \log\left(dx + c\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(9*b^5*c^5 - 35*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3
+ 5*a^4*b*c*d^4 + a^5*d^5 + 10*(b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2
*d^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6) + 1/
6*(2*b^5*d^2*x^3 - 3*(3*b^5*c*d - 5*a*b^4*d^2)*x^2 + 6*(6*b^5*c^2 - 15*a*b^
```

```
4*c*d + 10*a^2*b^3*d^2)*x)/d^5 - 10*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*
d^2 - a^3*b^2*d^3)*log(d*x + c)/d^6
```

mupad [B] time = 0.10, size = 291, normalized size = 2.19

$$x^2\left(\frac{5\,a\,b^4}{2\,d^3}-\frac{3\,b^5\,c}{2\,d^4}\right)-\frac{\frac{a^5\,d^5+5\,a^4\,b\,c\,d^4-30\,a^3\,b^2\,c^2\,d^3+50\,a^2\,b^3\,c^3\,d^2-35\,a\,b^4\,c^4\,d+9\,b^5\,c^5}{2\,d}+x\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)}{c^2\,d^5+2\,c\,d^6\,x+d^7\,x^2}-x\,\left(\frac{3\,c\,\left(\frac{5\,a\,b^4}{d^3}-\frac{3\,b^5\,c}{d^4}\right)}{d}-\frac{10\,a^2\,b^3}{d^3}+\frac{3\,b^5\,c^2}{d^5}\right)-\frac{\ln\left(c+d\,x\right)\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)}{d^6}+\frac{b^5\,x^3}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^3,x)
```

```
[Out] x^2*((5*a*b^4)/(2*d^3) - (3*b^5*c)/(2*d^4)) - ((a^5*d^5 + 9*b^5*c^5 + 50*a^
2*b^3*c^3*d^2 - 30*a^3*b^2*c^2*d^3 - 35*a*b^4*c^4*d + 5*a^4*b*c*d^4)/(2*d)
+ x*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a
*b^4*c^3*d))/(c^2*d^5 + d^7*x^2 + 2*c*d^6*x) - x*((3*c*((5*a*b^4)/d^3 - (3*
b^5*c)/d^4))/d - (10*a^2*b^3)/d^3 + (3*b^5*c^2)/d^5) - (log(c + d*x)*(10*b^
5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d))/d^6 + (b^5*x^3
)/(3*d^3)
```

sympy [B] time = 1.65, size = 258, normalized size = 1.94

$$\frac{b^5x^3}{3d^3}+\frac{10b^2\left(ad-bc\right)^3\log\left(c+dx\right)}{d^6}+x^2\left(\frac{5ab^4}{2d^3}-\frac{3b^5c}{2d^4}\right)+x\left(\frac{10a^2b^3}{d^3}-\frac{15ab^4c}{d^4}+\frac{6b^5c^2}{d^5}\right)+\frac{-a^5d^5-5a^4bcd^4+30a^3b^2c^2d^3-50a^2b^3c^3d^2+35ab^4c^4d-9b^5c^5+x\left(-10a^4bd^5+40a^3b^2cd^4-60a^2b^3c^2d^3+40ab^4c^3d^2-10b^5c^4d\right)}{2c^2d^6+4cd^7x+2d^8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**3,x)
```

```
[Out] b**5*x**3/(3*d**3) + 10*b**2*(a*d - b*c)**3*log(c + d*x)/d**6 + x**2*(5*a*b
**4/(2*d**3) - 3*b**5*c/(2*d**4)) + x*(10*a**2*b**3/d**3 - 15*a*b**4*c/d**4
 + 6*b**5*c**2/d**5) + (-a**5*d**5 - 5*a**4*b*c*d**4 + 30*a**3*b**2*c**2*d*
*3 - 50*a**2*b**3*c**3*d**2 + 35*a*b**4*c**4*d - 9*b**5*c**5 + x*(-10*a**4*
b*d**5 + 40*a**3*b**2*c*d**4 - 60*a**2*b**3*c**2*d**3 + 40*a*b**4*c**3*d**2
 - 10*b**5*c**4*d))/(2*c**2*d**6 + 4*c*d**7*x + 2*d**8*x**2)
```

3.1248 $\int \frac{(a+bx)^4}{(c+dx)^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{b^3x(3bc-4ad)}{d^4}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}+\frac{4b(bc-ad)^3}{d^5(c+dx)}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{b^4x^2}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] -((b^3*(3*b*c - 4*a*d)*x)/d^4) + (b^4*x^2)/(2*d^3) - (b*c - a*d)^4/(2*d^5*(
c + d*x)^2) + (4*b*(b*c - a*d)^3)/(d^5*(c + d*x)) + (6*b^2*(b*c - a*d)^2*Lo
g[c + d*x])/d^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^3}\,dx &= \int \left(-\frac{b^3(3bc-4ad)}{d^4}+\frac{b^4x}{d^3}+\frac{(-bc+ad)^4}{d^4(c+dx)^3}-\frac{4b(bc-ad)^3}{d^4(c+dx)^2}+\frac{6b^2(bc-ad)^2}{d^4(c+dx)}\right)dx\\ &= -\frac{b^3(3bc-4ad)x}{d^4}+\frac{b^4x^2}{2d^3}-\frac{(bc-ad)^4}{2d^5(c+dx)^2}+\frac{4b(bc-ad)^3}{d^5(c+dx)}+\frac{6b^2(bc-ad)^2\log(c+dx)}{d^5}\end{aligned}$$

Mathematica [A] time = 0.06, size = 167, normalized size = 1.62

$$\frac{-a^4d^4-4a^3bd^3(c+2dx)+6a^2b^2cd^2(3c+4dx)+4ab^3d\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)+12b^2(c+dx)^2(bc-ad)^2\log(c+dx)+b^4\left(7c^4+2c^3dx-11c^2d^2x^2-4cd^3x^3+d^4x^4\right)}{2d^5(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^3,x]
```

```
[Out] (-(a^4*d^4) - 4*a^3*b*d^3*(c + 2*d*x) + 6*a^2*b^2*c*d^2*(3*c + 4*d*x) + 4*a
*b^3*d*(-5*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) + b^4*(7*c^4 + 2*c^3*
d*x - 11*c^2*d^2*x^2 - 4*c*d^3*x^3 + d^4*x^4) + 12*b^2*(b*c - a*d)^2*(c + d
*x)^2*Log[c + d*x])/(2*d^5*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^3,x]
```

[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^3, x]

fricas [B] time = 0.77, size = 291, normalized size = 2.83

$$\frac{b^{4} d^{4} x^{4} + 7 \, b^{4} c^{4} - 20 \, a b^{3} c^{3} d + 18 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} - a^{4} d^{4} - 4 \, {\left(b^{4} c d^{3} - 2 \, a b^{3} d^{4}\right)} x^{3} - {\left(11 \, b^{4} c^{2} d^{2} - 16 \, a b^{3} c d^{3}\right)} x^{2} + 2 \, {\left(b^{4} c^{3} d - 8 \, a b^{3} c^{2} d^{2} + 12 \, a^{2} b^{2} c d^{3} - 4 \, a^{3} b d^{4}\right)} x + 12 \, {\left(b^{4} c^{4} - 2 \, a b^{3} c^{3} d + a^{2} b^{2} c^{2} d^{2} + {\left(b^{4} c^{2} d^{2} - 2 \, a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right)} x^{2} + 2 \, {\left(b^{4} c^{3} d - 2 \, a b^{3} c^{2} d^{2} + a^{2} b^{2} c d^{3}\right)} x\right)} \log\left(d x + c\right)}{2 \, {\left(d^{7} x^{2} + 2 \, c d^{6} x + c^{2} d^{5}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 - a^4*d^4 - 4*(b^4*c*d^3 - 2*a*b^3*d^4)*x^3 - (11*b^4*c^2*d^2 - 16*
a*b^3*c*d^3)*x^2 + 2*(b^4*c^3*d - 8*a*b^3*c^2*d^2 + 12*a^2*b^2*c*d^3 - 4*a^
3*b*d^4)*x + 12*(b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2 + (b^4*c^2*d^2 -
 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(b^4*c^3*d - 2*a*b^3*c^2*d^2 + a^2*b^
2*c*d^3)*x)*log(d*x + c))/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)
```

giac [A] time = 1.35, size = 183, normalized size = 1.78

$$\frac{6 \, {\left(b^{4} c^{2} - 2 \, a b^{3} c d + a^{2} b^{2} d^{2}\right)} \log\left({\left| d x + c \right|}\right)}{d^{5}} + \frac{b^{4} d^{3} x^{2} - 6 \, b^{4} c d^{2} x + 8 \, a b^{3} d^{3} x}{2 \, d^{6}} + \frac{7 \, b^{4} c^{4} - 20 \, a b^{3} c^{3} d + 18 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} - a^{4} d^{4} + 8 \, {\left(b^{4} c^{3} d - 3 \, a b^{3} c^{2} d^{2} + 3 \, a^{2} b^{2} c d^{3} - a^{3} b d^{4}\right)} x}{2 \, {\left(d x + c\right)}^{2} d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(abs(d*x + c))/d^5 + 1/2*(b^4*d^
3*x^2 - 6*b^4*c*d^2*x + 8*a*b^3*d^3*x)/d^6 + 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*
d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*d^4 + 8*(b^4*c^3*d - 3*a*b^3*c
^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/((d*x + c)^2*d^5)
```

maple [B] time = 0.01, size = 245, normalized size = 2.38

$$-\frac{a^{4}}{2 \left(d x +c \right)^{2} d}+\frac{2 a^{3} b c}{\left(d x +c \right)^{2} d^{2}}-\frac{3 a^{2} b^{2} c^{2}}{\left(d x +c \right)^{2} d^{3}}+\frac{2 a \,b^{3} c^{3}}{\left(d x +c \right)^{2} d^{4}}-\frac{b^{4} c^{4}}{2 \left(d x +c \right)^{2} d^{5}}+\frac{b^{4} x^{2}}{2 d^{3}}-\frac{4 a^{3} b}{\left(d x +c \right) d^{2}}+\frac{12 a^{2} b^{2} c}{\left(d x +c \right) d^{3}}+\frac{6 a^{2} b^{2} \ln \left(d x +c \right)}{d^{3}}-\frac{12 a \,b^{3} c^{2}}{\left(d x +c \right) d^{4}}-\frac{12 a \,b^{3} c \ln \left(d x +c \right)}{d^{4}}+\frac{4 a \,b^{3} x}{d^{3}}+\frac{4 b^{4} c^{3}}{\left(d x +c \right) d^{5}}+\frac{6 b^{4} c^{2} \ln \left(d x +c \right)}{d^{5}}-\frac{3 b^{4} c x}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^3,x)
```

```
[Out] 1/2*b^4*x^2/d^3+4*a*b^3*x/d^3-3*b^4*c*x/d^4-4*b/d^2/(d*x+c)*a^3+12*b^2/d^3/
(d*x+c)*a^2*c-12*b^3/d^4/(d*x+c)*a*c^2+4*b^4/d^5/(d*x+c)*c^3+6*b^2/d^3*ln(d
*x+c)*a^2-12*b^3/d^4*ln(d*x+c)*a*c+6*b^4/d^5*ln(d*x+c)*c^2-1/2/d/(d*x+c)^2*
a^4+2/d^2/(d*x+c)^2*a^3*b*c-3/d^3/(d*x+c)^2*a^2*b^2*c^2+2/d^4/(d*x+c)^2*a*b
^3*c^3-1/2/d^5/(d*x+c)^2*b^4*c^4
```

maxima [A] time = 1.39, size = 191, normalized size = 1.85

$$\frac{7 \, b^{4} c^{4} - 20 \, a b^{3} c^{3} d + 18 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} - a^{4} d^{4} + 8 \, {\left(b^{4} c^{3} d - 3 \, a b^{3} c^{2} d^{2} + 3 \, a^{2} b^{2} c d^{3} - a^{3} b d^{4}\right)} x}{2 \, {\left(d^{7} x^{2} + 2 \, c d^{6} x + c^{2} d^{5}\right)}} + \frac{b^{4} d x^{2} - 2 \, {\left(3 \, b^{4} c - 4 \, a b^{3} d\right)} x}{2 \, d^{4}} + \frac{6 \, {\left(b^{4} c^{2} - 2 \, a b^{3} c d + a^{2} b^{2} d^{2}\right)} \log\left(d x + c\right)}{d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(7*b^4*c^4 - 20*a*b^3*c^3*d + 18*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 - a^4*
d^4 + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*x)/(d^7
*x^2 + 2*c*d^6*x + c^2*d^5) + 1/2*(b^4*d*x^2 - 2*(3*b^4*c - 4*a*b^3*d)*x)/d
^4 + 6*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*log(d*x + c)/d^5
```

mupad [B] time = 0.10, size = 196, normalized size = 1.90

$$x\,\left(\frac{4\,a\,b^3}{d^3}-\frac{3\,b^4\,c}{d^4}\right)-\frac{\frac{a^4\,d^4+4\,a^3\,b\,c\,d^3-18\,a^2\,b^2\,c^2\,d^2+20\,a\,b^3\,c^3\,d-7\,b^4\,c^4}{2\,d}-x\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)}{c^2\,d^4+2\,c\,d^5\,x+d^6\,x^2}+\frac{b^4\,x^2}{2\,d^3}+\frac{\ln\left(c+d\,x\right)\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^3,x)
```

```
[Out] x*((4*a*b^3)/d^3 - (3*b^4*c)/d^4) - ((a^4*d^4 - 7*b^4*c^4 - 18*a^2*b^2*c^2*
d^2 + 20*a*b^3*c^3*d + 4*a^3*b*c*d^3)/(2*d) - x*(4*b^4*c^3 - 4*a^3*b*d^3 +
12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d))/(c^2*d^4 + d^6*x^2 + 2*c*d^5*x) + (b^4*
x^2)/(2*d^3) + (log(c + d*x)*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d))/d^
5
```

sympy [A] time = 1.25, size = 185, normalized size = 1.80

$$\frac{b^4x^2}{2d^3} + \frac{6b^2\,(ad-bc)^2\log{(c+dx)}}{d^5} + x\left(\frac{4ab^3}{d^3} - \frac{3b^4c}{d^4}\right) + \frac{-a^4d^4 - 4a^3bcd^3 + 18a^2b^2c^2d^2 - 20ab^3c^3d + 7b^4c^4 + x\left(-8a^3bd^4 + 24a^2b^2cd^3 - 24ab^3c^2d^2 + 8b^4c^3d\right)}{2c^2d^5 + 4cd^6x + 2d^7x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**3,x)
```

```
[Out] b**4*x**2/(2*d**3) + 6*b**2*(a*d - b*c)**2*log(c + d*x)/d**5 + x*(4*a*b**3/
d**3 - 3*b**4*c/d**4) + (-a**4*d**4 - 4*a**3*b*c*d**3 + 18*a**2*b**2*c**2*d
**2 - 20*a*b**3*c**3*d + 7*b**4*c**4 + x*(-8*a**3*b*d**4 + 24*a**2*b**2*c*d
**3 - 24*a*b**3*c**2*d**2 + 8*b**4*c**3*d))/(2*c**2*d**5 + 4*c*d**6*x + 2*d
**7*x**2)
```

3.1249 $\int \frac{(a+bx)^3}{(c+dx)^3} dx$

Optimal. Leaf size=78

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}-\frac{3b(bc-ad)^2}{d^4(c+dx)}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}+\frac{b^3x}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (b^3*x)/d^3 + (b*c - a*d)^3/(2*d^4*(c + d*x)^2) - (3*b*(b*c - a*d)^2)/(d^4*
(c + d*x)) - (3*b^2*(b*c - a*d)*Log[c + d*x])/d^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^3}\,dx &= \int \left(\frac{b^3}{d^3}+\frac{(-bc+ad)^3}{d^3(c+dx)^3}+\frac{3b(bc-ad)^2}{d^3(c+dx)^2}-\frac{3b^2(bc-ad)}{d^3(c+dx)}\right)dx\\ &= \frac{b^3x}{d^3}+\frac{(bc-ad)^3}{2d^4(c+dx)^2}-\frac{3b(bc-ad)^2}{d^4(c+dx)}-\frac{3b^2(bc-ad)\log(c+dx)}{d^4}\end{aligned}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 1.46

$$\frac{-a^3d^3-3a^2bd^2(c+2dx)+3ab^2cd(3c+4dx)-6b^2(c+dx)^2(bc-ad)\log(c+dx)+b^3\left(-5c^3-4c^2dx+4cd^2x^2+2d^3x^3\right)}{2d^4(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] (-(a^3*d^3) - 3*a^2*b*d^2*(c + 2*d*x) + 3*a*b^2*c*d*(3*c + 4*d*x) + b^3*(-5
*c^3 - 4*c^2*d*x + 4*c*d^2*x^2 + 2*d^3*x^3) - 6*b^2*(b*c - a*d)*(c + d*x)^2
*Log[c + d*x])/(2*d^4*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^3, x]
```

fricas [B] time = 1.40, size = 188, normalized size = 2.41

$$\frac{2 b^3 d^3 x^3 + 4 b^3 c d^2 x^2 - 5 b^3 c^3 + 9 a b^2 c^2 d - 3 a^2 b c d^2 - a^3 d^3 - 2 \left(2 b^3 c^2 d - 6 a b^2 c d^2 + 3 a^2 b d^3\right) x - 6 \left(b^3 c^3 - a b^2 c^2 d + \left(b^3 c d^2 - a b^2 d^3\right) x^2 + 2 \left(b^3 c^2 d - a b^2 c d^2\right) x\right) \log\left(d x + c\right)}{2 \left(d^6 x^2 + 2 c d^5 x + c^2 d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 4*b^3*c*d^2*x^2 - 5*b^3*c^3 + 9*a*b^2*c^2*d - 3*a^2*b*
c*d^2 - a^3*d^3 - 2*(2*b^3*c^2*d - 6*a*b^2*c*d^2 + 3*a^2*b*d^3)*x - 6*(b^3*
c^3 - a*b^2*c^2*d + (b^3*c*d^2 - a*b^2*d^3)*x^2 + 2*(b^3*c^2*d - a*b^2*c*d^
2)*x)*log(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

giac [A] time = 1.28, size = 112, normalized size = 1.44

$$\frac{b^3 x}{d^3} - \frac{3 \left(b^3 c - a b^2 d\right) \log\left(|d x + c|\right)}{d^4} - \frac{5 b^3 c^3 - 9 a b^2 c^2 d + 3 a^2 b c d^2 + a^3 d^3 + 6 \left(b^3 c^2 d - 2 a b^2 c d^2 + a^2 b d^3\right) x}{2 \left(d x + c\right)^2 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*x/d^3 - 3*(b^3*c - a*b^2*d)*log(abs(d*x + c))/d^4 - 1/2*(5*b^3*c^3 - 9*
a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*
b*d^3)*x)/((d*x + c)^2*d^4)
```

maple [B] time = 0.01, size = 160, normalized size = 2.05

$$-\frac{a^3}{2 \left(d x + c\right)^2 d} + \frac{3 a^2 b c}{2 \left(d x + c\right)^2 d^2} - \frac{3 a b^2 c^2}{2 \left(d x + c\right)^2 d^3} + \frac{b^3 c^3}{2 \left(d x + c\right)^2 d^4} - \frac{3 a^2 b}{\left(d x + c\right) d^2} + \frac{6 a b^2 c}{\left(d x + c\right) d^3} + \frac{3 a b^2 \ln\left(d x + c\right)}{d^3} - \frac{3 b^3 c^2}{\left(d x + c\right) d^4} - \frac{3 b^3 c \ln\left(d x + c\right)}{d^4} + \frac{b^3 x}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] b^3/d^3*x-3*b/d^2/(d*x+c)*a^2+6*b^2/d^3/(d*x+c)*a*c-3*b^3/d^4/(d*x+c)*c^2+3
*b^2/d^3*ln(d*x+c)*a-3*b^3/d^4*ln(d*x+c)*c-1/2/d/(d*x+c)^2*a^3+3/2/d^2/(d*x
+c)^2*a^2*b*c-3/2/d^3/(d*x+c)^2*a*b^2*c^2+1/2/d^4/(d*x+c)^2*b^3*c^3
```

maxima [A] time = 1.34, size = 125, normalized size = 1.60

$$\frac{b^3 x}{d^3} - \frac{5 b^3 c^3 - 9 a b^2 c^2 d + 3 a^2 b c d^2 + a^3 d^3 + 6 \left(b^3 c^2 d - 2 a b^2 c d^2 + a^2 b d^3\right) x}{2 \left(d^6 x^2 + 2 c d^5 x + c^2 d^4\right)} - \frac{3 \left(b^3 c - a b^2 d\right) \log\left(d x + c\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^3*x/d^3 - 1/2*(5*b^3*c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2 + a^3*d^3 + 6*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4) -
3*(b^3*c - a*b^2*d)*log(d*x + c)/d^4
```

mupad [B] time = 0.11, size = 130, normalized size = 1.67

$$\frac{b^3\,x}{d^3} - \frac{\ln\left(c + d\,x\right)\,\left(3\,b^3\,c - 3\,a\,b^2\,d\right)}{d^4} - \frac{\frac{a^3\,d^3 + 3\,a^2\,b\,c\,d^2 - 9\,a\,b^2\,c^2\,d + 5\,b^3\,c^3}{2\,d} + x\,\left(3\,a^2\,b\,d^2 - 6\,a\,b^2\,c\,d + 3\,b^3\,c^2\right)}{c^2\,d^3 + 2\,c\,d^4\,x + d^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] (b^3*x)/d^3 - (log(c + d*x)*(3*b^3*c - 3*a*b^2*d))/d^4 - ((a^3*d^3 + 5*b^3*
c^3 - 9*a*b^2*c^2*d + 3*a^2*b*c*d^2)/(2*d) + x*(3*b^3*c^2 + 3*a^2*b*d^2 - 6
*a*b^2*c*d))/(c^2*d^3 + d^5*x^2 + 2*c*d^4*x)
```

sympy [A] time = 0.83, size = 128, normalized size = 1.64

$$\frac{b^3x}{d^3} + \frac{3b^2\left(ad - bc\right)\log\left(c + dx\right)}{d^4} + \frac{-a^3d^3 - 3a^2bcd^2 + 9ab^2c^2d - 5b^3c^3 + x\left(-6a^2bd^3 + 12ab^2cd^2 - 6b^3c^2d\right)}{2c^2d^4 + 4cd^5x + 2d^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] b**3*x/d**3 + 3*b**2*(a*d - b*c)*log(c + d*x)/d**4 + (-a**3*d**3 - 3*a**2*b
*c*d**2 + 9*a*b**2*c**2*d - 5*b**3*c**3 + x*(-6*a**2*b*d**3 + 12*a*b**2*c*d
**2 - 6*b**3*c**2*d))/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2)
```

3.1250 $\int \frac{(a+bx)^2}{(c+dx)^3} dx$

Optimal. Leaf size=59

$$\frac{2b(bc-ad)}{d^3(c+dx)} - \frac{(bc-ad)^2}{2d^3(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3}$$

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(bc-ad)}{d^3(c+dx)} - \frac{(bc-ad)^2}{2d^3(c+dx)^2} + \frac{b^2\log(c+dx)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] -(b*c - a*d)^2/(2*d^3*(c + d*x)^2) + (2*b*(b*c - a*d))/(d^3*(c + d*x)) + (b
^2*Log[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^3}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^3} - \frac{2b(bc-ad)}{d^2(c+dx)^2} + \frac{b^2}{d^2(c+dx)}\right)dx\\ &= -\frac{(bc-ad)^2}{2d^3(c+dx)^2} + \frac{2b(bc-ad)}{d^3(c+dx)} + \frac{b^2\log(c+dx)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 0.81

$$\frac{\frac{(bc-ad)(ad+3bc+4bdx)}{(c+dx)^2} + 2b^2\log(c+dx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] (((b*c - a*d)*(3*b*c + a*d + 4*b*d*x))/(c + d*x)^2 + 2*b^2*Log[c + d*x])/(2
*d^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^3, x]
```

fricas [A] time = 1.44, size = 100, normalized size = 1.69

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\left(b^2cd - abd^2\right)x + 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\log\left(dx + c\right)}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [A] time = 1.30, size = 69, normalized size = 1.17

$$\frac{b^2\log\left(|dx + c|\right)}{d^3} + \frac{4\left(b^2c - abd\right)x + \frac{3\,b^2c^2 - 2\,abcd - a^2d^2}{d}}{2\left(dx + c\right)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^2*log(abs(d*x + c))/d^3 + 1/2*(4*(b^2*c - a*b*d)*x + (3*b^2*c^2 - 2*a*b*c
*d - a^2*d^2)/d)/((d*x + c)^2*d^2)
```

maple [A] time = 0.01, size = 92, normalized size = 1.56

$$-\frac{a^2}{2\left(dx + c\right)^2 d} + \frac{abc}{\left(dx + c\right)^2 d^2} - \frac{b^2c^2}{2\left(dx + c\right)^2 d^3} - \frac{2ab}{\left(dx + c\right)d^2} + \frac{2b^2c}{\left(dx + c\right)d^3} + \frac{b^2\ln\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -2*b/d^2/(d*x+c)*a+2*b^2/d^3/(d*x+c)*c+b^2/d^3*ln(d*x+c)-1/2/d/(d*x+c)^2*a^
2+1/d^2/(d*x+c)^2*a*b*c-1/2/d^3/(d*x+c)^2*b^2*c^2
```

maxima [A] time = 1.33, size = 80, normalized size = 1.36

$$\frac{3\,b^2c^2 - 2\,abcd - a^2d^2 + 4\left(b^2cd - abd^2\right)x}{2\left(d^5x^2 + 2\,cd^4x + c^2d^3\right)} + \frac{b^2\log\left(dx + c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(3*b^2*c^2 - 2*a*b*c*d - a^2*d^2 + 4*(b^2*c*d - a*b*d^2)*x)/(d^5*x^2 +
2*c*d^4*x + c^2*d^3) + b^2*log(d*x + c)/d^3
```

mupad [B] time = 0.23, size = 77, normalized size = 1.31

$$\frac{b^2\,\ln\left(c + d\,x\right)}{d^3} - \frac{\frac{a^2\,d^2 + 2\,a\,b\,c\,d - 3\,b^2\,c^2}{2\,d^3} + \frac{2\,b\,x\,\left(a\,d - b\,c\right)}{d^2}}{c^2 + 2\,c\,d\,x + d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] (b^2*log(c + d*x))/d^3 - ((a^2*d^2 - 3*b^2*c^2 + 2*a*b*c*d)/(2*d^3) + (2*b*
x*(a*d - b*c))/d^2)/(c^2 + d^2*x^2 + 2*c*d*x)
```

sympy [A] time = 0.45, size = 80, normalized size = 1.36

$$\frac{b^2\log(c+dx)}{d^3}+\frac{-a^2d^2-2abcd+3b^2c^2+x\left(-4abd^2+4b^2cd\right)}{2c^2d^3+4cd^4x+2d^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] b**2*log(c + d*x)/d**3 + (-a**2*d**2 - 2*a*b*c*d + 3*b**2*c**2 + x*(-4*a*b*
d**2 + 4*b**2*c*d))/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2)
```

3.1251 $\int \frac{a+bx}{(c+dx)^3} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$\frac{(a+bx)^2}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] (a + b*x)^2/(2*(b*c - a*d)*(c + d*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{a+bx}{(c+dx)^3} dx = \frac{(a+bx)^2}{2(bc-ad)(c+dx)^2}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.93

$$-\frac{ad+b(c+2dx)}{2d^2(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] -1/2*(a*d + b*(c + 2*d*x))/(d^2*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^3,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^3, x]
```

fricas [A] time = 0.99, size = 38, normalized size = 1.36

$$-\frac{2\,bdx+bc+ad}{2\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

giac [A] time = 1.25, size = 24, normalized size = 0.86

$$-\frac{2\,bdx + bc + ad}{2\,(dx + c)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/((d*x + c)^2*d^2)
```

maple [A] time = 0.00, size = 35, normalized size = 1.25

$$-\frac{b}{(dx + c)\,d^2} - \frac{ad - bc}{2\,(dx + c)^2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/(d*x+c)*b/d^2-1/2*(a*d-b*c)/d^2/(d*x+c)^2
```

maxima [A] time = 1.36, size = 38, normalized size = 1.36

$$-\frac{2\,bdx + bc + ad}{2\left(d^4x^2 + 2\,cd^3x + c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*d*x + b*c + a*d)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

mupad [B] time = 0.03, size = 39, normalized size = 1.39

$$-\frac{\frac{a\,d+b\,c}{2\,d^2} + \frac{b\,x}{d}}{c^2 + 2\,c\,d\,x + d^2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^3,x)
```

```
[Out] -((a*d + b*c)/(2*d^2) + (b*x)/d)/(c^2 + d^2*x^2 + 2*c*d*x)
```

sympy [A] time = 0.26, size = 39, normalized size = 1.39

$$\frac{-ad - bc - 2bdx}{2c^2d^2 + 4cd^3x + 2d^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**3,x)
```

```
[Out] (-a*d - b*c - 2*b*d*x)/(2*c**2*d**2 + 4*c*d**3*x + 2*d**4*x**2)
```

3.1252 $\int \frac{1}{(c+dx)^3} dx$

Optimal. Leaf size=14

$$-\frac{1}{2d(c+dx)^2}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3),x]
```

```
[Out] -1/(2*d*(c + d*x)^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^3} dx = -\frac{1}{2d(c+dx)^2}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3),x]
```

```
[Out] -1/2*1/(d*(c + d*x)^2)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-3),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-3), x]
```

fricas [A] time = 1.24, size = 24, normalized size = 1.71

$$-\frac{1}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

giac [A] time = 1.20, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="giac")
[Out] -1/2/((d*x + c)^2*d)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{2\,(dx+c)^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^3,x)
[Out] -1/2/d/(d*x+c)^2
```

maxima [A] time = 1.34, size = 12, normalized size = 0.86

$$-\frac{1}{2\,(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^3,x, algorithm="maxima")
[Out] -1/2/((d*x + c)^2*d)
```

mupad [B] time = 0.02, size = 26, normalized size = 1.86

$$-\frac{1}{2\,c^2\,d+4\,c\,d^2\,x+2\,d^3\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^3,x)
[Out] -1/(2*c^2*d + 2*d^3*x^2 + 4*c*d^2*x)
```

sympy [B] time = 0.18, size = 26, normalized size = 1.86

$$-\frac{1}{2c^2d+4cd^2x+2d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**3,x)
[Out] -1/(2*c**2*d + 4*c*d**2*x + 2*d**3*x**2)
```

3.1253 $\int \frac{1}{(a+bx)(c+dx)^3} dx$

Optimal. Leaf size=82

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] 1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*Log[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)^3}\,dx &= \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{d}{(bc-ad)(c+dx)^3}-\frac{bd}{(bc-ad)^2(c+dx)^2}-\frac{b^2d}{(bc-ad)^3(c+dx)}\right)dx\\ &= \frac{1}{2(bc-ad)(c+dx)^2}+\frac{b}{(bc-ad)^2(c+dx)}+\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.82

$$\frac{2b^2\log(a+bx)+\frac{(bc-ad)(-ad+3bc+2bdx)}{(c+dx)^2}-2b^2\log(c+dx)}{2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] (((b*c - a*d)*(3*b*c - a*d + 2*b*d*x))/(c + d*x)^2 + 2*b^2*Log[a + b*x] - 2*b^2*Log[c + d*x])/(2*(b*c - a*d)^3)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^3), x]
```

fricas [B] time = 1.20, size = 242, normalized size = 2.95

$$\frac{3 b^2c^2 - 4 abcd + a^2d^2 + 2 \left(b^2cd - abd^2\right)x + 2 \left(b^2d^2x^2 + 2 b^2cdx + b^2c^2\right)\log\left(bx + a\right) - 2 \left(b^2d^2x^2 + 2 b^2cdx + b^2c^2\right)\log\left(dx + c\right)}{2 \left(b^3c^5 - 3 ab^2c^4d + 3 a^2bc^3d^2 - a^3c^2d^3 + \left(b^3c^3d^2 - 3 ab^2c^2d^3 + 3 a^2bcd^4 - a^3d^5\right)x^2 + 2 \left(b^3c^4d - 3 ab^2c^3d^2 + 3 a^2bc^2d^3 - a^3cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^
2*d^3 + (b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - a^3*d^5)*x^2 + 2*(
b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^3 - a^3*c*d^4)*x)
```

giac [B] time = 1.36, size = 165, normalized size = 2.01

$$\frac{b^3\log\left(|bx + a|\right)}{b^4c^3 - 3 ab^3c^2d + 3 a^2b^2cd^2 - a^3bd^3} - \frac{b^2d\log\left(|dx + c|\right)}{b^3c^3d - 3 ab^2c^2d^2 + 3 a^2bcd^3 - a^3d^4} + \frac{3 b^2c^2 - 4 abcd + a^2d^2 + 2 \left(b^2cd - abd^2\right)x}{2 \left(bc - ad\right)^3\left(dx + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^
3) - b^2*d*log(abs(d*x + c))/(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
 a^3*d^4) + 1/2*(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x)
/((b*c - a*d)^3*(d*x + c)^2)
```

maple [A] time = 0.01, size = 81, normalized size = 0.99

$$-\frac{b^2\ln\left(bx + a\right)}{\left(ad - bc\right)^3} + \frac{b^2\ln\left(dx + c\right)}{\left(ad - bc\right)^3} + \frac{b}{\left(ad - bc\right)^2\left(dx + c\right)} - \frac{1}{2\left(ad - bc\right)\left(dx + c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^3,x)
```

```
[Out] -1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/(d*x+c)-b^
2/(a*d-b*c)^3*ln(b*x+a)
```

maxima [B] time = 1.45, size = 202, normalized size = 2.46

$$\frac{b^2\log\left(bx + a\right)}{b^3c^3 - 3 ab^2c^2d + 3 a^2bcd^2 - a^3d^3} - \frac{b^2\log\left(dx + c\right)}{b^3c^3 - 3 ab^2c^2d + 3 a^2bcd^2 - a^3d^3} + \frac{2 bdx + 3 bc - ad}{2 \left(b^2c^4 - 2 abc^3d + a^2c^2d^2 + \left(b^2c^2d^2 - 2 abcd^3 + a^2d^4\right)x^2 + 2 \left(b^2c^3d - 2 abc^2d^2 + a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] b^2*log(b*x + a)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) - b^2*
log(d*x + c)/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3) + 1/2*(2*b
*d*x + 3*b*c - a*d)/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

mupad [B] time = 0.30, size = 183, normalized size = 2.23

$$-\frac{\frac{a\,d-3\,b\,c}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{b\,d\,x}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}}{c^2+2\,c\,d\,x+d^2\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^3\,d^3-a^2\,b\,c\,d^2-a\,b^2\,c^2\,d+b^3\,c^3}{{\left(a\,d-b\,c\right)}^3}+\frac{2\,b\,d\,x\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3}\right)}{{\left(a\,d-b\,c\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^3),x)
```

```
[Out] - ((a*d - 3*b*c)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (b*d*x)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d))/(c^2 + d^2*x^2 + 2*c*d*x) - (2*b^2*atanh((a^3*d^3 + b^
3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2)/(a*d - b*c)^3 + (2*b*d*x*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(a*d - b*c)^3
```

sympy [B] time = 1.07, size = 381, normalized size = 4.65

$$\frac{b^2\log\left(x+\frac{-\frac{a^4b^2d^4}{(ad-bc)^3}+\frac{4a^3b^3cd^3}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2}{(ad-bc)^3}+\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d-\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{(ad-bc)^3}-\frac{b^2\log\left(x+\frac{\frac{a^4b^2d^4}{(ad-bc)^3}-\frac{4a^3b^3cd^3}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2}{(ad-bc)^3}-\frac{4ab^5c^3d}{(ad-bc)^3}+ab^2d+\frac{b^6c^4}{(ad-bc)^3}+b^3c}{2b^3d}\right)}{(ad-bc)^3}+\frac{-ad+3bc+2bdx}{2a^2c^2d^2-4abc^3d+2b^2c^4+x^2\left(2a^2d^4-4abcd^3+2b^2c^2d^2\right)+x\left(4a^2cd^3-8abc^2d^2+4b^2c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**3,x)
```

```
[Out] b**2*log(x + (-a**4*b**2*d**4/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3/(a*d - b*
c)**3 - 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 + 4*a*b**5*c**3*d/(a*d - b*c)*
*3 + a*b**2*d - b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b*c)*
*3 - b**2*log(x + (a**4*b**2*d**4/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3/(a*d
- b*c)**3 + 6*a**2*b**4*c**2*d**2/(a*d - b*c)**3 - 4*a*b**5*c**3*d/(a*d - b
*c)**3 + a*b**2*d + b**6*c**4/(a*d - b*c)**3 + b**3*c)/(2*b**3*d))/(a*d - b
*c)**3 + (-a*d + 3*b*c + 2*b*d*x)/(2*a**2*c**2*d**2 - 4*a*b*c**3*d + 2*b**2
*c**4 + x**2*(2*a**2*d**4 - 4*a*b*c*d**3 + 2*b**2*c**2*d**2) + x*(4*a**2*c*
d**3 - 8*a*b*c**2*d**2 + 4*b**2*c**3*d))
```

3.1254 $\int \frac{1}{(a+bx)^2(c+dx)^3} dx$

Optimal. Leaf size=110

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^2}{(a+bx)(bc-ad)^3}-\frac{3b^2d\log(a+bx)}{(bc-ad)^4}+\frac{3b^2d\log(c+dx)}{(bc-ad)^4}-\frac{2bd}{(c+dx)(bc-ad)^3}-\frac{d}{2(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -(b^2/((b*c - a*d)^3*(a + b*x))) - d/(2*(b*c - a*d)^2*(c + d*x)^2) - (2*b*d
)/((b*c - a*d)^3*(c + d*x)) - (3*b^2*d*Log[a + b*x])/(b*c - a*d)^4 + (3*b^2
*d*Log[c + d*x])/(b*c - a*d)^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^3}dx = \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^2}-\frac{3b^3d}{(bc-ad)^4(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^3}+\frac{2bd^2}{(bc-ad)^3(c\cdot\right.$$
$$=-\frac{b^2}{(bc-ad)^3(a+bx)}-\frac{d}{2(bc-ad)^2(c+dx)^2}-\frac{2bd}{(bc-ad)^3(c+dx)}-\frac{3b^2d\log(a+bx}{(bc-ad)^4}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 0.88

$$-\frac{\frac{2b^2(bc-ad)}{a+bx}+6b^2d\log(a+bx)+\frac{4bd(bc-ad)}{c+dx}+\frac{d(bc-ad)^2}{(c+dx)^2}-6b^2d\log(c+dx)}{2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] -1/2*((2*b^2*(b*c - a*d))/(a + b*x) + (d*(b*c - a*d)^2)/(c + d*x)^2 + (4*b*
d*(b*c - a*d))/(c + d*x) + 6*b^2*d*Log[a + b*x] - 6*b^2*d*Log[c + d*x])/(b*
c - a*d)^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^3}dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^3), x]
```

fricas [B] time = 1.61, size = 495, normalized size = 4.50

$$-\frac{2 b^3c^3 + 3 ab^2c^2d - 6 a^2bcd^2 + a^3d^3 + 6 \left(b^3cd^2 - ab^2d^3\right)x^2 + 3 \left(3 b^3c^2d - 2 ab^2cd^2 - a^2bd^3\right)x + 6 \left(b^3d^3x^3 + ab^2c^2d + \left(2 b^3cd^2 + ab^2d^3\right)x^2 + \left(b^3c^2d + 2 ab^2cd^2\right)x\right)\log\left(bx + a\right) - 6 \left(b^3d^3x^3 + ab^2c^2d + \left(2 b^3cd^2 + ab^2d^3\right)x^2 + \left(b^3c^2d + 2 ab^2cd^2\right)x\right)\log\left(dx + c\right)}{2 \left(ab^4c^6 - 4 a^2b^3c^5d + 6 a^3b^2c^4d^2 - 4 a^4bc^3d^3 + a^5c^2d^4 + \left(b^5c^4d^2 - 4 ab^4c^3d^3 + 6 a^2b^3c^2d^4 - 4 a^3b^2cd^5 + a^4bd^6\right)x^3 + \left(2 b^5c^5d - 7 ab^4c^4d^2 + 8 a^2b^3c^3d^3 - 2 a^3b^2c^2d^4 - 2 a^4bcd^5 + a^5d^6\right)x^2 + \left(b^5c^6 - 2 ab^4c^5d - 2 a^2b^3c^4d^2 + 8 a^3b^2c^3d^3 - 7 a^4bc^2d^4 + 2 a^5cd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*c^3 + 3*a*b^2*c^2*d - 6*a^2*b*c*d^2 + a^3*d^3 + 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 + 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x + 6*(b^3*d^3
*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c
*d^2)*x)*log(b*x + a) - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2
*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(d*x + c))/(a*b^4*c^6 - 4*a^2
*b^3*c^5*d + 6*a^3*b^2*c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4 + (b^5*c^4*d
^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 + a^4*b*d^6)*x^3
 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 -
 2*a^4*b*c*d^5 + a^5*d^6)*x^2 + (b^5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^
2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 + 2*a^5*c*d^5)*x)
```

giac [B] time = 1.27, size = 217, normalized size = 1.97

$$\frac{3 b^3d \log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^5c^4 - 4 ab^4c^3d + 6 a^2b^3c^2d^2 - 4 a^3b^2cd^3 + a^4bd^4} - \frac{b^5}{\left(b^6c^3 - 3 ab^5c^2d + 3 a^2b^4cd^2 - a^3b^3d^3\right)(bx + a)} + \frac{5 b^2d^3 + \frac{6\left(b^4cd^2 - ab^3d^3\right)}{(bx+a)b}}{2 (bc - ad)^4\left(\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 3*b^3*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^5*c^4 - 4*a*b^4*c^3*
d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - b^5/((b^6*c^3 - 3*a*
b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*(b*x + a)) + 1/2*(5*b^2*d^3 + 6*
(b^4*c*d^2 - a*b^3*d^3)/((b*x + a)*b))/((b*c - a*d)^4*(b*c/(b*x + a) - a*d/
(b*x + a) + d)^2)
```

maple [A] time = 0.01, size = 108, normalized size = 0.98

$$-\frac{3b^2d\ln(bx + a)}{(ad - bc)^4} + \frac{3b^2d\ln(dx + c)}{(ad - bc)^4} + \frac{b^2}{(ad - bc)^3 (bx + a)} + \frac{2bd}{(ad - bc)^3 (dx + c)} - \frac{d}{2 (ad - bc)^2 (dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] -1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/(a*d-b*c)^3*
b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a)
```

maxima [B] time = 1.59, size = 386, normalized size = 3.51

$$-\frac{3 b^2d \log\left(bx + a\right)}{b^4c^4 - 4 ab^3c^3d + 6 a^2b^2c^2d^2 - 4 a^3bcd^3 + a^4d^4} + \frac{3 b^2d \log\left(dx + c\right)}{b^4c^4 - 4 ab^3c^3d + 6 a^2b^2c^2d^2 - 4 a^3bcd^3 + a^4d^4} - \frac{6 b^2d^2x^2 + 2 b^2c^2 + 5 abcd - a^2d^2 + 3 \left(3 b^2cd + abd^2\right)x}{2 \left(ab^3c^5 - 3 a^2b^2c^4d + 3 a^3bc^3d^2 - a^4c^2d^3 + \left(b^4c^3d^2 - 3 ab^3c^2d^3 + 3 a^2b^2cd^4 - a^3bd^5\right)x^3 + \left(2 b^4c^4d - 5 ab^3c^3d^2 + 3 a^2b^2c^2d^3 + a^3bcd^4 - a^4d^5\right)x^2 + \left(b^4c^5 - ab^3c^4d - 3 a^2b^2c^3d^2 + 5 a^3bc^2d^3 - 2 a^4cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -3*b^2*d*log(b*x + a)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4) + 3*b^2*d*log(d*x + c)/(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4) - 1/2*(6*b^2*d^2*x^2 + 2*b^2*c^2 + 5
*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d^2)*x)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
```

```
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

mupad [B] time = 0.40, size = 329, normalized size = 2.99

$$\frac{\frac{-a^2\,d^2+5\,a\,b\,c\,d+2\,b^2\,c^2}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b\,x\,\left(a\,d^2+3\,b\,c\,d\right)}{2\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}+\frac{3\,b^2\,d^2\,x^2}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}}{x\,\left(b\,c^2+2\,a\,d\,c\right)+a\,c^2+x^2\,\left(a\,d^2+2\,b\,c\,d\right)+b\,d^2\,x^3}-\frac{6\,b^2\,d\,\mathrm{atanh}\left(\frac{a^4\,d^4-2\,a^3\,b\,c\,d^3+2\,a\,b^3\,c^3\,d-b^4\,c^4}{{\left(a\,d-b\,c\right)}^4}+\frac{2\,b\,d\,x\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^4}\right)}{{\left(a\,d-b\,c\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^3),x)
```

```
[Out] ((2*b^2*c^2 - a^2*d^2 + 5*a*b*c*d)/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d -
3*a^2*b*c*d^2)) + (3*b*x*(a*d^2 + 3*b*c*d))/(2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2)) + (3*b^2*d^2*x^2)/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(x*(b*c^2 + 2*a*c*d) + a*c^2 + x^2*(a*d^2 + 2*b*c*d) +
 b*d^2*x^3) - (6*b^2*d*atanh((a^4*d^4 - b^4*c^4 + 2*a*b^3*c^3*d - 2*a^3*b*c
*d^3)/(a*d - b*c)^4 + (2*b*d*x*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))/(a*d - b*c)^4))/(a*d - b*c)^4
```

sympy [B] time = 1.72, size = 632, normalized size = 5.75

$$\frac{3b^2d\log\left(x+\frac{-\frac{3a^5b^2d^6}{(ad-bc)^4}+\frac{15a^4b^3cd^5}{(ad-bc)^4}-\frac{30a^3b^4c^2d^4}{(ad-bc)^4}+\frac{30a^2b^5c^3d^3}{(ad-bc)^4}-\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2+\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}-\frac{3b^2d\log\left(x+\frac{\frac{3a^5b^2d^6}{(ad-bc)^4}-\frac{15a^4b^3cd^5}{(ad-bc)^4}+\frac{30a^3b^4c^2d^4}{(ad-bc)^4}-\frac{30a^2b^5c^3d^3}{(ad-bc)^4}+\frac{15ab^6c^4d^2}{(ad-bc)^4}+3ab^2d^2-\frac{3b^7c^5d}{(ad-bc)^4}+3b^3cd}{6b^3d^2}\right)}{(ad-bc)^4}+\frac{-a^2d^2+5abcd+2b^2c^2+6b^2d^2x^2+x\left(3abd^2+9b^2cd\right)}{2a^4c^2d^3-6a^3bc^3d^2+6a^2b^2c^4d-2ab^3c^5+x^3\left(2a^3bd^5-6a^2b^2cd^4+6ab^3c^2d^3-2b^4c^3d^2\right)+x^2\left(2a^4d^5-2a^3bcd^4-6a^2b^2c^2d^3+10ab^3c^3d^2-4b^4c^4d\right)+x\left(4a^4cd^4-10a^3bc^2d^3+6a^2b^2c^3d^2+2ab^3c^4d-2b^4c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] 3*b**2*d*log(x + (-3*a**5*b**2*d**6/(a*d - b*c)**4 + 15*a**4*b**3*c*d**5/(a
*d - b*c)**4 - 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 + 30*a**2*b**5*c**3*d*
*3/(a*d - b*c)**4 - 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 + 3*
b**7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 - 3*
b**2*d*log(x + (3*a**5*b**2*d**6/(a*d - b*c)**4 - 15*a**4*b**3*c*d**5/(a*d
- b*c)**4 + 30*a**3*b**4*c**2*d**4/(a*d - b*c)**4 - 30*a**2*b**5*c**3*d**3/
(a*d - b*c)**4 + 15*a*b**6*c**4*d**2/(a*d - b*c)**4 + 3*a*b**2*d**2 - 3*b**
7*c**5*d/(a*d - b*c)**4 + 3*b**3*c*d)/(6*b**3*d**2))/(a*d - b*c)**4 + (-a**
2*d**2 + 5*a*b*c*d + 2*b**2*c**2 + 6*b**2*d**2*x**2 + x*(3*a*b*d**2 + 9*b**
2*c*d))/(2*a**4*c**2*d**3 - 6*a**3*b*c**3*d**2 + 6*a**2*b**2*c**4*d - 2*a*b
**3*c**5 + x**3*(2*a**3*b*d**5 - 6*a**2*b**2*c*d**4 + 6*a*b**3*c**2*d**3 -
2*b**4*c**3*d**2) + x**2*(2*a**4*d**5 - 2*a**3*b*c*d**4 - 6*a**2*b**2*c**2*
d**3 + 10*a*b**3*c**3*d**2 - 4*b**4*c**4*d) + x*(4*a**4*c*d**4 - 10*a**3*b*
c**2*d**3 + 6*a**2*b**2*c**3*d**2 + 2*a*b**3*c**4*d - 2*b**4*c**5))
```

3.1255 $\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$

Optimal. Leaf size=143

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(}$$

Rubi [A] time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$\frac{6b^2d^2\log(a+bx)}{(bc-ad)^5}-\frac{6b^2d^2\log(c+dx)}{(bc-ad)^5}+\frac{3b^2d}{(a+bx)(bc-ad)^4}-\frac{b^2}{2(a+bx)^2(bc-ad)^3}+\frac{3bd^2}{(c+dx)(bc-ad)^4}+\frac{d^2}{2(c+dx)^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] -b^2/(2*(b*c - a*d)^3*(a + b*x)^2) + (3*b^2*d)/((b*c - a*d)^4*(a + b*x)) +
d^2/(2*(b*c - a*d)^3*(c + d*x)^2) + (3*b*d^2)/((b*c - a*d)^4*(c + d*x)) + (
6*b^2*d^2*Log[a + b*x])/(b*c - a*d)^5 - (6*b^2*d^2*Log[c + d*x])/(b*c - a*d
)^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx = \int \left(\frac{b^3}{(bc-ad)^3(a+bx)^3}-\frac{3b^3d}{(bc-ad)^4(a+bx)^2}+\frac{6b^3d^2}{(bc-ad)^5(a+bx)}-\frac{d^3}{(bc-ad)^3(c+d\right.$$

$$= -\frac{b^2}{2(bc-ad)^3(a+bx)^2}+\frac{3b^2d}{(bc-ad)^4(a+bx)}+\frac{d^2}{2(bc-ad)^3(c+dx)^2}+\frac{3bd^2}{(bc-ad)^4(c+d}$$

Mathematica [A] time = 0.12, size = 128, normalized size = 0.90

$$\frac{\frac{6b^2d(bc-ad)}{a+bx}-\frac{b^2(bc-ad)^2}{(a+bx)^2}+12b^2d^2\log(a+bx)+\frac{6bd^2(bc-ad)}{c+dx}+\frac{d^2(bc-ad)^2}{(c+dx)^2}-12b^2d^2\log(c+dx)}{2(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] (-((b^2*(b*c - a*d)^2)/(a + b*x)^2) + (6*b^2*d*(b*c - a*d))/(a + b*x) + (d^
2*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*d^2*(b*c - a*d))/(c + d*x) + 12*b^2*d^2
*Log[a + b*x] - 12*b^2*d^2*Log[c + d*x])/(2*(b*c - a*d)^5)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^3),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^3), x]
```

fricas [**B**] time = 1.18, size = 760, normalized size = 5.31

$$-\frac{b^{4} c^{4} - 8 a b^{3} c^{3} d + 8 a^{3} b c d^{3} - a^{4} d^{4} - 12 \left(b^{4} c d^{3} - a b^{3} d^{4}\right) x^{3} - 18 \left(b^{4} c^{2} d^{2} - a^{2} b^{2} d^{4}\right) x^{2} - 4 \left(b^{4} c^{3} d + 6 a b^{3} c^{2} d^{2} - 6 a^{2} b^{2} c d^{3} - a^{3} b d^{4}\right) x - 12 \left(b^{4} d^{4} x^{4} + a^{2} b^{2} c^{2} d^{2} + 2 \left(b^{4} c d^{3} + a b^{3} d^{4}\right) x^{3} + \left(b^{4} c^{2} d^{2} + 4 a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right) x^{2} + 2 \left(a b^{3} c^{2} d^{2} + a^{2} b^{2} c d^{3}\right) x\right) \log\left(b x + a\right) + 12 \left(b^{4} d^{4} x^{4} + a^{2} b^{2} c^{2} d^{2} + 2 \left(b^{4} c d^{3} + a b^{3} d^{4}\right) x^{3} + \left(b^{4} c^{2} d^{2} + 4 a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right) x^{2} + 2 \left(a b^{3} c^{2} d^{2} + a^{2} b^{2} c d^{3}\right) x\right) \log\left(d x + c\right)}{2 \left(a^{2} b^{5} c^{7} - 5 a^{3} b^{4} c^{6} d + 10 a^{4} b^{3} c^{5} d^{2} - 10 a^{5} b^{2} c^{4} d^{3} + 5 a^{6} b c^{3} d^{4} - a^{7} c^{2} d^{5} + \left(b^{7} c^{5} d^{2} - 5 a b^{6} c^{4} d^{3} + 10 a^{2} b^{5} c^{3} d^{4} - 10 a^{3} b^{4} c^{2} d^{5} + 5 a^{4} b^{3} c d^{6} - a^{5} b^{2} d^{7}\right) x^{4} + 2 \left(b^{7} c^{6} d - 4 a b^{6} c^{5} d^{2} + 5 a^{2} b^{5} c^{4} d^{3} - 5 a^{4} b^{3} c^{2} d^{5} + 4 a^{5} b^{2} c d^{6} - a^{6} b d^{7}\right) x^{3} + \left(b^{7} c^{7} - a b^{6} c^{6} d - 9 a^{2} b^{5} c^{5} d^{2} + 25 a^{3} b^{4} c^{4} d^{3} - 25 a^{4} b^{3} c^{3} d^{4} + 9 a^{5} b^{2} c^{2} d^{5} + a^{6} b c d^{6} - a^{7} d^{7}\right) x^{2} + 2 \left(a b^{6} c^{7} - 4 a^{2} b^{5} c^{6} d + 5 a^{3} b^{4} c^{5} d^{2} - 5 a^{5} b^{2} c^{3} d^{4} + 4 a^{6} b c^{2} d^{5} - a^{7} c d^{6}\right) x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^4*c^4 - 8*a*b^3*c^3*d + 8*a^3*b*c*d^3 - a^4*d^4 - 12*(b^4*c*d^3 - a
*b^3*d^4)*x^3 - 18*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^2 - 4*(b^4*c^3*d + 6*a*b^3
*c^2*d^2 - 6*a^2*b^2*c*d^3 - a^3*b*d^4)*x - 12*(b^4*d^4*x^4 + a^2*b^2*c^2*d
^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2
*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a) + 12*(b^4*d^4
*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a
*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)*x)*log(d*
x + c))/(a^2*b^5*c^7 - 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^
4*d^3 + 5*a^6*b*c^3*d^4 - a^7*c^2*d^5 + (b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10
*a^2*b^5*c^3*d^4 - 10*a^3*b^4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*x^4
+ 2*(b^7*c^6*d - 4*a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 +
4*a^5*b^2*c*d^6 - a^6*b*d^7)*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d
^2 + 25*a^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*
d^6 - a^7*d^7)*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 - 5
*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*x)
```

giac [**B**] time = 1.28, size = 345, normalized size = 2.41

$$\frac{6 \, b^{3} d^{2} \log\left({\left| b x + a \right|}\right)}{b^{6} c^{5} - 5 \, a b^{5} c^{4} d + 10 \, a^{2} b^{4} c^{3} d^{2} - 10 \, a^{3} b^{3} c^{2} d^{3} + 5 \, a^{4} b^{2} c d^{4} - a^{5} b d^{5}} - \frac{6 \, b^{2} d^{3} \log\left({\left| d x + c \right|}\right)}{b^{5} c^{5} d - 5 \, a b^{4} c^{4} d^{2} + 10 \, a^{2} b^{3} c^{3} d^{3} - 10 \, a^{3} b^{2} c^{2} d^{4} + 5 \, a^{4} b c d^{5} - a^{5} d^{6}} + \frac{12 \, b^{3} d^{3} x^{3} + 18 \, b^{3} c d^{2} x^{2} + 18 \, a b^{2} d^{3} x^{2} + 4 \, b^{3} c^{2} d x + 28 \, a b^{2} c d^{2} x + 4 \, a^{2} b d^{3} x - b^{3} c^{3} + 7 \, a b^{2} c^{2} d + 7 \, a^{2} b c d^{2} - a^{3} d^{3}}{2 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} {\left(b d x^{2} + b c x + a d x + a c\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 6*b^3*d^2*log(abs(b*x + a))/(b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5) - 6*b^2*d^3*log(abs(d*x
+ c))/(b^5*c^5*d - 5*a*b^4*c^4*d^2 + 10*a^2*b^3*c^3*d^3 - 10*a^3*b^2*c^2*d^
4 + 5*a^4*b*c*d^5 - a^5*d^6) + 1/2*(12*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*
a*b^2*d^3*x^2 + 4*b^3*c^2*d*x + 28*a*b^2*c*d^2*x + 4*a^2*b*d^3*x - b^3*c^3
+ 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*d*x^2 + b*c*x + a*d*x + a*c)^2)
```

maple [**A**] time = 0.01, size = 140, normalized size = 0.98

$$-\frac{6 b^{2} d^{2} \ln \left(b x +a \right)}{\left(a d -b c \right)^{5}}+\frac{6 b^{2} d^{2} \ln \left(d x +c \right)}{\left(a d -b c \right)^{5}}+\frac{3 b^{2} d}{\left(a d -b c \right)^{4} \left(b x +a \right)}+\frac{3 b \,d^{2}}{\left(a d -b c \right)^{4} \left(d x +c \right)}+\frac{b^{2}}{2 \left(a d -b c \right)^{3} \left(b x +a \right)^{2}}-\frac{d^{2}}{2 \left(a d -b c \right)^{3} \left(d x +c \right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] -1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+3*d^2/(a*d-b
*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^5*b^2*ln(b*x+
a)+3*b^2/(a*d-b*c)^4*d/(b*x+a)
```

maxima [**B**] time = 1.55, size = 594, normalized size = 4.15

$$\frac{6 \, b^{2} d^{2} \log\left(b x + a\right)}{b^{5} c^{5} - 5 \, a b^{4} c^{4} d + 10 \, a^{2} b^{3} c^{3} d^{2} - 10 \, a^{3} b^{2} c^{2} d^{3} + 5 \, a^{4} b c d^{4} - a^{5} d^{5}} - \frac{6 \, b^{2} d^{2} \log\left(d x + c\right)}{b^{5} c^{5} - 5 \, a b^{4} c^{4} d + 10 \, a^{2} b^{3} c^{3} d^{2} - 10 \, a^{3} b^{2} c^{2} d^{3} + 5 \, a^{4} b c d^{4} - a^{5} d^{5}} + \frac{12 \, b^{3} d^{3} x^{3} - b^{3} c^{3} + 7 \, a b^{2} c^{2} d + 7 \, a^{2} b c d^{2} - a^{3} d^{3} + 18 \, {\left(b^{3} c d^{2} + a b^{2} d^{3}\right)} x^{2} + 4 \, {\left(b^{3} c^{2} d + 7 \, a b^{2} c d^{2} + a^{2} b d^{3}\right)} x}{2 \, {\left(a^{2} b^{4} c^{6} - 4 \, a^{3} b^{3} c^{5} d + 6 \, a^{4} b^{2} c^{4} d^{2} - 4 \, a^{5} b c^{3} d^{3} + a^{6} c^{2} d^{4} + {\left(b^{6} c^{4} d^{2} - 4 \, a b^{5} c^{3} d^{3} + 6 \, a^{2} b^{4} c^{2} d^{4} - 4 \, a^{3} b^{3} c d^{5} + a^{4} b^{2} d^{6}\right)} x^{4} + 2 \, {\left(b^{6} c^{5} d - 3 \, a b^{5} c^{4} d^{2} + 2 \, a^{2} b^{4} c^{3} d^{3} + 2 \, a^{3} b^{3} c^{2} d^{4} - 3 \, a^{4} b^{2} c d^{5} + a^{5} b d^{6}\right)} x^{3} + {\left(b^{6} c^{6} - 9 \, a^{2} b^{4} c^{4} d^{2} + 16 \, a^{3} b^{3} c^{3} d^{3} - 9 \, a^{4} b^{2} c^{2} d^{4} + a^{6} d^{6}\right)} x^{2} + 2 \, {\left(a b^{5} c^{6} - 3 \, a^{2} b^{4} c^{5} d + 2 \, a^{3} b^{3} c^{4} d^{2} + 2 \, a^{4} b^{2} c^{3} d^{3} - 3 \, a^{5} b c^{2} d^{4} + a^{6} c d^{5}\right)} x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 6*b^2*d^2*log(b*x + a)/(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a
^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5) - 6*b^2*d^2*log(d*x + c)/(b^5*c^5
 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
- a^5*d^5) + 1/2*(12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
- a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 +
 a^2*b*d^3)*x)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b
*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4
 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*
a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b
^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d
^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^
3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

mupad [B] time = 0.53, size = 542, normalized size = 3.79

$$\frac{\frac{6\,b^3\,d^3\,x^3}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}-\frac{a^3\,d^3-7\,a^2\,b\,c\,d^2-7\,a\,b^2\,c^2\,d+b^3\,c^3}{2\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}+\frac{9\,b\,d\,x^2\,\left(c\,b^2\,d+a\,b\,d^2\right)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}+\frac{2\,b\,d\,x\,\left(a^2\,d^2+7\,a\,b\,c\,d+b^2\,c^2\right)}{a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4}}{x\,\left(2\,d\,a^2\,c+2\,b\,a\,c^2\right)+x^2\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)+x^3\,\left(2\,c\,b^2\,d+2\,a\,b\,d^2\right)+a^2\,c^2+b^2\,d^2\,x^4}-\frac{12\,b^2\,d^2\,\mathrm{atanh}\left(\frac{a^5\,d^5-3\,a^4\,b\,c\,d^4+2\,a^3\,b^2\,c^2\,d^3+2\,a^2\,b^3\,c^3\,d^2-3\,a\,b^4\,c^4\,d+b^5\,c^5}{{\left(a\,d-b\,c\right)}^5}+\frac{2\,b\,d\,x\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{{\left(a\,d-b\,c\right)}^5}\right)}{{\left(a\,d-b\,c\right)}^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^3),x)
```

```
[Out] ((6*b^3*d^3*x^3)/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3) - (a^3*d^3 + b^3*c^3 - 7*a*b^2*c^2*d - 7*a^2*b*c*d^2)/(2*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (9*b
*d*x^2*(a*b*d^2 + b^2*c*d))/(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^
3*c^3*d - 4*a^3*b*c*d^3) + (2*b*d*x*(a^2*d^2 + b^2*c^2 + 7*a*b*c*d))/(a^4*d
^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(x*(2*a*
b*c^2 + 2*a^2*c*d) + x^2*(a^2*d^2 + b^2*c^2 + 4*a*b*c*d) + x^3*(2*a*b*d^2 +
 2*b^2*c*d) + a^2*c^2 + b^2*d^2*x^4) - (12*b^2*d^2*atanh((a^5*d^5 + b^5*c^5
 + 2*a^2*b^3*c^3*d^2 + 2*a^3*b^2*c^2*d^3 - 3*a*b^4*c^4*d - 3*a^4*b*c*d^4)/(
a*d - b*c)^5 + (2*b*d*x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^
3*d - 4*a^3*b*c*d^3))/(a*d - b*c)^5))/(a*d - b*c)^5
```

sympy [B] time = 2.42, size = 881, normalized size = 6.16

$$\frac{6b^2d^2\log\left(x+\frac{-\frac{6a^6b^2d^8}{(ad-bc)^5}+\frac{36a^5b^3cd^7}{(ad-bc)^5}-\frac{90a^4b^4c^2d^6}{(ad-bc)^5}+\frac{120a^3b^5c^3d^5}{(ad-bc)^5}-\frac{90a^2b^6c^4d^4}{(ad-bc)^5}+\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3-\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}-\frac{6b^2d^2\log\left(x+\frac{\frac{6a^6b^2d^8}{(ad-bc)^5}-\frac{36a^5b^3cd^7}{(ad-bc)^5}+\frac{90a^4b^4c^2d^6}{(ad-bc)^5}-\frac{120a^3b^5c^3d^5}{(ad-bc)^5}+\frac{90a^2b^6c^4d^4}{(ad-bc)^5}-\frac{36ab^7c^5d^3}{(ad-bc)^5}+6ab^2d^3+\frac{6b^8c^6d^2}{(ad-bc)^5}+6b^3cd^2}{12b^3d^3}\right)}{(ad-bc)^5}+\frac{-a^3d^3+7a^2bcd^2+7ab^2c^2d-b^3c^3+12b^3d^3x^3+x^2\left(18ab^2d^3+18b^3cd^2\right)+x\left(4a^2bd^3+28ab^2cd^2+4b^3c^2d\right)}{2a^6c^2d^4-8a^5bc^3d^3+12a^4b^2c^4d^2-8a^3b^3c^5d+2a^2b^4c^6+x^4\left(2a^4b^2d^6-8a^3b^3cd^5+12a^2b^4c^2d^4-8ab^5c^3d^3+2b^6c^4d^2\right)+x^3\left(4a^5bd^6-12a^4b^2cd^5+8a^3b^3c^2d^4+8a^2b^4c^3d^3-12ab^5c^4d^2+4b^6c^5d\right)+x^2\left(2a^6d^6-18a^4b^2c^2d^4+32a^3b^3c^3d^3-18a^2b^4c^4d^2+2b^6c^6\right)+x\left(4a^6cd^5-12a^5bc^2d^4+8a^4b^2c^3d^3+8a^3b^3c^4d^2-12a^2b^4c^5d+4ab^5c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] 6*b**2*d**2*log(x + (-6*a**6*b**2*d**8/(a*d - b*c)**5 + 36*a**5*b**3*c*d**7
/(a*d - b*c)**5 - 90*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*a**3*b**5*c**
3*d**5/(a*d - b*c)**5 - 90*a**2*b**6*c**4*d**4/(a*d - b*c)**5 + 36*a*b**7*c
**5*d**3/(a*d - b*c)**5 + 6*a*b**2*d**3 - 6*b**8*c**6*d**2/(a*d - b*c)**5 +
 6*b**3*c*d**2)/(12*b**3*d**3))/(a*d - b*c)**5 - 6*b**2*d**2*log(x + (6*a**
6*b**2*d**8/(a*d - b*c)**5 - 36*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*a**4*b
**4*c**2*d**6/(a*d - b*c)**5 - 120*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*
a**2*b**6*c**4*d**4/(a*d - b*c)**5 - 36*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6
*a*b**2*d**3 + 6*b**8*c**6*d**2/(a*d - b*c)**5 + 6*b**3*c*d**2)/(12*b**3*d*
*3))/(a*d - b*c)**5 + (-a**3*d**3 + 7*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**
3*c**3 + 12*b**3*d**3*x**3 + x**2*(18*a*b**2*d**3 + 18*b**3*c*d**2) + x*(4*
a**2*b*d**3 + 28*a*b**2*c*d**2 + 4*b**3*c**2*d))/(2*a**6*c**2*d**4 - 8*a**5
*b*c**3*d**3 + 12*a**4*b**2*c**4*d**2 - 8*a**3*b**3*c**5*d + 2*a**2*b**4*c*
*6 + x**4*(2*a**4*b**2*d**6 - 8*a**3*b**3*c*d**5 + 12*a**2*b**4*c**2*d**4 -
 8*a*b**5*c**3*d**3 + 2*b**6*c**4*d**2) + x**3*(4*a**5*b*d**6 - 12*a**4*b**
2*c*d**5 + 8*a**3*b**3*c**2*d**4 + 8*a**2*b**4*c**3*d**3 - 12*a*b**5*c**4*d
**2 + 4*b**6*c**5*d) + x**2*(2*a**6*d**6 - 18*a**4*b**2*c**2*d**4 + 32*a**3
*b**3*c**3*d**3 - 18*a**2*b**4*c**4*d**2 + 2*b**6*c**6) + x*(4*a**6*c*d**5
- 12*a**5*b*c**2*d**4 + 8*a**4*b**2*c**3*d**3 + 8*a**3*b**3*c**4*d**2 - 12*
a**2*b**4*c**5*d + 4*a*b**5*c**6))
```

3.1256 $\int \frac{(a+bx)^9}{(c+dx)^8} dx$

Optimal. Leaf size=232

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad}{d^{10}(c+dx)^4}$$

Rubi [A] time = 0.36, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {43}

$$-\frac{b^8x(8bc-9ad)}{d^9}+\frac{84b^6(bc-ad)^3}{d^{10}(c+dx)}-\frac{63b^5(bc-ad)^4}{d^{10}(c+dx)^2}+\frac{42b^4(bc-ad)^5}{d^{10}(c+dx)^3}-\frac{21b^3(bc-ad)^6}{d^{10}(c+dx)^4}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}+\frac{36b^7(bc-ad)^2\log(c+dx)}{d^{10}}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}+\frac{b^9x^2}{2d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -((b^8*(8*b*c - 9*a*d)*x)/d^9) + (b^9*x^2)/(2*d^8) + (b*c - a*d)^9/(7*d^10*
(c + d*x)^7) - (3*b*(b*c - a*d)^8)/(2*d^10*(c + d*x)^6) + (36*b^2*(b*c - a*
d)^7)/(5*d^10*(c + d*x)^5) - (21*b^3*(b*c - a*d)^6)/(d^10*(c + d*x)^4) + (4
2*b^4*(b*c - a*d)^5)/(d^10*(c + d*x)^3) - (63*b^5*(b*c - a*d)^4)/(d^10*(c +
 d*x)^2) + (84*b^6*(b*c - a*d)^3)/(d^10*(c + d*x)) + (36*b^7*(b*c - a*d)^2*
Log[c + d*x])/d^10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^9}{(c+dx)^8} dx = \int \left(-\frac{b^8(8bc-9ad)}{d^9}+\frac{b^9x}{d^8}+\frac{(-bc+ad)^9}{d^9(c+dx)^8}+\frac{9b(bc-ad)^8}{d^9(c+dx)^7}-\frac{36b^2(bc-ad)^7}{d^9(c+dx)^6}+\frac{84b^3(bc-ad)^6}{d^9(c+dx)^5}\right.$$

$$= -\frac{b^8(8bc-9ad)x}{d^9}+\frac{b^9x^2}{2d^8}+\frac{(bc-ad)^9}{7d^{10}(c+dx)^7}-\frac{3b(bc-ad)^8}{2d^{10}(c+dx)^6}+\frac{36b^2(bc-ad)^7}{5d^{10}(c+dx)^5}-\frac{21b^3(bc-ad}{d^{10}(c+dx)^4}$$

Mathematica [B] time = 0.27, size = 584, normalized size = 2.52

$$-\frac{10a^9d^9+15a^8bd^8(c+7dx)+24a^7b^2d^7(c^2+7cdx+21d^2x^2)+42a^6b^3d^6(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+84a^5b^4d^5(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+210a^4b^5d^4(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+840a^3b^6d^3(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-6a^2b^7cd^2(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+6ab^8d(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)-2520b^7(c+dx)^7(bc-ad)^2\log(c+dx)-(b^9(3349c^9+20923c^8dx+53949c^7d^2x^2+72275c^6d^3x^3+50225c^5d^4x^4+12495c^4d^5x^5-4655c^3d^6x^6-3185c^2d^7x^7-315cd^8x^8+35d^9x^9))}{70d^{10}(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^9/(c + d*x)^8,x]
```

```
[Out] -1/70*(10*a^9*d^9 + 15*a^8*b*d^8*(c + 7*d*x) + 24*a^7*b^2*d^7*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 42*a^6*b^3*d^6*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x
^3) + 84*a^5*b^4*d^5*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*
d^4*x^4) + 210*a^4*b^5*d^4*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x
^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 840*a^3*b^6*d^3*(c^6 + 7*c^5*d*x + 21*c^4
*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 6*
a^2*b^7*c*d^2*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*
x^3 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + 6*a*b^8*d*(1443
*c^8 + 9261*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4
*x^4 + 11025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) -
 b^9*(3349*c^9 + 20923*c^8*d*x + 53949*c^7*d^2*x^2 + 72275*c^6*d^3*x^3 + 50
```

225*c^5*d^4*x^4 + 12495*c^4*d^5*x^5 - 4655*c^3*d^6*x^6 - 3185*c^2*d^7*x^7 -
 315*c*d^8*x^8 + 35*d^9*x^9) - 2520*b^7*(b*c - a*d)^2*(c + d*x)^7*Log[c + d
x])/(d^10(c + d*x)^7)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^9}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^9/(c + d*x)^8,x]

[Out] IntegrateAlgebraic[(a + b*x)^9/(c + d*x)^8, x]

fricas [B] time = 0.96, size = 1093, normalized size = 4.71

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="fricas")

[Out] 1/70*(35*b^9*d^9*x^9 + 3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d
^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^
6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 - 315*(b^9
*c*d^8 - 2*a*b^8*d^9)*x^8 - 245*(13*b^9*c^2*d^7 - 18*a*b^8*c*d^8)*x^7 - 245
*(19*b^9*c^3*d^6 + 18*a*b^8*c^2*d^7 - 72*a^2*b^7*c*d^8 + 24*a^3*b^6*d^9)*x^
6 + 735*(17*b^9*c^4*d^5 - 90*a*b^8*c^3*d^6 + 108*a^2*b^7*c^2*d^7 - 24*a^3*b
^6*c*d^8 - 6*a^4*b^5*d^9)*x^5 + 245*(205*b^9*c^5*d^4 - 690*a*b^8*c^4*d^5 +
660*a^2*b^7*c^3*d^6 - 120*a^3*b^6*c^2*d^7 - 30*a^4*b^5*c*d^8 - 12*a^5*b^4*d
^9)*x^4 + 245*(295*b^9*c^6*d^3 - 870*a*b^8*c^5*d^4 + 750*a^2*b^7*c^4*d^5 -
120*a^3*b^6*c^3*d^6 - 30*a^4*b^5*c^2*d^7 - 12*a^5*b^4*c*d^8 - 6*a^6*b^3*d^9
)*x^3 + 21*(2569*b^9*c^7*d^2 - 7098*a*b^8*c^6*d^3 + 5754*a^2*b^7*c^5*d^4 -
840*a^3*b^6*c^4*d^5 - 210*a^4*b^5*c^3*d^6 - 84*a^5*b^4*c^2*d^7 - 42*a^6*b^3
*c*d^8 - 24*a^7*b^2*d^9)*x^2 + 7*(2989*b^9*c^8*d - 7938*a*b^8*c^7*d^2 + 617
4*a^2*b^7*c^6*d^3 - 840*a^3*b^6*c^5*d^4 - 210*a^4*b^5*c^4*d^5 - 84*a^5*b^4*
c^3*d^6 - 42*a^6*b^3*c^2*d^7 - 24*a^7*b^2*c*d^8 - 15*a^8*b*d^9)*x + 2520*(b
^9*c^9 - 2*a*b^8*c^8*d + a^2*b^7*c^7*d^2 + (b^9*c^2*d^7 - 2*a*b^8*c*d^8 + a
^2*b^7*d^9)*x^7 + 7*(b^9*c^3*d^6 - 2*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 2
1*(b^9*c^4*d^5 - 2*a*b^8*c^3*d^6 + a^2*b^7*c^2*d^7)*x^5 + 35*(b^9*c^5*d^4 -
 2*a*b^8*c^4*d^5 + a^2*b^7*c^3*d^6)*x^4 + 35*(b^9*c^6*d^3 - 2*a*b^8*c^5*d^4
 + a^2*b^7*c^4*d^5)*x^3 + 21*(b^9*c^7*d^2 - 2*a*b^8*c^6*d^3 + a^2*b^7*c^5*d
^4)*x^2 + 7*(b^9*c^8*d - 2*a*b^8*c^7*d^2 + a^2*b^7*c^6*d^3)*x)*log(d*x + c)
)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^14*x^4 + 35*c^4*d^1
3*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10)

giac [B] time = 1.32, size = 723, normalized size = 3.12

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="giac")

[Out] 36*(b^9*c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*log(abs(d*x + c))/d^10 + 1/2*(b^9*
d^8*x^2 - 16*b^9*c*d^7*x + 18*a*b^8*d^8*x)/d^16 + 1/70*(3349*b^9*c^9 - 8658
*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*c^6*d^3 - 210*a^4*b^5*c^5
*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 24*a^7*b^2*c^2*d^7 - 15*a^
8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b^8*c^2*d^7 + 3*a^2*b^7*c*
d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*a*b^8*c^3*d^6 + 18*a^2*b^
7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1470*(47*b^9*c^5*d^4 - 130

```
*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*c^2*d^7 - 5*a^4*b^5*c*d^8
 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154*a*b^8*c^5*d^4 + 125*a^2*
b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d^7 - 2*a^5*b^4*c*d^8 - a^
6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8*c^6*d^3 + 959*a^2*b^7*c^
5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 - 14*a^5*b^4*c^2*d^7 - 7*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c^8*d - 2676*a*b^8*c^7*d^2
 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70*a^4*b^5*c^4*d^5 - 28*a^5
*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^8 - 5*a^8*b*d^9)*x)/((d*x
 + c)^7*d^10)
```

maple [**B**] time = 0.02, size = 1035, normalized size = 4.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^9/(d*x+c)^8,x)
```

```
[Out] 1/2*b^9*x^2/d^8-21*b^9/d^10/(d*x+c)^4*c^6+36/5*b^9/d^10/(d*x+c)^5*c^7+1/7/d
^10/(d*x+c)^7*b^9*c^9-36/5*b^2/d^3/(d*x+c)^5*a^7+9*b^8/d^8*a*x-8*b^9/d^9*x*
c-84*b^6/d^7/(d*x+c)*a^3+84*b^9/d^10/(d*x+c)*c^3+36*b^7/d^8*ln(d*x+c)*a^2+3
6*b^9/d^10*ln(d*x+c)*c^2-3/2*b/d^2/(d*x+c)^6*a^8-3/2*b^9/d^10/(d*x+c)^6*c^8
-42*b^4/d^5/(d*x+c)^3*a^5+42*b^9/d^10/(d*x+c)^3*c^5-63*b^5/d^6/(d*x+c)^2*a^
4-63*b^9/d^10/(d*x+c)^2*c^4-21*b^3/d^4/(d*x+c)^4*a^6-1/7/d/(d*x+c)^7*a^9+25
2*b^6/d^7/(d*x+c)^2*a^3*c-378*b^7/d^8/(d*x+c)^2*a^2*c^2+252*b^8/d^9/(d*x+c)
^2*a*c^3-36/7/d^3/(d*x+c)^7*a^7*b^2*c^2+126*b^4/d^5/(d*x+c)^4*a^5*c-315*b^5
/d^6/(d*x+c)^4*a^4*c^2+420*b^6/d^7/(d*x+c)^4*a^3*c^3-315*b^7/d^8/(d*x+c)^4*
a^2*c^4+126*b^8/d^9/(d*x+c)^4*a*c^5+9/7/d^2/(d*x+c)^7*a^8*b*c+252/5*b^3/d^4
/(d*x+c)^5*a^6*c-756/5*b^4/d^5/(d*x+c)^5*a^5*c^2+252*b^5/d^6/(d*x+c)^5*a^4*
c^3-252*b^6/d^7/(d*x+c)^5*a^3*c^4+756/5*b^7/d^8/(d*x+c)^5*a^2*c^5-252/5*b^8
/d^9/(d*x+c)^5*a*c^6+252*b^7/d^8/(d*x+c)*a^2*c-252*b^8/d^9/(d*x+c)*a*c^2-72
*b^8/d^9*ln(d*x+c)*a*c+12*b^2/d^3/(d*x+c)^6*a^7*c-42*b^3/d^4/(d*x+c)^6*a^6*
c^2+84*b^4/d^5/(d*x+c)^6*a^5*c^3-105*b^5/d^6/(d*x+c)^6*a^4*c^4+84*b^6/d^7/(
d*x+c)^6*a^3*c^5+12/d^4/(d*x+c)^7*a^6*b^3*c^3+18/d^6/(d*x+c)^7*a^4*b^5*c^5-
12/d^7/(d*x+c)^7*a^3*b^6*c^6+36/7/d^8/(d*x+c)^7*a^2*b^7*c^7-9/7/d^9/(d*x+c)
^7*a*b^8*c^8-42*b^7/d^8/(d*x+c)^6*a^2*c^6+12*b^8/d^9/(d*x+c)^6*a*c^7+210*b^
5/d^6/(d*x+c)^3*a^4*c-420*b^6/d^7/(d*x+c)^3*a^3*c^2+420*b^7/d^8/(d*x+c)^3*a
^2*c^3-210*b^8/d^9/(d*x+c)^3*a*c^4-18/d^5/(d*x+c)^7*a^5*b^4*c^4
```

maxima [**B**] time = 2.20, size = 786, normalized size = 3.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^9/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/70*(3349*b^9*c^9 - 8658*a*b^8*c^8*d + 6534*a^2*b^7*c^7*d^2 - 840*a^3*b^6*
c^6*d^3 - 210*a^4*b^5*c^5*d^4 - 84*a^5*b^4*c^4*d^5 - 42*a^6*b^3*c^3*d^6 - 2
4*a^7*b^2*c^2*d^7 - 15*a^8*b*c*d^8 - 10*a^9*d^9 + 5880*(b^9*c^3*d^6 - 3*a*b
^8*c^2*d^7 + 3*a^2*b^7*c*d^8 - a^3*b^6*d^9)*x^6 + 4410*(7*b^9*c^4*d^5 - 20*
a*b^8*c^3*d^6 + 18*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 - a^4*b^5*d^9)*x^5 + 1
470*(47*b^9*c^5*d^4 - 130*a*b^8*c^4*d^5 + 110*a^2*b^7*c^3*d^6 - 20*a^3*b^6*
c^2*d^7 - 5*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 + 1470*(57*b^9*c^6*d^3 - 154
*a*b^8*c^5*d^4 + 125*a^2*b^7*c^4*d^5 - 20*a^3*b^6*c^3*d^6 - 5*a^4*b^5*c^2*d
^7 - 2*a^5*b^4*c*d^8 - a^6*b^3*d^9)*x^3 + 126*(459*b^9*c^7*d^2 - 1218*a*b^8
*c^6*d^3 + 959*a^2*b^7*c^5*d^4 - 140*a^3*b^6*c^4*d^5 - 35*a^4*b^5*c^3*d^6 -
 14*a^5*b^4*c^2*d^7 - 7*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 + 21*(1023*b^9*c
^8*d - 2676*a*b^8*c^7*d^2 + 2058*a^2*b^7*c^6*d^3 - 280*a^3*b^6*c^5*d^4 - 70
*a^4*b^5*c^4*d^5 - 28*a^5*b^4*c^3*d^6 - 14*a^6*b^3*c^2*d^7 - 8*a^7*b^2*c*d^
8 - 5*a^8*b*d^9)*x)/(d^17*x^7 + 7*c*d^16*x^6 + 21*c^2*d^15*x^5 + 35*c^3*d^1
4*x^4 + 35*c^4*d^13*x^3 + 21*c^5*d^12*x^2 + 7*c^6*d^11*x + c^7*d^10) + 1/2*
```

```
(b^9*d*x^2 - 2*(8*b^9*c - 9*a*b^8*d)*x)/d^9 + 36*(b^9*c^2 - 2*a*b^8*c*d + a
^2*b^7*d^2)*log(d*x + c)/d^10
```

mupad [B] time = 0.26, size = 784, normalized size = 3.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^9/(c + d*x)^8,x)
```

```
[Out] x*((9*a*b^8)/d^8 - (8*b^9*c)/d^9) - ((10*a^9*d^9 - 3349*b^9*c^9 - 6534*a^2*
b^7*c^7*d^2 + 840*a^3*b^6*c^6*d^3 + 210*a^4*b^5*c^5*d^4 + 84*a^5*b^4*c^4*d^
5 + 42*a^6*b^3*c^3*d^6 + 24*a^7*b^2*c^2*d^7 + 8658*a*b^8*c^8*d + 15*a^8*b*c
*d^8)/(70*d) + x*((3*a^8*b*d^8)/2 - (3069*b^9*c^8)/10 + (12*a^7*b^2*c*d^7)/
5 - (3087*a^2*b^7*c^6*d^2)/5 + 84*a^3*b^6*c^5*d^3 + 21*a^4*b^5*c^4*d^4 + (4
2*a^5*b^4*c^3*d^5)/5 + (21*a^6*b^3*c^2*d^6)/5 + (4014*a*b^8*c^7*d)/5) + x^3
*(21*a^6*b^3*d^8 - 1197*b^9*c^6*d^2 + 3234*a*b^8*c^5*d^3 + 42*a^5*b^4*c*d^7
 - 2625*a^2*b^7*c^4*d^4 + 420*a^3*b^6*c^3*d^5 + 105*a^4*b^5*c^2*d^6) + x^2*
((36*a^7*b^2*d^8)/5 - (4131*b^9*c^7*d)/5 + (10962*a*b^8*c^6*d^2)/5 + (63*a^
6*b^3*c*d^7)/5 - (8631*a^2*b^7*c^5*d^3)/5 + 252*a^3*b^6*c^4*d^4 + 63*a^4*b^
5*c^3*d^5 + (126*a^5*b^4*c^2*d^6)/5) + x^5*(63*a^4*b^5*d^8 - 441*b^9*c^4*d^
4 + 1260*a*b^8*c^3*d^5 + 252*a^3*b^6*c*d^7 - 1134*a^2*b^7*c^2*d^6) + x^4*(4
2*a^5*b^4*d^8 - 987*b^9*c^5*d^3 + 2730*a*b^8*c^4*d^4 + 105*a^4*b^5*c*d^7 -
2310*a^2*b^7*c^3*d^5 + 420*a^3*b^6*c^2*d^6) + x^6*(84*a^3*b^6*d^8 - 84*b^9*
c^3*d^5 + 252*a*b^8*c^2*d^6 - 252*a^2*b^7*c*d^7))/(c^7*d^9 + d^16*x^7 + 7*c
^6*d^10*x + 7*c*d^15*x^6 + 21*c^5*d^11*x^2 + 35*c^4*d^12*x^3 + 35*c^3*d^13*
x^4 + 21*c^2*d^14*x^5) + (b^9*x^2)/(2*d^8) + (log(c + d*x)*(36*b^9*c^2 + 36
*a^2*b^7*d^2 - 72*a*b^8*c*d))/d^10
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**9/(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.1257 $\int \frac{(a+bx)^8}{(c+dx)^8} \, dx$

Optimal. Leaf size=209

$$-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}-$$

Rubi [A] time = 0.28, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{28b^6(bc-ad)^2}{d^9(c+dx)}+\frac{28b^5(bc-ad)^3}{d^9(c+dx)^2}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}-\frac{8b^7(bc-ad)\log(c+dx)}{d^9}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{b^8x}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] (b^8*x)/d^8 - (b*c - a*d)^8/(7*d^9*(c + d*x)^7) + (4*b*(b*c - a*d)^7)/(3*d^
9*(c + d*x)^6) - (28*b^2*(b*c - a*d)^6)/(5*d^9*(c + d*x)^5) + (14*b^3*(b*c
- a*d)^5)/(d^9*(c + d*x)^4) - (70*b^4*(b*c - a*d)^4)/(3*d^9*(c + d*x)^3) +
(28*b^5*(b*c - a*d)^3)/(d^9*(c + d*x)^2) - (28*b^6*(b*c - a*d)^2)/(d^9*(c +
 d*x)) - (8*b^7*(b*c - a*d)*Log[c + d*x])/d^9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^8}{(c+dx)^8} \, dx = \int \left(\frac{b^8}{d^8}+\frac{(-bc+ad)^8}{d^8(c+dx)^8}-\frac{8b(bc-ad)^7}{d^8(c+dx)^7}+\frac{28b^2(bc-ad)^6}{d^8(c+dx)^6}-\frac{56b^3(bc-ad)^5}{d^8(c+dx)^5}+\frac{70b^4(bc-ad)^4}{d^8(c+dx)^4}-\right.$$
$$=\frac{b^8x}{d^8}-\frac{(bc-ad)^8}{7d^9(c+dx)^7}+\frac{4b(bc-ad)^7}{3d^9(c+dx)^6}-\frac{28b^2(bc-ad)^6}{5d^9(c+dx)^5}+\frac{14b^3(bc-ad)^5}{d^9(c+dx)^4}-\frac{70b^4(bc-ad)^4}{3d^9(c+dx)^3}+$$

Mathematica [B] time = 0.20, size = 474, normalized size = 2.27

$$-\frac{15a^8d^8+20a^7bd^7(c+7dx)+28a^6b^2d^6(c^2+7cdx+21d^2x^2)+42a^5b^3d^5(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+70a^4b^4d^4(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+140a^3b^5d^3(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+420a^2b^6d^2(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)-2ab^7cd(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6)+b^8(1443c^8+9261c^7dx+24843c^6d^2x^2+35525c^5d^3x^3+28175c^4d^4x^4+11025c^3d^5x^5+735c^2d^6x^6-735cd^7x^7-105d^8x^8)+840b^7(bc-ad)(c+dx)^7\log(c+dx)}{105d^9(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^8*d^8 + 20*a^7*b*d^7*(c + 7*d*x) + 28*a^6*b^2*d^6*(c^2 + 7*c*d
*x + 21*d^2*x^2) + 42*a^5*b^3*d^5*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*
x^3) + 70*a^4*b^4*d^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35
*d^4*x^4) + 140*a^3*b^5*d^3*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*
x^3 + 35*c*d^4*x^4 + 21*d^5*x^5) + 420*a^2*b^6*d^2*(c^6 + 7*c^5*d*x + 21*c^
4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6) - 2
*a*b^7*c*d*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3
 + 26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6) + b^8*(1443*c^8 + 92
61*c^7*d*x + 24843*c^6*d^2*x^2 + 35525*c^5*d^3*x^3 + 28175*c^4*d^4*x^4 + 11
025*c^3*d^5*x^5 + 735*c^2*d^6*x^6 - 735*c*d^7*x^7 - 105*d^8*x^8) + 840*b^7*
(b*c - a*d)*(c + d*x)^7*Log[c + d*x])/(d^9*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^8}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^8/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^8/(c + d*x)^8, x]
```

fricas [B] time = 0.97, size = 852, normalized size = 4.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/105*(105*b^8*d^8*x^8 + 735*b^8*c*d^7*x^7 - 1443*b^8*c^8 + 2178*a*b^7*c^7*
d - 420*a^2*b^6*c^6*d^2 - 140*a^3*b^5*c^5*d^3 - 70*a^4*b^4*c^4*d^4 - 42*a^5
*b^3*c^3*d^5 - 28*a^6*b^2*c^2*d^6 - 20*a^7*b*c*d^7 - 15*a^8*d^8 - 735*(b^8*
c^2*d^6 - 8*a*b^7*c*d^7 + 4*a^2*b^6*d^8)*x^6 - 735*(15*b^8*c^3*d^5 - 36*a*b
^7*c^2*d^6 + 12*a^2*b^6*c*d^7 + 4*a^3*b^5*d^8)*x^5 - 1225*(23*b^8*c^4*d^4 -
 44*a*b^7*c^3*d^5 + 12*a^2*b^6*c^2*d^6 + 4*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x
^4 - 245*(145*b^8*c^5*d^3 - 250*a*b^7*c^4*d^4 + 60*a^2*b^6*c^3*d^5 + 20*a^3
*b^5*c^2*d^6 + 10*a^4*b^4*c*d^7 + 6*a^5*b^3*d^8)*x^3 - 147*(169*b^8*c^6*d^2
 - 274*a*b^7*c^5*d^3 + 60*a^2*b^6*c^4*d^4 + 20*a^3*b^5*c^3*d^5 + 10*a^4*b^4
*c^2*d^6 + 6*a^5*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 - 7*(1323*b^8*c^7*d - 2058*
a*b^7*c^6*d^2 + 420*a^2*b^6*c^5*d^3 + 140*a^3*b^5*c^4*d^4 + 70*a^4*b^4*c^3*
d^5 + 42*a^5*b^3*c^2*d^6 + 28*a^6*b^2*c*d^7 + 20*a^7*b*d^8)*x - 840*(b^8*c^
8 - a*b^7*c^7*d + (b^8*c*d^7 - a*b^7*d^8)*x^7 + 7*(b^8*c^2*d^6 - a*b^7*c*d^
7)*x^6 + 21*(b^8*c^3*d^5 - a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 - a*b^7*c^3
*d^5)*x^4 + 35*(b^8*c^5*d^3 - a*b^7*c^4*d^4)*x^3 + 21*(b^8*c^6*d^2 - a*b^7*
c^5*d^3)*x^2 + 7*(b^8*c^7*d - a*b^7*c^6*d^2)*x)*log(d*x + c))/(d^16*x^7 + 7
*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*
d^11*x^2 + 7*c^6*d^10*x + c^7*d^9)
```

giac [B] time = 1.27, size = 581, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*x/d^8 - 8*(b^8*c - a*b^7*d)*log(abs(d*x + c))/d^9 - 1/105*(1443*b^8*c^8
 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 + 20*a^7*b*c*d^7 + 15*a
^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 + a^2*b^6*d^8)*x^6 + 2940*(5*b^8
*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 2450*(13*
b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*
b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5
 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 294*(87*b^8*
c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a
^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2*d^8)*x^2 + 14*(669*b^8*c^7*d -
 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 + 70*a^3*b^5*c^4*d^4 + 35*a^4*b^4
*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x)/((d*x +
 c)^7*d^9)
```

maple [B] time = 0.01, size = 845, normalized size = 4.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^8/(d*x+c)^8,x)
```

```
[Out] -14*b^3/d^4/(d*x+c)^4*a^5+14*b^8/d^9/(d*x+c)^4*c^5-28*b^6/d^7/(d*x+c)*a^2-2
8*b^8/d^9/(d*x+c)*c^2+8*b^7/d^8*ln(d*x+c)*a-8*b^8/d^9*ln(d*x+c)*c-28/5*b^2/
d^3/(d*x+c)^5*a^6-28/5*b^8/d^9/(d*x+c)^5*c^6-28*b^5/d^6/(d*x+c)^2*a^3+28*b^
8/d^9/(d*x+c)^2*c^3-1/7/d^9/(d*x+c)^7*b^8*c^8-4/3*b/d^2/(d*x+c)^6*a^7+4/3*b
^8/d^9/(d*x+c)^6*c^7-70/3*b^4/d^5/(d*x+c)^3*a^4-70/3*b^8/d^9/(d*x+c)^3*c^4+
b^8*x/d^8+8/7/d^2/(d*x+c)^7*a^7*b*c-4/d^3/(d*x+c)^7*a^6*b^2*c^2-1/7/d/(d*x+
c)^7*a^8+8/d^4/(d*x+c)^7*a^5*b^3*c^3-10/d^5/(d*x+c)^7*a^4*b^4*c^4+8/d^6/(d*
x+c)^7*a^3*b^5*c^5-4/d^7/(d*x+c)^7*a^2*b^6*c^6+8/7/d^8/(d*x+c)^7*a*b^7*c^7+
140/3*b^4/d^5/(d*x+c)^6*a^4*c^3-140/3*b^5/d^6/(d*x+c)^6*a^3*c^4+28*b^6/d^7/
(d*x+c)^6*a^2*c^5-28/3*b^7/d^8/(d*x+c)^6*a*c^6+280/3*b^5/d^6/(d*x+c)^3*a^3*
c-140*b^6/d^7/(d*x+c)^3*a^2*c^2+280/3*b^7/d^8/(d*x+c)^3*a*c^3+70*b^4/d^5/(d
*x+c)^4*a^4*c-140*b^5/d^6/(d*x+c)^4*a^3*c^2+140*b^6/d^7/(d*x+c)^4*a^2*c^3-7
0*b^7/d^8/(d*x+c)^4*a*c^4+56*b^7/d^8/(d*x+c)*a*c+168/5*b^3/d^4/(d*x+c)^5*a^
5*c-84*b^4/d^5/(d*x+c)^5*a^4*c^2+112*b^5/d^6/(d*x+c)^5*a^3*c^3-84*b^6/d^7/(
d*x+c)^5*a^2*c^4+168/5*b^7/d^8/(d*x+c)^5*a*c^5+84*b^6/d^7/(d*x+c)^2*a^2*c-8
4*b^7/d^8/(d*x+c)^2*a*c^2+28/3*b^2/d^3/(d*x+c)^6*a^6*c-28*b^3/d^4/(d*x+c)^6
*a^5*c^2
```

maxima [B] time = 1.96, size = 649, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^8/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^8*x/d^8 - 1/105*(1443*b^8*c^8 - 2178*a*b^7*c^7*d + 420*a^2*b^6*c^6*d^2 +
140*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 + 20*a^7*b*c*d^7 + 15*a^8*d^8 + 2940*(b^8*c^2*d^6 - 2*a*b^7*c*d^7 +
 a^2*b^6*d^8)*x^6 + 2940*(5*b^8*c^3*d^5 - 9*a*b^7*c^2*d^6 + 3*a^2*b^6*c*d^7
 + a^3*b^5*d^8)*x^5 + 2450*(13*b^8*c^4*d^4 - 22*a*b^7*c^3*d^5 + 6*a^2*b^6*c
^2*d^6 + 2*a^3*b^5*c*d^7 + a^4*b^4*d^8)*x^4 + 490*(77*b^8*c^5*d^3 - 125*a*b
^7*c^4*d^4 + 30*a^2*b^6*c^3*d^5 + 10*a^3*b^5*c^2*d^6 + 5*a^4*b^4*c*d^7 + 3*
a^5*b^3*d^8)*x^3 + 294*(87*b^8*c^6*d^2 - 137*a*b^7*c^5*d^3 + 30*a^2*b^6*c^4
*d^4 + 10*a^3*b^5*c^3*d^5 + 5*a^4*b^4*c^2*d^6 + 3*a^5*b^3*c*d^7 + 2*a^6*b^2
*d^8)*x^2 + 14*(669*b^8*c^7*d - 1029*a*b^7*c^6*d^2 + 210*a^2*b^6*c^5*d^3 +
70*a^3*b^5*c^4*d^4 + 35*a^4*b^4*c^3*d^5 + 21*a^5*b^3*c^2*d^6 + 14*a^6*b^2*c
*d^7 + 10*a^7*b*d^8)*x)/(d^16*x^7 + 7*c*d^15*x^6 + 21*c^2*d^14*x^5 + 35*c^3
*d^13*x^4 + 35*c^4*d^12*x^3 + 21*c^5*d^11*x^2 + 7*c^6*d^10*x + c^7*d^9) - 8
*(b^8*c - a*b^7*d)*log(d*x + c)/d^9
```

mupad [B] time = 0.43, size = 649, normalized size = 3.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^8/(c + d*x)^8,x)
```

```
[Out] (b^8*x)/d^8 - (log(c + d*x)*(8*b^8*c - 8*a*b^7*d))/d^9 - (x^4*((70*a^4*b^4*
d^7)/3 + (910*b^8*c^4*d^3)/3 - (1540*a*b^7*c^3*d^4)/3 + (140*a^3*b^5*c*d^6)
/3 + 140*a^2*b^6*c^2*d^5) + x^6*(28*a^2*b^6*d^7 + 28*b^8*c^2*d^5 - 56*a*b^7
*c*d^6) + (15*a^8*d^8 + 1443*b^8*c^8 + 420*a^2*b^6*c^6*d^2 + 140*a^3*b^5*c^
5*d^3 + 70*a^4*b^4*c^4*d^4 + 42*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 2178
*a*b^7*c^7*d + 20*a^7*b*c*d^7)/(105*d) + x*((446*b^8*c^7)/5 + (4*a^7*b*d^7)
/3 + (28*a^6*b^2*c*d^6)/15 + 28*a^2*b^6*c^5*d^2 + (28*a^3*b^5*c^4*d^3)/3 +
(14*a^4*b^4*c^3*d^4)/3 + (14*a^5*b^3*c^2*d^5)/5 - (686*a*b^7*c^6*d)/5) + x^
3*(14*a^5*b^3*d^7 + (1078*b^8*c^5*d^2)/3 - (1750*a*b^7*c^4*d^3)/3 + (70*a^4
```

```
*b^4*c*d^6)/3 + 140*a^2*b^6*c^3*d^4 + (140*a^3*b^5*c^2*d^5)/3) + x^2*((1218
*b^8*c^6*d)/5 + (28*a^6*b^2*d^7)/5 - (1918*a*b^7*c^5*d^2)/5 + (42*a^5*b^3*c
*d^6)/5 + 84*a^2*b^6*c^4*d^3 + 28*a^3*b^5*c^3*d^4 + 14*a^4*b^4*c^2*d^5) + x
^5*(28*a^3*b^5*d^7 + 140*b^8*c^3*d^4 - 252*a*b^7*c^2*d^5 + 84*a^2*b^6*c*d^6
))/(c^7*d^8 + d^15*x^7 + 7*c^6*d^9*x + 7*c*d^14*x^6 + 21*c^5*d^10*x^2 + 35*
c^4*d^11*x^3 + 35*c^3*d^12*x^4 + 21*c^2*d^13*x^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**8/(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.1258 $\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$

Optimal. Leaf size=194

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+$$

Rubi [A] time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{7b^6(bc-ad)}{d^8(c+dx)}-\frac{21b^5(bc-ad)^2}{2d^8(c+dx)^2}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{(bc-ad)^7}{7d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^7/(7*d^8*(c + d*x)^7) - (7*b*(b*c - a*d)^6)/(6*d^8*(c + d*x)^6)
 + (21*b^2*(b*c - a*d)^5)/(5*d^8*(c + d*x)^5) - (35*b^3*(b*c - a*d)^4)/(4*d
^8*(c + d*x)^4) + (35*b^4*(b*c - a*d)^3)/(3*d^8*(c + d*x)^3) - (21*b^5*(b*c
 - a*d)^2)/(2*d^8*(c + d*x)^2) + (7*b^6*(b*c - a*d))/(d^8*(c + d*x)) + (b^7
*Log[c + d*x])/d^8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^7}{(c+dx)^8}\,dx = \int \left(\frac{(-bc+ad)^7}{d^7(c+dx)^8}+\frac{7b(bc-ad)^6}{d^7(c+dx)^7}-\frac{21b^2(bc-ad)^5}{d^7(c+dx)^6}+\frac{35b^3(bc-ad)^4}{d^7(c+dx)^5}-\frac{35b^4(bc-ad)^3}{d^7(c+dx)^4}+\frac{21b}{d^7}\right.$$

$$=\frac{(bc-ad)^7}{7d^8(c+dx)^7}-\frac{7b(bc-ad)^6}{6d^8(c+dx)^6}+\frac{21b^2(bc-ad)^5}{5d^8(c+dx)^5}-\frac{35b^3(bc-ad)^4}{4d^8(c+dx)^4}+\frac{35b^4(bc-ad)^3}{3d^8(c+dx)^3}-\frac{21b^5(b}{2d^8(c}$$

Mathematica [A] time = 0.16, size = 308, normalized size = 1.59

$$\frac{(bc-ad)\left(60a^6d^6+10a^5bd^5(13c+49dx)+2a^4b^2d^4\left(107c^2+539cdx+882d^2x^2\right)+a^3b^3d^3\left(319c^3+1813c^2dx+3969cd^2x^2+3675d^3x^3\right)+a^2b^4d^2\left(459c^4+2793c^3dx+6909c^2d^2x^2+8575cd^3x^3+4900d^4x^4\right)+ab^5d\left(669c^5+4263c^4dx+11319c^3d^2x^2+15925c^2d^3x^3+12250cd^4x^4+4410d^5x^5\right)+b^6\left(1089c^6+7203c^5dx+20139c^4d^2x^2+30625c^3d^3x^3+26950c^2d^4x^4+13230cd^5x^5+2940d^6x^6\right)\right)}{420d^8(c+dx)^7}+\frac{b^7\log(c+dx)}{d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] ((b*c - a*d)*(60*a^6*d^6 + 10*a^5*b*d^5*(13*c + 49*d*x) + 2*a^4*b^2*d^4*(10
7*c^2 + 539*c*d*x + 882*d^2*x^2) + a^3*b^3*d^3*(319*c^3 + 1813*c^2*d*x + 39
69*c*d^2*x^2 + 3675*d^3*x^3) + a^2*b^4*d^2*(459*c^4 + 2793*c^3*d*x + 6909*c
^2*d^2*x^2 + 8575*c*d^3*x^3 + 4900*d^4*x^4) + a*b^5*d*(669*c^5 + 4263*c^4*d
*x + 11319*c^3*d^2*x^2 + 15925*c^2*d^3*x^3 + 12250*c*d^4*x^4 + 4410*d^5*x^5
) + b^6*(1089*c^6 + 7203*c^5*d*x + 20139*c^4*d^2*x^2 + 30625*c^3*d^3*x^3 +
26950*c^2*d^4*x^4 + 13230*c*d^5*x^5 + 2940*d^6*x^6)))/(420*d^8*(c + d*x)^7)
 + (b^7*Log[c + d*x])/d^8
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^7}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^7/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^7/(c + d*x)^8, x]
```

fricas [B] time = 1.12, size = 625, normalized size = 3.22

$$\frac{1089\,b^7c^7 - 420\,ab^6c^6d - 210\,a^2b^5c^5d^2 - 140\,a^3b^4c^4d^3 - 105\,a^4b^3c^3d^4 - 84\,a^5b^2c^2d^5 - 70\,a^6bcd^6 - 60\,a^7d^7 + 2940\left(b^7cd^6 - ab^6d^7\right)x^6 + 4410\left(3\,b^7c^2d^5 - 2\,ab^6cd^6 - a^2b^5d^7\right)x^5 + 2450\left(11\,b^7c^3d^4 - 6\,ab^6c^2d^5 - 3\,a^2b^5cd^6 - 2\,a^3b^4d^7\right)x^4 + 1225\left(25\,b^7c^4d^3 - 12\,ab^6c^3d^4 - 6\,a^2b^5c^2d^5 - 4\,a^3b^4cd^6 - 3\,a^4b^3d^7\right)x^3 + 147\left(137\,b^7c^5d^2 - 60\,ab^6c^4d^3 - 30\,a^2b^5c^3d^4 - 20\,a^3b^4c^2d^5 - 15\,a^4b^3cd^6 - 12\,a^5b^2d^7\right)x^2 + 49\left(147\,b^7c^6d - 60\,ab^6c^5d^2 - 30\,a^2b^5c^4d^3 - 20\,a^3b^4c^3d^4 - 15\,a^4b^3c^2d^5 - 12\,a^5b^2cd^6 - 10\,a^6bd^7\right)x + 420\left(b^7d^7x^7 + 7\,b^7cd^6x^6 + 21\,b^7c^2d^5x^5 + 35\,b^7c^3d^4x^4 + 35\,b^7c^4d^3x^3 + 21\,b^7c^5d^2x^2 + 7\,b^7c^6dx + b^7c^7\right)\log\left(dx + c\right)}{420\left(d^{15}x^7 + 7\,cd^{14}x^6 + 21\,c^2d^{13}x^5 + 35\,c^3d^{12}x^4 + 35\,c^4d^{11}x^3 + 21\,c^5d^{10}x^2 + 7\,c^6d^9x + c^7d^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 21*b^7*c^2*d^5*x^5
+ 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^2*x^2 + 7*b^7*c^6*
d*x + b^7*c^7)*log(d*x + c))/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 3
5*c^3*d^12*x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8)
```

giac [B] time = 1.30, size = 467, normalized size = 2.41

$$\frac{b^7\log\left(|dx + c|\right)}{d^8} + \frac{2940\left(b^7cd^5 - ab^6d^6\right)x^6 + 4410\left(3\,b^7c^2d^4 - 2\,ab^6cd^5 - a^2b^5d^6\right)x^5 + 2450\left(11\,b^7c^3d^3 - 6\,ab^6c^2d^4 - 3\,a^2b^5cd^5 - 2\,a^3b^4d^6\right)x^4 + 1225\left(25\,b^7c^4d^2 - 12\,ab^6c^3d^3 - 6\,a^2b^5c^2d^4 - 4\,a^3b^4cd^5 - 3\,a^4b^3d^6\right)x^3 + 147\left(137\,b^7c^5d - 60\,ab^6c^4d^2 - 30\,a^2b^5c^3d^3 - 20\,a^3b^4c^2d^4 - 15\,a^4b^3cd^5 - 12\,a^5b^2d^6\right)x^2 + 49\left(147\,b^7c^6 - 60\,ab^6c^5d - 30\,a^2b^5c^4d^2 - 20\,a^3b^4c^3d^3 - 15\,a^4b^3c^2d^4 - 12\,a^5b^2cd^5 - 10\,a^6bd^6\right)x + \frac{1089\,b^7c^7 - 420\,ab^6c^6d - 210\,a^2b^5c^5d^2 - 140\,a^3b^4c^4d^3 - 105\,a^4b^3c^3d^4 - 84\,a^5b^2c^2d^5 - 70\,a^6bcd^6 - 60\,a^7d^7}{d}}{420\left(dx + c\right)^7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^7*log(abs(d*x + c))/d^8 + 1/420*(2940*(b^7*c*d^5 - a*b^6*d^6)*x^6 + 4410*
(3*b^7*c^2*d^4 - 2*a*b^6*c*d^5 - a^2*b^5*d^6)*x^5 + 2450*(11*b^7*c^3*d^3 -
6*a*b^6*c^2*d^4 - 3*a^2*b^5*c*d^5 - 2*a^3*b^4*d^6)*x^4 + 1225*(25*b^7*c^4*d
^2 - 12*a*b^6*c^3*d^3 - 6*a^2*b^5*c^2*d^4 - 4*a^3*b^4*c*d^5 - 3*a^4*b^3*d^6
)*x^3 + 147*(137*b^7*c^5*d - 60*a*b^6*c^4*d^2 - 30*a^2*b^5*c^3*d^3 - 20*a^3
*b^4*c^2*d^4 - 15*a^4*b^3*c*d^5 - 12*a^5*b^2*d^6)*x^2 + 49*(147*b^7*c^6 - 6
0*a*b^6*c^5*d - 30*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 - 15*a^4*b^3*c^2*d^
4 - 12*a^5*b^2*c*d^5 - 10*a^6*b*d^6)*x + (1089*b^7*c^7 - 420*a*b^6*c^6*d -
210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^
2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7*d^7)/d)/((d*x + c)^7*d^7)
```

maple [B] time = 0.01, size = 672, normalized size = 3.46

$$-\frac{a^7}{7\left(dx+c\right)^7d}+\frac{a^6bc}{\left(dx+c\right)^7d^2}-\frac{3a^5b^2c^2}{\left(dx+c\right)^7d^3}+\frac{5a^4b^3c^3}{\left(dx+c\right)^7d^4}-\frac{5a^3b^4c^4}{\left(dx+c\right)^7d^5}+\frac{3a^2b^5c^5}{\left(dx+c\right)^7d^6}-\frac{a\,b^6c^6}{\left(dx+c\right)^7d^7}+\frac{b^7c^7}{7\left(dx+c\right)^7d^8}-\frac{7a^6b}{6\left(dx+c\right)^6d^2}+\frac{7a^5b^2c}{\left(dx+c\right)^6d^3}-\frac{35a^4b^3c^2}{2\left(dx+c\right)^6d^4}+\frac{70a^3b^4c^3}{3\left(dx+c\right)^6d^5}-\frac{35a^2b^5c^4}{2\left(dx+c\right)^6d^6}+\frac{7a\,b^6c^5}{\left(dx+c\right)^6d^7}-\frac{7b^7c^6}{6\left(dx+c\right)^6d^8}-\frac{21a^5b^2}{5\left(dx+c\right)^5d^3}+\frac{21a^4b^3c}{\left(dx+c\right)^5d^4}-\frac{42a^3b^4c^2}{\left(dx+c\right)^5d^5}+\frac{42a^2b^5c^3}{\left(dx+c\right)^5d^6}-\frac{21a\,b^6c^4}{\left(dx+c\right)^5d^7}+\frac{21b^7c^5}{5\left(dx+c\right)^5d^8}-\frac{35a^4b^3}{4\left(dx+c\right)^4d^4}+\frac{35a^3b^4c}{\left(dx+c\right)^4d^5}-\frac{105a^2b^5c^2}{2\left(dx+c\right)^4d^6}+\frac{35a\,b^6c^3}{\left(dx+c\right)^4d^7}-\frac{35b^7c^4}{4\left(dx+c\right)^4d^8}-\frac{35a^3b^4}{3\left(dx+c\right)^3d^5}+\frac{35a^2b^5c}{\left(dx+c\right)^3d^6}-\frac{35a\,b^6c^2}{\left(dx+c\right)^3d^7}+\frac{35b^7c^3}{3\left(dx+c\right)^3d^8}-\frac{21a^2b^5}{2\left(dx+c\right)^2d^6}+\frac{21a\,b^6c}{\left(dx+c\right)^2d^7}-\frac{21b^7c^2}{2\left(dx+c\right)^2d^8}-\frac{7a\,b^6}{\left(dx+c\right)d^7}+\frac{7b^7c}{\left(dx+c\right)d^8}+\frac{b^7\ln\left(dx+c\right)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^7/(d*x+c)^8,x)
```

```
[Out] -35/3*b^4/d^5/(d*x+c)^3*a^3+35/3*b^7/d^8/(d*x+c)^3*c^3-21/2*b^5/d^6/(d*x+c)
^2*a^2-21/2*b^7/d^8/(d*x+c)^2*c^2+1/7/d^8/(d*x+c)^7*b^7*c^7-7*b^6/d^7/(d*x+
c)*a+7*b^7/d^8/(d*x+c)*c-21/5*b^2/d^3/(d*x+c)^5*a^5+21/5*b^7/d^8/(d*x+c)^5*
c^5-7/6*b/d^2/(d*x+c)^6*a^6-7/6*b^7/d^8/(d*x+c)^6*c^6-35/4*b^3/d^4/(d*x+c)^
4*a^4-35/4*b^7/d^8/(d*x+c)^4*c^4-21*b^6/d^7/(d*x+c)^5*a*c^4+35*b^6/d^7/(d*x
+c)^4*a*c^3+35*b^5/d^6/(d*x+c)^3*a^2*c-35*b^6/d^7/(d*x+c)^3*a*c^2+21*b^6/d^
7/(d*x+c)^2*a*c+21*b^3/d^4/(d*x+c)^5*a^4*c-42*b^4/d^5/(d*x+c)^5*a^3*c^2+42*
b^5/d^6/(d*x+c)^5*a^2*c^3+b^7*ln(d*x+c)/d^8-1/7/d/(d*x+c)^7*a^7-3/d^3/(d*x+
```

```
c)^7*a^5*b^2*c^2+5/d^4/(d*x+c)^7*a^4*b^3*c^3-5/d^5/(d*x+c)^7*a^3*b^4*c^4+3/
d^6/(d*x+c)^7*a^2*b^5*c^5-1/d^7/(d*x+c)^7*a*b^6*c^6+7*b^2/d^3/(d*x+c)^6*a^5
*c-35/2*b^3/d^4/(d*x+c)^6*a^4*c^2+70/3*b^4/d^5/(d*x+c)^6*a^3*c^3-35/2*b^5/d
^6/(d*x+c)^6*a^2*c^4+7*b^6/d^7/(d*x+c)^6*a*c^5+35*b^4/d^5/(d*x+c)^4*a^3*c-1
05/2*b^5/d^6/(d*x+c)^4*a^2*c^2+1/d^2/(d*x+c)^7*a^6*b*c
```

maxima [**B**] time = 1.66, size = 535, normalized size = 2.76

$$\frac{1089\,b^7c^7 - 420\,ab^6c^6d - 210\,a^2b^5c^5d^2 - 140\,a^3b^4c^4d^3 - 105\,a^4b^3c^3d^4 - 84\,a^5b^2c^2d^5 - 70\,a^6bcd^6 - 60\,a^7d^7 + 2940\left(b^7cd^6 - ab^6d^7\right)x^6 + 4410\left(3\,b^7c^2d^5 - 2\,ab^6cd^6 - a^2b^5d^7\right)x^5 + 2450\left(11\,b^7c^3d^4 - 6\,ab^6c^2d^5 - 3\,a^2b^5cd^6 - 2\,a^3b^4d^7\right)x^4 + 1225\left(25\,b^7c^4d^3 - 12\,ab^6c^3d^4 - 6\,a^2b^5c^2d^5 - 4\,a^3b^4cd^6 - 3\,a^4b^3d^7\right)x^3 + 147\left(137\,b^7c^5d^2 - 60\,ab^6c^4d^3 - 30\,a^2b^5c^3d^4 - 20\,a^3b^4c^2d^5 - 15\,a^4b^3cd^6 - 12\,a^5b^2d^7\right)x^2 + 49\left(147\,b^7c^6d - 60\,ab^6c^5d^2 - 30\,a^2b^5c^4d^3 - 20\,a^3b^4c^3d^4 - 15\,a^4b^3c^2d^5 - 12\,a^5b^2cd^6 - 10\,a^6bd^7\right)x}{420\left(d^{15}x^7 + 7\,cd^{14}x^6 + 21\,c^2d^{13}x^5 + 35\,c^3d^{12}x^4 + 35\,c^4d^{11}x^3 + 21\,c^5d^{10}x^2 + 7\,c^6d^9x + c^7d^8\right)} + \frac{b^7\log\left(dx + c\right)}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^7/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 1/420*(1089*b^7*c^7 - 420*a*b^6*c^6*d - 210*a^2*b^5*c^5*d^2 - 140*a^3*b^4*c
^4*d^3 - 105*a^4*b^3*c^3*d^4 - 84*a^5*b^2*c^2*d^5 - 70*a^6*b*c*d^6 - 60*a^7
*d^7 + 2940*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 4410*(3*b^7*c^2*d^5 - 2*a*b^6*c*d
^6 - a^2*b^5*d^7)*x^5 + 2450*(11*b^7*c^3*d^4 - 6*a*b^6*c^2*d^5 - 3*a^2*b^5*
c*d^6 - 2*a^3*b^4*d^7)*x^4 + 1225*(25*b^7*c^4*d^3 - 12*a*b^6*c^3*d^4 - 6*a^
2*b^5*c^2*d^5 - 4*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 147*(137*b^7*c^5*d^2
 - 60*a*b^6*c^4*d^3 - 30*a^2*b^5*c^3*d^4 - 20*a^3*b^4*c^2*d^5 - 15*a^4*b^3*
c*d^6 - 12*a^5*b^2*d^7)*x^2 + 49*(147*b^7*c^6*d - 60*a*b^6*c^5*d^2 - 30*a^2
*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 - 15*a^4*b^3*c^2*d^5 - 12*a^5*b^2*c*d^6 -
 10*a^6*b*d^7)*x)/(d^15*x^7 + 7*c*d^14*x^6 + 21*c^2*d^13*x^5 + 35*c^3*d^12*
x^4 + 35*c^4*d^11*x^3 + 21*c^5*d^10*x^2 + 7*c^6*d^9*x + c^7*d^8) + b^7*log(
d*x + c)/d^8
```

mupad [**B**] time = 0.38, size = 460, normalized size = 2.37

$$\frac{b^7\,\ln\left(c+d\,x\right)}{d^8}-\frac{x\,\left(\frac{7\,a^6\,b\,d^7}{6}+\frac{7\,a^5\,b^2\,c\,d^6}{5}+\frac{7\,a^4\,b^3\,c^2\,d^5}{4}+\frac{7\,a^3\,b^4\,c^3\,d^4}{3}+\frac{7\,a^2\,b^5\,c^4\,d^3}{2}+7\,a\,b^6\,c^5\,d^2-\frac{343\,b^7\,c^6\,d}{20}\right)+x^6\,\left(7\,a\,b^6\,d^7-7\,b^7\,c\,d^6\right)+x^3\,\left(\frac{35\,a^4\,b^3\,d^7}{4}+\frac{35\,a^3\,b^4\,c\,d^6}{3}+\frac{35\,a^2\,b^5\,c^2\,d^5}{2}+35\,a\,b^6\,c^3\,d^4-\frac{875\,b^7\,c^4\,d^3}{12}\right)+x^5\,\left(\frac{21\,a^2\,b^5\,d^7}{2}+21\,a\,b^6\,c\,d^6-\frac{63\,b^7\,c^2\,d^5}{2}\right)+x^2\,\left(\frac{21\,a^5\,b^2\,d^7}{5}+\frac{21\,a^4\,b^3\,c\,d^6}{4}+7\,a^3\,b^4\,c^2\,d^5+\frac{21\,a^2\,b^5\,c^3\,d^4}{2}+21\,a\,b^6\,c^4\,d^3-\frac{959\,b^7\,c^5\,d^2}{20}\right)+\frac{a^7\,d^7}{7}-\frac{363\,b^7\,c^7}{140}+x^4\,\left(\frac{35\,a^3\,b^4\,d^7}{3}+\frac{35\,a^2\,b^5\,c\,d^6}{2}+35\,a\,b^6\,c^2\,d^5-\frac{385\,b^7\,c^3\,d^4}{6}\right)+\frac{a^2\,b^5\,c^5\,d^2}{2}+\frac{a^3\,b^4\,c^4\,d^3}{3}+\frac{a^4\,b^3\,c^3\,d^4}{4}+\frac{a^5\,b^2\,c^2\,d^5}{5}+a\,b^6\,c^6\,d+\frac{a^6\,b\,c\,d^6}{6}}{d^8\,{\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^7/(c + d*x)^8,x)
```

```
[Out] (b^7*log(c + d*x))/d^8 - (x*((7*a^6*b*d^7)/6 - (343*b^7*c^6*d)/20 + 7*a*b^6
*c^5*d^2 + (7*a^5*b^2*c*d^6)/5 + (7*a^2*b^5*c^4*d^3)/2 + (7*a^3*b^4*c^3*d^4
)/3 + (7*a^4*b^3*c^2*d^5)/4) + x^6*(7*a*b^6*d^7 - 7*b^7*c*d^6) + x^3*((35*a
^4*b^3*d^7)/4 - (875*b^7*c^4*d^3)/12 + 35*a*b^6*c^3*d^4 + (35*a^3*b^4*c*d^6
)/3 + (35*a^2*b^5*c^2*d^5)/2) + x^5*((21*a^2*b^5*d^7)/2 - (63*b^7*c^2*d^5)/
2 + 21*a*b^6*c*d^6) + x^2*((21*a^5*b^2*d^7)/5 - (959*b^7*c^5*d^2)/20 + 21*a
*b^6*c^4*d^3 + (21*a^4*b^3*c*d^6)/4 + (21*a^2*b^5*c^3*d^4)/2 + 7*a^3*b^4*c^
2*d^5) + (a^7*d^7)/7 - (363*b^7*c^7)/140 + x^4*((35*a^3*b^4*d^7)/3 - (385*b
^7*c^3*d^4)/6 + 35*a*b^6*c^2*d^5 + (35*a^2*b^5*c*d^6)/2) + (a^2*b^5*c^5*d^2
)/2 + (a^3*b^4*c^4*d^3)/3 + (a^4*b^3*c^3*d^4)/4 + (a^5*b^2*c^2*d^5)/5 + a*b
^6*c^6*d + (a^6*b*c*d^6)/6)/(d^8*(c + d*x)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**7/(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.1259 $\int \frac{(a+bx)^6}{(c+dx)^8} dx$

Optimal. Leaf size=28

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {37}

$$\frac{(a+bx)^7}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^7/(7*(b*c - a*d)*(c + d*x)^7)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^6}{(c+dx)^8} dx = \frac{(a+bx)^7}{7(bc-ad)(c+dx)^7}$$

Mathematica [B] time = 0.09, size = 271, normalized size = 9.68

$$-\frac{a^6d^6+a^5bd^5(c+7dx)+a^4b^2d^4(c^2+7cdx+21d^2x^2)+a^3b^3d^3(c^3+7c^2dx+21cd^2x^2+35d^3x^3)+a^2b^4d^2(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4)+ab^5d(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5)+b^6(c^6+7c^5dx+21c^4d^2x^2+35c^3d^3x^3+35c^2d^4x^4+21cd^5x^5+7d^6x^6)}{7d^7(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] -1/7*(a^6*d^6 + a^5*b*d^5*(c + 7*d*x) + a^4*b^2*d^4*(c^2 + 7*c*d*x + 21*d^2
*x^2) + a^3*b^3*d^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) + a^2*b^4
*d^2*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4) + a*b^5
*d*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 21*d
^5*x^5) + b^6*(c^6 + 7*c^5*d*x + 21*c^4*d^2*x^2 + 35*c^3*d^3*x^3 + 35*c^2*d
^4*x^4 + 21*c*d^5*x^5 + 7*d^6*x^6))/(d^7*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^6}{(c+dx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^6/(c + d*x)^8, x]
```

fricas [B] time = 1.38, size = 398, normalized size = 14.21

$$-\frac{7 b^{6} d^{6} x^{6} + b^{6} c^{6} + a b^{5} c^{5} d + a^{2} b^{4} c^{4} d^{2} + a^{3} b^{3} c^{3} d^{3} + a^{4} b^{2} c^{2} d^{4} + a^{5} b c d^{5} + a^{6} d^{6} + 21 \left(b^{6} c d^{5} + a b^{5} d^{6}\right) x^{5} + 35 \left(b^{6} c^{2} d^{4} + a b^{5} c d^{5} + a^{2} b^{4} d^{6}\right) x^{4} + 35 \left(b^{6} c^{3} d^{3} + a b^{5} c^{2} d^{4} + a^{2} b^{4} c d^{5} + a^{3} b^{3} d^{6}\right) x^{3} + 21 \left(b^{6} c^{4} d^{2} + a b^{5} c^{3} d^{3} + a^{2} b^{4} c^{2} d^{4} + a^{3} b^{3} c d^{5} + a^{4} b^{2} d^{6}\right) x^{2} + 7 \left(b^{6} c^{5} d + a b^{5} c^{4} d^{2} + a^{2} b^{4} c^{3} d^{3} + a^{3} b^{3} c^{2} d^{4} + a^{4} b^{2} c d^{5} + a^{5} b d^{6}\right) x}{7 \left(d^{14} x^{7} + 7 c d^{13} x^{6} + 21 c^{2} d^{12} x^{5} + 35 c^{3} d^{11} x^{4} + 35 c^{4} d^{10} x^{3} + 21 c^{5} d^{9} x^{2} + 7 c^{6} d^{8} x + c^{7} d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

giac [B] time = 1.28, size = 369, normalized size = 13.18

$$-\frac{7 b^{6} d^{6} x^{6} + 21 b^{6} c d^{5} x^{5} + 21 a b^{5} d^{6} x^{5} + 35 b^{6} c^{2} d^{4} x^{4} + 35 a b^{5} c d^{5} x^{4} + 35 a^{2} b^{4} d^{6} x^{4} + 35 b^{6} c^{3} d^{3} x^{3} + 35 a b^{5} c^{2} d^{4} x^{3} + 35 a^{2} b^{4} c d^{5} x^{3} + 35 a^{3} b^{3} d^{6} x^{3} + 21 b^{6} c^{4} d^{2} x^{2} + 21 a b^{5} c^{3} d^{3} x^{2} + 21 a^{2} b^{4} c^{2} d^{4} x^{2} + 21 a^{3} b^{3} c d^{5} x^{2} + 21 a^{4} b^{2} d^{6} x^{2} + 7 b^{6} c^{5} d x + 7 a b^{5} c^{4} d^{2} x + 7 a^{2} b^{4} c^{3} d^{3} x + 7 a^{3} b^{3} c^{2} d^{4} x + 7 a^{4} b^{2} c d^{5} x + 7 a^{5} b d^{6} x + b^{6} c^{6} + a b^{5} c^{5} d + a^{2} b^{4} c^{4} d^{2} + a^{3} b^{3} c^{3} d^{3} + a^{4} b^{2} c^{2} d^{4} + a^{5} b c d^{5} + a^{6} d^{6}}{7 (d x + c)^{7} d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + 21*b^6*c*d^5*x^5 + 21*a*b^5*d^6*x^5 + 35*b^6*c^2*d^4*
x^4 + 35*a*b^5*c*d^5*x^4 + 35*a^2*b^4*d^6*x^4 + 35*b^6*c^3*d^3*x^3 + 35*a*b
^5*c^2*d^4*x^3 + 35*a^2*b^4*c*d^5*x^3 + 35*a^3*b^3*d^6*x^3 + 21*b^6*c^4*d^2
*x^2 + 21*a*b^5*c^3*d^3*x^2 + 21*a^2*b^4*c^2*d^4*x^2 + 21*a^3*b^3*c*d^5*x^2
 + 21*a^4*b^2*d^6*x^2 + 7*b^6*c^5*d*x + 7*a*b^5*c^4*d^2*x + 7*a^2*b^4*c^3*d
^3*x + 7*a^3*b^3*c^2*d^4*x + 7*a^4*b^2*c*d^5*x + 7*a^5*b*d^6*x + b^6*c^6 +
a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c
*d^5 + a^6*d^6)/((d*x + c)^7*d^7)
```

maple [B] time = 0.01, size = 357, normalized size = 12.75

$$-\frac{b^{6}}{(d x + c) d^{7}} - \frac{3 (a d - b c) b^{5}}{(d x + c)^{2} d^{7}} - \frac{5 \left(a^{2} d^{2} - 2 a b c d + b^{2} c^{2}\right) b^{4}}{(d x + c)^{3} d^{7}} - \frac{5 \left(a^{3} d^{3} - 3 a^{2} b c d^{2} + 3 a b^{2} c^{2} d - b^{3} c^{3}\right) b^{3}}{(d x + c)^{4} d^{7}} - \frac{3 \left(a^{4} d^{4} - 4 a^{3} b c d^{3} + 6 a^{2} b^{2} c^{2} d^{2} - 4 a b^{3} c^{3} d + b^{4} c^{4}\right) b^{2}}{(d x + c)^{5} d^{7}} - \frac{\left(a^{5} d^{5} - 5 a^{4} b c d^{4} + 10 a^{3} b^{2} c^{2} d^{3} - 10 a^{2} b^{3} c^{3} d^{2} + 5 a b^{4} c^{4} d - b^{5} c^{5}\right) b}{(d x + c)^{6} d^{7}} - \frac{a^{6} d^{6} - 6 a^{5} b c d^{5} + 15 a^{4} b^{2} c^{2} d^{4} - 20 a^{3} b^{3} c^{3} d^{3} + 15 a^{2} b^{4} c^{4} d^{2} - 6 a b^{5} c^{5} d + b^{6} c^{6}}{7 (d x + c)^{7} d^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^6/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^
4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)/d^7/(d*x+c)^7-5*b^4*(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^7/(d*x+c)^3-3*b^2*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)/d^7/(d*x+c)^5-b^6/d^7/(d*x+c)-b*(a^5*d^5-5*a^4*b*c*d^4+10*a^
3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)/d^7/(d*x+c)^6-5*b^3
*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^7/(d*x+c)^4-3*b^5*(a*d-b*c
)/d^7/(d*x+c)^2
```

maxima [B] time = 1.61, size = 398, normalized size = 14.21

$$-\frac{7 b^{6} d^{6} x^{6} + b^{6} c^{6} + a b^{5} c^{5} d + a^{2} b^{4} c^{4} d^{2} + a^{3} b^{3} c^{3} d^{3} + a^{4} b^{2} c^{2} d^{4} + a^{5} b c d^{5} + a^{6} d^{6} + 21 \left(b^{6} c d^{5} + a b^{5} d^{6}\right) x^{5} + 35 \left(b^{6} c^{2} d^{4} + a b^{5} c d^{5} + a^{2} b^{4} d^{6}\right) x^{4} + 35 \left(b^{6} c^{3} d^{3} + a b^{5} c^{2} d^{4} + a^{2} b^{4} c d^{5} + a^{3} b^{3} d^{6}\right) x^{3} + 21 \left(b^{6} c^{4} d^{2} + a b^{5} c^{3} d^{3} + a^{2} b^{4} c^{2} d^{4} + a^{3} b^{3} c d^{5} + a^{4} b^{2} d^{6}\right) x^{2} + 7 \left(b^{6} c^{5} d + a b^{5} c^{4} d^{2} + a^{2} b^{4} c^{3} d^{3} + a^{3} b^{3} c^{2} d^{4} + a^{4} b^{2} c d^{5} + a^{5} b d^{6}\right) x}{7 \left(d^{14} x^{7} + 7 c d^{13} x^{6} + 21 c^{2} d^{12} x^{5} + 35 c^{3} d^{11} x^{4} + 35 c^{4} d^{10} x^{3} + 21 c^{5} d^{9} x^{2} + 7 c^{6} d^{8} x + c^{7} d^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^6/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7*(7*b^6*d^6*x^6 + b^6*c^6 + a*b^5*c^5*d + a^2*b^4*c^4*d^2 + a^3*b^3*c^3
*d^3 + a^4*b^2*c^2*d^4 + a^5*b*c*d^5 + a^6*d^6 + 21*(b^6*c*d^5 + a*b^5*d^6)
*x^5 + 35*(b^6*c^2*d^4 + a*b^5*c*d^5 + a^2*b^4*d^6)*x^4 + 35*(b^6*c^3*d^3 +
 a*b^5*c^2*d^4 + a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^3 + 21*(b^6*c^4*d^2 + a*b^5
*c^3*d^3 + a^2*b^4*c^2*d^4 + a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^2 + 7*(b^6*c^5*
```

```
d + a*b^5*c^4*d^2 + a^2*b^4*c^3*d^3 + a^3*b^3*c^2*d^4 + a^4*b^2*c*d^5 + a^5
*b*d^6)*x)/(d^14*x^7 + 7*c*d^13*x^6 + 21*c^2*d^12*x^5 + 35*c^3*d^11*x^4 + 3
5*c^4*d^10*x^3 + 21*c^5*d^9*x^2 + 7*c^6*d^8*x + c^7*d^7)
```

mupad [B] time = 0.15, size = 378, normalized size = 13.50

$$-\frac{\frac{a^6\,d^6+a^5\,b\,c\,d^5+a^4\,b^2\,c^2\,d^4+a^3\,b^3\,c^3\,d^3+a^2\,b^4\,c^4\,d^2+a\,b^5\,c^5\,d+b^6\,c^6}{7\,d^7}+\frac{b^6\,x^6}{d}+\frac{5\,b^3\,x^3\,\left(a^3\,d^3+a^2\,b\,c\,d^2+a\,b^2\,c^2\,d+b^3\,c^3\right)}{d^4}+\frac{b\,x\,\left(a^5\,d^5+a^4\,b\,c\,d^4+a^3\,b^2\,c^2\,d^3+a^2\,b^3\,c^3\,d^2+a\,b^4\,c^4\,d+b^5\,c^5\right)}{d^6}+\frac{3\,b^5\,x^5\,\left(a\,d+b\,c\right)}{d^2}+\frac{3\,b^2\,x^2\,\left(a^4\,d^4+a^3\,b\,c\,d^3+a^2\,b^2\,c^2\,d^2+a\,b^3\,c^3\,d+b^4\,c^4\right)}{d^5}+\frac{5\,b^4\,x^4\,\left(a^2\,d^2+a\,b\,c\,d+b^2\,c^2\right)}{d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^6/(c + d*x)^8,x)
```

```
[Out] -((a^6*d^6 + b^6*c^6 + a^2*b^4*c^4*d^2 + a^3*b^3*c^3*d^3 + a^4*b^2*c^2*d^4
+ a*b^5*c^5*d + a^5*b*c*d^5)/(7*d^7) + (b^6*x^6)/d + (5*b^3*x^3*(a^3*d^3 +
b^3*c^3 + a*b^2*c^2*d + a^2*b*c*d^2))/d^4 + (b*x*(a^5*d^5 + b^5*c^5 + a^2*b
^3*c^3*d^2 + a^3*b^2*c^2*d^3 + a*b^4*c^4*d + a^4*b*c*d^4))/d^6 + (3*b^5*x^5
*(a*d + b*c))/d^2 + (3*b^2*x^2*(a^4*d^4 + b^4*c^4 + a^2*b^2*c^2*d^2 + a*b^3
*c^3*d + a^3*b*c*d^3))/d^5 + (5*b^4*x^4*(a^2*d^2 + b^2*c^2 + a*b*c*d))/d^3)
/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^
4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**6/(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.1260 $\int \frac{(a+bx)^5}{(c+dx)^8} \, dx$

Optimal. Leaf size=58

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {45, 37}

$$\frac{b(a+bx)^6}{42(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^6}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^6/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^6)/(42*(b*c - a*d)^2
*(c + d*x)^6)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^8} \, dx &= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b\int \frac{(a+bx)^5}{(c+dx)^7} \, dx}{7(bc-ad)}\\ &= \frac{(a+bx)^6}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^6}{42(bc-ad)^2(c+dx)^6}\end{aligned}$$

Mathematica [B] time = 0.06, size = 205, normalized size = 3.53

$$-\frac{6a^5d^5+5a^4bd^4(c+7dx)+4a^3b^2d^3\left(c^2+7cdx+21d^2x^2\right)+3a^2b^3d^2\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+2ab^4d\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)+b^5\left(c^5+7c^4dx+21c^3d^2x^2+35c^2d^3x^3+35cd^4x^4+21d^5x^5\right)}{42d^6(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a^5*d^5 + 5*a^4*b*d^4*(c + 7*d*x) + 4*a^3*b^2*d^3*(c^2 + 7*c*d*x +
 21*d^2*x^2) + 3*a^2*b^3*d^2*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3)
+ 2*a*b^4*d*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4)
```

```
+ b^5*(c^5 + 7*c^4*d*x + 21*c^3*d^2*x^2 + 35*c^2*d^3*x^3 + 35*c*d^4*x^4 + 2
1*d^5*x^5))/(d^6*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^5}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^8, x]
```

fricas [B] time = 1.31, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

giac [B] time = 1.32, size = 271, normalized size = 4.67

$$-\frac{21\,b^5d^5x^5+35\,b^5cd^4x^4+70\,ab^4d^5x^4+35\,b^5c^2d^3x^3+70\,ab^4cd^4x^3+105\,a^2b^3d^5x^3+21\,b^5c^3d^2x^2+42\,ab^4c^2d^3x^2+63\,a^2b^3cd^4x^2+84\,a^3b^2d^5x^2+7\,b^5c^4dx+14\,ab^4c^3d^2x+21\,a^2b^3c^2d^3x+28\,a^3b^2cd^4x+35\,a^4bd^5x+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5}{42\,(dx+c)^7d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + 35*b^5*c*d^4*x^4 + 70*a*b^4*d^5*x^4 + 35*b^5*c^2*d^
3*x^3 + 70*a*b^4*c*d^4*x^3 + 105*a^2*b^3*d^5*x^3 + 21*b^5*c^3*d^2*x^2 + 42*
a*b^4*c^2*d^3*x^2 + 63*a^2*b^3*c*d^4*x^2 + 84*a^3*b^2*d^5*x^2 + 7*b^5*c^4*d
*x + 14*a*b^4*c^3*d^2*x + 21*a^2*b^3*c^2*d^3*x + 28*a^3*b^2*c*d^4*x + 35*a^
4*b*d^5*x + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3*b^2*c^2*d^3
 + 5*a^4*b*c*d^4 + 6*a^5*d^5)/((d*x + c)^7*d^6)
```

maple [B] time = 0.01, size = 265, normalized size = 4.57

$$-\frac{b^5}{2\,(dx+c)^2d^6}-\frac{5\,(ad-bc)\,b^4}{3\,(dx+c)^3d^6}-\frac{5\,(a^2d^2-2abcd+b^2c^2)\,b^3}{2\,(dx+c)^4d^6}-\frac{2\,(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3)\,b^2}{(dx+c)^5d^6}-\frac{5\,(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4)\,b}{6\,(dx+c)^6d^6}-\frac{a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5}{7\,(dx+c)^7d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c
^4*d-b^5*c^5)/d^6/(d*x+c)^7-5/3*b^4*(a*d-b*c)/d^6/(d*x+c)^3-5/6*b*(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^6/(d*x+c)^6-2*b^2*
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^6/(d*x+c)^5-5/2*b^3*(a^2*d^
2-2*a*b*c*d+b^2*c^2)/d^6/(d*x+c)^4-1/2*b^5/d^6/(d*x+c)^2
```

maxima [B] time = 1.58, size = 326, normalized size = 5.62

$$-\frac{21\,b^5d^5x^5+b^5c^5+2\,ab^4c^4d+3\,a^2b^3c^3d^2+4\,a^3b^2c^2d^3+5\,a^4bcd^4+6\,a^5d^5+35\,(b^5cd^4+2\,ab^4d^5)x^4+35\,(b^5c^2d^3+2\,ab^4cd^4+3\,a^2b^3d^5)x^3+21\,(b^5c^3d^2+2\,ab^4c^2d^3+3\,a^2b^3cd^4+4\,a^3b^2d^5)x^2+7\,(b^5c^4d+2\,ab^4c^3d^2+3\,a^2b^3c^2d^3+4\,a^3b^2cd^4+5\,a^4bd^5)x}{42\,(d^{13}x^7+7\,cd^{12}x^6+21\,c^2d^{11}x^5+35\,c^3d^{10}x^4+35\,c^4d^9x^3+21\,c^5d^8x^2+7\,c^6d^7x+c^7d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(21*b^5*d^5*x^5 + b^5*c^5 + 2*a*b^4*c^4*d + 3*a^2*b^3*c^3*d^2 + 4*a^3
*b^2*c^2*d^3 + 5*a^4*b*c*d^4 + 6*a^5*d^5 + 35*(b^5*c*d^4 + 2*a*b^4*d^5)*x^4
 + 35*(b^5*c^2*d^3 + 2*a*b^4*c*d^4 + 3*a^2*b^3*d^5)*x^3 + 21*(b^5*c^3*d^2 +
 2*a*b^4*c^2*d^3 + 3*a^2*b^3*c*d^4 + 4*a^3*b^2*d^5)*x^2 + 7*(b^5*c^4*d + 2*
a*b^4*c^3*d^2 + 3*a^2*b^3*c^2*d^3 + 4*a^3*b^2*c*d^4 + 5*a^4*b*d^5)*x)/(d^13
*x^7 + 7*c*d^12*x^6 + 21*c^2*d^11*x^5 + 35*c^3*d^10*x^4 + 35*c^4*d^9*x^3 +
21*c^5*d^8*x^2 + 7*c^6*d^7*x + c^7*d^6)
```

mupad [B] time = 0.28, size = 39, normalized size = 0.67

$$\frac{(a+b\,x)^6\,\left(7\,b\,c-6\,a\,d+b\,d\,x\right)}{42\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^8,x)
```

```
[Out] ((a + b*x)^6*(7*b*c - 6*a*d + b*d*x))/(42*(a*d - b*c)^2*(c + d*x)^7)
```

sympy [B] time = 54.91, size = 354, normalized size = 6.10

$$\frac{-6a^5d^5 - 5a^4bcd^4 - 4a^3b^2c^2d^3 - 3a^2b^3c^3d^2 - 2ab^4c^4d - b^5c^5 - 21b^5d^5x^5 + x^4\left(-70ab^4d^5 - 35b^5cd^4\right) + x^3\left(-105a^2b^3d^5 - 70ab^4cd^4 - 35b^5c^2d^3\right) + x^2\left(-84a^3b^2d^5 - 63a^2b^3cd^4 - 42ab^4c^2d^3 - 21b^5c^3d^2\right) + x\left(-35a^4bd^5 - 28a^3b^2cd^4 - 21a^2b^3c^2d^3 - 14ab^4c^3d^2 - 7b^5c^4d\right)}{42c^7d^6 + 294c^6d^7x + 882c^5d^8x^2 + 1470c^4d^9x^3 + 1470c^3d^{10}x^4 + 882c^2d^{11}x^5 + 294cd^{12}x^6 + 42d^{13}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**8,x)
```

```
[Out] (-6*a**5*d**5 - 5*a**4*b*c*d**4 - 4*a**3*b**2*c**2*d**3 - 3*a**2*b**3*c**3*
d**2 - 2*a*b**4*c**4*d - b**5*c**5 - 21*b**5*d**5*x**5 + x**4*(-70*a*b**4*d
**5 - 35*b**5*c*d**4) + x**3*(-105*a**2*b**3*d**5 - 70*a*b**4*c*d**4 - 35*b
**5*c**2*d**3) + x**2*(-84*a**3*b**2*d**5 - 63*a**2*b**3*c*d**4 - 42*a*b**4
*c**2*d**3 - 21*b**5*c**3*d**2) + x*(-35*a**4*b*d**5 - 28*a**3*b**2*c*d**4
- 21*a**2*b**3*c**2*d**3 - 14*a*b**4*c**3*d**2 - 7*b**5*c**4*d))/(42*c**7*d
**6 + 294*c**6*d**7*x + 882*c**5*d**8*x**2 + 1470*c**4*d**9*x**3 + 1470*c**
3*d**10*x**4 + 882*c**2*d**11*x**5 + 294*c*d**12*x**6 + 42*d**13*x**7)
```

3.1261 $\int \frac{(a+bx)^4}{(c+dx)^8} \, dx$

Optimal. Leaf size=89

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {45, 37}

$$\frac{b^2(a+bx)^5}{105(c+dx)^5(bc-ad)^3}+\frac{b(a+bx)^5}{21(c+dx)^6(bc-ad)^2}+\frac{(a+bx)^5}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] (a + b*x)^5/(7*(b*c - a*d)*(c + d*x)^7) + (b*(a + b*x)^5)/(21*(b*c - a*d)^2
*(c + d*x)^6) + (b^2*(a + b*x)^5)/(105*(b*c - a*d)^3*(c + d*x)^5)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^4}{(c+dx)^8} \, dx &= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{(2b)\int \frac{(a+bx)^4}{(c+dx)^7} \, dx}{7(bc-ad)}\\
&= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2\int \frac{(a+bx)^4}{(c+dx)^6} \, dx}{21(bc-ad)^2}\\
&= \frac{(a+bx)^5}{7(bc-ad)(c+dx)^7}+\frac{b(a+bx)^5}{21(bc-ad)^2(c+dx)^6}+\frac{b^2(a+bx)^5}{105(bc-ad)^3(c+dx)^5}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 144, normalized size = 1.62

$$-\frac{15a^4d^4+10a^3bd^3(c+7dx)+6a^2b^2d^2\left(c^2+7cdx+21d^2x^2\right)+3ab^3d\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)+b^4\left(c^4+7c^3dx+21c^2d^2x^2+35cd^3x^3+35d^4x^4\right)}{105d^5(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^4*d^4 + 10*a^3*b*d^3*(c + 7*d*x) + 6*a^2*b^2*d^2*(c^2 + 7*c*d*
x + 21*d^2*x^2) + 3*a*b^3*d*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3) +
 b^4*(c^4 + 7*c^3*d*x + 21*c^2*d^2*x^2 + 35*c*d^3*x^3 + 35*d^4*x^4))/(d^5*(
c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^4}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^8, x]
```

fricas [B] time = 1.06, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4 + b^4c^4 + 3\,ab^3c^3d + 6\,a^2b^2c^2d^2 + 10\,a^3bcd^3 + 15\,a^4d^4 + 35\left(b^4cd^3 + 3\,ab^3d^4\right)x^3 + 21\left(b^4c^2d^2 + 3\,ab^3cd^3 + 6\,a^2b^2d^4\right)x^2 + 7\left(b^4c^3d + 3\,ab^3c^2d^2 + 6\,a^2b^2cd^3 + 10\,a^3bd^4\right)x}{105\left(d^{12}x^7 + 7\,cd^{11}x^6 + 21\,c^2d^{10}x^5 + 35\,c^3d^9x^4 + 35\,c^4d^8x^3 + 21\,c^5d^7x^2 + 7\,c^6d^6x + c^7d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

giac [B] time = 1.29, size = 184, normalized size = 2.07

$$-\frac{35\,b^4d^4x^4 + 35\,b^4cd^3x^3 + 105\,ab^3d^4x^3 + 21\,b^4c^2d^2x^2 + 63\,ab^3cd^3x^2 + 126\,a^2b^2d^4x^2 + 7\,b^4c^3dx + 21\,ab^3c^2d^2x + 42\,a^2b^2cd^3x + 70\,a^3bd^4x + b^4c^4 + 3\,ab^3c^3d + 6\,a^2b^2c^2d^2 + 10\,a^3bcd^3 + 15\,a^4d^4}{105\,(dx+c)^7d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + 35*b^4*c*d^3*x^3 + 105*a*b^3*d^4*x^3 + 21*b^4*c^2*
d^2*x^2 + 63*a*b^3*c*d^3*x^2 + 126*a^2*b^2*d^4*x^2 + 7*b^4*c^3*d*x + 21*a*b
^3*c^2*d^2*x + 42*a^2*b^2*c*d^3*x + 70*a^3*b*d^4*x + b^4*c^4 + 3*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 + 10*a^3*b*c*d^3 + 15*a^4*d^4)/((d*x + c)^7*d^5)
```

maple [B] time = 0.01, size = 186, normalized size = 2.09

$$-\frac{b^4}{3\,(dx+c)^3\,d^5} - \frac{(ad-bc)\,b^3}{(dx+c)^4\,d^5} - \frac{6\left(a^2d^2 - 2abcd + b^2c^2\right)b^2}{5\,(dx+c)^5\,d^5} - \frac{2\left(a^3d^3 - 3a^2bc\,d^2 + 3a\,b^2c^2d - b^3c^3\right)b}{3\,(dx+c)^6\,d^5} - \frac{a^4d^4 - 4a^3bc\,d^3 + 6a^2b^2c^2d^2 - 4a\,b^3c^3d + b^4c^4}{7\,(dx+c)^7\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/d^5/(d
*x+c)^7-1/3*b^4/d^5/(d*x+c)^3-b^3*(a*d-b*c)/d^5/(d*x+c)^4-2/3*b*(a^3*d^3-3*
a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^5/(d*x+c)^6-6/5*b^2*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/d^5/(d*x+c)^5
```

maxima [B] time = 1.52, size = 247, normalized size = 2.78

$$-\frac{35\,b^4d^4x^4 + b^4c^4 + 3\,ab^3c^3d + 6\,a^2b^2c^2d^2 + 10\,a^3bcd^3 + 15\,a^4d^4 + 35\left(b^4cd^3 + 3\,ab^3d^4\right)x^3 + 21\left(b^4c^2d^2 + 3\,ab^3cd^3 + 6\,a^2b^2d^4\right)x^2 + 7\left(b^4c^3d + 3\,ab^3c^2d^2 + 6\,a^2b^2cd^3 + 10\,a^3bd^4\right)x}{105\left(d^{12}x^7 + 7\,cd^{11}x^6 + 21\,c^2d^{10}x^5 + 35\,c^3d^9x^4 + 35\,c^4d^8x^3 + 21\,c^5d^7x^2 + 7\,c^6d^6x + c^7d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(35*b^4*d^4*x^4 + b^4*c^4 + 3*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 10*a
^3*b*c*d^3 + 15*a^4*d^4 + 35*(b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 21*(b^4*c^2*d^
2 + 3*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*x^2 + 7*(b^4*c^3*d + 3*a*b^3*c^2*d^2 + 6
*a^2*b^2*c*d^3 + 10*a^3*b*d^4)*x)/(d^12*x^7 + 7*c*d^11*x^6 + 21*c^2*d^10*x^
5 + 35*c^3*d^9*x^4 + 35*c^4*d^8*x^3 + 21*c^5*d^7*x^2 + 7*c^6*d^6*x + c^7*d^
5)
```

mupad [B] time = 0.11, size = 237, normalized size = 2.66

$$-\frac{\frac{15\,a^4\,d^4+10\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2+3\,a\,b^3\,c^3\,d+b^4\,c^4}{105\,d^5}+\frac{b^4\,x^4}{3\,d}+\frac{b^3\,x^3\,\left(3\,a\,d+b\,c\right)}{3\,d^2}+\frac{b\,x\,\left(10\,a^3\,d^3+6\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{15\,d^4}+\frac{b^2\,x^2\,\left(6\,a^2\,d^2+3\,a\,b\,c\,d+b^2\,c^2\right)}{5\,d^3}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^8,x)
```

```
[Out] -((15*a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 + 3*a*b^3*c^3*d + 10*a^3*b*c*d^
3)/(105*d^5) + (b^4*x^4)/(3*d) + (b^3*x^3*(3*a*d + b*c))/(3*d^2) + (b*x*(10
*a^3*d^3 + b^3*c^3 + 3*a*b^2*c^2*d + 6*a^2*b*c*d^2))/(15*d^4) + (b^2*x^2*(6
*a^2*d^2 + b^2*c^2 + 3*a*b*c*d))/(5*d^3))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21
*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x
)
```

sympy [B] time = 9.64, size = 267, normalized size = 3.00

$$\frac{-15a^4d^4 - 10a^3bcd^3 - 6a^2b^2c^2d^2 - 3ab^3c^3d - b^4c^4 - 35b^4d^4x^4 + x^3\left(-105ab^3d^4 - 35b^4cd^3\right) + x^2\left(-126a^2b^2d^4 - 63ab^3cd^3 - 21b^4c^2d^2\right) + x\left(-70a^3bd^4 - 42a^2b^2cd^3 - 21ab^3c^2d^2 - 7b^4c^3d\right)}{105c^7d^5 + 735c^6d^6x + 2205c^5d^7x^2 + 3675c^4d^8x^3 + 3675c^3d^9x^4 + 2205c^2d^{10}x^5 + 735cd^{11}x^6 + 105d^{12}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**8,x)
```

```
[Out] (-15*a**4*d**4 - 10*a**3*b*c*d**3 - 6*a**2*b**2*c**2*d**2 - 3*a*b**3*c**3*d
 - b**4*c**4 - 35*b**4*d**4*x**4 + x**3*(-105*a*b**3*d**4 - 35*b**4*c*d**3)
 + x**2*(-126*a**2*b**2*d**4 - 63*a*b**3*c*d**3 - 21*b**4*c**2*d**2) + x*(-
70*a**3*b*d**4 - 42*a**2*b**2*c*d**3 - 21*a*b**3*c**2*d**2 - 7*b**4*c**3*d)
)/(105*c**7*d**5 + 735*c**6*d**6*x + 2205*c**5*d**7*x**2 + 3675*c**4*d**8*x
**3 + 3675*c**3*d**9*x**4 + 2205*c**2*d**10*x**5 + 735*c*d**11*x**6 + 105*d
**12*x**7)
```

3.1262 $\int \frac{(a+bx)^3}{(c+dx)^8} dx$

Optimal. Leaf size=92

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

Rubi [A] time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b^3}{4d^4(c+dx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)^3/(7*d^4*(c + d*x)^7) - (b*(b*c - a*d)^2)/(2*d^4*(c + d*x)^6) +
 (3*b^2*(b*c - a*d))/(5*d^4*(c + d*x)^5) - b^3/(4*d^4*(c + d*x)^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^8} dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^8}+\frac{3b(bc-ad)^2}{d^3(c+dx)^7}-\frac{3b^2(bc-ad)}{d^3(c+dx)^6}+\frac{b^3}{d^3(c+dx)^5}\right) dx\\ &= \frac{(bc-ad)^3}{7d^4(c+dx)^7}-\frac{b(bc-ad)^2}{2d^4(c+dx)^6}+\frac{3b^2(bc-ad)}{5d^4(c+dx)^5}-\frac{b^3}{4d^4(c+dx)^4}\end{aligned}$$

Mathematica [A] time = 0.03, size = 94, normalized size = 1.02

$$-\frac{20a^3d^3+10a^2bd^2(c+7dx)+4ab^2d\left(c^2+7cdx+21d^2x^2\right)+b^3\left(c^3+7c^2dx+21cd^2x^2+35d^3x^3\right)}{140d^4(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] -1/140*(20*a^3*d^3 + 10*a^2*b*d^2*(c + 7*d*x) + 4*a*b^2*d*(c^2 + 7*c*d*x +
21*d^2*x^2) + b^3*(c^3 + 7*c^2*d*x + 21*c*d^2*x^2 + 35*d^3*x^3))/(d^4*(c +
d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^3}{(c+dx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^8, x]
```

fricas [B] time = 0.96, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\,\left(b^3cd^2+4\,ab^2d^3\right)x^2+7\,\left(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3\right)x}{140\,\left(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

giac [A] time = 1.24, size = 114, normalized size = 1.24

$$-\frac{35\,b^3d^3x^3+21\,b^3cd^2x^2+84\,ab^2d^3x^2+7\,b^3c^2dx+28\,ab^2cd^2x+70\,a^2bd^3x+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3}{140\,(dx+c)^7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + 21*b^3*c*d^2*x^2 + 84*a*b^2*d^3*x^2 + 7*b^3*c^2*d*
x + 28*a*b^2*c*d^2*x + 70*a^2*b*d^3*x + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*
c*d^2 + 20*a^3*d^3)/((d*x + c)^7*d^4)
```

maple [A] time = 0.01, size = 122, normalized size = 1.33

$$-\frac{b^3}{4\,(dx+c)^4\,d^4}-\frac{3\,(ad-bc)\,b^2}{5\,(dx+c)^5\,d^4}-\frac{\left(a^2d^2-2abcd+b^2c^2\right)b}{2\,(dx+c)^6\,d^4}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{7\,(dx+c)^7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/d^4/(d*x+c)^7-1/2*b*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/d^4/(d*x+c)^6-1/4*b^3/d^4/(d*x+c)^4-3/5*b^2*(a*d-b*
c)/d^4/(d*x+c)^5
```

maxima [B] time = 1.47, size = 182, normalized size = 1.98

$$-\frac{35\,b^3d^3x^3+b^3c^3+4\,ab^2c^2d+10\,a^2bcd^2+20\,a^3d^3+21\,\left(b^3cd^2+4\,ab^2d^3\right)x^2+7\,\left(b^3c^2d+4\,ab^2cd^2+10\,a^2bd^3\right)x}{140\,\left(d^{11}x^7+7\,cd^{10}x^6+21\,c^2d^9x^5+35\,c^3d^8x^4+35\,c^4d^7x^3+21\,c^5d^6x^2+7\,c^6d^5x+c^7d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/140*(35*b^3*d^3*x^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2 + 20*a^3*
d^3 + 21*(b^3*c*d^2 + 4*a*b^2*d^3)*x^2 + 7*(b^3*c^2*d + 4*a*b^2*c*d^2 + 10*
a^2*b*d^3)*x)/(d^11*x^7 + 7*c*d^10*x^6 + 21*c^2*d^9*x^5 + 35*c^3*d^8*x^4 +
35*c^4*d^7*x^3 + 21*c^5*d^6*x^2 + 7*c^6*d^5*x + c^7*d^4)
```

mupad [B] time = 0.10, size = 176, normalized size = 1.91

$$-\frac{\frac{20\,a^3\,d^3+10\,a^2\,b\,c\,d^2+4\,a\,b^2\,c^2\,d+b^3\,c^3}{140\,d^4}+\frac{b^3\,x^3}{4\,d}+\frac{b\,x\,\left(10\,a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{20\,d^3}+\frac{3\,b^2\,x^2\,\left(4\,a\,d+b\,c\right)}{20\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^8,x)
```

```
[Out] -((20*a^3*d^3 + b^3*c^3 + 4*a*b^2*c^2*d + 10*a^2*b*c*d^2)/(140*d^4) + (b^3*
x^3)/(4*d) + (b*x*(10*a^2*d^2 + b^2*c^2 + 4*a*b*c*d))/(20*d^3) + (3*b^2*x^2
*(4*a*d + b*c))/(20*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 3
5*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

sympy [B] time = 3.10, size = 196, normalized size = 2.13

$$\frac{-20a^3d^3 - 10a^2bcd^2 - 4ab^2c^2d - b^3c^3 - 35b^3d^3x^3 + x^2\left(-84ab^2d^3 - 21b^3cd^2\right) + x\left(-70a^2bd^3 - 28ab^2cd^2 - 7b^3c^2d\right)}{140c^7d^4 + 980c^6d^5x + 2940c^5d^6x^2 + 4900c^4d^7x^3 + 4900c^3d^8x^4 + 2940c^2d^9x^5 + 980cd^{10}x^6 + 140d^{11}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] (-20*a**3*d**3 - 10*a**2*b*c*d**2 - 4*a*b**2*c**2*d - b**3*c**3 - 35*b**3*d
**3*x**3 + x**2*(-84*a*b**2*d**3 - 21*b**3*c*d**2) + x*(-70*a**2*b*d**3 - 2
8*a*b**2*c*d**2 - 7*b**3*c**2*d))/(140*c**7*d**4 + 980*c**6*d**5*x + 2940*c
**5*d**6*x**2 + 4900*c**4*d**7*x**3 + 4900*c**3*d**8*x**4 + 2940*c**2*d**9*
x**5 + 980*c*d**10*x**6 + 140*d**11*x**7)
```

3.1263 $\int \frac{(a+bx)^2}{(c+dx)^8} dx$

Optimal. Leaf size=65

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{(bc-ad)^2}{7d^3(c+dx)^7}-\frac{b^2}{5d^3(c+dx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -(b*c - a*d)^2/(7*d^3*(c + d*x)^7) + (b*(b*c - a*d))/(3*d^3*(c + d*x)^6) -
b^2/(5*d^3*(c + d*x)^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^8}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^8}-\frac{2b(bc-ad)}{d^2(c+dx)^7}+\frac{b^2}{d^2(c+dx)^6}\right)dx\\ &= -\frac{(bc-ad)^2}{7d^3(c+dx)^7}+\frac{b(bc-ad)}{3d^3(c+dx)^6}-\frac{b^2}{5d^3(c+dx)^5}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.85

$$-\frac{15a^2d^2+5abd(c+7dx)+b^2\left(c^2+7cdx+21d^2x^2\right)}{105d^3(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] -1/105*(15*a^2*d^2 + 5*a*b*d*(c + 7*d*x) + b^2*(c^2 + 7*c*d*x + 21*d^2*x^2)
)/(d^3*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^2}{(c+dx)^8}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^8, x]
```

fricas [B] time = 1.26, size = 131, normalized size = 2.02

$$-\frac{21 \, b^{2} d^{2} x^{2} + b^{2} c^{2} + 5 \, a b c d + 15 \, a^{2} d^{2} + 7 \, {\left(b^{2} c d + 5 \, a b d^{2}\right)} x}{105 \, {\left(d^{10} x^{7} + 7 \, c d^{9} x^{6} + 21 \, c^{2} d^{8} x^{5} + 35 \, c^{3} d^{7} x^{4} + 35 \, c^{4} d^{6} x^{3} + 21 \, c^{5} d^{5} x^{2} + 7 \, c^{6} d^{4} x + c^{7} d^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

giac [A] time = 1.22, size = 61, normalized size = 0.94

$$-\frac{21 \, b^{2} d^{2} x^{2} + 7 \, b^{2} c d x + 35 \, a b d^{2} x + b^{2} c^{2} + 5 \, a b c d + 15 \, a^{2} d^{2}}{105 \, {\left(d x + c\right)}^{7} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + 7*b^2*c*d*x + 35*a*b*d^2*x + b^2*c^2 + 5*a*b*c*d +
 15*a^2*d^2)/((d*x + c)^7*d^3)
```

maple [A] time = 0.00, size = 71, normalized size = 1.09

$$-\frac{b^{2}}{5 \left(d x +c \right)^{5} d^{3}}-\frac{\left(a d -b c \right) b}{3 \left(d x +c \right)^{6} d^{3}}-\frac{a^{2} d^{2}-2 a b c d +b^{2} c^{2}}{7 \left(d x +c \right)^{7} d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3/(d*x+c)^7-1/5*b^2/d^3/(d*x+c)^5-1/3*b*
(a*d-b*c)/d^3/(d*x+c)^6
```

maxima [B] time = 1.46, size = 131, normalized size = 2.02

$$-\frac{21 \, b^{2} d^{2} x^{2} + b^{2} c^{2} + 5 \, a b c d + 15 \, a^{2} d^{2} + 7 \, {\left(b^{2} c d + 5 \, a b d^{2}\right)} x}{105 \, {\left(d^{10} x^{7} + 7 \, c d^{9} x^{6} + 21 \, c^{2} d^{8} x^{5} + 35 \, c^{3} d^{7} x^{4} + 35 \, c^{4} d^{6} x^{3} + 21 \, c^{5} d^{5} x^{2} + 7 \, c^{6} d^{4} x + c^{7} d^{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/105*(21*b^2*d^2*x^2 + b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + 7*(b^2*c*d + 5*
a*b*d^2)*x)/(d^10*x^7 + 7*c*d^9*x^6 + 21*c^2*d^8*x^5 + 35*c^3*d^7*x^4 + 35*
c^4*d^6*x^3 + 21*c^5*d^5*x^2 + 7*c^6*d^4*x + c^7*d^3)
```

mupad [B] time = 0.09, size = 129, normalized size = 1.98

$$-\frac{\frac{15\,a^2\,d^2+5\,a\,b\,c\,d+b^2\,c^2}{105\,d^3}+\frac{b^2\,x^2}{5\,d}+\frac{b\,x\,\left(5\,a\,d+b\,c\right)}{15\,d^2}}{c^7+7\,c^6\,d\,x+21\,c^5\,d^2\,x^2+35\,c^4\,d^3\,x^3+35\,c^3\,d^4\,x^4+21\,c^2\,d^5\,x^5+7\,c\,d^6\,x^6+d^7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^8,x)
```

```
[Out] -((15*a^2*d^2 + b^2*c^2 + 5*a*b*c*d)/(105*d^3) + (b^2*x^2)/(5*d) + (b*x*(5*
a*d + b*c))/(15*d^2))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^
4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

sympy [B] time = 1.39, size = 139, normalized size = 2.14

$$\frac{-15a^2d^2 - 5abcd - b^2c^2 - 21b^2d^2x^2 + x\left(-35abd^2 - 7b^2cd\right)}{105c^7d^3 + 735c^6d^4x + 2205c^5d^5x^2 + 3675c^4d^6x^3 + 3675c^3d^7x^4 + 2205c^2d^8x^5 + 735cd^9x^6 + 105d^{10}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] (-15*a**2*d**2 - 5*a*b*c*d - b**2*c**2 - 21*b**2*d**2*x**2 + x*(-35*a*b*d**
2 - 7*b**2*c*d))/(105*c**7*d**3 + 735*c**6*d**4*x + 2205*c**5*d**5*x**2 + 3
675*c**4*d**6*x**3 + 3675*c**3*d**7*x**4 + 2205*c**2*d**8*x**5 + 735*c*d**9
*x**6 + 105*d**10*x**7)
```

3.1264 $\int \frac{a+bx}{(c+dx)^8} dx$

Optimal. Leaf size=38

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] (b*c - a*d)/(7*d^2*(c + d*x)^7) - b/(6*d^2*(c + d*x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^8} dx &= \int \left(\frac{-bc+ad}{d(c+dx)^8}+\frac{b}{d(c+dx)^7}\right) dx\\ &= \frac{bc-ad}{7d^2(c+dx)^7}-\frac{b}{6d^2(c+dx)^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.71

$$-\frac{6ad+b(c+7dx)}{42d^2(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] -1/42*(6*a*d + b*(c + 7*d*x))/(d^2*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^8,x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/(c + d*x)^8, x]
```

fricas [B] time = 1.20, size = 94, normalized size = 2.47

$$-\frac{7\,bdx+bc+6\,ad}{42\left(d^9x^7+7\,cd^8x^6+21\,c^2d^7x^5+35\,c^3d^6x^4+35\,c^4d^5x^3+21\,c^5d^4x^2+7\,c^6d^3x+c^7d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

giac [A] time = 1.31, size = 25, normalized size = 0.66

$$-\frac{7\,bdx + bc + 6\,ad}{42\,(dx + c)^{7}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/((d*x + c)^7*d^2)
```

maple [A] time = 0.01, size = 35, normalized size = 0.92

$$-\frac{b}{6\,(dx + c)^{6}\,d^{2}} - \frac{ad - bc}{7\,(dx + c)^{7}\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7*(a*d-b*c)/d^2/(d*x+c)^7-1/6*b/d^2/(d*x+c)^6
```

maxima [B] time = 1.38, size = 94, normalized size = 2.47

$$-\frac{7\,bdx + bc + 6\,ad}{42\,\left(d^{9}x^{7} + 7\,cd^{8}x^{6} + 21\,c^{2}d^{7}x^{5} + 35\,c^{3}d^{6}x^{4} + 35\,c^{4}d^{5}x^{3} + 21\,c^{5}d^{4}x^{2} + 7\,c^{6}d^{3}x + c^{7}d^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/42*(7*b*d*x + b*c + 6*a*d)/(d^9*x^7 + 7*c*d^8*x^6 + 21*c^2*d^7*x^5 + 35*
c^3*d^6*x^4 + 35*c^4*d^5*x^3 + 21*c^5*d^4*x^2 + 7*c^6*d^3*x + c^7*d^2)
```

mupad [B] time = 0.23, size = 96, normalized size = 2.53

$$-\frac{\frac{6\,a\,d+b\,c}{42\,d^{2}} + \frac{b\,x}{6\,d}}{c^{7} + 7\,c^{6}\,d\,x + 21\,c^{5}\,d^{2}\,x^{2} + 35\,c^{4}\,d^{3}\,x^{3} + 35\,c^{3}\,d^{4}\,x^{4} + 21\,c^{2}\,d^{5}\,x^{5} + 7\,c\,d^{6}\,x^{6} + d^{7}\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^8,x)
```

```
[Out] -((6*a*d + b*c)/(42*d^2) + (b*x)/(6*d))/(c^7 + d^7*x^7 + 7*c*d^6*x^6 + 21*c
^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c^2*d^5*x^5 + 7*c^6*d*x)
```

sympy [B] time = 0.73, size = 100, normalized size = 2.63

$$\frac{-6ad - bc - 7bdx}{42c^{7}d^{2} + 294c^{6}d^{3}x + 882c^{5}d^{4}x^{2} + 1470c^{4}d^{5}x^{3} + 1470c^{3}d^{6}x^{4} + 882c^{2}d^{7}x^{5} + 294cd^{8}x^{6} + 42d^{9}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**8,x)
```

```
[Out] (-6*a*d - b*c - 7*b*d*x)/(42*c**7*d**2 + 294*c**6*d**3*x + 882*c**5*d**4*x*
*2 + 1470*c**4*d**5*x**3 + 1470*c**3*d**6*x**4 + 882*c**2*d**7*x**5 + 294*c
*d**8*x**6 + 42*d**9*x**7)
```

3.1265 $\int \frac{1}{(c+dx)^8} dx$

Optimal. Leaf size=14

$$-\frac{1}{7d(c+dx)^7}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-8),x]
```

```
[Out] -1/(7*d*(c + d*x)^7)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^8} dx = -\frac{1}{7d(c+dx)^7}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{1}{7d(c+dx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-8),x]
```

```
[Out] -1/7*1/(d*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^8} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-8),x]
```

```
[Out] IntegrateAlgebraic[(c + d*x)^(-8), x]
```

fricas [B] time = 1.17, size = 79, normalized size = 5.64

$$-\frac{1}{7\left(d^8x^7 + 7\,cd^7x^6 + 21\,c^2d^6x^5 + 35\,c^3d^5x^4 + 35\,c^4d^4x^3 + 21\,c^5d^3x^2 + 7\,c^6d^2x + c^7d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(d^8*x^7 + 7*c*d^7*x^6 + 21*c^2*d^6*x^5 + 35*c^3*d^5*x^4 + 35*c^4*d^4*
x^3 + 21*c^5*d^3*x^2 + 7*c^6*d^2*x + c^7*d)
```

giac [A] time = 1.26, size = 12, normalized size = 0.86

$$-\frac{1}{7(dx+c)^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{1}{7(dx+c)^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^8,x)
```

```
[Out] -1/7/d/(d*x+c)^7
```

maxima [A] time = 1.36, size = 12, normalized size = 0.86

$$-\frac{1}{7(dx+c)^7 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/7/((d*x + c)^7*d)
```

mupad [B] time = 0.22, size = 81, normalized size = 5.79

$$-\frac{1}{7c^7 d+49c^6 d^2 x+147c^5 d^3 x^2+245c^4 d^4 x^3+245c^3 d^5 x^4+147c^2 d^6 x^5+49c d^7 x^6+7d^8 x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^8,x)
```

```
[Out] -1/(7*c^7*d + 7*d^8*x^7 + 49*c^6*d^2*x + 49*c*d^7*x^6 + 147*c^5*d^3*x^2 + 2
45*c^4*d^4*x^3 + 245*c^3*d^5*x^4 + 147*c^2*d^6*x^5)
```

sympy [B] time = 0.46, size = 85, normalized size = 6.07

$$-\frac{1}{7c^7d+49c^6d^2x+147c^5d^3x^2+245c^4d^4x^3+245c^3d^5x^4+147c^2d^6x^5+49cd^7x^6+7d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**8,x)
```

```
[Out] -1/(7*c**7*d + 49*c**6*d**2*x + 147*c**5*d**3*x**2 + 245*c**4*d**4*x**3 + 2
45*c**3*d**5*x**4 + 147*c**2*d**6*x**5 + 49*c*d**7*x**6 + 7*d**8*x**7)
```

3.1266 $\int \frac{1}{(a+bx)(c+dx)^8} \, dx$

Optimal. Leaf size=202

$$\frac{b^7 \log(a+bx)}{(bc-ad)^8} - \frac{b^7 \log(c+dx)}{(bc-ad)^8} + \frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc}$$

Rubi [A] time = 0.17, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {44}

$$\frac{b^6}{(c+dx)(bc-ad)^7} + \frac{b^5}{2(c+dx)^2(bc-ad)^6} + \frac{b^4}{3(c+dx)^3(bc-ad)^5} + \frac{b^3}{4(c+dx)^4(bc-ad)^4} + \frac{b^2}{5(c+dx)^5(bc-ad)^3} + \frac{b^7 \log(a+bx)}{(bc-ad)^8} - \frac{b^7 \log(c+dx)}{(bc-ad)^8} + \frac{b}{6(c+dx)^6(bc-ad)^2} + \frac{1}{7(c+dx)^7(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] 1/(7*(b*c - a*d)*(c + d*x)^7) + b/(6*(b*c - a*d)^2*(c + d*x)^6) + b^2/(5*(b
*c - a*d)^3*(c + d*x)^5) + b^3/(4*(b*c - a*d)^4*(c + d*x)^4) + b^4/(3*(b*c
- a*d)^5*(c + d*x)^3) + b^5/(2*(b*c - a*d)^6*(c + d*x)^2) + b^6/((b*c - a*d
)^7*(c + d*x)) + (b^7*Log[a + b*x])/(b*c - a*d)^8 - (b^7*Log[c + d*x])/(b*c
 - a*d)^8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^8} \, dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)} - \frac{d}{(bc-ad)(c+dx)^8} - \frac{bd}{(bc-ad)^2(c+dx)^7} - \frac{b^2 d}{(bc-ad)^3(c+}\right.$$
$$= \frac{1}{7(bc-ad)(c+dx)^7} + \frac{b}{6(bc-ad)^2(c+dx)^6} + \frac{b^2}{5(bc-ad)^3(c+dx)^5} + \frac{b^3}{4(bc-ad)^4(c}$$

Mathematica [A] time = 0.10, size = 196, normalized size = 0.97

$$\frac{420b^7(c+dx)^7\log(a+bx) + 420b^6(c+dx)^6(bc-ad) + 210b^5(c+dx)^5(bc-ad)^2 + 140b^4(c+dx)^4(bc-ad)^3 + 105b^3(c+dx)^3(bc-ad)^4 + 84b^2(c+dx)^2(bc-ad)^5 + 70b(c+dx)(bc-ad)^6 + 60(bc-ad)^7 - 420b^7(c+dx)^7\log(c+dx)}{420(c+dx)^7(bc-ad)^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] (60*(b*c - a*d)^7 + 70*b*(b*c - a*d)^6*(c + d*x) + 84*b^2*(b*c - a*d)^5*(c
+ d*x)^2 + 105*b^3*(b*c - a*d)^4*(c + d*x)^3 + 140*b^4*(b*c - a*d)^3*(c + d
*x)^4 + 210*b^5*(b*c - a*d)^2*(c + d*x)^5 + 420*b^6*(b*c - a*d)*(c + d*x)^6
 + 420*b^7*(c + d*x)^7*Log[a + b*x] - 420*b^7*(c + d*x)^7*Log[c + d*x])/(42
0*(b*c - a*d)^8*(c + d*x)^7)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^8),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^8), x]
```

fricas [B] time = 1.54, size = 1589, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/420*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^
4*c^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 -
 60*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*
b^6*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21
*a^2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d
^4 + 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*
b^7*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5
 + 35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^
5*d^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 -
84*a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x + 420*(b^7*d^7*x^7 + 7*b^7*c*d^6*x^6 + 2
1*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21*b^7*c^5*d^
2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(b*x + a) - 420*(b^7*d^7*x^7 + 7*b^7*c*
d^6*x^6 + 21*b^7*c^2*d^5*x^5 + 35*b^7*c^3*d^4*x^4 + 35*b^7*c^4*d^3*x^3 + 21
*b^7*c^5*d^2*x^2 + 7*b^7*c^6*d*x + b^7*c^7)*log(d*x + c))/(b^8*c^15 - 8*a*b
^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4
 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8
 + (b^8*c^8*d^7 - 8*a*b^7*c^7*d^8 + 28*a^2*b^6*c^6*d^9 - 56*a^3*b^5*c^5*d^1
0 + 70*a^4*b^4*c^4*d^11 - 56*a^5*b^3*c^3*d^12 + 28*a^6*b^2*c^2*d^13 - 8*a^7
*b*c*d^14 + a^8*d^15)*x^7 + 7*(b^8*c^9*d^6 - 8*a*b^7*c^8*d^7 + 28*a^2*b^6*c
^7*d^8 - 56*a^3*b^5*c^6*d^9 + 70*a^4*b^4*c^5*d^10 - 56*a^5*b^3*c^4*d^11 + 2
8*a^6*b^2*c^3*d^12 - 8*a^7*b*c^2*d^13 + a^8*c*d^14)*x^6 + 21*(b^8*c^10*d^5
- 8*a*b^7*c^9*d^6 + 28*a^2*b^6*c^8*d^7 - 56*a^3*b^5*c^7*d^8 + 70*a^4*b^4*c^
6*d^9 - 56*a^5*b^3*c^5*d^10 + 28*a^6*b^2*c^4*d^11 - 8*a^7*b*c^3*d^12 + a^8*
c^2*d^13)*x^5 + 35*(b^8*c^11*d^4 - 8*a*b^7*c^10*d^5 + 28*a^2*b^6*c^9*d^6 -
56*a^3*b^5*c^8*d^7 + 70*a^4*b^4*c^7*d^8 - 56*a^5*b^3*c^6*d^9 + 28*a^6*b^2*c
^5*d^10 - 8*a^7*b*c^4*d^11 + a^8*c^3*d^12)*x^4 + 35*(b^8*c^12*d^3 - 8*a*b^7
*c^11*d^4 + 28*a^2*b^6*c^10*d^5 - 56*a^3*b^5*c^9*d^6 + 70*a^4*b^4*c^8*d^7 -
 56*a^5*b^3*c^7*d^8 + 28*a^6*b^2*c^6*d^9 - 8*a^7*b*c^5*d^10 + a^8*c^4*d^11)
*x^3 + 21*(b^8*c^13*d^2 - 8*a*b^7*c^12*d^3 + 28*a^2*b^6*c^11*d^4 - 56*a^3*b
^5*c^10*d^5 + 70*a^4*b^4*c^9*d^6 - 56*a^5*b^3*c^8*d^7 + 28*a^6*b^2*c^7*d^8
- 8*a^7*b*c^6*d^9 + a^8*c^5*d^10)*x^2 + 7*(b^8*c^14*d - 8*a*b^7*c^13*d^2 +
28*a^2*b^6*c^12*d^3 - 56*a^3*b^5*c^11*d^4 + 70*a^4*b^4*c^10*d^5 - 56*a^5*b^
3*c^9*d^6 + 28*a^6*b^2*c^8*d^7 - 8*a^7*b*c^7*d^8 + a^8*c^6*d^9)*x)
```

giac [B] time = 1.33, size = 703, normalized size = 3.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] b^8*log(abs(b*x + a))/(b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^
3*b^6*c^5*d^3 + 70*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^
6 - 8*a^7*b^2*c*d^7 + a^8*b*d^8) - b^7*d*log(abs(d*x + c))/(b^8*c^8*d - 8*a
*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4 + 70*a^4*b^4*c^4*d^5
 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c*d^8 + a^8*d^9) + 1/4
20*(1089*b^7*c^7 - 2940*a*b^6*c^6*d + 4410*a^2*b^5*c^5*d^2 - 4900*a^3*b^4*c
^4*d^3 + 3675*a^4*b^3*c^3*d^4 - 1764*a^5*b^2*c^2*d^5 + 490*a^6*b*c*d^6 - 60
*a^7*d^7 + 420*(b^7*c*d^6 - a*b^6*d^7)*x^6 + 210*(13*b^7*c^2*d^5 - 14*a*b^6
*c*d^6 + a^2*b^5*d^7)*x^5 + 70*(107*b^7*c^3*d^4 - 126*a*b^6*c^2*d^5 + 21*a^
```

```
2*b^5*c*d^6 - 2*a^3*b^4*d^7)*x^4 + 35*(319*b^7*c^4*d^3 - 420*a*b^6*c^3*d^4
+ 126*a^2*b^5*c^2*d^5 - 28*a^3*b^4*c*d^6 + 3*a^4*b^3*d^7)*x^3 + 21*(459*b^7
*c^5*d^2 - 700*a*b^6*c^4*d^3 + 350*a^2*b^5*c^3*d^4 - 140*a^3*b^4*c^2*d^5 +
35*a^4*b^3*c*d^6 - 4*a^5*b^2*d^7)*x^2 + 7*(669*b^7*c^6*d - 1260*a*b^6*c^5*d
^2 + 1050*a^2*b^5*c^4*d^3 - 700*a^3*b^4*c^3*d^4 + 315*a^4*b^3*c^2*d^5 - 84*
a^5*b^2*c*d^6 + 10*a^6*b*d^7)*x)/((b*c - a*d)^8*(d*x + c)^7)
```

maple [A] time = 0.02, size = 192, normalized size = 0.95

$$\frac{b^7\ln(bx+a)}{(ad-bc)^8}-\frac{b^7\ln(dx+c)}{(ad-bc)^8}-\frac{b^6}{(ad-bc)^7(dx+c)}+\frac{b^5}{2(ad-bc)^6(dx+c)^2}-\frac{b^4}{3(ad-bc)^5(dx+c)^3}+\frac{b^3}{4(ad-bc)^4(dx+c)^4}-\frac{b^2}{5(ad-bc)^3(dx+c)^5}+\frac{b}{6(ad-bc)^2(dx+c)^6}-\frac{1}{7(ad-bc)(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^8,x)
```

```
[Out] -1/7/(a*d-b*c)/(d*x+c)^7-1/5*b^2/(a*d-b*c)^3/(d*x+c)^5-1/3*b^4/(a*d-b*c)^5/
(d*x+c)^3-b^6/(a*d-b*c)^7/(d*x+c)+1/6*b/(a*d-b*c)^2/(d*x+c)^6+1/4*b^3/(a*d-
b*c)^4/(d*x+c)^4+1/2*b^5/(a*d-b*c)^6/(d*x+c)^2-b^7/(a*d-b*c)^8*ln(d*x+c)+b^
7/(a*d-b*c)^8*ln(b*x+a)
```

maxima [B] time = 2.97, size = 1418, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] b^7*log(b*x + a)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8) - b^7*log(d*x + c)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8) + 1/420*(420*b^6*d^6*x^6
+ 1089*b^6*c^6 - 1851*a*b^5*c^5*d + 2559*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3
*d^3 + 1334*a^4*b^2*c^2*d^4 - 430*a^5*b*c*d^5 + 60*a^6*d^6 + 210*(13*b^6*c*
d^5 - a*b^5*d^6)*x^5 + 70*(107*b^6*c^2*d^4 - 19*a*b^5*c*d^5 + 2*a^2*b^4*d^6
)*x^4 + 35*(319*b^6*c^3*d^3 - 101*a*b^5*c^2*d^4 + 25*a^2*b^4*c*d^5 - 3*a^3*
b^3*d^6)*x^3 + 21*(459*b^6*c^4*d^2 - 241*a*b^5*c^3*d^3 + 109*a^2*b^4*c^2*d^
4 - 31*a^3*b^3*c*d^5 + 4*a^4*b^2*d^6)*x^2 + 7*(669*b^6*c^5*d - 591*a*b^5*c^
4*d^2 + 459*a^2*b^4*c^3*d^3 - 241*a^3*b^3*c^2*d^4 + 74*a^4*b^2*c*d^5 - 10*a
^5*b*d^6)*x)/(b^7*c^14 - 7*a*b^6*c^13*d + 21*a^2*b^5*c^12*d^2 - 35*a^3*b^4*
c^11*d^3 + 35*a^4*b^3*c^10*d^4 - 21*a^5*b^2*c^9*d^5 + 7*a^6*b*c^8*d^6 - a^7
*c^7*d^7 + (b^7*c^7*d^7 - 7*a*b^6*c^6*d^8 + 21*a^2*b^5*c^5*d^9 - 35*a^3*b^4
*c^4*d^10 + 35*a^4*b^3*c^3*d^11 - 21*a^5*b^2*c^2*d^12 + 7*a^6*b*c*d^13 - a^
7*d^14)*x^7 + 7*(b^7*c^8*d^6 - 7*a*b^6*c^7*d^7 + 21*a^2*b^5*c^6*d^8 - 35*a^
3*b^4*c^5*d^9 + 35*a^4*b^3*c^4*d^10 - 21*a^5*b^2*c^3*d^11 + 7*a^6*b*c^2*d^1
2 - a^7*c*d^13)*x^6 + 21*(b^7*c^9*d^5 - 7*a*b^6*c^8*d^6 + 21*a^2*b^5*c^7*d^
7 - 35*a^3*b^4*c^6*d^8 + 35*a^4*b^3*c^5*d^9 - 21*a^5*b^2*c^4*d^10 + 7*a^6*b
*c^3*d^11 - a^7*c^2*d^12)*x^5 + 35*(b^7*c^10*d^4 - 7*a*b^6*c^9*d^5 + 21*a^2
*b^5*c^8*d^6 - 35*a^3*b^4*c^7*d^7 + 35*a^4*b^3*c^6*d^8 - 21*a^5*b^2*c^5*d^9
 + 7*a^6*b*c^4*d^10 - a^7*c^3*d^11)*x^4 + 35*(b^7*c^11*d^3 - 7*a*b^6*c^10*d
^4 + 21*a^2*b^5*c^9*d^5 - 35*a^3*b^4*c^8*d^6 + 35*a^4*b^3*c^7*d^7 - 21*a^5*
b^2*c^6*d^8 + 7*a^6*b*c^5*d^9 - a^7*c^4*d^10)*x^3 + 21*(b^7*c^12*d^2 - 7*a*
b^6*c^11*d^3 + 21*a^2*b^5*c^10*d^4 - 35*a^3*b^4*c^9*d^5 + 35*a^4*b^3*c^8*d^
6 - 21*a^5*b^2*c^7*d^7 + 7*a^6*b*c^6*d^8 - a^7*c^5*d^9)*x^2 + 7*(b^7*c^13*d
 - 7*a*b^6*c^12*d^2 + 21*a^2*b^5*c^11*d^3 - 35*a^3*b^4*c^10*d^4 + 35*a^4*b^
3*c^9*d^5 - 21*a^5*b^2*c^8*d^6 + 7*a^6*b*c^7*d^7 - a^7*c^6*d^8)*x)
```

mupad [B] time = 0.87, size = 1299, normalized size = 6.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^8),x)
```

```
[Out] (2*b^7*atanh((a^8*d^8 - b^8*c^8 - 14*a^2*b^6*c^6*d^2 + 14*a^3*b^5*c^5*d^3 -
 14*a^5*b^3*c^3*d^5 + 14*a^6*b^2*c^2*d^6 + 6*a*b^7*c^7*d - 6*a^7*b*c*d^7)/(
a*d - b*c)^8 + (2*b*d*x*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^
4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6
*b*c*d^6))/(a*d - b*c)^8))/(a*d - b*c)^8 - ((60*a^6*d^6 + 1089*b^6*c^6 + 25
59*a^2*b^4*c^4*d^2 - 2341*a^3*b^3*c^3*d^3 + 1334*a^4*b^2*c^2*d^4 - 1851*a*b
^5*c^5*d - 430*a^5*b*c*d^5)/(420*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 +
35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*
d - 7*a^6*b*c*d^6)) - (b^3*x^3*(3*a^3*d^6 - 319*b^3*c^3*d^3 + 101*a*b^2*c^2
*d^4 - 25*a^2*b*c*d^5))/(12*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^
3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7
*a^6*b*c*d^6)) + (b^6*d^6*x^6)/(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35
*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d
- 7*a^6*b*c*d^6) - (b^5*x^5*(a*d^6 - 13*b*c*d^5))/(2*(a^7*d^7 - b^7*c^7 - 2
1*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^
2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^2*x^2*(4*a^4*d^6 + 459*b^4*c^4
*d^2 - 241*a*b^3*c^3*d^3 + 109*a^2*b^2*c^2*d^4 - 31*a^3*b*c*d^5))/(20*(a^7*
d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^
4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)) + (b^4*x^4*(2*a^2*
d^6 + 107*b^2*c^2*d^4 - 19*a*b*c*d^5))/(6*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c
^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a
*b^6*c^6*d - 7*a^6*b*c*d^6)) - (b*x*(10*a^5*d^6 - 669*b^5*c^5*d + 591*a*b^4
*c^4*d^2 - 459*a^2*b^3*c^3*d^3 + 241*a^3*b^2*c^2*d^4 - 74*a^4*b*c*d^5))/(60
*(a^7*d^7 - b^7*c^7 - 21*a^2*b^5*c^5*d^2 + 35*a^3*b^4*c^4*d^3 - 35*a^4*b^3*
c^3*d^4 + 21*a^5*b^2*c^2*d^5 + 7*a*b^6*c^6*d - 7*a^6*b*c*d^6)))/(c^7 + d^7*
x^7 + 7*c*d^6*x^6 + 21*c^5*d^2*x^2 + 35*c^4*d^3*x^3 + 35*c^3*d^4*x^4 + 21*c
^2*d^5*x^5 + 7*c^6*d*x)
```

sympy [B] time = 4.49, size = 1776, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**8,x)
```

```
[Out] -b**7*log(x + (-a**9*b**7*d**9/(a*d - b*c)**8 + 9*a**8*b**8*c*d**8/(a*d - b
*c)**8 - 36*a**7*b**9*c**2*d**7/(a*d - b*c)**8 + 84*a**6*b**10*c**3*d**6/(a
*d - b*c)**8 - 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 + 126*a**4*b**12*c**
5*d**4/(a*d - b*c)**8 - 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 + 36*a**2*b*
*14*c**7*d**2/(a*d - b*c)**8 - 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d +
 b**16*c**9/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + b**7*log(
x + (a**9*b**7*d**9/(a*d - b*c)**8 - 9*a**8*b**8*c*d**8/(a*d - b*c)**8 + 36
*a**7*b**9*c**2*d**7/(a*d - b*c)**8 - 84*a**6*b**10*c**3*d**6/(a*d - b*c)**
8 + 126*a**5*b**11*c**4*d**5/(a*d - b*c)**8 - 126*a**4*b**12*c**5*d**4/(a*d
 - b*c)**8 + 84*a**3*b**13*c**6*d**3/(a*d - b*c)**8 - 36*a**2*b**14*c**7*d*
*2/(a*d - b*c)**8 + 9*a*b**15*c**8*d/(a*d - b*c)**8 + a*b**7*d - b**16*c**9
/(a*d - b*c)**8 + b**8*c)/(2*b**8*d))/(a*d - b*c)**8 + (-60*a**6*d**6 + 430
*a**5*b*c*d**5 - 1334*a**4*b**2*c**2*d**4 + 2341*a**3*b**3*c**3*d**3 - 2559
*a**2*b**4*c**4*d**2 + 1851*a*b**5*c**5*d - 1089*b**6*c**6 - 420*b**6*d**6*
x**6 + x**5*(210*a*b**5*d**6 - 2730*b**6*c*d**5) + x**4*(-140*a**2*b**4*d**
6 + 1330*a*b**5*c*d**5 - 7490*b**6*c**2*d**4) + x**3*(105*a**3*b**3*d**6 -
875*a**2*b**4*c*d**5 + 3535*a*b**5*c**2*d**4 - 11165*b**6*c**3*d**3) + x**2
*(-84*a**4*b**2*d**6 + 651*a**3*b**3*c*d**5 - 2289*a**2*b**4*c**2*d**4 + 50
61*a*b**5*c**3*d**3 - 9639*b**6*c**4*d**2) + x*(70*a**5*b*d**6 - 518*a**4*b
**2*c*d**5 + 1687*a**3*b**3*c**2*d**4 - 3213*a**2*b**4*c**3*d**3 + 4137*a*b
**5*c**4*d**2 - 4683*b**6*c**5*d))/(420*a**7*c**7*d**7 - 2940*a**6*b*c**8*d
**6 + 8820*a**5*b**2*c**9*d**5 - 14700*a**4*b**3*c**10*d**4 + 14700*a**3*b*
*4*c**11*d**3 - 8820*a**2*b**5*c**12*d**2 + 2940*a*b**6*c**13*d - 420*b**7*
c**14 + x**7*(420*a**7*d**14 - 2940*a**6*b*c*d**13 + 8820*a**5*b**2*c**2*d*
```

```
*12 - 14700*a**4*b**3*c**3*d**11 + 14700*a**3*b**4*c**4*d**10 - 8820*a**2*b
**5*c**5*d**9 + 2940*a*b**6*c**6*d**8 - 420*b**7*c**7*d**7) + x**6*(2940*a*
*7*c*d**13 - 20580*a**6*b*c**2*d**12 + 61740*a**5*b**2*c**3*d**11 - 102900*
a**4*b**3*c**4*d**10 + 102900*a**3*b**4*c**5*d**9 - 61740*a**2*b**5*c**6*d*
*8 + 20580*a*b**6*c**7*d**7 - 2940*b**7*c**8*d**6) + x**5*(8820*a**7*c**2*d
**12 - 61740*a**6*b*c**3*d**11 + 185220*a**5*b**2*c**4*d**10 - 308700*a**4*
b**3*c**5*d**9 + 308700*a**3*b**4*c**6*d**8 - 185220*a**2*b**5*c**7*d**7 +
61740*a*b**6*c**8*d**6 - 8820*b**7*c**9*d**5) + x**4*(14700*a**7*c**3*d**11
 - 102900*a**6*b*c**4*d**10 + 308700*a**5*b**2*c**5*d**9 - 514500*a**4*b**3
*c**6*d**8 + 514500*a**3*b**4*c**7*d**7 - 308700*a**2*b**5*c**8*d**6 + 1029
00*a*b**6*c**9*d**5 - 14700*b**7*c**10*d**4) + x**3*(14700*a**7*c**4*d**10
- 102900*a**6*b*c**5*d**9 + 308700*a**5*b**2*c**6*d**8 - 514500*a**4*b**3*c
**7*d**7 + 514500*a**3*b**4*c**8*d**6 - 308700*a**2*b**5*c**9*d**5 + 102900
*a*b**6*c**10*d**4 - 14700*b**7*c**11*d**3) + x**2*(8820*a**7*c**5*d**9 - 6
1740*a**6*b*c**6*d**8 + 185220*a**5*b**2*c**7*d**7 - 308700*a**4*b**3*c**8*
d**6 + 308700*a**3*b**4*c**9*d**5 - 185220*a**2*b**5*c**10*d**4 + 61740*a*b
**6*c**11*d**3 - 8820*b**7*c**12*d**2) + x*(2940*a**7*c**6*d**8 - 20580*a**
6*b*c**7*d**7 + 61740*a**5*b**2*c**8*d**6 - 102900*a**4*b**3*c**9*d**5 + 10
2900*a**3*b**4*c**10*d**4 - 61740*a**2*b**5*c**11*d**3 + 20580*a*b**6*c**12
*d**2 - 2940*b**7*c**13*d))
```

3.1267 $\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$

Optimal. Leaf size=231

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc$$

Rubi [A] time = 0.27, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{b^7}{(a+bx)(bc-ad)^8}-\frac{7b^6d}{(c+dx)(bc-ad)^8}-\frac{3b^5d}{(c+dx)^2(bc-ad)^7}-\frac{5b^4d}{3(c+dx)^3(bc-ad)^6}-\frac{b^3d}{(c+dx)^4(bc-ad)^5}-\frac{3b^2d}{5(c+dx)^5(bc-ad)^4}-\frac{8b^7d\log(a+bx)}{(bc-ad)^9}+\frac{8b^7d\log(c+dx)}{(bc-ad)^9}-\frac{bd}{3(c+dx)^6(bc-ad)^3}-\frac{d}{7(c+dx)^7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -(b^7/((b*c - a*d)^8*(a + b*x))) - d/(7*(b*c - a*d)^2*(c + d*x)^7) - (b*d)/
(3*(b*c - a*d)^3*(c + d*x)^6) - (3*b^2*d)/(5*(b*c - a*d)^4*(c + d*x)^5) - (
b^3*d)/((b*c - a*d)^5*(c + d*x)^4) - (5*b^4*d)/(3*(b*c - a*d)^6*(c + d*x)^3
) - (3*b^5*d)/((b*c - a*d)^7*(c + d*x)^2) - (7*b^6*d)/((b*c - a*d)^8*(c + d
*x)) - (8*b^7*d*Log[a + b*x])/(b*c - a*d)^9 + (8*b^7*d*Log[c + d*x])/(b*c -
 a*d)^9
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx &= \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^2}-\frac{8b^8d}{(bc-ad)^9(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)^8}+\frac{2bd^2}{(bc-ad)^3(c+d\right.\\ &= -\frac{b^7}{(bc-ad)^8(a+bx)}-\frac{d}{7(bc-ad)^2(c+dx)^7}-\frac{bd}{3(bc-ad)^3(c+dx)^6}-\frac{3b^2d}{5(bc-ad)^4(c+\end{aligned}$$

Mathematica [A] time = 0.24, size = 213, normalized size = 0.92

$$-\frac{\frac{105b^7(bc-ad)}{a+bx}+840b^7d\log(a+bx)+\frac{735b^6d(bc-ad)}{c+dx}+\frac{315b^5d(bc-ad)^2}{(c+dx)^2}+\frac{175b^4d(bc-ad)^3}{(c+dx)^3}+\frac{105b^3d(bc-ad)^4}{(c+dx)^4}+\frac{63b^2d(bc-ad)^5}{(c+dx)^5}+\frac{35bd(bc-ad)^6}{(c+dx)^6}-\frac{15d(ad-bc)^7}{(c+dx)^7}-840b^7d\log(c+dx)}{105(bc-ad)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^8),x]
```

```
[Out] -1/105*((105*b^7*(b*c - a*d))/(a + b*x) - (15*d*(-(b*c) + a*d)^7)/(c + d*x)
^7 + (35*b*d*(b*c - a*d)^6)/(c + d*x)^6 + (63*b^2*d*(b*c - a*d)^5)/(c + d*x
)^5 + (105*b^3*d*(b*c - a*d)^4)/(c + d*x)^4 + (175*b^4*d*(b*c - a*d)^3)/(c
+ d*x)^3 + (315*b^5*d*(b*c - a*d)^2)/(c + d*x)^2 + (735*b^6*d*(b*c - a*d))/
(c + d*x) + 840*b^7*d*Log[a + b*x] - 840*b^7*d*Log[c + d*x])/(b*c - a*d)^9
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2(c+dx)^8}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^8),x]

[Out] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^8), x]

fricas [B] time = 1.46, size = 2264, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="fricas")

[Out] -1/105*(105*b^8*c^8 + 1338*a*b^7*c^7*d - 2940*a^2*b^6*c^6*d^2 + 2940*a^3*b^5*c^5*d^3 - 2450*a^4*b^4*c^4*d^4 + 1470*a^5*b^3*c^3*d^5 - 588*a^6*b^2*c^2*d^6 + 140*a^7*b*c*d^7 - 15*a^8*d^8 + 840*(b^8*c*d^7 - a*b^7*d^8)*x^7 + 420*(13*b^8*c^2*d^6 - 12*a*b^7*c*d^7 - a^2*b^6*d^8)*x^6 + 140*(107*b^8*c^3*d^5 - 87*a*b^7*c^2*d^6 - 21*a^2*b^6*c*d^7 + a^3*b^5*d^8)*x^5 + 70*(319*b^8*c^4*d^4 - 206*a*b^7*c^3*d^5 - 126*a^2*b^6*c^2*d^6 + 14*a^3*b^5*c*d^7 - a^4*b^4*d^8)*x^4 + 14*(1377*b^8*c^5*d^3 - 505*a*b^7*c^4*d^4 - 1050*a^2*b^6*c^3*d^5 + 210*a^3*b^5*c^2*d^6 - 35*a^4*b^4*c*d^7 + 3*a^5*b^3*d^8)*x^3 + 14*(669*b^8*c^6*d^2 + 117*a*b^7*c^5*d^3 - 1050*a^2*b^6*c^4*d^4 + 350*a^3*b^5*c^3*d^5 - 105*a^4*b^4*c^2*d^6 + 21*a^5*b^3*c*d^7 - 2*a^6*b^2*d^8)*x^2 + 2*(1089*b^8*c^7*d + 1743*a*b^7*c^6*d^2 - 4410*a^2*b^6*c^5*d^3 + 2450*a^3*b^5*c^4*d^4 - 1225*a^4*b^4*c^3*d^5 + 441*a^5*b^3*c^2*d^6 - 98*a^6*b^2*c*d^7 + 10*a^7*b*d^8)*x + 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2)*x)*log(b*x + a) - 840*(b^8*d^8*x^8 + a*b^7*c^7*d + (7*b^8*c*d^7 + a*b^7*d^8)*x^7 + 7*(3*b^8*c^2*d^6 + a*b^7*c*d^7)*x^6 + 7*(5*b^8*c^3*d^5 + 3*a*b^7*c^2*d^6)*x^5 + 35*(b^8*c^4*d^4 + a*b^7*c^3*d^5)*x^4 + 7*(3*b^8*c^5*d^3 + 5*a*b^7*c^4*d^4)*x^3 + 7*(b^8*c^6*d^2 + 3*a*b^7*c^5*d^3)*x^2 + (b^8*c^7*d + 7*a*b^7*c^6*d^2)*x)*log(d*x + c))/(a*b^9*c^16 - 9*a^2*b^8*c^15*d + 36*a^3*b^7*c^14*d^2 - 84*a^4*b^6*c^13*d^3 + 126*a^5*b^5*c^12*d^4 - 126*a^6*b^4*c^11*d^5 + 84*a^7*b^3*c^10*d^6 - 36*a^8*b^2*c^9*d^7 + 9*a^9*b*c^8*d^8 - a^10*c^7*d^9 + (b^10*c^9*d^7 - 9*a*b^9*c^8*d^8 + 36*a^2*b^8*c^7*d^9 - 84*a^3*b^7*c^6*d^10 + 126*a^4*b^6*c^5*d^11 - 126*a^5*b^5*c^4*d^12 + 84*a^6*b^4*c^3*d^13 - 36*a^7*b^3*c^2*d^14 + 9*a^8*b^2*c*d^15 - a^9*b*d^16)*x^8 + (7*b^10*c^10*d^6 - 62*a*b^9*c^9*d^7 + 243*a^2*b^8*c^8*d^8 - 552*a^3*b^7*c^7*d^9 + 798*a^4*b^6*c^6*d^10 - 756*a^5*b^5*c^5*d^11 + 462*a^6*b^4*c^4*d^12 - 168*a^7*b^3*c^3*d^13 + 27*a^8*b^2*c^2*d^14 + 2*a^9*b*c*d^15 - a^10*d^16)*x^7 + 7*(3*b^10*c^11*d^5 - 26*a*b^9*c^10*d^6 + 99*a^2*b^8*c^9*d^7 - 216*a^3*b^7*c^8*d^8 + 294*a^4*b^6*c^7*d^9 - 252*a^5*b^5*c^6*d^10 + 126*a^6*b^4*c^5*d^11 - 24*a^7*b^3*c^4*d^12 - 9*a^8*b^2*c^3*d^13 + 6*a^9*b*c^2*d^14 - a^10*c*d^15)*x^6 + 7*(5*b^10*c^12*d^4 - 42*a*b^9*c^11*d^5 + 153*a^2*b^8*c^10*d^6 - 312*a^3*b^7*c^9*d^7 + 378*a^4*b^6*c^8*d^8 - 252*a^5*b^5*c^7*d^9 + 42*a^6*b^4*c^6*d^10 + 72*a^7*b^3*c^5*d^11 - 63*a^8*b^2*c^4*d^12 + 22*a^9*b*c^3*d^13 - 3*a^10*c^2*d^14)*x^5 + 35*(b^10*c^13*d^3 - 8*a*b^9*c^12*d^4 + 27*a^2*b^8*c^11*d^5 - 48*a^3*b^7*c^10*d^6 + 42*a^4*b^6*c^9*d^7 - 42*a^6*b^4*c^7*d^9 + 48*a^7*b^3*c^6*d^10 - 27*a^8*b^2*c^5*d^11 + 8*a^9*b*c^4*d^12 - a^10*c^3*d^13)*x^4 + 7*(3*b^10*c^14*d^2 - 22*a*b^9*c^13*d^3 + 63*a^2*b^8*c^12*d^4 - 72*a^3*b^7*c^11*d^5 - 42*a^4*b^6*c^10*d^6 + 252*a^5*b^5*c^9*d^7 - 378*a^6*b^4*c^8*d^8 + 312*a^7*b^3*c^7*d^9 - 153*a^8*b^2*c^6*d^10 + 42*a^9*b*c^5*d^11 - 5*a^10*c^4*d^12)*x^3 + 7*(b^10*c^15*d - 6*a*b^9*c^14*d^2 + 9*a^2*b^8*c^13*d^3 + 24*a^3*b^7*c^12*d^4 - 126*a^4*b^6*c^11*d^5 + 252*a^5*b^5*c^10*d^6 - 294*a^6*b^4*c^9*d^7 + 216*a^7*b^3*c^8*d^8 - 99*a^8*b^2*c^7*d^9 + 26*a^9*b*c^6*d^10 - 3*a^10*c^5*d^11)*x^2 + (b^10*c^16 - 2*a*b^9*c^15*d - 27*a^2*b^8*c^14*d^2 + 168*a^3*b^7*c^13*d^3 - 462*a^4*b^6*c^12*d^4 + 756*a^5*b^5*c^11*d^5 - 798*a^6*b^4*c^10*d^6 + 552*a^7*b^3*c^9*d^7 - 243*a^8*b^2*c^8*d^8 + 62*a^9*b*c^7*d^9 - 7*a^10*c^6*d^10)*x)

giac [B] time = 1.37, size = 714, normalized size = 3.09

$$-\frac{b^{15}}{\left(b^{16}c^{8}-8\,ab^{15}c^{7}d+28\,a^{2}b^{14}c^{6}d^{2}-56\,a^{3}b^{13}c^{5}d^{3}+70\,a^{4}b^{12}c^{4}d^{4}-56\,a^{5}b^{11}c^{3}d^{5}+28\,a^{6}b^{10}c^{2}d^{6}-8\,a^{7}b^{9}cd^{7}+a^{8}b^{8}d^{8}\right)(bx+a)}+\frac{8\,b^{8}d\log\left(\left|\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right|\right)}{b^{10}c^{9}-9\,ab^{9}c^{8}d+36\,a^{2}b^{8}c^{7}d^{2}-84\,a^{3}b^{7}c^{6}d^{3}+126\,a^{4}b^{6}c^{5}d^{4}-126\,a^{5}b^{5}c^{4}d^{5}+84\,a^{6}b^{4}c^{3}d^{6}-36\,a^{7}b^{3}c^{2}d^{7}+9\,a^{8}b^{2}cd^{8}-a^{9}bd^{9}}+\frac{1443\,b^{7}d^{8}+\frac{9366\left(b^{9}cd^{7}-ab^{8}d^{8}\right)}{(bx+a)b}+\frac{25578\left(b^{11}c^{2}d^{6}-2\,ab^{10}cd^{7}+a^{2}b^{9}d^{8}\right)}{(bx+a)^{2}b^{2}}+\frac{37730\left(b^{13}c^{3}d^{5}-3\,ab^{12}c^{2}d^{6}+3\,a^{2}b^{11}cd^{7}-a^{3}b^{10}d^{8}\right)}{(bx+a)^{3}b^{3}}+\frac{31850\left(b^{15}c^{4}d^{4}-4\,ab^{14}c^{3}d^{5}+6\,a^{2}b^{13}c^{2}d^{6}-4\,a^{3}b^{12}cd^{7}+a^{4}b^{11}d^{8}\right)}{(bx+a)^{4}b^{4}}+\frac{14700\left(b^{17}c^{5}d^{3}-5\,ab^{16}c^{4}d^{4}+10\,a^{2}b^{15}c^{3}d^{5}-10\,a^{3}b^{14}c^{2}d^{6}+5\,a^{4}b^{13}cd^{7}-a^{5}b^{12}d^{8}\right)}{(bx+a)^{5}b^{5}}+\frac{2940\left(b^{19}c^{6}d^{2}-6\,ab^{18}c^{5}d^{3}+15\,a^{2}b^{17}c^{4}d^{4}-20\,a^{3}b^{16}c^{3}d^{5}+15\,a^{4}b^{15}c^{2}d^{6}-6\,a^{5}b^{14}cd^{7}+a^{6}b^{13}d^{8}\right)}{(bx+a)^{6}b^{6}}}{105\,(bc-ad)^{9}\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -b^15/((b^16*c^8 - 8*a*b^15*c^7*d + 28*a^2*b^14*c^6*d^2 - 56*a^3*b^13*c^5*d
^3 + 70*a^4*b^12*c^4*d^4 - 56*a^5*b^11*c^3*d^5 + 28*a^6*b^10*c^2*d^6 - 8*a^
7*b^9*c*d^7 + a^8*b^8*d^8)*(b*x + a)) + 8*b^8*d*log(abs(b*c/(b*x + a) - a*d
/(b*x + a) + d))/(b^10*c^9 - 9*a*b^9*c^8*d + 36*a^2*b^8*c^7*d^2 - 84*a^3*b^
7*c^6*d^3 + 126*a^4*b^6*c^5*d^4 - 126*a^5*b^5*c^4*d^5 + 84*a^6*b^4*c^3*d^6
- 36*a^7*b^3*c^2*d^7 + 9*a^8*b^2*c*d^8 - a^9*b*d^9) + 1/105*(1443*b^7*d^8 +
 9366*(b^9*c*d^7 - a*b^8*d^8)/((b*x + a)*b) + 25578*(b^11*c^2*d^6 - 2*a*b^1
0*c*d^7 + a^2*b^9*d^8)/((b*x + a)^2*b^2) + 37730*(b^13*c^3*d^5 - 3*a*b^12*c
^2*d^6 + 3*a^2*b^11*c*d^7 - a^3*b^10*d^8)/((b*x + a)^3*b^3) + 31850*(b^15*c
^4*d^4 - 4*a*b^14*c^3*d^5 + 6*a^2*b^13*c^2*d^6 - 4*a^3*b^12*c*d^7 + a^4*b^1
1*d^8)/((b*x + a)^4*b^4) + 14700*(b^17*c^5*d^3 - 5*a*b^16*c^4*d^4 + 10*a^2*
b^15*c^3*d^5 - 10*a^3*b^14*c^2*d^6 + 5*a^4*b^13*c*d^7 - a^5*b^12*d^8)/((b*x
 + a)^5*b^5) + 2940*(b^19*c^6*d^2 - 6*a*b^18*c^5*d^3 + 15*a^2*b^17*c^4*d^4
- 20*a^3*b^16*c^3*d^5 + 15*a^4*b^15*c^2*d^6 - 6*a^5*b^14*c*d^7 + a^6*b^13*d
^8)/((b*x + a)^6*b^6))/((b*c - a*d)^9*(b*c/(b*x + a) - a*d/(b*x + a) + d)^7
)
```

maple [A] time = 0.02, size = 223, normalized size = 0.97

$$\frac{8b^{7}d\ln(bx+a)}{(ad-bc)^{9}}-\frac{8b^{7}d\ln(dx+c)}{(ad-bc)^{9}}-\frac{b^{7}}{(ad-bc)^{8}(bx+a)}-\frac{7b^{6}d}{(ad-bc)^{8}(dx+c)}+\frac{3b^{5}d}{(ad-bc)^{7}(dx+c)^{2}}-\frac{5b^{4}d}{3(ad-bc)^{6}(dx+c)^{3}}+\frac{b^{3}d}{(ad-bc)^{5}(dx+c)^{4}}-\frac{3b^{2}d}{5(ad-bc)^{4}(dx+c)^{5}}+\frac{bd}{3(ad-bc)^{3}(dx+c)^{6}}-\frac{d}{7(ad-bc)^{2}(dx+c)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^8,x)
```

```
[Out] -1/7*d/(a*d-b*c)^2/(d*x+c)^7-8*d/(a*d-b*c)^9*b^7*ln(d*x+c)-7*d/(a*d-b*c)^8*
b^6/(d*x+c)+3*d/(a*d-b*c)^7*b^5/(d*x+c)^2-5/3*d/(a*d-b*c)^6*b^4/(d*x+c)^3+d
/(a*d-b*c)^5*b^3/(d*x+c)^4-3/5*d/(a*d-b*c)^4*b^2/(d*x+c)^5+1/3*d/(a*d-b*c)^
3*b/(d*x+c)^6-b^7/(a*d-b*c)^8/(b*x+a)+8*d/(a*d-b*c)^9*b^7*ln(b*x+a)
```

maxima [B] time = 3.88, size = 1881, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -8*b^7*d*log(b*x + a)/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^
3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*
d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9) + 8*b^7*d*log(d*x + c)/
(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^
4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d
^7 + 9*a^8*b*c*d^8 - a^9*d^9) - 1/105*(840*b^7*d^7*x^7 + 105*b^7*c^7 + 1443
*a*b^6*c^6*d - 1497*a^2*b^5*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c
^3*d^4 + 463*a^5*b^2*c^2*d^5 - 125*a^6*b*c*d^6 + 15*a^7*d^7 + 420*(13*b^7*c
*d^6 + a*b^6*d^7)*x^6 + 140*(107*b^7*c^2*d^5 + 20*a*b^6*c*d^6 - a^2*b^5*d^7
)*x^5 + 70*(319*b^7*c^3*d^4 + 113*a*b^6*c^2*d^5 - 13*a^2*b^5*c*d^6 + a^3*b^
4*d^7)*x^4 + 14*(1377*b^7*c^4*d^3 + 872*a*b^6*c^3*d^4 - 178*a^2*b^5*c^2*d^5
 + 32*a^3*b^4*c*d^6 - 3*a^4*b^3*d^7)*x^3 + 14*(669*b^7*c^5*d^2 + 786*a*b^6*
c^4*d^3 - 264*a^2*b^5*c^3*d^4 + 86*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 2*a
^5*b^2*d^7)*x^2 + 2*(1089*b^7*c^6*d + 2832*a*b^6*c^5*d^2 - 1578*a^2*b^5*c^4
*d^3 + 872*a^3*b^4*c^3*d^4 - 353*a^4*b^3*c^2*d^5 + 88*a^5*b^2*c*d^6 - 10*a^
6*b*d^7)*x)/(a*b^8*c^15 - 8*a^2*b^7*c^14*d + 28*a^3*b^6*c^13*d^2 - 56*a^4*b
^5*c^12*d^3 + 70*a^5*b^4*c^11*d^4 - 56*a^6*b^3*c^10*d^5 + 28*a^7*b^2*c^9*d^
```

```
6 - 8*a^8*b*c^8*d^7 + a^9*c^7*d^8 + (b^9*c^8*d^7 - 8*a*b^8*c^7*d^8 + 28*a^2
*b^7*c^6*d^9 - 56*a^3*b^6*c^5*d^10 + 70*a^4*b^5*c^4*d^11 - 56*a^5*b^4*c^3*d
^12 + 28*a^6*b^3*c^2*d^13 - 8*a^7*b^2*c*d^14 + a^8*b*d^15)*x^8 + (7*b^9*c^9
*d^6 - 55*a*b^8*c^8*d^7 + 188*a^2*b^7*c^7*d^8 - 364*a^3*b^6*c^6*d^9 + 434*a
^4*b^5*c^5*d^10 - 322*a^5*b^4*c^4*d^11 + 140*a^6*b^3*c^3*d^12 - 28*a^7*b^2*
c^2*d^13 - a^8*b*c*d^14 + a^9*d^15)*x^7 + 7*(3*b^9*c^10*d^5 - 23*a*b^8*c^9*
d^6 + 76*a^2*b^7*c^8*d^7 - 140*a^3*b^6*c^7*d^8 + 154*a^4*b^5*c^6*d^9 - 98*a
^5*b^4*c^5*d^10 + 28*a^6*b^3*c^4*d^11 + 4*a^7*b^2*c^3*d^12 - 5*a^8*b*c^2*d^
13 + a^9*c*d^14)*x^6 + 7*(5*b^9*c^11*d^4 - 37*a*b^8*c^10*d^5 + 116*a^2*b^7*
c^9*d^6 - 196*a^3*b^6*c^8*d^7 + 182*a^4*b^5*c^7*d^8 - 70*a^5*b^4*c^6*d^9 -
28*a^6*b^3*c^5*d^10 + 44*a^7*b^2*c^4*d^11 - 19*a^8*b*c^3*d^12 + 3*a^9*c^2*d
^13)*x^5 + 35*(b^9*c^12*d^3 - 7*a*b^8*c^11*d^4 + 20*a^2*b^7*c^10*d^5 - 28*a
^3*b^6*c^9*d^6 + 14*a^4*b^5*c^8*d^7 + 14*a^5*b^4*c^7*d^8 - 28*a^6*b^3*c^6*d
^9 + 20*a^7*b^2*c^5*d^10 - 7*a^8*b*c^4*d^11 + a^9*c^3*d^12)*x^4 + 7*(3*b^9*
c^13*d^2 - 19*a*b^8*c^12*d^3 + 44*a^2*b^7*c^11*d^4 - 28*a^3*b^6*c^10*d^5 -
70*a^4*b^5*c^9*d^6 + 182*a^5*b^4*c^8*d^7 - 196*a^6*b^3*c^7*d^8 + 116*a^7*b^
2*c^6*d^9 - 37*a^8*b*c^5*d^10 + 5*a^9*c^4*d^11)*x^3 + 7*(b^9*c^14*d - 5*a*b
^8*c^13*d^2 + 4*a^2*b^7*c^12*d^3 + 28*a^3*b^6*c^11*d^4 - 98*a^4*b^5*c^10*d^
5 + 154*a^5*b^4*c^9*d^6 - 140*a^6*b^3*c^8*d^7 + 76*a^7*b^2*c^7*d^8 - 23*a^8
*b*c^6*d^9 + 3*a^9*c^5*d^10)*x^2 + (b^9*c^15 - a*b^8*c^14*d - 28*a^2*b^7*c^
13*d^2 + 140*a^3*b^6*c^12*d^3 - 322*a^4*b^5*c^11*d^4 + 434*a^5*b^4*c^10*d^5
 - 364*a^6*b^3*c^9*d^6 + 188*a^7*b^2*c^8*d^7 - 55*a^8*b*c^7*d^8 + 7*a^9*c^6
*d^9)*x)
```

mupad [B] time = 1.39, size = 1738, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^8),x)
```

```
[Out] (16*b^7*d*atanh((a^9*d^9 + b^9*c^9 + 20*a^2*b^7*c^7*d^2 - 28*a^3*b^6*c^6*d^
3 + 14*a^4*b^5*c^5*d^4 + 14*a^5*b^4*c^4*d^5 - 28*a^6*b^3*c^3*d^6 + 20*a^7*b
^2*c^2*d^7 - 7*a*b^8*c^8*d - 7*a^8*b*c*d^8)/(a*d - b*c)^9 + (2*b*d*x*(a^8*d
^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4
 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)
)/(a*d - b*c)^9))/(a*d - b*c)^9 - ((15*a^7*d^7 + 105*b^7*c^7 - 1497*a^2*b^5
*c^5*d^2 + 1443*a^3*b^4*c^4*d^3 - 1007*a^4*b^3*c^3*d^4 + 463*a^5*b^2*c^2*d^
5 + 1443*a*b^6*c^6*d - 125*a^6*b*c*d^6)/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*b^
6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 +
28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (4*b^5*x^5*(107*b^2*
c^2*d^5 - a^2*d^7 + 20*a*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d
^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*
b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^2*x^2*(2*a^5*d^7 + 669
*b^5*c^5*d^2 + 786*a*b^4*c^4*d^3 - 264*a^2*b^3*c^3*d^4 + 86*a^3*b^2*c^2*d^5
 - 19*a^4*b*c*d^6))/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^
5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 -
8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b^4*x^4*(a^3*d^7 + 319*b^3*c^3*d^4 + 1
13*a*b^2*c^2*d^5 - 13*a^2*b*c*d^6))/(3*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (2*b*x*(1089*b^6*c^6*d - 1
0*a^6*d^7 + 2832*a*b^5*c^5*d^2 - 1578*a^2*b^4*c^4*d^3 + 872*a^3*b^3*c^3*d^4
 - 353*a^4*b^2*c^2*d^5 + 88*a^5*b*c*d^6))/(105*(a^8*d^8 + b^8*c^8 + 28*a^2*
b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7)) + (8*b^7*d^7*x^7)/(a
^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4
*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*
d^7) + (4*b^6*x^6*(a*d^7 + 13*b*c*d^6))/(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6
*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^
6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7) + (2*b^3*x^3*(1377*b^4*c^4*d
```

```
^3 - 3*a^4*d^7 + 872*a*b^3*c^3*d^4 - 178*a^2*b^2*c^2*d^5 + 32*a^3*b*c*d^6))
/(15*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a
^7*b*c*d^7)))/(x^7*(a*d^7 + 7*b*c*d^6) + x^3*(35*a*c^4*d^3 + 21*b*c^5*d^2)
+ x^5*(21*a*c^2*d^5 + 35*b*c^3*d^4) + x^4*(35*a*c^3*d^4 + 35*b*c^4*d^3) + a
*c^7 + x*(b*c^7 + 7*a*c^6*d) + x^2*(21*a*c^5*d^2 + 7*b*c^6*d) + x^6*(21*b*c
^2*d^5 + 7*a*c*d^6) + b*d^7*x^8)
```

sympy [B] time = 7.75, size = 2336, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**8,x)
```

```
[Out] -8*b**7*d*log(x + (-8*a**10*b**7*d**11/(a*d - b*c)**9 + 80*a**9*b**8*c*d**1
0/(a*d - b*c)**9 - 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 + 960*a**7*b**10*
c**3*d**8/(a*d - b*c)**9 - 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 + 2016*
a**5*b**12*c**5*d**6/(a*d - b*c)**9 - 1680*a**4*b**13*c**6*d**5/(a*d - b*c)
**9 + 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 - 360*a**2*b**15*c**8*d**3/(a
*d - b*c)**9 + 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 - 8*b**1
7*c**10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + 8*b
**7*d*log(x + (8*a**10*b**7*d**11/(a*d - b*c)**9 - 80*a**9*b**8*c*d**10/(a*
d - b*c)**9 + 360*a**8*b**9*c**2*d**9/(a*d - b*c)**9 - 960*a**7*b**10*c**3*
d**8/(a*d - b*c)**9 + 1680*a**6*b**11*c**4*d**7/(a*d - b*c)**9 - 2016*a**5*
b**12*c**5*d**6/(a*d - b*c)**9 + 1680*a**4*b**13*c**6*d**5/(a*d - b*c)**9 -
 960*a**3*b**14*c**7*d**4/(a*d - b*c)**9 + 360*a**2*b**15*c**8*d**3/(a*d -
b*c)**9 - 80*a*b**16*c**9*d**2/(a*d - b*c)**9 + 8*a*b**7*d**2 + 8*b**17*c**
10*d/(a*d - b*c)**9 + 8*b**8*c*d)/(16*b**8*d**2))/(a*d - b*c)**9 + (-15*a**
7*d**7 + 125*a**6*b*c*d**6 - 463*a**5*b**2*c**2*d**5 + 1007*a**4*b**3*c**3*
d**4 - 1443*a**3*b**4*c**4*d**3 + 1497*a**2*b**5*c**5*d**2 - 1443*a*b**6*c*
*6*d - 105*b**7*c**7 - 840*b**7*d**7*x**7 + x**6*(-420*a*b**6*d**7 - 5460*b
**7*c*d**6) + x**5*(140*a**2*b**5*d**7 - 2800*a*b**6*c*d**6 - 14980*b**7*c*
*2*d**5) + x**4*(-70*a**3*b**4*d**7 + 910*a**2*b**5*c*d**6 - 7910*a*b**6*c*
*2*d**5 - 22330*b**7*c**3*d**4) + x**3*(42*a**4*b**3*d**7 - 448*a**3*b**4*c
*d**6 + 2492*a**2*b**5*c**2*d**5 - 12208*a*b**6*c**3*d**4 - 19278*b**7*c**4
*d**3) + x**2*(-28*a**5*b**2*d**7 + 266*a**4*b**3*c*d**6 - 1204*a**3*b**4*c
**2*d**5 + 3696*a**2*b**5*c**3*d**4 - 11004*a*b**6*c**4*d**3 - 9366*b**7*c*
*5*d**2) + x*(20*a**6*b*d**7 - 176*a**5*b**2*c*d**6 + 706*a**4*b**3*c**2*d*
*5 - 1744*a**3*b**4*c**3*d**4 + 3156*a**2*b**5*c**4*d**3 - 5664*a*b**6*c**5
*d**2 - 2178*b**7*c**6*d))/(105*a**9*c**7*d**8 - 840*a**8*b*c**8*d**7 + 294
0*a**7*b**2*c**9*d**6 - 5880*a**6*b**3*c**10*d**5 + 7350*a**5*b**4*c**11*d*
*4 - 5880*a**4*b**5*c**12*d**3 + 2940*a**3*b**6*c**13*d**2 - 840*a**2*b**7*
c**14*d + 105*a*b**8*c**15 + x**8*(105*a**8*b*d**15 - 840*a**7*b**2*c*d**14
 + 2940*a**6*b**3*c**2*d**13 - 5880*a**5*b**4*c**3*d**12 + 7350*a**4*b**5*c
**4*d**11 - 5880*a**3*b**6*c**5*d**10 + 2940*a**2*b**7*c**6*d**9 - 840*a*b*
*8*c**7*d**8 + 105*b**9*c**8*d**7) + x**7*(105*a**9*d**15 - 105*a**8*b*c*d*
*14 - 2940*a**7*b**2*c**2*d**13 + 14700*a**6*b**3*c**3*d**12 - 33810*a**5*b
**4*c**4*d**11 + 45570*a**4*b**5*c**5*d**10 - 38220*a**3*b**6*c**6*d**9 + 1
9740*a**2*b**7*c**7*d**8 - 5775*a*b**8*c**8*d**7 + 735*b**9*c**9*d**6) + x*
*6*(735*a**9*c*d**14 - 3675*a**8*b*c**2*d**13 + 2940*a**7*b**2*c**3*d**12 +
 20580*a**6*b**3*c**4*d**11 - 72030*a**5*b**4*c**5*d**10 + 113190*a**4*b**5
*c**6*d**9 - 102900*a**3*b**6*c**7*d**8 + 55860*a**2*b**7*c**8*d**7 - 16905
*a*b**8*c**9*d**6 + 2205*b**9*c**10*d**5) + x**5*(2205*a**9*c**2*d**13 - 13
965*a**8*b*c**3*d**12 + 32340*a**7*b**2*c**4*d**11 - 20580*a**6*b**3*c**5*d
**10 - 51450*a**5*b**4*c**6*d**9 + 133770*a**4*b**5*c**7*d**8 - 144060*a**3
*b**6*c**8*d**7 + 85260*a**2*b**7*c**9*d**6 - 27195*a*b**8*c**10*d**5 + 367
5*b**9*c**11*d**4) + x**4*(3675*a**9*c**3*d**12 - 25725*a**8*b*c**4*d**11 +
 73500*a**7*b**2*c**5*d**10 - 102900*a**6*b**3*c**6*d**9 + 51450*a**5*b**4*
c**7*d**8 + 51450*a**4*b**5*c**8*d**7 - 102900*a**3*b**6*c**9*d**6 + 73500*
```

```
a**2*b**7*c**10*d**5 - 25725*a*b**8*c**11*d**4 + 3675*b**9*c**12*d**3) + x*
*3*(3675*a**9*c**4*d**11 - 27195*a**8*b*c**5*d**10 + 85260*a**7*b**2*c**6*d
**9 - 144060*a**6*b**3*c**7*d**8 + 133770*a**5*b**4*c**8*d**7 - 51450*a**4*
b**5*c**9*d**6 - 20580*a**3*b**6*c**10*d**5 + 32340*a**2*b**7*c**11*d**4 -
13965*a*b**8*c**12*d**3 + 2205*b**9*c**13*d**2) + x**2*(2205*a**9*c**5*d**1
0 - 16905*a**8*b*c**6*d**9 + 55860*a**7*b**2*c**7*d**8 - 102900*a**6*b**3*c
**8*d**7 + 113190*a**5*b**4*c**9*d**6 - 72030*a**4*b**5*c**10*d**5 + 20580*
a**3*b**6*c**11*d**4 + 2940*a**2*b**7*c**12*d**3 - 3675*a*b**8*c**13*d**2 +
 735*b**9*c**14*d) + x*(735*a**9*c**6*d**9 - 5775*a**8*b*c**7*d**8 + 19740*
a**7*b**2*c**8*d**7 - 38220*a**6*b**3*c**9*d**6 + 45570*a**5*b**4*c**10*d**
5 - 33810*a**4*b**5*c**11*d**4 + 14700*a**3*b**6*c**12*d**3 - 2940*a**2*b**
7*c**13*d**2 - 105*a*b**8*c**14*d + 105*b**9*c**15))
```

3.1268 $\int \frac{1}{(a+bx)^3(c+dx)^8} \, dx$

Optimal. Leaf size=276

$$\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21}{2(c+dx}$$

Rubi [A] time = 0.36, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {44}

$$\frac{28b^6d^2}{(c+dx)(bc-ad)^9}+\frac{21b^5d^2}{2(c+dx)^2(bc-ad)^8}+\frac{5b^4d^2}{(c+dx)^3(bc-ad)^7}+\frac{5b^3d^2}{2(c+dx)^4(bc-ad)^6}+\frac{6b^2d^2}{5(c+dx)^5(bc-ad)^5}+\frac{36b^7d^2\log(a+bx)}{(bc-ad)^{10}}-\frac{36b^7d^2\log(c+dx)}{(bc-ad)^{10}}+\frac{8b^7d}{(a+bx)(bc-ad)^9}-\frac{b^7}{2(a+bx)^2(bc-ad)^8}+\frac{bd^2}{2(c+dx)^6(bc-ad)^4}+\frac{d^2}{7(c+dx)^7(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] -b^7/(2*(b*c - a*d)^8*(a + b*x)^2) + (8*b^7*d)/((b*c - a*d)^9*(a + b*x)) +
d^2/(7*(b*c - a*d)^3*(c + d*x)^7) + (b*d^2)/(2*(b*c - a*d)^4*(c + d*x)^6) +
 (6*b^2*d^2)/(5*(b*c - a*d)^5*(c + d*x)^5) + (5*b^3*d^2)/(2*(b*c - a*d)^6*(
c + d*x)^4) + (5*b^4*d^2)/((b*c - a*d)^7*(c + d*x)^3) + (21*b^5*d^2)/(2*(b*
c - a*d)^8*(c + d*x)^2) + (28*b^6*d^2)/((b*c - a*d)^9*(c + d*x)) + (36*b^7*
d^2*Log[a + b*x])/(b*c - a*d)^10 - (36*b^7*d^2*Log[c + d*x])/(b*c - a*d)^10
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^8} \, dx = \int \left(\frac{b^8}{(bc-ad)^8(a+bx)^3}-\frac{8b^8d}{(bc-ad)^9(a+bx)^2}+\frac{36b^8d^2}{(bc-ad)^{10}(a+bx)}-\frac{d^3}{(bc-ad)^3(c+}\right.$$
$$=-\frac{b^7}{2(bc-ad)^8(a+bx)^2}+\frac{8b^7d}{(bc-ad)^9(a+bx)}+\frac{d^2}{7(bc-ad)^3(c+dx)^7}+\frac{bd^2}{2(bc-ad)^4(c+}$$

Mathematica [A] time = 0.20, size = 254, normalized size = 0.92

$$\frac{\frac{560b^7d(bc-ad)}{a+bx}-\frac{35b^7(bc-ad)^2}{(a+bx)^2}+2520b^7d^2\log(a+bx)+\frac{1960b^6d^2(bc-ad)}{c+dx}+\frac{735b^5d^2(bc-ad)^2}{(c+dx)^2}+\frac{350b^4d^2(bc-ad)^3}{(c+dx)^3}+\frac{175b^3d^2(bc-ad)^4}{(c+dx)^4}+\frac{84b^2d^2(bc-ad)^5}{(c+dx)^5}+\frac{35bd^2(bc-ad)^6}{(c+dx)^6}+\frac{10d^2(bc-ad)^7}{(c+dx)^7}-2520b^7d^2\log(c+dx)}{70(bc-ad)^{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] ((-35*b^7*(b*c - a*d)^2)/(a + b*x)^2 + (560*b^7*d*(b*c - a*d))/(a + b*x) +
(10*d^2*(b*c - a*d)^7)/(c + d*x)^7 + (35*b*d^2*(b*c - a*d)^6)/(c + d*x)^6 +
 (84*b^2*d^2*(b*c - a*d)^5)/(c + d*x)^5 + (175*b^3*d^2*(b*c - a*d)^4)/(c +
d*x)^4 + (350*b^4*d^2*(b*c - a*d)^3)/(c + d*x)^3 + (735*b^5*d^2*(b*c - a*d)
^2)/(c + d*x)^2 + (1960*b^6*d^2*(b*c - a*d))/(c + d*x) + 2520*b^7*d^2*Log[a
 + b*x] - 2520*b^7*d^2*Log[c + d*x])/(70*(b*c - a*d)^10)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^3(c+dx)^8} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^8),x]
```

```
[Out] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^8), x]
```

fricas [B] time = 1.59, size = 3016, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/70*(35*b^9*c^9 - 630*a*b^8*c^8*d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c
^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 2940*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6
 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a
*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7
- 420*(107*b^9*c^3*d^6 - 48*a*b^8*c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^
9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*
a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 - 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^
5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b
^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3 + 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d
^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3
*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 + 6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4
 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a
^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2 - 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 +
 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^
5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x - 2
520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*
b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^
8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^
2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 + 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^
6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c
^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*
a^2*b^7*c^6*d^3)*x)*log(b*x + a) + 2520*(b^9*d^9*x^9 + a^2*b^7*c^7*d^2 + (7
*b^9*c*d^8 + 2*a*b^8*d^9)*x^8 + (21*b^9*c^2*d^7 + 14*a*b^8*c*d^8 + a^2*b^7*
d^9)*x^7 + 7*(5*b^9*c^3*d^6 + 6*a*b^8*c^2*d^7 + a^2*b^7*c*d^8)*x^6 + 7*(5*b
^9*c^4*d^5 + 10*a*b^8*c^3*d^6 + 3*a^2*b^7*c^2*d^7)*x^5 + 7*(3*b^9*c^5*d^4 +
 10*a*b^8*c^4*d^5 + 5*a^2*b^7*c^3*d^6)*x^4 + 7*(b^9*c^6*d^3 + 6*a*b^8*c^5*d
^4 + 5*a^2*b^7*c^4*d^5)*x^3 + (b^9*c^7*d^2 + 14*a*b^8*c^6*d^3 + 21*a^2*b^7*
c^5*d^4)*x^2 + (2*a*b^8*c^7*d^2 + 7*a^2*b^7*c^6*d^3)*x)*log(d*x + c))/(a^2*
b^10*c^17 - 10*a^3*b^9*c^16*d + 45*a^4*b^8*c^15*d^2 - 120*a^5*b^7*c^14*d^3
+ 210*a^6*b^6*c^13*d^4 - 252*a^7*b^5*c^12*d^5 + 210*a^8*b^4*c^11*d^6 - 120*
a^9*b^3*c^10*d^7 + 45*a^10*b^2*c^9*d^8 - 10*a^11*b*c^8*d^9 + a^12*c^7*d^10
+ (b^12*c^10*d^7 - 10*a*b^11*c^9*d^8 + 45*a^2*b^10*c^8*d^9 - 120*a^3*b^9*c^
7*d^10 + 210*a^4*b^8*c^6*d^11 - 252*a^5*b^7*c^5*d^12 + 210*a^6*b^6*c^4*d^13
 - 120*a^7*b^5*c^3*d^14 + 45*a^8*b^4*c^2*d^15 - 10*a^9*b^3*c*d^16 + a^10*b^
2*d^17)*x^9 + (7*b^12*c^11*d^6 - 68*a*b^11*c^10*d^7 + 295*a^2*b^10*c^9*d^8
- 750*a^3*b^9*c^8*d^9 + 1230*a^4*b^8*c^7*d^10 - 1344*a^5*b^7*c^6*d^11 + 966
*a^6*b^6*c^5*d^12 - 420*a^7*b^5*c^4*d^13 + 75*a^8*b^4*c^3*d^14 + 20*a^9*b^3
*c^2*d^15 - 13*a^10*b^2*c*d^16 + 2*a^11*b*d^17)*x^8 + (21*b^12*c^12*d^5 - 1
96*a*b^11*c^11*d^6 + 806*a^2*b^10*c^10*d^7 - 1900*a^3*b^9*c^9*d^8 + 2775*a^
4*b^8*c^8*d^9 - 2472*a^5*b^7*c^7*d^10 + 1092*a^6*b^6*c^6*d^11 + 168*a^7*b^5
*c^5*d^12 - 525*a^8*b^4*c^4*d^13 + 300*a^9*b^3*c^3*d^14 - 74*a^10*b^2*c^2*d
^15 + 4*a^11*b*c*d^16 + a^12*d^17)*x^7 + 7*(5*b^12*c^13*d^4 - 44*a*b^11*c^1
2*d^5 + 166*a^2*b^10*c^11*d^6 - 340*a^3*b^9*c^10*d^7 + 375*a^4*b^8*c^9*d^8
- 120*a^5*b^7*c^8*d^9 - 252*a^6*b^6*c^7*d^10 + 408*a^7*b^5*c^6*d^11 - 285*a
^8*b^4*c^5*d^12 + 100*a^9*b^3*c^4*d^13 - 10*a^10*b^2*c^3*d^14 - 4*a^11*b*c^
2*d^15 + a^12*c*d^16)*x^6 + 7*(5*b^12*c^14*d^3 - 40*a*b^11*c^13*d^4 + 128*a
^2*b^10*c^12*d^5 - 180*a^3*b^9*c^11*d^6 - 15*a^4*b^8*c^10*d^7 + 480*a^5*b^7
*c^9*d^8 - 840*a^6*b^6*c^8*d^9 + 744*a^7*b^5*c^7*d^10 - 345*a^8*b^4*c^6*d^1
```

```
1 + 40*a^9*b^3*c^5*d^12 + 40*a^10*b^2*c^4*d^13 - 20*a^11*b*c^3*d^14 + 3*a^1
2*c^2*d^15)*x^5 + 7*(3*b^12*c^15*d^2 - 20*a*b^11*c^14*d^3 + 40*a^2*b^10*c^1
3*d^4 + 40*a^3*b^9*c^12*d^5 - 345*a^4*b^8*c^11*d^6 + 744*a^5*b^7*c^10*d^7 -
 840*a^6*b^6*c^9*d^8 + 480*a^7*b^5*c^8*d^9 - 15*a^8*b^4*c^7*d^10 - 180*a^9*
b^3*c^6*d^11 + 128*a^10*b^2*c^5*d^12 - 40*a^11*b*c^4*d^13 + 5*a^12*c^3*d^14
)*x^4 + 7*(b^12*c^16*d - 4*a*b^11*c^15*d^2 - 10*a^2*b^10*c^14*d^3 + 100*a^3
*b^9*c^13*d^4 - 285*a^4*b^8*c^12*d^5 + 408*a^5*b^7*c^11*d^6 - 252*a^6*b^6*c
^10*d^7 - 120*a^7*b^5*c^9*d^8 + 375*a^8*b^4*c^8*d^9 - 340*a^9*b^3*c^7*d^10
+ 166*a^10*b^2*c^6*d^11 - 44*a^11*b*c^5*d^12 + 5*a^12*c^4*d^13)*x^3 + (b^12
*c^17 + 4*a*b^11*c^16*d - 74*a^2*b^10*c^15*d^2 + 300*a^3*b^9*c^14*d^3 - 525
*a^4*b^8*c^13*d^4 + 168*a^5*b^7*c^12*d^5 + 1092*a^6*b^6*c^11*d^6 - 2472*a^7
*b^5*c^10*d^7 + 2775*a^8*b^4*c^9*d^8 - 1900*a^9*b^3*c^8*d^9 + 806*a^10*b^2*
c^7*d^10 - 196*a^11*b*c^6*d^11 + 21*a^12*c^5*d^12)*x^2 + (2*a*b^11*c^17 - 1
3*a^2*b^10*c^16*d + 20*a^3*b^9*c^15*d^2 + 75*a^4*b^8*c^14*d^3 - 420*a^5*b^7
*c^13*d^4 + 966*a^6*b^6*c^12*d^5 - 1344*a^7*b^5*c^11*d^6 + 1230*a^8*b^4*c^1
0*d^7 - 750*a^9*b^3*c^9*d^8 + 295*a^10*b^2*c^8*d^9 - 68*a^11*b*c^7*d^10 + 7
*a^12*c^6*d^11)*x)
```

giac [B] time = 1.49, size = 1029, normalized size = 3.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 36*b^8*d^2*log(abs(b*x + a))/(b^11*c^10 - 10*a*b^10*c^9*d + 45*a^2*b^9*c^8*
d^2 - 120*a^3*b^8*c^7*d^3 + 210*a^4*b^7*c^6*d^4 - 252*a^5*b^6*c^5*d^5 + 210
*a^6*b^5*c^4*d^6 - 120*a^7*b^4*c^3*d^7 + 45*a^8*b^3*c^2*d^8 - 10*a^9*b^2*c*
d^9 + a^10*b*d^10) - 36*b^7*d^3*log(abs(d*x + c))/(b^10*c^10*d - 10*a*b^9*c
^9*d^2 + 45*a^2*b^8*c^8*d^3 - 120*a^3*b^7*c^7*d^4 + 210*a^4*b^6*c^6*d^5 - 2
52*a^5*b^5*c^5*d^6 + 210*a^6*b^4*c^4*d^7 - 120*a^7*b^3*c^3*d^8 + 45*a^8*b^2
*c^2*d^9 - 10*a^9*b*c*d^10 + a^10*d^11) - 1/70*(35*b^9*c^9 - 630*a*b^8*c^8*
d - 2754*a^2*b^7*c^7*d^2 + 5880*a^3*b^6*c^6*d^3 - 4410*a^4*b^5*c^5*d^4 + 29
40*a^5*b^4*c^4*d^5 - 1470*a^6*b^3*c^3*d^6 + 504*a^7*b^2*c^2*d^7 - 105*a^8*b
*c*d^8 + 10*a^9*d^9 - 2520*(b^9*c*d^8 - a*b^8*d^9)*x^8 - 1260*(13*b^9*c^2*d
^7 - 10*a*b^8*c*d^8 - 3*a^2*b^7*d^9)*x^7 - 420*(107*b^9*c^3*d^6 - 48*a*b^8*
c^2*d^7 - 57*a^2*b^7*c*d^8 - 2*a^3*b^6*d^9)*x^6 - 210*(319*b^9*c^4*d^5 + 8*
a*b^8*c^3*d^6 - 300*a^2*b^7*c^2*d^7 - 28*a^3*b^6*c*d^8 + a^4*b^5*d^9)*x^5 -
 42*(1377*b^9*c^5*d^4 + 1090*a*b^8*c^4*d^5 - 2080*a^2*b^7*c^3*d^6 - 420*a^3
*b^6*c^2*d^7 + 35*a^4*b^5*c*d^8 - 2*a^5*b^4*d^9)*x^4 - 42*(669*b^9*c^6*d^3
+ 1494*a*b^8*c^5*d^4 - 1555*a^2*b^7*c^4*d^5 - 700*a^3*b^6*c^3*d^6 + 105*a^4
*b^5*c^2*d^7 - 14*a^5*b^4*c*d^8 + a^6*b^3*d^9)*x^3 - 6*(1089*b^9*c^7*d^2 +
6426*a*b^8*c^6*d^3 - 3591*a^2*b^7*c^5*d^4 - 4900*a^3*b^6*c^4*d^5 + 1225*a^4
*b^5*c^3*d^6 - 294*a^5*b^4*c^2*d^7 + 49*a^6*b^3*c*d^8 - 4*a^7*b^2*d^9)*x^2
- 3*(105*b^9*c^8*d + 3516*a*b^8*c^7*d^2 + 546*a^2*b^7*c^6*d^3 - 5880*a^3*b^
6*c^5*d^4 + 2450*a^4*b^5*c^4*d^5 - 980*a^5*b^4*c^3*d^6 + 294*a^6*b^3*c^2*d^
7 - 56*a^7*b^2*c*d^8 + 5*a^8*b*d^9)*x)/((b*c - a*d)^10*(b*x + a)^2*(d*x + c
)^7)
```

maple [A] time = 0.02, size = 265, normalized size = 0.96

$$\frac{36b^7d^2\ln(bx+a)}{(ad-bc)^{10}}-\frac{36b^7d^2\ln(dx+c)}{(ad-bc)^{10}}-\frac{8b^7d}{(ad-bc)^9(bx+a)}-\frac{28b^6d^2}{(ad-bc)^9(dx+c)}-\frac{b^7}{2(ad-bc)^8(bx+a)^2}+\frac{21b^5d^2}{2(ad-bc)^8(dx+c)^2}-\frac{5b^4d^2}{(ad-bc)^7(dx+c)^3}+\frac{5b^3d^2}{2(ad-bc)^6(dx+c)^4}-\frac{6b^2d^2}{5(ad-bc)^5(dx+c)^5}+\frac{b\,d^2}{2(ad-bc)^4(dx+c)^6}-\frac{d^2}{7(ad-bc)^3(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^8,x)
```

```
[Out] -1/7*d^2/(a*d-b*c)^3/(d*x+c)^7-36*d^2/(a*d-b*c)^10*b^7*ln(d*x+c)-28*d^2/(a*
d-b*c)^9*b^6/(d*x+c)+21/2*d^2/(a*d-b*c)^8*b^5/(d*x+c)^2-5*d^2/(a*d-b*c)^7*b
^4/(d*x+c)^3+5/2*d^2/(a*d-b*c)^6*b^3/(d*x+c)^4-6/5*d^2/(a*d-b*c)^5*b^2/(d*x
```

+c)^5+1/2*d^2/(a*d-b*c)^4*b/(d*x+c)^6-1/2*b^7/(a*d-b*c)^8/(b*x+a)^2+36*d^2/(a*d-b*c)^10*b^7*ln(b*x+a)-8*b^7/(a*d-b*c)^9*d/(b*x+a)

maxima [B] time = 5.22, size = 2399, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] 36*b^7*d^2*log(b*x + a)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 -
120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b
^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^
10*d^10) - 36*b^7*d^2*log(d*x + c)/(b^10*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8
*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5
+ 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b
*c*d^9 + a^10*d^10) + 1/70*(2520*b^8*d^8*x^8 - 35*b^8*c^8 + 595*a*b^7*c^7*d
 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b^4*c^4*d^4 - 106
1*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 - 95*a^7*b*c*d^7 + 10*a^8*d^8 + 126
0*(13*b^8*c*d^7 + 3*a*b^7*d^8)*x^7 + 420*(107*b^8*c^2*d^6 + 59*a*b^7*c*d^7
+ 2*a^2*b^6*d^8)*x^6 + 210*(319*b^8*c^3*d^5 + 327*a*b^7*c^2*d^6 + 27*a^2*b^
6*c*d^7 - a^3*b^5*d^8)*x^5 + 42*(1377*b^8*c^4*d^4 + 2467*a*b^7*c^3*d^5 + 38
7*a^2*b^6*c^2*d^6 - 33*a^3*b^5*c*d^7 + 2*a^4*b^4*d^8)*x^4 + 42*(669*b^8*c^5
*d^3 + 2163*a*b^7*c^4*d^4 + 608*a^2*b^6*c^3*d^5 - 92*a^3*b^5*c^2*d^6 + 13*a
^4*b^4*c*d^7 - a^5*b^3*d^8)*x^3 + 6*(1089*b^8*c^6*d^2 + 7515*a*b^7*c^5*d^3
+ 3924*a^2*b^6*c^4*d^4 - 976*a^3*b^5*c^3*d^5 + 249*a^4*b^4*c^2*d^6 - 45*a^5
*b^3*c*d^7 + 4*a^6*b^2*d^8)*x^2 + 3*(105*b^8*c^7*d + 3621*a*b^7*c^6*d^2 + 4
167*a^2*b^6*c^5*d^3 - 1713*a^3*b^5*c^4*d^4 + 737*a^4*b^4*c^3*d^5 - 243*a^5*
b^3*c^2*d^6 + 51*a^6*b^2*c*d^7 - 5*a^7*b*d^8)*x)/(a^2*b^9*c^16 - 9*a^3*b^8*
c^15*d + 36*a^4*b^7*c^14*d^2 - 84*a^5*b^6*c^13*d^3 + 126*a^6*b^5*c^12*d^4 -
 126*a^7*b^4*c^11*d^5 + 84*a^8*b^3*c^10*d^6 - 36*a^9*b^2*c^9*d^7 + 9*a^10*b
*c^8*d^8 - a^11*c^7*d^9 + (b^11*c^9*d^7 - 9*a*b^10*c^8*d^8 + 36*a^2*b^9*c^7
*d^9 - 84*a^3*b^8*c^6*d^10 + 126*a^4*b^7*c^5*d^11 - 126*a^5*b^6*c^4*d^12 +
84*a^6*b^5*c^3*d^13 - 36*a^7*b^4*c^2*d^14 + 9*a^8*b^3*c*d^15 - a^9*b^2*d^16
)*x^9 + (7*b^11*c^10*d^6 - 61*a*b^10*c^9*d^7 + 234*a^2*b^9*c^8*d^8 - 516*a^
3*b^8*c^7*d^9 + 714*a^4*b^7*c^6*d^10 - 630*a^5*b^6*c^5*d^11 + 336*a^6*b^5*c
^4*d^12 - 84*a^7*b^4*c^3*d^13 - 9*a^8*b^3*c^2*d^14 + 11*a^9*b^2*c*d^15 - 2*
a^10*b*d^16)*x^8 + (21*b^11*c^11*d^5 - 175*a*b^10*c^10*d^6 + 631*a^2*b^9*c^
9*d^7 - 1269*a^3*b^8*c^8*d^8 + 1506*a^4*b^7*c^7*d^9 - 966*a^5*b^6*c^6*d^10
+ 126*a^6*b^5*c^5*d^11 + 294*a^7*b^4*c^4*d^12 - 231*a^8*b^3*c^3*d^13 + 69*a
^9*b^2*c^2*d^14 - 5*a^10*b*c*d^15 - a^11*d^16)*x^7 + 7*(5*b^11*c^12*d^4 - 3
9*a*b^10*c^11*d^5 + 127*a^2*b^9*c^10*d^6 - 213*a^3*b^8*c^9*d^7 + 162*a^4*b^
7*c^8*d^8 + 42*a^5*b^6*c^7*d^9 - 210*a^6*b^5*c^6*d^10 + 198*a^7*b^4*c^5*d^1
1 - 87*a^8*b^3*c^4*d^12 + 13*a^9*b^2*c^3*d^13 + 3*a^10*b*c^2*d^14 - a^11*c*
d^15)*x^6 + 7*(5*b^11*c^13*d^3 - 35*a*b^10*c^12*d^4 + 93*a^2*b^9*c^11*d^5 -
 87*a^3*b^8*c^10*d^6 - 102*a^4*b^7*c^9*d^7 + 378*a^5*b^6*c^8*d^8 - 462*a^6*
b^5*c^7*d^9 + 282*a^7*b^4*c^6*d^10 - 63*a^8*b^3*c^5*d^11 - 23*a^9*b^2*c^4*d
^12 + 17*a^10*b*c^3*d^13 - 3*a^11*c^2*d^14)*x^5 + 7*(3*b^11*c^14*d^2 - 17*a
*b^10*c^13*d^3 + 23*a^2*b^9*c^12*d^4 + 63*a^3*b^8*c^11*d^5 - 282*a^4*b^7*c^
10*d^6 + 462*a^5*b^6*c^9*d^7 - 378*a^6*b^5*c^8*d^8 + 102*a^7*b^4*c^7*d^9 +
87*a^8*b^3*c^6*d^10 - 93*a^9*b^2*c^5*d^11 + 35*a^10*b*c^4*d^12 - 5*a^11*c^3
*d^13)*x^4 + 7*(b^11*c^15*d - 3*a*b^10*c^14*d^2 - 13*a^2*b^9*c^13*d^3 + 87*
a^3*b^8*c^12*d^4 - 198*a^4*b^7*c^11*d^5 + 210*a^5*b^6*c^10*d^6 - 42*a^6*b^5
*c^9*d^7 - 162*a^7*b^4*c^8*d^8 + 213*a^8*b^3*c^7*d^9 - 127*a^9*b^2*c^6*d^10
 + 39*a^10*b*c^5*d^11 - 5*a^11*c^4*d^12)*x^3 + (b^11*c^16 + 5*a*b^10*c^15*d
 - 69*a^2*b^9*c^14*d^2 + 231*a^3*b^8*c^13*d^3 - 294*a^4*b^7*c^12*d^4 - 126*
a^5*b^6*c^11*d^5 + 966*a^6*b^5*c^10*d^6 - 1506*a^7*b^4*c^9*d^7 + 1269*a^8*b
^3*c^8*d^8 - 631*a^9*b^2*c^7*d^9 + 175*a^10*b*c^6*d^10 - 21*a^11*c^5*d^11)*
x^2 + (2*a*b^10*c^16 - 11*a^2*b^9*c^15*d + 9*a^3*b^8*c^14*d^2 + 84*a^4*b^7*
c^13*d^3 - 336*a^5*b^6*c^12*d^4 + 630*a^6*b^5*c^11*d^5 - 714*a^7*b^4*c^10*d
```

```
^6 + 516*a^8*b^3*c^9*d^7 - 234*a^9*b^2*c^8*d^8 + 61*a^10*b*c^7*d^9 - 7*a^11
*c^6*d^10)*x)
```

mupad [B] time = 1.91, size = 2224, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^8),x)
```

```
[Out] (72*b^7*d^2*atanh((a^10*d^10 - b^10*c^10 - 27*a^2*b^8*c^8*d^2 + 48*a^3*b^7*
c^7*d^3 - 42*a^4*b^6*c^6*d^4 + 42*a^6*b^4*c^4*d^6 - 48*a^7*b^3*c^3*d^7 + 27
*a^8*b^2*c^2*d^8 + 8*a*b^9*c^9*d - 8*a^9*b*c*d^9)/(a*d - b*c)^10 + (2*b*d*x
*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5
*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
9*a*b^8*c^8*d - 9*a^8*b*c*d^8))/(a*d - b*c)^10))/(a*d - b*c)^10 - ((10*a^8*
d^8 - 35*b^8*c^8 + 3349*a^2*b^6*c^6*d^2 - 2531*a^3*b^5*c^5*d^3 + 1879*a^4*b
^4*c^4*d^4 - 1061*a^5*b^3*c^3*d^5 + 409*a^6*b^2*c^2*d^6 + 595*a*b^7*c^7*d -
 95*a^7*b*c*d^7)/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (3*b^2*x^2*(4*a^6*d^8
 + 1089*b^6*c^6*d^2 + 7515*a*b^5*c^5*d^3 + 3924*a^2*b^4*c^4*d^4 - 976*a^3*b
^3*c^3*d^5 + 249*a^4*b^2*c^2*d^6 - 45*a^5*b*c*d^7))/(35*(a^9*d^9 - b^9*c^9
- 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b
^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^
8*b*c*d^8)) + (3*b^4*x^4*(2*a^4*d^8 + 1377*b^4*c^4*d^4 + 2467*a*b^3*c^3*d^5
 + 387*a^2*b^2*c^2*d^6 - 33*a^3*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^
7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5
- 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8))
 + (3*b*x*(105*b^7*c^7*d - 5*a^7*d^8 + 3621*a*b^6*c^6*d^2 + 4167*a^2*b^5*c^
5*d^3 - 1713*a^3*b^4*c^4*d^4 + 737*a^4*b^3*c^3*d^5 - 243*a^5*b^2*c^2*d^6 +
51*a^6*b*c*d^7))/(70*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c
^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 3
6*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) + (6*b^6*x^6*(2*a^2*d^8
 + 107*b^2*c^2*d^6 + 59*a*b*c*d^7))/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2
 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*
b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (3*b^3*
x^3*(669*b^5*c^5*d^3 - a^5*d^8 + 2163*a*b^4*c^4*d^4 + 608*a^2*b^3*c^3*d^5 -
 92*a^3*b^2*c^2*d^6 + 13*a^4*b*c*d^7))/(5*(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c
^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 8
4*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*d^8)) +
(3*b^5*x^5*(319*b^3*c^3*d^5 - a^3*d^8 + 327*a*b^2*c^2*d^6 + 27*a^2*b*c*d^7)
)/(a^9*d^9 - b^9*c^9 - 36*a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^
5*c^5*d^4 + 126*a^5*b^4*c^4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 +
 9*a*b^8*c^8*d - 9*a^8*b*c*d^8) + (36*b^8*d^8*x^8)/(a^9*d^9 - b^9*c^9 - 36*
a^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^
4*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c
*d^8) + (18*b^6*d*x^7*(13*b^2*c*d^6 + 3*a*b*d^7))/(a^9*d^9 - b^9*c^9 - 36*a
^2*b^7*c^7*d^2 + 84*a^3*b^6*c^6*d^3 - 126*a^4*b^5*c^5*d^4 + 126*a^5*b^4*c^4
*d^5 - 84*a^6*b^3*c^3*d^6 + 36*a^7*b^2*c^2*d^7 + 9*a*b^8*c^8*d - 9*a^8*b*c*
d^8))/(x^3*(7*b^2*c^6*d + 35*a^2*c^4*d^3 + 42*a*b*c^5*d^2) + x^6*(7*a^2*c*d
^6 + 35*b^2*c^3*d^4 + 42*a*b*c^2*d^5) + x*(7*a^2*c^6*d + 2*a*b*c^7) + x^2*(
b^2*c^7 + 21*a^2*c^5*d^2 + 14*a*b*c^6*d) + x^7*(a^2*d^7 + 21*b^2*c^2*d^5 +
14*a*b*c*d^6) + x^4*(35*a^2*c^3*d^4 + 21*b^2*c^5*d^2 + 70*a*b*c^4*d^3) + x^
5*(21*a^2*c^2*d^5 + 35*b^2*c^4*d^3 + 70*a*b*c^3*d^4) + x^8*(7*b^2*c*d^6 + 2
*a*b*d^7) + a^2*c^7 + b^2*d^7*x^9)
```

sympy [B] time = 20.66, size = 2917, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**8,x)
```

```
[Out] -36*b**7*d**2*log(x + (-36*a**11*b**7*d**13/(a*d - b*c)**10 + 396*a**10*b**
8*c*d**12/(a*d - b*c)**10 - 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 + 594
0*a**8*b**10*c**3*d**10/(a*d - b*c)**10 - 11880*a**7*b**11*c**4*d**9/(a*d -
 b*c)**10 + 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 - 16632*a**5*b**13*c
**6*d**7/(a*d - b*c)**10 + 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 - 594
0*a**3*b**15*c**8*d**5/(a*d - b*c)**10 + 1980*a**2*b**16*c**9*d**4/(a*d - b
*c)**10 - 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 + 36*b**1
8*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b*c)*
*10 + 36*b**7*d**2*log(x + (36*a**11*b**7*d**13/(a*d - b*c)**10 - 396*a**10
*b**8*c*d**12/(a*d - b*c)**10 + 1980*a**9*b**9*c**2*d**11/(a*d - b*c)**10 -
 5940*a**8*b**10*c**3*d**10/(a*d - b*c)**10 + 11880*a**7*b**11*c**4*d**9/(a
*d - b*c)**10 - 16632*a**6*b**12*c**5*d**8/(a*d - b*c)**10 + 16632*a**5*b**
13*c**6*d**7/(a*d - b*c)**10 - 11880*a**4*b**14*c**7*d**6/(a*d - b*c)**10 +
 5940*a**3*b**15*c**8*d**5/(a*d - b*c)**10 - 1980*a**2*b**16*c**9*d**4/(a*d
 - b*c)**10 + 396*a*b**17*c**10*d**3/(a*d - b*c)**10 + 36*a*b**7*d**3 - 36*
b**18*c**11*d**2/(a*d - b*c)**10 + 36*b**8*c*d**2)/(72*b**8*d**3))/(a*d - b
*c)**10 + (-10*a**8*d**8 + 95*a**7*b*c*d**7 - 409*a**6*b**2*c**2*d**6 + 106
1*a**5*b**3*c**3*d**5 - 1879*a**4*b**4*c**4*d**4 + 2531*a**3*b**5*c**5*d**3
 - 3349*a**2*b**6*c**6*d**2 - 595*a*b**7*c**7*d + 35*b**8*c**8 - 2520*b**8*
d**8*x**8 + x**7*(-3780*a*b**7*d**8 - 16380*b**8*c*d**7) + x**6*(-840*a**2*
b**6*d**8 - 24780*a*b**7*c*d**7 - 44940*b**8*c**2*d**6) + x**5*(210*a**3*b*
*5*d**8 - 5670*a**2*b**6*c*d**7 - 68670*a*b**7*c**2*d**6 - 66990*b**8*c**3*
d**5) + x**4*(-84*a**4*b**4*d**8 + 1386*a**3*b**5*c*d**7 - 16254*a**2*b**6*
c**2*d**6 - 103614*a*b**7*c**3*d**5 - 57834*b**8*c**4*d**4) + x**3*(42*a**5
*b**3*d**8 - 546*a**4*b**4*c*d**7 + 3864*a**3*b**5*c**2*d**6 - 25536*a**2*b
**6*c**3*d**5 - 90846*a*b**7*c**4*d**4 - 28098*b**8*c**5*d**3) + x**2*(-24*
a**6*b**2*d**8 + 270*a**5*b**3*c*d**7 - 1494*a**4*b**4*c**2*d**6 + 5856*a**
3*b**5*c**3*d**5 - 23544*a**2*b**6*c**4*d**4 - 45090*a*b**7*c**5*d**3 - 653
4*b**8*c**6*d**2) + x*(15*a**7*b*d**8 - 153*a**6*b**2*c*d**7 + 729*a**5*b**
3*c**2*d**6 - 2211*a**4*b**4*c**3*d**5 + 5139*a**3*b**5*c**4*d**4 - 12501*a
**2*b**6*c**5*d**3 - 10863*a*b**7*c**6*d**2 - 315*b**8*c**7*d))/(70*a**11*c
**7*d**9 - 630*a**10*b*c**8*d**8 + 2520*a**9*b**2*c**9*d**7 - 5880*a**8*b**
3*c**10*d**6 + 8820*a**7*b**4*c**11*d**5 - 8820*a**6*b**5*c**12*d**4 + 5880
*a**5*b**6*c**13*d**3 - 2520*a**4*b**7*c**14*d**2 + 630*a**3*b**8*c**15*d -
 70*a**2*b**9*c**16 + x**9*(70*a**9*b**2*d**16 - 630*a**8*b**3*c*d**15 + 25
20*a**7*b**4*c**2*d**14 - 5880*a**6*b**5*c**3*d**13 + 8820*a**5*b**6*c**4*d
**12 - 8820*a**4*b**7*c**5*d**11 + 5880*a**3*b**8*c**6*d**10 - 2520*a**2*b*
*9*c**7*d**9 + 630*a*b**10*c**8*d**8 - 70*b**11*c**9*d**7) + x**8*(140*a**1
0*b*d**16 - 770*a**9*b**2*c*d**15 + 630*a**8*b**3*c**2*d**14 + 5880*a**7*b*
*4*c**3*d**13 - 23520*a**6*b**5*c**4*d**12 + 44100*a**5*b**6*c**5*d**11 - 4
9980*a**4*b**7*c**6*d**10 + 36120*a**3*b**8*c**7*d**9 - 16380*a**2*b**9*c**
8*d**8 + 4270*a*b**10*c**9*d**7 - 490*b**11*c**10*d**6) + x**7*(70*a**11*d*
*16 + 350*a**10*b*c*d**15 - 4830*a**9*b**2*c**2*d**14 + 16170*a**8*b**3*c**
3*d**13 - 20580*a**7*b**4*c**4*d**12 - 8820*a**6*b**5*c**5*d**11 + 67620*a*
*5*b**6*c**6*d**10 - 105420*a**4*b**7*c**7*d**9 + 88830*a**3*b**8*c**8*d**8
 - 44170*a**2*b**9*c**9*d**7 + 12250*a*b**10*c**10*d**6 - 1470*b**11*c**11*
d**5) + x**6*(490*a**11*c*d**15 - 1470*a**10*b*c**2*d**14 - 6370*a**9*b**2*
c**3*d**13 + 42630*a**8*b**3*c**4*d**12 - 97020*a**7*b**4*c**5*d**11 + 1029
00*a**6*b**5*c**6*d**10 - 20580*a**5*b**6*c**7*d**9 - 79380*a**4*b**7*c**8*
d**8 + 104370*a**3*b**8*c**9*d**7 - 62230*a**2*b**9*c**10*d**6 + 19110*a*b*
*10*c**11*d**5 - 2450*b**11*c**12*d**4) + x**5*(1470*a**11*c**2*d**14 - 833
0*a**10*b*c**3*d**13 + 11270*a**9*b**2*c**4*d**12 + 30870*a**8*b**3*c**5*d*
*11 - 138180*a**7*b**4*c**6*d**10 + 226380*a**6*b**5*c**7*d**9 - 185220*a**
5*b**6*c**8*d**8 + 49980*a**4*b**7*c**9*d**7 + 42630*a**3*b**8*c**10*d**6 -
 45570*a**2*b**9*c**11*d**5 + 17150*a*b**10*c**12*d**4 - 2450*b**11*c**13*d
**3) + x**4*(2450*a**11*c**3*d**13 - 17150*a**10*b*c**4*d**12 + 45570*a**9*
b**2*c**5*d**11 - 42630*a**8*b**3*c**6*d**10 - 49980*a**7*b**4*c**7*d**9 +
185220*a**6*b**5*c**8*d**8 - 226380*a**5*b**6*c**9*d**7 + 138180*a**4*b**7*
```

c**10*d**6 - 30870*a**3*b**8*c**11*d**5 - 11270*a**2*b**9*c**12*d**4 + 8330 *a*b**10*c**13*d**3 - 1470*b**11*c**14*d**2) + x**3*(2450*a**11*c**4*d**12 - 19110*a**10*b*c**5*d**11 + 62230*a**9*b**2*c**6*d**10 - 104370*a**8*b**3* c**7*d**9 + 79380*a**7*b**4*c**8*d**8 + 20580*a**6*b**5*c**9*d**7 - 102900* a**5*b**6*c**10*d**6 + 97020*a**4*b**7*c**11*d**5 - 42630*a**3*b**8*c**12*d **4 + 6370*a**2*b**9*c**13*d**3 + 1470*a*b**10*c**14*d**2 - 490*b**11*c**15 *d) + x**2*(1470*a**11*c**5*d**11 - 12250*a**10*b*c**6*d**10 + 44170*a**9*b **2*c**7*d**9 - 88830*a**8*b**3*c**8*d**8 + 105420*a**7*b**4*c**9*d**7 - 67 620*a**6*b**5*c**10*d**6 + 8820*a**5*b**6*c**11*d**5 + 20580*a**4*b**7*c**1 2*d**4 - 16170*a**3*b**8*c**13*d**3 + 4830*a**2*b**9*c**14*d**2 - 350*a*b** 10*c**15*d - 70*b**11*c**16) + x*(490*a**11*c**6*d**10 - 4270*a**10*b*c**7* d**9 + 16380*a**9*b**2*c**8*d**8 - 36120*a**8*b**3*c**9*d**7 + 49980*a**7*b **4*c**10*d**6 - 44100*a**6*b**5*c**11*d**5 + 23520*a**5*b**6*c**12*d**4 - 5880*a**4*b**7*c**13*d**3 - 630*a**3*b**8*c**14*d**2 + 770*a**2*b**9*c**15* d - 140*a*b**10*c**16))

3.1269 $\int (a+bx)^5\sqrt{c+dx}\,dx$

Optimal. Leaf size=156

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-2(\ldots$$

Rubi [A] time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{11/2}(bc-ad)}{11d^6}+\frac{20b^3(c+dx)^{9/2}(bc-ad)^2}{9d^6}-\frac{20b^2(c+dx)^{7/2}(bc-ad)^3}{7d^6}+\frac{2b(c+dx)^{5/2}(bc-ad)^4}{d^6}-\frac{2(c+dx)^{3/2}(bc-ad)^5}{3d^6}+\frac{2b^5(c+dx)^{13/2}}{13d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(3/2))/(3*d^6) + (2*b*(b*c - a*d)^4*(c + d*x)^(
5/2))/d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(b*c -
 a*d)^2*(c + d*x)^(9/2))/(9*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(11/2))/(1
1*d^6) + (2*b^5*(c + d*x)^(13/2))/(13*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5\sqrt{c+dx}\,dx=\int\left(\frac{(-bc+ad)^5\sqrt{c+dx}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{3/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{5/2}}{d^5}+\frac{10\ldots}{\ldots}\right.$$

$$=-\frac{2(bc-ad)^5(c+dx)^{3/2}}{3d^6}+\frac{2b(bc-ad)^4(c+dx)^{5/2}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{7/2}}{7d^6}+\frac{20b\ldots}{\ldots}$$

Mathematica [A] time = 0.15, size = 123, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-4095b^4(c+dx)^4(bc-ad)+10010b^3(c+dx)^3(bc-ad)^2-12870b^2(c+dx)^2(bc-ad)^3+9009b(c+dx)(bc-ad)^4-3003(bc-ad)^5+693b^5(c+dx)^5\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3003*(b*c - a*d)^5 + 9009*b*(b*c - a*d)^4*(c + d*x) -
12870*b^2*(b*c - a*d)^3*(c + d*x)^2 + 10010*b^3*(b*c - a*d)^2*(c + d*x)^3 -
 4095*b^4*(b*c - a*d)*(c + d*x)^4 + 693*b^5*(c + d*x)^5))/(9009*d^6)
```

IntegrateAlgebraic [B] time = 0.10, size = 315, normalized size = 2.02

$$\frac{2(c+dx)^{3/2}\left(3003a^5d^5+9009a^4bd^4(c+dx)-15015a^4bcd^4+30030a^3b^2c^2d^3+12870a^3b^2d^3(c+dx)^2-36036a^3b^2cd^3(c+dx)-30030a^2b^3c^3d^2+54054a^2b^3c^2d^2(c+dx)+10010a^2b^3d^2(c+dx)^3-38610a^2b^3cd^2(c+dx)^2+15015ab^4c^4d-36036ab^4c^3d(c+dx)+38610ab^4c^2d(c+dx)^2+4095ab^4d(c+dx)^4-20020ab^4cd(c+dx)^3-3003b^5c^5+9009b^5c^4(c+dx)-12870b^5c^3(c+dx)^2+10010b^5c^2(c+dx)^3+693b^5(c+dx)^5-4095b^5c(c+dx)^4\right)}{9009d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3003*b^5*c^5 + 15015*a*b^4*c^4*d - 30030*a^2*b^3*c^3*d
^2 + 30030*a^3*b^2*c^2*d^3 - 15015*a^4*b*c*d^4 + 3003*a^5*d^5 + 9009*b^5*c^
```

4*(c + d*x) - 36036*a*b^4*c^3*d*(c + d*x) + 54054*a^2*b^3*c^2*d^2*(c + d*x) - 36036*a^3*b^2*c*d^3*(c + d*x) + 9009*a^4*b*d^4*(c + d*x) - 12870*b^5*c^3*(c + d*x)^2 + 38610*a*b^4*c^2*d*(c + d*x)^2 - 38610*a^2*b^3*c*d^2*(c + d*x)^2 + 12870*a^3*b^2*d^3*(c + d*x)^2 + 10010*b^5*c^2*(c + d*x)^3 - 20020*a*b^4*c*d*(c + d*x)^3 + 10010*a^2*b^3*d^2*(c + d*x)^3 - 4095*b^5*c*(c + d*x)^4 + 4095*a*b^4*d*(c + d*x)^4 + 693*b^5*(c + d*x)^5))/(9009*d^6)

fricas [B] time = 1.21, size = 338, normalized size = 2.17

$$\frac{2\left(693\,b^5d^6x^6-256\,b^5c^6+1664\,ab^4c^5d-4576\,a^2b^3c^4d^2+6864\,a^3b^2c^3d^3-6006\,a^4bc^2d^4+3003\,a^5cd^5+63\left(b^5cd^5+65\,ab^4d^6\right)x^5-35\left(2\,b^5c^2d^4-13\,ab^4cd^5-286\,a^2b^3d^6\right)x^4+10\left(8\,b^5c^3d^3-52\,ab^4c^2d^4+143\,a^2b^3cd^5+1287\,a^3b^2d^6\right)x^3-3\left(32\,b^5c^4d^2-208\,ab^4c^3d^3+572\,a^2b^3c^2d^4-858\,a^3b^2cd^5-3003\,a^4bd^6\right)x^2+\left(128\,b^5c^5d-832\,ab^4c^4d^2+2288\,a^2b^3c^3d^3-3432\,a^3b^2c^2d^4+3003\,a^4bcd^5+3003\,a^5d^6\right)x\right)\sqrt{dx+c}}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 2/9009*(693*b^5*d^6*x^6 - 256*b^5*c^6 + 1664*a*b^4*c^5*d - 4576*a^2*b^3*c^4*d^2 + 6864*a^3*b^2*c^3*d^3 - 6006*a^4*b*c^2*d^4 + 3003*a^5*c*d^5 + 63*(b^5*c*d^5 + 65*a*b^4*d^6)*x^5 - 35*(2*b^5*c^2*d^4 - 13*a*b^4*c*d^5 - 286*a^2*b^3*d^6)*x^4 + 10*(8*b^5*c^3*d^3 - 52*a*b^4*c^2*d^4 + 143*a^2*b^3*c*d^5 + 1287*a^3*b^2*d^6)*x^3 - 3*(32*b^5*c^4*d^2 - 208*a*b^4*c^3*d^3 + 572*a^2*b^3*c^2*d^4 - 858*a^3*b^2*c*d^5 - 3003*a^4*b*d^6)*x^2 + (128*b^5*c^5*d - 832*a*b^4*c^4*d^2 + 2288*a^2*b^3*c^3*d^3 - 3432*a^3*b^2*c^2*d^4 + 3003*a^4*b*c*d^5 + 3003*a^5*d^6)*x)*sqrt(d*x + c)/d^6

giac [B] time = 1.38, size = 641, normalized size = 4.11

$$\frac{2\left(9009\,\sqrt{dx+c}\,a^5c+3003\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^5+\frac{15015\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^4bc}{d}+\frac{6006\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^3b^2c}{d^2}+\frac{3003\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^4b}{d}+\frac{2574\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^2b^3c}{d^3}+\frac{2574\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^3b^2}{d^2}+\frac{143\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab^4c}{d^4}+\frac{286\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)a^2b^3}{d^3}+\frac{13\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)b^5c}{d^5}+\frac{65\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)ab^4}{d^4}+\frac{3\left(231\,(dx+c)^{\frac{13}{2}}-1638\,(dx+c)^{\frac{11}{2}}c+5005\,(dx+c)^{\frac{9}{2}}c^2-8580\,(dx+c)^{\frac{7}{2}}c^3+9009\,(dx+c)^{\frac{5}{2}}c^4-6006\,(dx+c)^{\frac{3}{2}}c^5+3003\,\sqrt{dx+c}\,c^6\right)b^5}{d^5}\right)}{9009\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="giac")

[Out] 2/9009*(9009*sqrt(d*x + c)*a^5*c + 3003*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^5 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c/d + 6006*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*b^2*c/d^2 + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4*b/d + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c/d^3 + 2574*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2/d^2 + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c/d^4 + 286*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^3/d^3 + 13*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c/d^5 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^4/d^4 + 3*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5/d^5)/d

maple [B] time = 0.01, size = 273, normalized size = 1.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(693b^5x^5d^5+4095a\,b^4d^5x^4-630b^5c\,d^4x^4+10010a^2b^3d^5x^3-3640a\,b^4c\,d^4x^3+560b^5c^2d^3x^3+12870a^3b^2d^5x^2-8580a^2b^3c\,d^4x^2+3120a\,b^4c^2d^3x^2-480b^5c^3d^2x^2+9009a^4b\,d^5x-10296a^3b^2c\,d^4x+6864a^2b^3c^2d^3x-2496a\,b^4c^3d^2x+384b^5c^4dx+3003a^5d^5-6006a^4bc\,d^4+6864a^3b^2c^2d^3-4576a^2b^3c^3d^2+1664a\,b^4c^4d-256b^5c^5\right)}{9009d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5*(d*x+c)^(1/2),x)

[Out] 2/9009*(d*x+c)^(3/2)*(693*b^5*d^5*x^5+4095*a*b^4*d^5*x^4-630*b^5*c*d^4*x^4+10010*a^2*b^3*d^5*x^3-3640*a*b^4*c*d^4*x^3+560*b^5*c^2*d^3*x^3+12870*a^3*b^2*d^5*x^2-8580*a^2*b^3*c*d^4*x^2+3120*a*b^4*c^2*d^3*x^2-480*b^5*c^3*d^2*x^2

```
+9009*a^4*b*d^5*x-10296*a^3*b^2*c*d^4*x+6864*a^2*b^3*c^2*d^3*x-2496*a*b^4*c
^3*d^2*x+384*b^5*c^4*d*x+3003*a^5*d^5-6006*a^4*b*c*d^4+6864*a^3*b^2*c^2*d^3
-4576*a^2*b^3*c^3*d^2+1664*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

maxima [A] time = 1.42, size = 259, normalized size = 1.66

$$\frac{2\left(693\,(dx+c)^{\frac{13}{2}}b^5-4095\left(b^5c-ab^4d\right)(dx+c)^{\frac{11}{2}}+10010\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{9}{2}}-12870\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{7}{2}}+9009\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)(dx+c)^{\frac{5}{2}}-3003\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)(dx+c)^{\frac{3}{2}}\right)}{9009\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9009*(693*(d*x + c)^(13/2)*b^5 - 4095*(b^5*c - a*b^4*d)*(d*x + c)^(11/2)
+ 10010*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(9/2) - 12870*(b^5*
c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(7/2) + 9009
*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*(d*x + c)^(5/2) - 3003*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10
*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(3/2))/d^6
```

mupad [B] time = 0.08, size = 137, normalized size = 0.88

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(13/2))/(13*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(11/
2))/(11*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(3/2))/(3*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(7/2))/(7*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(9/2))/(9*
d^6) + (2*b*(a*d - b*c)^4*(c + d*x)^(5/2))/d^6
```

sympy [B] time = 5.12, size = 314, normalized size = 2.01

$$\frac{2\left(\frac{b^5(c+dx)^{\frac{13}{2}}}{13d^5}+\frac{(c+dx)^{\frac{11}{2}}\left(5ab^4d-5b^5c\right)}{11d^5}+\frac{(c+dx)^{\frac{9}{2}}\left(10a^2b^3d^2-20ab^4cd+10b^5c^2\right)}{9d^5}+\frac{(c+dx)^{\frac{7}{2}}\left(10a^3b^2d^3-30a^2b^3cd^2+30ab^4c^2d-10b^5c^3\right)}{7d^5}+\frac{(c+dx)^{\frac{5}{2}}\left(5a^4bd^4-20a^3b^2cd^3+30a^2b^3c^2d^2-20ab^4c^3d+5b^5c^4\right)}{5d^5}+\frac{(c+dx)^{\frac{3}{2}}\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{3d^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**5*(c + d*x)**(13/2)/(13*d**5) + (c + d*x)**(11/2)*(5*a*b**4*d - 5*b**
5*c)/(11*d**5) + (c + d*x)**(9/2)*(10*a**2*b**3*d**2 - 20*a*b**4*c*d + 10*b
**5*c**2)/(9*d**5) + (c + d*x)**(7/2)*(10*a**3*b**2*d**3 - 30*a**2*b**3*c*d
**2 + 30*a*b**4*c**2*d - 10*b**5*c**3)/(7*d**5) + (c + d*x)**(5/2)*(5*a**4*
b*d**4 - 20*a**3*b**2*c*d**3 + 30*a**2*b**3*c**2*d**2 - 20*a*b**4*c**3*d +
5*b**5*c**4)/(5*d**5) + (c + d*x)**(3/2)*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*
a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)
/(3*d**5))/d
```

3.1270 $\int (a+bx)^4\sqrt{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)}{11d^5}$$

Rubi [A] time = 0.05, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{9/2}(bc-ad)}{9d^5}+\frac{12b^2(c+dx)^{7/2}(bc-ad)^2}{7d^5}-\frac{8b(c+dx)^{5/2}(bc-ad)^3}{5d^5}+\frac{2(c+dx)^{3/2}(bc-ad)^4}{3d^5}+\frac{2b^4(c+dx)^{11/2}}{11d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(3/2))/(3*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(5
/2))/(5*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^5) - (8*b^3*(b*c
 - a*d)*(c + d*x)^(9/2))/(9*d^5) + (2*b^4*(c + d*x)^(11/2))/(11*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^4\sqrt{c+dx}\,dx=\int \left(\frac{(-bc+ad)^4\sqrt{c+dx}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{3/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{5/2}}{d^4}-4b^3(bc\right)$$

$$=\frac{2(bc-ad)^4(c+dx)^{3/2}}{3d^5}-\frac{8b(bc-ad)^3(c+dx)^{5/2}}{5d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{7/2}}{7d^5}-8b^3(bc-$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{3/2}\left(-1540b^3(c+dx)^3(bc-ad)+2970b^2(c+dx)^2(bc-ad)^2-2772b(c+dx)(bc-ad)^3+1155(bc-ad)^4+315b^4(c+dx)^4\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*(b*c - a*d)^4 - 2772*b*(b*c - a*d)^3*(c + d*x) + 2
970*b^2*(b*c - a*d)^2*(c + d*x)^2 - 1540*b^3*(b*c - a*d)*(c + d*x)^3 + 315*
b^4*(c + d*x)^4))/(3465*d^5)
```

IntegrateAlgebraic [A] time = 0.07, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{3/2}\left(1155a^4d^4+2772a^3bd^3(c+dx)-4620a^3bcd^3+6930a^2b^2c^2d^2+2970a^2b^2d^2(c+dx)^2-8316a^2b^2cd^2(c+dx)-4620ab^3c^3d+8316ab^3c^2d(c+dx)+1540ab^3d(c+dx)^3-5940ab^3cd(c+dx)^2+1155b^4c^4-2772b^4c^3(c+dx)+2970b^4c^2(c+dx)^2+315b^4(c+dx)^4-1540b^4c(c+dx)^3\right)}{3465d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(1155*b^4*c^4 - 4620*a*b^3*c^3*d + 6930*a^2*b^2*c^2*d^2
- 4620*a^3*b*c*d^3 + 1155*a^4*d^4 - 2772*b^4*c^3*(c + d*x) + 8316*a*b^3*c^2
```

```
*d*(c + d*x) - 8316*a^2*b^2*c*d^2*(c + d*x) + 2772*a^3*b*d^3*(c + d*x) + 29
70*b^4*c^2*(c + d*x)^2 - 5940*a*b^3*c*d*(c + d*x)^2 + 2970*a^2*b^2*d^2*(c +
 d*x)^2 - 1540*b^4*c*(c + d*x)^3 + 1540*a*b^3*d*(c + d*x)^3 + 315*b^4*(c +
d*x)^4))/(3465*d^5)
```

fricas [B] time = 1.69, size = 245, normalized size = 1.90

$$\frac{2\left(315\,b^4d^5x^5+128\,b^4c^5-704\,ab^3c^4d+1584\,a^2b^2c^3d^2-1848\,a^3bc^2d^3+1155\,a^4cd^4+35\left(b^4cd^4+44\,ab^3d^5\right)x^4-10\left(4\,b^4c^2d^3-22\,ab^3cd^4-297\,a^2b^2d^5\right)x^3+6\left(8\,b^4c^3d^2-44\,ab^3c^2d^3+99\,a^2b^2cd^4+462\,a^3bd^5\right)x^2-\left(64\,b^4c^4d-352\,ab^3c^3d^2+792\,a^2b^2c^2d^3-924\,a^3bcd^4-1155\,a^4d^5\right)x\right)\sqrt{dx+c}}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*b^4*d^5*x^5 + 128*b^4*c^5 - 704*a*b^3*c^4*d + 1584*a^2*b^2*c^3*
d^2 - 1848*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 + 35*(b^4*c*d^4 + 44*a*b^3*d^5)*x
^4 - 10*(4*b^4*c^2*d^3 - 22*a*b^3*c*d^4 - 297*a^2*b^2*d^5)*x^3 + 6*(8*b^4*c
^3*d^2 - 44*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 + 462*a^3*b*d^5)*x^2 - (64*b^4
*c^4*d - 352*a*b^3*c^3*d^2 + 792*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 - 1155*a
^4*d^5)*x)*sqrt(d*x + c)/d^5
```

giac [B] time = 1.25, size = 470, normalized size = 3.64

$$\frac{2\left(3465\,\sqrt{dx+c}\,a^4c+1155\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^4+\frac{4620\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3bc}{d}+\frac{1386\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2c}{d^2}+\frac{924\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^3b}{d}+\frac{396\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3c}{d^3}+\frac{594\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)a^2b^2}{d^2}+\frac{11\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4c}{d^4}+\frac{44\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)ab^3}{d^3}+\frac{5\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^2-1386\,(dx+c)^{\frac{5}{2}}c^3+1155\,(dx+c)^{\frac{3}{2}}c^4-693\,\sqrt{dx+c}\,c^5\right)b^4}{d^4}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^4*c + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*
c)*a^4 + 4620*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c/d + 1386*(3*(d*
x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b^2*c/d^2 +
 924*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3*
b/d + 396*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^
2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c/d^3 + 594*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2/d^2 + 1
1*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 4
20*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c/d^4 + 44*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3/d^3 + 5*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4/d^4)/d
```

maple [A] time = 0.01, size = 186, normalized size = 1.44

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(315b^4x^4d^4+1540a\,b^3d^4x^3-280b^4c\,d^3x^3+2970a^2b^2d^4x^2-1320a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+2772a^3b\,d^4x-2376a^2b^2c\,d^3x+1056a\,b^3c^2d^2x-192b^4c^3dx+1155a^4d^4-1848a^3bc\,d^3+1584a^2b^2c^2d^2-704a\,b^3c^3d+128b^4c^4\right)}{3465d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(1/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(315*b^4*d^4*x^4+1540*a*b^3*d^4*x^3-280*b^4*c*d^3*x^3+
2970*a^2*b^2*d^4*x^2-1320*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+2772*a^3*b*d^
4*x-2376*a^2*b^2*c*d^3*x+1056*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+1155*a^4*d^4-
1848*a^3*b*c*d^3+1584*a^2*b^2*c^2*d^2-704*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

maxima [A] time = 1.36, size = 181, normalized size = 1.40

$$\frac{2\left(315\,(dx+c)^{\frac{11}{2}}b^4-1540\left(b^4c-ab^3d\right)(dx+c)^{\frac{9}{2}}+2970\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{7}{2}}-2772\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)(dx+c)^{\frac{5}{2}}+1155\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)(dx+c)^{\frac{3}{2}}\right)}{3465\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3465*(315*(d*x + c)^(11/2)*b^4 - 1540*(b^4*c - a*b^3*d)*(d*x + c)^(9/2) +
 2970*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(7/2) - 2772*(b^4*c^3
 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(5/2) + 1155*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x +
c)^(3/2))/d^5
```

mupad [B] time = 0.22, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(11/2))/(11*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(9/2))
/(9*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^5) + (12*b^2*(a*d - b*c)^
2*(c + d*x)^(7/2))/(7*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^5)
```

sympy [A] time = 4.19, size = 223, normalized size = 1.73

$$\frac{2\left(\frac{b^{4}(c+dx)^{\frac{11}{2}}}{11d^{4}}+\frac{(c+dx)^{\frac{9}{2}}\left(4ab^{3}d-4b^{4}c\right)}{9d^{4}}+\frac{(c+dx)^{\frac{7}{2}}\left(6a^{2}b^{2}d^{2}-12ab^{3}cd+6b^{4}c^{2}\right)}{7d^{4}}+\frac{(c+dx)^{\frac{5}{2}}\left(4a^{3}bd^{3}-12a^{2}b^{2}cd^{2}+12ab^{3}c^{2}d-4b^{4}c^{3}\right)}{5d^{4}}+\frac{(c+dx)^{\frac{3}{2}}\left(a^{4}d^{4}-4a^{3}bcd^{3}+6a^{2}b^{2}c^{2}d^{2}-4ab^{3}c^{3}d+b^{4}c^{4}\right)}{3d^{4}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**4*(c + d*x)**(11/2)/(11*d**4) + (c + d*x)**(9/2)*(4*a*b**3*d - 4*b**4
*c)/(9*d**4) + (c + d*x)**(7/2)*(6*a**2*b**2*d**2 - 12*a*b**3*c*d + 6*b**4*
c**2)/(7*d**4) + (c + d*x)**(5/2)*(4*a**3*b*d**3 - 12*a**2*b**2*c*d**2 + 12
*a*b**3*c**2*d - 4*b**4*c**3)/(5*d**4) + (c + d*x)**(3/2)*(a**4*d**4 - 4*a*
*3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*d**4)
)/d
```

3.1271 $\int (a+bx)^3\sqrt{c+dx}\,dx$

Optimal. Leaf size=100

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Rubi [A] time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{7/2}(bc-ad)}{7d^4}+\frac{6b(c+dx)^{5/2}(bc-ad)^2}{5d^4}-\frac{2(c+dx)^{3/2}(bc-ad)^3}{3d^4}+\frac{2b^3(c+dx)^{9/2}}{9d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
5/2))/(5*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^4) + (2*b^3*(c + d
*x)^(9/2))/(9*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3\sqrt{c+dx}\,dx=\int\left(\frac{(-bc+ad)^3\sqrt{c+dx}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{3/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{5/2}}{d^3}+\frac{b^3(c$$

$$=-\frac{2(bc-ad)^3(c+dx)^{3/2}}{3d^4}+\frac{6b(bc-ad)^2(c+dx)^{5/2}}{5d^4}-\frac{6b^2(bc-ad)(c+dx)^{7/2}}{7d^4}+\frac{2b^3(c$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}\left(-135b^2(c+dx)^2(bc-ad)+189b(c+dx)(bc-ad)^2-105(bc-ad)^3+35b^3(c+dx)^3\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-105*(b*c - a*d)^3 + 189*b*(b*c - a*d)^2*(c + d*x) - 13
5*b^2*(b*c - a*d)*(c + d*x)^2 + 35*b^3*(c + d*x)^3))/(315*d^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+189a^2bd^2(c+dx)-315a^2bcd^2+315ab^2c^2d+135ab^2d(c+dx)^2-378ab^2cd(c+dx)-105b^3c^3+189b^3c^2(c+dx)+35b^3(c+dx)^3-135b^3c(c+dx)^2\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-105*b^3*c^3 + 315*a*b^2*c^2*d - 315*a^2*b*c*d^2 + 105*
a^3*d^3 + 189*b^3*c^2*(c + d*x) - 378*a*b^2*c*d*(c + d*x) + 189*a^2*b*d^2*(
```

```
c + d*x) - 135*b^3*c*(c + d*x)^2 + 135*a*b^2*d*(c + d*x)^2 + 35*b^3*(c + d*
x)^3))/(315*d^4)
```

fricas [A] time = 1.16, size = 164, normalized size = 1.64

$$\frac{2\left(35\,b^{3}d^{4}x^{4}-16\,b^{3}c^{4}+72\,ab^{2}c^{3}d-126\,a^{2}bc^{2}d^{2}+105\,a^{3}cd^{3}+5\left(b^{3}cd^{3}+27\,ab^{2}d^{4}\right)x^{3}-3\left(2\,b^{3}c^{2}d^{2}-9\,ab^{2}cd^{3}-63\,a^{2}bd^{4}\right)x^{2}+\left(8\,b^{3}c^{3}d-36\,ab^{2}c^{2}d^{2}+63\,a^{2}bcd^{3}+105\,a^{3}d^{4}\right)x\right)\sqrt{dx+c}}{315\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^3*d^4*x^4 - 16*b^3*c^4 + 72*a*b^2*c^3*d - 126*a^2*b*c^2*d^2 + 1
05*a^3*c*d^3 + 5*(b^3*c*d^3 + 27*a*b^2*d^4)*x^3 - 3*(2*b^3*c^2*d^2 - 9*a*b^
2*c*d^3 - 63*a^2*b*d^4)*x^2 + (8*b^3*c^3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*
d^3 + 105*a^3*d^4)*x)*sqrt(d*x + c)/d^4
```

giac [B] time = 1.27, size = 322, normalized size = 3.22

$$\frac{2\left(315\,\sqrt{dx+c}\,a^{3}c+105\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{3}+\frac{315\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{2}bc}{d}+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)ab^{2}c}{d^{2}}+\frac{63\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)a^{2}b}{d}+\frac{9\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)b^{3}c}{d^{3}}+\frac{27\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)ab^{2}}{d^{2}}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\,\sqrt{dx+c}\,c^{4}\right)b^{3}}{d^{3}}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^3*c + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c/d^2 + 63*(3*(
d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*b/d + 9*(
5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt
(d*x + c)*c^3)*b^3*c/d^3 + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2/d^2 + (35*(d*x + c)^(9/
2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*
c^3 + 315*sqrt(d*x + c)*c^4)*b^3/d^3)/d
```

maple [A] time = 0.00, size = 116, normalized size = 1.16

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(35b^{3}x^{3}d^{3}+135a\,b^{2}d^{3}x^{2}-30b^{3}c\,d^{2}x^{2}+189a^{2}b\,d^{3}x-108a\,b^{2}c\,d^{2}x+24b^{3}c^{2}dx+105a^{3}d^{3}-126a^{2}bc\,d^{2}+72a\,b^{2}c^{2}d-16b^{3}c^{3}\right)}{315d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(35*b^3*d^3*x^3+135*a*b^2*d^3*x^2-30*b^3*c*d^2*x^2+189*
a^2*b*d^3*x-108*a*b^2*c*d^2*x+24*b^3*c^2*d*x+105*a^3*d^3-126*a^2*b*c*d^2+72
*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

maxima [A] time = 1.37, size = 118, normalized size = 1.18

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^{3}-135\left(b^{3}c-ab^{2}d\right)(dx+c)^{\frac{7}{2}}+189\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)(dx+c)^{\frac{5}{2}}-105\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)(dx+c)^{\frac{3}{2}}\right)}{315\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^3 - 135*(b^3*c - a*b^2*d)*(d*x + c)^(7/2) + 189
*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(5/2) - 105*(b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))/d^4
```

mupad [B] time = 0.07, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(9/2))/(9*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(7/2))/(
7*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^4) + (6*b*(a*d - b*c)^2*(c
+ d*x)^(5/2))/(5*d^4)
```

sympy [A] time = 3.34, size = 146, normalized size = 1.46

$$\frac{2\left(\frac{b^3(c+dx)^{\frac{9}{2}}}{9d^3}+\frac{(c+dx)^{\frac{7}{2}}\left(3ab^2d-3b^3c\right)}{7d^3}+\frac{(c+dx)^{\frac{5}{2}}\left(3a^2bd^2-6ab^2cd+3b^3c^2\right)}{5d^3}+\frac{(c+dx)^{\frac{3}{2}}\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3d^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**3*(c + d*x)**(9/2)/(9*d**3) + (c + d*x)**(7/2)*(3*a*b**2*d - 3*b**3*c
)/(7*d**3) + (c + d*x)**(5/2)*(3*a**2*b*d**2 - 6*a*b**2*c*d + 3*b**3*c**2)/
(5*d**3) + (c + d*x)**(3/2)*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3)/(3*d**3))/d
```

3.1272 $\int (a+bx)^2\sqrt{c+dx}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{5/2}(bc-ad)}{5d^3}+\frac{2(c+dx)^{3/2}(bc-ad)^2}{3d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(3/2))/(3*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(5/2
))/(5*d^3) + (2*b^2*(c + d*x)^(7/2))/(7*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2\sqrt{c+dx}\,dx &= \int \left(\frac{(-bc+ad)^2\sqrt{c+dx}}{d^2}-\frac{2b(bc-ad)(c+dx)^{3/2}}{d^2}+\frac{b^2(c+dx)^{5/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{3/2}}{3d^3}-\frac{4b(bc-ad)(c+dx)^{5/2}}{5d^3}+\frac{2b^2(c+dx)^{7/2}}{7d^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(3dx-2c)+b^2\left(8c^2-12cdx+15d^2x^2\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-2*c + 3*d*x) + b^2*(8*c^2 - 12*
c*d*x + 15*d^2*x^2)))/(105*d^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{3/2}\left(35a^2d^2+42abd(c+dx)-70abcd+35b^2c^2+15b^2(c+dx)^2-42b^2c(c+dx)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(35*b^2*c^2 - 70*a*b*c*d + 35*a^2*d^2 - 42*b^2*c*(c + d*
x) + 42*a*b*d*(c + d*x) + 15*b^2*(c + d*x)^2))/(105*d^3)
```

fricas [A] time = 1.39, size = 99, normalized size = 1.39

$$\frac{2\left(15\,b^{2}d^{3}x^{3}+8\,b^{2}c^{3}-28\,abc^{2}d+35\,a^{2}cd^{2}+3\left(b^{2}cd^{2}+14\,abd^{3}\right)x^{2}-\left(4\,b^{2}c^{2}d-14\,abcd^{2}-35\,a^{2}d^{3}\right)x\right)\sqrt{dx+c}}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^2*d^3*x^3 + 8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2 + 3*(b^2*c*
d^2 + 14*a*b*d^3)*x^2 - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*x)*sqrt(d
*x + c)/d^3
```

giac [B] time = 1.29, size = 200, normalized size = 2.82

$$\frac{2\left(105\,\sqrt{dx+c}\,a^{2}c+35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{2}+\frac{70\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)b^{2}c}{d^{2}}+\frac{14\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)ab}{d}+\frac{3\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)b^{2}}{d^{2}}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a^2*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c/d^2 + 14*(3*(d*x + c
)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b/d + 3*(5*(d*x +
c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)
*c^3)*b^2/d^2)/d
```

maple [A] time = 0.01, size = 63, normalized size = 0.89

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(15b^{2}x^{2}d^{2}+42ab\,d^{2}x-12b^{2}cdx+35a^{2}d^{2}-28abcd+8b^{2}c^{2}\right)}{105d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] 2/105*(d*x+c)^(3/2)*(15*b^2*d^2*x^2+42*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-28
*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.35, size = 68, normalized size = 0.96

$$\frac{2\left(15\,(dx+c)^{\frac{7}{2}}b^{2}-42\left(b^{2}c-abd\right)(dx+c)^{\frac{5}{2}}+35\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(dx+c)^{\frac{3}{2}}\right)}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(d*x + c)^(7/2)*b^2 - 42*(b^2*c - a*b*d)*(d*x + c)^(5/2) + 35*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))/d^3
```

mupad [B] time = 0.24, size = 68, normalized size = 0.96

$$\frac{2\,{(c+d\,x)}^{3/2}\,\left(15\,b^{2}\,{(c+d\,x)}^{2}+35\,a^{2}\,d^{2}+35\,b^{2}\,c^{2}-42\,b^{2}\,c\,(c+d\,x)+42\,a\,b\,d\,(c+d\,x)-70\,a\,b\,c\,d\right)}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(15*b^2*(c + d*x)^2 + 35*a^2*d^2 + 35*b^2*c^2 - 42*b^2*c
*(c + d*x) + 42*a*b*d*(c + d*x) - 70*a*b*c*d))/(105*d^3)
```

sympy [A] time = 2.69, size = 85, normalized size = 1.20

$$\frac{2\left(\frac{b^2(c+dx)^{\frac{7}{2}}}{7d^2}+\frac{(c+dx)^{\frac{5}{2}}\left(2abd-2b^2c\right)}{5d^2}+\frac{(c+dx)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}{3d^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b**2*(c + d*x)**(7/2)/(7*d**2) + (c + d*x)**(5/2)*(2*a*b*d - 2*b**2*c)/(
5*d**2) + (c + d*x)**(3/2)*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*d**2))/d
```

3.1273 $\int (a+bx)\sqrt{c+dx}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{5/2}}{5d^2}-\frac{2(c+dx)^{3/2}(bc-ad)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(3/2))/(3*d^2) + (2*b*(c + d*x)^(5/2))/(5*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)\sqrt{c+dx}\,dx&=\int\left(\frac{(-bc+ad)\sqrt{c+dx}}{d}+\frac{b(c+dx)^{3/2}}{d}\right)dx\\&=-\frac{2(bc-ad)(c+dx)^{3/2}}{3d^2}+\frac{2b(c+dx)^{5/2}}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{3/2}(5ad-2bc+3bdx)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x))/(15*d^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{3/2}(5ad+3b(c+dx)-5bc)}{15d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-5*b*c + 5*a*d + 3*b*(c + d*x)))/(15*d^2)
```

fricas [A] time = 1.43, size = 46, normalized size = 1.10

$$\frac{2\left(3\,bd^2x^2-2\,bc^2+5\,acd+\left(bcd+5\,ad^2\right)x\right)\sqrt{dx+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b*d^2*x^2 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x)*sqrt(d*x + c)/
d^2
```

giac [B] time = 1.37, size = 100, normalized size = 2.38

$$\frac{2\left(15\sqrt{dx+c}\,ac+5\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)a+\frac{5\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)bc}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)b}{d}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a*c + 5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a + 5*
((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c/d + (3*(d*x + c)^(5/2) - 10*(d*x
+ c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b/d)/d
```

maple [A] time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{3}{2}}\,(3bdx+5ad-2bc)}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(3*b*d*x+5*a*d-2*b*c)/d^2
```

maxima [A] time = 1.29, size = 33, normalized size = 0.79

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b-5\,(bc-ad)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(d*x + c)^(5/2)*b - 5*(b*c - a*d)*(d*x + c)^(3/2))/d^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.69

$$\frac{2\,(c+d\,x)^{3/2}\,\left(5\,a\,d-5\,b\,c+3\,b\,\left(c+d\,x\right)\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(3/2)*(5*a*d - 5*b*c + 3*b*(c + d*x)))/(15*d^2)
```

sympy [A] time = 2.12, size = 36, normalized size = 0.86

$$\frac{2\left(\frac{b(c+dx)^{\frac{5}{2}}}{5d}+\frac{(c+dx)^{\frac{3}{2}}(ad-bc)}{3d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] 2*(b*(c + d*x)**(5/2)/(5*d) + (c + d*x)**(3/2)*(a*d - b*c)/(3*d))/d
```

3.1274 $\int \sqrt{c+dx}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{3/2}}{3d}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{c+dx}\,dx = \frac{2(c+dx)^{3/2}}{3d}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x],x]
```

```
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

fricas [A] time = 1.28, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(d*x + c)^(3/2)/d
```

giac [A] time = 1.34, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(d*x + c)^(3/2)/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2),x)
[Out] 2/3*(d*x+c)^(3/2)/d
```

maxima [A] time = 1.35, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(d*x + c)^(3/2)/d
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(3/2))/(3*d)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2),x)
[Out] 2*(c + d*x)**(3/2)/(3*d)
```

3.1275 $\int \frac{\sqrt{c+dx}}{a+bx}\,dx$

Optimal. Leaf size=62

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{a+bx}\,dx &= \frac{2\sqrt{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b}\\
&= \frac{2\sqrt{c+dx}}{b}+\frac{(2(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{bd}\\
&= \frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{b}-\frac{2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b - (2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/b^(3/2)
```

IntegrateAlgebraic [A] time = 0.07, size = 72, normalized size = 1.16

$$\frac{2\sqrt{ad-bc}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}}+\frac{2\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x])/b + (2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a
*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(3/2)
```

fricas [A] time = 1.36, size = 143, normalized size = 2.31

$$\left[\frac{\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,\sqrt{dx+c}}{b},-\frac{2\left(\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\sqrt{dx+c}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*
c - a*d)/b))/(b*x + a)) + 2*sqrt(d*x + c))/b, -2*(sqrt(-(b*c - a*d)/b)*arct
an(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - sqrt(d*x + c))/b]
```

giac [A] time = 1.27, size = 62, normalized size = 1.00

$$\frac{2\,(bc-ad)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}+\frac{2\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b*c - a*d)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a
*b*d)*b) + 2*sqrt(d*x + c)/b
```

maple [A] time = 0.01, size = 92, normalized size = 1.48

$$-\frac{2ad\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}+\frac{2c\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}+\frac{2\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*x+c)^(1/2)/b-2/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)*b/((a*d-b*c)
*b)^(1/2))*a*d+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)*b/((a*d-b*c)*b)^(
1/2))*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.07, size = 50, normalized size = 0.81

$$\frac{2\sqrt{c+d\,x}}{b}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)\,\sqrt{a\,d-b\,c}}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(1/2))/b - (2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)
)*(a*d - b*c)^(1/2))/b^(3/2)
```

sympy [A] time = 4.39, size = 61, normalized size = 0.98

$$\frac{2\left(\frac{d\sqrt{c+dx}}{b}-\frac{d(ad-bc)\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^2\sqrt{\frac{ad-bc}{b}}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a),x)
```

```
[Out] 2*(d*sqrt(c + d*x)/b - d*(a*d - b*c)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b)
)/(b**2*sqrt((a*d - b*c)/b)))/d
```

3.1276 $\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx$

Optimal. Leaf size=70

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) - (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*
c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx &= -\frac{\sqrt{c+dx}}{b(a+bx)}+\frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b}\\
&= -\frac{\sqrt{c+dx}}{b(a+bx)}+\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b}\\
&= -\frac{\sqrt{c+dx}}{b(a+bx)}-\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 69, normalized size = 0.99

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{b^{3/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx}}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -(Sqrt[c + d*x]/(b*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b
*c) + a*d]])/(b^(3/2)*Sqrt[-(b*c) + a*d])
```

IntegrateAlgebraic [A] time = 0.22, size = 91, normalized size = 1.30

$$-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{3/2}\sqrt{ad-bc}}-\frac{d\sqrt{c+dx}}{b(ad+b(c+dx)-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^2,x]
```

```
[Out] -((d*Sqrt[c + d*x])/(b*(-(b*c) + a*d + b*(c + d*x)))) - (d*ArcTan[(Sqrt[b]*
Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(b^(3/2)*Sqrt[-(b*c) + a*d]
)
```

fricas [A] time = 1.44, size = 232, normalized size = 3.31

$$\left[\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x\right)},\frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(b^2c-abd\right)\sqrt{dx+c}}{ab^3c-a^2b^2d+\left(b^4c-ab^3d\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b
^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(b^2*c - a*b*d)*sqrt(d*x + c))/
(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x), (sqrt(-b^2*c + a*b*d)*(b*d*x +
 a*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (b^2*c - a
*b*d)*sqrt(d*x + c))/(a*b^3*c - a^2*b^2*d + (b^4*c - a*b^3*d)*x)]
```

giac [A] time = 1.38, size = 72, normalized size = 1.03

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b}-\frac{\sqrt{dx+c}\,d}{((dx+c)b-bc+ad)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b) - s
qrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*b)
```

maple [A] time = 0.01, size = 64, normalized size = 0.91

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}-\frac{\sqrt{dx+c}\;d}{(bdx+ad)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^2,x)
```

```
[Out] -d/b*(d*x+c)^(1/2)/(b*d*x+a*d)+d/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)
/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.24, size = 61, normalized size = 0.87

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{b^{3/2}\,\sqrt{a\,d-b\,c}}-\frac{d\,\sqrt{c+d\,x}}{d\,x\,b^2+a\,d\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^2,x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(3/2)*(a*d - b*c)^
(1/2)) - (d*(c + d*x)^(1/2))/(a*b*d + b^2*d*x)
```

sympy [**B**] time = 58.58, size = 573, normalized size = 8.19

$$-\frac{2ad^2\sqrt{c+dx}}{2a^2bd^2-2ab^2cd+2ab^2d^2x-2b^3cdx}+\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{ad^2\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2b}-\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(-a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}+2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}-b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{cd\sqrt{-\frac{1}{b(ad-bc)^3}}\log\left(a^2d^2\sqrt{-\frac{1}{b(ad-bc)^3}}-2abcd\sqrt{-\frac{1}{b(ad-bc)^3}}+b^2c^2\sqrt{-\frac{1}{b(ad-bc)^3}}+\sqrt{c+dx}\right)}{2}+\frac{2cd\sqrt{c+dx}}{2a^2d^2-2abcd+2abd^2x-2b^2cdx}+\frac{2d\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad}{b}-c}}\right)}{b^2\sqrt{\frac{ad}{b}-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x - 2
*b**3*c*d*x) + a*d**2*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b
*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(
-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*sqrt(-1/(b*(a*d - b*
c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a
*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*
b) - c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)*
*3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d -
b*c)**3)) + sqrt(c + d*x))/2 + c*d*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**
2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**
2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*c*d*sqrt(c + d*x)
/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2*d*atan(sqrt(c
+ d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c))
```

3.1277 $\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx$

Optimal. Leaf size=110

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)}-\frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Rubi [A] time = 0.08, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}-\frac{d\sqrt{c+dx}}{4b(a+bx)(bc-ad)}-\frac{\sqrt{c+dx}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -Sqrt[c + d*x]/(2*b*(a + b*x)^2) - (d*Sqrt[c + d*x])/(4*b*(b*c - a*d)*(a +
b*x)) + (d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(3/2)*(
b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^3} \, dx = -\frac{\sqrt{c+dx}}{2b(a+bx)^2} + \frac{d \int \frac{1}{(a+bx)^2 \sqrt{c+dx}} \, dx}{4b}$$
$$= -\frac{\sqrt{c+dx}}{2b(a+bx)^2} - \frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)} - \frac{d^2 \int \frac{1}{(a+bx)\sqrt{c+dx}} \, dx}{8b(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{2b(a+bx)^2} - \frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)} - \frac{d \, \text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}} \, dx, x, \sqrt{c+dx}\right)}{4b(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{2b(a+bx)^2} - \frac{d\sqrt{c+dx}}{4b(bc-ad)(a+bx)} + \frac{d^2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{3/2}(bc-ad)^{3/2}}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.47

$$\frac{2d^2(c+dx)^{3/2} \, {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; -\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 0.42, size = 125, normalized size = 1.14

$$-\frac{d^2 \tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{3/2}(ad-bc)^{3/2}} - \frac{d^2\sqrt{c+dx}\,(-ad+b(c+dx)+bc)}{4b(bc-ad)(-ad-b(c+dx)+bc)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^3,x]
```

```
[Out] -1/4*(d^2*Sqrt[c + d*x]*(b*c - a*d + b*(c + d*x)))/(b*(b*c - a*d)*(b*c - a*
d - b*(c + d*x))^2) - (d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x]
)/(b*c - a*d)])/(4*b^(3/2)*(-(b*c) + a*d)^(3/2))
```

fricas [B] time = 1.06, size = 456, normalized size = 4.15

$$\left[-\frac{\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\sqrt{b^2c - abd}\,\log\left(\frac{bdx + 2\,bc - ad - 2\,\sqrt{b^2c - abd}\,\sqrt{dx + c}}{bx + a}\right) + 2\left(2\,b^3c^2 - 3\,ab^2cd + a^2bd^2 + \left(b^3cd - ab^2d^2\right)x\right)\sqrt{dx + c}}{8\left(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2 + \left(b^6c^2 - 2\,ab^5cd + a^2b^4d^2\right)x^2 + 2\left(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2\right)x\right)}, -\frac{\left(b^2d^2x^2 + 2\,abd^2x + a^2d^2\right)\sqrt{-b^2c + abd}\,\arctan\left(\frac{\sqrt{-b^2c + abd}\,\sqrt{dx + c}}{bdx + bc}\right) + \left(2\,b^3c^2 - 3\,ab^2cd + a^2bd^2 + \left(b^3cd - ab^2d^2\right)x\right)\sqrt{dx + c}}{4\left(a^2b^4c^2 - 2\,a^3b^3cd + a^4b^2d^2 + \left(b^6c^2 - 2\,ab^5cd + a^2b^4d^2\right)x^2 + 2\left(ab^5c^2 - 2\,a^2b^4cd + a^3b^3d^2\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*x
 + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(2*b^3
*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x), -1/4*((b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*
d)*sqrt(d*x + c)/(b*d*x + b*c)) + (2*b^3*c^2 - 3*a*b^2*c*d + a^2*b*d^2 + (b
^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^
2*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^
4*c*d + a^3*b^3*d^2)*x)]
```

giac [A] time = 1.33, size = 126, normalized size = 1.15

$$-\frac{d^2 \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^2c-abd\right)\sqrt{-b^2c+abd}}-\frac{(dx+c)^{\frac{3}{2}}bd^2+\sqrt{dx+c}\,bcd^2-\sqrt{dx+c}\,ad^3}{4\left(b^2c-abd\right)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c - a*b*d)*sqrt
(-b^2*c + a*b*d)) - 1/4*((d*x + c)^(3/2)*b*d^2 + sqrt(d*x + c)*b*c*d^2 - sq
rt(d*x + c)*a*d^3)/((b^2*c - a*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

maple [A] time = 0.01, size = 111, normalized size = 1.01

$$\frac{d^2 \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)\,\sqrt{(ad-bc)\,b}\,b}+\frac{(dx+c)^{\frac{3}{2}}\,d^2}{4\,(bdx+ad)^2\,(ad-bc)}-\frac{\sqrt{dx+c}\,d^2}{4\,(bdx+ad)^2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^3,x)
```

```
[Out] 1/4*d^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)-1/4*d^2/(b*d*x+a*d)^2/b*(d*x+
c)^(1/2)+1/4*d^2/(a*d-b*c)/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d
-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.30, size = 135, normalized size = 1.23

$$\frac{d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{d^2\,\sqrt{c+d\,x}}{4\,b}-\frac{d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,\left(a\,d-b\,c\right)}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^3,x)
```

```
[Out] (d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(3/2)*(a*d - b
*c)^(3/2)) - ((d^2*(c + d*x)^(1/2))/(4*b) - (d^2*(c + d*x)^(3/2))/(4*(a*d -
 b*c)))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.1278 $\int \frac{\sqrt{c+dx}}{(a+bx)^4} dx$

Optimal. Leaf size=146

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}+\frac{d^2\sqrt{c+dx}}{8b(a+bx)(bc-ad)^2}-\frac{d\sqrt{c+dx}}{12b(a+bx)^2(bc-ad)}-\frac{\sqrt{c+dx}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] -Sqrt[c + d*x]/(3*b*(a + b*x)^3) - (d*Sqrt[c + d*x])/(12*b*(b*c - a*d)*(a +
 b*x)^2) + (d^2*Sqrt[c + d*x])/(8*b*(b*c - a*d)^2*(a + b*x)) - (d^3*ArcTanh
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(3/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx = -\frac{\sqrt{c+dx}}{3b(a+bx)^3} + \frac{d\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6b}$$

$$= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} - \frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} + \frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} + \frac{d^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{8b(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{3b(a+bx)^3} - \frac{d\sqrt{c+dx}}{12b(bc-ad)(a+bx)^2} + \frac{d^2\sqrt{c+dx}}{8b(bc-ad)^2(a+bx)} - \frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{3/2}(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.36

$$\frac{2d^3(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 0.75, size = 176, normalized size = 1.21

$$\frac{d^3\sqrt{c+dx}\left(3a^2d^2-8abd(c+dx)-6abcd+3b^2c^2-3b^2(c+dx)^2+8b^2c(c+dx)\right)}{24b(bc-ad)^2(-ad-b(c+dx)+bc)^3} - \frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{3/2}(bc-ad)^2\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^4,x]
```

```
[Out] (d^3*Sqrt[c + d*x]*(3*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2 + 8*b^2*c*(c + d*x) -
 8*a*b*d*(c + d*x) - 3*b^2*(c + d*x)^2))/(24*b*(b*c - a*d)^2*(b*c - a*d - b
*(c + d*x))^3) - (d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x]])/(b*
c - a*d)])/(8*b^(3/2)*(b*c - a*d)^2*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.09, size = 785, normalized size = 5.38

$$\left[\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(8b^4c^3-22ab^3c^2d+17a^2b^2cd^2-3a^3bd^3-3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^3-3a^4b^4c^2d+3a^5b^3cd^2-a^6b^2d^3+\left(b^8c^3-3ab^7c^2d+3a^2b^6cd^2-a^3b^5d^3\right)x^3+3\left(ab^7c^3-3a^2b^6c^2d+3a^3b^5cd^2-a^4b^4d^3\right)x^2+3\left(a^2b^6c^3-3a^3b^5c^2d+3a^4b^4cd^2-a^5b^3d^3\right)x\right)}, \frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)-\left(8b^4c^3-22ab^3c^2d+17a^2b^2cd^2-3a^3bd^3-3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^3-3a^4b^4c^2d+3a^5b^3cd^2-a^6b^2d^3+\left(b^8c^3-3ab^7c^2d+3a^2b^6cd^2-a^3b^5d^3\right)x^3+3\left(ab^7c^3-3a^2b^6c^2d+3a^3b^5cd^2-a^4b^4d^3\right)x^2+3\left(a^2b^6c^3-3a^3b^5c^2d+3a^4b^4cd^2-a^5b^3d^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) - 2*(8*b^4*c^3 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^
3 - 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^
2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^
```

```
3 + 3*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3
*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x), 1/24*(
3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a
*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3
 - 22*a*b^3*c^2*d + 17*a^2*b^2*c*d^2 - 3*a^3*b*d^3 - 3*(b^4*c*d^2 - a*b^3*d
^3)*x^2 + 2*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^3 + 3*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^2 + 3*(a^2*b^6*c^3 - 3*a^3*b^5*
c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x)]
```

giac [A] time = 1.35, size = 207, normalized size = 1.42

$$\frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2} c+abd}}\right)}{8 \left(b^{3} c^{2} - 2 \, a b^{2} c d + a^{2} b d^{2}\right) \sqrt{-b^{2} c + abd}} + \frac{3 \, (dx + c)^{\frac{5}{2}} b^{2} d^{3} - 8 \, (dx + c)^{\frac{3}{2}} b^{2} c d^{3} - 3 \, \sqrt{dx + c}\, b^{2} c^{2} d^{3} + 8 \, (dx + c)^{\frac{3}{2}} a b d^{4} + 6 \, \sqrt{dx + c}\, a b c d^{4} - 3 \, \sqrt{dx + c}\, a^{2} d^{5}}{24 \left(b^{3} c^{2} - 2 \, a b^{2} c d + a^{2} b d^{2}\right) \left((dx + c)b - bc + ad\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^2 - 2*a*b^2*c*
d + a^2*b*d^2)*sqrt(-b^2*c + a*b*d)) + 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 - 8*
(d*x + c)^(3/2)*b^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 + 8*(d*x + c)^(3/2)
*a*b*d^4 + 6*sqrt(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c^2 -
 2*a*b^2*c*d + a^2*b*d^2)*((d*x + c)*b - b*c + a*d)^3)
```

maple [A] time = 0.02, size = 170, normalized size = 1.16

$$\frac{(dx+c)^{\frac{5}{2}} b d^{3}}{8 (bdx+ad)^{3} \left(a^{2} d^{2} - 2abcd + b^{2} c^{2}\right)} + \frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{(ad-bc)b}}\right)}{8 \left(a^{2} d^{2} - 2abcd + b^{2} c^{2}\right) \sqrt{(ad-bc)b}\, b} + \frac{(dx+c)^{\frac{3}{2}} d^{3}}{3 (bdx+ad)^{3} (ad-bc)} - \frac{\sqrt{dx+c}\, d^{3}}{8 (bdx+ad)^{3} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+1/3*d^3/(
b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(3/2)-1/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(1/2)+
1/8*d^3/b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1
/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.37, size = 207, normalized size = 1.42

$$\frac{\frac{d^{3} (c+d\,x)^{3/2}}{3\,(a\,d-b\,c)} - \frac{d^{3} \sqrt{c+d\,x}}{8\,b} + \frac{b\,d^{3} (c+d\,x)^{5/2}}{8\,(a\,d-b\,c)^{2}}}{(c+d\,x)\left(3\,a^{2}\,b\,d^{2} - 6\,a\,b^{2}\,c\,d + 3\,b^{3}\,c^{2}\right) + b^{3}\,(c+d\,x)^{3} - \left(3\,b^{3}\,c - 3\,a\,b^{2}\,d\right)(c+d\,x)^{2} + a^{3}\,d^{3} - b^{3}\,c^{3} + 3\,a\,b^{2}\,c^{2}\,d - 3\,a^{2}\,b\,c\,d^{2}} + \frac{d^{3}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{3/2}\,(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^4,x)
```

```
[Out] ((d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)) - (d^3*(c + d*x)^(1/2))/(8*b) + (b*d
^3*(c + d*x)^(5/2))/(8*(a*d - b*c)^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2
- 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (d^3*atan((b^(1/2)*(c + d*
x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(3/2)*(a*d - b*c)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.1279 $\int \frac{\sqrt{c+dx}}{(a+bx)^5} \, dx$

Optimal. Leaf size=182

$$\frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}} - \frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3} + \frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2} - \frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)} - \frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Rubi [A] time = 0.12, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = $\{47, 51, 63, 208\}$

$$\frac{5d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{3/2}(bc-ad)^{7/2}} - \frac{5d^3\sqrt{c+dx}}{64b(a+bx)(bc-ad)^3} + \frac{5d^2\sqrt{c+dx}}{96b(a+bx)^2(bc-ad)^2} - \frac{d\sqrt{c+dx}}{24b(a+bx)^3(bc-ad)} - \frac{\sqrt{c+dx}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -Sqrt[c + d*x]/(4*b*(a + b*x)^4) - (d*Sqrt[c + d*x])/(24*b*(b*c - a*d)*(a +
 b*x)^3) + (5*d^2*Sqrt[c + d*x])/(96*b*(b*c - a*d)^2*(a + b*x)^2) - (5*d^3*
Sqrt[c + d*x])/(64*b*(b*c - a*d)^3*(a + b*x)) + (5*d^4*ArcTanh[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[b*c - a*d]])/(64*b^(3/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx &= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}+\frac{d\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8b}\\
&= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}-\frac{\left(5d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48b(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}+\frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b(bc-ad)^2}\\
&= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}-\\
&= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}-\\
&= -\frac{\sqrt{c+dx}}{4b(a+bx)^4}-\frac{d\sqrt{c+dx}}{24b(bc-ad)(a+bx)^3}+\frac{5d^2\sqrt{c+dx}}{96b(bc-ad)^2(a+bx)^2}-\frac{5d^3\sqrt{c+dx}}{64b(bc-ad)^3(a+bx)}+
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.29

$$
\frac{2d^4(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},5;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 5, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^5)
```

IntegrateAlgebraic [A] time = 1.07, size = 226, normalized size = 1.24

$$
-\frac{d^4\sqrt{c+dx}\left(-15a^3d^3+73a^2bd^2(c+dx)+45a^2bcd^2-45ab^2c^2d+55ab^2d(c+dx)^2-146ab^2cd(c+dx)+15b^3c^3+73b^3c^2(c+dx)+15b^3(c+dx)^3-55b^3c(c+dx)^2\right)}{192b(bc-ad)^3(-ad-b(c+dx)+bc)^4}-\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{3/2}(ad-bc)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^5,x]
```

```
[Out] -1/192*(d^4*Sqrt[c + d*x]*(15*b^3*c^3 - 45*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 1
5*a^3*d^3 + 73*b^3*c^2*(c + d*x) - 146*a*b^2*c*d*(c + d*x) + 73*a^2*b*d^2*(
c + d*x) - 55*b^3*c*(c + d*x)^2 + 55*a*b^2*d*(c + d*x)^2 + 15*b^3*(c + d*x)
^3))/(b*(b*c - a*d)^3*(b*c - a*d - b*(c + d*x))^4) - (5*d^4*ArcTan[(Sqrt[b]
*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(3/2)*(-(b*c) + a*d)
^(7/2))
```

fricas [B] time = 0.89, size = 1176, normalized size = 6.46

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 184*a*b^4*c^3*d + 254*
```

```
a^2*b^3*c^2*d^2 - 133*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*
d^4)*x^3 - 5*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5
*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3
 + a^8*b^2*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*
c*d^3 + a^4*b^6*d^4)*x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d
^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d
+ 6*a^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 -
 4*a^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x), -
1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)
/(b*d*x + b*c)) + (48*b^5*c^4 - 184*a*b^4*c^3*d + 254*a^2*b^3*c^2*d^2 - 133
*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*x^3 - 5*(2*b^5*c
^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (8*b^5*c^3*d - 44*a*b^4*c^2
*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4 + (b^1
0*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
x^4 + 4*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3
+ a^5*b^5*d^4)*x^3 + 6*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 -
 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^2 + 4*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*
a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x)]
```

giac [B] time = 1.39, size = 311, normalized size = 1.71

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\,\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4-55\,(dx+c)^{\frac{5}{2}}b^3cd^4+73\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15\,\sqrt{dx+c}\,b^3c^3d^4+55\,(dx+c)^{\frac{5}{2}}ab^2d^5-146\,(dx+c)^{\frac{3}{2}}ab^2cd^5-45\,\sqrt{dx+c}\,ab^2c^2d^5+73\,(dx+c)^{\frac{3}{2}}a^2bd^6+45\,\sqrt{dx+c}\,a^2bcd^6-15\,\sqrt{dx+c}\,a^3d^7}{192\,\left(b^4c^3-3\,ab^3c^2d+3\,a^2b^2cd^2-a^3bd^3\right)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x
 + c)^(7/2)*b^3*d^4 - 55*(d*x + c)^(5/2)*b^3*c*d^4 + 73*(d*x + c)^(3/2)*b^3
*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^4 + 55*(d*x + c)^(5/2)*a*b^2*d^5 - 14
6*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqrt(d*x + c)*a*b^2*c^2*d^5 + 73*(d*x +
c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^
7)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*((d*x + c)*b -
b*c + a*d)^4)
```

maple [A] time = 0.02, size = 248, normalized size = 1.36

$$\frac{5\,(dx+c)^{\frac{7}{2}}\,b^2d^4}{64\,(bdx+ad)^4\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{55\,(dx+c)^{\frac{5}{2}}\,b\,d^4}{192\,(bdx+ad)^4\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{5d^4\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{64\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\sqrt{(ad-bc)\,b}\;b}+\frac{73\,(dx+c)^{\frac{3}{2}}\,d^4}{192\,(bdx+ad)^4\,(ad-bc)}-\frac{5\sqrt{dx+c}\;d^4}{64\,(bdx+ad)^4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d
*x+c)^(7/2)+55/192*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^
(5/2)+73/192*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(3/2)-5/64*d^4/(b*d*x+a*d)
^4/b*(d*x+c)^(1/2)+5/64*d^4/b/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.22, size = 297, normalized size = 1.63

$$\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{3/2}}{192\,\left(a\,d-b\,c\right)}-\frac{5\,d^4\,\sqrt{c+d\,x}}{64\,b}+\frac{5\,b^2\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^3}+\frac{55\,b\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^5,x)
```

```
[Out] ((73*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (5*d^4*(c + d*x)^(1/2))/(64*b
) + (5*b^2*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^3) + (55*b*d^4*(c + d*x)^(5
/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x
)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*
d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*
c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3) + (5*d^4*atan((b^
(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(3/2)*(a*d - b*c)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.1280 $\int \frac{\sqrt{c+dx}}{(a+bx)^6} dx$

Optimal. Leaf size=218

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c}}{40b(a+bx}$$

Rubi [A] time = 0.15, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{7d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{3/2}(bc-ad)^{9/2}}+\frac{7d^4\sqrt{c+dx}}{128b(a+bx)(bc-ad)^4}-\frac{7d^3\sqrt{c+dx}}{192b(a+bx)^2(bc-ad)^3}+\frac{7d^2\sqrt{c+dx}}{240b(a+bx)^3(bc-ad)^2}-\frac{d\sqrt{c+dx}}{40b(a+bx)^4(bc-ad)}-\frac{\sqrt{c+dx}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] -Sqrt[c + d*x]/(5*b*(a + b*x)^5) - (d*Sqrt[c + d*x])/(40*b*(b*c - a*d)*(a +
 b*x)^4) + (7*d^2*Sqrt[c + d*x])/(240*b*(b*c - a*d)^2*(a + b*x)^3) - (7*d^3
*Sqrt[c + d*x])/(192*b*(b*c - a*d)^3*(a + b*x)^2) + (7*d^4*Sqrt[c + d*x])/(
128*b*(b*c - a*d)^4*(a + b*x)) - (7*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqr
t[b*c - a*d]])/(128*b^(3/2)*(b*c - a*d)^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^6}\,dx &= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} + \frac{d\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx}{10b} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} - \frac{\left(7d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b(bc-ad)} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} + \frac{\left(7d^3\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{96b(bc-ad)^2} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2} \\
&= -\frac{\sqrt{c+dx}}{5b(a+bx)^5} - \frac{d\sqrt{c+dx}}{40b(bc-ad)(a+bx)^4} + \frac{7d^2\sqrt{c+dx}}{240b(bc-ad)^2(a+bx)^3} - \frac{7d^3\sqrt{c+dx}}{192b(bc-ad)^3(a+bx)^2}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.24

$$
\frac{2d^5(c+dx)^{3/2}\,{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(ad-bc)^6}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(3/2)*Hypergeometric2F1[3/2, 6, 5/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(3*(-(b*c) + a*d)^6)
```

IntegrateAlgebraic [A] time = 1.52, size = 317, normalized size = 1.45

$$
\frac{d^5\sqrt{c+dx}\left(105a^4d^4-790a^3bd^3(c+dx)-420a^3bcd^3+630a^2b^2c^2d^2-896a^2b^2d^2(c+dx)^2+2370a^2b^2cd^2(c+dx)-420ab^3c^3d-2370ab^3c^2d(c+dx)-490ab^3d(c+dx)^3+1792ab^3cd(c+dx)^2+105b^4c^4+790b^4c^3(c+dx)-896b^4c^2(c+dx)^2-105b^4(c+dx)^4+490b^4c(c+dx)^3\right)}{1920b(bc-ad)^4(-ad-b(c+dx)+bc)^5}-\frac{7d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{3/2}(bc-ad)^4\sqrt{ad-bc}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^6,x]
```

```
[Out] (d^5*Sqrt[c + d*x]*(105*b^4*c^4 - 420*a*b^3*c^3*d + 630*a^2*b^2*c^2*d^2 - 4
20*a^3*b*c*d^3 + 105*a^4*d^4 + 790*b^4*c^3*(c + d*x) - 2370*a*b^3*c^2*d*(c
+ d*x) + 2370*a^2*b^2*c*d^2*(c + d*x) - 790*a^3*b*d^3*(c + d*x) - 896*b^4*c
^2*(c + d*x)^2 + 1792*a*b^3*c*d*(c + d*x)^2 - 896*a^2*b^2*d^2*(c + d*x)^2 +
 490*b^4*c*(c + d*x)^3 - 490*a*b^3*d*(c + d*x)^3 - 105*b^4*(c + d*x)^4))/(1
920*b*(b*c - a*d)^4*(b*c - a*d - b*(c + d*x))^5) - (7*d^5*ArcTan[(Sqrt[b]*S
qrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(3/2)*(b*c - a*d)^4*S
qrt[-(b*c) + a*d])
```

fricas [B] time = 1.34, size = 1673, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/3840*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(384*b^6*c^5
- 1872*a*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4
*b^2*c*d^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*
d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2
*d^3 + 87*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5
*c^3*d^2 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sq
rt(d*x + c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b
^4*c^2*d^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d +
10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x
^5 + 5*(a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2
*d^3 + 5*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^
4*d + 10*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d
^5)*x^3 + 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b
^6*c^2*d^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^
7*c^4*d + 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b
^3*d^5)*x), 1/1920*(105*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3
 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)*arcta
n(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (384*b^6*c^5 - 1872*a
*b^5*c^4*d + 3592*a^2*b^4*c^3*d^2 - 3314*a^3*b^3*c^2*d^3 + 1315*a^4*b^2*c*d
^4 - 105*a^5*b*d^5 - 105*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 70*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 - 14*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 8
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(24*b^6*c^4*d - 152*a*b^5*c^3*d^2
 + 417*a^2*b^4*c^2*d^3 - 684*a^3*b^3*c*d^4 + 395*a^4*b^2*d^5)*x)*sqrt(d*x +
 c))/(a^5*b^7*c^5 - 5*a^6*b^6*c^4*d + 10*a^7*b^5*c^3*d^2 - 10*a^8*b^4*c^2*d
^3 + 5*a^9*b^3*c*d^4 - a^10*b^2*d^5 + (b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b
^10*c^3*d^2 - 10*a^3*b^9*c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*x^5 + 5*(
a*b^11*c^5 - 5*a^2*b^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5
*a^5*b^7*c*d^4 - a^6*b^6*d^5)*x^4 + 10*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10
*a^4*b^8*c^3*d^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*x^3
+ 10*(a^3*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d
^3 + 5*a^7*b^5*c*d^4 - a^8*b^4*d^5)*x^2 + 5*(a^4*b^8*c^5 - 5*a^5*b^7*c^4*d
+ 10*a^6*b^6*c^3*d^2 - 10*a^7*b^5*c^2*d^3 + 5*a^8*b^4*c*d^4 - a^9*b^3*d^5)*
x)]
```

giac [B] time = 1.47, size = 432, normalized size = 1.98

$$\frac{7\,d^5\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{128\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{9}{2}}b^4d^5-490\,(dx+c)^{\frac{7}{2}}b^4cd^5+896\,(dx+c)^{\frac{5}{2}}b^4c^2d^5-790\,(dx+c)^{\frac{3}{2}}b^4c^3d^5-105\,\sqrt{dx+c}\,b^4c^4d^5+490\,(dx+c)^{\frac{7}{2}}ab^3d^6-1792\,(dx+c)^{\frac{5}{2}}ab^3cd^6+2370\,(dx+c)^{\frac{3}{2}}ab^3c^2d^6+420\,\sqrt{dx+c}\,ab^3c^3d^6+896\,(dx+c)^{\frac{5}{2}}a^2b^2d^7-2370\,(dx+c)^{\frac{3}{2}}a^2b^2cd^7-630\,\sqrt{dx+c}\,a^2b^2c^2d^7+790\,(dx+c)^{\frac{3}{2}}a^3bd^8+420\,\sqrt{dx+c}\,a^3bcd^8-105\,\sqrt{dx+c}\,a^4d^9}{1920\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)((dx+c)b-bc+ad)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 7/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(-b^2*c + a*b*
d)) + 1/1920*(105*(d*x + c)^(9/2)*b^4*d^5 - 490*(d*x + c)^(7/2)*b^4*c*d^5 +
 896*(d*x + c)^(5/2)*b^4*c^2*d^5 - 790*(d*x + c)^(3/2)*b^4*c^3*d^5 - 105*sq
rt(d*x + c)*b^4*c^4*d^5 + 490*(d*x + c)^(7/2)*a*b^3*d^6 - 1792*(d*x + c)^(5
/2)*a*b^3*c*d^6 + 2370*(d*x + c)^(3/2)*a*b^3*c^2*d^6 + 420*sqrt(d*x + c)*a*
b^3*c^3*d^6 + 896*(d*x + c)^(5/2)*a^2*b^2*d^7 - 2370*(d*x + c)^(3/2)*a^2*b^
2*c*d^7 - 630*sqrt(d*x + c)*a^2*b^2*c^2*d^7 + 790*(d*x + c)^(3/2)*a^3*b*d^8
 + 420*sqrt(d*x + c)*a^3*b*c*d^8 - 105*sqrt(d*x + c)*a^4*d^9)/((b^5*c^4 - 4
*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*((d*x + c)*
b - b*c + a*d)^5)
```

maple [A] time = 0.02, size = 337, normalized size = 1.55

$$\frac{7\,(dx+c)^{\frac{9}{2}}\,b^3d^5}{128\,(bdx+ad)^5\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}+\frac{49\,(dx+c)^{\frac{7}{2}}\,b^2d^5}{192\,(bdx+ad)^5\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{7\,(dx+c)^{\frac{5}{2}}\,b\,d^5}{15\,(bdx+ad)^5\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{7d^5\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)\sqrt{(ad-bc)\,b}\;b}+\frac{79\,(dx+c)^{\frac{3}{2}}\,d^5}{192\,(bdx+ad)^5\,(ad-bc)}-\frac{7\sqrt{dx+c}\;d^5}{128\,(bdx+ad)^5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^6,x)
```

```
[Out] 7/128*d^5/(b*d*x+a*d)^5*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^
3*c^3*d+b^4*c^4)*(d*x+c)^(9/2)+49/192*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(d*x+c)^(7/2)+7/15*d^5/(b*d*x+a*d)^5*b/(a^2*
d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(5/2)+79/192*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*
x+c)^(3/2)-7/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)+7/128*d^5/b/(a^4*d^4-4*a
^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)/((a*d-b*c)*b)^(1/2)*arc
tan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.49, size = 401, normalized size = 1.84

$$\frac{\frac{79\,d^5\,{\left(c+d\,x\right)}^{3/2}}{192\,\left(a\,d-b\,c\right)}-\frac{7\,d^5\,\sqrt{c+d\,x}}{128\,b}+\frac{49\,b^2\,d^5\,{\left(c+d\,x\right)}^{7/2}}{192\,{\left(a\,d-b\,c\right)}^3}+\frac{7\,b^3\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^4}+\frac{7\,b\,d^5\,{\left(c+d\,x\right)}^{5/2}}{15\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{7\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{3/2}\,{\left(a\,d-b\,c\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^6,x)
```

```
[Out] ((79*d^5*(c + d*x)^(3/2))/(192*(a*d - b*c)) - (7*d^5*(c + d*x)^(1/2))/(128*
b) + (49*b^2*d^5*(c + d*x)^(7/2))/(192*(a*d - b*c)^3) + (7*b^3*d^5*(c + d*x
)^(9/2))/(128*(a*d - b*c)^4) + (7*b*d^5*(c + d*x)^(5/2))/(15*(a*d - b*c)^2)
)/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*
c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5
*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)
*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^
4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*
b*c*d^4) + (7*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b
^(3/2)*(a*d - b*c)^(9/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

3.1281 $\int (a+bx)^5(c+dx)^{3/2}\,dx$

Optimal. Leaf size=158

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c$$

Rubi [A] time = 0.05, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{13/2}(bc-ad)}{13d^6}+\frac{20b^3(c+dx)^{11/2}(bc-ad)^2}{11d^6}-\frac{20b^2(c+dx)^{9/2}(bc-ad)^3}{9d^6}+\frac{10b(c+dx)^{7/2}(bc-ad)^4}{7d^6}-\frac{2(c+dx)^{5/2}(bc-ad)^5}{5d^6}+\frac{2b^5(c+dx)^{15/2}}{15d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(5/2))/(5*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(7/2))/(7*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(
b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(13
/2))/(13*d^6) + (2*b^5*(c + d*x)^(15/2))/(15*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{3/2}\,dx=\int\left(\frac{(-bc+ad)^5(c+dx)^{3/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{5/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{7/2}}{d^5}+\frac{1(}{}\right.$$
$$=-\frac{2(bc-ad)^5(c+dx)^{5/2}}{5d^6}+\frac{10b(bc-ad)^4(c+dx)^{7/2}}{7d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{9/2}}{9d^6}+\frac{20b}{}$$

Mathematica [A] time = 0.15, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(-17325b^4(c+dx)^4(bc-ad)+40950b^3(c+dx)^3(bc-ad)^2-50050b^2(c+dx)^2(bc-ad)^3+32175b(c+dx)(bc-ad)^4-9009(bc-ad)^5+3003b^5(c+dx)^5\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-9009*(b*c - a*d)^5 + 32175*b*(b*c - a*d)^4*(c + d*x) -
 50050*b^2*(b*c - a*d)^3*(c + d*x)^2 + 40950*b^3*(b*c - a*d)^2*(c + d*x)^3
- 17325*b^4*(b*c - a*d)*(c + d*x)^4 + 3003*b^5*(c + d*x)^5))/(45045*d^6)
```

IntegrateAlgebraic [A] time = 0.10, size = 315, normalized size = 1.99

$$\frac{2(c+dx)^{5/2}\left(9009a^5d^5+32175a^4bd^4(c+dx)-45045a^4bcd^4+90090a^3b^2c^2d^3+50050a^3b^2d^3(c+dx)^2-128700a^3b^2cd^3(c+dx)-90090a^2b^3c^3d^2+193050a^2b^3c^2d^2(c+dx)+40950a^2b^3d^2(c+dx)^3-150150a^2b^3cd^2(c+dx)^2+45045ab^4c^4d-128700ab^4c^3d(c+dx)+150150ab^4c^2d(c+dx)^2+17325ab^4d(c+dx)^4-81900ab^4cd(c+dx)^3-9009b^5c^5+32175b^5c^4(c+dx)-50050b^5c^3(c+dx)^2+40950b^5c^2(c+dx)^3+3003b^5(c+dx)^5-17325b^5c(c+dx)^4\right)}{45045d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-9009*b^5*c^5 + 45045*a*b^4*c^4*d - 90090*a^2*b^3*c^3*d
^2 + 90090*a^3*b^2*c^2*d^3 - 45045*a^4*b*c*d^4 + 9009*a^5*d^5 + 32175*b^5*c
```

```
^4*(c + d*x) - 128700*a*b^4*c^3*d*(c + d*x) + 193050*a^2*b^3*c^2*d^2*(c + d
*x) - 128700*a^3*b^2*c*d^3*(c + d*x) + 32175*a^4*b*d^4*(c + d*x) - 50050*b^
5*c^3*(c + d*x)^2 + 150150*a*b^4*c^2*d*(c + d*x)^2 - 150150*a^2*b^3*c*d^2*(
c + d*x)^2 + 50050*a^3*b^2*d^3*(c + d*x)^2 + 40950*b^5*c^2*(c + d*x)^3 - 81
900*a*b^4*c*d*(c + d*x)^3 + 40950*a^2*b^3*d^2*(c + d*x)^3 - 17325*b^5*c*(c
+ d*x)^4 + 17325*a*b^4*d*(c + d*x)^4 + 3003*b^5*(c + d*x)^5))/(45045*d^6)
```

fricas [B] time = 1.37, size = 418, normalized size = 2.65

$$\frac{2\left(3003\,b^5d^7x^7-256\,b^5c^7+1920\,ab^4c^6d-6240\,a^2b^3c^5d^2+11440\,a^3b^2c^4d^3-12870\,a^4bc^3d^4+9009\,a^5c^2d^5+231\left(16\,b^5cd^6+75\,ab^4d^7\right)x^6+63\left(b^5c^2d^5+350\,ab^4cd^6+650\,a^2b^3d^7\right)x^5-35\left(2\,b^5c^3d^4-15\,ab^4c^2d^5-1560\,a^2b^3cd^6-1430\,a^3b^2d^7\right)x^4+5\left(16\,b^5c^4d^3-120\,ab^4c^3d^4+390\,a^2b^3c^2d^5+14300\,a^3b^2cd^6+6435\,a^4bd^7\right)x^3-3\left(32\,b^5c^5d^2-240\,ab^4c^4d^3+780\,a^2b^3c^3d^4-1430\,a^3b^2c^2d^5-17160\,a^4bcd^6-3003\,a^5d^7\right)x^2+\left(128\,b^5c^6d-960\,ab^4c^5d^2+3120\,a^2b^3c^4d^3-5720\,a^3b^2c^3d^4+6435\,a^4bc^2d^5+18018\,a^5cd^6\right)x\right)\sqrt{dx+c}}{45045\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^5*d^7*x^7 - 256*b^5*c^7 + 1920*a*b^4*c^6*d - 6240*a^2*b^3*c
^5*d^2 + 11440*a^3*b^2*c^4*d^3 - 12870*a^4*b*c^3*d^4 + 9009*a^5*c^2*d^5 + 2
31*(16*b^5*c*d^6 + 75*a*b^4*d^7)*x^6 + 63*(b^5*c^2*d^5 + 350*a*b^4*c*d^6 +
650*a^2*b^3*d^7)*x^5 - 35*(2*b^5*c^3*d^4 - 15*a*b^4*c^2*d^5 - 1560*a^2*b^3*
c*d^6 - 1430*a^3*b^2*d^7)*x^4 + 5*(16*b^5*c^4*d^3 - 120*a*b^4*c^3*d^4 + 390
*a^2*b^3*c^2*d^5 + 14300*a^3*b^2*c*d^6 + 6435*a^4*b*d^7)*x^3 - 3*(32*b^5*c^
5*d^2 - 240*a*b^4*c^4*d^3 + 780*a^2*b^3*c^3*d^4 - 1430*a^3*b^2*c^2*d^5 - 17
160*a^4*b*c*d^6 - 3003*a^5*d^7)*x^2 + (128*b^5*c^6*d - 960*a*b^4*c^5*d^2 +
3120*a^2*b^3*c^4*d^3 - 5720*a^3*b^2*c^3*d^4 + 6435*a^4*b*c^2*d^5 + 18018*a^
5*c*d^6)*x)*sqrt(d*x + c)/d^6
```

giac [B] time = 1.53, size = 1084, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^5*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^5*c + 75075*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^5 +
 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
3*b^2*c^2/d^2 + 30030*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^4*b*c/d + 12870*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3*c^2/d^3 + 25740*(5*(
d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*
x + c)*c^3)*a^3*b^2*c/d^2 + 6435*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c
+ 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4*b/d + 715*(35*(d*x + c
)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(
3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^4*c^2/d^4 + 2860*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a^2*b^3*c/d^3 + 1430*(35*(d*x + c)^(9/2) - 180*(d
*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*s
qrt(d*x + c)*c^4)*a^3*b^2/d^2 + 65*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/
2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^
(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5*c^2/d^5 + 650*(63*(d*x + c)^(11/2) -
 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3
 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c/d^4 + 650*(63*
(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(
d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*
b^3/d^3 + 30*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x +
c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(
d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c/d^5 + 75*(231*(d*x + c)^
(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c
)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sq
rt(d*x + c)*c^6)*a*b^4/d^4 + 7*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2
)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x +
```

```
 c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 643
5*sqrt(d*x + c)*c^7)*b^5/d^5)/d
```

maple [B] time = 0.01, size = 273, normalized size = 1.73

$$\frac{2 (dx+c)^{\frac{5}{2}} \left(3003 b^5 x^5 d^5+17325 a b^4 d^5 x^4-2310 b^5 c d^4 x^4+40950 a^2 b^3 d^5 x^3-12600 a b^4 c d^4 x^3+1680 b^5 c^2 d^3 x^3+50050 a^3 b^2 d^5 x^2-27300 a^2 b^3 c d^4 x^2+8400 a b^4 c^2 d^3 x^2-1120 b^5 c^3 d^2 x^2+32175 a^4 b d^5 x-28600 a^3 b^2 c d^4 x+15600 a^2 b^3 c^2 d^3 x-4800 a b^4 c^3 d^2 x+640 b^5 c^4 d x+9009 a^5 d^5-12870 a^4 b c d^4+11440 a^3 b^2 c^2 d^3-6240 a^2 b^3 c^3 d^2+1920 a b^4 c^4 d-256 b^5 c^5\right)}{45045 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(3/2),x)
```

```
[Out] 2/45045*(d*x+c)^(5/2)*(3003*b^5*d^5*x^5+17325*a*b^4*d^5*x^4-2310*b^5*c*d^4*
x^4+40950*a^2*b^3*d^5*x^3-12600*a*b^4*c*d^4*x^3+1680*b^5*c^2*d^3*x^3+50050*
a^3*b^2*d^5*x^2-27300*a^2*b^3*c*d^4*x^2+8400*a*b^4*c^2*d^3*x^2-1120*b^5*c^3
*d^2*x^2+32175*a^4*b*d^5*x-28600*a^3*b^2*c*d^4*x+15600*a^2*b^3*c^2*d^3*x-48
00*a*b^4*c^3*d^2*x+640*b^5*c^4*d*x+9009*a^5*d^5-12870*a^4*b*c*d^4+11440*a^3
*b^2*c^2*d^3-6240*a^2*b^3*c^3*d^2+1920*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

maxima [A] time = 1.35, size = 259, normalized size = 1.64

$$\frac{2 \left(3003 (dx+c)^{\frac{15}{2}} b^5-17325 \left(b^5 c-a b^4 d\right)(dx+c)^{\frac{13}{2}}+40950 \left(b^5 c^2-2 a b^4 c d+a^2 b^3 d^2\right)(dx+c)^{\frac{11}{2}}-50050 \left(b^5 c^3-3 a b^4 c^2 d+3 a^2 b^3 c d^2-a^3 b^2 d^3\right)(dx+c)^{\frac{9}{2}}+32175 \left(b^5 c^4-4 a b^4 c^3 d+6 a^2 b^3 c^2 d^2-4 a^3 b^2 c d^3+a^4 b d^4\right)(dx+c)^{\frac{7}{2}}-9009 \left(b^5 c^5-5 a b^4 c^4 d+10 a^2 b^3 c^3 d^2-10 a^3 b^2 c^2 d^3+5 a^4 b c d^4-a^5 d^5\right)(dx+c)^{\frac{5}{2}}\right)}{45045 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^5 - 17325*(b^5*c - a*b^4*d)*(d*x + c)^(13/
2) + 40950*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(11/2) - 50050*(
b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(9/2) +
32175*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*
b*d^4)*(d*x + c)^(7/2) - 9009*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2
 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(5/2))/d^6
```

mupad [B] time = 0.24, size = 137, normalized size = 0.87

$$\frac{2 b^5 (c+d x)^{15/2}}{15 d^6}-\frac{\left(10 b^5 c-10 a b^4 d\right) (c+d x)^{13/2}}{13 d^6}+\frac{2 (a d-b c)^5 (c+d x)^{5/2}}{5 d^6}+\frac{20 b^2 (a d-b c)^3 (c+d x)^{9/2}}{9 d^6}+\frac{20 b^3 (a d-b c)^2 (c+d x)^{11/2}}{11 d^6}+\frac{10 b (a d-b c)^4 (c+d x)^{7/2}}{7 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(15/2))/(15*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(13/
2))/(13*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(5/2))/(5*d^6) + (20*b^2*(a*d - b
*c)^3*(c + d*x)^(9/2))/(9*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(11/2))/(1
1*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^6)
```

sympy [A] time = 26.42, size = 763, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(3/2),x)
```

```
[Out] a**5*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**5*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 10*a**4*b*c*(-c*(c
 + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 10*a**4*b*(c**2*(c + d*x)**(3
/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 20*a**3*b**2*c*
(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**
3 + 20*a**3*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*
c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**2*b**3*c*(-c**3*(c
+ d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c +
 d*x)**(9/2)/9)/d**4 + 20*a**2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c +
d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d
```

```
*x)**(11/2)/11)/d**4 + 10*a*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d
*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*
x)**(11/2)/11)/d**5 + 10*a*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**5*c*(-c**5*(c + d*x)**
(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c +
 d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**6 + 2*
b**5*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*
x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6
*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6
```

3.1282 $\int (a+bx)^4(c+dx)^{3/2}\,dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)}{13d^5}$$

Rubi [A] time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{11/2}(bc-ad)}{11d^5}+\frac{4b^2(c+dx)^{9/2}(bc-ad)^2}{3d^5}-\frac{8b(c+dx)^{7/2}(bc-ad)^3}{7d^5}+\frac{2(c+dx)^{5/2}(bc-ad)^4}{5d^5}+\frac{2b^4(c+dx)^{13/2}}{13d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(5/2))/(5*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(7
/2))/(7*d^5) + (4*b^2*(b*c - a*d)^2*(c + d*x)^(9/2))/(3*d^5) - (8*b^3*(b*c
- a*d)*(c + d*x)^(11/2))/(11*d^5) + (2*b^4*(c + d*x)^(13/2))/(13*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{3/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{5/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{7/2}}{d^4}-4b^3\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{5/2}}{5d^5}-\frac{8b(bc-ad)^3(c+dx)^{7/2}}{7d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{9/2}}{3d^5}-\frac{8b^3(bc}{}\end{aligned}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{5/2}\left(-5460b^3(c+dx)^3(bc-ad)+10010b^2(c+dx)^2(bc-ad)^2-8580b(c+dx)(bc-ad)^3+3003(bc-ad)^4+1155b^4(c+dx)^4\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*(b*c - a*d)^4 - 8580*b*(b*c - a*d)^3*(c + d*x) + 1
0010*b^2*(b*c - a*d)^2*(c + d*x)^2 - 5460*b^3*(b*c - a*d)*(c + d*x)^3 + 115
5*b^4*(c + d*x)^4))/(15015*d^5)
```

IntegrateAlgebraic [A] time = 0.07, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{5/2}\left(3003a^4d^4+8580a^3bd^3(c+dx)-12012a^3bcd^3+18018a^2b^2c^2d^2+10010a^2b^2d^2(c+dx)^2-25740a^2b^2cd^2(c+dx)-12012ab^3c^3d+25740ab^3c^2d(c+dx)+5460ab^3d(c+dx)^3-20020ab^3cd(c+dx)^2+3003b^4c^4-8580b^4c^3(c+dx)+10010b^4c^2(c+dx)^2+1155b^4(c+dx)^4-5460b^4c(c+dx)^3\right)}{15015d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(3003*b^4*c^4 - 12012*a*b^3*c^3*d + 18018*a^2*b^2*c^2*d^
2 - 12012*a^3*b*c*d^3 + 3003*a^4*d^4 - 8580*b^4*c^3*(c + d*x) + 25740*a*b^3
```

```
*c^2*d*(c + d*x) - 25740*a^2*b^2*c*d^2*(c + d*x) + 8580*a^3*b*d^3*(c + d*x)
 + 10010*b^4*c^2*(c + d*x)^2 - 20020*a*b^3*c*d*(c + d*x)^2 + 10010*a^2*b^2*
d^2*(c + d*x)^2 - 5460*b^4*c*(c + d*x)^3 + 5460*a*b^3*d*(c + d*x)^3 + 1155*
b^4*(c + d*x)^4))/(15015*d^5)
```

fricas [B] time = 1.07, size = 311, normalized size = 2.41

$$\frac{2\left(1155\,b^4d^6x^6+128\,b^4c^6-832\,ab^3c^5d+2288\,a^2b^2c^4d^2-3432\,a^3bc^3d^3+3003\,a^4c^2d^4+210\left(7\,b^4cd^5+26\,ab^3d^6\right)x^5+35\left(b^4c^2d^4+208\,ab^3cd^5+286\,a^2b^2d^6\right)x^4-20\left(2\,b^4c^3d^3-13\,ab^3c^2d^4-715\,a^2b^2cd^5-429\,a^3bd^6\right)x^3+3\left(16\,b^4c^4d^2-104\,ab^3c^3d^3+286\,a^2b^2c^2d^4+4576\,a^3bcd^5+1001\,a^4d^6\right)x^2-2\left(32\,b^4c^5d-208\,ab^3c^4d^2+572\,a^2b^2c^3d^3-858\,a^3bc^2d^4-3003\,a^4cd^5\right)x\right)\sqrt{dx+c}}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15015*(1155*b^4*d^6*x^6 + 128*b^4*c^6 - 832*a*b^3*c^5*d + 2288*a^2*b^2*c^
4*d^2 - 3432*a^3*b*c^3*d^3 + 3003*a^4*c^2*d^4 + 210*(7*b^4*c*d^5 + 26*a*b^3
*d^6)*x^5 + 35*(b^4*c^2*d^4 + 208*a*b^3*c*d^5 + 286*a^2*b^2*d^6)*x^4 - 20*(
2*b^4*c^3*d^3 - 13*a*b^3*c^2*d^4 - 715*a^2*b^2*c*d^5 - 429*a^3*b*d^6)*x^3 +
 3*(16*b^4*c^4*d^2 - 104*a*b^3*c^3*d^3 + 286*a^2*b^2*c^2*d^4 + 4576*a^3*b*c
*d^5 + 1001*a^4*d^6)*x^2 - 2*(32*b^4*c^5*d - 208*a*b^3*c^4*d^2 + 572*a^2*b^
2*c^3*d^3 - 858*a^3*b*c^2*d^4 - 3003*a^4*c*d^5)*x)*sqrt(d*x + c)/d^5
```

giac [B] time = 1.40, size = 807, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^2 + 30030*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^2/d + 3
003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4 +
 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^
2*b^2*c^2/d^2 + 24024*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^3*b*c/d + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3*c^2/d^3 + 15444*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x +
 c)*c^3)*a^2*b^2*c/d^2 + 5148*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 3
5*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b/d + 143*(35*(d*x + c)^(
9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2
)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^2/d^4 + 1144*(35*(d*x + c)^(9/2) - 180
*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 31
5*sqrt(d*x + c)*c^4)*a*b^3*c/d^3 + 858*(35*(d*x + c)^(9/2) - 180*(d*x + c)^
(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
+ c)*c^4)*a^2*b^2/d^2 + 130*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*b^4*c/d^4 + 260*(63*(d*x + c)^(11/2) - 385*(d*x
 + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(
d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^3/d^3 + 15*(231*(d*x + c)^(
13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)
^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqr
t(d*x + c)*c^6)*b^4/d^4)/d
```

maple [A] time = 0.01, size = 186, normalized size = 1.44

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}\left(1155b^4x^4d^4+5460a\,b^3d^4x^3-840b^4c\,d^3x^3+10010a^2b^2d^4x^2-3640a\,b^3c\,d^3x^2+560b^4c^2d^2x^2+8580a^3b\,d^4x-5720a^2b^2c\,d^3x+2080a\,b^3c^2d^2x-320b^4c^3dx+3003a^4d^4-3432a^3bc\,d^3+2288a^2b^2c^2d^2-832a\,b^3c^3d+128b^4c^4\right)}{15015d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(3/2),x)
```

```
[Out] 2/15015*(d*x+c)^(5/2)*(1155*b^4*d^4*x^4+5460*a*b^3*d^4*x^3-840*b^4*c*d^3*x^
3+10010*a^2*b^2*d^4*x^2-3640*a*b^3*c*d^3*x^2+560*b^4*c^2*d^2*x^2+8580*a^3*b
```

```
*d^4*x-5720*a^2*b^2*c*d^3*x+2080*a*b^3*c^2*d^2*x-320*b^4*c^3*d*x+3003*a^4*d
^4-3432*a^3*b*c*d^3+2288*a^2*b^2*c^2*d^2-832*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

maxima [A] time = 1.36, size = 181, normalized size = 1.40

$$\frac{2\left(1155\,(dx+c)^{\frac{13}{2}}b^4 - 5460\left(b^4c - ab^3d\right)(dx+c)^{\frac{11}{2}} + 10010\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)(dx+c)^{\frac{9}{2}} - 8580\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)(dx+c)^{\frac{7}{2}} + 3003\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)(dx+c)^{\frac{5}{2}}\right)}{15015\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*(d*x + c)^(13/2)*b^4 - 5460*(b^4*c - a*b^3*d)*(d*x + c)^(11/2
) + 10010*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(9/2) - 8580*(b^4
*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(7/2) + 3003*
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*
x + c)^(5/2))/d^5
```

mupad [B] time = 0.24, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(13/2))/(13*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(11/2)
)/(11*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(5/2))/(5*d^5) + (4*b^2*(a*d - b*c)
^2*(c + d*x)^(9/2))/(3*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^5)
```

sympy [A] time = 20.09, size = 559, normalized size = 4.33

$$a^4c\left(\begin{cases}\sqrt{c}\,x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^4\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{8a^3bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{8a^3b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{12a^2b^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{12a^2b^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{8ab^3c\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}+\frac{8ab^3\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^4}+\frac{2b^4c\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}+\frac{2b^4\left(-\frac{c^5(c+dx)^{\frac{3}{2}}}{3}+c^4\,(c+dx)^{\frac{5}{2}}-\frac{10c^3(c+dx)^{\frac{7}{2}}}{7}+\frac{10c^2(c+dx)^{\frac{9}{2}}}{9}-\frac{5c(c+dx)^{\frac{11}{2}}}{11}+\frac{(c+dx)^{\frac{13}{2}}}{13}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(3/2),x)
```

```
[Out] a**4*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**4*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 8*a**3*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 8*a**3*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 12*a**2*b**2*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3
+ 12*a**2*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*
(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 8*a*b**3*c*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 8*a*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2
)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2
)/11)/d**4 + 2*b**4*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
+ 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11
)/d**5 + 2*b**4*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3
*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11
 + (c + d*x)**(13/2)/13)/d**5
```

3.1283 $\int (a+bx)^3 (c+dx)^{3/2} \, dx$

Optimal. Leaf size=100

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^2(c+dx)^{9/2}(bc-ad)}{3d^4}+\frac{6b(c+dx)^{7/2}(bc-ad)^2}{7d^4}-\frac{2(c+dx)^{5/2}(bc-ad)^3}{5d^4}+\frac{2b^3(c+dx)^{11/2}}{11d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(b*c - a*d)^2*(c + d*x)^(
7/2))/(7*d^4) - (2*b^2*(b*c - a*d)*(c + d*x)^(9/2))/(3*d^4) + (2*b^3*(c + d
*x)^(11/2))/(11*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{3/2}\,dx = \int \left(\frac{(-bc+ad)^3(c+dx)^{3/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{5/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{7/2}}{d^3}+\right.$$

$$= -\frac{2(bc-ad)^3(c+dx)^{5/2}}{5d^4}+\frac{6b(bc-ad)^2(c+dx)^{7/2}}{7d^4}-\frac{2b^2(bc-ad)(c+dx)^{9/2}}{3d^4}+$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{5/2}\left(-385b^2(c+dx)^2(bc-ad)+495b(c+dx)(bc-ad)^2-231(bc-ad)^3+105b^3(c+dx)^3\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-231*(b*c - a*d)^3 + 495*b*(b*c - a*d)^2*(c + d*x) - 38
5*b^2*(b*c - a*d)*(c + d*x)^2 + 105*b^3*(c + d*x)^3))/(1155*d^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+495a^2bd^2(c+dx)-693a^2bcd^2+693ab^2c^2d+385ab^2d(c+dx)^2-990ab^2cd(c+dx)-231b^3c^3+495b^3c^2(c+dx)+105b^3(c+dx)^3-385b^3c(c+dx)^2\right)}{1155d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-231*b^3*c^3 + 693*a*b^2*c^2*d - 693*a^2*b*c*d^2 + 231*
a^3*d^3 + 495*b^3*c^2*(c + d*x) - 990*a*b^2*c*d*(c + d*x) + 495*a^2*b*d^2*(
```

```
c + d*x) - 385*b^3*c*(c + d*x)^2 + 385*a*b^2*d*(c + d*x)^2 + 105*b^3*(c + d
*x)^3))/(1155*d^4)
```

fricas [B] time = 0.78, size = 216, normalized size = 2.16

$$\frac{2\left(105 b^3 d^5 x^5 - 16 b^3 c^5 + 88 a b^2 c^4 d - 198 a^2 b c^3 d^2 + 231 a^3 c^2 d^3 + 35\left(4 b^3 c d^4 + 11 a b^2 d^5\right)x^4 + 5\left(b^3 c^2 d^3 + 110 a b^2 c d^4 + 99 a^2 b d^5\right)x^3 - 3\left(2 b^3 c^3 d^2 - 11 a b^2 c^2 d^3 - 264 a^2 b c d^4 - 77 a^3 d^5\right)x^2 + \left(8 b^3 c^4 d - 44 a b^2 c^3 d^2 + 99 a^2 b c^2 d^3 + 462 a^3 c d^4\right)x\right)\sqrt{dx+c}}{1155 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^3*d^5*x^5 - 16*b^3*c^5 + 88*a*b^2*c^4*d - 198*a^2*b*c^3*d^2 +
 231*a^3*c^2*d^3 + 35*(4*b^3*c*d^4 + 11*a*b^2*d^5)*x^4 + 5*(b^3*c^2*d^3 + 1
10*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - 3*(2*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3 -
264*a^2*b*c*d^4 - 77*a^3*d^5)*x^2 + (8*b^3*c^4*d - 44*a*b^2*c^3*d^2 + 99*a^
2*b*c^2*d^3 + 462*a^3*c*d^4)*x)*sqrt(d*x + c)/d^4
```

giac [B] time = 1.31, size = 566, normalized size = 5.66

$$\frac{2\left(3465\sqrt{dx+c}\,a^3c^2 + 2310\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a^3c + \frac{3465\left((dx+c)^{\frac{3}{2}} - 3\sqrt{dx+c}\,c\right)a^2bc^2}{d} + 231\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)a^3 + \frac{693\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)ab^2c^2}{d^2} + \frac{1386\left(3(dx+c)^{\frac{5}{2}} - 10(dx+c)^{\frac{3}{2}}c + 15\sqrt{dx+c}\,c^2\right)a^2bc}{d} + \frac{99\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)b^3c^2}{d^3} + \frac{594\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)ab^2c}{d^2} + \frac{297\left(5(dx+c)^{\frac{7}{2}} - 21(dx+c)^{\frac{5}{2}}c + 35(dx+c)^{\frac{3}{2}}c^2 - 35\sqrt{dx+c}\,c^3\right)a^2b}{d} + \frac{22\left(35(dx+c)^{\frac{9}{2}} - 180(dx+c)^{\frac{7}{2}}c + 378(dx+c)^{\frac{5}{2}}c^2 - 420(dx+c)^{\frac{3}{2}}c^3 + 315\sqrt{dx+c}\,c^4\right)b^3c}{d^3} + \frac{33\left(35(dx+c)^{\frac{9}{2}} - 180(dx+c)^{\frac{7}{2}}c + 378(dx+c)^{\frac{5}{2}}c^2 - 420(dx+c)^{\frac{3}{2}}c^3 + 315\sqrt{dx+c}\,c^4\right)ab^2}{d^2} + \frac{5\left(63(dx+c)^{\frac{11}{2}} - 385(dx+c)^{\frac{9}{2}}c + 990(dx+c)^{\frac{7}{2}}c^2 - 1386(dx+c)^{\frac{5}{2}}c^3 + 1155(dx+c)^{\frac{3}{2}}c^4 - 693\sqrt{dx+c}\,c^5\right)b^3}{d^3}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^3*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^3*c + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^2/d + 231*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3 + 693*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2*c^2
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b*c/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3*c^2/d^3 + 594*(5*(d*x + c)^(7/2) - 21
*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c
/d^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c
^2 - 35*sqrt(d*x + c)*c^3)*a^2*b/d + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)
^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x
 + c)*c^4)*b^3*c/d^3 + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378
*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b
^2/d^2 + 5*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/
2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x
 + c)*c^5)*b^3/d^3)/d
```

maple [A] time = 0.01, size = 116, normalized size = 1.16

$$\frac{2(dx+c)^{\frac{5}{2}}\left(105b^3x^3d^3 + 385a\,b^2d^3x^2 - 70b^3c\,d^2x^2 + 495a^2b\,d^3x - 220a\,b^2c\,d^2x + 40b^3c^2dx + 231a^3d^3 - 198a^2bc\,d^2 + 88a\,b^2c^2d - 16b^3c^3\right)}{1155d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(3/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(105*b^3*d^3*x^3+385*a*b^2*d^3*x^2-70*b^3*c*d^2*x^2+49
5*a^2*b*d^3*x-220*a*b^2*c*d^2*x+40*b^3*c^2*d*x+231*a^3*d^3-198*a^2*b*c*d^2+
88*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

maxima [A] time = 1.36, size = 118, normalized size = 1.18

$$\frac{2\left(105(dx+c)^{\frac{11}{2}}b^3 - 385\left(b^3c - ab^2d\right)(dx+c)^{\frac{9}{2}} + 495\left(b^3c^2 - 2ab^2cd + a^2bd^2\right)(dx+c)^{\frac{7}{2}} - 231\left(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3\right)(dx+c)^{\frac{5}{2}}\right)}{1155\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*(d*x + c)^(11/2)*b^3 - 385*(b^3*c - a*b^2*d)*(d*x + c)^(9/2) +
495*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(7/2) - 231*(b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(5/2))/d^4
```

mupad [B] time = 0.25, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{6\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(11/2))/(11*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(9/2))
/(9*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(5/2))/(5*d^4) + (6*b*(a*d - b*c)^2*(
c + d*x)^(7/2))/(7*d^4)
```

sympy [A] time = 14.38, size = 386, normalized size = 3.86

$$a^3c\left(\begin{cases}\sqrt{c}\,x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^3\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{6a^2bc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}+\frac{6a^2b\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2}+\frac{6ab^2c\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}+\frac{6ab^2\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^3}+\frac{2b^3c\left(-\frac{c^3(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^2(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}+\frac{2b^3\left(\frac{c^4(c+dx)^{\frac{3}{2}}}{3}-\frac{4c^3(c+dx)^{\frac{5}{2}}}{5}+\frac{6c^2(c+dx)^{\frac{7}{2}}}{7}-\frac{4c(c+dx)^{\frac{9}{2}}}{9}+\frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(3/2),x)
```

```
[Out] a**3*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**3*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 6*a**2*b*c*(-c*(c
+ d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 6*a**2*b*(c**2*(c + d*x)**(3/2
)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 6*a*b**2*c*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 6*
a*b**2*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x
)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**3*c*(-c**3*(c + d*x)**(3/2)/3
+ 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/
d**4 + 2*b**3*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2
*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**4
```

3.1284 $\int (a+bx)^2(c+dx)^{3/2}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{7/2}(bc-ad)}{7d^3}+\frac{2(c+dx)^{5/2}(bc-ad)^2}{5d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(7/2
))/(7*d^3) + (2*b^2*(c + d*x)^(9/2))/(9*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{3/2}\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^{3/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{5/2}}{d^2}+\frac{b^2(c+dx)^{7/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2(c+dx)^{5/2}}{5d^3}-\frac{4b(bc-ad)(c+dx)^{7/2}}{7d^3}+\frac{2b^2(c+dx)^{9/2}}{9d^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(5dx-2c)+b^2\left(8c^2-20cdx+35d^2x^2\right)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-2*c + 5*d*x) + b^2*(8*c^2 - 20*
c*d*x + 35*d^2*x^2)))/(315*d^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{5/2}\left(63a^2d^2+90abd(c+dx)-126abcd+63b^2c^2+35b^2(c+dx)^2-90b^2c(c+dx)\right)}{315d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(63*b^2*c^2 - 126*a*b*c*d + 63*a^2*d^2 - 90*b^2*c*(c + d
*x) + 90*a*b*d*(c + d*x) + 35*b^2*(c + d*x)^2))/(315*d^3)
```

fricas [B] time = 1.24, size = 137, normalized size = 1.93

$$\frac{2\left(35\,b^{2}d^{4}x^{4}+8\,b^{2}c^{4}-36\,abc^{3}d+63\,a^{2}c^{2}d^{2}+10\left(5\,b^{2}cd^{3}+9\,abd^{4}\right)x^{3}+3\left(b^{2}c^{2}d^{2}+48\,abcd^{3}+21\,a^{2}d^{4}\right)x^{2}-2\left(2\,b^{2}c^{3}d-9\,abc^{2}d^{2}-63\,a^{2}cd^{3}\right)x\right)\sqrt{dx+c}}{315\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^2*d^4*x^4 + 8*b^2*c^4 - 36*a*b*c^3*d + 63*a^2*c^2*d^2 + 10*(5*b
^2*c*d^3 + 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 + 48*a*b*c*d^3 + 21*a^2*d^4)*x^2
 - 2*(2*b^2*c^3*d - 9*a*b*c^2*d^2 - 63*a^2*c*d^3)*x)*sqrt(d*x + c)/d^3
```

giac [B] time = 1.41, size = 360, normalized size = 5.07

$$\frac{2\left(315\,\sqrt{dx+c}\,a^{2}c^{2}+210\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{2}c+\frac{210\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc^{2}}{d}+21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)a^{2}+\frac{21\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)b^{2}c^{2}}{d^{2}}+\frac{84\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)abc}{d}+\frac{18\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)b^{2}c}{d^{2}}+\frac{18\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)ab}{d}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\,\sqrt{dx+c}\,c^{4}\right)b^{2}}{d^{2}}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^2*c^2 + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c
)*a^2*c + 210*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^2/d + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2 + 21*(3*(d*x
+ c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^2/d^2 + 84*
(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d +
 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35
*sqrt(d*x + c)*c^3)*b^2*c/d^2 + 18*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*
c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b/d + (35*(d*x + c)^(9
/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)
*c^3 + 315*sqrt(d*x + c)*c^4)*b^2/d^2)/d
```

maple [A] time = 0.01, size = 63, normalized size = 0.89

$$\frac{2\,(dx+c)^{\frac{5}{2}}\left(35b^{2}x^{2}d^{2}+90ab\,d^{2}x-20b^{2}cdx+63a^{2}d^{2}-36abcd+8b^{2}c^{2}\right)}{315d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(3/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(35*b^2*d^2*x^2+90*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-36
*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.38, size = 68, normalized size = 0.96

$$\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^{2}-90\left(b^{2}c-abd\right)(dx+c)^{\frac{7}{2}}+63\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(dx+c)^{\frac{5}{2}}\right)}{315\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*(d*x + c)^(9/2)*b^2 - 90*(b^2*c - a*b*d)*(d*x + c)^(7/2) + 63*(b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(5/2))/d^3
```

mupad [B] time = 0.06, size = 68, normalized size = 0.96

$$\frac{2\,(c+d\,x)^{5/2}\,\left(35\,b^2\,(c+d\,x)^2+63\,a^2\,d^2+63\,b^2\,c^2-90\,b^2\,c\,(c+d\,x)+90\,a\,b\,d\,(c+d\,x)-126\,a\,b\,c\,d\right)}{315\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(35*b^2*(c + d*x)^2 + 63*a^2*d^2 + 63*b^2*c^2 - 90*b^2*c
*(c + d*x) + 90*a*b*d*(c + d*x) - 126*a*b*c*d))/(315*d^3)
```

sympy [A] time = 9.61, size = 240, normalized size = 3.38

$$a^{2}c\left(\begin{cases}\sqrt{c}\,x & \text{for}\ d=0\\ \frac{2(c+dx)^{\frac{3}{2}}}{3d} & \text{otherwise}\end{cases}\right)+\frac{2a^{2}\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d}+\frac{4abc\left(-\frac{c(c+dx)^{\frac{3}{2}}}{3}+\frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^{2}}+\frac{4ab\left(\frac{c^{2}(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{2}}+\frac{2b^{2}c\left(\frac{c^{2}(c+dx)^{\frac{3}{2}}}{3}-\frac{2c(c+dx)^{\frac{5}{2}}}{5}+\frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^{3}}+\frac{2b^{2}\left(-\frac{c^{3}(c+dx)^{\frac{3}{2}}}{3}+\frac{3c^{2}(c+dx)^{\frac{5}{2}}}{5}-\frac{3c(c+dx)^{\frac{7}{2}}}{7}+\frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(3/2),x)
```

```
[Out] a**2*c*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)) +
 2*a**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 4*a*b*c*(-c*(c + d
*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 -
2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*c*(c**2*(c + d*x
)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 2*b**2*(-c
**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7
 + (c + d*x)**(9/2)/9)/d**3
```

3.1285 $\int (a+bx)(c+dx)^{3/2}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{7/2}}{7d^2}-\frac{2(c+dx)^{5/2}(bc-ad)}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^2) + (2*b*(c + d*x)^(7/2))/(7*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{3/2}\,dx&=\int\left(\frac{(-bc+ad)(c+dx)^{3/2}}{d}+\frac{b(c+dx)^{5/2}}{d}\right)dx\\&=-\frac{2(bc-ad)(c+dx)^{5/2}}{5d^2}+\frac{2b(c+dx)^{7/2}}{7d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{5/2}(7ad-2bc+5bdx)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-2*b*c + 7*a*d + 5*b*d*x))/(35*d^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{5/2}(7ad+5b(c+dx)-7bc)}{35d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^(3/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-7*b*c + 7*a*d + 5*b*(c + d*x)))/(35*d^2)
```

fricas [B] time = 1.27, size = 69, normalized size = 1.64

$$\frac{2\left(5\,bd^3x^3-2\,bc^3+7\,ac^2d+\left(8\,bcd^2+7\,ad^3\right)x^2+\left(bc^2d+14\,acd^2\right)x\right)\sqrt{dx+c}}{35\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b*d^3*x^3 - 2*b*c^3 + 7*a*c^2*d + (8*b*c*d^2 + 7*a*d^3)*x^2 + (b*c^
2*d + 14*a*c*d^2)*x)*sqrt(d*x + c)/d^2
```

giac [B] time = 1.23, size = 192, normalized size = 4.57

$$\frac{2\left(105\,\sqrt{dx+c}\,ac^{2}+70\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ac+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)bc^{2}}{d}+7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)a+\frac{14\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)bc}{d}+\frac{3\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)b}{d}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*sqrt(d*x + c)*a*c^2 + 70*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
*c + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^2/d + 7*(3*(d*x + c)^(5/2
) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a + 14*(3*(d*x + c)^(5/2)
- 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c/d + 3*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b/
d)/d
```

maple [A] time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{5}{2}}\,(5bdx+7ad-2bc)}{35d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(3/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(5*b*d*x+7*a*d-2*b*c)/d^2
```

maxima [A] time = 1.37, size = 33, normalized size = 0.79

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}b-7\,(bc-ad)(dx+c)^{\frac{5}{2}}\right)}{35\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2)*b - 7*(b*c - a*d)*(d*x + c)^(5/2))/d^2
```

mupad [B] time = 0.21, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}\,\left(7\,a\,d-7\,b\,c+5\,b\,\left(c+d\,x\right)\right)}{35\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] (2*(c + d*x)^(5/2)*(7*a*d - 7*b*c + 5*b*(c + d*x)))/(35*d^2)
```

sympy [A] time = 0.67, size = 146, normalized size = 3.48

$$\begin{cases} \frac{2ac^{2}\sqrt{c+dx}}{5d}+\frac{4acx\sqrt{c+dx}}{5}+\frac{2adx^{2}\sqrt{c+dx}}{5}-\frac{4bc^{3}\sqrt{c+dx}}{35d^{2}}+\frac{2bc^{2}x\sqrt{c+dx}}{35d}+\frac{16bcx^{2}\sqrt{c+dx}}{35}+\frac{2bdx^{3}\sqrt{c+dx}}{7} & \text{for}\ d\neq 0\\ c^{\frac{3}{2}}\left(ax+\frac{bx^{2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((2*a*c**2*sqrt(c + d*x)/(5*d) + 4*a*c*x*sqrt(c + d*x)/5 + 2*a*d*x
**2*sqrt(c + d*x)/5 - 4*b*c**3*sqrt(c + d*x)/(35*d**2) + 2*b*c**2*x*sqrt(c
+ d*x)/(35*d) + 16*b*c*x**2*sqrt(c + d*x)/35 + 2*b*d*x**3*sqrt(c + d*x)/7,
Ne(d, 0)), (c**(3/2)*(a*x + b*x**2/2), True))
```

3.1286 $\int (c+dx)^{3/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{5/2}}{5d}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2),x]
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{3/2}\,dx = \frac{2(c+dx)^{5/2}}{5d}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2),x]
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2),x]
[Out] (2*(c + d*x)^(5/2))/(5*d)
```

fricas [B] time = 1.20, size = 28, normalized size = 1.75

$$\frac{2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{dx+c}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2),x, algorithm="fricas")
```

[Out] 2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(d*x + c)/d

giac [B] time = 1.24, size = 58, normalized size = 3.62

$$\frac{2\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+30\,\sqrt{dx+c}\,c^2+10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2),x, algorithm="giac")

[Out] 2/15*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 30*sqrt(d*x + c)*c^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c)/d

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(3/2),x)

[Out] 2/5*(d*x+c)^(5/2)/d

maxima [A] time = 1.36, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{5}{2}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^(3/2),x, algorithm="maxima")

[Out] 2/5*(d*x + c)^(5/2)/d

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^(3/2),x)

[Out] (2*(c + d*x)^(5/2))/(5*d)

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{5}{2}}}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)**(3/2),x)

[Out] 2*(c + d*x)**(5/2)/(5*d)

3.1287 $\int \frac{(c+dx)^{3/2}}{a+bx}\,dx$

Optimal. Leaf size=86

$$-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}+\frac{2(c+dx)^{3/2}}{3b}$$

Rubi [A] time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}\,(bc-ad)}{b^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}+\frac{2(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)*Sqrt[c + d*x])/b^2 + (2*(c + d*x)^(3/2))/(3*b) - (2*(b*c - a
*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{a+bx}\,dx &= \frac{2(c+dx)^{3/2}}{3b}+\frac{(bc-ad)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}+\frac{(bc-ad)^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^2}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}+\frac{\left(2(bc-ad)^2\right)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{b^2d}\\ &= \frac{2(bc-ad)\sqrt{c+dx}}{b^2}+\frac{2(c+dx)^{3/2}}{3b}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 0.90

$$\frac{2\sqrt{c+dx}\,(-3ad+4bc+bdx)}{3b^2}-\frac{2(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(4*b*c - 3*a*d + b*d*x))/(3*b^2) - (2*(b*c - a*d)^(3/2)*Ar
cTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

IntegrateAlgebraic [A] time = 0.12, size = 90, normalized size = 1.05

$$\frac{2\sqrt{c+dx}\,(-3ad+b(c+dx)+3bc)}{3b^2}-\frac{2(ad-bc)^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(3*b*c - 3*a*d + b*(c + d*x)))/(3*b^2) - (2*(-(b*c) + a*d)
^(3/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(5
/2)
```

fricas [A] time = 1.14, size = 188, normalized size = 2.19

$$\left[-\frac{3\,(bc-ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad+2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\,(bdx+4\,bc-3\,ad)\sqrt{dx+c}}{3\,b^2},\ -\frac{2\left(3\,(bc-ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(bdx+4\,bc-3\,ad)\sqrt{dx+c}\right)}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [-1/3*(3*(b*c - a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d + 2*sqrt(
d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) - 2*(b*d*x + 4*b*c - 3*a*d)*sqrt
(d*x + c))/b^2, -2/3*(3*(b*c - a*d)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x +
 c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (b*d*x + 4*b*c - 3*a*d)*sqrt(d*x
+ c))/b^2]
```

giac [A] time = 1.29, size = 105, normalized size = 1.22

$$\frac{2\left(b^2c^2-2\,abcd+a^2d^2\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}+\frac{2\left((dx+c)^{\frac{3}{2}}b^2+3\,\sqrt{dx+c}\,b^2c-3\,\sqrt{dx+c}\,abd\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*
d))/(sqrt(-b^2*c + a*b*d)*b^2) + 2/3*((d*x + c)^(3/2)*b^2 + 3*sqrt(d*x + c)
*b^2*c - 3*sqrt(d*x + c)*a*b*d)/b^3
```

maple [B] time = 0.01, size = 167, normalized size = 1.94

$$\frac{2a^2d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^2}-\frac{4acd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}+\frac{2c^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}-\frac{2\sqrt{dx+c}\,ad}{b^2}+\frac{2\sqrt{dx+c}\,c}{b}+\frac{2\,(dx+c)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a),x)
```

```
[Out] 2/3*(d*x+c)^(3/2)/b-2/b^2*a*d*(d*x+c)^(1/2)+2/b*(d*x+c)^(1/2)*c+2/b^2/((a*d
-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*d^2-4/b/((a*
d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c*d+2/((a*d-b
*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.07, size = 93, normalized size = 1.08

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{3\,b}-\frac{2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{b^2}+\frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(3/2))/(3*b) - (2*(a*d - b*c)*(c + d*x)^(1/2))/b^2 + (2*atan((
b^(1/2)*(a*d - b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
*(a*d - b*c)^(3/2))/b^(5/2)
```

sympy [A] time = 14.70, size = 82, normalized size = 0.95

$$\frac{2\left(c+dx\right)^{\frac{3}{2}}}{3b}+\frac{\sqrt{c+dx}\,\left(-2ad+2bc\right)}{b^2}+\frac{2\left(ad-bc\right)^2\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^3\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(3/2)/(3*b) + sqrt(c + d*x)*(-2*a*d + 2*b*c)/b**2 + 2*(a*d - b
*c)**2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**3*sqrt((a*d - b*c)/b))
```

3.1288 $\int \frac{(c+dx)^{3/2}}{(a+bx)^2} dx$

Optimal. Leaf size=85

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{3d\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{3d\sqrt{c+dx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/b^2 - (c + d*x)^(3/2)/(b*(a + b*x)) - (3*d*Sqrt[b*c - a
*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx &= -\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3d(bc-ad))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}+\frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{b^2}\\
&= \frac{3d\sqrt{c+dx}}{b^2}-\frac{(c+dx)^{3/2}}{b(a+bx)}-\frac{3d\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [C]   time = 0.01, size = 50, normalized size = 0.59

$$
\frac{2d(c+dx)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (2*d*(c + d*x)^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(5*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A]   time = 0.26, size = 107, normalized size = 1.26

$$
\frac{3d\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{5/2}}+\frac{d\sqrt{c+dx}\,(3ad+2b(c+dx)-3bc)}{b^2(ad+b(c+dx)-bc)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^2,x]
```

```
[Out] (d*Sqrt[c + d*x]*(-3*b*c + 3*a*d + 2*b*(c + d*x)))/(b^2*(-(b*c) + a*d + b*(
c + d*x))) + (3*d*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqr
t[c + d*x])/(b*c - a*d)])/b^(5/2)
```

fricas [A]   time = 1.74, size = 210, normalized size = 2.47

$$
\left[\frac{3\,(bdx+ad)\sqrt{\frac{bc-ad}{b}}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\,(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{2\,\left(b^3x+ab^2\right)},\ -\frac{3\,(bdx+ad)\sqrt{-\frac{bc-ad}{b}}\,\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-(2\,bdx-bc+3\,ad)\sqrt{dx+c}}{b^3x+ab^2}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d*x + a*d)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt
(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(2*b*d*x - b*c + 3*a*d)*sqr
t(d*x + c))/(b^3*x + a*b^2), -(3*(b*d*x + a*d)*sqrt(-(b*c - a*d)/b)*arctan(
-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (2*b*d*x - b*c + 3*a*d
)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

giac [A]   time = 1.30, size = 113, normalized size = 1.33

$$
\frac{2\,\sqrt{dx+c}\,d}{b^2}+\frac{3\,\left(bcd-ad^2\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^2}-\frac{\sqrt{dx+c}\,bcd-\sqrt{dx+c}\,ad^2}{((dx+c)b-bc+ad)b^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d/b^2 + 3*(b*c*d - a*d^2)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*
c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^2) - (sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*a*d^2)/(((d*x + c)*b - b*c + a*d)*b^2)
```

maple [B] time = 0.01, size = 148, normalized size = 1.74

$$-\frac{3a\,d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^2}+\frac{3cd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}+\frac{\sqrt{dx+c}\,a\,d^2}{(bdx+ad)\,b^2}-\frac{\sqrt{dx+c}\,cd}{(bdx+ad)\,b}+\frac{2\sqrt{dx+c}\,d}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^2,x)
```

```
[Out] 2*d*(d*x+c)^(1/2)/b^2+1/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*d^2-d/b*(d*x+c)^(1/
2)/(b*d*x+a*d)*c-3/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*
b)^(1/2)*b)*a*d^2+3*d/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)
*b)^(1/2)*b)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.11, size = 109, normalized size = 1.28

$$\frac{\left(a\,d^2-b\,c\,d\right)\,\sqrt{c+d\,x}}{b^3\,\left(c+d\,x\right)-b^3\,c+a\,b^2\,d}+\frac{2\,d\,\sqrt{c+d\,x}}{b^2}-\frac{3\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^2-b\,c\,d}\right)\,\sqrt{a\,d-b\,c}}{b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^2,x)
```

```
[Out] ((a*d^2 - b*c*d)*(c + d*x)^(1/2))/(b^3*(c + d*x) - b^3*c + a*b^2*d) + (2*d*
(c + d*x)^(1/2))/b^2 - (3*d*atan((b^(1/2)*d*(a*d - b*c)^(1/2)*(c + d*x)^(1/
2))/(a*d^2 - b*c*d))*(a*d - b*c)^(1/2))/b^(5/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.1289 $\int \frac{(c+dx)^{3/2}}{(a+bx)^3} dx$

Optimal. Leaf size=100

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Rubi [A] time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {47, 63, 208}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}-\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(4*b^2*(a + b*x)) - (c + d*x)^(3/2)/(2*b*(a + b*x)^2)
- (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(5/2)*Sqrt[
b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx &= -\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{4b} \\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{\left(3d^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^2} \\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}+\frac{(3d)\operatorname{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4b^2} \\
&= -\frac{3d\sqrt{c+dx}}{4b^2(a+bx)}-\frac{(c+dx)^{3/2}}{2b(a+bx)^2}-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{5/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.90

$$
\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{4b^{5/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx}\,(3ad+2bc+5bdx)}{4b^2(a+bx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(Sqrt[c + d*x]*(2*b*c + 3*a*d + 5*b*d*x))/(b^2*(a + b*x)^2) + (3*d^2*A
rcTan[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[-(b*c) + a*d]])/(4*b^(5/2)*Sqrt[-(b*c) +
 a*d])
```

IntegrateAlgebraic [A] time = 0.38, size = 116, normalized size = 1.16

$$
-\frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{5/2}\sqrt{ad-bc}}-\frac{d^2\sqrt{c+dx}\,(3ad+5b(c+dx)-3bc)}{4b^2(ad+b(c+dx)-bc)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^3,x]
```

```
[Out] -1/4*(d^2*Sqrt[c + d*x]*(-3*b*c + 3*a*d + 5*b*(c + d*x)))/(b^2*(-(b*c) + a*
d + b*(c + d*x))^2) - (3*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*
x])/(b*c - a*d)])/(4*b^(5/2)*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.00, size = 383, normalized size = 3.83

$$
\left[\frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2+5\left(b^3cd-ab^2d^2\right)x\right)\sqrt{dx+c}}{8\left(a^2b^4c-a^3b^3d+\left(b^6c-ab^5d\right)x^2+2\left(ab^5c-a^2b^4d\right)x\right)},\ \frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(2\,b^3c^2+ab^2cd-3\,a^2bd^2+5\left(b^3cd-ab^2d^2\right)x\right)\sqrt{dx+c}}{4\left(a^2b^4c-a^3b^3d+\left(b^6c-ab^5d\right)x^2+2\left(ab^5c-a^2b^4d\right)x\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 + a*b^2*c*d - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))
/(a^2*b^4*c - a^3*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x
), 1/4*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (2*b^3*c^2 + a*b^2*c*d
 - 3*a^2*b*d^2 + 5*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c - a^3
*b^3*d + (b^6*c - a*b^5*d)*x^2 + 2*(a*b^5*c - a^2*b^4*d)*x)]
```

giac [A] time = 1.36, size = 108, normalized size = 1.08

$$\frac{3\,d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\,\sqrt{-b^2c+abd}\,b^2}-\frac{5\,(dx+c)^{\frac{3}{2}}bd^2-3\,\sqrt{dx+c}\,bcd^2+3\,\sqrt{dx+c}\,ad^3}{4\,((dx+c)b-bc+ad)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^2) - 1/4*(5*(d*x + c)^(3/2)*b*d^2 - 3*sqrt(d*x + c)*b*c*d^2 + 3*sqrt(d*x
+ c)*a*d^3)/(((d*x + c)*b - b*c + a*d)^2*b^2)
```

maple [A] time = 0.01, size = 121, normalized size = 1.21

$$-\frac{3\sqrt{dx+c}\;a\,d^3}{4\,(bdx+ad)^2\,b^2}+\frac{3\sqrt{dx+c}\;c\,d^2}{4\,(bdx+ad)^2\,b}-\frac{5\,(dx+c)^{\frac{3}{2}}\,d^2}{4\,(bdx+ad)^2\,b}+\frac{3d^2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)\,b}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^3,x)
```

```
[Out] -5/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(3/2)-3/4*d^3/(b*d*x+a*d)^2/b^2*(d*x+c)^(1
/2)*a+3/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(1/2)*c+3/4*d^2/b^2/((a*d-b*c)*b)^(1/
2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.28, size = 135, normalized size = 1.35

$$\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,b^{5/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{5\,d^2\,{\left(c+d\,x\right)}^{3/2}}{4\,b}+\frac{3\,d^2\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{4\,b^2}}{b^2\,{\left(c+d\,x\right)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,\left(c+d\,x\right)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^3,x)
```

```
[Out] (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(4*b^(5/2)*(a*d -
 b*c)^(1/2)) - ((5*d^2*(c + d*x)^(3/2))/(4*b) + (3*d^2*(a*d - b*c)*(c + d*x
)^(1/2))/(4*b^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^
2 + b^2*c^2 - 2*a*b*c*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.1290 $\int \frac{(c+dx)^{3/2}}{(a+bx)^4} dx$

Optimal. Leaf size=136

$$\frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}} - \frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)} - \frac{d\sqrt{c+dx}}{4b^2(a+bx)^2} - \frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Rubi [A] time = 0.06, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{d^2\sqrt{c+dx}}{8b^2(a+bx)(bc-ad)} + \frac{d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}} - \frac{d\sqrt{c+dx}}{4b^2(a+bx)^2} - \frac{(c+dx)^{3/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(4*b^2*(a + b*x)^2) - (d^2*Sqrt[c + d*x])/(8*b^2*(b*c -
a*d)*(a + b*x)) - (c + d*x)^(3/2)/(3*b*(a + b*x)^3) + (d^3*ArcTanh[(Sqrt[b]
*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(5/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{2b} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^2\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8b^2} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^2(bc-ad)} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}-\frac{d^2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^2(bc-ad)} \\
&= -\frac{d\sqrt{c+dx}}{4b^2(a+bx)^2}-\frac{d^2\sqrt{c+dx}}{8b^2(bc-ad)(a+bx)}-\frac{(c+dx)^{3/2}}{3b(a+bx)^3}+\frac{d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{5/2}(bc-ad)^{3/2}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.38

$$
\frac{2d^3(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] (2*d^3*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^4)
```

IntegrateAlgebraic [A] time = 0.62, size = 166, normalized size = 1.22

$$
-\frac{d^3\sqrt{c+dx}\left(3a^2d^2+8abd(c+dx)-6abcd+3b^2c^2-3b^2(c+dx)^2-8b^2c(c+dx)\right)}{24b^2(bc-ad)(-ad-b(c+dx)+bc)^3}-\frac{d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{5/2}(ad-bc)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(d^3*Sqrt[c + d*x]*(3*b^2*c^2 - 6*a*b*c*d + 3*a^2*d^2 - 8*b^2*c*(c +
d*x) + 8*a*b*d*(c + d*x) - 3*b^2*(c + d*x)^2))/(b^2*(b*c - a*d)*(b*c - a*d
- b*(c + d*x))^3) - (d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/
(b*c - a*d)])/(8*b^(5/2)*(-(b*c) + a*d)^(3/2))
```

fricas [B] time = 1.37, size = 666, normalized size = 4.90

$$
\left[-\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(8b^4c^3-10ab^3c^2d-a^2b^2cd^2+3a^3bd^3+3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(7b^4c^2d-11ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^2-2a^4b^4cd+a^5b^3d^2+\left(b^8c^2-2ab^7cd+a^2b^6d^2\right)x^3+3\left(ab^7c^2-2a^2b^6cd+a^3b^5d^2\right)x^2+3\left(a^2b^6c^2-2a^3b^5cd+a^4b^4d^2\right)x\right)},\ -\frac{3\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)+\left(8b^4c^3-10ab^3c^2d-a^2b^2cd^2+3a^3bd^3+3\left(b^4cd^2-ab^3d^3\right)x^2+2\left(7b^4c^2d-11ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^2-2a^4b^4cd+a^5b^3d^2+\left(b^8c^2-2ab^7cd+a^2b^6d^2\right)x^3+3\left(ab^7c^2-2a^2b^6cd+a^3b^5d^2\right)x^2+3\left(a^2b^6c^2-2a^3b^5cd+a^4b^4d^2\right)x\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/48*(3*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*(8*b^4*c^3 - 10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3
+ 3*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b
^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*
b^5*d^2)*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*x), -1/24*(3*(
```

```
b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*
d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (8*b^4*c^3 -
10*a*b^3*c^2*d - a^2*b^2*c*d^2 + 3*a^3*b*d^3 + 3*(b^4*c*d^2 - a*b^3*d^3)*x^
2 + 2*(7*b^4*c^2*d - 11*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3
*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2 + (b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d
^2)*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*x^2 + 3*(a^2*b^6*c^2
- 2*a^3*b^5*c*d + a^4*b^4*d^2)*x)]
```

giac [**A**] time = 1.40, size = 185, normalized size = 1.36

$$-\frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2} c+abd}}\right)}{8 \left(b^{3} c - a b^{2} d\right) \sqrt{-b^{2} c + abd}} - \frac{3 \,(dx+c)^{\frac{5}{2}} b^{2} d^{3} + 8 \,(dx+c)^{\frac{3}{2}} b^{2} c d^{3} - 3 \sqrt{dx+c}\, b^{2} c^{2} d^{3} - 8 \,(dx+c)^{\frac{3}{2}} a b d^{4} + 6 \sqrt{dx+c}\, a b c d^{4} - 3 \sqrt{dx+c}\, a^{2} d^{5}}{24 \left(b^{3} c - a b^{2} d\right) \left((dx+c) b - bc + ad\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c - a*b^2*d)*sq
rt(-b^2*c + a*b*d)) - 1/24*(3*(d*x + c)^(5/2)*b^2*d^3 + 8*(d*x + c)^(3/2)*b
^2*c*d^3 - 3*sqrt(d*x + c)*b^2*c^2*d^3 - 8*(d*x + c)^(3/2)*a*b*d^4 + 6*sqrt
(d*x + c)*a*b*c*d^4 - 3*sqrt(d*x + c)*a^2*d^5)/((b^3*c - a*b^2*d)*((d*x + c
)*b - b*c + a*d)^3)
```

maple [**A**] time = 0.02, size = 163, normalized size = 1.20

$$-\frac{\sqrt{dx+c}\; a d^{4}}{8 \,(b d x + a d)^{3} b^{2}} + \frac{\sqrt{dx+c}\; c d^{3}}{8 \,(b d x + a d)^{3} b} + \frac{(dx+c)^{\frac{5}{2}} d^{3}}{8 \,(b d x + a d)^{3} (a d - b c)} - \frac{(dx+c)^{\frac{3}{2}} d^{3}}{3 \,(b d x + a d)^{3} b} + \frac{d^{3} \arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{(ad-bc)b}}\right)}{8 \,(a d - b c) \sqrt{(a d - b c) b}\; b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^4,x)
```

```
[Out] 1/8*d^3/(b*d*x+a*d)^3/(a*d-b*c)*(d*x+c)^(5/2)-1/3*d^3/(b*d*x+a*d)^3/b*(d*x+
c)^(3/2)-1/8*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a+1/8*d^3/(b*d*x+a*d)^3/b*
(d*x+c)^(1/2)*c+1/8*d^3/(a*d-b*c)/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1
/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.34, size = 209, normalized size = 1.54

$$\frac{d^{3} \,\mathrm{atan}\left(\frac{\sqrt{b}\, \sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\, b^{5/2} \,(a\,d-b\,c)^{3/2}} - \frac{\frac{d^{3}\,(c+d\,x)^{3/2}}{3\,b} - \frac{d^{3}\,(c+d\,x)^{5/2}}{8\,(a\,d-b\,c)} + \frac{d^{3}\,(a\,d-b\,c)\,\sqrt{c+d\,x}}{8\,b^{2}}}{(c+d\,x)\,\left(3\,a^{2}\,b\,d^{2} - 6\,a\,b^{2}\,c\,d + 3\,b^{3}\,c^{2}\right) + b^{3}\,(c+d\,x)^{3} - \left(3\,b^{3}\,c - 3\,a\,b^{2}\,d\right)\,(c+d\,x)^{2} + a^{3}\,d^{3} - b^{3}\,c^{3} + 3\,a\,b^{2}\,c^{2}\,d - 3\,a^{2}\,b\,c\,d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^4,x)
```

```
[Out] (d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(5/2)*(a*d - b
*c)^(3/2)) - ((d^3*(c + d*x)^(3/2))/(3*b) - (d^3*(c + d*x)^(5/2))/(8*(a*d -
 b*c)) + (d^3*(a*d - b*c)*(c + d*x)^(1/2))/(8*b^2))/((c + d*x)*(3*b^3*c^2 +
```

```
 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^2*d)*(c +
d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.1291 $\int \frac{(c+dx)^{3/2}}{(a+bx)^5} dx$

Optimal. Leaf size=172

$$-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}+\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Rubi [A] time = 0.07, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {47, 51, 63, 208}

$$\frac{3d^3\sqrt{c+dx}}{64b^2(a+bx)(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{32b^2(a+bx)^2(bc-ad)}-\frac{3d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{5/2}(bc-ad)^{5/2}}-\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3}-\frac{(c+dx)^{3/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] -(d*Sqrt[c + d*x])/(8*b^2*(a + b*x)^3) - (d^2*Sqrt[c + d*x])/(32*b^2*(b*c -
 a*d)*(a + b*x)^2) + (3*d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x))
 - (c + d*x)^(3/2)/(4*b*(a + b*x)^4) - (3*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(64*b^(5/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{d^2\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{16b^2}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{\left(3d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^2(bc-ad)}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^4\right)\int \cdots}{128b^2\cdots}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} + \frac{\left(3d^3\right)\text{Sub}\cdots}{\cdots}$$

$$= -\frac{d\sqrt{c+dx}}{8b^2(a+bx)^3} - \frac{d^2\sqrt{c+dx}}{32b^2(bc-ad)(a+bx)^2} + \frac{3d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)} - \frac{(c+dx)^{3/2}}{4b(a+bx)^4} - \frac{3d^4\tanh^{-1}\cdots}{64b^{5/2}(\cdots}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.30

$$\frac{2d^4(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^5)
```

IntegrateAlgebraic [A] time = 1.01, size = 226, normalized size = 1.31

$$\frac{d^4\sqrt{c+dx}\left(-3a^3d^3-11a^2bd^2(c+dx)+9a^2bcd^2-9ab^2c^2d+11ab^2d(c+dx)^2+22ab^2cd(c+dx)+3b^3c^3-11b^3c^2(c+dx)+3b^3(c+dx)^3-11b^3c(c+dx)^2\right)}{64b^2(bc-ad)^2(-ad-b(c+dx)+bc)^4} - \frac{3d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{5/2}(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^5,x]
```

```
[Out] (d^4*Sqrt[c + d*x]*(3*b^3*c^3 - 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - 3*a^3*d^3 -
 11*b^3*c^2*(c + d*x) + 22*a*b^2*c*d*(c + d*x) - 11*a^2*b*d^2*(c + d*x) - 1
1*b^3*c*(c + d*x)^2 + 11*a*b^2*d*(c + d*x)^2 + 3*b^3*(c + d*x)^3))/(64*b^2*
(b*c - a*d)^2*(b*c - a*d - b*(c + d*x))^4) - (3*d^4*ArcTan[(Sqrt[b]*Sqrt[-(
b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(5/2)*(-(b*c) + a*d)^(5/2))
```

fricas [B] time = 1.54, size = 1043, normalized size = 6.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [1/128*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*
x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c -
a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(16*b^5*c^4 - 40*a*b^4*c^3*d + 26*a^2*
b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 - a*b^4*d^4)*x^3 +
```

```
(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (24*b^5*c^3*d - 68
*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 + (b^10*c^3 - 3*a
*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*c^3 - 3*a^2*b^8*
c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3 - 3*a^3*b^7*c^2
*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 - 3*a^4*b^6*c^2*d
+ 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x), 1/64*(3*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3
 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (16*b^5*c^4 - 40*a*b^4
*c^3*d + 26*a^2*b^3*c^2*d^2 + a^3*b^2*c*d^3 - 3*a^4*b*d^4 - 3*(b^5*c*d^3 -
a*b^4*d^4)*x^3 + (2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 + (2
4*b^5*c^3*d - 68*a*b^4*c^2*d^2 + 55*a^2*b^3*c*d^3 - 11*a^3*b^2*d^4)*x)*sqrt
(d*x + c))/(a^4*b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3 +
 (b^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*x^4 + 4*(a*b^9*
c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*x^3 + 6*(a^2*b^8*c^3
 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*x^2 + 4*(a^3*b^7*c^3 -
3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*x)]
```

giac [**A**]   time = 1.47, size = 285, normalized size = 1.66

$$\frac{3\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{7}{2}}b^3d^4-11\,(dx+c)^{\frac{5}{2}}b^3cd^4-11\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+3\,\sqrt{dx+c}\,b^3c^3d^4+11\,(dx+c)^{\frac{5}{2}}ab^2d^5+22\,(dx+c)^{\frac{3}{2}}ab^2cd^5-9\,\sqrt{dx+c}\,ab^2c^2d^5-11\,(dx+c)^{\frac{3}{2}}a^2bd^6+9\,\sqrt{dx+c}\,a^2bcd^6-3\,\sqrt{dx+c}\,a^3d^7}{64\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 3/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/64*(3*(d*x + c)^(7/2)*b^3*d^4 -
 11*(d*x + c)^(5/2)*b^3*c*d^4 - 11*(d*x + c)^(3/2)*b^3*c^2*d^4 + 3*sqrt(d*x
 + c)*b^3*c^3*d^4 + 11*(d*x + c)^(5/2)*a*b^2*d^5 + 22*(d*x + c)^(3/2)*a*b^2
*c*d^5 - 9*sqrt(d*x + c)*a*b^2*c^2*d^5 - 11*(d*x + c)^(3/2)*a^2*b*d^6 + 9*s
qrt(d*x + c)*a^2*b*c*d^6 - 3*sqrt(d*x + c)*a^3*d^7)/((b^4*c^2 - 2*a*b^3*c*d
 + a^2*b^2*d^2)*((d*x + c)*b - b*c + a*d)^4)
```

maple [**A**]   time = 0.02, size = 222, normalized size = 1.29

$$\frac{3\,(dx+c)^{\frac{7}{2}}\,b\,d^4}{64\,(bdx+ad)^4\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{3\sqrt{dx+c}\;a\,d^5}{64\,(bdx+ad)^4\,b^2}+\frac{3\sqrt{dx+c}\;c\,d^4}{64\,(bdx+ad)^4\,b}+\frac{11\,(dx+c)^{\frac{5}{2}}\,d^4}{64\,(bdx+ad)^4\,(ad-bc)}-\frac{11\,(dx+c)^{\frac{3}{2}}\,d^4}{64\,(bdx+ad)^4\,b}+\frac{3d^4\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{64\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{(ad-bc)\,b}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^5,x)
```

```
[Out] 3/64*d^4/(b*d*x+a*d)^4*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(7/2)+11/64*d^
4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(5/2)-11/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
3/2)-3/64*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a+3/64*d^4/(b*d*x+a*d)^4/b*(d
*x+c)^(1/2)*c+3/64*d^4/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/((a*d-b*c)*b)^(1/2)*
arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.37, size = 296, normalized size = 1.72

$$\frac{3\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{11\,d^4\,{\left(c+d\,x\right)}^{3/2}}{64\,b}-\frac{11\,d^4\,{\left(c+d\,x\right)}^{5/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^4\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{64\,b^2}-\frac{3\,b\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^5,x)
```

```
[Out] (3*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(5/2)*(a*d
- b*c)^(5/2)) - ((11*d^4*(c + d*x)^(3/2))/(64*b) - (11*d^4*(c + d*x)^(5/2))
/(64*(a*d - b*c)) + (3*d^4*(a*d - b*c)*(c + d*x)^(1/2))/(64*b^2) - (3*b*d^4
*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c - 4*a*b^3
*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c*d^2 - 1
2*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2*b^2*d^2
 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.1292 $\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx$

Optimal. Leaf size=208

$$\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}-\frac{3d\sqrt{c+}}{40b^2(a+}$$

Rubi [A] time = 0.09, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {47, 51, 63, 208}

$$-\frac{3d^4\sqrt{c+dx}}{128b^2(a+bx)(bc-ad)^3}+\frac{d^3\sqrt{c+dx}}{64b^2(a+bx)^2(bc-ad)^2}-\frac{d^2\sqrt{c+dx}}{80b^2(a+bx)^3(bc-ad)}+\frac{3d^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{5/2}(bc-ad)^{7/2}}-\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4}-\frac{(c+dx)^{3/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (-3*d*Sqrt[c + d*x])/(40*b^2*(a + b*x)^4) - (d^2*Sqrt[c + d*x])/(80*b^2*(b*
c - a*d)*(a + b*x)^3) + (d^3*Sqrt[c + d*x])/(64*b^2*(b*c - a*d)^2*(a + b*x)
^2) - (3*d^4*Sqrt[c + d*x])/(128*b^2*(b*c - a*d)^3*(a + b*x)) - (c + d*x)^(
3/2)/(5*b*(a + b*x)^5) + (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c -
a*d]])/(128*b^(5/2)*(b*c - a*d)^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^6}\,dx = -\frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{(3d)\int \frac{\sqrt{c+dx}}{(a+bx)^5}\,dx}{10b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{\left(3d^2\right)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{80b^2}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} - \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{32b^2(bc-ad)}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{(c+dx)^{3/2}}{5b(a+bx)^5} + \frac{\left(3d^4\right)\int}{128}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

$$= -\frac{3d\sqrt{c+dx}}{40b^2(a+bx)^4} - \frac{d^2\sqrt{c+dx}}{80b^2(bc-ad)(a+bx)^3} + \frac{d^3\sqrt{c+dx}}{64b^2(bc-ad)^2(a+bx)^2} - \frac{3d^4\sqrt{c+dx}}{128b^2(bc-ad)^3(a+b}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.25

$$\frac{2d^5(c+dx)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};-\frac{b(c+dx)}{ad-bc}\right)}{5(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(5*(-(b*c) + a*d)^6)
```

IntegrateAlgebraic [A] time = 1.98, size = 317, normalized size = 1.52

$$\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{5/2}(bc-ad)^3\sqrt{ad-bc}} - \frac{d^5\sqrt{c+dx}\left(15a^4d^4+70a^3bd^3(c+dx)-60a^3bcd^3+90a^2b^2c^2d^2-128a^2b^2d^2(c+dx)^2-210a^2b^2cd^2(c+dx)-60ab^3c^3d+210ab^3c^2d(c+dx)-70ab^3d(c+dx)^3+256ab^3cd(c+dx)^2+15b^4c^4-70b^4c^3(c+dx)-128b^4c^2(c+dx)^2-15b^4(c+dx)^4+70b^4c(c+dx)^3\right)}{640b^2(bc-ad)^3(-ad-b(c+dx)+bc)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^6,x]
```

```
[Out] -1/640*(d^5*Sqrt[c + d*x]*(15*b^4*c^4 - 60*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2
 - 60*a^3*b*c*d^3 + 15*a^4*d^4 - 70*b^4*c^3*(c + d*x) + 210*a*b^3*c^2*d*(c
+ d*x) - 210*a^2*b^2*c*d^2*(c + d*x) + 70*a^3*b*d^3*(c + d*x) - 128*b^4*c^2
*(c + d*x)^2 + 256*a*b^3*c*d*(c + d*x)^2 - 128*a^2*b^2*d^2*(c + d*x)^2 + 70
*b^4*c*(c + d*x)^3 - 70*a*b^3*d*(c + d*x)^3 - 15*b^4*(c + d*x)^4))/(b^2*(b*
c - a*d)^3*(b*c - a*d - b*(c + d*x))^5) + (3*d^5*ArcTan[(Sqrt[b]*Sqrt[-(b*c
) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(5/2)*(b*c - a*d)^3*Sqrt[-(b*c
) + a*d])
```

fricas [B] time = 1.31, size = 1492, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [-1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b
^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*
c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(128*b^6*c^5
- 464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c
*d^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*
a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87
*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2
+ 489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c
))/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a
^9*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*
d^3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d
^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*
d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^
4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^
2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3
+ a^8*b^4*d^4)*x), -1/640*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d
^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(-b^2*c + a*b*d)
*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (128*b^6*c^5 -
464*a*b^5*c^4*d + 584*a^2*b^4*c^3*d^2 - 258*a^3*b^3*c^2*d^3 - 5*a^4*b^2*c*d
^4 + 15*a^5*b*d^5 + 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 - 10*(b^6*c^2*d^3 - 8*a*
b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(4*b^6*c^3*d^2 - 27*a*b^5*c^2*d^3 + 87*a
^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(88*b^6*c^4*d - 344*a*b^5*c^3*d^2 +
489*a^2*b^4*c^2*d^3 - 268*a^3*b^3*c*d^4 + 35*a^4*b^2*d^5)*x)*sqrt(d*x + c))
/(a^5*b^7*c^4 - 4*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9
*b^3*d^4 + (b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2 - 4*a^3*b^9*c*d^
3 + a^4*b^8*d^4)*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*d + 6*a^3*b^9*c^2*d^2
 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*x^4 + 10*(a^2*b^10*c^4 - 4*a^3*b^9*c^3*d
+ 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*d^4)*x^3 + 10*(a^3*b^9*c^4
- 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a^6*b^6*c*d^3 + a^7*b^5*d^4)*x^2
+ 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 +
a^8*b^4*d^4)*x)]
```

giac [B] time = 1.39, size = 410, normalized size = 1.97

$$-\frac{3\, d^{5} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2}c+abd}}\right)}{128\,\left(b^{5}c^{3} - 3\,ab^{4}c^{2}d + 3\,a^{2}b^{3}cd^{2} - a^{3}b^{2}d^{3}\right)\sqrt{-b^{2}c + abd}} - \frac{15\,(dx+c)^{\frac{9}{2}}b^{4}d^{5} - 70\,(dx+c)^{\frac{7}{2}}b^{4}cd^{5} + 128\,(dx+c)^{\frac{5}{2}}b^{4}c^{2}d^{5} + 70\,(dx+c)^{\frac{3}{2}}b^{4}c^{3}d^{5} - 15\,\sqrt{dx+c}\,b^{4}c^{4}d^{5} + 70\,(dx+c)^{\frac{7}{2}}ab^{3}d^{6} - 256\,(dx+c)^{\frac{5}{2}}ab^{3}cd^{6} - 210\,(dx+c)^{\frac{3}{2}}ab^{3}c^{2}d^{6} + 60\,\sqrt{dx+c}\,ab^{3}c^{3}d^{6} + 128\,(dx+c)^{\frac{5}{2}}a^{2}b^{2}d^{7} + 210\,(dx+c)^{\frac{3}{2}}a^{2}b^{2}cd^{7} - 90\,\sqrt{dx+c}\,a^{2}b^{2}c^{2}d^{7} - 70\,(dx+c)^{\frac{3}{2}}a^{3}bd^{8} + 60\,\sqrt{dx+c}\,a^{3}bcd^{8} - 15\,\sqrt{dx+c}\,a^{4}d^{9}}{640\,\left(b^{5}c^{3} - 3\,ab^{4}c^{2}d + 3\,a^{2}b^{3}cd^{2} - a^{3}b^{2}d^{3}\right)\left((dx+c)b - bc + ad\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] -3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^3 - 3*a*b^4
*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(-b^2*c + a*b*d)) - 1/640*(15*(
d*x + c)^(9/2)*b^4*d^5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 + 128*(d*x + c)^(5/2)
*b^4*c^2*d^5 + 70*(d*x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^
5 + 70*(d*x + c)^(7/2)*a*b^3*d^6 - 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d
*x + c)^(3/2)*a*b^3*c^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 + 128*(d*x + c
)^(5/2)*a^2*b^2*d^7 + 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*
a^2*b^2*c^2*d^7 - 70*(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d
^8 - 15*sqrt(d*x + c)*a^4*d^9)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
- a^3*b^2*d^3)*((d*x + c)*b - b*c + a*d)^5)
```

maple [A] time = 0.02, size = 300, normalized size = 1.44

$$\frac{3\,(dx+c)^{\frac{9}{2}}\, b^{2}d^{5}}{128\,(bdx+ad)^{5}\left(a^{3}d^{3} - 3a^{2}bc\,d^{2} + 3a\,b^{2}c^{2}d - b^{3}c^{3}\right)} + \frac{7\,(dx+c)^{\frac{7}{2}}\, b\,d^{5}}{64\,(bdx+ad)^{5}\left(a^{2}d^{2} - 2abcd + b^{2}c^{2}\right)} - \frac{3\sqrt{dx+c}\; a\,d^{6}}{128\,(bdx+ad)^{5}\, b^{2}} + \frac{3\sqrt{dx+c}\; c\,d^{5}}{128\,(bdx+ad)^{5}\, b} + \frac{(dx+c)^{\frac{5}{2}}\, d^{5}}{5\,(bdx+ad)^{5}\,(ad-bc)} - \frac{7\,(dx+c)^{\frac{3}{2}}\, d^{5}}{64\,(bdx+ad)^{5}\, b} + \frac{3d^{5}\arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^{3}d^{3} - 3a^{2}bc\,d^{2} + 3a\,b^{2}c^{2}d - b^{3}c^{3}\right)\sqrt{(ad-bc)\,b}\; b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*(
d*x+c)^(9/2)+7/64*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(
```

```
7/2)+1/5*d^5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(5/2)-7/64*d^5/(b*d*x+a*d)^5/b
*(d*x+c)^(3/2)-3/128*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(1/2)*a+3/128*d^5/(b*d*x
+a*d)^5/b*(d*x+c)^(1/2)*c+3/128*d^5/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.47, size = 398, normalized size = 1.91

$$\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,\left(a\,d-b\,c\right)}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{3/2}}{64\,b}+\frac{3\,b^2\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^3}-\frac{3\,d^5\,\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}{128\,b^2}+\frac{7\,b\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}+\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{5/2}\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^6,x)
```

```
[Out] ((d^5*(c + d*x)^(5/2))/(5*(a*d - b*c)) - (7*d^5*(c + d*x)^(3/2))/(64*b) + (
3*b^2*d^5*(c + d*x)^(9/2))/(128*(a*d - b*c)^3) - (3*d^5*(a*d - b*c)*(c + d*
x)^(1/2))/(128*b^2) + (7*b*d^5*(c + d*x)^(7/2))/(64*(a*d - b*c)^2))/(b^5*(c
 + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 3
0*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c
 + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 - 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c
^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d)
- 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
+ (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(5/2)*(a
*d - b*c)^(7/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

3.1293 $\int (a+bx)^5(c+dx)^{5/2}\,dx$

Optimal. Leaf size=158

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-$$

Rubi [A] time = 0.05, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{15/2}(bc-ad)}{3d^6}+\frac{20b^3(c+dx)^{13/2}(bc-ad)^2}{13d^6}-\frac{20b^2(c+dx)^{11/2}(bc-ad)^3}{11d^6}+\frac{10b(c+dx)^{9/2}(bc-ad)^4}{9d^6}-\frac{2(c+dx)^{7/2}(bc-ad)^5}{7d^6}+\frac{2b^5(c+dx)^{17/2}}{17d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^5*(c + d*x)^(7/2))/(7*d^6) + (10*b*(b*c - a*d)^4*(c + d*x)^
(9/2))/(9*d^6) - (20*b^2*(b*c - a*d)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3
*(b*c - a*d)^2*(c + d*x)^(13/2))/(13*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(1
5/2))/(3*d^6) + (2*b^5*(c + d*x)^(17/2))/(17*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^5(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^5(c+dx)^{5/2}}{d^5}+\frac{5b(bc-ad)^4(c+dx)^{7/2}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{9/2}}{d^5}+\right.$$
$$=-\frac{2(bc-ad)^5(c+dx)^{7/2}}{7d^6}+\frac{10b(bc-ad)^4(c+dx)^{9/2}}{9d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{11/2}}{11d^6}+$$

Mathematica [A] time = 0.11, size = 123, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(-51051b^4(c+dx)^4(bc-ad)+117810b^3(c+dx)^3(bc-ad)^2-139230b^2(c+dx)^2(bc-ad)^3+85085b(c+dx)(bc-ad)^4-21879(bc-ad)^5+9009b^5(c+dx)^5\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-21879*(b*c - a*d)^5 + 85085*b*(b*c - a*d)^4*(c + d*x)
- 139230*b^2*(b*c - a*d)^3*(c + d*x)^2 + 117810*b^3*(b*c - a*d)^2*(c + d*x)
^3 - 51051*b^4*(b*c - a*d)*(c + d*x)^4 + 9009*b^5*(c + d*x)^5))/(153153*d^6
)
```

IntegrateAlgebraic [A] time = 0.11, size = 315, normalized size = 1.99

$$\frac{2(c+dx)^{7/2}\left(21879a^5d^5+85085a^4bd^4(c+dx)-109395a^4bcd^4+218790a^3b^2c^2d^3+139230a^3b^2d^3(c+dx)^2-340340a^3b^2cd^3(c+dx)-218790a^2b^3c^3d^2+510510a^2b^3c^2d^2(c+dx)+117810a^2b^3d^2(c+dx)^3-417690a^2b^3cd^2(c+dx)^2+109395ab^4c^4d-340340ab^4c^3d(c+dx)+417690ab^4c^2d(c+dx)^2+51051ab^4d(c+dx)^4-235620ab^4cd(c+dx)^3-21879b^5c^5+85085b^5c^4(c+dx)-139230b^5c^3(c+dx)^2+117810b^5c^2(c+dx)^3+9009b^5(c+dx)^5-51051b^5c(c+dx)^4\right)}{153153d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-21879*b^5*c^5 + 109395*a*b^4*c^4*d - 218790*a^2*b^3*c^
3*d^2 + 218790*a^3*b^2*c^2*d^3 - 109395*a^4*b*c*d^4 + 21879*a^5*d^5 + 85085
```

```
*b^5*c^4*(c + d*x) - 340340*a*b^4*c^3*d*(c + d*x) + 510510*a^2*b^3*c^2*d^2*
(c + d*x) - 340340*a^3*b^2*c*d^3*(c + d*x) + 85085*a^4*b*d^4*(c + d*x) - 13
9230*b^5*c^3*(c + d*x)^2 + 417690*a*b^4*c^2*d*(c + d*x)^2 - 417690*a^2*b^3*
c*d^2*(c + d*x)^2 + 139230*a^3*b^2*d^3*(c + d*x)^2 + 117810*b^5*c^2*(c + d*
x)^3 - 235620*a*b^4*c*d*(c + d*x)^3 + 117810*a^2*b^3*d^2*(c + d*x)^3 - 5105
1*b^5*c*(c + d*x)^4 + 51051*a*b^4*d*(c + d*x)^4 + 9009*b^5*(c + d*x)^5))/(1
53153*d^6)
```

fricas [B] time = 1.29, size = 497, normalized size = 3.15

$$\frac{2\left(9009\,b^5d^8x^8-256\,b^5c^8+2176\,ab^4c^7d-8160\,a^2b^3c^6d^2+17680\,a^3b^2c^5d^3-24310\,a^4bc^4d^4+21879\,a^5c^3d^5+3003\left(7\,b^5cd^7+17\,ab^4d^8\right)x^7+231\left(55\,b^5c^2d^6+527\,ab^4cd^7+510\,a^2b^3d^8\right)x^6+63\left(b^5c^3d^5+1207\,ab^4c^2d^6+4590\,a^2b^3cd^7+2210\,a^3b^2d^8\right)x^5-35\left(2\,b^5c^4d^4-17\,ab^4c^3d^5-5406\,a^2b^3c^2d^6-10166\,a^3b^2cd^7-2431\,a^4bd^8\right)x^4+\left(80\,b^5c^5d^3-680\,ab^4c^4d^4+2550\,a^2b^3c^3d^5+249730\,a^3b^2c^2d^6+230945\,a^4bcd^7+21879\,a^5d^8\right)x^3-3\left(32\,b^5c^6d^2-272\,ab^4c^5d^3+1020\,a^2b^3c^4d^4-2210\,a^3b^2c^3d^5-60775\,a^4bc^2d^6-21879\,a^5cd^7\right)x^2+\left(128\,b^5c^7d-1088\,ab^4c^6d^2+4080\,a^2b^3c^5d^3-8840\,a^3b^2c^4d^4+12155\,a^4bc^3d^5+65637\,a^5c^2d^6\right)x\right)\sqrt{dx+c}}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/153153*(9009*b^5*d^8*x^8 - 256*b^5*c^8 + 2176*a*b^4*c^7*d - 8160*a^2*b^3*
c^6*d^2 + 17680*a^3*b^2*c^5*d^3 - 24310*a^4*b*c^4*d^4 + 21879*a^5*c^3*d^5 +
 3003*(7*b^5*c*d^7 + 17*a*b^4*d^8)*x^7 + 231*(55*b^5*c^2*d^6 + 527*a*b^4*c*
d^7 + 510*a^2*b^3*d^8)*x^6 + 63*(b^5*c^3*d^5 + 1207*a*b^4*c^2*d^6 + 4590*a^
2*b^3*c*d^7 + 2210*a^3*b^2*d^8)*x^5 - 35*(2*b^5*c^4*d^4 - 17*a*b^4*c^3*d^5
- 5406*a^2*b^3*c^2*d^6 - 10166*a^3*b^2*c*d^7 - 2431*a^4*b*d^8)*x^4 + (80*b^
5*c^5*d^3 - 680*a*b^4*c^4*d^4 + 2550*a^2*b^3*c^3*d^5 + 249730*a^3*b^2*c^2*d
^6 + 230945*a^4*b*c*d^7 + 21879*a^5*d^8)*x^3 - 3*(32*b^5*c^6*d^2 - 272*a*b^
4*c^5*d^3 + 1020*a^2*b^3*c^4*d^4 - 2210*a^3*b^2*c^3*d^5 - 60775*a^4*b*c^2*d
^6 - 21879*a^5*c*d^7)*x^2 + (128*b^5*c^7*d - 1088*a*b^4*c^6*d^2 + 4080*a^2*
b^3*c^5*d^3 - 8840*a^3*b^2*c^4*d^4 + 12155*a^4*b*c^3*d^5 + 65637*a^5*c^2*d^
6)*x)*sqrt(d*x + c)/d^6
```

giac [B] time = 1.52, size = 1599, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/765765*(765765*sqrt(d*x + c)*a^5*c^3 + 765765*((d*x + c)^(3/2) - 3*sqrt(d
*x + c)*c)*a^5*c^2 + 1276275*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^4*b*c^
3/d + 153153*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^5*c + 510510*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x
+ c)*c^2)*a^3*b^2*c^3/d^2 + 765765*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*
c + 15*sqrt(d*x + c)*c^2)*a^4*b*c^2/d + 21879*(5*(d*x + c)^(7/2) - 21*(d*x
+ c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^5 + 218790*
(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqr
t(d*x + c)*c^3)*a^2*b^3*c^3/d^3 + 656370*(5*(d*x + c)^(7/2) - 21*(d*x + c)^
(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3*b^2*c^2/d^2 +
328185*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 -
 35*sqrt(d*x + c)*c^3)*a^4*b*c/d + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c
)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*
x + c)*c^4)*a*b^4*c^3/d^4 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)
*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*
c^4)*a^2*b^3*c^2/d^3 + 72930*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c +
378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*
a^3*b^2*c/d^2 + 12155*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*
x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^4*b/d
 + 1105*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*
c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x +
c)*c^5)*b^5*c^3/d^5 + 16575*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c +
990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c
^4 - 693*sqrt(d*x + c)*c^5)*a*b^4*c^2/d^4 + 33150*(63*(d*x + c)^(11/2) - 38
5*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 +
1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^2*b^3*c/d^3 + 11050*(63
```

```
*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*
(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a^3
*b^2/d^2 + 765*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x
+ c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006
*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^5*c^2/d^5 + 3825*(231*(d*x
 + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d
*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3
003*sqrt(d*x + c)*c^6)*a*b^4*c/d^4 + 2550*(231*(d*x + c)^(13/2) - 1638*(d*x
 + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009
*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a
^2*b^3/d^3 + 357*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12285*(d
*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2)*c^4
- 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d*x + c
)*c^7)*b^5*c/d^5 + 595*(429*(d*x + c)^(15/2) - 3465*(d*x + c)^(13/2)*c + 12
285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/2)*c^3 + 32175*(d*x + c)^(7/2
)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x + c)^(3/2)*c^6 - 6435*sqrt(d
*x + c)*c^7)*a*b^4/d^4 + 7*(6435*(d*x + c)^(17/2) - 58344*(d*x + c)^(15/2)*
c + 235620*(d*x + c)^(13/2)*c^2 - 556920*(d*x + c)^(11/2)*c^3 + 850850*(d*x
 + c)^(9/2)*c^4 - 875160*(d*x + c)^(7/2)*c^5 + 612612*(d*x + c)^(5/2)*c^6 -
 291720*(d*x + c)^(3/2)*c^7 + 109395*sqrt(d*x + c)*c^8)*b^5/d^5)/d
```

maple [B] time = 0.00, size = 273, normalized size = 1.73

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(9009b^5x^5d^5 + 51051a\,b^4d^5x^4 - 6006b^5c\,d^4x^4 + 117810a^2b^3d^5x^3 - 31416a\,b^4c\,d^4x^3 + 3696b^5c^2d^3x^3 + 139230a^3b^2d^5x^2 - 64260a^2b^3c\,d^4x^2 + 17136a\,b^4c^2d^3x^2 - 2016b^5c^3d^2x^2 + 85085a^4b\,d^5x - 61880a^3b^2c\,d^4x + 28560a^2b^3c^2d^3x - 7616a\,b^4c^3d^2x + 896b^5c^4dx + 21879a^5d^5 - 24310a^4bc\,d^4 + 17680a^3b^2c^2d^3 - 8160a^2b^3c^3d^2 + 2176a\,b^4c^4d - 256b^5c^5\right)}{153153d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(d*x+c)^(5/2),x)
```

```
[Out] 2/153153*(d*x+c)^(7/2)*(9009*b^5*d^5*x^5+51051*a*b^4*d^5*x^4-6006*b^5*c*d^4
*x^4+117810*a^2*b^3*d^5*x^3-31416*a*b^4*c*d^4*x^3+3696*b^5*c^2*d^3*x^3+1392
30*a^3*b^2*d^5*x^2-64260*a^2*b^3*c*d^4*x^2+17136*a*b^4*c^2*d^3*x^2-2016*b^5
*c^3*d^2*x^2+85085*a^4*b*d^5*x-61880*a^3*b^2*c*d^4*x+28560*a^2*b^3*c^2*d^3*
x-7616*a*b^4*c^3*d^2*x+896*b^5*c^4*d*x+21879*a^5*d^5-24310*a^4*b*c*d^4+1768
0*a^3*b^2*c^2*d^3-8160*a^2*b^3*c^3*d^2+2176*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

maxima [A] time = 1.36, size = 259, normalized size = 1.64

$$\frac{2 \left(9009 (dx+c)^{\frac{17}{2}} b^5 - 51051 \left(b^5c - ab^4d\right)(dx+c)^{\frac{15}{2}} + 117810 \left(b^5c^2 - 2\,ab^4cd + a^2b^3d^2\right)(dx+c)^{\frac{13}{2}} - 139230 \left(b^5c^3 - 3\,ab^4c^2d + 3\,a^2b^3cd^2 - a^3b^2d^3\right)(dx+c)^{\frac{11}{2}} + 85085 \left(b^5c^4 - 4\,ab^4c^3d + 6\,a^2b^3c^2d^2 - 4\,a^3b^2cd^3 + a^4bd^4\right)(dx+c)^{\frac{9}{2}} - 21879 \left(b^5c^5 - 5\,ab^4c^4d + 10\,a^2b^3c^3d^2 - 10\,a^3b^2c^2d^3 + 5\,a^4bcd^4 - a^5d^5\right)(dx+c)^{\frac{7}{2}}\right)}{153153\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/153153*(9009*(d*x + c)^(17/2)*b^5 - 51051*(b^5*c - a*b^4*d)*(d*x + c)^(15
/2) + 117810*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(13/2) - 13923
0*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(11/2
) + 85085*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)*(d*x + c)^(9/2) - 21879*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(d*x + c)^(7/2))/d^6
```

mupad [B] time = 0.27, size = 137, normalized size = 0.87

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{17/2}}{17\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{15/2}}{15\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{13/2}}{13\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(17/2))/(17*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(15/
2))/(15*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(7/2))/(7*d^6) + (20*b^2*(a*d - b
```

```
*c)^3*(c + d*x)^(11/2))/(11*d^6) + (20*b^3*(a*d - b*c)^2*(c + d*x)^(13/2))/
(13*d^6) + (10*b*(a*d - b*c)^4*(c + d*x)^(9/2))/(9*d^6)
```

sympy [A] time = 43.08, size = 1292, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(d*x+c)**(5/2),x)
```

```
[Out] a**5*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**5*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**5*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 10*a*
*4*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 20*a**4*b*c*(
c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
 + 10*a**4*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c
 + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 20*a**3*b**2*c**2*(c**2*(c +
d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 40*a**3
*b**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*
x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 20*a**3*b**2*(c**4*(c + d*x)**(3/2
)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)
**(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 20*a**2*b**3*c**2*(-c**3*(c + d*x)
**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)*
*(9/2)/9)/d**4 + 40*a**2*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)
**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)*
*(11/2)/11)/d**4 + 20*a**2*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)*
*(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c +
 d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**4 + 10*a*b**4*c**2*(c**4*(c + d
*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*
(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/11)/d**5 + 20*a*b**4*c*(-c**5*(c + d
*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2
*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5
 + 10*a*b**4*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4
*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2
)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**5 + 2*b**5*c**2*
(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2
)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(1
3/2)/13)/d**6 + 4*b**5*c*(c**6*(c + d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)
/5 + 15*c**4*(c + d*x)**(7/2)/7 - 20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c +
 d*x)**(11/2)/11 - 6*c*(c + d*x)**(13/2)/13 + (c + d*x)**(15/2)/15)/d**6 +
2*b**5*(-c**7*(c + d*x)**(3/2)/3 + 7*c**6*(c + d*x)**(5/2)/5 - 3*c**5*(c +
d*x)**(7/2) + 35*c**4*(c + d*x)**(9/2)/9 - 35*c**3*(c + d*x)**(11/2)/11 + 2
1*c**2*(c + d*x)**(13/2)/13 - 7*c*(c + d*x)**(15/2)/15 + (c + d*x)**(17/2)/
17)/d**6
```

3.1294 $\int (a+bx)^4 (c+dx)^{5/2}\, dx$

Optimal. Leaf size=129

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

Rubi [A] time = 0.04, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{13/2}(bc-ad)}{13d^5}+\frac{12b^2(c+dx)^{11/2}(bc-ad)^2}{11d^5}-\frac{8b(c+dx)^{9/2}(bc-ad)^3}{9d^5}+\frac{2(c+dx)^{7/2}(bc-ad)^4}{7d^5}+\frac{2b^4(c+dx)^{15/2}}{15d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^4*(c + d*x)^(7/2))/(7*d^5) - (8*b*(b*c - a*d)^3*(c + d*x)^(9
/2))/(9*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(11/2))/(11*d^5) - (8*b^3*(b
*c - a*d)*(c + d*x)^(13/2))/(13*d^5) + (2*b^4*(c + d*x)^(15/2))/(15*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^4(c+dx)^{5/2}\,dx &= \int \left(\frac{(-bc+ad)^4(c+dx)^{5/2}}{d^4}-\frac{4b(bc-ad)^3(c+dx)^{7/2}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{9/2}}{d^4}-\right.\\ &= \frac{2(bc-ad)^4(c+dx)^{7/2}}{7d^5}-\frac{8b(bc-ad)^3(c+dx)^{9/2}}{9d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{11/2}}{11d^5}-8b\end{aligned}$$

Mathematica [A] time = 0.11, size = 101, normalized size = 0.78

$$\frac{2(c+dx)^{7/2}\left(-13860b^3(c+dx)^3(bc-ad)+24570b^2(c+dx)^2(bc-ad)^2-20020b(c+dx)(bc-ad)^3+6435(bc-ad)^4+3003b^4(c+dx)^4\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*(b*c - a*d)^4 - 20020*b*(b*c - a*d)^3*(c + d*x) +
24570*b^2*(b*c - a*d)^2*(c + d*x)^2 - 13860*b^3*(b*c - a*d)*(c + d*x)^3 + 3
003*b^4*(c + d*x)^4))/(45045*d^5)
```

IntegrateAlgebraic [A] time = 0.08, size = 213, normalized size = 1.65

$$\frac{2(c+dx)^{7/2}\left(6435a^4d^4+20020a^3bd^3(c+dx)-25740a^3bcd^3+38610a^2b^2c^2d^2+24570a^2b^2d^2(c+dx)^2-60060a^2b^2cd^2(c+dx)-25740ab^3c^3d+60060ab^3c^2d(c+dx)+13860ab^3d(c+dx)^3-49140ab^3cd(c+dx)^2+6435b^4c^4-20020b^4c^3(c+dx)+24570b^4c^2(c+dx)^2+3003b^4(c+dx)^4-13860b^4c(c+dx)^3\right)}{45045d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(6435*b^4*c^4 - 25740*a*b^3*c^3*d + 38610*a^2*b^2*c^2*d^
2 - 25740*a^3*b*c*d^3 + 6435*a^4*d^4 - 20020*b^4*c^3*(c + d*x) + 60060*a*b^
```

```
3*c^2*d*(c + d*x) - 60060*a^2*b^2*c*d^2*(c + d*x) + 20020*a^3*b*d^3*(c + d*
x) + 24570*b^4*c^2*(c + d*x)^2 - 49140*a*b^3*c*d*(c + d*x)^2 + 24570*a^2*b^
2*d^2*(c + d*x)^2 - 13860*b^4*c*(c + d*x)^3 + 13860*a*b^3*d*(c + d*x)^3 + 3
003*b^4*(c + d*x)^4))/(45045*d^5)
```

fricas [B] time = 1.38, size = 377, normalized size = 2.92

$$\frac{2\left(3003\,b^4d^7x^7 + 128\,b^4c^7 - 960\,ab^3c^6d + 3120\,a^2b^2c^5d^2 - 5720\,a^3bc^4d^3 + 6435\,a^4c^3d^4 + 231\left(31\,b^4cd^6 + 60\,ab^3d^7\right)x^6 + 63\left(71\,b^4c^2d^5 + 540\,ab^3cd^6 + 390\,a^2b^2d^7\right)x^5 + 35\left(b^4c^3d^4 + 636\,ab^3c^2d^5 + 1794\,a^2b^2cd^6 + 572\,a^3bd^7\right)x^4 - 5\left(8\,b^4c^4d^3 - 60\,ab^3c^3d^4 - 8814\,a^2b^2c^2d^5 - 10868\,a^3bcd^6 - 1287\,a^4d^7\right)x^3 + 3\left(16\,b^4c^5d^2 - 120\,ab^3c^4d^3 + 390\,a^2b^2c^3d^4 + 14300\,a^3bc^2d^5 + 6435\,a^4cd^6\right)x^2 - \left(64\,b^4c^6d - 480\,ab^3c^5d^2 + 1560\,a^2b^2c^4d^3 - 2860\,a^3bc^3d^4 - 19305\,a^4c^2d^5\right)x\right)\sqrt{dx+c}}{45045\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^4*d^7*x^7 + 128*b^4*c^7 - 960*a*b^3*c^6*d + 3120*a^2*b^2*c^
5*d^2 - 5720*a^3*b*c^4*d^3 + 6435*a^4*c^3*d^4 + 231*(31*b^4*c*d^6 + 60*a*b^
3*d^7)*x^6 + 63*(71*b^4*c^2*d^5 + 540*a*b^3*c*d^6 + 390*a^2*b^2*d^7)*x^5 +
35*(b^4*c^3*d^4 + 636*a*b^3*c^2*d^5 + 1794*a^2*b^2*c*d^6 + 572*a^3*b*d^7)*x
^4 - 5*(8*b^4*c^4*d^3 - 60*a*b^3*c^3*d^4 - 8814*a^2*b^2*c^2*d^5 - 10868*a^3
*b*c*d^6 - 1287*a^4*d^7)*x^3 + 3*(16*b^4*c^5*d^2 - 120*a*b^3*c^4*d^3 + 390*
a^2*b^2*c^3*d^4 + 14300*a^3*b*c^2*d^5 + 6435*a^4*c*d^6)*x^2 - (64*b^4*c^6*d
 - 480*a*b^3*c^5*d^2 + 1560*a^2*b^2*c^4*d^3 - 2860*a^3*b*c^3*d^4 - 19305*a^
4*c^2*d^5)*x)*sqrt(d*x + c)/d^5
```

giac [B] time = 1.45, size = 1204, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/45045*(45045*sqrt(d*x + c)*a^4*c^3 + 45045*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^4*c^2 + 60060*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^3*b*c^3/d +
 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^4
*c + 18018*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2
)*a^2*b^2*c^3/d^2 + 36036*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sq
rt(d*x + c)*c^2)*a^3*b*c^2/d + 1287*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)
*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^4 + 5148*(5*(d*x + c)
^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c
^3)*a*b^3*c^3/d^3 + 23166*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d
*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^2*c^2/d^2 + 15444*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a^3*b*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d
*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^4*c^
3/d^4 + 1716*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5
/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^3*c^2/d^3 +
2574*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2
- 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b^2*c/d^2 + 572*(35*
(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*
x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^3*b/d + 195*(63*(d*x + c)^(11/2
) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*
c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^4*c^2/d^4 + 780*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a
*b^3*c/d^3 + 390*(63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x +
c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sq
rt(d*x + c)*c^5)*a^2*b^2/d^2 + 45*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(1
1/2)*c + 5005*(d*x + c)^(9/2)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x +
c)^(5/2)*c^4 - 6006*(d*x + c)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^4*c/d^4
 + 60*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2
)*c^2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c
)^(3/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*a*b^3/d^3 + 7*(429*(d*x + c)^(15/2) -
 3465*(d*x + c)^(13/2)*c + 12285*(d*x + c)^(11/2)*c^2 - 25025*(d*x + c)^(9/
```

```
2)*c^3 + 32175*(d*x + c)^(7/2)*c^4 - 27027*(d*x + c)^(5/2)*c^5 + 15015*(d*x
 + c)^(3/2)*c^6 - 6435*sqrt(d*x + c)*c^7)*b^4/d^4)/d
```

maple [A] time = 0.01, size = 186, normalized size = 1.44

$$\frac{2 (dx+c)^{\frac{7}{2}} \left(3003b^4x^4d^4+13860a b^3d^4x^3-1848b^4c d^3x^3+24570a^2b^2d^4x^2-7560a b^3c d^3x^2+1008b^4c^2d^2x^2+20020a^3b d^4x-10920a^2b^2c d^3x+3360a b^3c^2d^2x-448b^4c^3dx+6435a^4d^4-5720a^3bc d^3+3120a^2b^2c^2d^2-960a b^3c^3d+128b^4c^4\right)}{45045d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4*(d*x+c)^(5/2),x)
```

```
[Out] 2/45045*(d*x+c)^(7/2)*(3003*b^4*d^4*x^4+13860*a*b^3*d^4*x^3-1848*b^4*c*d^3*
x^3+24570*a^2*b^2*d^4*x^2-7560*a*b^3*c*d^3*x^2+1008*b^4*c^2*d^2*x^2+20020*a
^3*b*d^4*x-10920*a^2*b^2*c*d^3*x+3360*a*b^3*c^2*d^2*x-448*b^4*c^3*d*x+6435*
a^4*d^4-5720*a^3*b*c*d^3+3120*a^2*b^2*c^2*d^2-960*a*b^3*c^3*d+128*b^4*c^4)/
d^5
```

maxima [A] time = 1.33, size = 181, normalized size = 1.40

$$\frac{2 \left(3003 (dx+c)^{\frac{15}{2}} b^4-13860 \left(b^4c-ab^3d\right)(dx+c)^{\frac{13}{2}}+24570 \left(b^4c^2-2 ab^3cd+a^2b^2d^2\right)(dx+c)^{\frac{11}{2}}-20020 \left(b^4c^3-3 ab^3c^2d+3 a^2b^2cd^2-a^3bd^3\right)(dx+c)^{\frac{9}{2}}+6435 \left(b^4c^4-4 ab^3c^3d+6 a^2b^2c^2d^2-4 a^3bcd^3+a^4d^4\right)(dx+c)^{\frac{7}{2}}\right)}{45045 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*(d*x + c)^(15/2)*b^4 - 13860*(b^4*c - a*b^3*d)*(d*x + c)^(13/
2) + 24570*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(11/2) - 20020*(
b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c)^(9/2) + 64
35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
(d*x + c)^(7/2))/d^5
```

mupad [B] time = 0.23, size = 112, normalized size = 0.87

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{15/2}}{15\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{13/2}}{13\,d^5}+\frac{2\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{11/2}}{11\,d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{9/2}}{9\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(15/2))/(15*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(13/2)
)/(13*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(7/2))/(7*d^5) + (12*b^2*(a*d - b*c
)^2*(c + d*x)^(11/2))/(11*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(9/2))/(9*d^5
)
```

sympy [A] time = 33.64, size = 960, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4*(d*x+c)**(5/2),x)
```

```
[Out] a**4*c**2*Piecewise((sqrt(c)*x, Eq(d, 0)), (2*(c + d*x)**(3/2)/(3*d), True)
) + 4*a**4*c*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*a**4*(c**2*
(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d + 8*a**
3*b*c**2*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d**2 + 16*a**3*b*c*(c
**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2
+ 8*a**3*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c +
 d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**2 + 12*a**2*b**2*c**2*(c**2*(c + d*
x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**3 + 24*a**2*b
**2*c*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)
**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 12*a**2*b**2*(c**4*(c + d*x)**(3/2)/
3 - 4*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**
```

```
(9/2)/9 + (c + d*x)**(11/2)/11)/d**3 + 8*a*b**3*c**2*(-c**3*(c + d*x)**(3/2
)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)
/9)/d**4 + 16*a*b**3*c*(c**4*(c + d*x)**(3/2)/3 - 4*c**3*(c + d*x)**(5/2)/5
 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)/9 + (c + d*x)**(11/2)/1
1)/d**4 + 8*a*b**3*(-c**5*(c + d*x)**(3/2)/3 + c**4*(c + d*x)**(5/2) - 10*c
**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9 - 5*c*(c + d*x)**(11/2)
/11 + (c + d*x)**(13/2)/13)/d**4 + 2*b**4*c**2*(c**4*(c + d*x)**(3/2)/3 - 4
*c**3*(c + d*x)**(5/2)/5 + 6*c**2*(c + d*x)**(7/2)/7 - 4*c*(c + d*x)**(9/2)
/9 + (c + d*x)**(11/2)/11)/d**5 + 4*b**4*c*(-c**5*(c + d*x)**(3/2)/3 + c**4
*(c + d*x)**(5/2) - 10*c**3*(c + d*x)**(7/2)/7 + 10*c**2*(c + d*x)**(9/2)/9
 - 5*c*(c + d*x)**(11/2)/11 + (c + d*x)**(13/2)/13)/d**5 + 2*b**4*(c**6*(c
+ d*x)**(3/2)/3 - 6*c**5*(c + d*x)**(5/2)/5 + 15*c**4*(c + d*x)**(7/2)/7 -
20*c**3*(c + d*x)**(9/2)/9 + 15*c**2*(c + d*x)**(11/2)/11 - 6*c*(c + d*x)**
(13/2)/13 + (c + d*x)**(15/2)/15)/d**5
```

3.1295 $\int (a+bx)^3(c+dx)^{5/2}\,dx$

Optimal. Leaf size=100

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Rubi [A] time = 0.03, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{11/2}(bc-ad)}{11d^4}+\frac{2b(c+dx)^{9/2}(bc-ad)^2}{3d^4}-\frac{2(c+dx)^{7/2}(bc-ad)^3}{7d^4}+\frac{2b^3(c+dx)^{13/2}}{13d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(b*c - a*d)^2*(c + d*x)^(
9/2))/(3*d^4) - (6*b^2*(b*c - a*d)*(c + d*x)^(11/2))/(11*d^4) + (2*b^3*(c +
 d*x)^(13/2))/(13*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3(c+dx)^{5/2}\,dx=\int \left(\frac{(-bc+ad)^3(c+dx)^{5/2}}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{7/2}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{9/2}}{d^3}+\ldots\right)$$

$$=-\frac{2(bc-ad)^3(c+dx)^{7/2}}{7d^4}+\frac{2b(bc-ad)^2(c+dx)^{9/2}}{3d^4}-\frac{6b^2(bc-ad)(c+dx)^{11/2}}{11d^4}+\ldots$$

Mathematica [A] time = 0.07, size = 79, normalized size = 0.79

$$\frac{2(c+dx)^{7/2}\left(-819b^2(c+dx)^2(bc-ad)+1001b(c+dx)(bc-ad)^2-429(bc-ad)^3+231b^3(c+dx)^3\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-429*(b*c - a*d)^3 + 1001*b*(b*c - a*d)^2*(c + d*x) - 8
19*b^2*(b*c - a*d)*(c + d*x)^2 + 231*b^3*(c + d*x)^3))/(3003*d^4)
```

IntegrateAlgebraic [A] time = 0.06, size = 132, normalized size = 1.32

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+1001a^2bd^2(c+dx)-1287a^2bcd^2+1287ab^2c^2d+819ab^2d(c+dx)^2-2002ab^2cd(c+dx)-429b^3c^3+1001b^3c^2(c+dx)+231b^3(c+dx)^3-819b^3c(c+dx)^2\right)}{3003d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-429*b^3*c^3 + 1287*a*b^2*c^2*d - 1287*a^2*b*c*d^2 + 42
9*a^3*d^3 + 1001*b^3*c^2*(c + d*x) - 2002*a*b^2*c*d*(c + d*x) + 1001*a^2*b*
```

```
d^2*(c + d*x) - 819*b^3*c*(c + d*x)^2 + 819*a*b^2*d*(c + d*x)^2 + 231*b^3*(
c + d*x)^3))/(3003*d^4)
```

fricas [B] time = 1.54, size = 268, normalized size = 2.68

$$\frac{2\left(231\,b^3d^6x^6 - 16\,b^3c^6 + 104\,ab^2c^5d - 286\,a^2bc^4d^2 + 429\,a^3c^3d^3 + 63\left(9\,b^3cd^5 + 13\,ab^2d^6\right)x^5 + 7\left(53\,b^3c^2d^4 + 299\,ab^2cd^5 + 143\,a^2bd^6\right)x^4 + \left(5\,b^3c^3d^3 + 1469\,ab^2c^2d^4 + 2717\,a^2bcd^5 + 429\,a^3d^6\right)x^3 - 3\left(2\,b^3c^4d^2 - 13\,ab^2c^3d^3 - 715\,a^2bc^2d^4 - 429\,a^3cd^5\right)x^2 + \left(8\,b^3c^5d - 52\,ab^2c^4d^2 + 143\,a^2bc^3d^3 + 1287\,a^3c^2d^4\right)x\right)\sqrt{dx+c}}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(231*b^3*d^6*x^6 - 16*b^3*c^6 + 104*a*b^2*c^5*d - 286*a^2*b*c^4*d^2
+ 429*a^3*c^3*d^3 + 63*(9*b^3*c*d^5 + 13*a*b^2*d^6)*x^5 + 7*(53*b^3*c^2*d^4
 + 299*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 + (5*b^3*c^3*d^3 + 1469*a*b^2*c^2*d
^4 + 2717*a^2*b*c*d^5 + 429*a^3*d^6)*x^3 - 3*(2*b^3*c^4*d^2 - 13*a*b^2*c^3*
d^3 - 715*a^2*b*c^2*d^4 - 429*a^3*c*d^5)*x^2 + (8*b^3*c^5*d - 52*a*b^2*c^4*
d^2 + 143*a^2*b*c^3*d^3 + 1287*a^3*c^2*d^4)*x)*sqrt(d*x + c)/d^4
```

giac [B] time = 1.58, size = 857, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15015*(15015*sqrt(d*x + c)*a^3*c^3 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x
+ c)*c)*a^3*c^2 + 15015*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b*c^3/d +
 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^3
*c + 3003*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)
*a*b^2*c^3/d^2 + 9009*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d
*x + c)*c^2)*a^2*b*c^2/d + 429*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c +
35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^3 + 429*(5*(d*x + c)^(7/2)
 - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^
3*c^3/d^3 + 3861*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(
3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^2*c^2/d^2 + 3861*(5*(d*x + c)^(7/2) -
21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b
*c/d + 143*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2
)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*b^3*c^2/d^3 + 429*
(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420
*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b^2*c/d^2 + 143*(35*(d*x +
c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^
(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a^2*b/d + 65*(63*(d*x + c)^(11/2) - 385*
(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 11
55*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^3*c/d^3 + 65*(63*(d*x + c
)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)
^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*a*b^2/d^2 +
5*(231*(d*x + c)^(13/2) - 1638*(d*x + c)^(11/2)*c + 5005*(d*x + c)^(9/2)*c^
2 - 8580*(d*x + c)^(7/2)*c^3 + 9009*(d*x + c)^(5/2)*c^4 - 6006*(d*x + c)^(3
/2)*c^5 + 3003*sqrt(d*x + c)*c^6)*b^3/d^3)/d
```

maple [A] time = 0.01, size = 116, normalized size = 1.16

$$\frac{2\,(dx+c)^{\frac{7}{2}}\left(231b^3x^3d^3 + 819a\,b^2d^3x^2 - 126b^3c\,d^2x^2 + 1001a^2b\,d^3x - 364a\,b^2c\,d^2x + 56b^3c^2dx + 429a^3d^3 - 286a^2bc\,d^2 + 104a\,b^2c^2d - 16b^3c^3\right)}{3003d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^(5/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(231*b^3*d^3*x^3+819*a*b^2*d^3*x^2-126*b^3*c*d^2*x^2+1
001*a^2*b*d^3*x-364*a*b^2*c*d^2*x+56*b^3*c^2*d*x+429*a^3*d^3-286*a^2*b*c*d^
2+104*a*b^2*c^2*d-16*b^3*c^3)/d^4
```

maxima [A] time = 1.40, size = 118, normalized size = 1.18

$$\frac{2\left(231\,(dx+c)^{\frac{13}{2}}b^3 - 819\left(b^3c - ab^2d\right)(dx+c)^{\frac{11}{2}} + 1001\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)(dx+c)^{\frac{9}{2}} - 429\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)(dx+c)^{\frac{7}{2}}\right)}{3003\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3003*(231*(d*x + c)^(13/2)*b^3 - 819*(b^3*c - a*b^2*d)*(d*x + c)^(11/2) +
 1001*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*(d*x + c)^(9/2) - 429*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(7/2))/d^4
```

mupad [B] time = 0.08, size = 87, normalized size = 0.87

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{13/2}}{13\,d^4} - \frac{\left(6\,b^3\,c - 6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{11/2}}{11\,d^4} + \frac{2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4} + \frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{9/2}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(13/2))/(13*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(11/2)
)/(11*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(7/2))/(7*d^4) + (2*b*(a*d - b*c)^2
*(c + d*x)^(9/2))/(3*d^4)
```

sympy [A] time = 4.61, size = 549, normalized size = 5.49

$$\begin{cases} \frac{2a^3c^3\sqrt{c+dx}}{7d} + \frac{6a^3c^2x\sqrt{c+dx}}{7} + \frac{6a^3cdx^2\sqrt{c+dx}}{7} + \frac{2a^3d^2x^3\sqrt{c+dx}}{7} - \frac{4a^2bc^4\sqrt{c+dx}}{21d^2} + \frac{2a^2bc^3x\sqrt{c+dx}}{21d} + \frac{10a^2bc^2x^2\sqrt{c+dx}}{7} + \frac{38a^2bcdx^3\sqrt{c+dx}}{21} + \frac{2a^2bd^2x^4\sqrt{c+dx}}{3} + \frac{16ab^2c^5\sqrt{c+dx}}{231d^3} - \frac{8ab^2c^4x\sqrt{c+dx}}{231d^2} + \frac{2ab^2c^3x^2\sqrt{c+dx}}{77d} + \frac{226ab^2c^2x^3\sqrt{c+dx}}{231} + \frac{46ab^2cdx^4\sqrt{c+dx}}{33} + \frac{6ab^2d^2x^5\sqrt{c+dx}}{11} - \frac{32b^3c^6\sqrt{c+dx}}{3003d^4} + \frac{16b^3c^5x\sqrt{c+dx}}{3003d^3} - \frac{4b^3c^4x^2\sqrt{c+dx}}{1001d^2} + \frac{10b^3c^3x^3\sqrt{c+dx}}{3003d} + \frac{106b^3c^2x^4\sqrt{c+dx}}{429} + \frac{54b^3cdx^5\sqrt{c+dx}}{143} + \frac{2b^3d^2x^6\sqrt{c+dx}}{13} & \text{for}\: d \neq 0 \\ c^{\frac{5}{2}}\left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**3*c**3*sqrt(c + d*x)/(7*d) + 6*a**3*c**2*x*sqrt(c + d*x)/7
+ 6*a**3*c*d*x**2*sqrt(c + d*x)/7 + 2*a**3*d**2*x**3*sqrt(c + d*x)/7 - 4*a*
*2*b*c**4*sqrt(c + d*x)/(21*d**2) + 2*a**2*b*c**3*x*sqrt(c + d*x)/(21*d) +
10*a**2*b*c**2*x**2*sqrt(c + d*x)/7 + 38*a**2*b*c*d*x**3*sqrt(c + d*x)/21 +
 2*a**2*b*d**2*x**4*sqrt(c + d*x)/3 + 16*a*b**2*c**5*sqrt(c + d*x)/(231*d**
3) - 8*a*b**2*c**4*x*sqrt(c + d*x)/(231*d**2) + 2*a*b**2*c**3*x**2*sqrt(c +
 d*x)/(77*d) + 226*a*b**2*c**2*x**3*sqrt(c + d*x)/231 + 46*a*b**2*c*d*x**4*
sqrt(c + d*x)/33 + 6*a*b**2*d**2*x**5*sqrt(c + d*x)/11 - 32*b**3*c**6*sqrt(
c + d*x)/(3003*d**4) + 16*b**3*c**5*x*sqrt(c + d*x)/(3003*d**3) - 4*b**3*c*
*4*x**2*sqrt(c + d*x)/(1001*d**2) + 10*b**3*c**3*x**3*sqrt(c + d*x)/(3003*d
) + 106*b**3*c**2*x**4*sqrt(c + d*x)/429 + 54*b**3*c*d*x**5*sqrt(c + d*x)/1
43 + 2*b**3*d**2*x**6*sqrt(c + d*x)/13, Ne(d, 0)), (c**(5/2)*(a**3*x + 3*a*
*2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), True))
```

3.1296 $\int (a+bx)^2(c+dx)^{5/2}\,dx$

Optimal. Leaf size=71

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Rubi [A] time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{9/2}(bc-ad)}{9d^3}+\frac{2(c+dx)^{7/2}(bc-ad)^2}{7d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^2*(c + d*x)^(7/2))/(7*d^3) - (4*b*(b*c - a*d)*(c + d*x)^(9/2
))/(9*d^3) + (2*b^2*(c + d*x)^(11/2))/(11*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^{5/2}\,dx&=\int\left(\frac{(-bc+ad)^2(c+dx)^{5/2}}{d^2}-\frac{2b(bc-ad)(c+dx)^{7/2}}{d^2}+\frac{b^2(c+dx)^{9/2}}{d^2}\right)dx\\&=\frac{2(bc-ad)^2(c+dx)^{7/2}}{7d^3}-\frac{4b(bc-ad)(c+dx)^{9/2}}{9d^3}+\frac{2b^2(c+dx)^{11/2}}{11d^3}\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 0.86

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(7dx-2c)+b^2\left(8c^2-28cdx+63d^2x^2\right)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-2*c + 7*d*x) + b^2*(8*c^2 - 28*
c*d*x + 63*d^2*x^2)))/(693*d^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 1.01

$$\frac{2(c+dx)^{7/2}\left(99a^2d^2+154abd(c+dx)-198abcd+99b^2c^2+63b^2(c+dx)^2-154b^2c(c+dx)\right)}{693d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(99*b^2*c^2 - 198*a*b*c*d + 99*a^2*d^2 - 154*b^2*c*(c +
d*x) + 154*a*b*d*(c + d*x) + 63*b^2*(c + d*x)^2))/(693*d^3)
```

fricas [B] time = 1.23, size = 174, normalized size = 2.45

$$\frac{2\left(63\,b^{2}d^{5}x^{5}+8\,b^{2}c^{5}-44\,abc^{4}d+99\,a^{2}c^{3}d^{2}+7\left(23\,b^{2}cd^{4}+22\,abd^{5}\right)x^{4}+\left(113\,b^{2}c^{2}d^{3}+418\,abcd^{4}+99\,a^{2}d^{5}\right)x^{3}+3\left(b^{2}c^{3}d^{2}+110\,abc^{2}d^{3}+99\,a^{2}cd^{4}\right)x^{2}-\left(4\,b^{2}c^{4}d-22\,abc^{3}d^{2}-297\,a^{2}c^{2}d^{3}\right)x\right)\sqrt{dx+c}}{693\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^2*d^5*x^5 + 8*b^2*c^5 - 44*a*b*c^4*d + 99*a^2*c^3*d^2 + 7*(23*b
^2*c*d^4 + 22*a*b*d^5)*x^4 + (113*b^2*c^2*d^3 + 418*a*b*c*d^4 + 99*a^2*d^5)
*x^3 + 3*(b^2*c^3*d^2 + 110*a*b*c^2*d^3 + 99*a^2*c*d^4)*x^2 - (4*b^2*c^4*d
- 22*a*b*c^3*d^2 - 297*a^2*c^2*d^3)*x)*sqrt(d*x + c)/d^3
```

giac [B] time = 1.76, size = 558, normalized size = 7.86

$$\frac{2\left(3465\,\sqrt{dx+c}\,a^{2}c^{3}+3465\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^{2}c^{2}+\frac{2310\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)abc^{3}}{d}+693\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)a^{2}c+\frac{231\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)b^{2}c^{3}}{d^{2}}+\frac{1386\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)abc^{2}}{d}+99\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)a^{2}+\frac{297\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)b^{2}c^{2}}{d^{2}}+\frac{594\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}-35\,\sqrt{dx+c}\,c^{3}\right)abc}{d}+\frac{33\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\,\sqrt{dx+c}\,c^{4}\right)b^{2}c}{d^{2}}+\frac{22\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^{2}-420\,(dx+c)^{\frac{3}{2}}c^{3}+315\,\sqrt{dx+c}\,c^{4}\right)ab}{d}+\frac{5\left(63\,(dx+c)^{\frac{11}{2}}-385\,(dx+c)^{\frac{9}{2}}c+990\,(dx+c)^{\frac{7}{2}}c^{2}-1386\,(dx+c)^{\frac{5}{2}}c^{3}+1155\,(dx+c)^{\frac{3}{2}}c^{4}-693\,\sqrt{dx+c}\,c^{5}\right)b^{2}}{d^{2}}\right)}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3465*(3465*sqrt(d*x + c)*a^2*c^3 + 3465*((d*x + c)^(3/2) - 3*sqrt(d*x + c
)*c)*a^2*c^2 + 2310*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b*c^3/d + 693*(
3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a^2*c + 23
1*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2*c^3
/d^2 + 1386*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a*b*c^2/d + 99*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^
(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2 + 297*(5*(d*x + c)^(7/2) - 21*(d*x +
c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^2*c^2/d^2 + 5
94*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*
sqrt(d*x + c)*c^3)*a*b*c/d + 33*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c
 + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^
4)*b^2*c/d^2 + 22*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x +
c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d + 5*(
63*(d*x + c)^(11/2) - 385*(d*x + c)^(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 138
6*(d*x + c)^(5/2)*c^3 + 1155*(d*x + c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b
^2/d^2)/d
```

maple [A] time = 0.01, size = 63, normalized size = 0.89

$$\frac{2\,(dx+c)^{\frac{7}{2}}\left(63b^{2}x^{2}d^{2}+154ab\,d^{2}x-28b^{2}cdx+99a^{2}d^{2}-44abcd+8b^{2}c^{2}\right)}{693d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^(5/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(63*b^2*d^2*x^2+154*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-4
4*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.38, size = 68, normalized size = 0.96

$$\frac{2\left(63\,(dx+c)^{\frac{11}{2}}b^{2}-154\left(b^{2}c-abd\right)(dx+c)^{\frac{9}{2}}+99\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)(dx+c)^{\frac{7}{2}}\right)}{693\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/693*(63*(d*x + c)^(11/2)*b^2 - 154*(b^2*c - a*b*d)*(d*x + c)^(9/2) + 99*(
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(d*x + c)^(7/2))/d^3
```

mupad [B] time = 0.07, size = 68, normalized size = 0.96

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(63\,b^2\,{\left(c+d\,x\right)}^2+99\,a^2\,d^2+99\,b^2\,c^2-154\,b^2\,c\,\left(c+d\,x\right)+154\,a\,b\,d\,\left(c+d\,x\right)-198\,a\,b\,c\,d\right)}{693\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(63*b^2*(c + d*x)^2 + 99*a^2*d^2 + 99*b^2*c^2 - 154*b^2*
c*(c + d*x) + 154*a*b*d*(c + d*x) - 198*a*b*c*d))/(693*d^3)
```

sympy [A] time = 3.58, size = 355, normalized size = 5.00

$$\begin{cases} \frac{2a^2c^3\sqrt{c+dx}}{7d} + \frac{6a^2c^2x\sqrt{c+dx}}{7} + \frac{6a^2cdx^2\sqrt{c+dx}}{7} + \frac{2a^2d^2x^3\sqrt{c+dx}}{7} - \frac{8abc^4\sqrt{c+dx}}{63d^2} + \frac{4abc^3x\sqrt{c+dx}}{63d} + \frac{20abc^2x^2\sqrt{c+dx}}{21} + \frac{76abcdx^3\sqrt{c+dx}}{63} + \frac{4abd^2x^4\sqrt{c+dx}}{9} + \frac{16b^2c^5\sqrt{c+dx}}{693d^3} - \frac{8b^2c^4x\sqrt{c+dx}}{693d^2} + \frac{2b^2c^3x^2\sqrt{c+dx}}{231d} + \frac{226b^2c^2x^3\sqrt{c+dx}}{693} + \frac{46b^2cdx^4\sqrt{c+dx}}{99} + \frac{2b^2d^2x^5\sqrt{c+dx}}{11} & \text{for}\ d \neq 0 \\ c^{\frac{5}{2}}\left(a^2x + abx^2 + \frac{b^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a**2*c**3*sqrt(c + d*x)/(7*d) + 6*a**2*c**2*x*sqrt(c + d*x)/7
+ 6*a**2*c*d*x**2*sqrt(c + d*x)/7 + 2*a**2*d**2*x**3*sqrt(c + d*x)/7 - 8*a*
b*c**4*sqrt(c + d*x)/(63*d**2) + 4*a*b*c**3*x*sqrt(c + d*x)/(63*d) + 20*a*b
*c**2*x**2*sqrt(c + d*x)/21 + 76*a*b*c*d*x**3*sqrt(c + d*x)/63 + 4*a*b*d**2
*x**4*sqrt(c + d*x)/9 + 16*b**2*c**5*sqrt(c + d*x)/(693*d**3) - 8*b**2*c**4
*x*sqrt(c + d*x)/(693*d**2) + 2*b**2*c**3*x**2*sqrt(c + d*x)/(231*d) + 226*
b**2*c**2*x**3*sqrt(c + d*x)/693 + 46*b**2*c*d*x**4*sqrt(c + d*x)/99 + 2*b*
*2*d**2*x**5*sqrt(c + d*x)/11, Ne(d, 0)), (c**(5/2)*(a**2*x + a*b*x**2 + b*
*2*x**3/3), True))
```

3.1297 $\int (a+bx)(c+dx)^{5/2}\,dx$

Optimal. Leaf size=42

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Rubi [A] time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{9/2}}{9d^2}-\frac{2(c+dx)^{7/2}(bc-ad)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (-2*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^2) + (2*b*(c + d*x)^(9/2))/(9*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^{5/2}\,dx&=\int \left(\frac{(-bc+ad)(c+dx)^{5/2}}{d}+\frac{b(c+dx)^{7/2}}{d}\right)dx\\&=-\frac{2(bc-ad)(c+dx)^{7/2}}{7d^2}+\frac{2b(c+dx)^{9/2}}{9d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.71

$$\frac{2(c+dx)^{7/2}(9ad-2bc+7bdx)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2)*(-2*b*c + 9*a*d + 7*b*d*x))/(63*d^2)
```

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 0.79

$$\frac{2(c+dx)^{7/2}(9ad+7b(c+dx)-9bc)}{63d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^(5/2),x]
[Out] (2*(c + d*x)^(7/2)*(-9*b*c + 9*a*d + 7*b*(c + d*x)))/(63*d^2)
```

fricas [B] time = 1.22, size = 93, normalized size = 2.21

$$\frac{2\left(7\,bd^4x^4-2\,bc^4+9\,ac^3d+\left(19\,bcd^3+9\,ad^4\right)x^3+3\left(5\,bc^2d^2+9\,acd^3\right)x^2+\left(bc^3d+27\,ac^2d^2\right)x\right)\sqrt{dx+c}}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b*d^4*x^4 - 2*b*c^4 + 9*a*c^3*d + (19*b*c*d^3 + 9*a*d^4)*x^3 + 3*(5
*b*c^2*d^2 + 9*a*c*d^3)*x^2 + (b*c^3*d + 27*a*c^2*d^2)*x)*sqrt(d*x + c)/d^2
```

giac [B] time = 1.61, size = 306, normalized size = 7.29

$$\frac{2\left(315\sqrt{dx+c}\,ac^3+315\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)ac^2+\frac{105\left((dx+c)^{\frac{3}{2}}-3\sqrt{dx+c}\,c\right)bc^3}{d}+63\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)ac+\frac{63\left(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}\,c^2\right)bc^2}{d}+9\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)a+\frac{27\left(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}\,c^3\right)bc}{d}+\frac{\left(35(dx+c)^{\frac{9}{2}}-180(dx+c)^{\frac{7}{2}}c+378(dx+c)^{\frac{5}{2}}c^2-420(dx+c)^{\frac{3}{2}}c^3+315\sqrt{dx+c}\,c^4\right)b}{d}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a*c^3 + 315*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*
a*c^2 + 105*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b*c^3/d + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*c + 63*(3*(d*x + c)
^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b*c^2/d + 9*(5*(d*x +
 c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c
)*c^3)*a + 27*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2
)*c^2 - 35*sqrt(d*x + c)*c^3)*b*c/d + (35*(d*x + c)^(9/2) - 180*(d*x + c)^(
7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x +
 c)*c^4)*b/d)/d
```

maple [A] time = 0.00, size = 27, normalized size = 0.64

$$\frac{2\,(dx+c)^{\frac{7}{2}}\,(7bdx+9ad-2bc)}{63d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(5/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(7*b*d*x+9*a*d-2*b*c)/d^2
```

maxima [A] time = 1.40, size = 33, normalized size = 0.79

$$\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b-9\,(bc-ad)(dx+c)^{\frac{7}{2}}\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/63*(7*(d*x + c)^(9/2)*b - 9*(b*c - a*d)*(d*x + c)^(7/2))/d^2
```

mupad [B] time = 0.05, size = 29, normalized size = 0.69

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}\,\left(9\,a\,d-9\,b\,c+7\,b\,\left(c+d\,x\right)\right)}{63\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2)*(9*a*d - 9*b*c + 7*b*(c + d*x)))/(63*d^2)
```

sympy [A] time = 2.36, size = 194, normalized size = 4.62

$$\begin{cases} \frac{2ac^3\sqrt{c+dx}}{7d}+\frac{6ac^2x\sqrt{c+dx}}{7}+\frac{6acdx^2\sqrt{c+dx}}{7}+\frac{2ad^2x^3\sqrt{c+dx}}{7}-\frac{4bc^4\sqrt{c+dx}}{63d^2}+\frac{2bc^3x\sqrt{c+dx}}{63d}+\frac{10bc^2x^2\sqrt{c+dx}}{21}+\frac{38bcdx^3\sqrt{c+dx}}{63}+\frac{2bd^2x^4\sqrt{c+dx}}{9} & \text{for } d \neq 0 \\ c^{\frac{5}{2}}\left(ax+\frac{bx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((2*a*c**3*sqrt(c + d*x)/(7*d) + 6*a*c**2*x*sqrt(c + d*x)/7 + 6*a*
c*d*x**2*sqrt(c + d*x)/7 + 2*a*d**2*x**3*sqrt(c + d*x)/7 - 4*b*c**4*sqrt(c
+ d*x)/(63*d**2) + 2*b*c**3*x*sqrt(c + d*x)/(63*d) + 10*b*c**2*x**2*sqrt(c
+ d*x)/21 + 38*b*c*d*x**3*sqrt(c + d*x)/63 + 2*b*d**2*x**4*sqrt(c + d*x)/9,
 Ne(d, 0)), (c**(5/2)*(a*x + b*x**2/2), True))
```

3.1298 $\int (c+dx)^{5/2}\,dx$

Optimal. Leaf size=16

$$\frac{2(c+dx)^{7/2}}{7d}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^{5/2}\,dx = \frac{2(c+dx)^{7/2}}{7d}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$\frac{2(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2),x]
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

fricas [B] time = 1.22, size = 39, normalized size = 2.44

$$\frac{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\sqrt{dx+c}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(d*x + c)/d
```

giac [B] time = 1.78, size = 95, normalized size = 5.94

$$\frac{2\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^{2}+35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)c^{2}+7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^{2}\right)c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 + 3
5*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*c^2 + 7*(3*(d*x + c)^(5/2) - 10*(d*
x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*c)/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2),x)
```

```
[Out] 2/7*(d*x+c)^(7/2)/d
```

maxima [A] time = 1.30, size = 12, normalized size = 0.75

$$\frac{2\,(dx+c)^{\frac{7}{2}}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(d*x + c)^(7/2)/d
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(7/2))/(7*d)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.75

$$\frac{2\,(c+dx)^{\frac{7}{2}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2),x)
```

```
[Out] 2*(c + d*x)**(7/2)/(7*d)
```

3.1299 $\int \frac{(c+dx)^{5/2}}{a+bx} \, dx$

Optimal. Leaf size=112

$$-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}+\frac{2(c+dx)^{5/2}}{5b}$$

Rubi [A] time = 0.06, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {50, 63, 208}

$$\frac{2\sqrt{c+dx}\,(bc-ad)^2}{b^3}+\frac{2(c+dx)^{3/2}(bc-ad)}{3b^2}-\frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/b^3 + (2*(b*c - a*d)*(c + d*x)^(3/2))/(3*b^
2) + (2*(c + d*x)^(5/2))/(5*b) - (2*(b*c - a*d)^(5/2)*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{a+bx}\,dx = \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{b}$$
$$= \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{b^2}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{(bc-ad)^3\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{b^3}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} + \frac{\left(2(bc-ad)^3\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+ \cdots}\right.}{b^3 d}$$
$$= \frac{2(bc-ad)^2\sqrt{c+dx}}{b^3} + \frac{2(bc-ad)(c+dx)^{3/2}}{3b^2} + \frac{2(c+dx)^{5/2}}{5b} - \frac{2(bc-ad)^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+d\cdots}}{\sqrt{bc-a\cdots}}\right.}{b^{7/2}}$$

Mathematica [A] time = 0.16, size = 105, normalized size = 0.94

$$\frac{2(bc-ad)\left(\sqrt{b}\sqrt{c+dx}\,(-3ad+4bc+bdx)-3(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\right)}{3b^{7/2}}+\frac{2(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) + (2*(b*c - a*d)*(Sqrt[b]*Sqrt[c + d*x]*(4*b*c -
3*a*d + b*d*x) - 3*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b
*c - a*d]]))/(3*b^(7/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 130, normalized size = 1.16

$$\frac{2\sqrt{c+dx}\left(15a^2d^2-5abd(c+dx)-30abcd+15b^2c^2+3b^2(c+dx)^2+5b^2c(c+dx)\right)}{15b^3}+\frac{2(ad-bc)^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x),x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 + 5*b^2*c*(c + d*x)
- 5*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(15*b^3) + (2*(-(b*c) + a*d)^(5/2
)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/b^(7/2)
```

fricas [A] time = 1.25, size = 290, normalized size = 2.59

$$\left[\frac{15\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(3b^2d^2x^2+23b^2c^2-35abcd+15a^2d^2+\left(11b^2cd-5abd^2\right)x\right)\sqrt{dx+c}}{15b^3}, -\frac{2\left(15\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(3b^2d^2x^2+23b^2c^2-35abcd+15a^2d^2+\left(11b^2cd-5abd^2\right)x\right)\sqrt{dx+c}\right)}{15b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d*x +
2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(3*b^2*
d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)*x
)*sqrt(d*x + c))/b^3, -2/15*(15*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c
- a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)) - (3*b^
2*d^2*x^2 + 23*b^2*c^2 - 35*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d - 5*a*b*d^2)
*x)*sqrt(d*x + c))/b^3]
```

giac [A] time = 1.60, size = 171, normalized size = 1.53

$$\frac{2\left(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,b^3} + \frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^4 + 5\,(dx+c)^{\frac{3}{2}}b^4c + 15\,\sqrt{dx+c}\,b^4c^2 - 5\,(dx+c)^{\frac{3}{2}}ab^3d - 30\,\sqrt{dx+c}\,ab^3cd + 15\,\sqrt{dx+c}\,a^2b^2d^2\right)}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(sqrt(d*x + c)*
b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) + 2/15*(3*(d*x + c)^(5/2
)*b^4 + 5*(d*x + c)^(3/2)*b^4*c + 15*sqrt(d*x + c)*b^4*c^2 - 5*(d*x + c)^(3
/2)*a*b^3*d - 30*sqrt(d*x + c)*a*b^3*c*d + 15*sqrt(d*x + c)*a^2*b^2*d^2)/b^
5
```

maple [B] time = 0.01, size = 263, normalized size = 2.35

$$-\frac{2a^3d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^3} + \frac{6a^2c\,d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b^2} - \frac{6a\,c^2d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b} + \frac{2c^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}} + \frac{2\sqrt{dx+c}\,a^2d^2}{b^3} - \frac{4\sqrt{dx+c}\,acd}{b^2} + \frac{2\sqrt{dx+c}\,c^2}{b} - \frac{2\,(dx+c)^{\frac{3}{2}}\,ad}{3b^2} + \frac{2\,(dx+c)^{\frac{3}{2}}\,c}{3b} + \frac{2\,(dx+c)^{\frac{5}{2}}}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a),x)
```

```
[Out] 2/5*(d*x+c)^(5/2)/b-2/3/b^2*(d*x+c)^(3/2)*a*d+2/3/b*(d*x+c)^(3/2)*c+2/b^3*a
^2*d^2*(d*x+c)^(1/2)-4/b^2*a*c*d*(d*x+c)^(1/2)+2/b*c^2*(d*x+c)^(1/2)-2/b^3/
((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^3*d^3+6/b
^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*c*d^
2-6/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c^2
*d+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.08, size = 130, normalized size = 1.16

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{5\,b} - \frac{2\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2} + \frac{2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{b^3} - \frac{2\,\mathrm{atan}\left(\frac{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{5/2}\,\sqrt{c+d\,x}}{a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3}\right)\,{\left(a\,d-b\,c\right)}^{5/2}}{b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x),x)
```

```
[Out] (2*(c + d*x)^(5/2))/(5*b) - (2*(a*d - b*c)*(c + d*x)^(3/2))/(3*b^2) + (2*(a
*d - b*c)^2*(c + d*x)^(1/2))/b^3 - (2*atan((b^(1/2)*(a*d - b*c)^(5/2)*(c +
d*x)^(1/2))/(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))*(a*d - b*c
)^(5/2))/b^(7/2)
```

sympy [A] time = 27.01, size = 121, normalized size = 1.08

$$\frac{2\,(c+dx)^{\frac{5}{2}}}{5b} + \frac{(c+dx)^{\frac{3}{2}}\,(-2ad+2bc)}{3b^2} + \frac{\sqrt{c+dx}\,\left(2a^2d^2 - 4abcd + 2b^2c^2\right)}{b^3} - \frac{2\,(ad-bc)^3\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{b^4\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a),x)
```

```
[Out] 2*(c + d*x)**(5/2)/(5*b) + (c + d*x)**(3/2)*(-2*a*d + 2*b*c)/(3*b**2) + sqr
t(c + d*x)*(2*a**2*d**2 - 4*a*b*c*d + 2*b**2*c**2)/b**3 - 2*(a*d - b*c)**3*
atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(b**4*sqrt((a*d - b*c)/b))
```

3.1300 $\int \frac{(c+dx)^{5/2}}{(a+bx)^2} dx$

Optimal. Leaf size=110

$$-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}+\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$\frac{5d\sqrt{c+dx}\,(bc-ad)}{b^3}-\frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}-\frac{(c+dx)^{5/2}}{b(a+bx)}+\frac{5d(c+dx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (5*d*(b*c - a*d)*Sqrt[c + d*x])/b^3 + (5*d*(c + d*x)^(3/2))/(3*b^2) - (c +
d*x)^(5/2)/(b*(a + b*x)) - (5*d*(b*c - a*d)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[b*c - a*d]])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^2}\,dx = -\frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{a+bx}\,dx}{2b}$$
$$= \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{(5d(bc-ad))\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{2b^2}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{\left(5d(bc-ad)^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2b^3}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{b^3}$$
$$= \frac{5d(bc-ad)\sqrt{c+dx}}{b^3} + \frac{5d(c+dx)^{3/2}}{3b^2} - \frac{(c+dx)^{5/2}}{b(a+bx)} - \frac{5d(bc-ad)^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.45

$$\frac{2d(c+dx)^{7/2}\,{}_2F_1\left(2,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (2*d*(c + d*x)^(7/2)*Hypergeometric2F1[2, 7/2, 9/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(7*(-(b*c) + a*d)^2)
```

IntegrateAlgebraic [A] time = 0.37, size = 187, normalized size = 1.70

$$\frac{d\sqrt{c+dx}\left(-15a^2d^2-10abd(c+dx)+30abcd-15b^2c^2+2b^2(c+dx)^2+10b^2c(c+dx)\right)}{3b^3(ad+b(c+dx)-bc)} + \frac{5\left(-a^3d^4+3a^2bcd^3-3ab^2c^2d^2+b^3c^3d\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{b^{7/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^2,x]
```

```
[Out] (d*Sqrt[c + d*x]*(-15*b^2*c^2 + 30*a*b*c*d - 15*a^2*d^2 + 10*b^2*c*(c + d*x
) - 10*a*b*d*(c + d*x) + 2*b^2*(c + d*x)^2))/(3*b^3*(-(b*c) + a*d + b*(c +
d*x))) + (5*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*ArcTan[
(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(b^(7/2)*(-(b*c) +
 a*d)^(3/2))
```

fricas [A] time = 1.25, size = 330, normalized size = 3.00

$$\left[-\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad+2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)-2\left(2b^2d^2x^2-3b^2c^2+20abcd-15a^2d^2+2\left(7b^2cd-5abd^2\right)x\right)\sqrt{dx+c}}{6\left(b^4x+ab^3\right)}, -\frac{15\left(abcd-a^2d^2+\left(b^2cd-abd^2\right)x\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(2b^2d^2x^2-3b^2c^2+20abcd-15a^2d^2+2\left(7b^2cd-5abd^2\right)x\right)\sqrt{dx+c}}{3\left(b^4x+ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(a*b*c*d - a^2*d^2 + (b^2*c*d - a*b*d^2)*x)*sqrt((b*c - a*d)/b)*l
og((b*d*x + 2*b*c - a*d + 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a))
 - 2*(2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*a^2*d^2 + 2*(7*b^2*c*d -
5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3), -1/3*(15*(a*b*c*d - a^2*d^2 +
 (b^2*c*d - a*b*d^2)*x)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-
```

```
(b*c - a*d)/b)/(b*c - a*d)) - (2*b^2*d^2*x^2 - 3*b^2*c^2 + 20*a*b*c*d - 15*
a^2*d^2 + 2*(7*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

giac [A] time = 1.28, size = 181, normalized size = 1.65

$$\frac{5\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\sqrt{-b^{2}c+abd}\,b^{3}}-\frac{\sqrt{dx+c}\,b^{2}c^{2}d-2\,\sqrt{dx+c}\,abcd^{2}+\sqrt{dx+c}\,a^{2}d^{3}}{((dx+c)b-bc+ad)b^{3}}+\frac{2\left((dx+c)^{\frac{3}{2}}b^{4}d+6\,\sqrt{dx+c}\,b^{4}cd-6\,\sqrt{dx+c}\,ab^{3}d^{2}\right)}{3\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 5*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c +
a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - (sqrt(d*x + c)*b^2*c^2*d - 2*sqrt(d*x
+ c)*a*b*c*d^2 + sqrt(d*x + c)*a^2*d^3)/(((d*x + c)*b - b*c + a*d)*b^3) + 2
/3*((d*x + c)^(3/2)*b^4*d + 6*sqrt(d*x + c)*b^4*c*d - 6*sqrt(d*x + c)*a*b^3
*d^2)/b^6
```

maple [B] time = 0.01, size = 258, normalized size = 2.35

$$\frac{5a^{2}d^{3}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b^{3}}-\frac{10ac\,d^{2}\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b^{2}}+\frac{5c^{2}d\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\;b}-\frac{\sqrt{dx+c}\;a^{2}d^{3}}{(bdx+ad)\,b^{3}}+\frac{2\sqrt{dx+c}\;ac\,d^{2}}{(bdx+ad)\,b^{2}}-\frac{\sqrt{dx+c}\;c^{2}d}{(bdx+ad)\,b}-\frac{4\sqrt{dx+c}\;a\,d^{2}}{b^{3}}+\frac{4\sqrt{dx+c}\;cd}{b^{2}}+\frac{2\,(dx+c)^{\frac{3}{2}}\,d}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^2,x)
```

```
[Out] 2/3*d*(d*x+c)^(3/2)/b^2-4/b^3*a*d^2*(d*x+c)^(1/2)+4*d/b^2*(d*x+c)^(1/2)*c-1
/b^3*(d*x+c)^(1/2)/(b*d*x+a*d)*a^2*d^3+2/b^2*(d*x+c)^(1/2)/(b*d*x+a*d)*a*c*
d^2-d/b*(d*x+c)^(1/2)/(b*d*x+a*d)*c^2+5/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x
+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a^2*d^3-10/b^2/((a*d-b*c)*b)^(1/2)*arctan(
(d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*a*c*d^2+5*d/b/((a*d-b*c)*b)^(1/2)*arct
an((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.12, size = 161, normalized size = 1.46

$$\frac{2\,d\,{\left(c+d\,x\right)}^{3/2}}{3\,b^2}-\frac{\sqrt{c+d\,x}\,\left(a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d\right)}{b^4\,\left(c+d\,x\right)-b^4\,c+a\,b^3\,d}+\frac{5\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,d\,{\left(a\,d-b\,c\right)}^{3/2}\,\sqrt{c+d\,x}}{a^2\,d^3-2\,a\,b\,c\,d^2+b^2\,c^2\,d}\right)\,{\left(a\,d-b\,c\right)}^{3/2}}{b^{7/2}}+\frac{2\,d\,\left(2\,b^2\,c-2\,a\,b\,d\right)\,\sqrt{c+d\,x}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^2,x)
```

```
[Out] (2*d*(c + d*x)^(3/2))/(3*b^2) - ((c + d*x)^(1/2)*(a^2*d^3 + b^2*c^2*d - 2*a
*b*c*d^2))/(b^4*(c + d*x) - b^4*c + a*b^3*d) + (5*d*atan((b^(1/2)*d*(a*d -
b*c)^(3/2)*(c + d*x)^(1/2))/(a^2*d^3 + b^2*c^2*d - 2*a*b*c*d^2))*(a*d - b*c
)^(3/2))/b^(7/2) + (2*d*(2*b^2*c - 2*a*b*d)*(c + d*x)^(1/2))/b^4
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.1301 $\int \frac{(c+dx)^{5/2}}{(a+bx)^3} dx$

Optimal. Leaf size=119

$$-\frac{15d^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 50, 63, 208}

$$-\frac{15d^2\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}-\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)}-\frac{(c+dx)^{5/2}}{2b(a+bx)^2}+\frac{15d^2\sqrt{c+dx}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x])/(4*b^3) - (5*d*(c + d*x)^(3/2))/(4*b^2*(a + b*x)) -
(c + d*x)^(5/2)/(2*b*(a + b*x)^2) - (15*d^2*Sqrt[b*c - a*d]*ArcTanh[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^3}\,dx = -\frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^2}\,dx}{4b}$$

$$= -\frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{\left(15d^2\right)\int \frac{\sqrt{c+dx}}{a+bx}\,dx}{8b^2}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{\left(15d^2(bc-ad)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} + \frac{(15d(bc-ad))\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4b^3}$$

$$= \frac{15d^2\sqrt{c+dx}}{4b^3} - \frac{5d(c+dx)^{3/2}}{4b^2(a+bx)} - \frac{(c+dx)^{5/2}}{2b(a+bx)^2} - \frac{15d^2\sqrt{bc-ad}\,\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4b^{7/2}}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.44

$$\frac{2d^2(c+dx)^{7/2}\,{}_2F_1\left(3,\frac{7}{2};\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (2*d^2*(c + d*x)^(7/2)*Hypergeometric2F1[3, 7/2, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^3)
```

IntegrateAlgebraic [A] time = 0.48, size = 155, normalized size = 1.30

$$\frac{d^2\sqrt{c+dx}\left(15a^2d^2+25abd(c+dx)-30abcd+15b^2c^2+8b^2(c+dx)^2-25b^2c(c+dx)\right)}{4b^3(ad+b(c+dx)-bc)^2} + \frac{15d^2\sqrt{ad-bc}\,\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^3,x]
```

```
[Out] (d^2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 25*b^2*c*(c + d*
x) + 25*a*b*d*(c + d*x) + 8*b^2*(c + d*x)^2))/(4*b^3*(-(b*c) + a*d + b*(c +
 d*x))^2) + (15*d^2*Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*S
qrt[c + d*x])/(b*c - a*d)])/(4*b^(7/2))
```

fricas [A] time = 1.56, size = 344, normalized size = 2.89

$$\left[\frac{15\left(b^2d^2x^2+2abd^2x+a^2d^2\right)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)+2\left(8b^2d^2x^2-2b^2c^2-5abcd+15a^2d^2-\left(9b^2cd-25abd^2\right)x\right)\sqrt{dx+c}}{8\left(b^5x^2+2ab^4x+a^2b^3\right)}, -\frac{15\left(b^2d^2x^2+2abd^2x+a^2d^2\right)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)-\left(8b^2d^2x^2-2b^2c^2-5abcd+15a^2d^2-\left(9b^2cd-25abd^2\right)x\right)\sqrt{dx+c}}{4\left(b^5x^2+2ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(15*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt((b*c - a*d)/b)*log((b*d
*x + 2*b*c - a*d - 2*sqrt(d*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)) + 2*(8
*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2
)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3), -1/4*(15*(b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*sqrt(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(
-(b*c - a*d)/b)/(b*c - a*d)) - (8*b^2*d^2*x^2 - 2*b^2*c^2 - 5*a*b*c*d + 15*
```

```
a^2*d^2 - (9*b^2*c*d - 25*a*b*d^2)*x)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x +
 a^2*b^3)]
```

giac [A] time = 1.24, size = 171, normalized size = 1.44

$$\frac{2\sqrt{dx+c}\,d^2}{b^3}+\frac{15\left(bcd^2-ad^3\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\sqrt{-b^2c+abd}\,b^3}-\frac{9\,(dx+c)^{\frac{3}{2}}b^2cd^2-7\sqrt{dx+c}\,b^2c^2d^2-9\,(dx+c)^{\frac{3}{2}}abd^3+14\sqrt{dx+c}\,abcd^3-7\sqrt{dx+c}\,a^2d^4}{4\left((dx+c)b-bc+ad\right)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*d^2/b^3 + 15/4*(b*c*d^2 - a*d^3)*arctan(sqrt(d*x + c)*b/sqr
t(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*b^3) - 1/4*(9*(d*x + c)^(3/2)*b^2*
c*d^2 - 7*sqrt(d*x + c)*b^2*c^2*d^2 - 9*(d*x + c)^(3/2)*a*b*d^3 + 14*sqrt(d
*x + c)*a*b*c*d^3 - 7*sqrt(d*x + c)*a^2*d^4)/(((d*x + c)*b - b*c + a*d)^2*b
^3)
```

maple [B] time = 0.02, size = 238, normalized size = 2.00

$$\frac{7\sqrt{dx+c}\;a^2d^4}{4\left(bdx+ad\right)^2b^3}-\frac{7\sqrt{dx+c}\;ac\,d^3}{2\left(bdx+ad\right)^2b^2}+\frac{7\sqrt{dx+c}\;c^2d^2}{4\left(bdx+ad\right)^2b}+\frac{9\,(dx+c)^{\frac{3}{2}}\,a\,d^3}{4\left(bdx+ad\right)^2b^2}-\frac{15a\,d^3\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)\,b}\;b^3}-\frac{9\,(dx+c)^{\frac{3}{2}}\,c\,d^2}{4\left(bdx+ad\right)^2b}+\frac{15c\,d^2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{4\sqrt{(ad-bc)\,b}\;b^2}+\frac{2\sqrt{dx+c}\;d^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^3,x)
```

```
[Out] 2*d^2*(d*x+c)^(1/2)/b^3+9/4*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(3/2)*a-9/4*d^2/b
/(b*d*x+a*d)^2*(d*x+c)^(3/2)*c+7/4*d^4/b^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a^2-
7/2*d^3/b^2/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a*c+7/4*d^2/b/(b*d*x+a*d)^2*(d*x+c)
^(1/2)*c^2-15/4*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)
*b)^(1/2)*b)*a+15/4*d^2/b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-
b*c)*b)^(1/2)*b)*c
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.16, size = 199, normalized size = 1.67

$$\frac{2\,d^2\,\sqrt{c+d\,x}}{b^3}-\frac{\left(\frac{9\,b^2\,c\,d^2}{4}-\frac{9\,a\,b\,d^3}{4}\right)\,{\left(c+d\,x\right)}^{3/2}-\sqrt{c+d\,x}\,\left(\frac{7\,a^2\,d^4}{4}-\frac{7\,a\,b\,c\,d^3}{2}+\frac{7\,b^2\,c^2\,d^2}{4}\right)}{b^5\,{\left(c+d\,x\right)}^2-\left(2\,b^5\,c-2\,a\,b^4\,d\right)\,\left(c+d\,x\right)+b^5\,c^2+a^2\,b^3\,d^2-2\,a\,b^4\,c\,d}-\frac{15\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,d^2\,\sqrt{a\,d-b\,c}\,\sqrt{c+d\,x}}{a\,d^3-b\,c\,d^2}\right)\,\sqrt{a\,d-b\,c}}{4\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^3,x)
```

```
[Out] (2*d^2*(c + d*x)^(1/2))/b^3 - (((9*b^2*c*d^2)/4 - (9*a*b*d^3)/4)*(c + d*x)^
(3/2) - (c + d*x)^(1/2)*((7*a^2*d^4)/4 + (7*b^2*c^2*d^2)/4 - (7*a*b*c*d^3)/
2))/(b^5*(c + d*x)^2 - (2*b^5*c - 2*a*b^4*d)*(c + d*x) + b^5*c^2 + a^2*b^3*
d^2 - 2*a*b^4*c*d) - (15*d^2*atan((b^(1/2)*d^2*(a*d - b*c)^(1/2)*(c + d*x)^
(1/2))/(a*d^3 - b*c*d^2))*(a*d - b*c)^(1/2))/(4*b^(7/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.1302 $\int \frac{(c+dx)^{5/2}}{(a+bx)^4} \, dx$

Optimal. Leaf size=126

$$-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Rubi [A] time = 0.05, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{47, 63, 208\}$

$$-\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(8*b^3*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(12*b^2*(a
 + b*x)^2) - (c + d*x)^(5/2)/(3*b*(a + b*x)^3) - (5*d^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/Sqrt[b*c - a*d]])/(8*b^(7/2)*Sqrt[b*c - a*d])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^4}\,dx &= -\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^3}\,dx}{6b} \\
&= -\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^2}\,dx}{8b^2} \\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^3\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{16b^3} \\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}+\frac{\left(5d^2\right)\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{8b^3} \\
&= -\frac{5d^2\sqrt{c+dx}}{8b^3(a+bx)}-\frac{5d(c+dx)^{3/2}}{12b^2(a+bx)^2}-\frac{(c+dx)^{5/2}}{3b(a+bx)^3}-\frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8b^{7/2}\sqrt{bc-ad}}
\end{aligned}
$$

Mathematica [A] time = 0.15, size = 119, normalized size = 0.94

$$
\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{8b^{7/2}\sqrt{ad-bc}}-\frac{\sqrt{c+dx}\left(15a^2d^2+10abd(c+4dx)+b^2\left(8c^2+26cdx+33d^2x^2\right)\right)}{24b^3(a+bx)^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + 4*d*x) + b^2*(8*c^2 + 26*c
*d*x + 33*d^2*x^2)))/(b^3*(a + b*x)^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[c + d*
x])/Sqrt[-(b*c) + a*d]])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

IntegrateAlgebraic [A] time = 0.62, size = 155, normalized size = 1.23

$$
-\frac{d^3\sqrt{c+dx}\left(15a^2d^2+40abd(c+dx)-30abcd+15b^2c^2+33b^2(c+dx)^2-40b^2c(c+dx)\right)}{24b^3(ad+b(c+dx)-bc)^3}-\frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8b^{7/2}\sqrt{ad-bc}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^4,x]
```

```
[Out] -1/24*(d^3*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 40*b^2*c*(
c + d*x) + 40*a*b*d*(c + d*x) + 33*b^2*(c + d*x)^2))/(b^3*(-(b*c) + a*d + b
*(c + d*x))^3) - (5*d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(
b*c - a*d)])/(8*b^(7/2)*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.20, size = 563, normalized size = 4.47

$$
\left[\frac{15\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)-2\left(8b^4c^3+2ab^3c^2d+5a^2b^2cd^2-15a^3bd^3+33\left(b^4cd^2-ab^3d^3\right)x^2+2\left(13b^4c^2d+7ab^3cd^2-20a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c-a^4b^4d+\left(b^8c-ab^7d\right)x^3+3\left(ab^7c-a^2b^6d\right)x^2+3\left(a^2b^6c-a^3b^5d\right)x\right)}, \frac{15\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)-\left(8b^4c^3+2ab^3c^2d+5a^2b^2cd^2-15a^3bd^3+33\left(b^4cd^2-ab^3d^3\right)x^2+2\left(13b^4c^2d+7ab^3cd^2-20a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c-a^4b^4d+\left(b^8c-ab^7d\right)x^3+3\left(ab^7c-a^2b^6d\right)x^2+3\left(a^2b^6c-a^3b^5d\right)x\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) - 2*(8*b^4*c^3 + 2*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^
3 + 33*(b^4*c*d^2 - a*b^3*d^3)*x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^
```

```
3 + 3*(a*b^7*c - a^2*b^6*d)*x^2 + 3*(a^2*b^6*c - a^3*b^5*d)*x), 1/24*(15*(b
^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(-b^2*c + a*b*d
)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (8*b^4*c^3 + 2
*a*b^3*c^2*d + 5*a^2*b^2*c*d^2 - 15*a^3*b*d^3 + 33*(b^4*c*d^2 - a*b^3*d^3)*
x^2 + 2*(13*b^4*c^2*d + 7*a*b^3*c*d^2 - 20*a^2*b^2*d^3)*x)*sqrt(d*x + c))/(
a^3*b^5*c - a^4*b^4*d + (b^8*c - a*b^7*d)*x^3 + 3*(a*b^7*c - a^2*b^6*d)*x^2
 + 3*(a^2*b^6*c - a^3*b^5*d)*x)]
```

giac [A] time = 0.98, size = 161, normalized size = 1.28

$$\frac{5\,d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\,\sqrt{-b^2c+abd}\,b^3}-\frac{33\,(dx+c)^{\frac{5}{2}}b^2d^3-40\,(dx+c)^{\frac{3}{2}}b^2cd^3+15\,\sqrt{dx+c}\,b^2c^2d^3+40\,(dx+c)^{\frac{3}{2}}abd^4-30\,\sqrt{dx+c}\,abcd^4+15\,\sqrt{dx+c}\,a^2d^5}{24\,((dx+c)b-bc+ad)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*
b^3) - 1/24*(33*(d*x + c)^(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 15
*sqrt(d*x + c)*b^2*c^2*d^3 + 40*(d*x + c)^(3/2)*a*b*d^4 - 30*sqrt(d*x + c)*
a*b*c*d^4 + 15*sqrt(d*x + c)*a^2*d^5)/(((d*x + c)*b - b*c + a*d)^3*b^3)
```

maple [A] time = 0.02, size = 204, normalized size = 1.62

$$-\frac{5\sqrt{dx+c}\;a^2d^5}{8\,(bdx+ad)^3\,b^3}+\frac{5\sqrt{dx+c}\;ac\,d^4}{4\,(bdx+ad)^3\,b^2}-\frac{5\sqrt{dx+c}\;c^2d^3}{8\,(bdx+ad)^3\,b}-\frac{5\,(dx+c)^{\frac{3}{2}}\,a\,d^4}{3\,(bdx+ad)^3\,b^2}+\frac{5\,(dx+c)^{\frac{3}{2}}\,c\,d^3}{3\,(bdx+ad)^3\,b}-\frac{11\,(dx+c)^{\frac{5}{2}}\,d^3}{8\,(bdx+ad)^3\,b}+\frac{5d^3\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{8\sqrt{(ad-bc)\,b}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^4,x)
```

```
[Out] -11/8*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(5/2)-5/3*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(
3/2)*a+5/3*d^3/(b*d*x+a*d)^3/b*(d*x+c)^(3/2)*c-5/8*d^5/(b*d*x+a*d)^3/b^3*(d
*x+c)^(1/2)*a^2+5/4*d^4/(b*d*x+a*d)^3/b^2*(d*x+c)^(1/2)*a*c-5/8*d^3/(b*d*x+
a*d)^3/b*(d*x+c)^(1/2)*c^2+5/8*d^3/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(
1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.36, size = 222, normalized size = 1.76

$$\frac{5\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,b^{7/2}\,\sqrt{a\,d-b\,c}}-\frac{\frac{11\,d^3\,{(c+d\,x)}^{5/2}}{8\,b}+\frac{5\,d^3\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{8\,b^3}+\frac{5\,d^3\,\left(a\,d-b\,c\right)\,{(c+d\,x)}^{3/2}}{3\,b^2}}{\left(c+d\,x\right)\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)+b^3\,{\left(c+d\,x\right)}^3-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^2+a^3\,d^3-b^3\,c^3+3\,a\,b^2\,c^2\,d-3\,a^2\,b\,c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^4,x)
```

```
[Out] (5*d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(7/2)*(a*d -
 b*c)^(1/2)) - ((11*d^3*(c + d*x)^(5/2))/(8*b) + (5*d^3*(c + d*x)^(1/2)*(a^
2*d^2 + b^2*c^2 - 2*a*b*c*d))/(8*b^3) + (5*d^3*(a*d - b*c)*(c + d*x)^(3/2))
/(3*b^2))/((c + d*x)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x
```

```
)^3 - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**4,x)
```

```
[Out] Timed out
```

3.1303 $\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx$

Optimal. Leaf size=162

$$\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Rubi [A] time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$-\frac{5d^3\sqrt{c+dx}}{64b^3(a+bx)(bc-ad)}-\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2}+\frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64b^{7/2}(bc-ad)^{3/2}}-\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3}-\frac{(c+dx)^{5/2}}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (-5*d^2*Sqrt[c + d*x])/(32*b^3*(a + b*x)^2) - (5*d^3*Sqrt[c + d*x])/(64*b^3
*(b*c - a*d)*(a + b*x)) - (5*d*(c + d*x)^(3/2))/(24*b^2*(a + b*x)^3) - (c +
 d*x)^(5/2)/(4*b*(a + b*x)^4) + (5*d^4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[b*c - a*d]])/(64*b^(7/2)*(b*c - a*d)^(3/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^5}\,dx = -\frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^4}\,dx}{8b}$$

$$= -\frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^3}\,dx}{16b^2}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{\left(5d^3\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{64b^3}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{\left(5d^4\right)\int \frac{1}{(a+bx)\sqrt{c \ldots}}}{128b^3(bc- \ldots}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} - \frac{\left(5d^3\right)\text{Subst}\left(\int \ldots\right.}{64 \ldots}$$

$$= -\frac{5d^2\sqrt{c+dx}}{32b^3(a+bx)^2} - \frac{5d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)} - \frac{5d(c+dx)^{3/2}}{24b^2(a+bx)^3} - \frac{(c+dx)^{5/2}}{4b(a+bx)^4} + \frac{5d^4\tanh^{-1}\left(\frac{\sqrt{b} \ldots}{\sqrt{\ldots}}\right.}{64b^{7/2}(bc-a \ldots}$$

Mathematica [C]   time = 0.02, size = 52, normalized size = 0.32

$$\frac{2d^4(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},5;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] (2*d^4*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 5, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^5)
```

IntegrateAlgebraic [A]   time = 1.08, size = 226, normalized size = 1.40

$$-\frac{d^4\sqrt{c+dx}\left(-15a^3d^3-55a^2bd^2(c+dx)+45a^2bcd^2-45ab^2c^2d-73ab^2d(c+dx)^2+110ab^2cd(c+dx)+15b^3c^3-55b^3c^2(c+dx)+15b^3(c+dx)^3+73b^3c(c+dx)^2\right)}{192b^3(bc-ad)(-ad-b(c+dx)+bc)^4}-\frac{5d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64b^{7/2}(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^5,x]
```

```
[Out] -1/192*(d^4*Sqrt[c + d*x]*(15*b^3*c^3 - 45*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 1
5*a^3*d^3 - 55*b^3*c^2*(c + d*x) + 110*a*b^2*c*d*(c + d*x) - 55*a^2*b*d^2*(
c + d*x) + 73*b^3*c*(c + d*x)^2 - 73*a*b^2*d*(c + d*x)^2 + 15*b^3*(c + d*x)
^3))/(b^3*(b*c - a*d)*(b*c - a*d - b*(c + d*x))^4) - (5*d^4*ArcTan[(Sqrt[b]
*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*b^(7/2)*(-(b*c) + a*d)
^(3/2))
```

fricas [B]   time = 1.42, size = 894, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="fricas")
```

```
[Out] [-1/384*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a^2
```

```
*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4)*
x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5*c
^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x +
 c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d +
 a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(a^
2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6*c
*d + a^5*b^5*d^2)*x), -1/192*(15*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2
*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
 + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + (48*b^5*c^4 - 56*a*b^4*c^3*d - 2*a
^2*b^3*c^2*d^2 - 5*a^3*b^2*c*d^3 + 15*a^4*b*d^4 + 15*(b^5*c*d^3 - a*b^4*d^4
)*x^3 + (118*b^5*c^2*d^2 - 191*a*b^4*c*d^3 + 73*a^2*b^3*d^4)*x^2 + (136*b^5
*c^3*d - 172*a*b^4*c^2*d^2 - 19*a^2*b^3*c*d^3 + 55*a^3*b^2*d^4)*x)*sqrt(d*x
 + c))/(a^4*b^6*c^2 - 2*a^5*b^5*c*d + a^6*b^4*d^2 + (b^10*c^2 - 2*a*b^9*c*d
 + a^2*b^8*d^2)*x^4 + 4*(a*b^9*c^2 - 2*a^2*b^8*c*d + a^3*b^7*d^2)*x^3 + 6*(
a^2*b^8*c^2 - 2*a^3*b^7*c*d + a^4*b^6*d^2)*x^2 + 4*(a^3*b^7*c^2 - 2*a^4*b^6
*c*d + a^5*b^5*d^2)*x)]
```

giac [A] time = 1.09, size = 259, normalized size = 1.60

$$-\frac{5\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c-ab^3d\right)\sqrt{-b^2c+abd}}-\frac{15\,(dx+c)^{\frac{7}{2}}b^3d^4+73\,(dx+c)^{\frac{5}{2}}b^3cd^4-55\,(dx+c)^{\frac{3}{2}}b^3c^2d^4+15\,\sqrt{dx+c}\,b^3c^3d^4-73\,(dx+c)^{\frac{5}{2}}ab^2d^5+110\,(dx+c)^{\frac{3}{2}}ab^2cd^5-45\,\sqrt{dx+c}\,ab^2c^2d^5-55\,(dx+c)^{\frac{3}{2}}a^2bd^6+45\,\sqrt{dx+c}\,a^2bcd^6-15\,\sqrt{dx+c}\,a^3d^7}{192\left(b^4c-ab^3d\right)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="giac")
```

```
[Out] -5/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c - a*b^3*d)*s
qrt(-b^2*c + a*b*d)) - 1/192*(15*(d*x + c)^(7/2)*b^3*d^4 + 73*(d*x + c)^(5/
2)*b^3*c*d^4 - 55*(d*x + c)^(3/2)*b^3*c^2*d^4 + 15*sqrt(d*x + c)*b^3*c^3*d^
4 - 73*(d*x + c)^(5/2)*a*b^2*d^5 + 110*(d*x + c)^(3/2)*a*b^2*c*d^5 - 45*sqr
t(d*x + c)*a*b^2*c^2*d^5 - 55*(d*x + c)^(3/2)*a^2*b*d^6 + 45*sqrt(d*x + c)*
a^2*b*c*d^6 - 15*sqrt(d*x + c)*a^3*d^7)/((b^4*c - a*b^3*d)*((d*x + c)*b - b
*c + a*d)^4)
```

maple [A] time = 0.02, size = 246, normalized size = 1.52

$$-\frac{5\sqrt{dx+c}\;a^2d^6}{64\,(bdx+ad)^4\,b^3}+\frac{5\sqrt{dx+c}\;ac\,d^5}{32\,(bdx+ad)^4\,b^2}-\frac{5\sqrt{dx+c}\;c^2d^4}{64\,(bdx+ad)^4\,b}-\frac{55\,(dx+c)^{\frac{3}{2}}\,a\,d^5}{192\,(bdx+ad)^4\,b^2}+\frac{55\,(dx+c)^{\frac{3}{2}}\,c\,d^4}{192\,(bdx+ad)^4\,b}+\frac{5\,(dx+c)^{\frac{7}{2}}\,d^4}{64\,(bdx+ad)^4\,(ad-bc)}-\frac{73\,(dx+c)^{\frac{5}{2}}\,d^4}{192\,(bdx+ad)^4\,b}+\frac{5d^4\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{64\,(ad-bc)\,\sqrt{(ad-bc)\,b}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^5,x)
```

```
[Out] 5/64*d^4/(b*d*x+a*d)^4/(a*d-b*c)*(d*x+c)^(7/2)-73/192*d^4/(b*d*x+a*d)^4/b*(
d*x+c)^(5/2)-55/192*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(3/2)*a+55/192*d^4/(b*d*x
+a*d)^4/b*(d*x+c)^(3/2)*c-5/64*d^6/(b*d*x+a*d)^4/b^3*(d*x+c)^(1/2)*a^2+5/32
*d^5/(b*d*x+a*d)^4/b^2*(d*x+c)^(1/2)*a*c-5/64*d^4/(b*d*x+a*d)^4/b*(d*x+c)^(
1/2)*c^2+5/64*d^4/(a*d-b*c)/b^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.41, size = 309, normalized size = 1.91

$$\frac{5\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{3/2}}-\frac{\frac{73\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,b}-\frac{5\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{5\,d^4\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{64\,b^3}+\frac{55\,d^4\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{192\,b^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^5,x)
```

```
[Out] (5*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(7/2)*(a*d
- b*c)^(3/2)) - ((73*d^4*(c + d*x)^(5/2))/(192*b) - (5*d^4*(c + d*x)^(7/2))
/(64*(a*d - b*c)) + (5*d^4*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
/(64*b^3) + (55*d^4*(a*d - b*c)*(c + d*x)^(3/2))/(192*b^2))/(b^4*(c + d*x)^
4 - (4*b^4*c - 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3
+ 12*a^2*b^2*c*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b
^4*c^2 + 6*a^2*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d
- 4*a^3*b*c*d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**5,x)
```

```
[Out] Timed out
```

3.1304 $\int \frac{(c+dx)^{5/2}}{(a+bx)^6} dx$

Optimal. Leaf size=198

$$-\frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}+\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+d}{5b(a+}$$

Rubi [A] time = 0.09, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {47, 51, 63, 208}

$$\frac{3d^4\sqrt{c+dx}}{128b^3(a+bx)(bc-ad)^2}-\frac{d^3\sqrt{c+dx}}{64b^3(a+bx)^2(bc-ad)}-\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3}-\frac{3d^5 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{128b^{7/2}(bc-ad)^{5/2}}-\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4}-\frac{(c+dx)^{5/2}}{5b(a+bx)^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] -(d^2*Sqrt[c + d*x])/(16*b^3*(a + b*x)^3) - (d^3*Sqrt[c + d*x])/(64*b^3*(b*
c - a*d)*(a + b*x)^2) + (3*d^4*Sqrt[c + d*x])/(128*b^3*(b*c - a*d)^2*(a + b
*x)) - (d*(c + d*x)^(3/2))/(8*b^2*(a + b*x)^4) - (c + d*x)^(5/2)/(5*b*(a +
b*x)^5) - (3*d^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(128*b^(
7/2)*(b*c - a*d)^(5/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^6} dx = -\frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^5} dx}{2b}$$

$$= -\frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{(3d^2)\int \frac{\sqrt{c+dx}}{(a+bx)^4} dx}{16b^2}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} + \frac{d^3\int \frac{1}{(a+bx)^3\sqrt{c+dx}} dx}{32b^3}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c+dx)^{5/2}}{5b(a+bx)^5} - \frac{(3d^4)\int \frac{1}{(a+bx)^2\sqrt{}}}{128b^3(bc-}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c}{5b(}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c}{5b(}$$

$$= -\frac{d^2\sqrt{c+dx}}{16b^3(a+bx)^3} - \frac{d^3\sqrt{c+dx}}{64b^3(bc-ad)(a+bx)^2} + \frac{3d^4\sqrt{c+dx}}{128b^3(bc-ad)^2(a+bx)} - \frac{d(c+dx)^{3/2}}{8b^2(a+bx)^4} - \frac{(c}{5b(}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.26

$$\frac{2d^5(c+dx)^{7/2}\,{}_2F_1\left(\frac{7}{2},6;\frac{9}{2};-\frac{b(c+dx)}{ad-bc}\right)}{7(ad-bc)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] (2*d^5*(c + d*x)^(7/2)*Hypergeometric2F1[7/2, 6, 9/2, -((b*(c + d*x))/(-(b*
c) + a*d))])/(7*(-(b*c) + a*d)^6)
```

IntegrateAlgebraic [A] time = 1.43, size = 307, normalized size = 1.55

$$\frac{d^5\sqrt{c+dx}\left(15a^4d^4+70a^3bd^3(c+dx)-60a^3bcd^3+90a^2b^2c^2d^2+128a^2b^2d^2(c+dx)^2-210a^2b^2cd^2(c+dx)-60ab^3c^3d+210ab^3c^2d(c+dx)-70ab^3d(c+dx)^3-256ab^3cd(c+dx)^2+15b^4c^4-70b^4c^3(c+dx)+128b^4c^2(c+dx)^2-15b^4(c+dx)^4+70b^4c(c+dx)^3\right)}{640b^3(bc-ad)^2(-ad-b(c+dx)+bc)^5}-\frac{3d^5\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{128b^{7/2}(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^6,x]
```

```
[Out] (d^5*Sqrt[c + d*x]*(15*b^4*c^4 - 60*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 - 60*a
^3*b*c*d^3 + 15*a^4*d^4 - 70*b^4*c^3*(c + d*x) + 210*a*b^3*c^2*d*(c + d*x)
- 210*a^2*b^2*c*d^2*(c + d*x) + 70*a^3*b*d^3*(c + d*x) + 128*b^4*c^2*(c + d
*x)^2 - 256*a*b^3*c*d*(c + d*x)^2 + 128*a^2*b^2*d^2*(c + d*x)^2 + 70*b^4*c*
(c + d*x)^3 - 70*a*b^3*d*(c + d*x)^3 - 15*b^4*(c + d*x)^4))/(640*b^3*(b*c -
 a*d)^2*(b*c - a*d - b*(c + d*x))^5) - (3*d^5*ArcTan[(Sqrt[b]*Sqrt[-(b*c) +
 a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(128*b^(7/2)*(-(b*c) + a*d)^(5/2))
```

fricas [B] time = 1.49, size = 1337, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*(b^5*d^5*x^5 + 5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^
2*d^5*x^2 + 5*a^4*b*d^5*x + a^5*d^5)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c
 - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(128*b^6*c^5 -
 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^
4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*x^4 + 10*(b^6*c^2*d^3 - 8*a*b
^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3*d^2 - 357*a*b^5*c^2*d^3 + 29
7*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168*b^6*c^4*d - 424*a*b^5*c^3*d^
2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 35*a^4*b^2*d^5)*x)*sqrt(d*x +
c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5*c*d^2 - a^8*b^4*d^3 + (b^12*
c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*b^9*d^3)*x^5 + 5*(a*b^11*c^3
- 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8*d^3)*x^4 + 10*(a^2*b^10*c^3
- 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*d^3)*x^3 + 10*(a^3*b^9*c^3 -
3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^3)*x^2 + 5*(a^4*b^8*c^3 - 3*a
^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*x), 1/640*(15*(b^5*d^5*x^5 +
5*a*b^4*d^5*x^4 + 10*a^2*b^3*d^5*x^3 + 10*a^3*b^2*d^5*x^2 + 5*a^4*b*d^5*x +
 a^5*d^5)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b
*d*x + b*c)) - (128*b^6*c^5 - 304*a*b^5*c^4*d + 184*a^2*b^4*c^3*d^2 + 2*a^3
*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - 15*a^5*b*d^5 - 15*(b^6*c*d^4 - a*b^5*d^5)*
x^4 + 10*(b^6*c^2*d^3 - 8*a*b^5*c*d^4 + 7*a^2*b^4*d^5)*x^3 + 2*(124*b^6*c^3
*d^2 - 357*a*b^5*c^2*d^3 + 297*a^2*b^4*c*d^4 - 64*a^3*b^3*d^5)*x^2 + 2*(168
*b^6*c^4*d - 424*a*b^5*c^3*d^2 + 279*a^2*b^4*c^2*d^3 + 12*a^3*b^3*c*d^4 - 3
5*a^4*b^2*d^5)*x)*sqrt(d*x + c))/(a^5*b^7*c^3 - 3*a^6*b^6*c^2*d + 3*a^7*b^5
*c*d^2 - a^8*b^4*d^3 + (b^12*c^3 - 3*a*b^11*c^2*d + 3*a^2*b^10*c*d^2 - a^3*
b^9*d^3)*x^5 + 5*(a*b^11*c^3 - 3*a^2*b^10*c^2*d + 3*a^3*b^9*c*d^2 - a^4*b^8
*d^3)*x^4 + 10*(a^2*b^10*c^3 - 3*a^3*b^9*c^2*d + 3*a^4*b^8*c*d^2 - a^5*b^7*
d^3)*x^3 + 10*(a^3*b^9*c^3 - 3*a^4*b^8*c^2*d + 3*a^5*b^7*c*d^2 - a^6*b^6*d^
3)*x^2 + 5*(a^4*b^8*c^3 - 3*a^5*b^7*c^2*d + 3*a^6*b^6*c*d^2 - a^7*b^5*d^3)*
x)]
```

giac [**B**] time = 1.25, size = 380, normalized size = 1.92

$$\frac{3\,d^5\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{128\,\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)\sqrt{-b^2c+abd}}+\frac{15\,(dx+c)^{\frac{9}{2}}b^4d^5-70\,(dx+c)^{\frac{7}{2}}b^4cd^5-128\,(dx+c)^{\frac{5}{2}}b^4c^2d^5+70\,(dx+c)^{\frac{3}{2}}b^4c^3d^5-15\,\sqrt{dx+c}\,b^4c^4d^5+70\,(dx+c)^{\frac{7}{2}}ab^3d^6+256\,(dx+c)^{\frac{5}{2}}ab^3cd^6-210\,(dx+c)^{\frac{3}{2}}ab^3c^2d^6+60\,\sqrt{dx+c}\,ab^3c^3d^6-128\,(dx+c)^{\frac{5}{2}}a^2b^2d^7+210\,(dx+c)^{\frac{3}{2}}a^2b^2cd^7-90\,\sqrt{dx+c}\,a^2b^2c^2d^7-70\,(dx+c)^{\frac{3}{2}}a^3bd^8+60\,\sqrt{dx+c}\,a^3bcd^8-15\,\sqrt{dx+c}\,a^4d^9}{640\,\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)\left((dx+c)b-bc+ad\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 3/128*d^5*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^2 - 2*a*b^4*
c*d + a^2*b^3*d^2)*sqrt(-b^2*c + a*b*d)) + 1/640*(15*(d*x + c)^(9/2)*b^4*d^
5 - 70*(d*x + c)^(7/2)*b^4*c*d^5 - 128*(d*x + c)^(5/2)*b^4*c^2*d^5 + 70*(d*
x + c)^(3/2)*b^4*c^3*d^5 - 15*sqrt(d*x + c)*b^4*c^4*d^5 + 70*(d*x + c)^(7/2
)*a*b^3*d^6 + 256*(d*x + c)^(5/2)*a*b^3*c*d^6 - 210*(d*x + c)^(3/2)*a*b^3*c
^2*d^6 + 60*sqrt(d*x + c)*a*b^3*c^3*d^6 - 128*(d*x + c)^(5/2)*a^2*b^2*d^7 +
 210*(d*x + c)^(3/2)*a^2*b^2*c*d^7 - 90*sqrt(d*x + c)*a^2*b^2*c^2*d^7 - 70*
(d*x + c)^(3/2)*a^3*b*d^8 + 60*sqrt(d*x + c)*a^3*b*c*d^8 - 15*sqrt(d*x + c)
*a^4*d^9)/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*((d*x + c)*b - b*c + a*d)^
5)
```

maple [**A**] time = 0.02, size = 305, normalized size = 1.54

$$-\frac{3\sqrt{dx+c}\;a^2d^7}{128\,(bdx+ad)^5\,b^3}+\frac{3\sqrt{dx+c}\;ac\,d^6}{64\,(bdx+ad)^5\,b^2}+\frac{3\,(dx+c)^{\frac{9}{2}}\,b\,d^5}{128\,(bdx+ad)^5\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{3\sqrt{dx+c}\;c^2d^5}{128\,(bdx+ad)^5\,b}-\frac{7\,(dx+c)^{\frac{3}{2}}\,a\,d^6}{64\,(bdx+ad)^5\,b^2}+\frac{7\,(dx+c)^{\frac{3}{2}}\,c\,d^5}{64\,(bdx+ad)^5\,b}+\frac{7\,(dx+c)^{\frac{7}{2}}\,d^5}{64\,(bdx+ad)^5\,(ad-bc)}-\frac{(dx+c)^{\frac{5}{2}}\,d^5}{5\,(bdx+ad)^5\,b}+\frac{3d^5\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{128\left(a^2d^2-2abcd+b^2c^2\right)\sqrt{(ad-bc)\,b}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^6,x)
```

```
[Out] 3/128*d^5/(b*d*x+a*d)^5*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x+c)^(9/2)+7/64*d^
5/(b*d*x+a*d)^5/(a*d-b*c)*(d*x+c)^(7/2)-1/5*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(5/
2)-7/64*d^6/(b*d*x+a*d)^5/b^2*(d*x+c)^(3/2)*a+7/64*d^5/(b*d*x+a*d)^5/b*(d*x
+c)^(3/2)*c-3/128*d^7/(b*d*x+a*d)^5/b^3*(d*x+c)^(1/2)*a^2+3/64*d^6/(b*d*x+a
*d)^5/b^2*(d*x+c)^(1/2)*a*c-3/128*d^5/(b*d*x+a*d)^5/b*(d*x+c)^(1/2)*c^2+3/1
```

```
28*d^5/b^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(
1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.50, size = 411, normalized size = 2.08

$$\frac{3\,d^5\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{128\,b^{7/2}\,{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{d^5\,{\left(c+d\,x\right)}^{5/2}}{5\,b}-\frac{7\,d^5\,{\left(c+d\,x\right)}^{7/2}}{64\,\left(a\,d-b\,c\right)}+\frac{3\,d^5\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{128\,b^3}+\frac{7\,d^5\,\left(a\,d-b\,c\right)\,{\left(c+d\,x\right)}^{3/2}}{64\,b^2}-\frac{3\,b\,d^5\,{\left(c+d\,x\right)}^{9/2}}{128\,{\left(a\,d-b\,c\right)}^2}}{b^5\,{\left(c+d\,x\right)}^5-{\left(c+d\,x\right)}^2\,\left(-10\,a^3\,b^2\,d^3+30\,a^2\,b^3\,c\,d^2-30\,a\,b^4\,c^2\,d+10\,b^5\,c^3\right)-\left(5\,b^5\,c-5\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^4+a^5\,d^5-b^5\,c^5+{\left(c+d\,x\right)}^3\,\left(10\,a^2\,b^3\,d^2-20\,a\,b^4\,c\,d+10\,b^5\,c^2\right)+\left(c+d\,x\right)\,\left(5\,a^4\,b\,d^4-20\,a^3\,b^2\,c\,d^3+30\,a^2\,b^3\,c^2\,d^2-20\,a\,b^4\,c^3\,d+5\,b^5\,c^4\right)-10\,a^2\,b^3\,c^3\,d^2+10\,a^3\,b^2\,c^2\,d^3+5\,a\,b^4\,c^4\,d-5\,a^4\,b\,c\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^6,x)
```

```
[Out] (3*d^5*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(128*b^(7/2)*(a*d
 - b*c)^(5/2)) - ((d^5*(c + d*x)^(5/2))/(5*b) - (7*d^5*(c + d*x)^(7/2))/(64
*(a*d - b*c)) + (3*d^5*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12
8*b^3) + (7*d^5*(a*d - b*c)*(c + d*x)^(3/2))/(64*b^2) - (3*b*d^5*(c + d*x)^
(9/2))/(128*(a*d - b*c)^2))/(b^5*(c + d*x)^5 - (c + d*x)^2*(10*b^5*c^3 - 10
*a^3*b^2*d^3 + 30*a^2*b^3*c*d^2 - 30*a*b^4*c^2*d) - (5*b^5*c - 5*a*b^4*d)*(
c + d*x)^4 + a^5*d^5 - b^5*c^5 + (c + d*x)^3*(10*b^5*c^2 + 10*a^2*b^3*d^2 -
 20*a*b^4*c*d) + (c + d*x)*(5*b^5*c^4 + 5*a^4*b*d^4 - 20*a^3*b^2*c*d^3 + 30
*a^2*b^3*c^2*d^2 - 20*a*b^4*c^3*d) - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^
3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**6,x)
```

```
[Out] Timed out
```

3.1305 $\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx$

Optimal. Leaf size=35

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}}{(1+x)^2}\,dx &= -\frac{\sqrt{-1+x}}{1+x}+\frac{1}{2}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx\\
&= -\frac{\sqrt{-1+x}}{1+x}+\text{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\
&= -\frac{\sqrt{-1+x}}{1+x}+\frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 1.46

$$\frac{-2x-\sqrt{2-2x}\,(x+1)\tanh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)+2}{2\sqrt{x-1}\,(x+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] (2 - 2*x - Sqrt[2 - 2*x]*(1 + x)*ArcTanh[Sqrt[1 - x]/Sqrt[2]])/(2*Sqrt[-1 +
 x]*(1 + x))
```

IntegrateAlgebraic [A] time = 0.05, size = 35, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\sqrt{x-1}}{x+1}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x]/(1 + x)^2,x]
```

```
[Out] -(Sqrt[-1 + x]/(1 + x)) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/Sqrt[2]
```

fricas [A] time = 1.37, size = 33, normalized size = 0.94

$$\frac{\sqrt{2}\,(x+1)\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-2\sqrt{x-1}}{2\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - 2*sqrt(x - 1))/(x +
1)
```

giac [A] time = 0.96, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

maple [A] time = 0.01, size = 30, normalized size = 0.86

$$\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{x-1}\,\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/(x+1)^2,x)
```

```
[Out] 1/2*arctan(1/2*(x-1)^(1/2)*2^(1/2))*2^(1/2)-(x-1)^(1/2)/(x+1)
```

maxima [A] time = 2.97, size = 29, normalized size = 0.83

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) - sqrt(x - 1)/(x + 1)
```

mupad [B] time = 0.06, size = 29, normalized size = 0.83

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sqrt{x-1}}{2}\right)}{2}-\frac{\sqrt{x-1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^2,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/2 - (x - 1)^(1/2)/(x + 1)
```

sympy [A] time = 1.50, size = 104, normalized size = 2.97

$$\begin{cases} \frac{\sqrt{2}\,i\,\mathrm{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2}+\frac{i}{\sqrt{-1+\frac{2}{x+1}}\,\sqrt{x+1}}-\frac{2i}{\sqrt{-1+\frac{2}{x+1}}\,(x+1)^{\frac{3}{2}}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{\sqrt{1-\frac{2}{x+1}}}{\sqrt{x+1}}-\frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**2,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/2 + I/(sqrt(-1 + 2/(x + 1))
*sqrt(x + 1)) - 2*I/(sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)), 2/Abs(x + 1) > 1
), (-sqrt(1 - 2/(x + 1))/sqrt(x + 1) - sqrt(2)*asin(sqrt(2)/sqrt(x + 1))/2,
 True))
```

3.1306 $\int \frac{\sqrt{-1+x}}{(1+x)^3} dx$

Optimal. Leaf size=56

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Rubi [A] time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {47, 51, 63, 203}

$$\frac{\sqrt{x-1}}{8(x+1)} - \frac{\sqrt{x-1}}{2(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] -Sqrt[-1 + x]/(2*(1 + x)^2) + Sqrt[-1 + x]/(8*(1 + x)) + ArcTan[Sqrt[-1 + x
]/Sqrt[2]]/(8*Sqrt[2])
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+x}}{(1+x)^3}\,dx = -\frac{\sqrt{-1+x}}{2(1+x)^2} + \frac{1}{4}\int \frac{1}{\sqrt{-1+x}\,(1+x)^2}\,dx$$
$$= -\frac{\sqrt{-1+x}}{2(1+x)^2} + \frac{\sqrt{-1+x}}{8(1+x)} + \frac{1}{16}\int \frac{1}{\sqrt{-1+x}\,(1+x)}\,dx$$
$$= -\frac{\sqrt{-1+x}}{2(1+x)^2} + \frac{\sqrt{-1+x}}{8(1+x)} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-1+x}\right)$$
$$= -\frac{\sqrt{-1+x}}{2(1+x)^2} + \frac{\sqrt{-1+x}}{8(1+x)} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Mathematica [C] time = 0.01, size = 28, normalized size = 0.50

$$\frac{1}{12}(x-1)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{1-x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] ((-1 + x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, (1 - x)/2])/12
```

IntegrateAlgebraic [A] time = 0.06, size = 43, normalized size = 0.77

$$\frac{\sqrt{x-1}\,(x-3)}{8(x+1)^2} + \frac{\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 + x]/(1 + x)^3,x]
```

```
[Out] ((-3 + x)*Sqrt[-1 + x])/(8*(1 + x)^2) + ArcTan[Sqrt[-1 + x]/Sqrt[2]]/(8*Sqr
t[2])
```

fricas [A] time = 1.12, size = 46, normalized size = 0.82

$$\frac{\sqrt{2}\left(x^2+2\,x+1\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+2\,\sqrt{x-1}\,(x-3)}{16\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(sqrt(2)*(x^2 + 2*x + 1)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x -
1)*(x - 3))/(x^2 + 2*x + 1)
```

giac [A] time = 1.04, size = 37, normalized size = 0.66

$$\frac{1}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\,\sqrt{x-1}}{8\,(x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/(x + 1)^2
```

maple [A] time = 0.01, size = 40, normalized size = 0.71

$$\frac{\sqrt{2}\ \arctan\left(\frac{\sqrt{x-1}\ \sqrt{2}}{2}\right)}{16}+\frac{\frac{(x-1)^{\frac{3}{2}}}{8}-\frac{\sqrt{x-1}}{4}}{(x+1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/2)/(x+1)^3,x)
```

```
[Out] 2*(1/16*(x-1)^(3/2)-1/8*(x-1)^(1/2))/(x+1)^2+1/16*2^(1/2)*arctan(1/2*(x-1)^
(1/2)*2^(1/2))
```

maxima [A] time = 3.03, size = 43, normalized size = 0.77

$$\frac{1}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+\frac{(x-1)^{\frac{3}{2}}-2\sqrt{x-1}}{8\left((x-1)^{2}+4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 1/8*((x - 1)^(3/2) - 2*sqrt(
x - 1))/((x - 1)^2 + 4*x)
```

mupad [B] time = 0.04, size = 45, normalized size = 0.80

$$\frac{\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\ \sqrt{x-1}}{2}\right)}{16}-\frac{\frac{\sqrt{x-1}}{4}-\frac{(x-1)^{3/2}}{8}}{4x+(x-1)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/2)/(x + 1)^3,x)
```

```
[Out] (2^(1/2)*atan((2^(1/2)*(x - 1)^(1/2))/2))/16 - ((x - 1)^(1/2)/4 - (x - 1)^(
3/2)/8)/(4*x + (x - 1)^2)
```

sympy [A] time = 2.61, size = 167, normalized size = 2.98

$$\begin{cases} \frac{\sqrt{2}i\,\mathrm{acosh}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16}-\frac{i}{8\sqrt{-1+\frac{2}{x+1}}\sqrt{x+1}}+\frac{3i}{4\sqrt{-1+\frac{2}{x+1}}(x+1)^{\frac{3}{2}}}-\frac{i}{\sqrt{-1+\frac{2}{x+1}}(x+1)^{\frac{5}{2}}} & \text{for}\ \frac{2}{|x+1|}>1 \\ -\frac{\sqrt{2}\,\mathrm{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right)}{16}+\frac{1}{8\sqrt{1-\frac{2}{x+1}}\sqrt{x+1}}-\frac{3}{4\sqrt{1-\frac{2}{x+1}}(x+1)^{\frac{3}{2}}}+\frac{1}{\sqrt{1-\frac{2}{x+1}}(x+1)^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x)**3,x)
```

```
[Out] Piecewise((sqrt(2)*I*acosh(sqrt(2)/sqrt(x + 1))/16 - I/(8*sqrt(-1 + 2/(x +
1))*sqrt(x + 1)) + 3*I/(4*sqrt(-1 + 2/(x + 1))*(x + 1)**(3/2)) - I/(sqrt(-1
 + 2/(x + 1))*(x + 1)**(5/2)), 2/Abs(x + 1) > 1), (-sqrt(2)*asin(sqrt(2)/sq
rt(x + 1))/16 + 1/(8*sqrt(1 - 2/(x + 1))*sqrt(x + 1)) - 3/(4*sqrt(1 - 2/(x
+ 1))*(x + 1)**(3/2)) + 1/(sqrt(1 - 2/(x + 1))*(x + 1)**(5/2)), True))
```

3.1307 $\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=154

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+}$$

Rubi [A] time = 0.05, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{9/2}(bc-ad)}{9d^6}+\frac{20b^3(c+dx)^{7/2}(bc-ad)^2}{7d^6}-\frac{4b^2(c+dx)^{5/2}(bc-ad)^3}{d^6}+\frac{10b(c+dx)^{3/2}(bc-ad)^4}{3d^6}-\frac{2\sqrt{c+dx}\,(bc-ad)^5}{d^6}+\frac{2b^5(c+dx)^{11/2}}{11d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^5*Sqrt[c + d*x])/d^6 + (10*b*(b*c - a*d)^4*(c + d*x)^(3/2))
/(3*d^6) - (4*b^2*(b*c - a*d)^3*(c + d*x)^(5/2))/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(7/2))/(7*d^6) - (10*b^4*(b*c - a*d)*(c + d*x)^(9/2))/(9*d^6) +
 (2*b^5*(c + d*x)^(11/2))/(11*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{\sqrt{c+dx}}\,dx = \int \left(\frac{(-bc+ad)^5}{d^5\sqrt{c+dx}}+\frac{5b(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{10b^2(bc-ad)^3(c+dx)^{3/2}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^5}{d^5}\right.$$
$$=-\frac{2(bc-ad)^5\sqrt{c+dx}}{d^6}+\frac{10b(bc-ad)^4(c+dx)^{3/2}}{3d^6}-\frac{4b^2(bc-ad)^3(c+dx)^{5/2}}{d^6}+\frac{20b^3(bc-ad)^2(}{7d^6}$$

Mathematica [A] time = 0.09, size = 123, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\left(-385b^4(c+dx)^4(bc-ad)+990b^3(c+dx)^3(bc-ad)^2-1386b^2(c+dx)^2(bc-ad)^3+1155b(c+dx)(bc-ad)^4-693(bc-ad)^5+63b^5(c+dx)^5\right)}{693d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-693*(b*c - a*d)^5 + 1155*b*(b*c - a*d)^4*(c + d*x) - 138
6*b^2*(b*c - a*d)^3*(c + d*x)^2 + 990*b^3*(b*c - a*d)^2*(c + d*x)^3 - 385*b
^4*(b*c - a*d)*(c + d*x)^4 + 63*b^5*(c + d*x)^5))/(693*d^6)
```

IntegrateAlgebraic [B] time = 0.10, size = 315, normalized size = 2.05

$$\frac{2\sqrt{c+dx}\left(693a^5d^5+1155a^4bd^4(c+dx)-3465a^4bcd^4+6930a^3b^2c^2d^3+1386a^3b^2d^3(c+dx)^2-4620a^3b^2cd^3(c+dx)-6930a^2b^3c^3d^2+6930a^2b^3c^2d^2(c+dx)+990a^2b^3d^2(c+dx)^3-4158a^2b^3cd^2(c+dx)^2+3465ab^4c^4d-4620ab^4c^3d(c+dx)+4158ab^4c^2d(c+dx)^2+385ab^4d(c+dx)^4-1980ab^4cd(c+dx)^3-693b^5c^5+1155b^5c^4(c+dx)-1386b^5c^3(c+dx)^2+990b^5c^2(c+dx)^3+63b^5(c+dx)^5-385b^5c(c+dx)^4\right)}{693d^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-693*b^5*c^5 + 3465*a*b^4*c^4*d - 6930*a^2*b^3*c^3*d^2 +
6930*a^3*b^2*c^2*d^3 - 3465*a^4*b*c*d^4 + 693*a^5*d^5 + 1155*b^5*c^4*(c + d
*x) - 4620*a*b^4*c^3*d*(c + d*x) + 6930*a^2*b^3*c^2*d^2*(c + d*x) - 4620*a^
3*b^2*c*d^3*(c + d*x) + 1155*a^4*b*d^4*(c + d*x) - 1386*b^5*c^3*(c + d*x)^2
 + 4158*a*b^4*c^2*d*(c + d*x)^2 - 4158*a^2*b^3*c*d^2*(c + d*x)^2 + 1386*a^3
*b^2*d^3*(c + d*x)^2 + 990*b^5*c^2*(c + d*x)^3 - 1980*a*b^4*c*d*(c + d*x)^3
 + 990*a^2*b^3*d^2*(c + d*x)^3 - 385*b^5*c*(c + d*x)^4 + 385*a*b^4*d*(c + d
*x)^4 + 63*b^5*(c + d*x)^5))/(693*d^6)
```

fricas [A] time = 1.26, size = 261, normalized size = 1.69

$$\frac{2\,{\left(63\,b^{5} d^{5} x^{5} - 256\,b^{5} c^{5} + 1408\,a b^{4} c^{4} d - 3168\,a^{2} b^{3} c^{3} d^{2} + 3696\,a^{3} b^{2} c^{2} d^{3} - 2310\,a^{4} b c d^{4} + 693\,a^{5} d^{5} - 35\,{\left(2\,b^{5} c d^{4} - 11\,a b^{4} d^{5}\right)} x^{4} + 10\,{\left(8\,b^{5} c^{2} d^{3} - 44\,a b^{4} c d^{4} + 99\,a^{2} b^{3} d^{5}\right)} x^{3} - 6\,{\left(16\,b^{5} c^{3} d^{2} - 88\,a b^{4} c^{2} d^{3} + 198\,a^{2} b^{3} c d^{4} - 231\,a^{3} b^{2} d^{5}\right)} x^{2} + {\left(128\,b^{5} c^{4} d - 704\,a b^{4} c^{3} d^{2} + 1584\,a^{2} b^{3} c^{2} d^{3} - 1848\,a^{3} b^{2} c d^{4} + 1155\,a^{4} b d^{5}\right)} x\right)} \sqrt{d x + c}}{693\,d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/693*(63*b^5*d^5*x^5 - 256*b^5*c^5 + 1408*a*b^4*c^4*d - 3168*a^2*b^3*c^3*d
^2 + 3696*a^3*b^2*c^2*d^3 - 2310*a^4*b*c*d^4 + 693*a^5*d^5 - 35*(2*b^5*c*d^
4 - 11*a*b^4*d^5)*x^4 + 10*(8*b^5*c^2*d^3 - 44*a*b^4*c*d^4 + 99*a^2*b^3*d^5
)*x^3 - 6*(16*b^5*c^3*d^2 - 88*a*b^4*c^2*d^3 + 198*a^2*b^3*c*d^4 - 231*a^3*
b^2*d^5)*x^2 + (128*b^5*c^4*d - 704*a*b^4*c^3*d^2 + 1584*a^2*b^3*c^2*d^3 -
1848*a^3*b^2*c*d^4 + 1155*a^4*b*d^5)*x)*sqrt(d*x + c)/d^6
```

giac [B] time = 1.07, size = 283, normalized size = 1.84

$$\frac{2\,{\left(693\,\sqrt{d x + c} a^{5} + \frac{1155\,{\left({\left(d x + c\right)}^{\frac{3}{2}} - 3\,\sqrt{d x + c} c\right)} a^{4} b}{d} + \frac{462\,{\left(3\,{\left(d x + c\right)}^{\frac{5}{2}} - 10\,{\left(d x + c\right)}^{\frac{3}{2}} c + 15\,\sqrt{d x + c} c^{2}\right)} a^{3} b^{2}}{d^{2}} + \frac{198\,{\left(5\,{\left(d x + c\right)}^{\frac{7}{2}} - 21\,{\left(d x + c\right)}^{\frac{5}{2}} c + 35\,{\left(d x + c\right)}^{\frac{3}{2}} c^{2} - 35\,\sqrt{d x + c} c^{3}\right)} a^{2} b^{3}}{d^{3}} + \frac{11\,{\left(35\,{\left(d x + c\right)}^{\frac{9}{2}} - 180\,{\left(d x + c\right)}^{\frac{7}{2}} c + 378\,{\left(d x + c\right)}^{\frac{5}{2}} c^{2} - 420\,{\left(d x + c\right)}^{\frac{3}{2}} c^{3} + 315\,\sqrt{d x + c} c^{4}\right)} a b^{4}}{d^{4}} + \frac{{\left(63\,{\left(d x + c\right)}^{\frac{11}{2}} - 385\,{\left(d x + c\right)}^{\frac{9}{2}} c + 990\,{\left(d x + c\right)}^{\frac{7}{2}} c^{2} - 1386\,{\left(d x + c\right)}^{\frac{5}{2}} c^{3} + 1155\,{\left(d x + c\right)}^{\frac{3}{2}} c^{4} - 693\,\sqrt{d x + c} c^{5}\right)} b^{5}}{d^{5}}\right)}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

maple [B] time = 0.01, size = 273, normalized size = 1.77

$$\frac{2 \sqrt{d x + c}\, \left(63 b^{5} x^{5} d^{5} + 385 a\, b^{4} d^{5} x^{4} - 70 b^{5} c\, d^{4} x^{4} + 990 a^{2} b^{3} d^{5} x^{3} - 440 a\, b^{4} c\, d^{4} x^{3} + 80 b^{5} c^{2} d^{3} x^{3} + 1386 a^{3} b^{2} d^{5} x^{2} - 1188 a^{2} b^{3} c\, d^{4} x^{2} + 528 a\, b^{4} c^{2} d^{3} x^{2} - 96 b^{5} c^{3} d^{2} x^{2} + 1155 a^{4} b\, d^{5} x - 1848 a^{3} b^{2} c\, d^{4} x + 1584 a^{2} b^{3} c^{2} d^{3} x - 704 a\, b^{4} c^{3} d^{2} x + 128 b^{5} c^{4} d x + 693 a^{5} d^{5} - 2310 a^{4} b c\, d^{4} + 3696 a^{3} b^{2} c^{2} d^{3} - 3168 a^{2} b^{3} c^{3} d^{2} + 1408 a\, b^{4} c^{4} d - 256 b^{5} c^{5}\right)}{693 d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 2/693*(d*x+c)^(1/2)*(63*b^5*d^5*x^5+385*a*b^4*d^5*x^4-70*b^5*c*d^4*x^4+990*
a^2*b^3*d^5*x^3-440*a*b^4*c*d^4*x^3+80*b^5*c^2*d^3*x^3+1386*a^3*b^2*d^5*x^2
-1188*a^2*b^3*c*d^4*x^2+528*a*b^4*c^2*d^3*x^2-96*b^5*c^3*d^2*x^2+1155*a^4*b
*d^5*x-1848*a^3*b^2*c*d^4*x+1584*a^2*b^3*c^2*d^3*x-704*a*b^4*c^3*d^2*x+128*
b^5*c^4*d*x+693*a^5*d^5-2310*a^4*b*c*d^4+3696*a^3*b^2*c^2*d^3-3168*a^2*b^3*
c^3*d^2+1408*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

maxima [B] time = 1.38, size = 283, normalized size = 1.84

$$\frac{2\,{\left(693\,\sqrt{d x + c} a^{5} + \frac{1155\,{\left({\left(d x + c\right)}^{\frac{3}{2}} - 3\,\sqrt{d x + c} c\right)} a^{4} b}{d} + \frac{462\,{\left(3\,{\left(d x + c\right)}^{\frac{5}{2}} - 10\,{\left(d x + c\right)}^{\frac{3}{2}} c + 15\,\sqrt{d x + c} c^{2}\right)} a^{3} b^{2}}{d^{2}} + \frac{198\,{\left(5\,{\left(d x + c\right)}^{\frac{7}{2}} - 21\,{\left(d x + c\right)}^{\frac{5}{2}} c + 35\,{\left(d x + c\right)}^{\frac{3}{2}} c^{2} - 35\,\sqrt{d x + c} c^{3}\right)} a^{2} b^{3}}{d^{3}} + \frac{11\,{\left(35\,{\left(d x + c\right)}^{\frac{9}{2}} - 180\,{\left(d x + c\right)}^{\frac{7}{2}} c + 378\,{\left(d x + c\right)}^{\frac{5}{2}} c^{2} - 420\,{\left(d x + c\right)}^{\frac{3}{2}} c^{3} + 315\,\sqrt{d x + c} c^{4}\right)} a b^{4}}{d^{4}} + \frac{{\left(63\,{\left(d x + c\right)}^{\frac{11}{2}} - 385\,{\left(d x + c\right)}^{\frac{9}{2}} c + 990\,{\left(d x + c\right)}^{\frac{7}{2}} c^{2} - 1386\,{\left(d x + c\right)}^{\frac{5}{2}} c^{3} + 1155\,{\left(d x + c\right)}^{\frac{3}{2}} c^{4} - 693\,\sqrt{d x + c} c^{5}\right)} b^{5}}{d^{5}}\right)}}{693\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/693*(693*sqrt(d*x + c)*a^5 + 1155*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a
^4*b/d + 462*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c
^2)*a^3*b^2/d^2 + 198*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x +
 c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a^2*b^3/d^3 + 11*(35*(d*x + c)^(9/2)
- 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3
 + 315*sqrt(d*x + c)*c^4)*a*b^4/d^4 + (63*(d*x + c)^(11/2) - 385*(d*x + c)^
(9/2)*c + 990*(d*x + c)^(7/2)*c^2 - 1386*(d*x + c)^(5/2)*c^3 + 1155*(d*x +
c)^(3/2)*c^4 - 693*sqrt(d*x + c)*c^5)*b^5/d^5)/d
```

mupad [B] time = 0.07, size = 137, normalized size = 0.89

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{11/2}}{11\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}+\frac{2\,{\left(a\,d-b\,c\right)}^5\,\sqrt{c+d\,x}}{d^6}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(11/2))/(11*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(9/2
))/(9*d^6) + (2*(a*d - b*c)^5*(c + d*x)^(1/2))/d^6 + (4*b^2*(a*d - b*c)^3*(
c + d*x)^(5/2))/d^6 + (20*b^3*(a*d - b*c)^2*(c + d*x)^(7/2))/(7*d^6) + (10*
b*(a*d - b*c)^4*(c + d*x)^(3/2))/(3*d^6)
```

sympy [A] time = 79.91, size = 728, normalized size = 4.73

$$\begin{cases} \frac{-\frac{2a^5c}{\sqrt{c+dx}} - 2a^5\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{10a^4bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{10a^4b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{20a^3b^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{20a^3b^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{20a^2b^3c\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{20a^2b^3\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{10ab^4c\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4} - \frac{10ab^4\left(-\frac{c^5}{\sqrt{c+dx}} - 5c^4\sqrt{c+dx} + \frac{10c^3(c+dx)^{\frac{3}{2}}}{3} - 2c^2(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4} - \frac{2b^5c\left(-\frac{c^5}{\sqrt{c+dx}} - 5c^4\sqrt{c+dx} + \frac{10c^3(c+dx)^{\frac{3}{2}}}{3} - 2c^2(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^5} - \frac{2b^5\left(\frac{c^6}{\sqrt{c+dx}} + 6c^5\sqrt{c+dx} - 5c^4(c+dx)^{\frac{3}{2}} + 4c^3(c+dx)^{\frac{5}{2}} - \frac{15c^2(c+dx)^{\frac{7}{2}}}{7} + \frac{2c(c+dx)^{\frac{9}{2}}}{3} - \frac{(c+dx)^{\frac{11}{2}}}{11}\right)}{d^5}}{d} & \text{for}\: d \neq 0 \\ \frac{\begin{cases} a^5x & \text{for}\: b = 0 \\ \frac{(a+bx)^6}{6b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**5*c/sqrt(c + d*x) - 2*a**5*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 10*a**4*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 10*a**4*b*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 20*a**3*b**2*c*(
c**2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 20*a**3
*b**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c
 + d*x)**(5/2)/5)/d**2 - 20*a**2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(
c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 20*a**2*b**3*(c*
*4/sqrt(c + d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c
+ d*x)**(5/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 10*a*b**4*c*(c**4/sqrt(c + d*x
) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5
 - (c + d*x)**(7/2)/7)/d**4 - 10*a*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(
c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c +
d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**4 - 2*b**5*c*(-c**5/sqrt(c + d*x) -
5*c**4*sqrt(c + d*x) + 10*c**3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2)
 + 5*c*(c + d*x)**(7/2)/7 - (c + d*x)**(9/2)/9)/d**5 - 2*b**5*(c**6/sqrt(c
+ d*x) + 6*c**5*sqrt(c + d*x) - 5*c**4*(c + d*x)**(3/2) + 4*c**3*(c + d*x)*
*(5/2) - 15*c**2*(c + d*x)**(7/2)/7 + 2*c*(c + d*x)**(9/2)/3 - (c + d*x)**(
11/2)/11)/d**5)/d, Ne(d, 0)), (Piecewise((a**5*x, Eq(b, 0)), ((a + b*x)**6/
(6*b), True))/sqrt(c), True))
```

3.1308 $\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=127

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+d\cdots}{9d^5}$$

Rubi [A] time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{7/2}(bc-ad)}{7d^5}+\frac{12b^2(c+dx)^{5/2}(bc-ad)^2}{5d^5}-\frac{8b(c+dx)^{3/2}(bc-ad)^3}{3d^5}+\frac{2\sqrt{c+dx}\,(bc-ad)^4}{d^5}+\frac{2b^4(c+dx)^{9/2}}{9d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^4*Sqrt[c + d*x])/d^5 - (8*b*(b*c - a*d)^3*(c + d*x)^(3/2))/(
3*d^5) + (12*b^2*(b*c - a*d)^2*(c + d*x)^(5/2))/(5*d^5) - (8*b^3*(b*c - a*d
)*(c + d*x)^(7/2))/(7*d^5) + (2*b^4*(c + d*x)^(9/2))/(9*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4\sqrt{c+dx}}-\frac{4b(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{6b^2(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{5/\cdots}}{d^4}\right.\\ &= \frac{2(bc-ad)^4\sqrt{c+dx}}{d^5}-\frac{8b(bc-ad)^3(c+dx)^{3/2}}{3d^5}+\frac{12b^2(bc-ad)^2(c+dx)^{5/2}}{5d^5}-\frac{8b^3(bc-ad)(c\cdots}{7d^5}\end{aligned}$$

Mathematica [A] time = 0.09, size = 101, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\left(-180b^3(c+dx)^3(bc-ad)+378b^2(c+dx)^2(bc-ad)^2-420b(c+dx)(bc-ad)^3+315(bc-ad)^4+35b^4(c+dx)^4\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*(b*c - a*d)^4 - 420*b*(b*c - a*d)^3*(c + d*x) + 378*b
^2*(b*c - a*d)^2*(c + d*x)^2 - 180*b^3*(b*c - a*d)*(c + d*x)^3 + 35*b^4*(c
+ d*x)^4))/(315*d^5)
```

IntegrateAlgebraic [A] time = 0.07, size = 213, normalized size = 1.68

$$\frac{2\sqrt{c+dx}\left(315a^4d^4+420a^3bd^3(c+dx)-1260a^3bcd^3+1890a^2b^2c^2d^2+378a^2b^2d^2(c+dx)^2-1260a^2b^2cd^2(c+dx)-1260ab^3c^3d+1260ab^3c^2d(c+dx)+180ab^3d(c+dx)^3-756ab^3cd(c+dx)^2+315b^4c^4-420b^4c^3(c+dx)+378b^4c^2(c+dx)^2+35b^4(c+dx)^4-180b^4c(c+dx)^3\right)}{315d^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*b^4*c^4 - 1260*a*b^3*c^3*d + 1890*a^2*b^2*c^2*d^2 - 1
260*a^3*b*c*d^3 + 315*a^4*d^4 - 420*b^4*c^3*(c + d*x) + 1260*a*b^3*c^2*d*(c
 + d*x) - 1260*a^2*b^2*c*d^2*(c + d*x) + 420*a^3*b*d^3*(c + d*x) + 378*b^4*
c^2*(c + d*x)^2 - 756*a*b^3*c*d*(c + d*x)^2 + 378*a^2*b^2*d^2*(c + d*x)^2 -
 180*b^4*c*(c + d*x)^3 + 180*a*b^3*d*(c + d*x)^3 + 35*b^4*(c + d*x)^4))/(31
5*d^5)
```

fricas [A] time = 1.18, size = 182, normalized size = 1.43

$$\frac{2\left(35\,b^4d^4x^4+128\,b^4c^4-576\,ab^3c^3d+1008\,a^2b^2c^2d^2-840\,a^3bcd^3+315\,a^4d^4-20\left(2\,b^4cd^3-9\,ab^3d^4\right)x^3+6\left(8\,b^4c^2d^2-36\,ab^3cd^3+63\,a^2b^2d^4\right)x^2-4\left(16\,b^4c^3d-72\,ab^3c^2d^2+126\,a^2b^2cd^3-105\,a^3bd^4\right)x\right)\sqrt{dx+c}}{315\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*d^4*x^4 + 128*b^4*c^4 - 576*a*b^3*c^3*d + 1008*a^2*b^2*c^2*d^
2 - 840*a^3*b*c*d^3 + 315*a^4*d^4 - 20*(2*b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 6*
(8*b^4*c^2*d^2 - 36*a*b^3*c*d^3 + 63*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 7
2*a*b^3*c^2*d^2 + 126*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(d*x + c)/d^5
```

giac [A] time = 0.96, size = 204, normalized size = 1.61

$$\frac{2\left(315\,\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d
```

maple [A] time = 0.01, size = 186, normalized size = 1.46

$$\frac{2\sqrt{dx+c}\,\left(35b^4x^4d^4+180a\,b^3d^4x^3-40b^4c\,d^3x^3+378a^2b^2d^4x^2-216a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+420a^3b\,d^4x-504a^2b^2c\,d^3x+288a\,b^3c^2d^2x-64b^4c^3dx+315a^4d^4-840a^3bc\,d^3+1008a^2b^2c^2d^2-576a\,b^3c^3d+128b^4c^4\right)}{315d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(35*b^4*d^4*x^4+180*a*b^3*d^4*x^3-40*b^4*c*d^3*x^3+378*
a^2*b^2*d^4*x^2-216*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+420*a^3*b*d^4*x-504*
a^2*b^2*c*d^3*x+288*a*b^3*c^2*d^2*x-64*b^4*c^3*d*x+315*a^4*d^4-840*a^3*b*c*
d^3+1008*a^2*b^2*c^2*d^2-576*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

maxima [A] time = 1.38, size = 204, normalized size = 1.61

$$\frac{2\left(315\,\sqrt{dx+c}\,a^4+\frac{420\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^3b}{d}+\frac{126\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)a^2b^2}{d^2}+\frac{36\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)ab^3}{d^3}+\frac{\left(35\,(dx+c)^{\frac{9}{2}}-180\,(dx+c)^{\frac{7}{2}}c+378\,(dx+c)^{\frac{5}{2}}c^2-420\,(dx+c)^{\frac{3}{2}}c^3+315\,\sqrt{dx+c}\,c^4\right)b^4}{d^4}\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(315*sqrt(d*x + c)*a^4 + 420*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^
3*b/d + 126*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^
2)*a^2*b^2/d^2 + 36*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c
)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b^3/d^3 + (35*(d*x + c)^(9/2) - 180*(
```

d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*
sqrt(d*x + c)*c^4)*b^4/d^4)/d

mupad [B] time = 0.24, size = 112, normalized size = 0.88

$$\frac{2\,b^4\,(c+d\,x)^{9/2}}{9\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,(c+d\,x)^{7/2}}{7\,d^5}+\frac{2\,(a\,d-b\,c)^4\,\sqrt{c+d\,x}}{d^5}+\frac{12\,b^2\,(a\,d-b\,c)^2\,(c+d\,x)^{5/2}}{5\,d^5}+\frac{8\,b\,(a\,d-b\,c)^3\,(c+d\,x)^{3/2}}{3\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(9/2))/(9*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(7/2))/(
7*d^5) + (2*(a*d - b*c)^4*(c + d*x)^(1/2))/d^5 + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(5/2))/(5*d^5) + (8*b*(a*d - b*c)^3*(c + d*x)^(3/2))/(3*d^5)
```

sympy [A] time = 56.90, size = 532, normalized size = 4.19

$$\begin{cases} \frac{-\frac{2a^4c}{\sqrt{c+dx}} - 2a^4\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{8a^3bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{8a^3b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{12a^2b^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{12a^2b^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{8ab^3c\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{8ab^3\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3} - \frac{2b^4c\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^4} - \frac{2b^4\left(-\frac{c^5}{\sqrt{c+dx}} - 5c^4\sqrt{c+dx} + \frac{10c^3(c+dx)^{\frac{3}{2}}}{3} - 2c^2(c+dx)^{\frac{5}{2}} + \frac{5c(c+dx)^{\frac{7}{2}}}{7} - \frac{(c+dx)^{\frac{9}{2}}}{9}\right)}{d^4}}{d} & \text{for}\: d \neq 0 \\ \frac{\begin{cases} a^4x & \text{for}\: b = 0 \\ \frac{(a+bx)^5}{5b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**4*c/sqrt(c + d*x) - 2*a**4*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 8*a**3*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 8*a**3*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 12*a**2*b**2*c*(c*
*2/sqrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 12*a**2*b
**2*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c +
 d*x)**(5/2)/5)/d**2 - 8*a*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*
x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 8*a*b**3*(c**4/sqrt(c
+ d*x) + 4*c**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5
/2)/5 - (c + d*x)**(7/2)/7)/d**3 - 2*b**4*c*(c**4/sqrt(c + d*x) + 4*c**3*sq
rt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c + d*x)*
*(7/2)/7)/d**4 - 2*b**4*(-c**5/sqrt(c + d*x) - 5*c**4*sqrt(c + d*x) + 10*c*
*3*(c + d*x)**(3/2)/3 - 2*c**2*(c + d*x)**(5/2) + 5*c*(c + d*x)**(7/2)/7 -
(c + d*x)**(9/2)/9)/d**4)/d, Ne(d, 0)), (Piecewise((a**4*x, Eq(b, 0)), ((a
+ b*x)**5/(5*b), True))/sqrt(c), True))
```

3.1309 $\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=96

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2(c+dx)^{5/2}(bc-ad)}{5d^4}+\frac{2b(c+dx)^{3/2}(bc-ad)^2}{d^4}-\frac{2\sqrt{c+dx}\,(bc-ad)^3}{d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (-2*(b*c - a*d)^3*Sqrt[c + d*x])/d^4 + (2*b*(b*c - a*d)^2*(c + d*x)^(3/2))/
d^4 - (6*b^2*(b*c - a*d)*(c + d*x)^(5/2))/(5*d^4) + (2*b^3*(c + d*x)^(7/2))
/(7*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3\sqrt{c+dx}}+\frac{3b(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{3/2}}{d^3}+\frac{b^3(c+dx)^{5/2}}{d^3}\right)dx\\ &= -\frac{2(bc-ad)^3\sqrt{c+dx}}{d^4}+\frac{2b(bc-ad)^2(c+dx)^{3/2}}{d^4}-\frac{6b^2(bc-ad)(c+dx)^{5/2}}{5d^4}+\frac{2b^3(c+dx)^{7/2}}{7d^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 79, normalized size = 0.82

$$\frac{2\sqrt{c+dx}\left(-21b^2(c+dx)^2(bc-ad)+35b(c+dx)(bc-ad)^2-35(bc-ad)^3+5b^3(c+dx)^3\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(-35*(b*c - a*d)^3 + 35*b*(b*c - a*d)^2*(c + d*x) - 21*b^2
*(b*c - a*d)*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*d^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 132, normalized size = 1.38

$$\frac{2\sqrt{c+dx}\left(35a^3d^3+35a^2bd^2(c+dx)-105a^2bcd^2+105ab^2c^2d+21ab^2d(c+dx)^2-70ab^2cd(c+dx)-35b^3c^3+35b^3c^2(c+dx)+5b^3(c+dx)^3-21b^3c(c+dx)^2\right)}{35d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/Sqrt[c + d*x],x]
```

[Out] (2*Sqrt[c + d*x]*(-35*b^3*c^3 + 105*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 35*a^3*d^3 + 35*b^3*c^2*(c + d*x) - 70*a*b^2*c*d*(c + d*x) + 35*a^2*b*d^2*(c + d*x) - 21*b^3*c*(c + d*x)^2 + 21*a*b^2*d*(c + d*x)^2 + 5*b^3*(c + d*x)^3))/(35*d^4)

fricas [A] time = 0.94, size = 115, normalized size = 1.20

$$\frac{2\left(5\,b^3d^3x^3-16\,b^3c^3+56\,ab^2c^2d-70\,a^2bcd^2+35\,a^3d^3-3\left(2\,b^3cd^2-7\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-28\,ab^2cd^2+35\,a^2bd^3\right)x\right)\sqrt{dx+c}}{35\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 2/35*(5*b^3*d^3*x^3 - 16*b^3*c^3 + 56*a*b^2*c^2*d - 70*a^2*b*c*d^2 + 35*a^3*d^3 - 3*(2*b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 28*a*b^2*c*d^2 + 35*a^2*b*d^3)*x)*sqrt(d*x + c)/d^4

giac [A] time = 1.01, size = 137, normalized size = 1.43

$$\frac{2\left(35\,\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")

[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d

maple [A] time = 0.01, size = 116, normalized size = 1.21

$$\frac{2\sqrt{dx+c}\,\left(5b^3x^3d^3+21a\,b^2d^3x^2-6b^3c\,d^2x^2+35a^2b\,d^3x-28a\,b^2c\,d^2x+8b^3c^2dx+35a^3d^3-70a^2bc\,d^2+56a\,b^2c^2d-16b^3c^3\right)}{35d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^3/(d*x+c)^(1/2),x)

[Out] 2/35*(d*x+c)^(1/2)*(5*b^3*d^3*x^3+21*a*b^2*d^3*x^2-6*b^3*c*d^2*x^2+35*a^2*b*d^3*x-28*a*b^2*c*d^2*x+8*b^3*c^2*d*x+35*a^3*d^3-70*a^2*b*c*d^2+56*a*b^2*c^2*d-16*b^3*c^3)/d^4

maxima [A] time = 1.38, size = 137, normalized size = 1.43

$$\frac{2\left(35\,\sqrt{dx+c}\,a^3+\frac{35\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)a^2b}{d}+\frac{7\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)ab^2}{d^2}+\frac{\left(5\,(dx+c)^{\frac{7}{2}}-21\,(dx+c)^{\frac{5}{2}}c+35\,(dx+c)^{\frac{3}{2}}c^2-35\,\sqrt{dx+c}\,c^3\right)b^3}{d^3}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 2/35*(35*sqrt(d*x + c)*a^3 + 35*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a^2*b/d + 7*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b^2/d^2 + (5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*b^3/d^3)/d

mupad [B] time = 0.26, size = 87, normalized size = 0.91

$$\frac{2\,b^3\,{\left(c+d\,x\right)}^{7/2}}{7\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^4}+\frac{2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^4}+\frac{2\,b\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(1/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(7/2))/(7*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(5/2))/(
5*d^4) + (2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^4 + (2*b*(a*d - b*c)^2*(c + d*
x)^(3/2))/d^4
```

sympy [A] time = 37.06, size = 366, normalized size = 3.81

$$\begin{cases} \frac{-\frac{2a^3c}{\sqrt{c+dx}} - 2a^3\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{6a^2bc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{6a^2b\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{6ab^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{6ab^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2} - \frac{2b^3c\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^3} - \frac{2b^3\left(\frac{c^4}{\sqrt{c+dx}} + 4c^3\sqrt{c+dx} - 2c^2(c+dx)^{\frac{3}{2}} + \frac{4c(c+dx)^{\frac{5}{2}}}{5} - \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^3}}{d} & \text{for } d \neq 0 \\ \frac{\begin{cases} a^3x & \text{for } b = 0 \\ \frac{(a+bx)^4}{4b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**3*c/sqrt(c + d*x) - 2*a**3*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 6*a**2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 6*a**2*b*(c**2/sqr
t(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 6*a*b**2*c*(c**2/s
qrt(c + d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 6*a*b**2*(-c*
*3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(
5/2)/5)/d**2 - 2*b**3*c*(-c**3/sqrt(c + d*x) - 3*c**2*sqrt(c + d*x) + c*(c
+ d*x)**(3/2) - (c + d*x)**(5/2)/5)/d**3 - 2*b**3*(c**4/sqrt(c + d*x) + 4*c
**3*sqrt(c + d*x) - 2*c**2*(c + d*x)**(3/2) + 4*c*(c + d*x)**(5/2)/5 - (c +
 d*x)**(7/2)/7)/d**3)/d, Ne(d, 0)), (Piecewise((a**3*x, Eq(b, 0)), ((a + b*
x)**4/(4*b), True))/sqrt(c), True))
```

3.1310 $\int \frac{(a+bx)^2}{\sqrt{c+dx}} dx$

Optimal. Leaf size=69

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b(c+dx)^{3/2}(bc-ad)}{3d^3}+\frac{2\sqrt{c+dx}\,(bc-ad)^2}{d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*(b*c - a*d)^2*Sqrt[c + d*x])/d^3 - (4*b*(b*c - a*d)*(c + d*x)^(3/2))/(3*
d^3) + (2*b^2*(c + d*x)^(5/2))/(5*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{\sqrt{c+dx}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2\sqrt{c+dx}}-\frac{2b(bc-ad)\sqrt{c+dx}}{d^2}+\frac{b^2(c+dx)^{3/2}}{d^2}\right)dx\\ &= \frac{2(bc-ad)^2\sqrt{c+dx}}{d^3}-\frac{4b(bc-ad)(c+dx)^{3/2}}{3d^3}+\frac{2b^2(c+dx)^{5/2}}{5d^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.87

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2+10abd(dx-2c)+b^2\left(8c^2-4cdx+3d^2x^2\right)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(-2*c + d*x) + b^2*(8*c^2 - 4*c*d*x
 + 3*d^2*x^2)))/(15*d^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 1.04

$$\frac{2\sqrt{c+dx}\,\left(15a^2d^2+10abd(c+dx)-30abcd+15b^2c^2+3b^2(c+dx)^2-10b^2c(c+dx)\right)}{15d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/Sqrt[c + d*x],x]
```

[Out] (2*Sqrt[c + d*x]*(15*b^2*c^2 - 30*a*b*c*d + 15*a^2*d^2 - 10*b^2*c*(c + d*x) + 10*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(15*d^3)

fricas [A] time = 1.46, size = 64, normalized size = 0.93

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-20\,abcd+15\,a^2d^2-2\left(2\,b^2cd-5\,abd^2\right)x\right)\sqrt{dx+c}}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 20*a*b*c*d + 15*a^2*d^2 - 2*(2*b^2*c*d - 5*a*b*d^2)*x)*sqrt(d*x + c)/d^3
```

giac [A] time = 1.10, size = 82, normalized size = 1.19

$$\frac{2\left(15\,\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^2)/d
```

maple [A] time = 0.00, size = 63, normalized size = 0.91

$$\frac{2\sqrt{dx+c}\,\left(3b^2x^2d^2+10ab\,d^2x-4b^2cdx+15a^2d^2-20abcd+8b^2c^2\right)}{15d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(3*b^2*d^2*x^2+10*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-20*a*b*c*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.37, size = 82, normalized size = 1.19

$$\frac{2\left(15\,\sqrt{dx+c}\,a^2+\frac{10\left((dx+c)^{\frac{3}{2}}-3\,\sqrt{dx+c}\,c\right)ab}{d}+\frac{\left(3\,(dx+c)^{\frac{5}{2}}-10\,(dx+c)^{\frac{3}{2}}c+15\,\sqrt{dx+c}\,c^2\right)b^2}{d^2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sqrt(d*x + c)*a^2 + 10*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b/d + (3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*b^2/d^2)/d
```

mupad [B] time = 0.07, size = 68, normalized size = 0.99

$$\frac{2\,\sqrt{c+d\,x}\,\left(3\,b^2\,{\left(c+d\,x\right)}^2+15\,a^2\,d^2+15\,b^2\,c^2-10\,b^2\,c\,\left(c+d\,x\right)+10\,a\,b\,d\,\left(c+d\,x\right)-30\,a\,b\,c\,d\right)}{15\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] (2*(c + d*x)^(1/2)*(3*b^2*(c + d*x)^2 + 15*a^2*d^2 + 15*b^2*c^2 - 10*b^2*c*
(c + d*x) + 10*a*b*d*(c + d*x) - 30*a*b*c*d))/(15*d^3)
```

sympy [A] time = 20.94, size = 231, normalized size = 3.35

$$\begin{cases} \frac{-\frac{2a^2c}{\sqrt{c+dx}} - 2a^2\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right) - \frac{4abc\left(-\frac{c}{\sqrt{c+dx}} - \sqrt{c+dx}\right)}{d} - \frac{4ab\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d} - \frac{2b^2c\left(\frac{c^2}{\sqrt{c+dx}} + 2c\sqrt{c+dx} - \frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d^2} - \frac{2b^2\left(-\frac{c^3}{\sqrt{c+dx}} - 3c^2\sqrt{c+dx} + c(c+dx)^{\frac{3}{2}} - \frac{(c+dx)^{\frac{5}{2}}}{5}\right)}{d^2}}{d} & \text{for } d \neq 0 \\ \frac{\begin{cases} a^2x & \text{for } b = 0 \\ \frac{(a+bx)^3}{3b} & \text{otherwise} \end{cases}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a**2*c/sqrt(c + d*x) - 2*a**2*(-c/sqrt(c + d*x) - sqrt(c + d
*x)) - 4*a*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 4*a*b*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d - 2*b**2*c*(c**2/sqrt(c +
d*x) + 2*c*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d**2 - 2*b**2*(-c**3/sqrt(c
+ d*x) - 3*c**2*sqrt(c + d*x) + c*(c + d*x)**(3/2) - (c + d*x)**(5/2)/5)/d*
*2)/d, Ne(d, 0)), (Piecewise((a**2*x, Eq(b, 0)), ((a + b*x)**3/(3*b), True)
)/sqrt(c), True))
```

3.1311 $\int \frac{a+bx}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=40

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2b(c+dx)^{3/2}}{3d^2}-\frac{2\sqrt{c+dx}\,(bc-ad)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/Sqrt[c + d*x],x]
[Out] (-2*(b*c - a*d)*Sqrt[c + d*x])/d^2 + (2*b*(c + d*x)^(3/2))/(3*d^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{\sqrt{c+dx}}\,dx &= \int \left(\frac{-bc+ad}{d\sqrt{c+dx}}+\frac{b\sqrt{c+dx}}{d}\right)dx\\ &= -\frac{2(bc-ad)\sqrt{c+dx}}{d^2}+\frac{2b(c+dx)^{3/2}}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.72

$$\frac{2\sqrt{c+dx}\,(3ad-2bc+bdx)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x]*(-2*b*c + 3*a*d + b*d*x))/(3*d^2)
```

IntegrateAlgebraic [A] time = 0.02, size = 32, normalized size = 0.80

$$\frac{2\sqrt{c+dx}\,(3ad+b(c+dx)-3bc)}{3d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/Sqrt[c + d*x],x]
[Out] (2*Sqrt[c + d*x]*(-3*b*c + 3*a*d + b*(c + d*x)))/(3*d^2)
```

fricas [A] time = 1.06, size = 25, normalized size = 0.62

$$\frac{2\,(bdx - 2\,bc + 3\,ad)\sqrt{dx + c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2/3*(b*d*x - 2*b*c + 3*a*d)*sqrt(d*x + c)/d^2
```

giac [A] time = 0.88, size = 39, normalized size = 0.98

$$\frac{2\left(3\,\sqrt{dx + c}\,a + \frac{\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

maple [A] time = 0.00, size = 26, normalized size = 0.65

$$\frac{2\sqrt{dx + c}\,\,(bdx + 3ad - 2bc)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(1/2),x)
[Out] 2/3*(d*x+c)^(1/2)*(b*d*x+3*a*d-2*b*c)/d^2
```

maxima [A] time = 1.35, size = 39, normalized size = 0.98

$$\frac{2\left(3\,\sqrt{dx + c}\,a + \frac{\left((dx+c)^{\frac{3}{2}} - 3\,\sqrt{dx+c}\,c\right)b}{d}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2/3*(3*sqrt(d*x + c)*a + ((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*b/d)/d
```

mupad [B] time = 0.05, size = 28, normalized size = 0.70

$$\frac{2\,\sqrt{c + d\,x}\,\,(3\,a\,d - 3\,b\,c + b\,(c + d\,x))}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(1/2)*(3*a*d - 3*b*c + b*(c + d*x)))/(3*d^2)
```

sympy [A] time = 4.78, size = 121, normalized size = 3.02

$$\begin{cases} \frac{-\frac{2ac}{\sqrt{c+dx}}-2a\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)-\frac{2bc\left(-\frac{c}{\sqrt{c+dx}}-\sqrt{c+dx}\right)}{d}-\frac{2b\left(\frac{c^2}{\sqrt{c+dx}}+2c\sqrt{c+dx}-\frac{(c+dx)^{\frac{3}{2}}}{3}\right)}{d}}{d} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((-2*a*c/sqrt(c + d*x) - 2*a*(-c/sqrt(c + d*x) - sqrt(c + d*x)) -
 2*b*c*(-c/sqrt(c + d*x) - sqrt(c + d*x))/d - 2*b*(c**2/sqrt(c + d*x) + 2*c
*sqrt(c + d*x) - (c + d*x)**(3/2)/3)/d)/d, Ne(d, 0)), ((a*x + b*x**2/2)/sqr
t(c), True))
```

3.1312 $\int \frac{1}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{c+dx}}{d}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{c+dx}} \, dx = \frac{2\sqrt{c+dx}}{d}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/Sqrt[c + d*x],x]
```

```
[Out] (2*Sqrt[c + d*x])/d
```

fricas [A] time = 1.22, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] 2*sqrt(d*x + c)/d
```

giac [A] time = 0.91, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 2*sqrt(d*x + c)/d
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(1/2),x)
[Out] 2*(d*x+c)^(1/2)/d
```

maxima [A] time = 1.30, size = 12, normalized size = 0.86

$$\frac{2\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 2*sqrt(d*x + c)/d
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{2\sqrt{c+d\,x}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(1/2),x)
[Out] (2*(c + d*x)^(1/2))/d
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{2\sqrt{c+dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(1/2),x)
[Out] 2*sqrt(c + d*x)/d
```

3.1313 $\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(Sqrt[b]*Sqrt[b*c - a
*d])
```

IntegrateAlgebraic [A] time = 0.05, size = 57, normalized size = 1.21

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(Sqrt[b
]*Sqrt[-(b*c) + a*d])
```

fricas [A] time = 1.20, size = 119, normalized size = 2.53

$$\left[\frac{\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)}{\sqrt{b^2c-abd}}, \frac{2\,\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)}{b^2c-abd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a))
/sqrt(b^2*c - a*b*d), 2*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sq
rt(d*x + c)/(b*d*x + b*c))/(b^2*c - a*b*d)]
```

giac [A] time = 0.88, size = 38, normalized size = 0.81

$$\frac{2\,\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/sqrt(-b^2*c + a*b*d)
```

maple [A] time = 0.01, size = 37, normalized size = 0.79

$$\frac{2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] 2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.27, size = 38, normalized size = 0.81

$$\frac{2\,\mathrm{atan}\left(\frac{b\,\sqrt{c+d\,x}}{\sqrt{a\,b\,d-b^2\,c}}\right)}{\sqrt{a\,b\,d-b^2\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/2)),x)
[Out] (2*atan((b*(c + d*x)^(1/2))/(a*b*d - b^2*c)^(1/2)))/(a*b*d - b^2*c)^(1/2)
```

sympy [A] time = 5.41, size = 44, normalized size = 0.94

$$-\frac{2\,\mathrm{atan}\left(\frac{1}{\sqrt{\frac{b}{ad-bc}}\sqrt{c+dx}}\right)}{\sqrt{\frac{b}{ad-bc}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/2),x)
[Out] -2*atan(1/(sqrt(b/(a*d - b*c))*sqrt(c + d*x)))/(sqrt(b/(a*d - b*c))*(a*d -
b*c))
```

3.1314 $\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[b*c - a*d]])/(Sqrt[b]*(b*c - a*d)^(3/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx &= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)}-\frac{d\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)}\\
&= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)}-\frac{\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{bc-ad}\\
&= -\frac{\sqrt{c+dx}}{(bc-ad)(a+bx)}+\frac{d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{\sqrt{b}\,(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.00

$$\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{ad-bc}}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}-\frac{\sqrt{c+dx}}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] -(Sqrt[c + d*x]/((b*c - a*d)*(a + b*x))) + (d*ArcTan[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[-(b*c) + a*d]])/(Sqrt[b]*(-(b*c) + a*d)^(3/2))
```

IntegrateAlgebraic [A] time = 0.20, size = 98, normalized size = 1.29

$$\frac{d\sqrt{c+dx}}{(bc-ad)(-ad-b(c+dx)+bc)}-\frac{d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{\sqrt{b}\,(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*Sqrt[c + d*x]),x]
```

```
[Out] (d*Sqrt[c + d*x])/((b*c - a*d)*(b*c - a*d - b*(c + d*x))) - (d*ArcTan[(Sqrt
[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(Sqrt[b]*(-(b*c) + a*d)
^(3/2))
```

fricas [B] time = 1.29, size = 280, normalized size = 3.68

$$\left[-\frac{\sqrt{b^2c-abd}\,(bdx+ad)\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)+2\left(b^2c-abd\right)\sqrt{dx+c}}{2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x\right)},\ -\frac{\sqrt{-b^2c+abd}\,(bdx+ad)\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)+\left(b^2c-abd\right)\sqrt{dx+c}}{ab^3c^2-2\,a^2b^2cd+a^3bd^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(b^2*c - a*b*d)*(b*d*x + a*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(
b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) + 2*(b^2*c - a*b*d)*sqrt(d*x + c))
/(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*x), -(sqrt(-b^2*c + a*b*d)*(b*d*x + a*d)*arctan(sqrt(-b^2*c + a*b*d)*s
qrt(d*x + c)/(b*d*x + b*c)) + (b^2*c - a*b*d)*sqrt(d*x + c))/(a*b^3*c^2 - 2
*a^2*b^2*c*d + a^3*b*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x)]
```

giac [A] time = 1.03, size = 87, normalized size = 1.14

$$-\frac{d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}-\frac{\sqrt{dx+c}\,d}{((dx+c)b-bc+ad)(bc-ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
- a*d)) - sqrt(d*x + c)*d/(((d*x + c)*b - b*c + a*d)*(b*c - a*d))
```

maple [A] time = 0.01, size = 77, normalized size = 1.01

$$\frac{d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\,d}{(ad-bc)\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] d*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)+d/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arcta
n((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.09, size = 74, normalized size = 0.97

$$\frac{d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{\sqrt{b}\,{\left(a\,d-b\,c\right)}^{3/2}}+\frac{d\,\sqrt{c+d\,x}}{\left(a\,d-b\,c\right)\,\left(a\,d-b\,c+b\,\left(c+d\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(1/2)),x)
```

```
[Out] (d*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(b^(1/2)*(a*d - b*c)^
(3/2)) + (d*(c + d*x)^(1/2))/((a*d - b*c)*(a*d - b*c + b*(c + d*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{2} \sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*sqrt(c + d*x)), x)
```

3.1315 $\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx$

Optimal. Leaf size=114

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Rubi [A] time = 0.04, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}+\frac{3d\sqrt{c+dx}}{4(a+bx)(bc-ad)^2}-\frac{\sqrt{c+dx}}{2(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(2*(b*c - a*d)*(a + b*x)^2) + (3*d*Sqrt[c + d*x])/(4*(b*c -
a*d)^2*(a + b*x)) - (3*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]
)/(4*Sqrt[b]*(b*c - a*d)^(5/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} - \frac{(3d)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{4(bc-ad)}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} + \frac{\left(3d^2\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} + \frac{(3d)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{4(bc-ad)^2}$$
$$= -\frac{\sqrt{c+dx}}{2(bc-ad)(a+bx)^2} + \frac{3d\sqrt{c+dx}}{4(bc-ad)^2(a+bx)} - \frac{3d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4\sqrt{b}\,(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.44

$$\frac{2d^2\sqrt{c+dx}\;{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^2*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 3, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^3
```

IntegrateAlgebraic [A] time = 0.23, size = 124, normalized size = 1.09

$$\frac{d^2\sqrt{c+dx}\,(5ad+3b(c+dx)-5bc)}{4(bc-ad)^2(-ad-b(c+dx)+bc)^2} - \frac{3d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4\sqrt{b}\,(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*Sqrt[c + d*x]),x]
```

```
[Out] (d^2*Sqrt[c + d*x]*(-5*b*c + 5*a*d + 3*b*(c + d*x)))/(4*(b*c - a*d)^2*(b*c
- a*d - b*(c + d*x))^2) - (3*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
+ d*x])/(b*c - a*d)])/(4*Sqrt[b]*(-(b*c) + a*d)^(5/2))
```

fricas [B] time = 1.57, size = 549, normalized size = 4.82

$$\left[\frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2\,bc-ad-2\,\sqrt{b^2c-abd}\,\sqrt{dx+c}}{bx+a}\right)-2\left(2\,b^3c^2-7\,ab^2cd+5\,a^2bd^2-3\left(b^3cd-ab^2d^2\right)x\right)\sqrt{dx+c}}{8\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^2+2\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x\right)}, \frac{3\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\,\sqrt{dx+c}}{bdx+bc}\right)-\left(2\,b^3c^2-7\,ab^2cd+5\,a^2bd^2-3\left(b^3cd-ab^2d^2\right)x\right)\sqrt{dx+c}}{4\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^2+2\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*sqrt(b^2*c - a*b*d)*log((b*d*
x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(2*b^
3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d - a*b^2*d^2)*x)*sqrt(d*x + c
))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3 + (b^6*c^3
- 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*x^2 + 2*(a*b^5*c^3 - 3*a^2
*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x), 1/4*(3*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x
 + c)/(b*d*x + b*c)) - (2*b^3*c^2 - 7*a*b^2*c*d + 5*a^2*b*d^2 - 3*(b^3*c*d
- a*b^2*d^2)*x)*sqrt(d*x + c))/(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c
```

```
*d^2 - a^5*b*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*x^2 + 2*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*x)]
```

giac [A] time = 0.93, size = 148, normalized size = 1.30

$$\frac{3\,d^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{3\,(dx+c)^{\frac{3}{2}}bd^2-5\,\sqrt{dx+c}\,bcd^2+5\,\sqrt{dx+c}\,ad^3}{4\left(b^2c^2-2\,abcd+a^2d^2\right)((dx+c)b-bc+ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 3/4*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 1/4*(3*(d*x + c)^(3/2)*b*d^2 - 5*sqrt(d*
x + c)*b*c*d^2 + 5*sqrt(d*x + c)*a*d^3)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((
d*x + c)*b - b*c + a*d)^2)
```

maple [A] time = 0.01, size = 115, normalized size = 1.01

$$\frac{3d^2\arctan\left(\frac{\sqrt{dx+c}\ b}{\sqrt{(ad-bc)b}}\right)}{4\,(ad-bc)^2\,\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\ d^2}{2\,(ad-bc)\,(bdx+ad)^2}+\frac{3\sqrt{dx+c}\ d^2}{4\,(ad-bc)^2\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*d^2*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^2+3/4*d^2/(a*d-b*c)^2*(d*x+c)^(
1/2)/(b*d*x+a*d)+3/4*d^2/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/
2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.33, size = 142, normalized size = 1.25

$$\frac{\frac{5\,d^2\,\sqrt{c+d\,x}}{4\,(a\,d-b\,c)}+\frac{3\,b\,d^2\,{(c+d\,x)}^{3/2}}{4\,{(a\,d-b\,c)}^2}}{b^2\,{(c+d\,x)}^2-\left(2\,b^2\,c-2\,a\,b\,d\right)\,(c+d\,x)+a^2\,d^2+b^2\,c^2-2\,a\,b\,c\,d}+\frac{3\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{4\,\sqrt{b}\,{(a\,d-b\,c)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(1/2)),x)
```

```
[Out] ((5*d^2*(c + d*x)^(1/2))/(4*(a*d - b*c)) + (3*b*d^2*(c + d*x)^(3/2))/(4*(a*
d - b*c)^2))/(b^2*(c + d*x)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (3*d^2*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/
2)))/(4*b^(1/2)*(a*d - b*c)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1316 $\int \frac{1}{(a+bx)^4 \sqrt{c+dx}} \, dx$

Optimal. Leaf size=147

$$\frac{5d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{5d^2\sqrt{c+dx}}{8(a+bx)(bc-ad)^3}+\frac{5d^3 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8\sqrt{b}\,(bc-ad)^{7/2}}+\frac{5d\sqrt{c+dx}}{12(a+bx)^2(bc-ad)^2}-\frac{\sqrt{c+dx}}{3(a+bx)^3(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(3*(b*c - a*d)*(a + b*x)^3) + (5*d*Sqrt[c + d*x])/(12*(b*c - a*d)^2*(a + b*x)^2) - (5*d^2*Sqrt[c + d*x])/(8*(b*c - a*d)^3*(a + b*x)) + (5*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*Sqrt[b]*(b*c - a*d)^(7/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} - \frac{(5d)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{6(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} + \frac{\left(5d^2\right)\int \frac{1}{(a+bx)^2\sqrt{c+dx}}\,dx}{8(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{\left(5d^3\right)\int \frac{1}{(a+bx)\sqrt{\ldots}}}{16(bc-ad\ldots}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} - \frac{\left(5d^2\right)\text{Subst}\left(\int \ldots\right.}{8\ldots}$$

$$= -\frac{\sqrt{c+dx}}{3(bc-ad)(a+bx)^3} + \frac{5d\sqrt{c+dx}}{12(bc-ad)^2(a+bx)^2} - \frac{5d^2\sqrt{c+dx}}{8(bc-ad)^3(a+bx)} + \frac{5d^3\tanh^{-1}\left(\frac{\sqrt{b}\ldots}{\sqrt{\ldots}}\right.}{8\sqrt{b}\,(bc-a\ldots}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.34

$$\frac{2d^3\sqrt{c+dx}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^3*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 4, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^4
```

IntegrateAlgebraic [A] time = 0.27, size = 173, normalized size = 1.18

$$\frac{d^3\sqrt{c+dx}\left(33a^2d^2+40abd(c+dx)-66abcd+33b^2c^2+15b^2(c+dx)^2-40b^2c(c+dx)\right)}{24(bc-ad)^3(-ad-b(c+dx)+bc)^3} + \frac{5d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8\sqrt{b}\,(bc-ad)^3\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*Sqrt[c + d*x]),x]
```

```
[Out] (d^3*Sqrt[c + d*x]*(33*b^2*c^2 - 66*a*b*c*d + 33*a^2*d^2 - 40*b^2*c*(c + d*
x) + 40*a*b*d*(c + d*x) + 15*b^2*(c + d*x)^2))/(24*(b*c - a*d)^3*(b*c - a*d
 - b*(c + d*x))^3) + (5*d^3*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x
])/(b*c - a*d)])/(8*Sqrt[b]*(b*c - a*d)^3*Sqrt[-(b*c) + a*d])
```

fricas [B] time = 1.38, size = 884, normalized size = 6.01

$$\left[-\frac{15\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{b^2c-abd}\,\log\left(\frac{bdx+2bc-ad-2\sqrt{b^2c-abd}\sqrt{dx+c}}{bx+a}\right)+2\left(8b^4c^3-34ab^3c^2d+59a^2b^2cd^2-33a^3bd^3+15\left(b^4cd^2-ab^3d^3\right)x^2-10\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{48\left(a^3b^5c^4-4a^4b^4c^3d+6a^5b^3c^2d^2-4a^6b^2cd^3+a^7bd^4+\left(b^8c^4-4ab^7c^3d+6a^2b^6c^2d^2-4a^3b^5cd^3+a^4b^4d^4\right)x^3+3\left(ab^7c^4-4a^2b^6c^3d+6a^3b^5c^2d^2-4a^4b^4cd^3+a^5b^3d^4\right)x^2+3\left(a^2b^6c^4-4a^3b^5c^3d+6a^4b^4c^2d^2-4a^5b^3cd^3+a^6b^2d^4\right)x\right)}, -\frac{15\left(b^3d^3x^3+3ab^2d^3x^2+3a^2bd^3x+a^3d^3\right)\sqrt{-b^2c+abd}\,\arctan\left(\frac{\sqrt{-b^2c+abd}\sqrt{dx+c}}{bdx+bc}\right)+\left(8b^4c^3-34ab^3c^2d+59a^2b^2cd^2-33a^3bd^3+15\left(b^4cd^2-ab^3d^3\right)x^2-10\left(b^4c^2d-5ab^3cd^2+4a^2b^2d^3\right)x\right)\sqrt{dx+c}}{24\left(a^3b^5c^4-4a^4b^4c^3d+6a^5b^3c^2d^2-4a^6b^2cd^3+a^7bd^4+\left(b^8c^4-4ab^7c^3d+6a^2b^6c^2d^2-4a^3b^5cd^3+a^4b^4d^4\right)x^3+3\left(ab^7c^4-4a^2b^6c^3d+6a^3b^5c^2d^2-4a^4b^4cd^3+a^5b^3d^4\right)x^2+3\left(a^2b^6c^4-4a^3b^5c^3d+6a^4b^4c^2d^2-4a^5b^3cd^3+a^6b^2d^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(b
^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c
))/(b*x + a)) + 2*(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b
*d^3 + 15*(b^4*c*d^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a
^2*b^2*d^3)*x)*sqrt(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*
```

```
c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3
*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^
4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)
, -1/24*(15*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*sqrt(
-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) +
(8*b^4*c^3 - 34*a*b^3*c^2*d + 59*a^2*b^2*c*d^2 - 33*a^3*b*d^3 + 15*(b^4*c*d
^2 - a*b^3*d^3)*x^2 - 10*(b^4*c^2*d - 5*a*b^3*c*d^2 + 4*a^2*b^2*d^3)*x)*sqr
t(d*x + c))/(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6*b^2*
c*d^3 + a^7*b*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^
5*c*d^3 + a^4*b^4*d^4)*x^3 + 3*(a*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2
*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^2 + 3*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*
d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*b^2*d^4)*x)]
```

giac [A] time = 1.02, size = 231, normalized size = 1.57

$$-\frac{5\,d^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{8\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{15\,(dx+c)^{\frac{5}{2}}b^{2}d^{3}-40\,(dx+c)^{\frac{3}{2}}b^{2}cd^{3}+33\,\sqrt{dx+c}\,b^{2}c^{2}d^{3}+40\,(dx+c)^{\frac{3}{2}}abd^{4}-66\,\sqrt{dx+c}\,abcd^{4}+33\,\sqrt{dx+c}\,a^{2}d^{5}}{24\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left((dx+c)b-bc+ad\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - 1/24*(15*(d*x + c)^
(5/2)*b^2*d^3 - 40*(d*x + c)^(3/2)*b^2*c*d^3 + 33*sqrt(d*x + c)*b^2*c^2*d^3
 + 40*(d*x + c)^(3/2)*a*b*d^4 - 66*sqrt(d*x + c)*a*b*c*d^4 + 33*sqrt(d*x +
c)*a^2*d^5)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)
*b - b*c + a*d)^3)
```

maple [A] time = 0.01, size = 147, normalized size = 1.00

$$\frac{5d^{3}\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{8\,(ad-bc)^{3}\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\,d^{3}}{3\,(ad-bc)\,(bdx+ad)^{3}}+\frac{5\sqrt{dx+c}\,d^{3}}{12\,(ad-bc)^{2}\,(bdx+ad)^{2}}+\frac{5\sqrt{dx+c}\,d^{3}}{8\,(ad-bc)^{3}\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*d^3*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^3+5/12*d^3/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^2+5/8*d^3/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5/8*d^3/(
a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.39, size = 218, normalized size = 1.48

$$\frac{\frac{11\,d^{3}\,\sqrt{c+d\,x}}{8\,(a\,d-b\,c)}+\frac{5\,b^{2}\,d^{3}\,(c+d\,x)^{5/2}}{8\,(a\,d-b\,c)^{3}}+\frac{5\,b\,d^{3}\,(c+d\,x)^{3/2}}{3\,(a\,d-b\,c)^{2}}}{(c+d\,x)\,\left(3\,a^{2}\,b\,d^{2}-6\,a\,b^{2}\,c\,d+3\,b^{3}\,c^{2}\right)+b^{3}\,(c+d\,x)^{3}-\left(3\,b^{3}\,c-3\,a\,b^{2}\,d\right)\,(c+d\,x)^{2}+a^{3}\,d^{3}-b^{3}\,c^{3}+3\,a\,b^{2}\,c^{2}\,d-3\,a^{2}\,b\,c\,d^{2}}+\frac{5\,d^{3}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{8\,\sqrt{b}\,(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(1/2)),x)
```

```
[Out] ((11*d^3*(c + d*x)^(1/2))/(8*(a*d - b*c)) + (5*b^2*d^3*(c + d*x)^(5/2))/(8*
(a*d - b*c)^3) + (5*b*d^3*(c + d*x)^(3/2))/(3*(a*d - b*c)^2))/((c + d*x)*(3
*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d) + b^3*(c + d*x)^3 - (3*b^3*c - 3*a*b^
2*d)*(c + d*x)^2 + a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + (5*
d^3*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(8*b^(1/2)*(a*d - b*
c)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1317 $\int \frac{1}{(a+bx)^5 \sqrt{c+dx}} \, dx$

Optimal. Leaf size=180

$$-\frac{35d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}(bc-ad)^{9/2}}+\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Rubi [A] time = 0.06, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{51, 63, 208\}$

$$\frac{35d^3\sqrt{c+dx}}{64(a+bx)(bc-ad)^4}-\frac{35d^2\sqrt{c+dx}}{96(a+bx)^2(bc-ad)^3}-\frac{35d^4 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{64\sqrt{b}(bc-ad)^{9/2}}+\frac{7d\sqrt{c+dx}}{24(a+bx)^3(bc-ad)^2}-\frac{\sqrt{c+dx}}{4(a+bx)^4(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] -Sqrt[c + d*x]/(4*(b*c - a*d)*(a + b*x)^4) + (7*d*Sqrt[c + d*x])/(24*(b*c -
 a*d)^2*(a + b*x)^3) - (35*d^2*Sqrt[c + d*x])/(96*(b*c - a*d)^3*(a + b*x)^2
) + (35*d^3*Sqrt[c + d*x])/(64*(b*c - a*d)^4*(a + b*x)) - (35*d^4*ArcTanh[(
Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(64*Sqrt[b]*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^5\sqrt{c+dx}}\,dx = -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} - \frac{(7d)\int \frac{1}{(a+bx)^4\sqrt{c+dx}}\,dx}{8(bc-ad)}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)^3\sqrt{c+dx}}\,dx}{48(bc-ad)^2}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} - \frac{\left(35d^3\right)\int \frac{}{(a+}}{64(bc}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c}}{64(bc-ad)^4}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c}}{64(bc-ad)^4}$$

$$= -\frac{\sqrt{c+dx}}{4(bc-ad)(a+bx)^4} + \frac{7d\sqrt{c+dx}}{24(bc-ad)^2(a+bx)^3} - \frac{35d^2\sqrt{c+dx}}{96(bc-ad)^3(a+bx)^2} + \frac{35d^3\sqrt{c}}{64(bc-ad)^4}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.28

$$\frac{2d^4\sqrt{c+dx}\;{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};-\frac{b(c+dx)}{ad-bc}\right)}{(ad-bc)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] (2*d^4*Sqrt[c + d*x]*Hypergeometric2F1[1/2, 5, 3/2, -((b*(c + d*x))/(-(b*c)
 + a*d))])/(-(b*c) + a*d)^5
```

IntegrateAlgebraic [A] time = 0.44, size = 223, normalized size = 1.24

$$\frac{d^4\sqrt{c+dx}\left(279a^3d^3+511a^2bd^2(c+dx)-837a^2bcd^2+837ab^2c^2d+385ab^2d(c+dx)^2-1022ab^2cd(c+dx)-279b^3c^3+511b^3c^2(c+dx)+105b^3(c+dx)^3-385b^3c(c+dx)^2\right)}{192(bc-ad)^4(-ad-b(c+dx)+bc)^4} - \frac{35d^4\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{64\sqrt{b}(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^5*Sqrt[c + d*x]),x]
```

```
[Out] (d^4*Sqrt[c + d*x]*(-279*b^3*c^3 + 837*a*b^2*c^2*d - 837*a^2*b*c*d^2 + 279*
a^3*d^3 + 511*b^3*c^2*(c + d*x) - 1022*a*b^2*c*d*(c + d*x) + 511*a^2*b*d^2*
(c + d*x) - 385*b^3*c*(c + d*x)^2 + 385*a*b^2*d*(c + d*x)^2 + 105*b^3*(c +
d*x)^3))/(192*(b*c - a*d)^4*(b*c - a*d - b*(c + d*x))^4) - (35*d^4*ArcTan[(
Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(64*Sqrt[b]*(-(b*c)
 + a*d)^(9/2))
```

fricas [B] time = 1.17, size = 1325, normalized size = 7.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/384*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^
4*x + a^4*d^4)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c
- a*b*d)*sqrt(d*x + c))/(b*x + a)) - 2*(48*b^5*c^4 - 248*a*b^4*c^3*d + 526*
```

```
a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*(b^5*c*d^3 - a*b^
4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b^3*d^4)*x^2 - 7*(
8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^3*b^2*d^4)*x)*sqr
t(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^3*d^2 - 10*a^7*b^
3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^
2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^4 + 4
*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5
*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a
^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^2 +
4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3
+ 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x), 1/192*(105*(b^4*d^4*x^4 + 4*a*b^3*d^4*
x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*sqrt(-b^2*c + a*b*d)*arc
tan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - (48*b^5*c^4 - 248*a
*b^4*c^3*d + 526*a^2*b^3*c^2*d^2 - 605*a^3*b^2*c*d^3 + 279*a^4*b*d^4 - 105*
(b^5*c*d^3 - a*b^4*d^4)*x^3 + 35*(2*b^5*c^2*d^2 - 13*a*b^4*c*d^3 + 11*a^2*b
^3*d^4)*x^2 - 7*(8*b^5*c^3*d - 44*a*b^4*c^2*d^2 + 109*a^2*b^3*c*d^3 - 73*a^
3*b^2*d^4)*x)*sqrt(d*x + c))/(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*c^
3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5 + (b^10*c^5 - 5*a*
b^9*c^4*d + 10*a^2*b^8*c^3*d^2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5
*b^5*d^5)*x^4 + 4*(a*b^9*c^5 - 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^
4*b^6*c^2*d^3 + 5*a^5*b^5*c*d^4 - a^6*b^4*d^5)*x^3 + 6*(a^2*b^8*c^5 - 5*a^3
*b^7*c^4*d + 10*a^4*b^6*c^3*d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^
7*b^3*d^5)*x^2 + 4*(a^3*b^7*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10
*a^6*b^4*c^2*d^3 + 5*a^7*b^3*c*d^4 - a^8*b^2*d^5)*x)]
```

giac [**B**] time = 1.13, size = 331, normalized size = 1.84

$$\frac{35\,d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{64\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}+\frac{105\,(dx+c)^{\frac{7}{2}}b^3d^4-385\,(dx+c)^{\frac{5}{2}}b^3cd^4+511\,(dx+c)^{\frac{3}{2}}b^3c^2d^4-279\,\sqrt{dx+c}\,b^3c^3d^4+385\,(dx+c)^{\frac{5}{2}}ab^2d^5-1022\,(dx+c)^{\frac{3}{2}}ab^2cd^5+837\,\sqrt{dx+c}\,ab^2c^2d^5+511\,(dx+c)^{\frac{3}{2}}a^2bd^6-837\,\sqrt{dx+c}\,a^2bcd^6+279\,\sqrt{dx+c}\,a^3d^7}{192\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)((dx+c)b-bc+ad)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 35/64*d^4*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d))
+ 1/192*(105*(d*x + c)^(7/2)*b^3*d^4 - 385*(d*x + c)^(5/2)*b^3*c*d^4 + 511*
(d*x + c)^(3/2)*b^3*c^2*d^4 - 279*sqrt(d*x + c)*b^3*c^3*d^4 + 385*(d*x + c)
^(5/2)*a*b^2*d^5 - 1022*(d*x + c)^(3/2)*a*b^2*c*d^5 + 837*sqrt(d*x + c)*a*b
^2*c^2*d^5 + 511*(d*x + c)^(3/2)*a^2*b*d^6 - 837*sqrt(d*x + c)*a^2*b*c*d^6
+ 279*sqrt(d*x + c)*a^3*d^7)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b - b*c + a*d)^4)
```

maple [**A**] time = 0.01, size = 179, normalized size = 0.99

$$\frac{35d^4\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{64\,(ad-bc)^4\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\,d^4}{4\,(ad-bc)\,(bdx+ad)^4}+\frac{7\sqrt{dx+c}\,d^4}{24\,(ad-bc)^2\,(bdx+ad)^3}+\frac{35\sqrt{dx+c}\,d^4}{96\,(ad-bc)^3\,(bdx+ad)^2}+\frac{35\sqrt{dx+c}\,d^4}{64\,(ad-bc)^4\,(bdx+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^5/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*d^4*(d*x+c)^(1/2)/(a*d-b*c)/(b*d*x+a*d)^4+7/24*d^4/(a*d-b*c)^2*(d*x+c)^
(1/2)/(b*d*x+a*d)^3+35/96*d^4/(a*d-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)^2+35/64
*d^4/(a*d-b*c)^4*(d*x+c)^(1/2)/(b*d*x+a*d)+35/64*d^4/(a*d-b*c)^4/((a*d-b*c)
*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^5/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.46, size = 307, normalized size = 1.71

$$\frac{\frac{93\,d^4\,\sqrt{c+d\,x}}{64\,\left(a\,d-b\,c\right)}+\frac{385\,b^2\,d^4\,{\left(c+d\,x\right)}^{5/2}}{192\,{\left(a\,d-b\,c\right)}^3}+\frac{35\,b^3\,d^4\,{\left(c+d\,x\right)}^{7/2}}{64\,{\left(a\,d-b\,c\right)}^4}+\frac{511\,b\,d^4\,{\left(c+d\,x\right)}^{3/2}}{192\,{\left(a\,d-b\,c\right)}^2}}{b^4\,{\left(c+d\,x\right)}^4-\left(4\,b^4\,c-4\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^3-\left(c+d\,x\right)\,\left(-4\,a^3\,b\,d^3+12\,a^2\,b^2\,c\,d^2-12\,a\,b^3\,c^2\,d+4\,b^4\,c^3\right)+a^4\,d^4+b^4\,c^4+{\left(c+d\,x\right)}^2\,\left(6\,a^2\,b^2\,d^2-12\,a\,b^3\,c\,d+6\,b^4\,c^2\right)+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d-4\,a^3\,b\,c\,d^3}+\frac{35\,d^4\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{64\,\sqrt{b}\,{\left(a\,d-b\,c\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^5*(c + d*x)^(1/2)),x)
```

```
[Out] ((93*d^4*(c + d*x)^(1/2))/(64*(a*d - b*c)) + (385*b^2*d^4*(c + d*x)^(5/2))/
(192*(a*d - b*c)^3) + (35*b^3*d^4*(c + d*x)^(7/2))/(64*(a*d - b*c)^4) + (51
1*b*d^4*(c + d*x)^(3/2))/(192*(a*d - b*c)^2))/(b^4*(c + d*x)^4 - (4*b^4*c -
 4*a*b^3*d)*(c + d*x)^3 - (c + d*x)*(4*b^4*c^3 - 4*a^3*b*d^3 + 12*a^2*b^2*c
*d^2 - 12*a*b^3*c^2*d) + a^4*d^4 + b^4*c^4 + (c + d*x)^2*(6*b^4*c^2 + 6*a^2
*b^2*d^2 - 12*a*b^3*c*d) + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^
3) + (35*d^4*atan((b^(1/2)*(c + d*x)^(1/2))/(a*d - b*c)^(1/2)))/(64*b^(1/2)
*(a*d - b*c)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**5/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1318 $\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc\cdots}{d^6\sqrt{c\cdots}}$$

Rubi [A] time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{10b^4(c+dx)^{7/2}(bc-ad)}{7d^6}+\frac{4b^3(c+dx)^{5/2}(bc-ad)^2}{d^6}-\frac{20b^2(c+dx)^{3/2}(bc-ad)^3}{3d^6}+\frac{10b\sqrt{c+dx}\,(bc-ad)^4}{d^6}+\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{2b^5(c+dx)^{9/2}}{9d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(d^6*Sqrt[c + d*x]) + (10*b*(b*c - a*d)^4*Sqrt[c + d*x])/
d^6 - (20*b^2*(b*c - a*d)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(b*c - a*d)^2
*(c + d*x)^(5/2))/d^6 - (10*b^4*(b*c - a*d)*(c + d*x)^(7/2))/(7*d^6) + (2*b
^5*(c + d*x)^(9/2))/(9*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(a+bx)^5}{(c+dx)^{3/2}}\,dx=\int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{3/2}}+\frac{5b(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{10b^2(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{10b^3(bc-ad)^2(c+dx)^{3/2}}{d^5}-\cdots\right.$$
$$=\frac{2(bc-ad)^5}{d^6\sqrt{c+dx}}+\frac{10b(bc-ad)^4\sqrt{c+dx}}{d^6}-\frac{20b^2(bc-ad)^3(c+dx)^{3/2}}{3d^6}+\frac{4b^3(bc-ad)^2(c+dx)^{5\cdots}}{d^6}$$

Mathematica [A] time = 0.12, size = 123, normalized size = 0.81

$$\frac{2\left(-45b^4(c+dx)^4(bc-ad)+126b^3(c+dx)^3(bc-ad)^2-210b^2(c+dx)^2(bc-ad)^3+315b(c+dx)(bc-ad)^4+63(bc-ad)^5+7b^5(c+dx)^5\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(63*(b*c - a*d)^5 + 315*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^
3*(c + d*x)^2 + 126*b^3*(b*c - a*d)^2*(c + d*x)^3 - 45*b^4*(b*c - a*d)*(c +
 d*x)^4 + 7*b^5*(c + d*x)^5))/(63*d^6*Sqrt[c + d*x])
```

IntegrateAlgebraic [B] time = 0.07, size = 315, normalized size = 2.07

$$\frac{2\left(-63a^5d^5+315a^4bd^4(c+dx)+315a^4bcd^4-630a^3b^2c^2d^3+210a^3b^2d^3(c+dx)^2-1260a^3b^2cd^3(c+dx)+630a^2b^3c^3d^2+1890a^2b^3c^2d^2(c+dx)+126a^2b^3d^2(c+dx)^3-630a^2b^3cd^2(c+dx)^2-315ab^4c^4d-1260ab^4c^3d(c+dx)+630ab^4c^2d(c+dx)^2+45ab^4d(c+dx)^4-252ab^4cd(c+dx)^3+63b^5c^5+315b^5c^4(c+dx)-210b^5c^3(c+dx)^2+126b^5c^2(c+dx)^3+7b^5(c+dx)^5-45b^5c(c+dx)^4\right)}{63d^6\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^(3/2),x]
```

```
[Out] (2*(63*b^5*c^5 - 315*a*b^4*c^4*d + 630*a^2*b^3*c^3*d^2 - 630*a^3*b^2*c^2*d^
3 + 315*a^4*b*c*d^4 - 63*a^5*d^5 + 315*b^5*c^4*(c + d*x) - 1260*a*b^4*c^3*d
*(c + d*x) + 1890*a^2*b^3*c^2*d^2*(c + d*x) - 1260*a^3*b^2*c*d^3*(c + d*x)
+ 315*a^4*b*d^4*(c + d*x) - 210*b^5*c^3*(c + d*x)^2 + 630*a*b^4*c^2*d*(c +
d*x)^2 - 630*a^2*b^3*c*d^2*(c + d*x)^2 + 210*a^3*b^2*d^3*(c + d*x)^2 + 126*
b^5*c^2*(c + d*x)^3 - 252*a*b^4*c*d*(c + d*x)^3 + 126*a^2*b^3*d^2*(c + d*x)
^3 - 45*b^5*c*(c + d*x)^4 + 45*a*b^4*d*(c + d*x)^4 + 7*b^5*(c + d*x)^5))/(6
3*d^6*Sqrt[c + d*x])
```

fricas [B] time = 1.09, size = 271, normalized size = 1.78

$$\frac{2\left(7\,b^5d^5x^5+256\,b^5c^5-1152\,ab^4c^4d+2016\,a^2b^3c^3d^2-1680\,a^3b^2c^2d^3+630\,a^4bcd^4-63\,a^5d^5-5\left(2\,b^5cd^4-9\,ab^4d^5\right)x^4+2\left(8\,b^5c^2d^3-36\,ab^4cd^4+63\,a^2b^3d^5\right)x^3-2\left(16\,b^5c^3d^2-72\,ab^4c^2d^3+126\,a^2b^3cd^4-105\,a^3b^2d^5\right)x^2+\left(128\,b^5c^4d-576\,ab^4c^3d^2+1008\,a^2b^3c^2d^3-840\,a^3b^2cd^4+315\,a^4bd^5\right)x\right)\sqrt{dx+c}}{63\left(d^7x+cd^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/63*(7*b^5*d^5*x^5 + 256*b^5*c^5 - 1152*a*b^4*c^4*d + 2016*a^2*b^3*c^3*d^2
 - 1680*a^3*b^2*c^2*d^3 + 630*a^4*b*c*d^4 - 63*a^5*d^5 - 5*(2*b^5*c*d^4 - 9
*a*b^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 36*a*b^4*c*d^4 + 63*a^2*b^3*d^5)*x^3 -
 2*(16*b^5*c^3*d^2 - 72*a*b^4*c^2*d^3 + 126*a^2*b^3*c*d^4 - 105*a^3*b^2*d^5
)*x^2 + (128*b^5*c^4*d - 576*a*b^4*c^3*d^2 + 1008*a^2*b^3*c^2*d^3 - 840*a^3
*b^2*c*d^4 + 315*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^7*x + c*d^6)
```

giac [B] time = 1.01, size = 350, normalized size = 2.30

$$\frac{2\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)}{\sqrt{dx+c}\,d^6}+\frac{2\left(7\,(dx+c)^{\frac{9}{2}}b^5d^{48}-45\,(dx+c)^{\frac{7}{2}}b^5cd^{48}+126\,(dx+c)^{\frac{5}{2}}b^5c^2d^{48}-210\,(dx+c)^{\frac{3}{2}}b^5c^3d^{48}+315\,\sqrt{dx+c}\,b^5c^4d^{48}+45\,(dx+c)^{\frac{7}{2}}ab^4d^{49}-252\,(dx+c)^{\frac{5}{2}}ab^4cd^{49}+630\,(dx+c)^{\frac{3}{2}}ab^4c^2d^{49}-1260\,\sqrt{dx+c}\,ab^4c^3d^{49}+126\,(dx+c)^{\frac{5}{2}}a^2b^3d^{50}-630\,(dx+c)^{\frac{3}{2}}a^2b^3cd^{50}+1890\,\sqrt{dx+c}\,a^2b^3c^2d^{50}+210\,(dx+c)^{\frac{3}{2}}a^3b^2d^{51}-1260\,\sqrt{dx+c}\,a^3b^2cd^{51}+315\,\sqrt{dx+c}\,a^4bd^{52}\right)}{63\,d^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^
4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^6) + 2/63*(7*(d*x + c)^(9/2)*b^5*d^48
 - 45*(d*x + c)^(7/2)*b^5*c*d^48 + 126*(d*x + c)^(5/2)*b^5*c^2*d^48 - 210*(
d*x + c)^(3/2)*b^5*c^3*d^48 + 315*sqrt(d*x + c)*b^5*c^4*d^48 + 45*(d*x + c)
^(7/2)*a*b^4*d^49 - 252*(d*x + c)^(5/2)*a*b^4*c*d^49 + 630*(d*x + c)^(3/2)*
a*b^4*c^2*d^49 - 1260*sqrt(d*x + c)*a*b^4*c^3*d^49 + 126*(d*x + c)^(5/2)*a^
2*b^3*d^50 - 630*(d*x + c)^(3/2)*a^2*b^3*c*d^50 + 1890*sqrt(d*x + c)*a^2*b^
3*c^2*d^50 + 210*(d*x + c)^(3/2)*a^3*b^2*d^51 - 1260*sqrt(d*x + c)*a^3*b^2*
c*d^51 + 315*sqrt(d*x + c)*a^4*b*d^52)/d^54
```

maple [B] time = 0.01, size = 273, normalized size = 1.80

$$-\frac{2\left(-7b^5x^5d^5-45a\,b^4d^5x^4+10b^5c\,d^4x^4-126a^2b^3d^5x^3+72a\,b^4c\,d^4x^3-16b^5c^2d^3x^3-210a^3b^2d^5x^2+252a^2b^3c\,d^4x^2-144a\,b^4c^2d^3x^2+32b^5c^3d^2x^2-315a^4b\,d^5x+840a^3b^2c\,d^4x-1008a^2b^3c^2d^3x+576a\,b^4c^3d^2x-128b^5c^4dx+63a^5d^5-630a^4bc\,d^4+1680a^3b^2c^2d^3-2016a^2b^3c^3d^2+1152a\,b^4c^4d-256b^5c^5\right)}{63\sqrt{dx+c}\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(3/2),x)
```

```
[Out] -2/63/(d*x+c)^(1/2)*(-7*b^5*d^5*x^5-45*a*b^4*d^5*x^4+10*b^5*c*d^4*x^4-126*a
^2*b^3*d^5*x^3+72*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+25
2*a^2*b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+32*b^5*c^3*d^2*x^2-315*a^4*b*d^5*
x+840*a^3*b^2*c*d^4*x-1008*a^2*b^3*c^2*d^3*x+576*a*b^4*c^3*d^2*x-128*b^5*c^
4*d*x+63*a^5*d^5-630*a^4*b*c*d^4+1680*a^3*b^2*c^2*d^3-2016*a^2*b^3*c^3*d^2+
1152*a*b^4*c^4*d-256*b^5*c^5)/d^6
```

maxima [A] time = 1.56, size = 267, normalized size = 1.76

$$\frac{2\left(\frac{7\,(dx+c)^{\frac{9}{2}}b^5-45\left(b^5c-ab^4d\right)(dx+c)^{\frac{7}{2}}+126\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)(dx+c)^{\frac{5}{2}}-210\left(b^5c^3-3\,ab^4c^2d+3\,a^2b^3cd^2-a^3b^2d^3\right)(dx+c)^{\frac{3}{2}}+315\left(b^5c^4-4\,ab^4c^3d+6\,a^2b^3c^2d^2-4\,a^3b^2cd^3+a^4bd^4\right)\sqrt{dx+c}}{d^5}+\frac{63\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)}{\sqrt{dx+c}\,d^5}\right)}{63\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/63*((7*(d*x + c)^(9/2)*b^5 - 45*(b^5*c - a*b^4*d)*(d*x + c)^(7/2) + 126*(
b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(5/2) - 210*(b^5*c^3 - 3*a*b
^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*(d*x + c)^(3/2) + 315*(b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*sqrt(d*x +
 c))/d^5 + 63*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)/(sqrt(d*x + c)*d^5))/d
```

mupad [B] time = 0.08, size = 192, normalized size = 1.26

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{9/2}}{9\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{2\,a^5\,d^5-10\,a^4\,b\,c\,d^4+20\,a^3\,b^2\,c^2\,d^3-20\,a^2\,b^3\,c^3\,d^2+10\,a\,b^4\,c^4\,d-2\,b^5\,c^5}{d^6\,\sqrt{c+d\,x}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}+\frac{4\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{5/2}}{d^6}+\frac{10\,b\,{\left(a\,d-b\,c\right)}^4\,\sqrt{c+d\,x}}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(9/2))/(9*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(7/2))
/(7*d^6) - (2*a^5*d^5 - 2*b^5*c^5 - 20*a^2*b^3*c^3*d^2 + 20*a^3*b^2*c^2*d^3
 + 10*a*b^4*c^4*d - 10*a^4*b*c*d^4)/(d^6*(c + d*x)^(1/2)) + (20*b^2*(a*d -
b*c)^3*(c + d*x)^(3/2))/(3*d^6) + (4*b^3*(a*d - b*c)^2*(c + d*x)^(5/2))/d^6
 + (10*b*(a*d - b*c)^4*(c + d*x)^(1/2))/d^6
```

sympy [A] time = 47.94, size = 243, normalized size = 1.60

$$\frac{2b^5\left(c+dx\right)^{\frac{9}{2}}}{9d^6}+\frac{\left(c+dx\right)^{\frac{7}{2}}\left(10ab^4d-10b^5c\right)}{7d^6}+\frac{\left(c+dx\right)^{\frac{5}{2}}\left(20a^2b^3d^2-40ab^4cd+20b^5c^2\right)}{5d^6}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3\right)}{3d^6}+\frac{\sqrt{c+dx}\left(10a^4bd^4-40a^3b^2cd^3+60a^2b^3c^2d^2-40ab^4c^3d+10b^5c^4\right)}{d^6}-\frac{2\left(ad-bc\right)^5}{d^6\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(9/2)/(9*d**6) + (c + d*x)**(7/2)*(10*a*b**4*d - 10*b**5*
c)/(7*d**6) + (c + d*x)**(5/2)*(20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5
*c**2)/(5*d**6) + (c + d*x)**(3/2)*(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2
 + 60*a*b**4*c**2*d - 20*b**5*c**3)/(3*d**6) + sqrt(c + d*x)*(10*a**4*b*d**
4 - 40*a**3*b**2*c*d**3 + 60*a**2*b**3*c**2*d**2 - 40*a*b**4*c**3*d + 10*b*
*5*c**4)/d**6 - 2*(a*d - b*c)**5/(d**6*sqrt(c + d*x))
```

3.1319 $\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{5/2}(bc-ad)}{5d^5}+\frac{4b^2(c+dx)^{3/2}(bc-ad)^2}{d^5}-\frac{8b\sqrt{c+dx}\,(bc-ad)^3}{d^5}-\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}+\frac{2b^4(c+dx)^{7/2}}{7d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(d^5*Sqrt[c + d*x]) - (8*b*(b*c - a*d)^3*Sqrt[c + d*x])/
d^5 + (4*b^2*(b*c - a*d)^2*(c + d*x)^(3/2))/d^5 - (8*b^3*(b*c - a*d)*(c + d
*x)^(5/2))/(5*d^5) + (2*b^4*(c + d*x)^(7/2))/(7*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{3/2}}-\frac{4b(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b^2(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{4b^3(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{b^4(c+\right.\\ &= -\frac{2(bc-ad)^4}{d^5\sqrt{c+dx}}-\frac{8b(bc-ad)^3\sqrt{c+dx}}{d^5}+\frac{4b^2(bc-ad)^2(c+dx)^{3/2}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{5/2}}{5d^5}+\end{aligned}$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.82

$$\frac{2\left(-28b^3(c+dx)^3(bc-ad)+70b^2(c+dx)^2(bc-ad)^2-140b(c+dx)(bc-ad)^3-35(bc-ad)^4+5b^4(c+dx)^4\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*(b*c - a*d)^4 - 140*b*(b*c - a*d)^3*(c + d*x) + 70*b^2*(b*c - a*d)^
2*(c + d*x)^2 - 28*b^3*(b*c - a*d)*(c + d*x)^3 + 5*b^4*(c + d*x)^4))/(35*d^
5*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.07, size = 213, normalized size = 1.73

$$\frac{2\left(-35a^4d^4+140a^3bd^3(c+dx)+140a^3bcd^3-210a^2b^2c^2d^2+70a^2b^2d^2(c+dx)^2-420a^2b^2cd^2(c+dx)+140ab^3c^3d+420ab^3c^2d(c+dx)+28ab^3d(c+dx)^3-140ab^3cd(c+dx)^2-35b^4c^4-140b^4c^3(c+dx)+70b^4c^2(c+dx)^2+5b^4(c+dx)^4-28b^4c(c+dx)^3\right)}{35d^5\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-35*b^4*c^4 + 140*a*b^3*c^3*d - 210*a^2*b^2*c^2*d^2 + 140*a^3*b*c*d^3 -
 35*a^4*d^4 - 140*b^4*c^3*(c + d*x) + 420*a*b^3*c^2*d*(c + d*x) - 420*a^2*b
^2*c*d^2*(c + d*x) + 140*a^3*b*d^3*(c + d*x) + 70*b^4*c^2*(c + d*x)^2 - 140
*a*b^3*c*d*(c + d*x)^2 + 70*a^2*b^2*d^2*(c + d*x)^2 - 28*b^4*c*(c + d*x)^3
+ 28*a*b^3*d*(c + d*x)^3 + 5*b^4*(c + d*x)^4))/(35*d^5*Sqrt[c + d*x])
```

fricas [A] time = 1.54, size = 192, normalized size = 1.56

$$\frac{2\left(5\,b^4d^4x^4 - 128\,b^4c^4 + 448\,ab^3c^3d - 560\,a^2b^2c^2d^2 + 280\,a^3bcd^3 - 35\,a^4d^4 - 4\left(2\,b^4cd^3 - 7\,ab^3d^4\right)x^3 + 2\left(8\,b^4c^2d^2 - 28\,ab^3cd^3 + 35\,a^2b^2d^4\right)x^2 - 4\left(16\,b^4c^3d - 56\,ab^3c^2d^2 + 70\,a^2b^2cd^3 - 35\,a^3bd^4\right)x\right)\sqrt{dx+c}}{35\left(d^6x + cd^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^4*d^4*x^4 - 128*b^4*c^4 + 448*a*b^3*c^3*d - 560*a^2*b^2*c^2*d^2 +
 280*a^3*b*c*d^3 - 35*a^4*d^4 - 4*(2*b^4*c*d^3 - 7*a*b^3*d^4)*x^3 + 2*(8*b^
4*c^2*d^2 - 28*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^2 - 4*(16*b^4*c^3*d - 56*a*b
^3*c^2*d^2 + 70*a^2*b^2*c*d^3 - 35*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^6*x + c*d
^5)
```

giac [B] time = 1.08, size = 240, normalized size = 1.95

$$-\frac{2\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)}{\sqrt{dx+c}\,d^5} + \frac{2\left(5\,(dx+c)^{\frac{7}{2}}b^4d^{30} - 28\,(dx+c)^{\frac{5}{2}}b^4cd^{30} + 70\,(dx+c)^{\frac{3}{2}}b^4c^2d^{30} - 140\,\sqrt{dx+c}\,b^4c^3d^{30} + 28\,(dx+c)^{\frac{5}{2}}ab^3d^{31} - 140\,(dx+c)^{\frac{3}{2}}ab^3cd^{31} + 420\,\sqrt{dx+c}\,ab^3c^2d^{31} + 70\,(dx+c)^{\frac{3}{2}}a^2b^2d^{32} - 420\,\sqrt{dx+c}\,a^2b^2cd^{32} + 140\,\sqrt{dx+c}\,a^3bd^{33}\right)}{35\,d^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/
(sqrt(d*x + c)*d^5) + 2/35*(5*(d*x + c)^(7/2)*b^4*d^30 - 28*(d*x + c)^(5/2)
*b^4*c*d^30 + 70*(d*x + c)^(3/2)*b^4*c^2*d^30 - 140*sqrt(d*x + c)*b^4*c^3*d
^30 + 28*(d*x + c)^(5/2)*a*b^3*d^31 - 140*(d*x + c)^(3/2)*a*b^3*c*d^31 + 42
0*sqrt(d*x + c)*a*b^3*c^2*d^31 + 70*(d*x + c)^(3/2)*a^2*b^2*d^32 - 420*sqrt
(d*x + c)*a^2*b^2*c*d^32 + 140*sqrt(d*x + c)*a^3*b*d^33)/d^35
```

maple [A] time = 0.01, size = 186, normalized size = 1.51

$$-\frac{2\left(-5b^4x^4d^4 - 28a\,b^3d^4x^3 + 8b^4c\,d^3x^3 - 70a^2b^2d^4x^2 + 56a\,b^3c\,d^3x^2 - 16b^4c^2d^2x^2 - 140a^3b\,d^4x + 280a^2b^2c\,d^3x - 224a\,b^3c^2d^2x + 64b^4c^3dx + 35a^4d^4 - 280a^3bc\,d^3 + 560a^2b^2c^2d^2 - 448a\,b^3c^3d + 128b^4c^4\right)}{35\sqrt{dx+c}\;d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2/35/(d*x+c)^(1/2)*(-5*b^4*d^4*x^4-28*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-70*a^2
*b^2*d^4*x^2+56*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2-140*a^3*b*d^4*x+280*a^2*
b^2*c*d^3*x-224*a*b^3*c^2*d^2*x+64*b^4*c^3*d*x+35*a^4*d^4-280*a^3*b*c*d^3+5
60*a^2*b^2*c^2*d^2-448*a*b^3*c^3*d+128*b^4*c^4)/d^5
```

maxima [A] time = 1.35, size = 189, normalized size = 1.54

$$\frac{2\left(\frac{5\,(dx+c)^{\frac{7}{2}}b^4 - 28\left(b^4c - ab^3d\right)(dx+c)^{\frac{5}{2}} + 70\left(b^4c^2 - 2\,ab^3cd + a^2b^2d^2\right)(dx+c)^{\frac{3}{2}} - 140\left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)\sqrt{dx+c}}{d^4} - \frac{35\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)}{\sqrt{dx+c}\,d^4}\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/35*((5*(d*x + c)^(7/2)*b^4 - 28*(b^4*c - a*b^3*d)*(d*x + c)^(5/2) + 70*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*(d*x + c)^(3/2) - 140*(b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*sqrt(d*x + c))/d^4 - 35*(b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(sqrt(d*x + c)*
d^4))/d
```

mupad [B] time = 0.06, size = 153, normalized size = 1.24

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{7/2}}{7\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{2\,a^4\,d^4-8\,a^3\,b\,c\,d^3+12\,a^2\,b^2\,c^2\,d^2-8\,a\,b^3\,c^3\,d+2\,b^4\,c^4}{d^5\,\sqrt{c+d\,x}}+\frac{4\,b^2\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{d^5}+\frac{8\,b\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(7/2))/(7*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(5/2))/(
5*d^5) - (2*a^4*d^4 + 2*b^4*c^4 + 12*a^2*b^2*c^2*d^2 - 8*a*b^3*c^3*d - 8*a^
3*b*c*d^3)/(d^5*(c + d*x)^(1/2)) + (4*b^2*(a*d - b*c)^2*(c + d*x)^(3/2))/d^
5 + (8*b*(a*d - b*c)^3*(c + d*x)^(1/2))/d^5
```

sympy [A] time = 32.87, size = 168, normalized size = 1.37

$$\frac{2b^4\left(c+dx\right)^{\frac{7}{2}}}{7d^5}+\frac{\left(c+dx\right)^{\frac{5}{2}}\left(8ab^3d-8b^4c\right)}{5d^5}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{3d^5}+\frac{\sqrt{c+dx}\left(8a^3bd^3-24a^2b^2cd^2+24ab^3c^2d-8b^4c^3\right)}{d^5}-\frac{2\left(ad-bc\right)^4}{d^5\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(7/2)/(7*d**5) + (c + d*x)**(5/2)*(8*a*b**3*d - 8*b**4*c)
/(5*d**5) + (c + d*x)**(3/2)*(12*a**2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c
**2)/(3*d**5) + sqrt(c + d*x)*(8*a**3*b*d**3 - 24*a**2*b**2*c*d**2 + 24*a*b
**3*c**2*d - 8*b**4*c**3)/d**5 - 2*(a*d - b*c)**4/(d**5*sqrt(c + d*x))
```

3.1320 $\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Rubi [A] time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{43\}$

$$-\frac{2b^2(c+dx)^{3/2}(bc-ad)}{d^4}+\frac{6b\sqrt{c+dx}\,(bc-ad)^2}{d^4}+\frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{2b^3(c+dx)^{5/2}}{5d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(d^4*Sqrt[c + d*x]) + (6*b*(b*c - a*d)^2*Sqrt[c + d*x])/d
^4 - (2*b^2*(b*c - a*d)*(c + d*x)^(3/2))/d^4 + (2*b^3*(c + d*x)^(5/2))/(5*d
^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{3/2}}+\frac{3b(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{3b^2(bc-ad)\sqrt{c+dx}}{d^3}+\frac{b^3(c+dx)^{3/2}}{d^3}\right)dx\\ &= \frac{2(bc-ad)^3}{d^4\sqrt{c+dx}}+\frac{6b(bc-ad)^2\sqrt{c+dx}}{d^4}-\frac{2b^2(bc-ad)(c+dx)^{3/2}}{d^4}+\frac{2b^3(c+dx)^{5/2}}{5d^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.83

$$\frac{2\left(-5b^2(c+dx)^2(bc-ad)+15b(c+dx)(bc-ad)^2+5(bc-ad)^3+b^3(c+dx)^3\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(5*(b*c - a*d)^3 + 15*b*(b*c - a*d)^2*(c + d*x) - 5*b^2*(b*c - a*d)*(c +
 d*x)^2 + b^3*(c + d*x)^3))/(5*d^4*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 131, normalized size = 1.39

$$\frac{2\left(-5a^3d^3+15a^2bd^2(c+dx)+15a^2bcd^2-15ab^2c^2d+5ab^2d(c+dx)^2-30ab^2cd(c+dx)+5b^3c^3+15b^3c^2(c+dx)+b^3(c+dx)^3-5b^3c(c+dx)^2\right)}{5d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^(3/2),x]
```

```
[Out] (2*(5*b^3*c^3 - 15*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 5*a^3*d^3 + 15*b^3*c^2*(c
 + d*x) - 30*a*b^2*c*d*(c + d*x) + 15*a^2*b*d^2*(c + d*x) - 5*b^3*c*(c + d*
x)^2 + 5*a*b^2*d*(c + d*x)^2 + b^3*(c + d*x)^3))/(5*d^4*Sqrt[c + d*x])
```

fricas [A] time = 1.26, size = 124, normalized size = 1.32

$$\frac{2\left(b^3d^3x^3+16\,b^3c^3-40\,ab^2c^2d+30\,a^2bcd^2-5\,a^3d^3-\left(2\,b^3cd^2-5\,ab^2d^3\right)x^2+\left(8\,b^3c^2d-20\,ab^2cd^2+15\,a^2bd^3\right)x\right)\sqrt{dx+c}}{5\left(d^5x+cd^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*d^3*x^3 + 16*b^3*c^3 - 40*a*b^2*c^2*d + 30*a^2*b*c*d^2 - 5*a^3*d^3
 - (2*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + (8*b^3*c^2*d - 20*a*b^2*c*d^2 + 15*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^5*x + c*d^4)
```

giac [A] time = 1.05, size = 152, normalized size = 1.62

$$\frac{2\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}{\sqrt{dx+c}\,d^4}+\frac{2\left((dx+c)^{\frac{5}{2}}b^3d^{16}-5\,(dx+c)^{\frac{3}{2}}b^3cd^{16}+15\,\sqrt{dx+c}\,b^3c^2d^{16}+5\,(dx+c)^{\frac{3}{2}}ab^2d^{17}-30\,\sqrt{dx+c}\,ab^2cd^{17}+15\,\sqrt{dx+c}\,a^2bd^{18}\right)}{5\,d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^4) +
 2/5*((d*x + c)^(5/2)*b^3*d^16 - 5*(d*x + c)^(3/2)*b^3*c*d^16 + 15*sqrt(d*x
 + c)*b^3*c^2*d^16 + 5*(d*x + c)^(3/2)*a*b^2*d^17 - 30*sqrt(d*x + c)*a*b^2*
c*d^17 + 15*sqrt(d*x + c)*a^2*b*d^18)/d^20
```

maple [A] time = 0.01, size = 116, normalized size = 1.23

$$-\frac{2\left(-b^3x^3d^3-5a\,b^2d^3x^2+2b^3c\,d^2x^2-15a^2b\,d^3x+20a\,b^2c\,d^2x-8b^3c^2dx+5a^3d^3-30a^2bc\,d^2+40a\,b^2c^2d-16b^3c^3\right)}{5\sqrt{dx+c}\;d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2/5/(d*x+c)^(1/2)*(-b^3*d^3*x^3-5*a*b^2*d^3*x^2+2*b^3*c*d^2*x^2-15*a^2*b*d
^3*x+20*a*b^2*c*d^2*x-8*b^3*c^2*d*x+5*a^3*d^3-30*a^2*b*c*d^2+40*a*b^2*c^2*d
-16*b^3*c^3)/d^4
```

maxima [A] time = 1.38, size = 125, normalized size = 1.33

$$\frac{2\left(\frac{(dx+c)^{\frac{5}{2}}b^3-5\left(b^3c-ab^2d\right)(dx+c)^{\frac{3}{2}}+15\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)\sqrt{dx+c}}{d^3}+\frac{5\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)}{\sqrt{dx+c}\,d^3}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(((d*x + c)^(5/2)*b^3 - 5*(b^3*c - a*b^2*d)*(d*x + c)^(3/2) + 15*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)*sqrt(d*x + c))/d^3 + 5*(b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)/(sqrt(d*x + c)*d^3))/d
```

mupad [B] time = 0.08, size = 114, normalized size = 1.21

$$\frac{2\,b^3\,(c+d\,x)^{5/2}}{5\,d^4}-\frac{\left(6\,b^3\,c-6\,a\,b^2\,d\right)\,(c+d\,x)^{3/2}}{3\,d^4}-\frac{2\,a^3\,d^3-6\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d-2\,b^3\,c^3}{d^4\,\sqrt{c+d\,x}}+\frac{6\,b\,(a\,d-b\,c)^2\,\sqrt{c+d\,x}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] (2*b^3*(c + d*x)^(5/2))/(5*d^4) - ((6*b^3*c - 6*a*b^2*d)*(c + d*x)^(3/2))/(
3*d^4) - (2*a^3*d^3 - 2*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(d^4*(c +
d*x)^(1/2)) + (6*b*(a*d - b*c)^2*(c + d*x)^(1/2))/d^4
```

sympy [A] time = 21.51, size = 109, normalized size = 1.16

$$\frac{2b^3(c+dx)^{\frac{5}{2}}}{5d^4}+\frac{(c+dx)^{\frac{3}{2}}\left(6ab^2d-6b^3c\right)}{3d^4}+\frac{\sqrt{c+dx}\left(6a^2bd^2-12ab^2cd+6b^3c^2\right)}{d^4}-\frac{2(ad-bc)^3}{d^4\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**3*(c + d*x)**(5/2)/(5*d**4) + (c + d*x)**(3/2)*(6*a*b**2*d - 6*b**3*c)
/(3*d**4) + sqrt(c + d*x)*(6*a**2*b*d**2 - 12*a*b**2*c*d + 6*b**3*c**2)/d**
4 - 2*(a*d - b*c)**3/(d**4*sqrt(c + d*x))
```

3.1321 $\int \frac{(a+bx)^2}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=67

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{4b\sqrt{c+dx}\,(bc-ad)}{d^3}-\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}+\frac{2b^2(c+dx)^{3/2}}{3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(d^3*Sqrt[c + d*x]) - (4*b*(b*c - a*d)*Sqrt[c + d*x])/d^
3 + (2*b^2*(c + d*x)^(3/2))/(3*d^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{3/2}}-\frac{2b(bc-ad)}{d^2\sqrt{c+dx}}+\frac{b^2\sqrt{c+dx}}{d^2}\right)dx\\ &= -\frac{2(bc-ad)^2}{d^3\sqrt{c+dx}}-\frac{4b(bc-ad)\sqrt{c+dx}}{d^3}+\frac{2b^2(c+dx)^{3/2}}{3d^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.88

$$\frac{2\left(-3a^2d^2+6abd(2c+dx)+b^2\left(-8c^2-4cdx+d^2x^2\right)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*a^2*d^2 + 6*a*b*d*(2*c + d*x) + b^2*(-8*c^2 - 4*c*d*x + d^2*x^2)))/(
3*d^3*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 71, normalized size = 1.06

$$\frac{2\left(-3a^2d^2+6abd(c+dx)+6abcd-3b^2c^2+b^2(c+dx)^2-6b^2c(c+dx)\right)}{3d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^(3/2),x]
```

```
[Out] (2*(-3*b^2*c^2 + 6*a*b*c*d - 3*a^2*d^2 - 6*b^2*c*(c + d*x) + 6*a*b*d*(c + d
*x) + b^2*(c + d*x)^2))/(3*d^3*Sqrt[c + d*x])
```

fricas [A] time = 1.13, size = 73, normalized size = 1.09

$$\frac{2\left(b^2d^2x^2 - 8\,b^2c^2 + 12\,abcd - 3\,a^2d^2 - 2\left(2\,b^2cd - 3\,abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^4x + cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^2*d^2*x^2 - 8*b^2*c^2 + 12*a*b*c*d - 3*a^2*d^2 - 2*(2*b^2*c*d - 3*a*
b*d^2)*x)*sqrt(d*x + c)/(d^4*x + c*d^3)
```

giac [A] time = 1.04, size = 84, normalized size = 1.25

$$-\frac{2\left(b^2c^2 - 2\,abcd + a^2d^2\right)}{\sqrt{dx+c}\,d^3} + \frac{2\left((dx+c)^{\frac{3}{2}}b^2d^6 - 6\,\sqrt{dx+c}\,b^2cd^6 + 6\,\sqrt{dx+c}\,abd^7\right)}{3\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^3) + 2/3*((d*x + c)^(3/
2)*b^2*d^6 - 6*sqrt(d*x + c)*b^2*c*d^6 + 6*sqrt(d*x + c)*a*b*d^7)/d^9
```

maple [A] time = 0.00, size = 63, normalized size = 0.94

$$-\frac{2\left(-b^2x^2d^2 - 6ab\,d^2x + 4b^2cdx + 3a^2d^2 - 12abcd + 8b^2c^2\right)}{3\sqrt{dx+c}\;d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2/3/(d*x+c)^(1/2)*(-b^2*d^2*x^2-6*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2-12*a*b*c
*d+8*b^2*c^2)/d^3
```

maxima [A] time = 1.30, size = 75, normalized size = 1.12

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^2 - 6\left(b^2c - abd\right)\sqrt{dx+c}}{d^2} - \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)}{\sqrt{dx+c}\,d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^2 - 6*(b^2*c - a*b*d)*sqrt(d*x + c))/d^2 - 3*(b^2*c
^2 - 2*a*b*c*d + a^2*d^2)/(sqrt(d*x + c)*d^2))/d
```

mupad [B] time = 0.26, size = 67, normalized size = 1.00

$$\frac{\frac{2\,b^2\,{\left(c+d\,x\right)}^2}{3} - 2\,a^2\,d^2 - 2\,b^2\,c^2 - 4\,b^2\,c\,\left(c+d\,x\right) + 4\,a\,b\,d\,\left(c+d\,x\right) + 4\,a\,b\,c\,d}{d^3\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(3/2),x)
```

```
[Out] ((2*b^2*(c + d*x)^2)/3 - 2*a^2*d^2 - 2*b^2*c^2 - 4*b^2*c*(c + d*x) + 4*a*b*
d*(c + d*x) + 4*a*b*c*d)/(d^3*(c + d*x)^(1/2))
```

sympy [A] time = 13.29, size = 65, normalized size = 0.97

$$\frac{2b^2(c+dx)^{\frac{3}{2}}}{3d^3}+\frac{\sqrt{c+dx}\left(4abd-4b^2c\right)}{d^3}-\frac{2(ad-bc)^2}{d^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] 2*b**2*(c + d*x)**(3/2)/(3*d**3) + sqrt(c + d*x)*(4*a*b*d - 4*b**2*c)/d**3
- 2*(a*d - b*c)**2/(d**3*sqrt(c + d*x))
```

3.1322 $\int \frac{a+bx}{(c+dx)^{3/2}} dx$

Optimal. Leaf size=38

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(3/2),x]
[Out] (2*(b*c - a*d))/(d^2*Sqrt[c + d*x]) + (2*b*Sqrt[c + d*x])/d^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{3/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{3/2}}+\frac{b}{d\sqrt{c+dx}}\right) dx\\ &= \frac{2(bc-ad)}{d^2\sqrt{c+dx}}+\frac{2b\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 0.71

$$\frac{2(-ad+2bc+bdx)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(3/2),x]
[Out] (2*(2*b*c - a*d + b*d*x))/(d^2*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 29, normalized size = 0.76

$$\frac{2(-ad+b(c+dx)+bc)}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^(3/2),x]
[Out] (2*(b*c - a*d + b*(c + d*x)))/(d^2*Sqrt[c + d*x])
```

fricas [A] time = 1.13, size = 35, normalized size = 0.92

$$\frac{2\,(bdx+2\,bc-ad)\sqrt{dx+c}}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(b*d*x + 2*b*c - a*d)*sqrt(d*x + c)/(d^3*x + c*d^2)
```

giac [A] time = 1.03, size = 34, normalized size = 0.89

$$\frac{2\sqrt{dx+c}\,b}{d^2}+\frac{2\,(bc-ad)}{\sqrt{dx+c}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b/d^2 + 2*(b*c - a*d)/(sqrt(d*x + c)*d^2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.68

$$-\frac{2\,(-bdx+ad-2bc)}{\sqrt{dx+c}\;d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] -2/(d*x+c)^(1/2)*(-b*d*x+a*d-2*b*c)/d^2
```

maxima [A] time = 1.33, size = 37, normalized size = 0.97

$$\frac{2\left(\frac{\sqrt{dx+c}\,b}{d}+\frac{bc-ad}{\sqrt{dx+c}\,d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(d*x + c)*b/d + (b*c - a*d)/(sqrt(d*x + c)*d))/d
```

mupad [B] time = 0.05, size = 25, normalized size = 0.66

$$\frac{4\,b\,c-2\,a\,d+2\,b\,d\,x}{d^2\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] (4*b*c - 2*a*d + 2*b*d*x)/(d^2*(c + d*x)^(1/2))
```

sympy [A] time = 0.61, size = 60, normalized size = 1.58

$$\begin{cases} -\frac{2a}{d\sqrt{c+dx}}+\frac{4bc}{d^2\sqrt{c+dx}}+\frac{2bx}{d\sqrt{c+dx}} & \text{for } d\neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Piecewise((-2*a/(d*sqrt(c + d*x)) + 4*b*c/(d**2*sqrt(c + d*x)) + 2*b*x/(d*s
qrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(3/2), True))
```

3.1323 $\int \frac{1}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=14

$$-\frac{2}{d\sqrt{c+dx}}$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{3/2}}\,dx = -\frac{2}{d\sqrt{c+dx}}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$-\frac{2}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(-3/2),x]
```

```
[Out] -2/(d*Sqrt[c + d*x])
```

fricas [A] time = 1.21, size = 20, normalized size = 1.43

$$-\frac{2\sqrt{dx+c}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(d*x + c)/(d^2*x + c*d)
```

giac [A] time = 1.02, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="giac")
[Out] -2/(sqrt(d*x + c)*d)
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$-\frac{2}{\sqrt{dx+c}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(3/2),x)
[Out] -2/d/(d*x+c)^(1/2)
```

maxima [A] time = 1.33, size = 12, normalized size = 0.86

$$-\frac{2}{\sqrt{dx+c}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(d*x + c)*d)
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2}{d\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(3/2),x)
[Out] -2/(d*(c + d*x)^(1/2))
```

sympy [A] time = 0.06, size = 12, normalized size = 0.86

$$-\frac{2}{d\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(3/2),x)
[Out] -2/(d*sqrt(c + d*x))
```

3.1324 $\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) - (2*Sqrt[b]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx &= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{b\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{bc-ad}\\
&= \frac{2}{(bc-ad)\sqrt{c+dx}}+\frac{(2b)\,\text{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d(bc-ad)}\\
&= \frac{2}{(bc-ad)\sqrt{c+dx}}-\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 46, normalized size = 0.67

$$-\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{\sqrt{c+dx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, (b*(c + d*x))/(b*c - a*d)])/((-(b*c) +
a*d)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 79, normalized size = 1.14

$$\frac{2}{\sqrt{c+dx}\,(bc-ad)}+\frac{2\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(3/2)),x]
```

```
[Out] 2/((b*c - a*d)*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*
d]*Sqrt[c + d*x])/(b*c - a*d)])/(-(b*c) + a*d)^(3/2)
```

fricas [A] time = 1.43, size = 214, normalized size = 3.10

$$\left[-\frac{(dx+c)\sqrt{\frac{b}{bc-ad}}\,\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\,\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x},\,-\frac{2\left((dx+c)\sqrt{-\frac{b}{bc-ad}}\,\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\,\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\sqrt{dx+c}\right)}{bc^2-acd+\left(bcd-ad^2\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-((d*x + c)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*s
qrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*sqrt(d*x + c))/(b*c^2 - a*
c*d + (b*c*d - a*d^2)*x), -2*((d*x + c)*sqrt(-b/(b*c - a*d))*arctan(-(b*c -
 a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - sqrt(d*x + c))/(b
*c^2 - a*c*d + (b*c*d - a*d^2)*x)]
```

giac [A] time = 0.94, size = 69, normalized size = 1.00

$$\frac{2\,b\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,(bc-ad)}+\frac{2}{(bc-ad)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*b*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*d)*(b*c
 - a*d)) + 2/((b*c - a*d)*sqrt(d*x + c))
```

maple [A] time = 0.01, size = 68, normalized size = 0.99

$$-\frac{2b\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)\,\sqrt{(ad-bc)\,b}}-\frac{2}{(ad-bc)\,\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] -2/(a*d-b*c)/(d*x+c)^(1/2)-2*b/(a*d-b*c)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)
^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.27, size = 57, normalized size = 0.83

$$-\frac{2}{(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}}{\sqrt{a\,d-b\,c}}\right)}{(a\,d-b\,c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(3/2)),x)
```

```
[Out] - 2/((a*d - b*c)*(c + d*x)^(1/2)) - (2*b^(1/2)*atan((b^(1/2)*(c + d*x)^(1/2
))/(a*d - b*c)^(1/2)))/(a*d - b*c)^(3/2)
```

sympy [A] time = 11.49, size = 60, normalized size = 0.87

$$-\frac{2}{\sqrt{c+dx}\,(ad-bc)}-\frac{2\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] -2/(sqrt(c + d*x)*(a*d - b*c)) - 2*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/
(sqrt((a*d - b*c)/b)*(a*d - b*c))
```

3.1325 $\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=99

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Rubi [A] time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{3d}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{(a+bx)\sqrt{c+dx}\,(bc-ad)}+\frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*d)/((b*c - a*d)^2*Sqrt[c + d*x]) - 1/((b*c - a*d)*(a + b*x)*Sqrt[c + d*
x]) + (3*Sqrt[b]*d*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c -
 a*d)^(5/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3d)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3bd)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} - \frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x,\right.}{(bc-ad)^2}$$

$$= -\frac{3d}{(bc-ad)^2\sqrt{c+dx}} - \frac{1}{(bc-ad)(a+bx)\sqrt{c+dx}} + \frac{3\sqrt{b}\,d\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.48

$$-\frac{2d\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-1/2, 2, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/((-
(b*c) + a*d)^2*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.29, size = 115, normalized size = 1.16

$$\frac{d(2ad+3b(c+dx)-2bc)}{\sqrt{c+dx}\,(bc-ad)^2(-ad-b(c+dx)+bc)} + \frac{3\sqrt{b}\,d\tan^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}\,\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^(3/2)),x]
```

```
[Out] (d*(-2*b*c + 2*a*d + 3*b*(c + d*x)))/((b*c - a*d)^2*Sqrt[c + d*x]*(b*c - a*
d - b*(c + d*x))) + (3*Sqrt[b]*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
+ d*x])/(b*c - a*d)])/(-(b*c) + a*d)^(5/2)
```

fricas [B] time = 1.32, size = 423, normalized size = 4.27

$$\left[\frac{3\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad+2\,(bc-ad)\sqrt{dx+c}\,\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\,(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}, \frac{3\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\,\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+bc+2\,ad)\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*
d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x +
 a)) - 2*(3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 - 2*a^2*b*c^2*d
+ a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x), (3*(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2
)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c -
a*d))/(b*d*x + b*c)) - (3*b*d*x + b*c + 2*a*d)*sqrt(d*x + c))/(a*b^2*c^3 -
```

```
2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (
b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)]
```

giac [A] time = 1.03, size = 143, normalized size = 1.44

$$-\frac{3\,bd\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}-\frac{3\,(dx+c)bd-2\,bcd+2\,ad^2}{\left(b^2c^2-2\,abcd+a^2d^2\right)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+\sqrt{dx+c}\,ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -3*b*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*sqrt(-b^2*c + a*b*d)) - (3*(d*x + c)*b*d - 2*b*c*d + 2*a*d^2)/((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + sqrt
(d*x + c)*a*d))
```

maple [A] time = 0.01, size = 101, normalized size = 1.02

$$-\frac{3bd\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^2\sqrt{(ad-bc)\,b}}-\frac{\sqrt{dx+c}\;bd}{(ad-bc)^2\,(bdx+ad)}-\frac{2d}{(ad-bc)^2\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] -2*d/(a*d-b*c)^2/(d*x+c)^(1/2)-d*b/(a*d-b*c)^2*(d*x+c)^(1/2)/(b*d*x+a*d)-3*
d*b/(a*d-b*c)^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.19, size = 123, normalized size = 1.24

$$-\frac{\frac{2\,d}{a\,d-b\,c}+\frac{3\,b\,d\,(c+d\,x)}{(a\,d-b\,c)^2}}{b\,(c+d\,x)^{3/2}+(a\,d-b\,c)\,\sqrt{c+d\,x}}-\frac{3\,\sqrt{b}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{(a\,d-b\,c)^{5/2}}\right)}{(a\,d-b\,c)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d)/(a*d - b*c) + (3*b*d*(c + d*x))/(a*d - b*c)^2)/(b*(c + d*x)^(3/2)
+ (a*d - b*c)*(c + d*x)^(1/2)) - (3*b^(1/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)
*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(3/2)), x)
```

3.1326 $\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.05, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$\frac{15d^2}{4\sqrt{c+dx}\,(bc-ad)^3}-\frac{15\sqrt{b}\,d^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{7/2}}+\frac{5d}{4(a+bx)\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{2(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (15*d^2)/(4*(b*c - a*d)^3*Sqrt[c + d*x]) - 1/(2*(b*c - a*d)*(a + b*x)^2*Sqr
t[c + d*x]) + (5*d)/(4*(b*c - a*d)^2*(a + b*x)*Sqrt[c + d*x]) - (15*Sqrt[b]
*d^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(7/2)
)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} - \frac{(5d)\int \frac{1}{(a+bx)^2(c+dx)^{3/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}} + \frac{\left(15d^2\right)\int \frac{1}{(a+bx)(c+}}{8(bc-ad)}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}}$$

$$= \frac{15d^2}{4(bc-ad)^3\sqrt{c+dx}} - \frac{1}{2(bc-ad)(a+bx)^2\sqrt{c+dx}} + \frac{5d}{4(bc-ad)^2(a+bx)\sqrt{c+dx}}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.36

$$-\frac{2d^2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-1/2, 3, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^3*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.49, size = 163, normalized size = 1.16

$$\frac{d^2\left(8a^2d^2+25abd(c+dx)-16abcd+8b^2c^2+15b^2(c+dx)^2-25b^2c(c+dx)\right)}{4\sqrt{c+dx}\,(bc-ad)^3(-ad-b(c+dx)+bc)^2}+\frac{15\sqrt{b}\,d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^(3/2)),x]
```

```
[Out] (d^2*(8*b^2*c^2 - 16*a*b*c*d + 8*a^2*d^2 - 25*b^2*c*(c + d*x) + 25*a*b*d*(c
 + d*x) + 15*b^2*(c + d*x)^2))/(4*(b*c - a*d)^3*Sqrt[c + d*x]*(b*c - a*d -
b*(c + d*x))^2) + (15*Sqrt[b]*d^2*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c
 + d*x])/(b*c - a*d)])/(4*(-(b*c) + a*d)^(7/2))
```

fricas [B] time = 1.28, size = 782, normalized size = 5.59

$$\left[-\frac{15\left(b^2d^3x^3+a^2cd^2+\left(b^2cd^2+2abd^3\right)x^2+\left(2abcd^2+a^2d^3\right)x\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2bc-ad+2(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)-2\left(15b^2d^2x^2-2b^2c^2+9abcd+8a^2d^2+5\left(b^2cd+5abd^2\right)x\right)\sqrt{dx+c}}{8\left(a^2b^3c^4-3a^3b^2c^3d+3a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3ab^4c^2d^2+3a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3a^2b^3c^2d^2+5a^3b^2cd^3-2a^4bd^4\right)x^2+\left(2ab^4c^4-5a^2b^3c^3d+3a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)},-\frac{15\left(b^2d^3x^3+a^2cd^2+\left(b^2cd^2+2abd^3\right)x^2+\left(2abcd^2+a^2d^3\right)x\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-\left(15b^2d^2x^2-2b^2c^2+9abcd+8a^2d^2+5\left(b^2cd+5abd^2\right)x\right)\sqrt{dx+c}}{4\left(a^2b^3c^4-3a^3b^2c^3d+3a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3ab^4c^2d^2+3a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3a^2b^3c^2d^2+5a^3b^2cd^3-2a^4bd^4\right)x^2+\left(2ab^4c^4-5a^2b^3c^3d+3a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c
*d^2 + a^2*d^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d + 2*(b*c -
a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) - 2*(15*b^2*d^2*x^2 - 2*
b^2*c^2 + 9*a*b*c*d + 8*a^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))
/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d
- 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c
```

```
^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^
4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x), -1/4*(
15*(b^2*d^3*x^3 + a^2*c*d^2 + (b^2*c*d^2 + 2*a*b*d^3)*x^2 + (2*a*b*c*d^2 +
a^2*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/
(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 - 2*b^2*c^2 + 9*a*b*c*d + 8*a
^2*d^2 + 5*(b^2*c*d + 5*a*b*d^2)*x)*sqrt(d*x + c))/(a^2*b^3*c^4 - 3*a^3*b^2
*c^3*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2
*b^3*c*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2
+ 5*a^3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a
^3*b^2*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)]
```

giac [B] time = 1.03, size = 234, normalized size = 1.67

$$\frac{15\,bd^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{-b^2c+abd}}+\frac{2\,d^2}{\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\sqrt{dx+c}}+\frac{7\,(dx+c)^{\frac{3}{2}}b^2d^2-9\,\sqrt{dx+c}\,b^2cd^2+9\,\sqrt{dx+c}\,abd^3}{4\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\left((dx+c)b-bc+ad\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 15/4*b*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) + 2*d^2/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(d*x + c)) + 1/4*(7*(d*x + c)
^(3/2)*b^2*d^2 - 9*sqrt(d*x + c)*b^2*c*d^2 + 9*sqrt(d*x + c)*a*b*d^3)/((b^3
*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c + a*d)^2
)
```

maple [A] time = 0.02, size = 179, normalized size = 1.28

$$-\frac{9\sqrt{dx+c}\;ab\,d^3}{4\left(ad-bc\right)^3\left(bdx+ad\right)^2}+\frac{9\sqrt{dx+c}\;b^2c\,d^2}{4\left(ad-bc\right)^3\left(bdx+ad\right)^2}-\frac{7\left(dx+c\right)^{\frac{3}{2}}\,b^2d^2}{4\left(ad-bc\right)^3\left(bdx+ad\right)^2}-\frac{15b\,d^2\arctan\left(\frac{\sqrt{dx+c}\;b}{\sqrt{\left(ad-bc\right)b}}\right)}{4\left(ad-bc\right)^3\sqrt{\left(ad-bc\right)b}}-\frac{2d^2}{\left(ad-bc\right)^3\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^2/(a*d-b*c)^3/(d*x+c)^(1/2)-7/4*d^2*b^2/(a*d-b*c)^3/(b*d*x+a*d)^2*(d*x
+c)^(3/2)-9/4*d^3*b/(a*d-b*c)^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*a+9/4*d^2*b^2/(
a*d-b*c)^3/(b*d*x+a*d)^2*(d*x+c)^(1/2)*c-15/4*d^2*b/(a*d-b*c)^3/((a*d-b*c)*
b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.44, size = 205, normalized size = 1.46

$$-\frac{\frac{2\,d^2}{a\,d-b\,c}+\frac{15\,b^2\,d^2\,{\left(c+d\,x\right)}^2}{4\,{\left(a\,d-b\,c\right)}^3}+\frac{25\,b\,d^2\,\left(c+d\,x\right)}{4\,{\left(a\,d-b\,c\right)}^2}}{b^2\,{\left(c+d\,x\right)}^{5/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,{\left(c+d\,x\right)}^{3/2}+\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{15\,\sqrt{b}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^{7/2}}\right)}{4\,{\left(a\,d-b\,c\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^2)/(a*d - b*c) + (15*b^2*d^2*(c + d*x)^2)/(4*(a*d - b*c)^3) + (25*b
*d^2*(c + d*x))/(4*(a*d - b*c)^2))/(b^2*(c + d*x)^(5/2) - (2*b^2*c - 2*a*b*
d)*(c + d*x)^(3/2) + (c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (15
*b^(1/2)*d^2*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(4*(a*d - b*c)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.1327 $\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.07, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35d^3}{8\sqrt{c+dx}\,(bc-ad)^4}-\frac{35d^2}{24(a+bx)\sqrt{c+dx}\,(bc-ad)^3}+\frac{35\sqrt{b}\,d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{9/2}}+\frac{7d}{12(a+bx)^2\sqrt{c+dx}\,(bc-ad)^2}-\frac{1}{3(a+bx)^3\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*Sqrt[c + d*x]) - 1/(3*(b*c - a*d)*(a + b*x)^3*Sq
rt[c + d*x]) + (7*d)/(12*(b*c - a*d)^2*(a + b*x)^2*Sqrt[c + d*x]) - (35*d^2
)/(24*(b*c - a*d)^3*(a + b*x)*Sqrt[c + d*x]) + (35*Sqrt[b]*d^3*ArcTanh[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(8*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{3/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} - \frac{(7d)\int \frac{1}{(a+bx)^3(c+dx)^{3/2}}\,dx}{6(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} + \frac{(35d^2)\int \frac{1}{(a+bx)^2 \cdots}}{24(bc-\cdots}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c+dx}} - \frac{35 \cdots}{24(bc-ad)^3(a \cdots}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c \cdots}}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c \cdots}}$$

$$= -\frac{35d^3}{8(bc-ad)^4\sqrt{c+dx}} - \frac{1}{3(bc-ad)(a+bx)^3\sqrt{c+dx}} + \frac{7d}{12(bc-ad)^2(a+bx)^2\sqrt{c \cdots}}$$

Mathematica [C] time = 0.02, size = 50, normalized size = 0.29

$$-\frac{2d^3\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{\sqrt{c+dx}\,(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-1/2, 4, 1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(
(-(b*c) + a*d)^4*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.68, size = 223, normalized size = 1.29

$$\frac{d^3\left(48a^3d^3+231a^2bd^2(c+dx)-144a^2bcd^2+144ab^2c^2d+280ab^2d(c+dx)^2-462ab^2cd(c+dx)-48b^3c^3+231b^3c^2(c+dx)+105b^3(c+dx)^3-280b^3c(c+dx)^2\right)}{24\sqrt{c+dx}\,(bc-ad)^4(-ad-b(c+dx)+bc)^3}+\frac{35\sqrt{b}\,d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*(c + d*x)^(3/2)),x]
```

```
[Out] (d^3*(-48*b^3*c^3 + 144*a*b^2*c^2*d - 144*a^2*b*c*d^2 + 48*a^3*d^3 + 231*b^
3*c^2*(c + d*x) - 462*a*b^2*c*d*(c + d*x) + 231*a^2*b*d^2*(c + d*x) - 280*b
^3*c*(c + d*x)^2 + 280*a*b^2*d*(c + d*x)^2 + 105*b^3*(c + d*x)^3))/(24*(b*c
 - a*d)^4*Sqrt[c + d*x]*(b*c - a*d - b*(c + d*x))^3) + (35*Sqrt[b]*d^3*ArcT
an[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(8*(-(b*c) + a*
d)^(9/2))
```

fricas [B] time = 1.53, size = 1204, normalized size = 6.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*x^3 + 3*(a*
b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(b/(b*c - a*d
```

```
))*log((b*d*x + 2*b*c - a*d + 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d
)))/(b*x + a)) - 2*(105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b
*c*d^2 + 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 1
4*a*b^2*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x), 1/24*(105*(b^3*d^4*x^4 + a^3*c*d^3 + (b^3*c*d^3 + 3*a*b^2*d^4)*
x^3 + 3*(a*b^2*c*d^3 + a^2*b*d^4)*x^2 + (3*a^2*b*c*d^3 + a^3*d^4)*x)*sqrt(-
b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*
x + b*c)) - (105*b^3*d^3*x^3 + 8*b^3*c^3 - 38*a*b^2*c^2*d + 87*a^2*b*c*d^2
+ 48*a^3*d^3 + 35*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 7*(2*b^3*c^2*d - 14*a*b^2
*c*d^2 - 33*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6
*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*
b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a
^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^
4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*
b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)
*x)]
```

giac [B] time = 1.31, size = 326, normalized size = 1.88

$$-\frac{35\,bd^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{-b^2c+abd}}-\frac{2\,d^3}{\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\sqrt{dx+c}}-\frac{57\,(dx+c)^{\frac{5}{2}}b^3d^3-136\,(dx+c)^{\frac{3}{2}}b^3cd^3+87\,\sqrt{dx+c}\,b^3c^2d^3+136\,(dx+c)^{\frac{3}{2}}ab^2d^4-174\,\sqrt{dx+c}\,ab^2cd^4+87\,\sqrt{dx+c}\,a^2bd^5}{24\left(b^4c^4-4\,ab^3c^3d+6\,a^2b^2c^2d^2-4\,a^3bcd^3+a^4d^4\right)\left((dx+c)b-bc+ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -35/8*b*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d)
) - 2*d^3/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(d*x + c)) - 1/24*(57*(d*x + c)^(5/2)*b^3*d^3 - 136*(d*x + c)^(
3/2)*b^3*c*d^3 + 87*sqrt(d*x + c)*b^3*c^2*d^3 + 136*(d*x + c)^(3/2)*a*b^2*d
^4 - 174*sqrt(d*x + c)*a*b^2*c*d^4 + 87*sqrt(d*x + c)*a^2*b*d^5)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*((d*x + c)*b
 - b*c + a*d)^3)
```

maple [B] time = 0.02, size = 292, normalized size = 1.69

$$-\frac{29\sqrt{dx+c}\,a^2b\,d^5}{8\,(ad-bc)^4\,(bdx+ad)^3}+\frac{29\sqrt{dx+c}\,a\,b^2c\,d^4}{4\,(ad-bc)^4\,(bdx+ad)^3}-\frac{29\sqrt{dx+c}\,b^3c^2d^3}{8\,(ad-bc)^4\,(bdx+ad)^3}-\frac{17\,(dx+c)^{\frac{3}{2}}\,a\,b^2d^4}{3\,(ad-bc)^4\,(bdx+ad)^3}+\frac{17\,(dx+c)^{\frac{3}{2}}\,b^3c\,d^3}{3\,(ad-bc)^4\,(bdx+ad)^3}-\frac{19\,(dx+c)^{\frac{5}{2}}\,b^3d^3}{8\,(ad-bc)^4\,(bdx+ad)^3}-\frac{35b\,d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{8\,(ad-bc)^4\,\sqrt{(ad-bc)\,b}}-\frac{2d^3}{(ad-bc)^4\,\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(3/2),x)
```

```
[Out] -2*d^3/(a*d-b*c)^4/(d*x+c)^(1/2)-19/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*
x+c)^(5/2)-17/3*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(3/2)*a+17/3*d^3/
(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(3/2)*c-29/8*d^5/(a*d-b*c)^4*b/(b*d*x
+a*d)^3*(d*x+c)^(1/2)*a^2+29/4*d^4/(a*d-b*c)^4*b^2/(b*d*x+a*d)^3*(d*x+c)^(1
/2)*a*c-29/8*d^3/(a*d-b*c)^4*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*c^2-35/8*d^3/(
a*d-b*c)^4*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.54, size = 294, normalized size = 1.70

$$-\frac{\frac{2\,d^3}{a\,d-b\,c}+\frac{35\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{8\,{\left(a\,d-b\,c\right)}^4}+\frac{77\,b\,d^3\,\left(c+d\,x\right)}{8\,{\left(a\,d-b\,c\right)}^2}}{\sqrt{c+d\,x}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{7/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{5/2}+{\left(c+d\,x\right)}^{3/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}-\frac{35\,\sqrt{b}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{{\left(a\,d-b\,c\right)}^{9/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(3/2)),x)
```

```
[Out] - ((2*d^3)/(a*d - b*c) + (35*b^2*d^3*(c + d*x)^2)/(3*(a*d - b*c)^3) + (35*b
^3*d^3*(c + d*x)^3)/(8*(a*d - b*c)^4) + (77*b*d^3*(c + d*x))/(8*(a*d - b*c)
^2))/((c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) +
 b^3*(c + d*x)^(7/2) - (3*b^3*c - 3*a*b^2*d)*(c + d*x)^(5/2) + (c + d*x)^(3
/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*b^2*c*d)) - (35*b^(1/2)*d^3*atan((b^(1/2
)*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d -
4*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(8*(a*d - b*c)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.1328 $\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b}{}$$

Rubi [A] time = 0.05, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{2b^4(c+dx)^{5/2}(bc-ad)}{d^6}+\frac{20b^3(c+dx)^{3/2}(bc-ad)^2}{3d^6}-\frac{20b^2\sqrt{c+dx}\,(bc-ad)^3}{d^6}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}+\frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}+\frac{2b^5(c+dx)^{7/2}}{7d^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^5)/(3*d^6*(c + d*x)^(3/2)) - (10*b*(b*c - a*d)^4)/(d^6*Sqrt[
c + d*x]) - (20*b^2*(b*c - a*d)^3*Sqrt[c + d*x])/d^6 + (20*b^3*(b*c - a*d)^
2*(c + d*x)^(3/2))/(3*d^6) - (2*b^4*(b*c - a*d)*(c + d*x)^(5/2))/d^6 + (2*b
^5*(c + d*x)^(7/2))/(7*d^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^5}{d^5(c+dx)^{5/2}}+\frac{5b(bc-ad)^4}{d^5(c+dx)^{3/2}}-\frac{10b^2(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{10b^3(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{5b^4(bc-ad)(}{d^5}\right.\\ &= \frac{2(bc-ad)^5}{3d^6(c+dx)^{3/2}}-\frac{10b(bc-ad)^4}{d^6\sqrt{c+dx}}-\frac{20b^2(bc-ad)^3\sqrt{c+dx}}{d^6}+\frac{20b^3(bc-ad)^2(c+dx)^{3/2}}{3d^6}-\frac{2b^4(b}{}\end{aligned}$$

Mathematica [A] time = 0.12, size = 123, normalized size = 0.81

$$\frac{2\left(-21b^4(c+dx)^4(bc-ad)+70b^3(c+dx)^3(bc-ad)^2-210b^2(c+dx)^2(bc-ad)^3-105b(c+dx)(bc-ad)^4+7(bc-ad)^5+3b^5(c+dx)^5\right)}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(7*(b*c - a*d)^5 - 105*b*(b*c - a*d)^4*(c + d*x) - 210*b^2*(b*c - a*d)^3
*(c + d*x)^2 + 70*b^3*(b*c - a*d)^2*(c + d*x)^3 - 21*b^4*(b*c - a*d)*(c + d
*x)^4 + 3*b^5*(c + d*x)^5))/(21*d^6*(c + d*x)^(3/2))
```

IntegrateAlgebraic [B] time = 0.07, size = 315, normalized size = 2.07

$$\frac{2\left(-7a^5d^5-105a^4bd^4(c+dx)+35a^4bcd^4-70a^3b^2c^2d^3+210a^3b^2d^3(c+dx)^2+420a^3b^2cd^3(c+dx)+70a^2b^3c^3d^2-630a^2b^3c^2d^2(c+dx)+70a^2b^3d^2(c+dx)^3-630a^2b^3cd^2(c+dx)^2-35ab^4c^4d+420ab^4c^3d(c+dx)+630ab^4c^2d(c+dx)^2+21ab^4d(c+dx)^4-140ab^4cd(c+dx)^3+7b^5c^5-105b^5c^4(c+dx)-210b^5c^3(c+dx)^2+70b^5c^2(c+dx)^3+3b^5(c+dx)^5-21b^5c(c+dx)^4\right)}{21d^6(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(c + d*x)^(5/2),x]
```

```
[Out] (2*(7*b^5*c^5 - 35*a*b^4*c^4*d + 70*a^2*b^3*c^3*d^2 - 70*a^3*b^2*c^2*d^3 +
35*a^4*b*c*d^4 - 7*a^5*d^5 - 105*b^5*c^4*(c + d*x) + 420*a*b^4*c^3*d*(c + d
*x) - 630*a^2*b^3*c^2*d^2*(c + d*x) + 420*a^3*b^2*c*d^3*(c + d*x) - 105*a^4
*b*d^4*(c + d*x) - 210*b^5*c^3*(c + d*x)^2 + 630*a*b^4*c^2*d*(c + d*x)^2 -
630*a^2*b^3*c*d^2*(c + d*x)^2 + 210*a^3*b^2*d^3*(c + d*x)^2 + 70*b^5*c^2*(c
 + d*x)^3 - 140*a*b^4*c*d*(c + d*x)^3 + 70*a^2*b^3*d^2*(c + d*x)^3 - 21*b^5
*c*(c + d*x)^4 + 21*a*b^4*d*(c + d*x)^4 + 3*b^5*(c + d*x)^5))/(21*d^6*(c +
d*x)^(3/2))
```

fricas [B] time = 1.36, size = 283, normalized size = 1.86

$$\frac{2\,{\left(3\,b^{5}d^{5}x^{5} - 256\,b^{5}c^{5} + 896\,ab^{4}c^{4}d - 1120\,a^{2}b^{3}c^{3}d^{2} + 560\,a^{3}b^{2}c^{2}d^{3} - 70\,a^{4}bcd^{4} - 7\,a^{5}d^{5} - 3\,{\left(2\,b^{5}cd^{4} - 7\,ab^{4}d^{5}\right)}x^{4} + 2\,{\left(8\,b^{5}c^{2}d^{3} - 28\,ab^{4}cd^{4} + 35\,a^{2}b^{3}d^{5}\right)}x^{3} - 6\,{\left(16\,b^{5}c^{3}d^{2} - 56\,ab^{4}c^{2}d^{3} + 70\,a^{2}b^{3}cd^{4} - 35\,a^{3}b^{2}d^{5}\right)}x^{2} - 3\,{\left(128\,b^{5}c^{4}d - 448\,ab^{4}c^{3}d^{2} + 560\,a^{2}b^{3}c^{2}d^{3} - 280\,a^{3}b^{2}cd^{4} + 35\,a^{4}bd^{5}\right)}x\right)}\sqrt{dx + c}}{21\,{\left(d^{8}x^{2} + 2\,cd^{7}x + c^{2}d^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*b^5*d^5*x^5 - 256*b^5*c^5 + 896*a*b^4*c^4*d - 1120*a^2*b^3*c^3*d^2
+ 560*a^3*b^2*c^2*d^3 - 70*a^4*b*c*d^4 - 7*a^5*d^5 - 3*(2*b^5*c*d^4 - 7*a*b
^4*d^5)*x^4 + 2*(8*b^5*c^2*d^3 - 28*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 6*(
16*b^5*c^3*d^2 - 56*a*b^4*c^2*d^3 + 70*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
- 3*(128*b^5*c^4*d - 448*a*b^4*c^3*d^2 + 560*a^2*b^3*c^2*d^3 - 280*a^3*b^2*
c*d^4 + 35*a^4*b*d^5)*x)*sqrt(d*x + c)/(d^8*x^2 + 2*c*d^7*x + c^2*d^6)
```

giac [B] time = 0.91, size = 335, normalized size = 2.20

$$-\frac{2\,{\left(15\,{\left(dx + c\right)}b^{5}c^{4} - b^{5}c^{5} - 60\,{\left(dx + c\right)}ab^{4}c^{3}d + 5\,ab^{4}c^{4}d + 90\,{\left(dx + c\right)}a^{2}b^{3}c^{2}d^{2} - 10\,a^{2}b^{3}c^{3}d^{2} - 60\,{\left(dx + c\right)}a^{3}b^{2}cd^{3} + 10\,a^{3}b^{2}c^{2}d^{3} + 15\,{\left(dx + c\right)}a^{4}bd^{4} - 5\,a^{4}bcd^{4} + a^{5}d^{5}\right)}}{3\,{\left(dx + c\right)}^{\frac{3}{2}}d^{6}} + \frac{2\,{\left(3\,{\left(dx + c\right)}^{\frac{7}{2}}b^{5}d^{36} - 21\,{\left(dx + c\right)}^{\frac{5}{2}}b^{5}cd^{36} + 70\,{\left(dx + c\right)}^{\frac{3}{2}}b^{5}c^{2}d^{36} - 210\,\sqrt{dx + c}\,b^{5}c^{3}d^{36} + 21\,{\left(dx + c\right)}^{\frac{5}{2}}ab^{4}d^{37} - 140\,{\left(dx + c\right)}^{\frac{3}{2}}ab^{4}cd^{37} + 630\,\sqrt{dx + c}\,ab^{4}c^{2}d^{37} + 70\,{\left(dx + c\right)}^{\frac{3}{2}}a^{2}b^{3}d^{38} - 630\,\sqrt{dx + c}\,a^{2}b^{3}cd^{38} + 210\,\sqrt{dx + c}\,a^{3}b^{2}d^{39}\right)}}{21\,d^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(15*(d*x + c)*b^5*c^4 - b^5*c^5 - 60*(d*x + c)*a*b^4*c^3*d + 5*a*b^4*c
^4*d + 90*(d*x + c)*a^2*b^3*c^2*d^2 - 10*a^2*b^3*c^3*d^2 - 60*(d*x + c)*a^3
*b^2*c*d^3 + 10*a^3*b^2*c^2*d^3 + 15*(d*x + c)*a^4*b*d^4 - 5*a^4*b*c*d^4 +
a^5*d^5)/((d*x + c)^(3/2)*d^6) + 2/21*(3*(d*x + c)^(7/2)*b^5*d^36 - 21*(d*x
 + c)^(5/2)*b^5*c*d^36 + 70*(d*x + c)^(3/2)*b^5*c^2*d^36 - 210*sqrt(d*x + c
)*b^5*c^3*d^36 + 21*(d*x + c)^(5/2)*a*b^4*d^37 - 140*(d*x + c)^(3/2)*a*b^4*
c*d^37 + 630*sqrt(d*x + c)*a*b^4*c^2*d^37 + 70*(d*x + c)^(3/2)*a^2*b^3*d^38
 - 630*sqrt(d*x + c)*a^2*b^3*c*d^38 + 210*sqrt(d*x + c)*a^3*b^2*d^39)/d^42
```

maple [B] time = 0.01, size = 273, normalized size = 1.80

$$-\frac{2\left(-3b^{5}x^{5}d^{5} - 21a\,b^{4}d^{5}x^{4} + 6b^{5}c\,d^{4}x^{4} - 70a^{2}b^{3}d^{5}x^{3} + 56a\,b^{4}c\,d^{4}x^{3} - 16b^{5}c^{2}d^{3}x^{3} - 210a^{3}b^{2}d^{5}x^{2} + 420a^{2}b^{3}c\,d^{4}x^{2} - 336a\,b^{4}c^{2}d^{3}x^{2} + 96b^{5}c^{3}d^{2}x^{2} + 105a^{4}b\,d^{5}x - 840a^{3}b^{2}c\,d^{4}x + 1680a^{2}b^{3}c^{2}d^{3}x - 1344a\,b^{4}c^{3}d^{2}x + 384b^{5}c^{4}dx + 7a^{5}d^{5} + 70a^{4}bc\,d^{4} - 560a^{3}b^{2}c^{2}d^{3} + 1120a^{2}b^{3}c^{3}d^{2} - 896a\,b^{4}c^{4}d + 256b^{5}c^{5}\right)}{21\left(dx + c\right)^{\frac{3}{2}}d^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(d*x+c)^(5/2),x)
```

```
[Out] -2/21/(d*x+c)^(3/2)*(-3*b^5*d^5*x^5-21*a*b^4*d^5*x^4+6*b^5*c*d^4*x^4-70*a^2
*b^3*d^5*x^3+56*a*b^4*c*d^4*x^3-16*b^5*c^2*d^3*x^3-210*a^3*b^2*d^5*x^2+420*
a^2*b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+96*b^5*c^3*d^2*x^2+105*a^4*b*d^5*x-
840*a^3*b^2*c*d^4*x+1680*a^2*b^3*c^2*d^3*x-1344*a*b^4*c^3*d^2*x+384*b^5*c^4
*d*x+7*a^5*d^5+70*a^4*b*c*d^4-560*a^3*b^2*c^2*d^3+1120*a^2*b^3*c^3*d^2-896*
a*b^4*c^4*d+256*b^5*c^5)/d^6
```

maxima [A] time = 1.38, size = 265, normalized size = 1.74

$$\frac{2\,{\left(\frac{3\,{\left(dx+c\right)}^{\frac{7}{2}}b^{5}-21\,{\left(b^{5}c-ab^{4}d\right)}{\left(dx+c\right)}^{\frac{5}{2}}+70\,{\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)}{\left(dx+c\right)}^{\frac{3}{2}}-210\,{\left(b^{5}c^{3}-3\,ab^{4}c^{2}d+3\,a^{2}b^{3}cd^{2}-a^{3}b^{2}d^{3}\right)}\sqrt{dx+c}}{d^{5}} + \frac{7\,{\left(b^{5}c^{5}-5\,ab^{4}c^{4}d+10\,a^{2}b^{3}c^{3}d^{2}-10\,a^{3}b^{2}c^{2}d^{3}+5\,a^{4}bcd^{4}-a^{5}d^{5}-15\,{\left(b^{5}c^{4}-4\,ab^{4}c^{3}d+6\,a^{2}b^{3}c^{2}d^{2}-4\,a^{3}b^{2}cd^{3}+a^{4}bd^{4}\right)}{\left(dx+c\right)}\right)}}{{\left(dx+c\right)}^{\frac{3}{2}}d^{5}}\right)}}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*((3*(d*x + c)^(7/2)*b^5 - 21*(b^5*c - a*b^4*d)*(d*x + c)^(5/2) + 70*(b
^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(d*x + c)^(3/2) - 210*(b^5*c^3 - 3*a*b^
4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt(d*x + c))/d^5 + 7*(b^5*c^5 -
5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a
^5*d^5 - 15*(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3
+ a^4*b*d^4)*(d*x + c))/((d*x + c)^(3/2)*d^5))/d
```

mupad [B] time = 0.08, size = 229, normalized size = 1.51

$$\frac{2\,b^5\,{\left(c+d\,x\right)}^{7/2}}{7\,d^6}-\frac{\left(10\,b^5\,c-10\,a\,b^4\,d\right)\,{\left(c+d\,x\right)}^{5/2}}{5\,d^6}-\frac{\frac{2\,a^5\,d^5}{3}-\frac{2\,b^5\,c^5}{3}+\left(c+d\,x\right)\,\left(10\,a^4\,b\,d^4-40\,a^3\,b^2\,c\,d^3+60\,a^2\,b^3\,c^2\,d^2-40\,a\,b^4\,c^3\,d+10\,b^5\,c^4\right)-\frac{20\,a^2\,b^3\,c^3\,d^2}{3}+\frac{20\,a^3\,b^2\,c^2\,d^3}{3}+\frac{10\,a\,b^4\,c^4\,d}{3}-\frac{10\,a^4\,b\,c\,d^4}{3}}{d^6\,{\left(c+d\,x\right)}^{3/2}}+\frac{20\,b^2\,{\left(a\,d-b\,c\right)}^3\,\sqrt{c+d\,x}}{d^6}+\frac{20\,b^3\,{\left(a\,d-b\,c\right)}^2\,{\left(c+d\,x\right)}^{3/2}}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^5*(c + d*x)^(7/2))/(7*d^6) - ((10*b^5*c - 10*a*b^4*d)*(c + d*x)^(5/2))
/(5*d^6) - ((2*a^5*d^5)/3 - (2*b^5*c^5)/3 + (c + d*x)*(10*b^5*c^4 + 10*a^4*
b*d^4 - 40*a^3*b^2*c*d^3 + 60*a^2*b^3*c^2*d^2 - 40*a*b^4*c^3*d) - (20*a^2*b
^3*c^3*d^2)/3 + (20*a^3*b^2*c^2*d^3)/3 + (10*a*b^4*c^4*d)/3 - (10*a^4*b*c*d
^4)/3)/(d^6*(c + d*x)^(3/2)) + (20*b^2*(a*d - b*c)^3*(c + d*x)^(1/2))/d^6 +
 (20*b^3*(a*d - b*c)^2*(c + d*x)^(3/2))/(3*d^6)
```

sympy [A] time = 59.93, size = 196, normalized size = 1.29

$$\frac{2b^5\left(c+dx\right)^{\frac{7}{2}}}{7d^6}-\frac{10b\left(ad-bc\right)^4}{d^6\sqrt{c+dx}}+\frac{\left(c+dx\right)^{\frac{5}{2}}\left(10ab^4d-10b^5c\right)}{5d^6}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(20a^2b^3d^2-40ab^4cd+20b^5c^2\right)}{3d^6}+\frac{\sqrt{c+dx}\left(20a^3b^2d^3-60a^2b^3cd^2+60ab^4c^2d-20b^5c^3\right)}{d^6}-\frac{2\left(ad-bc\right)^5}{3d^6\left(c+dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**5*(c + d*x)**(7/2)/(7*d**6) - 10*b*(a*d - b*c)**4/(d**6*sqrt(c + d*x))
 + (c + d*x)**(5/2)*(10*a*b**4*d - 10*b**5*c)/(5*d**6) + (c + d*x)**(3/2)*(
20*a**2*b**3*d**2 - 40*a*b**4*c*d + 20*b**5*c**2)/(3*d**6) + sqrt(c + d*x)*
(20*a**3*b**2*d**3 - 60*a**2*b**3*c*d**2 + 60*a*b**4*c**2*d - 20*b**5*c**3)
/d**6 - 2*(a*d - b*c)**5/(3*d**6*(c + d*x)**(3/2))
```

3.1329 $\int \frac{(a+bx)^4}{(c+dx)^{5/2}} \, dx$

Optimal. Leaf size=125

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Rubi [A] time = 0.04, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{8b^3(c+dx)^{3/2}(bc-ad)}{3d^5}+\frac{12b^2\sqrt{c+dx}\,(bc-ad)^2}{d^5}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}-\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{2b^4(c+dx)^{5/2}}{5d^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^4)/(3*d^5*(c + d*x)^(3/2)) + (8*b*(b*c - a*d)^3)/(d^5*Sqrt[
c + d*x]) + (12*b^2*(b*c - a*d)^2*Sqrt[c + d*x])/d^5 - (8*b^3*(b*c - a*d)*(
c + d*x)^(3/2))/(3*d^5) + (2*b^4*(c + d*x)^(5/2))/(5*d^5)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^4}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^4}{d^4(c+dx)^{5/2}}-\frac{4b(bc-ad)^3}{d^4(c+dx)^{3/2}}+\frac{6b^2(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{4b^3(bc-ad)\sqrt{c+dx}}{d^4}+\frac{b^4(c+dx)^{3/2}}{d^4}\right) \\ &= -\frac{2(bc-ad)^4}{3d^5(c+dx)^{3/2}}+\frac{8b(bc-ad)^3}{d^5\sqrt{c+dx}}+\frac{12b^2(bc-ad)^2\sqrt{c+dx}}{d^5}-\frac{8b^3(bc-ad)(c+dx)^{3/2}}{3d^5}+\frac{2b^4}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 101, normalized size = 0.81

$$\frac{2\left(-20b^3(c+dx)^3(bc-ad)+90b^2(c+dx)^2(bc-ad)^2+60b(c+dx)(bc-ad)^3-5(bc-ad)^4+3b^4(c+dx)^4\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*(b*c - a*d)^4 + 60*b*(b*c - a*d)^3*(c + d*x) + 90*b^2*(b*c - a*d)^2*
(c + d*x)^2 - 20*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4))/(15*d^5*
(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.08, size = 213, normalized size = 1.70

$$\frac{2\left(-5a^4d^4-60a^3bd^3(c+dx)+20a^3bcd^3-30a^2b^2c^2d^2+90a^2b^2d^2(c+dx)^2+180a^2b^2cd^2(c+dx)+20ab^3c^3d-180ab^3c^2d(c+dx)+20ab^3d(c+dx)^3-180ab^3cd(c+dx)^2-5b^4c^4+60b^4c^3(c+dx)+90b^4c^2(c+dx)^2+3b^4(c+dx)^4-20b^4c(c+dx)^3\right)}{15d^5(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^4/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-5*b^4*c^4 + 20*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 5*a
^4*d^4 + 60*b^4*c^3*(c + d*x) - 180*a*b^3*c^2*d*(c + d*x) + 180*a^2*b^2*c*d
^2*(c + d*x) - 60*a^3*b*d^3*(c + d*x) + 90*b^4*c^2*(c + d*x)^2 - 180*a*b^3*
c*d*(c + d*x)^2 + 90*a^2*b^2*d^2*(c + d*x)^2 - 20*b^4*c*(c + d*x)^3 + 20*a*
b^3*d*(c + d*x)^3 + 3*b^4*(c + d*x)^4))/(15*d^5*(c + d*x)^(3/2))
```

fricas [A] time = 1.13, size = 203, normalized size = 1.62

$$\frac{2\left(3\,b^{4}d^{4}x^{4}+128\,b^{4}c^{4}-320\,ab^{3}c^{3}d+240\,a^{2}b^{2}c^{2}d^{2}-40\,a^{3}bcd^{3}-5\,a^{4}d^{4}-4\left(2\,b^{4}cd^{3}-5\,ab^{3}d^{4}\right)x^{3}+6\left(8\,b^{4}c^{2}d^{2}-20\,ab^{3}cd^{3}+15\,a^{2}b^{2}d^{4}\right)x^{2}+12\left(16\,b^{4}c^{3}d-40\,ab^{3}c^{2}d^{2}+30\,a^{2}b^{2}cd^{3}-5\,a^{3}bd^{4}\right)x\right)\sqrt{dx+c}}{15\left(d^{7}x^{2}+2\,cd^{6}x+c^{2}d^{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^4*d^4*x^4 + 128*b^4*c^4 - 320*a*b^3*c^3*d + 240*a^2*b^2*c^2*d^2 -
 40*a^3*b*c*d^3 - 5*a^4*d^4 - 4*(2*b^4*c*d^3 - 5*a*b^3*d^4)*x^3 + 6*(8*b^4*
c^2*d^2 - 20*a*b^3*c*d^3 + 15*a^2*b^2*d^4)*x^2 + 12*(16*b^4*c^3*d - 40*a*b^
3*c^2*d^2 + 30*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(d*x + c)/(d^7*x^2 + 2*c
*d^6*x + c^2*d^5)
```

giac [B] time = 1.02, size = 229, normalized size = 1.83

$$\frac{2\left(12\,(dx+c)b^{4}c^{3}-b^{4}c^{4}-36\,(dx+c)ab^{3}c^{2}d+4\,ab^{3}c^{3}d+36\,(dx+c)a^{2}b^{2}cd^{2}-6\,a^{2}b^{2}c^{2}d^{2}-12\,(dx+c)a^{3}bd^{3}+4\,a^{3}bcd^{3}-a^{4}d^{4}\right)}{3\,(dx+c)^{\frac{3}{2}}d^{5}}+\frac{2\left(3\,(dx+c)^{\frac{5}{2}}b^{4}d^{20}-20\,(dx+c)^{\frac{3}{2}}b^{4}cd^{20}+90\,\sqrt{dx+c}\,b^{4}c^{2}d^{20}+20\,(dx+c)^{\frac{3}{2}}ab^{3}d^{21}-180\,\sqrt{dx+c}\,ab^{3}cd^{21}+90\,\sqrt{dx+c}\,a^{2}b^{2}d^{22}\right)}{15\,d^{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(12*(d*x + c)*b^4*c^3 - b^4*c^4 - 36*(d*x + c)*a*b^3*c^2*d + 4*a*b^3*c^
3*d + 36*(d*x + c)*a^2*b^2*c*d^2 - 6*a^2*b^2*c^2*d^2 - 12*(d*x + c)*a^3*b*d
^3 + 4*a^3*b*c*d^3 - a^4*d^4)/((d*x + c)^(3/2)*d^5) + 2/15*(3*(d*x + c)^(5/
2)*b^4*d^20 - 20*(d*x + c)^(3/2)*b^4*c*d^20 + 90*sqrt(d*x + c)*b^4*c^2*d^20
 + 20*(d*x + c)^(3/2)*a*b^3*d^21 - 180*sqrt(d*x + c)*a*b^3*c*d^21 + 90*sqrt
(d*x + c)*a^2*b^2*d^22)/d^25
```

maple [A] time = 0.01, size = 186, normalized size = 1.49

$$-\frac{2\left(-3b^{4}x^{4}d^{4}-20a\,b^{3}d^{4}x^{3}+8b^{4}c\,d^{3}x^{3}-90a^{2}b^{2}d^{4}x^{2}+120a\,b^{3}c\,d^{3}x^{2}-48b^{4}c^{2}d^{2}x^{2}+60a^{3}b\,d^{4}x-360a^{2}b^{2}c\,d^{3}x+480a\,b^{3}c^{2}d^{2}x-192b^{4}c^{3}dx+5a^{4}d^{4}+40a^{3}bc\,d^{3}-240a^{2}b^{2}c^{2}d^{2}+320a\,b^{3}c^{3}d-128b^{4}c^{4}\right)}{15\,(dx+c)^{\frac{3}{2}}\,d^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/15/(d*x+c)^(3/2)*(-3*b^4*d^4*x^4-20*a*b^3*d^4*x^3+8*b^4*c*d^3*x^3-90*a^2
*b^2*d^4*x^2+120*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2+60*a^3*b*d^4*x-360*a^2*
b^2*c*d^3*x+480*a*b^3*c^2*d^2*x-192*b^4*c^3*d*x+5*a^4*d^4+40*a^3*b*c*d^3-24
0*a^2*b^2*c^2*d^2+320*a*b^3*c^3*d-128*b^4*c^4)/d^5
```

maxima [A] time = 1.46, size = 187, normalized size = 1.50

$$\frac{2\left(\frac{3\,(dx+c)^{\frac{5}{2}}b^{4}-20\left(b^{4}c-ab^{3}d\right)(dx+c)^{\frac{3}{2}}+90\left(b^{4}c^{2}-2\,ab^{3}cd+a^{2}b^{2}d^{2}\right)\sqrt{dx+c}}{d^{4}}-\frac{5\left(b^{4}c^{4}-4\,ab^{3}c^{3}d+6\,a^{2}b^{2}c^{2}d^{2}-4\,a^{3}bcd^{3}+a^{4}d^{4}-12\left(b^{4}c^{3}-3\,ab^{3}c^{2}d+3\,a^{2}b^{2}cd^{2}-a^{3}bd^{3}\right)(dx+c)\right)}{(dx+c)^{\frac{3}{2}}d^{4}}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/15*((3*(d*x + c)^(5/2)*b^4 - 20*(b^4*c - a*b^3*d)*(d*x + c)^(3/2) + 90*(b
^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*sqrt(d*x + c))/d^4 - 5*(b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4 - 12*(b^4*c^3 - 3*a*
b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*(d*x + c))/((d*x + c)^(3/2)*d^4))/
d
```

mupad [B] time = 0.30, size = 175, normalized size = 1.40

$$\frac{2\,b^4\,{\left(c+d\,x\right)}^{5/2}}{5\,d^5}-\frac{\left(8\,b^4\,c-8\,a\,b^3\,d\right)\,{\left(c+d\,x\right)}^{3/2}}{3\,d^5}+\frac{\left(c+d\,x\right)\,\left(-8\,a^3\,b\,d^3+24\,a^2\,b^2\,c\,d^2-24\,a\,b^3\,c^2\,d+8\,b^4\,c^3\right)-\frac{2\,a^4\,d^4}{3}-\frac{2\,b^4\,c^4}{3}-4\,a^2\,b^2\,c^2\,d^2+\frac{8\,a\,b^3\,c^3\,d}{3}+\frac{8\,a^3\,b\,c\,d^3}{3}}{d^5\,{\left(c+d\,x\right)}^{3/2}}+\frac{12\,b^2\,{\left(a\,d-b\,c\right)}^2\,\sqrt{c+d\,x}}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^4/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^4*(c + d*x)^(5/2))/(5*d^5) - ((8*b^4*c - 8*a*b^3*d)*(c + d*x)^(3/2))/(
3*d^5) + ((c + d*x)*(8*b^4*c^3 - 8*a^3*b*d^3 + 24*a^2*b^2*c*d^2 - 24*a*b^3*
c^2*d) - (2*a^4*d^4)/3 - (2*b^4*c^4)/3 - 4*a^2*b^2*c^2*d^2 + (8*a*b^3*c^3*d
)/3 + (8*a^3*b*c*d^3)/3)/(d^5*(c + d*x)^(3/2)) + (12*b^2*(a*d - b*c)^2*(c +
 d*x)^(1/2))/d^5
```

sympy [A] time = 43.59, size = 136, normalized size = 1.09

$$\frac{2b^4\left(c+dx\right)^{\frac{5}{2}}}{5d^5}-\frac{8b\left(ad-bc\right)^3}{d^5\sqrt{c+dx}}+\frac{\left(c+dx\right)^{\frac{3}{2}}\left(8ab^3d-8b^4c\right)}{3d^5}+\frac{\sqrt{c+dx}\left(12a^2b^2d^2-24ab^3cd+12b^4c^2\right)}{d^5}-\frac{2\left(ad-bc\right)^4}{3d^5\left(c+dx\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] 2*b**4*(c + d*x)**(5/2)/(5*d**5) - 8*b*(a*d - b*c)**3/(d**5*sqrt(c + d*x))
+ (c + d*x)**(3/2)*(8*a*b**3*d - 8*b**4*c)/(3*d**5) + sqrt(c + d*x)*(12*a**
2*b**2*d**2 - 24*a*b**3*c*d + 12*b**4*c**2)/d**5 - 2*(a*d - b*c)**4/(3*d**5
*(c + d*x)**(3/2))
```

3.1330 $\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=96

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Rubi [A] time = 0.03, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$-\frac{6b^2\sqrt{c+dx}\,(bc-ad)}{d^4}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}+\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}+\frac{2b^3(c+dx)^{3/2}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d)^3)/(3*d^4*(c + d*x)^(3/2)) - (6*b*(b*c - a*d)^2)/(d^4*Sqrt[c
 + d*x]) - (6*b^2*(b*c - a*d)*Sqrt[c + d*x])/d^4 + (2*b^3*(c + d*x)^(3/2))/
(3*d^4)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^3}{(c+dx)^{5/2}}\,dx&=\int \left(\frac{(-bc+ad)^3}{d^3(c+dx)^{5/2}}+\frac{3b(bc-ad)^2}{d^3(c+dx)^{3/2}}-\frac{3b^2(bc-ad)}{d^3\sqrt{c+dx}}+\frac{b^3\sqrt{c+dx}}{d^3}\right)dx\\&=\frac{2(bc-ad)^3}{3d^4(c+dx)^{3/2}}-\frac{6b(bc-ad)^2}{d^4\sqrt{c+dx}}-\frac{6b^2(bc-ad)\sqrt{c+dx}}{d^4}+\frac{2b^3(c+dx)^{3/2}}{3d^4}\end{aligned}$$

Mathematica [A] time = 0.06, size = 76, normalized size = 0.79

$$\frac{2\left(-9b^2(c+dx)^2(bc-ad)-9b(c+dx)(bc-ad)^2+(bc-ad)^3+b^3(c+dx)^3\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*((b*c - a*d)^3 - 9*b*(b*c - a*d)^2*(c + d*x) - 9*b^2*(b*c - a*d)*(c + d*
x)^2 + b^3*(c + d*x)^3))/(3*d^4*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.07, size = 130, normalized size = 1.35

$$\frac{2\left(-a^3d^3-9a^2bd^2(c+dx)+3a^2bcd^2-3ab^2c^2d+9ab^2d(c+dx)^2+18ab^2cd(c+dx)+b^3c^3-9b^3c^2(c+dx)+b^3(c+dx)^3-9b^3c(c+dx)^2\right)}{3d^4(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^3/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*b^3*c^2*(c + d*x)
 + 18*a*b^2*c*d*(c + d*x) - 9*a^2*b*d^2*(c + d*x) - 9*b^3*c*(c + d*x)^2 + 9
*a*b^2*d*(c + d*x)^2 + b^3*(c + d*x)^3))/(3*d^4*(c + d*x)^(3/2))
```

fricas [A] time = 1.18, size = 136, normalized size = 1.42

$$\frac{2\left(b^3d^3x^3 - 16\,b^3c^3 + 24\,ab^2c^2d - 6\,a^2bcd^2 - a^3d^3 - 3\left(2\,b^3cd^2 - 3\,ab^2d^3\right)x^2 - 3\left(8\,b^3c^2d - 12\,ab^2cd^2 + 3\,a^2bd^3\right)x\right)\sqrt{dx+c}}{3\left(d^6x^2 + 2\,cd^5x + c^2d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b^3*d^3*x^3 - 16*b^3*c^3 + 24*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a^3*d^3 -
3*(2*b^3*c*d^2 - 3*a*b^2*d^3)*x^2 - 3*(8*b^3*c^2*d - 12*a*b^2*c*d^2 + 3*a^2
*b*d^3)*x)*sqrt(d*x + c)/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)
```

giac [A] time = 1.00, size = 141, normalized size = 1.47

$$-\frac{2\left(9\,(dx+c)b^3c^2 - b^3c^3 - 18\,(dx+c)ab^2cd + 3\,ab^2c^2d + 9\,(dx+c)a^2bd^2 - 3\,a^2bcd^2 + a^3d^3\right)}{3\,(dx+c)^{\frac{3}{2}}d^4} + \frac{2\left((dx+c)^{\frac{3}{2}}b^3d^8 - 9\,\sqrt{dx+c}\,b^3cd^8 + 9\,\sqrt{dx+c}\,ab^2d^9\right)}{3\,d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(9*(d*x + c)*b^3*c^2 - b^3*c^3 - 18*(d*x + c)*a*b^2*c*d + 3*a*b^2*c^2*
d + 9*(d*x + c)*a^2*b*d^2 - 3*a^2*b*c*d^2 + a^3*d^3)/((d*x + c)^(3/2)*d^4)
+ 2/3*((d*x + c)^(3/2)*b^3*d^8 - 9*sqrt(d*x + c)*b^3*c*d^8 + 9*sqrt(d*x + c
)*a*b^2*d^9)/d^12
```

maple [A] time = 0.01, size = 115, normalized size = 1.20

$$-\frac{2\left(-b^3x^3d^3 - 9a\,b^2d^3x^2 + 6b^3c\,d^2x^2 + 9a^2b\,d^3x - 36a\,b^2c\,d^2x + 24b^3c^2dx + a^3d^3 + 6a^2bc\,d^2 - 24a\,b^2c^2d + 16b^3c^3\right)}{3\,(dx+c)^{\frac{3}{2}}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-b^3*d^3*x^3-9*a*b^2*d^3*x^2+6*b^3*c*d^2*x^2+9*a^2*b*d^
3*x-36*a*b^2*c*d^2*x+24*b^3*c^2*d*x+a^3*d^3+6*a^2*b*c*d^2-24*a*b^2*c^2*d+16
*b^3*c^3)/d^4
```

maxima [A] time = 1.37, size = 122, normalized size = 1.27

$$\frac{2\left(\frac{(dx+c)^{\frac{3}{2}}b^3 - 9\left(b^3c - ab^2d\right)\sqrt{dx+c}}{d^3} + \frac{b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3 - 9\left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^3}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(((d*x + c)^(3/2)*b^3 - 9*(b^3*c - a*b^2*d)*sqrt(d*x + c))/d^3 + (b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - 9*(b^3*c^2 - 2*a*b^2*c*d + a
^2*b*d^2)*(d*x + c))/((d*x + c)^(3/2)*d^3))/d
```

mupad [B] time = 0.09, size = 128, normalized size = 1.33

$$\frac{2\,b^3\,(c+d\,x)^3 - 2\,a^3\,d^3 + 2\,b^3\,c^3 - 18\,b^3\,c\,(c+d\,x)^2 - 18\,b^3\,c^2\,(c+d\,x) + 18\,a\,b^2\,d\,(c+d\,x)^2 - 18\,a^2\,b\,d^2\,(c+d\,x) - 6\,a\,b^2\,c^2\,d + 6\,a^2\,b\,c\,d^2 + 36\,a\,b^2\,c\,d\,(c+d\,x)}{3\,d^4\,(c+d\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] (2*b^3*(c + d*x)^3 - 2*a^3*d^3 + 2*b^3*c^3 - 18*b^3*c*(c + d*x)^2 - 18*b^3*
c^2*(c + d*x) + 18*a*b^2*d*(c + d*x)^2 - 18*a^2*b*d^2*(c + d*x) - 6*a*b^2*c
^2*d + 6*a^2*b*c*d^2 + 36*a*b^2*c*d*(c + d*x))/(3*d^4*(c + d*x)^(3/2))
```

sympy [A] time = 1.44, size = 461, normalized size = 4.80

$$\begin{cases} -\frac{2a^3d^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{12a^2bcd^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{18a^2bd^3x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{48ab^2c^2d}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{72ab^2cd^2x}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{18ab^2d^3x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{32b^3c^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{48b^3c^2dx}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} - \frac{12b^3cd^2x^2}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} + \frac{2b^3d^3x^3}{3cd^4\sqrt{c+dx}+3d^5x\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{a^3x+\frac{3a^2bx^2}{2}+ab^2x^3+\frac{b^3x^4}{4}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**3*d**3/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) -
 12*a**2*b*c*d**2/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 18*a*
*2*b*d**3*x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 48*a*b**2*c
**2*d/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 72*a*b**2*c*d**2*
x/(3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) + 18*a*b**2*d**3*x**2/(
3*c*d**4*sqrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 32*b**3*c**3/(3*c*d**4*s
qrt(c + d*x) + 3*d**5*x*sqrt(c + d*x)) - 48*b**3*c**2*d*x/(3*c*d**4*sqrt(c
+ d*x) + 3*d**5*x*sqrt(c + d*x)) - 12*b**3*c*d**2*x**2/(3*c*d**4*sqrt(c + d
*x) + 3*d**5*x*sqrt(c + d*x)) + 2*b**3*d**3*x**3/(3*c*d**4*sqrt(c + d*x) +
3*d**5*x*sqrt(c + d*x)), Ne(d, 0)), ((a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**
3 + b**3*x**4/4)/c**(5/2), True))
```

3.1331 $\int \frac{(a+bx)^2}{(c+dx)^{5/2}} dx$

Optimal. Leaf size=67

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}} - \frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}} + \frac{2b^2\sqrt{c+dx}}{d^3}$$

Rubi [A] time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {43}

$$\frac{4b(bc-ad)}{d^3\sqrt{c+dx}} - \frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}} + \frac{2b^2\sqrt{c+dx}}{d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(b*c - a*d)^2)/(3*d^3*(c + d*x)^(3/2)) + (4*b*(b*c - a*d))/(d^3*Sqrt[c
+ d*x]) + (2*b^2*Sqrt[c + d*x])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^2}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{(-bc+ad)^2}{d^2(c+dx)^{5/2}} - \frac{2b(bc-ad)}{d^2(c+dx)^{3/2}} + \frac{b^2}{d^2\sqrt{c+dx}}\right) dx\\ &= -\frac{2(bc-ad)^2}{3d^3(c+dx)^{3/2}} + \frac{4b(bc-ad)}{d^3\sqrt{c+dx}} + \frac{2b^2\sqrt{c+dx}}{d^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 62, normalized size = 0.93

$$\frac{-2a^2d^2 - 4abd(2c+3dx) + 2b^2\left(8c^2+12cdx+3d^2x^2\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*a^2*d^2 - 4*a*b*d*(2*c + 3*d*x) + 2*b^2*(8*c^2 + 12*c*d*x + 3*d^2*x^2))
/(3*d^3*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.05, size = 72, normalized size = 1.07

$$\frac{2\left(-a^2d^2 - 6abd(c+dx) + 2abcd + b^2\left(-c^2\right) + 3b^2(c+dx)^2 + 6b^2c(c+dx)\right)}{3d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^2/(c + d*x)^(5/2),x]
```

```
[Out] (2*(-(b^2*c^2) + 2*a*b*c*d - a^2*d^2 + 6*b^2*c*(c + d*x) - 6*a*b*d*(c + d*x
) + 3*b^2*(c + d*x)^2))/(3*d^3*(c + d*x)^(3/2))
```

fricas [A] time = 1.24, size = 85, normalized size = 1.27

$$\frac{2\left(3\,b^2d^2x^2+8\,b^2c^2-4\,abcd-a^2d^2+6\left(2\,b^2cd-abd^2\right)x\right)\sqrt{dx+c}}{3\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(3*b^2*d^2*x^2 + 8*b^2*c^2 - 4*a*b*c*d - a^2*d^2 + 6*(2*b^2*c*d - a*b*d
^2)*x)*sqrt(d*x + c)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [A] time = 1.11, size = 72, normalized size = 1.07

$$\frac{2\,\sqrt{dx+c}\,b^2}{d^3}+\frac{2\left(6\,(dx+c)b^2c-b^2c^2-6\,(dx+c)abd+2\,abcd-a^2d^2\right)}{3\,(dx+c)^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(d*x + c)*b^2/d^3 + 2/3*(6*(d*x + c)*b^2*c - b^2*c^2 - 6*(d*x + c)*a*
b*d + 2*a*b*c*d - a^2*d^2)/((d*x + c)^(3/2)*d^3)
```

maple [A] time = 0.01, size = 62, normalized size = 0.93

$$-\frac{2\left(-3b^2x^2d^2+6ab\,d^2x-12b^2cdx+a^2d^2+4abcd-8b^2c^2\right)}{3\,(dx+c)^{\frac{3}{2}}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(-3*b^2*d^2*x^2+6*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2+4*a*b*c
*d-8*b^2*c^2)/d^3
```

maxima [A] time = 1.39, size = 72, normalized size = 1.07

$$\frac{2\left(\frac{3\,\sqrt{dx+c}\,b^2}{d^2}-\frac{b^2c^2-2\,abcd+a^2d^2-6\left(b^2c-abd\right)(dx+c)}{(dx+c)^{\frac{3}{2}}d^2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(d*x + c)*b^2/d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 6*(b^2*c -
a*b*d)*(d*x + c))/((d*x + c)^(3/2)*d^2))/d
```

mupad [B] time = 0.07, size = 68, normalized size = 1.01

$$\frac{6\,b^2\,{\left(c+d\,x\right)}^2-2\,a^2\,d^2-2\,b^2\,c^2+12\,b^2\,c\,\left(c+d\,x\right)-12\,a\,b\,d\,\left(c+d\,x\right)+4\,a\,b\,c\,d}{3\,d^3\,{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2/(c + d*x)^(5/2),x)
```

```
[Out] (6*b^2*(c + d*x)^2 - 2*a^2*d^2 - 2*b^2*c^2 + 12*b^2*c*(c + d*x) - 12*a*b*d*
(c + d*x) + 4*a*b*c*d)/(3*d^3*(c + d*x)^(3/2))
```

sympy [A] time = 1.27, size = 265, normalized size = 3.96

$$\begin{cases} -\frac{2a^2d^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} - \frac{8abcd}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} - \frac{12abd^2x}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{16b^2c^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{24b^2cdx}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} + \frac{6b^2d^2x^2}{3cd^3\sqrt{c+dx}+3d^4x\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{a^2x+abx^2+\frac{b^2x^3}{3}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a**2*d**2/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) -
 8*a*b*c*d/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) - 12*a*b*d**2*
x/(3*c*d**3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 16*b**2*c**2/(3*c*d**
3*sqrt(c + d*x) + 3*d**4*x*sqrt(c + d*x)) + 24*b**2*c*d*x/(3*c*d**3*sqrt(c
+ d*x) + 3*d**4*x*sqrt(c + d*x)) + 6*b**2*d**2*x**2/(3*c*d**3*sqrt(c + d*x)
 + 3*d**4*x*sqrt(c + d*x)), Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)/c
**(5/2), True))
```

3.1332 $\int \frac{a+bx}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Rubi [A] time = 0.01, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(b*c - a*d))/(3*d^2*(c + d*x)^(3/2)) - (2*b)/(d^2*Sqrt[c + d*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)^{5/2}}\,dx &= \int \left(\frac{-bc+ad}{d(c+dx)^{5/2}}+\frac{b}{d(c+dx)^{3/2}}\right)dx\\ &= \frac{2(bc-ad)}{3d^2(c+dx)^{3/2}}-\frac{2b}{d^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.72

$$-\frac{2(ad+2bc+3bdx)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(2*b*c + a*d + 3*b*d*x))/(3*d^2*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 0.80

$$-\frac{2(ad+3b(c+dx)-bc)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(-(b*c) + a*d + 3*b*(c + d*x)))/(3*d^2*(c + d*x)^(3/2))
```

fricas [A] time = 1.11, size = 46, normalized size = 1.15

$$-\frac{2\,(3\,bdx+2\,bc+ad)\sqrt{dx+c}}{3\,\left(d^4x^2+2\,cd^3x+c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(3*b*d*x + 2*b*c + a*d)*sqrt(d*x + c)/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

giac [A] time = 1.02, size = 28, normalized size = 0.70

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

maple [A] time = 0.00, size = 26, normalized size = 0.65

$$-\frac{2\,(3bdx+ad+2bc)}{3\,(dx+c)^{\frac{3}{2}}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(d*x+c)^(3/2)*(3*b*d*x+a*d+2*b*c)/d^2
```

maxima [A] time = 1.28, size = 28, normalized size = 0.70

$$-\frac{2\,(3\,(dx+c)b-bc+ad)}{3\,(dx+c)^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(d*x + c)*b - b*c + a*d)/((d*x + c)^(3/2)*d^2)
```

mupad [B] time = 0.25, size = 29, normalized size = 0.72

$$-\frac{2\,a\,d-2\,b\,c+6\,b\,(c+d\,x)}{3\,d^2\,{(c+d\,x)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/(c + d*x)^(5/2),x)
```

```
[Out] -(2*a*d - 2*b*c + 6*b*(c + d*x))/(3*d^2*(c + d*x)^(3/2))
```

sympy [A] time = 1.12, size = 124, normalized size = 3.10

$$\begin{cases} -\frac{2ad}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} - \frac{4bc}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} - \frac{6bdx}{3cd^2\sqrt{c+dx}+3d^3x\sqrt{c+dx}} & \text{for } d \neq 0 \\ \frac{ax+\frac{bx^2}{2}}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Piecewise((-2*a*d/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 4*b*c
/(3*c*d**2*sqrt(c + d*x) + 3*d**3*x*sqrt(c + d*x)) - 6*b*d*x/(3*c*d**2*sqrt
(c + d*x) + 3*d**3*x*sqrt(c + d*x)), Ne(d, 0)), ((a*x + b*x**2/2)/c**(5/2),
 True))
```

3.1333 $\int \frac{1}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=16

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(c+dx)^{5/2}}\,dx = -\frac{2}{3d(c+dx)^{3/2}}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(-5/2),x]
```

```
[Out] -2/(3*d*(c + d*x)^(3/2))
```

fricas [B] time = 1.32, size = 31, normalized size = 1.94

$$-\frac{2\sqrt{dx+c}}{3\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(d*x + c)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

giac [A] time = 0.96, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="giac")
[Out] -2/3/((d*x + c)^(3/2)*d)
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^(5/2),x)
[Out] -2/3/d/(d*x+c)^(3/2)
```

maxima [A] time = 1.36, size = 12, normalized size = 0.75

$$-\frac{2}{3\,(dx+c)^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^(5/2),x, algorithm="maxima")
[Out] -2/3/((d*x + c)^(3/2)*d)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.75

$$-\frac{2}{3\,d\,(c+d\,x)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c + d*x)^(5/2),x)
[Out] -2/(3*d*(c + d*x)^(3/2))
```

sympy [A] time = 0.07, size = 14, normalized size = 0.88

$$-\frac{2}{3d\,(c+dx)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**(5/2),x)
[Out] -2/(3*d*(c + d*x)**(3/2))
```

3.1334 $\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=93

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}+\frac{2b}{\sqrt{c+dx}\,(bc-ad)^2}+\frac{2}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] 2/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (2*b)/((b*c - a*d)^2*Sqrt[c + d*x]) - (
2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/
2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx = \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{b\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{bc-ad}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} + \frac{b^2\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{(bc-ad)^2}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} + \frac{(2b^2)\,\mathrm{Subst}\left(\int \frac{1}{a-\frac{bc}{d}+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d(bc-ad)^2}$$
$$= \frac{2}{3(bc-ad)(c+dx)^{3/2}} + \frac{2b}{(bc-ad)^2\sqrt{c+dx}} - \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.52

$$\frac{2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b(c+dx)}{bc-ad}\right)}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, (b*(c + d*x))/(b*c - a*d)])/(3*(b*c - a
*d)*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.13, size = 97, normalized size = 1.04

$$\frac{2(-ad+3b(c+dx)+bc)}{3(c+dx)^{3/2}(bc-ad)^2} - \frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(b*c - a*d + 3*b*(c + d*x)))/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - (2*b^(3
/2)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(-(b*c)
 + a*d)^(5/2)
```

fricas [B] time = 1.44, size = 398, normalized size = 4.28

$$\left[\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{\frac{b}{bc-ad}}\log\left(\frac{bdx+2\,bc-ad-2\,(bc-ad)\sqrt{dx+c}\sqrt{\frac{b}{bc-ad}}}{bx+a}\right)+2\,(3\,bdx+4\,bc-ad)\sqrt{dx+c}}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}, -\frac{2\left(3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-\frac{b}{bc-ad}}\arctan\left(-\frac{(bc-ad)\sqrt{dx+c}\sqrt{-\frac{b}{bc-ad}}}{bdx+bc}\right)-(3\,bdx+4\,bc-ad)\sqrt{dx+c}\right)}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*
b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2
*(3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^
2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^
2 + a^2*c*d^3)*x), -2/3*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/(b*c - a
*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c))
- (3*b*d*x + 4*b*c - a*d)*sqrt(d*x + c))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d
^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d
^2 + a^2*c*d^3)*x)]
```

giac [A] time = 1.16, size = 113, normalized size = 1.22

$$\frac{2\,b^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-b^2c+abd}}+\frac{2\,(3\,(dx+c)b+bc-ad)}{3\,\left(b^2c^2-2\,abcd+a^2d^2\right)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2*b^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*sqrt(-b^2*c + a*b*d)) + 2/3*(3*(d*x + c)*b + b*c - a*d)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(d*x + c)^(3/2))
```

maple [A] time = 0.01, size = 90, normalized size = 0.97

$$\frac{2b^2\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^2\sqrt{(ad-bc)\,b}}+\frac{2b}{(ad-bc)^2\sqrt{dx+c}}-\frac{2}{3\,(ad-bc)\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3/(a*d-b*c)/(d*x+c)^(3/2)+2*b/(a*d-b*c)^2/(d*x+c)^(1/2)+2*b^2/(a*d-b*c)^
2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.33, size = 100, normalized size = 1.08

$$\frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^{5/2}}\right)}{{\left(a\,d-b\,c\right)}^{5/2}}-\frac{\frac{2}{3\,\left(a\,d-b\,c\right)}-\frac{2\,b\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(5/2)),x)
```

```
[Out] (2*b^(3/2)*atan((b^(1/2)*(c + d*x)^(1/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(
a*d - b*c)^(5/2)))/(a*d - b*c)^(5/2) - (2/(3*(a*d - b*c)) - (2*b*(c + d*x))
/(a*d - b*c)^2)/(c + d*x)^(3/2)
```

sympy [A] time = 13.58, size = 83, normalized size = 0.89

$$\frac{2b}{\sqrt{c+dx}\,(ad-bc)^2}+\frac{2b\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{\sqrt{\frac{ad-bc}{b}}\,(ad-bc)^2}-\frac{2}{3\,(c+dx)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] 2*b/(sqrt(c + d*x)*(a*d - b*c)**2) + 2*b*atan(sqrt(c + d*x)/sqrt((a*d - b*c
)/b))/(sqrt((a*d - b*c)/b)*(a*d - b*c)**2) - 2/(3*(c + d*x)**(3/2)*(a*d - b
*c))
```

3.1335 $\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=124

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Rubi [A] time = 0.05, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = $\{51, 63, 208\}$

$$\frac{5b^{3/2}d\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{7/2}}-\frac{5bd}{\sqrt{c+dx}\,(bc-ad)^3}-\frac{1}{(a+bx)(c+dx)^{3/2}(bc-ad)}-\frac{5d}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-5*d)/(3*(b*c - a*d)^2*(c + d*x)^(3/2)) - 1/((b*c - a*d)*(a + b*x)*(c + d*
x)^(3/2)) - (5*b*d)/((b*c - a*d)^3*Sqrt[c + d*x]) + (5*b^(3/2)*d*ArcTanh[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(7/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx = -\frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5d)\int \frac{1}{(a+bx)(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{(5bd)\int \frac{1}{(a+bx)(c+dx)^{3/2}}\,dx}{2(bc-ad)^2}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5}{}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} - \frac{(5}{}$$

$$= -\frac{5d}{3(bc-ad)^2(c+dx)^{3/2}} - \frac{1}{(bc-ad)(a+bx)(c+dx)^{3/2}} - \frac{5bd}{(bc-ad)^3\sqrt{c+dx}} + \frac{5b}{}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.40

$$-\frac{2d\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d*Hypergeometric2F1[-3/2, 2, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/(3
*(-(b*c) + a*d)^2*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.36, size = 157, normalized size = 1.27

$$-\frac{d\left(2a^2d^2-10abd(c+dx)-4abcd+2b^2c^2-15b^2(c+dx)^2+10b^2c(c+dx)\right)}{3(c+dx)^{3/2}(bc-ad)^3(-ad-b(c+dx)+bc)} - \frac{5b^{3/2}d\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{(ad-bc)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^2*(c + d*x)^(5/2)),x]
```

```
[Out] -1/3*(d*(2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + 10*b^2*c*(c + d*x) - 10*a*b*d*
(c + d*x) - 15*b^2*(c + d*x)^2))/((b*c - a*d)^3*(c + d*x)^(3/2)*(b*c - a*d
- b*(c + d*x))) - (5*b^(3/2)*d*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c +
d*x])/(b*c - a*d)])/(-(b*c) + a*d)^(7/2)
```

fricas [B] time = 1.42, size = 782, normalized size = 6.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2
*d + 2*a*b*c*d^2)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c
- a*d)*sqrt(d*x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(15*b^2*d^2*x^2 +
3*b^2*c^2 + 14*a*b*c*d - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x +
 c))/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^
3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d -
```

```
5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^
5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x), 1/
3*(15*(b^2*d^3*x^3 + a*b*c^2*d + (2*b^2*c*d^2 + a*b*d^3)*x^2 + (b^2*c^2*d +
 2*a*b*c*d^2)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt(d*x + c)*sqr
t(-b/(b*c - a*d))/(b*d*x + b*c)) - (15*b^2*d^2*x^2 + 3*b^2*c^2 + 14*a*b*c*d
 - 2*a^2*d^2 + 10*(2*b^2*c*d + a*b*d^2)*x)*sqrt(d*x + c))/(a*b^3*c^5 - 3*a^
2*b^2*c^4*d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^
3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a
^2*b^2*c^2*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^
2*b^2*c^3*d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)]
```

giac [B] time = 1.11, size = 216, normalized size = 1.74

$$-\frac{5\,b^{2}d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^{2}c+abd}}\right)}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\sqrt{-b^{2}c+abd}}-\frac{\sqrt{dx+c}\,b^{2}d}{\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\left((dx+c)b-bc+ad\right)}-\frac{2\left(6\,(dx+c)bd+bcd-ad^{2}\right)}{3\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -5*b^2*d*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b^2*c + a*b*d)) - sqrt(d*x + c)*b^2*d
/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*((d*x + c)*b - b*c +
a*d)) - 2/3*(6*(d*x + c)*b*d + b*c*d - a*d^2)/((b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*(d*x + c)^(3/2))
```

maple [A] time = 0.02, size = 125, normalized size = 1.01

$$\frac{5b^{2}d\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{(ad-bc)^{3}\sqrt{(ad-bc)\,b}}+\frac{\sqrt{dx+c}\,b^{2}d}{(ad-bc)^{3}\,(bdx+ad)}+\frac{4bd}{(ad-bc)^{3}\sqrt{dx+c}}-\frac{2d}{3\,(ad-bc)^{2}\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d/(a*d-b*c)^2/(d*x+c)^(3/2)+4*d/(a*d-b*c)^3*b/(d*x+c)^(1/2)+d*b^2/(a*d
-b*c)^3*(d*x+c)^(1/2)/(b*d*x+a*d)+5*d*b^2/(a*d-b*c)^3/((a*d-b*c)*b)^(1/2)*a
rctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.38, size = 161, normalized size = 1.30

$$\frac{\frac{10\,b\,d\,(c+d\,x)}{3\,(a\,d-b\,c)^{2}}-\frac{2\,d}{3\,(a\,d-b\,c)}+\frac{5\,b^{2}\,d\,(c+d\,x)^{2}}{(a\,d-b\,c)^{3}}}{b\,(c+d\,x)^{5/2}+(a\,d-b\,c)\,(c+d\,x)^{3/2}}+\frac{5\,b^{3/2}\,d\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^{3}\,d^{3}-3\,a^{2}\,b\,c\,d^{2}+3\,a\,b^{2}\,c^{2}\,d-b^{3}\,c^{3}\right)}{(a\,d-b\,c)^{7/2}}\right)}{(a\,d-b\,c)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)^(5/2)),x)
```

```
[Out] ((10*b*d*(c + d*x))/(3*(a*d - b*c)^2) - (2*d)/(3*(a*d - b*c)) + (5*b^2*d*(c
 + d*x)^2)/(a*d - b*c)^3)/(b*(c + d*x)^(5/2) + (a*d - b*c)*(c + d*x)^(3/2))
 + (5*b^(3/2)*d*atan((b^(1/2)*(c + d*x)^(1/2)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*
c^2*d - 3*a^2*b*c*d^2))/(a*d - b*c)^(7/2)))/(a*d - b*c)^(7/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^2 (c+dx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**2*(c + d*x)**(5/2)), x)
```

3.1336 $\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{}{2(a}$$

Rubi [A] time = 0.06, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {51, 63, 208}

$$-\frac{35b^{3/2}d^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4(bc-ad)^{9/2}}+\frac{35bd^2}{4\sqrt{c+dx}\,(bc-ad)^4}+\frac{35d^2}{12(c+dx)^{3/2}(bc-ad)^3}+\frac{7d}{4(a+bx)(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{2(a+bx)^2(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (35*d^2)/(12*(b*c - a*d)^3*(c + d*x)^(3/2)) - 1/(2*(b*c - a*d)*(a + b*x)^2*
(c + d*x)^(3/2)) + (7*d)/(4*(b*c - a*d)^2*(a + b*x)*(c + d*x)^(3/2)) + (35*
b*d^2)/(4*(b*c - a*d)^4*Sqrt[c + d*x]) - (35*b^(3/2)*d^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/Sqrt[b*c - a*d]])/(4*(b*c - a*d)^(9/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx = -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} - \frac{(7d)\int \frac{1}{(a+bx)^2(c+dx)^{5/2}}\,dx}{4(bc-ad)}$$

$$= -\frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c+dx)^{3/2}} + \frac{\left(35d^2\right)\int \frac{1}{(a+bx)\ldots}}{8(bc-\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c\ldots}$$

$$= \frac{35d^2}{12(bc-ad)^3(c+dx)^{3/2}} - \frac{1}{2(bc-ad)(a+bx)^2(c+dx)^{3/2}} + \frac{7d}{4(bc-ad)^2(a+bx)(c\ldots}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.31

$$-\frac{2d^2\, {}_2F_1\left(-\frac{3}{2},3;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^2*Hypergeometric2F1[-3/2, 3, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^3*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.60, size = 223, normalized size = 1.34

$$-\frac{d^2\left(8a^3d^3-56a^2bd^2(c+dx)-24a^2bcd^2+24ab^2c^2d-175ab^2d(c+dx)^2+112ab^2cd(c+dx)-8b^3c^3-56b^3c^2(c+dx)-105b^3(c+dx)^3+175b^3c(c+dx)^2\right)}{12(c+dx)^{3/2}(bc-ad)^4(-ad-b(c+dx)+bc)^2} - \frac{35b^{3/2}d^2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4(ad-bc)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^3*(c + d*x)^(5/2)),x]
```

```
[Out] -1/12*(d^2*(-8*b^3*c^3 + 24*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 8*a^3*d^3 - 56*b
^3*c^2*(c + d*x) + 112*a*b^2*c*d*(c + d*x) - 56*a^2*b*d^2*(c + d*x) + 175*b
^3*c*(c + d*x)^2 - 175*a*b^2*d*(c + d*x)^2 - 105*b^3*(c + d*x)^3))/((b*c -
a*d)^4*(c + d*x)^(3/2)*(b*c - a*d - b*(c + d*x))^2) - (35*b^(3/2)*d^2*ArcTa
n[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(4*(-(b*c) + a*d
)^(9/2))
```

fricas [B] time = 1.36, size = 1226, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d^3 + a*b^2*d^4)*x^3 + (
b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*b^2*c^2*d^2 + a^2*b*c*d
^3)*x)*sqrt(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*
x + c)*sqrt(b/(b*c - a*d)))/(b*x + a)) + 2*(105*b^3*d^3*x^3 - 6*b^3*c^3 + 3
```

```
9*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2*d^3)
*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c))/(a^
2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2
*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5
 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 +
2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4
*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5
*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^
2*d^4 + a^6*c*d^5)*x), -1/12*(105*(b^3*d^4*x^4 + a^2*b*c^2*d^2 + 2*(b^3*c*d
^3 + a*b^2*d^4)*x^3 + (b^3*c^2*d^2 + 4*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (105*b^3*d^3*x^3 - 6*b^3*c^
3 + 39*a*b^2*c^2*d + 80*a^2*b*c*d^2 - 8*a^3*d^3 + 35*(4*b^3*c*d^2 + 5*a*b^2
*d^3)*x^2 + 7*(3*b^3*c^2*d + 34*a*b^2*c*d^2 + 8*a^2*b*d^3)*x)*sqrt(d*x + c)
)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^
6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*
c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4*c^3*d
^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*x^3 + (b^6*c^6 - 9*a^
2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^6*d^6)*x^2 + 2*(
a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 + 2*a^4*b^2*c^3*d^3 - 3*a^5
*b*c^2*d^4 + a^6*c*d^5)*x)]
```

giac [B] time = 1.20, size = 298, normalized size = 1.78

$$\frac{35 \, b^{2} d^{2} \arctan\left(\frac{\sqrt{dx+c}\, b}{\sqrt{-b^{2} c+abd}}\right)}{4 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} \sqrt{-b^{2} c + abd}} + \frac{2 \, {\left(9 \, {\left(dx + c\right)} b d^{2} + b c d^{2} - a d^{3}\right)}}{3 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} {\left(dx + c\right)}^{\frac{3}{2}}} + \frac{11 \, {\left(dx + c\right)}^{\frac{3}{2}} b^{3} d^{2} - 13 \, \sqrt{dx + c}\, b^{3} c d^{2} + 13 \, \sqrt{dx + c}\, a b^{2} d^{3}}{4 \, {\left(b^{4} c^{4} - 4 \, a b^{3} c^{3} d + 6 \, a^{2} b^{2} c^{2} d^{2} - 4 \, a^{3} b c d^{3} + a^{4} d^{4}\right)} {\left({\left(dx + c\right)} b - b c + a d\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 35/4*b^2*d^2*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^4*c^4 - 4*a*b
^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(-b^2*c + a*b*d
)) + 2/3*(9*(d*x + c)*b*d^2 + b*c*d^2 - a*d^3)/((b^4*c^4 - 4*a*b^3*c^3*d +
6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(d*x + c)^(3/2)) + 1/4*(11*(d*
x + c)^(3/2)*b^3*d^2 - 13*sqrt(d*x + c)*b^3*c*d^2 + 13*sqrt(d*x + c)*a*b^2*
d^3)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)*((d*x + c)*b - b*c + a*d)^2)
```

maple [A] time = 0.02, size = 206, normalized size = 1.23

$$\frac{13\sqrt{dx+c}\; a\, b^{2} d^{3}}{4 \left(ad-bc\right)^{4} \left(bdx+ad\right)^{2}}-\frac{13\sqrt{dx+c}\; b^{3} c\, d^{2}}{4 \left(ad-bc\right)^{4} \left(bdx+ad\right)^{2}}+\frac{11 \left(dx+c\right)^{\frac{3}{2}} b^{3} d^{2}}{4 \left(ad-bc\right)^{4} \left(bdx+ad\right)^{2}}+\frac{35 b^{2} d^{2} \arctan\left(\frac{\sqrt{dx+c}\; b}{\sqrt{\left(ad-bc\right) b}}\right)}{4 \left(ad-bc\right)^{4} \sqrt{\left(ad-bc\right) b}}+\frac{6 b\, d^{2}}{\left(ad-bc\right)^{4} \sqrt{dx+c}}-\frac{2 d^{2}}{3 \left(ad-bc\right)^{3} \left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^2/(a*d-b*c)^3/(d*x+c)^(3/2)+6*d^2/(a*d-b*c)^4*b/(d*x+c)^(1/2)+11/4*d
^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(3/2)+13/4*d^3/(a*d-b*c)^4*b^2/(b*
d*x+a*d)^2*(d*x+c)^(1/2)*a-13/4*d^2/(a*d-b*c)^4*b^3/(b*d*x+a*d)^2*(d*x+c)^(
1/2)*c+35/4*d^2/(a*d-b*c)^4*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((
a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 0.28, size = 243, normalized size = 1.46

$$\frac{\frac{175\,b^2\,d^2\,(c+d\,x)^2}{12\,(a\,d-b\,c)^3}-\frac{2\,d^2}{3\,(a\,d-b\,c)}+\frac{35\,b^3\,d^2\,(c+d\,x)^3}{4\,(a\,d-b\,c)^4}+\frac{14\,b\,d^2\,(c+d\,x)}{3\,(a\,d-b\,c)^2}}{b^2\,(c+d\,x)^{7/2}-\left(2\,b^2\,c-2\,a\,b\,d\right)\,(c+d\,x)^{5/2}+(c+d\,x)^{3/2}\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\frac{35\,b^{3/2}\,d^2\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^4\,d^4-4\,a^3\,b\,c\,d^3+6\,a^2\,b^2\,c^2\,d^2-4\,a\,b^3\,c^3\,d+b^4\,c^4\right)}{(a\,d-b\,c)^{9/2}}\right)}{4\,(a\,d-b\,c)^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^3*(c + d*x)^(5/2)),x)
```

```
[Out] ((175*b^2*d^2*(c + d*x)^2)/(12*(a*d - b*c)^3) - (2*d^2)/(3*(a*d - b*c)) + (
35*b^3*d^2*(c + d*x)^3)/(4*(a*d - b*c)^4) + (14*b*d^2*(c + d*x))/(3*(a*d -
b*c)^2))/(b^2*(c + d*x)^(7/2) - (2*b^2*c - 2*a*b*d)*(c + d*x)^(5/2) + (c +
d*x)^(3/2)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (35*b^(3/2)*d^2*atan((b^(1/2)
*(c + d*x)^(1/2)*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4
*a^3*b*c*d^3))/(a*d - b*c)^(9/2)))/(4*(a*d - b*c)^(9/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1337 $\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{}{4(a+}$$

Rubi [A] time = 0.13, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {51, 63, 208}

$$\frac{105b^{3/2}d^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{8(bc-ad)^{11/2}}-\frac{105bd^3}{8\sqrt{c+dx}\,(bc-ad)^5}-\frac{35d^3}{8(c+dx)^{3/2}(bc-ad)^4}-\frac{21d^2}{8(a+bx)(c+dx)^{3/2}(bc-ad)^3}+\frac{3d}{4(a+bx)^2(c+dx)^{3/2}(bc-ad)^2}-\frac{1}{3(a+bx)^3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-35*d^3)/(8*(b*c - a*d)^4*(c + d*x)^(3/2)) - 1/(3*(b*c - a*d)*(a + b*x)^3*
(c + d*x)^(3/2)) + (3*d)/(4*(b*c - a*d)^2*(a + b*x)^2*(c + d*x)^(3/2)) - (2
1*d^2)/(8*(b*c - a*d)^3*(a + b*x)*(c + d*x)^(3/2)) - (105*b*d^3)/(8*(b*c -
a*d)^5*Sqrt[c + d*x]) + (105*b^(3/2)*d^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sq
rt[b*c - a*d]])/(8*(b*c - a*d)^(11/2))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^4(c+dx)^{5/2}}\,dx = -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} - \frac{(3d)\int \frac{1}{(a+bx)^3(c+dx)^{5/2}}\,dx}{2(bc-ad)}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} + \frac{\left(21d^2\right)\int \frac{}{(a+b}}{8(bc-}$$

$$= -\frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c+dx)^{3/2}} - \frac{}{8(bc-ad)^3(a}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

$$= -\frac{35d^3}{8(bc-ad)^4(c+dx)^{3/2}} - \frac{1}{3(bc-ad)(a+bx)^3(c+dx)^{3/2}} + \frac{3d}{4(bc-ad)^2(a+bx)^2(c}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.26

$$-\frac{2d^3\,{}_2F_1\left(-\frac{3}{2},4;-\frac{1}{2};-\frac{b(c+dx)}{ad-bc}\right)}{3(c+dx)^{3/2}(ad-bc)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*d^3*Hypergeometric2F1[-3/2, 4, -1/2, -((b*(c + d*x))/(-(b*c) + a*d))])/
(3*(-(b*c) + a*d)^4*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.87, size = 304, normalized size = 1.52

$$-\frac{d^3\left(16a^4d^4-144a^3bd^3(c+dx)-64a^3bcd^3+96a^2b^2c^2d^2-693a^2b^2d^2(c+dx)^2+432a^2b^2cd^2(c+dx)-64ab^3c^3d-432ab^3c^2d(c+dx)-840ab^3d(c+dx)^3+1386ab^3cd(c+dx)^2+16b^4c^4+144b^4c^3(c+dx)-693b^4c^2(c+dx)^2-315b^4(c+dx)^4+840b^4c(c+dx)^3\right)}{24(c+dx)^{3/2}(bc-ad)^5(-ad-b(c+dx)+bc)^3}-\frac{105b^{3/2}d^3\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{8(ad-bc)^{11/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^4*(c + d*x)^(5/2)),x]
```

```
[Out] -1/24*(d^3*(16*b^4*c^4 - 64*a*b^3*c^3*d + 96*a^2*b^2*c^2*d^2 - 64*a^3*b*c*d
^3 + 16*a^4*d^4 + 144*b^4*c^3*(c + d*x) - 432*a*b^3*c^2*d*(c + d*x) + 432*a
^2*b^2*c*d^2*(c + d*x) - 144*a^3*b*d^3*(c + d*x) - 693*b^4*c^2*(c + d*x)^2
+ 1386*a*b^3*c*d*(c + d*x)^2 - 693*a^2*b^2*d^2*(c + d*x)^2 + 840*b^4*c*(c +
 d*x)^3 - 840*a*b^3*d*(c + d*x)^3 - 315*b^4*(c + d*x)^4))/((b*c - a*d)^5*(c
 + d*x)^(3/2)*(b*c - a*d - b*(c + d*x))^3) - (105*b^(3/2)*d^3*ArcTan[(Sqrt[
b]*Sqrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(8*(-(b*c) + a*d)^(11/2)
)
```

fricas [B] time = 1.43, size = 1840, normalized size = 9.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 + (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4
+ (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*
a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqr
t(b/(b*c - a*d))*log((b*d*x + 2*b*c - a*d - 2*(b*c - a*d)*sqrt(d*x + c)*sqr
t(b/(b*c - a*d)))/(b*x + a)) + 2*(315*b^4*d^4*x^4 + 8*b^4*c^4 - 50*a*b^3*c^
3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 420*(b^4*c*d^3 +
 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2
- 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^3*b*d^4)*x)*sqr
t(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^
2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3
+ 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*
x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d
^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7
+ a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^
4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9
*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4
+ 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3
*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 +
7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x), 1/24*(315*(b^4*d^5*x^5 + a^3*b*c^2*d^3 +
 (2*b^4*c*d^4 + 3*a*b^3*d^5)*x^4 + (b^4*c^2*d^3 + 6*a*b^3*c*d^4 + 3*a^2*b^2
*d^5)*x^3 + (3*a*b^3*c^2*d^3 + 6*a^2*b^2*c*d^4 + a^3*b*d^5)*x^2 + (3*a^2*b^
2*c^2*d^3 + 2*a^3*b*c*d^4)*x)*sqrt(-b/(b*c - a*d))*arctan(-(b*c - a*d)*sqrt
(d*x + c)*sqrt(-b/(b*c - a*d))/(b*d*x + b*c)) - (315*b^4*d^4*x^4 + 8*b^4*c^
4 - 50*a*b^3*c^3*d + 165*a^2*b^2*c^2*d^2 + 208*a^3*b*c*d^3 - 16*a^4*d^4 + 4
20*(b^4*c*d^3 + 2*a*b^3*d^4)*x^3 + 63*(b^4*c^2*d^2 + 18*a*b^3*c*d^3 + 11*a^
2*b^2*d^4)*x^2 - 18*(b^4*c^3*d - 10*a*b^3*c^2*d^2 - 53*a^2*b^2*c*d^3 - 8*a^
3*b*d^4)*x)*sqrt(d*x + c))/(a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*
d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5
*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6
- a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 1
0*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*
x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25
*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (
3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a
^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b
^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6
*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^6)*x)]
```

giac [B] time = 1.20, size = 432, normalized size = 2.16

$$-\frac{105\,b^2d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{8\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)\sqrt{-b^2c+abd}}-\frac{315\,(dx+c)^4b^4d^3-840\,(dx+c)^3b^4cd^3+693\,(dx+c)^2b^4c^2d^3-144\,(dx+c)b^4c^3d^3-16\,b^4c^4d^3+840\,(dx+c)^3ab^3d^4-1386\,(dx+c)^2ab^3cd^4+432\,(dx+c)ab^3c^2d^4+64\,ab^3c^3d^4+693\,(dx+c)^2a^2b^2d^5-432\,(dx+c)a^2b^2cd^5-96\,a^2b^2c^2d^5+144\,(dx+c)a^3bd^6+64\,a^3bcd^6-16\,a^4d^7}{24\left(b^5c^5-5\,ab^4c^4d+10\,a^2b^3c^3d^2-10\,a^3b^2c^2d^3+5\,a^4bcd^4-a^5d^5\right)\left((dx+c)^{\frac{3}{2}}b-\sqrt{dx+c}\,bc+\sqrt{dx+c}\,ad\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -105/8*b^2*d^3*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((b^5*c^5 - 5*a
*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*
d^5)*sqrt(-b^2*c + a*b*d)) - 1/24*(315*(d*x + c)^4*b^4*d^3 - 840*(d*x + c)^
3*b^4*c*d^3 + 693*(d*x + c)^2*b^4*c^2*d^3 - 144*(d*x + c)*b^4*c^3*d^3 - 16*
b^4*c^4*d^3 + 840*(d*x + c)^3*a*b^3*d^4 - 1386*(d*x + c)^2*a*b^3*c*d^4 + 43
2*(d*x + c)*a*b^3*c^2*d^4 + 64*a*b^3*c^3*d^4 + 693*(d*x + c)^2*a^2*b^2*d^5
- 432*(d*x + c)*a^2*b^2*c*d^5 - 96*a^2*b^2*c^2*d^5 + 144*(d*x + c)*a^3*b*d^
6 + 64*a^3*b*c*d^6 - 16*a^4*d^7)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*((d*x + c)^(3/2)*b - s
qrt(d*x + c)*b*c + sqrt(d*x + c)*a*d)^3)
```

maple [A] time = 0.02, size = 319, normalized size = 1.60

$$\frac{55\sqrt{dx+c}\,a^2b^2d^5}{8\,(ad-bc)^5\,(bdx+ad)^3}-\frac{55\sqrt{dx+c}\,a\,b^3c\,d^4}{4\,(ad-bc)^5\,(bdx+ad)^3}+\frac{55\sqrt{dx+c}\,b^4c^2d^3}{8\,(ad-bc)^5\,(bdx+ad)^3}+\frac{35\,(dx+c)^{\frac{3}{2}}\,a\,b^3d^4}{3\,(ad-bc)^5\,(bdx+ad)^3}-\frac{35\,(dx+c)^{\frac{3}{2}}\,b^4c\,d^3}{3\,(ad-bc)^5\,(bdx+ad)^3}+\frac{41\,(dx+c)^{\frac{5}{2}}\,b^4d^3}{8\,(ad-bc)^5\,(bdx+ad)^3}+\frac{105b^2d^3\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{8\,(ad-bc)^5\,\sqrt{(ad-bc)\,b}}+\frac{8b\,d^3}{(ad-bc)^5\,\sqrt{dx+c}}-\frac{2d^3}{3\,(ad-bc)^4\,(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^4/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*d^3/(a*d-b*c)^4/(d*x+c)^(3/2)+8*d^3/(a*d-b*c)^5*b/(d*x+c)^(1/2)+41/8*d
^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(5/2)+35/3*d^4/(a*d-b*c)^5*b^3/(b*
d*x+a*d)^3*(d*x+c)^(3/2)*a-35/3*d^3/(a*d-b*c)^5*b^4/(b*d*x+a*d)^3*(d*x+c)^(
3/2)*c+55/8*d^5/(a*d-b*c)^5*b^2/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a^2-55/4*d^4/(a
*d-b*c)^5*b^3/(b*d*x+a*d)^3*(d*x+c)^(1/2)*a*c+55/8*d^3/(a*d-b*c)^5*b^4/(b*d
*x+a*d)^3*(d*x+c)^(1/2)*c^2+105/8*d^3/(a*d-b*c)^5*b^2/((a*d-b*c)*b)^(1/2)*a
rctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^4/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.64, size = 334, normalized size = 1.67

$$\frac{\frac{231\,b^2\,d^3\,{\left(c+d\,x\right)}^2}{8\,{\left(a\,d-b\,c\right)}^3}-\frac{2\,d^3}{3\,\left(a\,d-b\,c\right)}+\frac{35\,b^3\,d^3\,{\left(c+d\,x\right)}^3}{{\left(a\,d-b\,c\right)}^4}+\frac{105\,b^4\,d^3\,{\left(c+d\,x\right)}^4}{8\,{\left(a\,d-b\,c\right)}^5}+\frac{6\,b\,d^3\,\left(c+d\,x\right)}{{\left(a\,d-b\,c\right)}^2}}{{\left(c+d\,x\right)}^{3/2}\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)+b^3\,{\left(c+d\,x\right)}^{9/2}-\left(3\,b^3\,c-3\,a\,b^2\,d\right)\,{\left(c+d\,x\right)}^{7/2}+{\left(c+d\,x\right)}^{5/2}\,\left(3\,a^2\,b\,d^2-6\,a\,b^2\,c\,d+3\,b^3\,c^2\right)}+\frac{105\,b^{3/2}\,d^3\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c+d\,x}\,\left(a^5\,d^5-5\,a^4\,b\,c\,d^4+10\,a^3\,b^2\,c^2\,d^3-10\,a^2\,b^3\,c^3\,d^2+5\,a\,b^4\,c^4\,d-b^5\,c^5\right)}{{\left(a\,d-b\,c\right)}^{11/2}}\right)}{8\,{\left(a\,d-b\,c\right)}^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^4*(c + d*x)^(5/2)),x)
```

```
[Out] ((231*b^2*d^3*(c + d*x)^2)/(8*(a*d - b*c)^3) - (2*d^3)/(3*(a*d - b*c)) + (3
5*b^3*d^3*(c + d*x)^3)/(a*d - b*c)^4 + (105*b^4*d^3*(c + d*x)^4)/(8*(a*d -
b*c)^5) + (6*b*d^3*(c + d*x))/(a*d - b*c)^2)/((c + d*x)^(3/2)*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2) + b^3*(c + d*x)^(9/2) - (3*b^3*c - 3
*a*b^2*d)*(c + d*x)^(7/2) + (c + d*x)^(5/2)*(3*b^3*c^2 + 3*a^2*b*d^2 - 6*a*
b^2*c*d)) + (105*b^(3/2)*d^3*atan((b^(1/2)*(c + d*x)^(1/2)*(a^5*d^5 - b^5*c
^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^
4))/(a*d - b*c)^(11/2)))/(8*(a*d - b*c)^(11/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**4/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1338 $\int (a+bx)^5(ac+bcx)^{3/2}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5(ac+bcx)^{3/2}\,dx &= \frac{\int (ac+bcx)^{13/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{15/2}}{15bc^6}\end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6(c(a+bx))^{3/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6*(c*(a + b*x))^(3/2))/(15*b)
```

IntegrateAlgebraic [A] time = 0.05, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{15/2}}{15bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(15/2))/(15*b*c^6)
```

fricas [B] time = 1.14, size = 95, normalized size = 4.32

$$\frac{2\left(b^{7}cx^{7}+7\,ab^{6}cx^{6}+21\,a^{2}b^{5}cx^{5}+35\,a^{3}b^{4}cx^{4}+35\,a^{4}b^{3}cx^{3}+21\,a^{5}b^{2}cx^{2}+7\,a^{6}bcx+a^{7}c\right)\sqrt{bcx+ac}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*(b^7*c*x^7 + 7*a*b^6*c*x^6 + 21*a^2*b^5*c*x^5 + 35*a^3*b^4*c*x^4 + 35*
a^4*b^3*c*x^3 + 21*a^5*b^2*c*x^2 + 7*a^6*b*c*x + a^7*c)*sqrt(b*c*x + a*c)/b
```

giac [B] time = 1.26, size = 637, normalized size = 28.95

$$\frac{2\left(6435\sqrt{bcx+ac}\,a^{7}c-15015\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^{6}+\frac{9009\left(15\sqrt{bcx+ac}\,a^{2}c^{2}-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^{5}}{c}-\frac{6435\left(35\sqrt{bcx+ac}\,a^{3}c^{3}-35\,(bcx+ac)^{\frac{3}{2}}a^{2}c^{2}+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^{4}}{c^{2}}+\frac{715\left(315\sqrt{bcx+ac}\,a^{4}c^{4}-420\,(bcx+ac)^{\frac{3}{2}}a^{3}c^{3}+378\,(bcx+ac)^{\frac{5}{2}}a^{2}c^{2}-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a^{3}}{c^{3}}-\frac{195\left(693\sqrt{bcx+ac}\,a^{5}c^{5}-1155\,(bcx+ac)^{\frac{3}{2}}a^{4}c^{4}+1386\,(bcx+ac)^{\frac{5}{2}}a^{3}c^{3}-990\,(bcx+ac)^{\frac{7}{2}}a^{2}c^{2}+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}\right)a^{2}}{c^{4}}+\frac{15\left(3003\sqrt{bcx+ac}\,a^{6}c^{6}-6006\,(bcx+ac)^{\frac{3}{2}}a^{5}c^{5}+9009\,(bcx+ac)^{\frac{5}{2}}a^{4}c^{4}-8580\,(bcx+ac)^{\frac{7}{2}}a^{3}c^{3}+5005\,(bcx+ac)^{\frac{9}{2}}a^{2}c^{2}-1638\,(bcx+ac)^{\frac{11}{2}}ac+231\,(bcx+ac)^{\frac{13}{2}}\right)a}{c^{5}}-\frac{6435\sqrt{bcx+ac}\,a^{7}c^{7}-15015\,(bcx+ac)^{\frac{3}{2}}a^{6}c^{6}+27027\,(bcx+ac)^{\frac{5}{2}}a^{5}c^{5}-32175\,(bcx+ac)^{\frac{7}{2}}a^{4}c^{4}+25025\,(bcx+ac)^{\frac{9}{2}}a^{3}c^{3}-12285\,(bcx+ac)^{\frac{11}{2}}a^{2}c^{2}+3465\,(bcx+ac)^{\frac{13}{2}}ac-429\,(bcx+ac)^{\frac{15}{2}}}{c^{6}}\right)}{6435\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="giac")
```

```
[Out] 2/6435*(6435*sqrt(b*c*x + a*c)*a^7*c - 15015*(3*sqrt(b*c*x + a*c)*a*c - (b*
c*x + a*c)^(3/2))*a^6 + 9009*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*
c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^5/c - 6435*(35*sqrt(b*c*x + a*c)*a^
3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*
c*x + a*c)^(7/2))*a^4/c^2 + 715*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x
 + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)
^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^3/c^3 - 195*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a^2/c^4 + 15*(3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*
x + a*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a
*c)^(7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(
11/2)*a*c + 231*(b*c*x + a*c)^(13/2))*a/c^5 - (6435*sqrt(b*c*x + a*c)*a^7*c
^7 - 15015*(b*c*x + a*c)^(3/2)*a^6*c^6 + 27027*(b*c*x + a*c)^(5/2)*a^5*c^5
- 32175*(b*c*x + a*c)^(7/2)*a^4*c^4 + 25025*(b*c*x + a*c)^(9/2)*a^3*c^3 - 1
2285*(b*c*x + a*c)^(11/2)*a^2*c^2 + 3465*(b*c*x + a*c)^(13/2)*a*c - 429*(b*
c*x + a*c)^(15/2))/c^6)/b
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^{6}\,(bcx+ac)^{\frac{3}{2}}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(3/2),x)
```

```
[Out] 2/15*(b*x+a)^6*(b*c*x+a*c)^(3/2)/b
```

maxima [A] time = 1.37, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{15}{2}}}{15\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15*(b*c*x + a*c)^(15/2)/(b*c^6)
```

mupad [B] time = 0.05, size = 17, normalized size = 0.77

$$\frac{2\,\left(c\,\left(a+b\,x\right)\right)^{15/2}}{15\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(3/2)*(a + b*x)^5,x)
```

```
[Out] (2*(c*(a + b*x))^(15/2))/(15*b*c^6)
```

sympy [A] time = 1.21, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{13}{2}}c^{\frac{3}{2}}\left(\frac{a}{b}+x\right)^{\frac{15}{2}}}{15} & \text{for } \left|\frac{a}{b}+x\right| < 1 \\ b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{17}{2} \\ \frac{15}{2} & 0 \end{matrix}\middle| \frac{a}{b}+x\right) + b^{\frac{13}{2}}c^{\frac{3}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{17}{2},1 & \\ & \frac{15}{2},0 \end{matrix}\middle| \frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(13/2)*c**(3/2)*(a/b + x)**(15/2)/15, Abs(a/b + x) < 1), (b
**(13/2)*c**(3/2)*meijerg(((1,), (17/2,)), ((15/2,), (0,)), a/b + x) + b**(
13/2)*c**(3/2)*meijerg(((17/2, 1), ()), ((), (15/2, 0)), a/b + x), True))
```

3.1339 $\int (a+bx)^5\sqrt{ac+bcx}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^5\sqrt{ac+bcx}\,dx &= \frac{\int (ac+bcx)^{11/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{13/2}}{13bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6\sqrt{c(a+bx)}}{13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
[Out] (2*(a + b*x)^6*Sqrt[c*(a + b*x)])/(13*b)
```

IntegrateAlgebraic [A] time = 0.04, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{13/2}}{13bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5*Sqrt[a*c + b*c*x],x]
[Out] (2*(a*c + b*c*x)^(13/2))/(13*b*c^6)
```

fricas [B] time = 1.16, size = 75, normalized size = 3.41

$$\frac{2 \left(b^{6} x^{6} + 6 \, a b^{5} x^{5} + 15 \, a^{2} b^{4} x^{4} + 20 \, a^{3} b^{3} x^{3} + 15 \, a^{4} b^{2} x^{2} + 6 \, a^{5} b x + a^{6}\right) \sqrt{b c x + a c}}{13 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/13*(b^6*x^6 + 6*a*b^5*x^5 + 15*a^2*b^4*x^4 + 20*a^3*b^3*x^3 + 15*a^4*b^2*
x^2 + 6*a^5*b*x + a^6)*sqrt(b*c*x + a*c)/b
```

giac [B] time = 1.27, size = 495, normalized size = 22.50

$$\frac{2 \left(3003 \, \sqrt{b c x + a c} a^{6} - \frac{6006 \left(3 \, \sqrt{b c x + a c} a c - (b c x + a c)^{\frac{3}{2}}\right) a^{5}}{c} + \frac{3003 \left(15 \, \sqrt{b c x + a c} a^{2} c^{2} - 10 \, (b c x + a c)^{\frac{3}{2}} a c + 3 \, (b c x + a c)^{\frac{5}{2}}\right) a^{4}}{c^{2}} - \frac{1716 \left(35 \, \sqrt{b c x + a c} a^{3} c^{3} - 35 \, (b c x + a c)^{\frac{3}{2}} a^{2} c^{2} + 21 \, (b c x + a c)^{\frac{5}{2}} a c - 5 \, (b c x + a c)^{\frac{7}{2}}\right) a^{3}}{c^{3}} + \frac{143 \left(315 \, \sqrt{b c x + a c} a^{4} c^{4} - 420 \, (b c x + a c)^{\frac{3}{2}} a^{3} c^{3} + 378 \, (b c x + a c)^{\frac{5}{2}} a^{2} c^{2} - 180 \, (b c x + a c)^{\frac{7}{2}} a c + 35 \, (b c x + a c)^{\frac{9}{2}}\right) a^{2}}{c^{4}} - \frac{26 \left(693 \, \sqrt{b c x + a c} a^{5} c^{5} - 1155 \, (b c x + a c)^{\frac{3}{2}} a^{4} c^{4} + 1386 \, (b c x + a c)^{\frac{5}{2}} a^{3} c^{3} - 990 \, (b c x + a c)^{\frac{7}{2}} a^{2} c^{2} + 385 \, (b c x + a c)^{\frac{9}{2}} a c - 63 \, (b c x + a c)^{\frac{11}{2}}\right) a}{c^{5}} + \frac{3003 \, \sqrt{b c x + a c} a^{6} c^{6} - 6006 \, (b c x + a c)^{\frac{3}{2}} a^{5} c^{5} + 9009 \, (b c x + a c)^{\frac{5}{2}} a^{4} c^{4} - 8580 \, (b c x + a c)^{\frac{7}{2}} a^{3} c^{3} + 5005 \, (b c x + a c)^{\frac{9}{2}} a^{2} c^{2} - 1638 \, (b c x + a c)^{\frac{11}{2}} a c + 231 \, (b c x + a c)^{\frac{13}{2}}}{c^{6}}\right)}{3003 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3003*(3003*sqrt(b*c*x + a*c)*a^6 - 6006*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x
 + a*c)^(3/2))*a^5/c + 3003*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c
)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^4/c^2 - 1716*(35*sqrt(b*c*x + a*c)*a
^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b
*c*x + a*c)^(7/2))*a^3/c^3 + 143*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*
x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c
)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a^2/c^4 - 26*(693*sqrt(b*c*x + a*c)*a
^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^
3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c
*x + a*c)^(11/2))*a/c^5 + (3003*sqrt(b*c*x + a*c)*a^6*c^6 - 6006*(b*c*x + a
*c)^(3/2)*a^5*c^5 + 9009*(b*c*x + a*c)^(5/2)*a^4*c^4 - 8580*(b*c*x + a*c)^(
7/2)*a^3*c^3 + 5005*(b*c*x + a*c)^(9/2)*a^2*c^2 - 1638*(b*c*x + a*c)^(11/2)
*a*c + 231*(b*c*x + a*c)^(13/2))/c^6)/b
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2 \left(b x + a\right)^{6} \sqrt{b c x + a c}}{13 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5*(b*c*x+a*c)^(1/2),x)
```

```
[Out] 2/13*(b*x+a)^6*(b*c*x+a*c)^(1/2)/b
```

maxima [A] time = 1.36, size = 18, normalized size = 0.82

$$\frac{2 \, {\left(b c x + a c\right)}^{\frac{13}{2}}}{13 \, b c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5*(b*c*x+a*c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/13*(b*c*x + a*c)^(13/2)/(b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{13/2}}{13\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*c + b*c*x)^(1/2)*(a + b*x)^5,x)
```

```
[Out] (2*(c*(a + b*x))^(13/2))/(13*b*c^6)
```

sympy [**A**] time = 1.06, size = 66, normalized size = 3.00

$$\begin{cases} \frac{2b^{\frac{11}{2}}\sqrt{c}\left(\frac{a}{b}+x\right)^{\frac{13}{2}}}{13} & \text{for } \left|\frac{a}{b}+x\right| < 1 \\ b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{15}{2} \\ \frac{13}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right) + b^{\frac{11}{2}}\sqrt{c}\,G_{2,2}^{0,2}\left(\begin{matrix} \frac{15}{2}, 1 & \\ & \frac{13}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5*(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(11/2)*sqrt(c)*(a/b + x)**(13/2)/13, Abs(a/b + x) < 1), (b*
*(11/2)*sqrt(c)*meijerg(((1,), (15/2,)), ((13/2,), (0,)), a/b + x) + b**(11
/2)*sqrt(c)*meijerg(((15/2, 1), ()), ((), (13/2, 0)), a/b + x), True))
```

3.1340 $\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{\sqrt{ac+bcx}}\,dx &= \frac{\int (ac+bcx)^{9/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{11/2}}{11bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{11b\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
[Out] (2*(a + b*x)^6)/(11*b*Sqrt[c*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.05, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{11/2}}{11bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/Sqrt[a*c + b*c*x],x]
```

[Out] (2*(a*c + b*c*x)^(11/2))/(11*b*c^6)

fricas [B] time = 0.83, size = 67, normalized size = 3.05

$$\frac{2\left(b^5x^5+5\,ab^4x^4+10\,a^2b^3x^3+10\,a^3b^2x^2+5\,a^4bx+a^5\right)\sqrt{bcx+ac}}{11\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="fricas")

[Out] 2/11*(b^5*x^5 + 5*a*b^4*x^4 + 10*a^2*b^3*x^3 + 10*a^3*b^2*x^2 + 5*a^4*b*x + a^5)*sqrt(b*c*x + a*c)/(b*c)

giac [B] time = 0.97, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^5-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2}-\frac{198\left(35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^2}{c^3}+\frac{11\left(315\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^4}-\frac{693\sqrt{bcx+ac}\,a^5c^5-1155\,(bcx+ac)^{\frac{3}{2}}a^4c^4+1386\,(bcx+ac)^{\frac{5}{2}}a^3c^3-990\,(bcx+ac)^{\frac{7}{2}}a^2c^2+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^5}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="giac")

[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^(11/2))/c^5)/(b*c)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^6}{11\sqrt{bcx+ac}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5/(b*c*x+a*c)^(1/2),x)

[Out] 2/11*(b*x+a)^6/b/(b*c*x+a*c)^(1/2)

maxima [B] time = 1.46, size = 374, normalized size = 17.00

$$\frac{2\left(693\sqrt{bcx+ac}\,a^5-\frac{1155\left(3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^4}{c}+\frac{462\left(15\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a^3}{c^2}-\frac{198\left(35\sqrt{bcx+ac}\,a^3c^3-35\,(bcx+ac)^{\frac{3}{2}}a^2c^2+21\,(bcx+ac)^{\frac{5}{2}}ac-5\,(bcx+ac)^{\frac{7}{2}}\right)a^2}{c^3}+\frac{11\left(315\sqrt{bcx+ac}\,a^4c^4-420\,(bcx+ac)^{\frac{3}{2}}a^3c^3+378\,(bcx+ac)^{\frac{5}{2}}a^2c^2-180\,(bcx+ac)^{\frac{7}{2}}ac+35\,(bcx+ac)^{\frac{9}{2}}\right)a}{c^4}-\frac{693\sqrt{bcx+ac}\,a^5c^5-1155\,(bcx+ac)^{\frac{3}{2}}a^4c^4+1386\,(bcx+ac)^{\frac{5}{2}}a^3c^3-990\,(bcx+ac)^{\frac{7}{2}}a^2c^2+385\,(bcx+ac)^{\frac{9}{2}}ac-63\,(bcx+ac)^{\frac{11}{2}}}{c^5}\right)}{693\,bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(1/2),x, algorithm="maxima")

[Out] 2/693*(693*sqrt(b*c*x + a*c)*a^5 - 1155*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^4/c + 462*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^3/c^2 - 198*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a^2/c^3 + 11*(315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))*a/c^4 - (693*sqrt(b*c*x + a*c)*a^5*c^5 - 1155*(b*c*x + a*c)^(3/2)*a^4*c^4 + 1386*(b*c*x + a*c)^(5/2)*a^3*c^3 - 990*(b*c*x + a*c)^(7/2)*a^2*c^2 + 385*(b*c*x + a*c)^(9/2)*a*c - 63*(b*c*x + a*c)^(11/2))/c^5)/(b*c)

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{11/2}}{11\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(1/2),x)
```

```
[Out] (2*(c*(a + b*x))^(11/2))/(11*b*c^6)
```

sympy [A] time = 1.54, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{9}{2}}\left(\frac{a}{b}+x\right)^{\frac{11}{2}}}{11\sqrt{c}} & \text{for } \left|\frac{a}{b}+x\right|>1 \vee \left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{9}{2}}G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{13}{2} \\ \frac{11}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}} + \frac{b^{\frac{9}{2}}G_{2,2}^{0,2}\left(\begin{matrix} \frac{13}{2},1 & \\ & \frac{11}{2},0 \end{matrix} \middle| \frac{a}{b}+x\right)}{\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(1/2),x)
```

```
[Out] Piecewise((2*b**(9/2)*(a/b + x)**(11/2)/(11*sqrt(c)), (Abs(a/b + x) > 1) |
(Abs(a/b + x) < 1)), (b**(9/2)*meijerg(((1,), (13/2,)), ((11/2,), (0,)), a/
b + x)/sqrt(c) + b**(9/2)*meijerg(((13/2, 1), ()), ((), (11/2, 0)), a/b + x
)/sqrt(c), True))
```

3.1341 $\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{3/2}}\,dx &= \frac{\int (ac+bcx)^{7/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{9/2}}{9bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{9b(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^6)/(9*b*(c*(a + b*x))^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{9/2}}{9bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(3/2),x]
```

[Out] (2*(a*c + b*c*x)^(9/2))/(9*b*c^6)

fricas [B] time = 1.40, size = 56, normalized size = 2.55

$$\frac{2\left(b^4x^4 + 4\,ab^3x^3 + 6\,a^2b^2x^2 + 4\,a^3bx + a^4\right)\sqrt{bcx + ac}}{9\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="fricas")

[Out] 2/9*(b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*sqrt(b*c*x + a*c)/(b*c^2)

giac [B] time = 1.18, size = 266, normalized size = 12.09

$$\frac{2\left(315\,\sqrt{bcx + ac}\,a^4 - \frac{420\left(3\,\sqrt{bcx+ac}\,ac - (bcx+ac)^{\frac{3}{2}}\right)a^3}{c} + \frac{126\left(15\,\sqrt{bcx+ac}\,a^2c^2 - 10\,(bcx+ac)^{\frac{3}{2}}ac + 3\,(bcx+ac)^{\frac{5}{2}}\right)a^2}{c^2} - \frac{36\left(35\,\sqrt{bcx+ac}\,a^3c^3 - 35\,(bcx+ac)^{\frac{3}{2}}a^2c^2 + 21\,(bcx+ac)^{\frac{5}{2}}ac - 5\,(bcx+ac)^{\frac{7}{2}}\right)a}{c^3} + \frac{315\,\sqrt{bcx+ac}\,a^4c^4 - 420\,(bcx+ac)^{\frac{3}{2}}a^3c^3 + 378\,(bcx+ac)^{\frac{5}{2}}a^2c^2 - 180\,(bcx+ac)^{\frac{7}{2}}ac + 35\,(bcx+ac)^{\frac{9}{2}}}{c^4}\right)}{315\,bc^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="giac")

[Out] 2/315*(315*sqrt(b*c*x + a*c)*a^4 - 420*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2))*a^3/c + 126*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c + 3*(b*c*x + a*c)^(5/2))*a^2/c^2 - 36*(35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2))*a/c^3 + (315*sqrt(b*c*x + a*c)*a^4*c^4 - 420*(b*c*x + a*c)^(3/2)*a^3*c^3 + 378*(b*c*x + a*c)^(5/2)*a^2*c^2 - 180*(b*c*x + a*c)^(7/2)*a*c + 35*(b*c*x + a*c)^(9/2))/c^4)/(b*c^2)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx + a)^6}{9\,(bcx + ac)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^5/(b*c*x+a*c)^(3/2),x)

[Out] 2/9*(b*x+a)^6/b/(b*c*x+a*c)^(3/2)

maxima [A] time = 1.43, size = 18, normalized size = 0.82

$$\frac{2\,(bcx + ac)^{\frac{9}{2}}}{9\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^5/(b*c*x+a*c)^(3/2),x, algorithm="maxima")

[Out] 2/9*(b*c*x + a*c)^(9/2)/(b*c^6)

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,{\left(c\,\left(a+b\,x\right)\right)}^{9/2}}{9\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^5/(a*c + b*c*x)^(3/2),x)

[Out] (2*(c*(a + b*x))^(9/2))/(9*b*c^6)

sympy [A] time = 1.65, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{7}{2}}\left(\frac{a}{b}+x\right)^{\frac{9}{2}}}{9c^{\frac{3}{2}}} & \text{for } \left|\frac{a}{b}+x\right|>1\vee\left|\frac{a}{b}+x\right|<1 \\ \frac{b^{\frac{7}{2}}G_{2,2}^{1,1}\left(\begin{matrix}1 & \frac{11}{2} \\ \frac{9}{2} & 0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{3}{2}}}+\frac{b^{\frac{7}{2}}G_{2,2}^{0,2}\left(\begin{matrix}\frac{11}{2},1 & \\ & \frac{9}{2},0\end{matrix}\middle|\frac{a}{b}+x\right)}{c^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(3/2),x)
```

```
[Out] Piecewise((2*b**(7/2)*(a/b + x)**(9/2)/(9*c**(3/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(7/2)*meijerg(((1,), (11/2,)), ((9/2,), (0,)), a/b
+ x)/c**(3/2) + b**(7/2)*meijerg(((11/2, 1), ()), ((), (9/2, 0)), a/b + x)/
c**(3/2), True))
```

3.1342 $\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{5/2}}\,dx &= \frac{\int (ac+bcx)^{5/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{7/2}}{7bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{7b(c(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
[Out] (2*(a + b*x)^6)/(7*b*(c*(a + b*x))^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{7/2}}{7bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(5/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(7/2))/(7*b*c^6)
```

fricas [B] time = 1.44, size = 45, normalized size = 2.05

$$\frac{2\left(b^{3}x^{3}+3\,ab^{2}x^{2}+3\,a^{2}bx+a^{3}\right)\sqrt{bcx+ac}}{7\,bc^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*c*x + a*c)/(b*c^3)
```

giac [B] time = 0.96, size = 178, normalized size = 8.09

$$\frac{2\left(35\,\sqrt{bcx+ac}\,a^{3}-\frac{35\left(3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a^{2}}{c}+\frac{7\left(15\,\sqrt{bcx+ac}\,a^{2}c^{2}-10\,(bcx+ac)^{\frac{3}{2}}\,ac+3\,(bcx+ac)^{\frac{5}{2}}\right)a}{c^{2}}-\frac{35\,\sqrt{bcx+ac}\,a^{3}c^{3}-35\,(bcx+ac)^{\frac{3}{2}}\,a^{2}c^{2}+21\,(bcx+ac)^{\frac{5}{2}}\,ac-5\,(bcx+ac)^{\frac{7}{2}}}{c^{3}}\right)}{35\,bc^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/35*(35*sqrt(b*c*x + a*c)*a^3 - 35*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a^2/c + 7*(15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a
*c + 3*(b*c*x + a*c)^(5/2))*a/c^2 - (35*sqrt(b*c*x + a*c)*a^3*c^3 - 35*(b*c
*x + a*c)^(3/2)*a^2*c^2 + 21*(b*c*x + a*c)^(5/2)*a*c - 5*(b*c*x + a*c)^(7/2
))/c^3)/(b*c^3)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^{6}}{7\,(bcx+ac)^{\frac{5}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(5/2),x)
```

```
[Out] 2/7*(b*x+a)^6/b/(b*c*x+a*c)^(5/2)
```

maxima [A] time = 1.38, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{7}{2}}}{7\,bc^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*c*x + a*c)^(7/2)/(b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,\left(c\,\left(a+b\,x\right)\right)^{7/2}}{7\,b\,c^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(5/2),x)
```

```
[Out] (2*(c*(a + b*x))^(7/2))/(7*b*c^6)
```

sympy [A] time = 1.62, size = 73, normalized size = 3.32

$$\begin{cases} \frac{2b^{\frac{5}{2}}\left(\frac{a}{b}+x\right)^{\frac{7}{2}}}{7c^{\frac{5}{2}}} & \text{for } \left|\frac{a}{b}+x\right| > 1 \vee \left|\frac{a}{b}+x\right| < 1 \\ \frac{b^{\frac{5}{2}} G_{2,2}^{1,1}\left(\begin{matrix} 1 & \frac{9}{2} \\ \frac{7}{2} & 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{5}{2}}} + \frac{b^{\frac{5}{2}} G_{2,2}^{0,2}\left(\begin{matrix} \frac{9}{2}, 1 & \\ & \frac{7}{2}, 0 \end{matrix} \middle| \frac{a}{b}+x\right)}{c^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(5/2),x)
```

```
[Out] Piecewise((2*b**(5/2)*(a/b + x)**(7/2)/(7*c**(5/2)), (Abs(a/b + x) > 1) | (
Abs(a/b + x) < 1)), (b**(5/2)*meijerg(((1,), (9/2,)), ((7/2,), (0,)), a/b +
 x)/c**(5/2) + b**(5/2)*meijerg(((9/2, 1), ()), ((), (7/2, 0)), a/b + x)/c*
*(5/2), True))
```

3.1343 $\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}} dx$

Optimal. Leaf size=22

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {21, 32}

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{7/2}}\,dx &= \frac{\int (ac+bcx)^{3/2}\,dx}{c^5}\\ &= \frac{2(ac+bcx)^{5/2}}{5bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.14

$$\frac{2(a+bx)^6}{5b(c(a+bx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

```
[Out] (2*(a + b*x)^6)/(5*b*(c*(a + b*x))^(7/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac+bcx)^{5/2}}{5bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(7/2),x]
```

[Out] (2*(a*c + b*c*x)^(5/2))/(5*b*c^6)

fricas [A] time = 1.26, size = 34, normalized size = 1.55

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bcx+ac}}{5\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*c*x + a*c)/(b*c^4)
```

giac [B] time = 0.99, size = 106, normalized size = 4.82

$$\frac{2\left(15\,\sqrt{bcx+ac}\,a^2-\frac{10\left(3\,\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}\right)a}{c}+\frac{15\,\sqrt{bcx+ac}\,a^2c^2-10\,(bcx+ac)^{\frac{3}{2}}\,ac+3\,(bcx+ac)^{\frac{5}{2}}}{c^2}\right)}{15\,bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(b*c*x + a*c)*a^2 - 10*(3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c
)^(3/2))*a/c + (15*sqrt(b*c*x + a*c)*a^2*c^2 - 10*(b*c*x + a*c)^(3/2)*a*c +
 3*(b*c*x + a*c)^(5/2))/c^2)/(b*c^4)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^6}{5\,(bcx+ac)^{\frac{7}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(7/2),x)
```

```
[Out] 2/5*(b*x+a)^6/b/(b*c*x+a*c)^(7/2)
```

maxima [A] time = 1.41, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{5}{2}}}{5\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*c*x + a*c)^(5/2)/(b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,(c\,\left(a+b\,x\right))^{5/2}}{5\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(7/2),x)
```

```
[Out] (2*(c*(a + b*x))^(5/2))/(5*b*c^6)
```

sympy [A] time = 4.11, size = 80, normalized size = 3.64

$$\begin{cases} \frac{2a^2\sqrt{ac+bcx}}{5bc^4} + \frac{4ax\sqrt{ac+bcx}}{5c^4} + \frac{2bx^2\sqrt{ac+bcx}}{5c^4} & \text{for } b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{7}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(7/2),x)
```

```
[Out] Piecewise((2*a**2*sqrt(a*c + b*c*x)/(5*b*c**4) + 4*a*x*sqrt(a*c + b*c*x)/(5
*c**4) + 2*b*x**2*sqrt(a*c + b*c*x)/(5*c**4), Ne(b, 0)), (a**5*x/(a*c)**(7/
2), True))
```

3.1344 $\int \frac{(a+bx)^5}{(ac+bcx)^{9/2}} \, dx$

Optimal. Leaf size=22

$$\frac{2(ac + bcx)^{3/2}}{3bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2(ac + bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx)^5}{(ac+bcx)^{9/2}} \, dx &= \frac{\int \sqrt{ac+bcx} \, dx}{c^5} \\ &= \frac{2(ac+bcx)^{3/2}}{3bc^6} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.18

$$\frac{2(a+bx)\sqrt{c(a+bx)}}{3bc^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a + b*x)*Sqrt[c*(a + b*x)])/(3*b*c^5)
```

IntegrateAlgebraic [A] time = 0.06, size = 22, normalized size = 1.00

$$\frac{2(ac + bcx)^{3/2}}{3bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(9/2),x]
```

```
[Out] (2*(a*c + b*c*x)^(3/2))/(3*b*c^6)
```

fricas [A] time = 1.31, size = 23, normalized size = 1.05

$$\frac{2\sqrt{bcx+ac}\,(bx+a)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*c*x + a*c)*(b*x + a)/(b*c^5)
```

giac [B] time = 1.07, size = 54, normalized size = 2.45

$$\frac{2\left(3\sqrt{bcx+ac}\,a-\frac{3\sqrt{bcx+ac}\,ac-(bcx+ac)^{\frac{3}{2}}}{c}\right)}{3\,bc^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="giac")
```

```
[Out] 2/3*(3*sqrt(b*c*x + a*c)*a - (3*sqrt(b*c*x + a*c)*a*c - (b*c*x + a*c)^(3/2)
)/c)/(b*c^5)
```

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{2\,(bx+a)^6}{3\,(bcx+ac)^{\frac{9}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(9/2),x)
```

```
[Out] 2/3*(b*x+a)^6/b/(b*c*x+a*c)^(9/2)
```

maxima [A] time = 1.39, size = 18, normalized size = 0.82

$$\frac{2\,(bcx+ac)^{\frac{3}{2}}}{3\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*c*x + a*c)^(3/2)/(b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,(c\,(a+b\,x))^{3/2}}{3\,b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(9/2),x)
```

```
[Out] (2*(c*(a + b*x))^(3/2))/(3*b*c^6)
```

sympy [A] time = 8.31, size = 53, normalized size = 2.41

$$\begin{cases} \frac{2a\sqrt{ac+bcx}}{3bc^5}+\frac{2x\sqrt{ac+bcx}}{3c^5} & \text{for } b\neq 0 \\ \frac{a^5x}{(ac)^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(9/2),x)
```

```
[Out] Piecewise((2*a*sqrt(a*c + b*c*x)/(3*b*c**5) + 2*x*sqrt(a*c + b*c*x)/(3*c**5
), Ne(b, 0)), (a**5*x/(a*c)**(9/2), True))
```

3.1345 $\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} \, dx$

Optimal. Leaf size=20

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{11/2}} \, dx &= \frac{\int \frac{1}{\sqrt{ac+bcx}} \, dx}{c^5}\\ &= \frac{2\sqrt{ac+bcx}}{bc^6}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.20

$$\frac{2(a+bx)}{bc^5\sqrt{c(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*(a + b*x))/(b*c^5*Sqrt[c*(a + b*x)])
```

IntegrateAlgebraic [A] time = 0.06, size = 20, normalized size = 1.00

$$\frac{2\sqrt{ac+bcx}}{bc^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[a*c + b*c*x])/(b*c^6)
```

fricas [A] time = 1.19, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="fricas")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

giac [A] time = 1.03, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="giac")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$\frac{2\,(bx+a)^6}{(bcx+ac)^{\frac{11}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(11/2),x)
[Out] 2*(b*x+a)^6/b/(b*c*x+a*c)^(11/2)
```

maxima [A] time = 1.38, size = 18, normalized size = 0.90

$$\frac{2\sqrt{bcx+ac}}{bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(11/2),x, algorithm="maxima")
[Out] 2*sqrt(b*c*x + a*c)/(b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.85

$$\frac{2\sqrt{c\,(a+b\,x)}}{b\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(11/2),x)
[Out] (2*(c*(a + b*x))^(1/2))/(b*c^6)
```

sympy [A] time = 15.48, size = 29, normalized size = 1.45

$$\begin{cases} \frac{2\sqrt{ac+bcx}}{bc^6} & \text{for}\ b \neq 0 \\ \frac{a^5x}{(ac)^{\frac{11}{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(11/2),x)
```

```
[Out] Piecewise((2*sqrt(a*c + b*c*x)/(b*c**6), Ne(b, 0)), (a**5*x/(a*c)**(11/2),
True))
```

3.1346 $\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx$

Optimal. Leaf size=20

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 32}

$$-\frac{2}{bc^6\sqrt{ac+bcx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] -2/(b*c^6*Sqrt[a*c + b*c*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^5}{(ac+bcx)^{13/2}}\,dx &= \frac{\int \frac{1}{(ac+bcx)^{3/2}}\,dx}{c^5}\\ &= -\frac{2}{bc^6\sqrt{ac+bcx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.20

$$-\frac{2(a+bx)}{bc^5(c(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] (-2*(a + b*x))/(b*c^5*(c*(a + b*x))^(3/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 27, normalized size = 1.35

$$-\frac{2\sqrt{ac+bcx}}{bc^7(a+bx)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^5/(a*c + b*c*x)^(13/2),x]
```

```
[Out] (-2*Sqrt[a*c + b*c*x])/(b*c^7*(a + b*x))
```

fricas [A] time = 1.10, size = 29, normalized size = 1.45

$$-\frac{2\sqrt{bcx+ac}}{b^2c^7x+abc^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="fricas")
[Out] -2*sqrt(b*c*x + a*c)/(b^2*c^7*x + a*b*c^7)
```

giac [A] time = 0.81, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="giac")
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$-\frac{2\,(bx+a)^6}{(bcx+ac)^{\frac{13}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^5/(b*c*x+a*c)^(13/2),x)
[Out] -2*(b*x+a)^6/b/(b*c*x+a*c)^(13/2)
```

maxima [A] time = 1.34, size = 18, normalized size = 0.90

$$-\frac{2}{\sqrt{bcx+ac}\,bc^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^5/(b*c*x+a*c)^(13/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*c*x + a*c)*b*c^6)
```

mupad [B] time = 0.03, size = 17, normalized size = 0.85

$$-\frac{2}{b\,c^6\,\sqrt{c\,\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^5/(a*c + b*c*x)^(13/2),x)
[Out] -2/(b*c^6*(c*(a + b*x))^(1/2))
```

sympy [A] time = 39.43, size = 48, normalized size = 2.40

$$\begin{cases} -\frac{2\sqrt{ac+bcx}}{abc^7+b^2c^7x} & \text{for}\ a \neq 0 \\ -\frac{2}{b^{\frac{3}{2}}c^{\frac{13}{2}}\sqrt{x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**5/(b*c*x+a*c)**(13/2),x)
```

```
[Out] Piecewise((-2*sqrt(a*c + b*c*x)/(a*b*c**7 + b**2*c**7*x), Ne(a, 0)), (-2/(b
**(3/2)*c**(13/2)*sqrt(x)), True))
```

3.1347 $\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx$

Optimal. Leaf size=14

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 207}

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)\sqrt{2+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{2+x}\right)\\ &= -\tanh^{-1}\left(\frac{\sqrt{2+x}}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x]),x]
```

```
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

IntegrateAlgebraic [A] time = 0.02, size = 14, normalized size = 1.00

$$-\tanh^{-1}\left(\frac{\sqrt{x+2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((-2 + x)*Sqrt[2 + x]),x]
[Out] -ArcTanh[Sqrt[2 + x]/2]
```

fricas [B] time = 1.22, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="fricas")
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

giac [B] time = 0.87, size = 22, normalized size = 1.57

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="giac")
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(abs(sqrt(x + 2) - 2))
```

maple [B] time = 0.01, size = 22, normalized size = 1.57

$$\frac{\ln\left(\sqrt{x+2}-2\right)}{2}-\frac{\ln\left(\sqrt{x+2}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-2)/(x+2)^(1/2),x)
[Out] 1/2*ln((x+2)^(1/2)-2)-1/2*ln((x+2)^(1/2)+2)
```

maxima [B] time = 1.35, size = 21, normalized size = 1.50

$$-\frac{1}{2}\log\left(\sqrt{x+2}+2\right)+\frac{1}{2}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)^(1/2),x, algorithm="maxima")
[Out] -1/2*log(sqrt(x + 2) + 2) + 1/2*log(sqrt(x + 2) - 2)
```

mupad [B] time = 0.05, size = 10, normalized size = 0.71

$$-\mathrm{atanh}\left(\frac{\sqrt{x+2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x - 2)*(x + 2)^(1/2)),x)
[Out] -atanh((x + 2)^(1/2)/2)
```

sympy [A] time = 0.66, size = 27, normalized size = 1.93

$$\begin{cases} -\operatorname{acoth}\left(\frac{\sqrt{x+2}}{2}\right) & \text{for}\: \frac{|x+2|}{4} > 1 \\ -\operatorname{atanh}\left(\frac{\sqrt{x+2}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(2+x)**(1/2),x)
```

```
[Out] Piecewise((-acoth(sqrt(x + 2)/2), Abs(x + 2)/4 > 1), (-atanh(sqrt(x + 2)/2)
, True))
```

3.1348 $\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx$

Optimal. Leaf size=25

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 203}

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+3x)\sqrt{1+5x}}\,dx &= \frac{2}{5}\text{Subst}\left(\int \frac{1}{\frac{7}{5}+\frac{3x^2}{5}}\,dx, x, \sqrt{1+5x}\right)\\ &= \frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{1+5x}\right)}{\sqrt{21}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

IntegrateAlgebraic [A]   time = 0.03, size = 25, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\sqrt{\frac{3}{7}}\sqrt{5x+1}\right)}{\sqrt{21}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((2 + 3*x)*Sqrt[1 + 5*x]),x]
[Out] (2*ArcTan[Sqrt[3/7]*Sqrt[1 + 5*x]])/Sqrt[21]
```

fricas [A]   time = 1.05, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="fricas")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

giac [A]   time = 0.98, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="giac")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

maple [A]   time = 0.01, size = 19, normalized size = 0.76

$$\frac{2\sqrt{21}\arctan\left(\frac{\sqrt{21}\sqrt{5x+1}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x+2)/(1+5*x)^(1/2),x)
[Out] 2/21*arctan(1/7*21^(1/2)*(1+5*x)^(1/2))*21^(1/2)
```

maxima [A]   time = 3.00, size = 18, normalized size = 0.72

$$\frac{2}{21}\sqrt{21}\arctan\left(\frac{1}{7}\sqrt{21}\sqrt{5x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)^(1/2),x, algorithm="maxima")
[Out] 2/21*sqrt(21)*arctan(1/7*sqrt(21)*sqrt(5*x + 1))
```

mupad [B]   time = 0.06, size = 15, normalized size = 0.60

$$\frac{2\sqrt{21}\operatorname{atan}\left(\frac{\sqrt{105x+21}}{7}\right)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3*x + 2)*(5*x + 1)^(1/2)),x)
```

```
[Out] (2*21^(1/2)*atan((105*x + 21)^(1/2)/7))/21
```

sympy [**A**] time = 1.12, size = 61, normalized size = 2.44

$$\begin{cases} \frac{2\sqrt{21}\, i \operatorname{acosh}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{for}\ \frac{7}{15\left|x+\frac{2}{3}\right|} > 1 \\ -\frac{2\sqrt{21}\operatorname{asin}\left(\frac{\sqrt{105}}{15\sqrt{x+\frac{2}{3}}}\right)}{21} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x)/(1+5*x)**(1/2),x)
```

```
[Out] Piecewise((2*sqrt(21)*I*acosh(sqrt(105)/(15*sqrt(x + 2/3)))/21, 7/(15*Abs(x
 + 2/3)) > 1), (-2*sqrt(21)*asin(sqrt(105)/(15*sqrt(x + 2/3)))/21, True))
```

3.1349 $\int \frac{\sqrt[3]{1-x}}{1+x}\,dx$

Optimal. Leaf size=84

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {50, 57, 617, 204, 31}

$$3\sqrt[3]{1-x}+\frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}}-\frac{\log(x+1)}{2^{2/3}}-\sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + (3*Log[2^(1/3) - (1 - x)^(1/3)])/2^(2/3) - Log[1 + x]/2^(2/3)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{\sqrt[3]{1-x}}{1+x}\,dx = 3\sqrt[3]{1-x} + 2\int \frac{1}{(1-x)^{2/3}(1+x)}\,dx$$
$$= 3\sqrt[3]{1-x} - \frac{\log(1+x)}{2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{2}-x}\,dx, x, \sqrt[3]{1-x}\right)}{2^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[3]{1-x}\right)}{\sqrt[3]{2}}$$
$$= 3\sqrt[3]{1-x} + \frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}} - \frac{\log(1+x)}{2^{2/3}} + \left(3\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2^{2/3}\sqrt[3]{1-x}\right)$$
$$= 3\sqrt[3]{1-x} - \sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{1+2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}\right) + \frac{3\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right)}{2^{2/3}} - \frac{\log(1+x)}{2^{2/3}}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.24

$$3\sqrt[3]{1-x} + \sqrt[3]{2}\log\left(\sqrt[3]{2}-\sqrt[3]{1-x}\right) - \frac{\log\left((1-x)^{2/3}+\sqrt[3]{2-2x}+2^{2/3}\right)}{2^{2/3}} - \sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[(1 + 2^(2/3)*(1 - x)^(1/3))/Sqrt[3
]] + 2^(1/3)*Log[2^(1/3) - (1 - x)^(1/3)] - Log[2^(2/3) + (2 - 2*x)^(1/3) +
 (1 - x)^(2/3)]/2^(2/3)
```

IntegrateAlgebraic [A] time = 0.12, size = 115, normalized size = 1.37

$$3\sqrt[3]{1-x} + \sqrt[3]{2}\log\left(2^{2/3}\sqrt[3]{1-x}-2\right) - \frac{\log\left(\sqrt[3]{2}(1-x)^{2/3}+2^{2/3}\sqrt[3]{1-x}+2\right)}{2^{2/3}} - \sqrt[3]{2}\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}\sqrt[3]{1-x}}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/3)/(1 + x),x]
```

```
[Out] 3*(1 - x)^(1/3) - 2^(1/3)*Sqrt[3]*ArcTan[1/Sqrt[3] + (2^(2/3)*(1 - x)^(1/3)
)/Sqrt[3]] + 2^(1/3)*Log[-2 + 2^(2/3)*(1 - x)^(1/3)] - Log[2 + 2^(2/3)*(1 -
 x)^(1/3) + 2^(1/3)*(1 - x)^(2/3)]/2^(2/3)
```

fricas [A] time = 1.33, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\,2^{\frac{2}{3}}(-x+1)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right) - \frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right) + 2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right) + 3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/3*sqrt(3)*2^(2/3)*(-x + 1)^(1/3) + 1/3*sqrt(3)) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

giac [A] time = 1.05, size = 87, normalized size = 1.04

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2(-x+1)^{\frac{1}{3}}\right)\right) - \frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right) + 2^{\frac{1}{3}}\log\left(\left|-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right|\right) + 3(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="giac")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(abs(-2^(1/3) + (-x + 1)^(1/3))) + 3*(-x + 1)^(1/3)
```

maple [A] time = 0.01, size = 84, normalized size = 1.00

$$-2^{\frac{1}{3}}\sqrt{3}\, \arctan\left(\frac{\left(1+2^{\frac{2}{3}}\left(-x+1\right)^{\frac{1}{3}}\right)\sqrt{3}}{3}\right)+2^{\frac{1}{3}}\ln\left(\left(-x+1\right)^{\frac{1}{3}}-2^{\frac{1}{3}}\right)-\frac{2^{\frac{1}{3}}\ln\left(\left(-x+1\right)^{\frac{2}{3}}+2^{\frac{1}{3}}\left(-x+1\right)^{\frac{1}{3}}+2^{\frac{2}{3}}\right)}{2}+3\left(-x+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/3)/(x+1),x)
```

```
[Out] 3*(-x+1)^(1/3)+2^(1/3)*ln((-x+1)^(1/3)-2^(1/3))-1/2*2^(1/3)*ln((-x+1)^(2/3)
+2^(1/3)*(-x+1)^(1/3)+2^(2/3))-2^(1/3)*arctan(1/3*(1+2^(2/3)*(-x+1)^(1/3))*
3^(1/2))*3^(1/2)
```

maxima [A] time = 3.00, size = 86, normalized size = 1.02

$$-\sqrt{3}\,2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}\,2^{\frac{2}{3}}\left(2^{\frac{1}{3}}+2\left(-x+1\right)^{\frac{1}{3}}\right)\right)-\frac{1}{2}\cdot 2^{\frac{1}{3}}\log\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}(-x+1)^{\frac{1}{3}}+(-x+1)^{\frac{2}{3}}\right)+2^{\frac{1}{3}}\log\left(-2^{\frac{1}{3}}+(-x+1)^{\frac{1}{3}}\right)+3\left(-x+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)/(1+x),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2^(1/3) + 2*(-x + 1)^(1/3))) -
 1/2*2^(1/3)*log(2^(2/3) + 2^(1/3)*(-x + 1)^(1/3) + (-x + 1)^(2/3)) + 2^(1/
3)*log(-2^(1/3) + (-x + 1)^(1/3)) + 3*(-x + 1)^(1/3)
```

mupad [B] time = 0.07, size = 104, normalized size = 1.24

$$2^{1/3}\ln\left(18\,(1-x)^{1/3}-18\,2^{1/3}\right)+3\,(1-x)^{1/3}+\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}-9\,2^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2}-\frac{2^{1/3}\ln\left(18\,(1-x)^{1/3}+9\,2^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)/(x + 1),x)
```

```
[Out] 2^(1/3)*log(18*(1 - x)^(1/3) - 18*2^(1/3)) + 3*(1 - x)^(1/3) + (2^(1/3)*log
(18*(1 - x)^(1/3) - 9*2^(1/3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/2 - (2^(1
/3)*log(18*(1 - x)^(1/3) + 9*2^(1/3)*(3^(1/2)*1i + 1))*(3^(1/2)*1i + 1))/2
```

sympy [C] time = 2.26, size = 170, normalized size = 2.02

$$\frac{4\sqrt[3]{-1}\,\sqrt[3]{x-1}\,\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{-\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{-2}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{i\pi}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-2}\,e^{\frac{i\pi}{3}}\log\left(-\frac{2^{\frac{2}{3}}\sqrt[3]{x-1}\,e^{\frac{5i\pi}{3}}}{2}+1\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)/(1+x),x)
```

```
[Out] 4*(-1)**(1/3)*(x - 1)**(1/3)*gamma(4/3)/gamma(7/3) + 4*(-2)**(1/3)*exp(-I*p
i/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi/3)/2 + 1)*gamma(4/3)/(3*ga
mma(7/3)) - 4*(-2)**(1/3)*log(-2**(2/3)*(x - 1)**(1/3)*exp_polar(I*pi)/2 +
1)*gamma(4/3)/(3*gamma(7/3)) + 4*(-2)**(1/3)*exp(I*pi/3)*log(-2**(2/3)*(x -
 1)**(1/3)*exp_polar(5*I*pi/3)/2 + 1)*gamma(4/3)/(3*gamma(7/3))
```

3.1350 $\int \sqrt[3]{3-2x}\,(7+x)\,dx$

Optimal. Leaf size=27

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Rubi [A] time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{3}{28}(3-2x)^{7/3}-\frac{51}{16}(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (-51*(3 - 2*x)^(4/3))/16 + (3*(3 - 2*x)^(7/3))/28
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{3-2x}\,(7+x)\,dx &= \int \left(\frac{17}{2}\sqrt[3]{3-2x}-\frac{1}{2}(3-2x)^{4/3}\right)dx\\ &= -\frac{51}{16}(3-2x)^{4/3}+\frac{3}{28}(3-2x)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.67

$$-\frac{3}{112}(3-2x)^{4/3}(8x+107)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (-3*(3 - 2*x)^(4/3)*(107 + 8*x))/112
```

IntegrateAlgebraic [A] time = 0.01, size = 22, normalized size = 0.81

$$\frac{3}{112}(4(3-2x)-119)(3-2x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(3 - 2*x)^(1/3)*(7 + x),x]
[Out] (3*(-119 + 4*(3 - 2*x))*(3 - 2*x)^(4/3))/112
```

fricas [A] time = 1.24, size = 19, normalized size = 0.70

$$\frac{3}{112}\left(16\,x^2+190\,x-321\right)(-2\,x+3)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="fricas")
```

```
[Out] 3/112*(16*x^2 + 190*x - 321)*(-2*x + 3)^(1/3)
```

giac [A] time = 0.94, size = 26, normalized size = 0.96

$$\frac{3}{28}\,(2\,x-3)^2(-2\,x+3)^{\frac{1}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="giac")
```

```
[Out] 3/28*(2*x - 3)^2*(-2*x + 3)^(1/3) - 51/16*(-2*x + 3)^(4/3)
```

maple [A] time = 0.00, size = 15, normalized size = 0.56

$$-\frac{3\,(8x+107)\,(-2x+3)^{\frac{4}{3}}}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3-2*x)^(1/3)*(7+x),x)
```

```
[Out] -3/112*(8*x+107)*(3-2*x)^(4/3)
```

maxima [A] time = 1.35, size = 19, normalized size = 0.70

$$\frac{3}{28}\,(-2\,x+3)^{\frac{7}{3}}-\frac{51}{16}\,(-2\,x+3)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)^(1/3)*(7+x),x, algorithm="maxima")
```

```
[Out] 3/28*(-2*x + 3)^(7/3) - 51/16*(-2*x + 3)^(4/3)
```

mupad [B] time = 0.26, size = 14, normalized size = 0.52

$$-\frac{3\,(3-2\,x)^{4/3}\,(8\,x+107)}{112}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3 - 2*x)^(1/3)*(x + 7),x)
```

```
[Out] -(3*(3 - 2*x)^(4/3)*(8*x + 107))/112
```

sympy [A] time = 1.09, size = 114, normalized size = 4.22

$$\begin{cases} \frac{3(x+7)^2\,\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{7}-\frac{51(x+7)\,\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{56}-\frac{2601\,\sqrt[3]{2x-3}\,e^{\frac{i\pi}{3}}}{112} & \text{for}\ \frac{2|x+7|}{17}>1 \\ \frac{3\,\sqrt[3]{3-2x}\,(x+7)^2}{7}-\frac{51\,\sqrt[3]{3-2x}\,(x+7)}{56}-\frac{2601\,\sqrt[3]{3-2x}}{112} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3-2*x)**(1/3)*(7+x),x)
```

```
[Out] Piecewise((3*(x + 7)**2*(2*x - 3)**(1/3)*exp(I*pi/3)/7 - 51*(x + 7)*(2*x -
3)**(1/3)*exp(I*pi/3)/56 - 2601*(2*x - 3)**(1/3)*exp(I*pi/3)/112, 2*Abs(x +
 7)/17 > 1), (3*(3 - 2*x)**(1/3)*(x + 7)**2/7 - 51*(3 - 2*x)**(1/3)*(x + 7)
/56 - 2601*(3 - 2*x)**(1/3)/112, True))
```

3.1351 $\int \sqrt[3]{1-x}\,(1+x)^2\,dx$

Optimal. Leaf size=38

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3}{10}(1-x)^{10/3}+\frac{12}{7}(1-x)^{7/3}-3(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^(1/3)*(1 + x)^2,x]
[Out] -3*(1 - x)^(4/3) + (12*(1 - x)^(7/3))/7 - (3*(1 - x)^(10/3))/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{1-x}\,(1+x)^2\,dx &= \int \left(4\sqrt[3]{1-x}-4(1-x)^{4/3}+(1-x)^{7/3}\right)dx\\ &= -3(1-x)^{4/3}+\frac{12}{7}(1-x)^{7/3}-\frac{3}{10}(1-x)^{10/3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.61

$$-\frac{3}{70}(1-x)^{4/3}\left(7x^2+26x+37\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^(1/3)*(1 + x)^2,x]
[Out] (-3*(1 - x)^(4/3)*(37 + 26*x + 7*x^2))/70
```

IntegrateAlgebraic [A] time = 0.02, size = 31, normalized size = 0.82

$$-\frac{3}{70}\left(7(1-x)^2-40(1-x)+70\right)(1-x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - x)^(1/3)*(1 + x)^2,x]
[Out] (-3*(70 - 40*(1 - x) + 7*(1 - x)^2)*(1 - x)^(4/3))/70
```

fricas [A] time = 1.32, size = 24, normalized size = 0.63

$$\frac{3}{70}\left(7\,x^3+19\,x^2+11\,x-37\right)(-x+1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="fricas")
```

```
[Out] 3/70*(7*x^3 + 19*x^2 + 11*x - 37)*(-x + 1)^(1/3)
```

giac [A] time = 0.86, size = 38, normalized size = 1.00

$$\frac{3}{10}(x-1)^3(-x+1)^{\frac{1}{3}}+\frac{12}{7}(x-1)^2(-x+1)^{\frac{1}{3}}-3(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="giac")
```

```
[Out] 3/10*(x - 1)^3*(-x + 1)^(1/3) + 12/7*(x - 1)^2*(-x + 1)^(1/3) - 3*(-x + 1)^
(4/3)
```

maple [A] time = 0.00, size = 20, normalized size = 0.53

$$-\frac{3\left(7x^2+26x+37\right)(-x+1)^{\frac{4}{3}}}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x+1)^(1/3)*(x+1)^2,x)
```

```
[Out] -3/70*(7*x^2+26*x+37)*(-x+1)^(4/3)
```

maxima [A] time = 1.36, size = 28, normalized size = 0.74

$$-\frac{3}{10}(-x+1)^{\frac{10}{3}}+\frac{12}{7}(-x+1)^{\frac{7}{3}}-3(-x+1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^(1/3)*(1+x)^2,x, algorithm="maxima")
```

```
[Out] -3/10*(-x + 1)^(10/3) + 12/7*(-x + 1)^(7/3) - 3*(-x + 1)^(4/3)
```

mupad [B] time = 0.05, size = 21, normalized size = 0.55

$$-\frac{3(1-x)^{4/3}\left(40x+7(x-1)^2+30\right)}{70}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x)^(1/3)*(x + 1)^2,x)
```

```
[Out] -(3*(1 - x)^(4/3)*(40*x + 7*(x - 1)^2 + 30))/70
```

sympy [A] time = 1.52, size = 146, normalized size = 3.84

$$\begin{cases} -\frac{3\sqrt[3]{x-1}(x+1)^3e^{-\frac{2i\pi}{3}}}{10}+\frac{3\sqrt[3]{x-1}(x+1)^2e^{-\frac{2i\pi}{3}}}{35}+\frac{9\sqrt[3]{x-1}(x+1)e^{-\frac{2i\pi}{3}}}{35}+\frac{54\sqrt[3]{x-1}e^{-\frac{2i\pi}{3}}}{35} & \text{for } \frac{|x+1|}{2}>1 \\ \frac{3\sqrt[3]{1-x}(x+1)^3}{10}-\frac{3\sqrt[3]{1-x}(x+1)^2}{35}-\frac{9\sqrt[3]{1-x}(x+1)}{35}-\frac{54\sqrt[3]{1-x}}{35} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**(1/3)*(1+x)**2,x)
```

```
[Out] Piecewise((-3*(x - 1)**(1/3)*(x + 1)**3*exp(-2*I*pi/3)/10 + 3*(x - 1)**(1/3
)*(x + 1)**2*exp(-2*I*pi/3)/35 + 9*(x - 1)**(1/3)*(x + 1)*exp(-2*I*pi/3)/35
 + 54*(x - 1)**(1/3)*exp(-2*I*pi/3)/35, Abs(x + 1)/2 > 1), (3*(1 - x)**(1/3
)*(x + 1)**3/10 - 3*(1 - x)**(1/3)*(x + 1)**2/35 - 9*(1 - x)**(1/3)*(x + 1)
/35 - 54*(1 - x)**(1/3)/35, True))
```

3.1352 $\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=139

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

Rubi [A] time = 0.11, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {55, 617, 204, 31}

$$-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[3]
])/(b^(2/3)*(b*c - a*d)^(1/3)) - Log[a + b*x]/(2*b^(2/3)*(b*c - a*d)^(1/3))
 + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(2/3)*(b*c - a
*d)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx = -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{bc-ad}\,x}{\sqrt[3]{b}}+x^2}\,dx, x, \sqrt[3]{c+dx}\right)}{2b} - \frac{3\,\text{Subst}\left(\int \cdots \right)}{2b^{2/3}\cdots}$$

$$= -\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\cdots}{\cdots}\right)}{b^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{bc-ad}} - \frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.08, size = 106, normalized size = 0.76

$$\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)-\log(a+bx)}{2b^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]] - Log[a + b*x] + 3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2
*b^(2/3)*(b*c - a*d)^(1/3))
```

IntegrateAlgebraic [A] time = 0.23, size = 191, normalized size = 1.37

$$-\frac{\log\left(\sqrt[3]{ad-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{b^{2/3}\sqrt[3]{ad-bc}} + \frac{\log\left(-\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad-bc}+(ad-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{2b^{2/3}\sqrt[3]{ad-bc}} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{ad-bc}}\right)}{b^{2/3}\sqrt[3]{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*(-(b*c)
+ a*d)^(1/3))])/(b^(2/3)*(-(b*c) + a*d)^(1/3))) - Log[(-(b*c) + a*d)^(1/3)
+ b^(1/3)*(c + d*x)^(1/3)]/(b^(2/3)*(-(b*c) + a*d)^(1/3)) + Log[(-(b*c) + a
*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x
)^(2/3)]/(2*b^(2/3)*(-(b*c) + a*d)^(1/3))
```

fricas [B] time = 1.27, size = 570, normalized size = 4.10

$$\left[\frac{\sqrt{3}(b^2c-abd)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\log\left(\frac{2b^2dx+3b^2c-abd-\sqrt{3}\left((b^3c-ab^2d)^{\frac{1}{3}}(bc-ad)+(b^2c-abd)(dx+c)^{\frac{1}{3}}-2(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{2}{3}}\right)\sqrt{-\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}-3(b^3c-ab^2d)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2(b^3c-ab^2d)}, \frac{2\sqrt{3}(b^2c-abd)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}\arctan\left(\frac{\sqrt{3}\left(2(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{1}{3}}\right)\sqrt{\frac{(b^3c-ab^2d)^{\frac{1}{3}}}{bc-ad}}}{3b}\right)-(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{2}{3}}b^2+(b^3c-ab^2d)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}b+(b^3c-ab^2d)^{\frac{2}{3}}\right)+2(b^3c-ab^2d)^{\frac{2}{3}}\log\left((dx+c)^{\frac{1}{3}}b-(b^3c-ab^2d)^{\frac{1}{3}}\right)}{2(b^3c-ab^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))*lo
g((2*b^2*d*x + 3*b^2*c - a*b*d - sqrt(3)*((b^3*c - a*b^2*d)^(1/3)*(b*c - a*
d) + (b^2*c - a*b*d)*(d*x + c)^(1/3) - 2*(b^3*c - a*b^2*d)^(2/3)*(d*x + c)^
(2/3))*sqrt(-(b^3*c - a*b^2*d)^(1/3)/(b*c - a*d)) - 3*(b^3*c - a*b^2*d)^(2/
```

```
3)*(d*x + c)^(1/3))/(b*x + a)) - (b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(2/3
)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)*b + (b^3*c - a*b^2*d)^(2/3)
) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/3)*b - (b^3*c - a*b^2*d)^(1/
3)))/(b^3*c - a*b^2*d), 1/2*(2*sqrt(3)*(b^2*c - a*b*d)*sqrt((b^3*c - a*b^2*
d)^(1/3)/(b*c - a*d))*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3)*b + (b^3*c - a*
b^2*d)^(1/3))*sqrt((b^3*c - a*b^2*d)^(1/3)/(b*c - a*d))/b) - (b^3*c - a*b^2
*d)^(2/3)*log((d*x + c)^(2/3)*b^2 + (b^3*c - a*b^2*d)^(1/3)*(d*x + c)^(1/3)
*b + (b^3*c - a*b^2*d)^(2/3)) + 2*(b^3*c - a*b^2*d)^(2/3)*log((d*x + c)^(1/
3)*b - (b^3*c - a*b^2*d)^(1/3)))/(b^3*c - a*b^2*d)]
```

giac [A] time = 1.10, size = 196, normalized size = 1.41

$$\frac{3\left(b^{3}c-ab^{2}d\right)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(dx+c\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{3}c-\sqrt{3}\,ab^{2}d}-\frac{\log\left(\left(dx+c\right)^{\frac{2}{3}}+\left(dx+c\right)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\log\left(\left|\left(dx+c\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] 3*(b^3*c - a*b^2*d)^(2/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c - a
*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^3*c - sqrt(3)*a*b^2*d) - 1/
2*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)/b)^(1/3) + ((b*c - a*d
)/b)^(2/3))/(b^3*c - a*b^2*d)^(1/3) + ((b*c - a*d)/b)^(2/3)*log(abs((d*x +
c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

maple [A] time = 0.01, size = 161, normalized size = 1.16

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\left(dx+c\right)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}b}-\frac{\ln\left(\left(dx+c\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}b}+\frac{\ln\left(\left(dx+c\right)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\left(dx+c\right)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(1/3),x)
```

```
[Out] -1/b/((a*d-b*c)/b)^(1/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))+1/2/b/((a*d-
b*c)/b)^(1/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)
/b)^(2/3))+3^(1/2)/b/((a*d-b*c)/b)^(1/3)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/b
)^(1/3)*(d*x+c)^(1/3)-1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.21, size = 204, normalized size = 1.47

$$\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)}{b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}+\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{{\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}-\frac{\ln\left(9\,b\,{\left(c+d\,x\right)}^{1/3}-\frac{{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^2\,\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{4\,b^{4/3}\,{\left(b\,c-a\,d\right)}^{2/3}}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{2\,b^{2/3}\,{\left(b\,c-a\,d\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(1/3)),x)
```

```
[Out] log(9*b*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(4/3)*(b*c - a*d)^(2/3))
)/(b^(2/3)*(b*c - a*d)^(1/3)) + (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i - 1
)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i - 1))
/(2*b^(2/3)*(b*c - a*d)^(1/3)) - (log(9*b*(c + d*x)^(1/3) - ((3^(1/2)*1i +
1)^2*(9*b^3*c - 9*a*b^2*d))/(4*b^(4/3)*(b*c - a*d)^(2/3)))*(3^(1/2)*1i + 1)
)/(2*b^(2/3)*(b*c - a*d)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(1/3)), x)
```

3.1353 $\int \frac{1}{(a+bx)(c+dx)^{2/3}} \, dx$

Optimal. Leaf size=140

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}$$

Rubi [A] time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = $\{57, 617, 204, 31\}$

$$-\frac{\log(a+bx)}{2\sqrt[3]{b}(bc-ad)^{2/3}}+\frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}(bc-ad)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{\sqrt[3]{b}(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/Sqrt[
3]])/(b^(1/3)*(b*c - a*d)^(2/3))) - Log[a + b*x]/(2*b^(1/3)*(b*c - a*d)^(2/
3)) + (3*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)])/(2*b^(1/3)*(b*c
- a*d)^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx)(c+dx)^{2/3}}\,dx = -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{b}}-x}\,dx, x, \sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{\frac{(bc-ad)^{2/3}}{b^{2/3}}+\frac{\sqrt[3]{b}}{}\cdots}\right.}{2b^{2/3}\sqrt[]{}\cdots}$$

$$= -\frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\cdots\right.}{\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}}{\sqrt{3}}\right)}{\sqrt[3]{b}\,(bc-ad)^{2/3}} - \frac{\log(a+bx)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}} + \frac{3\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

Mathematica [A] time = 0.07, size = 154, normalized size = 1.10

$$-\frac{\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{bc-ad}+(bc-ad)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)-2\log\left(\sqrt[3]{bc-ad}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{bc-ad}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{b}\,(bc-ad)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -1/2*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*(c + d*x)^(1/3))/(b*c - a*d)^(1/3))/
Sqrt[3]] - 2*Log[(b*c - a*d)^(1/3) - b^(1/3)*(c + d*x)^(1/3)] + Log[(b*c -
a*d)^(2/3) + b^(1/3)*(b*c - a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x)^
(2/3)])/(b^(1/3)*(b*c - a*d)^(2/3))
```

IntegrateAlgebraic [A] time = 0.21, size = 190, normalized size = 1.36

$$-\frac{\log\left(-\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad-bc}+(ad-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{2\sqrt[3]{b}\,(ad-bc)^{2/3}}+\frac{\log\left(\sqrt[3]{ad-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{\sqrt[3]{b}\,(ad-bc)^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{ad-bc}}\right)}{\sqrt[3]{b}\,(ad-bc)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*(-(b*c)
+ a*d)^(1/3))])/(b^(1/3)*(-(b*c) + a*d)^(2/3))) + Log[(-(b*c) + a*d)^(1/3)
+ b^(1/3)*(c + d*x)^(1/3)]/(b^(1/3)*(-(b*c) + a*d)^(2/3)) - Log[(-(b*c) + a
*d)^(2/3) - b^(1/3)*(-(b*c) + a*d)^(1/3)*(c + d*x)^(1/3) + b^(2/3)*(c + d*x
)^(2/3)]/(2*b^(1/3)*(-(b*c) + a*d)^(2/3))
```

fricas [B] time = 1.58, size = 900, normalized size = 6.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(3)*(b^2*c - a*b*d)*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1
/3)/b)*log(-(3*b^2*c^2 - 4*a*b*c*d + a^2*d^2 + 2*(b^2*c*d - a*b*d^2)*x + sq
rt(3)*(2*(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d
^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)
^(1/3))*sqrt(-(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b) - 3*(b^3*c^2 - 2
```

```
*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d)*(d*x + c)^(1/3))/(b*x + a)) + (b^
3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3)
 - (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b
^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2), -1/2*(2*sqrt(3)*
(b^2*c - a*b*d)*sqrt((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)/b)*arctan(1/
3*sqrt(3)*((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) + 2*(b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(2/3)*(d*x + c)^(1/3))*sqrt((b^3*c^2 - 2*a*b^
2*c*d + a^2*b*d^2)^(1/3)/b)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2)) + (b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)^(2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(2/3) - (b^3*c
^2 - 2*a*b^2*c*d + a^2*b*d^2)^(1/3)*(b*c - a*d) - (b^3*c^2 - 2*a*b^2*c*d +
a^2*b*d^2)^(2/3)*(d*x + c)^(1/3)) - 2*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)^(
2/3)*log(-(b^2*c - a*b*d)*(d*x + c)^(1/3) + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*
d^2)^(2/3)))/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)]
```

giac [A] time = 0.96, size = 207, normalized size = 1.48

$$-\frac{3\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(dx+c\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}\,b^{2}c-\sqrt{3}\,abd}-\frac{\left(b^{3}c-ab^{2}d\right)^{\frac{1}{3}}\log\left(\left(dx+c\right)^{\frac{2}{3}}+\left(dx+c\right)^{\frac{1}{3}}\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}+\left(\frac{bc-ad}{b}\right)^{\frac{2}{3}}\right)}{2\left(b^{2}c-abd\right)}+\frac{\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\log\left(\left|\left(dx+c\right)^{\frac{1}{3}}-\left(\frac{bc-ad}{b}\right)^{\frac{1}{3}}\right|\right)}{bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] -3*(b^3*c - a*b^2*d)^(1/3)*arctan(1/3*sqrt(3)*(2*(d*x + c)^(1/3) + ((b*c -
a*d)/b)^(1/3))/((b*c - a*d)/b)^(1/3))/(sqrt(3)*b^2*c - sqrt(3)*a*b*d) - 1/2
*(b^3*c - a*b^2*d)^(1/3)*log((d*x + c)^(2/3) + (d*x + c)^(1/3)*((b*c - a*d)
/b)^(1/3) + ((b*c - a*d)/b)^(2/3))/(b^2*c - a*b*d) + ((b*c - a*d)/b)^(1/3)*
log(abs((d*x + c)^(1/3) - ((b*c - a*d)/b)^(1/3)))/(b*c - a*d)
```

maple [A] time = 0.01, size = 160, normalized size = 1.14

$$\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(dx+c)^{\frac{1}{3}}}{\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}+\frac{\ln\left((dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}-\frac{\ln\left((dx+c)^{\frac{2}{3}}-\left(\frac{ad-bc}{b}\right)^{\frac{1}{3}}(dx+c)^{\frac{1}{3}}+\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}\right)}{2\left(\frac{ad-bc}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)^(2/3),x)
```

```
[Out] 1/b/((a*d-b*c)/b)^(2/3)*ln((d*x+c)^(1/3)+((a*d-b*c)/b)^(1/3))-1/2/b/((a*d-b
*c)/b)^(2/3)*ln((d*x+c)^(2/3)-((a*d-b*c)/b)^(1/3)*(d*x+c)^(1/3)+((a*d-b*c)/
b)^(2/3))+1/b/((a*d-b*c)/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/((a*d-b*c)/
b)^(1/3)*(d*x+c)^(1/3)-1))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.37, size = 206, normalized size = 1.47

$$\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}-\frac{9\,b^3\,c-9\,a\,b^2\,d}{b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)}{b^{1/3}\,(a\,d-b\,c)^{2/3}}+\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}-\frac{\left(-1+\sqrt{3}\,1\mathrm{i}\right)\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}-\frac{\ln\left(9\,b^2\,(c+d\,x)^{1/3}+\frac{\left(1+\sqrt{3}\,1\mathrm{i}\right)\left(9\,b^3\,c-9\,a\,b^2\,d\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{2\,b^{1/3}\,(a\,d-b\,c)^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)*(c + d*x)^(2/3)),x)
```

```
[Out] log(9*b^2*(c + d*x)^(1/3) - (9*b^3*c - 9*a*b^2*d)/(b^(1/3)*(a*d - b*c)^(2/3
)))/(b^(1/3)*(a*d - b*c)^(2/3)) + (log(9*b^2*(c + d*x)^(1/3) - ((3^(1/2)*1i
 - 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i - 1
))/(2*b^(1/3)*(a*d - b*c)^(2/3)) - (log(9*b^2*(c + d*x)^(1/3) + ((3^(1/2)*1
i + 1)*(9*b^3*c - 9*a*b^2*d))/(2*b^(1/3)*(a*d - b*c)^(2/3)))*(3^(1/2)*1i +
1))/(2*b^(1/3)*(a*d - b*c)^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)*(c + d*x)**(2/3)), x)
```

3.1354 $\int (a+bx)^{7/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=230

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+\cdots}}{240bd\cdots}$$

Rubi [A] time = 0.16, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {50, 63, 217, 206}

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{3/2}d^{9/2}}-\frac{7\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128bd^4}+\frac{7(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{192bd^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{240bd^2}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40bd}+\frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] (-7*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b*d^4) + (7*(b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(192*b*d^3) - (7*(b*c - a*d)^2*(a + b*x)^
(5/2)*Sqrt[c + d*x])/(240*b*d^2) + ((b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*
x])/(40*b*d) + ((a + b*x)^(9/2)*Sqrt[c + d*x])/(5*b) + (7*(b*c - a*d)^5*Arc
Tanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(3/2)*d^(9/2)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{7/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} + \frac{(bc-ad)\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}dx}{10b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+dx}}{5b} - \frac{\left(7(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}dx}{80bd}$$

$$= -\frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40bd} + \frac{(a+bx)^{9/2}\sqrt{c+}}{5b}$$

$$= \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{240bd^2} + \frac{(bc-ad)(a+b}{40b}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+}{240}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+}{240}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+}{240}$$

$$= -\frac{7(bc-ad)^4\sqrt{a+bx}\sqrt{c+dx}}{128bd^4} + \frac{7(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{192bd^3} - \frac{7(bc-ad)^2(a+}{240}$$

Mathematica [A] time = 1.47, size = 194, normalized size = 0.84

$$\frac{(a+bx)^{9/2}\sqrt{c+dx}\left(\frac{70(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{9/2}(a+bx)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} - \frac{70(bc-ad)^4}{d^4(a+bx)^4} + \frac{140(bc-ad)^3}{3d^3(a+bx)^3} - \frac{112(bc-ad)^2}{3d^2(a+bx)^2} + \frac{32bc-32ad}{ad+bdx} + 256\right)}{1280b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] ((a + b*x)^(9/2)*Sqrt[c + d*x]*(256 - (70*(b*c - a*d)^4)/(d^4*(a + b*x)^4)
+ (140*(b*c - a*d)^3)/(3*d^3*(a + b*x)^3) - (112*(b*c - a*d)^2)/(3*d^2*(a +
 b*x)^2) + (32*b*c - 32*a*d)/(a*d + b*d*x) + (70*(b*c - a*d)^(9/2)*ArcSinh[
(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(9/2)*(a + b*x)^(9/2)*Sqrt[(b*
(c + d*x))/(b*c - a*d)])))/(1280*b)
```

IntegrateAlgebraic [A] time = 0.31, size = 198, normalized size = 0.86

$$\frac{7(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{128b^{3/2}d^{9/2}} - \frac{\sqrt{c+dx}\,(bc-ad)^5\left(\frac{105b^4(c+dx)^4}{(a+bx)^4} - \frac{490b^3d(c+dx)^3}{(a+bx)^3} + \frac{896b^2d^2(c+dx)^2}{(a+bx)^2} - \frac{790bd^3(c+dx)}{a+bx} - 105d^4\right)}{1920bd^4\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx} - d\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/1920*((b*c - a*d)^5*Sqrt[c + d*x]*(-105*d^4 - (790*b*d^3*(c + d*x))/(a +
 b*x) + (896*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 - (490*b^3*d*(c + d*x)^3)/(a
+ b*x)^3 + (105*b^4*(c + d*x)^4)/(a + b*x)^4))/(b*d^4*Sqrt[a + b*x]*(-d + (
b*(c + d*x))/(a + b*x))^5) + (7*(b*c - a*d)^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x
])/(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(3/2)*d^(9/2))
```

fricas [A] time = 1.74, size = 702, normalized size = 3.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^
2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*
a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*
x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 49
0*a*b^4*c^3*d^2 - 896*a^2*b^3*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 +
 48*(b^5*c*d^4 + 31*a*b^4*d^5)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 26
3*a^2*b^3*d^5)*x^2 + 2*(35*b^5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*
d^4 + 605*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^5), -1/3840*(
105*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*
a^4*b*c*d^4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*
d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)
*x)) - 2*(384*b^5*d^5*x^4 - 105*b^5*c^4*d + 490*a*b^4*c^3*d^2 - 896*a^2*b^3
*c^2*d^3 + 790*a^3*b^2*c*d^4 + 105*a^4*b*d^5 + 48*(b^5*c*d^4 + 31*a*b^4*d^5
)*x^3 - 8*(7*b^5*c^2*d^3 - 32*a*b^4*c*d^4 - 263*a^2*b^3*d^5)*x^2 + 2*(35*b^
5*c^3*d^2 - 161*a*b^4*c^2*d^3 + 289*a^2*b^3*c*d^4 + 605*a^3*b^2*d^5)*x)*sqr
t(b*x + a)*sqrt(d*x + c))/(b^2*d^5)]
```

giac [B] time = 1.84, size = 1107, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^4*abs(b)/
b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*
(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4
+ 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a
*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x +
 a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 3
5*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d -
a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*b*abs(b) + (sqrt(b^2*c + (b*x + a)*b*d - a*
b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^1
9*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/
(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 4
47*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3
*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^
8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*
a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x
+ a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*b^2*abs(b
) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a^3*abs(b)/b^2)/b
```

maple [B] time = 0.01, size = 858, normalized size = 3.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(7/2)*(d*x+c)^(3/2)+7/40/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)*a+7/48
/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a^2+7/64/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^3+2
1/64/d^3*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a*b^2*c^2+21/64/d^2*(d*x+c)^(1/2)*(b*x
+a)^(1/2)*a^2*c^2*b-7/32/d^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b^2-21/64/d^
2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2*b*c-7/24/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)*
a*b*c-7/40/d^2*(b*x+a)^(5/2)*(d*x+c)^(3/2)*b*c+7/48/d^3*(b*x+a)^(3/2)*(d*x+
c)^(3/2)*b^2*c^2-7/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b^3*c^3+7/128/b*(d*x+
c)^(1/2)*(b*x+a)^(1/2)*a^4-7/32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c+7/128/d
^4*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^4*b^3+35/256*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(a*d
+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^4*c+35/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(d
*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(d*x^2*b+(
a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^2*c^3*b^2-35/256/d^3*((b*x+a)*(d*x+c))
^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(
d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a*c^4*b^3-7/256*d/b*((b*x+a)*(d
*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(
1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^5-35/128/d*((b*x+a)*(d*
x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1
/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*a^3*c^2*b+7/256/d^4*((b*x+
a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b
*d)^(1/2)+(d*x^2*b+(a*d+b*c)*x+a*c)^(1/2))/(b*d)^(1/2)*c^5*b^4
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{7/2}\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)*(c + d*x)^(1/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1355 $\int (a+bx)^{5/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=192

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+}}{24bd}$$

Rubi [A] time = 0.09, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{3/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64bd^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96bd^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24bd}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b*d^3) - (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*b*d^2) + ((b*c - a*d)*(a + b*x)^(5/2)*S
qrt[c + d*x])/(24*b*d) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*b) - (5*(b*c -
a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(3/2
)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4b} - \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{48bd}$$

$$= -\frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24bd} + \frac{(a+bx)^{7/2}\sqrt{c+}}{4b}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)}{24bd}$$

$$= \frac{5(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64bd^3} - \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96bd^2} + \frac{(bc-ad)(a+bx)}{24bd}$$

Mathematica [A] time = 0.60, size = 190, normalized size = 0.99

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)\left(15a^3d^3+a^2bd^2(73c+118dx)+ab^2d\left(-55c^2+36cdx+136d^2x^2\right)+b^3\left(15c^3-10c^2dx+8cd^2x^2+48d^3x^3\right)\right)-15(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{192b^2d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 + a^2*b*d^2*(73*c + 118*d*x)
 + a*b^2*d*(-55*c^2 + 36*c*d*x + 136*d^2*x^2) + b^3*(15*c^3 - 10*c^2*d*x +
8*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^2*d^(7/2)*S
qrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.29, size = 176, normalized size = 0.92

$$\frac{\sqrt{c+dx}(bc-ad)^4\left(\frac{15b^3(c+dx)^3}{(a+bx)^3}-\frac{55b^2d(c+dx)^2}{(a+bx)^2}+\frac{73bd^2(c+dx)}{a+bx}+15d^3\right)}{192bd^3\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^4}-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{64b^{3/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)^4*Sqrt[c + d*x]*(15*d^3 + (73*b*d^2*(c + d*x))/(a + b*x) - (55
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (15*b^3*(c + d*x)^3)/(a + b*x)^3))/(192*b
*d^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^4) - (5*(b*c - a*d)^4*Arc
Tanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*b^(3/2)*d^(7/2))
```

fricas [A] time = 1.42, size = 540, normalized size = 2.81

$$\left[\frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8\left(b^4cd^3+17ab^3d^4\right)x^2-2\left(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768b^2d^4}, \frac{15\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(48b^4d^4x^3+15b^4c^3d-55ab^3c^2d^2+73a^2b^2cd^3+15a^3bd^4+8\left(b^4cd^3+17ab^3d^4\right)x^2-2\left(5b^4c^2d^2-18ab^3cd^3-59a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384b^2d^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2
 - 18*a*b^3*c*d^3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^
4), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d - 55*a*b^3*c^2*d^2 + 73*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(b^4*c*d^3 + 17*a*b^3*d^4)*x^2 - 2*(5*b^4*c^2*d^2 - 18*a*b^3*c*d^
3 - 59*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^4)]
```

giac [B] time = 1.64, size = 726, normalized size = 3.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*abs(b) - 192*((b^2*c - a*b*d)*lo
g(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt
(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*abs(b)/b^2 +
 (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a
)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b
^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^
2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*
(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^
4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*b^2*d^3))*b*abs(b) + 144*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(
2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
 - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)))/(sqrt(b*d)*d))*a^2*abs(b)/b^2)/b
```

maple [B] time = 0.01, size = 645, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(5/2)*(d*x+c)^(3/2)+5/24/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)*a-5/24
/d^2*(b*x+a)^(3/2)*(d*x+c)^(3/2)*b*c+5/32/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a^2
-5/16/d^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a*b*c+5/32/d^3*(b*x+a)^(1/2)*(d*x+c)^
(3/2)*b^2*c^2+5/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64/d*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2*b-5/64/d^3*
(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^3*b^2-5/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c-15/64/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^2*b+5/32/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
```

```
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^3*b^2-5/128/d^3*((b*x+a)*(d*x+c))^(1/2)
/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^
2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^4*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{5/2}\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1356 $\int (a+bx)^{3/2}\sqrt{c+dx}\,dx$

Optimal. Leaf size=154

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Rubi [A] time = 0.07, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{3/2}d^{5/2}}-\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8bd^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12bd}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b*d^2) + ((b*c - a*d)*(a +
b*x)^(3/2)*Sqrt[c + d*x])/(12*b*d) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*b)
+ ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(3/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}\sqrt{c+dx}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} + \frac{(bc-ad)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}dx}{6b}$$
$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b} - \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{8bd}$$
$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$
$$= -\frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8bd^2} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12bd} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3b}$$

Mathematica [A] time = 0.43, size = 151, normalized size = 0.98

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)\left(3a^2d^2+2abd(4c+7dx)+b^2\left(-3c^2+2cdx+8d^2x^2\right)\right)+3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^2d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 + 2*a*b*d*(4*c + 7*d*x) + b^2
*(-3*c^2 + 2*c*d*x + 8*d^2*x^2)) + 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^2*d^(5/
2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.26, size = 154, normalized size = 1.00

$$\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8b^{3/2}d^{5/2}} - \frac{\sqrt{c+dx}(bc-ad)^3\left(\frac{3b^2(c+dx)^2}{(a+bx)^2}-\frac{8bd(c+dx)}{a+bx}-3d^2\right)}{24bd^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*Sqrt[c + d*x],x]
```

```
[Out] -1/24*((b*c - a*d)^3*Sqrt[c + d*x]*(-3*d^2 - (8*b*d*(c + d*x))/(a + b*x) +
(3*b^2*(c + d*x)^2)/(a + b*x)^2))/(b*d^2*Sqrt[a + b*x]*(-d + (b*(c + d*x))/
(a + b*x))^3) + ((b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqr
t[a + b*x])])/(8*b^(3/2)*d^(5/2))
```

fricas [A] time = 1.37, size = 410, normalized size = 2.66

$$\left[-\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2-3b^3c^2d+8ab^2cd^2+3a^2bd^3+2\left(b^3cd^2+7ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^2d^3}, -\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(8b^3d^3x^2-3b^3c^2d+8ab^2cd^2+3a^2bd^3+2\left(b^3cd^2+7ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^2d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b*d^3 + 2*(b^3*c*d^2 + 7*a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3), -1/48*(3*(b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a
*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 - 3*b^3*c^2*d + 8*a*b^2*c*d^2 + 3*a^2*b
*d^3 + 2*(b^3*c*d^2 + 7*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^3
)]
```

giac [B] time = 1.38, size = 438, normalized size = 2.84

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{b^2}+\frac{b^6cd^3-13\,ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2\,ab^6cd^3-11\,a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)|b|-\frac{24\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\right)a^2|b|}{b^2}+\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2\,ab^2cd-3\,a^2bd^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)a|b|}{b^2}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*((sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*abs(b) - 24*((b^2*c - a*b*d)*log(abs(-
sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(b*d) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*abs(b)/b^2 + 12*(sq
rt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqr
t(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a*abs(b)/b^
2)/b
```

maple [B] time = 0.01, size = 460, normalized size = 2.99

$$-\frac{\sqrt{(bx+a)(dx+c)}\,a^3d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{3\sqrt{(bx+a)(dx+c)}\,a^2c\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{(bx+a)(dx+c)}\,ab\,c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{\sqrt{(bx+a)(dx+c)}\,b^2c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,a^2}{8b}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,ac}{4d}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,b\,c^2}{8d^2}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}a}{4d}-\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}bc}{4d^2}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(3/2)*(d*x+c)^(3/2)+1/4/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)*a-1/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(3/2)*b*c+1/8/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2-1/
4/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c+1/8/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
*b-1/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1
/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^
3+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*
d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c-3
/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d
+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^2*b+
1/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*
a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^3*b
^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b x)^{3/2} \sqrt{c + d x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1357 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Rubi [A] time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,d\right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\right.}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.28, size = 118, normalized size = 1.02

$$\frac{b\sqrt{d}\sqrt{a+bx}\,(c+dx)(ad+b(c+2dx)) - (bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4b^2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(a*d + b*(c + 2*d*x)) - (b*c - a*d)^(5/2
)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(4*b^2*d^(3/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.21, size = 129, normalized size = 1.11

$$\frac{\sqrt{c+dx}\,(bc-ad)^2\left(\frac{b(c+dx)}{a+bx}+d\right)}{4bd\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}-d\right)^2} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4b^{3/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[c + d*x]*(d + (b*(c + d*x))/(a + b*x)))/(4*b*d*Sqrt[a +
 b*x]*(-d + (b*(c + d*x))/(a + b*x))^2) - ((b*c - a*d)^2*ArcTanh[(Sqrt[b]*S
qrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*b^(3/2)*d^(3/2))
```

fricas [A] time = 1.50, size = 300, normalized size = 2.59

$$\left[\frac{(b^2c^2-2abcd+a^2d^2)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8(b^2cd+abd^2)x\right)+4\left(2b^2d^2x+b^2cd+abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{16b^2d^2}, \frac{(b^2c^2-2abcd+a^2d^2)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)+2\left(2b^2d^2x+b^2cd+abd^2\right)\sqrt{bx+a}\sqrt{dx+c}}{8b^2d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
```

```
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

giac [B] time = 1.25, size = 232, normalized size = 2.00

$$-\frac{\frac{4\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)a|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c^2+2ab^2cd-3a^2bd^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)|b|}{b^2}}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*a*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*abs(b)/b^2)/b
```

maple [B] time = 0.01, size = 305, normalized size = 2.63

$$-\frac{\sqrt{(bx+a)(dx+c)}\,a^2d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{\sqrt{(bx+a)(dx+c)}\,ac\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{4\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{\sqrt{(bx+a)(dx+c)}\,bc^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,a}{4b}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,c}{4d}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a-1/4/d
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^2+1/4*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x
+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^
(1/2))/(b*d)^(1/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^
(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2
))/(b*d)^(1/2)*c^2*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.14, size = 88, normalized size = 0.76

$$\left(\frac{x}{2}+\frac{a\,d+b\,c}{4\,b\,d}\right)\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}-\frac{\ln\left(a\,d+b\,c+2\,b\,d\,x+2\,\sqrt{b}\,\sqrt{d}\,\sqrt{a+b\,x}\,\sqrt{c+d\,x}\right)\,{\left(a\,d-b\,c\right)}^2}{8\,b^{3/2}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(1/2),x)
```

```
[Out] (x/2 + (a*d + b*c)/(4*b*d))*(a + b*x)^(1/2)*(c + d*x)^(1/2) - (log(a*d + b*
c + 2*b*d*x + 2*b^(1/2)*d^(1/2)*(a + b*x)^(1/2)*(c + d*x)^(1/2))*(a*d - b*c
)^2/(8*b^(3/2)*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+bx}\,\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

3.1358 $\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=72

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}+\frac{\sqrt{a+bx}\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/b + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(b^(3/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{b} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}\sqrt{d}}$$

Mathematica [A] time = 0.10, size = 117, normalized size = 1.62

$$\frac{\sqrt{c+dx}\left(\sqrt{d}\,\sqrt{a+bx}\,\sqrt{\frac{b(c+dx)}{bc-ad}} + \sqrt{bc-ad}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{b\sqrt{d}\,\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)] + Sqr
t[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b*Sqrt[d]*
Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

IntegrateAlgebraic [A] time = 0.31, size = 104, normalized size = 1.44

$$\frac{\sqrt{c+dx}\,\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}}{b} - \frac{\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/b - (Sqrt[b/d]*(b*c - a
*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/
b^2
```

fricas [A] time = 1.32, size = 236, normalized size = 3.28

$$\left[\frac{4\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2-4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{4\,b^2d}, \frac{2\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{2\,b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b^2*d), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b^2*d)]
```

giac [A] time = 1.10, size = 93, normalized size = 1.29

$$-\frac{\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)|b|}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
a))*abs(b)/b^3
```

maple [A] time = 0.01, size = 107, normalized size = 1.49

$$-\frac{(ad-bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{2\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/b-1/2*(a*d-b*c)/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 4.01, size = 260, normalized size = 3.61

$$\frac{\frac{(2\,a\,d+2\,b\,c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)}+\frac{(2\,a\,d+2\,b\,c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{b\,d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}-\frac{8\,\sqrt{a}\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}-\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)(a\,d-b\,c)}{b^{3/2}\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2)))/(d^2*((c + d*x)^(1/2) - c^(1
/2))) + ((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(b*d*((c + d*x)^(1/
2) - c^(1/2))^3) - (8*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c
 + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d*x)^(1/2
) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*((c + d*x
)^(1/2) - c^(1/2))^2)) - (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/2)))/(b^
(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(3/2)*d^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/sqrt(a + b*x), x)
```

3.1359 $\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx &= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{d\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{b}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^2}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^2}\\
&= -\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}+\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.23, size = 99, normalized size = 1.50

$$
\frac{2\left(\sqrt{d}\,\sqrt{bc-ad}\,\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-\frac{b(c+dx)}{\sqrt{a+bx}}\right)}{b^2\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (2*(-((b*(c + d*x))/Sqrt[a + b*x]) + Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d
*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(b^2*S
qrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.10, size = 66, normalized size = 1.00

$$
\frac{2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{b^{3/2}}-\frac{2\sqrt{c+dx}}{b\sqrt{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(b*Sqrt[a + b*x]) + (2*Sqrt[d]*ArcTanh[(Sqrt[b]*Sqrt[c +
 d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(3/2)
```

fricas [B] time = 1.48, size = 241, normalized size = 3.65

$$
\left[\frac{(bx+a)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2x+ab\right)}, -\frac{(bx+a)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\sqrt{bx+a}\,\sqrt{dx+c}}{b^2x+ab}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*x + a)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*x + a*b), -((b
*x + a)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(b^2*x + a*b)]
```

giac [B] time = 1.19, size = 131, normalized size = 1.98

$$-\frac{\left(\frac{\sqrt{bd}\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b}+\frac{4\left(\sqrt{bd}\,bc-\sqrt{bd}\,ad\right)}{b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2}\right)|b|}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(sqrt(b*d)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^2)/b + 4*(sqrt(b*d)*b*c - sqrt(b*d)*a*d)/(b^2*c - a*b*d - (sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2))*abs(b)/b^2
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{dx+c}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(1/2)/(b*x+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c+d\,x}}{{(a+b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(1/2)/(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(3/2), x)
```

3.1360 $\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3(bc-ad)(a+bx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{3/2}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(5/2),x]
[Out] (-2*(c + d*x)^(3/2))/(3*(b*c - a*d)*(a + b*x)^(3/2))
```

fricas [B] time = 1.53, size = 65, normalized size = 2.03

$$-\frac{2\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}}{3\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x + a)*(d*x + c)^(3/2)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x^2
 + 2*(a*b^2*c - a^2*b*d)*x)
```

giac [B] time = 1.43, size = 152, normalized size = 4.75

$$-\frac{4\left(\sqrt{bd}\,b^4c^2d - 2\sqrt{bd}\,ab^3cd^2 + \sqrt{bd}\,a^2b^2d^3 + 3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^4 d\right)|b|}{3\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)^3 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -4/3*(sqrt(b*d)*b^4*c^2*d - 2*sqrt(b*d)*a*b^3*c*d^2 + sqrt(b*d)*a^2*b^2*d^3
 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*d)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^2)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\,(dx+c)^{\frac{3}{2}}}{3\,(bx+a)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(5/2),x)
```

```
[Out] 2/3/(b*x+a)^(3/2)*(d*x+c)^(3/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.72, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{3/2}}{\left(3\,a\,d-3\,b\,c\right)\,{\left(a+b\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(5/2),x)
```

```
[Out] (2*(c + d*x)^(3/2))/((3*a*d - 3*b*c)*(a + b*x)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral(sqrt(c + d*x)/(a + b*x)**(5/2), x)
```

3.1361 $\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}} dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2} - \frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{4d(c+dx)^{3/2}}{15(a+bx)^{3/2}(bc-ad)^2} - \frac{2(c+dx)^{3/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (4*d*(c + d*x)^(3/2)
)/(15*(b*c - a*d)^2*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}} dx &= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} - \frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}} dx}{5(bc-ad)}\\ &= -\frac{2(c+dx)^{3/2}}{5(bc-ad)(a+bx)^{5/2}} + \frac{4d(c+dx)^{3/2}}{15(bc-ad)^2(a+bx)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{3/2}(5ad-3bc+2bdx)}{15(a+bx)^{5/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-3*b*c + 5*a*d + 2*b*d*x))/(15*(b*c - a*d)^2*(a + b*x)^
(5/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 57, normalized size = 0.86

$$-\frac{2\left(\frac{3b(c+dx)^{5/2}}{(a+bx)^{5/2}}-\frac{5d(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(7/2),x]
```

```
[Out] (-2*((-5*d*(c + d*x)^(3/2))/(a + b*x)^(3/2) + (3*b*(c + d*x)^(5/2))/(a + b*
x)^(5/2)))/(15*(b*c - a*d)^2)
```

fricas [B] time = 2.32, size = 175, normalized size = 2.65

$$\frac{2\left(2\,bd^2x^2-3\,bc^2+5\,acd-\left(bcd-5\,ad^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{15\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3+3\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2+3\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*b*d^2*x^2 - 3*b*c^2 + 5*a*c*d - (b*c*d - 5*a*d^2)*x)*sqrt(b*x + a)*
sqrt(d*x + c)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 + 3*(
a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

giac [B] time = 1.44, size = 447, normalized size = 6.77

$$\frac{8\left(\sqrt{bd}\,b^7c^3d^2-3\,\sqrt{bd}\,ab^6c^2d^3+3\,\sqrt{bd}\,a^2b^5cd^4-\sqrt{bd}\,a^3b^4d^5-5\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^5c^2d^2+10\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^4cd^3-5\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^3d^4-5\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^3cd^2+5\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^2d^3-15\,\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6bd^2\right)|b|}{15\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^5b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(sqrt(b*d)*b^7*c^3*d^2 - 3*sqrt(b*d)*a*b^6*c^2*d^3 + 3*sqrt(b*d)*a^2*b
^5*c*d^4 - sqrt(b*d)*a^3*b^4*d^5 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c^2*d^2 + 10*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^4*c*d^3 - 5*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
2*b^3*d^4 - 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*b^3*c*d^2 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a*b^2*d^3 - 15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b*d^2)*abs(b)/((b^2*c - a*b*d
 - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^2
)
```

maple [A] time = 0.01, size = 54, normalized size = 0.82

$$\frac{2\,(dx+c)^{\frac{3}{2}}\,(2bdx+5ad-3bc)}{15\,(bx+a)^{\frac{5}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/15*(d*x+c)^(3/2)*(2*b*d*x+5*a*d-3*b*c)/(b*x+a)^(5/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.82, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(10\,a\,d^2-2\,b\,c\,d\right)}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{6\,b\,c^2-10\,a\,c\,d}{15\,b^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,d^2\,x^2}{15\,b\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(7/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(10*a*d^2 - 2*b*c*d))/(15*b^2*(a*d - b*c)^2) - (6*b*c^
2 - 10*a*c*d)/(15*b^2*(a*d - b*c)^2) + (4*d^2*x^2)/(15*b*(a*d - b*c)^2)))/(
x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*x)^(1/2))/b
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.1362 $\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}} dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{3/2}(bc-ad)^3}+\frac{8d(c+dx)^{3/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (8*d*(c + d*x)^(3/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}} dx &= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}-\frac{(4d)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}} dx}{7(bc-ad)}\\
&= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}+\frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{5/2}} dx}{35(bc-ad)^2}\\
&= -\frac{2(c+dx)^{3/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{8d(c+dx)^{3/2}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{3/2}\left(35a^2d^2+14abd(2dx-3c)+b^2\left(15c^2-12cdx+8d^2x^2\right)\right)}{105(a+bx)^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(35*a^2*d^2 + 14*a*b*d*(-3*c + 2*d*x) + b^2*(15*c^2 - 1
2*c*d*x + 8*d^2*x^2)))/(105*(b*c - a*d)^3*(a + b*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 83, normalized size = 0.82

$$-\frac{2\left(\frac{15b^2(c+dx)^{7/2}}{(a+bx)^{7/2}}+\frac{35d^2(c+dx)^{3/2}}{(a+bx)^{3/2}}-\frac{42bd(c+dx)^{5/2}}{(a+bx)^{5/2}}\right)}{105(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(9/2),x]
```

```
[Out] (-2*((35*d^2*(c + d*x)^(3/2))/(a + b*x)^(3/2) - (42*b*d*(c + d*x)^(5/2))/(a
 + b*x)^(5/2) + (15*b^2*(c + d*x)^(7/2))/(a + b*x)^(7/2)))/(105*(b*c - a*d)
^3)
```

fricas [B] time = 3.93, size = 337, normalized size = 3.34

$$-\frac{2\left(8b^2d^3x^3+15b^2c^3-42abc^2d+35a^2cd^2-4\left(b^2cd^2-7abd^3\right)x^2+\left(3b^2c^2d-14abcd^2+35a^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{105\left(a^4b^3c^3-3a^5b^2c^2d+3a^6bcd^2-a^7d^3+\left(b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3\right)x^3+6\left(a^2b^5c^3-3a^3b^4c^2d+3a^4b^3cd^2-a^5b^2d^3\right)x^2+4\left(a^3b^4c^3-3a^4b^3c^2d+3a^5b^2cd^2-a^6bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/105*(8*b^2*d^3*x^3 + 15*b^2*c^3 - 42*a*b*c^2*d + 35*a^2*c*d^2 - 4*(b^2*c
*d^2 - 7*a*b*d^3)*x^2 + (3*b^2*c^2*d - 14*a*b*c*d^2 + 35*a^2*d^3)*x)*sqrt(b
*x + a)*sqrt(d*x + c)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 - a^7*
d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 + 4*(a*
b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^2*b^5
*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b^4*c^
3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

giac [B] time = 1.58, size = 689, normalized size = 6.82

$$-\frac{32\left(\sqrt{bd}b^{10}c^4d^3-4\sqrt{bd}ab^9c^3d^4+6\sqrt{bd}a^2b^8c^2d^5-4\sqrt{bd}a^3b^7cd^6+\sqrt{bd}a^4b^6d^7-7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^8c^3d^3+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^7c^2d^4-21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^6cd^5+7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^5d^6+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^6c^2d^3-42\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^5cd^4+21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^4d^5+35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^4cd^3-35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6ab^3d^4+70\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8b^2d^3\right)|b|}{105\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^7b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] -32/105*(sqrt(b*d)*b^10*c^4*d^3 - 4*sqrt(b*d)*a*b^9*c^3*d^4 + 6*sqrt(b*d)*a
^2*b^8*c^2*d^5 - 4*sqrt(b*d)*a^3*b^7*c*d^6 + sqrt(b*d)*a^4*b^6*d^7 - 7*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8
*c^3*d^3 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^7*c^2*d^4 - 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*d^5 + 7*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^5*d^6 + 21*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6
*c^2*d^3 - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^5*c*d^4 + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^5 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^4*c*d^3 - 35*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^3*d^
4 + 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*b^2*d^3)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^2)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\left(dx+c\right)^{\frac{3}{2}}\left(8b^2x^2d^2+28ab\,d^2x-12b^2cdx+35a^2d^2-42abcd+15b^2c^2\right)}{105\left(bx+a\right)^{\frac{7}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/105*(d*x+c)^(3/2)*(8*b^2*d^2*x^2+28*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-42*
a*b*c*d+15*b^2*c^2)/(b*x+a)^(7/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.97, size = 203, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{70\,a^2\,c\,d^2-84\,a\,b\,c^2\,d+30\,b^2\,c^3}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{x\,\left(70\,a^2\,d^3-28\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^3\,x^3}{105\,b\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^3}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((30*b^2*c^3 + 70*a^2*c*d^2 - 84*a*b*c^2*d)/(105*b^3*(a*d
- b*c)^3) + (x*(70*a^2*d^3 + 6*b^2*c^2*d - 28*a*b*c*d^2))/(105*b^3*(a*d - b
*c)^3) + (16*d^3*x^3)/(105*b*(a*d - b*c)^3) + (8*d^2*x^2*(7*a*d - b*c))/(10
5*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (
3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.1363 $\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{3/2}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{3/2}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{4d(c+dx)^{3/2}}{21(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(3/2)
)/(21*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(3/2))/(105*(b*c -
 a*d)^3*(a + b*x)^(5/2)) + (32*d^3*(c + d*x)^(3/2))/(315*(b*c - a*d)^4*(a +
 b*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(2d)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{3(bc-ad)}\\ &= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}}+\frac{\left(8d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{7/2}}\,dx}{21(bc-ad)^2}\\ &= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}}-\frac{\left(16d^3\right)\int \frac{\sqrt{c+}}{(a+bx}}{105(bc-a}\\ &= -\frac{2(c+dx)^{3/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{3/2}}{21(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{3/2}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{32d^3(c+}{315(bc-ad)^4}\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{3/2}\left(105a^3d^3+63a^2bd^2(2dx-3c)+9ab^2d\left(15c^2-12cdx+8d^2x^2\right)+b^3\left(-35c^3+30c^2dx-24cd^2x^2+16d^3x^3\right)\right)}{315(a+bx)^{9/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(105*a^3*d^3 + 63*a^2*b*d^2*(-3*c + 2*d*x) + 9*a*b^2*d*(
15*c^2 - 12*c*d*x + 8*d^2*x^2) + b^3*(-35*c^3 + 30*c^2*d*x - 24*c*d^2*x^2 +
 16*d^3*x^3)))/(315*(b*c - a*d)^4*(a + b*x)^(9/2))
```

IntegrateAlgebraic [**A**] time = 0.11, size = 109, normalized size = 0.80

$$-\frac{2\left(\frac{35b^3(c+dx)^{9/2}}{(a+bx)^{9/2}}-\frac{135b^2d(c+dx)^{7/2}}{(a+bx)^{7/2}}-\frac{105d^3(c+dx)^{3/2}}{(a+bx)^{3/2}}+\frac{189bd^2(c+dx)^{5/2}}{(a+bx)^{5/2}}\right)}{315(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(11/2),x]
```

```
[Out] (-2*((-105*d^3*(c + d*x)^(3/2))/(a + b*x)^(3/2) + (189*b*d^2*(c + d*x)^(5/2
))/(a + b*x)^(5/2) - (135*b^2*d*(c + d*x)^(7/2))/(a + b*x)^(7/2) + (35*b^3*
(c + d*x)^(9/2))/(a + b*x)^(9/2)))/(315*(b*c - a*d)^4)
```

fricas [**B**] time = 13.04, size = 532, normalized size = 3.91

$$\frac{2\left(16b^3d^4x^4-35b^3c^4+135ab^2c^3d-189a^2bc^2d^2+105a^3cd^3-8\left(b^3cd^3-9ab^2d^4\right)x^3+6\left(b^3c^2d^2-6ab^2cd^3+21a^2bd^4\right)x^2-\left(5b^3c^3d-27ab^2c^2d^2+63a^2bcd^3-105a^3d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{315\left(a^5b^4c^4-4a^6b^3c^3d+6a^7b^2c^2d^2-4a^8bcd^3+a^9d^4+\left(b^9c^4-4ab^8c^3d+6a^2b^7c^2d^2-4a^3b^6cd^3+a^4b^5d^4\right)x^5+5\left(ab^8c^4-4a^2b^7c^3d+6a^3b^6c^2d^2-4a^4b^5cd^3+a^5b^4d^4\right)x^4+10\left(a^2b^7c^4-4a^3b^6c^3d+6a^4b^5c^2d^2-4a^5b^4cd^3+a^6b^3d^4\right)x^3+10\left(a^3b^6c^4-4a^4b^5c^3d+6a^5b^4c^2d^2-4a^6b^3cd^3+a^7b^2d^4\right)x^2+5\left(a^4b^5c^4-4a^5b^4c^3d+6a^6b^3c^2d^2-4a^7b^2cd^3+a^8bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/315*(16*b^3*d^4*x^4 - 35*b^3*c^4 + 135*a*b^2*c^3*d - 189*a^2*b*c^2*d^2 +
105*a^3*c*d^3 - 8*(b^3*c*d^3 - 9*a*b^2*d^4)*x^3 + 6*(b^3*c^2*d^2 - 6*a*b^2*
c*d^3 + 21*a^2*b*d^4)*x^2 - (5*b^3*c^3*d - 27*a*b^2*c^2*d^2 + 63*a^2*b*c*d^
3 - 105*a^3*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^4*c^4 - 4*a^6*b^3*c^
3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 - 4*a*b^8*c^3*
d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5*(a*b^8*c^4 -
 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^4 +
 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 +
a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a
^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

giac [**B**] time = 2.04, size = 989, normalized size = 7.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 64/315*(sqrt(b*d)*b^13*c^5*d^4 - 5*sqrt(b*d)*a*b^12*c^4*d^5 + 10*sqrt(b*d)*
a^2*b^11*c^3*d^6 - 10*sqrt(b*d)*a^3*b^10*c^2*d^7 + 5*sqrt(b*d)*a^4*b^9*c*d^
8 - sqrt(b*d)*a^5*b^8*d^9 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*b^11*c^4*d^4 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^10*c^3*d^5 - 54*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
b^9*c^2*d^6 + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^3*b^8*c*d^7 - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^7*d^8 + 36*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^9*c^3*d^4 - 108*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^8*c^2*d^5 + 108*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^4*a^2*b^7*c*d^6 - 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
```

```
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^6*d^7 - 84*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^7*c^2*d^4 + 168
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a*b^6*c*d^5 - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^6*a^2*b^5*d^6 - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^5*c*d^4 + 189*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^4*d^5 - 315*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b
^3*d^4)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)^9*b^2)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2\,(dx+c)^{\frac{3}{2}}\left(16b^3x^3d^3+72a\,b^2d^3x^2-24b^3c\,d^2x^2+126a^2b\,d^3x-108a\,b^2c\,d^2x+30b^3c^2dx+105a^3d^3-189a^2bc\,d^2+135a\,b^2c^2d-35b^3c^3\right)}{315\,(bx+a)^{\frac{9}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(3/2)*(16*b^3*d^3*x^3+72*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2+126*a
^2*b*d^3*x-108*a*b^2*c*d^2*x+30*b^3*c^2*d*x+105*a^3*d^3-189*a^2*b*c*d^2+135
*a*b^2*c^2*d-35*b^3*c^3)/(b*x+a)^(9/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.18, size = 292, normalized size = 2.15

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^4\,x^4}{315\,b\,{\left(a\,d-b\,c\right)}^4}-\frac{-210\,a^3\,c\,d^3+378\,a^2\,b\,c^2\,d^2-270\,a\,b^2\,c^3\,d+70\,b^3\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(210\,a^3\,d^4-126\,a^2\,b\,c\,d^3+54\,a\,b^2\,c^2\,d^2-10\,b^3\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^3\,{\left(a\,d-b\,c\right)}^4}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^4*x^4)/(315*b*(a*d - b*c)^4) - (70*b^3*c^4 - 210*a^
3*c*d^3 + 378*a^2*b*c^2*d^2 - 270*a*b^2*c^3*d)/(315*b^4*(a*d - b*c)^4) + (x
*(210*a^3*d^4 - 10*b^3*c^3*d + 54*a*b^2*c^2*d^2 - 126*a^2*b*c*d^3))/(315*b^
4*(a*d - b*c)^4) + (16*d^3*x^3*(9*a*d - b*c))/(315*b^2*(a*d - b*c)^4) + (4*
d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^3*(a*d - b*c)^4)))/(x^4*
(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b
^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

3.1364 $\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{}{11}$$

Rubi [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{256d^4(c+dx)^{3/2}}{3465(a+bx)^{3/2}(bc-ad)^5}+\frac{128d^3(c+dx)^{3/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{32d^2(c+dx)^{3/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{16d(c+dx)^{3/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{3/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (16*d*(c + d*x)^(3
/2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (32*d^2*(c + d*x)^(3/2))/(231*(b*
c - a*d)^3*(a + b*x)^(7/2)) + (128*d^3*(c + d*x)^(3/2))/(1155*(b*c - a*d)^4
*(a + b*x)^(5/2)) - (256*d^4*(c + d*x)^(3/2))/(3465*(b*c - a*d)^5*(a + b*x)
^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{\sqrt{c+dx}}{(a+bx)^{13/2}}\,dx = -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}-\frac{(8d)\int \frac{\sqrt{c+dx}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}+\frac{\left(16d^2\right)\int \frac{\sqrt{c+dx}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}}-\frac{\left(64d^3\right)\int}{231(b}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{128}{1155(bc}$$

$$= -\frac{2(c+dx)^{3/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{16d(c+dx)^{3/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{32d^2(c+dx)^{3/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{128}{1155(bc}$$

Mathematica [**A**]   time = 0.08, size = 170, normalized size = 0.99

$$-\frac{2(c+dx)^{3/2}\left(1155a^4d^4+924a^3bd^3(2dx-3c)+198a^2b^2d^2\left(15c^2-12cdx+8d^2x^2\right)+44ab^3d\left(-35c^3+30c^2dx-24cd^2x^2+16d^3x^3\right)+b^4\left(315c^4-280c^3dx+240c^2d^2x^2-192cd^3x^3+128d^4x^4\right)\right)}{3465(a+bx)^{11/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*a^4*d^4 + 924*a^3*b*d^3*(-3*c + 2*d*x) + 198*a^2*
b^2*d^2*(15*c^2 - 12*c*d*x + 8*d^2*x^2) + 44*a*b^3*d*(-35*c^3 + 30*c^2*d*x
- 24*c*d^2*x^2 + 16*d^3*x^3) + b^4*(315*c^4 - 280*c^3*d*x + 240*c^2*d^2*x^2
 - 192*c*d^3*x^3 + 128*d^4*x^4)))/(3465*(b*c - a*d)^5*(a + b*x)^(11/2))
```

IntegrateAlgebraic [**A**]   time = 0.14, size = 117, normalized size = 0.68

$$-\frac{2(c+dx)^{3/2}\left(\frac{315b^4(c+dx)^4}{(a+bx)^4}-\frac{1540b^3d(c+dx)^3}{(a+bx)^3}+\frac{2970b^2d^2(c+dx)^2}{(a+bx)^2}-\frac{2772bd^3(c+dx)}{a+bx}+1155d^4\right)}{3465(a+bx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[c + d*x]/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(1155*d^4 - (2772*b*d^3*(c + d*x))/(a + b*x) + (2970*b^
2*d^2*(c + d*x)^2)/(a + b*x)^2 - (1540*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (31
5*b^4*(c + d*x)^4)/(a + b*x)^4))/(3465*(b*c - a*d)^5*(a + b*x)^(3/2))
```

fricas [**B**]   time = 27.26, size = 781, normalized size = 4.57

$$-\frac{2\left(128b^4d^5x^5+315b^4c^5-1540ab^3c^4d+2970a^2b^2c^3d^2-2772a^3bc^2d^3+1155a^4cd^4-64\left(b^4cd^4-11ab^3d^5\right)x^4+16\left(3b^4c^2d^3-22ab^3cd^4+99a^2b^2d^5\right)x^3-8\left(5b^4c^3d^2-33ab^3c^2d^3+99a^2b^2cd^4-231a^3bd^5\right)x^2+\left(35b^4c^4d-220ab^3c^3d^2+594a^2b^2c^2d^3-924a^3bcd^4+1155a^4d^5\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3465\left(a^6b^5c^5-5a^7b^4c^4d+10a^8b^3c^3d^2-10a^9b^2c^2d^3+5a^{10}bcd^4-a^{11}d^5+\left(b^{11}c^5-5ab^{10}c^4d+10a^2b^9c^3d^2-10a^3b^8c^2d^3+5a^4b^7cd^4-a^5b^6d^5\right)x^6+6\left(ab^{10}c^5-5a^2b^9c^4d+10a^3b^8c^3d^2-10a^4b^7c^2d^3+5a^5b^6cd^4-a^6b^5d^5\right)x^5+15\left(a^2b^9c^5-5a^3b^8c^4d+10a^4b^7c^3d^2-10a^5b^6c^2d^3+5a^6b^5cd^4-a^7b^4d^5\right)x^4+20\left(a^3b^8c^5-5a^4b^7c^4d+10a^5b^6c^3d^2-10a^6b^5c^2d^3+5a^7b^4cd^4-a^8b^3d^5\right)x^3+15\left(a^4b^7c^5-5a^5b^6c^4d+10a^6b^5c^3d^2-10a^7b^4c^2d^3+5a^8b^3cd^4-a^9b^2d^5\right)x^2+6\left(a^5b^6c^5-5a^6b^5c^4d+10a^7b^4c^3d^2-10a^8b^3c^2d^3+5a^9b^2cd^4-a^{10}bd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/3465*(128*b^4*d^5*x^5 + 315*b^4*c^5 - 1540*a*b^3*c^4*d + 2970*a^2*b^2*c^
3*d^2 - 2772*a^3*b*c^2*d^3 + 1155*a^4*c*d^4 - 64*(b^4*c*d^4 - 11*a*b^3*d^5)
*x^4 + 16*(3*b^4*c^2*d^3 - 22*a*b^3*c*d^4 + 99*a^2*b^2*d^5)*x^3 - 8*(5*b^4*
c^3*d^2 - 33*a*b^3*c^2*d^3 + 99*a^2*b^2*c*d^4 - 231*a^3*b*d^5)*x^2 + (35*b^
4*c^4*d - 220*a*b^3*c^3*d^2 + 594*a^2*b^2*c^2*d^3 - 924*a^3*b*c*d^4 + 1155*
a^4*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^5*c^5 - 5*a^7*b^4*c^4*d + 10
*a^8*b^3*c^3*d^2 - 10*a^9*b^2*c^2*d^3 + 5*a^10*b*c*d^4 - a^11*d^5 + (b^11*c
^5 - 5*a*b^10*c^4*d + 10*a^2*b^9*c^3*d^2 - 10*a^3*b^8*c^2*d^3 + 5*a^4*b^7*c
*d^4 - a^5*b^6*d^5)*x^6 + 6*(a*b^10*c^5 - 5*a^2*b^9*c^4*d + 10*a^3*b^8*c^3*
d^2 - 10*a^4*b^7*c^2*d^3 + 5*a^5*b^6*c*d^4 - a^6*b^5*d^5)*x^5 + 15*(a^2*b^9
*c^5 - 5*a^3*b^8*c^4*d + 10*a^4*b^7*c^3*d^2 - 10*a^5*b^6*c^2*d^3 + 5*a^6*b^
5*c*d^4 - a^7*b^4*d^5)*x^4 + 20*(a^3*b^8*c^5 - 5*a^4*b^7*c^4*d + 10*a^5*b^6
*c^3*d^2 - 10*a^6*b^5*c^2*d^3 + 5*a^7*b^4*c*d^4 - a^8*b^3*d^5)*x^3 + 15*(a^
4*b^7*c^5 - 5*a^5*b^6*c^4*d + 10*a^6*b^5*c^3*d^2 - 10*a^7*b^4*c^2*d^3 + 5*a
^8*b^3*c*d^4 - a^9*b^2*d^5)*x^2 + 6*(a^5*b^6*c^5 - 5*a^6*b^5*c^4*d + 10*a^7
*b^4*c^3*d^2 - 10*a^8*b^3*c^2*d^3 + 5*a^9*b^2*c*d^4 - a^10*b*d^5)*x)
```

giac [**B**]   time = 2.38, size = 1345, normalized size = 7.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -512/3465*(sqrt(b*d)*b^16*c^6*d^5 - 6*sqrt(b*d)*a*b^15*c^5*d^6 + 15*sqrt(b*
d)*a^2*b^14*c^4*d^7 - 20*sqrt(b*d)*a^3*b^13*c^3*d^8 + 15*sqrt(b*d)*a^4*b^12
*c^2*d^9 - 6*sqrt(b*d)*a^5*b^11*c*d^10 + sqrt(b*d)*a^6*b^10*d^11 - 11*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14
```

```
*c^5*d^5 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a*b^13*c^4*d^6 - 110*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^3*d^7 + 110*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^2*d
^8 - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^2*a^4*b^10*c*d^9 + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*d^10 + 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^4*d^5 - 220*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^11
*c^3*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a^2*b^10*c^2*d^7 - 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c*d^8 + 55*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^8*d^9 - 1
65*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^10*c^3*d^5 + 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^9*c^2*d^6 - 495*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^8*c*d^7 + 165*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*d
^8 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^8*b^8*c^2*d^5 - 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^8*a*b^7*c*d^6 + 330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*d^7 + 924*sqrt(b*d)
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c*d
^5 - 924*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^10*a*b^5*d^6 + 1386*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^12*b^4*d^5)*abs(b)/((b^2*c - a*b*d - (sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^11*b^2)
```

maple [A] time = 0.01, size = 256, normalized size = 1.50

$$\frac{2 (dx+c)^{\frac{3}{2}} \left(128b^4x^4d^4+704a\,b^3d^4x^3-192b^4c\,d^3x^3+1584a^2b^2d^4x^2-1056a\,b^3c\,d^3x^2+240b^4c^2d^2x^2+1848a^3b\,d^4x-2376a^2b^2c\,d^3x+1320a\,b^3c^2d^2x-280b^4c^3dx+1155a^4d^4-2772a^3bc\,d^3+2970a^2b^2c^2d^2-1540a\,b^3c^3d+315b^4c^4\right)}{3465 (bx+a)^{\frac{11}{2}} \left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/3465*(d*x+c)^(3/2)*(128*b^4*d^4*x^4+704*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3+1
584*a^2*b^2*d^4*x^2-1056*a*b^3*c*d^3*x^2+240*b^4*c^2*d^2*x^2+1848*a^3*b*d^4
*x-2376*a^2*b^2*c*d^3*x+1320*a*b^3*c^2*d^2*x-280*b^4*c^3*d*x+1155*a^4*d^4-2
772*a^3*b*c*d^3+2970*a^2*b^2*c^2*d^2-1540*a*b^3*c^3*d+315*b^4*c^4)/(b*x+a)^
(11/2)/(a^5*d^5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4
*c^4*d-b^5*c^5)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.43, size = 397, normalized size = 2.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{2310\,a^4\,c\,d^4-5544\,a^3\,b\,c^2\,d^3+5940\,a^2\,b^2\,c^3\,d^2-3080\,a\,b^3\,c^4\,d+630\,b^4\,c^5}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(2310\,a^4\,d^5-1848\,a^3\,b\,c\,d^4+1188\,a^2\,b^2\,c^2\,d^3-440\,a\,b^3\,c^3\,d^2+70\,b^4\,c^4\,d\right)}{3465\,b^5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,d^5\,x^5}{3465\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{16\,d^2\,x^2\,\left(231\,a^3\,d^3-99\,a^2\,b\,c\,d^2+33\,a\,b^2\,c^2\,d-5\,b^3\,c^3\right)}{3465\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{3465\,b^2\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3465\,b^3\,{\left(a\,d-b\,c\right)}^5}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((630*b^4*c^5 + 2310*a^4*c*d^4 - 5544*a^3*b*c^2*d^3 + 5940
*a^2*b^2*c^3*d^2 - 3080*a*b^3*c^4*d)/(3465*b^5*(a*d - b*c)^5) + (x*(2310*a^
4*d^5 + 70*b^4*c^4*d - 440*a*b^3*c^3*d^2 + 1188*a^2*b^2*c^2*d^3 - 1848*a^3*
b*c*d^4))/(3465*b^5*(a*d - b*c)^5) + (256*d^5*x^5)/(3465*b*(a*d - b*c)^5) +
 (16*d^2*x^2*(231*a^3*d^3 - 5*b^3*c^3 + 33*a*b^2*c^2*d - 99*a^2*b*c*d^2))/(
3465*b^4*(a*d - b*c)^5) + (128*d^4*x^4*(11*a*d - b*c))/(3465*b^2*(a*d - b*c
)^5) + (32*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2 - 22*a*b*c*d))/(3465*b^3*(a*d -
b*c)^5)))/(x^5*(a + b*x)^(1/2) + (a^5*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a
 + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^
(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

3.1365 $\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=227

$$-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{5/2}\sqrt{c}}{80}$$

Rubi [A] time = 0.13, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^2d^3}-\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}-\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{80b^2d}+\frac{3(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}{40b^2}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] (3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^2*d^3) - ((b*c - a*d)^
3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^2*(a + b*x)^(5
/2)*Sqrt[c + d*x])/(80*b^2*d) + (3*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x
])/(40*b^2) + ((a + b*x)^(7/2)*(c + d*x)^(3/2))/(5*b) - (3*(b*c - a*d)^5*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2
))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{(3(bc-ad))\int (a+bx)^{5/2}\sqrt{c+dx}\,dx}{10b}$$

$$= \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)^{3/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{80b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}{40b^2} + \frac{(a+bx)^{7/2}(c+dx)}{5b}$$

$$= -\frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{80b^2d} + \frac{3(bc-ad)(a+bx}{40b^2}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/}}{80b^2d}$$

$$= \frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^2d^3} - \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{5/}}{80b^2d}$$

Mathematica [A] time = 1.78, size = 187, normalized size = 0.82

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(-\frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} + \frac{15(bc-ad)^4}{d^3(a+bx)^3} + \frac{10(ad-bc)^3}{d^2(a+bx)^2} + \frac{8(bc-ad)^2}{d(a+bx)} + 48(bc-ad) + 128b(c+dx)\right)}{640b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d) + (15*(b*c - a*d)^4)/(d^3*(a
 + b*x)^3) + (10*(-(b*c) + a*d)^3)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^2)/(d
*(a + b*x)) + 128*b*(c + d*x) - (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a + b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b
*c - a*d)])))/(640*b^2)
```

IntegrateAlgebraic [A] time = 0.41, size = 197, normalized size = 0.87

$$\frac{\sqrt{a+bx}\,(bc-ad)^5\left(-\frac{70b^3d(a+bx)}{c+dx} + \frac{128b^2d^2(a+bx)^2}{(c+dx)^2} - \frac{15d^4(a+bx)^4}{(c+dx)^4} + \frac{70bd^3(a+bx)^3}{(c+dx)^3} + 15b^4\right)}{640b^2d^3\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^5*Sqrt[a + b*x]*(15*b^4 - (15*d^4*(a + b*x)^4)/(c + d*x)^4 + (
70*b*d^3*(a + b*x)^3)/(c + d*x)^3 + (128*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 -
 (70*b^3*d*(a + b*x))/(c + d*x)))/(640*b^2*d^3*Sqrt[c + d*x]*(b - (d*(a + b
*x))/(c + d*x))^5) - (3*(b*c - a*d)^5*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt
[b]*Sqrt[c + d*x])])/(128*b^(5/2)*d^(7/2))
```

fricas [A] time = 1.52, size = 702, normalized size = 3.09

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(
11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2
*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^4), 1/1280*(15*(b^5*c^
5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4
 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(1
28*b^5*d^5*x^4 + 15*b^5*c^4*d - 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 + 70
*a^3*b^2*c*d^4 - 15*a^4*b*d^5 + 16*(11*b^5*c*d^4 + 21*a*b^4*d^5)*x^3 + 8*(b
^5*c^2*d^3 + 64*a*b^4*c*d^4 + 31*a^2*b^3*d^5)*x^2 - 2*(5*b^5*c^3*d^2 - 23*a
*b^4*c^2*d^3 - 233*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x
 + c))/(b^3*d^4)]
```

giac [B] time = 2.30, size = 1740, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1920*(240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c*abs(b) - 1920*((b^2*c - a*b*
d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c*abs(b
)/b^2 + 10*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(
6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^
4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9
*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x
 + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 -
 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
- a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c*abs(b) + 30*(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b
^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6
)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 -
93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b
*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d*ab
s(b) + 240*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*d*abs(b)/b + (sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*
c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 51
3*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^
2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4
```

```
 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^
19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b
^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3
*d^4))*b*d*abs(b) + 1440*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
+ (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d
^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*d))*a^2*c*abs(b)/b^2 + 480*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 853, normalized size = 3.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(5/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)*a+1/16/
d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a^2-1/8/d^2*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a*b*c
+3/64/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2*b+3/32/b*(d*x+c)^(1/2)*(b*x+a)^
(1/2)*a^3*c+3/32/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3*b-1/8/d^2*(b*x+a)^(3
/2)*(d*x+c)^(5/2)*b*c+1/16/d^3*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b^2*c^2+1/64/b*(
d*x+c)^(3/2)*(b*x+a)^(1/2)*a^3-3/64/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2*c-1/6
4/d^3*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^3*b^2-3/128*d/b^2*(d*x+c)^(1/2)*(b*x+a)
^(1/2)*a^4-9/64/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^2-3/128/d^3*(d*x+c)^(1/
2)*(b*x+a)^(1/2)*c^4*b^2+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+
a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(
1/2))/(b*d)^(1/2)*a^3*c^2+15/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)
*x)^(1/2))/(b*d)^(1/2)*a*c^4*b^2-15/256*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4*c+3/256*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(
d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+
a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^5-15/128/d*((b*x+a)*(d*x+c))^(1/2)/(d
*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a
*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^3*b-3/256/d^3*((b*x+a)*(d*x+c))^(1
/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d
*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^5*b^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}}(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(3/2), x)
```

3.1366 $\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx$

Optimal. Leaf size=189

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{8b^2}$$

Rubi [A] time = 0.09, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64b^2d^2}+\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{5/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^2d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-3*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^2*d^2) + ((b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^2*d) + ((b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(3/2))/(4*b) + (3*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(5
/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(3(bc-ad))\int (a+bx)^{3/2}\sqrt{c+dx}\,dx}{8b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{3/2}}{4b} + \frac{(bc-ad)^2\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{8b^2} + \frac{(a+bx)^{5/2}(c+d}{4b}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+b}{8b}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+b}{8b}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+b}{8b}$$

$$= -\frac{3(bc-ad)^3\sqrt{a+bx}\sqrt{c+dx}}{64b^2d^2} + \frac{(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^2d} + \frac{(bc-ad)(a+b}{8b}$$

Mathematica [A] time = 0.57, size = 193, normalized size = 1.02

$$\frac{3(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right) - b\sqrt{d}\sqrt{a+bx}(c+dx)\left(3a^3d^3 - a^2bd^2(11c+2dx) - ab^2d\left(11c^2+44cdx+24d^2x^2\right) + b^3\left(3c^3-2c^2dx-24cd^2x^2-16d^3x^3\right)\right)}{64b^3d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^3*d^3 - a^2*b*d^2*(11*c + 2*d*x)
- a*b^2*d*(11*c^2 + 44*c*d*x + 24*d^2*x^2) + b^3*(3*c^3 - 2*c^2*d*x - 24*c*
d^2*x^2 - 16*d^3*x^3))) + 3*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(64*b^3*d^(5/2)*Sqrt[c
 + d*x])
```

IntegrateAlgebraic [A] time = 0.34, size = 176, normalized size = 0.93

$$\frac{3(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{64b^{5/2}d^{5/2}} - \frac{\sqrt{c+dx}(bc-ad)^4\left(\frac{3b^3(c+dx)^3}{(a+bx)^3} - \frac{11b^2d(c+dx)^2}{(a+bx)^2} - \frac{11bd^2(c+dx)}{a+bx} + 3d^3\right)}{64b^2d^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx} - d\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(c + d*x)^(3/2),x]
```

```
[Out] -1/64*((b*c - a*d)^4*Sqrt[c + d*x]*(3*d^3 - (11*b*d^2*(c + d*x))/(a + b*x)
- (11*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (3*b^3*(c + d*x)^3)/(a + b*x)^3))/(b
^2*d^2*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^4) + (3*(b*c - a*d)^4*A
rcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*b^(5/2)*d^(5/2
))
```

fricas [A] time = 1.35, size = 534, normalized size = 2.83

$$\left[\frac{3\left(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4\right)\sqrt{bd}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4(2bdx + bc + ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c} + 8\left(b^2cd + abd^2\right)x\right) + 4\left(16b^4d^4x^3 - 3b^4c^3d + 11ab^3c^2d^2 + 11a^2b^2cd^3 - 3a^3bd^4 + 24\left(b^4cd^3 + ab^3d^4\right)x^2 + 2\left(b^4c^2d^2 + 22ab^3cd^3 + a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{256b^3d^3}, -\frac{3\left(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right) - 2\left(16b^4d^4x^3 - 3b^4c^3d + 11ab^3c^2d^2 + 11a^2b^2cd^3 - 3a^3bd^4 + 24\left(b^4cd^3 + ab^3d^4\right)x^2 + 2\left(b^4c^2d^2 + 22ab^3cd^3 + a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{128b^3d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^
4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b
*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(16*b^4*d^4*x^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c
*d^3 - 3*a^3*b*d^4 + 24*(b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a
*b^3*c*d^3 + a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3), -1/128
*(3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(
d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(16*b^4*d^4*x
^3 - 3*b^4*c^3*d + 11*a*b^3*c^2*d^2 + 11*a^2*b^2*c*d^3 - 3*a^3*b*d^4 + 24*(
b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(b^4*c^2*d^2 + 22*a*b^3*c*d^3 + a^2*b^2*d^4)
*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^3)]
```

giac [B] time = 1.89, size = 1071, normalized size = 5.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*(8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4
*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2
*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^
2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*abs(b) - 192*((b^2*c - a*b*d)*log
(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/sqrt(
b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c*abs(b)/b^2
+ (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x +
a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*
b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c
^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3
*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d
^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
)/(sqrt(b*d)*b^2*d^3))*d*abs(b) + 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d*abs
(b)/b + 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a
*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-s
qrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d
))*a*c*abs(b)/b^2 + 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
(b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*d))*a^2*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 640, normalized size = 3.39

$$\frac{3\sqrt{(bx+a)(dx+c)}\,a^4d^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{3\sqrt{(bx+a)(dx+c)}\,a^3cd\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{9\sqrt{(bx+a)(dx+c)}\,a^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{(bx+a)(dx+c)}\,abc^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{3\sqrt{(bx+a)(dx+c)}\,b^2c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d^2}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,a^3d}{64b^2}+\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,a^2c}{64b}-\frac{9\sqrt{dx+c}\,\sqrt{bx+a}\,ac^2}{64d}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,bc^3}{64d^2}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2}{32b}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ac}{16d}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,bc^2}{32d^2}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}a}{8d}-\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}bc}{8d^2}+\frac{(bx+a)^{\frac{3}{2}}(dx+c)^{\frac{5}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(3/2)*(d*x+c)^(5/2)+1/8/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)*a-1/8/d
^2*(b*x+a)^(1/2)*(d*x+c)^(5/2)*b*c+1/32/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2-1
/16/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c+1/32/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
c^2*b-3/64*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3+9/64/b*(d*x+c)^(1/2)*(b*x+
```

```
a)^(1/2)*a^2*c-9/64/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2+3/64/d^2*(d*x+c)^(1
/2)*(b*x+a)^(1/2)*c^3*b+3/128*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)
/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c
)*x)^(1/2))/(b*d)^(1/2)*a^4-3/32*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/
(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)
*x)^(1/2))/(b*d)^(1/2)*a^3*c+9/64*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*
x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)
^(1/2))/(b*d)^(1/2)*a^2*c^2-3/32/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b
*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*a*c^3*b+3/128/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*c^4*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(3/2), x)
```

3.1367 $\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx$

Optimal. Leaf size=151

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Rubi [A] time = 0.07, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}+\frac{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8b^2d}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^2*d) + ((b*c - a*d)*(a + b
*x)^(3/2)*Sqrt[c + d*x])/(4*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(3/2))/(3*b)
- ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/
(8*b^(5/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)\int \sqrt{a+bx}\sqrt{c+dx}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b} + \frac{(bc-ad)^2\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}dx}{8b^2}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

$$= \frac{(bc-ad)^2\sqrt{a+bx}\sqrt{c+dx}}{8b^2d} + \frac{(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4b^2} + \frac{(a+bx)^{3/2}(c+dx)^{3/2}}{3b}$$

Mathematica [A] time = 0.44, size = 152, normalized size = 1.01

$$\frac{-b\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(3a^2d^2-2abd(4c+dx)-\left(b^2\left(3c^2+14cdx+8d^2x^2\right)\right)\right)-3(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{24b^3d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] (-(b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(3*a^2*d^2 - 2*a*b*d*(4*c + d*x) - b^2
*(3*c^2 + 14*c*d*x + 8*d^2*x^2))) - 3*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*x))/
(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(24*b^3*d^(3
/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.24, size = 153, normalized size = 1.01

$$\frac{\sqrt{a+bx}\,(bc-ad)^3\left(-\frac{3d^2(a+bx)^2}{(c+dx)^2}+\frac{8bd(a+bx)}{c+dx}+3b^2\right)}{24b^2d\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{5/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*(c + d*x)^(3/2),x]
```

```
[Out] ((b*c - a*d)^3*Sqrt[a + b*x]*(3*b^2 - (3*d^2*(a + b*x)^2)/(c + d*x)^2 + (8*
b*d*(a + b*x))/(c + d*x)))/(24*b^2*d*Sqrt[c + d*x]*(b - (d*(a + b*x))/(c +
d*x))^3) - ((b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c +
 d*x])])/(8*b^(5/2)*d^(3/2))
```

fricas [A] time = 1.48, size = 410, normalized size = 2.72

$$\left[-\frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\,\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+3b^3c^2d+8ab^2cd^2-3a^2bd^3+2\left(7b^3cd^2+ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^3d^2}, \frac{3\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\,\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(8b^3d^3x^2+3b^3c^2d+8ab^2cd^2-3a^2bd^3+2\left(7b^3cd^2+ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^3d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sq
rt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d
^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*d^3 + 2*(7*b^3*c*d^2 + a*b^2
*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2), 1/48*(3*(b^3*c^3 - 3*a*b^2
*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*
d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
 + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 3*b^3*c^2*d + 8*a*b^2*c*d^2 - 3*a^2*b*
d^3 + 2*(7*b^3*c*d^2 + a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d^2)
]
```

giac [B] time = 1.61, size = 576, normalized size = 3.81

$$-\frac{\frac{24\left(\frac{(b^2c-abd)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)ac|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{b^2}+\frac{b^6cd^3-13ab^5d^4}{b^7d^4}\right)-\frac{3(b^7c^2d^2+2ab^6cd^3-11a^2b^5d^4)}{b^7d^4}\right)-\frac{3(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,bd^2}\right)d|b|}{b}-\frac{6\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c^2+2ab^2cd-3a^2bd^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)c|b|}{b^2}-\frac{6\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{(b^3c^2+2ab^2cd-3a^2bd^2)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)ad|b|}{b^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*a*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d*abs(b)/b - 6
*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)
*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*s
qrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*abs(b
)/b^2 - 6*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*
d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d)
)*a*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 459, normalized size = 3.04

$$\frac{\sqrt{(bx+a)(dx+c)}\,a^3d^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{3\sqrt{(bx+a)(dx+c)}\,a^2cd\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{3\sqrt{(bx+a)(dx+c)}\,a\,c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{\sqrt{(bx+a)(dx+c)}\,b\,c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,a^2d}{8b^2}+\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,ac}{4b}-\frac{\sqrt{dx+c}\,\sqrt{bx+a}\,c^2}{8d}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a}{12b}-\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c}{12d}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{5}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(3/2),x)
```

```
[Out] 1/3/d*(b*x+a)^(1/2)*(d*x+c)^(5/2)+1/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a-1/12
/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c-1/8*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2+
1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c-1/8/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c^2
+1/16*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x
+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*
a^3-3/16*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+
1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a
^2*c+3/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2
*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^
2-1/16/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*
a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^3*b
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.1368 $\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=113

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)}{4b^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}+\frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^2) + (Sqrt[a + b*x]*(c + d
*x)^(3/2))/(2*b) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b
]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{(3(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{8b^2}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \sqrt{}\right.}{4b^3}$$

$$= \frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{3/2}}{2b} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}}$$

Mathematica [A] time = 0.29, size = 109, normalized size = 0.96

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}\,(-3ad+5bc+2bdx)+\frac{3(bc-ad)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\,\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(5*b*c - 3*a*d + 2*b*d*x) + (3*(b*c - a*d)^(3
/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c +
 d*x))/(b*c - a*d)])))/(4*b^2)
```

IntegrateAlgebraic [A] time = 0.18, size = 134, normalized size = 1.19

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{5/2}\sqrt{d}} + \frac{(bc-ad)^2\left(\frac{5b\sqrt{a+bx}}{\sqrt{c+dx}}-\frac{3d(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{4b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/Sqrt[a + b*x],x]
```

```
[Out] ((b*c - a*d)^2*((-3*d*(a + b*x)^(3/2))/(c + d*x)^(3/2) + (5*b*Sqrt[a + b*x]
)/Sqrt[c + d*x]))/(4*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + (3*(b*c - a*d)^
2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*b^(5/2)*Sqrt
[d])
```

fricas [A] time = 1.02, size = 306, normalized size = 2.71

$$\left[\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(2\,b^2d^2x+5\,b^2cd-3\,abd^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{16\,b^3d}, -\frac{3\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(2\,b^2d^2x+5\,b^2cd-3\,abd^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{8\,b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
```

```
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + 5*b^2*c*d - 3*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x + 5*b^2*c*d - 3*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*d)]
```

giac [B] time = 1.25, size = 233, normalized size = 2.06

$$-\frac{\frac{4\left(\frac{\left(b^{2}c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^{2}c+(bx+a)bd-abd}\sqrt{bx+a}\right)c|b|}{b^{2}}-\frac{\left(\sqrt{b^{2}c+(bx+a)bd-abd}\left(2\,bx+2\,a+\frac{bcd-5\,ad^{2}}{d^{2}}\right)\sqrt{bx+a}+\frac{\left(b^{3}c^{2}+2\,ab^{2}cd-3\,a^{2}bd^{2}\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}\right)d|b|}{b^{3}}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt
(b*x + a))*c*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a
 + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*
d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*d))*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 308, normalized size = 2.73

$$\frac{3\sqrt{(bx+a)(dx+c)}\,a^{2}d^{2}\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^{2}}-\frac{3\sqrt{(bx+a)(dx+c)}\,acd\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)x}\right)}{4\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{3\sqrt{(bx+a)(dx+c)}\,c^{2}\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)x}\right)}{8\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,ad}{4b^{2}}+\frac{3\sqrt{dx+c}\,\sqrt{bx+a}\,c}{4b}+\frac{(dx+c)^{\frac{3}{2}}\sqrt{bx+a}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/2*(d*x+c)^(3/2)*(b*x+a)^(1/2)/b-3/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d+3
/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c+3/8/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*d^2-3/4/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*d*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{3/2}}{\sqrt{a+b\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{\sqrt{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(1/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/sqrt(a + b*x), x)
```

3.1369 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}} dx$

Optimal. Leaf size=98

$$\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}+\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (3*d*Sqrt[a + b*x]*Sqrt[c + d*x])/b^2 - (2*(c + d*x)^(3/2))/(b*Sqrt[a + b*x
]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c
 + d*x])])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx &= -\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^2}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{(3d(bc-ad))\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}\\
&= \frac{3d\sqrt{a+bx}\,\sqrt{c+dx}}{b^2}-\frac{2(c+dx)^{3/2}}{b\sqrt{a+bx}}+\frac{3\sqrt{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.72

$$
-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

IntegrateAlgebraic [A] time = 0.56, size = 159, normalized size = 1.62

$$
\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(-3ad^2\sqrt{c+dx}-bd(c+dx)^{3/2}+3bcd\sqrt{c+dx}\right)}{b^2(-ad-b(c+dx)+bc)}-\frac{3\sqrt{\frac{b}{d}}\left(bcd-ad^2\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(3/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(3*b*c*d*Sqrt[c + d*x] - 3*a*d^2*Sqrt[
c + d*x] - b*d*(c + d*x)^(3/2)))/(b^2*(b*c - a*d - b*(c + d*x))) - (3*Sqrt[
b/d]*(b*c*d - a*d^2)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b
*(c + d*x))/d]])/b^3
```

fricas [A] time = 1.39, size = 311, normalized size = 3.17

$$
\left[-\frac{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4(bdx-2bc+3ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\left(b^3x+ab^2\right)},-\frac{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)-2(bdx-2bc+3ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^3x+ab^2\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(a*b*c - a^2*d + (b^2*c - a*b*d)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)
```

```
*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x - 2*b*c + 3*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2), -1/2*(3*(a*b*c - a^2*d +
 (b^2*c - a*b*d)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(b
*d*x - 2*b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^3*x + a*b^2)]
```

giac [B] time = 1.50, size = 204, normalized size = 2.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,d|b|}{b^4}-\frac{3\left(\sqrt{bd}\,bc|b|-\sqrt{bd}\,ad|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2\,b^4}-\frac{4\left(\sqrt{bd}\,b^2c^2|b|-2\,\sqrt{bd}\,abcd|b|+\sqrt{bd}\,a^2d^2|b|\right)}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d*abs(b)/b^4 - 3/2*(sqrt(
b*d)*b*c*abs(b) - sqrt(b*d)*a*d*abs(b))*log((sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^4 - 4*(sqrt(b*d)*b^2*c^2*abs(b) - 2*s
qrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b))/((b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^3)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c+d\,x)}^{3/2}}{{(a+b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(3/2), x)
```

3.1370 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}} \, dx$

Optimal. Leaf size=92

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}-\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}}-\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*d*Sqrt[c + d*x])/(b^2*Sqrt[a + b*x]) - (2*(c + d*x)^(3/2))/(3*b*(a + b*
x)^(3/2)) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/b^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}} + \frac{d\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b}$$

$$= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}} - \frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}} + \frac{d^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^2}$$

$$= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}} - \frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b^3}$$

$$= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}} - \frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}} + \frac{(2d^2)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^3}$$

$$= -\frac{2d\sqrt{c+dx}}{b^2\sqrt{a+bx}} - \frac{2(c+dx)^{3/2}}{3b(a+bx)^{3/2}} + \frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.79

$$-\frac{2(c+dx)^{3/2}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*Hypergeometric2F1[-3/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 85, normalized size = 0.92

$$\frac{2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{5/2}} - \frac{2(c+dx)^{3/2}\left(\frac{3d(a+bx)}{c+dx}+b\right)}{3b^2(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(3/2)*(b + (3*d*(a + b*x))/(c + d*x)))/(3*b^2*(a + b*x)^(3/2)
) + (2*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^
(5/2)
```

fricas [B] time = 2.06, size = 325, normalized size = 3.53

$$\left[\frac{3\left(b^2dx^2+2abdx+a^2d\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(4bdx+bc+3ad\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^4x^2+2ab^3x+a^2b^2\right)}, -\frac{3\left(b^2dx^2+2abdx+a^2d\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\left(4bdx+bc+3ad\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(b^4x^2+2ab^3x+a^2b^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + b*c + 3*a*d)*
```

```
sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^3*x + a^2*b^2), -1/3*(3*(b^2*
d*x^2 + 2*a*b*d*x + a^2*d)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x))
 + 2*(4*b*d*x + b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x^2 + 2*a*b^
3*x + a^2*b^2)]
```

giac [B] time = 1.74, size = 455, normalized size = 4.95

$$-\frac{\sqrt{bd}\,d|b|\log\left(\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{b^4}-\frac{8\left(2\sqrt{bd}\,b^5c^3d|b|-6\sqrt{bd}\,ab^4c^2d^2|b|+6\sqrt{bd}\,a^2b^3cd^3|b|-2\sqrt{bd}\,a^3b^2d^4|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^3c^2d|b|+6\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^2cd^2|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2bd^3|b|+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4bcd|b|-3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ad^2|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2)/b^4 - 8/3*(2*sqrt(b*d)*b^5*c^3*d*abs(b) - 6*sqrt(b*d)*a*b^4
*c^2*d^2*abs(b) + 6*sqrt(b*d)*a^2*b^3*c*d^3*abs(b) - 2*sqrt(b*d)*a^3*b^2*d^
4*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^3*c^2*d*abs(b) + 6*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^2*c*d^2*abs(b) - 3*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b*d^3*abs(b
) + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*b*c*d*abs(b) - 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^4*a*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*b^3)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{3}{2}}}{(bx+a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(3/2)/(b*x+a)^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(c+d\,x)}^{3/2}}{{(a+b\,x)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(5/2),x)
```

```
[Out] int((c + d*x)^(3/2)/(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{3}{2}}}{(a+bx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(5/2),x)
```

```
[Out] Integral((c + d*x)**(3/2)/(a + b*x)**(5/2), x)
```

3.1371 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx = -\frac{2(c+dx)^{5/2}}{5(bc-ad)(a+bx)^{5/2}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{5/2}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(5*(b*c - a*d)*(a + b*x)^(5/2))
```

fricas [B] time = 2.26, size = 104, normalized size = 3.25

$$-\frac{2\left(d^2x^2+2\,cdx+c^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{5\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] -2/5*(d^2*x^2 + 2*c*d*x + c^2)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b*c - a^4*d
 + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c - a^3
*b*d)*x)
```

giac [B] time = 1.69, size = 374, normalized size = 11.69

$$-\frac{4\left(\sqrt{bd}\,b^{8}c^{4}d^{2}|b| - 4\sqrt{bd}\,ab^{7}c^{3}d^{3}|b| + 6\sqrt{bd}\,a^{2}b^{6}c^{2}d^{4}|b| - 4\sqrt{bd}\,a^{3}b^{5}cd^{5}|b| + \sqrt{bd}\,a^{4}b^{4}d^{6}|b| + 10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c + (bx+a)bd - abd}\right)^{4}b^{4}c^{2}d^{2}|b| - 20\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c + (bx+a)bd - abd}\right)^{4}ab^{3}cd^{3}|b| + 10\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c + (bx+a)bd - abd}\right)^{4}a^{2}b^{2}d^{4}|b| + 5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c + (bx+a)bd - abd}\right)^{8}d^{2}|b|\right)}{5\left(b^{2}c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c + (bx+a)bd - abd}\right)^{2}\right)^{5}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -4/5*(sqrt(b*d)*b^8*c^4*d^2*abs(b) - 4*sqrt(b*d)*a*b^7*c^3*d^3*abs(b) + 6*s
qrt(b*d)*a^2*b^6*c^2*d^4*abs(b) - 4*sqrt(b*d)*a^3*b^5*c*d^5*abs(b) + sqrt(b
*d)*a^4*b^4*d^6*abs(b) + 10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^4*b^4*c^2*d^2*abs(b) - 20*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d^3*abs(b) +
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^2*b^2*d^4*abs(b) + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*d^2*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*b^3)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}}{5\left(bx+a\right)^{\frac{5}{2}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(7/2),x)
```

```
[Out] 2/5/(b*x+a)^(5/2)*(d*x+c)^(5/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.80, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{5/2}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(a+b\,x\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(7/2),x)
```

```
[Out] (2*(c + d*x)^(5/2))/((5*a*d - 5*b*c)*(a + b*x)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.1372 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{5/2}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (4*d*(c + d*x)^(5/2)
)/(35*(b*c - a*d)^2*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}-\frac{(2d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{7(bc-ad)}\\ &= -\frac{2(c+dx)^{5/2}}{7(bc-ad)(a+bx)^{7/2}}+\frac{4d(c+dx)^{5/2}}{35(bc-ad)^2(a+bx)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{5/2}(7ad-5bc+2bdx)}{35(a+bx)^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(-5*b*c + 7*a*d + 2*b*d*x))/(35*(b*c - a*d)^2*(a + b*x)^
(7/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 51, normalized size = 0.77

$$\frac{2(c+dx)^{7/2}\left(\frac{7d(a+bx)}{c+dx}-5b\right)}{35(a+bx)^{7/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(9/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-5*b + (7*d*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^2*(a
 + b*x)^(7/2))
```

fricas [B] time = 3.92, size = 235, normalized size = 3.56

$$\frac{2\left(2\,bd^{3}x^{3}-5\,bc^{3}+7\,ac^{2}d-\left(bcd^{2}-7\,ad^{3}\right)x^{2}-2\left(4\,bc^{2}d-7\,acd^{2}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{35\left(a^{4}b^{2}c^{2}-2\,a^{5}bcd+a^{6}d^{2}+\left(b^{6}c^{2}-2\,ab^{5}cd+a^{2}b^{4}d^{2}\right)x^{4}+4\left(ab^{5}c^{2}-2\,a^{2}b^{4}cd+a^{3}b^{3}d^{2}\right)x^{3}+6\left(a^{2}b^{4}c^{2}-2\,a^{3}b^{3}cd+a^{4}b^{2}d^{2}\right)x^{2}+4\left(a^{3}b^{3}c^{2}-2\,a^{4}b^{2}cd+a^{5}bd^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/35*(2*b*d^3*x^3 - 5*b*c^3 + 7*a*c^2*d - (b*c*d^2 - 7*a*d^3)*x^2 - 2*(4*b*
c^2*d - 7*a*c*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^2*c^2 - 2*a^5*b*c*
d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*c^2 - 2*
a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

giac [B] time = 2.12, size = 1024, normalized size = 15.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] 8/35*(sqrt(b*d)*b^10*c^5*d^3*abs(b) - 5*sqrt(b*d)*a*b^9*c^4*d^4*abs(b) + 10
*sqrt(b*d)*a^2*b^8*c^3*d^5*abs(b) - 10*sqrt(b*d)*a^3*b^7*c^2*d^6*abs(b) + 5
*sqrt(b*d)*a^4*b^6*c*d^7*abs(b) - sqrt(b*d)*a^5*b^5*d^8*abs(b) - 7*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^8*c^4
*d^3*abs(b) + 28*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a*b^7*c^3*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c^2*d^5*abs(b) + 28*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^3*b^5*c*d^6*abs(b) - 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*a^4*b^4*d^7*abs(b) - 14*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^6*c^3*d^3*abs(b) + 42*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^5*c^2*d^4*abs(b) - 42*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^4*c*d^5*abs(b) + 14*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^3*d^6*abs(b) -
70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6*b^4*c^2*d^3*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*a*b^3*c*d^4*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^2*d^5*abs(b) -
 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*b^2*c*d^3*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^8*a*b*d^4*abs(b) - 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*d^3*abs(b))/((b^2*c - a*b
*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b
^2)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{2 (dx + c)^{\frac{5}{2}} (2bdx + 7ad - 5bc)}{35 (bx + a)^{\frac{7}{2}} \left(a^2d^2 - 2abcd + b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/35*(d*x+c)^(5/2)*(2*b*d*x+7*a*d-5*b*c)/(b*x+a)^(7/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.93, size = 178, normalized size = 2.70

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^3\,x^3}{35\,b^2\,{\left(a\,d-b\,c\right)}^2}-\frac{10\,b\,c^3-14\,a\,c^2\,d}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{x^2\,\left(14\,a\,d^3-2\,b\,c\,d^2\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,c\,d\,x\,\left(7\,a\,d-4\,b\,c\right)}{35\,b^3\,{\left(a\,d-b\,c\right)}^2}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(9/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^3*x^3)/(35*b^2*(a*d - b*c)^2) - (10*b*c^3 - 14*a*c^2
*d)/(35*b^3*(a*d - b*c)^2) + (x^2*(14*a*d^3 - 2*b*c*d^2))/(35*b^3*(a*d - b*
c)^2) + (4*c*d*x*(7*a*d - 4*b*c))/(35*b^3*(a*d - b*c)^2)))/(x^3*(a + b*x)^(
1/2) + (a^3*(a + b*x)^(1/2))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(
a + b*x)^(1/2))/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.1373 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{5/2}}{315(a+bx)^{5/2}(bc-ad)^3}+\frac{8d(c+dx)^{5/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (8*d*(c + d*x)^(5/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (16*d^2*(c + d*x)^(5/2))/(315*(b*c -
 a*d)^3*(a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(4d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}\\
&= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}+\frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{7/2}}\,dx}{63(bc-ad)^2}\\
&= -\frac{2(c+dx)^{5/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{8d(c+dx)^{5/2}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{16d^2(c+dx)^{5/2}}{315(bc-ad)^3(a+bx)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{5/2}\left(63a^2d^2+18abd(2dx-5c)+b^2\left(35c^2-20cdx+8d^2x^2\right)\right)}{315(a+bx)^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(63*a^2*d^2 + 18*a*b*d*(-5*c + 2*d*x) + b^2*(35*c^2 - 2
0*c*d*x + 8*d^2*x^2)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{9/2}\left(\frac{63d^2(a+bx)^2}{(c+dx)^2}-\frac{90bd(a+bx)}{c+dx}+35b^2\right)}{315(a+bx)^{9/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(35*b^2 + (63*d^2*(a + b*x)^2)/(c + d*x)^2 - (90*b*d*(a
 + b*x))/(c + d*x)))/(315*(b*c - a*d)^3*(a + b*x)^(9/2))
```

fricas [B] time = 13.56, size = 426, normalized size = 4.22

$$-\frac{2\left(8b^2d^4x^4+35b^2c^4-90abc^3d+63a^2c^2d^2-4\left(b^2cd^3-9abd^4\right)x^3+3\left(b^2c^2d^2-6abcd^3+21a^2d^4\right)x^2+2\left(25b^2c^3d-72abc^2d^2+63a^2cd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{315\left(a^5b^3c^3-3a^6b^2c^2d+3a^7bcd^2-a^8d^3+\left(b^8c^3-3ab^7c^2d+3a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3a^2b^6c^2d+3a^3b^5cd^2-a^4b^4d^3\right)x^4+10\left(a^2b^6c^3-3a^3b^5c^2d+3a^4b^4cd^2-a^5b^3d^3\right)x^3+10\left(a^3b^5c^3-3a^4b^4c^2d+3a^5b^3cd^2-a^6b^2d^3\right)x^2+5\left(a^4b^4c^3-3a^5b^3c^2d+3a^6b^2cd^2-a^7bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] -2/315*(8*b^2*d^4*x^4 + 35*b^2*c^4 - 90*a*b*c^3*d + 63*a^2*c^2*d^2 - 4*(b^2
*c*d^3 - 9*a*b*d^4)*x^3 + 3*(b^2*c^2*d^2 - 6*a*b*c*d^3 + 21*a^2*d^4)*x^2 +
2*(25*b^2*c^3*d - 72*a*b*c^2*d^2 + 63*a^2*c*d^3)*x)*sqrt(b*x + a)*sqrt(d*x
+ c)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3 + (b^8*c^3 -
3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7*c^3 - 3*a^2*b
^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c^3 - 3*a^3*b^5
*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3 - 3*a^4*b^4*c
^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 - 3*a^5*b^3*c^2*
d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

giac [B] time = 2.94, size = 1394, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] -32/315*(sqrt(b*d)*b^12*c^6*d^4*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^5*abs(b)
+ 15*sqrt(b*d)*a^2*b^10*c^4*d^6*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^7*abs(b
) + 15*sqrt(b*d)*a^4*b^8*c^2*d^8*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^9*abs(b)
+ sqrt(b*d)*a^6*b^6*d^10*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^10*c^5*d^4*abs(b) + 45*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^9*c^4*d^
5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*a^2*b^8*c^3*d^6*abs(b) + 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^7*c^2*d^7*abs(b) - 45*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^
4*b^6*c*d^8*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2*a^5*b^5*d^9*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^4*abs(b) - 144*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a*b^7*c^3*d^5*abs(b) + 216*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*a^2*b^6*c^2*d^6*abs(b) - 144*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^7*abs
(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^4*d^8*abs(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^6*c^3*d^4*abs(b) - 378*sqrt(b*d)*(sqr
```

```
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^5*c^2*d^5
*abs(b) + 378*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^6*a^2*b^4*c*d^6*abs(b) - 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^3*d^7*abs(b) + 441*sqrt(b*
d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^4*c^
2*d^4*abs(b) - 882*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^8*a*b^3*c*d^5*abs(b) + 441*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2*d^6*abs(b) + 315*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^
2*c*d^4*abs(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*a*b*d^5*abs(b) + 210*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*d^4*abs(b))/((b^2*c - a*b*d -
 (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^9*b)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\left(dx+c\right)^{\frac{5}{2}}\left(8b^{2}x^{2}d^{2}+36ab\,d^{2}x-20b^{2}cdx+63a^{2}d^{2}-90abcd+35b^{2}c^{2}\right)}{315\left(bx+a\right)^{\frac{9}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/315*(d*x+c)^(5/2)*(8*b^2*d^2*x^2+36*a*b*d^2*x-20*b^2*c*d*x+63*a^2*d^2-90*
a*b*c*d+35*b^2*c^2)/(b*x+a)^(9/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*
c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.11, size = 268, normalized size = 2.65

$$\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^2\,c^2\,d^2-180\,a\,b\,c^3\,d+70\,b^2\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{x^2\,\left(126\,a^2\,d^4-36\,a\,b\,c\,d^3+6\,b^2\,c^2\,d^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^4\,x^4}{315\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{4\,c\,d\,x\,\left(63\,a^2\,d^2-72\,a\,b\,c\,d+25\,b^2\,c^2\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^3}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((70*b^2*c^4 + 126*a^2*c^2*d^2 - 180*a*b*c^3*d)/(315*b^4*(
a*d - b*c)^3) + (x^2*(126*a^2*d^4 + 6*b^2*c^2*d^2 - 36*a*b*c*d^3))/(315*b^4
*(a*d - b*c)^3) + (16*d^4*x^4)/(315*b^2*(a*d - b*c)^3) + (8*d^3*x^3*(9*a*d
- b*c))/(315*b^3*(a*d - b*c)^3) + (4*c*d*x*(63*a^2*d^2 + 25*b^2*c^2 - 72*a*
b*c*d))/(315*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/
2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (
4*a^3*x*(a + b*x)^(1/2))/b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

3.1374 $\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{5/2}}{1155(a+bx)^{5/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{5/2}}{231(a+bx)^{7/2}(bc-ad)^3}+\frac{4d(c+dx)^{5/2}}{33(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{5/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (4*d*(c + d*x)^(5/
2))/(33*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(5/2))/(231*(b*c
 - a*d)^3*(a + b*x)^(7/2)) + (32*d^3*(c + d*x)^(5/2))/(1155*(b*c - a*d)^4*(
a + b*x)^(5/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{3/2}}{(a+bx)^{13/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}-\frac{(6d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{11/2}}\,dx}{11(bc-ad)}\\ &= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}}+\frac{\left(8d^2\right)\int \frac{(c+dx)^{3/2}}{(a+bx)^{9/2}}\,dx}{33(bc-ad)^2}\\ &= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}}-\frac{\left(16d^3\right)\int}{231(b}\\ &= -\frac{2(c+dx)^{5/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{4d(c+dx)^{5/2}}{33(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{5/2}}{231(bc-ad)^3(a+bx)^{7/2}}+\frac{32a}{1155(bc}\end{aligned}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{5/2}\left(231a^3d^3+99a^2bd^2(2dx-5c)+11ab^2d\left(35c^2-20cdx+8d^2x^2\right)+b^3\left(-105c^3+70c^2dx-40cd^2x^2+16d^3x^3\right)\right)}{1155(a+bx)^{11/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (2*(c + d*x)^(5/2)*(231*a^3*d^3 + 99*a^2*b*d^2*(-5*c + 2*d*x) + 11*a*b^2*d*
(35*c^2 - 20*c*d*x + 8*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*x - 40*c*d^2*x^2
 + 16*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(11/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 95, normalized size = 0.70

$$\frac{2(c+dx)^{11/2}\left(\frac{385b^2d(a+bx)}{c+dx}+\frac{231d^3(a+bx)^3}{(c+dx)^3}-\frac{495bd^2(a+bx)^2}{(c+dx)^2}-105b^3\right)}{1155(a+bx)^{11/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(3/2)/(a + b*x)^(13/2),x]
```

```
[Out] (2*(c + d*x)^(11/2)*(-105*b^3 + (231*d^3*(a + b*x)^3)/(c + d*x)^3 - (495*b*
d^2*(a + b*x)^2)/(c + d*x)^2 + (385*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
 - a*d)^4*(a + b*x)^(11/2))
```

fricas [B] time = 28.36, size = 649, normalized size = 4.77

$$\frac{2\left(16b^3d^5x^5-105b^3c^5+385ab^2c^4d-495a^2bc^3d^2+231a^3c^2d^3-8\left(b^3cd^4-11ab^2d^5\right)x^4+2\left(3b^3c^2d^3-22ab^2cd^4+99a^2bd^5\right)x^3-\left(5b^3c^3d^2-33ab^2c^2d^3+99a^2bcd^4-231a^3d^5\right)x^2-2\left(70b^3c^4d-275ab^2c^3d^2+396a^2bc^2d^3-231a^3cd^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{1155\left(a^6b^4c^4-4a^7b^3c^3d+6a^8b^2c^2d^2-4a^9bcd^3+a^{10}d^4+\left(b^{10}c^4-4ab^9c^3d+6a^2b^8c^2d^2-4a^3b^7cd^3+a^4b^6d^4\right)x^6+6\left(ab^9c^4-4a^2b^8c^3d+6a^3b^7c^2d^2-4a^4b^6cd^3+a^5b^5d^4\right)x^5+15\left(a^2b^8c^4-4a^3b^7c^3d+6a^4b^6c^2d^2-4a^5b^5cd^3+a^6b^4d^4\right)x^4+20\left(a^3b^7c^4-4a^4b^6c^3d+6a^5b^5c^2d^2-4a^6b^4cd^3+a^7b^3d^4\right)x^3+15\left(a^4b^6c^4-4a^5b^5c^3d+6a^6b^4c^2d^2-4a^7b^3cd^3+a^8b^2d^4\right)x^2+6\left(a^5b^5c^4-4a^6b^4c^3d+6a^7b^3c^2d^2-4a^8b^2cd^3+a^9bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(16*b^3*d^5*x^5 - 105*b^3*c^5 + 385*a*b^2*c^4*d - 495*a^2*b*c^3*d^2
+ 231*a^3*c^2*d^3 - 8*(b^3*c*d^4 - 11*a*b^2*d^5)*x^4 + 2*(3*b^3*c^2*d^3 - 2
2*a*b^2*c*d^4 + 99*a^2*b*d^5)*x^3 - (5*b^3*c^3*d^2 - 33*a*b^2*c^2*d^3 + 99*
a^2*b*c*d^4 - 231*a^3*d^5)*x^2 - 2*(70*b^3*c^4*d - 275*a*b^2*c^3*d^2 + 396*
a^2*b*c^2*d^3 - 231*a^3*c*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b^4*c^4
- 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 + a^10*d^4 + (b^10*c^
4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*x^6
+ 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^
5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*
a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a^
5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*(a^4*b^6*c^4 - 4*a^
5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*x^2 + 6*(a
^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 + a^9*b*
d^4)*x)
```

giac [B] time = 3.28, size = 1823, normalized size = 13.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] 64/1155*(sqrt(b*d)*b^14*c^7*d^5*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^6*abs(b)
+ 21*sqrt(b*d)*a^2*b^12*c^5*d^7*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^8*abs(
b) + 35*sqrt(b*d)*a^4*b^10*c^3*d^9*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^10*a
bs(b) + 7*sqrt(b*d)*a^6*b^8*c*d^11*abs(b) - sqrt(b*d)*a^7*b^7*d^12*abs(b) -
 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^12*c^6*d^5*abs(b) + 66*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^6*abs(b) - 165*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^
7*abs(b) + 220*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2*a^3*b^9*c^3*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^9*abs(b) + 66*s
```

```
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a^5*b^7*c*d^10*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2*a^6*b^6*d^11*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^5*abs(b) -
 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^9*c^4*d^6*abs(b) + 550*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^7*abs(b) - 550*sqrt(b*d)*(s
qrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2
*d^8*abs(b) + 275*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^4*a^4*b^6*c*d^9*abs(b) - 55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^10*abs(b) - 165*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^4*d^5*abs(b) + 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^7*c^3*d^6*abs(b) - 990*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^7*abs(b)
+ 660*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^6*a^3*b^5*c*d^8*abs(b) - 165*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^9*abs(b) - 825*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^5*ab
s(b) + 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^8*a*b^5*c^2*d^6*abs(b) - 2475*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^7*abs(b) + 825*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b
^3*d^8*abs(b) - 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^10*b^4*c^2*d^5*abs(b) + 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^6*abs(b) - 254
1*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^10*a^2*b^2*d^7*abs(b) - 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^12*b^2*c*d^5*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^6*abs(b) -
1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^14*d^5*abs(b))/(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2)^11
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2\,(dx+c)^{\frac{5}{2}}\left(16b^3x^3d^3+88a\,b^2d^3x^2-40b^3c\,d^2x^2+198a^2b\,d^3x-220a\,b^2c\,d^2x+70b^3c^2dx+231a^3d^3-495a^2bc\,d^2+385a\,b^2c^2d-105b^3c^3\right)}{1155\,(bx+a)^{\frac{11}{2}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/1155*(d*x+c)^(5/2)*(16*b^3*d^3*x^3+88*a*b^2*d^3*x^2-40*b^3*c*d^2*x^2+198*
a^2*b*d^3*x-220*a*b^2*c*d^2*x+70*b^3*c^2*d*x+231*a^3*d^3-495*a^2*b*c*d^2+38
5*a*b^2*c^2*d-105*b^3*c^3)/(b*x+a)^(11/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*
c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.33, size = 376, normalized size = 2.76

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(462\,a^3\,d^5-198\,a^2\,b\,c\,d^4+66\,a\,b^2\,c^2\,d^3-10\,b^3\,c^3\,d^2\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}-\frac{-462\,a^3\,c^2\,d^3+990\,a^2\,b\,c^3\,d^2-770\,a\,b^2\,c^4\,d+210\,b^3\,c^5}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{x\,\left(924\,a^3\,c\,d^4-1584\,a^2\,b\,c^2\,d^3+1100\,a\,b^2\,c^3\,d^2-280\,b^3\,c^4\,d\right)}{1155\,b^5\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^5\,x^5}{1155\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{1155\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^3\,x^3\,\left(99\,a^2\,d^2-22\,a\,b\,c\,d+3\,b^2\,c^2\right)}{1155\,b^4\,{\left(a\,d-b\,c\right)}^4}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(3/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(462*a^3*d^5 - 10*b^3*c^3*d^2 + 66*a*b^2*c^2*d^3 - 1
98*a^2*b*c*d^4))/(1155*b^5*(a*d - b*c)^4) - (210*b^3*c^5 - 462*a^3*c^2*d^3
+ 990*a^2*b*c^3*d^2 - 770*a*b^2*c^4*d)/(1155*b^5*(a*d - b*c)^4) + (x*(924*a
^3*c*d^4 - 280*b^3*c^4*d + 1100*a*b^2*c^3*d^2 - 1584*a^2*b*c^2*d^3))/(1155*
b^5*(a*d - b*c)^4) + (32*d^5*x^5)/(1155*b^2*(a*d - b*c)^4) + (16*d^4*x^4*(1
1*a*d - b*c))/(1155*b^3*(a*d - b*c)^4) + (4*d^3*x^3*(99*a^2*d^2 + 3*b^2*c^2
 - 22*a*b*c*d))/(1155*b^4*(a*d - b*c)^4)))/(x^5*(a + b*x)^(1/2) + (a^5*(a +
 b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a + b*x)
^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

3.1375 $\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=262

$$-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}+\frac{(a+bx)^{5/2}\sqrt{c}}{19}$$

Rubi [A] time = 0.15, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^5}{512b^3d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}{768b^3d^2}-\frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{512b^{7/2}d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}{192b^3d}+\frac{(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x])/(512*b^3*d^3) - (5*(b*c - a*d
)^4*(a + b*x)^(3/2)*Sqrt[c + d*x])/(768*b^3*d^2) + ((b*c - a*d)^3*(a + b*x)
^(5/2)*Sqrt[c + d*x])/(192*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(7/2)*Sqrt[c +
 d*x])/(32*b^3) + ((b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2))/(12*b^2) +
((a + b*x)^(7/2)*(c + d*x)^(5/2))/(6*b) - (5*(b*c - a*d)^6*ArcTanh[(Sqrt[d]
*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(512*b^(7/2)*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{5/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(5(bc-ad))\int (a+bx)^{5/2}(c+dx)^{3/2}\,dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)^{7/2}(c+dx)^{5/2}}{6b} + \frac{(bc-ad)^2\int (a+bx)^{5/2}\sqrt{c}}{8b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}}{12b^2} + \frac{(a+bx)^{7/2}(c+dx)}{6b}$$

$$= \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}}{32b^3} + \frac{(bc-ad)(a+bx)^{7/2}}{12b^2}$$

$$= -\frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)^{5/2}\sqrt{c+dx}}{192b^3d} + \frac{(bc-ad)^2(a+b}{32b}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)}{192b^3d}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)}{192b^3d}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)}{192b^3d}$$

$$= \frac{5(bc-ad)^5\sqrt{a+bx}\sqrt{c+dx}}{512b^3d^3} - \frac{5(bc-ad)^4(a+bx)^{3/2}\sqrt{c+dx}}{768b^3d^2} + \frac{(bc-ad)^3(a+bx)}{192b^3d}$$

Mathematica [A] time = 2.53, size = 209, normalized size = 0.80

$$\frac{(a+bx)^{7/2}\sqrt{c+dx}\left(-\frac{15(bc-ad)^{11/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{7/2}(a+bx)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} + \frac{15(bc-ad)^5}{d^3(a+bx)^3} - \frac{10(bc-ad)^4}{d^2(a+bx)^2} + \frac{8(bc-ad)^3}{d(a+bx)} + 128b(c+dx)(bc-ad) + 48(bc-ad)^2 + 256b^2(c+dx)^2\right)}{1536b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(7/2)*Sqrt[c + d*x]*(48*(b*c - a*d)^2 + (15*(b*c - a*d)^5)/(d^3*
(a + b*x)^3) - (10*(b*c - a*d)^4)/(d^2*(a + b*x)^2) + (8*(b*c - a*d)^3)/(d*
(a + b*x)) + 128*b*(b*c - a*d)*(c + d*x) + 256*b^2*(c + d*x)^2 - (15*(b*c -
 a*d)^(11/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(7/2)*(a
+ b*x)^(7/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)])))/(1536*b^3)
```

IntegrateAlgebraic [A] time = 0.50, size = 220, normalized size = 0.84

$$\frac{\sqrt{c+dx}\,(bc-ad)^6\left(\frac{15b^5(c+dx)^5}{(a+bx)^5} - \frac{85b^4d(c+dx)^4}{(a+bx)^4} + \frac{198b^3d^2(c+dx)^3}{(a+bx)^3} + \frac{198b^2d^3(c+dx)^2}{(a+bx)^2} - \frac{85bd^4(c+dx)}{a+bx} + 15d^5\right)}{1536b^3d^3\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx} - d\right)^6} - \frac{5(bc-ad)^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{512b^{7/2}d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((b*c - a*d)^6*Sqrt[c + d*x]*(15*d^5 - (85*b*d^4*(c + d*x))/(a + b*x) + (19
8*b^2*d^3*(c + d*x)^2)/(a + b*x)^2 + (198*b^3*d^2*(c + d*x)^3)/(a + b*x)^3
- (85*b^4*d*(c + d*x)^4)/(a + b*x)^4 + (15*b^5*(c + d*x)^5)/(a + b*x)^5))/(
1536*b^3*d^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a + b*x))^6) - (5*(b*c - a*
```

```
d)^6*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(512*b^(7/2)
*d^(7/2))
```

fricas [B] time = 1.43, size = 882, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6144*(15*(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*sqrt(b*d)*log(8*b^2*d^2
*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sq
rt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(256*b^6*d^6*x^5 +
 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^3*d^3 + 198*a^3*b^3*c^2*d^
4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*d^5 + a*b^5*d^6)*x^4 + 16*
(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^6)*x^3 + 8*(b^6*c^3*d^3 +
159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d^6)*x^2 - 2*(5*b^6*c^4*d^2
 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^3*b^3*c*d^5 + 5*a^4*b^2*d^
6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4), 1/3072*(15*(b^6*c^6 - 6*a*b^5
*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a
^5*b*c*d^5 + a^6*d^6)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d
)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)) + 2*(256*b^6*d^6*x^5 + 15*b^6*c^5*d - 85*a*b^5*c^4*d^2 + 198*a^2*b^4*c^
3*d^3 + 198*a^3*b^3*c^2*d^4 - 85*a^4*b^2*c*d^5 + 15*a^5*b*d^6 + 640*(b^6*c*
d^5 + a*b^5*d^6)*x^4 + 16*(27*b^6*c^2*d^4 + 106*a*b^5*c*d^5 + 27*a^2*b^4*d^
6)*x^3 + 8*(b^6*c^3*d^3 + 159*a*b^5*c^2*d^4 + 159*a^2*b^4*c*d^5 + a^3*b^3*d
^6)*x^2 - 2*(5*b^6*c^4*d^2 - 28*a*b^5*c^3*d^3 - 594*a^2*b^4*c^2*d^4 - 28*a^
3*b^3*c*d^5 + 5*a^4*b^2*d^6)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^4)]
```

giac [B] time = 3.53, size = 3120, normalized size = 11.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/7680*(960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)
*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2
+ 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3
*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*c^2*abs(b) - 7680*((b^2*c - a*
b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^3*c^2*a
bs(b)/b^2 + 40*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x +
a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^
2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3
 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt
(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d
^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*
b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*b*c^2*abs(b) + 240*(sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 -
 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b
^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c
*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)
*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))
*a*c*d*abs(b) + 1920*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*
(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7
*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*
c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(
```

```
b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^2*c*d*abs(b)/b + 8*(s
qrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a
)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b
^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c
^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(
7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^
7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c
^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
(sqrt(b*d)*b^3*d^4))*b*c*d*abs(b) + 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^
8)/(b^23*d^8)) - (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^2
3*d^8)) + 5*(7*b^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a
^3*b^19*d^8)/(b^23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5
 + 18*a^2*b^21*c^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*
sqrt(b*x + a) - 15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*
b^2*c^2*d^3 + 35*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a)
 + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*a*d^2*abs(b)
+ 320*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b
*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*
b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b
*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x +
a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a^3*d^2*abs(b)/b^2 + 120*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*
c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 16
3*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2
*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^
2*d^3))*a^2*d^2*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(2*(b
*x + a)*(8*(b*x + a)*(10*(b*x + a)/b^5 + (b^30*c*d^9 - 61*a*b^29*d^10)/(b^3
4*d^10)) - 3*(3*b^31*c^2*d^8 + 14*a*b^30*c*d^9 - 417*a^2*b^29*d^10)/(b^34*d
^10)) + (21*b^32*c^3*d^7 + 77*a*b^31*c^2*d^8 + 183*a^2*b^30*c*d^9 - 3481*a^
3*b^29*d^10)/(b^34*d^10))*(b*x + a) - 5*(21*b^33*c^4*d^6 + 56*a*b^32*c^3*d^
7 + 106*a^2*b^31*c^2*d^8 + 176*a^3*b^30*c*d^9 - 2279*a^4*b^29*d^10)/(b^34*d
^10))*(b*x + a) + 15*(21*b^34*c^5*d^5 + 35*a*b^33*c^4*d^6 + 50*a^2*b^32*c^3
*d^7 + 70*a^3*b^31*c^2*d^8 + 105*a^4*b^30*c*d^9 - 793*a^5*b^29*d^10)/(b^34*
d^10))*sqrt(b*x + a) + 15*(21*b^6*c^6 + 14*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 + 20*a^3*b^3*c^3*d^3 + 35*a^4*b^2*c^2*d^4 + 126*a^5*b*c*d^5 - 231*a^6*d^6)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b^4*d^5))*b*d^2*abs(b) + 5760*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d
)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*
c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a
)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^2*c^2*abs(b)/b^2 + 3840*(sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (
b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqr
t(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*a^3*c*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 1089, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/6/d*(b*x+a)^(5/2)*(d*x+c)^(7/2)+25/256*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1
/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+
b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c^3-1/16/d^2*(b*x+a)^(1/2)*(d*x+c)^(7/2)*a*b
*c+5/192/d^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c^3*b*a+25/512/d^2*(d*x+c)^(1/2)*(
b*x+a)^(1/2)*a*c^4*b+1/64/d^2*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c^2*b*a-25/512*d/
```

```
b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4*c+1/192/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a
^3+1/12/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)*a+1/32/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)*
a^2-1/12/d^2*(b*x+a)^(3/2)*(d*x+c)^(7/2)*b*c-5/128/d*(d*x+c)^(3/2)*(b*x+a)^
(1/2)*a^2*c^2-25/256/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^2*c^3-5/768/d^3*(d*x+c
)^(3/2)*(b*x+a)^(1/2)*c^4*b^2-1/64/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a^2*c+25/2
56/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c^2+5/192/b*(d*x+c)^(3/2)*(b*x+a)^(1/2
)*a^3*c+5/512*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^5-5/512/d^3*(d*x+c)^(1/
2)*(b*x+a)^(1/2)*c^5*b^2-1/192/d^3*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c^3*b^2-5/76
8*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^4+1/32/d^3*(b*x+a)^(1/2)*(d*x+c)^(7/2
)*b^2*c^2-75/1024*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*l
n((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d
)^(1/2)*a^4*c^2+15/512*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a
)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1
/2))/(b*d)^(1/2)*a^5*c+15/512/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*
x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)
^(1/2))/(b*d)^(1/2)*a*c^5*b^2-75/1024/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b
*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^4*b-5/1024/d^3*((b*x+a)*(d*x+c))^(1/2)/(d*x
+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c
+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^6*b^3-5/1024*d^3/b^3*((b*x+a)*(d*x+c))^(
1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*
d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^6
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)*(c + d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{2}} (c+dx)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(5/2)*(c + d*x)**(5/2), x)
```

3.1376 $\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx$

Optimal. Leaf size=224

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{16b^3}$$

Rubi [A] time = 0.12, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^4}{128b^3d^2}+\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{128b^{7/2}d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}{16b^3}+\frac{(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}{8b^2}+\frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] (-3*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])/(128*b^3*d^2) + ((b*c - a*d)
^3*(a + b*x)^(3/2)*Sqrt[c + d*x])/(64*b^3*d) + ((b*c - a*d)^2*(a + b*x)^(5/
2)*Sqrt[c + d*x])/(16*b^3) + ((b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2))/
(8*b^2) + ((a + b*x)^(5/2)*(c + d*x)^(5/2))/(5*b) + (3*(b*c - a*d)^5*ArcTan
h[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(128*b^(7/2)*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int (a+bx)^{3/2}(c+dx)^{5/2}\,dx = \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{(bc-ad)\int (a+bx)^{3/2}(c+dx)^{3/2}\,dx}{2b}$$

$$= \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+dx)^{5/2}}{5b} + \frac{\left(3(bc-ad)^2\right)\int (a+bx)}{16b^2}$$

$$= \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}{8b^2} + \frac{(a+bx)^{5/2}(c+}{5b}$$

$$= \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}{16b^3} + \frac{(bc-ad)(a+bx}{8b}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+}{16}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+}{16}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+}{16}$$

$$= -\frac{3(bc-ad)^4\sqrt{a+bx}\,\sqrt{c+dx}}{128b^3d^2} + \frac{(bc-ad)^3(a+bx)^{3/2}\sqrt{c+dx}}{64b^3d} + \frac{(bc-ad)^2(a+}{16}$$

Mathematica [A] time = 1.47, size = 187, normalized size = 0.83

$$\frac{(a+bx)^{5/2}\sqrt{c+dx}\left(\frac{15(bc-ad)^{9/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{5/2}(a+bx)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}} - \frac{15(bc-ad)^4}{d^2(a+bx)^2} + \frac{10(bc-ad)^3}{d(a+bx)} + 80b(c+dx)(bc-ad) + 40(bc-ad)^2 + 128b^2(c+dx)^2\right)}{640b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] ((a + b*x)^(5/2)*Sqrt[c + d*x]*(40*(b*c - a*d)^2 - (15*(b*c - a*d)^4)/(d^2*
(a + b*x)^2) + (10*(b*c - a*d)^3)/(d*(a + b*x)) + 80*b*(b*c - a*d)*(c + d*x
) + 128*b^2*(c + d*x)^2 + (15*(b*c - a*d)^(9/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[b*c - a*d]])/(d^(5/2)*(a + b*x)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a
*d)]])))/(640*b^3)
```

IntegrateAlgebraic [A] time = 0.43, size = 198, normalized size = 0.88

$$\frac{3(bc-ad)^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{128b^{7/2}d^{5/2}} - \frac{\sqrt{c+dx}\,(bc-ad)^5\left(\frac{15b^4(c+dx)^4}{(a+bx)^4} - \frac{70b^3d(c+dx)^3}{(a+bx)^3} - \frac{128b^2d^2(c+dx)^2}{(a+bx)^2} + \frac{70bd^3(c+dx)}{a+bx} - 15d^4\right)}{640b^3d^2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx} - d\right)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)*(c + d*x)^(5/2),x]
```

```
[Out] -1/640*((b*c - a*d)^5*Sqrt[c + d*x]*(-15*d^4 + (70*b*d^3*(c + d*x))/(a + b*
x) - (128*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 - (70*b^3*d*(c + d*x)^3)/(a + b*
x)^3 + (15*b^4*(c + d*x)^4)/(a + b*x)^4))/(b^3*d^2*Sqrt[a + b*x]*(-d + (b*(
c + d*x))/(a + b*x))^5) + (3*(b*c - a*d)^5*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/
(Sqrt[d]*Sqrt[a + b*x])])/(128*b^(7/2)*d^(5/2))
```

fricas [A] time = 0.89, size = 702, normalized size = 3.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2560*(15*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2
*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x
 + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a
*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 70*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(
21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2
*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5
*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^3), -1/1280*(15*(b^5*c
^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^
4 - a^5*d^5)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*
x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(
128*b^5*d^5*x^4 - 15*b^5*c^4*d + 70*a*b^4*c^3*d^2 + 128*a^2*b^3*c^2*d^3 - 7
0*a^3*b^2*c*d^4 + 15*a^4*b*d^5 + 16*(21*b^5*c*d^4 + 11*a*b^4*d^5)*x^3 + 8*(
31*b^5*c^2*d^3 + 64*a*b^4*c*d^4 + a^2*b^3*d^5)*x^2 + 2*(5*b^5*c^3*d^2 + 233
*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 5*a^3*b^2*d^5)*x)*sqrt(b*x + a)*sqrt(d*
x + c))/(b^4*d^3)]
```

giac [B] time = 2.33, size = 1962, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1920*(80*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*
(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*
a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b
*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c^2*abs(b) - 1920*((b^2*c - a*b*d
)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/
sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a))*a^2*c^2*abs(
b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*
(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d
^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 +
9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*
x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3
- 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)))/(sqrt(b*d)*b^2*d^3))*c*d*abs(b) + 320*(sqrt(b^2*c + (b*x + a)*b
*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a
*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^
7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*
d^2))*a*c*d*abs(b)/b + (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(4*(b*x + a)
*(6*(b*x + a)*(8*(b*x + a)/b^4 + (b^20*c*d^7 - 41*a*b^19*d^8)/(b^23*d^8)) -
 (7*b^21*c^2*d^6 + 26*a*b^20*c*d^7 - 513*a^2*b^19*d^8)/(b^23*d^8)) + 5*(7*b
^22*c^3*d^5 + 19*a*b^21*c^2*d^6 + 37*a^2*b^20*c*d^7 - 447*a^3*b^19*d^8)/(b^
23*d^8))*(b*x + a) - 15*(7*b^23*c^4*d^4 + 12*a*b^22*c^3*d^5 + 18*a^2*b^21*c
^2*d^6 + 28*a^3*b^20*c*d^7 - 193*a^4*b^19*d^8)/(b^23*d^8))*sqrt(b*x + a) -
15*(7*b^5*c^5 + 5*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 35
*a^4*b*c*d^4 - 63*a^5*d^5)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^3*d^4))*d^2*abs(b) + 80*(sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*
c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*
```

```
b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)
*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(
sqrt(b*d)*b*d^2))*a^2*d^2*abs(b)/b^2 + 20*(sqrt(b^2*c + (b*x + a)*b*d - a*b
*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*d^5 - 25*a*b^11*d^
6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*a^2*b^11*d^6)/(b^1
4*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b^12*c*d^5 - 93*a^3
*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*d^3))*a*d^2*abs(b)
/b + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d
^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))
*a*c^2*abs(b)/b^2 + 960*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*a^2*c*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 848, normalized size = 3.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/5/d*(b*x+a)^(3/2)*(d*x+c)^(7/2)+3/64/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2*c-
3/32*d/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3*c+9/64/b*(d*x+c)^(1/2)*(b*x+a)^(
1/2)*a^2*c^2+3/40/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)*a-1/64*d/b^2*(d*x+c)^(3/2)*
(b*x+a)^(1/2)*a^3-3/64/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c^2+1/64/d^2*(d*x+c)
^(3/2)*(b*x+a)^(1/2)*c^3*b+3/128*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^4-3/
32/d*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^3+3/128/d^2*(d*x+c)^(1/2)*(b*x+a)^(1/2
)*c^4*b-3/40/d^2*(b*x+a)^(1/2)*(d*x+c)^(7/2)*b*c+1/80/b*(d*x+c)^(5/2)*(b*x+
a)^(1/2)*a^2-1/40/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a*c+1/80/d^2*(d*x+c)^(5/2)*
(b*x+a)^(1/2)*c^2*b+15/128*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))
/(b*d)^(1/2)*a^2*c^3+15/256*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(
b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*
x)^(1/2))/(b*d)^(1/2)*a^4*c-15/128*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^3*c^2-15/256/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(
1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d
+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^4*b+3/256/d^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+
c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+
(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^5*b^2-3/256*d^3/b^3*((b*x+a)*(d*x+c))^(1/
2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*
x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/2}\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)*(c + d*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{2}} (c+dx)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)*(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)*(c + d*x)**(5/2), x)
```

3.1377 $\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx$

Optimal. Leaf size=186

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(}{}$$

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {50, 63, 217, 206}

$$-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}+\frac{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}{64b^3d}+\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{32b^3}+\frac{5(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}{24b^2}+\frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*b^3*d) + (5*(b*c - a*d)^2
*(a + b*x)^(3/2)*Sqrt[c + d*x])/(32*b^3) + (5*(b*c - a*d)*(a + b*x)^(3/2)*(
c + d*x)^(3/2))/(24*b^2) + ((a + b*x)^(3/2)*(c + d*x)^(5/2))/(4*b) - (5*(b*
c - a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^
(7/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+bx}\,(c+dx)^{5/2}\,dx = \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{(5(bc-ad))\int \sqrt{a+bx}\,(c+dx)^{3/2}\,dx}{8b}$$
$$= \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+dx)^{5/2}}{4b} + \frac{\left(5(bc-ad)^2\right)\int \sqrt{a+bx}\,\sqrt{}}{16b^2}$$
$$= \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}{24b^2} + \frac{(a+bx)^{3/2}(c+d}{4b}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/}}{24b^2}$$
$$= \frac{5(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64b^3d} + \frac{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{32b^3} + \frac{5(bc-ad)(a+bx)^{3/}}{24b^2}$$

Mathematica [A] time = 0.58, size = 191, normalized size = 1.03

$$\frac{b\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(15a^3d^3-5a^2bd^2(11c+2dx)+ab^2d\left(73c^2+36cdx+8d^2x^2\right)+b^3\left(15c^3+118c^2dx+136cd^2x^2+48d^3x^3\right)\right)-15(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{192b^4d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(15*a^3*d^3 - 5*a^2*b*d^2*(11*c + 2*d*x)
 + a*b^2*d*(73*c^2 + 36*c*d*x + 8*d^2*x^2) + b^3*(15*c^3 + 118*c^2*d*x + 13
6*c*d^2*x^2 + 48*d^3*x^3)) - 15*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))/(b*c -
 a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(192*b^4*d^(3/2)*S
qrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.28, size = 175, normalized size = 0.94

$$\frac{\sqrt{a+bx}\,(bc-ad)^4\left(\frac{73b^2d(a+bx)}{c+dx}+\frac{15d^3(a+bx)^3}{(c+dx)^3}-\frac{55bd^2(a+bx)^2}{(c+dx)^2}+15b^3\right)}{192b^3d\sqrt{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{5(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64b^{7/2}d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]*(c + d*x)^(5/2),x]
```

```
[Out] ((b*c - a*d)^4*Sqrt[a + b*x]*(15*b^3 + (15*d^3*(a + b*x)^3)/(c + d*x)^3 - (
55*b*d^2*(a + b*x)^2)/(c + d*x)^2 + (73*b^2*d*(a + b*x))/(c + d*x)))/(192*b
^3*d*Sqrt[c + d*x]*(b - (d*(a + b*x))/(c + d*x))^4) - (5*(b*c - a*d)^4*ArcT
anh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*b^(7/2)*d^(3/2))
```

fricas [A] time = 1.30, size = 540, normalized size = 2.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/768*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*
b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b
*d^2)*x) + 4*(48*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2
*c*d^3 + 15*a^3*b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^
2 + 18*a*b^3*c*d^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^
2), 1/384*(15*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
+ a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(48
*b^4*d^4*x^3 + 15*b^4*c^3*d + 73*a*b^3*c^2*d^2 - 55*a^2*b^2*c*d^3 + 15*a^3*
b*d^4 + 8*(17*b^4*c*d^3 + a*b^3*d^4)*x^2 + 2*(59*b^4*c^2*d^2 + 18*a*b^3*c*d
^3 - 5*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d^2)]
```

giac [B] time = 1.93, size = 1083, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(192*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*
sqrt(b*x + a))*a*c^2*abs(b)/b^2 - 16*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*s
qrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^
7*d^4)) - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(
b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(
b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*c*d*abs
(b)/b - 8*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(
4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4)) - 3*(b^7*c^2*d^2 +
2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a
^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*
x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*a*d^2*abs(b)/b^2 - (sqrt(b^2*c + (
b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)/b^3 + (b^12*c*
d^5 - 25*a*b^11*d^6)/(b^14*d^6)) - (5*b^13*c^2*d^4 + 14*a*b^12*c*d^5 - 163*
a^2*b^11*d^6)/(b^14*d^6)) + 3*(5*b^14*c^3*d^3 + 9*a*b^13*c^2*d^4 + 15*a^2*b
^12*c*d^5 - 93*a^3*b^11*d^6)/(b^14*d^6))*sqrt(b*x + a) + 3*(5*b^4*c^4 + 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 + 20*a^3*b*c*d^3 - 35*a^4*d^4)*log(abs(-sqrt
(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b^2*
d^3))*d^2*abs(b)/b - 48*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a +
 (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^
2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d))*c^2*abs(b)/b^2 - 96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2
*b*x + 2*a + (b*c*d - 5*a*d^2)/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d
- 3*a^2*b*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d))*a*c*d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 641, normalized size = 3.45

$$-\frac{5\sqrt{(bx+a)(dx+c)}\, a^4 d^3 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^3}+\frac{5\sqrt{(bx+a)(dx+c)}\, a^3 c\, d^2 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\, a^2 c^2 d \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{64\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{5\sqrt{(bx+a)(dx+c)}\, a\, c^3 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{32\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}-\frac{5\sqrt{(bx+a)(dx+c)}\, b\, c^4 \ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{128\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,d}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,a^3 d^2}{64b^3}-\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a^2 c d}{64b^2}+\frac{15\sqrt{dx+c}\,\sqrt{bx+a}\,a c^2}{64b}-\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,c^3}{64d}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,a^2 d}{96b^2}+\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ac}{48b}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c^2}{96d}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,a}{24b}-\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}\,c}{24d}+\frac{\sqrt{bx+a}\,(dx+c)^{\frac{7}{2}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(5/2),x)
```

```
[Out] 1/4/d*(b*x+a)^(1/2)*(d*x+c)^(7/2)+1/24/b*(d*x+c)^(5/2)*(b*x+a)^(1/2)*a-1/24
/d*(d*x+c)^(5/2)*(b*x+a)^(1/2)*c-5/96*d/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a^2
+5/48/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*c-5/96/d*(d*x+c)^(3/2)*(b*x+a)^(1/2)*
c^2+5/64*d^2/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^3-15/64*d/b^2*(d*x+c)^(1/2)*
(b*x+a)^(1/2)*a^2*c+15/64/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*c^2-5/64/d*(d*x+c
)^(1/2)*(b*x+a)^(1/2)*c^3-5/128*d^3/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/
```

```
2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b
*c)*x)^(1/2))/(b*d)^(1/2)*a^4+5/32*d^2/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^
(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*
d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*c-15/64*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c
)^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(
a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*c^2+5/32*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*c^3-5/128/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)
^(1/2)/(b*x+a)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a
*d+b*c)*x)^(1/2))/(b*d)^(1/2)*c^4*b
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+b\,x}\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)*(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(1/2)*(c + d*x)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1378 $\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}} \, dx$

Optimal. Leaf size=148

$$\frac{5(bc-ad)^3 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

Rubi [A] time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8b^3}+\frac{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}{12b^2}+\frac{5(bc-ad)^3 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}}+\frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*b^3) + (5*(b*c - a*d)*Sqrt
[a + b*x]*(c + d*x)^(3/2))/(12*b^2) + (Sqrt[a + b*x]*(c + d*x)^(5/2))/(3*b)
 + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*b^(7/2)*Sqrt[d])
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{\sqrt{a+bx}}\,dx = \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{(5(bc-ad))\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{6b}$$

$$= \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{8b^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc - \cdots}{\cdots}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc - \cdots}{\cdots}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{\left(5(bc - \cdots}{\cdots}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8b^3} + \frac{5(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}{12b^2} + \frac{\sqrt{a+bx}\,(c+dx)^{5/2}}{3b} + \frac{5(bc - \cdots}{\cdots}$$

Mathematica [A] time = 0.41, size = 139, normalized size = 0.94

$$\frac{\sqrt{c+dx}\left(\sqrt{a+bx}\left(15a^2d^2-10abd(4c+dx)+b^2\left(33c^2+26cdx+8d^2x^2\right)\right)+\frac{15(bc-ad)^{5/2}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{\frac{b(c+dx)}{bc-ad}}}\right)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[a + b*x]*(15*a^2*d^2 - 10*a*b*d*(4*c + d*x) + b^2*(33*
c^2 + 26*c*d*x + 8*d^2*x^2)) + (15*(b*c - a*d)^(5/2)*ArcSinh[(Sqrt[d]*Sqrt[
a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[(b*(c + d*x))/(b*c - a*d)])))/(24
*b^3)
```

IntegrateAlgebraic [A] time = 0.20, size = 160, normalized size = 1.08

$$\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8b^{7/2}\sqrt{d}} + \frac{(bc-ad)^3\left(\frac{33b^2\sqrt{a+bx}}{\sqrt{c+dx}}+\frac{15d^2(a+bx)^{5/2}}{(c+dx)^{5/2}}-\frac{40bd(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{24b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/Sqrt[a + b*x],x]
```

```
[Out] ((b*c - a*d)^3*((15*d^2*(a + b*x)^(5/2))/(c + d*x)^(5/2) - (40*b*d*(a + b*x
)^(3/2))/(c + d*x)^(3/2) + (33*b^2*Sqrt[a + b*x])/Sqrt[c + d*x]))/(24*b^3*(
b - (d*(a + b*x))/(c + d*x))^3) + (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*b^(7/2)*Sqrt[d])
```

fricas [A] time = 1.25, size = 412, normalized size = 2.78

$$\left[-\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+33b^3c^2d-40ab^2cd^2+15a^2bd^3+2\left(13b^3cd^2-5ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96b^4d}, -\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(8b^3d^3x^2+33b^3c^2d-40ab^2cd^2+15a^2bd^3+2\left(13b^3cd^2-5ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48b^4d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 + 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 -
5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*d), -1/48*(15*(b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b
*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (
b^2*c*d + a*b*d^2)*x)) - 2*(8*b^3*d^3*x^2 + 33*b^3*c^2*d - 40*a*b^2*c*d^2 +
 15*a^2*b*d^3 + 2*(13*b^3*c*d^2 - 5*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x +
c))/(b^4*d)]
```

giac [B] time = 1.43, size = 446, normalized size = 3.01

$$-\frac{\frac{24\left(\frac{\left(b^2c-abd\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}}-\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\right)c^2|b|}{b^2}-\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(2(bx+a)\left(\frac{4(bx+a)}{b^2}+\frac{b^6cd^3-13ab^5d^4}{b^7d^4}\right)-\frac{3\left(b^7c^2d^2+2ab^6cd^3-11a^2b^5d^4\right)}{b^7d^4}\right)-\frac{3\left(b^3c^3+ab^2c^2d+3a^2bcd^2-5a^3d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}bd^2}\right)d^2|b|}{b^2}-\frac{12\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2bx+2a+\frac{bcd-5ad^2}{d^2}\right)\sqrt{bx+a}+\frac{\left(b^3c^2+2ab^2cd-3a^2bd^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}d}\right)cd|b|}{b^3}}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*(24*((b^2*c - a*b*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c +
(b*x + a)*b*d - a*b*d)))/sqrt(b*d) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sq
rt(b*x + a))*c^2*abs(b)/b^2 - (sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x
 + a)*(2*(b*x + a)*(4*(b*x + a)/b^2 + (b^6*c*d^3 - 13*a*b^5*d^4)/(b^7*d^4))
 - 3*(b^7*c^2*d^2 + 2*a*b^6*c*d^3 - 11*a^2*b^5*d^4)/(b^7*d^4)) - 3*(b^3*c^3
 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a
) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*b*d^2))*d^2*abs(b)/b^2
 - 12*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*b*x + 2*a + (b*c*d - 5*a*d^2)
/d^2)*sqrt(b*x + a) + (b^3*c^2 + 2*a*b^2*c*d - 3*a^2*b*d^2)*log(abs(-sqrt(b
*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d))*c*
d*abs(b)/b^3)/b
```

maple [B] time = 0.01, size = 465, normalized size = 3.14

$$-\frac{5\sqrt{(bx+a)(dx+c)}\,a^3d^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^3}+\frac{15\sqrt{(bx+a)(dx+c)}\,a^2cd^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b^2}-\frac{15\sqrt{(bx+a)(dx+c)}\,ac^2d\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}\,b}+\frac{5\sqrt{(bx+a)(dx+c)}\,c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)x}\right)}{16\sqrt{dx+c}\,\sqrt{bx+a}\,\sqrt{bd}}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,a^2d^2}{8b^3}-\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,acd}{4b^2}+\frac{5\sqrt{dx+c}\,\sqrt{bx+a}\,c^2}{8b}-\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,ad}{12b^2}+\frac{5(dx+c)^{\frac{3}{2}}\sqrt{bx+a}\,c}{12b}+\frac{(dx+c)^{\frac{5}{2}}\sqrt{bx+a}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(5/2)*(b*x+a)^(1/2)/b-5/12/b^2*(d*x+c)^(3/2)*(b*x+a)^(1/2)*a*d+
5/12/b*(d*x+c)^(3/2)*(b*x+a)^(1/2)*c+5/8/b^3*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a^
2*d^2-5/4/b^2*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a*d*c+5/8/b*(d*x+c)^(1/2)*(b*x+a)
^(1/2)*c^2-5/16/b^3*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)*ln(
(b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^
(1/2)*a^3*d^3+15/16/b^2*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1/2)
*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b
*d)^(1/2)*a^2*d^2*c-15/16/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(
1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2)
)/(b*d)^(1/2)*a*d*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^(1
/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))
/(b*d)^(1/2)*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d x)^{5/2}}{\sqrt{a+b x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(1/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

3.1379 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Rubi [A] time = 0.06, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2}+\frac{15d\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4b^3}+\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4b^{7/2}}-\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (15*d*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b^3) + (5*d*Sqrt[a + b*x]
*(c + d*x)^(3/2))/(2*b^2) - (2*(c + d*x)^(5/2))/(b*Sqrt[a + b*x]) + (15*Sqr
t[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])]
)/(4*b^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{3/2}}\,dx = -\frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{\sqrt{a+bx}}\,dx}{b}$$

$$= \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{(15d(bc-ad))\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{4b^2}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\int \frac{}{\sqrt{a}}}{8b^3}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{\left(15d(bc-ad)^2\right)\mathrm{Sub}}{}$$

$$= \frac{15d(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4b^3} + \frac{5d\sqrt{a+bx}\,(c+dx)^{3/2}}{2b^2} - \frac{2(c+dx)^{5/2}}{b\sqrt{a+bx}} + \frac{15\sqrt{d}\,(bc-ad)^2\tanh}{4b^{7/2}}$$

Mathematica [C] time = 0.07, size = 71, normalized size = 0.51

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{1}{2};\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt{a+bx}\,\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -1/2, 1/2, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*Sqrt[a + b*x]*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 151, normalized size = 1.09

$$\frac{15\sqrt{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4b^{7/2}} - \frac{\sqrt{c+dx}\,(bc-ad)^2\left(\frac{8b^2(c+dx)^2}{(a+bx)^2}-\frac{25bd(c+dx)}{a+bx}+15d^2\right)}{4b^3\sqrt{a+bx}\,\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(3/2),x]
```

```
[Out] -1/4*((b*c - a*d)^2*Sqrt[c + d*x]*(15*d^2 - (25*b*d*(c + d*x))/(a + b*x) +
(8*b^2*(c + d*x)^2)/(a + b*x)^2))/(b^3*Sqrt[a + b*x]*(-d + (b*(c + d*x))/(a
 + b*x))^2) + (15*Sqrt[d]*(b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sq
rt[d]*Sqrt[a + b*x])])/(4*b^(7/2))
```

fricas [A] time = 1.42, size = 439, normalized size = 3.18

$$\left[\frac{15\left(ab^2c^2 - 2a^2bcd + a^3d^2 + \left(b^3c^2 - 2ab^2cd + a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4\left(2b^2dx + b^2c + abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}} + 8\left(b^2cd + abd^2\right)x\right) + 4\left(2b^2d^2x^2 - 8b^2c^2 + 25abcd - 15a^2d^2 + \left(9b^2cd - 5abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{16\left(b^4x + ab^3\right)}, -\frac{15\left(ab^2c^2 - 2a^2bcd + a^3d^2 + \left(b^3c^2 - 2ab^2cd + a^2bd^2\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right) - 2\left(2b^2d^2x^2 - 8b^2c^2 + 25abcd - 15a^2d^2 + \left(9b^2cd - 5abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{8\left(b^4x + ab^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2
*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2
+ (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3),
-1/8*(15*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*
b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) - 2*(2*b^2*d^2*x
^2 - 8*b^2*c^2 + 25*a*b*c*d - 15*a^2*d^2 + (9*b^2*c*d - 5*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(b^4*x + a*b^3)]
```

giac [B] time = 2.02, size = 287, normalized size = 2.08

$$\frac{1}{4}\sqrt{b^2c + (bx+a)bd - abd}\sqrt{bx+a}\left(\frac{2(bx+a)d^2|b|}{b^5} + \frac{9\left(b^{10}cd^3|b| - ab^9d^4|b|\right)}{b^{14}d^2}\right) - \frac{15\left(\sqrt{bd}\,b^2c^2|b| - 2\sqrt{bd}\,abcd|b| + \sqrt{bd}\,a^2d^2|b|\right)\log\left(\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)}{8b^5} - \frac{4\left(\sqrt{bd}\,b^3c^3|b| - 3\sqrt{bd}\,ab^2c^2d|b| + 3\sqrt{bd}\,a^2bcd^2|b| - \sqrt{bd}\,a^3d^3|b|\right)}{\left(b^2c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^2c + (bx+a)bd - abd}\right)^2\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*d^2*abs(
b)/b^5 + 9*(b^10*c*d^3*abs(b) - a*b^9*d^4*abs(b))/(b^14*d^2)) - 15/8*(sqrt(
b*d)*b^2*c^2*abs(b) - 2*sqrt(b*d)*a*b*c*d*abs(b) + sqrt(b*d)*a^2*d^2*abs(b)
)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^
5 - 4*(sqrt(b*d)*b^3*c^3*abs(b) - 3*sqrt(b*d)*a*b^2*c^2*d*abs(b) + 3*sqrt(b
*d)*a^2*b*c*d^2*abs(b) - sqrt(b*d)*a^3*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*b^4)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/2}}{{\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(3/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{2}}}{(a+bx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(3/2),x)
```

```
[Out] Integral((c + d*x)**(5/2)/(a + b*x)**(3/2), x)
```

3.1380 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}} dx$

Optimal. Leaf size=128

$$\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}+\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5d^2\sqrt{a+bx}\sqrt{c+dx}}{b^3}+\frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}}-\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (5*d^2*Sqrt[a + b*x]*Sqrt[c + d*x])/b^3 - (10*d*(c + d*x)^(3/2))/(3*b^2*Sqr
t[a + b*x]) - (2*(c + d*x)^(5/2))/(3*b*(a + b*x)^(3/2)) + (5*d^(3/2)*(b*c -
 a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{5/2}}\,dx = -\frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{(5d)\int \frac{(c+dx)^{3/2}}{(a+bx)^{3/2}}\,dx}{3b}$$

$$= -\frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2\right)\int \frac{\sqrt{c+dx}}{\sqrt{a+bx}}\,dx}{b^2}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2b^3}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{\left(5d^2(bc-ad)\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{b^4}$$

$$= \frac{5d^2\sqrt{a+bx}\,\sqrt{c+dx}}{b^3} - \frac{10d(c+dx)^{3/2}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/2}}{3b(a+bx)^{3/2}} + \frac{5d^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{b^{7/2}}$$

Mathematica [C] time = 0.06, size = 73, normalized size = 0.57

$$-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{3}{2};-\frac{1}{2};\frac{d(a+bx)}{ad-bc}\right)}{3b(a+bx)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -3/2, -1/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(3*b*(a + b*x)^(3/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

IntegrateAlgebraic [A] time = 0.99, size = 227, normalized size = 1.77

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(15a^2d^4\sqrt{c+dx}+20abd^3(c+dx)^{3/2}-30abcd^3\sqrt{c+dx}+15b^2c^2d^2\sqrt{c+dx}+3b^2d^2(c+dx)^{5/2}-20b^2cd^2(c+dx)^{3/2}\right)}{3b^3(-ad-b(c+dx)+bc)^2}-\frac{5\sqrt{\frac{b}{d}}\left(bcd^2-ad^3\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(5/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(15*b^2*c^2*d^2*Sqrt[c + d*x] - 30*a*b
*c*d^3*Sqrt[c + d*x] + 15*a^2*d^4*Sqrt[c + d*x] - 20*b^2*c*d^2*(c + d*x)^(3
/2) + 20*a*b*d^3*(c + d*x)^(3/2) + 3*b^2*d^2*(c + d*x)^(5/2)))/(3*b^3*(b*c
- a*d - b*(c + d*x))^2) - (5*Sqrt[b/d]*(b*c*d^2 - a*d^3)*Log[-(Sqrt[b/d]*Sq
rt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/b^4
```

fricas [B] time = 1.93, size = 475, normalized size = 3.71

$$\left[-\frac{15\left(a^2bcd-a^3d^2+\left(b^3cd-ab^2d^2\right)x^2+2\left(ab^2cd-a^2bd^2\right)x\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2-4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(3b^2d^2x^2-2b^2c^2-10abcd+15a^2d^2-2\left(7b^2cd-10abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{12\left(b^5x^2+2ab^4x+a^2b^3\right)}, -\frac{15\left(a^2bcd-a^3d^2+\left(b^3cd-ab^2d^2\right)x^2+2\left(ab^2cd-a^2bd^2\right)x\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)-2\left(3b^2d^2x^2-2b^2c^2-10abcd+15a^2d^2-2\left(7b^2cd-10abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(b^5x^2+2ab^4x+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^2)*x^2 + 2*(a*b^2*c*d
- a^2*b*d^2)*x)*sqrt(d/b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 - 4*(2*b^2*d*x + b^2*c + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2
*d^2 - 2*(7*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2
+ 2*a*b^4*x + a^2*b^3), -1/6*(15*(a^2*b*c*d - a^3*d^2 + (b^3*c*d - a*b^2*d^
2)*x^2 + 2*(a*b^2*c*d - a^2*b*d^2)*x)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c
+ a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d +
 a*d^2)*x)) - 2*(3*b^2*d^2*x^2 - 2*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 2*(7
*b^2*c*d - 10*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^5*x^2 + 2*a*b^4*x
 + a^2*b^3)]
```

giac [**B**] time = 2.19, size = 650, normalized size = 5.08

$$\frac{\sqrt{b^2c+(bx+a)bd-abd}\,\sqrt{bx+a}\,d^2|b|}{b^5}-\frac{5\left(\sqrt{bd}\,bcd|b|-\sqrt{bd}\,ad^2|b|\right)\log\left(\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)}{2b^5}-\frac{4\left(7\sqrt{bd}\,b^6c^4d|b|-28\sqrt{bd}\,ab^5c^3d^2|b|+42\sqrt{bd}\,a^2b^4c^2d^3|b|-28\sqrt{bd}\,a^3b^3cd^4|b|+7\sqrt{bd}\,a^4b^2d^5|b|-12\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^4c^3d|b|+36\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^3c^2d^2|b|-36\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^2cd^3|b|+12\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3bd^4|b|+9\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^2c^2d|b|-18\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4abcd^2|b|+9\sqrt{bd}\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2d^3|b|\right)}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*d^2*abs(b)/b^5 - 5/2*(sqr
t(b*d)*b*c*d*abs(b) - sqrt(b*d)*a*d^2*abs(b))*log((sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)/b^5 - 4/3*(7*sqrt(b*d)*b^6*c^4*d*
abs(b) - 28*sqrt(b*d)*a*b^5*c^3*d^2*abs(b) + 42*sqrt(b*d)*a^2*b^4*c^2*d^3*a
bs(b) - 28*sqrt(b*d)*a^3*b^3*c*d^4*abs(b) + 7*sqrt(b*d)*a^4*b^2*d^5*abs(b)
- 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^4*c^3*d*abs(b) + 36*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a*b^3*c^2*d^2*abs(b) - 36*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*c*d^3*abs(b
) + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^3*b*d^4*abs(b) + 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*b^2*c^2*d*abs(b) - 18*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b*c*d^2*abs(b) + 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
a^2*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2)^3*b^4)
```

maple [**F**] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(5/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/2}}{(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(5/2),x)
[Out] int((c + d*x)^(5/2)/(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(5/2),x)
[Out] Timed out
```

3.1381 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}} dx$

Optimal. Leaf size=120

$$\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Rubi [A] time = 0.05, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$-\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*d^2*Sqrt[c + d*x])/(b^3*Sqrt[a + b*x]) - (2*d*(c + d*x)^(3/2))/(3*b^2*(
a + b*x)^(3/2)) - (2*(c + d*x)^(5/2))/(5*b*(a + b*x)^(5/2)) + (2*d^(5/2)*Ar
cTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/b^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{7/2}}\,dx &= -\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d\int \frac{(c+dx)^{3/2}}{(a+bx)^{5/2}}\,dx}{b}\\
&= -\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^2\int \frac{\sqrt{c+dx}}{(a+bx)^{3/2}}\,dx}{b^2}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{d^3\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{b^3}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{\left(2d^3\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b^4}\\
&= -\frac{2d^2\sqrt{c+dx}}{b^3\sqrt{a+bx}}-\frac{2d(c+dx)^{3/2}}{3b^2(a+bx)^{3/2}}-\frac{2(c+dx)^{5/2}}{5b(a+bx)^{5/2}}+\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{b^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.61

$$
-\frac{2(c+dx)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*Hypergeometric2F1[-5/2, -5/2, -3/2, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(5/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 119, normalized size = 0.99

$$
\frac{2d^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{b^{7/2}}-\frac{2\left(\frac{3b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{15d^2\sqrt{c+dx}}{\sqrt{a+bx}}+\frac{5bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15b^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(7/2),x]
```

```
[Out] (-2*((15*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] + (5*b*d*(c + d*x)^(3/2))/(a + b*
x)^(3/2) + (3*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(15*b^3) + (2*d^(5/2)*
ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/b^(7/2)
```

fricas [B] time = 2.69, size = 463, normalized size = 3.86

$$
\left[\frac{15\left(b^3d^2x^3+3ab^2d^2x^2+3a^2bd^2x+a^3d^2\right)\sqrt{\frac{d}{b}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2b^2dx+b^2c+abd\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{d}{b}}+8\left(b^2cd+abd^2\right)x\right)-4\left(23b^2d^2x^2+3b^2c^2+5abcd+15a^2d^2+\left(11b^2cd+35abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{30\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)},-\frac{15\left(b^3d^2x^3+3ab^2d^2x^2+3a^2bd^2x+a^3d^2\right)\sqrt{-\frac{d}{b}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{d}{b}}}{2\left(bd^2x^2+acd+\left(bcd+ad^2\right)x\right)}\right)+2\left(23b^2d^2x^2+3b^2c^2+5abcd+15a^2d^2+\left(11b^2cd+35abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(b^6x^3+3ab^5x^2+3a^2b^4x+a^3b^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x + a^3*d^2)*sqrt(d/
b)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b^2*d*x + b^2*c
```

```
 + a*b*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(d/b) + 8*(b^2*c*d + a*b*d^2)*x)
- 4*(23*b^2*d^2*x^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^
4*x + a^3*b^3), -1/15*(15*(b^3*d^2*x^3 + 3*a*b^2*d^2*x^2 + 3*a^2*b*d^2*x +
a^3*d^2)*sqrt(-d/b)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
 + c)*sqrt(-d/b)/(b*d^2*x^2 + a*c*d + (b*c*d + a*d^2)*x)) + 2*(23*b^2*d^2*x
^2 + 3*b^2*c^2 + 5*a*b*c*d + 15*a^2*d^2 + (11*b^2*c*d + 35*a*b*d^2)*x)*sqrt
(b*x + a)*sqrt(d*x + c))/(b^6*x^3 + 3*a*b^5*x^2 + 3*a^2*b^4*x + a^3*b^3)]
```

giac [B] time = 2.48, size = 1025, normalized size = 8.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] -sqrt(b*d)*d^2*abs(b)*log((sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^2)/b^5 - 4/15*(23*sqrt(b*d)*b^9*c^5*d^2*abs(b) - 115*sqrt(b*
d)*a*b^8*c^4*d^3*abs(b) + 230*sqrt(b*d)*a^2*b^7*c^3*d^4*abs(b) - 230*sqrt(b
*d)*a^3*b^6*c^2*d^5*abs(b) + 115*sqrt(b*d)*a^4*b^5*c*d^6*abs(b) - 23*sqrt(b
*d)*a^5*b^4*d^7*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*b^7*c^4*d^2*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^6*c^3*d^3*abs(b)
 - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^2*a^2*b^5*c^2*d^4*abs(b) + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^4*c*d^5*abs(b) - 70*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^3*d^
6*abs(b) + 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^5*c^3*d^2*abs(b) - 420*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^4*c^2*d^3*abs(b) + 420*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b
^3*c*d^4*abs(b) - 140*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^4*a^3*b^2*d^5*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^3*c^2*d^2*abs(b) + 180*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a
*b^2*c*d^3*abs(b) - 90*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a^2*b*d^4*abs(b) + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b*c*d^2*abs(b) - 45*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*d^3*a
bs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^2)^5*b^4)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{2}}}{(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

```
[Out] int((d*x+c)^(5/2)/(b*x+a)^(7/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^{5/2}}{(a + b\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(7/2),x)
```

```
[Out] int((c + d*x)^(5/2)/(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.1382 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx$

Optimal. Leaf size=32

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx = -\frac{2(c+dx)^{7/2}}{7(bc-ad)(a+bx)^{7/2}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.06, size = 32, normalized size = 1.00

$$-\frac{2(c+dx)^{7/2}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(9/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(7*(b*c - a*d)*(a + b*x)^(7/2))
```

fricas [B] time = 3.85, size = 138, normalized size = 4.31

$$-\frac{2\left(d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3\right)\sqrt{bx+a}\,\sqrt{dx+c}}{7\left(a^4bc-a^5d+\left(b^5c-ab^4d\right)x^4+4\left(ab^4c-a^2b^3d\right)x^3+6\left(a^2b^3c-a^3b^2d\right)x^2+4\left(a^3b^2c-a^4bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -2/7*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sqrt(b*x + a)*sqrt(d*x + c)/
(a^4*b*c - a^5*d + (b^5*c - a*b^4*d)*x^4 + 4*(a*b^4*c - a^2*b^3*d)*x^3 + 6*
(a^2*b^3*c - a^3*b^2*d)*x^2 + 4*(a^3*b^2*c - a^4*b*d)*x)
```

giac [B] time = 2.54, size = 706, normalized size = 22.06

$$-\frac{4\left(\sqrt{bd}\,b^{12}c^{6}d^{3}|b| - 6\sqrt{bd}\,ab^{11}c^{5}d^{4}|b| + 15\sqrt{bd}\,a^{2}b^{10}c^{4}d^{5}|b| - 20\sqrt{bd}\,a^{3}b^{9}c^{3}d^{6}|b| + 15\sqrt{bd}\,a^{4}b^{8}c^{2}d^{7}|b| - 6\sqrt{bd}\,a^{5}b^{7}cd^{8}|b| + \sqrt{bd}\,a^{6}b^{6}d^{9}|b| + 21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{8}c^{4}d^{3}|b| - 84\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}ab^{7}c^{3}d^{4}|b| + 126\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}a^{2}b^{6}c^{2}d^{5}|b| - 84\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}a^{3}b^{5}cd^{6}|b| + 21\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}a^{4}b^{4}d^{7}|b| + 35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{8}b^{4}c^{2}d^{3}|b| - 70\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{8}ab^{3}cd^{4}|b| + 35\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{8}a^{2}b^{2}d^{5}|b| + 7\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{12}d^{3}|b|\right)}{7\left(b^{2}c - abd - \left(\sqrt{bd}\sqrt{bx+a} - \sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{7}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] -4/7*(sqrt(b*d)*b^12*c^6*d^3*abs(b) - 6*sqrt(b*d)*a*b^11*c^5*d^4*abs(b) + 1
5*sqrt(b*d)*a^2*b^10*c^4*d^5*abs(b) - 20*sqrt(b*d)*a^3*b^9*c^3*d^6*abs(b) +
 15*sqrt(b*d)*a^4*b^8*c^2*d^7*abs(b) - 6*sqrt(b*d)*a^5*b^7*c*d^8*abs(b) + s
qrt(b*d)*a^6*b^6*d^9*abs(b) + 21*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^8*c^4*d^3*abs(b) - 84*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^7*c^3*d^4*ab
s(b) + 126*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^2*b^6*c^2*d^5*abs(b) - 84*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^5*c*d^6*abs(b) + 21*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^4
*d^7*abs(b) + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*b^4*c^2*d^3*abs(b) - 70*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^3*c*d^4*abs(b) + 35*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^2
*d^5*abs(b) + 7*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)
*b*d - a*b*d))^12*d^3*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7*b^4)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{2\left(dx+c\right)^{\frac{7}{2}}}{7\left(bx+a\right)^{\frac{7}{2}}\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(9/2),x)
```

```
[Out] 2/7/(b*x+a)^(7/2)*(d*x+c)^(7/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.97, size = 27, normalized size = 0.84

$$\frac{2\,{\left(c+d\,x\right)}^{7/2}}{\left(7\,a\,d-7\,b\,c\right)\,{\left(a+b\,x\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(9/2),x)
```

```
[Out] (2*(c + d*x)^(7/2))/((7*a*d - 7*b*c)*(a + b*x)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.1383 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d(c+dx)^{7/2}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (4*d*(c + d*x)^(7/2)
)/(63*(b*c - a*d)^2*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx &= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(2d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}}\,dx}{9(bc-ad)}\\ &= -\frac{2(c+dx)^{7/2}}{9(bc-ad)(a+bx)^{9/2}}+\frac{4d(c+dx)^{7/2}}{63(bc-ad)^2(a+bx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{2(c+dx)^{7/2}(9ad-7bc+2bdx)}{63(a+bx)^{9/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-7*b*c + 9*a*d + 2*b*d*x))/(63*(b*c - a*d)^2*(a + b*x)^
(9/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 57, normalized size = 0.86

$$-\frac{2\left(\frac{7b(c+dx)^{9/2}}{(a+bx)^{9/2}}-\frac{9d(c+dx)^{7/2}}{(a+bx)^{7/2}}\right)}{63(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(11/2),x]
```

```
[Out] (-2*((-9*d*(c + d*x)^(7/2))/(a + b*x)^(7/2) + (7*b*(c + d*x)^(9/2))/(a + b*
x)^(9/2)))/(63*(b*c - a*d)^2)
```

fricas [B] time = 13.62, size = 295, normalized size = 4.47

$$\frac{2\left(2\,bd^4x^4-7\,bc^4+9\,ac^3d-\left(bcd^3-9\,ad^4\right)x^3-3\left(5\,bc^2d^2-9\,acd^3\right)x^2-\left(19\,bc^3d-27\,ac^2d^2\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{63\left(a^5b^2c^2-2\,a^6bcd+a^7d^2+\left(b^7c^2-2\,ab^6cd+a^2b^5d^2\right)x^5+5\left(ab^6c^2-2\,a^2b^5cd+a^3b^4d^2\right)x^4+10\left(a^2b^5c^2-2\,a^3b^4cd+a^4b^3d^2\right)x^3+10\left(a^3b^4c^2-2\,a^4b^3cd+a^5b^2d^2\right)x^2+5\left(a^4b^3c^2-2\,a^5b^2cd+a^6bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="fricas")
```

```
[Out] 2/63*(2*b*d^4*x^4 - 7*b*c^4 + 9*a*c^3*d - (b*c*d^3 - 9*a*d^4)*x^3 - 3*(5*b*
c^2*d^2 - 9*a*c*d^3)*x^2 - (19*b*c^3*d - 27*a*c^2*d^2)*x)*sqrt(b*x + a)*sqr
t(d*x + c)/(a^5*b^2*c^2 - 2*a^6*b*c*d + a^7*d^2 + (b^7*c^2 - 2*a*b^6*c*d +
a^2*b^5*d^2)*x^5 + 5*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*x^4 + 10*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*x^3 + 10*(a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*x^2 + 5*(a^4*b^3*c^2 - 2*a^5*b^2*c*d + a^6*b*d^2)*x)
```

giac [B] time = 3.80, size = 1826, normalized size = 27.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="giac")
```

```
[Out] 8/63*(sqrt(b*d)*b^14*c^7*d^4*abs(b) - 7*sqrt(b*d)*a*b^13*c^6*d^5*abs(b) + 2
1*sqrt(b*d)*a^2*b^12*c^5*d^6*abs(b) - 35*sqrt(b*d)*a^3*b^11*c^4*d^7*abs(b)
+ 35*sqrt(b*d)*a^4*b^10*c^3*d^8*abs(b) - 21*sqrt(b*d)*a^5*b^9*c^2*d^9*abs(b
) + 7*sqrt(b*d)*a^6*b^8*c*d^10*abs(b) - sqrt(b*d)*a^7*b^7*d^11*abs(b) - 9*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
b^12*c^6*d^4*abs(b) + 54*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a*b^11*c^5*d^5*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^10*c^4*d^6*abs
(b) + 180*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^9*c^3*d^7*abs(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^8*c^2*d^8*abs(b) + 54*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b
^7*c*d^9*abs(b) - 9*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^2*a^6*b^6*d^10*abs(b) - 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^5*d^4*abs(b) + 135*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a
*b^9*c^4*d^5*abs(b) - 270*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^4*a^2*b^8*c^3*d^6*abs(b) + 270*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c^2*d^7*ab
s(b) - 135*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^4*a^4*b^6*c*d^8*abs(b) + 27*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^5*d^9*abs(b) - 189*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^8*c^4*d^
4*abs(b) + 756*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^6*a*b^7*c^3*d^5*abs(b) - 1134*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c^2*d^6*abs(b) + 756*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
```

```
a^3*b^5*c*d^7*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^4*b^4*d^8*abs(b) - 189*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^6*c^3*d^4*abs(b) +
567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^8*a*b^5*c^2*d^5*abs(b) - 567*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b
^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*c*d^6*abs(b) + 189*sqrt(b*d)*(sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^3*d^7*ab
s(b) - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^10*b^4*c^2*d^4*abs(b) + 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^3*c*d^5*abs(b) - 315*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^2*
d^6*abs(b) - 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^12*b^2*c*d^4*abs(b) + 105*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b*d^5*abs(b) - 63*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*d^4*abs(b
))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
- a*b*d))^2)^9*b^3)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{2\,(dx+c)^{\frac{7}{2}}\,(2bdx+9ad-7bc)}{63\,(bx+a)^{\frac{9}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(11/2),x)
```

```
[Out] 2/63*(d*x+c)^(7/2)*(2*b*d*x+9*a*d-7*b*c)/(b*x+a)^(9/2)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(11/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.14, size = 229, normalized size = 3.47

$$\frac{\sqrt{c+d\,x}\,\left(\frac{4\,d^4\,x^4}{63\,b^3\,{\left(a\,d-b\,c\right)}^2}-\frac{14\,b\,c^4-18\,a\,c^3\,d}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{x^3\,\left(18\,a\,d^4-2\,b\,c\,d^3\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c^2\,d\,x\,\left(27\,a\,d-19\,b\,c\right)}{63\,b^4\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,c\,d^2\,x^2\,\left(9\,a\,d-5\,b\,c\right)}{21\,b^4\,{\left(a\,d-b\,c\right)}^2}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(11/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((4*d^4*x^4)/(63*b^3*(a*d - b*c)^2) - (14*b*c^4 - 18*a*c^3
*d)/(63*b^4*(a*d - b*c)^2) + (x^3*(18*a*d^4 - 2*b*c*d^3))/(63*b^4*(a*d - b*
c)^2) + (2*c^2*d*x*(27*a*d - 19*b*c))/(63*b^4*(a*d - b*c)^2) + (2*c*d^2*x^2
*(9*a*d - 5*b*c))/(21*b^4*(a*d - b*c)^2)))/(x^4*(a + b*x)^(1/2) + (a^4*(a +
 b*x)^(1/2))/b^4 + (6*a^2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/
2))/b + (4*a^3*x*(a + b*x)^(1/2))/b^3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(11/2),x)
```

```
[Out] Timed out
```

3.1384 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}} \, dx$

Optimal. Leaf size=101

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{16d^2(c+dx)^{7/2}}{693(a+bx)^{7/2}(bc-ad)^3}+\frac{8d(c+dx)^{7/2}}{99(a+bx)^{9/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{11(a+bx)^{11/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(11*(b*c - a*d)*(a + b*x)^(11/2)) + (8*d*(c + d*x)^(7/
2))/(99*(b*c - a*d)^2*(a + b*x)^(9/2)) - (16*d^2*(c + d*x)^(7/2))/(693*(b*c
 - a*d)^3*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}} \, dx &= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}-\frac{(4d) \int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}} \, dx}{11(bc-ad)}\\ &= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}+\frac{\left(8d^2\right) \int \frac{(c+dx)^{5/2}}{(a+bx)^{9/2}} \, dx}{99(bc-ad)^2}\\ &= -\frac{2(c+dx)^{7/2}}{11(bc-ad)(a+bx)^{11/2}}+\frac{8d(c+dx)^{7/2}}{99(bc-ad)^2(a+bx)^{9/2}}-\frac{16d^2(c+dx)^{7/2}}{693(bc-ad)^3(a+bx)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.76

$$-\frac{2(c+dx)^{7/2}\left(99a^2d^2+22abd(2dx-7c)+b^2\left(63c^2-28cdx+8d^2x^2\right)\right)}{693(a+bx)^{11/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*a^2*d^2 + 22*a*b*d*(-7*c + 2*d*x) + b^2*(63*c^2 - 2
8*c*d*x + 8*d^2*x^2)))/(693*(b*c - a*d)^3*(a + b*x)^(11/2))
```

IntegrateAlgebraic [A] time = 0.17, size = 73, normalized size = 0.72

$$-\frac{2(c+dx)^{7/2}\left(\frac{63b^2(c+dx)^2}{(a+bx)^2}-\frac{154bd(c+dx)}{a+bx}+99d^2\right)}{693(a+bx)^{7/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(13/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(99*d^2 - (154*b*d*(c + d*x))/(a + b*x) + (63*b^2*(c +
d*x)^2)/(a + b*x)^2))/(693*(b*c - a*d)^3*(a + b*x)^(7/2))
```

fricas [B] time = 31.13, size = 513, normalized size = 5.08

$$-\frac{2\left(8b^2d^5x^5+63b^2c^5-154abc^4d+99a^2c^3d^2-4\left(b^2cd^4-11abd^5\right)x^4+\left(3b^2c^2d^3-22abcd^4+99a^2d^5\right)x^3+\left(113b^2c^3d^2-330abc^2d^3+297a^2cd^4\right)x^2+\left(161b^2c^4d-418abc^3d^2+297a^2c^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{693\left(a^6b^3c^3-3a^7b^2c^2d+3a^8bcd^2-a^9d^3+\left(b^9c^3-3ab^8c^2d+3a^2b^7cd^2-a^3b^6d^3\right)x^6+6\left(ab^8c^3-3a^2b^7c^2d+3a^3b^6cd^2-a^4b^5d^3\right)x^5+15\left(a^2b^7c^3-3a^3b^6c^2d+3a^4b^5cd^2-a^5b^4d^3\right)x^4+20\left(a^3b^6c^3-3a^4b^5c^2d+3a^5b^4cd^2-a^6b^3d^3\right)x^3+15\left(a^4b^5c^3-3a^5b^4c^2d+3a^6b^3cd^2-a^7b^2d^3\right)x^2+6\left(a^5b^4c^3-3a^6b^3c^2d+3a^7b^2cd^2-a^8bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="fricas")
```

```
[Out] -2/693*(8*b^2*d^5*x^5 + 63*b^2*c^5 - 154*a*b*c^4*d + 99*a^2*c^3*d^2 - 4*(b^
2*c*d^4 - 11*a*b*d^5)*x^4 + (3*b^2*c^2*d^3 - 22*a*b*c*d^4 + 99*a^2*d^5)*x^3
 + (113*b^2*c^3*d^2 - 330*a*b*c^2*d^3 + 297*a^2*c*d^4)*x^2 + (161*b^2*c^4*d
 - 418*a*b*c^3*d^2 + 297*a^2*c^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^6*b
^3*c^3 - 3*a^7*b^2*c^2*d + 3*a^8*b*c*d^2 - a^9*d^3 + (b^9*c^3 - 3*a*b^8*c^2
*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*x^6 + 6*(a*b^8*c^3 - 3*a^2*b^7*c^2*d +
3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*x^5 + 15*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*
a^4*b^5*c*d^2 - a^5*b^4*d^3)*x^4 + 20*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2 - a^6*b^3*d^3)*x^3 + 15*(a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*
b^3*c*d^2 - a^7*b^2*d^3)*x^2 + 6*(a^5*b^4*c^3 - 3*a^6*b^3*c^2*d + 3*a^7*b^2
*c*d^2 - a^8*b*d^3)*x)
```

giac [B] time = 4.59, size = 2316, normalized size = 22.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="giac")
```

```
[Out] -32/693*(sqrt(b*d)*b^16*c^8*d^5*abs(b) - 8*sqrt(b*d)*a*b^15*c^7*d^6*abs(b)
+ 28*sqrt(b*d)*a^2*b^14*c^6*d^7*abs(b) - 56*sqrt(b*d)*a^3*b^13*c^5*d^8*abs(
b) + 70*sqrt(b*d)*a^4*b^12*c^4*d^9*abs(b) - 56*sqrt(b*d)*a^5*b^11*c^3*d^10*
abs(b) + 28*sqrt(b*d)*a^6*b^10*c^2*d^11*abs(b) - 8*sqrt(b*d)*a^7*b^9*c*d^12
*abs(b) + sqrt(b*d)*a^8*b^8*d^13*abs(b) - 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
+ a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^14*c^7*d^5*abs(b) + 77*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b
^13*c^6*d^6*abs(b) - 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^2*a^2*b^12*c^5*d^7*abs(b) + 385*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^11*c^4*d^8*a
bs(b) - 385*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^2*a^4*b^10*c^3*d^9*abs(b) + 231*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^9*c^2*d^10*abs(b) - 77*sq
rt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a
^6*b^8*c*d^11*abs(b) + 11*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^7*b^7*d^12*abs(b) + 55*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^12*c^6*d^5*abs(b) -
330*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
```

```
))^4*a*b^11*c^5*d^6*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^10*c^4*d^7*abs(b) - 1100*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^9*c
^3*d^8*abs(b) + 825*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
+ a)*b*d - a*b*d))^4*a^4*b^8*c^2*d^9*abs(b) - 330*sqrt(b*d)*(sqrt(b*d)*sqrt
(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^7*c*d^10*abs(b) +
55*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^4*a^6*b^6*d^11*abs(b) + 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^6*b^10*c^5*d^5*abs(b) - 1485*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^9*c^4*d^6*ab
s(b) + 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d))^6*a^2*b^8*c^3*d^7*abs(b) - 2970*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^7*c^2*d^8*abs(b) + 1485*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*
a^4*b^6*c*d^9*abs(b) - 297*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^6*a^5*b^5*d^10*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^8*c^4*d^5*abs(b)
- 5940*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^8*a*b^7*c^3*d^6*abs(b) + 8910*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^6*c^2*d^7*abs(b) - 5940*sqrt(b*d
)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^5
*c*d^8*abs(b) + 1485*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d))^8*a^4*b^4*d^9*abs(b) + 2079*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^6*c^3*d^5*abs(b) - 6237
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
10*a*b^5*c^2*d^6*abs(b) + 6237*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^4*c*d^7*abs(b) - 2079*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^3*d^8*
abs(b) + 2541*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^12*b^4*c^2*d^5*abs(b) - 5082*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a
) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^3*c*d^6*abs(b) + 2541*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2
*b^2*d^7*abs(b) + 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^14*b^2*c*d^5*abs(b) - 1155*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b*d^6*abs(b) + 462*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*d
^5*abs(b))/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
 a)*b*d - a*b*d))^2)^11*b^2)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\left(dx+c\right)^{\frac{7}{2}}\left(8b^{2}x^{2}d^{2}+44ab\,d^{2}x-28b^{2}cdx+99a^{2}d^{2}-154abcd+63b^{2}c^{2}\right)}{693\left(bx+a\right)^{\frac{11}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(13/2),x)
```

```
[Out] 2/693*(d*x+c)^(7/2)*(8*b^2*d^2*x^2+44*a*b*d^2*x-28*b^2*c*d*x+99*a^2*d^2-154
*a*b*c*d+63*b^2*c^2)/(b*x+a)^(11/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(13/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.35, size = 333, normalized size = 3.30

$$\frac{\sqrt{c+d\,x}\,\left(\frac{198\,a^2\,c^3\,d^2-308\,a\,b\,c^4\,d+126\,b^2\,c^5}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{x^3\,\left(198\,a^2\,d^5-44\,a\,b\,c\,d^4+6\,b^2\,c^2\,d^3\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,d^5\,x^5}{693\,b^3\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d^4\,x^4\,\left(11\,a\,d-b\,c\right)}{693\,b^4\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c\,d^2\,x^2\,\left(297\,a^2\,d^2-330\,a\,b\,c\,d+113\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}+\frac{2\,c^2\,d\,x\,\left(297\,a^2\,d^2-418\,a\,b\,c\,d+161\,b^2\,c^2\right)}{693\,b^5\,{\left(a\,d-b\,c\right)}^3}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^5\,\sqrt{a+b\,x}}{b^5}+\frac{10\,a^2\,x^3\,\sqrt{a+b\,x}}{b^2}+\frac{10\,a^3\,x^2\,\sqrt{a+b\,x}}{b^3}+\frac{5\,a\,x^4\,\sqrt{a+b\,x}}{b}+\frac{5\,a^4\,x\,\sqrt{a+b\,x}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(13/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((126*b^2*c^5 + 198*a^2*c^3*d^2 - 308*a*b*c^4*d)/(693*b^5*
(a*d - b*c)^3) + (x^3*(198*a^2*d^5 + 6*b^2*c^2*d^3 - 44*a*b*c*d^4))/(693*b^
5*(a*d - b*c)^3) + (16*d^5*x^5)/(693*b^3*(a*d - b*c)^3) + (8*d^4*x^4*(11*a*
d - b*c))/(693*b^4*(a*d - b*c)^3) + (2*c*d^2*x^2*(297*a^2*d^2 + 113*b^2*c^2
 - 330*a*b*c*d))/(693*b^5*(a*d - b*c)^3) + (2*c^2*d*x*(297*a^2*d^2 + 161*b^
2*c^2 - 418*a*b*c*d))/(693*b^5*(a*d - b*c)^3)))/(x^5*(a + b*x)^(1/2) + (a^5
*(a + b*x)^(1/2))/b^5 + (10*a^2*x^3*(a + b*x)^(1/2))/b^2 + (10*a^3*x^2*(a +
 b*x)^(1/2))/b^3 + (5*a*x^4*(a + b*x)^(1/2))/b + (5*a^4*x*(a + b*x)^(1/2))/
b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(13/2),x)
```

```
[Out] Timed out
```

3.1385 $\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3(c+dx)^{7/2}}{3003(a+bx)^{7/2}(bc-ad)^4}-\frac{16d^2(c+dx)^{7/2}}{429(a+bx)^{9/2}(bc-ad)^3}+\frac{12d(c+dx)^{7/2}}{143(a+bx)^{11/2}(bc-ad)^2}-\frac{2(c+dx)^{7/2}}{13(a+bx)^{13/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2))/(13*(b*c - a*d)*(a + b*x)^(13/2)) + (12*d*(c + d*x)^(7
/2))/(143*(b*c - a*d)^2*(a + b*x)^(11/2)) - (16*d^2*(c + d*x)^(7/2))/(429*(
b*c - a*d)^3*(a + b*x)^(9/2)) + (32*d^3*(c + d*x)^(7/2))/(3003*(b*c - a*d)^
4*(a + b*x)^(7/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/2}}{(a+bx)^{15/2}}\,dx &= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}-\frac{(6d)\int \frac{(c+dx)^{5/2}}{(a+bx)^{13/2}}\,dx}{13(bc-ad)}\\
&= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}+\frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}}+\frac{\left(24d^2\right)\int \frac{(c+dx)^{5/2}}{(a+bx)^{11/2}}\,dx}{143(bc-ad)^2}\\
&= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}+\frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}}-\frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}}-\frac{\left(16d^3\right)\int}{429(bc}\\
&= -\frac{2(c+dx)^{7/2}}{13(bc-ad)(a+bx)^{13/2}}+\frac{12d(c+dx)^{7/2}}{143(bc-ad)^2(a+bx)^{11/2}}-\frac{16d^2(c+dx)^{7/2}}{429(bc-ad)^3(a+bx)^{9/2}}+\frac{32d}{3003(bc-}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{2(c+dx)^{7/2}\left(429a^3d^3+143a^2bd^2(2dx-7c)+13ab^2d\left(63c^2-28cdx+8d^2x^2\right)+b^3\left(-231c^3+126c^2dx-56cd^2x^2+16d^3x^3\right)\right)}{3003(a+bx)^{13/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(429*a^3*d^3 + 143*a^2*b*d^2*(-7*c + 2*d*x) + 13*a*b^2*d
*(63*c^2 - 28*c*d*x + 8*d^2*x^2) + b^3*(-231*c^3 + 126*c^2*d*x - 56*c*d^2*x
^2 + 16*d^3*x^3)))/(3003*(b*c - a*d)^4*(a + b*x)^(13/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 95, normalized size = 0.70

$$-\frac{2(c+dx)^{7/2}\left(\frac{231b^3(c+dx)^3}{(a+bx)^3}-\frac{819b^2d(c+dx)^2}{(a+bx)^2}+\frac{1001bd^2(c+dx)}{a+bx}-429d^3\right)}{3003(a+bx)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/2)/(a + b*x)^(15/2),x]
```

```
[Out] (-2*(c + d*x)^(7/2)*(-429*d^3 + (1001*b*d^2*(c + d*x))/(a + b*x) - (819*b^2
*d*(c + d*x)^2)/(a + b*x)^2 + (231*b^3*(c + d*x)^3)/(a + b*x)^3))/(3003*(b*
c - a*d)^4*(a + b*x)^(7/2))
```

fricas [B] time = 55.76, size = 765, normalized size = 5.62

$$\frac{2\left(16\,b^{3}d^{6}x^{6}-231\,b^{3}c^{6}+819\,ab^{2}c^{5}d-1001\,a^{2}bc^{4}d^{2}+429\,a^{3}c^{3}d^{3}-8\left(b^{3}cd^{5}-13\,ab^{2}d^{6}\right)x^{5}+2\left(3\,b^{3}c^{2}d^{4}-26\,ab^{2}cd^{5}+143\,a^{2}bd^{6}\right)x^{4}-\left(5\,b^{3}c^{3}d^{3}-39\,ab^{2}c^{2}d^{4}+143\,a^{2}bcd^{5}-429\,a^{3}d^{6}\right)x^{3}-\left(371\,b^{3}c^{4}d^{2}-1469\,ab^{2}c^{3}d^{3}+2145\,a^{2}bc^{2}d^{4}-1287\,a^{3}cd^{5}\right)x^{2}-\left(567\,b^{3}c^{5}d-2093\,ab^{2}c^{4}d^{2}+2717\,a^{2}bc^{3}d^{3}-1287\,a^{3}c^{2}d^{4}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3003\left(a^{7}b^{4}c^{4}-4\,a^{8}b^{3}c^{3}d+6\,a^{9}b^{2}c^{2}d^{2}-4\,a^{10}bcd^{3}+a^{11}d^{4}+\left(b^{11}c^{4}-4\,ab^{10}c^{3}d+6\,a^{2}b^{9}c^{2}d^{2}-4\,a^{3}b^{8}cd^{3}+a^{4}b^{7}d^{4}\right)x^{7}+7\left(ab^{10}c^{4}-4\,a^{2}b^{9}c^{3}d+6\,a^{3}b^{8}c^{2}d^{2}-4\,a^{4}b^{7}cd^{3}+a^{5}b^{6}d^{4}\right)x^{6}+21\left(a^{2}b^{9}c^{4}-4\,a^{3}b^{8}c^{3}d+6\,a^{4}b^{7}c^{2}d^{2}-4\,a^{5}b^{6}cd^{3}+a^{6}b^{5}d^{4}\right)x^{5}+35\left(a^{3}b^{8}c^{4}-4\,a^{4}b^{7}c^{3}d+6\,a^{5}b^{6}c^{2}d^{2}-4\,a^{6}b^{5}cd^{3}+a^{7}b^{4}d^{4}\right)x^{4}+35\left(a^{4}b^{7}c^{4}-4\,a^{5}b^{6}c^{3}d+6\,a^{6}b^{5}c^{2}d^{2}-4\,a^{7}b^{4}cd^{3}+a^{8}b^{3}d^{4}\right)x^{3}+21\left(a^{5}b^{6}c^{4}-4\,a^{6}b^{5}c^{3}d+6\,a^{7}b^{4}c^{2}d^{2}-4\,a^{8}b^{3}cd^{3}+a^{9}b^{2}d^{4}\right)x^{2}+7\left(a^{6}b^{5}c^{4}-4\,a^{7}b^{4}c^{3}d+6\,a^{8}b^{3}c^{2}d^{2}-4\,a^{9}b^{2}cd^{3}+a^{10}bd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="fricas")
```

```
[Out] 2/3003*(16*b^3*d^6*x^6 - 231*b^3*c^6 + 819*a*b^2*c^5*d - 1001*a^2*b*c^4*d^2
 + 429*a^3*c^3*d^3 - 8*(b^3*c*d^5 - 13*a*b^2*d^6)*x^5 + 2*(3*b^3*c^2*d^4 -
26*a*b^2*c*d^5 + 143*a^2*b*d^6)*x^4 - (5*b^3*c^3*d^3 - 39*a*b^2*c^2*d^4 + 1
43*a^2*b*c*d^5 - 429*a^3*d^6)*x^3 - (371*b^3*c^4*d^2 - 1469*a*b^2*c^3*d^3 +
 2145*a^2*b*c^2*d^4 - 1287*a^3*c*d^5)*x^2 - (567*b^3*c^5*d - 2093*a*b^2*c^4
*d^2 + 2717*a^2*b*c^3*d^3 - 1287*a^3*c^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
)/(a^7*b^4*c^4 - 4*a^8*b^3*c^3*d + 6*a^9*b^2*c^2*d^2 - 4*a^10*b*c*d^3 + a^1
1*d^4 + (b^11*c^4 - 4*a*b^10*c^3*d + 6*a^2*b^9*c^2*d^2 - 4*a^3*b^8*c*d^3 +
a^4*b^7*d^4)*x^7 + 7*(a*b^10*c^4 - 4*a^2*b^9*c^3*d + 6*a^3*b^8*c^2*d^2 - 4*
a^4*b^7*c*d^3 + a^5*b^6*d^4)*x^6 + 21*(a^2*b^9*c^4 - 4*a^3*b^8*c^3*d + 6*a^
4*b^7*c^2*d^2 - 4*a^5*b^6*c*d^3 + a^6*b^5*d^4)*x^5 + 35*(a^3*b^8*c^4 - 4*a^
4*b^7*c^3*d + 6*a^5*b^6*c^2*d^2 - 4*a^6*b^5*c*d^3 + a^7*b^4*d^4)*x^4 + 35*(
a^4*b^7*c^4 - 4*a^5*b^6*c^3*d + 6*a^6*b^5*c^2*d^2 - 4*a^7*b^4*c*d^3 + a^8*b
^3*d^4)*x^3 + 21*(a^5*b^6*c^4 - 4*a^6*b^5*c^3*d + 6*a^7*b^4*c^2*d^2 - 4*a^8
*b^3*c*d^3 + a^9*b^2*d^4)*x^2 + 7*(a^6*b^5*c^4 - 4*a^7*b^4*c^3*d + 6*a^8*b^
3*c^2*d^2 - 4*a^9*b^2*c*d^3 + a^10*b*d^4)*x)
```

giac [B] time = 6.09, size = 2868, normalized size = 21.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="giac")
```

```
[Out] 64/3003*(sqrt(b*d)*b^18*c^9*d^6*abs(b) - 9*sqrt(b*d)*a*b^17*c^8*d^7*abs(b)
+ 36*sqrt(b*d)*a^2*b^16*c^7*d^8*abs(b) - 84*sqrt(b*d)*a^3*b^15*c^6*d^9*abs(
b) + 126*sqrt(b*d)*a^4*b^14*c^5*d^10*abs(b) - 126*sqrt(b*d)*a^5*b^13*c^4*d^
11*abs(b) + 84*sqrt(b*d)*a^6*b^12*c^3*d^12*abs(b) - 36*sqrt(b*d)*a^7*b^11*c
^2*d^13*abs(b) + 9*sqrt(b*d)*a^8*b^10*c*d^14*abs(b) - sqrt(b*d)*a^9*b^9*d^1
5*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^2*b^16*c^8*d^6*abs(b) + 104*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^7*d^7*abs(b) - 364*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*
```

b^14*c^6*d^8*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^13*c^5*d^9*abs(b) - 910*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^4*d^10*abs(b) + 728*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^3*d^11*abs(b) - 364*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^6*b^10*c^2*d^12*abs(b) + 104*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*c*d^13*abs(b) - 13*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^8*b^8*d^14*abs(b) + 78*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^7*d^6*abs(b) - 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^13*c^6*d^7*abs(b) + 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^5*d^8*abs(b) - 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^4*d^9*abs(b) + 2730*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^10*c^3*d^10*abs(b) - 1638*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c^2*d^11*abs(b) + 546*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*c*d^12*abs(b) - 78*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^7*b^7*d^13*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^12*c^6*d^6*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^11*c^5*d^7*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^4*d^8*abs(b) + 5720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^9*c^3*d^9*abs(b) - 4290*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c^2*d^10*abs(b) + 1716*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*c*d^11*abs(b) - 286*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^6*b^6*d^12*abs(b) - 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10*c^5*d^6*abs(b) + 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^9*c^4*d^7*abs(b) - 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^3*d^8*abs(b) + 22880*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c^2*d^9*abs(b) - 11440*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^4*b^6*c*d^10*abs(b) + 2288*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^5*b^5*d^11*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^4*d^6*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^3*d^7*abs(b) - 61776*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^2*b^6*c^2*d^8*abs(b) + 41184*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^3*b^5*c*d^9*abs(b) - 10296*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a^4*b^4*d^10*abs(b) - 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^3*d^6*abs(b) + 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^5*c^2*d^7*abs(b) - 48906*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^2*b^4*c*d^8*abs(b) + 16302*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a^3*b^3*d^9*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c^2*d^6*abs(b) + 36036*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*c*d^7*abs(b) - 18018*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a^2*b^2*d^8*abs(b) - 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2*c*d^6*abs(b) + 9009*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*a*b*d^7*abs(b) - 3003*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^18*d^6*abs(b))/((b^2*c - a*

```
b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^13
*b)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2\left(dx+c\right)^{\frac{7}{2}}\left(16b^{3}x^{3}d^{3}+104a\,b^{2}d^{3}x^{2}-56b^{3}c\,d^{2}x^{2}+286a^{2}b\,d^{3}x-364a\,b^{2}c\,d^{2}x+126b^{3}c^{2}dx+429a^{3}d^{3}-1001a^{2}bc\,d^{2}+819a\,b^{2}c^{2}d-231b^{3}c^{3}\right)}{3003\left(bx+a\right)^{\frac{13}{2}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)/(b*x+a)^(15/2),x)
```

```
[Out] 2/3003*(d*x+c)^(7/2)*(16*b^3*d^3*x^3+104*a*b^2*d^3*x^2-56*b^3*c*d^2*x^2+286
*a^2*b*d^3*x-364*a*b^2*c*d^2*x+126*b^3*c^2*d*x+429*a^3*d^3-1001*a^2*b*c*d^2
+819*a*b^2*c^2*d-231*b^3*c^3)/(b*x+a)^(13/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)/(b*x+a)^(15/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.62, size = 459, normalized size = 3.38

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x^2\,\left(2574\,a^3\,c\,d^5-4290\,a^2\,b\,c^2\,d^4+2938\,a\,b^2\,c^3\,d^3-742\,b^3\,c^4\,d^2\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}-\frac{-858\,a^3\,c^3\,d^3+2002\,a^2\,b\,c^4\,d^2-1638\,a\,b^2\,c^5\,d+462\,b^3\,c^6}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{x^3\,\left(858\,a^3\,d^6-286\,a^2\,b\,c\,d^5+78\,a\,b^2\,c^2\,d^4-10\,b^3\,c^3\,d^3\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,d^6\,x^6}{3003\,b^3\,{\left(a\,d-b\,c\right)}^4}-\frac{x\,\left(-2574\,a^3\,c^2\,d^4+5434\,a^2\,b\,c^3\,d^3-4186\,a\,b^2\,c^4\,d^2+1134\,b^3\,c^5\,d\right)}{3003\,b^6\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^5\,x^5\,\left(13\,a\,d-b\,c\right)}{3003\,b^4\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d^4\,x^4\,\left(143\,a^2\,d^2-26\,a\,b\,c\,d+3\,b^2\,c^2\right)}{3003\,b^5\,{\left(a\,d-b\,c\right)}^4}\right)}{x^6\,\sqrt{a+b\,x}+\frac{a^6\,\sqrt{a+b\,x}}{b^6}+\frac{15\,a^2\,x^4\,\sqrt{a+b\,x}}{b^2}+\frac{20\,a^3\,x^3\,\sqrt{a+b\,x}}{b^3}+\frac{15\,a^4\,x^2\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a\,x^5\,\sqrt{a+b\,x}}{b}+\frac{6\,a^5\,x\,\sqrt{a+b\,x}}{b^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/2)/(a + b*x)^(15/2),x)
```

```
[Out] ((c + d*x)^(1/2)*((x^2*(2574*a^3*c*d^5 - 742*b^3*c^4*d^2 + 2938*a*b^2*c^3*d
^3 - 4290*a^2*b*c^2*d^4))/(3003*b^6*(a*d - b*c)^4) - (462*b^3*c^6 - 858*a^3
*c^3*d^3 + 2002*a^2*b*c^4*d^2 - 1638*a*b^2*c^5*d)/(3003*b^6*(a*d - b*c)^4)
+ (x^3*(858*a^3*d^6 - 10*b^3*c^3*d^3 + 78*a*b^2*c^2*d^4 - 286*a^2*b*c*d^5))
/(3003*b^6*(a*d - b*c)^4) + (32*d^6*x^6)/(3003*b^3*(a*d - b*c)^4) - (x*(113
4*b^3*c^5*d - 2574*a^3*c^2*d^4 - 4186*a*b^2*c^4*d^2 + 5434*a^2*b*c^3*d^3))/
(3003*b^6*(a*d - b*c)^4) + (16*d^5*x^5*(13*a*d - b*c))/(3003*b^4*(a*d - b*c
)^4) + (4*d^4*x^4*(143*a^2*d^2 + 3*b^2*c^2 - 26*a*b*c*d))/(3003*b^5*(a*d -
b*c)^4)))/(x^6*(a + b*x)^(1/2) + (a^6*(a + b*x)^(1/2))/b^6 + (15*a^2*x^4*(a
 + b*x)^(1/2))/b^2 + (20*a^3*x^3*(a + b*x)^(1/2))/b^3 + (15*a^4*x^2*(a + b*
x)^(1/2))/b^4 + (6*a*x^5*(a + b*x)^(1/2))/b + (6*a^5*x*(a + b*x)^(1/2))/b^5
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)/(b*x+a)**(15/2),x)
```

```
[Out] Timed out
```

3.1386 $\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=183

$$\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}-\frac{35\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c}}{24}$$

Rubi [A] time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{35\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^3}{64d^4}+\frac{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}{96d^3}-\frac{7(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}{24d^2}+\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{64\sqrt{b}\,d^{9/2}}+\frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (-35*(b*c - a*d)^3*Sqrt[a + b*x]*Sqrt[c + d*x])/(64*d^4) + (35*(b*c - a*d)^
2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(96*d^3) - (7*(b*c - a*d)*(a + b*x)^(5/2)*
Sqrt[c + d*x])/(24*d^2) + ((a + b*x)^(7/2)*Sqrt[c + d*x])/(4*d) + (35*(b*c
- a*d)^4*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(64*Sqrt
[b]*d^(9/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} + \frac{\left(35(bc-ad)^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{48d^2}$$

$$= \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}{24d^2} + \frac{(a+bx)^{7/2}\sqrt{c+dx}}{4d} - \cdots$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}}{24d^2}\cdots$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}}{24d^2}\cdots$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}}{24d^2}\cdots$$

$$= -\frac{35(bc-ad)^3\sqrt{a+bx}\,\sqrt{c+dx}}{64d^4} + \frac{35(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}{96d^3} - \frac{7(bc-ad)(a+bx)^{5/2}}{24d^2}\cdots$$

Mathematica [A] time = 0.66, size = 189, normalized size = 1.03

$$\frac{\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(279a^3d^3+a^2bd^2(326dx-511c)+ab^2d\left(385c^2-252cdx+200d^2x^2\right)+b^3\left(-105c^3+70c^2dx-56cd^2x^2+48d^3x^3\right)\right)+\frac{105(bc-ad)^{9/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{192d^{9/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(279*a^3*d^3 + a^2*b*d^2*(-511*c + 326*d*x
) + a*b^2*d*(385*c^2 - 252*c*d*x + 200*d^2*x^2) + b^3*(-105*c^3 + 70*c^2*d*
x - 56*c*d^2*x^2 + 48*d^3*x^3)) + (105*(b*c - a*d)^(9/2)*Sqrt[(b*(c + d*x))
/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(192*d^(
9/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.20, size = 172, normalized size = 0.94

$$\frac{\sqrt{c+dx}\,(ad-bc)^4\left(-\frac{105b^3(c+dx)^3}{(a+bx)^3}+\frac{385b^2d(c+dx)^2}{(a+bx)^2}-\frac{511bd^2(c+dx)}{a+bx}+279d^3\right)}{192d^4\sqrt{a+bx}\left(d-\frac{b(c+dx)}{a+bx}\right)^4}+\frac{35(bc-ad)^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{64\sqrt{b}\,d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^4*Sqrt[c + d*x]*(279*d^3 - (511*b*d^2*(c + d*x))/(a + b*x)
+ (385*b^2*d*(c + d*x)^2)/(a + b*x)^2 - (105*b^3*(c + d*x)^3)/(a + b*x)^3))
/(192*d^4*Sqrt[a + b*x]*(d - (b*(c + d*x))/(a + b*x))^4) + (35*(b*c - a*d)^
4*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(64*Sqrt[b]*d^(
9/2))
```

fricas [A] time = 1.10, size = 542, normalized size = 2.96

$$\left[\frac{105\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{bd}\,\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(48b^4d^4x^3-105b^4c^3d+385ab^3c^2d^2-511a^2b^2cd^3+279a^3bd^4-8\left(7b^4cd^3-25ab^3d^4\right)x^2+2\left(35b^4c^2d^2-126ab^3cd^3+163a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{768bd^5}, -\frac{105\left(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(48b^4d^4x^3-105b^4c^3d+385ab^3c^2d^2-511a^2b^2cd^3+279a^3bd^4-8\left(7b^4cd^3-25ab^3d^4\right)x^2+2\left(35b^4c^2d^2-126ab^3cd^3+163a^2b^2d^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{384bd^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2
*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*
b*d^2)*x) + 4*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2
*b^2*c*d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4
*c^2*d^2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^5), -1/384*(105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)
*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x
)) - 2*(48*b^4*d^4*x^3 - 105*b^4*c^3*d + 385*a*b^3*c^2*d^2 - 511*a^2*b^2*c*
d^3 + 279*a^3*b*d^4 - 8*(7*b^4*c*d^3 - 25*a*b^3*d^4)*x^2 + 2*(35*b^4*c^2*d^
2 - 126*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d
^5)]
```

giac [A] time = 1.23, size = 268, normalized size = 1.46

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\left(2(bx+a)\left(4(bx+a)\left(\frac{6(bx+a)}{bd}-\frac{7(bcd^5-ad^6)}{bd^7}\right)+\frac{35(b^2c^2d^4-2abcd^5+a^2d^6)}{bd^7}\right)-\frac{105(b^3c^3d^3-3ab^2c^2d^4+3a^2bcd^5-a^3d^6)}{bd^7}\right)\sqrt{bx+a}-\frac{105(b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^4}\right)b}{192|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*
x + a)/(b*d) - 7*(b*c*d^5 - a*d^6)/(b*d^7)) + 35*(b^2*c^2*d^4 - 2*a*b*c*d^5
 + a^2*d^6)/(b*d^7)) - 105*(b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d^5 -
 a^3*d^6)/(b*d^7))*sqrt(b*x + a) - 105*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2
*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt
(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4))*b/abs(b)
```

maple [B] time = 0.01, size = 650, normalized size = 3.55

$$\frac{35\sqrt{(bx+a)(dx+c)}\,a^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bdx^2+ac+(ad+bc)x}\right)}{128\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}}-\frac{35\sqrt{(bx+a)(dx+c)}\,a^3bc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bdx^2+ac+(ad+bc)x}\right)}{32\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{105\sqrt{(bx+a)(dx+c)}\,a^2b^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bdx^2+ac+(ad+bc)x}\right)}{64\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^2}-\frac{35\sqrt{(bx+a)(dx+c)}\,ab^3c^3\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bdx^2+ac+(ad+bc)x}\right)}{32\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^3}+\frac{35\sqrt{(bx+a)(dx+c)}\,b^4c^4\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bdx^2+ac+(ad+bc)x}\right)}{128\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d^4}+\frac{35\sqrt{bx+a}\,\sqrt{dx+c}\,a^3}{64d}-\frac{105\sqrt{bx+a}\,\sqrt{dx+c}\,a^2bc}{64d^2}+\frac{105\sqrt{bx+a}\,\sqrt{dx+c}\,ab^2c^2}{64d^3}-\frac{35\sqrt{bx+a}\,\sqrt{dx+c}\,b^3c^3}{64d^4}+\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,a^2}{96d}-\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,abc}{48d^2}+\frac{35(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\,b^2c^2}{96d^3}+\frac{7(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,a}{24d}-\frac{7(bx+a)^{\frac{5}{2}}\sqrt{dx+c}\,bc}{24d^2}+\frac{(bx+a)^{\frac{7}{2}}\sqrt{dx+c}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/4*(b*x+a)^(7/2)*(d*x+c)^(1/2)/d+7/24/d*(b*x+a)^(5/2)*(d*x+c)^(1/2)*a-7/24
/d^2*(b*x+a)^(5/2)*(d*x+c)^(1/2)*b*c+35/96/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a^
2-35/48/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a*b*c+35/96/d^3*(b*x+a)^(3/2)*(d*x+
c)^(1/2)*b^2*c^2+35/64/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^3-105/64/d^2*(b*x+a)
^(1/2)*(d*x+c)^(1/2)*a^2*b*c+105/64/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b^2*c
^2-35/64/d^4*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b^3*c^3+35/128*((b*x+a)*(d*x+c))^(
1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*
d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^4-35/32/d*((b*x+a)*(d*x+c))^(1/
2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*
x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^3*b*c+105/64/d^2*((b*x+a)*(d*x+c)
)^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+
(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a^2*b^2*c^2-35/32/d^3*((b*x+a)
*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d
)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*a*b^3*c^3+35/128/d^4*(
(b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*
c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*b^4*c^4
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1387 $\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=148

$$-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Rubi [A] time = 0.07, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{5\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^3}-\frac{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^2}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8\sqrt{b}\,d^{7/2}}+\frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (5*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^3) - (5*(b*c - a*d)*(a +
 b*x)^(3/2)*Sqrt[c + d*x])/(12*d^2) + ((a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d)
 - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(8*Sqrt[b]*d^(7/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \frac{(5(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d}$$

$$= -\frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} + \frac{\left(5(bc-ad)^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{8d^2}$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \cdots$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \cdots$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \cdots$$

$$= \frac{5(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^3} - \frac{5(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^2} + \frac{(a+bx)^{5/2}\sqrt{c+dx}}{3d} - \cdots$$

Mathematica [A] time = 0.52, size = 150, normalized size = 1.01

$$\frac{\sqrt{d}\sqrt{a+bx}\,(c+dx)\left(33a^2d^2+2abd(13dx-20c)+b^2\left(15c^2-10cdx+8d^2x^2\right)\right)-\frac{15(bc-ad)^{7/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{24d^{7/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(33*a^2*d^2 + 2*a*b*d*(-20*c + 13*d*x) + b
^2*(15*c^2 - 10*c*d*x + 8*d^2*x^2)) - (15*(b*c - a*d)^(7/2)*Sqrt[(b*(c + d*
x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/b)/(24*d
^(7/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.21, size = 160, normalized size = 1.08

$$\frac{(ad-bc)^3\left(\frac{15b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{33d^2\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{40bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{24d^3\left(d-\frac{b(c+dx)}{a+bx}\right)^3}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8\sqrt{b}\,d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^3*((33*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] - (40*b*d*(c + d*x)
^(3/2))/(a + b*x)^(3/2) + (15*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(24*d^
3*(d - (b*(c + d*x))/(a + b*x))^3) - (5*(b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*Sqrt[b]*d^(7/2))
```

fricas [A] time = 1.02, size = 412, normalized size = 2.78

$$\left[-\frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{bd}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)-4\left(8b^3d^3x^2+15b^3c^2d-40ab^2cd^2+33a^2bd^3-2\left(5b^3cd^2-13ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96bd^4}, \frac{15\left(b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3\right)\sqrt{-bd}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)+2\left(8b^3d^3x^2+15b^3c^2d-40ab^2cd^2+33a^2bd^3-2\left(5b^3cd^2-13ab^2d^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48bd^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(b*d)*lo
g(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*s
qrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*
d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 + 33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 1
3*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^4), 1/48*(15*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*
c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)) + 2*(8*b^3*d^3*x^2 + 15*b^3*c^2*d - 40*a*b^2*c*d^2 +
33*a^2*b*d^3 - 2*(5*b^3*c*d^2 - 13*a*b^2*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c
))/(b*d^4)]
```

giac [A] time = 1.27, size = 198, normalized size = 1.34

$$\frac{\left(\sqrt{b^{2}c+(bx+a)bd-abd}\,\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)}{bd}-\frac{5\left(bcd^{3}-ad^{4}\right)}{bd^{5}}\right)+\frac{15\left(b^{2}c^{2}d^{2}-2\,abcd^{3}+a^{2}d^{4}\right)}{bd^{5}}\right)+\frac{15\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^{3}}\right)b}{24\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*
x + a)/(b*d) - 5*(b*c*d^3 - a*d^4)/(b*d^5)) + 15*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)/(b*d^5)) + 15*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))
/(sqrt(b*d)*d^3))*b/abs(b)
```

maple [B] time = 0.01, size = 465, normalized size = 3.14

$$\frac{5\sqrt{(bx+a)\,(dx+c)}\;a^{3}\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)\,x}\right)}{16\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}}-\frac{15\sqrt{(bx+a)\,(dx+c)}\;a^{2}bc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)\,x}\right)}{16\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}\;d}+\frac{15\sqrt{(bx+a)\,(dx+c)}\;a\,b^{2}c^{2}\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)\,x}\right)}{16\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}\;d^{2}}-\frac{5\sqrt{(bx+a)\,(dx+c)}\;b^{3}c^{3}\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^{2}+ac+(ad+bc)\,x}\right)}{16\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}\;d^{3}}+\frac{5\sqrt{bx+a}\;\sqrt{dx+c}\;a^{2}}{8d}-\frac{5\sqrt{bx+a}\;\sqrt{dx+c}\;abc}{4d^{2}}+\frac{5\sqrt{bx+a}\;\sqrt{dx+c}\;b^{2}c^{2}}{8d^{3}}+\frac{5\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\;a}{12d}-\frac{5\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}\;bc}{12d^{2}}+\frac{(bx+a)^{\frac{5}{2}}\sqrt{dx+c}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(b*x+a)^(5/2)*(d*x+c)^(1/2)/d+5/12/d*(b*x+a)^(3/2)*(d*x+c)^(1/2)*a-5/12
/d^2*(b*x+a)^(3/2)*(d*x+c)^(1/2)*b*c+5/8/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a^2-
5/4/d^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a*b*c+5/8/d^3*(b*x+a)^(1/2)*(d*x+c)^(1/
2)*b^2*c^2+5/16*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d
*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2
)*a^3-15/16/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x
+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)*
a^2*b*c+15/16/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b
*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1
/2)*a*b^2*c^2-5/16/d^3*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*
ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*
d)^(1/2)*b^3*c^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1388 $\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=113

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Rubi [A] time = 0.05, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$-\frac{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^2}+\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (-3*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^2) + ((a + b*x)^(3/2)*Sqr
t[c + d*x])/(2*d) + (3*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[
b]*Sqrt[c + d*x])])/(4*Sqrt[b]*d^(5/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} - \frac{(3(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{8d^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,d\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{\left(3(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x,\right.}{4bd^2}$$

$$= -\frac{3(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^2} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2d} + \frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4\sqrt{b}\,d^{5/2}}$$

Mathematica [**A**] time = 0.37, size = 119, normalized size = 1.05

$$\frac{\sqrt{d}\,\sqrt{a+bx}\,(c+dx)(5ad-3bc+2bdx) + \frac{3(bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b}}{4d^{5/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(-3*b*c + 5*a*d + 2*b*d*x) + (3*(b*c - a*d
)^(5/2)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqr
t[b*c - a*d]])/b)/(4*d^(5/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [**A**] time = 0.19, size = 134, normalized size = 1.19

$$\frac{3(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{4\sqrt{b}\,d^{5/2}} + \frac{(ad-bc)^2\left(\frac{5d\sqrt{c+dx}}{\sqrt{a+bx}} - \frac{3b(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{4d^2\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/Sqrt[c + d*x],x]
```

```
[Out] ((-(b*c) + a*d)^2*((5*d*Sqrt[c + d*x])/Sqrt[a + b*x] - (3*b*(c + d*x)^(3/2)
)/(a + b*x)^(3/2)))/(4*d^2*(d - (b*(c + d*x))/(a + b*x))^2) + (3*(b*c - a*d
)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*Sqrt[b]*d^
(5/2))
```

fricas [**A**] time = 1.36, size = 306, normalized size = 2.71

$$\left[\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{bd}\,\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4(2bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)+4\left(2b^2d^2x-3b^2cd+5abd^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{16bd^3}, -\frac{3\left(b^2c^2-2abcd+a^2d^2\right)\sqrt{-bd}\,\arctan\left(\frac{(2bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)-2\left(2b^2d^2x-3b^2cd+5abd^2\right)\sqrt{bx+a}\,\sqrt{dx+c}}{8bd^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*
c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)
```

```
*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x - 3*b^2*c*d + 5*
a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3), -1/8*(3*(b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x
 + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) - 2*(2
*b^2*d^2*x - 3*b^2*c*d + 5*a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b*d^3)]
```

giac [A] time = 0.95, size = 139, normalized size = 1.23

$$\frac{\left(\sqrt{b^2c + (bx+a)bd - abd}\,\sqrt{bx+a}\left(\frac{2\,(bx+a)}{bd} - \frac{3\left(bcd - ad^2\right)}{bd^3}\right) - \frac{3\left(b^2c^2 - 2\,abcd + a^2d^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a} + \sqrt{b^2c + (bx+a)bd - abd}\right|\right)}{\sqrt{bd}\,d^2}\right)b}{4\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b*d) -
 3*(b*c*d - a*d^2)/(b*d^3)) - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(-sq
rt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^
2))*b/abs(b)
```

maple [B] time = 0.01, size = 308, normalized size = 2.73

$$\frac{3\sqrt{(bx+a)\,(dx+c)}\;a^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}} + \sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{8\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}} - \frac{3\sqrt{(bx+a)\,(dx+c)}\;abc\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}} + \sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{4\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}\;d} + \frac{3\sqrt{(bx+a)\,(dx+c)}\;b^2c^2\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}} + \sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{8\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}\;d^2} + \frac{3\sqrt{bx+a}\;\sqrt{dx+c}\;a}{4d} - \frac{3\sqrt{bx+a}\;\sqrt{dx+c}\;bc}{4d^2} + \frac{(bx+a)^{\frac{3}{2}}\;\sqrt{dx+c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(b*x+a)^(3/2)*(d*x+c)^(1/2)/d+3/4/d*(b*x+a)^(1/2)*(d*x+c)^(1/2)*a-3/4/d
^2*(b*x+a)^(1/2)*(d*x+c)^(1/2)*b*c+3/8*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2
)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*
c)*x)^(1/2))/(b*d)^(1/2)*a^2-3/4/d*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*a*b*c+3/8/d^2*((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d
*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x
)^(1/2))/(b*d)^(1/2)*b^2*c^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(1/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/sqrt(c + d*x), x)
```

3.1389 $\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=73

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {50, 63, 217, 206}

$$\frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[a + b*x]*Sqrt[c + d*x])/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*
x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx = \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt{a+bx}\,\sqrt{c+dx}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,d^{3/2}}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 1.41

$$\frac{b\sqrt{d}\,\sqrt{a+bx}\,(c+dx)-(bc-ad)^{3/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{bd^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x) - (b*c - a*d)^(3/2)*Sqrt[(b*(c + d*x))/(
b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*d^(3/2)*Sq
rt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.29, size = 106, normalized size = 1.45

$$\frac{\sqrt{c+dx}\,\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}}{d}+\frac{\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[c + d*x]*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/d + (Sqrt[b/d]*(b*c - a
*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/
(b*d)
```

fricas [A] time = 0.77, size = 235, normalized size = 3.22

$$\left[\frac{4\sqrt{bx+a}\,\sqrt{dx+c}\,bd-(bc-ad)\sqrt{bd}\,\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\,\sqrt{bx+a}\,\sqrt{dx+c}+8\,(b^2cd+abd^2)x\right)}{4\,bd^2},\ \frac{2\sqrt{bx+a}\,\sqrt{dx+c}\,bd+(bc-ad)\sqrt{-bd}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\,(b^2d^2x^2+abcd+(b^2cd+abd^2)x)}\right)}{2\,bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(b*x + a)*sqrt(d*x + c)*b*d - (b*c - a*d)*sqrt(b*d)*log(8*b^2*d
^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*
sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x))/(b*d^2), 1/2*(2*sqr
t(b*x + a)*sqrt(d*x + c)*b*d + (b*c - a*d)*sqrt(-b*d)*arctan(1/2*(2*b*d*x +
 b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d +
 (b^2*c*d + a*b*d^2)*x)))/(b*d^2)]
```

giac [A] time = 1.12, size = 97, normalized size = 1.33

$$\frac{b\left(\frac{(bc-ad)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d}+\frac{\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}}{bd}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] b*((b*c - a*d)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*
d - a*b*d)))/(sqrt(b*d)*d) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x +
 a)/(b*d))/abs(b)
```

maple [A] time = 0.01, size = 107, normalized size = 1.47

$$-\frac{(-ad+bc)\sqrt{(bx+a)(dx+c)}\,\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{2\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{bd}\,d}+\frac{\sqrt{bx+a}\,\sqrt{dx+c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (b*x+a)^(1/2)*(d*x+c)^(1/2)/d-1/2*(-a*d+b*c)/d*((b*x+a)*(d*x+c))^(1/2)/(b*x
+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b*c)/(b*d)^(1/2)+(b*d*x^2+a*c
+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 3.80, size = 261, normalized size = 3.58

$$\frac{\frac{(2\,a\,d+2\,b\,c)\left(\sqrt{a+b\,x}-\sqrt{a}\right)^3}{d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^3}+\frac{\left(2\,c\,b^2+2\,a\,d\,b\right)\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{d^3\left(\sqrt{c+d\,x}-\sqrt{c}\right)}-\frac{8\,\sqrt{a}\,b\,\sqrt{c}\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d^2\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}{\frac{\left(\sqrt{a+b\,x}-\sqrt{a}\right)^4}{\left(\sqrt{c+d\,x}-\sqrt{c}\right)^4}+\frac{b^2}{d^2}-\frac{2\,b\left(\sqrt{a+b\,x}-\sqrt{a}\right)^2}{d\left(\sqrt{c+d\,x}-\sqrt{c}\right)^2}}+\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\left(\sqrt{a+b\,x}-\sqrt{a}\right)}{\sqrt{b}\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(1/2),x)
```

```
[Out] (((2*a*d + 2*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/(d^2*((c + d*x)^(1/2) - c^
(1/2))^3) + ((2*b^2*c + 2*a*b*d)*((a + b*x)^(1/2) - a^(1/2)))/(d^3*((c + d*
x)^(1/2) - c^(1/2))) - (8*a^(1/2)*b*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/
(d^2*((c + d*x)^(1/2) - c^(1/2))^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + d
*x)^(1/2) - c^(1/2))^4 + b^2/d^2 - (2*b*((a + b*x)^(1/2) - a^(1/2))^2)/(d*(
(c + d*x)^(1/2) - c^(1/2))^2)) + (2*atanh((d^(1/2)*((a + b*x)^(1/2) - a^(1/
2)))/(b^(1/2)*((c + d*x)^(1/2) - c^(1/2))))*(a*d - b*c))/(b^(1/2)*d^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/sqrt(c + d*x), x)
```

3.1390 $\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {63, 217, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{b}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{b}\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{\sqrt{b}\,\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 77, normalized size = 1.83

$$\frac{2\sqrt{c+dx}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d)])
```

IntegrateAlgebraic [A] time = 0.09, size = 42, normalized size = 1.00

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[
d])
```

fricas [B] time = 1.11, size = 178, normalized size = 4.24

$$\left[\frac{\sqrt{bd}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\left(b^2cd+abd^2\right)x\right)}{2\,bd}, -\frac{\sqrt{-bd}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{-bd}\sqrt{bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2+abcd+\left(b^2cd+abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d
*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d + a*b*d^
2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b
*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d
)]
```

giac [A] time = 1.10, size = 50, normalized size = 1.19

$$-\frac{2\,b\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
))/(sqrt(b*d)*abs(b))
```

maple [B] time = 0.01, size = 76, normalized size = 1.81

$$\frac{\sqrt{(bx+a)\,(dx+c)}\;\ln\left(\frac{bdx+\frac{1}{2}ad+\frac{1}{2}bc}{\sqrt{bd}}+\sqrt{bd\,x^2+ac+(ad+bc)\,x}\right)}{\sqrt{bx+a}\;\sqrt{dx+c}\;\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+a)*(d*x+c))^(1/2)/(b*x+a)^(1/2)/(d*x+c)^(1/2)*ln((b*d*x+1/2*a*d+1/2*b
*c)/(b*d)^(1/2)+(b*d*x^2+a*c+(a*d+b*c)*x)^(1/2))/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.29, size = 45, normalized size = 1.07

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}{\sqrt{-b\,d}\,\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((c + d*x)^(1/2) - c^(1/2)))/((-b*d)^(1/2)*((a + b*x)^(1/2) - a
^(1/2)))))/(-b*d)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx}\,\sqrt{c + dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(c + d*x)), x)
```

3.1391 $\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx = -\frac{2\sqrt{c+dx}}{(bc-ad)\sqrt{a+bx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x])/((-(b*c) + a*d)*Sqrt[a + b*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2\sqrt{c+dx}}{\sqrt{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/((b*c - a*d)*Sqrt[a + b*x])
```

fricas [A] time = 0.92, size = 42, normalized size = 1.40

$$-\frac{2\sqrt{bx+a}\,\sqrt{dx+c}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(b*x + a)*sqrt(d*x + c)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [B] time = 1.04, size = 66, normalized size = 2.20

$$-\frac{4\sqrt{bd}\,b}{\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^2)*abs(b))
```

maple [A] time = 0.01, size = 27, normalized size = 0.90

$$\frac{2\sqrt{dx+c}}{\sqrt{bx+a}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/(b*x+a)^(1/2)*(d*x+c)^(1/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.73, size = 26, normalized size = 0.87

$$\frac{2\sqrt{c+d\,x}}{(a\,d-b\,c)\,\sqrt{a+b\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (2*(c + d*x)^(1/2))/((a*d - b*c)*(a + b*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*sqrt(c + d*x)), x)
```

3.1392 $\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4d\sqrt{c+dx}}{3\sqrt{a+bx}\,(bc-ad)^2}-\frac{2\sqrt{c+dx}}{3(a+bx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(3*(b*c - a*d)*(a + b*x)^(3/2)) + (4*d*Sqrt[c + d*x])/(3
*(b*c - a*d)^2*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}-\frac{(2d)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{3(bc-ad)}\\ &= -\frac{2\sqrt{c+dx}}{3(bc-ad)(a+bx)^{3/2}}+\frac{4d\sqrt{c+dx}}{3(bc-ad)^2\sqrt{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{2\sqrt{c+dx}\,(3ad-bc+2bdx)}{3(a+bx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*c) + 3*a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(a + b*x)^(3/
2))
```

IntegrateAlgebraic [A] time = 0.10, size = 56, normalized size = 0.85

$$-\frac{2\left(\frac{b(c+dx)^{3/2}}{(a+bx)^{3/2}}-\frac{3d\sqrt{c+dx}}{\sqrt{a+bx}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((-3*d*Sqrt[c + d*x])/Sqrt[a + b*x] + (b*(c + d*x)^(3/2))/(a + b*x)^(3/
2)))/(3*(b*c - a*d)^2)
```

fricas [B] time = 0.92, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx-bc+3\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x - b*c + 3*a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^2*c^2 - 2*a^
3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^3*c^
2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

giac [B] time = 1.08, size = 121, normalized size = 1.83

$$\frac{8\left(b^2c-abd-3\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)\sqrt{bd}\,b^2d}{3\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(b^2*c - a*b*d - 3*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2)*sqrt(b*d)*b^2*d/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3*abs(b))
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{2\sqrt{dx+c}\,\left(2bdx+3ad-bc\right)}{3\,(bx+a)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/3*(d*x+c)^(1/2)*(2*b*d*x+3*a*d-b*c)/(b*x+a)^(3/2)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?

mupad [B] time = 0.89, size = 71, normalized size = 1.08

$$\frac{\left(\frac{4\,d\,x}{3\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d-2\,b\,c}{3\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{a\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(1/2)),x)

[Out] (((4*d*x)/(3*(a*d - b*c)^2) + (6*a*d - 2*b*c)/(3*b*(a*d - b*c)^2))*(c + d*x
)^(1/2))/(x*(a + b*x)^(1/2) + (a*(a + b*x)^(1/2))/b)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(1/2),x)

[Out] Integral(1/((a + b*x)**(5/2)*sqrt(c + d*x)), x)

3.1393 $\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d^2\sqrt{c+dx}}{15\sqrt{a+bx}\,(bc-ad)^3}+\frac{8d\sqrt{c+dx}}{15(a+bx)^{3/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{5(a+bx)^{5/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(5*(b*c - a*d)*(a + b*x)^(5/2)) + (8*d*Sqrt[c + d*x])/(1
5*(b*c - a*d)^2*(a + b*x)^(3/2)) - (16*d^2*Sqrt[c + d*x])/(15*(b*c - a*d)^3
*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}-\frac{(4d)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{5(bc-ad)}\\
&= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}+\frac{\left(8d^2\right)\int \frac{1}{(a+bx)^{3/2}\sqrt{c+dx}}\,dx}{15(bc-ad)^2}\\
&= -\frac{2\sqrt{c+dx}}{5(bc-ad)(a+bx)^{5/2}}+\frac{8d\sqrt{c+dx}}{15(bc-ad)^2(a+bx)^{3/2}}-\frac{16d^2\sqrt{c+dx}}{15(bc-ad)^3\sqrt{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.74

$$-\frac{2\sqrt{c+dx}\left(15a^2d^2-10abd(c-2dx)+b^2\left(3c^2-4cdx+8d^2x^2\right)\right)}{15(a+bx)^{5/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(15*a^2*d^2 - 10*a*b*d*(c - 2*d*x) + b^2*(3*c^2 - 4*c*d*x
 + 8*d^2*x^2)))/(15*(b*c - a*d)^3*(a + b*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.10, size = 83, normalized size = 0.82

$$-\frac{2\left(\frac{3b^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{15d^2\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{10bd(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{15(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((15*d^2*Sqrt[c + d*x])/Sqrt[a + b*x] - (10*b*d*(c + d*x)^(3/2))/(a + b
*x)^(3/2) + (3*b^2*(c + d*x)^(5/2))/(a + b*x)^(5/2)))/(15*(b*c - a*d)^3)
```

fricas [B] time = 1.32, size = 251, normalized size = 2.49

$$-\frac{2\left(8b^2d^2x^2+3b^2c^2-10abcd+15a^2d^2-4\left(b^2cd-5abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3+\left(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3a^2b^4c^2d+3a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3a^3b^3c^2d+3a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(8*b^2*d^2*x^2 + 3*b^2*c^2 - 10*a*b*c*d + 15*a^2*d^2 - 4*(b^2*c*d - 5
*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3
*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^
3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*x)
```

giac [B] time = 1.27, size = 227, normalized size = 2.25

$$-\frac{32\left(b^4c^2-2ab^3cd+a^2b^2d^2-5\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^2c+5\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2abd+10\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4\right)\sqrt{bd}\,b^3d^2}{15\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^5|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -32/15*(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2 - 5*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^2*c + 5*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b*d + 10*(sqrt(b*d)*sqrt(b*x + a)
 - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4)*sqrt(b*d)*b^3*d^2/((b^2*c - a*b*
d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5*ab
s(b))
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{2\sqrt{dx+c}\,\left(8b^2x^2d^2+20ab\,d^2x-4b^2cdx+15a^2d^2-10abcd+3b^2c^2\right)}{15\,(bx+a)^{\frac{5}{2}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/15*(d*x+c)^(1/2)*(8*b^2*d^2*x^2+20*a*b*d^2*x-4*b^2*c*d*x+15*a^2*d^2-10*a*
b*c*d+3*b^2*c^2)/(b*x+a)^(5/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.01, size = 133, normalized size = 1.32

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d^2\,x^2}{15\,{\left(a\,d-b\,c\right)}^3}+\frac{30\,a^2\,d^2-20\,a\,b\,c\,d+6\,b^2\,c^2}{15\,b^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,d\,x\,\left(5\,a\,d-b\,c\right)}{15\,b\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a^2\,\sqrt{a+b\,x}}{b^2}+\frac{2\,a\,x\,\sqrt{a+b\,x}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*d^2*x^2)/(15*(a*d - b*c)^3) + (30*a^2*d^2 + 6*b^2*c^2
 - 20*a*b*c*d)/(15*b^2*(a*d - b*c)^3) + (8*d*x*(5*a*d - b*c))/(15*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/2) + (a^2*(a + b*x)^(1/2))/b^2 + (2*a*x*(a + b*
x)^(1/2))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*sqrt(c + d*x)), x)
```

3.1394 $\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{32d^3\sqrt{c+dx}}{35\sqrt{a+bx}\,(bc-ad)^4}-\frac{16d^2\sqrt{c+dx}}{35(a+bx)^{3/2}(bc-ad)^3}+\frac{12d\sqrt{c+dx}}{35(a+bx)^{5/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{7(a+bx)^{7/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(7*(b*c - a*d)*(a + b*x)^(7/2)) + (12*d*Sqrt[c + d*x])/(
35*(b*c - a*d)^2*(a + b*x)^(5/2)) - (16*d^2*Sqrt[c + d*x])/(35*(b*c - a*d)^
3*(a + b*x)^(3/2)) + (32*d^3*Sqrt[c + d*x])/(35*(b*c - a*d)^4*Sqrt[a + b*x]
)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}-\frac{(6d)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{7(bc-ad)}\\ &= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}+\frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}}+\frac{\left(24d^2\right)\int \frac{1}{(a+bx)^{5/2}\sqrt{c+dx}}\,dx}{35(bc-ad)^2}\\ &= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}+\frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}-\frac{(16d}{}\\ &= -\frac{2\sqrt{c+dx}}{7(bc-ad)(a+bx)^{7/2}}+\frac{12d\sqrt{c+dx}}{35(bc-ad)^2(a+bx)^{5/2}}-\frac{16d^2\sqrt{c+dx}}{35(bc-ad)^3(a+bx)^{3/2}}+\frac{}{35(}\end{aligned}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.85

$$\frac{2\sqrt{c+dx}\left(35a^3d^3-35a^2bd^2(c-2dx)+7ab^2d\left(3c^2-4cdx+8d^2x^2\right)+b^3\left(-5c^3+6c^2dx-8cd^2x^2+16d^3x^3\right)\right)}{35(a+bx)^{7/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[c + d*x]*(35*a^3*d^3 - 35*a^2*b*d^2*(c - 2*d*x) + 7*a*b^2*d*(3*c^2
- 4*c*d*x + 8*d^2*x^2) + b^3*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*x^2 + 16*d^3*x^3
)))/(35*(b*c - a*d)^4*(a + b*x)^(7/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 109, normalized size = 0.80

$$-\frac{2\left(\frac{5b^3(c+dx)^{7/2}}{(a+bx)^{7/2}}-\frac{21b^2d(c+dx)^{5/2}}{(a+bx)^{5/2}}-\frac{35d^3\sqrt{c+dx}}{\sqrt{a+bx}}+\frac{35bd^2(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{35(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((-35*d^3*Sqrt[c + d*x])/Sqrt[a + b*x] + (35*b*d^2*(c + d*x)^(3/2))/(a
+ b*x)^(3/2) - (21*b^2*d*(c + d*x)^(5/2))/(a + b*x)^(5/2) + (5*b^3*(c + d*x
)^(7/2))/(a + b*x)^(7/2)))/(35*(b*c - a*d)^4)
```

fricas [B] time = 2.89, size = 419, normalized size = 3.08

$$\frac{2\left(16\,b^{3}d^{3}x^{3}-5\,b^{3}c^{3}+21\,ab^{2}c^{2}d-35\,a^{2}bcd^{2}+35\,a^{3}d^{3}-8\left(b^{3}cd^{2}-7\,ab^{2}d^{3}\right)x^{2}+2\left(3\,b^{3}c^{2}d-14\,ab^{2}cd^{2}+35\,a^{2}bd^{3}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{35\left(a^{4}b^{4}c^{4}-4\,a^{5}b^{3}c^{3}d+6\,a^{6}b^{2}c^{2}d^{2}-4\,a^{7}bcd^{3}+a^{8}d^{4}+\left(b^{8}c^{4}-4\,ab^{7}c^{3}d+6\,a^{2}b^{6}c^{2}d^{2}-4\,a^{3}b^{5}cd^{3}+a^{4}b^{4}d^{4}\right)x^{4}+4\left(ab^{7}c^{4}-4\,a^{2}b^{6}c^{3}d+6\,a^{3}b^{5}c^{2}d^{2}-4\,a^{4}b^{4}cd^{3}+a^{5}b^{3}d^{4}\right)x^{3}+6\left(a^{2}b^{6}c^{4}-4\,a^{3}b^{5}c^{3}d+6\,a^{4}b^{4}c^{2}d^{2}-4\,a^{5}b^{3}cd^{3}+a^{6}b^{2}d^{4}\right)x^{2}+4\left(a^{3}b^{5}c^{4}-4\,a^{4}b^{4}c^{3}d+6\,a^{5}b^{3}c^{2}d^{2}-4\,a^{6}b^{2}cd^{3}+a^{7}bd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*d^3*x^3 - 5*b^3*c^3 + 21*a*b^2*c^2*d - 35*a^2*b*c*d^2 + 35*a^3
*d^3 - 8*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 2*(3*b^3*c^2*d - 14*a*b^2*c*d^2 +
35*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^4*c^4 - 4*a^5*b^3*c^3*d
 + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b^7*c^3*d +
 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^7*c^4 - 4*
a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4)*x^3 + 6*
(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*d^3 + a^6*
b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d^2 - 4*a^6
*b^2*c*d^3 + a^7*b*d^4)*x)
```

giac [B] time = 1.47, size = 386, normalized size = 2.84

$$\frac{64\left(b^{6}c^{3}-3\,ab^{5}c^{2}d+3\,a^{2}b^{4}cd^{2}-a^{3}b^{3}d^{3}-7\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{4}c^{2}+14\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{3}cd-7\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}a^{2}b^{2}d^{2}+21\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{2}c-21\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}abd-35\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{6}\right)\sqrt{bd}\,b^{4}d^{3}}{35\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{7}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 64/35*(b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3 - 7*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^4*c^2 + 14*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^3*c*d - 7
*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^2*
d^2 + 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*
b^2*c - 21*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
4*a*b*d - 35*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^6)*sqrt(b*d)*b^4*d^3/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^
2*c + (b*x + a)*b*d - a*b*d))^2)^7*abs(b))
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{2\sqrt{dx+c}\,\left(16b^{3}x^{3}d^{3}+56a\,b^{2}d^{3}x^{2}-8b^{3}c\,d^{2}x^{2}+70a^{2}b\,d^{3}x-28a\,b^{2}c\,d^{2}x+6b^{3}c^{2}dx+35a^{3}d^{3}-35a^{2}bc\,d^{2}+21a\,b^{2}c^{2}d-5b^{3}c^{3}\right)}{35\,(bx+a)^{\frac{7}{2}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/35*(d*x+c)^(1/2)*(16*b^3*d^3*x^3+56*a*b^2*d^3*x^2-8*b^3*c*d^2*x^2+70*a^2*
b*d^3*x-28*a*b^2*c*d^2*x+6*b^3*c^2*d*x+35*a^3*d^3-35*a^2*b*c*d^2+21*a*b^2*c
^2*d-5*b^3*c^3)/(b*x+a)^(7/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*
b^3*c^3*d+b^4*c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.19, size = 209, normalized size = 1.54

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,d^3\,x^3}{35\,{\left(a\,d-b\,c\right)}^4}+\frac{70\,a^3\,d^3-70\,a^2\,b\,c\,d^2+42\,a\,b^2\,c^2\,d-10\,b^3\,c^3}{35\,b^3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,d\,x\,\left(35\,a^2\,d^2-14\,a\,b\,c\,d+3\,b^2\,c^2\right)}{35\,b^2\,{\left(a\,d-b\,c\right)}^4}+\frac{16\,d^2\,x^2\,\left(7\,a\,d-b\,c\right)}{35\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^3\,\sqrt{a+b\,x}}{b^3}+\frac{3\,a\,x^2\,\sqrt{a+b\,x}}{b}+\frac{3\,a^2\,x\,\sqrt{a+b\,x}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*d^3*x^3)/(35*(a*d - b*c)^4) + (70*a^3*d^3 - 10*b^3*c^
3 + 42*a*b^2*c^2*d - 70*a^2*b*c*d^2)/(35*b^3*(a*d - b*c)^4) + (4*d*x*(35*a^
2*d^2 + 3*b^2*c^2 - 14*a*b*c*d))/(35*b^2*(a*d - b*c)^4) + (16*d^2*x^2*(7*a*
d - b*c))/(35*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^3*(a + b*x)^(1/2
))/b^3 + (3*a*x^2*(a + b*x)^(1/2))/b + (3*a^2*x*(a + b*x)^(1/2))/b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*sqrt(c + d*x)), x)
```

3.1395 $\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=171

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c}}{9(a+bx)^{9}}$$

Rubi [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{256d^4\sqrt{c+dx}}{315\sqrt{a+bx}\,(bc-ad)^5}+\frac{128d^3\sqrt{c+dx}}{315(a+bx)^{3/2}(bc-ad)^4}-\frac{32d^2\sqrt{c+dx}}{105(a+bx)^{5/2}(bc-ad)^3}+\frac{16d\sqrt{c+dx}}{63(a+bx)^{7/2}(bc-ad)^2}-\frac{2\sqrt{c+dx}}{9(a+bx)^{9/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x])/(9*(b*c - a*d)*(a + b*x)^(9/2)) + (16*d*Sqrt[c + d*x])/(
63*(b*c - a*d)^2*(a + b*x)^(7/2)) - (32*d^2*Sqrt[c + d*x])/(105*(b*c - a*d)
^3*(a + b*x)^(5/2)) + (128*d^3*Sqrt[c + d*x])/(315*(b*c - a*d)^4*(a + b*x)^
(3/2)) - (256*d^4*Sqrt[c + d*x])/(315*(b*c - a*d)^5*Sqrt[a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{11/2}\sqrt{c+dx}}\,dx &= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}-\frac{(8d)\int \frac{1}{(a+bx)^{9/2}\sqrt{c+dx}}\,dx}{9(bc-ad)}\\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}+\frac{\left(16d^2\right)\int \frac{1}{(a+bx)^{7/2}\sqrt{c+dx}}\,dx}{21(bc-ad)^2}\\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}-\frac{(64d}{}\\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{}{315(}\\
&= -\frac{2\sqrt{c+dx}}{9(bc-ad)(a+bx)^{9/2}}+\frac{16d\sqrt{c+dx}}{63(bc-ad)^2(a+bx)^{7/2}}-\frac{32d^2\sqrt{c+dx}}{105(bc-ad)^3(a+bx)^{5/2}}+\frac{}{315(}\end{aligned}$$

Mathematica [A] time = 0.07, size = 168, normalized size = 0.98

$$-\frac{2\sqrt{c+dx}\left(315a^4d^4-420a^3bd^3(c-2dx)+126a^2b^2d^2\left(3c^2-4cdx+8d^2x^2\right)+36ab^3d\left(-5c^3+6c^2dx-8cd^2x^2+16d^3x^3\right)+b^4\left(35c^4-40c^3dx+48c^2d^2x^2-64cd^3x^3+128d^4x^4\right)\right)}{315(a+bx)^{9/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[c + d*x]*(315*a^4*d^4 - 420*a^3*b*d^3*(c - 2*d*x) + 126*a^2*b^2*d^
2*(3*c^2 - 4*c*d*x + 8*d^2*x^2) + 36*a*b^3*d*(-5*c^3 + 6*c^2*d*x - 8*c*d^2*
x^2 + 16*d^3*x^3) + b^4*(35*c^4 - 40*c^3*d*x + 48*c^2*d^2*x^2 - 64*c*d^3*x^
3 + 128*d^4*x^4)))/(315*(b*c - a*d)^5*(a + b*x)^(9/2))
```

IntegrateAlgebraic [A] time = 0.11, size = 135, normalized size = 0.79

$$-\frac{2\left(\frac{35b^4(c+dx)^{9/2}}{(a+bx)^{9/2}}-\frac{180b^3d(c+dx)^{7/2}}{(a+bx)^{7/2}}+\frac{378b^2d^2(c+dx)^{5/2}}{(a+bx)^{5/2}}+\frac{315d^4\sqrt{c+dx}}{\sqrt{a+bx}}-\frac{420bd^3(c+dx)^{3/2}}{(a+bx)^{3/2}}\right)}{315(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/2)*Sqrt[c + d*x]),x]
```

```
[Out] (-2*((315*d^4*Sqrt[c + d*x])/Sqrt[a + b*x] - (420*b*d^3*(c + d*x)^(3/2))/(a
 + b*x)^(3/2) + (378*b^2*d^2*(c + d*x)^(5/2))/(a + b*x)^(5/2) - (180*b^3*d*
(c + d*x)^(7/2))/(a + b*x)^(7/2) + (35*b^4*(c + d*x)^(9/2))/(a + b*x)^(9/2)
))/(315*(b*c - a*d)^5)
```

fricas [B] time = 11.40, size = 638, normalized size = 3.73

$$-\frac{2\left(128b^4d^4x^4+35b^4c^4-180ab^3c^3d+378a^2b^2c^2d^2-420a^3bcd^3+315a^4d^4-64\left(b^4cd^3-9ab^3d^4\right)x^3+48\left(b^4c^2d^2-6ab^3cd^3+21a^2b^2d^4\right)x^2-8\left(5b^4c^3d-27ab^3c^2d^2+63a^2b^2cd^3-105a^3bd^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{315\left(a^5b^5c^5-5a^6b^4c^4d+10a^7b^3c^3d^2-10a^8b^2c^2d^3+5a^9bcd^4-a^{10}d^5+\left(b^{10}c^5-5ab^9c^4d+10a^2b^8c^3d^2-10a^3b^7c^2d^3+5a^4b^6cd^4-a^5b^5d^5\right)x^5+5\left(ab^9c^5-5a^2b^8c^4d+10a^3b^7c^3d^2-10a^4b^6c^2d^3+5a^5b^5cd^4-a^6b^4d^5\right)x^4+10\left(a^2b^8c^5-5a^3b^7c^4d+10a^4b^6c^3d^2-10a^5b^5c^2d^3+5a^6b^4cd^4-a^7b^3d^5\right)x^3+10\left(a^3b^7c^5-5a^4b^6c^4d+10a^5b^5c^3d^2-10a^6b^4c^2d^3+5a^7b^3cd^4-a^8b^2d^5\right)x^2+5\left(a^4b^6c^5-5a^5b^5c^4d+10a^6b^4c^3d^2-10a^7b^3c^2d^3+5a^8b^2cd^4-a^9bd^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*d^4*x^4 + 35*b^4*c^4 - 180*a*b^3*c^3*d + 378*a^2*b^2*c^2*d^
2 - 420*a^3*b*c*d^3 + 315*a^4*d^4 - 64*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 48*(
b^4*c^2*d^2 - 6*a*b^3*c*d^3 + 21*a^2*b^2*d^4)*x^2 - 8*(5*b^4*c^3*d - 27*a*b
^3*c^2*d^2 + 63*a^2*b^2*c*d^3 - 105*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x +
c)/(a^5*b^5*c^5 - 5*a^6*b^4*c^4*d + 10*a^7*b^3*c^3*d^2 - 10*a^8*b^2*c^2*d^3
 + 5*a^9*b*c*d^4 - a^10*d^5 + (b^10*c^5 - 5*a*b^9*c^4*d + 10*a^2*b^8*c^3*d^
2 - 10*a^3*b^7*c^2*d^3 + 5*a^4*b^6*c*d^4 - a^5*b^5*d^5)*x^5 + 5*(a*b^9*c^5
- 5*a^2*b^8*c^4*d + 10*a^3*b^7*c^3*d^2 - 10*a^4*b^6*c^2*d^3 + 5*a^5*b^5*c*d
^4 - a^6*b^4*d^5)*x^4 + 10*(a^2*b^8*c^5 - 5*a^3*b^7*c^4*d + 10*a^4*b^6*c^3*
d^2 - 10*a^5*b^5*c^2*d^3 + 5*a^6*b^4*c*d^4 - a^7*b^3*d^5)*x^3 + 10*(a^3*b^7
*c^5 - 5*a^4*b^6*c^4*d + 10*a^5*b^5*c^3*d^2 - 10*a^6*b^4*c^2*d^3 + 5*a^7*b^
3*c*d^4 - a^8*b^2*d^5)*x^2 + 5*(a^4*b^6*c^5 - 5*a^5*b^5*c^4*d + 10*a^6*b^4*
c^3*d^2 - 10*a^7*b^3*c^2*d^3 + 5*a^8*b^2*c*d^4 - a^9*b*d^5)*x)
```

giac [B] time = 1.57, size = 596, normalized size = 3.49

$$-\frac{512\left(b^8c^4-4ab^7c^3d+6a^2b^6c^2d^2-4a^3b^5cd^3+a^4b^4d^4-9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^6c^3+27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^5c^2d-27\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^4cd^2+9\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^3d^3+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^4c^2-72\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^3cd+36\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^2d^2-84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^2c+84\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6abd+126\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8\right)\sqrt{bd}\,b^5d^4}{315\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^9|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -512/315*(b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a
^4*b^4*d^4 - 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^2*b^6*c^3 + 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^5*c^2*d - 27*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
```

```
)*b*d - a*b*d))^2*a^2*b^4*c*d^2 + 9*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^2*a^3*b^3*d^3 + 36*(sqrt(b*d)*sqrt(b*x + a) - sqrt
(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^4*c^2 - 72*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^3*c*d + 36*(sqrt(b*d)*sqrt(b*x +
 a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^2*d^2 - 84*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^2*c + 84*(sqrt(b*d)*
sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b*d + 126*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8)*sqrt(b*d)*b^5*d^
4/((b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2)^9*abs(b))
```

maple [A]   time = 0.01, size = 256, normalized size = 1.50

$$\frac{2\sqrt{dx+c}\,\left(128b^4x^4d^4+576a\,b^3d^4x^3-64b^4c\,d^3x^3+1008a^2b^2d^4x^2-288a\,b^3c\,d^3x^2+48b^4c^2d^2x^2+840a^3b\,d^4x-504a^2b^2c\,d^3x+216a\,b^3c^2d^2x-40b^4c^3dx+315a^4d^4-420a^3bc\,d^3+378a^2b^2c^2d^2-180a\,b^3c^3d+35b^4c^4\right)}{315\left(bx+a\right)^{\frac{9}{2}}\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x)
```

```
[Out] 2/315*(d*x+c)^(1/2)*(128*b^4*d^4*x^4+576*a*b^3*d^4*x^3-64*b^4*c*d^3*x^3+100
8*a^2*b^2*d^4*x^2-288*a*b^3*c*d^3*x^2+48*b^4*c^2*d^2*x^2+840*a^3*b*d^4*x-50
4*a^2*b^2*c*d^3*x+216*a*b^3*c^2*d^2*x-40*b^4*c^3*d*x+315*a^4*d^4-420*a^3*b*
c*d^3+378*a^2*b^2*c^2*d^2-180*a*b^3*c^3*d+35*b^4*c^4)/(b*x+a)^(9/2)/(a^5*d^
5-5*a^4*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5
)
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B]   time = 1.37, size = 303, normalized size = 1.77

$$\frac{\sqrt{c+d\,x}\,\left(\frac{256\,d^4\,x^4}{315\,{\left(a\,d-b\,c\right)}^5}+\frac{630\,a^4\,d^4-840\,a^3\,b\,c\,d^3+756\,a^2\,b^2\,c^2\,d^2-360\,a\,b^3\,c^3\,d+70\,b^4\,c^4}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(1680\,a^3\,b\,d^4-1008\,a^2\,b^2\,c\,d^3+432\,a\,b^3\,c^2\,d^2-80\,b^4\,c^3\,d\right)}{315\,b^4\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^3\,x^3\,\left(9\,a\,d-b\,c\right)}{315\,b\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d^2\,x^2\,\left(21\,a^2\,d^2-6\,a\,b\,c\,d+b^2\,c^2\right)}{105\,b^2\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^4\,\sqrt{a+b\,x}}{b^4}+\frac{6\,a^2\,x^2\,\sqrt{a+b\,x}}{b^2}+\frac{4\,a\,x^3\,\sqrt{a+b\,x}}{b}+\frac{4\,a^3\,x\,\sqrt{a+b\,x}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(1/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((256*d^4*x^4)/(315*(a*d - b*c)^5) + (630*a^4*d^4 + 70*b^4
*c^4 + 756*a^2*b^2*c^2*d^2 - 360*a*b^3*c^3*d - 840*a^3*b*c*d^3)/(315*b^4*(a
*d - b*c)^5) + (x*(1680*a^3*b*d^4 - 80*b^4*c^3*d + 432*a*b^3*c^2*d^2 - 1008
*a^2*b^2*c*d^3))/(315*b^4*(a*d - b*c)^5) + (128*d^3*x^3*(9*a*d - b*c))/(315
*b*(a*d - b*c)^5) + (32*d^2*x^2*(21*a^2*d^2 + b^2*c^2 - 6*a*b*c*d))/(105*b^
2*(a*d - b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^4*(a + b*x)^(1/2))/b^4 + (6*a^
2*x^2*(a + b*x)^(1/2))/b^2 + (4*a*x^3*(a + b*x)^(1/2))/b + (4*a^3*x*(a + b*
x)^(1/2))/b^3)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

3.1396 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}+\frac{35b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^4}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^3}+\frac{7b(a+b}{}$$

Rubi [A] time = 0.09, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2}-\frac{35b(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{12d^3}+\frac{35b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}{8d^4}-\frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}-\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(d*Sqrt[c + d*x]) + (35*b*(b*c - a*d)^2*Sqrt[a + b*x]*
Sqrt[c + d*x])/(8*d^4) - (35*b*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(
12*d^3) + (7*b*(a + b*x)^(5/2)*Sqrt[c + d*x])/(3*d^2) - (35*Sqrt[b]*(b*c -
a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(9/2)
)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} - \frac{(35b(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{6d^2}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a+bx)^{5/2}\sqrt{c+dx}}{3d^2} + \frac{\left(35b(bc-ad)\right.}{8d}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a}{}$$

$$= -\frac{2(a+bx)^{7/2}}{d\sqrt{c+dx}} + \frac{35b(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^4} - \frac{35b(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{12d^3} + \frac{7b(a}{}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.42

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 173, normalized size = 0.99

$$-\frac{\sqrt{a+bx}\,(ad-bc)^3\left(\frac{280b^2d(a+bx)}{c+dx}+\frac{48d^3(a+bx)^3}{(c+dx)^3}-\frac{231bd^2(a+bx)^2}{(c+dx)^2}-105b^3\right)}{24d^4\sqrt{c+dx}\left(\frac{d(a+bx)}{c+dx}-b\right)^3} - \frac{35\sqrt{b}\,(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{8d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/(c + d*x)^(3/2),x]
```

```
[Out] -1/24*((-(b*c) + a*d)^3*Sqrt[a + b*x]*(-105*b^3 + (48*d^3*(a + b*x)^3)/(c +
 d*x)^3 - (231*b*d^2*(a + b*x)^2)/(c + d*x)^2 + (280*b^2*d*(a + b*x))/(c +
d*x)))/(d^4*Sqrt[c + d*x]*(-b + (d*(a + b*x))/(c + d*x))^3) - (35*Sqrt[b]*(
```

```
b*c - a*d)^3*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d
^(9/2))
```

fricas [B] time = 1.73, size = 603, normalized size = 3.47

$$\left[-\frac{105 \left(b^3c^4 - 3ab^2c^3d + 3a^2bc^2d^2 - a^3cd^3 + \left(b^3c^3d - 3ab^2c^2d^2 + 3a^2bcd^3 - a^3d^4\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4\left(2bd^2x + bcd + ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}} + 8\left(b^2cd + abd^2\right)x\right) - 4\left(8b^3d^3x^3 + 105b^3c^3 - 280ab^2c^2d + 231a^2bcd^2 - 48a^3d^3 - 2\left(7b^3cd^2 - 19ab^2d^3\right)x^2 + \left(35b^3c^2d - 98ab^2cd^2 + 87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{96\left(d^5x + cd^4\right)}, \frac{105\left(b^3c^4 - 3ab^2c^3d + 3a^2bc^2d^2 - a^3cd^3 + \left(b^3c^3d - 3ab^2c^2d^2 + 3a^2bcd^3 - a^3d^4\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{\left(2bdx+bc+ad\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right) + 2\left(8b^3d^3x^3 + 105b^3c^3 - 280ab^2c^2d + 231a^2bcd^2 - 48a^3d^3 - 2\left(7b^3cd^2 - 19ab^2d^3\right)x^2 + \left(35b^3c^2d - 98ab^2cd^2 + 87a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{48\left(d^5x + cd^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(105*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c
^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(b/d)*log(8*b^2*d^
2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(
b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8*b^3*d^3*
x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48*a^3*d^3 - 2*(7*b
^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*d^2 + 87*a^2*b*d^
3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4), 1/48*(105*(b^3*c^4 - 3*
a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3 + (b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqr
t(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)
) + 2*(8*b^3*d^3*x^3 + 105*b^3*c^3 - 280*a*b^2*c^2*d + 231*a^2*b*c*d^2 - 48
*a^3*d^3 - 2*(7*b^3*c*d^2 - 19*a*b^2*d^3)*x^2 + (35*b^3*c^2*d - 98*a*b^2*c*
d^2 + 87*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x + c*d^4)]
```

giac [B] time = 1.38, size = 279, normalized size = 1.60

$$\frac{\left(\left(bx+a\right)\left(2\left(bx+a\right)\left(\frac{4(bx+a)b^2}{d|b|} - \frac{7\left(b^3cd^5 - ab^2d^6\right)}{d^7|b|}\right) + \frac{35\left(b^4c^2d^4 - 2ab^3cd^5 + a^2b^2d^6\right)}{d^7|b|}\right) + \frac{105\left(b^5c^3d^3 - 3ab^4c^2d^4 + 3a^2b^3cd^5 - a^3b^2d^6\right)}{d^7|b|}\right)\sqrt{bx+a}}{24\sqrt{b^2c+(bx+a)bd-abd}} + \frac{35\left(b^5c^3 - 3ab^4c^2d + 3a^2b^3cd^2 - a^3b^2d^3\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a} + \sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\sqrt{bd}\,d^4|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*((b*x + a)*(2*(b*x + a)*(4*(b*x + a)*b^2/(d*abs(b)) - 7*(b^3*c*d^5 - a
*b^2*d^6)/(d^7*abs(b))) + 35*(b^4*c^2*d^4 - 2*a*b^3*c*d^5 + a^2*b^2*d^6)/(d
^7*abs(b))) + 105*(b^5*c^3*d^3 - 3*a*b^4*c^2*d^4 + 3*a^2*b^3*c*d^5 - a^3*b^
2*d^6)/(d^7*abs(b)))*sqrt(b*x + a)/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 35
/8*(b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*log(abs(-sqrt(
b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^4*a
bs(b))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(7/2)/(c + d*x)**(3/2), x)
```

3.1397 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Rubi [A] time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2}-\frac{15b\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}{4d^3}+\frac{15\sqrt{b}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{4d^{7/2}}-\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(d*Sqrt[c + d*x]) - (15*b*(b*c - a*d)*Sqrt[a + b*x]*Sq
rt[c + d*x])/(4*d^3) + (5*b*(a + b*x)^(3/2)*Sqrt[c + d*x])/(2*d^2) + (15*Sq
rt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])
])/(4*d^(7/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx = -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{(5b)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} - \frac{(15b(bc-ad))\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^2}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15b(bc-ad)^2\right)}{8}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right) S}{}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{\left(15(bc-ad)^2\right) S}{}$$

$$= -\frac{2(a+bx)^{5/2}}{d\sqrt{c+dx}} - \frac{15b(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4d^3} + \frac{5b(a+bx)^{3/2}\sqrt{c+dx}}{2d^2} + \frac{15\sqrt{b}(bc-ad)^2}{4}$$

Mathematica [C] time = 0.06, size = 73, normalized size = 0.53

$$\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2} {}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.24, size = 151, normalized size = 1.09

$$\frac{15\sqrt{b}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{7/2}} - \frac{\sqrt{a+bx}(ad-bc)^2\left(\frac{8d^2(a+bx)^2}{(c+dx)^2} - \frac{25bd(a+bx)}{c+dx} + 15b^2\right)}{4d^3\sqrt{c+dx}\left(\frac{d(a+bx)}{c+dx} - b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/(c + d*x)^(3/2),x]
```

```
[Out] -1/4*((-(b*c) + a*d)^2*Sqrt[a + b*x]*(15*b^2 + (8*d^2*(a + b*x)^2)/(c + d*x
)^2 - (25*b*d*(a + b*x))/(c + d*x)))/(d^3*Sqrt[c + d*x]*(-b + (d*(a + b*x))
/(c + d*x))^2) + (15*Sqrt[b]*(b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/
(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(7/2))
```

fricas [A] time = 1.50, size = 441, normalized size = 3.20

$$\left[\frac{15\left(b^2c^3 - 2abc^2d + a^2cd^2 + \left(b^2c^2d - 2abcd^2 + a^2d^3\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2 + b^2c^2 + 6abcd + a^2d^2 + 4\left(2bd^2x + bcd + ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}} + 8\left(b^2cd + abd^2\right)x\right) + 4\left(2b^2d^2x^2 - 15b^2c^2 + 25abcd - 8a^2d^2 - \left(5b^2cd - 9abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{16\left(d^4x + cd^3\right)}, -\frac{15\left(b^2c^3 - 2abc^2d + a^2cd^2 + \left(b^2c^2d - 2abcd^2 + a^2d^3\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx + bc + ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2 + abc + \left(b^2c + abd\right)x\right)}\right) - 2\left(2b^2d^2x^2 - 15b^2c^2 + 25abcd - 8a^2d^2 - \left(5b^2cd - 9abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{8\left(d^4x + cd^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*
(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*
c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2
- (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3),
-1/8*(15*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*
x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(2*b^2*d^2*x
^2 - 15*b^2*c^2 + 25*a*b*c*d - 8*a^2*d^2 - (5*b^2*c*d - 9*a*b*d^2)*x)*sqrt(
b*x + a)*sqrt(d*x + c))/(d^4*x + c*d^3)]
```

giac [A] time = 1.26, size = 201, normalized size = 1.46

$$\frac{\sqrt{bx+a}\left((bx+a)\left(\frac{2\,(bx+a)b^2}{d|b|}-\frac{5\left(b^3cd^3-ab^2d^4\right)}{d^5|b|}\right)-\frac{15\left(b^4c^2d^2-2\,ab^3cd^3+a^2b^2d^4\right)}{d^5|b|}\right)}{4\,\sqrt{b^2c+(bx+a)bd-abd}}-\frac{15\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{4\,\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*((b*x + a)*(2*(b*x + a)*b^2/(d*abs(b)) - 5*(b^3*c*d^3 - a
*b^2*d^4)/(d^5*abs(b))) - 15*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)/(d
^5*abs(b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) - 15/4*(b^4*c^2 - 2*a*b^3*c
*d + a^2*b^2*d^2)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)
*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{5/2}}{{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(3/2),x)
[Out] Integral((a + b*x)**(5/2)/(c + d*x)**(3/2), x)
```

3.1398 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Rubi [A] time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(d*Sqrt[c + d*x]) + (3*b*Sqrt[a + b*x]*Sqrt[c + d*x])/
d^2 - (3*Sqrt[b]*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[
c + d*x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx &= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{(3b)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3b(bc-ad))\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+bx}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{(3(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}\\
&= -\frac{2(a+bx)^{3/2}}{d\sqrt{c+dx}}+\frac{3b\sqrt{a+bx}\,\sqrt{c+dx}}{d^2}-\frac{3\sqrt{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.74

$$\frac{2(a+bx)^{5/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] (2*(a + b*x)^(5/2)*((b*(c + d*x))/(b*c - a*d))^(3/2)*Hypergeometric2F1[3/2,
 5/2, 7/2, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.56, size = 120, normalized size = 1.22

$$\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\,(-2ad+b(c+dx)+2bc)}{d^2\sqrt{c+dx}}+\frac{3\sqrt{\frac{b}{d}}\,(bc-ad)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/(c + d*x)^(3/2),x]
```

```
[Out] ((2*b*c - 2*a*d + b*(c + d*x))*Sqrt[a - (b*c)/d + (b*(c + d*x))/d])/(d^2*Sq
rt[c + d*x]) + (3*Sqrt[b/d]*(b*c - a*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sq
rt[a - (b*c)/d + (b*(c + d*x))/d]])/d^2
```

fricas [A] time = 1.05, size = 311, normalized size = 3.17

$$\left[-\frac{3\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{4\left(d^3x+cd^2\right)},\ \frac{3\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\,(bdx+3\,bc-2\,ad)\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(d^3x+cd^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 +
b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)
```

```
*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(b*d*x + 3*b*c - 2*
a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2), 1/2*(3*(b*c^2 - a*c*d +
(b*c*d - a*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a
)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(b*
d*x + 3*b*c - 2*a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^3*x + c*d^2)]
```

giac [A] time = 1.27, size = 137, normalized size = 1.40

$$\frac{\sqrt{bx+a}\left(\frac{(bx+a)b^2}{d|b|}+\frac{3\left(b^3cd-ab^2d^2\right)}{d^3|b|}\right)}{\sqrt{b^2c+(bx+a)bd-abd}}+\frac{3\left(b^3c-ab^2d\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x + a)*((b*x + a)*b^2/(d*abs(b)) + 3*(b^3*c*d - a*b^2*d^2)/(d^3*abs(
b)))/sqrt(b^2*c + (b*x + a)*b*d - a*b*d) + 3*(b^3*c - a*b^2*d)*log(abs(-sqr
t(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^2
*abs(b))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{(a+b\,x)}^{3/2}}{{(c+d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(3/2), x)
```

3.1399 $\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{3/2}}-\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx &= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{b\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{d} \\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d} \\
&= -\frac{2\sqrt{a+bx}}{d\sqrt{c+dx}} + \frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}}
\end{aligned}
$$

Mathematica [A] time = 0.35, size = 95, normalized size = 1.44

$$
\frac{2\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\,\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)-2\sqrt{d}\,\sqrt{a+bx}}{d^{3/2}\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x] + 2*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x))/(b*c - a*d
)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[c + d*x]
)
```

IntegrateAlgebraic [A] time = 0.10, size = 66, normalized size = 1.00

$$
\frac{2\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{3/2}} - \frac{2\sqrt{a+bx}}{d\sqrt{c+dx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a + b*x])/(d*Sqrt[c + d*x]) + (2*Sqrt[b]*ArcTanh[(Sqrt[d]*Sqrt[a +
 b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(3/2)
```

fricas [B] time = 1.73, size = 241, normalized size = 3.65

$$
\left[\frac{(dx+c)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\,\sqrt{bx+a}\,\sqrt{dx+c}}{2\left(d^2x+cd\right)}, -\frac{(dx+c)\sqrt{-\frac{b}{d}}\,\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\,\sqrt{bx+a}\,\sqrt{dx+c}}{d^2x+cd}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((d*x + c)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*sqrt(b*x + a)*sqrt(d*x + c))/(d^2*x + c*d), -((d
*x + c)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x
+ c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*sqrt(b*x + a)*
sqrt(d*x + c))/(d^2*x + c*d)]
```

giac [A] time = 1.22, size = 96, normalized size = 1.45

$$-\frac{2\,b^{2}\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d|b|}-\frac{2\,\sqrt{bx+a}\,b^{2}}{\sqrt{b^{2}c+(bx+a)bd-abd}\,d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*b^2*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d*abs(b)) - 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d
 - a*b*d)*d*abs(b))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx+a}}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] int((b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x}}{{\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(3/2),x)
```

```
[Out] int((a + b*x)^(1/2)/(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(3/2), x)
```

3.1400 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx = \frac{2\sqrt{a+bx}}{(bc-ad)\sqrt{c+dx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.03, size = 30, normalized size = 1.00

$$\frac{2\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*(c + d*x)^(3/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/((b*c - a*d)*Sqrt[c + d*x])
```

fricas [A] time = 1.29, size = 42, normalized size = 1.40

$$\frac{2\sqrt{bx+a}\,\sqrt{dx+c}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)*sqrt(d*x + c)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

giac [A] time = 1.04, size = 47, normalized size = 1.57

$$\frac{2\sqrt{bx+a}\,b^2}{\sqrt{b^2c+(bx+a)bd-abd}\,(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2/(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*(b*c*abs(b) - a*d*
abs(b)))
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$-\frac{2\sqrt{bx+a}}{\sqrt{dx+c}\,\left(ad-bc\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.74, size = 26, normalized size = 0.87

$$-\frac{2\sqrt{a+b\,x}}{\left(a\,d-b\,c\right)\,\sqrt{c+d\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(2*(a + b*x)^(1/2))/((a*d - b*c)*(c + d*x)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(3/2)), x)
```

3.1401 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{4d\sqrt{a+bx}}{\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x]) - (4*d*Sqrt[a + b*x])/((b*c -
a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}-\frac{(2d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{bc-ad}\\ &= -\frac{2}{(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}-\frac{4d\sqrt{a+bx}}{(bc-ad)^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.68

$$-\frac{2(ad+b(c+2dx))}{\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(a*d + b*(c + 2*d*x)))/((b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.11, size = 46, normalized size = 0.74

$$-\frac{2\sqrt{a+bx}\left(\frac{b(c+dx)}{a+bx}+d\right)}{\sqrt{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[a + b*x]*(d + (b*(c + d*x))/(a + b*x)))/((b*c - a*d)^2*Sqrt[c + d*
x])
```

fricas [B] time = 1.55, size = 125, normalized size = 2.02

$$-\frac{2\,(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^2*c^3 - 2*a^2*b*c
^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 -
 a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

giac [B] time = 1.22, size = 142, normalized size = 2.29

$$-\frac{2\sqrt{bx+a}\,b^2d}{\left(b^2c^2|b|-2\,abcd|b|+a^2d^2|b|\right)\sqrt{b^2c+(bx+a)bd-abd}}-\frac{4\sqrt{bd}\,b^2}{\left(b^2c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)(bc|b|-ad|b|)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b)
)*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) - 4*sqrt(b*d)*b^2/((b^2*c - a*b*d -
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)*(b*c*abs
(b) - a*d*abs(b)))
```

maple [A] time = 0.00, size = 52, normalized size = 0.84

$$-\frac{2\,(2bdx+ad+bc)}{\sqrt{bx+a}\,\sqrt{dx+c}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2*(2*b*d*x+a*d+b*c)/(b*x+a)^(1/2)/(d*x+c)^(1/2)/(a^2*d^2-2*a*b*c*d+b^2*c^2
)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.86, size = 71, normalized size = 1.15

$$-\frac{\left(\frac{4\,b\,x}{{\left(a\,d-b\,c\right)}^2}+\frac{2\,a\,d+2\,b\,c}{d\,{\left(a\,d-b\,c\right)}^2}\right)\,\sqrt{c+d\,x}}{x\,\sqrt{a+b\,x}+\frac{c\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -(((4*b*x)/(a*d - b*c)^2 + (2*a*d + 2*b*c)/(d*(a*d - b*c)^2))*(c + d*x)^(1/
2))/(x*(a + b*x)^(1/2) + (c*(a + b*x)^(1/2))/d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{2}}\,(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(3/2)), x)
```

3.1402 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}+\frac{8d}{3\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x]) + (8*d)/(3*(b*c - a*d)^2*S
qrt[a + b*x]*Sqrt[c + d*x]) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}-\frac{(4d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{3/2}}\,dx}{3(bc-ad)}\\ &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}+\frac{\left(8d^2\right)\int \frac{1}{\sqrt{a+bx}\,(c+}}{3(bc-ad)}\\ &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}+\frac{8d}{3(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}+\frac{16d^2\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.74

$$\frac{2\left(3a^2d^2+6abd(c+2dx)+b^2\left(-c^2+4cdx+8d^2x^2\right)\right)}{3(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(3*a^2*d^2 + 6*a*b*d*(c + 2*d*x) + b^2*(-c^2 + 4*c*d*x + 8*d^2*x^2)))/(3
*(b*c - a*d)^3*(a + b*x)^(3/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.72

$$\frac{2(c+dx)^{3/2}\left(\frac{3d^2(a+bx)^2}{(c+dx)^2}+\frac{6bd(a+bx)}{c+dx}-b^2\right)}{3(a+bx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(c + d*x)^(3/2)*(-b^2 + (3*d^2*(a + b*x)^2)/(c + d*x)^2 + (6*b*d*(a + b*
x))/(c + d*x)))/(3*(b*c - a*d)^3*(a + b*x)^(3/2))
```

fricas [B] time = 1.92, size = 273, normalized size = 2.70

$$\frac{2\left(8b^2d^2x^2-b^2c^2+6abcd+3a^2d^2+4\left(b^2cd+3abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(a^2b^3c^4-3a^3b^2c^3d+3a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3ab^4c^2d^2+3a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3a^2b^3c^2d^2+5a^3b^2cd^3-2a^4bd^4\right)x^2+\left(2ab^4c^4-5a^2b^3c^3d+3a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*b^2*d^2*x^2 - b^2*c^2 + 6*a*b*c*d + 3*a^2*d^2 + 4*(b^2*c*d + 3*a*b*d
^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a^4*b
*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3
*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^
3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^2 +
 a^4*b*c*d^3 - a^5*d^4)*x)
```

giac [B] time = 1.54, size = 368, normalized size = 3.64

$$\frac{2\sqrt{bx+a}\,b^2d^2}{\left(b^3c^3|b|-3ab^2c^2d|b|+3a^2bcd^2|b|-a^3d^3|b|\right)\sqrt{b^2c+(bx+a)bd-abd}}+\frac{4\left(5\sqrt{bd}\,b^6c^2d-10\sqrt{bd}\,ab^5cd^2+5\sqrt{bd}\,a^2b^4d^3-12\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^4cd+12\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^3d^2+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^2d\right)}{3\left(b^2c^2|b|-2abcd|b|+a^2d^2|b|\right)\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^2/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c
*d^2*abs(b) - a^3*d^3*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/3*(5
*sqrt(b*d)*b^6*c^2*d - 10*sqrt(b*d)*a*b^5*c*d^2 + 5*sqrt(b*d)*a^2*b^4*d^3 -
 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^2*b^4*c*d + 12*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
a)*b*d - a*b*d))^2*a*b^3*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*b^2*d)/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b
) + a^2*d^2*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^2)^3)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{2\left(8b^2x^2d^2+12ab\,d^2x+4b^2cdx+3a^2d^2+6abcd-b^2c^2\right)}{3\left(bx+a\right)^{\frac{3}{2}}\sqrt{dx+c}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/3*(8*b^2*d^2*x^2+12*a*b*d^2*x+4*b^2*c*d*x+3*a^2*d^2+6*a*b*c*d-b^2*c^2)/(
b*x+a)^(3/2)/(d*x+c)^(1/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.06, size = 141, normalized size = 1.40

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{8\,x\,\left(3\,a\,d+b\,c\right)}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{16\,b\,d\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{6\,a^2\,d^2+12\,a\,b\,c\,d-2\,b^2\,c^2}{3\,b\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{a\,c\,\sqrt{a+b\,x}}{b\,d}+\frac{x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((8*x*(3*a*d + b*c))/(3*(a*d - b*c)^3) + (16*b*d*x^2)/(3*
(a*d - b*c)^3) + (6*a^2*d^2 - 2*b^2*c^2 + 12*a*b*c*d)/(3*b*d*(a*d - b*c)^3)
))/(x^2*(a + b*x)^(1/2) + (a*c*(a + b*x)^(1/2))/(b*d) + (x*(a*d + b*c)*(a +
 b*x)^(1/2))/(b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{5}{2}} \left(c + dx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(3/2)), x)
```

3.1403 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{32d^3\sqrt{a+bx}}{5\sqrt{c+dx}\,(bc-ad)^4}-\frac{16d^2}{5\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^3}+\frac{4d}{5(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*Sqrt[c + d*x]) + (4*d)/(5*(b*c - a*d)^2*(
a + b*x)^(3/2)*Sqrt[c + d*x]) - (16*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*x]*Sqr
t[c + d*x]) - (32*d^3*Sqrt[a + b*x])/(5*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx &= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}-\frac{(6d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{3/2}}\,dx}{5(bc-ad)}\\
&= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}+\frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}+\frac{\left(8d^2\right)\int \frac{}{(a+b}}{5(bc}\\
&= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}+\frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}-\frac{}{5(bc-ad)^3}\\
&= -\frac{2}{5(bc-ad)(a+bx)^{5/2}\sqrt{c+dx}}+\frac{4d}{5(bc-ad)^2(a+bx)^{3/2}\sqrt{c+dx}}-\frac{}{5(bc-ad)^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 114, normalized size = 0.84

$$-\frac{2\left(5a^3d^3+15a^2bd^2(c+2dx)+5ab^2d\left(-c^2+4cdx+8d^2x^2\right)+b^3\left(c^3-2c^2dx+8cd^2x^2+16d^3x^3\right)\right)}{5(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(5*a^3*d^3 + 15*a^2*b*d^2*(c + 2*d*x) + 5*a*b^2*d*(-c^2 + 4*c*d*x + 8*d
^2*x^2) + b^3*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3)))/(5*(b*c - a*d)
^4*(a + b*x)^(5/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.13, size = 93, normalized size = 0.68

$$-\frac{2(c+dx)^{5/2}\left(-\frac{5b^2d(a+bx)}{c+dx}+\frac{5d^3(a+bx)^3}{(c+dx)^3}+\frac{15bd^2(a+bx)^2}{(c+dx)^2}+b^3\right)}{5(a+bx)^{5/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(5/2)*(b^3 + (5*d^3*(a + b*x)^3)/(c + d*x)^3 + (15*b*d^2*(a +
 b*x)^2)/(c + d*x)^2 - (5*b^2*d*(a + b*x))/(c + d*x)))/(5*(b*c - a*d)^4*(a
+ b*x)^(5/2))
```

fricas [B] time = 2.45, size = 455, normalized size = 3.35

$$-\frac{2\left(16b^3d^3x^3+b^3c^3-5ab^2c^2d+15a^2bcd^2+5a^3d^3+8\left(b^3cd^2+5ab^2d^3\right)x^2-2\left(b^3c^2d-10ab^2cd^2-15a^2bd^3\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{5\left(a^3b^4c^5-4a^4b^3c^4d+6a^5b^2c^3d^2-4a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4ab^6c^3d^2+6a^2b^5c^2d^3-4a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6a^2b^5c^3d^2+14a^3b^4c^2d^3-11a^4b^3cd^4+3a^5b^2d^5\right)x^3+3\left(ab^6c^5-3a^2b^5c^4d+2a^3b^4c^3d^2+2a^4b^3c^2d^3-3a^5b^2cd^4+a^6bd^5\right)x^2+\left(3a^2b^5c^5-11a^3b^4c^4d+14a^4b^3c^3d^2-6a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/5*(16*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 5*a^3*d^3
 + 8*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 10*a*b^2*c*d^2 - 15*a^2
*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a
^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^6*c^3*d^2
 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c^5 - a*b^
6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5
*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*
b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*x
)
```

giac [B] time = 2.46, size = 830, normalized size = 6.10

$$-\frac{2\sqrt{bx+a}\,b^2d^3}{\left(b^4c^4|b|-4ab^3c^3d|b|+6a^2b^2c^2d^2|b|-4a^3bcd^3|b|+a^4d^4|b|\right)\sqrt{b^2c+(bx+a)bd-abd}}-\frac{4\left(11\sqrt{bd}\,b^{10}c^4d^2-44\sqrt{bd}\,ab^9c^3d^3+66\sqrt{bd}\,a^2b^8c^2d^4-44\sqrt{bd}\,a^3b^7cd^5+11\sqrt{bd}\,a^4b^6d^6-50\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2b^8c^3d^2+150\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2ab^7c^2d^3-150\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^2b^6cd^4+50\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2a^3b^5d^5+80\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4b^6c^2d^2-160\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4ab^5cd^3+80\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^4a^2b^4d^4-30\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6b^4cd^2+30\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^6ab^3d^3+5\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^8b^2d^2\right)}{5\left(b^3c^3|b|-3ab^2c^2d|b|+3a^2bcd^2|b|-a^3d^3|b|\right)\left(b^2c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^2c+(bx+a)bd-abd}\right)^2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^3/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^
2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*sqrt(b^2*c + (b*x
 + a)*b*d - a*b*d)) - 4/5*(11*sqrt(b*d)*b^10*c^4*d^2 - 44*sqrt(b*d)*a*b^9*c
^3*d^3 + 66*sqrt(b*d)*a^2*b^8*c^2*d^4 - 44*sqrt(b*d)*a^3*b^7*c*d^5 + 11*sqr
t(b*d)*a^4*b^6*d^6 - 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^2*b^8*c^3*d^2 + 150*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x +
a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^7*c^2*d^3 - 150*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^6*c*
d^4 + 50*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a^3*b^5*d^5 + 80*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^4*b^6*c^2*d^2 - 160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^5*c*d^3 + 80*sqrt(b*d)*(
sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^4*d^
4 - 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
```

```
b*d))^6*b^4*c*d^2 + 30*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b
*x + a)*b*d - a*b*d))^6*a*b^3*d^3 + 5*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^2*d^2)/((b^3*c^3*abs(b) - 3*a*b^2*
c^2*d*abs(b) + 3*a^2*b*c*d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

maple [**A**] time = 0.01, size = 170, normalized size = 1.25

$$-\frac{2\left(16b^3x^3d^3+40a\,b^2d^3x^2+8b^3c\,d^2x^2+30a^2b\,d^3x+20a\,b^2c\,d^2x-2b^3c^2dx+5a^3d^3+15a^2bc\,d^2-5a\,b^2c^2d+b^3c^3\right)}{5\left(bx+a\right)^{\frac{5}{2}}\sqrt{dx+c}\,\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/5*(16*b^3*d^3*x^3+40*a*b^2*d^3*x^2+8*b^3*c*d^2*x^2+30*a^2*b*d^3*x+20*a*b
^2*c*d^2*x-2*b^3*c^2*d*x+5*a^3*d^3+15*a^2*b*c*d^2-5*a*b^2*c^2*d+b^3*c^3)/(b
*x+a)^(5/2)/(d*x+c)^(1/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*
c^3*d+b^4*c^4)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.31, size = 227, normalized size = 1.67

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{16\,d\,x^2\,\left(5\,a\,d+b\,c\right)}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{2\,a^3\,d^3+6\,a^2\,b\,c\,d^2-2\,a\,b^2\,c^2\,d+\frac{2\,b^3\,c^3}{5}}{b^2\,d\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b\,d^2\,x^3}{5\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(15\,a^2\,d^2+10\,a\,b\,c\,d-b^2\,c^2\right)}{5\,b\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a^2\,c\,\sqrt{a+b\,x}}{b^2\,d}+\frac{x^2\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a\,x\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((16*d*x^2*(5*a*d + b*c))/(5*(a*d - b*c)^4) + (2*a^3*d^3
+ (2*b^3*c^3)/5 - 2*a*b^2*c^2*d + 6*a^2*b*c*d^2)/(b^2*d*(a*d - b*c)^4) + (3
2*b*d^2*x^3)/(5*(a*d - b*c)^4) + (4*x*(15*a^2*d^2 - b^2*c^2 + 10*a*b*c*d))/
(5*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/2) + (a^2*c*(a + b*x)^(1/2))/(b^2*d
) + (x^2*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a*x*(a*d + 2*b*c)*(a + b*x
)^(1/2))/(b^2*d))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{7}{2}}\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(3/2)), x)
```

3.1404 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(bc}$$

Rubi [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{256d^4\sqrt{a+bx}}{35\sqrt{c+dx}\,(bc-ad)^5}+\frac{128d^3}{35\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{35(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{16d}{35(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*Sqrt[c + d*x]) + (16*d)/(35*(b*c - a*d)^2
*(a + b*x)^(5/2)*Sqrt[c + d*x]) - (32*d^2)/(35*(b*c - a*d)^3*(a + b*x)^(3/2
)*Sqrt[c + d*x]) + (128*d^3)/(35*(b*c - a*d)^4*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (256*d^4*Sqrt[a + b*x])/(35*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx=-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}-\frac{(8d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{3/2}}\,dx}{7(bc-ad)}$$

$$=-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}+\frac{\left(48d^2\right)\int \frac{}{(a+bx}}{35(bc}$$

$$=-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}-\frac{}{35(bc-ad)^3(}$$

$$=-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}-\frac{}{35(bc-ad)^3(}$$

$$=-\frac{2}{7(bc-ad)(a+bx)^{7/2}\sqrt{c+dx}}+\frac{16d}{35(bc-ad)^2(a+bx)^{5/2}\sqrt{c+dx}}-\frac{}{35(bc-ad)^3(}$$

Mathematica [A] time = 0.06, size = 166, normalized size = 0.97

$$\frac{2\left(35a^4d^4+140a^3bd^3(c+2dx)+70a^2b^2d^2\left(-c^2+4cdx+8d^2x^2\right)+28ab^3d\left(c^3-2c^2dx+8cd^2x^2+16d^3x^3\right)+b^4\left(-5c^4+8c^3dx-16c^2d^2x^2+64cd^3x^3+128d^4x^4\right)\right)}{35(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(35*a^4*d^4 + 140*a^3*b*d^3*(c + 2*d*x) + 70*a^2*b^2*d^2*(-c^2 + 4*c*d*x
 + 8*d^2*x^2) + 28*a*b^3*d*(c^3 - 2*c^2*d*x + 8*c*d^2*x^2 + 16*d^3*x^3) + b
^4*(-5*c^4 + 8*c^3*d*x - 16*c^2*d^2*x^2 + 64*c*d^3*x^3 + 128*d^4*x^4)))/(35
*(b*c - a*d)^5*(a + b*x)^(7/2)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.14, size = 117, normalized size = 0.68

$$\frac{2(c+dx)^{7/2}\left(\frac{28b^3d(a+bx)}{c+dx}-\frac{70b^2d^2(a+bx)^2}{(c+dx)^2}+\frac{35d^4(a+bx)^4}{(c+dx)^4}+\frac{140bd^3(a+bx)^3}{(c+dx)^3}-5b^4\right)}{35(a+bx)^{7/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (2*(c + d*x)^(7/2)*(-5*b^4 + (35*d^4*(a + b*x)^4)/(c + d*x)^4 + (140*b*d^3*
(a + b*x)^3)/(c + d*x)^3 - (70*b^2*d^2*(a + b*x)^2)/(c + d*x)^2 + (28*b^3*d
*(a + b*x))/(c + d*x)))/(35*(b*c - a*d)^5*(a + b*x)^(7/2))
```

fricas [B] time = 8.19, size = 689, normalized size = 4.03

$$\frac{2\left(128b^4d^4x^4-5b^4c^4+28ab^3c^3d-70a^2b^2c^2d^2+140a^3bcd^3+35a^4d^4+64\left(b^4cd^3+7ab^3d^4\right)x^3-16\left(b^4c^2d^2-14ab^3cd^3-35a^2b^2d^4\right)x^2+8\left(b^4c^3d-7ab^3c^2d^2+35a^2b^2cd^3+35a^3bd^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{35\left(a^4b^5c^6-5a^5b^4c^5d+10a^6b^3c^4d^2-10a^7b^2c^3d^3+5a^8bc^2d^4-a^9cd^5+\left(b^9c^5d-5ab^8c^4d^2+10a^2b^7c^3d^3-10a^3b^6c^2d^4+5a^4b^5cd^5-a^5b^4d^6\right)x^5+\left(b^9c^6-ab^8c^5d-10a^2b^7c^4d^2+30a^3b^6c^3d^3-35a^4b^5c^2d^4+19a^5b^4cd^5-4a^6b^3d^6\right)x^4+2\left(2ab^8c^6-7a^2b^7c^5d+5a^3b^6c^4d^2+10a^4b^5c^3d^3-20a^5b^4c^2d^4+13a^6b^3cd^5-3a^7b^2d^6\right)x^3+2\left(3a^2b^7c^6-13a^3b^6c^5d+20a^4b^5c^4d^2-10a^5b^4c^3d^3-5a^6b^3c^2d^4+7a^7b^2cd^5-2a^8bd^6\right)x^2+\left(4a^3b^6c^6-19a^4b^5c^5d+35a^5b^4c^4d^2-30a^6b^3c^3d^3+10a^7b^2c^2d^4+a^8bcd^5-a^9d^6\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/35*(128*b^4*d^4*x^4 - 5*b^4*c^4 + 28*a*b^3*c^3*d - 70*a^2*b^2*c^2*d^2 + 1
40*a^3*b*c*d^3 + 35*a^4*d^4 + 64*(b^4*c*d^3 + 7*a*b^3*d^4)*x^3 - 16*(b^4*c^
2*d^2 - 14*a*b^3*c*d^3 - 35*a^2*b^2*d^4)*x^2 + 8*(b^4*c^3*d - 7*a*b^3*c^2*d
^2 + 35*a^2*b^2*c*d^3 + 35*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b
^5*c^6 - 5*a^5*b^4*c^5*d + 10*a^6*b^3*c^4*d^2 - 10*a^7*b^2*c^3*d^3 + 5*a^8*
b*c^2*d^4 - a^9*c*d^5 + (b^9*c^5*d - 5*a*b^8*c^4*d^2 + 10*a^2*b^7*c^3*d^3 -
 10*a^3*b^6*c^2*d^4 + 5*a^4*b^5*c*d^5 - a^5*b^4*d^6)*x^5 + (b^9*c^6 - a*b^8
*c^5*d - 10*a^2*b^7*c^4*d^2 + 30*a^3*b^6*c^3*d^3 - 35*a^4*b^5*c^2*d^4 + 19*
a^5*b^4*c*d^5 - 4*a^6*b^3*d^6)*x^4 + 2*(2*a*b^8*c^6 - 7*a^2*b^7*c^5*d + 5*a
^3*b^6*c^4*d^2 + 10*a^4*b^5*c^3*d^3 - 20*a^5*b^4*c^2*d^4 + 13*a^6*b^3*c*d^5
 - 3*a^7*b^2*d^6)*x^3 + 2*(3*a^2*b^7*c^6 - 13*a^3*b^6*c^5*d + 20*a^4*b^5*c^
4*d^2 - 10*a^5*b^4*c^3*d^3 - 5*a^6*b^3*c^2*d^4 + 7*a^7*b^2*c*d^5 - 2*a^8*b*
d^6)*x^2 + (4*a^3*b^6*c^6 - 19*a^4*b^5*c^5*d + 35*a^5*b^4*c^4*d^2 - 30*a^6*
b^3*c^3*d^3 + 10*a^7*b^2*c^2*d^4 + a^8*b*c*d^5 - a^9*d^6)*x)
```

giac [B] time = 4.77, size = 1518, normalized size = 8.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)*b^2*d^4/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^
3*c^3*d^2*abs(b) - 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d
^5*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)) + 4/35*(93*sqrt(b*d)*b^14*c
^6*d^3 - 558*sqrt(b*d)*a*b^13*c^5*d^4 + 1395*sqrt(b*d)*a^2*b^12*c^4*d^5 - 1
860*sqrt(b*d)*a^3*b^11*c^3*d^6 + 1395*sqrt(b*d)*a^4*b^10*c^2*d^7 - 558*sqrt
```

```
(b*d)*a^5*b^9*c*d^8 + 93*sqrt(b*d)*a^6*b^8*d^9 - 616*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^12*c^5*d^3 + 3080*s
qrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*
a*b^11*c^4*d^4 - 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^2*a^2*b^10*c^3*d^5 + 6160*sqrt(b*d)*(sqrt(b*d)*sqrt(b*
x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^9*c^2*d^6 - 3080*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4
*b^8*c*d^7 + 616*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^2*a^5*b^7*d^8 + 1673*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^10*c^4*d^3 - 6692*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a*b^9*c^3*d^4 +
10038*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^4*a^2*b^8*c^2*d^5 - 6692*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^7*c*d^6 + 1673*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^4*b^6*d^7 - 2240*sqr
t(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^
8*c^3*d^3 + 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^6*a*b^7*c^2*d^4 - 6720*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
 sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^6*c*d^5 + 2240*sqrt(b*d)*(sqr
t(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^3*b^5*d^6 +
 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b
*d))^8*b^6*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^8*a*b^5*c*d^4 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x
 + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^4*d^5 - 280*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*b^4*c*d^
3 + 280*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a
*b*d))^10*a*b^3*d^4 + 35*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
(b*x + a)*b*d - a*b*d))^12*b^2*d^3)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b)
 + 6*a^2*b^2*c^2*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c
 - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
2)^7)
```

maple [A] time = 0.01, size = 256, normalized size = 1.50

$$-\frac{2\left(128b^4x^4d^4+448a\,b^3d^4x^3+64b^4c\,d^3x^3+560a^2b^2d^4x^2+224a\,b^3c\,d^3x^2-16b^4c^2d^2x^2+280a^3b\,d^4x+280a^2b^2c\,d^3x-56a\,b^3c^2d^2x+8b^4c^3dx+35a^4d^4+140a^3bc\,d^3-70a^2b^2c^2d^2+28a\,b^3c^3d-5b^4c^4\right)}{35\,(bx+a)^{\frac{7}{2}}\sqrt{dx+c}\,\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/35*(128*b^4*d^4*x^4+448*a*b^3*d^4*x^3+64*b^4*c*d^3*x^3+560*a^2*b^2*d^4*x
^2+224*a*b^3*c*d^3*x^2-16*b^4*c^2*d^2*x^2+280*a^3*b*d^4*x+280*a^2*b^2*c*d^3
*x-56*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+35*a^4*d^4+140*a^3*b*c*d^3-70*a^2*b^2*c
^2*d^2+28*a*b^3*c^3*d-5*b^4*c^4)/(b*x+a)^(7/2)/(d*x+c)^(1/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.50, size = 337, normalized size = 1.97

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{256\,b\,d^3\,x^4}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,d^2\,x^3\,\left(7\,a\,d+b\,c\right)}{35\,{\left(a\,d-b\,c\right)}^5}+\frac{70\,a^4\,d^4+280\,a^3\,b\,c\,d^3-140\,a^2\,b^2\,c^2\,d^2+56\,a\,b^3\,c^3\,d-10\,b^4\,c^4}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(560\,a^3\,b\,d^4+560\,a^2\,b^2\,c\,d^3-112\,a\,b^3\,c^2\,d^2+16\,b^4\,c^3\,d\right)}{35\,b^3\,d\,{\left(a\,d-b\,c\right)}^5}+\frac{32\,d\,x^2\,\left(35\,a^2\,d^2+14\,a\,b\,c\,d-b^2\,c^2\right)}{35\,b\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{a^3\,c\,\sqrt{a+b\,x}}{b^3\,d}+\frac{x^3\,\left(3\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{3\,a\,x^2\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^2\,x\,\left(a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((256*b*d^3*x^4)/(35*(a*d - b*c)^5) + (128*d^2*x^3*(7*a*d
 + b*c))/(35*(a*d - b*c)^5) + (70*a^4*d^4 - 10*b^4*c^4 - 140*a^2*b^2*c^2*d^
2 + 56*a*b^3*c^3*d + 280*a^3*b*c*d^3)/(35*b^3*d*(a*d - b*c)^5) + (x*(560*a^
3*b*d^4 + 16*b^4*c^3*d - 112*a*b^3*c^2*d^2 + 560*a^2*b^2*c*d^3))/(35*b^3*d*
(a*d - b*c)^5) + (32*d*x^2*(35*a^2*d^2 - b^2*c^2 + 14*a*b*c*d))/(35*b*(a*d
- b*c)^5)))/(x^4*(a + b*x)^(1/2) + (a^3*c*(a + b*x)^(1/2))/(b^3*d) + (x^3*(
3*a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (3*a*x^2*(a*d + b*c)*(a + b*x)^(1/2))
/(b^2*d) + (a^2*x*(a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{2}}(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x)**(9/2)*(c + d*x)**(3/2)), x)
```

3.1405 $\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=206

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(}$$

Rubi [A] time = 0.06, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^6}-\frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^5}+\frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4}-\frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3}+\frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2}-\frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) - (256*d^4)/(63*(b*c - a*d)^5*Sqrt[a + b*x]*Sqrt[c + d*x]) - (512*d^5*Sqr
t[a + b*x])/(63*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{11/2}(c+dx)^{3/2}}\,dx = -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} - \frac{(10d)\int \frac{1}{(a+bx)^{9/2}(c+dx)^{3/2}}\,dx}{9(bc-ad)}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} + \frac{\left(80d^2\right)\int}{63}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-a}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-a}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-a}$$

$$= -\frac{2}{9(bc-ad)(a+bx)^{9/2}\sqrt{c+dx}} + \frac{20d}{63(bc-ad)^2(a+bx)^{7/2}\sqrt{c+dx}} - \frac{}{63(bc-a}$$

Mathematica [A] time = 0.08, size = 226, normalized size = 1.10

$$\frac{512d^5\sqrt{a+bx}}{63\sqrt{c+dx}\,(bc-ad)^5(ad-bc)} + \frac{256d^4}{63\sqrt{a+bx}\,\sqrt{c+dx}\,(bc-ad)^4(ad-bc)} + \frac{64d^3}{63(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)^4} - \frac{32d^2}{63(a+bx)^{5/2}\sqrt{c+dx}\,(bc-ad)^3} + \frac{20d}{63(a+bx)^{7/2}\sqrt{c+dx}\,(bc-ad)^2} - \frac{2}{9(a+bx)^{9/2}\sqrt{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -2/(9*(b*c - a*d)*(a + b*x)^(9/2)*Sqrt[c + d*x]) + (20*d)/(63*(b*c - a*d)^2
*(a + b*x)^(7/2)*Sqrt[c + d*x]) - (32*d^2)/(63*(b*c - a*d)^3*(a + b*x)^(5/2
)*Sqrt[c + d*x]) + (64*d^3)/(63*(b*c - a*d)^4*(a + b*x)^(3/2)*Sqrt[c + d*x]
) + (256*d^4)/(63*(b*c - a*d)^4*(-(b*c) + a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])
 + (512*d^5*Sqrt[a + b*x])/(63*(b*c - a*d)^5*(-(b*c) + a*d)*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.15, size = 139, normalized size = 0.67

$$-\frac{2(c+dx)^{9/2}\left(-\frac{45b^4d(a+bx)}{c+dx} + \frac{126b^3d^2(a+bx)^2}{(c+dx)^2} - \frac{210b^2d^3(a+bx)^3}{(c+dx)^3} + \frac{63d^5(a+bx)^5}{(c+dx)^5} + \frac{315bd^4(a+bx)^4}{(c+dx)^4} + 7b^5\right)}{63(a+bx)^{9/2}(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*(c + d*x)^(9/2)*(7*b^5 + (63*d^5*(a + b*x)^5)/(c + d*x)^5 + (315*b*d^4*
(a + b*x)^4)/(c + d*x)^4 - (210*b^2*d^3*(a + b*x)^3)/(c + d*x)^3 + (126*b^3
*d^2*(a + b*x)^2)/(c + d*x)^2 - (45*b^4*d*(a + b*x))/(c + d*x)))/(63*(b*c -
 a*d)^6*(a + b*x)^(9/2))
```

fricas [B] time = 15.84, size = 955, normalized size = 4.64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/63*(256*b^5*d^5*x^5 + 7*b^5*c^5 - 45*a*b^4*c^4*d + 126*a^2*b^3*c^3*d^2 -
 210*a^3*b^2*c^2*d^3 + 315*a^4*b*c*d^4 + 63*a^5*d^5 + 128*(b^5*c*d^4 + 9*a*
b^4*d^5)*x^4 - 32*(b^5*c^2*d^3 - 18*a*b^4*c*d^4 - 63*a^2*b^3*d^5)*x^3 + 16*
```

```
(b^5*c^3*d^2 - 9*a*b^4*c^2*d^3 + 63*a^2*b^3*c*d^4 + 105*a^3*b^2*d^5)*x^2 -
2*(5*b^5*c^4*d - 36*a*b^4*c^3*d^2 + 126*a^2*b^3*c^2*d^3 - 420*a^3*b^2*c*d^4
 - 315*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^5*b^6*c^7 - 6*a^6*b^5*c
^6*d + 15*a^7*b^4*c^5*d^2 - 20*a^8*b^3*c^4*d^3 + 15*a^9*b^2*c^3*d^4 - 6*a^1
0*b*c^2*d^5 + a^11*c*d^6 + (b^11*c^6*d - 6*a*b^10*c^5*d^2 + 15*a^2*b^9*c^4*
d^3 - 20*a^3*b^8*c^3*d^4 + 15*a^4*b^7*c^2*d^5 - 6*a^5*b^6*c*d^6 + a^6*b^5*d
^7)*x^6 + (b^11*c^7 - a*b^10*c^6*d - 15*a^2*b^9*c^5*d^2 + 55*a^3*b^8*c^4*d^
3 - 85*a^4*b^7*c^3*d^4 + 69*a^5*b^6*c^2*d^5 - 29*a^6*b^5*c*d^6 + 5*a^7*b^4*
d^7)*x^5 + 5*(a*b^10*c^7 - 4*a^2*b^9*c^6*d + 3*a^3*b^8*c^5*d^2 + 10*a^4*b^7
*c^4*d^3 - 25*a^5*b^6*c^3*d^4 + 24*a^6*b^5*c^2*d^5 - 11*a^7*b^4*c*d^6 + 2*a
^8*b^3*d^7)*x^4 + 10*(a^2*b^9*c^7 - 5*a^3*b^8*c^6*d + 9*a^4*b^7*c^5*d^2 - 5
*a^5*b^6*c^4*d^3 - 5*a^6*b^5*c^3*d^4 + 9*a^7*b^4*c^2*d^5 - 5*a^8*b^3*c*d^6
+ a^9*b^2*d^7)*x^3 + 5*(2*a^3*b^8*c^7 - 11*a^4*b^7*c^6*d + 24*a^5*b^6*c^5*d
^2 - 25*a^6*b^5*c^4*d^3 + 10*a^7*b^4*c^3*d^4 + 3*a^8*b^3*c^2*d^5 - 4*a^9*b^
2*c*d^6 + a^10*b*d^7)*x^2 + (5*a^4*b^7*c^7 - 29*a^5*b^6*c^6*d + 69*a^6*b^5*
c^5*d^2 - 85*a^7*b^4*c^4*d^3 + 55*a^8*b^3*c^3*d^4 - 15*a^9*b^2*c^2*d^5 - a^
10*b*c*d^6 + a^11*d^7)*x)
```

giac [B] time = 8.71, size = 2438, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(b*x + a)*b^2*d^5/((b^6*c^6*abs(b) - 6*a*b^5*c^5*d*abs(b) + 15*a^2*b
^4*c^4*d^2*abs(b) - 20*a^3*b^3*c^3*d^3*abs(b) + 15*a^4*b^2*c^2*d^4*abs(b) -
 6*a^5*b*c*d^5*abs(b) + a^6*d^6*abs(b))*sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
) - 4/63*(193*sqrt(b*d)*b^18*c^8*d^4 - 1544*sqrt(b*d)*a*b^17*c^7*d^5 + 5404
*sqrt(b*d)*a^2*b^16*c^6*d^6 - 10808*sqrt(b*d)*a^3*b^15*c^5*d^7 + 13510*sqrt
(b*d)*a^4*b^14*c^4*d^8 - 10808*sqrt(b*d)*a^5*b^13*c^3*d^9 + 5404*sqrt(b*d)*
a^6*b^12*c^2*d^10 - 1544*sqrt(b*d)*a^7*b^11*c*d^11 + 193*sqrt(b*d)*a^8*b^10
*d^12 - 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*b^16*c^7*d^4 + 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^2*a*b^15*c^6*d^5 - 35154*sqrt(b*d)*(sqrt(
b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^14*c^5*d^
6 + 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^2*a^3*b^13*c^4*d^7 - 58590*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^4*b^12*c^3*d^8 + 35154*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^11*c^2
*d^9 - 11718*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^2*a^6*b^10*c*d^10 + 1674*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - s
qrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^7*b^9*d^11 + 6318*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^14*c^6*d^4 - 3
7908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*
d))^4*a*b^13*c^5*d^5 + 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^4*a^2*b^12*c^4*d^6 - 126360*sqrt(b*d)*(sqrt(b*d
)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^11*c^3*d^7 +
 94770*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*
b*d))^4*a^4*b^10*c^2*d^8 - 37908*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^5*b^9*c*d^9 + 6318*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^6*b^8*d^10 - 1331
4*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))
^6*b^12*c^5*d^4 + 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (
b*x + a)*b*d - a*b*d))^6*a*b^11*c^4*d^5 - 133140*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2*b^10*c^3*d^6 + 133140
*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^
6*a^3*b^9*c^2*d^7 - 66570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c +
 (b*x + a)*b*d - a*b*d))^6*a^4*b^8*c*d^8 + 13314*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^5*b^7*d^9 + 16128*sqrt(
```

```
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^10
*c^4*d^4 - 64512*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a
)*b*d - a*b*d))^8*a*b^9*c^3*d^5 + 96768*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^2*b^8*c^2*d^6 - 64512*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a^3*b^7*c
*d^7 + 16128*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*
d - a*b*d))^8*a^4*b^6*d^8 - 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^10*b^8*c^3*d^4 + 24570*sqrt(b*d)*(sqrt(b*d)
*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^7*c^2*d^5 - 24
570*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d
))^10*a^2*b^6*c*d^6 + 8190*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d))^10*a^3*b^5*d^7 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b
*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^6*c^2*d^4 - 5796*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*a*b^
5*c*d^5 + 2898*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^12*a^2*b^4*d^6 - 630*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqr
t(b^2*c + (b*x + a)*b*d - a*b*d))^14*b^4*c*d^4 + 630*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^14*a*b^3*d^5 + 63*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^16*b^2
*d^4)/((b^5*c^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) -
 10*a^3*b^2*c^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c
- a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
)^9)
```

maple [B] time = 0.01, size = 356, normalized size = 1.73

$$-\frac{2\left(256b^5x^5d^5+1152a\,b^4d^5x^4+128b^5c\,d^4x^4+2016a^2b^3d^5x^3+576a\,b^4c\,d^4x^3-32b^5c^2d^3x^3+1680a^3b^2d^5x^2+1008a^2b^3c\,d^4x^2-144a\,b^4c^2d^3x^2+16b^5c^3d^2x^2+630a^4b\,d^5x+840a^3b^2c\,d^4x-252a^2b^3c^2d^3x+72a\,b^4c^3d^2x-10b^5c^4dx+63a^5d^5+315a^4bc\,d^4-210a^3b^2c^2d^3+126a^2b^3c^3d^2-45a\,b^4c^4d+7b^5c^5\right)}{63\left(bx+a\right)^{\frac{9}{2}}\sqrt{dx+c}\,\left(d^6a^6-6b\,d^5c\,a^5+15b^2d^4c^2a^4-20b^3d^3c^3a^3+15b^4d^2c^4a^2-6b^5d\,c^5a+b^6c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x)
```

```
[Out] -2/63*(256*b^5*d^5*x^5+1152*a*b^4*d^5*x^4+128*b^5*c*d^4*x^4+2016*a^2*b^3*d^
5*x^3+576*a*b^4*c*d^4*x^3-32*b^5*c^2*d^3*x^3+1680*a^3*b^2*d^5*x^2+1008*a^2*
b^3*c*d^4*x^2-144*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2+630*a^4*b*d^5*x+840*
a^3*b^2*c*d^4*x-252*a^2*b^3*c^2*d^3*x+72*a*b^4*c^3*d^2*x-10*b^5*c^4*d*x+63*
a^5*d^5+315*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+126*a^2*b^3*c^3*d^2-45*a*b^4*c^
4*d+7*b^5*c^5)/(b*x+a)^(9/2)/(d*x+c)^(1/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^
2*c^2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.96, size = 454, normalized size = 2.20

$$-\frac{\sqrt{c+d\,x}\,\left(\frac{126\,a^5\,d^5+630\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+252\,a^2\,b^3\,c^3\,d^2-90\,a\,b^4\,c^4\,d+14\,b^5\,c^5}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b\,d^4\,x^5}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,d^3\,x^4\,\left(9\,a\,d+b\,c\right)}{63\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(1260\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4-504\,a^2\,b^3\,c^2\,d^3+144\,a\,b^4\,c^3\,d^2-20\,b^5\,c^4\,d\right)}{63\,b^4\,d\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d^2\,x^3\,\left(63\,a^2\,d^2+18\,a\,b\,c\,d-b^2\,c^2\right)}{63\,b\,{\left(a\,d-b\,c\right)}^6}+\frac{32\,d\,x^2\,\left(105\,a^3\,d^3+63\,a^2\,b\,c\,d^2-9\,a\,b^2\,c^2\,d+b^3\,c^3\right)}{63\,b^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{a^4\,c\,\sqrt{a+b\,x}}{b^4\,d}+\frac{x^4\,\left(4\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{2\,a\,x^3\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d}+\frac{a^3\,x\,\left(a\,d+4\,b\,c\right)\,\sqrt{a+b\,x}}{b^4\,d}+\frac{2\,a^2\,x^2\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/2)*(c + d*x)^(3/2)),x)
```

```
[Out] -((c + d*x)^(1/2)*((126*a^5*d^5 + 14*b^5*c^5 + 252*a^2*b^3*c^3*d^2 - 420*a^
3*b^2*c^2*d^3 - 90*a*b^4*c^4*d + 630*a^4*b*c*d^4)/(63*b^4*d*(a*d - b*c)^6)
+ (512*b*d^4*x^5)/(63*(a*d - b*c)^6) + (256*d^3*x^4*(9*a*d + b*c))/(63*(a*d
 - b*c)^6) + (x*(1260*a^4*b*d^5 - 20*b^5*c^4*d + 144*a*b^4*c^3*d^2 + 1680*a
^3*b^2*c*d^4 - 504*a^2*b^3*c^2*d^3))/(63*b^4*d*(a*d - b*c)^6) + (64*d^2*x^3
*(63*a^2*d^2 - b^2*c^2 + 18*a*b*c*d))/(63*b*(a*d - b*c)^6) + (32*d*x^2*(105
*a^3*d^3 + b^3*c^3 - 9*a*b^2*c^2*d + 63*a^2*b*c*d^2))/(63*b^2*(a*d - b*c)^6
)))/(x^5*(a + b*x)^(1/2) + (a^4*c*(a + b*x)^(1/2))/(b^4*d) + (x^4*(4*a*d +
b*c)*(a + b*x)^(1/2))/(b*d) + (2*a*x^3*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b^
2*d) + (a^3*x*(a*d + 4*b*c)*(a + b*x)^(1/2))/(b^4*d) + (2*a^2*x^2*(2*a*d +
3*b*c)*(a + b*x)^(1/2))/(b^3*d))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

3.1406 $\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=204

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{11/2}}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\cdots$$

Rubi [A] time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}-\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}\,(bc-ad)}{4d^4}+\frac{105b^2\sqrt{a+bx}\sqrt{c+dx}\,(bc-ad)^2}{8d^5}-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{8d^{11/2}}-\frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(9/2))/(3*d*(c + d*x)^(3/2)) - (6*b*(a + b*x)^(7/2))/(d^2*Sqr
t[c + d*x]) + (105*b^2*(b*c - a*d)^2*Sqrt[a + b*x]*Sqrt[c + d*x])/(8*d^5) -
 (35*b^2*(b*c - a*d)*(a + b*x)^(3/2)*Sqrt[c + d*x])/(4*d^4) + (7*b^2*(a + b
*x)^(5/2)*Sqrt[c + d*x])/d^3 - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[d]*
Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(8*d^(11/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{9/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} + \frac{(3b)\int \frac{(a+bx)^{7/2}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{(21b^2)\int \frac{(a+bx)^{5/2}}{\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3} - \frac{(35b^2(bc-ad))\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{2d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} - \frac{35b^2(bc-ad)(a+bx)^{3/2}\sqrt{c+dx}}{4d^4} + \frac{7b^2(a+bx)^{5/2}\sqrt{c+dx}}{d^3}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

$$= -\frac{2(a+bx)^{9/2}}{3d(c+dx)^{3/2}} - \frac{6b(a+bx)^{7/2}}{d^2\sqrt{c+dx}} + \frac{105b^2(bc-ad)^2\sqrt{a+bx}\,\sqrt{c+dx}}{8d^5} - \frac{35b^2(bc-ad)(a+bx)^{3/}}{4d^4}$$

Mathematica [C] time = 0.10, size = 73, normalized size = 0.36

$$\frac{2(a+bx)^{11/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2} {}_2F_1\left(\frac{5}{2},\frac{11}{2};\frac{13}{2};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(11/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2
, 11/2, 13/2, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.34, size = 194, normalized size = 0.95

$$-\frac{105b^{3/2}(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{8d^{11/2}} - \frac{(a+bx)^{3/2}(ad-bc)^3\left(-\frac{315b^4(c+dx)^4}{(a+bx)^4}+\frac{840b^3d(c+dx)^3}{(a+bx)^3}-\frac{693b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{144bd^3(c+dx)}{a+bx}+16d^4\right)}{24d^5(c+dx)^{3/2}\left(d-\frac{b(c+dx)}{a+bx}\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(9/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/24*((-(b*c) + a*d)^3*(a + b*x)^(3/2)*(16*d^4 + (144*b*d^3*(c + d*x))/(a
+ b*x) - (693*b^2*d^2*(c + d*x)^2)/(a + b*x)^2 + (840*b^3*d*(c + d*x)^3)/(a
```

```
 + b*x)^3 - (315*b^4*(c + d*x)^4)/(a + b*x)^4))/(d^5*(c + d*x)^(3/2)*(d - (
b*(c + d*x))/(a + b*x))^3) - (105*b^(3/2)*(b*c - a*d)^3*ArcTanh[(Sqrt[b]*Sq
rt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(8*d^(11/2))
```

fricas [B] time = 2.90, size = 879, normalized size = 4.31

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 +
(b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*
c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 - a^3*b*c*d^4)*x)*sqrt(b/d)*log
(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d
^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5), 1/48*(315*(b^4*c^5 - 3*a*b^3*c^4*d + 3*a
^2*b^2*c^3*d^2 - a^3*b*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2
*c*d^4 - a^3*b*d^5)*x^2 + 2*(b^4*c^4*d - 3*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^
3 - a^3*b*c*d^4)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x +
a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) + 2*(8
*b^4*d^4*x^4 + 315*b^4*c^4 - 840*a*b^3*c^3*d + 693*a^2*b^2*c^2*d^2 - 144*a^
3*b*c*d^3 - 16*a^4*d^4 - 2*(9*b^4*c*d^3 - 25*a*b^3*d^4)*x^3 + 3*(21*b^4*c^2
*d^2 - 60*a*b^3*c*d^3 + 55*a^2*b^2*d^4)*x^2 + 2*(210*b^4*c^3*d - 567*a*b^3*
c^2*d^2 + 477*a^2*b^2*c*d^3 - 104*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)
)/(d^7*x^2 + 2*c*d^6*x + c^2*d^5)]
```

giac [B] time = 2.42, size = 500, normalized size = 2.45

$$\frac{\left(\left(\left(2\,(bx+a)\left(\frac{4\,(b^6cd^8-ab^5d^9)(bx+a)}{b^2cd^9|b|-abd^{10}|b|}-\frac{9\,(b^7c^2d^7-2\,ab^6cd^8+a^2b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)+\frac{63\,(b^8c^3d^6-3\,ab^7c^2d^7+3\,a^2b^6cd^8-a^3b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{420\,(b^9c^4d^5-4\,ab^8c^3d^6+6\,a^2b^7c^2d^7-4\,a^3b^6cd^8+a^4b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)(bx+a)+\frac{315\,(b^{10}c^5d^4-5\,ab^9c^4d^5+10\,a^2b^8c^3d^6-10\,a^3b^7c^2d^7+5\,a^4b^6cd^8-a^5b^5d^9)}{b^2cd^9|b|-abd^{10}|b|}\right)\sqrt{bx+a}}{24\,(b^2c+(bx+a)bd-abd)^{\frac{3}{2}}}+\frac{105\,(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{8\sqrt{bd}\,d^5|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((2*(b*x + a)*(4*(b^6*c*d^8 - a*b^5*d^9)*(b*x + a)/(b^2*c*d^9*abs(b)
- a*b*d^10*abs(b)) - 9*(b^7*c^2*d^7 - 2*a*b^6*c*d^8 + a^2*b^5*d^9)/(b^2*c*d
^9*abs(b) - a*b*d^10*abs(b))) + 63*(b^8*c^3*d^6 - 3*a*b^7*c^2*d^7 + 3*a^2*b
^6*c*d^8 - a^3*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 4
20*(b^9*c^4*d^5 - 4*a*b^8*c^3*d^6 + 6*a^2*b^7*c^2*d^7 - 4*a^3*b^6*c*d^8 + a
^4*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*(b*x + a) + 315*(b^10*c^5
*d^4 - 5*a*b^9*c^4*d^5 + 10*a^2*b^8*c^3*d^6 - 10*a^3*b^7*c^2*d^7 + 5*a^4*b^
6*c*d^8 - a^5*b^5*d^9)/(b^2*c*d^9*abs(b) - a*b*d^10*abs(b)))*sqrt(b*x + a)/
(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 105/8*(b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c
+ (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^5*abs(b))
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{9}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{9/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(9/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(9/2)/(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1407 $\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=170

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}$$

Rubi [A] time = 0.08, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{35b^2\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4d^4}+\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4d^{9/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(7/2))/(3*d*(c + d*x)^(3/2)) - (14*b*(a + b*x)^(5/2))/(3*d^2*
Sqrt[c + d*x]) - (35*b^2*(b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*d^4) +
 (35*b^2*(a + b*x)^(3/2)*Sqrt[c + d*x])/(6*d^3) + (35*b^(3/2)*(b*c - a*d)^2
*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(4*d^(9/2))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{7/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}+\frac{(7b)\int \frac{(a+bx)^{5/2}}{(c+dx)^{3/2}}\,dx}{3d}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}+\frac{\left(35b^2\right)\int \frac{(a+bx)^{3/2}}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}-\frac{\left(35b^2(bc-ad)\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4d^3}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}\\
&= -\frac{2(a+bx)^{7/2}}{3d(c+dx)^{3/2}}-\frac{14b(a+bx)^{5/2}}{3d^2\sqrt{c+dx}}-\frac{35b^2(bc-ad)\sqrt{a+bx}\,\sqrt{c+dx}}{4d^4}+\frac{35b^2(a+bx)^{3/2}\sqrt{c+dx}}{6d^3}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 73, normalized size = 0.43

$$\frac{2(a+bx)^{9/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{9}{2};\frac{11}{2};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(9/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 9/2, 11/2, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 172, normalized size = 1.01

$$\frac{35b^{3/2}(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{4d^{9/2}}-\frac{(a+bx)^{3/2}(ad-bc)^2\left(\frac{105b^3(c+dx)^3}{(a+bx)^3}-\frac{175b^2d(c+dx)^2}{(a+bx)^2}+\frac{56bd^2(c+dx)}{a+bx}+8d^3\right)}{12d^4(c+dx)^{3/2}\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/2)/(c + d*x)^(5/2),x]
```

```
[Out] -1/12*((-(b*c) + a*d)^2*(a + b*x)^(3/2)*(8*d^3 + (56*b*d^2*(c + d*x))/(a +
b*x) - (175*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (105*b^3*(c + d*x)^3)/(a + b*x
)^3))/(d^4*(c + d*x)^(3/2)*(d - (b*(c + d*x))/(a + b*x))^2) + (35*b^(3/2)*(
b*c - a*d)^2*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(4*d
^(9/2))
```

fricas [B] time = 2.24, size = 657, normalized size = 3.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3*c^2*d^2 - 2*a*b^
2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2 + a^2*b*c*d^3)*x)
*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x
 + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*
d^2)*x) + 4*(6*b^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2
 - 8*a^3*d^3 - 3*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a
*b^2*c*d^2 + 40*a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d
^5*x + c^2*d^4), -1/24*(105*(b^3*c^4 - 2*a*b^2*c^3*d + a^2*b*c^2*d^2 + (b^3
*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*x^2 + 2*(b^3*c^3*d - 2*a*b^2*c^2*d^2
+ a^2*b*c*d^3)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(b*x + a)
*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x)) - 2*(6*b
^3*d^3*x^3 - 105*b^3*c^3 + 175*a*b^2*c^2*d - 56*a^2*b*c*d^2 - 8*a^3*d^3 - 3
*(7*b^3*c*d^2 - 13*a*b^2*d^3)*x^2 - 2*(70*b^3*c^2*d - 119*a*b^2*c*d^2 + 40*
a^2*b*d^3)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^6*x^2 + 2*c*d^5*x + c^2*d^4)]
```

giac [B] time = 2.14, size = 380, normalized size = 2.24

$$\frac{\left(\left(3\,(bx+a)\left(\frac{2\left(b^{6}cd^{6}-ab^{5}d^{7}\right)(bx+a)}{b^{2}cd^{7}|b|-abd^{8}|b|}-\frac{7\left(b^{7}c^{2}d^{5}-2\,ab^{6}cd^{6}+a^{2}b^{5}d^{7}\right)}{b^{2}cd^{7}|b|-abd^{8}|b|}\right)-\frac{140\left(b^{8}c^{3}d^{4}-3\,ab^{7}c^{2}d^{5}+3\,a^{2}b^{6}cd^{6}-a^{3}b^{5}d^{7}\right)}{b^{2}cd^{7}|b|-abd^{8}|b|}\right)(bx+a)-\frac{105\left(b^{9}c^{4}d^{3}-4\,ab^{8}c^{3}d^{4}+6\,a^{2}b^{7}c^{2}d^{5}-4\,a^{3}b^{6}cd^{6}+a^{4}b^{5}d^{7}\right)}{b^{2}cd^{7}|b|-abd^{8}|b|}\right)\sqrt{bx+a}}{12\left(b^{2}c+(bx+a)bd-abd\right)^{\frac{3}{2}}}-\frac{35\left(b^{5}c^{2}-2\,ab^{4}cd+a^{2}b^{3}d^{2}\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^{2}c+(bx+a)bd-abd}\right|\right)}{4\sqrt{bd}\,d^{4}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((3*(b*x + a)*(2*(b^6*c*d^6 - a*b^5*d^7)*(b*x + a)/(b^2*c*d^7*abs(b) -
 a*b*d^8*abs(b)) - 7*(b^7*c^2*d^5 - 2*a*b^6*c*d^6 + a^2*b^5*d^7)/(b^2*c*d^7
*abs(b) - a*b*d^8*abs(b))) - 140*(b^8*c^3*d^4 - 3*a*b^7*c^2*d^5 + 3*a^2*b^6
*c*d^6 - a^3*b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*(b*x + a) - 105*
(b^9*c^4*d^3 - 4*a*b^8*c^3*d^4 + 6*a^2*b^7*c^2*d^5 - 4*a^3*b^6*c*d^6 + a^4*
b^5*d^7)/(b^2*c*d^7*abs(b) - a*b*d^8*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x +
 a)*b*d - a*b*d)^(3/2) - 35/4*(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*log(abs
(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d
)*d^4*abs(b))
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{2}}}{(dx+c)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(7/2)/(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1408 $\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=128

$$-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}+\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {47, 50, 63, 217, 206}

$$\frac{5b^2\sqrt{a+bx}\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{7/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}-\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(5/2))/(3*d*(c + d*x)^(3/2)) - (10*b*(a + b*x)^(3/2))/(3*d^2*
Sqrt[c + d*x]) + (5*b^2*Sqrt[a + b*x]*Sqrt[c + d*x])/d^3 - (5*b^(3/2)*(b*c
- a*d)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*x])])/d^(7/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/2}}\,dx &= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}+\frac{(5b)\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/2}}\,dx}{3d}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{\left(5b^2\right)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{d^2}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{\left(5b^2(bc-ad)\right)\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{2d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\right.}{d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{(5b(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\right.}{d^3}\\
&= -\frac{2(a+bx)^{5/2}}{3d(c+dx)^{3/2}}-\frac{10b(a+bx)^{3/2}}{3d^2\sqrt{c+dx}}+\frac{5b^2\sqrt{a+bx}\,\sqrt{c+dx}}{d^3}-\frac{5b^{3/2}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.08, size = 73, normalized size = 0.57

$$
\frac{2(a+bx)^{7/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},\frac{7}{2};\frac{9}{2};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(7/2)*((b*(c + d*x))/(b*c - a*d))^(5/2)*Hypergeometric2F1[5/2,
 7/2, 9/2, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(5/2))
```

IntegrateAlgebraic [A] time = 0.98, size = 166, normalized size = 1.30

$$
\frac{\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}\left(-2a^2d^2-14abd(c+dx)+4abcd-2b^2c^2+3b^2(c+dx)^2+14b^2c(c+dx)\right)}{3d^3(c+dx)^{3/2}}+\frac{5\sqrt{\frac{b}{d}}\left(b^2c-abd\right)\log\left(\sqrt{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/2)/(c + d*x)^(5/2),x]
```

```
[Out] (Sqrt[a - (b*c)/d + (b*(c + d*x))/d]*(-2*b^2*c^2 + 4*a*b*c*d - 2*a^2*d^2 +
14*b^2*c*(c + d*x) - 14*a*b*d*(c + d*x) + 3*b^2*(c + d*x)^2))/(3*d^3*(c + d
*x)^(3/2)) + (5*Sqrt[b/d]*(b^2*c - a*b*d)*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) +
Sqrt[a - (b*c)/d + (b*(c + d*x))/d]])/d^3
```

fricas [B] time = 1.62, size = 475, normalized size = 3.71

$$
\left[-\frac{15\left(b^2c^3-abc^2d+\left(b^2cd^2-abd^3\right)x^2+2\left(b^2c^2d-abcd^2\right)x\right)\sqrt{\frac{b}{d}}\log\left(8b^2d^2x^2+b^2c^2+6abcd+a^2d^2+4\left(2bd^2x+bcd+ad^2\right)\sqrt{bx+a}\sqrt{dx+c}\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\left(3b^2d^2x^2+15b^2c^2-10abcd-2a^2d^2+2\left(10b^2cd-7abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{12\left(d^5x^2+2cd^4x+c^2d^3\right)}, \frac{15\left(b^2c^3-abc^2d+\left(b^2cd^2-abd^3\right)x^2+2\left(b^2c^2d-abcd^2\right)x\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2bdx+bc+ad)\sqrt{bx+a}\sqrt{dx+c}\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\left(3b^2d^2x^2+15b^2c^2-10abcd-2a^2d^2+2\left(10b^2cd-7abd^2\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{6\left(d^5x^2+2cd^4x+c^2d^3\right)}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3)*x^2 + 2*(b^2*c^2*d
- a*b*c*d^2)*x)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2
 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(b/d) + 8*
(b^2*c*d + a*b*d^2)*x) - 4*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2
*d^2 + 2*(10*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2
+ 2*c*d^4*x + c^2*d^3), 1/6*(15*(b^2*c^3 - a*b*c^2*d + (b^2*c*d^2 - a*b*d^3
)*x^2 + 2*(b^2*c^2*d - a*b*c*d^2)*x)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c +
 a*d)*sqrt(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c +
a*b*d)*x)) + 2*(3*b^2*d^2*x^2 + 15*b^2*c^2 - 10*a*b*c*d - 2*a^2*d^2 + 2*(10
*b^2*c*d - 7*a*b*d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c))/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)]
```

giac [**B**] time = 1.97, size = 276, normalized size = 2.16

$$\frac{\left((bx+a)\left(\frac{3\left(b^6cd^4-ab^5d^5\right)(bx+a)}{b^2cd^5|b|-abd^6|b|}+\frac{20\left(b^7c^2d^3-2\,ab^6cd^4+a^2b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)+\frac{15\left(b^8c^3d^2-3\,ab^7c^2d^3+3\,a^2b^6cd^4-a^3b^5d^5\right)}{b^2cd^5|b|-abd^6|b|}\right)\sqrt{bx+a}}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{5\left(b^4c-ab^3d\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^3|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((b*x + a)*(3*(b^6*c*d^4 - a*b^5*d^5)*(b*x + a)/(b^2*c*d^5*abs(b) - a*b
*d^6*abs(b)) + 20*(b^7*c^2*d^3 - 2*a*b^6*c*d^4 + a^2*b^5*d^5)/(b^2*c*d^5*ab
s(b) - a*b*d^6*abs(b))) + 15*(b^8*c^3*d^2 - 3*a*b^7*c^2*d^3 + 3*a^2*b^6*c*d
^4 - a^3*b^5*d^5)/(b^2*c*d^5*abs(b) - a*b*d^6*abs(b)))*sqrt(b*x + a)/(b^2*c
 + (b*x + a)*b*d - a*b*d)^(3/2) + 5*(b^4*c - a*b^3*d)*log(abs(-sqrt(b*d)*sq
rt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)*d^3*abs(b))
```

maple [**F**] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/2}}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(5/2)/(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1409 $\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=92

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {47, 63, 217, 206}

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{d^{5/2}}-\frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}}-\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2))/(3*d*(c + d*x)^(3/2)) - (2*b*Sqrt[a + b*x])/(d^2*Sqrt[
c + d*x]) + (2*b^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[c + d*
x])])/d^(5/2)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/2}}\,dx = -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}} + \frac{b\int \frac{\sqrt{a+bx}}{(c+dx)^{3/2}}\,dx}{d}$$

$$= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}} - \frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}} + \frac{b^2\int \frac{1}{\sqrt{a+bx}\,\sqrt{c+dx}}\,dx}{d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}} - \frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}} - \frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{d^2}$$

$$= -\frac{2(a+bx)^{3/2}}{3d(c+dx)^{3/2}} - \frac{2b\sqrt{a+bx}}{d^2\sqrt{c+dx}} + \frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{b}\,\sqrt{c+dx}}\right)}{d^{5/2}}$$

Mathematica [A] time = 0.55, size = 111, normalized size = 1.21

$$\frac{6(bc-ad)^{3/2}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/2}\sinh^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right) - 2\sqrt{d}\,\sqrt{a+bx}\,(ad+3bc+4bdx)}{3d^{5/2}(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[d]*Sqrt[a + b*x]*(3*b*c + a*d + 4*b*d*x) + 6*(b*c - a*d)^(3/2)*((b
*(c + d*x))/(b*c - a*d))^(3/2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a
*d]])/(3*d^(5/2)*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 85, normalized size = 0.92

$$\frac{2b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{a+bx}}\right)}{d^{5/2}} - \frac{2(a+bx)^{3/2}\left(\frac{3b(c+dx)}{a+bx}+d\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/2)/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d + (3*b*(c + d*x))/(a + b*x)))/(3*d^2*(c + d*x)^(3/2)
) + (2*b^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/d^
(5/2)
```

fricas [B] time = 1.49, size = 325, normalized size = 3.53

$$\left[\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{\frac{b}{d}}\log\left(8\,b^2d^2x^2+b^2c^2+6\,abcd+a^2d^2+4\left(2\,bd^2x+bcd+ad^2\right)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}+8\left(b^2cd+abd^2\right)x\right)-4\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{6\left(d^4x^2+2\,cd^3x+c^2d^2\right)}, -\frac{3\left(bd^2x^2+2\,bcdx+bc^2\right)\sqrt{-\frac{b}{d}}\arctan\left(\frac{(2\,bdx+bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}}{2\left(b^2dx^2+abc+\left(b^2c+abd\right)x\right)}\right)+2\,(4\,bdx+3\,bc+ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(d^4x^2+2\,cd^3x+c^2d^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(b/d)*log(8*b^2*d^2*x^2 + b^2*c
^2 + 6*a*b*c*d + a^2*d^2 + 4*(2*b*d^2*x + b*c*d + a*d^2)*sqrt(b*x + a)*sqrt
(d*x + c)*sqrt(b/d) + 8*(b^2*c*d + a*b*d^2)*x) - 4*(4*b*d*x + 3*b*c + a*d)*
```

```
sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2), -1/3*(3*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sqrt(-b/d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt
(b*x + a)*sqrt(d*x + c)*sqrt(-b/d)/(b^2*d*x^2 + a*b*c + (b^2*c + a*b*d)*x))
 + 2*(4*b*d*x + 3*b*c + a*d)*sqrt(b*x + a)*sqrt(d*x + c))/(d^4*x^2 + 2*c*d^
3*x + c^2*d^2)]
```

giac [**B**] time = 1.43, size = 181, normalized size = 1.97

$$-\frac{2\,b^3\log\left(\left|-\sqrt{bd}\,\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}\,d^2|b|}-\frac{2\,\sqrt{bx+a}\left(\frac{4\left(b^5cd^2-ab^4d^3\right)(bx+a)}{bcd^3|b|-ad^4|b|}+\frac{3\left(b^6c^2d-2\,ab^5cd^2+a^2b^4d^3\right)}{bcd^3|b|-ad^4|b|}\right)}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2*b^3*log(abs(-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*
d)))/(sqrt(b*d)*d^2*abs(b)) - 2/3*sqrt(b*x + a)*(4*(b^5*c*d^2 - a*b^4*d^3)*
(b*x + a)/(b*c*d^3*abs(b) - a*d^4*abs(b)) + 3*(b^6*c^2*d - 2*a*b^5*c*d^2 +
a^2*b^4*d^3)/(b*c*d^3*abs(b) - a*d^4*abs(b)))/(b^2*c + (b*x + a)*b*d - a*b*
d)^(3/2)
```

maple [**F**] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{2}}}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] int((b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{3/2}}{{\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/2)/(c + d*x)^(5/2),x)
```

```
[Out] int((a + b*x)^(3/2)/(c + d*x)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{2}}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral((a + b*x)**(3/2)/(c + d*x)**(5/2), x)
```

3.1410 $\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/2}}\,dx = \frac{2(a+bx)^{3/2}}{3(bc-ad)(c+dx)^{3/2}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*(b*c - a*d)*(c + d*x)^(3/2))
```

fricas [B] time = 1.15, size = 65, normalized size = 2.03

$$\frac{2\,(bx+a)^{\frac{3}{2}}\sqrt{dx+c}}{3\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)*sqrt(d*x + c)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2
+ 2*(b*c^2*d - a*c*d^2)*x)
```

giac [A] time = 1.15, size = 51, normalized size = 1.59

$$\frac{2\,(bx+a)^{\frac{3}{2}}b^4d}{3\left(bcd|b|-ad^2|b|\right)\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)*b^4*d/((b*c*d*abs(b) - a*d^2*abs(b))*(b^2*c + (b*x + a)
*b*d - a*b*d)^(3/2))
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$-\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,(dx+c)^{\frac{3}{2}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(3/2)/(d*x+c)^(3/2)/(a*d-b*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{2\,a\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{2\,b\,x\,\sqrt{a+b\,x}}{3\,a\,d^3-3\,b\,c\,d^2}\right)\,\sqrt{c+d\,x}}{x^2-\frac{3\,b\,c^3-3\,a\,c^2\,d}{3\,a\,d^3-3\,b\,c\,d^2}+\frac{6\,c\,d\,x\,\left(a\,d-b\,c\right)}{3\,a\,d^3-3\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/2)/(c + d*x)^(5/2),x)
```

```
[Out] -(((2*a*(a + b*x)^(1/2))/(3*a*d^3 - 3*b*c*d^2) + (2*b*x*(a + b*x)^(1/2))/(3
*a*d^3 - 3*b*c*d^2))*(c + d*x)^(1/2))/(x^2 - (3*b*c^3 - 3*a*c^2*d)/(3*a*d^3
 - 3*b*c*d^2) + (6*c*d*x*(a*d - b*c))/(3*a*d^3 - 3*b*c*d^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a + b*x)/(c + d*x)**(5/2), x)
```

3.1411 $\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=66

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{4b\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^2}+\frac{2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x])/(3*(b*c - a*d)*(c + d*x)^(3/2)) + (4*b*Sqrt[a + b*x])/(3*
(b*c - a*d)^2*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx &= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{(2b)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)}\\ &= \frac{2\sqrt{a+bx}}{3(bc-ad)(c+dx)^{3/2}}+\frac{4b\sqrt{a+bx}}{3(bc-ad)^2\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{2\sqrt{a+bx}\,(-ad+3bc+2bdx)}{3(c+dx)^{3/2}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*Sqrt[a + b*x]*(3*b*c - a*d + 2*b*d*x))/(3*(b*c - a*d)^2*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.09, size = 57, normalized size = 0.86

$$\frac{2\left(\frac{3b\sqrt{a+bx}}{\sqrt{c+dx}}-\frac{d(a+bx)^{3/2}}{(c+dx)^{3/2}}\right)}{3(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(-((d*(a + b*x)^(3/2))/(c + d*x)^(3/2)) + (3*b*Sqrt[a + b*x])/Sqrt[c + d
*x]))/(3*(b*c - a*d)^2)
```

fricas [B] time = 1.32, size = 118, normalized size = 1.79

$$\frac{2\,(2\,bdx+3\,bc-ad)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*b*d*x + 3*b*c - a*d)*sqrt(b*x + a)*sqrt(d*x + c)/(b^2*c^4 - 2*a*b*c^
3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*c^3*
d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

giac [B] time = 1.02, size = 126, normalized size = 1.91

$$\frac{2\left(\frac{2\,(bx+a)b^4d^2}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}+\frac{3\left(b^5cd-ab^4d^2\right)}{b^2c^2d|b|-2\,abcd^2|b|+a^2d^3|b|}\right)\sqrt{bx+a}}{3\left(b^2c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*(b*x + a)*b^4*d^2/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a^2*d^3*a
bs(b)) + 3*(b^5*c*d - a*b^4*d^2)/(b^2*c^2*d*abs(b) - 2*a*b*c*d^2*abs(b) + a
^2*d^3*abs(b)))*sqrt(b*x + a)/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

maple [A] time = 0.00, size = 53, normalized size = 0.80

$$-\frac{2\sqrt{bx+a}\;(-2bdx+ad-3bc)}{3\,(dx+c)^{\frac{3}{2}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-2*b*d*x+a*d-3*b*c)/(d*x+c)^(3/2)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 0.90, size = 127, normalized size = 1.92

$$\frac{\sqrt{c+d\,x}\,\left(\frac{x\,\left(6\,c\,b^2+2\,a\,d\,b\right)}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,a^2\,d-6\,a\,b\,c}{3\,d^2\,{\left(a\,d-b\,c\right)}^2}+\frac{4\,b^2\,x^2}{3\,d\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((x*(6*b^2*c + 2*a*b*d))/(3*d^2*(a*d - b*c)^2) - (2*a^2*d
- 6*a*b*c)/(3*d^2*(a*d - b*c)^2) + (4*b^2*x^2)/(3*d*(a*d - b*c)^2)))/(x^2*(
a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(1/2))/d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*(c + d*x)**(5/2)), x)
```

3.1412 $\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16bd\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^3}-\frac{8d\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/((b*c - a*d)*Sqrt[a + b*x]*(c + d*x)^(3/2)) - (8*d*Sqrt[a + b*x])/(3*(b*
c - a*d)^2*(c + d*x)^(3/2)) - (16*b*d*Sqrt[a + b*x])/(3*(b*c - a*d)^3*Sqrt[
c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx &= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}-\frac{(4d)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{5/2}}\,dx}{bc-ad}\\ &= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{(8bd)\int \frac{1}{\sqrt{a+bx}\,(c+dx)^{3/2}}\,dx}{3(bc-ad)^2}\\ &= -\frac{2}{(bc-ad)\sqrt{a+bx}\,(c+dx)^{3/2}}-\frac{8d\sqrt{a+bx}}{3(bc-ad)^2(c+dx)^{3/2}}-\frac{16bd\sqrt{a+bx}}{3(bc-ad)^3\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 78, normalized size = 0.80

$$\frac{2a^2d^2-4abd(3c+2dx)-2b^2\left(3c^2+12cdx+8d^2x^2\right)}{3\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*a^2*d^2 - 4*a*b*d*(3*c + 2*d*x) - 2*b^2*(3*c^2 + 12*c*d*x + 8*d^2*x^2))/
(3*(b*c - a*d)^3*Sqrt[a + b*x]*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.74

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^2(c+dx)^2}{(a+bx)^2}+\frac{6bd(c+dx)}{a+bx}-d^2\right)}{3(c+dx)^{3/2}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-d^2 + (6*b*d*(c + d*x))/(a + b*x) + (3*b^2*(c + d*x)^
2)/(a + b*x)^2))/(3*(b*c - a*d)^3*(c + d*x)^(3/2))
```

fricas [B] time = 2.08, size = 273, normalized size = 2.79

$$-\frac{2\left(8\,b^{2}d^{2}x^{2}+3\,b^{2}c^{2}+6\,abcd-a^{2}d^{2}+4\left(3\,b^{2}cd+abd^{2}\right)x\right)\sqrt{bx+a}\,\sqrt{dx+c}}{3\left(ab^{3}c^{5}-3\,a^{2}b^{2}c^{4}d+3\,a^{3}bc^{3}d^{2}-a^{4}c^{2}d^{3}+\left(b^{4}c^{3}d^{2}-3\,ab^{3}c^{2}d^{3}+3\,a^{2}b^{2}cd^{4}-a^{3}bd^{5}\right)x^{3}+\left(2\,b^{4}c^{4}d-5\,ab^{3}c^{3}d^{2}+3\,a^{2}b^{2}c^{2}d^{3}+a^{3}bcd^{4}-a^{4}d^{5}\right)x^{2}+\left(b^{4}c^{5}-ab^{3}c^{4}d-3\,a^{2}b^{2}c^{3}d^{2}+5\,a^{3}bc^{2}d^{3}-2\,a^{4}cd^{4}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/3*(8*b^2*d^2*x^2 + 3*b^2*c^2 + 6*a*b*c*d - a^2*d^2 + 4*(3*b^2*c*d + a*b*
d^2)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a^3*b*
c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 -
a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b
*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a^3*
b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

giac [B] time = 1.48, size = 373, normalized size = 3.81

$$-\frac{4\sqrt{bd}\,b^{3}}{\left(b^{2}c^{2}|b|-2\,abcd|b|+a^{2}d^{2}|b|\right)\left(b^{2}c-abd-\left(\sqrt{bd}\,\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)}-\frac{2\sqrt{bx+a}\left(\frac{5\left(b^{6}c^{2}d^{3}|b|-2\,ab^{5}cd^{4}|b|+a^{2}b^{4}d^{5}|b|\right)(bx+a)}{b^{7}c^{5}d-5\,ab^{6}c^{4}d^{2}+10\,a^{2}b^{5}c^{3}d^{3}-10\,a^{3}b^{4}c^{2}d^{4}+5\,a^{4}b^{3}cd^{5}-a^{5}b^{2}d^{6}}+\frac{6\left(b^{7}c^{3}d^{2}|b|-3\,ab^{6}c^{2}d^{3}|b|+3\,a^{2}b^{5}cd^{4}|b|-a^{3}b^{4}d^{5}|b|\right)}{b^{7}c^{5}d-5\,ab^{6}c^{4}d^{2}+10\,a^{2}b^{5}c^{3}d^{3}-10\,a^{3}b^{4}c^{2}d^{4}+5\,a^{4}b^{3}cd^{5}-a^{5}b^{2}d^{6}}\right)}{3\left(b^{2}c+(bx+a)bd-abd\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -4*sqrt(b*d)*b^3/((b^2*c^2*abs(b) - 2*a*b*c*d*abs(b) + a^2*d^2*abs(b))*(b^2
*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2)) - 2/3*sqrt(b*x + a)*(5*(b^6*c^2*d^3*abs(b) - 2*a*b^5*c*d^4*abs(b) + a
^2*b^4*d^5*abs(b))*(b*x + a)/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*
d^3 - 10*a^3*b^4*c^2*d^4 + 5*a^4*b^3*c*d^5 - a^5*b^2*d^6) + 6*(b^7*c^3*d^2*
abs(b) - 3*a*b^6*c^2*d^3*abs(b) + 3*a^2*b^5*c*d^4*abs(b) - a^3*b^4*d^5*abs(
b))/(b^7*c^5*d - 5*a*b^6*c^4*d^2 + 10*a^2*b^5*c^3*d^3 - 10*a^3*b^4*c^2*d^4
+ 5*a^4*b^3*c*d^5 - a^5*b^2*d^6))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2)
```

maple [A] time = 0.01, size = 104, normalized size = 1.06

$$-\frac{2\left(-8b^{2}x^{2}d^{2}-4ab\,d^{2}x-12b^{2}cdx+a^{2}d^{2}-6abcd-3b^{2}c^{2}\right)}{3\sqrt{bx+a}\,\left(dx+c\right)^{\frac{3}{2}}\left(a^{3}d^{3}-3a^{2}bc\,d^{2}+3a\,b^{2}c^{2}d-b^{3}c^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-8*b^2*d^2*x^2-4*a*b*d^2*x-12*b^2*c*d*x+a^2*d^2-6*a*b*c*d-3*b^2*c^2)/
(b*x+a)^(1/2)/(d*x+c)^(3/2)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.03, size = 132, normalized size = 1.35

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b^2\,x^2}{3\,{\left(a\,d-b\,c\right)}^3}+\frac{-2\,a^2\,d^2+12\,a\,b\,c\,d+6\,b^2\,c^2}{3\,d^2\,{\left(a\,d-b\,c\right)}^3}+\frac{8\,b\,x\,\left(a\,d+3\,b\,c\right)}{3\,d\,{\left(a\,d-b\,c\right)}^3}\right)}{x^2\,\sqrt{a+b\,x}+\frac{c^2\,\sqrt{a+b\,x}}{d^2}+\frac{2\,c\,x\,\sqrt{a+b\,x}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b^2*x^2)/(3*(a*d - b*c)^3) + (6*b^2*c^2 - 2*a^2*d^2 +
 12*a*b*c*d)/(3*d^2*(a*d - b*c)^3) + (8*b*x*(a*d + 3*b*c))/(3*d*(a*d - b*c)
^3)))/(x^2*(a + b*x)^(1/2) + (c^2*(a + b*x)^(1/2))/d^2 + (2*c*x*(a + b*x)^(
1/2))/d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{3}{2}}\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(3/2)*(c + d*x)**(5/2)), x)
```

3.1413 $\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{32bd^2\sqrt{a+bx}}{3\sqrt{c+dx}\,(bc-ad)^4}+\frac{16d^2\sqrt{a+bx}}{3(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(3*(b*c - a*d)*(a + b*x)^(3/2)*(c + d*x)^(3/2)) + (4*d)/((b*c - a*d)^2*S
qrt[a + b*x]*(c + d*x)^(3/2)) + (16*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^3*(c
+ d*x)^(3/2)) + (32*b*d^2*Sqrt[a + b*x])/(3*(b*c - a*d)^4*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{(2d)\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/2}}\,dx}{bc-ad}\\ &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}}+\frac{\left(8d^2\right)\int \frac{1}{\sqrt{a+bx}\,(}{(bc-ad}\\ &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}}+\frac{16d^2\sqrt{a+}}{3(bc-ad)^3(c-}\\ &= -\frac{2}{3(bc-ad)(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{4d}{(bc-ad)^2\sqrt{a+bx}\,(c+dx)^{3/2}}+\frac{16d^2\sqrt{a+}}{3(bc-ad)^3(c-}\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.87

$$\frac{-2a^3d^3+6a^2bd^2(3c+2dx)+6ab^2d\left(3c^2+12cdx+8d^2x^2\right)+b^3\left(-2c^3+12c^2dx+48cd^2x^2+32d^3x^3\right)}{3(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*a^3*d^3 + 6*a^2*b*d^2*(3*c + 2*d*x) + 6*a*b^2*d*(3*c^2 + 12*c*d*x + 8*d
^2*x^2) + b^3*(-2*c^3 + 12*c^2*d*x + 48*c*d^2*x^2 + 32*d^3*x^3))/(3*(b*c -
a*d)^4*(a + b*x)^(3/2)*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.14, size = 92, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(\frac{b^3(c+dx)^3}{(a+bx)^3}-\frac{9b^2d(c+dx)^2}{(a+bx)^2}-\frac{9bd^2(c+dx)}{a+bx}+d^3\right)}{3(c+dx)^{3/2}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(d^3 - (9*b*d^2*(c + d*x))/(a + b*x) - (9*b^2*d*(c + d*
x)^2)/(a + b*x)^2 + (b^3*(c + d*x)^3)/(a + b*x)^3))/(3*(b*c - a*d)^4*(c + d
*x)^(3/2))
```

fricas [B] time = 2.42, size = 447, normalized size = 3.31

$$\frac{2\left(16\,b^{3}d^{3}x^{3}-b^{3}c^{3}+9\,ab^{2}c^{2}d+9\,a^{2}bcd^{2}-a^{3}d^{3}+24\left(b^{3}cd^{2}+ab^{2}d^{3}\right)x^{2}+6\left(b^{3}c^{2}d+6\,ab^{2}cd^{2}+a^{2}bd^{3}\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{3\left(a^{2}b^{4}c^{6}-4\,a^{3}b^{3}c^{5}d+6\,a^{4}b^{2}c^{4}d^{2}-4\,a^{5}bc^{3}d^{3}+a^{6}c^{2}d^{4}+\left(b^{6}c^{4}d^{2}-4\,ab^{5}c^{3}d^{3}+6\,a^{2}b^{4}c^{2}d^{4}-4\,a^{3}b^{3}cd^{5}+a^{4}b^{2}d^{6}\right)x^{4}+2\left(b^{6}c^{5}d-3\,ab^{5}c^{4}d^{2}+2\,a^{2}b^{4}c^{3}d^{3}+2\,a^{3}b^{3}c^{2}d^{4}-3\,a^{4}b^{2}cd^{5}+a^{5}bd^{6}\right)x^{3}+\left(b^{6}c^{6}-9\,a^{2}b^{4}c^{4}d^{2}+16\,a^{3}b^{3}c^{3}d^{3}-9\,a^{4}b^{2}c^{2}d^{4}+a^{6}d^{6}\right)x^{2}+2\left(ab^{5}c^{6}-3\,a^{2}b^{4}c^{5}d+2\,a^{3}b^{3}c^{4}d^{2}+2\,a^{4}b^{2}c^{3}d^{3}-3\,a^{5}bc^{2}d^{4}+a^{6}cd^{5}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(16*b^3*d^3*x^3 - b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 - a^3*d^3 + 2
4*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 6*(b^3*c^2*d + 6*a*b^2*c*d^2 + a^2*b*d^3)*x
)*sqrt(b*x + a)*sqrt(d*x + c)/(a^2*b^4*c^6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^
4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4 + (b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*
a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*x^4 + 2*(b^6*c^5*d - 3*a*b
^5*c^4*d^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*
b*d^6)*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
 + 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*x)
```

giac [B] time = 2.06, size = 670, normalized size = 4.96

$$\frac{2\sqrt{bx+a}\left(\frac{8\left(b^{7}c^{3}d^{4}|b|-3\,ab^{6}c^{2}d^{5}|b|+3\,a^{2}b^{5}cd^{6}|b|-a^{3}b^{4}d^{7}|b|\right)(bx+a)}{b^{9}c^{7}d-7\,ab^{8}c^{6}d^{2}+21\,a^{2}b^{7}c^{5}d^{3}-35\,a^{3}b^{6}c^{4}d^{4}+35\,a^{4}b^{5}c^{3}d^{5}-21\,a^{5}b^{4}c^{2}d^{6}+7\,a^{6}b^{3}cd^{7}-a^{7}b^{2}d^{8}}+\frac{9\left(b^{8}c^{4}d^{3}|b|-4\,ab^{7}c^{3}d^{4}|b|+6\,a^{2}b^{6}c^{2}d^{5}|b|-4\,a^{3}b^{5}cd^{6}|b|+a^{4}b^{4}d^{7}|b|\right)}{b^{9}c^{7}d-7\,ab^{8}c^{6}d^{2}+21\,a^{2}b^{7}c^{5}d^{3}-35\,a^{3}b^{6}c^{4}d^{4}+35\,a^{4}b^{5}c^{3}d^{5}-21\,a^{5}b^{4}c^{2}d^{6}+7\,a^{6}b^{3}cd^{7}-a^{7}b^{2}d^{8}}\right)}{3\left(b^{2}c+(bx+a)bd-abd\right)^{\frac{3}{2}}}+\frac{8\left(4\sqrt{bd}\,b^{7}c^{2}d-8\sqrt{bd}\,ab^{6}cd^{2}+4\sqrt{bd}\,a^{2}b^{5}d^{3}-9\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}b^{5}cd+9\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}ab^{4}d^{2}+3\sqrt{bd}\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{4}b^{3}d\right)}{3\left(b^{3}c^{3}|b|-3\,ab^{2}c^{2}d|b|+3\,a^{2}bcd^{2}|b|-a^{3}d^{3}|b|\right)\left(b^{2}c-abd-\left(\sqrt{bd}\sqrt{bx+a}-\sqrt{b^{2}c+(bx+a)bd-abd}\right)^{2}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(8*(b^7*c^3*d^4*abs(b) - 3*a*b^6*c^2*d^5*abs(b) + 3*a^2*b
^5*c*d^6*abs(b) - a^3*b^4*d^7*abs(b))*(b*x + a)/(b^9*c^7*d - 7*a*b^8*c^6*d^
2 + 21*a^2*b^7*c^5*d^3 - 35*a^3*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b
^4*c^2*d^6 + 7*a^6*b^3*c*d^7 - a^7*b^2*d^8) + 9*(b^8*c^4*d^3*abs(b) - 4*a*b
^7*c^3*d^4*abs(b) + 6*a^2*b^6*c^2*d^5*abs(b) - 4*a^3*b^5*c*d^6*abs(b) + a^4
*b^4*d^7*abs(b))/(b^9*c^7*d - 7*a*b^8*c^6*d^2 + 21*a^2*b^7*c^5*d^3 - 35*a^3
*b^6*c^4*d^4 + 35*a^4*b^5*c^3*d^5 - 21*a^5*b^4*c^2*d^6 + 7*a^6*b^3*c*d^7 -
a^7*b^2*d^8))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) + 8/3*(4*sqrt(b*d)*b^7*
c^2*d - 8*sqrt(b*d)*a*b^6*c*d^2 + 4*sqrt(b*d)*a^2*b^5*d^3 - 9*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^5*c*d + 9*
sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2
*a*b^4*d^2 + 3*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*b^3*d)/((b^3*c^3*abs(b) - 3*a*b^2*c^2*d*abs(b) + 3*a^2*b*c*
d^2*abs(b) - a^3*d^3*abs(b))*(b^2*c - a*b*d - (sqrt(b*d)*sqrt(b*x + a) - sq
rt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^3)
```

maple [A] time = 0.01, size = 169, normalized size = 1.25

$$-\frac{2\left(-16b^{3}x^{3}d^{3}-24a\,b^{2}d^{3}x^{2}-24b^{3}c\,d^{2}x^{2}-6a^{2}b\,d^{3}x-36a\,b^{2}c\,d^{2}x-6b^{3}c^{2}dx+a^{3}d^{3}-9a^{2}bc\,d^{2}-9a\,b^{2}c^{2}d+b^{3}c^{3}\right)}{3\left(bx+a\right)^{\frac{3}{2}}\left(dx+c\right)^{\frac{3}{2}}\left(a^{4}d^{4}-4a^{3}bc\,d^{3}+6a^{2}b^{2}c^{2}d^{2}-4a\,b^{3}c^{3}d+b^{4}c^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/3*(-16*b^3*d^3*x^3-24*a*b^2*d^3*x^2-24*b^3*c*d^2*x^2-6*a^2*b*d^3*x-36*a*
b^2*c*d^2*x-6*b^3*c^2*d*x+a^3*d^3-9*a^2*b*c*d^2-9*a*b^2*c^2*d+b^3*c^3)/(b*x
+a)^(3/2)/(d*x+c)^(3/2)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^
3*d+b^4*c^4)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.29, size = 224, normalized size = 1.66

$$\frac{\sqrt{c+d\,x}\,\left(\frac{16\,b\,x^2\,\left(a\,d+b\,c\right)}{{\left(a\,d-b\,c\right)}^4}-\frac{2\,a^3\,d^3-18\,a^2\,b\,c\,d^2-18\,a\,b^2\,c^2\,d+2\,b^3\,c^3}{3\,b\,d^2\,{\left(a\,d-b\,c\right)}^4}+\frac{32\,b^2\,d\,x^3}{3\,{\left(a\,d-b\,c\right)}^4}+\frac{4\,x\,\left(a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{d\,{\left(a\,d-b\,c\right)}^4}\right)}{x^3\,\sqrt{a+b\,x}+\frac{a\,c^2\,\sqrt{a+b\,x}}{b\,d^2}+\frac{x^2\,\left(a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{c\,x\,\left(2\,a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((16*b*x^2*(a*d + b*c))/(a*d - b*c)^4 - (2*a^3*d^3 + 2*b^3
*c^3 - 18*a*b^2*c^2*d - 18*a^2*b*c*d^2)/(3*b*d^2*(a*d - b*c)^4) + (32*b^2*d
*x^3)/(3*(a*d - b*c)^4) + (4*x*(a^2*d^2 + b^2*c^2 + 6*a*b*c*d))/(d*(a*d - b
*c)^4)))/(x^3*(a + b*x)^(1/2) + (a*c^2*(a + b*x)^(1/2))/(b*d^2) + (x^2*(a*d
 + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (c*x*(2*a*d + b*c)*(a + b*x)^(1/2))/(b*d
^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{5}{2}} \left(c + dx\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(5/2)*(c + d*x)**(5/2)), x)
```

3.1414 $\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=172

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-$$

Rubi [A] time = 0.04, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{256bd^3\sqrt{a+bx}}{15\sqrt{c+dx}\,(bc-ad)^5}-\frac{128d^3\sqrt{a+bx}}{15(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{5\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^3}+\frac{16d}{15(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{5(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(5*(b*c - a*d)*(a + b*x)^(5/2)*(c + d*x)^(3/2)) + (16*d)/(15*(b*c - a*d)
^2*(a + b*x)^(3/2)*(c + d*x)^(3/2)) - (32*d^2)/(5*(b*c - a*d)^3*Sqrt[a + b*
x]*(c + d*x)^(3/2)) - (128*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^4*(c + d*x)^(
3/2)) - (256*b*d^3*Sqrt[a + b*x])/(15*(b*c - a*d)^5*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx = -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}-\frac{(8d)\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/2}}\,dx}{5(bc-ad)}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}+\frac{\left(16d^2\right)}{}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{}{5(bc-}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{}{5(bc-}$$

$$= -\frac{2}{5(bc-ad)(a+bx)^{5/2}(c+dx)^{3/2}}+\frac{16d}{15(bc-ad)^2(a+bx)^{3/2}(c+dx)^{3/2}}-\frac{}{5(bc-}$$

Mathematica [A] time = 0.07, size = 170, normalized size = 0.99

$$-\frac{2\left(-5a^4d^4+20a^3bd^3(3c+2dx)+30a^2b^2d^2\left(3c^2+12cdx+8d^2x^2\right)+20ab^3d\left(-c^3+6c^2dx+24cd^2x^2+16d^3x^3\right)+b^4\left(3c^4-8c^3dx+48c^2d^2x^2+192cd^3x^3+128d^4x^4\right)\right)}{15(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(-5*a^4*d^4 + 20*a^3*b*d^3*(3*c + 2*d*x) + 30*a^2*b^2*d^2*(3*c^2 + 12*c
*d*x + 8*d^2*x^2) + 20*a*b^3*d*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3*x^
3) + b^4*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128*d^4*x^4)
))/(15*(b*c - a*d)^5*(a + b*x)^(5/2)*(c + d*x)^(3/2))
```

IntegrateAlgebraic [A] time = 0.15, size = 117, normalized size = 0.68

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^4(c+dx)^4}{(a+bx)^4}-\frac{20b^3d(c+dx)^3}{(a+bx)^3}+\frac{90b^2d^2(c+dx)^2}{(a+bx)^2}+\frac{60bd^3(c+dx)}{a+bx}-5d^4\right)}{15(c+dx)^{3/2}(bc-ad)^5}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(-5*d^4 + (60*b*d^3*(c + d*x))/(a + b*x) + (90*b^2*d^2*
(c + d*x)^2)/(a + b*x)^2 - (20*b^3*d*(c + d*x)^3)/(a + b*x)^3 + (3*b^4*(c +
 d*x)^4)/(a + b*x)^4))/(15*(b*c - a*d)^5*(c + d*x)^(3/2))
```

fricas [B] time = 8.26, size = 715, normalized size = 4.16

$$-\frac{2\left(128b^4d^4x^4+3b^4c^4-20ab^3c^3d+90a^2b^2c^2d^2+60a^3bcd^3-5a^4d^4+64\left(3b^4cd^3+5ab^3d^4\right)x^3+48\left(b^4c^2d^2+10ab^3cd^3+5a^2b^2d^4\right)x^2-8\left(b^4c^3d-15ab^3c^2d^2-45a^2b^2cd^3-5a^3bd^4\right)x\right)\sqrt{bx+a}\sqrt{dx+c}}{15\left(a^3b^5c^7-5a^4b^4c^6d+10a^5b^3c^5d^2-10a^6b^2c^4d^3+5a^7bc^3d^4-a^8c^2d^5+\left(b^8c^5d^2-5ab^7c^4d^3+10a^2b^6c^3d^4-10a^3b^5c^2d^5+5a^4b^4cd^6-a^5b^3d^7\right)x^5+\left(2b^8c^6d-7ab^7c^5d^2+5a^2b^6c^4d^3+10a^3b^5c^3d^4-20a^4b^4c^2d^5+13a^5b^3cd^6-3a^6b^2d^7\right)x^4+\left(b^8c^7+ab^7c^6d-17a^2b^6c^5d^2+35a^3b^5c^4d^3-25a^4b^4c^3d^4-a^5b^3c^2d^5+9a^6b^2cd^6-3a^7bd^7\right)x^3+\left(3ab^7c^7-9a^2b^6c^6d+a^3b^5c^5d^2+25a^4b^4c^4d^3-35a^5b^3c^3d^4+17a^6b^2c^2d^5-a^7bcd^6-a^8d^7\right)x^2+\left(3a^2b^6c^7-13a^3b^5c^6d+20a^4b^4c^5d^2-10a^5b^3c^4d^3-5a^6b^2c^3d^4+7a^7bc^2d^5-2a^8cd^6\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -2/15*(128*b^4*d^4*x^4 + 3*b^4*c^4 - 20*a*b^3*c^3*d + 90*a^2*b^2*c^2*d^2 +
60*a^3*b*c*d^3 - 5*a^4*d^4 + 64*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 48*(b^4*c
^2*d^2 + 10*a*b^3*c*d^3 + 5*a^2*b^2*d^4)*x^2 - 8*(b^4*c^3*d - 15*a*b^3*c^2*
d^2 - 45*a^2*b^2*c*d^3 - 5*a^3*b*d^4)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^3*b
^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^6*b^2*c^4*d^3 + 5*a^7*
b*c^3*d^4 - a^8*c^2*d^5 + (b^8*c^5*d^2 - 5*a*b^7*c^4*d^3 + 10*a^2*b^6*c^3*d
^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^3*d^7)*x^5 + (2*b^8*c^6*d
 - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 + 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^
2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7)*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17
*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^
5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*d^7)*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a
^3*b^5*c^5*d^2 + 25*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d
^5 - a^7*b*c*d^6 - a^8*d^7)*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^
4*b^4*c^5*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 -
2*a^8*c*d^6)*x)
```

giac [B] time = 3.92, size = 1203, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b*x + a)*(11*(b^8*c^4*d^5*abs(b) - 4*a*b^7*c^3*d^6*abs(b) + 6*a^2
*b^6*c^2*d^7*abs(b) - 4*a^3*b^5*c*d^8*abs(b) + a^4*b^4*d^9*abs(b))*(b*x + a
)/(b^11*c^9*d - 9*a*b^10*c^8*d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4
+ 126*a^4*b^7*c^5*d^5 - 126*a^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b
^4*c^2*d^8 + 9*a^8*b^3*c*d^9 - a^9*b^2*d^10) + 12*(b^9*c^5*d^4*abs(b) - 5*a
```

```
*b^8*c^4*d^5*abs(b) + 10*a^2*b^7*c^3*d^6*abs(b) - 10*a^3*b^6*c^2*d^7*abs(b)
 + 5*a^4*b^5*c*d^8*abs(b) - a^5*b^4*d^9*abs(b))/(b^11*c^9*d - 9*a*b^10*c^8*
d^2 + 36*a^2*b^9*c^7*d^3 - 84*a^3*b^8*c^6*d^4 + 126*a^4*b^7*c^5*d^5 - 126*a
^5*b^6*c^4*d^6 + 84*a^6*b^5*c^3*d^7 - 36*a^7*b^4*c^2*d^8 + 9*a^8*b^3*c*d^9
- a^9*b^2*d^10))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/2) - 4/15*(73*sqrt(b*d)
*b^11*c^4*d^2 - 292*sqrt(b*d)*a*b^10*c^3*d^3 + 438*sqrt(b*d)*a^2*b^9*c^2*d^
4 - 292*sqrt(b*d)*a^3*b^8*c*d^5 + 73*sqrt(b*d)*a^4*b^7*d^6 - 320*sqrt(b*d)*
(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*b^9*c^3*d
^2 + 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*a*b^8*c^2*d^3 - 960*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^7*c*d^4 + 320*sqrt(b*d)*(sqrt(b*d)*sqr
t(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^3*b^6*d^5 + 490*sqrt(
b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^7*
c^2*d^2 - 980*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b
*d - a*b*d))^4*a*b^6*c*d^3 + 490*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(
b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^5*d^4 - 240*sqrt(b*d)*(sqrt(b*d)*sq
rt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*b^5*c*d^2 + 240*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^4
*d^3 + 45*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
 a*b*d))^8*b^3*d^2)/((b^4*c^4*abs(b) - 4*a*b^3*c^3*d*abs(b) + 6*a^2*b^2*c^2
*d^2*abs(b) - 4*a^3*b*c*d^3*abs(b) + a^4*d^4*abs(b))*(b^2*c - a*b*d - (sqrt
(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^5)
```

maple [A] time = 0.01, size = 256, normalized size = 1.49

$$-\frac{2\left(-128b^4x^4d^4-320a\,b^3d^4x^3-192b^4c\,d^3x^3-240a^2b^2d^4x^2-480a\,b^3c\,d^3x^2-48b^4c^2d^2x^2-40a^3b\,d^4x-360a^2b^2c\,d^3x-120a\,b^3c^2d^2x+8b^4c^3dx+5a^4d^4-60a^3bc\,d^3-90a^2b^2c^2d^2+20a\,b^3c^3d-3b^4c^4\right)}{15\left(bx+a\right)^{\frac{5}{2}}\left(dx+c\right)^{\frac{3}{2}}\left(a^5d^5-5a^4bc\,d^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5a\,b^4c^4d-b^5c^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/15*(-128*b^4*d^4*x^4-320*a*b^3*d^4*x^3-192*b^4*c*d^3*x^3-240*a^2*b^2*d^4
*x^2-480*a*b^3*c*d^3*x^2-48*b^4*c^2*d^2*x^2-40*a^3*b*d^4*x-360*a^2*b^2*c*d^
3*x-120*a*b^3*c^2*d^2*x+8*b^4*c^3*d*x+5*a^4*d^4-60*a^3*b*c*d^3-90*a^2*b^2*c
^2*d^2+20*a*b^3*c^3*d-3*b^4*c^4)/(b*x+a)^(5/2)/(d*x+c)^(3/2)/(a^5*d^5-5*a^4
*b*c*d^4+10*a^3*b^2*c^2*d^3-10*a^2*b^3*c^3*d^2+5*a*b^4*c^4*d-b^5*c^5)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [B] time = 1.53, size = 346, normalized size = 2.01

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(5\,a^2\,d^2+10\,a\,b\,c\,d+b^2\,c^2\right)}{5\,{\left(a\,d-b\,c\right)}^5}+\frac{256\,b^2\,d^2\,x^4}{15\,{\left(a\,d-b\,c\right)}^5}+\frac{-10\,a^4\,d^4+120\,a^3\,b\,c\,d^3+180\,a^2\,b^2\,c^2\,d^2-40\,a\,b^3\,c^3\,d+6\,b^4\,c^4}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{x\,\left(80\,a^3\,b\,d^4+720\,a^2\,b^2\,c\,d^3+240\,a\,b^3\,c^2\,d^2-16\,b^4\,c^3\,d\right)}{15\,b^2\,d^2\,{\left(a\,d-b\,c\right)}^5}+\frac{128\,b\,d\,x^3\,\left(5\,a\,d+3\,b\,c\right)}{15\,{\left(a\,d-b\,c\right)}^5}\right)}{x^4\,\sqrt{a+b\,x}+\frac{x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+4\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{2\,x^3\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^2\,c^2\,\sqrt{a+b\,x}}{b^2\,d^2}+\frac{2\,a\,c\,x\,\left(a\,d+b\,c\right)\,\sqrt{a+b\,x}}{b^2\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(5*a^2*d^2 + b^2*c^2 + 10*a*b*c*d))/(5*(a*d - b*c
)^5) + (256*b^2*d^2*x^4)/(15*(a*d - b*c)^5) + (6*b^4*c^4 - 10*a^4*d^4 + 180
```

```
*a^2*b^2*c^2*d^2 - 40*a*b^3*c^3*d + 120*a^3*b*c*d^3)/(15*b^2*d^2*(a*d - b*c
)^5) + (x*(80*a^3*b*d^4 - 16*b^4*c^3*d + 240*a*b^3*c^2*d^2 + 720*a^2*b^2*c*
d^3))/(15*b^2*d^2*(a*d - b*c)^5) + (128*b*d*x^3*(5*a*d + 3*b*c))/(15*(a*d -
 b*c)^5)))/(x^4*(a + b*x)^(1/2) + (x^2*(a + b*x)^(1/2)*(a^2*d^2 + b^2*c^2 +
 4*a*b*c*d))/(b^2*d^2) + (2*x^3*(a*d + b*c)*(a + b*x)^(1/2))/(b*d) + (a^2*c
^2*(a + b*x)^(1/2))/(b^2*d^2) + (2*a*c*x*(a*d + b*c)*(a + b*x)^(1/2))/(b^2*
d^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{2}}(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/2)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(1/((a + b*x)**(7/2)*(c + d*x)**(5/2)), x)
```

3.1415 $\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=207

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-a}$$

Rubi [A] time = 0.06, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512bd^4\sqrt{a+bx}}{21\sqrt{c+dx}\,(bc-ad)^6}+\frac{256d^4\sqrt{a+bx}}{21(c+dx)^{3/2}(bc-ad)^5}+\frac{64d^3}{7\sqrt{a+bx}\,(c+dx)^{3/2}(bc-ad)^4}-\frac{32d^2}{21(a+bx)^{3/2}(c+dx)^{3/2}(bc-ad)^3}+\frac{4d}{7(a+bx)^{5/2}(c+dx)^{3/2}(bc-ad)^2}-\frac{2}{7(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] -2/(7*(b*c - a*d)*(a + b*x)^(7/2)*(c + d*x)^(3/2)) + (4*d)/(7*(b*c - a*d)^2
*(a + b*x)^(5/2)*(c + d*x)^(3/2)) - (32*d^2)/(21*(b*c - a*d)^3*(a + b*x)^(3
/2)*(c + d*x)^(3/2)) + (64*d^3)/(7*(b*c - a*d)^4*Sqrt[a + b*x]*(c + d*x)^(3
/2)) + (256*d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^5*(c + d*x)^(3/2)) + (512*b*
d^4*Sqrt[a + b*x])/(21*(b*c - a*d)^6*Sqrt[c + d*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int \frac{1}{(a+bx)^{9/2}(c+dx)^{5/2}}\,dx = -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} - \frac{(10d)\int \frac{1}{(a+bx)^{7/2}(c+dx)^{5/2}}\,dx}{7(bc-ad)}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} + \frac{(16d^2)\int \frac{}{(a\cdots}}{7(b\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(bc-ad)\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(bc-ad)\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(bc-ad)\cdots}$$

$$= -\frac{2}{7(bc-ad)(a+bx)^{7/2}(c+dx)^{3/2}} + \frac{4d}{7(bc-ad)^2(a+bx)^{5/2}(c+dx)^{3/2}} - \frac{}{21(bc-ad)\cdots}$$

Mathematica [**A**] time = 0.08, size = 233, normalized size = 1.13

$$\frac{2\left(-7a^5d^5+35a^4bd^4(3c+2dx)+70a^3b^2d^3\left(3c^2+12cdx+8d^2x^2\right)+70a^2b^3d^2\left(-c^3+6c^2dx+24cd^2x^2+16d^3x^3\right)+7ab^4d\left(3c^4-8c^3dx+48c^2d^2x^2+192cd^3x^3+128d^4x^4\right)+b^5\left(-3c^5+6c^4dx-16c^3d^2x^2+96c^2d^3x^3+384cd^4x^4+256d^5x^5\right)\right)}{21(a+bx)^{7/2}(c+dx)^{3/2}(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (2*(-7*a^5*d^5 + 35*a^4*b*d^4*(3*c + 2*d*x) + 70*a^3*b^2*d^3*(3*c^2 + 12*c*
d*x + 8*d^2*x^2) + 70*a^2*b^3*d^2*(-c^3 + 6*c^2*d*x + 24*c*d^2*x^2 + 16*d^3
*x^3) + 7*a*b^4*d*(3*c^4 - 8*c^3*d*x + 48*c^2*d^2*x^2 + 192*c*d^3*x^3 + 128
*d^4*x^4) + b^5*(-3*c^5 + 6*c^4*d*x - 16*c^3*d^2*x^2 + 96*c^2*d^3*x^3 + 384
*c*d^4*x^4 + 256*d^5*x^5)))/(21*(b*c - a*d)^6*(a + b*x)^(7/2)*(c + d*x)^(3/
2))
```

IntegrateAlgebraic [**A**] time = 0.16, size = 139, normalized size = 0.67

$$-\frac{2(a+bx)^{3/2}\left(\frac{3b^5(c+dx)^5}{(a+bx)^5}-\frac{21b^4d(c+dx)^4}{(a+bx)^4}+\frac{70b^3d^2(c+dx)^3}{(a+bx)^3}-\frac{210b^2d^3(c+dx)^2}{(a+bx)^2}-\frac{105bd^4(c+dx)}{a+bx}+7d^5\right)}{21(c+dx)^{3/2}(bc-ad)^6}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x]
```

```
[Out] (-2*(a + b*x)^(3/2)*(7*d^5 - (105*b*d^4*(c + d*x))/(a + b*x) - (210*b^2*d^3
*(c + d*x)^2)/(a + b*x)^2 + (70*b^3*d^2*(c + d*x)^3)/(a + b*x)^3 - (21*b^4*
d*(c + d*x)^4)/(a + b*x)^4 + (3*b^5*(c + d*x)^5)/(a + b*x)^5))/(21*(b*c - a
*d)^6*(c + d*x)^(3/2))
```

fricas [**B**] time = 19.33, size = 999, normalized size = 4.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/21*(256*b^5*d^5*x^5 - 3*b^5*c^5 + 21*a*b^4*c^4*d - 70*a^2*b^3*c^3*d^2 + 2
10*a^3*b^2*c^2*d^3 + 105*a^4*b*c*d^4 - 7*a^5*d^5 + 128*(3*b^5*c*d^4 + 7*a*b
^4*d^5)*x^4 + 32*(3*b^5*c^2*d^3 + 42*a*b^4*c*d^4 + 35*a^2*b^3*d^5)*x^3 - 16
```

```
*(b^5*c^3*d^2 - 21*a*b^4*c^2*d^3 - 105*a^2*b^3*c*d^4 - 35*a^3*b^2*d^5)*x^2
+ 2*(3*b^5*c^4*d - 28*a*b^4*c^3*d^2 + 210*a^2*b^3*c^2*d^3 + 420*a^3*b^2*c*d
^4 + 35*a^4*b*d^5)*x)*sqrt(b*x + a)*sqrt(d*x + c)/(a^4*b^6*c^8 - 6*a^5*b^5*
c^7*d + 15*a^6*b^4*c^6*d^2 - 20*a^7*b^3*c^5*d^3 + 15*a^8*b^2*c^4*d^4 - 6*a^
9*b*c^3*d^5 + a^10*c^2*d^6 + (b^10*c^6*d^2 - 6*a*b^9*c^5*d^3 + 15*a^2*b^8*c
^4*d^4 - 20*a^3*b^7*c^3*d^5 + 15*a^4*b^6*c^2*d^6 - 6*a^5*b^5*c*d^7 + a^6*b^
4*d^8)*x^6 + 2*(b^10*c^7*d - 4*a*b^9*c^6*d^2 + 3*a^2*b^8*c^5*d^3 + 10*a^3*b
^7*c^4*d^4 - 25*a^4*b^6*c^3*d^5 + 24*a^5*b^5*c^2*d^6 - 11*a^6*b^4*c*d^7 + 2
*a^7*b^3*d^8)*x^5 + (b^10*c^8 + 2*a*b^9*c^7*d - 27*a^2*b^8*c^6*d^2 + 64*a^3
*b^7*c^5*d^3 - 55*a^4*b^6*c^4*d^4 - 6*a^5*b^5*c^3*d^5 + 43*a^6*b^4*c^2*d^6
- 28*a^7*b^3*c*d^7 + 6*a^8*b^2*d^8)*x^4 + 4*(a*b^9*c^8 - 3*a^2*b^8*c^7*d -
2*a^3*b^7*c^6*d^2 + 19*a^4*b^6*c^5*d^3 - 30*a^5*b^5*c^4*d^4 + 19*a^6*b^4*c^
3*d^5 - 2*a^7*b^3*c^2*d^6 - 3*a^8*b^2*c*d^7 + a^9*b*d^8)*x^3 + (6*a^2*b^8*c
^8 - 28*a^3*b^7*c^7*d + 43*a^4*b^6*c^6*d^2 - 6*a^5*b^5*c^5*d^3 - 55*a^6*b^4
*c^4*d^4 + 64*a^7*b^3*c^3*d^5 - 27*a^8*b^2*c^2*d^6 + 2*a^9*b*c*d^7 + a^10*d
^8)*x^2 + 2*(2*a^3*b^7*c^8 - 11*a^4*b^6*c^7*d + 24*a^5*b^5*c^6*d^2 - 25*a^6
*b^4*c^5*d^3 + 10*a^7*b^3*c^4*d^4 + 3*a^8*b^2*c^3*d^5 - 4*a^9*b*c^2*d^6 + a
^10*c*d^7)*x)
```

giac [B] time = 7.76, size = 1964, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(b*x + a)*(14*(b^9*c^5*d^6*abs(b) - 5*a*b^8*c^4*d^7*abs(b) + 10*a^2
*b^7*c^3*d^8*abs(b) - 10*a^3*b^6*c^2*d^9*abs(b) + 5*a^4*b^5*c*d^10*abs(b) -
 a^5*b^4*d^11*abs(b))*(b*x + a)/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*
b^11*c^9*d^3 - 165*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6
*d^6 + 462*a^6*b^7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55
*a^9*b^4*c^2*d^10 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12) + 15*(b^10*c^6*d^5*
abs(b) - 6*a*b^9*c^5*d^6*abs(b) + 15*a^2*b^8*c^4*d^7*abs(b) - 20*a^3*b^7*c^
3*d^8*abs(b) + 15*a^4*b^6*c^2*d^9*abs(b) - 6*a^5*b^5*c*d^10*abs(b) + a^6*b^
4*d^11*abs(b))/(b^13*c^11*d - 11*a*b^12*c^10*d^2 + 55*a^2*b^11*c^9*d^3 - 16
5*a^3*b^10*c^8*d^4 + 330*a^4*b^9*c^7*d^5 - 462*a^5*b^8*c^6*d^6 + 462*a^6*b^
7*c^5*d^7 - 330*a^7*b^6*c^4*d^8 + 165*a^8*b^5*c^3*d^9 - 55*a^9*b^4*c^2*d^10
 + 11*a^10*b^3*c*d^11 - a^11*b^2*d^12))/(b^2*c + (b*x + a)*b*d - a*b*d)^(3/
2) + 8/21*(79*sqrt(b*d)*b^15*c^6*d^3 - 474*sqrt(b*d)*a*b^14*c^5*d^4 + 1185*
sqrt(b*d)*a^2*b^13*c^4*d^5 - 1580*sqrt(b*d)*a^3*b^12*c^3*d^6 + 1185*sqrt(b*
d)*a^4*b^11*c^2*d^7 - 474*sqrt(b*d)*a^5*b^10*c*d^8 + 79*sqrt(b*d)*a^6*b^9*d
^9 - 511*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^2*b^13*c^5*d^3 + 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2
*c + (b*x + a)*b*d - a*b*d))^2*a*b^12*c^4*d^4 - 5110*sqrt(b*d)*(sqrt(b*d)*s
qrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^2*b^11*c^3*d^5 + 51
10*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^2*a^3*b^10*c^2*d^6 - 2555*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c
 + (b*x + a)*b*d - a*b*d))^2*a^4*b^9*c*d^7 + 511*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2*a^5*b^8*d^8 + 1344*sqrt(b
*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*b^11*
c^4*d^3 - 5376*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*
b*d - a*b*d))^4*a*b^10*c^3*d^4 + 8064*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^2*b^9*c^2*d^5 - 5376*sqrt(b*d)*(sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^4*a^3*b^8*c*d^
6 + 1344*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d -
a*b*d))^4*a^4*b^7*d^7 - 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*
c + (b*x + a)*b*d - a*b*d))^6*b^9*c^3*d^3 + 5250*sqrt(b*d)*(sqrt(b*d)*sqrt(
b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a*b^8*c^2*d^4 - 5250*sqrt
(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^6*a^2
*b^7*c*d^5 + 1750*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x +
```

```
a)*b*d - a*b*d))^6*a^3*b^6*d^6 + 1015*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) -
sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*b^7*c^2*d^3 - 2030*sqrt(b*d)*(sqrt(b
*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^8*a*b^6*c*d^4 + 10
15*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d)
)^8*a^2*b^5*d^5 - 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*
x + a)*b*d - a*b*d))^10*b^5*c*d^3 + 315*sqrt(b*d)*(sqrt(b*d)*sqrt(b*x + a)
- sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^10*a*b^4*d^4 + 42*sqrt(b*d)*(sqrt(b*
d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^12*b^3*d^3)/((b^5*c
^5*abs(b) - 5*a*b^4*c^4*d*abs(b) + 10*a^2*b^3*c^3*d^2*abs(b) - 10*a^3*b^2*c
^2*d^3*abs(b) + 5*a^4*b*c*d^4*abs(b) - a^5*d^5*abs(b))*(b^2*c - a*b*d - (sq
rt(b*d)*sqrt(b*x + a) - sqrt(b^2*c + (b*x + a)*b*d - a*b*d))^2)^7)
```

maple [**B**] time = 0.02, size = 356, normalized size = 1.72

$$-\frac{2\left(-256b^5x^5d^5-896a\,b^4d^5x^4-384b^5c\,d^4x^4-1120a^2b^3d^5x^3-1344a\,b^4c\,d^4x^3-96b^5c^2d^3x^3-560a^3b^2d^5x^2-1680a^2b^3c\,d^4x^2-336a\,b^4c^2d^3x^2+16b^5c^3d^2x^2-70a^4b\,d^5x-840a^3b^2c\,d^4x-420a^2b^3c^2d^3x+56a\,b^4c^3d^2x-6b^5c^4dx+7a^5d^5-105a^4bc\,d^4-210a^3b^2c^2d^3+70a^2b^3c^3d^2-21a\,b^4c^4d+3b^5c^5\right)}{21\left(bx+a\right)^{\frac{7}{2}}\left(dx+c\right)^{\frac{3}{2}}\left(d^6a^6-6b\,d^5c\,a^5+15b^2d^4c^2a^4-20b^3d^3c^3a^3+15b^4d^2c^4a^2-6b^5d\,c^5a+b^6c^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x)
```

```
[Out] -2/21*(-256*b^5*d^5*x^5-896*a*b^4*d^5*x^4-384*b^5*c*d^4*x^4-1120*a^2*b^3*d^
5*x^3-1344*a*b^4*c*d^4*x^3-96*b^5*c^2*d^3*x^3-560*a^3*b^2*d^5*x^2-1680*a^2*
b^3*c*d^4*x^2-336*a*b^4*c^2*d^3*x^2+16*b^5*c^3*d^2*x^2-70*a^4*b*d^5*x-840*a
^3*b^2*c*d^4*x-420*a^2*b^3*c^2*d^3*x+56*a*b^4*c^3*d^2*x-6*b^5*c^4*d*x+7*a^5
*d^5-105*a^4*b*c*d^4-210*a^3*b^2*c^2*d^3+70*a^2*b^3*c^3*d^2-21*a*b^4*c^4*d+
3*b^5*c^5)/(b*x+a)^(7/2)/(d*x+c)^(3/2)/(a^6*d^6-6*a^5*b*c*d^5+15*a^4*b^2*c^
2*d^4-20*a^3*b^3*c^3*d^3+15*a^2*b^4*c^4*d^2-6*a*b^5*c^5*d+b^6*c^6)
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/2)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 1.91, size = 478, normalized size = 2.31

$$\frac{\sqrt{c+d\,x}\,\left(\frac{32\,x^2\,\left(35\,a^3\,d^3+105\,a^2\,b\,c\,d^2+21\,a\,b^2\,c^2\,d-b^3\,c^3\right)}{21\,b\,{\left(a\,d-b\,c\right)}^6}-\frac{14\,a^5\,d^5-210\,a^4\,b\,c\,d^4-420\,a^3\,b^2\,c^2\,d^3+140\,a^2\,b^3\,c^3\,d^2-42\,a\,b^4\,c^4\,d+6\,b^5\,c^5}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}+\frac{64\,d\,x^3\,\left(35\,a^2\,d^2+42\,a\,b\,c\,d+3\,b^2\,c^2\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{512\,b^2\,d^3\,x^5}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{256\,b\,d^2\,x^4\,\left(7\,a\,d+3\,b\,c\right)}{21\,{\left(a\,d-b\,c\right)}^6}+\frac{x\,\left(140\,a^4\,b\,d^5+1680\,a^3\,b^2\,c\,d^4+840\,a^2\,b^3\,c^2\,d^3-112\,a\,b^4\,c^3\,d^2+12\,b^5\,c^4\,d\right)}{21\,b^3\,d^2\,{\left(a\,d-b\,c\right)}^6}\right)}{x^5\,\sqrt{a+b\,x}+\frac{x^3\,\sqrt{a+b\,x}\,\left(3\,a^2\,d^2+6\,a\,b\,c\,d+b^2\,c^2\right)}{b^2\,d^2}+\frac{x^4\,\left(3\,a\,d+2\,b\,c\right)\,\sqrt{a+b\,x}}{b\,d}+\frac{a^3\,c^2\,\sqrt{a+b\,x}}{b^3\,d^2}+\frac{a\,x^2\,\sqrt{a+b\,x}\,\left(a^2\,d^2+6\,a\,b\,c\,d+3\,b^2\,c^2\right)}{b^3\,d^2}+\frac{a^2\,c\,x\,\left(2\,a\,d+3\,b\,c\right)\,\sqrt{a+b\,x}}{b^3\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/2)*(c + d*x)^(5/2)),x)
```

```
[Out] ((c + d*x)^(1/2)*((32*x^2*(35*a^3*d^3 - b^3*c^3 + 21*a*b^2*c^2*d + 105*a^2*
b*c*d^2))/(21*b*(a*d - b*c)^6) - (14*a^5*d^5 + 6*b^5*c^5 + 140*a^2*b^3*c^3*
d^2 - 420*a^3*b^2*c^2*d^3 - 42*a*b^4*c^4*d - 210*a^4*b*c*d^4)/(21*b^3*d^2*(
a*d - b*c)^6) + (64*d*x^3*(35*a^2*d^2 + 3*b^2*c^2 + 42*a*b*c*d))/(21*(a*d -
 b*c)^6) + (512*b^2*d^3*x^5)/(21*(a*d - b*c)^6) + (256*b*d^2*x^4*(7*a*d + 3
*b*c))/(21*(a*d - b*c)^6) + (x*(140*a^4*b*d^5 + 12*b^5*c^4*d - 112*a*b^4*c^
3*d^2 + 1680*a^3*b^2*c*d^4 + 840*a^2*b^3*c^2*d^3))/(21*b^3*d^2*(a*d - b*c)^
6)))/(x^5*(a + b*x)^(1/2) + (x^3*(a + b*x)^(1/2)*(3*a^2*d^2 + b^2*c^2 + 6*a
*b*c*d))/(b^2*d^2) + (x^4*(3*a*d + 2*b*c)*(a + b*x)^(1/2))/(b*d) + (a^3*c^2
*(a + b*x)^(1/2))/(b^3*d^2) + (a*x^2*(a + b*x)^(1/2)*(a^2*d^2 + 3*b^2*c^2 +
 6*a*b*c*d))/(b^3*d^2) + (a^2*c*x*(2*a*d + 3*b*c)*(a + b*x)^(1/2))/(b^3*d^2
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/2)/(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

3.1416 $\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+bx}\sqrt{4+a+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{a+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[a + b*x]/2])/b
```

IntegrateAlgebraic [A] time = 0.05, size = 28, normalized size = 1.47

$$-\frac{2\log\left(\sqrt{a+bx+4}-\sqrt{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a + b*x]*Sqrt[4 + a + b*x]),x]
[Out] (-2*Log[-Sqrt[a + b*x] + Sqrt[4 + a + b*x]])/b
```

fricas [B] time = 1.05, size = 31, normalized size = 1.63

$$-\frac{\log\left(-bx + \sqrt{bx + a + 4}\sqrt{bx + a} - a - 2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + a + 4)*sqrt(b*x + a) - a - 2)/b
```

giac [A] time = 1.02, size = 24, normalized size = 1.26

$$-\frac{2\log\left(\sqrt{bx + a + 4} - \sqrt{bx + a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + a + 4) - sqrt(b*x + a))/b
```

maple [B] time = 0.01, size = 86, normalized size = 4.53

$$\frac{\sqrt{(bx + a)(bx + a + 4)}\ \ln\left(\frac{b^2x + \frac{ab}{2} + \frac{(a+4)b}{2}}{\sqrt{b^2}} + \sqrt{b^2x^2 + (a + 4)a + (ab + (a + 4)b)x}\right)}{\sqrt{bx + a}\ \sqrt{bx + a + 4}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x)
[Out] ((b*x+a)*(b*x+a+4))^(1/2)/(b*x+a)^(1/2)/(b*x+a+4)^(1/2)*ln((1/2*a*b+1/2*b*(
a+4)+b^2*x)/(b^2)^(1/2)+(b^2*x^2+(a*b+b*(a+4))*x+a*(a+4))^(1/2))/(b^2)^(1/2
)
```

maxima [B] time = 1.36, size = 48, normalized size = 2.53

$$\frac{\log\left(2b^2x + 2ab + 2\sqrt{b^2x^2 + a^2 + 2(ab + 2b)x + 4a}\,b + 4b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2)/(b*x+a+4)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*a*b + 2*sqrt(b^2*x^2 + a^2 + 2*(a*b + 2*b)*x + 4*a)*b + 4*b
)/b
```

mupad [B] time = 0.31, size = 50, normalized size = 2.63

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{a+4}-\sqrt{a+b\,x+4}\right)}{\sqrt{-b^2}\left(\sqrt{a+b\,x}-\sqrt{a}\right)}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2)*(a + b*x + 4)^(1/2)),x)
```

```
[Out] (4*atan((b*((a + 4)^(1/2) - (a + b*x + 4)^(1/2)))/((-b^2)^(1/2)*((a + b*x)^
(1/2) - a^(1/2)))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx}\,\sqrt{a+bx+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2)/(b*x+a+4)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x)*sqrt(a + b*x + 4)), x)
```

3.1417 $\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+bx}\,\sqrt{6+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{2+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+2}\,\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-2}\right)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
```

```
[Out] (2*Sqrt[2 + b*x]*ArcSin[Sqrt[-2 - b*x]/2])/(b*Sqrt[-2 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+6}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 + b*x]*Sqrt[6 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[6 + b*x]/Sqrt[2 + b*x]])/b
```

fricas [A] time = 0.81, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx + \sqrt{bx+6}\,\sqrt{bx+2} - 4\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 6)*sqrt(b*x + 2) - 4)/b
```

giac [A] time = 1.05, size = 23, normalized size = 1.21

$$-\frac{2\,\log\left(\sqrt{bx+6} - \sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 6) - sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx+2)\,(bx+6)}\;\ln\left(\frac{b^2x+4b}{\sqrt{b^2}} + \sqrt{b^2x^2+8bx+12}\right)}{\sqrt{bx+2}\;\sqrt{bx+6}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x)
[Out] ((b*x+2)*(b*x+6))^(1/2)/(b*x+2)^(1/2)/(b*x+6)^(1/2)*ln((b^2*x+4*b)/(b^2)^(1
/2)+(b^2*x^2+8*b*x+12)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.37, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 + 8\,bx + 12}\,b + 8\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+6)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 8*b*x + 12)*b + 8*b)/b
```

mupad [B] time = 0.34, size = 47, normalized size = 2.47

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{6}-\sqrt{b\,x+6}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 6)^(1/2)),x)
[Out] -(4*atan((b*(6^(1/2) - (b*x + 6)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+2}\,\sqrt{bx+6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+6)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 6)), x)
```

3.1418 $\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{63, 215\}$

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{5+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx+1}\,\sin^{-1}\left(\frac{1}{2}\sqrt{-bx-1}\right)}{b\sqrt{-bx-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*ArcSin[Sqrt[-1 - b*x]/2])/(b*Sqrt[-1 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+5}}{\sqrt{bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + b*x]*Sqrt[5 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[5 + b*x]/Sqrt[1 + b*x]])/b
```

fricas [A] time = 0.88, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx + \sqrt{bx+5}\,\sqrt{bx+1} - 3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 5)*sqrt(b*x + 1) - 3)/b
```

giac [A] time = 0.96, size = 23, normalized size = 1.21

$$-\frac{2\,\log\left(\sqrt{bx+5} - \sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 5) - sqrt(b*x + 1))/b
```

maple [B] time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx+1)\,(bx+5)}\;\ln\left(\frac{b^2x+3b}{\sqrt{b^2}} + \sqrt{b^2x^2+6bx+5}\right)}{\sqrt{bx+1}\,\sqrt{bx+5}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x)
```

```
[Out] ((b*x+1)*(b*x+5))^(1/2)/(b*x+1)^(1/2)/(b*x+5)^(1/2)*ln((b^2*x+3*b)/(b^2)^(1
/2)+(b^2*x^2+6*b*x+5)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.39, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 + 6\,bx + 5}\,b + 6\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 6*b*x + 5)*b + 6*b)/b
```

mupad [B] time = 0.33, size = 43, normalized size = 2.26

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{5}-\sqrt{b\,x+5}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 5)^(1/2)),x)
```

```
[Out] (4*atan((b*(5^(1/2) - (b*x + 5)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+1}\,\sqrt{bx+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+5)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 5)), x)
```

3.1419 $\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=17

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{4+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 2.00

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{2}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/2])/(Sqrt[b]*Sqrt[b*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 1.47

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[b*x]*Sqrt[4 + b*x]),x]
[Out] (-2*Log[-Sqrt[b*x] + Sqrt[4 + b*x]])/b
```

fricas [A] time = 0.95, size = 25, normalized size = 1.47

$$-\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx}-2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x) - 2)/b
```

giac [A] time = 0.94, size = 21, normalized size = 1.24

$$-\frac{2\log\left(\sqrt{bx+4}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x))/b
```

maple [B] time = 0.01, size = 60, normalized size = 3.53

$$\frac{\sqrt{(bx+4)\,bx}\;\ln\left(\frac{b^2x+2b}{\sqrt{b^2}}+\sqrt{b^2x^2+4bx}\right)}{\sqrt{bx}\;\sqrt{bx+4}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+4)^(1/2),x)
[Out] (x*b*(b*x+4))^(1/2)/(b*x)^(1/2)/(b*x+4)^(1/2)*ln((b^2*x+2*b)/(b^2)^(1/2)+(b
^2*x^2+4*b*x)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.33, size = 32, normalized size = 1.88

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+4\,bx}\,b+4\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 4*b*x)*b + 4*b)/b
```

mupad [B] time = 0.31, size = 33, normalized size = 1.94

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x+4}-2\right)}{\sqrt{b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x + 4)^(1/2) - 2))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)^(1/2
)
```

sympy [A] time = 1.27, size = 15, normalized size = 0.88

$$\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] 2*asinh(sqrt(b)*sqrt(x)/2)/b
```

3.1420 $\int \frac{1}{\sqrt{-1+bx}\sqrt{3+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-1+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-1}\,\sin^{-1}\left(\frac{1}{2}\sqrt{1-bx}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/2])/(b*Sqrt[1 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + b*x]*Sqrt[3 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[-1 + b*x]])/b
```

fricas [A] time = 0.97, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+3}\sqrt{bx-1}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(b*x + 3)*sqrt(b*x - 1) - 1)/b
```

giac [A] time = 1.03, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+3}-\sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x - 1))/b
```

maple [B] time = 0.01, size = 64, normalized size = 3.37

$$\frac{\sqrt{(bx-1)(bx+3)}\,\ln\left(\frac{b^2x+b}{\sqrt{b^2}}+\sqrt{b^2x^2+2bx-3}\right)}{\sqrt{bx-1}\,\sqrt{bx+3}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x)
[Out] ((b*x-1)*(b*x+3))^(1/2)/(b*x-1)^(1/2)/(b*x+3)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2
)+(b^2*x^2+2*b*x-3)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.32, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+2\,bx-3}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x - 3)*b + 2*b)/b
```

mupad [B] time = 0.32, size = 44, normalized size = 2.32

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{3}-\sqrt{b\,x+3}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 3)^(1/2)),x)
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((3^(1/2) - (b*x + 3)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-1}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 3)), x)
```

3.1421 $\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}} dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\sqrt{2+bx}} dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

IntegrateAlgebraic [B] time = 0.05, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

fricas [B] time = 1.12, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx + \sqrt{bx+2}\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

giac [B] time = 1.08, size = 23, normalized size = 2.09

$$-\frac{2 \log\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

maple [B] time = 0.01, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-2)(bx+2)}\ \ln\left(\frac{b^2 x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{bx-2}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)*ln(b^2*x/(b^2)^(1/2)+(b
^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.33, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

mupad [B] time = 0.30, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

sympy [C] time = 4.20, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{4 e^{2 i \pi}}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} - \frac{1}{2}, - \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ - \frac{1}{4}, \frac{1}{4} & - \frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{4}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

3.1422 $\int \frac{1}{\sqrt{-3+bx}\sqrt{1+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{63, 215\}$

$$\frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{bx-3}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/2])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\sqrt{1+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{4+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{1}{2}\sqrt{-3+bx}\right)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 39, normalized size = 2.05

$$\frac{2\sqrt{bx-3}\,\sin^{-1}\left(\frac{1}{2}\sqrt{3-bx}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]
```

```
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/2])/(b*Sqrt[3 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+1}}{\sqrt{bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-3 + b*x]*Sqrt[1 + b*x]),x]

[Out] (2*ArcTanh[Sqrt[1 + b*x]/Sqrt[-3 + b*x]])/b

fricas [A] time = 0.73, size = 27, normalized size = 1.42

$$-\frac{\log\left(-bx+\sqrt{bx+1}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="fricas")

[Out] -log(-b*x + sqrt(b*x + 1)*sqrt(b*x - 3) + 1)/b

giac [A] time = 1.00, size = 23, normalized size = 1.21

$$-\frac{2\log\left(\sqrt{bx+1}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="giac")

[Out] -2*log(sqrt(b*x + 1) - sqrt(b*x - 3))/b

maple [B] time = 0.01, size = 66, normalized size = 3.47

$$\frac{\sqrt{(bx-3)(bx+1)}\,\ln\left(\frac{b^2x-b}{\sqrt{b^2}}+\sqrt{b^2x^2-2bx-3}\right)}{\sqrt{bx-3}\,\sqrt{bx+1}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x)

[Out] ((b*x-3)*(b*x+1))^(1/2)/(b*x-3)^(1/2)/(b*x+1)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2
)+(b^2*x^2-2*b*x-3)^(1/2))/(b^2)^(1/2)

maxima [B] time = 1.38, size = 33, normalized size = 1.74

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-2\,bx-3}\,b-2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x-3)^(1/2)/(b*x+1)^(1/2),x, algorithm="maxima")

[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x - 3)*b - 2*b)/b

mupad [B] time = 0.29, size = 46, normalized size = 2.42

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{b\,x+1}-1\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x + 1)^(1/2)*(b*x - 3)^(1/2)),x)

[Out] (4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1
/2))))/(-b^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 1)), x)
```

3.1423 $\int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2+bx}\sqrt{3+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{2+bx}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\sqrt{bx+2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[2 + b*x]])/b
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 + b*x]*Sqrt[3 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[3 + b*x]/Sqrt[2 + b*x]])/b
```

fricas [B] time = 1.13, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{bx + 3}\,\sqrt{bx + 2} - 5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 3)*sqrt(b*x + 2) - 5)/b
```

giac [A] time = 1.01, size = 23, normalized size = 1.53

$$-\frac{2\,\log\left(\sqrt{bx + 3} - \sqrt{bx + 2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 66, normalized size = 4.40

$$\frac{\sqrt{(bx + 2)\,(bx + 3)}\;\ln\left(\frac{b^2x + \frac{5}{2}b}{\sqrt{b^2}} + \sqrt{b^2x^2 + 5bx + 6}\right)}{\sqrt{bx + 2}\;\sqrt{bx + 3}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x)
[Out] ((b*x+2)*(b*x+3))^(1/2)/(b*x+2)^(1/2)/(b*x+3)^(1/2)*ln((5/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+5*b*x+6)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.38, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 + 5\,bx + 6}\,b + 5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)^(1/2)/(b*x+3)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 5*b*x + 6)*b + 5*b)/b
```

mupad [B] time = 0.29, size = 47, normalized size = 3.13

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{b\,x+3}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x + 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (b*x + 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2
)^(1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+2}\,\sqrt{bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2)**(1/2)/(b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 2)*sqrt(b*x + 3)), x)
```

3.1424 $\int \frac{1}{2+bx} \, dx$

Optimal. Leaf size=10

$$\frac{\log(bx+2)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + b*x)^(-1),x]
[Out] Log[2 + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+bx} \, dx = \frac{\log(2+bx)}{b}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + b*x)^(-1),x]
[Out] Log[2 + b*x]/b
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2+bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 + b*x)^(-1),x]
[Out] IntegrateAlgebraic[(2 + b*x)^(-1), x]
```

fricas [A] time = 1.00, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="fricas")
[Out] log(b*x + 2)/b
```

giac [A] time = 0.95, size = 11, normalized size = 1.10

$$\frac{\log(|bx+2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="giac")
[Out] log(abs(b*x + 2))/b
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$\frac{\ln(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+2),x)
[Out] ln(b*x+2)/b
```

maxima [A] time = 1.32, size = 10, normalized size = 1.00

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x, algorithm="maxima")
[Out] log(b*x + 2)/b
```

mupad [B] time = 0.26, size = 10, normalized size = 1.00

$$\frac{\ln(b\,x+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x + 2),x)
[Out] log(b*x + 2)/b
```

sympy [A] time = 0.06, size = 7, normalized size = 0.70

$$\frac{\log(bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+2),x)
[Out] log(b*x + 2)/b
```

3.1425 $\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=15

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\sqrt{1+bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{2\sinh^{-1}\left(\sqrt{bx+1}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[Sqrt[1 + b*x]])/b
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.67

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx+1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[1 + b*x]])/b
```

fricas [B] time = 0.95, size = 28, normalized size = 1.87

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx+1}\,-3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x + 1) - 3)/b
```

giac [A] time = 0.92, size = 23, normalized size = 1.53

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x + 1))/b
```

maple [B] time = 0.01, size = 66, normalized size = 4.40

$$\frac{\sqrt{(bx+1)\,(bx+2)}\,\ln\left(\frac{b^2x+\frac{3}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+3bx+2}\right)}{\sqrt{bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((b*x+1)*(b*x+2))^(1/2)/(b*x+1)^(1/2)/(b*x+2)^(1/2)*ln((3/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+3*b*x+2)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.39, size = 33, normalized size = 2.20

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+3\,bx+2}\,b+3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 3*b*x + 2)*b + 3*b)/b
```

mupad [B] time = 0.29, size = 43, normalized size = 2.87

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{b\,x+1}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((b*x + 1)^(1/2) - 1)*(-b^2)^(1/2)
)))/(-b^2)^(1/2)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x + 1)*sqrt(b*x + 2)), x)
```

3.1426 $\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=19

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcSinh[Sqrt[b*x]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx}\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.89

$$\frac{2\sqrt{x}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
[Out] (2*Sqrt[x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[b*x])
```

IntegrateAlgebraic [A] time = 0.04, size = 25, normalized size = 1.32

$$-\frac{2\log\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Log[-Sqrt[b*x] + Sqrt[2 + b*x]])/b
```

fricas [A] time = 0.98, size = 25, normalized size = 1.32

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x) - 1)/b
```

giac [A] time = 1.07, size = 21, normalized size = 1.11

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x))/b
```

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$\frac{\sqrt{(bx+2)\,bx}\;\ln\left(\frac{b^{2}x+b}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}+2bx}\right)}{\sqrt{bx}\;\sqrt{bx+2}\;\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] (x*b*(b*x+2))^(1/2)/(b*x)^(1/2)/(b*x+2)^(1/2)*ln((b^2*x+b)/(b^2)^(1/2)+(b^2
*x^2+2*b*x)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.38, size = 32, normalized size = 1.68

$$\frac{\log\left(2\,b^{2}x+2\,\sqrt{b^{2}x^{2}+2\,bx}\,b+2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + 2*b*x)*b + 2*b)/b
```

mupad [B] time = 0.28, size = 37, normalized size = 1.95

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{b\,x}\,\sqrt{-b^{2}}}\right)}{\sqrt{-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

[Out] (4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((b*x)^(1/2)*(-b^2)^(1/2))))/(-b^2)
^(1/2)

sympy [A] time = 1.35, size = 20, normalized size = 1.05

$$\frac{2\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x)**(1/2)/(b*x+2)**(1/2),x)

[Out] 2*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b

3.1427 $\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-1 + b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-1+bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-1}\,\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b\sqrt{1-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[-1 + b*x]*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/(b*Sqrt[1 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-1 + b*x]])/b
```

fricas [A] time = 1.08, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{bx+2}\,\sqrt{bx-1} - 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 1) - 1)/b
```

giac [A] time = 1.02, size = 23, normalized size = 1.10

$$-\frac{2\,\log\left(\sqrt{bx+2} - \sqrt{bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 1))/b
```

maple [B] time = 0.01, size = 65, normalized size = 3.10

$$\frac{\sqrt{(bx-1)\,(bx+2)}\,\ln\left(\frac{b^2x+\frac{1}{2}b}{\sqrt{b^2}} + \sqrt{b^2x^2+bx-2}\right)}{\sqrt{bx-1}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-1)*(b*x+2))^(1/2)/(b*x-1)^(1/2)/(b*x+2)^(1/2)*ln((1/2*b+b^2*x)/(b^2)^
(1/2)+(b^2*x^2+b*x-2)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.68, size = 30, normalized size = 1.43

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2+bx-2}\,b + b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 2)*b + b)/b
```

mupad [B] time = 0.29, size = 44, normalized size = 2.10

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 1)*sqrt(b*x + 2)), x)
```

3.1428 $\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/2]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2+bx}\,\sqrt{2+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-2}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-2 + b*x]/Sqrt[2 + b*x]])/b
```

IntegrateAlgebraic [B] time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-2 + b*x]])/b
```

fricas [B] time = 0.87, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+2}\,\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 2)*sqrt(b*x - 2))/b
```

giac [B] time = 0.96, size = 23, normalized size = 2.09

$$-\frac{2 \log\left(\sqrt{bx+2}-\sqrt{bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 2))/b
```

maple [B] time = 0.00, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-2)(bx+2)}\, \ln\left(\frac{b^2 x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{bx-2}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((b*x-2)*(b*x+2))^(1/2)/(b*x-2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*ln(1/(b^2)^
(1/2)*b^2*x+(b^2*x^2-4)^(1/2))
```

maxima [B] time = 1.36, size = 26, normalized size = 2.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

mupad [B] time = 0.00, size = 50, normalized size = 4.55

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{b\,x-2}+\sqrt{2}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 2)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2)*1i - (b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

sympy [C] time = 4.28, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| \frac{4e^{2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, 0, 0, 0 \end{matrix} \middle| \frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] meijerg((((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4*e
xp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg((((-1/2, -1/4, 0,
1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4/(b**2*x**2))/(4*pi**(3
/2)*b)
```

3.1429 $\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*ArcSinh[Sqrt[-3 + b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+bx}\,\sqrt{2+bx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{-3+bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.95

$$\frac{2\sqrt{bx-3}\,\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b\sqrt{3-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[-3 + b*x]*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/(b*Sqrt[3 - b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 25, normalized size = 1.19

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 + b*x]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTanh[Sqrt[2 + b*x]/Sqrt[-3 + b*x]])/b
```

fricas [A] time = 1.05, size = 28, normalized size = 1.33

$$-\frac{\log\left(-2\,bx+2\,\sqrt{bx+2}\,\sqrt{bx-3}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(b*x + 2)*sqrt(b*x - 3) + 1)/b
```

giac [A] time = 0.96, size = 23, normalized size = 1.10

$$-\frac{2\,\log\left(\sqrt{bx+2}-\sqrt{bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -2*log(sqrt(b*x + 2) - sqrt(b*x - 3))/b
```

maple [B] time = 0.01, size = 66, normalized size = 3.14

$$\frac{\sqrt{(bx-3)\,(bx+2)}\;\ln\left(\frac{b^{2}x-\frac{1}{2}b}{\sqrt{b^{2}}}+\sqrt{b^{2}x^{2}-bx-6}\right)}{\sqrt{bx-3}\,\sqrt{bx+2}\,\sqrt{b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((b*x-3)*(b*x+2))^(1/2)/(b*x-3)^(1/2)/(b*x+2)^(1/2)*ln((-1/2*b+b^2*x)/(b^2)
^(1/2)+(b^2*x^2-b*x-6)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.24, size = 33, normalized size = 1.57

$$\frac{\log\left(2\,b^{2}x+2\,\sqrt{b^{2}x^{2}-bx-6}\,b-b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 6)*b - b)/b
```

mupad [B] time = 0.28, size = 50, normalized size = 2.38

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{b\,x-3}+\sqrt{3}\,1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{-b^{2}}}\right)}{\sqrt{-b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(b*x - 3)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2)*1i - (b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(-
b^2)^(1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*x - 3)*sqrt(b*x + 2)), x)
```

3.1430 $\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(1 - 2*b*x)/5]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{3-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{6+bx-b^2x^2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{25b^2}}}\,dx, x, b-2b^2x\right)}{5b^2}\\
&= -\frac{\sin^{-1}\left(\frac{1}{5}(1-2bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcSin[Sqrt[3 - b*x]/Sqrt[5]])/b
```

IntegrateAlgebraic [A] time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{3-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[3 - b*x]/Sqrt[2 + b*x]])/b
```

fricas [B] time = 0.83, size = 44, normalized size = 2.75

$$-\frac{\arctan\left(\frac{(2\,bx-1)\sqrt{bx+2}\,\sqrt{-bx+3}}{2\left(b^2x^2-bx-6\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x - 1)*sqrt(b*x + 2)*sqrt(-b*x + 3)/(b^2*x^2 - b*x - 6))/b
```

giac [A] time = 1.05, size = 18, normalized size = 1.12

$$\frac{2\arcsin\left(\frac{1}{5}\sqrt{5}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(1/5*sqrt(5)*sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 65, normalized size = 4.06

$$\frac{\sqrt{(-bx+3)(bx+2)}\,\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx+6}}\right)}{\sqrt{-bx+3}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+3)*(b*x+2))^(1/2)/(-b*x+3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x+6)^(1/2))
```

maxima [A] time = 2.99, size = 21, normalized size = 1.31

$$-\frac{\arcsin\left(-\frac{2\,b^2x-b}{5\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/5*(2*b^2*x - b)/b)/b
```

mupad [B] time = 0.08, size = 44, normalized size = 2.75

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+3)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 3)*sqrt(b*x + 2)), x)
```

3.1431 $\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {41, 216}

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{4-b^2x^2}}\,dx\\ &= \frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\sin^{-1}\left(\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[(b*x)/2]/b
```

IntegrateAlgebraic [B] time = 0.05, size = 26, normalized size = 2.36

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[2 - b*x]/Sqrt[2 + b*x]])/b
```

fricas [B] time = 0.75, size = 31, normalized size = 2.82

$$-\frac{2 \arctan\left(\frac{\sqrt{bx+2}\,\sqrt{-bx+2}-2}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan((sqrt(b*x + 2)*sqrt(-b*x + 2) - 2)/(b*x))/b
```

giac [A] time = 0.91, size = 15, normalized size = 1.36

$$\frac{2 \arcsin\left(\frac{1}{2}\,\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 56, normalized size = 5.09

$$\frac{\sqrt{(-bx+2)\,(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\,x}{\sqrt{-b^2x^2+4}}\right)}{\sqrt{-bx+2}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((-b*x+2)*(b*x+2))^(1/2)/(-b*x+2)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*x/(-b^2*x^2+4)^(1/2))
```

maxima [A] time = 3.03, size = 9, normalized size = 0.82

$$\frac{\arcsin\left(\frac{1}{2}\,bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] arcsin(1/2*b*x)/b
```

mupad [B] time = 0.08, size = 44, normalized size = 4.00

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)
^(1/2))))/(b^2)^(1/2)
```

sympy [C] time = 4.44, size = 76, normalized size = 6.91

$$-\frac{iG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{4}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}+\frac{G_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{4e^{-2i\pi}}{b^2x^2}\right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -I*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()),
4/(b**2*x**2))/(4*pi**(3/2)*b) + meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

3.1432 $\int \frac{1}{\sqrt{1-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$-\frac{\sin^{-1}\left(\frac{1}{3}(-2bx-1)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] -(ArcSin[(-1 - 2*b*x)/3]/b)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{1-bx}\,\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{2-bx-b^2x^2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9b^2}}}\,dx, x, -b-2b^2x\right)}{3b^2}\\
&= -\frac{\sin^{-1}\left(\frac{1}{3}(-1-2bx)\right)}{b}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.38

$$-\frac{2\sin^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcSin[Sqrt[1 - b*x]/Sqrt[3]])/b
```

IntegrateAlgebraic [A] time = 0.05, size = 26, normalized size = 1.62

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{1-bx}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - b*x]*Sqrt[2 + b*x]),x]
[Out] (-2*ArcTan[Sqrt[1 - b*x]/Sqrt[2 + b*x]])/b
```

fricas [B] time = 1.07, size = 43, normalized size = 2.69

$$-\frac{\arctan\left(\frac{(2\,bx+1)\sqrt{bx+2}\,\sqrt{-bx+1}}{2\,(b^2x^2+bx-2)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -arctan(1/2*(2*b*x + 1)*sqrt(b*x + 2)*sqrt(-b*x + 1)/(b^2*x^2 + b*x - 2))/b
```

giac [A] time = 1.05, size = 18, normalized size = 1.12

$$\frac{2\,\arcsin\left(\frac{1}{3}\,\sqrt{3}\,\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/3*sqrt(3)*sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 66, normalized size = 4.12

$$\frac{\sqrt{(-bx+1)\,(bx+2)}\;\arctan\left(\frac{\sqrt{b^2}\,\left(x+\frac{1}{2b}\right)}{\sqrt{-b^2x^2-bx+2}}\right)}{\sqrt{-bx+1}\,\sqrt{bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x)
[Out] ((-b*x+1)*(b*x+2))^(1/2)/(-b*x+1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+1/2/b)/(-b^2*x^2-b*x+2)^(1/2))
```

maxima [A] time = 3.01, size = 19, normalized size = 1.19

$$-\frac{\arcsin\left(-\frac{2\,b^2x+b}{3\,b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-1/3*(2*b^2*x + b)/b)/b
```

mupad [B] time = 0.32, size = 40, normalized size = 2.50

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\left(\sqrt{1-b\,x}-1\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(b^2)^(1/2)
)))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(b*x + 2)), x)
```

3.1433 $\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx$

Optimal. Leaf size=10

$$\frac{\sin^{-1}(bx+1)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{53, 619, 216\}$

$$\frac{\sin^{-1}(bx+1)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[1 + b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-bx}\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{-2bx-b^2x^2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4b^2}}}\,dx, x, -2b-2b^2x\right)}{2b^2}\\ &= \frac{\sin^{-1}(1+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 51, normalized size = 5.10

$$\frac{2\sqrt{x}\sqrt{bx+2}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\sqrt{-bx(bx+2)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[x]*Sqrt[2 + b*x]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[
-(b*x*(2 + b*x))])
```

IntegrateAlgebraic [B] time = 0.04, size = 24, normalized size = 2.40

$$\frac{2\tan^{-1}\left(\frac{\sqrt{bx+2}}{\sqrt{-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-(b*x)]*Sqrt[2 + b*x]),x]
[Out] (2*ArcTan[Sqrt[2 + b*x]/Sqrt[-(b*x)]])/b
```

fricas [B] time = 1.12, size = 26, normalized size = 2.60

$$-\frac{2\arctan\left(\frac{\sqrt{bx+2}\sqrt{-bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(b*x + 2)*sqrt(-b*x)/(b*x))/b
```

giac [A] time = 0.93, size = 18, normalized size = 1.80

$$\frac{2\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(2)*sqrt(b*x + 2))/b
```

maple [B] time = 0.00, size = 58, normalized size = 5.80

$$\frac{\sqrt{-(bx+2)bx}\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{1}{b}\right)}{\sqrt{-b^2x^2-2bx}}\right)}{\sqrt{-bx}\sqrt{bx+2}\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x)
[Out] (-(b*x+2)*b*x)^(1/2)/(-b*x)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b^2)^(1
/2)*(x+1/b)/(-b^2*x^2-2*b*x)^(1/2))
```

maxima [A] time = 3.14, size = 18, normalized size = 1.80

$$-\frac{\arcsin\left(-\frac{b^2x+b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] -arcsin(-(b^2*x + b)/b)/b
```

mupad [B] time = 0.29, size = 34, normalized size = 3.40

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{b\,x+2}\right)}{\sqrt{-b\,x}\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (b*x + 2)^(1/2)))/((-b*x)^(1/2)*(b^2)^(1/2))))/(b^2)
^(1/2)
```

sympy [C] time = 1.28, size = 24, normalized size = 2.40

$$-\frac{2i\,\mathrm{asinh}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{x}}{2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] -2*I*asinh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b
```

3.1434 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\sin^{-1}(2bx+3)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {53, 619, 216}

$$\frac{\sin^{-1}(2bx+3)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ArcSin[3 + 2*b*x]/b
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2+bx}}\,dx &= \int \frac{1}{\sqrt{-2-3bx-b^2x^2}}\,dx\\ &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{b^2}}}\,dx, x, -3b-2b^2x\right)}{b^2}\\ &= \frac{\sin^{-1}(3+2bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 49, normalized size = 4.45

$$\frac{2\sqrt{bx+1}\,\sqrt{bx+2}\,\sinh^{-1}\left(\sqrt{bx+1}\right)}{b\sqrt{-((bx+1)(bx+2))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (2*Sqrt[1 + b*x]*Sqrt[2 + b*x]*ArcSinh[Sqrt[1 + b*x]])/(b*Sqrt[-((1 + b*x)*
(2 + b*x))])
```

IntegrateAlgebraic [B] time = 0.05, size = 26, normalized size = 2.36

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-1 - b*x]/Sqrt[2 + b*x]])/b
```

fricas [B] time = 1.09, size = 44, normalized size = 4.00

$$-\frac{\arctan\left(\frac{(2\,bx+3)\sqrt{bx+2}\,\sqrt{-bx-1}}{2\left(b^2x^2+3\,bx+2\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 3)*sqrt(b*x + 2)*sqrt(-b*x - 1)/(b^2*x^2 + 3*b*x + 2))
/b
```

giac [A] time = 1.17, size = 13, normalized size = 1.18

$$\frac{2\arcsin\left(\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arcsin(sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 66, normalized size = 6.00

$$\frac{\sqrt{(-bx-1)(bx+2)}\;\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{3}{2b}\right)}{\sqrt{-b^2x^2-3bx-2}}\right)}{\sqrt{-bx-1}\;\sqrt{bx+2}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(b*x+2))^(1/2)/(-b*x-1)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+3/2/b)/(-b^2*x^2-3*b*x-2)^(1/2))
```

maxima [A] time = 3.01, size = 21, normalized size = 1.91

$$-\frac{\arcsin\left(-\frac{2\,b^2x+3\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-(2*b^2*x + 3*b)/b)/b
```

mupad [B] time = 0.30, size = 41, normalized size = 3.73

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(b*x + 2)^(1/2)),x)
```

```
[Out] (4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)^(1/
2))))/(b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(b*x + 2)), x)
```

3.1435 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=29

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

Rubi [A] time = 0.00, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {23, 31}

$$\frac{\sqrt{bx+2}\,\log(bx+2)}{b\sqrt{-bx-2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (Sqrt[2 + b*x]*Log[2 + b*x])/(b*Sqrt[-2 - b*x])
```

Rule 23

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> D
ist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && EqQ[b*c - a*d, 0] &&  !(IntegerQ[m] || IntegerQ[n] ||
GtQ[b/d, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx &= \frac{\sqrt{2+bx}\,\int \frac{1}{2+bx}\,dx}{\sqrt{-2-bx}}\\ &= \frac{\sqrt{2+bx}\,\log(2+bx)}{b\sqrt{-2-bx}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.97

$$\frac{(bx+2)\log(bx+2)}{b\sqrt{-(bx+2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] ((2 + b*x)*Log[2 + b*x])/(b*Sqrt[-(2 + b*x)^2])
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2+bx}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]),x]
[Out] Defer[IntegrateAlgebraic][1/(Sqrt[-2 - b*x]*Sqrt[2 + b*x]), x]
```

fricas [A] time = 0.99, size = 1, normalized size = 0.03

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
[Out] 0
```

giac [C] time = 0.97, size = 12, normalized size = 0.41

$$-\frac{i \log\left(|bx+2|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
[Out] -I*log(abs(b*x + 2))/b
```

maple [A] time = 0.00, size = 26, normalized size = 0.90

$$\frac{\sqrt{bx+2}\,\ln\left(bx+2\right)}{\sqrt{-bx-2}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x)
[Out] ln(b*x+2)*(b*x+2)^(1/2)/b/(-b*x-2)^(1/2)
```

maxima [A] time = 1.35, size = 16, normalized size = 0.55

$$\sqrt{-\frac{1}{b^2}}\log\left(x+\frac{2}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
[Out] sqrt(-1/b^2)*log(x + 2/b)
```

mupad [B] time = 0.07, size = 47, normalized size = 1.62

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-2}+\sqrt{2}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x + 2)^(1/2)*(- b*x - 2)^(1/2)),x)
[Out] -(4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*
(b^2)^(1/2))))/(b^2)^(1/2)
```

sympy [C] time = 1.98, size = 53, normalized size = 1.83

$$\begin{cases} -\frac{i\log\left(x+\frac{2}{b}\right)}{b} & \text{for } \left|x+\frac{2}{b}\right|<1 \\ \frac{i\log\left(\frac{1}{x+\frac{2}{b}}\right)}{b} & \text{for } \frac{1}{\left|x+\frac{2}{b}\right|}<1 \\ \frac{iG_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|x+\frac{2}{b}\right)}{b}-\frac{iG_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle|x+\frac{2}{b}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-I*log(x + 2/b)/b, Abs(x + 2/b) < 1), (I*log(1/(x + 2/b))/b, 1/A
bs(x + 2/b) < 1), (I*meijerg(((), (1, 1)), ((0, 0), ()), x + 2/b)/b - I*mei
jerg(((1, 1), ()), ((), (0, 0)), x + 2/b)/b, True))
```

3.1436 $\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2+bx}}\,dx$

Optimal. Leaf size=26

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2+bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{2+bx}}\right)}{b}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 53, normalized size = 2.04

$$-\frac{2\sqrt{-bx-3}\,\sqrt{-bx-2}\,\sin^{-1}\left(\sqrt{bx+3}\right)}{b\sqrt{bx+2}\,\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*Sqrt[-3 - b*x]*Sqrt[-2 - b*x]*ArcSin[Sqrt[3 + b*x]])/(b*Sqrt[2 + b*x]*S
qrt[3 + b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 26, normalized size = 1.00

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{bx+2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - b*x]*Sqrt[2 + b*x]),x]
```

```
[Out] (-2*ArcTan[Sqrt[-3 - b*x]/Sqrt[2 + b*x]])/b
```

fricas [A] time = 0.82, size = 44, normalized size = 1.69

$$-\frac{\arctan\left(\frac{(2\,bx+5)\sqrt{bx+2}\,\sqrt{-bx-3}}{2\left(b^2x^2+5\,bx+6\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*(2*b*x + 5)*sqrt(b*x + 2)*sqrt(-b*x - 3)/(b^2*x^2 + 5*b*x + 6))
/b
```

giac [C] time = 1.07, size = 23, normalized size = 0.88

$$\frac{2i\,\log\left(\sqrt{bx+3}-\sqrt{bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*I*log(sqrt(b*x + 3) - sqrt(b*x + 2))/b
```

maple [B] time = 0.01, size = 66, normalized size = 2.54

$$\frac{\sqrt{(-bx-3)(bx+2)}\ \arctan\left(\frac{\sqrt{b^2}\left(x+\frac{5}{2b}\right)}{\sqrt{-b^2x^2-5bx-6}}\right)}{\sqrt{-bx-3}\ \sqrt{bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-3)*(b*x+2))^(1/2)/(-b*x-3)^(1/2)/(b*x+2)^(1/2)/(b^2)^(1/2)*arctan((b
^2)^(1/2)*(x+5/2/b)/(-b^2*x^2-5*b*x-6)^(1/2))
```

maxima [A] time = 3.01, size = 21, normalized size = 0.81

$$-\frac{\arcsin\left(-\frac{2\,b^2x+5\,b}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(b*x+2)^(1/2),x, algorithm="maxima")
```

[Out] -arcsin(-(2*b^2*x + 5*b)/b)/b

mupad [B] time = 0.30, size = 47, normalized size = 1.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{b\,x+2}\right)\,\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x + 2)^(1/2)*(- b*x - 3)^(1/2)),x)

[Out] -(4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (b*x + 2)^(1/2))*(b^2)^(1/2))))/(b^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-3}\,\sqrt{bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b*x-3)**(1/2)/(b*x+2)**(1/2),x)

[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(b*x + 2)), x)

3.1437 $\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2-bx}\sqrt{3-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{2-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sinh^{-1}\left(\sqrt{2-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[2 - b*x]])/b
```

IntegrateAlgebraic [B] time = 0.06, size = 59, normalized size = 3.69

$$\frac{\log\left(\frac{\sqrt{3-bx}}{\sqrt{2-bx}}-1\right)}{b}-\frac{\log\left(\frac{b\sqrt{3-bx}}{\sqrt{2-bx}}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[2 - b*x]*Sqrt[3 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[3 - b*x]/Sqrt[2 - b*x]]/b - Log[b + (b*Sqrt[3 - b*x])/Sqrt[2
- b*x]]/b
```

fricas [B] time = 1.07, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+3}\,\sqrt{-bx+2}\,+5\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 3)*sqrt(-b*x + 2) + 5)/b
```

giac [A] time = 1.04, size = 25, normalized size = 1.56

$$\frac{2\,\log\left(\sqrt{-bx+3}-\sqrt{-bx+2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 3) - sqrt(-b*x + 2))/b
```

maple [B] time = 0.01, size = 70, normalized size = 4.38

$$\frac{\sqrt{(-bx+2)\,(-bx+3)}\;\ln\left(\frac{b^2x-\frac{5}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2-5bx+6}\right)}{\sqrt{-bx+2}\;\sqrt{-bx+3}\;\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x)
```

```
[Out] ((-b*x+2)*(-b*x+3))^(1/2)/(-b*x+2)^(1/2)/(-b*x+3)^(1/2)*ln((-5/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-5*b*x+6)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.36, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-5\,bx+6}\,b-5\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)^(1/2)/(-b*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 5*b*x + 6)*b - 5*b)/b
```

mupad [B] time = 0.31, size = 49, normalized size = 3.06

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{3}-\sqrt{3-b\,x}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(3 - b*x)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2) - (3 - b*x)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)
^(1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+2}\,\sqrt{-bx+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2)**(1/2)/(-b*x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 2)*sqrt(-b*x + 3)), x)
```

3.1438 $\int \frac{1}{2-bx} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(2-bx)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {31}

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - b*x)^(-1),x]
[Out] -(Log[2 - b*x]/b)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2-bx} \, dx = -\frac{\log(2-bx)}{b}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{\log(2-bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - b*x)^(-1),x]
[Out] -(Log[2 - b*x]/b)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{2-bx} \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(2 - b*x)^(-1),x]
[Out] IntegrateAlgebraic[(2 - b*x)^(-1), x]
```

fricas [A] time = 0.67, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="fricas")
[Out] -log(b*x - 2)/b
```

giac [**A**] time = 1.10, size = 12, normalized size = 1.00

$$-\frac{\log(|bx-2|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="giac")
[Out] -log(abs(b*x - 2))/b
```

maple [**A**] time = 0.00, size = 13, normalized size = 1.08

$$-\frac{\ln(-bx+2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+2),x)
[Out] -ln(-b*x+2)/b
```

maxima [**A**] time = 1.42, size = 11, normalized size = 0.92

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x, algorithm="maxima")
[Out] -log(b*x - 2)/b
```

mupad [**B**] time = 0.03, size = 11, normalized size = 0.92

$$-\frac{\ln(b\,x-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(b*x - 2),x)
[Out] -log(b*x - 2)/b
```

sympy [**A**] time = 0.07, size = 8, normalized size = 0.67

$$-\frac{\log(bx-2)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+2),x)
[Out] -log(b*x - 2)/b
```

3.1439 $\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=16

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, \sqrt{1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{2\sinh^{-1}\left(\sqrt{1-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[1 - b*x]])/b
```

IntegrateAlgebraic [B] time = 0.06, size = 59, normalized size = 3.69

$$\frac{\log\left(\frac{\sqrt{2-bx}}{\sqrt{1-bx}}-1\right)}{b}-\frac{\log\left(\frac{b\sqrt{2-bx}}{\sqrt{1-bx}}+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] Log[-1 + Sqrt[2 - b*x]/Sqrt[1 - b*x]]/b - Log[b + (b*Sqrt[2 - b*x])/Sqrt[1
- b*x]]/b
```

fricas [B] time = 0.91, size = 30, normalized size = 1.88

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{-bx + 2}\,\sqrt{-bx + 1} + 3\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x + 1) + 3)/b
```

giac [A] time = 0.85, size = 25, normalized size = 1.56

$$\frac{2\,\log\left(\sqrt{-bx + 2} - \sqrt{-bx + 1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x + 1))/b
```

maple [B] time = 0.01, size = 70, normalized size = 4.38

$$\frac{\sqrt{(-bx + 1)\,(-bx + 2)}\ \ln\left(\frac{b^2x - \frac{3}{2}b}{\sqrt{b^2}} + \sqrt{b^2x^2 - 3bx + 2}\right)}{\sqrt{-bx + 1}\,\sqrt{-bx + 2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x+1)*(-b*x+2))^(1/2)/(-b*x+1)^(1/2)/(-b*x+2)^(1/2)*ln((-3/2*b+b^2*x)/(
b^2)^(1/2)+(b^2*x^2-3*b*x+2)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.33, size = 33, normalized size = 2.06

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 3\,bx + 2}\,b - 3\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 3*b*x + 2)*b - 3*b)/b
```

mupad [B] time = 0.31, size = 45, normalized size = 2.81

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\left(\sqrt{1-b\,x}-1\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/(((1 - b*x)^(1/2) - 1)*(-b^2)^(1/2
))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx+1}\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x + 1)*sqrt(-b*x + 2)), x)
```

3.1440 $\int \frac{1}{\sqrt{-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=20

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcSinh[Sqrt[-(b*x)]/Sqrt[2]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{2+x^2}}\,dx, x, \sqrt{-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx}}{\sqrt{2}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.85

$$\frac{2\sqrt{x}\,\sin^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{2}}\right)}{\sqrt{b}\,\sqrt{-bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
```

```
[Out] (2*Sqrt[x]*ArcSin[(Sqrt[b]*Sqrt[x])/Sqrt[2]])/(Sqrt[b]*Sqrt[-(b*x)])
```

IntegrateAlgebraic [A] time = 0.04, size = 27, normalized size = 1.35

$$\frac{2\log\left(\sqrt{2-bx}-\sqrt{-bx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-(b*x)]*Sqrt[2 - b*x]),x]
[Out] (2*Log[-Sqrt[-(b*x)] + Sqrt[2 - b*x]])/b
```

fricas [A] time = 0.70, size = 27, normalized size = 1.35

$$-\frac{\log\left(-bx + \sqrt{-bx+2}\,\sqrt{-bx} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x) + 1)/b
```

giac [A] time = 1.03, size = 23, normalized size = 1.15

$$\frac{2\,\log\left(\sqrt{-bx+2} - \sqrt{-bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x))/b
```

maple [B] time = 0.01, size = 64, normalized size = 3.20

$$\frac{\sqrt{-\left(-bx+2\right)bx}\,\ln\left(\frac{b^2x-b}{\sqrt{b^2}} + \sqrt{b^2x^2-2bx}\right)}{\sqrt{-bx}\,\sqrt{-bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x)
[Out] (-x*b*(-b*x+2))^(1/2)/(-b*x)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-b)/(b^2)^(1/2)+
(b^2*x^2-2*b*x)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.36, size = 32, normalized size = 1.60

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2 - 2\,bx}\,b - 2\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 2*b*x)*b - 2*b)/b
```

mupad [B] time = 0.28, size = 39, normalized size = 1.95

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{2}-\sqrt{2-b\,x}\right)}{\sqrt{-b\,x}\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*(2^(1/2) - (2 - b*x)^(1/2)))/((-b*x)^(1/2)*(-b^2)^(1/2))))/(-b^
2)^(1/2)
```

sympy [A] time = 1.34, size = 53, normalized size = 2.65

$$\begin{cases} -\frac{2\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b} & \text{for}\ \frac{|bx|}{2} > 1 \\ -\frac{2i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{x}}{2}\right)}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Piecewise((-2*acosh(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, Abs(b*x)/2 > 1), (-2*I*as
in(sqrt(2)*sqrt(b)*sqrt(x)/2)/b, True))
```

3.1441 $\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-1}}{\sqrt{3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcSinh[Sqrt[-1 - b*x]/Sqrt[3]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-1-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-1-bx}}{\sqrt{3}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-1}\,\sin^{-1}\left(\frac{\sqrt{bx+1}}{\sqrt{3}}\right)}{b\sqrt{bx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*Sqrt[-1 - b*x]*ArcSin[Sqrt[1 + b*x]/Sqrt[3]])/(b*Sqrt[1 + b*x])
```

IntegrateAlgebraic [A] time = 0.06, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-1}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-1 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-1 - b*x]])/b
```

fricas [A] time = 0.77, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx + 2\,\sqrt{-bx+2}\,\sqrt{-bx-1} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 1) + 1)/b
```

giac [A] time = 1.10, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2} - \sqrt{-bx-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 1))/b
```

maple [B] time = 0.01, size = 70, normalized size = 3.18

$$\frac{\sqrt{(-bx-1)\,(-bx+2)}\,\ln\left(\frac{b^2x-\frac{1}{2}b}{\sqrt{b^2}} + \sqrt{b^2x^2-bx-2}\right)}{\sqrt{-bx-1}\,\sqrt{-bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-1)*(-b*x+2))^(1/2)/(-b*x-1)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x-1/2*b)/(b
^2)^(1/2)+(b^2*x^2-b*x-2)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.38, size = 33, normalized size = 1.50

$$\frac{\log\left(2\,b^2x + 2\,\sqrt{b^2x^2-bx-2}\,b - b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x - 2)*b - b)/b
```

mupad [B] time = 0.28, size = 46, normalized size = 2.09

$$-\frac{4\,\mathrm{atan}\left(\frac{b\,\left(\sqrt{-b\,x-1}-\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((- b*x - 1)^(1/2)*(2 - b*x)^(1/2)),x)
```

```
[Out] -(4*atan((b*((- b*x - 1)^(1/2) - 1i))/((2^(1/2) - (2 - b*x)^(1/2))*(-b^2)^(
1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-1}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-1)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 1)*sqrt(-b*x + 2)), x)
```

3.1442 $\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=12

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {52}

$$-\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] -(ArcCosh[-(b*x)/2]/b)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-2-bx}\,\sqrt{2-bx}}\,dx = -\frac{\cosh^{-1}\left(-\frac{bx}{2}\right)}{b}$$

Mathematica [B] time = 0.00, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{-bx-2}}{\sqrt{2-bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[-2 - b*x]/Sqrt[2 - b*x]])/b
```

IntegrateAlgebraic [B] time = 0.05, size = 27, normalized size = 2.25

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-2}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-2 - b*x]*Sqrt[2 - b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-2 - b*x]])/b
```

fricas [B] time = 1.02, size = 28, normalized size = 2.33

$$-\frac{\log\left(-bx+\sqrt{-bx+2}\,\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(-b*x + 2)*sqrt(-b*x - 2))/b
```

giac [B] time = 1.10, size = 25, normalized size = 2.08

$$\frac{2 \log\left(\sqrt{-bx+2}-\sqrt{-bx-2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 2))/b
```

maple [B] time = 0.01, size = 61, normalized size = 5.08

$$\frac{\sqrt{(-bx-2)(-bx+2)}\ \ln\left(\frac{b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-4}\right)}{\sqrt{-bx-2}\ \sqrt{-bx+2}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x)
```

```
[Out] ((-b*x-2)*(-b*x+2))^(1/2)/(-b*x-2)^(1/2)/(-b*x+2)^(1/2)*ln(1/(b^2)^(1/2)*b^
2*x+(b^2*x^2-4)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.27, size = 26, normalized size = 2.17

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-4}\,b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 4)*b)/b
```

mupad [B] time = 0.29, size = 52, normalized size = 4.33

$$\frac{4\,\mathrm{atan}\left(\frac{b\left(-\sqrt{-b\,x-2}+\sqrt{2}\ 1\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 2)^(1/2)),x)
```

```
[Out] (4*atan((b*(2^(1/2)*1i - (- b*x - 2)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

sympy [C] time = 4.63, size = 78, normalized size = 6.50

$$-\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| {\frac{4}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| {\frac{4e^{-2i\pi}}{b^2x^2}} \right)}{4\pi^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-2)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] -meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 4/
(b**2*x**2))/(4*pi**(3/2)*b) - I*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ())
, ((-1/4, 1/4), (-1/2, 0, 0, 0)), 4*exp_polar(-2*I*pi)/(b**2*x**2))/(4*pi**
(3/2)*b)
```

3.1443 $\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2-bx}}\,dx$

Optimal. Leaf size=22

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {63, 215}

$$-\frac{2\sinh^{-1}\left(\frac{\sqrt{-bx-3}}{\sqrt{5}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcSinh[Sqrt[-3 - b*x]/Sqrt[5]])/b
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3-bx}\,\sqrt{2-bx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, \sqrt{-3-bx}\right)}{b}\\ &= -\frac{2\sinh^{-1}\left(\frac{\sqrt{-3-bx}}{\sqrt{5}}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.82

$$-\frac{2\sqrt{-bx-3}\,\sin^{-1}\left(\frac{\sqrt{bx+3}}{\sqrt{5}}\right)}{b\sqrt{bx+3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*Sqrt[-3 - b*x]*ArcSin[Sqrt[3 + b*x]/Sqrt[5]])/(b*Sqrt[3 + b*x])
```

IntegrateAlgebraic [A] time = 0.05, size = 27, normalized size = 1.23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{2-bx}}{\sqrt{-bx-3}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 - b*x]*Sqrt[2 - b*x]),x]
[Out] (-2*ArcTanh[Sqrt[2 - b*x]/Sqrt[-3 - b*x]])/b
```

fricas [A] time = 1.12, size = 30, normalized size = 1.36

$$-\frac{\log\left(-2\,bx+2\,\sqrt{-bx+2}\,\sqrt{-bx-3}\,-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="fricas")
[Out] -log(-2*b*x + 2*sqrt(-b*x + 2)*sqrt(-b*x - 3) - 1)/b
```

giac [A] time = 1.28, size = 25, normalized size = 1.14

$$\frac{2\,\log\left(\sqrt{-bx+2}-\sqrt{-bx-3}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="giac")
[Out] 2*log(sqrt(-b*x + 2) - sqrt(-b*x - 3))/b
```

maple [B] time = 0.01, size = 69, normalized size = 3.14

$$\frac{\sqrt{(-bx-3)\,(-bx+2)}\,\ln\left(\frac{b^2x+\frac{1}{2}b}{\sqrt{b^2}}+\sqrt{b^2x^2+bx-6}\right)}{\sqrt{-bx-3}\,\sqrt{-bx+2}\,\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x)
[Out] ((-b*x-3)*(-b*x+2))^(1/2)/(-b*x-3)^(1/2)/(-b*x+2)^(1/2)*ln((b^2*x+1/2*b)/(b
^2)^(1/2)+(b^2*x^2+b*x-6)^(1/2))/(b^2)^(1/2)
```

maxima [A] time = 1.37, size = 30, normalized size = 1.36

$$\frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx-6}\,b+b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)^(1/2)/(-b*x+2)^(1/2),x, algorithm="maxima")
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x - 6)*b + b)/b
```

mupad [B] time = 0.29, size = 52, normalized size = 2.36

$$\frac{4\,\mathrm{atan}\left(\frac{b\,\left(-\sqrt{-b\,x-3}+\sqrt{3}\,1{}\mathrm{i}\right)}{\left(\sqrt{2}-\sqrt{2-b\,x}\right)\,\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2 - b*x)^(1/2)*(- b*x - 3)^(1/2)),x)
```

```
[Out] (4*atan((b*(3^(1/2)*1i - (- b*x - 3)^(1/2)))/((2^(1/2) - (2 - b*x)^(1/2))*(
-b^2)^(1/2))))/(-b^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx-3}\,\sqrt{-bx+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x-3)**(1/2)/(-b*x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-b*x - 3)*sqrt(-b*x + 2)), x)
```

3.1444 $\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx$

Optimal. Leaf size=11

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {52}

$$\frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] ArcCosh[(b*x)/4]/b
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{-4+bx}\sqrt{4+bx}}\,dx = \frac{\cosh^{-1}\left(\frac{bx}{4}\right)}{b}$$

Mathematica [B] time = 0.00, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx-4}}{\sqrt{bx+4}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[-4 + b*x]/Sqrt[4 + b*x]])/b
```

IntegrateAlgebraic [B] time = 0.05, size = 25, normalized size = 2.27

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{bx+4}}{\sqrt{bx-4}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-4 + b*x]*Sqrt[4 + b*x]),x]
```

```
[Out] (2*ArcTanh[Sqrt[4 + b*x]/Sqrt[-4 + b*x]])/b
```

fricas [B] time = 0.84, size = 26, normalized size = 2.36

$$-\frac{\log\left(-bx+\sqrt{bx+4}\sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-b*x + sqrt(b*x + 4)*sqrt(b*x - 4))/b
```

giac [B] time = 1.04, size = 23, normalized size = 2.09

$$-\frac{2 \log\left(\sqrt{bx+4} - \sqrt{bx-4}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(sqrt(b*x + 4) - sqrt(b*x - 4))/b
```

maple [B] time = 0.01, size = 57, normalized size = 5.18

$$\frac{\sqrt{(bx-4)(bx+4)}\ \ln\left(\frac{b^2 x}{\sqrt{b^2}} + \sqrt{b^2 x^2 - 16}\right)}{\sqrt{bx-4}\ \sqrt{bx+4}\ \sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x)
```

```
[Out] ((b*x-4)*(b*x+4))^(1/2)/(b*x-4)^(1/2)/(b*x+4)^(1/2)*ln(1/(b^2)^(1/2)*b^2*x+
(b^2*x^2-16)^(1/2))/(b^2)^(1/2)
```

maxima [B] time = 1.38, size = 26, normalized size = 2.36

$$\frac{\log\left(2\, b^2 x + 2\, \sqrt{b^2 x^2 - 16}\, b\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)^(1/2)/(b*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*b^2*x + 2*sqrt(b^2*x^2 - 16)*b)/b
```

mupad [B] time = 0.32, size = 40, normalized size = 3.64

$$-\frac{4\,\mathrm{atan}\left(\frac{b\left(\sqrt{b\,x-4}-2\mathrm{i}\right)}{\left(\sqrt{b\,x+4}-2\right)\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - 4)^(1/2)*(b*x + 4)^(1/2)),x)
```

```
[Out] -(4*atan((b*((b*x - 4)^(1/2) - 2i))/(((b*x + 4)^(1/2) - 2)*(-b^2)^(1/2))))/
(-b^2)^(1/2)
```

sympy [C] time = 4.21, size = 75, normalized size = 6.82

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 & \end{matrix} \middle| {\frac{16 e^{2 i \pi}}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b} + \frac{i G_{6,6}^{2,6}\left(\begin{matrix} - \frac{1}{2}, - \frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 & \\ - \frac{1}{4}, \frac{1}{4} & - \frac{1}{2}, 0, 0, 0 \end{matrix} \middle| {\frac{16}{b^{2} x^{2}}} \right)}{4 \pi^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x-4)**(1/2)/(b*x+4)**(1/2),x)
```

```
[Out] meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 16*
exp_polar(2*I*pi)/(b**2*x**2))/(4*pi**(3/2)*b) + I*meijerg(((-1/2, -1/4, 0,
 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), 16/(b**2*x**2))/(4*pi**
(3/2)*b)
```

3.1445 $\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 215}

$$\frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{bx-\frac{b(1-c)}{d}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcSinh[(Sqrt[d]*Sqrt[-((b*(1 - c))/d) + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{-b+bc}{d}+bx}\,\sqrt{c+dx}}\,dx &= \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{-b+bc}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{-b+bc}{d}+bx}\right)}{b}\\ &= \frac{2\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{-\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.95

$$\frac{2\sqrt{c+dx-1}\,\sinh^{-1}\left(\sqrt{c+dx-1}\right)}{d\sqrt{\frac{b(c+dx-1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-1 + c + d*x]*ArcSinh[Sqrt[-1 + c + d*x]])/(d*Sqrt[(b*(-1 + c + d*x
))/d])
```

IntegrateAlgebraic [A] time = 0.11, size = 57, normalized size = 1.33

$$-\frac{2\sqrt{\frac{b}{d}}\log\left(\sqrt{\frac{b(c+dx)}{d}-\frac{b}{d}}-\sqrt{\frac{b}{d}}\sqrt{c+dx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[(-b + b*c)/d + b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*Sqrt[b/d]*Log[-(Sqrt[b/d]*Sqrt[c + d*x]) + Sqrt[-(b/d) + (b*(c + d*x))/
d]])/b
```

fricas [B] time = 0.90, size = 175, normalized size = 4.07

$$\left[\frac{\sqrt{bd}\log\left(8\,bd^2x^2+8\,bc^2+8\,(2\,bc-b)dx+4\,\sqrt{bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}\,(2\,dx+2\,c-1)\sqrt{dx+c}\,\sqrt{\frac{bdx+bc-b}{d}}}{2\left(bd^2x^2+bc^2+(2\,bc-b)dx-bc\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b*d)*log(8*b*d^2*x^2 + 8*b*c^2 + 8*(2*b*c - b)*d*x + 4*sqrt(b*d)*
(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((b*d*x + b*c - b)/d) - 8*b*c + b)/(b*d
), -sqrt(-b*d)*arctan(1/2*sqrt(-b*d)*(2*d*x + 2*c - 1)*sqrt(d*x + c)*sqrt((
b*d*x + b*c - b)/d)/(b*d^2*x^2 + b*c^2 + (2*b*c - b)*d*x - b*c))/(b*d)]
```

giac [A] time = 0.97, size = 57, normalized size = 1.33

$$-\frac{2\,b\log\left(-\sqrt{bd}\,\sqrt{\frac{bdx+bc-b}{d}}+\sqrt{(bdx+bc-b)b+b^2}\right)}{\sqrt{bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(b*d)*sqrt((b*d*x + b*c - b)/d) + sqrt((b*d*x + b*c - b)*b +
b^2))/(sqrt(b*d)*abs(b))
```

maple [B] time = 0.02, size = 100, normalized size = 2.33

$$\frac{\sqrt{\left(bx+\frac{(c-1)b}{d}\right)(dx+c)}\;\ln\left(\frac{bdx+\frac{bc}{2}+\frac{(c-1)b}{2}}{\sqrt{bd}}+\sqrt{bd\,x^2+\frac{(c-1)bc}{d}+(bc+(c-1)\,b)\,x}\right)}{\sqrt{bx+\frac{(c-1)b}{d}}\;\sqrt{dx+c}\;\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((b*x+b*(c-1)/d)*(d*x+c))^(1/2)/(b*x+b*(c-1)/d)^(1/2)/(d*x+c)^(1/2)*ln((1/2
*b*(c-1)+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(b*(c-1)+b*c)*x+b*(c-1)/d*c)^(
1/2))/(b*d)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)Is 2*c-1 zero or nonzero?
```

mupad [B] time = 0.50, size = 66, normalized size = 1.53

$$\frac{4\,\mathrm{atan}\left(-\frac{d\,\left(\sqrt{b\,x-\frac{b-b\,c}{d}}-\sqrt{-\frac{b-b\,c}{d}}\right)}{\sqrt{-b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{-b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x - (b - b*c)/d)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] (4*atan(-(d*((b*x - (b - b*c)/d)^(1/2) - (-(b - b*c)/d)^(1/2)))/((-b*d)^(1/
2)*((c + d*x)^(1/2) - c^(1/2)))))/(-b*d)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(\frac{c}{d}+x-\frac{1}{d}\right)}\,\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*c-b)/d+b*x)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(c/d + x - 1/d))*sqrt(c + d*x)), x)
```

3.1446 $\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx$

Optimal. Leaf size=22

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {54, 215}

$$\sqrt{2}\sinh^{-1}\left(\frac{\sqrt{2x-3}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] Sqrt[2]*ArcSinh[Sqrt[-3 + 2*x]/Sqrt[3]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{-3+2x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{3+x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{2}\sinh^{-1}\left(\frac{\sqrt{-3+2x}}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.41

$$\frac{\sqrt{4x-6}\sin^{-1}\left(\sqrt{1-\frac{2x}{3}}\right)}{\sqrt{3-2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] (Sqrt[-6 + 4*x]*ArcSin[Sqrt[1 - (2*x)/3]])/Sqrt[3 - 2*x]
```

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.36

$$-\sqrt{2}\log\left(\sqrt{2x-3}-\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[-3 + 2*x]),x]
```

```
[Out] -(Sqrt[2]*Log[-(Sqrt[2]*Sqrt[x]) + Sqrt[-3 + 2*x]])
```

fricas [**A**] time = 0.98, size = 26, normalized size = 1.18

$$\frac{1}{2}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{2x-3}\sqrt{x}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-2*sqrt(2)*sqrt(2*x - 3)*sqrt(x) - 4*x + 3)
```

giac [**A**] time = 0.92, size = 23, normalized size = 1.05

$$-\sqrt{2}\log\left(\sqrt{2}\sqrt{x}-\sqrt{2x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(sqrt(2)*sqrt(x) - sqrt(2*x - 3))
```

maple [**B**] time = 0.01, size = 48, normalized size = 2.18

$$\frac{\sqrt{(2x-3)x}\,\sqrt{2}\,\ln\left(\frac{\left(2x-\frac{3}{2}\right)\sqrt{2}}{2}+\sqrt{2x^{2}-3x}\right)}{2\sqrt{2x-3}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(-3+2*x)^(1/2),x)
```

```
[Out] 1/2*(x*(-3+2*x))^(1/2)/x^(1/2)/(-3+2*x)^(1/2)*ln(1/2*(-3/2+2*x)*2^(1/2)+(2*
x^2-3*x)^(1/2))*2^(1/2)
```

maxima [**B**] time = 2.87, size = 41, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2x-3}}{\sqrt{x}}}{\sqrt{2}+\frac{\sqrt{2x-3}}{\sqrt{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-3+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sqrt(2*x - 3)/sqrt(x))/(sqrt(2) + sqrt(2*x - 3
)/sqrt(x)))
```

mupad [**B**] time = 0.44, size = 30, normalized size = 1.36

$$-2\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(2*x - 3)^(1/2)),x)
```

```
[Out] -2*2^(1/2)*atanh((2^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*x^(1/2)))
```

sympy [A] time = 1.03, size = 44, normalized size = 2.00

$$\begin{cases} \sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{for } \frac{2|x|}{3} > 1 \\ -\sqrt{2}\,i\operatorname{asin}\left(\frac{\sqrt{6}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-3+2*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (-sqrt(2)*I*a
sin(sqrt(6)*sqrt(x)/3), True))
```

3.1447 $\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {54, 215}

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+2x}\,\sqrt{2+3x}}\,dx &= \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{13+3x^2}}\,dx, x, \sqrt{-3+2x}\right)\\ &= \sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{-3+2x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\sqrt{\frac{2}{3}}\sinh^{-1}\left(\sqrt{\frac{3}{13}}\sqrt{2x-3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcSinh[Sqrt[3/13]*Sqrt[-3 + 2*x]]
```

IntegrateAlgebraic [A] time = 0.07, size = 35, normalized size = 1.35

$$\sqrt{\frac{2}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}\sqrt{3x+2}}{\sqrt{2x-3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[-3 + 2*x]*Sqrt[2 + 3*x]),x]
```

```
[Out] Sqrt[2/3]*ArcTanh[(Sqrt[2/3]*Sqrt[2 + 3*x])/Sqrt[-3 + 2*x]]
```

fricas [B] time = 1.07, size = 46, normalized size = 1.77

$$\frac{1}{12}\sqrt{3}\sqrt{2}\log\left(4\sqrt{3}\sqrt{2}(12x-5)\sqrt{3x+2}\sqrt{2x-3}+288x^2-240x-119\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*sqrt(2)*log(4*sqrt(3)*sqrt(2)*(12*x - 5)*sqrt(3*x + 2)*sqrt(2*
x - 3) + 288*x^2 - 240*x - 119)
```

giac [A] time = 1.26, size = 30, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\sqrt{2}\log\left(\left|-\sqrt{2}\sqrt{3x+2}+\sqrt{6x-9}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*sqrt(2)*log(abs(-sqrt(2)*sqrt(3*x + 2) + sqrt(6*x - 9)))
```

maple [B] time = 0.01, size = 57, normalized size = 2.19

$$\frac{\sqrt{(2x-3)(3x+2)}\,\sqrt{6}\,\ln\left(\frac{\left(6x-\frac{5}{2}\right)\sqrt{6}}{6}+\sqrt{6x^2-5x-6}\right)}{6\sqrt{2x-3}\,\sqrt{3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x-3)^(1/2)/(3*x+2)^(1/2),x)
```

```
[Out] 1/6*((2*x-3)*(3*x+2))^(1/2)/(2*x-3)^(1/2)/(3*x+2)^(1/2)*ln(1/6*(-5/2+6*x)*6
^(1/2)+(6*x^2-5*x-6)^(1/2))*6^(1/2)
```

maxima [A] time = 3.04, size = 28, normalized size = 1.08

$$\frac{1}{6}\sqrt{6}\log\left(2\sqrt{6}\sqrt{6x^2-5x-6}+12x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*log(2*sqrt(6)*sqrt(6*x^2 - 5*x - 6) + 12*x - 5)
```

mupad [B] time = 0.12, size = 43, normalized size = 1.65

$$\frac{2\sqrt{6}\,\mathrm{atanh}\left(\frac{\sqrt{6}\left(-\sqrt{2x-3}+\sqrt{3}\,1\mathrm{i}\right)}{2\left(\sqrt{2}-\sqrt{3x+2}\right)}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x - 3)^(1/2)*(3*x + 2)^(1/2)),x)
```

```
[Out] (2*6^(1/2)*atanh((6^(1/2)*(3^(1/2)*1i - (2*x - 3)^(1/2)))/(2*(2^(1/2) - (3*
x + 2)^(1/2)))))/3
```

sympy [A] time = 1.09, size = 58, normalized size = 2.23

$$\begin{cases} \frac{\sqrt{6}\operatorname{acosh}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{for } \frac{6\left|x+\frac{2}{3}\right|}{13} > 1 \\ -\frac{\sqrt{6}\,i\operatorname{asin}\left(\frac{\sqrt{78}\sqrt{x+\frac{2}{3}}}{13}\right)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+2*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((sqrt(6)*acosh(sqrt(78)*sqrt(x + 2/3)/13)/3, 6*Abs(x + 2/3)/13 >
1), (-sqrt(6)*I*asin(sqrt(78)*sqrt(x + 2/3)/13)/3, True))
```

3.1448 $\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx$

Optimal. Leaf size=42

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {63, 216}

$$\frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*ArcSin[(Sqrt[d]*Sqrt[(b*(1 - c))/d + b*x])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{b-bc}{d}+bx}\sqrt{c-dx}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{b-bc}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{\frac{b-bc}{d}+bx}\right)}{b}\\
&= \frac{2\sin^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{b(1-c)}{d}+bx}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 1.60

$$\frac{2\sqrt{-d}\sqrt{-c+dx+1}\,\sinh^{-1}\left(\frac{\sqrt{-d}\sqrt{c-dx}}{\sqrt{d}}\right)}{d^{3/2}\sqrt{\frac{b(-c+dx+1)}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (2*Sqrt[-d]*Sqrt[1 - c + d*x]*ArcSinh[(Sqrt[-d]*Sqrt[c - d*x])/Sqrt[d]])/(d
^(3/2)*Sqrt[(b*(1 - c + d*x))/d])
```

IntegrateAlgebraic [A] time = 0.12, size = 61, normalized size = 1.45

$$-\frac{2\sqrt{-\frac{b}{d}}\log\left(\sqrt{\frac{b}{d}-\frac{b(c-dx)}{d}}-\sqrt{-\frac{b}{d}}\sqrt{c-dx}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[(b - b*c)/d + b*x]*Sqrt[c - d*x]),x]
```

```
[Out] (-2*Sqrt[-(b/d)]*Log[-(Sqrt[-(b/d)]*Sqrt[c - d*x]) + Sqrt[b/d - (b*(c - d*x
))/d]])/b
```

fricas [B] time = 0.85, size = 176, normalized size = 4.19

$$\left[-\frac{\sqrt{-bd}\log\left(8\,bd^2x^2+8\,bc^2-8\,(2\,bc-b)dx-4\,\sqrt{-bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}-8\,bc+b\right)}{2\,bd},-\frac{\sqrt{bd}\arctan\left(\frac{\sqrt{bd}\,(2\,dx-2\,c+1)\sqrt{-dx+c}\,\sqrt{\frac{bdx-bc+b}{d}}}{2\left(bd^2x^2+bc^2-(2\,bc-b)dx-bc\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b*d^2*x^2 + 8*b*c^2 - 8*(2*b*c - b)*d*x - 4*sqrt(-b*
d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqrt((b*d*x - b*c + b)/d) - 8*b*c + b)/
(b*d), -sqrt(b*d)*arctan(1/2*sqrt(b*d)*(2*d*x - 2*c + 1)*sqrt(-d*x + c)*sqr
t((b*d*x - b*c + b)/d)/(b*d^2*x^2 + b*c^2 - (2*b*c - b)*d*x - b*c))/(b*d)]
```

giac [A] time = 1.08, size = 58, normalized size = 1.38

$$-\frac{2\,b\log\left(-\sqrt{-bd}\,\sqrt{\frac{bdx-bc+b}{d}}+\sqrt{-(bdx-bc+b)b+b^2}\right)}{\sqrt{-bd}\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*b*log(-sqrt(-b*d)*sqrt((b*d*x - b*c + b)/d) + sqrt(-(b*d*x - b*c + b)*b
+ b^2))/(sqrt(-b*d)*abs(b))
```

maple [B] time = 0.04, size = 118, normalized size = 2.81

$$\frac{\sqrt{\left(bx+\frac{(-c+1)b}{d}\right)(-dx+c)}\,\arctan\left(\frac{\sqrt{bd}\left(x-\frac{bc-(-c+1)b}{2bd}\right)}{\sqrt{-bd\,x^2+\frac{(-c+1)bc}{d}+(bc-(-c+1)b)x}}\right)}{\sqrt{bx+\frac{(-c+1)b}{d}}\,\sqrt{-dx+c}\,\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x)
```

```
[Out] ((b*(1-c)/d+b*x)*(-d*x+c))^(1/2)/(b*(1-c)/d+b*x)^(1/2)/(-d*x+c)^(1/2)/(b*d)
^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-b*(1-c)+b*c)/b/d)/(-b*d*x^2+(-b*(1-c)+b*
c)*x+b*(1-c)/d*c)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)^(1/2)/(-d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c-1>0)', see `assume?` for more d
etails)Is 2*c-1 zero or nonzero?
```

mupad [B] time = 0.51, size = 63, normalized size = 1.50

$$-\frac{4\,\mathrm{atan}\left(-\frac{d\,\left(\sqrt{\frac{b-b\,c}{d}+b\,x}-\sqrt{\frac{b-b\,c}{d}}\right)}{\sqrt{b\,d}\,\left(\sqrt{c-d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(((b - b*c)/d + b*x)^(1/2)*(c - d*x)^(1/2)),x)
```

```
[Out] -(4*atan(-(d*(((b - b*c)/d + b*x)^(1/2) - ((b - b*c)/d)^(1/2)))/((b*d)^(1/2
)*((c - d*x)^(1/2) - c^(1/2)))))/(b*d)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\left(-\frac{c}{d}+x+\frac{1}{d}\right)}\,\sqrt{c-dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*c+b)/d+b*x)**(1/2)/(-d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*(-c/d + x + 1/d))*sqrt(c - d*x)), x)
```

3.1449 $\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] -ArcSin[1 - x/2]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4-x}\sqrt{x}}\,dx &= \int \frac{1}{\sqrt{4x-x^2}}\,dx\\ &= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx,x,4-2x\right)\right)\\ &= -\sin^{-1}\left(1-\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.40

$$-2\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 - x]*Sqrt[x]),x]
```

```
[Out] -2*ArcSin[Sqrt[1 - x/4]]
```

IntegrateAlgebraic [C] time = 0.04, size = 24, normalized size = 2.40

$$2i\log\left(\sqrt{4-x}-i\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[4 - x]*Sqrt[x]),x]
[Out] (2*I)*Log[Sqrt[4 - x] - I*Sqrt[x]]
```

fricas [B] time = 0.98, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

giac [A] time = 1.10, size = 8, normalized size = 0.80

$$2\arcsin\left(\frac{1}{2}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] 2*arcsin(1/2*sqrt(x))
```

maple [B] time = 0.00, size = 27, normalized size = 2.70

$$\frac{\sqrt{(-x+4)x}\,\arcsin\left(\frac{x}{2}-1\right)}{\sqrt{-x+4}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-x)^(1/2)/x^(1/2),x)
[Out] ((4-x)*x)^(1/2)/(4-x)^(1/2)/x^(1/2)*arcsin(-1+1/2*x)
```

maxima [B] time = 2.99, size = 14, normalized size = 1.40

$$-2\arctan\left(\frac{\sqrt{-x+4}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -2*arctan(sqrt(-x + 4)/sqrt(x))
```

mupad [B] time = 0.29, size = 16, normalized size = 1.60

$$-4\,\mathrm{atan}\left(\frac{\sqrt{4-x}-2}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(4 - x)^(1/2)),x)
```

```
[Out] -4*atan(((4 - x)^(1/2) - 2)/x^(1/2))
```

sympy [**A**] time = 0.99, size = 26, normalized size = 2.60

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{x}}{2}\right) & \text{for } \frac{|x|}{4} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{x}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(x)/2), Abs(x)/4 > 1), (2*asin(sqrt(x)/2), True))
```

3.1450 $\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {54, 216}

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{3-2x^2}}\,dx, x, \sqrt{x}\right)\\ &= \sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\sqrt{2}\sin^{-1}\left(\sqrt{\frac{2}{3}}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
[Out] Sqrt[2]*ArcSin[Sqrt[2/3]*Sqrt[x]]
```

IntegrateAlgebraic [A] time = 0.07, size = 38, normalized size = 1.90

$$-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{3}-\sqrt{3-2x}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 2*x]*Sqrt[x]),x]
[Out] -2*Sqrt[2]*ArcTan[(Sqrt[2]*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 2*x])]
```

fricas [A] time = 1.07, size = 21, normalized size = 1.05

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="fricas")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

giac [A] time = 1.09, size = 13, normalized size = 0.65

$$\sqrt{2}\arcsin\left(\frac{1}{3}\sqrt{6}\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="giac")
[Out] sqrt(2)*arcsin(1/3*sqrt(6)*sqrt(x))
```

maple [B] time = 0.01, size = 31, normalized size = 1.55

$$\frac{\sqrt{(-2x+3)x}\,\sqrt{2}\,\arcsin\left(\frac{4x}{3}-1\right)}{2\sqrt{-2x+3}\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(1/2)/x^(1/2),x)
[Out] 1/2*((-2*x+3)*x)^(1/2)/(-2*x+3)^(1/2)/x^(1/2)*2^(1/2)*arcsin(4/3*x-1)
```

maxima [A] time = 3.12, size = 21, normalized size = 1.05

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x+3}}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/x^(1/2),x, algorithm="maxima")
[Out] -sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*x + 3)/sqrt(x))
```

mupad [B] time = 0.30, size = 27, normalized size = 1.35

$$2\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\left(\sqrt{3}-\sqrt{3-2x}\right)}{2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(3 - 2*x)^(1/2)),x)
[Out] 2*2^(1/2)*atan((2^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*x^(1/2)))
```

sympy [A] time = 1.00, size = 44, normalized size = 2.20

$$\begin{cases} -\sqrt{2}\, i \operatorname{acosh}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{for}\ \frac{2|x|}{3} > 1 \\ \sqrt{2}\, \operatorname{asin}\left(\frac{\sqrt{6}\,\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/x**(1/2),x)
```

```
[Out] Piecewise((-sqrt(2)*I*acosh(sqrt(6)*sqrt(x)/3), 2*Abs(x)/3 > 1), (sqrt(2)*a
sin(sqrt(6)*sqrt(x)/3), True))
```

3.1451 $\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}} dx$

Optimal. Leaf size=26

$$\sqrt{\frac{2}{5}} \sin^{-1}\left(\sqrt{\frac{2}{21}} \sqrt{5x+3}\right)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {54, 216}

$$\sqrt{\frac{2}{5}} \sin^{-1}\left(\sqrt{\frac{2}{21}} \sqrt{5x+3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] Sqrt[2/5]*ArcSin[Sqrt[2/21]*Sqrt[3 + 5*x]]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{3-2x}\sqrt{3+5x}} dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{21-2x^2}} dx, x, \sqrt{3+5x}\right)}{\sqrt{5}}\\ &= \sqrt{\frac{2}{5}} \sin^{-1}\left(\sqrt{\frac{2}{21}} \sqrt{3+5x}\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 45, normalized size = 1.73

$$-\frac{\sqrt{\frac{2}{5}}\sqrt{3-2x}\sinh^{-1}\left(\sqrt{\frac{5}{21}}\sqrt{2x-3}\right)}{\sqrt{2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -((Sqrt[2/5]*Sqrt[3 - 2*x]*ArcSinh[Sqrt[5/21]*Sqrt[-3 + 2*x]])/Sqrt[-3 + 2*
x])
```

IntegrateAlgebraic [A] time = 0.05, size = 36, normalized size = 1.38

$$-\sqrt{\frac{2}{5}} \tan^{-1}\left(\frac{\sqrt{\frac{5}{2}}\sqrt{3-2x}}{\sqrt{5x+3}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[3 - 2*x]*Sqrt[3 + 5*x]),x]
```

```
[Out] -(Sqrt[2/5]*ArcTan[(Sqrt[5/2]*Sqrt[3 - 2*x])/Sqrt[3 + 5*x]])
```

fricas [B] time = 0.77, size = 44, normalized size = 1.69

$$-\frac{1}{5}\sqrt{5}\sqrt{2}\arctan\left(\frac{\sqrt{5}\sqrt{2}\sqrt{5x+3}\sqrt{-2x+3}-3\sqrt{5}\sqrt{2}}{10x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*sqrt(5)*sqrt(2)*arctan(1/10*(sqrt(5)*sqrt(2)*sqrt(5*x + 3)*sqrt(-2*x +
 3) - 3*sqrt(5)*sqrt(2))/x)
```

giac [A] time = 0.93, size = 21, normalized size = 0.81

$$\frac{1}{5}\sqrt{5}\sqrt{2}\arcsin\left(\frac{1}{21}\sqrt{42}\sqrt{5x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*sqrt(2)*arcsin(1/21*sqrt(42)*sqrt(5*x + 3))
```

maple [B] time = 0.01, size = 39, normalized size = 1.50

$$\frac{\sqrt{(-2x+3)(5x+3)}\,\sqrt{10}\,\arcsin\left(\frac{20x}{21}-\frac{3}{7}\right)}{10\sqrt{-2x+3}\,\sqrt{5x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x+3)^(1/2)/(3+5*x)^(1/2),x)
```

```
[Out] 1/10*((-2*x+3)*(3+5*x))^(1/2)/(-2*x+3)^(1/2)/(3+5*x)^(1/2)*10^(1/2)*arcsin(
20/21*x-3/7)
```

maxima [A] time = 3.00, size = 11, normalized size = 0.42

$$-\frac{1}{10}\sqrt{10}\arcsin\left(-\frac{20}{21}x+\frac{3}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)^(1/2)/(3+5*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(10)*arcsin(-20/21*x + 3/7)
```

mupad [B] time = 0.08, size = 40, normalized size = 1.54

$$-\frac{2\sqrt{10}\,\mathrm{atan}\left(\frac{\sqrt{10}\left(\sqrt{3}-\sqrt{3-2x}\right)}{2\left(\sqrt{3}-\sqrt{5x+3}\right)}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - 2*x)^(1/2)*(5*x + 3)^(1/2)),x)
```

```
[Out] -(2*10^(1/2)*atan((10^(1/2)*(3^(1/2) - (3 - 2*x)^(1/2)))/(2*(3^(1/2) - (5*x
 + 3)^(1/2)))))/5
```

sympy [A] time = 1.07, size = 58, normalized size = 2.23

$$\begin{cases} -\frac{\sqrt{10}\, i \operatorname{acosh}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{for}\ \frac{10\left|x+\frac{3}{5}\right|}{21} > 1 \\ \frac{\sqrt{10}\operatorname{asin}\left(\frac{\sqrt{210}\sqrt{x+\frac{3}{5}}}{21}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-2*x)**(1/2)/(3+5*x)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(10)*I*acosh(sqrt(210)*sqrt(x + 3/5)/21)/5, 10*Abs(x + 3/5)
/21 > 1), (sqrt(10)*asin(sqrt(210)*sqrt(x + 3/5)/21)/5, True))
```

3.1452 $\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=43

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {63, 217, 203}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[c + d*x])])/(Sqrt[b]*Sqrt[
d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-bx}\sqrt{c+dx}}\,dx &= -\frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{c+\frac{ad}{b}-\frac{dx^2}{b}}}\,dx, x, \sqrt{a-bx}\right)}{b}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{1+\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a-bx}}{\sqrt{c+dx}}\right)}{b}\\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [B] time = 0.08, size = 103, normalized size = 2.40

$$\frac{2\sqrt{-b}\sqrt{-ad-bc}\sqrt{\frac{b(c+dx)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{-b}\sqrt{d}\sqrt{a-bx}}{\sqrt{b}\sqrt{-ad-bc}}\right)}{b^{3/2}\sqrt{d}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*Sqrt[-b]*Sqrt[-(b*c) - a*d]*Sqrt[(b*(c + d*x))/(b*c + a*d)]*ArcSin[(Sqrt
[-b]*Sqrt[d]*Sqrt[a - b*x])/(Sqrt[b]*Sqrt[-(b*c) - a*d])])/(b^(3/2)*Sqrt[d]
*Sqrt[c + d*x])
```

IntegrateAlgebraic [A] time = 0.09, size = 43, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a-bx}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(Sqrt[a - b*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a - b*x])])/(Sqrt[b]*Sqrt[d
])
```

fricas [B] time = 0.73, size = 185, normalized size = 4.30

$$\left[-\frac{\sqrt{-bd}\log\left(8b^2d^2x^2+b^2c^2-6abcd+a^2d^2-4(2bdx+bc-ad)\sqrt{-bd}\sqrt{-bx+a}\sqrt{dx+c}+8\left(b^2cd-abd^2\right)x\right)}{2bd}, -\frac{\sqrt{bd}\arctan\left(\frac{(2bdx+bc-ad)\sqrt{bd}\sqrt{-bx+a}\sqrt{dx+c}}{2\left(b^2d^2x^2-abcd+\left(b^2cd-abd^2\right)x\right)}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*(2*b
*d*x + b*c - a*d)*sqrt(-b*d)*sqrt(-b*x + a)*sqrt(d*x + c) + 8*(b^2*c*d - a*
b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*(2*b*d*x + b*c - a*d)*sqrt(b*d)*sqrt
(-b*x + a)*sqrt(d*x + c)/(b^2*d^2*x^2 - a*b*c*d + (b^2*c*d - a*b*d^2)*x))/(
b*d)]
```

giac [A] time = 1.16, size = 54, normalized size = 1.26

$$\frac{2b\log\left(\left|-\sqrt{-bd}\sqrt{-bx+a}+\sqrt{b^2c+(bx-a)bd+abd}\right|\right)}{\sqrt{-bd}|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 2*b*log(abs(-sqrt(-b*d)*sqrt(-b*x + a) + sqrt(b^2*c + (b*x - a)*b*d + a*b*d
)))/(sqrt(-b*d)*abs(b))
```

maple [B] time = 0.01, size = 84, normalized size = 1.95

$$\frac{\sqrt{(-bx+a)(dx+c)}\arctan\left(\frac{\sqrt{bd}\left(x-\frac{ad-bc}{2bd}\right)}{\sqrt{-bd\,x^2+ac+(ad-bc)x}}\right)}{\sqrt{-bx+a}\sqrt{dx+c}\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] ((-b*x+a)*(d*x+c))^(1/2)/(-b*x+a)^(1/2)/(d*x+c)^(1/2)/(b*d)^(1/2)*arctan((b
*d)^(1/2)*(x-1/2*(a*d-b*c)/b/d)/(-b*d*x^2+(a*d-b*c)*x+a*c)^(1/2))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c zero or nonzero?
```

mupad [**B**] time = 0.34, size = 44, normalized size = 1.02

$$-\frac{4\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{a-b\,x}-\sqrt{a}\right)}{\sqrt{b\,d}\,\left(\sqrt{c+d\,x}-\sqrt{c}\right)}\right)}{\sqrt{b\,d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x)^(1/2)*(c + d*x)^(1/2)),x)
```

```
[Out] -(4*atan((d*((a - b*x)^(1/2) - a^(1/2)))/((b*d)^(1/2)*((c + d*x)^(1/2) - c^
(1/2)))))/(b*d)^(1/2)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - bx}\,\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x+a)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a - b*x)*sqrt(c + d*x)), x)
```

3.1453 $\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx$

Optimal. Leaf size=219

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+ \dots}}{6bd \dots}$$

Rubi [A] time = 0.09, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)^2\log(c+dx)}{18b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{6b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}(bc-ad)}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*b*d) + ((a + b*x)^(5/3)*(c
 + d*x)^(1/3))/(2*b) + ((b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*
x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) +
 ((b*c - a*d)^2*Log[c + d*x])/(18*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[-1
+ (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(4/3)*d^(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{2/3}\sqrt[3]{c+dx}\,dx &= \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}+\frac{(bc-ad)\int\frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6b}\\ &= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}-\frac{(bc-ad)^2\int\frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx \dots}{9bd \dots}\\ &= \frac{(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6bd}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2b}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d} \dots}{\sqrt{3}\sqrt[3]{\dots}}\right.}{3\sqrt{3}b^{4/3}d^{5/3}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.33

$$\frac{3(a+bx)^{5/3}\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 5/3, 8/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

IntegrateAlgebraic [A] time = 0.49, size = 294, normalized size = 1.34

$$\frac{(bc-ad)^2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{9b^{4/3}d^{5/3}}-\frac{(bc-ad)^2\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{18b^{4/3}d^{5/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}d^{5/3}}+\frac{(bc-ad)^2\left(\frac{b(c+dx)^{4/3}}{(a+bx)^{4/3}}+\frac{2d\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{6bd\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)*(c + d*x)^(1/3),x]
```

```
[Out] ((b*c - a*d)^2*((2*d*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b*(c + d*x)^(4/3))
/(a + b*x)^(4/3)))/(6*b*d*(-d + (b*(c + d*x))/(a + b*x))^2) - ((b*c - a*d)^
2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)
^(1/3))])/(3*Sqrt[3]*b^(4/3)*d^(5/3)) + ((b*c - a*d)^2*Log[d^(1/3) - (b^(1/
3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)])/(9*b^(4/3)*d^(5/3)) - ((b*c - a*d)^2*
Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*
(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(18*b^(4/3)*d^(5/3))
```

fricas [A] time = 0.89, size = 717, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(b*d^2)^(1
/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1/3)*(b*x + a)^(2/3)*(
d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (b*
d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(1/3)*(b*d*x + a*d))*s
qrt(-(b*d^2)^(1/3)/b)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*lo
g(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)
) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(
1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x + b^2*c*d^2 + 2*a*b*d^3)*(b
*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3), -1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqrt(
(b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*
d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x +
a))/(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*d^2)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3
) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) - 3*(3*b^2*d^3*x + b^2*c*d^2 +
2*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(2/3)*(d*x+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)*(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(2/3)*(d*x + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{2/3}\,(c+d\,x)^{1/3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)*(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(2/3)*(c + d*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{2}{3}} \sqrt[3]{c+dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)*(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(2/3)*(c + d*x)**(1/3), x)
```

3.1454 $\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$

Optimal. Leaf size=172

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Rubi [A] time = 0.05, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/b - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^
(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(4/3)
*d^(2/3)) - ((b*c - a*d)*Log[c + d*x])/(6*b^(4/3)*d^(2/3)) - ((b*c - a*d)*L
og[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3)*d^
(2/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3b}\\ &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{b}-\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,b^{4/3}d^{2/3}}-\frac{(bc-ad)\log(c+dx)}{6b^{4/3}d^{2/3}}-\frac{(bc-ad)\,\mathrm{l}}{}\end{aligned}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.42

$$\frac{3(a+bx)^{2/3}\sqrt[3]{c+dx}\;{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(2/3)*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, (d*(a
+ b*x))/(-(b*c) + a*d)])/(2*b*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

IntegrateAlgebraic [A] time = 7.15, size = 300, normalized size = 1.74

$$\frac{\sqrt[3]{ad+bdx}\left(\frac{(ad-bc)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{3b^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{6b^{4/3}d^{2/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}b^{4/3}d^{2/3}}+\frac{\sqrt[3]{c+dx}(ad+b(c+dx)-bc)^{2/3}}{bd^{2/3}}\right)}{\sqrt[3]{d}\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(1/3),x]
```

```
[Out] ((a*d + b*d*x)^(1/3)*(((c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(2/3))/
(b*d^(2/3)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3
)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(4/3
)*d^(2/3)) + ((-(b*c) + a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d
 + b*(c + d*x))^(1/3)])/(3*b^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[b^(2/3)*(c +
 d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) +
(-(b*c) + a*d + b*(c + d*x))^(2/3)])/(6*b^(4/3)*d^(2/3))))/(d^(1/3)*(a + b*
x)^(1/3))
```

fricas [B] time = 0.98, size = 596, normalized size = 3.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt(-(b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 + 3*(b*d^2)^(1
/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d + 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x
 + c)^(2/3)*b*d - (b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^2)^(
1/3)*(b*d*x + a*d))*sqrt(-(b*d^2)^(1/3)/b)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*l
og(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a
)) + (b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (
b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))
/(b*x + a)))/(b^2*d^2), 1/6*(6*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 + 6*sq
rt(1/3)*(b^2*c*d - a*b*d^2)*sqrt((b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*(2*(b*d^
2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))*sqr
t((b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(b*d^2)^(2/3)*(b*c - a*d)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + (
b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (b*d^2)
^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^2)^(1/3)*(b*d*x + a*d))/(b*x
+ a)))/(b^2*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
[Out] int((d*x+c)^(1/3)/(b*x+a)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(1/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(1/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(1/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(1/3), x)
```

3.1455 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}} dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

Rubi [A] time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{d}\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2b^{4/3}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) - (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[c + d*x])/(2*b^(4/3)) - (3*d^(1/3)*Log[-1 + (d^(1/3)*(a + b
*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{4/3}} dx &= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}+\frac{d\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}} dx}{b}\\ &= -\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}-\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{b^{4/3}}-\frac{\sqrt[3]{d}\log(c+dx)}{2b^{4/3}}-\frac{3\sqrt[3]{d}\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2b^{4/3}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.48

$$-\frac{3\sqrt[3]{c+dx}\,{}_2F_1\left(-\frac{1}{3},-\frac{1}{3};\frac{2}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*Hypergeometric2F1[-1/3, -1/3, 2/3, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3))
```

IntegrateAlgebraic [A] time = 0.16, size = 200, normalized size = 1.34

$$\frac{\sqrt[3]{d}\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{2b^{4/3}}-\frac{\sqrt[3]{d}\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{b^{4/3}}+\frac{\sqrt{3}\sqrt[3]{d}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{4/3}}-\frac{3\sqrt[3]{c+dx}}{b\sqrt[3]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(4/3),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(b*(a + b*x)^(1/3)) + (Sqrt[3]*d^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/b^(4/3)
 - (d^(1/3)*Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)])/b^(4/
3) + (d^(1/3)*Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/
3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)])/(2*b^(4/3))
```

fricas [B] time = 1.07, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}b\left(-\frac{d}{b}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)+(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{2}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bx+a}\right)-2(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}\log\left(\frac{(bx+a)\left(-\frac{d}{b}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}\right)+6(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{2\left(b^2x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(b*x + a)*(-d/b)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)*b*(-d/b)^(2/3) + sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) +
(b*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(2/3) - (b*x + a)^(2/3)*(d*x +
 c)^(1/3)*(-d/b)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(b*x + a)) - 2*(b
*x + a)*(-d/b)^(1/3)*log(((b*x + a)*(-d/b)^(1/3) + (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a)) + 6*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b^2*x + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

```
[Out] int((d*x+c)^(1/3)/(b*x+a)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(4/3),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{1/3}}{(a+b\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(4/3),x)
[Out] int((c + d*x)^(1/3)/(a + b*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(4/3),x)
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(4/3), x)
```

3.1456 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx = -\frac{3(c+dx)^{4/3}}{4(bc-ad)(a+bx)^{4/3}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{4/3}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(7/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(4*(b*c - a*d)*(a + b*x)^(4/3))
```

fricas [B] time = 0.99, size = 65, normalized size = 2.03

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}{4\left(a^2bc-a^3d+\left(b^3c-ab^2d\right)x^2+2\left(ab^2c-a^2bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4*(b*x + a)^(2/3)*(d*x + c)^(4/3)/(a^2*b*c - a^3*d + (b^3*c - a*b^2*d)*x
^2 + 2*(a*b^2*c - a^2*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{3(dx+c)^{\frac{4}{3}}}{4(bx+a)^{\frac{4}{3}}(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(7/3),x)
```

```
[Out] 3/4/(b*x+a)^(4/3)*(d*x+c)^(4/3)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(7/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(7/3), x)
```

mupad [B] time = 0.71, size = 92, normalized size = 2.88

$$-\frac{\left(\frac{3c}{4b^2c-4abd}+\frac{3dx}{4b^2c-4abd}\right)(c+dx)^{1/3}}{x(a+bx)^{1/3}-\frac{\left(4a^2d-4abc\right)(a+bx)^{1/3}}{4b^2c-4abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(7/3),x)
```

```
[Out] -(((3*c)/(4*b^2*c - 4*a*b*d) + (3*d*x)/(4*b^2*c - 4*a*b*d))*(c + d*x)^(1/3)
)/(x*(a + b*x)^(1/3) - ((4*a^2*d - 4*a*b*c)*(a + b*x)^(1/3))/(4*b^2*c - 4*a
*b*d))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(7/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(7/3), x)
```

3.1457 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx$

Optimal. Leaf size=66

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{9d(c+dx)^{4/3}}{28(a+bx)^{4/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(4/3)
)/(28*(b*c - a*d)^2*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx &= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}-\frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{7(bc-ad)}\\ &= -\frac{3(c+dx)^{4/3}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d(c+dx)^{4/3}}{28(bc-ad)^2(a+bx)^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{4/3}(7ad-4bc+3bdx)}{28(a+bx)^{7/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(-4*b*c + 7*a*d + 3*b*d*x))/(28*(b*c - a*d)^2*(a + b*x)^
(7/3))
```

IntegrateAlgebraic [A] time = 0.11, size = 57, normalized size = 0.86

$$-\frac{3\left(\frac{4b(c+dx)^{7/3}}{(a+bx)^{7/3}}-\frac{7d(c+dx)^{4/3}}{(a+bx)^{4/3}}\right)}{28(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(10/3),x]
```

```
[Out] (-3*((-7*d*(c + d*x)^(4/3))/(a + b*x)^(4/3) + (4*b*(c + d*x)^(7/3))/(a + b*
x)^(7/3)))/(28*(b*c - a*d)^2)
```

fricas [B] time = 1.08, size = 175, normalized size = 2.65

$$\frac{3\left(3\,bd^2x^2-4\,bc^2+7\,acd-\left(bcd-7\,ad^2\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{28\left(a^3b^2c^2-2\,a^4bcd+a^5d^2+\left(b^5c^2-2\,ab^4cd+a^2b^3d^2\right)x^3+3\left(ab^4c^2-2\,a^2b^3cd+a^3b^2d^2\right)x^2+3\left(a^2b^3c^2-2\,a^3b^2cd+a^4bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 3/28*(3*b*d^2*x^2 - 4*b*c^2 + 7*a*c*d - (b*c*d - 7*a*d^2)*x)*(b*x + a)^(2/3
)*(d*x + c)^(1/3)/(a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2 + (b^5*c^2 - 2*a*b^4
*c*d + a^2*b^3*d^2)*x^3 + 3*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x^2 +
 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{4}{3}}\,(3bdx+7ad-4bc)}{28\,(bx+a)^{\frac{7}{3}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(10/3),x)
```

```
[Out] 3/28*(d*x+c)^(4/3)*(3*b*d*x+7*a*d-4*b*c)/(b*x+a)^(7/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(10/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(10/3), x)
```

mupad [B] time = 1.03, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{x\,\left(21\,a\,d^2-3\,b\,c\,d\right)}{28\,b^2\,(a\,d-b\,c)^2}-\frac{12\,b\,c^2-21\,a\,c\,d}{28\,b^2\,(a\,d-b\,c)^2}+\frac{9\,d^2\,x^2}{28\,b\,(a\,d-b\,c)^2}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(10/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((x*(21*a*d^2 - 3*b*c*d))/(28*b^2*(a*d - b*c)^2) - (12*b*c
^2 - 21*a*c*d)/(28*b^2*(a*d - b*c)^2) + (9*d^2*x^2)/(28*b*(a*d - b*c)^2)))/
(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*x)^(1/3))/
b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(10/3),x)
```

```
[Out] Integral((c + d*x)**(1/3)/(a + b*x)**(10/3), x)
```

3.1458 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{4/3}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{9d(c+dx)^{4/3}}{35(a+bx)^{7/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (9*d*(c + d*x)^(4/
3))/(35*(b*c - a*d)^2*(a + b*x)^(7/3)) - (27*d^2*(c + d*x)^(4/3))/(140*(b*c
 - a*d)^3*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx &= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}-\frac{(3d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{5(bc-ad)}\\
&= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}+\frac{\left(9d^2\right)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{7/3}}\,dx}{35(bc-ad)^2}\\
&= -\frac{3(c+dx)^{4/3}}{10(bc-ad)(a+bx)^{10/3}}+\frac{9d(c+dx)^{4/3}}{35(bc-ad)^2(a+bx)^{7/3}}-\frac{27d^2(c+dx)^{4/3}}{140(bc-ad)^3(a+bx)^{4/3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{4/3}\left(35a^2d^2+10abd(3dx-4c)+b^2\left(14c^2-12cdx+9d^2x^2\right)\right)}{140(a+bx)^{10/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*a^2*d^2 + 10*a*b*d*(-4*c + 3*d*x) + b^2*(14*c^2 - 1
2*c*d*x + 9*d^2*x^2)))/(140*(b*c - a*d)^3*(a + b*x)^(10/3))
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{4/3}\left(\frac{14b^2(c+dx)^2}{(a+bx)^2}-\frac{40bd(c+dx)}{a+bx}+35d^2\right)}{140(a+bx)^{4/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(13/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(35*d^2 - (40*b*d*(c + d*x))/(a + b*x) + (14*b^2*(c + d
*x)^2)/(a + b*x)^2))/(140*(b*c - a*d)^3*(a + b*x)^(4/3))
```

fricas [B] time = 0.87, size = 337, normalized size = 3.34

$$-\frac{3\left(9b^2d^3x^3+14b^2c^3-40abc^2d+35a^2cd^2-3\left(b^2cd^2-10abd^3\right)x^2+\left(2b^2c^2d-10abcd^2+35a^2d^3\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\left(a^4b^3c^3-3a^5b^2c^2d+3a^6bcd^2-a^7d^3+\left(b^7c^3-3ab^6c^2d+3a^2b^5cd^2-a^3b^4d^3\right)x^4+4\left(ab^6c^3-3a^2b^5c^2d+3a^3b^4cd^2-a^4b^3d^3\right)x^3+6\left(a^2b^5c^3-3a^3b^4c^2d+3a^4b^3cd^2-a^5b^2d^3\right)x^2+4\left(a^3b^4c^3-3a^4b^3c^2d+3a^5b^2cd^2-a^6bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="fricas")
```

```
[Out] -3/140*(9*b^2*d^3*x^3 + 14*b^2*c^3 - 40*a*b*c^2*d + 35*a^2*c*d^2 - 3*(b^2*c
*d^2 - 10*a*b*d^3)*x^2 + (2*b^2*c^2*d - 10*a*b*c*d^2 + 35*a^2*d^3)*x)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^3*c^3 - 3*a^5*b^2*c^2*d + 3*a^6*b*c*d^2 -
 a^7*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^4 +
4*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^3 + 6*(a^
2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x^2 + 4*(a^3*b
^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{4}{3}}\left(9b^2x^2d^2+30ab\,d^2x-12b^2cdx+35a^2d^2-40abcd+14b^2c^2\right)}{140\,(bx+a)^{\frac{10}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(13/3),x)
```

```
[Out] 3/140*(d*x+c)^(4/3)*(9*b^2*d^2*x^2+30*a*b*d^2*x-12*b^2*c*d*x+35*a^2*d^2-40*
a*b*c*d+14*b^2*c^2)/(b*x+a)^(10/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3
*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(13/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(13/3), x)
```

mupad [B] time = 1.02, size = 203, normalized size = 2.01

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{105\,a^2\,c\,d^2-120\,a\,b\,c^2\,d+42\,b^2\,c^3}{140\,b^3\,(a\,d-b\,c)^3}+\frac{x\,\left(105\,a^2\,d^3-30\,a\,b\,c\,d^2+6\,b^2\,c^2\,d\right)}{140\,b^3\,(a\,d-b\,c)^3}+\frac{27\,d^3\,x^3}{140\,b\,(a\,d-b\,c)^3}+\frac{9\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(13/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((42*b^2*c^3 + 105*a^2*c*d^2 - 120*a*b*c^2*d)/(140*b^3*(a*
d - b*c)^3) + (x*(105*a^2*d^3 + 6*b^2*c^2*d - 30*a*b*c*d^2))/(140*b^3*(a*d
- b*c)^3) + (27*d^3*x^3)/(140*b*(a*d - b*c)^3) + (9*d^2*x^2*(10*a*d - b*c))
/(140*b^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/3) + (a^3*(a + b*x)^(1/3))/b^3
 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(13/3),x)
```

```
[Out] Timed out
```

3.1459 $\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx$

Optimal. Leaf size=136

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{4/3}}{1820(a+bx)^{4/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{4/3}}{455(a+bx)^{7/3}(bc-ad)^3}+\frac{27d(c+dx)^{4/3}}{130(a+bx)^{10/3}(bc-ad)^2}-\frac{3(c+dx)^{4/3}}{13(a+bx)^{13/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3))/(13*(b*c - a*d)*(a + b*x)^(13/3)) + (27*d*(c + d*x)^(4
/3))/(130*(b*c - a*d)^2*(a + b*x)^(10/3)) - (81*d^2*(c + d*x)^(4/3))/(455*(
b*c - a*d)^3*(a + b*x)^(7/3)) + (243*d^3*(c + d*x)^(4/3))/(1820*(b*c - a*d)
^4*(a + b*x)^(4/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{16/3}}\,dx &= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}-\frac{(9d)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{13/3}}\,dx}{13(bc-ad)}\\
&= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}+\frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}}+\frac{\left(27d^2\right)\int \frac{\sqrt[3]{c+dx}}{(a+bx)^{10/3}}\,dx}{65(bc-ad)^2}\\
&= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}+\frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}}-\frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}}-\frac{\left(81d^3\right)\int
\end{aligned}$$

$$= -\frac{3(c+dx)^{4/3}}{13(bc-ad)(a+bx)^{13/3}}+\frac{27d(c+dx)^{4/3}}{130(bc-ad)^2(a+bx)^{10/3}}-\frac{81d^2(c+dx)^{4/3}}{455(bc-ad)^3(a+bx)^{7/3}}+\frac{243a}{1820(bc-}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{4/3}\left(455a^3d^3+195a^2bd^2(3dx-4c)+39ab^2d\left(14c^2-12cdx+9d^2x^2\right)+b^3\left(-140c^3+126c^2dx-108cd^2x^2+81d^3x^3\right)\right)}{1820(a+bx)^{13/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (3*(c + d*x)^(4/3)*(455*a^3*d^3 + 195*a^2*b*d^2*(-4*c + 3*d*x) + 39*a*b^2*d
*(14*c^2 - 12*c*d*x + 9*d^2*x^2) + b^3*(-140*c^3 + 126*c^2*d*x - 108*c*d^2*
x^2 + 81*d^3*x^3)))/(1820*(b*c - a*d)^4*(a + b*x)^(13/3))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{4/3}\left(\frac{140b^3(c+dx)^3}{(a+bx)^3}-\frac{546b^2d(c+dx)^2}{(a+bx)^2}+\frac{780bd^2(c+dx)}{a+bx}-455d^3\right)}{1820(a+bx)^{4/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/3)/(a + b*x)^(16/3),x]
```

```
[Out] (-3*(c + d*x)^(4/3)*(-455*d^3 + (780*b*d^2*(c + d*x))/(a + b*x) - (546*b^2*
d*(c + d*x)^2)/(a + b*x)^2 + (140*b^3*(c + d*x)^3)/(a + b*x)^3))/(1820*(b*c
 - a*d)^4*(a + b*x)^(4/3))
```

fricas [B] time = 1.11, size = 533, normalized size = 3.92

$$\frac{3\left(81\,b^3d^4x^4-140\,b^3c^4+546\,ab^2c^3d-780\,a^2bc^2d^2+455\,a^3cd^3-27\left(b^3cd^3-13\,ab^2d^4\right)x^3+9\left(2\,b^3c^2d^2-13\,ab^2cd^3+65\,a^2bd^4\right)x^2-\left(14\,b^3c^3d-78\,ab^2c^2d^2+195\,a^2bcd^3-455\,a^3d^4\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{1820\left(a^5b^4c^4-4\,a^6b^3c^3d+6\,a^7b^2c^2d^2-4\,a^8bcd^3+a^9d^4+\left(b^9c^4-4\,ab^8c^3d+6\,a^2b^7c^2d^2-4\,a^3b^6cd^3+a^4b^5d^4\right)x^5+5\left(ab^8c^4-4\,a^2b^7c^3d+6\,a^3b^6c^2d^2-4\,a^4b^5cd^3+a^5b^4d^4\right)x^4+10\left(a^2b^7c^4-4\,a^3b^6c^3d+6\,a^4b^5c^2d^2-4\,a^5b^4cd^3+a^6b^3d^4\right)x^3+10\left(a^3b^6c^4-4\,a^4b^5c^3d+6\,a^5b^4c^2d^2-4\,a^6b^3cd^3+a^7b^2d^4\right)x^2+5\left(a^4b^5c^4-4\,a^5b^4c^3d+6\,a^6b^3c^2d^2-4\,a^7b^2cd^3+a^8bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 3/1820*(81*b^3*d^4*x^4 - 140*b^3*c^4 + 546*a*b^2*c^3*d - 780*a^2*b*c^2*d^2
+ 455*a^3*c*d^3 - 27*(b^3*c*d^3 - 13*a*b^2*d^4)*x^3 + 9*(2*b^3*c^2*d^2 - 13
*a*b^2*c*d^3 + 65*a^2*b*d^4)*x^2 - (14*b^3*c^3*d - 78*a*b^2*c^2*d^2 + 195*a
^2*b*c*d^3 - 455*a^3*d^4)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^5*b^4*c^4 -
 4*a^6*b^3*c^3*d + 6*a^7*b^2*c^2*d^2 - 4*a^8*b*c*d^3 + a^9*d^4 + (b^9*c^4 -
 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^5 + 5
*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b
^4*d^4)*x^4 + 10*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5
*b^4*c*d^3 + a^6*b^3*d^4)*x^3 + 10*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x^2 + 5*(a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{4}{3}}\left(81b^3d^3x^3+351a\,b^2d^3x^2-108b^3c\,d^2x^2+585a^2b\,d^3x-468a\,b^2c\,d^2x+126b^3c^2dx+455a^3d^3-780a^2bc\,d^2+546a\,b^2c^2d-140b^3c^3\right)}{1820\,(bx+a)^{\frac{13}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/3)/(b*x+a)^(16/3),x)
```

```
[Out] 3/1820*(d*x+c)^(4/3)*(81*b^3*d^3*x^3+351*a*b^2*d^3*x^2-108*b^3*c*d^2*x^2+58
5*a^2*b*d^3*x-468*a*b^2*c*d^2*x+126*b^3*c^2*d*x+455*a^3*d^3-780*a^2*b*c*d^2
+546*a*b^2*c^2*d-140*b^3*c^3)/(b*x+a)^(13/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b
^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/3)/(b*x+a)^(16/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/3)/(b*x + a)^(16/3), x)
```

mupad [B] time = 1.15, size = 293, normalized size = 2.15

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^4\,x^4}{1820\,b\,(a\,d-b\,c)^4}-\frac{-1365\,a^3\,c\,d^3+2340\,a^2\,b\,c^2\,d^2-1638\,a\,b^2\,c^3\,d+420\,b^3\,c^4}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{x\left(1365\,a^3\,d^4-585\,a^2\,b\,c\,d^3+234\,a\,b^2\,c^2\,d^2-42\,b^3\,c^3\,d\right)}{1820\,b^4\,(a\,d-b\,c)^4}+\frac{81\,d^3\,x^3\,(13\,a\,d-b\,c)}{1820\,b^2\,(a\,d-b\,c)^4}+\frac{27\,d^2\,x^2\left(65\,a^2\,d^2-13\,a\,b\,c\,d+2\,b^2\,c^2\right)}{1820\,b^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/3}+\frac{a^4\,(a+b\,x)^{1/3}}{b^4}+\frac{6\,a^2\,x^2\,(a+b\,x)^{1/3}}{b^2}+\frac{4\,a\,x^3\,(a+b\,x)^{1/3}}{b}+\frac{4\,a^3\,x\,(a+b\,x)^{1/3}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/3)/(a + b*x)^(16/3),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^4*x^4)/(1820*b*(a*d - b*c)^4) - (420*b^3*c^4 - 136
5*a^3*c*d^3 + 2340*a^2*b*c^2*d^2 - 1638*a*b^2*c^3*d)/(1820*b^4*(a*d - b*c)^
4) + (x*(1365*a^3*d^4 - 42*b^3*c^3*d + 234*a*b^2*c^2*d^2 - 585*a^2*b*c*d^3)
)/(1820*b^4*(a*d - b*c)^4) + (81*d^3*x^3*(13*a*d - b*c))/(1820*b^2*(a*d - b
*c)^4) + (27*d^2*x^2*(65*a^2*d^2 + 2*b^2*c^2 - 13*a*b*c*d))/(1820*b^3*(a*d
- b*c)^4)))/(x^4*(a + b*x)^(1/3) + (a^4*(a + b*x)^(1/3))/b^4 + (6*a^2*x^2*(
a + b*x)^(1/3))/b^2 + (4*a*x^3*(a + b*x)^(1/3))/b + (4*a^3*x*(a + b*x)^(1/3
))/b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/3)/(b*x+a)**(16/3),x)
```

```
[Out] Timed out
```

3.1460 $\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=216

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+$$

Rubi [A] time = 0.09, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{(bc-ad)^2\log(a+bx)}{9b^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{3b^{2/3}d^{7/3}}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}d^{7/3}}-\frac{2\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (-2*(b*c - a*d)*(a + b*x)^(1/3)*(c + d*x)^(2/3))/(3*d^2) + ((a + b*x)^(4/3)
*(c + d*x)^(2/3))/(2*d) - (2*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c
 + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/
3)) - ((b*c - a*d)^2*Log[a + b*x])/(9*b^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log
[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*b^(2/3)*d^(7
/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx=\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}-\frac{(2(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d}$$

$$=-\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}+\frac{\left(2(bc-ad)^2\right)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{9d^2}$$

$$=-\frac{2(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^2}+\frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d}-\frac{2(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{}}\right.}{3\sqrt{3}b^{2/3}d^{7/3}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{7/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.50, size = 292, normalized size = 1.35

$$-\frac{2(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{9b^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{9b^{2/3}d^{7/3}}+\frac{2(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,b^{2/3}d^{7/3}}+\frac{(ad-bc)^2\left(\frac{7d(a+bx)^{4/3}}{(c+dx)^{4/3}}-\frac{4b\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{6d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((-(b*c) + a*d)^2*((7*d*(a + b*x)^(4/3))/(c + d*x)^(4/3) - (4*b*(a + b*x)^(
1/3))/(c + d*x)^(1/3)))/(6*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) + (2*(b*c
- a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c
 + d*x)^(1/3))])/(3*Sqrt[3]*b^(2/3)*d^(7/3)) - (2*(b*c - a*d)^2*Log[b^(1/3)
 - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*b^(2/3)*d^(7/3)) + ((b*c
- a*d)^2*Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)
*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*b^(2/3)*d^(7/3))
```

fricas [B] time = 1.07, size = 740, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b^2*d)^(1
/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)^(1/3)*(b*d*x + b*c))
*sqrt((-b^2*d)^(1/3)/d)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)
*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 4*(b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d
 - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*b^3*d^2*x - 4*b^3*c*d + 7*a*
b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*d^3), 1/18*(12*sqrt(1/3)*(b^
3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*
x + b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + 2*(b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b
^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))
/(d*x + c)) - 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b^2*d)^(2/3)*log(((b*x +
a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)) + 3*(3*
b^3*d^2*x - 4*b^3*c*d + 7*a*b^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(b^2*
d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

```
[Out] int((b*x+a)^(4/3)/(d*x+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{4/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(1/3),x)
```

```
[Out] int((a + b*x)^(4/3)/(c + d*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(1/3), x)
```

3.1461 $\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=171

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

Rubi [A] time = 0.04, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] ((a + b*x)^(1/3)*(c + d*x)^(2/3))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^
(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*b^(2/3)
*d^(4/3)) + ((b*c - a*d)*Log[a + b*x])/(6*b^(2/3)*d^(4/3)) + ((b*c - a*d)*L
og[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3)*d^
(4/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx=\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}-\frac{(bc-ad)\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d}$$

$$=\frac{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,b^{2/3}d^{4/3}}+\frac{(bc-ad)\log(a+bx)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)}{}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{4/3}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 7.30, size = 297, normalized size = 1.74

$$\frac{\sqrt[3]{d}\sqrt[3]{a+bx}\left(\frac{(bc-ad)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{3b^{2/3}d^{4/3}}+\frac{(ad-bc)\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{6b^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}b^{2/3}d^{4/3}}+\frac{(c+dx)^{2/3}\sqrt[3]{ad+b(c+dx)-bc}}{d^{4/3}}\right)}{\sqrt[3]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/(c + d*x)^(1/3),x]
```

```
[Out] (d^(1/3)*(a + b*x)^(1/3)*(((c + d*x)^(2/3)*(-(b*c) + a*d + b*(c + d*x))^(1/
3))/d^(4/3) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3
)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d +
b*(c + d*x))^(1/3)])/(3*b^(2/3)*d^(4/3)) + ((-(b*c) + a*d)*Log[b^(2/3)*(c +
 d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) +
(-(b*c) + a*d + b*(c + d*x))^(2/3)])/(6*b^(2/3)*d^(4/3))))/(a*d + b*d*x)^(1
/3)
```

fricas [B] time = 1.13, size = 618, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d + 3*(-b^2*d)^(1
/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + 3*sqrt(1/3)*(2*(b*x + a)^(2/3)*(d*x
 + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (-b^2*d)
^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) - (-b^2*d)^(2/3)*(b*c - a*d)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(
d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)^(2/3
)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*
x + c))/(d*x + c)))/(b^2*d^2), 1/6*(6*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2*d
 - 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b^2*d)^(1/3)/d)*arctan(sqrt(1/3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) - (-b^2*d)^(2/3)*(b*c -
a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1
/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) + 2*(-b^2*d)
^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3
)*(d*x + c))/(d*x + c)))/(b^2*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
[Out] int((b*x+a)^(1/3)/(d*x+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(1/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(1/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(1/3),x)
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(1/3), x)
```

3.1462 $\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Rubi [A] time = 0.01, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*
(a + b*x)^(1/3))])/(b^(2/3)*d^(1/3))) - Log[a + b*x]/(2*b^(2/3)*d^(1/3)) -
(3*Log[-1 + (b^(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*b^(2/3
)*d^(1/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{b^{2/3}\sqrt[3]{d}}-\frac{\log(a+bx)}{2b^{2/3}\sqrt[3]{d}}-\frac{3\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2b^{2/3}\sqrt[3]{d}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.56

$$\frac{3\sqrt[3]{a+bx}\sqrt[3]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(a + b*x)^(1/3)*((b*(c + d*x))/(b*c - a*d))^(1/3)*Hypergeometric2F1[1/3,
 1/3, 4/3, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.14, size = 177, normalized size = 1.40

$$\frac{\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{2b^{2/3}\sqrt[3]{d}}-\frac{\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{b^{2/3}\sqrt[3]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}\sqrt[3]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c
 + d*x)^(1/3))])/(b^(2/3)*d^(1/3)) - Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3)]/(b^(2/3)*d^(1/3)) + Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/
3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)]/(2
*b^(2/3)*d^(1/3))
```

fricas [B] time = 0.71, size = 519, normalized size = 4.12

$$\left[\frac{\sqrt{3}\, b d \sqrt{\frac{\left(-b^{2} d\right)^{\frac{1}{3}}}{d}} \log\left(3 \, b^{2} d x + b^{2} c + 2 \, a b d + 3 \, \left(-b^{2} d\right)^{\frac{1}{3}} {\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} b + \sqrt{3} \left(2 \, {\left(b x + a\right)}^{\frac{2}{3}} {\left(d x + c\right)}^{\frac{1}{3}} b d - \left(-b^{2} d\right)^{\frac{2}{3}} {\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} + \left(-b^{2} d\right)^{\frac{1}{3}} {\left(b d x + b c\right)}\right) \sqrt{\frac{\left(-b^{2} d\right)^{\frac{1}{3}}}{d}}\right) + \left(-b^{2} d\right)^{\frac{2}{3}} \log\left(\frac{{\left(b x + a\right)}^{\frac{2}{3}} {\left(d x + c\right)}^{\frac{1}{3}} b d + \left(-b^{2} d\right)^{\frac{2}{3}} {\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} - \left(-b^{2} d\right)^{\frac{1}{3}} {\left(b d x + b c\right)}}{d x + c}\right) - 2 \, \left(-b^{2} d\right)^{\frac{2}{3}} \log\left(\frac{{\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} b d - \left(-b^{2} d\right)^{\frac{2}{3}} {\left(d x + c\right)}}{d x + c}\right)}{2 \, b^{2} d}, \frac{2 \, \sqrt{3}\, b d \sqrt{-\frac{\left(-b^{2} d\right)^{\frac{1}{3}}}{d}} \arctan\left(\frac{\sqrt{3} \left(2 \, \left(-b^{2} d\right)^{\frac{2}{3}} {\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} - \left(-b^{2} d\right)^{\frac{1}{3}} {\left(b d x + b c\right)}\right) \sqrt{-\frac{\left(-b^{2} d\right)^{\frac{1}{3}}}{d}}}{3 \, {\left(b^{2} d x + b^{2} c\right)}}\right) + \left(-b^{2} d\right)^{\frac{2}{3}} \log\left(\frac{{\left(b x + a\right)}^{\frac{2}{3}} {\left(d x + c\right)}^{\frac{1}{3}} b d + \left(-b^{2} d\right)^{\frac{2}{3}} {\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} - \left(-b^{2} d\right)^{\frac{1}{3}} {\left(b d x + b c\right)}}{d x + c}\right) - 2 \, \left(-b^{2} d\right)^{\frac{2}{3}} \log\left(\frac{{\left(b x + a\right)}^{\frac{1}{3}} {\left(d x + c\right)}^{\frac{2}{3}} b d - \left(-b^{2} d\right)^{\frac{2}{3}} {\left(d x + c\right)}}{d x + c}\right)}{2 \, b^{2} d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b^2*d)^(1/3)/d)*log(3*b^2*d*x + b^2*c + 2*a*b*d +
3*(-b^2*d)^(1/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b + sqrt(3)*(2*(b*x + a)^(
2/3)*(d*x + c)^(1/3)*b*d - (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) +
 (-b^2*d)^(1/3)*(b*d*x + b*c))*sqrt((-b^2*d)^(1/3)/d)) + (-b^2*d)^(2/3)*log
(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x
 + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*l
og(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x +
c)))/(b^2*d), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b^2*d)^(1/3)/d)*arctan(1/3*sqrt(3)
*(2*(-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (-b^2*d)^(1/3)*(b*d*x
+ b*c))*sqrt(-(-b^2*d)^(1/3)/d)/(b^2*d*x + b^2*c)) + (-b^2*d)^(2/3)*log(((b
*x + a)^(2/3)*(d*x + c)^(1/3)*b*d + (-b^2*d)^(2/3)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (-b^2*d)^(1/3)*(b*d*x + b*c))/(d*x + c)) - 2*(-b^2*d)^(2/3)*log((
(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (-b^2*d)^(2/3)*(d*x + c))/(d*x + c)))
/(b^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b x + a\right)}^{\frac{2}{3}} {\left(d x + c\right)}^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b x +a \right)^{\frac{2}{3}} \left(d x +c \right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

```
[Out] int(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(b x + a\right)}^{\frac{2}{3}} {\left(d x + c\right)}^{\frac{1}{3}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{2/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(1/3)), x)
```

3.1463 $\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx = -\frac{3(c+dx)^{2/3}}{2(bc-ad)(a+bx)^{2/3}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{3(c+dx)^{2/3}}{2(a+bx)^{2/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(2*(b*c - a*d)*(a + b*x)^(2/3))
```

fricas [A] time = 1.32, size = 42, normalized size = 1.31

$$-\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(abc-a^2d+\left(b^2c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="fricas")
[Out] -3/2*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{3\,(dx+c)^{\frac{2}{3}}}{2\,(bx+a)^{\frac{2}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x)
[Out] 3/2/(b*x+a)^(2/3)*(d*x+c)^(2/3)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{5/3}\,{(c+d\,x)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(1/3),x)
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(1/3)), x)
```

3.1464 $\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d(c+dx)^{2/3}}{10(a+bx)^{2/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{5(a+bx)^{5/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(5*(b*c - a*d)*(a + b*x)^(5/3)) + (9*d*(c + d*x)^(2/3)
)/(10*(b*c - a*d)^2*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx &= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{5(bc-ad)}\\ &= -\frac{3(c+dx)^{2/3}}{5(bc-ad)(a+bx)^{5/3}}+\frac{9d(c+dx)^{2/3}}{10(bc-ad)^2(a+bx)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{3(c+dx)^{2/3}(5ad-2bc+3bdx)}{10(a+bx)^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(-2*b*c + 5*a*d + 3*b*d*x))/(10*(b*c - a*d)^2*(a + b*x)^
(5/3))
```

IntegrateAlgebraic [A] time = 0.16, size = 51, normalized size = 0.77

$$\frac{3(c+dx)^{5/3}\left(\frac{5d(a+bx)}{c+dx}-2b\right)}{10(a+bx)^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(-2*b + (5*d*(a + b*x))/(c + d*x)))/(10*(b*c - a*d)^2*(a
 + b*x)^(5/3))
```

fricas [B] time = 1.40, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx-2\,bc+5\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{10\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/10*(3*b*d*x - 2*b*c + 5*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{2}{3}}\,(3bdx+5ad-2bc)}{10\,(bx+a)^{\frac{5}{3}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/10*(d*x+c)^(2/3)*(3*b*d*x+5*a*d-2*b*c)/(b*x+a)^(5/3)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x)^{8/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(1/3)), x)
```

3.1465 $\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2(c+dx)^{2/3}}{40(a+bx)^{2/3}(bc-ad)^3}+\frac{9d(c+dx)^{2/3}}{20(a+bx)^{5/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{8(a+bx)^{8/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(8*(b*c - a*d)*(a + b*x)^(8/3)) + (9*d*(c + d*x)^(2/3)
)/(20*(b*c - a*d)^2*(a + b*x)^(5/3)) - (27*d^2*(c + d*x)^(2/3))/(40*(b*c -
a*d)^3*(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx &= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{4(bc-ad)}\\ &= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}+\frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{5/3}\sqrt[3]{c+dx}}\,dx}{20(bc-ad)^2}\\ &= -\frac{3(c+dx)^{2/3}}{8(bc-ad)(a+bx)^{8/3}}+\frac{9d(c+dx)^{2/3}}{20(bc-ad)^2(a+bx)^{5/3}}-\frac{27d^2(c+dx)^{2/3}}{40(bc-ad)^3(a+bx)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.76

$$-\frac{3(c+dx)^{2/3}\left(20a^2d^2+8abd(3dx-2c)+b^2\left(5c^2-6cdx+9d^2x^2\right)\right)}{40(a+bx)^{8/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*(20*a^2*d^2 + 8*a*b*d*(-2*c + 3*d*x) + b^2*(5*c^2 - 6*c
*d*x + 9*d^2*x^2)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

IntegrateAlgebraic [A] time = 0.17, size = 73, normalized size = 0.72

$$-\frac{3(c+dx)^{8/3}\left(\frac{20d^2(a+bx)^2}{(c+dx)^2}-\frac{16bd(a+bx)}{c+dx}+5b^2\right)}{40(a+bx)^{8/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^2 + (20*d^2*(a + b*x)^2)/(c + d*x)^2 - (16*b*d*(a
+ b*x))/(c + d*x)))/(40*(b*c - a*d)^3*(a + b*x)^(8/3))
```

fricas [B] time = 1.59, size = 251, normalized size = 2.49

$$-\frac{3\left(9b^2d^2x^2+5b^2c^2-16abcd+20a^2d^2-6\left(b^2cd-4abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\left(a^3b^3c^3-3a^4b^2c^2d+3a^5bcd^2-a^6d^3+\left(b^6c^3-3ab^5c^2d+3a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3a^2b^4c^2d+3a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3a^3b^3c^2d+3a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/40*(9*b^2*d^2*x^2 + 5*b^2*c^2 - 16*a*b*c*d + 20*a^2*d^2 - 6*(b^2*c*d - 4
*a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a
^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^
2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3
)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{2}{3}}\left(9b^2x^2d^2+24ab\,d^2x-6b^2cdx+20a^2d^2-16abcd+5b^2c^2\right)}{40\,(bx+a)^{\frac{8}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/40*(d*x+c)^(2/3)*(9*b^2*d^2*x^2+24*a*b*d^2*x-6*b^2*c*d*x+20*a^2*d^2-16*a*
b*c*d+5*b^2*c^2)/(b*x+a)^(8/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(1/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}\sqrt[3]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(1/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(1/3)), x)
```

3.1466 $\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3(c+dx)^{2/3}}{440(a+bx)^{2/3}(bc-ad)^4}-\frac{81d^2(c+dx)^{2/3}}{220(a+bx)^{5/3}(bc-ad)^3}+\frac{27d(c+dx)^{2/3}}{88(a+bx)^{8/3}(bc-ad)^2}-\frac{3(c+dx)^{2/3}}{11(a+bx)^{11/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3))/(11*(b*c - a*d)*(a + b*x)^(11/3)) + (27*d*(c + d*x)^(2
/3))/(88*(b*c - a*d)^2*(a + b*x)^(8/3)) - (81*d^2*(c + d*x)^(2/3))/(220*(b*
c - a*d)^3*(a + b*x)^(5/3)) + (243*d^3*(c + d*x)^(2/3))/(440*(b*c - a*d)^4*
(a + b*x)^(2/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{14/3}\sqrt[3]{c+dx}}\,dx &= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}-\frac{(9d)\int \frac{1}{(a+bx)^{11/3}\sqrt[3]{c+dx}}\,dx}{11(bc-ad)}\\
&= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}+\frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}}+\frac{\left(27d^2\right)\int \frac{1}{(a+bx)^{8/3}\sqrt[3]{c+dx}}\,dx}{44(bc-ad)^2}\\
&= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}+\frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}}-\frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}}-\frac{(81}{}\\
&= -\frac{3(c+dx)^{2/3}}{11(bc-ad)(a+bx)^{11/3}}+\frac{27d(c+dx)^{2/3}}{88(bc-ad)^2(a+bx)^{8/3}}-\frac{81d^2(c+dx)^{2/3}}{220(bc-ad)^3(a+bx)^{5/3}}+\frac{}{44}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.87

$$\frac{3(c+dx)^{2/3}\left(220a^3d^3+132a^2bd^2(3dx-2c)+33ab^2d\left(5c^2-6cdx+9d^2x^2\right)+b^3\left(-40c^3+45c^2dx-54cd^2x^2+81d^3x^3\right)\right)}{440(a+bx)^{11/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(2/3)*(220*a^3*d^3 + 132*a^2*b*d^2*(-2*c + 3*d*x) + 33*a*b^2*d
*(5*c^2 - 6*c*d*x + 9*d^2*x^2) + b^3*(-40*c^3 + 45*c^2*d*x - 54*c*d^2*x^2 +
 81*d^3*x^3)))/(440*(b*c - a*d)^4*(a + b*x)^(11/3))
```

IntegrateAlgebraic [A] time = 0.18, size = 95, normalized size = 0.70

$$\frac{3(c+dx)^{11/3}\left(\frac{165b^2d(a+bx)}{c+dx}+\frac{220d^3(a+bx)^3}{(c+dx)^3}-\frac{264bd^2(a+bx)^2}{(c+dx)^2}-40b^3\right)}{440(a+bx)^{11/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x]
```

```
[Out] (3*(c + d*x)^(11/3)*(-40*b^3 + (220*d^3*(a + b*x)^3)/(c + d*x)^3 - (264*b*d
^2*(a + b*x)^2)/(c + d*x)^2 + (165*b^2*d*(a + b*x))/(c + d*x)))/(440*(b*c -
 a*d)^4*(a + b*x)^(11/3))
```

fricas [B] time = 1.41, size = 420, normalized size = 3.09

$$\frac{3\left(81\,b^3d^3x^3-40\,b^3c^3+165\,ab^2c^2d-264\,a^2bcd^2+220\,a^3d^3-27\left(2\,b^3cd^2-11\,ab^2d^3\right)x^2+9\left(5\,b^3c^2d-22\,ab^2cd^2+44\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{440\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/440*(81*b^3*d^3*x^3 - 40*b^3*c^3 + 165*a*b^2*c^2*d - 264*a^2*b*c*d^2 + 22
0*a^3*d^3 - 27*(2*b^3*c*d^2 - 11*a*b^2*d^3)*x^2 + 9*(5*b^3*c^2*d - 22*a*b^2
*c*d^2 + 44*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{2}{3}}\left(81b^3d^3x^3+297a\,b^2d^3x^2-54b^3c\,d^2x^2+396a^2b\,d^3x-198a\,b^2c\,d^2x+45b^3c^2dx+220a^3d^3-264a^2bc\,d^2+165a\,b^2c^2d-40b^3c^3\right)}{440\,(bx+a)^{\frac{11}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x)
```

```
[Out] 3/440*(d*x+c)^(2/3)*(81*b^3*d^3*x^3+297*a*b^2*d^3*x^2-54*b^3*c*d^2*x^2+396*
a^2*b*d^3*x-198*a*b^2*c*d^2*x+45*b^3*c^2*d*x+220*a^3*d^3-264*a^2*b*c*d^2+16
```

```
5*a*b^2*c^2*d-40*b^3*c^3)/(b*x+a)^(11/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{14}{3}}(dx+c)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(14/3)/(d*x+c)^(1/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(14/3)*(d*x + c)^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{14/3}\,(c+d\,x)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)),x)
[Out] int(1/((a + b*x)^(14/3)*(c + d*x)^(1/3)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(14/3)/(d*x+c)**(1/3),x)
[Out] Timed out
```

3.1467 $\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}} dx$

Optimal. Leaf size=216

$$-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}-\frac{5(a+bx)^{2/3}\cdots}{}$$

Rubi [A] time = 0.08, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$-\frac{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}{6d^2}-\frac{5(bc-ad)^2\log(c+dx)}{18\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{b}\,\sqrt[3]{c+dx}}-1\right)}{6\sqrt[3]{b}\,d^{8/3}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3))/(6*d^2) + ((a + b*x)^(5/3)
*(c + d*x)^(1/3))/(2*d) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a
 + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(3*Sqrt[3]*b^(1/3)*d^(8/
3)) - (5*(b*c - a*d)^2*Log[c + d*x])/(18*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^
2*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(6*b^(1/3)
*d^(8/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/3}}{(c+dx)^{2/3}}\,dx &= \frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}-\frac{(5(bc-ad))\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx}{6d}\\
&= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}+\frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{9d^2}\\
&= -\frac{5(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}{6d^2}+\frac{(a+bx)^{5/3}\sqrt[3]{c+dx}}{2d}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\,\sqrt[3]{a+\cdots}}{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+\cdots}}\right)}{3\sqrt{3}\,\sqrt[3]{b}\,d^{8/3}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.34

$$\frac{3(a+bx)^{8/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{d(a+bx)}{ad-bc}\right)}{8b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(8/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 8/3, 11/3, (d*(a + b*x))/(-(b*c) + a*d)])/(8*b*(c + d*x)^(2/3))
```

IntegrateAlgebraic [A] time = 0.30, size = 285, normalized size = 1.32

$$\frac{5(bc-ad)^2\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{18\sqrt[3]{b}d^{8/3}}-\frac{5(bc-ad)^2\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{9\sqrt[3]{b}d^{8/3}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}d^{8/3}}+\frac{\sqrt[3]{c+dx}(ad-bc)^2\left(8d-\frac{5b(c+dx)}{a+bx}\right)}{6d^2\sqrt[3]{a+bx}\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((-(b*c) + a*d)^2*(c + d*x)^(1/3)*(8*d - (5*b*(c + d*x))/(a + b*x)))/(6*d^2
*(a + b*x)^(1/3)*(d - (b*(c + d*x))/(a + b*x))^2) + (5*(b*c - a*d)^2*ArcTan
[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))]
)/(3*Sqrt[3]*b^(1/3)*d^(8/3)) - (5*(b*c - a*d)^2*Log[d^(1/3) - (b^(1/3)*(c
+ d*x)^(1/3))/(a + b*x)^(1/3)])/(9*b^(1/3)*d^(8/3)) + (5*(b*c - a*d)^2*Log[
d^(2/3) + (b^(1/3)*d^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c +
 d*x)^(2/3))/(a + b*x)^(2/3)])/(18*b^(1/3)*d^(8/3))
```

fricas [B] time = 1.58, size = 741, normalized size = 3.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/18*(15*sqrt(1/3)*(b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt((-b*d^2)^(
1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)
*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2)^(1/3)*(b*d*x + a*d
))*sqrt((-b*d^2)^(1/3)/b)) - 10*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2
/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b
*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
- (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(3*b^2*d^3*x - 5*b^2*c*d^2 +
 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*d^4), 1/18*(30*sqrt(1/3)*(b
^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1
/3)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d
*x + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 10*(b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(-b*d^2)^(2/3)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (
-b*d^2)^(2/3)*(b*x + a))/(b*x + a)) + 5*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(-b
*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a)) + 3*(
3*b^2*d^3*x - 5*b^2*c*d^2 + 8*a*b*d^3)*(b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*
d^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(5/3)/(d*x+c)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/3)/(d*x + c)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/3}}{(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(5/3)/(c + d*x)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{3}}}{(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(5/3)/(c + d*x)**(2/3), x)
```

3.1468 $\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=169

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Rubi [A] time = 0.04, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {50, 59}

$$\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{\sqrt[3]{b}\,d^{5/3}}+\frac{2(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] ((a + b*x)^(2/3)*(c + d*x)^(1/3))/d + (2*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*
d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))])/(Sqrt[3]*b^(1/
3)*d^(5/3)) + ((b*c - a*d)*Log[c + d*x])/(3*b^(1/3)*d^(5/3)) + ((b*c - a*d)
*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(b^(1/3)*d^
(5/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{2/3}}{(c+dx)^{2/3}}\,dx &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}-\frac{(2(bc-ad))\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{2/3}}\,dx}{3d}\\ &= \frac{(a+bx)^{2/3}\sqrt[3]{c+dx}}{d}+\frac{2(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-ad)\log(c+dx)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{(bc-a}{}\end{aligned}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.43

$$\frac{3(a+bx)^{5/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] (3*(a + b*x)^(5/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 5/3, 8/3, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(2/3))
```

IntegrateAlgebraic [A] time = 7.90, size = 298, normalized size = 1.76

$$\frac{d^{2/3}(a+bx)^{2/3}\left(\frac{(ad-bc)\log\left(\sqrt[3]{b}\sqrt[3]{c+dx}\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{3\sqrt[3]{b}\,d^{5/3}}+\frac{\sqrt[3]{c+dx}\,(ad+b(c+dx)-bc)^{2/3}}{d^{5/3}}+\frac{2(bc-ad)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\sqrt[3]{c+dx}\right)}{3\sqrt[3]{b}\,d^{5/3}}-\frac{2(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt{3}\sqrt[3]{b}\,d^{5/3}}\right)}{(ad+bdx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(2/3)/(c + d*x)^(2/3),x]
```

```
[Out] (d^(2/3)*(a + b*x)^(2/3)*(((c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(2/
3))/d^(5/3) - (2*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1
/3)*(c + d*x)^(1/3) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*b^(1
/3)*d^(5/3)) + (2*(b*c - a*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*
d + b*(c + d*x))^(1/3)])/(3*b^(1/3)*d^(5/3)) + ((-(b*c) + a*d)*Log[b^(2/3)*
(c + d*x)^(2/3) + b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3
) + (-(b*c) + a*d + b*(c + d*x))^(2/3)])/(3*b^(1/3)*d^(5/3))))/(a*d + b*d*x
)^(2/3)
```

fricas [B] time = 1.53, size = 619, normalized size = 3.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2 - 3*sqrt(1/3)*(b^2*c*d - a*b*
d^2)*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 - 3*(-b*d^2)^(
1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - 3*sqrt(1/3)*(2*(b*x + a)^(1/3)*(d*
x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (-b*d^2
)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) + 2*(-b*d^2)^(2/3)*(b*c - a*
d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*
x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b
*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))/(b*x + a)))/(b*d^3), 1/3*(3*(b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d^2
- 6*sqrt(1/3)*(b^2*c*d - a*b*d^2)*sqrt(-(-b*d^2)^(1/3)/b)*arctan(sqrt(1/3)*
(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x +
 a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) + 2*(-b*d^2)^(2/3)*(b*c -
 a*d)*log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/
(b*x + a)) - (-b*d^2)^(2/3)*(b*c - a*d)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3
)*b*d + (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*
d*x + a*d))/(b*x + a)))/(b*d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
[Out] int((b*x+a)^(2/3)/(d*x+c)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{2}{3}}}{(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(2/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(2/3)/(d*x + c)^(2/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{2/3}}{(c+d\,x)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(2/3)/(c + d*x)^(2/3),x)
[Out] int((a + b*x)^(2/3)/(c + d*x)^(2/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{2}{3}}}{(c+dx)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(2/3)/(d*x+c)**(2/3),x)
[Out] Integral((a + b*x)**(2/3)/(c + d*x)**(2/3), x)
```

3.1469 $\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=126

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}$$

Rubi [A] time = 0.02, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {59}

$$-\frac{3\log\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}-1\right)}{2\sqrt[3]{b}d^{2/3}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*
(c + d*x)^(1/3))])/(b^(1/3)*d^(2/3))) - Log[c + d*x]/(2*b^(1/3)*d^(2/3)) -
(3*Log[-1 + (d^(1/3)*(a + b*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3))])/(2*b^(1/3
)*d^(2/3))
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx}(c+dx)^{2/3}}\,dx=-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{\sqrt[3]{b}d^{2/3}}-\frac{\log(c+dx)}{2\sqrt[3]{b}d^{2/3}}-\frac{3\log\left(-1+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{c+dx}}\right)}{2\sqrt[3]{b}d^{2/3}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.58

$$\frac{3(a+bx)^{2/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{2}{3};\frac{5}{3};\frac{d(a+bx)}{ad-bc}\right)}{2b(c+dx)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(a + b*x)^(2/3)*((b*(c + d*x))/(b*c - a*d))^(2/3)*Hypergeometric2F1[2/3,
 2/3, 5/3, (d*(a + b*x))/(-(b*c) + a*d)])/(2*b*(c + d*x)^(2/3))
```

IntegrateAlgebraic [A] time = 0.17, size = 177, normalized size = 1.40

$$\frac{\log\left(\frac{b^{2/3}(c+dx)^{2/3}}{(a+bx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+d^{2/3}\right)}{2\sqrt[3]{b}d^{2/3}}-\frac{\log\left(\sqrt[3]{d}-\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{\sqrt[3]{b}d^{2/3}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a
 + b*x)^(1/3))])/(b^(1/3)*d^(2/3)) - Log[d^(1/3) - (b^(1/3)*(c + d*x)^(1/3)
)/(a + b*x)^(1/3)]/(b^(1/3)*d^(2/3)) + Log[d^(2/3) + (b^(1/3)*d^(1/3)*(c +
d*x)^(1/3))/(a + b*x)^(1/3) + (b^(2/3)*(c + d*x)^(2/3))/(a + b*x)^(2/3)]/(2
*b^(1/3)*d^(2/3))
```

fricas [B] time = 1.24, size = 521, normalized size = 4.13

$$\left[\frac{\sqrt{3}\, bd \sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}} \log\left(-3\, bd^2 x - 2\, bcd - ad^2 - 3\left(-bd^2\right)^{\frac{1}{3}} (bx+a)^{\frac{2}{3}} (dx+c)^{\frac{1}{3}} d - \sqrt{3}\left(2\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}} bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} + \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}\right) - 2\left(-bd^2\right)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + \left(-bd^2\right)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}} bd + \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\, bd^2}, \frac{2\sqrt{3}\, bd \sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}} \arctan\left(\frac{\sqrt{3}\left(2\left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)\right)\sqrt{-\frac{\left(-bd^2\right)^{\frac{1}{3}}}{b}}}{3\left(bd^2x+ad^2\right)}\right) - 2\left(-bd^2\right)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} bd - \left(-bd^2\right)^{\frac{2}{3}}(bx+a)}{bx+a}\right) + \left(-bd^2\right)^{\frac{2}{3}} \log\left(\frac{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}} bd + \left(-bd^2\right)^{\frac{2}{3}}(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}} - \left(-bd^2\right)^{\frac{1}{3}}(bdx+ad)}{bx+a}\right)}{2\, bd^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(3)*b*d*sqrt((-b*d^2)^(1/3)/b)*log(-3*b*d^2*x - 2*b*c*d - a*d^2 -
 3*(-b*d^2)^(1/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)*d - sqrt(3)*(2*(b*x + a)^
(1/3)*(d*x + c)^(2/3)*b*d - (-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (-b*d^2)^(1/3)*(b*d*x + a*d))*sqrt((-b*d^2)^(1/3)/b)) - 2*(-b*d^2)^(2/3)*
log(((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x +
 a)) + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(
2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x +
 a)))/(b*d^2), 1/2*(2*sqrt(3)*b*d*sqrt(-(-b*d^2)^(1/3)/b)*arctan(1/3*sqrt(3
)*(2*(-b*d^2)^(2/3)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x
 + a*d))*sqrt(-(-b*d^2)^(1/3)/b)/(b*d^2*x + a*d^2)) - 2*(-b*d^2)^(2/3)*log(
((b*x + a)^(2/3)*(d*x + c)^(1/3)*b*d - (-b*d^2)^(2/3)*(b*x + a))/(b*x + a))
 + (-b*d^2)^(2/3)*log(((b*x + a)^(1/3)*(d*x + c)^(2/3)*b*d + (-b*d^2)^(2/3)
*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (-b*d^2)^(1/3)*(b*d*x + a*d))/(b*x + a))
)/(b*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}\,(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

```
[Out] int(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/3)*(d*x + c)^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{1/3}\,(c+d\,x)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)),x)
[Out] int(1/((a + b*x)^(1/3)*(c + d*x)^(2/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a+bx}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(1/3)*(c + d*x)**(2/3)), x)
```

3.1470 $\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=30

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx = -\frac{3\sqrt[3]{c+dx}}{(bc-ad)\sqrt[3]{a+bx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
[Out] (3*(c + d*x)^(1/3))/((-(b*c) + a*d)*(a + b*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$-\frac{3\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x]
[Out] (-3*(c + d*x)^(1/3))/((b*c - a*d)*(a + b*x)^(1/3))
```

fricas [A] time = 0.74, size = 42, normalized size = 1.40

$$-\frac{3\,(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="fricas")
[Out] -3*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$\frac{3\,(dx+c)^{\frac{1}{3}}}{(bx+a)^{\frac{1}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x)
[Out] 3/(b*x+a)^(1/3)*(d*x+c)^(1/3)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{4}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(4/3)/(d*x+c)^(2/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(4/3)*(d*x + c)^(2/3)), x)
```

mupad [B] time = 0.83, size = 26, normalized size = 0.87

$$\frac{3\,{\left(c+d\,x\right)}^{1/3}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(4/3)*(c + d*x)^(2/3)),x)
[Out] (3*(c + d*x)^(1/3))/((a*d - b*c)*(a + b*x)^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{4}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(4/3)/(d*x+c)**(2/3),x)
[Out] Integral(1/((a + b*x)**(4/3)*(c + d*x)**(2/3)), x)
```

3.1471 $\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=66

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{9d\sqrt[3]{c+dx}}{4\sqrt[3]{a+bx}\,(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{4(a+bx)^{4/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(4*(b*c - a*d)*(a + b*x)^(4/3)) + (9*d*(c + d*x)^(1/3)
)/(4*(b*c - a*d)^2*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}-\frac{(3d)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{4(bc-ad)}\\ &= -\frac{3\sqrt[3]{c+dx}}{4(bc-ad)(a+bx)^{4/3}}+\frac{9d\sqrt[3]{c+dx}}{4(bc-ad)^2\sqrt[3]{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{3\sqrt[3]{c+dx}\,(4ad-bc+3bdx)}{4(a+bx)^{4/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(-(b*c) + 4*a*d + 3*b*d*x))/(4*(b*c - a*d)^2*(a + b*x)^(
4/3))
```

IntegrateAlgebraic [A] time = 0.12, size = 56, normalized size = 0.85

$$-\frac{3\left(\frac{b(c+dx)^{4/3}}{(a+bx)^{4/3}}-\frac{4d\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}\right)}{4(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((-4*d*(c + d*x)^(1/3))/(a + b*x)^(1/3) + (b*(c + d*x)^(4/3))/(a + b*x)
^(4/3)))/(4*(b*c - a*d)^2)
```

fricas [B] time = 2.11, size = 118, normalized size = 1.79

$$\frac{3\,(3\,bdx-bc+4\,ad)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{4\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/4*(3*b*d*x - b*c + 4*a*d)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{3\,(dx+c)^{\frac{1}{3}}\,(3bdx+4ad-bc)}{4\,(bx+a)^{\frac{4}{3}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/4*(d*x+c)^(1/3)*(3*b*d*x+4*a*d-b*c)/(b*x+a)^(4/3)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/3)*(d*x + c)^(2/3)), x)
```

mupad [B] time = 0.98, size = 71, normalized size = 1.08

$$\frac{\left(\frac{9\,d\,x}{4\,{\left(a\,d-b\,c\right)}^2}+\frac{12\,a\,d-3\,b\,c}{4\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/3}}{x\,{\left(a+b\,x\right)}^{1/3}+\frac{a\,{\left(a+b\,x\right)}^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/3)*(c + d*x)^(2/3)),x)
```

```
[Out] (((9*d*x)/(4*(a*d - b*c)^2) + (12*a*d - 3*b*c)/(4*b*(a*d - b*c)^2))*(c + d*
x)^(1/3))/(x*(a + b*x)^(1/3) + (a*(a + b*x)^(1/3))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+bx\right)^{\frac{7}{3}}\left(c+dx\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(7/3)*(c + d*x)**(2/3)), x)
```

3.1472 $\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=101

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{27d^2\sqrt[3]{c+dx}}{14\sqrt[3]{a+bx}\,(bc-ad)^3}+\frac{9d\sqrt[3]{c+dx}}{14(a+bx)^{4/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{7(a+bx)^{7/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(7*(b*c - a*d)*(a + b*x)^(7/3)) + (9*d*(c + d*x)^(1/3)
)/(14*(b*c - a*d)^2*(a + b*x)^(4/3)) - (27*d^2*(c + d*x)^(1/3))/(14*(b*c -
a*d)^3*(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}-\frac{(6d)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{7(bc-ad)}\\
&= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}+\frac{\left(9d^2\right)\int \frac{1}{(a+bx)^{4/3}(c+dx)^{2/3}}\,dx}{14(bc-ad)^2}\\
&= -\frac{3\sqrt[3]{c+dx}}{7(bc-ad)(a+bx)^{7/3}}+\frac{9d\sqrt[3]{c+dx}}{14(bc-ad)^2(a+bx)^{4/3}}-\frac{27d^2\sqrt[3]{c+dx}}{14(bc-ad)^3\sqrt[3]{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.74

$$-\frac{3\sqrt[3]{c+dx}\left(14a^2d^2-7abd(c-3dx)+b^2\left(2c^2-3cdx+9d^2x^2\right)\right)}{14(a+bx)^{7/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(14*a^2*d^2 - 7*a*b*d*(c - 3*d*x) + b^2*(2*c^2 - 3*c*d*
x + 9*d^2*x^2)))/(14*(b*c - a*d)^3*(a + b*x)^(7/3))
```

IntegrateAlgebraic [A] time = 0.12, size = 83, normalized size = 0.82

$$-\frac{3\left(\frac{2b^2(c+dx)^{7/3}}{(a+bx)^{7/3}}+\frac{14d^2\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}-\frac{7bd(c+dx)^{4/3}}{(a+bx)^{4/3}}\right)}{14(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*((14*d^2*(c + d*x)^(1/3))/(a + b*x)^(1/3) - (7*b*d*(c + d*x)^(4/3))/(a
+ b*x)^(4/3) + (2*b^2*(c + d*x)^(7/3))/(a + b*x)^(7/3)))/(14*(b*c - a*d)^3)
```

fricas [B] time = 1.11, size = 251, normalized size = 2.49

$$-\frac{3\left(9\,b^2d^2x^2+2\,b^2c^2-7\,abcd+14\,a^2d^2-3\left(b^2cd-7\,abd^2\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{14\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/14*(9*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 14*a^2*d^2 - 3*(b^2*c*d - 7*
a*b*d^2)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
+ 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^
3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2
*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)
*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{3\,(dx+c)^{\frac{1}{3}}\left(9b^2x^2d^2+21ab\,d^2x-3b^2cdx+14a^2d^2-7abcd+2b^2c^2\right)}{14\,(bx+a)^{\frac{7}{3}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/14*(d*x+c)^(1/3)*(9*b^2*d^2*x^2+21*a*b*d^2*x-3*b^2*c*d*x+14*a^2*d^2-7*a*b
*c*d+2*b^2*c^2)/(b*x+a)^(7/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{10}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(10/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(10/3)*(d*x + c)^(2/3)), x)
```

mupad [B] time = 1.51, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/3}\,\left(\frac{27\,d^2\,x^2}{14\,(a\,d-b\,c)^3}+\frac{42\,a^2\,d^2-21\,a\,b\,c\,d+6\,b^2\,c^2}{14\,b^2\,(a\,d-b\,c)^3}+\frac{9\,d\,x\,(7\,a\,d-b\,c)}{14\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/3}+\frac{a^2\,(a+b\,x)^{1/3}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/3}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(10/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((27*d^2*x^2)/(14*(a*d - b*c)^3) + (42*a^2*d^2 + 6*b^2*c^2
 - 21*a*b*c*d)/(14*b^2*(a*d - b*c)^3) + (9*d*x*(7*a*d - b*c))/(14*b*(a*d -
b*c)^3)))/(x^2*(a + b*x)^(1/3) + (a^2*(a + b*x)^(1/3))/b^2 + (2*a*x*(a + b*
x)^(1/3))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{10}{3}}\,(c+dx)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(10/3)/(d*x+c)**(2/3),x)
```

```
[Out] Integral(1/((a + b*x)**(10/3)*(c + d*x)**(2/3)), x)
```

3.1473 $\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx$

Optimal. Leaf size=136

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{243d^3\sqrt[3]{c+dx}}{140\sqrt[3]{a+bx}\,(bc-ad)^4}-\frac{81d^2\sqrt[3]{c+dx}}{140(a+bx)^{4/3}(bc-ad)^3}+\frac{27d\sqrt[3]{c+dx}}{70(a+bx)^{7/3}(bc-ad)^2}-\frac{3\sqrt[3]{c+dx}}{10(a+bx)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3))/(10*(b*c - a*d)*(a + b*x)^(10/3)) + (27*d*(c + d*x)^(1
/3))/(70*(b*c - a*d)^2*(a + b*x)^(7/3)) - (81*d^2*(c + d*x)^(1/3))/(140*(b*
c - a*d)^3*(a + b*x)^(4/3)) + (243*d^3*(c + d*x)^(1/3))/(140*(b*c - a*d)^4*
(a + b*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{13/3}(c+dx)^{2/3}}\,dx &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}-\frac{(9d)\int \frac{1}{(a+bx)^{10/3}(c+dx)^{2/3}}\,dx}{10(bc-ad)}\\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}+\frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}}+\frac{\left(27d^2\right)\int \frac{1}{(a+bx)^{7/3}(c+dx)^{2/3}}\,dx}{35(bc-ad)^2}\\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}+\frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}}-\frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}}-\\ &= -\frac{3\sqrt[3]{c+dx}}{10(bc-ad)(a+bx)^{10/3}}+\frac{27d\sqrt[3]{c+dx}}{70(bc-ad)^2(a+bx)^{7/3}}-\frac{81d^2\sqrt[3]{c+dx}}{140(bc-ad)^3(a+bx)^{4/3}}+\end{aligned}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.85

$$\frac{3\sqrt[3]{c+dx}\left(140a^3d^3-105a^2bd^2(c-3dx)+30ab^2d\left(2c^2-3cdx+9d^2x^2\right)+b^3\left(-14c^3+18c^2dx-27cd^2x^2+81d^3x^3\right)\right)}{140(a+bx)^{10/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (3*(c + d*x)^(1/3)*(140*a^3*d^3 - 105*a^2*b*d^2*(c - 3*d*x) + 30*a*b^2*d*(2
*c^2 - 3*c*d*x + 9*d^2*x^2) + b^3*(-14*c^3 + 18*c^2*d*x - 27*c*d^2*x^2 + 81
*d^3*x^3)))/(140*(b*c - a*d)^4*(a + b*x)^(10/3))
```

IntegrateAlgebraic [**A**] time = 0.14, size = 95, normalized size = 0.70

$$-\frac{3\sqrt[3]{c+dx}\left(\frac{14b^3(c+dx)^3}{(a+bx)^3}-\frac{60b^2d(c+dx)^2}{(a+bx)^2}+\frac{105bd^2(c+dx)}{a+bx}-140d^3\right)}{140\sqrt[3]{a+bx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x]
```

```
[Out] (-3*(c + d*x)^(1/3)*(-140*d^3 + (105*b*d^2*(c + d*x))/(a + b*x) - (60*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (14*b^3*(c + d*x)^3)/(a + b*x)^3))/(140*(b*c -
a*d)^4*(a + b*x)^(1/3))
```

fricas [**B**] time = 0.83, size = 419, normalized size = 3.08

$$\frac{3\left(81\,b^3d^3x^3-14\,b^3c^3+60\,ab^2c^2d-105\,a^2bcd^2+140\,a^3d^3-27\left(b^3cd^2-10\,ab^2d^3\right)x^2+9\left(2\,b^3c^2d-10\,ab^2cd^2+35\,a^2bd^3\right)x\right)(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{140\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="fricas")
```

```
[Out] 3/140*(81*b^3*d^3*x^3 - 14*b^3*c^3 + 60*a*b^2*c^2*d - 105*a^2*b*c*d^2 + 140
*a^3*d^3 - 27*(b^3*c*d^2 - 10*a*b^2*d^3)*x^2 + 9*(2*b^3*c^2*d - 10*a*b^2*c*
d^2 + 35*a^2*b*d^3)*x)*(b*x + a)^(2/3)*(d*x + c)^(1/3)/(a^4*b^4*c^4 - 4*a^5
*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a*b
^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*b^
7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d^4
)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*c*
d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2*d
^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

maple [**A**] time = 0.01, size = 171, normalized size = 1.26

$$\frac{3\,(dx+c)^{\frac{1}{3}}\left(81b^3d^3x^3+270a\,b^2d^3x^2-27b^3c\,d^2x^2+315a^2b\,d^3x-90a\,b^2c\,d^2x+18b^3c^2dx+140a^3d^3-105a^2bc\,d^2+60a\,b^2c^2d-14b^3c^3\right)}{140\,(bx+a)^{\frac{10}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x)
```

```
[Out] 3/140*(d*x+c)^(1/3)*(81*b^3*d^3*x^3+270*a*b^2*d^3*x^2-27*b^3*c*d^2*x^2+315*
a^2*b*d^3*x-90*a*b^2*c*d^2*x+18*b^3*c^2*d*x+140*a^3*d^3-105*a^2*b*c*d^2+60*
```

```
a*b^2*c^2*d-14*b^3*c^3)/(b*x+a)^(10/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2
*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{3}}(dx+c)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/3)/(d*x+c)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/3)*(d*x + c)^(2/3)), x)
```

mupad [B] time = 1.27, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/3}\left(\frac{243\,d^3\,x^3}{140\,(a\,d-b\,c)^4}+\frac{420\,a^3\,d^3-315\,a^2\,b\,c\,d^2+180\,a\,b^2\,c^2\,d-42\,b^3\,c^3}{140\,b^3\,(a\,d-b\,c)^4}+\frac{27\,d\,x\,\left(35\,a^2\,d^2-10\,a\,b\,c\,d+2\,b^2\,c^2\right)}{140\,b^2\,(a\,d-b\,c)^4}+\frac{81\,d^2\,x^2\,(10\,a\,d-b\,c)}{140\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/3}+\frac{a^3\,(a+b\,x)^{1/3}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/3}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/3}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/3)*(c + d*x)^(2/3)),x)
```

```
[Out] ((c + d*x)^(1/3)*((243*d^3*x^3)/(140*(a*d - b*c)^4) + (420*a^3*d^3 - 42*b^3
*c^3 + 180*a*b^2*c^2*d - 315*a^2*b*c*d^2)/(140*b^3*(a*d - b*c)^4) + (27*d*x
*(35*a^2*d^2 + 2*b^2*c^2 - 10*a*b*c*d))/(140*b^2*(a*d - b*c)^4) + (81*d^2*x
^2*(10*a*d - b*c))/(140*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/3) + (a^3*(a +
 b*x)^(1/3))/b^3 + (3*a*x^2*(a + b*x)^(1/3))/b + (3*a^2*x*(a + b*x)^(1/3))/
b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/3)/(d*x+c)**(2/3),x)
```

```
[Out] Timed out
```

3.1474 $\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=241

$$-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-$$

Rubi [A] time = 0.11, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{47, 50, 59\}$

$$\frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2}-\frac{14b\sqrt[3]{a+bx}\,(c+dx)^{2/3}(bc-ad)}{3d^3}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log(a+bx)}{9d^{10/3}}-\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{d}\,\sqrt[3]{a+bx}}-1\right)}{3d^{10/3}}-\frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}-\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(7/3))/(d*(c + d*x)^(1/3)) - (14*b*(b*c - a*d)*(a + b*x)^(1/3
)*(c + d*x)^(2/3))/(3*d^3) + (7*b*(a + b*x)^(4/3)*(c + d*x)^(2/3))/(2*d^2)
- (14*b^(1/3)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^(1/3))/
(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(3*Sqrt[3]*d^(10/3)) - (7*b^(1/3)*(b*c
- a*d)^2*Log[a + b*x])/(9*d^(10/3)) - (7*b^(1/3)*(b*c - a*d)^2*Log[-1 + (b^
(1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(3*d^(10/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{(7b)\int \frac{(a+bx)^{4/3}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{(14b(bc-ad))\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} + \frac{\left(14b(bc-ad)^2\right)}{}$$

$$= -\frac{3(a+bx)^{7/3}}{d\sqrt[3]{c+dx}} - \frac{14b(bc-ad)\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{3d^3} + \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14\sqrt[3]{b}\,(bc-ad)^2}{}$$

Mathematica [C] time = 0.06, size = 73, normalized size = 0.30

$$\frac{3(a+bx)^{10/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{10}{3};\frac{13}{3};\frac{d(a+bx)}{ad-bc}\right)}{10b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]

[Out] (3*(a + b*x)^(10/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3
, 10/3, 13/3, (d*(a + b*x))/(-(b*c) + a*d)])/(10*b*(c + d*x)^(4/3))

IntegrateAlgebraic [A] time = 0.50, size = 318, normalized size = 1.32

$$\frac{7\sqrt[3]{b}\,(bc-ad)^2\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{9d^{10/3}} - \frac{(ad-bc)^2\left(\frac{28b^2\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\frac{18d^2(a+bx)^{7/3}}{(c+dx)^{7/3}}-\frac{49bd(a+bx)^{4/3}}{(c+dx)^{4/3}}\right)}{6d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2} - \frac{14\sqrt[3]{b}\,(bc-ad)^2\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{9d^{10/3}} + \frac{14\sqrt[3]{b}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{3\sqrt{3}\,d^{10/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x)^(7/3)/(c + d*x)^(4/3),x]

[Out] -1/6*((-(b*c) + a*d)^2*((18*d^2*(a + b*x)^(7/3))/(c + d*x)^(7/3) - (49*b*d*
(a + b*x)^(4/3))/(c + d*x)^(4/3) + (28*b^2*(a + b*x)^(1/3))/(c + d*x)^(1/3)
))/(d^3*(-b + (d*(a + b*x))/(c + d*x))^2) + (14*b^(1/3)*(b*c - a*d)^2*ArcTa
n[1/Sqrt[3] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))
])/(3*Sqrt[3]*d^(10/3)) - (14*b^(1/3)*(b*c - a*d)^2*Log[b^(1/3) - (d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*d^(10/3)) + (7*b^(1/3)*(b*c - a*d)^2*
Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^(1/3)*d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(9*d^(10/3))

fricas [B] time = 1.04, size = 423, normalized size = 1.76

$$-\frac{28\sqrt{3}\left(b^2c^3-2abc^2d+a^2cd^2+\left(b^2c^2d-2abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right)+14\left(b^2c^3-2abc^2d+a^2cd^2+\left(b^2c^2d-2abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-28\left(b^2c^3-2abc^2d+a^2cd^2+\left(b^2c^2d-2abcd^2+a^2d^3\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)-3\left(3b^2d^2x^2-28b^2c^2+49abcd-18a^2d^2-\left(7b^2cd-13abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{18\left(d^4x+cd^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="fricas")

[Out] -1/18*(28*sqrt(3)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) + 14*(b^2
*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b
/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b

```
/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 28*(b^2*c^3 - 2*a
*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*x)*(-b/d)^(1/3)*
log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) -
 3*(3*b^2*d^2*x^2 - 28*b^2*c^2 + 49*a*b*c*d - 18*a^2*d^2 - (7*b^2*c*d - 13*
a*b*d^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^4*x + c*d^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(7/3)/(d*x+c)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/3)/(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(7/3)/(c + d*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/3)/(d*x+c)**(4/3),x)
[Out] Integral((a + b*x)**(7/3)/(c + d*x)**(4/3), x)
```

3.1475 $\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}} dx$

Optimal. Leaf size=195

$$\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}+\frac{4b\sqrt[3]{a+}}{}$$

Rubi [A] time = 0.07, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = $\{47, 50, 59\}$

$$\frac{4b\sqrt[3]{a+bx}(c+dx)^{2/3}}{d^2}+\frac{2\sqrt[3]{b}(bc-ad)\log(a+bx)}{3d^{7/3}}+\frac{2\sqrt[3]{b}(bc-ad)\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{d^{7/3}}+\frac{4\sqrt[3]{b}(bc-ad)\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}d^{7/3}}-\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(4/3))/(d*(c + d*x)^(1/3)) + (4*b*(a + b*x)^(1/3)*(c + d*x)^(
2/3))/d^2 + (4*b^(1/3)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*b^(1/3)*(c + d*x)^
(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/(Sqrt[3]*d^(7/3)) + (2*b^(1/3)*(
b*c - a*d)*Log[a + b*x])/(3*d^(7/3)) + (2*b^(1/3)*(b*c - a*d)*Log[-1 + (b^(
1/3)*(c + d*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/d^(7/3)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{4/3}}\,dx = -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{(4b)\int \frac{\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\,dx}{d}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} - \frac{(4b(bc-ad))\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{3d^2}$$

$$= -\frac{3(a+bx)^{4/3}}{d\sqrt[3]{c+dx}} + \frac{4b\sqrt[3]{a+bx}\,(c+dx)^{2/3}}{d^2} + \frac{4\sqrt[3]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}} + \frac{2\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt{3}\,\sqrt[3]{d}\,\sqrt[3]{a+bx}}\right)}{\sqrt{3}\,d^{7/3}} + 2\sqrt[3]{b}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.37

$$\frac{3(a+bx)^{7/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3} {}_2F_1\left(\frac{4}{3},\frac{7}{3};\frac{10}{3};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(7/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 7/3, 10/3, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(4/3))
```

IntegrateAlgebraic [A] time = 11.02, size = 326, normalized size = 1.67

$$\frac{d^{4/3}(a+bx)^{4/3}\left(\frac{4\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{ad+b(c+dx)-bc}-\sqrt[3]{b}\,\sqrt[3]{c+dx}\right)}{3d^{7/3}} - \frac{2\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\log\left(\sqrt[3]{b}\,\sqrt[3]{c+dx}\,\sqrt[3]{ad+b(c+dx)-bc}+(ad+b(c+dx)-bc)^{2/3}+b^{2/3}(c+dx)^{2/3}\right)}{3d^{7/3}} + \frac{4\left(b^{4/3}c-a\sqrt[3]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{b}\,\sqrt[3]{c+dx}}{2\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}\right)}{\sqrt{3}\,d^{7/3}} + \frac{\sqrt[3]{ad+b(c+dx)-bc}\,(-3ad+b(c+dx)+3bc)}{d^{7/3}\sqrt[3]{c+dx}}\right)}{(ad+bdx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(4/3)/(c + d*x)^(4/3),x]
```

```
[Out] (d^(4/3)*(a + b*x)^(4/3)*(((3*b*c - 3*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/3))/(d^(7/3)*(c + d*x)^(1/3)) + (4*(b^(4/3)*c - a*b^(1/3)*d)*
ArcTan[(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))/(b^(1/3)*(c + d*x)^(1/3) + 2*(-(b*
c) + a*d + b*(c + d*x))^(1/3))])/(Sqrt[3]*d^(7/3)) + (4*(b^(4/3)*c - a*b^(1
/3)*d)*Log[-(b^(1/3)*(c + d*x)^(1/3)) + (-(b*c) + a*d + b*(c + d*x))^(1/3)]
)/(3*d^(7/3)) - (2*(b^(4/3)*c - a*b^(1/3)*d)*Log[b^(2/3)*(c + d*x)^(2/3) +
b^(1/3)*(c + d*x)^(1/3)*(-(b*c) + a*d + b*(c + d*x))^(1/3) + (-(b*c) + a*d
+ b*(c + d*x))^(2/3)])/(3*d^(7/3))))/(a*d + b*d*x)^(4/3)
```

fricas [A] time = 1.22, size = 306, normalized size = 1.57

$$\frac{4\sqrt{3}\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right)+2\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-4\left(bc^2-acd+\left(bcd-ad^2\right)x\right)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+3(bdx+4bc-3ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{3\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(3)*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)*arctan(1/3*
(2*sqrt(3)*(b*x + a)^(1/3)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x
+ b*c))/(b*d*x + b*c)) + 2*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)*(-b/d)^(1/3)
*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*(-b/d)^(1/3)
 + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 4*(b*c^2 - a*c*d + (b*c*d
- a*d^2)*x)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x
 + c)^(2/3))/(d*x + c)) + 3*(b*d*x + 4*b*c - 3*a*d)*(b*x + a)^(1/3)*(d*x +
c)^(2/3))/(d^3*x + c*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
[Out] int((b*x+a)^(4/3)/(d*x+c)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{4}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(4/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(4/3)/(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,x\right)}^{4/3}}{{\left(c+d\,x\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(4/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(4/3)/(c + d*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{4}{3}}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(4/3)/(d*x+c)**(4/3),x)
[Out] Integral((a + b*x)**(4/3)/(c + d*x)**(4/3), x)
```

3.1476 $\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=149

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

Rubi [A] time = 0.03, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {47, 59}

$$-\frac{3\sqrt[3]{b}\log\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}-1\right)}{2d^{4/3}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) - (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*b^(1/3)*(c + d*x)^(1/3))/(Sqrt[3]*d^(1/3)*(a + b*x)^(1/3))])/d^(4/3)
 - (b^(1/3)*Log[a + b*x])/(2*d^(4/3)) - (3*b^(1/3)*Log[-1 + (b^(1/3)*(c + d
*x)^(1/3))/(d^(1/3)*(a + b*x)^(1/3))])/(2*d^(4/3))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{4/3}}\,dx &= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{2/3}\sqrt[3]{c+dx}}\,dx}{d}\\ &= -\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}-\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt{3}\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{d^{4/3}}-\frac{\sqrt[3]{b}\log(a+bx)}{2d^{4/3}}-\frac{3\sqrt[3]{b}\log\left(-1+\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{d}\sqrt[3]{a+bx}}\right)}{2d^{4/3}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.49

$$\frac{3(a+bx)^{4/3}\left(\frac{b(c+dx)}{bc-ad}\right)^{4/3}{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};\frac{d(a+bx)}{ad-bc}\right)}{4b(c+dx)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (3*(a + b*x)^(4/3)*((b*(c + d*x))/(b*c - a*d))^(4/3)*Hypergeometric2F1[4/3,
 4/3, 7/3, (d*(a + b*x))/(-(b*c) + a*d)])/(4*b*(c + d*x)^(4/3))
```

IntegrateAlgebraic [A] time = 0.16, size = 200, normalized size = 1.34

$$\frac{\sqrt[3]{b}\log\left(\frac{d^{2/3}(a+bx)^{2/3}}{(c+dx)^{2/3}}+\frac{\sqrt[3]{b}\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+b^{2/3}\right)}{2d^{4/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{b}-\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\right)}{d^{4/3}}+\frac{\sqrt{3}\sqrt[3]{b}\tan^{-1}\left(\frac{2\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{4/3}}-\frac{3\sqrt[3]{a+bx}}{d\sqrt[3]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/3)/(c + d*x)^(4/3),x]
```

```
[Out] (-3*(a + b*x)^(1/3))/(d*(c + d*x)^(1/3)) + (Sqrt[3]*b^(1/3)*ArcTan[1/Sqrt[3
] + (2*d^(1/3)*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*(c + d*x)^(1/3))]])/d^(4/3)
 - (b^(1/3)*Log[b^(1/3) - (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/d^(4/
3) + (b^(1/3)*Log[b^(2/3) + (d^(2/3)*(a + b*x)^(2/3))/(c + d*x)^(2/3) + (b^
(1/3)*d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)])/(2*d^(4/3))
```

fricas [B] time = 1.46, size = 233, normalized size = 1.56

$$-\frac{2\sqrt{3}(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}d\left(-\frac{b}{d}\right)^{\frac{2}{3}}+\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right)+(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{2}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{dx+c}\right)-2(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}\log\left(\frac{(dx+c)\left(-\frac{b}{d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}\right)+6(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*sqrt(3)*(d*x + c)*(-b/d)^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x + a)^(1/3
)*(d*x + c)^(2/3)*d*(-b/d)^(2/3) + sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) +
(d*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(2/3) - (b*x + a)^(1/3)*(d*x +
 c)^(2/3)*(-b/d)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(d*x + c)) - 2*(d
*x + c)*(-b/d)^(1/3)*log(((d*x + c)*(-b/d)^(1/3) + (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c)) + 6*(b*x + a)^(1/3)*(d*x + c)^(2/3))/(d^2*x + c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

```
[Out] int((b*x+a)^(1/3)/(d*x+c)^(4/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/3)/(d*x + c)^(4/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/3}}{(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/3)/(c + d*x)^(4/3),x)
[Out] int((a + b*x)^(1/3)/(c + d*x)^(4/3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+bx}}{(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/3)/(d*x+c)**(4/3),x)
[Out] Integral((a + b*x)**(1/3)/(c + d*x)**(4/3), x)
```

3.1477 $\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=30

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx = \frac{3\sqrt[3]{a+bx}}{(bc-ad)\sqrt[3]{c+dx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$\frac{3\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x]
[Out] (3*(a + b*x)^(1/3))/((b*c - a*d)*(c + d*x)^(1/3))
```

fricas [A] time = 0.76, size = 42, normalized size = 1.40

$$\frac{3\,(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="fricas")
[Out] 3*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$-\frac{3\,(bx+a)^{\frac{1}{3}}}{(dx+c)^{\frac{1}{3}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x)
[Out] -3*(b*x+a)^(1/3)/(d*x+c)^(1/3)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(2/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(2/3)*(d*x + c)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{2/3}\,{(c+d\,x)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(2/3)*(c + d*x)^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{2}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(2/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(2/3)*(c + d*x)**(4/3)), x)
```

3.1478 $\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=66

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{9d\sqrt[3]{a+bx}}{2\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(2*(b*c - a*d)*(a + b*x)^(2/3)*(c + d*x)^(1/3)) - (9*d*(a + b*x)^(1/3))/
(2*(b*c - a*d)^2*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx &= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{(3d)\int \frac{1}{(a+bx)^{2/3}(c+dx)^{4/3}}\,dx}{2(bc-ad)}\\ &= -\frac{3}{2(bc-ad)(a+bx)^{2/3}\sqrt[3]{c+dx}}-\frac{9d\sqrt[3]{a+bx}}{2(bc-ad)^2\sqrt[3]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.68

$$-\frac{3(2ad+b(c+3dx))}{2(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(2*a*d + b*(c + 3*d*x)))/(2*(b*c - a*d)^2*(a + b*x)^(2/3)*(c + d*x)^(1/
3))
```

IntegrateAlgebraic [A] time = 0.11, size = 49, normalized size = 0.74

$$-\frac{3(c+dx)^{2/3}\left(\frac{2d(a+bx)}{c+dx}+b\right)}{2(a+bx)^{2/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(2/3)*(b + (2*d*(a + b*x))/(c + d*x)))/(2*(b*c - a*d)^2*(a +
b*x)^(2/3))
```

fricas [B] time = 1.21, size = 126, normalized size = 1.91

$$-\frac{3\,(3\,bdx+bc+2\,ad)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{2\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/2*(3*b*d*x + b*c + 2*a*d)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

maple [A] time = 0.00, size = 53, normalized size = 0.80

$$-\frac{3\,(3bdx+2ad+bc)}{2\,(bx+a)^{\frac{2}{3}}\,(dx+c)^{\frac{1}{3}}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/2*(3*b*d*x+2*a*d+b*c)/(b*x+a)^(2/3)/(d*x+c)^(1/3)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/3)*(d*x + c)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x)^{5/3}\,(c+d\,x)^{4/3}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(5/3)*(c + d*x)^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(5/3)*(c + d*x)**(4/3)), x)
```

3.1479 $\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=101

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{27d^2\sqrt[3]{a+bx}}{5\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{9d}{5(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(5*(b*c - a*d)*(a + b*x)^(5/3)*(c + d*x)^(1/3)) + (9*d)/(5*(b*c - a*d)^2
*(a + b*x)^(2/3)*(c + d*x)^(1/3)) + (27*d^2*(a + b*x)^(1/3))/(5*(b*c - a*d)
^3*(c + d*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx &= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{(6d)\int \frac{1}{(a+bx)^{5/3}(c+dx)^{4/3}}\,dx}{5(bc-ad)}\\ &= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{\left(9d^2\right)\int \frac{1}{(a+b}{5(bc}\\ &= -\frac{3}{5(bc-ad)(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{9d}{5(bc-ad)^2(a+bx)^{2/3}\sqrt[3]{c+dx}}+\frac{27d^2\sqrt[3]{a}}{5(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.74

$$\frac{3\left(5a^2d^2+5abd(c+3dx)+b^2\left(-c^2+3cdx+9d^2x^2\right)\right)}{5(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(5*a^2*d^2 + 5*a*b*d*(c + 3*d*x) + b^2*(-c^2 + 3*c*d*x + 9*d^2*x^2)))/(5
*(b*c - a*d)^3*(a + b*x)^(5/3)*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.72

$$\frac{3(c+dx)^{5/3}\left(\frac{5d^2(a+bx)^2}{(c+dx)^2}+\frac{5bd(a+bx)}{c+dx}-b^2\right)}{5(a+bx)^{5/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (3*(c + d*x)^(5/3)*(-b^2 + (5*d^2*(a + b*x)^2)/(c + d*x)^2 + (5*b*d*(a + b*
x))/(c + d*x)))/(5*(b*c - a*d)^3*(a + b*x)^(5/3))
```

fricas [B] time = 1.11, size = 273, normalized size = 2.70

$$\frac{3\left(9\,b^2d^2x^2-b^2c^2+5\,abcd+5\,a^2d^2+3\left(b^2cd+5\,abd^2\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{5\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4\right)x^2+\left(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/5*(9*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 5*a^2*d^2 + 3*(b^2*c*d + 5*a*b*d
^2)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*a
^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 -
 a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*
c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d
^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{3\left(9b^2x^2d^2+15ab\,d^2x+3b^2cdx+5a^2d^2+5abcd-b^2c^2\right)}{5\,(bx+a)^{\frac{5}{3}}\,(dx+c)^{\frac{1}{3}}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/5*(9*b^2*d^2*x^2+15*a*b*d^2*x+3*b^2*c*d*x+5*a^2*d^2+5*a*b*c*d-b^2*c^2)/(
b*x+a)^(5/3)/(d*x+c)^(1/3)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{8}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(8/3)/(d*x+c)^(4/3),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(8/3)*(d*x + c)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{8/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)),x)
[Out] int(1/((a + b*x)^(8/3)*(c + d*x)^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{8}{3}}\,(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(8/3)/(d*x+c)**(4/3),x)
[Out] Integral(1/((a + b*x)**(8/3)*(c + d*x)**(4/3)), x)
```

3.1480 $\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx$

Optimal. Leaf size=136

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{243d^3\sqrt[3]{a+bx}}{40\sqrt[3]{c+dx}\,(bc-ad)^4}-\frac{81d^2}{40(a+bx)^{2/3}\sqrt[3]{c+dx}\,(bc-ad)^3}+\frac{27d}{40(a+bx)^{5/3}\sqrt[3]{c+dx}\,(bc-ad)^2}-\frac{3}{8(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] -3/(8*(b*c - a*d)*(a + b*x)^(8/3)*(c + d*x)^(1/3)) + (27*d)/(40*(b*c - a*d)
^2*(a + b*x)^(5/3)*(c + d*x)^(1/3)) - (81*d^2)/(40*(b*c - a*d)^3*(a + b*x)^
(2/3)*(c + d*x)^(1/3)) - (243*d^3*(a + b*x)^(1/3))/(40*(b*c - a*d)^4*(c + d
*x)^(1/3))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{11/3}(c+dx)^{4/3}}\,dx &= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}-\frac{(9d)\int \frac{1}{(a+bx)^{8/3}(c+dx)^{4/3}}\,dx}{8(bc-ad)}\\
&= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}+\frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}}+\frac{\left(27d^2\right)\int \frac{}{(a+b}}{20(bc}\\
&= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}+\frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{}{40(bc-ad)^3(}\\
&= -\frac{3}{8(bc-ad)(a+bx)^{8/3}\sqrt[3]{c+dx}}+\frac{27d}{40(bc-ad)^2(a+bx)^{5/3}\sqrt[3]{c+dx}}-\frac{}{40(bc-ad)^3(}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.85

$$-\frac{3\left(40a^3d^3+60a^2bd^2(c+3dx)+24ab^2d\left(-c^2+3cdx+9d^2x^2\right)+b^3\left(5c^3-9c^2dx+27cd^2x^2+81d^3x^3\right)\right)}{40(a+bx)^{8/3}\sqrt[3]{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(40*a^3*d^3 + 60*a^2*b*d^2*(c + 3*d*x) + 24*a*b^2*d*(-c^2 + 3*c*d*x + 9
*d^2*x^2) + b^3*(5*c^3 - 9*c^2*d*x + 27*c*d^2*x^2 + 81*d^3*x^3)))/(40*(b*c
- a*d)^4*(a + b*x)^(8/3)*(c + d*x)^(1/3))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.70

$$-\frac{3(c+dx)^{8/3}\left(-\frac{24b^2d(a+bx)}{c+dx}+\frac{40d^3(a+bx)^3}{(c+dx)^3}+\frac{60bd^2(a+bx)^2}{(c+dx)^2}+5b^3\right)}{40(a+bx)^{8/3}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x]
```

```
[Out] (-3*(c + d*x)^(8/3)*(5*b^3 + (40*d^3*(a + b*x)^3)/(c + d*x)^3 + (60*b*d^2*(
a + b*x)^2)/(c + d*x)^2 - (24*b^2*d*(a + b*x))/(c + d*x)))/(40*(b*c - a*d)^
4*(a + b*x)^(8/3))
```

fricas [B] time = 1.52, size = 456, normalized size = 3.35

$$-\frac{3\left(81\,b^3d^3x^3+5\,b^3c^3-24\,ab^2c^2d+60\,a^2bcd^2+40\,a^3d^3+27\left(b^3cd^2+8\,ab^2d^3\right)x^2-9\left(b^3c^2d-8\,ab^2cd^2-20\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{40\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5\right)x^3+3\left(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5\right)x^2+\left(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="fricas")
```

```
[Out] -3/40*(81*b^3*d^3*x^3 + 5*b^3*c^3 - 24*a*b^2*c^2*d + 60*a^2*b*c*d^2 + 40*a^
3*d^3 + 27*(b^3*c*d^2 + 8*a*b^2*d^3)*x^2 - 9*(b^3*c^2*d - 8*a*b^2*c*d^2 - 2
0*a^2*b*d^3)*x)*(b*x + a)^(1/3)*(d*x + c)^(2/3)/(a^3*b^4*c^5 - 4*a^4*b^3*c^
4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4*a*b^
6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (b^7*c
^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^
4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2
 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c^5 -
11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a
^7*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{3\left(81b^3d^3x^3+216a\,b^2d^3x^2+27b^3c\,d^2x^2+180a^2b\,d^3x+72a\,b^2c\,d^2x-9b^3c^2dx+40a^3d^3+60a^2bc\,d^2-24a\,b^2c^2d+5b^3c^3\right)}{40\,(bx+a)^{\frac{8}{3}}\,(dx+c)^{\frac{1}{3}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x)
```

```
[Out] -3/40*(81*b^3*d^3*x^3+216*a*b^2*d^3*x^2+27*b^3*c*d^2*x^2+180*a^2*b*d^3*x+72
*a*b^2*c*d^2*x-9*b^3*c^2*d*x+40*a^3*d^3+60*a^2*b*c*d^2-24*a*b^2*c^2*d+5*b^3
```

```
*c^3)/(b*x+a)^(8/3)/(d*x+c)^(1/3)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-
4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{3}}(dx+c)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/3)/(d*x+c)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/3)*(d*x + c)^(4/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/3}\,(c+d\,x)^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)),x)
```

```
[Out] int(1/((a + b*x)^(11/3)*(c + d*x)^(4/3)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{3}}(c+dx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/3)/(d*x+c)**(4/3),x)
```

```
[Out] Integral(1/((a + b*x)**(11/3)*(c + d*x)**(4/3)), x)
```

3.1481 $\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=77

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {50, 59}

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{1}{3}\log(x-1)+\log\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x+1}}{\sqrt{3}\sqrt[3]{x-1}}+\frac{1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[1/Sqrt[3] + (2*(1 + x)^(1/3))/(Sqr
t[3]*(-1 + x)^(1/3))])/Sqrt[3] + Log[-1 + x]/3 + Log[-1 + (1 + x)^(1/3)/(-1
 + x)^(1/3)]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 59

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, -Simp[(Sqrt[3]*q*ArcTan[(2*q*(a + b*x)^(1/3))/(Sqrt
[3]*(c + d*x)^(1/3)) + 1/Sqrt[3]])/d, x] + (-Simp[(3*q*Log[(q*(a + b*x)^(1/
3))/(c + d*x)^(1/3) - 1])/(2*d), x] - Simp[(q*Log[c + d*x])/(2*d), x])] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[d/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{-1+x}}{\sqrt[3]{1+x}}\,dx &= \sqrt[3]{-1+x}\,(1+x)^{2/3}-\frac{2}{3}\int \frac{1}{(-1+x)^{2/3}\sqrt[3]{1+x}}\,dx\\ &= \sqrt[3]{-1+x}\,(1+x)^{2/3}+\frac{2\tan^{-1}\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{-1+x}}\right)}{\sqrt{3}}+\frac{1}{3}\log(-1+x)+\log\left(-1+\frac{\sqrt[3]{1+x}}{\sqrt[3]{-1+x}}\right)\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.62

$$\frac{3\left(\frac{x-1}{x+1}\right)^{4/3}(x+1)^{4/3}\,{}_2F_1\left(\frac{1}{3},\frac{4}{3};\frac{7}{3};\frac{1-x}{2}\right)}{4\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (3*((-1 + x)/(1 + x))^(4/3)*(1 + x)^(4/3)*Hypergeometric2F1[1/3, 4/3, 7/3, (1 - x)/2])/(4*2^(1/3))
```

IntegrateAlgebraic [A] time = 0.22, size = 113, normalized size = 1.47

$$\sqrt[3]{x-1}\,(x+1)^{2/3}+\frac{2}{3}\log\left(\sqrt[3]{x-1}-\sqrt[3]{x+1}\right)-\frac{1}{3}\log\left((x-1)^{2/3}+\sqrt[3]{x+1}\,\sqrt[3]{x-1}+(x+1)^{2/3}\right)+\frac{2\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{x+1}}{2\sqrt[3]{x-1}+\sqrt[3]{x+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x)^(1/3)/(1 + x)^(1/3),x]
```

```
[Out] (-1 + x)^(1/3)*(1 + x)^(2/3) + (2*ArcTan[(Sqrt[3]*(1 + x)^(1/3))/(2*(-1 + x
)^(1/3) + (1 + x)^(1/3))])/Sqrt[3] + (2*Log[(-1 + x)^(1/3) - (1 + x)^(1/3)]
)/3 - Log[(-1 + x)^(2/3) + (-1 + x)^(1/3)*(1 + x)^(1/3) + (1 + x)^(2/3)]/3
```

fricas [A] time = 1.35, size = 107, normalized size = 1.39

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(x+1)+2\sqrt{3}(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}}{3(x+1)}\right)+(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-\frac{1}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}+(x+1)^{\frac{1}{3}}(x-1)^{\frac{2}{3}}+x+1}{x+1}\right)+\frac{2}{3}\log\left(\frac{(x+1)^{\frac{2}{3}}(x-1)^{\frac{1}{3}}-x-1}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(sqrt(3)*(x + 1) + 2*sqrt(3)*(x + 1)^(2/3)*(x - 1)^
(1/3))/(x + 1)) + (x + 1)^(2/3)*(x - 1)^(1/3) - 1/3*log(((x + 1)^(2/3)*(x -
 1)^(1/3) + (x + 1)^(1/3)*(x - 1)^(2/3) + x + 1)/(x + 1)) + 2/3*log(((x + 1
)^(2/3)*(x - 1)^(1/3) - x - 1)/(x + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

maple [C] time = 0.38, size = 573, normalized size = 7.44

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x-1)^(1/3)/(x+1)^(1/3),x)
```

```
[Out] (x-1)^(1/3)*(x+1)^(2/3)+(2/3*RootOf(_Z^2+_Z+1)*ln(-(2*RootOf(_Z^2+_Z+1)^2*x
^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1
)^(1/3)*x-2*RootOf(_Z^2+_Z+1)^2*x+5*RootOf(_Z^2+_Z+1)*x^2-3*RootOf(_Z^2+_Z+
1)*(x^3-x^2-x+1)^(1/3)-4*RootOf(_Z^2+_Z+1)*x+2*x^2-RootOf(_Z^2+_Z+1)-2)/(x-
1))-2/3*ln((-2*RootOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(2
/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+2*RootOf(_Z^2+_Z+1)^2*x+RootO
f(_Z^2+_Z+1)*x^2+3*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1
/3)+3*(x^3-x^2-x+1)^(1/3)*x+x^2-3*(x^3-x^2-x+1)^(1/3)-RootOf(_Z^2+_Z+1)-2*x
+1)/(x-1))*RootOf(_Z^2+_Z+1)-2/3*ln((-2*RootOf(_Z^2+_Z+1)^2*x^2+3*RootOf(_Z
```

```
^2+_Z+1)*(x^3-x^2-x+1)^(2/3)+3*RootOf(_Z^2+_Z+1)*(x^3-x^2-x+1)^(1/3)*x+2*Ro
otOf(_Z^2+_Z+1)^2*x+RootOf(_Z^2+_Z+1)*x^2+3*(x^3-x^2-x+1)^(2/3)-3*RootOf(_Z
^2+_Z+1)*(x^3-x^2-x+1)^(1/3)+3*(x^3-x^2-x+1)^(1/3)*x+x^2-3*(x^3-x^2-x+1)^(1
/3)-RootOf(_Z^2+_Z+1)-2*x+1)/(x-1)))/(x-1)^(2/3)*((x-1)^2*(x+1))^(1/3)/(x+1
)^(1/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)^{\frac{1}{3}}}{(x+1)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/3)/(1+x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(x-1)^{1/3}}{(x+1)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 1)^(1/3)/(x + 1)^(1/3),x)
```

```
[Out] int((x - 1)^(1/3)/(x + 1)^(1/3), x)
```

sympy [C] time = 2.55, size = 39, normalized size = 0.51

$$\frac{2^{\frac{2}{3}}(x-1)^{\frac{4}{3}}\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{(x-1)e^{i\pi}}{2}\right)}{2\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/3)/(1+x)**(1/3),x)
```

```
[Out] 2**(2/3)*(x - 1)**(4/3)*gamma(4/3)*hyper((1/3, 4/3), (7/3,), (x - 1)*exp_po
lar(I*pi)/2)/(2*gamma(7/3))
```

3.1482 $\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx$

Optimal. Leaf size=205

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+}}{24b^2}$$

Rubi [A] time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}-\frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{64b^{9/4}d^{7/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}{96b^2d}+\frac{5(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}{24b^2}+\frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (5*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(96*b^2*d) + (5*(b*c - a*
d)*(a + b*x)^(7/4)*(c + d*x)^(1/4))/(24*b^2) + ((a + b*x)^(7/4)*(c + d*x)^(
5/4))/(3*b) + (5*(b*c - a*d)^3*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c
 + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c - a*d)^3*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(64*b^(9/4)*d^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int (a+bx)^{3/4}(c+dx)^{5/4}\,dx = \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{(5(bc-ad))\int (a+bx)^{3/4}\sqrt[4]{c+dx}\,dx}{12b}$$

$$= \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+dx)^{5/4}}{3b} + \frac{\left(5(bc-ad)^2\right)\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}}{96b^2}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

$$= \frac{5(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}{96b^2d} + \frac{5(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}{24b^2} + \frac{(a+bx)^{7/4}(c+}{3b}$$

Mathematica [C] time = 0.06, size = 73, normalized size = 0.36

$$\frac{4(a+bx)^{7/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 7/4, 11/4, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

IntegrateAlgebraic [A] time = 0.53, size = 218, normalized size = 1.06

$$-\frac{5(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{64b^{9/4}d^{7/4}} - \frac{5(bc-ad)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{64b^{9/4}d^{7/4}} + \frac{(bc-ad)^3\left(\frac{5b^2(c+dx)^{9/4}}{(a+bx)^{9/4}} - \frac{15d^2\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}} + \frac{42bd(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{96b^2d\left(\frac{b(c+dx)}{a+bx}-d\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/4)*(c + d*x)^(5/4),x]
```

```
[Out] ((b*c - a*d)^3*((-15*d^2*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (42*b*d*(c + d*
x)^(5/4))/(a + b*x)^(5/4) + (5*b^2*(c + d*x)^(9/4))/(a + b*x)^(9/4)))/(96*b
```

```
^2*d*(-d + (b*(c + d*x))/(a + b*x))^3) - (5*(b*c - a*d)^3*ArcTan[(b^(1/4)*(
c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(64*b^(9/4)*d^(7/4)) - (5*(b*c
- a*d)^3*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(64*
b^(9/4)*d^(7/4))
```

fricas [B] time = 1.56, size = 2151, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/384*(60*b^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*arc
tan(((b^10*c^3*d^5 - 3*a*b^9*c^2*d^6 + 3*a^2*b^8*c*d^7 - a^3*b^7*d^8)*(b*x
+ a)^(3/4)*(d*x + c)^(1/4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^1
0*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 9
24*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^
3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))
^(3/4) + (b^8*d^5*x + a*b^7*d^5)*sqrt(((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^
4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d
^6)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^4*x + a*b^4*d^4)*sqrt((b^12*c^12 -
 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^
8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7
 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^
11*b*c*d^11 + a^12*d^12)/(b^9*d^7)))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^9*d^7))^(3/4))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^1
0*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^
5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a
^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^
13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 +
495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b
^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^
10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) + 15*b^2*d*((b^12*c^12 - 12*a*b^
11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^
4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a
^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d
^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*
b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d^2*x + a*b^2*d^2
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4))/(b*x + a)) - 15*b
^2*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7))^(1/4)*log(-5*((b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(b*x + a)^(3/4)*(d*x + c)^(1/4
) - (b^3*d^2*x + a*b^2*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^9*d^7)
)^(1/4))/(b*x + a)) - 4*(32*b^2*d^2*x^2 + 5*b^2*c^2 + 42*a*b*c*d - 15*a^2*d
^2 + 4*(13*b^2*c*d + 3*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
[Out] int((b*x+a)^(3/4)*(d*x+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)*(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(3/4)*(d*x + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{3/4}\,(c+d\,x)^{5/4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)*(c + d*x)^(5/4),x)
[Out] int((a + b*x)^(3/4)*(c + d*x)^(5/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)*(d*x+c)**(5/4),x)
[Out] Integral((a + b*x)**(3/4)*(c + d*x)**(5/4), x)
```

3.1483 $\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx$

Optimal. Leaf size=167

$$-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+d}{2b}$$

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}}+\frac{5(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8b^2}+\frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*b^2) + ((a + b*x)^(3/4)*
(c + d*x)^(5/4))/(2*b) - (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/
(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh
[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{\sqrt[4]{a+bx}}\,dx = \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{8b}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{32b^2}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{\cdots}}\cdots\right)}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x\cdots\right)}{8b^3}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,\cdots\right)}{16b^2\sqrt{d}}$$

$$= \frac{5(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8b^2} + \frac{(a+bx)^{3/4}(c+dx)^{5/4}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{9/4}d^{3/4}} + \cdots$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{3/4}(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(3/4)*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, 3/4, 7/4, (d*(a
+ b*x))/(-(b*c) + a*d)])/(3*b*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

IntegrateAlgebraic [A] time = 0.38, size = 189, normalized size = 1.13

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{16b^{9/4}d^{3/4}} + \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{16b^{9/4}d^{3/4}} + \frac{(bc-ad)^2\left(\frac{9b(c+dx)^{5/4}}{(a+bx)^{5/4}} - \frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{8b^2\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(1/4),x]
```

```
[Out] ((b*c - a*d)^2*((-5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (9*b*(c + d*x)^(5/
4))/(a + b*x)^(5/4)))/(8*b^2*(-d + (b*(c + d*x))/(a + b*x))^2) + (5*(b*c -
a*d)^2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(16*b^(
```

```
9/4)*d^(3/4)) + (5*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)
*(a + b*x)^(1/4))])/(16*b^(9/4)*d^(3/4))
```

fricas [B] time = 1.34, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*arctan(-((b^9*c^2*d^2 - 2*a*b^8*c*d^
3 + a^2*b^7*d^4)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4) -
 (b^8*d^2*x + a*b^7*d^2)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*d^2*x + a*b^
4*d^2)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^9*d^3)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(3/4))/(a*b^8*
c^8 - 8*a^2*b^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^
4*c^4*d^4 - 56*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d
^8 + (b^9*c^8 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 7
0*a^4*b^5*c^4*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d
^7 + a^8*b*d^8)*x)) - 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*d*x + a*b^2*d)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^9*d^3))^(1/4))/(b*x + a)) + 5*b^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b^3*d*x +
a*b^2*d)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^9*d^3))^(1/4))/(b*x + a)) - 4*(4*b*d*x + 9*b*c - 5*a*d)
*(b*x + a)^(3/4)*(d*x + c)^(1/4))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^{5/4}}{(a+b\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(1/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{\sqrt[4]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(1/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(1/4), x)
```

3.1484 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Rubi [A] time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 331, 298, 205, 208}

$$\frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2}-\frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}+\frac{5\sqrt[4]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}}-\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (5*d*(a + b*x)^(3/4)*(c + d*x)^(1/4))/b^2 - (4*(c + d*x)^(5/4))/(b*(a + b*x
)^(1/4)) - (5*d^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)
*(c + d*x)^(1/4))])/(2*b^(9/4)) + (5*d^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*(
a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{5/4}}\,dx = -\frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d)\int \frac{\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\,dx}{b}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4b^2}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{(5d(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} + \frac{\left(5\sqrt{d}\,(bc-ad)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2b^2} -$$

$$= \frac{5d(a+bx)^{3/4}\sqrt[4]{c+dx}}{b^2} - \frac{4(c+dx)^{5/4}}{b\sqrt[4]{a+bx}} - \frac{5\sqrt[4]{d}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{9/4}} + \frac{5\sqrt[4]{d}\,(bc-ad)\,\mathrm{ta}}{2}$$

Mathematica [C] time = 0.05, size = 71, normalized size = 0.47

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{4};\frac{3}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -1/4, 3/4, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

IntegrateAlgebraic [A] time = 13.91, size = 244, normalized size = 1.61

$$\frac{(ad+bdx)^{5/4}\left(\frac{5\left(bc\sqrt[4]{d}-ad^{5/4}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{9/4}}+\frac{5\left(bc\sqrt[4]{d}-ad^{5/4}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{9/4}}+\frac{(ad+b(c+dx)-bc)^{3/4}\left(-5ad^{5/4}\sqrt[4]{c+dx}-b\sqrt[4]{d}\,(c+dx)^{5/4}+5bc\sqrt[4]{d}\,\sqrt[4]{c+dx}\right)}{b^2(-ad-b(c+dx)+bc)}\right)}{d^{5/4}(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(5/4),x]
```

```
[Out] ((a*d + b*d*x)^(5/4)*(((-(b*c) + a*d + b*(c + d*x))^(3/4)*(5*b*c*d^(1/4)*(c
 + d*x)^(1/4) - 5*a*d^(5/4)*(c + d*x)^(1/4) - b*d^(1/4)*(c + d*x)^(5/4)))/(
b^2*(b*c - a*d - b*(c + d*x))) + (5*(b*c*d^(1/4) - a*d^(5/4))*ArcTan[(b^(1/
4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(9/4)) + (5*(
b*c*d^(1/4) - a*d^(5/4))*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d +
b*(c + d*x))^(1/4)])/(2*b^(9/4))))/(d^(5/4)*(a + b*x)^(5/4))
```

fricas [B] time = 1.04, size = 857, normalized size = 5.64

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="fricas")
```

```
[Out] 1/4*(20*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 -
 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*arctan(((b^8*c - a*b^7*d)*(b*x + a)^(3
/4)*(d*x + c)^(1/4)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a
^3*b*c*d^4 + a^4*d^5)/b^9)^(3/4) + (b^8*x + a*b^7)*sqrt(((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^5*x + a*b^4)*sqrt((b^4*c^4*d
 - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9))/(b*
x + a))*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 +
 a^4*d^5)/b^9)^(3/4))/(a*b^4*c^4*d - 4*a^2*b^3*c^3*d^2 + 6*a^3*b^2*c^2*d^3
- 4*a^4*b*c*d^4 + a^5*d^5 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d^
3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*
b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-
5*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b^3*x + a*b^2)*((b^4*c^4*
d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/
4))/(b*x + a)) - 5*(b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^
2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
a)^(3/4)*(d*x + c)^(1/4) - (b^3*x + a*b^2)*((b^4*c^4*d - 4*a*b^3*c^3*d^2 +
6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)/b^9)^(1/4))/(b*x + a)) + 4*(b*
d*x - 4*b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^3*x + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(5/4),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/4}}{{\left(a+b\,x\right)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(5/4),x)
[Out] int((c + d*x)^(5/4)/(a + b*x)^(5/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(5/4),x)
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(5/4), x)
```

3.1485 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}} \, dx$

Optimal. Leaf size=134

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 331, 298, 205, 208}

$$-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}-\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*d*(c + d*x)^(1/4))/(b^2*(a + b*x)^(1/4)) - (4*(c + d*x)^(5/4))/(5*b*(a
+ b*x)^(5/4)) - (2*d^(5/4)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d
*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/b^(9/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}}\,dx &= -\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{d\int \frac{\sqrt[4]{c+dx}}{(a+bx)^{5/4}}\,dx}{b}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{d^2\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{b^2}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(4d^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx,x,\sqrt[4]{a+bx}\right)}{b^3}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(4d^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^3}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}+\frac{\left(2d^{3/2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b^2}-\frac{\left(2d^{3/2}\right)\text{Subst}\left(\right.}{}\\
&= -\frac{4d\sqrt[4]{c+dx}}{b^2\sqrt[4]{a+bx}}-\frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}}-\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{9/4}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.54

$$-\frac{4(c+dx)^{5/4}\,{}_2F_1\left(-\frac{5}{4},-\frac{5}{4};-\frac{1}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*(c + d*x)^(5/4)*Hypergeometric2F1[-5/4, -5/4, -1/4, (d*(a + b*x))/(-(b*
c) + a*d)])/(5*b*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4))
```

IntegrateAlgebraic [A] time = 0.21, size = 134, normalized size = 1.00

$$\frac{2d^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{b^{9/4}}+\frac{2d^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{b^{9/4}}-\frac{4\left(\frac{b(c+dx)^{5/4}}{(a+bx)^{5/4}}+\frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{5b^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(9/4),x]
```

```
[Out] (-4*((5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (b*(c + d*x)^(5/4))/(a + b*x)^
(5/4)))/(5*b^2) + (2*d^(5/4)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a +
```

```
 b*x)^(1/4))])/b^(9/4) + (2*d^(5/4)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1
/4)*(a + b*x)^(1/4))])/b^(9/4)
```

fricas [B] time = 1.17, size = 368, normalized size = 2.75

$$-\frac{20\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}b^{7}d\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}-\left(b^{8}x+ab^{7}\right)\sqrt{\frac{\sqrt{bx+a}\,\sqrt{dx+c}\,d^{2}+\left(b^{5}x+ab^{4}\right)\sqrt{\frac{d^{5}}{b^{9}}}}{bx+a}}\left(\frac{d^{5}}{b^{9}}\right)^{\frac{3}{4}}}{bd^{5}x+ad^{5}}\right)-5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d+\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}\log\left(\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}d-\left(b^{3}x+ab^{2}\right)\left(\frac{d^{5}}{b^{9}}\right)^{\frac{1}{4}}}{bx+a}\right)+4\,(6\,bdx+bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(b^{4}x^{2}+2\,ab^{3}x+a^{2}b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="fricas")
```

```
[Out] -1/5*(20*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*arctan(-((b*x + a)
^(3/4)*(d*x + c)^(1/4)*b^7*d*(d^5/b^9)^(3/4) - (b^8*x + a*b^7)*sqrt((sqrt(b
*x + a)*sqrt(d*x + c)*d^2 + (b^5*x + a*b^4)*sqrt(d^5/b^9))/(b*x + a))*(d^5/
b^9)^(3/4))/(b*d^5*x + a*d^5)) - 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9
)^(1/4)*log(((b*x + a)^(3/4)*(d*x + c)^(1/4)*d + (b^3*x + a*b^2)*(d^5/b^9)^
(1/4))/(b*x + a)) + 5*(b^4*x^2 + 2*a*b^3*x + a^2*b^2)*(d^5/b^9)^(1/4)*log((
(b*x + a)^(3/4)*(d*x + c)^(1/4)*d - (b^3*x + a*b^2)*(d^5/b^9)^(1/4))/(b*x +
 a)) + 4*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4))/(b^4*x^2
+ 2*a*b^3*x + a^2*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

```
[Out] int((d*x+c)^(5/4)/(b*x+a)^(9/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(9/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^{5/4}}{{\left(a+b\,x\right)}^{9/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(9/4),x)
```

```
[Out] int((c + d*x)^(5/4)/(a + b*x)^(9/4), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{4}}}{(a+bx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(9/4),x)
```

```
[Out] Integral((c + d*x)**(5/4)/(a + b*x)**(9/4), x)
```

3.1486 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx = -\frac{4(c+dx)^{9/4}}{9(bc-ad)(a+bx)^{9/4}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

IntegrateAlgebraic [A] time = 0.06, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{9/4}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(13/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(9*(b*c - a*d)*(a + b*x)^(9/4))
```

fricas [B] time = 1.03, size = 104, normalized size = 3.25

$$-\frac{4\left(d^2x^2+2\,cdx+c^2\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{9\left(a^3bc-a^4d+\left(b^4c-ab^3d\right)x^3+3\left(ab^3c-a^2b^2d\right)x^2+3\left(a^2b^2c-a^3bd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="fricas")
```

```
[Out] -4/9*(d^2*x^2 + 2*c*d*x + c^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b*c - a
^4*d + (b^4*c - a*b^3*d)*x^3 + 3*(a*b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b^2*c -
 a^3*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{4\,(dx+c)^{\frac{9}{4}}}{9\,(bx+a)^{\frac{9}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(13/4),x)
```

```
[Out] 4/9/(b*x+a)^(9/4)*(d*x+c)^(9/4)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(13/4), x)
```

mupad [B] time = 0.81, size = 99, normalized size = 3.09

$$\frac{4\,c^2\,(c+d\,x)^{1/4}+4\,d^2\,x^2\,(c+d\,x)^{1/4}+8\,c\,d\,x\,(c+d\,x)^{1/4}}{(a+b\,x)^{1/4}\,\left(9\,d\,a^3+18\,d\,a^2\,b\,x-9\,c\,a^2\,b+9\,d\,a\,b^2\,x^2-18\,c\,a\,b^2\,x-9\,c\,b^3\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(13/4),x)
```

```
[Out] (4*c^2*(c + d*x)^(1/4) + 4*d^2*x^2*(c + d*x)^(1/4) + 8*c*d*x*(c + d*x)^(1/4
))/((a + b*x)^(1/4)*(9*a^3*d - 9*b^3*c*x^2 - 9*a^2*b*c - 18*a*b^2*c*x + 18*
a^2*b*d*x + 9*a*b^2*d*x^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(13/4),x)
```

```
[Out] Timed out
```

3.1487 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx$

Optimal. Leaf size=66

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{16d(c+dx)^{9/4}}{117(a+bx)^{9/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(9
/4))/(117*(b*c - a*d)^2*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx &= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}}-\frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}}\,dx}{13(bc-ad)}\\ &= -\frac{4(c+dx)^{9/4}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d(c+dx)^{9/4}}{117(bc-ad)^2(a+bx)^{9/4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{9/4}(13ad-9bc+4bdx)}{117(a+bx)^{13/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(-9*b*c + 13*a*d + 4*b*d*x))/(117*(b*c - a*d)^2*(a + b*x
)^(13/4))
```

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 0.77

$$-\frac{4(c+dx)^{9/4}\left(\frac{9b(c+dx)}{a+bx}-13d\right)}{117(a+bx)^{9/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(17/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(-13*d + (9*b*(c + d*x))/(a + b*x)))/(117*(b*c - a*d)^2
*(a + b*x)^(9/4))
```

fricas [B] time = 1.24, size = 235, normalized size = 3.56

$$\frac{4\left(4\,bd^3x^3-9\,bc^3+13\,ac^2d-\left(bcd^2-13\,ad^3\right)x^2-2\left(7\,bc^2d-13\,acd^2\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{117\left(a^4b^2c^2-2\,a^5bcd+a^6d^2+\left(b^6c^2-2\,ab^5cd+a^2b^4d^2\right)x^4+4\left(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2\right)x^3+6\left(a^2b^4c^2-2\,a^3b^3cd+a^4b^2d^2\right)x^2+4\left(a^3b^3c^2-2\,a^4b^2cd+a^5bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="fricas")
```

```
[Out] 4/117*(4*b*d^3*x^3 - 9*b*c^3 + 13*a*c^2*d - (b*c*d^2 - 13*a*d^3)*x^2 - 2*(7
*b*c^2*d - 13*a*c*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^2*c^2 - 2*
a^5*b*c*d + a^6*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^4 + 4*(a*b^5*
c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^3 + 6*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a
^4*b^2*d^2)*x^2 + 4*(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{9}{4}}\,(4bdx+13ad-9bc)}{117\,(bx+a)^{\frac{13}{4}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(17/4),x)
```

```
[Out] 4/117*(d*x+c)^(9/4)*(4*b*d*x+13*a*d-9*b*c)/(b*x+a)^(13/4)/(a^2*d^2-2*a*b*c*
d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{17}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(17/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(17/4), x)
```

mupad [B] time = 0.95, size = 178, normalized size = 2.70

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{16\,d^3\,x^3}{117\,b^2\,{(a\,d-b\,c)}^2}-\frac{36\,b\,c^3-52\,a\,c^2\,d}{117\,b^3\,{(a\,d-b\,c)}^2}+\frac{x^2\,\left(52\,a\,d^3-4\,b\,c\,d^2\right)}{117\,b^3\,{(a\,d-b\,c)}^2}+\frac{8\,c\,d\,x\,(13\,a\,d-7\,b\,c)}{117\,b^3\,{(a\,d-b\,c)}^2}\right)}{x^3\,{(a+b\,x)}^{1/4}+\frac{a^3\,{(a+b\,x)}^{1/4}}{b^3}+\frac{3\,a\,x^2\,{(a+b\,x)}^{1/4}}{b}+\frac{3\,a^2\,x\,{(a+b\,x)}^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(17/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((16*d^3*x^3)/(117*b^2*(a*d - b*c)^2) - (36*b*c^3 - 52*a*c
^2*d)/(117*b^3*(a*d - b*c)^2) + (x^2*(52*a*d^3 - 4*b*c*d^2))/(117*b^3*(a*d
- b*c)^2) + (8*c*d*x*(13*a*d - 7*b*c))/(117*b^3*(a*d - b*c)^2)))/(x^3*(a +
b*x)^(1/4) + (a^3*(a + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a
^2*x*(a + b*x)^(1/4))/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(17/4),x)
```

```
[Out] Timed out
```

3.1488 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}} dx$

Optimal. Leaf size=101

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{9/4}}{1989(a+bx)^{9/4}(bc-ad)^3}+\frac{32d(c+dx)^{9/4}}{221(a+bx)^{13/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{17(a+bx)^{17/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(17*(b*c - a*d)*(a + b*x)^(17/4)) + (32*d*(c + d*x)^(9
/4))/(221*(b*c - a*d)^2*(a + b*x)^(13/4)) - (128*d^2*(c + d*x)^(9/4))/(1989
*(b*c - a*d)^3*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}} dx &= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}-\frac{(8d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}} dx}{17(bc-ad)}\\
&= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}+\frac{\left(32d^2\right)\int \frac{(c+dx)^{5/4}}{(a+bx)^{13/4}} dx}{221(bc-ad)^2}\\
&= -\frac{4(c+dx)^{9/4}}{17(bc-ad)(a+bx)^{17/4}}+\frac{32d(c+dx)^{9/4}}{221(bc-ad)^2(a+bx)^{13/4}}-\frac{128d^2(c+dx)^{9/4}}{1989(bc-ad)^3(a+bx)^{9/4}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{9/4}\left(221a^2d^2+34abd(4dx-9c)+b^2\left(117c^2-72cdx+32d^2x^2\right)\right)}{1989(a+bx)^{17/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*a^2*d^2 + 34*a*b*d*(-9*c + 4*d*x) + b^2*(117*c^2 -
 72*c*d*x + 32*d^2*x^2)))/(1989*(b*c - a*d)^3*(a + b*x)^(17/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{9/4}\left(\frac{117b^2(c+dx)^2}{(a+bx)^2}-\frac{306bd(c+dx)}{a+bx}+221d^2\right)}{1989(a+bx)^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(21/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(221*d^2 - (306*b*d*(c + d*x))/(a + b*x) + (117*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(1989*(b*c - a*d)^3*(a + b*x)^(9/4))
```

fricas [B] time = 1.52, size = 426, normalized size = 4.22

$$-\frac{4\left(32\,b^2d^4x^4+117\,b^2c^4-306\,abc^3d+221\,a^2c^2d^2-8\left(b^2cd^3-17\,abd^4\right)x^3+\left(5\,b^2c^2d^2-34\,abcd^3+221\,a^2d^4\right)x^2+2\left(81\,b^2c^3d-238\,abc^2d^2+221\,a^2cd^3\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{1989\left(a^5b^3c^3-3\,a^6b^2c^2d+3\,a^7bcd^2-a^8d^3+\left(b^8c^3-3\,ab^7c^2d+3\,a^2b^6cd^2-a^3b^5d^3\right)x^5+5\left(ab^7c^3-3\,a^2b^6c^2d+3\,a^3b^5cd^2-a^4b^4d^3\right)x^4+10\left(a^2b^6c^3-3\,a^3b^5c^2d+3\,a^4b^4cd^2-a^5b^3d^3\right)x^3+10\left(a^3b^5c^3-3\,a^4b^4c^2d+3\,a^5b^3cd^2-a^6b^2d^3\right)x^2+5\left(a^4b^4c^3-3\,a^5b^3c^2d+3\,a^6b^2cd^2-a^7bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="fricas")
```

```
[Out] -4/1989*(32*b^2*d^4*x^4 + 117*b^2*c^4 - 306*a*b*c^3*d + 221*a^2*c^2*d^2 - 8
*(b^2*c*d^3 - 17*a*b*d^4)*x^3 + (5*b^2*c^2*d^2 - 34*a*b*c*d^3 + 221*a^2*d^4
)*x^2 + 2*(81*b^2*c^3*d - 238*a*b*c^2*d^2 + 221*a^2*c*d^3)*x)*(b*x + a)^(3/
4)*(d*x + c)^(1/4)/(a^5*b^3*c^3 - 3*a^6*b^2*c^2*d + 3*a^7*b*c*d^2 - a^8*d^3
 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^5 + 5*(a*b^7
*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^4 + 10*(a^2*b^6*c
^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^3 + 10*(a^3*b^5*c^3
 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x^2 + 5*(a^4*b^4*c^3 -
3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{9}{4}}\left(32b^2x^2d^2+136ab\,d^2x-72b^2cdx+221a^2d^2-306abcd+117b^2c^2\right)}{1989\,(bx+a)^{\frac{17}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(21/4),x)
```

```
[Out] 4/1989*(d*x+c)^(9/4)*(32*b^2*d^2*x^2+136*a*b*d^2*x-72*b^2*c*d*x+221*a^2*d^2
-306*a*b*c*d+117*b^2*c^2)/(b*x+a)^(17/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{21}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(21/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(21/4), x)
```

mupad [B] time = 1.13, size = 268, normalized size = 2.65

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{884\,a^2\,c^2\,d^2-1224\,a\,b\,c^3\,d+468\,b^2\,c^4}{1989\,b^4\,{(a\,d-b\,c)}^3}+\frac{x^2\,\left(884\,a^2\,d^4-136\,a\,b\,c\,d^3+20\,b^2\,c^2\,d^2\right)}{1989\,b^4\,{(a\,d-b\,c)}^3}+\frac{128\,d^4\,x^4}{1989\,b^2\,{(a\,d-b\,c)}^3}+\frac{32\,d^3\,x^3\,(17\,a\,d-b\,c)}{1989\,b^3\,{(a\,d-b\,c)}^3}+\frac{8\,c\,d\,x\,\left(221\,a^2\,d^2-238\,a\,b\,c\,d+81\,b^2\,c^2\right)}{1989\,b^4\,{(a\,d-b\,c)}^3}\right)}{x^4\,(a+b\,x)^{1/4}+\frac{a^4\,{(a+b\,x)}^{1/4}}{b^4}+\frac{6\,a^2\,x^2\,{(a+b\,x)}^{1/4}}{b^2}+\frac{4\,a\,x^3\,{(a+b\,x)}^{1/4}}{b}+\frac{4\,a^3\,x\,{(a+b\,x)}^{1/4}}{b^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(21/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((468*b^2*c^4 + 884*a^2*c^2*d^2 - 1224*a*b*c^3*d)/(1989*b^
4*(a*d - b*c)^3) + (x^2*(884*a^2*d^4 + 20*b^2*c^2*d^2 - 136*a*b*c*d^3))/(19
89*b^4*(a*d - b*c)^3) + (128*d^4*x^4)/(1989*b^2*(a*d - b*c)^3) + (32*d^3*x^
3*(17*a*d - b*c))/(1989*b^3*(a*d - b*c)^3) + (8*c*d*x*(221*a^2*d^2 + 81*b^2
*c^2 - 238*a*b*c*d))/(1989*b^4*(a*d - b*c)^3)))/(x^4*(a + b*x)^(1/4) + (a^4
*(a + b*x)^(1/4))/b^4 + (6*a^2*x^2*(a + b*x)^(1/4))/b^2 + (4*a*x^3*(a + b*x
)^(1/4))/b + (4*a^3*x*(a + b*x)^(1/4))/b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(21/4),x)
```

```
[Out] Timed out
```

3.1489 $\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx$

Optimal. Leaf size=136

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{9/4}}{13923(a+bx)^{9/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{9/4}}{1547(a+bx)^{13/4}(bc-ad)^3}+\frac{16d(c+dx)^{9/4}}{119(a+bx)^{17/4}(bc-ad)^2}-\frac{4(c+dx)^{9/4}}{21(a+bx)^{21/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4))/(21*(b*c - a*d)*(a + b*x)^(21/4)) + (16*d*(c + d*x)^(9
/4))/(119*(b*c - a*d)^2*(a + b*x)^(17/4)) - (128*d^2*(c + d*x)^(9/4))/(1547
*(b*c - a*d)^3*(a + b*x)^(13/4)) + (512*d^3*(c + d*x)^(9/4))/(13923*(b*c -
a*d)^4*(a + b*x)^(9/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{5/4}}{(a+bx)^{25/4}}\,dx &= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}-\frac{(4d)\int \frac{(c+dx)^{5/4}}{(a+bx)^{21/4}}\,dx}{7(bc-ad)}\\
&= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}+\frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}}+\frac{\left(32d^2\right)\int \frac{(c+dx)^{5/4}}{(a+bx)^{17/4}}\,dx}{119(bc-ad)^2}\\
&= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}+\frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}}-\frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}-\frac{\left(128d^3\right)\ldots}{154\ldots}\\
&= -\frac{4(c+dx)^{9/4}}{21(bc-ad)(a+bx)^{21/4}}+\frac{16d(c+dx)^{9/4}}{119(bc-ad)^2(a+bx)^{17/4}}-\frac{128d^2(c+dx)^{9/4}}{1547(bc-ad)^3(a+bx)^{13/4}}+\frac{51\ldots}{13923(\ldots}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{9/4}\left(1547a^3d^3+357a^2bd^2(4dx-9c)+21ab^2d\left(117c^2-72cdx+32d^2x^2\right)+b^3\left(-663c^3+468c^2dx-288cd^2x^2+128d^3x^3\right)\right)}{13923(a+bx)^{21/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (4*(c + d*x)^(9/4)*(1547*a^3*d^3 + 357*a^2*b*d^2*(-9*c + 4*d*x) + 21*a*b^2*
d*(117*c^2 - 72*c*d*x + 32*d^2*x^2) + b^3*(-663*c^3 + 468*c^2*d*x - 288*c*d
^2*x^2 + 128*d^3*x^3)))/(13923*(b*c - a*d)^4*(a + b*x)^(21/4))
```

IntegrateAlgebraic [A] time = 0.19, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{9/4}\left(\frac{663b^3(c+dx)^3}{(a+bx)^3}-\frac{2457b^2d(c+dx)^2}{(a+bx)^2}+\frac{3213bd^2(c+dx)}{a+bx}-1547d^3\right)}{13923(a+bx)^{9/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/4)/(a + b*x)^(25/4),x]
```

```
[Out] (-4*(c + d*x)^(9/4)*(-1547*d^3 + (3213*b*d^2*(c + d*x))/(a + b*x) - (2457*b
^2*d*(c + d*x)^2)/(a + b*x)^2 + (663*b^3*(c + d*x)^3)/(a + b*x)^3))/(13923*
(b*c - a*d)^4*(a + b*x)^(9/4))
```

fricas [B] time = 1.66, size = 649, normalized size = 4.77

$$\frac{4\left(128\,b^3d^5x^5-663\,b^3c^5+2457\,ab^2c^4d-3213\,a^2bc^3d^2+1547\,a^3c^2d^3-32\left(b^3cd^4-21\,ab^2d^5\right)x^4+4\left(5\,b^3c^2d^3-42\,ab^2cd^4+357\,a^2bd^5\right)x^3-\left(15\,b^3c^3d^2-105\,ab^2c^2d^3+357\,a^2bcd^4-1547\,a^3d^5\right)x^2-2\left(429\,b^3c^4d-1701\,ab^2c^3d^2+2499\,a^2bc^2d^3-1547\,a^3cd^4\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{13923\left(a^6b^4c^4-4\,a^7b^3c^3d+6\,a^8b^2c^2d^2-4\,a^9bcd^3+a^{10}d^4+\left(b^{10}c^4-4\,ab^9c^3d+6\,a^2b^8c^2d^2-4\,a^3b^7cd^3+a^4b^6d^4\right)x^6+6\left(ab^9c^4-4\,a^2b^8c^3d+6\,a^3b^7c^2d^2-4\,a^4b^6cd^3+a^5b^5d^4\right)x^5+15\left(a^2b^8c^4-4\,a^3b^7c^3d+6\,a^4b^6c^2d^2-4\,a^5b^5cd^3+a^6b^4d^4\right)x^4+20\left(a^3b^7c^4-4\,a^4b^6c^3d+6\,a^5b^5c^2d^2-4\,a^6b^4cd^3+a^7b^3d^4\right)x^3+15\left(a^4b^6c^4-4\,a^5b^5c^3d+6\,a^6b^4c^2d^2-4\,a^7b^3cd^3+a^8b^2d^4\right)x^2+6\left(a^5b^5c^4-4\,a^6b^4c^3d+6\,a^7b^3c^2d^2-4\,a^8b^2cd^3+a^9bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="fricas")
```

```
[Out] 4/13923*(128*b^3*d^5*x^5 - 663*b^3*c^5 + 2457*a*b^2*c^4*d - 3213*a^2*b*c^3*
d^2 + 1547*a^3*c^2*d^3 - 32*(b^3*c*d^4 - 21*a*b^2*d^5)*x^4 + 4*(5*b^3*c^2*d
^3 - 42*a*b^2*c*d^4 + 357*a^2*b*d^5)*x^3 - (15*b^3*c^3*d^2 - 105*a*b^2*c^2*
d^3 + 357*a^2*b*c*d^4 - 1547*a^3*d^5)*x^2 - 2*(429*b^3*c^4*d - 1701*a*b^2*c
^3*d^2 + 2499*a^2*b*c^2*d^3 - 1547*a^3*c*d^4)*x)*(b*x + a)^(3/4)*(d*x + c)^
(1/4)/(a^6*b^4*c^4 - 4*a^7*b^3*c^3*d + 6*a^8*b^2*c^2*d^2 - 4*a^9*b*c*d^3 +
a^10*d^4 + (b^10*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3
+ a^4*b^6*d^4)*x^6 + 6*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4
*a^4*b^6*c*d^3 + a^5*b^5*d^4)*x^5 + 15*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a
^4*b^6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*x^4 + 20*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*x^3 + 15*
(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*
b^2*d^4)*x^2 + 6*(a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8
*b^2*c*d^3 + a^9*b*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{9}{4}}\left(128b^3d^3x^3+672a\,b^2d^3x^2-288b^3c\,d^2x^2+1428a^2b\,d^3x-1512a\,b^2c\,d^2x+468b^3c^2dx+1547a^3d^3-3213a^2bc\,d^2+2457a\,b^2c^2d-663b^3c^3\right)}{13923\,(bx+a)^{\frac{21}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/4)/(b*x+a)^(25/4),x)
```

```
[Out] 4/13923*(d*x+c)^(9/4)*(128*b^3*d^3*x^3+672*a*b^2*d^3*x^2-288*b^3*c*d^2*x^2+
1428*a^2*b*d^3*x-1512*a*b^2*c*d^2*x+468*b^3*c^2*d*x+1547*a^3*d^3-3213*a^2*b
*c*d^2+2457*a*b^2*c^2*d-663*b^3*c^3)/(b*x+a)^(21/4)/(a^4*d^4-4*a^3*b*c*d^3+
6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{4}}}{(bx+a)^{\frac{25}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/4)/(b*x+a)^(25/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(5/4)/(b*x + a)^(25/4), x)
```

mupad [B] time = 1.36, size = 376, normalized size = 2.76

$$\frac{(c+d\,x)^{1/4}\left(\frac{x^2\left(6188\,a^3\,d^5-1428\,a^2\,b\,c\,d^4+420\,a\,b^2\,c^2\,d^3-60\,b^3\,c^3\,d^2\right)}{13923\,b^5\,(a\,d-b\,c)^4}-\frac{-6188\,a^3\,c^2\,d^3+12852\,a^2\,b\,c^3\,d^2-9828\,a\,b^2\,c^4\,d+2652\,b^3\,c^5}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{x\left(12376\,a^3\,c\,d^4-19992\,a^2\,b\,c^2\,d^3+13608\,a\,b^2\,c^3\,d^2-3432\,b^3\,c^4\,d\right)}{13923\,b^5\,(a\,d-b\,c)^4}+\frac{512\,d^5\,x^5}{13923\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^4\,x^4\,(21\,a\,d-b\,c)}{13923\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d^3\,x^3\left(357\,a^2\,d^2-42\,a\,b\,c\,d+5\,b^2\,c^2\right)}{13923\,b^4\,(a\,d-b\,c)^4}\right)}{x^5\,(a+b\,x)^{1/4}+\frac{a^5\,(a+b\,x)^{1/4}}{b^5}+\frac{10\,a^2\,x^3\,(a+b\,x)^{1/4}}{b^2}+\frac{10\,a^3\,x^2\,(a+b\,x)^{1/4}}{b^3}+\frac{5\,a\,x^4\,(a+b\,x)^{1/4}}{b}+\frac{5\,a^4\,x\,(a+b\,x)^{1/4}}{b^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/4)/(a + b*x)^(25/4),x)
```

```
[Out] ((c + d*x)^(1/4)*((x^2*(6188*a^3*d^5 - 60*b^3*c^3*d^2 + 420*a*b^2*c^2*d^3 -
 1428*a^2*b*c*d^4))/(13923*b^5*(a*d - b*c)^4) - (2652*b^3*c^5 - 6188*a^3*c^
2*d^3 + 12852*a^2*b*c^3*d^2 - 9828*a*b^2*c^4*d)/(13923*b^5*(a*d - b*c)^4) +
 (x*(12376*a^3*c*d^4 - 3432*b^3*c^4*d + 13608*a*b^2*c^3*d^2 - 19992*a^2*b*c
^2*d^3))/(13923*b^5*(a*d - b*c)^4) + (512*d^5*x^5)/(13923*b^2*(a*d - b*c)^4
) + (128*d^4*x^4*(21*a*d - b*c))/(13923*b^3*(a*d - b*c)^4) + (16*d^3*x^3*(3
57*a^2*d^2 + 5*b^2*c^2 - 42*a*b*c*d))/(13923*b^4*(a*d - b*c)^4)))/(x^5*(a +
 b*x)^(1/4) + (a^5*(a + b*x)^(1/4))/b^5 + (10*a^2*x^3*(a + b*x)^(1/4))/b^2
+ (10*a^3*x^2*(a + b*x)^(1/4))/b^3 + (5*a*x^4*(a + b*x)^(1/4))/b + (5*a^4*x
*(a + b*x)^(1/4))/b^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/4)/(b*x+a)**(25/4),x)
```

```
[Out] Timed out
```

3.1490 $\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=167

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+}{2d}$$

Rubi [A] time = 0.10, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}+\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}}-\frac{5\sqrt[4]{a+bx}\,(c+dx)^{3/4}(bc-ad)}{8d^2}+\frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (-5*(b*c - a*d)*(a + b*x)^(1/4)*(c + d*x)^(3/4))/(8*d^2) + ((a + b*x)^(5/4)
*(c + d*x)^(3/4))/(2*d) + (5*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))
/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTan
h[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{(a+bx)^{5/4}}{\sqrt[4]{c+dx}}\,dx = \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} - \frac{(5(bc-ad))\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{8d}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{32d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\right.}{8bd^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x,\right.}{8bd^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx\right.}{16\sqrt{b}\,d^2}$$

$$= -\frac{5(bc-ad)\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{8d^2} + \frac{(a+bx)^{5/4}(c+dx)^{3/4}}{2d} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{5(}{}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{9/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.40, size = 189, normalized size = 1.13

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{16b^{3/4}d^{9/4}} + \frac{(ad-bc)^2\left(\frac{9d(a+bx)^{5/4}}{(c+dx)^{5/4}} - \frac{5b\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{8d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((-(b*c) + a*d)^2*((9*d*(a + b*x)^(5/4))/(c + d*x)^(5/4) - (5*b*(a + b*x)^(
1/4))/(c + d*x)^(1/4)))/(8*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) + (5*(b*c
- a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b
```

```
^(3/4)*d^(9/4)) + (5*(b*c - a*d)^2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/
4)*(c + d*x)^(1/4))])/(16*b^(3/4)*d^(9/4))
```

fricas [B] time = 1.15, size = 1468, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/32*(20*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*arctan(-((b^4*c^2*d^7 - 2*a*b^3*c*d^
8 + a^2*b^2*d^9)*(b*x + a)^(1/4)*(d*x + c)^(3/4)*((b^8*c^8 - 8*a*b^7*c^7*d
+ 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3
*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4) -
 (b^2*d^8*x + b^2*c*d^7)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^5*x + b^2*
c*d^4)*sqrt((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*
b*c*d^7 + a^8*d^8)/(b^3*d^9)))/(d*x + c))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^
2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^
5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(3/4))/(b^8*c^
9 - 8*a*b^7*c^8*d + 28*a^2*b^6*c^7*d^2 - 56*a^3*b^5*c^6*d^3 + 70*a^4*b^4*c^
5*d^4 - 56*a^5*b^3*c^4*d^5 + 28*a^6*b^2*c^3*d^6 - 8*a^7*b*c^2*d^7 + a^8*c*d
^8 + (b^8*c^8*d - 8*a*b^7*c^7*d^2 + 28*a^2*b^6*c^6*d^3 - 56*a^3*b^5*c^5*d^4
 + 70*a^4*b^4*c^4*d^5 - 56*a^5*b^3*c^3*d^6 + 28*a^6*b^2*c^2*d^7 - 8*a^7*b*c
*d^8 + a^8*d^9)*x)) - 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^3*x + b*c*d^2)*((b^
8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d
^8)/(b^3*d^9))^(1/4))/(d*x + c)) + 5*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4)*log(5*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/4)*(d*x + c)^(3/4) - (b*d^3*x +
b*c*d^2)*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^
3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*
c*d^7 + a^8*d^8)/(b^3*d^9))^(1/4))/(d*x + c)) - 4*(4*b*d*x - 5*b*c + 9*a*d)
*(b*x + a)^(1/4)*(d*x + c)^(3/4))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(5/4)/(c + d*x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(1/4), x)
```

3.1491 $\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}} dx$

Optimal. Leaf size=127

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

Rubi [A] time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 240, 212, 208, 205}

$$-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}+\frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] ((a + b*x)^(1/4)*(c + d*x)^(3/4))/d - ((b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x
)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/4)) - ((b*c - a*d)*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(3/4)*d^(5/
4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx &= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d}\\
&= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}\\
&= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}\\
&= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2\sqrt{b}\,d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\right.}{2\sqrt{b}\,d}\\
&= \frac{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2b^{3/4}d^{5/4}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{5/4}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 7.84, size = 176, normalized size = 1.39

$$\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}\left(\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{3/4}d^{5/4}} + \frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2b^{3/4}d^{5/4}} + \frac{(c+dx)^{3/4}\sqrt[4]{ad+b(c+dx)-bc}}{d^{5/4}}\right)}{\sqrt[4]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/4)/(c + d*x)^(1/4),x]
```

```
[Out] (d^(1/4)*(a + b*x)^(1/4)*(((c + d*x)^(3/4)*(-(b*c) + a*d + b*(c + d*x))^(1/
4))/d^(5/4) + ((b*c - a*d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d +
 b*(c + d*x))^(1/4)])/(2*b^(3/4)*d^(5/4)) + ((-(b*c) + a*d)*ArcTanh[(b^(1/4
)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(3/4)*d^(5/4))
))/(a*d + b*d*x)^(1/4)
```

fricas [B] time = 1.13, size = 814, normalized size = 6.41

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b^3*d^5))^(1/4)*arctan(((b^3*c*d^4 - a*b^2*d^5)*(b*x + a)^(1/4)*(d
*x + c)^(3/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(b^3*d^5))^(3/4) + (b^2*d^5*x + b^2*c*d^4)*sqrt(((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b^2*d^3*x + b^2*c*d^2)*sqr
t((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(
b^3*d^5)))/(d*x + c))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(3/4))/(b^4*c^5 - 4*a*b^3*c^4*d + 6*a^2*b^2*
c^3*d^2 - 4*a^3*b*c^2*d^3 + a^4*c*d^4 + (b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^
2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*x)) + d*((b^4*c^4 - 4*a*b^3*c^3*d
+ 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c
- a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/
(d*x + c)) - d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)/(b^3*d^5))^(1/4)*log(-((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(
3/4) - (b*d^2*x + b*c*d)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)/(b^3*d^5))^(1/4))/(d*x + c)) - 4*(b*x + a)^(1/4)*(d*
x + c)^(3/4))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(1/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(1/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(1/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(1/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(1/4), x)
```

3.1492 $\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Rubi [A] time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 240, 212, 208, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{b}}$$
$$= \frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.84

$$\frac{4\sqrt[4]{a+bx}\sqrt[4]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4)*((b*(c + d*x))/(b*c - a*d))^(1/4)*Hypergeometric2F1[1/4,
 1/4, 5/4, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{b^{3/4}\sqrt[4]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(3/4)*d^
(1/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(
b^(3/4)*d^(1/4))
```

fricas [B] time = 0.75, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}b^2d\left(\frac{1}{b^3d}\right)^{\frac{3}{4}} - \left(b^2d^2x+b^2cd\right)\sqrt{\frac{\left(b^2dx+b^2c\right)\sqrt{\frac{1}{b^3d}}+\sqrt{bx+a}\,\sqrt{dx+c}}{dx+c}}\left(\frac{1}{b^3d}\right)^{\frac{3}{4}}}{dx+c}\right) + \left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}} + (bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right) - \left(\frac{1}{b^3d}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^3d}\right)^{\frac{1}{4}} - (bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b^3*d))^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*b^2*d*(1/(b^3
*d))^(3/4) - (b^2*d^2*x + b^2*c*d)*sqrt(((b^2*d*x + b^2*c)*sqrt(1/(b^3*d))
+ sqrt(b*x + a)*sqrt(d*x + c))/(d*x + c))*(1/(b^3*d))^(3/4))/(d*x + c)) + (
1/(b^3*d))^(1/4)*log(((b*d*x + b*c)*(1/(b^3*d))^(1/4) + (b*x + a)^(1/4)*(d*
x + c)^(3/4))/(d*x + c)) - (1/(b^3*d))^(1/4)*log(-((b*d*x + b*c)*(1/(b^3*d)
)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
[Out] int(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{3/4}\,{\left(c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\,\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(1/4)), x)
```

3.1493 $\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=32

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx = -\frac{4(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/4}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$-\frac{4(c+dx)^{3/4}}{3(a+bx)^{3/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x]
[Out] (-4*(c + d*x)^(3/4))/(3*(b*c - a*d)*(a + b*x)^(3/4))
```

fricas [A] time = 1.10, size = 42, normalized size = 1.31

$$-\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(abc-a^2d+\left(b^2c-abd\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="fricas")
[Out] -4/3*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$\frac{4\,(dx+c)^{\frac{3}{4}}}{3\,(bx+a)^{\frac{3}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x)
[Out] 4/3/(b*x+a)^(3/4)*(d*x+c)^(3/4)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{7/4}\,{(c+d\,x)}^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\sqrt[4]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(1/4)), x)
```

3.1494 $\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=66

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d(c+dx)^{3/4}}{21(a+bx)^{3/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{7(a+bx)^{7/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(7*(b*c - a*d)*(a + b*x)^(7/4)) + (16*d*(c + d*x)^(3/4
))/(21*(b*c - a*d)^2*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx &= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}-\frac{(4d)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{7(bc-ad)}\\ &= -\frac{4(c+dx)^{3/4}}{7(bc-ad)(a+bx)^{7/4}}+\frac{16d(c+dx)^{3/4}}{21(bc-ad)^2(a+bx)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{4(c+dx)^{3/4}(7ad-3bc+4bdx)}{21(a+bx)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(-3*b*c + 7*a*d + 4*b*d*x))/(21*(b*c - a*d)^2*(a + b*x)^
(7/4))
```

IntegrateAlgebraic [A] time = 0.16, size = 51, normalized size = 0.77

$$\frac{4(c+dx)^{7/4}\left(\frac{7d(a+bx)}{c+dx}-3b\right)}{21(a+bx)^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(7/4)*(-3*b + (7*d*(a + b*x))/(c + d*x)))/(21*(b*c - a*d)^2*(a
 + b*x)^(7/4))
```

fricas [B] time = 1.30, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx-3\,bc+7\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/21*(4*b*d*x - 3*b*c + 7*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a
*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{3}{4}}\,(4bdx+7ad-3bc)}{21\,(bx+a)^{\frac{7}{4}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/21*(d*x+c)^(3/4)*(4*b*d*x+7*a*d-3*b*c)/(b*x+a)^(7/4)/(a^2*d^2-2*a*b*c*d+b
^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left (a+b\,x\right)}^{11/4}\,{\left (c+d\,x\right)}^{1/4}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(1/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + bx\right)^{\frac{11}{4}} \sqrt[4]{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(1/4),x)
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(1/4)), x)
```

3.1495 $\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=101

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2(c+dx)^{3/4}}{231(a+bx)^{3/4}(bc-ad)^3}+\frac{32d(c+dx)^{3/4}}{77(a+bx)^{7/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{11(a+bx)^{11/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(11*(b*c - a*d)*(a + b*x)^(11/4)) + (32*d*(c + d*x)^(3
/4))/(77*(b*c - a*d)^2*(a + b*x)^(7/4)) - (128*d^2*(c + d*x)^(3/4))/(231*(b
*c - a*d)^3*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx &= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}-\frac{(8d)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{11(bc-ad)}\\ &= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{7/4}\sqrt[4]{c+dx}}\,dx}{77(bc-ad)^2}\\ &= -\frac{4(c+dx)^{3/4}}{11(bc-ad)(a+bx)^{11/4}}+\frac{32d(c+dx)^{3/4}}{77(bc-ad)^2(a+bx)^{7/4}}-\frac{128d^2(c+dx)^{3/4}}{231(bc-ad)^3(a+bx)^{3/4}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 0.76

$$-\frac{4(c+dx)^{3/4}\left(77a^2d^2+22abd(4dx-3c)+b^2\left(21c^2-24cdx+32d^2x^2\right)\right)}{231(a+bx)^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4)*(77*a^2*d^2 + 22*a*b*d*(-3*c + 4*d*x) + b^2*(21*c^2 - 2
4*c*d*x + 32*d^2*x^2)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

IntegrateAlgebraic [A] time = 0.17, size = 73, normalized size = 0.72

$$-\frac{4(c+dx)^{11/4}\left(\frac{77d^2(a+bx)^2}{(c+dx)^2}-\frac{66bd(a+bx)}{c+dx}+21b^2\right)}{231(a+bx)^{11/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(21*b^2 + (77*d^2*(a + b*x)^2)/(c + d*x)^2 - (66*b*d*(
a + b*x))/(c + d*x)))/(231*(b*c - a*d)^3*(a + b*x)^(11/4))
```

fricas [B] time = 2.55, size = 252, normalized size = 2.50

$$-\frac{4\left(32\,b^2d^2x^2+21\,b^2c^2-66\,abcd+77\,a^2d^2-8\left(3\,b^2cd-11\,abd^2\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{231\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] -4/231*(32*b^2*d^2*x^2 + 21*b^2*c^2 - 66*a*b*c*d + 77*a^2*d^2 - 8*(3*b^2*c*
d - 11*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^3*c^3 - 3*a^4*b^2
*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d
^2 - a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 -
a^4*b^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5
*b*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{3}{4}}\left(32b^2x^2d^2+88ab\,d^2x-24b^2cdx+77a^2d^2-66abcd+21b^2c^2\right)}{231\,(bx+a)^{\frac{11}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/231*(d*x+c)^(3/4)*(32*b^2*d^2*x^2+88*a*b*d^2*x-24*b^2*c*d*x+77*a^2*d^2-66
*a*b*c*d+21*b^2*c^2)/(b*x+a)^(11/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)),x)
```

```
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(1/4),x)
```

```
[Out] Timed out
```

3.1496 $\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx$

Optimal. Leaf size=136

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3(c+dx)^{3/4}}{1155(a+bx)^{3/4}(bc-ad)^4}-\frac{128d^2(c+dx)^{3/4}}{385(a+bx)^{7/4}(bc-ad)^3}+\frac{16d(c+dx)^{3/4}}{55(a+bx)^{11/4}(bc-ad)^2}-\frac{4(c+dx)^{3/4}}{15(a+bx)^{15/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4))/(15*(b*c - a*d)*(a + b*x)^(15/4)) + (16*d*(c + d*x)^(3
/4))/(55*(b*c - a*d)^2*(a + b*x)^(11/4)) - (128*d^2*(c + d*x)^(3/4))/(385*(
b*c - a*d)^3*(a + b*x)^(7/4)) + (512*d^3*(c + d*x)^(3/4))/(1155*(b*c - a*d)
^4*(a + b*x)^(3/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{19/4}\sqrt[4]{c+dx}}\,dx &= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}-\frac{(4d)\int \frac{1}{(a+bx)^{15/4}\sqrt[4]{c+dx}}\,dx}{5(bc-ad)}\\
&= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}+\frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{11/4}\sqrt[4]{c+dx}}\,dx}{55(bc-ad)^2}\\
&= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}+\frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}}-\frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}}-\Big(1\\
&= -\frac{4(c+dx)^{3/4}}{15(bc-ad)(a+bx)^{15/4}}+\frac{16d(c+dx)^{3/4}}{55(bc-ad)^2(a+bx)^{11/4}}-\frac{128d^2(c+dx)^{3/4}}{385(bc-ad)^3(a+bx)^{7/4}}+\frac{}{1}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 118, normalized size = 0.87

$$\frac{4(c+dx)^{3/4}\left(385a^3d^3+165a^2bd^2(4dx-3c)+15ab^2d\left(21c^2-24cdx+32d^2x^2\right)+b^3\left(-77c^3+84c^2dx-96cd^2x^2+128d^3x^3\right)\right)}{1155(a+bx)^{15/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(3/4)*(385*a^3*d^3 + 165*a^2*b*d^2*(-3*c + 4*d*x) + 15*a*b^2*d
*(21*c^2 - 24*c*d*x + 32*d^2*x^2) + b^3*(-77*c^3 + 84*c^2*d*x - 96*c*d^2*x^
2 + 128*d^3*x^3)))/(1155*(b*c - a*d)^4*(a + b*x)^(15/4))
```

IntegrateAlgebraic [A] time = 0.18, size = 95, normalized size = 0.70

$$\frac{4(c+dx)^{15/4}\left(\frac{315b^2d(a+bx)}{c+dx}+\frac{385d^3(a+bx)^3}{(c+dx)^3}-\frac{495bd^2(a+bx)^2}{(c+dx)^2}-77b^3\right)}{1155(a+bx)^{15/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x]
```

```
[Out] (4*(c + d*x)^(15/4)*(-77*b^3 + (385*d^3*(a + b*x)^3)/(c + d*x)^3 - (495*b*d
^2*(a + b*x)^2)/(c + d*x)^2 + (315*b^2*d*(a + b*x))/(c + d*x)))/(1155*(b*c
- a*d)^4*(a + b*x)^(15/4))
```

fricas [B] time = 5.10, size = 419, normalized size = 3.08

$$\frac{4\left(128\,b^3d^3x^3-77\,b^3c^3+315\,ab^2c^2d-495\,a^2bcd^2+385\,a^3d^3-96\left(b^3cd^2-5\,ab^2d^3\right)x^2+12\left(7\,b^3c^2d-30\,ab^2cd^2+55\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{1155\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/1155*(128*b^3*d^3*x^3 - 77*b^3*c^3 + 315*a*b^2*c^2*d - 495*a^2*b*c*d^2 +
385*a^3*d^3 - 96*(b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 12*(7*b^3*c^2*d - 30*a*b^2
*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*
a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a
*b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*
d^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3
*c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^
2*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{19}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{3}{4}}\left(128b^3d^3x^3+480a\,b^2d^3x^2-96b^3c\,d^2x^2+660a^2b\,d^3x-360a\,b^2c\,d^2x+84b^3c^2dx+385a^3d^3-495a^2bc\,d^2+315a\,b^2c^2d-77b^3c^3\right)}{1155\,(bx+a)^{\frac{15}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x)
```

```
[Out] 4/1155*(d*x+c)^(3/4)*(128*b^3*d^3*x^3+480*a*b^2*d^3*x^2-96*b^3*c*d^2*x^2+66
0*a^2*b*d^3*x-360*a*b^2*c*d^2*x+84*b^3*c^2*d*x+385*a^3*d^3-495*a^2*b*c*d^2+
```

```
315*a*b^2*c^2*d-77*b^3*c^3)/(b*x+a)^(15/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2
*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{19}{4}}(dx+c)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(19/4)/(d*x+c)^(1/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(19/4)*(d*x + c)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{19/4}\,(c+d\,x)^{1/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)),x)
[Out] int(1/((a + b*x)^(19/4)*(c + d*x)^(1/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(19/4)/(d*x+c)**(1/4),x)
[Out] Timed out
```

3.1497 $\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=167

$$-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}+\frac{(a+bx)^{7/4}}{2d}$$

Rubi [A] time = 0.11, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$-\frac{7(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}{8d^2}-\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}}+\frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4))/(8*d^2) + ((a + b*x)^(7/4)
*(c + d*x)^(1/4))/(2*d) - (21*(b*c - a*d)^2*ArcTan[(d^(1/4)*(a + b*x)^(1/4)
)/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(11/4)) + (21*(b*c - a*d)^2*Arc
Tanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(16*b^(1/4)*d^(1
1/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{(7(bc-ad))\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx}{8d}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{32d^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\right.}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \right.}{8bd^2}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} + \frac{\left(21(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx,\right.}{16d^{5/2}}$$

$$= -\frac{7(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}{8d^2} + \frac{(a+bx)^{7/4}\sqrt[4]{c+dx}}{2d} - \frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{21(}{}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.44

$$\frac{4(a+bx)^{11/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{11}{4};\frac{15}{4};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(11/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4
, 11/4, 15/4, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.27, size = 182, normalized size = 1.09

$$\frac{21(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{21(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sqrt[4]{c+dx}}{\sqrt[4]{d}\sqrt[4]{a+bx}}\right)}{16\sqrt[4]{b}\,d^{11/4}} + \frac{\sqrt[4]{c+dx}\,(ad-bc)^2\left(11d-\frac{7b(c+dx)}{a+bx}\right)}{8d^2\sqrt[4]{a+bx}\left(d-\frac{b(c+dx)}{a+bx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((-(b*c) + a*d)^2*(c + d*x)^(1/4)*(11*d - (7*b*(c + d*x))/(a + b*x)))/(8*d^
2*(a + b*x)^(1/4)*(d - (b*(c + d*x))/(a + b*x))^2) + (21*(b*c - a*d)^2*ArcT
an[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(16*b^(1/4)*d^(11/
```

```
4)) + (21*(b*c - a*d)^2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x
)^(1/4))])/(16*b^(1/4)*d^(11/4))
```

fricas [B] time = 1.03, size = 1457, normalized size = 8.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/32*(84*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c
^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a
^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*arctan(-((b^3*c^2*d^8 - 2*a*b^2*c*d^9
 + a^2*b*d^10)*(b*x + a)^(3/4)*(d*x + c)^(1/4)*((b^8*c^8 - 8*a*b^7*c^7*d +
28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c
^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4) - (b
^2*d^8*x + a*b*d^8)*sqrt(((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^6*x + a*d^6)*sqrt
((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(b*d^11)))/(b*x + a))*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^
2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b
^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(3/4))/(a*b^8*c^8 - 8*a^2*b
^7*c^7*d + 28*a^3*b^6*c^6*d^2 - 56*a^4*b^5*c^5*d^3 + 70*a^5*b^4*c^4*d^4 - 5
6*a^6*b^3*c^3*d^5 + 28*a^7*b^2*c^2*d^6 - 8*a^8*b*c*d^7 + a^9*d^8 + (b^9*c^8
 - 8*a*b^8*c^7*d + 28*a^2*b^7*c^6*d^2 - 56*a^3*b^6*c^5*d^3 + 70*a^4*b^5*c^4
*d^4 - 56*a^5*b^4*c^3*d^5 + 28*a^6*b^3*c^2*d^6 - 8*a^7*b^2*c*d^7 + a^8*b*d^
8)*x)) - 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5
*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8
*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) + (b*d^3*x + a*d^3)*((b^8*c^8 - 8*a*b^
7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56
*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^
(1/4))/(b*x + a)) + 21*d^2*((b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 -
 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*
c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(b*d^11))^(1/4)*log(21*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^3*x + a*d^3)*((b^8*c
^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c
^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)
/(b*d^11))^(1/4))/(b*x + a)) - 4*(4*b*d*x - 7*b*c + 11*a*d)*(b*x + a)^(3/4)
*(d*x + c)^(1/4))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
[Out] int((b*x+a)^(7/4)/(d*x+c)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/4)/(d*x + c)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{7/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(7/4)/(c + d*x)^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(7/4)/(c + d*x)**(3/4), x)
```

3.1498 $\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=127

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Rubi [A] time = 0.08, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {50, 63, 331, 298, 205, 208}

$$\frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}-\frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}+\frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] ((a + b*x)^(3/4)*(c + d*x)^(1/4))/d + (3*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b
*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)
*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*b^(1/4)*d
^(7/4))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}}\,dx = \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx}{4d}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} - \frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^{3/2}} + \frac{(3(bc-ad))\,\mathrm{Subst}\left(\int \ldots\right.}{2d^{3\ldots}}$$

$$= \frac{(a+bx)^{3/4}\sqrt[4]{c+dx}}{d} + \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2\sqrt[4]{b}\,d^{7/4}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.57

$$\frac{4(a+bx)^{7/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{7}{4};\frac{11}{4};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (4*(a + b*x)^(7/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 7/4, 11/4, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(3/4))
```

IntegrateAlgebraic [A] time = 8.42, size = 176, normalized size = 1.39

$$\frac{d^{3/4}(a+bx)^{3/4}\left(\frac{\sqrt[4]{c+dx}\,(ad+b(c+dx)-bc)^{3/4}}{d^{7/4}} - \frac{3(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2\sqrt[4]{b}\,d^{7/4}} - \frac{3(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2\sqrt[4]{b}\,d^{7/4}}\right)}{(ad+bdx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(3/4)/(c + d*x)^(3/4),x]
```

```
[Out] (d^(3/4)*(a + b*x)^(3/4)*(((c + d*x)^(1/4)*(-(b*c) + a*d + b*(c + d*x))^(3/
4))/d^(7/4) - (3*(b*c - a*d)*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d
 + b*(c + d*x))^(1/4)])/(2*b^(1/4)*d^(7/4)) - (3*(b*c - a*d)*ArcTanh[(b^(1/
4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*b^(1/4)*d^(7/4)
)))/(a*d + b*d*x)^(3/4)
```

fricas [B] time = 1.42, size = 808, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -1/4*(12*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(1/4)*arctan(((b^2*c*d^5 - a*b*d^6)*(b*x + a)^(3/4)*(d*x
+ c)^(1/4)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(b*d^7))^(3/4) + (b^2*d^5*x + a*b*d^5)*sqrt(((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (b*d^4*x + a*d^4)*sqrt((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7)))/(b*x
 + a))*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*
d^4)/(b*d^7))^(3/4))/(a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a
^4*b*c*d^3 + a^5*d^4 + (b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3
*b^2*c*d^3 + a^4*b*d^4)*x)) + 3*d*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2
*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a
)^(3/4)*(d*x + c)^(1/4) + (b*d^2*x + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 3*d*
((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(b
*d^7))^(1/4)*log(-3*((b*c - a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4) - (b*d^2*x
 + a*d^2)*((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(b*d^7))^(1/4))/(b*x + a)) - 4*(b*x + a)^(3/4)*(d*x + c)^(1/4))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

```
[Out] int((b*x+a)^(3/4)/(d*x+c)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{3}{4}}}{(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(3/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(3/4)/(d*x + c)^(3/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{3/4}}{(c+d\,x)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(3/4)/(c + d*x)^(3/4),x)
[Out] int((a + b*x)^(3/4)/(c + d*x)^(3/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{3}{4}}}{(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(3/4)/(d*x+c)**(3/4),x)
[Out] Integral((a + b*x)**(3/4)/(c + d*x)**(3/4), x)
```

3.1499 $\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=85

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Rubi [A] time = 0.07, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {63, 331, 298, 205, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}-\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-2*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(b^(1/4)*d
^(3/4)) + (2*ArcTanh[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/
(b^(1/4)*d^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{3/4}}\,dx = \frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(c-\frac{ad}{b}+\frac{dx^4}{b}\right)^{3/4}}\,dx, x, \sqrt[4]{a+bx}\right)}{b}$$
$$= \frac{4\,\text{Subst}\left(\int \frac{x^2}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{b}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{\sqrt{d}}$$
$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Mathematica [C] time = 0.03, size = 73, normalized size = 0.86

$$\frac{4(a+bx)^{3/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{3/4} {}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{d(a+bx)}{ad-bc}\right)}{3b(c+dx)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(a + b*x)^(3/4)*((b*(c + d*x))/(b*c - a*d))^(3/4)*Hypergeometric2F1[3/4,
 3/4, 7/4, (d*(a + b*x))/(-(b*c) + a*d)])/(3*b*(c + d*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 1.00

$$\frac{2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}\,d^{3/4}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{d}\,\sqrt[4]{a+bx}}\right)}{\sqrt[4]{b}\,d^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (2*ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(b^(1/4)*d^
(3/4)) + (2*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(d^(1/4)*(a + b*x)^(1/4))])/(
b^(1/4)*d^(3/4))
```

fricas [B] time = 1.24, size = 234, normalized size = 2.75

$$-4\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}bd^2\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}-\left(b^2d^2x+abd^2\right)\sqrt{\frac{\left(bd^2x+ad^2\right)\sqrt{\frac{1}{bd^3}}+\sqrt{bx+a}\,\sqrt{dx+c}}{bx+a}}\left(\frac{1}{bd^3}\right)^{\frac{3}{4}}}{bx+a}\right)+\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}+(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)-\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}\log\left(-\frac{(bdx+ad)\left(\frac{1}{bd^3}\right)^{\frac{1}{4}}-(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(1/(b*d^3))^(1/4)*arctan(-((b*x + a)^(3/4)*(d*x + c)^(1/4)*b*d^2*(1/(b*d
^3))^(3/4) - (b^2*d^2*x + a*b*d^2)*sqrt(((b*d^2*x + a*d^2)*sqrt(1/(b*d^3))
+ sqrt(b*x + a)*sqrt(d*x + c))/(b*x + a))*(1/(b*d^3))^(3/4))/(b*x + a)) + (
1/(b*d^3))^(1/4)*log(((b*d*x + a*d)*(1/(b*d^3))^(1/4) + (b*x + a)^(3/4)*(d*
```

```
x + c)^(1/4))/(b*x + a)) - (1/(b*d^3))^(1/4)*log(-((b*d*x + a*d)*(1/(b*d^3)
)^(1/4) - (b*x + a)^(3/4)*(d*x + c)^(1/4))/(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}\,(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
[Out] int(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/4)*(d*x + c)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,x\right)}^{1/4}\,{\left(c+d\,x\right)}^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)),x)
[Out] int(1/((a + b*x)^(1/4)*(c + d*x)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a+bx}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(1/4)*(c + d*x)**(3/4)), x)
```

3.1500 $\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}} dx$

Optimal. Leaf size=30

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {37}

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx = -\frac{4\sqrt[4]{c+dx}}{(bc-ad)\sqrt[4]{a+bx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4))/((-(b*c) + a*d)*(a + b*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$-\frac{4\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/((b*c - a*d)*(a + b*x)^(1/4))
```

fricas [A] time = 1.17, size = 42, normalized size = 1.40

$$-\frac{4\,(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="fricas")
[Out] -4*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$\frac{4\,(dx+c)^{\frac{1}{4}}}{(bx+a)^{\frac{1}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x)
[Out] 4/(b*x+a)^(1/4)*(d*x+c)^(1/4)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/4)/(d*x+c)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/4)*(d*x + c)^(3/4)), x)
```

mupad [B] time = 0.71, size = 26, normalized size = 0.87

$$\frac{4\,{\left(c+d\,x\right)}^{1/4}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/4)*(c + d*x)^(3/4)),x)
[Out] (4*(c + d*x)^(1/4))/((a*d - b*c)*(a + b*x)^(1/4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/4)/(d*x+c)**(3/4),x)
[Out] Integral(1/((a + b*x)**(5/4)*(c + d*x)**(3/4)), x)
```

3.1501 $\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=66

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{16d\sqrt[4]{c+dx}}{5\sqrt[4]{a+bx}\,(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{5(a+bx)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(5*(b*c - a*d)*(a + b*x)^(5/4)) + (16*d*(c + d*x)^(1/4
))/(5*(b*c - a*d)^2*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}-\frac{(4d)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{5(bc-ad)}\\ &= -\frac{4\sqrt[4]{c+dx}}{5(bc-ad)(a+bx)^{5/4}}+\frac{16d\sqrt[4]{c+dx}}{5(bc-ad)^2\sqrt[4]{a+bx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{4\sqrt[4]{c+dx}\,(5ad-bc+4bdx)}{5(a+bx)^{5/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(-(b*c) + 5*a*d + 4*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(
5/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 56, normalized size = 0.85

$$-\frac{4\left(\frac{b(c+dx)^{5/4}}{(a+bx)^{5/4}}-\frac{5d\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}\right)}{5(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((-5*d*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (b*(c + d*x)^(5/4))/(a + b*x)
^(5/4)))/(5*(b*c - a*d)^2)
```

fricas [B] time = 1.21, size = 118, normalized size = 1.79

$$\frac{4\,(4\,bdx-bc+5\,ad)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{5\left(a^2b^2c^2-2\,a^3bcd+a^4d^2+\left(b^4c^2-2\,ab^3cd+a^2b^2d^2\right)x^2+2\left(ab^3c^2-2\,a^2b^2cd+a^3bd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/5*(4*b*d*x - b*c + 5*a*d)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^2*b^2*c^2 -
2*a^3*b*c*d + a^4*d^2 + (b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 2*(a*b^
3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$\frac{4\,(dx+c)^{\frac{1}{4}}\,(4bdx+5ad-bc)}{5\,(bx+a)^{\frac{5}{4}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/5*(d*x+c)^(1/4)*(4*b*d*x+5*a*d-b*c)/(b*x+a)^(5/4)/(a^2*d^2-2*a*b*c*d+b^2*
c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{9}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(9/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(9/4)*(d*x + c)^(3/4)), x)
```

mupad [B] time = 0.87, size = 71, normalized size = 1.08

$$\frac{\left(\frac{16\,d\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{20\,a\,d-4\,b\,c}{5\,b\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/4}}{x\,{\left(a+b\,x\right)}^{1/4}+\frac{a\,{\left(a+b\,x\right)}^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(9/4)*(c + d*x)^(3/4)),x)
```

```
[Out] (((16*d*x)/(5*(a*d - b*c)^2) + (20*a*d - 4*b*c)/(5*b*(a*d - b*c)^2))*(c + d
*x)^(1/4))/(x*(a + b*x)^(1/4) + (a*(a + b*x)^(1/4))/b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{9}{4}}\,(c+dx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(9/4)/(d*x+c)**(3/4),x)
```

```
[Out] Integral(1/((a + b*x)**(9/4)*(c + d*x)**(3/4)), x)
```

3.1502 $\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=101

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{128d^2\sqrt[4]{c+dx}}{45\sqrt[4]{a+bx}\,(bc-ad)^3}+\frac{32d\sqrt[4]{c+dx}}{45(a+bx)^{5/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{9(a+bx)^{9/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(9*(b*c - a*d)*(a + b*x)^(9/4)) + (32*d*(c + d*x)^(1/4
))/(45*(b*c - a*d)^2*(a + b*x)^(5/4)) - (128*d^2*(c + d*x)^(1/4))/(45*(b*c
- a*d)^3*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}-\frac{(8d)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{9(bc-ad)}\\
&= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{5/4}(c+dx)^{3/4}}\,dx}{45(bc-ad)^2}\\
&= -\frac{4\sqrt[4]{c+dx}}{9(bc-ad)(a+bx)^{9/4}}+\frac{32d\sqrt[4]{c+dx}}{45(bc-ad)^2(a+bx)^{5/4}}-\frac{128d^2\sqrt[4]{c+dx}}{45(bc-ad)^3\sqrt[4]{a+bx}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 0.74

$$-\frac{4\sqrt[4]{c+dx}\left(45a^2d^2-18abd(c-4dx)+b^2\left(5c^2-8cdx+32d^2x^2\right)\right)}{45(a+bx)^{9/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4)*(45*a^2*d^2 - 18*a*b*d*(c - 4*d*x) + b^2*(5*c^2 - 8*c*d
*x + 32*d^2*x^2)))/(45*(b*c - a*d)^3*(a + b*x)^(9/4))
```

IntegrateAlgebraic [A] time = 0.12, size = 83, normalized size = 0.82

$$-\frac{4\left(\frac{5b^2(c+dx)^{9/4}}{(a+bx)^{9/4}}+\frac{45d^2\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}-\frac{18bd(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{45(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((45*d^2*(c + d*x)^(1/4))/(a + b*x)^(1/4) - (18*b*d*(c + d*x)^(5/4))/(a
 + b*x)^(5/4) + (5*b^2*(c + d*x)^(9/4))/(a + b*x)^(9/4)))/(45*(b*c - a*d)^3
)
```

fricas [B] time = 0.88, size = 251, normalized size = 2.49

$$-\frac{4\left(32\,b^2d^2x^2+5\,b^2c^2-18\,abcd+45\,a^2d^2-8\left(b^2cd-9\,abd^2\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{45\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3+\left(b^6c^3-3\,ab^5c^2d+3\,a^2b^4cd^2-a^3b^3d^3\right)x^3+3\left(ab^5c^3-3\,a^2b^4c^2d+3\,a^3b^3cd^2-a^4b^2d^3\right)x^2+3\left(a^2b^4c^3-3\,a^3b^3c^2d+3\,a^4b^2cd^2-a^5bd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] -4/45*(32*b^2*d^2*x^2 + 5*b^2*c^2 - 18*a*b*c*d + 45*a^2*d^2 - 8*(b^2*c*d -
9*a*b*d^2)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^3*b^3*c^3 - 3*a^4*b^2*c^2*
d + 3*a^5*b*c*d^2 - a^6*d^3 + (b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 -
a^3*b^3*d^3)*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b
^2*d^3)*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^
3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$\frac{4\,(dx+c)^{\frac{1}{4}}\left(32b^2x^2d^2+72ab\,d^2x-8b^2cdx+45a^2d^2-18abcd+5b^2c^2\right)}{45\,(bx+a)^{\frac{9}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/45*(d*x+c)^(1/4)*(32*b^2*d^2*x^2+72*a*b*d^2*x-8*b^2*c*d*x+45*a^2*d^2-18*a
*b*c*d+5*b^2*c^2)/(b*x+a)^(9/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^
3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{13}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(13/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(13/4)*(d*x + c)^(3/4)), x)
```

mupad [B] time = 1.02, size = 133, normalized size = 1.32

$$\frac{(c+d\,x)^{1/4}\,\left(\frac{128\,d^2\,x^2}{45\,(a\,d-b\,c)^3}+\frac{180\,a^2\,d^2-72\,a\,b\,c\,d+20\,b^2\,c^2}{45\,b^2\,(a\,d-b\,c)^3}+\frac{32\,d\,x\,(9\,a\,d-b\,c)}{45\,b\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/4}+\frac{a^2\,(a+b\,x)^{1/4}}{b^2}+\frac{2\,a\,x\,(a+b\,x)^{1/4}}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(13/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((128*d^2*x^2)/(45*(a*d - b*c)^3) + (180*a^2*d^2 + 20*b^2*
c^2 - 72*a*b*c*d)/(45*b^2*(a*d - b*c)^3) + (32*d*x*(9*a*d - b*c))/(45*b*(a*
d - b*c)^3)))/(x^2*(a + b*x)^(1/4) + (a^2*(a + b*x)^(1/4))/b^2 + (2*a*x*(a
+ b*x)^(1/4))/b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(13/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

3.1503 $\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx$

Optimal. Leaf size=136

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{512d^3\sqrt[4]{c+dx}}{195\sqrt[4]{a+bx}\,(bc-ad)^4}-\frac{128d^2\sqrt[4]{c+dx}}{195(a+bx)^{5/4}(bc-ad)^3}+\frac{16d\sqrt[4]{c+dx}}{39(a+bx)^{9/4}(bc-ad)^2}-\frac{4\sqrt[4]{c+dx}}{13(a+bx)^{13/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*(c + d*x)^(1/4))/(13*(b*c - a*d)*(a + b*x)^(13/4)) + (16*d*(c + d*x)^(1
/4))/(39*(b*c - a*d)^2*(a + b*x)^(9/4)) - (128*d^2*(c + d*x)^(1/4))/(195*(b
*c - a*d)^3*(a + b*x)^(5/4)) + (512*d^3*(c + d*x)^(1/4))/(195*(b*c - a*d)^4
*(a + b*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{17/4}(c+dx)^{3/4}}\,dx &= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}-\frac{(12d)\int \frac{1}{(a+bx)^{13/4}(c+dx)^{3/4}}\,dx}{13(bc-ad)}\\
&= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}}+\frac{\left(32d^2\right)\int \frac{1}{(a+bx)^{9/4}(c+dx)^{3/4}}\,dx}{39(bc-ad)^2}\\
&= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}}-\frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}}-\\
&= -\frac{4\sqrt[4]{c+dx}}{13(bc-ad)(a+bx)^{13/4}}+\frac{16d\sqrt[4]{c+dx}}{39(bc-ad)^2(a+bx)^{9/4}}-\frac{128d^2\sqrt[4]{c+dx}}{195(bc-ad)^3(a+bx)^{5/4}}+\end{aligned}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.85

$$\frac{4\sqrt[4]{c+dx}\left(195a^3d^3-117a^2bd^2(c-4dx)+13ab^2d\left(5c^2-8cdx+32d^2x^2\right)+b^3\left(-15c^3+20c^2dx-32cd^2x^2+128d^3x^3\right)\right)}{195(a+bx)^{13/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (4*(c + d*x)^(1/4)*(195*a^3*d^3 - 117*a^2*b*d^2*(c - 4*d*x) + 13*a*b^2*d*(5
*c^2 - 8*c*d*x + 32*d^2*x^2) + b^3*(-15*c^3 + 20*c^2*d*x - 32*c*d^2*x^2 + 1
28*d^3*x^3)))/(195*(b*c - a*d)^4*(a + b*x)^(13/4))
```

IntegrateAlgebraic [**A**] time = 0.13, size = 109, normalized size = 0.80

$$-\frac{4\left(\frac{15b^3(c+dx)^{13/4}}{(a+bx)^{13/4}}-\frac{65b^2d(c+dx)^{9/4}}{(a+bx)^{9/4}}-\frac{195d^3\sqrt[4]{c+dx}}{\sqrt[4]{a+bx}}+\frac{117bd^2(c+dx)^{5/4}}{(a+bx)^{5/4}}\right)}{195(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x]
```

```
[Out] (-4*((-195*d^3*(c + d*x)^(1/4))/(a + b*x)^(1/4) + (117*b*d^2*(c + d*x)^(5/4
))/(a + b*x)^(5/4) - (65*b^2*d*(c + d*x)^(9/4))/(a + b*x)^(9/4) + (15*b^3*(
c + d*x)^(13/4))/(a + b*x)^(13/4)))/(195*(b*c - a*d)^4)
```

fricas [**B**] time = 1.32, size = 419, normalized size = 3.08

$$\frac{4\left(128\,b^3d^3x^3-15\,b^3c^3+65\,ab^2c^2d-117\,a^2bcd^2+195\,a^3d^3-32\left(b^3cd^2-13\,ab^2d^3\right)x^2+4\left(5\,b^3c^2d-26\,ab^2cd^2+117\,a^2bd^3\right)x\right)(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{1}{4}}}{195\left(a^4b^4c^4-4\,a^5b^3c^3d+6\,a^6b^2c^2d^2-4\,a^7bcd^3+a^8d^4+\left(b^8c^4-4\,ab^7c^3d+6\,a^2b^6c^2d^2-4\,a^3b^5cd^3+a^4b^4d^4\right)x^4+4\left(ab^7c^4-4\,a^2b^6c^3d+6\,a^3b^5c^2d^2-4\,a^4b^4cd^3+a^5b^3d^4\right)x^3+6\left(a^2b^6c^4-4\,a^3b^5c^3d+6\,a^4b^4c^2d^2-4\,a^5b^3cd^3+a^6b^2d^4\right)x^2+4\left(a^3b^5c^4-4\,a^4b^4c^3d+6\,a^5b^3c^2d^2-4\,a^6b^2cd^3+a^7bd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="fricas")
```

```
[Out] 4/195*(128*b^3*d^3*x^3 - 15*b^3*c^3 + 65*a*b^2*c^2*d - 117*a^2*b*c*d^2 + 19
5*a^3*d^3 - 32*(b^3*c*d^2 - 13*a*b^2*d^3)*x^2 + 4*(5*b^3*c^2*d - 26*a*b^2*c
*d^2 + 117*a^2*b*d^3)*x)*(b*x + a)^(3/4)*(d*x + c)^(1/4)/(a^4*b^4*c^4 - 4*a
^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4 + (b^8*c^4 - 4*a
*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*x^4 + 4*(a*
b^7*c^4 - 4*a^2*b^6*c^3*d + 6*a^3*b^5*c^2*d^2 - 4*a^4*b^4*c*d^3 + a^5*b^3*d
^4)*x^3 + 6*(a^2*b^6*c^4 - 4*a^3*b^5*c^3*d + 6*a^4*b^4*c^2*d^2 - 4*a^5*b^3*
c*d^3 + a^6*b^2*d^4)*x^2 + 4*(a^3*b^5*c^4 - 4*a^4*b^4*c^3*d + 6*a^5*b^3*c^2
*d^2 - 4*a^6*b^2*c*d^3 + a^7*b*d^4)*x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{17}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

maple [**A**] time = 0.01, size = 171, normalized size = 1.26

$$\frac{4\,(dx+c)^{\frac{1}{4}}\left(128b^3d^3x^3+416a\,b^2d^3x^2-32b^3c\,d^2x^2+468a^2b\,d^3x-104a\,b^2c\,d^2x+20b^3c^2dx+195a^3d^3-117a^2bc\,d^2+65a\,b^2c^2d-15b^3c^3\right)}{195\,(bx+a)^{\frac{13}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x)
```

```
[Out] 4/195*(d*x+c)^(1/4)*(128*b^3*d^3*x^3+416*a*b^2*d^3*x^2-32*b^3*c*d^2*x^2+468
*a^2*b*d^3*x-104*a*b^2*c*d^2*x+20*b^3*c^2*d*x+195*a^3*d^3-117*a^2*b*c*d^2+6
```

```
5*a*b^2*c^2*d-15*b^3*c^3)/(b*x+a)^(13/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c
^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{17}{4}}(dx+c)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(17/4)/(d*x+c)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(17/4)*(d*x + c)^(3/4)), x)
```

mupad [B] time = 1.26, size = 209, normalized size = 1.54

$$\frac{(c+d\,x)^{1/4}\left(\frac{512\,d^3\,x^3}{195\,(a\,d-b\,c)^4}+\frac{780\,a^3\,d^3-468\,a^2\,b\,c\,d^2+260\,a\,b^2\,c^2\,d-60\,b^3\,c^3}{195\,b^3\,(a\,d-b\,c)^4}+\frac{16\,d\,x\left(117\,a^2\,d^2-26\,a\,b\,c\,d+5\,b^2\,c^2\right)}{195\,b^2\,(a\,d-b\,c)^4}+\frac{128\,d^2\,x^2\,(13\,a\,d-b\,c)}{195\,b\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/4}+\frac{a^3\,(a+b\,x)^{1/4}}{b^3}+\frac{3\,a\,x^2\,(a+b\,x)^{1/4}}{b}+\frac{3\,a^2\,x\,(a+b\,x)^{1/4}}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(17/4)*(c + d*x)^(3/4)),x)
```

```
[Out] ((c + d*x)^(1/4)*((512*d^3*x^3)/(195*(a*d - b*c)^4) + (780*a^3*d^3 - 60*b^3
*c^3 + 260*a*b^2*c^2*d - 468*a^2*b*c*d^2)/(195*b^3*(a*d - b*c)^4) + (16*d*x
*(117*a^2*d^2 + 5*b^2*c^2 - 26*a*b*c*d))/(195*b^2*(a*d - b*c)^4) + (128*d^2
*x^2*(13*a*d - b*c))/(195*b*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/4) + (a^3*(a
 + b*x)^(1/4))/b^3 + (3*a*x^2*(a + b*x)^(1/4))/b + (3*a^2*x*(a + b*x)^(1/4)
)/b^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(17/4)/(d*x+c)**(3/4),x)
```

```
[Out] Timed out
```

3.1504 $\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=152

$$-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Rubi [A] time = 0.09, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {47, 50, 63, 240, 212, 208, 205}

$$\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(5/4))/(d*(c + d*x)^(1/4)) + (5*b*(a + b*x)^(1/4)*(c + d*x)^(
3/4))/d^2 - (5*b^(1/4)*(b*c - a*d)*ArcTan[(d^(1/4)*(a + b*x)^(1/4))/(b^(1/4
)*(c + d*x)^(1/4))])/(2*d^(9/4)) - (5*b^(1/4)*(b*c - a*d)*ArcTanh[(d^(1/4)*
(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/(2*d^(9/4))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}}\,dx &= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{(5b)\int \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{(5b(bc-ad))\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{4d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx,x,\sqrt[4]{a+bx}\right)}{d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d^2}\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{\left(5\sqrt{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx,x,\frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{2d^2}-\cdots\\
&= -\frac{4(a+bx)^{5/4}}{d\sqrt[4]{c+dx}}+\frac{5b\sqrt[4]{a+bx}\,(c+dx)^{3/4}}{d^2}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan^{-1}\left(\frac{\sqrt[4]{d}\,\sqrt[4]{a+bx}}{\sqrt[4]{b}\,\sqrt[4]{c+dx}}\right)}{2d^{9/4}}-\frac{5\sqrt[4]{b}\,(bc-ad)\tan\cdots}{2d^{9\cdots}}\end{aligned}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.48

$$\frac{4(a+bx)^{9/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}\,{}_2F_1\left(\frac{5}{4},\frac{9}{4};\frac{13}{4};\frac{d(a+bx)}{ad-bc}\right)}{9b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(9/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 9/4, 13/4, (d*(a + b*x))/(-(b*c) + a*d)])/(9*b*(c + d*x)^(5/4))
```

IntegrateAlgebraic [A] time = 14.16, size = 200, normalized size = 1.32

$$\frac{d^{5/4}(a+bx)^{5/4}\left(\frac{5\left(b^{5/4}c-a\sqrt[4]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2d^{9/4}}-\frac{5\left(b^{5/4}c-a\sqrt[4]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt[4]{c+dx}}{\sqrt[4]{ad+b(c+dx)-bc}}\right)}{2d^{9/4}}+\frac{\sqrt[4]{ad+b(c+dx)-bc}\,(-4ad+b(c+dx)+4bc)}{d^{9/4}\sqrt[4]{c+dx}}\right)}{(ad+bdx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/4)/(c + d*x)^(5/4),x]
```

```
[Out] (d^(5/4)*(a + b*x)^(5/4)*(((4*b*c - 4*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/4))/(d^(9/4)*(c + d*x)^(1/4)) + (5*(b^(5/4)*c - a*b^(1/4)*d)*
ArcTan[(b^(1/4)*(c + d*x)^(1/4))/(-(b*c) + a*d + b*(c + d*x))^(1/4)])/(2*d^
(9/4)) - (5*(b^(5/4)*c - a*b^(1/4)*d)*ArcTanh[(b^(1/4)*(c + d*x)^(1/4))/(-(
b*c) + a*d + b*(c + d*x))^(1/4)])/(2*d^(9/4))))/(a*d + b*d*x)^(5/4)
```

fricas [B] time = 1.22, size = 857, normalized size = 5.64

$$-\frac{20\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(bcd^{7}-ad^{8}\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}+\left(d^{8}x+cd^{7}\right)\sqrt{\frac{\left(b^{2}c^{2}-2abcd+a^{2}d^{2}\right)\sqrt{bx+a}\sqrt{dx+c}+\left(d^{5}x+cd^{4}\right)\sqrt{\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}}}{dx+c}}\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{3}{4}}}{b^{5}c^{5}-4ab^{4}c^{4}d+6a^{2}b^{3}c^{3}d^{2}-4a^{3}b^{2}c^{2}d^{3}+a^{4}bcd^{4}+\left(b^{5}c^{4}d-4ab^{4}c^{3}d^{2}+6a^{2}b^{3}c^{2}d^{3}-4a^{3}b^{2}cd^{4}+a^{4}bd^{5}\right)x}\right)+5\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-5\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\log\left(-\frac{5\left((bc-ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b^{5}c^{4}-4ab^{4}c^{3}d+6a^{2}b^{3}c^{2}d^{2}-4a^{3}b^{2}cd^{3}+a^{4}bd^{4}}{d^{9}}\right)^{\frac{1}{4}}\right)}{dx+c}\right)-4(bdx+5bc-4ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{4\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -1/4*(20*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*
a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*arctan(((b*c*d^7 - a*d^8)*(b*x + a)^(
1/4)*(d*x + c)^(3/4)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*
b^2*c*d^3 + a^4*b*d^4)/d^9)^(3/4) + (d^8*x + c*d^7)*sqrt(((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*sqrt(b*x + a)*sqrt(d*x + c) + (d^5*x + c*d^4)*sqrt((b^5*c^4
- 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9))/(d
*x + c))*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 +
a^4*b*d^4)/d^9)^(3/4))/(b^5*c^5 - 4*a*b^4*c^4*d + 6*a^2*b^3*c^3*d^2 - 4*a^3
*b^2*c^2*d^3 + a^4*b*c*d^4 + (b^5*c^4*d - 4*a*b^4*c^3*d^2 + 6*a^2*b^3*c^2*d
^3 - 4*a^3*b^2*c*d^4 + a^4*b*d^5)*x)) + 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b
^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(
-5*((b*c - a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4) + (d^3*x + c*d^2)*((b^5*c^4
 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1
/4))/(d*x + c)) - 5*(d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c
^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4)*log(-5*((b*c - a*d)*(b*x +
 a)^(1/4)*(d*x + c)^(3/4) - (d^3*x + c*d^2)*((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)/d^9)^(1/4))/(d*x + c)) - 4*(b
*d*x + 5*b*c - 4*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4))/(d^3*x + c*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(5/4)/(d*x+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/4)/(d*x + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{5/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(5/4)/(c + d*x)^(5/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{4}}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(5/4)/(c + d*x)**(5/4), x)
```

3.1505 $\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=108

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Rubi [A] time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {47, 63, 240, 212, 208, 205}

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{5/4}}\,dx &= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{b\int \frac{1}{(a+bx)^{3/4}\sqrt[4]{c+dx}}\,dx}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{c-\frac{ad}{b}+\frac{dx^4}{b}}}\,dx, x, \sqrt[4]{a+bx}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{4\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^4}{b}}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}-\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}+\frac{\left(2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b}+\sqrt{d}\,x^2}\,dx, x, \frac{\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}}\right)}{d}\\
&= -\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}+\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.68

$$\frac{4(a+bx)^{5/4}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/4}{}_2F_1\left(\frac{5}{4},\frac{5}{4};\frac{9}{4};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (4*(a + b*x)^(5/4)*((b*(c + d*x))/(b*c - a*d))^(5/4)*Hypergeometric2F1[5/4,
 5/4, 9/4, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 108, normalized size = 1.00

$$\frac{2\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}+\frac{2\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{d}\sqrt[4]{a+bx}}{\sqrt[4]{b}\sqrt[4]{c+dx}}\right)}{d^{5/4}}-\frac{4\sqrt[4]{a+bx}}{d\sqrt[4]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/4)/(c + d*x)^(5/4),x]
```

```
[Out] (-4*(a + b*x)^(1/4))/(d*(c + d*x)^(1/4)) + (2*b^(1/4)*ArcTan[(d^(1/4)*(a +
b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4) + (2*b^(1/4)*ArcTanh[(d^(1/
4)*(a + b*x)^(1/4))/(b^(1/4)*(c + d*x)^(1/4))])/d^(5/4)
```

fricas [B] time = 1.72, size = 273, normalized size = 2.53

$$-\frac{4\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\arctan\left(-\frac{(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}d^{4}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}-\left(d^{5}x+cd^{4}\right)\sqrt{\frac{\left(d^{3}x+cd^{2}\right)\sqrt{\frac{b}{d^{5}}}+\sqrt{bx+a}\,\sqrt{dx+c}}{dx+c}}\left(\frac{b}{d^{5}}\right)^{\frac{3}{4}}}{bdx+bc}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}+(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}\log\left(-\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{5}}\right)^{\frac{1}{4}}-(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{dx+c}\right)+4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -(4*(d^2*x + c*d)*(b/d^5)^(1/4)*arctan(-((b*x + a)^(1/4)*(d*x + c)^(3/4)*d^
4*(b/d^5)^(3/4) - (d^5*x + c*d^4)*sqrt(((d^3*x + c*d^2)*sqrt(b/d^5) + sqrt(
b*x + a)*sqrt(d*x + c))/(d*x + c))*(b/d^5)^(3/4))/(b*d*x + b*c)) - (d^2*x +
 c*d)*(b/d^5)^(1/4)*log(((d^2*x + c*d)*(b/d^5)^(1/4) + (b*x + a)^(1/4)*(d*x
 + c)^(3/4))/(d*x + c)) + (d^2*x + c*d)*(b/d^5)^(1/4)*log(-((d^2*x + c*d)*(
b/d^5)^(1/4) - (b*x + a)^(1/4)*(d*x + c)^(3/4))/(d*x + c)) + 4*(b*x + a)^(1
/4)*(d*x + c)^(3/4))/(d^2*x + c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

```
[Out] int((b*x+a)^(1/4)/(d*x+c)^(5/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/4)/(d*x + c)^(5/4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x)^{1/4}}{(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/4)/(c + d*x)^(5/4),x)
```

```
[Out] int((a + b*x)^(1/4)/(c + d*x)^(5/4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a+bx}}{(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral((a + b*x)**(1/4)/(c + d*x)**(5/4), x)
```

3.1506 $\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=30

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx = \frac{4\sqrt[4]{a+bx}}{(bc-ad)\sqrt[4]{c+dx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$\frac{4\sqrt[4]{a+bx}}{\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(a + b*x)^(1/4))/((b*c - a*d)*(c + d*x)^(1/4))
```

fricas [A] time = 0.77, size = 42, normalized size = 1.40

$$\frac{4\,(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="fricas")
[Out] 4*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$-\frac{4\,(bx+a)^{\frac{1}{4}}}{(dx+c)^{\frac{1}{4}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x)
[Out] -4*(b*x+a)^(1/4)/(d*x+c)^(1/4)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{3}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(3/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(3/4)*(d*x + c)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{3/4}\,{(c+d\,x)}^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(3/4)*(c + d*x)^(5/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{3}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(3/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(3/4)*(c + d*x)**(5/4)), x)
```

3.1507 $\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=66

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{16d\sqrt[4]{a+bx}}{3\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(3*(b*c - a*d)*(a + b*x)^(3/4)*(c + d*x)^(1/4)) - (16*d*(a + b*x)^(1/4))
/(3*(b*c - a*d)^2*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx &= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{(4d)\int \frac{1}{(a+bx)^{3/4}(c+dx)^{5/4}}\,dx}{3(bc-ad)}\\ &= -\frac{4}{3(bc-ad)(a+bx)^{3/4}\sqrt[4]{c+dx}}-\frac{16d\sqrt[4]{a+bx}}{3(bc-ad)^2\sqrt[4]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.68

$$-\frac{4(3ad+b(c+4dx))}{3(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(3*a*d + b*(c + 4*d*x)))/(3*(b*c - a*d)^2*(a + b*x)^(3/4)*(c + d*x)^(1/
4))
```

IntegrateAlgebraic [A] time = 0.11, size = 49, normalized size = 0.74

$$-\frac{4(c+dx)^{3/4}\left(\frac{3d(a+bx)}{c+dx}+b\right)}{3(a+bx)^{3/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(3/4)*(b + (3*d*(a + b*x))/(c + d*x)))/(3*(b*c - a*d)^2*(a +
b*x)^(3/4))
```

fricas [B] time = 1.17, size = 126, normalized size = 1.91

$$-\frac{4\,(4\,bdx+bc+3\,ad)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{3\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/3*(4*b*d*x + b*c + 3*a*d)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a*b^2*c^3 - 2
*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b
^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.80

$$-\frac{4\,(4bdx+3ad+bc)}{3\,(bx+a)^{\frac{3}{4}}\,(dx+c)^{\frac{1}{4}}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/3*(4*b*d*x+3*a*d+b*c)/(b*x+a)^(3/4)/(d*x+c)^(1/4)/(a^2*d^2-2*a*b*c*d+b^2
*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/4)*(d*x + c)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+b\,x)^{7/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(7/4)*(c + d*x)^(5/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/4)/(d*x+c)**(5/4),x)
[Out] Integral(1/((a + b*x)**(7/4)*(c + d*x)**(5/4)), x)
```

3.1508 $\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=101

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{128d^2\sqrt[4]{a+bx}}{21\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{32d}{21(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{7(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(7*(b*c - a*d)*(a + b*x)^(7/4)*(c + d*x)^(1/4)) + (32*d)/(21*(b*c - a*d)
^2*(a + b*x)^(3/4)*(c + d*x)^(1/4)) + (128*d^2*(a + b*x)^(1/4))/(21*(b*c -
a*d)^3*(c + d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx &= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{(8d)\int \frac{1}{(a+bx)^{7/4}(c+dx)^{5/4}}\,dx}{7(bc-ad)}\\ &= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{\left(32d^2\right)\int \frac{1}{(a+b}{21(bc}\\ &= -\frac{4}{7(bc-ad)(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{32d}{21(bc-ad)^2(a+bx)^{3/4}\sqrt[4]{c+dx}}+\frac{128d^2\sqrt[4]{a}}{21(bc-ad)^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 76, normalized size = 0.75

$$\frac{84a^2d^2+56abd(c+4dx)+4b^2\left(-3c^2+8cdx+32d^2x^2\right)}{21(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (84*a^2*d^2 + 56*a*b*d*(c + 4*d*x) + 4*b^2*(-3*c^2 + 8*c*d*x + 32*d^2*x^2))
/(21*(b*c - a*d)^3*(a + b*x)^(7/4)*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 0.72

$$\frac{4(c+dx)^{7/4}\left(\frac{21d^2(a+bx)^2}{(c+dx)^2}+\frac{14bd(a+bx)}{c+dx}-3b^2\right)}{21(a+bx)^{7/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (4*(c + d*x)^(7/4)*(-3*b^2 + (21*d^2*(a + b*x)^2)/(c + d*x)^2 + (14*b*d*(a
+ b*x))/(c + d*x)))/(21*(b*c - a*d)^3*(a + b*x)^(7/4))
```

fricas [B] time = 1.39, size = 273, normalized size = 2.70

$$\frac{4\left(32\,b^2d^2x^2-3\,b^2c^2+14\,abcd+21\,a^2d^2+8\left(b^2cd+7\,abd^2\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{21\left(a^2b^3c^4-3\,a^3b^2c^3d+3\,a^4bc^2d^2-a^5cd^3+\left(b^5c^3d-3\,ab^4c^2d^2+3\,a^2b^3cd^3-a^3b^2d^4\right)x^3+\left(b^5c^4-ab^4c^3d-3\,a^2b^3c^2d^2+5\,a^3b^2cd^3-2\,a^4bd^4\right)x^2+\left(2\,ab^4c^4-5\,a^2b^3c^3d+3\,a^3b^2c^2d^2+a^4bcd^3-a^5d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] 4/21*(32*b^2*d^2*x^2 - 3*b^2*c^2 + 14*a*b*c*d + 21*a^2*d^2 + 8*(b^2*c*d + 7
*a*b*d^2)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3 + (b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c
*d^3 - a^3*b^2*d^4)*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^
3*b^2*c*d^3 - 2*a^4*b*d^4)*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{4\left(32b^2x^2d^2+56ab\,d^2x+8b^2cdx+21a^2d^2+14abcd-3b^2c^2\right)}{21\,(bx+a)^{\frac{7}{4}}\,(dx+c)^{\frac{1}{4}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/21*(32*b^2*d^2*x^2+56*a*b*d^2*x+8*b^2*c*d*x+21*a^2*d^2+14*a*b*c*d-3*b^2*
c^2)/(b*x+a)^(7/4)/(d*x+c)^(1/4)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c
^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{11}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(11/4)/(d*x+c)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(11/4)*(d*x + c)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{11/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)),x)
```

```
[Out] int(1/((a + b*x)^(11/4)*(c + d*x)^(5/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{11}{4}}\,(c+dx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(11/4)/(d*x+c)**(5/4),x)
```

```
[Out] Integral(1/((a + b*x)**(11/4)*(c + d*x)**(5/4)), x)
```

3.1509 $\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx$

Optimal. Leaf size=136

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c-}}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{512d^3\sqrt[4]{a+bx}}{77\sqrt[4]{c+dx}\,(bc-ad)^4}-\frac{128d^2}{77(a+bx)^{3/4}\sqrt[4]{c+dx}\,(bc-ad)^3}+\frac{48d}{77(a+bx)^{7/4}\sqrt[4]{c+dx}\,(bc-ad)^2}-\frac{4}{11(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] -4/(11*(b*c - a*d)*(a + b*x)^(11/4)*(c + d*x)^(1/4)) + (48*d)/(77*(b*c - a*
d)^2*(a + b*x)^(7/4)*(c + d*x)^(1/4)) - (128*d^2)/(77*(b*c - a*d)^3*(a + b*
x)^(3/4)*(c + d*x)^(1/4)) - (512*d^3*(a + b*x)^(1/4))/(77*(b*c - a*d)^4*(c
+ d*x)^(1/4))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{15/4}(c+dx)^{5/4}}\,dx &= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}-\frac{(12d)\int \frac{1}{(a+bx)^{11/4}(c+dx)^{5/4}}\,dx}{11(bc-ad)}\\ &= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}+\frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}}+\frac{\left(96d^2\right)\ldots}{7\ldots}\\ &= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}+\frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{\ldots}{77(bc-\ldots}\\ &= -\frac{4}{11(bc-ad)(a+bx)^{11/4}\sqrt[4]{c+dx}}+\frac{48d}{77(bc-ad)^2(a+bx)^{7/4}\sqrt[4]{c+dx}}-\frac{\ldots}{77(bc-\ldots}\end{aligned}$$

Mathematica [A] time = 0.04, size = 116, normalized size = 0.85

$$-\frac{4\left(77a^3d^3+77a^2bd^2(c+4dx)+11ab^2d\left(-3c^2+8cdx+32d^2x^2\right)+b^3\left(7c^3-12c^2dx+32cd^2x^2+128d^3x^3\right)\right)}{77(a+bx)^{11/4}\sqrt[4]{c+dx}\,(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(77*a^3*d^3 + 77*a^2*b*d^2*(c + 4*d*x) + 11*a*b^2*d*(-3*c^2 + 8*c*d*x +
 32*d^2*x^2) + b^3*(7*c^3 - 12*c^2*d*x + 32*c*d^2*x^2 + 128*d^3*x^3)))/(77*
(b*c - a*d)^4*(a + b*x)^(11/4)*(c + d*x)^(1/4))
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 0.70

$$-\frac{4(c+dx)^{11/4}\left(-\frac{33b^2d(a+bx)}{c+dx}+\frac{77d^3(a+bx)^3}{(c+dx)^3}+\frac{77bd^2(a+bx)^2}{(c+dx)^2}+7b^3\right)}{77(a+bx)^{11/4}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x]
```

```
[Out] (-4*(c + d*x)^(11/4)*(7*b^3 + (77*d^3*(a + b*x)^3)/(c + d*x)^3 + (77*b*d^2*
(a + b*x)^2)/(c + d*x)^2 - (33*b^2*d*(a + b*x))/(c + d*x)))/(77*(b*c - a*d)
^4*(a + b*x)^(11/4))
```

fricas [B] time = 2.58, size = 457, normalized size = 3.36

$$-\frac{4\left(128\,b^3d^3x^3+7\,b^3c^3-33\,ab^2c^2d+77\,a^2bcd^2+77\,a^3d^3+32\left(b^3cd^2+11\,ab^2d^3\right)x^2-4\left(3\,b^3c^2d-22\,ab^2cd^2-77\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{4}}(dx+c)^{\frac{3}{4}}}{77\left(a^3b^4c^5-4\,a^4b^3c^4d+6\,a^5b^2c^3d^2-4\,a^6bc^2d^3+a^7cd^4+\left(b^7c^4d-4\,ab^6c^3d^2+6\,a^2b^5c^2d^3-4\,a^3b^4cd^4+a^4b^3d^5\right)x^4+\left(b^7c^5-ab^6c^4d-6\,a^2b^5c^3d^2+14\,a^3b^4c^2d^3-11\,a^4b^3cd^4+3\,a^5b^2d^5\right)x^3+3\left(ab^6c^5-3\,a^2b^5c^4d+2\,a^3b^4c^3d^2+2\,a^4b^3c^2d^3-3\,a^5b^2cd^4+a^6bd^5\right)x^2+\left(3\,a^2b^5c^5-11\,a^3b^4c^4d+14\,a^4b^3c^3d^2-6\,a^5b^2c^2d^3-a^6bcd^4+a^7d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="fricas")
```

```
[Out] -4/77*(128*b^3*d^3*x^3 + 7*b^3*c^3 - 33*a*b^2*c^2*d + 77*a^2*b*c*d^2 + 77*a
^3*d^3 + 32*(b^3*c*d^2 + 11*a*b^2*d^3)*x^2 - 4*(3*b^3*c^2*d - 22*a*b^2*c*d^
2 - 77*a^2*b*d^3)*x)*(b*x + a)^(1/4)*(d*x + c)^(3/4)/(a^3*b^4*c^5 - 4*a^4*b
^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2*d^3 + a^7*c*d^4 + (b^7*c^4*d - 4
*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*x^4 + (
b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^
3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*x^2 + (3*a^2*b^5*c
^5 - 11*a^3*b^4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^
4 + a^7*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{4\left(128b^3d^3x^3+352a\,b^2d^3x^2+32b^3c\,d^2x^2+308a^2b\,d^3x+88a\,b^2c\,d^2x-12b^3c^2dx+77a^3d^3+77a^2bc\,d^2-33a\,b^2c^2d+7b^3c^3\right)}{77\,(bx+a)^{\frac{11}{4}}\,(dx+c)^{\frac{1}{4}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x)
```

```
[Out] -4/77*(128*b^3*d^3*x^3+352*a*b^2*d^3*x^2+32*b^3*c*d^2*x^2+308*a^2*b*d^3*x+8
8*a*b^2*c*d^2*x-12*b^3*c^2*d*x+77*a^3*d^3+77*a^2*b*c*d^2-33*a*b^2*c^2*d+7*b
```

```
^3*c^3)/(b*x+a)^(11/4)/(d*x+c)^(1/4)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d
^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{15}{4}}(dx+c)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(15/4)/(d*x+c)^(5/4),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(15/4)*(d*x + c)^(5/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{15/4}\,(c+d\,x)^{5/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)),x)
[Out] int(1/((a + b*x)^(15/4)*(c + d*x)^(5/4)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(15/4)/(d*x+c)**(5/4),x)
[Out] Timed out
```

3.1510 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx$

Optimal. Leaf size=279

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}\cdot$$

Rubi [A] time = 0.30, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{bx+1}}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{bx+1}}+\sqrt{a}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{bx+1}}+1\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*b^(1/4)*(1 - a*x)^(1/4))/(a^(1/4)*(1 + b*x)^(1
/4))])/(a^(1/4)*b^(3/4)) - (Sqrt[2]*ArcTan[1 + (Sqrt[2]*b^(1/4)*(1 - a*x)^(
1/4))/(a^(1/4)*(1 + b*x)^(1/4))])/(a^(1/4)*b^(3/4)) - Log[Sqrt[a] + (Sqrt[b
]*Sqrt[1 - a*x])/Sqrt[1 + b*x] - (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/
(1 + b*x)^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4)) + Log[Sqrt[a] + (Sqrt[b]*Sqrt[1
- a*x])/Sqrt[1 + b*x] + (Sqrt[2]*a^(1/4)*b^(1/4)*(1 - a*x)^(1/4))/(1 + b*x)
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(3/4))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+bx)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(1+\frac{b}{a}-\frac{bx^4}{a}\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$

$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}-\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}} - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a}+\sqrt{b}\,x^2}{1+\frac{bx^4}{a}}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{a\sqrt{b}}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{b}}+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{b} - \ldots$$

$$= -\frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} + \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}}+\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{1+bx}}\right)}{\sqrt{2}\sqrt[4]{a}\,b^{3/4}} \ldots$$

$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt[4]{1-ax}}{\sqrt[4]{a}\sqrt[4]{1+bx}}\right)}{\sqrt[4]{a}\,b^{3/4}} - \frac{\log\left(\sqrt{a}+\frac{\sqrt{b}\sqrt{1-ax}}{\sqrt{1+bx}} \ldots\right)}{\sqrt{2}\sqrt[4]{\ldots}}$$

Mathematica [C] time = 0.04, size = 65, normalized size = 0.23

$$-\frac{4(1-ax)^{3/4}\left(\frac{abx+a}{a+b}\right)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{b-abx}{a+b}\right)}{3a(bx+1)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (-4*(1 - a*x)^(3/4)*((a + a*b*x)/(a + b))^(3/4)*Hypergeometric2F1[3/4, 3/4,
 7/4, (b - a*b*x)/(a + b)])/(3*a*(1 + b*x)^(3/4))
```

IntegrateAlgebraic [A] time = 0.22, size = 176, normalized size = 0.63

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{1-ax}\left(\frac{\sqrt[4]{a}\sqrt{bx+1}}{\sqrt{2}\sqrt[4]{b}\sqrt{1-ax}}-\frac{\sqrt[4]{b}}{\sqrt{2}\sqrt[4]{a}}\right)}{\sqrt[4]{bx+1}}\right)}{\sqrt[4]{a}\,b^{3/4}}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{bx+1}}{\sqrt[4]{1-ax}\left(\frac{\sqrt{a}\sqrt{bx+1}}{\sqrt{1-ax}}+\sqrt{b}\right)}\right)}{\sqrt[4]{a}\,b^{3/4}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - a*x)^(1/4)*(1 + b*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[((1 - a*x)^(1/4)*(-(b^(1/4)/(Sqrt[2]*a^(1/4))) + (a^(1/4)*S
qrt[1 + b*x])/(Sqrt[2]*b^(1/4)*Sqrt[1 - a*x])))/(1 + b*x)^(1/4)])/(a^(1/4)*
b^(3/4)) + (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*(1 + b*x)^(1/4))/((1 -
 a*x)^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[1 + b*x])/Sqrt[1 - a*x]))])/(a^(1/4)*b
^(3/4))
```

fricas [A] time = 1.11, size = 247, normalized size = 0.89

$$-4\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\arctan\left(-\frac{(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}-\left(a^2b^2x-ab^2\right)\sqrt{\frac{\left(ab^2x-b^2\right)\sqrt{-\frac{1}{ab^3}}-\sqrt{-ax+1}\sqrt{bx+1}}{ax-1}}\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}}{ax-1}\right)-\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}+(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)+\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-\frac{(abx-b)\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}-(-ax+1)^{\frac{3}{4}}(bx+1)^{\frac{1}{4}}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] -4*(-1/(a*b^3))^(1/4)*arctan(-((-a*x + 1)^(3/4)*(b*x + 1)^(1/4)*a*b^2*(-1/(
a*b^3))^(3/4) - (a^2*b^2*x - a*b^2)*sqrt(((a*b^2*x - b^2)*sqrt(-1/(a*b^3))
- sqrt(-a*x + 1)*sqrt(b*x + 1))/(a*x - 1))*(-1/(a*b^3))^(3/4))/(a*x - 1)) -
 (-1/(a*b^3))^(1/4)*log(((a*b*x - b)*(-1/(a*b^3))^(1/4) + (-a*x + 1)^(3/4)*
(b*x + 1)^(1/4))/(a*x - 1)) + (-1/(a*b^3))^(1/4)*log(-((a*b*x - b)*(-1/(a*b
^3))^(1/4) - (-a*x + 1)^(3/4)*(b*x + 1)^(1/4))/(a*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}\,(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-ax+1)^{\frac{1}{4}}(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(b*x+1)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((-a*x + 1)^(1/4)*(b*x + 1)^(3/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(1-a\,x)^{1/4}\,(b\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)),x)
[Out] int(1/((1 - a*x)^(1/4)*(b*x + 1)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(bx+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(b*x+1)**(3/4),x)
[Out] Integral(1/((-a*x + 1)**(1/4)*(b*x + 1)**(3/4)), x)
```

3.1511 $\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx$

Optimal. Leaf size=193

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {63, 331, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{\sqrt{2}a}+\frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}\right)}{a}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{ax+1}}+1\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[1 - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - (Sqrt[2
]*ArcTan[1 + (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)])/a - Log[1 + Sqrt[1
 - a*x]/Sqrt[1 + a*x] - (Sqrt[2]*(1 - a*x)^(1/4))/(1 + a*x)^(1/4)]/(Sqrt[2]
*a) + Log[1 + Sqrt[1 - a*x]/Sqrt[1 + a*x] + (Sqrt[2]*(1 - a*x)^(1/4))/(1 +
a*x)^(1/4)]/(Sqrt[2]*a)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1-ax}\,(1+ax)^{3/4}}\,dx = -\frac{4\,\text{Subst}\left(\int \frac{x^2}{\left(2-x^4\right)^{3/4}}\,dx, x, \sqrt[4]{1-ax}\right)}{a}$$
$$= -\frac{4\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{2\,\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} + \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{\sqrt{2}a} - \frac{\sqrt{2}\,\text{Subst}\left(\int \frac{1}{-1-x}\right.}{}$$
$$= \frac{\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\sqrt{2}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{1-ax}}{\sqrt[4]{1+ax}}\right)}{a} - \frac{\log\left(1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}\sqrt[4]{1-a}}{\sqrt[4]{1+ax}}\right.}{\sqrt{2}a}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.22

$$-\frac{2\sqrt[4]{2}\,(1-ax)^{3/4}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1}{2}(1-ax)\right)}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (-2*2^(1/4)*(1 - a*x)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, (1 - a*x)/2])/
(3*a)
```

IntegrateAlgebraic [A] time = 0.12, size = 123, normalized size = 0.64

$$\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{1-ax}\left(\frac{\sqrt{ax+1}}{\sqrt{2}\sqrt{1-ax}}-\frac{1}{\sqrt{2}}\right)}{\sqrt[4]{ax+1}}\right)}{a}+\frac{\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{ax+1}}{\sqrt[4]{1-ax}\left(\frac{\sqrt{ax+1}}{\sqrt{1-ax}}+1\right)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((1 - a*x)^(1/4)*(1 + a*x)^(3/4)),x]
```

```
[Out] (Sqrt[2]*ArcTan[((1 - a*x)^(1/4)*(-(1/Sqrt[2]) + Sqrt[1 + a*x]/(Sqrt[2]*Sqr
t[1 - a*x])))/(1 + a*x)^(1/4)])/a + (Sqrt[2]*ArcTanh[(Sqrt[2]*(1 + a*x)^(1/
4))/((1 - a*x)^(1/4)*(1 + Sqrt[1 + a*x]/Sqrt[1 - a*x]))])/a
```

fricas [B] time = 0.92, size = 448, normalized size = 2.32

$$2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\left(a^4x-a^3\right)\sqrt{\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}+ax-1}{ax-1}\right)+2\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a^3\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-\sqrt{2}\left(a^4x-a^3\right)\sqrt{-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}}\left(\frac{1}{a^4}\right)^{\frac{3}{4}}-ax+1}{ax-1}\right)-\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}+\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}-\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)+\frac{1}{2}\sqrt{2}\left(\frac{1}{a^4}\right)^{\frac{1}{4}}\log\left(-\frac{\sqrt{2}(ax+1)^{\frac{1}{4}}(-ax+1)^{\frac{3}{4}}a\left(\frac{1}{a^4}\right)^{\frac{1}{4}}-\left(a^3x-a^2\right)\sqrt{\frac{1}{a^4}}+\sqrt{ax+1}\sqrt{-ax+1}}{ax-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="fricas")
```

```
[Out] 2*sqrt(2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a^3*(a^(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt((sqrt(2)*(a*x + 1)^(1/4)*(-
a*x + 1)^(3/4)*a*(a^(-4))^(1/4) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1
)*sqrt(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) + a*x - 1)/(a*x - 1)) + 2*sqrt(
2)*(a^(-4))^(1/4)*arctan(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a^3*(a^
(-4))^(3/4) - sqrt(2)*(a^4*x - a^3)*sqrt(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x +
1)^(3/4)*a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt
(-a*x + 1))/(a*x - 1))*(a^(-4))^(3/4) - a*x + 1)/(a*x - 1)) - 1/2*sqrt(2)*(
a^(-4))^(1/4)*log((sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*a*(a^(-4))^(1/4
) + (a^3*x - a^2)*sqrt(a^(-4)) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x - 1)) +
 1/2*sqrt(2)*(a^(-4))^(1/4)*log(-(sqrt(2)*(a*x + 1)^(1/4)*(-a*x + 1)^(3/4)*
a*(a^(-4))^(1/4) - (a^3*x - a^2)*sqrt(a^(-4)) + sqrt(a*x + 1)*sqrt(-a*x + 1
))/(a*x - 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int\frac{1}{(-ax+1)^{\frac{1}{4}}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

```
[Out] int(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(ax+1)^{\frac{3}{4}}(-ax+1)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)^(1/4)/(a*x+1)^(3/4),x, algorithm="maxima")
[Out] integrate(1/((a*x + 1)^(3/4)*(-a*x + 1)^(1/4)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(1-a\,x)^{1/4}\,(a\,x+1)^{3/4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)),x)
[Out] int(1/((1 - a*x)^(1/4)*(a*x + 1)^(3/4)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{-ax+1}\,(ax+1)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a*x+1)**(1/4)/(a*x+1)**(3/4),x)
[Out] Integral(1/((-a*x + 1)**(1/4)*(a*x + 1)**(3/4)), x)
```

3.1512 $\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2}{}$$

Rubi [A] time = 0.64, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{11/6}d^{7/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}}+\frac{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}{12bd}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (5*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b*d) + ((a + b*x)^(7/6)
*(c + d*x)^(5/6))/(2*b) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(
7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(11/6)*d^(7/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(11/6)*d^(7/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \sqrt[6]{a+bx}\,(c+dx)^{5/6}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{(5(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12b}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}}{72bd}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt[6]{c-}}\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\right.}{12b^2d}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{b}-}\right.}{36b^{11}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right.}{36b^{11/6}d^{7/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right.}{36b^{11/6}d^{7/6}}$$

$$= \frac{5(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12bd} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2b} + \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right.}{24\sqrt{3}\,b^{11/6}d^{7/6}}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{7/6}(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 7/6, 13/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(7*b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

IntegrateAlgebraic [A] time = 0.77, size = 364, normalized size = 0.85

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{11/6}d^{7/6}} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{11/6}d^{7/6}} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{11/6}d^{7/6}} + \frac{(bc-ad)^2\left(\frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}+\frac{5b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12bd\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)*(c + d*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*((d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (5*b*(a + b*x)^(1/6))
/(c + d*x)^(1/6)))/(12*b*d*(b - (d*(a + b*x))/(c + d*x))^2) + (5*(b*c - a*d
)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*
x)^(1/6))])/(24*Sqrt[3]*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[
```

```
3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sq
rt[3]*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6)
)/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(11/6)*d^(7/6)) - (5*(b*c - a*d)^2*ArcT
anh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*
(a + b*x)^(1/3))/(c + d*x)^(1/3)))])/(72*b^(11/6)*d^(7/6))
```

fricas [B] time = 1.54, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d^7 + a^2*b^9*d^8)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x
 + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x
 + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(5/6) + sqrt(3)*(b
^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 +
 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*
b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d
^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2
+ 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^
5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4
*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^1
2*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*
b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*
d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*
a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*
b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^
8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 4
95*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*
b*c*d^12 + a^12*d^13)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^11*d^7))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^11*c^2*d^6 - 2*a*b^10*c*d
^7 + a^2*b^9*d^8)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
```

```
+ a^12*d^12)/(b^11*d^7))^(5/6) - 2*sqrt(3)*(b^9*d^7*x + b^9*c*d^6)*sqrt(-((
b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
 c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^11*d^7))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^
7))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 -
220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*
b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d
^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^
4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5
*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^
10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*
d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220
*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12
+ (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^
9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 7
92*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b
^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(25*((b^4*c^2*d - 2*a*b^3*c*d^2 + a
^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^11*d^7))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d^3*x + b^4*
c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/3))/(d*x + c))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6)*log(-25*((b
^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^11*d^7))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x +
c)^(2/3) - (b^4*d^3*x + b^4*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^11*d^7))^(1/3))/(d*x + c)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
```

```
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)*(d
*x + c)^(5/6) + (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^11*d^7))^(1/6))/(d*x + c)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^11*d^7))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6) - (b^2*d^2*x + b^2*c*d)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^11*d^7))^(1/6))/(d*x + c)) + 12*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(1
/6)*(d*x + c)^(5/6))/(b*d)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(1/6)*(d*x+c)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)*(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)*(d*x + c)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{1/6}\,(c+d\,x)^{5/6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)*(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(1/6)*(c + d*x)^(5/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)*(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)*(c + d*x)**(5/6), x)
```

3.1513 $\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=378

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan}{2}$$

Rubi [A] time = 0.50, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}-\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}b^{5/6}d^{7/6}}-\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}}+\frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/d + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/
6)*d^(7/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(5/6)*d^(7/6)) - ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(5/6)
*d^(7/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(
7/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(5/6)*d^(7/6)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{5/6}d} - \frac{(bc-ad)\,\text{Subst}\Big(}{}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\right.}{12b^{5/6}d^{7/6}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{5/6}d^{7/6}} + \frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}}{\sqrt[6]{c}}\right.}{12b^{5/6}d^{7/6}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d} + \frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}} - \frac{(bc-}{}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{7/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(1/6))
```

IntegrateAlgebraic [A] time = 12.20, size = 385, normalized size = 1.02

$$\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}\left(-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{5/6}d^{7/6}}+\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3b^{5/6}d^{7/6}}+\frac{(ad-bc)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}\,\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6b^{5/6}d^{7/6}}+\frac{(c+dx)^{5/6}\sqrt[6]{ad+b(c+dx)-bc}}{d^{7/6}}\right)}{\sqrt[6]{ad+bdx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(1/6),x]
```

```
[Out] (d^(1/6)*(a + b*x)^(1/6)*(((c + d*x)^(5/6)*(-(b*c) + a*d + b*(c + d*x))^(1/
6))/d^(7/6) - ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6
)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(5
/6)*d^(7/6)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/
6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(
5/6)*d^(7/6)) + ((-(b*c) + a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) +
 a*d + b*(c + d*x))^(1/6)])/(3*b^(5/6)*d^(7/6)) + ((-(b*c) + a*d)*ArcTanh[(
```

```
b^(1/3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c +
 d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(5/6)*d^(7/6))))/(a*
d + b*d*x)^(1/6)
```

fricas [B] time = 1.39, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*
b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + 2*sqrt(3)*
(b^4*d^7*x + b^4*c*d^6)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)
^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^3*x + b^2*
c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)
)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*
a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(5/6) + sqrt(3)*(b^6*
c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*
c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a
^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4
*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*
b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5
 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 4*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^5*d^7))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^5*c*d^6 - a*b^4*d^7)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^
5*d^7))^(5/6) + 2*sqrt(3)*(b^4*d^7*x + b^4*c*d^6)*sqrt(-((b^2*c*d - a*b*d^2
)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c
)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b^5*d^7))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*
d^7))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^
3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6
*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2
*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b
^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^
6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^
3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + d*((b^6*c^6 - 6
*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x
+ a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3)
 + (b^2*d^3*x + b^2*c*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^
7))^(1/3))/(d*x + c)) - d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7
))^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^
```

```
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6) - (b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^2*d^3*x + b^2*c*d^2)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/3))/(d*x + c)) + 2*d*((b^6*c
^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c
^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6) + (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^5*d^7))^(1/6))/(d*x + c)) - 2*d*((b^6*c^6 - 6*a*b^5*c^5*d +
15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^
5 + a^6*d^6)/(b^5*d^7))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(
5/6) - (b*d^2*x + b*c*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^5*d^7)
)^(1/6))/(d*x + c)) - 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(1/6)/(d*x+c)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(1/6), x)
```

3.1514 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2}{\sqrt{}}\right.}{d^{7/6}}$$

Rubi [A] time = 0.49, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{b}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}+\frac{\sqrt[6]{b}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{7/6}}-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}+\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}}+\frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}}-\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) - (b^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*d^(7/6)) + (b^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*d^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx = -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{b\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{6\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{\left(2\sqrt[6]{b}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d} + \frac{\left(2\sqrt[6]{b}\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\cdots}\cdots\right)}{d}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \cdots$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} - \frac{\sqrt[6]{b}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{7/6}} + \frac{\sqrt[6]{b}\,\mathrm{l}\cdots}{\cdots}$$

$$= -\frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}} - \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{7/6}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\cdots\right)}{d^{7/6}}$$

Mathematica [C] time = 0.06, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{7/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}{}_2F_1\left(\frac{7}{6},\frac{7}{6};\frac{13}{6};\frac{d(a+bx)}{ad-bc}\right)}{7b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6,
 7/6, 13/6, (d*(a + b*x))/(-(b*c) + a*d)])/(7*b*(c + d*x)^(7/6))
```

IntegrateAlgebraic [A] time = 0.24, size = 256, normalized size = 0.77

$$-\frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} + \frac{\sqrt{3}\sqrt[6]{b}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{7/6}} + \frac{2\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{7/6}} + \frac{\sqrt[6]{b}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{d^{7/6}} - \frac{6\sqrt[6]{a+bx}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(1/6))/(d*(c + d*x)^(1/6)) - (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6)
 + (Sqrt[3]*b^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (2*b^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(7/6) + (b^(1/6)*ArcTanh[(b^(1/6)*d^(
```

```
1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3)))])/d^(7/6)
```

fricas [B] time = 1.00, size = 663, normalized size = 2.00

$$-\frac{4\sqrt{3}\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d^{6}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(d^{7}x+cd^{6}\right)\sqrt{\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}+\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right)+4\sqrt{3}\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d^{6}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(d^{7}x+cd^{6}\right)\sqrt{-\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{b}{d^{7}}\right)^{\frac{5}{6}}-\sqrt{3}(bdx+bc)}{3(bdx+bc)}\right)-\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)+\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}d\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-\left(d^{3}x+cd^{2}\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)-2\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}+(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)+2\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{\left(d^{2}x+cd\right)\left(\frac{b}{d^{7}}\right)^{\frac{1}{6}}-(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)+12(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{2\left(d^{2}x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sqrt(3)*(d^7*x + c*d^6)*sqrt(
((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3*x + c*d^2)*(b/d^7)^
(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(b/d^7)^(5/6) + sqrt(3)
*(b*d*x + b*c))/(b*d*x + b*c)) + 4*sqrt(3)*(d^2*x + c*d)*(b/d^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*d^6*(b/d^7)^(5/6) - 2*sq
rt(3)*(d^7*x + c*d^6)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6
) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x +
 c))*(b/d^7)^(5/6) - sqrt(3)*(b*d*x + b*c))/(b*d*x + b*c)) - (d^2*x + c*d)*
(b/d^7)^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7)^(1/6) + (d^3
*x + c*d^2)*(b/d^7)^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c)) + (
d^2*x + c*d)*(b/d^7)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*d*(b/d^7
)^(1/6) - (d^3*x + c*d^2)*(b/d^7)^(1/3) - (b*x + a)^(1/3)*(d*x + c)^(2/3))/
(d*x + c)) - 2*(d^2*x + c*d)*(b/d^7)^(1/6)*log(((d^2*x + c*d)*(b/d^7)^(1/6)
 + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) + 2*(d^2*x + c*d)*(b/d^7)^(1
/6)*log(-((d^2*x + c*d)*(b/d^7)^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d
*x + c)) + 12*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^2*x + c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

```
[Out] int((b*x+a)^(1/6)/(d*x+c)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{1/6}}{(c+d\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(7/6),x)
[Out] int((a + b*x)^(1/6)/(c + d*x)^(7/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(7/6),x)
[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(7/6), x)
```

3.1515 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx=\frac{6(a+bx)^{7/6}}{7(bc-ad)(c+dx)^{7/6}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{7/6}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(13/6),x]
[Out] (6*(a + b*x)^(7/6))/(7*(b*c - a*d)*(c + d*x)^(7/6))
```

fricas [B] time = 1.17, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}{7\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="fricas")

[Out] 6/7*(b*x + a)^(7/6)*(d*x + c)^(5/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*x^2 + 2*(b*c^2*d - a*c*d^2)*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="giac")

[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{7}{6}}}{7\,(dx+c)^{\frac{7}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^(1/6)/(d*x+c)^(13/6),x)

[Out] -6/7*(b*x+a)^(7/6)/(d*x+c)^(7/6)/(a*d-b*c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(1/6)/(d*x+c)^(13/6),x, algorithm="maxima")

[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(13/6), x)

mupad [B] time = 0.56, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,(a+b\,x)^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{6\,b\,x\,(a+b\,x)^{1/6}}{7\,a\,d^3-7\,b\,c\,d^2}\right)\,(c+d\,x)^{5/6}}{x^2-\frac{7\,b\,c^3-7\,a\,c^2\,d}{7\,a\,d^3-7\,b\,c\,d^2}+\frac{14\,c\,d\,x\,(a\,d-b\,c)}{7\,a\,d^3-7\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(1/6)/(c + d*x)^(13/6),x)

[Out] -(((6*a*(a + b*x)^(1/6))/(7*a*d^3 - 7*b*c*d^2) + (6*b*x*(a + b*x)^(1/6))/(7*a*d^3 - 7*b*c*d^2))*(c + d*x)^(5/6))/(x^2 - (7*b*c^3 - 7*a*c^2*d)/(7*a*d^3 - 7*b*c*d^2) + (14*c*d*x*(a*d - b*c))/(7*a*d^3 - 7*b*c*d^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(1/6)/(d*x+c)**(13/6),x)

[Out] Integral((a + b*x)**(1/6)/(c + d*x)**(13/6), x)

3.1516 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{7/6}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (36*b*(a + b*x)^(7/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx &= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{(6b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{13(bc-ad)}\\ &= \frac{6(a+bx)^{7/6}}{13(bc-ad)(c+dx)^{13/6}}+\frac{36b(a+bx)^{7/6}}{91(bc-ad)^2(c+dx)^{7/6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}(-7ad+13bc+6bdx)}{91(c+dx)^{13/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b*c - 7*a*d + 6*b*d*x))/(91*(b*c - a*d)^2*(c + d*x)^
(13/6))
```

IntegrateAlgebraic [A] time = 0.15, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{7/6}\left(13b-\frac{7d(a+bx)}{c+dx}\right)}{91(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(13*b - (7*d*(a + b*x))/(c + d*x)))/(91*(b*c - a*d)^2*(c
 + d*x)^(7/6))
```

fricas [B] time = 0.93, size = 175, normalized size = 2.65

$$\frac{6\left(6\,b^2dx^2+13\,abc-7\,a^2d+\left(13\,b^2c-abd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\left(b^2c^5-2\,abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2\,abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2\,abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2\,abc^3d^2+a^2c^2d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(6*b^2*d*x^2 + 13*a*b*c - 7*a^2*d + (13*b^2*c - a*b*d)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3 - 2
*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*x^2
 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{7}{6}}\,(-6bdx+7ad-13bc)}{91\,(dx+c)^{\frac{13}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(7/6)*(-6*b*d*x+7*a*d-13*b*c)/(d*x+c)^(13/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(19/6), x)
```

mupad [B] time = 0.75, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{5/6}\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{1/6}}{91\,d^2\,(a\,d-b\,c)^2}-\frac{\left(42\,a^2\,d-78\,a\,b\,c\right)\,(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}+\frac{x\left(78\,b^2\,c-6\,a\,b\,d\right)\,(a+b\,x)^{1/6}}{91\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(19/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((36*b^2*x^2*(a + b*x)^(1/6))/(91*d^2*(a*d - b*c)^2) - ((4
2*a^2*d - 78*a*b*c)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2) + (x*(78*b^2*c
- 6*a*b*d)*(a + b*x)^(1/6))/(91*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 + (3*c*
x^2)/d + (3*c^2*x)/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

3.1517 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{7/6}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{72b(a+bx)^{7/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (72*b*(a + b*x)^(7/
6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (432*b^2*(a + b*x)^(7/6))/(1729*
(b*c - a*d)^3*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(12b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}\\ &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{\left(72b^2\right)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{13/6}}\,dx}{247(bc-ad)^2}\\ &= \frac{6(a+bx)^{7/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{72b(a+bx)^{7/6}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{432b^2(a+bx)^{7/6}}{1729(bc-ad)^3(c+dx)^{7/6}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{7/6}\left(91a^2d^2-14abd(19c+6dx)+b^2\left(247c^2+228cdx+72d^2x^2\right)\right)}{1729(c+dx)^{19/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(91*a^2*d^2 - 14*a*b*d*(19*c + 6*d*x) + b^2*(247*c^2 + 2
28*c*d*x + 72*d^2*x^2)))/(1729*(b*c - a*d)^3*(c + d*x)^(19/6))
```

IntegrateAlgebraic [A] time = 0.17, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{7/6}\left(\frac{91d^2(a+bx)^2}{(c+dx)^2}-\frac{266bd(a+bx)}{c+dx}+247b^2\right)}{1729(c+dx)^{7/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(247*b^2 + (91*d^2*(a + b*x)^2)/(c + d*x)^2 - (266*b*d*(
a + b*x))/(c + d*x)))/(1729*(b*c - a*d)^3*(c + d*x)^(7/6))
```

fricas [B] time = 0.75, size = 338, normalized size = 3.35

$$\frac{6\left(72\,b^3d^2x^3+247\,ab^2c^2-266\,a^2bcd+91\,a^3d^2+12\left(19\,b^3cd-ab^2d^2\right)x^2+\left(247\,b^3c^2-38\,ab^2cd+7\,a^2bd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6\right)x^3+6\left(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5\right)x^2+4\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(72*b^3*d^2*x^3 + 247*a*b^2*c^2 - 266*a^2*b*c*d + 91*a^3*d^2 + 12*(1
9*b^3*c*d - a*b^2*d^2)*x^2 + (247*b^3*c^2 - 38*a*b^2*c*d + 7*a^2*b*d^2)*x)*
(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5*d^2
- a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*x
^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x^3 +
6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2 + 4*(
b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{7}{6}}\left(72b^2x^2d^2-84ab\,d^2x+228b^2cdx+91a^2d^2-266abcd+247b^2c^2\right)}{1729\,(dx+c)^{\frac{19}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(7/6)*(72*b^2*d^2*x^2-84*a*b*d^2*x+228*b^2*c*d*x+91*a^2*d^2
-266*a*b*c*d+247*b^2*c^2)/(d*x+c)^(19/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2
*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(25/6), x)
```

mupad [B] time = 0.95, size = 213, normalized size = 2.11

$$-\frac{(c+d\,x)^{5/6}\left(\frac{(a+b\,x)^{1/6}\,\left(546\,a^3\,d^2-1596\,a^2\,b\,c\,d+1482\,a\,b^2\,c^2\right)}{1729\,d^4\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{1/6}}{1729\,d^2\,(a\,d-b\,c)^3}+\frac{x\,(a+b\,x)^{1/6}\,\left(42\,a^2\,b\,d^2-228\,a\,b^2\,c\,d+1482\,b^3\,c^2\right)}{1729\,d^4\,(a\,d-b\,c)^3}-\frac{72\,b^2\,x^2\,(a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{1729\,d^3\,(a\,d-b\,c)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(546*a^3*d^2 + 1482*a*b^2*c^2 - 1596*a^
2*b*c*d))/(1729*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(1/6))/(1729*d^
2*(a*d - b*c)^3) + (x*(a + b*x)^(1/6)*(1482*b^3*c^2 + 42*a^2*b*d^2 - 228*a*
b^2*c*d))/(1729*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(a*d - 19*b*c)*(a + b*x)^(
1/6))/(1729*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d
^3 + (6*c^2*x^2)/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

3.1518 $\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{7/6}}{43225(c+dx)^{7/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{7/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{108b(a+bx)^{7/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{7/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (108*b*(a + b*x)^(7
/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (1296*b^2*(a + b*x)^(7/6))/(617
5*(b*c - a*d)^3*(c + d*x)^(13/6)) + (7776*b^3*(a + b*x)^(7/6))/(43225*(b*c
- a*d)^4*(c + d*x)^(7/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{31/6}}\,dx &= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{(18b)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}\\ &= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{\left(216b^2\right)\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}\\ &= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}}+\frac{\left(1296b^3\right.}{6175}\\ &= \frac{6(a+bx)^{7/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{108b(a+bx)^{7/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{1296b^2(a+bx)^{7/6}}{6175(bc-ad)^3(c+dx)^{13/6}}+\frac{777}{43225(b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{7/6}\left(-1729a^3d^3+273a^2bd^2(25c+6dx)-21ab^2d\left(475c^2+300cdx+72d^2x^2\right)+b^3\left(6175c^3+8550c^2dx+5400cd^2x^2+1296d^3x^3\right)\right)}{43225(c+dx)^{25/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(-1729*a^3*d^3 + 273*a^2*b*d^2*(25*c + 6*d*x) - 21*a*b^2
*d*(475*c^2 + 300*c*d*x + 72*d^2*x^2) + b^3*(6175*c^3 + 8550*c^2*d*x + 5400
*c*d^2*x^2 + 1296*d^3*x^3)))/(43225*(b*c - a*d)^4*(c + d*x)^(25/6))
```

IntegrateAlgebraic [A] time = 0.17, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{7/6}\left(-\frac{9975b^2d(a+bx)}{c+dx}-\frac{1729d^3(a+bx)^3}{(c+dx)^3}+\frac{6825bd^2(a+bx)^2}{(c+dx)^2}+6175b^3\right)}{43225(c+dx)^{7/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(1/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(7/6)*(6175*b^3 - (1729*d^3*(a + b*x)^3)/(c + d*x)^3 + (6825*b
*d^2*(a + b*x)^2)/(c + d*x)^2 - (9975*b^2*d*(a + b*x))/(c + d*x)))/(43225*(
b*c - a*d)^4*(c + d*x)^(7/6))
```

fricas [B] time = 1.10, size = 533, normalized size = 3.92

$$\frac{6\left(1296b^4d^3x^4+6175ab^3c^3-9975a^2b^2c^2d+6825a^3bcd^2-1729a^4d^3+216\left(25b^4cd^2-ab^3d^3\right)x^3+18\left(475b^4c^2d-50ab^3cd^2+7a^2b^2d^3\right)x^2+\left(6175b^4c^3-1425ab^3c^2d+525a^2b^2cd^2-91a^3bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{43225\left(b^4c^9-4ab^3c^8d+6a^2b^2c^7d^2-4a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4ab^3c^3d^6+6a^2b^2c^2d^7-4a^3bcd^8+a^4d^9\right)x^5+5\left(b^4c^5d^4-4ab^3c^4d^5+6a^2b^2c^3d^6-4a^3bc^2d^7+a^4cd^8\right)x^4+10\left(b^4c^6d^3-4ab^3c^5d^4+6a^2b^2c^4d^5-4a^3bc^3d^6+a^4c^2d^7\right)x^3+10\left(b^4c^7d^2-4ab^3c^6d^3+6a^2b^2c^5d^4-4a^3bc^4d^5+a^4c^3d^6\right)x^2+5\left(b^4c^8d-4ab^3c^7d^2+6a^2b^2c^6d^3-4a^3bc^5d^4+a^4c^4d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/43225*(1296*b^4*d^3*x^4 + 6175*a*b^3*c^3 - 9975*a^2*b^2*c^2*d + 6825*a^3*
b*c*d^2 - 1729*a^4*d^3 + 216*(25*b^4*c*d^2 - a*b^3*d^3)*x^3 + 18*(475*b^4*c
^2*d - 50*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + (6175*b^4*c^3 - 1425*a*b^3*c^2
*d + 525*a^2*b^2*c*d^2 - 91*a^3*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(
b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 + a^4*c^5*d^4
 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4
*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*
c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^
4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 - 4*a*b^3*c^6*
d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 + 5*(b^4*c^8*d
 - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6(bx+a)^{\frac{7}{6}}\left(-1296b^3d^3x^3+1512ab^2d^3x^2-5400b^3cd^2x^2-1638a^2bd^3x+6300ab^2cd^2x-8550b^3c^2dx+1729a^3d^3-6825a^2bcd^2+9975ab^2c^2d-6175b^3c^3\right)}{43225(dx+c)^{\frac{25}{6}}\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/43225*(b*x+a)^(7/6)*(-1296*b^3*d^3*x^3+1512*a*b^2*d^3*x^2-5400*b^3*c*d^2
*x^2-1638*a^2*b*d^3*x+6300*a*b^2*c*d^2*x-8550*b^3*c^2*d*x+1729*a^3*d^3-6825
*a^2*b*c*d^2+9975*a*b^2*c^2*d-6175*b^3*c^3)/(d*x+c)^(25/6)/(a^4*d^4-4*a^3*b
*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(1/6)/(d*x + c)^(31/6), x)
```

mupad [B] time = 1.15, size = 302, normalized size = 2.22

$$\frac{(c+d\,x)^{5/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{1/6}}{43225\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\left(10374\,a^4\,d^3-40950\,a^3\,b\,c\,d^2+59850\,a^2\,b^2\,c^2\,d-37050\,a\,b^3\,c^3\right)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\left(-546\,a^3\,b\,d^3+3150\,a^2\,b^2\,c\,d^2-8550\,a\,b^3\,c^2\,d+37050\,b^4\,c^3\right)}{43225\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{1/6}\left(7\,a^2\,d^2-50\,a\,b\,c\,d+475\,b^2\,c^2\right)}{43225\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{43225\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(1/6)/(c + d*x)^(31/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^4*x^4*(a + b*x)^(1/6))/(43225*d^2*(a*d - b*c)^4)
- ((a + b*x)^(1/6)*(10374*a^4*d^3 - 37050*a*b^3*c^3 + 59850*a^2*b^2*c^2*d -
 40950*a^3*b*c*d^2))/(43225*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(37050*
b^4*c^3 - 546*a^3*b*d^3 + 3150*a^2*b^2*c*d^2 - 8550*a*b^3*c^2*d))/(43225*d^
5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(1/6)*(7*a^2*d^2 + 475*b^2*c^2 -
50*a*b*c*d))/(43225*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(a*d - 25*b*c)*(a +
b*x)^(1/6))/(43225*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*x^4)/d + (5*c
^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

3.1519 $\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx$

Optimal. Leaf size=427

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad}{}$$

Rubi [A] time = 0.64, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}+\frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}}-\frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{7/6}d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}(bc-ad)}{12bd}+\frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b*d) + ((a + b*x)^(11/6)*
(c + d*x)^(1/6))/(2*b) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
+ b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11
/6)) + (5*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqr
t[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a
*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(
7/6)*d^(11/6)) + (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(
c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b
^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(7/6)*d^(11/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int (a+bx)^{5/6}\sqrt[6]{c+dx}\,dx = \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} + \frac{(bc-ad)\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}dx}{12b}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5}}}{72bd}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\left(c-\frac{ad}{b}\right.}\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\right.}{12b^2d}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{\left(5(bc-ad)^2\right)\text{Subst}\left(\int \frac{}{\sqrt[3]{b}-}\right.}{36b^{7/6}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+b}}{\sqrt[6]{b}\sqrt[6]{c+d}}\right.}{36b^{7/6}d^{11/6}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+b}}{\sqrt[6]{b}\sqrt[6]{c+d}}\right.}{36b^{7/6}d^{11/6}}$$

$$= \frac{(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12bd} + \frac{(a+bx)^{11/6}\sqrt[6]{c+dx}}{2b} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+}}{\sqrt[6]{b}\sqrt[6]{c+d}}}{\sqrt{3}}\right.}{24\sqrt{3}\,b^{7/6}d^{11/6}}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{11/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 11/6, 17/6, (d*
(a + b*x))/(-(b*c) + a*d)])/(11*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

IntegrateAlgebraic [A] time = 0.75, size = 365, normalized size = 0.85

$$\frac{5(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}} - \frac{5(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{7/6}d^{11/6}} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{36b^{7/6}d^{11/6}} - \frac{5(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{7/6}d^{11/6}} + \frac{(bc-ad)^2\left(\frac{b(c+dx)^{7/6}}{(a+bx)^{7/6}}+\frac{5d\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\right)}{12bd\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)*(c + d*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((5*d*(c + d*x)^(1/6))/(a + b*x)^(1/6) + (b*(c + d*x)^(7/6))
/(a + b*x)^(7/6)))/(12*b*d*(-d + (b*(c + d*x))/(a + b*x))^2) + (5*(b*c - a*
d)^2*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b
*x)^(1/6))])/(24*Sqrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*ArcTan[1/Sqrt
```

```
[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(24*S
qrt[3]*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6
))/(d^(1/6)*(a + b*x)^(1/6))])/(36*b^(7/6)*d^(11/6)) - (5*(b*c - a*d)^2*Arc
Tanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)
*(c + d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(7/6)*d^(11/6))
```

fricas [B] time = 1.47, size = 5633, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/144*(20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2
 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^
6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3
*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^10 + a^2*b^6*d^11)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
 + a^12*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x
 + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x
 + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(5/6) + sqrt(3)*(a
*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^
3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a
^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c
^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66
*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b
^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^
8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*
b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*
a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c
^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 +
 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^
12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d
^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*
a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*
c*d^11 + a^12*b*d^12)*x)) + 20*sqrt(3)*b*d*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^7*d^11))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^8*c^2*d^9 - 2*a*b^7*c*d^1
0 + a^2*b^6*d^11)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c
^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b
^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
```

```
+ a^12*d^12)/(b^7*d^11))^(5/6) - 2*sqrt(3)*(b^7*d^9*x + a*b^6*d^9)*sqrt(-((
b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((
b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*
b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d
^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*
c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
 c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^7*d^11))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^1
1))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^
2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a
^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c
^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 -
 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b
^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^
7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a
^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^
3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c
^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 -
220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12
+ (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d
^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*
a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c
^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 5*b*d*((b^12*c^12 - 12*a*
b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*
d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495
*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c
*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(25*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 +
 a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^7*d^11))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^4*x + a*b^
2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/3))/(b*x + a))
+ 5*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9
*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6
- 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10
*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6)*log(-25*((b
^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b
^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 +
495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b
^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^
10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^7*d^11))^(1/6) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x +
c)^(1/3) - (b^3*d^4*x + a*b^2*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*
b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7
*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 22
0*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b
^7*d^11))^(1/3))/(b*x + a)) - 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^
```

```
2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c
^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/
(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)*(d
*x + c)^(1/6) + (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a
^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*
c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 -
 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)
/(b^7*d^11))^(1/6))/(b*x + a)) + 10*b*d*((b^12*c^12 - 12*a*b^11*c^11*d + 66
*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^
7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8
 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^1
2)/(b^7*d^11))^(1/6)*log(5*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(5/6)
*(d*x + c)^(1/6) - (b^2*d^2*x + a*b*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^7*d^11))^(1/6))/(b*x + a)) + 12*(6*b*d*x + b*c + 5*a*d)*(b*x + a)^(5
/6)*(d*x + c)^(1/6))/(b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(5/6)*(d*x+c)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)*(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)*(d*x + c)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,x)^{5/6}\,(c+d\,x)^{1/6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)*(c + d*x)^(1/6),x)
[Out] int((a + b*x)^(5/6)*(c + d*x)^(1/6), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{5}{6}} \sqrt[6]{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)*(d*x+c)**(1/6),x)
[Out] Integral((a + b*x)**(5/6)*(c + d*x)**(1/6), x)
```

3.1520 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan}{2}$$

Rubi [A] time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/d - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
1/6)*d^(11/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(1/6)*d^(11/6)) - (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(1/6)*d^(11/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(1/6)*d^(11/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}}\,dx = \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6d}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{bd}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3d^{5/3}} - \frac{(5(bc-ad))\,\text{Subst}\left(\int \ldots\right.}{3\ldots}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{(5(bc-ad))\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\ldots\right.}{12\sqrt[6]{b}\,d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\ldots}{\ldots}\right.}{12\sqrt[6]{b}\,d^{11/6}}$$

$$= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{d} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}} - \frac{5(bc\ldots}{\ldots}$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.19

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}\,{}_2F_1\left(\frac{5}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6
, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(5/6))
```

IntegrateAlgebraic [A] time = 17.90, size = 385, normalized size = 1.02

$$\frac{d^{5/6}(a+bx)^{5/6}\left(\frac{\sqrt[6]{c+dx}\,(ad+b(c+dx)-bc)^{5/6}}{d^{11/6}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3\sqrt[6]{b}\,d^{11/6}}-\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}\,\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6\sqrt[6]{b}\,d^{11/6}}\right)}{(ad+bdx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(5/6),x]
```

```
[Out] (d^(5/6)*(a + b*x)^(5/6)*(((c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(5/
6))/d^(11/6) + (5*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(
1/6)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b
^(1/6)*d^(11/6)) - (5*(b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/
(b^(1/6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[
3]*b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(
```

```
b*c) + a*d + b*(c + d*x))^(1/6)])/(3*b^(1/6)*d^(11/6)) - (5*(b*c - a*d)*Arc
Tanh[(b^(1/3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6
)*(c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(1/6)*d^(11/6)
)))/(a*d + b*d*x)^(5/6)
```

fricas [B] time = 1.25, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="fricas")

[Out] -1/12*(20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3
*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6
)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2*sqrt(3)*(b
^2*d^9*x + a*b*d^9)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*
c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*
c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*((b^6*c^6 -
6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^
4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + sqrt(3)*(a*b^6*c^6 - 6*a^2*b
^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6
*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a
*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a
^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^
2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
 a^6*b*d^6)*x)) + 20*sqrt(3)*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d
^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*
d^11))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^2*c*d^9 - a*b*d^10)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*
c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) + 2
*sqrt(3)*(b^2*d^9*x + a*b*d^9)*sqrt(-((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*
x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^
3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) - (b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b*d^4*x + a
*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a))*
((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^
4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(5/6) - sqrt(3)*(a*b^6*c
^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2
*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*
c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b
*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^
3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c
^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5
*b^2*c*d^5 + a^6*b*d^6)*x)) + 5*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^
4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/
(b*d^11))^(1/6)*log(25*((b*c*d^2 - a*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*(
(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4
*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6) + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b*d^4*x + a*d^4)*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/3))/(b*x + a)) - 5*d*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-25*((b*c*d^2 - a*d^3)
```

```
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b*d^11))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^
(1/3) - (b*d^4*x + a*d^4)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11)
)^(1/3))/(b*x + a)) + 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11
))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b*d^2*x + a
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a))
- 10*d*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3
+ 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6)*log(-5*((b*
c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b*d^2*x + a*d^2)*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b*d^11))^(1/6))/(b*x + a)) - 12*(b*x + a)^(5/6)
*(d*x + c)^(1/6))/d
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
[Out] int((b*x+a)^(5/6)/(d*x+c)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{5/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(5/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(5/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(5/6), x)
```

3.1521 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=334

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{a}}{\sqrt{3}\sqrt[6]{}}\right.}{d^{11/6}}$$

Rubi [A] time = 0.56, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{b^{5/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{b^{5/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{11/6}}+\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}}+\frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{11/6}}-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11
/6) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt
[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(d^(1/6)*(a +
b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/d^(11/6) - (b^(5/6)*Log[b^(1/3) + (
d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6)
)/(c + d*x)^(1/6)]])/(2*d^(11/6)) + (b^(5/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)]])/(2*d^(11/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
```

```
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}}\,dx = -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{b\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{5/3}} + \frac{\left(2b^{5/6}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{}}{2}}{\sqrt[3]{b}+\sqrt[6]{b}}\right.}{d^5}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} +$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{11/6}} - \frac{b^{5/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2d^{11/6}} + \frac{b^{5/6}}{}$$

$$= -\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} - \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{11/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\right.}{d^{11/6}}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.22

$$\frac{6(a+bx)^{11/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}{}_2F_1\left(\frac{11}{6},\frac{11}{6};\frac{17}{6};\frac{d(a+bx)}{ad-bc}\right)}{11b(c+dx)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*((b*(c + d*x))/(b*c - a*d))^(11/6)*Hypergeometric2F1[11
/6, 11/6, 17/6, (d*(a + b*x))/(-(b*c) + a*d)])/(11*b*(c + d*x)^(11/6))
```

IntegrateAlgebraic [A] time = 0.33, size = 258, normalized size = 0.77

$$-\frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{d^{11/6}} + \frac{\sqrt{3}\,b^{5/6}\tan^{-1}\left(\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{d^{11/6}} + \frac{2b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{d^{11/6}} + \frac{b^{5/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{d^{11/6}} - \frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(11/6),x]
```

```
[Out] (-6*(a + b*x)^(5/6))/(5*d*(c + d*x)^(5/6)) - (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt
[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/d^(11
/6) + (Sqrt[3]*b^(5/6)*ArcTan[1/Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt
[3]*d^(1/6)*(a + b*x)^(1/6))])/d^(11/6) + (2*b^(5/6)*ArcTanh[(b^(1/6)*(c +
d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/d^(11/6) + (b^(5/6)*ArcTanh[(b^(1/6
)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c + d*x)^(
1/3))/(a + b*x)^(1/3)))])/d^(11/6)
```

fricas [B] time = 1.18, size = 755, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -1/10*(20*sqrt(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*
x + a)^(5/6)*(d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x +
a*d^9)*sqrt(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) + (b*x
+ a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x +
 a))*(b^5/d^11)^(5/6) + sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) + 20*sqrt
(3)*(d^2*x + c*d)*(b^5/d^11)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(
d*x + c)^(1/6)*b*d^9*(b^5/d^11)^(5/6) - 2*sqrt(3)*(b*d^9*x + a*d^9)*sqrt(-(
(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d
*x + c)^(1/3)*b^2 - (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a))*(b^5/d^1
1)^(5/6) - sqrt(3)*(b^6*x + a*b^5))/(b^6*x + a*b^5)) - 5*(d^2*x + c*d)*(b^5
/d^11)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6)
+ (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 + (b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))
/(b*x + a)) + 5*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x
 + c)^(1/6)*b*d^2*(b^5/d^11)^(1/6) - (b*x + a)^(2/3)*(d*x + c)^(1/3)*b^2 -
(b*d^4*x + a*d^4)*(b^5/d^11)^(1/3))/(b*x + a)) - 10*(d^2*x + c*d)*(b^5/d^11
)^(1/6)*log(((b*x + a)^(5/6)*(d*x + c)^(1/6)*b + (b*d^2*x + a*d^2)*(b^5/d^1
1)^(1/6))/(b*x + a)) + 10*(d^2*x + c*d)*(b^5/d^11)^(1/6)*log(((b*x + a)^(5/
6)*(d*x + c)^(1/6)*b - (b*d^2*x + a*d^2)*(b^5/d^11)^(1/6))/(b*x + a)) + 12*
(b*x + a)^(5/6)*(d*x + c)^(1/6))/(d^2*x + c*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

```
[Out] int((b*x+a)^(5/6)/(d*x+c)^(11/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(11/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{5/6}}{(c+d\,x)^{11/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(11/6),x)
[Out] int((a + b*x)^(5/6)/(c + d*x)^(11/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{5}{6}}}{(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(11/6),x)
[Out] Integral((a + b*x)**(5/6)/(c + d*x)**(11/6), x)
```

3.1522 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx = \frac{6(a+bx)^{11/6}}{11(bc-ad)(c+dx)^{11/6}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

IntegrateAlgebraic [A] time = 0.05, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{11/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(17/6),x]
[Out] (6*(a + b*x)^(11/6))/(11*(b*c - a*d)*(c + d*x)^(11/6))
```

fricas [B] time = 0.77, size = 65, normalized size = 2.03

$$\frac{6\,(bx+a)^{\frac{11}{6}}(dx+c)^{\frac{1}{6}}}{11\left(bc^3-ac^2d+\left(bcd^2-ad^3\right)x^2+2\left(bc^2d-acd^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/11*(b*x + a)^(11/6)*(d*x + c)^(1/6)/(b*c^3 - a*c^2*d + (b*c*d^2 - a*d^3)*
x^2 + 2*(b*c^2*d - a*c*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{11}{6}}}{11\,(dx+c)^{\frac{11}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/11*(b*x+a)^(11/6)/(d*x+c)^(11/6)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(17/6), x)
```

mupad [B] time = 0.59, size = 130, normalized size = 4.06

$$-\frac{\left(\frac{6\,a\,(a+b\,x)^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{6\,b\,x\,(a+b\,x)^{5/6}}{11\,a\,d^3-11\,b\,c\,d^2}\right)\,(c+d\,x)^{1/6}}{x^2-\frac{11\,b\,c^3-11\,a\,c^2\,d}{11\,a\,d^3-11\,b\,c\,d^2}+\frac{22\,c\,d\,x\,(a\,d-b\,c)}{11\,a\,d^3-11\,b\,c\,d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(17/6),x)
```

```
[Out] -(((6*a*(a + b*x)^(5/6))/(11*a*d^3 - 11*b*c*d^2) + (6*b*x*(a + b*x)^(5/6))/
(11*a*d^3 - 11*b*c*d^2))*(c + d*x)^(1/6))/(x^2 - (11*b*c^3 - 11*a*c^2*d)/(1
1*a*d^3 - 11*b*c*d^2) + (22*c*d*x*(a*d - b*c))/(11*a*d^3 - 11*b*c*d^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

3.1523 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{11/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (36*b*(a + b*x)^(1
1/6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx &= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{(6b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx}{17(bc-ad)}\\ &= \frac{6(a+bx)^{11/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{36b(a+bx)^{11/6}}{187(bc-ad)^2(c+dx)^{11/6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{11/6}(-11ad+17bc+6bdx)}{187(c+dx)^{17/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(17*b*c - 11*a*d + 6*b*d*x))/(187*(b*c - a*d)^2*(c + d*
x)^(17/6))
```

IntegrateAlgebraic [A] time = 0.17, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{17/6}\left(\frac{17b(c+dx)}{a+bx}-11d\right)}{187(c+dx)^{17/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(23/6),x]
```

```
[Out] (6*(a + b*x)^(17/6)*(-11*d + (17*b*(c + d*x))/(a + b*x)))/(187*(b*c - a*d)^
2*(c + d*x)^(17/6))
```

fricas [B] time = 1.14, size = 175, normalized size = 2.65

$$\frac{6\left(6b^2dx^2+17abc-11a^2d+\left(17b^2c-5abd\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{187\left(b^2c^5-2abc^4d+a^2c^3d^2+\left(b^2c^2d^3-2abcd^4+a^2d^5\right)x^3+3\left(b^2c^3d^2-2abc^2d^3+a^2cd^4\right)x^2+3\left(b^2c^4d-2abc^3d^2+a^2c^2d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/187*(6*b^2*d*x^2 + 17*a*b*c - 11*a^2*d + (17*b^2*c - 5*a*b*d)*x)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)/(b^2*c^5 - 2*a*b*c^4*d + a^2*c^3*d^2 + (b^2*c^2*d^3
 - 2*a*b*c*d^4 + a^2*d^5)*x^3 + 3*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)
*x^2 + 3*(b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\,(-6bdx+11ad-17bc)}{187\,(dx+c)^{\frac{17}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/187*(b*x+a)^(11/6)*(-6*b*d*x+11*a*d-17*b*c)/(d*x+c)^(17/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(23/6), x)
```

mupad [B] time = 0.74, size = 137, normalized size = 2.08

$$\frac{(c+d\,x)^{1/6}\left(\frac{36\,b^2\,x^2\,(a+b\,x)^{5/6}}{187\,d^2\,(a\,d-b\,c)^2}-\frac{\left(66\,a^2\,d-102\,a\,b\,c\right)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}+\frac{x\left(102\,b^2\,c-30\,a\,b\,d\right)\,(a+b\,x)^{5/6}}{187\,d^3\,(a\,d-b\,c)^2}\right)}{x^3+\frac{c^3}{d^3}+\frac{3\,c\,x^2}{d}+\frac{3\,c^2\,x}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(23/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((36*b^2*x^2*(a + b*x)^(5/6))/(187*d^2*(a*d - b*c)^2) - ((
66*a^2*d - 102*a*b*c)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2) + (x*(102*b^
2*c - 30*a*b*d)*(a + b*x)^(5/6))/(187*d^3*(a*d - b*c)^2)))/(x^3 + c^3/d^3 +
 (3*c*x^2)/d + (3*c^2*x)/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

3.1524 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}} dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{11/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{72b(a+bx)^{11/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (72*b*(a + b*x)^(1
1/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (432*b^2*(a + b*x)^(11/6))/(43
01*(b*c - a*d)^3*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}} dx &= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{(12b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}} dx}{23(bc-ad)}\\
&= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{\left(72b^2\right)\int \frac{(a+bx)^{5/6}}{(c+dx)^{17/6}} dx}{391(bc-ad)^2}\\
&= \frac{6(a+bx)^{11/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{72b(a+bx)^{11/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{432b^2(a+bx)^{11/6}}{4301(bc-ad)^3(c+dx)^{11/6}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{11/6}\left(187a^2d^2-22abd(23c+6dx)+b^2\left(391c^2+276cdx+72d^2x^2\right)\right)}{4301(c+dx)^{23/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(187*a^2*d^2 - 22*a*b*d*(23*c + 6*d*x) + b^2*(391*c^2 +
 276*c*d*x + 72*d^2*x^2)))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

IntegrateAlgebraic [A] time = 0.18, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{23/6}\left(\frac{391b^2(c+dx)^2}{(a+bx)^2}-\frac{506bd(c+dx)}{a+bx}+187d^2\right)}{4301(c+dx)^{23/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(29/6),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(187*d^2 - (506*b*d*(c + d*x))/(a + b*x) + (391*b^2*(c
+ d*x)^2)/(a + b*x)^2))/(4301*(b*c - a*d)^3*(c + d*x)^(23/6))
```

fricas [B] time = 1.16, size = 338, normalized size = 3.35

$$\frac{6\left(72\,b^3d^2x^3+391\,ab^2c^2-506\,a^2bcd+187\,a^3d^2+12\left(23\,b^3cd-5\,ab^2d^2\right)x^2+\left(391\,b^3c^2-230\,ab^2cd+55\,a^2bd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{4301\left(b^3c^7-3\,ab^2c^6d+3\,a^2bc^5d^2-a^3c^4d^3+\left(b^3c^3d^4-3\,ab^2c^2d^5+3\,a^2bcd^6-a^3d^7\right)x^4+4\left(b^3c^4d^3-3\,ab^2c^3d^4+3\,a^2bc^2d^5-a^3cd^6\right)x^3+6\left(b^3c^5d^2-3\,ab^2c^4d^3+3\,a^2bc^3d^4-a^3c^2d^5\right)x^2+4\left(b^3c^6d-3\,ab^2c^5d^2+3\,a^2bc^4d^3-a^3c^3d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/4301*(72*b^3*d^2*x^3 + 391*a*b^2*c^2 - 506*a^2*b*c*d + 187*a^3*d^2 + 12*(
23*b^3*c*d - 5*a*b^2*d^2)*x^2 + (391*b^3*c^2 - 230*a*b^2*c*d + 55*a^2*b*d^2
)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^7 - 3*a*b^2*c^6*d + 3*a^2*b*c^5
*d^2 - a^3*c^4*d^3 + (b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d
^7)*x^4 + 4*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*x
^3 + 6*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*x^2
+ 4*(b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\left(72b^2x^2d^2-132ab\,d^2x+276b^2cdx+187a^2d^2-506abcd+391b^2c^2\right)}{4301\,(dx+c)^{\frac{23}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/4301*(b*x+a)^(11/6)*(72*b^2*d^2*x^2-132*a*b*d^2*x+276*b^2*c*d*x+187*a^2*
d^2-506*a*b*c*d+391*b^2*c^2)/(d*x+c)^(23/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^(5/6)/(d*x+c)^(29/6),x, algorithm="maxima")

[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(29/6), x)

mupad [B] time = 0.94, size = 214, normalized size = 2.12

$$-\frac{(c+d\,x)^{1/6}\left(\frac{(a+b\,x)^{5/6}\,\left(1122\,a^3\,d^2-3036\,a^2\,b\,c\,d+2346\,a\,b^2\,c^2\right)}{4301\,d^4\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3\,(a+b\,x)^{5/6}}{4301\,d^2\,(a\,d-b\,c)^3}+\frac{x\,(a+b\,x)^{5/6}\,\left(330\,a^2\,b\,d^2-1380\,a\,b^2\,c\,d+2346\,b^3\,c^2\right)}{4301\,d^4\,(a\,d-b\,c)^3}-\frac{72\,b^2\,x^2\,(5\,a\,d-23\,b\,c)\,(a+b\,x)^{5/6}}{4301\,d^3\,(a\,d-b\,c)^3}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x)^(5/6)/(c + d*x)^(29/6),x)

[Out] -((c + d*x)^(1/6)*(((a + b*x)^(5/6)*(1122*a^3*d^2 + 2346*a*b^2*c^2 - 3036*a^2*b*c*d))/(4301*d^4*(a*d - b*c)^3) + (432*b^3*x^3*(a + b*x)^(5/6))/(4301*d^2*(a*d - b*c)^3) + (x*(a + b*x)^(5/6)*(2346*b^3*c^2 + 330*a^2*b*d^2 - 1380*a*b^2*c*d))/(4301*d^4*(a*d - b*c)^3) - (72*b^2*x^2*(5*a*d - 23*b*c)*(a + b*x)^(5/6))/(4301*d^3*(a*d - b*c)^3)))/(x^4 + c^4/d^4 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**(5/6)/(d*x+c)**(29/6),x)

[Out] Timed out

3.1525 $\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{11/6}}{124729(c+dx)^{11/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{11/6}}{11339(c+dx)^{17/6}(bc-ad)^3}+\frac{108b(a+bx)^{11/6}}{667(c+dx)^{23/6}(bc-ad)^2}+\frac{6(a+bx)^{11/6}}{29(c+dx)^{29/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6))/(29*(b*c - a*d)*(c + d*x)^(29/6)) + (108*b*(a + b*x)^(
11/6))/(667*(b*c - a*d)^2*(c + d*x)^(23/6)) + (1296*b^2*(a + b*x)^(11/6))/(
11339*(b*c - a*d)^3*(c + d*x)^(17/6)) + (7776*b^3*(a + b*x)^(11/6))/(124729
*(b*c - a*d)^4*(c + d*x)^(11/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{5/6}}{(c+dx)^{35/6}}\,dx &= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{(18b)\int \frac{(a+bx)^{5/6}}{(c+dx)^{29/6}}\,dx}{29(bc-ad)}\\ &= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}}+\frac{\left(216b^2\right)\int \frac{(a+bx)^{5/6}}{(c+dx)^{23/6}}\,dx}{667(bc-ad)^2}\\ &= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}}+\frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}}+\frac{(129\ldots}{1\ldots}\\ &= \frac{6(a+bx)^{11/6}}{29(bc-ad)(c+dx)^{29/6}}+\frac{108b(a+bx)^{11/6}}{667(bc-ad)^2(c+dx)^{23/6}}+\frac{1296b^2(a+bx)^{11/6}}{11339(bc-ad)^3(c+dx)^{17/6}}+\frac{\ldots}{1247\ldots}\end{aligned}$$

Mathematica [A] time = 0.09, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{11/6}\left(-4301a^3d^3+561a^2bd^2(29c+6dx)-33ab^2d\left(667c^2+348cdx+72d^2x^2\right)+b^3\left(11339c^3+12006c^2dx+6264cd^2x^2+1296d^3x^3\right)\right)}{124729(c+dx)^{29/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(-4301*a^3*d^3 + 561*a^2*b*d^2*(29*c + 6*d*x) - 33*a*b^
2*d*(667*c^2 + 348*c*d*x + 72*d^2*x^2) + b^3*(11339*c^3 + 12006*c^2*d*x + 6
264*c*d^2*x^2 + 1296*d^3*x^3)))/(124729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

IntegrateAlgebraic [A] time = 0.20, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{29/6}\left(\frac{11339b^3(c+dx)^3}{(a+bx)^3}-\frac{22011b^2d(c+dx)^2}{(a+bx)^2}+\frac{16269bd^2(c+dx)}{a+bx}-4301d^3\right)}{124729(c+dx)^{29/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(5/6)/(c + d*x)^(35/6),x]
```

```
[Out] (6*(a + b*x)^(29/6)*(-4301*d^3 + (16269*b*d^2*(c + d*x))/(a + b*x) - (22011
*b^2*d*(c + d*x)^2)/(a + b*x)^2 + (11339*b^3*(c + d*x)^3)/(a + b*x)^3))/(12
4729*(b*c - a*d)^4*(c + d*x)^(29/6))
```

fricas [B] time = 1.18, size = 533, normalized size = 3.92

$$\frac{6\left(1296\,b^4d^3x^4+11339\,ab^3c^3-22011\,a^2b^2c^2d+16269\,a^3bcd^2-4301\,a^4d^3+216\left(29\,b^4cd^2-5\,ab^3d^3\right)x^3+18\left(667\,b^4c^2d-290\,ab^3cd^2+55\,a^2b^2d^3\right)x^2+\left(11339\,b^4c^3-10005\,ab^3c^2d+4785\,a^2b^2cd^2-935\,a^3bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{124729\left(b^4c^9-4\,ab^3c^8d+6\,a^2b^2c^7d^2-4\,a^3bc^6d^3+a^4c^5d^4+\left(b^4c^4d^5-4\,ab^3c^3d^6+6\,a^2b^2c^2d^7-4\,a^3bcd^8+a^4d^9\right)x^5+5\left(b^4c^5d^4-4\,ab^3c^4d^5+6\,a^2b^2c^3d^6-4\,a^3bc^2d^7+a^4cd^8\right)x^4+10\left(b^4c^6d^3-4\,ab^3c^5d^4+6\,a^2b^2c^4d^5-4\,a^3bc^3d^6+a^4c^2d^7\right)x^3+10\left(b^4c^7d^2-4\,ab^3c^6d^3+6\,a^2b^2c^5d^4-4\,a^3bc^4d^5+a^4c^3d^6\right)x^2+5\left(b^4c^8d-4\,ab^3c^7d^2+6\,a^2b^2c^6d^3-4\,a^3bc^5d^4+a^4c^4d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="fricas")
```

```
[Out] 6/124729*(1296*b^4*d^3*x^4 + 11339*a*b^3*c^3 - 22011*a^2*b^2*c^2*d + 16269*
a^3*b*c*d^2 - 4301*a^4*d^3 + 216*(29*b^4*c*d^2 - 5*a*b^3*d^3)*x^3 + 18*(667
*b^4*c^2*d - 290*a*b^3*c*d^2 + 55*a^2*b^2*d^3)*x^2 + (11339*b^4*c^3 - 10005
*a*b^3*c^2*d + 4785*a^2*b^2*c*d^2 - 935*a^3*b*d^3)*x)*(b*x + a)^(5/6)*(d*x
+ c)^(1/6)/(b^4*c^9 - 4*a*b^3*c^8*d + 6*a^2*b^2*c^7*d^2 - 4*a^3*b*c^6*d^3 +
 a^4*c^5*d^4 + (b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b
*c*d^8 + a^4*d^9)*x^5 + 5*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^
6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*x^4 + 10*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 +
6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*x^3 + 10*(b^4*c^7*d^2 -
4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*x^2 +
5*(b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*
c^4*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{11}{6}}\left(-1296b^3d^3x^3+2376a\,b^2d^3x^2-6264b^3c\,d^2x^2-3366a^2b\,d^3x+11484a\,b^2c\,d^2x-12006b^3c^2dx+4301a^3d^3-16269a^2bc\,d^2+22011a\,b^2c^2d-11339b^3c^3\right)}{124729\,(dx+c)^{\frac{29}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(5/6)/(d*x+c)^(35/6),x)
```

```
[Out] -6/124729*(b*x+a)^(11/6)*(-1296*b^3*d^3*x^3+2376*a*b^2*d^3*x^2-6264*b^3*c*d
^2*x^2-3366*a^2*b*d^3*x+11484*a*b^2*c*d^2*x-12006*b^3*c^2*d*x+4301*a^3*d^3-
16269*a^2*b*c*d^2+22011*a*b^2*c^2*d-11339*b^3*c^3)/(d*x+c)^(29/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{5}{6}}}{(dx+c)^{\frac{35}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(5/6)/(d*x+c)^(35/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(5/6)/(d*x + c)^(35/6), x)
```

mupad [B] time = 1.16, size = 303, normalized size = 2.23

$$\frac{(c+d\,x)^{1/6}\left(\frac{7776\,b^4\,x^4\,(a+b\,x)^{5/6}}{124729\,d^2\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{5/6}\left(25806\,a^4\,d^3-97614\,a^3\,b\,c\,d^2+132066\,a^2\,b^2\,c^2\,d-68034\,a\,b^3\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{5/6}\left(-5610\,a^3\,b\,d^3+28710\,a^2\,b^2\,c\,d^2-60030\,a\,b^3\,c^2\,d+68034\,b^4\,c^3\right)}{124729\,d^5\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,(a+b\,x)^{5/6}\left(55\,a^2\,d^2-290\,a\,b\,c\,d+667\,b^2\,c^2\right)}{124729\,d^4\,(a\,d-b\,c)^4}-\frac{1296\,b^3\,x^3\,(5\,a\,d-29\,b\,c)\,(a+b\,x)^{5/6}}{124729\,d^3\,(a\,d-b\,c)^4}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(5/6)/(c + d*x)^(35/6),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4*(a + b*x)^(5/6))/(124729*d^2*(a*d - b*c)^4)
 - ((a + b*x)^(5/6)*(25806*a^4*d^3 - 68034*a*b^3*c^3 + 132066*a^2*b^2*c^2*d
 - 97614*a^3*b*c*d^2))/(124729*d^5*(a*d - b*c)^4) + (x*(a + b*x)^(5/6)*(680
34*b^4*c^3 - 5610*a^3*b*d^3 + 28710*a^2*b^2*c*d^2 - 60030*a*b^3*c^2*d))/(12
4729*d^5*(a*d - b*c)^4) + (108*b^2*x^2*(a + b*x)^(5/6)*(55*a^2*d^2 + 667*b^
2*c^2 - 290*a*b*c*d))/(124729*d^4*(a*d - b*c)^4) - (1296*b^3*x^3*(5*a*d - 2
9*b*c)*(a + b*x)^(5/6))/(124729*d^3*(a*d - b*c)^4)))/(x^5 + c^5/d^5 + (5*c*
x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(5/6)/(d*x+c)**(35/6),x)
```

```
[Out] Timed out
```

3.1526 $\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)}{}$$

Rubi [A] time = 0.55, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{5/6}d^{13/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{5/6}d^{13/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}}-\frac{7\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12d^2}+\frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (-7*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*d^2) + ((a + b*x)^(7/6
)*(c + d*x)^(5/6))/(2*d) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(
a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(
13/6)) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(S
qrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c -
 a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b
^(5/6)*d^(13/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144
*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(1
44*b^(5/6)*d^(13/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{\sqrt[6]{c+dx}}\,dx = \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{(7(bc-ad))\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{12d}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}} \ldots\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \ldots\right.}{12bd^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\ldots}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}x+\ldots} \ldots\right.}{36b^{5/6}d^2}$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - \ldots$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} - \ldots$$

$$= -\frac{7(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12d^2} + \frac{(a+bx)^{7/6}(c+dx)^{5/6}}{2d} - \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}} + \ldots$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.17

$$\frac{6(a+bx)^{13/6}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\;{}_2F_1\left(\frac{1}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(1/6))
```

IntegrateAlgebraic [A] time = 0.78, size = 363, normalized size = 0.86

$$-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{5/6}d^{13/6}} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{5/6}d^{13/6}} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{5/6}d^{13/6}} + \frac{(ad-bc)^2\left(\frac{13d(a+bx)^{7/6}}{(c+dx)^{7/6}}-\frac{7b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{12d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(1/6),x]
```

```
[Out] ((-(b*c) + a*d)^2*((13*d*(a + b*x)^(7/6))/(c + d*x)^(7/6) - (7*b*(a + b*x)^
(1/6))/(c + d*x)^(1/6)))/(12*d^2*(-b + (d*(a + b*x))/(c + d*x))^2) - (7*(b*
c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*ArcTan[
```

```
1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])
/(24*Sqrt[3]*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x
)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))]])/(36*b^(5/6)*d^(13/6)) + (7*(b*c - a*d)
^2*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d
^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))]])/(72*b^(5/6)*d^(13/6))
```

fricas [B] time = 1.56, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/144*(28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*c*d^12 + a^2*b^4*d^13)*(
b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d
^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*sqrt(((b^3*c^2*d^2 - 2*a*
b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d
^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/
(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(5/6) + sqrt(3
)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d
^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*
a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c
^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*
d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 79
2*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4
*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 +
 a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*
a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*
c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 +
 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 1
2*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^
8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8
 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a
^11*b*c*d^12 + a^12*d^13)*x)) + 28*sqrt(3)*d^2*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^5*d^13))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^6*c^2*d^11 - 2*a*b^5*
c*d^12 + a^2*b^4*d^13)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
```

d^11 + a^12*d^12)/(b^5*d^13))^(5/6) - 2*sqrt(3)*(b^4*d^12*x + b^4*c*d^11)*s
qrt(-((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(
5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)
*(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^5*d^13))^(1/3))/(d*x + c))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^5*d^13))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11
*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 9
24*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^
3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12
*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 +
495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*
b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*
d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c^13 - 12*a*b^11*c^12*d + 66
*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^4 - 792*a^5*b
^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a^8*b^4*c^5*d^
8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2*d^11 + a^12*
c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3
*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*
d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66
*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x)) - 7*d^2*((b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(49*((b^3*c^2*d^2 - 2*a*b^2*
c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^5*d^13))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^2*d^5*x
+ b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/3))/(d*x
+ c)) + 7*d^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6)*log(
-49*((b^3*c^2*d^2 - 2*a*b^2*c*d^3 + a^2*b*d^4)*(b*x + a)^(1/6)*(d*x + c)^(5
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^5*d^13))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*
(d*x + c)^(2/3) - (b^2*d^5*x + b^2*c*d^4)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d
^12)/(b^5*d^13))^(1/3))/(d*x + c)) - 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d

```
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) + (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^5*d^13))^(1/6))/(d*x + c)) + 14*d^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^5*d^13))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(1/6)*(d*x + c)^(5/6) - (b*d^3*x + b*c*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^5*d^13))^(1/6))/(d*x + c)) - 12*(6*b*d*x - 7*b*c + 13*a*d)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6))/d^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(1/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{7/6}}{{\left(c+d\,x\right)}^{1/6}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(1/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(1/6), x)
```

3.1527 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}+\frac{7\sqrt[6]{\ldots}}{}$$

Rubi [A] time = 0.53, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2}+\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12d^{13/6}}+\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}}-\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(d*(c + d*x)^(1/6)) + (7*b*(a + b*x)^(1/6)*(c + d*x)^(
5/6))/d^2 + (7*b^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6
)*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/
6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*b^(1/6)*(b*c - a*d)*ArcTanh
[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*d^(13/6)) + (7*b^
(1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
 (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6)) - (7*b^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) +
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*d^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}}\,dx = -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{(7b)\int \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7b(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{(7(bc-ad))\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{\left(7\sqrt[6]{b}\,(bc-ad)\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \right.}{3d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{\left(7\sqrt[6]{b}\,(bc-a\right.}{}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} - \frac{7\sqrt[6]{b}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3d^{13/6}} + \frac{7\sqrt[6]{b}\,(bc-ad}{}$$

$$= -\frac{6(a+bx)^{7/6}}{d\sqrt[6]{c+dx}} + \frac{7b\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{d^2} + \frac{7\sqrt[6]{b}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,d^{13/6}} - \frac{7\sqrt[6]{b}\,(bc-ad}{}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.18

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}\,{}_2F_1\left(\frac{7}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(7/6)*Hypergeometric2F1[7/6
, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(7/6))
```

IntegrateAlgebraic [A] time = 23.49, size = 417, normalized size = 1.03

$$\frac{d^{7/6}(a+bx)^{7/6}\left(-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\,d^{13/6}}+\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,d^{13/6}}-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3d^{13/6}}-\frac{7\left(b^{7/6}c-a\sqrt[6]{b}\,d\right)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}\,\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6d^{13/6}}+\frac{\sqrt[6]{ad+b(c+dx)-bc}\,(-6ad+b(c+dx)+6bc)}{d^{13/6}\sqrt[6]{c+dx}}\right)}{(ad+bdx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(7/6),x]
```

```
[Out] (d^(7/6)*(a + b*x)^(7/6)*(((6*b*c - 6*a*d + b*(c + d*x))*(-(b*c) + a*d + b*
(c + d*x))^(1/6))/(d^(13/6)*(c + d*x)^(1/6)) - (7*(b^(7/6)*c - a*b^(1/6)*d)
*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6) - 2*(-(b
```

```
*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*d^(13/6)) + (7*(b^(7/6)*c - a*
b^(1/6)*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6
) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*d^(13/6)) - (7*(b^(7
/6)*c - a*b^(1/6)*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) + a*d + b*(c
 + d*x))^(1/6)])/(3*d^(13/6)) - (7*(b^(7/6)*c - a*b^(1/6)*d)*ArcTanh[(b^(1/
3)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c + d*x)
^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*d^(13/6))))/(a*d + b*d*x)^(
7/6)
```

fricas [B] time = 1.43, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/12*(28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^
4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d
^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*d^11 - a*d^12)*(b*x + a)^(1/6)*(
d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + 2
*sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c
)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*(
(b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4
*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c))*((b^7*c
^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c
^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(5/6) + sqrt(3)*(b^7*c^7 - 6*a*
b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 -
6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5
*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*
b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d
^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*
a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^
5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(d^3*x + c*d^2)*((b^7*c^6
 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2
*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b*c*
d^11 - a*d^12)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d +
15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*
d^5 + a^6*b*d^6)/d^13)^(5/6) + 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*c*d^2
- a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2
*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 +
a^6*b*d^6)/d^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d
*x + c)^(2/3) - (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/3))/(d*x + c))*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(5/6) - sqrt(3)*(b^7*c^7 - 6*a*b^6*c^6*d + 15*a^2*b^5*c^5*d^2 - 20*a^3*b^
4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 + (b^7*c^6
*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3
*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(b^7*c^7 - 6*a*b^6*c^6*d + 15*a
^2*b^5*c^5*d^2 - 20*a^3*b^4*c^4*d^3 + 15*a^4*b^3*c^3*d^4 - 6*a^5*b^2*c^2*d^
5 + a^6*b*c*d^6 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^
3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(
d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*
c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log
(49*((b*c*d^2 - a*d^3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*
c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^
5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
```

```
 + a)^(1/3)*(d*x + c)^(2/3) + (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 1
5*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d
^5 + a^6*b*d^6)/d^13)^(1/3))/(d*x + c)) - 7*(d^3*x + c*d^2)*((b^7*c^6 - 6*a
*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 -
 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6)*log(-49*((b*c*d^2 - a*d^3)*(b*x +
 a)^(1/6)*(d*x + c)^(5/6)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 -
20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13
)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) -
 (d^5*x + c*d^4)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^
4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/3))/
(d*x + c)) + 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*
d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6
)/d^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (d^3*x
+ c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^
3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c
)) - 14*(d^3*x + c*d^2)*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20
*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^
(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (d^3*x + c*d^2)
*((b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a
^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)/d^13)^(1/6))/(d*x + c)) - 12*
(b*d*x + 7*b*c - 6*a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6))/(d^3*x + c*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
[Out] int((b*x+a)^(7/6)/(d*x+c)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,x\right)}^{7/6}}{{\left(c+d\,x\right)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(7/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(7/6), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)^{\frac{7}{6}}}{(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(7/6),x)
```

```
[Out] Integral((a + b*x)**(7/6)/(c + d*x)**(7/6), x)
```

3.1528 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=358

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\right.}{d^{13/6}}$$

Rubi [A] time = 0.50, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {47, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{b^{7/6}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}+\frac{b^{7/6}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2d^{13/6}}-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}+\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}}+\frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{d^{13/6}}-\frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}}-\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*(a + b*x)^(7/6))/(7*d*(c + d*x)^(7/6)) - (6*b*(a + b*x)^(1/6))/(d^2*(c
+ d*x)^(1/6)) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1
/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcTan
[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))]
)/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x
)^(1/6))])/d^(13/6) - (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c +
 d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/
6)) + (b^(7/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2*d^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
```

```
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{13/6}}\,dx = -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} + \frac{b\int \frac{\sqrt[6]{a+bx}}{(c+dx)^{7/6}}\,dx}{d}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{b^2\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{(6b)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{(6b)\operatorname{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{(2b^{7/6})\operatorname{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^2} + \frac{(2b^{7/6})}{}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}} - \frac{b^{7/6}\operatorname{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\right.}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} + \frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}} - \frac{b^{7/6}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} - \right.}{2d^{13/6}}$$

$$= -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b\sqrt[6]{a+bx}}{d^2\sqrt[6]{c+dx}} - \frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}} + \frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{d^{13/6}}$$

Mathematica [C] time = 0.09, size = 73, normalized size = 0.20

$$\frac{6(a+bx)^{13/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6} {}_2F_1\left(\frac{13}{6},\frac{13}{6};\frac{19}{6};\frac{d(a+bx)}{ad-bc}\right)}{13b(c+dx)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*((b*(c + d*x))/(b*c - a*d))^(13/6)*Hypergeometric2F1[13
/6, 13/6, 19/6, (d*(a + b*x))/(-(b*c) + a*d)])/(13*b*(c + d*x)^(13/6))
```

IntegrateAlgebraic [A] time = 0.33, size = 282, normalized size = 0.79

$$-\frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}} + \frac{\sqrt{3}\,b^{7/6}\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{d^{13/6}} + \frac{2b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{d^{13/6}} + \frac{b^{7/6}\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{d^{13/6}} - \frac{6\left(\frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}+\frac{7b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{7d^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(13/6),x]
```

```
[Out] (-6*((d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (7*b*(a + b*x)^(1/6))/(c + d*x)^
(1/6)))/(7*d^2) - (Sqrt[3]*b^(7/6)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^
(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/d^(13/6) + (Sqrt[3]*b^(7/6)*ArcT
```

```
an[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6)
)])/d^(13/6) + (2*b^(7/6)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d
*x)^(1/6))])/d^(13/6) + (b^(7/6)*ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/
((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))])/d
^(13/6)
```

fricas [B] time = 1.46, size = 855, normalized size = 2.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] -1/14*(28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*arctan(-
1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/6) - 2*
sqrt(3)*(d^12*x + c*d^11)*sqrt(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/
d^13)^(1/6) + (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^
13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) + sqrt(3)*(b^7*d*x + b^7*c))/(b^7*d*
x + b^7*c)) + 28*sqrt(3)*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*a
rctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^11*(b^7/d^13)^(5/
6) - 2*sqrt(3)*(d^12*x + c*d^11)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d
^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 - (d^5*x + c*d^4)
*(b^7/d^13)^(1/3))/(d*x + c))*(b^7/d^13)^(5/6) - sqrt(3)*(b^7*d*x + b^7*c))
/(b^7*d*x + b^7*c)) - 7*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*lo
g(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) + (b*x + a)^(1/
3)*(d*x + c)^(2/3)*b^2 + (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) + 7*(
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b^7/d^13)^(1/6)*log(-4*((b*x + a)^(1/6)*(d*
x + c)^(5/6)*b*d^2*(b^7/d^13)^(1/6) - (b*x + a)^(1/3)*(d*x + c)^(2/3)*b^2 -
 (d^5*x + c*d^4)*(b^7/d^13)^(1/3))/(d*x + c)) - 14*(d^4*x^2 + 2*c*d^3*x + c
^2*d^2)*(b^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b + (d^3*x +
c*d^2)*(b^7/d^13)^(1/6))/(d*x + c)) + 14*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*(b
^7/d^13)^(1/6)*log(((b*x + a)^(1/6)*(d*x + c)^(5/6)*b - (d^3*x + c*d^2)*(b^
7/d^13)^(1/6))/(d*x + c)) + 12*(8*b*d*x + 7*b*c + a*d)*(b*x + a)^(1/6)*(d*x
 + c)^(5/6))/(d^4*x^2 + 2*c*d^3*x + c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

```
[Out] int((b*x+a)^(7/6)/(d*x+c)^(13/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(13/6), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,x)^{7/6}}{(c+d\,x)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(13/6),x)
```

```
[Out] int((a + b*x)^(7/6)/(c + d*x)^(13/6), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

3.1529 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx = \frac{6(a+bx)^{13/6}}{13(bc-ad)(c+dx)^{13/6}}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

IntegrateAlgebraic [A] time = 0.06, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{13/6}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(19/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(13*(b*c - a*d)*(c + d*x)^(13/6))
```

fricas [B] time = 1.33, size = 104, normalized size = 3.25

$$\frac{6\left(b^2x^2+2\,abx+a^2\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{13\left(bc^4-ac^3d+\left(bcd^3-ad^4\right)x^3+3\left(bc^2d^2-acd^3\right)x^2+3\left(bc^3d-ac^2d^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/13*(b^2*x^2 + 2*a*b*x + a^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^4 - a*c
^3*d + (b*c*d^3 - a*d^4)*x^3 + 3*(b*c^2*d^2 - a*c*d^3)*x^2 + 3*(b*c^3*d - a
*c^2*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{13}{6}}}{13\,(dx+c)^{\frac{13}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/13*(b*x+a)^(13/6)/(d*x+c)^(13/6)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(19/6), x)
```

mupad [B] time = 0.76, size = 199, normalized size = 6.22

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{6\,a^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{6\,b^2\,x^2\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}}{13\,a\,d^4-13\,b\,c\,d^3}\right)}{x^3-\frac{13\,b\,c^4-13\,a\,c^3\,d}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c\,d^2\,x^2\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}+\frac{39\,c^2\,d\,x\,(a\,d-b\,c)}{13\,a\,d^4-13\,b\,c\,d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(19/6),x)
```

```
[Out] -((c + d*x)^(5/6)*((6*a^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (6*b^2
*x^2*(a + b*x)^(1/6))/(13*a*d^4 - 13*b*c*d^3) + (12*a*b*x*(a + b*x)^(1/6))/
(13*a*d^4 - 13*b*c*d^3)))/(x^3 - (13*b*c^4 - 13*a*c^3*d)/(13*a*d^4 - 13*b*c
*d^3) + (39*c*d^2*x^2*(a*d - b*c))/(13*a*d^4 - 13*b*c*d^3) + (39*c^2*d*x*(a
*d - b*c))/(13*a*d^4 - 13*b*c*d^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

3.1530 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{36b(a+bx)^{13/6}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (36*b*(a + b*x)^(1
3/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx &= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(6b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{19(bc-ad)}\\ &= \frac{6(a+bx)^{13/6}}{19(bc-ad)(c+dx)^{19/6}}+\frac{36b(a+bx)^{13/6}}{247(bc-ad)^2(c+dx)^{13/6}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}(-13ad+19bc+6bdx)}{247(c+dx)^{19/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(19*b*c - 13*a*d + 6*b*d*x))/(247*(b*c - a*d)^2*(c + d*
x)^(19/6))
```

IntegrateAlgebraic [A] time = 0.18, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{13/6}\left(19b-\frac{13d(a+bx)}{c+dx}\right)}{247(c+dx)^{13/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(25/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(19*b - (13*d*(a + b*x))/(c + d*x)))/(247*(b*c - a*d)^2
*(c + d*x)^(13/6))
```

fricas [B] time = 1.34, size = 235, normalized size = 3.56

$$\frac{6\left(6\,b^3dx^3+19\,a^2bc-13\,a^3d+\left(19\,b^3c-ab^2d\right)x^2+2\left(19\,ab^2c-10\,a^2bd\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{247\left(b^2c^6-2\,abc^5d+a^2c^4d^2+\left(b^2c^2d^4-2\,abcd^5+a^2d^6\right)x^4+4\left(b^2c^3d^3-2\,abc^2d^4+a^2cd^5\right)x^3+6\left(b^2c^4d^2-2\,abc^3d^3+a^2c^2d^4\right)x^2+4\left(b^2c^5d-2\,abc^4d^2+a^2c^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/247*(6*b^3*d*x^3 + 19*a^2*b*c - 13*a^3*d + (19*b^3*c - a*b^2*d)*x^2 + 2*(
19*a*b^2*c - 10*a^2*b*d)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^6 - 2*a*
b*c^5*d + a^2*c^4*d^2 + (b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*x^4 + 4*(b^2*
c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*x^3 + 6*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 +
 a^2*c^2*d^4)*x^2 + 4*(b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\,(-6bdx+13ad-19bc)}{247\,(dx+c)^{\frac{19}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/247*(b*x+a)^(13/6)*(-6*b*d*x+13*a*d-19*b*c)/(d*x+c)^(19/6)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(25/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(25/6), x)
```

mupad [B] time = 0.91, size = 189, normalized size = 2.86

$$-\frac{(c+d\,x)^{5/6}\,\left(\frac{\left(78\,a^3\,d-114\,a^2\,b\,c\right)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}-\frac{36\,b^3\,x^3\,(a+b\,x)^{1/6}}{247\,d^3\,(a\,d-b\,c)^2}-\frac{x^2\,\left(114\,b^3\,c-6\,a\,b^2\,d\right)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}+\frac{12\,a\,b\,x\,(10\,a\,d-19\,b\,c)\,(a+b\,x)^{1/6}}{247\,d^4\,(a\,d-b\,c)^2}\right)}{x^4+\frac{c^4}{d^4}+\frac{4\,c\,x^3}{d}+\frac{4\,c^3\,x}{d^3}+\frac{6\,c^2\,x^2}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(25/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((78*a^3*d - 114*a^2*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d
 - b*c)^2) - (36*b^3*x^3*(a + b*x)^(1/6))/(247*d^3*(a*d - b*c)^2) - (x^2*(1
14*b^3*c - 6*a*b^2*d)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2) + (12*a*b*x*
(10*a*d - 19*b*c)*(a + b*x)^(1/6))/(247*d^4*(a*d - b*c)^2)))/(x^4 + c^4/d^4
 + (4*c*x^3)/d + (4*c^3*x)/d^3 + (6*c^2*x^2)/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(25/6),x)
```

```
[Out] Timed out
```

3.1531 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{13/6}}{6175(c+dx)^{13/6}(bc-ad)^3}+\frac{72b(a+bx)^{13/6}}{475(c+dx)^{19/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{25(c+dx)^{25/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(25*(b*c - a*d)*(c + d*x)^(25/6)) + (72*b*(a + b*x)^(1
3/6))/(475*(b*c - a*d)^2*(c + d*x)^(19/6)) + (432*b^2*(a + b*x)^(13/6))/(61
75*(b*c - a*d)^3*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{(12b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{25(bc-ad)}\\ &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{\left(72b^2\right)\int \frac{(a+bx)^{7/6}}{(c+dx)^{19/6}}\,dx}{475(bc-ad)^2}\\ &= \frac{6(a+bx)^{13/6}}{25(bc-ad)(c+dx)^{25/6}}+\frac{72b(a+bx)^{13/6}}{475(bc-ad)^2(c+dx)^{19/6}}+\frac{432b^2(a+bx)^{13/6}}{6175(bc-ad)^3(c+dx)^{13/6}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{13/6}\left(247a^2d^2-26abd(25c+6dx)+b^2\left(475c^2+300cdx+72d^2x^2\right)\right)}{6175(c+dx)^{25/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(247*a^2*d^2 - 26*a*b*d*(25*c + 6*d*x) + b^2*(475*c^2 +
 300*c*d*x + 72*d^2*x^2)))/(6175*(b*c - a*d)^3*(c + d*x)^(25/6))
```

IntegrateAlgebraic [A] time = 0.20, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{13/6}\left(\frac{247d^2(a+bx)^2}{(c+dx)^2}-\frac{650bd(a+bx)}{c+dx}+475b^2\right)}{6175(c+dx)^{13/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(31/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(475*b^2 + (247*d^2*(a + b*x)^2)/(c + d*x)^2 - (650*b*d
*(a + b*x))/(c + d*x)))/(6175*(b*c - a*d)^3*(c + d*x)^(13/6))
```

fricas [B] time = 1.37, size = 427, normalized size = 4.23

$$\frac{6\left(72b^4d^2x^4+475a^2b^2c^2-650a^3bcd+247a^4d^2+12\left(25b^4cd-ab^3d^2\right)x^3+\left(475b^4c^2-50ab^3cd+7a^2b^2d^2\right)x^2+2\left(475ab^3c^2-500a^2b^2cd+169a^3bd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{6175\left(b^3c^8-3ab^2c^7d+3a^2bc^6d^2-a^3c^5d^3+\left(b^3c^3d^5-3ab^2c^2d^6+3a^2bcd^7-a^3d^8\right)x^5+5\left(b^3c^4d^4-3ab^2c^3d^5+3a^2bc^2d^6-a^3cd^7\right)x^4+10\left(b^3c^5d^3-3ab^2c^4d^4+3a^2bc^3d^5-a^3c^2d^6\right)x^3+10\left(b^3c^6d^2-3ab^2c^5d^3+3a^2bc^4d^4-a^3c^3d^5\right)x^2+5\left(b^3c^7d-3ab^2c^6d^2+3a^2bc^5d^3-a^3c^4d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="fricas")
```

```
[Out] 6/6175*(72*b^4*d^2*x^4 + 475*a^2*b^2*c^2 - 650*a^3*b*c*d + 247*a^4*d^2 + 12
*(25*b^4*c*d - a*b^3*d^2)*x^3 + (475*b^4*c^2 - 50*a*b^3*c*d + 7*a^2*b^2*d^2
)*x^2 + 2*(475*a*b^3*c^2 - 500*a^2*b^2*c*d + 169*a^3*b*d^2)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^3*c^8 - 3*a*b^2*c^7*d + 3*a^2*b*c^6*d^2 - a^3*c^5*d^
3 + (b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*x^5 + 5*(b^3*
c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*x^4 + 10*(b^3*c^5*
d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*x^3 + 10*(b^3*c^6*d^
2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*x^2 + 5*(b^3*c^7*d - 3
*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\left(72b^2x^2d^2-156ab\,d^2x+300b^2cdx+247a^2d^2-650abcd+475b^2c^2\right)}{6175\,(dx+c)^{\frac{25}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(31/6),x)
```

```
[Out] -6/6175*(b*x+a)^(13/6)*(72*b^2*d^2*x^2-156*a*b*d^2*x+300*b^2*c*d*x+247*a^2*
d^2-650*a*b*c*d+475*b^2*c^2)/(d*x+c)^(25/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*
c^2*d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{31}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(31/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(31/6), x)
```

mupad [B] time = 1.14, size = 278, normalized size = 2.75

$$-\frac{(c+d\,x)^{5/6}\left(\frac{(a+b\,x)^{1/6}\left(1482\,a^4\,d^2-3900\,a^3\,b\,c\,d+2850\,a^2\,b^2\,c^2\right)}{6175\,d^5\,(a\,d-b\,c)^3}+\frac{432\,b^4\,x^4\,(a+b\,x)^{1/6}}{6175\,d^3\,(a\,d-b\,c)^3}+\frac{x^2\,(a+b\,x)^{1/6}\left(42\,a^2\,b^2\,d^2-300\,a\,b^3\,c\,d+2850\,b^4\,c^2\right)}{6175\,d^5\,(a\,d-b\,c)^3}-\frac{72\,b^3\,x^3\,(a\,d-25\,b\,c)\,(a+b\,x)^{1/6}}{6175\,d^4\,(a\,d-b\,c)^3}+\frac{12\,a\,b\,x\,(a+b\,x)^{1/6}\left(169\,a^2\,d^2-500\,a\,b\,c\,d+475\,b^2\,c^2\right)}{6175\,d^5\,(a\,d-b\,c)^3}\right)}{x^5+\frac{c^5}{d^5}+\frac{5\,c\,x^4}{d}+\frac{5\,c^4\,x}{d^4}+\frac{10\,c^2\,x^3}{d^2}+\frac{10\,c^3\,x^2}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(31/6),x)
```

```
[Out] -((c + d*x)^(5/6)*(((a + b*x)^(1/6)*(1482*a^4*d^2 + 2850*a^2*b^2*c^2 - 3900
*a^3*b*c*d))/(6175*d^5*(a*d - b*c)^3) + (432*b^4*x^4*(a + b*x)^(1/6))/(6175
*d^3*(a*d - b*c)^3) + (x^2*(a + b*x)^(1/6)*(2850*b^4*c^2 + 42*a^2*b^2*d^2 -
 300*a*b^3*c*d))/(6175*d^5*(a*d - b*c)^3) - (72*b^3*x^3*(a*d - 25*b*c)*(a +
 b*x)^(1/6))/(6175*d^4*(a*d - b*c)^3) + (12*a*b*x*(a + b*x)^(1/6)*(169*a^2*
d^2 + 475*b^2*c^2 - 500*a*b*c*d))/(6175*d^5*(a*d - b*c)^3)))/(x^5 + c^5/d^5
 + (5*c*x^4)/d + (5*c^4*x)/d^4 + (10*c^2*x^3)/d^2 + (10*c^3*x^2)/d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(31/6),x)
```

```
[Out] Timed out
```

3.1532 $\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{13/6}}{191425(c+dx)^{13/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{13/6}}{14725(c+dx)^{19/6}(bc-ad)^3}+\frac{108b(a+bx)^{13/6}}{775(c+dx)^{25/6}(bc-ad)^2}+\frac{6(a+bx)^{13/6}}{31(c+dx)^{31/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6))/(31*(b*c - a*d)*(c + d*x)^(31/6)) + (108*b*(a + b*x)^(
13/6))/(775*(b*c - a*d)^2*(c + d*x)^(25/6)) + (1296*b^2*(a + b*x)^(13/6))/(
14725*(b*c - a*d)^3*(c + d*x)^(19/6)) + (7776*b^3*(a + b*x)^(13/6))/(191425
*(b*c - a*d)^4*(c + d*x)^(13/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx)^{7/6}}{(c+dx)^{37/6}}\,dx &= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{(18b)\int \frac{(a+bx)^{7/6}}{(c+dx)^{31/6}}\,dx}{31(bc-ad)}\\
&= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}}+\frac{\left(216b^2\right)\int \frac{(a+bx)^{7/6}}{(c+dx)^{25/6}}\,dx}{775(bc-ad)^2}\\
&= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}}+\frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}}+\cdots\\
&= \frac{6(a+bx)^{13/6}}{31(bc-ad)(c+dx)^{31/6}}+\frac{108b(a+bx)^{13/6}}{775(bc-ad)^2(c+dx)^{25/6}}+\frac{1296b^2(a+bx)^{13/6}}{14725(bc-ad)^3(c+dx)^{19/6}}+\cdots
\end{aligned}$$

Mathematica [A] time = 0.09, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{13/6}\left(-6175a^3d^3+741a^2bd^2(31c+6dx)-39ab^2d\left(775c^2+372cdx+72d^2x^2\right)+b^3\left(14725c^3+13950c^2dx+6696cd^2x^2+1296d^3x^3\right)\right)}{191425(c+dx)^{31/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(-6175*a^3*d^3 + 741*a^2*b*d^2*(31*c + 6*d*x) - 39*a*b^
2*d*(775*c^2 + 372*c*d*x + 72*d^2*x^2) + b^3*(14725*c^3 + 13950*c^2*d*x + 6
696*c*d^2*x^2 + 1296*d^3*x^3)))/(191425*(b*c - a*d)^4*(c + d*x)^(31/6))
```

IntegrateAlgebraic [A] time = 0.23, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{13/6}\left(-\frac{30225b^2d(a+bx)}{c+dx}-\frac{6175d^3(a+bx)^3}{(c+dx)^3}+\frac{22971bd^2(a+bx)^2}{(c+dx)^2}+14725b^3\right)}{191425(c+dx)^{13/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x)^(7/6)/(c + d*x)^(37/6),x]
```

```
[Out] (6*(a + b*x)^(13/6)*(14725*b^3 - (6175*d^3*(a + b*x)^3)/(c + d*x)^3 + (2297
1*b*d^2*(a + b*x)^2)/(c + d*x)^2 - (30225*b^2*d*(a + b*x))/(c + d*x)))/(191
425*(b*c - a*d)^4*(c + d*x)^(13/6))
```

fricas [B] time = 1.39, size = 649, normalized size = 4.77

$$\frac{6\left(1296\,b^5d^3x^5+14725\,a^2b^3c^3-30225\,a^3b^2c^2d+22971\,a^4bcd^2-6175\,a^5d^3+216\left(31\,b^5cd^2-ab^4d^3\right)x^4+18\left(775\,b^5c^2d-62\,ab^4cd^2+7\,a^2b^3d^3\right)x^3+\left(14725\,b^5c^3-2325\,ab^4c^2d+651\,a^2b^3cd^2-91\,a^3b^2d^3\right)x^2+2\left(14725\,ab^4c^3-23250\,a^2b^3c^2d+15717\,a^3b^2cd^2-3952\,a^4bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{191425\left(b^4c^{10}-4\,ab^3c^9d+6\,a^2b^2c^8d^2-4\,a^3bc^7d^3+a^4c^6d^4+\left(b^4c^4d^6-4\,ab^3c^3d^7+6\,a^2b^2c^2d^8-4\,a^3bcd^9+a^4d^{10}\right)x^6+6\left(b^4c^5d^5-4\,ab^3c^4d^6+6\,a^2b^2c^3d^7-4\,a^3bc^2d^8+a^4cd^9\right)x^5+15\left(b^4c^6d^4-4\,ab^3c^5d^5+6\,a^2b^2c^4d^6-4\,a^3bc^3d^7+a^4c^2d^8\right)x^4+20\left(b^4c^7d^3-4\,ab^3c^6d^4+6\,a^2b^2c^5d^5-4\,a^3bc^4d^6+a^4c^3d^7\right)x^3+15\left(b^4c^8d^2-4\,ab^3c^7d^3+6\,a^2b^2c^6d^4-4\,a^3bc^5d^5+a^4c^4d^6\right)x^2+6\left(b^4c^9d-4\,ab^3c^8d^2+6\,a^2b^2c^7d^3-4\,a^3bc^6d^4+a^4c^5d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="fricas")
```

```
[Out] 6/191425*(1296*b^5*d^3*x^5 + 14725*a^2*b^3*c^3 - 30225*a^3*b^2*c^2*d + 2297
1*a^4*b*c*d^2 - 6175*a^5*d^3 + 216*(31*b^5*c*d^2 - a*b^4*d^3)*x^4 + 18*(775
*b^5*c^2*d - 62*a*b^4*c*d^2 + 7*a^2*b^3*d^3)*x^3 + (14725*b^5*c^3 - 2325*a*
b^4*c^2*d + 651*a^2*b^3*c*d^2 - 91*a^3*b^2*d^3)*x^2 + 2*(14725*a*b^4*c^3 -
23250*a^2*b^3*c^2*d + 15717*a^3*b^2*c*d^2 - 3952*a^4*b*d^3)*x)*(b*x + a)^(1
/6)*(d*x + c)^(5/6)/(b^4*c^10 - 4*a*b^3*c^9*d + 6*a^2*b^2*c^8*d^2 - 4*a^3*b
*c^7*d^3 + a^4*c^6*d^4 + (b^4*c^4*d^6 - 4*a*b^3*c^3*d^7 + 6*a^2*b^2*c^2*d^8
 - 4*a^3*b*c*d^9 + a^4*d^10)*x^6 + 6*(b^4*c^5*d^5 - 4*a*b^3*c^4*d^6 + 6*a^2
*b^2*c^3*d^7 - 4*a^3*b*c^2*d^8 + a^4*c*d^9)*x^5 + 15*(b^4*c^6*d^4 - 4*a*b^3
*c^5*d^5 + 6*a^2*b^2*c^4*d^6 - 4*a^3*b*c^3*d^7 + a^4*c^2*d^8)*x^4 + 20*(b^4
*c^7*d^3 - 4*a*b^3*c^6*d^4 + 6*a^2*b^2*c^5*d^5 - 4*a^3*b*c^4*d^6 + a^4*c^3*
d^7)*x^3 + 15*(b^4*c^8*d^2 - 4*a*b^3*c^7*d^3 + 6*a^2*b^2*c^6*d^4 - 4*a^3*b*
c^5*d^5 + a^4*c^4*d^6)*x^2 + 6*(b^4*c^9*d - 4*a*b^3*c^8*d^2 + 6*a^2*b^2*c^7
*d^3 - 4*a^3*b*c^6*d^4 + a^4*c^5*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{37}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{13}{6}}\left(-1296b^3d^3x^3+2808a\,b^2d^3x^2-6696b^3c\,d^2x^2-4446a^2b\,d^3x+14508a\,b^2c\,d^2x-13950b^3c^2dx+6175a^3d^3-22971a^2bc\,d^2+30225a\,b^2c^2d-14725b^3c^3\right)}{191425\,(dx+c)^{\frac{31}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(7/6)/(d*x+c)^(37/6),x)
```

```
[Out] -6/191425*(b*x+a)^(13/6)*(-1296*b^3*d^3*x^3+2808*a*b^2*d^3*x^2-6696*b^3*c*d
^2*x^2-4446*a^2*b*d^3*x+14508*a*b^2*c*d^2*x-13950*b^3*c^2*d*x+6175*a^3*d^3-
22971*a^2*b*c*d^2+30225*a*b^2*c^2*d-14725*b^3*c^3)/(d*x+c)^(31/6)/(a^4*d^4-
4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{\frac{7}{6}}}{(dx+c)^{\frac{37}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(7/6)/(d*x+c)^(37/6),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(7/6)/(d*x + c)^(37/6), x)
```

mupad [B] time = 1.43, size = 385, normalized size = 2.83

$$\frac{(c+d\,x)^{5/6}\,\left(\frac{7776\,b^5\,x^5\,(a+b\,x)^{1/6}}{191425\,d^3\,(a\,d-b\,c)^4}-\frac{(a+b\,x)^{1/6}\,\left(37050\,a^5\,d^3-137826\,a^4\,b\,c\,d^2+181350\,a^3\,b^2\,c^2\,d-88350\,a^2\,b^3\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x^2\,(a+b\,x)^{1/6}\,\left(-546\,a^3\,b^2\,d^3+3906\,a^2\,b^3\,c\,d^2-13950\,a\,b^4\,c^2\,d+88350\,b^5\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{x\,(a+b\,x)^{1/6}\,\left(-47424\,a^4\,b\,d^3+188604\,a^3\,b^2\,c\,d^2-279000\,a^2\,b^3\,c^2\,d+176700\,a\,b^4\,c^3\right)}{191425\,d^6\,(a\,d-b\,c)^4}+\frac{108\,b^3\,x^3\,(a+b\,x)^{1/6}\,\left(7\,a^2\,d^2-62\,a\,b\,c\,d+775\,b^2\,c^2\right)}{191425\,d^5\,(a\,d-b\,c)^4}-\frac{1296\,b^4\,x^4\,(a\,d-31\,b\,c)\,(a+b\,x)^{1/6}}{191425\,d^4\,(a\,d-b\,c)^4}\right)}{x^6+\frac{c^6}{d^6}+\frac{6\,c\,x^5}{d}+\frac{6\,c^5\,x}{d^5}+\frac{15\,c^2\,x^4}{d^2}+\frac{20\,c^3\,x^3}{d^3}+\frac{15\,c^4\,x^2}{d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(7/6)/(c + d*x)^(37/6),x)
```

```
[Out] ((c + d*x)^(5/6)*((7776*b^5*x^5*(a + b*x)^(1/6))/(191425*d^3*(a*d - b*c)^4)
 - ((a + b*x)^(1/6)*(37050*a^5*d^3 - 88350*a^2*b^3*c^3 + 181350*a^3*b^2*c^2
*d - 137826*a^4*b*c*d^2))/(191425*d^6*(a*d - b*c)^4) + (x^2*(a + b*x)^(1/6)
*(88350*b^5*c^3 - 546*a^3*b^2*d^3 + 3906*a^2*b^3*c*d^2 - 13950*a*b^4*c^2*d)
)/(191425*d^6*(a*d - b*c)^4) + (x*(a + b*x)^(1/6)*(176700*a*b^4*c^3 - 47424
*a^4*b*d^3 - 279000*a^2*b^3*c^2*d + 188604*a^3*b^2*c*d^2))/(191425*d^6*(a*d
 - b*c)^4) + (108*b^3*x^3*(a + b*x)^(1/6)*(7*a^2*d^2 + 775*b^2*c^2 - 62*a*b
*c*d))/(191425*d^5*(a*d - b*c)^4) - (1296*b^4*x^4*(a*d - 31*b*c)*(a + b*x)^
(1/6))/(191425*d^4*(a*d - b*c)^4)))/(x^6 + c^6/d^6 + (6*c*x^5)/d + (6*c^5*x
)/d^5 + (15*c^2*x^4)/d^2 + (20*c^3*x^3)/d^3 + (15*c^4*x^2)/d^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(7/6)/(d*x+c)**(37/6),x)
```

```
[Out] Timed out
```

3.1533 $\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx$

Optimal. Leaf size=424

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)}{}$$

Rubi [A] time = 0.61, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{7(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^2}+\frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (7*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^2) + ((a + b*x)^(5/6)
*(c + d*x)^(7/6))/(2*b) + (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a
 + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(
5/6)) - (7*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sq
rt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c -
a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^
(13/6)*d^(5/6)) - (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*
b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3)
)/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(14
4*b^(13/6)*d^(5/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx &= \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{(7(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b} \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{72b^2} \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\cdots\right.}{12b^3} \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \cdots\right.}{12b^3} \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{\left(7(bc-ad)^2\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\cdots}\cdots\right.}{36b^{13/6}d^{2/3}} \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \cdots \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{13/6}d^{5/6}} - \cdots \\
&= \frac{7(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^2} + \frac{(a+bx)^{5/6}(c+dx)^{7/6}}{2b} + \frac{7(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}} - \cdots
\end{aligned}
$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.17

$$
\frac{6(a+bx)^{5/6}(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

IntegrateAlgebraic [A] time = 0.79, size = 363, normalized size = 0.86

$$
-\frac{7(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{36b^{13/6}d^{5/6}}+\frac{7(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{13/6}d^{5/6}}+\frac{(bc-ad)^2\left(\frac{13b(c+dx)^{7/6}}{(a+bx)^{7/6}}-\frac{7d\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\right)}{12b^2\left(\frac{b(c+dx)}{a+bx}-d\right)^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(7/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((b*c - a*d)^2*((-7*d*(c + d*x)^(1/6))/(a + b*x)^(1/6) + (13*b*(c + d*x)^(7
/6))/(a + b*x)^(7/6)))/(12*b^2*(-d + (b*(c + d*x))/(a + b*x))^2) - (7*(b*c
- a*d)^2*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a
```

```
 + b*x)^(1/6))])/(24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*ArcTan[1/
Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(
24*Sqrt[3]*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^
(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/(36*b^(13/6)*d^(5/6)) + (7*(b*c - a*d)^2
*ArcTanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(
1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(13/6)*d^(5/6))
```

fricas [B] time = 2.01, size = 5633, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] -1/144*(28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^
2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a
^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^
3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12*c*d^5 + a^2*b^11*d^6)*(
b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10
*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5
 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^
9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*
d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*sqrt(((b^4*c^2*d - 2*a*b^
3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d
^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/
(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*
b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d
^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a
^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(5/6) + sqrt(3
)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^
9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 7
92*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b
^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d
+ 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a
^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^
4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a
^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*c^10*d^2 -
220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5 + 924*a^7*b
^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^10*b^3*c^3*d
^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13*c^12 - 12*
a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 495*a^4*b^9*c
^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6*c^5*d^7 +
495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10 - 12*a^11*
b^2*c*d^11 + a^12*b*d^12)*x)) + 28*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^13*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^13*c^2*d^4 - 2*a*b^12
*c*d^5 + a^2*b^11*d^6)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
```

a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(b^13*d^5))^(5/6) - 2*sqrt(3)*(b^12*d^4*x + a*b^11*d^4)*s
qrt(-((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(
1/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^
9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 7
92*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^
2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)
*(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/3))/(b*x + a))*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2
*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^
7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 2
20*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(
b^13*d^5))^(5/6) - sqrt(3)*(a*b^12*c^12 - 12*a^2*b^11*c^11*d + 66*a^3*b^10*
c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^6*b^7*c^7*d^5
+ 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4*d^8 - 220*a^1
0*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^13*d^12 + (b^13
*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^10*c^9*d^3 + 49
5*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6 - 792*a^7*b^6
*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^10*b^3*c^2*d^10
- 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x))/(a*b^12*c^12 - 12*a^2*b^11*c^11*d
+ 66*a^3*b^10*c^10*d^2 - 220*a^4*b^9*c^9*d^3 + 495*a^5*b^8*c^8*d^4 - 792*a^
6*b^7*c^7*d^5 + 924*a^7*b^6*c^6*d^6 - 792*a^8*b^5*c^5*d^7 + 495*a^9*b^4*c^4
*d^8 - 220*a^10*b^3*c^3*d^9 + 66*a^11*b^2*c^2*d^10 - 12*a^12*b*c*d^11 + a^1
3*d^12 + (b^13*c^12 - 12*a*b^12*c^11*d + 66*a^2*b^11*c^10*d^2 - 220*a^3*b^1
0*c^9*d^3 + 495*a^4*b^9*c^8*d^4 - 792*a^5*b^8*c^7*d^5 + 924*a^6*b^7*c^6*d^6
- 792*a^7*b^6*c^5*d^7 + 495*a^8*b^5*c^4*d^8 - 220*a^9*b^4*c^3*d^9 + 66*a^1
0*b^3*c^2*d^10 - 12*a^11*b^2*c*d^11 + a^12*b*d^12)*x)) - 7*b^2*((b^12*c^12
- 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b
^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^
7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a
^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(49*((b^4*c^2*d - 2*a*b^3*c*
d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12 - 12*a*b^11*
c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 -
792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*
b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11
+ a^12*d^12)/(b^13*d^5))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*
d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*d^2*x
+ a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a
^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^
6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 6
6*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/3))/(b*x
+ a)) + 7*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6)*log(
-49*((b^4*c^2*d - 2*a*b^3*c*d^2 + a^2*b^2*d^3)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9
*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 79
2*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2
*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^13*d^5))^(1/6) - (b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*
(d*x + c)^(1/3) - (b^5*d^2*x + a*b^4*d^2)*((b^12*c^12 - 12*a*b^11*c^11*d +
66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*
b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d
^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d

```
^12)/(b^13*d^5))^(1/3))/(b*x + a)) - 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
5/6)*(d*x + c)^(1/6) + (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d +
 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5
*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*
d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*
d^12)/(b^13*d^5))^(1/6))/(b*x + a)) + 14*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^13*d^5))^(1/6)*log(7*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^
(5/6)*(d*x + c)^(1/6) - (b^3*d*x + a*b^2*d)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^13*d^5))^(1/6))/(b*x + a)) - 12*(6*b*d*x + 13*b*c - 7*a*d)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(1/6),x)
```

```
[Out] int((c + d*x)^(7/6)/(a + b*x)^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(1/6),x)
```

```
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(1/6), x)
```

3.1534 $\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}} \, dx$

Optimal. Leaf size=378

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\,\mathrm{ta}}{}$$

Rubi [A] time = 0.56, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}+\frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a + b*x)^(5/6)*(c + d*x)^(1/6))/b + ((b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^
(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/
6)*d^(5/6)) - ((b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(
Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(7/6)*d^(5/6)) + ((b*c - a*
d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(7/6)
*d^(5/6)) - ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(
5/6)) + ((b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3
) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(7/6)*d^(5/6)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx &= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx,x,\sqrt[6]{a+bx}\right)}{b^2}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx,x,\frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{2/3}}+\frac{(bc-ad)\,\mathrm{Subst}\Big(}{}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx\right.}{12b^{7/6}d^{5/6}}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{7/6}d^{5/6}}-\frac{(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}}{\sqrt[6]{c}}\right.}{12b^{7/6}d^{5/6}}\\
&= \frac{(a+bx)^{5/6}\sqrt[6]{c+dx}}{b}+\frac{(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}-\frac{(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}+\frac{(bc-}{}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 73, normalized size = 0.19

$$
\frac{6(a+bx)^{5/6}\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, 5/6, 11/6, (d*(a
 + b*x))/(-(b*c) + a*d)])/(5*b*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

IntegrateAlgebraic [A] time = 16.57, size = 388, normalized size = 1.03

$$
\frac{\sqrt[6]{ad+bdx}\left(-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}-2\sqrt[6]{ad+b(c+dx)-bc}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}{2\sqrt[6]{ad+b(c+dx)-bc}+\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{ad+b(c+dx)-bc}}\right)}{3b^{7/6}d^{5/6}}+\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt[3]{ad+b(c+dx)-bc}+\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}\,\sqrt[6]{ad+b(c+dx)-bc}}\right)}{6b^{7/6}d^{5/6}}+\frac{\sqrt[6]{c+dx}\,(ad+b(c+dx)-bc)^{5/6}}{bd^{5/6}}\right)}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/6)/(a + b*x)^(1/6),x]
```

```
[Out] ((a*d + b*d*x)^(1/6)*(((c + d*x)^(1/6)*(-(b*c) + a*d + b*(c + d*x))^(5/6))/
(b*d^(5/6)) - ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/6
)*(c + d*x)^(1/6) - 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(7
/6)*d^(5/6)) + ((b*c - a*d)*ArcTan[(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))/(b^(1/
6)*(c + d*x)^(1/6) + 2*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(2*Sqrt[3]*b^(
7/6)*d^(5/6)) + ((b*c - a*d)*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(-(b*c) + a*
```

```
d + b*(c + d*x))^(1/6)])/(3*b^(7/6)*d^(5/6)) + ((b*c - a*d)*ArcTanh[(b^(1/3
)*(c + d*x)^(1/3) + (-(b*c) + a*d + b*(c + d*x))^(1/3))/(b^(1/6)*(c + d*x)^
(1/6)*(-(b*c) + a*d + b*(c + d*x))^(1/6))])/(6*b^(7/6)*d^(5/6))))/(d^(1/6)*
(a + b*x)^(1/6))
```

fricas [B] time = 1.85, size = 3025, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b
^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)
*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b*x + a)^(5/6)*(d*x + c)^(1
/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 1
5*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + 2*sqrt(3)*(
b^7*d^4*x + a*b^6*d^4)*sqrt(((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^
(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^3*d^2*x + a*b^2
*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +
 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a))
*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a
^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(5/6) + sqrt(3)*(a*b^6
*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b
^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^
5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6
*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20*a^4*b^3*
c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6 - 6*a*b^6
*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*d^4 - 6*a
^5*b^2*c*d^5 + a^6*b*d^6)*x)) + 4*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*
a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 +
 a^6*d^6)/(b^7*d^5))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^7*c*d^4 - a*b^6*d^5)*(b
*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^
2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7
*d^5))^(5/6) + 2*sqrt(3)*(b^7*d^4*x + a*b^6*d^4)*sqrt(-((b^2*c*d - a*b*d^2)
*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)
^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4
*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(
b^7*d^5))^(1/3))/(b*x + a))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2
- 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d
^5))^(5/6) - sqrt(3)*(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^3*b^4*c^4*d^2 - 20
*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a^7*d^6 + (b^7*c^6
- 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3 + 15*a^4*b^3*c^2*
d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x))/(a*b^6*c^6 - 6*a^2*b^5*c^5*d + 15*a^
3*b^4*c^4*d^2 - 20*a^4*b^3*c^3*d^3 + 15*a^5*b^2*c^2*d^4 - 6*a^6*b*c*d^5 + a
^7*d^6 + (b^7*c^6 - 6*a*b^6*c^5*d + 15*a^2*b^5*c^4*d^2 - 20*a^3*b^4*c^3*d^3
 + 15*a^4*b^3*c^2*d^4 - 6*a^5*b^2*c*d^5 + a^6*b*d^6)*x)) + b*((b^6*c^6 - 6*
a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4
- 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(((b^2*c*d - a*b*d^2)*(b*x +
 a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 -
20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5
))^(1/3))/(b*x + a)) - b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 2
0*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5)
```

```
)^(1/6)*log(-((b^2*c*d - a*b*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) - (b^3*d^2*x + a*b^2*d^2)*((b^6*c^6
 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2
*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/3))/(b*x + a)) + 2*b*((b^6*c^
6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^
2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6) + (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15
*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
+ a^6*d^6)/(b^7*d^5))^(1/6))/(b*x + a)) - 2*b*((b^6*c^6 - 6*a*b^5*c^5*d + 1
5*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5
 + a^6*d^6)/(b^7*d^5))^(1/6)*log(-((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1
/6) - (b^2*d*x + a*b*d)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^7*d^5))
^(1/6))/(b*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
[Out] int((d*x+c)^(1/6)/(b*x+a)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(1/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(1/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(1/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(1/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(1/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(1/6), x)
```

3.1535 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Rubi [A] time = 0.51, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2\sqrt[6]{b}\,d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c
 + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) - (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^(5/6)) + (
2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(1/6)*d^
(5/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)
*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6)) + Log[b^(1/3
) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^
(1/6))/(c + d*x)^(1/6)]/(2*b^(1/6)*d^(5/6))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{d^{2/3}} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{2/3}} + \cdots$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\text{Subst}\left(\int \frac{-\sqrt[6]{b}\,\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}} + \frac{\text{Subst}\left(\int \frac{\sqrt[6]{b}\,\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\,\cdots}\cdots\right)}{2\cdots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2\sqrt[6]{b}\,d^{5/6}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}\cdots\right.}{2\sqrt[6]{b}\,d^{5/6}\cdots}$$

$$= \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\sqrt{3}\,\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{\sqrt[6]{b}\,d^{5/6}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{\sqrt[6]{b}\,d^{5/6}} - \frac{\log\left(\sqrt[3]{b}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.05, size = 73, normalized size = 0.24

$$\frac{6(a+bx)^{5/6}\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6} {}_2F_1\left(\frac{5}{6},\frac{5}{6};\frac{11}{6};\frac{d(a+bx)}{ad-bc}\right)}{5b(c+dx)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*((b*(c + d*x))/(b*c - a*d))^(5/6)*Hypergeometric2F1[5/6,
 5/6, 11/6, (d*(a + b*x))/(-(b*c) + a*d)])/(5*b*(c + d*x)^(5/6))
```

IntegrateAlgebraic [A] time = 0.22, size = 233, normalized size = 0.75

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{\sqrt[6]{b}d^{5/6}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[6]{b}d^{5/6}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{\sqrt[6]{b}d^{5/6}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{\sqrt[6]{b}d^{5/6}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*
(a + b*x)^(1/6))]])/(b^(1/6)*d^(5/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1
/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))]])/(b^(1/6)*d^(5/6))
+ (2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(1/6)*(a + b*x)^(1/6))]])/(b^(1/6)
*d^(5/6)) + ArcTanh[(b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(
1/3) + (b^(1/3)*(c + d*x)^(1/3))/(a + b*x)^(1/3)))]]/(b^(1/6)*d^(5/6))
```

fricas [B] time = 0.93, size = 620, normalized size = 2.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x +
c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqrt(3)*(b^2*d^4*x + a*b*d^4)*sqrt(((b
*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/6) + (b*d^2*x + a*d^2)*(1/(b
*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6
) + sqrt(3)*(b*x + a))/(b*x + a)) - 2*sqrt(3)*(1/(b*d^5))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b*d^4*(1/(b*d^5))^(5/6) - 2*sqr
t(3)*(b^2*d^4*x + a*b*d^4)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d
^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3)*(d*x + c
)^(1/3))/(b*x + a))*(1/(b*d^5))^(5/6) - sqrt(3)*(b*x + a))/(b*x + a)) + 1/2
*(1/(b*d^5))^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*d*(1/(b*d^5))^(1/
6) + (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))
/(b*x + a)) - 1/2*(1/(b*d^5))^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)
*d*(1/(b*d^5))^(1/6) - (b*d^2*x + a*d^2)*(1/(b*d^5))^(1/3) - (b*x + a)^(2/3
)*(d*x + c)^(1/3))/(b*x + a)) + (1/(b*d^5))^(1/6)*log(((b*d*x + a*d)*(1/(b*
d^5))^(1/6) + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) - (1/(b*d^5))^(1/
6)*log(-((b*d*x + a*d)*(1/(b*d^5))^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))
/(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

```
[Out] int(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}} (dx+c)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(5/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{1/6}\,(c+d\,x)^{5/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)),x)
```

```
[Out] int(1/((a + b*x)^(1/6)*(c + d*x)^(5/6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(5/6),x)
```

```
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(5/6)), x)
```

3.1536 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=32

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx = \frac{6(a+bx)^{5/6}}{5(bc-ad)(c+dx)^{5/6}}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

IntegrateAlgebraic [A] time = 0.04, size = 32, normalized size = 1.00

$$\frac{6(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(5*(b*c - a*d)*(c + d*x)^(5/6))
```

fricas [A] time = 0.70, size = 42, normalized size = 1.31

$$\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\left(bc^2-acd+\left(bcd-ad^2\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="fricas")
[Out] 6/5*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.84

$$-\frac{6\,(bx+a)^{\frac{5}{6}}}{5\,(dx+c)^{\frac{5}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x)
[Out] -6/5*(b*x+a)^(5/6)/(d*x+c)^(5/6)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(11/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(11/6)), x)
```

mupad [B] time = 0.76, size = 27, normalized size = 0.84

$$-\frac{6\,{\left(a+b\,x\right)}^{5/6}}{\left(5\,a\,d-5\,b\,c\right)\,{\left(c+d\,x\right)}^{5/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(11/6)),x)
[Out] -(6*(a + b*x)^(5/6))/((5*a*d - 5*b*c)*(c + d*x)^(5/6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(11/6),x)
[Out] Integral(1/((a + b*x)**(1/6)*(c + d*x)**(11/6)), x)
```

3.1537 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(11*(b*c - a*d)*(c + d*x)^(11/6)) + (36*b*(a + b*x)^(5/
6))/(55*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx &= \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{(6b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{11(bc-ad)}\\ &= \frac{6(a+bx)^{5/6}}{11(bc-ad)(c+dx)^{11/6}}+\frac{36b(a+bx)^{5/6}}{55(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.70

$$\frac{6(a+bx)^{5/6}(-5ad+11bc+6bdx)}{55(c+dx)^{11/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(11*b*c - 5*a*d + 6*b*d*x))/(55*(b*c - a*d)^2*(c + d*x)^
(11/6))
```

IntegrateAlgebraic [A] time = 0.11, size = 51, normalized size = 0.77

$$\frac{6(a+bx)^{11/6}\left(\frac{11b(c+dx)}{a+bx}-5d\right)}{55(c+dx)^{11/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (6*(a + b*x)^(11/6)*(-5*d + (11*b*(c + d*x))/(a + b*x)))/(55*(b*c - a*d)^2*
(c + d*x)^(11/6))
```

fricas [B] time = 1.38, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+11\,bc-5\,ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\,\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\,\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] 6/55*(6*b*d*x + 11*b*c - 5*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^2*c^4 -
2*a*b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(
b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

maple [A] time = 0.00, size = 54, normalized size = 0.82

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\,(-6bdx+5ad-11bc)}{55\,(dx+c)^{\frac{11}{6}}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/55*(b*x+a)^(5/6)*(-6*b*d*x+5*a*d-11*b*c)/(d*x+c)^(11/6)/(a^2*d^2-2*a*b*c
*d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(17/6)), x)
```

mupad [B] time = 0.86, size = 127, normalized size = 1.92

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{x\,\left(66\,c\,b^2+6\,a\,d\,b\right)}{55\,d^2\,{\left(a\,d-b\,c\right)}^2}-\frac{30\,a^2\,d-66\,a\,b\,c}{55\,d^2\,{\left(a\,d-b\,c\right)}^2}+\frac{36\,b^2\,x^2}{55\,d\,{\left(a\,d-b\,c\right)}^2}\right)}{x^2\,{\left(a+b\,x\right)}^{1/6}+\frac{c^2\,{\left(a+b\,x\right)}^{1/6}}{d^2}+\frac{2\,c\,x\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((x*(66*b^2*c + 6*a*b*d))/(55*d^2*(a*d - b*c)^2) - (30*a^2
*d - 66*a*b*c)/(55*d^2*(a*d - b*c)^2) + (36*b^2*x^2)/(55*d*(a*d - b*c)^2)))
/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(a + b*x)^(1/6))
/d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

3.1538 $\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^3}+\frac{72b(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(17*(b*c - a*d)*(c + d*x)^(17/6)) + (72*b*(a + b*x)^(5/
6))/(187*(b*c - a*d)^2*(c + d*x)^(11/6)) + (432*b^2*(a + b*x)^(5/6))/(935*(
b*c - a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx &= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{(12b)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}}\,dx}{17(bc-ad)}\\
&= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{\left(72b^2\right)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{187(bc-ad)^2}\\
&= \frac{6(a+bx)^{5/6}}{17(bc-ad)(c+dx)^{17/6}}+\frac{72b(a+bx)^{5/6}}{187(bc-ad)^2(c+dx)^{11/6}}+\frac{432b^2(a+bx)^{5/6}}{935(bc-ad)^3(c+dx)^{5/6}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 0.76

$$\frac{6(a+bx)^{5/6}\left(55a^2d^2-10abd(17c+6dx)+b^2\left(187c^2+204cdx+72d^2x^2\right)\right)}{935(c+dx)^{17/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(55*a^2*d^2 - 10*a*b*d*(17*c + 6*d*x) + b^2*(187*c^2 + 2
04*c*d*x + 72*d^2*x^2)))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

IntegrateAlgebraic [A] time = 0.12, size = 73, normalized size = 0.72

$$\frac{6(a+bx)^{17/6}\left(\frac{187b^2(c+dx)^2}{(a+bx)^2}-\frac{170bd(c+dx)}{a+bx}+55d^2\right)}{935(c+dx)^{17/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (6*(a + b*x)^(17/6)*(55*d^2 - (170*b*d*(c + d*x))/(a + b*x) + (187*b^2*(c +
 d*x)^2)/(a + b*x)^2))/(935*(b*c - a*d)^3*(c + d*x)^(17/6))
```

fricas [B] time = 1.37, size = 252, normalized size = 2.50

$$\frac{6\left(72\,b^2d^2x^2+187\,b^2c^2-170\,abcd+55\,a^2d^2+12\left(17\,b^2cd-5\,abd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2+3\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] 6/935*(72*b^2*d^2*x^2 + 187*b^2*c^2 - 170*a*b*c*d + 55*a^2*d^2 + 12*(17*b^2
*c*d - 5*a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^3*c^6 - 3*a*b^2*c^5
*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2
*b*c*d^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^
4 - a^3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3
*c^2*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\left(72b^2x^2d^2-60ab\,d^2x+204b^2cdx+55a^2d^2-170abcd+187b^2c^2\right)}{935\,(dx+c)^{\frac{17}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(b*x+a)^(5/6)*(72*b^2*d^2*x^2-60*a*b*d^2*x+204*b^2*c*d*x+55*a^2*d^2-
170*a*b*c*d+187*b^2*c^2)/(d*x+c)^(17/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*
d-b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(23/6)), x)
```

mupad [B] time = 1.03, size = 203, normalized size = 2.01

$$-\frac{(c+d\,x)^{1/6}\,\left(\frac{330\,a^3\,d^2-1020\,a^2\,b\,c\,d+1122\,a\,b^2\,c^2}{935\,d^3\,(a\,d-b\,c)^3}+\frac{x\,\left(-30\,a^2\,b\,d^2+204\,a\,b^2\,c\,d+1122\,b^3\,c^2\right)}{935\,d^3\,(a\,d-b\,c)^3}+\frac{432\,b^3\,x^3}{935\,d\,(a\,d-b\,c)^3}+\frac{72\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d^2\,(a\,d-b\,c)^3}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((330*a^3*d^2 + 1122*a*b^2*c^2 - 1020*a^2*b*c*d)/(935*d^3
*(a*d - b*c)^3) + (x*(1122*b^3*c^2 - 30*a^2*b*d^2 + 204*a*b^2*c*d))/(935*d^
3*(a*d - b*c)^3) + (432*b^3*x^3)/(935*d*(a*d - b*c)^3) + (72*b^2*x^2*(a*d +
 17*b*c))/(935*d^2*(a*d - b*c)^3)))/(x^3*(a + b*x)^(1/6) + (c^3*(a + b*x)^(
1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*x)^(1/6))/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

3.1539 $\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{29/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3(a+bx)^{5/6}}{21505(c+dx)^{5/6}(bc-ad)^4}+\frac{1296b^2(a+bx)^{5/6}}{4301(c+dx)^{11/6}(bc-ad)^3}+\frac{108b(a+bx)^{5/6}}{391(c+dx)^{17/6}(bc-ad)^2}+\frac{6(a+bx)^{5/6}}{23(c+dx)^{23/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6))/(23*(b*c - a*d)*(c + d*x)^(23/6)) + (108*b*(a + b*x)^(5
/6))/(391*(b*c - a*d)^2*(c + d*x)^(17/6)) + (1296*b^2*(a + b*x)^(5/6))/(430
1*(b*c - a*d)^3*(c + d*x)^(11/6)) + (7776*b^3*(a + b*x)^(5/6))/(21505*(b*c
- a*d)^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{29/6}}\,dx &= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{(18b)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{23/6}}\,dx}{23(bc-ad)}\\ &= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{\left(216b^2\right)\int \frac{1}{\sqrt[6]{a+bx}(c+dx)^{17/6}}\,dx}{391(bc-ad)^2}\\ &= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}}\\ &= \frac{6(a+bx)^{5/6}}{23(bc-ad)(c+dx)^{23/6}}+\frac{108b(a+bx)^{5/6}}{391(bc-ad)^2(c+dx)^{17/6}}+\frac{1296b^2(a+bx)^{5/6}}{4301(bc-ad)^3(c+dx)^{11/6}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 118, normalized size = 0.87

$$\frac{6(a+bx)^{5/6}\left(-935a^3d^3+165a^2bd^2(23c+6dx)-15ab^2d\left(391c^2+276cdx+72d^2x^2\right)+b^3\left(4301c^3+7038c^2dx+4968cd^2x^2+1296d^3x^3\right)\right)}{21505(c+dx)^{23/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(5/6)*(-935*a^3*d^3 + 165*a^2*b*d^2*(23*c + 6*d*x) - 15*a*b^2*
d*(391*c^2 + 276*c*d*x + 72*d^2*x^2) + b^3*(4301*c^3 + 7038*c^2*d*x + 4968*
c*d^2*x^2 + 1296*d^3*x^3)))/(21505*(b*c - a*d)^4*(c + d*x)^(23/6))
```

IntegrateAlgebraic [A] time = 0.12, size = 95, normalized size = 0.70

$$\frac{6(a+bx)^{23/6}\left(\frac{4301b^3(c+dx)^3}{(a+bx)^3}-\frac{5865b^2d(c+dx)^2}{(a+bx)^2}+\frac{3795bd^2(c+dx)}{a+bx}-935d^3\right)}{21505(c+dx)^{23/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x]
```

```
[Out] (6*(a + b*x)^(23/6)*(-935*d^3 + (3795*b*d^2*(c + d*x))/(a + b*x) - (5865*b^
2*d*(c + d*x)^2)/(a + b*x)^2 + (4301*b^3*(c + d*x)^3)/(a + b*x)^3))/(21505*
(b*c - a*d)^4*(c + d*x)^(23/6))
```

fricas [B] time = 1.44, size = 420, normalized size = 3.09

$$\frac{6\left(1296\,b^3d^3x^3+4301\,b^3c^3-5865\,ab^2c^2d+3795\,a^2bcd^2-935\,a^3d^3+216\left(23\,b^3cd^2-5\,ab^2d^3\right)x^2+18\left(391\,b^3c^2d-230\,ab^2cd^2+55\,a^2bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{21505\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7\right)x^3+6\left(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6\right)x^2+4\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="fricas")
```

```
[Out] 6/21505*(1296*b^3*d^3*x^3 + 4301*b^3*c^3 - 5865*a*b^2*c^2*d + 3795*a^2*b*c*
d^2 - 935*a^3*d^3 + 216*(23*b^3*c*d^2 - 5*a*b^2*d^3)*x^2 + 18*(391*b^3*c^2*
d - 230*a*b^2*c*d^2 + 55*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(b^4
*c^8 - 4*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 +
(b^4*c^4*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^
8)*x^4 + 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2
*d^6 + a^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^
4 - 4*a^3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6
*a^2*b^2*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{5}{6}}\left(-1296b^3d^3x^3+1080a\,b^2d^3x^2-4968b^3c\,d^2x^2-990a^2b\,d^3x+4140a\,b^2c\,d^2x-7038b^3c^2dx+935a^3d^3-3795a^2bc\,d^2+5865a\,b^2c^2d-4301b^3c^3\right)}{21505\,(dx+c)^{\frac{23}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x)
```

```
[Out] -6/21505*(b*x+a)^(5/6)*(-1296*b^3*d^3*x^3+1080*a*b^2*d^3*x^2-4968*b^3*c*d^2
*x^2-990*a^2*b*d^3*x+4140*a*b^2*c*d^2*x-7038*b^3*c^2*d*x+935*a^3*d^3-3795*a
^2*b*c*d^2+5865*a*b^2*c^2*d-4301*b^3*c^3)/(d*x+c)^(23/6)/(a^4*d^4-4*a^3*b*c
*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{29}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/6)/(d*x+c)^(29/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(1/6)*(d*x + c)^(29/6)), x)
```

mupad [B] time = 1.20, size = 292, normalized size = 2.15

$$\frac{(c+d\,x)^{1/6}\left(\frac{7776\,b^4\,x^4}{21505\,d\,(a\,d-b\,c)^4}-\frac{5610\,a^4\,d^3-22770\,a^3\,b\,c\,d^2+35190\,a^2\,b^2\,c^2\,d-25806\,a\,b^3\,c^3}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{x\,\left(330\,a^3\,b\,d^3-2070\,a^2\,b^2\,c\,d^2+7038\,a\,b^3\,c^2\,d+25806\,b^4\,c^3\right)}{21505\,d^4\,(a\,d-b\,c)^4}+\frac{1296\,b^3\,x^3\,(a\,d+23\,b\,c)}{21505\,d^2\,(a\,d-b\,c)^4}+\frac{108\,b^2\,x^2\,\left(-5\,a^2\,d^2+46\,a\,b\,c\,d+391\,b^2\,c^2\right)}{21505\,d^3\,(a\,d-b\,c)^4}\right)}{x^4\,(a+b\,x)^{1/6}+\frac{c^4\,(a+b\,x)^{1/6}}{d^4}+\frac{6\,c^2\,x^2\,(a+b\,x)^{1/6}}{d^2}+\frac{4\,c\,x^3\,(a+b\,x)^{1/6}}{d}+\frac{4\,c^3\,x\,(a+b\,x)^{1/6}}{d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/6)*(c + d*x)^(29/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((7776*b^4*x^4)/(21505*d*(a*d - b*c)^4) - (5610*a^4*d^3 -
25806*a*b^3*c^3 + 35190*a^2*b^2*c^2*d - 22770*a^3*b*c*d^2)/(21505*d^4*(a*d
- b*c)^4) + (x*(25806*b^4*c^3 + 330*a^3*b*d^3 - 2070*a^2*b^2*c*d^2 + 7038*a
*b^3*c^2*d))/(21505*d^4*(a*d - b*c)^4) + (1296*b^3*x^3*(a*d + 23*b*c))/(215
05*d^2*(a*d - b*c)^4) + (108*b^2*x^2*(391*b^2*c^2 - 5*a^2*d^2 + 46*a*b*c*d)
)/(21505*d^3*(a*d - b*c)^4)))/(x^4*(a + b*x)^(1/6) + (c^4*(a + b*x)^(1/6))/
d^4 + (6*c^2*x^2*(a + b*x)^(1/6))/d^2 + (4*c*x^3*(a + b*x)^(1/6))/d + (4*c^
3*x*(a + b*x)^(1/6))/d^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/6)/(d*x+c)**(29/6),x)
```

```
[Out] Timed out
```

3.1540 $\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}} \, dx$

Optimal. Leaf size=424

$$-\frac{55(bc-ad)^2 \log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2 \log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}-\frac{55(bc-$$

Rubi [A] time = 0.55, antiderivative size = 424, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$\frac{11\sqrt[6]{a+bx}(c+dx)^{5/6}(bc-ad)}{12b^2}-\frac{55(bc-ad)^2 \log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2 \log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{17/6}\sqrt[6]{d}}-\frac{55(bc-ad)^2 \tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2 \tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}b^{17/6}\sqrt[6]{d}}+\frac{55(bc-ad)^2 \tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}(c+dx)^{11/6}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (11*(b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6))/(12*b^2) + ((a + b*x)^(1/6
)*(c + d*x)^(11/6))/(2*b) - (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)
*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*
d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))
/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b
*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(
36*b^(17/6)*d^(1/6)) - (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(
1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])
/(144*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x
)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6
)])/(144*b^(17/6)*d^(1/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{(11(bc-ad))\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx}{12b}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{72b^2}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}} \ldots\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x \ldots\right.}{12b^3}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{\left(55(bc-ad)^2\right)\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d} \ldots}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x \ldots} \ldots\right.}{36b^{17/6}}$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} + \ldots$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} - \ldots$$

$$= \frac{11(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{12b^2} + \frac{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}{2b} - \frac{55(bc-ad)^2\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}} + \ldots$$

Mathematica [C] time = 0.06, size = 71, normalized size = 0.17

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{11/6}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{11/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(11/6)*Hypergeometric2F1[-11/6, 1/6, 7/6, (d*(
a + b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(11/6))
```

IntegrateAlgebraic [A] time = 0.43, size = 356, normalized size = 0.84

$$-\frac{55(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}} + \frac{55(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{17/6}\sqrt[6]{d}} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{36b^{17/6}\sqrt[6]{d}} + \frac{55(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{72b^{17/6}\sqrt[6]{d}} + \frac{\sqrt[6]{a+bx}\,(bc-ad)^2\left(17b-\frac{11d(a+bx)}{c+dx}\right)}{12b^2\sqrt[6]{c+dx}\left(b-\frac{d(a+bx)}{c+dx}\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(11/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1/6)*(17*b - (11*d*(a + b*x))/(c + d*x)))/(12*b^2
*(c + d*x)^(1/6)*(b - (d*(a + b*x))/(c + d*x))^2) - (55*(b*c - a*d)^2*ArcTa
n[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))
```

```
])/(24*Sqrt[3]*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^
(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1
/6)*(c + d*x)^(1/6))])/(36*b^(17/6)*d^(1/6)) + (55*(b*c - a*d)^2*ArcTanh[(b
^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(1/3) + (d^(1/3)*(a + b
*x)^(1/3))/(c + d*x)^(1/3)))])/(72*b^(17/6)*d^(1/6))
```

fricas [B] time = 2.18, size = 5591, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

```
[Out] -1/144*(220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d
^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*
a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c
^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/
6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*x
 + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^
10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b
^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))
^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(((b^5*c^2 - 2*a*b^4*c*d + a
^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^6*d*x + b^6*c)*(
(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3
+ 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7
*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*
d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c^1
2 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4
*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*
d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12
*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) + sqrt(3)*(b^12*c^13 - 12*a*b^1
1*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*d^
4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495*a
^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c^2
*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924
*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*
c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12*c
^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*
a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c
^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 -
 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*
a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7
*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
- 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^1
3)*x)) + 220*sqrt(3)*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*
d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924
*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*
c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1
/6)*arctan(-1/3*(2*sqrt(3)*(b^16*c^2*d - 2*a*b^15*c*d^2 + a^2*b^14*d^3)*(b*
x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c
^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 +
 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*
```

b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d)
)^(5/6) - 2*sqrt(3)*(b^14*d^2*x + b^14*c*d)*sqrt(-((b^5*c^2 - 2*a*b^4*c*d +
a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*
d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*
a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c
^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^
12*d^12)/(b^17*d))^(1/6) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)
*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^
3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a
^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^
2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/3))/(d*x + c))*((b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(5/6) - sqrt(3)*(b^12*c^13 - 12*a*b
^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 495*a^4*b^8*c^9*
d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5*c^6*d^7 + 495
*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10 - 12*a^11*b*c
^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^1
0*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 9
24*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^
3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)*x))/(b^12
*c^13 - 12*a*b^11*c^12*d + 66*a^2*b^10*c^11*d^2 - 220*a^3*b^9*c^10*d^3 + 49
5*a^4*b^8*c^9*d^4 - 792*a^5*b^7*c^8*d^5 + 924*a^6*b^6*c^7*d^6 - 792*a^7*b^5
*c^6*d^7 + 495*a^8*b^4*c^5*d^8 - 220*a^9*b^3*c^4*d^9 + 66*a^10*b^2*c^3*d^10
- 12*a^11*b*c^2*d^11 + a^12*c*d^12 + (b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)*x)) - 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 2
20*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^
6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6)*log
(3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6
)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d
^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*
a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c
^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) + (b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(1/3)*(d*x
+ c)^(2/3) + (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17
*d))^(1/3))/(d*x + c)) + 55*b^2*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^1
0*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^
5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a
^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17
*d))^(1/6)*log(-3025*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(1/6)
*(d*x + c)^(5/6)*((b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 22
0*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6
*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9
+ 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(b^17*d))^(1/6) - (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x
+ a)^(1/3)*(d*x + c)^(2/3) - (b^6*d*x + b^6*c)*((b^12*c^12 - 12*a*b^11*c^11
*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792
*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*
c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a
^12*d^12)/(b^17*d))^(1/3))/(d*x + c)) - 110*b^2*((b^12*c^12 - 12*a*b^11*c^1

```
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a
)^(1/6)*(d*x + c)^(5/6) + (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d
+ 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^
5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4
*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12
*d^12)/(b^17*d))^(1/6))/(d*x + c)) + 110*b^2*((b^12*c^12 - 12*a*b^11*c^11*d
 + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a
^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^
4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^1
2*d^12)/(b^17*d))^(1/6)*log(55*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(
1/6)*(d*x + c)^(5/6) - (b^3*d*x + b^3*c)*((b^12*c^12 - 12*a*b^11*c^11*d + 6
6*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b
^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^
8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^
12)/(b^17*d))^(1/6))/(d*x + c)) - 12*(6*b*d*x + 17*b*c - 11*a*d)*(b*x + a)^
(1/6)*(d*x + c)^(5/6))/b^2
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

```
[Out] int((d*x+c)^(11/6)/(b*x+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{11}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(11/6)/(b*x+a)^(5/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(11/6)/(b*x + a)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{11/6}}{(a+b\,x)^{5/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(11/6)/(a + b*x)^(5/6),x)
```

```
[Out] int((c + d*x)^(11/6)/(a + b*x)^(5/6), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(11/6)/(b*x+a)**(5/6),x)
```

```
[Out] Timed out
```

3.1541 $\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx$

Optimal. Leaf size=378

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad}{}$$

Rubi [A] time = 0.48, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {50, 63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{5(bc-ad)\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{11/6}\sqrt[6]{d}}-\frac{5(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{2\sqrt{3}b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}b^{11/6}\sqrt[6]{d}}+\frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}}+\frac{\sqrt[6]{a+bx}(c+dx)^{5/6}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] ((a + b*x)^(1/6)*(c + d*x)^(5/6))/b - (5*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*
d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(
11/6)*d^(1/6)) + (5*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/
6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(11/6)*d^(1/6)) + (5*(
b*c - a*d)*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3
*b^(11/6)*d^(1/6)) - (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))
/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*
b^(11/6)*d^(1/6)) + (5*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/
(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b
^(11/6)*d^(1/6))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx = \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx}{6b}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{3b^{11/6}} + \frac{(5(bc-ad))}{}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} + \frac{(5(bc-ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+}\right.}{4b^{5/3}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} + \frac{5(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{11/6}\sqrt[6]{d}} - \frac{5(bc-ad)\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}} - \right.}{12b^{11/6}\sqrt[6]{d}}$$

$$= \frac{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}{b} - \frac{5(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5(bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{11/6}\sqrt[6]{d}} + \frac{5}{}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.19

$$\frac{6\sqrt[6]{a+bx}\,(c+dx)^{5/6}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\left(\frac{b(c+dx)}{bc-ad}\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(c + d*x)^(5/6)*Hypergeometric2F1[-5/6, 1/6, 7/6, (d*(a
+ b*x))/(-(b*c) + a*d)])/(b*((b*(c + d*x))/(b*c - a*d))^(5/6))
```

IntegrateAlgebraic [F] time = 106.34, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(5/6)/(a + b*x)^(5/6),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^(5/6)/(a + b*x)^(5/6), x]
```

fricas [B] time = 1.83, size = 2997, normalized size = 7.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="fricas")
```

[Out] 1/12*(20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 20*sqrt(3)*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*arctan(1/3*(2*sqrt(3)*(b^10*c*d - a*b^9*d^2)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) + 2*sqrt(3)*(b^9*d^2*x + b^9*c*d)*sqrt(-((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c))*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(5/6) - sqrt(3)*(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x))/(b^6*c^7 - 6*a*b^5*c^6*d + 15*a^2*b^4*c^5*d^2 - 20*a^3*b^3*c^4*d^3 + 15*a^4*b^2*c^3*d^4 - 6*a^5*b*c^2*d^5 + a^6*c*d^6 + (b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)*x)) + 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) + (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) - 5*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-25*((b^3*c - a*b^2*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(1/3)*(d*x + c)^(2/3) - (b^4*d*x + b^4*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/3))/(d*x + c)) + 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) + (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) - 10*b*((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 +

```
15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6)*log(-5*((b*c
 - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6) - (b^2*d*x + b^2*c)*((b^6*c^6 - 6*a
*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 -
 6*a^5*b*c*d^5 + a^6*d^6)/(b^11*d))^(1/6))/(d*x + c)) + 12*(b*x + a)^(1/6)*
(d*x + c)^(5/6))/b
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="giac")
[Out] Timed out
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
[Out] int((d*x+c)^(5/6)/(b*x+a)^(5/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{5}{6}}}{(bx+a)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/6)/(b*x+a)^(5/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(5/6)/(b*x + a)^(5/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{5/6}}{(a+b\,x)^{5/6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(5/6)/(a + b*x)^(5/6),x)
[Out] int((c + d*x)^(5/6)/(a + b*x)^(5/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{5}{6}}}{(a+bx)^{\frac{5}{6}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/6)/(b*x+a)**(5/6),x)
[Out] Integral((c + d*x)**(5/6)/(a + b*x)**(5/6), x)
```

3.1542 $\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx$

Optimal. Leaf size=309

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+$$

Rubi [A] time = 0.44, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {63, 240, 210, 634, 618, 204, 628, 208}

$$-\frac{\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}+\frac{\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{5/6}\sqrt[6]{d}}-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) - Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1
/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6)) + Log[b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*
x)^(1/6))/(c + d*x)^(1/6)]/(2*b^(5/6)*d^(1/6))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}\sqrt[6]{c+dx}}\,dx = \frac{6\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{c-\frac{ad}{b}+\frac{dx^6}{b}}}\,dx, x, \sqrt[6]{a+bx}\right)}{b}$$

$$= \frac{6\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{5/6}} + \frac{2\,\text{Subst}\left(\int \frac{\sqrt[6]{b}+\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}+\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \ldots\right)}{b^{5/6}}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{2/3}} + \frac{3\,\text{Subst}\left(\int \ldots\right)}{\ldots}$$

$$= \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{5/6}\sqrt[6]{d}} + \frac{\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+\ldots}}{\sqrt[3]{c+dx}}\ldots\right)}{2b^{5/6}\ldots}$$

$$= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}} + \frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}} - \frac{\log \ldots}{\ldots}$$

Mathematica [C] time = 0.04, size = 71, normalized size = 0.23

$$\frac{6\sqrt[6]{a+bx}\,\sqrt[6]{\frac{b(c+dx)}{bc-ad}}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6)*Hypergeometric2F1[1/6,
 1/6, 7/6, (d*(a + b*x))/(-(b*c) + a*d)])/(b*(c + d*x)^(1/6))
```

IntegrateAlgebraic [A] time = 0.22, size = 233, normalized size = 0.75

$$-\frac{\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{2\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{5/6}\sqrt[6]{d}}+\frac{\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\left(\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}\right)}{b^{5/6}\sqrt[6]{d}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x]
```

```
[Out] -((Sqrt[3]*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*
(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))) + (Sqrt[3]*ArcTan[1/Sqrt[3] + (2*d^(1
/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)*d^(1/6))
+ (2*ArcTanh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(b^(5/6)
*d^(1/6)) + ArcTanh[(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/((c + d*x)^(1/6)*(b^(
1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3)))]/(b^(5/6)*d^(1/6))
```

fricas [B] time = 1.68, size = 620, normalized size = 2.01

$$-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^4d^2x+b^4cd\right)\sqrt{\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}+\sqrt{3}(dx+c)}{3(dx+c)}\right)-2\sqrt{3}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b^4d\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^4d^2x+b^4cd\right)\sqrt{-\frac{(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}}{dx+c}}\left(\frac{1}{b^5d}\right)^{\frac{5}{6}}-\sqrt{3}(dx+c)}{3(dx+c)}\right)+\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}+(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)-\frac{1}{2}\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}b\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-\left(b^2dx+b^2c\right)\left(\frac{1}{b^5d}\right)^{\frac{1}{3}}-(bx+a)^{\frac{1}{3}}(dx+c)^{\frac{2}{3}}\right)}{dx+c}\right)+\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}+(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)-\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}\log\left(-\frac{(bdx+bc)\left(\frac{1}{b^5d}\right)^{\frac{1}{6}}-(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{dx+c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="fricas")
```

```
[Out] -2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x +
c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqrt(3)*(b^4*d^2*x + b^4*c*d)*sqrt(((b
*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/6) + (b^2*d*x + b^2*c)*(1/(b
^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6
) + sqrt(3)*(d*x + c))/(d*x + c)) - 2*sqrt(3)*(1/(b^5*d))^(1/6)*arctan(-1/3
*(2*sqrt(3)*(b*x + a)^(1/6)*(d*x + c)^(5/6)*b^4*d*(1/(b^5*d))^(5/6) - 2*sqr
t(3)*(b^4*d^2*x + b^4*c*d)*sqrt(-((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5
*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3)*(d*x + c
)^(2/3))/(d*x + c))*(1/(b^5*d))^(5/6) - sqrt(3)*(d*x + c))/(d*x + c)) + 1/2
*(1/(b^5*d))^(1/6)*log(4*((b*x + a)^(1/6)*(d*x + c)^(5/6)*b*(1/(b^5*d))^(1/
6) + (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) + (b*x + a)^(1/3)*(d*x + c)^(2/3))
/(d*x + c)) - 1/2*(1/(b^5*d))^(1/6)*log(-4*((b*x + a)^(1/6)*(d*x + c)^(5/6)
*b*(1/(b^5*d))^(1/6) - (b^2*d*x + b^2*c)*(1/(b^5*d))^(1/3) - (b*x + a)^(1/3
)*(d*x + c)^(2/3))/(d*x + c)) + (1/(b^5*d))^(1/6)*log(((b*d*x + b*c)*(1/(b^
5*d))^(1/6) + (b*x + a)^(1/6)*(d*x + c)^(5/6))/(d*x + c)) - (1/(b^5*d))^(1/
6)*log(-((b*d*x + b*c)*(1/(b^5*d))^(1/6) - (b*x + a)^(1/6)*(d*x + c)^(5/6))
/(d*x + c))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}\,(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

```
[Out] int(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(1/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(1/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{1/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(1/6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\sqrt[6]{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(1/6),x)
```

```
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(1/6)), x)
```

3.1543 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx$

Optimal. Leaf size=30

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx = \frac{6\sqrt[6]{a+bx}}{(bc-ad)\sqrt[6]{c+dx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x]
[Out] (6*(a + b*x)^(1/6))/((b*c - a*d)*(c + d*x)^(1/6))
```

fricas [A] time = 1.35, size = 42, normalized size = 1.40

$$\frac{6\,(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{bc^2-acd+\left(bcd-ad^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="fricas")
[Out] 6*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b*c^2 - a*c*d + (b*c*d - a*d^2)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

maple [A] time = 0.00, size = 27, normalized size = 0.90

$$-\frac{6\,(bx+a)^{\frac{1}{6}}}{(dx+c)^{\frac{1}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x)
[Out] -6*(b*x+a)^(1/6)/(d*x+c)^(1/6)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(7/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(7/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{{(a+b\,x)}^{5/6}\,{(c+d\,x)}^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(7/6)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{5}{6}}\,(c+dx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(7/6),x)
[Out] Integral(1/((a + b*x)**(5/6)*(c + d*x)**(7/6)), x)
```

3.1544 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx$

Optimal. Leaf size=66

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{36b\sqrt[6]{a+bx}}{7\sqrt[6]{c+dx}\,(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{7(c+dx)^{7/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(7*(b*c - a*d)*(c + d*x)^(7/6)) + (36*b*(a + b*x)^(1/6)
)/(7*(b*c - a*d)^2*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx &= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{(6b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{7(bc-ad)}\\ &= \frac{6\sqrt[6]{a+bx}}{7(bc-ad)(c+dx)^{7/6}}+\frac{36b\sqrt[6]{a+bx}}{7(bc-ad)^2\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 0.70

$$\frac{6\sqrt[6]{a+bx}\,(-ad+7bc+6bdx)}{7(c+dx)^{7/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*b*c - a*d + 6*b*d*x))/(7*(b*c - a*d)^2*(c + d*x)^(7/6
))
```

IntegrateAlgebraic [A] time = 0.12, size = 57, normalized size = 0.86

$$\frac{6\left(\frac{7b\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}-\frac{d(a+bx)^{7/6}}{(c+dx)^{7/6}}\right)}{7(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x]
```

```
[Out] (6*(-((d*(a + b*x)^(7/6))/(c + d*x)^(7/6)) + (7*b*(a + b*x)^(1/6))/(c + d*x
)^(1/6)))/(7*(b*c - a*d)^2)
```

fricas [B] time = 1.44, size = 118, normalized size = 1.79

$$\frac{6\,(6\,bdx+7\,bc-ad)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{7\left(b^2c^4-2\,abc^3d+a^2c^2d^2+\left(b^2c^2d^2-2\,abcd^3+a^2d^4\right)x^2+2\left(b^2c^3d-2\,abc^2d^2+a^2cd^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="fricas")
```

```
[Out] 6/7*(6*b*d*x + 7*b*c - a*d)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^2*c^4 - 2*a*
b*c^3*d + a^2*c^2*d^2 + (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*x^2 + 2*(b^2*
c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.80

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\,(-6bdx+ad-7bc)}{7\,(dx+c)^{\frac{7}{6}}\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x)
```

```
[Out] -6/7*(b*x+a)^(1/6)*(-6*b*d*x+a*d-7*b*c)/(d*x+c)^(7/6)/(a^2*d^2-2*a*b*c*d+b^
2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{13}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(13/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(13/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(13/6)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(13/6),x)
```

```
[Out] Timed out
```

3.1545 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx$

Optimal. Leaf size=101

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{432b^2\sqrt[6]{a+bx}}{91\sqrt[6]{c+dx}\,(bc-ad)^3}+\frac{72b\sqrt[6]{a+bx}}{91(c+dx)^{7/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{13(c+dx)^{13/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(13*(b*c - a*d)*(c + d*x)^(13/6)) + (72*b*(a + b*x)^(1/
6))/(91*(b*c - a*d)^2*(c + d*x)^(7/6)) + (432*b^2*(a + b*x)^(1/6))/(91*(b*c
 - a*d)^3*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx &= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{(12b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,dx}{13(bc-ad)}\\ &= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{\left(72b^2\right)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{7/6}}\,dx}{91(bc-ad)^2}\\ &= \frac{6\sqrt[6]{a+bx}}{13(bc-ad)(c+dx)^{13/6}}+\frac{72b\sqrt[6]{a+bx}}{91(bc-ad)^2(c+dx)^{7/6}}+\frac{432b^2\sqrt[6]{a+bx}}{91(bc-ad)^3\sqrt[6]{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.76

$$\frac{6\sqrt[6]{a+bx}\left(7a^2d^2-2abd(13c+6dx)+b^2\left(91c^2+156cdx+72d^2x^2\right)\right)}{91(c+dx)^{13/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(7*a^2*d^2 - 2*a*b*d*(13*c + 6*d*x) + b^2*(91*c^2 + 156*
c*d*x + 72*d^2*x^2)))/(91*(b*c - a*d)^3*(c + d*x)^(13/6))
```

IntegrateAlgebraic [A] time = 0.12, size = 83, normalized size = 0.82

$$\frac{6\left(\frac{91b^2\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{7d^2(a+bx)^{13/6}}{(c+dx)^{13/6}}-\frac{26bd(a+bx)^{7/6}}{(c+dx)^{7/6}}\right)}{91(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x]
```

```
[Out] (6*((7*d^2*(a + b*x)^(13/6))/(c + d*x)^(13/6) - (26*b*d*(a + b*x)^(7/6))/(c
 + d*x)^(7/6) + (91*b^2*(a + b*x)^(1/6))/(c + d*x)^(1/6)))/(91*(b*c - a*d)^
3)
```

fricas [B] time = 1.24, size = 252, normalized size = 2.50

$$\frac{6\left(72\,b^2d^2x^2+91\,b^2c^2-26\,abcd+7\,a^2d^2+12\left(13\,b^2cd-abd^2\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{91\left(b^3c^6-3\,ab^2c^5d+3\,a^2bc^4d^2-a^3c^3d^3+\left(b^3c^3d^3-3\,ab^2c^2d^4+3\,a^2bcd^5-a^3d^6\right)x^3+3\left(b^3c^4d^2-3\,ab^2c^3d^3+3\,a^2bc^2d^4-a^3cd^5\right)x^2+3\left(b^3c^5d-3\,ab^2c^4d^2+3\,a^2bc^3d^3-a^3c^2d^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="fricas")
```

```
[Out] 6/91*(72*b^2*d^2*x^2 + 91*b^2*c^2 - 26*a*b*c*d + 7*a^2*d^2 + 12*(13*b^2*c*d
 - a*b*d^2)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^3*c^6 - 3*a*b^2*c^5*d + 3
*a^2*b*c^4*d^2 - a^3*c^3*d^3 + (b^3*c^3*d^3 - 3*a*b^2*c^2*d^4 + 3*a^2*b*c*d
^5 - a^3*d^6)*x^3 + 3*(b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^
3*c*d^5)*x^2 + 3*(b^3*c^5*d - 3*a*b^2*c^4*d^2 + 3*a^2*b*c^3*d^3 - a^3*c^2*d
^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.04

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\left(72b^2x^2d^2-12ab\,d^2x+156b^2cdx+7a^2d^2-26abcd+91b^2c^2\right)}{91\,(dx+c)^{\frac{13}{6}}\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x)
```

```
[Out] -6/91*(b*x+a)^(1/6)*(72*b^2*d^2*x^2-12*a*b*d^2*x+156*b^2*c*d*x+7*a^2*d^2-26
*a*b*c*d+91*b^2*c^2)/(d*x+c)^(13/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^
3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{19}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(19/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(19/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{19/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)),x)
```

```
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(19/6)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(19/6),x)
```

```
[Out] Timed out
```

3.1546 $\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx$

Optimal. Leaf size=136

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{7776b^3\sqrt[6]{a+bx}}{1729\sqrt[6]{c+dx}\,(bc-ad)^4}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(c+dx)^{7/6}(bc-ad)^3}+\frac{108b\sqrt[6]{a+bx}}{247(c+dx)^{13/6}(bc-ad)^2}+\frac{6\sqrt[6]{a+bx}}{19(c+dx)^{19/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6))/(19*(b*c - a*d)*(c + d*x)^(19/6)) + (108*b*(a + b*x)^(1
/6))/(247*(b*c - a*d)^2*(c + d*x)^(13/6)) + (1296*b^2*(a + b*x)^(1/6))/(172
9*(b*c - a*d)^3*(c + d*x)^(7/6)) + (7776*b^3*(a + b*x)^(1/6))/(1729*(b*c -
a*d)^4*(c + d*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{5/6}(c+dx)^{25/6}}\,dx &= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{(18b)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{19/6}}\,dx}{19(bc-ad)}\\ &= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{\left(216b^2\right)\int \frac{1}{(a+bx)^{5/6}(c+dx)^{13/6}}\,d}{247(bc-ad)^2}\\ &= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}+\\ &= \frac{6\sqrt[6]{a+bx}}{19(bc-ad)(c+dx)^{19/6}}+\frac{108b\sqrt[6]{a+bx}}{247(bc-ad)^2(c+dx)^{13/6}}+\frac{1296b^2\sqrt[6]{a+bx}}{1729(bc-ad)^3(c+dx)^{7/6}}+\end{aligned}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.87

$$\frac{6\sqrt[6]{a+bx}\left(-91a^3d^3+21a^2bd^2(19c+6dx)-3ab^2d\left(247c^2+228cdx+72d^2x^2\right)+b^3\left(1729c^3+4446c^2dx+4104cd^2x^2+1296d^3x^3\right)\right)}{1729(c+dx)^{19/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*(a + b*x)^(1/6)*(-91*a^3*d^3 + 21*a^2*b*d^2*(19*c + 6*d*x) - 3*a*b^2*d*(
247*c^2 + 228*c*d*x + 72*d^2*x^2) + b^3*(1729*c^3 + 4446*c^2*d*x + 4104*c*d
^2*x^2 + 1296*d^3*x^3)))/(1729*(b*c - a*d)^4*(c + d*x)^(19/6))
```

IntegrateAlgebraic [A] time = 0.13, size = 109, normalized size = 0.80

$$\frac{6\left(\frac{1729b^3\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}-\frac{741b^2d(a+bx)^{7/6}}{(c+dx)^{7/6}}-\frac{91d^3(a+bx)^{19/6}}{(c+dx)^{19/6}}+\frac{399bd^2(a+bx)^{13/6}}{(c+dx)^{13/6}}\right)}{1729(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x]
```

```
[Out] (6*((-91*d^3*(a + b*x)^(19/6))/(c + d*x)^(19/6) + (399*b*d^2*(a + b*x)^(13/
6))/(c + d*x)^(13/6) - (741*b^2*d*(a + b*x)^(7/6))/(c + d*x)^(7/6) + (1729*
b^3*(a + b*x)^(1/6))/(c + d*x)^(1/6)))/(1729*(b*c - a*d)^4)
```

fricas [B] time = 1.11, size = 420, normalized size = 3.09

$$\frac{6\left(1296\,b^3d^3x^3+1729\,b^3c^3-741\,ab^2c^2d+399\,a^2bcd^2-91\,a^3d^3+216\left(19\,b^3cd^2-ab^2d^3\right)x^2+18\left(247\,b^3c^2d-38\,ab^2cd^2+7\,a^2bd^3\right)x\right)(bx+a)^{\frac{1}{6}}(dx+c)^{\frac{5}{6}}}{1729\left(b^4c^8-4\,ab^3c^7d+6\,a^2b^2c^6d^2-4\,a^3bc^5d^3+a^4c^4d^4+\left(b^4c^4d^4-4\,ab^3c^3d^5+6\,a^2b^2c^2d^6-4\,a^3bcd^7+a^4d^8\right)x^4+4\left(b^4c^5d^3-4\,ab^3c^4d^4+6\,a^2b^2c^3d^5-4\,a^3bc^2d^6+a^4cd^7\right)x^3+6\left(b^4c^6d^2-4\,ab^3c^5d^3+6\,a^2b^2c^4d^4-4\,a^3bc^3d^5+a^4c^2d^6\right)x^2+4\left(b^4c^7d-4\,ab^3c^6d^2+6\,a^2b^2c^5d^3-4\,a^3bc^4d^4+a^4c^3d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="fricas")
```

```
[Out] 6/1729*(1296*b^3*d^3*x^3 + 1729*b^3*c^3 - 741*a*b^2*c^2*d + 399*a^2*b*c*d^2
 - 91*a^3*d^3 + 216*(19*b^3*c*d^2 - a*b^2*d^3)*x^2 + 18*(247*b^3*c^2*d - 38
*a*b^2*c*d^2 + 7*a^2*b*d^3)*x)*(b*x + a)^(1/6)*(d*x + c)^(5/6)/(b^4*c^8 - 4
*a*b^3*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (b^4*c^4
*d^4 - 4*a*b^3*c^3*d^5 + 6*a^2*b^2*c^2*d^6 - 4*a^3*b*c*d^7 + a^4*d^8)*x^4 +
 4*(b^4*c^5*d^3 - 4*a*b^3*c^4*d^4 + 6*a^2*b^2*c^3*d^5 - 4*a^3*b*c^2*d^6 + a
^4*c*d^7)*x^3 + 6*(b^4*c^6*d^2 - 4*a*b^3*c^5*d^3 + 6*a^2*b^2*c^4*d^4 - 4*a^
3*b*c^3*d^5 + a^4*c^2*d^6)*x^2 + 4*(b^4*c^7*d - 4*a*b^3*c^6*d^2 + 6*a^2*b^2
*c^5*d^3 - 4*a^3*b*c^4*d^4 + a^4*c^3*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.26

$$-\frac{6\,(bx+a)^{\frac{1}{6}}\left(-1296b^3d^3x^3+216a\,b^2d^3x^2-4104b^3c\,d^2x^2-126a^2b\,d^3x+684a\,b^2c\,d^2x-4446b^3c^2dx+91a^3d^3-399a^2bc\,d^2+741a\,b^2c^2d-1729b^3c^3\right)}{1729\,(dx+c)^{\frac{19}{6}}\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x)
```

```
[Out] -6/1729*(b*x+a)^(1/6)*(-1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2-4104*b^3*c*d^2*x
^2-126*a^2*b*d^3*x+684*a*b^2*c*d^2*x-4446*b^3*c^2*d*x+91*a^3*d^3-399*a^2*b*
```

```
c*d^2+741*a*b^2*c^2*d-1729*b^3*c^3)/(d*x+c)^(19/6)/(a^4*d^4-4*a^3*b*c*d^3+6
*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{25}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(5/6)/(d*x+c)^(25/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(5/6)*(d*x + c)^(25/6)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\,x)^{5/6}\,(c+d\,x)^{25/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)),x)
[Out] int(1/((a + b*x)^(5/6)*(c + d*x)^(25/6)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(5/6)/(d*x+c)**(25/6),x)
[Out] Timed out
```

3.1547 $\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=449

$$-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right.}{144b^{19/6}}$$

Rubi [A] time = 0.66, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2}+\frac{91d(a+bx)^{5/6}\sqrt[6]{c+dx}\,(bc-ad)}{12b^3}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{144b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{24\sqrt{3}\,b^{19/6}}-\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{36b^{19/6}}-\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (91*d*(b*c - a*d)*(a + b*x)^(5/6)*(c + d*x)^(1/6))/(12*b^3) + (13*d*(a + b*
x)^(5/6)*(c + d*x)^(7/6))/(2*b^2) - (6*(c + d*x)^(13/6))/(b*(a + b*x)^(1/6)
) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(1/6)
)/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) - (91*d^(1/6)*(
b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcT
anh[(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(36*b^(19/6)) - (
91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^
(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6))
+ (91*d^(1/6)*(b*c - a*d)^2*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*
x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(144*b^(19/6
))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(13d)\int \frac{(c+dx)^{7/6}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{(91d(bc-ad))\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{12b^2}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\big(91d(bc-\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\big(91d(bc-\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\big(91d(bc-\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{\big(91\sqrt[3]{d}\,(bc\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-\cdots}{\cdots}$$

$$= \frac{91d(bc-ad)(a+bx)^{5/6}\sqrt[6]{c+dx}}{12b^3} + \frac{13d(a+bx)^{5/6}(c+dx)^{7/6}}{2b^2} - \frac{6(c+dx)^{13/6}}{b\sqrt[6]{a+bx}} + \frac{91\sqrt[6]{d}\,(bc-\cdots}{\cdots}$$

Mathematica [C] time = 0.08, size = 71, normalized size = 0.16

$$-\frac{6(c+dx)^{13/6}\,{}_2F_1\left(-\frac{13}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{13/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(13/6)*Hypergeometric2F1[-13/6, -1/6, 5/6, (d*(a + b*x))/(-(b
*c) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(13/6))
```

IntegrateAlgebraic [A] time = 0.81, size = 389, normalized size = 0.87

$$-\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tan^{-1}\left(\frac{2\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt{3}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{24\sqrt{3}\,b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{c+dx}}{\sqrt[6]{d}\,\sqrt[6]{a+bx}}\right)}{36b^{19/6}}+\frac{91\sqrt[6]{d}\,(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\,\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{72b^{19/6}}-\frac{(bc-ad)^2\left(\frac{72b^2(c+dx)^{13/6}}{(a+bx)^{13/6}}+\frac{91d^2\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}-\frac{169bd(c+dx)^{7/6}}{(a+bx)^{7/6}}\right)}{12b^3\left(\frac{b(c+dx)}{a+bx}-d\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(13/6)/(a + b*x)^(7/6),x]
```

```
[Out] -1/12*((b*c - a*d)^2*((91*d^2*(c + d*x)^(1/6))/(a + b*x)^(1/6) - (169*b*d*(
c + d*x)^(7/6))/(a + b*x)^(7/6) + (72*b^2*(c + d*x)^(13/6))/(a + b*x)^(13/6
)))/(b^3*(-d + (b*(c + d*x))/(a + b*x))^2) - (91*d^(1/6)*(b*c - a*d)^2*ArcT
an[1/Sqrt[3] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6)
)])/(24*Sqrt[3]*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTan[1/Sqrt[3] + (2
*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/(24*Sqrt[3]*b
^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTanh[(b^(1/6)*(c + d*x)^(1/6))/(d^(
1/6)*(a + b*x)^(1/6))])/(36*b^(19/6)) + (91*d^(1/6)*(b*c - a*d)^2*ArcTanh[(
b^(1/6)*d^(1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c +
d*x)^(1/3))/(a + b*x)^(1/3)))])/(72*b^(19/6))
```

fricas [B] time = 2.22, size = 5690, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/144*(364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66
*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^
7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9
 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^
13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2 - 2*a*b^17*c*d + a^2*b^16*
d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 6
6*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b
^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^
9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d
^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sqrt(((b^5*c^2 - 2*a*b^4*c*d
 + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c
^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5
- 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8
*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^
12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^7*x + a*b^6
)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c
^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 -
792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*
b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/(b*x + a))*((b^12
*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 +
 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*
b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*
d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) + sqrt(3)*(a*b^12*c^12*d -
 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^
5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5
*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 -
 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*
b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^
7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 2
20*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^
13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 220*
a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^6*c
^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^10
+ 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 12*a
*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b^9*
c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^8 +
 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*a^1
1*b^2*c*d^12 + a^12*b*d^13)*x)) + 364*sqrt(3)*(b^4*x + a*b^3)*((b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^
4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5
*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
```

12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b^18*c^2
- 2*a*b^17*c*d + a^2*b^16*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^12*
d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a
^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^
5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 -
12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6) - 2*sqrt(3)*(b^17*x + a*b^16)*sq
rt(-((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*
((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9
*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 79
2*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^
2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) - (b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x
+ c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^
10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d
^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*
a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^1
9)^(1/3))/(b*x + a))*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*
d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924
*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*
c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(5/6)
- sqrt(3)*(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 + 66*a^3*b^10*c^10*d^3 - 2
20*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*b^7*c^7*d^6 + 924*a^7*b^
6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d^9 - 220*a^10*b^3*c^3*d^
10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13*d^13 + (b^13*c^12*d - 1
2*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*b^10*c^9*d^4 + 495*a^4*b
^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*d^7 - 792*a^7*b^6*c^5*d^
8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66*a^10*b^3*c^2*d^11 - 12*
a^11*b^2*c*d^12 + a^12*b*d^13)*x))/(a*b^12*c^12*d - 12*a^2*b^11*c^11*d^2 +
66*a^3*b^10*c^10*d^3 - 220*a^4*b^9*c^9*d^4 + 495*a^5*b^8*c^8*d^5 - 792*a^6*
b^7*c^7*d^6 + 924*a^7*b^6*c^6*d^7 - 792*a^8*b^5*c^5*d^8 + 495*a^9*b^4*c^4*d
^9 - 220*a^10*b^3*c^3*d^10 + 66*a^11*b^2*c^2*d^11 - 12*a^12*b*c*d^12 + a^13
*d^13 + (b^13*c^12*d - 12*a*b^12*c^11*d^2 + 66*a^2*b^11*c^10*d^3 - 220*a^3*
b^10*c^9*d^4 + 495*a^4*b^9*c^8*d^5 - 792*a^5*b^8*c^7*d^6 + 924*a^6*b^7*c^6*
d^7 - 792*a^7*b^6*c^5*d^8 + 495*a^8*b^5*c^4*d^9 - 220*a^9*b^4*c^3*d^10 + 66
*a^10*b^3*c^2*d^11 - 12*a^11*b^2*c*d^12 + a^12*b*d^13)*x)) - 91*(b^4*x + a*
b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^
9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7
- 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^
10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(8281*((b^5*
c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^12*c^1
2*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
- 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) + (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*x + a)^(2/3)*(d*x + c)^(1/3)
+ (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^
3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a
^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^
3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/3))/
(b*x + a)) + 91*(b^4*x + a*b^3)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2
*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^
7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 2
20*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/
b^19)^(1/6)*log(-8281*((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*(b*x + a)^(5/6
)*(d*x + c)^(1/6)*((b^12*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3
- 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^
6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3
*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*(b*

```
x + a)^(2/3)*(d*x + c)^(1/3) - (b^7*x + a*b^6)*((b^12*c^12*d - 12*a*b^11*c^
11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*c^8*d^5 -
 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 + 495*a^8*
b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^11*b*c*d^1
2 + a^12*d^13)/b^19)^(1/3))/(b*x + a)) - 182*(b^4*x + a*b^3)*((b^12*c^12*d
 - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4
*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*
d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 1
2*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^4*x + a*b^3)*((b^12*c^12*d - 12*
a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495*a^4*b^8*
c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*c^5*d^8 +
 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11 - 12*a^1
1*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) + 182*(b^4*x + a*b^3)*((b^1
2*c^12*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4
+ 495*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7
*b^5*c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2
*d^11 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6)*log(91*((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^4*x + a*b^3)*((b^12*c^1
2*d - 12*a*b^11*c^11*d^2 + 66*a^2*b^10*c^10*d^3 - 220*a^3*b^9*c^9*d^4 + 495
*a^4*b^8*c^8*d^5 - 792*a^5*b^7*c^7*d^6 + 924*a^6*b^6*c^6*d^7 - 792*a^7*b^5*
c^5*d^8 + 495*a^8*b^4*c^4*d^9 - 220*a^9*b^3*c^3*d^10 + 66*a^10*b^2*c^2*d^11
 - 12*a^11*b*c*d^12 + a^12*d^13)/b^19)^(1/6))/(b*x + a)) - 12*(6*b^2*d^2*x^
2 - 72*b^2*c^2 + 169*a*b*c*d - 91*a^2*d^2 + (25*b^2*c*d - 13*a*b*d^2)*x)*(b
*x + a)^(5/6)*(d*x + c)^(1/6))/(b^4*x + a*b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(13/6)/(b*x+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{13}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(13/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(13/6)/(b*x + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d\,x)^{13/6}}{(a + b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(13/6)/(a + b*x)^(7/6),x)
```

```
[Out] int((c + d*x)^(13/6)/(a + b*x)^(7/6), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(13/6)/(b*x+a)**(7/6),x)
```

```
[Out] Timed out
```

3.1548 $\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=403

$$-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}}{}$$

Rubi [A] time = 0.60, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {47, 50, 63, 331, 296, 634, 618, 204, 628, 208}

$$\frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2}-\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(-\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\log\left(\frac{\sqrt[6]{b}\,\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\,\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{12b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{2\sqrt{3}\,b^{13/6}}-\frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt{3}\,\sqrt[6]{b}\,\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}}+\frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}}-\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (7*d*(a + b*x)^(5/6)*(c + d*x)^(1/6))/b^2 - (6*(c + d*x)^(7/6))/(b*(a + b*x
)^(1/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTan[1/Sqrt[3] - (2*d^(1/6)*(a + b*x)^(
1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) - (7*d^(1/6)
*(b*c - a*d)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6
)*(c + d*x)^(1/6))])/(2*Sqrt[3]*b^(13/6)) + (7*d^(1/6)*(b*c - a*d)*ArcTanh[
(d^(1/6)*(a + b*x)^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/(3*b^(13/6)) - (7*d^(
1/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) -
(b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6)) + (7*d^(1
/6)*(b*c - a*d)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) + (
b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(12*b^(13/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx = -\frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d)\int \frac{\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}}\,dx}{b}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{6b^2}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{(7d(bc-ad))\,\mathrm{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^3}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{\left(7\sqrt[3]{d}\,(bc-ad)\right)\mathrm{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\,\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+b}}{\sqrt[6]{c+c}}\right.}{3b^{13/6}}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{\left(7\sqrt[3]{d}\,(bc-ad)\right)\mathrm{S}}{}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tanh^{-1}\left(\frac{\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}\right)}{3b^{13/6}} - \frac{7\sqrt[6]{d}\,(bc-ad)\log}{}$$

$$= \frac{7d(a+bx)^{5/6}\sqrt[6]{c+dx}}{b^2} - \frac{6(c+dx)^{7/6}}{b\sqrt[6]{a+bx}} + \frac{7\sqrt[6]{d}\,(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\,\sqrt[6]{a+bx}}{\sqrt[6]{b}\,\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{2\sqrt{3}\,b^{13/6}} - \frac{7\sqrt[6]{d}\,(bc-ad)\tan}{2\sqrt{3}}$$

Mathematica [C] time = 0.06, size = 71, normalized size = 0.18

$$-\frac{6(c+dx)^{7/6}\,{}_2F_1\left(-\frac{7}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\left(\frac{b(c+dx)}{bc-ad}\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(7/6)*Hypergeometric2F1[-7/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(7/6))
```

IntegrateAlgebraic [F] time = 156.14, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{7/6}}{(a+bx)^{7/6}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(7/6)/(a + b*x)^(7/6),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^(7/6)/(a + b*x)^(7/6), x]
```

fricas [B] time = 2.01, size = 3084, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] 1/12*(28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4
*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^
7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*c - a*b^11*d)*(b*x + a)^(5/6)*(d
*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + 2*
sqrt(3)*(b^12*x + a*b^11)*sqrt(((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)
^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*
d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2
- 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((
b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*
a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a))*((b^6*c^
6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^
2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(5/6) + sqrt(3)*(a*b^6*c^6*d - 6
*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2
*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*
c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b
*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^
3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a
*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5
 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 28*sqrt(3)*(b^3*x + a*b^2)*((b^6*c^6*
d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*
c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*arctan(1/3*(2*sqrt(3)*(b^12*
c - a*b^11*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c
*d^6 + a^6*d^7)/b^13)^(5/6) + 2*sqrt(3)*(b^12*x + a*b^11)*sqrt(-((b^3*c - a
*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*
a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 +
 a^6*d^7)/b^13)^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*
x + c)^(1/3) - (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/3))/(b*x + a))*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(5/6) - sqrt(3)*(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2 + 15*a^3*b^4*c^4*d^3 - 20
*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c*d^6 + a^7*d^7 + (b^7*c^6*
d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3*b^4*c^3*d^4 + 15*a^4*b^3*
c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x))/(a*b^6*c^6*d - 6*a^2*b^5*c^5*d^2
 + 15*a^3*b^4*c^4*d^3 - 20*a^4*b^3*c^3*d^4 + 15*a^5*b^2*c^2*d^5 - 6*a^6*b*c
*d^6 + a^7*d^7 + (b^7*c^6*d - 6*a*b^6*c^5*d^2 + 15*a^2*b^5*c^4*d^3 - 20*a^3
*b^4*c^3*d^4 + 15*a^4*b^3*c^2*d^5 - 6*a^5*b^2*c*d^6 + a^6*b*d^7)*x)) + 7*(b
^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b
^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(
49*((b^3*c - a*b^2*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5
*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6
*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x
+ a)^(2/3)*(d*x + c)^(1/3) + (b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2
+ 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*
d^6 + a^6*d^7)/b^13)^(1/3))/(b*x + a)) - 7*(b^3*x + a*b^2)*((b^6*c^6*d - 6*
a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^
5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6)*log(-49*((b^3*c - a*b^2*d)*(b*x +
a)^(5/6)*(d*x + c)^(1/6)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3
 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)
^(1/6) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(b*x + a)^(2/3)*(d*x + c)^(1/3) -
(b^5*x + a*b^4)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3
*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/3))/(
```

```
b*x + a)) + 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c
^4*d^3 - 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)
/b^13)^(1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) + (b^3*x +
 a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3
*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)
) - 14*(b^3*x + a*b^2)*((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 -
 20*a^3*b^3*c^3*d^4 + 15*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(
1/6)*log(-7*((b*c - a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6) - (b^3*x + a*b^2)*
((b^6*c^6*d - 6*a*b^5*c^5*d^2 + 15*a^2*b^4*c^4*d^3 - 20*a^3*b^3*c^3*d^4 + 1
5*a^4*b^2*c^2*d^5 - 6*a^5*b*c*d^6 + a^6*d^7)/b^13)^(1/6))/(b*x + a)) + 12*(
b*d*x - 6*b*c + 7*a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^3*x + a*b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="giac")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
[Out] int((d*x+c)^(7/6)/(b*x+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{7}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(7/6)/(b*x+a)^(7/6),x, algorithm="maxima")
[Out] integrate((d*x + c)^(7/6)/(b*x + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{7/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(7/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(7/6)/(a + b*x)^(7/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{\frac{7}{6}}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(7/6)/(b*x+a)**(7/6),x)
```

```
[Out] Integral((c + d*x)**(7/6)/(a + b*x)**(7/6), x)
```

3.1549 $\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx$

Optimal. Leaf size=332

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{a}}{\sqrt{3}\sqrt[6]{}}\right.}{b^{7/6}}$$

Rubi [A] time = 0.54, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {47, 63, 331, 296, 634, 618, 204, 628, 208}

$$-\frac{\sqrt[6]{d}\log\left(-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt[6]{d}\log\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}+\sqrt[3]{b}\right)}{2b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt{3}\sqrt[6]{b}\sqrt[6]{c+dx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6)
 - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*d^(1/6)*(a + b*x)^(1/6))/(Sqrt[3]
*b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) + (2*d^(1/6)*ArcTanh[(d^(1/6)*(a + b*x)
^(1/6))/(b^(1/6)*(c + d*x)^(1/6))])/b^(7/6) - (d^(1/6)*Log[b^(1/3) + (d^(1/
3)*(a + b*x)^(1/3))/(c + d*x)^(1/3) - (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c
+ d*x)^(1/6)])/(2*b^(7/6)) + (d^(1/6)*Log[b^(1/3) + (d^(1/3)*(a + b*x)^(1/3
))/(c + d*x)^(1/3) + (b^(1/6)*d^(1/6)*(a + b*x)^(1/6))/(c + d*x)^(1/6)])/(2
*b^(7/6))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{7/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{d\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}}\,dx}{b}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{\left(c-\frac{ad}{b}+\frac{dx^6}{b}\right)^{5/6}}\,dx, x, \sqrt[6]{a+bx}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{(6d)\,\text{Subst}\left(\int \frac{x^4}{1-\frac{dx^6}{b}}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^2}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}-\frac{\sqrt[6]{d}\,x}{2}}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{b^{7/6}} + \frac{\left(2\sqrt[3]{d}\right)\text{Subst}\left(\int \frac{-\frac{\sqrt[6]{b}}{2}+\cdots}{\sqrt[3]{b}+\sqrt[6]{b}\sqrt[6]{d}\,x\cdots}\right.}{b^{7/6}}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\,\text{Subst}\left(\int \frac{-\sqrt[6]{b}\sqrt[6]{d}+2\sqrt[3]{d}\,x}{\sqrt[3]{b}-\sqrt[6]{b}\sqrt[6]{d}\,x+\sqrt[3]{d}\,x^2}\,dx, x, \frac{\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \frac{\sqrt[6]{d}\cdots}{\cdots}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}\right)}{b^{7/6}} - \frac{\sqrt[6]{d}\log\left(\sqrt[3]{b}+\frac{\sqrt[3]{d}\sqrt[3]{a+bx}}{\sqrt[3]{c+dx}}-\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{c+dx}}\right)}{2b^{7/6}} + \frac{\sqrt[6]{d}\log\cdots}{\cdots}$$

$$= -\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}} + \frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1-\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} - \frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1+\frac{2\sqrt[6]{d}\sqrt[6]{a+bx}}{\sqrt[6]{b}\sqrt[6]{c+dx}}}{\sqrt{3}}\right)}{b^{7/6}} + \frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{d}\cdots}{\sqrt[6]{b}\cdots}\right.}{b^{7/6}}$$

Mathematica [C] time = 0.03, size = 71, normalized size = 0.21

$$-\frac{6\sqrt[6]{c+dx}\;{}_2F_1\left(-\frac{1}{6},-\frac{1}{6};\frac{5}{6};\frac{d(a+bx)}{ad-bc}\right)}{b\sqrt[6]{a+bx}\sqrt[6]{\frac{b(c+dx)}{bc-ad}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6)*Hypergeometric2F1[-1/6, -1/6, 5/6, (d*(a + b*x))/(-(b*c
) + a*d)])/(b*(a + b*x)^(1/6)*((b*(c + d*x))/(b*c - a*d))^(1/6))
```

IntegrateAlgebraic [A] time = 0.26, size = 256, normalized size = 0.77

$$-\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{b^{7/6}}+\frac{\sqrt{3}\sqrt[6]{d}\tan^{-1}\left(\frac{2\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt{3}\sqrt[6]{d}\sqrt[6]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{7/6}}+\frac{2\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{c+dx}}{\sqrt[6]{d}\sqrt[6]{a+bx}}\right)}{b^{7/6}}+\frac{\sqrt[6]{d}\tanh^{-1}\left(\frac{\sqrt[6]{b}\sqrt[6]{d}\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\left(\frac{\sqrt[3]{b}\sqrt[3]{c+dx}}{\sqrt[3]{a+bx}}+\sqrt[3]{d}\right)}\right)}{b^{7/6}}-\frac{6\sqrt[6]{c+dx}}{b\sqrt[6]{a+bx}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c + d*x)^(1/6)/(a + b*x)^(7/6),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/(b*(a + b*x)^(1/6)) - (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3
] - (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]*d^(1/6)*(a + b*x)^(1/6))])/b^(7/6)
 + (Sqrt[3]*d^(1/6)*ArcTan[1/Sqrt[3] + (2*b^(1/6)*(c + d*x)^(1/6))/(Sqrt[3]
*d^(1/6)*(a + b*x)^(1/6))])/b^(7/6) + (2*d^(1/6)*ArcTanh[(b^(1/6)*(c + d*x)
^(1/6))/(d^(1/6)*(a + b*x)^(1/6))])/b^(7/6) + (d^(1/6)*ArcTanh[(b^(1/6)*d^(
```

```
1/6)*(c + d*x)^(1/6))/((a + b*x)^(1/6)*(d^(1/3) + (b^(1/3)*(c + d*x)^(1/3))
/(a + b*x)^(1/3)))])/b^(7/6)
```

fricas [B] time = 1.58, size = 663, normalized size = 2.00

$$-\frac{4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}+\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)+4\sqrt{3}\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\arctan\left(-\frac{2\sqrt{3}(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b^{6}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-2\sqrt{3}\left(b^{7}x+ab^{6}\right)\sqrt{-\frac{(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}}{bx+a}}\left(\frac{d}{b^{7}}\right)^{\frac{5}{6}}-\sqrt{3}(bdx+ad)}{3(bdx+ad)}\right)-\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}+(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)+\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{4\left((bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}b\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-\left(b^{3}x+ab^{2}\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{3}}-(bx+a)^{\frac{2}{3}}(dx+c)^{\frac{1}{3}}\right)}{bx+a}\right)-2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}+(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+2\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}\log\left(-\frac{\left(b^{2}x+ab\right)\left(\frac{d}{b^{7}}\right)^{\frac{1}{6}}-(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{bx+a}\right)+12(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{2\left(b^{2}x+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="fricas")
```

```
[Out] -1/2*(4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arctan(-1/3*(2*sqrt(3)*(b*x + a
)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sqrt(3)*(b^7*x + a*b^6)*sqrt(
((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3*x + a*b^2)*(d/b^7)^
(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a))*(d/b^7)^(5/6) + sqrt(3)
*(b*d*x + a*d))/(b*d*x + a*d)) + 4*sqrt(3)*(b^2*x + a*b)*(d/b^7)^(1/6)*arct
an(-1/3*(2*sqrt(3)*(b*x + a)^(5/6)*(d*x + c)^(1/6)*b^6*(d/b^7)^(5/6) - 2*sq
rt(3)*(b^7*x + a*b^6)*sqrt(-((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6
) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x +
 a))*(d/b^7)^(5/6) - sqrt(3)*(b*d*x + a*d))/(b*d*x + a*d)) - (b^2*x + a*b)*
(d/b^7)^(1/6)*log(4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7)^(1/6) + (b^3
*x + a*b^2)*(d/b^7)^(1/3) + (b*x + a)^(2/3)*(d*x + c)^(1/3))/(b*x + a)) + (
b^2*x + a*b)*(d/b^7)^(1/6)*log(-4*((b*x + a)^(5/6)*(d*x + c)^(1/6)*b*(d/b^7
)^(1/6) - (b^3*x + a*b^2)*(d/b^7)^(1/3) - (b*x + a)^(2/3)*(d*x + c)^(1/3))/
(b*x + a)) - 2*(b^2*x + a*b)*(d/b^7)^(1/6)*log(((b^2*x + a*b)*(d/b^7)^(1/6)
 + (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b*x + a)) + 2*(b^2*x + a*b)*(d/b^7)^(1
/6)*log(-((b^2*x + a*b)*(d/b^7)^(1/6) - (b*x + a)^(5/6)*(d*x + c)^(1/6))/(b
*x + a)) + 12*(b*x + a)^(5/6)*(d*x + c)^(1/6))/(b^2*x + a*b)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

maple [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

```
[Out] int((d*x+c)^(1/6)/(b*x+a)^(7/6),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/6)/(b*x+a)^(7/6),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^(1/6)/(b*x + a)^(7/6), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^{1/6}}{(a+b\,x)^{7/6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/6)/(a + b*x)^(7/6),x)
[Out] int((c + d*x)^(1/6)/(a + b*x)^(7/6), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[6]{c+dx}}{(a+bx)^{\frac{7}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/6)/(b*x+a)**(7/6),x)
[Out] Integral((c + d*x)**(1/6)/(a + b*x)**(7/6), x)
```

3.1550 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx$

Optimal. Leaf size=30

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^{7/6}(c+dx)^{5/6}}\,dx = -\frac{6\sqrt[6]{c+dx}}{(bc-ad)\sqrt[6]{a+bx}}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.00

$$\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (6*(c + d*x)^(1/6))/((-(b*c) + a*d)*(a + b*x)^(1/6))
```

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.00

$$-\frac{6\sqrt[6]{c+dx}}{\sqrt[6]{a+bx}\,(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x]
```

```
[Out] (-6*(c + d*x)^(1/6))/((b*c - a*d)*(a + b*x)^(1/6))
```

fricas [A] time = 1.43, size = 42, normalized size = 1.40

$$-\frac{6\,(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{abc-a^2d+\left(b^2c-abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="fricas")
[Out] -6*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="giac")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

maple [A] time = 0.01, size = 27, normalized size = 0.90

$$\frac{6\,(dx+c)^{\frac{1}{6}}}{(bx+a)^{\frac{1}{6}}\,(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x)
[Out] 6/(b*x+a)^(1/6)*(d*x+c)^(1/6)/(a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(5/6),x, algorithm="maxima")
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(5/6)), x)
```

mupad [B] time = 0.68, size = 26, normalized size = 0.87

$$\frac{6\,{\left(c+d\,x\right)}^{1/6}}{\left(a\,d-b\,c\right)\,{\left(a+b\,x\right)}^{1/6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(5/6)),x)
[Out] (6*(c + d*x)^(1/6))/((a*d - b*c)*(a + b*x)^(1/6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(5/6),x)
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(5/6)), x)
```

3.1551 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx$

Optimal. Leaf size=64

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{36d(a+bx)^{5/6}}{5(c+dx)^{5/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(5/6)) - (36*d*(a + b*x)^(5/6))/(
5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)^{7/6}(c+dx)^{11/6}}\,dx &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}-\frac{(6d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}}\,dx}{bc-ad}\\ &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{5/6}}-\frac{36d(a+bx)^{5/6}}{5(bc-ad)^2(c+dx)^{5/6}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.70

$$-\frac{6(ad+5bc+6bdx)}{5\sqrt[6]{a+bx}\,(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(5*b*c + a*d + 6*b*d*x))/(5*(b*c - a*d)^2*(a + b*x)^(1/6)*(c + d*x)^(5/
6))
```

IntegrateAlgebraic [A] time = 0.12, size = 49, normalized size = 0.77

$$-\frac{6(a+bx)^{5/6}\left(\frac{5b(c+dx)}{a+bx}+d\right)}{5(c+dx)^{5/6}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x]
```

```
[Out] (-6*(a + b*x)^(5/6)*(d + (5*b*(c + d*x))/(a + b*x)))/(5*(b*c - a*d)^2*(c + d*x)^(5/6))
```

fricas [B] time = 1.33, size = 126, normalized size = 1.97

$$-\frac{6\,(6\,bdx+5\,bc+ad)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{5\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)x^2+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="fricas")
```

```
[Out] -6/5*(6*b*d*x + 5*b*c + a*d)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.83

$$-\frac{6\,(6bdx+ad+5bc)}{5\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{5}{6}}\,\left(a^2d^2-2abcd+b^2c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x)
```

```
[Out] -6/5*(6*b*d*x+a*d+5*b*c)/(b*x+a)^(1/6)/(d*x+c)^(5/6)/(a^2*d^2-2*a*b*c*d+b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(11/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(11/6)), x)
```

mupad [B] time = 0.83, size = 72, normalized size = 1.12

$$-\frac{\left(\frac{36\,b\,x}{5\,{\left(a\,d-b\,c\right)}^2}+\frac{6\,a\,d+30\,b\,c}{5\,d\,{\left(a\,d-b\,c\right)}^2}\right)\,{\left(c+d\,x\right)}^{1/6}}{x\,{\left(a+b\,x\right)}^{1/6}+\frac{c\,{\left(a+b\,x\right)}^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(11/6)),x)
```

```
[Out] -(((36*b*x)/(5*(a*d - b*c)^2) + (6*a*d + 30*b*c)/(5*d*(a*d - b*c)^2))*(c +
d*x)^(1/6))/(x*(a + b*x)^(1/6) + (c*(a + b*x)^(1/6))/d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)^{\frac{7}{6}}\,(c+dx)^{\frac{11}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(11/6),x)
```

```
[Out] Integral(1/((a + b*x)**(7/6)*(c + d*x)**(11/6)), x)
```

3.1552 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}} \, dx$

Optimal. Leaf size=98

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{432bd(a+bx)^{5/6}}{55(c+dx)^{5/6}(bc-ad)^3}-\frac{72d(a+bx)^{5/6}}{11(c+dx)^{11/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(11/6)) - (72*d*(a + b*x)^(5/6))/
(11*(b*c - a*d)^2*(c + d*x)^(11/6)) - (432*b*d*(a + b*x)^(5/6))/(55*(b*c -
a*d)^3*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/6}(c+dx)^{17/6}} \, dx &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}}-\frac{(12d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}} \, dx}{bc-ad}\\
&= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{(72bd)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{11}}}{11(bc-ad)^2}\\
&= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{11/6}}-\frac{72d(a+bx)^{5/6}}{11(bc-ad)^2(c+dx)^{11/6}}-\frac{432bd(a+bx)^{5/6}}{55(bc-ad)^3(c+dx)^{5}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.79

$$-\frac{6\left(-5a^2d^2+2abd(11c+6dx)+b^2\left(55c^2+132cdx+72d^2x^2\right)\right)}{55\sqrt[6]{a+bx}\,(c+dx)^{11/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(-5*a^2*d^2 + 2*a*b*d*(11*c + 6*d*x) + b^2*(55*c^2 + 132*c*d*x + 72*d^2
*x^2)))/(55*(b*c - a*d)^3*(a + b*x)^(1/6)*(c + d*x)^(11/6))
```

IntegrateAlgebraic [A] time = 0.13, size = 73, normalized size = 0.74

$$-\frac{6(a+bx)^{11/6}\left(\frac{55b^2(c+dx)^2}{(a+bx)^2}+\frac{22bd(c+dx)}{a+bx}-5d^2\right)}{55(c+dx)^{11/6}(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x]
```

```
[Out] (-6*(a + b*x)^(11/6)*(-5*d^2 + (22*b*d*(c + d*x))/(a + b*x) + (55*b^2*(c +
d*x)^2)/(a + b*x)^2))/(55*(b*c - a*d)^3*(c + d*x)^(11/6))
```

fricas [B] time = 1.57, size = 273, normalized size = 2.79

$$-\frac{6\left(72\,b^2d^2x^2+55\,b^2c^2+22\,abcd-5\,a^2d^2+12\left(11\,b^2cd+abd^2\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{55\left(ab^3c^5-3\,a^2b^2c^4d+3\,a^3bc^3d^2-a^4c^2d^3+\left(b^4c^3d^2-3\,ab^3c^2d^3+3\,a^2b^2cd^4-a^3bd^5\right)x^3+\left(2\,b^4c^4d-5\,ab^3c^3d^2+3\,a^2b^2c^2d^3+a^3bcd^4-a^4d^5\right)x^2+\left(b^4c^5-ab^3c^4d-3\,a^2b^2c^3d^2+5\,a^3bc^2d^3-2\,a^4cd^4\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="fricas")
```

```
[Out] -6/55*(72*b^2*d^2*x^2 + 55*b^2*c^2 + 22*a*b*c*d - 5*a^2*d^2 + 12*(11*b^2*c*
d + a*b*d^2)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3 + (b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*
b^2*c*d^4 - a^3*b*d^5)*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2
*d^3 + a^3*b*c*d^4 - a^4*d^5)*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*
d^2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

maple [A] time = 0.01, size = 105, normalized size = 1.07

$$-\frac{6\left(-72b^2x^2d^2-12ab\,d^2x-132b^2cdx+5a^2d^2-22abcd-55b^2c^2\right)}{55\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{11}{6}}\,\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x)
```

```
[Out] -6/55*(-72*b^2*d^2*x^2-12*a*b*d^2*x-132*b^2*c*d*x+5*a^2*d^2-22*a*b*c*d-55*b
^2*c^2)/(b*x+a)^(1/6)/(d*x+c)^(11/6)/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b
^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{17}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(17/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(17/6)), x)
```

mupad [B] time = 0.96, size = 132, normalized size = 1.35

$$\frac{(c+d\,x)^{1/6}\,\left(\frac{432\,b^2\,x^2}{55\,(a\,d-b\,c)^3}+\frac{-30\,a^2\,d^2+132\,a\,b\,c\,d+330\,b^2\,c^2}{55\,d^2\,(a\,d-b\,c)^3}+\frac{72\,b\,x\,(a\,d+11\,b\,c)}{55\,d\,(a\,d-b\,c)^3}\right)}{x^2\,(a+b\,x)^{1/6}+\frac{c^2\,(a+b\,x)^{1/6}}{d^2}+\frac{2\,c\,x\,(a+b\,x)^{1/6}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(17/6)),x)
```

```
[Out] ((c + d*x)^(1/6)*((432*b^2*x^2)/(55*(a*d - b*c)^3) + (330*b^2*c^2 - 30*a^2*
d^2 + 132*a*b*c*d)/(55*d^2*(a*d - b*c)^3) + (72*b*x*(a*d + 11*b*c))/(55*d*(
a*d - b*c)^3)))/(x^2*(a + b*x)^(1/6) + (c^2*(a + b*x)^(1/6))/d^2 + (2*c*x*(
a + b*x)^(1/6))/d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(17/6),x)
```

```
[Out] Timed out
```

3.1553 $\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx$

Optimal. Leaf size=134

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

Rubi [A] time = 0.03, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{7776b^2d(a+bx)^{5/6}}{935(c+dx)^{5/6}(bc-ad)^4}-\frac{1296bd(a+bx)^{5/6}}{187(c+dx)^{11/6}(bc-ad)^3}-\frac{108d(a+bx)^{5/6}}{17(c+dx)^{17/6}(bc-ad)^2}-\frac{6}{\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] -6/((b*c - a*d)*(a + b*x)^(1/6)*(c + d*x)^(17/6)) - (108*d*(a + b*x)^(5/6))
/(17*(b*c - a*d)^2*(c + d*x)^(17/6)) - (1296*b*d*(a + b*x)^(5/6))/(187*(b*c
 - a*d)^3*(c + d*x)^(11/6)) - (7776*b^2*d*(a + b*x)^(5/6))/(935*(b*c - a*d)
^4*(c + d*x)^(5/6))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx)^{7/6}(c+dx)^{23/6}}\,dx &= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{(18d)\int \frac{1}{\sqrt[6]{a+bx}\,(c+dx)^{23/6}}\,dx}{bc-ad}\\
&= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}}-\frac{(216bd)\int \frac{1}{\sqrt[6]{a+bx}\,(c+
\end{aligned}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}}-\frac{1296bd(a+bx}{187(bc-ad)^3(c+}$$

$$= -\frac{6}{(bc-ad)\sqrt[6]{a+bx}\,(c+dx)^{17/6}}-\frac{108d(a+bx)^{5/6}}{17(bc-ad)^2(c+dx)^{17/6}}-\frac{1296bd(a+bx}{187(bc-ad)^3(c+}$$

Mathematica [A] time = 0.06, size = 118, normalized size = 0.88

$$-\frac{6\left(55a^3d^3-15a^2bd^2(17c+6dx)+3ab^2d\left(187c^2+204cdx+72d^2x^2\right)+b^3\left(935c^3+3366c^2dx+3672cd^2x^2+1296d^3x^3\right)\right)}{935\sqrt[6]{a+bx}\,(c+dx)^{17/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(55*a^3*d^3 - 15*a^2*b*d^2*(17*c + 6*d*x) + 3*a*b^2*d*(187*c^2 + 204*c*
d*x + 72*d^2*x^2) + b^3*(935*c^3 + 3366*c^2*d*x + 3672*c*d^2*x^2 + 1296*d^3
*x^3)))/(935*(b*c - a*d)^4*(a + b*x)^(1/6)*(c + d*x)^(17/6))
```

IntegrateAlgebraic [A] time = 0.14, size = 95, normalized size = 0.71

$$-\frac{6(a+bx)^{17/6}\left(\frac{935b^3(c+dx)^3}{(a+bx)^3}+\frac{561b^2d(c+dx)^2}{(a+bx)^2}-\frac{255bd^2(c+dx)}{a+bx}+55d^3\right)}{935(c+dx)^{17/6}(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x]
```

```
[Out] (-6*(a + b*x)^(17/6)*(55*d^3 - (255*b*d^2*(c + d*x))/(a + b*x) + (561*b^2*d
*(c + d*x)^2)/(a + b*x)^2 + (935*b^3*(c + d*x)^3)/(a + b*x)^3))/(935*(b*c -
 a*d)^4*(c + d*x)^(17/6))
```

fricas [B] time = 1.49, size = 457, normalized size = 3.41

$$-\frac{6\left(1296\,b^3d^3x^3+935\,b^3c^3+561\,ab^2c^2d-255\,a^2bcd^2+55\,a^3d^3+216\left(17\,b^3cd^2+ab^2d^3\right)x^2+18\left(187\,b^3c^2d+34\,ab^2cd^2-5\,a^2bd^3\right)x\right)(bx+a)^{\frac{5}{6}}(dx+c)^{\frac{1}{6}}}{935\left(ab^4c^7-4\,a^2b^3c^6d+6\,a^3b^2c^5d^2-4\,a^4bc^4d^3+a^5c^3d^4+\left(b^5c^4d^3-4\,ab^4c^3d^4+6\,a^2b^3c^2d^5-4\,a^3b^2cd^6+a^4bd^7\right)x^4+\left(3\,b^5c^5d^2-11\,ab^4c^4d^3+14\,a^2b^3c^3d^4-6\,a^3b^2c^2d^5-a^4bcd^6+a^5d^7\right)x^3+3\left(b^5c^6d-3\,ab^4c^5d^2+2\,a^2b^3c^4d^3+2\,a^3b^2c^3d^4-3\,a^4bc^2d^5+a^5cd^6\right)x^2+\left(b^5c^7-ab^4c^6d-6\,a^2b^3c^5d^2+14\,a^3b^2c^4d^3-11\,a^4bc^3d^4+3\,a^5c^2d^5\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="fricas")
```

```
[Out] -6/935*(1296*b^3*d^3*x^3 + 935*b^3*c^3 + 561*a*b^2*c^2*d - 255*a^2*b*c*d^2
+ 55*a^3*d^3 + 216*(17*b^3*c*d^2 + a*b^2*d^3)*x^2 + 18*(187*b^3*c^2*d + 34*
a*b^2*c*d^2 - 5*a^2*b*d^3)*x)*(b*x + a)^(5/6)*(d*x + c)^(1/6)/(a*b^4*c^7 -
4*a^2*b^3*c^6*d + 6*a^3*b^2*c^5*d^2 - 4*a^4*b*c^4*d^3 + a^5*c^3*d^4 + (b^5*
c^4*d^3 - 4*a*b^4*c^3*d^4 + 6*a^2*b^3*c^2*d^5 - 4*a^3*b^2*c*d^6 + a^4*b*d^7
)*x^4 + (3*b^5*c^5*d^2 - 11*a*b^4*c^4*d^3 + 14*a^2*b^3*c^3*d^4 - 6*a^3*b^2*
c^2*d^5 - a^4*b*c*d^6 + a^5*d^7)*x^3 + 3*(b^5*c^6*d - 3*a*b^4*c^5*d^2 + 2*a
^2*b^3*c^4*d^3 + 2*a^3*b^2*c^3*d^4 - 3*a^4*b*c^2*d^5 + a^5*c*d^6)*x^2 + (b^
5*c^7 - a*b^4*c^6*d - 6*a^2*b^3*c^5*d^2 + 14*a^3*b^2*c^4*d^3 - 11*a^4*b*c^3
*d^4 + 3*a^5*c^2*d^5)*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

maple [A] time = 0.01, size = 171, normalized size = 1.28

$$-\frac{6\left(1296b^3d^3x^3+216a\,b^2d^3x^2+3672b^3c\,d^2x^2-90a^2b\,d^3x+612a\,b^2c\,d^2x+3366b^3c^2dx+55a^3d^3-255a^2bc\,d^2+561a\,b^2c^2d+935b^3c^3\right)}{935\,(bx+a)^{\frac{1}{6}}\,(dx+c)^{\frac{17}{6}}\,\left(a^4d^4-4a^3bc\,d^3+6a^2b^2c^2d^2-4a\,b^3c^3d+b^4c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x)
```

```
[Out] -6/935*(1296*b^3*d^3*x^3+216*a*b^2*d^3*x^2+3672*b^3*c*d^2*x^2-90*a^2*b*d^3*
x+612*a*b^2*c*d^2*x+3366*b^3*c^2*d*x+55*a^3*d^3-255*a^2*b*c*d^2+561*a*b^2*c
```

```
^2*d+935*b^3*c^3)/(b*x+a)^(1/6)/(d*x+c)^(17/6)/(a^4*d^4-4*a^3*b*c*d^3+6*a^2
*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{7}{6}}(dx+c)^{\frac{23}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(7/6)/(d*x+c)^(23/6),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^(7/6)*(d*x + c)^(23/6)), x)
```

mupad [B] time = 1.15, size = 209, normalized size = 1.56

$$-\frac{(c+d\,x)^{1/6}\left(\frac{7776\,b^3\,x^3}{935\,(a\,d-b\,c)^4}+\frac{330\,a^3\,d^3-1530\,a^2\,b\,c\,d^2+3366\,a\,b^2\,c^2\,d+5610\,b^3\,c^3}{935\,d^3\,(a\,d-b\,c)^4}+\frac{108\,b\,x\left(-5\,a^2\,d^2+34\,a\,b\,c\,d+187\,b^2\,c^2\right)}{935\,d^2\,(a\,d-b\,c)^4}+\frac{1296\,b^2\,x^2\,(a\,d+17\,b\,c)}{935\,d\,(a\,d-b\,c)^4}\right)}{x^3\,(a+b\,x)^{1/6}+\frac{c^3\,(a+b\,x)^{1/6}}{d^3}+\frac{3\,c\,x^2\,(a+b\,x)^{1/6}}{d}+\frac{3\,c^2\,x\,(a+b\,x)^{1/6}}{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(7/6)*(c + d*x)^(23/6)),x)
```

```
[Out] -((c + d*x)^(1/6)*((7776*b^3*x^3)/(935*(a*d - b*c)^4) + (330*a^3*d^3 + 5610
*b^3*c^3 + 3366*a*b^2*c^2*d - 1530*a^2*b*c*d^2)/(935*d^3*(a*d - b*c)^4) + (
108*b*x*(187*b^2*c^2 - 5*a^2*d^2 + 34*a*b*c*d))/(935*d^2*(a*d - b*c)^4) + (
1296*b^2*x^2*(a*d + 17*b*c))/(935*d*(a*d - b*c)^4)))/(x^3*(a + b*x)^(1/6) +
 (c^3*(a + b*x)^(1/6))/d^3 + (3*c*x^2*(a + b*x)^(1/6))/d + (3*c^2*x*(a + b*
x)^(1/6))/d^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(7/6)/(d*x+c)**(23/6),x)
```

```
[Out] Timed out
```

3.1554 $\int (a+bx)^m(a+b(2+m)x)\,dx$

Optimal. Leaf size=11

$$x(a+bx)^{m+1}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {34}

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

Rule 34

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_)), x_Symbol] :> Simp[(d*x*(
a + b*x)^(m + 1))/(b*(m + 2)), x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[a*d -
 b*c*(m + 2), 0]
```

Rubi steps

$$\int (a+bx)^m(a+b(2+m)x)\,dx = x(a+bx)^{1+m}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$x(a+bx)^{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] x*(a + b*x)^(1 + m)
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^m(a+b(2+m)x)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(a + b*(2 + m)*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(a + b*(2 + m)*x), x]
```

fricas [A] time = 1.19, size = 17, normalized size = 1.55

$$\left(bx^2+ax\right)(bx+a)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="fricas")
```

```
[Out] (b*x^2 + a*x)*(b*x + a)^m
```

giac [**B**] time = 1.00, size = 23, normalized size = 2.09

$$(bx+a)^m bx^2 + (bx+a)^m ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="giac")
```

```
[Out] (b*x + a)^m*b*x^2 + (b*x + a)^m*a*x
```

maple [**A**] time = 0.00, size = 12, normalized size = 1.09

$$x(bx+a)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a+b*(m+2)*x),x)
```

```
[Out] x*(b*x+a)^(m+1)
```

maxima [**B**] time = 1.12, size = 106, normalized size = 9.64

$$\frac{\left(b^2(m+1)x^2 + abmx - a^2\right)(bx+a)^m m}{\left(m^2+3\,m+2\right)b} + \frac{2\left(b^2(m+1)x^2 + abmx - a^2\right)(bx+a)^m}{\left(m^2+3\,m+2\right)b} + \frac{(bx+a)^{m+1}a}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a+b*(2+m)*x),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*m/((m^2 + 3*m + 2)*b) + 2*(b^
2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m/((m^2 + 3*m + 2)*b) + (b*x + a)^
(m + 1)*a/(b*(m + 1))
```

mupad [**B**] time = 0.46, size = 11, normalized size = 1.00

$$x(a+b\,x)^{m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x*(m + 2))*(a + b*x)^m,x)
```

```
[Out] x*(a + b*x)^(m + 1)
```

sympy [**B**] time = 0.27, size = 20, normalized size = 1.82

$$ax(a+bx)^m + bx^2(a+bx)^m$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a+b*(2+m)*x),x)
```

```
[Out] a*x*(a + b*x)**m + b*x**2*(a + b*x)**m
```

3.1555 $\int (a+bx)^m(c+dx)^3\,dx$

Optimal. Leaf size=110

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Rubi [A] time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{3d^2(bc-ad)(a+bx)^{m+3}}{b^4(m+3)}+\frac{(bc-ad)^3(a+bx)^{m+1}}{b^4(m+1)}+\frac{3d(bc-ad)^2(a+bx)^{m+2}}{b^4(m+2)}+\frac{d^3(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((b*c - a*d)^3*(a + b*x)^(1 + m))/(b^4*(1 + m)) + (3*d*(b*c - a*d)^2*(a + b
*x)^(2 + m))/(b^4*(2 + m)) + (3*d^2*(b*c - a*d)*(a + b*x)^(3 + m))/(b^4*(3
+ m)) + (d^3*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^3\,dx &= \int \left(\frac{(bc-ad)^3(a+bx)^m}{b^3}+\frac{3d(bc-ad)^2(a+bx)^{1+m}}{b^3}+\frac{3d^2(bc-ad)(a+bx)^{2+m}}{b^3}+\frac{d^3(a\right.\\ &= \frac{(bc-ad)^3(a+bx)^{1+m}}{b^4(1+m)}+\frac{3d(bc-ad)^2(a+bx)^{2+m}}{b^4(2+m)}+\frac{3d^2(bc-ad)(a+bx)^{3+m}}{b^4(3+m)}+\frac{d^3(a+}{b^4(4}\end{aligned}$$

Mathematica [A] time = 0.11, size = 94, normalized size = 0.85

$$\frac{(a+bx)^{m+1}\left(\frac{3d^2(a+bx)^2(bc-ad)}{m+3}+\frac{3d(a+bx)(bc-ad)^2}{m+2}+\frac{(bc-ad)^3}{m+1}+\frac{d^3(a+bx)^3}{m+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^3/(1 + m) + (3*d*(b*c - a*d)^2*(a + b*x))/(
2 + m) + (3*d^2*(b*c - a*d)*(a + b*x)^2)/(3 + m) + (d^3*(a + b*x)^3)/(4 + m
)))/b^4
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^3\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^3,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^3, x]
```

fricas [B] time = 1.26, size = 497, normalized size = 4.52

$$\frac{\left(ab^3c^3m^3 + 24\,ab^3c^3 - 36\,a^2b^2c^2d + 24\,a^3bcd^2 - 6\,a^4d^3 + \left(b^4d^3m^3 + 6\,b^4d^3m^2 + 11\,b^4d^3m + 6\,b^4d^3\right)x^4 + \left(24\,b^4cd^2 + \left(3\,b^4cd^2 + ab^3d^3\right)m^3 + 3\left(7\,b^4cd^2 + ab^3d^3\right)m^2 + 2\left(21\,b^4cd^2 + ab^3d^3\right)m\right)x^3 + 3\left(3\,ab^3c^3 - a^2b^2c^2d\right)m^2 + 3\left(12\,b^4c^2d + \left(b^4c^2d + ab^3cd^2\right)m^3 + \left(8\,b^4c^2d + 5\,ab^3cd^2 - a^2b^2d^3\right)m^2 + \left(19\,b^4c^2d + 4\,ab^3cd^2 - a^2b^2d^3\right)m\right)x^2 + \left(26\,ab^3c^3 - 21\,a^2b^2c^2d + 6\,a^3bcd^2\right)m + \left(24\,b^4c^3 + \left(b^4c^3 + 3\,ab^3c^2d\right)m^3 + 3\left(3\,b^4c^3 + 7\,ab^3c^2d - 2\,a^2b^2cd^2\right)m^2 + 2\left(13\,b^4c^3 + 18\,ab^3c^2d - 12\,a^2b^2cd^2 + 3\,a^3bd^3\right)m\right)x\right)(bx+a)^m}{b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (a*b^3*c^3*m^3 + 24*a*b^3*c^3 - 36*a^2*b^2*c^2*d + 24*a^3*b*c*d^2 - 6*a^4*d
^3 + (b^4*d^3*m^3 + 6*b^4*d^3*m^2 + 11*b^4*d^3*m + 6*b^4*d^3)*x^4 + (24*b^4
*c*d^2 + (3*b^4*c*d^2 + a*b^3*d^3)*m^3 + 3*(7*b^4*c*d^2 + a*b^3*d^3)*m^2 +
2*(21*b^4*c*d^2 + a*b^3*d^3)*m)*x^3 + 3*(3*a*b^3*c^3 - a^2*b^2*c^2*d)*m^2 +
 3*(12*b^4*c^2*d + (b^4*c^2*d + a*b^3*c*d^2)*m^3 + (8*b^4*c^2*d + 5*a*b^3*c
*d^2 - a^2*b^2*d^3)*m^2 + (19*b^4*c^2*d + 4*a*b^3*c*d^2 - a^2*b^2*d^3)*m)*x
^2 + (26*a*b^3*c^3 - 21*a^2*b^2*c^2*d + 6*a^3*b*c*d^2)*m + (24*b^4*c^3 + (b
^4*c^3 + 3*a*b^3*c^2*d)*m^3 + 3*(3*b^4*c^3 + 7*a*b^3*c^2*d - 2*a^2*b^2*c*d^
2)*m^2 + 2*(13*b^4*c^3 + 18*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 3*a^3*b*d^3)*m
)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

giac [B] time = 1.04, size = 833, normalized size = 7.57

$$\frac{(bx+a)^m b^4d^3m^3x^4 + 3(bx+a)^m b^4cd^2m^3x^3 + (bx+a)^m ab^3d^3m^3x^3 + 6(bx+a)^m b^4d^3m^2x^4 + 3(bx+a)^m b^4c^2dm^3x^2 + 3(bx+a)^m ab^3cd^2m^3x^2 + 21(bx+a)^m b^4cd^2m^2x^3 + 3(bx+a)^m ab^3d^3m^2x^3 + 11(bx+a)^m b^4d^3mx^4 + (bx+a)^m b^4c^3m^3x + 3(bx+a)^m ab^3c^2dm^3x + 24(bx+a)^m b^4c^2dm^2x^2 + 15(bx+a)^m ab^3cd^2m^2x^2 - 3(bx+a)^m a^2b^2d^3m^2x^2 + 42(bx+a)^m b^4cd^2mx^3 + 2(bx+a)^m ab^3d^3mx^3 + 6(bx+a)^m b^4d^3x^4 + (bx+a)^m ab^3c^3m^3 + 9(bx+a)^m b^4c^3m^2x + 21(bx+a)^m ab^3c^2dm^2x - 6(bx+a)^m a^2b^2cd^2m^2x + 57(bx+a)^m b^4c^2dmx^2 + 12(bx+a)^m ab^3cd^2mx^2 - 3(bx+a)^m a^2b^2d^3mx^2 + 24(bx+a)^m b^4cd^2x^3 + 9(bx+a)^m ab^3c^3m^2 - 3(bx+a)^m a^2b^2c^2dm^2 + 26(bx+a)^m b^4c^3mx + 36(bx+a)^m ab^3c^2dmx - 24(bx+a)^m a^2b^2cd^2mx + 6(bx+a)^m a^3bd^3mx + 36(bx+a)^m b^4c^2dx^2 + 26(bx+a)^m ab^3c^3m - 21(bx+a)^m a^2b^2c^2dm + 6(bx+a)^m a^3bcd^2m + 24(bx+a)^m b^4c^3x + 24(bx+a)^m ab^3c^3 - 36(bx+a)^m a^2b^2c^2d + 24(bx+a)^m a^3bcd^2 - 6(bx+a)^m a^4d^3}{b^4m^4 + 10\,b^4m^3 + 35\,b^4m^2 + 50\,b^4m + 24\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d^3*m^3*x^4 + 3*(b*x + a)^m*b^4*c*d^2*m^3*x^3 + (b*x + a)^
m*a*b^3*d^3*m^3*x^3 + 6*(b*x + a)^m*b^4*d^3*m^2*x^4 + 3*(b*x + a)^m*b^4*c^2
*d*m^3*x^2 + 3*(b*x + a)^m*a*b^3*c*d^2*m^3*x^2 + 21*(b*x + a)^m*b^4*c*d^2*m
^2*x^3 + 3*(b*x + a)^m*a*b^3*d^3*m^2*x^3 + 11*(b*x + a)^m*b^4*d^3*m*x^4 + (
b*x + a)^m*b^4*c^3*m^3*x + 3*(b*x + a)^m*a*b^3*c^2*d*m^3*x + 24*(b*x + a)^m
*b^4*c^2*d*m^2*x^2 + 15*(b*x + a)^m*a*b^3*c*d^2*m^2*x^2 - 3*(b*x + a)^m*a^2
*b^2*d^3*m^2*x^2 + 42*(b*x + a)^m*b^4*c*d^2*m*x^3 + 2*(b*x + a)^m*a*b^3*d^3
*m*x^3 + 6*(b*x + a)^m*b^4*d^3*x^4 + (b*x + a)^m*a*b^3*c^3*m^3 + 9*(b*x + a
)^m*b^4*c^3*m^2*x + 21*(b*x + a)^m*a*b^3*c^2*d*m^2*x - 6*(b*x + a)^m*a^2*b^
2*c*d^2*m^2*x + 57*(b*x + a)^m*b^4*c^2*d*m*x^2 + 12*(b*x + a)^m*a*b^3*c*d^2
*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d^3*m*x^2 + 24*(b*x + a)^m*b^4*c*d^2*x^3 + 9
*(b*x + a)^m*a*b^3*c^3*m^2 - 3*(b*x + a)^m*a^2*b^2*c^2*d*m^2 + 26*(b*x + a)
^m*b^4*c^3*m*x + 36*(b*x + a)^m*a*b^3*c^2*d*m*x - 24*(b*x + a)^m*a^2*b^2*c*
d^2*m*x + 6*(b*x + a)^m*a^3*b*d^3*m*x + 36*(b*x + a)^m*b^4*c^2*d*x^2 + 26*(
b*x + a)^m*a*b^3*c^3*m - 21*(b*x + a)^m*a^2*b^2*c^2*d*m + 6*(b*x + a)^m*a^3
*b*c*d^2*m + 24*(b*x + a)^m*b^4*c^3*x + 24*(b*x + a)^m*a*b^3*c^3 - 36*(b*x
+ a)^m*a^2*b^2*c^2*d + 24*(b*x + a)^m*a^3*b*c*d^2 - 6*(b*x + a)^m*a^4*d^3)/
(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50*b^4*m + 24*b^4)
```

maple [B] time = 0.01, size = 389, normalized size = 3.54

$$-\frac{\left(-b^3d^3m^3x^3 - 3b^3cd^2m^3x^2 - 6b^3d^3m^2x^3 + 3ab^2d^3m^2x^2 - 3b^3c^2dm^3x - 21b^3cd^2m^2x^2 - 11b^3d^3mx^3 + 6ab^2cd^2m^2x + 9ab^2d^3mx^2 - b^3c^3m^3 - 24b^3c^2dm^2x - 42b^3cd^2mx^2 - 6b^3d^3x^3 - 6a^2bd^3mx + 3ab^2c^2dm^2 + 30ab^2cd^2mx + 6ab^2d^3x^2 - 9b^3c^3m^2 - 57b^3c^2dmx - 24b^3cd^2x^2 - 6a^2bcd^2m - 6a^2bd^3x + 21ab^2c^2dm + 24ab^2cd^2x - 26b^3c^3m - 36b^3c^2dx + 6a^3d^3 - 24a^2bcd^2 + 36ab^2c^2d - 24b^3c^3\right)(bx+a)^{m+1}}{\left(m^4 + 10m^3 + 35m^2 + 50m + 24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^3,x)
```

```
[Out] -(b*x+a)^(m+1)*(-b^3*d^3*m^3*x^3-3*b^3*c*d^2*m^3*x^2-6*b^3*d^3*m^2*x^3+3*a*
b^2*d^3*m^2*x^2-3*b^3*c^2*d*m^3*x-21*b^3*c*d^2*m^2*x^2-11*b^3*d^3*m*x^3+6*a
*b^2*c*d^2*m^2*x+9*a*b^2*d^3*m*x^2-b^3*c^3*m^3-24*b^3*c^2*d*m^2*x-42*b^3*c*
d^2*m*x^2-6*b^3*d^3*x^3-6*a^2*b*d^3*m*x+3*a*b^2*c^2*d*m^2+30*a*b^2*c*d^2*m*
x+6*a*b^2*d^3*x^2-9*b^3*c^3*m^2-57*b^3*c^2*d*m*x-24*b^3*c*d^2*x^2-6*a^2*b*c
*d^2*m-6*a^2*b*d^3*x+21*a*b^2*c^2*d*m+24*a*b^2*c*d^2*x-26*b^3*c^3*m-36*b^3*
```

```
c^2*d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/b^4/(m^4+10*m^3
+35*m^2+50*m+24)
```

maxima [B] time = 1.17, size = 246, normalized size = 2.24

$$\frac{3\left(b^2(m+1)x^2+abmx-a^2\right)(bx+a)^m c^2 d}{\left(m^2+3m+2\right)b^2}+\frac{(bx+a)^{m+1}c^3}{b(m+1)}+\frac{3\left(\left(m^2+3m+2\right)b^3x^3+\left(m^2+m\right)ab^2x^2-2a^2bmx+2a^3\right)(bx+a)^m cd^2}{\left(m^3+6m^2+11m+6\right)b^3}+\frac{\left(\left(m^3+6m^2+11m+6\right)b^4x^4+\left(m^3+3m^2+2m\right)ab^3x^3-3\left(m^2+m\right)a^2b^2x^2+6a^3bmx-6a^4\right)(bx+a)^m d^3}{\left(m^4+10m^3+35m^2+50m+24\right)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 3*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c^2*d/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c^3/(b*(m + 1)) + 3*((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*d^2/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m^2 + 2*m)*a*b^3*x
^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x + a)^m*d^3/((m^4 +
 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

mupad [B] time = 0.94, size = 478, normalized size = 4.35

$$\frac{d^3x^4(a+bx)^m\left(m^3+6m^2+11m+6\right)}{m^4+10m^3+35m^2+50m+24}+\frac{a(a+bx)^m\left(-6a^3d^3+6a^2bcd^2m+24a^2bcd^2-3ab^2c^2dm^2-21ab^2c^2dm-36ab^2c^2d+b^3c^3m^3+9b^3c^3m^2+26b^3c^3m+24b^3c^3\right)}{b^4\left(m^4+10m^3+35m^2+50m+24\right)}+\frac{x(a+bx)^m\left(6a^3bd^3m-6a^2b^2cd^2m^2-24a^2b^2cd^2m+3ab^3c^2dm^3+21ab^3c^2dm^2+36ab^3c^2dm+b^4c^3m^3+9b^4c^3m^2+26b^4c^3m+24b^4c^3\right)}{b^4\left(m^4+10m^3+35m^2+50m+24\right)}+\frac{3dx^2(m+1)(a+bx)^m\left(-a^2d^2m+abcdm^2+4abcdm+b^2c^2m^2+7b^2c^2m+12b^2c^2\right)}{b^2\left(m^4+10m^3+35m^2+50m+24\right)}+\frac{d^2x^3(a+bx)^m(12bc+adm+3bcm)\left(m^2+3m+2\right)}{b\left(m^4+10m^3+35m^2+50m+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^3,x)
```

```
[Out] (d^3*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m^
4 + 24) + (a*(a + b*x)^m*(24*b^3*c^3 - 6*a^3*d^3 + 26*b^3*c^3*m + 9*b^3*c^3
*m^2 + b^3*c^3*m^3 - 36*a*b^2*c^2*d + 24*a^2*b*c*d^2 - 21*a*b^2*c^2*d*m + 6
*a^2*b*c*d^2*m - 3*a*b^2*c^2*d*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 2
4)) + (x*(a + b*x)^m*(24*b^4*c^3 + 26*b^4*c^3*m + 9*b^4*c^3*m^2 + b^4*c^3*m
^3 + 6*a^3*b*d^3*m + 36*a*b^3*c^2*d*m - 24*a^2*b^2*c*d^2*m + 21*a*b^3*c^2*d
*m^2 + 3*a*b^3*c^2*d*m^3 - 6*a^2*b^2*c*d^2*m^2))/(b^4*(50*m + 35*m^2 + 10*m
^3 + m^4 + 24)) + (3*d*x^2*(m + 1)*(a + b*x)^m*(12*b^2*c^2 - a^2*d^2*m + 7*
b^2*c^2*m + b^2*c^2*m^2 + 4*a*b*c*d*m + a*b*c*d*m^2))/(b^2*(50*m + 35*m^2 +
 10*m^3 + m^4 + 24)) + (d^2*x^3*(a + b*x)^m*(12*b*c + a*d*m + 3*b*c*m)*(3*m
 + m^2 + 2))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy [A] time = 4.67, size = 4058, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**3,x)
```

```
[Out] Piecewise((a**m*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), Eq(
b, 0)), (6*a**3*d**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 11*a**3*d**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 6*a**2*b*c*d**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d**3*x*log(a/b + x)/(6*a**3*b**4 +
18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d**3*x/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*a*b**2*c**2*d/(6*a
**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 18*a*b**2*c*d**
2*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b*
*2*d**3*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
6*b**7*x**3) + 18*a*b**2*d**3*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) - 2*b**3*c**3/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b*
*6*x**2 + 6*b**7*x**3) - 9*b**3*c**2*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18
*a*b**6*x**2 + 6*b**7*x**3) - 18*b**3*c*d**2*x**2/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d**3*x**3*log(a/b + x)/(6*a**
3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), Eq(m, -4)), (-6*a*
*3*d**3*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d**3
```

```
/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 6*a**2*b*c*d**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 9*a**2*b*c*d**2/(2*a**2*b**4 + 4*a
*b**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b
**5*x + 2*b**6*x**2) - 12*a**2*b*d**3*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*
x**2) - 3*a*b**2*c**2*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 12*a*b**
2*c*d**2*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d**3*x**2*lo
g(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c**3/(2*a**2*b**
4 + 4*a*b**5*x + 2*b**6*x**2) - 6*b**3*c**2*d*x/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 6*b**3*c*d**2*x**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x +
 2*b**6*x**2) + 2*b**3*d**3*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2),
Eq(m, -3)), (6*a**3*d**3*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d**3/(
2*a*b**4 + 2*b**5*x) - 12*a**2*b*c*d**2*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
- 12*a**2*b*c*d**2/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d**3*x*log(a/b + x)/(2*
a*b**4 + 2*b**5*x) + 6*a*b**2*c**2*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6
*a*b**2*c**2*d/(2*a*b**4 + 2*b**5*x) - 12*a*b**2*c*d**2*x*log(a/b + x)/(2*a
*b**4 + 2*b**5*x) - 3*a*b**2*d**3*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c**3/
(2*a*b**4 + 2*b**5*x) + 6*b**3*c**2*d*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*b**3*c*d**2*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d**3*x**3/(2*a*b**4 + 2*b
**5*x), Eq(m, -2)), (-a**3*d**3*log(a/b + x)/b**4 + 3*a**2*c*d**2*log(a/b +
 x)/b**3 + a**2*d**3*x/b**3 - 3*a*c**2*d*log(a/b + x)/b**2 - 3*a*c*d**2*x/b
**2 - a*d**3*x**2/(2*b**2) + c**3*log(a/b + x)/b + 3*c**2*d*x/b + 3*c*d**2*
x**2/(2*b) + d**3*x**3/(3*b), Eq(m, -1)), (-6*a**4*d**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*a**3*b*c*d**2
*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 24*a**3*b*c*d**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m*
*2 + 50*b**4*m + 24*b**4) + 6*a**3*b*d**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*c**2*d*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 21*a**2*b**2*c**2*d*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) - 36*a**2*b**2*c**2*d*(a + b*x)**m/(b**4*m**4 +
10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 6*a**2*b**2*c*d**2*m**
2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) - 24*a**2*b**2*c*d**2*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35
*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d**3*m**2*x**2*(a + b*x)**m
/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b
**2*d**3*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + a*b**3*c**3*m**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c**3*m**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*a*b**3*c
**3*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 24*a*b**3*c**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c**2*d*m**3*x*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*a*b**3*c**2*d*m*
*2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 36*a*b**3*c**2*d*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 3*a*b**3*c*d**2*m**3*x**2*(a + b*x)**m/(b
**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 15*a*b**3*c
*d**2*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*d**2*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b*
*4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d**3*m**3*x**3*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3
*a*b**3*d**3*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**
2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d**3*m*x**3*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c**3*m**3*x*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*
b**4*c**3*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50
*b**4*m + 24*b**4) + 26*b**4*c**3*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**
3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c**3*x*(a + b*x)**m/(b**4
```

```
*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 3*b**4*c**2*d*
m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) + 24*b**4*c**2*d*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 57*b**4*c**2*d*m*x**2*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 36*b**
4*c**2*d*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b*
*4*m + 24*b**4) + 3*b**4*c*d**2*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 21*b**4*c*d**2*m**2*x**3*(a +
 b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
42*b**4*c*d**2*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
 + 50*b**4*m + 24*b**4) + 24*b**4*c*d**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d**3*m**3*x**4*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6
*b**4*d**3*m**2*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 11*b**4*d**3*m*x**4*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d**3*x**4*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4), True))
```

3.1556 $\int (a+bx)^m (c+dx)^2 \, dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(a+bx)^{m+1}}{b^3(m+1)}+\frac{2d(bc-ad)(a+bx)^{m+2}}{b^3(m+2)}+\frac{d^2(a+bx)^{m+3}}{b^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((b*c - a*d)^2*(a + b*x)^(1 + m))/(b^3*(1 + m)) + (2*d*(b*c - a*d)*(a + b*x
)^(2 + m))/(b^3*(2 + m)) + (d^2*(a + b*x)^(3 + m))/(b^3*(3 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)^2\,dx &= \int \left(\frac{(bc-ad)^2(a+bx)^m}{b^2}+\frac{2d(bc-ad)(a+bx)^{1+m}}{b^2}+\frac{d^2(a+bx)^{2+m}}{b^2}\right)dx\\ &= \frac{(bc-ad)^2(a+bx)^{1+m}}{b^3(1+m)}+\frac{2d(bc-ad)(a+bx)^{2+m}}{b^3(2+m)}+\frac{d^2(a+bx)^{3+m}}{b^3(3+m)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.86

$$\frac{(a+bx)^{m+1}\left(\frac{2d(a+bx)(bc-ad)}{m+2}+\frac{(bc-ad)^2}{m+1}+\frac{d^2(a+bx)^2}{m+3}\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] ((a + b*x)^(1 + m)*((b*c - a*d)^2/(1 + m) + (2*d*(b*c - a*d)*(a + b*x))/(2
+ m) + (d^2*(a + b*x)^2)/(3 + m)))/b^3
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)^2\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^2, x]
```

fricas [B] time = 1.18, size = 235, normalized size = 3.01

$$\frac{\left(ab^2c^2m^2 + 6\,ab^2c^2 - 6\,a^2bcd + 2\,a^3d^2 + \left(b^3d^2m^2 + 3\,b^3d^2m + 2\,b^3d^2\right)x^3 + \left(6\,b^3cd + \left(2\,b^3cd + ab^2d^2\right)m^2 + \left(8\,b^3cd + ab^2d^2\right)m\right)x^2 + \left(5\,ab^2c^2 - 2\,a^2bcd\right)m + \left(6\,b^3c^2 + \left(b^3c^2 + 2\,ab^2cd\right)m^2 + \left(5\,b^3c^2 + 6\,ab^2cd - 2\,a^2bd^2\right)m\right)x\right)(bx+a)^m}{b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^2*c^2*m^2 + 6*a*b^2*c^2 - 6*a^2*b*c*d + 2*a^3*d^2 + (b^3*d^2*m^2 + 3*b
^3*d^2*m + 2*b^3*d^2)*x^3 + (6*b^3*c*d + (2*b^3*c*d + a*b^2*d^2)*m^2 + (8*b
^3*c*d + a*b^2*d^2)*m)*x^2 + (5*a*b^2*c^2 - 2*a^2*b*c*d)*m + (6*b^3*c^2 + (
b^3*c^2 + 2*a*b^2*c*d)*m^2 + (5*b^3*c^2 + 6*a*b^2*c*d - 2*a^2*b*d^2)*m)*x)*
(b*x + a)^m/(b^3*m^3 + 6*b^3*m^2 + 11*b^3*m + 6*b^3)
```

giac [B] time = 0.97, size = 385, normalized size = 4.94

$$\frac{(bx+a)^m b^3d^2m^2x^3 + 2\,(bx+a)^m b^3cdm^2x^2 + (bx+a)^m ab^2d^2m^2x^2 + 3\,(bx+a)^m b^3d^2mx^3 + (bx+a)^m b^3c^2m^2x + 2\,(bx+a)^m ab^2cdm^2x + 8\,(bx+a)^m b^3cdmx^2 + (bx+a)^m ab^2d^2mx^2 + 2\,(bx+a)^m b^3d^2x^3 + (bx+a)^m ab^2c^2m^2 + 5\,(bx+a)^m b^3c^2mx + 6\,(bx+a)^m ab^2cdmx - 2\,(bx+a)^m a^2bd^2mx + 6\,(bx+a)^m b^3cdx^2 + 5\,(bx+a)^m ab^2c^2m - 2\,(bx+a)^m a^2bcdm + 6\,(bx+a)^m b^3c^2x + 6\,(bx+a)^m ab^2c^2 - 6\,(bx+a)^m a^2bcd + 2\,(bx+a)^m a^3d^2}{b^3m^3 + 6\,b^3m^2 + 11\,b^3m + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^3*d^2*m^2*x^3 + 2*(b*x + a)^m*b^3*c*d*m^2*x^2 + (b*x + a)^m*
a*b^2*d^2*m^2*x^2 + 3*(b*x + a)^m*b^3*d^2*m*x^3 + (b*x + a)^m*b^3*c^2*m^2*x
 + 2*(b*x + a)^m*a*b^2*c*d*m^2*x + 8*(b*x + a)^m*b^3*c*d*m*x^2 + (b*x + a)^
m*a*b^2*d^2*m*x^2 + 2*(b*x + a)^m*b^3*d^2*x^3 + (b*x + a)^m*a*b^2*c^2*m^2 +
 5*(b*x + a)^m*b^3*c^2*m*x + 6*(b*x + a)^m*a*b^2*c*d*m*x - 2*(b*x + a)^m*a^
2*b*d^2*m*x + 6*(b*x + a)^m*b^3*c*d*x^2 + 5*(b*x + a)^m*a*b^2*c^2*m - 2*(b*
x + a)^m*a^2*b*c*d*m + 6*(b*x + a)^m*b^3*c^2*x + 6*(b*x + a)^m*a*b^2*c^2 -
6*(b*x + a)^m*a^2*b*c*d + 2*(b*x + a)^m*a^3*d^2)/(b^3*m^3 + 6*b^3*m^2 + 11*
b^3*m + 6*b^3)
```

maple [B] time = 0.01, size = 159, normalized size = 2.04

$$\frac{\left(b^2d^2m^2x^2 + 2b^2cd\,m^2x + 3b^2d^2m\,x^2 - 2ab\,d^2mx + b^2c^2m^2 + 8b^2cdmx + 2b^2x^2d^2 - 2abcdm - 2ab\,d^2x + 5b^2c^2m + 6b^2cdx + 2a^2d^2 - 6abcd + 6b^2c^2\right)(bx+a)^{m+1}}{\left(m^3+6m^2+11m+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^2,x)
```

```
[Out] (b*x+a)^(m+1)*(b^2*d^2*m^2*x^2+2*b^2*c*d*m^2*x+3*b^2*d^2*m*x^2-2*a*b*d^2*m*
x+b^2*c^2*m^2+8*b^2*c*d*m*x+2*b^2*d^2*x^2-2*a*b*c*d*m-2*a*b*d^2*x+5*b^2*c^2
*m+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/b^3/(m^3+6*m^2+11*m+6)
```

maxima [A] time = 1.15, size = 138, normalized size = 1.77

$$\frac{2\left(b^2(m+1)x^2 + abmx - a^2\right)(bx+a)^m cd}{\left(m^2+3\,m+2\right)b^2} + \frac{(bx+a)^{m+1}c^2}{b(m+1)} + \frac{\left(\left(m^2+3\,m+2\right)b^3x^3 + \left(m^2+m\right)ab^2x^2 - 2\,a^2bmx + 2\,a^3\right)(bx+a)^m d^2}{\left(m^3+6\,m^2+11\,m+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*d/((m^2 + 3*m + 2)*b^2) +
 (b*x + a)^(m + 1)*c^2/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a
*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d^2/((m^3 + 6*m^2 + 11*m + 6)*b
^3)
```

mupad [B] time = 0.66, size = 226, normalized size = 2.90

$$(a+b\,x)^m\left(\frac{a\,\left(2\,a^2\,d^2-2\,a\,b\,c\,d\,m-6\,a\,b\,c\,d+b^2\,c^2\,m^2+5\,b^2\,c^2\,m+6\,b^2\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d^2\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6}+\frac{x\,\left(-2\,a^2\,b\,d^2\,m+2\,a\,b^2\,c\,d\,m^2+6\,a\,b^2\,c\,d\,m+b^3\,c^2\,m^2+5\,b^3\,c^2\,m+6\,b^3\,c^2\right)}{b^3\,\left(m^3+6\,m^2+11\,m+6\right)}+\frac{d\,x^2\,\left(m+1\right)\,\left(6\,b\,c+a\,d\,m+2\,b\,c\,m\right)}{b\,\left(m^3+6\,m^2+11\,m+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x)^2,x)
```

```
[Out] (a + b*x)^m*((a*(2*a^2*d^2 + 6*b^2*c^2 + 5*b^2*c^2*m + b^2*c^2*m^2 - 6*a*b*
c*d - 2*a*b*c*d*m))/(b^3*(11*m + 6*m^2 + m^3 + 6)) + (d^2*x^3*(3*m + m^2 +
2))/(11*m + 6*m^2 + m^3 + 6) + (x*(6*b^3*c^2 + 5*b^3*c^2*m + b^3*c^2*m^2 -
2*a^2*b*d^2*m + 2*a*b^2*c*d*m^2 + 6*a*b^2*c*d*m))/(b^3*(11*m + 6*m^2 + m^3
+ 6)) + (d*x^2*(m + 1)*(6*b*c + a*d*m + 2*b*c*m))/(b*(11*m + 6*m^2 + m^3 +
6)))
```

sympy [A] time = 2.14, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**2,x)
```

```
[Out] Piecewise((a**m*(c**2*x + c*d*x**2 + d**2*x**3/3), Eq(b, 0)), (2*a**2*d**2*
log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 3*a**2*d**2/(2*a**2
*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*a*b*c*d/(2*a**2*b**3 + 4*a*b**4*x + 2
*b**5*x**2) + 4*a*b*d**2*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*
x**2) + 4*a*b*d**2*x/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c**2/(
2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 4*b**2*c*d*x/(2*a**2*b**3 + 4*a*b
**4*x + 2*b**5*x**2) + 2*b**2*d**2*x**2*log(a/b + x)/(2*a**2*b**3 + 4*a*b**
4*x + 2*b**5*x**2), Eq(m, -3)), (-2*a**2*d**2*log(a/b + x)/(a*b**3 + b**4*x
) - 2*a**2*d**2/(a*b**3 + b**4*x) + 2*a*b*c*d*log(a/b + x)/(a*b**3 + b**4*x
) + 2*a*b*c*d/(a*b**3 + b**4*x) - 2*a*b*d**2*x*log(a/b + x)/(a*b**3 + b**4*
x) - b**2*c**2/(a*b**3 + b**4*x) + 2*b**2*c*d*x*log(a/b + x)/(a*b**3 + b**4
*x) + b**2*d**2*x**2/(a*b**3 + b**4*x), Eq(m, -2)), (a**2*d**2*log(a/b + x)
/b**3 - 2*a*c*d*log(a/b + x)/b**2 - a*d**2*x/b**2 + c**2*log(a/b + x)/b + 2
*c*d*x/b + d**2*x**2/(2*b), Eq(m, -1)), (2*a**3*d**2*(a + b*x)**m/(b**3*m**
3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*c*d*m*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 6*a**2*b*c*d*(a + b*x)**m/(b**3*m
**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) - 2*a**2*b*d**2*m*x*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*c**2*m**2*(a + b*x)**
m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 5*a*b**2*c**2*m*(a + b*x
)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c**2*(a + b*
x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*a*b**2*c*d*m**2*x*
(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 6*a*b**2*c*d*
m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + a*b**2*d*
*2*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) +
a*b**2*d**2*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b*
*3) + b**3*c**2*m**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m +
6*b**3) + 5*b**3*c**2*m*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*m
 + 6*b**3) + 6*b**3*c**2*x*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 + 11*b**3*
m + 6*b**3) + 2*b**3*c*d*m**2*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m**2 +
11*b**3*m + 6*b**3) + 8*b**3*c*d*m*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3*m*
*2 + 11*b**3*m + 6*b**3) + 6*b**3*c*d*x**2*(a + b*x)**m/(b**3*m**3 + 6*b**3
*m**2 + 11*b**3*m + 6*b**3) + b**3*d**2*m**2*x**3*(a + b*x)**m/(b**3*m**3 +
 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 3*b**3*d**2*m*x**3*(a + b*x)**m/(b**3*
m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3) + 2*b**3*d**2*x**3*(a + b*x)**m/(b
**3*m**3 + 6*b**3*m**2 + 11*b**3*m + 6*b**3), True))
```

3.1557 $\int (a+bx)^m(c+dx)\,dx$

Optimal. Leaf size=46

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{(bc-ad)(a+bx)^{m+1}}{b^2(m+1)}+\frac{d(a+bx)^{m+2}}{b^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((b*c - a*d)*(a + b*x)^(1 + m))/(b^2*(1 + m)) + (d*(a + b*x)^(2 + m))/(b^2*
(2 + m))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^m(c+dx)\,dx &= \int \left(\frac{(bc-ad)(a+bx)^m}{b}+\frac{d(a+bx)^{1+m}}{b}\right)dx\\ &= \frac{(bc-ad)(a+bx)^{1+m}}{b^2(1+m)}+\frac{d(a+bx)^{2+m}}{b^2(2+m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.89

$$\frac{(a+bx)^{m+1}(-ad+bc(m+2)+bd(m+1)x)}{b^2(m+1)(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(-(a*d) + b*c*(2 + m) + b*d*(1 + m)*x))/(b^2*(1 + m)*(2
+ m))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x), x]
```

fricas [A] time = 0.73, size = 83, normalized size = 1.80

$$\frac{\left(abcm+2\,abc-a^{2}d+\left(b^{2}dm+b^{2}d\right)x^{2}+\left(2\,b^{2}c+\left(b^{2}c+abd\right)m\right)x\right)(bx+a)^{m}}{b^{2}m^{2}+3\,b^{2}m+2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="fricas")
```

```
[Out] (a*b*c*m + 2*a*b*c - a^2*d + (b^2*d*m + b^2*d)*x^2 + (2*b^2*c + (b^2*c + a*
b*d)*m)*x)*(b*x + a)^m/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

giac [B] time = 0.86, size = 132, normalized size = 2.87

$$\frac{(bx+a)^{m}b^{2}dmx^{2}+(bx+a)^{m}b^{2}cmx+(bx+a)^{m}abdmx+(bx+a)^{m}b^{2}dx^{2}+(bx+a)^{m}abcm+2\,(bx+a)^{m}b^{2}cx+2\,(bx+a)^{m}abc-(bx+a)^{m}a^{2}d}{b^{2}m^{2}+3\,b^{2}m+2\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^2*d*m*x^2 + (b*x + a)^m*b^2*c*m*x + (b*x + a)^m*a*b*d*m*x +
(b*x + a)^m*b^2*d*x^2 + (b*x + a)^m*a*b*c*m + 2*(b*x + a)^m*b^2*c*x + 2*(b*
x + a)^m*a*b*c - (b*x + a)^m*a^2*d)/(b^2*m^2 + 3*b^2*m + 2*b^2)
```

maple [A] time = 0.00, size = 49, normalized size = 1.07

$$-\frac{(-bdmx-bcm-bdx+ad-2bc)\,(bx+a)^{m+1}}{\left(m^{2}+3m+2\right)b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c),x)
```

```
[Out] -(b*x+a)^(m+1)*(-b*d*m*x-b*c*m-b*d*x+a*d-2*b*c)/b^2/(m^2+3*m+2)
```

maxima [A] time = 1.06, size = 63, normalized size = 1.37

$$\frac{\left(b^{2}(m+1)x^{2}+abmx-a^{2}\right)(bx+a)^{m}d}{\left(m^{2}+3\,m+2\right)b^{2}}+\frac{(bx+a)^{m+1}c}{b(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d/((m^2 + 3*m + 2)*b^2) + (b*
x + a)^(m + 1)*c/(b*(m + 1))
```

mupad [B] time = 0.48, size = 88, normalized size = 1.91

$$(a+b\,x)^{m}\left(\frac{a\,\left(2\,b\,c-a\,d+b\,c\,m\right)}{b^{2}\,\left(m^{2}+3\,m+2\right)}+\frac{x\,\left(2\,b^{2}\,c+b^{2}\,c\,m+a\,b\,d\,m\right)}{b^{2}\,\left(m^{2}+3\,m+2\right)}+\frac{d\,x^{2}\,\left(m+1\right)}{m^{2}+3\,m+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m*(c + d*x),x)
```

```
[Out] (a + b*x)^m*((a*(2*b*c - a*d + b*c*m))/(b^2*(3*m + m^2 + 2)) + (x*(2*b^2*c
+ b^2*c*m + a*b*d*m))/(b^2*(3*m + m^2 + 2)) + (d*x^2*(m + 1))/(3*m + m^2 +
2))
```

sympy [A] time = 0.86, size = 377, normalized size = 8.20

$$\begin{cases} a^m\left(cx+\frac{dx^2}{2}\right) & \text{for } b=0 \\ \frac{ad\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x}+\frac{ad}{ab^2+b^3x}-\frac{bc}{ab^2+b^3x}+\frac{bdx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } m=-2 \\ -\frac{ad\log\left(\frac{a}{b}+x\right)}{b^2}+\frac{c\log\left(\frac{a}{b}+x\right)}{b}+\frac{dx}{b} & \text{for } m=-1 \\ -\frac{a^2d(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abcm(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2abc(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{abdmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2cmx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{2b^2cx(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2dmx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2}+\frac{b^2dx^2(a+bx)^m}{b^2m^2+3b^2m+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c),x)
```

```
[Out] Piecewise((a**m*(c*x + d*x**2/2), Eq(b, 0)), (a*d*log(a/b + x)/(a*b**2 + b*
*3*x) + a*d/(a*b**2 + b**3*x) - b*c/(a*b**2 + b**3*x) + b*d*x*log(a/b + x)/
(a*b**2 + b**3*x), Eq(m, -2)), (-a*d*log(a/b + x)/b**2 + c*log(a/b + x)/b +
 d*x/b, Eq(m, -1)), (-a**2*d*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 a*b*c*m*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + 2*a*b*c*(a + b*x)**
m/(b**2*m**2 + 3*b**2*m + 2*b**2) + a*b*d*m*x*(a + b*x)**m/(b**2*m**2 + 3*b
**2*m + 2*b**2) + b**2*c*m*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) +
 2*b**2*c*x*(a + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*m*x**2*(a
 + b*x)**m/(b**2*m**2 + 3*b**2*m + 2*b**2) + b**2*d*x**2*(a + b*x)**m/(b**2
*m**2 + 3*b**2*m + 2*b**2), True))
```

3.1558 $\int (a+bx)^3(c+dx)^n\,dx$

Optimal. Leaf size=111

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Rubi [A] time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$-\frac{3b^2(bc-ad)(c+dx)^{n+3}}{d^4(n+3)}-\frac{(bc-ad)^3(c+dx)^{n+1}}{d^4(n+1)}+\frac{3b(bc-ad)^2(c+dx)^{n+2}}{d^4(n+2)}+\frac{b^3(c+dx)^{n+4}}{d^4(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)^3*(c + d*x)^(1 + n))/(d^4*(1 + n))) + (3*b*(b*c - a*d)^2*(c
+ d*x)^(2 + n))/(d^4*(2 + n)) - (3*b^2*(b*c - a*d)*(c + d*x)^(3 + n))/(d^4*
(3 + n)) + (b^3*(c + d*x)^(4 + n))/(d^4*(4 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^3(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^3(c+dx)^n}{d^3}+\frac{3b(bc-ad)^2(c+dx)^{1+n}}{d^3}-\frac{3b^2(bc-ad)(c+dx)^{2+n}}{d^3}+\frac{b^3\right.\\ &= -\frac{(bc-ad)^3(c+dx)^{1+n}}{d^4(1+n)}+\frac{3b(bc-ad)^2(c+dx)^{2+n}}{d^4(2+n)}-\frac{3b^2(bc-ad)(c+dx)^{3+n}}{d^4(3+n)}+\frac{b^3(c}{d^4}\end{aligned}$$

Mathematica [A] time = 0.11, size = 95, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{3b^2(c+dx)^2(bc-ad)}{n+3}+\frac{3b(c+dx)(bc-ad)^2}{n+2}-\frac{(bc-ad)^3}{n+1}+\frac{b^3(c+dx)^3}{n+4}\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-((b*c - a*d)^3/(1 + n)) + (3*b*(b*c - a*d)^2*(c + d*x)
)/(2 + n) - (3*b^2*(b*c - a*d)*(c + d*x)^2)/(3 + n) + (b^3*(c + d*x)^3)/(4
+ n)))/d^4
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^3(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^3*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^3*(c + d*x)^n, x]
```

fricas [B] time = 1.04, size = 496, normalized size = 4.47

$$\frac{\left(a^3cd^3n^3 - 6\,b^3c^4 + 24\,ab^2c^3d - 36\,a^2bc^2d^2 + 24\,a^3cd^3 + \left(b^3d^4n^3 + 6\,b^3d^4n^2 + 11\,b^3d^4n + 6\,b^3d^4\right)x^4 + \left(24\,ab^2d^4 + \left(b^3cd^3 + 3\,ab^2d^4\right)n^3 + 3\left(b^3cd^3 + 7\,ab^2d^4\right)n^2 + 2\left(b^3cd^3 + 21\,ab^2d^4\right)n\right)x^3 - 3\left(a^2bc^2d^2 - 3\,a^3cd^3\right)n^2 + 3\left(12\,a^2bd^4 + \left(ab^2cd^3 + a^2bd^4\right)n^3 - \left(b^3c^2d^2 - 5\,ab^2cd^3 - 8\,a^2bd^4\right)n^2 - \left(b^3c^2d^2 - 4\,ab^2cd^3 - 19\,a^2bd^4\right)n\right)x^2 + \left(6\,ab^2c^3d - 21\,a^2bc^2d^2 + 26\,a^3cd^3\right)n + \left(24\,a^3d^4 + \left(3\,a^2bcd^3 + a^3d^4\right)n^3 - 3\left(2\,ab^2c^2d^2 - 7\,a^2bcd^3 - 3\,a^3d^4\right)n^2 + 2\left(3\,b^3c^3d - 12\,ab^2c^2d^2 + 18\,a^2bcd^3 + 13\,a^3d^4\right)n\right)x\right)(dx+c)^n}{d^4n^4 + 10\,d^4n^3 + 35\,d^4n^2 + 50\,d^4n + 24\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^3*c*d^3*n^3 - 6*b^3*c^4 + 24*a*b^2*c^3*d - 36*a^2*b*c^2*d^2 + 24*a^3*c*d
^3 + (b^3*d^4*n^3 + 6*b^3*d^4*n^2 + 11*b^3*d^4*n + 6*b^3*d^4)*x^4 + (24*a*b
^2*d^4 + (b^3*c*d^3 + 3*a*b^2*d^4)*n^3 + 3*(b^3*c*d^3 + 7*a*b^2*d^4)*n^2 +
2*(b^3*c*d^3 + 21*a*b^2*d^4)*n)*x^3 - 3*(a^2*b*c^2*d^2 - 3*a^3*c*d^3)*n^2 +
 3*(12*a^2*b*d^4 + (a*b^2*c*d^3 + a^2*b*d^4)*n^3 - (b^3*c^2*d^2 - 5*a*b^2*c
*d^3 - 8*a^2*b*d^4)*n^2 - (b^3*c^2*d^2 - 4*a*b^2*c*d^3 - 19*a^2*b*d^4)*n)*x
^2 + (6*a*b^2*c^3*d - 21*a^2*b*c^2*d^2 + 26*a^3*c*d^3)*n + (24*a^3*d^4 + (3
*a^2*b*c*d^3 + a^3*d^4)*n^3 - 3*(2*a*b^2*c^2*d^2 - 7*a^2*b*c*d^3 - 3*a^3*d^
4)*n^2 + 2*(3*b^3*c^3*d - 12*a*b^2*c^2*d^2 + 18*a^2*b*c*d^3 + 13*a^3*d^4)*n
)*x)*(d*x + c)^n/(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

giac [B] time = 0.98, size = 833, normalized size = 7.50

$$\frac{(dx+c)^n b^3d^4n^3x^4 + (dx+c)^n b^3cd^3n^3x^3 + 3\,(dx+c)^n ab^2d^4n^3x^3 + 6\,(dx+c)^n b^3d^4n^2x^4 + 3\,(dx+c)^n ab^2cd^3n^3x^2 + 3\,(dx+c)^n a^2bd^4n^3x^2 + 3\,(dx+c)^n b^3cd^3n^2x^3 + 21\,(dx+c)^n ab^2d^4n^2x^3 + 11\,(dx+c)^n b^3d^4nx^4 + 3\,(dx+c)^n a^2bcd^3n^3x + (dx+c)^n a^3d^4n^3x - 3\,(dx+c)^n b^3c^2d^2n^2x^2 + 15\,(dx+c)^n ab^2cd^3n^2x^2 + 24\,(dx+c)^n a^2bd^4n^2x^2 + 2\,(dx+c)^n b^3cd^3nx^3 + 42\,(dx+c)^n ab^2d^4nx^3 + 6\,(dx+c)^n b^3d^4x^4 + (dx+c)^n a^3cd^3n^3 - 6\,(dx+c)^n ab^2c^2d^2n^2x + 21\,(dx+c)^n a^2bcd^3n^2x + 9\,(dx+c)^n a^3d^4n^2x - 3\,(dx+c)^n b^3c^2d^2nx^2 + 12\,(dx+c)^n ab^2cd^3nx^2 + 57\,(dx+c)^n a^2bd^4nx^2 + 24\,(dx+c)^n ab^2d^4x^3 - 3\,(dx+c)^n a^2bc^2d^2n^2 + 9\,(dx+c)^n a^3cd^3n^2 + 6\,(dx+c)^n b^3c^3dnx - 24\,(dx+c)^n ab^2c^2d^2nx + 36\,(dx+c)^n a^2bcd^3nx + 26\,(dx+c)^n a^3d^4nx + 36\,(dx+c)^n a^2bd^4x^2 + 6\,(dx+c)^n ab^2c^3dn - 21\,(dx+c)^n a^2bc^2d^2n + 26\,(dx+c)^n a^3cd^3n + 24\,(dx+c)^n a^3d^4x - 6\,(dx+c)^n b^3c^4 + 24\,(dx+c)^n ab^2c^3d - 36\,(dx+c)^n a^2bc^2d^2 + 24\,(dx+c)^n a^3cd^3}{d^4n^4 + 10\,d^4n^3 + 35\,d^4n^2 + 50\,d^4n + 24\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^3*d^4*n^3*x^4 + (d*x + c)^n*b^3*c*d^3*n^3*x^3 + 3*(d*x + c)^
n*a*b^2*d^4*n^3*x^3 + 6*(d*x + c)^n*b^3*d^4*n^2*x^4 + 3*(d*x + c)^n*a*b^2*c
*d^3*n^3*x^2 + 3*(d*x + c)^n*a^2*b*d^4*n^3*x^2 + 3*(d*x + c)^n*b^3*c*d^3*n^
2*x^3 + 21*(d*x + c)^n*a*b^2*d^4*n^2*x^3 + 11*(d*x + c)^n*b^3*d^4*n*x^4 + 3
*(d*x + c)^n*a^2*b*c*d^3*n^3*x + (d*x + c)^n*a^3*d^4*n^3*x - 3*(d*x + c)^n*
b^3*c^2*d^2*n^2*x^2 + 15*(d*x + c)^n*a*b^2*c*d^3*n^2*x^2 + 24*(d*x + c)^n*a
^2*b*d^4*n^2*x^2 + 2*(d*x + c)^n*b^3*c*d^3*n*x^3 + 42*(d*x + c)^n*a*b^2*d^4
*n*x^3 + 6*(d*x + c)^n*b^3*d^4*x^4 + (d*x + c)^n*a^3*c*d^3*n^3 - 6*(d*x + c
)^n*a*b^2*c^2*d^2*n^2*x + 21*(d*x + c)^n*a^2*b*c*d^3*n^2*x + 9*(d*x + c)^n*
a^3*d^4*n^2*x - 3*(d*x + c)^n*b^3*c^2*d^2*n*x^2 + 12*(d*x + c)^n*a*b^2*c*d^
3*n*x^2 + 57*(d*x + c)^n*a^2*b*d^4*n*x^2 + 24*(d*x + c)^n*a*b^2*d^4*x^3 - 3
*(d*x + c)^n*a^2*b*c^2*d^2*n^2 + 9*(d*x + c)^n*a^3*c*d^3*n^2 + 6*(d*x + c)^
n*b^3*c^3*d*n*x - 24*(d*x + c)^n*a*b^2*c^2*d^2*n*x + 36*(d*x + c)^n*a^2*b*c
*d^3*n*x + 26*(d*x + c)^n*a^3*d^4*n*x + 36*(d*x + c)^n*a^2*b*d^4*x^2 + 6*(d
*x + c)^n*a*b^2*c^3*d*n - 21*(d*x + c)^n*a^2*b*c^2*d^2*n + 26*(d*x + c)^n*a
^3*c*d^3*n + 24*(d*x + c)^n*a^3*d^4*x - 6*(d*x + c)^n*b^3*c^4 + 24*(d*x + c
)^n*a*b^2*c^3*d - 36*(d*x + c)^n*a^2*b*c^2*d^2 + 24*(d*x + c)^n*a^3*c*d^3)/
(d^4*n^4 + 10*d^4*n^3 + 35*d^4*n^2 + 50*d^4*n + 24*d^4)
```

maple [B] time = 0.01, size = 386, normalized size = 3.48

$$\frac{\left(b^3d^3n^3x^3 + 3ab^2d^3n^3x^2 + 6b^3d^3n^2x^3 + 3a^2bd^3n^3x + 21ab^2d^3n^2x^2 - 3b^3cd^2n^2x^2 + 11b^3d^3nx^3 + a^3d^3n^3 + 24a^2bd^3n^2x - 6ab^2cd^2n^2x + 42ab^2d^3nx^2 - 9b^3cd^2nx^2 + 6b^3d^3x^3 + 9a^3d^3n^2 - 3a^2bcd^2n^2 + 57a^2bd^3nx - 30ab^2cd^2nx + 24ab^2d^3x^2 + 6b^3c^2dnx - 6b^3cd^2x^2 + 26a^3d^3n - 21a^2bcd^2n + 36a^2bd^3x + 6ab^2c^2dn - 24ab^2cd^2x + 6b^3c^2dx + 24a^3d^3 - 36a^2bcd^2 + 24ab^2c^2d - 6b^3c^3\right)(dx+c)^{n+1}}{\left(n^4 + 10n^3 + 35n^2 + 50n + 24\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)*(b^3*d^3*n^3*x^3+3*a*b^2*d^3*n^3*x^2+6*b^3*d^3*n^2*x^3+3*a^2*
b*d^3*n^3*x+21*a*b^2*d^3*n^2*x^2-3*b^3*c*d^2*n^2*x^2+11*b^3*d^3*n*x^3+a^3*d
^3*n^3+24*a^2*b*d^3*n^2*x-6*a*b^2*c*d^2*n^2*x+42*a*b^2*d^3*n*x^2-9*b^3*c*d^
2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-3*a^2*b*c*d^2*n^2+57*a^2*b*d^3*n*x-30*a
*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2+6*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*
n-21*a^2*b*c*d^2*n+36*a^2*b*d^3*x+6*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x+6*b^3*c^
```

```
2*d*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/d^4/(n^4+10*n^3+3
5*n^2+50*n+24)
```

maxima [B] time = 1.30, size = 246, normalized size = 2.22

$$\frac{3\left(d^2(n+1)x^2+cdnx-c^2\right)(dx+c)^n a^2 b}{\left(n^2+3n+2\right)d^2}+\frac{(dx+c)^{n+1}a^3}{d(n+1)}+\frac{3\left(\left(n^2+3n+2\right)d^3x^3+\left(n^2+n\right)cd^2x^2-2c^2dnx+2c^3\right)(dx+c)^n ab^2}{\left(n^3+6n^2+11n+6\right)d^3}+\frac{\left(\left(n^3+6n^2+11n+6\right)d^4x^4+\left(n^3+3n^2+2n\right)cd^3x^3-3\left(n^2+n\right)c^2d^2x^2+6c^3dnx-6c^4\right)(dx+c)^n b^3}{\left(n^4+10n^3+35n^2+50n+24\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 3*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a^2*b/((n^2 + 3*n + 2)*d^2)
 + (d*x + c)^(n + 1)*a^3/(d*(n + 1)) + 3*((n^2 + 3*n + 2)*d^3*x^3 + (n^2 +
n)*c*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*a*b^2/((n^3 + 6*n^2 + 11*n
+ 6)*d^3) + ((n^3 + 6*n^2 + 11*n + 6)*d^4*x^4 + (n^3 + 3*n^2 + 2*n)*c*d^3*x
^3 - 3*(n^2 + n)*c^2*d^2*x^2 + 6*c^3*d*n*x - 6*c^4)*(d*x + c)^n*b^3/((n^4 +
 10*n^3 + 35*n^2 + 50*n + 24)*d^4)
```

mupad [B] time = 0.91, size = 478, normalized size = 4.31

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^4\,n^3+9\,a^3\,d^4\,n^2+26\,a^3\,d^4\,n+24\,a^3\,d^4+3\,a^2\,b\,c\,d^3\,n^3+21\,a^2\,b\,c\,d^3\,n^2+36\,a^2\,b\,c\,d^3\,n-6\,a\,b^2\,c^2\,d^2\,n^2-24\,a\,b^2\,c^2\,d^2\,n+6\,b^3\,c^3\,d\,n\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^3\,x^4\,(c+d\,x)^n\,\left(n^3+6\,n^2+11\,n+6\right)}{n^4+10\,n^3+35\,n^2+50\,n+24}+\frac{c\,(c+d\,x)^n\,\left(a^3\,d^3\,n^3+9\,a^3\,d^3\,n^2+26\,a^3\,d^3\,n+24\,a^3\,d^3-3\,a^2\,b\,c\,d^2\,n^2-21\,a^2\,b\,c\,d^2\,n-36\,a^2\,b\,c\,d^2+6\,a\,b^2\,c^2\,d\,n+24\,a\,b^2\,c^2\,d-6\,b^3\,c^3\right)}{d^4\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{3\,b\,x^2\,(n+1)\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2+a\,b\,c\,d\,n^2+4\,a\,b\,c\,d\,n-b^2\,c^2\,n\right)}{d^2\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}+\frac{b^2\,x^3\,(c+d\,x)^n\,(12\,a\,d+3\,a\,d\,n+b\,c\,n)\,\left(n^2+3\,n+2\right)}{d\,\left(n^4+10\,n^3+35\,n^2+50\,n+24\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^3*(c + d*x)^n,x)
```

```
[Out] (x*(c + d*x)^n*(24*a^3*d^4 + 26*a^3*d^4*n + 9*a^3*d^4*n^2 + a^3*d^4*n^3 + 6
*b^3*c^3*d*n + 36*a^2*b*c*d^3*n - 24*a*b^2*c^2*d^2*n + 21*a^2*b*c*d^3*n^2 +
 3*a^2*b*c*d^3*n^3 - 6*a*b^2*c^2*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n^3 + n
^4 + 24)) + (b^3*x^4*(c + d*x)^n*(11*n + 6*n^2 + n^3 + 6))/(50*n + 35*n^2 +
 10*n^3 + n^4 + 24) + (c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n
 + 9*a^3*d^3*n^2 + a^3*d^3*n^3 + 24*a*b^2*c^2*d - 36*a^2*b*c*d^2 + 6*a*b^2*
c^2*d*n - 21*a^2*b*c*d^2*n - 3*a^2*b*c*d^2*n^2))/(d^4*(50*n + 35*n^2 + 10*n
^3 + n^4 + 24)) + (3*b*x^2*(n + 1)*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n -
b^2*c^2*n + a^2*d^2*n^2 + 4*a*b*c*d*n + a*b*c*d*n^2))/(d^2*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24)) + (b^2*x^3*(c + d*x)^n*(12*a*d + 3*a*d*n + b*c*n)*(3*n
 + n^2 + 2))/(d*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))
```

sympy [A] time = 4.44, size = 4058, normalized size = 36.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**3*x + 3*a**2*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4), Eq(
d, 0)), (-2*a**3*d**3/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d*
*7*x**3) - 3*a**2*b*c*d**2/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 +
 6*d**7*x**3) - 9*a**2*b*d**3*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x
**2 + 6*d**7*x**3) - 6*a*b**2*c**2*d/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d
**6*x**2 + 6*d**7*x**3) - 18*a*b**2*c*d**2*x/(6*c**3*d**4 + 18*c**2*d**5*x
+ 18*c*d**6*x**2 + 6*d**7*x**3) - 18*a*b**2*d**3*x**2/(6*c**3*d**4 + 18*c**
2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*c**3*log(c/d + x)/(6*c**3
*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 11*b**3*c**3/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c**2*d*x
*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3)
 + 27*b**3*c**2*d*x/(6*c**3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7
*x**3) + 18*b**3*c*d**2*x**2*log(c/d + x)/(6*c**3*d**4 + 18*c**2*d**5*x + 1
8*c*d**6*x**2 + 6*d**7*x**3) + 18*b**3*c*d**2*x**2/(6*c**3*d**4 + 18*c**2*d
**5*x + 18*c*d**6*x**2 + 6*d**7*x**3) + 6*b**3*d**3*x**3*log(c/d + x)/(6*c*
*3*d**4 + 18*c**2*d**5*x + 18*c*d**6*x**2 + 6*d**7*x**3), Eq(n, -4)), (-a**
3*d**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 3*a**2*b*c*d**2/(2*c**2*d
```

```
**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*a**2*b*d**3*x/(2*c**2*d**4 + 4*c*d**5*x
 + 2*d**6*x**2) + 6*a*b**2*c**2*d*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x +
2*d**6*x**2) + 9*a*b**2*c**2*d/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 1
2*a*b**2*c*d**2*x/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) + 6*a*b**2*d**3*
x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 6*b**3*c**3*lo
g(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2) - 9*b**3*c**3/(2*c**2*d
**4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x*log(c/d + x)/(2*c**2*d**
4 + 4*c*d**5*x + 2*d**6*x**2) - 12*b**3*c**2*d*x/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) - 6*b**3*c*d**2*x**2*log(c/d + x)/(2*c**2*d**4 + 4*c*d**5*x
+ 2*d**6*x**2) + 2*b**3*d**3*x**3/(2*c**2*d**4 + 4*c*d**5*x + 2*d**6*x**2),
 Eq(n, -3)), (-2*a**3*d**3/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2*log(c/d
+ x)/(2*c*d**4 + 2*d**5*x) + 6*a**2*b*c*d**2/(2*c*d**4 + 2*d**5*x) + 6*a**2
*b*d**3*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d*log(c/d + x
)/(2*c*d**4 + 2*d**5*x) - 12*a*b**2*c**2*d/(2*c*d**4 + 2*d**5*x) - 12*a*b**
2*c*d**2*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*a*b**2*d**3*x**2/(2*c*d**
4 + 2*d**5*x) + 6*b**3*c**3*log(c/d + x)/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**
3/(2*c*d**4 + 2*d**5*x) + 6*b**3*c**2*d*x*log(c/d + x)/(2*c*d**4 + 2*d**5*x
) - 3*b**3*c*d**2*x**2/(2*c*d**4 + 2*d**5*x) + b**3*d**3*x**3/(2*c*d**4 + 2
*d**5*x), Eq(n, -2)), (a**3*log(c/d + x)/d - 3*a**2*b*c*log(c/d + x)/d**2 +
 3*a**2*b*x/d + 3*a*b**2*c**2*log(c/d + x)/d**3 - 3*a*b**2*c*x/d**2 + 3*a*b
**2*x**2/(2*d) - b**3*c**3*log(c/d + x)/d**4 + b**3*c**2*x/d**3 - b**3*c*x*
*2/(2*d**2) + b**3*x**3/(3*d), Eq(n, -1)), (a**3*c*d**3*n**3*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9*a**3*c*d
**3*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n
 + 24*d**4) + 26*a**3*c*d**3*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d
**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*c*d**3*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + a**3*d**4*n**3*x*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 9
*a**3*d**4*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 5
0*d**4*n + 24*d**4) + 26*a**3*d**4*n*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n*
*3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a**3*d**4*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*a**2*b*c**2
*d**2*n**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*
n + 24*d**4) - 21*a**2*b*c**2*d**2*n*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 36*a**2*b*c**2*d**2*(c + d*x)**n/(
d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**2*b*c
*d**3*n**3*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**
4*n + 24*d**4) + 21*a**2*b*c*d**3*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*
n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 36*a**2*b*c*d**3*n*x*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a**
2*b*d**4*n**3*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
50*d**4*n + 24*d**4) + 24*a**2*b*d**4*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 1
0*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 57*a**2*b*d**4*n*x**2*(
c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4)
 + 36*a**2*b*d**4*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**
2 + 50*d**4*n + 24*d**4) + 6*a*b**2*c**3*d*n*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 24*a*b**2*c**3*d*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 6*a*b
**2*c**2*d**2*n**2*x*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) - 24*a*b**2*c**2*d**2*n*x*(c + d*x)**n/(d**4*n**4 +
10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2*c*d**3*n**3*x
**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*
d**4) + 15*a*b**2*c*d**3*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 +
 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 12*a*b**2*c*d**3*n*x**2*(c + d*x)**n
/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*a*b**2
*d**4*n**3*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*
d**4*n + 24*d**4) + 21*a*b**2*d**4*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 42*a*b**2*d**4*n*x**3*(c +
```

```
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) +
24*a*b**2*d**4*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 +
 50*d**4*n + 24*d**4) - 6*b**3*c**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3
+ 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*c**3*d*n*x*(c + d*x)**n/(d**
4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) - 3*b**3*c**2*d
**2*n**2*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d*
*4*n + 24*d**4) - 3*b**3*c**2*d**2*n*x**2*(c + d*x)**n/(d**4*n**4 + 10*d**4
*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*c*d**3*n**3*x**3*(c + d*
x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 3*b
**3*c*d**3*n**2*x**3*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 2*b**3*c*d**3*n*x**3*(c + d*x)**n/(d**4*n**4 + 10*
d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + b**3*d**4*n**3*x**4*(c +
d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6
*b**3*d**4*n**2*x**4*(c + d*x)**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2
+ 50*d**4*n + 24*d**4) + 11*b**3*d**4*n*x**4*(c + d*x)**n/(d**4*n**4 + 10*d
**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4) + 6*b**3*d**4*x**4*(c + d*x)
**n/(d**4*n**4 + 10*d**4*n**3 + 35*d**4*n**2 + 50*d**4*n + 24*d**4), True))
```

3.1559 $\int (a+bx)^2(c+dx)^n\,dx$

Optimal. Leaf size=78

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {43}

$$\frac{(bc-ad)^2(c+dx)^{n+1}}{d^3(n+1)}-\frac{2b(bc-ad)(c+dx)^{n+2}}{d^3(n+2)}+\frac{b^2(c+dx)^{n+3}}{d^3(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((b*c - a*d)^2*(c + d*x)^(1 + n))/(d^3*(1 + n)) - (2*b*(b*c - a*d)*(c + d*x
)^(2 + n))/(d^3*(2 + n)) + (b^2*(c + d*x)^(3 + n))/(d^3*(3 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^2(c+dx)^n\,dx &= \int \left(\frac{(-bc+ad)^2(c+dx)^n}{d^2}-\frac{2b(bc-ad)(c+dx)^{1+n}}{d^2}+\frac{b^2(c+dx)^{2+n}}{d^2}\right)dx\\ &= \frac{(bc-ad)^2(c+dx)^{1+n}}{d^3(1+n)}-\frac{2b(bc-ad)(c+dx)^{2+n}}{d^3(2+n)}+\frac{b^2(c+dx)^{3+n}}{d^3(3+n)}\end{aligned}$$

Mathematica [A] time = 0.10, size = 67, normalized size = 0.86

$$\frac{(c+dx)^{n+1}\left(-\frac{2b(c+dx)(bc-ad)}{n+2}+\frac{(bc-ad)^2}{n+1}+\frac{b^2(c+dx)^2}{n+3}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*((b*c - a*d)^2/(1 + n) - (2*b*(b*c - a*d)*(c + d*x))/(2
+ n) + (b^2*(c + d*x)^2)/(3 + n)))/d^3
```

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (a+bx)^2(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^2*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^2*(c + d*x)^n, x]
```

fricas [**B**]   time = 1.35, size = 237, normalized size = 3.04

$$\frac{\left(a^2cd^2n^2 + 2\,b^2c^3 - 6\,abc^2d + 6\,a^2cd^2 + \left(b^2d^3n^2 + 3\,b^2d^3n + 2\,b^2d^3\right)x^3 + \left(6\,abd^3 + \left(b^2cd^2 + 2\,abd^3\right)n^2 + \left(b^2cd^2 + 8\,abd^3\right)n\right)x^2 - \left(2\,abc^2d - 5\,a^2cd^2\right)n + \left(6\,a^2d^3 + \left(2\,abcd^2 + a^2d^3\right)n^2 - \left(2\,b^2c^2d - 6\,abcd^2 - 5\,a^2d^3\right)n\right)x\right)(dx+c)^n}{d^3n^3 + 6\,d^3n^2 + 11\,d^3n + 6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a^2*c*d^2*n^2 + 2*b^2*c^3 - 6*a*b*c^2*d + 6*a^2*c*d^2 + (b^2*d^3*n^2 + 3*b
^2*d^3*n + 2*b^2*d^3)*x^3 + (6*a*b*d^3 + (b^2*c*d^2 + 2*a*b*d^3)*n^2 + (b^2
*c*d^2 + 8*a*b*d^3)*n)*x^2 - (2*a*b*c^2*d - 5*a^2*c*d^2)*n + (6*a^2*d^3 + (
2*a*b*c*d^2 + a^2*d^3)*n^2 - (2*b^2*c^2*d - 6*a*b*c*d^2 - 5*a^2*d^3)*n)*x)*
(d*x + c)^n/(d^3*n^3 + 6*d^3*n^2 + 11*d^3*n + 6*d^3)
```

giac [**B**]   time = 0.93, size = 385, normalized size = 4.94

$$\frac{(dx+c)^n b^2d^3n^2x^3 + (dx+c)^n b^2cd^2n^2x^2 + 2\,(dx+c)^n abd^3n^2x^2 + 3\,(dx+c)^n b^2d^3nx^3 + 2\,(dx+c)^n abcd^2n^2x + (dx+c)^n a^2d^3n^2x + (dx+c)^n b^2cd^2nx^2 + 8\,(dx+c)^n abd^3nx^2 + 2\,(dx+c)^n b^2d^3x^3 + (dx+c)^n a^2cd^2n^2 - 2\,(dx+c)^n b^2c^2dnx + 6\,(dx+c)^n abcd^2nx + 5\,(dx+c)^n a^2d^3nx + 6\,(dx+c)^n abd^3x^2 - 2\,(dx+c)^n abc^2dn + 5\,(dx+c)^n a^2cd^2n + 6\,(dx+c)^n a^2d^3x + 2\,(dx+c)^n b^2c^3 - 6\,(dx+c)^n abc^2d + 6\,(dx+c)^n a^2cd^2}{d^3n^3 + 6\,d^3n^2 + 11\,d^3n + 6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b^2*d^3*n^2*x^3 + (d*x + c)^n*b^2*c*d^2*n^2*x^2 + 2*(d*x + c)^
n*a*b*d^3*n^2*x^2 + 3*(d*x + c)^n*b^2*d^3*n*x^3 + 2*(d*x + c)^n*a*b*c*d^2*n
^2*x + (d*x + c)^n*a^2*d^3*n^2*x + (d*x + c)^n*b^2*c*d^2*n*x^2 + 8*(d*x + c
)^n*a*b*d^3*n*x^2 + 2*(d*x + c)^n*b^2*d^3*x^3 + (d*x + c)^n*a^2*c*d^2*n^2 -
 2*(d*x + c)^n*b^2*c^2*d*n*x + 6*(d*x + c)^n*a*b*c*d^2*n*x + 5*(d*x + c)^n*
a^2*d^3*n*x + 6*(d*x + c)^n*a*b*d^3*x^2 - 2*(d*x + c)^n*a*b*c^2*d*n + 5*(d*
x + c)^n*a^2*c*d^2*n + 6*(d*x + c)^n*a^2*d^3*x + 2*(d*x + c)^n*b^2*c^3 - 6*
(d*x + c)^n*a*b*c^2*d + 6*(d*x + c)^n*a^2*c*d^2)/(d^3*n^3 + 6*d^3*n^2 + 11*
d^3*n + 6*d^3)
```

maple [**B**]   time = 0.01, size = 159, normalized size = 2.04

$$\frac{\left(b^2d^2n^2x^2 + 2ab\,d^2n^2x + 3b^2d^2n\,x^2 + a^2d^2n^2 + 8ab\,d^2nx - 2b^2cdnx + 2b^2x^2d^2 + 5a^2d^2n - 2abcdn + 6ab\,d^2x - 2b^2cdx + 6a^2d^2 - 6abcd + 2b^2c^2\right)(dx+c)^{n+1}}{\left(n^3 + 6n^2 + 11n + 6\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)*(b^2*d^2*n^2*x^2+2*a*b*d^2*n^2*x+3*b^2*d^2*n*x^2+a^2*d^2*n^2+
8*a*b*d^2*n*x-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-2*a*b*c*d*n+6*a*b*d^2
*x-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/d^3/(n^3+6*n^2+11*n+6)
```

maxima [**A**]   time = 1.17, size = 138, normalized size = 1.77

$$\frac{2\left(d^2(n+1)x^2 + cdnx - c^2\right)(dx+c)^n ab}{\left(n^2 + 3\,n + 2\right)d^2} + \frac{(dx+c)^{n+1}a^2}{d(n+1)} + \frac{\left(\left(n^2 + 3\,n + 2\right)d^3x^3 + \left(n^2 + n\right)cd^2x^2 - 2\,c^2dnx + 2\,c^3\right)(dx+c)^n b^2}{\left(n^3 + 6\,n^2 + 11\,n + 6\right)d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] 2*(d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*a*b/((n^2 + 3*n + 2)*d^2) +
 (d*x + c)^(n + 1)*a^2/(d*(n + 1)) + ((n^2 + 3*n + 2)*d^3*x^3 + (n^2 + n)*c
*d^2*x^2 - 2*c^2*d*n*x + 2*c^3)*(d*x + c)^n*b^2/((n^3 + 6*n^2 + 11*n + 6)*d
^3)
```

mupad [**B**]   time = 0.62, size = 226, normalized size = 2.90

$$(c+d\,x)^n\left(\frac{c\left(a^2\,d^2\,n^2 + 5\,a^2\,d^2\,n + 6\,a^2\,d^2 - 2\,a\,b\,c\,d\,n - 6\,a\,b\,c\,d + 2\,b^2\,c^2\right)}{d^3\left(n^3 + 6\,n^2 + 11\,n + 6\right)} + \frac{b^2\,x^3\left(n^2 + 3\,n + 2\right)}{n^3 + 6\,n^2 + 11\,n + 6} + \frac{x\left(a^2\,d^3\,n^2 + 5\,a^2\,d^3\,n + 6\,a^2\,d^3 + 2\,a\,b\,c\,d^2\,n^2 + 6\,a\,b\,c\,d^2\,n - 2\,b^2\,c^2\,d\,n\right)}{d^3\left(n^3 + 6\,n^2 + 11\,n + 6\right)} + \frac{b\,x^2\,(n+1)\,(6\,a\,d + 2\,a\,d\,n + b\,c\,n)}{d\left(n^3 + 6\,n^2 + 11\,n + 6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^2*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(6*a^2*d^2 + 2*b^2*c^2 + 5*a^2*d^2*n + a^2*d^2*n^2 - 6*a*b*
c*d - 2*a*b*c*d*n))/(d^3*(11*n + 6*n^2 + n^3 + 6)) + (b^2*x^3*(3*n + n^2 +
2))/(11*n + 6*n^2 + n^3 + 6) + (x*(6*a^2*d^3 + 5*a^2*d^3*n + a^2*d^3*n^2 -
2*b^2*c^2*d*n + 2*a*b*c*d^2*n^2 + 6*a*b*c*d^2*n))/(d^3*(11*n + 6*n^2 + n^3
+ 6)) + (b*x^2*(n + 1)*(6*a*d + 2*a*d*n + b*c*n))/(d*(11*n + 6*n^2 + n^3 +
6)))
```

sympy [A] time = 2.09, size = 1506, normalized size = 19.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a**2*x + a*b*x**2 + b**2*x**3/3), Eq(d, 0)), (-a**2*d**2/(
2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) - 2*a*b*c*d/(2*c**2*d**3 + 4*c*d**4
*x + 2*d**5*x**2) - 4*a*b*d**2*x/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) +
 2*b**2*c**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 3*b**2
*c**2/(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x*log(c/d + x)/
(2*c**2*d**3 + 4*c*d**4*x + 2*d**5*x**2) + 4*b**2*c*d*x/(2*c**2*d**3 + 4*c*
d**4*x + 2*d**5*x**2) + 2*b**2*d**2*x**2*log(c/d + x)/(2*c**2*d**3 + 4*c*d*
*4*x + 2*d**5*x**2), Eq(n, -3)), (-a**2*d**2/(c*d**3 + d**4*x) + 2*a*b*c*d*
log(c/d + x)/(c*d**3 + d**4*x) + 2*a*b*c*d/(c*d**3 + d**4*x) + 2*a*b*d**2*x
*log(c/d + x)/(c*d**3 + d**4*x) - 2*b**2*c**2*log(c/d + x)/(c*d**3 + d**4*x
) - 2*b**2*c**2/(c*d**3 + d**4*x) - 2*b**2*c*d*x*log(c/d + x)/(c*d**3 + d**
4*x) + b**2*d**2*x**2/(c*d**3 + d**4*x), Eq(n, -2)), (a**2*log(c/d + x)/d -
 2*a*b*c*log(c/d + x)/d**2 + 2*a*b*x/d + b**2*c**2*log(c/d + x)/d**3 - b**2
*c*x/d**2 + b**2*x**2/(2*d), Eq(n, -1)), (a**2*c*d**2*n**2*(c + d*x)**n/(d*
*3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*c*d**2*n*(c + d*x)**n/
(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*c*d**2*(c + d*x)**n
/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + a**2*d**3*n**2*x*(c + d*x
)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 5*a**2*d**3*n*x*(c +
d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a**2*d**3*x*(c +
 d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 2*a*b*c**2*d*n*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) - 6*a*b*c**2*d*(c
 + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*b*c*d**2*n*
*2*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 6*a*b*c*
d**2*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*a*
b*d**3*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3
) + 8*a*b*d**3*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6
*d**3) + 6*a*b*d**3*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n
+ 6*d**3) + 2*b**2*c**3*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**3*n +
 6*d**3) - 2*b**2*c**2*d*n*x*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2 + 11*d**
3*n + 6*d**3) + b**2*c*d**2*n**2*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**3*n**2
 + 11*d**3*n + 6*d**3) + b**2*c*d**2*n*x**2*(c + d*x)**n/(d**3*n**3 + 6*d**
3*n**2 + 11*d**3*n + 6*d**3) + b**2*d**3*n**2*x**3*(c + d*x)**n/(d**3*n**3
+ 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 3*b**2*d**3*n*x**3*(c + d*x)**n/(d**3
*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3) + 2*b**2*d**3*x**3*(c + d*x)**n/(
d**3*n**3 + 6*d**3*n**2 + 11*d**3*n + 6*d**3), True))
```

3.1560 $\int (a+bx)(c+dx)^n\,dx$

Optimal. Leaf size=47

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Rubi [A] time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{b(c+dx)^{n+2}}{d^2(n+2)}-\frac{(bc-ad)(c+dx)^{n+1}}{d^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] -(((b*c - a*d)*(c + d*x)^(1 + n))/(d^2*(1 + n))) + (b*(c + d*x)^(2 + n))/(d
^2*(2 + n))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx)(c+dx)^n\,dx&=\int\left(\frac{(-bc+ad)(c+dx)^n}{d}+\frac{b(c+dx)^{1+n}}{d}\right)dx\\&=-\frac{(bc-ad)(c+dx)^{1+n}}{d^2(1+n)}+\frac{b(c+dx)^{2+n}}{d^2(2+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.87

$$\frac{(c+dx)^{n+1}(ad(n+2)-bc+bd(n+1)x)}{d^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] ((c + d*x)^(1 + n)*(-(b*c) + a*d*(2 + n) + b*d*(1 + n)*x))/(d^2*(1 + n)*(2
+ n))
```

IntegrateAlgebraic [F] time = 0.03, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)*(c + d*x)^n, x]
```

fricas [A] time = 1.28, size = 83, normalized size = 1.77

$$\frac{\left(acdn - bc^2 + 2\,acd + \left(bd^2n + bd^2\right)x^2 + \left(2\,ad^2 + \left(bcd + ad^2\right)n\right)x\right)(dx + c)^n}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (a*c*d*n - b*c^2 + 2*a*c*d + (b*d^2*n + b*d^2)*x^2 + (2*a*d^2 + (b*c*d + a*
d^2)*n)*x)*(d*x + c)^n/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

giac [B] time = 1.00, size = 132, normalized size = 2.81

$$\frac{(dx+c)^n bd^2nx^2 + (dx+c)^n bcdnx + (dx+c)^n ad^2nx + (dx+c)^n bd^2x^2 + (dx+c)^n acdn + 2\,(dx+c)^n ad^2x - (dx+c)^n bc^2 + 2\,(dx+c)^n acd}{d^2n^2 + 3\,d^2n + 2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] ((d*x + c)^n*b*d^2*n*x^2 + (d*x + c)^n*b*c*d*n*x + (d*x + c)^n*a*d^2*n*x +
(d*x + c)^n*b*d^2*x^2 + (d*x + c)^n*a*c*d*n + 2*(d*x + c)^n*a*d^2*x - (d*x
+ c)^n*b*c^2 + 2*(d*x + c)^n*a*c*d)/(d^2*n^2 + 3*d^2*n + 2*d^2)
```

maple [A] time = 0.00, size = 46, normalized size = 0.98

$$\frac{(bdnx + adn + bdx + 2ad - bc)\,(dx + c)^{n+1}}{\left(n^2 + 3n + 2\right)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^n,x)
```

```
[Out] (d*x+c)^(n+1)*(b*d*n*x+a*d*n+b*d*x+2*a*d-b*c)/d^2/(n^2+3*n+2)
```

maxima [A] time = 1.17, size = 63, normalized size = 1.34

$$\frac{\left(d^2(n+1)x^2 + cdnx - c^2\right)(dx + c)^n b}{\left(n^2 + 3\,n + 2\right)d^2} + \frac{(dx + c)^{n+1}a}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] (d^2*(n + 1)*x^2 + c*d*n*x - c^2)*(d*x + c)^n*b/((n^2 + 3*n + 2)*d^2) + (d*
x + c)^(n + 1)*a/(d*(n + 1))
```

mupad [B] time = 0.49, size = 88, normalized size = 1.87

$$(c + d\,x)^n\left(\frac{c\,(2\,a\,d - b\,c + a\,d\,n)}{d^2\,\left(n^2 + 3\,n + 2\right)} + \frac{b\,x^2\,(n+1)}{n^2 + 3\,n + 2} + \frac{x\,\left(2\,a\,d^2 + a\,d^2\,n + b\,c\,d\,n\right)}{d^2\,\left(n^2 + 3\,n + 2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)*(c + d*x)^n,x)
```

```
[Out] (c + d*x)^n*((c*(2*a*d - b*c + a*d*n))/(d^2*(3*n + n^2 + 2)) + (b*x^2*(n +
1))/(3*n + n^2 + 2) + (x*(2*a*d^2 + a*d^2*n + b*c*d*n))/(d^2*(3*n + n^2 + 2
)))
```

sympy [A] time = 0.82, size = 377, normalized size = 8.02

$$\begin{cases} c^n\left(ax+\frac{bx^2}{2}\right) & \text{for } d=0 \\ -\frac{ad}{cd^2+d^3x}+\frac{bc\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x}+\frac{bc}{cd^2+d^3x}+\frac{bdx\log\left(\frac{c}{d}+x\right)}{cd^2+d^3x} & \text{for } n=-2 \\ \frac{a\log\left(\frac{c}{d}+x\right)}{d}-\frac{bc\log\left(\frac{c}{d}+x\right)}{d^2}+\frac{bx}{d} & \text{for } n=-1 \\ \frac{acdn(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2acd(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{ad^2nx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{2ad^2x(c+dx)^n}{d^2n^2+3d^2n+2d^2}-\frac{bc^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bcdnx(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2nx^2(c+dx)^n}{d^2n^2+3d^2n+2d^2}+\frac{bd^2x^2(c+dx)^n}{d^2n^2+3d^2n+2d^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**n,x)
```

```
[Out] Piecewise((c**n*(a*x + b*x**2/2), Eq(d, 0)), (-a*d/(c*d**2 + d**3*x) + b*c*
log(c/d + x)/(c*d**2 + d**3*x) + b*c/(c*d**2 + d**3*x) + b*d*x*log(c/d + x)
/(c*d**2 + d**3*x), Eq(n, -2)), (a*log(c/d + x)/d - b*c*log(c/d + x)/d**2 +
 b*x/d, Eq(n, -1)), (a*c*d*n*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 2*a*c*d*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + a*d**2*n*x*(c + d*x
)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + 2*a*d**2*x*(c + d*x)**n/(d**2*n**2 +
 3*d**2*n + 2*d**2) - b*c**2*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) +
 b*c*d*n*x*(c + d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*n*x**2*(c
+ d*x)**n/(d**2*n**2 + 3*d**2*n + 2*d**2) + b*d**2*x**2*(c + d*x)**n/(d**2*
n**2 + 3*d**2*n + 2*d**2), True))
```

3.1561 $\int (c+dx)^n \, dx$

Optimal. Leaf size=18

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(c+dx)^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^n,x]
```

```
[Out] (c + d*x)^(1 + n)/(d*(1 + n))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (c+dx)^n \, dx = \frac{(c+dx)^{1+n}}{d(1+n)}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{(c+dx)^{n+1}}{dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^n,x]
```

```
[Out] (c + d*x)^(1 + n)/(d + d*n)
```

IntegrateAlgebraic [F] time = 0.01, size = 0, normalized size = 0.00

$$\int (c+dx)^n \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^n, x]
```

fricas [A] time = 1.25, size = 20, normalized size = 1.11

$$\frac{(dx+c)(dx+c)^n}{dn+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (d*x + c)*(d*x + c)^n/(d*n + d)
```

giac [A] time = 1.02, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="giac")
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

maple [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{(dx+c)^{n+1}}{(n+1)\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^n,x)
[Out] (d*x+c)^(n+1)/d/(n+1)
```

maxima [A] time = 1.12, size = 18, normalized size = 1.00

$$\frac{(dx+c)^{n+1}}{d(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^n,x, algorithm="maxima")
[Out] (d*x + c)^(n + 1)/(d*(n + 1))
```

mupad [B] time = 0.38, size = 18, normalized size = 1.00

$$\frac{(c+d\,x)^{n+1}}{d\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n,x)
[Out] (c + d*x)^(n + 1)/(d*(n + 1))
```

sympy [A] time = 0.06, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(c+dx)^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log(c+dx) & \text{otherwise} \end{cases}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**n,x)
[Out] Piecewise(((c + d*x)**(n + 1)/(n + 1), Ne(n, -1)), (log(c + d*x), True))/d
```

3.1562 $\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=143

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Rubi [A] time = 0.06, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(3-n)(bc-ad)^3}-\frac{(a+bx)^{n-3}(c+dx)^{1-n}}{(3-n)(bc-ad)}+\frac{2d(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(3-n)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(3 - n))) + (2*d*(a +
 b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(2 - n)*(3 - n)) - (2*d^2*
(a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^3*(1 - n)*(2 - n)*(3 - n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}-\frac{(2d)\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx}{(bc-ad)(3-n)}\\ &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}+\frac{\left(2d^2\right)\int (a+bx)^{-2+n}(c+\\ &= -\frac{(a+bx)^{-3+n}(c+dx)^{1-n}}{(bc-ad)(3-n)}+\frac{2d(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)^2(2-n)(3-n)}-\frac{2d^2(a+bx)^{-1+n}(c+dx}{(bc-ad)^3(1-n)(2-n)(}\end{aligned}$$

Mathematica [A] time = 0.10, size = 112, normalized size = 0.78

$$\frac{(a+bx)^{n-3}(c+dx)^{1-n}\left(a^2d^2\left(n^2-5n+6\right)-2abd(n-3)(c(n-1)+dx)+b^2\left(c^2\left(n^2-3n+2\right)+2cd(n-1)x+2d^2x^2\right)\right)}{(n-3)(n-2)(n-1)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-3 + n)*(c + d*x)^(1 - n)*(a^2*d^2*(6 - 5*n + n^2) - 2*a*b*d*(-
3 + n)*(c*(-1 + n) + d*x) + b^2*(c^2*(2 - 3*n + n^2) + 2*c*d*(-1 + n)*x + 2
*d^2*x^2)))/((b*c - a*d)^3*(-3 + n)*(-2 + n)*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-4+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-4 + n)/(c + d*x)^n, x]
```

fricas [B] time = 1.43, size = 512, normalized size = 3.58

$$-\frac{\left(2\,b^{3}d^{3}x^{4}+2\,ab^{2}c^{3}-6\,a^{2}bc^{2}d+6\,a^{3}cd^{2}+2\left(4\,ab^{2}d^{3}+\left(b^{3}cd^{2}-ab^{2}d^{3}\right)n\right)x^{3}+\left(ab^{2}c^{3}-2\,a^{2}bc^{2}d+a^{3}cd^{2}\right)n^{2}+\left(12\,a^{2}bd^{3}+\left(b^{3}c^{2}d-2\,ab^{2}cd^{2}+a^{2}bd^{3}\right)n^{2}-\left(b^{3}c^{2}d-8\,ab^{2}cd^{2}+7\,a^{2}bd^{3}\right)n\right)x^{2}-\left(3\,ab^{2}c^{3}-8\,a^{2}bc^{2}d+5\,a^{3}cd^{2}\right)n+\left(2\,b^{3}c^{3}-6\,ab^{2}c^{2}d+6\,a^{2}bcd^{2}+6\,a^{3}d^{3}+\left(b^{3}c^{3}-ab^{2}c^{2}d-a^{2}bcd^{2}+a^{3}d^{3}\right)n^{2}-\left(3\,b^{3}c^{3}-7\,ab^{2}c^{2}d-a^{2}bcd^{2}+5\,a^{3}d^{3}\right)n\right)x\right)(bx+a)^{n-4}}{\left(6\,b^{3}c^{3}-18\,ab^{2}c^{2}d+18\,a^{2}bcd^{2}-6\,a^{3}d^{3}-\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n^{3}+6\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n^{2}-11\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)n\right)(dx+c)^{n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 - (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 - (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 - (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(n - 4)/((6*b^3*c^3 - 18*
a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*
d^3)*n^2 - 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)*(d*x +
 c)^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

maple [B] time = 0.01, size = 322, normalized size = 2.25

$$-\frac{(dx+c)\left(a^{2}d^{2}n^{2}-2abcd\,n^{2}-2ab\,d^{2}nx+b^{2}c^{2}n^{2}+2b^{2}cdnx+2b^{2}x^{2}d^{2}-5a^{2}d^{2}n+8abcdn+6ab\,d^{2}x-3b^{2}c^{2}n-2b^{2}cdx+6a^{2}d^{2}-6abcd+2b^{2}c^{2}\right)(bx+a)^{n-3}\,(dx+c)^{-n}}{a^{3}d^{3}n^{3}-3a^{2}bc\,d^{2}n^{3}+3a\,b^{2}c^{2}d\,n^{3}-b^{3}c^{3}n^{3}-6a^{3}d^{3}n^{2}+18a^{2}bc\,d^{2}n^{2}-18a\,b^{2}c^{2}d\,n^{2}+6b^{3}c^{3}n^{2}+11a^{3}d^{3}n-33a^{2}bc\,d^{2}n+33a\,b^{2}c^{2}dn-11b^{3}c^{3}n-6a^{3}d^{3}+18a^{2}bc\,d^{2}-18a\,b^{2}c^{2}d+6b^{3}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4+n)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-3)*(d*x+c)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^2
+2*b^2*c*d*n*x+2*b^2*d^2*x^2-5*a^2*d^2*n+8*a*b*c*d*n+6*a*b*d^2*x-3*b^2*c^2*
n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3
+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3-6*a^3*d^3*n^2+18*a^2*b*c*d^2*n^2-18*a*b^2*c^
2*d*n^2+6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^3
*c^3*n-6*a^3*d^3+18*a^2*b*c*d^2-18*a*b^2*c^2*d+6*b^3*c^3)/((d*x+c)^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-4}}{(dx+c)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 4)/(d*x + c)^n, x)
```

mupad [B] time = 1.10, size = 528, normalized size = 3.69

$$-\frac{x\,(a+b\,x)^{n-4}\,\left(a^3\,d^3\,n^2-5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2+a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2+7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2-3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(c+d\,x)^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{a\,c\,(a+b\,x)^{n-4}\,\left(a^2\,d^2\,n^2-5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2-3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(c+d\,x)^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^3\,d^3\,x^4\,(a+b\,x)^{n-4}}{(a\,d-b\,c)^3\,(c+d\,x)^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{b\,d\,x^2\,(a+b\,x)^{n-4}\,\left(a^2\,d^2\,n^2-7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2+8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2-b^2\,c^2\,n\right)}{(a\,d-b\,c)^3\,(c+d\,x)^n\,\left(n^3-6\,n^2+11\,n-6\right)}-\frac{2\,b^2\,d^2\,x^3\,(a+b\,x)^{n-4}\,(4\,a\,d-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^3\,(c+d\,x)^n\,\left(n^3-6\,n^2+11\,n-6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 4)/(c + d*x)^n,x)
```

```
[Out] - (x*(a + b*x)^(n - 4)*(6*a^3*d^3 + 2*b^3*c^3 - 5*a^3*d^3*n - 3*b^3*c^3*n +
 a^3*d^3*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 7*a*b^2*c^2*d*
n + a^2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c +
 d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (a*c*(a + b*x)^(n - 4)*(6*a^2*d^2 + 2*b
^2*c^2 - 5*a^2*d^2*n - 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d
+ 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 +
n^3 - 6)) - (2*b^3*d^3*x^4*(a + b*x)^(n - 4))/((a*d - b*c)^3*(c + d*x)^n*(1
1*n - 6*n^2 + n^3 - 6)) - (b*d*x^2*(a + b*x)^(n - 4)*(12*a^2*d^2 - 7*a^2*d^
2*n - b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 + 8*a*b*c*d*n - 2*a*b*c*d*n^2))
/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6*n^2 + n^3 - 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^(n - 4)*(4*a*d - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^n*(11*n - 6
*n^2 + n^3 - 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

3.1563 $\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=86

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(2-n)(bc-ad)^2}-\frac{(a+bx)^{n-2}(c+dx)^{1-n}}{(2-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(2 - n))) + (d*(a + b
*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)^2*(1 - n)*(2 - n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}-\frac{d\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx}{(bc-ad)(2-n)}\\ &= -\frac{(a+bx)^{-2+n}(c+dx)^{1-n}}{(bc-ad)(2-n)}+\frac{d(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)^2(1-n)(2-n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.69

$$\frac{(a+bx)^{n-2}(c+dx)^{1-n}(-ad(n-2)+bc(n-1)+bdx)}{(n-2)(n-1)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 + n)*(c + d*x)^(1 - n)*(-(a*d*(-2 + n)) + b*c*(-1 + n) + b*d
*x))/((b*c - a*d)^2*(-2 + n)*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-3+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-3 + n)/(c + d*x)^n, x]
```

fricas [B] time = 1.38, size = 206, normalized size = 2.40

$$\frac{\left(b^2d^2x^3 - abc^2 + 2\,a^2cd + \left(3\,abd^2 + \left(b^2cd - abd^2\right)n\right)x^2 + \left(abc^2 - a^2cd\right)n - \left(b^2c^2 - 2\,abcd - 2\,a^2d^2 - \left(b^2c^2 - a^2d^2\right)n\right)x\right)(bx+a)^{n-3}}{\left(2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + \left(b^2c^2 - 2\,abcd + a^2d^2\right)n^2 - 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)n\right)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 - (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(n - 3)/((2*b^2*c^2 - 4*a*b*c*d + 2*a^2*d^2 + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*n^2 - 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n)*(d*x
+ c)^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

maple [A] time = 0.01, size = 127, normalized size = 1.48

$$-\frac{(dx+c)\,(adn - bcn - bdx - 2ad + bc)\,(bx+a)^{n-2}\,(dx+c)^{-n}}{a^2d^2n^2 - 2abcd\,n^2 + b^2c^2n^2 - 3a^2d^2n + 6abcdn - 3b^2c^2n + 2a^2d^2 - 4abcd + 2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(n-3)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-2)*(d*x+c)*(a*d*n-b*c*n-b*d*x-2*a*d+b*c)/(a^2*d^2*n^2-2*a*b*c*d
*n^2+b^2*c^2*n^2-3*a^2*d^2*n+6*a*b*c*d*n-3*b^2*c^2*n+2*a^2*d^2-4*a*b*c*d+2*
b^2*c^2)/((d*x+c)^n)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-3}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 3)/(d*x + c)^n, x)
```

mupad [B] time = 0.77, size = 220, normalized size = 2.56

$$(a+b\,x)^{n-3}\left(\frac{x\,\left(2\,a^2\,d^2 - b^2\,c^2 - a^2\,d^2\,n + b^2\,c^2\,n + 2\,a\,b\,c\,d\right)}{(a\,d-b\,c)^2\,(c+d\,x)^n\,\left(n^2-3\,n+2\right)} + \frac{b^2\,d^2\,x^3}{(a\,d-b\,c)^2\,(c+d\,x)^n\,\left(n^2-3\,n+2\right)} + \frac{a\,c\,(2\,a\,d - b\,c - a\,d\,n + b\,c\,n)}{(a\,d-b\,c)^2\,(c+d\,x)^n\,\left(n^2-3\,n+2\right)} + \frac{b\,d\,x^2\,(3\,a\,d - a\,d\,n + b\,c\,n)}{(a\,d-b\,c)^2\,(c+d\,x)^n\,\left(n^2-3\,n+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 3)/(c + d*x)^n,x)
```

```
[Out] (a + b*x)^(n - 3)*((x*(2*a^2*d^2 - b^2*c^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*
c*d))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b^2*d^2*x^3)/((a*d - b
*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (a*c*(2*a*d - b*c - a*d*n + b*c*n))/((
a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)) + (b*d*x^2*(3*a*d - a*d*n + b*c*n
))/((a*d - b*c)^2*(c + d*x)^n*(n^2 - 3*n + 2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3+n)/((d*x+c)**n),x)
```

```
[Out] Timed out
```

3.1564 $\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$

Optimal. Leaf size=39

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(1-n)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(1 - n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx = -\frac{(a+bx)^{-1+n}(c+dx)^{1-n}}{(bc-ad)(1-n)}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.92

$$\frac{(a+bx)^{n-1}(c+dx)^{1-n}}{(n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-1 + n)*(c + d*x)^(1 - n))/((b*c - a*d)*(-1 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-2+n}(c+dx)^{-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2 + n)/(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-2 + n)/(c + d*x)^n, x]
```

fricas [A] time = 1.32, size = 60, normalized size = 1.54

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{n-2}}{(bc-ad-(bc-ad)n)(dx+c)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(n - 2)/((b*c - a*d - (b*c - a*d
)*n)*(d*x + c)^n)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

maple **[A]** time = 0.00, size = 45, normalized size = 1.15

$$-\frac{(dx+c)\,(bx+a)^{n-1}\,(dx+c)^{-n}}{adn-bcn-ad+bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(n-2)/((d*x+c)^n),x)
```

```
[Out] -(b*x+a)^(n-1)*(d*x+c)/(a*d*n-b*c*n-a*d+b*c)/((d*x+c)^n)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx+a)^{n-2}}{(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2+n)/((d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(n - 2)/(d*x + c)^n, x)
```

mupad **[B]** time = 0.56, size = 102, normalized size = 2.62

$$-(a+b\,x)^{n-2}\left(\frac{a\,c}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{x\,(a\,d+b\,c)}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}+\frac{b\,d\,x^2}{(a\,d-b\,c)\,(n-1)\,(c+d\,x)^n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^(n - 2)/(c + d*x)^n,x)
```

```
[Out] -(a + b*x)^(n - 2)*((a*c)/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (x*(a*d + b*c
))/((a*d - b*c)*(n - 1)*(c + d*x)^n) + (b*d*x^2)/((a*d - b*c)*(n - 1)*(c +
d*x)^n))
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2+n)/((d*x+c)**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1565 $\int (a+bx)^{-2-n}(c+dx)^n\,dx$

Optimal. Leaf size=37

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$-\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
[Out] -(((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(1 + n)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx = -\frac{(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)(1+n)}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.03

$$\frac{(a+bx)^{-n-1}(c+dx)^{n+1}}{(-n-1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
[Out] ((a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(-1 - n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-2-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-2 - n)*(c + d*x)^n,x]
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-2 - n)*(c + d*x)^n, x]
```

fricas [A] time = 1.33, size = 59, normalized size = 1.59

$$-\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^{-n-2}(dx+c)^n}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^(-n - 2)*(d*x + c)^n/(b*c - a*d
+ (b*c - a*d)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

maple [A] time = 0.00, size = 41, normalized size = 1.11

$$\frac{(bx+a)^{-n-1}(dx+c)^{n+1}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-n-2)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-n-1)*(d*x+c)^(n+1)/(a*d*n-b*c*n+a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-2}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-2-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 2)*(d*x + c)^n, x)
```

mupad [B] time = 0.53, size = 97, normalized size = 2.62

$$\frac{\frac{a\,c\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{x\,\left(a\,d+b\,c\right)\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}+\frac{b\,d\,x^2\,{\left(c+d\,x\right)}^n}{\left(a\,d-b\,c\right)\,\left(n+1\right)}}{{\left(a+b\,x\right)}^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 2),x)
```

```
[Out] ((a*c*(c + d*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(c + d*x)^n)/((a*
d - b*c)*(n + 1)) + (b*d*x^2*(c + d*x)^n)/((a*d - b*c)*(n + 1)))/(a + b*x)^
(n + 2)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-2-n)*(d*x+c)**n,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.1566 $\int (a+bx)^{-3-n}(c+dx)^n\,dx$

Optimal. Leaf size=80

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Rubi [A] time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{d(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(bc-ad)^2}-\frac{(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(2 + n))) + (d*(a + b
*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-3-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}-\frac{d\int (a+bx)^{-2-n}(c+dx)^n\,dx}{(bc-ad)(2+n)}\\ &= -\frac{(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)(2+n)}+\frac{d(a+bx)^{-1-n}(c+dx)^{1+n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 60, normalized size = 0.75

$$\frac{(a+bx)^{-n-2}(c+dx)^{n+1}(ad(n+2)-b(cn+c-dx))}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-2 - n)*(c + d*x)^(1 + n)*(a*d*(2 + n) - b*(c + c*n - d*x)))/((
b*c - a*d)^2*(1 + n)*(2 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-3-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-3 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-3 - n)*(c + d*x)^n, x]
```

fricas [B] time = 1.28, size = 207, normalized size = 2.59

$$\frac{\left(b^2d^2x^3 - abc^2 + 2\,a^2cd + \left(3\,abd^2 - \left(b^2cd - abd^2\right)n\right)x^2 - \left(abc^2 - a^2cd\right)n - \left(b^2c^2 - 2\,abcd - 2\,a^2d^2 + \left(b^2c^2 - a^2d^2\right)n\right)x\right)(bx+a)^{-n-3}(dx+c)^n}{2\,b^2c^2 - 4\,abcd + 2\,a^2d^2 + \left(b^2c^2 - 2\,abcd + a^2d^2\right)n^2 + 3\left(b^2c^2 - 2\,abcd + a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 - a*b*c^2 + 2*a^2*c*d + (3*a*b*d^2 - (b^2*c*d - a*b*d^2)*n)*x^
2 - (a*b*c^2 - a^2*c*d)*n - (b^2*c^2 - 2*a*b*c*d - 2*a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^(-n - 3)*(d*x + c)^n/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

maple [A] time = 0.00, size = 123, normalized size = 1.54

$$\frac{(adn - bcn + bdx + 2ad - bc)\,(bx+a)^{-n-2}\,(dx+c)^{n+1}}{a^2d^2n^2 - 2abcd\,n^2 + b^2c^2n^2 + 3a^2d^2n - 6abcdn + 3b^2c^2n + 2a^2d^2 - 4abcd + 2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-3-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-n-2)*(d*x+c)^(n+1)*(a*d*n-b*c*n+b*d*x+2*a*d-b*c)/(a^2*d^2*n^2-2*a
*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b*
c*d+2*b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-3}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-3-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 3)*(d*x + c)^n, x)
```

mupad [B] time = 0.74, size = 214, normalized size = 2.68

$$\frac{\frac{x\,(c+d\,x)^n\,\left(2\,a^2\,d^2-b^2\,c^2+a^2\,d^2\,n-b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{a\,c\,(c+d\,x)^n\,(2\,a\,d-b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,(c+d\,x)^n}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,(3\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}}{(a+b\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 3),x)
```

```
[Out] ((x*(c + d*x)^n*(2*a^2*d^2 - b^2*c^2 + a^2*d^2*n - b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) + (a*c*(c + d*x)^n*(2*a*d - b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(c + d*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(c + d*x)^n*(3*a*d + a*d*n - b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(a + b*x)^(n + 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-3-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

3.1567 $\int (a+bx)^{-4-n}(c+dx)^n\,dx$

Optimal. Leaf size=131

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Rubi [A] time = 0.05, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$-\frac{2d^2(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(bc-ad)^3}-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(bc-ad)}+\frac{2d(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(3 + n))) + (2*d*(a +
 b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(2 + n)*(3 + n)) - (2*d^2*
(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n
))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-4-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}-\frac{(2d)\int (a+bx)^{-3-n}(c+dx)^n\,dx}{(bc-ad)(3+n)}\\ &= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}+\frac{\left(2d^2\right)\int (a+bx)^{-2-n}(c}{(bc-ad)^2(2+n)(3}\\ &= -\frac{(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)(3+n)}+\frac{2d(a+bx)^{-2-n}(c+dx)^{1+n}}{(bc-ad)^2(2+n)(3+n)}-\frac{2d^2(a+bx)^{-1-n}(c+}{(bc-ad)^3(1+n)(2+n}\end{aligned}$$

Mathematica [A] time = 0.08, size = 113, normalized size = 0.86

$$-\frac{(a+bx)^{-n-3}(c+dx)^{n+1}\left(a^2d^2\left(n^2+5n+6\right)-2abd(n+3)(cn+c-dx)+b^2\left(c^2\left(n^2+3n+2\right)-2cd(n+1)x+2d^2x^2\right)\right)}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-3 - n)*(c + d*x)^(1 + n)*(a^2*d^2*(6 + 5*n + n^2) - 2*a*b*d*
(3 + n)*(c + c*n - d*x) + b^2*(c^2*(2 + 3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^
2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n)))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-4-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-4 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-4 - n)*(c + d*x)^n, x]
```

fricas [B] time = 1.01, size = 509, normalized size = 3.89

$$-\frac{\left(2\,b^3d^3x^4+2\,ab^2c^3-6\,a^2bc^2d+6\,a^3cd^2+2\left(4\,ab^2d^3-\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,a^2bd^3+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)n^2+\left(b^3c^2d-8\,ab^2cd^2+7\,a^2bd^3\right)n\right)x^2+\left(3\,ab^2c^3-8\,a^2bc^2d+5\,a^3cd^2\right)n+\left(2\,b^3c^3-6\,ab^2c^2d+6\,a^2bcd^2+6\,a^3d^3+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)n^2+\left(3\,b^3c^3-7\,ab^2c^2d-a^2bcd^2+5\,a^3d^3\right)n\right)x\right)(bx+a)^{-n-4}(dx+c)^n}{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3+\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^3+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^2+11\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] -(2*b^3*d^3*x^4 + 2*a*b^2*c^3 - 6*a^2*b*c^2*d + 6*a^3*c*d^2 + 2*(4*a*b^2*d^
3 - (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*n^2 + (12*a^2*b*d^3 + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (b^3*
c^2*d - 8*a*b^2*c*d^2 + 7*a^2*b*d^3)*n)*x^2 + (3*a*b^2*c^3 - 8*a^2*b*c^2*d
+ 5*a^3*c*d^2)*n + (2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 + 6*a^3*d^3 +
 (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (3*b^3*c^3 - 7*a*b^2
*c^2*d - a^2*b*c*d^2 + 5*a^3*d^3)*n)*x)*(b*x + a)^(-n - 4)*(d*x + c)^n/(6*b
^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*
c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

maple [B] time = 0.01, size = 318, normalized size = 2.43

$$\frac{\left(a^2d^2n^2-2abcd\,n^2+2ab\,d^2nx+b^2c^2n^2-2b^2cdnx+2b^2x^2d^2+5a^2d^2n-8abcdn+6ab\,d^2x+3b^2c^2n-2b^2cdx+6a^2d^2-6abcd+2b^2c^2\right)(bx+a)^{-n-3}(dx+c)^{n+1}}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c^3n+6a^3d^3-18a^2bc\,d^2+18a\,b^2c^2d-6b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-4-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-3-n)*(d*x+c)^(n+1)*(a^2*d^2*n^2-2*a*b*c*d*n^2+2*a*b*d^2*n*x+b^2*c
^2*n^2-2*b^2*c*d*n*x+2*b^2*d^2*x^2+5*a^2*d^2*n-8*a*b*c*d*n+6*a*b*d^2*x+3*b^
2*c^2*n-2*b^2*c*d*x+6*a^2*d^2-6*a*b*c*d+2*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d
^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*
b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-
11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-4}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-4-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 4)*(d*x + c)^n, x)
```

mupad [B] time = 0.99, size = 525, normalized size = 4.01

$$\frac{x\,(c+d\,x)^n\,\left(a^3\,d^3\,n^2+5\,a^3\,d^3\,n+6\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-a^2\,b\,c\,d^2\,n+6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-7\,a\,b^2\,c^2\,d\,n-6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+3\,b^3\,c^3\,n+2\,b^3\,c^3\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{a\,c\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+5\,a^2\,d^2\,n+6\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+3\,b^2\,c^2\,n+2\,b^2\,c^2\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^3\,d^3\,x^4\,(c+d\,x)^n}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{b\,d\,x^2\,(c+d\,x)^n\,\left(a^2\,d^2\,n^2+7\,a^2\,d^2\,n+12\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+b^2\,c^2\,n\right)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}+\frac{2\,b^2\,d^2\,x^3\,(c+d\,x)^n\,(4\,a\,d+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^3\,(a+b\,x)^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 4),x)
```

```
[Out] (x*(c + d*x)^n*(6*a^3*d^3 + 2*b^3*c^3 + 5*a^3*d^3*n + 3*b^3*c^3*n + a^3*d^3
*n^2 + b^3*c^3*n^2 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 7*a*b^2*c^2*d*n - a^2*
b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(a + b*x)^(n
 + 4)*(11*n + 6*n^2 + n^3 + 6)) + (a*c*(c + d*x)^n*(6*a^2*d^2 + 2*b^2*c^2 +
 5*a^2*d^2*n + 3*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*b*
c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^
3 + 6)) + (2*b^3*d^3*x^4*(c + d*x)^n)/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*
n + 6*n^2 + n^3 + 6)) + (b*d*x^2*(c + d*x)^n*(12*a^2*d^2 + 7*a^2*d^2*n + b^
2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d -
 b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (2*b^2*d^2*x^3*(c + d
*x)^n*(4*a*d + a*d*n - b*c*n))/((a*d - b*c)^3*(a + b*x)^(n + 4)*(11*n + 6*n
^2 + n^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-4-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

3.1568 $\int (a+bx)^{-5-n}(c+dx)^n\,dx$

Optimal. Leaf size=186

$$\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-}}{(n+3)(n+4}$$

Rubi [A] time = 0.09, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$-\frac{6d^2(a+bx)^{-n-2}(c+dx)^{n+1}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6d^3(a+bx)^{-n-1}(c+dx)^{n+1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}-\frac{(a+bx)^{-n-4}(c+dx)^{n+1}}{(n+4)(bc-ad)}+\frac{3d(a+bx)^{-n-3}(c+dx)^{n+1}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] -(((a + b*x)^(-4 - n)*(c + d*x)^(1 + n))/((b*c - a*d)*(4 + n))) + (3*d*(a +
 b*x)^(-3 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^2*(3 + n)*(4 + n)) - (6*d^2*
(a + b*x)^(-2 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n
)) + (6*d^3*(a + b*x)^(-1 - n)*(c + d*x)^(1 + n))/((b*c - a*d)^4*(1 + n)*(2
 + n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-5-n}(c+dx)^n\,dx &= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}-\frac{(3d)\int (a+bx)^{-4-n}(c+dx)^n\,dx}{(bc-ad)(4+n)}\\ &= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}+\frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)}+\frac{\left(6d^2\right)\int (a+bx)^{-3-n}(c+}{(bc-ad)^2(3+n)(4+}\\ &= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}+\frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)}-\frac{6d^2(a+bx)^{-2-n}(c+dx}{(bc-ad)^3(2+n)(3+n)(}\\ &= -\frac{(a+bx)^{-4-n}(c+dx)^{1+n}}{(bc-ad)(4+n)}+\frac{3d(a+bx)^{-3-n}(c+dx)^{1+n}}{(bc-ad)^2(3+n)(4+n)}-\frac{6d^2(a+bx)^{-2-n}(c+dx}{(bc-ad)^3(2+n)(3+n)(}\end{aligned}$$

Mathematica [A] time = 0.12, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{-n-4}(c+dx)^{n+1}\left(a^3d^3\left(n^3+9n^2+26n+24\right)-3a^2bd^2\left(n^2+7n+12\right)(cn+c-dx)+3ab^2d(n+4)\left(c^2\left(n^2+3n+2\right)-2cd(n+1)x+2d^2x^2\right)-\left(b^3\left(c^3\left(n^3+6n^2+11n+6\right)-3c^2d\left(n^2+3n+2\right)x+6cd^2(n+1)x^2-6d^3x^3\right)\right)\right)}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] ((a + b*x)^(-4 - n)*(c + d*x)^(1 + n)*(a^3*d^3*(24 + 26*n + 9*n^2 + n^3) -
3*a^2*b*d^2*(12 + 7*n + n^2)*(c + c*n - d*x) + 3*a*b^2*d*(4 + n)*(c^2*(2 +
3*n + n^2) - 2*c*d*(1 + n)*x + 2*d^2*x^2) - b^3*(c^3*(6 + 11*n + 6*n^2 + n^
3) - 3*c^2*d*(2 + 3*n + n^2)*x + 6*c*d^2*(1 + n)*x^2 - 6*d^3*x^3)))/((b*c -
 a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^{-5-n}(c+dx)^n\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-5 - n)*(c + d*x)^n,x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-5 - n)*(c + d*x)^n, x]
```

fricas [B] time = 1.40, size = 959, normalized size = 5.16

$$\frac{\left(6 b^{4} d^{4} x^{5} - 6 a b^{3} c^{4} + 24 a^{2} b^{2} c^{3} d - 36 a^{3} b c^{2} d^{2} + 24 a^{4} c d^{3} + 6 \left(5 a b^{3} d^{4} - \left(b^{4} c d^{3} - a b^{3} d^{4}\right) n\right) x^{4} - \left(a b^{3} c^{4} - 3 a^{2} b^{2} c^{3} d + 3 a^{3} b c^{2} d^{2} - a^{4} c d^{3}\right) n^{3} + 3 \left(20 a^{2} b^{2} d^{4} + \left(b^{4} c^{2} d^{2} - 2 a b^{3} c d^{3} + a^{2} b^{2} d^{4}\right) n^{2} + \left(b^{4} c^{2} d^{2} - 10 a b^{3} c d^{3} + 9 a^{2} b^{2} d^{4}\right) n\right) x^{3} - 3 \left(2 a b^{3} c^{4} - 7 a^{2} b^{2} c^{3} d + 8 a^{3} b c^{2} d^{2} - 3 a^{4} c d^{3}\right) n^{2} + \left(60 a^{3} b d^{4} - \left(b^{4} c^{3} d - 3 a b^{3} c^{2} d^{2} + 3 a^{2} b^{2} c d^{3} - a^{3} b d^{4}\right) n^{3} - 3 \left(b^{4} c^{3} d - 6 a b^{3} c^{2} d^{2} + 9 a^{2} b^{2} c d^{3} - 4 a^{3} b d^{4}\right) n^{2} - \left(2 b^{4} c^{3} d - 15 a b^{3} c^{2} d^{2} + 60 a^{2} b^{2} c d^{3} - 47 a^{3} b d^{4}\right) n\right) x^{2} - \left(11 a b^{3} c^{4} - 42 a^{2} b^{2} c^{3} d + 57 a^{3} b c^{2} d^{2} - 26 a^{4} c d^{3}\right) n - \left(6 b^{4} c^{4} - 24 a b^{3} c^{3} d + 36 a^{2} b^{2} c^{2} d^{2} - 24 a^{3} b c d^{3} - 24 a^{4} d^{4} + \left(b^{4} c^{4} - 2 a b^{3} c^{3} d + 2 a^{3} b c d^{3} - a^{4} d^{4}\right) n^{3} + 3 \left(2 b^{4} c^{4} - 6 a b^{3} c^{3} d + 3 a^{2} b^{2} c^{2} d^{2} + 4 a^{3} b c d^{3} - 3 a^{4} d^{4}\right) n^{2} + \left(11 b^{4} c^{4} - 40 a b^{3} c^{3} d + 45 a^{2} b^{2} c^{2} d^{2} + 10 a^{3} b c d^{3} - 26 a^{4} d^{4}\right) n\right) x\right) (bx+a)^{-n-5} (dx+c)^{n}}{24 b^{4} c^{4} - 96 a b^{3} c^{3} d + 144 a^{2} b^{2} c^{2} d^{2} - 96 a^{3} b c d^{3} + 24 a^{4} d^{4} + \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) n^{4} + 10 \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) n^{3} + 35 \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) n^{2} + 50 \left(b^{4} c^{4} - 4 a b^{3} c^{3} d + 6 a^{2} b^{2} c^{2} d^{2} - 4 a^{3} b c d^{3} + a^{4} d^{4}\right) n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 - 6*a*b^3*c^4 + 24*a^2*b^2*c^3*d - 36*a^3*b*c^2*d^2 + 24*a^4
*c*d^3 + 6*(5*a*b^3*d^4 - (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 - (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*a^2*b^2*d^4 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (b^4*c^2*d^2 - 10*a*b^3*c*d^3
+ 9*a^2*b^2*d^4)*n)*x^3 - 3*(2*a*b^3*c^4 - 7*a^2*b^2*c^3*d + 8*a^3*b*c^2*d^
2 - 3*a^4*c*d^3)*n^2 + (60*a^3*b*d^4 - (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 - 3*(b^4*c^3*d - 6*a*b^3*c^2*d^2 + 9*a^2*b^2*c*
d^3 - 4*a^3*b*d^4)*n^2 - (2*b^4*c^3*d - 15*a*b^3*c^2*d^2 + 60*a^2*b^2*c*d^3
 - 47*a^3*b*d^4)*n)*x^2 - (11*a*b^3*c^4 - 42*a^2*b^2*c^3*d + 57*a^3*b*c^2*d
^2 - 26*a^4*c*d^3)*n - (6*b^4*c^4 - 24*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 - 2
4*a^3*b*c*d^3 - 24*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(2*b^4*c^4 - 6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^
3 - 3*a^4*d^4)*n^2 + (11*b^4*c^4 - 40*a*b^3*c^3*d + 45*a^2*b^2*c^2*d^2 + 10
*a^3*b*c*d^3 - 26*a^4*d^4)*n)*x)*(b*x + a)^(-n - 5)*(d*x + c)^n/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-5}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

maple [B] time = 0.01, size = 661, normalized size = 3.55

$$\frac{\left(a^{3} d^{3} n^{3} - 3 a^{2} b c d^{2} n^{3} + 3 a^{2} b d^{3} n^{2} x + 3 a b^{2} c^{2} d n^{3} - 6 a b^{2} c d^{2} n^{2} x + 6 a b^{2} d^{3} n x^{2} - b^{3} c^{3} n^{3} + 3 b^{3} c^{2} d n^{2} x - 6 b^{3} c d^{2} n x^{2} + 6 b^{3} d^{3} x^{3} + 9 a^{3} d^{3} n^{2} - 24 a^{2} b c d^{2} n^{2} + 21 a^{2} b d^{3} n x + 21 a b^{2} c^{2} d n^{2} - 30 a b^{2} c d^{2} n x + 24 a b^{2} d^{3} x^{2} - 6 b^{3} c^{3} n^{2} + 9 b^{3} c^{2} d n x - 6 b^{3} c d^{2} x^{2} + 26 a^{3} d^{3} n - 57 a^{2} b c d^{2} n + 36 a^{2} b d^{3} x + 42 a b^{2} c^{2} d n - 24 a b^{2} c d^{2} x - 11 b^{3} c^{3} n + 6 b^{3} c^{2} d x + 24 a^{3} d^{3} - 36 a^{2} b c d^{2} + 24 a b^{2} c^{2} d - 6 b^{3} c^{3}\right) (bx+a)^{-n-4} (dx+c)^{n+1}}{a^{4} d^{4} n^{4} - 4 a^{3} b c d^{3} n^{4} + 6 a^{2} b^{2} c^{2} d^{2} n^{4} - 4 a b^{3} c^{3} d n^{4} + b^{4} c^{4} n^{4} + 10 a^{4} d^{4} n^{3} - 40 a^{3} b c d^{3} n^{3} + 60 a^{2} b^{2} c^{2} d^{2} n^{3} - 40 a b^{3} c^{3} d n^{3} + 10 b^{4} c^{4} n^{3} + 35 a^{4} d^{4} n^{2} - 140 a^{3} b c d^{3} n^{2} + 210 a^{2} b^{2} c^{2} d^{2} n^{2} - 140 a b^{3} c^{3} d n^{2} + 35 b^{4} c^{4} n^{2} + 50 a^{4} d^{4} n - 200 a^{3} b c d^{3} n + 300 a^{2} b^{2} c^{2} d^{2} n - 200 a b^{3} c^{3} d n + 50 b^{4} c^{4} n + 24 a^{4} d^{4} - 96 a^{3} b c d^{3} + 144 a^{2} b^{2} c^{2} d^{2} - 96 a b^{3} c^{3} d + 24 b^{4} c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-5-n)*(d*x+c)^n,x)
```

```
[Out] (b*x+a)^(-4-n)*(d*x+c)^(n+1)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a^2*b*d^3*n^2
*x+3*a*b^2*c^2*d*n^3-6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3+3*b^
3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2+6*b^3*d^3*x^3+9*a^3*d^3*n^2-24*a^2*b*c*d^2*
n^2+21*a^2*b*d^3*n*x+21*a*b^2*c^2*d*n^2-30*a*b^2*c*d^2*n*x+24*a*b^2*d^3*x^2
-6*b^3*c^3*n^2+9*b^3*c^2*d*n*x-6*b^3*c*d^2*x^2+26*a^3*d^3*n-57*a^2*b*c*d^2*
n+36*a^2*b*d^3*x+42*a*b^2*c^2*d*n-24*a*b^2*c*d^2*x-11*b^3*c^3*n+6*b^3*c^2*d
*x+24*a^3*d^3-36*a^2*b*c*d^2+24*a*b^2*c^2*d-6*b^3*c^3)/(a^4*d^4*n^4-4*a^3*b
*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*n
^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4*
n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c^
3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*n
-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d
^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-n-5}(dx+c)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-5-n)*(d*x+c)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^(-n - 5)*(d*x + c)^n, x)
```

mupad [B] time = 1.64, size = 944, normalized size = 5.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^n/(a + b*x)^(n + 5),x)
```

```
[Out] (a*c*(c + d*x)^n*(24*a^3*d^3 - 6*b^3*c^3 + 26*a^3*d^3*n - 11*b^3*c^3*n + 9*
a^3*d^3*n^2 - 6*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 24*a*b^2*c^2*d -
36*a^2*b*c*d^2 + 42*a*b^2*c^2*d*n - 57*a^2*b*c*d^2*n + 21*a*b^2*c^2*d*n^2 -
 24*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) - (x*(c + d*x)^n*(
6*b^4*c^4 - 24*a^4*d^4 - 26*a^4*d^4*n + 11*b^4*c^4*n - 9*a^4*d^4*n^2 + 6*b^
4*c^4*n^2 - a^4*d^4*n^3 + b^4*c^4*n^3 + 36*a^2*b^2*c^2*d^2 - 24*a*b^3*c^3*d
 - 24*a^3*b*c*d^3 - 40*a*b^3*c^3*d*n + 10*a^3*b*c*d^3*n + 9*a^2*b^2*c^2*d^2
*n^2 - 18*a*b^3*c^3*d*n^2 + 12*a^3*b*c*d^3*n^2 - 2*a*b^3*c^3*d*n^3 + 2*a^3*
b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n
 + 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^4*d^4*x^5*(c + d*x)^n)/((a*d - b*c)^
4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
c + d*x)^n*(20*a^2*d^2 + 9*a^2*d^2*n + b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(c + d*x)^n*(5*a*d + a*d*n -
 b*c*n))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(c + d*x)^n*(60*a^3*d^3 + 47*a^3*d^3*n - 2*b^3*c^3*n + 12*a^
3*d^3*n^2 - 3*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^3*n^3 + 15*a*b^2*c^2*d*n -
60*a^2*b*c*d^2*n + 18*a*b^2*c^2*d*n^2 - 27*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*
n^3 - 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(a + b*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-5-n)*(d*x+c)**n,x)
```

```
[Out] Timed out
```

3.1569 $\int (a+bx)^n (c+dx)^{-2-n}\,dx$

Optimal. Leaf size=36

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int (a+bx)^n (c+dx)^{-2-n}\,dx = \frac{(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)(1+n)}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)*(1 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n (c+dx)^{-2-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-2 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-2 - n), x]
```

fricas [A] time = 1.32, size = 58, normalized size = 1.61

$$\frac{\left(bdx^2+ac+(bc+ad)x\right)(bx+a)^n(dx+c)^{-n-2}}{bc-ad+(bc-ad)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="fricas")
```

```
[Out] (b*d*x^2 + a*c + (b*c + a*d)*x)*(b*x + a)^n*(d*x + c)^(-n - 2)/(b*c - a*d +
 (b*c - a*d)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

maple [A] time = 0.00, size = 42, normalized size = 1.17

$$-\frac{(bx+a)^{n+1}(dx+c)^{-n-1}}{adn-bcn+ad-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-n-2),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-1)/(a*d*n-b*c*n+a*d-b*c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-2-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 2), x)
```

mupad [B] time = 0.56, size = 98, normalized size = 2.72

$$-\frac{\frac{a\,c\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}+\frac{x\,(a\,d+b\,c)\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}+\frac{b\,d\,x^2\,(a+b\,x)^n}{(a\,d-b\,c)\,(n+1)}}{(c+d\,x)^{n+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 2),x)
```

```
[Out] -((a*c*(a + b*x)^n)/((a*d - b*c)*(n + 1)) + (x*(a*d + b*c)*(a + b*x)^n)/((a
*d - b*c)*(n + 1)) + (b*d*x^2*(a + b*x)^n)/((a*d - b*c)*(n + 1)))/(c + d*x)
^(n + 2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-2-n),x)
```

```
[Out] Timed out
```

3.1570 $\int (a+bx)^n(c+dx)^{-3-n}\,dx$

Optimal. Leaf size=79

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Rubi [A] time = 0.02, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(bc-ad)}+\frac{b(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)*(2 + n)) + (b*(a + b*x)
^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-3-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)(2+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)(2+n)}+\frac{b(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 59, normalized size = 0.75

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-2}(-ad(n+1)+bc(n+2)+bdx)}{(n+1)(n+2)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-2 - n)*(-(a*d*(1 + n)) + b*c*(2 + n) + b*d*x
))/((b*c - a*d)^2*(1 + n)*(2 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n (c+dx)^{-3-n}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-3 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-3 - n), x]
```

fricas [B] time = 1.29, size = 205, normalized size = 2.59

$$\frac{\left(b^2d^2x^3+2\,abc^2-a^2cd+\left(3\,b^2cd+\left(b^2cd-abd^2\right)n\right)x^2+\left(abc^2-a^2cd\right)n+\left(2\,b^2c^2+2\,abcd-a^2d^2+\left(b^2c^2-a^2d^2\right)n\right)x\right)(bx+a)^n(dx+c)^{-n-3}}{2\,b^2c^2-4\,abcd+2\,a^2d^2+\left(b^2c^2-2\,abcd+a^2d^2\right)n^2+3\left(b^2c^2-2\,abcd+a^2d^2\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^3 + 2*a*b*c^2 - a^2*c*d + (3*b^2*c*d + (b^2*c*d - a*b*d^2)*n)*x^
2 + (a*b*c^2 - a^2*c*d)*n + (2*b^2*c^2 + 2*a*b*c*d - a^2*d^2 + (b^2*c^2 - a
^2*d^2)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 3)/(2*b^2*c^2 - 4*a*b*c*d + 2*a^2
*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*n^2 + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

maple [A] time = 0.00, size = 124, normalized size = 1.57

$$-\frac{(adn-bcn-bdx+ad-2bc)\,(bx+a)^{n+1}\,(dx+c)^{-n-2}}{a^2d^2n^2-2abcd\,n^2+b^2c^2n^2+3a^2d^2n-6abcdn+3b^2c^2n+2a^2d^2-4abcd+2b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-n-3),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-2)*(a*d*n-b*c*n-b*d*x+a*d-2*b*c)/(a^2*d^2*n^2-2*
a*b*c*d*n^2+b^2*c^2*n^2+3*a^2*d^2*n-6*a*b*c*d*n+3*b^2*c^2*n+2*a^2*d^2-4*a*b
*c*d+2*b^2*c^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-3-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 3), x)
```

mupad [B] time = 0.74, size = 214, normalized size = 2.71

$$\frac{\frac{x\,(a+b\,x)^n\,\left(2\,b^2\,c^2-a^2\,d^2-a^2\,d^2\,n+b^2\,c^2\,n+2\,a\,b\,c\,d\right)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}-\frac{a\,c\,(a+b\,x)^n\,(a\,d-2\,b\,c+a\,d\,n-b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b^2\,d^2\,x^3\,(a+b\,x)^n}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}+\frac{b\,d\,x^2\,(a+b\,x)^n\,(3\,b\,c-a\,d\,n+b\,c\,n)}{(a\,d-b\,c)^2\,\left(n^2+3\,n+2\right)}}{(c+d\,x)^{n+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 3),x)
```

```
[Out] ((x*(a + b*x)^n*(2*b^2*c^2 - a^2*d^2 - a^2*d^2*n + b^2*c^2*n + 2*a*b*c*d))/
((a*d - b*c)^2*(3*n + n^2 + 2)) - (a*c*(a + b*x)^n*(a*d - 2*b*c + a*d*n - b
*c*n))/((a*d - b*c)^2*(3*n + n^2 + 2)) + (b^2*d^2*x^3*(a + b*x)^n)/((a*d -
b*c)^2*(3*n + n^2 + 2)) + (b*d*x^2*(a + b*x)^n*(3*b*c - a*d*n + b*c*n))/((a
*d - b*c)^2*(3*n + n^2 + 2)))/(c + d*x)^(n + 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-3-n),x)
```

```
[Out] Timed out
```

3.1571 $\int (a+bx)^n (c+dx)^{-4-n}\,dx$

Optimal. Leaf size=130

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Rubi [A] time = 0.04, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {45, 37}

$$\frac{2b^2(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(bc-ad)}+\frac{2b(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)*(3 + n)) + (2*b*(a + b*
x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^2*(2 + n)*(3 + n)) + (2*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-4-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{(2b)\int (a+bx)^n(c+dx)^{-3-n}\,dx}{(bc-ad)(3+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{\left(2b^2\right)\int (a+bx)^n(c+dx)^{-2-n}\,dx}{(bc-ad)^2(2+n)(3+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)(3+n)}+\frac{2b(a+bx)^{1+n}(c+dx)^{-2-n}}{(bc-ad)^2(2+n)(3+n)}+\frac{2b^2(a+bx)^{1+n}(c+dx)^{-1-n}}{(bc-ad)^3(1+n)(2+n)(3+n)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 112, normalized size = 0.86

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-3}\left(a^2d^2\left(n^2+3n+2\right)-2abd(n+1)(c(n+3)+dx)+b^2\left(c^2\left(n^2+5n+6\right)+2cd(n+3)x+2d^2x^2\right)\right)}{(n+1)(n+2)(n+3)(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-4 - n),x]
```

[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-3 - n)*(a^2*d^2*(2 + 3*n + n^2) - 2*a*b*d*(1 + n)*(c*(3 + n) + d*x) + b^2*(c^2*(6 + 5*n + n^2) + 2*c*d*(3 + n)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + n)*(2 + n)*(3 + n))

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n(c+dx)^{-4-n}\,dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-4 - n),x]

[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-4 - n), x]

fricas [B] time = 1.30, size = 507, normalized size = 3.90

$$\frac{\left(2\,b^3d^3x^4+6\,ab^2c^3-6\,a^2bc^2d+2\,a^3cd^2+2\left(4\,b^3cd^2+\left(b^3cd^2-ab^2d^3\right)n\right)x^3+\left(ab^2c^3-2\,a^2bc^2d+a^3cd^2\right)n^2+\left(12\,b^3c^2d+\left(b^3c^2d-2\,ab^2cd^2+a^2bd^3\right)n^2+\left(7\,b^3c^2d-8\,ab^2cd^2+a^2bd^3\right)n\right)x^2+\left(5\,ab^2c^3-8\,a^2bc^2d+3\,a^3cd^2\right)n+\left(6\,b^3c^3+6\,ab^2c^2d-6\,a^2bcd^2+2\,a^3d^3+\left(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3\right)n^2+\left(5\,b^3c^3-ab^2c^2d-7\,a^2bcd^2+3\,a^3d^3\right)n\right)x\right)(bx+a)^n(dx+c)^{-n-4}}{6\,b^3c^3-18\,ab^2c^2d+18\,a^2bcd^2-6\,a^3d^3+\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^3+6\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n^2+11\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="fricas")

[Out] (2*b^3*d^3*x^4 + 6*a*b^2*c^3 - 6*a^2*b*c^2*d + 2*a^3*c*d^2 + 2*(4*b^3*c*d^2 + (b^3*c*d^2 - a*b^2*d^3)*n)*x^3 + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*n^2 + (12*b^3*c^2*d + (b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*n^2 + (7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*n)*x^2 + (5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*n + (6*b^3*c^3 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + 2*a^3*d^3 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*n^2 + (5*b^3*c^3 - a*b^2*c^2*d - 7*a^2*b*c*d^2 + 3*a^3*d^3)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n^2 + 11*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="giac")

[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)

maple [B] time = 0.01, size = 319, normalized size = 2.45

$$-\frac{\left(a^2d^2n^2-2abcd\,n^2-2ab\,d^2nx+b^2c^2n^2+2b^2cdnx+2b^2x^2d^2+3a^2d^2n-8abcdn-2ab\,d^2x+5b^2c^2n+6b^2cdx+2a^2d^2-6abcd+6b^2c^2\right)(bx+a)^{n+1}(dx+c)^{-n-3}}{a^3d^3n^3-3a^2bc\,d^2n^3+3a\,b^2c^2d\,n^3-b^3c^3n^3+6a^3d^3n^2-18a^2bc\,d^2n^2+18a\,b^2c^2d\,n^2-6b^3c^3n^2+11a^3d^3n-33a^2bc\,d^2n+33a\,b^2c^2dn-11b^3c^3n+6a^3d^3-18a^2bc\,d^2+18a\,b^2c^2d-6b^3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^n*(d*x+c)^(-n-4),x)

[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-3)*(a^2*d^2*n^2-2*a*b*c*d*n^2-2*a*b*d^2*n*x+b^2*c^2*n^2+2*b^2*c*d*n*x+2*b^2*d^2*x^2+3*a^2*d^2*n-8*a*b*c*d*n-2*a*b*d^2*x+5*b^2*c^2*n+6*b^2*c*d*x+2*a^2*d^2-6*a*b*c*d+6*b^2*c^2)/(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3+3*a*b^2*c^2*d*n^3-b^3*c^3*n^3+6*a^3*d^3*n^2-18*a^2*b*c*d^2*n^2+18*a*b^2*c^2*d*n^2-6*b^3*c^3*n^2+11*a^3*d^3*n-33*a^2*b*c*d^2*n+33*a*b^2*c^2*d*n-11*b^3*c^3*n+6*a^3*d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n(dx+c)^{-n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-4-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 4), x)
```

mupad [B] time = 1.02, size = 528, normalized size = 4.06

$$-\frac{x\,(a+b\,x)^n\,\left(a^3\,d^3\,n^2+3\,a^3\,d^3\,n+2\,a^3\,d^3-a^2\,b\,c\,d^2\,n^2-7\,a^2\,b\,c\,d^2\,n-6\,a^2\,b\,c\,d^2-a\,b^2\,c^2\,d\,n^2-a\,b^2\,c^2\,d\,n+6\,a\,b^2\,c^2\,d+b^3\,c^3\,n^2+5\,b^3\,c^3\,n+6\,b^3\,c^3\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{a\,c\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+3\,a^2\,d^2\,n+2\,a^2\,d^2-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n-6\,a\,b\,c\,d+b^2\,c^2\,n^2+5\,b^2\,c^2\,n+6\,b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^3\,d^3\,x^4\,(a+b\,x)^n}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{b\,d\,x^2\,(a+b\,x)^n\,\left(a^2\,d^2\,n^2+a^2\,d^2\,n-2\,a\,b\,c\,d\,n^2-8\,a\,b\,c\,d\,n+b^2\,c^2\,n^2+7\,b^2\,c^2\,n+12\,b^2\,c^2\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}-\frac{2\,b^2\,d^2\,x^3\,(a+b\,x)^n\,\left(4\,b\,c-a\,d\,n+b\,c\,n\right)}{{\left(a\,d-b\,c\right)}^3\,{\left(c+d\,x\right)}^{n+4}\,\left(n^3+6\,n^2+11\,n+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 4),x)
```

```
[Out] - (x*(a + b*x)^n*(2*a^3*d^3 + 6*b^3*c^3 + 3*a^3*d^3*n + 5*b^3*c^3*n + a^3*d
^3*n^2 + b^3*c^3*n^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 - a*b^2*c^2*d*n - 7*a^
2*b*c*d^2*n - a*b^2*c^2*d*n^2 - a^2*b*c*d^2*n^2))/((a*d - b*c)^3*(c + d*x)^
(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (a*c*(a + b*x)^n*(2*a^2*d^2 + 6*b^2*c^2
 + 3*a^2*d^2*n + 5*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 6*a*b*c*d - 8*a*
b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 +
n^3 + 6)) - (2*b^3*d^3*x^4*(a + b*x)^n)/((a*d - b*c)^3*(c + d*x)^(n + 4)*(1
1*n + 6*n^2 + n^3 + 6)) - (b*d*x^2*(a + b*x)^n*(12*b^2*c^2 + a^2*d^2*n + 7*
b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^2 - 8*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d
 - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*d^2*x^3*(a +
 b*x)^n*(4*b*c - a*d*n + b*c*n))/((a*d - b*c)^3*(c + d*x)^(n + 4)*(11*n + 6
*n^2 + n^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-4-n),x)
```

```
[Out] Timed out
```

3.1572 $\int (a+bx)^n (c+dx)^{-5-n}\,dx$

Optimal. Leaf size=185

$$\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}}{(n+3)(n+}$$

Rubi [A] time = 0.06, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {45, 37}

$$\frac{6b^2(a+bx)^{n+1}(c+dx)^{-n-2}}{(n+2)(n+3)(n+4)(bc-ad)^3}+\frac{6b^3(a+bx)^{n+1}(c+dx)^{-n-1}}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}+\frac{(a+bx)^{n+1}(c+dx)^{-n-4}}{(n+4)(bc-ad)}+\frac{3b(a+bx)^{n+1}(c+dx)^{-n-3}}{(n+3)(n+4)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n))/((b*c - a*d)*(4 + n)) + (3*b*(a + b*
x)^(1 + n)*(c + d*x)^(-3 - n))/((b*c - a*d)^2*(3 + n)*(4 + n)) + (6*b^2*(a
+ b*x)^(1 + n)*(c + d*x)^(-2 - n))/((b*c - a*d)^3*(2 + n)*(3 + n)*(4 + n))
+ (6*b^3*(a + b*x)^(1 + n)*(c + d*x)^(-1 - n))/((b*c - a*d)^4*(1 + n)*(2 +
n)*(3 + n)*(4 + n))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^n(c+dx)^{-5-n}\,dx &= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{(3b)\int (a+bx)^n(c+dx)^{-4-n}\,dx}{(bc-ad)(4+n)}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)}+\frac{\left(6b^2\right)\int (a+bx)^n(c+dx)^{-3}}{(bc-ad)^2(3+n)(4+n}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)}+\frac{6b^2(a+bx)^{1+n}(c+dx)^{-2}}{(bc-ad)^3(2+n)(3+n)(4}\\ &= \frac{(a+bx)^{1+n}(c+dx)^{-4-n}}{(bc-ad)(4+n)}+\frac{3b(a+bx)^{1+n}(c+dx)^{-3-n}}{(bc-ad)^2(3+n)(4+n)}+\frac{6b^2(a+bx)^{1+n}(c+dx)^{-2}}{(bc-ad)^3(2+n)(3+n)(4}\end{aligned}$$

Mathematica [A] time = 0.13, size = 195, normalized size = 1.05

$$\frac{(a+bx)^{n+1}(c+dx)^{-n-4}\left(-a^3d^3\left(n^3+6n^2+11n+6\right)+3a^2bd^2\left(n^2+3n+2\right)(c(n+4)+dx)-3ab^2d(n+1)\left(c^2\left(n^2+7n+12\right)+2cd(n+4)x+2d^2x^2\right)+b^3\left(c^3\left(n^3+9n^2+26n+24\right)+3c^2d\left(n^2+7n+12\right)x+6cd^2(n+4)x^2+6d^3x^3\right)\right)}{(n+1)(n+2)(n+3)(n+4)(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] ((a + b*x)^(1 + n)*(c + d*x)^(-4 - n)*(-(a^3*d^3*(6 + 11*n + 6*n^2 + n^3))
+ 3*a^2*b*d^2*(2 + 3*n + n^2)*(c*(4 + n) + d*x) - 3*a*b^2*d*(1 + n)*(c^2*(1
2 + 7*n + n^2) + 2*c*d*(4 + n)*x + 2*d^2*x^2) + b^3*(c^3*(24 + 26*n + 9*n^2
 + n^3) + 3*c^2*d*(12 + 7*n + n^2)*x + 6*c*d^2*(4 + n)*x^2 + 6*d^3*x^3)))/(
(b*c - a*d)^4*(1 + n)*(2 + n)*(3 + n)*(4 + n))
```

IntegrateAlgebraic [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (a+bx)^n (c+dx)^{-5-n}\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x)^(-5 - n),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^n*(c + d*x)^(-5 - n), x]
```

fricas [B] time = 0.92, size = 954, normalized size = 5.16

$$\frac{\left(6\,b^4d^4x^5 + 24\,ab^3c^4 - 36\,a^2b^2c^3d + 24\,a^3bc^2d^2 - 6\,a^4cd^3 + 6\left(5\,b^4cd^3 + \left(b^4cd^3 - ab^3d^4\right)n\right)x^4 + \left(ab^3c^4 - 3\,a^2b^2c^3d + 3\,a^3bc^2d^2 - a^4cd^3\right)n^3 + 3\left(20\,b^4c^2d^2 + \left(b^4c^2d^2 - 2\,ab^3cd^3 + a^2b^2d^4\right)n^2 + \left(9\,b^4c^2d^2 - 10\,ab^3cd^3 + a^2b^2d^4\right)n\right)x^3 + 3\left(3\,ab^3c^4 - 8\,a^2b^2c^3d + 7\,a^3bc^2d^2 - 2\,a^4cd^3\right)n^2 + \left(60\,b^4c^3d + \left(b^4c^3d - 3\,ab^3c^2d^2 + 3\,a^2b^2cd^3 - a^3bd^4\right)n^3 + 3\left(4\,b^4c^3d - 9\,ab^3c^2d^2 + 6\,a^2b^2cd^3 - a^3bd^4\right)n^2 + \left(47\,b^4c^3d - 60\,ab^3c^2d^2 + 15\,a^2b^2cd^3 - 2\,a^3bd^4\right)n\right)x^2 + \left(26\,ab^3c^4 - 57\,a^2b^2c^3d + 42\,a^3bc^2d^2 - 11\,a^4cd^3\right)n + \left(24\,b^4c^4 + 24\,ab^3c^3d - 36\,a^2b^2c^2d^2 + 24\,a^3bcd^3 - 6\,a^4d^4 + \left(b^4c^4 - 2\,ab^3c^3d + 2\,a^3bcd^3 - a^4d^4\right)n^3 + 3\left(3\,b^4c^4 - 4\,ab^3c^3d - 3\,a^2b^2c^2d^2 + 6\,a^3bcd^3 - 2\,a^4d^4\right)n^2 + \left(26\,b^4c^4 - 10\,ab^3c^3d - 45\,a^2b^2c^2d^2 + 40\,a^3bcd^3 - 11\,a^4d^4\right)n\right)x\right)(bx+a)^n(dx+c)^{-n-5}}{24\,b^4c^4 - 96\,ab^3c^3d + 144\,a^2b^2c^2d^2 - 96\,a^3bcd^3 + 24\,a^4d^4 + \left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)n^4 + 10\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)n^3 + 35\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)n^2 + 50\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="fricas")
```

```
[Out] (6*b^4*d^4*x^5 + 24*a*b^3*c^4 - 36*a^2*b^2*c^3*d + 24*a^3*b*c^2*d^2 - 6*a^4
*c*d^3 + 6*(5*b^4*c*d^3 + (b^4*c*d^3 - a*b^3*d^4)*n)*x^4 + (a*b^3*c^4 - 3*a
^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*n^3 + 3*(20*b^4*c^2*d^2 + (b^4*
c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*n^2 + (9*b^4*c^2*d^2 - 10*a*b^3*c*d^
3 + a^2*b^2*d^4)*n)*x^3 + 3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^
2 - 2*a^4*c*d^3)*n^2 + (60*b^4*c^3*d + (b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*n^3 + 3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*
c*d^3 - a^3*b*d^4)*n^2 + (47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2*b^2*c*d^
3 - 2*a^3*b*d^4)*n)*x^2 + (26*a*b^3*c^4 - 57*a^2*b^2*c^3*d + 42*a^3*b*c^2*d
^2 - 11*a^4*c*d^3)*n + (24*b^4*c^4 + 24*a*b^3*c^3*d - 36*a^2*b^2*c^2*d^2 +
24*a^3*b*c*d^3 - 6*a^4*d^4 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4
*d^4)*n^3 + 3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c*d^
3 - 2*a^4*d^4)*n^2 + (26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40
*a^3*b*c*d^3 - 11*a^4*d^4)*n)*x)*(b*x + a)^n*(d*x + c)^(-n - 5)/(24*b^4*c^4
 - 96*a*b^3*c^3*d + 144*a^2*b^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^
4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*n^4 +
10*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
n^3 + 35*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4
*d^4)*n^2 + 50*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

maple [B] time = 0.01, size = 662, normalized size = 3.58

$$-\frac{\left(a^3d^3n^3 - 3a^2bc\,d^2n^3 - 3a^2b\,d^3n^2x + 3a\,b^2c^2dn^3 + 6a\,b^2c\,d^2n^2x + 6a\,b^2d^3n\,x^2 - b^3c^3n^3 - 3b^3c^2dn^2x - 6b^3c\,d^2n\,x^2 - 6b^3d^3x^3 + 6a^3d^3n^2 - 21a^2bc\,d^2n^2 - 9a^2b\,d^3nx + 24a\,b^2c^2dn^2 + 30a\,b^2c\,d^2nx + 6a\,b^2d^3x^2 - 9b^3c^3n^2 - 21b^3c^2dnx - 24b^3c\,d^2x^2 + 11a^3d^3n - 42a^2bc\,d^2n - 6a^2b\,d^3x + 57a\,b^2c^2dn + 24a\,b^2c\,d^2x - 26b^3c^3n - 36b^3c^2dx + 6a^3d^3 - 24a^2bc\,d^2 + 36a\,b^2c^2d - 24b^3c^3\right)(bx+a)^{n+1}(dx+c)^{-n-4}}{a^4d^4n^4 - 4a^3bc\,d^3n^4 + 6a^2b^2c^2d^2n^4 - 4a\,b^3c^3dn^4 + b^4c^4n^4 + 10a^4d^4n^3 - 40a^3bc\,d^3n^3 + 60a^2b^2c^2d^2n^3 - 40a\,b^3c^3dn^3 + 10b^4c^4n^3 + 35a^4d^4n^2 - 140a^3bc\,d^3n^2 + 210a^2b^2c^2d^2n^2 - 140a\,b^3c^3d\,n^2 + 35b^4c^4n^2 + 50a^4d^4n - 200a^3bc\,d^3n + 300a^2b^2c^2d^2n - 200a\,b^3c^3dn + 50b^4c^4n + 24a^4d^4 - 96a^3bc\,d^3 + 144a^2b^2c^2d^2 - 96a\,b^3c^3d + 24b^4c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^n*(d*x+c)^(-5-n),x)
```

```
[Out] -(b*x+a)^(n+1)*(d*x+c)^(-n-4)*(a^3*d^3*n^3-3*a^2*b*c*d^2*n^3-3*a^2*b*d^3*n^
2*x+3*a*b^2*c^2*d*n^3+6*a*b^2*c*d^2*n^2*x+6*a*b^2*d^3*n*x^2-b^3*c^3*n^3-3*b
^3*c^2*d*n^2*x-6*b^3*c*d^2*n*x^2-6*b^3*d^3*x^3+6*a^3*d^3*n^2-21*a^2*b*c*d^2
*n^2-9*a^2*b*d^3*n*x+24*a*b^2*c^2*d*n^2+30*a*b^2*c*d^2*n*x+6*a*b^2*d^3*x^2-
9*b^3*c^3*n^2-21*b^3*c^2*d*n*x-24*b^3*c*d^2*x^2+11*a^3*d^3*n-42*a^2*b*c*d^2
*n-6*a^2*b*d^3*x+57*a*b^2*c^2*d*n+24*a*b^2*c*d^2*x-26*b^3*c^3*n-36*b^3*c^2*
d*x+6*a^3*d^3-24*a^2*b*c*d^2+36*a*b^2*c^2*d-24*b^3*c^3)/(a^4*d^4*n^4-4*a^3*
b*c*d^3*n^4+6*a^2*b^2*c^2*d^2*n^4-4*a*b^3*c^3*d*n^4+b^4*c^4*n^4+10*a^4*d^4*
n^3-40*a^3*b*c*d^3*n^3+60*a^2*b^2*c^2*d^2*n^3-40*a*b^3*c^3*d*n^3+10*b^4*c^4
*n^3+35*a^4*d^4*n^2-140*a^3*b*c*d^3*n^2+210*a^2*b^2*c^2*d^2*n^2-140*a*b^3*c
^3*d*n^2+35*b^4*c^4*n^2+50*a^4*d^4*n-200*a^3*b*c*d^3*n+300*a^2*b^2*c^2*d^2*
n-200*a*b^3*c^3*d*n+50*b^4*c^4*n+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*
d^2-96*a*b^3*c^3*d+24*b^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^n (dx+c)^{-n-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x+c)^(-5-n),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)^n*(d*x + c)^(-n - 5), x)
```

mupad [B] time = 1.61, size = 945, normalized size = 5.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^n/(c + d*x)^(n + 5),x)
```

```
[Out] (6*b^4*d^4*x^5*(a + b*x)^n)/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2
 + 10*n^3 + n^4 + 24)) - (a*c*(a + b*x)^n*(6*a^3*d^3 - 24*b^3*c^3 + 11*a^3*
d^3*n - 26*b^3*c^3*n + 6*a^3*d^3*n^2 - 9*b^3*c^3*n^2 + a^3*d^3*n^3 - b^3*c^
3*n^3 + 36*a*b^2*c^2*d - 24*a^2*b*c*d^2 + 57*a*b^2*c^2*d*n - 42*a^2*b*c*d^2
*n + 24*a*b^2*c^2*d*n^2 - 21*a^2*b*c*d^2*n^2 + 3*a*b^2*c^2*d*n^3 - 3*a^2*b*
c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4
+ 24)) - (x*(a + b*x)^n*(6*a^4*d^4 - 24*b^4*c^4 + 11*a^4*d^4*n - 26*b^4*c^4
*n + 6*a^4*d^4*n^2 - 9*b^4*c^4*n^2 + a^4*d^4*n^3 - b^4*c^4*n^3 + 36*a^2*b^2
*c^2*d^2 - 24*a*b^3*c^3*d - 24*a^3*b*c*d^3 + 10*a*b^3*c^3*d*n - 40*a^3*b*c*
d^3*n + 9*a^2*b^2*c^2*d^2*n^2 + 12*a*b^3*c^3*d*n^2 - 18*a^3*b*c*d^3*n^2 + 2
*a*b^3*c^3*d*n^3 - 2*a^3*b*c*d^3*n^3 + 45*a^2*b^2*c^2*d^2*n))/((a*d - b*c)^
4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (3*b^2*d^2*x^3*(
a + b*x)^n*(20*b^2*c^2 + a^2*d^2*n + 9*b^2*c^2*n + a^2*d^2*n^2 + b^2*c^2*n^
2 - 10*a*b*c*d*n - 2*a*b*c*d*n^2))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n +
 35*n^2 + 10*n^3 + n^4 + 24)) + (6*b^3*d^3*x^4*(a + b*x)^n*(5*b*c - a*d*n +
 b*c*n))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 + 10*n^3 + n^4 + 2
4)) + (b*d*x^2*(a + b*x)^n*(60*b^3*c^3 - 2*a^3*d^3*n + 47*b^3*c^3*n - 3*a^3
*d^3*n^2 + 12*b^3*c^3*n^2 - a^3*d^3*n^3 + b^3*c^3*n^3 - 60*a*b^2*c^2*d*n +
15*a^2*b*c*d^2*n - 27*a*b^2*c^2*d*n^2 + 18*a^2*b*c*d^2*n^2 - 3*a*b^2*c^2*d*
n^3 + 3*a^2*b*c*d^2*n^3))/((a*d - b*c)^4*(c + d*x)^(n + 5)*(50*n + 35*n^2 +
 10*n^3 + n^4 + 24))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**n*(d*x+c)**(-5-n),x)
```

```
[Out] Timed out
```

3.1573 $\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$

Optimal. Leaf size=57

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7, 44}

$$-\frac{1}{(a+bx)(bc-ad)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] -(1/((b*c - a*d)*(a + b*x))) - (d*Log[a + b*x])/(b*c - a*d)^2 + (d*Log[c +
d*x])/(b*c - a*d)^2
```

Rule 7

```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> Int[u*Px^Simplify[p], x] /; PolyQ[Px, x]
 &&  !RationalQ[p] && FreeQ[p, x] && RationalQ[Simplify[p]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx &= \int \frac{1}{(a+bx)^2(c+dx)} dx \\
&= \int \left(\frac{b}{(bc-ad)(a+bx)^2}-\frac{bd}{(bc-ad)^2(a+bx)}+\frac{d^2}{(bc-ad)^2(c+dx)}\right) dx \\
&= -\frac{1}{(bc-ad)(a+bx)}-\frac{d\log(a+bx)}{(bc-ad)^2}+\frac{d\log(c+dx)}{(bc-ad)^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.93

$$\frac{d(a+bx)\log(c+dx)-d(a+bx)\log(a+bx)+ad-bc}{(a+bx)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
```

```
[Out] (-(b*c) + a*d - d*(a + b*x)*Log[a + b*x] + d*(a + b*x)*Log[c + d*x])/((b*c
- a*d)^2*(a + b*x))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^{1+2n-2(1+n)}}{(a+bx)^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2,x]
[Out] IntegrateAlgebraic[(c + d*x)^(1 + 2*n - 2*(1 + n))/(a + b*x)^2, x]
```

fricas [A] time = 1.49, size = 93, normalized size = 1.63

$$-\frac{bc - ad + (bdx + ad)\log(bx + a) - (bdx + ad)\log(dx + c)}{ab^2c^2 - 2\,a^2bcd + a^3d^2 + \left(b^3c^2 - 2\,ab^2cd + a^2bd^2\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] -(b*c - a*d + (b*d*x + a*d)*log(b*x + a) - (b*d*x + a*d)*log(d*x + c))/(a*b
^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*x)
```

giac [A] time = 0.92, size = 78, normalized size = 1.37

$$\frac{bd\log\left(\left|\frac{bc}{bx+a} - \frac{ad}{bx+a} + d\right|\right)}{b^3c^2 - 2\,ab^2cd + a^2bd^2} - \frac{b}{\left(b^2c - abd\right)(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] b*d*log(abs(b*c/(b*x + a) - a*d/(b*x + a) + d))/(b^3*c^2 - 2*a*b^2*c*d + a^
2*b*d^2) - b/((b^2*c - a*b*d)*(b*x + a))
```

maple [A] time = 0.01, size = 57, normalized size = 1.00

$$-\frac{d\ln(bx + a)}{(ad - bc)^2} + \frac{d\ln(dx + c)}{(ad - bc)^2} + \frac{1}{(ad - bc)(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2/(d*x+c),x)
[Out] d/(a*d-b*c)^2*ln(d*x+c)+1/(a*d-b*c)/(b*x+a)-d/(a*d-b*c)^2*ln(b*x+a)
```

maxima [A] time = 1.16, size = 92, normalized size = 1.61

$$-\frac{d\log(bx + a)}{b^2c^2 - 2\,abcd + a^2d^2} + \frac{d\log(dx + c)}{b^2c^2 - 2\,abcd + a^2d^2} - \frac{1}{abc - a^2d + \left(b^2c - abd\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] -d*log(b*x + a)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) + d*log(d*x + c)/(b^2*c^2 -
 2*a*b*c*d + a^2*d^2) - 1/(a*b*c - a^2*d + (b^2*c - a*b*d)*x)
```

mupad [B] time = 0.44, size = 46, normalized size = 0.81

$$\frac{1}{(a\,d - b\,c)\,(a + b\,x)} - \frac{d\,\ln\left(\frac{a+b\,x}{c+d\,x}\right)}{(a\,d - b\,c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^2*(c + d*x)),x)
```

```
[Out] 1/((a*d - b*c)*(a + b*x)) - (d*log((a + b*x)/(c + d*x)))/(a*d - b*c)^2
```

sympy [B] time = 0.69, size = 233, normalized size = 4.09

$$\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{(ad-bc)^2}+\frac{1}{a^2d-abc+x\left(abd-b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2/(d*x+c),x)
```

```
[Out] d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a
*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*
d)/(2*b*d**2))/(a*d - b*c)**2 - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**
2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b*
*3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(a*d - b*c)**2 + 1/(a**2*d -
a*b*c + x*(a*b*d - b**2*c))
```

3.1574 $\int (a+bx)^m (ac(1+m)+bc(2+m)x)^{-3-m}\,dx$

Optimal. Leaf size=95

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Rubi [A] time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {45, 37}

$$\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-1}}{a^2bc^2(m+1)(m+2)}-\frac{(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m-2}}{abc(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] -(((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-2 - m))/(a*b*c*(2 + m)
)) + ((a + b*x)^(1 + m)*(a*c*(1 + m) + b*c*(2 + m)*x)^(-1 - m))/(a^2*b*c^2*
(1 + m)*(2 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\int (a+bx)^m(ac(1+m)+bc(2+m)x)^{-3-m}\,dx=-\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}-\frac{\int (a+bx)^m(ac(1}{}$$
$$=-\frac{(a+bx)^{1+m}(ac(1+m)+bc(2+m)x)^{-2-m}}{abc(2+m)}+\frac{(a+bx)^{1+m}(ac(}{a^2bc^2}$$

Mathematica [A] time = 0.07, size = 54, normalized size = 0.57

$$\frac{x(a+bx)^{m+1}(ac(m+1)+bc(m+2)x)^{-m}}{a^2c^3(m+1)(a(m+1)+b(m+2)x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
```

```
[Out] (x*(a + b*x)^(1 + m))/(a^2*c^3*(1 + m)*(a*(1 + m) + b*(2 + m)*x)^2*(a*c*(1
+ m) + b*c*(2 + m)*x)^m)
```

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int (a+bx)^m (ac(1+m)+bc(2+m)x)^{-3-m}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m),x]
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(a*c*(1 + m) + b*c*(2 + m)*x)^(-3 - m
), x]
```

fricas [A] time = 1.32, size = 85, normalized size = 0.89

$$\frac{\left(\left(b^2m+2\,b^2\right)x^3+(2\,abm+3\,ab)x^2+\left(a^2m+a^2\right)x\right)(acm+ac+(bcm+2\,bc)x)^{-m-3}(bx+a)^m}{a^2m+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="fricas")
[Out] ((b^2*m + 2*b^2)*x^3 + (2*a*b*m + 3*a*b)*x^2 + (a^2*m + a^2)*x)*(a*c*m + a*
c + (b*c*m + 2*b*c)*x)^(-m - 3)*(b*x + a)^m/(a^2*m + a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bc(m+2)x+ac(m+1))^{-m-3}(bx+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="giac")
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

maple [A] time = 0.01, size = 57, normalized size = 0.60

$$\frac{(bxm+am+2bx+a)\,x\,(bx+a)^{m+1}\,(bcxm+acm+2bcx+ac)^{-m-3}}{(m+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(a*c*(m+1)+b*c*(m+2)*x)^(-3-m),x)
[Out] (b*x+a)^(m+1)*(b*m*x+a*m+2*b*x+a)/a^2/(m+1)*x*(b*c*m*x+a*c*m+2*b*c*x+a*c)^(
-3-m)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bc(m+2)x+ac(m+1))^{-m-3}(bx+a)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(a*c*(1+m)+b*c*(2+m)*x)^(-3-m),x, algorithm="maxima")
[Out] integrate((b*c*(m + 2)*x + a*c*(m + 1))^(-m - 3)*(b*x + a)^m, x)
```

mupad [B] time = 1.04, size = 81, normalized size = 0.85

$$\frac{x\,{\left(a+b\,x\right)}^m+\frac{b\,x^2\,\left(2\,m+3\right)\,{\left(a+b\,x\right)}^m}{a\,\left(m+1\right)}+\frac{b^2\,x^3\,\left(m+2\right)\,{\left(a+b\,x\right)}^m}{a^2\,\left(m+1\right)}}{{\left(a\,c\,\left(m+1\right)+b\,c\,x\,\left(m+2\right)\right)}^{m+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^m/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3),x)
```

```
[Out] (x*(a + b*x)^m + (b*x^2*(2*m + 3)*(a + b*x)^m)/(a*(m + 1)) + (b^2*x^3*(m +
2)*(a + b*x)^m)/(a^2*(m + 1)))/(a*c*(m + 1) + b*c*x*(m + 2))^(m + 3)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(a*c*(1+m)+b*c*(2+m)*x)**(-3-m),x)
```

```
[Out] Timed out
```

3.1575 $\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$

Optimal. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Rubi [A] time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a*d)),x
]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}-\frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx}{bc}\\ &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)/(b*c - a
*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

IntegrateAlgebraic [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (a+bx)^{-1-\frac{bc}{bc-ad}}(c+dx)^{-1+\frac{ad}{bc-ad}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1 + (a*d)
/(b*c - a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^(-1 - (b*c)/(b*c - a*d))*(c + d*x)^(-1
+ (a*d)/(b*c - a*d)), x]
```

fricas [A] time = 1.39, size = 84, normalized size = 0.87

$$\frac{bdx^3+acx+(bc+ad)x^2}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="giac")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

maple [A] time = 0.00, size = 66, normalized size = 0.68

$$\frac{x\,(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] (b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))/a/c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)^{-\frac{bc}{bc-ad}-1}(dx+c)^{\frac{ad}{bc-ad}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(-1-b*c/(-a*d+b*c))*(d*x+c)^(-1+a*d/(-a*d+b*c)),x, algori
thm="maxima")
```

```
[Out] integrate((b*x + a)^(-b*c/(b*c - a*d) - 1)*(d*x + c)^(a*d/(b*c - a*d) - 1),
 x)
```

mupad [B] time = 2.14, size = 119, normalized size = 1.23

$$\frac{x\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}+\frac{x^2\,(a\,d+b\,c)\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}+\frac{b\,d\,x^3\,(a+b\,x)^{\frac{b\,c}{a\,d-b\,c}-1}}{a\,c}}{(c+d\,x)^{\frac{a\,d}{a\,d-b\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)^((b*c)/(a*d - b*c) - 1)/(c + d*x)^((a*d)/(a*d - b*c) + 1),x)
```

```
[Out] (x*(a + b*x)^((b*c)/(a*d - b*c) - 1) + (x^2*(a*d + b*c)*(a + b*x)^((b*c)/(a
*d - b*c) - 1))/(a*c) + (b*d*x^3*(a + b*x)^((b*c)/(a*d - b*c) - 1))/(a*c))/
(c + d*x)^((a*d)/(a*d - b*c) + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(-1-b*c/(-a*d+b*c))*(d*x+c)**(-1+a*d/(-a*d+b*c)),x)
```

```
[Out] Timed out
```

3.1576 $\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$

Optimal. Leaf size=97

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Rubi [A] time = 0.02, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.039, Rules used = {45, 37}

$$\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}-\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(-(b*c)
 + a*d)),x]
```

```
[Out] -((c + d*x)^((a*d)/(b*c - a*d))/(b*c*(a + b*x)^((b*c)/(b*c - a*d)))) + (c +
 d*x)^((a*d)/(b*c - a*d))/(a*b*c*(a + b*x)^((a*d)/(b*c - a*d)))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}-\frac{d\int (a+bx)^{\frac{bc}{-bc+ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx}{bc}\\ &= -\frac{(a+bx)^{-\frac{bc}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{bc}+\frac{(a+bx)^{-\frac{ad}{bc-ad}}(c+dx)^{\frac{ad}{bc-ad}}}{abc}\end{aligned}$$

Mathematica [A] time = 0.07, size = 46, normalized size = 0.47

$$\frac{x(a+bx)^{\frac{bc}{ad-bc}}(c+dx)^{\frac{ad}{bc-ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c - 2*a*d)/(
-(b*c) + a*d)),x]
```

```
[Out] (x*(a + b*x)^((b*c)/(-(b*c) + a*d))*(c + d*x)^((a*d)/(b*c - a*d)))/(a*c)
```

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a+bx)^{\frac{-2bc+ad}{bc-ad}}(c+dx)^{\frac{bc-2ad}{-bc+ad}}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^((b*c -
 2*a*d)/(-(b*c) + a*d)),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^((-2*b*c + a*d)/(b*c - a*d))*(c + d*x)^
((b*c - 2*a*d)/(-(b*c) + a*d)), x]
```

fricas [A] time = 1.40, size = 84, normalized size = 0.87

$$\frac{bdx^3+acx+(bc+ad)x^2}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="fricas")
```

```
[Out] (b*d*x^3 + a*c*x + (b*c + a*d)*x^2)/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*
(d*x + c)^((b*c - 2*a*d)/(b*c - a*d))*a*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

maple [A] time = 0.00, size = 66, normalized size = 0.68

$$\frac{x\,(bx+a)^{1-\frac{ad-2bc}{ad-bc}}(dx+c)^{1-\frac{2ad-bc}{ad-bc}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c)),x)
```

```
[Out] 1/a/c*x*(b*x+a)^(1-(a*d-2*b*c)/(a*d-b*c))*(d*x+c)^(1-(2*a*d-b*c)/(a*d-b*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx+a)^{\frac{2\,bc-ad}{bc-ad}}(dx+c)^{\frac{bc-2\,ad}{bc-ad}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)^((-2*a*d+b*c)/(a*d-b*c))
,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x + a)^((2*b*c - a*d)/(b*c - a*d))*(d*x + c)^((b*c - 2*a*d)
/(b*c - a*d))), x)
```

mupad [B] time = 0.85, size = 142, normalized size = 1.46

$$\frac{\frac{x}{(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{x^2\,(a\,d+b\,c)}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}+\frac{b\,d\,x^3}{a\,c\,(a+b\,x)^{\frac{a\,d-2\,b\,c}{a\,d-b\,c}}}}{(c+d\,x)^{\frac{2\,a\,d-b\,c}{a\,d-b\,c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^((a*d - 2*b*c)/(a*d - b*c))*(c + d*x)^((2*a*d - b*c)/(a*d
- b*c))),x)
```

```
[Out] (x/(a + b*x)^((a*d - 2*b*c)/(a*d - b*c)) + (x^2*(a*d + b*c))/(a*c*(a + b*x)
^((a*d - 2*b*c)/(a*d - b*c))) + (b*d*x^3)/(a*c*(a + b*x)^((a*d - 2*b*c)/(a*
d - b*c))))/(c + d*x)^((2*a*d - b*c)/(a*d - b*c))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**((a*d-2*b*c)/(-a*d+b*c))*(d*x+c)**((-2*a*d+b*c)/(a*d-b*c
)),x)
```

```
[Out] Timed out
```

3.1577 $\int \left(a+bx+cx^2+dx^3\right)dx$

Optimal. Leaf size=28

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

Rubi steps

$$\int \left(a+bx+cx^2+dx^3\right)dx=ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$ax+\frac{bx^2}{2}+\frac{cx^3}{3}+\frac{dx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*x + c*x^2 + d*x^3,x]
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+bx+cx^2+dx^3\right)dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + b*x + c*x^2 + d*x^3,x]
[Out] IntegrateAlgebraic[a + b*x + c*x^2 + d*x^3, x]
```

fricas [A] time = 1.17, size = 22, normalized size = 0.79

$$\frac{1}{4}x^4d+\frac{1}{3}x^3c+\frac{1}{2}x^2b+xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="fricas")
[Out] 1/4*x^4*d + 1/3*x^3*c + 1/2*x^2*b + x*a
```

giac [A] time = 1.01, size = 22, normalized size = 0.79

$$\frac{1}{4}dx^4+\frac{1}{3}cx^3+\frac{1}{2}bx^2+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="giac")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

maple [A] time = 0.00, size = 23, normalized size = 0.82

$$\frac{1}{4}d\,x^{4}+\frac{1}{3}c\,x^{3}+\frac{1}{2}b\,x^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d*x^3+c*x^2+b*x+a,x)
```

```
[Out] a*x+1/2*b*x^2+1/3*c*x^3+1/4*d*x^4
```

maxima [A] time = 1.07, size = 22, normalized size = 0.79

$$\frac{1}{4}\,dx^{4}+\frac{1}{3}\,cx^{3}+\frac{1}{2}\,bx^{2}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x^3+c*x^2+b*x+a,x, algorithm="maxima")
```

```
[Out] 1/4*d*x^4 + 1/3*c*x^3 + 1/2*b*x^2 + a*x
```

mupad [B] time = 0.04, size = 22, normalized size = 0.79

$$\frac{d\,x^4}{4}+\frac{c\,x^3}{3}+\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*x + c*x^2 + d*x^3,x)
```

```
[Out] a*x + (b*x^2)/2 + (c*x^3)/3 + (d*x^4)/4
```

sympy [A] time = 0.06, size = 22, normalized size = 0.79

$$ax+\frac{bx^{2}}{2}+\frac{cx^{3}}{3}+\frac{dx^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d*x**3+c*x**2+b*x+a,x)
```

```
[Out] a*x + b*x**2/2 + c*x**3/3 + d*x**4/4
```

3.1578 $\int (-x^3 + x^4)\, dx$

Optimal. Leaf size=15

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[-x^3 + x^4,x]
[Out] -x^4/4 + x^5/5
```

Rubi steps

$$\int (-x^3 + x^4)\, dx = -\frac{x^4}{4} + \frac{x^5}{5}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^3 + x^4,x]
[Out] -1/4*x^4 + x^5/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^3 + x^4)\, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-x^3 + x^4,x]
[Out] IntegrateAlgebraic[-x^3 + x^4, x]
```

fricas [A] time = 1.04, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="fricas")
[Out] 1/5*x^5 - 1/4*x^4
```

giac [A] time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="giac")
[Out] 1/5*x^5 - 1/4*x^4
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{1}{5}x^5 - \frac{1}{4}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4-x^3,x)
[Out] -1/4*x^4+1/5*x^5
```

maxima [A] time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{5}\,x^5 - \frac{1}{4}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4-x^3,x, algorithm="maxima")
[Out] 1/5*x^5 - 1/4*x^4
```

mupad [B] time = 0.02, size = 10, normalized size = 0.67

$$\frac{x^4\,\left(4\,x-5\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4 - x^3,x)
[Out] (x^4*(4*x - 5))/20
```

sympy [A] time = 0.05, size = 8, normalized size = 0.53

$$\frac{x^5}{5} - \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4-x**3,x)
[Out] x**5/5 - x**4/4
```

3.1579 $\int \left(-1+x^5\right)\,dx$

Optimal. Leaf size=11

$$\frac{x^6}{6}-x$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Int[-1 + x^5,x]
```

```
[Out] -x + x^6/6
```

Rubi steps

$$\int \left(-1+x^5\right)\,dx=-x+\frac{x^6}{6}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^6}{6}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 + x^5,x]
```

```
[Out] -x + x^6/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-1+x^5\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1 + x^5,x]
```

```
[Out] IntegrateAlgebraic[-1 + x^5, x]
```

fricas [A] time = 0.97, size = 9, normalized size = 0.82

$$\frac{1}{6}x^6-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="fricas")
```

```
[Out] 1/6*x^6 - x
```

giac [A] time = 0.94, size = 9, normalized size = 0.82

$$\frac{1}{6}\,x^6-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="giac")
```

```
[Out] 1/6*x^6 - x
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{1}{6}x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5-1,x)
```

```
[Out] -x+1/6*x^6
```

maxima [A] time = 0.96, size = 9, normalized size = 0.82

$$\frac{1}{6}\,x^6 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5-1,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 - x
```

mupad [B] time = 0.02, size = 8, normalized size = 0.73

$$\frac{x\,\left(x^5-6\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5 - 1,x)
```

```
[Out] (x*(x^5 - 6))/6
```

sympy [A] time = 0.05, size = 5, normalized size = 0.45

$$\frac{x^6}{6} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5-1,x)
```

```
[Out] x**6/6 - x
```

3.1580 $\int (7+4x)\,dx$

Optimal. Leaf size=9

$$2x^2 + 7x$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$2x^2 + 7x$$

Antiderivative was successfully verified.

```
[In] Int[7 + 4*x,x]
```

```
[Out] 7*x + 2*x^2
```

Rubi steps

$$\int (7+4x)\,dx = 7x + 2x^2$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$2x^2 + 7x$$

Antiderivative was successfully verified.

```
[In] Integrate[7 + 4*x,x]
```

```
[Out] 7*x + 2*x^2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (7+4x)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[7 + 4*x,x]
```

```
[Out] IntegrateAlgebraic[7 + 4*x, x]
```

fricas [A] time = 0.97, size = 9, normalized size = 1.00

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="fricas")
```

```
[Out] 2*x^2 + 7*x
```

giac [A] time = 0.96, size = 9, normalized size = 1.00

$$2\,x^2 + 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="giac")
```

```
[Out] 2*x^2 + 7*x
```

maple [A] time = 0.00, size = 10, normalized size = 1.11

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7+4*x,x)
```

```
[Out] 2*x^2+7*x
```

maxima [A] time = 1.01, size = 9, normalized size = 1.00

$$2\,x^2 + 7\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x, algorithm="maxima")
```

```
[Out] 2*x^2 + 7*x
```

mupad [B] time = 0.03, size = 7, normalized size = 0.78

$$x\,\left(2\,x+7\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + 7,x)
```

```
[Out] x*(2*x + 7)
```

sympy [A] time = 0.05, size = 7, normalized size = 0.78

$$2x^2 + 7x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7+4*x,x)
```

```
[Out] 2*x**2 + 7*x
```

3.1581 $\int \left(4x+\pi x^3\right)\,dx$

Optimal. Leaf size=14

$$\frac{\pi x^4}{4}+2x^2$$

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{\pi x^4}{4}+2x^2$$

Antiderivative was successfully verified.

```
[In] Int[4*x + Pi*x^3,x]
[Out] 2*x^2 + (Pi*x^4)/4
```

Rubi steps

$$\int \left(4x+\pi x^3\right)\,dx=2x^2+\frac{\pi x^4}{4}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{\pi x^4}{4}+2x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[4*x + Pi*x^3,x]
[Out] 2*x^2 + (Pi*x^4)/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(4x+\pi x^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[4*x + Pi*x^3,x]
[Out] IntegrateAlgebraic[4*x + Pi*x^3, x]
```

fricas [A] time = 0.71, size = 12, normalized size = 0.86

$$\frac{1}{4}\,\pi x^4+2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="fricas")
[Out] 1/4*pi*x^4 + 2*x^2
```

giac [A] time = 1.06, size = 12, normalized size = 0.86

$$\frac{1}{4}\,\pi x^4+2\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="giac")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$\frac{1}{4}\pi x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(Pi*x^3+4*x,x)
```

```
[Out] 2*x^2+1/4*Pi*x^4
```

maxima [A] time = 1.00, size = 12, normalized size = 0.86

$$\frac{1}{4}\pi x^4 + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x^3+4*x,x, algorithm="maxima")
```

```
[Out] 1/4*pi*x^4 + 2*x^2
```

mupad [B] time = 0.02, size = 12, normalized size = 0.86

$$\frac{\Pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x + Pi*x^3,x)
```

```
[Out] (Pi*x^4)/4 + 2*x^2
```

sympy [A] time = 0.06, size = 10, normalized size = 0.71

$$\frac{\pi x^4}{4} + 2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(pi*x**3+4*x,x)
```

```
[Out] pi*x**4/4 + 2*x**2
```

3.1582 $\int \left(2x+5x^2\right) dx$

Optimal. Leaf size=11

$$\frac{5x^3}{3}+x^2$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{5x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

Rubi steps

$$\int \left(2x+5x^2\right) dx = x^2+\frac{5x^3}{3}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{5x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + 5*x^2,x]
```

```
[Out] x^2 + (5*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(2x+5x^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[2*x + 5*x^2,x]
```

```
[Out] IntegrateAlgebraic[2*x + 5*x^2, x]
```

fricas [A] time = 0.72, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="fricas")
```

```
[Out] 5/3*x^3 + x^2
```

giac [A] time = 1.10, size = 9, normalized size = 0.82

$$\frac{5}{3}x^3+x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="giac")
[Out] 5/3*x^3 + x^2
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{5}{3}x^{3}+x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x^2+2*x,x)
[Out] x^2+5/3*x^3
```

maxima [A] time = 1.10, size = 9, normalized size = 0.82

$$\frac{5}{3}\,x^{3}+x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x^2+2*x,x, algorithm="maxima")
[Out] 5/3*x^3 + x^2
```

mupad [B] time = 0.02, size = 10, normalized size = 0.91

$$\frac{x^{2}\,\left(5\,x+3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 5*x^2,x)
[Out] (x^2*(5*x + 3))/3
```

sympy [A] time = 0.05, size = 8, normalized size = 0.73

$$\frac{5x^{3}}{3}+x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x**2+2*x,x)
[Out] 5*x**3/3 + x**2
```

3.1583 $\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$

Optimal. Leaf size=15

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

Rubi steps

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx=\frac{x^3}{6}+\frac{x^4}{12}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^4}{12}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/2 + x^3/3,x]
```

```
[Out] x^3/6 + x^4/12
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{x^2}{2}+\frac{x^3}{3}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2/2 + x^3/3,x]
```

```
[Out] IntegrateAlgebraic[x^2/2 + x^3/3, x]
```

fricas [A] time = 1.02, size = 11, normalized size = 0.73

$$\frac{1}{12}x^4+\frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="fricas")
```

```
[Out] 1/12*x^4 + 1/6*x^3
```

giac [A] time = 0.89, size = 11, normalized size = 0.73

$$\frac{1}{12}x^4+\frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="giac")
[Out] 1/12*x^4 + 1/6*x^3
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{1}{12}x^4 + \frac{1}{6}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*x^2+1/3*x^3,x)
[Out] 1/6*x^3+1/12*x^4
```

maxima [A] time = 1.05, size = 11, normalized size = 0.73

$$\frac{1}{12}\,x^4 + \frac{1}{6}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x^2+1/3*x^3,x, algorithm="maxima")
[Out] 1/12*x^4 + 1/6*x^3
```

mupad [B] time = 0.02, size = 8, normalized size = 0.53

$$\frac{x^3\,\left(x+2\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/2 + x^3/3,x)
[Out] (x^3*(x + 2))/12
```

sympy [A] time = 0.06, size = 8, normalized size = 0.53

$$\frac{x^4}{12} + \frac{x^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*x**2+1/3*x**3,x)
[Out] x**4/12 + x**3/6
```

3.1584 $\int (3-5x+2x^2) \, dx$

Optimal. Leaf size=18

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Rubi [A] time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Int[3 - 5*x + 2*x^2,x]
```

```
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

Rubi steps

$$\int (3-5x+2x^2) \, dx = 3x-\frac{5x^2}{2}+\frac{2x^3}{3}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Antiderivative was successfully verified.

```
[In] Integrate[3 - 5*x + 2*x^2,x]
```

```
[Out] 3*x - (5*x^2)/2 + (2*x^3)/3
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (3-5x+2x^2) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[3 - 5*x + 2*x^2,x]
```

```
[Out] IntegrateAlgebraic[3 - 5*x + 2*x^2, x]
```

fricas [A] time = 0.49, size = 14, normalized size = 0.78

$$\frac{2}{3}x^3-\frac{5}{2}x^2+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="fricas")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

giac [A] time = 0.85, size = 14, normalized size = 0.78

$$\frac{2}{3}x^3-\frac{5}{2}x^2+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="giac")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

maple [A] time = 0.00, size = 15, normalized size = 0.83

$$\frac{2}{3}x^3-\frac{5}{2}x^2+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2-5*x+3,x)
```

```
[Out] 3*x-5/2*x^2+2/3*x^3
```

maxima [A] time = 0.97, size = 14, normalized size = 0.78

$$\frac{2}{3}\,x^3-\frac{5}{2}\,x^2+3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x^2-5*x+3,x, algorithm="maxima")
```

```
[Out] 2/3*x^3 - 5/2*x^2 + 3*x
```

mupad [B] time = 0.02, size = 13, normalized size = 0.72

$$\frac{x\,\left(4\,x^2-15\,x+18\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x^2 - 5*x + 3,x)
```

```
[Out] (x*(4*x^2 - 15*x + 18))/6
```

sympy [A] time = 0.06, size = 15, normalized size = 0.83

$$\frac{2x^3}{3}-\frac{5x^2}{2}+3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x**2-5*x+3,x)
```

```
[Out] 2*x**3/3 - 5*x**2/2 + 3*x
```

3.1585 $\int (-2x + x^2 + x^3) \, dx$

Optimal. Leaf size=20

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int (-2x + x^2 + x^3) \, dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]
[Out] -x^2 + x^3/3 + x^4/4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-2x + x^2 + x^3) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2*x + x^2 + x^3,x]
[Out] IntegrateAlgebraic[-2*x + x^2 + x^3, x]
```

fricas [A] time = 1.09, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

giac [A] time = 1.10, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x)
[Out] -x^2+1/3*x^3+1/4*x^4
```

maxima [A] time = 0.97, size = 16, normalized size = 0.80

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

mupad [B] time = 0.03, size = 15, normalized size = 0.75

$$\frac{x^2\left(3x^2+4x-12\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2 - 2*x + x^3,x)
[Out] (x^2*(4*x + 3*x^2 - 12))/12
```

sympy [A] time = 0.05, size = 12, normalized size = 0.60

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
[Out] x**4/4 + x**3/3 - x**2
```

3.1586 $\int (1 - x^2 - 3x^5) \, dx$

Optimal. Leaf size=16

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Antiderivative was successfully verified.

```
[In] Int[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

Rubi steps

$$\int (1 - x^2 - 3x^5) \, dx = x - \frac{x^3}{3} - \frac{x^6}{2}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$-\frac{x^6}{2} - \frac{x^3}{3} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - x^2 - 3*x^5,x]
[Out] x - x^3/3 - x^6/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - x^2 - 3x^5) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1 - x^2 - 3*x^5,x]
[Out] IntegrateAlgebraic[1 - x^2 - 3*x^5, x]
```

fricas [A] time = 0.99, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="fricas")
[Out] -1/2*x^6 - 1/3*x^3 + x
```

giac [A] time = 0.96, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^6 - \frac{1}{3}x^3 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="giac")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

maple [A] time = 0.00, size = 13, normalized size = 0.81

$$-\frac{1}{2}x^{6}-\frac{1}{3}x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3*x^5-x^2+1,x)
```

```
[Out] x-1/3*x^3-1/2*x^6
```

maxima [A] time = 0.98, size = 12, normalized size = 0.75

$$-\frac{1}{2}x^{6}-\frac{1}{3}x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x^5-x^2+1,x, algorithm="maxima")
```

```
[Out] -1/2*x^6 - 1/3*x^3 + x
```

mupad [B] time = 0.02, size = 12, normalized size = 0.75

$$-\frac{x^{6}}{2}-\frac{x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1 - 3*x^5 - x^2,x)
```

```
[Out] x - x^3/3 - x^6/2
```

sympy [A] time = 0.06, size = 10, normalized size = 0.62

$$-\frac{x^{6}}{2}-\frac{x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3*x**5-x**2+1,x)
```

```
[Out] -x**6/2 - x**3/3 + x
```

3.1587 $\int \left(5+2x+3x^2+4x^3\right)\,dx$

Optimal. Leaf size=13

$$x^4+x^3+x^2+5x$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

```
[In] Int[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

Rubi steps

$$\int \left(5+2x+3x^2+4x^3\right)\,dx=5x+x^2+x^3+x^4$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$x^4+x^3+x^2+5x$$

Antiderivative was successfully verified.

```
[In] Integrate[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] 5*x + x^2 + x^3 + x^4
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(5+2x+3x^2+4x^3\right)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[5 + 2*x + 3*x^2 + 4*x^3,x]
[Out] IntegrateAlgebraic[5 + 2*x + 3*x^2 + 4*x^3, x]
```

fricas [A] time = 0.53, size = 13, normalized size = 1.00

$$x^4+x^3+x^2+5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="fricas")
[Out] x^4 + x^3 + x^2 + 5*x
```

giac [A] time = 1.00, size = 13, normalized size = 1.00

$$x^4+x^3+x^2+5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="giac")
```

```
[Out] x^4 + x^3 + x^2 + 5*x
```

maple [A] time = 0.00, size = 14, normalized size = 1.08

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4*x^3+3*x^2+2*x+5,x)
[Out] x^4+x^3+x^2+5*x
```

maxima [A] time = 1.14, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x^3+3*x^2+2*x+5,x, algorithm="maxima")
[Out] x^4 + x^3 + x^2 + 5*x
```

mupad [B] time = 0.03, size = 13, normalized size = 1.00

$$x^4 + x^3 + x^2 + 5\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 3*x^2 + 4*x^3 + 5,x)
[Out] 5*x + x^2 + x^3 + x^4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.92

$$x^4 + x^3 + x^2 + 5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(4*x**3+3*x**2+2*x+5,x)
[Out] x**4 + x**3 + x**2 + 5*x
```

3.1588 $\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$

Optimal. Leaf size=22

$$ax+b\log(x)-\frac{c}{x}-\frac{d}{2x^2}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$ax+b\log(x)-\frac{c}{x}-\frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[a + d/x^3 + c/x^2 + b/x,x]
[Out] -d/(2*x^2) - c/x + a*x + b*Log[x]
```

Rubi steps

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx=-\frac{d}{2x^2}-\frac{c}{x}+ax+b\log(x)$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$ax+b\log(x)-\frac{c}{x}-\frac{d}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + d/x^3 + c/x^2 + b/x,x]
[Out] -1/2*d/x^2 - c/x + a*x + b*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+\frac{d}{x^3}+\frac{c}{x^2}+\frac{b}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[a + d/x^3 + c/x^2 + b/x,x]
[Out] IntegrateAlgebraic[a + d/x^3 + c/x^2 + b/x, x]
```

fricas [A] time = 1.20, size = 27, normalized size = 1.23

$$\frac{2\,ax^3+2\,bx^2\log(x)-2\,cx-d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="fricas")
[Out] 1/2*(2*a*x^3 + 2*b*x^2*log(x) - 2*c*x - d)/x^2
```

giac [A] time = 0.85, size = 21, normalized size = 0.95

$$ax+b\log\left(|x|\right)-\frac{c}{x}-\frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="giac")
[Out] a*x + b*log(abs(x)) - c/x - 1/2*d/x^2
```

maple [A] time = 0.00, size = 21, normalized size = 0.95

$$ax + b\ln(x) - \frac{c}{x} - \frac{d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+d/x^3+c/x^2+b/x,x)
[Out] -1/2*d/x^2-c/x+a*x+b*ln(x)
```

maxima [A] time = 1.03, size = 20, normalized size = 0.91

$$ax + b\log(x) - \frac{c}{x} - \frac{d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x^3+c/x^2+b/x,x, algorithm="maxima")
[Out] a*x + b*log(x) - c/x - 1/2*d/x^2
```

mupad [B] time = 0.04, size = 20, normalized size = 0.91

$$a\,x - \frac{\frac{d}{2} + c\,x}{x^2} + b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b/x + c/x^2 + d/x^3,x)
[Out] a*x - (d/2 + c*x)/x^2 + b*log(x)
```

sympy [A] time = 0.16, size = 20, normalized size = 0.91

$$ax + b\log{(x)} + \frac{-2cx - d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+d/x**3+c/x**2+b/x,x)
[Out] a*x + b*log(x) + (-2*c*x - d)/(2*x**2)
```

3.1589 $\int \left(\frac{1}{x^5} + x + x^5\right) dx$

Optimal. Leaf size=22

$$\frac{x^6}{6} - \frac{1}{4x^4} + \frac{x^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^6}{6} + \frac{x^2}{2} - \frac{1}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-5) + x + x^5,x]
```

```
[Out] -1/(4*x^4) + x^2/2 + x^6/6
```

Rubi steps

$$\int \left(\frac{1}{x^5} + x + x^5\right) dx = -\frac{1}{4x^4} + \frac{x^2}{2} + \frac{x^6}{6}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{x^6}{6} - \frac{1}{4x^4} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-5) + x + x^5,x]
```

```
[Out] -1/4*1/x^4 + x^2/2 + x^6/6
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x^5} + x + x^5\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-5) + x + x^5,x]
```

```
[Out] IntegrateAlgebraic[x^(-5) + x + x^5, x]
```

fricas [A] time = 1.28, size = 17, normalized size = 0.77

$$\frac{2\,x^{10} + 6\,x^6 - 3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^10 + 6*x^6 - 3)/x^4
```

giac [A] time = 0.81, size = 16, normalized size = 0.73

$$\frac{1}{6}\,x^6 + \frac{1}{2}\,x^2 - \frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="giac")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

maple [A] time = 0.00, size = 17, normalized size = 0.77

$$\frac{x^6}{6} + \frac{x^2}{2} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5+x+x^5,x)
```

```
[Out] -1/4/x^4+1/2*x^2+1/6*x^6
```

maxima [A] time = 1.08, size = 16, normalized size = 0.73

$$\frac{1}{6}\,x^6 + \frac{1}{2}\,x^2 - \frac{1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5+x+x^5,x, algorithm="maxima")
```

```
[Out] 1/6*x^6 + 1/2*x^2 - 1/4/x^4
```

mupad [B] time = 0.03, size = 17, normalized size = 0.77

$$\frac{2\,x^{10} + 6\,x^6 - 3}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x^5 + x^5,x)
```

```
[Out] (6*x^6 + 2*x^10 - 3)/(12*x^4)
```

sympy [A] time = 0.08, size = 15, normalized size = 0.68

$$\frac{x^6}{6} + \frac{x^2}{2} - \frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5+x+x**5,x)
```

```
[Out] x**6/6 + x**2/2 - 1/(4*x**4)
```

3.1590 $\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$

Optimal. Leaf size=15

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3) + x^(-2) + x^(-1),x]
[Out] -1/(2*x^2) - x^(-1) + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx = -\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{x}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3) + x^(-2) + x^(-1),x]
[Out] -1/2*1/x^2 - x^(-1) + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-3) + x^(-2) + x^(-1),x]
[Out] IntegrateAlgebraic[x^(-3) + x^(-2) + x^(-1), x]
```

fricas [A] time = 1.23, size = 17, normalized size = 1.13

$$\frac{2\,x^2\log(x)-2\,x-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="fricas")
[Out] 1/2*(2*x^2*log(x) - 2*x - 1)/x^2
```

giac [A] time = 1.08, size = 14, normalized size = 0.93

$$-\frac{1}{x}-\frac{1}{2\,x^2}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="giac")
```

```
[Out] -1/x - 1/2/x^2 + log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$\ln(x) - \frac{1}{x} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3+1/x^2+1/x,x)
```

```
[Out] -1/2/x^2-1/x+ln(x)
```

maxima [A] time = 1.04, size = 13, normalized size = 0.87

$$-\frac{1}{x} - \frac{1}{2\,x^2} + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3+1/x^2+1/x,x, algorithm="maxima")
```

```
[Out] -1/x - 1/2/x^2 + log(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.73

$$\ln(x) - \frac{x + \frac{1}{2}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x + 1/x^2 + 1/x^3,x)
```

```
[Out] log(x) - (x + 1/2)/x^2
```

sympy [A] time = 0.09, size = 14, normalized size = 0.93

$$\log(x) + \frac{-2x - 1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3+1/x**2+1/x,x)
```

```
[Out] log(x) + (-2*x - 1)/(2*x**2)
```

3.1591 $\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$

Optimal. Leaf size=10

$$\frac{2}{x}+3\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx=\frac{2}{x}+3\log(x)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{2}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x^2 + 3/x,x]
```

```
[Out] 2/x + 3*Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{2}{x^2}+\frac{3}{x}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-2/x^2 + 3/x,x]
```

```
[Out] IntegrateAlgebraic[-2/x^2 + 3/x, x]
```

fricas [A] time = 1.02, size = 11, normalized size = 1.10

$$\frac{3\,x\log(x)+2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="fricas")
```

```
[Out] (3*x*log(x) + 2)/x
```

giac [A] time = 1.12, size = 11, normalized size = 1.10

$$\frac{2}{x}+3\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="giac")
```

```
[Out] 2/x + 3*log(abs(x))
```

maple [A] time = 0.00, size = 11, normalized size = 1.10

$$3\ln(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x^2+3/x,x)
```

```
[Out] 2/x+3*ln(x)
```

maxima [A] time = 1.03, size = 10, normalized size = 1.00

$$\frac{2}{x} + 3\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x^2+3/x,x, algorithm="maxima")
```

```
[Out] 2/x + 3*log(x)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$3\,\ln(x) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/x - 2/x^2,x)
```

```
[Out] 3*log(x) + 2/x
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$3\log\left(x\right) + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x**2+3/x,x)
```

```
[Out] 3*log(x) + 2/x
```

3.1592 $\int \left(-\frac{1}{7x^6}+x^6\right) dx$

Optimal. Leaf size=15

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[-1/(7*x^6) + x^6,x]
[Out] 1/(35*x^5) + x^7/7
```

Rubi steps

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx=\frac{1}{35x^5}+\frac{x^7}{7}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[-1/7*1/x^6 + x^6,x]
[Out] 1/(35*x^5) + x^7/7
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{1}{7x^6}+x^6\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-1/7*1/x^6 + x^6,x]
[Out] IntegrateAlgebraic[-1/7*1/x^6 + x^6, x]
```

fricas [A] time = 1.31, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="fricas")
[Out] 1/35*(5*x^12 + 1)/x^5
```

giac [A] time = 1.08, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^7+\frac{1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="giac")
[Out] 1/7*x^7 + 1/35/x^5
```

maple [A] time = 0.00, size = 12, normalized size = 0.80

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/7/x^6+x^6,x)
[Out] 1/35/x^5+1/7*x^7
```

maxima [A] time = 0.97, size = 11, normalized size = 0.73

$$\frac{1}{7}\,x^7+\frac{1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x^6+x^6,x, algorithm="maxima")
[Out] 1/7*x^7 + 1/35/x^5
```

mupad [B] time = 0.03, size = 12, normalized size = 0.80

$$\frac{5\,x^{12}+1}{35\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6 - 1/(7*x^6),x)
[Out] (5*x^12 + 1)/(35*x^5)
```

sympy [A] time = 0.08, size = 10, normalized size = 0.67

$$\frac{x^7}{7}+\frac{1}{35x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/7/x**6+x**6,x)
[Out] x**7/7 + 1/(35*x**5)
```

3.1593 $\int \left(1+\frac{1}{x}+x\right) \, dx$

Optimal. Leaf size=11

$$\frac{x^2}{2}+x+\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

Rubi steps

$$\int \left(1+\frac{1}{x}+x\right) \, dx=x+\frac{x^2}{2}+\log(x)$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{x^2}{2}+x+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x^(-1) + x,x]
```

```
[Out] x + x^2/2 + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(1+\frac{1}{x}+x\right) \, dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1 + x^(-1) + x,x]
```

```
[Out] IntegrateAlgebraic[1 + x^(-1) + x, x]
```

fricas [A] time = 1.19, size = 9, normalized size = 0.82

$$\frac{1}{2}\,x^2+x+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + log(x)
```

giac [A] time = 0.89, size = 10, normalized size = 0.91

$$\frac{1}{2}\,x^2+x+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="giac")
[Out] 1/2*x^2 + x + log(abs(x))
```

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$\frac{x^2}{2} + x + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+1/x+x,x)
[Out] x+1/2*x^2+ln(x)
```

maxima [A] time = 0.92, size = 9, normalized size = 0.82

$$\frac{1}{2}x^2 + x + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x, algorithm="maxima")
[Out] 1/2*x^2 + x + log(x)
```

mupad [B] time = 0.03, size = 9, normalized size = 0.82

$$x + \ln(x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x + 1/x + 1,x)
[Out] x + log(x) + x^2/2
```

sympy [A] time = 0.08, size = 8, normalized size = 0.73

$$\frac{x^2}{2} + x + \log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+1/x+x,x)
[Out] x**2/2 + x + log(x)
```

3.1594 $\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$

Optimal. Leaf size=13

$$\frac{3}{2x^2}-\frac{4}{x}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Int[-3/x^3 + 4/x^2,x]
[Out] 3/(2*x^2) - 4/x
```

Rubi steps

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx=\frac{3}{2x^2}-\frac{4}{x}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{3}{2x^2}-\frac{4}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[-3/x^3 + 4/x^2,x]
[Out] 3/(2*x^2) - 4/x
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\frac{3}{x^3}+\frac{4}{x^2}\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[-3/x^3 + 4/x^2,x]
[Out] IntegrateAlgebraic[-3/x^3 + 4/x^2, x]
```

fricas [A] time = 1.23, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="fricas")
[Out] -1/2*(8*x - 3)/x^2
```

giac [A] time = 0.90, size = 11, normalized size = 0.85

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="giac")
[Out] -4/x + 3/2/x^2
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$-\frac{4}{x}+\frac{3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-3/x^3+4/x^2,x)
[Out] 3/2/x^2-4/x
```

maxima [A] time = 0.97, size = 11, normalized size = 0.85

$$-\frac{4}{x}+\frac{3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x^3+4/x^2,x, algorithm="maxima")
[Out] -4/x + 3/2/x^2
```

mupad [B] time = 0.03, size = 10, normalized size = 0.77

$$-\frac{8\,x-3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(4/x^2 - 3/x^3,x)
[Out] -(8*x - 3)/(2*x^2)
```

sympy [A] time = 0.08, size = 8, normalized size = 0.62

$$\frac{3-8x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-3/x**3+4/x**2,x)
[Out] (3 - 8*x)/(2*x**2)
```

3.1595 $\int \left(\frac{1}{x}+2x+x^2\right) dx$

Optimal. Leaf size=13

$$\frac{x^3}{3}+x^2+\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

Rubi steps

$$\int \left(\frac{1}{x}+2x+x^2\right) dx = x^2+\frac{x^3}{3}+\log(x)$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{x^3}{3}+x^2+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1) + 2*x + x^2,x]
```

```
[Out] x^2 + x^3/3 + Log[x]
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{1}{x}+2x+x^2\right) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^(-1) + 2*x + x^2,x]
```

```
[Out] IntegrateAlgebraic[x^(-1) + 2*x + x^2, x]
```

fricas [A] time = 1.29, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3+x^2+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + x^2 + log(x)
```

giac [A] time = 1.01, size = 12, normalized size = 0.92

$$\frac{1}{3}x^3+x^2+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="giac")
[Out] 1/3*x^3 + x^2 + log(abs(x))
```

maple [A] time = 0.00, size = 12, normalized size = 0.92

$$\frac{x^3}{3} + x^2 + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x+2*x+x^2,x)
[Out] x^2+1/3*x^3+ln(x)
```

maxima [A] time = 1.08, size = 11, normalized size = 0.85

$$\frac{1}{3}x^3 + x^2 + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x^2,x, algorithm="maxima")
[Out] 1/3*x^3 + x^2 + log(x)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.85

$$\ln(x) + x^2 + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x + 1/x + x^2,x)
[Out] log(x) + x^2 + x^3/3
```

sympy [A] time = 0.08, size = 10, normalized size = 0.77

$$\frac{x^3}{3} + x^2 + \log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x+2*x+x**2,x)
[Out] x**3/3 + x**2 + log(x)
```

3.1596 $\int \left(x^{5/6}-x^3\right) dx$

Optimal. Leaf size=17

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Int[x^(5/6) - x^3,x]

[Out] (6*x^(11/6))/11 - x^4/4

Rubi steps

$$\int \left(x^{5/6}-x^3\right) dx=\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{6x^{11/6}}{11}-\frac{x^4}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(5/6) - x^3,x]

[Out] (6*x^(11/6))/11 - x^4/4

IntegrateAlgebraic [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{1}{44}\left(24x^{11/6}-11x^4\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(5/6) - x^3,x]

[Out] (24*x^(11/6) - 11*x^4)/44

fricas [A] time = 1.28, size = 11, normalized size = 0.65

$$-\frac{1}{4}x^4+\frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(5/6)-x^3,x, algorithm="fricas")

[Out] -1/4*x^4 + 6/11*x^(11/6)

giac [A] time = 1.06, size = 11, normalized size = 0.65

$$-\frac{1}{4}x^4+\frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="giac")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

maple [A] time = 0.00, size = 12, normalized size = 0.71

$$-\frac{x^4}{4}+\frac{6x^{\frac{11}{6}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6)-x^3,x)
```

```
[Out] 6/11*x^(11/6)-1/4*x^4
```

maxima [A] time = 1.07, size = 11, normalized size = 0.65

$$-\frac{1}{4}x^4+\frac{6}{11}x^{\frac{11}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/6)-x^3,x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + 6/11*x^(11/6)
```

mupad [B] time = 0.03, size = 11, normalized size = 0.65

$$\frac{6\,x^{11/6}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/6) - x^3,x)
```

```
[Out] (6*x^(11/6))/11 - x^4/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{6x^{\frac{11}{6}}}{11}-\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/6)-x**3,x)
```

```
[Out] 6*x**(11/6)/11 - x**4/4
```

3.1597 $\int \left(33+\sqrt[33]{x}\right) dx$

Optimal. Leaf size=13

$$\frac{33x^{34/33}}{34}+33x$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] Int[33 + x^(1/33),x]
[Out] 33*x + (33*x^(34/33))/34
```

Rubi steps

$$\int \left(33+\sqrt[33]{x}\right) dx = 33x+\frac{33x^{34/33}}{34}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] Integrate[33 + x^(1/33),x]
[Out] 33*x + (33*x^(34/33))/34
```

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{33x^{34/33}}{34}+33x$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[33 + x^(1/33),x]
[Out] 33*x + (33*x^(34/33))/34
```

fricas [A] time = 1.20, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}}+33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="fricas")
[Out] 33/34*x^(34/33) + 33*x
```

giac [A] time = 0.85, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}}+33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="giac")
[Out] 33/34*x^(34/33) + 33*x
```

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(33+x^(1/33),x)
[Out] 33*x+33/34*x^(34/33)
```

maxima [A] time = 1.00, size = 9, normalized size = 0.69

$$\frac{33}{34}\,x^{\frac{34}{33}} + 33\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x^(1/33),x, algorithm="maxima")
[Out] 33/34*x^(34/33) + 33*x
```

mupad [B] time = 0.02, size = 8, normalized size = 0.62

$$\frac{33\,x\,\left(x^{1/33}+34\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/33) + 33,x)
[Out] (33*x*(x^(1/33) + 34))/34
```

sympy [A] time = 0.06, size = 10, normalized size = 0.77

$$\frac{33x^{\frac{34}{33}}}{34} + 33x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(33+x**(1/33),x)
[Out] 33*x**(34/33)/34 + 33*x
```

3.1598 $\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right)\,dx$

Optimal. Leaf size=15

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{4x^{3/2}}{3}+\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] Sqrt[x] + (4*x^(3/2))/3
```

Rubi steps

$$\int \left(\frac{1}{2\sqrt{x}}+2\sqrt{x}\right)\,dx=\sqrt{x}+\frac{4x^{3/2}}{3}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.93

$$\frac{1}{3}\sqrt{x}\,(4x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] (Sqrt[x]*(3 + 4*x))/3
```

IntegrateAlgebraic [A] time = 0.01, size = 19, normalized size = 1.27

$$\frac{1}{3}\left(4x^{3/2}+3\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(2*Sqrt[x]) + 2*Sqrt[x],x]
[Out] (3*Sqrt[x] + 4*x^(3/2))/3
```

fricas [A] time = 0.97, size = 10, normalized size = 0.67

$$\frac{1}{3}\,(4\,x+3)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="fricas")
[Out] 1/3*(4*x + 3)*sqrt(x)
```

giac [A] time = 1.08, size = 9, normalized size = 0.60

$$\frac{4}{3}\,x^{\frac{3}{2}}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="giac")
[Out] 4/3*x^(3/2) + sqrt(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.73

$$\frac{(4x+3)\sqrt{x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2/x^(1/2)+2*x^(1/2),x)
[Out] 1/3*x^(1/2)*(4*x+3)
```

maxima [A] time = 1.11, size = 9, normalized size = 0.60

$$\frac{4}{3}x^{\frac{3}{2}} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x^(1/2)+2*x^(1/2),x, algorithm="maxima")
[Out] 4/3*x^(3/2) + sqrt(x)
```

mupad [B] time = 0.02, size = 10, normalized size = 0.67

$$\frac{\sqrt{x}\ (4\,x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^(1/2)) + 2*x^(1/2),x)
[Out] (x^(1/2)*(4*x + 3))/3
```

sympy [A] time = 0.06, size = 12, normalized size = 0.80

$$\frac{4x^{\frac{3}{2}}}{3} + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2/x**(1/2)+2*x**(1/2),x)
[Out] 4*x**(3/2)/3 + sqrt(x)
```

3.1599 $\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx$

Optimal. Leaf size=15

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

Rubi steps

$$\int \left(-\frac{1}{x^2}+\frac{10}{x}+6\sqrt{x}\right) dx = \frac{1}{x}+4x^{3/2}+10\log(x)$$

Mathematica [A] time = 0.01, size = 15, normalized size = 1.00

$$4x^{3/2}+\frac{1}{x}+10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] x^(-1) + 4*x^(3/2) + 10*Log[x]
```

IntegrateAlgebraic [A] time = 0.02, size = 22, normalized size = 1.47

$$\frac{4x^{5/2}+1}{x}+20\log\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-x^(-2) + 10/x + 6*Sqrt[x],x]
```

```
[Out] (1 + 4*x^(5/2))/x + 20*Log[Sqrt[x]]
```

fricas [A] time = 1.25, size = 18, normalized size = 1.20

$$\frac{4\,x^{\frac{5}{2}}+20\,x\log\left(\sqrt{x}\right)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="fricas")
```

```
[Out] (4*x^(5/2) + 20*x*log(sqrt(x)) + 1)/x
```

giac [A] time = 0.95, size = 14, normalized size = 0.93

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="giac")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(abs(x))
```

maple [A] time = 0.00, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}}+10\ln(x)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/x^2+10/x+6*x^(1/2),x)
```

```
[Out] 1/x+4*x^(3/2)+10*ln(x)
```

maxima [A] time = 1.02, size = 13, normalized size = 0.87

$$4\,x^{\frac{3}{2}}+\frac{1}{x}+10\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x^2+10/x+6*x^(1/2),x, algorithm="maxima")
```

```
[Out] 4*x^(3/2) + 1/x + 10*log(x)
```

mupad [B] time = 0.29, size = 15, normalized size = 1.00

$$20\,\ln\left(\sqrt{x}\right)+\frac{1}{x}+4\,x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/x - 1/x^2 + 6*x^(1/2),x)
```

```
[Out] 20*log(x^(1/2)) + 1/x + 4*x^(3/2)
```

sympy [A] time = 0.06, size = 14, normalized size = 0.93

$$4x^{\frac{3}{2}}+10\log{(x)}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/x**2+10/x+6*x**(1/2),x)
```

```
[Out] 4*x**(3/2) + 10*log(x) + 1/x
```

3.1600 $\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{5/2}}{5}-\frac{2}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3/2) + x^(3/2),x]
[Out] -2/Sqrt[x] + (2*x^(5/2))/5
```

Rubi steps

$$\int \left(\frac{1}{x^{3/2}}+x^{3/2}\right) dx = -\frac{2}{\sqrt{x}}+\frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3/2) + x^(3/2),x]
[Out] (2*(-5 + x^3))/(5*Sqrt[x])
```

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 0.82

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^(-3/2) + x^(3/2),x]
[Out] (2*(-5 + x^3))/(5*Sqrt[x])
```

fricas [A] time = 1.20, size = 10, normalized size = 0.59

$$\frac{2\left(x^3-5\right)}{5\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="fricas")
[Out] 2/5*(x^3 - 5)/sqrt(x)
```

giac [A] time = 1.14, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

maple [A] time = 0.00, size = 11, normalized size = 0.65

$$\frac{\frac{2x^3}{5} - 2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)+x^(3/2),x)
[Out] 2/5*(x^3-5)/x^(1/2)
```

maxima [A] time = 1.00, size = 11, normalized size = 0.65

$$\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)+x^(3/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) - 2/sqrt(x)
```

mupad [B] time = 0.03, size = 12, normalized size = 0.71

$$\frac{2\,x^3 - 10}{5\,\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2) + x^(3/2),x)
[Out] (2*x^3 - 10)/(5*x^(1/2))
```

sympy [A] time = 0.06, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} - \frac{2}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)+x**(3/2),x)
[Out] 2*x**(5/2)/5 - 2/sqrt(x)
```

3.1601 $\int \left(-5x^{3/2}+7x^{5/2}\right) dx$

Optimal. Leaf size=15

$$2x^{7/2}-2x^{5/2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$2x^{7/2}-2x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[-5*x^(3/2) + 7*x^(5/2),x]
[Out] -2*x^(5/2) + 2*x^(7/2)
```

Rubi steps

$$\int \left(-5x^{3/2}+7x^{5/2}\right) dx = -2x^{5/2}+2x^{7/2}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 0.67

$$2(x-1)x^{5/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[-5*x^(3/2) + 7*x^(5/2),x]
[Out] 2*(-1 + x)*x^(5/2)
```

IntegrateAlgebraic [A] time = 0.01, size = 10, normalized size = 0.67

$$2(x-1)x^{5/2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-5*x^(3/2) + 7*x^(5/2),x]
[Out] 2*(-1 + x)*x^(5/2)
```

fricas [A] time = 1.23, size = 14, normalized size = 0.93

$$2\left(x^3-x^2\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="fricas")
[Out] 2*(x^3 - x^2)*sqrt(x)
```

giac [A] time = 0.89, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="giac")
```

```
[Out] 2*x^(7/2) - 2*x^(5/2)
```

maple [A] time = 0.00, size = 9, normalized size = 0.60

$$2\left(x-1\right)x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-5*x^(3/2)+7*x^(5/2),x)
[Out] 2*x^(5/2)*(x-1)
```

maxima [A] time = 1.01, size = 11, normalized size = 0.73

$$2\,x^{\frac{7}{2}}-2\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x^(3/2)+7*x^(5/2),x, algorithm="maxima")
[Out] 2*x^(7/2) - 2*x^(5/2)
```

mupad [B] time = 0.03, size = 8, normalized size = 0.53

$$2\,x^{5/2}\,\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7*x^(5/2) - 5*x^(3/2),x)
[Out] 2*x^(5/2)*(x - 1)
```

sympy [A] time = 0.06, size = 12, normalized size = 0.80

$$2x^{\frac{7}{2}}-2x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-5*x**(3/2)+7*x**(5/2),x)
[Out] 2*x**(7/2) - 2*x**(5/2)
```

3.1602 $\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx$

Optimal. Leaf size=24

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

Rubi [A] time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$-\frac{x^2}{4}+\frac{2x^{3/2}}{3}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

Rubi steps

$$\int \left(\frac{2}{\sqrt{x}}+\sqrt{x}-\frac{x}{2}\right) dx = 4\sqrt{x}+\frac{2x^{3/2}}{3}-\frac{x^2}{4}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{2x^{3/2}}{3}-\frac{x^2}{4}+4\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] 4*Sqrt[x] + (2*x^(3/2))/3 - x^2/4
```

IntegrateAlgebraic [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{1}{12}\left(8x^{3/2}-3x^2+48\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[2/Sqrt[x] + Sqrt[x] - x/2,x]
[Out] (48*Sqrt[x] + 8*x^(3/2) - 3*x^2)/12
```

fricas [A] time = 1.19, size = 14, normalized size = 0.58

$$-\frac{1}{4}x^2+\frac{2}{3}(x+6)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="fricas")
[Out] -1/4*x^2 + 2/3*(x + 6)*sqrt(x)
```

giac [A] time = 1.09, size = 16, normalized size = 0.67

$$-\frac{1}{4}x^2+\frac{2}{3}x^{\frac{3}{2}}+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="giac")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

maple [A] time = 0.00, size = 17, normalized size = 0.71

$$-\frac{x^2}{4}+\frac{2x^{\frac{3}{2}}}{3}+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/2*x+2/x^(1/2)+x^(1/2),x)
[Out] 2/3*x^(3/2)-1/4*x^2+4*x^(1/2)
```

maxima [A] time = 1.05, size = 16, normalized size = 0.67

$$-\frac{1}{4}x^2+\frac{2}{3}x^{\frac{3}{2}}+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x^(1/2)+x^(1/2),x, algorithm="maxima")
[Out] -1/4*x^2 + 2/3*x^(3/2) + 4*sqrt(x)
```

mupad [B] time = 0.03, size = 15, normalized size = 0.62

$$\frac{\sqrt{x}\,\left(8\,x-3\,x^{3/2}+48\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2/x^(1/2) - x/2 + x^(1/2),x)
[Out] (x^(1/2)*(8*x - 3*x^(3/2) + 48))/12
```

sympy [A] time = 0.06, size = 19, normalized size = 0.79

$$\frac{2x^{\frac{3}{2}}}{3}+4\sqrt{x}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/2*x+2/x**(1/2)+x**(1/2),x)
[Out] 2*x**(3/2)/3 + 4*sqrt(x) - x**2/4
```

3.1603 $\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx$

Optimal. Leaf size=23

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-2/x + Sqrt[x]/5 + x^(3/2),x]
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

Rubi steps

$$\int \left(-\frac{2}{x}+\frac{\sqrt{x}}{5}+x^{3/2}\right) dx=\frac{2x^{3/2}}{15}+\frac{2x^{5/2}}{5}-2\log(x)$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{15}-2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-2/x + Sqrt[x]/5 + x^(3/2),x]
[Out] (2*x^(3/2))/15 + (2*x^(5/2))/5 - 2*Log[x]
```

IntegrateAlgebraic [A] time = 0.01, size = 26, normalized size = 1.13

$$\frac{2}{15}\left(3x^{5/2}+x^{3/2}\right)-4\log\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[-2/x + Sqrt[x]/5 + x^(3/2),x]
[Out] (2*(x^(3/2) + 3*x^(5/2)))/15 - 4*Log[Sqrt[x]]
```

fricas [A] time = 1.27, size = 19, normalized size = 0.83

$$\frac{2}{15}\left(3x^2+x\right)\sqrt{x}-4\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*x^2 + x)*sqrt(x) - 4*log(sqrt(x))
```

giac [A] time = 0.82, size = 16, normalized size = 0.70

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{15}x^{\frac{3}{2}}-2\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="giac")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(abs(x))
```

maple [A] time = 0.00, size = 16, normalized size = 0.70

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{15}-2\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2/x+x^(3/2)+1/5*x^(1/2),x)
[Out] 2/15*x^(3/2)+2/5*x^(5/2)-2*ln(x)
```

maxima [A] time = 1.03, size = 15, normalized size = 0.65

$$\frac{2}{5}\,x^{\frac{5}{2}}+\frac{2}{15}\,x^{\frac{3}{2}}-2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x^(3/2)+1/5*x^(1/2),x, algorithm="maxima")
[Out] 2/5*x^(5/2) + 2/15*x^(3/2) - 2*log(x)
```

mupad [B] time = 0.28, size = 17, normalized size = 0.74

$$\frac{2\,x^{3/2}}{15}-4\,\ln\left(\sqrt{x}\right)+\frac{2\,x^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/5 - 2/x + x^(3/2),x)
[Out] (2*x^(3/2))/15 - 4*log(x^(1/2)) + (2*x^(5/2))/5
```

sympy [A] time = 0.06, size = 20, normalized size = 0.87

$$\frac{2x^{\frac{5}{2}}}{5}+\frac{2x^{\frac{3}{2}}}{15}-2\log{(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2/x+x**(3/2)+1/5*x**(1/2),x)
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/15 - 2*log(x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```